

SMITH | ELMS | ROLAND

JACARANDA
MATHS QUEST

8

STAGE 4 | NSW AUSTRALIAN CURRICULUM | SECOND EDITION



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OVERVIEW

Jacaranda Maths Quest 8 Stage 4 for the NSW Australian curriculum has been completely revised to help teachers and students navigate the NSW Mathematics syllabus. The suite of resources in the Maths Quest series is designed to enrich the learning experience and improve learning outcomes for all students.

Maths Quest is designed to cater for students of all abilities: no student is left behind and none is held back. Maths Quest is written with the specific purpose of helping students deeply understand mathematical concepts. The content is organised around a number of features, in both print and online through Jacaranda's learnON platform, to allow for seamless sequencing through material to scaffold every student's learning.

Topic introductions put the topic into a real-world context.

Start each topic with an engaging Discussion point.

The learning sequence at a glance

NSW Mathematics Syllabus references are linked to contents of the topic.

Fully worked examples throughout the text demonstrate key concepts.

Your FREE online learnON resources contain hundreds of videos, interactivities and traditional WorkSHEETs and SkillSHEETs to support and enhance learning.

Carefully graded questions cater for all abilities. Question types are classified according to the Working Mathematically strand of the NSW Mathematics Syllabus.

TOPIC 13
Data analysis

13.1 Overview

13.1.1 Why learn this?

DISCUSSION

LEARNING SEQUENCE

LEARNING OUTCOMES

CONTENT DESCRIPTIONS

Source: NSW Syllabus for the Australian curriculum

13.2 Samples and populations

13.2.1 Collecting data

DISCUSSION

WORKED EXAMPLE 1

14.8 Review

Investigation | Rich task

Exercise 11.9 Review questions

Visit your learnON title to watch videos which tell the story of mathematics.

An extensive glossary of mathematical terms in print, and as a hover-over feature in your learnON title

Expand the conversation with embedded Discussion points.

Engaging Investigations at the end of each topic to deepen conceptual understanding

Each topic concludes with comprehensive Review questions, in both print and online.

Fully worked solutions are provided online, and answers are provided at the end of each print topic.

Individual pathway interactivities in each sub-topic ensure consolidation of learning for every skill level.

Fully worked solutions to every question are provided online, and answers are provided at the end of each print topic.

LearnON is Jacaranda's immersive and flexible digital learning platform that transforms trusted Jacaranda content to make learning more visible, personalised and social. Hundreds of engaging videos and interactivities are embedded just where you need them — at the point of learning. At Jacaranda, our 'learning made visible' framework ensures immediate feedback for students and teachers, with customisation and collaboration to drive engagement with learning.

Maths Quest contains a free activation code for *learnON* (please see instructions on the inside front cover), so students and teachers can take advantage of the benefits of both print and digital, and see how *learnON* enhances their digital learning and teaching journey.

learnon includes:

- Students and teachers connected in a class group
- Hundreds of videos and interactivities to bring concepts to life
- Fully worked solutions to every question
- Immediate feedback for students
- Immediate insight into student progress and performance for teachers
- Dashboards to track progress
- Collaboration in real time through class discussions
- Comprehensive summaries for each topic
- Code puzzles and dynamic interactivities to help students engage with and work through challenging concepts
- Formative and summative assessments
- And much more ...



Also available for purchase is **SpyClass**. An online game combining comic book-style art with problem based learning

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TOPIC 1

NAPLAN practice

1.1 Overview

Numerous **videos** and **interactivities** are embedded just where you need them, at the point of learning, in your learnON title at www.jacplus.com.au. They will help you to learn the concepts covered in this topic.

1.1.1 Why learn this?

All students in Australia are required to sit NAPLAN tests in May of Years 3, 5, 7 and 9.

NAPLAN is an acronym for National Assessment Program — Literacy and Numeracy.

There are two Numeracy tests — one in which students may use a calculator and one in which they may not. Both tests contain 32 questions, most of which are multiple choice. Each test lasts for 40 minutes.

In the exercises, there are NAPLAN practice tests to help you prepare for when you have to sit the actual tests. It is best to complete one practice test per week before you undertake the formal tests.



LEARNING SEQUENCE

- 1.1 Overview
- 1.2 NAPLAN practice 1
- 1.3 NAPLAN practice 2
- 1.4 NAPLAN practice 3
- 1.5 Review

learnon RESOURCES — ONLINE ONLY

eLesson: NAPLAN — Strategies and tips (eles-1688)

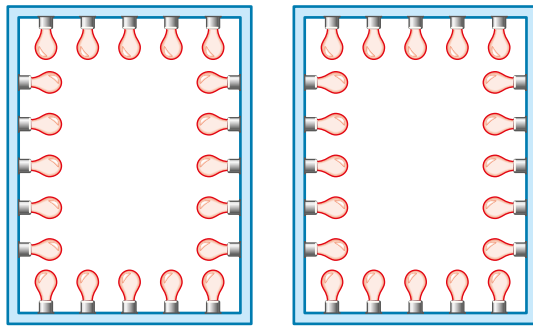
1.2 NAPLAN practice 1

1.2.1 Set A (Calculator allowed)

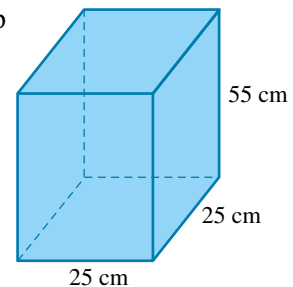
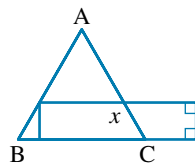
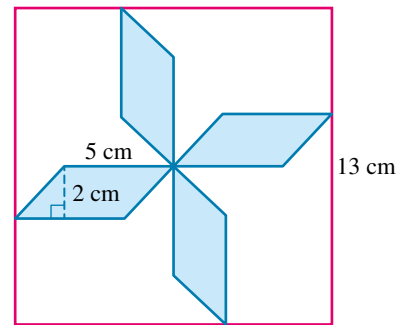
1. The original Ferris Wheel opened to the public in Chicago in 1893. The distance around the outside of the ferris wheel is approximately three times the diameter. This ferris wheel has a diameter of 76.2 m. If you were to string lights to the outside edge of the ferris wheel, what would be the approximate length of lights needed to the nearest metre?
A. 119 m **B.** 120 m
C. 200 m **D.** 229 m



2. You want to string lights around two windows that each measure 90 cm by 120 cm. What is the minimum length of lights needed?



- A.** 2.5 m **B.** 3.5 m
C. 6.5 m **D.** 8.5 m
3. This quilt design is a square with four identical parallelograms. What is the unshaded area of the quilt design?
- A.** 65 cm^2 **B.** 129 cm^2
C. 149 cm^2 **D.** 159 cm^2
4. During the January holidays, Anna works in a café in Newcastle. She saves 75% of her earnings. If Anna earns \$750, what is the best estimate of the amount of money that she saves?
- A.** \$550 **B.** \$300
C. \$150 **D.** \$100
5. The measure of the interior angles of a triangle are $2x$, $6x$ and $10x$. What is the measure in degrees of the largest angle?
- A.** 20 **B.** 100
C. 200 **D.** 250
6. You have a rectangular storage box that is 55 cm high. You cut off a 5-cm strip around the top of the box. What will be the new volume of the box in cubic centimetres?
- A.** $31\,250 \text{ cm}^3$ **B.** $27\,500 \text{ cm}^3$
C. $22\,000 \text{ cm}^3$ **D.** $20\,000 \text{ cm}^3$
7. Triangle ABC is equilateral. What is the measure of angle x ?



8. A shop reduces the price of sports shoes by 40%. The new price is \$72. What was the original price of the sports shoes?
- A.** \$120 **B.** \$115.20
C. \$100.80 **D.** \$100
9. High-speed label applicators can put labels onto envelopes at a rate of 200 per second. Which of the following represents the number in a day?
- A.** 1.728×10^5 **B.** 1.728×10^6
C. 1.728×10^7 **D.** 1.728×10^8



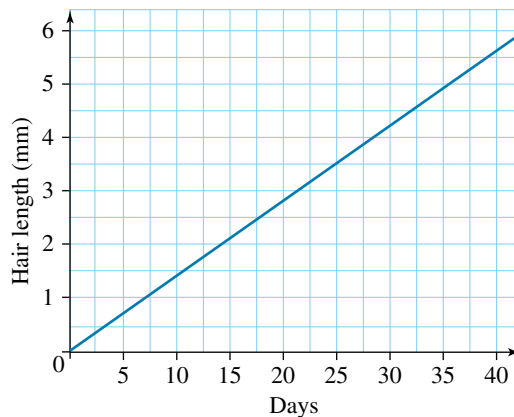
10. Alex and his six friends played a game of laser tag in which the person with the lowest final score wins. The table shows the final scores for each person except Alice.

Player	Score
Alex	151
Ben	153
Julie	149
Lee	139
Alice	
Aysha	135
Keta	143



If Alice won the game and the range between the highest and lowest scores was 19, what was Alice's score?

- A. 132 B. 134 C. 170 D. 172
11. The following graph shows the growth rate of hair.

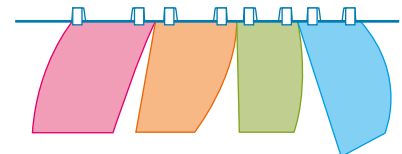


Based on the information in the graph, which figure best represents the number of millimetres that hair grows in 30 days?

- A. 2.4 mm B. 3.6 mm C. 4.2 mm D. 5.7 mm
12. A Frisbee fits inside a cube. The top of the cube has a perimeter of 72 cm. If the Frisbee occupies 250 cm^3 , how much space is left in the cube?

13. You are hanging towels on a clothes line. You use two pegs per towel.

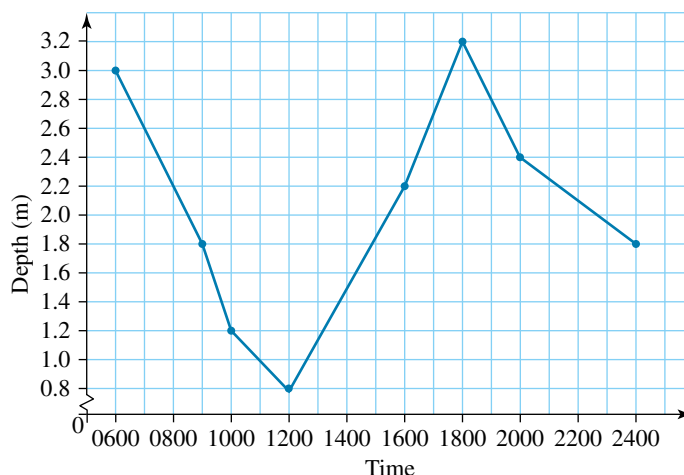
However, as you continue you realise that you will run out of pegs. Instead, you attach the end of the new towel to the old towel. In this way the towels share a peg.



If t represents the number of towels and P represents the number of pegs, which of these equations represents the number of pegs needed after you have reattached the pegs?

- A. $P = 2 \times t + 1$ B. $P = 2 \times t + 2$ C. $P = 3 \times t$ D. $P = t + 1$
14. A cyclist is competing in an 80-km race. The record time for the race is 2 hours and 40 minutes. What will his speed, s (in km/h), need to be to beat this record?
- A. $s < 30$ B. $s > 30$ C. $s < 33$ D. $s > 20$

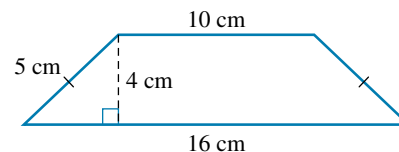
15. The depth of the water inside Blue Lagoon Bay has been recorded over a time period as shown on the following graph.



What is the depth at 10 pm?

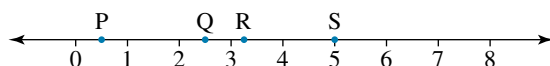
16. Sing and Nam each buy a health bar at lunchtime from the vending machine. There are four different types to choose from: ANZAC, chocolate chip, triple fruit and yoghurt. What is the probability that they choose exactly the same type of bar?
17. A triangle with two identical sides and an angle of 119° is:
- A. scalene and acute
 - B. isosceles and acute
 - C. isosceles and obtuse
 - D. isosceles and right-angled

18. Alex wants to find the perimeter of the isosceles trapezium at right. Which equation could Alex use to find the perimeter of the trapezoid?



- A. $P = 10 + 16 + 4 + 5$
- B. $P = 10 + 16 + (2 \times 5)$
- C. $P = 10 + 16 + 4 + 5$
- D. $P = (10 + 16) \times 4 + 2$

19. Which point on the number line could represent the value of $\sqrt{10}$?



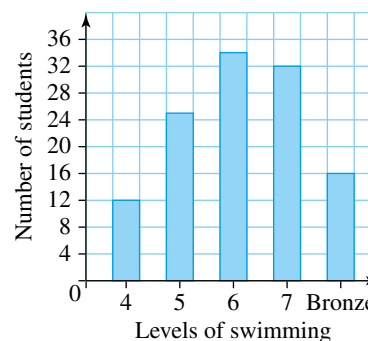
- A. S
- B. P
- C. R
- D. Q

20. Lightning quickly heats the air, causing it to expand. This produces the sound of thunder. Sound travels approximately 1 km in 3 seconds. How far away is a thunderstorm when you notice a 2-second delay between the lightning and the sound of thunder?

- A. 1 km away
- B. $\frac{1}{3}$ km away
- C. $\frac{1}{2}$ km away
- D. $\frac{2}{3}$ km away

21. The bar graph shows 120 students in Year 8 and their different swimming levels. What percentage of students, to the nearest whole number, have reached the bronze level?

- A. 13%
- B. 15%
- C. 17%
- D. 19%



22. A reservoir has a total capacity of 1 068 000 megalitres and is full. Suppose the water is to be drained by a pump at a constant daily rate. If $\frac{9}{10}$ of the volume of the reservoir remains after the first day's pumping, how many megalitres (to the nearest megalitre) have been lost over 3 days? (Round to the nearest megalitre.)

A. 772 740 B. 320 400 C. 289 428 D. 1068

23. The diagram at right shows the current storage level of a dam.

If the total capacity is 1 670 500 megalitres, how much water is available (to the nearest ten thousand megalitres)?

A. 700 000 B. 670 000
C. 669 000 D. 660 000

24. One student in the Japanese language class is to make name tags, 50 mm × 50 mm square, from stiff card. The sheet of card measures 25 cm by 30 cm. What is the maximum number of name tags that can be made from one sheet?

A. 10 B. 20
C. 30 D. 40

25. The table shows the cost of hiring a band called The Hotshots.

The Hotshots band for hire	
Monday to Friday	\$55 per hour
Saturday	\$110 per hour
Acoustic rental	\$60 per booking
Deposit 20% of total cost	

A booking is made for Saturday for 4 hours, along with acoustic rental.

Which of the following represents the deposit?

A. $(110 \times 4 + 60) \times \frac{20}{100}$ B. $(110 \times 4 + 60) \times \frac{100}{20}$
C. $(110 + 60) \times 4 \times \frac{20}{100}$ D. $(110 + 60) \times 4 \times \frac{100}{20}$

26. The Queenscliff Marine Centre hires a boat for 12 biologists to go diving for six days. The cost for hiring the boat for six days is usually \$880. The marine centre obtains a 10% discount. What is the cost per person with the discount?

A. \$88 B. \$74 C. \$70 D. \$66

27. The manager of a cinema complex records the number of people attending the 2 pm session at the cinema from Monday to Friday.

	Monday	Tuesday	Wednesday	Thursday	Friday
Number of adults	120	170	147	160	183
Number of children	37	42	52	62	85

What is the mean number of adults attending the 2 pm session for that week?

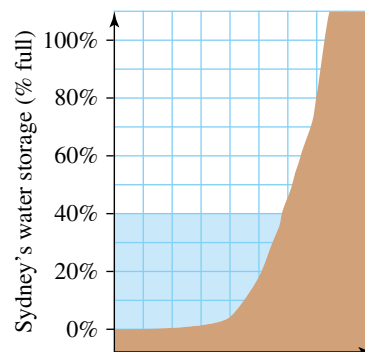
A. 120 B. 150 C. 156 D. 160

28. Two cars begin at the same time and travel the same distance of 160 km. One car travels at 80 km per hour and the other car travels at 100 km per hour. How many minutes after the faster car will the slower car complete the journey?

A. 20 minutes B. 24 minutes C. 30 minutes D. 36 minutes

29. You can usually stay in the sun for 9 minutes before burning. Using a sun protecting lotion with an SPF 20 rating means that you can stay in the sun for 9×20 minutes before burning. Your friend burns in 15 minutes. What sun protection factor would she need to use so that you can both stay for the same amount of time out in the sun? (Of course remember to wear a hat!)

A. 8 B. 10 C. 12 D. 15



1.2.2 Set B (Non-calculator)

1. The breadth of the rectangular phone screen shown is 8 cm, and its area is 80 cm^2 .
The perimeter of the screen is:

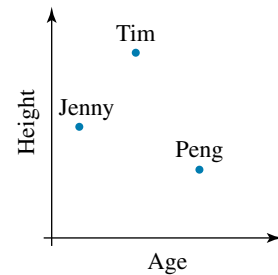
A. 32 cm B. 36 cm
C. 38 cm D. 40 cm

2. The sum of $3x$ dollars and $3x$ cents, in cents, is:

A. $3x + 3$ B. $3x$
C. $303x$ D. $3x + 3x$

3. The graph shows the ages and heights of three Year 8 students.
Which one of the following statements is true?

A. Tim is the eldest and the tallest.
B. Jenny is older than Peng and younger than Tim.
C. Peng and Tim are the same age.
D. Peng is the shortest.



4. Two angles of a triangle are 63° and 57° . Which of the following could not be the measure of an exterior angle of the triangle?

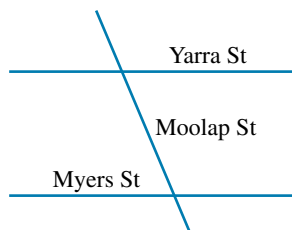
A. 110° B. 117° C. 120° D. 123°

5. Susan claims that the weight of her cat is at most 8 kg. What inequality represents her claim?

A. $w < 8$ B. $w > 8$
C. $w \leq 8$ D. $w \geq 8$



6. The following diagram shows two parallel streets, Yarra Street and Myers Street, intersected by Moolap St. The obtuse angle that Myers Street forms with Moolap Street is four times the measure of the acute angle that Yarra Street makes with Moolap Street. What is the measure of the acute angle at Yarra Street and Moolap Street?



A. 30° B. 36° C. 108° D. 144°

7. Ann and Jack are playing a game where Ann gives an Input number for Jack to put in to the same expression to give an Output number.

Input	1	2	3	4	?
Output	2	5	8	11	59

What is the Input number that Ann gave Jack for an Output number of 59?

A. 18 B. 20 C. 22 D. 24

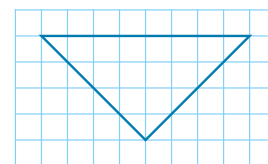
8. Five students competed in a 200-m race. Their finishing times were 47.5 s, 46.8 s, 47.3 s, 48.0 s and 48.2 s. What is their average time for running a 200-m race, correct to 2 decimal places?



9. A triangle has been drawn on 1-cm grid paper.

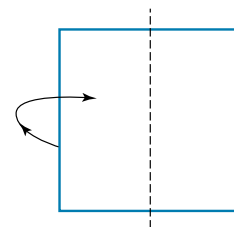
Which statement is incorrect?

- A. The triangle is isosceles. B. The triangle is right-angled.
C. The perimeter is 24 cm. D. The area is 16 cm^2 .



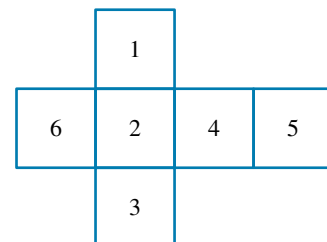
10. Fold a paper square in half vertically and then cut it along the fold line. What is the ratio of the perimeter of one of the resulting two smaller rectangles to the large square?

- A. $\frac{1}{2}$ B. $\frac{2}{3}$
C. $\frac{3}{4}$ D. $\frac{5}{6}$



11. Imagine that you fold the figure shown into a cube. Three faces meet at each corner. What is the largest sum of the three numbers whose faces meet at a corner?

- A. 15 B. 14
C. 13 D. 12



12. Anne has to put 2 drops from an eyedropper in her eye twice a day. If the bottle of eye drops contains 20 mL and there are 8 drops in a millilitre, how many days will the bottle of eye drops last?

- A. 10 days B. 20 days
C. 30 days D. 40 days



13. Two hundred Year 8 students are doing a science experiment in which 25 mL of an alkaline solution will be used by each student. How much solution is needed in total?

- A. 50 litres B. 5 litres
C. 0.5 litres D. 0.05 litres

14. You have made 15 muffins for your class. You realise that this is only $\frac{3}{5}$ of the total that you need? How many more do you need to make?

- A. 3 B. 5
C. 9 D. 10



15. A market gardener noted that 6 boxes of cherry tomatoes and 2 kg weighed the same amount as 5 boxes of cherry tomatoes and 4 kg. If x represents the weight of a box of cherry tomatoes in kg, which equation best represents the information?

- A. $8x = 9x$ B. $12x = 20x$
C. $6x + 2 = 5x + 4$ D. $6x - 2 = 5x + 4$



16. Mulching the garden in the summertime is a great way of saving water. A bale of pea straw costs \$5.00 and the delivery charge is \$15.00. Olivia spent \$90 in total. How many bales of pea straw did she buy for the garden?

- A. 18 B. 15
C. 12 D. 9

17. In Yen's class, the ratio of the number of students who walked to school on Tuesday to the number of students who took some form of transport was 12 : 18. Which fraction is an equivalent form of this ratio?

- A. $\frac{2}{8}$ B. $\frac{4}{9}$
C. $\frac{2}{3}$ D. $\frac{3}{4}$



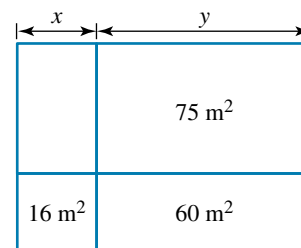
18. The Cheetahs and the Leopards are two school netball teams. The table shows the scores of their games.

	Game 1	Game 2	Game 3	Game 4	Game 5
Cheetahs	45	40	35	49	64
Leopards	28	50	27	52	63

Based on the scores in the table, which statement is true?

- A. The Cheetahs won 20% of the games. B. The Cheetahs won 30% of the games.
C. The Leopards won 40% of the games. D. The Leopards won 60% of the games.

19. The diagram shows a proposed floor plan for an office space. The proposed plan has four areas. Three of the areas are rectangular and the fourth is a square. What is the length of y ?



- A. 3 m B. 5 m
C. 15 m D. 25 m

20. Adults, on average, have 5.5 litres of blood in their bodies. How many millilitres of blood are in the kidneys (see the graph below right)?

- A. 25 B. 500
C. 1000 D. 1375

21. Anne and her friends decided to watch a DVD. They started it at 8.30 pm and it ran for 105 minutes. At what time did the DVD end?

- A. 9.30 pm B. 9.35 pm
C. 10.05 pm D. 10.15 pm

22. A 900-car parking lot is divided into 3 sections. There are 330 spots in Section 1. Section 2 holds 160 more than will fit into Section 3. How many spots are in Section 3?

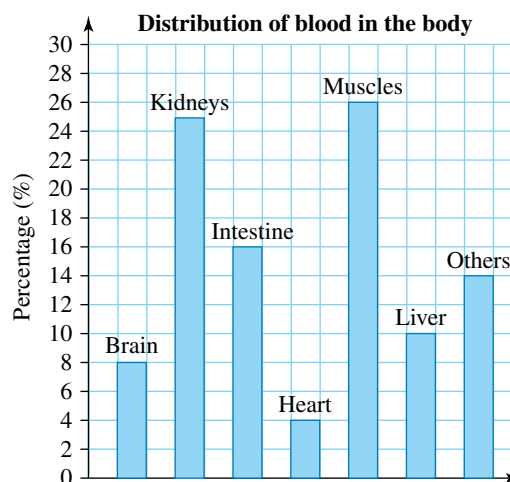
23. A chef assembles a cake in $\frac{2}{3}$ of an hour. If he works for $7\frac{1}{2}$ hours, how many cakes will he fully assemble?

- A. $10\frac{3}{4}$ B. 11 C. $11\frac{1}{4}$ D. 12

24. A roll of material contains 6 metres of cloth. Four lengths, each of x centimetres, are cut from the cloth. What length of material (in centimetres) remains?

- A. $6 - 4x$ B. $4x - 6$
C. $600 - 4x$ D. $100(6 - 4x)$

25. Two straight roads cross. What is the size of angle A ?



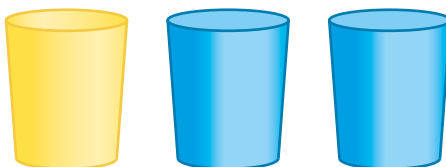
26. A survey is taken of a Year 8 group to determine the length of time students spend on homework each night. What percentage of the group spend 60 minutes or more on homework each night?

Time spent in minutes	Number of pupils
0	6
15	10
30	14
40	2
50	4
60	8
70	10
80	5
90	1

27. In this diagram of yellow and blue buckets, each bucket of the same colour contains the same number of pegs. How many pegs are in each of the given coloured buckets?



Total number of pegs in 4 yellow buckets = 28 pegs



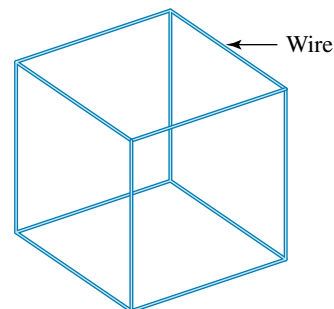
Total number of pegs in 1 yellow bucket and 2 blue buckets = 43 pegs

28. Which of these is the lightest?

- A. 25000 grams
- B. 2.50 kilograms
- C. 25000000 milligrams
- D. 2.5 tonnes

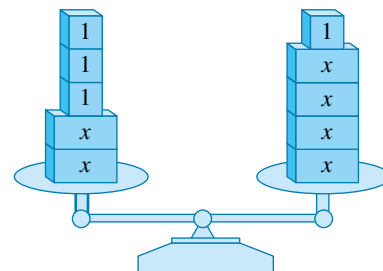
29. A piece of wire is cut into the exact number of pieces needed to create the edges of a cube. If the volume of the cube is 125cm^3 , what was the length of wire to begin with?

- A. 5 cm
- B. 30 cm
- C. 50 cm
- D. 60 cm



30. Given the balance, what is the value for x ?

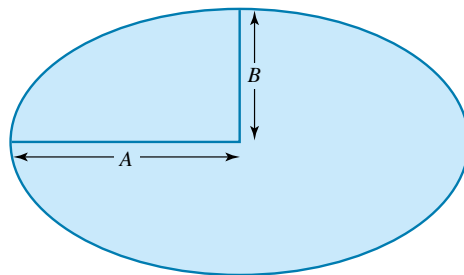
- A. $\frac{1}{2}$
- B. 1
- C. 2
- D. 4



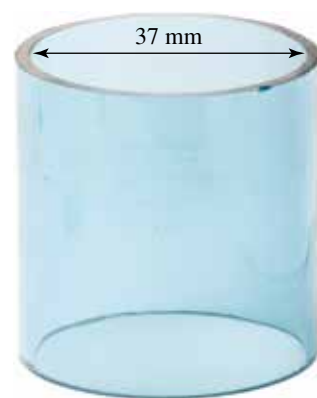
1.3 NAPLAN practice 2

1.3.1 Set C (Calculator allowed)

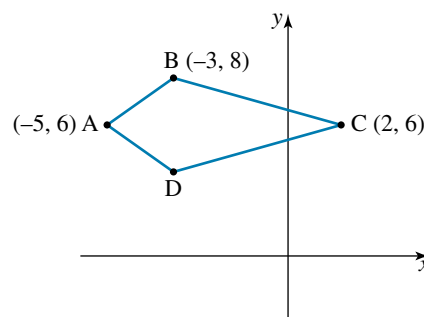
1. Each year, the area of tree logging in the Otway Ranges is approximately equivalent to clearing 200 football ovals. The area of a football oval is given by $\text{Area} = 3.142 \times A \times B$ (see the diagram). If the width of the football oval is 110 m and the length is 160 m, approximately how many square metres of trees are felled each year in the Otways?



- A.** 1.9×10^4 **B.** 2.76×10^4
C. 1.9×10^6 **D.** 2.76×10^6
2. A designer wants to enlarge a cylinder to 150% of its original dimensions. If the diameter of the original cylinder is 37 mm, what will be the radius of the enlarged cylinder?
- A.** 27.75 mm **B.** 46.25 mm
C. 55.5 mm **D.** 92.5 mm
3. A teenager is to receive 750 mL of saline solution. The drip rate is adjusted to 60 mL per hour. When will it be necessary to change the saline solution if the drip rate begins at 10.30 am?
4. The area of a square is 73 square metres. Which is the closest to the length of each side?
- A.** 8.4 m **B.** 8.5 m
C. 8.6 m **D.** 8.7 m

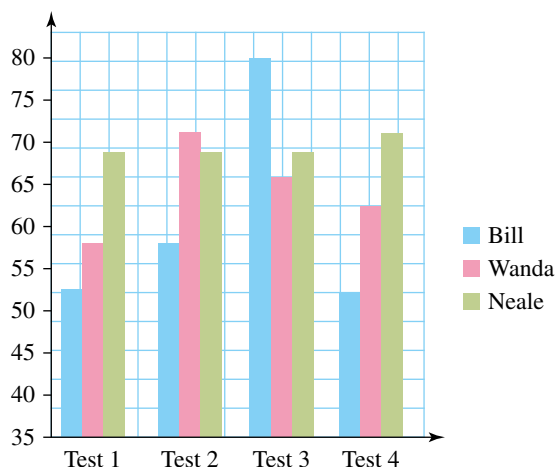


5. A kite is drawn with the coordinates of D being omitted. What are the coordinates of D?



- A.** (-3, -3) **B.** (-3, -8)
C. (-3, 4) **D.** (-3, 6)
6. Your friend is planning a trip of 2320 km. The plan is to drive between 400 km and 480 km each day. At this rate, which of the following would be a reasonable number of days to complete the trip?
- A.** Fewer than 4 days **B.** Between 4 and 6 days
C. Between 6 and 8 days **D.** More than 8 days
7. Last year there were 225 students at Top End High School. This year there are 20 per cent fewer students than there were last year. Approximately how many students are at Top End High School this year?
- A.** 205 **B.** 245
C. 270 **D.** 180

8. This column graph shows four test results for Bill, Wanda and Neale. Which of the following options represents the test results?
- A.** Bill has a higher average than Neale but a lower average than Wanda.
B. Bill has a lower average than Wanda but a higher average than Neale.
C. Wanda has a higher average than Bill but a lower average than Neale.
D. Wanda has a higher average than Neale and Bill.



9. A prepared workout on a treadmill consists of intervals of walking at various rates and angles of incline. A 3% incline means 3 units of vertical rise for every 100 units of horizontal run. My treadmill, when set at a 3% incline, has a horizontal run of 1.6 m. What will be the vertical rise?

A. 4.8 m B. 48 cm C. 48 mm D. 4.8 mm

10. You are about to play your final game in a computer tournament. Your previous scores have been 134, 99, 109, 117 and 101. To win the tournament your average must be at least 114. What is the minimum score you must achieve in this game to win?

11. Suppose your heart rate is 72 beats per minute. How many days will it take your heart to beat 1 000 000 times? Round your answer to the nearest number of days.

12. A gardener wants to put 12 cm of mulch on his garden, the dimensions of which are 20 m by 13 m. How many trailer loads will he require if his trailer holds 1.5 m^3 .

A. 20 B. 21 C. 46 D. 47

13. A machine packs grain at a rate of $1\frac{1}{5}$ tonnes of grain per hour. How long will the machine take to pack 18 000 kg of grain?

A. 15 hours B. 21 hours 6 minutes
C. 21 hours 24 minutes D. 216 hours

14. In the diagram below right, the shape is made from four identical rectangles. Each rectangle has a width of x and a length of $2x$. If the perimeter of the shape is 48 cm, what is the area of the shape?

A. 112.5 cm^2 B. 72 cm^2 C. 60 cm^2 D. 180 cm^2

15. A student recorded the times for 25 people running a 100-metre race. The stem-and-leaf diagram shows the results.

Key: 13|7 represents 13.7 seconds

Stem	Leaf
13	7
14	2 3 4 4
14	5 5 7 7 8 9
15	0 1 2 2 3 4
15	5 5 6 7 9
16	0 1 2

What percentage of students ran for 14.8 seconds or less?

A. 36% B. 40% C. 45% D. 48%

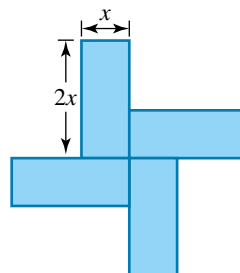
16. A student recorded the times for 25 people running a 200-metre race. The stem-and-leaf diagram shows the results.

Key: 13|7 represents 13.7 seconds

Stem	Leaf
13	7
14	2 3 4 4
14	5 5 7 7 8 9
15	0 1 2 2 3 4
15	5 5 6 7 9
16	0 1 2

What percentage of students ran for more than 14.8 seconds but less than 15.5 seconds?

A. 28% B. 32% C. 36% D. 40%



17. Tsing works in a bakery. One of her chores is to take cardboard sheets and to fold them into small and large trays. It takes 2 minutes to fold a small tray and 3 minutes to fold a large tray. Can she complete 80 small and 45 large trays in the allocated time of 3.5 hours?

A. Yes, Tsing will finish in 2.6 hours. **B.** No, Tsing will take 4 hours and 25 minutes.
C. Yes, Tsing will finish exactly in 3.5 hours. **D.** No, Tsing will take 4 hours and 55 minutes.

18. The maximum quantity of air that can fill your lungs is called Force Vital Capacity (L). This can be modelled by the formula $L = 4.43 \times H - 0.026 \times A - 2.89$ where H = your height (metres) and A = your age (years). If you are 165 cm tall and 15 years old, what is your capacity (to the nearest litre)?

A. 4 **B.** 5
C. 70 **D.** 106



19. What is the surface area of a rectangular prism that has a length of 11 cm, a width of 6 cm and a height of 50 millimetres?

A. 170 cm^2 **B.** 302 cm^2 **C.** 330 cm^2 **D.** 1832 cm^2

20. You have a piece of string 3.7 metres long. You cut it into 6 pieces of equal length to tie to 6 balloons, and there is 22 cm of string left. How long were each of the 6 pieces?

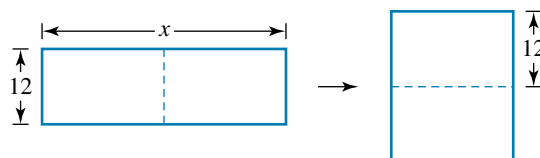
21. My teacher and I boarded a tram together. At the second stop, three people got on. At the third stop, three people got on and one got off. At the fourth stop, three got off. At the fifth stop, six people got off. At the sixth stop, one-half of the passengers got off and I was the only passenger left on the tram. How many passengers were on the tram when my teacher and I got on?

22. Examine the expression $3k^2 + 6k - 5 + 6k^2 + 2$. When it is simplified, which of the following is the equivalent expression?

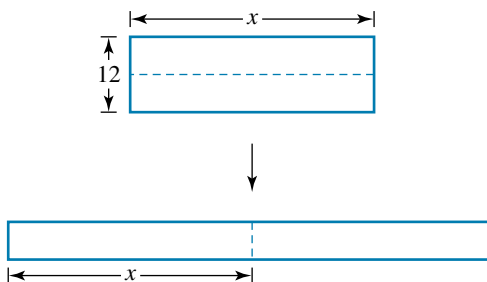
A. $3k^2 + 12k - 3$ **B.** $9k^2 + 6k - 3$ **C.** $9k^2 + 6k + 7$ **D.** $15k^2 + 6k - 3$

23. Jane and Lance each have a rectangular piece of paper of the same dimensions. The length is labelled x cm and the width is 12 cm. Jane and Lance each cut their paper in half in two different ways and then join the halves as illustrated.

Jane's cutting



Lance's cutting



Which of the following represents the sum of the perimeters of the two new designs?

A. $P = 5x + 48$ **B.** $P = 5(x + 12)$ **C.** $P = 6x + 60$ **D.** $P = 6(x + 12)$

24. The process for making a Chinese dish called ‘Dragon’s beard noodles’ by hand is as follows.

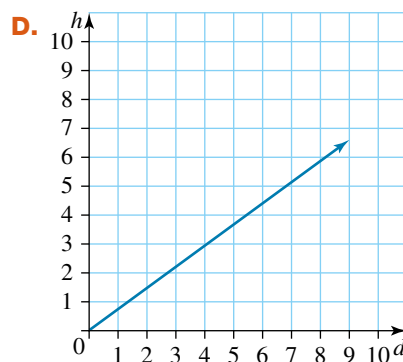
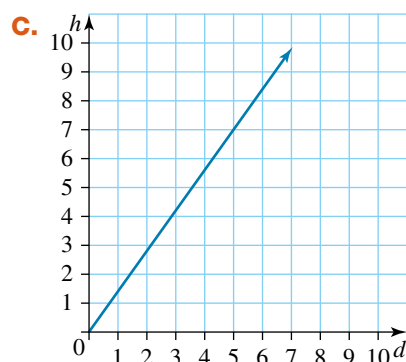
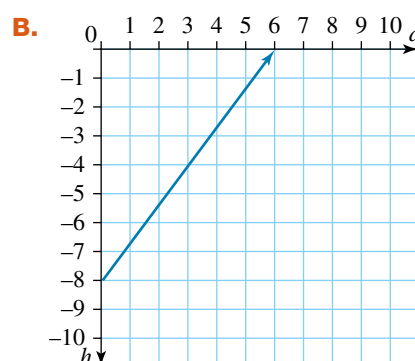
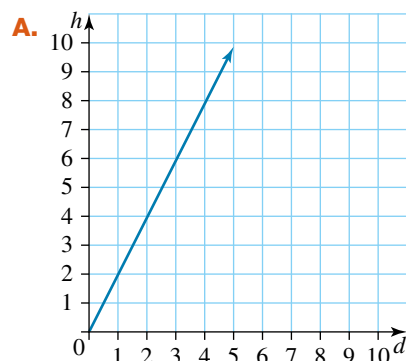
Take a 100-cm strand of dough and fold it in half. Stretch the dough back to its original length so that the two thinner strands are formed. Repeat this process over and over increasing the number of noodles as they get progressively thinner.

After the 6th fold how many noodles strands will you have?

- A. 12 B. 32
C. 64 D. 128
25. The steps on the foreshore have a horizontal width of 24 cm and a height of 18 cm, as shown.



For someone climbing the steps, which of the following graphs models the height above the ground (h) against the distance from the first step (d)?



26. A recipe for pizza dough requires a $20\text{ cm} \times 40\text{ cm}$ rectangular pan. However, you have only a square pan, with dimensions of $25\text{ cm} \times 25\text{ cm}$. What will be the approximate difference in the size of your pizza due to using the square pan?

A. Smaller by 22% B. Larger by 22% C. Smaller by 28% D. Larger by 28%

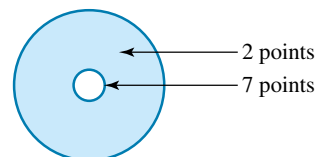
27. Alice ripped a piece of paper into three parts, and tore each of those parts into three more parts. If she repeated this action 3 more times, how many pieces of paper would she have?

A. 15 B. 81 C. 243 D. 729

28. Ning was throwing darts at a target as shown at right. When his dart landed on or inside the circle (bullseye) of the target board, he earned 7 points. However, when his dart landed outside the circle he earned 2 points. After 50 throws his friend reported his score to be 140 points. Ning wanted to know how many bullseyes he had hit, but his friend did not know.

Let x = number of hits on the bullseye. Which of the following equations could be used to solve for x ?

A. $7x + 50 - x = 140$ B. $7x + 2(50 - x) = 140$
C. $7x + 2x = 140$ D. $2x + 2(50 - 7x) = 140$

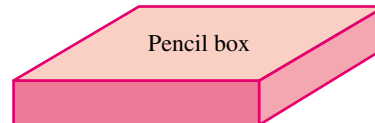
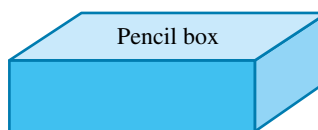


29. John bushwalked 36 km through the Blue Mountains in 3 days. On the first day he hiked 50% of the total distance. On the second day he hiked 25% of the distance that remained. How many kilometres did he hike on the third day?

A. $11\frac{1}{2}$ B. $13\frac{1}{2}$ C. $22\frac{1}{2}$ D. 27

30. Two pencil boxes as shown have the following information. The volume of one pencil box is 24 cm^2 more than the volume of the other pencil box. One box has two of its sides measuring 3 cm and 4 cm, while the other box has sides that measure 1.5 cm and 6 cm. The third sides of the boxes are the same length. What is that length?

A. 2 cm B. 4 cm
C. 6 cm D. 8 cm



1.3.2 Set D (Non-calculator)

1. A 130-cm long strand of wire is cut into three pieces.

The longest piece is three times as long as the second shortest piece.

The second longest piece is three times as long as the shortest piece.

How long is the shortest piece?

A. 18 cm B. 10 cm C. 17.5 cm D. 15 cm

2. One litre of paint covers an area of 20 square metres.

How much paint will cover one square metre?

A. 0.005 litre B. 0.002 litre
C. 0.05 litre D. 0.02 litre

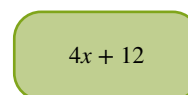
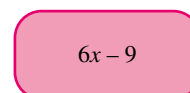
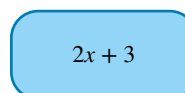
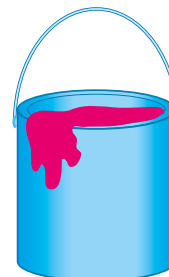
3. A pancake recipe requires $4\frac{1}{2}$ cups of milk. If you wish to make one-fifth of the recipe, how many cups of milk will you need?

A. 0.2 B. 0.5
C. 0.8 D. 0.9

4. Each card pictured is labelled with a value.

What is the mean value of these cards?

A. $12x + 6$
B. $12x + 24$
C. $4x + 6$
D. $4x + 2$



5. In order to purchase a new iPod you must save at least \$260. What inequality represents the amount of money, m , that you must save?

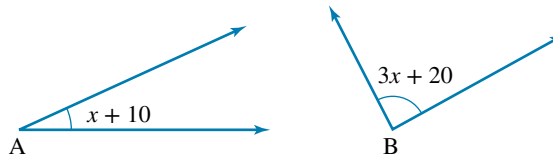
A. $m \leq 260$

B. $m < 260$

C. $m \geq 260$

D. $m > 260$

6. In the diagram below, $\angle A$ and $\angle B$ are complementary.



What is the measure of $\angle B$?

A. 65°

B. 45°

C. 30°

D. 15°

7. A swimming pool is being filled with water. The pool already contained 5000 litres of water. The table shows the number of litres of water in the pool after t hours.

Litres of water in pool (L)	Number of hours (t)
5000	0
7500	1
10000	2
12500	3
15000	4

Which rule can be used to determine the number of litres, L , of water in the pool after t hours?

A. $L = 2500t$

B. $L = 5000t$

C. $L = 5000t + 2500$

D. $L = 5000 + 2500t$

8. Anne wants to solve the equation shown.

$$2x - 3 = 13.$$

Which steps could she use to find the solution?

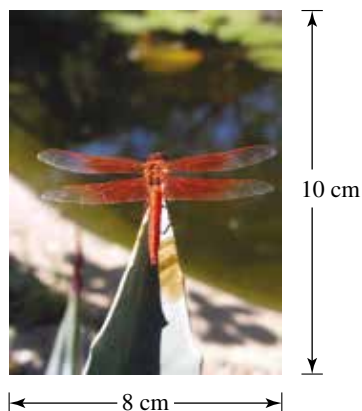
A. Add 3 to both sides, then divide both sides by 2.

B. Subtract 3 from both sides, then divide both sides by 2.

C. Divide both sides by 2, then add 3 to both sides.

D. Multiply both sides by 2, then subtract 3 from both sides.

9. The photograph of a dragonfly is shown with its dimensions.



You decide to enlarge the photograph, and the new width is 12 cm. What is the new length?

A. 14 cm

B. 15 cm

C. 16 cm

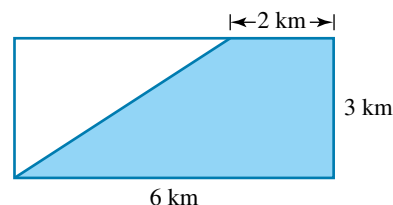
D. 17 cm

10. Madeline noticed that in one minute she blinked 50 times. At this rate, approximately how many days will it take her to blink 1 000 000 times?

A. 8 B. 10 C. 12 D. 14

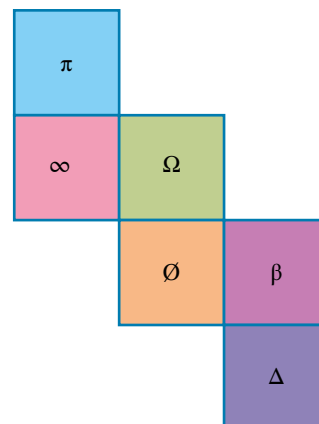
11. A parcel of land is to be subdivided as shown below. The shaded area is to be sold. What percentage of the total area does this represent?

A. $33\frac{1}{3}$ B. 50
C. $66\frac{2}{3}$ D. 75



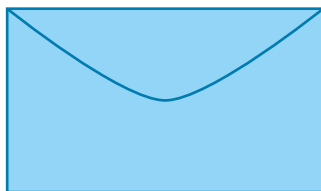
12. When the diagram shown is folded to make a cube, what symbol is on the face opposite the face marked Δ ?

A. β B. Ω
C. $\emptyset$ D. ∞



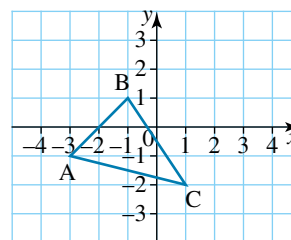
13. You have an envelope that has a perimeter of 35 cm. If the ratio of the length to the width is 4 : 3, what are the dimensions of the envelope?

A. $L = 20$ cm, $W = 15$ cm B. $L = 10$ cm, $W = 7.5$ cm
C. $L = 15$ cm, $W = 20$ cm D. $L = 7.5$ cm, $W = 10$ cm



14. A triangle ABC was drawn on coordinate axes as shown. What would be the coordinates of the triangle reflected in the y-axis?

A. (1, 3) (1, 1) (-1, -3) B. (3, -1) (1, 1) (-1, -2)
C. (-3, 1) (-1, -1) (1, 3) D. (3, 1) (1, 1) (1, -3)



15. Evaluate the following expression.

$$2\frac{2}{5} - 1\frac{2}{3}$$

16. An electronic device counted 4500 vehicles passing through an intersection during an 8-hour period. If the number of vehicles passing through the intersection per hour remains the same, what proportion can be used to find x , the number of vehicles that would be counted during a 10-hour period?

A. $\frac{4500}{8} = \frac{x}{10}$ B. $\frac{8}{4500} = \frac{x}{10}$
C. $\frac{8}{x} = \frac{10}{4500}$ D. $\frac{8}{4500} = \frac{10}{x}$



17. John has five fewer marbles than Liam, and Tang has three times as many as John. If Liam has n marbles, which of the following represents the number of marbles that Tang has?

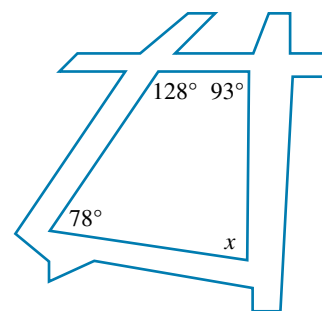
A. $3n - 5$ B. $3n$ C. $5 - 3n$ D. $3(n - 5)$

18. The directions for using a concentrated cleaning product say to add 3 capfuls of the product to every 4 litres of water. Which of the following equations can be used to calculate c , the number of capfuls of the product needed for 7 litres of water?

A. $\frac{3}{4} = \frac{c}{7}$ B. $\frac{3}{4} = \frac{c}{11}$ C. $\frac{4}{3} = \frac{c}{7}$ D. $\frac{4}{3} = \frac{c}{11}$

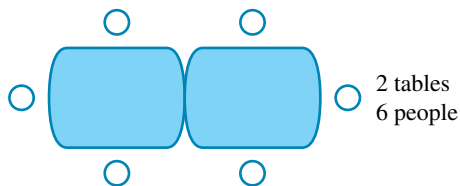
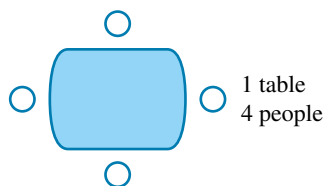
19. Intersecting paths have been constructed to surround a decorative garden bed. What is the angle measurement for x ?

A. 128° B. 102°
C. 87° D. 61°



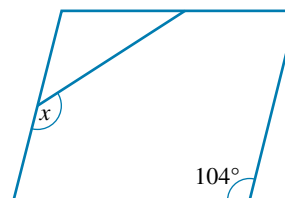
20. The diagrams below show the seating arrangements that will be used if tables are placed end to end. Which formula represents the relationship between the number of people (P) that can be seated and the number of tables (t) placed end to end?

A. $P = 4t$ B. $P = 3t$
C. $P = 4t - 2$ D. $P = 2t + 2$



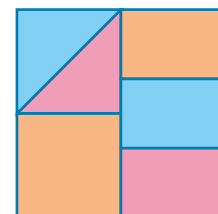
21. The diagram at right shows a rhombus. The midpoints of two of its sides are joined with a straight line. What is the measure of angle x ?

A. 76° B. 104°
C. 142° D. 152°



22. A square dart board is shown at right. Suppose a dart, thrown randomly, hits the board. Determine the probability of the dart's landing on a blue segment.

A. $\frac{7}{12}$ B. $\frac{1}{8}$
C. $\frac{7}{24}$ D. $\frac{1}{14}$



23. In how many different ways could these plastic bottles be arranged in a line?



24. What is the solution for the following expression?

$$5 + \frac{70}{10} \times (1 + 2)^2 - 1$$

A. 95 B. 71 C. 67 D. 46

25. A helicopter has a rotor that moves at a rate of 720 revolutions per minute. Through how many degrees does the rotor turn per second?

A. 0.03 B. 12
C. 4320 D. 259200



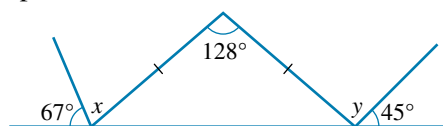
26. You counted a total of 40 goldfish in a pond in the Botanical Gardens. The gardener told you that the ratio of female fish to male fish was 3 : 5. What was the total of the number of male goldfish?

A. 24 B. 25 C. 26 D. 30

27. Based on the table, which statement is true?

Polygons	
Number of sides (n)	Sum of interior angle measures (S)
3	180°
4	360°
5	540°
6	720°
7	900°

- A. The sum of the interior angle measures decreases by $\frac{1}{2}$ for each side increase of 1.
 B. The sum of the interior angle measures increases by 180° for each side increase of 1.
 C. The sum of the interior angle measures doubles for each side increase of 1.
 D. The sum of the interior angle measures is a whole number multiple of 360° .
28. The sum of x and y is:
 A. 196° B. 180°
 C. 135° D. 113°



29. A box is displayed on the top of a shop counter as shown in the photograph. Its dimensions are length = $1\frac{1}{4}$ m, width = $1\frac{2}{5}$ m and height = 0.8 m.



- What is the volume of the box?
 A. 1.2 m^3 B. 1.25 m^3
 C. 1.4 m^3 D. 1.65 m^3
30. If two sides of a triangle are 12 cm and 20 cm, the third side must be:
 A. between and including 8 cm and 32 cm.
 B. between but not including 8 cm and 32 cm.
 C. greater than 8 cm.
 D. less than 32 cm.

1.4 NAPLAN practice 3

1.4.1 Set E (Calculator allowed)

1. The students in a class measure their heights in cm. The following stem-and-leaf plot shows their heights.

Key: 13|5 means 135 cm.

Stem	Leaf
13	5 8
14	0 4 9
15	3 5 5
16	3

What is the mean height?

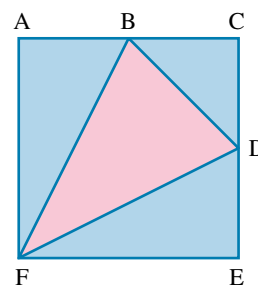
- A. 148 cm B. 149 cm C. 155 cm D. 163 cm

2. You have a rectangular box with a lid. The top of the lid has an area of 392 square centimetres. The ratio of the width to the length of the lid is 1 : 8. What are the dimensions of the lid?

A. 4 cm by 98 cm B. 7 cm by 56 cm C. 8 cm by 49 cm D. 8 cm by 64 cm

3. At right is a design of a quilt square (ACEF) whose sides are 5 cm long. B and D are the midpoints of sides AC and CE respectively. What is the area of the triangle BDF?

A. 6.25 B. 9.375
C. 12.5 D. 15.625



4. A photograph is placed on a card measuring 8.5 cm by 10 cm. A 1.5-cm edging is left all around the photograph. What area (in square cm) does the photograph cover?

A. 59.5 cm² B. 49 cm²
C. 46.75 cm² D. 38.5 cm²

5. A student recorded the temperature outside her classroom every hour for one school day. She drew up the following table.

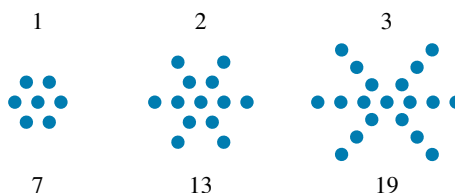
Time	9 am	10 am	11 am	12 pm	1 pm	2 pm	3 pm
Temperature °C	11	13		34	36	33	29

If the temperature increased from 11 am to midday by $41\frac{2}{3}\%$, what was the temperature at 11 am?

A. 22° B. 23° C. 24° D. 25°

6. The school bus took $1\frac{1}{4}$ hours to travel 80 km. How far did it travel in $2\frac{1}{2}$ hours?

7. Consider the following pattern.



Which one of the following equations could model the design, where C represents the total number of dots (the numbers shown below) and n represents the series number (the numbers shown above)?

A. $C = 7n - 1$ B. $C = 6n$ C. $C = 6n + 1$ D. $C = 6n - 1$

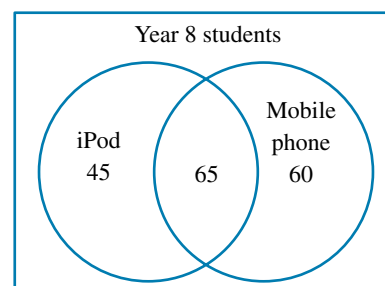
8. A bag of fertiliser is made by mixing nitrates, potash and phosphates in the ratio of 3 : 2 : 5. How much fertiliser will be produced if 15 kg of nitrates are used?

A. 22 kg B. 25 kg C. 45 kg D. 50 kg

9. Fuel for a two-stroke engine is made by mixing petrol and oil in the ratio 30 : 1. If you have 250 mL of oil, how much petrol needs to be added to make a two-stroke mixture?

A. 0.12 litre B. 1.2 litres C. 7.5 litres D. 8.3 litres

10. The Venn diagram shows how many of the 200 Year 8 students have an iPod only, a mobile phone only, or both an iPod and a mobile phone. Use the information in the diagram to find the probability that a student chosen at random has neither an iPod nor a mobile phone.



A. $\frac{1}{30}$ B. $\frac{3}{20}$
C. $\frac{6}{10}$ D. $\frac{1}{20}$

11. The Orion Nebula is approximately 1.5×10^3 light-years away from Earth. It is visible with the naked eye. One light-year is approximately 9.5×10^{12} km. What is the approximate distance in kilometres between Earth and the Orion Nebula?

A. 1.425×10^{15} B. 1.425×10^{16}
C. 1.1×10^{11} D. 6.33×10^9

12. An express train leaves Geelong at 8.55 am and arrives in Altona station at 9.37 am. If the train travelled 52 km, what was the train's average speed in km/h?

A. 98 km/h B. 84 km/h
C. 74 km/h D. 71 km/h

13. A packet of M&Ms contains red, orange, blue, green, yellow and brown chocolates. The probability of choosing a colour from the packet is shown in the table below.

Colour	Red	Orange	Blue	Green	Yellow	Brown
Probability	0.3	0.13	0.12		0.25	0.09

What is the probability of choosing a green M&M?

A. 0.9 B. 0.38
C. 0.11 D. 0.1

14. The following table shows the number of M&Ms of different colours in a particular bag.

Colour	Number	Expression
Red		x
Orange		$x - 2$
Blue		$x - 1$
Green		$x + 1$
Yellow		$x - 3$
Brown		$x + 2$
Total	33	

How many yellow M&Ms are in the bag?

A. 30 B. 20 C. 10 D. 3

15. In a bag of 200 jelly beans, the following distribution of colours was found.

Brown 13%, yellow 14%, red 13%, blue 24%, orange 20%, green 16%

How many blue jelly beans are in the bag?

A. 20 B. 24 C. 48 D. 40

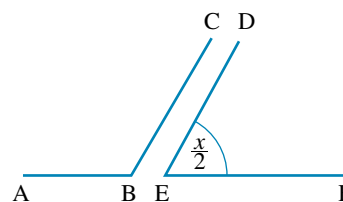
16. 'Dragon beards' is a special Chinese dish. The noodles are hand-pulled until they are extraordinarily fine. It has been calculated that a piece of pasta prepared with 1.5 kilograms of flour can make 144 000 hair-thin noodles, each 20 centimetres long. If you joined all the strands together, what would be their total length?

A. 28.8 km B. 288 km C. 28 800 km D. 288 000 km

17. The angles below are supplementary. The measure of $\angle DEF$ is $\frac{x}{2}$.

What expression represents the measure of $\angle ABC$?

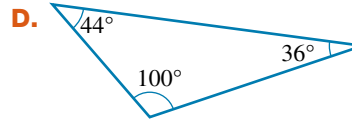
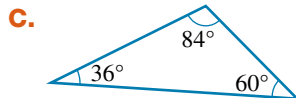
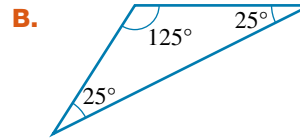
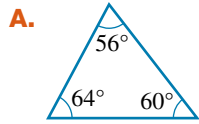
A. $90 - \frac{x}{2}$ B. $90 + \frac{x}{2}$
C. $180 - \frac{x}{2}$ D. $180 + \frac{x}{2}$



18. A driver drives 8 km south, then 6 km east followed by 2 km south. He then travels 3 km west. Next, in order to avoid a traffic jam, he turns and travels 6 km north. How many kilometres is the driver south of his starting point?
19. If the ratios of the sides of a quadrilateral are given in order as 3 : 3 : 4 : 4, what type of quadrilateral must it be?

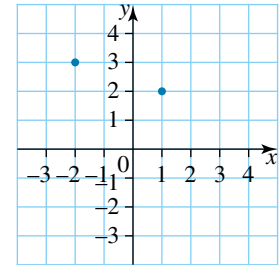
A. Rectangle **B.** Square **C.** Kite **D.** Parallelogram

20. The angles of a triangle are in the ratio 3 : 5 : 7. Which diagram has angles in this ratio?



21. A trapezium was created on a Cartesian plane using the coordinates A (2, 0), B (−2, 0), C (−3, 4) and D (x, 4). What is the value for x if the area of the trapezium is 24 units?

22. Ann is drawing a right-angled isosceles triangle on a coordinate grid. She has plotted two of the corners of the triangle at (−2, 3) and (1, 2). Which of the following coordinates could be the third corner of the right-angled isosceles triangle?



A. (−1, 4) **B.** (4, 3)
C. (0, −1) **D.** (3, 5)

23. Ann recorded the highest temperature each day over a 7-day period in February. Which statement describes the data?

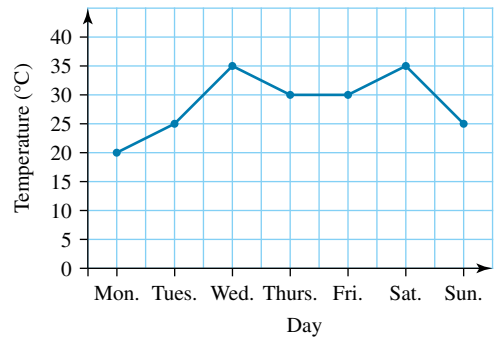
A. Median > mean **B.** Mean = mode
C. Median = mean **D.** Median < mean

24. Ethan wants to solve for x in this equation.

$$1\frac{2}{5} - x \times \frac{2}{3} = 1\frac{2}{3}$$

Which step should he perform first?

A. Divide both sides by $\frac{2}{3}$.
B. Multiply both sides by $\frac{2}{3}$.
C. Subtract $1\frac{2}{5}$ from both sides.
D. Add $1\frac{2}{5}$ to both sides.



25. Al-Samaw'al was simplifying the expression $6x - 3(x - 5)$. Which is the equivalent expression?

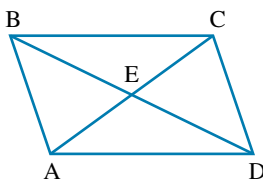
A. $3x - 5$ **B.** $3x + 5$ **C.** $3x - 15$ **D.** $3x + 15$

26. In the parallelogram ABCD below, AC and DB intersect at E.

AE = $3x - 3$ and EC = $x + 13$.

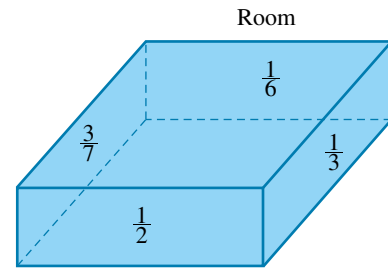
What is the value of x?

A. 8 **B.** 16 **C.** 20 **D.** 40

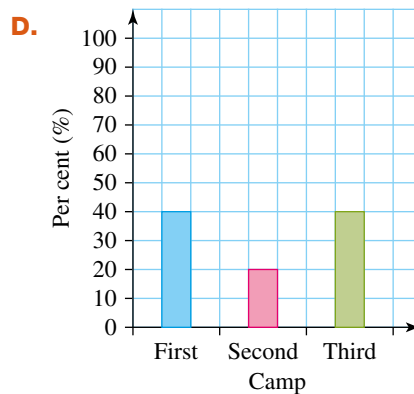
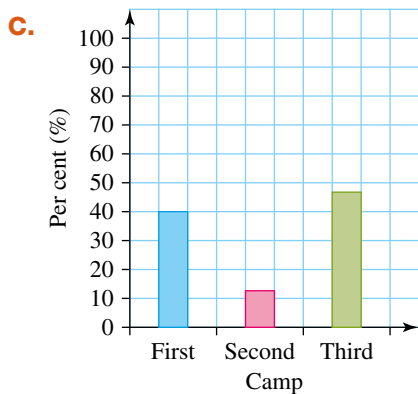
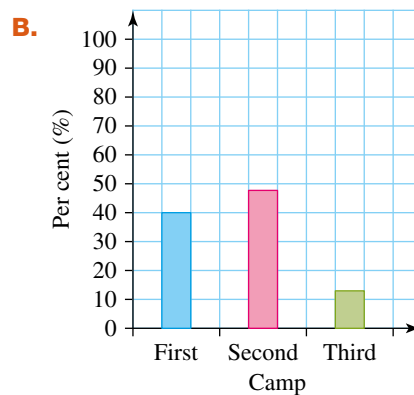
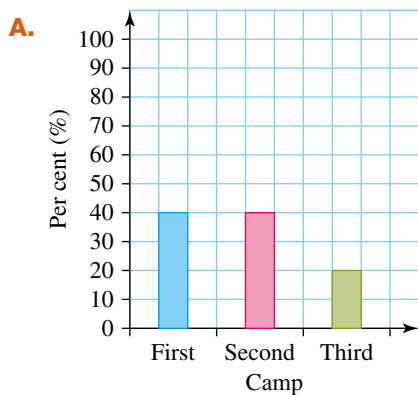


1.4.2 Set F (Non-calculator)

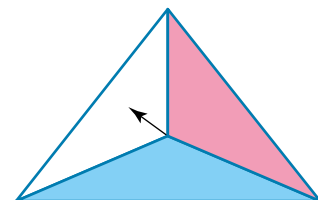
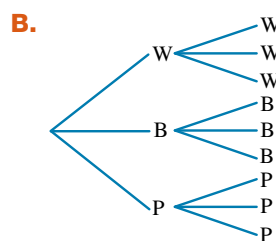
1. A room has four identical walls that are being painted. For decorative purposes, each of the four walls is being painted in a variety of colours. One of the colours is pink, and the four walls will each have different areas that are painted this colour. The fraction of each wall that is painted pink is shown on the diagram. What fraction of all 4 walls will be painted with this colour?

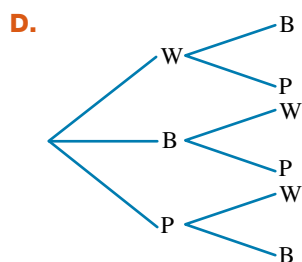
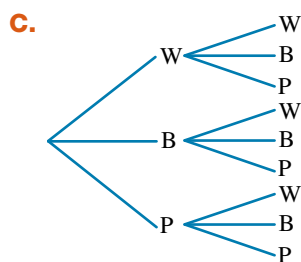


- A. $\frac{5}{7}$ B. $\frac{6}{7}$
 C. $\frac{5}{14}$ D. $1\frac{5}{7}$
2. Five friends earned money by helping to build a wall in a garden. After they divided the money equally, they each received \$280. Which of the following equations could be used to determine x , the total amount (in dollars) that the five friends earned?
- A. $5x = 280$ B. $x - 5 = 280$ C. $x + 5 = 280$ D. $\frac{x}{5} = 280$
3. There are 150 Scouts at a camp. For 40% of them, this is their first camp. One-fifth of the remainder have been to one other camp. The others have been to three camps. Which of the following represents the percentages of students on camp for the first, second and third times?



4. Ash spins the arrow twice on the spinner. Which tree diagram shows all the possible outcomes?





5. The human heart beats an average of 37 800 000 times in one year. Which of the following is an equivalent value?
- A.** 3.78×10^{-7} **B.** 3.78×10^{-6} **C.** 3.78×10^6 **D.** 3.78×10^7
6. Which unit of measurement would be most appropriate for measuring the area of a page of newspaper?
- A.** mm^2 **B.** cm^2 **C.** m^2 **D.** km^2
7. A hockey team played n games, losing five of them and winning the rest. The ratio of games won to games lost is:
- A.** $\frac{n}{5}$ **B.** $\frac{n-5}{5}$ **C.** $\frac{5}{n}$ **D.** $\frac{5}{n-5}$

8. What is the value of the angle x in the diagram at right?

- A.** 85° **B.** 70°
C. 110° **D.** 130°

9. An ironing board with the measurements shown is advertised online. When the ironing board is set up, two similar triangles are formed. How long is the top?

- A.** 90 cm **B.** 62.5 cm
C. 51.2 cm **D.** 48 cm

10. The stem-and-leaf plot below shows the age of each member of a bicycle club.

Bike club members' ages

Key: 1|8 represents 18

Stem	Leaf
1	8 8 9
2	3 4 6 6 6 7 9
3	1 2 5 7 8
4	0 2 5
5	2 7

What is the range of the ages of the bicycle club members?

- A.** 57 **B.** 39 **C.** 36 **D.** 30

11. The stem-and-leaf plot below shows the age of each member of a running club.

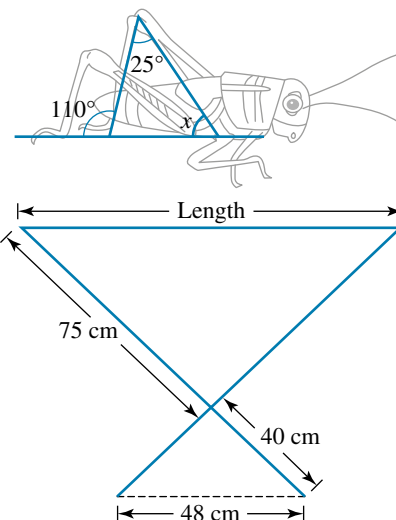
Running club members' ages

Key: 1|3 represents 13

Stem	Leaf
1	3 5 6 7 8 8 9
2	0 1 2 3 4 4 4 5
3	0 2 3 4 5

What is the median age of the club members?

- A.** 22 **B.** 22.5
C. 24 **D.** 24.5



12. The *Chef and Cook* television show produces two square-based cakes. The small cake has a side length of 8 cm while the large cake has a side length of 16 cm. Which of the following statements is true?
- A. The area of the base of the large cake is 2 times the area of the small cake.
 B. The area of the base of the large cake is 4 times the area of the small cake.
 C. The area of the base of the large cake is 8 times the area of the small cake.
 D. The area of the base of the large cake is 16 times the area of the small cake.

13. Calculate the value of the following.

$$25 \times (2768 + 2768 + 2768 + 2768)$$

14. The stage in the hall at League School has the shape of a quadrilateral, as shown. Which of these are the most likely values of x and y ?



- A. $x = 30, y = 160$ B. $x = 45, y = 145$
 C. $x = 63, y = 117$ D. $x = 72, y = 118$

15. The numbers of passengers on a train over a 20-day period were recorded as follows:

59, 65, 73, 83, 90, 83, 71, 92, 60, 58, 96, 66, 75, 76, 85, 77, 86, 79, 87, 79

The data are displayed below on a stem-and-leaf plot.

Key: 5|9 represents 59

Stem	Leaf
5	8 9
6	0 5 ?
7	1 3 5 6 7 9 9
8	3 3 5 6 7
9	0 2 6

What is the missing number?

16. The table shows the results of a survey, which asked drivers how many accidents they had over the previous 5 years. What is the median number of accidents per year?

Number of accidents	0	1	2	3	4	5	6
Number of drivers	16	14	21	4	3	1	1

- A. 0.5 B. 1 C. 1.5 D. 2

17. Alex wants to know the answer to the following expression.

$$(5 \times 10^4) + (2 \times 10^2) + (4 \times 10) = \square$$

Which of the following is correct?

- A. 502400 B. 52400 C. 50240 D. 5240

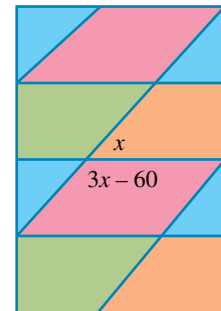
18. What is the missing number in the following equation?

$$\frac{4}{3} \div \frac{?}{3} = \frac{1}{8}$$

19. My mother is 4 times as old as I am. My sister is 75% of my age and 10% of my grandfather's age. My father is 50, which is 2 years older than my mother. How old are my sister and grandfather?

20. In the quilt design at right, what is the measure of x ?

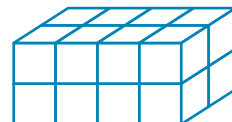
- A. 10° B. 30°
 C. 40° D. 60°



21. Is the following equation true or false?

$$12 + (5 \times 9) - (108 \div 2) = 3$$

22. The solid brick shown is made of small cubic bricks of sides 1 unit. When the large brick is disassembled into its component small bricks, the total surface area of all the small bricks is how much greater than the surface area of the large brick?

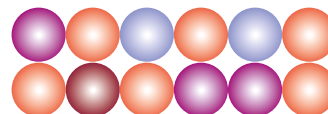


- A. 32 B. 40 C. 56 D. 96

23. Jane ate 0.2 parts of pizza and her friend ate 0.02 parts of the same pizza. Find the ratio between the parts they ate.
- A. 10 : 1 B. 5 : 2
C. 1 : 1 D. 1 : 10

24. A marble is selected at random. What is the probability that the marble will not be blue?

- A. $\frac{5}{6}$ B. $\frac{1}{6}$
C. $\frac{3}{14}$ D. $\frac{3}{7}$



25. Write 70 as a percentage of 200.

26. If $4x + 3 = 22$, what is the value of $4x - 3$?

- A. -3 B. 0 C. 16 D. 19

27. The following pattern consists of 10 cubes. If this pattern continues, how many cubes will it take to make 10 layers?



28. The first number in a pattern is 48. To go from one number to the next the rule is to divide by 4. What is the fourth number in the pattern?

- A. $\frac{4}{3}$ B. $\frac{3}{4}$ C. $\frac{1}{3}$ D. $\frac{1}{4}$

29. Which of the following is equivalent to the expression $2^6 \times 2^4$?

- A. 2^{24} B. 2^{10} C. 4^{24} D. 4^{10}

30. Andrew wants to find the value for the expression $\frac{6x + 5}{8x + 5}$ when $x = -10$.

What is the value?

- A. $\frac{2}{3}$ B. $\frac{22}{3}$ C. $\frac{11}{15}$ D. $\frac{211}{15}$

1.5 Review

Throughout this topic you have practised completing NAPLAN tests. It is important to be familiar with how the NAPLAN tests are run and what to expect on the day of the tests.

Answers

Topic 1 NAPLAN practice

Exercise 1.2.1 Set A (Calculator allowed)

1. D	2. D	3. B	4. A	5. B	6. A
7. 120°	8. A	9. C	10. B	11. C	12. 5582 cm^3
13. D	14. B	15. 2.1 m	16. $\frac{1}{4}$	17. C	18. B
19. C	20. D	21. A	22. B	23. B	24. C
25. A	26. D	27. C	28. B	29. C	

Exercise 1.2.2 Set B (Non-calculator)

1. B	2. C	3. D	4. A	5. C	6. B
7. B	8. 47.56 s	9. C	10. C	11. B	12. D
13. B	14. D	15. C	16. B	17. C	18. C
19. C	20. D	21. D	22. 205	23. B	24. C
25. 107°	26. 40%	27. 7 pegs in each yellow bucket; 18 pegs in each blue bucket.			
28. B	29. D	30. B			

Exercise 1.3.1 Set C (Calculator allowed)

1. D	2. A	3. 11 pm	4. B	5. C	6. B
7. D	8. C	9. C	10. 124	11. 10 days	12. B
13. A	14. B	15. B	16. A	17. D	18. A
19. B	20. 58 cm	21. 4	22. B	23. B	24. C
25. D	26. A	27. C	28. B	29. B	30. D

Exercise 1.3.2 Set D (Non-calculator)

1. B	2. C	3. D	4. D	5. C	6. A
7. D	8. A	9. B	10. D	11. C	12. B
13. B	14. B	15. $\frac{11}{15}$	16. A	17. D	18. A
19. D	20. D	21. C	22. C	23. 24	24. C
25. C	26. B	27. B	28. A	29. C	30. B

Exercise 1.4.1 Set E (Calculator allowed)

1. A	2. B	3. B	4. D	5. C	6. 160 km
7. C	8. D	9. C	10. B	11. B	12. C
13. C	14. D	15. C	16. A	17. C	18. 4 km
19. C	20. C	21. 5	22. C	23. A	24. C
25. D	26. A				

Exercise 1.4.2 Set F (Non-calculator)

1. C	2. D	3. C	4. C	5. D	6. B
7. B	8. A	9. A	10. B	11. B	12. B
13. 276800	14. C	15. 6	16. C	17. C	18. 32
19. 9, 90	20. D	21. True	22. C	23. A	24. A
25. 35%	27. 220	29. B			
26. C	28. B	30. C			

TOPIC 2

Computation with integers

2.1 Overview

Numerous **videos** and **interactivities** are embedded just where you need them, at the point of learning, in your learnON title at www.jacplus.com.au. They will help you to learn the concepts covered in this topic.

2.1.1 Why learn this?

Integers are whole numbers that can be positive, negative or zero. There are many examples in everyday life where an understanding of integers is useful, such as money transactions and temperatures. A number line gives a picture of the world of numbers: positive, zero and negative. Many problems can be worked out using a number line.



LEARNING SEQUENCE

- 2.1 Overview
- 2.2 Adding and subtracting integers
- 2.3 Multiplying integers
- 2.4 Dividing integers
- 2.5 Combined operations on integers
- 2.6 Review

LEARNING OUTCOMES

A student:

- communicates and connects mathematical ideas using appropriate terminology, diagrams and symbols MA4-1WM
- applies appropriate mathematical techniques to solve problems MA4-2WM
- recognises and explains mathematical relationships using reasoning MA4-3WM
- compares, orders and calculates with integers, applying a range of strategies to aid computation MA4-4NA

CONTENT DESCRIPTIONS

Students:

Compare, order, add and subtract integers (ACMNA280)

Carry out the four operations with rational numbers and integers, using efficient mental and written strategies and appropriate digital technologies (ACMNA183)

Source: NSW Syllabus for the Australian curriculum

learn**on** RESOURCES — ONLINE ONLY

eLesson: The story of mathematics — Amalie Noether (eles-2021)

Note: Your teacher may now set you a pre-test to determine how familiar you are with the content in this topic.

2.2 Adding and subtracting integers

2.2.1 Integers

- **Integers** are positive whole numbers, negative whole numbers and zero.
- The group of integers is often referred to as the set Z .
- $Z = \{\dots -4, -3, -2, -1, 0, 1, 2, 3, 4\dots\}$
- When a number is given with a positive or negative sign (e.g. $+3$ or -5) it is known as a directed number.
- The direction of an integer indicates whether it is positive or negative. The magnitude of an integer indicates the size of the integer as measured by the distance from 0.

2.2.2 Addition of integers

- A number line can be used to add integers.
- To add a positive integer, move to the right.
- To add a negative integer, move to the left.

WORKED EXAMPLE 1

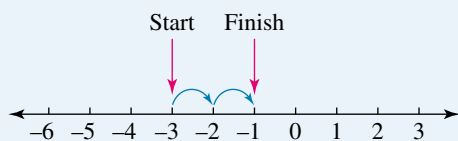
Calculate the value of each of the following.

a $-3 + +2$

b $-3 + -2$

THINK

- a 1 Start at -3 and move 2 units to the right, as this is the addition of a positive integer.



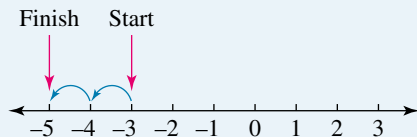
- 2 Write the answer.

WRITE

a $-3 + +2$

$$-3 + +2 = -1$$

- b 1 Start at -3 and move 2 units to the left, as this is the addition of a negative integer.



- 2 Write the answer.

b $-3 + -2$

$$-3 + -2 = -5$$

DISCUSSION

Negative numbers are used to describe many real-life situations, including temperatures. What limitations would be placed on our ability to describe certain situations if we could not go below zero?

2.2.3 Subtraction of integers

- Subtracting an integer gives the same result as adding its opposite.
For example, $-3 - 5 = -3 + +5 = -3 + -5 = -8$.
Note that $+5$ and -5 are opposites.
- By developing and extending a pattern, we can show that subtracting a negative has the same effect as adding a positive.
Look at the pattern shown at right.
It can be seen from the table that subtracting a negative is the same as adding its inverse. For example, $8 - -4 = 8 + +4 = 12$.
- In mathematics, a number without a positive or negative sign is considered to be positive.
So $8 + +4$ can be written as $8 + 4$ and $-5 - +1$ can be written as $-5 - 1$.

$8 - 3 =$	5
$8 - 2 =$	6
$8 - 1 =$	7
$8 - 0 =$	8
$8 - -1 =$	9
$8 - -2 =$	10
$8 - -3 =$	11

WORKED EXAMPLE 2

Calculate the value of each of the following.

a $-7 - +1$

b $-2 - -3$

THINK

a 1 Subtracting an integer gives the same result as adding its opposite.

2 Using a number line, start at -7 and move 1 unit to the left.

3 Write the answer.

WRITE

a $-7 - +1 = -7 + -1$

$= -8$

b 1 Subtracting an integer gives the same result as adding its opposite.

2 Using a number line, start at -2 and move 3 units to the right.

3 Write the answer.

b $-2 - -3 = -2 + +3$

$= +1$

ACTIVITY: WALK THE LINE

Equipment: A4 paper (If doing this activity outside, you will also need chalk and a pen.)

- Mark a number line from -12 to $+12$ on the floor using sheets of paper or on the ground using chalk.
- Your teacher will call out a question. With a partner, find your starting point and have one person walk the steps along the number line to the answer, while the other remains at the starting point. Compare the start and finish positions.
- Repeat step 2 until each pair of students has solved a question.
- As a pair, on an A4 sheet of paper, write an addition or subtraction question that can be solved using your number line. Write the answer and your names on the other side of the sheet. Hand your completed sheet to the teacher.
- As pairs, you will now walk the line for another group's question.
- Are there any relationships or shortcuts you can use when adding and subtracting positive and negative numbers on a number line?

Interactivity: Addition and subtraction of integers (int-3703)

Interactivity: Directed number target (int-0074)

eLesson: Directed numbers (eles-0040)

eLesson: Addition and subtraction of positive and negative numbers (eles-1869)

Digital doc: SkillsSHEET Arranging numbers in order (doc-6389)

Digital doc: SkillsSHEET Integers on the number line (doc-6387)

Digital doc: SkillsSHEET Adding and subtracting integers (doc-6388)

Exercise 2.2 Adding and subtracting integers

Individual pathways

PRACTISE

Questions:

1–3, 4a–d, 5a–d, 6a–i, 11, 12, 17–19

CONSOLIDATE

Questions:

1–6, 7a, b, 9–14, 17–19

MASTER

Questions:

3a–h, 4e–h, 5e–h, 6k–r, 7c, d, 8–11, 13–20

Individual pathway interactivity: int-4397

learnon ONLINE ONLY

To answer questions online and to receive **immediate feedback** and **fully worked solutions** for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Understanding and fluency

1. Which of the following numbers are integers?

$3, \frac{1}{2}, -4, 201, 20.1, -4.5, -62, -3\frac{2}{5}$

2. Copy and complete the following addition and subtraction number patterns by placing the correct integers in the boxes.

a. 6, 4, 2, , ,

b. -5, -10, -15, , ,

c. , , , -1, -3, -5

d. , , , -2, 0, 2

3. **WE1** Calculate the value of each of the following.

a. $-3 + 2$

b. $-7 + -3$

c. $6 + -7$

d. $-8 + -5$

e. $13 + +6$

f. $12 + -5$

g. $-25 + +10$

h. $16 + -16$

4. **WE2** Calculate the value of each of the following.

a. $7 - +2$

b. $-18 - +6$

c. $3 - +8$

d. $11 - +6$

e. $17 - -9$

f. $-28 - -12$

g. $14 - -8$

h. $-17 - -28$

5. Calculate the value of each of the following.

a. $-3 + -5$

b. $6 - -5$

c. $-17 + +3$

d. $-14 - -13$

e. $28 - -23$

f. $-48 + -3$

g. $-57 - -18$

h. $-32 - -40$

6. Calculate the value of each of the following.

a. $-4 + -3$

b. $-6 - +3$

c. $5 + -2$

d. $17 - +5$

e. $-13 - -3$

f. $10 - -3$

g. $-26 + -14$

h. $25 + -7$

i. $32 - +5$

- j. $-16 - +18$ k. $-26 - -15$ l. $124 - -26$
m. $-3 + -4 - -6$ n. $27 + -5 - -3$ o. $-10 + +3 - +6$
p. $23 + -15 - -14$ q. $15 - -4 + -10$ r. $-37 - -5 - -10$

7. Copy and complete the following tables. For the subtraction table, subtract the number on the side from the number at the top.

a.

+	-8	+25	-18	+32
-6	$-8 + -6 = -14$			
-13				
-16				
-19				

b.

-	+15	-17	-27	+57
+7	$+15 - +7 = 8$			
-6				
-9				
+12				

c.

+	-11		+13	
	-16			
+17		36		
		18	12	
-28				-35

d.

-	+9			+42
	-17			
-14			-1	
		23		
+23		-2		

8. Design your own tables for addition and subtraction of integers like those in question 7. Fill in all answers in your tables.

9. In a kitchen, some food is stored at -18°C in a freezer and some at 4°C in the fridge. A roast is cooking in the oven at a temperature of 180°C . Before answering each of the following questions, draw a number line to show the positions of the temperatures.

- a. What is the difference in temperature between the food stored in the freezer and the food stored in the fridge?
(Hint: Difference = largest value - smallest value)
b. What is the difference in temperature between the food stored in the fridge and the roast cooking in the oven?
c. What is the difference in temperature between the food stored in the freezer and the roast cooking in the oven?

10. Calculate the difference between the two extreme temperatures recorded at Mawson Station in Antarctica in recent times.



11. Locate the button on your calculator that allows you to enter negative numbers. Use it to answer the following.

a. $-458 + 157$

b. $-5487 - 476$

c. $-248 - -658 - -120$

d. $-42 + 57 - -68 + -11$

12. Write out these equations, filling in the missing numbers.

a. $-7 + \square = 6$

b. $8 - \square = 12$

c. $-15 - \square = -26$

d. $42 - \square + -17 = 35$

e. $-7 - \square - -31 = -28$

f. $\square - 13 + 21 = 79$

13. The following is a homework sheet done by a student in Year 8. Correct her work for her and give her a mark out of six. Make sure you include the correct answer if her answer is wrong.

a. $-3 + -7 = -10$

b. $-4 - -10 = -6$

c. $-7 - 8 = 15$

d. $9 - -8 + -7 = 10$

e. $42 + 7 - -11 = 60$

f. $-17 + 4 - 8 = 21$



Communicating, reasoning and problem solving

14. Evaluate and compare the following pairs of expressions.

a. $-4 + 1$ and $+1 - 4$

b. $-7 + 5$ and $+5 - 7$

c. $-8 + 3$ and $+3 - 8$

15. What did you notice about the answers in question 14? Use a number line to help you explain why this is the case.

16. Evaluate and compare the following pairs of expressions.

a. $-2 + -5$ and $-(2 + 5)$

b. $-3 + -8$ and $-(3 + 8)$

c. $-7 + -6$ and $-(7 + 6)$

17. What did you notice about the answers in question 16? Explain why this is the case.

18. Igor visited the Macau Casino. The currency used in the casino is the Hong Kong dollar (HK\$).

Arriving at the casino at 10 pm, Igor went straight to the baccarat table, where he won HK\$270. The roulette wheel that he played next was a disaster, costing HK\$340 in a very short space of time.

Igor moved to the 'vingt-et-un' table where he cleaned up, making HK\$1450 after a run of winning hands.

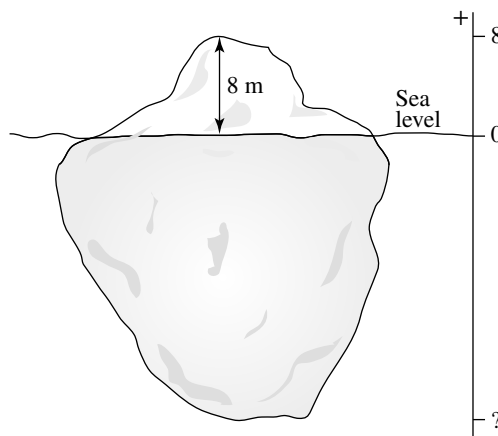
Little did Igor know that when he sat down at the stud poker table it would signal the end of his night at the casino. He lost everything in one hand, HK\$2750, not even leaving enough for a taxi to his hotel.

- a. By writing a loss as a negative number and a win as a positive number, write a directed number sentence to represent this situation.

- b. Find out how much money Igor had when he arrived at the casino.

19. What strategy will you use to remember how to add and subtract integers?

20. The tip of this iceberg is 8 metres above sea level. If one-tenth of its total height is above the surface, represent the depth of its lowest point as an integer.



2.3 Multiplying integers

2.3.1 Multiplication and powers

- Patterns in the answers in multiplication tables can be used to determine the sign of the product when two directed numbers are multiplied. Consider the following patterns.

$3 \times 3 = 9$	$-3 \times 3 = -9$
$3 \times 2 = 6$	$-3 \times 2 = -6$
$3 \times 1 = 3$	$-3 \times 1 = -3$
$3 \times 0 = 0$	$-3 \times 0 = 0$
$3 \times -1 = -3$	$-3 \times -1 = 3$
$3 \times -2 = -6$	$-3 \times -2 = 6$
$3 \times -3 = -9$	$-3 \times -3 = 9$
Answers go down by 3.	Answers go up by 3.

- When multiplying two directed numbers:
 - If their signs are the same, the answer is positive
 - If their signs are different, the answer is negative.

$+\times+=+$	$+\times-= -$
$-\times-=+$	$-\times+= -$

WORKED EXAMPLE 3

Evaluate each of the following.

a $-3 \times +7$

b -8×-7

THINK

a The two numbers have different signs, so the answer is negative ($7 \times 3 = 21$).

b The two numbers have the same signs, so the answer is positive ($8 \times 7 = 56$).

WRITE

a $-3 \times +7 = -21$

b $-8 \times -7 = 56$ (or $+56$)

2.3.2 Powers and square roots of directed numbers

- Calculating powers of negative numbers uses the same process as calculating powers of positive numbers.
- There are two possible answers when you take the square root of a number. For example:

$$\begin{array}{lcl} 4^2 = 4 \times 4 & & (-4)^2 = (-4) \times (-4) \\ = 16 & & = 16 \end{array}$$

- Therefore when asked to take the square root of 16, the answer could be $\pm\sqrt{16}$.
 $\pm\sqrt{16} = -4$ or $+4$, which can also be written as ± 4 (positive or negative 4).

WORKED EXAMPLE 4

Evaluate each of the following.

a $(-5)^3$

b The square root of 64

THINK

a 1 Write the question in expanded form.

2 Evaluate by working from left to right beginning with $-5 \times -5 = +25$.

b Look for the numbers that, when squared, result in 64 ($8 \times 8 = 64$ and $-8 \times -8 = 64$).

WRITE

a $(-5)^3 = (-5) \times (-5) \times (-5)$

$$= +25 \times (-5) \\ = -125$$

b $\pm\sqrt{64} = +8 \text{ or } -8 \\ = \pm 8$

learnon RESOURCES – ONLINE ONLY

Interactivity: Multiplying integers (int-3704)

Digital doc: SkillSHEET Multiplying integers (doc-6390)

Digital doc: WorkSHEET Integers I (doc-6400)

Exercise 2.3 Multiplying integers

Individual pathways

PRACTISE

Questions:

1–3, 4a–c, 5a–d, 7, 8a–c, 9, 13, 15, 16

CONSOLIDATE

Questions:

2, 3, 4a–f, 5a–f, 7, 8a–d, 9, 10, 12, 13, 15, 16

MASTER

Questions:

2, 3c–e, 4f–i, 5f–i, 6–14, 16–17

Individual pathway interactivity: int-4398

learnon ONLINE ONLY

To answer questions online and to receive **immediate feedback** and **fully worked solutions** for every question, go to your learn ON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Understanding and fluency

1. Copy and complete the following times tables.

$4 \times 4 =$	$-5 \times 4 =$	$-6 \times -4 =$
$4 \times 3 =$	$-5 \times 3 =$	$-6 \times -3 =$
$4 \times 2 =$	$-5 \times 2 =$	$-6 \times -2 =$
$4 \times 1 =$	$-5 \times 1 =$	$-6 \times -1 =$
$4 \times 0 =$	$-5 \times 0 =$	$-6 \times 0 =$
$4 \times -1 =$	$-5 \times -1 =$	$-6 \times 1 =$
$4 \times -2 =$	$-5 \times -2 =$	$-6 \times 2 =$
$4 \times -3 =$	$-5 \times -3 =$	$-6 \times 3 =$
$4 \times -4 =$	$-5 \times -4 =$	$-6 \times 4 =$

2. **WE3** Evaluate each of the following.

a. -2×5

b. 3×-8

c. -6×-7

d. 2×-13

e. -8×-6

f. -7×6

g. -10×75

h. -115×-10

i. -7×9

j. $+9 \times -8$

k. -11×-5

l. 150×-2

3. Use an appropriate method to evaluate the following.

a. $-2 \times 5 \times -8 \times -10$

b. $8 \times -1 \times 7 \times -2 \times 1$

c. $8 \times -4 \times -1 \times -1 \times 6$

d. $-3 \times -7 \times -2 \times -1 \times -1 \times -1$

e. $-5 \times -8 \times -2 \times -2$

4. Complete the following equations.

a. $7 \times \underline{\hspace{1cm}} = -63$

b. $-3 \times \underline{\hspace{1cm}} = -21$

c. $16 \times \underline{\hspace{1cm}} = -32$

d. $\underline{\hspace{1cm}} \times -3 = 36$

e. $\underline{\hspace{1cm}} \times 7 = -42$

f. $\underline{\hspace{1cm}} \times -9 = -72$

g. $\underline{\hspace{1cm}} \times -4 = 80$

h. $-10 \times \underline{\hspace{1cm}} = 60$

i. $-11 \times \underline{\hspace{1cm}} = 121$

5. **WE4a** Evaluate each of the following.

a. $(-2)^3$

b. $(-3)^2$

c. $(-2)^4$

d. $(-3)^4$

e. $(-2)^5$

f. $(-4)^2$

g. $(-5)^3$

h. $(-4)^4$

i. $(-5)^4$

j. $(-6)^3$

6. Use your answers to question 5 to help complete the following statements.

a. If a negative number is raised to an even power, the answer is (positive/negative).

b. If a negative number is raised to an odd power, the answer is (positive/negative).

7. **WE4b** Evaluate the square root of the following numbers.

a. 25

b. 81

c. 49

d. 121

e. 100

8. If $a = -2$, $b = -6$, $c = 4$ and $d = -3$, calculate the values of the following expressions.

a. $a \times b \times c$

b. $a \times -b \times -d$

c. $b \times -c \times -d$

d. $c \times -a \times -a$

e. $d \times -(-c)$

f. $a \times d \times b \times c^2$

9. For each of the following, write three possible sets of integers that can be placed in the boxes to make the equation a true statement.

a. $\square \times \square \times \square = -12$

b. $\square \times \square \times \square = 36$

c. $\square \times \square \times \square = -36$

10. For each of the following, determine whether the result is a positive or negative value. You do not have to work out the value.

a. $-25 \times 54 \times -47$

b. $-56 \times -120 \times -145$

c. $-a \times -b \times -c \times -d \times -e$

Note: In part c a , b , c , d and e are positive integers.

11. What happens when a number is multiplied by -1 ? Use some examples to illustrate your answer.

12. The notation $-(-3)$ is a short way of writing -1×-3 .

Write a similar expression for each of the following and then use an appropriate method to determine the answer.

a. $-(-2)$

b. $-(+3)$

c. $-(-5)$

d. $-(-(+5))$

e. $-(-(-7))$

f. $-(-(+4))$

Communicating, reasoning and problem solving

13. Explain why the answer to this question is negative. $-2 \times -4 \times +3 \times -6 \times +4 \times +3$

14. If the answer to $(-a)^n$ is negative, where a is an integer and n is a positive integer, establish whether a is positive or negative and whether n is odd or even. Give a reasoned explanation for your answer.

15. If $a = -3$ and $b = -4$, evaluate $a^3 \times b^2$.

16. Evaluate $(-1)^n \times (-1)^{n+1}$ if:

a. n is even

b. n is odd.

17. In a Year 12 mathematics examination there are 30 multiple choice questions. A student scores 2 marks for a correct answer, -1 mark for an incorrect answer and zero marks for an unanswered question.

Mary scores a total of 33 marks in the multiple choice section. How could she have reached this total?

2.4 Dividing integers

2.4.1 Division of integers

- Division is the inverse operation of multiplication. We can use the multiplication facts for directed numbers to discover the division facts for directed numbers.

Multiplication fact	Division fact	Pattern
$2 \times 3 = 6$	$6 \div 3 = 2$ or $\frac{6}{3} = 2$ and $6 \div 2 = 3$ or $\frac{6}{2} = 3$	$\frac{\text{positive}}{\text{positive}} = \text{positive}$
$-2 \times -3 = 6$	$6 \div -3 = -2$ or $\frac{6}{-3} = -2$ and $6 \div -2 = -3$ or $\frac{6}{-2} = -3$	$\frac{\text{positive}}{\text{negative}} = \text{negative}$
$-2 \times 3 = -6$	$-6 \div 3 = -2$ or $\frac{-6}{3} = -2$ and $-6 \div -2 = 3$ or $\frac{-6}{-2} = 3$	$\frac{\text{negative}}{\text{positive}} = \text{negative}$ and $\frac{\text{negative}}{\text{negative}} = \text{positive}$

- When dividing two directed numbers:
 - if their signs are the same, the answer is positive
 - if their signs are different, the answer is negative
- Remember that division statements can be written as fractions and then simplified. For example,

$$\begin{aligned}
 -12 \div -4 &= \frac{-12}{-4} \\
 &= \frac{12 \times \cancel{1}}{4 \times \cancel{1}} \\
 &= 3
 \end{aligned}$$

$\frac{+}{+} = +$	$\frac{+}{-} = -$
$\frac{-}{-} = +$	$\frac{-}{+} = -$

WORKED EXAMPLE 5

Evaluate each of the following.

a $-56 \div 8$ b $\frac{-36}{-9}$

THINK

- a The two numbers have different signs, so the answer is negative.
- b Cancel the common factors (-1). The two numbers have the same signs, so the answer is positive.

WRITE

$$\begin{aligned}
 \text{a } -56 \div 8 &= -7 \\
 \text{b } \frac{-36}{-9} &= \frac{-1 \times 36}{-1 \times 9} \\
 &= \frac{36}{9} \\
 &= 4
 \end{aligned}$$

WORKED EXAMPLE 6

Evaluate the following.

a $234 \div -6$

b $-182 \div -14$

THINK

a 1 Complete the division as if both numbers were positive numbers.

2 Determine the sign of the answer. A positive number divided by a negative number is a negative number.

b 1 Complete the division as if both numbers were positive numbers.

2 Determine the sign of the answer. A negative number divided by a negative number is a positive number.

WRITE

a
$$\begin{array}{r} 39 \\ 6 \overline{)234} \end{array}$$

$$234 \div -6 = -39$$

b
$$\begin{array}{r} 13 \\ 14 \overline{)182} \\ \underline{14} \\ 42 \\ \underline{42} \\ 0 \end{array}$$

$$-182 \div -14 = 13$$

ACTIVITY: DIVISION OF THE NUMBER LINE

Equipment: paper, pen, hat or small container, Blu Tack

1. In pairs, think of three integers between -50 and $+50$.
2. Write division of integers equations for each of these numbers.
3. Put all equations into a hat.
4. As a pair, select three other equations from the hat.
5. A volunteer might draw a large number line -50 to $+50$ on the board.
6. With your partner, solve the equations and place them in their correct positions on the number line.
7. Check where your original questions have been placed. Has the class done this correctly? Discuss any inaccuracies and work together to ensure that the number line is correct.

learnON RESOURCES — ONLINE ONLY

Interactivity: Division of integers (int-3706)

Digital doc: SkillsHEET Dividing integers (doc-6391)

Exercise 2.4 Dividing integers

Individual pathways

PRACTISE

Questions:

1, 2, 3a–f, 5, 6, 7a–c, 8a–c, 13, 16

CONSOLIDATE

Questions:

1a–h, 2, 3a–i, 5a–f, 6, 7a–d, 8, 9a, 10a, 11, 13, 15, 16

MASTER

Questions:

1g–l, 2, 3g–l, 4, 5c–h, 6, 7c–f, 8c–f, 9–12, 14, 15, 17

Individual pathway interactivity: int-4399

learnON ONLINE ONLY

To answer questions online and to receive **immediate feedback** and **fully worked solutions** for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Understanding and fluency

1. **WE5a** Evaluate each of the following.

a. $-63 \div 9$

b. $8 \div -2$

c. $-8 \div 2$

d. $-6 \div -1$

e. $88 \div -11$

f. $0 \div -5$

g. $48 \div -3$

h. $-129 \div 3$

i. $-56 \div -7$

j. $184 \div -4$

k. $-55 \div -11$

l. $304 \div -8$

2. **WE5b** Evaluate each of the following.

a. $\frac{-121}{-11}$

b. $\frac{-12}{3}$

c. $\frac{-36}{-12}$

d. $\frac{21}{-7}$

e. $\frac{-100}{-50}$

f. $-3 \times \frac{2}{-3}$

3. **WE6** Evaluate each of the following.

a. $960 \div -8$

b. $-243 \div 9$

c. $-266 \div -7$

d. $-132 \div -4$

e. $-282 \div 6$

f. $1440 \div -9$

g. $324 \div -12$

h. $-3060 \div 17$

i. $-6000 \div -24$

j. $-2294 \div -37$

k. $4860 \div 15$

l. $-5876 \div -26$

4. Write three different division statements, each of which has an answer of -8 .

5. Copy and complete the following by placing the correct integer in the box.

a. $-27 \div \square = -9$

b. $-68 \div \square = 34$

c. $72 \div \square = -8$

d. $-18 \div \square = -6$

e. $\square \div 7 = -5$

f. $\square \div -4 = -6$

g. $-132 \div \square = 11$

h. $-270 \div \square = 27$

6. Calculate the value of each of the following by working from left to right.

a. $-30 \div 6 \div -5$

b. $-120 \div 4 \div -5$

c. $-800 \div -4 \div -5 \div 2$

7. If $a = -12$, $b = 3$, $c = -4$ and $d = -6$, calculate the value of each of the following expressions.

a. $a \div c$

b. $a \div b$

c. $a \div d$

d. $b \div c$

e. $b \div d$

f. $a \div b \div d$

8. If $a = -24$, $b = 2$, $c = -4$ and $d = -12$, calculate the value of each of the following expressions.

a. $a \div b \times c$

b. $d \times c \div b \div c$

c. $b \div c \div d \times a$

d. $c \times a \div d \div b$

e. $a \times b \div d \div d$

f. $a \div d \times c \div b$

9. Copy and complete the following tables:

a.

$\times$			-6	+8
			18	
-10		-40		
	10		30	
-7				-56

b.

$\times$			-9	
+6	30			-42
		36		
	-55		99	
		-6	-18	

10. Copy and complete the following tables. Divide the number on the top by the number on the side.

a.

$\div$	+4	-10	+12	-8
-2				
+7				
-3				
-10				

b.

$\div$				-4
		-2		
-8	-4	3		
+6			-6	
				1

Communicating, reasoning and problem solving

11. The answer to $\frac{p \times q}{2 \times -5}$ is negative. What can you deduce about p and q ?

12. The answer to $\frac{(-a)^2}{(-b)^3 \div (-c)^4}$ is positive. What can you deduce about b ?

13. If $a = 2$ and $b = -6$, evaluate $\frac{(-a)^3}{(-b)^4}$.

14. Evaluate $\frac{(-1)^{n+1}}{(-1)^{n+2}}$, $n > 0$, if:

a. n is even

b. n is odd.

15. *Acceleration* is a measure of a change in speed or velocity. If you are increasing speed, acceleration is positive; if you are decreasing speed (slowing down) acceleration is negative. (You will need to do some research to answer this question.)

a. Is it possible to have positive acceleration with a negative velocity? If so, give an example.

b. Is it possible to have negative acceleration with a positive velocity? If so, give an example.



16. My last five scores in a very silly computer game were $+6$, -9 , -15 , $+8$ and -4 . What is my average score?

17. Can you list four areas in real life where directed numbers are used?

2.5 Combined operations on integers

2.5.1 Using BIDMAS

- The mathematical rules about order of operations apply when we work with directed numbers.
- BIDMAS helps us to remember the correct order in which we should perform the various operations. This means we do the brackets (B) first; then indices (I); then division (D) and multiplication (M), working from left to right; and finally addition (A) and subtraction (S), working from left to right.

DISCUSSION

What effect do directed numbers have on the order of operations? Use some examples to help your explanation.

WORKED EXAMPLE 7

Calculate the value of each of the following.

a $54 \div -6 + 8 \times -9 \div -4$

b $-8 \div 2 + (-2)^3$

THINK

a 1 Write the question.

2 There are no brackets or powers, so, working from left to right, complete all multiplication and division, then the addition.

3 Write the answer.

b 1 Write the question.

2 Simplify the cubed term.

3 Complete the division.

4 Write the answer.

WRITE

a $54 \div -6 + 8 \times -9 \div -4$

$$= -9 + 18$$

$$= 9$$

b $-8 \div 2 + (-2)^3$

$$= -8 \div 2 + -8$$

$$= -4 + -8$$

$$= -12$$

DISCUSSION

Use a calculator to perform the calculations in Worked example 7. Did you find it easier to perform these calculations using mental strategies or with a calculator? In which situations would it be more appropriate to use mental strategies or a calculator?

learnon RESOURCES — ONLINE ONLY

Interactivity: Order of operations (int-3707)

eLesson: Using BIDMAS (eles-1883)

Digital doc: SkillsSHEET Order of operations and directed numbers (doc-6392)

Digital doc: WorkSHEET Integers II (doc-6407)

Exercise 2.5 Combined operations on integers

Individual pathways

PRACTISE

Questions:
1–3, 4a, 8, 10

CONSOLIDATE

Questions:
1–3, 4a, b, 8–10

MASTER

Questions:
1–7, 9, 10

Individual pathway interactivity: int-4400

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To answer questions online and to receive **immediate feedback** and **fully worked solutions** for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Understanding and fluency

1. **WE7a** Calculate the values of the following expressions.

a. $-4 - 6 - 2$

b. $-4 \times 2 + 1$

c. $8 \div (2 - 4) + 1$

d. $7 - (3 - 1) + 4$

e. $6 \times (4 + 1)$

f. $-3 - 40 \div 8 + 2$

g. $-4 + 5 - 6 - 7$

h. $-5 \times 12 + 2$

i. $12 \div (2 - 4) - 6$

j. $13 - (4 - 6) + 2$

k. $7 \times (6 + 2)$

l. $-6 - 36 \div 9 + 3$

2. **WE7b** Evaluate each of the following.

a. $-7 + 6 \times -2$

b. $-9 - 15 + 3$

c. $(-63 \div -7) \times -3 + -2$

d. $(-3)^3 - 3 \times -5$

e. $-5 \times -7 - [5 + (-8)^2]$

f. $[(-48 \div 8)^2 \times 36] \div -4$

3. Calculate the values of the following expressions.

a. $-3 + 15 - 26 - 27$

b. $-8 \times 11 + 12$

c. $52 \div (-9 - 4) - 8$

d. $23 - (16 - 4) + 7 - 3$

e. $15 \times (-6 + 2)$

f. $-6 - 64 \div -16 + 8$

g. $-3 \times -4 \times -1 \times 5$

h. $-6 \times (-13 + 5) + -4 + 2$

4. A class of Year 8 students were given the following question to evaluate.

$$4 + 8 \div -(2)^2 - 7 \times 2$$

- a. A number of different answers were obtained, including -8 , -12 and -17 . Which one of these is the correct answer?

- b. Using only brackets, change the question in two ways so that the other two answers would be correct.

5. In a particular adventure video game, a player loses and gains points based on who or what they come in contact with during the game. See the list below right for the number of 'hit' points associated with each contact. Use the table to calculate the number of points the player has at the end of each round of the game.



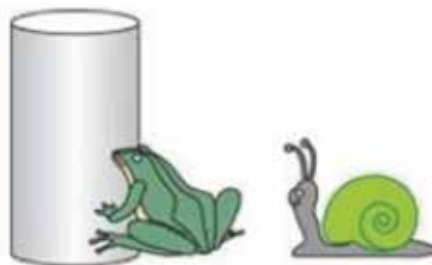
Round number	Points at the start of the round	Contacts during the round	Points at the end of the round
1	100	20 gnomes, 10 goblins and 3 healing potions	
2		3 gnomes, 5 goblins, 6 orcs and 5 healing potions	
3		3 orcs, 6 trolls and a cleric	
4		5 trolls, 1 balrog and a cleric	

Character	'Hit' points
Balrog	-100
Troll	-10
Orc	-5
Goblin	-2
Gnome	-1
Healing potion	+20
Cleric	+50

Communicating, reasoning and problem solving

6. Two numbers p and q have the same numerical value but the opposite sign; that is, one is positive and the other negative. If $-3 \times p + 4 \times q$ is positive, what can you say about p and q ?
7. Test your answer to question 6 if the numerical values of p and q , written as $|p|$ and $|q|$, are both equal to 7.

8. A frog and a snail are climbing an empty vertical pipe. The snail is 30 cm from the top of the pipe while the frog is 30 cm below the snail. At the start of the first hour, the frog climbs 40 cm up the pipe and rests. The snail crawls 20 cm during this time then also rests. While they are resting, both the frog and the snail slip back down the pipe. The frog slips back 20 cm while the snail slips back 10 cm. At the start of the second hour, the frog and the snail set off again and repeat the same process of climbing and resting until they reach the top of the pipe.



- a. Which reaches the top of the pipe first, the frog or the snail?
- b. During which hour do they reach the top of the pipe?
9. A classmate is recording the weather during July for a school project and wants help calculating information. He records the following data for one week.

	Mon	Tue	Wed	Thu	Fri	Sat	Sun
Max (°C)	12.2	14.5	16.7	12.8	11.3	7.2	-0.3
Min (°C)	3.0	2.1	4.6	3.2	6.4	-2.9	-6.0

Round all answers correct to 1 decimal place.

- a. What is the difference between the lowest temperature and the highest temperature recorded during this week?
- b. What is the average (mean) minimum temperature?
- c. Your classmate tries to predict temperatures and

says that the minimum temperature $= \frac{1}{4} \times$
maximum temperature. He says this is the
same as dividing by -4 . Explain why this
may not be correct.

- d. He then tries to make another prediction that involves taking the square root of the maximum temperature. Why might this not be a good idea?
- e. On the Monday after this week, the temperature rises by 6.5°C from Sunday's minimum temperature. It then drops by 3.2°C overnight and rises by 8.9°C on the Tuesday. What is the minimum temperature on the Monday night and the maximum temperature on the Tuesday?
- f. His last prediction involves subtracting 6 from the maximum temperature, then dividing by 2 to predict the minimum temperature. Calculate the predicted values for the minimum temperature for each of his three methods and discuss which method may be most accurate.



10. What effect do directed numbers have on the order of operations?

2.6 Review

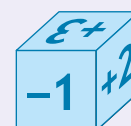
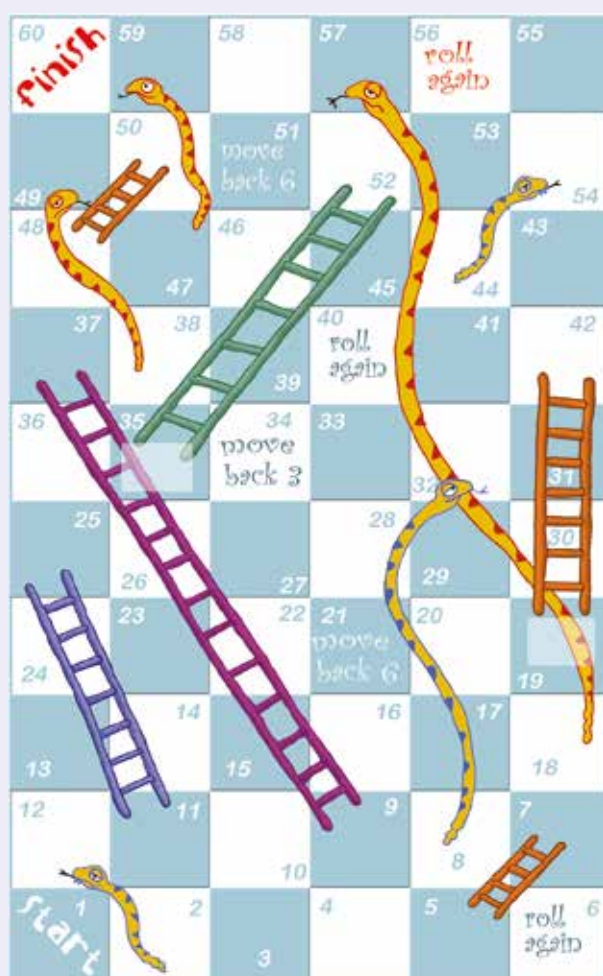
Investigation | Rich task

Directed numbers board game

Many board games engage the players in racing each other to the end of the board. Snakes and Ladders is an example of this style of game. You can climb ladders to get to the end quicker, but sliding down a snake means you get further away from the end.

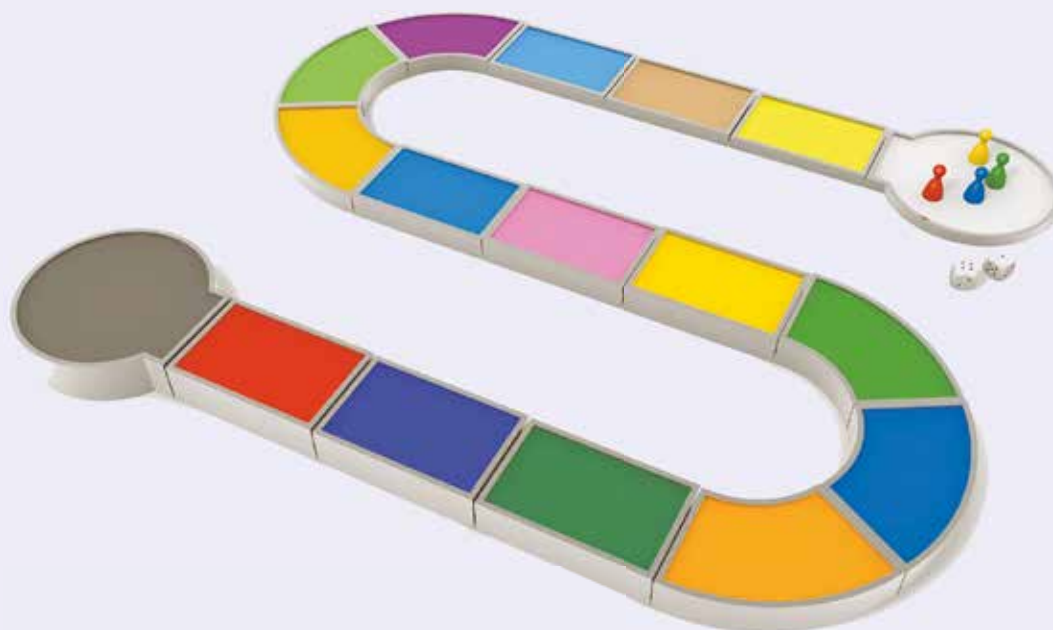
You are going to make a board game that will help you to practise addition and subtraction of directed numbers. This game is played with 2 dice and is a race to the end of the number line provided on the board. The diagram below shows part of a sample game, and can be used to explain the rules and requirements for your board game.

	10	9	8	7	6	
					5	
					4	
-2	-1	0	1	2	3	
-3						
-4	-5	-6	-7	-8	-9	-10



- Imagine that you are at 0, and that you roll an N and +2. N means that you face the negative numbers; +2 means that you move forward 2 places.
- If you roll a P and +1, this means that you face the positive numbers and move forward 1 place.
- If you roll a P and -2, this means that you face the positive numbers and move *backwards* 2 places.

1. For the three examples listed above, state which square you would end up in if you started at 0 each time.
2. Suppose that your first five turns at this game produced the following results on the dice.
P and +1, P and +3, P and -2, N and -1, N and +2.
If you started at 0, where were you after these five turns?



- The game is to be played with the two dice described earlier.
- Two or more players are required, taking turns to throw the dice.
- Start at 0 in the centre of the board. The race is on to get to either end of the board. The first person to reach an end is the winner.
- For 0 to be at the centre, your board will require an odd number of columns.
- At least 8 squares must have specific instructions — for example, ‘go back 5 squares’ or ‘miss a turn’. You may even wish to include ‘snakes and ladders’ style obstacles.
- Use a sensible number of squares. If there are very few squares, the game will end too quickly; if there are many squares, they will be very small.

3. Work with a partner to make the two required dice and to design your board. Play with others to test it, and make necessary improvements if they are required. Be certain that all the requirements of the game are included.
4. Present your game as a package that includes instructions and a list of contents. Include examples within your instructions to help players understand how to move throughout the game.

Digital doc: Investigation — Directed numbers board game (doc-13929)

Digital doc: Code puzzle — Why did the teacher wear sunglasses? (doc-13930)

Digital doc: Summary Topic 2 (doc-23238)

Interactivity: Word search Topic 2 (int-2724)

Interactivity: Crossword Topic 2 (int-2723)

Interactivity: Sudoku Topic 2 (int-3182)

Exercise 2.6 Review questions

To answer questions online and to receive **immediate feedback** and **fully worked solutions** for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Understanding and fluency

- True or false? The number -2.5 is called an integer.
- True or false? $-6 < -2$
- List the integers between -11 and -7 .
- Arrange these numbers in ascending order: $7, 0, -3, 10, -15$.
- Calculate the value of each of the following.
 - $-6 + -8$
 - $16 - -5$
 - $-3 - +7 + -2$
 - $-1 - -5 - +4$
- Write out the following equations and fill in the missing numbers.
 - $7 - \square = -14$
 - $-19 + \square = 2$
 - $\square - 13 - -12 = 10$
 - $-28 - \square = -17$
- MC** Which of the following statements is true?
 - Multiplying an even number of negative numbers together gives a negative answer.
 - The only square root of 16 is $+4$.
 - Dividing a negative number by another negative number gives a positive answer.
 - Adding two negative numbers together gives a positive answer.
- Evaluate each of the following.
 - -12×-5
 - $-(-10) \times 3 \times -2$
 - $-24 \div -3$
 - $-48 \div -4 \div -3$
 - $6 \times -3 \div -2$
 - $-36 \div 3 \div -4 \times -9$
 - $-8 \times -3 - (4 - -1) + -63 \div 7$
 - $-9 + -9 \div -9 \times -9 - -9$

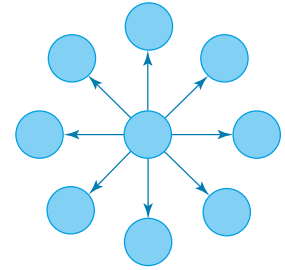
Communicating, reasoning and problem solving

- Give an example of two numbers that fit each description that follows. If no numbers fit the description, explain why.
 - Both the sum and the product of two numbers are negative.
 - The sum of two numbers is positive and the quotient is negative.
 - The sum of two numbers is 0 and the product is positive.
- On a test, each correct answer scores 5 points, each incorrect answer scores -2 points and each question left unanswered scores 0 points.
 - Suppose a student answers 16 questions correctly and 3 questions incorrectly, and does not answer 1 question. Write an expression for the student's score and find the score.
 - Suppose you answered all 20 questions on the test. What is the greatest number of questions you can answer incorrectly and still get a positive score? Explain your reasoning.
- Write each of the following problems as equations using directed numbers and then find the answer.
 - You have $\$25$ and you spend $\$8$ on lollies. You then spend another $\$6$ on lunch. A friend gives you $\$5$ to buy lunch, which comes to only $\$3.50$. You then find another $\$10$ in your pocket and buy an ice-cream for $\$3$. How much money do you have left in total before you return your friend's change from lunch?

- b. Two friends are on holiday; one decides to go skydiving and the other decides to go scuba diving. If the skydiving plane climbs to 4405 m above sea level, and the scuba diver goes to the ocean floor, which is 26 m below the surface, what is the vertical distance between the two friends?

12. Insert the integers from -6 to $+2$ into the circles on the figure at right so that each line of three circles has each of the following totals.

- a. -6 b. -3 c. -9

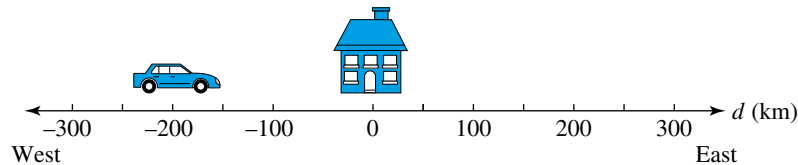


13. For each of the following:

- i. represent the situation using directed numbers
- ii. solve the problem.
 - a. You receive several letters in the mail: two cheques worth \$100 each, three bills worth \$75 each and a voucher for \$20. How much money do you end up with?
 - b. You earn \$150 each time you work at the local races. If you work at three race meetings in one month, how much do you earn that month?
 - c. For your birthday, you get three cards with \$40 in each of them. As a present, your brother tears up the IOUs you gave him when he did your chores. There were four IOUs worth \$10 each. In total, how much more money do you have after your birthday?

In science, directed numbers are often used to describe a direction or an increase or decrease in a measurement.

14. Directed numbers can describe the distance of an object from a reference point (known as the *displacement*, d , of the object). For example, if we are 200 km west of a town, and west is defined as a negative direction, we are -200 km from the town.



- a. If a car travels 150 km in an easterly direction from -200 km, describe the displacement of the car from the town.
 - b. If a car travels from 300 km east of the town, describe the displacement of the car after it has travelled 450 km in an easterly direction.
15. Directed numbers can describe the direction that an object is travelling. For example, in question 14, travelling towards the east is travelling in a positive direction; towards the west is a negative direction. A car travelling west at 100 km/h goes at -100 km/h. Scientists use the term *velocity*, v , to mean a speed in a particular direction.
- a. If a car travels past the town at -100 km/h, where will it be in 2 hours' time?
 - b. If a car goes past the town travelling at -100 km/h, where was the car an hour ago?

Answers

Topic 2 Computation with integers

Exercise 2.2 Adding and subtracting integers

1. 3, -4, 201, -62

2. a. 0, -2, -4

b. -20, -25, -30

c. 5, 3, 1

d. -8, -6, -4

3. a. -1

b. -10

c. -1

d. -13

e. 19

f. 7

g. -15

h. 0

4. a. 5

b. -24

c. -5

d. 5

e. 26

f. -16

g. 22

h. 11

5. a. -8

b. 11

c. -14

d. -1

e. 51

f. -51

g. -39

h. 8

6. a. -7

b. -9

c. 3

d. 12

e. -10

f. 13

g. -40

h. 18

i. 27

j. -34

k. -11

l. 150

m. -1

n. 25

o. -13

p. 22

q. 9

r. -22

7. a.

+	-8	25	-18	32
-6	-14	19	-24	26
-13	-21	12	-31	19
-16	-24	9	-34	16
-19	-27	6	-37	13

b.

-	15	-17	-27	57
7	8	-24	-34	50
-6	21	-11	-21	63
-9	24	-8	-18	66
12	3	-29	-39	45

c.

+	-11	19	13	-7
-5	-16	14	8	-12
17	6	36	30	10
-1	-12	18	12	-8
-28	-39	-9	-15	-35

d.

-	9	21	-15	42
26	-17	-5	-41	16
-14	23	35	-1	56
-2	11	23	-13	44
23	-14	-2	-38	19

8. Check with your teacher.

9. a. 22°C

b. 176°C

c. 198°C

10. 37°C

11. a. -301

b. -5963

c. 530

d. 72

12. a. 13

b. -4

c. 11

d. -10

e. 52

f. 71

13. a. Correct

b. Incorrect; 6

c. Incorrect; -15

d. Correct

e. Correct

f. Incorrect; -21

14. a. -3

b. -2

c. -5

15. The answers are the same.

16. a. $-2 + -5 = -7$; $-(2 + 5) = -7$

b. $-3 + -8 = -11$; $-(3 + 8) = -11$

c. $-7 + -6 = -13$; $-(7 + 6) = -13$

17. For each pair of expressions, the answers are the same: each number is negative, so putting brackets around the sum and then attaching a negative sign gives the same outcome.

18. a. $270 - 340 + 1450 = 2750$

b. HK\$1370

19. The number line. Go right on the number line when adding positive numbers, and left when adding negative numbers.

20. -72 metres

Exercise 2.3 Multiplying integers

1.

$4 \times 4 = 16$	$-5 \times 4 = -20$	$-6 \times -4 = 24$
$4 \times 3 = 12$	$-5 \times 3 = -15$	$-6 \times -3 = 18$
$4 \times 2 = 8$	$-5 \times 2 = -10$	$-6 \times -2 = 12$
$4 \times 1 = 4$	$-5 \times 1 = -5$	$-6 \times -1 = 6$
$4 \times 0 = 0$	$-5 \times 0 = 0$	$-6 \times 0 = 0$
$4 \times -1 = -4$	$-5 \times -1 = 5$	$-6 \times 1 = -6$
$4 \times -2 = -8$	$-5 \times -2 = 10$	$-6 \times 2 = -12$
$4 \times -3 = -12$	$-5 \times -3 = 15$	$-6 \times 3 = -18$
$4 \times -4 = -16$	$-5 \times -4 = 20$	$-6 \times 4 = -24$

2. a. -10 b. -24 c. 42 d. -26 e. 48 f. -42
 g. -750 h. 1150 i. -63 j. -72 k. 55 l. -300
3. a. -800 b. 112 c. -192 d. 42 e. 160
4. a. -9 b. 7 c. -2 d. -12 e. -6
 f. 8 g. -20 h. -6 i. -11
5. a. -8 b. 9 c. 16 d. 81 e. -32
 f. 16 g. -125 h. 256 i. 625 j. -216
6. a. Positive b. Negative
7. a. ± 5 b. ± 9 c. ± 7 d. ± 11 e. ± 10
8. a. 48 b. -36 c. 72 d. 16 e. -12 f. -576

9. Check with your teacher.

10. a. Positive b. Negative c. Negative

11. If a positive number is multiplied by -1 , the number becomes negative.

If a negative number is multiplied by -1 , the number becomes positive.

12. a. 2 b. -3 c. 5 d. 5 e. -7 f. 4

13. Because there is an odd number of negatives.

14. $-a$ must be negative for the answer to be negative, thus a must be positive. The power must be odd to give a negative result, thus n is odd.

15. -432

16. a. -1 b. -1

17. One of the following: 17 correct, 1 incorrect; 18 correct, 3 incorrect; 19 correct, 5 incorrect; 20 correct, 7 incorrect; 21 correct, 9 incorrect

Exercise 2.4 Dividing integers

1. a. -7 b. -4 c. -4 d. 6 e. -8 f. 0
 g. -16 h. -43 i. 8 j. -46 k. 5 l. -38
2. a. 11 b. -4 c. 3 d. -3 e. 2 f. 2
3. a. -120 b. -27 c. 38 d. 33 e. -47 f. -160
 g. -27 h. -180 i. 250 j. 62 k. 324 l. 226

4. Check with your teacher.

5. a. 3 b. -2 c. -9 d. 3
 e. -35 f. 24 g. -12 h. -10
6. a. 1 b. 6 c. -20
7. a. 3 b. -4 c. 2 d. $-\frac{3}{4}$ e. $-\frac{1}{2}$ f. $\frac{2}{3}$
 d. -4 e. $-\frac{1}{3}$ f. -4
8. a. 48 b. -6 c. -1

9. a.

$\times$	-2	4	-6	8
-3	6	-12	18	-24
-10	20	-40	60	-80
-5	10	-20	30	-40
-7	14	-28	42	-56

b.

$\times$	5	-3	-9	-7
6	30	-18	-54	-42
-12	-60	36	108	84
-11	-55	33	99	77
2	10	-6	-18	-14

10. a.

$\div$	4	-10	12	-8
-2	-2	5	-6	4
7	$\frac{4}{7}$	$-\frac{10}{7}$	$\frac{12}{7}$	$-\frac{8}{7}$
-3	$-\frac{4}{3}$	$\frac{10}{3}$	-4	$\frac{8}{3}$
-10	$-\frac{2}{5}$	1	$-\frac{6}{5}$	$\frac{4}{5}$

b.

$\div$	32	-24	-36	-4
12	$\frac{8}{3}$	-2	-3	$-\frac{1}{3}$
-8	-4	3	$\frac{9}{2}$	$\frac{1}{2}$
6	$\frac{16}{3}$	-4	-6	$-\frac{2}{3}$
-4	-8	6	9	1

11. Same sign

12. Negative

13. $-\frac{1}{162}$

14. a. -1 b. -1

15. a. Yes; for example, a car is getting faster while travelling towards the east.

b. Yes; for example, a car is slowing down (negative acceleration) while travelling towards the west.

16. -2.8

17. Discuss with your teacher

Exercise 2.5 Combined operations on integers

1. a. -12 b. -7 c. -3 d. 9 e. 30 f. -6
g. -12 h. -58 i. -12 j. 17 k. 56 l. -7
2. a. -19 b. -21 c. -29 d. -12 e. -34 f. -324
3. a. -41 b. -76 c. -12 d. 15 e. -60 f. 6 g. -60 h. 46
4. a. -12 b. $(4 + 8) \div -(2)^2 - 7 \times 2 = -17$ c. $4 + 8 \div -(2)^2 - 7 \times 2 = -8$

5.

Round number	Points at the start of the round	Contacts during the round	Points at the end of the round
1	100	20 gnomes, 10 goblins and 3 healing potions	120
2	120	3 gnomes, 5 goblins, 6 orcs and 5 healing potions	177
3	177	3 orcs, 6 trolls and a cleric	152
4	152	5 trolls, 1 balrog and a cleric	52

6. $p < 0$, $q > 0$

7. Check with your teacher.

8. a. The frog and the snail get out of the pipe at exactly the same time.

b. They make it out of the pipe during the second hour.

9. a. 22.7 °C

b. 1.5 °C

c. Multiplying by $\frac{1}{4}$ is not the same as dividing by -4. However, multiplying by $\frac{1}{4}$ is the same as dividing by 4.

d. This is not a good idea because the square root ($\sqrt{}$) of a number can be both positive and negative. Also, you cannot take the square root of a negative number, such as Sunday's maximum temperature.

e. Monday maximum = 0.5 °C; Monday minimum = -2.7 °C; Tuesday maximum = 6.2 °C

f.

	Mon	Tue	Wed	Thu	Fri	Sat	Sun
Max (°C)	12.2	14.5	16.7	12.8	11.3	7.2	-0.3
Min (°C)	3.0	2.1	4.6	3.2	6.4	-2.9	-6.0
Max $\times 1/4$	3.1	3.6	4.2	3.2	2.8	1.8	-0.1
$\sqrt{\text{Max}}$	± 3.5	± 3.8	± 4.1	± 3.6	± 3.4	± 2.7	
(Max - 6)/2	3.1	4.3	5.4	3.4	2.7	0.6	-3.2

The method of multiplying the maximum temperature by $\frac{1}{4}$ gave the most accurate prediction of minimum temperature, but none of the methods is very accurate. Any prediction method should be tested over a number of weeks before deciding if it is accurate enough.

10. Directed numbers are placed in brackets and applied first in operations. See the BIDMAS rule.

Investigation | Rich task

1. a. -2 b. 1 c. -2

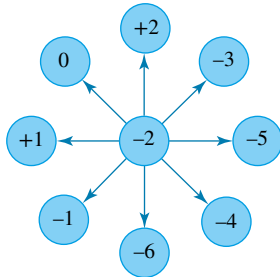
2. 1

3. Check with your teacher.

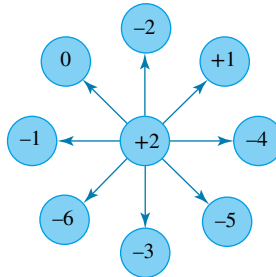
4. Check with your teacher.

Exercise 2.6 Review questions

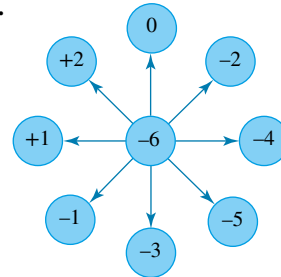
1. False
2. True
3. $-10, -9, -8$
4. $-15, -3, 0, 7, 10$
5. a. -14 b. 21 c. -12 d. 0
6. a. 21 b. 21 c. 11 d. -11
7. C
8. a. 60 b. -60 c. 8 d. -4 e. 9 f. -27 g. 10 h. -9
9. Check with your teacher.
10. a. 74 b. 14
11. a. $\$19.50$ b. 4431 m
12. a.



b.



c.



13. a. i. $2 \times 100 - 3 \times 75 + 20$
 b. i. 3×150
 c. i. $3 \times 40 + 4 \times 10$
14. a. -50 km
15. a. -200 km
- ii. $-\$5$
- ii. $\$450$
- ii. $\$160$
- b. 750 km
- b. 100 km

TOPIC 3

Fractions, decimals and percentages

3.1 Overview

Numerous **videos** and **interactivities** are embedded just where you need them, at the point of learning, in your learnON title at www.jacplus.com.au. They will help you to learn the concepts covered in this topic.



3.1.1 Why learn this?

Whole numbers cannot be used to measure everything. Instead, fractions and decimals must be used when you need to divide things up more finely than whole numbers allow.

How will you count dollars and cents if you do not know about decimals? How will you share anything equally among friends if you do not know about fractions?

Percentages are used to describe many different kinds of information. They are so common that they have their own symbol, %. A *per cent* is a hundredth, so using percentages is an alternative to using decimals and fractions. Percentages are a convenient way to describe how much of something you have and how meaningful information is. For example, you might see an item advertised for sale at 10% discount.

LEARNING SEQUENCE

- 3.1 Overview
- 3.2 Addition and subtraction of fractions
- 3.3 Multiplication and division of fractions
- 3.4 Terminating and recurring decimals
- 3.5 Addition and subtraction of decimals
- 3.6 Multiplication and division of decimals
- 3.7 Estimation
- 3.8 Percentages, fractions and decimals
- 3.9 Finding percentages of an amount
- 3.10 Review

LEARNING OUTCOMES

A student:

- communicates and connects mathematical ideas using appropriate terminology, diagrams and symbols MA4-1WM
- applies appropriate mathematical techniques to solve problems MA4-2WM
- recognises and explains mathematical relationships using reasoning MA4-3WM
- operates with fractions, decimals and percentages MA4-5NA

CONTENT DESCRIPTIONS

Students:

Compare fractions using equivalence; locate and represent positive and negative fractions and mixed numerals on a number line (ACMNA152)

Solve problems involving addition and subtraction of fractions, including those with unrelated denominators (ACMNA153)
 Multiply and divide fractions and decimals using efficient written strategies and digital technologies (ACMNA154)
 Investigate terminating and recurring decimals (ACMNA184)
 Connect fractions, decimals and percentages and carry out simple conversions (ACMNA157)
 Find percentages of quantities and express one quantity as a percentage of another, with and without the use of digital technologies (ACMNA158)
 Solve problems involving the use of percentages, including percentage increases and decreases, with and without the use of digital technologies (ACMNA187)

Source: NSW Syllabus for the Australian curriculum

learn**on** RESOURCES — ONLINE ONLY

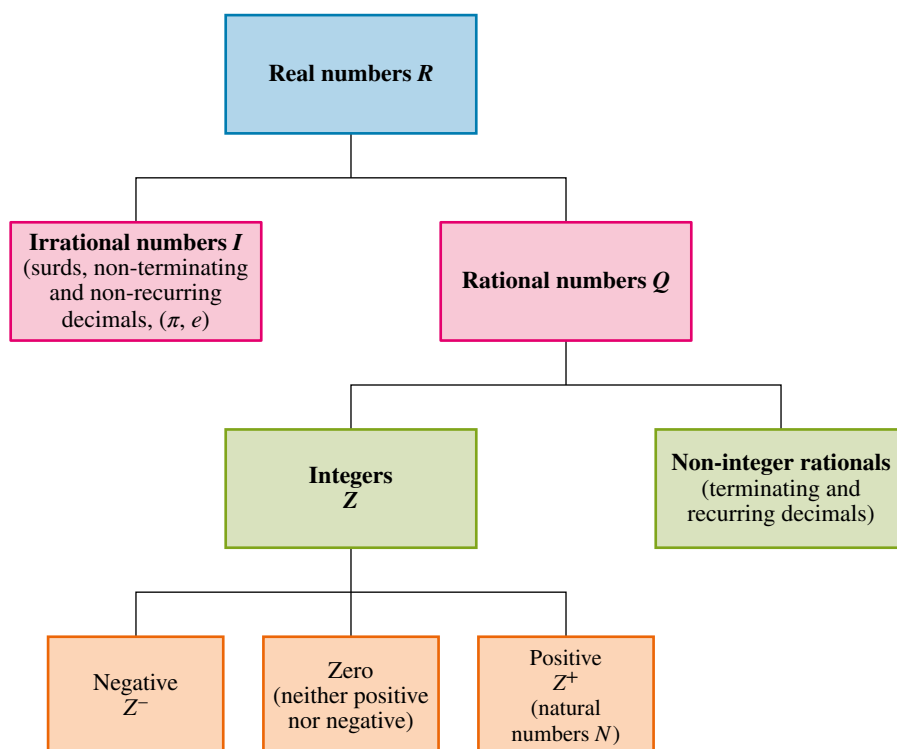
eLesson: The story of mathematics — Adelard of Bath (eles-1846)

Note: Your teacher may now set you a pre-test to determine how familiar you are with the content in this topic.

3.2 Addition and subtraction of fractions

3.2.1 Introduction to real numbers

- Number systems have become more sophisticated as societies have developed.
- Different types of numbers can be classified into specific groups.
- The **real number** system contains the set of **rational** and **irrational** numbers. It can be denoted by the symbol R .
- The set of real numbers contains a number of groups or subsets that can be classified as shown in the following chart.



3.2.2 Rational and irrational numbers

Rational numbers

- A rational number is any number that can be expressed as a ratio of two whole numbers in the form $\frac{a}{b}$, where a and b are whole numbers and $b \neq 0$.
- The set of rational numbers is given the symbol Q . Examples include $\frac{1}{4}$, $-3\frac{4}{7}$, 100, -63 , 1.5, -0.8 and $1.\overline{45}$.
- Rational numbers include all integers (symbol Z), as integers can be expressed in the form $\frac{a}{b}$. For example, $100 = \frac{100}{1}$, $-3 = \frac{-3}{1}$.
- Rational numbers include all fractions, both those that can be expressed as terminating decimals and those that can be expressed as recurring decimals. For example, $\frac{2}{5} = 0.4$ is a terminating decimal and $\frac{1}{3} = 0.33\ldots = 0.\overline{3}$ is a recurring decimal. These are rational numbers as they can be expressed in the form $\frac{a}{b}$, where a and b are whole numbers and $b \neq 0$.

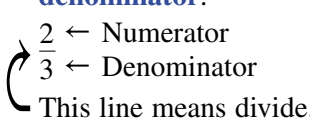
Irrational numbers

- Numbers that cannot be expressed in the form $\frac{a}{b}$, where a and b are whole numbers and $b \neq 0$, are called irrational numbers. Examples of these numbers are $\sqrt{5}$, $\sqrt{7}$ and $2\sqrt{11}$. These numbers can be expressed as decimals, but their decimal values do not terminate or repeat in any pattern.
- A special irrational number is π (pi). This is the distance around the circumference of a circle with a diameter of 1. It can be approximated as a decimal that is non-terminating and non-recurring.
- In decimal form, $\pi = 3.141592\ldots$
- The value of π has been calculated to many millions of decimal places with the aid of a computer, but it is still non-terminating and non-recurring.

ACTIVITY: INVESTIGATING IRRATIONAL NUMBERS

Use the internet to carry out an investigation on irrational numbers. Look into the history of numbers such as π and $\sqrt{2}$ and prepare a report for your class.

3.2.3 Review of addition and subtraction of proper fractions

- As can be seen from the diagram on the previous page, fractions are rational numbers.
- A fraction has two parts. The top part is called the **numerator** and the bottom part is called the **denominator**.

This line means divide.
- A **proper fraction** has a numerator that is less than the denominator; for example, $\frac{3}{5}$.
- Equivalent fractions** are fractions that are equal in value; for example, $\frac{1}{2} = \frac{2}{4}$.
- When we add and subtract fractions, the denominators *must* be the same.

WORKED EXAMPLE 1

Simplify $\frac{4}{5} - \frac{3}{4}$.

THINK

1 Write the question.

WRITE

$$\frac{4}{5} - \frac{3}{4}$$



2 Find the lowest common denominator, that is, the lowest multiple common to both.	$= \frac{4 \times 4}{5 \times 4} - \frac{3 \times 5}{4 \times 5}$
3 Write both fractions with the same denominator, that is, the lowest common denominator.	$= \frac{16}{20} - \frac{15}{20}$
4 Subtract the fractions.	$= \frac{1}{20}$
5 Write the answer.	

DISCUSSION

When is knowing how to add and subtract fractions useful in life outside of school?

3.2.4 Addition and subtraction of mixed numbers

- An **improper fraction** has a numerator greater than the denominator; for example, $\frac{7}{3}$.
- A **mixed number** contains a whole number part and a proper fraction part; for example, $7\frac{5}{8}$.
- When we are adding and subtracting mixed numbers, we must first change them to improper fractions, and then solve them using the method shown in Worked example 1. However, it is often better to add or subtract the whole number parts first and then the fraction parts.

WORKED EXAMPLE 2

Calculate $2\frac{2}{3} + 3\frac{1}{2}$.

THINK

- 1 Write the question.
- 2 Change each mixed number to an improper fraction.
- 3 Write both fractions with the same denominator using equivalent fractions.
- 4 Add the fractions.
- 5 Write the answer as a mixed number if appropriate.

WRITE

$$\begin{aligned}
 &2\frac{2}{3} + 3\frac{1}{2} \\
 &= \frac{8}{3} + \frac{7}{2} \\
 &= \frac{8 \times 2}{3 \times 2} + \frac{7 \times 3}{2 \times 3} = \frac{16}{6} + \frac{21}{6} \\
 &= \frac{37}{6} \\
 &= 6\frac{1}{6}
 \end{aligned}$$

- A rough estimate can be found by adding or subtracting the whole number parts. For example, $2 + 3 = 5$, so $2\frac{2}{3} + 3\frac{1}{2} > 5$.

WORKED EXAMPLE 3

Calculate $4\frac{1}{5} - 1\frac{1}{2}$.

THINK

- 1 Write the question.
- 2 Change each mixed number to an improper fraction.

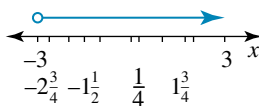
WRITE

$$\begin{aligned}
 &4\frac{1}{5} - 1\frac{1}{2} \\
 &= \frac{21}{5} - \frac{3}{2}
 \end{aligned}$$

- 3 Write both fractions with the same denominator using equivalent fractions.
- $$= \frac{21 \times 2}{5 \times 2} - \frac{3 \times 5}{2 \times 5}$$
- $$= \frac{42}{10} - \frac{15}{10}$$
- 4 Subtract the second fraction from the first.
- $$= \frac{27}{10}$$
- 5 Write the answer as a mixed number if appropriate.
- $$= 2\frac{7}{10}$$

3.2.5 Negative fractions

- Positive and negative numbers have both size and direction, so they are often called **directed numbers**.
- Directed numbers include zero, integers and fractions.
- To place a fraction or mixed number on a number line, divide the space between the whole numbers into an equal number of parts, determined by the denominator of the fraction.



WORKED EXAMPLE 4

Graph each of the following sets of directed numbers on a number line.

a $x > -2\frac{1}{2}$

b $x \leq 1\frac{3}{4}$

c $-2\frac{1}{3} < x \leq 3.4$

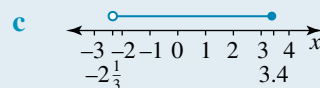
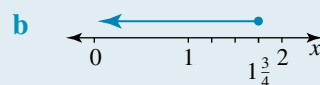
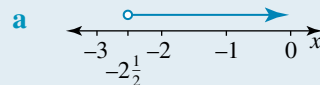
THINK

a $>$ means greater than, so x takes on all values to the right of $-2\frac{1}{2}$. Use an *open* or *unshaded* circle to indicate that the number itself is not included.

b $\leq$ means less than or equal to, so x takes on all values to the left of and including $1\frac{3}{4}$. Show all points fitting the description. Use a *closed* or *shaded* circle to show that the number itself is included.

c $-2\frac{1}{3} < x \leq 3.4$ means all numbers lying between the given boundaries, including 3.4 itself. Use an open dot to show that $-2\frac{1}{3}$ is not included and a closed dot to show that 3.4 is included. Use estimation to locate the boundary points.

DRAW



3.2.6 Addition and subtraction of positive and negative fractions

- The rules for adding and subtracting integers apply to the addition and subtraction of positive and negative fractions.
 - When adding same signs, add and keep the sign.
 - When adding different signs, find the difference between the 'sign-less' numbers (that is, ignoring the sign) then use the sign of the number furthest from zero.
 - When subtracting, add the opposite.
- The rules for addition and subtraction of fractions apply.

- Write all fractions with the same denominator, then add or subtract the numerators:

$$\frac{2}{3} + \frac{1}{2} = \frac{4}{6} + \frac{3}{6} = \frac{7}{6} = 1\frac{1}{6}$$

WORKED EXAMPLE 5

Calculate $-\frac{1}{3} + \frac{1}{2}$.

THINK

- Write the expression.
- Write both fractions with the same denominator.
- Add the numerators.
Note: They are different signs, so find the difference and use the sign of the number furthest from zero.
- Write the answer.

WRITE

$$\begin{aligned} &-\frac{1}{3} + \frac{1}{2} \\ &= -\frac{2}{6} + \frac{3}{6} \\ &= \frac{1}{6} \end{aligned}$$

WORKED EXAMPLE 6

Calculate $-\frac{3}{4} - \frac{1}{6}$.

THINK

- Write the expression.
- To subtract, add the opposite.
- Write both fractions with the same denominator.
- Add the numerators.
Note: They are the same sign, so add and keep the sign.
- Write the answer.

WRITE

$$\begin{aligned} &-\frac{3}{4} - \frac{1}{6} \\ &= -\frac{3}{4} + -\frac{1}{6} \\ &= -\frac{9}{12} + -\frac{2}{12} \\ &= -\frac{11}{12} \end{aligned}$$

learn on RESOURCES — ONLINE ONLY

Interactivity: Adding and subtracting fractions (int-2357)

Interactivity: Real numbers (int-3717)

Interactivity: Addition and subtraction of proper fractions (int-3718)

Interactivity: Addition and subtraction of mixed numbers (int-3719)

Interactivity: The number line (int-3720)

Interactivity: Addition and subtraction of fractions (int-3721)

eLesson: Addition and subtraction of fractions (eles-1862)

Digital doc: SkillsSHEET Order of operations I (doc-6854)

Digital doc: SkillsSHEET Factors (doc-6855)

Digital doc: SkillsSHEET Multiples (doc-6856)

Digital doc: SkillsSHEET Adding and subtracting fractions I (doc-6857)

Exercise 3.2 Addition and subtraction of fractions

Individual pathways

PRACTISE

Questions:
1–11, 13, 19

CONSOLIDATE

Questions:
1–3, 4d–j, 5–9, 10d–l, 11–14, 17–19

MASTER

Questions:
1–3, 4g–j, 5–9, 10g–l, 11–18, 20, 21

Individual pathway interactivity: int-4406

learnON ONLINE ONLY

To answer questions online and to receive **immediate feedback** and **fully worked solutions** for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Understanding and fluency

1. Simplify the following fractions, working from left to right.

a. $\frac{3}{17} + \frac{6}{17}$

b. $\frac{21}{27} - \frac{16}{27}$

c. $\frac{6}{17} + \frac{2}{17} + \frac{4}{17}$

d. $\frac{3}{15} + \frac{11}{15} - \frac{2}{15}$

2. Simplify the following fractions, writing the answer as a mixed number if appropriate.

a. $\frac{3}{5} + \frac{4}{5}$

b. $\frac{7}{8} + \frac{3}{8}$

c. $\frac{7}{8} + \frac{3}{8} + \frac{6}{8}$

d. $\frac{41}{50} - \frac{24}{50} + \frac{6}{50}$

3. **WE1** Simplify the following.

a. $\frac{2}{5} + \frac{1}{4}$

b. $\frac{3}{4} + \frac{5}{8}$

c. $\frac{6}{10} - \frac{2}{5}$

d. $\frac{8}{25} + \frac{34}{50} - \frac{7}{25}$

e. $\frac{3}{4} + \frac{5}{6}$

f. $\frac{9}{10} - \frac{2}{7}$

g. $\frac{8}{12} + \frac{5}{7} - \frac{1}{2}$

h. $\frac{21}{30} + \frac{5}{6} + \frac{9}{10}$

4. **WE2** Calculate the following and check answers with the aid of a calculator.

a. $4\frac{1}{5} - 2\frac{3}{5}$

b. $6\frac{7}{9} - 3\frac{5}{9}$

c. $8\frac{4}{5} - 4\frac{1}{5}$

d. $8\frac{5}{6} - 4\frac{1}{6}$

e. $6\frac{7}{8} - 4\frac{3}{8}$

f. $1\frac{4}{9} + 5\frac{5}{9}$

g. $6\frac{1}{4} + 3\frac{2}{8}$

h. $12\frac{2}{5} + 8\frac{7}{9}$

i. $4\frac{3}{4} - 5\frac{1}{6} + 3\frac{2}{12}$

j. $1\frac{2}{5} + 3\frac{1}{3} - 2\frac{4}{15}$

5. **WE3** Calculate the following and check answers with the aid of a calculator.

a. $5\frac{3}{5} - 2\frac{3}{10}$

b. $6\frac{1}{2} - 3\frac{5}{6}$

c. $10\frac{1}{4} - 5\frac{2}{5}$

d. $4\frac{1}{8} - 2\frac{3}{5}$

6. a. Put the following fractions, mixed numbers and decimals on a number line: $-\frac{1}{2}$, $2\frac{1}{3}$, 2.5, -1.4 , $-1\frac{3}{4}$, $\frac{2}{3}$.

b. Use the number line from part a to put the fractions, mixed numbers and decimals in ascending order.

7. **WE4** Graph each of the following sets of directed numbers on a number line.

a. $x > -3\frac{1}{2}$

b. $x < -2\frac{1}{3}$

c. $x \geq -4.5$

d. $x \leq 6.25$

e. $-2 < x < 1$

f. $-1\frac{2}{3} \leq x \leq \frac{1}{2}$

g. $-6\frac{3}{4} \leq x \leq -4\frac{1}{2}$

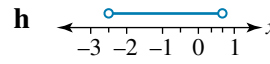
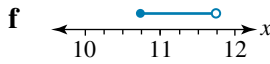
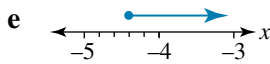
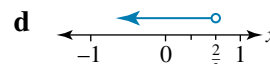
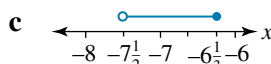
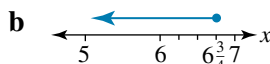
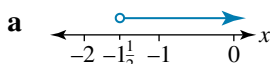
h. $x \geq 2.75$

i. $x < -1\frac{4}{5}$

j. $2\frac{3}{5} \leq x \leq 3.8$

8. Describe the directed numbers graphed on each number line. Use number sentences such as:

$x > -2\frac{1}{2}$, $x \leq 4.8$ and so on.



9. Arrange the following in ascending order.

a. 6, -3, 0, 5.8, $-4\frac{1}{2}$

b. $3\frac{2}{3}$, -1, -4.2, $1\frac{1}{2}$, $-\frac{3}{4}$

c. 0, $-\frac{2}{3}$, $\frac{1}{4}$, -6.8, -8.6

10. **WE5** Calculate the following.

a. $-\frac{3}{5} + \frac{1}{5}$

b. $-\frac{3}{8} + -\frac{5}{8}$

c. $\frac{1}{4} + -\frac{1}{2}$

d. $-\frac{1}{6} + -\frac{2}{3}$

e. $\frac{1}{2} + -\frac{3}{8}$

f. $-\frac{1}{2} + -\frac{1}{3}$

g. $-\frac{3}{5} + \frac{1}{2}$

h. $\frac{3}{4} + -\frac{1}{3}$

i. $-2\frac{1}{2} + -1\frac{3}{4}$

j. $1\frac{2}{3} + -2\frac{3}{5}$

k. $-2\frac{1}{2} - 3\frac{1}{3}$

l. $3\frac{1}{4} + 1\frac{3}{5}$

11. **WE6** Calculate the following.

a. $\frac{1}{2} - \frac{3}{4}$

b. $-\frac{1}{2} - \frac{1}{3}$

c. $-\frac{1}{3} - \frac{2}{5}$

d. $-\frac{3}{4} - \frac{2}{5}$

e. $2\frac{1}{2} - 3\frac{1}{4}$

f. $-3\frac{3}{5} - -1\frac{1}{3}$

12. If Mary eats $\frac{5}{8}$ of a block of chocolate for afternoon tea and $\frac{3}{8}$ of the block after dinner, how much of the block has she eaten altogether?

13. Seven bottles of soft drink were put out onto the table at a birthday party. How much soft drink was left over after $5\frac{2}{9}$ bottles were consumed?



Communicating, reasoning and problem solving

14. Frank has a part-time job at the local newsagency. If he spends $\frac{1}{3}$ of his pay on comic books and $\frac{2}{5}$ on lollies, what fraction of his pay does he have left over? Show your working.

15. In my class, $\frac{1}{3}$ of the students ride their bikes to school, $\frac{1}{4}$ catch the bus and the rest get a lift. What fraction of my class get a lift to school? Show your working.

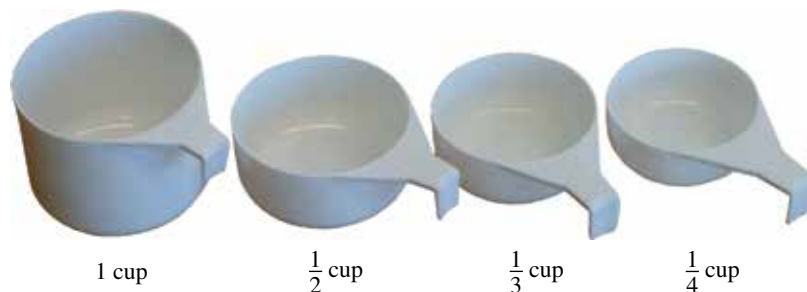
16. A Year 8 class organised a cake stall to raise some money. If they had 10 whole cakes to start with, sold $2\frac{3}{4}$ cakes at recess and then $5\frac{7}{8}$ at lunch time, how much cake was left over? Show your working.

17. Polly and Neda had divided up some coins. Neda was upset as Polly had more coins than Neda. Polly said 'Here's one-third of my coins'. Neda was moved by Polly's generosity and gave back one-half of her total. Polly gave her one-quarter of her new total and an extra coin.

a. Assuming that Polly started with x coins and Neda started with y coins, show that after their final exchange of coins Polly had $\frac{15x + 9y - 24}{24}$ coins and Neda had $\frac{9x + 15y + 24}{24}$ coins.

b. Show that the total number of coins Polly and Neda had between them after their final exchange was the same total with which they started.

18. You need to fill a container with exactly $2\frac{5}{12}$ cups of water. If you have cups that measure $\frac{1}{4}$, $\frac{1}{3}$, $\frac{1}{2}$ and 1 cup of water, what is the quickest way to measure this amount?



19. A tray used to cook a muesli bar slice holds 3 cups of ingredients. For the recipe shown, determine whether or not the mixture will fit in the tray.
20. Some fractions, when added, have the following property: The numerator of the sum of the fractions is equal to the sum of the numerators, and the denominator of the sum of the fractions is equal to the sum of the denominators. Find two pairs of fractions that have this property.
21. Why do we normally leave the answer as a mixed number instead of an improper fraction?

Muesli bar slice

Ingredients

$\frac{1}{2}$ cup of dried fruit
 $\frac{3}{4}$ cup of grated apple
 $1\frac{1}{4}$ cups of muesli
 $\frac{2}{3}$ cup of apple juice
 1 tablespoon ($\frac{2}{25}$ cup) of canola margarine

3.3 Multiplication and division of fractions

3.3.1 Multiplication of fractions

Step 1: Change mixed numbers to improper fractions before multiplying.

Step 2: Cancel if possible.

Step 3: Multiply the numerators and multiply the denominators.

WORKED EXAMPLE 7

Simplify $2\frac{1}{4} \times 1\frac{5}{7}$.

THINK

- 1 Write the question.
- 2 Change the mixed numbers to improper fractions and cancel if possible.
- 3 Multiply the numerators and then multiply the denominators.
- 4 Change to a mixed number and simplify if appropriate.

WRITE

$$\begin{aligned} 2\frac{1}{4} \times 1\frac{5}{7} \\ &= \frac{9}{4^1} \times \frac{12^3}{7} \\ &= \frac{27}{7} \\ &= 3\frac{6}{7} \end{aligned}$$

- To divide fractions, multiply the first fraction by the reciprocal of the second fraction. Cancel where possible, then multiply the numerators and multiply the denominators.

WORKED EXAMPLE 8

Find $2\frac{1}{4} \div \frac{3}{8}$.

THINK

- 1 Write the question.
- 2 Change mixed numbers to improper fractions.
- 3 Change $\div$ to $\times$ and invert the second fraction, ($\times$ and invert), and cancel if appropriate.
- 4 Multiply the numerators and then multiply the denominators.
- 5 Simplify if appropriate.

WRITE

$$\begin{aligned} 2\frac{1}{4} \div \frac{3}{8} \\ &= \frac{9}{4} \div \frac{3}{8} \\ &= \frac{9^3}{4^1} \times \frac{8^2}{3^1} \\ &= \frac{3}{1} \times \frac{2}{1} \\ &= 6 \end{aligned}$$

3.3.2 Multiplication and division of positive and negative fractions

- To multiply fractions, express any mixed numbers as improper fractions. Cancel where possible. Multiply the numerators and denominators, applying the rules for multiplying positive and negative numbers.

WORKED EXAMPLE 9

Simplify $\frac{2}{5} \times -\frac{5}{8}$.

THINK

- Write the expression and cancel the common factors in numerators and denominators.
- Multiply the numerators and then multiply the denominators.
Note: positive $\times$ negative = negative
- Write the answer.

WRITE

$$\begin{aligned}\frac{2^1}{5^1} \times -\frac{5^1}{8^1} \\&= \frac{1}{1} \times -\frac{1}{4} \\&= -\frac{1}{4}\end{aligned}$$

- When dividing by a fraction, express any mixed numbers as improper fractions and multiply by the reciprocal.

WORKED EXAMPLE 10

Evaluate $-\frac{3}{4} \div -1\frac{1}{2}$.

THINK

- Write the expression using a division sign.
- Change the divisor to an improper fraction.
- Change $\div$ to $\times$, invert the divisor and cancel common factors in numerators and denominators.
- Multiply the numerators and then multiply the denominators.
Note: negative $\times$ negative = positive
- Write the answer.

WRITE

$$\begin{aligned}-\frac{3}{4} \div -1\frac{1}{2} \\&= -\frac{3}{4} \div -\frac{3}{2} \\&= -\frac{3^1}{4^1} \times -\frac{2^1}{3^1} \\&= -\frac{1}{2} \times -\frac{1}{1} \\&= \frac{1}{2}\end{aligned}$$



RESOURCES – ONLINE ONLY

Interactivity: Dividing fractions (int-2358)

Interactivity: Multiplication of fractions (int-3722)

Interactivity: Multiplication and division of negative fractions (int-3723)

eLesson: Multiplication and division of fractions (eles-1867)

Digital doc: SkillsSHEET Multiplying and dividing fractions (doc-6858)

Digital doc: SpreadSHEET Four operations with fractions (doc-2130)

Digital doc: WorkSHEET Real numbers I (doc-6870)

Exercise 3.3 Multiplication and division of fractions

Individual pathways

PRACTISE

Questions:
1–7, 9, 14

CONSOLIDATE

Questions:
1d–l, 2d–i, 3d–i, 4d–i, 5d–i, 6d–i,
7–9, 10, 13, 14, 18

MASTER

Questions:
1g–l, 2f–i, 3f–i, 4f–i, 5d–i, 6f–i, 7–12,
14–18

Individual pathway interactivity: int-4407

learnON ONLINE ONLY

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Understanding and fluency

1. Simplify the following.

a. $\frac{3}{4} \times \frac{1}{2}$

b. $\frac{1}{8} \times \frac{1}{7}$

c. $\frac{2}{5} \times \frac{3}{5}$

d. $\frac{5}{7} \times \frac{1}{3}$

e. $\frac{1}{2} \times \frac{5}{6}$

f. $\frac{3}{7} \times \frac{7}{9}$

g. $\frac{11}{20} \times \frac{2}{3}$

h. $\frac{1}{3} \times \frac{3}{5}$

i. $\frac{5}{8} \times \frac{11}{20}$

j. $\frac{2}{3} \times \frac{9}{10}$

k. $\frac{6}{7} \times \frac{14}{15}$

l. $\frac{5}{6} \times \frac{3}{10}$

2. **WE7** Simplify the following and check your answers with the aid of a calculator.

a. $3\frac{1}{2} \times 1\frac{3}{5}$

b. $1\frac{2}{10} \times 1\frac{1}{5}$

c. $2\frac{2}{3} \times 1\frac{1}{2}$

d. $3\frac{2}{4} \times 2\frac{1}{2}$

e. $8\frac{9}{10} \times \frac{7}{10}$

f. $5\frac{3}{4} \times 2\frac{2}{5}$

g. $6 \times 2\frac{1}{6}$

h. $1\frac{3}{5} \times \frac{5}{8}$

i. $4\frac{3}{4} \times 2\frac{1}{2}$

3. Simplify the following.

a. $\frac{1}{3} \div \frac{1}{2}$

b. $\frac{7}{8} \div \frac{3}{2}$

c. $\frac{4}{14} \div \frac{1}{3}$

d. $\frac{2}{5} \div \frac{1}{4}$

e. $\frac{3}{4} \div \frac{7}{8}$

f. $\frac{5}{6} \div \frac{8}{9}$

g. $\frac{12}{15} \div \frac{4}{3}$

h. $\frac{1}{5} \div \frac{10}{12}$

i. $\frac{3}{4} \div \frac{3}{8}$

4. **WE8** Find the following.

a. $1\frac{6}{10} \div 1\frac{3}{5}$

b. $3\frac{5}{7} \div 2\frac{1}{6}$

c. $1\frac{1}{6} \div \frac{2}{1}$

d. $1\frac{5}{7} \div \frac{1}{3}$

e. $1\frac{1}{3} \div \frac{5}{6}$

f. $\frac{7}{9} \div 1\frac{7}{18}$

g. $3\frac{1}{2} \div 1\frac{3}{5}$

h. $10\frac{4}{5} \div 2\frac{1}{2}$

i. $7\frac{8}{9} \div 7\frac{1}{2}$

5. **WE9** Simplify the following.

a. $-\frac{1}{2} \times \frac{1}{3}$

b. $-\frac{3}{4} \times -\frac{1}{5}$

c. $-\frac{2}{3} \times 7$

d. $\frac{1}{3} \times -\frac{3}{4}$

e. $-\frac{3}{4} \times \frac{5}{6}$

f. $-\frac{5}{6} \times \frac{3}{10}$

g. $-\frac{8}{9} \times 1\frac{3}{4}$

h. $-3\frac{1}{7} \times -\frac{7}{8}$

i. $-2\frac{1}{2} \times \frac{4}{5}$

6. **WE10** Evaluate the following.

a. $-\frac{1}{5} \div \frac{1}{2}$

b. $\frac{2}{3} \div -\frac{3}{4}$

c. $-\frac{7}{4} \div -\frac{2}{1}$

d. $\frac{3}{2} \div -4$

e. $-\frac{1}{8} \div \frac{3}{4}$

f. $-1\frac{4}{5} \div 6$

g. $-2\frac{1}{4} \div -\frac{1}{2}$

h. $2\frac{2}{3} \div -1\frac{1}{9}$

i. $-\frac{3}{5} \div 2\frac{5}{8}$

7. Find $\frac{3}{4}$ of 16. ('Of' has the same meaning as multiplying.)

8. An assortment of 75 lollies is to be divided evenly among 5 children.

- What fraction of the total number of lollies will each child receive?
- How many lollies will each child receive?



9. Evaluate the following.

a. $-\frac{2}{3} + \frac{1}{6} \times -\frac{2}{5}$

b. $1\frac{1}{2} \times -\frac{5}{6} \div \frac{4}{7}$

c. $-\frac{7}{8} \div -1\frac{3}{4} - \frac{1}{2}$

d. $\left(\frac{2}{5} - \frac{6}{7}\right) \times -3\frac{1}{3}$

e. $\left(-1\frac{1}{2} - 3\frac{4}{5}\right) \div \frac{3}{5}$

f. $\frac{9}{10} \times -\frac{5}{3} \div \left(1\frac{2}{7} - 2\frac{1}{2}\right)$

Communicating, reasoning and problem solving

10. Sam has been collecting caps from all around the world. If he has a total of 160 caps and $\frac{1}{4}$ of them are from the USA, how many non-USA caps does he have? Show your working.

11. In the staff room there is $\frac{7}{8}$ of a cake left over from a meeting. If 14 members of staff would all like a piece, what fraction of the original cake will they each receive? Show how you reached your answer.

12. Year 8's cake stall raised \$120. If they plan to give $\frac{1}{4}$ to a children's charity, and $\frac{2}{3}$ to a charity for the prevention of cruelty to animals, how much will each group receive and how much is left over? Show your working.

13. The distance between Melbourne and Adelaide is 728 km. The Overland train takes $10\frac{1}{2}$ hours to travel from Melbourne to Adelaide. The driving time by car is $9\frac{3}{4}$ hours, and it takes $\frac{5}{6}$ hours to fly. If the average speed is found by dividing the total distance by the travelling time, calculate the average speed when travelling from Melbourne to Adelaide by:

a. train b. car c. plane.

14. You come across a fantastic recipe for chocolate pudding that you want to try, but it says that the recipe feeds five people. Explain what you would do to modify the recipe so that it feeds only you.

15. Fractions of the type $\frac{1}{a + \frac{1}{b + \frac{1}{c}}}$ are called continued fractions.

a. Investigate continued fractions and write an explanation.

b. Write $\frac{7}{30}$ as a continued fraction. What are the values of a , b and c ?

16. A wealthy merchant died and left 17 camels to be shared among his three children. The eldest was to have half of the camels, the second child one-third, and the third child one-ninth. 'It's not possible!', protested the eldest. A wise man lent his camel to the children, raising the total to 18. The eldest child then took half (nine camels); the second child took one-third (six camels); and the youngest child took one-ninth (two camels). The wise man then departed on his own camel and everyone was happy.

a. Explain what the eldest child meant when they said 'It's not possible!'.

b. Calculate the value of $\frac{1}{2} + \frac{1}{3} + \frac{1}{9}$ and comment on the relationship between the numerator, the denominator and the number of camels.

17. Do the rules of operation order apply to positive and negative fractions?

18. When the numerator and denominator of a certain fraction are placed together in a particular order, the resulting number is a two-digit prime number. The sum of these two digits is 11 and the digit in the tens place is the higher. If the numerator is less than the denominator, find the fraction and the prime number.



Chocolate pudding (serves 5)

Ingredients

$1\frac{1}{2}$ cups plain flour
1 teaspoon baking soda
 $\frac{1}{2}$ cup sugar
1 teaspoon vanilla essence
 $1\frac{1}{2}$ cups melted chocolate
 $\frac{1}{3}$ cup cocoa powder
 $\frac{1}{2}$ cup canola oil
 $\frac{1}{2}$ cup water
1 cup milk

3.4 Terminating and recurring decimals

3.4.1 Fractions to decimals

- Numbers that can be written as fractions are called rational numbers.
- To write a fraction as a decimal, divide the numerator by the denominator.
- Fractions expressed as decimals will be **terminating decimals**, which have a fixed number of decimal places; or **recurring decimals**, which have an infinite number of decimal places.
- All terminating and recurring decimals can be expressed as fractions in the form $\frac{a}{b}$, where a and b are integers and $b \neq 0$; therefore, they are rational numbers.
- Recurring decimals that have one recurring digit are written with a dot above the recurring digit. Recurring decimals with more than one digit are written with dots above the first and last digits of the repeating pattern:

$$\begin{aligned} 0.333\ 333\dots &= 0.\dot{3} \\ 0.484\ 848\dots &= 0.4\dot{8} \end{aligned}$$

(Sometimes a line is used instead of two dots, with the line drawn above the recurring digits; for example, $0.484\ 848\dots = 0.\overline{48}$.)

- If the only factors of the denominator are powers of 2 or 5, you will be able to find an exact decimal equivalent.
- Unless you are told otherwise, round to 2 decimal places.
- Negative fractions are converted using the same method (remember that a negative fraction will result in a negative decimal).
- A calculator can also be used to convert fractions to decimals.

WORKED EXAMPLE 11

Convert the following fractions to decimals. State if the decimal is a recurring decimal or a terminating decimal.

a $\frac{1}{5}$

b $4\frac{5}{12}$

c $-\frac{5}{8}$

d $\frac{2}{3}$

THINK

a 1 Write the question.

2 Rewrite the question using division.

3 Divide, adding zeros as required.

4 Write the answer.

b 1 Write the question.

2 Convert mixed numbers to improper fractions.

3 Rewrite the question using division.

4 Since 12 is not a power of 2 or 5, it will be a recurring decimal.

5 Divide, adding zeros until a pattern occurs.

WRITE/DRAW

a $\frac{1}{5}$

$$= 1 \div 5$$

$$\begin{array}{r} 0.2 \\ 5 \overline{)1.0} \end{array}$$

$$\frac{1}{5} = 0.2$$

This is a terminating decimal.

b $4\frac{5}{12}$

$$= \frac{53}{12}$$

$$= 53 \div 12$$

$$\begin{array}{r} 4.41666 \\ 12 \overline{)53.00000} \end{array}$$

6 Write the answer with a dot above the recurring number.

$$4\frac{5}{12} = 4.41\dot{6}$$

This is a recurring decimal.

c 1 Write the question.

$$c \quad -\frac{5}{8}$$

2 The size of the decimal will be the same as for $\frac{5}{8}$.

$$\frac{5}{8}$$

3 Rewrite the fraction using division.

$$= 5 \div 8$$

4 $8 = 2^3$, so an exact answer will be found. Divide, adding zeros as required.

$$\begin{array}{r} 0.625 \\ 8 \overline{)5.000} \end{array}$$

5 Write the answer, remembering that the original fraction was negative.

$$-\frac{5}{8} = -0.625$$

This is a terminating decimal.

d 1 Rewrite the question using division.

$$d \quad \frac{2}{3} = 2 \div 3$$

2 Since 3 is not a power of 2 or 5, it will be a recurring decimal.

$$\begin{array}{r} 0.666 \\ 3 \overline{)2.000} \end{array}$$

3 Write the answer, remembering to place a dot above the recurring number.

$$\frac{2}{3} = 0.\dot{6}$$

This is a recurring decimal.

- When changing a decimal to a fraction, rewrite the decimal as a fraction with the same number of zeros in the denominator as there are decimal places in the question. Simplify the fraction by cancelling.
- If the decimal has a whole part, it is easier to write it in expanded form; for example, $2.365 = 2 + 0.365$
- Negative decimals are converted using the same method (remember that a negative decimal will result in a negative fraction).

WORKED EXAMPLE 12

Convert the following decimals to fractions in simplest form.

a 0.25

b 1.342

c -0.8

THINK

a 1 Write the question.

2 Rewrite as a fraction with the same number of zeros in the denominator as there are decimal places in the question. Simplify the fraction by cancelling.

3 Write the answer.

WRITE

a 0.25

$$= \frac{25^1}{100^4}$$

$$= \frac{1}{4}$$

b 1 Write the question.

2 Rewrite the decimal in expanded form.

3 Write as a mixed number with the same number of zeros in the denominator as there are decimal places in the question and cancel.

b 1.342

$$= 1 + 0.342$$

$$= 1 + \frac{342^{171}}{1000^{500}}$$

4 Write the answer.

$$= 1\frac{171}{500}$$

c 1 Write the question.

$$c -0.8$$

2 Rewrite as a fraction with the same number of zeros in the denominator as there are decimal places in the question. Simplify the fraction by cancelling.

$$= -\frac{8^4}{10^5}$$

3 Write the answer.

$$= -\frac{4}{5}$$

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RESOURCES — ONLINE ONLY

Digital doc: SpreadSHEET Converting fractions to decimals (doc-2131)

Digital doc: SpreadSHEET Converting decimals to fractions (doc-2132)

Exercise 3.4 Terminating and recurring decimals

Individual pathways

PRACTISE

Questions:

1–4, 6, 10, 14

CONSOLIDATE

Questions:

1–6, 8, 10, 11, 13, 14

MASTER

Questions:

1f–j, 2f–j, 3f–j, 4k–r, 5–13, 15, 16

Individual pathway interactivity: int-4408

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Understanding and fluency

1. **WE11a** Convert the following fractions to decimals, giving exact answers or using the correct notation for recurring decimals where appropriate.

a. $\frac{4}{5}$

b. $\frac{1}{4}$

c. $\frac{3}{4}$

d. $\frac{5}{12}$

e. $\frac{9}{11}$

f. $\frac{21}{25}$

g. $\frac{7}{4}$

h. $\frac{13}{6}$

i. $\frac{7}{15}$

j. $\frac{2}{3}$

2. **WE11b** Convert the following mixed numbers to decimal numbers, giving exact answers, or using the correct notation for recurring decimals where appropriate. Check your answers using a calculator.

a. $1\frac{5}{6}$

b. $1\frac{3}{4}$

c. $3\frac{2}{5}$

d. $8\frac{4}{5}$

e. $12\frac{9}{10}$

f. $6\frac{3}{4}$

g. $5\frac{2}{3}$

h. $11\frac{11}{15}$

i. $6\frac{1}{2}$

j. $4\frac{1}{3}$

3. **WE11c** Convert the following fractions to decimal numbers, giving exact answers, or correct to 2 decimal places where appropriate.

a. $-\frac{4}{5}$

b. $-\frac{7}{9}$

c. $-\frac{8}{5}$

d. $-\frac{4}{15}$

e. $-\frac{3}{8}$

f. $-2\frac{1}{4}$

g. $-6\frac{3}{5}$

h. $-3\frac{1}{7}$

i. $-1\frac{5}{6}$

j. $-5\frac{8}{9}$

4. **WE12** Convert the following decimal numbers to fractions in simplest form.

a. 0.4

b. 0.8

c. 1.2

d. 3.2

e. 5.6

f. 0.75

g. 1.30

h. 7.14

i. 4.21

j. 10.04

k. 1.333

l. 8.05

m. 7.312

n. 9.940

o. 12.045

p. 84.126

q. 73.90

r. 0.0042

5. Of the people at a school social $\frac{3}{4}$ were boys. Write this fraction as a decimal number.
6. On a recent science test Katarina worked the bonus question correctly as well as everything else, and her score was $\frac{110}{100}$.
What is this as a decimal value?
7. Alison sold the greatest number of chocolates in her Scouting group. She sold $\frac{5}{9}$ of all chocolates sold by the group. Write this as a decimal number, correct to 2 decimal places.
8. Alfonzo ordered a pizza to share with three friends, but he ate 0.6 of it. What fraction was left for his friends?
9. Using examples, explain the difference between rational and irrational numbers.



Communicating, reasoning and problem solving

10. By converting to decimal fractions, arrange the following in order from lowest to highest, showing all of your working.
 $\frac{6}{10}, \frac{3}{4}, \frac{2}{3}, \frac{5}{8}, \frac{6}{7}$
11. To change $0.\dot{1}\dot{4}$ from a recurring decimal to a vulgar fraction, put the digits after the decimal place (14) on the numerator. The denominator has the same number of digits as the numerator (2), with all of these digits being 9's. Thus $0.\dot{1}\dot{4} = \frac{14}{99}$.
 - a. Write $0.\dot{7}$ as a vulgar fraction, showing all of your working.
 - b. Write $0.\dot{3}0\dot{6}$ as a vulgar fraction, showing all of your working.
 - c. Write $2.\dot{2}$ as a proper fraction, showing all of your working.
12.
 - a. When you add two recurring decimals, do you always get a recurring decimal? Explain.
 - b. When you add a recurring decimal and a terminating decimal, do you always get a recurring decimal? Explain.
 - c. When you subtract two recurring decimals, do you always get a recurring decimal? Explain.
 - d. When you subtract a recurring decimal from a terminating decimal, do you always get a recurring decimal? Explain.
 - e. When you subtract a terminating decimal from a recurring decimal, do you get a recurring decimal? Explain.
13. Robin Hoot, the captain of a motley pirate crew known throughout the Seven Seas as 'The Ferry Men', kept $0.\dot{3}$ of all stolen loot for himself. The rest was split between his crew.
 - a. If there were 20 Ferry Men, what fraction of the loot did each Ferry Man receive?
 - b. Little George, one of the Ferry Men, received 15 gold doubloons. How much loot was there altogether?
 - c. If there were p pirates not including Robin Hoot, and each of them received d doubloons, how much loot was there altogether?
14. The pig ate $\frac{2}{5}$ of the chocolate cake the animals had stolen from Mrs Brown's window sill where it had been cooling. The goat and the cow ate $0.\dot{2}$ and 0.35 of the cake respectively.
 - a. If the duck and the turkey finished the rest, having equal shares, what fraction did the duck eat?
 - b. If the duck and the turkey had r grams of the cake each, how much of the cake did the pig have?
15. James had 18 litres of water shared unequally between three buckets. Then he:
 - a. poured three quarters of the water in bucket 1 into bucket 2.
 - b. poured half the water that was now in bucket 2 into bucket 3.
 - c. poured a third of the water that was now in bucket 3 into bucket 1. After the pouring, all the buckets contained equal amounts of water. How much water did each bucket start with?
16. How would you decide whether it is better to write a number as a fraction or a decimal?



3.5 Addition and subtraction of decimals

3.5.1 Addition and subtraction of positive decimals

- When decimals are being added or subtracted, the decimal points are lined up one underneath the other.

WORKED EXAMPLE 13

Find $4.622 + 38 + 210.07 + 21.309$.

THINK

Write the numbers one underneath the other with the decimal points lined up and fill the spaces with zeros. Then add as for whole numbers putting the decimal point in the answer directly under the decimal points in the question.

WRITE

$$\begin{array}{r} 4.622 \\ 38.000 \\ 210.070 \\ + 21.309 \\ \hline 274.001 \end{array}$$

WORKED EXAMPLE 14

Find $37.6 - 12.043$.

THINK

Write the numbers one under the other with the first number on top and the decimal points lined up. Add in the required zeros and subtract.

WRITE

$$\begin{array}{r} 37.600 \\ -12.043 \\ \hline 25.557 \end{array}$$

3.5.2 Addition and subtraction of positive and negative decimals

- The rules for addition and subtraction of negative numbers apply.
 - To add two numbers with the same sign, add and keep the sign.
 - To add two numbers with different signs, find the difference between the signed numbers, then use the sign of the number further from 0.
 - To subtract a number, add the opposite.

WORKED EXAMPLE 15

Calculate:

a $-3.64 + -2.9$

b $-5.7 + 2.4$

and check each answer by using estimation.

THINK

a 1 Write the question in columns with the decimal points directly beneath each other. Include the zeros.

2 They have the same sign, so add and keep the sign.

3 Check the answer by using estimation.

b 1 Repeat step 1 from part a.

2 They have different signs, so subtract the smaller number from the larger number and use the sign of the number furthest from zero.

3 Check the answer by using estimation.

WRITE

a
$$\begin{array}{r} -3.64 \\ + -2.90 \\ \hline -6.54 \end{array}$$

$$-4 + -3 = -7$$

b
$$\begin{array}{r} 5.7 \\ -2.4 \\ \hline 3.3 \end{array}$$

$$-5.7 + 2.4 = -3.3$$

$$-6 + 2 = -4$$

WORKED EXAMPLE 16

Calculate $-5.307 - 0.62$ and check the answer by using estimation.

THINK

- 1 Write the question.
- 2 Change to addition of the opposite.
- 3 Rewrite the question in columns with the decimal points directly beneath each other. Include the zeros.
- 4 Evaluate.
- 5 Check the answer by using estimation.

WRITE

$$\begin{aligned} & -5.307 - 0.62 \\ &= -5.307 + -0.62 \\ & \quad \begin{array}{r} -5.307 \\ + -0.620 \\ \hline -5.927 \end{array} \\ & -5 - 1 = -6 \end{aligned}$$

learnon RESOURCES — ONLINE ONLY

Interactivity: Addition and decimal numbers (int-3725)

Interactivity: Addition and subtraction of negative decimals (int-3776)

Digital doc: SkillsHEET Adding and subtracting decimals (doc-6859)

Digital doc: SpreadSHEET Adding decimals (doc-2133)

Digital doc: SpreadSHEET Subtracting decimals (doc-2134)

Exercise 3.5 Addition and subtraction of decimals

Individual pathways

PRACTISE

Questions:
1–6, 8, 11, 14

CONSOLIDATE

Questions:
1–8, 11, 12, 14, 15

MASTER

Questions:
1g–n, 2g–l, 3, 4e–j, 5–16

Individual pathway interactivity: int-4409

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Understanding and fluency

1. **WE13** Find the following.

a. $8.3 + 4.6$	b. $7.2 + 5.8$	c. $16.45 + 3.23$
d. $7.9 + 12.4$	e. $13.06 + 4.2$	f. $5.34 + 2.80$
g. $128.09 + 4.35$	h. $5.308 + 33.671 + 3.74$	i. $0.93 + 4.009 + 1.3$
j. $5.67 + 3 + 12.002$	k. $56.830 + 2.504 + 0.1$	l. $306 + 5.2 + 6.032 + 76.9$
m. $25.3 + 89 + 4.087 + 7.77$	n. $34.2 + 7076 + 2.056 + 1.3$	

2. **WE14** Find the following.

a. $4.56 - 2.32$	b. $19.97 - 12.65$	c. $124.99 - 3.33$
d. $63.872 - 9.051$	e. $43.58 - 1.25$	f. $1709.53 - 34.6$
g. $87.25 - 34.09$	h. $125.006 - 0.04$	i. $24.86 - 1.963$
j. $35 - 8.97$	k. $42.1 - 9.072$	l. $482 - 7.896$

3. **MC** a. The difference between 47.09 and 21.962 is:

A. 17.253	B. 26.93	C. 25.932	D. 26.128	E. 25.128
b. The sum of 31.5 and 129.62 is:				
A. 98.12	B. 161.12	C. 150.12	D. 444.62	E. 132.77

- b. Your friend started work at a milk bar and was trying to round numbers in his head. The total came to \$12.97 and the person gave your friend \$15.00. Your friend realised that the change would come to \$2.03. Because 3 is less than 5, your friend rounded it down and gave the person \$2.00 back. Explain why the customer might discuss their change with your friend.
15. By writing each of these recurring decimals correct to 4 decimal places and showing your working, find:
- a. $0.\dot{3} + 0.\dot{5}$ b. $0.\dot{3} + 0.\dot{8}$ c. $0.\dot{3} + 0.\dot{7}$ d. $0.\dot{4} + 0.\dot{5}\dot{3}$
 e. $0.\dot{7} - 0.\dot{6}$ f. $1.1\dot{2} - 0.0\dot{3}$ g. $2.\dot{5}3\dot{4} - 0.1\dot{2}$.
16. When decimals are being added, why are the decimal points lined up one underneath the other?

3.6 Multiplication and division of decimals

3.6.1 Multiplication of positive decimals

- To multiply decimals, ignore the decimal point and multiply as for whole numbers. Count the number of digits after the decimal point in each of the multiplying numbers, then add these together to find the number of decimal places in the answer.
- It is a good idea to estimate the answer to make sure that your answer makes sense.

WORKED EXAMPLE 17

Calculate, giving an exact answer, 125.678×0.23 .

THINK

- 1 Write the numbers with the larger one on top. Multiply, starting with the last digit and ignoring the decimal point.

WRITE

$$\begin{array}{r} 1111 \\ 1222 \\ 12\,5678 \\ \times 023 \\ \hline 37\,7034 \\ 251\,3560 \\ \hline 289\,0594 \end{array}$$

- 2 Count the number of digits after the decimal point in each of the multiplying numbers and use this total as the number of decimal places in the answer. There are 3 decimal places in 125.678 and 2 in 0.23, so there will be 5 decimal places in the answer.

$$125.678 \times 0.23 = 28.90594$$

ACTIVITY: DECIMAL DECISIONS

- From pairs and write five questions involving times tables, but change them slightly so that one or both of the numbers are decimals: for example, 5×0.9 .
- Swap your questions with another pair and solve their questions. As a group of four, discuss the answers and any difficulties that you may have had.
- In your original pairs, write five questions using division based on times tables. Again, ensure that one or both of the numbers are decimals.
- This time, swap your questions with a different pair and solve their questions.
- As a class, discuss how your answers change when dividing and multiplying by a decimal rather than a whole number.

3.6.2 Division of positive decimals

- When dividing decimals, make sure that the divisor (the number you are dividing by) is a whole number.
- If the divisor is not a whole number, make it a whole number by either:
 - writing the question as a fraction and multiplying the numerator and denominator by an appropriate power of 10, or
 - multiplying the dividend and divisor by an appropriate power of 10.
- Once the divisor is a whole number, divide the numbers as usual, making sure the decimal point in the answer is directly above the decimal point in the question.
- Extra zeros can be placed after the decimal point in the dividend if needed.

WORKED EXAMPLE 18

Calculate:

a $54.6 \div 8$

b $89.356 \div 0.06$.

Give answers correct to 2 decimal places.

THINK

a 1 Write the question as shown, adding zeros to one more decimal place than is required. Write the decimal point in the answer directly above the decimal point in the question and divide as for short division.

2 Write the question and answer, rounded to the required number of decimal places.

b 1 Write the question.

2 Multiply both parts by an appropriate multiple of 10 so that the divisor is a whole number (in this case, 100).

3 Divide, adding zeros to one more decimal place than required. Write the decimal point in the answer directly above the decimal point in the question and divide as for short division.

4 Write the question and answer, rounded to the required number of decimal places.

WRITE

a
$$\begin{array}{r} 6.825 \\ 8 \overline{)54.620} \end{array}$$

$54.6 \div 8 \approx 6.83$ (2 decimal places)

b $89.356 \div 0.06$

$= (89.356 \times 100) \div (0.06 \times 100)$
 $= 8935.6 \div 6$

$$\begin{array}{r} 1489.266 \\ 6 \overline{)8935.600} \end{array}$$

$89.356 \div 0.06 \approx 1489.27$ (2 decimal places)

3.6.3 Multiplication and division of positive and negative decimals

- When positive and negative decimals are being multiplied and divided, the rules for multiplying and dividing positive and numbers apply.
 - If both numbers are positive or negative, the result is positive.
 - If one number is positive and the other is negative, the result is negative.

WORKED EXAMPLE 19

Simplify -3.8×0.05 .

THINK

1 Multiply as for whole numbers.

WRITE

$$\begin{array}{r} 38 \\ \times 5 \\ \hline 190 \end{array}$$

- 2 Count the number of decimal places in the question and insert the decimal point. The signs are different, so insert a negative sign in the answer.
- $$\begin{aligned} -3.8 \times 0.05 &= -0.190 \\ &= -0.19 \end{aligned}$$

WORKED EXAMPLE 20

Find the quotient of $-0.015 \div -0.4$, giving an exact answer.

THINK

- 1 Write the question.
- 2 Multiply both parts by 10 to produce a whole number divisor.
- 3 Divide until an exact answer is achieved or until a recurring pattern is evident.
- 4 The signs are the same, so the answer is positive.

WRITE

$$\begin{aligned} &-0.015 \div -0.4 \\ &= -(0.015 \times 10) \div -(0.4 \times 10) \\ &= -0.15 \div -4 \\ &\quad \begin{array}{r} 0.0375 \\ 4 \overline{)0.1530} \end{array} \\ &-0.015 \div -0.4 = 0.0375 \end{aligned}$$

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Interactivity: Multiplication of positive decimals (int-3726)

Interactivity: Division of positive decimals (int-3727)

Interactivity: Division of negative decimals (int-3728)

eLesson: Multiplication of decimals (eles-2311)

eLesson: Division of decimals (eles-1877)

Digital doc: SkillSHEET Multiplying and dividing decimals (doc-6860)

Digital doc: SpreadSHEET Multiplying decimals (doc-2135)

Digital doc: SpreadSHEET Dividing decimals (doc-2136)

Exercise 3.6 Multiplication and division of decimals

Individual pathways

PRACTISE

Questions:
1–5, 6, 8, 13, 16

CONSOLIDATE

Questions:
1–7, 11, 13, 15, 16

MASTER

Questions:
1g–o, 2, 3g–l, 4g–l, 5g–l, 6–17

Individual pathway interactivity: int-4410

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Understanding and fluency

1. **WE17** Calculate the following, giving an exact answer.

a. 6.2×0.8	b. 7.9×1.2	c. 65.7×3.2	d. 109.5×5.6
e. 5.09×0.4	f. 32.76×2.4	g. 123.97×4.7	h. 576.98×2
i. 3.4×642.1	j. 0.6×67.9	k. 23.4×6.7	l. 0.006×43.6
m. 52.003×12	n. 22.97×0.015	o. 13.42×0.011	

2. **WE18a** Calculate the following. Give answers correct to 2 decimal places.
- a. $43.2 \div 7$ b. $523.9 \div 4$ c. $6321.09 \div 8$
d. $286.634 \div 3$ e. $76.96 \div 12$ f. $27.8403 \div 11$
3. **WE18b** Calculate the following. Give answers correct to 2 decimal places, where appropriate.
- a. $53.3 \div 0.6$ b. $960.43 \div 0.5$ c. $21.42 \div 0.004$ d. $3219.09 \div 0.006$
e. $478.94 \div 0.016$ f. $76.327 \div 0.00008$ g. $25.865 \div 0.004$ h. $26.976 \div 0.0003$
i. $0.0673 \div 0.0005$ j. $12.00053 \div 0.007$ k. $35.064 \div 0.005$ l. $0.059 \div 0.009$
4. **WE19** Simplify each of the following.
- a. 0.3×-0.2 b. -0.8×0.9 c. -0.4×-0.06 d. $(-0.6)^2$
e. $(-0.3)^2$ f. -0.004×40 g. 4000×-0.5 h. -0.02×-0.4
i. $(-0.05)^2$ j. -4.9×0.06 k. $(0.2)^2 \times -40$ l. $(1.2)^2 \times (0.3)^2$
5. **WE20** Find the quotient of each of the following, giving an exact answer.
- a. $-8.4 \div 0.2$ b. $0.15 \div -0.5$ c. $-0.0405 \div -0.3$ d. $-15 \div 0.5$
e. $0.049 \div -0.07$ f. $-3.2 \div -0.008$ g. $-0.0036 \div 0.06$ h. $270 \div -0.03$
i. $-0.04 \div -800$ j. $0.8 \div -0.16$ k. $(1.2)^2 \div 0.04$ l. $(1.5)^2 \div 0.05$
6. Evaluate the following, giving answers correct to 1 decimal place.
- a. $4.6 \times 2.1 + 1.2 \times 3.5$ b. $5.9 \times 1.8 - 2.4 \times 3.8$
c. $6.2 + 4.5 \div 0.5 - 7.6$ d. $11.4 - 7.6 \times 1.5 + 2$
7. **MC** a. $\frac{1}{2}(3.6 + 1.4 \times 7.5)$ is equal to:
- A. 18.75 B. 14.1 C. 9.375 D. 7.05 E. 28.2
- b. Rounded to 2 decimal places, $\frac{3}{4}(10.5 - 5.8 \div 4 \times 1.2)$ is equal to:
- A. 1.06 B. 6.57 C. 0.73 D. 11.68 E. 2.19
8. Round each of the following to the nearest whole number to find an estimate.
- a. $3.5 \times 24.9 + 33.2$ b. $4.8 \times 19.6 - 10.4$
c. $15.6 + 50.1 \times 9.5 - 15.4$ d. $49.8 - 20.3 \div 4.7$
9. A group of 21 Year 8 students were going on an excursion to the planetarium. If the total cost is \$111.30, how much would each student have to pay?
10. Simplify $\frac{0.16}{0.28} \times \frac{56}{3.2}$.
11. Your car's tank holds 55 litres of fuel. The car consumes 7.6L of fuel for each 100km travelled. You make a journey from Melbourne to Traralgon, a distance of 145km. How much fuel is left in the tank after the journey?
12. Find the decimal exactly halfway between 2.01 and 2.02.

Communicating, reasoning and problem solving

13. A shoebox with a mass of 150g measures 30.1 cm by 16.5cm along the bottom with a height of 10.3cm.
- a. The volume of the shoebox is found by multiplying its three linear dimensions together. What is its volume?
b. Density is defined as mass divided by volume. What is the density of the shoebox in g/cm^3 ?
14. A square of grass has a side length of 3.4 metres. There is a path 1.2 metres wide around the outside of the square of grass.
- a. Find the area of the grassed area.
b. Find the area of the path, explaining your method.



15. The prestigious Collins Street lawyers, Messrs Waffle, Pockett and Runne, have been retained to secure the patent on the Global Coffee Constant, or GCC as it is better known. If t represents all the tea in China, c represents all the coffee in Brazil and g represents the total number of tea and coffee drinkers globally, find a simple expression for the GCC in terms of t , c and g , given the following information: the GCC equals 0.4 of all the tea in China multiplied by $\frac{5}{9}$ of all the coffee in Brazil divided by 0.5 of the number of tea and coffee drinkers globally.
16. Speed equals distance travelled divided by the time taken.
- If Septimus Squirrel travels 27.9 m in 8.4 seconds and Fortescue Fieldmouse travels 18.7 m in 6.4 seconds, which of them travels at the greater speed?
 - Travelling at 5 metres per second, how long (in hours) will it take Genevieve Goose to travel round the equator, assuming the circumference of the Earth at the equator is approximately 40075 kilometres?
17. Why is it a good idea to make an estimate of the answer when you are multiplying decimals?

3.7 Estimation

3.7.1 Estimation

- Sometimes an **estimate** of the answer is all that is required (an estimate is answer close to the actual answer, but found using easier numbers).
- Some estimation methods are detailed in this section.
- If the calculation involves similar values, the values can be rounded to the same number.

WORKED EXAMPLE 21

Marilyn and Kim disagree about the answer to the following calculation:

$$7.3 + 7.1 + 6.9 + 6.8 + 7.2 + 7.3 + 7.4 + 6.6.$$

Marilyn says the answer is 56.6, but Kim thinks it is 46.6. Obtain an estimate for the calculation and determine who is correct.

THINK

- Carefully analyse the values and devise a method to estimate the total.
- Perform the calculation using the rounded numbers.
- Answer the question.

WRITE

Each of the values can be approximated to 7 and there are 8 values.

$$7 \times 8 = 56$$

Marilyn is correct, because the approximate value is very close to 56.6.

- When we are **rounding** to a given place value:
 - if the next lower place value digit is less than 5, leave the place value digit as it is, and add zeros to all lower place values, if necessary
 - if the next lower place value digit is 5 or greater, increase the given place value digit by 1 and add zeros to all lower place values, if necessary.
- When rounding 25354:
 - to the nearest thousand, the result is 25000
 - to the nearest hundred, the result is 25400.
- When **rounding up**, the digit in the desired place value is increased by 1 regardless of the digits in the lower place positions (as long as they are not all zeros). Zeros are added to the lower place positions to retain the place value.

- When rounding up 3100:
 - to the nearest thousand, the result is 4000
 - to the nearest hundred, the result is 3100.
- When **rounding down**, all digits following the desired place value are replaced by zeros, leaving the given place value unchanged.
- When rounding down 635 to the nearest ten, the result is 630.

WORKED EXAMPLE 22

Consider the number 39461 and perform the following.

- Round to the nearest thousand.
- Round up to the nearest hundred.
- Round down to the nearest ten.

THINK

- Consider the digit in the thousands place position and the digit in the next lower place position.
 - Write the answer, adding the required number of zeros.
- Consider the digit in the hundreds place position and the digits in the lower place positions.
 - Write the answer, adding the required number of zeros.
- Consider the digit in the tens place position and the digit in the lower place position.
 - Write the answer, adding the required number of zeros.

WRITE

- The digit 9 lies in the thousands position. The digit 4, which is less than 5, lies in the hundreds position.
The number 39461 rounded to the nearest thousand is 39000.
- The digit 4 lies in the hundreds position. The digits in the lower place positions are not all zeros (they are 6 and 1). When rounding up to the nearest hundred, the 4 will increase to 5.
The number 39461 rounded up to the nearest hundred is 39500.
- The digit 6 lies in the tens position and will remain unchanged. The digit 1 lies in the units position. When rounding down to the nearest ten, the 1 will be converted to 0.
The number 39461 rounded down to the nearest ten is 39460.

3.7.2 Estimating by rounding to the first digit

- When estimating answers to calculations, sometimes it is simplest to round all numbers in the calculation to the first digit and then perform the operation.

WORKED EXAMPLE 23

Provide an estimate to the following calculations by first rounding each number to its first digit. Check your estimate with a calculator. Comment on the accuracy of your estimate.

a $394 + 76 - 121$

b $\frac{692 \times 32}{19 \times 87}$

THINK

- Round each of the numbers to the first digit.
 - Perform the calculation using the rounded numbers.

WRITE

- Rounded to the first digit, 394 becomes 400, 76 becomes 80 and 121 becomes 100.
 $394 + 76 - 121 \approx 400 + 80 - 100 \approx 380$

3 Check using a calculator. Comment on how the rounded result compares with the actual answer.

Using a calculator, the result is 349. The estimate compares well to the actual (calculator) value.

b 1 Round each of the numbers to the first digit.

b Rounded to the first digit, 692 becomes 700, 32 becomes 30, 19 becomes 20 and 87 becomes 90.

2 Perform the calculation using the rounded numbers.

$$\begin{aligned}\frac{692 \times 32}{19 \times 87} &\approx \frac{700^{35} \times 30^1}{20_1 \times 90_3} \\ &\approx \frac{35}{3} \\ &\approx 12\end{aligned}$$

3 Check using a calculator. Comment on how the rounded result compares with the actual answer.

Using a calculator, the result is 13.4 (rounded to 1 decimal place). The estimate is close to the actual (calculator) value.

3.7.3 Estimating by rounding the dividend to a multiple of the divisor

- To make division easier, the dividend can be rounded to a multiple of the divisor. For example, in $20\,532 \div 7$, the 20532 (the dividend) could be rounded to 21000. Because we know that 21 is a multiple of 7, we could see, through mental approximation, that the answer is close to 3000 (the exact answer is 2933).

WORKED EXAMPLE 24

Provide estimates for the calculation $\frac{537}{40}$ by:

- rounding the dividend up to the nearest hundred
- rounding the dividend to the nearest ten
- rounding the dividend to a multiple of the divisor.

THINK

a 1 Round the dividend up to the nearest hundred.

WRITE

a 537 rounded up to the nearest hundred is 600.

2 Perform the division. Write the estimation.

$$\frac{537}{40} \approx \frac{600^{15}}{40_1} \approx 15$$

b 1 Round the dividend to the nearest ten.

b 537 rounded up to the nearest ten is 540.

2 Perform the division. Write the estimation.

$$\frac{537}{40} \approx \frac{540^{27}}{40^2} \approx 13.5$$

c 1 Round the dividend to a multiple of the divisor.

c 520 is a multiple of 40.

2 Perform the division. Write the estimation.

$$\frac{537}{40} \approx \frac{520^{13}}{40_1} \approx 13$$

Different methods will give slightly different estimates.

WORKED EXAMPLE 25

The exact answer to $\frac{132 \times 77}{55}$ has the digits 1848. Use any estimation technique to locate the position of the decimal point.

THINK

- 1 Round each of the numbers to the first digit.
- 2 Perform the calculation using the rounded numbers and write the estimate, ignoring the decimal.
- 3 Use the estimate obtained to locate the position of the decimal point. Write the correct answer.

WRITE

Rounded to the first digit, 132 becomes 100, 77 becomes 80 and 55 becomes 60.

$$\begin{aligned}\frac{132 \times 77}{55} &\approx \frac{100 \times 80}{60} \\ &\approx \frac{400}{3} \\ &\approx 133\end{aligned}$$

The estimate gives an answer between 100 and 200. This indicates that the decimal point should be between the last two digits. The correct answer is 184.8.

DISCUSSION

When is estimating used in everyday life?

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RESOURCES — ONLINE ONLY

Interactivity: Rounding (int-3730)

Interactivity: Rounding to the first digit (int-3731)

Digital doc: SkillsHEET Rounding to the first (leading) digit (doc-6861)

Digital doc: SkillsHEET Operations with fractions (doc-6862)

Digital doc: SkillsHEET Operations with decimals (doc-6863)

Digital doc: WorkSHEET Real numbers II (doc-11438)

Exercise 3.7 Estimation

Individual pathways

PRACTISE

Questions:
1–7, 10, 12

CONSOLIDATE

Questions:
1–14

MASTER

Questions:
1, 2, 4, 5i–p, 6–16

Individual pathway interactivity: int-4411

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Understanding and fluency

1. **WE21** Marilyn and Kim disagree about the answer to the following calculation:

$$8.6 + 9.2 + 8.7 + 8.8 + 8.9 + 9.3 + 9.4 + 8.6.$$

Marilyn says the answer is 81.5, but Kim thinks it is 71.5. Obtain an estimate for the calculation and determine who is correct.

2. **WE22** For each of the following numbers:

i. round to the first digit

ii. round up to the first digit

iii. round down to the first digit.

a. 239

b. 4522

c. 21

d. 53624

e. 592

f. 1044

3. Round each of the numbers in question 2 down to the nearest ten.

4. Round each of the numbers in question 2 up to the nearest hundred.

5. **WE23** Find an estimate for each of the following by rounding each number to the first digit.

a. $78 \div 21$

b. $297 + 36$

c. $587 - 78$

d. $235 + 67 + 903$

e. $1256 - 678$

f. 789×34

g. 56×891

h. $1108 \div 53$

i. $345 + 8906 - 23 + 427$

j. $907 \div 88$

k. $326 \times 89 \times 4$

l. $2378 \div 109$

m. $(426 + 1076) \times 21$

n. $7 \times 211 - 832$

o. $977 \div 10 \times 37$

p. $(12\,384 - 6910) \times (214 + 67)$

6. **WE24** Provide estimates for each of the following by first rounding the dividend to a multiple of the divisor.

a. $35\,249 \div 9$

b. $2396 \div 5$

c. $526\,352 \div 7$

d. $145\,923 \div 12$

e. $92\,487 \div 11$

f. $5249 \div 13$

7. **WE25** Use any of the estimation techniques to locate the position of the decimal point in each of the following calculations. The correct digits for each one are shown in brackets.

a. $\frac{369 \times 16}{288}$ (205)

b. $\frac{42\,049}{14 \times 20}$ (150 175)

c. $\frac{99 \times 270}{1320}$ (2025)

d. $\frac{285 \times 36}{16 \times 125}$ (513)

e. $\frac{256 \times 680}{32 \times 100}$ (544)

f. $\frac{7290 \times 84}{27 \times 350}$ (648)

8. If 127 people came to a school social and each paid \$5 admission, find an estimate for the amount of money collected.

9. Estimate the whole numbers between which each of the following will lie.

a. $\sqrt{20}$

b. $\sqrt{120}$

c. $\sqrt{180}$

d. $\sqrt{240}$

10. Complete the table below with the rounded question, the estimated answer and the exact answer. Use rounding to the first digit. The first one has been completed. Calculate the exact answers using a written method, and then check by using a calculator.



	Question	Rounded question	Estimated answer	Exact answer
a	789×56	800×60	48 000	44 184
b	$124 \div 5$			
c	$678 + 98 + 46$			
d	235×209			
e	$7863 - 908$			
f	63×726			
g	$39\,654 \div 227$			
h	$1809 - 786 + 467$			
i	$21 \times 78 \times 234$			
j	$942 \div 89$			
k	$\frac{492 \times 94}{38 \times 49}$			
l	$\frac{54\,296}{97 \times 184}$			

Communicating, reasoning and problem solving

- 11.** Find an approximate answer to each of the worded problems below by rounding to the first digit. Remember to write each answer in a sentence.
- A company predicted that it would sell 13 cars in a month at \$28999 each. About how much money would they take in sales?
 - A tap was leaking 8 mL of water each hour. Approximately how many millilitres of water would be lost if the tap was allowed to leak for 78 hours?
 - The Year 8 cake stall sold 176 pieces of cake for 95 cents each. How much money did they make?
 - Steven swam 124 laps of a 50m pool and, on average, each lap took him 47 seconds. If he swam non-stop, for approximately how many seconds was he swimming?
 - An audience of 11 784 people attended a recent Kylie concert at Rod Laver Arena and paid \$89 each for their tickets. How much money was taken at the door?
 - A shop sold 4289 articles at \$4.20 each. How much money was paid altogether?
 - On Clean Up Australia Day, 19863 people volunteered to help. If they each picked up 196 pieces of rubbish, how many pieces of litter were collected altogether?
- 12.** Sports commentators often estimate the crowd size at sports events. They estimate the percentage of the seats that are occupied, and then use the venue's seating capacity to estimate the crowd size.
- The photograph below right shows the crowd at an AFL match between Collingwood and Essendon at the Melbourne Cricket Ground. Use the information given with the photograph to calculate the estimated number of people in attendance.
 - Choose two well-known sports venues and research the maximum seating capacity for each.
 - If you estimate that three out of every four seats are occupied at each of the venues chosen in part **b** for particular sports events, how many people are in attendance at each venue?
- 13.** For each of the following multiplications, calculate the product when only:
- the first number is rounded to the nearest 10
 - the second number is rounded to the nearest 10.
- a. 18×45 b. 18×46 c. 18×47 d. 18×48 e. 18×49
- 14.** Describe any patterns that you notice in your answers to question **13**.
- 15.** Explore the rounding error patterns that occur in calculating the following multiplications when only:
- the first number is rounded to the nearest 10
 - the second number is rounded to the nearest 10.
- a. 15×44 b. 16×44 c. 17×44
- 16.** Can you think of some examples of when an estimate rather than the exact answer is required?



3.8 Percentages, fractions and decimals

3.8.1 Writing percentages in different ways

- The term **per cent** means 'per hundred'.
- The symbol for percentage is **%**. For example, 60% means 60 parts out of 100 parts.
- Percentages, fractions and decimals are different ways of expressing the same quantity.

- Percentage is another way of writing a fraction with a denominator of 100, or of writing the number of hundredths in a decimal.

$$60\% = \frac{60}{100} = 0.60$$

- There are a number of common percentages, and their fraction and decimal equivalents, with which you should be familiar.

Percentage	Fraction	Decimal
50%	$\frac{1}{2}$	0.5
25%	$\frac{1}{4}$	0.25
75%	$\frac{3}{4}$	0.75
$33\frac{1}{3}\%$	$\frac{1}{3}$	0. $\dot{3}$
100%	1	1

WORKED EXAMPLE 26

Convert the following percentages to fractions and then to decimals.

a 67%

b 55%

THINK

a 1 To convert to a fraction, write the percentage, then change it to a fraction with a denominator of 100.

2 To convert 67% to a decimal, think of it as 67.0%, then divide it by 100 by moving the decimal point two places to the left.

b 1 To convert 55% to a fraction, write the percentage, then change it to a fraction by adding a denominator of 100.

2 This is not in simplest form, so cancel by dividing the numerator and the denominator by 5.

3 To convert 55% to a decimal, think of it as 55.0%, then divide it by 100 by moving the decimal point two places to the left.

WRITE

$$\mathbf{a} \quad 67\% = \frac{67}{100}$$

$$67\% = 0.67$$

$$\mathbf{b} \quad 55\% = \frac{55}{100}$$

$$55\% = \frac{\cancel{55}}{\cancel{100}} = \frac{11}{20}$$

$$55\% = 0.55$$

- The easiest method of comparing percentages, fractions and decimals is to convert all of them to their decimal form and use place values to compare them.

ACTIVITY: CLASS PERCENTAGES

Equipment: paper, pen

- As a class, brainstorm ways to classify members of the class, such as hair colour and gender.
- A volunteer might draw a table on the board with column headings 'Category', 'Number' and 'Percentage'. Fill in the 'Category' column with at least *five* different categories. Try to use specific categories, such as 'blue eyes' rather than 'eye colour'.
- In small groups, count the number of people in the class who fit the categories and fill in the second column.
- Fill in the third column by calculating the percentages for each category. Discuss the results as a class and resolve any inconsistencies.

WORKED EXAMPLE 27

Place the following quantities in ascending order, and then place them on a number line.

45%, $\frac{7}{10}$, 0.36, 80%, $2\frac{1}{2}$, 110%, 1.54

THINK

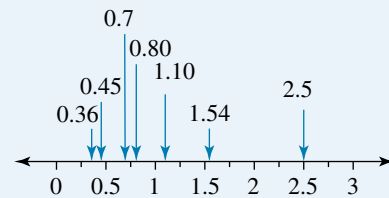
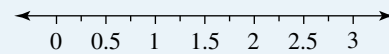
- 1 Convert all of the quantities into their decimal equivalents.
- 2 Place them in ascending order.
- 3 Place them in ascending order in their original form.
- 4 Draw a number line from 0 to 3, with increments of 0.25.
- 5 Place the numbers on the number line.

WRITE/DRAW

0.45, 0.7, 0.36, 0.80, 2.5, 1.10, 1.54

0.36, 0.45, 0.7, 0.80, 1.10, 1.54, 2.5

0.36, 45%, $\frac{7}{10}$, 80%, 110%, 1.54, $2\frac{1}{2}$



DISCUSSION

What strategies could be used to memorise the percentage, fraction and decimal forms of $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$ and $\frac{1}{8}$?

3.8.2 Percentage increases and decreases

- Percentage increases and decreases can be used to calculate and compare prices, markups, discounts, population changes, company profits and many other quantities.

WORKED EXAMPLE 28

Calculate the percentage increase when 52 increases to 72.

THINK

- 1 The difference between 52 and 72 is 20.
- 2 The percentage increase can be calculated by creating the fraction 20 out of 52 and then multiplying by 100.
- 3 Write the answer.

WRITE

$$72 - 52 = 20$$

$$\frac{20}{52} \times 100 = 38.46$$

The percentage increase is 38.46%.

WORKED EXAMPLE 29

Calculate the percentage decrease when the population of a town falls from 62 000 people to 48 000 people.

THINK

- 1 The difference between 62 000 and 48 000 is 14 000.
- 2 The percentage decrease can be calculated by creating the fraction 14 000 out of 62 000 and then multiplying by 100.
- 3 Write the answer.

WRITE

$$62\,000 - 48\,000 = 14\,000$$

$$\frac{14\,000}{62\,000} \times 100 = 22.58$$

The percentage decrease is 22.58%.

- *Note:* The percentage increase or decrease can be more than 100%; for example, the increase from 3 to 7.5 is an increase of 150%.

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RESOURCES – ONLINE ONLY

Interactivity: Percentages, fractions and decimals (int-3741)

Interactivity: Percentage increase and decrease (int-3742)

eLesson: Percentages, fractions and decimals (eles-1868)

Digital doc: SkillsHEET Rounding money to the nearest 5 cents (doc-6897)

Digital doc: SkillsHEET Converting a percentage to a decimal fraction (doc-6898)

Exercise 3.8 Percentages, fractions and decimals

Individual pathways

PRACTISE

Questions:
1–10, 12, 14

CONSOLIDATE

Questions:
1–14

MASTER

Questions:
1–16

Individual pathway interactivity: int-4419

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Understanding and fluency

1. Express each of the following fractions as a percentage.
a. $\frac{7}{8}$ b. $\frac{3}{5}$ c. $\frac{5}{6}$ d. $2\frac{1}{3}$
2. Express each of the following decimals as a percentage.
a. 0.15 b. 0.85 c. 3.10 d. 0.024
3. **WE26** Express the following percentages as fractions in simplest form.
a. 20% b. 35% c. 61% d. 105%
4. **WE26** Express the following percentages as decimals.
a. 24% b. 13% c. 1.5% d. 250%
5. **WE27** For the following sets of numbers, write them in ascending order and then place them on a number line.
a. 1.6, 25%, $\frac{7}{8}$, 75%, 10%, $3\frac{1}{2}$, 2.4 b. $3\frac{4}{5}$, 330%, 4.5%, 150%, 3, $2\frac{1}{3}$, 2.8

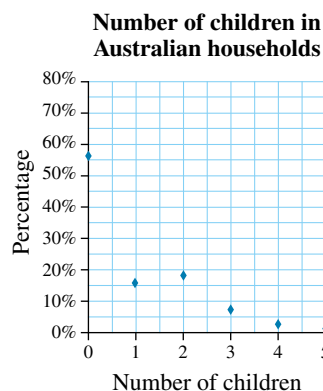
6. **WE28** Calculate the percentage increase when 250 increases to 325.
7. **WE29** Calculate the percentage decrease, correct to 2 decimal places, when the population of fish in a pond decreases from 1500 to 650.
8. Express \$120 as a percentage of \$400.
9. In a library, there are 24 children, 36 women and 42 men. What is the percentage of women visiting the library? Give your answer to 2 decimal places.
10. During a sale, a jacket originally priced at \$79.99 is decreased in price to \$55.99. What is the percentage decrease?

Communicating, reasoning and problem solving

11. A group of students was practising basketball free throws. Each student had four shots and the results are displayed in the table.

Free throw results	Number of students	Percentage of students
No shots in	3	
One shot in, three misses	11	
Two shots in, two misses	10	
Three shots in, one miss	4	
All shots in	2	

- a. How many students participated in the game?
 - b. Complete the table to show the percentage of students for each result.
 - c. How many students made exactly 25% of their shots?
 - d. What percentage of students made less than 50% of their shots?
12. The graph at right shows the percentage of households with 0 to 5 children.
 - a. What percentage of these households have six or more children?
 - b. What percentage of these households have fewer than two children?
 - c. What fraction of these households have no children?
 - d. What fraction of these households have between one and three children?
 13. Use the bunch of flowers shown to answer these questions.
 - a. What percentage of the flowers are yellow?
 - b. What fraction of the flowers are pink?
 - c. Write two of your own questions and swap with a classmate.
 14. Survey your classmates on the brand of mobile phone that they have. Present your results in a table showing the percentage, fraction and decimal amount of each brand.
 15. The Australian government capped the level of ethanol in petrol at 10%, because petrol with 20% or more ethanol may cause engine problems in some older vehicles.
 - a. Explain what is meant by the expression 'capped at 10%'.
 - b. What is the highest fraction of ethanol allowed in Australian petrol?
 - c. Above what fraction of ethanol can car engines experience problems?
 16. Where might it be necessary in daily life to convert between percentages and fractions or decimals?



3.9 Finding percentages of an amount

3.9.1 Percentages of an amount

- Sally wants to buy a book but is not sure whether she has enough money. The original price was \$35 but the sale notice says that all books have been reduced by 20%.
- If you were the sales assistant, could you help her solve the problem?
- To find a percentage of an amount (for example 20% of \$35):
 - write the percentage as a fraction with a denominator of 100
 - change the 'of' to a $\times$
 - write the amount over 1 if it is not already a fraction
 - cancel
 - multiply the numerators and multiply the denominators
 - divide the numerator by the denominator.



WORKED EXAMPLE 30

Find 20% of 35.

THINK

- Write the question.
- Write the percentage as a fraction with a denominator of 100, change the 'of' to a $\times$, write the amount as a fraction over 1 and cancel.
- Cancel again.
- Multiply numerators and multiply denominators.
- Simplify by dividing the numerator by the denominator.

WRITE

20% of 35

$$\begin{aligned} &= \frac{20}{100} \times \frac{35}{1} \\ &= \frac{2\cancel{0}^1}{\cancel{10}^1} \times \frac{7}{1} \\ &= \frac{7}{1} \\ &= 7 \end{aligned}$$

- If Sally has \$30, does she have enough money to buy the book? The price has been reduced by \$7. The sale price is \$28 so she will have enough.

WORKED EXAMPLE 31

Find 46% of 75 and write the answer as a mixed numeral.

THINK

- Write the question.
- Write the percentage as a fraction, change the 'of' to a $\times$, write the number as a fraction over 1 and cancel.
- Cancel again.
- Multiply numerators and multiply denominators.
- Write the answer as a mixed numeral by dividing the denominator into the numerator.

WRITE

46% of 75

$$\begin{aligned} &= \frac{46}{100} \times \frac{75}{1} \\ &= \frac{4\cancel{6}^2}{\cancel{10}^2} \times \frac{3}{1} \\ &= \frac{69}{2} \\ &= 34\frac{1}{2} \end{aligned}$$

- To find a percentage of an amount using decimals:
 1. write the percentage as a decimal
 2. change the 'of' to $\times$ (multiplication)
 3. multiply.

WORKED EXAMPLE 32

Of the 250 students selected at random to complete a survey, 16% were in Year 11. How many of the students were in Year 11?

THINK

- 1 Decide what percentage of the total is required and write an expression to find the percentage of the total.
- 2 Write the percentage as a decimal. Change the 'of' to a ' $\times$ '.
- 3 Multiply.
- 4 Answer the question by writing a sentence.

WRITE

16% of 250

$$= 0.16 \times 250$$

$$= 40$$

40 of the 250 students were in Year 11.

ACTIVITY: NUTRITIONAL INFORMATION

Look at the nutritional information panels on a variety of different foods. See the recommended percentage daily values for fat, carbohydrates and energy, and prepare a one-day diet that ensures that you do not exceed the recommended daily values for any of these.

3.9.2 Increasing or decreasing a quantity by $x\%$

- To increase a quantity by $x\%$, multiply it by $(100 + x)\%$.
- To decrease a quantity by $x\%$, multiply it by $(100 - x)\%$.

WORKED EXAMPLE 33

a A newborn baby weighed 3.5 kg. After 1 month the baby's weight had increased by 20%. Calculate the weight of the baby after 1 month.

b Carlos went for a run on Tuesday evening and ran for 10.2 km. When he next went for a run on Thursday evening, he ran 15% less than he did on Tuesday. How far did he run on Thursday?

THINK

- 1 Add the percentage increase to 100%.
- 2 Express the percentage as a fraction and multiply by the amount to be increased.
- 3 Write the answer.
- 1 Subtract the percentage decrease from 100%.

WRITE

$$\mathbf{a} \quad 100\% + 20\% = 120\%$$

$$\begin{aligned} \frac{120}{100} \times 3.5 &= \frac{120^{12}}{100^{20}} \times \frac{35^7}{10^1} \\ &= \frac{12^3}{20^5} \times 7 \\ &= \frac{21}{5} \\ &= 4.2 \text{ kg} \end{aligned}$$

The weight of the baby after 1 month is 4.2 kg.

$$\mathbf{b} \quad 100\% - 15\% = 85\%$$

- 2 Express the percentage as a fraction and multiply by the amount to be increased.

$$\frac{85}{100} \times 10.2 = \frac{85^{17}}{100^{50}} \times \frac{102^{51}}{10^2}$$

$$= \frac{867}{100}$$

$$= 8.67 \text{ km}$$

- 3 Write the answer.

Carlos ran 8.67 km on Tuesday.

learnON RESOURCES — ONLINE ONLY

Interactivity: Percent of an amount (int-3743)

eLesson: Percentages of an amount (eles-1882)

Digital doc: WorkSHEET Applications of percentages I (doc-6912)

Exercise 3.9 Finding percentages of an amount

Individual pathways

PRACTISE

Questions:

1–10, 18, 22, 29

CONSOLIDATE

Questions:

1a–e, 2a–l, 3a–i, 4, 5a–g, 6a–l,
7–18, 22–24, 29–31

MASTER

Questions:

1e–j, 2j–u, 3g–o, 4, 5i–l, 6i–r, 7–31

Individual pathway interactivity: int-4420

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Understanding and fluency

1. Copy each of the following problems, then find the answers by completing the working.

a. $90\% \text{ of } 200 = \frac{90}{100} \times \frac{200}{1} =$

b. $8\% \text{ of } 50 = \frac{8}{100} \times \frac{50}{1} =$

c. $50\% \text{ of } 120 = \frac{50}{100} \times \frac{120}{1} =$

d. $20\% \text{ of } 90 = \frac{20}{100} \times \frac{90}{1} =$

e. $30\% \text{ of } 150 = \frac{30}{100} \times \frac{150}{1} =$

f. $75\% \text{ of } 16 = \frac{75}{100} \times \frac{16}{1} =$

g. $5\% \text{ of } 30 = \frac{5}{100} \times \frac{30}{1} =$

h. $80\% \text{ of } 55 = \frac{80}{100} \times \frac{55}{1} =$

i. $15\% \text{ of } 70 = \frac{15}{100} \times \frac{70}{1} =$

j. $65\% \text{ of } 120 = \frac{65}{100} \times \frac{120}{1} =$

2. **WE30** Find the following.

a. 50% of 20

b. 20% of 80

c. 5% of 60

d. 10% of 30

e. 9% of 200

f. 31% of 300

g. 40% of 15

h. 12% of 50

i. 35% of 80

j. 70% of 110

k. 52% of 75

l. 90% of 70

m. 80% of 5000

n. 44% of 150

o. 68% of 25

p. 24% of 175

q. 38% of 250

r. 95% of 200

s. 110% of 50

t. 150% of 8

u. 125% of 20

3. **WE31** Find the following and write the answer as a mixed numeral.

a. 18% of 20

b. 16% of 30

c. 11% of 70

d. 8% of 120

e. 74% of 25

f. 66% of 20

g. 2% of 95

h. 55% of 45

i. 15% of 74

j. 32% of 220

k. 95% of 62

l. 32% of 65

m. 18% of 80

n. 82% of 120

o. 27% of 60

4. **MC** a. 45% written as a fraction is:
- A. $\frac{45}{100}$ B. $\frac{45}{1}$ C. $\frac{450}{1}$ D. $\frac{1}{45}$
- b. When finding 17% of 22, the 'of' will be changed to:
- A. $\div$ B. from C. $+$ D. $\times$
- c. Which of the following would find 15% of 33?
- A. 15 of 33 B. $\frac{15}{1} \times 33$ C. $\frac{15}{100} \times \frac{33}{100}$ D. $\frac{15}{100} \times \frac{33}{1}$
- d. 60% of 30 is:
- A. $19\frac{4}{5}$ B. $\frac{31}{5}$ C. 186 D. 18
5. For each of the following, express the percentage as a decimal first and then solve, remembering to round your answer to the nearest 5c.
- a. 15% of \$12.00 b. 15% of \$8.00 c. 15% of \$20.00 d. 15% of \$60.00
e. 25% of \$30.00 f. 25% of \$45.00 g. 25% of \$90.00 h. 25% of \$220.00
i. 30% of \$15.00 j. 30% of \$25.00 k. 30% of \$47.50 l. 30% of \$102.20
6. Find 1% of the following. Round the answers to the nearest 5c.
- a. \$268 b. \$713 c. \$573 d. \$604 e. \$5.60 f. \$12
g. \$13 h. \$14.80 i. \$21.70 j. \$81.75 k. \$19.89 l. \$429.50
m. \$4.25 n. \$6.49 o. \$9.99 p. \$0.24 q. \$0.77 r. \$1264.37
7. Use a calculator to find the following. Round the answers to the nearest 5c.
- a. 12% of \$11 b. 21% of \$50 c. 11% of \$30 d. 3% of \$22
e. 6% of \$40 f. 22% of \$10 g. 13% of \$14 h. 35% of \$210
i. 12% of \$150 j. 9% of \$17 k. 2% of \$53 l. 7% of \$29
m. 45% of \$71.50 n. 33% of \$14.50 o. 42% of \$3.80 p. 31% of \$1.45
q. 64% of \$22.50 r. 41% of \$1200
8. **MC** a. 10% of \$7.25 equals:
- A. \$725 B. \$7.30 C. \$72.50 D. \$0.73 E. \$7250
- b. 1% of \$31.48 equals:
- A. \$3.14 B. \$0.31 C. \$0.32 D. \$31.50 E. \$0.03
- c. 15% of \$124 equals:
- A. \$18.60 B. \$1.24 C. \$6.20 D. \$13.64 E. \$15.24
- d. 22% of \$5050 equals:
- A. \$60.60 B. \$50.50 C. \$1111 D. \$43.56 E. \$1010
9. **WE32** Thirty per cent of residents in the shire of Booroondara are over the age of 65. If there are 180000 residents, how many are over the age of 65?
10. **WE33** a. The grocery bill for Mika's shopping was \$250. The following week, Mika spent 7% more on his groceries. How much did he spend in the following week?
- b. A long-distance runner completed a 15-mile run in 120 minutes. The next time she ran 15 miles she reduced her time by 5%. How fast did she complete the 15 miles on the second occasion?
11. Maria is buying a new set of golf clubs. The clubs are marked at \$950, but if Maria pays cash, the shop will take 10% off the marked price. How much will the clubs cost if Maria pays cash?
12. a. When you multiply a quantity by 1.16, by what percentage are you increasing the quantity by?
- b. When you multiply a quantity by 0.77, by what percentage are you decreasing the quantity by?
13. Increase the following quantities by the given percentages without using a calculator, and then check your answers with a calculator.
- a. 33 kg by 10% b. 50 lb by 20% c. 83 cm by 100%



14. Decrease the following quantities by the given percentages without using a calculator, and then check your answers with a calculator.
 - a. 25 kg by 10%
 - b. 40 km by 20%
 - c. \$96 by 90%
15. Jay is buying a new lounge suite worth \$2150. Jay has to leave a 15% deposit and then pay the balance in monthly instalments. How much deposit does Jay have to pay?
16. Ninety per cent of students at a school were present for school photographs. If the school has 1100 students, how many were absent on the day the photographs were taken?
17. Jim can swim 50 m in 31 seconds. If he improves his time by 10%, what will Jim's time for 50 m be?
18. In a survey, 40 people were asked if they liked or disliked Vegemite. Of the people surveyed, 5% said they disliked Vegemite. How many people:
 - a. disliked Vegemite?
 - b. liked Vegemite?
19. Thirty-two thousand four hundred people went to the SCG to watch a Sydney versus Collingwood football match. Of the crowd, 42% went to the game by car and 55% caught public transport. How many people:
 - a. arrived by car
 - b. caught public transport?
20. Explain how to use the shortcut method (that is, without using a calculator) to leave a $17\frac{1}{2}\%$ tip for a bill of \$76.
21. If a train fare of \$12 was increased by 10%, how much will you pay for a return trip? (Assume that the fare is the same each way.)



Communicating, reasoning and problem solving

22. When I am 5% older than I am now I will be 21 years old. How old am I now?
23. The price of bread has increased to 250% of its price 20 years ago. If a loaf of bread costs \$2.00 now, how much would it have cost 20 years ago? Show your working.
24. I am six months old. If I gain 10% of my current mass I will be three times my birth mass. If my birth mass was 3 kg, what is my mass now? Round your answer to one decimal place. Show your working.
25. I am 33 years old. I have lived in England for 8 years. If I stay in England, how old will I be when the number of years I have lived there is 50% of my age? Show your working.
26. My mother is four times older than I am. My sister is 75% of my age, and 10% of my grandfather's age. My father is 50, 2 years older than my mother. How old are my sister and grandfather? Justify your answer.
27. On average, 38% of the energy that is used is in Australian households in on heating and cooling.
 - a. If a household consumed 19.8kWh of energy in a day, how much would they have used on heating and cooling if they used an average amount? Give your answer correct to 2 decimal places.
 - b. If they cut their heating and cooling use by 25%, how much energy would they now consume over the day? Give your answer correct to 2 decimal places.
28. Broadcasting regulations specify that 55% of television programs shown between 6 pm and midnight must be Australian and that, between 6 pm and midnight, there should be no more than 13 minutes per hour of advertising.
 - a. How many minutes of advertising are allowed between 6 pm and midnight?
 - b. For how many minutes are programs screened between 6 pm and midnight?
 - c. What is the maximum percentage time spent screening advertising?
 - d. How many minutes of Australian content must be screened between 6 pm and midnight?
29. In a Maths competition, the top 8% of students across the state achieve a score of 40 or more out of a possible 50.
 - a. In a school where 175 students are entered in the Maths competition, how many scores of more than 40 would you expect?
 - b. In one school, there were 17 scores of 40 or more, and 204 scores that were less than 40. Did the students perform better than state average?

30. I am 27 years old and have lived in Australia for 12 years. If I continue to live in Australia, how old will I be when the number of years I have lived here is 75% of my age?
31. In what situations would it be useful to use the shortcut methods for common percentages?

3.10 Review

Investigation | Rich task

A growing nation

Details about a population are collected in a census. Australia's most recent census took place in August 2016, on a day known as 'census day'. The information provided by the population is collected and then analysed by the Australian Bureau of Statistics over a period of 2 years. After this, the information is released to the public.

The following table displays some selected characteristics for Australia based on the information collected in the 2011 census.

Characteristics	Total persons
Males in Australia	10 634 013
Females in Australia	10 873 704
People aged 15 years and over	17 363 694
People aged 65 years and over	3 012 289
People born in Australia	15 017 847
People who speak English only	16 509 291



1. What was the total population of Australia on census day 2011?
2. According to the values in the table, how many people were not born in Australia?
3. Express the male population as a proportion of the entire population. Give your answer as a decimal. (Divide the male population by the total population.) Repeat this for the female population.
4. How many people were aged 14 years and younger?

The table below shows the same characteristics from the first table, but it relates to Victoria only.

Characteristics	Total persons
Males in Victoria	2 632 619
Females in Victoria	2 721 423
People aged 15 years and over	4 355 249
People aged 65 years and over	761 580
People born in Australia	3 670 933
People who speak English only	3 874 861



5. What was the population of Victoria on census day 2011?
6. Express both the male and female population of Victoria as a proportion of the total population of the state.
7. How does the proportion of males and females in Victoria compare with the proportion of males and females in the entire Australian population?
8. On a separate page, compare the other characteristics in the table with those for the entire population.

Australia's population has increased since Federation in 1901. The table below shows some of the information collected in the 1901 census.

Characteristics	Total persons
Males in Australia	1977928
Females in Australia	1795873
People living in Victoria	1201070
People born in Australia	2908303

9. How much did Australia's population grow in the 105 years since 1901?
10. Compare the proportion of people born in Australia at the 1901 census with the 2011 census.
11. The Australian Bureau of Statistics (ABS) website (www.abs.gov.au) can be used to investigate other characteristics from the 2011 census. Use the ABS website to investigate another characteristic from the 2011 census. State your findings and include all working to justify your conclusions.



learnON RESOURCES — ONLINE ONLY

Digital doc: Investigation — A growing nation (doc-13933)

Digital doc: Code puzzle — Funny definitions (doc-13934)

Digital doc: Summary Topic 3 (doc-23239)

Interactivity: Word search Topic 3 (int-2752)

Interactivity: Crossword Topic 3 (int-2753)

Interactivity: Sudoku Topic 3 (int-3184)

Exercise 3.10 Review questions

To answer questions online and to receive **immediate feedback** and **fully worked solutions** for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Understanding and fluency

1. Simplify the following.

a. $\frac{2}{3} + \frac{6}{7}$

b. $\frac{3}{5} + 4\frac{1}{2}$

c. $2\frac{3}{4} - 1\frac{1}{8}$

d. $\frac{5}{6} + \frac{3}{12} + \frac{4}{15}$

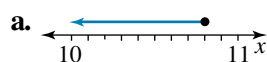
e. $\frac{127}{64} - \frac{5}{8} + 2\frac{3}{4}$

f. $2\frac{1}{2} + 3\frac{1}{2} - 1\frac{3}{5}$

2. Write the next 3 numbers of the sequence.

-3.6, -2.4, -1.2, _____, _____, _____

3. Describe the directed numbers graphed on each number line.



4. Graph the directed numbers $x > -2\frac{3}{4}$ on a number line.

5. Calculate the following.

a. $-1\frac{19}{60} + \frac{1}{4}$

b. $-\frac{3}{5} - \frac{7}{10}$

6. Simplify the following.

a. $\frac{2}{5} \times \frac{7}{8}$

b. $\frac{3}{4} \div \frac{7}{8}$

c. $\frac{22}{6} \times \frac{8}{11}$

d. $4\frac{1}{3} \times 9\frac{1}{2}$

e. $7\frac{1}{5} \div \frac{8}{20}$

f. $\frac{9}{4} \div 8\frac{1}{2}$

g. $-\frac{7}{8} \times \frac{5}{14}$

h. $-2\frac{3}{4} \div -\frac{3}{8}$

7. Evaluate the following.

a. $2.4 + 3.7$

b. $11.62 - 4.89$

c. $12.04 + 2.9$

d. $5.63 - 0.07$

e. $34.2 - 4.008$

f. $34.09 + 1.2 + 3479.3 + 0.0003$

g. $-2.48 + 1.903$

h. $-1.63 - 2.54$

8. Evaluate the following, correct to 2 decimal places where appropriate.

a. 432.9×2

b. 78.02×3.4

c. $543.7 \div 0.12$

d. $9.65 \div 1.1$

e. 923.06×0.00045

f. $74.23 \div 0.0007$

g. $0.08 \div -0.4$

h. $-1.02 \div -0.5$

9. For each of the following numbers:

i. round to the first digit

ii. round up to the first digit

iii. round down to the first digit.

a. 39260

b. 222

c. 3001

10. Provide estimates for each of the following by first rounding the dividend to a multiple of the divisor.

a. $809 \div 11$

b. $7143 \div 9$

c. $13\,216 \div 12$

11. The answer to $\frac{99 \times 1560}{132 \times 312}$ contains the digits 375, in that order. Use an estimating technique to determine the position of the decimal point and write the true answer.

12. Use your estimation skills to find approximate answers for the following.

a. 306×12

b. $268 + 3075 + 28 + 98\,031$

c. $4109 \div 21$

d. $19\,328 - 4811$

13. Find 10% of each of the following. Round your answers to the nearest 5 cents.

a. \$63.00

b. \$42.00

c. \$105.00

d. \$216

e. \$3.45

f. \$42.68

g. \$118.55

h. \$2125.85

14. Find 5% of the following by finding 10% and halving your answer. Round your answers to the nearest 5 cents.

a. \$8.00

b. \$21.00

c. \$64.00

d. \$104.00

e. \$35.00

f. \$52.00

g. \$205.50

h. \$77.30

15. Calculate the following using 'shortcuts'. Round your answer to the nearest 5 cents.

a. 1% of \$16.00

b. 1% of \$28.00

c. 12% of \$42.00

d. 30% of \$90.00

e. 22% of \$220.00

f. 43% of \$27

g. 15% of \$19.50

h. 8% of \$37

16. Mentally calculate 12% of \$15.

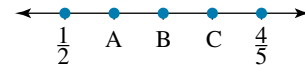
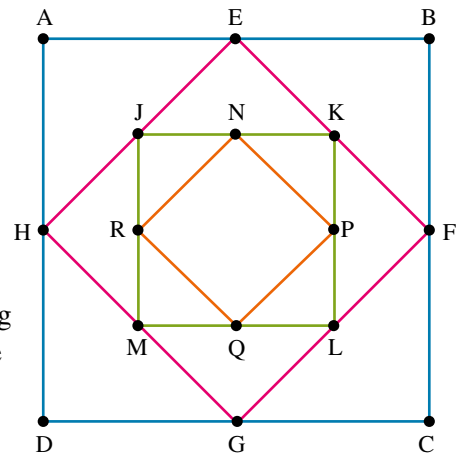
17. Use a shortcut to calculate 1.5% of \$20.

18. Use a shortcut to calculate 5.5% of \$50.

19. Use a shortcut to calculate 10.1% of \$18, rounding the answer to the nearest 5c.

Communicating, reasoning and problem solving

- 20.** In order to raise money for charity, a Year 8 class organised a cake stall. They started with 9 whole cakes, sold $2\frac{1}{4}$ cakes at recess and $4\frac{5}{8}$ at lunchtime.
- How much cake was left over?
 - If there are 20 students in the class, would they be able to equally share the leftover cake if it had been cut up into eighths?
 - This cake stall raised \$150. If they plan to give $\frac{1}{5}$ to the Red Cross and $\frac{2}{3}$ to World Vision, how much will each group receive and how much is left over?
- 21.** A number is rounded to give 2.15.
- What could the number have been?
 - What is the smallest the number could have been?
 - Is it possible to write down the largest number that can be rounded to give 2.15?
- 22.** Find the fraction that is equivalent to $\frac{3}{4}$ and:
- the difference between the denominator and the numerator is 7
 - the product of the denominator and numerator is 300.
- 23.** At Teagan's Farm, there are 24 horses. One sixth of them are brown and one quarter of them are black. If half of the remaining horses are red and the other half grey, how many grey horses are there?
- 24.** On the diagram above right, EFGH is half the area of ABCD, JKLM is half the area of EFGH and NPQR is half the area of JKLM. What fraction of ABCD is NPQR?
- 25.** If a shend is 15.36 plombs (pl) long and 1 plomb is equivalent to 6.309 sharns (sh), find the length of 2.25 shends in sharns.
- 26.** The five points shown on the number line are evenly spaced.
- What is the value for B?
- 27.** You and a friend have decided to test the combinations of cordial and water shown in the following table. The mixture with the highest cordial content will be the one you and your friend will give to 180 Year 8 students.
- Which mix will be used?
 - If each student will drink $\frac{1}{2}$ of a cup each, how much will you need to make?
 - How much water and how much cordial will be needed?



Mix A	Mix B
2 cups cordial 3 cups water	1 cup cordial 2 cups water
Mix C	Mix D
5 cups cordial 9 cups water	3 cups cordial 5 cups water

- 28.** The skin of a banana weighs about $\frac{1}{8}$ the mass of the whole banana. The cost of a bundle of bananas was \$5.12. They were peeled and the fruit was found to weigh 1.4kg. What was the price/kg of the whole bananas, including fruit and skin?
Use mathematical reasoning to justify your answer.
- 29.** James took a loan from a bank to pay a debt of \$14200. He pays \$670.50 per month for 2 years. How much interest does he pay in total?
- 30.** Shannon buys a used car for \$7500. The dealer requires a 15% deposit. Calculate the deposit Shannon has to pay.
- 31.** Ann has \$100 on 30 March. This increased by 10% on 30 April. The total amount increased by 10% again on 30 May.
a. How much did Ann have on 30 May?
b. Compare your answer with a 20% increase on \$100. Do they get the same answer?
- 32.** Magic squares show a grid of numbers which have the same sum horizontally, vertically and diagonally. It is not necessary for the numbers to be integers. Complete this magic square, indicating the magic sum.

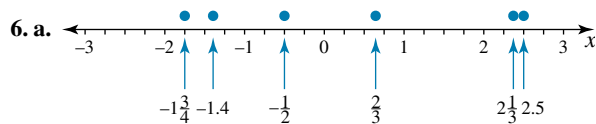
1.7×0.2			$(0.4)^2$	$\frac{1}{10} + \frac{1}{5}$
$\frac{19}{25} - \frac{3}{10}$	$0.01 \div 0.1$		$0.14 \div 0.5$	$\frac{2^3}{5^2}$
	$\frac{8}{25} - \frac{1}{5}$	$\frac{1}{100} + \frac{1}{4}$		$\frac{33}{100} \div \frac{3}{4}$
	0.4×0.6	$\frac{39}{50} - \frac{2}{5}$	0.6×0.7	
$\frac{11}{100} \div \frac{1}{2}$	$\left(\frac{3}{5}\right)^2$			$\frac{17}{25} - \frac{1}{2}$

Answers

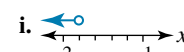
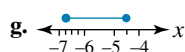
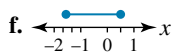
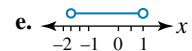
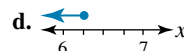
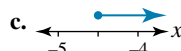
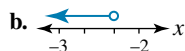
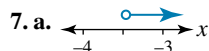
Topic 3 Fractions, decimals and percentages

Exercise 3.2 Addition and subtraction of fractions

1. a. $\frac{9}{17}$ b. $\frac{5}{27}$ c. $\frac{12}{17}$ d. $\frac{4}{5}$
2. a. $1\frac{2}{5}$ b. $1\frac{1}{4}$ c. 2 d. $\frac{23}{50}$
3. a. $\frac{13}{20}$ b. $1\frac{3}{8}$ c. $\frac{1}{5}$ d. $\frac{18}{25}$ e. $1\frac{7}{12}$
- f. $\frac{43}{70}$ g. $\frac{37}{42}$ h. $2\frac{13}{30}$
4. a. $1\frac{3}{5}$ b. $3\frac{2}{9}$ c. $4\frac{3}{5}$ d. $4\frac{2}{3}$ e. $2\frac{1}{2}$
- f. 7 g. $9\frac{1}{2}$ h. $21\frac{8}{45}$ i. $2\frac{3}{4}$ j. $2\frac{7}{15}$
5. a. $3\frac{3}{10}$ b. $2\frac{2}{3}$ c. $4\frac{17}{20}$ d. $1\frac{21}{40}$



b. $-1\frac{3}{4}$, -1.4 , $-\frac{1}{2}$, $\frac{2}{3}$, $2\frac{1}{3}$, 2.5



8. a. $x > -1\frac{1}{2}$ b. $x \leq 6\frac{3}{4}$ c. $-7\frac{1}{2} < x \leq -6\frac{1}{3}$ d. $x < \frac{2}{3}$ e. $x \geq -4\frac{2}{5}$
- f. $10\frac{3}{4} \leq x < 11\frac{3}{4}$ g. $x > -8.7$ h. $-2\frac{1}{2} < x < \frac{2}{3}$
9. a. $-4\frac{1}{2}$, -3 , 0 , 5 , 8 , 6 b. -4.2 , -1 , $-\frac{3}{4}$, $1\frac{1}{2}$, $3\frac{2}{3}$ c. -8.6 , -6.8 , $-\frac{2}{3}$, 0 , $\frac{1}{4}$
10. a. $-\frac{2}{5}$ b. $-\frac{8}{8}$ or -1 c. $-\frac{1}{4}$ d. $-\frac{5}{6}$ e. $\frac{1}{8}$
- f. $-\frac{5}{6}$ g. $-\frac{1}{10}$ h. $\frac{5}{12}$ i. $4\frac{1}{4}$ j. $-\frac{14}{15}$
- k. $-5\frac{5}{6}$ l. $-2\frac{4}{15}$
11. a. $-\frac{1}{4}$ b. $-\frac{5}{6}$ c. $-\frac{11}{15}$
- d. $-\frac{7}{20}$ e. $-\frac{3}{4}$ f. $-2\frac{4}{15}$

12. All of it

13. $1\frac{7}{9}$ bottles were left over.

14. He has $\frac{4}{15}$ of his pay left.

15. $\frac{5}{12}$ of the class get a lift.

16. $\frac{11}{8}$ or $1\frac{3}{8}$ of the cake was left.

17. Check with your teacher.

18. $1 \text{ cup} + \frac{1}{2} \text{ cup} + \frac{1}{4} \text{ cup} + \text{two } \frac{1}{3} \text{ cups}$

19. $\frac{1}{2} + \frac{3}{4} + 1\frac{1}{4} + \frac{2}{3} + \frac{2}{25} = 3\frac{37}{150}$

The tray holds 3 cups of ingredients, which means the mixture will not fit.

20. Answers will vary.

21. When adding or subtracting mixed numbers, it is better to add or subtract whole number first and then the fraction parts.

Exercise 3.3 Multiplication and division of fractions

1. a. $\frac{3}{8}$ b. $\frac{1}{56}$ c. $\frac{6}{25}$ d. $\frac{5}{21}$ e. $\frac{5}{12}$ f. $\frac{1}{3}$ g. $\frac{11}{30}$ h. $\frac{1}{5}$ i. $\frac{11}{32}$ j. $\frac{3}{5}$
 k. $\frac{4}{5}$ l. $\frac{1}{4}$
2. a. $5\frac{3}{5}$ b. $1\frac{11}{25}$ c. 4 d. $8\frac{3}{4}$ e. $6\frac{23}{100}$ f. $13\frac{4}{5}$ g. 13 h. 1 i. $11\frac{7}{8}$
3. a. $\frac{2}{3}$ b. $\frac{7}{12}$ c. $\frac{6}{7}$ d. $1\frac{3}{5}$ e. $\frac{6}{7}$ f. $\frac{15}{16}$ g. $\frac{3}{5}$ h. $\frac{6}{25}$ i. 2
4. a. 1 b. $1\frac{5}{7}$ c. $\frac{7}{12}$ d. $5\frac{1}{7}$ e. $1\frac{3}{5}$ f. $\frac{14}{25}$ g. $2\frac{3}{16}$ h. $4\frac{8}{25}$ i. $1\frac{7}{135}$
5. a. $-\frac{1}{6}$ b. $\frac{3}{20}$ c. $-4\frac{2}{3}$ d. $-\frac{1}{4}$ e. $-\frac{5}{8}$ f. $-\frac{1}{4}$ g. $-1\frac{5}{9}$ h. $2\frac{3}{4}$ i. -2
6. a. $-\frac{2}{5}$ b. $-\frac{8}{9}$ c. $\frac{7}{8}$ d. $-\frac{3}{8}$ e. $-\frac{1}{6}$ f. $-\frac{3}{10}$ g. $4\frac{1}{2}$ h. $-2\frac{2}{5}$ i. $-\frac{8}{35}$
7. 12
8. a. $\frac{1}{5}$ b. 15 lollies each
9. a. $-\frac{11}{15}$ b. $-2\frac{3}{16}$ c. 0 d. $1\frac{11}{21}$ e. $-8\frac{5}{6}$ f. $1\frac{4}{17}$
10. 120 non-USA caps
11. $\frac{1}{16}$ of the cake each
12. \$30 to the children's charity, \$80 to the charity for the prevention of cruelty to animals, \$10 left over
13. a. $69\frac{1}{3}$ km/h b. $74\frac{2}{3}$ km/h c. $873\frac{3}{5}$ km/h
14. To modify the recipe so that it feeds only you, divide all the ingredient amounts by 5. This will create a portion that is $\frac{1}{5}$ of the original size.
15. a. Answers will vary. b. $a = 4$, $b = 3$, $c = 2$
16. a. It is not possible to evenly divide 17 by $\frac{1}{2}$, $\frac{1}{3}$ or $\frac{1}{9}$. The eldest child meant that it was not possible to have a portion of a camel.
 b. $\frac{17}{18}$. Half of the 18 camels = 9; one-third of the 18 camels = 6; one-ninth of the 18 camels = 2; $9 + 6 + 2 = 17$. The numerator, 17, is the number of camels that the merchant had; the denominator, 18, is the total number of camels after the wise man lent one camel.
17. Yes the rules operation order apply to fractions. However, there are invisible brackets around all numerators, so these operations need to be performed first.
18. Fraction is $\frac{3}{8}$; prime number is 83.

Exercise 3.4 Terminating and recurring decimals

1. a. 0.8 b. 0.25 c. 0.75 d. $0.41\dot{6}$ e. $0.8\dot{1}$
 f. 0.84 g. 1.75 h. $2.1\dot{6}$ i. $0.4\dot{6}$ j. $0.\dot{6}$
2. a. $1.8\dot{3}$ b. 1.75 c. 3.4 d. 8.8 e. 12.9
 f. 6.75 g. $5.\dot{6}$ h. $11.7\dot{3}$ i. 6.5 j. $4.\dot{3}$
3. a. -0.8 b. -0.78 c. -1.6 d. -0.27 e. -0.375
 f. -2.25 g. -6.6 h. -3.14 i. -1.83 j. -5.89
4. a. $\frac{2}{5}$ b. $\frac{4}{5}$ c. $1\frac{1}{5}$ d. $3\frac{1}{5}$ e. $5\frac{3}{5}$ f. $\frac{3}{4}$
 g. $1\frac{3}{10}$ h. $7\frac{7}{50}$ i. $4\frac{21}{100}$ j. $10\frac{1}{25}$ k. $1\frac{333}{1000}$ l. $8\frac{1}{20}$
 m. $7\frac{39}{125}$ n. $9\frac{47}{50}$ o. $12\frac{9}{200}$ p. $84\frac{63}{500}$ q. $73\frac{9}{10}$ r. $\frac{21}{5000}$

5. 0.75
6. 1.1
7. 0.56
8. $\frac{2}{5}$
9. Check with your teacher.
10. $\frac{6}{10}, \frac{5}{8}, \frac{2}{3}, \frac{3}{4}, \frac{6}{7}$
11. a. $\frac{7}{9}$ b. $\frac{34}{111}$ c. $2\frac{2}{9}$
12. a. No, the sum of two recurring decimals may be an exact number; for example, $\frac{1}{3} + \frac{2}{3} = \frac{3}{3} = 1$.
- b. Yes, the recurring part is maintained; for example, $\frac{1}{3} + \frac{1}{2} = \frac{5}{6} = 0.8\bar{3}$.
- c. No, it could result in zero; for example, $\frac{1}{3} - \frac{1}{3} = 0$.
- d. Yes, the recurring part is maintained.
- e. Yes, the recurring part is maintained.
13. a. $\frac{1}{30}$ b. 450 gold doubloons c. 1.5pd
14. a. $\frac{1}{72}$ b. $\frac{144r}{5}$ g
15. Bucket 1: 12 L, bucket 2: 3 L, bucket 3: 3 L
16. Discuss with your teacher.

Exercise 3.5 Addition and subtraction of decimals

1. a. 12.9 b. 13 c. 19.68 d. 20.3 e. 17.26 f. 8.14
- g. 132.44 h. 42.719 i. 6.239 j. 20.672 k. 59.434 l. 394.132
- m. 126.157 n. 7113.556
2. a. 2.24 b. 7.32 c. 121.66 d. 54.821 e. 42.33 f. 1674.93
- g. 53.16 h. 124.966 i. 22.897 j. 26.03 k. 33.028 l. 474.104
3. a. E b. B
4. a. 115.09 b. 31.953 c. 821.48 d. 1954.291 e. 1493.875 f. 433.98
- g. 1553.589 h. 8.884 i. 598.018 j. 4224.296
5. a. -0.9 b. -0.7 c. -0.57 d. -0.991 e. 7.606 f. 0.6343
6. a. -0.7 b. -1.32 c. 3.4 d. -4.3 e. -2.55 f. -0.818
7. a. 27 b. 46 c. 21 d. 129
8. a. Salmah spent \$157.22. b. He would have \$42.78 left.
9. 41.37 seconds
10. 9.88 km 11. 763.4 km
12. a. 1.2 km b. 10.3 km c. 15.8 km in total
13. Spending decreased by \$10.5 billion.
14. a. The third decimal place is 5, and so should be rounded up. The friend made the mistake of rounding down. 23.92538 rounded to 2 decimal places is 23.93.
- b. \$2.03 should be rounded to the nearest 5 cents, but the friend rounded to the nearest 10 cents. \$2.03 rounded to the nearest 5 cents is \$2.05.
15. a. 0.8889 b. 1.2222 c. 1.1111 d. 0.9798 e. 0.1111
- f. 1.0889 g. 2.4133
16. Aligning decimal points when adding or subtracting decimals is to make sure numbers are evaluated correctly. When the decimals are aligned the place values of each digit in the numbers are also aligned.

Exercise 3.6 Multiplication and division of decimals

1. a. 4.96 b. 9.48 c. 210.24 d. 613.2 e. 2.036 f. 78.624
- g. 582.659 h. 1153.96 i. 2183.14 j. 40.74 k. 156.78 l. 0.2616
- m. 624.036 n. 0.34455 o. 0.14762
2. a. 6.17 b. 130.98 c. 790.14 d. 95.54 e. 6.41 f. 2.53
3. a. 88.83 b. 1920.86 c. 5355 d. 536515 e. 29933.75 f. 954087.50
- g. 6466.25 h. 89920 i. 134.60 j. 1714.36 k. 7012.80 l. 6.56

4. a. -0.06 b. -0.72 c. 0.024 d. 0.36 e. 0.09 f. -0.16
 g. -2000 h. 0.008 i. 0.0025 j. -0.294 k. -1.6 l. 0.1296
 5. a. -42 b. -0.3 c. 0.135 d. -30 e. -0.7 f. 400
 g. -0.06 h. -9000 i. 0.00005 j. -5 k. 36 l. 45
 6. a. 13.9 b. 1.5 c. 7.6 d. 2.0
 7. a. D b. B
 8. a. 133 b. 90 c. 501 d. 46
 9. $\$5.30$ 10. 10 11. 43.98 L 12. 2.015
 13. a. 5115.495 cm^3 b. 0.0293 g/cm^3
 14. a. 11.56 m^2 b. $5.8^2 - 11.56 = 22.08 \text{ m}^2$
 15. $\frac{2ct}{5g}$
 16. a. Septimus is faster, with a speed of 3.32 m/s ; Fortescue's speed is 2.92 m/s .
 b. 2226.39 hours

17. Multiplying only the whole number first can be a quick way of checking the answer makes sense prior to multiplying the decimal.

Exercise 3.7 Estimation

1. Estimate: 72. Kim is correct.
 2. a. i. 200 ii. 300 iii. 200
 b. i. 5000 ii. 5000 iii. 4000
 c. i. 20 ii. 30 iii. 20
 d. i. 50000 ii. 60000 iii. 50000
 e. i. 600 ii. 600 iii. 500
 f. i. 1000 ii. 2000 iii. 1000
 3. a. 230 b. 4520 c. 20 d. 53620 e. 590 f. 1040
 4. a. 300 b. 4600 c. 100 d. 53700 e. 600 f. 1100
 5. a. 4 b. 340 c. 520 d. 1170 e. 300 f. 24000
 g. 54000 h. 20 i. 9680 j. 10 k. 108000 l. 20
 m. 28000 n. 600 o. 4000 p. 810000
 6. a. 4000 b. 500 c. 70000 d. 12000 e. 8000 f. 400
 7. a. 20.5 b. 150.175 c. 20.25 d. 5.13 e. 54.4 f. 64.8
 8. $\$500$
 9. a. 4 and 5 b. 10 and 11 c. 13 and 14 d. 15 and 16

10.

	Question	Simplified question	Estimated answer	Exact answer
A	789×56	800×60	48000	44184
B	$124 \div 5$	$100 \div 5$	20	24.8
C	$678 + 98 + 46$	$700 + 100 + 50$	850	822
D	235×209	200×200	40000	49115
E	$7863 - 908$	$8000 - 900$	7100	6955
F	63×726	60×700	42000	45738
G	$39654 \div 227$	$40\,000 \div 200$	200	174.69
H	$1809 - 786 + 467$	$2000 - 800 + 500$	1700	1490
I	$21 \times 78 \times 234$	$20 \times 80 \times 200$	320000	383292
J	$942 \div 89$	$900 \div 90$	10	10.58
K	$\frac{492 \times 94}{38 \times 49}$	$\frac{500 \times 100}{40 \times 50}$	25	24.84
L	$\frac{54296}{97 \times 184}$	$\frac{50000}{100 \times 200}$	2.5	3.04

11. a. $\$300000$ b. 640 mL c. $\$200$
 d. 5000 seconds e. $\$900000$ f. $\$16000$
 g. 4000000 pieces of litter
 12. a. 85500 b, c. Answers will vary.

13. a. i. 900 ii. 900 b. i. 920 ii. 900
 c. i. 940 ii. 900 d. i. 960 ii. 900
 e. i. 980 ii. 900

14. When the first number is rounded, the answers increase. When the second number is rounded, the answers remain constant.

15. a. i. 880 ii. 600 b. i. 880 ii. 640
 c. i. 880 ii. 680

The error increases if the smaller number is rounded. The error decreases when the rounded number is closer to the actual number.

16. Discuss with your teacher.

Exercise 3.8 Percentages, fractions and decimals

1. a. 87.5% b. 60%
 c. 83.33% (correct to 2 decimal places) d. 233.33% (correct to 2 decimal places)
2. a. 15% b. 85% c. 310% d. 2.4%
3. a. $\frac{1}{5}$ b. $\frac{7}{20}$ c. $\frac{61}{100}$ d. $\frac{21}{20}$
4. a. 0.24 b. 0.13 c. 0.015 d. 2.50
5. a. 10%, 25%, 75%, $\frac{7}{8}$, 1.6, 2.4, $3\frac{1}{2}$ b. 150%, $2\frac{1}{3}$, 2.8, 3, 330%, $3\frac{4}{5}$, 4.5
6. 30%
7. 56.67%
8. 30%
9. 35.29%
10. 30%
11. a. 30

b.

Free throw results	Number of students	Percentage of students
No shots in	3	10%
One shot in, three misses	11	36.6%
Two shots in, two misses	10	33.3%
Three shots in, one miss	4	13.3%
All shots in	2	6.6%

- c. 11 d. 46.6%
12. a. 0% b. 72% c. $\frac{14}{25}$ d. $\frac{41}{100}$
13. a. 38% b. $\frac{8}{21}$ c. Answers will vary.
14. Answers will vary.
15. a. The expression 'capped at 10%' means that petrol can contain a maximum of 10% ethanol.
 b. $\frac{1}{10}$ c. $\frac{1}{5}$

16. Discuss with your teacher.

Exercise 3.9 Finding percentages of an amount

1. a. 180 b. 4 c. 60 d. 18 e. 45 f. 12
 g. $1\frac{1}{2}$ h. 44 i. $10\frac{1}{2}$ j. 78
2. a. 10 b. 16 c. 3 d. 3 e. 18 f. 93
 g. 6 h. 6 i. 28 j. 77 k. 39 l. 63
 m. 4000 n. 66 o. 17 p. 42 q. 95 r. 190
 s. 55 t. 12 u. 25
3. a. $3\frac{3}{5}$ b. $4\frac{4}{5}$ c. $7\frac{7}{10}$ d. $9\frac{3}{5}$ e. $18\frac{1}{2}$ f. $13\frac{1}{5}$
 g. $1\frac{9}{10}$ h. $24\frac{3}{4}$ i. $11\frac{1}{10}$ j. $70\frac{2}{5}$ k. $58\frac{9}{10}$ l. $20\frac{4}{5}$
 m. $14\frac{2}{5}$ n. $98\frac{2}{5}$ o. $16\frac{1}{5}$

4. a. A b. D c. D d. D
5. a. \$1.80 b. \$1.20 c. \$3.00 d. \$9.00 e. \$7.50 f. \$11.25
- g. \$22.50 h. \$55.00 i. \$4.50 j. \$7.50 k. \$14.25 l. \$30.65
6. a. \$2.70 b. \$7.15 c. \$5.75 d. \$6.05 e. \$0.05 f. \$0.10
- g. \$0.15 h. \$0.15 i. \$0.20 j. \$0.80 k. \$0.20 l. \$4.30
- m. \$0.05 n. \$0.05 o. \$0.10 p. \$0.00 q. \$0.00 r. \$12.65
7. a. \$1.30 b. \$10.50 c. \$3.30 d. \$0.65 e. \$2.40 f. \$2.20
- g. \$1.80 h. \$73.50 i. \$18.00 j. \$1.55 k. \$1.05 l. \$2.05
- m. \$32.20 n. \$4.80 o. \$1.60 p. \$0.45 q. \$14.40 r. \$492.00
8. a. D b. B c. A d. C
9. 54000 residents
10. a. \$267.50 b. 114 minutes
11. \$855
12. a. 16% b. 23%
13. a. 36.3kg b. 60lb c. 166cm
14. a. 22.5kg b. 32km c. \$9.60
15. \$322.50
16. 110 students
17. 27.9 seconds
18. a. 2 people b. 38 people
19. a. 13608 people b. 17820 people
20. $10\% + 5\% + 2\frac{1}{2}\% = \$7.60 + \$3.80 + \$1.90 = \$13.30$
21. \$26.40
22. 20 years old
23. \$0.80
24. 8.2 kg
25. 50 years old
26. 9 years old, 90 years old
27. a. 7.52 kWh b. 17.92 kWh
28. a. 78 minutes b. 282 minutes or 4 hours 42 minutes c. $21\frac{2}{3}\%$
- d. 155.1 minutes or 155 minutes 6 seconds
29. a. 14
- b. 7.69% of students achieved a score of 40 or more, which is just below the state average.
30. 60
31. Discuss with your teacher.

Investigation | Rich task

- 21507717
- 6489870
- Male: 0.494; female: 0.506
- 4144023
- 5354042
- Male: 0.492; female: 0.508
- The proportions of males and females in Victoria are similar to those in the whole of Australia.
- Answers will vary; check with your teacher.
- The population of Australia grew by 17733916 in the years between 1901 and 2011.
- In 1901, the proportion of people born in Australia was 0.771.
In 2011, the proportion of people born in Australia was 0.698.
The proportion of people born in Australia in 1901 is greater than the proportion born in Australia in 2011.
- Answers will vary; check with your teacher.

Exercise 3.10 Review questions

1. a. $1\frac{11}{21}$ b. $5\frac{1}{10}$ c. $1\frac{5}{8}$ d. $1\frac{7}{20}$ e. $4\frac{7}{64}$ f. $4\frac{2}{5}$
2. 0, 1.2, 2.4
3. a. $x \leq 10\frac{4}{5}$ (or 10.8) b. $-3.9 \leq x < -3.2$
4. 
5. a. $-1\frac{1}{15}$ b. $-1\frac{3}{10}$
6. a. $\frac{7}{20}$ b. $\frac{6}{7}$ c. $2\frac{2}{3}$ d. $41\frac{1}{6}$ e. 18 f. $\frac{9}{34}$
- g. $-\frac{5}{16}$ h. $7\frac{1}{3}$
7. a. 6.1 b. 6.73 c. 14.94 d. 5.56 e. 30.192 f. 3514.5903
- g. -0.577 h. -4.17
8. a. 865.8 b. 265.27 c. 4530.83 d. 8.77 e. 0.42 f. 106042.86
- g. -0.2 h. 2.04
9. a. i. 40000 ii. 40000 iii. 30000
b. i. 200 ii. 300 iii. 200
c. i. 3000 ii. 4000 iii. 3000
10. a. 70 b. 800 c. 1000
11. 3.75
12. a. 3600 b. 103330 c. 200 d. 15000
13. a. \$6.30 b. \$4.20 c. \$10.50 d. \$21.60 e. \$0.35 f. \$4.25
- g. \$11.85 h. \$212.60
14. a. \$0.40 b. \$1.05 c. \$3.20 d. \$5.20 e. \$1.75 f. \$2.60
- g. \$10.30 h. \$3.85
15. a. \$0.15 b. \$0.30 c. \$5.05 d. \$27.00 e. \$48.40
- f. \$11.60 g. \$2.95 h. \$2.95
16. \$1.80 17. \$0.30 18. \$2.75 19. \$1.80
20. a. $2\frac{1}{8}$
b. No. 3 pieces short
c. Red Cross \$30, World Vision \$100, \$20 left over
21. a. Check with your teacher.
b. 2.145
c. No because the number could have any number of decimal places, but must be less than 2.155.
22. a. $\frac{21}{28}$ b. $\frac{15}{20}$
23. 7 grey horses
24. NPQR is $\frac{1}{8}$ the area of ABCD.
25. 218.04
26. $\frac{13}{20}$
27. a. Mix A b. 90 cups
c. 36 cups of cordial, 54 cups of water
28. \$3.20/kg
29. \$1892
30. \$1125
31. a. \$121 b. \$120. No
32. The magic sum is 1.3.

0.34	0.48	0.02	0.16	0.3
0.46	0.1	0.14	0.28	0.32
0.08	0.12	0.26	0.4	0.44
0.2	0.24	0.38	0.42	0.06
0.22	0.36	0.5	0.04	0.18

TOPIC 4

Financial mathematics

4.1 Overview

Numerous **videos** and **interactivities** are embedded just where you need them, at the point of learning, in your learnON title at www.jacplus.com.au. They will help you to learn the concepts covered in this topic.

4.1.1 Why learn this?

Finance is an important part of all of our lives. By understanding the basics of finance you can put yourself in a better position to manage and make the most of your income. Making smart decisions with your money can help you to lead a more fulfilling life.



DISCUSSION

There is an old proverb, 'a fool and his money are soon parted'. Interpret the meaning behind the proverb and use it to explain the necessity of becoming financially literate.

LEARNING SEQUENCE

- 4.1 Overview
- 4.2 Discount
- 4.3 The unitary method
- 4.4 Profit and loss
- 4.5 Goods and Services Tax (GST)
- 4.6 Review

LEARNING OUTCOMES

A student:

- communicates and connects mathematical ideas using appropriate terminology, diagrams and symbols MA4-1WM
- applies appropriate mathematical techniques to solve problems MA4-2WM
- recognises and explains mathematical relationships using reasoning MA4-3WM
- solves financial problems involving purchasing goods MA4-6NA

CONTENT DESCRIPTIONS

Students:

Investigate and calculate the Goods and Services Tax (GST), with and without the use of digital technologies
Investigate and calculate 'best buys', with and without the use of digital technologies (ACMNA174)
Solve problems involving profit and loss, with and without the use of digital technologies (ACMNA189)

Source: NSW Syllabus for the Australian curriculum

Note: Your teacher may now set you a pre-test to determine how familiar you are with the content in this topic.

4.2 Discount

4.2.1 Applying discount

- To get rid of old stock (e.g. out-of-date fashions at the end of a season), store managers often reduce prices by giving discounts.
- A discount is a reduction in price.
- A discount can be shown as an amount in dollars.



- A discount can be shown as a percentage of the marked price (i.e. the price marked on the article).
- If the discount is expressed as a percentage, to find the actual amount of a discount, we calculate the percentage of the marked price by multiplying the marked price by the percentage. For example, a 10% discount on an item marked at \$120 gives a discount amount of \$12.



ACTIVITY: LET'S GO SHOPPING!

Equipment: sales catalogues from nearby shops, paper, pen, calculator

Part A

1. As a class, brainstorm percentage discounts that we see advertised in sales. Pick three common ones.
2. Each person should think of an item they want to buy and its current price. A volunteer might like to draw a table on the board with the column headings 'Item' and then the three common percentage discounts.
3. Each person should then calculate the new prices of the selected item assuming the discount shown on the board. Repeat this process for the item listed underneath yours.
4. As a class, fill in the table and discuss the results.

Part B

1. Work in groups of three or four. Select a page from one of the sales catalogues and calculate the percentage discount on *five* items.
2. Discuss the results as a class. How would you calculate the average percentage discount shown on the items in the catalogue?

4.2.2 Calculating selling price of a discounted item

• Method 1

Use the percentage remaining after the percentage discounted has been subtracted from 100%; that is, if an item for sale has a 10% discount then the price must be 90% of the marked price.

WORKED EXAMPLE 1

Find the sale price on a hat marked \$72 if a 10% discount is given.



THINK

- 1 Find the percentage of the marked price that is paid, by subtracting the percentage discount.
- 2 Find the sale price of the hat.
- 3 Write the answer in a sentence.

WRITE

$$100\% - 10\% = 90\%$$

$$\begin{aligned} 90\% \text{ of } \$72 &= 0.9 \times \$72 \\ &= \$64.80 \end{aligned}$$

The sale price of the hat is \$64.80.

• Method 2

The new sale price of the item can be solved by calculating the amount of the discount, then subtracting the discount from the marked price.

Alternative solution:

$$\begin{aligned} \text{Discount} &= 10\% \text{ of } \$72.00 \\ &= \$7.20 \end{aligned}$$

$$\begin{aligned} \text{Sale price} &= \text{marked price} - \text{discount} \\ &= \$72.00 - \$7.20 \\ &= \$64.80 \end{aligned}$$

WORKED EXAMPLE 2

Peddles is a bicycle store that has offered a discount of 15% on all goods.

Find:

a the cash discount allowed on a bicycle costing \$260

b the sale price of the bicycle.

THINK

a Find the discount, which is 15% of the marked price.

b 1 To find the sale price, subtract the discount from the marked price.

2 Answer the question in a sentence.

WRITE

$$\begin{aligned}\text{a Discount} &= 15\% \text{ of } \$260 \\ &= 0.15 \times \$260 \\ &= \$39\end{aligned}$$

The cash discount allowed is \$39.

$$\begin{aligned}\text{b Sale price} &= \text{marked price} - \text{discount} \\ &= \$260 - \$39 \\ &= \$221\end{aligned}$$

The sale price of the bicycle is \$221.

- To calculate the percentage discount, write the monetary amount as a percentage of the original price.

$$\text{Percentage discount} = \frac{\text{cash discount}}{\text{original price}} \times \frac{100}{1}\%$$

WORKED EXAMPLE 3

At Peddles, the price of a bicycle is reduced from \$260 to \$200. Calculate the percentage discount.

THINK

1 Calculate the amount of the discount.

2 Write the discount as a percentage of the original price.

3 Answer the question in a sentence.

WRITE

$$\begin{aligned}\text{Discount} &= \$260 - \$200 \\ &= \$60\end{aligned}$$

$$\begin{aligned}\text{Percentage discount} &= \frac{60}{260} \times 100\% \\ &= 23\%\end{aligned}$$

The percentage discount is about 23%.

DISCUSSION

Give some examples, other than discount situations, where you could use percentage calculations.

learn on RESOURCES — ONLINE ONLY

Interactivity: Discount (int-3744)

Interactivity: Selling price (int-3745)

Digital doc: SkillsHEET Decreasing a quantity by a percentage (doc-6899)

Digital doc: SkillsHEET Finding a percentage of a quantity (money) (doc-6900)

Digital doc: Investigation — Successive discounts (doc-2228)

Exercise 4.2 Discount

Individual pathways

PRACTISE

Questions:
1–7, 12, 13, 20, 28

CONSOLIDATE

Questions:
1–20, 22, 26, 28

MASTER

Questions:
1–29

Individual pathway interactivity: int-4421

learnon ONLINE ONLY

To answer questions online and to receive **immediate feedback** and **fully worked solutions** for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Understanding and fluency

1. Calculate the discount on each of the items in the table, using the percentage shown.

	Item	Marked price	Discount
a.	MP3 player	\$210	20%
b.	Skateboard	\$185	25%
c.	Rollerblades	\$330	15%
d.	Mobile phone	\$190	40%

2. Without the use of a calculator, calculate the percentage discount for each of the following.

	Marked price	Discount
a.	\$100	\$10
b.	\$250	\$125
c.	\$90	\$30
d.	\$80	\$20

3. **WE1** Find the sale price of each article when the marked price and discount are shown as in this table.

	Marked price (RRP)	Discount
a.	\$1000	15%
b.	\$250	20%
c.	\$95	12%
d.	\$156	$33\frac{1}{3}\%$
e.	\$69.95	$7\frac{1}{2}\%$

4. Decrease the amount by the percentages.

- a. \$50 by 10%
- b. \$90 by 50%
- c. \$45 by 20%

5. Find the percentage discount given on the items shown in the table. Round to the nearest per cent.

	Original price	Selling price
a.	\$25	\$15
b.	\$100	\$72
c.	\$69	\$50
d.	\$89.95	\$70

6. A tablet computer that usually sells for \$599 was advertised with a saving of \$148. Estimate the percentage discount being offered.

7. The following items are all discounted.



\$380

25% discount

\$450

20% discount

\$260

$33\frac{1}{3}\%$ discount

\$600

15% discount

- Which has the largest dollar discount?
 - Which have the same dollar discount?
 - What is the difference between the largest and the smallest dollar discount?
 - If the surfboard has a discount of 20%, would \$470 be enough to buy it?
8. **WE2** A sale discount of 20% was offered by the music store Solid Sound. Find:
- the cash discount allowed on a \$350 sound system
 - the sale price of the system.
9. A wristwatch is advertised at \$69.95, less 10% discount. Find the sale price.
10. A store-wide clearance sale advertised 15% off everything.
- What would be the selling price of a pair of jeans marked at \$49?
 - If a camera marked at \$189 was sold for \$160.65, was the correct percentage deducted?
11. T-shirts are advertised at \$15.95 less 5% discount. How much would Jim pay for five T-shirts?
12. **WE3** Calculators were advertised at \$20, discounted from \$25. What percentage discount was given?
13. A tennis racquet marked at \$79.95 sells for \$60. What percentage discount is this, to the nearest whole number?
14. CDs normally selling for \$28.95 were cleared for \$23.95. What percentage discount was given (correct to 1 decimal place)?
15. A shirt was reduced from \$90 to \$63. Express the reduction as a percentage of the original price.
16. At a sale, Ann bought a \$120 jacket for \$48. What percentage of the original price did she save?

17. You bought a mobile phone priced \$199.95 and signed up for a 1-year plan. You received a 10% discount on the telephone and 15% discount on the \$75 connection fee. How much did you pay altogether (correct to the nearest 5 cents)?
18. Aanh bought two hairdryers for \$128 each. She sold one at a loss of 5% and the other for a profit of 10%.
 - a. Find the selling price of each.
 - b. Will she have made a profit or a loss?
19. **MC** Kristen's car insurance was \$670, but she had a 'No claim bonus' discount of 12%. Which of the following will not give the amount she must pay?
 - A. First find 12% of \$670 and add your answer to \$670.
 - B. Calculate $88 \div 100 \times 670$.
 - C. Find 88% of \$670.
 - D. First find 12% of \$670, and subtract your answer from \$670.
 - E. Calculate $0.12 \times \$670$ and subtract your answer from \$670.

Communicating, reasoning and problem solving

20. Movie tickets sell for \$12.00 each, but if you buy 4 or more you get \$1.00 off each ticket. What percentage discount is this (correct to 2 decimal places)? Show your working. (*Hint: Find \$1 as a percentage of \$12.*)
21. **MC** I am allowed a discount of 10% off the total price of 6 articles that cost \$ x each. The price finally paid is:
A. \$60 x **B.** \$5.4 x **C.** \$0.06 x **D.** \$0.6 x **E.** \$6 x
22. **a.** At the local market there is a 'buy two, get one free' offer on hand-made soaps. What percentage discount is this equivalent to?
b. At a rival market there is a 'buy one, get another half price' offer on soaps. Is this deal the same, better or worse than the discount offered in part **a**?
23. You are in a surf shop and you hear 'For today only: take fifty per cent off the original price and then a further forty per cent off that.' You hear a customer say 'This is fantastic! You get ninety per cent off the original price!' Is this statement correct? Explain why.
24. Is there a difference between 75% off \$200 and 75% of \$200? Explain.
25. Henry buys a computer priced at \$1060, but with a 10% discount. Sancha finds the same computer selling at \$840 plus a tax of 18%. Who has the best buy? Explain.
26. A classmate was completing a discount problem where she needed to calculate a 25% discount on \$79. She misread the question and calculated a 20% discount to get \$63.20. She then realised her mistake and took a further 5% from \$63.20. Is this the same as taking 25% off \$79? Use calculations to support your answer.
27. A store had to increase its prices by 10% to cover increasing expenses. A particular DVD player was originally priced at \$220. Use the questions below to help you calculate the new price of the DVD player using two different methods.
 - a. **i.** Calculate the price increase and add it to the original price.
ii. Add the percentage amount to 100% and multiply your answer by the original price.
 - b. What do you notice about the answers to part **a**?
 - c. Describe this alternative method in your own words.
28. What would you multiply the original prices of items by to get their new prices with:
 - a. a 20% discount b. a 15% discount c. a 25% increase
 - d. a 5% increase e. a 35% discount f. an 11% increase
 - g. a 6% discount h. a 100% increase?
29. How are discounts used to encourage people to purchase?

4.3 The unitary method

4.3.1 Unit price

- Many consumers assume that if they buy the larger quantity of a product, they will be getting better value for money. This is not always the case.
- The **unitary method** can be used to work out which item is the best buy by comparing the unit price. Supermarkets in Australia are now obliged to show the unit cost on most products.
- The **unit price** is the price per unit. This may mean the price per 100mL, the price per 100g or the price of a single item.
- Remember to represent the same units when you are making comparisons.

WORKED EXAMPLE 4

Three shampoos are sold in the following quantities.

Brand A: 200mL for \$5.38

Brand B: 300mL for \$5.98

Brand C: 400mL for \$8.04

Which shampoo is the best buy?

THINK

- 1 Determine the number of 100-mL units for each shampoo.

- 2 Determine the price per unit for each shampoo.

$$\text{Price per unit} = \frac{\text{price}}{100\text{-mL units}}$$

- 3 Answer the question for the best buy.

WRITE

$$\text{Brand A} = \frac{200}{2} = 2 \text{ units}$$

$$\text{Brand B} = \frac{300}{3} = 3 \text{ units}$$

$$\text{Brand C} = \frac{400}{4} = 4 \text{ units}$$

$$\text{Brand A} = \frac{5.38}{2} = \$2.69 \text{ per } 100 \text{ mL}$$

$$\text{Brand B} = \frac{5.98}{3} = \$1.99 \text{ per } 100 \text{ mL}$$

$$\text{Brand C} = \frac{8.04}{4} = \$2.01 \text{ per } 100 \text{ mL}$$

Brand B: 300mL of shampoo for \$5.98

- The smaller the unit cost, the better the value of the item.

4.3.2 Unitary method in percentages

- The first step is to find 1% of the amount.
- The next step is to multiply the value of the goods by 100 to get the whole amount (100%).

WORKED EXAMPLE 5

Find the whole amount if 5% is represented by \$25.

THINK

- 1 Five per cent of a number equals 25.
Find 1% by dividing the number by 5.

- 2 Find 100% by multiplying the amount by 100.

WRITE

$$5\% = \$25$$

$$1\% = \$25 \div 5 \\ = \$5$$

$$100\% = \$5 \times 100 \\ = \$500$$

WORKED EXAMPLE 6

Ronan operates a sports store at a fixed profit margin of 65%. Ronan sells a tracksuit for \$99.95. What would Ronan have paid for the tracksuit?

THINK

- 1 Write the selling price as 165% of the cost price.
- 2 Find 1% of the cost price by dividing both sides by 165.
- 3 Find 100% of the cost price by multiplying both sides by 100. Give your answer to the nearest cent.
- 4 Answer in a sentence.

WRITE

$$165\% \text{ of cost price} = \$99.95$$

$$1\% \text{ of cost price} = \$0.6057576$$

$$100\% \text{ of cost price} = \$60.58$$

Ronan would have paid \$60.58 for the tracksuit.

4.3.3 Percentage composition

- If we are given the percentage breakdown spent on a group of items, and we are given the total cost of the whole group of items, we can use the unitary method to calculate the amount spent on each item within the group.

WORKED EXAMPLE 7

Georgina spent \$55.40 at the supermarket. The percentage composition of her shopping was:

Fruit and vegetables — 40%

Beverages — 10%

Frozen foods — 35%

Toiletries — 15%

Determine how much Georgina spent in each category.

THINK

- 1 Determine the amount spent for 1% of Georgina's shopping.
- 2 Multiply this quantity by the percentage in each category.
- 3 Write the answer.

WRITE

$$\frac{\$55.40}{100} = \$0.554$$

Fruit and vegetables:

$$\$0.554 \times 40 = \$22.16$$

Beverages:

$$\$0.554 \times 10 = \$5.54$$

Frozen foods:

$$\$0.554 \times 35 = \$19.39$$

Toiletries:

$$\$0.554 \times 15 = \$8.31$$

Georgina spent the following amounts in each category:

Fruit and vegetables — \$22.16

Beverages — \$5.54

Frozen foods — \$19.39

Fruit and vegetables — \$8.31

ACTIVITY: PRICE COMPARISON WEBSITES

Use the internet to research different price comparison websites. Investigate what goods and services price comparison websites display and present your research to your class.

learnon RESOURCES — ONLINE ONLY

Interactivity: Unitary method (int-2365)

Interactivity: Unitary method (int-4154)

Interactivity: Unitary method in percentages (int-4155)

Digital doc: WorkSHEET Money (doc-6617)

Exercise 4.3 The unitary method

Individual pathways

PRACTISE

Questions:

1, 2, 3, 4, 5, 6, 7, 8, 9, 13, 20, 22, 23

CONSOLIDATE

Questions:

1, 2, 3, 4b, c, e, i, 5, 6, 7, 8, 10, 11, 14, 15, 16, 20, 22, 23, 27

MASTER

Questions:

1, 2, 3, 4c, f, i, 5, 6, 7, 11, 12, 15, 17, 18, 19, 20, 21, 23, 24, 25, 26, 27, 28, 29

Individual pathway interactivity: int-4396

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Understanding and fluency

- How much will you pay for 4.5 kg of apples, given that 2 kg cost \$3.80?
- Compare the cost of 400 g of biscuits for \$2.48 and 500 g for \$3.10. Which is the better buy?
- WE4** Which of the following is the best buy?

a.

Chocolate weight	Cost
150 g	\$3.25
250 g	\$4.75
325 g	\$5.50

b.

No. of pages	Cost
80	\$1.98
160	\$3.38
200	\$3.98

- WE5** Calculate the original amount if:
 - 10% is \$18
 - 20% is \$6
 - 25% is \$60
 - 40% is \$900
 - 34% is \$60
 - 90% is \$380
 - 200% is \$800
 - 120% is \$420
 - 9% is \$54.
- Calculate:
 - 60%, if 40% is \$120
 - 38%, if 20% is \$6
 - 50%, if 25% is \$60
 - 150%, if 50% is \$900
 - 12%, if 75% is \$250
 - 86%, if 14% is \$4200.

6. **WE6** The profit on an MP3 player is \$240. If this is 60% of the cost price, what is:
- the cost price
 - the selling price?
7. **WE7** Pablo spent \$82.20 at the supermarket. The percentage composition of his shopping was:
- Fruit and vegetables — 25%
 - Beverages — 15%
 - Frozen food — 45%
 - Toiletries — 10%
 - Baked goods — 5%
- Determine how much Pablo spent in each category.
8. You spend 40% of your monthly allowance on your mobile phone. What is your monthly allowance if you spend \$20 on your mobile phone?
9. Hanh saves 32% of his wages each week. If he saves \$220, what is his weekly wage?
10. The retail price of in-line skates is \$320. This represents 200% of the cost price. What is the cost price?
11. Alex receives a 15% pay rise. He now receives \$97 290 a year. How much was his annual income before the pay rise?
12. Which is the best buy?
- 1.2 kg of apples for \$5.20 or 0.75 kg of apples for \$3.50?
 - 250 mL of orange juice for \$2.95 or 400 mL of orange juice for \$4.20?
13. Which is the best buy?
- 500 g of cheese for \$12.80, 200 g of cheese for \$5.62 or 80 g of cheese for \$2.10?
14. A company made a profit of \$238 000. This represents a 10% increase in profit on the previous year. What was the profit that year?
15. A camping goods shop operates on a profit margin of 85%. How much would the shop have paid for a sleeping bag that sells for \$89.95?
16. Annie spent \$100.10 at the supermarket. If 33% of her shop was on fresh food and 21% of her shop was on frozen food, how much did she spend on each category? Give your answers correct to the nearest cent.
17. After a discount of 15%, an mp3 player was worth \$183. What was its value before the discount?
18. During a sale, a retailer allows a discount of 15% off the marked price. His sale price of \$60 still gives him a profit of 10%.
- What did the article cost him?
 - What was the marked price?
19. A discount of 15% reduced the price of a CD by \$3.20.
- What was the original price of the CD?
 - What was its reduced selling price?
20. A discount of 22% reduced the price of an outfit by \$48.
- What was its original price?
 - At what reduced price was it selling?
21. You buy ten pairs of headphones for a total of \$150. At what price should you sell six pairs if you wish to make a profit of 25% on each pair?
22. Supermarkets are obliged by law to display the unit price when selling the same items in different quantities. Calculate the price per 100 g for the following amounts of chocolate. Give your answers correct to the nearest cent.
- 250 g for \$5.95
 - 400 g for \$8.50
 - 60 g for \$2.00



Communicating, reasoning and problem solving

23. An art dealer sold two paintings at an auction. The first painting sold for \$7600, making a 22% loss on its cost. The second painting was sold for \$5500, making a profit of 44%. Did the art dealer make an overall profit or loss? Show your working.
24. You love to make bracelets. To make them, you need to buy some beads. Brand A costs \$7.50 for a box that contains 1000 beads; Brand B costs \$17.50 for a box that contains 2500 beads. Which brand is the better buy? Explain how you decide.
25. While shopping in a sale, Carly spots a sign that says 'Buy 2 items, get the third half price (the least expensive item will be counted as the third item)'. If she finds a shirt for \$25.99, a skirt for \$87.99 and a belt for \$15.97 and pays cash for the items, how much will Carly end up paying?
26. Some players from a soccer club went to a café to celebrate their win. Each had a burger and a drink. The bill came to \$80. When it came time to pay the bill, 3 people had left without paying, so the remaining members had to pay an extra \$2 each to cover the bill. If there were originally n people in the group, and the burger and drink deal cost $\$b$, write an equation using these pronumerals to show how the bill was settled. Do not attempt to solve your equation.
27. In some situations it is not always practical to go for the best buy. Why might this be? List all of the possible reasons that you can think of.
28. What are the different situations in which you would use the unitary method?
29. The price of entry into a theme park has increased by 10% every year since the theme park opened. If the latest price rise increased the tickets to \$8.80, what was the price of a ticket 2 years ago?



4.4 Profit and loss

4.4.1 Cost prices and selling prices

- When a manufacturer produces a product, it is usually sold to a wholesaler who subsequently sells the product on to retail outlets. At each stage the product is marked up by a certain percentage.
- When a retailer calculates the price to be marked on an article (the *selling price*, SP), many overhead costs must be taken into account (staff wages, rent, store improvements, electricity, advertising and so on).
- The total price the retail shop owner pays for the product including overhead costs is the *cost price*, CP.
- The *profit* is the difference between the total of the retailer's costs (cost price) and the price for which the goods actually sell (selling price).
 - If $SP > CP$, then a profit is made.



$$\text{Profit} = \text{selling price} - \text{cost price}$$

- If $SP < CP$, then a loss is made.

$$\text{Loss} = \text{cost price} - \text{selling price}$$

4.4.2 Calculating the selling price

- To calculate the selling price of an item given the cost price and the percentage profit, increase the cost price by the given percentage.

Selling price = (100% + percentage profit) of cost price

- To calculate the selling price of an item given the cost price and the percentage loss, decrease the cost price by the given percentage.

Selling price = (100% – percentage loss) of cost price

WORKED EXAMPLE 8

**Ronan operates a sports store at a fixed profit margin of 65%.
For how much would he sell a pair of running shoes that cost him \$40?**



THINK

- Find the selling price by first adding the percentage profit to 100% then finding this percentage of the cost price.
- Write the answer in a sentence.

WRITE

$$\begin{aligned}\text{Selling price} &= 165\% \text{ of } \$40 \\ &= 1.65 \times \$40 \\ &= \$66\end{aligned}$$

The running shoes would sell for \$66.

WORKED EXAMPLE 9

**David bought a surfboard for \$300 and sold it at a 20% loss a year later.
What was the selling price?**

THINK

- Find the selling price by first subtracting the percentage loss from 100% then finding this percentage of the cost price.
- Write the answer in a sentence.

WRITE

$$\begin{aligned}\text{Selling price} &= 80\% \text{ of } \$300 \\ &= 0.80 \times \$300 \\ &= \$240\end{aligned}$$

David sold the surfboard for \$240.

- Profit or loss is usually calculated as a percentage of the cost price.

$$\text{Percentage profit} = \frac{\text{profit}}{\text{cost}} \times 100\%$$

$$\text{Percentage loss} = \frac{\text{loss}}{\text{cost}} \times 100\%$$

WORKED EXAMPLE 10

**A music store buys CDs at \$15 each and sells them for \$28.95 each.
What is the percentage profit made on the sale of a CD?**

THINK

- Calculate the profit on each CD:
selling price – cost.

WRITE

$$\begin{aligned}\text{Profit} &= \$28.95 - \$15 \\ &= \$13.95\end{aligned}$$

2 Calculate the percentage profit: $\frac{\text{profit}}{\text{cost}} \times 100\%$

$$\begin{aligned}\text{Percentage profit} &= \frac{13.95}{15} \times 100\% \\ &= 93\%\end{aligned}$$

3 Write the answer in a sentence, rounding to the nearest per cent if applicable.

The profit is 93% of the cost price.

- Modern accounting practice favours calculating profit or loss as a percentage of the selling price. This is because commissions, discounts, taxes and other items of expense are commonly based on the selling price.

$$\text{Percentage profit} = \frac{\text{profit}}{\text{selling price}} \times 100\%$$

$$\text{Percentage loss} = \frac{\text{loss}}{\text{selling price}} \times 100\%$$

4.4.3 Calculating the cost price

- If you are given the selling price and the percentage profit or loss, you can work backwards to calculate the cost price.

WORKED EXAMPLE 11

A fashion store sells a pair of jeans for \$180. If they made a percentage profit of 80% of the selling price, determine the cost price of the pair of jeans.

THINK

1 Enter the given information into the percentage profit selling price formula.

2 Rearrange the formula to make profit the subject.

3 Complete the calculation to determine the profit.

4 Subtract the profit from the selling price to determine the cost price.

WRITE

$$\begin{aligned}\text{Percentage profit} &= \frac{\text{profit}}{\text{selling price}} \times 100\% \\ 80 &= \frac{\text{profit}}{180} \times 100\%\end{aligned}$$

$$\begin{aligned}\frac{80}{100} &= \frac{\text{profit}}{180} \\ \frac{80}{100} \times 180 &= \text{profit} \\ \text{profit} &= \frac{80}{100} \times 180\end{aligned}$$

$$\text{Profit} = \$144$$

$$\begin{aligned}\text{Cost price} &= \$180 - \$144 \\ &= \$36\end{aligned}$$

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RESOURCES — ONLINE ONLY

Interactivity: Profit and loss (int-3746)

Digital doc: SkillsHEET Expressing one quantity by a percentage (doc-6901)

Digital doc: SkillsHEET Increasing a quantity by a percentage (doc-6902)

Digital doc: WorkSHEET Applications of percentages II (doc-6913)

Exercise 4.4 Profit and loss

Individual pathways

PRACTISE

Questions:
1–9, 12, 21

CONSOLIDATE

Questions:
1–17, 20, 21

MASTER

Questions:
1–24

Individual pathway interactivity: int-4422

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Assume percentage profit or loss is calculated on the cost price unless otherwise stated.

To answer questions online and to receive **immediate feedback** and **fully worked solutions** for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Understanding and fluency

1. **WE8,9** Find the selling price for each of the following:

	Cost price	%	Profit/loss
a.	\$18	40%	profit
b.	\$116	25%	loss
c.	\$1300	30%	profit
d.	\$213	75%	loss
e.	\$699	$33\frac{1}{3}\%$	profit

2. **WE10** For each of the following items, find the percentage profit or loss.

	Cost price	Selling price
a.	\$15	\$20
b.	\$40	\$50
c.	\$40	\$30
d.	\$75	\$85
e.	\$38.50	\$29.95

3. A supermarket buys frozen chickens for \$3.50 each and sells them for \$5.60. What is the percentage profit made on the sale of each chicken?
4. A restored motorbike was bought for \$350 and later sold for \$895.
- How much profit was made?
 - What percentage was profit? Give your answer correct to the nearest whole number.
5. **WE11** A music store sold a drum set for \$480. If they made a percentage profit of 75% of the selling price, determine the cost price of the drum set.
6. James' Second Hand Bookshop buys second hand books for \$4.80 and sells them for \$6.00.
- What is the ratio of the profit to the cost price?
 - What is the percentage profit on the cost price?
 - What is ratio of the profit to the selling price?
 - What is the percentage profit on the selling price?
 - Discuss how the answers to parts **a** and **b** are related.



7. A retailer bought a laptop for \$1200 and advertised it for \$1525.
 - a. How much profit was made?
 - b. What is the percentage profit (to the nearest whole number) on the cost price?
 - c. What is the percentage profit (to the nearest whole number) on the selling price?
 - d. Compare the differences between the answers to parts **b** and **c**.
8. Rollerblades bought for \$139.95 were sold after six months for \$60.
 - a. How much was the loss?
 - b. What was the percentage loss? Give your answer to the nearest whole number.
9. A sports card collection costing \$80 was sold for \$65. What was the percentage loss?
10. Running shoes bought by a sports store for \$30 per pair were sold at \$79.95. What percentage profit was made?
11. Kyle runs a jewellery business that uses a fixed profit margin of 98%. For how much would he sell a necklace that cost him \$830?
12. Find the selling price for each item.
 - a. Jeans costing \$20 are sold with a profit margin of 95%.
 - b. A soccer ball costing \$15 is sold with a profit margin of 80%.
 - c. A sound system costing \$499 is sold at a loss of 45%.
 - d. A skateboard costing \$30 is sold with a profit margin of 120%.
13. Determine the cost price for the following items.
 - a. A diamond ring sold for \$2400 with a percentage profit of 60% of the selling price.
 - b. A cricket bat sold for \$69 with a percentage profit of 25% of the selling price.
 - c. A 3-seater sofa sold for \$1055 with a percentage profit of 35% of the selling price.
14. A fruit-and-vegetable shop bought 500 kg of tomatoes for \$900 and sold them for \$2.80 per kg.
 - a. What is the profit per kilogram?
 - b. Calculate the profit as a percentage of the cost price (round to 1 decimal place).
 - c. Calculate the profit as a percentage of the selling price (round to 1 decimal place).
 - d. Compare the answers to parts **b** and **c**.
15. Sonja bought an old bike for \$20. She spent \$47 on parts and paint and renovated it. She then sold it for \$115 through her local newspaper. The advertisement cost \$10.
 - a. What were her total costs?
 - b. What percentage profit (to the nearest whole number) did she make on costs?
 - c. What percentage profit (to the nearest whole number) did she make on the selling price?
16. **MC** A clothing store operates on a profit margin of 150%. The selling price of an article bought for \$ p is:
 - A.** \$151 p
 - B.** \$150 p
 - C.** \$2.5 p
 - D.** \$1.5 p
 - E.** \$0.15 p



Communicating, reasoning and problem solving

17. A fruit-and-vegetable retailer buys potatoes by the tonne for \$180 and sells them in 5-kg bags for \$2.45. What percentage profit does he make (to the nearest whole number)? Show your working.
18. Two business partners bought a business for \$158 000 and sold it for \$213 000. The profit was to be shared between the two business partners in the ratio of 3 : 2. What percentage share (to the nearest whole number) does each person receive? How much does each receive?
19. What discount can a retailer offer on her marked price of \$100 so that she ends up selling at no profit and no loss, if she had initially marked her goods up by \$50? Justify your answer.
20. To produce a set of crockery consisting of a dinner plate, soup bowl, bread plate and coffee mug, the costs per item are \$0.98, \$0.89, \$0.72 and \$0.69 respectively. These items are packaged in boxes of 4 sets and sell for \$39. If the company sells 4000 boxes in a month, what is their total profit?
21. Copy and complete the table below.

Cost per item	Items sold	Sale price	Total profit
\$4.55	504	\$7.99	
\$20.00		\$40.00	\$8 040.00
\$6.06	64 321		\$225 123.50
	672	\$89.95	\$28 425.60

22. The method used to calculate profits can make a difference when comparing different profits.



Cost = \$20.00

Price = \$120.00



Cost = \$26 500.00

Price = \$32 000.00



Cost = \$1.00 (Homemade)

Price = \$3.50

- a
 - i. Describe the profits on each of the items above as a raw amount.
 - ii. List the items from largest profit to smallest profit.
 - iii. Discuss whether this is a fair method of comparing the profits.
 - b
 - i. Express the profit on each of the items as a percentage of its cost.
 - ii. List the items from largest profit to smallest profit.
 - iii. Discuss whether this is a fair method of comparing the profits.
 - c
 - i. Express the profit on each of the items as a percentage of its price.
 - ii. List the items from largest profit to smallest profit.
 - iii. Discuss whether this is a fair method of comparing the profits.
23. How can you tell if an item is being sold for a profit or a loss?
 24. Max bought a car for \$6000.00. He sold it to Janine for 80% of the price he paid for it. Janine sold it to Jennifer at a 10% loss. Jennifer then sold it to James for 75% of the price she paid. What did James pay for the car? What was the total percentage loss on the car from Max to James?



4.5 Goods and Services Tax (GST)

4.5.1 Investigating GST

- **GST** is a tax levied by the Australian federal government on goods and services. (As with all taxes, there are exemptions, but these will not be considered here.)
 - Goods: A tax of 10% is added to new items that are purchased, such as petrol, clothes and some foods.
 - Services: A tax of 10% is added to services that are paid for, such as work performed by plumbers, painters and accountants.



WORKED EXAMPLE 12

A packet of potato chips costs \$1.84 before GST.

Find:

- a the GST charged on the packet of chips
- b the total price the customer has to pay.



THINK

- a GST is 10%.
- b Total equals pre-GST price plus GST.

WRITE

$$\begin{aligned}10\% \text{ of } \$1.84 &= \$0.18 \text{ (rounded)} \\ \$1.84 + \$0.18 &= \$2.02 \\ &\text{(rounded up by the seller to } \$2.05)\end{aligned}$$

- To find the pre-GST amount when the total you are given includes GST, divide the GST-inclusive amount by 110 and multiply by 100.

WORKED EXAMPLE 13

A plumber's hourly charge includes GST. If she worked for 5 hours and the total bill including GST was \$580, what was her hourly price before GST?

THINK

- 1 Find the hourly price including GST.
- 2 Find the hourly price excluding GST.
- 3 Write the answer.

WRITE

$$\begin{aligned}\frac{\$580}{5} &= \$116 \\ 110\% \text{ of pre-GST hourly rate} &= \$116 \\ \text{pre-GST hourly rate} &= \frac{\$116}{110} \times \frac{100}{1} \\ &= \$105.45 \\ \text{The plumber's hourly rate is } \$105.45 \text{ before GST.}\end{aligned}$$

ACTIVITY: INTERPRETING RECEIPTS

Collect some receipts from a variety of different shops. Look for the section on the receipt where it details the GST information. Are any items on your receipts exempt from GST?

Exercise 4.5 Goods and Services Tax (GST)

Individual pathways

PRACTISE

Questions:
1–13, 17

CONSOLIDATE

Questions:
1–17

MASTER

Questions:
1–19

Individual pathway interactivity: int-4423

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Understanding and fluency

- Explain the GST in your own words.
- Does GST apply below? Answer yes or no for each example.
 - Petrol
 - A lawyer's fee
 - Hotel accommodation
 - Lounge room carpet
 - Floor tiling
 - Wages at a fast-food restaurant
- WE12** With or without a calculator:
 - calculate the GST payable on each of the following pre-GST prices
 - calculate the total price including GST.
 - 500 g laundry powder at \$4.50
 - 400-g tin of canned peaches at \$2.12
- WE13** The prices below are inclusive of GST. What is the pre-GST price of each?
 - 1 kg apples at \$3.85
 - 500 g laundry powder at \$4.50
 - 400-g tin of canned peaches at \$2.20
 - 5 kg potatoes at \$6.50
- The telephone company Ringtel charges home customers \$42.50 per month plus \$0.24 per local call. Find the monthly phone bill, including GST, if a customer makes 51 local calls in a month.
- All car rental agencies use similar charging plans. Drivo charges \$44 per day plus \$0.47 per kilometre travelled. A customer wishes to rent a car for four days and travels 1600 km. What is her total bill, including GST?
- Expresso is a company that operates in the 'we-visit-you' car repair business. It charges \$85 per hour plus a flat \$40 visiting fee.
 - Set up an expression, which includes GST, for the cost of a repair that takes t hours.
 - If the repair takes 3 hours and 30 minutes, what is the final cost?
- A company that installs floor tiles charges \$35 per square metre for the actual tiles, and a fee of \$100 plus \$10 per square metre to install the tiles in a home. Let the area of the floor to be tiled be x m².
 - Find an expression, including GST, that represents the total cost of tiling in terms of x .
 - What would be the total cost for a 20 m² floor?



9. A new bicycle costs \$450, including GST. How much is the GST?
10. To buy my new super-duper mobile phone outright I must pay \$30 per month, including GST, for 3 years. How much GST will I pay?
11. I buy a pack of batteries and pay 25 cents GST. How much did I pay in total for the batteries?
12. In the United Kingdom a similar tax, called the value added tax or VAT, is levied at 20%. If I paid A\$67 for a jumper purchased in a shop in Bond Street, London:
 - a. how much VAT did I pay
 - b. what was the pre-VAT price of the jumper?
13. In New Zealand GST is levied at 15% of the purchase price of goods. If I buy a pair of jeans and pay NZ\$12 in GST, what total price did I pay for the jeans in NZ dollars?

Communicating, reasoning and problem solving

14. Explain why the amount of GST on an item is not equivalent to 10% of the GST-inclusive price.
15. Explain why the pre-GST price of an item is not equivalent to 10% off the GST-inclusive price.
16. The GST rate is 10%. This means that when a business sells something or provides a service it must charge an extra $\frac{1}{10}$ of the price/cost. That extra money then must be sent to the tax office. For example, an item that would otherwise be worth \$100 now has GST of \$10 added. So the price tag will show \$110. The business will then send to the tax office that \$10 with all the other GST it has collected on behalf of the government.
 - a. Suppose a shopkeeper made sales totalling \$15400. How much GST must he put aside?
 - b. Is there a number he can quickly divide by to figure out the GST?
17. In the country Snowdonia, GST is 12.5%. Igor's girlfriend Karla has purchased a new hair drier that cost her in total, including GST, 111 Kopeks. There are 100 Plens in 1 Kopek.
 - a. How much GST did Karla pay?
 - b. If 1 Australian dollar = 2 Kopeks, how much GST would Karla have paid if she had purchased the hair drier in Melbourne, where GST is currently 10%?
18. Taking GST to be 10%:
 - a. what is the GST payable on an item whose pre-GST price is \$P, and what is the price payable
 - b. what is the pre-GST price of an item for which I paid \$A, and how much GST did I pay?
19. What do the terms *inclusive* and *exclusive of GST* mean?

4.6 Review

Investigation | Rich task

The composition of gold in jewellery

You may be aware that most gold jewellery is not made of pure gold. It is actually an alloy, or mixture of metals. The finest gold used in jewellery is 24 carat and is known as fine gold. Gold in this form is very soft and is easily scratched. Most metals will form an alloy with gold, the most common being silver, copper and zinc in jewellery making. Other metals may be used to create coloured gold. A table of the composition of some of the common gold alloys used in jewellery pieces is shown on the next page.



Gold name	Composition	
Gold (24 carat)	Gold	100%
Yellow gold (22 carat)	Gold	91.67%
	Silver	5%
	Copper	2%
	Zinc	1.33%
Pink gold (18 carat)	Gold	75%
	Copper	20%
	Silver	5%
Rose gold (18 carat)	Gold	75%
	Copper	22.25%
	Silver	2.75%
Red gold (18 carat)	Gold	75%
	Copper	25%
White gold (18 carat)	Gold	75%
	Palladium	10%
	Nickel	10%
	Zinc	5%
Grey–white gold (18 carat)	Gold	75%
	Iron	17%
	Copper	8%
Green gold (18 carat)	Gold	75%
	Silver	20%
	Copper	5%
Blue gold (18 carat)	Gold	75%
	Iron	25%
Purple gold (18 carat)	Gold	80%
	Aluminium	20%

Use the table to answer the following questions.

1. Study the table and list the metals used to create the alloys of gold mentioned.
2. A particular rose-gold bracelet weighs 36 grams. Calculate the masses of the various components in the bracelet.

3. How much more gold would be in a yellow-gold bracelet of the same mass? What fraction is this of the mass of the bracelet?
4. Pink, rose and red gold all contain 75% gold. In addition, they each contain copper, and pink and rose gold also contain silver. Describe the effect you feel the composition of the alloy has on the colour of the gold.
5. Why does white gold not contain any copper?
6. Compare the composition of the alloys in red gold and blue gold.
7. 24-carat gold is classed as 100% gold. On this basis, an alloy of gold containing 75% gold has a carat value of 18 carat. Note this fact in the table above. The purple gold is 80% gold. What would its carat value be?
8. Just as there are various qualities of gold used in jewellery making, the same is true of silver jewellery. Sterling silver, which is commonly used, is actually not pure silver. Find out about the composition of silver used in jewellery making. Write a short report on your findings on a separate sheet of paper.



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Digital doc: Investigation — The composition of gold in jewellery (doc-13937)

Digital doc: Code puzzle — What did the grass say to the dirt? (doc-13938)

Digital doc: Summary Topic 4 (doc-23240)

Interactivity: Word search Topic 4 (int-2625)

Interactivity: Crossword Topic 4 (int-2626)

Interactivity: Sudoku Topic 4 (int-3186)

Exercise 4.6 Review questions

To answer questions online and to receive **immediate feedback** and **fully worked solutions** for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Understanding and fluency

1. Calculate these amounts.
 - a. $\$2.45 + \$13.20 + \$6.05$
 - b. $\$304.60 - \$126.25$
 - c. $\$9.65 \times 7$
2. What is $\$65.50 \div 11$? (Round your answer to the nearest 5 cents.)
3. Jill purchased a hand bag for \$250 and later sold it on eBay for \$330.
 - a. What is the percentage profit on the cost price?
 - b. What is the percentage profit on the selling price?
 - c. Compare the answers to parts a and b.

4. Natalie went shopping and bought a pair of bathers for \$38.95, a football for \$75.50, four pot plants at \$8.75 each and a photograph album for \$14.90. How much money did Natalie spend in total?
5. A discount of 18% on a tennis racquet reduced its price by \$16.91. What was the sale price?
6. A portable CD player bought for \$129 was later sold for \$85. What percentage loss was made on the sale?
7. Find the percentage profit on a sound bar purchased for \$320 and later sold for \$350.
8. A 15% discount reduced the price of a basketball by \$4.83. What was the original price?
9. Tim works in a sports shop. He purchased wholesale golf shirts for \$55 each. For how much did he sell the shirts, if he made 163% profit?

Communicating, reasoning and problem solving

10. Jacques' furniture shop had a sale with $\frac{1}{3}$ off the usual price of lounge suites. If the original price of a suite was \$5689 including GST, what will the sale price be including GST?
11. Goods listed at \$180 were discounted by 22%.
 - a. What was the sale price?
 - b. If they had sold for \$100, what would the percentage discount have been?
12. After a 5% discount, a telephone bill is \$79.50. How much was the bill originally?
13. Steve Smith buys a cricket bat for \$85, signs it and donates it for a sport's auction. If it sells for \$500:
 - a. what is the percentage increase in the bat's value
 - b. what is the dollar value of the signature?
14. Sets of 90-minute CDs are sold as in the following packs. Which is the best buy?

Cost (\$)	Number of CDs
\$3.25	6
\$4.99	10
\$7.50	15

15. Which is a better buy — 400 g of biscuits costing \$2.98, or a pack of biscuits with 400 g + 25% extra, costing \$3.28?
16. Antwert buys a pair of jeans for \$59.95. The original price tag was covered by a 30% sticker but the sign on top of the rack said an 'Additional 15% off already reduced prices.'
 - a. What was the original price of the jeans? Give your answer correct to the nearest 5 cents.
 - b. What percentage of the original cost did he end up saving?
17. Café Noir charges a 1% levy on the bill for trading on Sundays. If the final bill is \$55.55, what was the original price, taking into account that the levy has been charged and then 10% GST has been added?



Answers

Topic 4 Financial mathematics

Exercise 4.2 Discount

1. a. \$42 b. \$46.25 c. \$49.50 d. \$76
2. a. 10% b. 50% c. $33\frac{1}{3}\%$ d. 25%
3. a. \$850 b. \$200 c. \$83.60 d. \$104 e. \$64.70
4. a. \$45 b. \$45 c. \$36
5. a. 40% b. 28% c. 28% d. 22%
6. Estimate 25%
7. a. Mobile phone \$95 b. Surfboard and bike c. \$8.35 d. No
8. a. \$70 b. \$280
9. \$62.96
10. a. \$41.65 b. Yes
11. \$75.76
12. 20%
13. 25%
14. 17.3%
15. 30%
16. 60%
17. \$243.70
18. a. \$121.60; \$140.80 b. Profit
19. A
20. $\$1.00/\$12.00 \times 100\% = 8.33\%$, so this is a 8.33% discount.
21. B
22. a. 33.33%
 - b. This deal is worse than the deal offered in part a as it is equivalent to only a 25% discount; however, it should be used if you only want to buy two soaps.
23. No, the statement is not correct. For example, if you have a cost of \$100, a 50% discount = \$50 and a 40% discount (on that \$50) = \$20.

Total discount = \$70; this represents a 70% discount, not 90%.
24. Yes, there is a difference in the meanings. 75% off \$200 = \$150 off the price so would pay only \$50.

$75\% \text{ of } \$200 = \$150$, i.e. $\frac{3}{4}$ of \$200.
25. Henry pays \$954; Sancha pays \$991.20. Henry has the best buy.
26. 95% of \$63.20 = \$60.05; 75% of \$79 = \$59.25. The two methods calculate percentages of different amounts so result in different answers.
27. a. i. 10% of \$220 = \$22
\$220 + \$22 = \$242 ii. 110% of \$220 = \$242
- b. The answers are the same.
- c. The price increase is 10%. Add this to 100% to get 110% and then multiply 110% by the original price to give the new price.
28. a. 80% b. 85% c. 125% d. 105%
- e. 65% f. 111% g. 94% h. 200%
29. Discounts are a reduction in price

Exercise 4.3 Unitary method

1. \$8.55
2. Neither is better (they are both 62 cents per 100 g).
3. a. The 325-g option is the best buy. b. The 200-page option is the best buy.
4. a. \$180 b. \$30 c. \$240 d. \$2250 e. \$176.47
- f. \$422.22 g. \$400 h. \$350 i. \$600

5. a. \$180 b. \$11.40 c. \$120 d. \$2700 e. \$40 f. \$25800
6. a. \$400 b. \$640
7. Fruit and vegetables — \$20.55, Beverages — \$12.33, Frozen food — \$36.99, Toiletries — \$8.22, Baked goods — \$4.11
8. \$50
9. \$687.50
10. \$160
11. \$84600
12. a. 1.2 kg apples is the best buy
b. 400 mL of orange juice is the best buy
13. 500 g of cheese is the best buy.
14. \$216364
15. \$48.62
16. Fresh food — \$33.03, frozen food — \$21.02
17. \$215.29
18. a. \$54.55 b. \$70.59
19. a. \$21.33 b. \$18.13
20. a. \$218.18 b. \$170.18
21. \$112.50
22. a. \$2.38 b. \$2.13 c. \$3.33
23. Loss of \$463
24. Brand B. The cost for each bead for Brand B is cheaper (7 c compared to 7.5 c).
25. \$121.97
26. $(n - 3)(b + 2) = 80$
27. The amount required should always be considered, as the best buy will often be for the largest quantity. Waste due to spoilage and the best before date of the item should also be taken into account. The size of the item and storage space may also need to be considered. The total cost should also be considered, particularly if shopping on a budget.
28. Discuss with your teacher.
29. \$7.27

Exercise 4.4 Profit and loss

1. a. \$25.20 b. \$87 c. \$1690 d. \$53.25 e. \$932
2. a. $33\frac{1}{3}\%$ profit b. 25% profit c. 25% loss d. $13\frac{1}{3}\%$ profit e. 22.2% loss
3. 60%
4. a. \$545 b. 156%
5. \$120
6. a. 1 : 4 b. 25% c. 1 : 5 d. 20%
e. The ratio of the profit to the cost price as a fraction is the same as the percentage profit on the cost price.
7. a. \$325 b. 27% c. 21%
d. The percentage profit is greater on the cost price.
8. a. \$79.95 b. 57%
9. 18.75%
10. 166.5%
11. \$1643.40
12. a. \$39 b. \$27 c. \$274.45 d. \$66
13. a. \$960 b. \$51.75 c. \$685.75
14. a. \$1.00 profit per kg b. 55.6% c. 35.7%
d. The percentage profit is greater on the cost price.
15. a. \$77 b. 49% c. 33%
16. C
17. 172%

18. 60%, 40%; \$33 000, \$22 000

19. 50%

20. \$103 520

21.

Cost per item	Items sold	Sale price	Total profit
\$4.55	504	\$7.99	\$1 733.76
\$20.00	402	\$40.00	\$8 040.00
\$6.06	64 321	\$9.56	\$225 123.50
\$47.65	672	\$89.95	\$28 425.60

22. a. i. \$100, \$5500, \$2.50

ii. Car, shoes, cake

iii. Not fair; profit should be compared as a proportion of cost.

b. i. 500%, 20.75%, 250%

ii. Shoes, cake, car

iii. Fairer than in part a

c. i. 83.3̄, 17%, 71.4%

ii. Shoes, cake, car

iii. Not fair; the profit should be calculated on the cost.

23. Calculate the profit or loss of the item as a percentage of the cost price.

24. James paid \$3240. The total percentage loss was 46%.

Exercise 4.5 Goods and Services Tax (GST)

1. GST is a tax of 10% levied by the Australian federal government on goods and services.

2. a–e. Yes f. No

3. a. i. 45 cents ii. \$4.95

b. i. 21 cents ii. \$2.33

4. a. \$3.50 b. \$4.09 c. \$2.00 d. \$5.91

5. \$60.21

6. \$1020.80

7. a. $1.1(85t + 40)$ b. \$371.25

8. a. $1.1(45x + 100)$ b. \$1100

9. \$40.91

10. \$98.18

11. \$2.75

12. a. \$11.17 b. \$55.83

13. NZ\$92

14. GST is equal to 10% of the pre-GST price, which is less than 10% of the GST-inclusive price. For example, if an item costs \$100 pre-GST, the GST would be \$10 and the GST-inclusive price would be \$110. 10% of the GST-inclusive price would be \$11, not \$10.

15. 10% off the GST-inclusive price is equal to 99% of the pre-GST price. For example, if an item costs \$100 pre-GST, the GST would be \$10 and the GST-inclusive price would be \$110. Taking 10% off the GST-inclusive price would be \$99, not \$100.

16. a. \$1400 b. 11

17. a. 12 Kopeks, 33 Plens b. \$4.94 GST, total price $\approx$ \$54.27

18. a. $\frac{P}{10}$, $\frac{11P}{10}$ b. $\frac{10A}{11}$, $\frac{A}{11}$

19. Inclusive includes GST in the total price and exclusive excludes GST from the total price.

Investigation | Rich task

1. Metals used as alloying elements with gold are silver, copper, zinc, palladium, nickel, iron and aluminium.

2. 27 g gold, 8.01 g copper, 0.99 g silver

3. 6 g, $\frac{1}{6}$

4. From pink to rose to red gold the percentage of silver decreases, causing the gold alloy to darken in colour. At the same time, the percentage of copper increases, also contributing to the darker colour.
5. The copper would colour the gold with its familiar reddish colour so that it would not be white.
6. Red gold and blue gold each have 75% gold and 25% of another metal. In the case of red gold the contributing metal is copper; blue gold contains iron.
7. 19 carat
8. Answers will vary but should include that sterling silver is 92.5% silver and 7.5% copper. Silver from Mexico is of a lower quality. Teacher to check student answers.

Exercise 4.6 Review questions

1. a. \$21.70 b. \$178.35 c. \$67.55
2. \$5.95
3. a. 32% b. 24.24% c. The percentage profit is greater on the cost price.
4. \$164.35
5. \$77.03
6. 34%
7. 9.375%
8. \$32.20
9. \$144.65
10. \$3792.67
11. a. \$140.40 b. 44.4%
12. \$83.68
13. a. 488% b. \$415
14. Options are 50c per CD, 49.9c per CD, 54.17c per CD.
Therefore the \$4.99 pack is the best buy.
15. First pack costs 76c per 100 g; second pack costs 66c per 100 g; so the second pack is the best buy.
16. a. \$100.75 b. 40.5% saved
17. \$50

TOPIC 5

Ratios and rates

5.1 Overview

Numerous **videos** and **interactivities** are embedded just where you need them, at the point of learning, in your learnON title at www.jacplus.com.au. They will help you to learn the concepts covered in this topic.

5.1.1 Why learn this?

Ratios are used to compare different quantities. For example, they can be used to describe money exchanges, ingredients used in cooking or perspective in drawings. Some ratios have been found to occur in nature, and in ancient times some philosophers believed that numbers could explain the nature of the universe.

Rates enable us to compare quantities expressed in different units. For example, how far will the family car travel on 30 litres of unleaded petrol when travelling in traffic at an average speed of 55 km/h?



LEARNING SEQUENCE

- 5.1 Overview
- 5.2 Introduction to ratios
- 5.3 Simplifying ratios
- 5.4 Proportion
- 5.5 Comparing ratios
- 5.6 Dividing in a given ratio
- 5.7 Rates
- 5.8 Interpreting graphs
- 5.9 Review

LEARNING OUTCOMES

A student:

- communicates and connects mathematical ideas using appropriate terminology, diagrams and symbols MA4-1WM
- applies appropriate mathematical techniques to solve problems MA4-2WM
- recognises and explains mathematical relationships using reasoning MA4-3WM
- operates with ratios and rates, and explores their graphical representations MA4-7NA

CONTENT DESCRIPTIONS

Students:

Recognise and solve problems involving simple ratios (ACMNA173)

Solve a range of problems involving ratios and rates, with and without the use of digital technologies (ACMNA188)

Investigate, interpret and analyse graphs from authentic data (ACMNA180)

Source: NSW Syllabus for the Australian curriculum

Note: Your teacher may now set you a pre-test to determine how familiar you are with the content in this topic.

5.2 Introduction to ratios

5.2.1 Ratios

- **Ratios** are used to compare quantities of the same kind.
- Ratios do not have names or units of measurement.
- The order of the numbers in a ratio is important. If the ratio of flour to water is 1 : 4, the 1 corresponds to the quantity of flour and the 4 corresponds to the quantity of water.
- Ratios can also be written in fractional form: $1 : 4 \Leftrightarrow \frac{1}{4}$.
- Ratios can also be written as percentages; for example, if 20% of a class of Year 8 students walked to school, then 80% did not walk to school — a ratio of 1 : 4.
- Before ratios are written, the numbers must be expressed in the *same* units of measurement. Once the units are the same, they can be omitted.
- Ratios contain only whole numbers.

ACTIVITY: RATIO STATIONS

Equipment: coloured blocks or counters (at least three different colours), paper, pen

1. Set up four stations around the room, each with at least 10 blocks of at least three different colours.
2. Divide the board into four sections, one for each station so that people can write their ratios on the board.
3. In small groups, begin at any station and write at least *three* different ratios to describe the relationships between the blocks. At least one of these ratios must be a ratio of a part to a whole. Record these ratios in your book, simplifying where possible. Make sure that you write down what your ratio represents, such as red blocks to blue blocks.
4. Move around the stations and repeat step 3, visiting all stations if time permits.
5. Share your findings by writing the ratios on the board.
6. What is the difference between comparing a part to a whole and comparing a part to another part?

WORKED EXAMPLE 1

Look at the completed game of noughts and crosses at right and write the ratios of:

a crosses to noughts

b noughts to unmarked spaces.

X	O	
X	X	X
O		O

THINK

a Count the number of crosses and the number of noughts. Write the 2 numbers as a ratio (the number of crosses must be written first).

b Count the number of noughts and the number of unmarked spaces. Write the 2 numbers as a ratio, putting them in the order required (the number of noughts must be written first).

WRITE

a 4 : 3

b 3 : 2

WORKED EXAMPLE 2

Rewrite the following statement as a ratio: 7 mm to 1 cm.

THINK

- 1 Express both quantities in the same units. To obtain whole numbers, convert 1 cm to mm (rather than 7 mm to cm).
- 2 Omit the units and write the 2 numbers as a ratio.

WRITE

7 mm to 1 cm
7 mm to 10 mm
7 : 10

5.2.2 Expressing one part of a ratio as a fraction of the whole

- If you are given a ratio where the parts of the ratio together represent the whole, you can express one part of the ratio as a fraction of the whole.

WORKED EXAMPLE 3

**The number of tries scored by two teams in a rugby match was in the ratio 1 : 3.
Express the amount of tries the first team scored as a fraction of the whole number of tries scored.**

THINK

- 1 Determine the total amount of parts in the ratio by summing the parts.
- 2 Express the first part (1) as a fraction of the whole (4).
- 3 Write the answer

WRITE

$1 + 3 = 4$
 $\frac{1}{4}$
The first team scored $\frac{1}{4}$ of the whole number of tries scored.

5.2.3 Ratios involving more than two numbers

- Ratios can be extended to compare more than two quantities of the same kind. Each quantity being compared should be split by a : symbol.
- For example, if a family has 1 dog, 2 cats and 3 goldfish, the ratio of dogs to cats to goldfish is 1 : 2 : 3.

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Interactivity: Fruit salad ratio (int-0076)

Interactivity: Introduction to ratios (int-3733)

Digital doc: SkillsHEET Converting units of length, capacity and time (doc-6888)

Digital doc: SkillsHEET Highest common factor (doc-6889)

Exercise 5.2 Introduction to ratios

Individual pathways

PRACTISE

Questions:
1–8, 10, 14–16

CONSOLIDATE

Questions:
1–4, 5a–p, 6–9, 12, 14–17

MASTER

Questions:
1–4, 5k–t, 7–19

   Individual pathway interactivity: int-4412

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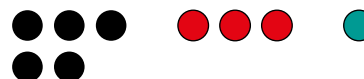
Understanding and fluency

1. **WE1** Look at the completed game of noughts and crosses and write the ratios of:

- noughts to crosses
- crosses to noughts
- crosses to total number of spaces
- total number of spaces to noughts
- noughts in the top row to crosses in the bottom row.

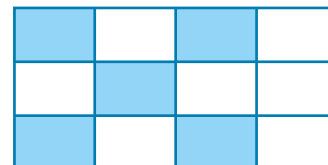
X	X	O
O	O	X
X	X	O

2. Look at the coloured circles at right and then write the following ratios.



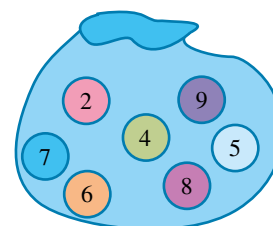
- Black : red
- Red : black
- Aqua : black
- Black : aqua
- Aqua : red
- Black : (red and aqua)
- Aqua : (black and red)
- Black : total circles
- Red : total circles
- Aqua : total circles

3. For the diagram shown at right, write the following ratios.



- Shaded parts : unshaded parts
- Unshaded parts : shaded parts
- Shaded parts : total parts

4. In the bag of numbers shown at right, write the ratios of:



- even numbers to odd numbers
- prime numbers to composite numbers
- numbers greater than 3 to numbers less than 3
- multiples of 2 to multiples of 5
- numbers divisible by 3 to numbers not divisible by 3.

5. **WE2** Rewrite each of the following statements as a ratio.

a. 3 mm to 5 mm	b. 6 s to 19 s
c. \$4 to \$11	d. 7 teams to 9 teams
e. 1 goal to 5 goals	f. 9 boys to 4 boys
g. 3 weeks to 1 month	h. 3 mm to 1 cm
i. 17 seconds to 1 minute	j. 53 cents to \$1
k. 11 cm to 1 m	l. 1 g to 1 kg
m. 1 L to 2 kL	n. 7 hours to 1 day
o. 5 months to 1 year	p. 1 km to 27 m
q. 7 apples to 1 dozen apples	r. 13 pears to 2 dozen pears
s. 3 females to 5 males	t. 1 teacher to 22 students



6. Out of 100 people selected for a school survey, 59 were junior students, 3 were teachers and the rest were senior students. Write the following ratios.

- Teachers : juniors
- Juniors : seniors
- Seniors : teachers
- Teachers : students
- Juniors : other members of the survey

7. **WE3** Julia purchased 6 apples and 11 oranges from the local farmers' market. Express the number of apples as a fraction of the whole number of pieces of fruit purchased.

8. Write each of the following as a mathematical ratio.
- In their chess battles, Lynda has won 24 games and Karen has won 17.
 - There are 21 first-division teams and 17 second-division teams.
 - Nathan could long jump twice as far as Rachel.
 - On the camp there were 4 teachers and 39 students.
 - In the mixture there were 4 cups of flour and 1 cup of milk.
 - Elena and Alex ran the 400 m in the same time.
 - The radius and diameter of a circle were measured.
 - The length of a rectangle is three times its width.
 - On Friday night 9 out of every 10 people enjoyed the movie.
 - The length of one side of an equilateral triangle compared to its perimeter.
 - The length of a regular hexagon compared to its perimeter.
 - The number of correct options compared to incorrect options in a multiple-choice question containing five options.



9. A pair of jeans originally priced at \$215 was purchased for \$179. What is the ratio of:
- the original price compared to the selling price
 - the original price compared to the discount?
10. Matthew received a score of 97% for his Maths test. What is the ratio of:
- the marks received compared to marks lost
 - the marks lost compared to total marks possible?
11. Given the following fractions, express the first number in the ratio as a fraction of the whole.
- 3 : 7
 - 4 : 11
 - 13 : 12
 - 7 : 2

12. For each comparison that follows, state whether a ratio could be written and give a reason for your answer. (*Remember: Before ratios are written, numbers must be expressed in the same unit of measurement.*)

- Anna's mass is 55 kg. Her cat has a mass of 7 kg.
 - Brian can throw a cricket ball 40 metres, and John can throw the same ball 35 metres.
 - The cost of painting the wall was \$55; its area is 10 m².
 - For a trip, the car's average speed was 85 km/h; the trip took 4 h.
 - Brett's height is 2.1 m. Matt's height is 150 cm.
 - Jonathon apples cost \$2.40 per dozen; Delicious apples cost \$3.20 per dozen.
 - Mary is paid \$108; she works 3 days a week.
 - David kicked 5 goals and 3 behinds (33 points); his team scored 189 points.
13. a. If 17% of students in a class have sports training once a week, write as a ratio the number of students who have sports training once a week compared to the students who do not.
- b. A survey found that only 3% of workers take their lunch to work on a regular basis. Write as a ratio the number of workers who do not take their lunch to work compared to those who do.
14. Rewrite the following statements as ratios.
- \$3 to \$5 to \$9
 - 2 days to 1 week to 2 weeks
 - 2 shares to 3 shares to 5 shares
 - 1 principal to 5 teachers to 95 students



Communicating, reasoning and problem solving

15. A recipe to make enough spaghetti bolognese to feed four people needs 500 grams of mincemeat. According to this recipe, how much mincemeat would be needed to make enough spaghetti bolognese to feed a group of nine people? Show all working.
16. Out of 100 people selected for a school survey, 68 were students, 5 were teachers and the rest were support staff.
- Write the ratio of teachers to students.
 - Write the ratio of support staff to teachers.
 - Write the ratio of students to people surveyed.
 - Write the number of students as both a fraction and a percentage of the number of people surveyed.
 - Write the number of students as both a fraction and a percentage of the number of teachers surveyed.
17. In class 8A, there are 4 boys and 5 girls. In class 8B, there are 12 boys and 15 girls. In class 8C, there are 8 boys and 6 girls.
- For each class, write the ratio of boys to girls.
 - In which two classes are the proportions of boys and girls the same?
 - Class 8A joined class 8B to watch a program for English. What was the resulting boy-to-girl ratio?
 - Class 8C joins class 8B for PE lessons. What is the boy-to-girl ratio in the combined PE class?
 - Is the ratio of boys to girls watching the English program equivalent to the ratio of boys to girls in the combined PE class?
18. Why doesn't a simplified ratio have units?
19. The ratio of gold coins to silver coins in a purse is 2 : 5. If there are 10 gold coins in the purse, what is the smallest number of coins that needs to be added to the purse so that the ratio of gold coins to silver coins changes to 3 : 4?



5.3 Simplifying ratios

5.3.1 Simplifying ratios

- When the numbers in a ratio are multiplied or divided by the same number to obtain another ratio, these two ratios are said to be equivalent. (This is a similar process to obtaining equivalent fractions.)
- Like fractions, ratios are usually written in simplest form; that is, reduced to lowest terms.

WORKED EXAMPLE 4

Express the ratio 16 : 24 in simplest form.

THINK

- Write the ratio.
- Determine the largest number by which both 16 and 24 can be divided (i.e. what is the highest common factor?). It is 8.
- Divide both 16 and 24 by 8 to obtain an equivalent ratio in simplest form.

WRITE

$$\begin{array}{c} 16 : 24 \\ \div 8 \quad \div 8 \\ \hline 2 : 3 \end{array}$$

WORKED EXAMPLE 5

Write the ratio of 45 cm to 1.5 m in simplest form.

THINK

- 1 Write the question.
- 2 Express both quantities in the same units by changing 1.5 m into cm. (1 m = 100 cm)
- 3 Omit the units and write the 2 numbers as a ratio.
- 4 Simplify the ratio by dividing both 45 and 150 by 15 — the HCF.

WRITE

45 cm to 1.5 m

45 cm to 150 cm

$$\begin{array}{ccc} & 45 : 150 & \\ \div 15 \swarrow & & \searrow \div 15 \\ & 3 : 10 & \end{array}$$

WORKED EXAMPLE 6

Simplify the following ratios.

a $\frac{2}{5} : \frac{7}{10}$

b $\frac{5}{6} : \frac{5}{8}$

THINK

- a 1 Write the fractions in ratio form.
- 2 Write equivalent fractions using the lowest common denominator (in this case, 10).
 - 3 Multiply both fractions by 10.
 - 4 Check if the remaining whole numbers that form the ratio can be simplified. In this case they cannot.
- b 1 Write the fractions in ratio form.
- 2 Write equivalent fractions using the lowest common denominator (in this case, 24).
 - 3 Multiply both fractions by 24.
 - 4 Check if the remaining whole numbers that form the ratio can be simplified. In this case divide each by the HCF of 5.

WRITE

a $\frac{2}{5} : \frac{7}{10}$

$$\begin{array}{ccc} & \frac{4}{10} : \frac{7}{10} & \\ \times 10 \swarrow & & \searrow \times 10 \\ & 4 : 7 & \end{array}$$

b $\frac{5}{6} : \frac{5}{8}$

$$\begin{array}{ccc} & \frac{20}{24} : \frac{15}{24} & \\ \times 24 \swarrow & & \searrow \times 24 \\ & 20 : 15 & \\ \div 5 \swarrow & & \searrow \div 5 \\ & 4 : 3 & \end{array}$$

- If the ratio uses decimals, we multiply by the smallest power of 10 that will produce a whole number for both parts of the ratio.

WORKED EXAMPLE 7

Write the following ratios in simplest form.

a 2.1 to 3.5

b 1.4 : 0.75

THINK

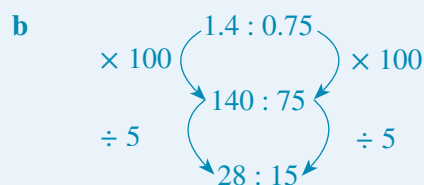
- a 1 Write the decimals in ratio form.
- 2 Both decimals have one decimal place, so multiplying each by 10 will produce whole numbers.
 - 3 Simplify by dividing both numbers by the HCF of 7.

WRITE

a

$$\begin{array}{ccc} & 2.1 : 3.5 & \\ \times 10 \swarrow & & \searrow \times 10 \\ & 21 : 35 & \\ \div 7 \swarrow & & \searrow \div 7 \\ & 3 : 5 & \end{array}$$

- b** 1 Write the decimals in ratio form.
- 2 Because 0.75 has two decimal places, we multiply each decimal by 100 to produce whole numbers.
- 3 Simplify by dividing both numbers by the HCF of 5.



- If the ratio contains algebraic terms, divide both parts of the ratio by the highest common factor including common algebraic terms.

WORKED EXAMPLE 8

Simplify the ratios:

a $10a^2b : 15ab^2$

b $3mn : 6mn$

THINK

- a** 1 Write the ratios.
- 2 Simplify the ratio by dividing both sides by $5ab$ (the HCF).
- 3 Cancel common factors to obtain the ratio in simplest form.
- b** 1 Write the ratios.
- 2 Simplify the ratio by dividing both sides by $3mn$ (the HCF).
- 3 Cancel common factors to obtain the ratio in simplest form.

WRITE

a $10a^2b : 15ab^2$

$$\frac{10a^2b}{5ab} : \frac{15ab^2}{5ab}$$

$$2a : 3b$$

b $3mn : 6mn$

$$\frac{3mn}{3mn} : \frac{6mn}{3mn}$$

$$1 : 2$$

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Interactivity: Simplifying ratios (int-3734)

Digital doc: SkillsHEET Simplifying fractions (doc-6890)

Digital doc: SkillsHEET Finding and converting to the lowest common denominator (doc-6891)

Digital doc: SpreadSHEET Simplifying ratios (doc-2213)

Exercise 5.3 Simplifying ratios

Individual pathways

PRACTISE

Questions:
1–7, 9, 12, 14, 15

CONSOLIDATE

Questions:
1, 2, 3a–l, 4a–f, 5, 6–11, 14, 15

MASTER

Questions:
1m–t, 2, 3m–t, 4f–j, 5–10, 13–18

Individual pathway interactivity: int-4413

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To answer questions online and to receive **immediate feedback** and **fully worked solutions** for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Understanding and fluency

1. **WE4** Express each ratio in simplest form.

a. 2 : 4	b. 3 : 9	c. 5 : 10	d. 6 : 18
e. 12 : 16	f. 15 : 18	g. 24 : 16	h. 21 : 14
i. 25 : 15	j. 13 : 26	k. 15 : 35	l. 27 : 36
m. 36 : 45	n. 42 : 28	o. 45 : 54	p. 50 : 15
q. 56 : 64	r. 75 : 100	s. 84 : 144	t. 88 : 132

2. Complete the patterns of equivalent ratios.

a. 1 : 3	b. 2 : 1	c. 2 : 3	d. 64 : 32	e. 48 : 64
2 : 6	4 : 2	4 : 6	___ : 16	24 : ___
___ : 9	___ : 4	6 : ___	___ : 8	12 : ___
___ : 12	___ : 8	___ : 12	8 : ___	___ : 8
5 : ___	20 : ___	___ : 24	___ : 1	___ : ___

3. **WE5** Write the following ratios in simplest form.

a. 8 cm to 12 cm	b. \$6 to \$18
c. 50 s to 30 s	d. 80 cm to 2 m
e. 75 cents to \$3	f. 2 h to 45 min
g. 300 mL to 4 L	h. 500 g to 2.5 kg
i. 45 mm to 2 cm	j. \$4 to \$6.50
k. 2500 L to 2 kL	l. 2500 m to 2 km
m. 30 cents to \$1.50	n. 2 h 45 min to 30 min
o. 200 m to 0.5 km	p. 0.8 km to 450 m
q. $1\frac{1}{2}$ min to 300 s	r. 1.8 cm to 12 mm
s. 3500 mg to 1.5 g	t. \$1.75 to \$10.50

4. Compare each of the following, using a mathematical ratio (in simplest form).

- The Panthers won 8 games and the Eels won 10 games.
- This jar of coffee costs \$4 but that one costs \$6.
- While Joanne made 12 hits, Holly made 8 hits.
- In the first innings, Ian scored 48 runs and Adam scored 12 runs.
- During the race, Rebecca's average speed was 200 km/h and Donna's average speed was 150 km/h.
- In the basketball match, the Tigers beat the Magic by 105 points to 84 points.
- The capacity of the plastic bottle is 250 mL and the capacity of the glass container is 2 L.
- Joseph ran the 600 m in 2 minutes while Maya ran the same distance in 96 seconds.
- In the movie audience, there were 280 children and 35 adults.
- On a page in the novel *Moby Dick* there are 360 words. Of these, 80 begin with a vowel.

5. A serving of a particular breakfast cereal contains:

3.6 g of protein	0.4 g of fat
20 g of carbohydrate	1 g of sugar
3.3 g of dietary fibre	84 mg of sodium.

Find the following ratios in simplest form:

- sugar to carbohydrate
- fat to protein
- protein to fibre
- sodium to protein.



6. **WE6** Write the following ratios in simplest form.

a. $\frac{1}{3}$ to $\frac{2}{3}$	b. $\frac{5}{7}$ to $\frac{6}{7}$	c. $\frac{1}{4}$: $\frac{1}{2}$	d. $\frac{2}{3}$: $\frac{5}{6}$
e. $\frac{4}{5}$ to $\frac{9}{20}$	f. $\frac{2}{3}$: $\frac{3}{5}$	g. $\frac{3}{10}$: 1	h. $1\frac{2}{3}$ to $\frac{1}{3}$
i. $1\frac{1}{4}$: $1\frac{1}{2}$	j. $3\frac{1}{3}$ to $2\frac{1}{2}$	k. 1 : $1\frac{3}{5}$	l. $3\frac{1}{4}$ to $2\frac{4}{5}$

7. **WE7** Write the following ratios in simplest form.

a. 0.7 to 0.9	b. 0.3 : 2.1	c. 0.05 to 0.15
d. 0.8 : 1	e. 0.25 : 1.5	f. 0.375 to 0.8
g. 0.95 : 0.095	h. 1 to 1.25	i. 0.01 : 0.1
j. 1.2 : 0.875	k. 0.004 to 0.08	l. 0.5 : 0.92

8. **WE8** Simplify these ratios.

a. $2a : 10b$	b. $6p : 3p$	c. $2x^2 : 3x$	d. $7xy^2 : 14xy$
e. $36m^3n^2 : 48m^2n^2$	f. $ab : 4ab^2$	g. $10^3x : 10x^3$	h. $50(cd)^2 : 25c$

9. **MC** Toowoomba's population is 80 000 and Brisbane's population is 1.8 million. The ratio of Toowoomba's population to that of Brisbane is:

- A. 2 : 45 B. 4 : 9 C. 1 : 1.8 D. 9 : 4 E. none of these

10. **MC** When he was born, Samuel was 30 cm long. Now, on his 20th birthday, he is 2.1 m tall. The ratio of his birth height to his present height is:

- A. 3 : 7 B. 1 : 21 C. 7 : 10 D. 1 : 7 E. none of these

11. **MC** The cost of tickets to two different concerts is in the ratio 3 : 5. If the more expensive ticket is \$110, the cheaper ticket is:

- A. \$180 B. \$80 C. \$50 D. \$45 E. \$66

12. **MC** A coin was tossed 100 times and Tails appeared 60 times. The ratio of Heads to Tails was:

- A. 2 : 3 B. 3 : 5 C. 3 : 2 D. 5 : 3 E. 2 : 5

13. **MC** Out of a 1.25 L bottle of soft drink, I have drunk 500 mL. The ratio of soft drink remaining to the original amount is:

- A. 2 : 3 B. 3 : 5 C. 3 : 2 D. 5 : 3 E. 2 : 5

Communicating, reasoning and problem solving

14. The total attendance at a large outdoor concert was 32 200 people. Of this total, 13 800 people were female.

- How many males attended the concert?
- Compare the number of females to males by writing the ratio in simplest form.
- Write the number of females as a fraction of the number of males.
- What percentage of the crowd was female? How does this compare with your answer to part c?

15. In a primary school that has 910 students, 350 students are in the senior school and the remainder are in the junior school. Of the senior school students, 140 are females. There are the same number of junior males and junior females. Write the following ratios in simplest form:

- senior students to junior students
- senior females to senior males
- senior males to total senior students
- junior males to senior males
- junior females to whole school population.

16. Compare the following, using a mathematical ratio (in simplest form).

- Of the 90 000 people who attended the test match, 23 112 were females. Compare the number of males to females.
- A Concorde jet cruises at 2170 km/h but a Cessna cruises at 220 km/h. Compare their speeds.
- A house and land package is sold for \$250 000. If the land was valued at \$90 000, compare the land and house values.
- In a kilogram of fertiliser, there are 550 g of phosphorus. Compare the amount of phosphorus to other components of the fertiliser.
- Sasha saves \$120 out of his take-home pay of \$700 each fortnight. Compare his savings with his expenses.

17. The table at right represents the selling price of a house over a period of time.

- Compare the price of the house purchased in December 2010 with the purchase price in April 2014 as a ratio in simplest form.
- Compare the price of the house purchased in December 2010 with the purchase price in December 2014 as a ratio in simplest form.
- Compare the price of the house purchased in December 2010 with the purchase price in March 2018 as a ratio in simplest form.
- How much has the value of the house increased from December 2010 to March 2018?
 - Compare the increase obtained in part **d i** to the price of the house purchased in December 2010 as a ratio in simplest form.
- Comment on the results obtained in part **d**.

Date of sale	Selling price
March 2018	\$1.275 million
December 2014	\$1.207 million
April 2014	\$1.03 million
December 2010	\$850 000

18. In what ways is simplifying ratios similar to simplifying fractions?

5.4 Proportion

5.4.1 Proportion

- A **proportion** is a statement of equality of two ratios.
- In general, if $\frac{a}{b} = \frac{c}{d}$, then, using cross-multiplication, $a \times d = c \times b$.

WORKED EXAMPLE 9

Use the cross-multiplication method to determine whether the following pair of ratios is in proportion:

6 : 9, 24 : 36.

THINK

- Write the ratios in fraction form.
- Perform a cross-multiplication.
- Check whether the products are equal.

WRITE

$$\frac{6}{9}, \frac{24}{36}$$

$$6 \times 36 = 216, \quad 24 \times 9 = 216$$

$$216 = 216$$

Therefore, the ratios are in proportion.

WORKED EXAMPLE 10

Find the value of a in the following proportion: $\frac{a}{3} = \frac{6}{9}$.

THINK

- Write the proportion statement.
- Cross-multiply and equate the products.
- Solve for a by dividing both sides of the equation by 9.

WRITE

$$\frac{a}{3} = \frac{6}{9}$$

$$a \times 9 = 6 \times 3$$

$$9a = 18$$

$$\frac{9a}{9} = \frac{18}{9}$$

$$a = 2$$

DISCUSSION

Provide an example to explain how proportion statements can be applied to other subject areas.

WORKED EXAMPLE 11

The ratio of girls to boys on the school bus was 4 : 3. If there were 28 girls, how many boys were there?

THINK

1 Let the number of boys be b and write a proportion statement. (Since the first number in the ratio represents girls, place 28 (the number of girls) as the numerator.)

2 Cross-multiply and equate the products.

3 Solve for b by dividing both sides by 4.

4 Write the answer.

WRITE

$$\frac{4}{3} = \frac{28}{b}$$

$$4 \times b = 28 \times 3$$

$$4b = 84$$

$$\frac{4b}{4} = \frac{84}{4}$$

$$b = 21$$

There are 21 boys.



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Interactivity: Proportion (int-3735)

Digital doc: SpreadSHEET Proportion (doc-2214)

Digital doc: WorkSHEET Ratios and rates I (doc-2216)

Exercise 5.4 Proportion

Individual pathways

PRACTISE

Questions:

1–4, 7, 8, 11, 13

CONSOLIDATE

Questions:

1a–h, 2a–h, 3–6, 8, 9, 11, 13, 14

MASTER

Questions:

1g–l, 2g–l, 3f–j, 4f–j, 5–10, 12–15

Individual pathway interactivity: int-4414

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Understanding and fluency

1. **WE9** Use the cross-multiplication method to determine whether the following pairs of ratio are in proportion.

a. $2 : 3$; $8 : 12$

b. $4 : 7$; $8 : 14$

c. $5 : 7$; $10 : 14$

d. $5 : 8$; $10 : 16$

e. $\frac{7}{9}$; $\frac{21}{25}$

f. $\frac{3}{8}$; $\frac{12}{32}$

g. $\frac{14}{16}$; $\frac{5}{9}$

h. $\frac{11}{12}$; $\frac{7}{8}$

i. $\frac{13}{15}$; $\frac{6}{7}$

j. $\frac{8}{9}$; $\frac{24}{27}$

k. $\frac{3}{5}$; $\frac{6}{8}$

l. $\frac{21}{18}$; $\frac{49}{42}$

2. **WE10** Find the value of a in each of the following proportions.

a. $\frac{a}{2} = \frac{4}{8}$

b. $\frac{a}{6} = \frac{8}{12}$

c. $\frac{a}{9} = \frac{2}{3}$

d. $\frac{3}{a} = \frac{9}{12}$

e. $\frac{7}{a} = \frac{14}{48}$

f. $\frac{10}{a} = \frac{3}{15}$

g. $\frac{3}{7} = \frac{a}{28}$

h. $\frac{12}{10} = \frac{a}{5}$

i. $\frac{8}{12} = \frac{a}{9}$

j. $\frac{35}{7} = \frac{5}{a}$

k. $\frac{24}{16} = \frac{6}{a}$

l. $\frac{30}{45} = \frac{2}{a}$

3. **WE11** Solve each of the following, using a proportion statement and the cross-multiplication method.

a. The ratio of boys to girls in a class is 3 : 4. If there are 12 girls, how many boys are in the class?

b. In a room the ratio of length to width is 5 : 4. If the width is 8 m, what is the length?

c. The team's win-loss ratio is 7 : 5. How many wins has it had if it has had 15 losses?

d. A canteen made ham and chicken sandwiches in the ratio 5 : 6. If 20 ham sandwiches were made, how many chicken sandwiches were made?

e. The ratio of concentrated cordial to water in a mixture is 1 : 5. How much concentrated cordial is needed for 25 litres of water?

f. The ratio of chairs to tables is 6 : 1. If there are 42 chairs, how many tables are there?

g. The ratio of flour to milk in a mixture is 7 : 2. If 14 cups of flour are used, how much milk is required?

h. The ratio of protein to fibre in a cereal is 12 : 11. If there are 36 grams of protein, what is the mass of fibre?

i. In a supermarket, the ratio of 600 mL cartons of milk to litre cartons is 4 : 5. If there are sixty 600 mL cartons, how many litre cartons are there?

j. In a crowd of mobile-phone users, the ratio of men to women is 7 : 8. How many women are there if there are 2870 men?

4. Although we know that only whole numbers are used in ratios, sometimes in a proportion statement the answer can be a fraction or a mixed number. Consider the following proportion:

$$\begin{aligned}\frac{a}{6} &= \frac{7}{4} \\ a \times 4 &= 7 \times 6 \\ 4a &= 42 \\ a &= 10.5 \left(\text{or } 10\frac{1}{2} \right)\end{aligned}$$

Calculate the value of a in each of the following proportion statements. Write your answer correct to 1 decimal place.

a. $\frac{a}{7} = \frac{8}{5}$

b. $\frac{a}{6} = \frac{4}{5}$

c. $\frac{a}{3} = \frac{7}{10}$

d. $\frac{a}{9} = \frac{9}{10}$

e. $\frac{5}{a} = \frac{7}{10}$

f. $\frac{8}{a} = \frac{6}{7}$

g. $\frac{9}{7} = \frac{a}{6}$

h. $\frac{13}{6} = \frac{a}{5}$

i. $\frac{9}{15} = \frac{7}{a}$

j. $\frac{7}{8} = \frac{9}{a}$

5. Write a proportion statement for each situation and then solve the problem. If necessary, write your answer correct to 1 decimal place.

a. A rice recipe uses the ratio of 1 cup of rice to 3 cups of water. How many cups of rice can be cooked in 5 cups of water?

b. Another recipe states that 2 cups of rice are required to serve 6 people. If you have invited 11 people, how many cups of rice will you need?

c. In a chemical compound there should be 15 g of chemical A to every 4 g of chemical B. If my compound contains 50 g of chemical A, how many grams of chemical B should it contain?

d. A saline solution contains 2 parts of salt to 17 parts of water. How much water should be added to 5 parts of salt?

e. To mix concrete, 2 buckets of sand are needed for every 3 buckets of blue metal. For a big job, how much blue metal will be needed for 15 buckets of sand?



6. Decide whether a proportion statement could be made for each of the following ratios.
- Height : age
 - Mass : age
 - Intelligence : age
 - Distance : time
 - Cost : number
 - Age : shoe size
 - Sausages cooked : number of people
 - Eggs : milk (in a recipe)
 - Number of words : pages typed
 - Length : area (of a square)
7. **MC** a. If $\frac{p}{q} = \frac{l}{m}$, then:
- $p \times q = l \times m$
 - $p \times l = q \times m$
 - $p \times m = l \times q$
 - $\frac{p}{m} = \frac{l}{q}$
 - none of these is true.
- b. If $\frac{x}{3} = \frac{y}{6}$, then:
- $x = 2$ and $y = 4$
 - $x = 1$ and $y = 2$
 - $x = 3$ and $y = 6$
 - $x = 6$ and $y = 12$
 - all of these are true.
- c. If $\frac{23}{34} = \frac{x}{19}$ then, correct to the nearest whole number, x equals:
- 13
 - 12
 - 34
 - 28
 - 17
- d. The directions on a cordial bottle suggest mixing 25 mL of cordial with 250 mL of water. How much cordial should be mixed with 5.5 L of water?
- 0.55 mL
 - 5.5 mL
 - 55 mL
 - 550 mL
 - 5500 mL

Communicating, reasoning and problem solving

8. In a family, 3 children receive their allowances in the ratio of their ages, which are 16 years, 14 years and 10 years. If the oldest child receives \$32, how much do the other two children receive? Show your working.
9. In jewellery, gold is often combined with other metals. 'Pink gold' is a mixture of pure gold, copper and silver in the ratio of 15 : 4 : 1. 'White gold' is a mixture of pure gold and platinum in the ratio of 3 : 1.
- What fraction of both pink and white gold jewellery is pure gold?
Pure gold is 24 carats and is not mixed with other metals. For most jewellery, however, 18-carat gold is used.
 - Using your answer to part a, show why jewellery gold is labelled 18 carats.
 - If the copper and silver in an 18-carat bracelet weigh a combined 2 grams, what is the weight of gold in the bracelet?
 - If the price of gold is \$35 per gram, what is the cost of the gold in the bracelet?
10. Two classes each contain 8 boys. In one class, the ratio of boys to girls is 1 : 2; in the other it is 2 : 1. If the two classes combine, what will the new ratio be? Justify your answer.



11. The nutritional panel for a certain cereal is shown at right. Sugars are one form of carbohydrate. Calculate the ratio of sugars to total carbohydrates in the cereal.
12. The solution to this question contains some gaps. Rewrite the solution, replacing the empty boxes with the appropriate numbers.
On a map for which the scale is 1 : 20 000, what distance in cm represents 2.4 km on the ground?

	Quantity per 30-g serving	Percentage daily intake per 30-g serving	Quantity per 30-g serving with $\frac{1}{2}$ cup skim milk
Energy	480 kJ	5.5%	670 kJ
Protein	6.6 g	13.1%	11.2 g
Fat			
— total	0.2 g	0.3%	0.3 g
— saturated	0.1 g	0.1%	0.2 g
Carbohydrate			
— total	20.8 g	6.7%	27.3 g
— sugars	9.6 g	10.7%	16.1 g

$$1 : 20\,000 = x \text{ cm} : 2.4 \text{ km}$$

$$1 : 20\,000 = x \text{ cm} : 2.4 \times 1000 \times 100 \text{ cm}$$

$$1 : 20\,000 = x : 240\,000$$

$$\frac{1}{20\,000} = \frac{x}{\square}$$

$$\frac{1}{20\,000} \times \square = \frac{x}{240\,000} \times \square$$

$$x = \square$$

So, $\square$ cm on the map represents 2.4 km on the ground.

13. Simplify the following three-part ratios.

a. $50 : 20 : 15$

b. $0.4 : 1.8 : 2.2$

14. A concreter needs to make 1.0 m^3 of concrete for the base of a garden shed. How many cubic metres of each of the components should be used?

To make concrete mix: 1 part cement, 2 parts sand and 4 parts gravel.

15. Can you think of an example of how a proportion statement might be used?



5.5 Comparing ratios

5.5.1 Comparison of ratios

- In some cases it is necessary to know which of two given ratios is the larger.
- In some cases it is necessary to know if two given ratios are equal.
- To compare ratios, write them as fractions with a common denominator.

WORKED EXAMPLE 12

Which is the larger ratio in the following pair?

$3 : 5$, $2 : 3$

THINK

- 1 Write each ratio in fraction form.
- 2 Change each fraction to the lowest common denominator (which is 15).
- 3 Compare the fractions: since both fractions have a denominator of 15, the larger the numerator, the larger the fraction.
- 4 The second fraction is larger and it corresponds to the second ratio in the pair. State your conclusion.

WRITE

$$\frac{3}{5} \quad \frac{2}{3}$$

$$\frac{9}{15} \quad \frac{10}{15}$$

$$\frac{9}{15} < \frac{10}{15}$$

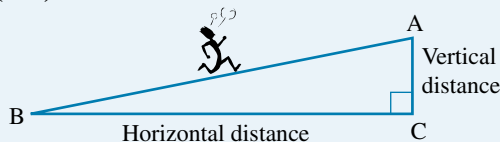
Therefore, $2 : 3$ is the larger ratio.

5.5.2 The gradient

- When measuring the steepness of various slopes or hills, we need to compare the gradient.
- **Gradient** is calculated by finding the ratio $\frac{\text{vertical distance}}{\text{horizontal distance}}$ between any 2 points on a hill. This is also known as calculating $\frac{\text{rise}}{\text{run}}$.
- The larger the gradient, the steeper the hill.

WORKED EXAMPLE 13

Find the gradient of the hill (AB) if AC = 2 m and BC = 10 m.



THINK

- 1 Write the rule for finding the gradient.
- 2 The vertical distance is 2 m and the horizontal distance is 10 m. Substitute these values into the formula.
- 3 Simplify by dividing both numerator and denominator by 2.

WRITE

$$\begin{aligned}\text{Gradient} &= \frac{\text{vertical distance}}{\text{horizontal distance}} \\ &= \frac{2}{10} \\ &= \frac{1}{5}\end{aligned}$$

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Interactivity: Equivalent ratios (int-3736)

Digital doc: SpreadSHEET Comparing ratios (doc-2215)

Exercise 5.5 Comparing ratios

Individual pathways

PRACTISE

Questions:
1–5, 8

CONSOLIDATE

Questions:
1–6, 8, 9

MASTER

Questions:
1–10

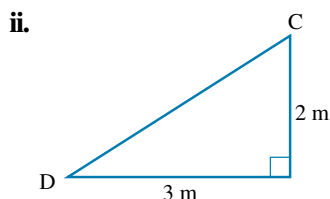
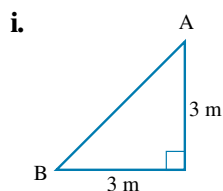
Individual pathway interactivity: int-4415

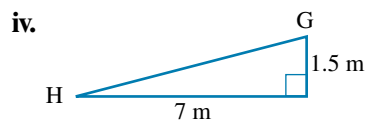
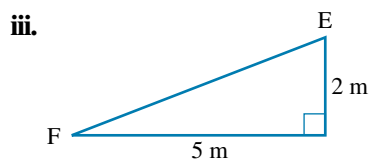
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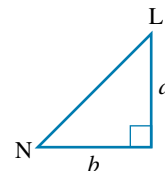
Understanding and fluency

1. **WE12** Which is the greater ratio in each of the following pairs?
 - a. 1 : 4; 3 : 4
 - b. 5 : 9; 7 : 9
 - c. 6 : 5; 2 : 5
 - d. 3 : 5; 7 : 10
 - e. 7 : 9; 2 : 3
 - f. 2 : 5; 1 : 3
 - g. 2 : 3; 3 : 4
 - h. 5 : 6; 7 : 8
 - i. 5 : 9; 7 : 12
 - j. 9 : 8; 6 : 5
2. In each of the following cases, decide which netball team has the better record.
 - a. Yarra Valley won 2 matches out of 5. Peninsula won 5 out of 10.
 - b. Boroondara won 13 out of 18. Geelong won 7 out of 12.
 - c. City West won 12 out of 20. Monash won 14 out of 25.
 - d. VU Western won 8 out of 13. DC North won 13 out of 20.
3. In a cricket match, Jenny bowled 5 wides in her 7 overs and Lisa bowled 4 wides in her 6 overs. Which bowler had the higher wides per over ratio?
4. a. **WE13** Find the gradient of each of the hills represented by the following triangles.





- b. Which slope has the largest gradient?
 d. List the hills in order of increasing steepness.
5. Draw triangles that demonstrate a gradient of:
- a. $\frac{2}{1}$ b. $\frac{3}{1}$ c. $\frac{4}{3}$ d. $\frac{3}{2}$ e. $\frac{2}{5}$
6. **MC** a. If the gradient of LN in the triangle at right is 1, then:
 A. $a > b$ B. $a < b$ C. $a = b$ D. $a = 1$ E. $b = 1$
- b. If $\frac{5}{6} > \frac{a}{5}$, then a could be:
 A. 4 B. 5 C. 6 D. 7
 E. all of these numbers.



Communicating, reasoning and problem solving

7. Draw a right-angled triangle on a piece of graph paper so that the two sides at right angles to each other are 6 cm and 8 cm. Measure the third side length of the triangle, which should be 10 cm.
- a. What is the ratio of the three sides of this triangle?
 b. If you change the size of your triangle but keep the shape the same, what happens to the ratio of the three sides of the triangle?
 c. A piece of string is used to mark out a triangle with its sides in the same ratio as the one you have drawn. The smallest side of the triangle is 75 cm long. How long are the other two sides?
8. An internet search for a homemade lemonade recipe yielded the following results. All of the recipes had water added, but the sweetness of the lemonade is determined by the ratio of lemon juice to sugar. For each of the recipe ratios shown below, determine the mixtures that have the same taste. If they do not have the same taste, determine which site has the sweeter lemonade recipe.
- a. Website 1: 3 tablespoons of sugar for every 15 tablespoons of lemon juice
 Website 2: 4 tablespoons of sugar for every 20 tablespoons of lemon juice
- b. Website 3: 3 tablespoons of sugar for every 9 tablespoons of lemon juice
 Website 4: 5 tablespoons of sugar for every 16 tablespoons of lemon juice
- c. Website 5: 2 cups of sugar for every 3 cups of lemon juice
 Website 6: 5 tablespoons of sugar for every 8 tablespoons of lemon juice
- d. Website 7: 3 tablespoons of sugar for every 8 tablespoons of lemon juice
 Website 8: 7 tablespoons of sugar for every 12 tablespoons of lemon juice
9. The steepness of a hill can be written as a ratio of the vertical distance (height) to the horizontal distance. Hill A has a ratio of 1 : 5 and hill B has a ratio of 7 : 36.
- a. Use diagrams to determine which hill has the steeper slope. Is this an accurate method of determining the slope?
 b. Without using diagrams, investigate another method to determine which ratio produces the steeper slope.
 c. Will your method from part b always work when you want to compare ratios?
10. What does it mean if the gradient is 0?

5.6 Dividing in a given ratio

5.6.1 Dividing in a given ratio

- When something is shared, we often use ratios to ensure that the sharing is fair.
- Consider this situation. Two people buy a lottery ticket for \$3. They win a prize of \$60. How is the prize divided fairly?

Each person contributes \$1.50	One person contributes \$1 and the other \$2
- The contribution for the ticket is in the ratio 1 : 1. - The prize is divided in the ratio 1 : 1. - Each person receives \$30.	- The contribution for the ticket is in the ratio 1 : 2. - The prize is divided in the ratio 1 : 2. - The person who contributed \$1 receives \$20 and the other receives \$40.

In the situation above:

- the 1 : 1 ratio has $1 + 1 = 2$ total parts. Each person receives $\frac{1}{2}$ of the prize money (\$30).
- the 1 : 2 ratio has $1 + 2 = 3$ total parts. Person 1 paid for 1 part of the ticket so receives $\frac{1}{3}$ of the prize money (\$20); person 2 paid for 2 parts of the ticket so receives $\frac{2}{3}$ of the prize money (\$40).



ACTIVITY: SHARING AND SHARE SIZE

- As a pair, discuss the answers to the following questions.
 - The lollies in the picture are to be shared between two people. How do you divide the lollies so that:
 - each person receives the same amount.
 - one person gets twice as many as the other person
 - for every one lolly that the first person receives, the other person receives three lollies (i.e. the lollies are divided in a ratio of 1 : 3)
 - they are divided in a ratio of 2 : 3?
 - The lollies are now to be shared among three people.
 - How do you divide the lollies so that they are divided in a ratio of 1 : 2 : 3?
 - What fraction does each person receive in part i above?
- As a class discuss the relationship between the ratio in which the lollies were shared and the fraction of lollies that each person received as a result.
- How can you determine the fraction of the whole amount each person will receive from the ratio by which the whole will be divided?



WORKED EXAMPLE 14

Share the amount of \$500 000 in the ratio 3 : 7.

THINK

- Determine how many parts are in the ratio.
- The first share represents 3 parts out of a total of 10, so find $\frac{3}{10}$ of the total amount.
- The second share is the remainder, so subtract the first share amount from the total amount.

WRITE

$$\text{Total number of parts} = 3 + 7 \\ = 10$$

$$\text{First share} = \frac{3}{10} \times \$500\,000 \\ = \$150\,000$$

$$\text{Second share} = \$500\,000 - \$150\,000 \\ = \$350\,000$$

- In Worked example 14, the second share represents 7 parts out of the total of 10.

Another way to calculate the size of the second share would be to find $\frac{7}{10}$ of the total amount.

$$\text{Second share} = \frac{7}{10} \times \$500\,000 \\ = \$350\,000$$

DISCUSSION

Think of an example where sharing things on the basis of ratios might be considered unfair.

WORKED EXAMPLE 15

Concrete mixture for a footpath was made up of 1 part of cement, 2 parts of sand and 4 parts of blue metal. How much sand was used to make 4.2 m^3 of concrete?

THINK

- 1 Find the total number of parts.
- 2 There are 2 parts of sand to be used in the mixture, so find $\frac{2}{7}$ of the total amount of concrete made.

WRITE

$$\begin{aligned}\text{Total number of parts} &= 1 + 2 + 4 \\ &= 7\end{aligned}$$

$$\begin{aligned}\text{Amount of sand} &= \frac{2}{7} \times 4.2 \text{ m}^3 \\ &= 1.2 \text{ m}^3\end{aligned}$$

5.6.2 Using the unitary method to solve ratio problems

- The unitary method can also be used to solve ratio problems. To use the unitary method divide the quantity that you are sharing by the total amount of equal parts. Then multiply each share of the ratio by the quantity that one part represents.

WORKED EXAMPLE 16

Elena, Christina and Megan contribute towards a lottery pool in the ratio 1 : 2 : 3. If they win \$1740 between them, use the unitary method to divide the winnings according to how much they contributed.

THINK

- 1 Find the total number of parts.
- 2 Divide the quantity by the total amount of equal parts.
- 3 Multiply each person's share by the quantity that one part represents.
- 4 Write the answer.

WRITE

$$\begin{aligned}\text{Total number of parts} &= 1 + 2 + 3 \\ &= 6\end{aligned}$$

$$\frac{1740}{6} = \$290$$

Elena:

$$290 \times 1 = \$290$$

Christina:

$$290 \times 2 = \$580$$

Megan:

$$290 \times 3 = \$870$$

Elena wins \$290, Christina wins \$580 and Megan wins \$870.

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Interactivity: Dividing in a given ratio (int-3737)

eLesson: Dividing in ratios (eles-0041)

Digital doc: SpreadSHEET Dividing in a given ratio (doc-2489)

Exercise 5.6 Dividing in a given ratio

Individual pathways

PRACTISE

Questions:
1–7, 9, 13

CONSOLIDATE

Questions:
1–9, 11, 13, 14

MASTER

Questions:
1–13, 16

Individual pathway interactivity: int-4416

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Understanding and fluency

- Write the total number of parts for each of the following ratios.
 - 1 : 2
 - 2 : 3
 - 3 : 1
 - 3 : 5
 - 4 : 9
 - 5 : 8
 - 6 : 7
 - 9 : 10
 - 1 : 2 : 3
 - 3 : 4 : 5
- WE14** Share the amount of \$1000 in the following ratios.
 - 2 : 3
 - 3 : 1
 - 1 : 4
 - 1 : 1
 - 3 : 5
 - 5 : 3
 - 3 : 7
 - 9 : 1
 - 7 : 13
 - 9 : 11
- If Nat and Sam decided to share their lottery winnings of \$10 000 in the following ratios, how much would each receive?
 - 1 : 1
 - 2 : 3
 - 3 : 2
 - 3 : 7
 - 7 : 3
 - 1 : 4
 - 9 : 1
 - 3 : 5
 - 12 : 13
 - 23 : 27
- Rosa and Mila bought a lottery ticket costing \$10. How should they share the first prize of \$50 000 if their respective contributions were:
 - \$2 and \$8
 - \$3 and \$7
 - \$4 and \$6
 - \$5 and \$5
 - \$2.50 and \$7.50?
- WE15** Concrete mixture is made up of 1 part cement, 2 parts sand and 4 parts blue metal.
 - How much sand is needed for 7 m³ of concrete?
 - How much cement is needed for 3.5 m³ of concrete?
 - How much blue metal is required for 2.8 m³ of concrete?
 - How much sand is used for 5.6 m³ of concrete?
 - How much cement is needed to make 8.4 m³ of concrete?
- WE16** In a family, 3 children receive their allowances in the ratio of their ages, which are 15 years, 12 years and 9 years. If the total of the allowances is \$60, use the unitary method to determine how much each child receives?
- Three of your teachers buy a Lotto ticket costing \$20. How should they share the first prize of \$600 000 if they each contribute:
 - \$3, \$7 and \$10
 - \$6, \$6 and \$8
 - \$1, \$8 and \$11
 - \$5, \$6 and \$9
 - \$5, \$7.50 and \$7.50?
- In a school, the ratio of girls in Years 8, 9 and 10 is 6 : 7 : 11. If there is a total of 360 girls in the three year levels:
 - how many Year 8 girls are there
 - how many more Year 10 girls are there than Year 8 girls?
- In a moneybox, there are 5-cent, 10-cent and 20-cent coins in the ratio 8 : 5 : 2. If there are 225 coins altogether:
 - how many 5-cent coins are there
 - how many more 10-cent coins than 20-cent coins are there



- c. what is the total value of the 5-cent coins
 d. what is the total value of the coins in the moneybox?
10. **MC** a. A square of side length 4 cm has its area divided into two sections in the ratio 3 : 5. The area of the larger section is:
 A. 3 cm^2 B. 5 cm^2 C. 8 cm^2 D. 10 cm^2 E. 16 cm^2
- b. A block of cheese is cut in the ratio 2 : 3. If the smaller piece is 150 g, the mass of the original block was:
 A. 75 g B. 200 g C. 300 g D. 375 g E. 450 g
- c. Contributions to the cost of a lottery ticket were \$1.75 and \$1.25. What fraction of the prize should the larger share be?
 A. $\frac{7}{12}$ B. $\frac{5}{7}$ C. $\frac{7}{5}$ D. $\frac{3}{5}$ E. $\frac{5}{12}$
- d. A television channel that telecasts only news, movies and sport does so in the ratio 2 : 3 : 4 respectively. How many movies, averaging a length of $1\frac{1}{2}$ hours, would be shown during a 24-hour period?
 A. 2 B. 3 C. 4 D. 5 E. 6

Communicating, reasoning and problem solving

11. Three angles of a triangle are in the ratio 1 : 2 : 3. What is the magnitude of each angle? Justify your answer.
12. The angles of a quadrilateral are in the ratio 2 : 3 : 4 : 6. What is the difference in magnitude between the smallest and largest angles? Show your working.
13. The ratio of boys to girls in Year 8 at a school is 3 : 2.
 a. If there are 75 boys in Year 8, how many girls are there
 Each class in Year 8 contains 25 students. One class contains boys only, but the remaining classes are a mixture of boys and girls.
 b. How many classes in Year 8 contain a mixture of boys and girls?
 c. What is the ratio of mixed classes to boys only classes?
14. The wing of a model aeroplane is 4 cm long, and the scale of the model is 1 : 300.
 a. What is the ratio of the length of the model's wing to the length of the wing of the actual aeroplane?
 b. How long is the wing of the actual aeroplane?
15. a. There are apples, oranges and bananas in a fruit bowl. The ratio of apples to oranges is 3 : 4, and the number of apples is $\frac{3}{8}$ of the total number of pieces of fruit. How many bananas are there?
 b. Next to the fruit bowl is a bowl of nuts. The ratio of cashews to peanuts is 15 : 4, the ratio of peanuts to walnuts is 4 : 7 and the ratio of hazelnuts to peanuts is 11 : 4. Calculate the minimum number of nuts there can be in the bowl.
 c. Somebody added some macadamias to the nut bowl in part b. The ratio of peanuts to macadamias is now 1 : 3. Calculate:
 i. the number of macadamia nuts added ii. the total number of nuts now in the bowl.
16. Think of some examples of instances where you need to divide in a ratio other than 1 : 1.

5.7 Rates

5.7.1 Rates

- A rate is used to compare how quantities change.
- Unlike ratios, which compare quantities measured in the same unit, a rate compares quantities measured in different units.
- Rates have units. An example of a rate is speed (measured in km/h or m/s).
- A rate is in its simplest form if it is *per one unit*.



WORKED EXAMPLE 17

Express the following statement using a rate in simplest form:
The 30-litre container was filled in 3 minutes.

THINK

- 1 A suitable rate would be litres per minute (L/min). Put the capacity of the container in the numerator and the time in which it was filled in the denominator of the fraction.
- 2 Simplify the fraction.

WRITE

$$\begin{aligned}\text{Rate} &= \frac{30 \text{ L}}{3 \text{ min}} \\ &= \frac{10 \text{ L}}{1 \text{ min}} \\ &= 10 \text{ L/min}\end{aligned}$$

WORKED EXAMPLE 18

Joseph is paid \$8.50 per hour as a casual worker. At this rate, how much does he receive for 6 hours of work?

THINK

- 1 The rate is given in \$ per hour. So it actually tells us the amount of money earned in each hour; that is, the hourly payment.
- 2 State the number of hours worked.
- 3 To find the total payment, multiply the hourly payment by the total number of hours worked.

WRITE

$$\begin{aligned}\text{Payment per 1 hour} &= \$8.50 \\ \text{Hours worked} &= 6 \\ \text{Total payment} &= \$8.50 \times 6 \\ &= \$51\end{aligned}$$

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Interactivity: Rates (int-3738)

Digital doc: SkillsHEET Converting a mixed number to an improper fraction (doc-6892)

Digital doc: SkillsHEET Multiplying decimals by 10, 100 and 1000 (doc-6893)

Digital doc: SkillsHEET Multiplying a whole number by a fraction (doc-6894)

Digital doc: SkillsHEET Converting minutes to a fraction of an hour (doc-6895)

Digital doc: WorksHEET Ratios and rates II (doc-2217)

Exercise 5.7 Rates

Individual pathways

PRACTISE

Questions:
1–7, 9, 12, 14, 15, 20

CONSOLIDATE

Questions:
1, 2, 3a–l, 4a–f, 5, 6–11, 14, 15, 18,
20, 24

MASTER

Questions:
1m–t, 2, 3m–t, 4f–j, 5–10, 13–26

Individual pathway interactivity: int-4417

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Understanding and fluency

1. **WE17** Express each of the following statements using a rate in simplest form.
- A lawn of 600 m^2 was mown in 60 min.
 - A tank of capacity 350 kL is filled in 70 min.
 - A balloon of volume 4500 cm^3 was inflated in 15 s.
 - The cost of 10 L of fuel was \$13.80.
 - A car used 16 litres of petrol in travelling 200 km.
 - A 12 m length of material cost \$30.
 - There were 20 cows grazing in a paddock that was 5000 m^2 in area.
 - The gate receipts for a crowd of 20 000 people were \$250 000.
 - The cost of painting a 50 m^2 area was \$160.
 - The cost of a 12 minute phone call was \$3.00.
 - The team scored 384 points in 24 games.
 - Last year 75 kg of fertiliser cost \$405.
 - The winner ran the 100 m in 12 s.
 - To win, Australia needs to make 260 runs in 50 overs.
 - For 6 hours work, Bill received \$159.
 - The 5.5 kg parcel cost \$19.25 to post.
 - Surprisingly, 780 words were typed in 15 minutes.
 - From 6 am to noon, the temperature changed from 10°C to 22°C .
 - When Naoum was 10 years old he was 120 cm tall. When he was 18 years old he was 172 cm tall.
 - A cyclist left home at 8.30 am and at 11.00 am had travelled 40 km.
2. **WE18** Sima is paid \$15.50 per hour. At this rate, how much does she earn in a day on which she works 7 hours?
3. A basketball player scores, on average, 22 points per match. How many points will he score in a season in which he plays 18 matches?
4. A car's fuel consumption is 11 L/100 km. How much fuel would it use in travelling 550 km?
5. To make a solution of fertiliser, the directions recommend mixing 3 capfuls of fertiliser with 5 L of water. How many capfuls of fertiliser should be used to make 35 L of solution?
6. Anne can type 60 words per minute. How long will she take to type 4200 words?
7. Marie is paid \$42 per day. For how long will she have to work to earn \$504?
8. The rate of 1 teacher per 16 students is used to staff a school. How many teachers will be required for a school with 784 students?
9. Land is valued at \$42 per m^2 . How much land could be bought for \$63 000?
10. On average, a test bowler took 1 wicket every 4.5 overs. How many wickets did he take in a season in which he bowled 189 overs?
11. What quantities (such as distance, time, volume) are changing if the units of rate are:
- | | | | | |
|-----------|-----------------------------|-----------------|-------------|--------------------|
| a. km/h | b. cm^3/sec | c. L/km | d. \$ per h | e. \$ per cm |
| f. kL/min | g. cents/litre | h. \$ per dozen | i. kg/year | j. cattle/hectare? |
12. What units would you use to measure the changes taking place in each of the following situations?
- | | |
|----------------------------------|---------------------------------|
| a. A rainwater tank being filled | b. A girl running a sprint race |
| c. A boy getting taller | d. A snail moving across a path |
| e. An ink blot getting larger | f. A car consuming fuel |
| g. A batsman scoring runs | h. A typist typing a letter |
13. Water flows from a hose at a rate of 3 L/min. How much water will flow in 2 h?
14. Tea bags in a supermarket can be bought for \$1.45 per pack (pack of 10) or for \$3.85 per pack (pack of 25). Which is the cheaper way of buying the tea bags?



15. Car A uses 41 L of petrol in travelling 500 km. Car B uses 34 L of petrol in travelling 400 km. Which car is the more economical?
16. Coffee can be bought in 250 g jars for \$9.50 or in 100 g jars for \$4.10. Which is the cheaper way of buying the coffee and how large is the saving?
17. **MC** a. A case containing 720 apples was bought for \$180. The cost could be written as:

- A. 30 cents each B. 20 cents each C. \$3.00 per dozen
D. \$2.00 per dozen E. \$2.80 for 10

- b. Mark, a test cricketer, has a batting strike rate of 68, which means he has made 68 runs for every 100 balls faced. What is Steve's strike rate if he has faced 65 overs and has made 280 runs? (Note: Each over contains 6 balls.)

- A. 65 B. 68.2 C. 71.8 D. 73.2 E. 74.1

- c. A carport measuring $8\text{ m} \times 4\text{ m}$ is to be paved. The paving tiles cost \$36 per m^2 and the tradesperson charges \$12 per m^2 to lay the tiles. How much will it cost to pave the carport (to the nearest \$50)?

- A. \$1400 B. \$1450 C. \$1500 D. \$1550 E. \$1600

- d. A tank of capacity 50 kL is to be filled by a hose whose flow rate is 150 L/min. If the tap is turned on at 8 am, when will the tank be filled?

- A. Between 1.00 pm and 1.30 pm
B. Between 1.30 pm and 2.00 pm
C. Between 2.00 pm and 2.30 pm
D. Between 2.30 pm and 3.00 pm
E. Between 3.00 pm and 3.30 pm



Communicating, reasoning and problem solving

18. Explain the difference between a rate and a ratio.
19. A chiropractor sees 160 patients every week.
- What is his rate of seeing patients each hour if he works a 40-hour week?
 - How long, on average, does he spend with each patient?
 - Using these rates, if the chiropractor wants to make at least \$10 000 every week, what is the minimum he must charge each patient? Justify your answer.
20. If 4 monkeys eat 4 bananas in 4 minutes, how long does it take 12 monkeys to eat 12 bananas? Show your working.
21. If Bill takes 3 hours to paint a room and James takes 5 hours to paint a room, how long will it take to paint a room if they work together? Show your working.
22. Beaches are sometimes unfit for swimming if heavy rain has washed pollution into the water. A beach is declared unsafe for swimming if the concentration of bacteria is more than 5000 organisms per litre. A sample of 20 millilitres was tested and found to contain 55 organisms. Calculate the concentration in the sample (in organisms/litre) and state whether or not the beach should be closed.
23. a. The Earth is a sphere with a mass of 6.0×10^{24} kg and radius 6.4×10^6 m. Use the formula $V = \frac{4}{3}\pi r^3$ to calculate the volume of the Earth. Hence calculate the density of the Earth.
- The planet Jupiter has a mass of 1.9×10^{27} kg and a radius of 7.2×10^7 m. Calculate the volume of Jupiter. Calculate the density of Jupiter.
 - Different substances have their own individual density. Does it seem that the Earth and Jupiter are made of the same substance? Explain.



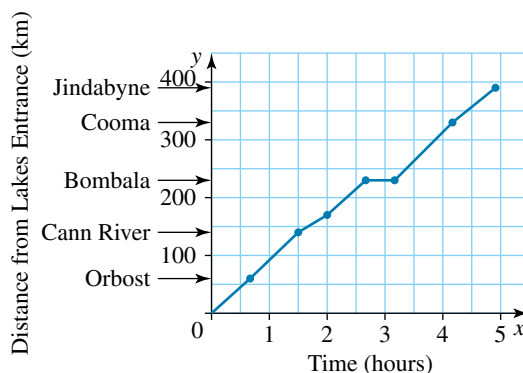
24. The Stawell Gift is a 120-m handicap footrace. Runners who start from scratch run the full 120 m; for other runners, their handicap is how far in front of scratch that they start. Joshua Ross has won the race twice. In 2003, with a handicap of 7 m, his time was 11.92 seconds. In 2005, from scratch, he won in 12.36 seconds. In which race did he run faster?
25. Why do we need units with rates?
26. Jennifer's work is 40 km from her home. On the way to work one morning, her average speed was 80 km/h. Due to bad weather and roadworks, her average speed for the trip home was only 30 km/h. How much time did Jennifer spend travelling to and from work on this day?



5.8 Interpreting graphs

5.8.1 Interpretation of graphs

- All graphs tell a story or describe a situation. They compare two related quantities against each other. As one quantity changes, it affects the other.
- Graphs are used to represent all sorts of data in everyday life and are frequently used in fields outside mathematics, including science, geography and economics.
- The gradient (also called slope) of a straight line is a measure of how steep a straight line is.
- In a distance–time graph, the gradient of the graph represents the speed.
- The units used for distance and time are also used for speed.
- The greater the gradient (slope), the greater the speed; and vice-versa.
- On a distance–time graph, time is represented on the horizontal axis (x -axis).
- On a distance–time graph, distance from a reference point (e.g. the starting point) is represented on the vertical axis (y -axis).
- A change in direction can be represented on a distance–time graph by a change in the direction of the line (i.e. from upward to downward or downward to upward).
- The graph at right shows the story of a car trip from Lakes Entrance to Jindabyne (distances are approximate). Describe the trip.
 - The trip could be described like this: *'We left Lakes Entrance and passed through Orbost 40 minutes into our trip. We reached Cann River approximately 1.5 hours after we set off. The trip out of Cann River was slow at first until we reached the top of the mountain. We then travelled faster on our way to Bombala where we stopped for a 30 minute break. We reached Cooma an hour after leaving Bombala, then the trip from Cooma to Jindabyne was slow due to snow traffic. The whole trip took us just under 5 hours.'*
 - Estimated travel times are determined by reading points along the graph.
 - A line's gradient can indicate whether distance is being travelled quickly (speed is fast), distance is being travelled slowly (speed is slow), or no distance is being travelled (speed is non-existent).
 - The line's gradient in this case is directly linked to the speed of the car.



5.8.2 The use of units

- The gradient of a line describes the change in one variable with respect to the other variable.
- When calculations of the gradient are done using the units from the graph, interpretation of the gradient becomes easier.

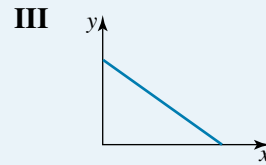
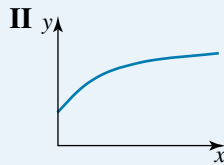
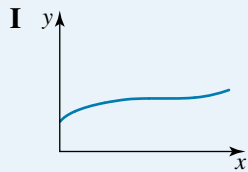
- The gradient of the graph between Orbost and Cann River is calculated below.

$$\begin{aligned}
 m &= \frac{\text{rise}}{\text{run}} \\
 &= \frac{\text{distance}_{\text{Orbost} \rightarrow \text{Cann River}}}{\text{time}_{\text{Orbost} \rightarrow \text{Cann River}}} \\
 &= \frac{140 \text{ km} - 60 \text{ km}}{1.5 \text{ h} - 0.6667 \text{ h}} \\
 &= \frac{80 \text{ km}}{0.8333 \text{ h}} \\
 &= 96 \text{ km/h}
 \end{aligned}$$

- The gradient of the line is the average speed of the car over this section.
- Each straight line section of graph indicates a constant speed.

WORKED EXAMPLE 19

Three graphs (I, II and III) and three descriptions (a, b and c) are shown below. Match each graph to the description that best suits it and explain your choice.



- a The circumference of a tree trunk over a calendar year
- b The distance run versus the distance from the finishing line for a runner in a 400-m race
- c The weight of a newborn baby in the first year of life

THINK

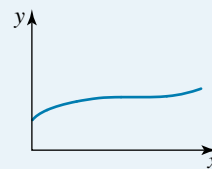
- a How does a tree grow in a normal year? You could expect that the growth rate of a tree will change with the seasons; the faster it grows, the steeper the slope. Over a year:

Summer	Autumn	Winter	Spring	Summer
Quickest growth	Growth slowing	Growth very slow	Growth increasing	Quickest growth

- b As the runner runs further, the distance to the finish line becomes less. The gradient of the curve should be decreasing. In fact:

Distance run (m)	0	100	200	300	400
Distance to finish (m)	400	300	200	100	0

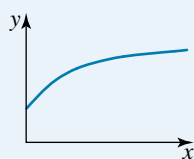
WRITE



The circumference of a tree is best represented by graph I.

As the runner runs, the distance (x) from the starting line becomes greater and the distance (y) to the finish line becomes less. The graph should be nearing $y = 0$. Graph III is the only graph which does this. The distance run versus the distance to run is best represented by graph III.

- c Babies grow rapidly at first, then this growth rate begins to slow.

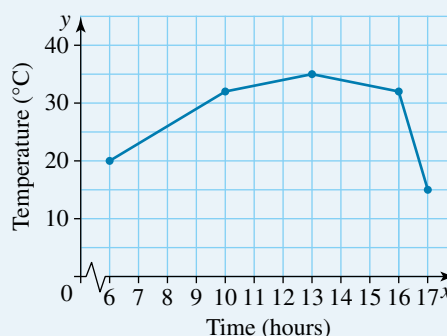


The growth rate of babies is quick at first then slows but continues to increase. Graph II shows an increasing gradient that becomes less steep; therefore, graph II best represents the growth of a baby in the first year.

WORKED EXAMPLE 20

Use the following graph to answer the following questions.

- What units are shown on the graph?
- What does a change in the y -value represent?
- What does a change in the x -value represent?
- Calculate the gradient of the graph over the section in which the temperature was rising the fastest.
- What does the gradient represent?



THINK

- Units are displayed on axis labels.
- The y -axis shows the temperature.
- The x -axis shows time.
- The temperature is rising the fastest when the graph is increasing most steeply. This occurs between 6 and 10am, when the temperature increases from 20 °C to ~32 °C.
- The gradient has units of °C/h, which are read as degrees Celsius per hour.

WRITE

The units shown on the graph are °C (degrees Celsius) for temperature and hours for time.

A change in the y -value indicates a change in temperature.

A change in the x -value indicates a change in time.

$$\begin{aligned}
 m &= \frac{\text{rise}}{\text{run}} \\
 &= \frac{12\text{ °C}}{4\text{ h}} \\
 &= 3\text{ °C/h}
 \end{aligned}$$

The gradient represents the rate at which the temperature is changing per hour.

ACTIVITY: CREATING A DISTANCE–TIME GRAPH

Walk the length of your house in 30 seconds, changing speed every 5 seconds and briefly stopping at least once. If you want to be adventurous walk backwards for one of the 5 second intervals!

Create a distance–time graph to represent the distance from one end of your house. Compare the graph that you've drawn with those of your classmates.

Interactivity: Interpreting graphs (int-3739)

Interactivity: Gradient (int-3740)

Exercise 5.8 Interpreting graphs

Individual pathways

PRACTISE

Questions:
1–9, 12, 16

CONSOLIDATE

Questions:
1–13, 16, 17

MASTER

Questions:
1–19

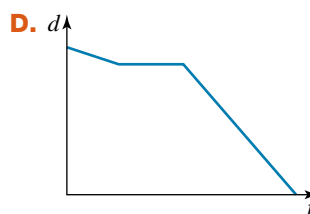
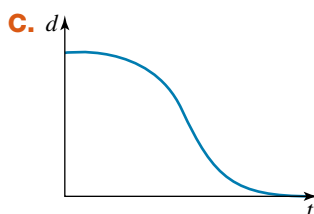
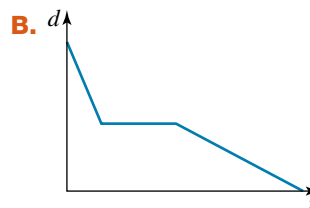
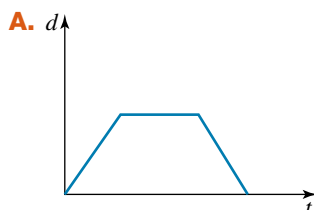
Individual pathway interactivity: int-4418

learnon ONLINE ONLY

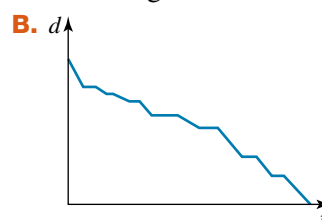
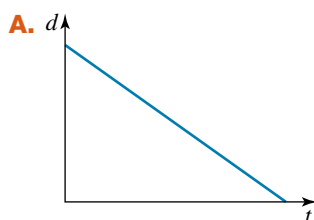
To answer questions online and to receive **immediate feedback** and **fully worked solutions** for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

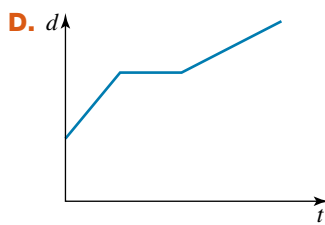
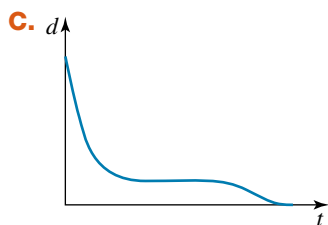
Understanding and fluency

- Which axis represents distance and which axis represents time on a distance–time graph?
- If distance is measured in metres (vertical axis) and time is measured in seconds (horizontal axis), how is speed measured?
- The express train through country New South Wales is delayed for one hour because of a herd of cattle on the line. How is this event graphed on a distance–time graph?
- MC** Which of the following graphs best describes a student walking to school who dawdles at first, meets up with some friends and drops in at the corner shop before stepping up the pace to get to school on time?



- Explain your reasons for choosing your answer in question 4.
- MC** Which of the following graphs best describes a student travelling to school on a bus?

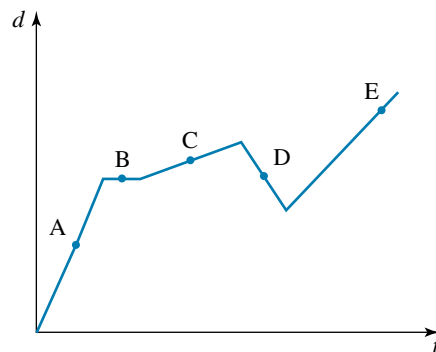




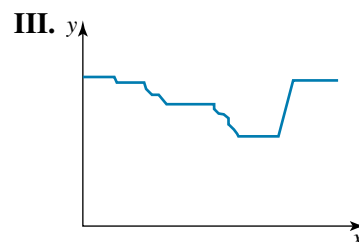
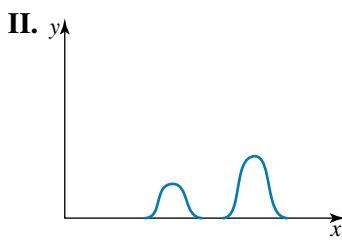
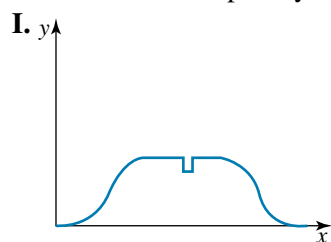
7. Explain your reasons for choosing your answer in question 6.

8. The graph at right shows the distance Andrea's car is from her house. Answer true or false for the following statements.

- Andrea is travelling faster at point C than at point A.
- Andrea is not moving at point B.
- Andrea is further away from home at point B compared to point E.
- At point D Andrea is travelling back towards home
- Andrea is travelling slower at point E than at point C.



9. **WE19** Three descriptions (**a**, **b** and **c**) and three graphs (**I**, **II** and **III**) are shown below. Match each graph to the description that best suits it and explain your choice.



- The number of students in the canteen line over a school day
- The number of cans in a school's drink vending machine over a school day
- The number of students on school grounds over a school day

Communication, reasoning and problem solving

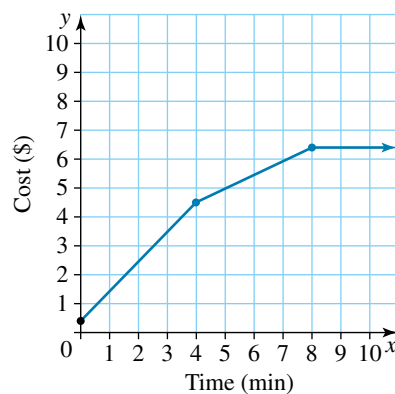
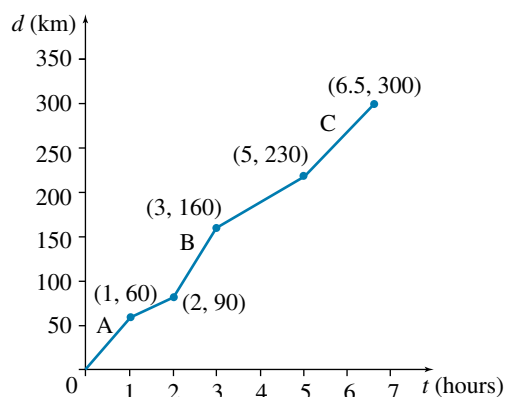
10. What do different gradients of straight-line segments of a distance–time graph represent?

11. What does a horizontal line segment on a distance–time graph represent?

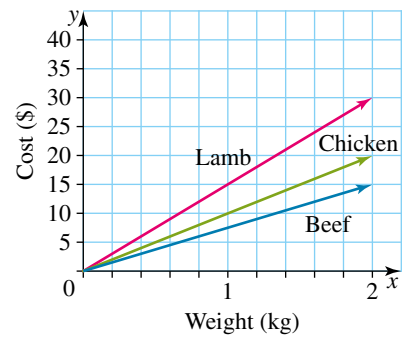
12. Calculate the speed for segments A, B and C of the distance–time graph shown above right. If necessary, give your answers correct to 1 decimal place.

13. **WE20** The graph at right shows the cost of a mobile telephone call. Use the graph to answer the following questions.

- What units are shown on the graph?
- What does a change in the y-value represent?
- What does a change in the x-value represent?
- Calculate the gradient for each straight line section of the graph using units.
- What does the gradient represent?

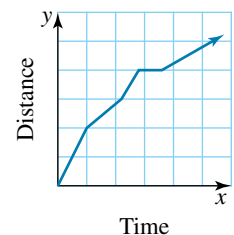


14. The price per kilogram for 3 different types of meat is illustrated in the graph at right.
- Calculate the gradient (using units) for each graph.
 - What is the cost of 1 kg of each type of meat?
 - What is the cost of purchasing:
 - 1 kg of lamb
 - 0.5 kg of chicken
 - 2 kg of beef?
 - What is the total cost of the order in part c?
 - Copy and complete the table below to confirm your answer from part d.



Meat type	Cost per kilogram (\$/kg)	Weight required (kg)	Cost = \$/kg × kg
Lamb		1	
Chicken		0.5	
Beef		2	
		Total cost	

15. The graph at right represents the distance travelled by a vehicle versus time. Copy the graph onto some grid paper and, using red pen, show what the graph would look like if the vehicle at any point in time was travelling twice as fast.
16. Write a story for each of the graphs in question 9.
17. Draw a distance–time graph to represent the following story.
Jordan decides to take his dog for a walk. He starts off at a steady, fast pace and continues at this pace for 2 minutes. After 2 minutes he breaks out into a gentle jog for 3 minutes, before taking a rest for 30 seconds. Following his rest he walks back home in 10 minutes.
18. Draw sketches to represent each of the stories below and explain your sketches.
- The mass of a pig over its lifetime
 - The altitude of a plane during a flight
 - The height of water in a bath tub as it is emptied
19. How can a graph be used to tell a story?



5.9 Review

Investigation | Rich task

The golden ratio

The Greeks believed that using a special ratio of numbers in building designs, paintings, sculpture etc. would automatically make them beautiful. This number is known as the golden ratio or golden number. The human body has many examples of the golden ratio.

$$\text{The golden ratio} = \frac{\sqrt{5} + 1}{2}$$

Work out this number as a decimal to 3 decimal places.

Part A

The golden ratio is often represented by phi (ϕ). One of the interesting relationships of this ratio to the design of the human body is that there are:

- five appendages on the torso: arms, legs and head
- five appendages on each of these: fingers, toes and five openings on the face
- five senses: sight, hearing, touch, taste and smell.

The golden number is also based on 5 because the number phi can be written as:

$$5^{0.5} \times 0.5 + 0.5.$$

Leonardo da Vinci's drawings of the human body emphasised its proportions. The ratios of the following distances equal the golden ratio:

- foot to navel : navel to head
- length of forearm : length of hand
- length of upper arm : length of hand and forearm.

Your task is to explore the golden ratio as it applies to your body.

Work in pairs to measure parts of your body. In the table below are some of the measurements you could take. Choose other parts of your body to measure and find as many golden ratios as possible.

Body measurement (cm)	Ratio	Decimal value
Foot to navel : navel to head		
Length of forearm : length of hand		
Length of upper arm : length of hand and forearm		

Part B

Another way to find the golden ratio is using the Fibonacci sequence:

1, 1, 2, 3, 5, 8, 13, ...

1. Write the first 20 terms of the sequence in the table.
2. Take the terms two at a time and divide the larger number by the smaller (e.g. divide 2 by 1, 3 by 2, 5 by 3 etc.), working as accurately as possible. Record your answers in the table on the following page.
3. Use the internet to discover other ways of finding the golden ratio.



[illegible]

learn **RESOURCES — ONLINE ONLY**

Digital doc: Investigation — The golden ratio (doc-13935)

Digital doc: Code puzzle — Who created Sherlock Holmes? (doc-13936)

Digital doc: Summary Topic 5 (doc-23241)

Interactivity: Word search Topic 5 (int-2623)

Interactivity: Crossword Topic 5 (int-2624)

Interactivity: Sudoku Topic 5 (int-3185)

Exercise 5.9 Review questions

To answer questions online and to receive **immediate feedback** and **fully worked solutions** for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

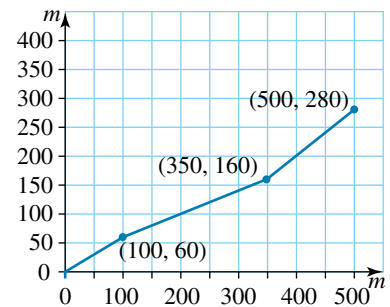
Understanding and fluency

- On a farm there are 5 dogs, 3 cats, 17 cows and 1 horse. Write the following ratios.
 - cats : dogs
 - horses : cows
 - cows : cats
 - dogs : horses
 - dogs : other animals
- Express each of the following ratios in simplest form.
 - 8 : 16
 - 24 : 36
 - 35 mm : 10 cm
 - \$2 : 60 cents
 - 20 s : $1\frac{1}{2}$ min
 - $\frac{1}{12} : \frac{1}{3}$
 - 4 : 10
 - 56 : 80
 - 2 hours : 40 min
 - 1.5 km : 400 m
- Find the value of n in each of the following proportions.
 - $\frac{n}{3} = \frac{20}{5}$
 - $\frac{n}{28} = \frac{5}{7}$
 - $\frac{2}{3} = \frac{8}{n}$
 - $\frac{4}{5} = \frac{12}{n}$
 - $\frac{6}{n} = \frac{5}{8}$
 - $\frac{3}{10} = \frac{n}{4}$
- The directions for making lime cordial require the mixing of 1 part cordial to 6 parts water.
 - Express this as a ratio.
 - How much cordial would you have to mix with 9 L of water?
- Which is the larger ratio?
 - $\frac{4}{5}, \frac{2}{3}$
 - $\frac{7}{12}, \frac{5}{8}$
- Place a number in the box to make a ratio greater than 3 : 2 but less than 2 : 1.
 : 6
- The horizontal and the vertical distances between the top and bottom points of slide A are 3 m and 2 m respectively. For slide B the horizontal distance between the top and bottom points is 10 m, and the vertical distance is 4 m.
 - Calculate the gradients of slide A and slide B.
 - Which slide is steeper? Justify your answer.
- Divide \$25 in the ratio 2 : 3.
 - Share \$720 in the ratio 7 : 5.
- Three people share a Lotto prize of \$6600 in the ratio 4 : 5 : 6. What is the difference between the smallest and largest shares?
- A car travels 840 km on 72 litres of petrol. Find the fuel consumption of the car in L/100 km.
- David's car has a fuel consumption rate of 12 km/L, and Susan's car has a fuel consumption rate of 11 km/L.
 - Which car is more economical?
 - How far can David's car travel on 36 L of fuel?
 - How much fuel (to the nearest litre) would Susan's car use in travelling 460 km?
- A 1 kg packet of flour costs \$2.80 and a 750 g packet costs \$2.20. Which is the cheaper way of buying flour?

Communicating, reasoning and problem solving

- The sides of a triangle are in the ratio 3 : 4 : 5. If the longest side of the triangle measures 40 cm, what is the perimeter of the triangle?
- To make two $\frac{2}{3}$ -cup servings of cooked rice, you add $\frac{3}{4}$ of a cup of rice, $\frac{1}{4}$ teaspoon of salt and 1 teaspoon of butter to $1\frac{1}{2}$ cups of water. How many $\frac{2}{3}$ -cup servings of cooked rice can you make from a bag containing 12 cups of rice?

15. Lachlan was driven from Newcastle to Branxton, a distance of 60 km, at an average speed of 80 km/h. He cycled back at an average speed of 20 km/h. What was his average speed for the whole journey? (*Hint: It is not 50 km/h.*)
16. The speed of the *Discovery* space shuttle while in orbit was 17 400 miles per hour. What is this in km/h? (1 kilometre = 0.62 miles)
17. The rate of ascent for the *Discovery* space shuttle was 71 miles in 8.5 minutes.
- What speed is this in km/min?
 - What speed is this in km/h?
18. A cyclist riding at 12 km/h completes a race in 3 h 45 min.
- What is the distance of the race?
 - At what speed would she have to ride to complete the race in 3 h?
19. You have a plastic bag that contains 80 tennis balls. This bag of balls weighs 4 kg (the weight of the plastic bag is insignificant). You add 10 more balls to your bag. How much does your bag of balls weigh now?
20. The steps of a staircase are to have a ratio of rise to run that is to be $\frac{2}{3}$. If the run is 30 cm, what is the rise?
21. Travelling from Noort to Bastion takes me 1 hour and 30 minutes by car at an average speed of 72 km per hour. I stop for 15 minutes in Bastion before travelling to Smoop, which is 163 km away. The trip from Bastion to Smoop takes me 2 hours and 12 minutes. What is my average speed for the whole trip?
22. A sum of money is divided in the ratio 2 : 5 : 7. Given that the smallest share is \$200, calculate the largest share.
23. It takes me 2 hours to mow my lawn. My son takes 2.5 hours to mow the same lawn. If we work together using two lawnmowers, how long will it take us to mow the lawn? Give your answer in hours, minutes and seconds.
24. The diagram at right represents the path of a ski lift.
- Calculate the gradients for each section.
 - Where is the gradient the steepest?
 - What is the average gradient of the ski lift?



Answers

Topic 5 Ratios and rates

Exercise 5.2 Introduction to ratios

1. a. 4 : 5 b. 5 : 4 c. 5 : 9 d. 9 : 4 e. 1 : 2
2. a. 5 : 3 b. 3 : 5 c. 1 : 5 d. 5 : 1 e. 1 : 3
f. 5 : 4 g. 1 : 8 h. 5 : 9 i. 1 : 9 j. 1 : 3
3. a. 5 : 7 b. 7 : 5 c. 5 : 12
4. a. 4 : 3 b. 3 : 4 c. 6 : 1 d. 4 : 1 e. 2 : 5
5. a. 3 : 5 b. 6 : 19 c. 4 : 11 d. 7 : 9 e. 1 : 5
f. 9 : 4 g. 3 : 4 h. 3 : 10 i. 17 : 60 j. 53 : 100
k. 11 : 100 l. 1 : 1000 m. 1 : 2000 n. 7 : 24 o. 5 : 12
p. 1000 : 27 q. 7 : 12 r. 13 : 24 s. 3 : 5 t. 1 : 22
6. a. 3 : 59 b. 59 : 38 c. 38 : 3 d. 3 : 97 e. 59 : 41
7. $\frac{6}{17}$
8. a. 24 : 17 b. 21 : 17 c. 2 : 1 d. 4 : 39 e. 4 : 1 f. 1 : 1
g. 1 : 2 h. 3 : 1 i. 9 : 1 j. 1 : 3 k. 1 : 6 l. 1 : 4
9. a. 215 : 179 b. 215 : 36
10. a. 97 : 3 b. 3 : 100
11. a. $\frac{3}{10}$ b. $\frac{4}{15}$ c. $\frac{13}{25}$ d. $\frac{7}{9}$
12. a. Yes (same units) b. Yes (same units) c. No (different units) d. No (different units)
e. Yes (same units) f. Yes (same units) g. No (different units) h. Yes (same units)
13. a. 17 : 83 b. 97 : 3
14. a. 3 : 5 : 9 b. 2 : 7 : 14 c. 2 : 3 : 5 d. 1 : 5 : 95
15. 1125 g
16. a. 5 : 68 b. 27 : 5 c. 68 : 100 d. $\frac{17}{25}$, 68% e. $\frac{68}{5}$, 1360%
17. a. Class 8A — 4 : 5; class 8B — 4 : 5; class 8C — 4 : 3
b. Classes 8A and 8B c. 4 : 5 d. 20 : 21 e. No
18. Ratios compare quantities of the same unit, so units are not required
19. Fourteen coins need to be added to the purse. Three silver coins and 11 gold coins will change the ratio of gold to silver to 3 : 4.

Exercise 5.3 Simplifying ratios

1. a. 1 : 2 b. 1 : 3 c. 1 : 2 d. 1 : 3 e. 3 : 4 f. 5 : 6 g. 3 : 2 h. 3 : 2 i. 5 : 3 j. 1 : 2
k. 3 : 7 l. 3 : 4 m. 4 : 5 n. 3 : 2 o. 5 : 6 p. 10 : 3 q. 7 : 8 r. 3 : 4 s. 7 : 12 t. 2 : 3
2. a. 1 : 3 b. 2 : 1 c. 2 : 3 d. 64 : 32 e. 48 : 64
2 : 6 4 : 2 4 : 6 32 : 16 24 : 32
3 : 9 8 : 4 6 : 9 16 : 8 12 : 16
4 : 12 16 : 8 8 : 12 8 : 4 6 : 8
5 : 15 20 : 10 16 : 24 2 : 1 3 : 4
3. a. 2 : 3 b. 1 : 3 c. 5 : 3 d. 2 : 5 e. 1 : 4 f. 8 : 3 g. 3 : 40 h. 1 : 5 i. 9 : 4 j. 8 : 13
k. 5 : 4 l. 5 : 4 m. 1 : 5 n. 11 : 2 o. 2 : 5 p. 16 : 9 q. 3 : 10 r. 3 : 2 s. 7 : 3 t. 1 : 6
4. a. 4 : 5 b. 2 : 3 c. 3 : 2 d. 4 : 1 e. 4 : 3 f. 5 : 4 g. 1 : 8 h. 5 : 4 i. 8 : 1 j. 9 : 2
5. a. 1 : 20 b. 1 : 9 c. 12 : 11 d. 7 : 300
6. a. 1 : 2 b. 5 : 6 c. 1 : 2 d. 4 : 5 e. 16 : 9 f. 10 : 9
g. 3 : 10 h. 5 : 1 i. 5 : 6 j. 4 : 3 k. 5 : 8 l. 65 : 56
7. a. 7 : 9 b. 1 : 7 c. 1 : 3 d. 4 : 5 e. 1 : 6 f. 15 : 32
g. 10 : 1 h. 4 : 5 i. 1 : 10 j. 48 : 35 k. 1 : 20 l. 25 : 46
8. a. $a : 5b$ b. 2 : 1 c. $2x : 3$ d. $y : 2$ e. $3m : 4$ f. $1 : 4b$ g. $10^2 : x^2$ h. $2cd^2 : 1$
9. A

10. D

11. E

12. A

13. B

14. a. 18 400 b. 3 : 4 c. $\frac{3}{4}$

d. 42.86%. In part c the number of females was compared with the number of males (part-to-part comparison), whereas in part d the number of females was compared with the total number in the crowd (part-to-whole comparison).

15. a. 5 : 8 b. 2 : 3 c. 3 : 5 d. 4 : 3 e. 4 : 13

16. a. 929 : 321 b. 217 : 22 c. 9 : 16 d. 11 : 9 e. 6 : 29

17. a. 85 : 103 b. 50 : 71 c. 2 : 3

d. i. \$425 000 ii. 1 : 2

e. In a period of just over 6 years, the price of the house has increased by half its purchase price (in 2010).

18. Both ratios and fractions are simplified by multiplying or dividing by the same number to obtain the lowest form.

Exercise 5.4 Proportion

1. a. Yes b. Yes c. Yes d. Yes e. No f. Yes

g. No h. No i. No j. Yes k. No l. Yes

2. a. $a = 1$ b. $a = 4$ c. $a = 6$ d. $a = 4$ e. $a = 24$ f. $a = 50$

g. $a = 12$ h. $a = 6$ i. $a = 6$ j. $a = 1$ k. $a = 4$ l. $a = 3$

3. a. 9 boys b. 10 m c. 21 wins d. 24 chicken sandwiches

e. 5 litres f. 7 tables g. 4 cups h. 33 g

i. 75 cartons j. 3280 women

4. a. 11.2 b. 4.8 c. 2.1 d. 8.1 e. 7.1 f. 9.3 g. 7.7 h. 10.8 i. 11.7 j. 10.3

5. a. $\frac{1}{3} = \frac{n}{5}$ b. $\frac{2}{6} = \frac{n}{11}$ c. $\frac{15}{4} = \frac{50}{n}$ d. $\frac{2}{17} = \frac{5}{n}$ e. $\frac{2}{3} = \frac{15}{n}$
 $n = 1.7$ $n = 3.7$ $n = 13.3$ $n = 42.5$ $n = 22.5$

6. a. No b. No c. No d. Yes e. Yes f. No g. Yes h. Yes i. Yes j. No

7. a. C b. E c. A d. D

8. The 14-year-old receives \$28 and the 10-year-old receives \$20.

9. a. White gold: $\frac{3}{4}$, pink gold: $\frac{3}{4}$

b. Because $\frac{18}{24} = \frac{3}{4}$ c. 6 g d. \$210

10. 4 : 5

11. $9.6 : 20.8 = 6 : 13$

12. $1 : 20\,000 = x \text{ cm} : 2.4 \text{ km}$

$1 : 20\,000 = x \text{ cm} : 2.4 \times 1000 \times 100 \text{ cm}$

$1 : 20\,000 = x : 240\,000$

$$\frac{1}{20\,000} = \frac{x}{240\,000}$$

$$\frac{1}{20\,000} \times 240\,000 = \frac{x}{240\,000} \times 240\,000$$
$$x = 12$$

So, 12 cm on the map represents 2.4 km on the ground.

13. a. 10 : 4 : 3 b. 2 : 9 : 11

14. 0.14 m^3 cement, 0.29 m^3 sand, 0.57 m^3 gravel

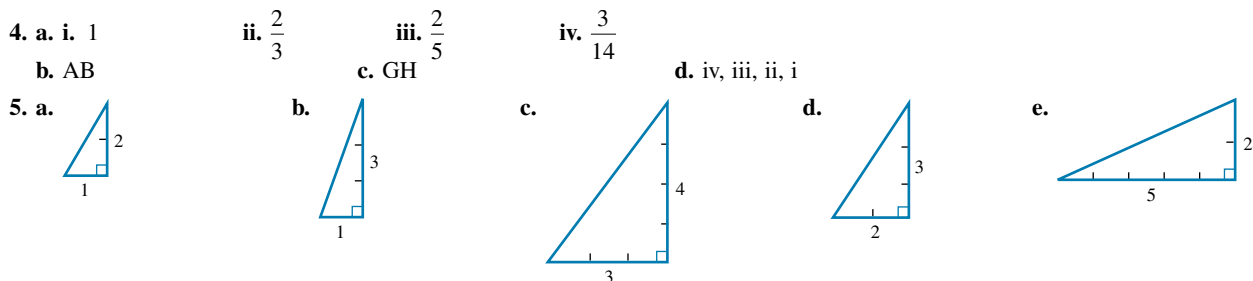
15. Discuss with your teacher.

Exercise 5.5 Comparing ratios

1. a. 3 : 4 b. 7 : 9 c. 6 : 5 d. 7 : 10 e. 7 : 9 f. 2 : 5 g. 3 : 4 h. 7 : 8 i. 7 : 12 j. 6 : 5

2. a. Peninsula b. Boroondara c. City West d. DC North

3. Jenny



6. a. C b. A

7. a. 3 : 4 : 5

c. 1 m, 1.25 m

8. a. $\frac{3}{15} = \frac{4}{20}$; same taste

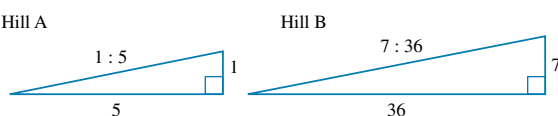
c. $\frac{2}{3} \neq \frac{5}{8}$; website 5 has the sweeter recipe.

b. The ratio of the 3 sides stays the same.

b. $\frac{3}{9} \neq \frac{5}{16}$; website 3 has the sweeter recipe.

d. $\frac{3}{8} \neq \frac{7}{12}$; website 8 has the sweeter recipe.

9. a. Hill A



Hill A is steeper. It is difficult to determine which hill has the steeper slope from these diagrams because the difference in slope is slight. Diagrams are accurate only if a scale drawing is used.

b. Answers will vary.

c. Answers will vary.

10. If the gradient is zero, the line is horizontal.

Exercise 5.6 Dividing in a given ratio

1. a. 3	b. 5	c. 4	d. 8	e. 13	f. 13	g. 13	h. 19	i. 6	j. 12		
2. a. \$400, \$600	b. \$750, \$250	c. \$200, \$800	d. \$500, \$500	e. \$375, \$625	f. \$625, \$375	g. \$300, \$700	h. \$900, \$100	i. \$350, \$650	j. \$450, \$550		
3. a. \$5000, \$5000	b. \$4000, \$6000	c. \$6000, \$4000	d. \$3000, \$7000	e. \$7000, \$3000	f. \$2000, \$8000	g. \$9000, \$1000	h. \$3750, \$6250	i. \$4800, \$5200	j. \$4600, \$5400		
4. a. \$10 000, \$40 000	b. \$15 000, \$35 000	c. \$20 000, \$30 000	d. \$25 000, \$25 000	e. \$12 500, \$37 500	5. a. 2 m ³	b. 0.5 m ³	c. 1.6 m ³	d. 1.6 m ³	e. 1.2 m ³		
6. \$25, \$20, \$15	a. \$90 000, \$210 000, \$300 000	b. \$180 000, \$180 000, \$240 000	c. \$30 000, \$240 000, \$330 000	d. \$150 000, \$180 000, \$270 000	e. \$150 000, \$225 000, \$225 000	8. a. 90	b. 75	9. a. 120	b. 45	c. \$6	d. \$19.50
10. a. D	b. D	c. A	d. D	11. 30°, 60°, 90°. Sum the ratio parts (1 + 2 + 3 = 6), divide the angle sum of a triangle by the total number of ratio parts to calculate the value of 1 ratio part (180 ÷ 6 = 30°), then multiply this by the number of ratio parts.							

12. 96°

13. a. 50 b. 4 c. 4:1

14. a. 1 : 300 b. 1200 cm

15. a. 1 b. 37
c. i. 12 ii. 49

16. Discuss with your teacher.

Exercise 5.7 Rates

1. a. 10 m ² /min	b. 5 kL/min	c. 300 cm ³ /s
d. \$1.38/L	e. 8 L/100 km or 12.5 km/L	f. \$2.50/m
g. 40 cows/hectare or (250 m ² /cow)	h. \$12.50/person	i. \$3.20/m ²
j. 25 c/min	k. 16 points/game	l. \$5.40/kg
m. $8\frac{1}{3}$ m/s	n. 5.2 runs/over	o. \$26.50/h

- p. \$3.50/kg
s. 6.5 cm/year
- q. 52 words/min
t. 16 km/h
- r. 2 °C/h
- \$108.50
 - 396 points
 - 60.5 L
 - 21
 - 70 min
 - 12 days
 - 49
 - 1500 m²
 - 42
 - $\frac{\text{distance}}{\text{time}}$
 - $\frac{\text{volume}}{\text{time}}$
 - $\frac{\text{capacity}}{\text{distance}}$
 - $\frac{\text{money}}{\text{time}}$
 - $\frac{\text{money}}{\text{length}}$
 - $\frac{\text{capacity}}{\text{time}}$
 - $\frac{\text{money}}{\text{capacity}}$
 - $\frac{\text{money}}{\text{number}}$
 - $\frac{\text{mass}}{\text{time}}$
 - $\frac{\text{number}}{\text{area}}$
 - L/min
 - m/s
 - cm/year
 - cm/h
 - mm²/sec
 - L/km
 - Runs/over
 - Words/min
 - 360 L
 - Packs of 10
 - Car A
 - 250 g jar; 75c
 - C
 - C
 - D
 - B
 - A ratio compares two quantities measured in the same units, where a rate compares two quantities measured in different units.
 - 4 patients per hour
 - 15 min
 - \$62.50 per patient
 - 4 min
 - $1\frac{7}{8}$ hours or 1 hour 52 minutes 30 seconds
 - 2750 organisms/litre. The beach should not be closed.
 - $V = 1.098\,07 \times 10^{21} \text{ cm}^3$
 - $V = 1.563\,46 \times 10^{24} \text{ cm}^3$
 - 5464.13 kg/m³
 - 1215.25 kg/m³
 - Earth and Jupiter do not seem to be made of the same substance, because Jupiter's density is much less than that of Earth.
 - The 2005 race
 - Rates require units as they measure how one unit changes per another unit.
 - To work: 30 mins.
From work: $1\frac{1}{3}$ hours (1 hour and 20 minutes)
Total travel time: $1\frac{5}{6}$ hours (1 hour and 50 minutes)

Exercise 5.8 Interpreting graphs

- The vertical axis is distance and the horizontal axis is time.
- m/s
- Horizontal line stretching for 60 minutes (with time measured horizontally)
- D
- The graph shows how the student dawdles at first and the rate of movement is slow. Then there is no change at all as the student goes into the shop. Finally the speed increases as the student hurries to school.
- B
- The graph shows how the bus makes its way to the school. The graph is not a completely straight line because there would be unevenness in relation to traffic conditions and pick-up and drop-off stops.
- False
 - True
 - False
 - True
 - False
- II
 - III
 - I
- A steeper gradient represents a faster speed, whereas a shallow gradient represents a slower speed.
- Horizontal line segments represent when and where an object is not moving.

12. **A:** 60 km/h
B: 70 km/h
C: 46.7 km/h

13. **a.** Min (time) and \$ (cost) **b.** Change in cost **c.** Change in time
d. $m = \$1$ per min, $m = 50c$ per min, $m = 0c$ (i.e. free calls)
e. The gradient represents the increase in cost for each increase in time.

14. **a.** Lamb: $m = \$15/\text{kg}$ **b.** Lamb: \$15 **c. i.** \$15
Chicken: $m = \$10/\text{kg}$ Chicken: \$10 **ii.** \$5
Beef: $m = \$7.50/\text{kg}$ Beef: \$7.50 **iii.** \$15
d. \$35

e.

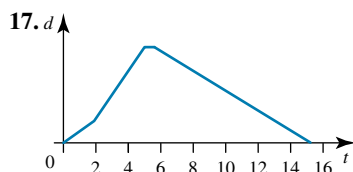
Meat type	Cost per kilogram (\$/kg)	Weight required (kg)	Cost = \$/kg $\times$ kg
Lamb	15	1	15
Chicken	10	0.5	5
Beef	7.50	2	15
		Total cost	35

15. Answers will vary.

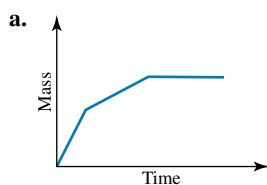
Students should understand that the distance travelled at the end of each leg of the journey should not change, but that the time taken to reach the end of each leg should be half the original time taken.

The amount of time taken for a rest is not affected and, as such, the overall amount of time taken for the whole journey will not be exactly half the original time.

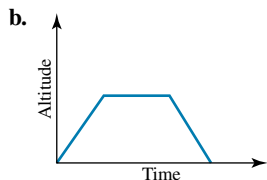
16. Answers will vary.



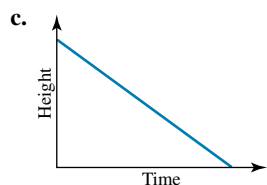
18. Answers will vary. Example:



The mass of a pig will steadily increase over time.



The altitude of a plane will steadily rise until cruising height, then stay at that height, before descending back to ground.



As a bath tub is being emptied the height of the water will steadily decrease.

19. Graphs can tell a story by showing how quantities of one variable differ in relation to another variable.

Investigation | Rich task

$$\frac{\sqrt{5} + 1}{2} = 1.618$$

Part A: Teacher to check

Part B:

1. 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584, 4181, 6765

2.

Sequence	Ratio	Decimal value
1	—	—
1	$\frac{1}{1}$	1
2	$\frac{2}{1}$	2
3	$\frac{3}{2}$	1.5
5	$\frac{5}{3}$	1.6
8	$\frac{8}{5}$	1.6
13	$\frac{13}{8}$	1.625
21	$\frac{21}{13}$	1.61538
34	$\frac{34}{21}$	1.61905
55	$\frac{55}{34}$	1.61765

Sequence	Ratio	Decimal value
89	$\frac{89}{55}$	1.61818
144	$\frac{144}{89}$	1.61798
233	$\frac{233}{144}$	1.61806
377	$\frac{377}{233}$	1.61803
610	$\frac{610}{377}$	1.61804
987	$\frac{987}{610}$	1.61803
1597	$\frac{1597}{987}$	1.61803
2584	$\frac{2584}{1597}$	1.61803
4181	$\frac{4181}{2584}$	1.61803
6765	$\frac{6765}{4181}$	1.61803

3. Teacher to check.

Exercise 5.9 Review questions

1. a. 3 : 5 b. 1 : 17 c. 17 : 3 d. 5 : 1 e. 5 : 21
2. a. 1 : 2 b. 2 : 3 c. 7 : 20 d. 10 : 3 e. 2 : 9
f. 1 : 4 g. 2 : 5 h. 7 : 10 i. 3 : 1 j. 15 : 4
3. a. $n = 12$ b. $n = 20$ c. $n = 12$ d. $n = 15$ e. $n = 9.6$ f. $n = 1.2$
4. a. 1 : 6 b. 1.5 L
5. a. $\frac{4}{5}$ b. $\frac{5}{8}$
6. 10 or 11
7. a. Slide A: $\frac{2}{3}$, slide B: $\frac{2}{5}$ b. Slide A
8. a. \$10, \$15 b. \$420, \$300
9. \$880
10. 8.57 L/100 km
11. a. David's b. 432 km c. 42 L
12. 1 kg packet
13. 96 cm 14. 32 15. 32 km/h 16. 28 064.5 km/h
17. a. 13.5 km/min b. 808.3 km/h
18. a. 45 km b. 15 km/h
19. 4.5 kg 20. 20 cm 21. 68.61 km/h 22. \$700 23. 1 h 6 min 40 s
24. a. 0.6; 0.4; 0.8 b. The third segment c. Average gradient is 0.56.

TOPIC 6

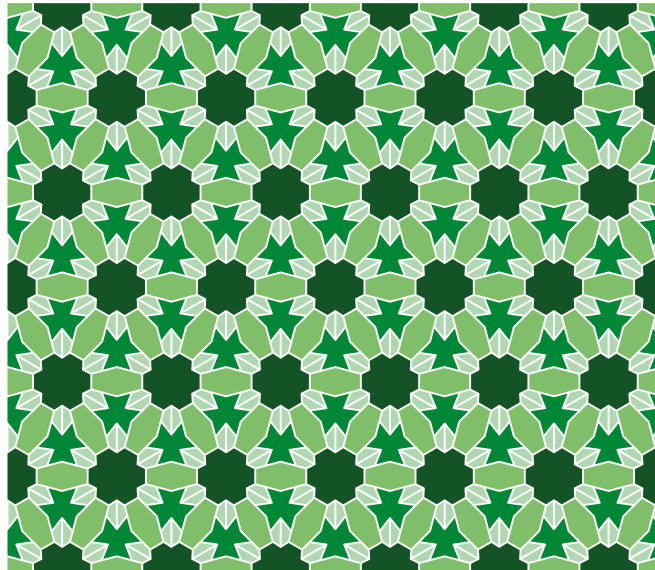
Congruence

6.1 Overview

Numerous **videos** and **interactivities** are embedded just where you need them, at the point of learning, in your learnON title at www.jacplus.com.au. They will help you to learn the concepts covered in this topic.

6.1.1 Why learn this?

From backyard sheds and home extensions to architecture itself, the design of buildings is founded on congruence. Many structures, from the modern to the very old, were created by people who had the chance to read a chapter similar to this one. Grand structures such as the Queen Victoria Building, and even the very house in which you are living, are all based on the principles of congruence.



DISCUSSION

What is the difference between axioms and theorems? Give some examples.

LEARNING SEQUENCE

- 6.1 Overview
- 6.2 Congruent figures
- 6.3 Congruent triangles
- 6.4 Quadrilaterals
- 6.5 Review

LEARNING OUTCOMES

A student:

- communicates and connects mathematical ideas using appropriate terminology, diagrams and symbols MA4-1WM
- applies appropriate mathematical techniques to solve problems MA4-2WM
- recognises and explains mathematical relationships using reasoning MA4-3WM
- classifies, describes and uses the properties of triangles and quadrilaterals, and determines congruent triangles to find unknown side lengths and angles MA4-17MG

CONTENT DESCRIPTIONS

Students:

Define congruence of plane shapes using transformations (ACMMG200)

Develop the conditions for congruence of triangles (ACMMG201)

Establish properties of quadrilaterals using congruent triangles and angle properties, and solve related numerical problems using reasoning (ACMMG202)

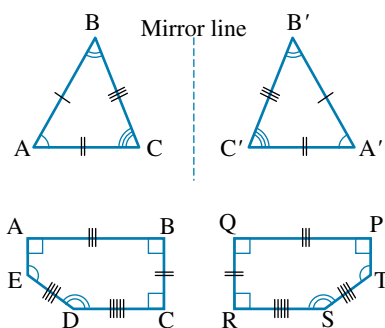
Source: NSW Syllabus for the Australian curriculum

Note: Your teacher may now set you a pre-test to determine how familiar you are with the content in this topic.

6.2 Congruent figures

6.2.1 Identifying congruent shapes

- **Congruent figures** are identical in size and shape.
- The transformations of reflection, rotation and translation do not change the shape and size of a figure. The original and transformed figure are said to be congruent.
- Repeated transformations of a congruent shape or a group of congruent shapes, so that an entire surface is covered with a regular pattern, is called a tessellation.
- The symbol used for congruence is $\equiv$.
- When writing congruence statements, we name the vertices of the figures in corresponding (or matching) order.
- Sometimes you will need to rotate, reflect or translate the figures for them to be orientated the same way.
- If, through one or more translations, rotations and reflections, one shape can lie exactly on top of the other, then these two figures are congruent.
- The figures below are congruent. For the triangle, we can write $ABC \equiv A'B'C'$; for the pentagon, we can write $ABCDE \equiv PQRST$.



Note that equivalent sides and angles are labelled with the same symbol.

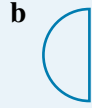
ACTIVITY: TESSELLATIONS IN ART

MC Escher is generally regarded as the master of modern tessellations. He created many amazing artworks using tessellations.

1. Find some examples of Escher's work. Create a poster containing at least five of his artworks, including some of his earlier work. Share this with your classmates.
2. Investigate some of Escher's techniques and try using them to create a tessellation of your own.

WORKED EXAMPLE 1

Select a pair of congruent shapes from the following set.



THINK

Figures **a**, **b** and **c** have the same shape (i.e. a semicircle). Figure **d** is not a semicircle and thus is not congruent to any other figures. Figure **c** is larger than figures **a** and **b** and so is not congruent to either of them. Figures **a** and **b** are identical in shape and size (**b** is a reflection of **a**), and therefore are congruent to each other.

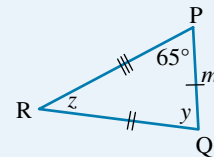
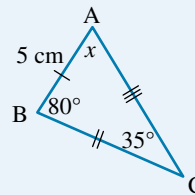
WRITE

Shape **a** $\equiv$ Shape **b**

- From Worked example 1 we can see that any two semicircles with the same size are congruent. This means that any two semicircles with equal radii are congruent. Likewise, if any two circles have the same radii then they must be congruent.

WORKED EXAMPLE 2

Find the value of the variables in the pair of congruent triangles at right.



THINK

- Since $\triangle ABC \equiv \triangle PQR$ the corresponding angles are equal in size. Corresponding angles are included between the sides of equal length. So, by looking at the markings on the sides of the triangles, we can conclude that:

$\angle BAC$ corresponds to $\angle QPR$

$\angle ABC$ corresponds to $\angle PQR$

$\angle ACB$ corresponds to $\angle PRQ$

So, match the variables with the corresponding angles whose sizes are given.

- In congruent triangles, the corresponding sides are equal in length. Using the markings on the sides of the triangles, observe that the unknown side PQ corresponds to side AB. State the value of the variable (which represents the length of PQ).

$$x = 65^\circ$$

$$y = 80^\circ$$

$$z = 35^\circ$$

$$m = 5 \text{ cm}$$

- The principle of congruence is applied in the construction of many everyday objects. For example, the frame of the bicycle shown is partly made up of two congruent triangles; the wheels are congruent circles.



DISCUSSION

Which regular polygons will tessellate on their own without any spaces or overlaps? Using mathematics, can you explain why not all regular polygons will tessellate?

learnon RESOURCES — ONLINE ONLY

Interactivity: Transformations and the Cartesian plane (int-2368)

Interactivity: Congruent figures (int-3749)

Digital doc: SkillsHEET Naming angles and shapes (doc-6915)

Digital doc: SkillsHEET Calculating angles in triangles (doc-6916)

Exercise 6.2 Congruent figures

Individual pathways

PRACTISE

Questions:
1–5, 7

CONSOLIDATE

Questions:
1–5, 7, 9

MASTER

Questions:
1–9

Individual pathway interactivity: int-4424

learnon ONLINE ONLY

To answer questions online and to receive **immediate feedback** and **fully worked solutions** for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Understanding and fluency

- Plot the following coordinates on a Cartesian plane: $(1, -1)$, $(1, 0)$, $(1, 1)$, $(2, 1)$, $(1, 2)$, $(1, 3)$, $(2, 3)$, $(3, 3)$. Join the points with straight lines to make the shape of the letter F.
 - Reflect this figure in the y -axis and draw the translated figure on a Cartesian plane.
 - Translate the original figure by three units to the right and two units down, and draw the translated figure on a Cartesian plane.
 - Rotate the original figure through 90° about the origin and draw the translated figure on a Cartesian plane.
 - Look at the transformations in parts **a**, **b** and **c**. Are the shapes congruent? Why?
- WE1** Select a pair of congruent shapes from each of the following sets.

a. i.



ii.



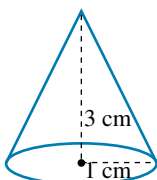
iii.



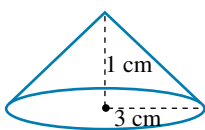
iv.



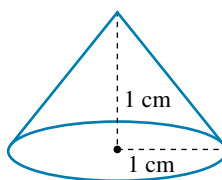
b. i.



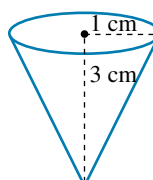
ii.



iii.



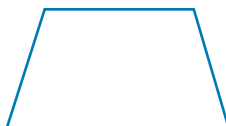
iv.



c. i.



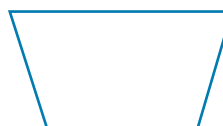
ii.



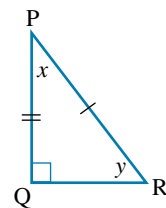
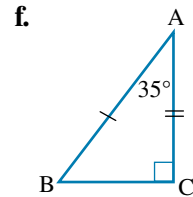
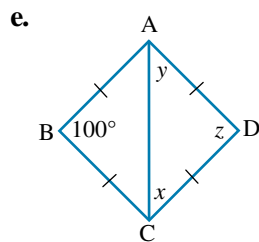
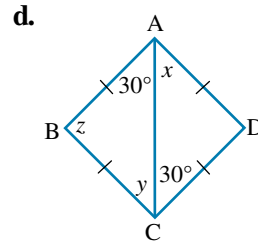
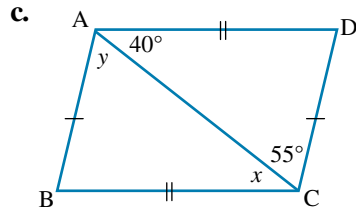
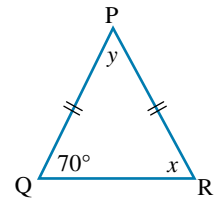
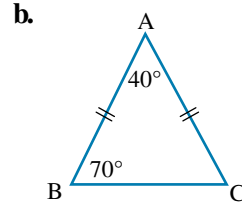
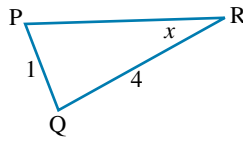
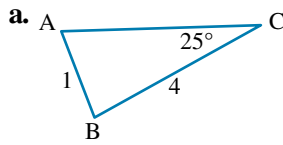
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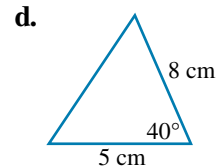
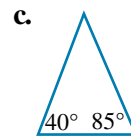
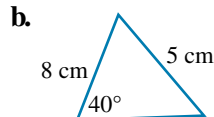
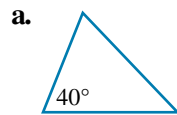
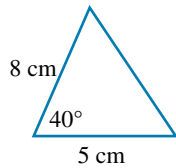
iv.



3. **WE2** All of these pairs of triangles are congruent. Name the congruent triangles and find the value of the variables in each case. (Remember: The vertices must be listed in corresponding order.)



4. **MC** Which of the following is congruent to the triangle shown below left?



A. a only

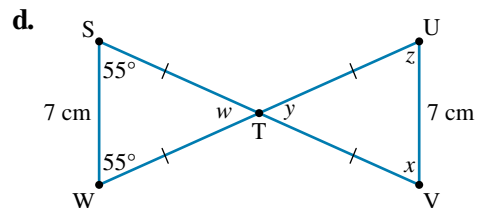
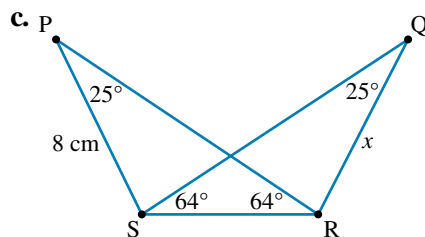
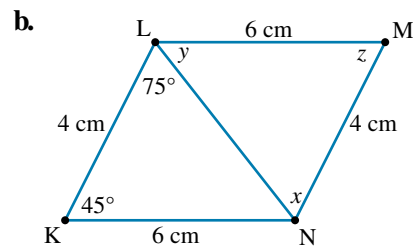
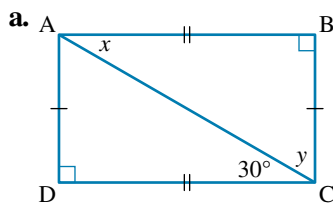
B. a and b

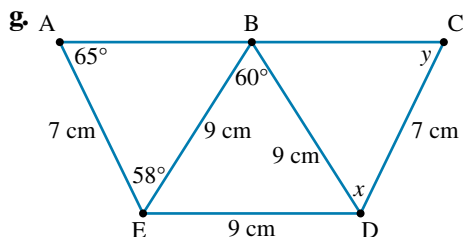
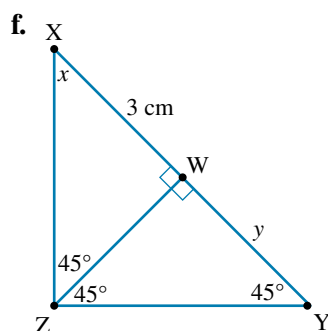
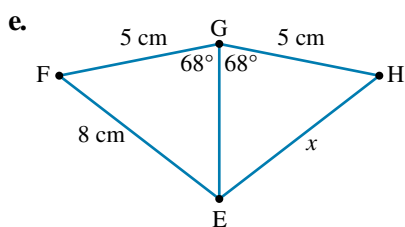
C. d only

D. b and d

E. None of the above

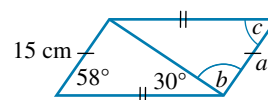
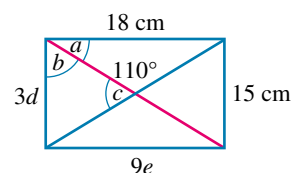
5. Name the congruent triangles in each question and find the value of the variable(s).





Communicating, reasoning and problem solving

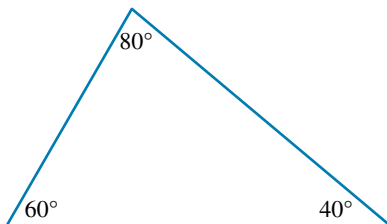
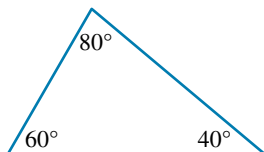
- Give an example to show that triangles with two angles of equal size and a pair of non-corresponding sides of equal length may not be congruent.
- Given that the four larger triangles in the figure at right are right-angled and congruent, find the values of the pronumerals.
- Given that the right-hand triangle is congruent to the left-hand triangle in the figure at right, find the values of the pronumerals.
- When we write a congruence statement, why are the vertices listed in corresponding order?



6.3 Congruent triangles

6.3.1 Congruent triangles

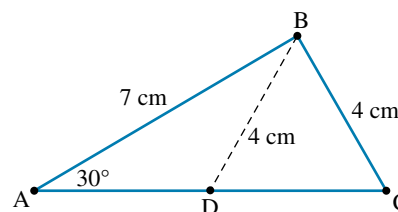
- A triangle can be described with three pieces of information about side length and/or angle size.
- If three angles are known, an infinite number of triangles can be drawn. For example, the triangles below have angle sizes of 60° , 40° and 80° .



$\therefore$ If only three angles are known, this is not a test for congruency.

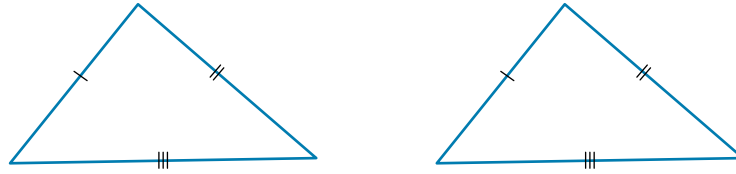
- There can be two possible triangles drawn from two sides and the non-included angle. In the diagram at right, triangles ABC and ABD both have sides of 7 cm and 4 cm, and an angle of 30° between the 7-cm side and the unknown side.

$\therefore$ If two sides and the non-included triangle are given, this is not a test for congruence.



6.3.2 Congruency tests

- There are five tests that can be used to demonstrate that two triangles are congruent.
- It is not enough for the diagrams to look the same; you need evidence!
- Two triangles are congruent if:
 - three sides of one triangle are equal to the three sides of the other (SSS)



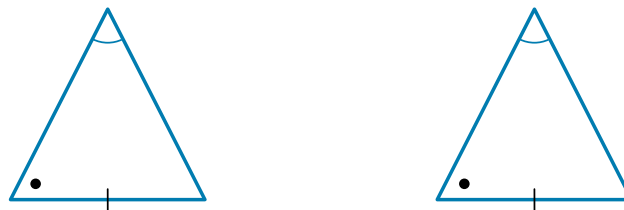
- two sides and the included angle of one triangle are equal to two sides and the included angle of the other (SAS)



- two angles and a side of one triangle are equal to the two angles and the corresponding side of the other (AAS)



- a pair of corresponding angles and a non-included side are equal (ASA)

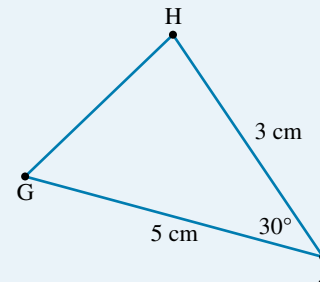
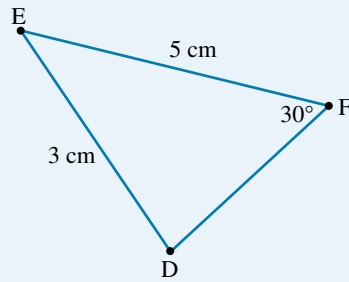
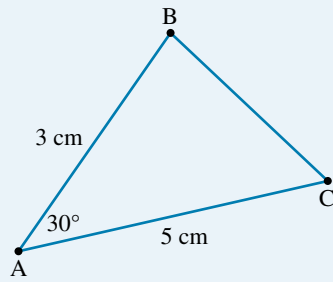


- the triangles are right-angled and the hypotenuse and a side of one are equal to the hypotenuse and a side of the other (RHS).



WORKED EXAMPLE 3

Which of the following triangles are definitely congruent? Give a reason for your answer.



THINK

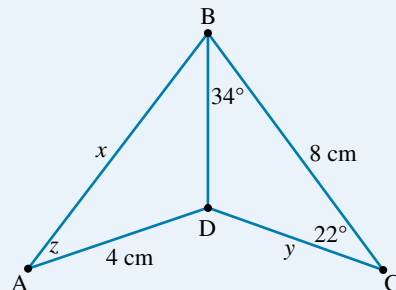
In all three triangles, two given sides are of equal length (3 cm and 5 cm). In triangles ABC and JHG, the included angle is 30° . In triangle EFD, 30° is not the included angle.

WRITE

$$\triangle ABC \equiv \triangle JHG \text{ (SAS)}$$

WORKED EXAMPLE 4

Given that $\triangle ABD \equiv \triangle CBD$, find the value of the variables.



THINK

1 The corresponding sides are equal in length.

$$\triangle ABD \equiv \triangle CBD.$$

AB and CD are unknown.

AB corresponds to CB.

CD corresponds to AD.

2 The corresponding angles are equal.

$\angle BAD$ is unknown.

$\angle BAD$ corresponds to $\angle BCD$.

WRITE

$$\triangle ABD \equiv \triangle CBD$$

$$AB = CB$$

$$x = 8 \text{ cm}$$

$$CD = AD$$

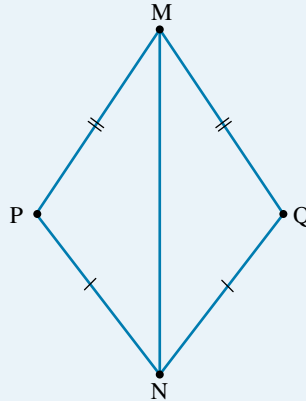
$$y = 4 \text{ cm}$$

$$\angle BAD = \angle BCD$$

$$z = 22^\circ$$

WORKED EXAMPLE 5

Prove that $\triangle MNP$ is congruent to $\triangle MNQ$.



THINK

- 1 Study the diagram and state which sides and/or angles are equal.
- 2 Select the appropriate congruency test (in this case SSS, as the triangles have all corresponding sides congruent in length).

WRITE

$NP = NQ$ (given)
 $PM = QM$ (given)
 MN is common

$\triangle NPM \equiv \triangle NQM$ (SSS)

ACTIVITY: SOFTWARE AND CONGRUENT TRIANGLES

Use dynamic geometry software to investigate what information is needed to show that two triangles are congruent.

learnON RESOURCES — ONLINE ONLY

Interactivity: Congruent triangles (int-3754)

Interactivity: Congruency tests (int-3755)

Digital doc: WorkSHEET Congruence and transformations I (doc-2250)

Exercise 6.3 Congruent triangles

Individual pathways

PRACTISE

Questions:
1–3

CONSOLIDATE

Questions:
1–6, 9, 11

MASTER

Questions:
1–11

Individual pathway interactivity: int-4426

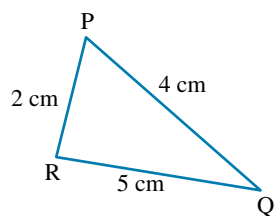
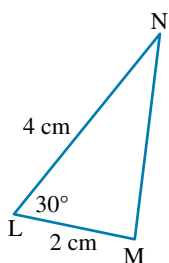
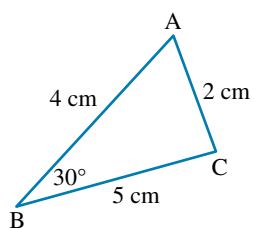
learnON ONLINE ONLY

To answer questions online and to receive **immediate feedback** and **fully worked solutions** for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

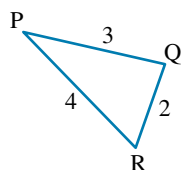
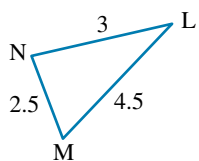
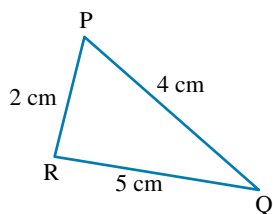
Understanding and fluency

1. **WE3** In each part of the question, which of the triangles are congruent? Give a reason for your answer.

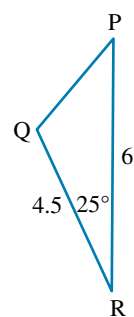
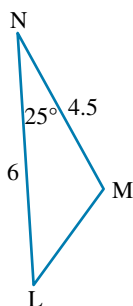
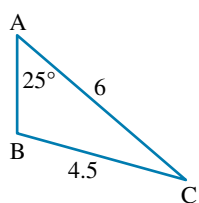
a.



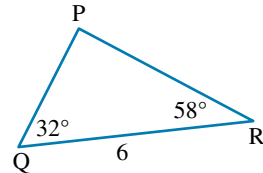
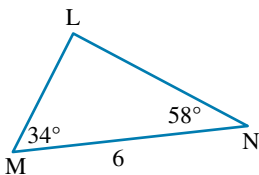
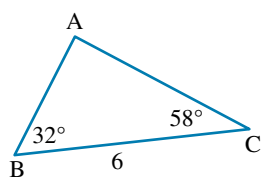
b.



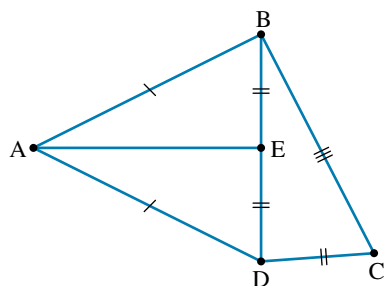
c.



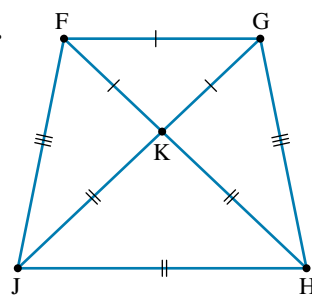
d.



e.

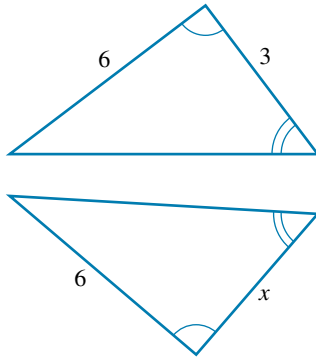


f.

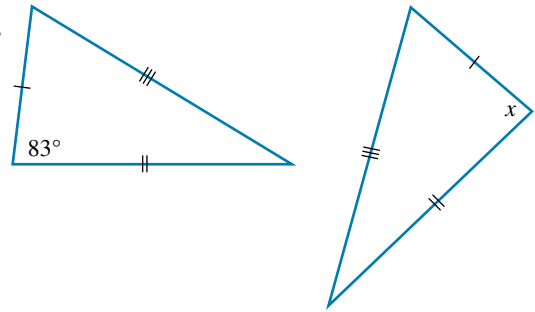


2. **WE4** Find the values of the variables in each of the following pairs of congruent triangles. All side lengths are in centimetres.

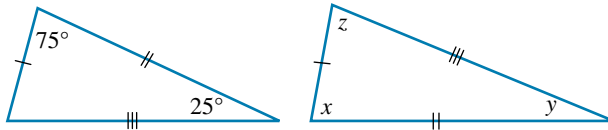
a.



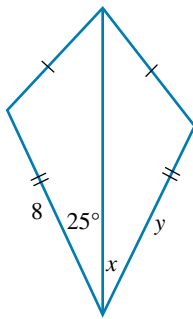
b.



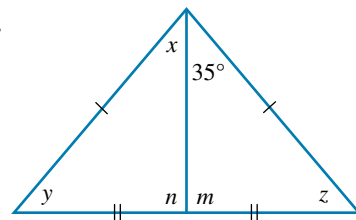
c.



d.

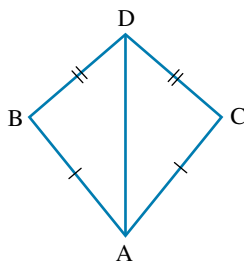


e.



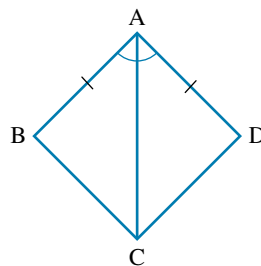
3. **WE5** For each of the following, prove that:

a.



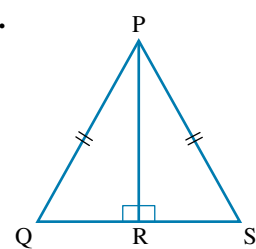
$$\triangle DBA \equiv \triangle DCA$$

b.



$$\triangle ABC \equiv \triangle ADC$$

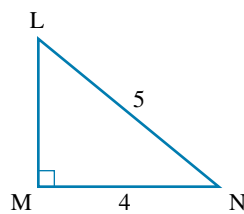
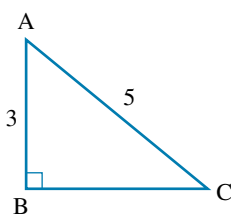
c.

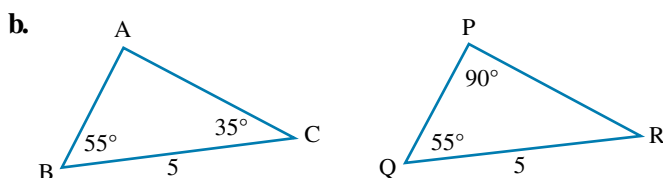


$$\triangle PQR \equiv \triangle PSR$$

4. Draw an example of two different triangles that have two corresponding sides equal and one pair of equal angles.
5. Draw an example of two different triangles that have three pairs of equal angles.
6. Are the following triangles congruent? Give a reason for your answer. (*Hint*: Pythagoras' theorem (Topic 14) is needed for this question.)

a.





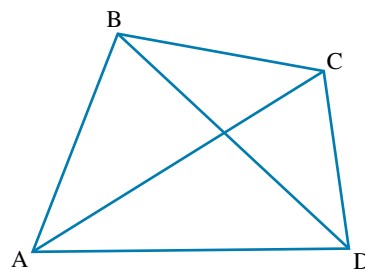
Communicating, reasoning and problem solving

7. In an equilateral triangle ABC, the midpoints of the sides are labelled D, E and F. Prove that triangle DEF is an equilateral triangle.
8. All triangles have three sides and three angles. Using mathematical reasoning, including examples, give the minimum information required to construct a unique triangle.
9. Prove that the construction of a diagonal in a parallelogram creates two congruent triangles.
10. From the right angle of a right-angled isosceles triangle, a straight line is that which bisects the hypotenuse. Prove that two congruent triangles are created.
11. When using the congruency rule ASA, why is it not necessary for the side to be between the two angles?

6.4 Quadrilaterals

6.4.1 Review of terms and rules

- The figure ABCD is a quadrilateral.
- A **quadrilateral** is a four-sided figure.
- The sum of the interior angles at the four vertices of a quadrilateral is 360° .
- **Opposite angles** do not have rays in common. $\angle ABC$ and $\angle ADC$ are opposite angles. $\angle BCD$ and $\angle BAD$ are also opposite angles.
- **Opposite sides** do not intersect. BC and AD are opposite sides. AB and CD are opposite sides.
- **Diagonals** connect opposite angles. AC is one diagonal and BD is the other.
- In some quadrilaterals, the *diagonals bisect each other*. In the diagram above, BD would bisect AC only if it divided AC into two equal sections. AC would bisect BD only if it divided it into two equal sections. In the diagram, the diagonals do not bisect each other.
- In some quadrilaterals, the *diagonals bisect the angles*. Diagonal BD bisects $\angle ABC$ if $\angle ABD$ and $\angle DBC$ are congruent. In the diagram, BD does not bisect $\angle ABC$.
- There are several special quadrilaterals.
 - A **parallelogram** is a quadrilateral where the opposite pairs of sides are parallel.
 - A **rhombus** is a parallelogram in which all sides are equal. (Because a rhombus is a parallelogram, the properties of a parallelogram are also properties of a rhombus.)
 - A **rectangle** is a parallelogram in which all angles are 90° . (Because a rectangle is a parallelogram, the properties of a parallelogram are also properties of a rectangle.)
 - A **square** is a rectangle with all sides equal. (Because rectangles are parallelograms and a rhombus is a parallelogram with all sides congruent, a square is also a right-angled rhombus.)
 - A **kite** is a quadrilateral that has two pairs of adjacent sides that are equal.
 - A **trapezium** is a quadrilateral with one pair of unequal parallel sides.
- By considering triangles in quadrilaterals, the properties of quadrilaterals can be determined.

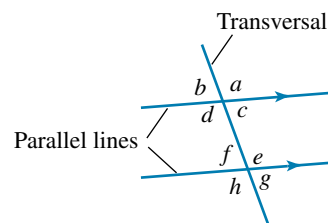


DISCUSSION

Which professions would use the knowledge of angles in triangles and quadrilaterals?

6.4.2 Angles and parallel lines

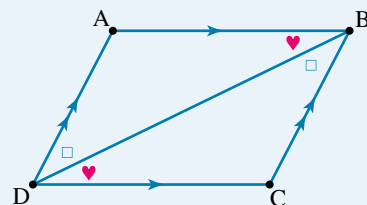
- Parallel lines are denoted by identical arrowheads.
- A straight line cutting a parallel line is called a **transversal**.
- Corresponding angles** are equal in magnitude. For example, in the diagram, $a = e$, $c = g$, $b = f$ and $d = h$.
- Vertically opposite angles** are equal in magnitude. For example, in the diagram, $a = d$, $b = c$, $f = g$ and $e = h$.
- Alternate angles** are equal in magnitude. For example, in the diagram, $c = f$ and $d = e$.
- Co-interior angles** (also known as allied angles) sum to 180° . For example, in the diagram, $c + e = 180^\circ$ and $d + f = 180^\circ$.
- Angles and parallel lines can be used to establish congruency of triangles in parallelograms.



WORKED EXAMPLE 6

When a diagonal is drawn in a parallelogram, two pairs of equal angles are formed, as shown (alternate angles).

- State the congruency relationship between the two triangles formed, giving a reason.
- Hence write a relationship between:
 - AB and DC
 - DA and CB
 - $\angle BAD$ and $\angle BCD$.
- What conclusion can be drawn about:
 - opposite sides of a parallelogram
 - opposite angles of a parallelogram?



THINK

- There are two equal angles, and BD is in both triangles.
- AB and DC are corresponding sides in the triangles.
 - DA and CB are corresponding sides in the triangles.
 - $\angle BAD$ and $\angle BCD$ are corresponding angles.
- AB and DC are opposite sides. DA and CB are opposite sides.
 - $\angle BAD$ and $\angle BCD$ are opposite angles. $\angle ABC$ and $\angle ADC$ are opposite angles.

WRITE

- $\angle ABD = \angle CDB$ (given)
 $\angle ADB = \angle CBD$ (given)
 BD is common to both
 $\therefore \triangle ABD \equiv \triangle CDB$ (ASA)
- $AB = DC$
 - $DA = CB$
 - $\angle BAD = \angle BCD$
- $AB = DC$ and $DA = CB$; therefore, opposite sides are equal.
 - $\angle BAD = \angle BCD$
 $\angle ABD = \angle CDB$ (given)
 $\angle ADB = \angle CBD$ (given)
 $\therefore \angle ABC = \angle ADC$
 Opposite angles are equal.

Interactivity: Quadrilaterals (int-3756)

eLesson: Parallel lines (eles-2309)

Digital doc: SkillsHEET Angles and parallel lines (doc-6917)

Digital doc: WorkSHEET Congruence and transformations II (doc-2251)

Exercise 6.4 Quadrilaterals

Individual pathways

PRACTISE

Questions:
1–5, 9, 10, 12

CONSOLIDATE

Questions:
1–6, 9, 10, 12

MASTER

Questions:
1–12

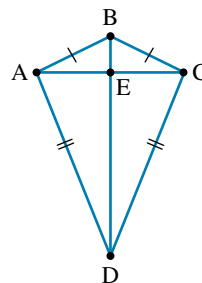
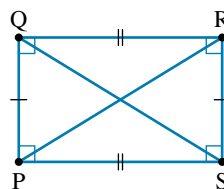
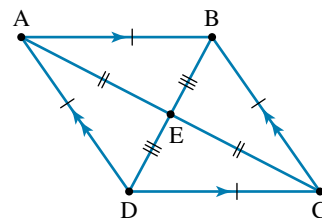
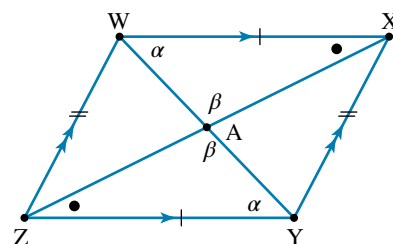
Individual pathway interactivity: int-4427

learnon ONLINE ONLY

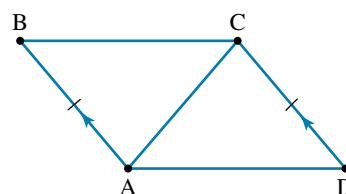
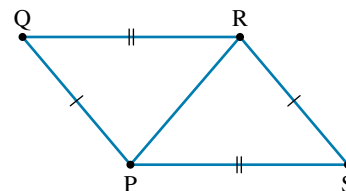
To answer questions online and to receive **immediate feedback** and **fully worked solutions** for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Understanding and fluency

- WE6a,b** Consider the parallelogram WXYZ. From Worked example 6, we know that opposite sides of a parallelogram are equal. Both diagonals are drawn giving 3 pairs of angles as shown (alternate angles and opposite angles).
 - State the congruency relationship between $\triangle WXA$ and $\triangle YZA$, giving a reason.
 - Hence state the relationship between:
 - WA and YA
 - XA and ZA.
 - Do the diagonals bisect each other?
- Consider the rhombus ABCD. Because the rhombus is also a parallelogram, we know that the diagonals bisect each other.
 - State the congruency relationship between $\triangle ABE$ and $\triangle BEC$, giving a reason.
 - Hence state the relationship between:
 - $\angle AEB$ and $\angle CEB$
 - $\angle ABE$ and $\angle CBE$.
 - What is the magnitude of $\angle AEB$?
 - Name all the triangles congruent to $\triangle ABE$.
 - Do the diagonals bisect the angles?
- Consider the rectangle PQRS.
 - Show that $\triangle PQR$ and $\triangle SRP$ are congruent.
 - What can you say about the diagonals of a rectangle?
- Consider the kite ABCD.
 - Show that $\triangle ABD$ and $\triangle CBD$ are congruent.
 - What can you say about:
 - $\angle ABD$ and $\angle CBD$
 - $\angle BAD$ and $\angle BCD$
 - $\angle BDA$ and $\angle BDC$?
 - Use part b to show that $\triangle ABD \equiv \triangle CBD$.
 - What can you say about:
 - AE and CE
 - $\angle AEB$ and $\angle CEB$?
 - Are $\triangle BEC$ and $\triangle DEC$ congruent?

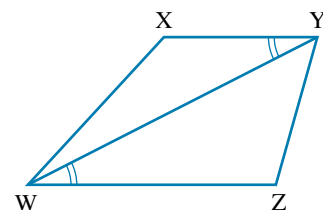


- f. Use the above results to answer the following questions about kites.
- Are the opposite sides equal?
 - Are the opposite angles equal?
 - Do the diagonals bisect each other?
 - Do the diagonals intersect at 90° ?
 - Do the diagonals bisect the angles?
5. The quadrilateral PQRS has opposite sides that are equal.
- Show that the triangles are congruent.
 - Complete the following.
 - $\angle QPR =$ _____
 - $\angle QRP =$ _____
 - Complete the following.
 - QP is parallel to _____.
 - QR is parallel to _____.
 - What type of quadrilateral is PQRS?
6. In quadrilateral ABCD, AB and DC are both parallel and equal in length.
- Identify one pair of alternate angles.
 - Show that the two triangles are congruent.
 - Hence demonstrate that BC and AD are parallel.
 - What type of quadrilateral is ABCD?
7. In a quadrilateral MNOP, use 2 pairs of adjacent triangles to show that the sum of the interior angles of a quadrilateral is 360° .



Communicating, reasoning and problem solving

8. WXYZ is a quadrilateral, where $\angle XYW = \angle YWZ$. Use mathematical reasoning to determine the type of quadrilateral.
9. The frame of the swing shown in the photograph is a trapezium. The diagonal sides make an angle of 73° with the ground. Evaluate the angle, x , that the horizontal crossbar makes with the sides.



10. The pattern for a storm sail for a sailboard is shown in the photograph. Calculate the size of each interior angle.
11. a. Prove that a quadrilateral can have no more than one interior angle that is greater than 180° .
 b. If all interior angles are integers, what is the largest possible interior angle that a quadrilateral can have?
12. Why is it important to be able to show congruency using mathematical reasoning?



6.5 Review

Investigation | Rich task

Puzzling constructions

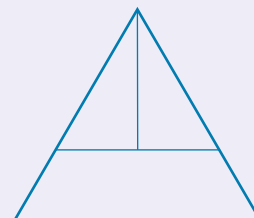


How well do you know the properties of the different triangles and quadrilaterals? This activity will challenge you to create as many triangles and quadrilaterals as you can from three different types of puzzles.

Follow the instructions in this activity to create your three puzzles. Draw each shape onto a firm sheet of A4 paper. Carefully space the puzzles, so they will all fit onto the one sheet.

Three-piece triangle

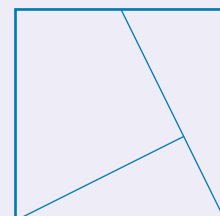
- Construct an equilateral triangle with side lengths of 10 cm. Mark the centre of each side.
- Lightly rule a line from each corner to the midpoint on the opposite side. These lines will intersect at one point.
- From this point, rule a line to the top corner. Rule another line parallel to the base side that goes through the centre point. Cut out the three shapes.



1. Name the three shapes created in this puzzle. List the main properties of each shape.
2. Use any number of pieces to construct as many triangles and quadrilaterals as you can. Draw each solution and list the main properties of each shape.

Three-piece square

- Draw a square with side lengths of 10 cm. Mark the centre of the top and right sides.
 - Rule a line from the centre of the top side to the bottom right corner.
 - Rule another line from the bottom left corner towards the centre on the right side. Stop when you meet the previous line. Cut out the three shapes.
3. Name the three shapes created in this puzzle. List the main properties of each shape.
 4. Repeat question 2 using the pieces from this puzzle.



Four-piece rectangle

- Draw a rectangle measuring 20 cm by 5 cm.
- Mark the centre of the long side and divide the rectangle into two.
- Rule a diagonal line in each smaller rectangle. Cut out the four shapes.



- What do you notice about the shapes you created? List their main properties.
- Repeat question 2 using the pieces from this puzzle.
- What other types of polygons can you construct from each individual puzzle?



learnon RESOURCES — ONLINE ONLY

Digital doc: Investigation — Puzzling constructions (doc-13939)

Digital doc: Code puzzle — What did the pencil say to the eraser? (doc-13940)

Digital doc: Summary Topic 6 (doc-23242)

Interactivity: Word search Topic 6 (int-2627)

Interactivity: Crossword Topic 6 (int-2728)

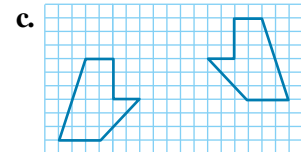
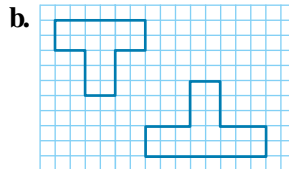
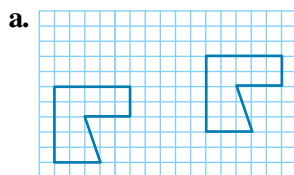
Interactivity: Sudoku Topic 6 (int-3187)

Exercise 6.5 Review questions

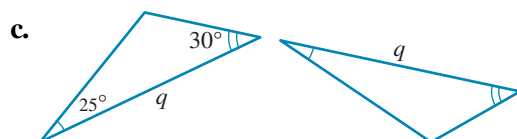
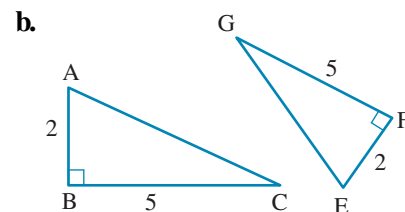
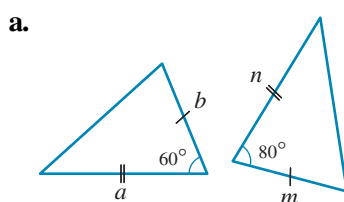
To answer questions online and to receive **immediate feedback** and **fully worked solutions** for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Understanding and fluency

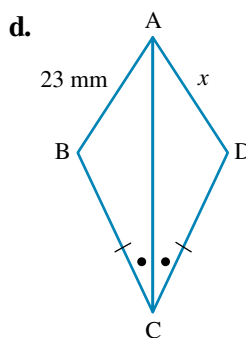
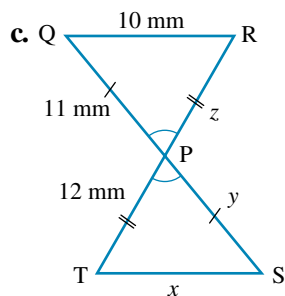
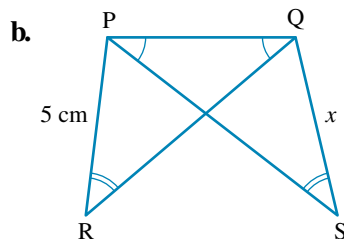
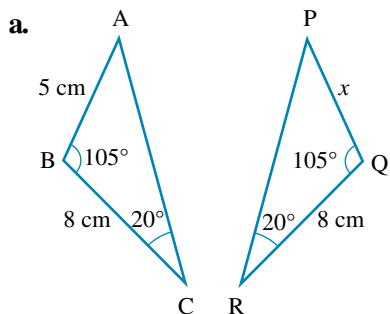
- Which of the following pairs of shapes are congruent?



- State whether the following shapes are congruent. Give a reason for your answer.



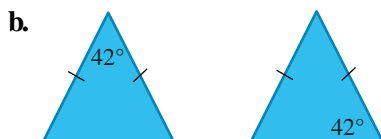
3. Name each pair of congruent triangles giving a reason. Hence, find the length of the side(s) marked with a variable.



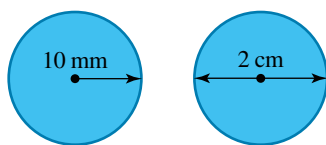
4. Complete the following table about the properties of quadrilaterals.

Properties of quadrilaterals	Parallelogram	Rhombus	Rectangle	Square	Kite
Opposite sides parallel					
Opposite sides one equal					
All sides equal					
All angles are 90°					
Diagonals are equal					
Diagonals bisect each other					
Diagonals intersect at 90°					
Diagonals bisect angles					

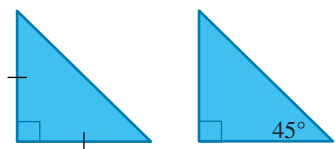
5. The following figures have not been drawn to scale and appear to be congruent. Use the measurements given to determine whether they are congruent or not.



c.



d.



Communicating, reasoning and problem solving

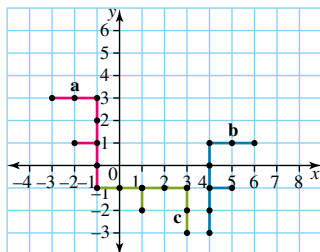
6. If two triangles look identical, name two things that you could do to check whether they are congruent.
7. Two rectangles both have an area of 20 m^2 . Are they necessarily congruent?
8. A classmate had two right-angled triangles and measured an angle of each. One triangle had an angle of 43° , and the other had an angle of 47° . She decided that they were not congruent. Is she correct? Explain.
9. In a quadrilateral PQRS, $PQ \equiv RS$ and $\angle PQR \equiv \angle RSP$. Suzie claims that the quadrilateral must be a parallelogram. Is her claim correct? If yes, give evidence to justify the claim. If no, draw a diagram to demonstrate how her claim is wrong.
10. In a rectangle WXYZ, M is the midpoint of WX. Show that $MZ \equiv MY$.
11. ABCD is a square. E is a point on AB. F is a point on BC. G is a point on CD and H is a point on DA. If $AE \equiv BF \equiv CG \equiv DH$, what type of quadrilateral is EFGH? Justify your answer.
12. Are squares and rhombuses special examples of kites? Explain your answer using mathematical reasoning.

Answers

Topic 6 Congruence

Exercise 6.2 Congruent figures

1. a–c.



d. Yes

2. a. i and iii

b. i and iv

c. ii and iv

3. a. $\triangle ABC \equiv \triangle PQR$, $x = 25^\circ$

c. $\triangle ABC \equiv \triangle CDA$, $x = 40^\circ$, $y = 55^\circ$

e. $\triangle ABC \equiv \triangle ADC$, $x = 40^\circ$, $y = 40^\circ$, $z = 100^\circ$

b. $\triangle ABC \equiv \triangle PQR$, $x = 70^\circ$, $y = 40^\circ$

d. $\triangle ABC \equiv \triangle ADC$, $x = 30^\circ$, $y = 30^\circ$, $z = 120^\circ$

f. $\triangle ABC \equiv \triangle PRQ$, $x = 35^\circ$, $y = 55^\circ$

4. C

5. a. $\triangle ABC \equiv \triangle ADC$, $x = 30^\circ$, $y = 60^\circ$

c. $\triangle PSR \equiv \triangle QRS$, $x = 8$ cm

e. $\triangle EFG \equiv \triangle EHG$, $x = 8$ cm

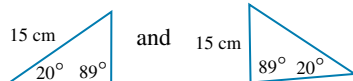
g. $\triangle ABE \equiv \triangle CDB$, $x = 58^\circ$, $y = 65^\circ$

b. $\triangle KLN \equiv \triangle MNL$, $x = 75^\circ$, $y = 60^\circ$, $z = 45^\circ$

d. $\triangle SWT \equiv \triangle UVT$, $w = y = 70^\circ$, $x = z = 55^\circ$

f. $\triangle WXZ \equiv \triangle WYZ$, $x = 45^\circ$, $y = 3$ cm

6.



7. $a = 35^\circ$, $b = 55^\circ$, $c = 70^\circ$, $d = 5$ cm, $e = 2$ cm

8. $a = 15$ cm, $b = 92^\circ$, $c = 58^\circ$

9. To indicate which vertices match on each shape.

Exercise 6.3 Congruent triangles

1. a. $\triangle ABC \equiv \triangle PQR$ (SSS)

c. $\triangle LMN \equiv \triangle PQR$ (SAS)

e. $\triangle ABE \equiv \triangle ADE$ (SSS)

b. $\triangle ABC \equiv \triangle PQR$ (SSS)

d. $\triangle ABC \equiv \triangle PQR$ (ASA)

f. $\triangle FKJ \equiv \triangle GKH$ (SSS)

2. a. $x = 3$ cm

d. $x = 25^\circ$, $y = 8$ cm

b. $x = 83^\circ$

e. $x = 35^\circ$, $y = z = 55^\circ$, $n = m = 90^\circ$

c. $x = 75^\circ$, $y = 25^\circ$, $z = 80^\circ$

3. a. $AB = AC$

$BD = CD$

AD is common.

$\triangle DBA \equiv \triangle DCA$ (SSS)

b. $AB = AD$

AC is common.

$\angle BAC = \angle CAD$

$\triangle ABC \equiv \triangle ADC$ (SAS)

c. $PQ = PS$

PR is common.

$\angle PRQ = \angle PRS = 90^\circ$

$\triangle PQR \equiv \triangle PSR$ (RHS)

4, 5. Check with your teacher.

6. a. Congruent (SSS)

b. Congruent (ASA)

7 to 11. Check with your teacher.

Exercise 6.4 Quadrilaterals

1. a. $\triangle WXA \equiv \triangle YZA$ (ASA)

b. i. $WA = YA$

c. Diagonals bisect each other.

ii. $XA = ZA$

2. a. $\triangle ABE \equiv \triangle CBE$ (SSS)

b. i. $\angle AEB = \angle CEB$

c. 90°

ii. $\angle ABE = \angle CBE$

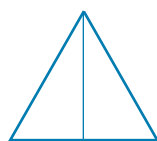
d. $\triangle CBE$, $\triangle ADE$, $\triangle CDE$

e. Diagonals bisect the angles.

3. a. $\triangle PQR \equiv \triangle SRP$ (SAS) b. Diagonals of a rectangle are equal.
4. a. $\triangle ABD \equiv \triangle CBD$ (SSS) ii. $\angle BAD = \angle BCD$ ii. $\angle BDA = \angle BDC$
 b. i. $\angle ABD = \angle CBD$ c. $\triangle ABD \equiv \triangle CBD$ (SAS)
 d. i. $AE = CE$ ii. $\angle AEB = \angle CEB$
 e. No
 f. i. No ii. No (one pair of opposite angles are equal)
 iii. No (one diagonal bisects the other) iv. Yes
 v. No (one diagonal bisects the unequal angles)
5. a. $\triangle QPR \equiv \triangle SRP$ (SSS)
 b. i. $\angle SRP$ ii. $\angle SPR$
 c. i. RS ii. PS
 d. Parallelogram
6. a. $\angle BAC = \angle DCA$ b. $\triangle ABC \equiv \triangle CDA$ (SAS)
 c. $\angle BCA = \angle DAC$ (alternate angles) d. Parallelogram
7. All quadrilaterals can be divided into two adjacent triangles. The interior angle sum of a triangle is 180° , so the interior angle sum of a quadrilateral is equal to 360° ($180^\circ \times 2$).
8. Trapezium
9. $x = 107^\circ$
10. $50^\circ, 70^\circ, 110^\circ, 130^\circ$
11. a. Check with your teacher. b. 357°
12. It is not enough for diagrams to look the same. Mathematical reasoning is evidence for congruence.

Investigation | Rich task

- Two right-angled triangles and one trapezium with two parallel sides
- Some possible solutions are shown; teacher to check for others.



Equilateral triangle
with two pieces;
all angles = 60°



Irregular quadrilateral
with two pieces; all sides
have different lengths

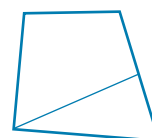
- One large and one smaller right-angled triangle and an irregular quadrilateral
- Some possible solutions are shown; teacher to check for others.



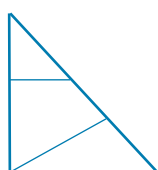
Right-angled triangle
with two pieces



Irregular quadrilateral
with two pieces



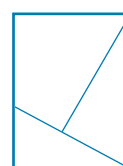
Trapezium with
two pieces



Right-angled triangle
with three pieces



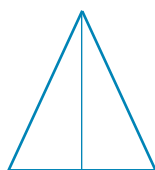
Parallelogram with
three pieces



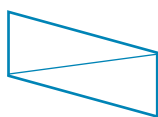
Rectangle with
three pieces

- Four identical right-angled triangles

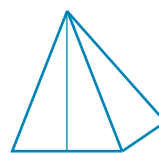
6. Some possible solutions are shown; teacher to check for others.



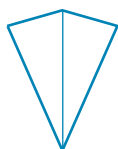
Isosceles triangle with two pieces



Parallelogram with two pieces



Irregular quadrilateral with three pieces



Kite with two pieces



Rhombus with all four pieces

7. Teacher to check.

Exercise 6.5 Review questions

1. a, c.

2. a. No, because the included angles are different sizes.

b. Yes, RHS

c. Yes, AAS

3. a. $\triangle ABC \equiv \triangle PQR$ (ASA), $x = 5$ cm

b. $\triangle PQR \equiv \triangle QPS$ (ASA), $x = 5$ cm

c. $\triangle PQR \equiv \triangle PST$ (SAS), $x = 10$ mm
 $y = 11$ mm, $z = 12$ mm

d. $\triangle ABC \equiv \triangle ADC$ (SAS), $x = 23$ mm

Properties of quadrilaterals	Parallelogram	Rhombus	Rectangle	Square	Kite
Opposite sides parallel	✓	✓	✓	✓	x
Opposite sides equal	✓	✓	✓	✓	x
All sides equal	x	✓	x	✓	x
All angles are 90°	x	x	✓	✓	x
Diagonals equal	x	x	✓	✓	x
Diagonals bisect each other	✓	✓	✓	✓	x
Diagonals intersect at 90°	x	✓	x	✓	✓
Diagonals bisect angles	x	✓	x	✓	x

5. a. Congruent

b. Not congruent

c. Congruent

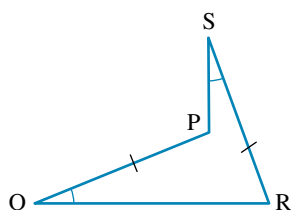
d. Congruent

6. Check the corresponding angles and side lengths.

7. No, for example, one rectangle could have dimensions of 2 m by 10 m and the other 4 m by 5 m.

8. She may be correct. The angles are identical, but the side lengths may be different.

9. Claim is not correct.



10. Check with your teacher.

11. Square

12. Check with your teacher.

TOPIC 7

Algebraic techniques

7.1 Overview

Numerous **videos** and **interactivities** are embedded just where you need them, at the point of learning, in your learnON title at www.jacplus.com.au. They will help you to learn the concepts covered in this topic.

7.1.1 Why learn this?

Humankind could not have travelled to the moon without algebra. Without algebra there would be no television, no iPod, no iPad — nothing electronic at all. A knowledge of algebra also makes possible complex geometry, which is reflected in structures ranging from the Colosseum to the simple suburban house. Algebra is the fundamental building block of mathematics.

You need a knowledge of algebra to succeed in mathematics at school. The further you advance in your studies, the more useful you will find algebra.



LEARNING SEQUENCE

- 7.1** Overview
- 7.2** Using variables
- 7.3** Substitution
- 7.4** Working with brackets
- 7.5** Substituting positive and negative numbers
- 7.6** Number laws and variables
- 7.7** Simplifying expressions
- 7.8** Multiplying and dividing expressions with variables
- 7.9** Expanding brackets
- 7.10** Factorising
- 7.11** Review

LEARNING OUTCOMES

A student:

- communicates and connects mathematical ideas using appropriate terminology, diagrams and symbols MA4-1WM
- recognises and explains mathematical relationships using reasoning MA4-3WM
- generalises number properties to operate with algebraic expressions MA4-8NA

CONTENT DESCRIPTIONS

Students:

Create algebraic expressions and evaluate them by substituting a given value for each variable (ACMNA176)

Extend and apply the laws and properties of arithmetic to algebraic terms and expressions (ACMNA177)

Extend and apply the distributive law to the expansion of algebraic expressions (ACMNA190)

Factorise algebraic expressions by identifying numerical factors (ACMNA191)

Simplify algebraic expressions involving the four operations (ACMNA192)

Source: NSW Syllabus for the Australian curriculum

Note: Your teacher may now set you a pre-test to determine how familiar you are with the content in this topic.

7.2 Using variables

7.2.1 Variables

- A **variable** (or **pronumeral**) is a letter or symbol that represents a value in an algebraic expression or equation.
- In algebraic expressions such as $a + b$, the variables represent any number.
- In algebraic equations such as $x + y = 9$, variables are referred to as **unknowns** because the variable represents a specific value that is not yet known.
- When we write expressions with variables, the multiplication sign is omitted. For example, $8n$ means ' $8 \times n$ ' and $12ab$ means ' $12 \times a \times b$ '.
- The division sign is rarely used. For example, $y \div 6$ is usually written as $\frac{y}{6}$.

WORKED EXAMPLE 1

Suppose we use b to represent the number of ants in a nest.

- Write an expression for the number of ants in the nest if 25 ants died.
- Write an expression for the number of ants in the nest if the original ant population doubled.
- Write an expression for the number of ants in the nest if the original population increased by 50.
- What would it mean if we said that a nearby nest contained $b + 100$ ants?
- What would it mean if we said that another nest contained $b - 1000$ ants?
- Another nest in very poor soil contains $\frac{b}{2}$ ants. How much smaller than the original is this nest?

THINK

- The original number of ants (b) must be reduced by 25.
- The original number of ants (b) must be multiplied by 2. It is not necessary to show the $\times$ sign.
- 50 must be added to the original number of ants (b).
- This expression tells us that the nearby nest has 100 more ants.
- This expression tells us that the nest has 1000 fewer ants.
- The expression $\frac{b}{2}$ means $b \div 2$, so this nest is half the size of the original nest.

WRITE

- $b - 25$
- $2b$
- $b + 50$
- The nearby nest has 100 more ants.
- This nest has 1000 fewer ants.
- This nest is half the size of the original nest.

ACTIVITY: ALGEBRAIC SYMBOLS IN OTHER CONTEXTS

Use the internet to research where algebraic symbols are used in other contexts; for example, when representing cells in a spreadsheet. Present your research to the class.

Interactivity: Using variables (int-3762)

eLesson: Using variables (eles-0042)

Digital doc: SkillsHEET Alternative expressions used to describe the four operations (doc-6922)

Exercise 7.2 Using variables

Individual pathways

PRACTISE

Questions:
1–8, 12

CONSOLIDATE

Questions:
1–9, 12, 13

MASTER

Questions:
1–16

Individual pathway interactivity: int-4429

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To answer questions online and to receive **immediate feedback** and **fully worked solutions** for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Understanding and fluency

1. **WE1** Suppose we use x to represent the original number of ants in a nest.
 - a. Write an expression for the number of ants in the nest if 420 ants were born.
 - b. Write an expression for the number of ants in the nest if the original ant population tripled.
 - c. Write an expression for the number of ants in the nest if the original ant population decreased by 130.
 - d. What would it mean if we said that a nearby nest contained $x + 60$ ants?
 - e. What would it mean if we said that a nearby nest contained $x - 90$ ants?
 - f. Another nest in very poor soil contains $\frac{x}{4}$ ants. How much smaller than the original is this nest?
2. Suppose x people are in attendance at the start of an Aussie Rules football match.
 - a. If a further y people arrive during the first quarter, write an expression for the number of people at the ground.
 - b. Write an expression for the number of people at the ground if a further 260 people arrive prior to the second quarter commencing.
 - c. At half-time 170 people leave. Write an expression for the number of people at the ground after they have left.
 - d. In the final quarter a further 350 people leave. Write an expression for the number of people at the ground after they have left.

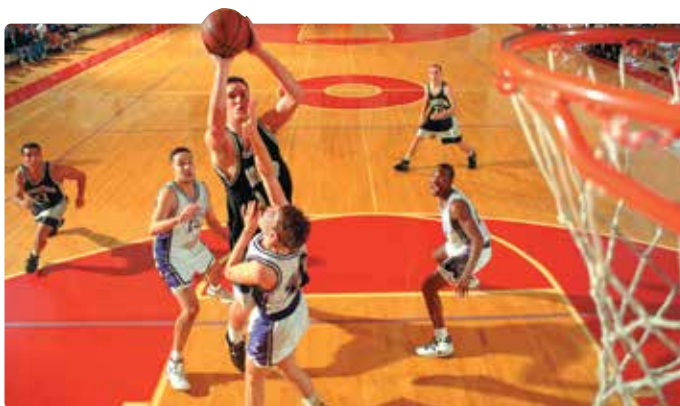


3. The canteen manager at Browning Industries orders m Danish pastries each day. Write a paragraph that could explain the table below.

Time	Number of Danish pastries available for sale
9.00 am	m
9.15 am	$m - 1$
10.45 am	$m - 12$
12.30 pm	$m - 12$
1.00 pm	$m - 30$
5.30 pm	$m - 30$



4. Imagine that your cutlery drawer contains a knives, b forks and c spoons.
- Write an expression for the total number of knives and forks you have.
 - Write an expression for the total number of items in the drawer.
 - You put 4 more forks in the drawer. Write an expression for the number of forks now.
 - Write an expression for the number of knives in the drawer after 6 knives are removed.
5. If y represents a certain number, write expressions for the following numbers.
- A number 7 more than y .
 - A number 8 less than y .
 - A number that is equal to five times y .
 - The number formed when y is subtracted from 14.
 - The number formed when y is divided by 3.
 - The number formed when y is multiplied by 8 and 3 is added to the result.
6. Using a and b to represent numbers, write expressions for:
- the sum of a and b
 - the difference between a and b
 - three times a subtracted from two times b
 - the product of a and b
 - twice the product of a and b
 - the sum of $3a$ and $7b$
 - a multiplied by itself
 - a multiplied by itself and the result divided by 5.
7. If tickets to a basketball match cost \$27 for adults and \$14 for children, write an expression for the cost of:
- y adult tickets
 - d child tickets
 - r adult and h child tickets.
8. Naomi is now t years old.
- Write an expression for her age in 2 years' time.
 - Write an expression for Steve's age if he is g years older than Naomi.
 - How old was Naomi 5 years ago?
 - Naomi's father is twice her age. How old is he?



9. James is travelling by train into town one particular evening and observes that there are t passengers in his carriage. He continues to take note of the number of people in his carriage each time the train departs from a station, which occurs every 3 minutes. The table below shows the number of passengers.

Time	Number of passengers
7.10 pm	t
7.13 pm	$2t$
7.16 pm	$2t + 12$
7.19 pm	$4t + 12$
7.22 pm	$4t + 7$
7.25 pm	t
7.28 pm	$t + 1$
7.31 pm	$t - 8$
7.34 pm	$t - 12$



- Write a paragraph explaining what happened.
- When did passengers first begin to alight the train?
- At what time did the carriage have the most number of passengers?
- At what time did the carriage have the least number of passengers?

Communicating, reasoning and problem solving

10. A microbiologist places m bacteria onto an agar plate. She counts the number of bacteria at approximately 3-hour intervals. The results are shown in the table below.

Time	Number of bacteria
9.00 am	m
Noon	$2m$
3.18 pm	$4m$
6.20 pm	$8m$
9.05 pm	$16m$
Midnight	$32m - 1240$



- Explain what happens to the number of bacteria in the first 5 intervals.
 - What might be causing the number of bacteria to increase in this way?
 - What is different about the last bacteria count?
 - What may have happened to cause this?
11. n represents an even number.
- Is the number $n + 1$ odd or even?
 - Is $3n$ odd or even?
 - Write expressions for:
 - the next three even numbers that are greater than n
 - the even number that is 2 less than n .
12. If the side of a square tile box is x cm long and the height is h cm, write expressions for the total surface area and the volume of the tile box.
13. If a rectangular tile box has the same width and height as the square tile box in question 12 but is one and a half times as long, write expressions for the total surface area and the volume of the tile box.
14. If the square tile box in question 12 has a side length of 20 cm and both boxes in questions 12 and 13 have a height of 15 cm, calculate the surface area and volume of the square tile box and the surface area and volume of the rectangular tile box using your expressions.

15. List some reasons for using variables instead of numbers
16. Bill was describing his age to a group of people using two different algebraic expressions. Four times a certain number minus one and three times a certain number plus three both reveal Bill's age. What is the number used and what is Bill's age?

7.3 Substitution

7.3.1 Substitution of values

- If the value of a variable (or variables) is known, it is possible to **evaluate** (work out the value of) an expression by using **substitution**. The variable is replaced with the number.
- Substitution can also be used with a formula or rule.

WORKED EXAMPLE 2

Find the value of the following expressions if $a = 3$ and $b = 15$.

a $6a$

b $7a - \frac{2b}{3}$

THINK

a 1 Substitute the correct value for the variable (a) and replace the multiplication sign.

2 Evaluate and write the answer.

b 1 Substitute the correct values for each variable and replace the multiplication signs.

2 Perform the first multiplication.

3 Perform the next multiplication.

4 Perform the division.

5 Perform the subtraction and write the answer.

WRITE

$$a \quad 6a = 6 \times 3$$

$$= 18$$

$$b \quad 7a - \frac{2b}{3} = 7 \times 3 - \frac{2 \times 15}{3}$$

$$= 21 - \frac{2 \times 15}{3}$$

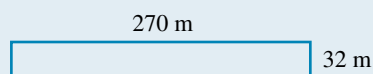
$$= 21 - \frac{30}{3}$$

$$= 21 - 10$$

$$= 11$$

WORKED EXAMPLE 3

The formula for finding the area (A) of a rectangle of length l and breadth b is $A = l \times b$. Use this formula to find the area of the rectangle at right.



THINK

1 Write the formula.

2 Substitute the value for each variable.

3 Perform the multiplication and state the correct units.

WRITE

$$A = l \times b$$

$$= 270 \times 32$$

$$= 8640 \text{ m}^2$$

DISCUSSION

Why might mathematicians, scientists, accountants and other professionals use variables in their work instead of numbers?

Interactivity: Substitution (int-3763)

eLesson: Substitution (eles-1892)

Digital doc: SpreadSHEET Substitution (doc-2287)

Exercise 7.3 Substitution

Individual pathways

PRACTISE

Questions:
1–4, 6, 8, 13

CONSOLIDATE

Questions:
1a–l, 2a–l, 3a–l, 4–8, 11, 13

MASTER

Questions:
1m–t, 2i–p, 3i–p, 4–13

Individual pathway interactivity: int-4430

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Understanding and fluency

1. **WE2** Find the value of the following expressions, if $a = 2$ and $b = 5$.

a. $3a$	b. $7a$	c. $6b$	d. $\frac{a}{2}$
e. $a + 7$	f. $b - 4$	g. $a + b$	h. $b - a$
i. $5 + \frac{b}{5}$	j. $3a + 9$	k. $2a + 3b$	l. $\frac{8}{a}$
m. $\frac{25}{b}$	n. ab	o. $2ab$	p. $7b - 30$
q. $6b - 4a$	r. $\frac{ab}{5}$	s. $\frac{15}{a} + \frac{7}{b}$	t. $\frac{9}{a} - \frac{3}{b}$

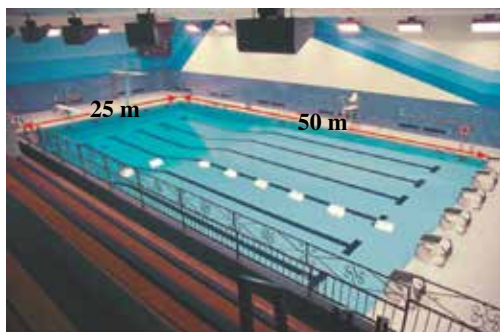
2. Substitute $x = 6$ and $y = 3$ into the following expressions and evaluate.

a. $6x + 2y$	b. $\frac{x}{3} + \frac{y}{3}$	c. $3xy$	d. $\frac{24}{x} - \frac{9}{y}$
e. $\frac{12}{x} + 4 + y$	f. $3x - y$	g. $2.5x$	h. $\frac{7x}{2}$
i. $3.2x + 1.7y$	j. $11y - 2x$	k. $\frac{13y}{3} - 2x$	l. $\frac{4xy}{15}$
m. $4.8x - 3.5y$	n. $8.7y - 3x$	o. $12.3x - 9.6x$	p. $\frac{3x}{9} - \frac{y}{12}$

3. Evaluate the following expressions, if $d = 5$ and $m = 2$.

a. $d + m$	b. $m + d$	c. $m - d$	d. $d - m$
e. $2m$	f. md	g. $5dm$	h. $\frac{md}{10}$
i. $-3d$	j. $-2m$	k. $6m + 5d$	l. $\frac{3md}{2}$
m. $25m - 2d$	n. $\frac{7d}{15}$	o. $4dm - 21$	p. $\frac{15}{d} - m$

4. **WE3** The formula for finding the perimeter (P) of a rectangle of length l and breadth b is $P = 2l + 2b$. Use this formula to find the perimeter of the rectangular swimming pool at right.
5. The formula for the perimeter (P) of a square of side length l is $P = 4l$. Use this formula to find the perimeter of a square of side length 2.5 cm.
6. The formula $c = 0.1a + 42$ is used to calculate the cost in dollars (c) of renting a car for one day from Poole's Car Hire Ltd, where a is the number of kilometres travelled on that day. Find the cost of renting a car for one day if the distance travelled is 220 kilometres.
7. The area (A) of a rectangle of length l and breadth b can be found using the formula $A = lb$. Find the area of the rectangles below:
- length 12 cm, breadth 4 cm
 - length 200 m, breadth 42 m
 - length 4.3 m, breadth 104 cm.
8. The formula $F = \frac{9}{5}C + 32$ is used to convert temperatures measured in degrees Celsius to an approximate Fahrenheit value. F represents the temperature in degrees Fahrenheit ($^{\circ}\text{F}$) and C the temperature in degrees Celsius ($^{\circ}\text{C}$).
- Find F when $C = 100^{\circ}\text{C}$.
 - Convert 28°C to Fahrenheit.
 - Water freezes at 0°C . What is the freezing temperature of water in Fahrenheit?
9. The formula $D = 0.6T$ can be used to convert distances in kilometres (T) to the approximate equivalent in miles (D). Use this rule to convert the following distances to miles:
- 100 kilometres
 - 248 kilometres
 - 12.5 kilometres.



Communicating, reasoning and problem solving

10. Ben says that $\frac{4x^2}{2x} = 2x$. Emma says that is not correct if $x = 0$. Explain Emma's reasoning.
11. If $s = ut + \frac{1}{2}at^2$, evaluate s if $u = 5$, $t = 10$ and $a = 9.81$.
12. The width of a cuboid is x cm.
- If the length is 5 cm more than the width and the height is 2 cm less than the width, find the volume, $V \text{ cm}^3$, of the cuboid in terms of x .
 - Evaluate V if x equals 10.
 - Explain why x cannot equal 1.5.
13. On the space battleship RAN *Fantasie*, there are p Pletons, each with 2 legs, $(p - 50)$ Argors, each with 3 legs, and $(2p + 35)$ Kleptors, each with 4 legs.
- Find the total number of legs, L , on board the *Fantasie*, in terms of p , in simplified form.
 - If $p = 200$, find L .

7.4 Working with brackets

7.4.1 Brackets

- Brackets are grouping symbols. The expression $3(a + 5)$ can be thought of as ‘three groups of $(a + 5)$ ’, or $(a + 5) + (a + 5) + (a + 5)$.
- When substituting into an expression with brackets, remember to place a multiplication sign ($\times$) next to the brackets. For example, $3(a + 5)$ is thought of as $3 \times (a + 5)$.
- Following operation order, evaluate the brackets first and then multiply by the value outside of the brackets.

WORKED EXAMPLE 4

- a** Substitute $r = 4$ and $s = 5$ into the expression $5(s + r)$ and evaluate.
b Substitute $t = 4$, $x = 3$ and $y = 5$ into the expression $2x(3t - y)$ and evaluate.

THINK

- a** 1 Place the multiplication sign back into the expression.

2 Substitute the correct values for the variables.

3 Evaluate the expression in the pair of brackets first.

4 Perform the multiplication and write the answer.

- b** 1 Place the multiplication signs back into the expression.

2 Substitute the correct values for the variables.

3 Perform the multiplication inside the pair of brackets.

4 Perform the subtraction inside the pair of brackets.

5 Perform the multiplication and write the answer.

WRITE

a $5(s + r) = 5 \times (s + r)$

$$= 5 \times (5 + 4)$$

$$= 5 \times 9$$

$$= 45$$

b $2x(3t - y) = 2 \times x \times (3 \times t - y)$

$$= 2 \times 3 \times (3 \times 4 - 5)$$

$$= 2 \times 3 \times (12 - 5)$$

$$= 2 \times 3 \times 7$$

$$= 42$$

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Interactivity: Working with brackets (int-3764)

Exercise 7.4 Working with brackets

Individual pathways

PRACTISE

Questions:
1–4, 7

CONSOLIDATE

Questions:
1a–l, 2a–l, 3–5, 7

MASTER

Questions:
1j–p, 2j–p, 3–9

Individual pathway interactivity: int-4431

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Understanding and fluency

1. **WE4** Substitute $r = 5$ and $s = 7$ into the following expressions and evaluate.

a. $3(r + s)$	b. $2(s - r)$	c. $7(r + s)$	d. $9(s - r)$
e. $s(r + 3)$	f. $s(2r - 5)$	g. $3r(r + 1)$	h. $rs(3 + s)$
i. $11r(s - 6)$	j. $2r(s - r)$	k. $s(4 + 3r)$	l. $7s(r - 2)$
m. $s(3rs + 7)$	n. $5r(24 - 2s)$	o. $5sr(sr + 3s)$	p. $8r(12 - s)$

2. Evaluate each of the expressions below, if $x = 3$, $y = 5$ and $z = 9$.

a. $xy(z - 3)$	b. $\frac{12}{x}(z - y)$	c. $\frac{z}{3}\left(\frac{2y}{10} + x - 2\right)$	d. $(x + y)(z - y)$
e. $(z - 3)4x$	f. $zy(17 - xy)$	g. $\frac{y}{5}(7 - x + 3)$	h. $(8 - y)(z + x)$
i. $\left(7 - \frac{12}{x}\right)4y$	j. $\frac{6}{x}(xz + y - 3)$	k. $(y + 2)\frac{z}{x}$	l. $2x(xyz - 105)$
m. $12(y - 1)(z + 3)$	n. $(3x - 7)\left(\frac{27}{x} + 7\right)$	o. $-2(4x + 1)\left(\frac{36}{z} - 3\right)$	p. $-3(2y - 11)\left(\frac{z}{x} + 8\right)$

3. The formula for the perimeter (P) of a rectangle of length l and width w is $P = 2l + 2w$. This rule can also be written as $P = 2(l + w)$. Use the rule to find the perimeter of rectangular comic covers with the following measurements.

- a. $l = 20$ cm, $w = 11$ cm
b. $l = 27.5$ cm, $w = 21.4$ cm

4. **MC** When $a = 8$ and $b = 12$ are substituted into the expression

$\frac{a}{6}(15 - b + 9)$, the expression is equal to:

- A. 32 B. 16 C. $21\frac{1}{3}$
D. 24 E. 27

5. A rule for finding the sum of the interior angles in a many-sided figure such as a pentagon is $S = 180(n - 2)^\circ$, where S represents the sum of the angles inside the figure and n represents the number of sides. The diagram below right shows the interior angles in a pentagon.

Use the rule to find the sum of the interior angles for the following figures:

a. a hexagon (6 sides)	b. a pentagon
c. a triangle	d. a quadrilateral (4 sides)
e. a 20-sided figure.	



Communicating, reasoning and problem solving

6. The dimensions of the figure below right are given in terms of m and n .

Write, in terms of m and n , an expression for:

- a. the length of CD b. the length of BC
c. the perimeter of the figure.

Show all of your working.

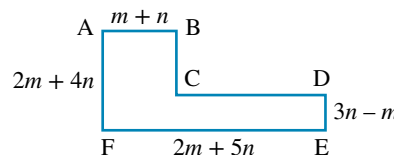
7. a. Find an expression for the area of a triangle whose base length is $(m + n)$ cm and whose height is $(m - n)$ cm.

- b. If $m = 15$ and $n = 6$, find the area of the triangle.
c. Show that $m > n$.
d. Explain what happens to the triangle as m and n move closer in value.

8. It can be shown that $(x - a)(x - a) = x^2 - 2ax + a^2$. By substitution, show that this is true if:

- a. $x = 4$, $a = 1$ b. $x = 3p$, $a = 2p$ c. $x = 0$.

9. Is operation order followed when substituting values for variables?



7.5 Substituting positive and negative numbers

7.5.1 Substituting integers

- If the variable you are substituting for has a negative value, simply remember the following rules for directed numbers:

- For addition and subtraction, signs that occur together can be combined.

Same signs positive for example, $7 + +3 = 7 + 3$
and $7 - -3 = 7 + 3$

Different signs negative for example, $7 - +3 = 7 - 3$
and $7 + -3 = 7 - 3$

- For multiplication and division:

Same signs positive for example, $+7 \times +3 = +21$
and $-7 \times -3 = +21$

Different signs negative for example, $+7 \times -3 = -21$
and $-7 \times +3 = -21$

WORKED EXAMPLE 5

- a** Substitute $m = 5$ and $n = -3$ into the expression $m - n$ and evaluate.
b Substitute $m = -2$ and $n = -1$ into the expression $2n - m$ and evaluate.
c Substitute $a = 4$ and $b = -3$ into the expression $5ab - \frac{12}{b}$ and evaluate.

THINK

- a** 1 Substitute the correct values for the variables.

- 2 Combine the two negative signs and add.

- 3 Write the answer.

- b** 1 Replace the multiplication sign.

- 2 Substitute the correct values for the variables.

- 3 Perform the multiplication.

- 4 Combine the two negative signs and add.

- 5 Write the answer.

- c** 1 Replace the multiplication signs.

- 2 Substitute the correct values for the variables.

- 3 Perform the multiplications.

- 4 Perform the division.

- 5 Combine the two negative signs and add.

- 6 Write the answer.

WRITE

a $m - n = 5 - -3$

$$= 5 + 3$$

$$= 8$$

b $2n - m = 2 \times n - m$

$$= 2 \times -1 - -2$$

$$= -2 - -2$$

$$= -2 + 2$$

$$= 0$$

c $5ab - \frac{12}{b} = 5 \times a \times b - \frac{12}{b}$

$$= 5 \times 4 \times -3 - \frac{12}{-3}$$

$$= -60 - \frac{12}{-3}$$

$$= -60 - -4$$

$$= -60 + 4$$

$$= -56$$

Interactivity: Substituting positive and negative numbers (int-3765)

Digital doc: SkillsSHEET Order of operations II (doc-6923)

Digital doc: SkillsSHEET Order of operations with brackets (doc-6924)

Digital doc: SkillsSHEET Operations with directed numbers (doc-6925)

Digital doc: WorkSHEET Algebra I (doc-2290)

Exercise 7.5 Substituting positive and negative numbers

Individual pathways

PRACTISE

Questions:
1–4, 6

CONSOLIDATE

Questions:
1a–l, 2, 3a–l, 4, 6, 7, 8

MASTER

Questions:
1m–t, 2g–l, 3m–t, 4–10

Individual pathway interactivity: int-4432

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Understanding and fluency

1. **WE5a** Substitute $m = 6$ and $n = -3$ into the following expressions and evaluate.

a. $m + n$	b. $m - n$	c. $n - m$	d. $n + m$	e. $3n$
f. $-2m$	g. $2n - m$	h. $n + 5$	i. $2m + n - 4$	j. $11n + 20$
k. $-5n - m$	l. $\frac{m}{2}$	m. $\frac{mn}{9}$	n. $\frac{4m}{n-5}$	o. $\frac{4m}{n}$
p. $\frac{12}{2n}$	q. $\frac{9}{n} + \frac{m}{2}$	r. $6mn - 1$	s. $-\frac{3n}{2} + 1.5$	t. $14 - \frac{mn}{9}$

2. **WE5b** Substitute $x = 8$ and $y = -3$ into the following expressions and evaluate.

a. $3(x - 2)$	b. $x(7 + y)$	c. $5y(x - 7)$	d. $2(3 - y)$
e. $(y + 5)x$	f. $xy(7 - x)$	g. $(3 + x)(5 + y)$	h. $5(7 - xy)$
i. $\frac{x}{2}(5 - y)$	j. $\left(\frac{x}{4} - 1\right)\left(\frac{2y}{6} + 4\right)$	k. $\frac{9}{y}(6 - x)$	l. $3(x - 1)\left(\frac{y}{3} + 2\right)$

3. **WE5c** Substitute $a = -4$ and $b = -5$ into the following expressions and evaluate.

a. $a + b$	b. $a - b$	c. $b - 2a$	d. $2ab$
e. $12 - ab$	f. $-2(b - a)$	g. $a - b - 4$	h. $3a(b + 4)$
i. $\frac{4}{b}$	j. $\frac{8}{a}$	k. $\frac{16}{4a}$	l. $\frac{6b}{5}$
m. $45 + 4ab$	n. $8ab - 3b$	o. $\frac{a}{2} + \frac{3b}{5}$	p. $2.5b$
q. $11a + 6b$	r. $(a - 5)(8 - b)$	s. $(9 - a)(b - 3)$	t. $1.5b + 2a$

4. If $p = -2$ and $q = -3$, evaluate $\frac{3(-pq - p^2)}{q + 2p}$.

Communicating, reasoning and problem solving

5. Consider the expression $1 - 5x$. If x is a negative integer, explain why the expression will have a positive value.

6. a. Complete the following table by substituting the values of x into x^2 .

x	-4	-3	-2	-1	0	1	2	3	4
x^2									

b. What do you notice about the values of x^2 for values of x with the same magnitude but opposite sign?

7. Consider the equation $(a - b)(a + b) = a^2 - b^2$.

a. By substituting $a = -3$ and $b = -2$, show that this is true.

b. By substituting $a = -q$ and $b = -2q$, show that this is true.

8. If $x = -2r$ is substituted into $\frac{(r - x)(x + r)}{(r - 2x)}$, will the answer be positive or negative if:

a. $r > 0$

b. $r < 0$?

9. A circle is cut out of a square.

a. If the side length of the square is x , the radius of the circle is $0.25x$ and the area of the circle is approximately $0.196x^2$, find an expression for the remaining area.

b. Calculate the area when $x = 2$ by substituting into your expression. Give your answer to 3 decimal places.

c. What is the largest radius the circle can have?

10. What can you say about the sign of x^2 ?

7.6 Number laws and variables

7.6.1 Commutative Law

- When dealing with any type of number, we must obey particular rules.
- The **Commutative Law** refers to the order in which two numbers may be added, subtracted, multiplied or divided.
- The Commutative Law holds true for addition and multiplication because the order in which two numbers are added or multiplied does not affect the result.

$$3 + 2 = 2 + 3$$

$$3 \times 2 = 2 \times 3$$

- Since variables take the place of numbers, the Commutative Law holds true for the addition and multiplication of variables.

$$x + y = y + x$$

$$x \times y = y \times x$$

- The Commutative Law does not hold true for subtraction or division because the results obtained are different.

$$3 - 2 \neq 2 - 3$$

$$3 \div 2 \neq 2 \div 3$$

- Since variables take the place of numbers, the Commutative Law does not hold true for the subtraction and division of variables.

$$x - y \neq y - x$$

$$x \div y \neq y \div x$$

WORKED EXAMPLE 6

Find the value of the following expressions if $x = 4$ and $y = 7$. Comment on the results obtained.

a i $x + y$

ii $y + x$

b i $x - y$

ii $y - x$

c i $x \times y$

ii $y \times x$

d i $x \div y$

ii $y \div x$

THINK

- a i** 1 Substitute the correct value for each variable.
2 Evaluate and write the answer.
- ii** 1 Substitute the correct value for each variable.
2 Evaluate and write the answer.
3 Compare the result with the answer obtained in part **a i**.
- b i** 1 Substitute the correct value for each variable.
2 Evaluate and write the answer.
- ii** 1 Substitute the correct value for each variable.
2 Evaluate and write the answer.
3 Compare the result with the answer obtained in part **b i**.
- c i** 1 Substitute the correct value for each variable.
2 Evaluate and write the answer.
- ii** 1 Substitute the correct value for each variable.
2 Evaluate and write the answer.
3 Compare the result with the answer obtained in part **c i**.
- d i** 1 Substitute the correct value for each variable.
2 Evaluate and write the answer.
- ii** 1 Substitute the correct value for each variable.
2 Evaluate and write the answer.
3 Compare the result with the answer obtained in part **d i**.

WRITE

a i $x + y = 4 + 7$
 $= 11$

ii $y + x = 7 + 4$
 $= 11$

The same result is obtained; therefore, order is not important when adding two terms.

b i $x - y = 4 - 7$
 $= -3$

ii $y - x = 7 - 4$
 $= 3$

Two different results are obtained; therefore, order is important when subtracting two terms.

c i $x \times y = 4 \times 7$
 $= 28$

ii $y \times x = 7 \times 4$
 $= 28$

The same result is obtained; therefore, order is not important when multiplying two terms.

d i $x \div y = 4 \div 7$
 $= \frac{4}{7} (\approx 0.57)$

ii $y \div x = 7 \div 4$
 $= \frac{7}{4} (1.75)$

Two different results are obtained; therefore, order is important when dividing two terms.

7.6.2 Associative Law

- The **Associative Law** refers to the order in which three numbers may be added, subtracted, multiplied or divided, taking two at a time.

Note: The Associative Law refers to the order in which the addition (or other operation) is performed, and this order is indicated by the use of brackets. The order in which the variables are written does not change.

- Like the Commutative Law, the Associative Law holds true for addition and multiplication of numbers.

$$5 + (10 + 3) = (5 + 10) + 3$$

$$5 \times (10 \times 3) = (5 \times 10) \times 3$$

- Since variables take the place of numbers, the Associative Law holds true for the addition and multiplication of variables.

$$x + (y + z) = (x + y) + z$$

$$x \times (y \times z) = (x \times y) \times z$$

- Like the Commutative Law, the Associative Law does not hold for subtraction and division of numbers.

$$5 - (10 - 3) \neq (5 - 10) - 3$$

$$5 \div (10 \div 3) \neq (5 \div 10) \div 3$$

- Since variables take the place of numbers, the Associative law does not hold true for the subtraction and division of variables.

$$x - (y - z) \neq (x - y) - z$$

$$x \div (y \div z) \neq (x \div y) \div z$$

WORKED EXAMPLE 7

Find the value of the following expressions if $x = 12$, $y = 6$ and $z = 2$. Comment on the results obtained.

a i $x + (y + z)$

ii $(x + y) + z$

b i $x - (y - z)$

ii $(x - y) - z$

c i $x \times (y \times z)$

ii $(x \times y) \times z$

d i $x \div (y \div z)$

ii $(x \div y) \div z$

THINK

- a i** 1 Substitute the correct value for each variable.
 2 Evaluate the expression in the pair of brackets.
 3 Perform the addition and write the answer.
- ii** 1 Substitute the correct value for each variable.
 2 Evaluate the expression in the pair of brackets.
 3 Perform the addition and write the answer.
 4 Compare the result with the answer obtained in part **a i**.
- b i** 1 Substitute the correct value for each variable.
 2 Evaluate the expression in the pair of brackets.
 3 Perform the subtraction and write the answer.
- ii** 1 Substitute the correct value for each variable.
 2 Evaluate the expression in the pair of brackets.
 3 Perform the subtraction and write the answer.
 4 Compare the result with the answer obtained in part **b i**.

WRITE

a i $x + (y + z) = 12 + (6 + 2)$
 $= 12 + 8$
 $= 20$

ii $(x + y) + z = (12 + 6) + 2$
 $= 18 + 2$
 $= 20$

The same result is obtained; therefore, order is not important when adding 3 terms.

b i $x - (y - z) = 12 - (6 - 2)$
 $= 12 - 4$
 $= 8$

ii $(x - y) - z = (12 - 6) - 2$
 $= 6 - 2$
 $= 4$

Two different results are obtained; therefore, order is important when subtracting 3 terms.

- c i** 1 Substitute the correct value for each variable.
2 Evaluate the expression in the pair of brackets.
3 Perform the multiplication and write the answer.

$$\begin{aligned} \mathbf{c \ i} \quad x \times (y \times z) &= 12 \times (6 \times 2) \\ &= 12 \times 12 \\ &= 144 \end{aligned}$$

- ii** 1 Substitute the correct value for each variable.
2 Evaluate the expression in the pair of brackets.
3 Perform the multiplication and write the answer.
4 Compare the result with the answer obtained in part **c i**.

$$\begin{aligned} \mathbf{ii} \quad (x \times y) \times z &= (12 \times 6) \times 2 \\ &= 72 \times 2 \\ &= 144 \end{aligned}$$

The same result is obtained; therefore, order is not important when multiplying 3 terms.

- d i** 1 Substitute the correct value for each variable.
2 Evaluate the expression in the pair of brackets.
3 Perform the division and write the answer.

$$\begin{aligned} \mathbf{d \ i} \quad x \div (y \div z) &= 12 \div (6 \div 2) \\ &= 12 \div 3 \\ &= 4 \end{aligned}$$

- ii** 1 Substitute the correct value for each variable.
2 Evaluate the expression in the pair of brackets.
3 Perform the division and write the answer.
4 Compare the result with the answer obtained in part **d i**.

$$\begin{aligned} \mathbf{ii} \quad (x \div y) \div z &= (12 \div 6) \div 2 \\ &= 2 \div 2 \\ &= 1 \end{aligned}$$

Two different results are obtained; therefore, order is important when dividing 3 terms.

7.6.3 Identity Law

- The **Identity Law for addition** states that when zero is added to any number, the original number remains unchanged. For example, $5 + 0 = 0 + 5 = 5$.
- The **Identity Law for multiplication** states that when any number is multiplied by 1, the original number remains unchanged. For example, $3 \times 1 = 1 \times 3 = 3$.
- Since variables take the place of numbers:

$$\begin{aligned} x + 0 &= 0 + x = x \\ x \times 1 &= 1 \times x = x \end{aligned}$$

7.6.4 Inverse Law

- The **Inverse Law for addition** states that when a number is added to its additive inverse (opposite sign), the result is 0. For example, $5 + -5 = 0$.
- The **Inverse Law for multiplication** states that when a number is multiplied by its multiplicative inverse (reciprocal), the result is 1. For example, $3 \times \frac{1}{3} = 1$.
- Since variables take the place of numbers:

$$\begin{aligned} x + -x &= -x + x = 0 \\ x \times \frac{1}{x} &= \frac{1}{x} \times x = 1 \end{aligned}$$

Interactivity: The Associative Law (int-2370)

Interactivity: Commutative Law (int-3766)

Interactivity: Associative Law (int-3767)

Interactivity: Identity Law (int-3768)

Interactivity: Inverse Law (int-3769)

Exercise 7.6 Number laws and variables

Individual pathways

PRACTISE

Questions:
1–6, 10

CONSOLIDATE

Questions:
1a–e, 2a–e, 3a–h, 4a–d, 5a–d, 6, 7,
9, 10

MASTER

Questions:
1e–h, 2e–h, 3g–l, 4c–f, 5c–f, 6–12

Individual pathway interactivity: int-4433

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Understanding and fluency

1. **WE6a,b** Find the values of the following expressions if $x = 3$ and $y = 8$. Comment on the results obtained.

a. i. $x + y$

ii. $y + x$

b. i. $3x + 2y$

ii. $2y + 3x$

c. i. $5x + 2y$

ii. $2y + 5x$

d. i. $8x + y$

ii. $y + 8x$

e. i. $x - y$

ii. $y - x$

f. i. $2x - 3y$

ii. $3y - 2x$

g. i. $4x - 5y$

ii. $5y - 4x$

h. i. $3x - y$

ii. $y - 3x$

2. **WE6c,d** Find the values of the following expressions if $x = -2$ and $y = 5$. Comment on the results obtained.

a. i. $x \times y$

ii. $y \times x$

b. i. $6x \times 3y$

ii. $3y \times 6x$

c. i. $4x \times y$

ii. $y \times 4x$

d. i. $7x \times 5y$

ii. $5y \times 7x$

e. i. $x \div y$

ii. $y \div x$

f. i. $10x \div 4y$

ii. $4y \div 10x$

g. i. $6x \div 3y$

ii. $3y \div 6x$

h. i. $7x \div 9y$

ii. $9y \div 7x$

3. Indicate whether each of the following is true or false for all values of the variables.

a. $a + 5b = 5b + a$

b. $6x - 2y = 2y - 6x$

c. $7c + 3d = -3d + 7c$

d. $5 \times 2x \times x = 10x^2$

e. $4x \times -y = -y \times 4x$

f. $4 \times 3x \times x = 12x \times x$

g. $\frac{5p}{3r} = \frac{3r}{5p}$

h. $-7i - 2j = 2j + 7i$

i. $-3y \div 4x = 4x \div -3y$

j. $-2c + 3d = 3d - 2c$

k. $\frac{0}{3s} = \frac{3s}{0}$

l. $15 \times -\frac{2x}{3} = \frac{2x}{3} \times -15$

4. **WE7a,b** Find the values of the following expressions if $x = 3$, $y = 8$ and $z = 2$. Comment on the results obtained.

a. i. $x + (y + z)$

ii. $(x + y) + z$

b. i. $2x + (y + 5z)$

ii. $(2x + y) + 5z$

c. i. $6x + (2y + 3z)$

ii. $(6x + 2y) + 3z$

- d. i. $x - (y - z)$ ii. $(x - y) - z$
 e. i. $x - (7y - 9z)$ ii. $(x - 7y) - 9z$
 f. i. $3x - (8y - 6z)$ ii. $(3x - 8y) - 6z$
5. **WE7c,d** Find the values of the following expressions if $x = 8$, $y = 4$ and $z = -2$. Comment on the results obtained.
- a. i. $x \times (y \times z)$ ii. $(x \times y) \times z$
 b. i. $x \times (-3y \times 4z)$ ii. $(x \times -3y) \times 4z$
 c. i. $2x \times (3y \times 4z)$ ii. $(2x \times 3y) \times 4z$
 d. i. $x \div (y \div z)$ ii. $(x \div y) \div z$
 e. i. $x \div (2y \div 3z)$ ii. $(x \div 2y) \div 3z$
 f. i. $-x \div (5y \div 2z)$ ii. $(-x \div 5y) \div 2z$
6. Indicate whether each of the following is true or false for all values of the variables except 0.
- a. $a - 0 = 0$ b. $a \times 1000\ 000 = 0$ c. $15t \times -\frac{1}{15t} = 1$
 d. $3d \times \frac{1}{3d} = 1$ e. $\frac{8x}{9y} \div \frac{8x}{9y} = 1$ f. $\frac{11t}{0} = 0$
7. **MC** The value of the expression $x \times (-3y \times 4z)$ when $x = 4$, $y = 3$ and $z = -3$ is:
A. 108 **B.** -432 **C.** 432 **D.** 112 **E.** -108
8. **MC** The value of the expression $(x - 8y) - 10z$ when $x = 6$, $y = 5$ and $z = -4$ is:
A. -74 **B.** 74 **C.** -6 **D.** 6 **E.** -36

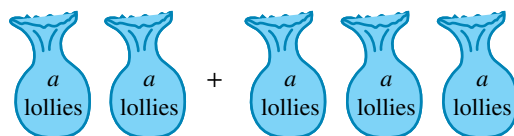
Communicating, reasoning and problem solving

9. a. If $x = -1$, $y = -2$ and $z = -3$, find the value of:
 i. $-x - y - z$ ii. $-x - (y + z)$
 b. Comment on the answers with special reference to the Associative Law.
10. Evaluate each of the following expressions for $x = -3$, $y = 2$ and $z = -1$.
 a. $2x - (3y + 2z)$ b. $x \times (y - 2z)$
11. a. State the additive inverse of $(3p - 4q)$.
 b. State the multiplicative inverse of $(3p - 4q)$.
 c. Evaluate the answers to parts **a** and **b** when $p = -1$ and $q = 3$.
 d. What happens if you multiply a term or pronumeral by its multiplicative inverse?
12. The Commutative Law does not hold for subtraction. What can you say about the results of $x - a$ and $a - x$?

7.7 Simplifying expressions

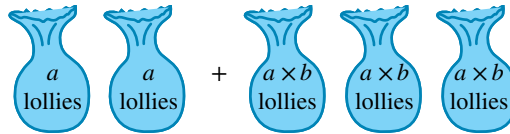
7.7.1 Simplification of algebraic expressions

- Expressions can often be written in a more simple form by collecting (adding or subtracting) like terms.
- Like terms** are terms that contain exactly the same variables, raised to the same power.
- To understand why $2a + 3a$ can be added but $2a + 3ab$ can *not* be added, consider the following identical bags of lollies, each containing a lollies.



- So, $2a + 3a = 5a$. Therefore we have 5 bags each containing a lollies.

- Then consider the following 2 bags containing a lollies and 3 bags containing $a \times b$ lollies.



- So $2a + 3ab$ cannot be added as they are not identical and we do not have any further information.
- So, $2a + 3ab = 2a + 3ab$. Therefore all we can say is that we have 2 bags containing a lollies and 3 bags containing ab lollies.
- For example:

$3x$ and $4x$ are like terms.

$3x$ and $3y$ are not like terms.

$3ab$ and $7ab$ are like terms.

$7ab$ and $8a$ are not like terms.

$2bc$ and $4cb$ are like terms.

$8a$ and $3a^2$ are not like terms.

$3g^2$ and $45g^2$ are like terms.

WORKED EXAMPLE 8

Simplify the following expressions.

a $3a + 5a$

b $7ab - 3a - 4ab$

c $2c - 6 + 4c + 15$

THINK

a 1 Write the expression and check that the two terms are like terms; that is, they contain the same variables.

2 Add the like terms and write the answer.

b 1 Write the expression and check for like terms.

2 Rearrange the terms so that the like terms are together.
Remember to keep the correct sign in front of each term.

3 Subtract the like terms and write the answer.

c 1 Write the expression and check for like terms.

2 Rearrange the terms so that the like terms are together.
Remember to keep the correct sign in front of each term.

3 Simplify by collecting like terms and write the answer.

WRITE

a $3a + 5a$

$= 8a$

b $7ab - 3a - 4ab$

$= 7ab - 4ab - 3a$

$= 3ab - 3a$

c $2c - 6 + 4c + 15$

$= 2c + 4c - 6 + 15$

$= 6c + 9$

DISCUSSION

How might you explain the important points to remember when showing someone else how to collect like terms in an algebraic expression?

learnon RESOURCES — ONLINE ONLY

Interactivity: Simplifying expressions (int-3771)

eLesson: Simplification of expressions (eles-1884)

Digital doc: SkillsHEET Combining like terms (doc-6926)

Exercise 7.7 Simplifying expressions

Individual pathways

PRACTISE

Questions:
1, 2, 3a–i, 4

CONSOLIDATE

Questions:
1, 2a–o, 3a–l, 4, 7

MASTER

Questions:
1m–x, 2m–v, 3m–u, 4–9

Individual pathway interactivity: int-4434

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Understanding and fluency

1. **WE8a** Simplify the following expressions.

a. $4c + 2c$

d. $6q - 5q$

g. $3a - 7a - 2a$

j. $-3h + 4h$

m. $9m + 5m - m$

p. $5p + 3p + 2p$

s. $13t - 4t + 5t$

v. $13m - 2m - 4m + m$

b. $2c - 5c$

e. $-h - 2h$

h. $-3f + 7f$

k. $11b + 2b + 5b$

n. $x - 2x$

q. $9g + 12g - 4g$

t. $-11j + 4j$

w. $m + 3m - 4m$

c. $3a + 5a - 4a$

f. $7x - 5x$

i. $4p - 7p$

l. $7t - 8t + 4t$

o. $7z + 13z$

r. $18b - 4b - 11b$

u. $-12l + 2l - 5l$

x. $t + 2t - t + 8t$

2. **WE8b,c** Simplify the following expressions.

a. $3x + 7x - 2y$

d. $3u - 4u + 6$

g. $11a - 5b + 6a$

j. $6m + 4m - 3n + n$

m. $2b - 6 - 4b + 18$

p. $8h - 6 + 3h - 2$

s. $3h + 4k - 16h - k + 7$

v. $17f - 3k + 2f - 7k$

b. $3x + 4x - 12$

e. $2m + 3p + 5m$

h. $9t - 7 + 5$

k. $5k - 5 + 2k - 7$

n. $11 - 12h + 9$

q. $11s - 6t + 4t - 7s$

t. $13 + 5t - 9t - 8$

c. $11 + 5f - 7f$

f. $-3h + 4r - 2h$

i. $12 - 3g + 5$

l. $3n - 4 + n - 5$

o. $12y - 3y - 7g + 5g - 6$

r. $2m + 13l - 7m + l$

u. $2g + 5 + 5g - 7$

3. Simplify the following expressions.

a. $x^2 + 2x^2$

d. $d^2 + 6d^2$

g. $2b^2 + 5b^2$

j. $a^2 + 4 + 3a^2 + 5$

m. $3a^2 + 2a + 5a^2 + 3a$

p. $11g^3 + 17 - 3g^3 + 5 - g^2$

s. $4fg + 2s - fg + s$

b. $3y^2 + 2y^2$

e. $7g^2 - 8g^2$

h. $4a^2 - 3a^2$

k. $11x^2 - 6 + 12x^2 + 6$

n. $11b - 3b^2 + 4b^2 + 12b$

q. $12ab + 3 + 6ab$

t. $11ab + ab - 5$

c. $a^3 + 3a^3$

f. $3y^3 + 7y^3$

i. $g^2 - 2g^2$

l. $12s^2 - 3 + 7 - s^2$

o. $6t^2 - 6g - 5t^2 + 2g - 7$

r. $14xy + 3xy - xy - 5xy$

u. $18ab^2 - 4ac + 2ab^2 - 10ac$

Communicating, reasoning and problem solving

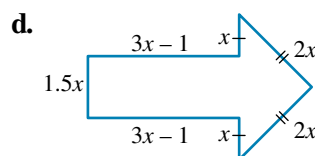
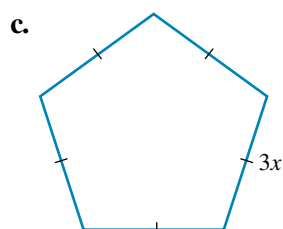
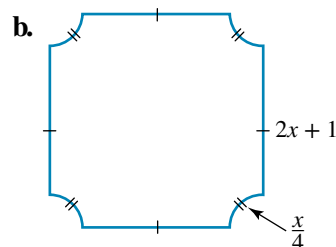
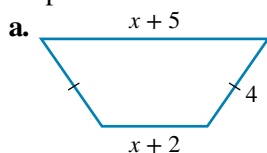
4. Rose owns an art gallery and sells items supplied to her by various artists. She receives a commission for all items sold. She uses the following method to keep track of the money she owes the artists when their items are sold.
- Ask the artist how much they want for the item.
 - Add 50% to that price, then mark the item for sale at this new price.
 - When the item sells, take one-third of the sale price as commission, then return the balance to the artist.

Use algebra to show that this method does return the correct amount to the artist.

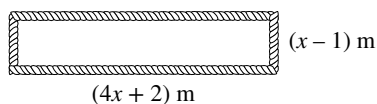
5. Explain, using mathematical reasoning and with diagrams if necessary, why the expression $2x + 2x^2$ cannot be simplified.
6. Three members of a fundraising committee are making books of tickets to sell for a raffle. Each book of tickets contains t tickets. If the first person has 14 books of tickets, the second person has 12 books of tickets and the third person has 13 books of tickets, write an expression for:
- the number of tickets that the first person has
 - the number of tickets that the second person has
 - the number of tickets that the third person has
 - the total number of tickets for the raffle.



7. Write an expression for the total perimeter of the following shapes.



8. What do you need to remember when checking for like terms?
9. If 32 metres of rope are required to make the rectangular shape shown in the figure, what is the value of x ?



7.8 Multiplying and dividing expressions with variables

7.8.1 Multiplying expressions with variables

- When we multiply variables (as already stated) the Commutative Law holds, so order is not important. For example:

$$3 \times 6 = 6 \times 3$$

$$6 \times w = w \times 6$$

$$a \times b = b \times a$$

- The multiplication sign ($\times$) is usually omitted for reasons of convention.

$$3 \times g \times h = 3gh$$

$$2 \times x^2 \times y = 2x^2y$$

- Although order is not important, conventionally the variables in each term are written in alphabetical order. For example,

$$2 \times b^2 \times a \times c = 2ab^2c$$

WORKED EXAMPLE 9

Simplify the following.

a $5 \times 4g$

THINK

1 Write the expression and replace the hidden multiplication signs.

2 Multiply the numbers.

3 Remove the multiplication sign.

b **1** Write the expression and replace the hidden multiplication signs.

2 Place the numbers at the front.

3 Multiply the numbers.

4 Remove the multiplication signs and place the variables in alphabetical order.

b $-3d \times 6ab \times 7$

WRITE

a $5 \times 4g = 5 \times 4 \times g$

$$= 20 \times g$$

$$= 20g$$

b $-3d \times 6ab \times 7$
 $= -3 \times d \times 6 \times a \times b \times 7$

$$= -3 \times 6 \times 7 \times d \times a \times b$$

$$= -126 \times d \times a \times b$$

$$= -126abd$$

7.8.2 Dividing expressions with variables

- When dividing expressions with variables, rewrite the expression as a fraction and simplify by cancelling.
- Remember that when the same variable appears as a factor on both the numerator and denominator, it may be cancelled.

WORKED EXAMPLE 10

a Simplify $\frac{16f}{4}$.

THINK

1 Write the expression.

2 Simplify the fraction by cancelling 16 with 4 (divide both by 4).

3 No need to write the denominator since we are dividing by 1.

b **1** Write the expression and then rewrite it as a fraction.

2 Simplify the fraction by cancelling 15 with 3 and n with n .

3 No need to write the denominator since we are dividing by 1.

WRITE

a $\frac{16f}{4} = \frac{\cancel{16}^4 f}{\cancel{4}_1}$

$$= \frac{4f}{1}$$

$$= 4f$$

b $15n \div 3n = \frac{15n}{3n}$
 $= \frac{\cancel{15}^5 \cancel{n}^1}{\cancel{3}_1 \cancel{n}_1}$

$$= \frac{5}{1}$$

$$= 5$$

WORKED EXAMPLE 11

Simplify $-12xy \div 27y$.

THINK

- 1 Write the expression and then rewrite it as a fraction.
- 2 Simplify the fraction by cancelling 12 with 27 (divide both by 3) and y with y .

WRITE

$$\begin{aligned}-12xy \div 27y &= -\frac{12xy}{27y} \\ &= -\frac{\overset{4}{\cancel{12}}x\cancel{y}}{\overset{9}{\cancel{27}}\cancel{y}} \\ &= -\frac{4x}{9}\end{aligned}$$

WORKED EXAMPLE 12

Simplify the following.

a $3m^3 \times 2m$ b $5p^{10} \times 3p^3$ c $36x^7 \div 12x^4$ d $\frac{6y^3 4y^8}{12y^4}$

THINK

- a
- 1 Write the problem.
 - 2 The order is not important when multiplying, so place the numbers first.
 - 3 Multiply the numbers.
 - 4 Check to see if the bases are the same. They are both m .
 - 5 Simplify by adding the indices.

WRITE

a

$$\begin{aligned}3m^3 \times 2m &= 3 \times 2 \times m^3 \times m \\ &= 6 \times m^3 \times m \\ &= 6 \times m^{3+1} \\ &= 6m^4\end{aligned}$$

- b
- 1 Write the problem.
 - 2 The order is not important when multiplying, so place the numbers first.
 - 3 Multiply the numbers.
 - 4 Check to see if the bases are the same. They are both p .
 - 5 Simplify by adding the indices.

b

$$\begin{aligned}5p^{10} \times 3p^3 &= 5 \times 3 \times p^{10} \times p^3 \\ &= 15 \times p^{10} \times p^3 \\ &= 15 \times p^{10+3} \\ &= 15p^{13}\end{aligned}$$

- c
- 1 Write the problem and express it as a fraction.
 - 2 Divide the numbers.
 - 3 Check to see if the bases are the same. They are both x .

c

$$\begin{aligned}36x^7 \div 12x^4 &= \frac{36x^7}{12x^4} \\ &= \frac{3x^7}{x^4}\end{aligned}$$

4 Subtract the powers.

$$= 3x^{7-4} = 3x^3$$

d 1 Write the problem.

$$\text{d } \frac{6y^3 \times 4y^8}{12y^4}$$

2 Multiply the numbers in the numerator. Simplify the numbers in index form in the numerator.

$$= \frac{24y^{11}}{12y^4}$$

3 Divide the numbers and subtract the powers.

$$= 2y^{11-4} \\ = 2y^7$$

ACTIVITY: COMMUNICATE!

1. Split the class into pairs. Each pair should have a scribe and a communicator. The scribe can do only what the communicator tells them and cannot speak. The communicator can tell the scribe only the steps required to solve the problem. They cannot provide the scribe with the actual answer.
2. Following the guidelines in step 1, the communicator should list the steps required to simplify the expression $-12cd^2 \div 9d$, and the scribe should complete the question without speaking (the scribe must strictly follow the instructions given by the communicator). Once finished, the pair should discuss whether the answer is correct and adjust their working where required.
3. Each pair should swap roles and simplify the second expression $60m^2n \div 55mnp$ using the same procedure.
4. Discuss the difficulties faced when acting as the communicator and then as the scribe. Has this activity strengthened your understanding of dividing pronumerals?

7.8.3 The order of operations

- When working with algebra, the order of operations (BIDMAS) still applies.
- Remember that when dealing with fractions, the numerator and denominator should be evaluated first before simplifying the fraction.

WORKED EXAMPLE 13

Simplify $\frac{16g \times (8g + 3g)}{5g - g}$.

THINK

- 1 Evaluate the expression inside the brackets.
- 2 Carry out the multiplication on the numerator.
- 3 Carry out the subtraction on the denominator.
- 4 Divide the numbers and subtract the powers.

WRITE

$$\begin{aligned} \frac{16g \times (8g + 3g)}{5g - g} &= \frac{16g \times 11g}{5g - g} \\ &= \frac{176g^2}{5g - g} \\ &= \frac{176g^2}{4g} \\ &= 44g^{2-1} \\ &= 44g \end{aligned}$$

Interactivity: Multiplying variables (int-3772)

Interactivity: Dividing expressions with variables (int-3773)

Exercise 7.8 Multiplying and dividing expressions with variables

Individual pathways

PRACTISE

Questions:

1–5, 8–10

CONSOLIDATE

Questions:

1a–p, 2a–l, 3a–l, 4a–l, 5a–l, 6a–l,
7a–e, 8–11

MASTER

Questions:

1m–z, 2m–t, 3m–t, 4m–u, 5m–t,
6m–r, 7e–j, 8–11, 13–14

   Individual pathway interactivity: int-4435

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Understanding and fluency

1. **WE9** Simplify the following.

a. $4 \times 3g$

b. $7 \times 3h$

c. $4d \times 6$

d. $3z \times 5$

e. $6 \times 5r$

f. $5t \times 7$

g. $4 \times 3u$

h. $7 \times 6p$

i. $7gy \times 3$

j. $2 \times 11ht$

k. $4x \times 6g$

l. $10a \times 7h$

m. $9m \times 4d$

n. $3c \times 5h$

o. $9g \times 2x$

p. $2.5t \times 5b$

q. $13m \times 12n$

r. $6a \times 12d$

s. $2ab \times 3c$

t. $4f \times 3gh$

u. $2 \times 8w \times 3x$

v. $11ab \times 3d \times 7$

w. $16xy \times 1.5$

x. $3.5x \times 3y$

y. $11q \times 4s \times 3$

z. $4a \times 3b \times 2c$

2. **WE10** Simplify the following.

a. $\frac{8f}{2}$

b. $\frac{6h}{3}$

c. $\frac{15x}{3}$

d. $9g \div 3$

e. $10r \div 5$

f. $4x \div 2x$

g. $8r \div 4r$

h. $\frac{16m}{8m}$

i. $14q \div 21q$

j. $\frac{3x}{6x}$

k. $\frac{12h}{14h}$

l. $50g \div 75g$

m. $\frac{8f}{24f}$

n. $35x \div 70x$

o. $24m \div 36m$

p. $y \div 34y$

q. $27h \div 3h$

r. $\frac{20d}{48d}$

s. $\frac{64q}{44q}$

t. $81l \div 27l$

3. Simplify the following.

a. $\frac{15fg}{3}$

b. $12cd \div 4$

c. $\frac{8xy}{12}$

d. $24cg \div 24$

e. $\frac{11xy}{11x}$

f. $\frac{9pq}{18pq}$

g. $\frac{21ab}{28b}$

h. $\frac{9dg}{12g}$

i. $\frac{5jk}{kj}$

j. $55rt \div 77t$

k. $\frac{10mxy}{35mx}$

l. $36bc \div 27c$

m. $13xy \div x$

n. $\frac{16cd}{40cd}$

o. $14abc \div 7bc$

p. $3gh \div 6h$

q. $\frac{132mnp}{60np}$

r. $\frac{11ad}{66ad}$

s. $18adg \div 45ag$

t. $\frac{bh}{7h}$

4. Simplify the following.

a. $3 \times -5f$

d. $-9t \times -3g$

g. $-3 \times -2w \times 7d$

j. $3as \times -3b \times -2x$

m. $-7a \times 3b \times g$

p. $5h \times 8j \times -k$

s. $2ab \times 3c \times 5$

b. $-6 \times -2d$

e. $-5t \times -4dh$

h. $-4a \times -3b \times 2c \times e$

k. $-5h \times -5t \times -3q$

n. $17ab \times -3gh$

q. $75x \times 1.5y$

t. $-4w \times 34x \times 3$

c. $11a \times -3g$

f. $6 \times -3st$

i. $11ab \times -3f$

l. $4 \times -3w \times -2 \times 6p$

o. $-3.5g \times 2h \times 7$

r. $12rt \times -3z \times 4p$

u. $-3ab \times -5cd \times -6ef$

5. **WE11** Simplify the following.

a. $\frac{-4a}{8}$

b. $\frac{-11ab}{33b}$

c. $60jk \div -5k$

d. $-3h \div -6dh$

e. $\frac{-32g}{40gl}$

f. $-12xy \div 48y$

g. $\frac{12ab}{-14ab}$

h. $\frac{6fgh}{30ghj}$

i. $-4xyz \div 6yz$

j. $\frac{-rt}{6rt}$

k. $-5mn \div 20n$

l. $-14st \div -28$

m. $34ab \div -17ab$

n. $\frac{-2ab}{-3a}$

o. $\frac{-7dg}{35gh}$

p. $-60mn \div 55mnp$

q. $\frac{28def}{18d}$

r. $-72xyz \div 28yz$

s. $\frac{54pq}{36pqr}$

t. $-\frac{121oc}{132oct}$

6. **WE12** Simplify the following.

a. $2a \times a$

d. $ab \times 7a$

g. $7pq \times 3p \times 2q$

j. $-7a \times -3b \times -2c^2$

m. $2a^4 \times 3a^7$

p. $\frac{25p^{12} \times 4q^7}{15p^2 \times 8q^2}$

b. $-5p \times -5p$

e. $3b^2 \times 2cd$

h. $5m \times n \times 6nt \times -t$

k. $2mn \times -3 \times 2n \times 0$

n. $2x^2y^3 \times x^3$

q. $\frac{8x^3 \times 7y^2 \times 2z^2}{6x \times 14y}$

c. $-5 \times 3x \times 2x$

f. $-5xy \times 4 \times 8x$

i. $-3 \times xyz \times -3z \times -2y$

l. $w^2x \times -9z^2 \times 2xy^2$

o. $20m^{12} \div 2m^3$

r. $\frac{a \times ab \times 3b^2}{5a^2b^2}$

7. Simplify the following.

a. $\frac{3}{a} \times \frac{2}{a}$

b. $\frac{5b}{2} \times \frac{4b}{3}$

c. $w \times \frac{5}{w^2}$

d. $\frac{3rk}{2s} \times \frac{6st}{5rt}$

e. $-\frac{15gt}{10ag} \times \frac{2g}{5t}$

f. $-\frac{4ht}{3dk} \times \frac{-12hk}{9dt}$

g. $\frac{5t}{gn} \div \frac{1}{g}$

h. $\frac{-9th}{4g} \div \frac{tg}{6h}$

i. $\frac{4xy}{7wz} \div \frac{x}{14z}$

j. $\frac{-10f}{3w} \div \frac{5}{-9wz}$

8. **WE13** Simplify the following.

a. $2x^2 + 3x \times 4x$

b. $4p^3 \times (5p^3 - 3p^3)$

c. $\frac{5s \times (11s - 5s)}{6s - 3s}$

d. $\frac{5x^3 \times (4y - 2y)}{x^2y}$

e. $\frac{(2s^2 + 6s^2) \times (6t - 3t)}{6}$

f. $\frac{(3a - 2a) \times (5bc + 2bc)}{abc}$

Communicating, reasoning and problem solving

9. Explain why $x \div yz$ is equivalent to $\frac{x}{yz}$ and $x \div (y \times z)$, but is not equivalent

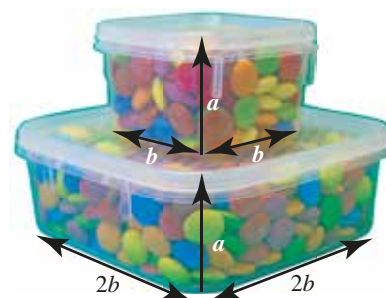
to $x \div y \times z$ or $\frac{x}{y} \times z$.

10. The student's working at right shows incorrect cancelling. Explain why the working is not correct.

11. Evaluate $\frac{2x - (3y + 2z)}{-(-x - y) + z}$ if $x = 2$, $y = -1$ and $z = -4$.

12. Evaluate $\frac{x \times (y - 2z)}{3x - (y - z)}$ if $x = -2$, $y = 5$ and $z = -1$.

13. a. Write an expression for the volume of each of the containers shown. (*Hint: The volume of a container is found by multiplying the length by the width by the height.*)
 b. How many times will the contents of the smaller container fit inside the larger container? (*Hint: Divide the volumes.*)
14. How are multiplication and division of expressions with variables similar to multiplication and division of numbers?



7.9 Expanding brackets

7.9.1 The Distributive Law

- The **Distributive Law** is the name given to the following process.

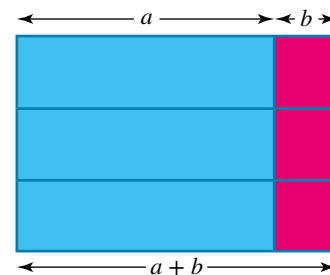
$$3(5 + 8) = 3 \times 5 + 3 \times 8$$

- This is because the number out in front is *distributed* to each of the terms in the brackets.
- Since variables take the place of numbers, the Distributive Law also holds true for algebraic expressions.

$$a(b + c) = ab + ac$$

- The Distributive Law can be demonstrated using the concept of area. As can be seen in the diagram at right, $3(a + b) = 3a + 3b$
- We can think of $3(a + b) = (a + b) + (a + b) + (a + b)$
- Collecting like terms, $a + a + a + b + b + b = 3a + 3b$
- An expression containing a bracket multiplied by a number can be written in expanded or factorised form.

$$\begin{array}{cc} \text{Factorised form} & \text{Expanded form} \\ 3(a + b) & = 3a + 3b \end{array}$$



- Expanding and **factorising** are the inverse of each other.
 - The Distributive Law can be used when the terms inside the brackets are either added or subtracted.
- $$a(b - c) = ab - ac$$
- The Distributive Law is not used when the terms inside the brackets are multiplied or divided. You can see this with numbers $2(4 \times 5) = 2 \times 4 \times 5$; *not* $(2 \times 4) \times (2 \times 5)$.
 - When simplifying expressions, we can leave the result in either factorised form or expanded form, but not a combination of both.

WORKED EXAMPLE 14

Use the Distributive Law to expand the following expressions.

a $3(a + 2)$

b $x(x - 5)$

THINK

- a 1 Write the expression.
 2 Use the Distributive Law to expand the brackets.
 3 Simplify by multiplying.
- b 1 Write the expression.
 2 Use the Distributive Law to expand the brackets.
 3 Simplify by multiplying.

WRITE

$$\begin{aligned} \text{a } 3(a + 2) &= 3(a + 2) \\ &= 3 \times a + 3 \times 2 \\ &= 3a + 6 \end{aligned}$$

$$\begin{aligned} \text{b } x(x - 5) &= x(x - 5) \\ &= x \times x + x \times -5 \\ &= x^2 - 5x \end{aligned}$$

- Some expressions can be simplified further by collecting like terms after any brackets have been expanded.

WORKED EXAMPLE 15

Expand the expressions below and then simplify by collecting any like terms.

a $3(x - 5) + 4$

b $4(3x + 4) + 7x + 12$

c $2x(3y + 3) + 3x(y + 1)$

d $4x(2x - 1) - 3(2x - 1)$

THINK

a 1 Write the expression.

2 Expand the brackets.

3 Collect the like terms (-15 and 4).

b 1 Write the expression.

2 Expand the brackets.

3 Rearrange so that the like terms are together. (Optional)

4 Collect the like terms.

c 1 Write the expression.

2 Expand the brackets.

3 Rearrange so that the like terms are together. (Optional)

4 Simplify by collecting the like terms.

d 1 Write the expression.

2 Expand the brackets. Take care with negative terms.

3 Simplify by collecting the like terms.

WRITE

a $3(x - 5) + 4$

$$\begin{aligned} &= 3 \times x + 3 \times -5 + 4 \\ &= 3x - 15 + 4 \\ &= 3x - 11 \end{aligned}$$

b $4(3x + 4) + 7x + 12$

$$\begin{aligned} &= 4 \times 3x + 4 \times 4 + 7x + 12 \\ &= 12x + 16 + 7x + 12 \\ &= 12x + 7x + 16 + 12 \\ &= 19x + 28 \end{aligned}$$

c $2x(3y + 3) + 3x(y + 1)$

$$\begin{aligned} &= 2x \times 3y + 2x \times 3 + 3x \times y + 3x \times 1 \\ &= 6xy + 6x + 3xy + 3x \\ &= 6xy + 3xy + 6x + 3x \\ &= 9xy + 9x \end{aligned}$$

d $4x(2x - 1) - 3(2x - 1)$

$$\begin{aligned} &= 4x \times 2x + 4x \times -1 - 3 \times 2x - 3 \times -1 \\ &= 8x^2 - 4x - 6x + 3 \\ &= 8x^2 - 10x + 3 \end{aligned}$$

learnon

RESOURCES — ONLINE ONLY

Interactivity: Expanding brackets: Distributive Law (int-3774)

eLesson: Expanding brackets (eles-1888)

Digital doc: SpreadSHEET Expanding brackets (doc-2288)

Exercise 7.9 Expanding brackets

Individual pathways

PRACTISE

Questions:
1–4, 8

CONSOLIDATE

Questions:
1a–l, 2a–l, 3a–l, 4a–f, 5a–f, 8

MASTER

Questions:
1m–t, 2m–r, 3j–p, 4f–j, 5f–j, 6–10

Individual pathway interactivity: int-4436

learnON ONLINE ONLY

To answer questions online and to receive **immediate feedback** and **fully worked solutions** for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Understanding and fluency

1. **WE14** Use the Distributive Law to expand the following expressions.

a. $3(d + 4)$	b. $2(a + 5)$	c. $4(x + 2)$
d. $5(r + 7)$	e. $6(g + 6)$	f. $2(t - 3)$
g. $7(d + 8)$	h. $9(2x - 6)$	i. $12(4 + c)$
j. $7(6 + 3x)$	k. $45(2g + 3)$	l. $1.5(t + 6)$
m. $11(t - 2)$	n. $3(2t - 6)$	o. $t(t + 3)$
p. $x(x + 4)$	q. $g(g + 7)$	r. $2g(g + 5)$
s. $3f(g + 3)$	t. $6m(n - 2m)$	

2. Expand the following.

a. $3(3x - 2)$	b. $3x(x - 6y)$	c. $5y(3x - 9y)$
d. $50(2y - 5)$	e. $-3(c + 3)$	f. $-5(3x + 4)$
g. $-5x(x + 6)$	h. $-2y(6 + y)$	i. $-6(t - 3)$
j. $-4f(5 - 2f)$	k. $9x(3y - 2)$	l. $-3h(2b - 6h)$
m. $4a(5b + 3c)$	n. $-3a(2g - 7a)$	o. $5a(3b + 6c)$
p. $-2w(9w - 5z)$	q. $12m(4m + 10)$	r. $-3k(-2k + 5)$

3. **WE15** Expand the expressions below and then simplify by collecting any like terms.

a. $7(5x + 4) + 21$	b. $3(c - 2) + 2$	c. $2c(5 - c) + 12c$
d. $6(v + 4) + 6$	e. $3d(d - 4) + 2d^2$	f. $3y + 4(2y + 3)$
g. $24r + r(2 + r)$	h. $5 - 3g + 6(2g - 7)$	i. $4(2f - 3g) + 3f - 7$
j. $3(3x - 4) + 12$	k. $-2(k + 5) - 3k$	l. $3x(3 + 4r) + 9x - 6xr$
m. $12 + 5(r - 5) + 3r$	n. $12gh + 3g(2h - 9) + 3g$	o. $3(2t + 8) + 5t - 23$
p. $24 + 3r(2 - 3r) - 2r^2 + 5r$		

4. Expand the following and then simplify by collecting like terms.

a. $3(x + 2) + 2(x + 1)$	b. $5(x + 3) + 4(x + 2)$	c. $2(y + 1) + 4(y + 6)$
d. $4(d + 7) - 3(d + 2)$	e. $6(2h + 1) + 2(h - 3)$	f. $3(3m + 2) + 2(6m - 5)$
g. $9(4f + 3) - 4(2f + 7)$	h. $2a(a + 2) - 5(a^2 + 7)$	i. $3(2 - t^2) + 2t(t + 1)$
j. $m(n + 4) - mn + 3m$		

5. Simplify the following expressions by removing the brackets and then collecting like terms.

a. $3h(2k + 7) + 4k(h + 5)$	b. $6n(3y + 7) - 3n(8y + 9)$
c. $4g(5m + 6) - 6(2gm + 3)$	d. $11b(3a + 5) + 3b(4 - 5a)$
e. $5a(2a - 7) - 5(a^2 + 7)$	f. $7c(2f - 3) + 3c(8 - f)$
g. $7x(4 - y) + 2xy - 29$	h. $11v(2w + 5) - 3(8 - 5vw)$
i. $3x(3 - 2y) + 6x(2y - 9)$	j. $8m(7n - 2) + 3n(4 + 7m)$

Communicating, reasoning and problem solving

6. **a.** Using the concept of area as shown in section 7.9, explain with diagrams and mathematical reasoning why $5(6 - 2) = 5 \times 6 - 5 \times 2$.
- b.** Using the concept of area as shown in section 7.9, explain with diagrams and mathematical reasoning why $4(x - y) = 4 \times x - 4 \times y$.
7. Expressions of the form $(a + b)(c + d)$ can be expanded by using the Distributive Law twice.

Distribute one of the factors over the other; for example, $(a + b)(c + d)$. The expression can then be fully expanded following Worked example 14.

- a.** $(x + 1)(x + 2)$ **b.** $(a + 3)(a + 4)$ **c.** $(c + 2)(c - 3)$
d. $(y + 4)(y - 4)$ **e.** $(u - 2)(u - 3)$ **f.** $(k - 5)(k - 2)$
8. The price of a pair of jeans is \$50. During a sale, the price of the jeans is discounted by \$ d .
- a.** Write an expression to represent the sale price of the jeans.
- b.** If you buy three pairs of jeans during the sale, write an expression to represent the total purchase price:
- i.** containing brackets
- ii.** in expanded form (without brackets).
- c.** Write an expression to represent the total change you would receive from \$200 for the three pairs of jeans purchased during the sale.
9. A triptych is a piece of art that is divided into three sections or panels. The middle panel is usually the largest panel and is flanked by two related panels.



- a.** Write a simplified expression for the area of each of the three paintings (excluding the frame).
- b.** Write a simplified expression for the combined area of the triptych.
- c.** The value of f is $m + 102.5$. Substitute $(m + 102.5)$ into your combined area formula and simplify the expression.
- d.** The actual value of m is 122.5 cm. Sketch the shape of the three paintings in your workbook and show the actual measurements of each, including length, width and area.
10. Why doesn't the Distributive Law apply when there is a multiplication sign inside the brackets; that is, for $a(b \times c)$?

7.10 Factorising

7.10.1 Factorising algebraic expressions

- Factorising is the opposite process to expanding.
- To factorise a single algebraic term, the term must be broken up into all the factors of its individual parts.
- For example, $10xy = 2 \times 5 \times x \times y$ and $6abc = 2 \times 3 \times a \times b \times c$.
- Factorising more than one algebraic term involves identifying the highest common factors of the algebraic terms.
- To find the highest common factor of the algebraic terms:
 1. Find the highest common factor of the number parts.
 2. Find the highest common factor of the variable parts.
 3. Multiply these together.

WORKED EXAMPLE 16

Find the highest common factor of $6x$ and 10 .

THINK

- 1 Find the highest common factor of the number parts.
Break 6 down into factors.
Break 10 down into factors.
The highest common factor is 2.
- 2 Find the highest common factor of the variable parts.
There isn't one, because only the first term has a variable part.

WRITE

$$\begin{aligned}6 &= 3 \times 2 \\10 &= 5 \times 2 \\ \text{HCF} &= 2\end{aligned}$$

The HCF of $6x$ and 10 is 2.

WORKED EXAMPLE 17

Find the highest common factor of $14fg$ and $21gh$.

THINK

- 1 Find the highest common factor of the number parts.
Break 14 down into factors.
Break 21 down into factors.
The highest common factor is 7.
- 2 Find the highest common factor of the variable parts.
Break fg down into factors.
Break gh down into factors.
Both contain a factor of g .
- 3 Multiply these together.

WRITE

$$\begin{aligned}14 &= 7 \times 2 \\21 &= 7 \times 3 \\ \text{HCF} &= 7\end{aligned}$$

$$\begin{aligned}fg &= f \times g \\gh &= g \times h \\ \text{HCF} &= g\end{aligned}$$

The HCF of $14fg$ and $21gh$ is $7g$.

- To factorise an expression, place the highest common factor of the terms outside the brackets and the remaining factors for each term inside the brackets.

WORKED EXAMPLE 18

Factorise the expression $2x + 6$.

THINK

- 1 Break down each term into its factors.
- 2 Write the highest common factor outside the brackets.
Write the other factors inside the brackets.
- 3 Remove the multiplication sign.

WRITE

$$\begin{aligned}2x + 6 &= 2 \times x + 2 \times 3 \\&= 2 \times (x + 3) \\&= 2(x + 3)\end{aligned}$$

WORKED EXAMPLE 19

Factorise $12gh - 8g$.

THINK

- 1 Break down each term into its factors.
- 2 Write the highest common factor outside the brackets.
Write the other factors inside the brackets.
- 3 Remove the multiplication signs.

WRITE

$$\begin{aligned}12gh - 8g &= 4 \times 3 \times g \times h - 4 \times 2 \times g \\&= 4 \times g \times (3 \times h - 2) \\&= 4g(3h - 2)\end{aligned}$$

learnon RESOURCES — ONLINE ONLY

Interactivity: Factorising (int-3775)

eLesson: Factorisation (eles-1887)

Digital doc: SpreadSHEET Finding the HCF (doc-2289)

Digital doc: WorkSHEET Algebra II (doc-2291)

Exercise 7.10 Factorising

Individual pathways

PRACTISE

Questions:
1–4, 12

CONSOLIDATE

Questions:
1, 2, 3a–l, 4a–l, 5, 6, 12, 13

MASTER

Questions:
1, 2, 3j–r, 4m–v, 5–14

Individual pathway interactivity int-4437

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Understanding and fluency

1. **WE16** Find the highest common factor of the following.
 - a. 4 and 6
 - b. 6 and 9
 - c. 12 and 18
 - d. 13 and 26
 - e. 14 and 21
 - f. $2x$ and 4
 - g. $3x$ and 9
 - h. $12a$ and 16

2. **WE17** Find the highest common factor of the following.

a. $2gh$ and $6g$	b. $3mn$ and $6mp$	c. $11a$ and $22b$	d. $4ma$ and $6m$
e. $12ab$ and $14ac$	f. $24fg$ and $36gh$	g. $20dg$ and $18ghq$	h. $11gl$ and $33lp$
i. $16mnp$ and $20mn$	j. $28bc$ and $12c$	k. $4c$ and $12cd$	l. x and $3xz$

3. **WE18** Factorise the following expressions.

a. $3x + 6$	b. $2y + 4$	c. $5g + 10$	d. $8x + 12$	e. $6f + 9$	f. $12c + 20$
g. $2d + 8$	h. $2x - 4$	i. $12g - 18$	j. $11h + 121$	k. $4s - 16$	l. $8x - 20$
m. $12g - 24$	n. $14 - 4b$	o. $16a + 64$	p. $48 - 12q$	q. $16 + 8f$	r. $12 - 12d$

4. **WE19** Factorise the following. Check your answers by expanding the factorised expression.

a. $3gh + 12$	b. $2xy + 6y$	c. $12pq + 4p$	d. $14g - 7gh$
e. $16jk - 2k$	f. $12eg + 2g$	g. $12k + 16$	h. $7mn + 6m$
i. $14ab + 7b$	j. $5a - 15abc$	k. $8r + 14rt$	l. $24mab + 12ab$
m. $4b - 6ab$	n. $12fg - 16gh$	o. $ab - 2bc$	p. $14x - 21xy$
q. $11jk + 3k$	r. $3p + 27pq$	s. $12ac - 4c + 3dc$	t. $4g + 8gh - 16$
u. $28s + 14st$	v. $15uv + 27vw$		

5. Find the highest common factor of $4ab$, $6a^2b^3$ and $12a^3b$.

6. Find the lowest common multiple of $3ab$, $4a^3bc$ and $6a^2b^2$.

7. Simplify $\frac{4x - 4}{10x - 20} \times \frac{15x + 15}{3x - 3} \times \frac{6x - 12}{20x + 20}$.

8. Simplify $\frac{3x + 6}{x - 1} \div \frac{12x + 24}{6(x + 1)}$.

9. Factorise and hence simplify $\frac{2x^2y - 6xy^2}{a + 2b} \times \frac{4a + 8b}{3xy} \div \frac{2x - 6y}{7}$.

Communicating, reasoning and problem solving

10. Simplify $(5ax^2y - 6bxy + 2ax^2y - bxy) \div (ax^2 - bx)$. Show your working.

11. A cuboid measures $(3x - 6)$ cm by $(2x + 8)$ cm by $(ax - 5a)$ cm.

a. Write an expression for its volume.

b. If the cuboid weighs $(8x - 16)$ g, find a factorised expression for its density in g/cm^3 .

12. A farmer's paddock is a rectangle of length $(2x - 6)$ m and width $(3x + 6)$ m.

a. Find the area of the paddock in factorised form.

b. State the smallest possible value of x . Explain your reasoning.

13. Factorise and hence simplify:

a. $\frac{3x^2y}{z^2} \times \frac{z^2}{12xy} \times \frac{6y^3}{z}$

b. $\frac{4x^2(y - 1)}{(z - 1)^2} \times \frac{z - 1}{24x(y - 1)} \times \frac{3(y - 1)^2}{9z}$

c. $\frac{5xz^2}{y^2} \times \frac{y^2}{15xz} \div \frac{5y^3}{xz^2}$

d. $\frac{24z(x + 1)}{(y - 1)^2} \div \frac{6y^2(x + 1)}{5x} \div \frac{15(y - 1)}{z^2(x + 1)^2}$

14. What strategies will you use to find the highest common factor?

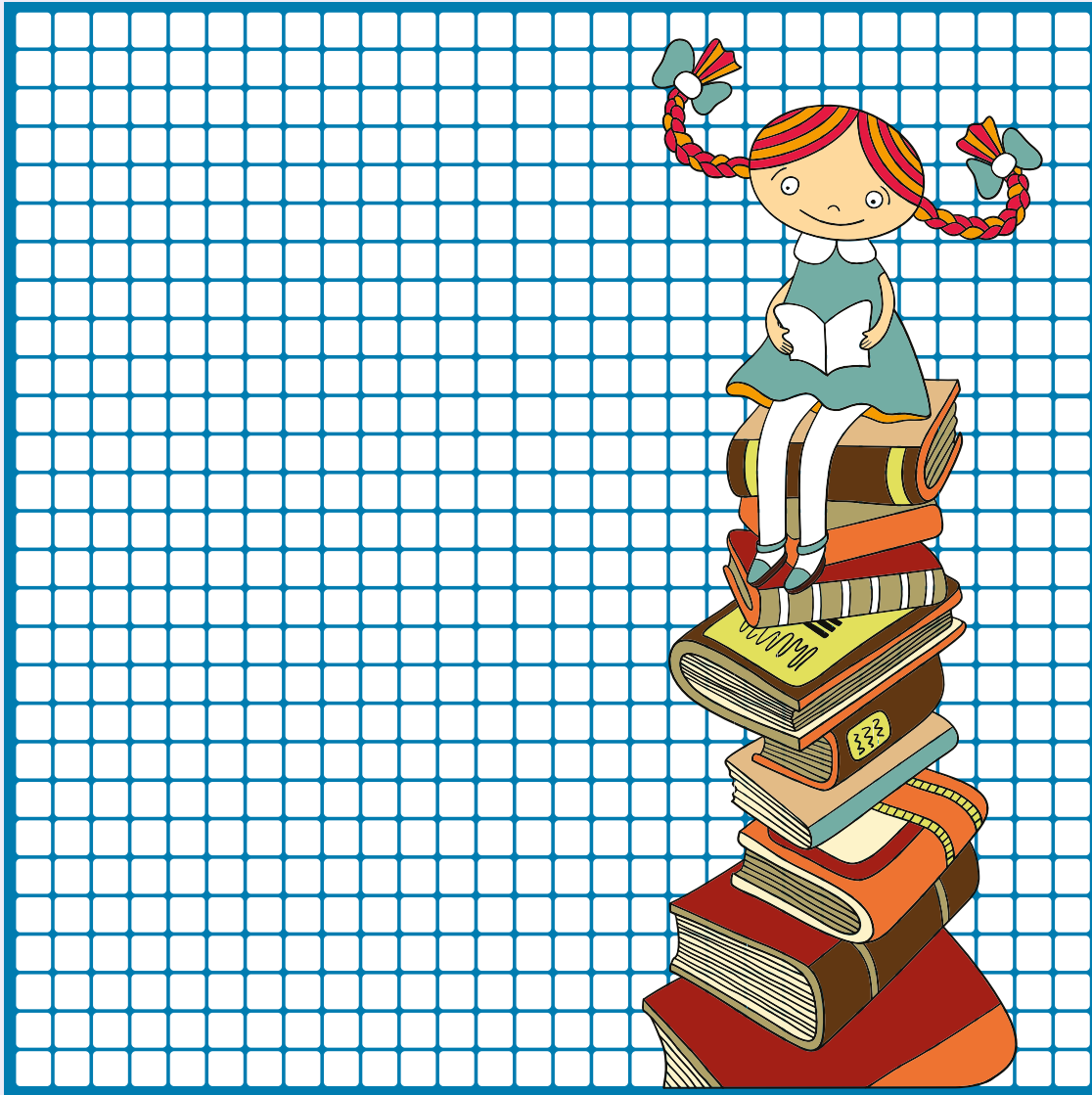


7.11 Review

Investigation | Rich task

Readability index

Since you first learned how to read, you have probably read many books. These books would have ranged from picture books with simple words to books with short sentences. As you learned more words, you read short stories and more challenging books. Have you ever picked up a book and put it down straight away because you thought there were too many ‘difficult words’ in it?



The reading difficulty of a text can be described by a readability index. There are several different methods used to calculate reading difficulty, and one of these methods is known as the Rix index. The Rix index is obtained by dividing the number of long words by the number of sentences.

1. Use a variable to represent the number of long words and another to represent the number of sentences. Write a formula that can be used to calculate the Rix index.

When using the formula to determine the readability index, follow these guidelines:

- A long word is a word that contains seven or more letters.
- A sentence is a group of words that ends with a full stop, question mark, exclamation mark, colon or semi-colon.
- Headings and numbers are not included and hyphenated words count as one word.

Consider this passage from a Science textbook.

2. How many sentences and long words appear in this passage of text?
3. Use your formula to calculate the Rix index for this passage. Round your answer to 2 decimal places.

A fatal fall . . . or was it murder?

In 1991, some German hikers found a body preserved in ice near the Italy–Austria border. Scientists used radiometric dating and found that the body was about 5300 years old! They thought that the person, known now as the Iceman, had died of hypothermia (extreme cold). Ten years later, another group of scientists using high-tech X-rays found the remains of an arrowhead lodged near his left lung. Specialists have not yet confirmed whether the Iceman fell back onto his arrow, or if he was murdered. And without any witnesses to question, the truth may never be known!

Once you have calculated the Rix index, the table below can be used to work out the equivalent year level of the passage of text.

Rix index	Equivalent year level	Rix index	Equivalent year level
Below 0.2	1	3.0–3.69	8
0.2–0.49	2	3.7–4.49	9
0.5–0.79	3	4.5–5.29	10
0.8–1.29	4	5.3–6.19	11
1.3–1.79	5	6.2–7.19	12
1.8–2.39	6	Above 7.2	University level
2.4–2.99	7		

4. What year level is the passage of text equivalent to?

When testing the reading difficulty of a book, it is not necessary to consider the entire book. Choose a section of text with at least **10** sentences and collect the required information for the formula.

5. Choose a passage of text from one of your school books, a magazine or newspaper. Calculate the Rix index and use the table above to determine the equivalent year level.
6. Repeat question **5** using another section of the book, magazine or newspaper. Did the readability level change?
7. On a separate page, rewrite the passage from question **5** with minimum changes, so that it is now suitable for a higher or lower year level. Explain the method you used to achieve this. Provide a Rix index calculation to prove that you changed the level of reading difficulty.

Digital doc: Investigation — Readability index (doc-13925)

Digital doc: Code puzzle — Doctor, I've swallowed the film out of my camera (doc-13926)

Digital doc: Summary Topic 7 (doc-23243)

Interactivity: Word search Topic 7 (int-2629)

Interactivity: Crossword Topic 7 (int-2630)

Interactivity: Sudoku Topic 7 (int-3188)

Exercise 7.11 Review questions

To answer questions online and to receive **immediate feedback** and **fully worked solutions** for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Understanding and fluency

- Using x and y to represent numbers, write expressions for:
 - the sum of x and y
 - the difference between y and x
 - five times y subtracted from three times x
 - the product of 5 and x
 - twice the product of x and y
 - the sum of $6x$ and $7y$
 - y multiplied by itself
 - twice a number is decreased by 7
 - the sum of x and y is tripled.
- If tickets to the school play cost \$15 for adults and \$9 for children, write an expression for the cost of:
 - x adult tickets
 - y child tickets
 - k adult tickets and m child tickets.
- Jake is now m years old.
 - Write an expression for his age in 5 years' time.
 - Write an expression for Jo's age if she is p years younger than Jake.
 - Jake's mother is 5 times his age. How old is she?
- Find the value of the following expressions if $a = 2$ and $b = 6$.

a. $2a$	b. $6a$	c. $5b$	d. $\frac{a}{2}$	e. $a + 8$	f. $b - 2$
g. $a + b$	h. $b - a$	i. $5 + \frac{b}{2}$	j. $3a + 7$	k. $2a + 3b$	l. $\frac{20}{a}$
m. $3b - 2a$	n. $\frac{b}{a}$	o. $\frac{a}{b}$	p. $-3ab$	q. $\frac{5a}{b}$	r. $\frac{2b}{9a}$
- The formula $C = 2.2k + 4$ can be used to calculate the cost in dollars, C , of travelling by taxi for a distance of k kilometres. Find the cost of travelling 4.5 km by taxi.
- The area (A) of a rectangle of length l and breadth b can be found using the formula $A = lb$. Find the breadth of a rectangle if $A = 65 \text{ cm}^2$ and $l = 13 \text{ cm}$.
- Substitute $r = 3$ and $s = 5$ into the following expressions and evaluate.

a. $2(r + s)$	b. $2(s - r)$	c. $5(r + s)$	d. $8(s - r)$
e. $s(r + 4)$	f. $s(2r - 3)$	g. $2r(r + 1)$	h. $rs(7 + s)$
i. $r^2(5 - r)$	j. $s^2(s + 15)$	k. $4r(s + r)$	l. $12r(r - s)$
- Find the value of the following expressions if $a = 2$ and $b = -5$.

a. $a + b$	b. $b + a$	c. ab	d. $\frac{ab}{5}$	e. $2ab$	f. $5 - a$
g. $12 - ab$	h. $a^2 - 2$	i. $3(a + 2)$	j. $b(a - 4)$	k. $12 - a(b - 3)$	l. $5a + 6b$
- Indicate whether each of the following is true or false for all values of the variables.

a. $3a + 5b = 5b + 2a$	b. $7x - 10y = 10y - 7x$	c. $8c + d = d + 8c$
d. $16 \times 2x \times x = 32x^2$	e. $9x \times -y = -y \times 9x$	f. $-4 \times 3x \times x = 12x \times x$

- g. $\frac{11p}{5r} = \frac{5r}{11p}$ h. $7i + 2j = 2j + 7i$ i. $-3y \div 7x = 7x \div -3y$
 j. $-8c + 5d = 5d - 8c$ k. $\frac{0}{5k} = \frac{5k}{0}$ l. $21 \times -\frac{7x}{3} = \frac{7x}{3} \times -21$
10. Simplify the following by collecting like terms.
 a. $4d + 3d$ b. $3c - 5c$ c. $3d + 5a - 4a$ d. $6g - 4g$
 e. $4x + 11 - 2x$ f. $2g + 5 - g - 6$ g. $2xy + 7xy$ h. $12t^2 + 3t + 3t^2 - t$
11. Simplify the following.
 a. $3 \times 7g$ b. $6 \times 3y$ c. $7d \times 6$ d. $-3z \times 8$
12. Simplify the following.
 a. $\frac{2a}{8}$ b. $\frac{11b}{44b}$ c. $6rt \div -2t$ d. $-3gh \div -6g$
 e. $\frac{32t}{40stv}$ f. $-36xy \div -12y$ g. $\frac{12ab}{-14ab}$ h. $\frac{5egh}{30ghj}$
13. Use the Distributive Law to expand the following expressions.
 a. $2(x + 3)$ b. $5(2x - 1)$ c. $-2(f + 7)$
 d. $3m(b - m)$ e. $-3y(7 - y)$ f. $9b(c - 2)$
14. Expand the following and then simplify by collecting like terms.
 a. $3(4v + 5) - 15$ b. $6t + 5(2t - 7)$ c. $23 + 5(3p - 4) + 2p$
 d. $2(x + 5) + 5(x + 1)$ e. $2g(g - 6) + 3g(g - 7)$ f. $3(3t - 4) - 6(2t - 9)$
15. Factorise the following expressions.
 a. $3g + 12$ b. $xy + 5y$ c. $5n - 20$
 d. $12mn + 4pn$ e. $12g - 6gh$ f. $12xy - 36yz$
16. Show that the Distributive Law holds for the following.
 $10(16 - 6) = 10 \times 16 - 10 \times 6$

Communicating, reasoning and problem solving

17. Using only +, -, $\times$, (), complete the following equations to demonstrate the Distributive Law.
 a. $3 \ 2 \ 1 = 3 \ 2 \ 3 \ 1$ b. $-10 \ 8 \ -6 \ -10 = -10 \ 8 \ -6$ c. $8 \ 6 \ 5 = 8 \ 5 \ 6 \ 8$
 d. $-4 \ 7 \ 1 \ 7 = -4 \ 1 \ 7$ e. $2 \ x \ y = 2 \ x \ 2 \ y$ f. $3 \ a \ a \ b = 3 \ a \ a \ 3 \ a \ b$
18. A builder has a 120-cm section of wood and wants to cut it into two unequal pieces.
 a. If the length of one of the sections is a cm, what is the length of the other piece?
 b. He has a second section of wood that is b cm long, and he cuts it into four unequal pieces: one is 36 cm, another 60 cm and another is $12a$ cm.
 What is the length of the fourth part of this second section of wood? (*Hint: Draw a diagram.*)
 c. If the total length of four of the six sections of wood is $(96 + 13a)$ cm, what is the total length of the other two pieces in terms of a and b ?
19. The base of a box has a length of $(x + 4)$ cm and a width of x cm.
 a. Draw a diagram, labelling the length and the width.
 b. Write an expression for the area of the base of the box.
 c. Expand part b.
 d. If $x = 3$, what is the area of the base of the box?
 e. If the height of the box is x cm, find an expression for the volume of the box.
 f. Find the volume of the box if x is 3 cm.
20. Aussie Rules Football is played in many Australian states. The scoring for the game is in Goals (G) and Behinds (B). Each Goal (G) scores six points and each behind (B) scores one point. To calculate the total number of points (P) scored by a team use the following rule: $P = 6G + B$
 a. Name the variables in the rule.
 b. State the expression in the rule.

- c. A team scored 11 goals and 10 behinds. How many points is this?
- d. A second team scored nine goals and 18 behinds. How many points is this?
- e. How many goals and behinds might a team have scored if its total points score was 87 points and the team scored more than six goals?
21. Stephanie bought some clothes from Target during their annual sale. She spent \$79.00. She bought a skirt, a T-shirt and a pair of shorts. She paid \$9 more for the T-shirt than for the shorts, and \$7 more for the skirt than for the T-shirt. How much did the skirt cost her?
22. Bobby the painter has two partially used 10-L tins of paint, A and B. There is more paint in Tin A than in Tin B. He mixes the paint in the following fashion.
- He pours paint from Tin A into Tin B until the volume of paint in Tin B is doubled.
 - He pours paint from Tin B into Tin A until the volume of paint in Tin A is doubled.
 - He pours paint from Tin A into Tin B until the volume of paint in Tin B is doubled.
- If Tin A originally contained x litres of paint and Tin B contained y litres of paint, find an expression in terms of x and y for the volume of paint in Tin A after Bobby finished mixing.



23. Two positive integers have a sum of 7 and a product of 12. Find the sum of their reciprocals.
24. If you add the first and last of any three consecutive integers together, can you find a relationship to the middle number?
25. The Flesch-Kincaid Grade Level formula is used to determine the readability of a piece of text. It produces a 0 to 100 score that can be used to determine the number of years of education generally required to understand a particular piece of text. The formula is as follows:

$$0.39 \left(\frac{\text{Total words}}{\text{Total sentences}} \right) + 11.8 \left(\frac{\text{Total syllables}}{\text{Total words}} \right) - 15.59$$

Text suitable for a Year 8 student should have a value of roughly 8. A passage of text contains 30 sentences, with 500 words and 730 syllables. Would it be suitable for a Year 8 student? Explain your answer.

26. Consider the expression

$$x^{(x+1)^{(x+2)}}$$

This is called a *power tower*.

What is the value of the last digit of the resulting number when $x = 2$?

Note: You will have to look at patterns to determine the answer, as a calculator will not give you an exact answer to the power.

Answers

Topic 7 Algebraic techniques

Exercise 7.2 Using variables

1. **a.** $x + 420$ **b.** $3x$ **c.** $x - 130$
d. The nearby nest has 60 more ants.
e. The nearby nest has 90 fewer ants.
f. The nest is one-quarter of the size of the original nest.
2. **a.** $x + y$ **b.** $x + y + 260$ **c.** $x + y + 90$ **d.** $x + y - 260$
3. Between 9.00 am and 9.15 am one Danish pastry was sold. In the next hour-and-a-half, a further 11 Danish pastries were sold. No more Danish pastries had been sold at 12.30 pm, but in the next half-hour 18 more were sold. No Danish pastries were sold after 1.00 pm.
4. **a.** $a + b$ **b.** $a + b + c$ **c.** $b + 4$ **d.** $a - 6$
5. **a.** $y + 7$ **b.** $y - 8$ **c.** $5y$ **d.** $14 - y$
e. $\frac{y}{3}$ **f.** $8y + 3$
6. **a.** $a + b$ **b.** $a - b$ **c.** $2b - 3a$ **d.** ab
e. $2ab$ **f.** $3a + 7b$ **g.** a^2 **h.** $\frac{a^2}{5}$
7. **a.** $\$27y$ **b.** $\$14d$ **c.** $\$(27r + 14h)$
8. **a.** $t + 2$ **b.** $t + g$ **c.** $t - 5$ **d.** $2t$
9. **a.** Various answers are possible; an example is shown. The number of passengers doubled at the next stop and continued to increase, more than quadrupling in the first nine minutes. At 7.22 pm, 5 people alighted the train, and by 7.25 pm the same number of passengers were on the train as there were at the beginning. By 7.34 pm there were 12 fewer passengers than there were at the beginning.
b. 7.22 pm **c.** 7.19 pm **d.** 7.34 pm
10. **a.** The number of bacteria in each of these intervals is double the number of bacteria in the previous interval.
b. The bacteria could be dividing in two.
c. It is lower than expected, based on the previous pattern of growth.
d. Some of the bacteria may have died, or failed to divide and reproduce.
11. **a.** Odd **b.** Even
c. i. $n + 2$, $n + 4$ and $n + 6$ **ii.** $n - 2$
12. $TSA = 2x^2 + 4xh$; $V = x^2h$
13. $TSA = 3x^2 + 5xh$; $V = \frac{3}{2}x^2h$
14. $TSA_{\text{square box}} = 2000 \text{ cm}^2$; $V_{\text{square box}} = 6000 \text{ cm}^3$
 $TSA_{\text{rectangular box}} = 2700 \text{ cm}^2$; $V_{\text{rectangular box}} = 9000 \text{ cm}^3$
15. Discuss with your teacher.
16. The number is 4; Bill's age is 15.

Exercise 7.3 Substitution

1. **a.** 6 **b.** 14 **c.** 30 **d.** 1 **e.** 9
f. 1 **g.** 7 **h.** 3 **i.** 6 **j.** 15
k. 19 **l.** 4 **m.** 5 **n.** 10 **o.** 20
p. 5 **q.** 22 **r.** 2 **s.** $\frac{89}{10} \left(8\frac{9}{10} \right)$ **t.** $\frac{39}{10} \left(3\frac{9}{10} \right)$
2. **a.** 42 **b.** 3 **c.** 54 **d.** 1 **e.** 9 **f.** 15 **g.** 15 **h.** 21
i. 24.3 **j.** 21 **k.** 1 **l.** 4.8 **m.** 18.3 **n.** 8.1 **o.** 16.2 **p.** $\frac{7}{4} \left(1\frac{3}{4} \right)$
3. **a.** 7 **b.** 7 **c.** -3 **d.** 3 **e.** 4 **f.** 10 **g.** 50 **h.** 1
i. -15 **j.** -4 **k.** 37 **l.** 15 **m.** 40 **n.** $2\frac{1}{3}$ **o.** 19 **p.** 1
4. 150 m

5. 10 cm

6. $c = \$64$

7. a. 48 cm^2

b. 8400 m^2

c. 4.472 m^2 or $44\,720 \text{ cm}^2$

8. a. $F = 212^\circ\text{F}$

b. $28^\circ\text{C} = 82.4^\circ\text{F}$

c. 32°F

9. a. $100 \text{ km} \approx 60 \text{ miles}$

b. $248 \text{ km} \approx 148.8 \text{ miles}$

c. $12.5 \text{ km} \approx 7.5 \text{ miles}$

10. If $x = 0$, then the expression becomes $\frac{0}{0}$, which is indeterminate.

11. 540.5

12. a. $V = x(x + 5)(x - 2)$

b. 1200

c. Because $1.5 - 2 < 0$

13. a. $13p - 10$

b. 2590

Exercise 7.4 Working with brackets

1. a. 36

b. 4

c. 84

d. 18

e. 56

f. 35

g. 90

h. 350

i. 55

j. 20

k. 133

l. 147

m. 784

n. 250

o. 9800

p. 200

2. a. 90

b. 16

c. 6

d. 32

e. 72

f. 90

g. 7

h. 36

i. 60

j. 58

k. 21

l. 180

m. 576

n. 32

o. -26

p. 33

3. a. 62 cm

b. 97.8 cm

4. B

5. a. 720°

b. 540°

c. 180°

d. 360°

e. 3240°

6. a. $CD = m + 4n$

b. $BC = 3m + n$

c. Perimeter $= 8m + 18n$

7. a. $A = \frac{1}{2}(m + n)(m - n)$

b. $A = \frac{1}{2}(21 \times 9) = 94.5 \text{ cm}^2$

c. If $m < n$, then $(m - n) < 0$. Negative height is impossible. Also $m \neq n$ as the height can't equal 0.

d. If m and n move closer in value, then the length of the base of the triangle gets closer to $2m$ (or $2n$) and the height gets closer to zero.

8. Check with your teacher.

9. Yes

Exercise 7.5 Substituting positive and negative numbers

1. a. 3

b. 9

c. -9

d. 3

e. -9

f. -12

g. -12

h. 2

i. 5

j. -13

k. 9

l. 3

m. -2

n. -3

o. -8

p. -2

q. 0

r. -109

s. 6

t. 16

2. a. 18

b. 32

c. -15

d. 12

e. 16

f. 24

g. 22

h. 155

i. 32

j. 3

k. 6

l. 21

3. a. -9

b. 1

c. 3

d. 40

e. -8

f. 2

g. -3

h. 12

i. $-\frac{4}{5}$

j. -2

k. -1

l. -6

m. 125

n. 175

o. -5

p. -12.5

q. -74

r. -117

s. -104

t. -15.5

4. $\frac{30}{7}$ or $4\frac{2}{7}$

5. If x is negative then $5x$ will also be a negative integer (less than or equal to -5). Subtracting this number is equivalent to adding a positive integer. The result will be positive.

6. a.

x	-4	-3	-2	-1	0	1	2	3	4
y	16	9	4	1	0	1	4	9	16

b. For two values of x with the same magnitude but different signs (e.g. -2 and 2) the values of x^2 are identical.

7. Check with your teacher.

8. a. Negative

b. Positive

9. a. $\approx 0.804x^2$

b. 3.216

c. $\frac{x}{2}$

10. Discuss with your teacher.

Exercise 7.6 Number laws and variables

1. a. i. 11

ii. 11, same

b. i. 25

ii. 25, same

c. i. 31

ii. 31, same

d. i. 32

ii. 32, same

- e. i. -5
 f. i. -18
 g. i. -28
 h. i. 1
 2. a. i. -10
 b. i. -180
 c. i. -40
 d. i. -350
 e. i. $-\frac{2}{5}$
 f. i. -1
 g. i. $-\frac{4}{5}$
 h. i. $-\frac{14}{45}$
 3. a. True
 g. False
 4. a. i. 13
 b. i. 24
 c. i. 40
 d. i. -3
 e. i. -35
 f. i. -43
 5. a. i. -64
 b. i. 768
 c. i. -1536
 d. i. -4
 e. i. -6
 f. i. $\frac{8}{5}$
 6. a. False
 7. C
 8. D
 9. a. i. 6
 ii. 5 , different
 ii. 18 , different
 ii. 28 , different
 ii. -1 , different
 ii. -10 , same
 ii. -180 , same
 ii. -40 , same
 ii. -350 , same
 ii. $-\frac{5}{2}$, different
 ii. -1 , same
 ii. $-\frac{5}{4}$, different
 ii. $-\frac{45}{14}$, different
 b. False
 h. False
 ii. 13 , same
 ii. 24 , same
 ii. 40 , same
 ii. -7 , different
 ii. -71 , different
 ii. -67 , different
 ii. -64 , same
 ii. 768 , same
 ii. -1536 , same
 ii. -1 , different
 ii. $-\frac{1}{6}$, different
 ii. $\frac{1}{10}$, different
 c. False
 i. False
 d. True
 j. True
 e. True
 k. False
 f. True
 l. True

b. The answers are equal because of the use of the Associative Law.

10. a. -10
 b. -12

11. a. $(-3p + 4q)$
 b. $\frac{1}{(3p - 4q)}$
 c. $15, -\frac{1}{15}$

d. The result is the identity, which is 1.

12. These expressions will have different results unless $x = a$.

Exercise 7.7 Simplifying expressions

1. a. $6c$	b. $-3c$	c. $4a$	d. q	e. $-3h$	f. $2x$
g. $-6a$	h. $4f$	i. $-3p$	j. h	k. $18b$	l. $3t$
m. $13m$	n. $-x$	o. $20z$	p. $10p$	q. $17g$	r. $3b$
s. $14t$	t. $-7j$	u. $-15l$	v. $8m$	w. 0	x. $10t$
2. a. $10x - 2y$	b. $7x - 12$	c. $11 - 2f$	d. $6 - u$	e. $7m + 3p$	f. $4r - 5h$
g. $17a - 5b$	h. $9t - 2$	i. $17 - 3g$	j. $10m - 2n$	k. $7k - 12$	l. $4n - 9$
m. $12 - 2b$	n. $20 - 12h$	o. $9y - 2g - 6$	p. $11h - 8$	q. $4s - 2t$	r. $14l - 5m$
s. $3k - 13h + 7$	t. $5 - 4t$	u. $7g - 2$	v. $19f - 10k$		
3. a. $3x^2$	b. $5y^2$	c. $4a^3$	d. $7d^2$	e. $-g^2$	
f. $10y^3$	g. $7b^2$	h. a^2	i. $-g^2$	j. $4a^2 + 9$	

k. $23x^2$	l. $11s^2 + 4$	m. $8a^2 + 5a$	n. $b^2 + 23b$	o. $t^2 - 4g - 7$
p. $7g^3 - g^2 + 22$	q. $18ab + 3$	r. $11xy$	s. $3fg + 3s$	t. $12ab - 5$
u. $20ab^2 - 14ac$				

4. Check with your teacher.

5. Check with your teacher.

6. a. $14t$	b. $12t$	c. $13t$	d. $39t$
7. a. $2x + 15$	b. $9x + 4$	c. $15x$	d. $13.5x - 2$

8. The terms contain the exact same variables, raised to the same power.

9. $x = 3$

Exercise 7.8 Multiplying and dividing expressions with variables

1. a. $12g$	b. $21h$	c. $24d$	d. $15z$	e. $30r$	f. $35t$	g. $12u$	h. $42p$
i. $21gy$	j. $22ht$	k. $24gx$	l. $70ah$	m. $36dm$	n. $15ch$	o. $18gx$	p. $12.5bt$
q. $156mn$	r. $72ad$	s. $6abc$	t. $12fgh$	u. $48wx$	v. $231abd$	w. $24xy$	x. $10.5xy$
y. $132qs$	z. $24abc$						
2. a. $4f$	b. $2h$	c. $5x$	d. $3g$	e. $2r$	f. 2	g. 2	h. 2
i. $\frac{2}{3}$	j. $\frac{1}{2}$	k. $\frac{6}{7}$	l. $\frac{2}{3}$	m. $\frac{1}{3}$	n. $\frac{1}{2}$	o. $\frac{2}{3}$	p. $\frac{1}{34}$
q. 9	r. $\frac{5}{12}$	s. $\frac{16}{11}\left(1\frac{5}{11}\right)$	t. 3				
3. a. $5fg$	b. $3cd$	c. $\frac{2xy}{3}$	d. cg	e. y	f. $\frac{1}{2}$	g. $\frac{3a}{4}$	h. $\frac{3d}{4}$
i. 5	j. $\frac{5r}{7}$	k. $\frac{2y}{7}$	l. $\frac{4b}{3}$	m. $13y$	n. $\frac{2}{5}$	o. $2a$	p. $\frac{g}{2}$
q. $\frac{11m}{5}$	r. $\frac{1}{6}$	s. $\frac{2d}{5}$	t. $\frac{b}{7}$				
4. a. $-15f$	b. $12d$	c. $-33ag$	d. $27gt$	e. $20dht$	f. $-18st$	g. $42dw$	h. $24abce$
i. $-33abf$	j. $18absx$	k. $-75hqt$	l. $144pw$	m. $-21abg$	n. $-51abgh$	o. $-49gh$	p. $-40hjk$
q. $112.5xy$	r. $-144prt$	s. $30abc$	t. $-408wx$	u. $-90abcdef$			
5. a. $\frac{a}{2}$	b. $\frac{a}{3}$	c. $-12j$	d. $\frac{1}{2d}$	e. $\frac{4}{5l}$	f. $\frac{x}{4}$	g. $\frac{6}{7}$	h. $\frac{f}{5j}$
i. $\frac{2x}{3}$	j. $\frac{1}{6}$	k. $\frac{m}{4}$	l. $\frac{st}{2}$	m. -2	n. $\frac{2b}{3}$	o. $\frac{d}{5h}$	p. $\frac{12}{11p}$
q. $\frac{14ef}{9}$	r. $\frac{18x}{7}$	s. $\frac{3}{2r}$	t. $\frac{11}{12t}$				
6. a. $2a^2$	b. $25p^2$	c. $-30x^2$	d. $7a^2b$	e. $6b^2cd$	f. $-160x^2y$	g. $42p^2q^2$	
h. $-30mn^2t^2$	i. $-18xy^2z^2$	j. $-42abc^2$	k. 0	l. $-18w^2x^2y^2z^2$	m. $6a^{11}$	n. $2x^5y^3$	
o. $10m^9$	p. $\frac{5p^{10}q^5}{6}$	q. $\frac{4x^2yz^2}{3}$	r. $\frac{3b}{5}$				
7. a. $\frac{6}{a^2}$	b. $\frac{10b^2}{3}$	c. $\frac{5}{w}$	d. $\frac{9k}{5}$	e. $\frac{3g}{5a}$	f. $\frac{16h^2}{9d^2}$	g. $\frac{5t}{n}$	h. $\frac{27h^2}{2g^2}$
i. $\frac{8y}{w}$	j. $6fz$						
8. a. $14x^2$	b. $8p^6$	c. $10s$	d. $10x$	e. $4s^2t$	f. 7		

9. The order of operations needs to be applied when dealing with algebraic expressions. Also, terms that contain more than one pronumeral should be dealt with as a single term. The first expressions are all equivalent to $\frac{x}{yz}$ and the final two terms are equivalent to $\frac{xz}{y}$.

10. The pronumeral must be a common factor of every term of the numerator and every term of the denominator before it can be cancelled.

11. -5

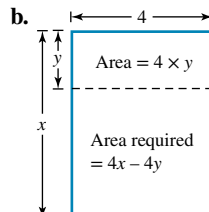
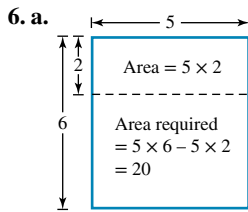
12. $\frac{7}{6}$

13. **a.** $V_{\text{small container}} = ab^2$, $V_{\text{large container}} = 4ab^2$ **b.** 4 times

14. They use the same principles. Multiplication of variables uses the Commutative Law, where order is not important. Division of variables is resolved the same way as numbers, where expressions are converted to fractions and then simplified and resolved.

Exercise 7.9 Expanding brackets

1. a. $3d + 12$	b. $2a + 10$	c. $4x + 8$	d. $5r + 35$	e. $6g + 36$	f. $2t - 6$
g. $7d + 56$	h. $18x - 54$	i. $48 + 12c$	j. $42 + 21x$	k. $90g + 135$	l. $1.5t + 9$
m. $11t - 22$	n. $6t - 18$	o. $t^2 + 3t$	p. $x^2 + 4x$	q. $g^2 + 7g$	r. $2g^2 + 10g$
s. $3fg + 9f$	t. $6mn - 12m^2$				
2. a. $9x - 6$	b. $3x^2 - 18xy$	c. $15xy - 45y^2$	d. $100y - 250$	e. $-3c - 9$	f. $-15x - 20$
g. $-5x^2 - 30x$	h. $-12y - 2y^2$	i. $-6t + 18$	j. $-20f + 8f^2$	k. $27xy - 18x$	l. $-6bh + 18h^2$
m. $20ab + 12ac$	n. $-6ag + 21a^2$	o. $15ab + 30ac$	p. $-18w^2 + 10wz$	q. $48m^2 + 120m$	r. $6k^2 - 15k$
3. a. $35x + 49$	b. $3c - 4$	c. $22c - 2c^2$	d. $6v + 30$	e. $5d^2 - 12d$	f. $11y + 12$
g. $26r + r^2$	h. $9g - 37$	i. $11f - 12g - 7$	j. $9x$	k. $-5k - 10$	l. $18x + 6rx$
m. $8r - 13$	n. $18gh - 24g$	o. $11t + 1$	p. $24 + 11r - 11r^2$		
4. a. $5x + 8$	b. $9x + 23$	c. $6y + 26$	d. $d + 22$	e. $14h$	f. $21m - 4$
g. $28f - 1$	h. $4a - 3a^2 - 35$	i. $6 - t^2 + 2t$	j. $7m$		
5. a. $10hk + 21h + 20k$	b. $15n - 6ny$	c. $8gm + 24g - 18$	d. $18ab + 67b$		
e. $5a^2 - 35a - 35$	f. $11cf + 3c$	g. $28x - 5xy - 29$	h. $37vw + 55v - 24$		
i. $6xy - 45x$	j. $77mn - 16m + 12n$				



7. a. $x^2 + 3x + 2$ b. $a^2 + 7a + 12$ c. $c^2 - c - 6$ d. $y^2 - 16$ e. $u^2 - 5u + 6$ f. $k^2 - 7k + 10$

8. a. $\$(50 - d)$

b. i. Sale price (P) = $3(50 - d)$

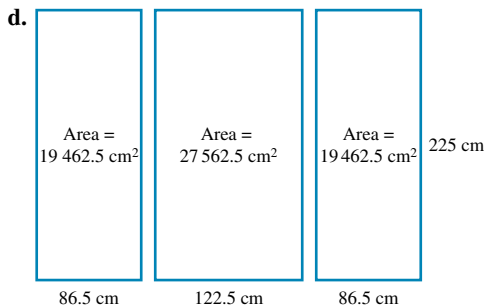
ii. $P = 150 - 3d$

c. Amount of change (C) = $200 - (150 - 3d) = 50 + 3d$

9. a. $A_{\text{left}} = fm - 36f$; $A_{\text{centre}} = fm$; $A_{\text{right}} = fm - 36f$

b. $A = 3fm - 72f$

c. $A = 3m^2 + 235.5m - 7380$



10. Multiplication of variables follows the Commutative Law where order doesn't matter; therefore brackets are not required.

Exercise 7.10 Factorising

1. a. 2	b. 3	c. 6	d. 13	e. 7	f. 2
g. 3	h. 4				
2. a. $2g$	b. $3m$	c. 11	d. $2m$	e. $2a$	f. $12g$
g. $2g$	h. $11l$	i. $4mn$	j. $4c$	k. $4c$	l. x

3. a. $3(x + 2)$ b. $2(y + 2)$ c. $5(g + 2)$ d. $4(2x + 3)$ e. $3(2f + 3)$ f. $4(3c + 5)$
 g. $2(d + 4)$ h. $2(x - 2)$ i. $6(2g - 3)$ j. $11(h + 11)$ k. $4(s - 4)$ l. $4(2x - 5)$
 m. $12(g - 2)$ n. $2(7 - 2b)$ o. $16(a + 4)$ p. $12(4 - q)$ q. $8(2 + f)$ r. $12(1 - d)$
 4. a. $3(gh + 4)$ b. $2y(x + 3)$ c. $4p(3q + 1)$ d. $7g(2 - h)$ e. $2k(8j - 1)$ f. $2g(6e + 1)$
 g. $4(3k + 4)$ h. $m(7n + 6)$ i. $7b(2a + 1)$ j. $5a(1 - 3bc)$ k. $2r(4 + 7t)$ l. $12ab(2m + 1)$
 m. $2b(2 - 3a)$ n. $4g(3f - 4h)$ o. $b(a - 2c)$ p. $7x(2 - 3y)$ q. $k(11j + 3)$ r. $3p(1 + 9q)$
 s. $c(12a - 4 + 3d)$ t. $4(g + 2h - 4)$ u. $14s(2 + t)$ v. $3v(5u + 9w)$
 5. $2ab$
 6. $12a^3b^2c$
 7. $\frac{3}{5}$
 8. $\frac{3(x + 1)}{2(x - 1)}$
 9. $\frac{28}{3}$ or $9\frac{1}{3}$
 10. $7y$
 11. a. $(3x - 6)(2x + 8)(ax - 5a)$ b. $\frac{4}{3a(x + 4)(x - 5)}$
 12. a. $6(x - 3)(x + 2)$
 b. $x > 3$. If x was smaller the area would be negative, which is not possible.
 13. a. $\frac{3xy^3}{2z}$ b. $\frac{x(y - 1)^2}{18z(z - 1)}$ c. $\frac{xz^3}{15y^3}$ d. $\frac{4xz^3(x + 1)^2}{3y^2(y - 1)^3}$
 14. Discuss with your teacher.

Investigation | Rich task

- Let l represent the number of long words and s represent the number of sentences. The Rix index = $\frac{l}{s}$.
- Six sentences and 19 long words
- 3.17
- Grade 8
- to 7. Check with your teacher.

Exercise 7.11 Review questions

1. a. $x + y$ b. $y - x$ or $x - y$ c. $3x - 5y$ d. $5x$ e. $2xy$
 f. $6x + 7y$ g. y^2 h. $2x - 7$ i. $3(x + y)$
 2. a. $15x$ b. $9y$ c. $15k + 9m$
 3. a. $m + 5$ b. $m - p$ c. $5m$
 4. a. 4 b. 12 c. 30 d. 1 e. 10 f. 4
 g. 8 h. 4 i. 8 j. 13 k. 22 l. 10
 m. 14 n. 3 o. $\frac{1}{3}$ p. -36 q. $\frac{5}{3}\left(1\frac{2}{3}\right)$ r. $\frac{2}{3}$
 5. \$13.90
 6. 5 cm
 7. a. 16 b. 4 c. 40 d. 16 e. 35 f. 15
 g. 24 h. 180 i. 18 j. 500 k. 96 l. -72
 8. a. -3 b. -3 c. -10 d. -2 e. -20 f. 3
 g. 22 h. 2 i. 12 j. 10 k. 28 l. -20
 9. a. False b. False c. True d. True e. True f. False
 g. False h. True i. False j. True k. False l. True
 10. a. $7d$ b. $-2c$ c. $3d + a$ d. $2g$ e. $2x + 11$ f. $g - 1$
 g. $9xy$ h. $15t^2 + 2t$
 11. a. $21g$ b. $18y$ c. $42d$ d. $-24z$
 12. a. $\frac{a}{4}$ b. $\frac{1}{4}$ c. $-3r$ d. $\frac{h}{2}$ e. $\frac{4}{5sv}$ f. $3x$
 g. $-\frac{6}{7}$ h. $\frac{e}{6j}$

13. a. $2x + 6$ b. $10x - 5$ c. $-2f - 14$ d. $3bm - 3m^2$ e. $3y^2 - 21y$ f. $9bc - 18b$
 14. a. $12v$ b. $16t - 35$ c. $17p + 3$ d. $7x + 15$ e. $5g^2 - 33g$ f. $42 - 3t$
 15. a. $3(g + 4)$ b. $y(x + 5)$ c. $5(n - 4)$ d. $4n(3m + p)$ e. $6g(2 - h)$ f. $12y(x - 3z)$

16. Check with your teacher.

17. a. $3(2 + 1) = 3 \times 2 + 3 \times 1$ or $3(2 - 1) = 3 \times 2 - 3 \times 1$
 b. $-10 \times 8 + -10 \times -6 = -10(8 + -6)$ or $-10 \times 8 - -10 \times -6 = -10(8 - -6)$
 c. $8(6 + 5) = 8 \times 6 + 8 \times 5$ or $8(6 - 5) = 8 \times 6 - 8 \times 5$
 d. $-4 \times 7 + 1 \times 7 = (-4 + 1)7$ or $-4 \times 7 - 1 \times 7 = (-4 - 1)7$
 e. $2(x + y) = 2 \times x + 2 \times y$ or $2(x - y) = 2 \times x - 2 \times y$
 f. $3 \times a(a + b) = 3 \times a \times a + 3 \times a \times b$ or $3 \times a(a - b) = 3 \times a \times a - 3 \times a \times b$

18. a. $(120 - a)$ cm b. $(b - 12a - 96)$ cm c. $(b - 13a + 24)$ cm

19. a.  b. $x(x + 4)$ cm² c. $x^2 + 4x$ d. 21 cm²

- e. $\text{Volume} = x^3 + 4x^2 \text{ cm}^3$ f. 63 cm²

20. a. P (points), G (goals) and B (behinds) b. $6G + B$

- c. 76 d. 72

- e. (G, B) : $(7, 45)$, $(8, 39)$, $(9, 33)$, $(10, 17)$, $(11, 21)$, $(12, 15)$, $(13, 9)$, $(14, 3)$

21. \$34

22. $(3x - 5y)$ L

23. $\frac{7}{12}$

24. The sum is twice the middle number.

25. Yes

26. The final digit of 2^{81} is 2.

TOPIC 8

Indices

8.1 Overview

Numerous **videos** and **interactivities** are embedded just where you need them, at the point of learning, in your learnON title at www.jacplus.com.au. They will help you to learn the concepts covered in this topic.

8.1.1 Why learn this?

Indices (the plural of index) can be thought of as the lazy mathematician's way of abbreviating operations such as multiplication and division. You do not have to use this technique if you have the time to do otherwise. For example, the lazy mathematician might write x^6 , whereas you could write $x \times x \times x \times x \times x \times x$.

Why use indices at all? Because doing so will save you time and effort in more difficult calculations later. Will you use indices later in life? That depends on what work you decide to do. However, if you are contemplating any work associated with computers, cars, engines, science, design or apprenticeships, then the answer is almost certainly yes.



LEARNING SEQUENCE

- 8.1** Overview
- 8.2** Review of index form
- 8.3** First Index Law (multiplying numbers in index form with the same base)
- 8.4** Second Index Law (dividing numbers in index form with the same base)
- 8.5** Third Index Law (the power of zero)
- 8.6** Fourth Index Law (raising a power to another power)
- 8.7** Review

LEARNING OUTCOMES

A student:

- communicates and connects mathematical ideas using appropriate terminology, diagrams and symbols MA4-1WM
- applies appropriate mathematical techniques to solve problems MA4-2WM
- recognises and explains mathematical relationships using reasoning MA4-3WM
- operates with positive-integer and zero indices of numerical bases MA4-9NA

CONTENT DESCRIPTIONS

Students:

Investigate index notation and represent whole numbers as products of powers of prime numbers (ACMNA149)

Use index notation with numbers to establish the index laws with positive-integer indices and the zero index (ACMNA182)

Source: NSW Syllabus for the Australian curriculum

Note: Your teacher may now set you a pre-test to determine how familiar you are with the content in this topic.

8.2 Review of index form

8.2.1 Index notation

- If a number or a variable is multiplied by itself several times, it can be written using shortcut notation referred to as **index form**.
- When written in index form, the number or the variable that is being multiplied is written once only. To indicate how many times it is being multiplied by itself, a small number is written above and to the right of it.
- For example, if the number 2 is multiplied by itself 4 times, it can be written as $2 \times 2 \times 2 \times 2$ (factor form) or as 2^4 (index form).

ACTIVITY: INVESTIGATING POWERS

1. As a class, create a table on the board to show the numbers 1 to 10 raised to the power of 1 to 6.
2. Compare the results of raising 2 and 4 to different powers.
3. How can the table be used to show the value of different roots of numbers?
4. Investigate how the table would look if the numbers that were raised to different powers were unit fractions.
5. Investigate how the table would look if the numbers that were raised to different powers were decimals, such as 0.1, 0.2 and 0.3.
6. What is the difference between raising a number that is greater than 1 to a power and raising a number that is less than 1 to a power?

- When written in index form, the number or variable that is being multiplied is called the **base**, while the number showing how many times it is being multiplied is called the **power**, **index** or **exponent**. For example, in the number 2^4 , 2 is the base and 4 is the power or index.
- When the base is multiplied by itself the number of times indicated by the power, the answer is called a *basic numeral*. For example,

$$2^4 = 2 \times 2 \times 2 \times 2 = 16$$

Index form	Factor form	Basic numeral

DISCUSSION

What strategy will you use to memorise the squares, cubes and other simple powers of 2, 3 and 5?

WORKED EXAMPLE 1

State the base and power for the number 5^{14} .

THINK

- 1 Write the number.
- 2 The base is the number below the power.
- 3 The power or index is the small number just above and to the right of the base.

WRITE

$$5^{14}$$

The base is 5.

The power is 14.

WORKED EXAMPLE 2

Write 12^4 in factor form.

THINK

- 1 Write the number.
- 2 The base is 12, so this is what will be multiplied.
- 3 The power is 4, so this is how many times 12 should be written and multiplied.

WRITE

$$\begin{aligned}12^4 \\ = 12 \times 12 \times 12 \times 12\end{aligned}$$

WORKED EXAMPLE 3

Write $2 \times 5 \times 2 \times 2 \times 5 \times 2 \times 5$ in index form.

THINK

- 1 Write the problem.
- 2 Write the factors in numerical order.
- 3 The number 2 has been written 4 times and multiplied.
The number 5 has been written 3 times and multiplied.

WRITE

$$\begin{aligned}2 \times 5 \times 2 \times 2 \times 5 \times 2 \times 5 \\ = 2 \times 2 \times 2 \times 2 \times 5 \times 5 \times 5 \\ = 2^4 \times 5^3\end{aligned}$$

WORKED EXAMPLE 4

Write $7 \times 5^3 \times 6^5$ in factor form.

THINK

- 1 Write the problem.
- 2 List the factors: 7 is written once, 5 is written 3 times and multiplied, and 6 is written 5 times and multiplied.

WRITE

$$\begin{aligned}7 \times 5^3 \times 6^5 \\ = 7 \times 5 \times 5 \times 5 \times 6 \times 6 \times 6 \times 6 \times 6\end{aligned}$$

learnON RESOURCES — ONLINE ONLY

Interactivity: Review of index form (int-3708)

Digital doc: SkillsHEET Factor trees (doc-6834)

Digital doc: SkillsHEET Squaring numbers (doc-6835)

Exercise 8.2 Review of index form

Individual pathways

PRACTISE

Questions:
1–8, 13

CONSOLIDATE

Questions:
1–9, 11–13

MASTER

Questions:
1f–j, 2, 3e–h, 4e–h, 5, 6e–h, 7, 8a,
c, f, 9–12, 14, 15

   Individual pathway interactivity: int-4401

learnON ONLINE ONLY

To answer questions online and to receive **immediate feedback** and **fully worked solutions** for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Understanding and fluency

- WE1** State the base and power for each of the following.
 - 8^4
 - 7^{10}
 - 20^{11}
 - 19^0
 - 78^{12}
 - 3^{100}
 - m^5
 - c^{24}
 - n^{36}
 - d^{42}
- Write the following in index form.
 - $2 \times 2 \times 2 \times 2 \times 2 \times 2$
 - $4 \times 4 \times 4 \times 4$
 - $x \times x \times x \times x \times x$
 - $9 \times 9 \times 9$
 - $11 \times l \times l \times l \times l \times l \times l \times l$
 - $44 \times m \times m \times m \times m \times m$
- WE2** Write the following in factor form.
 - 4^2
 - 5^4
 - 7^5
 - 6^3
 - 3^6
 - n^7
 - a^4
 - k^{10}
- Write each of the following as a basic numeral.
 - 3^5
 - 4^4
 - 2^8
 - 11^3
 - 7^4
 - 6^3
 - 1^{10}
 - 5^4
- MC** a. What does 6^3 mean?
 - 6×3
 - $6 \times 6 \times 6$
 - $3 \times 3 \times 3 \times 3 \times 3 \times 3$
 - $6 + 6 + 6$
 - 3×6
 b. What does 3^5 mean?
 - 3×5
 - 5×5
 - $3 + 3 + 3 + 3 + 3$
 - $3 \times 3 \times 3 \times 3 \times 3$
 - 5×3
- WE3** Write each of the following in index form.
 - $6 \times 2 \times 2 \times 4 \times 4 \times 4 \times 4$
 - $7 \times 7 \times 7 \times 7 \times 3 \times 3 \times 3 \times 3$
 - $19 \times 19 \times 19 \times 19 \times 19 \times 2 \times 2 \times 2$
 - $13 \times 13 \times 4 \times 4 \times 4 \times 4$
 - $66 \times p \times p \times m \times m \times m \times m \times m \times s \times s$
 - $21 \times n \times n \times 3 \times i \times i \times i \times 6 \times r \times r \times r$
 - $16 \times k \times e \times e \times e \times 12 \times p \times p$
 - $11 \times j \times j \times j \times j \times j \times 9 \times p \times p \times l$
- WE4** Write each of the following in factor form.
 - $15f^3j^4$
 - $7k^6s^2$
 - $4b^3c^5$
 - $19a^4mn^3$
 - $8l^4r^2t^2$
- Write each of the following numbers as a product of its prime factors, using indices.
 - 64
 - 40
 - 36
 - 400
 - 225
 - 2000
- Some basic numerals are written as the product of their prime factors. Identify each of these basic numerals.
 - $2^3 \times 3 \times 5$
 - $2^2 \times 5^2$
 - $2^3 \times 3^3$
 - $2^2 \times 7 \times 11$
 - $3^2 \times 5^2 \times 7$
 - $2^6 \times 5^4 \times 19$
- a. Write each of the following numbers in index form with base 10.
 - 10
 - 100
 - 1000
 - 1 000 000
 b. Use your knowledge of place value to rewrite each of the following basic numerals in expanded form using powers of 10. The first number has been done for you.

	Basic numeral	Expanded form
	230	$2 \times 10^2 + 3 \times 10^1$
i.	500	
ii.	470	
iii.	2360	
iv.	1980	
v.	5430	

- Write each of the following as a basic numeral.
 - $7 \times 10^4 + 5 \times 10^3$
 - $3 \times 10^4 + 6 \times 10^2$
 - $5 \times 10^6 + 2 \times 10^5 + 4 \times 10^2 + 8 \times 10^1$

Communicating, reasoning, problem solving

11. Explain what a^3b^4 means. Write a^3b^4 as a basic numeral in factor form as part of your explanation.
12. a , b and c are prime numbers.
 - a. Write $8 \times a \times a \times b \times b \times b \times c \times c \times c \times c$ as a product of prime factors in index form.
 - b. If $a = 2$, $b = 3$ and $c = 7$, calculate the value of the basic numeral represented by your answer in part a.
13.
 - a. Rewrite the numbers 140 and 680 in expanded form using powers of 10.
 - b. Add the numbers in expanded form.
 - c. Convert your answer for part b into a basic numeral.
 - d. Check your answer for part c by adding 140 and 680.
 - e. Try the question again, this time with two numbers of your own. Choose numbers between 1000 and 10 000.
14.
 - a. Rewrite the numbers 10 and 14 in expanded form with powers of 2 using, as appropriate, 2^3 , 2^2 and 2^1 .
 - b. Add the numbers in expanded form.
 - c. Convert your answer for part b into a basic numeral.
 - d. Check your answer for part c by adding 10 and 14.
 - e. Try the question again, this time with two numbers of your own. Choose numbers between 16 and 63.
15. How will you remember the meaning of the base and that of the index?

8.3 First Index Law (multiplying numbers in index form with the same base)

8.3.1 Multiplying numbers in index form with the same base

- The numbers in index form with the same base can be multiplied together by being written in factor form first. For example:

$$5^3 \times 5^2 = (5 \times 5 \times 5) \times (5 \times 5) = 5^5$$

- The simpler and faster way to multiply numbers or variables in index form with the same base is to use the First Index Law. The First Index Law states: $a^m \times a^n = a^{m+n}$. This means that when numbers in index form with the same base are multiplied by each other, the powers (indices) are added together. For example:

$$5^3 \times 5^2 = 5^{3+2} = 5^5 \text{ (as above)}$$

- If the variables in index form that are being multiplied have coefficients, the coefficients are multiplied together and the variables in index form are multiplied, and simplified using the First Index Law. For example:

$$2a^4 \times 3a^5 = (2 \times 3) \times (a^4 \times a^5) = 6a^9.$$

$\uparrow$ $\uparrow$
Coefficients Variables multiplied, which
multiplied means indices are added

WORKED EXAMPLE 5

Simplify $2^3 \times 2^6$ after first writing in factor form, giving the answer in index form.

THINK

- 1 Write the problem.
- 2 Write in factor form.
- 3 Simplify by writing in index form.

WRITE

$$\begin{aligned} &2^3 \times 2^6 \\ &= (2 \times 2 \times 2) \times (2 \times 2 \times 2 \times 2 \times 2 \times 2) \\ &= 2^9 \end{aligned}$$

WORKED EXAMPLE 6

Simplify $7^4 \times 7 \times 7^3$, giving your answer in index form.

THINK

- 1 Write the problem.
- 2 Show that the 7 in the middle has an index of 1.
- 3 Check to see if the bases are the same. They are all 7.
- 4 Simplify by using the First Index Law (add indices).

WRITE

$$\begin{aligned}7^4 \times 7 \times 7^3 \\&= 7^4 \times 7^1 \times 7^3 \\&= 7^{4+1+3} \\&= 7^8\end{aligned}$$

WORKED EXAMPLE 7

Simplify $5e^{10} \times 2e^3$.

THINK

- 1 Write the problem.
- 2 The order is not important when multiplying, so place the numbers first.
- 3 Multiply the numbers.
- 4 Check to see if the bases are the same. They are both e .
- 5 Simplify by using the First Index Law (add indices).

WRITE

$$\begin{aligned}5e^{10} \times 2e^3 \\&= 5 \times 2 \times e^{10} \times e^3 \\&= 10 \times e^{10} \times e^3 \\&= 10e^{10+3} \\&= 10e^{13}\end{aligned}$$

8.3.2 Multiplying expressions containing numbers in index form with different bases

- When more than one variable is involved in the multiplication question, the First Index Law is applied to each variable separately.

WORKED EXAMPLE 8

Simplify $7m^3 \times 3n^5 \times 2m^8 \times n^4$.

THINK

- 1 Write the problem.
- 2 The order is not important when multiplying, so place numbers first and group the same variables together.
- 3 Simplify by multiplying the numbers and using the First Index Law for bases that are the same (add indices).

WRITE

$$\begin{aligned}7m^3 \times 3n^5 \times 2m^8 \times n^4 \\&= 7 \times 3 \times 2 \times m^3 \times m^8 \times n^5 \times n^4 \\&= 42 \times m^{3+8} \times n^{5+4} \\&= 42m^{11}n^9\end{aligned}$$



RESOURCES — ONLINE ONLY

Interactivity: First Index Law (int-3709)

Interactivity: Multiplying expressions containing numbers in index form with different bases (int-3710)

Digital doc: SpreadSHEET Multiplying with indices (doc-2160)

Exercise 8.3 First Index Law (multiplying numbers in index form with the same base)

Individual pathways

PRACTISE

Questions:
1, 2, 3a–f, 4, 5, 6a–f, 11

CONSOLIDATE

Questions:
1–7, 10, 11, 13

MASTER

Questions:
2g–l, 3g–l, 4, 5, 6f–j, 7–13

Individual pathway interactivity: int-4402

learnON ONLINE ONLY

To answer questions online and to receive **immediate feedback** and **fully worked solutions** for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Understanding and fluency

1. **WE5** Simplify the following after first writing in factor form.

a. $2^2 \times 2^4 = (2 \times 2) \times (\square \times \square \times \square \times \square)$
 $= 2^{\square}$

b. $5^3 \times 5^5 = (5 \times 5 \times 5) \times (\square \times \square \times \square \times \square \times \square)$
 $= 5^{\square}$

c. $f^6 \times f \times f^2 = (\square \times \square \times \square \times \square \times \square \times \square) \times \square \times (\square \times \square)$
 $= f^{\square}$

2. Simplify each of the following.

a. $3^7 \times 3^2$

b. $61^4 \times 6^3$

c. $10^6 \times 10^4$

d. $11^3 \times 11^3$

e. $7^8 \times 7$

f. $2^{11} \times 2^3$

g. $5^2 \times 5^2$

h. $8^9 \times 8^2$

i. $13^7 \times 13^8$

j. $q^{23} \times q^{24}$

k. $x^7 \times x^7$

l. $e \times e^3$

3. **WE6** Simplify each of the following, giving your answer in index form.

a. $3^4 \times 3^6 \times 3^2$

b. $2^{10} \times 2^3 \times 2^5$

c. $5^4 \times 5^4 \times 5^9$

d. $6^8 \times 6 \times 6^2$

e. $10 \times 10 \times 10^4$

f. $17^2 \times 17^4 \times 17^6$

g. $p^7 \times p^8 \times p^7$

h. $e^{11} \times e^{10} \times e^2$

i. $g^{15} \times g \times g^{12}$

j. $e^{20} \times e^{12} \times e^6$

k. $3 \times b^2 \times b^{10} \times b$

l. $5 \times d^4 \times d^5 \times d^7$

4. a. **MC** What does $6 \times e^3 \times b^2 \times b^4 \times e$ equal?

A. $6b^6e^4$

B. $6b^6e^3$

C. $6b^9e$

D. $6b^{10}e$

E. $6b^8e^3$

b. What does $3 \times f^2 \times f^{10} \times 2 \times e^3 \times e^8$ equal?

A. $32e^{11}f^{12}$

B. $6e^{11}f^{12}$

C. $6e^{23}f$

D. $6e^{24}f^{20}$

E. $3e^{24}f^{12}$

5. **WE7** Simplify each of the following.

a. $4p^7 \times 5p^4$

b. $2x^2 \times 3x^6$

c. $8y^6 \times 7y^4$

d. $3p \times 7p^7$

e. $12t^3 \times t^2 \times 7t$

f. $6q^2 \times q^5 \times 5q^8$

6. **WE8** Simplify each of the following.

a. $2a^2 \times 3a^4 \times e^3 \times e^4$

b. $4p^3 \times 2h^7 \times h^5 \times p^3$

c. $2m^3 \times 5m^2 \times 8m^4$

d. $2gh \times 3g^2h^5$

e. $5p^4q^2 \times 6p^2q^7$

f. $8u^3w \times 3uw^2 \times 2u^5w^4$

g. $9dy^8 \times d^3y^5 \times 3d^7y^4$

h. $7b^3c^2 \times 2b^6c^4 \times 3b^5c^3$

i. $4r^2s^2 \times 3r^6s^{12} \times 2r^8s^4$

j. $10h^{10}v^2 \times 2h^8v^6 \times 3h^{20}v^{12}$

7. Simplify each of the following.

a. $3^x \times 3^4$

b. $3^y \times 3^{y+2}$

c. $3^{2y+1} \times 3^{4y-6}$

d. $3^{\frac{1}{2}} \times 3^{\frac{2}{3}} \times 3^{\frac{3}{4}}$

8. a. Express the following basic numerals in index form: 9, 27 and 81.
 b. Use your answers to part a to help you simplify each of the following expressions. (Give each answer in index form.)
 i. $3^4 \times 81 \times 9$
 ii. $27 \times 3^n \times 3^{n-1}$

Communicating, reasoning, problem solving

9. Explain why $2^x \times 3^y$ does not equal $6^{(x+y)}$.
10. Step 1: The prime number 5 is multiplied by itself n times.
 Step 2: The prime number 5 is multiplied by itself m times.
 Step 3: The answers from steps 1 and step 2 are multiplied together.
 Explain how you arrive at your final answer. What is your answer?
11. One dollar is placed on a square of a chess board, two dollars on the next square, four dollars on the next square, eight dollars on the next square and so on.
 a. Write the number of dollars on the 10th square in index form.
 b. Write the number of dollars on the r th square in index form.
 c. How much money is on the 6th and 7th squares in total?
 d. How much money is on the 14th and 15th squares in total? Write your answer in index form.
 e. Simplify your answer to part d by first taking out a common factor.
 f. Compare the numerical value of the number of dollars on the 64th square of the chess board to the numerical value of the distance in kilometres to the nearest star, Proxima Centauri.
12. a. If $x^2 = x \times x$, what does $(x^3)^2$ equal?
 b. If the sides of a cube are 2^4 cm long, what is the volume of the cube in index form?
 (Hint: The volume of a cube of side length s cm is s^3 cm³.)
 c. What is the side length of a cube of volume 5^6 mm³?
 d. What is the side length of a cube of volume $(a^n)^{3p}$ mm³?
13. If I square a certain number, then multiply the result by three times the cube of the certain number before adding one, the result is 97. What is the certain number?

8.4 Second Index Law (dividing numbers in index form with the same base)

8.4.1 Dividing numbers in index form with the same base

- The numbers in index form with the same base can be divided by first being written in factor form. For example:

$$\begin{aligned}
 2^6 \div 2^4 &= \frac{2^6}{2^4} \\
 &= \frac{2 \times 2 \times 2 \times 2 \times 2 \times 2}{2 \times 2 \times 2 \times 2} \\
 &= \frac{2 \times 2 \times 2 \times 2 \times 2 \times 2}{2 \times 2 \times 2 \times 2} \\
 &= 2 \times 2 \\
 &= 2^2
 \end{aligned}$$

- The simpler and faster way to divide the numbers in index form is to apply the Second Index Law. The Second Index Law states: $a^m \div a^n = a^{m-n}$. This means that when the numbers or variables in index form with the same base are divided, the powers are subtracted. For example:

$$\begin{aligned}
 2^6 \div 2^4 &= 2^{6-4} \\
 &= 2^2 \text{ (as above)}
 \end{aligned}$$

WORKED EXAMPLE 9

Simplify $\frac{5^{10}}{5^3}$ after first writing in factor form, leaving your answer in index form.

THINK

- 1 Write the problem.
- 2 Write in factor form.
- 3 Cancel 5s.
- 4 Write in index form.

WRITE

$$\begin{aligned}\frac{5^{10}}{5^3} &= \frac{5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5}{5 \times 5 \times 5} \\ &= 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \\ &= 5^7\end{aligned}$$

WORKED EXAMPLE 10

Simplify $d^{12} \div d^4$ using an index law.

THINK

- 1 Write the problem and express it as a fraction.
- 2 Check to see if the bases are the same. They are both d .
- 3 Simplify by using the Second Index Law (subtract indices).

WRITE

$$\begin{aligned}d^{12} \div d^4 &= \frac{d^{12}}{d^4} \\ &= d^{12-4} \\ &= d^8\end{aligned}$$

8.4.2 Dividing with coefficients

- When the coefficients are present, we divide them as we would divide any other numbers and then apply the Second Index Law to the variables.
- In examples where the coefficients do not divide evenly, we simplify the fraction that is formed by them.
- When there is more than one variable involved in the division question, the Second Index Law is applied to each variable separately.

WORKED EXAMPLE 11

Simplify $36d^7 \div 12d^3$ giving your answer in index form.

THINK

- 1 Write the problem and express it as a fraction.
- 2 Divide the numbers (or coefficients) and apply the Second Index Law to the variables.
- 3 Simplify by using the Second Index Law (subtract indices).

WRITE

$$\begin{aligned}36d^7 \div 12d^3 &= \frac{36d^7}{12d^3} \\ &= \frac{3d^7}{d^3} \\ &= 3d^{7-3} \\ &= 3d^4\end{aligned}$$

WORKED EXAMPLE 12

Simplify $\frac{7t^3 \times 4t^8}{12t^4}$.

THINK

- 1 Write the problem.
- 2 Multiply the numbers in the numerator and apply the First Index Law (add indices) in the numerator.
- 3 Simplify the fraction formed and apply the Second Index Law for the variable (subtract indices).

WRITE

$$\begin{aligned} & \frac{7t^3 \times 4t^8}{12t^4} \\ &= \frac{28t^{11}}{12t^4} \\ &= \frac{7t^7}{3} \end{aligned}$$

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Interactivity: Second Index Law (int-3711)

Interactivity: Dividing with coefficients (int-3712)

Digital doc: SkillsHEET Equivalent fractions (doc-6836)

Digital doc: SkillsHEET Simplifying algebraic expressions (doc-6837)

Digital doc: SkillsHEET Simplifying algebraic fractions (doc-6838)

Digital doc: SpreadSHEET Dividing with indices (doc-2161)

Digital doc: WorkSHEET Index laws I (doc-6851)

Exercise 8.4 Second Index Law (dividing numbers in index form with the same base)

Individual pathways

PRACTISE

Questions:
1–6, 7a, b, 14

CONSOLIDATE

Questions:
1–8, 11, 12, 14

MASTER

Questions:
1, 2g–l, 3g–l, 4, 5, 6e–i, 7–15

Individual pathway interactivity: int-4403

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Understanding and fluency

1. **WE9** Simplify each of the following after first writing in factor form, leaving your answer in index form.
 - a. $\frac{2^5}{2^2}$
 - b. $\frac{7^7}{7^3}$
 - c. $\frac{10^8}{10^5}$
2. **WE10** Simplify each of the following using the index law, leaving your answer in index form.
 - a. $3^3 \div 3^2$
 - b. $11^9 \div 11^2$
 - c. $5^8 \div 5^4$
 - d. $12^6 \div 12$
 - e. $3^{45} \div 3^{42}$
 - f. $13^{75} \div 13^{74}$
 - g. $6^{23} \div 6^{19}$
 - h. $\frac{10^{13}}{10^9}$
 - i. $\frac{15^{456}}{15^{423}}$
 - j. $\frac{h^{78}}{h}$
 - k. $\frac{b^{77}}{b^7}$
 - l. $\frac{f^{1000}}{f^{100}}$
3. **WE11** Simplify each of the following, giving your answer in index form.
 - a. $3x^5 \div x^3$
 - b. $6y^7 \div y^5$
 - c. $8w^{12} \div w^5$
 - d. $12q^{34} \div 4q^{30}$

- e. $16f^{12} \div 2f^3$ f. $100h^{100} \div 10h^{10}$ g. $80j^{15} \div 20j^5$ h. $\frac{45p^{14}}{9p^4}$
- i. $\frac{48g^8}{6g^5}$ j. $\frac{12b^7}{8b}$ k. $\frac{81m^6}{18m^2}$ l. $\frac{100n^{95}}{40n^5}$
4. **MC** a. What does $21r^{20} \div 14r^{10}$ equal?
 A. $7r^{10}$ B. $\frac{3r^2}{2}$ C. $7r^2$ D. $\frac{3r^{10}}{2}$ E. $\frac{2}{3}r^{10}$
- b. What does $\frac{2m^{33}}{16m^{11}}$ equal?
 A. $\frac{m^{22}}{8}$ B. $\frac{8}{m^{22}}$ C. $8m^{22}$ D. $\frac{m^3}{8}$
 E. None of the above
5. **WE12** Simplify each of the following.
 a. $\frac{15p^{12}}{5p^8}$ b. $\frac{18r^6}{3r^2}$ c. $\frac{45a^5}{5a^2}$ d. $\frac{60b^7}{20b}$ e. $\frac{100r^{10}}{5r^6}$ f. $\frac{9q^2}{q}$
6. **WE12** Simplify each of the following.
 a. $\frac{8p^6 \times 3p^4}{16p^5}$ b. $\frac{12b^5 \times 4b^2}{18b^2}$ c. $\frac{25m^{12} \times 4n^7}{15m^2 \times 8n}$
 d. $\frac{27x^9y^3}{12xy^2}$ e. $\frac{16h^7k^4}{12h^6k}$ f. $\frac{12j^8 \times 6f^5}{8j^3 \times 3f^2}$
 g. $\frac{8p^3 \times 7r^2 \times 2s}{6p \times 14r}$ h. $\frac{27a^9 \times 18b^5 \times 4c^2}{18a^4 \times 12b^2 \times 2c}$ i. $\frac{81f^{15} \times 25g^{12} \times 16h^{34}}{27f^9 \times 15g^{10} \times 12h^{30}}$
7. Simplify each of the following.
 a. $2^{10} \div 2^p$ b. $2^{7e} \div 2^{3e-4}$ c. $\frac{5^{4x} \times 5^{3y}}{5^{2y} \times 5^x}$ d. $\frac{3^{2-3m} \times 3^{7m}}{3^{5m} \times 3}$
8. Consider the fraction $\frac{8 \times 16 \times 4}{2 \times 32}$.
 a. Rewrite the fraction, expressing each basic numeral as power of 2.
 b. Simplify by giving your answer:
 i. in index form ii. as a basic numeral.
 c. Now check your answer by cancelling and evaluating the fraction in the ordinary way.
9. Consider the fraction $\frac{6 \times 27 \times 36}{12 \times 81}$.
 a. Rewrite the fraction, expressing each basic numeral as the product of its prime factors.
 b. Simplify, giving the answer:
 i. in index form ii. as a basic numeral.

Communicating, reasoning, problem solving

10. Explain why $\frac{12^x}{3^y}$ does not equal $4^{(x-y)}$.
11. Step 1: The prime number 3 is multiplied by itself p times.
 Step 2: The prime number 3 is multiplied by itself q times.
 Step 3: The answer from step 1 is divided by the answer from step 2.
 Explain how you arrive at your final answer. What is your answer?
12. By considering $a^p \div a^p$, show that any base raised to the power of zero equals 1.
13. By considering $m^6 \div m^8$, show that $m^{-2} = \frac{1}{m^2}$.
14. I cube a certain number, then multiply the result by six. I now divide the result by the certain number to the power of five. The result is 216. What is the certain number?
15. How will you remember that when numbers in index form are divided, powers are subtracted but coefficients are divided?

8.5 Third Index Law (the power of zero)

8.5.1 The power of zero

- Consider the following two different methods of simplifying $2^3 \div 2^3$.

Method 1	Method 2
$2^3 \div 2^3 = \frac{2 \times 2 \times 2}{2 \times 2 \times 2}$ $= \frac{8}{8}$ $= 1$	$2^3 \div 2^3 = \frac{2^3}{2^3}$ $= 2^{3-3} \text{ (using the Second Index Law)}$ $= 2^0$

- As the two results should be the same, 2^0 must equal 1.
- Any base that has an index power of 0 is equal to 1.
- This can also be seen by looking at patterns of indices; for example, the following table shows the powers of 4 from 4^5 to 4^0 .

4^5	4^4	4^3	4^2	4^1	4^0
1024	256	64	16	4	1

- You can see that you are dividing by 4 as you move through the columns from left to right.
- The Third Index Law states: $a^0 = 1$. This means that any base that is raised to the power of zero is equal to 1.
- If it is in brackets, any numeric or algebraic expression that is raised to the power of zero is equal to 1.
For example:

$$(2 \times 3)^0 = 1, (2abc^2)^0 = 1$$

WORKED EXAMPLE 13

Find the value of 15^0 .

THINK

- Write the problem.
- Any base with an index of zero is equal to one.

WRITE

$$15^0$$

$$= 1$$

WORKED EXAMPLE 14

Find the value of $(25 \times 36)^0$.

THINK

- Write the problem.
- Everything within the brackets has an index of zero, so the answer is 1.

WRITE

$$(25 \times 36)^0$$

$$= 1$$

WORKED EXAMPLE 15

Find the value of $19e^5a^0$.

THINK

- Write the problem.
- Only a has a power of zero, so replace it with a 1 and simplify.

WRITE

$$19e^5a^0$$

$$= 19e^5 \times 1$$

$$= 19e^5$$

WORKED EXAMPLE 16

Simplify $\frac{6m^3 \times 11m^{14}}{3m^{10} \times 2m^7}$.

THINK

- 1 Write the problem.
- 2 Multiply the numbers and apply the First Index Law in both the numerator and denominator.
- 3 Divide the numbers and simplify using the Second Index Law.
- 4 Simplify using the Third Index Law.

WRITE

$$\begin{aligned} & \frac{6m^3 \times 11m^{14}}{3m^{10} \times 2m^7} \\ &= \frac{66m^{17}}{6m^{17}} \\ &= 11m^{17-17} \\ &= 11m^0 \\ &= 11 \times 1 \\ &= 11 \end{aligned}$$

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Interactivity: Third Index Law (int-3713)

Exercise 8.5 Third Index Law (the power of zero)

Individual pathways

PRACTISE

Questions:
1–8, 12

CONSOLIDATE

Questions:
1–8, 10, 12

MASTER

Questions:
1–7, 8f–j, 9–13

Individual pathway interactivity: int-4404

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Understanding and fluency

1. **WE13** Find the value for each of the following.
 - a. 16^0
 - b. 44^0
 - c. f^0
 - d. h^0
2. **WE14** Find the value of each of the following.
 - a. $(23 \times 8)^0$
 - b. $7^0 \times 6^0$
 - c. $(35z^4)^0$
 - d. $(12w^7)^0$
3. Find the value of each of the following. Check your answers to parts **a** and **b** with a calculator.
 - a. 4×3^0
 - b. $9^0 + 11$
 - c. $c^0 - 10$
 - d. $3p^0 + 19$
4. **WE15** Find the value of each of the following.
 - a. $12m^3k^0$
 - b. $7c^0 + 14m^0$
 - c. $32g^0 + 40h^0$
 - d. $\frac{8k^0}{7j^0}$
 - e. $\frac{16t^0}{8y^0}$
 - f. $\frac{6b^2 \times 5c^0}{3s^0}$
 - g. $\frac{4d^0 \times 9p^2}{12q^0}$
 - h. $\frac{3p \times 4d^0}{2z^0 \times 6p}$
5. Find each of the following.
 - a. $e^{10} \div e^{10}$
 - b. $a^{12} \div a^{12}$
 - c. $(4b^3)^0 \div (4b^3)^0$
 - d. $84f^{11} \div 12f^{11}$
 - e. $30z^9 \div 10z^9$
 - f. $99t^{13} \div 33t^{13}$

6. Simplify each of the following.

a. $\frac{21p^4}{21p^4}$

b. $\frac{40f^{33}}{10f^{33}}$

c. $\frac{54p^6q^8}{27p^6q^8}$

d. $\frac{16p^{11}q^{10}}{8p^2q^{10}}$

e. $\frac{24a^9e^{10}}{16a^9e^6}$

f. $\frac{x^4y^2z^{11}}{x^4yz^{11}}$

g. $\frac{7i^7m^6r^4}{21i^7m^3r^4}$

h. $\frac{3c^5d^3l^9}{12c^2d^3l^9}$

7. **MC** a. You are told that there is an error in the statement $3p^7q^3r^5s^6 = 3p^7s^6$. To make the statement correct, what should the left-hand side be?

A. $(3p^7q^3r^5s^6)^0$

B. $(3p^7)^0q^3r^5s^6$

C. $3p^7(q^3r^5s^6)^0$

D. $3p^7(q^3r^5)^0s^6$

E. $3(p^7q^3r^5)^0s^6$

b. You are told that there is an error in the statement $\frac{8f^6g^7h^3}{6f^4g^2h} = \frac{8f^2}{g^2}$. To make the statement correct, what should the left-hand side be?

A. $\frac{8f^6(g^7h^3)^0}{(6)^0f^4g^2(h)^0}$

B. $\frac{8(f^6g^7h^3)^0}{(6f^4g^2h)^0}$

C. $\frac{8(f^6g^7)^0h^3}{(6f^4)^0g^2h}$

D. $\frac{8f^6g^7h^3}{(6f^4g^2h)^0}$

E. None of the above

c. What does $\frac{6k^7m^2n^8}{4k^7(m^6n)^0}$ equal?

A. $\frac{3}{2}$

B. $\frac{3n^8}{2}$

C. $\frac{3m^2}{2}$

D. $\frac{3m^2n^8}{2}$

E. None of the above

8. **WE16** Simplify each of the following.

a. $\frac{2a^3 \times 6a^2}{12a^5}$

b. $\frac{3c^6 \times 6c^3}{9c^9}$

c. $\frac{5b^7 \times 10b^5}{25b^{12}}$

d. $\frac{8f^3 \times 3f^7}{4f^5 \times 3f^5}$

e. $\frac{9k^{12} \times 4k^{10}}{18k^4 \times k^{18}}$

f. $\frac{2h^4 \times 5k^2}{20h^2 \times k^2}$

g. $\frac{p^3 \times q^4}{5p^3}$

h. $\frac{m^7 \times n^3}{5m^3 \times m^4}$

i. $\frac{8u^9 \times v^2}{2u^5 \times 4u^4}$

j. $\frac{9x^6 \times 2y^{12}}{3y^{10} \times 3y^2}$

Communicating, reasoning, problem solving

9. Explain why $x^0 = 1$.

10. Simplify $\frac{2^0x^2}{2^2x^0}$, explaining each step of your method.

11. I raise a certain number to the power of three, then multiply the answer by three to the power of zero. I then multiply the result by the certain number to the power of four and divide the answer by three times the certain number to the power of seven. If the final answer is three multiplied by the certain number squared, find the certain number. Show each line of your method.

12. A Mathematics class is asked to simplify $\frac{64x^6y^6z^3}{16x^2y^6z^3}$. Peter's answer is $4x^3$. Explain why this is incorrect, pointing out Peter's error. What is the correct answer? Can you identify another source of possible error involving indices?

13. Explain how the Second and Third Index Laws are connected.

8.6 Fourth Index Law (raising a power to another power)

8.6.1 Raising a power to another power

- The Fourth Index Law states that when raising a power to another power, the indices are multiplied; that is, $(a^m)^n = a^{m \times n}$. For example:

$$(5^3)^2 = 5^{3 \times 2} = 5^6$$

- Every number and variable inside the brackets should have its index multiplied by the power outside the brackets. That is:

$$(a \times b)^m = a^m \times b^m$$

$$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$$

(These are sometimes called the Fifth Index Law and the Sixth Index Law.)

- Any number or variable that does not appear to have an index really has an index of 1; that is, $2 = 2^1$, $a = a^1$.
- Every number or variable inside the brackets must be raised to the power outside the brackets. For example:

$$(3 \times 2)^4 = 3^4 \times 2^4 \text{ and } (2a^4)^3 = 2^3 \times a^{4 \times 3} = 8a^{12}$$

WORKED EXAMPLE 17

Simplify the following, leaving answers in index form.

a $(7^4)^8$ **b** $\left(\frac{3^2}{5^3}\right)^3$

THINK

- a** 1 Write the problem.
2 Simplify using the Fourth Index Law (multiply the indices).

- b** 1 Write the problem.

- 2 Multiply the indices.

- 3 Simplify.

WRITE

a $(7^4)^8$
 $= 7^{4 \times 8}$
 $= 7^{32}$

b $\left(\frac{3^2}{5^3}\right)^3$
 $= \frac{3^{2 \times 3}}{5^{3 \times 3}}$
 $= \frac{3^6}{5^9}$

WORKED EXAMPLE 18

Simplify $(2b^5)^2 \times (5b^8)^3$.

THINK

- 1 Write the problem.
2 Simplify using the Fourth Index Law.
3 Calculate the coefficient.
4 Simplify using the First Index Law.

WRITE

$(2b^5)^2 \times (5b^8)^3$
 $= 2^1 \times 2b^{5 \times 2} \times 5^{1 \times 3} b^{8 \times 3}$
 $= 2^2 b^{10} \times 5^3 b^{24}$
 $= 4b^{10} \times 125b^{24}$
 $= 500b^{10} \times b^{24}$
 $= 500b^{34}$

WORKED EXAMPLE 19

Simplify $\left(\frac{2a^5}{d^2}\right)^3$.

THINK

- 1 Write the problem.
- 2 Simplify using the Fourth Index Law for each term inside the grouping symbols.
- 3 Calculate the coefficient.

WRITE

$$\begin{aligned}\left(\frac{2a^5}{d^2}\right)^3 &= \frac{2^{1 \times 3} a^{5 \times 3}}{d^{2 \times 3}} \\ &= \frac{2^3 a^{15}}{d^6} \\ &= \frac{8a^{15}}{d^6}\end{aligned}$$

DISCUSSION

What does the 'I' in BIDMAS represent? Is there a relationship between BIDMAS and the Fourth Index Law?

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RESOURCES — ONLINE ONLY

Interactivity: Indices (int-2360)

Interactivity: Fourth Index Law (int-3715)

Digital doc: SpreadSHEET Raising a power to another power (doc-2162)

Digital doc: WorkSHEET Index laws II (doc-6852)

Exercise 8.6 Fourth Index Law (raising a power to another power)

Individual pathways

PRACTISE

Questions:

1–4, 5a–i, 6a–c, 8, 12, 13

CONSOLIDATE

Questions:

1–8, 10, 12, 13

MASTER

Questions:

1d–i, 2d–i, 3d–i, 4, 5g–i, 6, 7i–o, 8, 9g–o, 10–12, 14, 15

Individual pathway interactivity: int-4405

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Understanding and fluency

1. **WE17** Simplify each of the following, leaving your answers in index form.

a. $(3^2)^3$

b. $(6^8)^{10}$

c. $(11^{25})^4$

d. $(5^{12})^{12}$

e. $(3^2 \times 10^3)^4$

f. $(13 \times 17^3)^5$

g. $\left(\frac{3^3}{2^2}\right)^{10}$

h. $(3w^9q^2)^4$

i. $\left(\frac{7e^5}{r^2q^4}\right)^2$

2. **WE18** Simplify each of the following.

a. $(p^4)^2 \times (q^3)^2$ b. $(r^5)^3 \times (w^3)^3$ c. $(b^5)^2 \times (n^3)^6$ d. $(j^6)^3 \times (g^4)^3$ e. $(q^2)^2 \times (r^4)^5$
 f. $(h^3)^8 \times (j^2)^8$ g. $(f^4)^4 \times (a^7)^3$ h. $(t^5)^2 \times (u^4)^2$ i. $(i^3)^5 \times (j^2)^6$

3. Simplify each of the following.

a. $(2^3)^4 \times (2^4)^2$ b. $(t^7)^3 \times (t^3)^4$ c. $(a^4)^0 \times (a^3)^7$ d. $(b^6)^2 \times (b^4)^3$ e. $(e^7)^8 \times (e^5)^2$
 f. $(g^7)^3 \times (g^9)^2$ g. $(3a^2)^4 \times (2a^6)^2$ h. $(2d^7)^3 \times (3d^2)^3$ i. $(10r^{12})^4 \times (2r^3)^2$

4. **MC** a. What does $(p^7)^2 \div p^2$ equal?

A. p^7 B. p^{12} C. p^{16} D. $p^{4.5}$ E. p^{11}

b. What does $\frac{(w^5)^2 \times (p^7)^3}{(w^2)^2 \times (p^3)^5}$ equal?

A. w^2p^6 B. $(wp)^6$ C. $w^{14}p^{36}$ D. w^2p^2 E. w^6p^{19}

c. What does $(r^6)^3 \div (r^4)^2$ equal?

A. r^3 B. r^4 C. r^8 D. r^{10} E. r^{12}

5. Simplify each of the following.

a. $(a^3)^4 \div (a^2)^3$ b. $(m^8)^2 \div (m^3)^4$ c. $(n^5)^3 \div (n^6)^2$ d. $(b^4)^5 \div (b^6)^2$
 e. $(f^7)^3 \div (f^2)^2$ f. $(g^8)^2 \div (g^5)^2$ g. $(p^9)^3 \div (p^6)^3$ h. $(y^4)^4 \div (y^7)^2$
 i. $\frac{(c^6)^5}{(c^5)^2}$ j. $\frac{(f^5)^3}{(f^2)^4}$ k. $\frac{(k^3)^{10}}{(k^2)^8}$ l. $\frac{(p^{12})^3}{(p^{10})^2}$

6. **WE19** Simplify each of the following.

a. $\left(\frac{3b^4}{d^3}\right)^2$ b. $\left(\frac{5h^{10}}{2j^2}\right)^2$ c. $\left(\frac{2k^5}{3t^8}\right)^3$
 d. $\left(\frac{7p^9}{8q^{22}}\right)^2$ e. $\left(\frac{5y^7}{3z^{13}}\right)^3$ f. $\left(\frac{4a^3}{7c^5}\right)^4$

7. Simplify each of the following using the index laws.

a. $g^3 \times 2g^5$ b. $2p^6 \times 4p^2$ c. $(w^3)^6$ d. $12x^6 \div 2x$
 e. $(2d^3)^2$ f. $5a^6 \times 3a^2 \times a^2$ g. $15s^8 \div 5s^2$ h. $4bc^6 \times 3b^3 \times 5c^2$
 i. $\frac{14x^8}{7x^4}$ j. $(f^4g^3)^2$ k. $\frac{16u^6v^5}{6u^3v}$ l. $x^2y^4 \times xy^3$
 m. $5a^6b^2 \times a^2 \times 3ab^3$ n. $x^2y^4 \div xy^3$ o. $(4p^2q^5)^3$

Communicating, reasoning, problem solving

8. a. Simplify $(4^3)^2$, leaving your answer in index form.

b. Use a calculator to determine the value of your answer to part a.

c. Use a calculator to determine the value of $(4^3)^2$.

9. Simplify each of the following, giving your answer in index form. Justify your answer in each case.

a. $(w^3)^4 \div w^2$ b. $\frac{4x^5 \times 3x}{2x^4}$ c. $(2a^3)^2 \times 3a^5$
 d. $12x^6 \times 2x \div 3x^5$ e. $2d^3 + d^2 + 5d^3$ f. $\frac{(2k^3)^2}{4k^4}$
 g. $\frac{4p^5}{p^4 \times 6p}$ h. $15s^8t^3 \div 5s^2t^2 \times 2st^4$ i. $12b^4c^6 \div 3b^3 \div 4c^2$
 j. $(f^4g^3)^2 - fg^3 \times f^7g^3$ k. $\frac{(3p^3)^2 \times 4p^7}{2(p^4)^3}$ l. $2(x^2y)^4 \times 8xy^3$
 m. $5a^6b^2 + a^2 \times 3a^4b^2$ n. $24x^2y^4 \div 12xy^3 - xy$ o. $\frac{4p^2q^7 \times (3p^3q)^2}{6(pq)^3 \times p^5q^4}$

10. Explain why $((a^b)^c)^0 = 1$.
11. A Mathematics class is asked to simplify $(r^4)^3 \div (r^3)^2$. Karla's answer is r . Explain why this is incorrect, pointing out Karla's error. What is the correct answer?
12. Show that $2^{(2^{(2^2)})}$ is not equal to $((2^2)^2)^2$.
13. Arrange these numbers in ascending order.
 3^{2^2} , $2^{2^{2^2}}$, 2^{3^3} , 5^2 , 2^{5^2} , 2^{2^5}
14. **a.** Identify as many different expressions as possible that when raised to a power will result in $16x^8y^{12}$.
b. Identify as many different expressions as possible that when raised to a power will result in $3^{12n}a^{6n}b^{12n}$.
15. How will you remember to raise all coefficients to the power outside the brackets?



8.7.1 Review

Investigation | Rich task

Scientific notation and standard form



Scientists work with many extremely large (and small) numbers, and it is not easy to use them in their basic numeral form. For example, the distance to the nearest star outside the solar system, Proxima Centauri, is 40 000 000 000 000 000 m, and the radius of a hydrogen atom is 0.000 000 000 025 m.

Such numbers can look a little clumsy. Counting the zeros can be hard on the eye, and it's easy to miss one. Furthermore, your calculator would not be able to fit all the digits on its screen!

Scientists use powers of 10 in a number system called **scientific notation** or **standard form**. They have also come up with prefixes that stand for certain powers of 10. There is a prefix for every third power.

Work with a partner and use either the internet or a science encyclopaedia to complete the following table, which shows the scientific notation prefixes and abbreviations for a wide range of numbers.

Standard form	Basic numeral	Name	SI prefix	SI symbol
1.0×10^{12}	1 000 000 000 000		tera	
1.0×10^9		Billion		
1.0×10^6	1 000 000		mega	M
1.0×10^3				
	100	Hundred	hecto	
1.0×10^1			deca	da
1.0×10^{-1}	0.1			
1.0×10^{-2}	0			
1.0×10^{-3}	0.001	Thousandth		
1.0×10^{-6}		Millionth	micro	μ
1.0×10^{-9}			nano	
1.0×10^{-12}	0.000 000 000 001	Trillionth		p

Note: SI is the abbreviation for International System of Units.

Your calculator will accept very large or very small numbers when they are entered because it uses scientific notation. Use the following steps to write the number 825 460 in scientific notation.

Step 1: Place a decimal point so that the number appears to be between 1 and 10.

8.25460

Step 2: Count how many decimal places the decimal point is from its old position. (*Note:* For whole numbers, this is at the right-hand end of the number.) In this case, it is five places away.

8.25460

Step 3: Multiply the number in step 1 by the power of 10 equal to the number of places in step 2.

8.25460×10^5

Note: If your number was made smaller in step 1, multiply it by a positive power to increase it to its true value. If your number was made larger in step 1, multiply it by a negative power to reduce it to its true value.

Proxima Centauri, near the Southern Cross, is the closest star to Earth and is 4.2 light-years away. A light-year is the distance that light travels in 1 year. Light travels at 300 000 kilometres per second.

1. Write 300 000 km/s in scientific notation.
2. Find the distance travelled by light in 1 minute.
3. Find the distance travelled by light in 1 hour.

- Find the distance travelled by light in 1 day.
- Multiply your answer in question 4 by 365.25 to find the length of a light-year in kilometres. (Why do we multiply by 365.25?) Write this distance in scientific notation.
- Calculate the distance from Earth to Proxima Centauri in kilometres.
- Calculate the distance from Earth to some other stars in both light-years and kilometres.



learnON RESOURCES — ONLINE ONLY

Digital doc: Investigation — Scientific notation and standard form (doc-13931)

Digital doc: Code puzzle — The largest gland in the body (doc-13932)

Digital doc: Summary Topic 8 (doc-23244)

Interactivity: Word search Topic 8 (int-2619)

Interactivity: Crossword Topic 8 (int-2620)

Interactivity: Sudoku Topic 8 (int-3183)

Exercise 8.7 Review questions

To answer questions online and to receive **immediate feedback** and **fully worked solutions** for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Understanding and fluency

- State the base for each of the following.
 - 5^{10}
 - 9^4
 - x^8
 - w^7
- State the power or index for each of the following.
 - 11^6
 - 23^5
 - C^{17}
 - L^{100}
- Write the following in index form.
 - $7 \times 7 \times 7 \times 7$
 - $3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$
 - $m \times m \times m \times m \times m$
 - $k \times k \times k \times n \times n \times n \times n \times n$
- Write each of the following as a basic numeral.
 - 6^2
 - 8^2
 - 3^4
 - 2^7
 - 5^3
- Evaluate each of the following.
 - $7^2 - 4^2$
 - $9^2 + 3^3 - 5^2$
- Simplify each of the following.
 - $3^5 \times 3^6$
 - $10^{11} \times 10^4$
 - $7^3 \times 7^6$
 - $j^4 \times j^6 \times j^9$
 - $t^4 \times t^5 \times t$
 - $2z^5 \times 6z \times z$
 - $5w^3 \times 7w^{12} \times w^{14}$
 - $2e^2p^3 \times 6e^3p^5$
- Simplify each of the following.
 - $6^5 \div 6^2$
 - $12^{10} \div 12$
 - $5^{24} \div 5^{14}$
 - $2^6 \div 2^2$
 - $\frac{3^{20}}{3^{11}}$
 - $\frac{m^{99}}{m^{66}}$
 - $\frac{p^{15}}{p}$
 - $\frac{h^7 \times h^{11}}{h^5}$
 - $\frac{L^6 \times L^2 \times L^4}{L^8}$
 - $\frac{y^5 \times y^7 \times y^2}{y^8}$
 - $\frac{a^7 \times a \times a^5}{a^3 \times a^6}$
 - $\frac{c^4 \times c^2 \times c \times c^7}{c^3 \times c^8 \times c^4}$

8. Simplify the following.

a. 4^0	b. 9^0	c. 1966^0	d. m^0
e. n^0	f. zb^0	g. $7w^0$	h. $8q^0 - 2q^0$
i. $4s^0 + 60t^0$	j. v^0w^5	k. $(x^3y^6)^0$	l. klm^0
m. $d^2e^6f^0$	n. $r^4s^0u^9$		

9. Raise each of the following to the given power.

a. $(2^4)^3$	b. $(6^9)^2$	c. $(7^4)^{10}$	d. $(n^{21})^6$
e. $(r^{16}i^{12})^2$	f. $(b^2d^8)^{20}$	g. $(2pm^3)^3$	h. $(9wz^4)^2$

10. **MC** What does $\left(\frac{4b^4}{d^2}\right)^3$ equal?

A. $\frac{4b^3}{d^3}$	B. $\frac{12b^{12}}{d^6}$	C. $\frac{64b^{12}}{d^6}$	D. $\frac{64b^7}{d^5}$	E. $\frac{12b}{d^5}$

Communicating, reasoning and problem solving

11. a. Evaluate each of the following.

i. $(-1)^1$	ii. $(-1)^2$	iii. $(-1)^3$	iv. $(-1)^4$	v. $(-1)^5$	vi. $(-1)^6$

b. Use your answers to part a to complete the following sentence:

‘If negative one is raised to the even power, the result is ...; if it is raised to the odd power, the result is ...’

c. Consider the expression $(-1)^k + (-1)^l$.

d. Find all possible values of the above expression. Specify the values of k and l for which each result occurs.

12. At 9 am there were 10 bacteria in a Petri dish.

a. If the number of bacteria doubles every minute, find out how many bacteria were in a Petri dish after:

i. 1 minute	ii. 2 minutes	iii. 3 minutes	iv. 10 minutes.

b. Develop the rule that connects the number of bacteria, N , and the time, t (in minutes) after 9 am.

c. Use your answer to part b to find the number of bacteria in a dish at 10 am. Give your answer in index form (do not evaluate).



13. Lena receives an email containing a chain letter. She is asked to forward this letter to 5 friends (or else she will have a lot of bad luck!). Lena promptly sends 5 letters as required. (Let's call it the *first round* of letters.) Each of Lena's 5 friends also sends 5 letters. (Call this the *second round* of letters.)

a. How many letters were sent in the second round? Give your answer as:

i. a basic numeral	ii. in index form.

b. How many letters were sent in the third round? Give your answer as:

i. a basic numeral	ii. in index form.

c. Assuming the chain is not broken, and each recipient sent out 5 letters, what was the total number of letters sent in the first four rounds? Give your answer as:

i. a basic numeral	ii. in index form.

- 14.** Nathan is considering participating in the Premier's Reading Challenge. He decided to test himself first by trying to read a 400-page book in 6 days. Nathan read 7 pages on Day 1. After performing some basic arithmetic computations he realised that he needed to increase that amount to be able to finish the book on time. Nathan decided to double the number of pages read every day.
- How many pages did Nathan read on
 - Day 2
 - Day 3?
 - Develop the formula connecting the number of pages P read per day and the number of days d .
 - Use your answer to part **b** to find the number of pages Nathan will read on Day 6.
 - If Nathan continues according to plan, will he finish the book in 6 days? Justify your answer with mathematical calculations.



- 15.** Alex bought a second-hand car for \$25 000. Each year the car depreciates by 20% (i.e. each year it loses 20% of its value).
- Find the value of the car at the end of the first year.
 - Find the value of the car at the end of the second year.
 - The value, V , of the car can be found using the formula $V = 25\,000 \times 0.8^t$, where t is the number of years after purchase. Explain the meaning of the numbers 25 000 and 0.8.
 - Use the formula to find the value of the car after 5 years.
 - Alex decided that he will sell his car when its value falls below \$5000. How soon will he be able to do that?
- 16.** The number, E , of employees in a large firm grows according to the rule $E = 60 \times 1.15^t$, where t is the number of years from the year 2010.
- How many people did the firm employ in the year 2010?
 - How many employees were there in:
 - 2011
 - 2012?
 - How many years will it take for the number of employees to exceed 200?
- 17.** When money is invested in a bank, it earns interest. Interest is calculated as a percentage of the money invested. The term *compound interest* means that the interest earned in one year is added to the account at the end of that year, and that next year the interest is calculated on the larger amount. Compound interest can be found using the formula $A = P\left(1 + \frac{r}{100}\right)^t$, where P is the principal (the invested amount), r is the yearly interest rate and t is the number of years the money has been invested. A is the total amount of money (principal plus interest) accumulated after t years.
- Find the total amount of money accumulated if \$2000 was invested at 4.5% p.a. for 6 years.
(Note: p.a. stands for 'per annum', which means 'per year'.)
 - Find the total amount of money accumulated if \$10 000 was invested at 7% p.a. for 3 years.
 - For part **b**, calculate the total amount of interest earned on the investment over the 3 year period.

18. Four rabbits were accidentally introduced to a small island. The population of rabbits doubled every 4 weeks.
- How many rabbits were on the island:
 - 8 weeks later
 - 24 weeks later
 - 1 year later?
 - After one year, to cope with the rabbit problem, some foxes were brought to the island. As a result, the population of rabbits started declining by 10% each week. After the foxes had been brought in, how many rabbits were left after:
 - 1 week
 - 2 weeks
 - 10 weeks?



19. There were 50 bacteria of Type X and 30 bacteria of Type Y in a Petri dish. The number of bacteria of Type X doubles every 4 hours; the number of bacteria of Type Y quadruples every 6 hours. Find the total number of bacteria in the dish after:
- 12 hours
 - 1 day
 - 2 days.
20. A basic numeral can be expressed in a standard form (also called a scientific notation) by writing it as a number between 1 and 10 multiplied by power of 10. For example, number 4000 in standard form is written as 4.0×10^3 . For each of the following situations express the basic numeral in standard form.
- A company declares an annual profit of 3 billion dollars.
 - The diameter of Earth is (approximately) 40 000 km.
 - The half-life of a certain radioactive element is 5 000 000 years.
 - Light travels at a speed of 300 000 km/s.
21. a. Express the basic numerals 4, 8 and 16 as powers of 2.
 b. Use your answers from part a to simplify the following:
- $\frac{4^x \times 8^y}{16}$
 - $\frac{8^{2m} \times 16^m}{4^{3m} \times 2^{4m}}$
22. If $a^2 = 7$, find the value of:
- a^{4+1}
 - $2a^6$
 - $3a^6 - 4a^4$.
23. A rubber ball is dropped from a balcony which is 10 m above the ground. The ball bounces to $\frac{3}{4}$ of its previous height after each bounce.
- Find the greatest height of the ball above the ground after
 - 1 bounce
 - 3 bounces
 - 5 bounces.
 - When will the height of the ball above the ground be less than 1 m?

Topic 8 Indices

1. **a.** Base = 8, power = 4
e. Base = 78, power = 12
i. Base = n , power = 36
2. **a.** 2^6
b. 4^4
c. x^5
3. **a.** 4×4
c. $7 \times 7 \times 7 \times 7 \times 7$
e. $3 \times 3 \times 3 \times 3 \times 3 \times 3$
g. $a \times a \times a \times a$
4. **a.** 243
b. 256
c. 256
d. 1331
5. **a.** B
b. D
6. **a.** $2^2 \times 4^4 \times 6$
e. $66m^5p^2s^2$
7. **a.** $15 \times f \times f \times f \times j \times j \times j \times j$
c. $4 \times b \times b \times b \times c \times c \times c \times c \times c$
e. $8 \times l \times l \times l \times l \times r \times r \times t \times t$
8. **a.** $64 = 2^6$
d. $400 = 2^4 \times 5^2$
9. **a.** 120
b. 100
c. 216
10. **a. i.** 10^1
b. ii. 5×10^2
iv. $2 \times 10^3 + 3 \times 10^2 + 6 \times 10^1$
vi. $5 \times 10^3 + 4 \times 10^2 + 3 \times 10^1$
c. i. 75 000
ii. 30 600
11. Factors multiplied together in 'shorthand' form: $a \times a \times a \times b \times b \times b \times b$
12. **a.** $2^3a^2b^3c^4$
b. 2 074 464
13. **a.** $1 \times 10^2 + 4 \times 10^1, 6 \times 10^2 + 8 \times 10^1$
c. 820
e. Check with your teacher.
14. **a.** $1 \times 2^3 + 1 \times 2^1, 1 \times 2^3 + 1 \times 2^2 + 1 \times 2^1$
c. 24
e. Check with your teacher.
15. Discuss with your teacher.
- b.** Base = 7, power = 10
f. Base = 3, power = 100
j. Base = d , power = 42
- c.** Base = 20, power = 11
g. Base = m , power = 5
- d.** Base = 19, power = 0
h. Base = c , power = 24
- d.** 9^3
e. $11l^7$
f. $44m^5$
- b.** $5 \times 5 \times 5 \times 5$
d. $6 \times 6 \times 6$
f. $n \times n \times n \times n \times n \times n \times n$
h. $k \times k \times k \times k \times k \times k \times k \times k \times k \times k$
- e.** 2401
f. 216
g. 1
h. 625
- c.** $2^3 \times 19^5$
g. $192e^3kp^2$
- d.** $4^4 \times 13^2$
h. $99j^5lp^2$
- b.** $7 \times k \times k \times k \times k \times k \times k \times k \times s \times s$
d. $19 \times a \times a \times a \times a \times m \times n \times n \times n$
- c.** $36 = 2^2 \times 3^2$
f. $2000 = 2^4 \times 5^3$
- d.** 308
e. 1575
f. 760 000
- iii.** 10^3
iv. 10^6
- iii.** $4 \times 10^2 + 7 \times 10^1$
v. $1 \times 10^3 + 9 \times 10^2 + 8 \times 10^1$
- iii.** 5 200 480
- b.** $7 \times 10^2 + 12 \times 10^1 = 8 \times 10^2 + 2 \times 10^1$
d. 820
- b.** $2 \times 2^3 + 1 \times 2^2 + 2 \times 2^1 = 1 \times 2^4 + 1 \times 2^2 + 1 \times 2^2 = 1 \times 2^4 + 2 \times 2^2 = 1 \times 2^4 + 1 \times 2^3$
d. 24

1. a. $2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^6$			b. $5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 = 5^8$		
c. $f \times f \times f \times f \times f \times f \times f \times f \times f = f^9$					
2. a. 3^9	b. 6^{17}	c. 10^{10}	d. 11^6	e. 7^9	f. 2^{14}
g. 5^4	h. 8^{11}	i. 13^{15}	j. q^{47}	k. x^{14}	l. e^4
3. a. 3^{12}	b. 2^{18}	c. 5^{17}	d. 6^{11}	e. 10^6	f. 17^{12}
g. p^{22}	h. e^{23}	i. g^{28}	j. e^{38}	k. $3b^{13}$	l. $5d^{16}$
4. a. A	b. B				
5. a. $20p^{11}$	b. $6x^8$	c. $56y^{10}$	d. $21p^8$	e. $84t^6$	f. $30q^{15}$
6. a. $6a^6e^7$	b. $8h^{12}p^6$	c. $80m^9$	d. $6g^3h^6$	e. $30p^6q^9$	
f. $48u^9w^7$	g. $27d^{11}y^{17}$	h. $42b^{14}c^9$	i. $24r^{16}s^{18}$	j. $60h^{38}v^{20}$	
7. a. 3^{x+4}	b. 3^{2y+2}	c. 3^{6y-5}	d. $3^{\frac{23}{12}}$		

8. a. $9 = 3^2$; $27 = 3^3$; $81 = 3^4$

b. i. 3^{10}

ii. 3^{2n+2}

9. The bases, 2 and 3, are different. The index laws do not apply.

10. Step 1: 5^n

Step 2: 5^m

Step 3: 5^{n+m}

11. a. 2^9

b. 2^{r-1}

c. $2^5 + 2^6 = \$96$

d. $2^{13} + 2^{14}$

e. $2^{13}(1 + 2) = 3(2^{13})$

f. Check with your teacher.

12. a. x^6

b. 2^{12}

c. 5^2

d. $(a^n)^p = a^{np}$

13. 2

Exercise 8.4 Second Index Law (dividing numbers in index form with the same base)

1. a. 2^3

b. 7^4

c. 10^3

2. a. 3

b. 11^7

c. 5^4

d. 12^5

e. 3^3

f. 13

g. 6^4

h. 10^4

i. 15^{33}

j. h^{77}

k. b^{70}

l. f^{900}

3. a. $3x^2$

b. $6y^2$

c. $8w^7$

d. $3q^4$

e. $8f^9$

f. $10h^{90}$

g. $4j^{10}$

h. $5p^{10}$

i. $8g^3$

j. $\frac{3b^6}{2}$

k. $\frac{9m^4}{2}$

l. $\frac{5n^{90}}{2}$

4. a. D

b. A

5. a. $3p^4$

b. $6r^4$

c. $9a^3$

d. $3b^6$

e. $20r^4$

f. $9q$

6. a. $\frac{3p^5}{2}$

b. $\frac{8b^5}{3}$

c. $\frac{5m^{10}n^6}{6}$

d. $\frac{9x^8y}{4}$

e. $\frac{4hk^3}{3}$

f. $3f^3j^5$

g. $\frac{4p^2rs}{3}$

h. $\frac{9a^5b^3c}{2}$

i. $\frac{20f^6g^2h^4}{3}$

7. a. 2^{10-p}

b. 2^{4e+4}

c. 5^{3x+y}

d. 3^{1-m}

8. a. $\frac{2^3 \times 2^4 \times 2^2}{2^1 \times 2^5}$

b. i. 2^3

ii. 8

c. 8

9. a. $\frac{2 \times 3 \times 3^3 \times 2^2 \times 3^2}{2^2 \times 3 \times 3^4}$

b. i. $2^1 \times 3^1$

ii. 6

10. The bases, 12 and 3, are different. The laws of indices do not apply.

11. 3^{p-q}

12. $a^p \div a^p = 1$ and $a^p \div a^p = a^{p-p} = a^0 \Rightarrow a^0 = 1$

13. $m^6 \div m^8 = m^{6-8} = m^{-2}$ but $m^6 \div m^8 = \frac{m^6}{m^8} = \frac{1}{m^2} \Rightarrow m^{-2} = \frac{1}{m^2}$

14. $\frac{1}{6}$

15. Discuss with your teacher.

Exercise 8.5 Third Index Law (the power of zero)

1. a. 1

b. 1

c. 1

d. 1

2. a. 1

b. 1

c. 1

d. 1

3. a. 4

b. 12

c. -9

d. 22

4. a. $12m^3$

b. 21

c. 72

d. $\frac{8}{7}$

e. 2

f. $10b^2$

g. $3p^2$

h. 1

5. a. 1

b. 1

c. 1

d. 7

e. 3

f. 3

6. a. 1

b. 4

c. 2

d. $2p^9$

e. $\frac{3e^4}{2}$

f. y

g. $\frac{m^3}{3}$

h. $\frac{c^3}{4}$

7. a. D

b. A

c. D

8. a. 1

b. 2

c. 2

d. 2

e. 2

f. $\frac{h^2}{2}$

g. $\frac{q^4}{5}$

h. $\frac{n^3}{5}$

i. v^2

j. $2x^6$

9. Any base raised to the power of zero equals one.

10. $2^0 = 1$, $x^0 = 1 \Rightarrow \frac{2^0 x^2}{2^2 x^0} = \frac{x^2}{2^2} = \frac{x^2}{4}$

11. $\frac{1}{3}$

12. Peter has treated the x 's incorrectly. He has calculated $x^{6 \div 2}$ instead of x^{6-2} .

The answer is $4x^4$.

If 64 and 16 are converted to base 2 or base 4, and the indices divided not subtracted, an error will occur.

13. The Second Index Law proves why any base to the power of zero is equal to 1. For example:

$$2^4 \div 2^4 = 2^0 \\ = 1$$

Exercise 8.6 Fourth Index Law (raising a power to another power)

1. a. 3^6 b. 6^{80} c. 11^{100} d. 5^{144} e. $3^8 \times 10^{12}$
 f. $13^5 \times 17^{15}$ g. $\frac{3^{30}}{2^{20}}$ h. $3^4 w^{36} q^8$ i. $\frac{7^2 e^{10}}{r^4 q^8}$
2. a. $p^8 q^6$ b. $r^{15} w^9$ c. $b^{10} n^{18}$ d. $j^{18} g^{12}$ e. $q^4 r^{20}$
 f. $h^{24} j^{16}$ g. $f^{16} a^{21}$ h. $t^{10} u^8$ i. $i^{15} j^{12}$
3. a. 2^{20} b. 3^{33} c. a^{21} d. b^{24} e. e^{66}
 f. g^{39} g. $324a^{20}$ h. $216d^{27}$ i. $40\,000r^{54}$
4. a. B b. B c. D
5. a. a^6 b. m^4 c. n^3 d. b^8 e. f^{17} f. g^6
 g. p^9 h. y^2 i. c^{20} j. f^7 k. k^{14} l. p^{16}
6. a. $\frac{9b^8}{d^6}$ b. $\frac{25h^{20}}{4j^4}$ c. $\frac{8k^{15}}{27t^{24}}$ d. $\frac{49p^{18}}{64q^{44}}$ e. $\frac{125y^{21}}{27z^{39}}$ f. $\frac{256a^{12}}{2401c^{20}}$
7. a. $2g^8$ b. $8p^8$ c. w^{18} d. $6x^5$ e. $4d^6$ f. $15a^{10}$ g. $3s^6$ h. $60b^4c^8$
 i. $2x^4$ j. $f^8 g^6$ k. $\frac{8u^3 v^4}{3}$ l. $x^3 y^7$ m. $15a^9 b^5$ n. xy o. $64p^6 q^{15}$
8. a. 4^6 b. 4096 c. 4096
9. a. w^{10} b. $6x^2$ c. $12a^{11}$ d. $8x^2$ e. $7d^3 + d^2$ f. k^2 g. $\frac{2}{3}$ h. $6s^7 t^5$
 i. bc^4 j. 0 k. $18p$ l. $16x^9 y^7$ m. $8a^6 b^2$ n. xy o. $6q^2$

10. Any base, even a complex one like this, raised to the power of one equals zero.

11. Karla has added the powers instead of multiplying.

$$r^7 \div r^6 = r^1 = r$$

The correct answer is r^6 .

12. $2^{(2^{(2^2)})} = 2^{2^4} = 2^{16}; (((2^2)^2)^2) = 2^8$

13. $2^{2^5}; 2^{3^3}; 2^{5^2}; 2^{2^{2^2}}; 3^{2^2}; 5^2$

14. a. $(16x^8 y^{12})^1, (4x^4 y^6)^2, (2x^2 y^3)^4$
 b. $(3^{12n} a^{6n} b^{12n})^1, (3^{12} a^6 b^{12})^n, (3^6 a^3 b^6)^{2n}, (3^2 a^1 b^2)^{6n},$
 $(3^{6n} a^{3n} b^{6n})^2, (3^{2n} a^n b^{2n})^6$

15. Discuss with your teacher.

Investigation | Rich task

Standard form	Basic numeral	Name	SI prefix	SI symbol
1.0×10^{12}	1 000 000 000 000	Trillion	tera	T
1.0×10^9	1 000 000 000	Billion	giga	G
1.0×10^6	1 000 000	Million	mega	M
1.0×10^3	1000	Thousand	kilo	k
1.0×10^2	100	Hundred	hecto	h
1.0×10^1	10	Ten	deca	da
1.0×10^{-1}	0.1	Tenth	deci	d
1.0×10^{-2}	0.01	Hundredth	centi	c
1.0×10^{-3}	0.001	Thousandth	milli	m
1.0×10^{-6}	0.000001	Millionth	micro	μ
1.0×10^{-9}	0.000000001	Billionth	nano	n
1.0×10^{-12}	0.000000000001	Trillionth	pico	p

1. 3.0×10^5 km/s
2. 1.8×10^7 km/min
3. 1.08×10^9 km/h
4. 2.592×10^{10} km/day
5. 9.47×10^{12} km. On average, there are 365.25 days in one year.
6. 4.07×10^{13} km
7. Check with your teacher.

Exercise 8.7 Review questions

1. a. 5 b. 9 c. x d. w
2. a. 6 b. 5 c. 17 d. 100
3. a. 7^4 b. 3^7 c. m^5 d. k^3n^5
4. a. 36 b. 64 c. 81 d. 128 e. 125
5. a. 33 b. 83
6. a. 3^{11} b. 10^{15} c. 7^9 d. j^{19} e. t^{10} f. $12z^7$ g. $35w^{29}$ h. $12e^5p^8$
7. a. 6^3 b. 12^9 c. 5^{10} d. 2^4 e. 3^9 f. m^{33} g. p^{14} h. h^{13}
8. a. L^4 b. y^6 c. a^4 l. $\frac{1}{c}$
8. a. 1 b. 1 c. 1 d. 1 e. 1 f. z g. 7 h. 6
9. a. 64 b. w^5 k. 1 l. kl m. d^2e^6 n. r^4u^9
9. a. 2^{12} b. 6^{18} c. 7^{40} d. n^{126} e. $i^{24}r^{32}$ f. $b^{40}d^{160}$ g. $8p^3m^9$ h. $81w^2z^8$
10. C
11. a. i. -1 ii. 1 iii. -1 iv. 1 v. -1 vi. 1
11. b. Positive one; negative one
11. c. -2 if k and l are both odd; 0 if one of the powers is odd and one is even; 2 if both k and l are even.
12. a. i. 20 ii. 40 iii. 80 iv. 10 240
12. b. $N = 10 \times 2^t$ c. 10×2^{60}
13. a. i. 25 ii. 5^2
13. b. i. 125 ii. 5^3
13. c. i. 780 ii. $5 + 5^2 + 5^3 + 5^4$
14. a. i. 14 ii. 28
14. b. $P = 7 \times 2^{d-1}$
14. c. 224
14. d. Yes, the total for 6 days is 441, which is more than 400.
15. a. \$20 000 b. \$16 000
15. c. 25 000 represents the purchase price of the car; 0.8 means 80% (expressed as a decimal) — this is the portion of the value that the car retains after each year.
15. d. \$8192
15. e. After 8 years
16. a. 60
16. b. i. 69 ii. 79
16. c. 9 years
17. a. \$2604.52 b. \$12 250.43 c. \$2250.43
18. a. i. 16 ii. 256 iii. 32 768 b. i. 29 491 ii. 26 542 iii. 11 425
19. a. 880 b. 10 880 c. 2 170 880
20. a. 3.0×10^9 b. 4.0×10^4 c. 5.0×10^6 d. 3.0×10^5
21. a. $2^2, 2^3, 2^4$
21. b. i. $2^{2x+3y-4}$ ii. 1
22. a. 50 b. 686 c. 833
23. a. i. 7.5 m ii. 4.218 75 m iii. 2.373 05 m
23. b. After 9 bounces.

TOPIC 9

Measurement

9.1 Overview

Numerous **videos** and **interactivities** are embedded just where you need them, at the point of learning, in your learnON title at www.jacplus.com.au. They will help you to learn the concepts covered in this topic.

9.1.1 Why learn this?

Measurement is important in many aspects of everyday life. If you do not have an understanding of metric measurement — length, area and volume — many professions and jobs will be closed to you. Imagine a designer, dressmaker, architect or builder who did not understand measurement. Their job would be impossible!



LEARNING SEQUENCE

- 9.1 Overview
- 9.2 Perimeter
- 9.3 Circumference
- 9.4 Area of rectangles, triangles, parallelograms, rhombuses and kites
- 9.5 Area of a circle
- 9.6 Area of trapeziums
- 9.7 Volume of prisms and other solids
- 9.8 Time
- 9.9 24-hour clock and time zones
- 9.10 Review

LEARNING OUTCOMES

A student:

- communicates and connects mathematical ideas using appropriate terminology, diagrams and symbols MA4-1WM
- applies appropriate mathematical techniques to solve problems MA4-2WM
- calculates the perimeters of plane shapes and the circumferences of circles MA4-12MG
- uses formulas to calculate the areas of quadrilaterals and circles, and converts between units of area MA4-13MG
- performs calculations of time that involve mixed units, and interprets time zones MA4-15MG

CONTENT DESCRIPTIONS

Students:

Find perimeters of parallelograms, trapeziums, rhombuses and kites (ACMMG196)

Investigate the concept of irrational numbers, including π (ACMNA186)

Investigate the relationship between features of circles, such as the circumference, radius and diameter; use formulas to solve problems involving circumference (ACMMG197)

Choose appropriate units of measurement for area and convert from one unit to another (ACMMG195)



Establish the formulas for areas of rectangles, triangles and parallelograms and use these in problem solving (ACMMG159)

Find areas of trapeziums, rhombuses and kites (ACMMG196)

Investigate the relationship between features of circles, such as the area and the radius; use formulas to solve problems involving area (ACMMG197)

Develop the formulas for the volumes of rectangular and triangular prisms and of prisms in general; use formulas to solve problems involving volume (ACMMG198)

Solve problems involving duration, including using 12-hour and 24-hour time within a single time zone (ACMMG199)

Solve problems involving international time zones (NSW)

Calculate the volumes of cylinders and solve related problems (ACMMG217)

Source: NSW Syllabus for the Australian curriculum

learnon RESOURCES — ONLINE ONLY

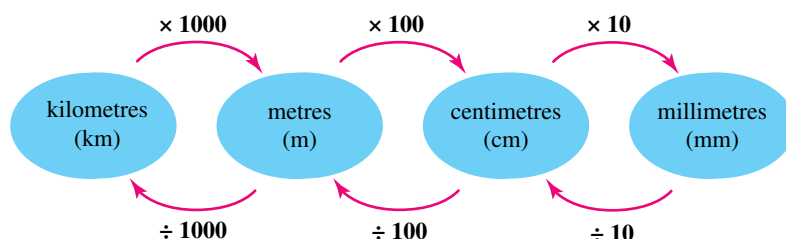
eLesson: The story of mathematics — Khayyam (eles-1843)

Note: Your teacher may now set you a pre-test to determine how familiar you are with the content in this topic.

9.2 Perimeter

9.2.1 Units of length

- Metric units of length include millimetres (mm), centimetres (cm), metres (m) and kilometres (km).
- To convert between the units of length, we use the following conversion chart:



- When converting from a larger unit to a smaller unit, multiply by the conversion factor; when converting from a smaller unit to a larger unit, divide by the conversion factor.

WORKED EXAMPLE 1

Complete the following metric length conversions.

- a $1.027 \text{ m} = \underline{\hspace{1cm}} \text{ cm}$
- b $0.0034 \text{ km} = \underline{\hspace{1cm}} \text{ m}$
- c $76\,500 \text{ m} = \underline{\hspace{1cm}} \text{ km}$
- d $3.069 \text{ m} = \underline{\hspace{1cm}} \text{ mm}$



THINK

- a Look at the conversion chart above. To convert metres to centimetres, we need to multiply by 100. So, move the decimal point two places to the right.

WRITE

a $1.027 \times 100 = 102.7 \text{ cm}$

THINK

- b** To convert kilometres to metres, we need to multiply by 1000. So, move the decimal point three places to the right.
- c** To convert metres to kilometres, divide by 1000. This can be done by moving the decimal point three places to the left.
- d** To convert metres to millimetres, we need to multiply by 100 and then by 10. This is the same as multiplying by 1000, so move the decimal point three places to the right.

WRITE

$$\text{b } 0.0034 \times 1000 = 3.4 \text{ m}$$

$$\text{c } 76500 \div 1000 = 76.5 \text{ km}$$

$$\text{d } 3.069 \times 100 \times 10 = 3069 \text{ mm}$$

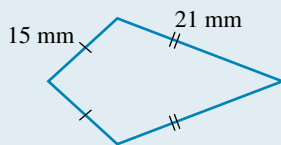
9.2.2 The perimeter

- The **perimeter** of a shape is the total distance around the shape.
- To find the perimeter of a shape:
 - identify the length of each side
 - ensure that all measurements are in the same units
 - add all side lengths together and include units with your answer.

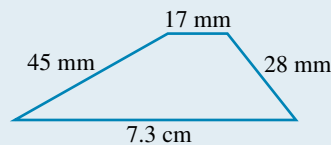
WORKED EXAMPLE 2

Find the perimeter of each of the shapes below.

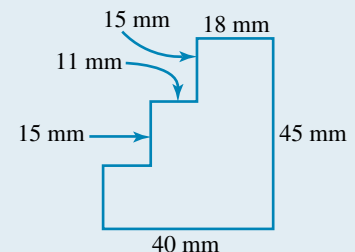
a A kite



b A trapezium



c An irregular shape

**THINK**

- a 1** Make sure that all the measurements are in the same units and add them together.
- 2** Write the answer in words, including the units.
- b 1** Notice that the measurements are not all in the same metric units. Convert to the smaller unit (in this case convert 7.3 cm to mm).
- 2** Add the measurements.
- 3** Write the answer in words, including the units.

WRITE/DRAW

$$\text{a } P = 21 \times 2 + 15 \times 2 \\ = 72$$

The perimeter of the kite shown is 72 mm.

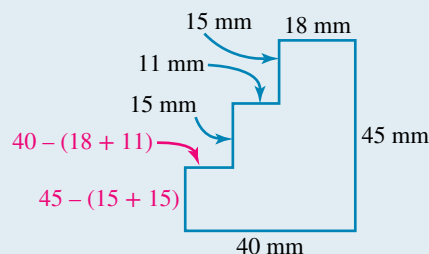
$$\text{b } 7.3 \text{ cm} = 73 \text{ mm}$$

$$P = 45 + 17 + 28 + 73 \\ = 163$$

The perimeter of the trapezium shown is 163 mm.

c 1 Make sure that given measurements are in the same units.

2 Determine the lengths of the unknown sides and label them on the diagram.



3 Add the measurements.

$$\begin{aligned} P &= 45 + 18 + 15 + 11 + 15 + [40 - (18 + 11)] + [45 - (15 + 15)] + 40 \\ &= 45 + 18 + 15 + 11 + 15 + 11 + 15 + 40 \\ &= 170 \text{ mm} \end{aligned}$$

4 Write the answer in words, including the units.

The perimeter of the irregular shape shown is 170 mm.

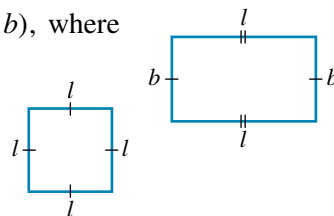
ACTIVITY: DESK PERIMETER

Equipment: measuring tape

1. Estimate the length of the outside edge of the top of your desk.
2. Individually or in pairs, use the measuring tape to measure the length of the top of your desk.
3. Are all the desks in the classroom the same size? If so, compare the length you found with that found by classmates.
4. How accurate was your estimate? What was the percentage error of your estimate?
5. As a class, discuss ways of improving the accuracy of your estimates.

9.2.3 Finding the perimeter of a square and a rectangle

- The perimeter (P) of a rectangle is given by the formula $P = 2(l + b)$, where l is the length and b is the breadth of the rectangle.
- The perimeter (P) of a square is given by the formula $P = 4l$, where l is the side length of the square.
- The perimeter of a rhombus can be calculated using the formula for the perimeter of a square, as a rhombus also has four sides of equal length.



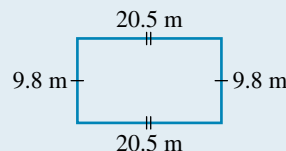
WORKED EXAMPLE 3

Find the perimeter of a rectangular block of land that is 20.5 m long and 9.8 m wide.

THINK

1 Draw a diagram of the block of land and write in the measurements.

WRITE/DRAW



2 Write the formula for the perimeter of a rectangle.

$$P = 2(l + b)$$

3 Substitute the values of l and b into the formula, and calculate.

$$\begin{aligned} P &= 2 \times (20.5 + 9.8) \\ &= 2 \times 30.3 \\ &= 60.6 \text{ m} \end{aligned}$$

4 Write the worded answer with the correct units.

The perimeter of the block of land is 60.6 m.

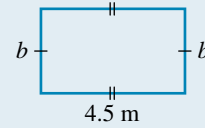
WORKED EXAMPLE 4

A rectangular billboard advertising country New South Wales has a perimeter of 16 m. Calculate its breadth if the length is 4.5 m.

THINK

- 1 Draw a diagram of the rectangular billboard and write in the measurements.
- 2 Write the formula for the perimeter of a rectangle.
- 3 Substitute the values of P and l into the formula, and solve the equation:
 - (a) subtract 9 from both sides
 - (b) divide both sides by 2
 - (c) simplify if appropriate.
- 4 Write the worded answer with the correct units.

WRITE/DRAW



$$P = 16 \text{ m}$$

$$\begin{aligned} P &= 2(l + b) \\ &= 2l + 2b \end{aligned}$$

$$16 = 2 \times 4.5 + 2b$$

$$16 = 9 + 2b$$

$$16 - 9 = 9 - 9 + 2b$$

$$7 = 2b$$

$$\frac{7}{2} = \frac{2b}{2}$$

$$3.5 = b$$

$$b = 3.5$$

The breadth of the rectangular billboard is 3.5 m.

learnON RESOURCES — ONLINE ONLY

Interactivity: Units of length (int-3779)

Interactivity: Perimeter (int-3780)

Interactivity: Perimeter of squares and rectangles (int-3781)

eLesson: Perimeter (eles-1874)

Digital doc: SkillsHEET Multiplying and dividing by powers (doc-6956)

Digital doc: SkillsHEET Converting units of length (doc-6957)

Exercise 9.2 Perimeter

Individual pathways

PRACTISE

Questions:

1, 2, 3, 7, 8, 9, 15, 18

CONSOLIDATE

Questions:

1a, c, e, g, i, k, m, 2a, c, d, f, g, h, i, j, 3, 5, 8, 9, 15, 17–19

MASTER

Questions:

1b, d, f, h, j, l, n, 2b, d, f, h, i, k, l, 4, 6, 7, 9–21

Individual pathway interactivity: int-4438

learnON ONLINE ONLY

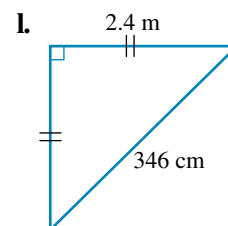
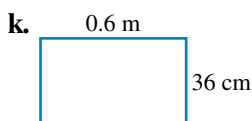
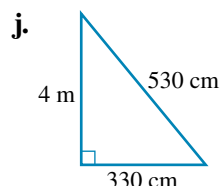
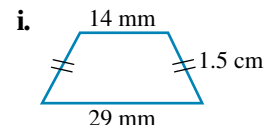
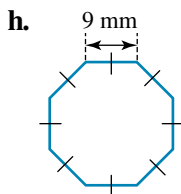
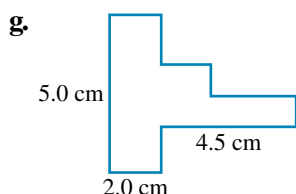
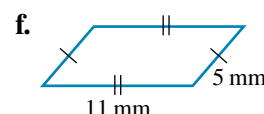
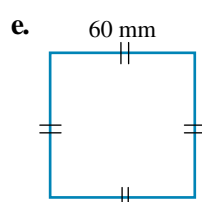
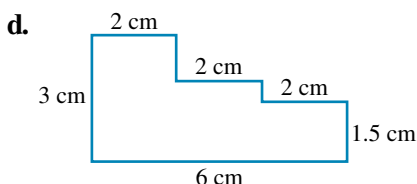
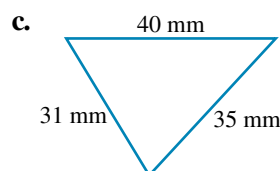
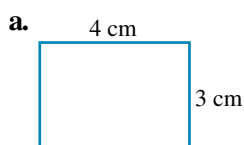
To answer questions online and to receive **immediate feedback** and **fully worked solutions** for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Understanding and fluency

1. **WE1** Fill in the gaps for each of the following.

a. 20 mm = _____ cm	b. 13 mm = _____ cm
c. 130 mm = _____ cm	d. 1.5 cm = _____ mm
e. 0.03 cm = _____ mm	f. 2.8 km = _____ m
g. 0.034 m = _____ cm	h. 2400 mm = _____ cm = _____ m
i. 1375 mm = _____ cm = _____ m	j. 2.7 m = _____ cm = _____ mm
k. 0.08 m = _____ mm	l. 6.071 km = _____ m
m. 670 cm = _____ m	n. 0.0051 km = _____ m

2. **WE2** Find the perimeter of the shapes below.



3. Chipboard sheets are sold in three sizes. Convert each of the measurements below into centimetres and then into metres:

- a. 1800 mm $\times$ 900 mm b. 2400 mm $\times$ 900 mm c. 2700 mm $\times$ 1200 mm.

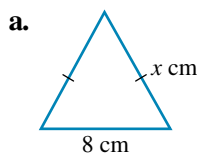
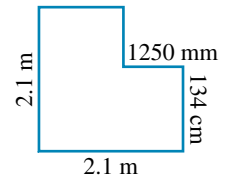
4. A particular type of chain is sold for \$2.25 per metre. What is the cost of 2.4 m of this chain?
5. Fabric is sold for \$7.95 per metre. How much will 4.8 m of this fabric cost?
6. The standard marathon distance is 42.2 km. If a marathon race starts and finishes with one lap of a stadium which is 400 m in length, what distance is run on the road outside the stadium?

7. Maria needs 3 pieces of timber of lengths 2100 mm, 65 cm and 4250 mm to construct a clothes rack.

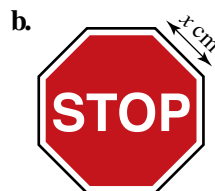
- a. What is the total length of timber required, in metres?
- b. How much will the timber cost at \$3.80 per metre?



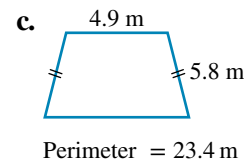
8. **WE3** Find the perimeter of a basketball court, which is 28 m long and 15 m wide.
9. A piece of modern art is in the shape of a rhombus with a side length of 65 cm. Find the perimeter of the piece of art in metres.
10. A woven rectangular rug is 175 cm wide and 315 cm long. Find the perimeter of the rug.
11. A line is drawn to form a border 2 cm from each edge of a piece of A4 paper. If the paper is 30 cm long and 21 cm wide, what is the length of the border line?
12. A rectangular paddock 144 m long and 111 m wide requires a new single-strand wire fence.
- What length of fencing wire is required to complete the fence?
 - How much will it cost to rewire the fence if the wire cost \$1.47 per metre.
13. A computer desk with the dimensions shown at right needs to have table edging. If the edging cost \$1.89 per metre, find the cost of the table edging required for the desk.
14. Calculate the unknown side lengths in each of the given shapes.



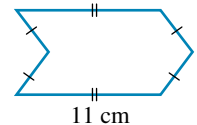
Perimeter = 30 cm



Perimeter = 176 cm

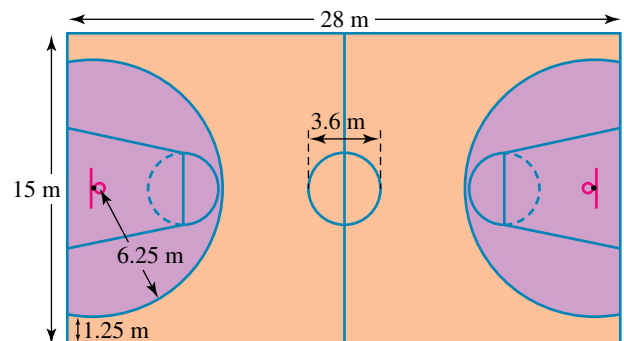


15. **WE4** A rectangular billboard has a perimeter of 25 m. Calculate its width if the length is 7 m.
16. The ticket at right has a perimeter of 42 cm.
- Calculate the unknown side length.
 - Olivia wishes to decorate the ticket by placing a gold line along the slanted sides. How long is the line on each ticket?
 - A bottle of gold ink will supply enough ink to draw 20 m of line. How many bottles of ink should be purchased if 200 tickets are to be decorated?



Communicating, reasoning and problem solving

17. A square and an equilateral triangle have the same perimeter. The side of the triangle is 3 cm longer than the side of the square. How long is the side of the square? Show your working.
18. As a warm-up activity for PE class, you are required to run laps of the basketball court. The dimensions of a full-size basketball court are shown.
- How far will you run if you run three full laps of the court?
 - How far will you run if you run three laps of half the court?
19. a. A pool fence is to be placed around a rectangular pool that is 6 m by 8 m. If the fence is to be 1 m away from the edge of the pool and also rectangular in shape, what length of fencing is required? (Include the width of the gate in the length of the fence.)
- Write a rule that relates the length of the fence to the length and width of any rectangular pool.



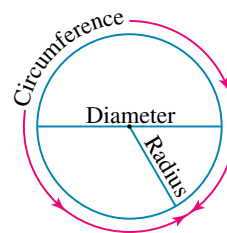
20. A present is to be wrapped using a box, as shown at right.
- If the dimensions of the box are 30 cm long by 25 cm wide by 18 cm high, what is the total length of ribbon that would be needed if the bow used 35 cm of ribbon? (Assume no overlap at the start or finish of the ribbon.)
 - Write a rule that will allow you to calculate the amount of ribbon required for any rectangular box.
21. What is the best way to remember the units conversion chart?



9.3 Circumference

9.3.1 Circumference

- The **circumference** (C) is another term for the perimeter of a circle.
- The **diameter** (D) of a circle is the name given to the straight-line distance across a circle through its centre.
- The straight-line distance from the centre of the circle to the circumferences is called the **radius**.
- There is a relationship between the diameter and the circumference of a circle.



ACTIVITY: THE DIAMETER AND CIRCUMFERENCE OF A CIRCLE

Measure the diameters and circumferences of a variety of different circles and cylinders and record your results in a table. Can you identify the relationship between the diameter and circumference of a circle?

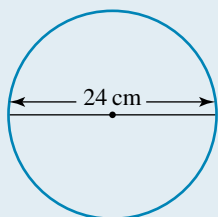
- The ratio $\frac{C}{D}$ is approximately 3. When this ratio is calculated exactly, it is called *pi* or π .
- The symbol π represents the ratio of the circumference of a circle to its diameter. π is a constant numerical value, and is an infinite, non-recurring and non-terminating decimal that begins as 3.141 592 653 5...
- The circumference of a circle is given by the formula $C = \pi D$.
- A diameter of a circle is twice as long as its radius, that is, $D = 2r$. Therefore, the other way to write the formula for the circumference is $C = 2\pi r$, where r is the radius of a circle.

WORKED EXAMPLE 5

Find the circumference of each of the following circles, giving answers

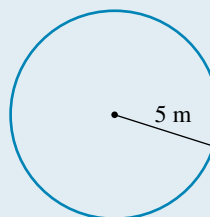
i in terms of π

a



ii correct to 2 decimal places.

b



THINK

- a i 1** Write the formula for the circumference of a circle.
Note: Since the diameter of the circle is given, use the formula that relates the circumference to the diameter.

2 Substitute the value $D = 24$ into the formula.

3 Write the answer and include the correct units.

- ii 1** Write the formula for the circumference of a circle.

2 Substitute the values $D = 24$ and $\pi = 3.14\dots$ into the formula.

3 Evaluate and include the correct units.

- b i 1** Write the formula for the circumference of a circle.
Note: Since the radius of the circle is given, use the formula that relates the circumference to the radius.

2 Substitute the value $r = 5$ into the formula.

3 Write the answer and include the correct units.

- ii 1** Write the formula for the circumference of a circle.

2 Substitute the values $r = 5$ and $\pi = 3.14\dots$ into the formula.

3 Evaluate and include the correct units.

WRITE

a i $C = \pi D$

$$= \pi \times 24$$

$$= 24\pi \text{ cm}$$

ii $C = \pi D$

$$= 3.14\dots \times 24$$

$$= 75.398\dots$$

$$= 75.40 \text{ cm}$$

b i $C = 2\pi r$

$$= 2 \times \pi \times 5$$

$$= 10\pi \text{ m}$$

ii $C = 2\pi r$

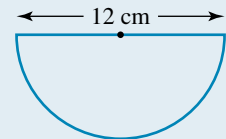
$$= 2 \times 3.14\dots \times 5$$

$$= 31.415\dots$$

$$= 31.42 \text{ m}$$

WORKED EXAMPLE 6

Find the perimeter of the following shape, correct to 2 decimal places.

**THINK**

- 1** Identify the parts that constitute the perimeter of the given shape.
- 2** Write the formula for the circumference of a circle.
Note: If the circle were complete, the straight-line segment shown would be its diameter. So the formula that relates the circumference to the diameter is used.
- 3** Substitute the values $D = 12$ and $\pi = 3.14\dots$ into the formula.
- 4** To find the perimeter of the given shape, halve the value of the circumference and add the length of the straight section.
- 5** Evaluate and include the correct units.

WRITE

$$P = \frac{1}{2} \text{ circumference} + \text{straight-line section}$$

$$P = \frac{1}{2} \pi D + \text{straight-line section}$$

$$= \frac{1}{2} \times 3.14\dots \times 12 + 12$$

$$= 18.849\dots + 12$$

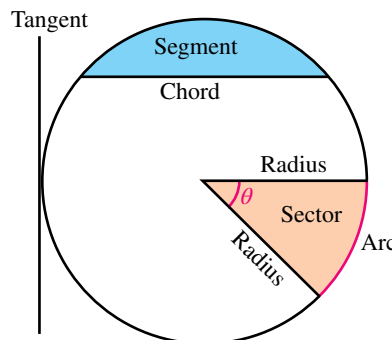
$$= 30.85 \text{ cm}$$

ACTIVITY: π THROUGHOUT THE AGES

Use the internet to create a poster on various approximations that have been used in place of π throughout history. Be sure to include any relevant historical information you find in your research.

9.3.2 Parts of a circle

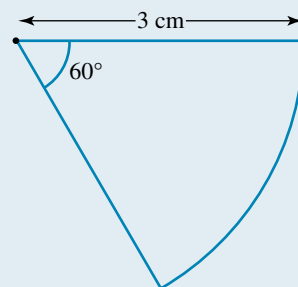
- If you draw two radii inside a circle, they divide the circle into two regions called **sectors**.
- An **arc** is a section of the circumference of a circle.
- A **chord** is a straight line joining any two points on the circumference of a circle. The diameter is a type of chord.
- A **segment** of a circle is a section bounded by a chord and an arc.
- A **tangent** is a straight line that touches the circumference of a circle only at one given point.
- The length of an arc can be calculated as $\frac{\theta}{360}$ of the circumference of the circle, where θ is the angle which defines the length of the arc.
- The perimeter of a sector consists of 2 radii and an arc. If the circumference of a circle = C , then the perimeter of the sector, P , will be given by:



$$\begin{aligned} P &= 2r + \frac{\theta}{360} \times C \\ &= 2r + \frac{\theta}{360} \times 2\pi r \end{aligned}$$

WORKED EXAMPLE 7

Find the perimeter of the following sector, correct to 2 decimal places.



THINK

- 1 Identify the values of θ and r .
- 2 Substitute these values into the formula for the perimeter of a sector.
- 3 Evaluate the formula.
- 4 Round the answer to 2 decimal places, remembering to include the units.

WRITE

$$\begin{aligned} \theta &= 60, r = 3 \\ P &= 2 \times 3 + \frac{60}{360} \times 2\pi \times 3 \\ &= 9.141... \\ &\approx 9.14 \text{ cm} \end{aligned}$$

learnon RESOURCES — ONLINE ONLY

Interactivity: Circumference (int-3792)

Digital doc: Investigation — The diameter of a circle and its circumference — any connection? (doc-2318)

Exercise 9.3 Circumference

Individual pathways

PRACTISE

Questions:
1–6, 9, 12, 13, 17, 19

CONSOLIDATE

Questions:
1a, c, e, 2a, c, e, 3a, c, e, 4b, d, 5,
6, 7, 9, 12, 13, 16, 17, 19–21

MASTER

Questions:
1a, d, 2a, b, f, 3c, e, f, 4b, c, d, f, i,
5–8, 10, 12c, 13c, 14–18, 20–25

Individual pathway interactivity: int-4439

learnon ONLINE ONLY

To answer questions online and to receive **immediate feedback** and **fully worked solutions** for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

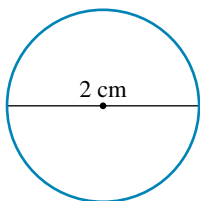
Understanding and fluency

1. **WE5a** Find the circumference of each of these circles, giving answers:

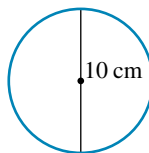
i. in terms of π

ii. correct to 2 decimal places.

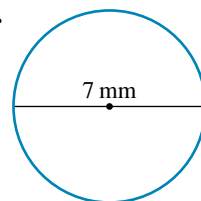
a.



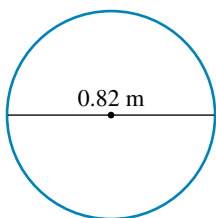
b.



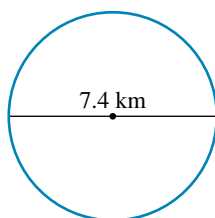
c.



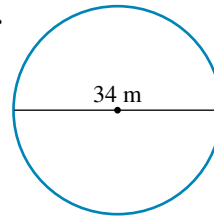
d.



e.



f.

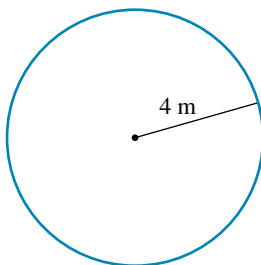


2. **WE5b** Find the circumference of each of the following circles, giving answers:

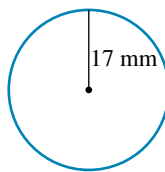
i. in terms of π

ii. correct to 2 decimal places.

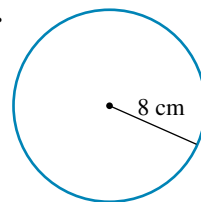
a.



b.



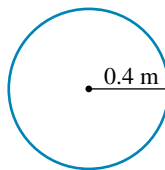
c.



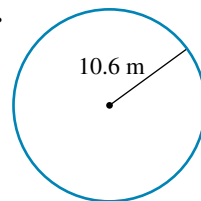
d.



e.

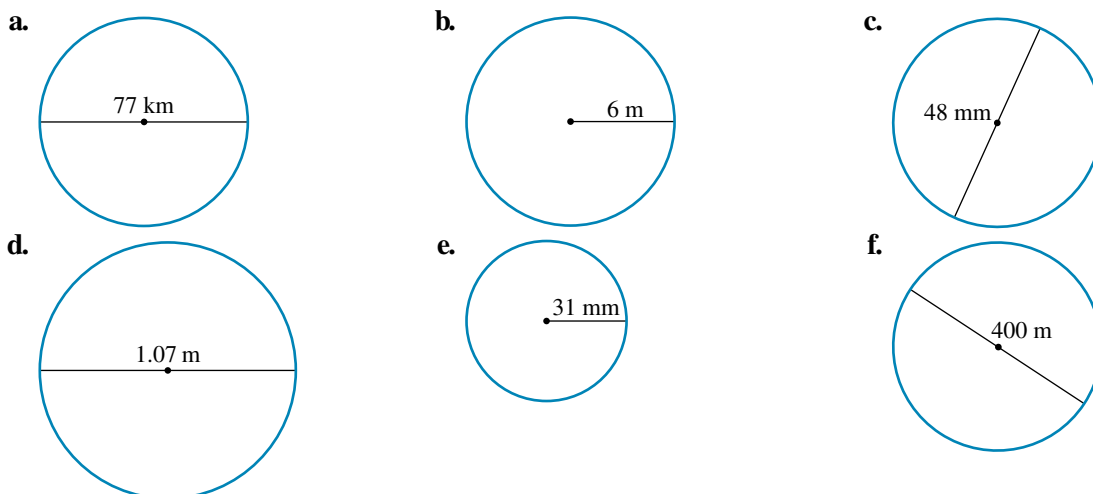


f.

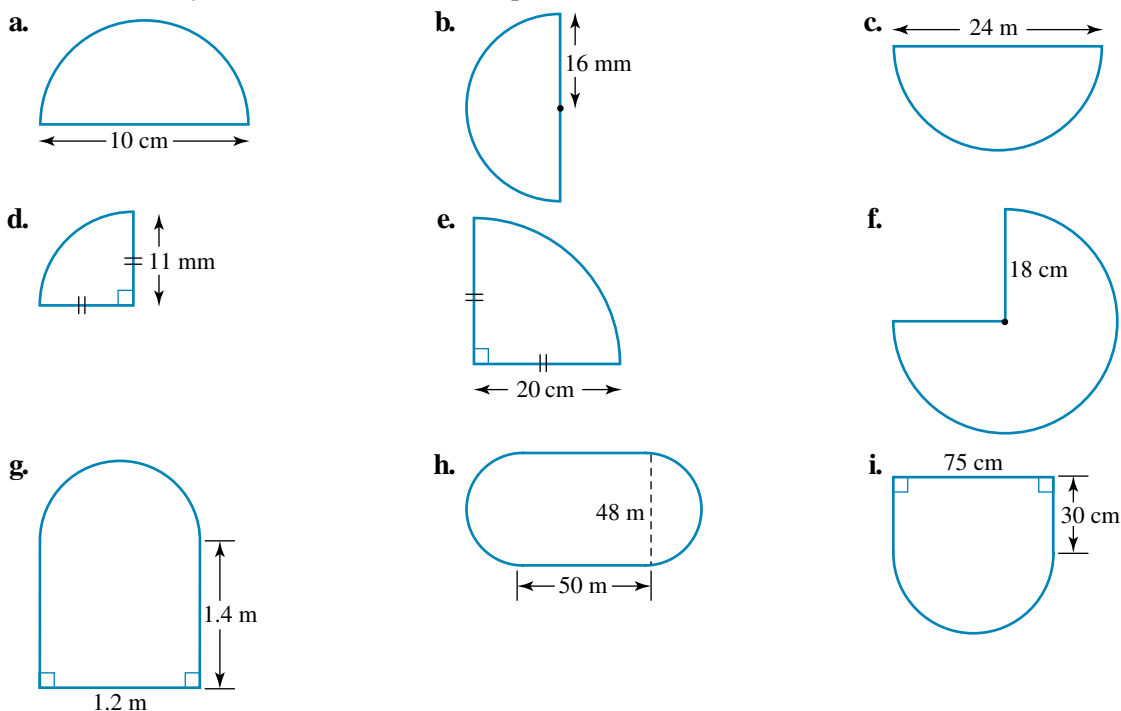


3. Estimate the circumference of these circles using:

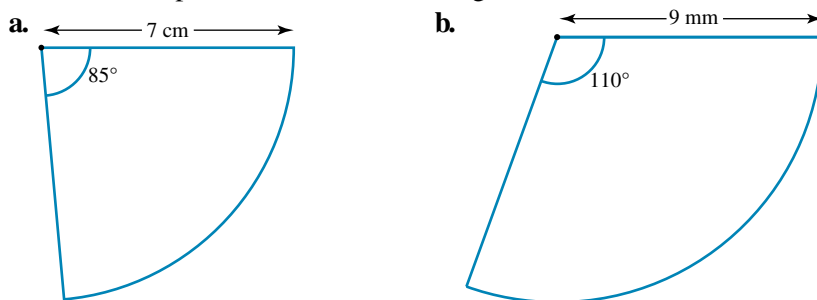
- i. 3 as an approximation of π
- ii. 3.1 as an approximation of π
- iii. 3.14 as an approximation of π .

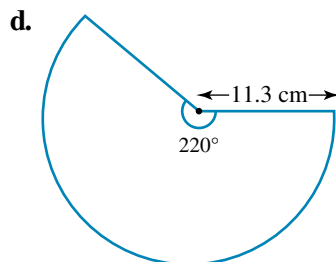
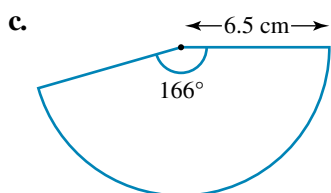


4. **WE6** Find the perimeter of each of the shapes below. (Remember to add the lengths of the straight sections.) Give your answers to 2 decimal places.

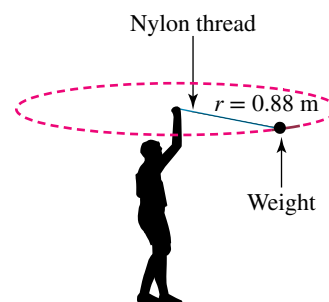


5. **MC** The circumference of a circle with a radius of 12 cm is:
A. $\pi \times 12$ cm **B.** $2 \times \pi \times 12$ cm **C.** $2 \times \pi \times 24$ cm **D.** $\pi \times 6$ cm **E.** $\pi \times 18$ cm
6. **MC** The circumference of a circle with a diameter of 55 m is:
A. $2 \times \pi \times 55$ m **B.** $\pi \times \frac{55}{2}$ m **C.** $\pi \times 55$ m
D. $\pi \times 110 \times 2$ m **E.** $2 \times \pi \times 110$ m
7. **WE7** Find the perimeter of the following sectors, correct to 2 decimal places.

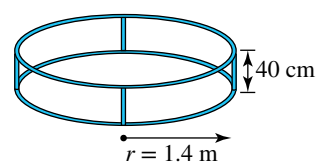




8. In a Physics experiment, students spin a metal weight around on the end of a nylon thread. How far does the metal weight travel if it completes 10 revolutions on the end of a 0.88 m thread? Give your answer to 2 decimal places.
9. A scooter tyre has a diameter of 32 cm. What is the circumference of the tyre? Give your answer to 2 decimal places.
10. Find the circumference of the seaweed around the outside of this sushi roll, correct to 2 decimal places.



11. Find the circumference of the Ferris wheel shown at right, correct to 2 decimal places.
12. Calculate the diameter of a circle (correct to 2 decimal places where appropriate) with a circumference of:
 - a. 18.84 m
 - b. 64.81 cm
 - c. 74.62 mm.
13. Calculate the radius of a circle (correct to 2 decimal places where appropriate) with a circumference of:
 - a. 12.62 cm
 - b. 47.35 m
 - c. 157 mm.
14. Calculate the radius of a tyre with a circumference of 135.56 cm. Give your answer to 2 decimal places.
15. Calculate the total length of metal pipe needed to assemble the wading pool frame shown at right. Give your answer in metres to 2 decimal places.
16. Nathan runs around the inside lane of a circular track that has a radius of 29 m. Rachel runs in the outer lane, which is 2.5 m further from the centre of the track. How much longer is the distance Rachel runs each lap? Give your answer to 2 decimal places.

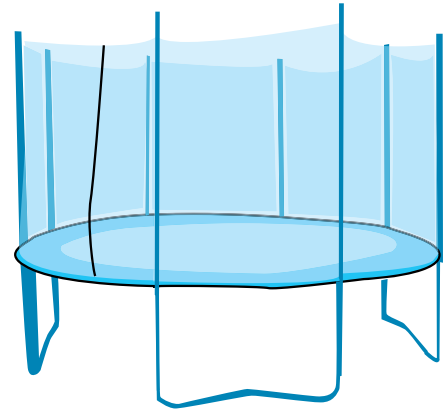


Communicating, reasoning and problem solving

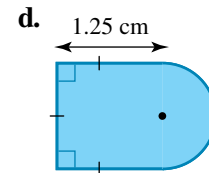
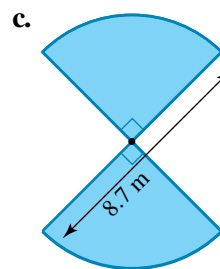
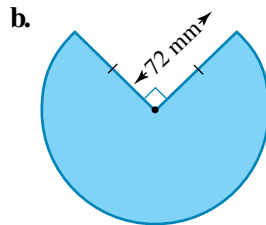
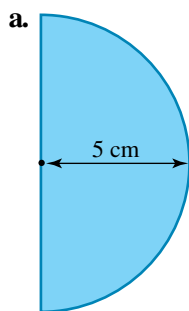
17. In *Around the world in eighty days* by Jules Verne, Phileas Fogg boasts that he can travel around the world in 80 days or fewer. This was in the 1800s, so he couldn't take a plane. What average speed is needed to go around the Earth at the equator in 80 days? Assume you travel for 12 hours each day and that the radius of the Earth is approximately 6390 km. Give your answer in km/h to 2 decimal places.
18. Liesel's bicycle covers 19 m in 10 revolutions of her bicycle wheel while Jared's bicycle covers 20 m in 8 revolutions of his bicycle wheel. What is the difference between the radii of the two bicycle wheels? Give your answer in cm to 2 decimal places.
19. A shop sells circular trampolines of four different sizes. Safety nets that go around the trampoline are optional and can be purchased separately. The entrance to the trampoline is via a zip in the net.

The following table shows the diameters of all available trampolines and their net lengths. Which safety net matches which trampoline?

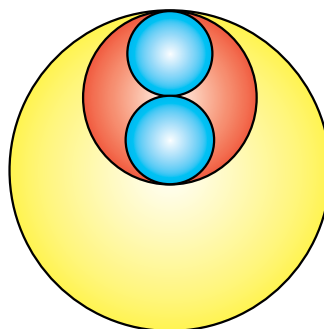
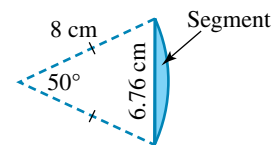
	Diameter of trampoline	Length of safety net
a	1.75 m	i 6.03 m
b	1.92 m	ii 9.86 m
c	2.46 m	iii 5.50 m
d	3.14 m	iv 7.73 m



20. Calculate the perimeter of each of the following shapes. Give your answers correct to 2 decimal places.



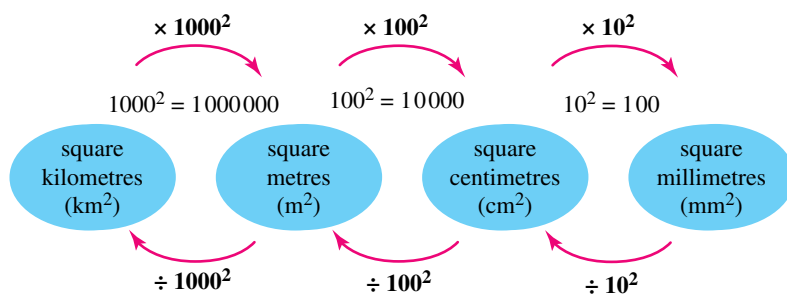
21. To cover a total distance of 1.5 km, a student needs to run around a circular track three times. Calculate the radius of the track correct to the nearest metre.
22. Investigate how to get the perimeter of the segment shown at right.
23. π is an irrational number. Investigate the concept of irrational numbers.
24. If you needed to estimate the value of π using any round object, how would you do it?
25. The two small circles in the diagram have a diameter that is equal to the radius of the medium-sized circle. The diameter of the medium-sized circle is equal to the radius of the large circle. If the large circle has a radius of 8 cm, determine the area of the red-shaded section.



9.4 Area of rectangles, triangles, parallelograms, rhombuses and kites

9.4.1 Area

- The area of a shape is the amount of flat surface enclosed by the shape.
- Area is measured in square units, such as square millimetres (mm^2), square centimetres (cm^2), square metres (m^2) and square kilometres (km^2).
- Area units can be converted using the chart below.



- Area units are the squares of those for the corresponding linear units.
- Large areas of land can be measured in hectares (ha). $1 \text{ ha} = 10\,000 \text{ m}^2$.

WORKED EXAMPLE 8

Complete the following metric conversions.

a $0.081 \text{ km}^2 = \underline{\hspace{2cm}} \text{ m}^2$

b $19\,645 \text{ mm}^2 = \underline{\hspace{2cm}} \text{ m}^2$

THINK

- a Look at the metric conversion chart. To convert square kilometres to square metres, multiply by 1 000 000; that is, move the decimal point 6 places to the right.
- b Look at the metric conversion chart. To convert square millimetres to square metres, divide by 1 000 000; that is, move the decimal point 6 places to the left.

WRITE

a $0.081 \text{ km}^2 = 0.081 \times 1\,000\,000 \text{ m}^2$
 $= 81\,000 \text{ m}^2$

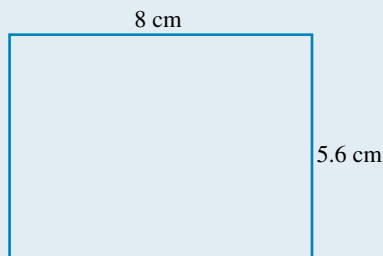
b $19\,645 \text{ mm}^2 = 19\,645 \div 1\,000\,000 \text{ m}^2$
 $= 0.019\,645 \text{ m}^2$

9.4.2 Area of a rectangle

- The area of a rectangle can be found using the formula $A_R = lb$, where l is the length and b is the breadth (or width) of the rectangle.
- The area of a square can be found using the formula $A_s = s^2$, where s is the side length of the square.

WORKED EXAMPLE 9

Find the area of a rectangle with the dimensions shown below.



THINK

- 1 Write the formula for the area of a rectangle.
- 2 Identify the values of l and b .
- 3 Substitute the values of l and b into the formula and evaluate. Include the appropriate units.

WRITE

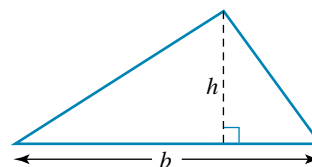
$$A = lb$$

$$l = 8 \text{ and } b = 5.6$$

$$\begin{aligned} A &= 8 \times 5.6 \\ &= 44.8 \text{ cm}^2 \end{aligned}$$

9.4.3 Area of a triangle

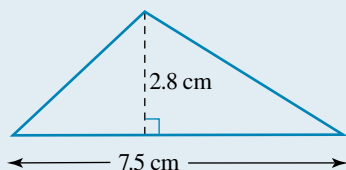
- The area of a triangle, A_T , is given by the formula $A_T = \frac{1}{2} \times b \times h$, where b is the base and h is the height of the triangle.
- The base and the height of a triangle are perpendicular (at right angles) to each other.



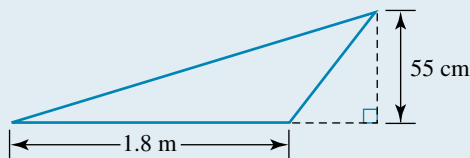
WORKED EXAMPLE 10

Find the area of each of these triangles in the smaller unit of measurement.

a



b

**THINK**

- a**
- 1 Write the formula for the area of a triangle.
 - 2 Identify the values of b and h .
 - 3 Substitute the values of b and h into the formula.
 - 4 Evaluate. Remember to include the correct units (cm^2).
- b**
- 1 Write the formula for the area of a triangle.
 - 2 Convert measurements to cm.
 - 3 Identify the values of b and h .
 - 4 Substitute the values of b and h into the formula.
 - 5 Evaluate. Remember to include the correct units (cm^2).

WRITE

a $A = \frac{1}{2}bh$

$$b = 7.5, h = 2.8$$

$$\begin{aligned} A &= \frac{1}{2} \times 7.5 \times 2.8 \\ &= 3.75 \times 2.8 \\ &= 10.5 \text{ cm}^2 \end{aligned}$$

b $A = \frac{1}{2}bh$

$$\begin{aligned} 1.8 \text{ m} &= 1.8 \times 100 \text{ cm} \\ &= 180 \text{ cm} \end{aligned}$$

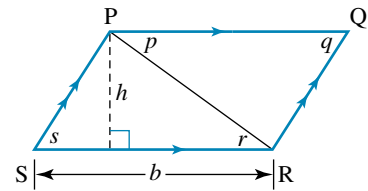
$$b = 180, h = 55$$

$$\begin{aligned} A &= \frac{1}{2} \times 180 \times 55 \\ &= 90 \times 55 \\ &= 4950 \text{ cm}^2 \end{aligned}$$

9.4.4 Area of a parallelogram

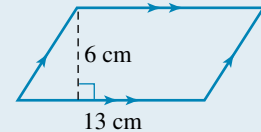
- A **parallelogram** is a quadrilateral with two pairs of parallel sides. Each parallel pair is of equal length.
- A parallelogram can be divided into two triangles by drawing a diagonal.

- In the parallelogram PQRS at right, are $\triangle PQR$ and $\triangle RSP$ congruent? As their corresponding sides are of equal length, then $\triangle PQR \equiv \triangle RSP$ (SSS congruency condition).
- As the area of a triangle is given as $A_T = \frac{1}{2} \times b \times h$, then the area of a parallelogram must be twice the area of each triangle, so A_P , area of a parallelogram, is given by the formula $A_P = b \times h$, where b is the base and h is the height of a parallelogram.
- The base and the height of a parallelogram are perpendicular to each other.



WORKED EXAMPLE 11

Find the area of the parallelogram shown.



THINK

- 1 Write the formula for the area of a parallelogram.
- 2 Identify the values of b and h .
- 3 Substitute 6 for h and 13 for b .
- 4 Multiply the numbers together and include the correct units.

WRITE

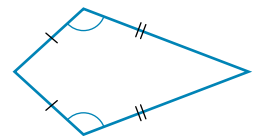
$$\begin{aligned}
 A &= bh \\
 b &= 13, h = 6 \\
 A &= 13 \times 6 \\
 &= 78 \text{ cm}^2
 \end{aligned}$$

9.4.5 Area of a rhombus

- A **rhombus** is a parallelogram with all four sides of equal length and each pair of opposite sides parallel.
- The area of a rhombus can be determined by multiplying the length of the diagonals and dividing by 2: $A_{RH} = \frac{d_1 \times d_2}{2}$.

9.4.6 Area of a kite

- A **kite** is a quadrilateral with two pairs of equal, adjacent sides and one pair of equal angles.
- The area of a kite can be determined by dividing the kite into two equal triangles and using the formula for the area of a triangle: $A_K = \frac{d_1 \times d_2}{2}$.



9.4.7 Area of composite figures

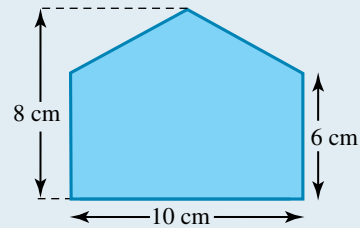
- The area of simple composite figures made up of rectangles, squares, parallelograms and triangles can be found by finding the area of the smaller shapes and adding them together.
- When you are breaking a composite figure down into smaller shapes, be sure to break it into shapes for which you know the area formulas.

WORKED EXAMPLE 12

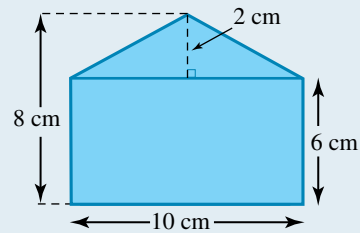
Calculate the area of the following shape.

THINK

- 1 The shape can be divided into a rectangle and a triangle.
The height of the triangle is $(8 - 6) = 2$ cm.
- 2 Write the formula for the area of a rectangle.
 - Identify the values of the pronumerals. Substitute these values into the formula and evaluate.
 - Write the formula for the area of a triangle.
 - Identify the values of the pronumerals. Substitute these values into the formula and evaluate.
- 3 Add the area of the rectangle and that of the triangle together to find the total area of the composite shape. Remember to include the units with the answer.



WRITE



$$A_{\text{rectangle}} = l \times b$$

$$l = 10 \text{ cm}; b = 6 \text{ cm}$$

$$\begin{aligned} A_{\text{rectangle}} &= 10 \times 6 \\ &= 60 \text{ cm}^2 \end{aligned}$$

$$A_{\text{triangle}} = \frac{1}{2} \times b \times h$$

$$b = 10 \text{ cm}; h = 2 \text{ cm}$$

$$\begin{aligned} A_{\text{triangle}} &= \frac{1}{2} \times 10 \times 2 \\ &= 10 \text{ cm}^2 \end{aligned}$$

$$\begin{aligned} \text{Total area} &= A_{\text{rectangle}} + A_{\text{triangle}} \\ &= 60 + 10 \\ &= 70 \text{ cm}^2 \end{aligned}$$

The area of the composite shape is 70 cm^2 .

DISCUSSION

Develop some strategies to help you memorise the formulas for areas of various shapes.

learn on RESOURCES — ONLINE ONLY

Interactivity: Conversion chart for area (int-3783)

Interactivity: Area of rectangles (int-3784)

Interactivity: Area of parallelograms (int-3786)

Interactivity: Area of rhombuses (int-3787)

Digital doc: Investigation — Area of a parallelogram (doc-2319)

Digital doc: SkillSHEET Areas of squares, rectangles and triangles (doc-6958)

Weblink: Area of a rhombus

Exercise 9.4 Area of rectangles, triangles, parallelograms, rhombuses and kites

Individual pathways

PRACTISE

Questions:

1–6, 7a, c, e, g, i, 8–10, 12, 14, 18, 19, 28

CONSOLIDATE

Questions:

1a, c, e, g, i, 2a, e, f, 3b, 4, 5, 6a, d, f, 7a, e, g, i, 8–11, 13, 15, 16, 19, 21, 26, 28, 29

MASTER

Questions:

1b, d, f, h, j, 2c, e, f, 3c, 4, 5, 6c, e, f, 8–12, 14, 17–31

Individual pathway interactivity: int-4440

learnON ONLINE ONLY

To answer questions online and to receive **immediate feedback** and **fully worked solutions** for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Understanding and fluency

1. **WE8** Complete the following metric conversions.

a. $0.53 \text{ km}^2 = \text{_____ m}^2$

c. $2540 \text{ cm}^2 = \text{_____ mm}^2$

e. $74\,000 \text{ mm}^2 = \text{_____ m}^2$

g. $98\,563 \text{ m}^2 = \text{_____ ha}$

i. $0.987 \text{ m}^2 = \text{_____ mm}^2$

b. $235 \text{ mm}^2 = \text{_____ cm}^2$

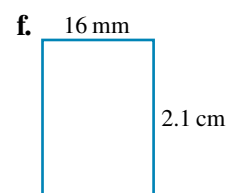
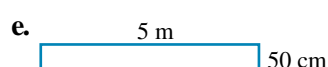
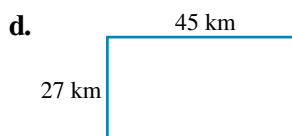
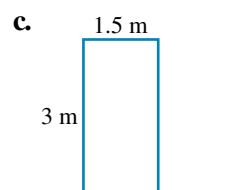
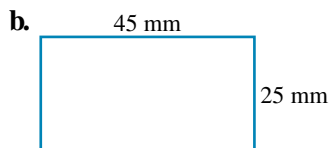
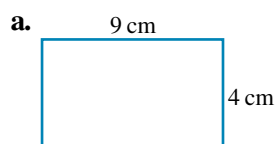
d. $542\,000 \text{ cm}^2 = \text{_____ m}^2$

f. $3\,000\,000 \text{ m}^2 = \text{_____ km}^2$

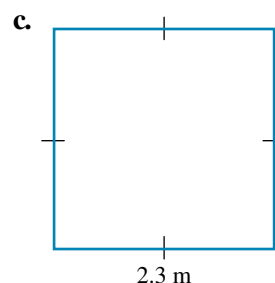
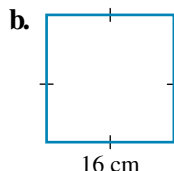
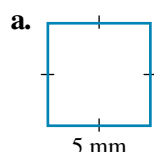
h. $1.78 \text{ ha} = \text{_____ m}^2$

j. $0.000\,127\,5 \text{ km}^2 = \text{_____ cm}^2$

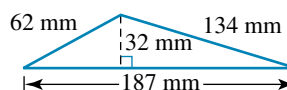
2. **WE9** Find the area of each of the rectangles below.



3. Find the area of each of the squares below.



Questions 4 and 5 relate to the diagram at right.



4. **MC** The height and base respectively of the triangle are:

A. 32 mm and 62 mm

B. 32 mm and 134 mm

C. 32 mm and 187 mm

D. 62 mm and 187 mm

E. 134 mm and 187 mm

5. **MC** The area of the triangle is:

A. 2992 mm

B. 2992 mm²

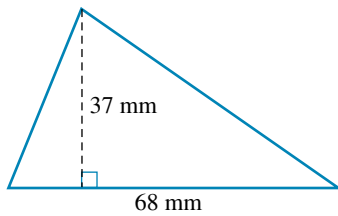
C. 5984 mm

D. 5984 mm²

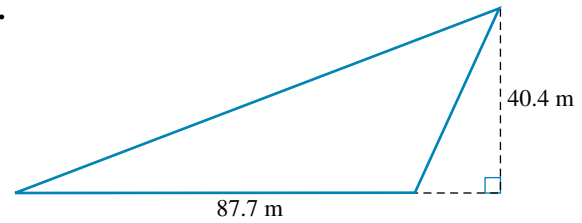
E. 6128 mm²

6. **WE10** Find the areas of the following triangles. Where there are two units given, answer using the smaller units.

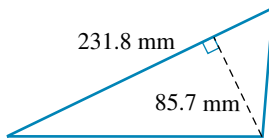
a.



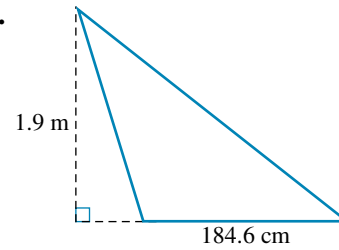
b.



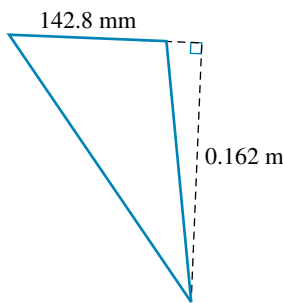
c.



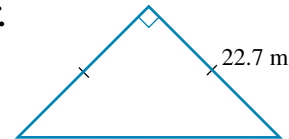
d.



e.

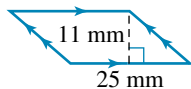


f.

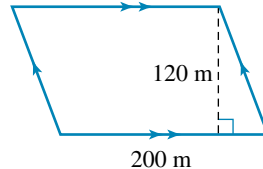


7. **WE11** Find the areas of the following parallelograms.

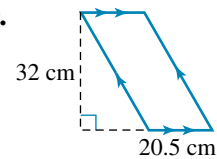
a.



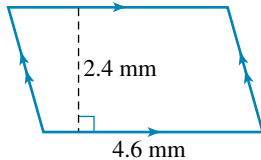
b.



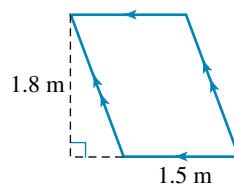
c.



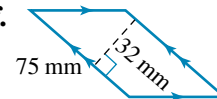
d.



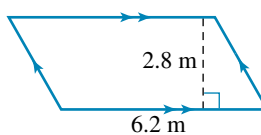
e.



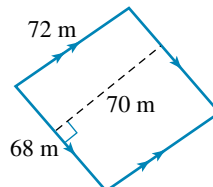
f.



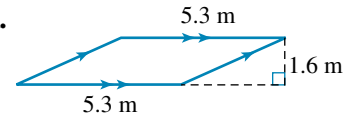
g.



h.



i.

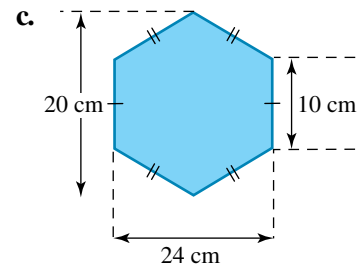
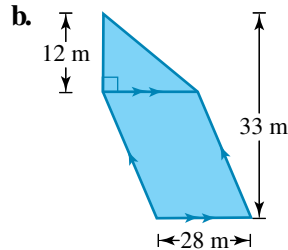
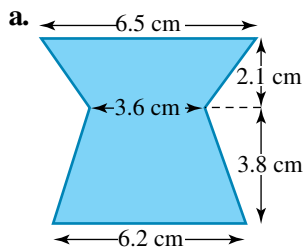


8. a. Find the area of a rhombus whose diagonals are 10 cm and 6 cm.

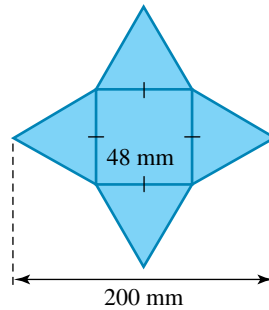
b. Find the area of a rhombus whose diagonals are 8 cm and 6 cm.

c. Find the area of a kite whose diagonals are 20 cm and 9 cm.

9. **WE11** Find the areas of the following composite shapes.



10. A triangular pyramid can be constructed from this net. Calculate the total area of the net.



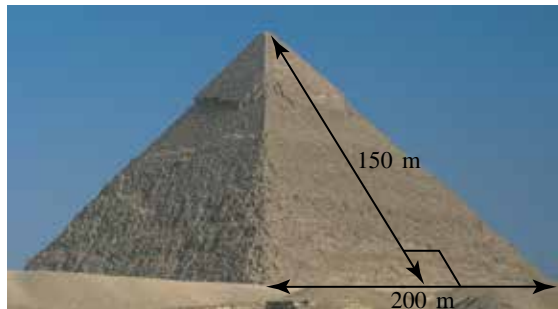
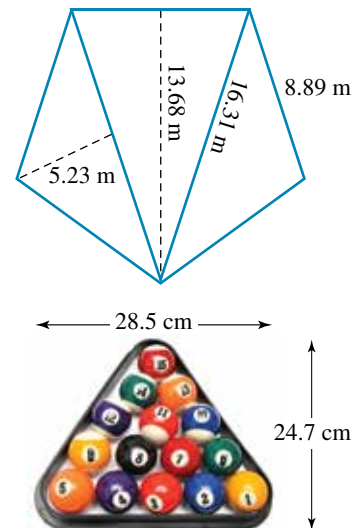
11. Zorko has divided his vegetable patch, which is in the shape of a regular (all sides equal) pentagon, into 3 sections as shown in the diagram at right.

- Calculate the area of each individual section, correct to 2 decimal places.
- Calculate the area of the vegetable patch, correct to 2 decimal places.

12. Find the area of the triangle used to rack up the pool balls below right.

13. The pyramid below has 4 identical triangular faces with the dimensions shown. Calculate:

- the area of one of the triangular faces
- the total area of the 4 faces.



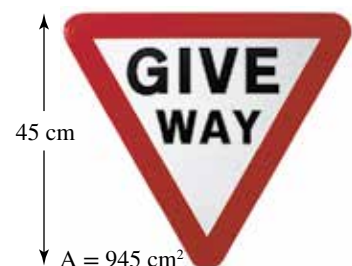
14. Georgia is planning to create a feature wall in her lounge room by painting it a different colour. The wall is 4.6 m wide and 3.4 m high.

- Calculate the area of the wall to be painted.
- Georgia knows that a 4 litre can of paint is sufficient to cover 12 square metres of wall. How many cans must she purchase if she needs to apply two coats of paint?

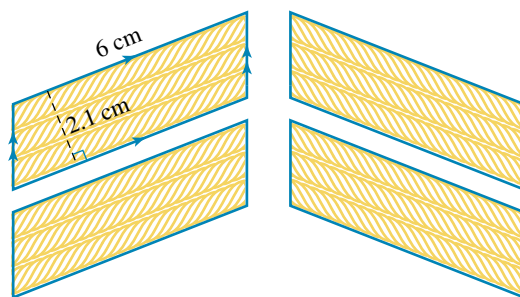
15. Calculate the base length of the give-way sign at right.

16. a. Calculate the width of a rectangular sportsground if it has an area of 30 ha and a length of 750 m.

- The watering system at the sportsground covers 8000 square metres in 10 minutes. How long does it take to water the sportsground?



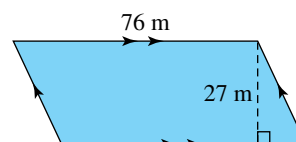
17. Find the area of gold braid needed to make the four military stripes shown.



18. What is the area of the block of land in the figure at right?

19. **MC** Which statement about a parallelogram is false?

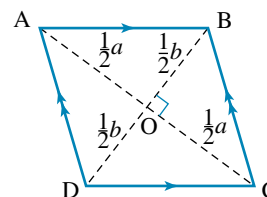
- A. The opposite sides of a parallelogram are parallel.
- B. The height of the parallelogram is perpendicular to its base.
- C. The area of a parallelogram is equal to the area of the rectangle whose length is the same as the base and whose breadth is the same as the height of the parallelogram.
- D. The perimeter of the parallelogram is given by the formula $P = 2(b + h)$.
- E. The area of a parallelogram is given by the formula $A = bh$.



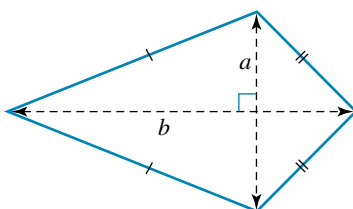
20. The base of a parallelogram is 3 times as long as its height. Find the area of the parallelogram, given that its height is 2.4 cm long.
21. A designer vase has a square base of side length 12 cm and four identical sides, each of which is a parallelogram. If the vertical height of the vase is 30 cm, find the total area of the glass used to make this vase. (Assume no waste and do not forget to include the base.)
22. a. Find the length of the base of a parallelogram whose height is 5.2 cm and whose area is 18.72 cm^2 .
b. Find the height of a parallelogram whose base is 7.5 cm long and whose area is 69 cm^2 .
23. The length of the base of a parallelogram is equal to its height. If the area of the parallelogram is 90.25 cm^2 , find its dimensions.

Communicating, reasoning and problem solving

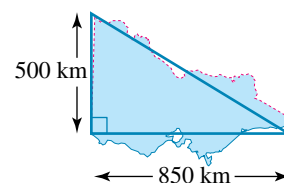
24. a. If you increase the side lengths of a rectangle by a factor of 2, what effect does this have on the area of the rectangle?
b. If you decrease the side lengths of a rectangle by a factor of 2, what effect does this have on the area of the rectangle?
c. If you square the side lengths of a rectangle, what effect does this have on the area of the rectangle?
25. a. If you increase the side lengths of a rectangle by factor of n , what effect does this have on the area of the rectangle?
b. If you decrease the side lengths of a rectangle by a factor of n , what effect does this have on the area of the rectangle?
26. If the diagonals of a rhombus bisect each other and intersect at right angles, use mathematical reasoning to show and explain that $A_{\text{RH}} = \frac{1}{2} \times a \times b$, where a and b are the lengths of the diagonals of the rhombus.



27. Using mathematical reasoning, determine the area of the kite below in terms of a and b .



28. Show possible dimensions for each of the following shapes so that they each have an area of 36cm^2 .
- Rectangle
 - Parallelogram
 - Rhombus
29. A rectangular flower bed measures 20 m by 16 m. A gravel path 2 m wide surrounds it.
- Draw a diagram representing the flower bed and path.
 - Calculate the area of the flower bed.
 - Calculate the area of the gravel path.
 - If gravel costs \$5 per square metre, how much will it cost to cover the path?
30. The area of Victoria can be approximated using a right-angled triangle with the measurements shown at right.
- Estimate the area of Victoria by calculating the area of the triangle.
 - Use your atlas or the internet to compare your estimate with the actual area of Victoria.
 - Explain why the answer you obtained in part a can be regarded only as an estimate.
 - How could a more accurate estimate of the area of Victoria be made?
31. Why is it important to know that the height and the base of any triangle and any parallelogram are perpendicular?



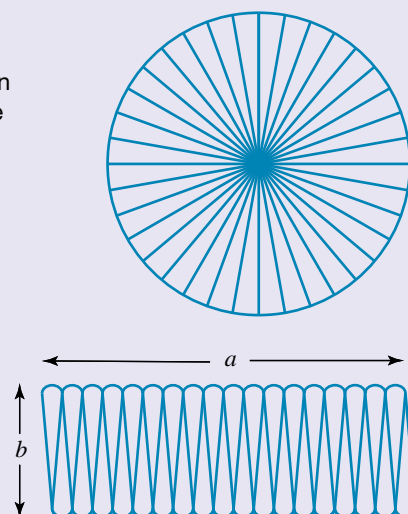
9.5 Area of a circle

9.5.1 The area of a circle

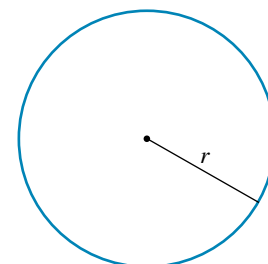
ACTIVITY: FINDING A FORMULA TO CALCULATE THE AREA OF A CIRCLE

Equipment: paper, pair of compasses, protractor, scissors, pencil

- Use your compasses to draw a circle with a radius of 10 cm.
- Use a protractor to mark off 10° angles along the circumference. Join the markings to the centre of the circle with straight lines. Your circle will be separated into 36 equal sectors.
- Cut out the sectors and arrange them in a pattern as shown. Half of the sectors will be pointing up and half pointing down.
- The resultant shape resembles a _____.
- Express the values of a and b shown on the diagram in terms of r , the radius of the original uncut circle. Hence, find the area.
- Explain why the area of the shape is the same as the area of the original circle. Hence state the formula for the area of the circle.



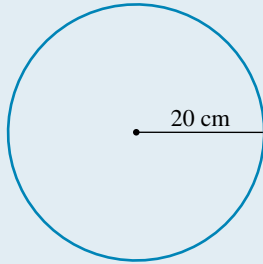
- The area of a circle, A , can be found using the formula $A = \pi r^2$, where r is the radius of the circle.
- The radius of a circle, r , is equal to a half of its diameter, D : $r = \frac{D}{2}$.



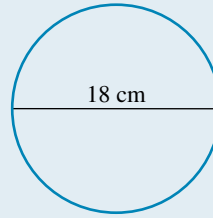
WORKED EXAMPLE 13

Find the area of each of the following circles correct to 2 decimal places.

a



b



THINK

- a**
- 1 Write the formula for the area of a circle.
 - 2 Substitute 20 for r and $3.14\dots$ for π .
 - 3 Evaluate (square the radius first) and include the correct units.
- b**
- 1 Write the formula for the area of a circle.
 - 2 We need radius, but are given the diameter. State the relation between the radius and the diameter.
 - 3 Halve the value of the diameter to get the radius.
 - 4 Substitute 9 for r and $3.14\dots$ for π .
 - 5 Evaluate (square the radius first) and include the correct units.

WRITE

a $A = \pi r^2$

$$\begin{aligned} A &= 3.14\dots \times 20^2 \\ &= 3.14\dots \times 400 \\ &= 1256.637\dots \\ &= 1256.64 \text{ cm}^2 \end{aligned}$$

b $A = \pi r^2$

$$\begin{aligned} D &= 18; r = D \div 2 \\ r &= 18 \div 2 \\ &= 9 \\ A &= 3.14\dots \times 9^2 \\ &= 3.14\dots \times 81 \\ &= 254.469\dots \\ &= 254.47 \text{ cm}^2 \end{aligned}$$

9.5.2 The area of quadrants, semicircles and sectors

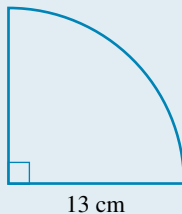
- We can use the formula for the area of a circle to calculate the area of quadrants, semicircles and sectors.
- A quadrant is a quarter circle, so the area of a quadrant is a quarter of the area of a circle. Similarly, the area of a semicircle is half the area of a circle.
- The area of a sector is $\frac{\theta}{360} \times$ the area of the circle.

$$A_{\text{sector}} = \frac{\theta}{360} \times \pi r^2$$

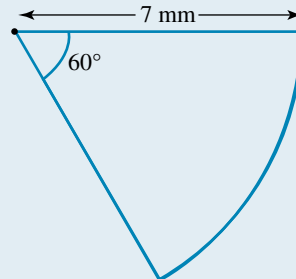
WORKED EXAMPLE 14

Find the area of each of the following shapes correct to 2 decimal places.

a



b



THINK

a 1 This shape is a quadrant, so the formula for the area is a quarter of the formula for the area of a circle.

2 Substitute $r = 13$ into the formula.

3 Evaluate and round the answer correct to 2 decimal places. Include the units

b 1 This sector is $\frac{60}{360}$ of a circle. Write the formula for its area.

2 Substitute $r = 7$ into the formula.

3 Evaluate and round the answer correct to 2 decimal places. Include the units

WRITE

$$\mathbf{a} \quad A = \frac{1}{4} \times \pi r^2$$

$$A = \frac{1}{4} \times \pi \times 13^2$$

$$= 132.732\dots$$

$$= 132.73 \text{ cm}^2$$

$$\mathbf{b} \quad A = \frac{60}{360} \times \pi r^2$$

$$A = \frac{60}{360} \times \pi \times 7^2$$

$$= 25.656\dots$$

$$= 25.66 \text{ mm}^2$$



RESOURCES — ONLINE ONLY

Interactivity: Area of circles (int-3788)

Digital doc: Investigation — Area of a circle (doc-2321)

Exercise 9.5 Area of a circle

Individual pathways

PRACTISE

Questions:
1–3, 5a–d, 7, 10, 11

CONSOLIDATE

Questions:
1–4, 5a–f, 6, 7, 10, 11, 13

MASTER

Questions:
1a, c, f, 2b, d, 3, 4a, 5–15

Individual pathway interactivity: int-4441

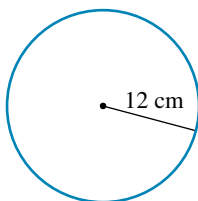
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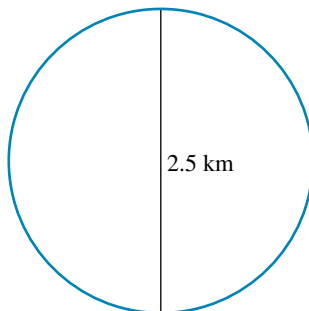
Understanding and fluency

1. WE13 Find the area of each of the following circles correct to 2 decimal places.

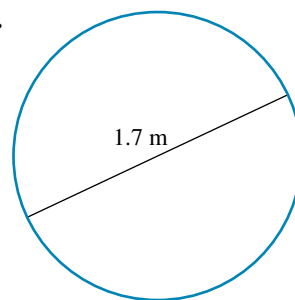
a.

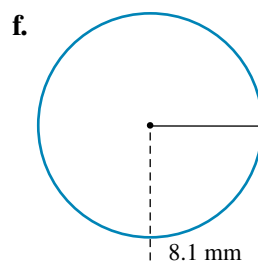
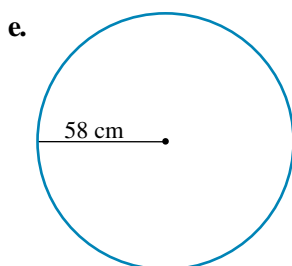
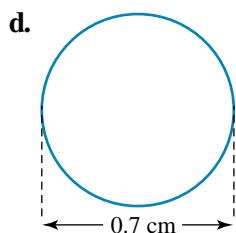


b.



c.

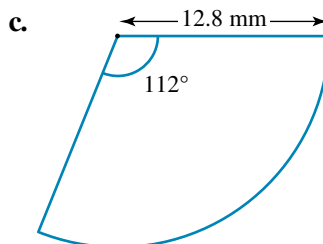
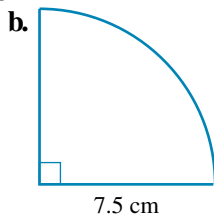
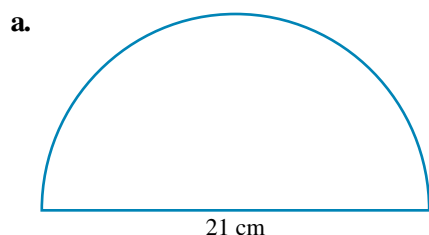




2. Find the area, correct to 2 decimal places, of:

- a. a circle of radius 5 cm
- b. a circle of radius 12.4 mm
- c. a circle of diameter 28 m
- d. a circle of diameter 18 cm.

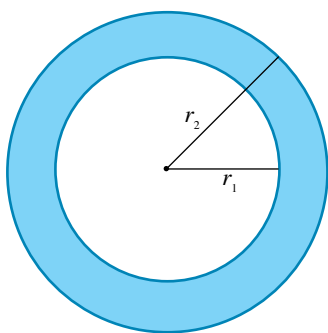
3. Find the area of each of the following shapes correct to 2 decimal places.



4. Find the radius, correct to 2 decimal places, of:

- a. a circle of area 50 cm^2
- b. a circle of area 75 mm^2
- c. a circle of area 333 cm^2 .

5. The word annulus is the Latin word for *ring*. An annulus is the shape formed between two circles with a common centre (called concentric circles). To find the area of an annulus, calculate the area of the smaller circle and subtract it from the area of the larger circle. Find the area of the annulus, correct to 2 decimal places, for each of the following sets of concentric circles.

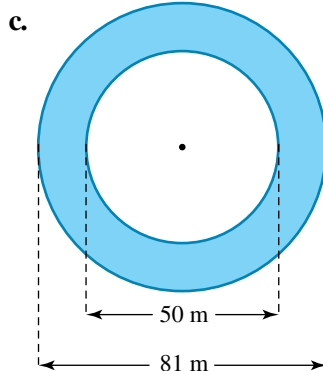
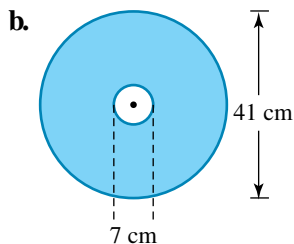
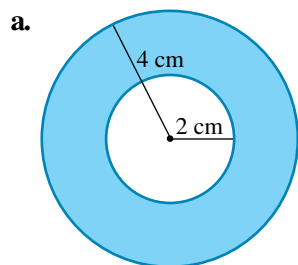


An annulus is the shaded area between the concentric circles.

$$r_1 = \text{radius of smaller circle}$$

$$r_2 = \text{radius of larger circle}$$

$$\text{Area}_{\text{annulus}} = \pi r_2^2 - \pi r_1^2$$

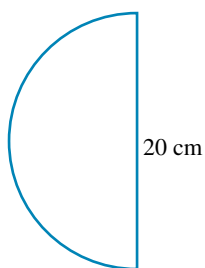


6. Find the area of each of the following shapes. Give your answers:

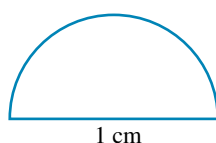
i. in terms of π .

ii. to 2 decimal places.

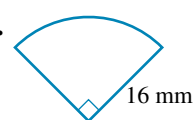
a.



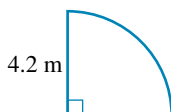
b.



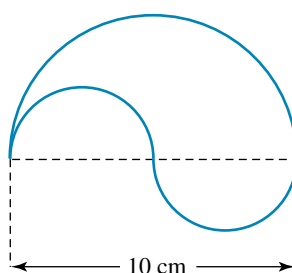
c.



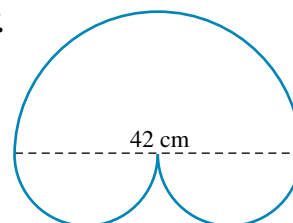
d.



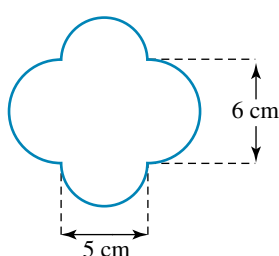
e.



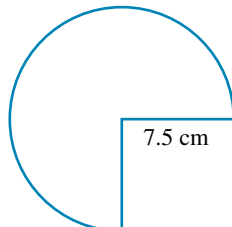
f.



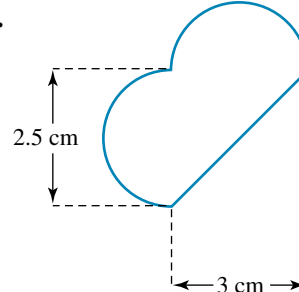
g.



h.



i.



7. Find the minimum area of aluminium foil, correct to 2 decimal places, that could be used to cover the top of the circular tray with diameter 38 cm.

8. What is the area of material in a circular mat of diameter 2.4 m? Give your answer correct to 2 decimal places.

9. How many packets of lawn seed should Joanne buy to sow a circular bed of diameter 27 m, if each packet of seed covers 23 m^2 ?

10. A landscape gardener wishes to spread fertiliser on a semicircular garden bed that has a diameter of 4.7 m. How much fertiliser is required if the fertiliser is applied at the rate of 20 g per square metre? Give your answer to the nearest gram.

Communicating, reasoning and problem solving

11. Investigate what happens to the diameter of a circle if its area is:

a. doubled

b. quadrupled

c. halved

d. changed by factor of n .

12. Investigate what happens to the circumference of a circle if its area is:

a. doubled

b. quadrupled

c. halved

d. changed by factor of n .

13. The total area of wood used to make a set of six identical round coasters is 425 cm^2 .

a. Calculate the area of the wood used in each coaster.

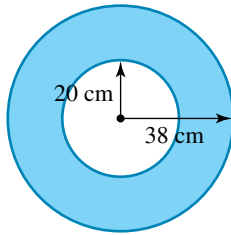
b. Calculate the radius of each coaster correct to 2 decimal places.

c. An 8.5 cm wide cylindrical coffee mug is placed on one of the coasters. Use your answer to part b to decide whether it will fit entirely within the coaster's surface.

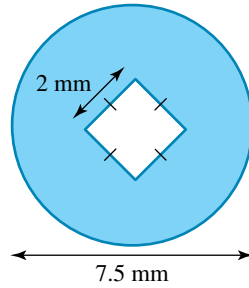


14. Calculate the shaded area in each of the following shapes.

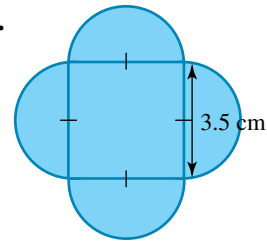
a.



b.



c.

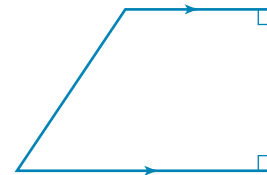
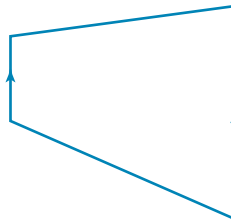


15. Think of a way to remember the difference between the formula for the circumference and the formula for the area of a circle.

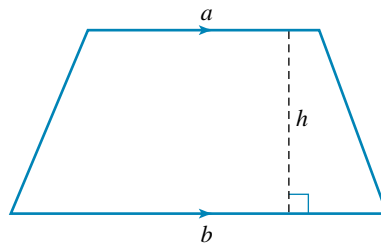
9.6 Area of trapeziums

9.6.1 The area of trapeziums

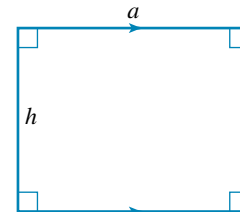
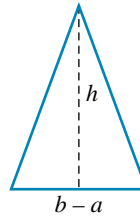
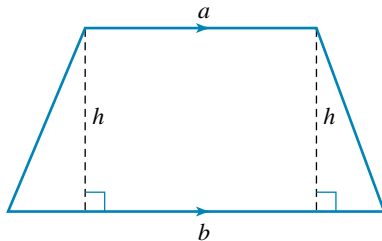
- A **trapezium** is a quadrilateral with one pair of parallel unequal sides.
- The following figures are all trapeziums.



- The height of the trapezium is perpendicular to each of its parallel bases.



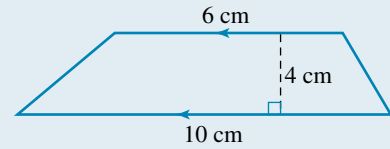
- To determine the area of a trapezium, draw two lines to create two triangles and one rectangle as shown.



- Placing the two triangles together creates one triangle and one rectangle.
- The total area of the two shapes, A , will give $A = a \times h + \frac{1}{2} \times h \times (b - a)$.
- Simplifying this gives $A = \frac{1}{2}(a + b) \times h$.
- The area of a trapezium, A , is given by the formula $A = \frac{1}{2}(a + b) \times h$, where a and b are the lengths of parallel sides and h is the height of the trapezium.

WORKED EXAMPLE 15

Find the area of the trapezium at right.



THINK

- 1 Write the formula for the area of the trapezium.
- 2 Identify the values of a , b and h .
Note: It does not matter which of the parallel sides is a and which one is b , since we will need to add them together.
- 3 Substitute the values of a , b and h into the formula.
- 4 Evaluate (work out the brackets first) and include the correct units.

WRITE

$$A = \frac{1}{2}(a + b) \times h$$

$$a = 10, b = 6 \text{ and } h = 4$$

$$A = \frac{1}{2} \times (10 + 6) \times 4$$

$$\begin{aligned} A &= \frac{1}{2} \times 16 \times 4 \\ &= 32 \text{ cm}^2 \end{aligned}$$

learnon RESOURCES — ONLINE ONLY

Interactivity: Area of trapeziums (int-3789)

Digital doc: Investigation — Area of a trapezium (doc-2320)

Digital doc: WorkSHEET Measurement I (doc-2322)

Exercise 9.6 Area of trapeziums

Individual pathways

PRACTISE

Questions:
1–4, 8, 10

CONSOLIDATE

Questions:
1b, d, e, f, 2, 3, 5, 7, 8, 10, 11

MASTER

Questions:
1b, e, f, 2, 3–11

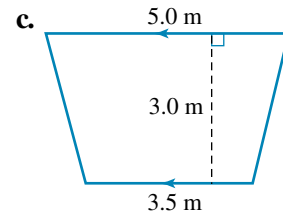
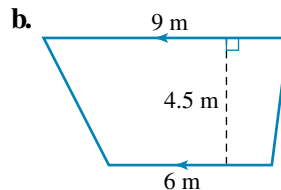
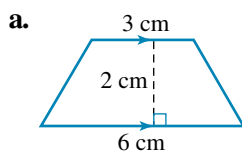
Individual pathway interactivity: int-4442

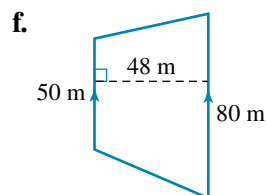
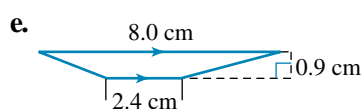
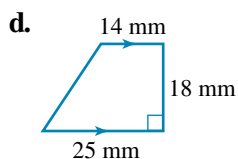
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Understanding and fluency

1. **WE15** Find the area of each of the following trapeziums.





2. **MC** Which of the following is the correct way to calculate the area of the trapezium shown?

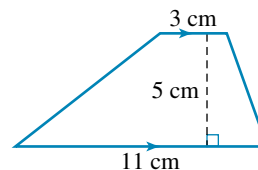
A. $\frac{1}{2} \times (3 + 5) \times 11$

B. $\frac{1}{2} \times (3 + 5 + 11)$

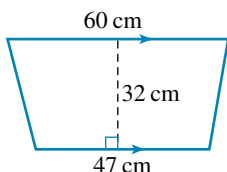
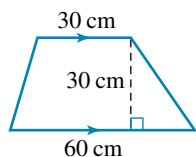
C. $\frac{1}{2} \times (11 - 3) \times 5$

D. $\frac{1}{2} \times (11 + 5) \times 3$

E. $\frac{1}{2} \times (3 + 11) \times 5$



3. A dress pattern contains these two pieces.



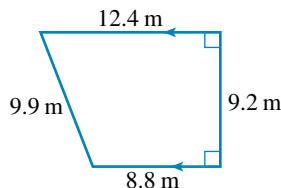
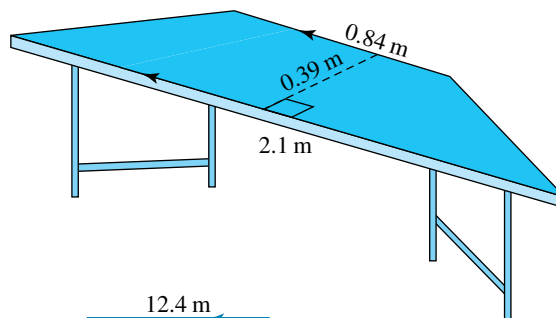
Find the total area of material needed to make both pieces.

4. A science laboratory has four benches with the dimensions shown at right.

What would be the cost, to the nearest 5 cents, of covering all four benches with a protective coating that costs \$38.50 per square metre?

5. Stavros has accepted a contract to concrete and edge the yard, the dimensions of which are shown in the figure below right.

- What will be the cost, to the nearest 5 cents, of concreting the yard if concrete costs \$28.00 per square metre?
- The yard must be surrounded by edging strips, which cost \$8.25 per metre. Find:
 - the cost of the edging strips, to the nearest 5 cents
 - the total cost of materials for the job.



6. The side wall of this shed is in the shape of a trapezium and has an area of 4.6 m^2 . Find the perpendicular distance between the parallel sides if one side of the wall is 2.6 m high and the other 2 m high.

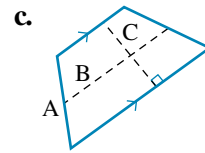
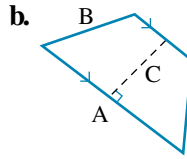
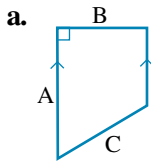
7. **MC** Two trapeziums have corresponding parallel sides of equal length. The height of the first trapezium is twice as large as the height of the second. The area of the second trapezium is:

- twice the area of the first trapezium
- half the area of the first trapezium
- quarter of the area of the first trapezium
- four times the area of the first trapezium
- impossible to say



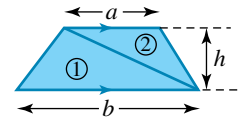
Communicating, reasoning and problem solving

8. Which side (A, B or C) represents the height of the following trapeziums?



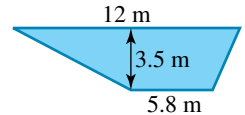
9. The formula for the area of a trapezium can be proved by dividing it into two triangles as shown.

- Find the area of triangle 1 in terms of the pronumerals shown in the diagram.
- Find the area of triangle 2 in terms of the pronumerals shown in the diagram.
- Find the area of the trapezium by adding the areas of the two triangles together.



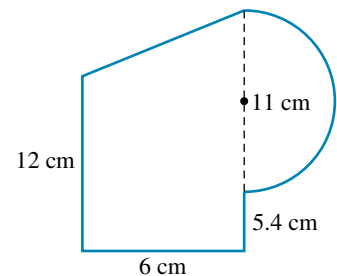
10. A section of a garden is in the shape of a trapezium as shown at right.

- Calculate the area of this section of the garden.
- This section of the garden is to be covered with mulch. According to the information on the pack, each bag contains enough mulch to cover 5 m^2 of surface. How many bags of mulch are needed to complete the job?
- If each bag costs \$8.95, calculate the total cost of the mulch.



11. Calculate the area of the shape in the figure at right as:

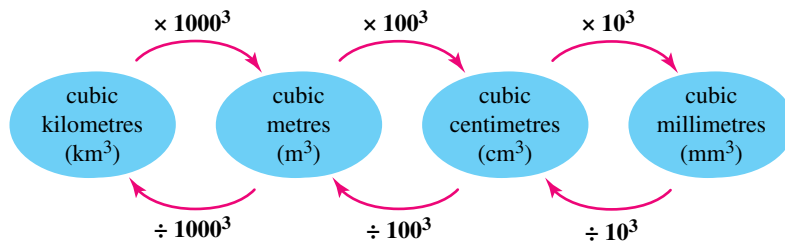
- an exact value
- rounded to 5 significant digits.



9.7 Volume of prisms and other solids

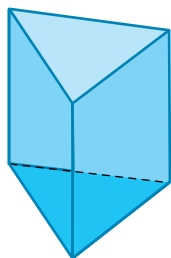
9.7.1 Volume

- Volume is the amount of space inside a three-dimensional object.
- Volume is measured in cubic units such as mm^3 , cm^3 or m^3 .
- Volume units can be converted using the chart below.

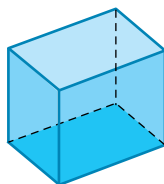


9.7.2 Prisms

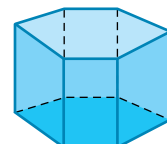
- Prisms are solid shapes with identical opposite polygonal ends which are joined by straight edges. They are three-dimensional objects that can be cut into identical slices, called cross-sections.
- Prisms are named according to the shape of their cross-section. The objects below are all prisms.



Triangular prism

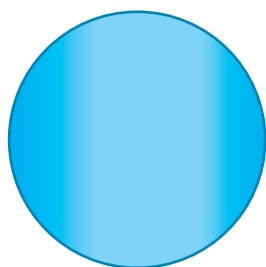


Rectangular prism

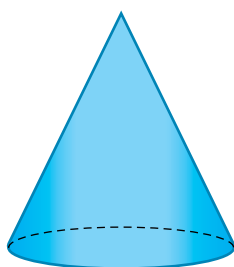


Hexagonal prism

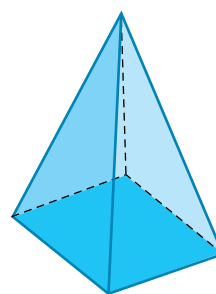
- Objects that do not have a uniform cross-section can not be classified as prisms. For example, the objects below are not prisms.



Sphere

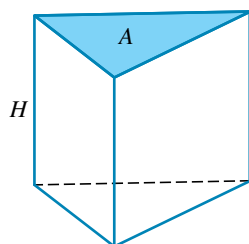


Cone



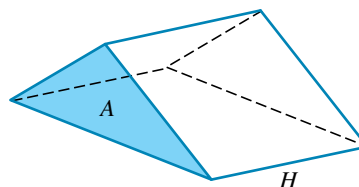
Square pyramid

- The volume of any prism is given by the formula $V = A \times H$, where A is the cross-sectional area of a prism and H is the height of a prism.
- The volume of a rectangular prism is given by the formula $V = lbh$, where l is the length, b is the breadth and h is the height of the prism.
- A rectangular prism can be cut to form two equal triangular prisms.
- The volume of a cube is given by the formula $V = l^3$, where l is the side length of a cube.
- The height of the prism is not necessarily the height of the object in a true sense of the word. It is just the dimension perpendicular to the cross-section.



$$V = A \times H$$

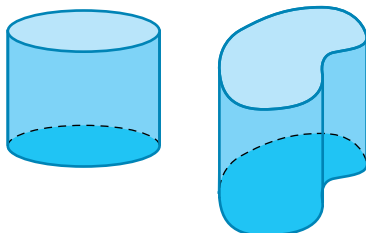
The base of this prism, A , is its cross-section;
 H is the height of the prism.



The base of this prism *is not* its cross-section.
 H represents the depth (or length) of the prism;
 A is its cross-section.

9.7.3 Volume of solids with uniform cross-section that are not prisms

- The objects with a uniform cross-section whose ends are not polygons (i.e. those whose ends are not joined by straight edges) can not be classified as prisms. For example, the shapes below are not prisms, even though they have uniform cross-sections.

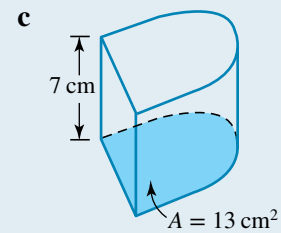
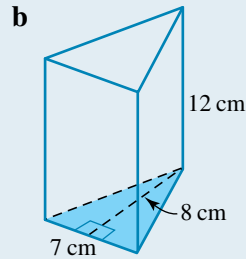
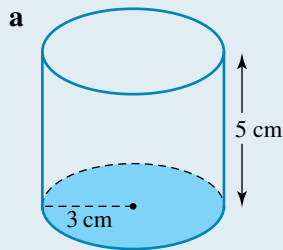


- The formula $V = AH$, where A is the cross-sectional area and H is the dimension perpendicular to it, will give the volume of any solid with a uniform cross-section, even if it is not a prism.

- The cross-section of a cylinder is a circle; therefore the volume for a cylinder is given by the formula $V = \pi r^2 h$, where r is the radius of the base and h is the perpendicular height.

WORKED EXAMPLE 16

Find the volume of each of the following. Give answers correct to 2 decimal places where appropriate.



THINK

- a**
- 1 Write the formula for the volume of a cylinder.
 - 2 State the value of r and h .
 - 3 Substitute the values of r and h into the formula and evaluate. Include the correct units in your answer.
- b**
- 1 Write the formula for the volume of a prism.
 - 2 Identify the shape of the cross-section and, hence, write the formula to find its area.
 - 3 State the values of the variables.
(Note: h is the height of the triangle, *not* of the prism.)
 - 4 Substitute the values of b and h into the formula and evaluate.
 - 5 State the value of H , the height of the prism.
 - 6 To find the volume of the prism, multiply the cross-sectional area by the height and include the correct units.
- c**
- 1 Write the formula for the volume of the given shape.
 - 2 State the values of the cross-sectional area and the height of the shape.
 - 3 Multiply the cross-sectional area by the height and include the correct units.

WRITE

a $V = \pi r^2 h$

$$r = 3, h = 5$$

$$V = 3.14... \times 3^2 \times 5$$

$$= 141.371...$$

$$= 141.37 \text{ cm}^3$$

b $V = A \times H$

$$A_{\text{triangle}} = \frac{1}{2}bh$$

$$b = 7, h = 8$$

$$A = \frac{1}{2} \times 7 \times 8$$

$$= 28 \text{ cm}^2$$

$$H = 12$$

$$V = 28 \times 12$$

$$= 336 \text{ cm}^3$$

c $V = A \times H$

$$A = 13, H = 7$$

$$V = 13 \times 7$$

$$= 91 \text{ cm}^3$$

Interactivity: Volumes of prisms (int-2754)

Interactivity: Conversion chart for volume (int-3791)

Interactivity: Prisms? (int-3792)

Interactivity: Volumes of solids (int-3794)

Digital doc: SkillSHEET Volume of cubes and rectangular prisms (doc-6959)

Digital doc: SpreadSHEET Volume of a prism (doc-2324)

Exercise 9.7 Volume of prisms and other solids

Individual pathways

PRACTISE

Questions:

1, 2a, d, g, j, 3, 4, 7, 12

CONSOLIDATE

Questions:

1, 2b, e, h, k, 3, 5, 6, 8, 10, 12, 13

MASTER

Questions:

1, 2b, c, i, l, 4–15

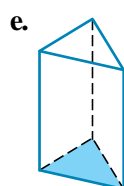
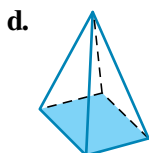
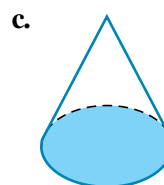
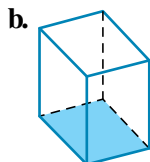
Individual pathway interactivity: int-4443

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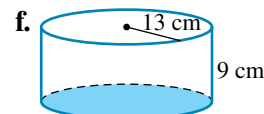
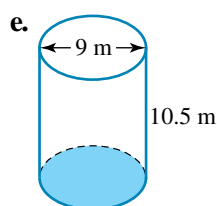
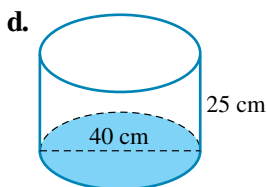
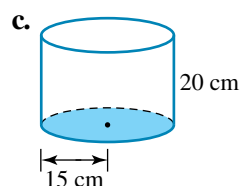
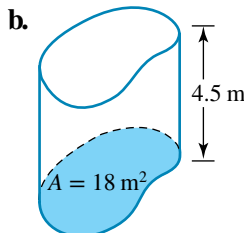
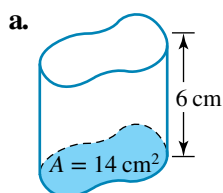
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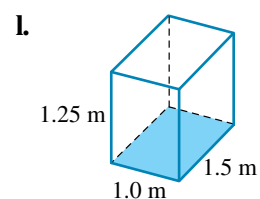
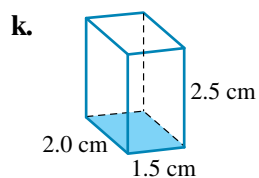
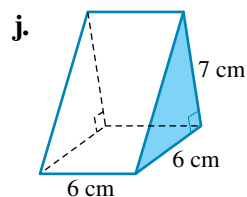
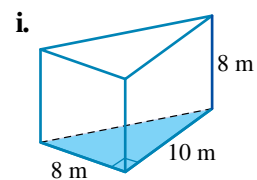
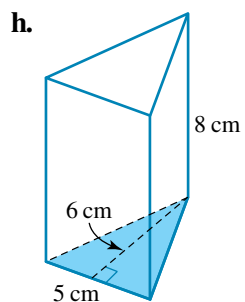
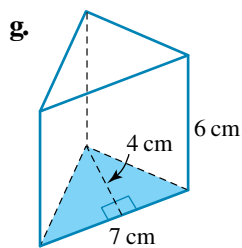
Understanding and fluency

1. Which of the three-dimensional shapes below are prisms?



2. **WE16** Find the volume of each of the following. Give your answers to 2 decimal places where appropriate.

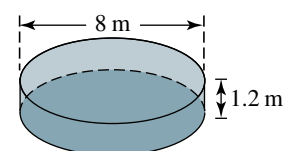




3. What volume of water will a rectangular swimming pool with dimensions shown in the following photograph hold if it is completely filled? The pool has no shallow or deep end. It is all the same depth.

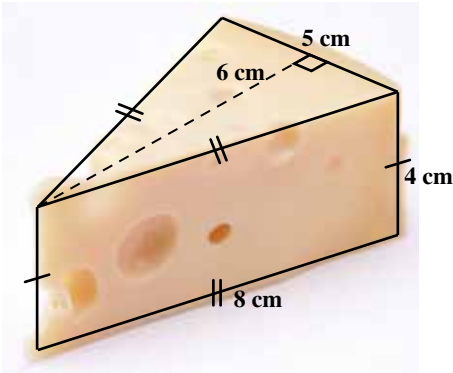


4. How many cubic metres of cement will be needed to make the cylindrical foundation shown in the figure at right? Give your answer to 2 decimal places.

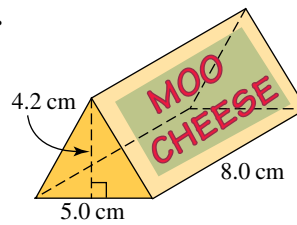


5. What are the volumes of these pieces of cheese?

a.

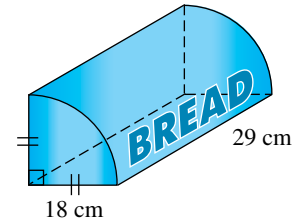
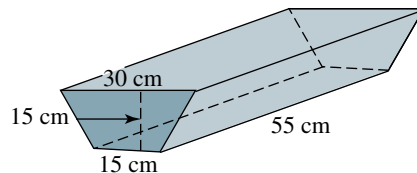


b.



6. What is the volume of the bread bin shown at right? Give your answer to the nearest whole number.

7. How much water will this pig trough, with dimensions shown in the figure below, hold if it is completely filled? Give your answer in litres correct to 1 decimal place. (*Hint*: 1 litre = 1000 cm³.)

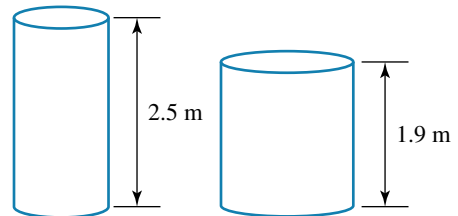


8. A cylindrical water bottle has a radius of 4 cm and a perpendicular height of 20 cm. Find the capacity of the water bottle in mL correct to the nearest whole number. (*Hint*: 1 mL = 1 cm³.)

Communicating, reasoning and problem solving

9. Explain the similarities between the volume formulas for cylinders and prisms.

10. A rectangular metal sheet with length 2.5 m and width 1.9 m is used to make a cylinder. The metal sheet can be rolled on its length or on its width. The two cylinders that can be formed are shown in the diagram.



a. Calculate the radius to one decimal place for:

i. the taller cylinder

ii. the shorter cylinder.

b. Calculate the two volumes.

c. Which volume is larger? Explain why this is the case.

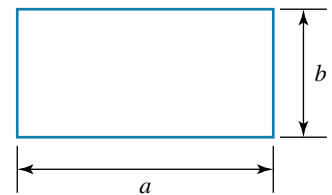
11. Consider the same situation as in question 10 but this time the metal sheet has length a and width b ($a > b$).

a. Write a formula for the radius of base of each cylinder.

b. Write a formula for each of the volumes.

c. Explain which volume is always going to be larger.

d. Does this explain your answer in the previous question?



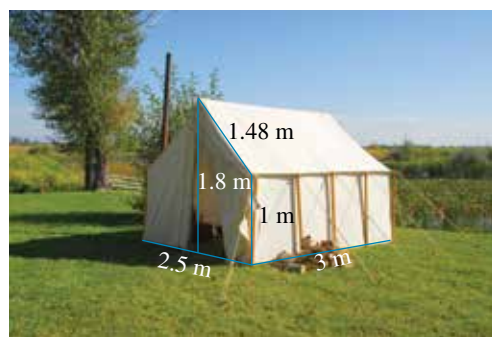
12. A new (unsharpened) pencil is in the shape of a hexagonal prism. If the area of the hexagonal end is 80 mm² and the volume of the whole pencil is 14.4 cm³, calculate the length of the pencil.



- 13.** Consider a set of three food containers of different sizes, each of which is in the shape of a rectangular prism.
- The smallest container is 6 cm long, 4 cm wide and 3 cm high. All the dimensions of the medium container are double those of the smallest one; all the dimensions of the largest container are triple those of the smallest one.



- Calculate the volume of the smallest container.
 - State the dimensions and hence calculate the volume of:
 - the medium container
 - the largest container.
 - Calculate the ratio of the volumes of:
 - the smallest container to the medium container
 - the smallest container to the largest container.
 - Consider your answers to part **c** and use them to copy and complete the following: If all dimensions of a rectangular prism are increased by a factor of n , the volume of the prism is increased by a factor of ____.
- 14.** A set of cabin tents is being made for a historical documentary. The dimensions of each tent are shown.
- Calculate the area of material required to make each tent, including the floor.
 - Calculate the amount of space inside the tent.
- 15.** How can you say if a solid has a uniform cross-section?



9.8 Time

9.8.1 Time — an introduction

- Time is one of our most useful systems of measurement. We can use it to work out how long we have been doing something, or how long we have until we must do something. We can use it to make future arrangements.
- Time is something we all use, every day.
- Time is divided into units. There are:

60 seconds	in	1 minute	12 months	in	1 year
60 minutes	in	1 hour	about 365 days	in	1 year
24 hours	in	1 day	10 years	in	1 decade
7 days	in	1 week	100 years	in	1 century
2 weeks	in	1 fortnight	1000 years	in	1 millennium.
about 4 weeks	in	1 month			
- We also use the word time to refer to an instant (e.g. 3 o'clock) rather than a period.



- A clock or watch can display the time in one of two ways.
 - Some clocks display the time in analogue form using hour, minute and second hands that move continuously as they point to numbers on the clock face.
 - Other clocks and watches display a set of digits that change in steps as time passes.

9.8.2 Time calculations

WORKED EXAMPLE 17

How many minutes are there in $4\frac{1}{4}$ hours?

THINK

1 Convert the mixed number to an improper fraction.

WRITE

$$\begin{aligned} 4\frac{1}{4} \text{ hours} &= \frac{4 \times 4 + 1}{4} \\ &= \frac{17}{4} \text{ hours} \end{aligned}$$

2 Multiply the improper fraction by the number of minutes in 1 hour.

$$\begin{aligned} \frac{17}{4} \text{ hours} &= \frac{17}{4} \times 60 \text{ minutes} \\ &= \frac{1020}{4} \text{ minutes} \end{aligned}$$

3 Evaluate.

$$= 255 \text{ minutes}$$

WORKED EXAMPLE 18

How many minutes are there in 5 days?

THINK

1 Convert the number of days to hours; that is, multiply 5 by 24 hours.

WRITE

$$\begin{aligned} 5 \text{ days} &= 5 \times 24 \text{ hours} \\ &= 120 \text{ hours} \end{aligned}$$

2 Evaluate.

3 Convert the number of hours to minutes; that is, multiply 120 by 60 minutes.

$$120 \text{ hours} = 120 \times 60$$

4 Evaluate.

$$= 7200 \text{ minutes}$$

WORKED EXAMPLE 19

Change the following into hours and minutes.

a 300 minutes

b 425 minutes

THINK

WRITE

a 1 Convert minutes to hours; that is, divide 300 by 60.

$$\begin{aligned} \text{a } 300 \text{ minutes} &= 300 \div 60 \\ &= 5 \text{ hours} \end{aligned}$$

2 Evaluate.

b 1 Convert minutes to hours; that is, divide 425 by 60.

$$\begin{aligned} \text{b } 425 \text{ minutes} &= 425 \div 60 \\ &= \frac{425}{60} \\ &= 7 \text{ remainder } 5 \end{aligned}$$

2 Evaluate.

3 Write the answer in hours and minutes.

$$425 \text{ minutes} = 7 \text{ hours } 5 \text{ minutes}$$

- The suffixes am and pm are used to indicate morning and afternoon, and are derived from *ante meridiem* (meaning before midday) and *post meridiem* (after midday).
- *Note:* Noon (or midday) may be written as 12.00 pm, and midnight may be written as 12.00 am.
- One way to find time differences is to mentally calculate how long it is to the next hour, then to the next noon or midnight. The following worked example illustrates this technique.



WORKED EXAMPLE 20

Find the time difference between:

a 7.15 pm and 10.25 pm

b 2.50 am and 8.20 pm

c 3.40 am on Tuesday and 5.10 pm on Wednesday (next day).

THINK

- 1 Construct a vertical column starting at 7.15 pm and ending at 10.25 pm. Set up two columns with the headings 'hours' and 'minutes' next to the first time.
- 2 Write key times between the start and finish times.
- 3 Calculate the times added to get each key time, and write them under the hours and minutes headings as shown.
- 4 Add the time differences to find the total time difference.
- 5 Convert any minute values over 60 to hours and minutes and adjust the answer.
- 6 Answer the question.

b Repeat steps 1 to 6 from part **a**.

c Repeat steps 1 to 6 from part **a**.

WRITE

a

	Hours	Minutes
7.15 pm		
8.00 pm		45
10.25 pm	2	25
Total time	2	70
	= 3	10

The time difference is 3 hours 10 minutes.

b

	Hours	Minutes
2.50 am		
3.00 am		10
12.00 pm	9	00
8.20 pm	8	20
Total time	17	30

The time difference is 17 hours 30 minutes.

c

	Hours	Minutes
3.40 am Tuesday		
4.00 am Tuesday		20
4.00 am Wednesday	24	00
12.00 pm Wednesday	8	00
5.00 pm Wednesday	5	10
Total time	37	30

The time difference is 37 hours 30 minutes.

ACTIVITY: ADDING AND SUBTRACTING TIME USING A CALCULATOR

Most calculators contain a DMS button, which stands for degrees, minutes and seconds. You can enter time in hours, minutes and seconds using this button. Enter the number of hours and press the button, then enter the number of minutes and press the button again, finally enter the number of seconds.

Try adding and subtracting time using the button on your DMS calculator.

learnon RESOURCES — ONLINE ONLY

Interactivity: Time conversion (int-3795)

Interactivity: Time calculations (int-3795)

Digital doc: SkillsHEET Reading the time (doc-6960)

Digital doc: SpreadSHEET Time difference (doc-3445)

Exercise 9.8 Time

Individual pathways

PRACTISE

Questions:

1–7, 8a, d, g, 9a, d, g, 10, 11, 12a, c, 13–16, 24, 27, 29, 31, 32

CONSOLIDATE

Questions:

1–7, 8b, e, h, 9b, e, h, 10a, d, g, i, l, 11, 12b, d, 13–18, 22, 25, 27–33

MASTER

Questions:

1–5, 6b, e, h, 7c, f, i, 8c, f, i, 9c, g, h, i, 10c, h, i, k, 11a, g, k, 12b, d, 13–34

Individual pathway interactivity: int-4444

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Understanding and fluency

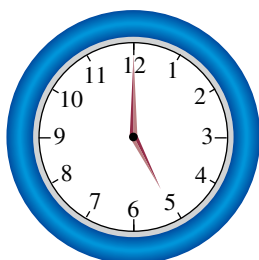
1. Match the following activities with the most likely amount of time.

a. a cricket test match	1 year
b. writing your name	10 seconds
c. eating breakfast	15 minutes
d. building a house	5 days
e. flying time from Melbourne to Hong Kong	7 hours
f. being in Year 7	6 months

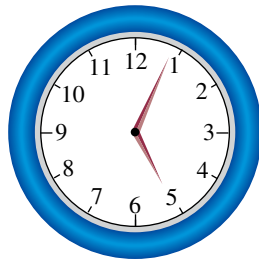


2. What are the times shown on each of the following analogue clocks?

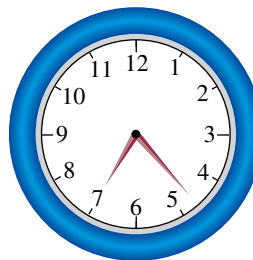
a.

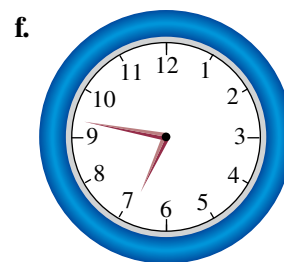
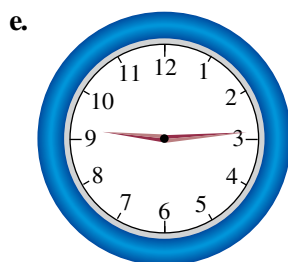
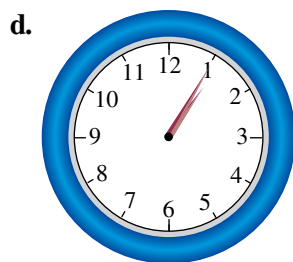


b.



c.





3. For each of the following, draw a 12-hour clock face and show the time.

a. 8.20 am	b. 8.45 pm	c. 10.50 am	d. 12.00 am	e. 11.05 pm
f. 5.11 pm	g. 7.32 pm	h. 9.24 am	i. 11.16 am	

4. Today we use clocks and watches to tell the time. Name two other devices that have been used in the past to measure time.

5. Explain the difference between 2 hours 25 minutes and 2.25 hours.

6. **WE17,18** How many minutes are there in each of the following periods?

a. 2 hours	b. $2\frac{1}{2}$ hours	c. $3\frac{1}{4}$ hours	d. $\frac{1}{2}$ hour
e. $\frac{3}{4}$ hour	f. $7\frac{3}{4}$ hours	g. 1 day	h. 9 days

7. **WE19** Change the following to hours and minutes.

a. 200 minutes	b. 185 minutes	c. 160 minutes	d. 230 minutes	e. 405 minutes
f. 95 minutes	g. 610 minutes	h. 72 minutes	i. 305 minutes	

8. Change the following to minutes.

a. 1 hour 15 minutes	b. 2 hours 10 minutes	c. 1 hour 50 minutes
d. 4 hours 25 minutes	e. 7 hours 35 minutes	f. 3 hours 12 minutes
g. $16\frac{3}{4}$ hours	h. $5\frac{1}{2}$ hours	i. $6\frac{1}{4}$ hours

9. How many:

a. days in a week	b. weeks in a year	c. seconds in a minute
d. seconds in an hour	e. seconds in a day	f. hours in a day
g. hours in a year	h. minutes in a day	i. minutes in a year?

Note: Assume 1 year = 365 days.

10. What is the time:

a. 1 hour after 4.00 pm	b. 1 hour before 4.00 pm
c. 1 hour after 5.30 am	d. 1 hour 20 minutes after half past 10 am
e. 1 hour 20 minutes before 10.00 am	f. 3 hours 18 minutes after 2.00 pm
g. 2 hours 30 minutes before 4.30 am	h. 4 hours 35 minutes after quarter to 10 am
i. 4 hours 35 minutes before 10.05 pm	j. 5 hours 27 minutes after 1.08 am
k. 5 hours 27 minutes before 1.08 am	l. 1 hour 10 minutes before 11.30 am?

11. **WE20a,b** Find the time difference between:

a. 8.20 pm and 8.35 pm	b. 4.15 pm and 5.20 pm
c. 7.15 am and 8.28 am	d. 5.17 am and 6.32 am
e. 9.15 pm and 10.08 pm	f. 3.16 pm and 5.09 pm
g. 11.28 pm and midnight	h. 9.21 am and 1.06 pm
i. 11.10 am and 4.25 pm	j. 6.05 am and 6.05 pm
k. half past 6 pm and quarter to 3 am.	

12. **WE20c** Find the time difference between:

- | |
|---|
| a. 7.20 am on Monday and 6.30 pm the next day, Tuesday |
| b. 4.38 am on Saturday and 1.25 pm the next day, Sunday. |
| c. 8.45 pm on Wednesday and 10.16 am the next day, Thursday |
| d. 1.20 pm on Wednesday and 9.09 am the first Friday. |

13. **MC** 225 minutes is the same as:
A. 2 hours 25 minutes **B.** 2 hours 15 minutes **C.** 3 hours 45 minutes
D. 3 hours 25 minutes **E.** 2 hours 45 minutes
14. **MC** If the time was 8.45 pm, what would be the time 5 hours 20 minutes later?
A. 1.05 am **B.** 1.05 pm **C.** 2.05 pm **D.** 2.05 am **E.** 1.10 pm
15. **MC** If a train takes 2 hours and 18 minutes to arrive at its destination and it arrives at 6.03 pm, at what time did it leave?
A. 3.45 pm **B.** 8.21 pm **C.** 4.15 pm **D.** 3.21 pm **E.** 3.45 am
16. If it takes Joanne 16 minutes to write one page of a letter, how long will it take her to write a letter of three pages?
17. Mathew spends half an hour doing homework each week night. How many hours of homework has he done after 3 weeks?
18. If the time is now 7.55 am in Sydney, what is the time in Adelaide if South Australian time is half an hour behind New South Wales time?
19. If the time is now 8.30 am in Sydney, what is the time in Perth if Western Australia is two hours behind New South Wales?
20. If the time is 9.45 pm in Perth, what is the time in Sydney? (See question 19.)
21. If the time is 10.05 pm in Adelaide, what is the time in Sydney? (See question 18.)
22. If Susan takes 25 minutes to walk to the station, waits for the train for 7 minutes, travels on the train for 12 minutes and then takes 8 minutes to walk to her friend's house, how long has she taken altogether?
23. If James spends 35 minutes on Friday, 2 hours and 12 minutes on Saturday and $1\frac{1}{4}$ hours on Sunday to complete his assignment, how much time did he take altogether?
24. Erika completes her mathematics homework, consisting of 13 questions, in 44 minutes. How long on average did it take her to complete each questions? Round your answer to the nearest minute.
25. Julio is a basketball fanatic and practises shooting hoops for 11 hours each week. How much is this on average per day? Round your answer to the nearest minute.
26. In the month of July Sandra spends 115 hours testing the new platform for her company intranet. If Sandra worked for 21 days in July, on average how much time each day did Sandra spend testing the new platform? Round your answer to the nearest hour.

Communicating, reasoning and problem solving

27. Tommy completes a calculation involving time on his calculator, and the answer is displayed as 3.25 hours. Tommy gives his answer as 3 hours, 25 minutes. Was Tommy correct? If not, what should Tommy's answer have been?
28. The first day of a school term starts at 9.00 am on 2 February and the last day of the term finishes at 3.30 pm on 6 April. How many months, days, hours and minutes did the term last?
29. In a mirror reflection, the time on an analogue clock face appears to be 25 minutes to 2. What time is it?
30. When would an analogue clock and its mirror image appear exactly the same? Explain your answer.
31. You have a training schedule for tennis, shown at right. If you follow the schedule, how long will you train each week?
32. You are allowed to stay up until 10:30 pm on Saturday night. What time will you need to start a DVD if you watch:
a. *Finding Nemo* (97 minutes)
b. *Aladdin* (86 minutes)
c. *Star Wars* (124 minutes)?
33. A classmate attempted to calculate some time differences. Explain where he may have gone wrong in his calculations and then find the correct answers.
- | | |
|---------|-----------|
| a. 2240 | b. 9 : 22 |
| -2005 | -7 : 38 |
| 235 min | 1 : 84 |
34. The battery on your new MP3 player is supposed to last for 10 hours. If you listen to music on your MP3 player for 40 minutes per day, how many days will the battery last until you have to charge it again?



9.9 24-hour clock and time zones

9.9.1 The 24-hour clock

- Sometimes the time is given using a 24-hour system or 24-hour clock.
- The hours are numbered from 1 to 24, beginning at midnight.
- The times from midnight to 12.59 pm look very similar to the 12-hour time. At 1.00 pm you need to add 12 hours, making the time 1300 hours.
- Times in the 24-hour system are written as ‘fourteen hundred hours’ (1400) or ‘twenty-three thirty hours’ (2330).
- Notice that in 24-hour time, we always use 4 digits. Some examples are given in the table below.

12-hour time	24-hour time
6.00 am	0600 hours
10.00 am	1000 hours
12.30 pm	1230 hours
2.00 pm	1400 hours (add 12 hours to 2.00)
8.00 pm	2000 hours (add 12 hours to 8.00)
11.30 pm	2330 hours (add 12 hours to 11.30)

WORKED EXAMPLE 21

Find the difference in hours and minutes between the following 24-hour times:

- a** 0635 and 2150 **b** 1055 and 1543.

THINK

- a** 1 Set up two columns with the headings ‘hours’ and ‘minutes’.
- 2 Write the values, putting the highest hour value first.
- 3 Calculate by subtraction.
- 4 Write as hours and minutes.
- b** 1 Set up two columns with the headings ‘hours’ and ‘minutes’.
- 2 Write the values, putting the highest hour value first.
- 3 You cannot take 55 minutes from 43 minutes.
Convert 1 hour into 60 minutes and add it to the minutes column.
Then rewrite 1543 as 14 103.
(Using 14 103 helps us to do the subtraction, but it is not a real time.)
- 4 Calculate by subtraction.
- 5 Write as hours and minutes.

WRITE

- a** Hours Minutes

$$\begin{array}{r} 21 \quad 50 \\ 06 \quad 35 \\ \hline 15 \quad 15 \end{array}$$

15 hours 15 minutes

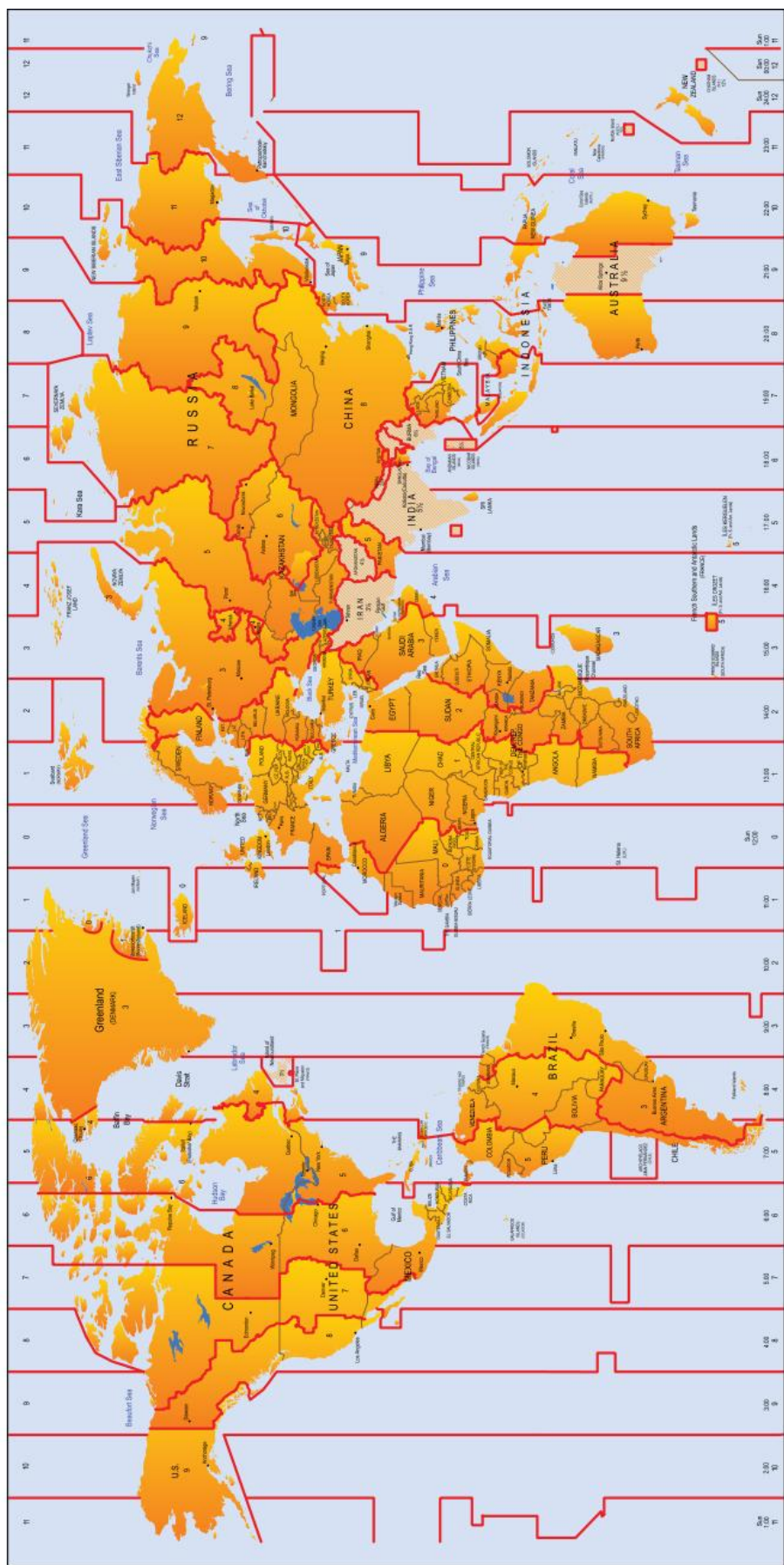
- b** Hours Minutes

$$\begin{array}{r} 15 \quad 43 \\ 10 \quad 55 \\ \hline 14 \quad 103 \\ 10 \quad 55 \\ \hline 4 \quad 48 \end{array}$$

4 hours 48 minutes

9.9.2 Time zones

- Time changes as we travel around the world. In Europe it is night time when in Australia it is day and it is day time in Europe when we have night.
- To help everyone know what time it is in different countries, the world is divided into 24 time zones. (See the map on the next page.) Most of the time zones are offset by one hour, but a few locations use different offsets, changing their times to be 30 or 45 minutes from the standard one-hour zone.



Time in Perth	Time in Melbourne	Time in Jakarta
3.00 pm	5.00 pm	2.00 pm
6.00 pm	8.00 pm	5.00 pm
10.00 am	Midday	9.00 am
10.00 pm	Midnight	9.00 pm

- If we travel east, we need to set our watches ahead.
- If we travel west, we need to set our watches back.
- Australia is divided into three time zones.
 - Queensland, New South Wales, the Australian Capital Territory, Victoria and Tasmania observe Eastern Standard Time (EST).
 - The Northern Territory and South Australia observe Central Standard Time (CST), which is $\frac{1}{2}$ hour behind EST.
 - Western Australia observes Western Standard Time (WST), which is 2 hours behind EST.
- The time in England is called Greenwich Mean Time (GMT) and is 10 hours behind EST.

Time in London (GMT)	Time in Jakarta	Time in Perth	Time in Melbourne (EST)
7.00 am	2.00 pm	3.00 pm	5.00 pm
4.00 pm	11.00 pm	Midnight	2.00 am (next day)
Midday	7.00 pm	8.00 pm	10.00 pm
Midnight	7.00 am	8.00 am	10.00 am
3.00 am	10.00 am	11.00 am	1.00 pm

WORKED EXAMPLE 22

a What is the time in Melbourne if the time in London (GMT) is 8.00 am?

b What is the time in London (GMT) if the time in Melbourne is 8.00 am?

THINK

- a** 1 Melbourne is 10 hours ahead of London. Add 10 hours to the London time.
- 2 Calculate.
- b** 1 London is 10 hours behind Melbourne. Subtract 10 hours from the Melbourne time.
- 2 Calculate.

WRITE

- a** London time: 8.00 am
Melbourne time: 8.00 am + 10 hours
It is 6.00 pm.
- b** Melbourne time: 8.00 am
London time: 8.00 am – 10 hours
It is 10.00 pm the previous day.

9.9.3 Daylight-saving time

- Many countries around the world have daylight-saving time during summer so that people can make the most of the warm weather.
- When daylight-saving time begins, clocks are turned forward by 1 hour at 2.00 am.
- At the end of daylight-saving time, the clocks are then turned back by 1 hour at 3.00 am.
- In Australia, the states Queensland and Western Australia, and the Northern Territory do not observe daylight-saving time, while the Australian Capital Territory and other states do.
- During daylight-saving, the time in Tasmania, Victoria, New South Wales and the Australian Capital Territory is known as Eastern Daylight-Saving Time (EDT). The time in South Australia is known as Central Daylight-Saving Time (CDT).

- During daylight-saving time:
 - Queensland (EST) is 1 hour behind EDT
 - Northern Territory (CST) is $1\frac{1}{2}$ hours behind EDT
 - South Australia (CDT) is $\frac{1}{2}$ hour behind EDT
 - Western Australia (WST) is 3 hours behind EDT.

WORKED EXAMPLE 23

You call your friend in Darwin on Christmas Day from your home in Sydney. It is 7 pm in Sydney. What time is it in Darwin?

THINK

- 1 Because it is Christmas Day (the middle of summer), Sydney time will be at EDT, which is $1\frac{1}{2}$ hours ahead of Darwin time (CST).
- 2 Subtract $1\frac{1}{2}$ hours from the Sydney time to give the time in Darwin.

WRITE

Sydney time is EDT and Darwin time is CST. The time in Sydney is $1\frac{1}{2}$ hours ahead of the time in Darwin.

$$\begin{aligned}\text{Time in Darwin} &= 7 \text{ pm} - 1 \text{ h } 30 \text{ min} \\ &= 5.30 \text{ pm}\end{aligned}$$

learnon RESOURCES — ONLINE ONLY

Interactivity: Time zones (int-0006)

Interactivity: Analogue clock (int-3797)

Interactivity: World time zones (int-3798)

Interactivity: Daylight saving (int-3799)

Digital doc: SkillsHEET 24-hour clock (doc-6961)

Digital doc: SpreadSHEET 12-h to 24-h time (doc-3447)

Digital doc: SpreadSHEET 24-h to 12-h time (doc-3448)

Digital doc: SpreadSHEET Time zones (doc-3449)

Digital doc: WorkSHEET Measurement II (doc-6932)

Exercise 9.9 24-hour clock and time zones

Individual pathways

PRACTISE

Questions:

1, 2a, d, g, j, 3a, d, g, j, 4a, c, e, g, i, 5–11, 12a, d, g, j, 13a, d, g, j, 14, 16, 17, 21, 26

CONSOLIDATE

Questions:

1, 2b, e, h, k, 3b, e, h, k, 4b, d, f, h, 5–11, 12b, e, h, k, 13b, e, h, k, 14, 15, 17, 18, 20, 24, 26, 27

MASTER

Questions:

1, 2c, f, i, l, 3c, f, i, l, 4a, e, h, 5–11, 12c, f, i, l, 13c, f, i, l, 14, 15, 19–27

Individual pathway interactivity: int-4445

learnon ONLINE ONLY

To answer questions online and to receive **immediate feedback** and **fully worked solutions** for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Understanding and fluency

- Match the 12-hour times with the appropriate 24-hour times.

10.12 am	1212
Noon	1120
12.12 pm	1200
11.20 pm	0000
Midnight	1012
11.20 am	2320
- Write each of the following times using the 24-hour clock.

a. 10.20 am	b. 11.30 am	c. 5.10 am	d. 4.15 am
e. 5.15 pm	f. 6.30 pm	g. 8.30 am	h. 8.40 pm
i. Midday	j. 11.30 pm	k. 4.35 am	l. 2.30 pm
- Convert each of the following 24-hour times to 12-hour time.

a. 2315	b. 1310	c. 0815	d. 0115
e. 1818	f. 1220	g. 0005	h. 1005
i. 2005	j. 1135	k. 1520	l. 1414
- WE21** Find the difference between each of the following times. (The first time is the earlier time in each case.)

a. 1005 and 2315	b. 1000 and 1215	c. 1430 and 1615
d. 1530 and 1615	e. 1023 and 2312	f. 1135 and 1440
g. 0820 and 1550	h. 0712 and 2008	i. 1455 and 0015
- Mary-Jane always arrived at school at 0855 and left at 1526. How long was she at school?
- Peter wished to record a movie on video. He had a 180-minute tape and a 240-minute tape. The film he wished to record started at 2030 and finished at 2345.
 - How long was the movie?
 - Which tape should Peter use?
- An aircraft left the airport at 0920 and arrived at its destination at 1305. How long was the flight?
- MC** A clock shows the time as 1543. What is the correct time if it is known that the clock is 33 minutes slow?

A. 1576	B. 1616	C. 1510	D. 1516	E. 1606
- MC** A clock shows the time as 2345. What is the correct time if it is known that the clock is 27 minutes slow?

A. 2318	B. 2372	C. 2412	D. 0042	E. 0012
- MC** A clock shows the time 0857. What is the correct time if it is known that the clock is 48 minutes fast?

A. 0809	B. 0811	C. 0945	D. 0805	E. 0905
- MC** A clock shows the time 1004. What is the correct time if it is known that the clock is 31 minutes fast?

A. 1035	B. 0935	C. 0973	D. 1013	E. 0933
- WE22a** What is the time in Sydney if the time in London (GMT) is:

a. 11.00 am	b. 12.30 pm
c. 10.20 am	d. Midnight
e. Noon	f. 2.15 am
g. 3.30 pm	h. 5.45 pm
i. 8.10 am	j. 9.35 pm
k. 1.12 pm	l. 1.12 am?



13. **WE22b** What is the time in London (GMT) if the time in Sydney is:
- | | | | |
|------------|------------|-------------|------------|
| a. 3.30 am | b. Noon | c. Midnight | d. 5.50 pm |
| e. 6.40 am | f. 7.20 pm | g. 11.10 am | h. 1325 |
| i. 1550 | j. 0615 | k. 1855 | l. 1935? |
14. **WE23** You call your friend in Adelaide on Christmas Day from your home in Cairns, Queensland. It is 5 pm in Cairns. What time is it in Adelaide?
15. Jamie has made a schedule for football training. He will start exercising at 0615 and stop at 0705. He will also train from 1600 until 1810. For how long will Jamie train each day?
16. **MC** If Suzie wanted to telephone her friend in England before he went to work for the day, what would be the best time for her to ring from Melbourne, Australia?
- | | | |
|---------|---------|---------|
| A. 0700 | B. 1700 | C. 1900 |
| D. 2100 | E. 2300 | |
17. If the time was 5.00 am in London, what would the time be in:
- | | | |
|----------|--------------|-------------|
| a. Perth | b. Melbourne | c. Jakarta? |
|----------|--------------|-------------|
18. If the time was 8.00 pm in Perth, what would the time be in:
- | | | |
|-----------|--------------|-------------|
| a. London | b. Melbourne | c. Jakarta? |
|-----------|--------------|-------------|
19. If the time was 3.30 pm in Jakarta, what would the time be in:
- | | | |
|-----------|----------|---------------|
| a. London | b. Perth | c. Melbourne? |
|-----------|----------|---------------|
20. Joe lives in Perth. He wants to ring his brother who lives in Hobart, and his parents who live in Darwin at 1 minute to midnight on New Year's Eve. What time (Perth time) should he make each call?
21. Using the internet and the preceding map of time zones of the world, investigate other countries in the world that are in the same time zones as Australia. List three cities in other countries that are in the same time zone as Australian capital cities.



Communicating, reasoning and problem solving

22. Erin was born on 5 July 1999 at 1300. How old, in years, months, days and hours, was Erin on 21 March 2018 at 1600?
23. Christine signed up for a 10 km fun run on 13 February at 0930. If the fun run was scheduled for 7 June at 11 am, how long did Christine have to wait between signing up and starting the fun run? Give your answer in months, days, hours and minutes. *Note:* Assume that it is not a leap year.
24. Jessica and Connor have to catch the airport train at 8.20 am so they will be on time to catch their plane at the airport. Jessica's watch is 10 minutes fast, but she thinks it is 5 minutes slow. Connor's watch is 10 minutes slow, but he thinks it is 10 minutes fast. Each leaves home expecting to arrive at the station just in time to catch the train. Do they both catch the train? Show your working.
25. A flight from Melbourne to Perth can take 4 hours. If your flight departs Melbourne airport at 2:10 pm (EST), at what time will you arrive in Perth (WST)?
26. A plane flew a direct route from Melbourne to London. Some of the details of the trip are listed below.
- Departure date: Friday 5 August
 - Departure time: 8:00 am
 - Flight time: 25 hours
- | |
|--|
| a. What was the time and date in Melbourne when the plane arrived in London? |
| b. What was the local time (UTC) when the plane arrived? |
27. Why is it important that arrival and departure times at airports are in 24-hour time?



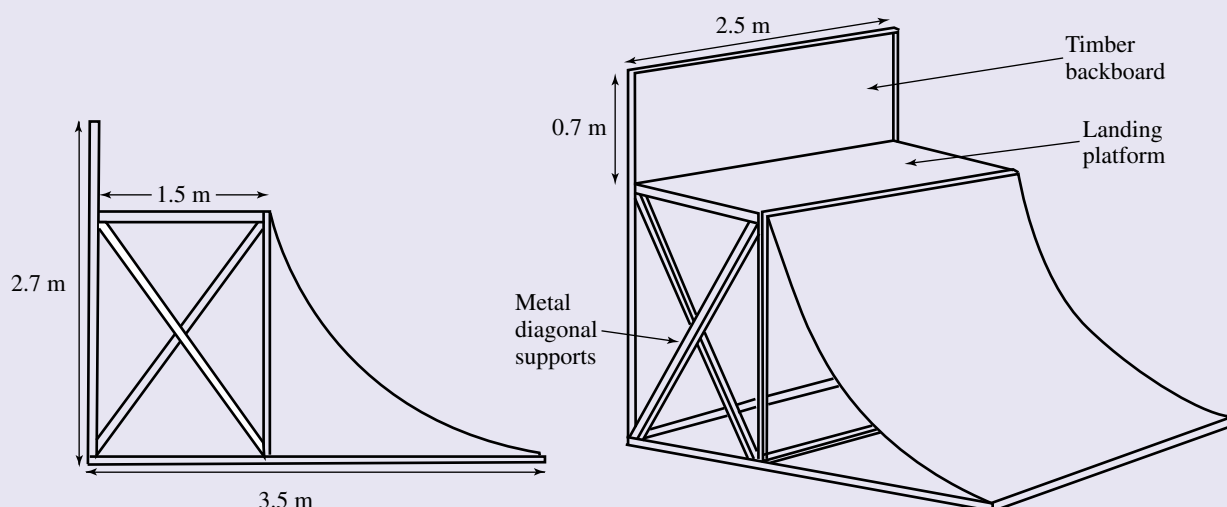
9.10 Review

Investigation | Rich task

Designing a skate park

Skateboarding is an extremely popular activity. Skateboard parks are constructed in many areas for enthusiasts to demonstrate their skills and practise new moves. Two popular pieces of equipment in skate parks are ramps known as the half-pipe and the quarter-pipe. Your local council has decided to build a skate park in your area. The park will have, among the collection of permanent equipment, a half-pipe ramp and a quarter-pipe ramp. You have been asked to provide precise diagrams and measurements to assist in the building of these two structures.

Two different views of your quarter-pipe design are shown below with measurements. The ramp section is made from metal and its length is equivalent to the arc length of a quarter circle with a radius of 2 m.



1. What is the length of the curved part of the ramp?
2. Use a scale diagram to estimate the length of the diagonal metal supports within the frame.
3. Calculate the combined area of the timber backboard, landing platform and the curved section of the ramp.
4. The frame is made from strong square metal piping. If there are 10 horizontal supports in the frame, what is the total length of metal piping used in the construction?
5. How many times will a wheel of a skateboard turn on the curved section of the ramp, given that the diameter of the skateboard wheel is 55 mm?

Your design for the half-pipe contains two quarter-pipes separated by a 2-metre flat section. The quarter-pipes contain the same measurements shown earlier.

6. Draw a side-on view of your half-pipe showing all relevant measurements.
7. What is the total area of metal required to construct the skating section of the half-pipe and landing platforms?
8. Construct a scale model of your design. Your teacher will provide you with materials to use.



Digital doc: Investigation — Designing a skate park (doc-13927)

Digital doc: Code puzzle — Two Australians who have won at Wimbledon (doc-13928)

Digital doc: Summary Topic 9 (doc-23245)

Interactivity: Word search Topic 9 (int-2756)

Interactivity: Crossword Topic 9 (int-2757)

Interactivity: Sudoku Topic 9 (int-3189)

Exercise 9.10 Review questions

To answer questions online and to receive **immediate feedback** and **fully worked solutions** for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Understanding and fluency

1. Convert each of the following to the units shown in brackets.

a. 5.3 mm (cm)

b. 7.6 cm (mm)

c. 15 cm (m)

d. 4.6 m (cm)

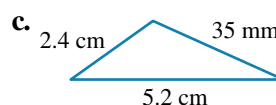
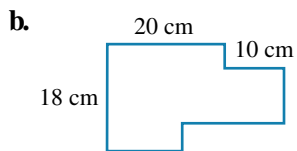
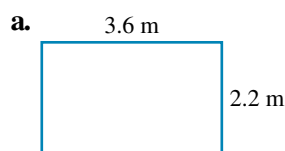
e. 250 m (km)

f. 6.5 km (m)

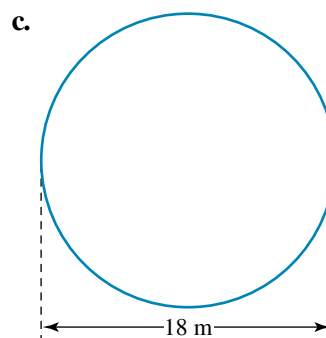
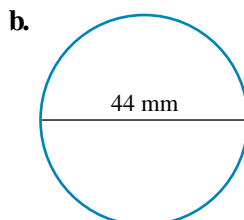
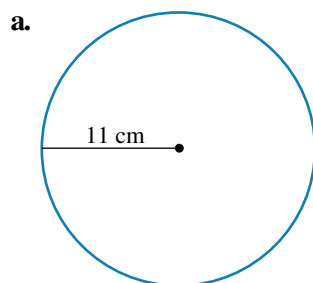
g. 1.5 m (mm)

h. 12 500 cm (km)

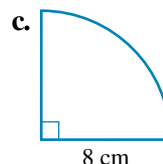
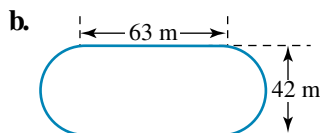
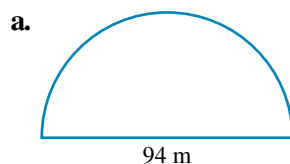
2. Find the perimeter of the shapes below. Where necessary, change to the smaller unit.



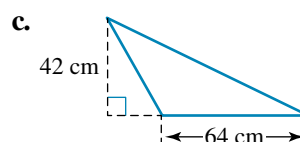
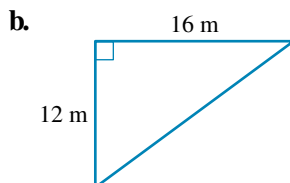
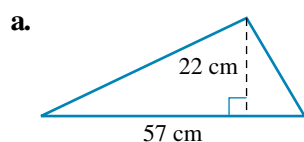
3. Find the circumference of each of these circles correct to 2 decimal places.



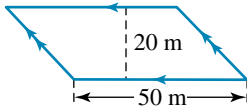
4. Find the perimeter of these shapes correct to 2 decimal places.



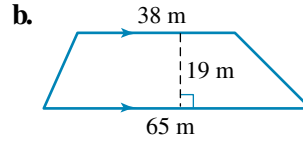
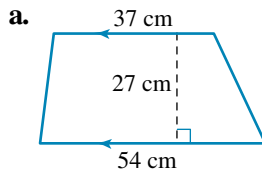
5. Find the area of the following triangles.



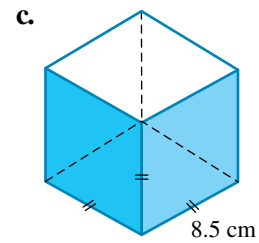
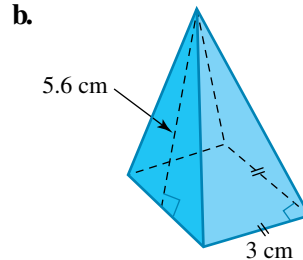
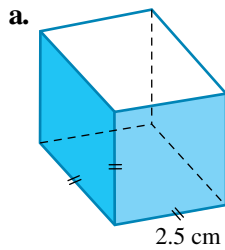
6. Find the area of this parallelogram.



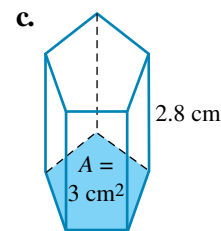
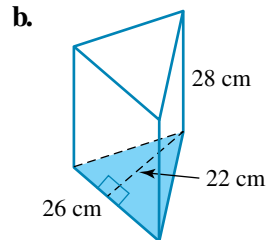
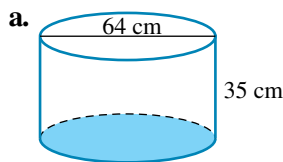
7. Find the area of these trapeziums.



8. Find the total surface area (TSA) of the following three-dimensional objects.



9. Find the volume of each of the following. Where necessary give your answers to 1 decimal place.



10. State how many minutes are in each of the following times.

a. 3 hours

b. $5\frac{1}{4}$ hours

c. $7\frac{1}{2}$ hours

d. $1\frac{3}{4}$ hours

e. 1 day

f. 3 days

11. Change the following times to hours and minutes.

a. 165 minutes

b. 140 minutes

c. 210 minutes

d. 220 minutes

e. 800 minutes

f. 75 minutes

12. Change the following times to minutes.

a. 1 hour 20 minutes

b. 2 hours 40 minutes

c. 3 hours 10 minutes

d. 4 hours 18 minutes

e. 10 hours 35 minutes

f. 3 hours 42 minutes

13. Jamie and his family are preparing to visit relatives in Canberra. They begin their journey at 8.15 am and arrive at their destination at 4.30 pm. For how long did they travel?

14. Write each of the following using 24-hour clock time.

a. 10.35 pm

b. 7.15 am

c. 3.20 am

d. 4.42 pm

e. 9.50 pm

f. 1.05 am

15. Convert each of the following 24-hour times to 12-hour times.

a. 1240

b. 0725

c. 1550

d. 0909

e. 2121

f. 1120

16. Find the difference between the first time shown and the second time (on the following day).

a. 1840 and 0920

b. 2112 and 1115

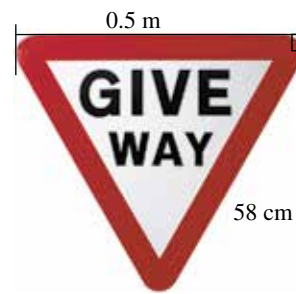
c. 2205 and 0627

d. 1833 and 1158

17. What is the time in Sydney if the GMT (Greenwich Mean Time) is:
 a. 11.20 am b. 3.30 pm c. 9.15 pm d. 6.23 am?
18. What is the GMT (Greenwich Mean Time) when the time in Sydney is:
 a. 11.30 pm b. 11.30 am c. 1450 hours d. 0835 hours?

Communicating, reasoning and problem solving

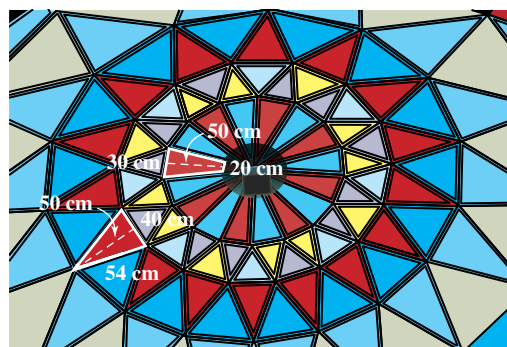
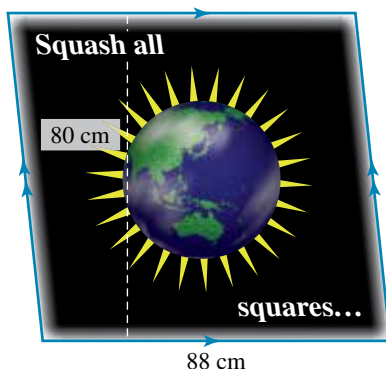
19. A give-way sign is in the shape of a triangle with a base of 0.5 m. If the sign is 58 cm high, find the amount (in m^2) of aluminium needed to make 20 such signs. Assume no waste.



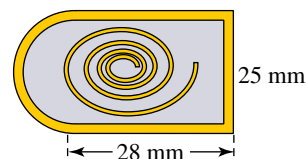
20. Restaurant owners want a dome like the one shown below right over their new kitchen. Red glass is more expensive and they want to estimate how much red glass is needed for the dome. The small sides of the red triangles are 40 cm, the longer sides are 54 cm and their heights are 50 cm. The trapeziums around the central light are also 50 cm high. The lengths of their parallel sides are 30 cm and 20 cm. Calculate the area of:

- the red triangles
- the red trapeziums
- the total area of red glass in m^2 .

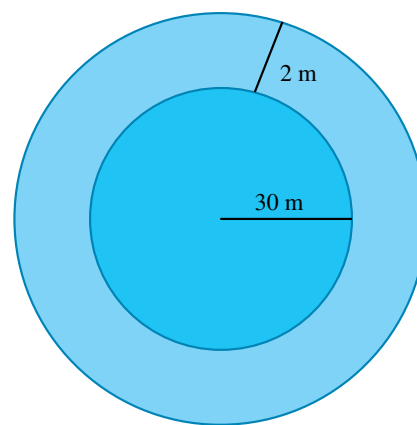
21. What area of cardboard would be required to make the poster shown below?



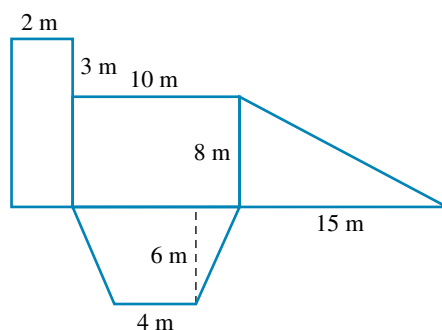
22. Find the area of each of the circles in question 3. Give your answers to 2 decimal places.
23. Find the area that the 12-mm-long minute hand of a watch sweeps out in one revolution. Give your answer correct to 2 decimal places.
24. Of the two parallel sides of a trapezium, one is 5 cm longer than the other. Find the height of the trapezium if the longer side is 12 cm and its area is 57 cm^2 .
25. The diagram shows a design for a brooch.
- What is the total area, correct to 2 decimal places, of the brooch in square millimetres?
 - If the brooch were to be edged with gold, what length of gold strip would be needed for the edge? Give your answers correct to 2 decimal places.
26. A rectangular toy box with no lid is to be painted all over (inside and outside). If the box is 1.2 m long, 60 cm wide and 80 cm tall, find the total area that needs to be painted.
27. A narrow cylindrical vase is 33 cm tall and has a volume of 2592 cm^3 . Find (to the nearest cm) the radius of the base of the vase.



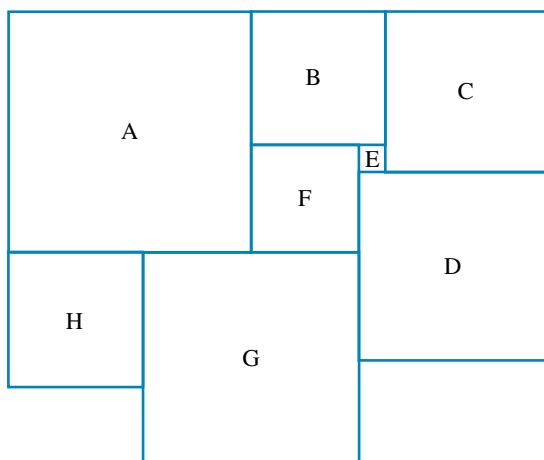
28. Nathan and Rachel ride around a circular track that has an inner radius of 30 m. The track is 2 m wide and Rachel rides along the outer lane.



- How much further does Rachel ride than Nathan in one lap? Give your answer correct to 2 decimal places.
 - The track area needs to be repaved before the next big race. How much bitumen will need to be laid (in m^2)? Give your answer correct to 2 decimal places.
 - The centre of the circular field will be grass except for a rectangular area in the centre, which will be a large shed used to store extra bikes and equipment. If the shed sits on a slab of concrete that is 18 m by 10 m, how much area will be grass? Give your answer correct to 2 decimal places.
 - How many packets of lawn seed will be required if each packet covers 25 m^2 .
29. A sand pit is to be built in the grounds of a kindergarten. As the garden already has various play equipment in it, the sand pit is shaped as shown below.

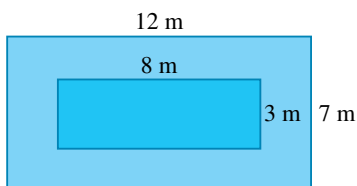
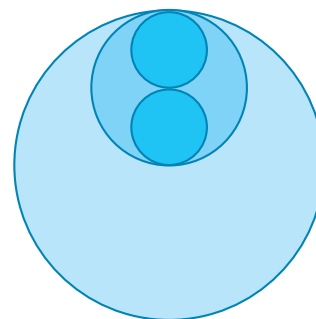
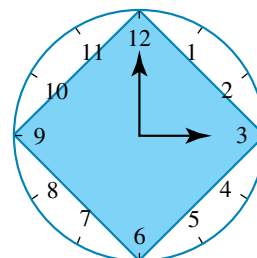


- Calculate the perimeter of the sand pit correct to 2 decimal places.
 - Find the area of the sandpit correct to 2 decimal places.
 - How much sand (in m^3) will be needed if the sandpit is to be 30 cm deep?
30. A cylindrical petrol drum has a base diameter of 40 cm and is 80 cm high. The drum was full on Friday morning but then 30 L of petrol was used for the mower and 5 L was used for the whipper snipper. Find the height of the petrol left in the drum correct to 2 decimal places. (*Hint*: $1 \text{ cm}^3 = 1 \text{ mL}$)
31. Below is a picture of squares. The area of square F is 16 square units. The area of square B is 25 square units. The area of square H is 25 square units.
- Find the area of all the other squares and explain how you got your answer.
 - Find the area of the total shape.
 - Find the perimeter of the shape.



32. A cube with edges 36 cm long is cut into smaller cubes.
- How many small cubes will there be if the smaller cubes have an edge of 12 cm?
 - What is the length of each side of the smaller cube if there are 64 cubes?

33. Elena is 162 cm tall. If she could walk around the earth along the equator (and the equator can be approximated as a circle), then the top of Elena's head would travel further than her feet would travel. How much further is this? Round your answer to 1 decimal place.
34. The width of a rectangle is 6 cm and its perimeter is 26 cm. What is the area of the rectangle?
35. While practising his karate skills, Alex accidentally made a large round hole in the wall of his office. The area of the hole is about 154 cm^2 . To cover up the hole, Alex plans to use a square shaped photo frame (with a photo of himself in full uniform including his black belt). What is the smallest side length the photo frame can have? (Use $\pi = \frac{22}{7}$.)
36. The Great Wall of China stretches from the east to the west of China. It is the largest of a number of walls that were built bit by bit over thousands of years to serve as protection against military invasions. Some estimates are that the Great Wall itself is about 6300 km long. Assume your pace length is 70 cm. At 10 000 paces a day, how many days would it take you to walk the length of the Great Wall?
37. This clock face is circular, with a radius of 10 cm. It has a decorative mother-of-pearl square inset, connecting the numbers 12, 3, 6 and 9. What fraction of the area of the circular face does the square inset represent? Give your answer as an exact fraction in terms of π .
38. The two smaller circles in this diagram have a diameter that is equal to the radius of the medium-sized circle. The diameter of the medium-sized circle is equal to the radius of the large circle. What fraction of the large circle do the two small circles represent?
39. A square field is enclosed by a square fence, using 48 posts. The posts are 5 m apart, with one at each corner of the field. Determine the area bounded by the fence.
40. Pete's Pizzas come in four sizes:
 small (12 cm diameter) \$8
 medium (25 cm diameter) \$19
 large (30 cm diameter) \$23.50
 party size (22 cm by 40 cm) \$27.
 Which one is the better buy?
41. Cameron was mowing his back yard when he stubbed his toe and had to come inside. If he'd mowed a 3 m strip in the middle before he stopped, what area did his sister need to mow to finish the job for him?



42. Polly divided her rectangular vegetable garden into four plots as shown. Three of the plots were rectangular and one was square. Find the area of the shaded plot.
43. Joe wants to make a kite. He has 0.5625 m^2 of material and wants to use it all. What lengths should Joe make his cross rods for his kite, if he wants them to be in the ratio 2 : 1? Explain.

15 m^2	9 m^2
	12 m^2

Answers

Topic 9 Measurement

Exercise 9.2 Perimeter

1. a. $20 \text{ mm} = 2 \text{ cm}$
d. $1.5 \text{ cm} = 15 \text{ mm}$
g. $0.034 \text{ m} = 3.4 \text{ cm}$
j. $2.7 \text{ m} = 270 \text{ cm} = 2700 \text{ mm}$
m. $670 \text{ cm} = 6.7 \text{ m}$
- b. $13 \text{ mm} = 1.3 \text{ cm}$
e. $0.03 \text{ cm} = 0.3 \text{ mm}$
h. $2400 \text{ mm} = 240 \text{ cm} = 2.4 \text{ m}$
k. $0.08 \text{ m} = 80 \text{ mm}$
n. $0.0051 \text{ km} = 5.1 \text{ m}$
- c. $130 \text{ mm} = 13 \text{ cm}$
f. $2.8 \text{ km} = 2800 \text{ m}$
i. $1375 \text{ mm} = 137.5 \text{ cm} = 1.375 \text{ m}$
l. $6.071 \text{ km} = 6071 \text{ m}$
2. a. 14 cm
d. 18 cm
g. 23 cm
j. $1260 \text{ cm} (12.6 \text{ m})$
- b. 12 cm
e. 240 mm
h. 72 mm
k. $192 \text{ cm} (1.92 \text{ m})$
- c. 106 mm
f. 32 mm
i. 73 mm
l. 826 cm
3. a. $1800 \text{ mm} \times 900 \text{ mm} = 180 \text{ cm} \times 90 \text{ cm} = 1.8 \text{ m} \times 0.9 \text{ m}$
b. $2400 \text{ mm} \times 900 \text{ mm} = 240 \text{ cm} \times 90 \text{ cm} = 2.4 \text{ m} \times 0.9 \text{ m}$
c. $2700 \text{ mm} \times 1200 \text{ mm} = 270 \text{ cm} \times 120 \text{ cm} = 2.7 \text{ m} \times 1.2 \text{ m}$
4. $\$5.40$
5. $\$38.16$
6. $41\,400 \text{ m}$ or 41.4 km
7. a. 7 m
b. $\$26.60$
8. 86 m
9. 2.6 m
10. 980 cm
11. 86 cm
12. a. 510 m
b. $\$749.70$
13. $\$15.88$
14. a. 11 cm
b. 22 cm
c. 6.9 m
15. 5.5 m
16. a. 5 cm
b. 20 cm
c. 2 bottles
17. 9 cm
18. a. 258 m
b. 174 m
19. a. Fence length (l) = 10 m and width (w) = 8 m , so perimeter = 36 m .
b. Fence length = $2(l + w + 4)$
20. a. 217 cm
b. Length = $2w + 2l + 4h + \text{bow}$
21. Discuss with your teacher.

Exercise 9.3 Circumference

1. a. i. $2\pi \text{ cm}$
b. i. $10\pi \text{ cm}$
c. i. $7\pi \text{ mm}$
d. i. $0.82\pi \text{ m}$
e. i. $7.4\pi \text{ km}$
f. i. $34\pi \text{ m}$
- ii. 6.28 cm
ii. 31.42 cm
ii. 21.99 mm
ii. 2.58 m
ii. 23.25 km
ii. 106.81 m
2. a. i. $8\pi \text{ m}$
b. i. $34\pi \text{ mm}$
c. i. $16\pi \text{ cm}$
d. i. $2.86\pi \text{ km}$
e. i. $0.8\pi \text{ m}$
f. i. $21.2\pi \text{ m}$
- ii. 25.13 m
ii. 106.81 mm
ii. 50.27 cm
ii. 8.98 km
ii. 2.51 m
ii. 66.60 m

3. a. i. 231 km ii. 238.7 km iii. 241.78 km
 b. i. 36 m ii. 37.2 m iii. 37.68 m
 c. i. 144 mm ii. 148.8 mm iii. 150.72 mm
 d. i. 3.21 m ii. 3.317 m iii. 3.3598 m
 e. i. 186 mm ii. 192.2 mm iii. 194.68 mm
 f. i. 1200 m ii. 1240 m iii. 1256 m
4. a. 25.71 cm b. 82.27 mm c. 61.70 m d. 39.28 mm e. 71.42 cm
 f. 120.82 cm g. 5.88 m h. 250.80 m i. 252.81 cm
5. B
 6. C
7. a. 24.38 cm b. 35.28 mm c. 31.83 cm d. 65.99 cm
8. 55.29 m
 9. 100.53 cm
 10. 119.38 mm
 11. 25.13 m
12. a. 6.00 m b. 20.63 cm c. 23.75 mm
13. a. 2.01 cm b. 7.54 m c. 24.99 mm
14. 21.58 cm
 15. 19.19 m
 16. 15.71 m
 17. 41.82 km/h
 18. 9.55 cm
19. a. iii b. i c. iv d. ii
20. a. 25.71 cm b. 483.29 mm c. 31.07 m d. 5.71 cm
21. 80 m
22. Length of arc = 6.98 cm; perimeter = 13.74 cm.
23. Irrational numbers are numbers that can't be expressed as fractions.
24. Rearrange the formula $C = \pi D$ to $\pi = \frac{C}{D}$.
25. 25.13 cm²

Exercise 9.4 Area of rectangles, triangles, parallelograms, rhombuses and kites

1. a. 530 000 m² b. 2.35 cm² c. 254 000 mm² d. 54.2 m² e. 0.074 m²
 f. 3 km² g. 9.8563 ha h. 17 800 m² i. 987 000 mm² j. 1 275 000 cm²
2. a. 36 cm² b. 1125 mm² c. 4.5 m² d. 1215 km² e. 2.5 m² f. 336 mm²
3. a. 25 mm² b. 256 cm² c. 5.29 m²
4. C
 5. B
6. a. 1258 mm² b. 1771.54 m² c. 9932.63 mm² d. 17 537 cm² e. 11 566.8 mm² f. 257.645 m²
 g. 24 000 mm² h. 656 cm² i. 11.04 mm² j. 2.7 m²
 f. 2400 mm² g. 17.36 m² h. 4760 m² i. 8.48 m²
8. a. 30 cm² b. 24 cm² c. 90 cm²
9. a. 29.23 cm² b. 756 m² c. 360 cm²
10. 9600 mm²
11. a. 42.65 m², 60.81 m², 42.65 m² b. 146.11 m²
12. 351.98 cm²
13. a. 15 000 m² b. 60 000 m²
14. a. 15.64 m² b. 3
15. 42 cm
16. a. 400 m b. 375 min or $6\frac{1}{4}$ h
17. 50.4 cm²

18. 2052 m^2

19. D

20. 17.28 cm^2

21. 1584 cm^2

22. a. 3.6 cm b. 9.2 cm

23. $b = h = 9.5 \text{ cm}$

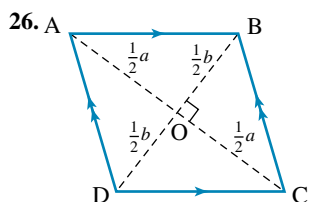
24. a. The area will increase by a factor of 4.

b. The area will decrease by a factor of 4.

c. The area will be squared.

25. a. The area will increase by a factor of n^2 .

b. The area will decrease by a factor of n^2 .



$$\begin{aligned} \text{Area of } \triangle ABC &= \frac{1}{2} \times \text{base} \times \text{height} \\ &= \frac{1}{2} \times a \times \frac{1}{2}b \\ &= \frac{1}{4} \times a \times b \end{aligned}$$

$$\begin{aligned} \text{Area of rhombus ABCD} &= 2 \times \text{area of } \triangle ABC \\ &= 2 \times \frac{1}{4} \times a \times b \\ &= \frac{1}{2} \times a \times b \end{aligned}$$

27. Split the triangle up into two congruent triangles, each with height $\frac{a}{2}$ and base length b . The area of each triangle will then be:

$$\frac{1}{2} \times \frac{a}{2} \times b = \frac{ab}{4}$$

Therefore the area of the kite is:

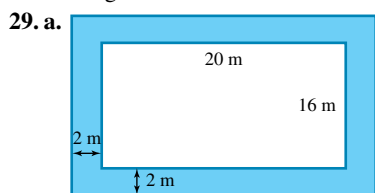
$$\frac{ab}{4} \times 2 = \frac{ab}{2} \text{ or } A_K = \frac{1}{2} \times a \times b.$$

28. Answers will vary. Examples are given.

a. Length = 12 cm, breadth = 3 cm

b. Base = 4 cm, height = 9 cm

c. Diagonals 12 cm and 6 cm



b. 320 m^2

c. 160 m^2

d. \$800

30. a. $212\,500 \text{ km}^2$

b. $227\,600 \text{ km}^2$

c. Because Victoria is not a triangle

d. Divide Victoria into smaller regular shapes.

31. Areas of triangles and parallelograms can only be calculated when the base and height are perpendicular.

Exercise 9.5 Area of a circle

1. a. 452.39 cm^2	b. 4.91 km^2	c. 2.27 m^2	d. 0.38 cm^2	e. $10\,568.32 \text{ cm}^2$	f. 206.12 mm^2
2. a. 78.54 cm^2	b. 483.05 mm^2	c. 615.75 m^2	d. 254.47 cm^2		
3. a. 173.18 cm^2	b. 44.18 cm^2	c. 160.13 mm^2			

4. a. 3.99 cm b. 4.89 mm c. 10.30 cm
5. a. 37.70 cm² b. 1281.77 cm² c. 3189.50 m²
6. a. 157.08 cm² b. 0.39 cm² c. 201.06 mm² d. 13.85 m² e. 39.27 cm²
- f. 1039.08 cm² g. 77.91 cm² h. 132.54 cm² i. 9.74 cm²
7. 1134.11 cm²
8. 4.52 m²
9. 25 packets
10. 173 g
11. a. Diameter changes by a factor of 1.414 ($= \sqrt{2}$).
 b. Diameter changes by a factor of 2.
 c. Diameter changes by a factor of 0.707 ($= \sqrt{\frac{1}{2}}$).
 d. Diameter changes by a factor of $\sqrt{n}$.
12. a. Circumference changes by a factor of 1.414 ($= \sqrt{2}$).
 b. Circumference changes by a factor of 2.
 c. Circumference changes by a factor of 0.707 ($= \sqrt{\frac{1}{2}}$).
 d. Circumference changes by a factor of $\sqrt{n}$.
13. a. $A = 70.83 \text{ cm}^2$
 b. $r = 4.75 \text{ cm}$
 c. The diameter of the coaster is larger than that of the mug, so the coffee mug will fit within the coaster's surface.
14. a. 3279.82 cm² b. 40.18 mm² c. 31.49 cm²
15. Discuss with your teacher.

Exercise 9.6 Area of trapeziums

1. a. 9 cm² b. 33.75 m² c. 12.75 m²
 d. 351 mm² (3.51 cm²) e. 4.68 cm² f. 3120 m²
2. E
3. 3062 cm²
4. \$88.30
5. a. \$2730.55
 b. i. \$332.50 ii. \$3063.05
6. 2 m
7. B
8. a. B b. C c. C
9. a. $A_1 = \frac{1}{2}bh$ b. $A_2 = \frac{1}{2}ah$ c. $A = A_1 + A_2 = \frac{1}{2}ah + \frac{1}{2}bh$
10. a. 31.15 m² b. 7 c. \$62.65
11. a. $85.2 + 15.125\pi \text{ cm}^2$ b. 132.72 cm²

Exercise 9.7 Volume of prisms and other solids

1. b and e
2. a. 84 cm³ b. 81 m³ c. 14 137.17 cm³ d. 31 415.93 cm³ e. 667.98 m³ f. 4778.36 cm³
 g. 84 cm³ h. 120 cm³ i. 320 m³ j. 126 cm³ k. 7.5 cm³ l. 1.875 m³
3. 1200 m³
4. 60.32 m³
5. a. 60 cm³ b. 84 cm³
6. 7380 cm³
7. 18.6 L
8. 1005 mL
9. Both formulas consist of the base of the prism being multiplied by the height.

4. Sundial and hourglass

5. 2.25 hours = $2\frac{1}{4}$ hours, which is 2 hours and 15 minutes

6. a. 120

b. 150

c. 195

d. 30

e. 45

f. 465

g. 1440

h. 12 960

7. a. 3 h 20 min

b. 3 h 5 min

c. 2 h 40 min

d. 3 h 50 min

e. 6 h 45 min

f. 1 h 35 min

g. 10 h 10 min

h. 1 h 12 min

i. 5 h 5 min

8. a. 75 min

b. 130 min

c. 110 min

d. 265 min

e. 455 min

f. 192 min

g. 1005 min

h. 330 min

i. 375 min

9. a. 7

b. 52

c. 60

d. 3600

e. 86 400

f. 24

g. 8760

h. 1440

i. 525 600

10. a. 5 pm

b. 3 pm

c. 6.30 am

d. 11.50 am

e. 8.40 am

f. 5.18 pm

g. 2.00 am

h. 2.20 pm

i. 5.30 pm

j. 6.35 am

k. 7.41 pm

l. 10.20 am

11. a. 15 min

b. 1 h 5 min

c. 1 h 13 min

d. 1 h 15 min

e. 53 min

f. 1 h 53 min

g. 32 min

h. 3 h 45 min

i. 5 h 15 min

j. 12 h

k. 8 h 15 min

12. a. 35 h 10 min

b. 32 h 47 min

c. 13 h 31 min

d. 43 h 49 min

13. C

14. D

15. A

16. 48 min

17. $7\frac{1}{2}$ h

18. 7.25 am

19. 6.30 am

20. 11.45 pm

21. 10.35 pm

22. 52 min

23. 4 h 2 min

24. 3 minutes

25. 1 hour, 34 minutes

26. 5 hours

27. Tommy was not correct. Tommy's answer should have been 3 hours, 15 minutes.

28. 2 months, 4 days, 6 hours and 30 minutes

29. 25 minutes past 10

30. 12 o'clock and 6 o'clock

31. 2 hours 50 minutes

32. a. 8.53 pm

b. 9.04 pm

c. 8.26 pm

33. a. The calculation is correct, but the interpretation of the answer is incorrect. It should be 2 hours and 35 minutes rather than 235 minutes.

b. The calculation is incorrect; the times have been subtracted as if they were decimals. The correct answer is 1 hour 44 minutes.

34. 15 days

Exercise 9.9 24-hour clock and time zones

1. 10.12 am = 1012, 12.12 pm = 1212, Midnight = 0000, Noon = 1200, 11.20 pm = 2320, 11.20 am = 1120

2. a. 1020

b. 1130

c. 0510

d. 0415

e. 1715

f. 1830

g. 0830

h. 2040

i. 1200

j. 2330

k. 0435

l. 1430

3. a. 11.15 pm

b. 1.10 pm

c. 8.15 am

d. 1.15 am

e. 6.18 pm

f. 12.20 pm

g. 12.05 am

h. 10.05 am

i. 8.05 pm

j. 11.35 am

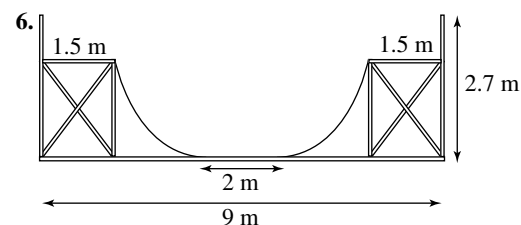
k. 3.20 pm

l. 2.14 pm

4. a. 13 h 10 min b. 2 h 15 min c. 1 h 45 min d. 45 min e. 12 h 49 min
 f. 3 h 5 min g. 7 h 30 min h. 12 h 56 min i. 9 h 20 min
5. 6 h 31 min
6. a. 3 h 15 min b. 240-min tape
7. 3 h 45 min
8. B
9. E
10. A
11. E
12. a. 9.00 pm b. 10.30 pm c. 8.20 pm d. 10 am (next day)
 e. 10 pm f. 12.15 pm g. 1.30 am (next day) h. 3.45 am (next day)
 i. 6.10 pm j. 7.35 am (next day) k. 11.12 pm l. 11.12 am
13. a. 5.30 pm (previous day) b. 2.00 am c. 2.00 pm d. 7.50 am
 e. 8.40 pm (previous day) f. 9.20 am g. 1.10 am h. 0325
 i. 0550 j. 2015 (previous day) k. 0855 l. 0935
14. 5.30 pm
15. 3 hours
16. B
17. a. 1 pm b. 3 pm b. 12 pm
18. a. Midday b. 10 pm c. 7 pm
19. a. 8.30 am b. 4.30 pm c. 6.30 pm
20. Brother 8.59 pm, parents 10.29 pm
21. Answers could include:
 Perth — Singapore, Kuala Lumpur, Cebu City in the Philippines, Denpasar in Bali Brisbane — Port Moresby in Papua New Guinea.
22. 18 years, 8 months, 16 days and 3 hours
23. 3 months, 25 days, 1 hour and 30 minutes
24. Connor misses the train by 20 minutes.
25. 4.10 pm
26. a. 9.00 am on 6 August b. 11.00 pm on 5 August
27. Discuss with your teacher.

Investigation | Rich task

1. 3.14 m
 2. 2.5 m
 3. 13.35 m²
 4. 44.4 m
 5. 18.18 times



7. 28.2 m²
 8. Teacher to check.

Exercise 9.10 Review questions

1. a. 0.53 m b. 76 mm c. 0.15 m d. 460 cm
 e. 0.25 km f. 6500 m g. 1500 mm h. 0.125 km

2. a. 11.6 m b. 96 cm c. 111 mm
3. a. 69.12 cm b. 138.23 mm c. 56.55 m
4. a. 241.65 m b. 257.95 m c. 28.57 cm
5. a. 627 cm^2 b. 96 m^2 c. 1344 cm^2
6. 1000 m^2
7. a. 1228.5 cm^2 b. 978.5 m^2
8. a. 37.5 cm^2 b. 42.6 cm^2 c. 433.5 cm^2
9. a. $112\,594.7 \text{ cm}^3$ b. 8008 cm^3 c. 8.4 cm^3
10. a. 180 min b. 315 min c. 450 min d. 105 min e. 1440 min f. 4320 min
11. a. 2 h 45 min b. 2 h 20 min c. 3 h 30 min d. 3 h 40 min e. 13 h 20 min f. 1 h 15 min
12. a. 80 min b. 160 min c. 190 min d. 258 min e. 635 min f. 222 min
13. 8 h 15 min
14. a. 2235 b. 0715 c. 0320 d. 1642 e. 2150 f. 0105
15. a. 12.40 pm b. 7.25 am c. 3.50 pm d. 9.09 am e. 9.21 pm f. 11.20 am
16. a. 14 h 40 min b. 14 h 3 min c. 8 h 22 min d. 17 h 25 min
17. a. 9.20 pm b. 1.30 am (next day) c. 7.15 am (next day) d. 4.23 pm
18. a. 1.30 pm b. 1.30 am c. 0450 d. 2235 (previous day)
19. 2.9 m^2
20. a. $16\,000 \text{ cm}^2$ b. $10\,000 \text{ cm}^2$ c. 2.6 m^2
21. 7040 cm^2
22. a. 380.13 cm^2 b. 1520.53 mm^2 c. 254.47 m^2
23. 452.39 mm^2
24. 6 cm
25. a. 945.44 mm^2 b. 120.27 mm
26. $72\,000 \text{ cm}^2$ (or 7.2 m^2)
27. 5 cm
28. a. 12.57 m further b. 389.56 m^2 c. 2647.43 m^2 d. 106 packets
29. a. 77.42 m b. 204 m^2 c. 61.2 m^3
30. 52.15 cm
31. a. If B is 25 square units, then each side is 5 units long. If F is 16 square units then each side is 4 units long. Then E is $1 \times 1 = 1$ square unit, C is $6 \times 6 = 36$ square units and D is $7 \times 7 = 49$ square units. A is made up of B and F, which means it is $9 \times 9 = 81$ square units, and then G is $8 \times 8 = 64$ square units.
- b. Total area: $A = 81$, $B = 25$, $C = 36$, $D = 49$, $E = 1$, $F = 16$, $G = 64$, $H = 25$.
Sum of all areas = 297 square units.
- c. 74 units
32. a. 27 cubes. b. 9 cm
33. 10.2 m
34. 42 cm^2
35. 14 cm
36. 900 days
37. $\frac{2}{\pi}$
38. $\frac{1}{8}$
39. 3600 m^2
40. The best buy is the party size (at $\$0.031$ per cm^2).
41. 60 m^2
42. The shaded plot has dimensions $5 \text{ m} \times 4 \text{ m}$ and an area of 20 m^2 .
43. 1.50 m and 0.75 m

TOPIC 10

Equations

10.1 Overview

Numerous **videos** and **interactivities** are embedded just where you need them, at the point of learning, in your learnON title at www.jacplus.com.au. They will help you to learn the concepts covered in this topic.

10.1.1 Why learn this?

Linear equations are a form of algebra, and they are often used to describe everyday situations using mathematics. One of the most useful skills that you learn in algebra is how to solve equations. Solving equations means finding numbers whose values you do not know. Once you learn the techniques of solving equations, you will find that many problems can be expressed and solved using linear equations.



LEARNING SEQUENCE

- 10.1 Overview
- 10.2 Keeping equations balanced
- 10.3 Using algebra to solve problems
- 10.4 Equations with the unknown on both sides
- 10.5 Quadratic equations
- 10.6 Review

LEARNING OUTCOMES

A student:

- communicates and connects mathematical ideas using appropriate terminology, diagrams and symbols MA4-1WM
- applies appropriate mathematical techniques to solve problems MA4-2WM
- recognises and explains mathematical relationships using reasoning MA4-3WM
- uses algebraic techniques to solve simple linear and quadratic equations MA4-10NA

CONTENT DESCRIPTIONS

Students:

Solve simple linear equations (ACMNA179)

Solve linear equations using algebraic techniques and verify solutions by substitution (ACMNA194)

Solve simple quadratic equations (NSW)

Source: NSW Syllabus for the Australian curriculum

learnon RESOURCES — ONLINE ONLY

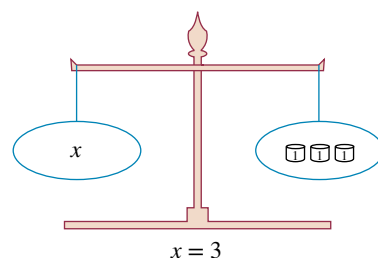
eLesson: The story of mathematics — Mary Somerville (eles-2020)

Note: Your teacher may now set you a pre-test to determine how familiar you are with the content in this topic.

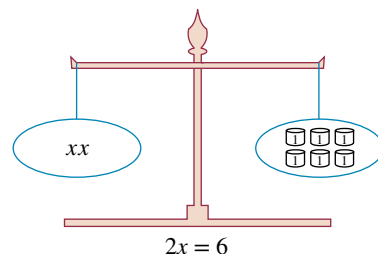
10.2 Keeping equations balanced

10.2.1 Balancing a set of scales

- As an equation can be thought of as two expressions with an equals sign between them, an equation can be thought of as a balanced scale. The diagram at right represents the simple equation $x = 3$.

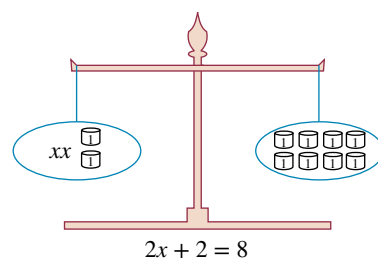


- If the amount on the left-hand side (LHS) is doubled, the scale will stay balanced provided that the amount on the right-hand side (RHS) is doubled.



- Similarly, the scale will stay balanced if we add a quantity to both sides.

- The scales will remain balanced as long as we do the same to both sides.



WORKED EXAMPLE 1

Starting with the equation $x = 4$, write the new equation when we:

a multiply both sides by 4

b take 6 from both sides

c divide both sides by $\frac{2}{5}$.

THINK

- a**
- 1 Write the equation.
 - 2 Multiply both sides by 4.
 - 3 Simplify by removing the multiplication signs. Write numbers before variables.
- b**
- 1 Write the equation.
 - 2 Subtract 6 from both sides.
 - 3 Simplify.
- c**
- 1 Write the equation.
 - 2 Dividing by a fraction is the same as multiplying by its reciprocal. Multiply both sides by $\frac{5}{2}$.
 - 3 Simplify.

WRITE

a $x = 4$
 $x \times 4 = 4 \times 4$
 $4x = 16$

b $x = 4$
 $x - 6 = 4 - 6$
 $x - 6 = -2$

c $x = 4$
 $x \div \frac{2}{5} = 4 \div \frac{2}{5}$
 $x \times \frac{5}{2} = 4 \times \frac{5}{2}$
 $\frac{5x}{2} = \frac{20}{2}$
 $\frac{5x}{2} = 10$

Interactivity: Balancing equations (int-0077)

Interactivity: Keeping equations balanced (int-3804)

Digital doc: WorkSHEET Linear equations I (doc-2351)

Exercise 10.2 Keeping equations balanced

Individual pathways

PRACTISE

Questions:
1–7, 10–13

CONSOLIDATE

Questions:
1–13

MASTER

Questions:
1–15

Individual pathway interactivity: int-4448

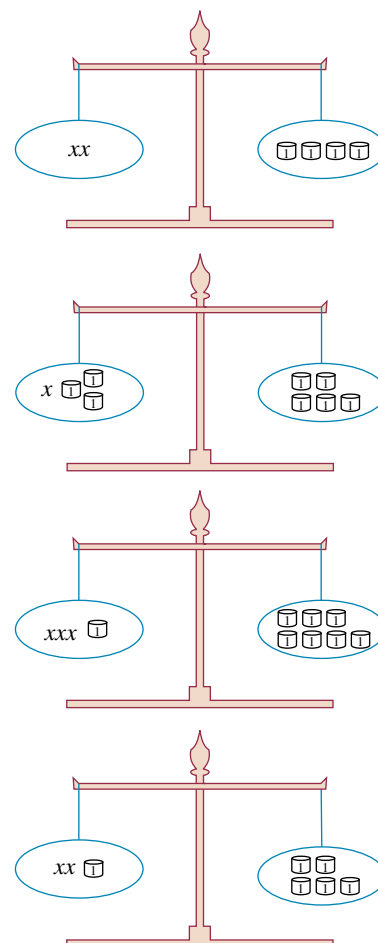
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Understanding and fluency

- WE1** Starting with the equation $x = 6$, write the new equation when we:

 - add 5 to both sides
 - multiply both sides by 7
 - take 4 from both sides
 - divide both sides by 3
 - multiply both sides by -4
 - multiply both sides by -1
 - divide both sides by -1
 - take 9 from both sides
 - divide both sides by $\frac{2}{3}$
 - take $\frac{2}{3}$ from both sides.
 - multiply both sides by $\frac{2}{3}$
- a.** Write the equation that is represented by the diagram at right.
b. Show what happens when you halve the amount on both sides. Write the new equation.
- a.** Write the equation that is represented by the diagram at right.
b. Show what happens when you take three from both sides. Write the new equation.
- a.** Write the equation that is represented by the diagram at right.
b. Show what happens when you add three to both sides. Write the new equation.
- a.** Write the equation that is represented by the diagram at right.
b. Show what happens when you double the amount on each side. Write the new equation.



6. **MC** If we start with $x = 5$, which of these equations is not true?
A. $x + 2 = 7$ **B.** $3x = 8$ **C.** $-2x = -10$ **D.** $\frac{x}{5} = 1$ **E.** $x - 2 = 3$
7. **MC** If we start with $x = 3$, which of these equations is not true?
A. $\frac{2x}{3} = 2$ **B.** $-2x = -6$ **C.** $2x - 6 = 0$ **D.** $\frac{x}{5} = \frac{3}{5}$ **E.** $x - 5 = 2$
8. **MC** If we start with $x = -6$, which of these equations is not true?
A. $-x = 6$ **B.** $2x = -12$ **C.** $x - 6 = 0$ **D.** $x + 4 = -2$ **E.** $x - 2 = -8$
9. **MC** If we start with $2x = 12$, which of these equations is not true?
A. $\frac{2x}{3} = 4$ **B.** $-2x = -12$ **C.** $2x - 6 = 2$ **D.** $4x = 24$ **E.** $2x + 5 = 17$

Communicating, reasoning and problem solving

10. Make an equivalent equation for each of the equations listed below by performing the operation given in brackets to both sides.
- | | | |
|---------------------------------------|--------------------------------------|--|
| a. $m + 8 = 9$ (+ 8) | b. $n - 3 = 5$ (- 2) | c. $2p + 3 = 9$ (+ 3) |
| d. $3m = 12$ ($\times 4$) | e. $12n = 36$ ($\div 6$) | f. $\frac{p}{3} = 5$ ($\times 6$) |
| g. $3p + 7 = 2$ ($\times 4$) | h. $5m - 3 = -1$ ($\div 3$) | i. $x + 5 = 7$ ($\times a$) |
11. Some simple equations are hard to represent on a balance scale. What would you do to balance the following equations?
- | | | |
|-----------------------------|----------------------------------|----------------------------------|
| a. $x - 4 = 10$ | b. $x - 6 = 3$ | c. $\frac{w}{3} = 7$ |
| d. $\frac{e}{2} = 9$ | e. $\frac{q}{4} - 2 = 10$ | f. $\frac{q - 2}{4} = 10$ |
12. **a.** Why is it difficult to represent the problems in question 11 on a balance scale?
b. What is the difference between parts **e** and **f** in question 11?
13. Solve the equations in question 11.
14. Represent the following problems on balance scales and solve for the unknown quantity.
- Five bags of sugar weigh three kilos. What does one bag of sugar weigh?
 - Four chocolate bars and an ice-cream cost \$9.90. If the ice-cream costs \$2.70, what does a chocolate bar cost?
 - A shopkeeper weighed three apples at 920 g. He thought this was heavy, and realised somebody had left a 500-g weight on his balance scale. What was the average weight of one apple?
15. You have eight \$1 coins, one of which is heavier than the rest. Using a set of balance scales, describe how you can find the heavy coin in the least number of weighings.



10.3 Using algebra to solve problems

10.3.1 Using inverse operations

- A **linear equation** is an equation where the variable has an index (power) of 1. This means that it never contains terms like x^2 or $\sqrt{x}$.
- To solve a linear equation, perform the same operations on both sides until the variable or unknown is left by itself.
- Sometimes the variable or unknown is called a pronumeral.
- A flowchart is useful to show you the order of operations applied to x , so that the reverse order and inverse operation can be used to solve the equation. As you become confident with solving equations algebraically, you can leave out the flowchart steps.

WORKED EXAMPLE 2

Solve these one-step equations by doing the same to both sides.

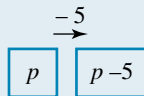
a $p - 5 = 11$

b $\frac{x}{16} = -2$

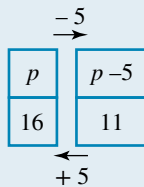
THINK

a 1 Write the equation.

2 Draw a flowchart and fill in the arrow to show what has been done to p .



3 Backtrack from 11.

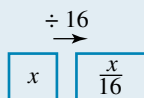


4 Add 5 to both sides.

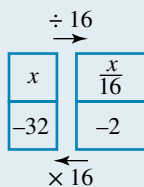
5 Give the solution.

b 1 Write the equation.

2 Draw a flowchart and fill in the arrow to show what has been done to x .



3 Backtrack from -2 .



4 Multiply both sides by 16.

5 Give the solution.

WRITE

a $p - 5 = 11$

$$p - 5 + 5 = 11 + 5$$

$$p = 16$$

b $\frac{x}{16} = -2$

$$\frac{x}{16} \times 16 = -2 \times 16$$

$$x = -32$$

- The equations in Worked example 2 are called one-step equations because only one operation needs to be undone to obtain the value of the unknown.

WORKED EXAMPLE 3

Solve these two-step equations by doing the same to both sides.

a $2(x + 5) = 18$

b $\frac{x}{3} + 1 = 7$

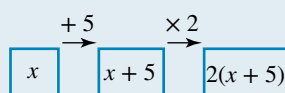
THINK

a 1 Write the equation.

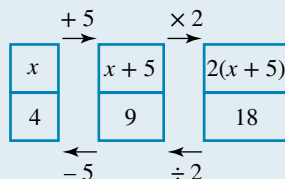
WRITE

a $2(x + 5) = 18$

- 2 Draw a flowchart and fill in the arrow to show what has been done to x .



- 3 Backtrack from 18.



- 4 Divide both sides by 2.

$$\frac{2(x+5)}{2} = \frac{18}{2}$$

$$x+5=9$$

- 5 Subtract 5 from both sides.

$$x+5-5=9-5$$

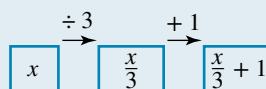
- 6 Give the solution.

$$x=4$$

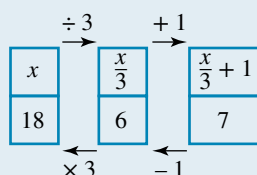
- b** 1 Write the equation.

$$\frac{x}{3}+1=7$$

- 2 Draw a flowchart and fill in the arrows to show what has been done to x .



- 3 Backtrack from 7.



- 4 Subtract 1 from both sides.

$$\frac{x}{3}+1-1=7-1$$

$$\frac{x}{3}=6$$

- 5 Multiply both sides by 3.

$$\frac{x}{3}\times 3=6\times 3$$

- 6 Give the solution.

$$x=18$$

- The equations in Worked example 3 are called two-step equations because two operations need to be undone to obtain the value of the unknown.

WORKED EXAMPLE 4

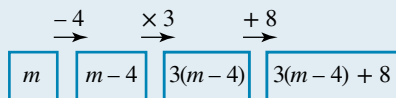
Solve the following equations by doing the same to both sides.

a $3(m - 4) + 8 = 5$ **b** $6\left(\frac{x}{2} + 5\right) = -18$

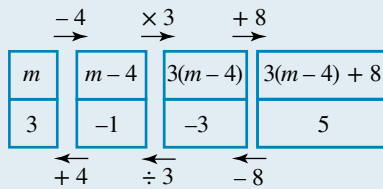
THINK

a 1 Write the equation.

2 Draw a flowchart and fill in the arrows to show what has been done to m .



3 Backtrack from 5.



4 Subtract 8 from both sides.

$$3(m - 4) + 8 - 8 = 5 - 8$$

$$3(m - 4) = -3$$

5 Divide both sides by 3.

$$\frac{3(m - 4)}{3} = \frac{-3}{3}$$

$$m - 4 = -1$$

6 Add 4 to both sides.

$$m - 4 + 4 = -1 + 4$$

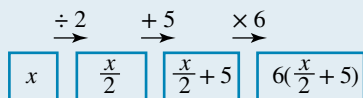
7 Give the solution.

$$m = 3$$

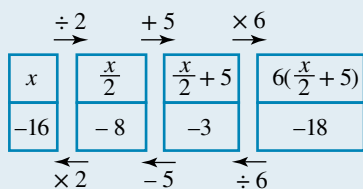
b 1 Write the equation.

b $6\left(\frac{x}{2} + 5\right) = -18$

2 Draw a flowchart and fill in the arrows to show what has been done to x .



3 Backtrack from -18.



4 Divide both sides by 6.

$$\frac{6\left(\frac{x}{2} + 5\right)}{6} = \frac{-18}{6}$$

5 Subtract 5 from both sides.

$$\frac{x}{2} + 5 = -3$$

$$\frac{x}{2} + 5 - 5 = -3 - 5$$

$$\frac{x}{2} = -8$$

6 Multiply both sides by 2.

$$\frac{x}{2} \times 2 = -8 \times 2$$

7 Give the solution.

$$x = -16$$

DISCUSSION

How do you decide which operation is the next to be undone?

learnon RESOURCES — ONLINE ONLY

Interactivity: Using algebra to solve problems (int-3805)

eLesson: Solving linear equations (eles-1895)

Digital doc: SpreadSHEET 2-step equations (doc-2353)

Digital doc: SpreadSHEET 3-step equations (doc-2354)

Exercise 10.3 Using algebra to solve problems

Individual pathways

PRACTISE

Questions:

1, 2a, b, e, f, i, j, 3a, e, i, m, q, r, 4:
columns 1 and 2; 5: columns 1 and
2; 6, 7: column 1; 8, 11, 12

CONSOLIDATE

Questions:

1, 2, 3a–1, 4, 5a–l, 6–12

MASTER

Questions:

1i–l, 2i–l, 3i–l, 4i–l, 6–13

Individual pathway interactivity: int-4449

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Understanding and fluency

1. **WE2a** Solve these one-step equations by doing the same to both sides.

a. $x + 8 = 7$

b. $12 + r = 7$

c. $31 = t + 7$

d. $w + 4.2 = 6.9$

e. $\frac{5}{8} = m + \frac{1}{8}$

f. $\frac{2}{7} = j + 3$

g. $q - 8 = 11$

h. $-16 + r = -7$

i. $21 = t - 11$

j. $y - 5.7 = 8.8$

k. $-\frac{11}{7} = z - \frac{2}{3}$

l. $-\frac{9}{13} = f - 1$

2. **WE2b** Solve these one-step equations by doing the same to both sides.

$$\begin{array}{llllll} \text{a. } 11d = 88 & \text{b. } 7p = -98 & \text{c. } 5u = 4 & \text{d. } 2.5g = 12.5 & \text{e. } 8m = \frac{1}{4} & \text{f. } -\frac{3}{5} = 9j \\ \text{g. } \frac{t}{8} = 3 & \text{h. } \frac{k}{5} = -12 & \text{i. } -5.3 = \frac{l}{4} & \text{j. } \frac{v}{6} = \frac{2}{3} & \text{k. } \frac{c}{9} = -\frac{5}{27} & \text{l. } -\frac{7}{12} = \frac{h}{5} \end{array}$$

3. **WE3a** Solve these two-step equations by doing the same to both sides.

$$\begin{array}{llll} \text{a. } 3m + 5 = 14 & \text{b. } -2w + 6 = 16 & \text{c. } -5k - 12 = 8 & \text{d. } 4t - 3 = -15 \\ \text{e. } 2(m - 4) = -6 & \text{f. } -3(n + 12) = 18 & \text{g. } 5(k + 6) = -15 & \text{h. } -6(s + 11) = -24 \\ \text{i. } 2m + 3 = 10 & \text{j. } 40 = -5(p + 6) & \text{k. } 5 - 3g = 14 & \text{l. } 11 - 4f = -9 \\ \text{m. } 2q - 4.9 = 13.2 & \text{n. } 7.6 + 5r = -8.4 & \text{o. } 13.6 = 4t - 0.8 & \text{p. } -6k + 7.3 = 8.5 \\ \text{q. } -4g - \frac{1}{5} = \frac{4}{5} & \text{r. } -\frac{3}{8} = 2f - \frac{18}{8} \end{array}$$

4. **WE3b** Solve these two-step equations by doing the same to both sides.

$$\begin{array}{llll} \text{a. } \frac{x}{3} + 2 = 9 & \text{b. } \frac{x - 5}{4} = 1 & \text{c. } \frac{m + 3}{2} = -7 & \text{d. } \frac{h}{-3} + 1 = 5 \\ \text{e. } \frac{-m}{5} - 3 = 1 & \text{f. } \frac{2w}{5} = -4 & \text{g. } \frac{-3m}{7} = -1 & \text{h. } \frac{c - 7}{3} = -2 \\ \text{i. } \frac{-5m}{4} = 10 & \text{j. } \frac{t + 2}{7} = -5 & \text{k. } \frac{c - 21}{9} = -4.5 & \text{l. } \frac{x}{8} - 3.2 = -5.8 \end{array}$$

5. **WE4** Solve these equations by doing the same to both sides. They will need more than two steps.

$$\begin{array}{lll} \text{a. } 2(m + 3) + 7 = 3 & \text{b. } \frac{-2(x + 5)}{5} = 6 & \text{c. } \frac{5m + 6}{3} = 4 \\ \text{d. } \frac{3x - 2}{7} = 1 & \text{e. } \frac{4 - 2x}{3} = 6 & \text{f. } \frac{-x + 3}{2} = -4 \\ \text{g. } \frac{3x}{7} - 2 = 1 & \text{h. } \frac{4b}{5} - 3 = 5 & \text{i. } \frac{7f}{9} + 2 = -5 \\ \text{j. } 6 - \frac{4z}{3} = -2 & \text{k. } 8 - \frac{6m}{5} = 2 & \text{l. } -9 - \frac{5u}{11} = -4 \\ \text{m. } \frac{3m - 5}{-2} = 7 & \text{n. } -7(5w + 3) = 35 & \text{o. } 5\left(\frac{x}{2} - 6\right) = -10 \\ \text{p. } \frac{d - 7}{2} + 10 = 8 & \text{q. } \frac{3n + 1}{4} - 5 = 2 & \text{r. } \frac{3(t - 5)}{7} + 9 = 6 \end{array}$$

6. Below is Alex's working to solve the equation $2x + 3 = 14$.

$$\begin{array}{l} 2x + 3 = 14 \\ \frac{2x}{2} + 3 = \frac{14}{2} \\ x + 3 = 7 \\ x + 3 - 3 = 7 - 3 \\ x = 4 \end{array}$$

a. Is the solution correct?

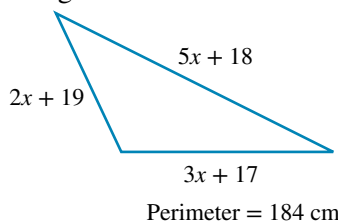
b. If not, can you find where the error is and correct it?

7. Simplify the left-hand side of the following equations by collecting like terms, and then solve.

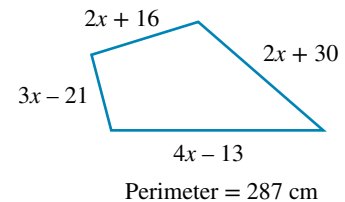
$$\begin{array}{ll} \text{a. } 3x + 5 + 2x + 4 = 19 & \text{b. } 13v - 4v + 2v = -22 \\ \text{c. } -3m + 6 - 5m + 1 = 15 & \text{d. } -3y + 7 + 4y - 2 = 9 \\ \text{e. } -3y - 7y + 4 = 64 & \text{f. } 5t + 4 - 8t = 19 \\ \text{g. } 5w + 3w - 7 + w = 13 & \text{h. } w + 7 + w - 15 + w + 1 = -5 \\ \text{i. } 7 - 3u + 4 + 2u = 15 & \text{j. } 7c - 4 - 11 + 3c - 7c + 5 = 8 \end{array}$$

Communicating, reasoning and problem solving

8. A repair person calculates his service fee using the equation $F = 40t + 55$, where F is the service fee in dollars and t is the number of hours spent on the job.
 - a. How long did a particular job take if the service fee was \$155?
 - b. Explain what the numbers 40 and 55 could represent as costs in the service fee equation.
9. Lyn and Peta together raised \$517 from their cake stalls at the school fete. If Lyn raised l dollars and Peta raised \$286, write an equation that represents the situation and determine the amount Lyn raised.
10. a. Write an equation that represents the perimeter of the figure below and then solve for x .



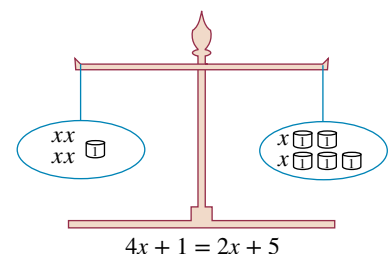
- b. Write an equation that represents the perimeter of the figure at right and then solve for x .
11. If four times a certain number equals nine minus a half of the number, find the number.
12. Tom is 5 years old and his dad is 10 times his age, being 50 years old. Is it possible, at any stage, for Tom's dad to be twice the age of his son? Explain your answer.
13. Lauren earns the same amount for mowing four of the neighbours' lawns every month. Each month she saves all her pay except \$30, which she spends on her mobile phone. If she has \$600 at the end of the year, how much did she earn each month? (Write an equation to solve for this situation first.)



10.4 Equations with the unknown on both sides

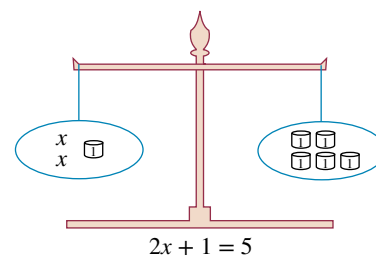
10.4.1 Solving equations and checking solutions

- Some equations have unknowns on both sides of the equation.
- If an equation has unknowns on both sides, eliminate the unknowns from one side and then solve as usual.
- Consider the equation $4x + 1 = 2x + 5$.
 - Drawing the equation on a pair of scales looks like this:



- The scales remain balanced if $2x$ is eliminated from both sides:
- Writing this algebraically, we have:

$$\begin{aligned}
 4x + 1 &= 2x + 5 \\
 4x + 1 - 2x &= 2x + 5 - 2x \\
 2x + 1 &= 5 \\
 2x + 1 - 1 &= 5 - 1 \\
 2x &= 4 \\
 \frac{2x}{2} &= \frac{4}{2} \\
 x &= 2
 \end{aligned}$$



WORKED EXAMPLE 5

Solve the equation $5t - 8 = 3t + 12$ and check your solution.

THINK

- 1 Write the equation.
- 2 Subtract the smaller unknown (i.e. $3t$) from both sides and simplify.
- 3 Add 8 to both sides and simplify.
- 4 Divide both sides by 2 and simplify.
- 5 Check the solution by substituting $t = 10$ into the left-hand side and then the right-hand side of the equation.
- 6 Comment on the answers obtained.

WRITE

$$\begin{aligned}
 5t - 8 &= 3t + 12 \\
 5t - 8 - 3t &= 3t + 12 - 3t \\
 2t - 8 &= 12 \\
 2t - 8 + 8 &= 12 + 8 \\
 2t &= 20 \\
 \frac{2t}{2} &= \frac{20}{2} \\
 t &= 10
 \end{aligned}$$

If $t = 10$,

$$\begin{aligned}
 \text{LHS} &= 5t - 8 \\
 &= 50 - 8 \\
 &= 42
 \end{aligned}$$

If $t = 10$,

$$\begin{aligned}
 \text{RHS} &= 3t + 12 \\
 &= 30 + 12 \\
 &= 42
 \end{aligned}$$

Since the LHS and RHS are equal, the equation is true when $t = 10$.

WORKED EXAMPLE 6

Solve the equation $3n + 11 = 6 - 2n$ and check your solution.

THINK

- 1 Write the equation.
- 2 The inverse of $-2n$ is $+2n$.
Therefore, add $2n$ to both sides and simplify.
- 3 Subtract 11 from both sides and simplify.
- 4 Divide both sides by 5 and simplify.

WRITE

$$\begin{aligned}
 3n + 11 &= 6 - 2n \\
 3n + 11 + 2n &= 6 - 2n + 2n \\
 5n + 11 &= 6 \\
 5n + 11 - 11 &= 6 - 11 \\
 5n &= -5 \\
 \frac{5n}{5} &= \frac{-5}{5} \\
 n &= -1
 \end{aligned}$$

- 5 Check the solution by substituting $n = -1$ into the left-hand side and then the right-hand side of the equation.

If $n = -1$,

$$\begin{aligned}\text{LHS} &= 3n + 11 \\ &= -3 + 11 \\ &= 8\end{aligned}$$

If $n = -1$,

$$\begin{aligned}\text{RHS} &= 6 - 2n \\ &= 6 - -2 \\ &= 6 + 2 \\ &= 8\end{aligned}$$

- 6 Comment on the answers obtained.

Since the LHS and RHS are equal, the equation is true when $n = -1$.

WORKED EXAMPLE 7

Expand the brackets and then solve the following equation, checking your solution.

a $3(s + 2) = 2(s + 7) + 4$

b $4(d + 3) - 2(d + 7) + 4 = 5(d + 2) + 7$

THINK

- a** 1 Write the equation.
- 2 Expand the brackets on each side of the equation first and then simplify.
- 3 Subtract the smaller unknown term (i.e. $2s$) from both sides and simplify.
- 4 Subtract 6 from both sides and simplify.
- 5 Check the solution by substituting $s = 12$ into the left-hand side and then the right-hand side of the equation.

WRITE

a $3(s + 2) = 2(s + 7) + 4$

$$\begin{aligned}3s + 6 &= 2s + 14 + 4 \\ 3s + 6 &= 2s + 18\end{aligned}$$

$$\begin{aligned}3s + 6 - 2s &= 2s + 18 - 2s \\ s + 6 &= 18\end{aligned}$$

$$\begin{aligned}s + 6 - 6 &= 18 - 6 \\ s &= 12\end{aligned}$$

If $s = 12$,

$$\begin{aligned}\text{LHS} &= 3(s + 2) \\ &= 3(12 + 2) \\ &= 3(14) \\ &= 42\end{aligned}$$

If $s = 12$,

$$\begin{aligned}\text{RHS} &= 2(s + 7) + 4 \\ &= 2(12 + 7) + 4 \\ &= 2(19) + 4 \\ &= 38 + 4 \\ &= 42\end{aligned}$$

Since the LHS and RHS are equal, the equation is true when $s = 12$.

- 6 Comment on the answers obtained.

- b** 1 Write the equation.

b $4(d + 3) - 2(d + 7) + 4 = 5(d + 2) + 7$

- 2 Expand the brackets on each side of the equation first, and then simplify.

$$\begin{aligned}4d + 12 - 2d - 14 + 4 &= 5d + 10 + 7 \\ 2d + 2 &= 5d + 17\end{aligned}$$

3 Subtract the smaller unknown term (i.e. $2d$) from both sides and simplify.	$2d - 2d + 2 = 5d - 2d + 17$ $2 = 3d + 17$
4 Rearrange the equation so that the unknown is on the left-hand side of the equation.	$3d + 17 = 2$
5 Subtract 17 from both sides and simplify.	$3d + 17 - 17 = 2 - 17$ $3d = -15$
6 Divide both sides by 3 and simplify.	$\frac{3d}{3} = \frac{-15}{3}$ $d = -5$
7 Check the solution by substituting $d = -5$ into the left-hand side and then the right-hand side of the equation.	If $d = -5$, $\begin{aligned} \text{LHS} &= 4(d + 3) - 2(d + 7) + 4 \\ &= 4(-5 + 3) - 2(-5 + 7) + 4 \\ &= 4(-2) - 2(2) + 4 \\ &= -8 - 4 + 4 \\ &= -8 \end{aligned}$ If $d = -5$, $\begin{aligned} \text{RHS} &= 5(-5 + 2) + 7 \\ &= 5(-3) + 7 \\ &= -15 + 7 \\ &= -8 \end{aligned}$
8 Comment on the answers obtained.	Since the LHS and RHS are equal, the equation is true when $d = -5$.

Note: When solving equations with the unknown on both sides, it is good practice to remove the unknown with the smaller coefficient from the relevant side.

ACTIVITY: A HATFUL OF EQUATIONS

Equipment: paper, pen, small container or hat

- In groups, write two equations similar to $4(w + 1) = 3w - 2$ and $3(a + 9) - 2(4a + 7) = 5a + 1$ and place them in the hat.
- When the class is ready, each group randomly selects an equation to solve.
 - First, use a trial-and-error process to solve the equation. The group might like to nominate a scribe to record the process in a table.
 - Next, solve the equation using inverse operations. Record your steps in solving the problem and check against your answer from the trial-and-error process.
- When you have recorded the solution to the equation for *both* processes, return the equation to the hat and select another.
- Repeat the activity and see if you can reduce the number of steps you take in the trial-and-error process.
- You might like to make the task more challenging by setting time limit or perhaps having a race — the first group to solve five equations wins.
- Discuss your results as a class. Which process did you find easier? Which process was quicker?

DISCUSSION

Does it really matter which side of the equation you eliminate the variable from?

Interactivity: Solving equations (int-2373)

Interactivity: Equations with the unknown on both sides (int-3806)

eLesson: Solving linear equations with the pronumeral on both sides (eles-1901)

Digital doc: SkillSHEET Expanding expressions containing brackets (doc-6985)

Digital doc: SkillSHEET Checking solutions by substitution (doc-6986)

Digital doc: SkillSHEET Writing equations from worded statements (doc-6987)

Digital doc: SpreadSHEET Unknowns on both sides (doc-2355)

Digital doc: WorkSHEET Linear equations II (doc-2352)

Exercise 10.4 Equations with the unknown on both sides

Individual pathways

PRACTISE

Questions:

1–4, 6, 8, 9, 13, 15

CONSOLIDATE

Questions:

1, 2, 3a–l, 4–10, 13–15

MASTER

Questions:

1g–i, 2g–i, 3k–p, 4–17

Individual pathway interactivity: int-4450

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To answer questions online and to receive **immediate feedback** and **fully worked solutions** for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Understanding and fluency

1. **WE5** Solve the following equations and check your solutions.

a. $8x + 5 = 6x + 11$

b. $5y - 5 = 2y + 7$

c. $11n - 1 = 6n + 19$

d. $6t + 5 = 3t + 17$

e. $2w + 6 = w + 11$

f. $4y - 2 = y + 9$

g. $3z - 15 = 2z - 11$

h. $5a + 2 = 2a - 10$

i. $2s + 9 = 5s + 3$

j. $k + 5 = 7k - 19$

k. $4w + 9 = 2w + 3$

l. $7v + 5 = 3v - 11$

2. **WE6** Solve the following equations and check your solutions.

a. $3w + 1 = 11 - 2w$

b. $2b + 7 = 13 - b$

c. $4n - 3 = 17 - 6n$

d. $3s + 1 = 16 - 2s$

e. $5a + 12 = -6 - a$

f. $7m + 2 = -3m + 22$

g. $p + 7 = -p + 15$

h. $3 + 2d = 15 - 2d$

i. $5 + m = 5 - m$

j. $7s + 3 = 15 - 5s$

k. $3t - 7 = -17 - 2t$

l. $16 - 2x = x + 4$

3. **WE7** Expand the brackets and then solve the following equations, checking your solutions.

a. $3(2x + 1) + 3x = 30$

b. $2(4m - 7) + m = 76$

c. $3(2n - 1) = 4(n + 5) + 1$

d. $t + 4 = 3(2t - 7)$

e. $3d - 5 = 3(4 - d)$

f. $4(3 - w) = 5w + 1$

g. $2(k + 5) - 3(k - 1) = k - 7$

h. $4(2 - s) = -2(3s - 1)$

i. $-3(z + 3) = 2(4 - z)$

j. $5(v + 2) = 7(v + 1)$

k. $2m + 3(2m - 7) = 4 + 5(m + 2)$

l. $3d + 2(d + 1) = 5(3d - 7)$

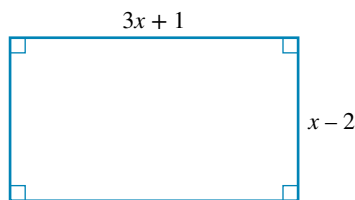
m. $4(d + 3) - 2(d + 7) + 5 = 5(d + 12)$

n. $5(k + 11) + 2(k - 3) - 7 = 2(k - 4)$

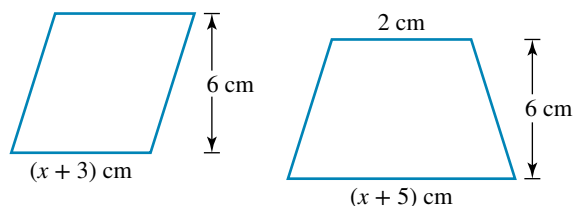
o. $7(v - 3) - 2(5 - v) + 25 = 4(v + 3) - 8$

p. $3(l - 7) + 4(8 - 2l) - 7 = -4(l + 2) - 6$

4. Solve the equation $\frac{-3(x+4)}{7} - \frac{5(1-3x)}{3} = 3x - 4$.
5. Solve the equation $\frac{(x-2)}{3} + 5 = 2x$.
6. Find the value of x given the perimeter of the rectangle is 48 cm.

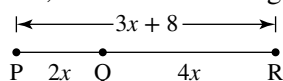


7. The two shapes shown have the same area.
- Write an equation to show that the parallelogram and the trapezium have the same area.
 - Solve the equation for x .
 - State the dimensions of the shapes.



Communicating, reasoning and problem solving

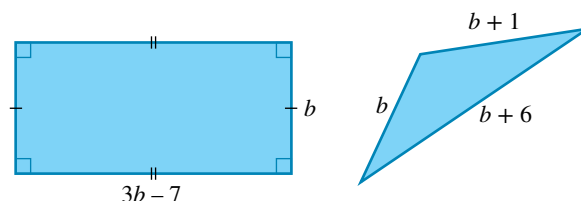
8. A maths class has equal numbers of boys and girls. Eight of the girls left early to play in a netball match. This left 3 times as many boys in the class as girls. How many students are in the class? Show your working.
9. Judy is thinking of a number. First she doubles it and adds 2. She realises that if she multiplies it by 3 and subtracts 1, she gets the same result. Find the number.
10. Mick's father was 28 years old when Mick was born. If his father is now 3 times as old as Mick is, how old are they both now?
11. Given the following number line for a line segment PR, determine the length of PR.



12. In 8 years' time, Tess will be 5 times as old as her age 8 years ago. How old is Tess now?
13. You have 12 more than three times the number of marker pens in your pencil case than your friend has in his pencil case. The teacher has 5 more than four times the number of marker pens in your friend's pencil case.



- Write an expression for the number of marker pens in:
 - your pencil case
 - your teacher's pencil case.
 - You have the same number of marker pens as the teacher. Write an equation to show this.
 - Calculate the number of marker pens in your friend's pencil case by solving the equation from part b.
14. The length of the rectangle at right is 7 centimetres less than three times its breadth. If the perimeter of the rectangle is the same as the perimeter of the triangle, calculate the side lengths of the rectangle and triangle.



Could this be your friend's pencil case?

15. a. How many rides do you need so that option 1 and option 2, at right, cost the same?
 b. If you were planning on having only two rides, explain which option you would choose and why.
16. On her birthday today, a mother is three times as old as her daughter will be in eight years time. It just so happens that the mother's age today is the same as their house number, which is four times the value of thirteen minus the daughter's age.
- Write each sentence above as an expression. (*Hint: Make the daughter's age the unknown variable.*)
 - Use these two expressions to make an equation and solve it.
 - How old are the mother and the daughter?
 - What is the number of their house?
17. A balanced scale contains boxes of Smarties and loose Smarties in the pans. There are three full boxes of Smarties and another box with four Smarties missing in one pan. The other pan contains four empty boxes and 48 loose Smarties. How many Smarties are in a full box?



10.5 Quadratic equations

10.5.1 Solving simple quadratic equations

- A **quadratic equation** is an equation in which the pronumeral is squared.
- Simple quadratic equations are quadratic equations in the form $x^2 = c$.
- When taking the square root of a positive number there are two solutions: a positive value and a negative value. For example, $3^2 = 9$ and $(-3)^2 = 9$ so $\sqrt{9} = +3$ and $\sqrt{9} = -3$. This is written as $\sqrt{9} = \pm 3$, where ± 3 is read as positive or negative 3.
- When solving quadratic equations there are either zero, one or two possible solutions.
- When a quadratic equation represents a real-life situation, it is possible that one of the values of the pronumeral is not valid.

WORKED EXAMPLE 8

Solve the quadratic equation $x^2 = 16$.

THINK

- Write the equation.
- Take the square root of both sides.
- Write the answer.

WRITE

$$x^2 = 16$$

$$x = \pm\sqrt{16}$$

$$x = \pm 4$$

WORKED EXAMPLE 9

Solve the quadratic equation $s^2 = 27$, leaving your answer in exact form.

THINK

- 1 Write the equation.
- 2 Take the square root of both sides.

WRITE

$$s^2 = 27$$

$$s = \pm\sqrt{27}$$

Exercise 10.5 Quadratic equations

Individual pathways

PRACTISE

Questions:
1, 2, 3, 7, 8

CONSOLIDATE

Questions:
1, 2, 3, 4, 5, 6, 7

MASTER

Questions:
1, 2, 3, 4, 5, 6, 7, 8, 9

Individual pathway interactivity: int-7048

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Understanding and fluency

1. **WE8** Solve the following quadratic equations without using a calculator.
a. $a^2 = 36$ b. $b^2 = 169$ c. $c^2 = 64$ d. $d^2 = 81$ e. $e^2 = 121$
2. Solve the following quadratic equations, giving your answers correct to 2 decimal places if necessary.
a. $a^2 = 14$ b. $b^2 = 5$ c. $c^2 = 17$ d. $d^2 = 24$ e. $e^2 = 144$
3. **WE9** Solve the following equations, leaving your answers in exact form.
a. $a^2 = 26$ b. $b^2 = 48$ c. $c^2 = 70$ d. $d^2 = 35$ e. $e^2 = 99$
4. Solve the following equations, giving your answers correct to 2 decimal places. (*Hint:* First divide both sides by the coefficient of x^2 .)
a. $3x^2 = 45$ b. $7y^2 = 21$ c. $8p^2 = 80$ d. $9s^2 = 85$ e. $4t^2 = 110$

Communicating, reasoning and problem solving

5. How many solutions does the quadratic equation $x^2 = 0$ have?
6. Explain why the equation $a^2 = -1$ has no solutions.
7. A square paving stone has an area of 144 cm^2 . By solving a quadratic equation, determine the side length of the paving stone.
8. The square sign shown has an area of 5329 cm^2 . What is the length of one side of the sign?



9. Give an example of a simple quadratic equation that has no solutions.

10.6 Review

Investigation | Rich task

The Olympic freestyle final

Swimming is one of the most popular sports of the Olympic Games. Over the years the Olympics have been held, competitors have been swimming more quickly and their times have been correspondingly reduced.

The table below displays the winning times for the men's and women's 100-metre freestyle final for the Olympic Games from 1960 to 2012.



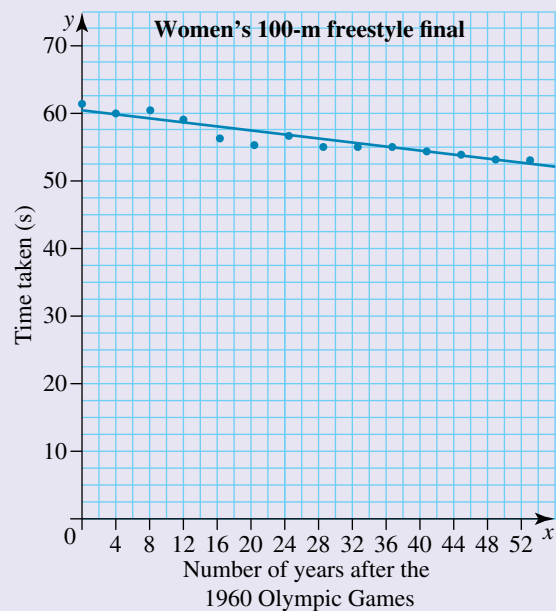
Men's and women's 100-metre freestyle results — Olympic Games 1960–2012

Year	Men's time (seconds)	Women's time (seconds)
1960	55.2	61.2
1964	53.4	59.5
1968	52.2	60.0
1972	51.2	58.6
1976	50.0	55.7
1980	50.4	54.8
1984	49.8	55.9
1988	48.6	54.9
1992	49.0	54.7
1996	48.7	54.5
2000	48.3	53.8
2004	48.2	53.8
2008	47.2	53.1
2012	47.5	53.0

Note: All times have been rounded to 1 decimal place.

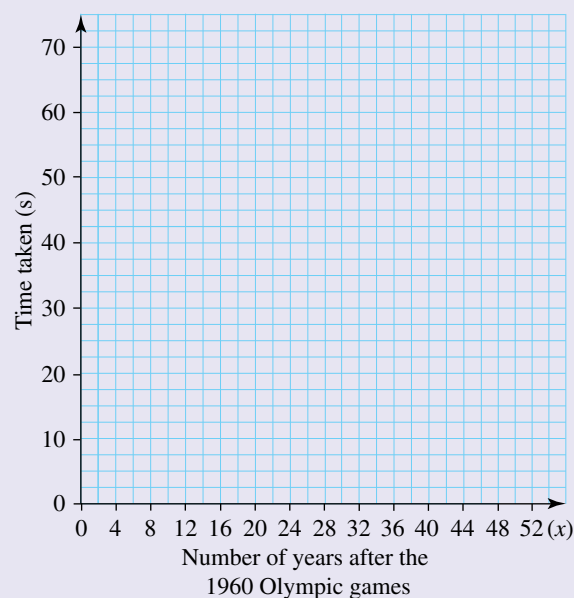
The women's times have been graphed on the set of axes on the next page. There is no straight line that passes through all the points, so a line of best fit has been selected to approximate the swimming times. The equation of this line is given by $t = 59.9 - 0.1504x$, where t represents the time taken and x represents the number of years after 1956.

1. What year is represented by $x = 16$?
2. Substitute $x = 16$ into the equation to find an approximation for the time taken. How close is this approximation to the actual time given in the table?
3. What x -value would you use for the 2016 Olympics? Use the equation to predict the women's 100-metre freestyle final time for the 2016 Olympics.
4. The women's time in 2016 was 52.7 seconds. How does your prediction compare with the actual time?
5. Use backtracking to solve the equation to the nearest whole number when $t = 55.7$ seconds. What year does your solution represent?



An equation to approximate the men's times is given by $t = 53.4 - 0.1299x$. Use this equation to answer the following questions.

6. The men's time in 1984 was 49.8 seconds. How does this compare with the time obtained from the equation?
7. Use the equation to predict the men's time for the 2016 Olympic Games.
8. Plot the men's times on the set of axes provided. Between the points, draw in a line of best fit.
9. Extend your line of best fit so that it passes the x -value that represents the year 2016. How does this value compare with the prediction you obtained in question 7 above?
10. The men's time in 2016 was 47.6 seconds. How does your prediction compare with the actual time?
11. Use your graphs to compare both the men's and women's times with the actual results obtained during the 2004 Athens Olympics.



How long will it be before the men and women are swimming identical times?

12. Investigate this by plotting the times given on the same set of axes and drawing a line of best fit. Extend both lines until they intersect. The point of intersection represents the time when the swimming times are identical. Present your findings on graph paper.

Digital doc: Investigation — The Olympic freestyle final (doc-13941)

Digital doc: Code puzzle — Kidneys (doc-13942)

Digital doc: Summary Topic 10 (doc-23246)

Interactivity: Word search Topic 10 (int-2633)

Interactivity: Crossword Topic 10 (int-2634)

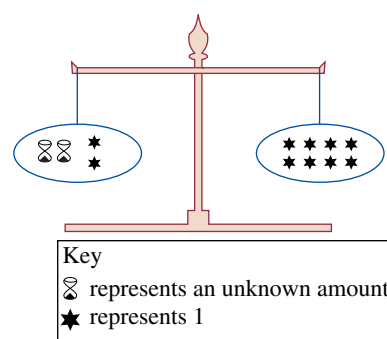
Interactivity: Sudoku Topic 10 (int-3190)

Exercise 10.6 Review questions

To answer questions online and to receive **immediate feedback** and **fully worked solutions** for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Understanding and fluency

- Write an equation that is represented by the diagram at right.
 - Show what happens when you take 2 from both sides, and write the new equation.
- MC** If we start with $x = 5$, which of these equations is not true?
 - $x + 2 = 7$
 - $3x = 12$
 - $-2x = -10$
 - $\frac{x}{5} = 1$
 - $x - 2 = 3$
- MC** If we start with $x = 3$, which of these equations is not true?
 - $\frac{2x}{3} = 2$
 - $-2x = -6$
 - $2x - 6 = 0$
 - $\frac{x}{5} = \frac{3}{5}$
 - $x - 5 = 2$
- Solve these equations by doing the same to both sides.
 - $z + 7 = 18$
 - $-25 + b = -18$
 - $9t = \frac{1}{3}$
 - $-8.7 = \frac{l}{5}$
- Solve these equations by doing the same to both sides.
 - $5v + 3 = 18$
 - $5(s + 11) = 35$
 - $-2(r + 5) - 3 = 5$
 - $\frac{2y - 3}{7} = 9$
- Solve the following equations and check each solution.
 - $5k + 7 = k + 19$
 - $4s - 8 = 2s - 12$
 - $3t - 11 = 5 - t$
 - $5x + 2 = -2x + 16$
- Expand the brackets first and then solve the following equations.
 - $5(2v + 3) - 7v = 21$
 - $3(m - 4) + 2m = m + 8$
- Solve the following quadratic equations, leaving your answers in exact form.
 - $x^2 = 55$
 - $s^2 = 30$
 - $t^2 = 101$
 - $v^2 = 200$
- Solve the following quadratic equations, giving your answers correct to 2 decimal places.
 - $a^2 = 11$
 - $c^2 = 46$
 - $e^2 = 154$
 - $f^2 = 77$



$$c. -\frac{8}{9} = z - \frac{4}{3}$$

$$f. -\frac{6}{13} = \frac{h}{8}$$

$$c. \frac{d - 7}{4} = 10$$

$$f. \frac{x}{5} - 3 = 2$$

Communicating, reasoning and problem solving

10. Ray the electrician charges \$80 for a call-out visit and then \$65 per half hour.
- Write an equation for his fees, where C is his call-out cost and t is the number of 30-minute periods he spent on the job.
 - How long did a particular job take if he charged \$275.00?
 - Ray's sister Rhonda is a plumber and uses the equation $C = 54t + 86$ to calculate her costs. She also charged \$275 for one job. How long did Rhonda spend at this particular job?
 - Explain what the numbers 54 and 86 could mean.



11. At the end of the term, Katie's teacher gave the class their average scores. They had done four tests for the term. Katie's average was 76%. She had a mark of 83% for Probability, 72% for Geometry and 91% for Measurement but had forgotten what she got for Algebra. Write an equation to show how Katie would work out her Algebra test score and then solve this equation.
12. My three daughters were each born 2 years apart. Their combined ages come to 63 years. What is the age of the eldest?
13. A truck carrying 50 bags of cement weighs 7.43 tonnes. After delivering 15 bags of cement, the truck weighs 6.755 tonnes. How much would an empty truck weigh?
14. You are 4 times as old as your sister. In 8 years time you will be twice as old as your sister. What are your ages now?
15. What whole number must be added to both the numerator and the denominator of a fraction of $\frac{1}{4}$ to obtain an answer of $\frac{2}{3}$?
16. You lend three friends a total of \$45. You lend the first friend x dollars. To the second friend you lend \$5 more than you do for the first friend. To the third friend you lend three times as much as for the second friend. How much does each person receive?
17. What is the greatest possible perimeter of a triangle with sides $5x + 20$, $3x + 76$, $x + 196$, given that the triangle is isosceles? All sides are in mm.
18. A square painting had an area of 3500 cm^2 . What is the side length of the painting correct to 2 decimal places?
19. Penny was looking at the following phone plans to see which one would be the most economical for her phone habits. Penny's average phone conversation was approximately 2 minutes long.

1. 15 cents connection fee and 30 cents per minute
 2. No connection fee and 40 cents per minute
 3. 40 cents connection fee and 19 cents per minute
- a.** Help Penny by writing an algebraic equation for each phone plan.
 - b.** Use the equations to find the most economical phone plan for Penny if her conversations average:
 - 1 minute, 2 minutes, 3 minutes.
 - c.** At what average talking time should Penny switch to Plan 3?
- 20.** A rectangular vegetable patch is $(3x + 4)$ metres long and $(2x - 5)$ metres wide. Its perimeter is 58 metres. What are the dimensions of the vegetable plot?
- 21.** Knitting involves following patterns, often with outstanding designs resulting. Consider a simple pattern to knit a triangular scarf. The pattern might read like this.
- Start with 3 stitches.
 - Increase by 1 stitch at both ends of each row.
 - Continue in this manner until the scarf is as long as you would like.
- a.** If r represents the row number, and n the number of stitches in that row, write an equation showing the relationship between n and r .
 - b.** Use your equation to calculate the number of rows you would have to knit to have 65 stitches.
- 22.** While on holidays Amy hired a bicycle for \$9 an hour and paid \$3 for the use of a helmet. Her brother, Ben, found a cheaper hire place, which charged \$6 per hour, but the hire of the helmet was \$5 and he had to pay \$5 for insurance. Each hire place measures the time and charges in 20-minute blocks. They both ended up paying the same amount and were gone for the same number of hours. Construct a table of values to determine how long they were gone and how much it cost.



- 23.** Michael checked his bank balance before going shopping. He had \$450. While shopping he paid with his bank card. He bought two suits, which cost the same, and three pairs of shoes, each of which cost half the price of a suit. He also had lunch for \$12. When he checked his balance again he was \$33 overdrawn. What did one suit cost?
- 24.** Shannon is saving to buy a new computer, which costs \$3299. So far he has \$449 in the bank and he wants to make regular deposits each month until he reaches his target of \$3299. If he wants to buy the computer in 8 months' time, how much does he need to save as a monthly deposit?

Answers

Topic 10 Equations

Exercise 10.2 Keeping equations balanced

1. a. $x + 5 = 11$ b. $7x = 42$ c. $x - 4 = 2$ d. $\frac{x}{3} = 2$
e. $-4x = -24$ f. $-x = -6$ g. $-x = -6$ h. $x - 9 = -3$
i. $\frac{2x}{3} = 4$ j. $\frac{3x}{2} = 9$ k. $x - \frac{2}{3} = 5\frac{1}{3}$
2. a. $2x = 4$ b. $x = 2$
3. a. $x + 3 = 5$ b. $x = 2$
4. a. $3x + 1 = 7$ b. $3x + 4 = 10$
5. a. $2x + 1 = 5$ b. $4x + 2 = 10$
6. B
7. E
8. C
9. C
10. a. $m + 16 = 17$ b. $n - 5 = 3$ c. $2p + 6 = 12$ d. $12m = 48$
e. $2n = 6$ f. $2p = 30$ g. $12p + 28 = 8$ h. $\frac{5m - 3}{3} = \frac{-1}{3}$
i. $ax + 5a = 7a$
11. a. Add 4 to both sides. b. Add 6 to both sides.
c. Multiply both sides by 3. d. Multiply both sides by 2.
e. Add 2 to both sides, then multiply both sides by 4. f. Multiply both sides by 4, then add 2 to both sides.
12. a. They have subtraction and division in the expressions and these cannot be represented on a pan balance scale.
b. In part e, the pronumeral is divided by 4 and then 2 is subtracted from the result. In part f, 4 is subtracted from the pronumeral and then the result is divided by 2.
13. a. $x = 14$ b. $x = 9$ c. $w = 21$ d. $e = 18$
e. $q = 48$ f. $q = 42$
14. a. $5x = 3$
 $x = \frac{3}{5}$. One bag of sugar weighs $\frac{3}{5}$ kg.
b. $4x + 2.70 = 9.90$
 $x = 1.8$. One chocolate bar costs \$1.80.
c. $3x + 500 = 920$
 $x = 140$. One apple weighs 140 g.
15. Two weighings are needed.

The least number of weighings is two. Take 6 coins and place 3 coins on each side of the balance. There is a remainder of 2 unweighed coins. If the 6 coins balance, the heavy coin must be one of the 2 unweighed coins. If the heavy coin is one of the 2 unweighed coins, place 1 coin on each side of the balance to determine which is the heaviest. If the 6 coins do not balance, the heavy coin must be one of the 3 coins on the heavy side of the balance (the side that drops lower than the other). Take the 3 coins, place one coin on each side of the empty balance, so that there are 2 coins on the scale at a time. If the coins balance, the heavy coin is the coin not on the balance. If the coins do not balance, the heavy coin is on the heavy side of the balance.

Exercise 10.3 Using algebra to solve problems

1. a. $x = -1$ b. $r = -5$ c. $t = 24$ d. $w = 2.7$
e. $m = \frac{1}{2}$ f. $j = -\frac{19}{7}$ or $-2\frac{5}{7}$ g. $q = 19$ h. $r = 9$
i. $t = 32$ j. $y = 14.5$ k. $z = \frac{19}{21}$ l. $f = \frac{4}{13}$
2. a. $d = 8$ b. $p = -14$ c. $u = \frac{4}{5}$ or 0.8 d. $g = 5$
e. $m = \frac{1}{32}$ f. $j = -\frac{1}{15}$ g. $t = 24$ h. $k = -60$
i. $l = -21.2$ j. $v = 4$ k. $c = -\frac{5}{3}$ or $-1\frac{2}{3}$ l. $h = -\frac{35}{12}$ or $-2\frac{11}{12}$

3. a. $m = 3$ b. $w = -5$ c. $k = -4$ d. $t = -3$
 e. $m = 1$ f. $n = -18$ g. $k = -9$ h. $s = -7$
 i. $m = 3.5$ j. $p = -14$ k. $g = -3$ l. $f = 5$
 m. $q = 9.05$ n. $r = -3.2$ o. $t = 3.6$ p. $k = -0.2$
 q. $g = -\frac{1}{4}$ or -0.25 r. $f = \frac{15}{16}$
4. a. $x = 21$ b. $x = 9$ c. $m = -17$ d. $h = -12$
 e. $m = -20$ f. $w = -10$ g. $m = 2\frac{1}{3}$ h. $c = 1$
 i. $m = -8$ j. $t = -37$ k. $c = -19.5$ l. $x = -20.8$
5. a. $m = -5$ b. $x = -20$ c. $m = 1\frac{1}{5}$ d. $x = 3$
 e. $x = -7$ f. $x = 11$ g. $x = 7$ h. $b = 10$
 i. $f = -9$ j. $z = 6$ k. $m = 5$ l. $u = -11$
 m. $m = -3$ n. $w = -1\frac{3}{5}$ o. $x = 8$ p. $d = 3$
 q. $n = 9$ r. $t = -2$

6. a. The solution is not correct.

b. Alex should have subtracted 3 from both sides first before dividing both sides by 2. The solution should be $x = 5\frac{1}{2}$. The error made was that he didn't divide all parts of the equation by 2.

7. a. $x = 2$ b. $v = -2$ c. $m = -1$ d. $y = 4$
 e. $y = -6$ f. $t = -5$ g. $w = 2\frac{2}{9}$ h. $w = \frac{2}{3}$
 i. $u = -4$ j. $c = 6$

8. a. $2\frac{1}{2}$ hours

b. 40 could be the charge per hour; 55 could be the flat fee covering travel and other expenses.

9. $l + 286 = 517, l = \$231$

10. a. $10x + 54 = 184, x = 13$ cm b. $11x + 12 = 287, x = 25$ cm

11. 2

12. Yes, when Tom is 45 and his dad is 90.

13. \$80 per month

Exercise 10.4 Equations with the unknown on both sides

1. a. $x = 3$ b. $y = 4$ c. $n = 4$ d. $t = 4$
 e. $w = 5$ f. $y = 3\frac{2}{3}$ g. $z = 4$ h. $a = -4$
 i. $s = 2$ j. $k = 4$ k. $w = -3$ l. $v = -4$
2. a. $w = 2$ b. $b = 2$ c. $n = 2$ d. $s = 3$
 e. $a = -3$ f. $m = 2$ g. $p = 4$ h. $d = 3$
 i. $m = 0$ j. $s = 1$ k. $t = -2$ l. $x = 4$
3. a. $x = 3$ b. $m = 10$ c. $n = 12$ d. $t = 5$
 e. $d = 2\frac{5}{6}$ f. $w = 1\frac{2}{9}$ g. $k = 10$ h. $s = -3$
 i. $z = -17$ j. $v = 1\frac{1}{2}$ k. $m = 11\frac{2}{3}$ l. $d = 3\frac{7}{10}$
 m. $d = -19$ n. $k = -10$ o. $v = 2$ p. $l = 18$

4. $x = -\frac{13}{33}$

5. $x = \frac{13}{5}$ or $2\frac{3}{5}$

6. 6.25 cm

7. a. $6(x + 3) = \frac{1}{2} \times 6 \times (x + 5 + 2)$

b. $x = 1$ cm

c. Parallelogram: base 4 cm, height 6 cm, Trapezium: base 6 cm, top 2 cm, height 6 cm.

8. 24

9. 3

10. Mick is 14, his father is 42.

11. 16

12. 12

13. a. i. $3x + 12$

ii. $4x + 5$

b. $3x + 12 = 4x + 5$

c. $x = 7$

14. $2(3b - 7) + 2b = 3b + 7$

$$6b - 14 + 2b = 3b + 7$$

$$5b = 21$$

$$b = 4.2$$

Rectangle lengths are 4.2 cm and 5.6 cm; triangle lengths are 4.2 cm, 5.2 cm and 10.2 cm.

15. a. $2.5x + 10 = 3.5x + 5$

$$x = 5$$

Both options cost the same for 5 rides.

b. Option 1: $2.5 \times 2 + 10 = \$15$; option 2: $3.5 \times 2 + 5 = \$12$. Option 2 is cheaper for 2 rides.

16. a. Let daughter's age = x . Mother's age = $3(x + 8)$ and $= 4(13 - x)$.

b. $3(x + 8) = 4(13 - x)$

c. $3x + 24 = 52 - 4x$

d. $x = 4$

e. The daughter is 4 and the mother is 36.

f. The number of their house is 36.

17. 13 Smarties

Exercise 10.5 Quadratic equations

1. a. $a = \pm 6$

b. $b = \pm 13$

c. $c = \pm 8$

d. $d = \pm 9$

e. $e = \pm 11$

2. a. $a = \pm 3.74$

b. $b = \pm 2.24$

c. $c = \pm 4.12$

d. $d = \pm 4.90$

e. $e = \pm 12$

3. a. $a = \pm\sqrt{26}$

b. $b = \pm\sqrt{48}$

c. $c = \pm\sqrt{70}$

d. $d = \pm\sqrt{35}$

e. $e = \pm\sqrt{99}$

4. a. $x = \pm 3.87$

b. $y = \pm 1.73$

c. $p = \pm 3.16$

d. $s = \pm 3.07$

e. $e = \pm 5.24$

5. One ($x = 0$)

6. There are no numbers that when squared equal -1 .

7. 12 cm

8. 73 cm

9. Answers will vary.

Investigation | Rich task

1. 1976

2. $t = 57.5$ s; difference of 1.8 s

3. $x = 56$, $t = 51.5$ s

4. Check with your teacher.

5. $x = 28$; year is 1988

6. $t = 50.3$ s; difference of 0.5 s

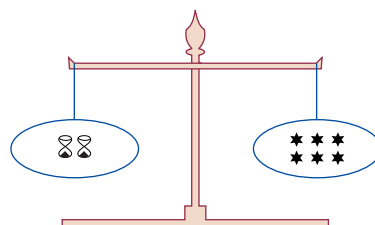
7. $t = 46.1$ s

8. to 12. Check with your teacher.

Exercise 10.6 Review questions

1. a. $2x + 2 = 8$

b. $2x = 6$



2. B

3. E

4. a. 11

b. 7

c. $\frac{4}{9}$

d. $\frac{1}{27}$

e. -43.5

f. $-\frac{48}{13}$ or $-3\frac{9}{13}$

5. a. $v = 3$

b. $s = -4$

c. $d = 47$

d. $r = -9$

e. $y = 33$

f. $x = 25$

6. a. $k = 3$

b. $s = -2$

c. $t = 4$

d. $x = 2$

7. a. $v = 2$

b. $m = 5$

8. a. $x = \pm\sqrt{55}$

b. $s = \pm\sqrt{30}$

c. $t = \pm\sqrt{101}$

d. $v = \pm\sqrt{200}$

9. a. $a = \pm 3.32$

b. $c = \pm 6.78$

c. $e = \pm 12.41$

d. $f = \pm 8.77$

10. a. $C = 65t + 80$

b. $1\frac{1}{2}$ hours

c. 1 hour 45 minutes

d. \$54: charge per half hour, \$86: flat call-out fee.

11. $\frac{83 + 72 + 91 + x}{4} = 76$

Katie scored 58% for her Algebra test.

12. Eldest is 23 years old.

13. The empty truck would weigh 5.18 tonnes.

14. You are 16 and your sister is 4.

15. 5

16. \$5, \$10, \$30

17. 832 mm

18. 59.16 cm

19. a. i. $y = 0.15 + 0.3x$

ii. $y = 0.4x$

iii. $y = 0.4 + 0.19x$

where x is the time of the call in minutes and y is the cost of the call.

b. 1 minute — choose Plan 2; 2 minutes — choose Plan 1; 3 minutes — choose Plan 3.

c. Switch to Plan 3 for calls over 2 minutes.

20. $22\text{ m} \times 7\text{ m}$

21. a. $n = 2r + 1$

b. 32 rows

22. 2 hours 20 minutes, \$24

Time (mins)	Amy	Ben
0	\$3	\$10
20	\$6	\$12
40	\$9	\$14
60	\$12	\$16
80	\$15	\$18
100	\$18	\$20
120	\$21	\$22
140	\$24	\$24

23. \$134.57

24. \$356.25

TOPIC 11

Linear relationships

11.1 Overview

Numerous **videos** and **interactivities** are embedded just where you need them, at the point of learning, in your learnON title at www.jacplus.com.au. They will help you to learn the concepts covered in this topic.



11.1.1 Why learn this?

Linear equations are a form of algebra, and they are often used to describe everyday situations using mathematics. One of the most useful skills that you learn in algebra is how to solve equations. Solving equations means finding numbers whose values you do not know. Once you learn the techniques of solving equations, you will find that many problems can be expressed and solved using linear equations.

Cartesian coordinates can be used to pinpoint where you are on a map or a graph. *Coordinate graphing* is a visual method of showing relationships between numbers. *Coordinate geometry* is the mathematical equivalent of latitude and longitude — it is a means of pinpointing your position. Wollongong, for example, is $34^{\circ}26' \text{ S}$, $150^{\circ}53' \text{ W}$. In coordinate geometry we define a point by stating how far across it is, and how far up or down it is.

LEARNING SEQUENCE

- 11.1** Overview
- 11.2** The Cartesian plane
- 11.3** Translations in the Cartesian plane
- 11.4** Identifying patterns
- 11.5** Linear patterns
- 11.6** Linear graphs
- 11.7** Graphing with technology
- 11.8** Solving equations graphically
- 11.9** Review

LEARNING OUTCOMES

A student:

- communicates and connects mathematical ideas using appropriate terminology, diagrams and symbols MA4-1WM
- recognises and explains mathematical relationships using reasoning MA4-3WM
- creates and displays number patterns; graphs and analyses linear relationships; and performs transformations on the Cartesian plane MA4-11NA

CONTENT DESCRIPTIONS

Students:

Given coordinates, plot points on the Cartesian plane, and find coordinates for a given point (ACMNA178)
Describe translations, reflections in an axis, and rotations of multiples of 90° on the Cartesian plane using coordinates (ACMMG181)



Plot linear relationships on the Cartesian plane, with and without the use of digital technologies (ACMNA193)
Solve linear equations using graphical techniques (ACMNA194)

Source: NSW Syllabus for the Australian curriculum

learnon RESOURCES — ONLINE ONLY

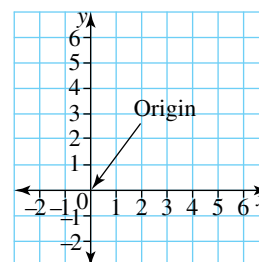
eLesson: The story of mathematics — Descartes (eles-1842)

Note: Your teacher may now set you a pre-test to determine how familiar you are with the content in this topic.

11.2 The Cartesian plane

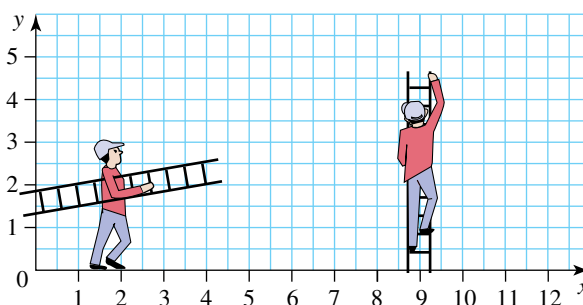
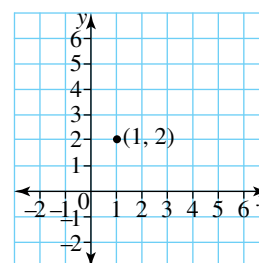
11.2.1 The Cartesian plane

- A **Cartesian plane** (named after its inventor René Descartes) is a visual means of describing locations on a plane by using two numbers as coordinates (rather than a letter and a number).
- The Cartesian plane is formed by two perpendicular lines, which are called the axes. The horizontal axis is called the **x-axis**; the vertical axis is called the **y-axis**.
- The centre of a Cartesian plane (where the x - and y -axis intersect) is called the **origin**.
- Both axes are evenly scaled and numbered, with 0 (zero) placed at the origin. On the x -axis the numbers increase from left to right, while on the y -axis the numbers increase from bottom to top.
- Arrows are placed on the end of each axis to show that they continue infinitely.



11.2.2 Cartesian coordinates

- To locate any point on the Cartesian plane we use a pair of numbers called *Cartesian coordinates*. A Cartesian coordinate is written as (x, y) , where x and y are any numbers. The first number refers to the horizontal position of the point and is called ‘the x -coordinate’ of a point. The second number refers to the vertical position of the point and is called ‘the y -coordinate’ of a point. The coordinates of the origin are $(0, 0)$.
- To locate a point on the Cartesian plane, move along the x -axis to the number indicated by the x -coordinate and then along the y -axis to the number indicated by the y -coordinate. For example, to locate the point with coordinates $(1, 2)$, move 1 unit to the right of the origin and then 2 units up.
- *Hint:* To help remember the order in which Cartesian coordinates are measured, think about using a ladder. Remember we must always walk across with our ladder and then climb up it.



WORKED EXAMPLE 1

Draw a Cartesian plane with axes extending from 0 to 6 units. Mark the following points with a dot, and label them.

a (2, 4)

b (5, 0)

c (0, 2)

d $(3\frac{1}{2}, 2)$

THINK

1 First rule up and label the axes.

2 Mark each point.

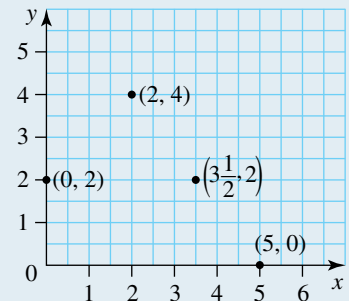
a (2, 4) means starting at the origin, go across 2 units, and then up 4 units.

b (5, 0) means go across 5 units and up 0 units. It lies on the x -axis.

c (0, 2) means go across 0 units and up 2 units. It lies on the y -axis.

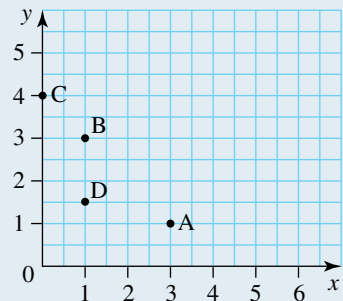
d $(3\frac{1}{2}, 2)$ means go across $3\frac{1}{2}$ units and up 2 units. Label each point.

DRAW



WORKED EXAMPLE 2

Find the Cartesian coordinates for each of the points A, B, C and D.



THINK

Point A is 3 units across and 1 unit up.

Point B is 1 unit across and 3 units up.

Point C is 0 units across and 4 units up.

Point D is 1 unit across and $1\frac{1}{2}$ units up.

WRITE

A is at (3, 1)

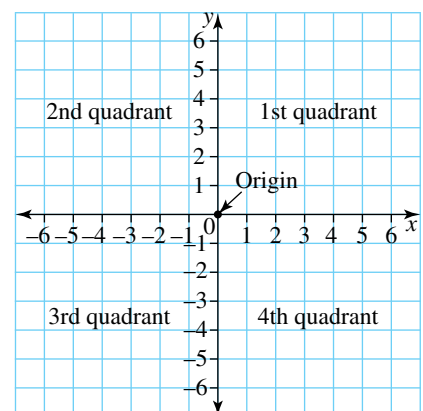
B is at (1, 3)

C is at (0, 4)

D is at $(1, 1\frac{1}{2})$

11.2.3 Extending the axes

- The Cartesian axes can extend infinitely in both directions.
- On the x -axis, the values to the left of the origin are negative and decreasing. Likewise, on the y -axis the values below the origin are negative and decreasing.
- The axes divide the Cartesian plane into four sections called **quadrants**. The quadrants are numbered in an anti-clockwise direction, starting with the top right corner.
- If both x - and y -coordinates of the point are positive, it will be located in the first quadrant. If the x -coordinate is negative but the y -coordinate is positive, the point will be in the second quadrant. If the point is in the third quadrant, both the



x - and y -coordinates of the point will be negative. Finally, if the point is in the fourth quadrant, its x -coordinate is positive, while its y -coordinate is negative.

- If the point is located on the x -axis, its y -coordinate is always 0. Likewise, if the point is on the y -axis, its x -coordinate is always 0.

ACTIVITY: CREATING A PICTURE

- Draw a Cartesian plane and create a picture of your choice on the plane that results from joining a series of points
- Write a list of instructions that details the order in which the points on the Cartesian plane are to be joined.
- Test your instructions on a classmate.

WORKED EXAMPLE 3

Plot the following points on the Cartesian plane.

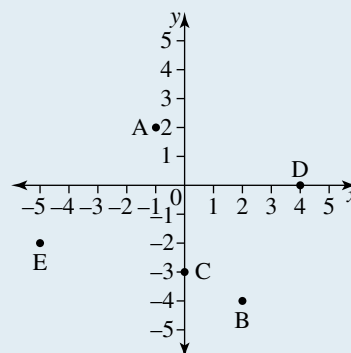
A(−1, 2), **B**(2, −4), **C**(0, −3), **D**(4, 0), **E**(−5, −2)

State the location of each point on the plane (i.e. the quadrant, or the axis on which it sits).

THINK

- Draw a set of axes, ensuring that they are long enough to fit all the values.
- Plot the points. The first point is one unit to the left and two units up from the origin. The second point is two units to the right and four units down from the origin (and so on).

WRITE



- Look at the plane and state the location of each point. Remember that the quadrants are numbered in an anti-clockwise direction, starting at the top right. If the point is on the axis, specify which axis it is.

Point A is in the second quadrant.
 Point B is in the fourth quadrant.
 Point C is on the y -axis.
 Point D is on the x -axis.
 Point E is in the third quadrant.

learnon RESOURCES — ONLINE ONLY

Interactivity: The Cartesian plane (int-3831)

Interactivity: Extending the axes (int-4137)

eLesson: Coordinates and the Cartesian plane (eles-0008)

Digital doc: SkillsSHEET Plotting coordinate points (doc-6936)

Exercise 11.2 The Cartesian plane

Individual pathways

PRACTISE

Questions:

1, 2, 3, 4, 5, 6, 8, 9, 11

CONSOLIDATE

Questions:

1, 2, 3, 4, 5, 6, 7, 9, 10, 12, 14

MASTER

Questions:

1, 2, 3, 4, 5, 6, 8–15

Individual pathway interactivity: int-4384

learnON ONLINE ONLY

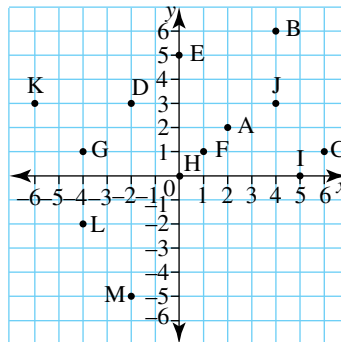
To answer questions online and to receive **immediate feedback** and **fully worked solutions** for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Understanding and fluency

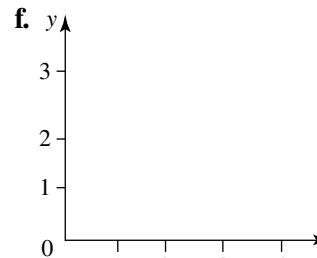
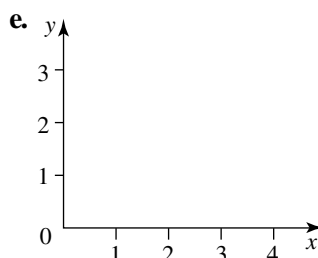
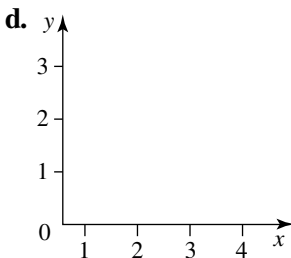
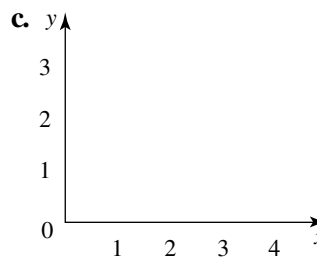
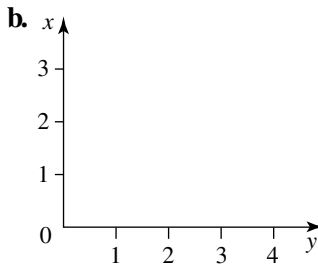
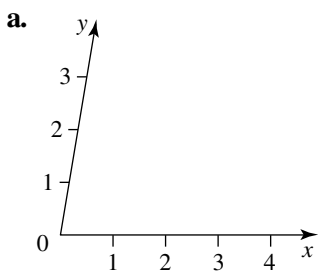
- WE1** Draw a Cartesian plane that extends from -6 to 6 on the x -axis and -6 to 6 on the y -axis, and plot and label the following points.

a. A(3, 3)	b. B(2, 5)	c. C(5, 1)	d. D(-1, 4)	e. E(-4, 2)
f. F(-2, 0)	g. G(-2, -3)	h. H(-4, -5)	i. I(0, -3)	j. J(1, -2)
k. K(3, $-1\frac{1}{2}$)	l. L($4\frac{1}{2}$, 0)			

- WE2** Write the Cartesian coordinates of the points A to M marked on the Cartesian plane.
- Each of the following sets of Cartesian axes (except one) has something wrong with it. From the list below, match the mistake in each diagram with one of the sentences.

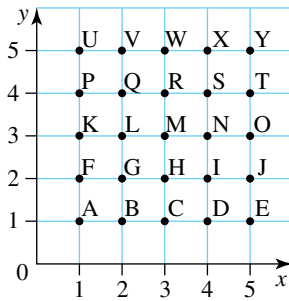


- The units are not marked evenly.
- The y -axis is not vertical.
- The axes are labelled incorrectly.
- The units are not marked on the axes.
- There is nothing wrong.



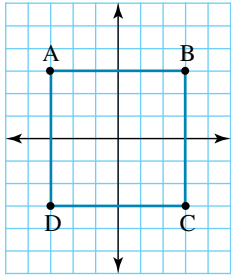
- MC** *Note:* There may be more than one correct answer.
 - The point (3, 4) gives a position on the Cartesian plane of:

A. 3 on the y -axis, 4 on the x -axis	B. 3 left, 4 up
C. 4 right, 3 up	D. 3 on the x -axis, 4 on the y -axis
E. 3 right, 4 up	

- b. The point $(-2, 0)$ gives a position on the Cartesian plane of:
- A. left 2, up 0
 C. -2 on the x -axis, 0 on the y -axis
 E. 2 on the x -axis, 0 on the y -axis
- B. left 0 , down 2
 D. -2 on the y -axis, 0 on the x -axis.
5. On 1-cm graph paper, draw a Cartesian plane with an x -axis from -6 to 6 and a y -axis from -6 to 6 . Connect these groups of points.
- START $(4, 6) (-4, 6) (-6, 0) (-4, -6) (4, -6) (6, 0) (4, 6) (-6, 0) (4, -6) (4, 6)$ STOP
- START $(-4, 6) (-4, -6) (6, 0) (-4, 6)$ STOP
- START $(4, 0) (2, 2.5) (-2, 2.5) (-4, 0) (-2, -2.5) (2, -2.5) (4, 0)$ STOP
- Colour the 6 triangles between the star and the hexagon. For example the triangle $(6, 0) (4, 6) (4, 1)$ could be coloured red. Colour the 6 triangles inside the star. For example $(4, 0) (4, 1) (2, 2.5)$ could be coloured green.
6. Draw a Cartesian plane using the following points. First check the coordinates to find the lowest and highest x - and y -values needed on each axis. Connect these groups of points.
- START $(4, 1) (3, 1) (4, 3) (5, 3) (5.5, 4) (5.5, 5) (6, 6) (6, 7) (5, 7.5) (4, 7.5) (3, 7.5) (2, 8) (1.5, 9) (1.5, 10) (2, 11) (3, 12) (4, 12.5) (5, 13) (6, 14) (7, 15) (8, 15) (10, 14) (11, 11) (11, 10) (10.5, 8.5) (10, 7.5) (9, 10.5) (8.5, 8) (7.5, 7) (8, 6.5) (8, 5) (9, 4) (10, 3) (10, 1) (4, 1)$ STOP
- START $(2, 8) (1.5, 7.5) (1, 8) (1.5, 9)$ STOP
- START $(4, 1) (4, 2) (5, 2.5) (6.5, 2.5) (5.5, 4)$ STOP
- START $(6, 6.5) (8, 6.5) (8, 6) (6, 6)$ STOP
- EYE AT $(5.5, 11)$
7. Draw a Cartesian plane. Check the following coordinates to find the lowest and highest x - and y -value needed on the axes. Then draw a cartoon character.
- START $(6, 7) (7.5, 9) (5, 9) (4.5, 12) (2, 11) (0, 13) (-1.5, 10) (-5, 11) (-5, 8) (-8, 6) (-6, 4) (-8, 2) (-6, 1) (-7, -2) (-4, -1.5) (-4, -3.5) (-1.5, -3) (-2, -4) (-4, -7) (-5, -8)$ STOP
- START $(-2, -9) (-1, -7) (1, -8) (3, -8) (4, -7.5) (5, -10)$ STOP
- START $(4, -7.5) (3.5, -6) (3.5, -4) (4, -3) (5, -2.5) (5, -2) (4, -1.5) (4, -1) (5, 0.5) (7, 1) (8, 2) (8, 2.5) (6.5, 3)$ STOP
- START $(4, -2.5) (2, -3) (0.5, -3) (0, -2) (1, -1) (2, -0.5) (3, 0) (7, 1)$ STOP
- START $(6, 2.5) (6.5, 3) (6.5, 4) (6, 4) (4, 3)$ STOP
- START $(6, 7) (5, 7.5) (4, 7) (3, 6) (1, 6) (0, 5) (-1, 4) (0, 2) (1.5, 1.5) (3, 2) (4, 4) (6.5, 4) (7, 5) (7, 6) (6, 7)$ STOP
- START $(4, 4) (4, 5) (3, 6)$ STOP
- START $(1, -1) (5, 0)$ STOP
- EYES AT $(1, 3)$ AND $(5, 5)$
- EYE LASHES $(-1, 4)$ TO $(-2, 4.5)$, $(0, 5)$ TO $(-0.5, 6)$, $(1, 6)$ TO $(0.5, 7)$, $(2, 6)$ TO $(2, 7)$, $(4, 7)$ TO $(3.5, 8)$, $(5, 7.5)$ TO $(5, 8.5)$, $(6, 7)$ TO $(6.5, 8)$, $(6.5, 6.5)$ TO $(7, 7)$
8. Messages can be sent in code using a grid like the one shown at right, where the letter B is represented by the coordinates $(2, 1)$. Use the diagram to decode the answer to the following riddle.
- Q** Where did they put the man who was run over by a steamroller?
- A** $(4, 2)(4, 3) (3, 2)(5, 3)(4, 4)(1, 4)(4, 2)(5, 4)(1, 1)(2, 3) (4, 2)(4, 3) (3, 5)(1, 1)(3, 4)(4, 1)(4, 4) (4, 4)(4, 2)(4, 5) (4, 4)(5, 1)(2, 5)(5, 1)(4, 3) (5, 1)(4, 2)(2, 2)(3, 2)(5, 4) (1, 1)(4, 3)(4, 1) (4, 3)(4, 2)(4, 3)(5, 1)$
9. Consider the following set of points: A $(2, 5)$, B $(-4, -12)$, C $(3, -7)$, D $(0, -2)$, E $(-10, 0)$, F $(0, 0)$, G $(-8, 15)$, H $(-9, -24)$, I $(18, -18)$, J $(24, 0)$.
- Which of the following statements are true?
- 

- a. Points A and J are in the first quadrant.
 - b. Points B and H are in the third quadrant.
 - c. Only point I is in the fourth quadrant.
 - d. Only one point is in the second quadrant.
 - e. Point F is at the origin.
 - f. Point J is not on the same axis as point E.
 - g. Point D is two units to the left of point F.
10. **WE3** Plot the following points on the Cartesian plane and state which quadrant the points lie in, or whether they sit on an axis.
 $A(2, 5)$, $B(-3, 2)$, $C(-1, -5)$, $D(-2, -5)$, $E(1, -1)$, $F(3, -4)$, $G(6, 1)$

Communicating, reasoning and problem solving

11. Consider the square ABCD shown.
- 
- a. State the coordinates of the vertices of the square ABCD.
 - b. Calculate the area of the square ABCD.
 - c. Extend sides AB and CD 3 squares to the right and 3 squares to the left.
 - d. Calculate the area of the new shape.
 - e. In your new diagram extend sides AD and BC 3 squares up and 3 squares down.
 - f. Calculate the area of the new shape.
 - g. Compare the three areas calculated in parts b, d and f. Explain the changes of area in relation to the change in side length.
12. A line passes through points $A(-5, 6)$ and $B(3, 6)$.
- a. Draw the line interval AB on a number plane.
 - b. What is the length of AB?
 - c. Find the coordinates of the middle point.
 - d. Draw another horizontal line and find the coordinates of the middle point.
 - e. Is there a formula to calculate the x -coordinate of the middle point of a horizontal line?
13. A line passes through points $C(1, 7)$ and $D(1, -5)$.
- a. Draw the line interval CD on a number plane.
 - b. What is the length of CD?
 - c. Find the coordinates of the middle point.
 - d. Draw another vertical line and find the coordinates of the middle point.
 - e. Is there a formula to calculate the y -coordinate of the middle point of a vertical line?
14. Why must the x -coordinate always be written first and the y -coordinate second?
15. What are the area and perimeter of a rectangle formed by connecting the points $(3, 2)$, $(9, 2)$, $(9, 5)$ and $(3, 5)$ on a Cartesian plane?

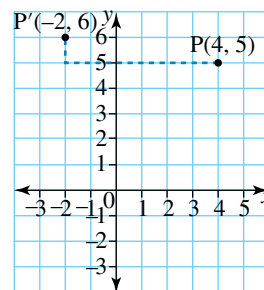
11.3 Translations in the Cartesian plane

11.3.1 Transformations

- Shapes can be shifted by an ordered set of moves from one place to another. Such ordered sets of moves are called transformations.
- Prior to transformation, the shape is usually called the *original object*; after the transformation, the shape is called the **image**.
- When dealing with various transformations, it is customary to denote the image with the same letters as the original shape, but with added apostrophes called primes. For example, if the original object is a rectangle, named ABCD, then its image after the transformation is usually named $A'B'C'D'$.
- If, after the transformation, the image has exactly the same size and shape as the original object, such a transformation is called **isometric**.
- Isometric transformations discussed in this topic include **translations**, **reflections** and **rotations**. They result in congruent figures.

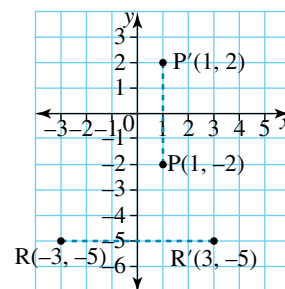
11.3.2 Translation

- A translation is the movement of a point or an object up, down, left or right (U, D, L or R) without flipping, turning or changing size.
- After a translation the object has been carried across a plane in any direction. Translations are also known as ‘slides’.
- Translations can also be described as the movement of coordinates on the Cartesian plane.
- We treat translations (i.e. the number of units) *to the right* as *positive numbers* and *translations to the left* as *negative*.
- We treat *upward moves* as *positive numbers* and *downward moves* as *negative numbers*. For example, if the point $P(4, 5)$ is translated 6 units to the left (negative) and 1 unit up (positive), the resulting point, usually named P' , is $P'(-2, 6)$.



11.3.3 Reflection

- A reflection is the image of an object, as seen in the mirror.
- Reflections are often called ‘mirror images’ and the lines in which the objects are reflected are called ‘mirror’ lines.
- Mirror images always have reverse orientation; that is, left appears to be right and vice versa.
- Points that are reflected always lie on the same line as the original point, which is perpendicular to the mirror line.
- Reflected points are the same distance from the mirror line as the original point on the other side of the mirror.
- Reflections are also known as ‘flips’.
- Any point P may be reflected in either axis. For example, if $P(1, -2)$ is reflected in the x -axis, the result is $P'(1, 2)$. For reflections in the x -axis the x -coordinate remains the same, and the y -coordinate has the same magnitude, but opposite sign.
- Alternatively, if $R(-3, -5)$ is reflected in the y -axis, the result is $R'(3, -5)$. So for reflections in the y -axis the y -coordinate remains the same, and the x -coordinate has the same magnitude, but opposite sign.



WORKED EXAMPLE 4

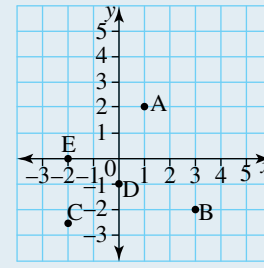
- a** Draw a Cartesian plane that goes from -3 to 5 on the x -axis and -3 to 5 on the y -axis and plot the following points.

$A(1, 2)$ $B(3, -2)$ $C(-2, -2\frac{1}{2})$ $D(0, -1)$ $E(-2, 0)$

- b** State the coordinates of each of the original five points if they are translated 2 units to the right and 3 units down. Label the new points A' , B' and so on.
- c** State the coordinates of each of the original five points if they are reflected in the x -axis, labelling the new points A'' , B'' and so on.

THINK

- a** **1** Draw a Cartesian plane that extends from -3 to 5 on both the x - and y -axes.
- 2** Find and plot the position of each point, and then label.
- A Across to 1 on the x -axis and then up to 2 on the y -axis.
- B Across to 3 on the x -axis and then down to -2 on the y -axis.
- C Across to -2 on the x -axis and then down to $-2\frac{1}{2}$ on the y -axis.
- D Stay at the origin (0 on the x -axis) and then down to -1 on the y -axis.
- E Across to -2 on the x -axis and then stay at 0 on the y -axis.
- b** To translate 2 units to the right, add 2 to x . To translate 3 units down, subtract 3 from y .
- c** For reflection in the x -axis, change the sign of y .

WRITE/DRAW

$$\begin{aligned}
 A(1, 2) &\Rightarrow A'(3, -1) \\
 B(3, -2) &\Rightarrow B'(5, -5) \\
 C\left(-2, -2\frac{1}{2}\right) &\Rightarrow C'\left(0, -5\frac{1}{2}\right) \\
 D(0, -1) &\Rightarrow D'(2, -4) \\
 E(-2, 0) &\Rightarrow E'(0, -3) \\
 A(1, 2) &\Rightarrow A''(1, -2) \\
 B(3, -2) &\Rightarrow B''(3, 2) \\
 C\left(-2, -2\frac{1}{2}\right) &\Rightarrow C''\left(-2, 2\frac{1}{2}\right) \\
 D(0, -1) &\Rightarrow D''(0, 1) \\
 E(-2, 0) &\Rightarrow E''(-2, 0)
 \end{aligned}$$

- If a point or a shape has been translated several times in both horizontal and vertical directions, its final position (relative to the original one) can be described by the total horizontal and total vertical translations.
- When finding the total horizontal translation, the number of units the object has been moved to the right and to the left (with appropriate signs) can be added together to give the total horizontal translation.
- To find the total vertical translation, we add together the number of units the object has been moved upward and downward (with appropriate signs).

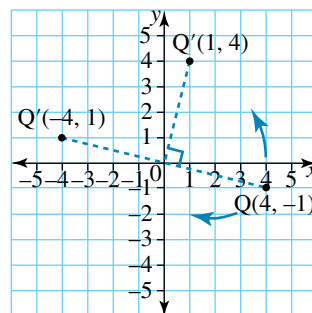
DISCUSSION

Think about what you have learned about reflections and translations of points on the Cartesian plane. Can a translation produce the same result as a single reflection? What about the reverse — can a reflection produce the same result as a single translation?

11.3.4 Rotation

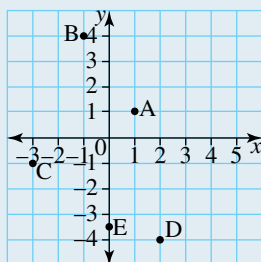
- A point or a shape can be rotated about a point called the **centre of rotation**. To specify the rotation, we need to give the angle through which the object is to be turned and the direction of the rotation (clockwise or anticlockwise).

- Rotations are also known as ‘turns’.
- Consider a centre of rotation at the origin (0, 0). Any point may be rotated about the origin in either direction, clockwise or anticlockwise.
- If $Q(4, -1)$ is rotated through 90° in an anticlockwise direction about the origin, the result is $Q'(1, 4)$.
- Alternatively, if $Q(4, -1)$ is rotated through 180° in a clockwise direction about the origin, the result is $Q'(-4, 1)$. Notice that the magnitudes of the coordinates of the original and rotated point are the same, but the signs are opposite.



WORKED EXAMPLE 5

- a** Write the Cartesian coordinates of the points A to E marked on the Cartesian plane.
- b** The points A to E are rotated through 90° in an anticlockwise direction about the origin. Find their new coordinates.



THINK

- a 1** Trace along the x -axis to find the first number, and then along the y -axis to find the second number.
- Point A is at 1 on the x -axis and 1 on the y -axis.
- Point B is at -1 on the x -axis and 4 on the y -axis.
- Point C is at -3 on the x -axis and -1 on the y -axis.
- Point D is at 2 on the x -axis and -4 on the y -axis.
- Point E is at 0 on the x -axis and $-3\frac{1}{2}$ on the y -axis.
- 2** Write each point as a pair of coordinates.

WRITE

- b** For anticlockwise rotation of 90° , interchange x and y and change the sign of the new x .

$A(1, 1), B(-1, 4), C(-3, -1),$
 $D(2, -4), E(0, -3\frac{1}{2})$

$A(1, 1) \Rightarrow A(-1, 1)$
 $B(-1, 4) \Rightarrow B(-4, -1)$
 $C(-3, -1) \Rightarrow C(1, -3)$
 $D(2, -4) \Rightarrow D(4, 2)$
 $E(0, -3\frac{1}{2}) \Rightarrow E(3\frac{1}{2}, 0)$

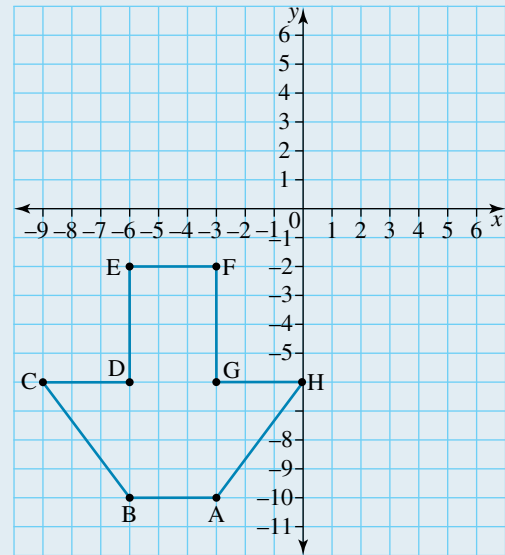
- When transforming a point on the Cartesian plane, a combination of translations and/or reflections can produce the same result as a single rotation. Likewise, a combination of rotations can produce the same results as a single translation and/or reflection.

11.3.5 Transforming a shape

- Shapes can be transformed in the same way as points. We consider the vertices of the shape to be individual points, which can be transformed and joined to form the transformed shape.
- The following worked examples demonstrate translation, reflection and rotation of a shape using the shape's vertices as points.

WORKED EXAMPLE 6

- Give the coordinates of the points A to H on the object shown at right.
- Translate the object 2 units left, 5 units up, 8 units right and 2 units up.
- Give the coordinates of the new points A' to H'.



THINK

- Write the coordinates of the points A to H.

- 1 Find the total horizontal shift.

- 2 Find the total vertical shift.

- 3 State the total translation.

- 4 Point A(-3, -10) can be translated to A' by either:

- moving the point 6R7U, or
- adding 6 to the x -coordinate and 7 to the y -coordinate.

Repeat for remaining points B–H.

Points A'–H' can then be joined to form the image.

(You may wish to highlight the image, using a different colour.)

WRITE

- The coordinates of the vertices are:

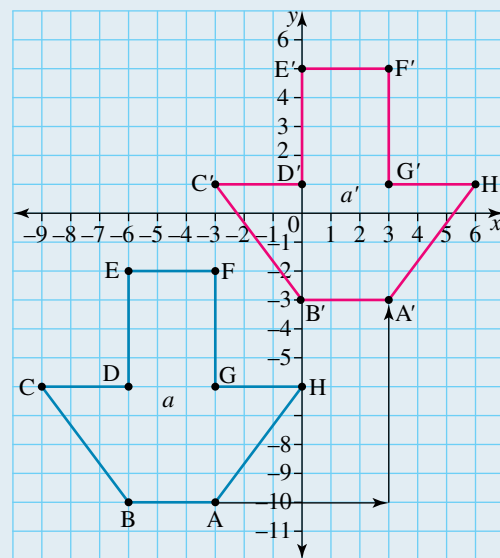
A(-3, -10), B(-6, -10), C(-9, -6),
D(-6, -6), E(-6, -2), F(-3, -2),
G(-3, -6), H(0, -6).

Horizontal:

2 to the left and 8 to the right = $-2 + 8$
= 6 (to the right)

- Vertical: 5 up and 2 up = $5 + 2$
= 7 (up)

Translation: 6R7U



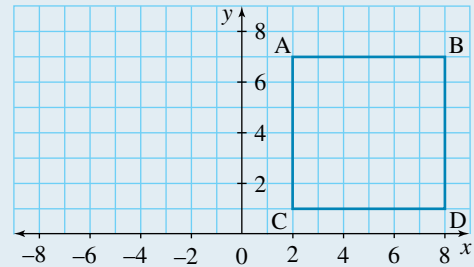
- Write the coordinates of the points A' to H'.

- The coordinates of the vertices are:

A'(3, -3), B'(0, -3), C'(-3, 1), D'(0, 1),
E'(0, 5), F'(3, 5), G'(3, 1), H'(6, 1)

WORKED EXAMPLE 7

Draw the image when the following shape is reflected in the line $x = 0$ (the y -axis), and state the coordinates of the vertices of the image.

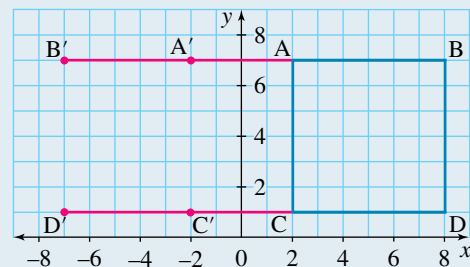


THINK

- a 1 Determine the coordinates of the vertices of the original shape.
- 2 Draw lines perpendicular to, and extending beyond, the mirror line $x = 0$.

WRITE/DRAW

- a $A(2, 7), B(8, 7), C(2, 1), D(8, 1)$



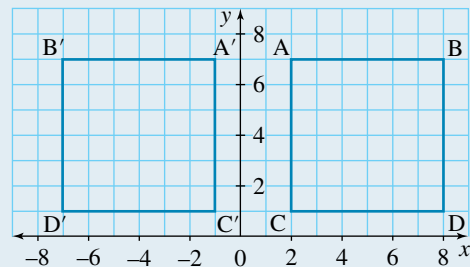
- 3 Points A and C are both 2 units to the right of the mirror line and have an x -coordinate of 2. Since the image will be reversed, the vertices A' and C' will be 2 units to the left-hand side of the mirror line.

Hence the x -coordinates of A' and C' are -2 .

Likewise, points B and D are 8 units to the right of the mirror line with x -coordinates of 8. Therefore the image points B' and D' will be 8 units to the left-hand side of the mirror line, with x -coordinates of -8 .

Note in a reflection through the y -axis ($x = 0$) the y -coordinates will not change.

- 4 Join the vertices A', B', C' and D' to complete the image.



- 5 Write the coordinates of the points A' to D' .

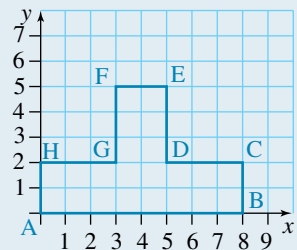
The vertices of the image are:

$A'(-2, 8), B'(-8, 7), C'(-2, 1)$ and $D'(-8, 1)$

- To perform a rotation, the following steps could be of assistance:
 - Step 1: Select any point on the object (vertices are most convenient to choose).
 - Step 2: Join the selected point and the centre of rotation with a straight line.
 - Step 3: Using a protractor and a ruler (or a pair of compasses), find the image of the selected point after the required rotation. The image point will be as far from the centre of rotation as is its corresponding point on the object.
 - Step 4: Rotate some other points if necessary.
 - Step 5: Complete the image, using previously rotated points as a guide. Label the image appropriately.

WORKED EXAMPLE 8

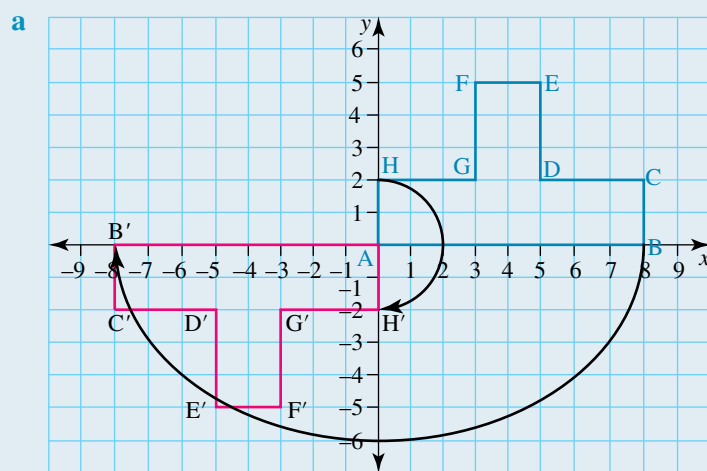
- a Draw the image of the following shape after a rotation of 180° (half turn) about the origin in a clockwise direction. Label the image appropriately.**
- b State the coordinates of the vertices of the image and comment on the relationship between the coordinates of each of the vertices before and after the rotation.**



THINK

- a** Place the protractor so that its centre is on A, its zero is on the line AB and its scale increases in an anticlockwise direction. Measure the 180° angle. The image point B' is as far from the centre of rotation, A, as is point B (8 units in this case). Rotate the vertices of the object 180° in a clockwise direction. Label the image of point B as B', the image of point C as C' and so on. Join the vertices appropriately to complete the image.

WRITE/DRAW



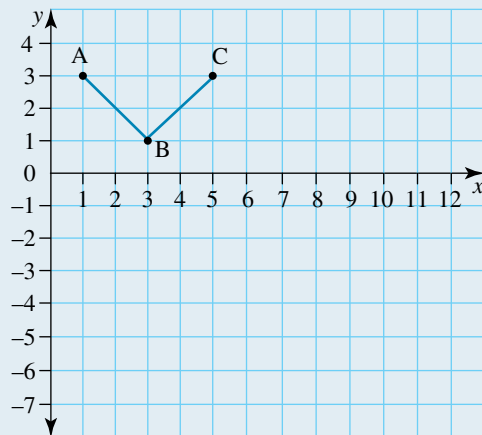
- b 1** Write the vertices of the image.
- 2** Consider the coordinates of each point and their image.
- b** The vertices of the image are:
 $A'(0, 0)$, $B'(-8, 0)$, $C'(-8, -2)$, $D'(-5, -2)$, $E'(-5, -5)$,
 $F'(-3, -5)$, $G'(-3, -2)$ and $H'(0, -2)$.
- Rotating an image 180° about the origin results in the x - and y -coordinates of each image point having the same magnitude but opposite sign as the original point.

11.3.6 Combined transformations in the Cartesian plane

- Transformations can be combined by performing the separate transformations, one after the other.

WORKED EXAMPLE 9

Reflect the V-shape in the line $y = 0$, translate the resulting image 6 units right and 3 units down, and then rotate the second image 90° anticlockwise. Write down the coordinates of the final image.



THINK

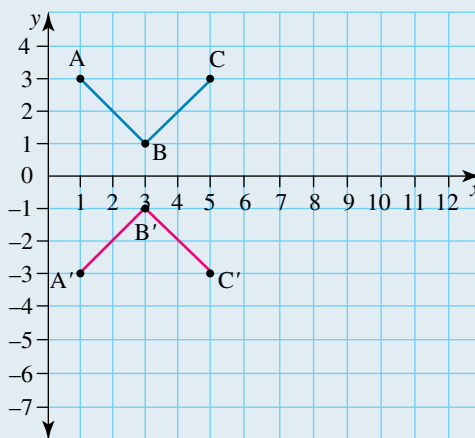
- 1 Reflect the V-shape in the line $y = 0$ and label the vertices A' , B' and C' .

$$A(1, 3) \rightarrow A'(1, -3)$$

$$B(3, 1) \rightarrow B'(3, -1)$$

$$C(5, 3) \rightarrow C'(5, -3)$$

DRAW



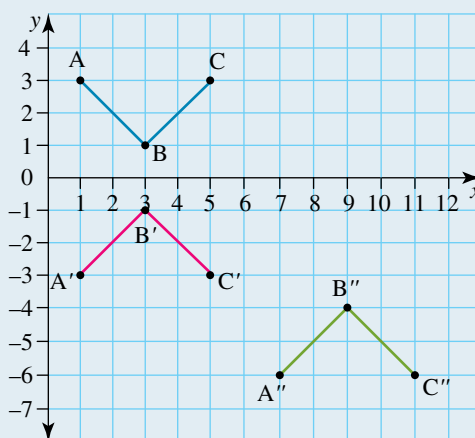
- 2 Translate the resulting image 6 units to the right and 3 units down by adding 6 to each x -value and subtracting 3 from each y -value.

Label this new image A'' , B'' and C'' .

$$A'(1, -3) \rightarrow A''(7, -6)$$

$$B'(3, -1) \rightarrow B''(9, -4)$$

$$C'(5, -3) \rightarrow C''(11, -6)$$

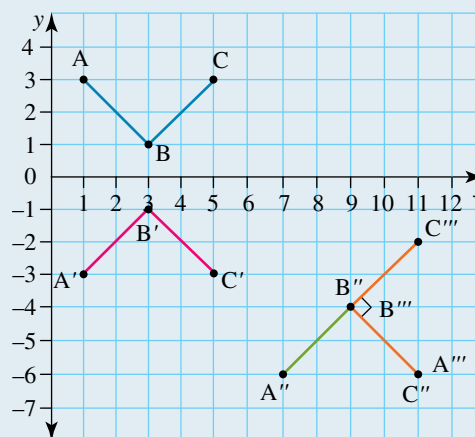


- 3 Rotate the second image 90° anticlockwise.
Label the final image A''' , B''' and C''' .

$$A''(7, -6) \rightarrow A'''(11, -6)$$

$$B''(9, -4) \rightarrow B'''(9, -4)$$

$$C''(11, -6) \rightarrow C'''(11, -2)$$



- 4 Write the coordinates of the final image.

$$A'''(11, -6), B'''(9, -4), C'''(11, -2)$$

learnon RESOURCES — ONLINE ONLY

Interactivity: Introduction to transformations (int-4143)

Interactivity: Translations (int-4142)

Interactivity: Reflections (int-4140)

Interactivity: Rotations (int-4144)

Interactivity: Combined transformations in the Cartesian plane (int-4145)

Digital doc: SkillsHEET Translation of a point (doc-6596)

Digital doc: SkillsHEET Reflection of a point (doc-6597)

Digital doc: SkillsHEET Reflection of an object (doc-6598)

Digital doc: SkillsHEET Rotation of a point (doc-6599)

Digital doc: WorkSHEET Transformations (doc-2025)

Exercise 11.3 Translations in the Cartesian plane

Individual pathways

PRACTISE

Questions:
1–5, 9, 10, 11, 12, 16

CONSOLIDATE

Questions:
1, 2, 4, 8, 11–13, 15

MASTER

Questions:
1, 2, 3, 6, 7, 13, 14, 17–21

Individual pathway interactivity: int-4388

learnon ONLINE ONLY

To answer questions online and to receive **immediate feedback** and **fully worked solutions** for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Understanding and fluency

1. **WE4 a.** Draw a Cartesian plane that goes from -5 to 6 on the x -axis and from -6 to 6 on the y -axis and plot the following points.

$$A(3, 1), B(5, -3), C(-2, -4), D\left(-2\frac{1}{2}, 0\right), E\left(-4, 3\frac{1}{2}\right)$$

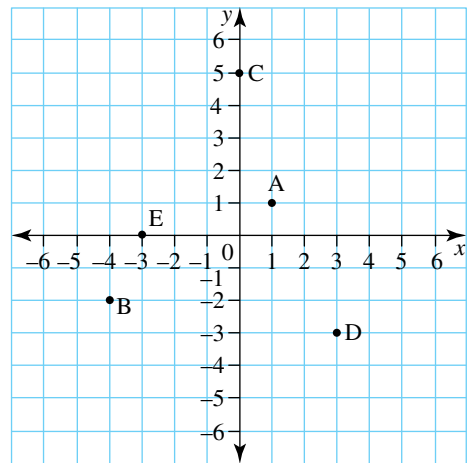
- b. State the new coordinates if the original five points are translated 2 units to the left and 1 unit down, and label the new points A' , B' and so on.
- c. State the new coordinates if the five points obtained in part **b** are reflected in the x -axis, labelling the new points A'' , B'' and so on.

2. **WE5** a. Write the Cartesian coordinates of the points A to E marked on the Cartesian plane shown at right.

- b. The points A to E are rotated 90° in a clockwise direction about the origin. Find their new coordinates.

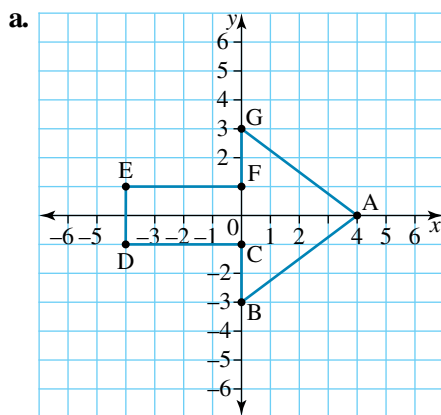
3. State the final position of an object (relative to its original position) after each of the following translations:

- a. 4L 2U, 3R 4U
 b. 8L 2U, 4R 3D, 6R 4U
 c. 12L 4D, 2L 3D, 8R 4D, 6L 2U
 d. 8L 2D, 4R 9U, 6L 2D, 4R 3U, 2L 9D

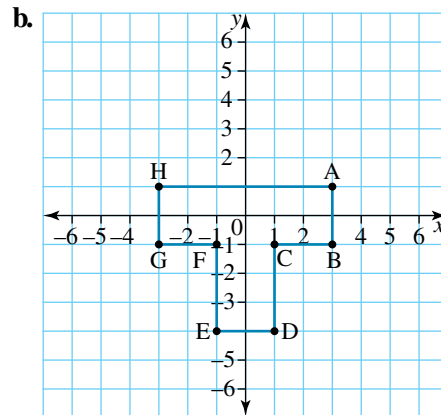


4. **WE6** i. Translate the following objects as specified under the figures.

- ii. Give the coordinates of the vertices of the objects after they have been translated.



4L 2D, 3R 4U



6L 2D, 8R 3D

5. **MC** If an object is translated 2L 3U, 5R 4D, 6L, its final position (relative to its original position) is:

- A. 3R 7U B. 3L 1U C. 3L 7U D. 3L 1D E. 3R 1D

6. **MC** If, relative to the original position, the final position of an object is 3L 4U, then the series of translations that could have occurred is:

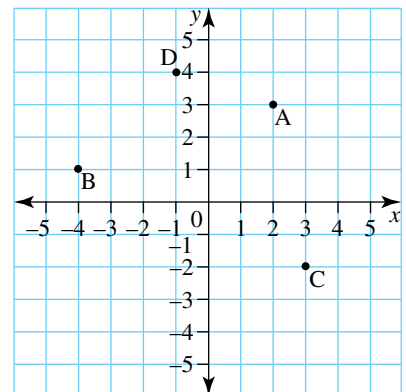
- A. 4R 2U, 3L 2U B. 4R 1U, 7L 3U C. 4R 1U, 7R 3U D. 4R 2U, 3L 3U E. 4R 1U, 4L 3U

7. **WE7** A triangle is formed by the points A(3, -2), B(-5, 1) and C(4, 7).

- a. If the points are reflected in the x -axis, find the coordinates of the vertices of the new triangle.
- b. The new triangle is now translated 5 units to the left and 1 unit down. Find the coordinates of the vertices of this new triangle.

8. **WE8** A quadrilateral whose vertices are given by the points P(1, 2), Q(5, -1), R(6, 5) and S(-1, -3) is rotated clockwise through 270° about the origin. Find the coordinates of the vertices of the rotated quadrilateral.

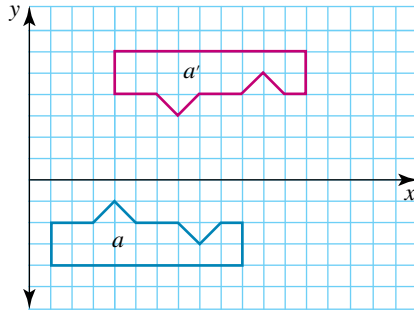
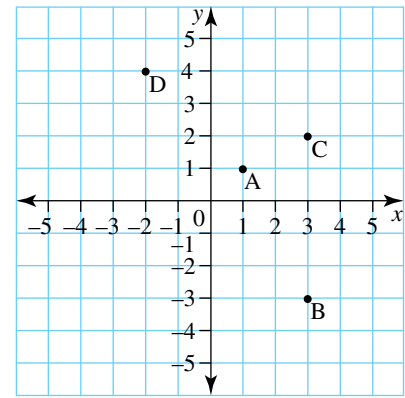
9. Reflect the points on the Cartesian plane at right in the x -axis.



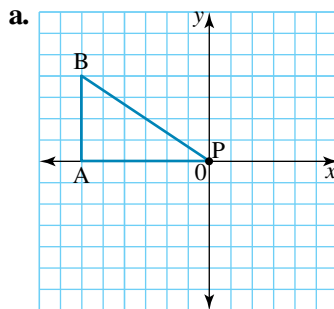
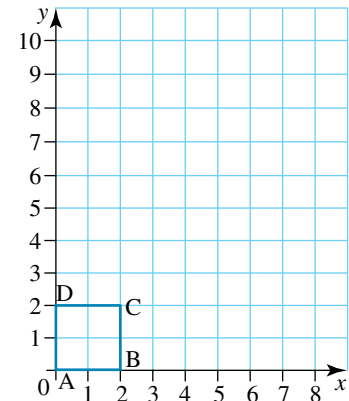
10. Reflect the points on the Cartesian plane at right in the y -axis.

11. **MC** The diagram below shows an object and its image after a certain set of transformations. The object was:

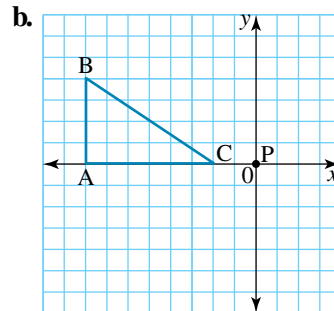
- A. reflected in the y -axis, then translated 3R 2D.
- B. reflected in the x -axis, then translated 4R 2D.
- C. translated 3R 2D, then reflected in the x -axis.
- D. translated 4R 2D, then reflected in the x -axis.
- E. translated 3R 2U, then reflected in the y -axis.



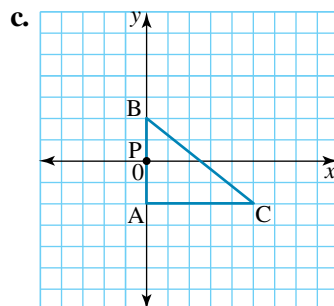
12. **WE9** a. Translate the square ABCD, shown at right, 3 units right and 1 unit up, then translate the resulting image 2 units left and 6 units up.
 b. Define a single translation that would have produced the final image.
13. Show the image of the following shapes after they have been rotated about point P(0, 0).



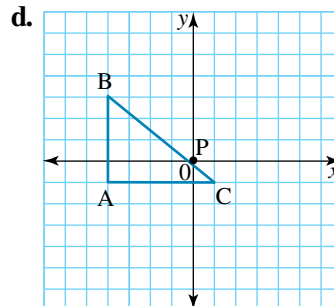
Rotated 90° clockwise



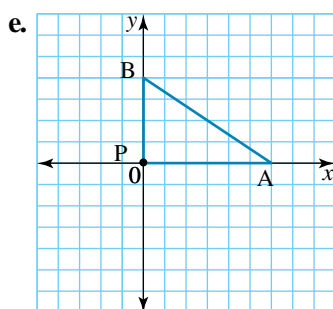
Rotated 90° anticlockwise



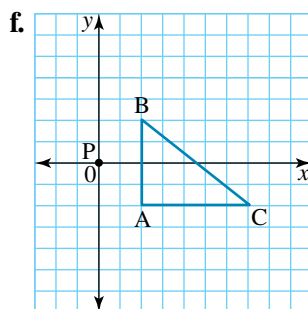
Rotated 180° anticlockwise



Rotated 270° ($\frac{3}{4}$ turn) clockwise



Rotated 270° anticlockwise



Rotated 450° ($1\frac{1}{4}$ turns) clockwise

Communicating, reasoning and problem solving

14. Alja likes to skip around her backyard.
- If her starting position on a number plane is at point A(1, 1) and she skips two steps forward and one step to the right, draw her next four locations after every set of three steps.
 - Do the five points form a pattern?
 - From her last location, Alja decides to skip one step backwards and two steps to the left. She skips in this pattern four times. Draw Alja's location after each set of steps.
 - Do these five points form a pattern?
15. a. Translate the point P(3, 1) 3 units up and 2 units to the left. Label the new point P' and give the coordinates of this point.
- b. Translate P' 2 units down and 3 units to the left. Label the new point P'' and give the coordinates of this point.
- c. What single translation would move P to P''?
16. The point P(a, -b) is reflected in the y-axis and then rotated through 180° about the origin. Find the coordinates of the new point. Show your working.
17. Using the point (x, -y), prove that a rotation of 180° gives an identical result to a rotation of -180° .
18. The point A(a, b) is rotated anticlockwise through 90° about the origin, then reflected in the x-axis, translated by 4 units in the negative x direction and finally reflected in the y-axis. What are the final coordinates of A?
19. Using the point R(-r, -q) and showing all working, prove that a rotation of 180° followed by a reflection in the y-axis is equivalent to a reflection in the x-axis.
20. An object was translated seven units to the right and four units down, rotated 180° clockwise about the initial position, then reflected in a vertical line passing through the initial position, and then translated seven units to the left and four units down. What is the new position of the object at the end of the transformations relative to the initial position? Draw a diagram to represent the transformations.
21. Think of an alternative way (i.e. other than Cartesian coordinates) to specify the position of a point on a plane.



11.4 Identifying patterns

11.4.1 Describing relationships and patterns

- Mathematics is used to describe *relationships* in the world around us.
- Mathematicians study *patterns* in numbers and shapes found in nature to discover rules.
- These rules can then be applied to other, more general situations.
- Looking at the number pattern 1, 4, 7, 10, ... we can see that by adding 3 to any of these numbers, we obtain the next number.

- This number pattern is called a **sequence**.
- Each number in the sequence is called a **term**.
- Each sequence has a rule that describes the pattern. For the sequence above, the rule is ‘add 3’.

WORKED EXAMPLE 10

Using the rule ‘start with 32 and subtract 3 each time’, write down the first five terms of the number pattern.

THINK

- 1 Start with 32 and subtract 3.
- 2 Keep subtracting 3 from the previous answer until five numbers have been calculated.
- 3 Write the answer.

WRITE

$$32 - 3 = 29$$

$$29 - 3 = 26$$

$$26 - 3 = 23$$

$$23 - 3 = 20$$

The first five numbers are 32, 29, 26, 23 and 20.

WORKED EXAMPLE 11

Describe the following number pattern in words then write down the next three terms in the pattern.

4, 8, 12, ...

THINK

- 1 The next number is found by adding 4 to the previous number.
- 2 Add 4 each time to get the next three numbers.
- 3 Write down the next three numbers.

WRITE

Next number = previous number + 4

$$12 + 4 = 16$$

$$16 + 4 = 20$$

$$20 + 4 = 24$$

The next three numbers are 16, 20, 24.

- We can also describe a number pattern in words by looking at the value of any particular term and relating it to its position in the number pattern.
- For example, we could use a table of values to show the relationship from the previous worked example.

Position in pattern	1	2	3	4	5	6
Value of term	4	8	12	16	20	24

- This number pattern can be described as ‘value of term = position in pattern $\times$ 4’.
- After a rule in the format for a number pattern has been identified, you can use it to find any future value in the pattern.

WORKED EXAMPLE 12

a Describe the following number pattern in words by relating the value of a term to its position in the number pattern.

b 6, 13, 20, ...

c What is the 15th term in the pattern?



THINK

- a** 1 Draw up a table relating the position in the pattern to the value of the term.
- 2 The value of the term is found by multiplying the position in the pattern by 7 and subtracting 1.
- b** 1 Substitute the position in the pattern into the formula and evaluate.
- 2 Write the answer.

WRITE

Position in pattern	1	2	3
Value of term	6	13	20

$$\text{Value of term} = \text{position in pattern} \times 7 - 1$$

$$\begin{aligned}\text{Value of term} &= 15 \times 7 - 1 \\ &= 104\end{aligned}$$

The 15th term in the pattern is 104.

- When the terms in a number pattern are decreasing the rule can be described by subtracting a multiple of the position in the pattern from a fixed number. For example, the number pattern 31, 25, 19, ... can be described by 'value of term = $37 - \text{position in pattern} \times 6$ '.

11.4.2 Geometric patterns

- Patterns can be also found in geometric shapes.
- If we examine the three shapes at right, we can see patterns by investigating the changes from one shape to the next. For example, look at the number of matchsticks in each set of triangles.
- By using a table of values we can see a number pattern developing:
- The pattern in the final row is 3, 6, 9, ...; we can see that the rule here is 'add 3'.
- We can also look for a relationship between the number of triangles and the number of matchsticks in each shape. If you examine the table, you will see that a relationship can be found. In words, the relationship is 'the number of matchsticks equals 3 times the number of triangles'.



Number of triangles	Number of matchsticks
1	3
2	6
3	9
4	12
5	15
6	18

WORKED EXAMPLE 13

Consider a set of hexagons constructed according to the pattern shown below.



- a** Using matches, pencils or similar objects, construct the above figures. Draw the next two figures in the series.
- b** Draw up a table showing the relationship between the number of hexagons in the figure and the number of matches used to construct the figure.
- c** Devise a rule to describe the number of matches required for each figure in terms of the number of hexagons in the figure.
- d** Use your rule to determine the number of matches required to make a figure consisting of 7 hexagons.

THINK

a 1 Construct the given figures with matches. Note the number of additional matches it takes to progress from one figure to the next — 5 in this case.

2 Draw the next two figures, adding another 5 matches each time.

b Draw up a table showing the number of matches needed for each figure in terms of the number of hexagons. Fill it in by looking at the figures.

c 1 Look at the pattern in the number of matches going from one figure to the next. It is increasing by 5 each time.

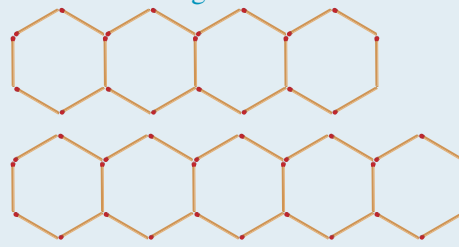
2 If we take the number of hexagons and multiply this number by 5, it does not give us the number of matches. However, if we add 1 to this number, it does give us the number of matches in each shape.

d 1 Use the rule to find the number of matches to make a figure with 7 hexagons.

2 Write the answer.

WRITE

a The next two figures are:



b

Number of hexagons	1	2	3	4	5
Number of matches	6	11	16	21	26

c The number of matches increased by 5 in going from one figure to the next.

$$\text{Number of matches} = \text{number of hexagons} \times 5 + 1$$

d Number of matches for 7 hexagons = $7 \times 5 + 1$
= 36

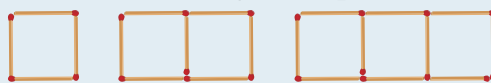
36 matches would be required to construct a figure consisting of 7 hexagons.

11.4.3 Using algebra to describe patterns

- Once a pattern has been identified we can use algebra to describe it.
- If we let n = the number of matches and h = the number of hexagons from the pattern in the previous worked example, the equation $n = 5h + 1$ would describe this pattern.

WORKED EXAMPLE 14

Consider the set of shapes constructed according to the pattern shown below.



a Draw up a table of values showing the number of matches needed for each figure in the pattern.

b Express the relationship between the number of matches (n) and the number of squares (s) in the pattern as an algebraic equation.

THINK

a Look at the figures in the pattern to complete a table of values.

b 1 Look at the pattern in the number of matches going from one figure to the next. It is increasing by 3 each time.

2 If we take the number of squares and multiply this number by 3, it does not give us the number of matches. However, if we add 1 to this number, it does give us the number of matches in each shape.

3 Replace 'number of matches' with ' n ' and 'number of squares' with ' s ' to create your algebraic equation, then simplify by removing the multiplication sign.

WRITE

a

Number of squares	1	2	3
Number of matches	4	7	10

b The number of matches increased by 3 in going from one figure to the next.

Number of matches =
number of squares $\times 3 + 1$

d $n = s \times 3 + 1$
 $n = 3s + 1$

learnon RESOURCES — ONLINE ONLY

Interactivity: Identifying patterns (int-3801)

Interactivity: Geometric patterns (int-3802)

Digital doc: SkillsHEET Number patterns (int-6978)

Digital doc: SkillsHEET Using tables to show number patterns (doc-6979)

Digital doc: SkillsHEET Describing a number pattern from a table (doc-6980)

Exercise 11.4 Identifying patterns

Individual pathways

PRACTISE

Questions:
1–7

CONSOLIDATE

Questions:
1–7, 9, 10

MASTER

Questions:
1–10

Individual pathway interactivity: int-4446

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To answer questions online and to receive **immediate feedback** and **fully worked solutions** for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Understanding and fluency

- WE10** Using the following rules, write down the first five terms of the number patterns.
 - Start with 1 and add 4 each time.
 - Start with 5 and add 9 each time.
 - Start with 50 and subtract 8 each time.
 - Start with 64 and subtract 11 each time.
- WE11** Copy the patterns below, describe the pattern in words and then write down the next three terms in the pattern.

2, 4, 6, 8, ... 27, 24, 21, 18, ...	3, 8, 13, 18, ... 48, 41, 34, 27, ...

3. Fill in the missing numbers in the following number patterns.
- a. 3, ..., 9, 12, ..., ... b. 8, ..., ..., 14, ...
 c. 4, 8, ..., 32, ... d. ..., ..., 13, 15, ...
 e. 66, 77, ..., 99, ..., 121 f. 100, ..., ..., 85, 80, ...
4. **WE12** i. Describe the following number patterns in words by relating the value of a term to its position in the number pattern.
 ii. What is the 15th term in the pattern?
 a. 5, 15, 25, ... b. 44, 41, 38, ...
 c. 102, 108, 114, ... d. 212, 199, 186, ...
5. **WE13** For each of the sets of shapes below, follow these instructions to investigate the pattern.
- Construct the shapes using matches. Draw the next two shapes in the series.
 - Construct a table to show the relationship between the number of shapes in each figure and the number of matchsticks used to construct it.
 - Devise a rule in words that describes the pattern relating the number of shapes in each figure and the number of matchsticks used to construct it.
 - Use your rule to work out the number of matchsticks required to construct a figure made up of 7 such shapes. Check your answers by drawing the figures and counting the number of matchsticks required.
- a. 
- b. 
- c. 
6. **WE14** Let m = the number of matches and n = the number of shapes for each of the patterns in question 5. Write an algebraic equation to describe each of these patterns using m and n .

Communicating, reasoning and problem solving

- Why is it more useful to describe a number pattern in terms of relating the value of a term to its position in the pattern as compared with describing a number pattern by the rule between terms?
- Are all patterns that can be described by written descriptions also able to be described using algebraic symbols? Explain your answer.
- Create a matchstick pattern that can be represented by the rule $m = 5n + 3$.
- Why is it useful to describe the rule for a pattern by drawing up a table and looking for the connection between the top row and the bottom row of the table?

11.5 Linear patterns

11.5.1 Plotting points from a table of values

- When we have created a table of values relating the value of a term to its position in a number pattern, we can plot these values on a Cartesian plane.
- The values in the top row of the table of values relate to the x -values on the Cartesian plane and the values in the bottom row of the table of values relate to the y -values on the Cartesian plane.

WORKED EXAMPLE 15

A number pattern is formed using the rule ‘start with 2 and add 1 each time’.

- Write down the first five terms of the number pattern.
- Draw up a table of values relating the value of a term to its position in the pattern.
- Plot the points from your table of values on a Cartesian plane.

THINK

- Start with 2 and add 1.
 - Keep adding 1 to the previous answer until five numbers have been calculated.
 - Write the answer.
- b Draw up a table relating the position in the pattern to the value of the term.
- c Plot the points from your table of values one at a time. Start with the first column of numbers, remembering that the top value relates to the x -coordinates and the bottom value relates to the y -coordinates.

WRITE/DRAW

$$2 + 1 = 3$$

$$3 + 1 = 4$$

$$4 + 1 = 5$$

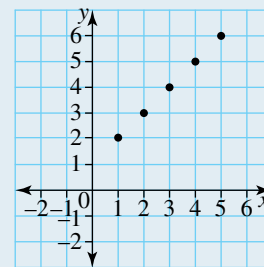
$$5 + 1 = 6$$

The first five numbers are 2, 3, 4, 5 and 6.

Position in pattern	1	2	3	4	5
Value of term	2	3	4	5	6

Points to plot are:

(1, 2), (2, 3), (3, 4), (4, 5) and (5, 6)



11.5.2 Straight-line patterns

- If the pattern formed by the set of points is a straight line, we refer to it as a **linear pattern**. For example, the points (1, 2), (2, 3), (3, 4), (4, 5) and (5, 6) when plotted will form a linear pattern, as shown in the previous worked example.
- A diagram formed by plotting a set of points on the Cartesian plane is referred to as a graph. If the points form a straight line, then the graph is linear (a straight-line graph).
- A set of coordinate pairs can be presented in the form of a table. For example, the points (−2, 4), (−1, 2), (0, 0), (1, −2) and (2, −4) can be presented as shown.

x	−2	−1	0	1	2
y	4	2	0	−2	−4

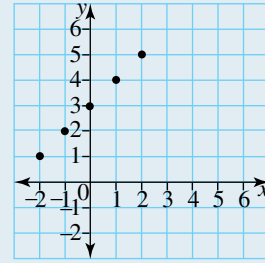
WORKED EXAMPLE 16

Plot the following points on a Cartesian plane and comment on any pattern formed. Check the lowest and highest values to help you decide the numbers to mark on the axes.

(−2, 1), (−1, 2), (0, 3), (1, 4) and (2, 5)

THINK

- 1 Look at the x - and y -values of the points and draw a Cartesian plane.
The lowest value for the x -axis is -2 , the highest is 2 .
The lowest value for the y -axis is 1 , the highest is 5 .
Extend each axis slightly beyond these values.

WRITE/DRAW

- 2 Plot each point.
- 3 Comment on any pattern formed.

A linear pattern is obtained as the points form a straight line.

WORKED EXAMPLE 17

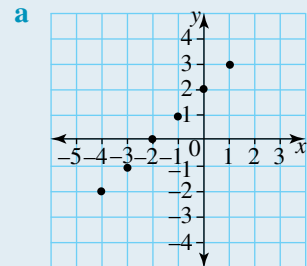
- a** Plot the points in the following table on a Cartesian plane.

x	-4	-3	-2	-1	0	1
y	-2	-1	0	1	2	3

- b** Do the points form a linear graph? If so, what would the next point in the pattern be?

THINK

- a**
- 1 Look at the x - and y -values of the points and draw a Cartesian plane.
 - 2 Plot each point.
- b**
- 1 Look at the position of the points and answer the question.
Note: The points form a straight line, so we have a linear graph.
 - 2 Study the pattern and answer the question.
Note: The pattern shows that the x -values increase by 1 , and the y -values increase by 1 . The next x -value will be 2 and the next y -value will be 4 .

WRITE/DRAW

- b** Yes, the points do form a linear graph.

The next point in the pattern is $(2, 4)$.

learn on

RESOURCES — ONLINE ONLY

Interactivity: Linear patterns (int-3833)

Digital doc: WorkSHEET Coordinates and linear graphs (doc-2416)

Exercise 11.5 Linear patterns

Individual pathways

PRACTISE

Questions:
1–8

CONSOLIDATE

Questions:
1–11

MASTER

Questions:
1–14

Individual pathway interactivity: int-4464

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To answer questions online and to receive **immediate feedback** and **fully worked solutions** for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Understanding and fluency

- WE15** A number pattern is formed using the rule ‘start with 9 and subtract 2 each time’.
 - Write down the first five terms of the number pattern.
 - Draw up a table of values relating the value of a term to its position in the pattern.
 - Plot the points from your table of values on a number grid.
- WE16** Plot the points on a Cartesian plane and comment on any pattern formed. (Check the lowest and highest values to help you decide what *scale* to mark on the axes.)
 - $(-2, -3), (-1, -2), (0, -1), (1, 0)$ and $(2, 1)$
 - $(-2, 0), (-1, 1), (0, 2), (1, 3)$ and $(2, 4)$
 - $(-2, -4), (-1, -2), (0, 0), (1, 2)$ and $(2, 4)$
 - $(-2, -5), (-1, -2), (0, 1), (1, 4)$ and $(2, 7)$
 - $(-2, 2), (-1, 1), (0, 0), (1, -1)$ and $(2, -2)$
 - $(-2, 0), (-1, -1), (0, -2), (1, -3)$ and $(2, -4)$

- WE17a** Plot the following points on a Cartesian plane.

a.

x	-2	-1	0	1	2
y	1	2	3	4	5

b.

x	-2	-1	0	1	2
y	-2	-1	0	1	2

c.

x	-2	-1	0	1	2
y	-7	-4	-1	2	5

d.

x	-2	-1	0	1	2
y	3	2	1	0	-1

e.

x	-2	-1	0	1	2
y	-1	-0.5	0	0.5	1

f.

x	-2	-1	0	1	2
y	4	2	0	-2	-4

- WE17b** Plot the following points on a Cartesian plane. Do the points form a linear graph? If so, state the next point in the pattern.
 - $(-3, -3), (-2, -1), (-1, 1), (0, 3), (1, 5), (2, 7)$ and $(3, 9)$
 - $(-3, -5), (-2, -3), (-1, 0), (0, 1), (1, 4), (2, 5)$ and $(3, 7)$

c.

x	-2	-1	0	1	2
y	3	-1	-2	-3	-4

d.

x	-2	-1	0	1	2
y	-6	-3	0	3	6

12. The latest version of a new computer operating system is offered for sale on 1 January. An expert software engineer claims there are 400 bugs in this release; however, the manufacturer claims that it can fix 29 bugs each month. When will the software be bug-free?
13. Consider the relationship $\frac{1}{y} = \frac{1}{x} + 1$. The relationship can be transposed as shown below.

$$\begin{aligned}\frac{1}{y} &= \frac{1}{x} + 1 \\ \frac{1}{y} &= \frac{1}{x} + \frac{x}{x} \\ \frac{1}{y} &= \frac{1+x}{x} \\ y &= \frac{x}{1+x}\end{aligned}$$

- Generate a table of values for x versus y , for $x = -3$ to 3 .
 - Plot a graph to show the points contained in the table of values.
 - Use your plot to confirm whether this relationship is linear.
14. Is it possible to say whether or not a set of points will form a straight line without actually plotting them?

11.6 Linear graphs

11.6.1 Plotting linear graphs

- A **linear graph** is a straight-line graph defined by a linear relationship.
- The straight line of a linear graph is continuous, meaning that all points on the linear graph satisfy the linear relationship.
- Each point on a linear graph is an **ordered pair**; that is, a pair of numbers written in order.
- To plot a linear graph whose equation is given, follow these steps.
 - Create a table of values first.
 - Draw a table.
 - Select some x -values.
 - Substitute the selected x -values into the rule to find corresponding y -values.
 - Draw a Cartesian plane.
 - Plot the points from the table and join them with the straight line.
 - Label the graph.
- When choosing x -values to draw up your table of values start with whole numbers close to 0. This will allow the graph to be seen in relation to the y -axis.

WORKED EXAMPLE 18

Draw a table of values and plot the graph of $y = 2x + 1$ and label the line.

THINK

- Write the rule.
- Draw a table and choose simple x -values.

WRITE/DRAW

$$y = 2x + 1$$

x	-2	-1	0	1	2
y	3	-1	1	3	5

- 3 Use the rule to find each y-value and enter them in the table.

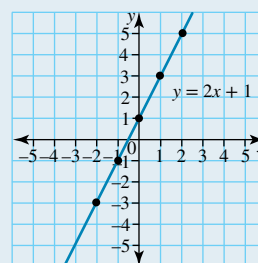
When $x = -2$, $y = 2 \times -2 + 1 = -3$.

When $x = -1$, $y = 2 \times -1 + 1 = -1$.

When $x = 0$, $y = 2 \times 0 + 1 = 1$.

When $x = 1$, $y = 2 \times 1 + 1 = 3$.

When $x = 2$, $y = 2 \times 2 + 1 = 5$.



- 4 Draw a Cartesian plane and plot the points.
- 5 Join the points to form a straight line and label the graph.

- The straight line representing a linear graph can be extended past the given points in both directions forever, meaning that there is an infinite number of ordered pairs that satisfy any given linear relationship.

ACTIVITY: READING COORDINATES FROM A LINEAR RELATIONSHIP GRAPH

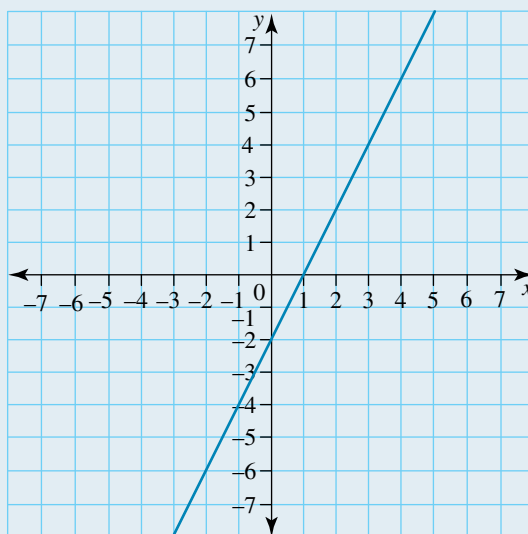
- Use a table of values to plot the graph of $y = 2x - 1$.
- List all of the points that lie on the graph.
- Compare your results with your classmates. Are there any points that you noted that they didn't?

11.6.2 Finding the rule

- We can find the rule for a linear equation by examining the table of values containing the points from a graph of the equation. Rules for linear equations can be written in the form $y = mx + b$.

WORKED EXAMPLE 19

Find the rule for the equation represented by the following linear graph.



THINK

- 1 Draw a table of values of points from the graph.
- 2 Identify how much the value of y changes as the value of x increases by 1.
- 3 This value is the coefficient (m) in front of the x -value in the rule.
- 4 Identify the value of the constant that needs to be added or subtracted so that the rule works for all values.
- 5 Write the answer.

WRITE/DRAW

x	-2	-1	0	1	2
y	-6	-4	-2	0	2

The y -value increase by 2 as the x -value increases by 1.

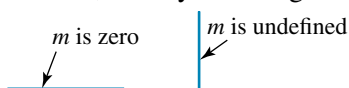
This means the rule is in the form $y = 2x + ?$.

2 needs to be subtracted in order for the rule to work.

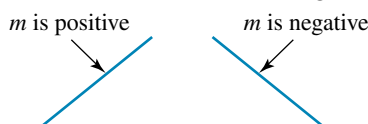
The rule for this equation is $y = 2x - 2$.

11.6.3 The gradient

- The **gradient** gives a measure of steepness. For example, the gradient of a road going up the hill would inform us of the steepness of the slope of the hill.
- The value of the gradient is represented by the pronumeral m .
- Since the gradient is the measure of steepness, the greater the value of the gradient, the steeper the line.
- If the line is horizontal, there is no slope; hence the value of the gradient (m) is zero. If the line is vertical, we say that its gradient is **infinite** or **undefined**.

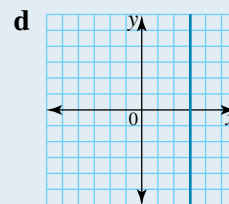
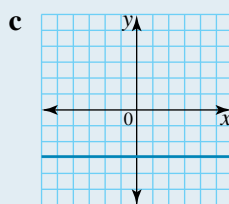
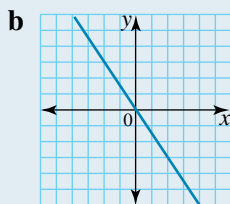
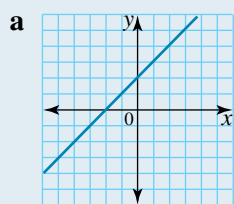


- If the line slopes upwards from left to right (i.e. it *rises*), the gradient is positive. If the line slopes downwards from left to right (i.e. it *falls*), the gradient is negative.



WORKED EXAMPLE 20

State whether these lines have a positive, negative, zero or undefined gradient.

**THINK**

- A line that rises from left to right has a positive gradient.
- A line that drops from left to right has a negative gradient.
- A line that is horizontal has a zero gradient.
- A line that is vertical has an undefined gradient.

WRITE

- Positive gradient
- Negative gradient
- Zero gradient
- Undefined gradient

Interactivity: Plotting linear graphs (int-3834)

Digital doc: SkillsHEET Substitution into rules (doc-6937)

Digital doc: SkillsHEET Completing a table of values for a given rule (doc-6938)

Digital doc: SkillsHEET Plotting a line using a table of values (doc-6939)

Digital doc: SkillsHEET Plotting graphs with tables (doc-2441)

Exercise 11.6 Linear graphs

Individual pathways

PRACTISE

Questions:
1–12, 14

CONSOLIDATE

Questions:
1–12, 14–17, 20

MASTER

Questions:
1–12, 14, 15, 17–21

Individual pathway interactivity: int-4465

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Understanding and fluency

1. Complete the following tables of values, plot the points on a Cartesian plane, and join them to make a linear graph. Label the graphs with the rules.

- a. Rule: $y = x + 3$

x	-2	-1	0	1	2
y	1	2	3		

- b. Rule: $y = x - 5$

x	-2	-1	0	1	2
y		-6			-3

- c. Rule: $y = 5x$

x	-2	-1	0	1	2
y	-10		0		

- d. Rule: $y = 2x + 4$

x	-2	-1	0	1	2
y	0			6	

- e. Rule: $y = 3x + 2$

x	-2	-1	0	1	2
y					

- f. Rule: $y = 2x - 2$

x	-2	-1	0	1	2
y		-4			

g. Rule: $y = 4x - 3$

x	-2	-1	0	1	2
y					

h. Rule: $y = -3x + 2$

x	-2	-1	0	1	2
y		5			-4

2. **MC** a. $y = 3x + 4$ means:

- A.** the y -value equals the x -value, add 3 and then times 4.
- B.** the y -value equals the x -value times 3 and then add 4.
- C.** the x -value equals the y -value times 3 and then add 4.
- D.** the y -value equals 4 times the x -value divided by 3.
- E.** the x -value equals 4 times the y -value and then add 3.

b. A table of values shows:

- A.** a rule.
- B.** coordinates.
- C.** a linear graph.
- D.** an axis.
- E.** that a rule continues forever.

3. **WE18** Draw a table of values and plot the graph for each of the following rules. Label each graph.

- a. $y = x + 2$
- b. $y = x - 4$
- c. $y = x - 1$
- d. $y = x + 5$
- e. $y = 3x$
- f. $y = 7x$
- g. $y = 4x + 1$
- h. $y = 2x - 3$
- i. $y = 3x - 5$
- j. $y = -2x$
- k. $y = -6x + 2$
- l. $y = -5x + 4$

4. Plot a graph of the following rules from the tables of values provided. Label the graphs. Then copy and complete the sentences.

a. $y = 4$

x	-2	-1	0	1	2
y	4	4	4	4	4

For the rule $y = 4$, the y -value of all coordinates is _____.

b. $y = 1$

x	-2	-1	0	1	2
y	1	1	1	1	1

For the rule $y = 1$, the y -value of all coordinates is _____.

c. $y = -2$

x	-2	-1	0	1	2
y	-2	-2	-2	-2	-2

For the rule $y = -2$, the y -value of all coordinates is _____.

d. $y = -5$

x	-2	-1	0	1	2
y	-5	-5	-5	-5	-5

For the rule $y = -5$, the y -value of all coordinates is _____.

5. Make a table of values and plot the graph for each of the following rules.

a. $y = 3$

b. $y = 2$

c. $y = -2$

d. $y = -4$

6. Plot the graph of each of the following rules from the table of values provided. Label the graph. Then copy and complete the sentence.

a. $x = 1$

x	1	1	1	1	1
y	-2	-1	0	1	2

For the rule $x = 1$, the x -value of all coordinates is _____.

b. $x = 3$

x	3	3	3	3	3
y	-2	-1	0	1	2

For the rule $x = 3$, the x -value of all coordinates is _____.

c. $x = -2$

x	-2	-2	-2	-2	-2
y	-2	-1	0	1	2

For the rule $x = -2$, the x -value of all coordinates is _____.

d. $x = -7$

x	-7	-7	-7	-7	-7
y	-2	-1	0	1	2

For the rule $x = -7$, the x -value of all coordinates is _____.

7. Make a table of values; then plot and label the graph for each of the following.

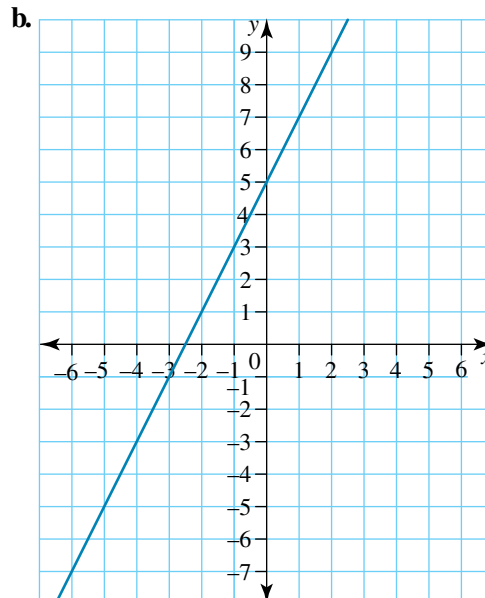
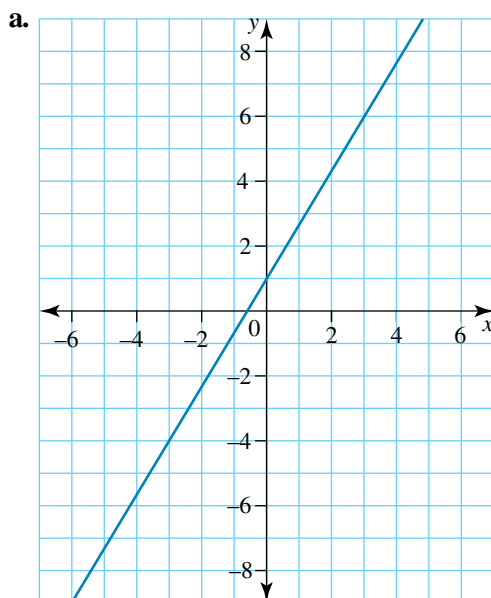
a. $x = 2$

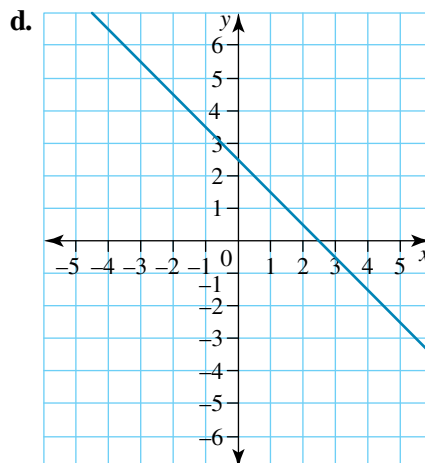
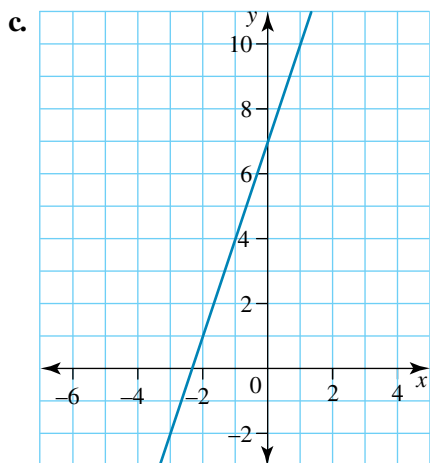
b. $x = 5$

c. $x = -5$

d. $x = 0$

8. **WE19** Find the rule for the equations represented by the following linear graphs.





9. Find the rule for each of the following sets of points.

a.

x	-2	-1	0	1	2
y	-11	-9	-7	-5	-3

b.

x	-2	-1	0	1	2
y	2	5	8	11	14

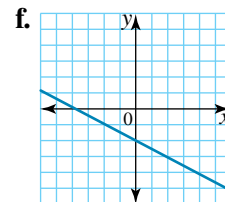
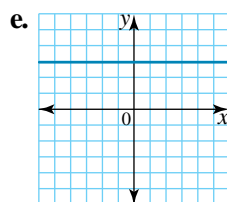
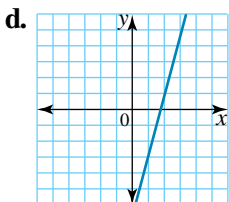
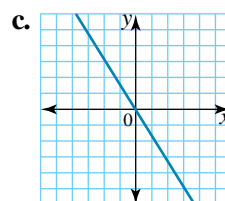
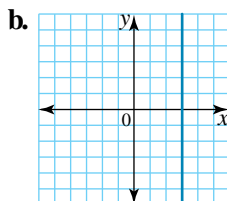
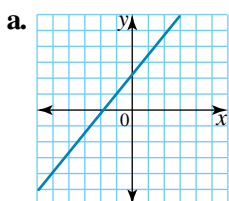
c.

x	-2	-1	0	1	2
y	-19	-9	1	11	21

d.

x	-2	-1	0	1	2
y	7	4	1	-2	-5

10. **WE20** State whether each of the following lines has a positive, negative, zero or undefined gradient.



Communicating, reasoning and problem solving

11. Why are 0, 1 and 2 frequently chosen as x -values in a table of values?

12. a. Create a table of values that shows the cost of lengths of fabric in the photo at right from 0 to 8 metres.
 b. Plot a graph showing the cost of 0 to 8 metres of fabric.
 c. Is this a linear relationship? Explain.



\$2.50
per meter

13. When you make a call from a mobile phone, you usually pay a flagfall (charge for just making the call) and then you are charged for how long you talk. One phone company offers 25 cents flagfall and 35 cents per 30 seconds.
 a. Create a table of values that shows the cost of a phone call between 0 and 5 minutes (in 30-second blocks).
 b. Plot a graph showing the cost of a phone call between 0 and 5 minutes.
14. Chris's fridge is not working. He called a repair company, and they are sending someone to repair the fridge.

Company charges are: \$55 to come to your home, plus

\$45 for every $\frac{1}{2}$ hour the repairer is there.

- a. Copy and complete the table to show how much it could cost Chris to have his fridge repaired.

Time (hours)	0	$\frac{1}{2}$	1	$1\frac{1}{2}$	2	$2\frac{1}{2}$	3
Cost (\$)	55	100	145				

- b. Draw a graph of this information. Place time on the x -axis and cost on the y -axis.
 c. Is there a linear relationship between cost and time?
 d. Find the rule for this graph using $y = mx + b$.
 e. In your equation replace y with cost and replace x with time:
 Cost = $\_\_ \times$ time + $\_\_$.
 f. Use your equation to find the cost if a repair person takes 4 hours to fix Chris's fridge.
15. Kyle was very bored on his holidays and decided to measure how much the grass grew every day for one week. His results are shown in the table.

Day number	0	1	2	3	4	5	6	7
Height of grass (mm)	10	12	14	16	18	20	22	24

- a. Kyle knew his dad would want the grass cut as soon as it was 2.5cm (25mm) long. On which day would this occur?
 b. Plot the points from the graph on a Cartesian plane putting day on the x -axis and height on the y -axis.
 c. Do the points form a linear graph?
 d. Find an equation for the height of the grass by filling in the blanks.
 Height = $\_\_ \times$ no. of days + $\_\_\_\_\_\_$ or $h = \_\_ d + \_\_\_\_\_\_$
 e. How long would the grass be after 14 days if it had not been cut?
16. a. If t represents the time in hours and C represents cost (\$), construct a table of values for 0–3 hours for the cost of playing ten-pin bowling at the new alley.
 b. Use your table of values to plot a graph of time versus cost.
 (Hint: Ensure your time axis (horizontal axis) extends to 6 hours and your cost axis (vertical axis) extends to \$40.)
 c. Write a linear equation to describe the relationship between cost and time.
 d. Use your linear equation from part c to calculate the cost of a 5-hour tournament.
 e. Use your graph to check your answer to part d.



Save \$\$\$ with Supa-Bowl!!!
 NEW Ten-Pin Bowling Alley
 Shoe rental just \$2 (fixed fee)
 Rent a lane for ONLY \$6/hour!

17. Find the equation for each of the following tables of values. Complete the tables and plot the linear graphs on separate Cartesian planes.

a.

x	-2	-1	0	1	2	3	4
y	-3		-2		-1	-0.5	0

b.

x	0	10	20	30	40	50
y	5	7	9		13	

18. a. Use the equation $y = 2x - 5$ to complete the table below and plot the linear graph.
b. What did you do differently to find the values for this table?

x				2			
y	-13	-9	-5		3	7	11

19. The line joining points on a linear graph is continuous, meaning that there are no breaks in the line. What does this mean in terms of finding ordered pairs that satisfy a linear relationship?
20. Determine which term of the rule $y = mx + b$ affects the gradient of a graph by looking at the rules and graphs of the linear relationships you have drawn in this exercise.
21. A certain linear pattern has its Cartesian coordinates in fraction form as:

$$(x, y) \left(\frac{1}{8}, \frac{7}{12} \right) \left(\frac{1}{4}, \frac{5}{6} \right) \left(\frac{3}{8}, \frac{13}{12} \right) \left(\frac{1}{2}, \frac{4}{3} \right).$$

Find the rule that satisfies these Cartesian coordinates.

11.7 Graphing with technology

11.7.1 Graphing using GeoGebra

- Graphs can be created using a variety of different technologies.
- In this subtopic we will create graphs using GeoGebra, a free online software package.

WORKED EXAMPLE 21

Use a digital technology of your choice to plot the graph of $y = 2x + 2$.

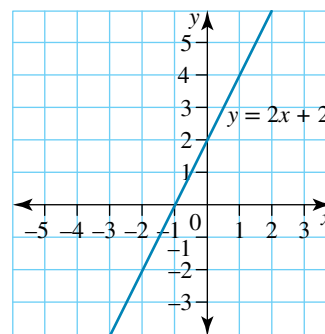
THINK

- Open GeoGebra and click on the 'Move Graphics View' button . Drag the screen until all four quadrants can be seen.
- Click on the keyboard icon at the bottom of the screen and enter in the equation as ' $y = 2x + 2$ '.
- Right click on the line in the Algebra bar on the left-hand side of the screen and click on 'Object Properties'.
- Change the 'Show Label' dropdown to 'Value'. In 'Object Properties' you can also change the colour and style of your line.
- Close the 'Object Properties' box by clicking on the grey cross in the corner and click on the 'Move' button . You can zoom in and out by using the scroll wheel on a mouse, or by right-clicking on the main screen away from your line.
- Click on the 'Move Graphics View' button  again and drag the screen to a position of your choice. The graph has been reproduced below this worked example.

You should have created a graph which looks like the one shown at right.

11.7.2 Comparing graphs

- We can compare two or more graphs by graphing them on the same set of axes.
- When comparing graphs, things to comment on are whether they are parallel (i.e. if their lines have the same gradient), whether they pass through the same point(s) and whether they represent linear relationships or not.
- If a graph is not a straight line then it does not represent a linear relationship.



WORKED EXAMPLE 22

Use a digital technology of your choice to plot the graphs of $y = 2x$ and $y = x^2$ on the same set of axes. Comment on the similarities and differences between the two graphs.

THINK

- 1 Follow the instructions from Worked example 21 to draw the first graph.
- 2 Click on the Input bar and enter in the equation of the second graph as ' $y = x^2$ '.
- 3 Right click on the lines in the Algebra bar and select Object Properties for each individual graph. Ensure that the two graphs are in different colours so they can be easily identified.
- 4 Comment on the similarities and differences between the two graphs.

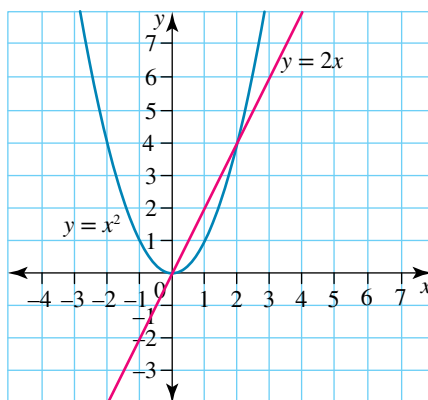
Note: The graphs have been reproduced below this worked example.

WRITE/DRAW

Both graphs pass through the origin $(0, 0)$ and the point $(2, 4)$.

The graphs are different shapes. The graph of $y = 2x$ is a straight line, so it is a linear relationship. The graph of $y = x^2$ is a curve, so it is not a linear relationship.

Your graphs will look like this:



Exercise 11.7 Graphing with technology

Individual pathways

PRACTISE

Questions:

1, 2a, c, 3a, 4–6, 8

CONSOLIDATE

Questions:

1a, c, e, 2a, c, e, 3–7, 9

MASTER

Questions:

1b, d, f, 2b, d, f, 3–10

Individual pathway interactivity: int-4504

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Understanding and fluency

- WE21** Sketch and label the following graphs using a digital technology of your choice.
 - $y = x + 1$
 - $y = x + 3$
 - $y = x - 3$
 - $y = x - 2$
 - $y = 2x + 2$
 - $y = 2x - 1$
- Sketch and label the following graphs using a digital technology of your choice.
 - $y = 4x - 2$
 - $y = 6x - 4$
 - $y = -x + 4$
 - $y = -x + 2$
 - $y = -x - 5$
 - $y = -2x + 3$
- Sketch and label the following graphs using a digital technology of your choice.
 - $y = 2x^2$
 - $y = -x^2$
 - $y = x^2 + 1$

Communicating, reasoning and problem solving

- WE22** Use a digital technology of your choice to plot the graphs of $y = 2x$, $y = 2x - 1$ and $y = 2x + 1$ on the same set on axes. Comment on the similarities and differences between the three graphs.
- Use a digital technology of your choice to plot the graphs of $y = x$, $y = 2x$ and $y = 3x$ on the same set of axes. Comment on the similarities and differences between the three graphs.
- Use a digital technology of your choice to plot the graphs of $y = x$ and $y = -x$ on the same set of axes. Comment on the similarities and differences between the two graphs.
- Explain the differences between equations that represent linear relationships and those that represent non-linear relationships.
- The monthly cost in dollars, C , of renting a mobile phone is given by the equation $C = 15 + 0.5x$, where x is the call time in minutes.
 - Use a digital technology of your choice to graph this equation.
 - If July's bill was \$100, what was your call time?
 - If August's bill was only \$50, what was your call time?
- The cost, C , in dollars, of a visit to an amusement park is given by the equation

$$C = 2R + 15$$

where R is the number of rides that you go on.

- Use a digital technology of your choice to graph this equation, plotting the cost (y -axis) against the number of rides (x -axis).
- What is the entry fee for this amusement park?



- c. What is the cost for each ride you take?
 - d. What does it cost, in total, if you go on five rides?
10. The total time, T , in minutes, that it takes a Year 8 student to complete a Maths homework sheet consisting of 20 questions is given by the equation

$$T = 4.5Q + 1.5$$

where Q is the number of questions that a student answers.

- a. Use a digital technology of your choice to graph this equation, with Q on the x -axis and T on the y -axis.
- b. How long does it take to complete all 20 questions?
- c. What is the possible significance of the number 1.5 in the equation given?

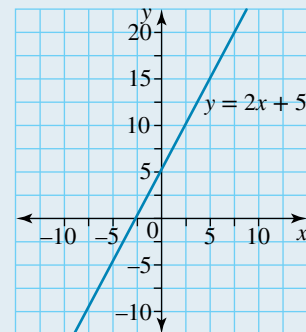
11.8 Solving equations graphically

11.8.1 Using linear graphs to solve linear equations

- When we substitute a y -value into the rule of a linear relationship it creates an equation that we can solve to determine the x -value.
- For example, by substituting $y = 6$ into the rule $y = 3x + 9$ we create the equation $6 = 3x + 9$.
- As each y -value on the line of a linear graph is unique, this means that each point on the graph of a linear relationship represents a solution to a particular linear equation.
- We can use therefore use the graph of a linear relationship to solve a corresponding linear equation.

WORKED EXAMPLE 23

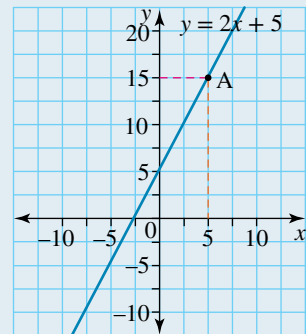
Use the following graph to solve the linear equation $2x + 5 = 15$.



THINK

- Rule a horizontal line (pink) at $y = 15$. This is the right-hand side of the original equation. This line meets the graph at point A. Rule a vertical line (orange) from point A to the x -axis. The line meets the x -axis at 5.

WRITE/DRAW



- Write the answer.

The solution to the linear equation $2x + 5 = 15$ is $x = 5$.

11.8.2 Solving equations using the point of interception

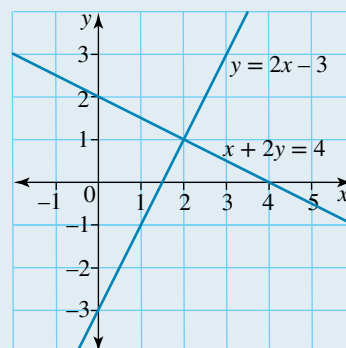
- Any two linear graphs will meet at a point unless they are parallel (i.e. unless they have the same gradient).
- Pairs of equations may be solved graphically.
- This method involves drawing the graph of each equation on the same set of axes, and then finding the point of intersection of these lines.
- The point of intersection is the point that is common to both equations and is therefore a solution to both the equations.

WORKED EXAMPLE 24

Use the graphs of the given straight lines to determine the point of intersection and, hence, the solution of the equations.

$$x + 2y = 4$$

$$y = 2x - 3$$



THINK

- Write the equations and number them.
- Locate the point of intersection of the two lines. This gives the solution.
- Check the solution by substituting $x = 2$ and $y = 1$ into the given equations. Comment on the results obtained.

WRITE

$$x + 2y = 4 \quad [1]$$

$$y = 2x - 3 \quad [2]$$

Point of intersection (2, 1)

The solution is $x = 2$ and $y = 1$.

Check equation [1]:

$$\begin{aligned} \text{LHS} &= x + 2y & \text{RHS} &= 4 \\ &= 2 + 2(1) \\ &= 4 \end{aligned}$$

LHS = RHS

Check equation [2]:

$$\begin{aligned} \text{LHS} &= y & \text{RHS} &= 2x - 3 \\ &= 1 & &= 2(2) - 3 \\ & & &= 4 - 3 \\ & & &= 1 \end{aligned}$$

LHS = RHS

In both cases LHS = RHS, therefore the solution set (2, 1) is correct.

WORKED EXAMPLE 25

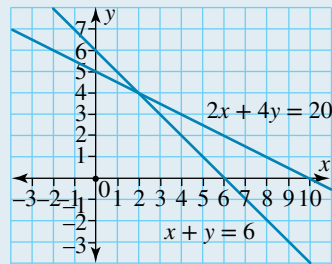
Solve the following pair of equations by graphing the equations using a digital technology of your choice and identifying the point of intersection.

$$x + y = 6$$

$$2x + 4y = 20$$

THINK

- 1 Plot both graphs on the same axes using a digital technology of your choice.

WRITE/DRAW

- 2 Locate the point of intersection of the lines.

The point of intersection is (2, 4).

- 3 Check the solution by substituting $x = 2$ and $y = 4$ into each equation.

$$\begin{aligned}\text{Check [1]: LHS} &= x + y & \text{RHS} &= 6 \\ &= 2 + 4 \\ &= 6\end{aligned}$$

LHS = RHS

$$\begin{aligned}\text{Check [2]: LHS} &= 2x + 4y & \text{RHS} &= 20 \\ &= 2(2) + 4(4) \\ &= 4 + 16 \\ &= 20\end{aligned}$$

LHS = RHS

- 4 State the solution.

In both cases, LHS = RHS. Therefore, the solution set (2, 4) is correct.

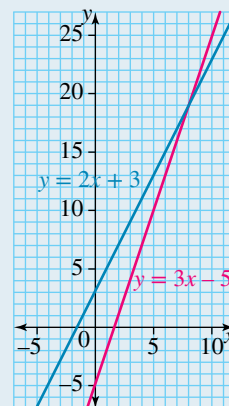
The solution is $x = 2, y = 4$.

WORKED EXAMPLE 26

Use a digital technology of your choice to solve the equation $2x + 3 = 3x - 5$ graphically.

THINK

- 1 We can let both sides of the equation = y , which gives us two equations:
 $y = 2x + 3$ and $y = 3x - 5$
 Sketch the graphs of $y = 2x + 3$ and $y = 3x - 5$ on the same set of axes using a digital technology of your choice.

WRITE/DRAW

- 2 Write the equations and number them.
- 3 Locate the point of intersection of the two lines.
This gives the solution.

$$\begin{aligned}y &= 3x - 5 & [1] \\ y &= 2x + 3 & [2]\end{aligned}$$

Point of intersection (8, 19)
Solution: $x = 8$ and $y = 19$

4 Check the solution by substituting $x = 8$ and $y = 19$ into each equation.

$$\begin{aligned}\text{Check [1]: LHS} &= 19 & \text{RHS} &= 3x - 5 \\ & & &= 3(8) - 5 \\ & & &= 24 - 5 \\ & & &= 19\end{aligned}$$

$$\text{LHS} = \text{RHS}$$

$$\begin{aligned}\text{Check [2]: LHS} &= 19 & \text{RHS} &= 3x - 5 \\ & & &= 3(8) - 5 \\ & & &= 24 - 5 \\ & & &= 19\end{aligned}$$

$$\text{LHS} = \text{RHS}$$

In both cases $\text{LHS} = \text{RHS}$. Therefore, the solution set $(8, 19)$ is correct.

The solution is $x = 8, y = 19$.

5 State the solution.

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Interactivity: Solving equations graphically (int-3842)

Exercise 11.8 Solving equations graphically

Individual pathways

PRACTISE

Questions:
1–5, 7, 10, 11

CONSOLIDATE

Questions:
1, 2, 4–7, 9–11

MASTER

Questions:
1–3, 4c–f, 5c–e, 6–12

Individual pathway interactivity: int-4507

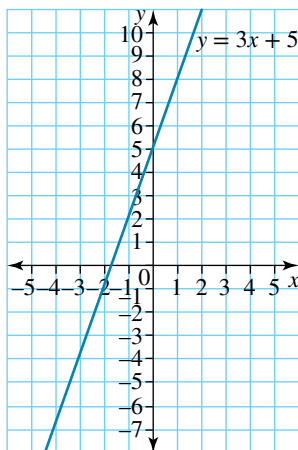
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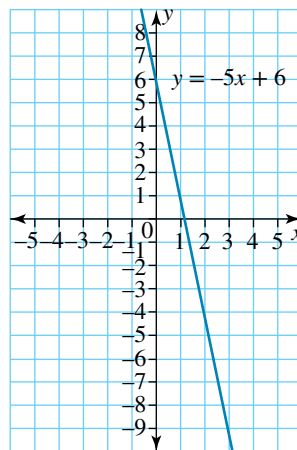
Understanding and fluency

1. **WE23** For each of the following, use the graph shown to solve the linear equation given.

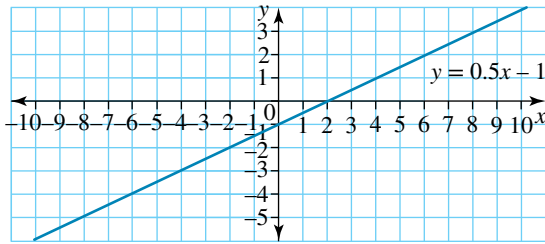
a. $3x + 5 = 8$



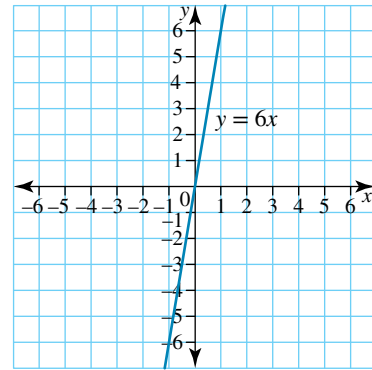
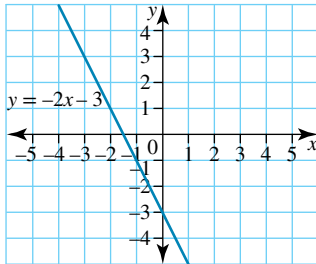
b. $-5x + 6 = -9$



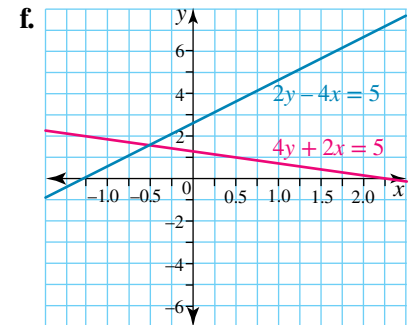
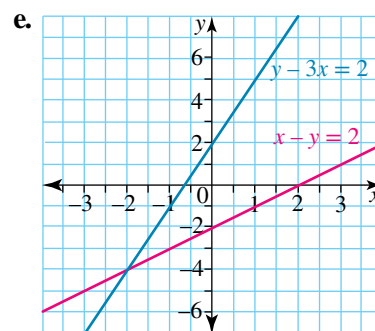
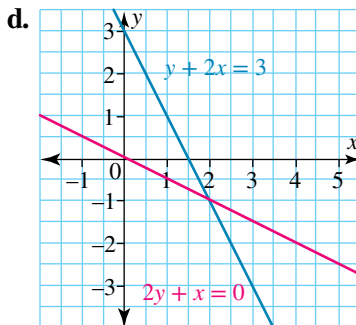
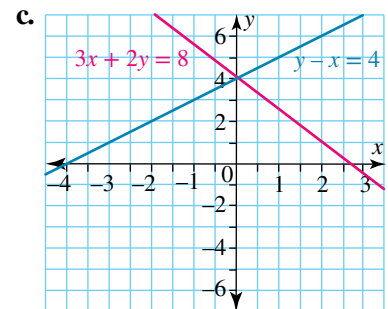
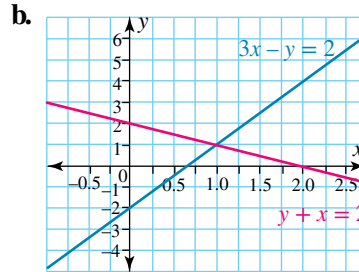
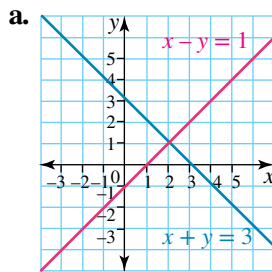
c. $0.5x - 1 = -4$



2. Use the graph of the line $y = -2x - 3$ below to solve the equation $-2x + 3 = 0$.



3. Use the graph of the line $y = 6x$ at right to solve the equation $6x + 1 = 0$.
4. **WE24** For each of the following graphs, determine the coordinates of the point of intersection.



5. **WE25** Solve each of the following pairs of equations by graphing the equations using a digital technology of your choice and identifying the point of intersection.

a. $y = 2x + 3$
 $y = 8 - 3x$

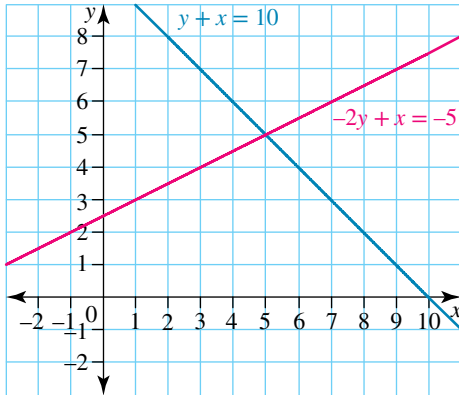
b. $y + 2x = -8$
 $y = 2x + 4$

c. $2y = 12x + 16$
 $3y = -6x - 24$

d. $2x - y = -1$
 $3x + y = 11$

e. $y = \frac{2x}{3} + 2$
 $y = 2x - 2$

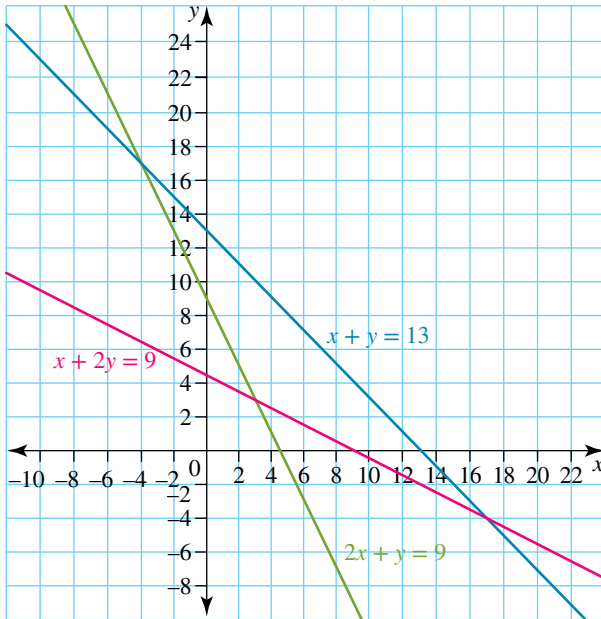
6. Solve the linear equations $y + x = 10$ and $-2y + x = -5$ graphically.



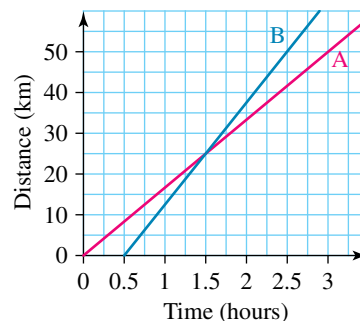
7. **WE26** Use a digital technology of your choice to solve the equation $3x - 7 = -2x + 3$ graphically.
 8. Use a digital technology of your choice to solve the equation $\frac{x}{3} + 1 = 2 - \frac{2x}{5}$ graphically.

Communicating, reasoning and problem solving

9. A triangle is formed by the graphs of $x + y = 13$, $x + 2y = 9$ and $2x + y = 9$. Determine the vertices of the triangle by identifying the coordinates of the points at which each pair of lines cross.



10. The photo below shows two friends during a race. The following graph shows the distance in kilometres covered by both friends in a given time in hours. At what time in the race was the photo taken?



11. A fire truck and an ambulance have been called to an accident scene. The ambulance starts at the hospital; the equation of the distance it travels with respect to time is $d = 1.8t$. The fire truck leaves

the garage and travels following the equation $d = 1.5t + 3.6$. In both equations, d is the distance travelled in kilometres, and t represents the time in minutes.

- a. Use a digital technology of your choice to plot the two graphs on the same set of axes with d on the vertical axis and t on the horizontal axis.
 - b. How far from the hospital was the ambulance after 5 minutes?
 - c. If the two vehicles meet at the scene of the accident, how long did it take the two vehicles to arrive?
 - d. What distance did the two vehicles travel to the scene of the accident?
12. You are given some money for your birthday and decide to save a certain amount per week to buy a bike. This information is represented in the equation $A = 17.5w + 110$, where A is the amount in dollars and w is the number of weeks.
- a. Use a digital technology of your choice to plot the graph of the equation.
 - b. Use the graph to determine how much you will have saved after 10 weeks.
 - c. How much money did you get for your birthday?
 - d. How many weeks would it take for you to save a total of \$670?
 - e. If the bike costs \$800, how long will it take you to save for it at this rate?

11.9 Review

Investigation | Rich task

Choosing the right hire car

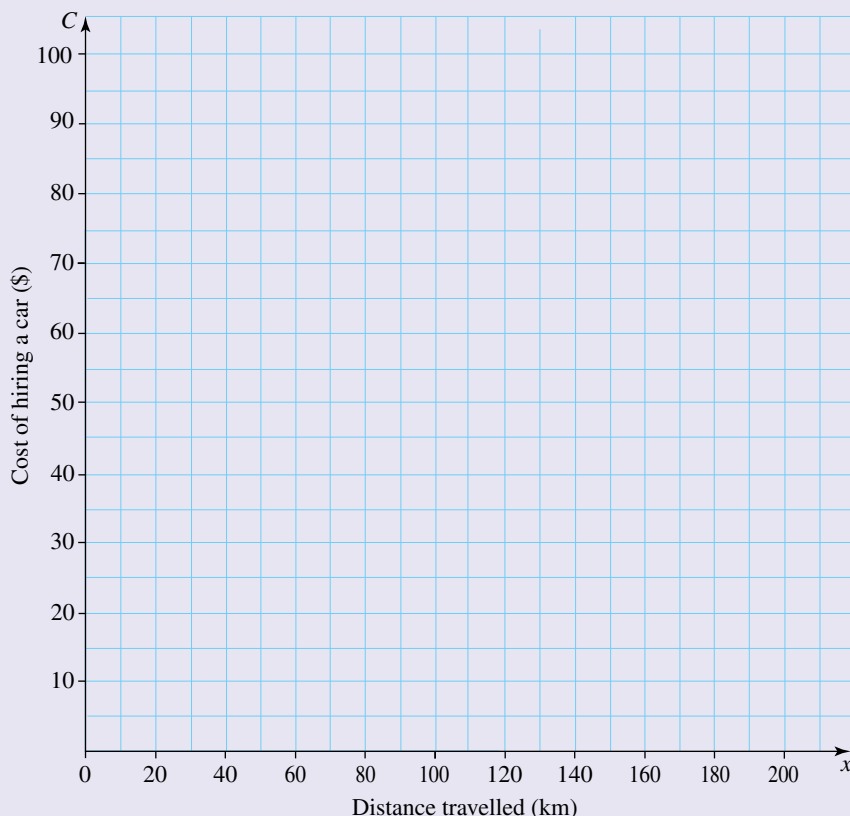
A group of tourists have just arrived at Sydney airport and are investigating the best hire car deals. They decide to study the different options offered by Bonza Car Rentals.

Option 1	\$60 per day unlimited kilometres
Option 2	\$30 per day and 25 cents/km
Option 3	\$40 per day and 35 cents for every kilometre over 100 each day



The group know that on their first day they will be visiting the local attractions close to Sydney, so they will not be travelling many kilometres.

1. How much would each option cost if the total kilometres travelled in a day was 90km?
2. Write an equation to show the cost of hiring a car for a day for options 1 and 2. Use C to represent the total cost of hiring a car for a day and x to represent the number of kilometres travelled in a day.
3. Use digital technology to plot the graphs of the three options on the set of axes provided to show the cost of hiring a car for a day to travel 200km.
4. Examine the graphs of the three options carefully. Write a brief statement to explain the costs associated with each option over 200km.



5. How much does it cost to travel 200km in one day with each option?

The group decide to drive to Melbourne and plan to spend 2 days travelling there, stopping overnight in Canberra. On arrival in Melbourne, they plan to stay for 4 days and return to Sydney on the seventh day, driving the whole distance in one day and returning the car to Bonza Car Rentals.

6. In their 4 days in Melbourne, the group travelled a total of 180km. What was the total distance travelled for this trip?
7. Work out how much it would cost to hire a car for each option to cover this trip. (Option 3 gives 100km 'free travel' per day. For a 7-day hire, you get 700km 'free'.)
8. The group took option 1. Was this the best deal? Comment.
9. Investigate some different companies' rates for car hire. Use digital technology to draw graphs to represent their rates and present your findings to the class.



Digital doc: Investigation — Choosing the right hire car (doc-13947)

Digital doc: Code puzzle — The first African country to win an Olympic gold medal! (doc-13948)

Digital doc: Summary Topic 11 (doc-23247)

Interactivity: Word search Topic 11 (int-2639)

Interactivity: Crossword Topic 11 (int-2640)

Interactivity: Sudoku Topic 11 (int-3193)

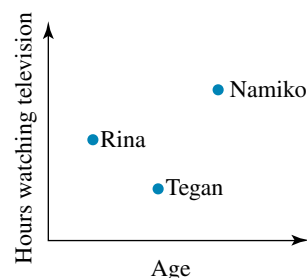
Exercise 11.9 Review questions

To answer questions online and to receive **immediate feedback** and **fully worked solutions** for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Understanding and fluency

1. Use the graph shown at right to answer the following.

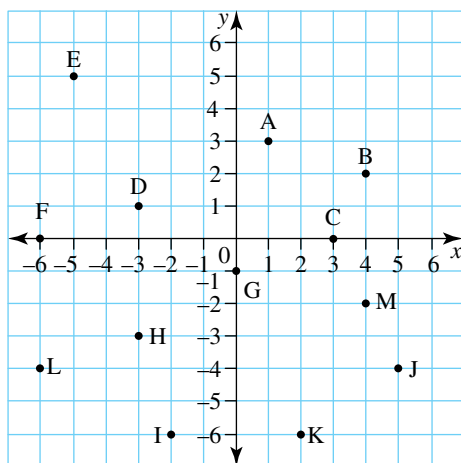
- Who watches the most television?
- Who is the youngest of the three?
- Does the youngest watch the least amount of television?



2. Place the following points on your Cartesian plane.

a. A(1, 4)	b. B(5, 3)	c. C(0, 2)
d. D(-2, 5)	e. E(-4, 1)	f. F(-5, 0)
g. G(-6, -6)	h. H(-5, -4)	i. I(0, -5)
j. J(2, -1)	k. K(2, -3)	l. L(2, 0)

3. Write down the coordinates of the points A to M, marked on the Cartesian plane below.



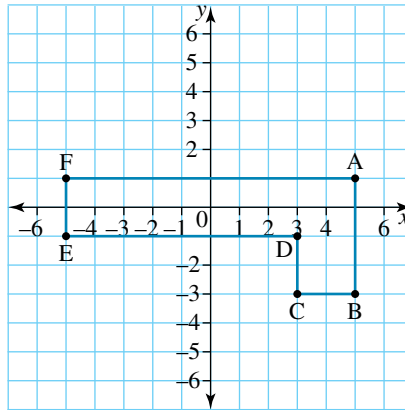
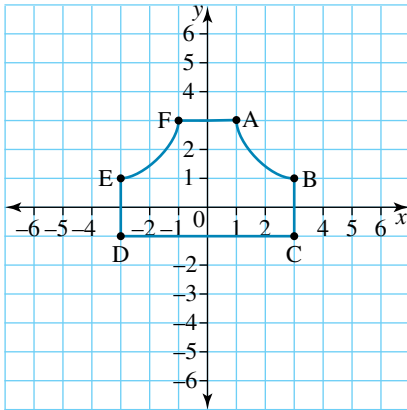
4. Plot the following points on a Cartesian plane.

- $(-3, -1)$, $(-2, 0)$, $(-1, 1)$, $(0, 2)$, $(1, 3)$, $(2, 4)$ and $(3, 5)$
- $(-3, -12)$, $(-2, -9)$, $(-1, -6)$, $(0, -3)$, $(1, 0)$, $(2, 3)$ and $(3, 6)$

5. State the final position of an object (relative to its original position) after each of the following translations:

- 6R 3D, 4L 5D
- 10R 3D, 4L 5D, 6R 8U
- 3U, 3R 4D, 6L 4D, 2R, 7L 2U, 3L
- 6R 2U, 3L 5U, 3R 4D, 6L 3D, 2R 4U, 6L 3U

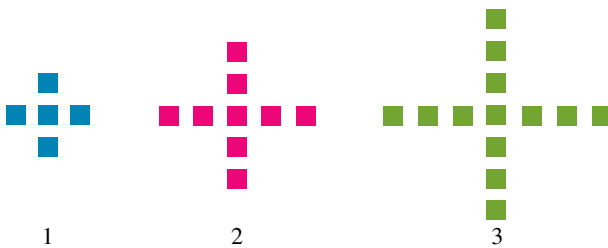
6. Transform the following shapes through the given translations.



2R 4D, 6R 3D, 2R 2D

7R 5U, 2R 3U, 8L 2D, 7L 1D

7. Follow the follow instructions for the pattern of shapes shown.



- Construct a table to show the relationship between the number of each figure and the number of squares used to construct it.
 - Devise a rule in words that describes the pattern relating the number each figure and the number of squares used to construct it.
 - Use your rule to work out the number of squares required to construct a figure made up of 20 such shapes.
8. Make a table of values and plot a graph for each of the following.
- $y = x - 2$
 - $y = x + 5$
 - $y = 4x - 2$
 - $y = 3x + 2$
9. Use digital technology to graph the line $y = 3x - 8$ and use this graph to solve the equation $3x - 8 = 19$.
10. Graph the following using a digital technology of your choice.
- $y = x + 7$
 - $y = 2x - 2$
 - $y = 3x - 5$
 - $y = -2x + 4$

Communicating, reasoning and problem solving

11. Chris has the newspaper delivered 7 days a week. He saves his newspapers for recycling. Over a month the newspaper pile grows very high. The table below shows the height of the newspaper recycling pile at the end of each week.

Time (weeks)	0	1	2	3	4	5	6
Height (cm)		35	70	105	140		

- Copy and complete the table.
- Draw a set of axes showing height on the y -axis and time on the x -axis. Plot the information from your table on the axes and join the points to form a linear graph.
- Find the rule for the graph using $y = mx + b$.
- What would the height of the pile be after 20 weeks?

12. James and his sister are going for a bike ride. They know they can ride 25 km in an hour (average).

a. Copy and complete the following table.

Distance (km)	0	25			100	
Time (hours)			2	3		5

b. Plot these points on a Cartesian plane.

c. Do the points form a linear graph?

d. Copy and complete the equation below.

$$\text{Distance} = \underline{\hspace{1cm}} \times \text{time} + \underline{\hspace{1cm}} \quad \text{or} \quad d = \underline{\hspace{1cm}} t + \underline{\hspace{1cm}}$$

e. Use the equation found in part d to work out how far they would have ridden after 7 hours.

f. If they leave home at 9.00 am and arrive home at 5.00 pm, how far would James and his sister have ridden?



13. Lara sells computers and is paid \$300 per week plus \$20 for every computer she sells.

a. Draw a table to show how much money Lara would be paid if she sold between 0 and 10 computers per week.

b. Plot the points in the table on a Cartesian plane.

c. Do the points form a linear graph?

d. Find an equation for the graph in the form $y = mx + b$.

e. If Lara sold 25 computers in a week, how much money would she be paid?

14. Lena and Alex have set up savings accounts. Each month they write down their savings. If the trend continues, who will be the first to save \$300?

Lena's account:

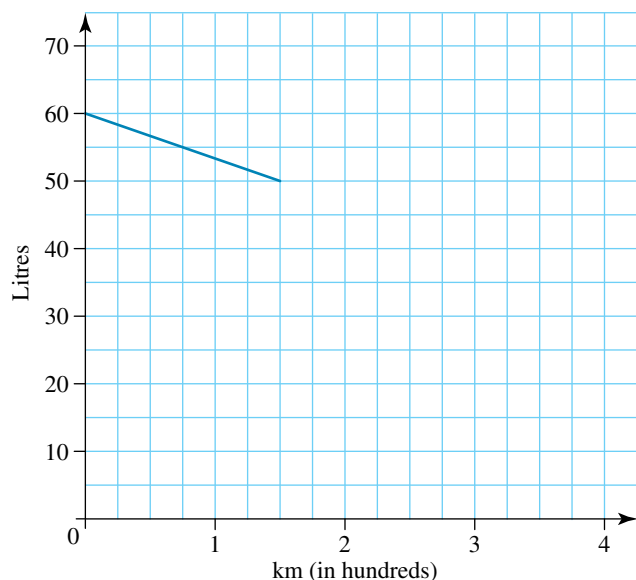
t (months)	0	1	2
A (\$)	100	120	140

Alex's account:

t (months)	0	1	2
A (\$)	150	160	170

15. A Fast Track company is fitting internet cable in the neighbourhood. It takes $1\frac{1}{2}$ hours to install 300 m of cable. In 2 hours, 450 m can be installed. If this is a linear relationship, how much can be laid in 4 hours?

16. As Rachel has been driving her new car she has kept a watch on her petrol usage. The graph below shows how the amount of petrol has changed.



- If this pattern continues, how much petrol will there be after 200 km?
 - The equation for this graph would be:
 Number of litres used = $\text{---} - \text{---} \times \text{number of km travelled (in hundreds)}$.
 This could be shortened to $l = \text{---} - \text{---} k$.
 - Using the equation from part **b**, find the amount of petrol left when $k = 5$.
 - How far will Rachel have travelled when she has used 60 litres of petrol?
17. Two yachts are sailing in Western Port Bay. At the end of a very windy day, rain begins to fall and visibility is markedly reduced. One yacht is travelling in the direction given by the equation $y = x + 6$ and the other in a direction given by $y = -2x + 3$.
- Could the yachts crash? (Assume both yachts are travelling towards the shore.) Graph both lines using digital technology to justify your answer.
 - If they could crash, state the coordinates of the point where they will meet.
 - Explain how you found your answer.

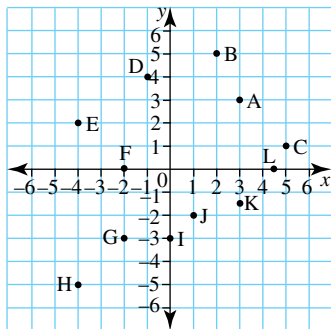


Answers

Topic 11 Linear relationships

Exercise 11.2 The Cartesian plane

1. a-l.



2. A(2, 2) B(4, 6) C(6, 1) D(-2, 3) E(0, 5) F(1, 1) G(-4, 1) H(0, 0) I(5, 0) J(4, 3) K(-6, 3) L(-4, -2) M(-2, -5)

3. a. B

b. C

c. D

d. A

e. E

f. A

4. a. D, E

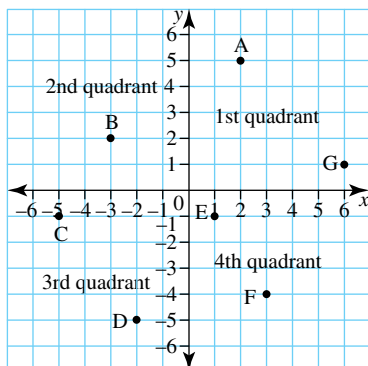
b. A, C

5. to 7. Check with your teacher.

8. In hospital in wards six, seven, eight and nine

9. b, d and e

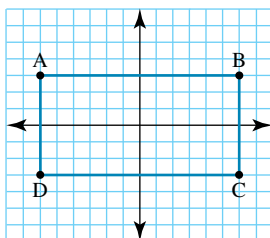
10. A: 1st quadrant, B: 2nd quadrant, C: 3rd quadrant, D: 3rd quadrant, E: 4th quadrant, F: 4th quadrant, G: 1st quadrant



11. a. A(-3, 3), B(3, 3), C(3, -3), D(-3, -3)

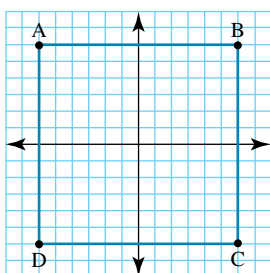
b. 36 square units.

c.



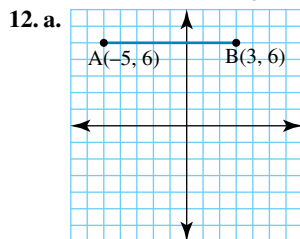
d. 72 squared units.

e.

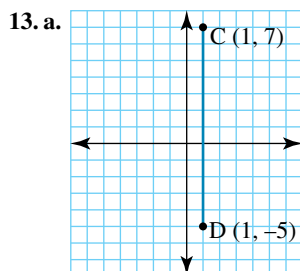


- f. 144 square units.
g. The length doubled, so the area doubled.

When both the length and the width doubled, the area became four times bigger.



- b. 8 units
c. $(-1, 6)$
d. Answers will vary.
e. $\frac{x_1 + x_2}{2}$

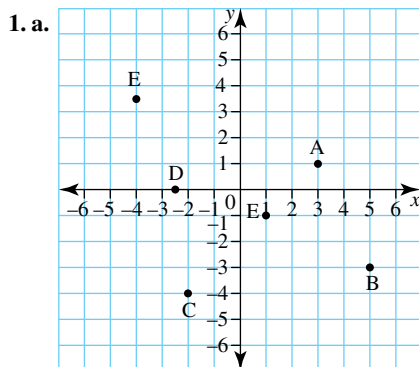


- b. 12 units
c. $(1, 1)$
d. Answers will vary.
e. $\frac{y_1 + y_2}{2}$

14. It is customary for the x -coordinate to be written first. This ensures that anyone reading a coordinate pair will know the correct location. If the x - and y -coordinates were reversed, the location would be incorrect.

15. Area is 18 square units, perimeter is 18 units.

Exercise 11.3 Translations in the Cartesian plane



b. $A'(1, 0), B'(3, -4), C'(-4, -5), D'\left(-4\frac{1}{2}, -1\right), E'\left(-6, 2\frac{1}{2}\right)$

c. $A''(1, 0), B''(3, 4), C''(-4, 5), D''\left(-4\frac{1}{2}, 1\right), E''\left(-6, -2\frac{1}{2}\right)$

2. a. $A(1, 1), B(-4, -2), C(0, 5), D(3, -3), E(-3, 0)$

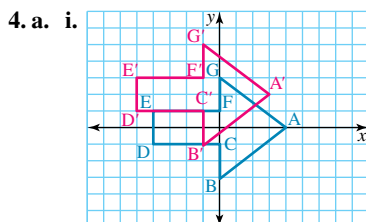
b. $A'(1, -1), B'(-2, 4), C'(5, 0), D'(-3, -3), E'(0, 3)$

3. a. 1L 6U

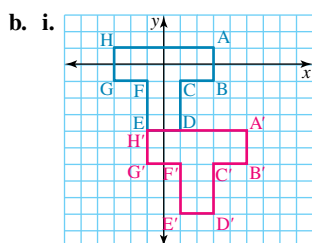
b. 2R 3U

c. 12L 9D

d. 8L 1D



ii. $A'(3, 2), B'(-1, -1), C'(-1, 1), D'(-5, 1), E'(-5, 3), F'(-1, 3), G'(-1, 5)$



ii. $A'(5, -4), B'(5, -6), C'(3, -6), D'(3, -9), E'(1, -9), F'(1, -6), G'(-1, -6), H'(-1, -4)$

5. D

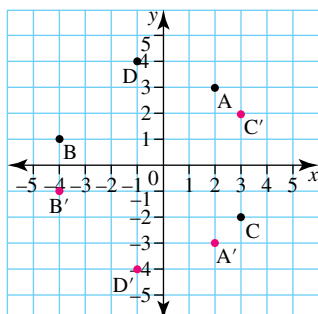
6. B

7. a. $A'(3, 2), B'(-5, -1), C'(4, -7)$

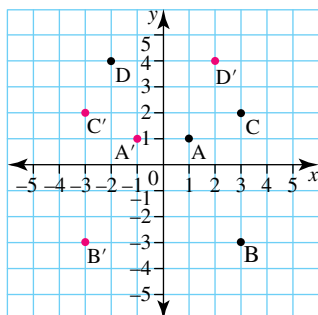
b. $A''(-2, 1), B''(-10, -2), C''(-1, -8)$

8. $A'(-2, 1), B'(1, 5), C'(-5, 6), D'(3, -1)$

9. $A'(2, -3), B'(-4, -1), C'(3, 2), D'(-1, -4)$

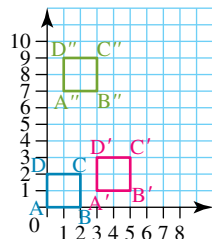


10. $A'(-1, 1), B'(-3, -3), C'(-3, 2), D'(2, 4)$



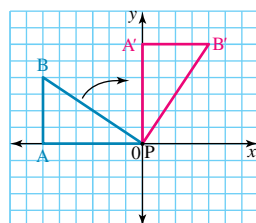
11. C

12. a.

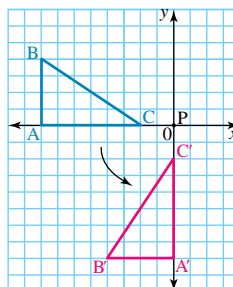


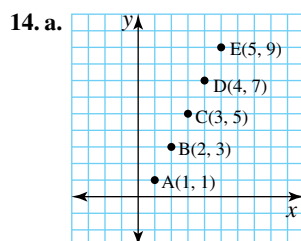
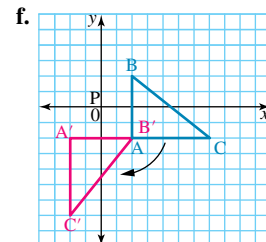
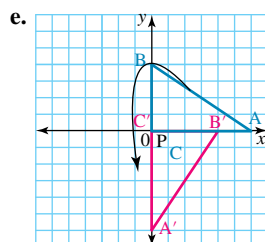
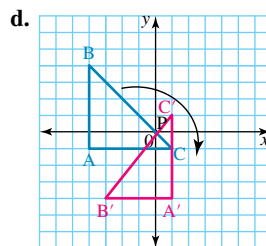
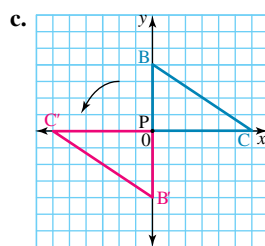
b. The combined transformation is the same as a translation of 1 unit right and 7 units up.

13. a.

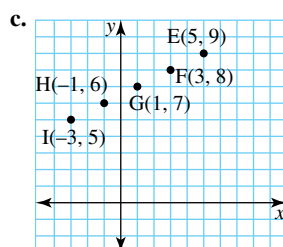


b.





b. The points form a straight line.



d. The points form a straight line.

15. a. $P'(1, 4)$

b. $P''(-2, 2)$

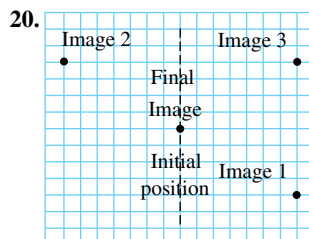
c. A translation of 1 unit up and 5 units to the left

16. $P(a, -b) \Rightarrow P'(-a, -b) \Rightarrow P''(a, b)$

17. Check with your teacher.

18. $(4 + b, -a)$

19. $(-r, -q) \Rightarrow (r, q) \Rightarrow (-r, q); (-r, -q) \Rightarrow (-r, q)$



Same position.

21. Alternatives to the Cartesian plane include latitude and longitude, which divide the earth into degrees of a sphere. Tourist maps often employ a local grid where east–west is divided into alphabetical labels, and north–south is divided into numerical labels, so a grid reference may read A2.

Exercise 11.4 Identifying patterns

1. a. 1, 5, 9, 13, 17 b. 5, 14, 23, 32, 41
 c. 50, 42, 34, 26, 18 d. 64, 53, 42, 31, 20
2. a. Add 2; 10, 12, 14 b. Add 5; 23, 28, 33
 c. Subtract 3; 15, 12, 9 d. Subtract 7; 20, 13, 6
3. a. 6, 15, 18 b. 10, 12, 16 c. 16, 64 d. 9, 11, 17 e. 88, 110 f. 95, 90, 75
4. a. i. Value of term = position in pattern $\times 10 - 5$ ii. 145
 b. i. Value of term = $47 - \text{position in pattern} \times 3$ ii. 2
 c. i. Value of term = position in pattern $\times 6 + 96$ ii. 186
 d. i. Value of term = $225 - \text{position in pattern} \times 13$ ii. 30



ii.

Number of triangles	1	2	3	4	5
Number of matches	3	5	7	9	11

- iii. Number of matches = number of triangles $\times 2 + 1$
 iv. 15



ii.

Number of houses	1	2	3	4	5
Number of matches	6	11	16	21	26

- iii. Number of matches = number of houses $\times 5 + 1$
 iv. 36



ii.

Number of fence units	1	2	3	4	5
Number of matches	4	7	10	13	16

- iii. Number of matches = number of fence units $\times 3 + 1$
 iv. 22

6. a. $m = 2n + 1$ b. $m = 5n + 1$ c. $m = 3n + 1$

7. When relating the value of a term to its position in the pattern, you can use that description to find any other future term in the number pattern.

8. Yes, you can substitute appropriate algebraic symbols for parts of the written descriptions to create an algebraic statement. For example, let the number of matches = m and the number of shapes = n .

9. Answers will vary. See a sample pattern below.



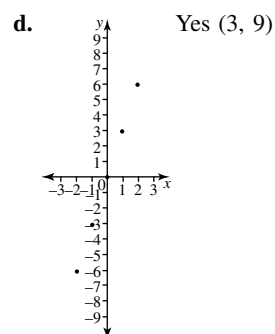
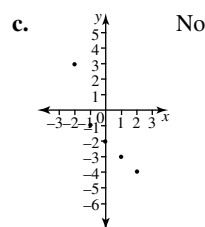
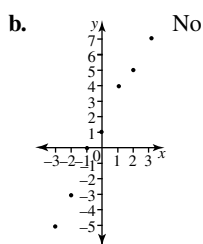
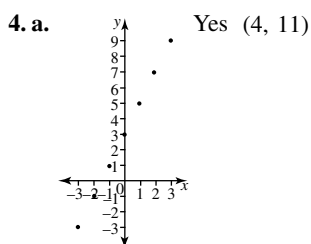
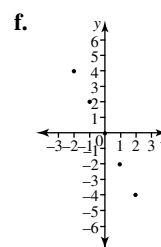
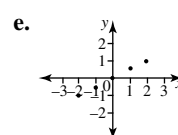
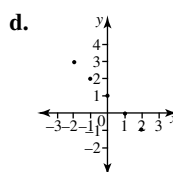
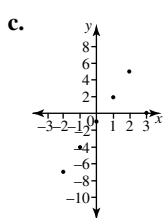
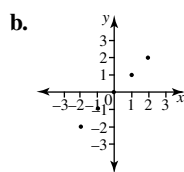
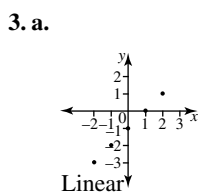
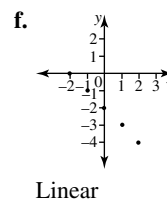
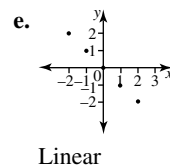
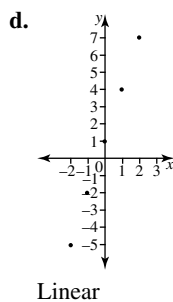
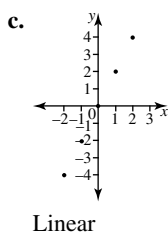
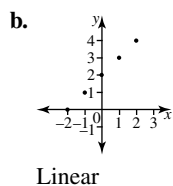
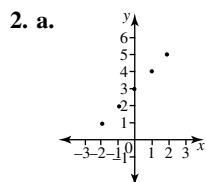
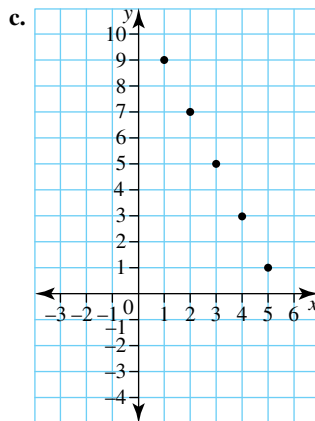
10. Once you have identified the connection between the top row and the bottom row in a table of values, you have identified the rule for the pattern.

Exercise 11.5 Linear patterns

1. a. 9, 7, 5, 3, 1

b.

Position in pattern	1	2	3	4	5
Value of term	9	7	5	3	1

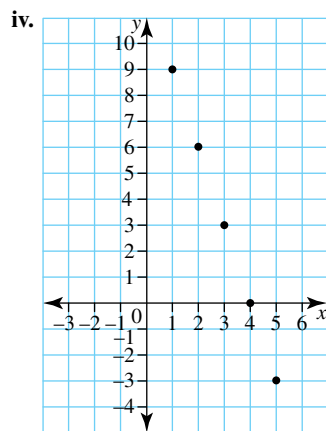


5. a. i.

Position in pattern	1	2	3	4	5
Value of term	9	6	3	0	-3

ii. Value of term = $12 - \text{position in pattern} \times 3$

iii. $y = 12 - 3x$

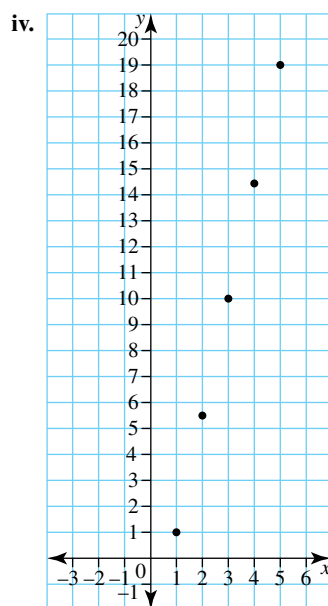


b. i.

Position in pattern	1	2	3	4	5
Value of term	1	5.5	10	14.5	19

ii. Value of term = position in pattern $\times 4.5 - 3.5$

iii. $y = 4.5x - 3.5$

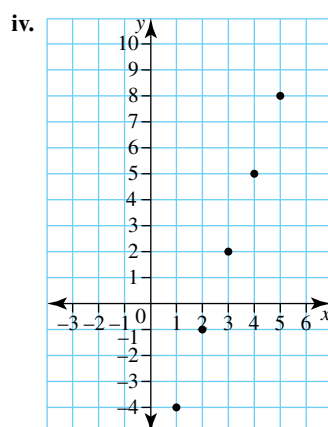


c. i.

Position in pattern	1	2	3	4	5
Value of term	-4	-1	2	5	8

ii. Value of term = position in pattern $\times 3 - 7$

iii. $y = 3x - 7$



6. a. D

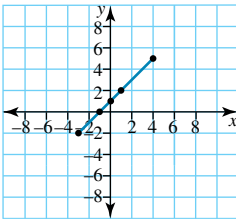
b. B

c. C

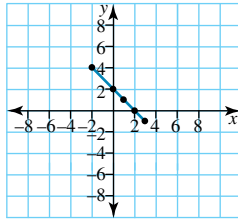
d. A, B, E

7. If the points of a number pattern go from the bottom left to the top right of a number grid the pattern is increasing. If the points go from the top left to the bottom right the pattern is decreasing.

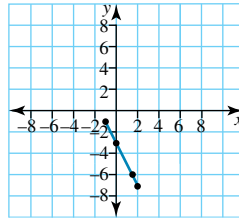
8. a.



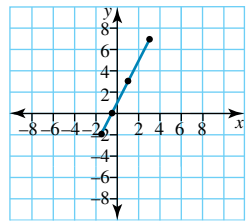
b.



c.



d.



9. Check with your teacher.

10. a.

Day (d)	Distance (D)
Monday (1)	1 km
Tuesday (2)	1.25 km
Wednesday (3)	1.5 km
Thursday (4)	1.75 km
Friday (5)	2 km
Saturday (6)	2.25 km
Sunday (7)	2.5 km

b. Linear

c. 0.25 km per day

d. 3.25 km

11. a. i. 12 minutes

ii. 24 minutes

iii. 82 minutes

b.

q	0	1	2	3	4	5	6	7	8	9	10
t	0	12	14	36	48	60	82	94	106	118	130
q	11	12	13	14	15	16	17	18	19	20	
t	152	164	176	188	200	222	234	246	258	270	

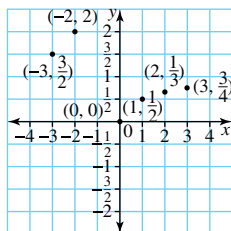
c. Not linear

12. 22 February of the following year

13. a.

x	-3	-2	-1	0	1	2	3
y	$\frac{3}{2}$	2	Undefined	0	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{3}{4}$

b.



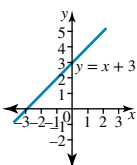
c. Not linear

14. Yes, if the points form a linear pattern then they will form a straight line when plotted.

Exercise 11.6 Linear graphs

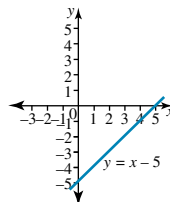
1. a.

x	-2	-1	0	1	2
y	1	2	3	4	5



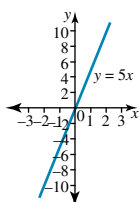
b.

x	-2	-1	0	1	2
y	-7	-6	-5	-4	-3



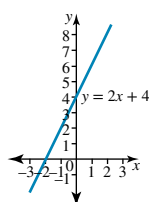
c.

x	-2	-1	0	1	2
y	-10	-5	0	5	10



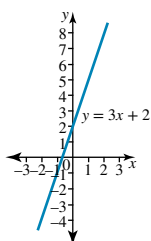
d.

x	-2	-1	0	1	2
y	0	2	4	6	8



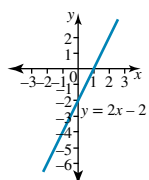
e.

x	-2	-1	0	1	2
y	-4	-1	2	5	8



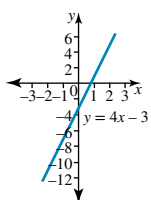
f.

x	-2	-1	0	1	2
y	-6	-4	-2	0	2



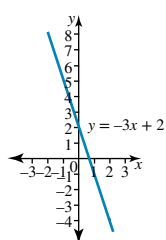
g.

x	-2	-1	0	1	2
y	-11	-7	-3	1	5



h.

x	-2	-1	0	1	2
y	8	5	2	-1	-4

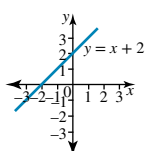


2. a. B

b. B

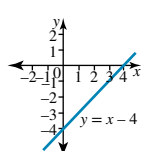
3. a.

x	-2	-1	0	1	2
y	0	1	2	3	4



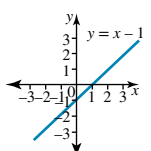
b.

x	-2	-1	0	1	2
y	-6	-5	-4	-3	-2



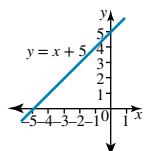
c.

x	-2	-1	0	1	2
y	-3	-2	-1	0	1



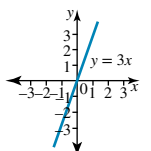
d.

x	-2	-1	0	1	2
y	3	4	5	6	7



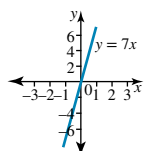
e.

x	-2	-1	0	1	2
y	-6	-3	0	3	6



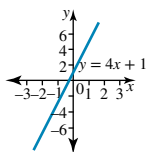
f.

x	-2	-1	0	1	2
y	-14	-7	0	7	14



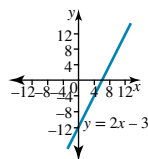
g.

x	-2	-1	0	1	2
y	-7	-3	1	5	9



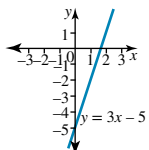
h.

x	-2	-1	0	1	2
y	-7	-5	-3	-1	1



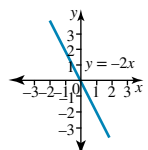
i.

x	-2	-1	0	1	2
y	-11	-8	-5	-2	1



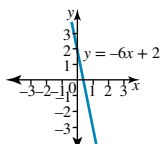
j.

x	-2	-1	0	1	2
y	4	2	0	-2	-4



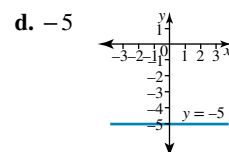
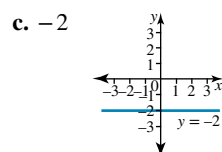
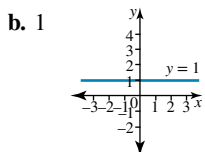
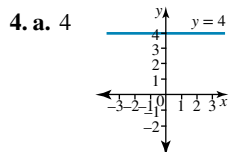
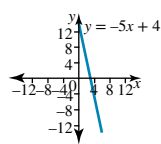
k.

x	-2	-1	0	1	2
y	14	8	2	-4	-10



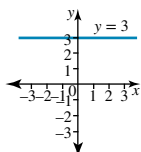
l.

x	-2	-1	0	1	2
y	14	9	4	-1	-6



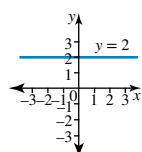
5. a.

x	-2	-1	0	1	2
y	3	3	3	3	3



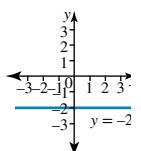
b.

x	-2	-1	0	1	2
y	2	2	2	2	2



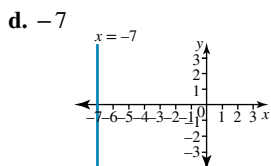
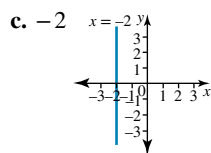
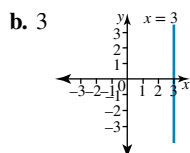
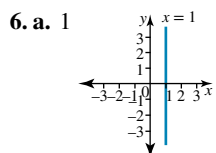
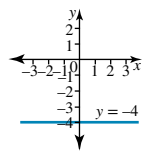
c.

x	-2	-1	0	1	2
y	-2	-2	-2	-2	-2



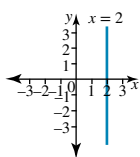
d.

x	-2	-1	0	1	2
y	-4	-4	-4	-4	-4



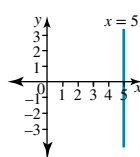
7. a.

x	2	2	2	2	2
y	-2	-1	0	1	2



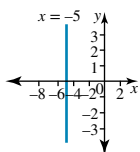
b.

x	5	5	5	5	5
y	-2	-1	0	1	2



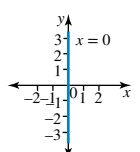
c.

x	-5	-5	-5	-5	-5
y	-2	-1	0	1	2



d.

x	0	0	0	0	0
y	-2	-1	0	1	2



8. a. $y = \frac{5}{3}x + 1$

b. $y = 2x + 5$

c. $y = 3x + 7$

d. $y = -x + 2.5$

9. a. $y = 2x - 7$

b. $y = 3x + 8$

c. $y = 10x + 1$

d. $y = -3x + 1$

10. a. Positive

b. Undefined

c. Negative

d. Positive

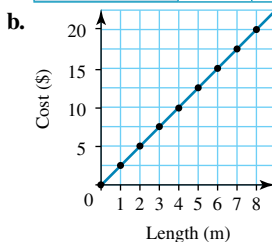
e. Zero

f. Negative

11. These values are all close to 0 (or are 0), so they are easy to join together to create a linear graph and they show where the graph crosses the y-axis

12. a.

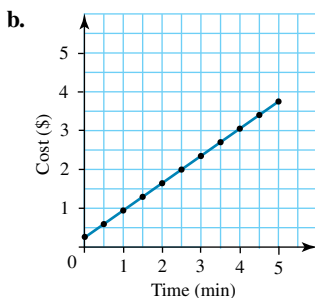
Length (m)	0	1	2	3	4	5	6	7	8
Cost (\$)	0	2.50	5.00	7.50	10.00	12.50	15.00	17.50	20.00



c. This is a linear relationship as the graph is a straight line.

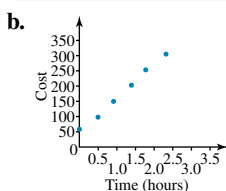
13. a.

Time (min)	0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0
Cost (\$)	0.25	0.60	0.95	1.30	1.65	2.00	2.35	2.70	3.05	3.40	3.75



14.

Times (hours)	0	$\frac{1}{2}$	1	$1\frac{1}{2}$	2	$2\frac{1}{2}$	3
Cost (\$)	55	100	145	190	235	280	325



c. Yes

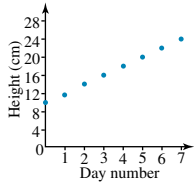
e. Cost = $90 \times \text{time} + 55$

d. $y = 90x + 55$

f. \$415

15. a. Day 8

b.



e. $h = 38 \text{ mm}$

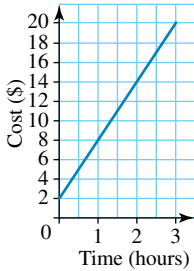
c. Yes

d. Height = $2 \times \text{no. of days} + 10$ or $h = 2d + 10$

16. a.

t	0	1	2	3
C	2	8	14	20

b.



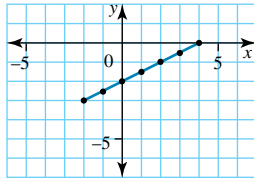
c. $C = 6t + 2$

d. \$32

e. Answers will vary.

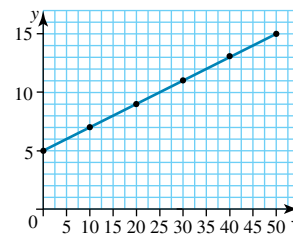
17. a.

x	-2	-1	0	1	2	3	4
y	-3	-2.5	-2	-1.5	-1	-0.5	0



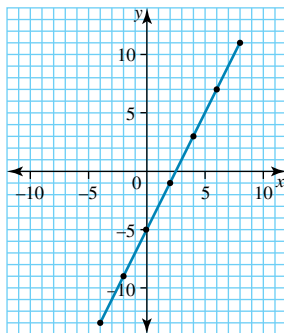
b.

x	0	10	20	30	40	50
y	5	7	9	11	13	15



18. a.

x	-4	-2	0	2	4	6	8
y	-13	-9	-5	-1	3	7	11



b. Look at the pattern of y-values to find the missing y-value. Use the y-values to calculate the x-values.

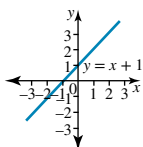
19. This means that there is an infinite number of ordered pairs that satisfy a linear relationship, as between any two given points on a linear graph there are more points that will also satisfy the relationship.

20. The value of the m determines the gradient of a graph.

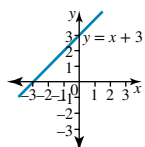
21. $y = 2x + \frac{1}{3}$

Exercise 11.7 Graphing with technology

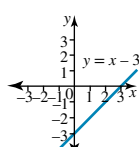
1. a.



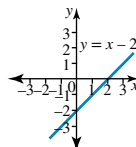
b.



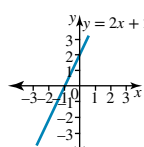
c.



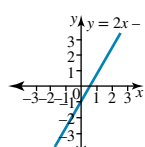
d.



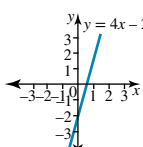
e.



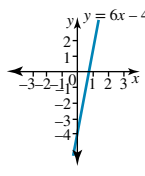
f.



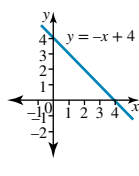
2. a.



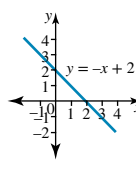
b.



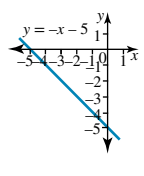
c.



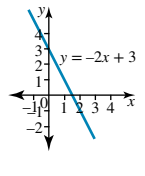
d.



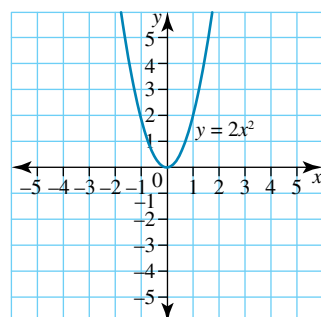
e.



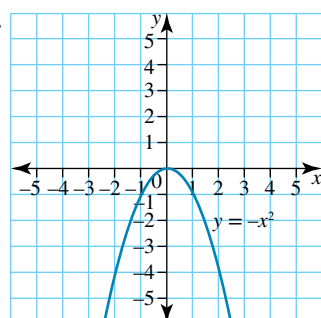
f.



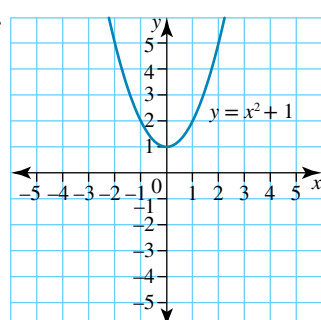
3. a.



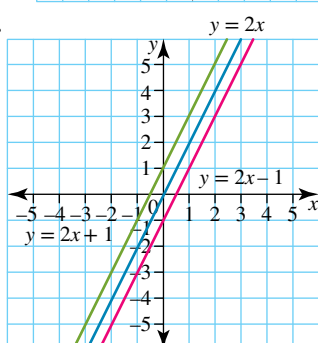
b.



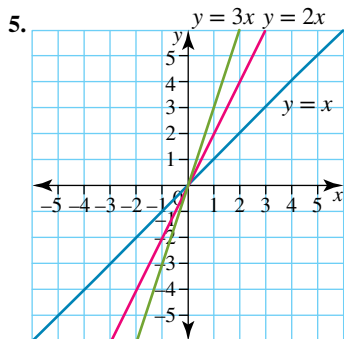
c.



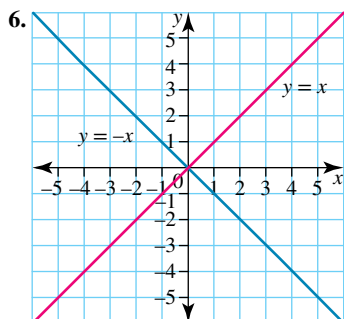
4.



The three graphs are all parallel and all represent linear relationships. None of them have any points in common.

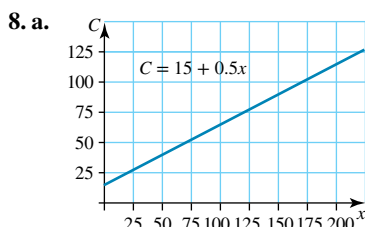


The three lines all pass through the origin $(0, 0)$ and have no other points in common. They all represent linear relationships.



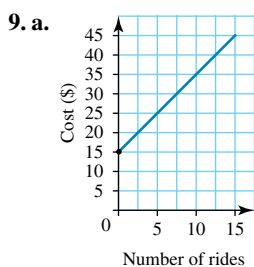
Both lines pass through the origin $(0, 0)$ and have no other points in common. They both represent linear relationships. The graph of $y = x$ has a positive gradient and the graph of $y = -x$ has a negative gradient.

7. Equations that represent linear relationships are of the form $y = mx + b$. Equations that represent non-linear relationships have a different power of x , for example $y = x^2$.



b. 170 minutes

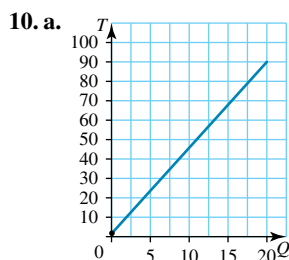
c. 70 minutes



b. \$15.00

c. \$2.00

d. \$25.00



b. 91.5 minutes

- c. Answers will vary. For example, 1.5 minutes might be the amount of time that it takes a student to set up their work and get ready.

Exercise 11.8 Solving equations graphically

1. a. $x = 1$

b. $x = 3$

c. $x = -6$

2. $x = \frac{3}{2}$

3. $x = -\frac{1}{6}$

4. a. $(2, 1)$

b. $(1, 1)$

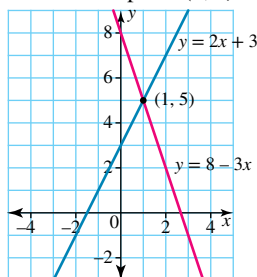
c. $(0, 4)$

d. $(2, -1)$

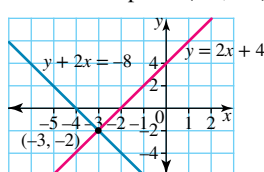
e. $(-2, -4)$

f. $(-0.5, 1.5)$

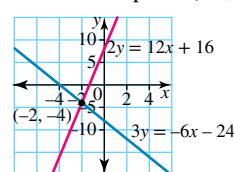
5. a. Intersection point $(1, 5)$



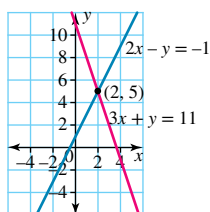
b. Intersection point $(-3, -2)$



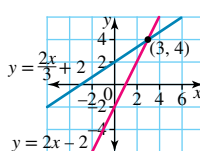
c. Intersection point $(-2, -4)$



d. Intersection point $(2, 5)$



e. Intersection point $(3, 4)$



6. $(5, 5)$

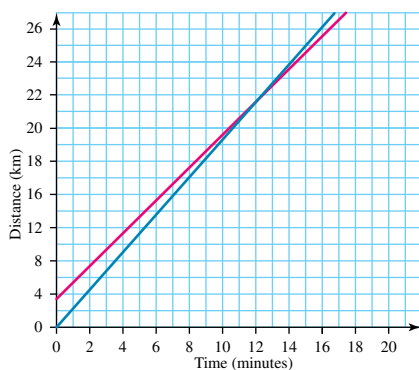
7. $x = 2$

8. $x = \frac{15}{11}$

9. $(3, 3), (-4, 17), (17, -4)$

10. 1.5 hours

11. a.

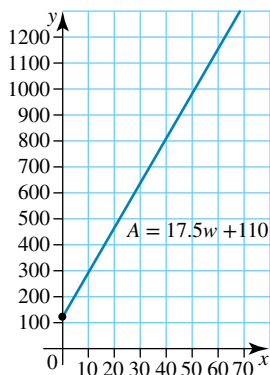


b. 9 km

c. 12 min

d. 21.6 km

12. a.



b. \$285

c. \$110

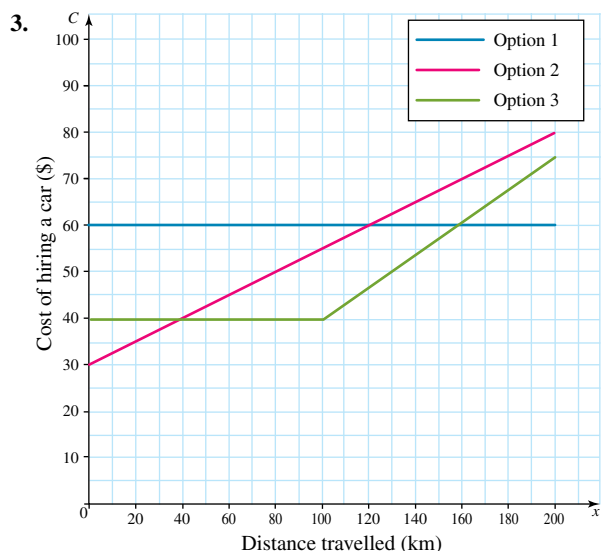
d. 32 weeks

e. 40 weeks

Investigation | Rich task

1. Option 1: \$60; Option 2: \$52.50; Option 3: \$40

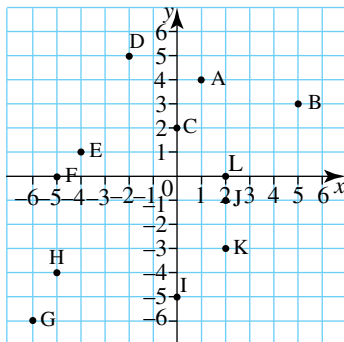
2. Option 1: $C = \$60$ Option 2: $C = \$(30 + 0.25x)$



4. Option 1 starts off the most expensive but by the end of the 200 km, it is the least expensive, remaining at \$60 for the day. Option 2 looks the least expensive at the start, but after travelling 200 km, its cost is the most expensive at \$80. Option 3's cost is \$40 up to 100 km travelled, but it increases to \$75 by the end of 200 km of travel.
5. Option 1: \$60; Option 2: \$80; Option 3: \$75
6. 205 km
7. Option 1: \$420; Option 2: \$723; Option 3: \$753.20
8. Yes. Teacher to check comments.
9. Teacher to check.

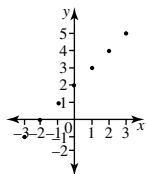
Exercise 11.9 Review questions

1. a. Namiko b. Rina c. No
2. a-l.

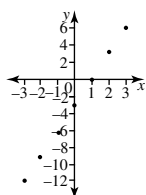


3. A(1, 3), B(4, 2), C(3, 0), D(-3, 1), E(-5, 5), F(-6, 0), G(0, -1), H(-3, -3), I(-2, -6), J(5, -4), K(2, -6), L(-6, -4), M(4, -2)

4. a.



b.



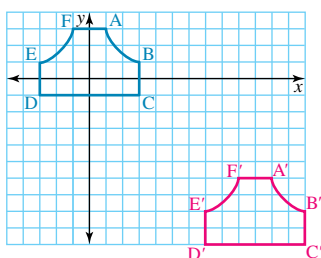
5. a. 2R 8D

b. 12R

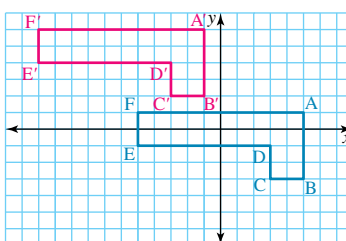
c. 11L 3D

d. 4L 7U

6. a.



b.



7. a.

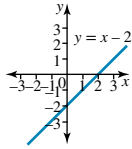
Number of figure	1	2	3
Number of squares	5	9	13

b. Number of squares = number of figure $\times 4 + 1$

c. 81

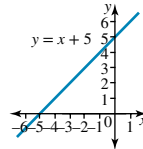
8. a.

x	-2	-1	0	1	2
y	-4	-3	-2	-1	0



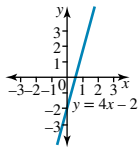
b.

x	-2	-1	0	1	2
$f(x)$	3	4	5	6	7



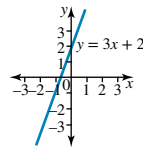
c.

x	-2	-1	0	1	2
$f(x)$	-10	-6	-2	2	6

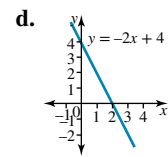
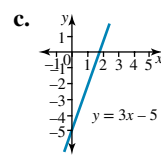
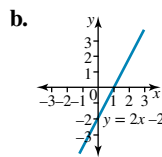
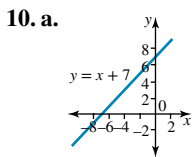


d.

x	-2	-1	0	1	2
$f(x)$	-4	-1	2	5	8

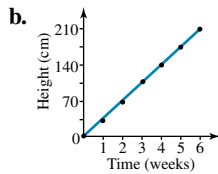


9. $x = 9$



11. a.

Times (weeks)	0	1	2	3	4	5	6
Height (cm)	0	35	70	105	140	175	210

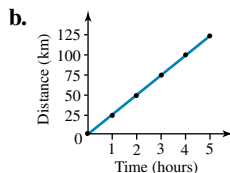


c. $y = 35x + 0$ or $y = 35x$

d. Height = 700 cm

12. a.

Distance (km)	0	25	50	75	100	125
Time (hours)	0	1	2	3	4	5



c. Yes

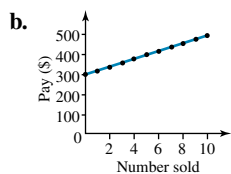
d. Distance = $25 \times \text{time} + 0$ or $d = 25t + 0$ or $d = 25t$

e. 175 km

f. 200 km

13. a.

Numbers sold (n)	0	1	2	3	4	5	6	7	8	9	10
Pay (\$)	300	320	340	360	380	400	420	440	460	480	500



c. Yes

d. $P = 20n + 300$

e. \$800

14. Lena (it will take her 10 months, while Alex will need 15 months).

15. 1050 m

16. a. 48 litres

b. Number of litres used = $60 - \frac{20}{3} \times \text{number of km travelled}$. $l = 60 - \frac{20}{3}k$ (l is number of litres; k is number of kilometres travelled (in hundreds).)

c. $26\frac{2}{3}$ litres

d. 1800 km

17. a. Yes they could crash, as the lines are not parallel.

b. They would meet at the point $(-1, 5)$.

c. By drawing the graphs of the two equations and finding the intersection point. Linear

TOPIC 12

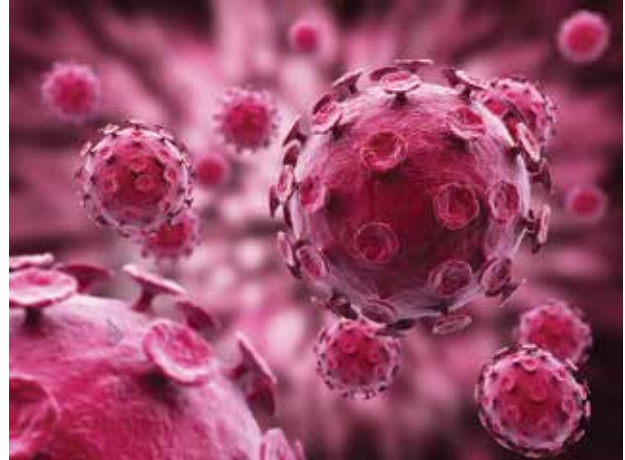
Probability

12.1 Overview

Numerous **videos** and **interactivities** are embedded just where you need them, at the point of learning, in your learnON title at www.jacplus.com.au. They will help you to learn the concepts covered in this topic.

12.1.1 Why learn this?

Probability is a branch of mathematics that uses numbers to represent the likelihood of certain events taking place. Whenever you use the words ‘unlikely’, ‘impossible’ or ‘certain’, you are using the language of probability. Probability is widely used to describe everyday events, such as the chance that a disease will be cured, the chance that a sporting team will win or the chance that Christmas day will be fine.



DISCUSSION

Of all the different groups who participate in the gambling industry in New South Wales, who do you think is the most aware of the mathematics behind the industry? What is the impact of this?

LEARNING SEQUENCE

- 12.1** Overview
- 12.2** Probability scale
- 12.3** Sample spaces and theoretical probability
- 12.4** Complementary events
- 12.5** Mutually exclusive and compound events
- 12.6** Venn diagrams and two-way tables
- 12.7** Review

LEARNING OUTCOMES

A student:

- communicates and connects mathematical ideas using appropriate terminology, diagrams and symbols MA4-1WM
- applies appropriate mathematical techniques to solve problems MA4-2WM
- recognises and explains mathematical relationships using reasoning MA4-3WM
- represents probabilities of simple and compound events MA4-21SP

CONTENT DESCRIPTIONS

Students:

Construct sample spaces for single-step experiments with equally likely outcomes (ACMSP167)

Assign probabilities to the outcomes of events and determine probabilities for events (ACMSP168)

Identify complementary events and use the sum of probabilities to solve problems (ACMSP204)

Describe events using language of ‘at least’, exclusive ‘or’ (A or B but not both), inclusive ‘or’ (A or B or both) and ‘and’ (ACMSP205)

Represent events in two-way tables and Venn diagrams and solve related problems (ACMSP292)

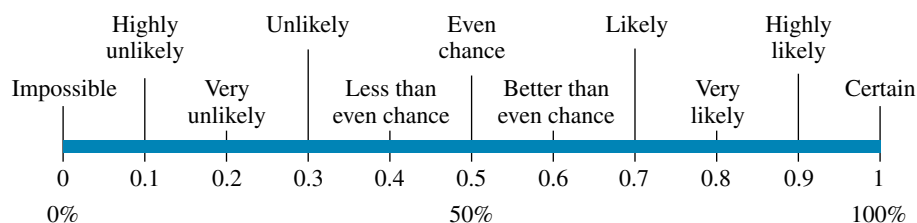
Source: NSW Syllabus for the Australian curriculum

Note: Your teacher may now set you a pre-test to determine how familiar you are with the content in this topic.

12.2 Probability scale

12.2.1 Assigning values to the likelihood of events

- **Probability** is defined as the chance of an event occurring.
- A scale from 0 to 1 inclusive is used to allocate the probability of an event as follows:



- A probability of 0 implies that the chance of an event happening is **impossible**.
- A probability of 1 implies that the chance of an event happening is **certain**.
- Probabilities may be written as fractions, decimals or percentages.

WORKED EXAMPLE 1

Describe the probability of each of the following events occurring, using a term from this list.

impossible	highly unlikely	very unlikely
less than even chance	even chance	
better than even chance	very likely	
highly likely	certain	

a February follows January.

b You draw the queen of diamonds from a standard deck of playing cards.

c You will compete in gymnastics at the Olympics.

d You roll a standard die and obtain an even number.

e Every Mathematics student will obtain a score of 99.95% as shown in an examination.



THINK

- a 1 Read the statement and associate the likelihood of the event occurring with one of the given words from the list.
Note: Provide reasoning.

2 Answer the question.

- b Repeat steps 1 and 2 of part a.

WRITE

- a This is a true statement. February always follows January.

It is *certain* this event will occur.

- b In a standard deck of 52 playing cards there is only one queen of diamonds. Thus, you have an extremely slim chance of drawing this particular card. It is *highly unlikely* this event will occur.

c Repeat steps 1 and 2 of part a. d Repeat steps 1 and 2 of part a. e Repeat steps 1 and 2 of part a.	c The chance of a person competing in the Olympics is very small. However, it could happen. It is <i>highly unlikely</i> this event will occur. d There are six possible outcomes when rolling a die, each of which are equally likely. Three of the outcomes are even while three are odd. There is an <i>even chance</i> this event will occur. e Due to each student having different capabilities, this situation could never occur. It is <i>highly unlikely</i> that this event will occur.

- It is important to note that the responses for particular situations, such as part **c** in Worked example 1, are not always straightforward and may differ for each individual. A careful analysis of each event is required before making any predictions about their future occurrences.

WORKED EXAMPLE 2

Assign a fraction to represent the estimated probability of each of the following events occurring.

- a** A high tide will be followed by a low tide.
- b** Everyone in the class will agree on every matter this year.
- c** A tossed coin lands Heads.
- d** A standard die is rolled and the number 5 appears uppermost.
- e** One of your 15 tickets in a 20-ticket raffle will win.



THINK

- a** **1** Determine the likelihood of an event occurring, with reasoning.
2 Answer the question.
- b** Repeat steps 1 and 2 of part **a**.
- c** Repeat steps 1 and 2 of part **a**.
- d** Repeat steps 1 and 2 of part **a**.
- e** Repeat steps 1 and 2 of part **a**.

WRITE

- a** The tide pattern occurs daily; this event seems *certain*.
The probability of this event occurring is equal to 1.
- b** Total agreement among many people on every subject over a long time is virtually *impossible*.
The probability of this event occurring is equal to 0.
- c** When tossing a coin there are two equally likely outcomes, a Head or a Tail.
The probability of this event occurring is equal to $\frac{1}{2}$.
- d** When rolling a die there are six equally likely outcomes: 1, 2, 3, 4, 5, 6.
The probability of this event occurring is equal to $\frac{1}{6}$.
- e** There are 15 chances out of 20 of winning. The probability of this event occurring is equal to $\frac{15}{20}$, which when simplified is equal to $\frac{3}{4}$.

Interactivity: Probability scale (int-3824)

Digital doc: SkillsHEET Understanding chance words (doc-6969)

Exercise 12.2 Probability scale

Individual pathways

PRACTISE

Questions:
1–6, 8

CONSOLIDATE

Questions:
1–8

MASTER

Questions:
1–11

Individual pathway interactivity: int-4457

learnon ONLINE ONLY

To answer questions online and to receive **immediate feedback** and **fully worked solutions** for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Understanding and fluency

1. **WE1** Describe the probability of each of the following events occurring using a term from the list below.

Impossible

Less than even chance

Very likely

Highly unlikely

Even chance

Highly likely

Very unlikely

Better than even chance

Certain

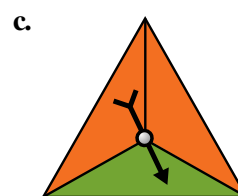
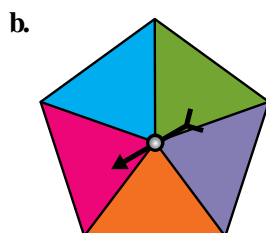
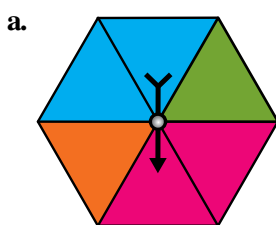
- a. The sun will set today.
 - b. Every student in this class will score 100% in the next Mathematics exam.
 - c. It will rain tomorrow.
 - d. Your shoelace will break next time you tie your shoes.
 - e. Commercial TV stations will reduce time devoted to ads.
 - f. A comet will collide with Earth this year.
 - g. The year 2020 will be a leap year.
 - h. You roll a standard die and an 8 appears uppermost.
 - i. A tossed coin lands on its edge.
 - j. World records will be broken at the next Olympics.
 - k. You roll a standard die and an odd number appears uppermost.
 - l. You draw the queen of hearts from a standard deck of playing cards.
 - m. You draw a heart or diamond card from a standard deck of playing cards.
 - n. One of your 11 tickets in a 20-ticket raffle will win.
 - o. A red marble will be drawn from a bag containing 1 white marble and 9 red marbles.
 - p. A red marble will be drawn from a bag containing 1 red and 9 white marbles.
2. Write two examples of events that are:
- a. impossible
 - b. certain
 - c. highly likely
 - d. highly unlikely
 - e. equally likely (even chance).



3. **WE2** Assign a fraction to represent the estimated probability of each of the following events occurring.
- A Head appears uppermost when a coin is tossed.
 - You draw a red marble from a bag containing 1 white and 9 red marbles.
 - A standard die shows a 7 when rolled.
 - You draw a yellow disk from a bag containing 8 yellow disks.
 - The next baby in a family will be a boy.
 - A standard die will show a 1 or a 2 when rolled.
 - You draw the queen of hearts from a standard deck of playing cards.
 - One of your 11 tickets in a 20-ticket raffle will win.
 - A standard die will show a number less than or equal to 5 when rolled.
 - You draw an ace from a standard deck of playing cards.
 - You draw a king or queen card from a standard deck of playing cards.
 - You spin a seven-sided spinner and obtain an odd number.
 - Heads or Tails will show uppermost when a coin is tossed.
4. **MC** The word that best describes the probability for a standard die to show a prime number is:
- A.** impossible **B.** very unlikely **C.** even chance
D. very likely **E.** certain
5. **MC** The probability of Darwin experiencing a white Christmas this year is closest to:
- A.** 1 **B.** 0.75 **C.** 0.5 **D.** 0.25 **E.** 0

Communicating, reasoning and problem solving

6. The letters of the word MATHEMATICS are each written on a small piece of card and placed in a bag. If one card is selected from the bag, what is the probability that it is:
- a vowel
 - a consonant
 - the letter M
 - the letter C?
7. Answer the following for each of the spinners shown.
- Is there an equal chance of landing on each colour? Explain.
 - List all the possible outcomes.
 - Find the probability of each outcome.



8. All the jelly beans in the photograph are placed in a bag for a simple probability experiment.
- Which colour jelly bean is most likely to be randomly selected from the bag? Explain.
 - Which colour jelly bean is least likely to be randomly selected from the bag? Explain.
 - Find the probability of randomly selecting each coloured jelly bean from the bag.



9. Draw a spinner with the following probabilities.

a. $P(\text{blue}) = \frac{1}{3}$ and $P(\text{white}) = \frac{2}{3}$

b. $P(\text{blue}) = \frac{1}{2}$, $P(\text{white}) = \frac{1}{4}$, $P(\text{green}) = \frac{1}{8}$ and $P(\text{red}) = \frac{1}{8}$

c. $P(\text{blue}) = 0.75$ and $P(\text{white}) = 0.25$

10. Can you think of some events that would have a probability of 0, 1 or $\frac{1}{2}$ of occurring?

11. Four different types of coloured card are enclosed in a bag: red, blue, green and yellow. There is a 40% chance of drawing a blue card and the probability of a red card is 0.25. There are 4 yellow cards. If there is a total of 20 cards in the bag, what is the probability of drawing a green card? How many green cards are in the bag?



12.3 Sample spaces and theoretical probability

12.3.1 Words associated with probability

- **Chance experiments** are performed to provide data, which can then be used to forecast the outcome of future similar events.
- An **outcome** is a possible result of a chance experiment.
- A **favourable outcome** is one that we are looking for.
- An **event** is either one or a collection of favourable outcomes.
- The **theoretical probability** (or empirical probability) of a particular event occurring is denoted by the symbol $P(\text{event})$.
- The **sample space**, S , is the set of all the possible outcomes.

$$P(\text{event}) = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$$

WORKED EXAMPLE 3

A standard 6-sided die is rolled.

a List the sample space for this chance experiment.

b Determine the probability of obtaining the following appearing uppermost:

- i 4 ii an odd number iii 5 or less.

THINK

a Write all the possible outcomes for the given chance experiment.

b i 1 Write the total number of possible outcomes.

2 Write the number of favourable outcomes.

Note: The favourable outcome is 4.

3 Write the rule for probability.

4 Substitute the known values into the rule and evaluate.

WRITE

a $S = \{1, 2, 3, 4, 5, 6\}$

b i Total number of outcomes = 6

Number of favourable outcomes = 1

$$P(\text{event}) = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$$

$$P(4) = \frac{1}{6}$$

5 Answer the question.

The probability of 4 appearing uppermost is $\frac{1}{6}$.

ii 1 Write the total number of possible outcomes.

ii Total number of outcomes = 6

2 Write the number of favourable outcomes. The favourable outcomes are 1, 3, 5.

Number of favourable outcomes = 3

3 Write the rule for probability.

$$P(\text{event}) = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$$

4 Substitute the known values into the rule.

$$P(\text{odd number}) = \frac{3}{6}$$

5 Evaluate and simplify.

$$= \frac{1}{2}$$

6 Answer the question.

The probability of an odd number appearing uppermost is $\frac{1}{2}$.

iii Repeat steps 1 to 5 of part b i.

Note: 5 or less means the favourable outcomes are 1, 2, 3, 4, 5. Therefore, the number of favourable outcomes is 5.

iii Total number of outcomes = 6

Number of favourable outcomes = 5

$$P(5 \text{ or less}) = \frac{5}{6}$$

The probability of obtaining 5 or less is $\frac{5}{6}$.

WORKED EXAMPLE 4

A card is drawn at random from a standard well-shuffled pack.

Find the probability of drawing:

a a club

b a king or an ace

c not a spade.

Express each answer as a fraction and as a percentage.

THINK

WRITE

a 1 Write the total number of outcomes in the sample space. There are 52 cards in a pack.

a Total number of outcomes = 52

2 Write the number of favourable outcomes. There are 13 cards in each suit.

Number of favourable outcomes = 13

3 Write the rule for probability.

$$P(\text{event}) = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$$

4 Substitute the known values into the rule and simplify.

$$\begin{aligned} P(\text{a club}) &= \frac{13}{52} \\ &= \frac{1}{4} \end{aligned}$$

5 Convert the fraction to a percentage; that is, multiply by 100%.

$$\begin{aligned} \text{Percentage} &= \frac{1}{4} \times 100\% \\ &= \frac{100}{4}\% \\ &= 25\% \end{aligned}$$

6 Answer the question.

The probability of drawing a club is $\frac{1}{4}$ or 25%.

b 1 Write the total number of outcomes in the sample space.

b Total number of outcomes = 52

2 Write the number of favourable outcomes.
There are 4 kings and 4 aces.

Number of favourable outcomes = 8

3 Write the rule for probability.

$$P(\text{event}) = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$$

4 Substitute the known values into the rule and simplify.

$$\begin{aligned} P(\text{a king or an ace}) &= \frac{8}{52} \\ &= \frac{2}{13} \end{aligned}$$

5 Convert the fraction to a percentage, rounded to one decimal place.

$$\begin{aligned} \text{Percentage} &= \frac{2}{13} \times 100\% \\ &= \frac{200}{13}\% \\ &\approx 15.4\% \end{aligned}$$

6 Answer the question.

The probability of drawing a king or an ace is $\frac{2}{13}$ or approximately 15.4%.

c Repeat steps 1 to 6 of part **a**.

Note: Not a spade means clubs, hearts or diamonds. Therefore, the number of favourable outcomes is 39.

c Total number of outcomes = 52

Number of favourable outcomes = 39

$$P(\text{not a spade}) = \frac{39}{52} = \frac{3}{4}$$

$$\text{Percentage} = \frac{3}{4} \times 100\% = \frac{300}{4} = 75\%$$

The probability of drawing a card that is not a spade is $\frac{3}{4}$ or 75%.

WORKED EXAMPLE 5

A shopping centre car park has spaces for 10 buses, 300 cars and 20 motorbikes. If all vehicles have an equal chance of leaving at any time, find the probability that the next vehicle to leave will be:

a a motorbike

b a bus or a car

c not a car.



THINK

a 1 Write the total number of outcomes in the sample space. There are 330 vehicles.

WRITE

a Total number of outcomes = 330

2 Write the number of favourable outcomes.
There are 20 motorbikes. Therefore, the number of favourable outcomes is 20.

Number of favourable outcomes = 20

3 Write the rule for probability.

$$P(\text{event}) = \frac{\text{number of favourable outcomes}}{\text{number of possible outcomes}}$$

4 Substitute the known values into the rule and simplify.

$$\begin{aligned}P(\text{a motorbike}) &= \frac{20}{330} \\&= \frac{2}{33}\end{aligned}$$

5 Answer the question.

The probability of a motorbike next leaving the car park is $\frac{2}{33}$.

b 1 Write the total number of outcomes in the sample space.

b Total number of outcomes = 330

2 Write the number of favourable outcomes. There are 10 buses and 300 cars. Therefore, the number of favourable outcomes is 310.

Number of favourable outcomes = 310

3 Write the rule for probability.

$$P(\text{event}) = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$$

4 Substitute the known values into the rule and simplify.

$$\begin{aligned}P(\text{a bus or a car}) &= \frac{310}{330} \\&= \frac{31}{33}\end{aligned}$$

5 Answer the question.

The probability of a bus or car next leaving the car park is $\frac{31}{33}$.

c Repeat steps 1 to 5 of part **a**.

Note: There are 10 buses and 20 motorbikes. Therefore, the number of favourable outcomes is 30.

c Total number of outcomes = 330

Number of favourable outcomes = 30

$$\begin{aligned}P(\text{not a car}) &= \frac{30}{330} \\&= \frac{1}{11}\end{aligned}$$

The probability of a vehicle that is not a bus next leaving the car park is $\frac{1}{11}$.

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Interactivity: Sample spaces and theoretical probability (int-3826)

Digital doc: SkillsHEET Converting a fraction into a decimal (doc-6972)

Digital doc: SkillsHEET Converting a fraction into a percentage (doc-6973)

Digital doc: SkillsHEET Multiplying a fraction by a whole number (doc-6974)

Exercise 12.3 Sample spaces and theoretical probability

Individual pathways

PRACTISE

Questions:
1–6, 8, 9, 14, 17, 21, 22

CONSOLIDATE

Questions:
1–5, 7, 9–17, 21–23

MASTER

Questions:
1a, e, h, k, 2–4, 5a, b, d, h, 6–8,
12–23

   Individual pathway interactivity: int-4459

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To answer questions online and to receive **immediate feedback** and **fully worked solutions** for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Understanding and fluency

- List the sample spaces for these chance experiments:
 - tossing a coin
 - selecting a vowel from the word ASTRONAUT
 - selecting a day of the week to go to the movies
 - drawing a marble from a bag containing 3 reds, 2 whites and 1 black
 - rolling a standard 6-sided die
 - drawing a picture card from a standard pack of playing cards
 - spinning an 8-sector circular spinner numbered from 1 to 8
 - selecting even numbers from the first 20 counting numbers
 - selecting a piece of fruit from a bowl containing 2 apples, 4 pears, 4 oranges and 4 bananas
 - selecting a magazine from a rack containing 3 *Dolly*, 2 *Girlfriend*, 1 *Smash Hits* and 2 *Mathsmag* magazines
 - selecting the correct answer from the options A, B, C, D, E on a multiple-choice test
 - winning a medal at the Olympic games.
- WE3** A standard 6-sided die is rolled.
 - List the sample space for this chance experiment.
 - Determine the probability of obtaining the following appearing uppermost:
 - a 6
 - an even number
 - at most, 4
 - a 1 or a 2
 - a prime number
 - a number greater than 4
 - a 7
 - a number that is a factor of 60.
- WE4** A card is drawn at random from a standard well-shuffled pack. Find the probability of drawing:

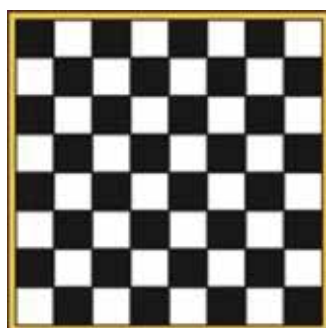
a. the king of spades	b. a 10	c. a jack or a queen	d. a club
e. a red card	f. an 8 or a diamond	g. an ace.	

Express each answer as a fraction and as a percentage, correct to 1 decimal place.
- WE5** A shopping centre car park has spaces for 8 buses, 160 cars and 12 motorbikes. If all vehicles have an equal chance of leaving at any time, find the probability that the next vehicle to leave will be:

a. a bus	b. a car
c. a motorbike or a bus	d. not a car.
- A bag contains 3 red, 2 black, 1 pink, 2 yellow, 3 green and 3 blue marbles. If a marble is drawn at random, calculate the chance that it is:

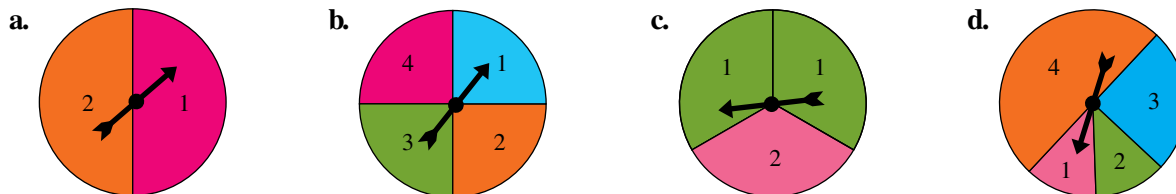
a. red	b. black	c. yellow
d. red or black	e. not blue	f. red or black or green
g. white	h. not pink.	
- A beetle drops onto *one* square of a chessboard. What are its chances of landing on a square that is:

a. black	b. white
c. neither black nor white	d. either black or white?



7. What chance is there that the next person you meet has his/her birthday:
- next Monday
 - sometime next week
 - in September
 - one particular day next year?

8. For each of the following spinners:



- state whether each of the outcomes are equally likely. Explain your answer.
 - find the probability of the pointer stopping on 1.
9. Hanna flipped a coin 5 times and each time a Tail showed. What are the chances of Tails showing on the *sixth* toss?
10. a. Design a circular spinner coloured pink, white, blue, orange and green so that each colour is equally likely to result from any trial.
b. What will be the angle between each sector in the spinner?
11. a. Design a circular spinner with the numerals 1, 2 and 3 so that 3 is twice as likely to occur as either 2 or 1 in any trial.
b. What will be the size of the angles in each sector at the centre of the spinner?
12. a. Design a circular spinner labelled A, B, C and D so that $P(A) = \frac{1}{4}$, $P(B) = \frac{1}{3}$, $P(C) = \frac{1}{6}$, $P(D) = \frac{1}{4}$.
b. What will be the size of the angles between each sector in the spinner?
13. a. What is the total of all the probabilities in question 12?
b. What is the angle sum of the sectors in question 12?
14. a. List all the outcomes for tossing a coin once, together with their individual probabilities.
b. Find the total.
15. a. List all the outcomes for tossing a coin twice, together with their individual probabilities.
b. Find the total.
16. a. List the probabilities for the elements of the sample space for rolling a 6-faced die.
b. Find the total of the probabilities.
c. Do the totals for questions 14b, 15b and 16b agree with that in 13a?
d. What conclusion can you draw?
17. **MC** If a circular spinner has 3 sectors, A, B and C, such that $P(A) = \frac{1}{2}$ and $P(B) = \frac{1}{3}$, then $P(C)$ must be:
- $\frac{1}{4}$
 - $\frac{2}{5}$
 - $\frac{1}{6}$
 - $\frac{5}{6}$
 - none of these
18. **MC** For an octagonal spinner with equal sectors, numbered from 1 to 8, the chance of getting a number between (but not including) 5 and 8 is:
- $\frac{1}{4}$
 - $\frac{1}{2}$
 - $\frac{3}{8}$
 - $\frac{5}{8}$
 - none of these
19. a. What is the sample space for rolling a standard 6-sided die?
b. How many elements are in the sample space for rolling 2 standard 6-sided dice (think of the dice having different colours)? *Hint:* The answer is not 12.
c. Complete the following sample space for rolling two dice.
Note: Each colour in the table corresponds to the colour of the die.

		Die 2					
		1	2	3	4	5	6
Die 1	1	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
	2						
	3						
	4						
	5						
	6						

d. Using the sample space, complete the following table.

Sum	2	3	4	5	6	7	8	9	10	11	12
Probability											

e. Do you notice a pattern involving the probabilities in the table? Explain your answer.

The pattern observed in part e relates to symmetry. Investigate the symmetry property of tossing three coins.

f. List the sample space for tossing three coins.

g. Using the sample space obtained in part f, complete the following table.

Number of Heads	0	1	2	3
Probability				

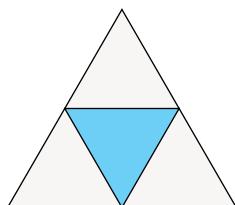
h. From the table above, which event is the probability of tossing three Tails the same as?

i. By symmetry, which event has the same probability as tossing Heads twice and Tails once?

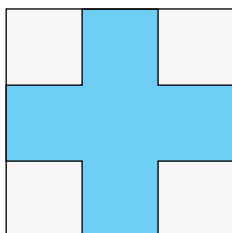
Communicating, reasoning and problem solving

20. The targets shown are an equilateral triangle, a square and a circle with coloured regions that are also formed from equilateral triangles, squares and circles. If a randomly thrown dart hits each target, find the probability that the dart hits each target's coloured region.

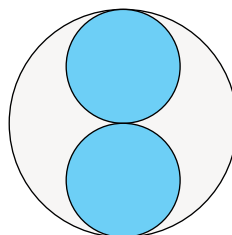
a.



b.



c.



21. A fair coin is flipped 3 times. Calculate the probability of obtaining:

a. at least two Heads or at least two Tails

b. exactly two Tails.

22. Two dice are rolled and the product of the two numbers is found. Calculate the probability that the product of the two numbers is:

a. an odd number

c. more than 1

b. a prime number

d. at most 36.

23. A fair die is rolled and a fair coin flipped. Calculate the probability of obtaining:

a. an even number and a Head

c. a prime number from the die

b. a Tail from the coin

d. a number less than 5 and a Head.

12.4 Complementary events

12.4.1 Considering two outcomes

- The sum of all of the probabilities of a single-step chance experiment is always 1.
- In some situations, there are only two possible outcomes.
- When there is nothing in common between these events and together they form the sample space, they are called **complementary events**.
- For example, when tossing a coin, there are only two possible outcomes, a Head or a Tail. They are complementary events.
- If an event is denoted by the letter A , its complement is denoted by the letter A' , and the sum of their probabilities is equal to 1.
- If we know the probability of one event, subtracting this probability from 1 will give us the probability of the complementary event.
- If events A and A' are complementary, then:

$$P(A) + P(A') = 1 \text{ and} \\ P(A') = 1 - P(A) \text{ or } P(A) = 1 - P(A').$$

WORKED EXAMPLE 6

Find the complement of each of the following events:

- a** selecting a red card from a standard deck
- b** rolling two dice and getting a total greater than 9
- c** selecting a red marble from a bag containing 50 marbles.

THINK

- a** Selecting a black card will complete the sample space for this chance experiment.
- b** When rolling two dice, rolling a total less than 10 will complete the sample space.
- c** Selecting a marble that is not red is the only way to define the rest of the sample space for this chance experiment.

WRITE

- a** The complement of selecting a red card is selecting a black card.
- b** The complement of rolling a total greater than 9 is rolling a total less than 10.
- c** The complement of selecting a red marble in this chance experiment is not selecting a red marble.

WORKED EXAMPLE 7

If a card is drawn from a pack of 52 cards, what is the probability that the card is not a diamond?

THINK

- 1** Determine the probability of drawing a diamond.

WRITE

Number of diamonds = 13

Number of cards = 52

$$P(\text{event}) = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$$

$$P(\text{diamond}) = \frac{13}{52} \\ = \frac{1}{4}$$

- 2** Write down the rule for obtaining the complement of drawing a diamond; that is, not drawing a diamond.

$$P(A') = 1 - P(A)$$

$$P(\text{not a diamond}) = 1 - P(\text{diamond})$$

- 3 Substitute the known values into the given rule and simplify.

$$\begin{aligned} &= 1 - \frac{1}{4} \\ &= \frac{3}{4} \end{aligned}$$

- 4 Answer the question.

The probability of drawing a card that is not a diamond is $\frac{3}{4}$.

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Interactivity: Complementary events (int-3827)

Exercise 12.4 Complementary events

Individual pathways

PRACTISE

Questions:
1–11

CONSOLIDATE

Questions:
1–13

MASTER

Questions:
1–15

Individual pathway interactivity: int-4460

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Understanding and fluency

- WE6** For each of the following, state the complementary event.
 - From a bag of numbered marbles selecting an even number
 - From the letters of the alphabet selecting a vowel
 - Tossing a coin and it landing Heads
 - Rolling a die and getting a number less than 3
 - Rolling two dice and getting a total less than 12
 - Selecting a diamond from a deck of cards
 - Selecting an E from the letters of the alphabet
 - Selecting a blue marble from a bag of marbles
- MC** *Note:* There may be more than one correct answer.

A student is to be chosen from a class of 30 students. Each student in the class is either 14 or 15 years old. Which of the following represent complementary events?

 - Selecting a 14-year-old boy and selecting a 14-year-old girl
 - Selecting a boy and selecting a girl
 - Selecting a 14-year-old and selecting a 15-year-old
 - Selecting a 14-year-old boy and selecting a 15-year-old girl
 - All of the above
- For each of the following, state whether the pair of events are complementary or not. Explain your answer.
 - Having Weet Bix or Corn Flakes for breakfast.
 - Walking to your friend's house or riding your bike to your friend's house.
 - Watching TV at night or listening to the radio.
 - Passing your Mathematics test or failing your Mathematics test.
 - Rolling a number less than 4 on a die or rolling a total greater than 4.

4. **WE7** If a card is drawn from a pack of 52 cards, what is the probability that the card is not a queen?
5. **MC** The statement that does *not* involve complementary events is:
- A. travelling to school by bus and travelling to school by car
 - B. drawing a red card from a pack of 52 playing cards and drawing a black card from 52 playing cards
 - C. drawing a vowel from cards representing the 26 letters of the alphabet or drawing a consonant
 - D. obtaining an even number on a six-sided die or obtaining an odd number on a die
 - E. All of the above
6. When a six-sided die is rolled 3 times, the probability of getting 3 sixes is $\frac{1}{216}$. What is the probability of not getting 3 sixes?
7. Eight athletes compete in a 100 m race. The probability that the athlete in lane 1 will win is $\frac{1}{5}$.

What is the probability that one of the other athletes wins? (Assume that there are no dead heats.)

8. A pencil case has 4 red pens, 3 blue pens and 5 black pens. If a pen is drawn randomly from the pencil case, find:
- a. $P(\text{drawing a blue pen})$
 - b. $P(\text{not drawing a blue pen})$
 - c. $P(\text{drawing a red or a black pen})$
 - d. $P(\text{drawing neither a red nor a black pen})$.
9. Holty is tossing two coins. He claims that getting two Heads and getting zero Heads are complementary events. Is he right? Explain your answer.
10. Seventy Year 9 students were surveyed. Their ages ranged from 13 years to 15 years, as shown in the following table.



Age	13	14	15	Total
Boys	10	20	9	39
Girls	7	15	9	31
Total	17	35	18	70

A student from the group is selected at random. Find:

- a. $P(\text{selecting a student of the age of 13 years})$
 - b. $P(\text{not selecting a student of the age of 13 years})$
 - c. $P(\text{selecting a 15-year-old boy})$
 - d. $P(\text{not selecting a 15-year-old boy})$.
- Communicating, reasoning and problem solving**
11. In a bag there are 4 red cubes and 7 green cubes. If Clementine picks a cube at random, what is the probability that it is not:
- a. red or green
 - b. red
 - c. green?
12. In a bag there are 4 red cubes and 7 green cubes. Clementine picks a cube at random, looks at it and notes that it is red. Without putting it back, she picks a second cube from the bag. What is the probability that it is not green? Show your working.
13. In a hand of n cards there are r red cards. I choose a card at random. What is the probability that it is:
- a. black
 - b. not black?
14. There are three cyclists in a road race. Cyclist A is twice as likely to win as cyclist B and three times as likely to win as cyclist C. Find the probability that:
- a. cyclist B wins
 - b. cyclist A does not win.
15. Explain why the probability of an event and the probability of the complement of the event always sum to 1.

12.5 Mutually exclusive and compound events

12.5.1 Mutually exclusive events

- Two events are **mutually exclusive** if one event happening excludes the other from happening.
- These events may or may not cover all the possible outcomes. For example, when selecting a card from a deck of cards selecting a heart excludes the possibility that the card is a spade, but together selecting a heart and selecting a spade doesn't cover all possible outcomes, as the card may also be a club or a diamond.

WORKED EXAMPLE 8

Determine whether the following events are mutually exclusive or not.

- a Rolling an odd number and rolling a 3 on a die.
- b Landing on Heads and landing on Tails when tossing a coin.
- c Selecting a jack and selecting a diamond from a pack of cards.

THINK

- a Are there any outcomes which satisfy both of these events?
- b Are there any outcomes which satisfy both of these events?
- c Are there any outcomes which satisfy both of these events?

WRITE/DRAW

- a Rolling a 3 satisfies both of these events; therefore they are not mutually exclusive.
- b There are no outcomes which satisfy both of these events; therefore they are mutually exclusive.
- c Selecting the jack of diamonds satisfies both of these events; therefore they are not mutually exclusive.

12.5.2 Compound events

- A **compound event** is an event that is made up of more than one individual event.
- We can use the language 'at least', 'at most', 'not' and 'and' to describe compound events.
- For example:
 - 'Rolling a 5 or 6 on a six-sided die' can be described as 'Rolling *at least* a 5 on a six-sided die'.
 - 'Rolling a 1, 2 or 3 on a six-sided die' can be described as 'Rolling *at most* a 3 on a six-sided die'.
 - 'Landing on Heads when tossing a coin' can be described as 'Not landing on Tails when tossing a coin.'
 - 'Selecting a red queen from a pack of cards' can be described as 'Selecting a queen *and* a red card from a pack of cards.'
- *Note:* When using the word *not* to describe a compound event, the event that has been described and the event that is *not* occurring must together cover the entire sample space.
- For example, if the event is selecting an odd number from the numbers 1–20, then the event *not* occurring is selecting an even number from the numbers 1–20. Together these events cover the entire sample space.



WORKED EXAMPLE 9

Describe the following using the language of compound events.

- a Rolling a 1 or 2 on a six-sided die
- b Selecting a black ace from a pack of cards
- c Rolling an even number on a six-sided die
- d Rolling a 8, 9 or 10 on a ten-sided die

THINK

- a 1 and 2 are the two lowest numbers on a six-sided die, so this event will use the words *at most*.
- b We are selecting a card which is both an ace and a black card, so this event will use the word *and*.
- c Even numbers and odd numbers make up the whole sample space when rolling a six-sided die, so this event will use the word *not*.
- d 8, 9 and 10 are the three highest numbers on a ten-sided die, so this event will use the words *at least*.

WRITE/DRAW

- a Rolling at most a 2 on a six-sided die
- b Selecting an ace and a black card from a pack of cards
- c Not rolling an odd number on a six-sided die.
- d Rolling at least an 8 on a ten-sided die.

ACTIVITY: IMPROVING YOUR INTERNET SEARCHES

The words 'not' and 'and' can be used to help refine internet searches. Conduct research into the effect of using both of these words when using internet search engines and present a report on your findings.

12.5.3 Inclusive and exclusive events

- When describing a compound event the event can be further categorised as either **inclusive** or **exclusive**.
- An inclusive event is an event where an outcome can satisfy both of the qualities being described.
- For example, if an event is 'selecting a three or selecting a diamond from a pack of cards' selecting the three of diamonds satisfies both qualities, so this is an inclusive event.
- An exclusive event is an event where an outcome cannot satisfy both of the qualities being described.
- For example, if an event is 'rolling a 3 or rolling a 4 on a six-sided die' there is no outcome that can satisfy both qualities, so this is an exclusive event.
- Sometimes we need to use the language 'both' or 'not both' to determine whether an event is inclusive or exclusive.
- For example, if the event is 'selecting a three or selecting a diamond from a pack of cards but not both', this indicates that the event must be exclusive.
- An inclusive event will always be made up of non-mutually exclusive individual events.

WORKED EXAMPLE 10

Determine whether the following compound events are inclusive or exclusive.

- a Rolling a multiple of 3 or rolling an even number on a six-sided die.
- b Rolling a 5 or rolling a number less than 3 on a six-sided die.
- c Rolling a multiple of 5 or rolling an even number on a ten-sided die, but not both.

THINK

- a** **1** Write out all of the outcomes of each individual event of the compound event. Highlight any numbers that appear in both lists.
- 2** If there are any outcomes that appear in both individual events then the compound event is inclusive.
If there are no outcomes that appear in both individual events then the compound event is exclusive.
- b** **1** Write out all of the outcomes of each individual event of the compound event. Highlight any numbers that appear in both lists.
- 2** If there are any outcomes that appear in both individual events then the compound event is inclusive.
If there are no outcomes that appear in both individual events then the compound event is exclusive.
- c** **1** Write out all of the outcomes of each individual event of the compound event. Note that as the compound event states that both individual events can't be satisfied, we can cross out any outcomes that appear in both lists.
- 2** If there are any outcomes that appear in both individual events then the compound event is inclusive.
If there are no outcomes that appear in both individual events then the compound event is exclusive.

WRITE

- a** **1** Rolling a multiple of 3: 3, 6
Rolling an even number: 2, 4, 6
- 2** This compound event is inclusive.
- b** **1** Rolling a 5: 5
Rolling a number less than 3: 1, 2
- 2** This compound event is exclusive.
- c** **1** Rolling a multiple of 5: 5, 10
Rolling an even number: 2, 4, 6, 8, 10
- 2** This compound event is exclusive.

12.5.4 Solving probability problems

- As mutually exclusive events do not cover any of the same outcomes, we can calculate the probability of an exclusive event comprised of mutually exclusive events by adding the probabilities of each individual event. That is:

if $P(A)$ and $P(B)$ are mutually exclusive, then $P(A \text{ or } B) = P(A) + P(B)$.

- If we added together the probabilities of the individual events of an inclusive event then we would not be calculating the probability of the inclusive event, because the outcomes that appear in both of the individual events would be added twice.
- Therefore to calculate the probability of an inclusive event we need to subtract the probability of the events that occur in both of the individual events from the sum of the individual events. That is:

if $P(A)$ and $P(B)$ are not mutually exclusive, then $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$.

WORKED EXAMPLE 11

Determine the probability of the following compound events.

a Rolling a 4 or rolling an odd number on a six-sided die.

b Selecting a heart or a jack from a pack of cards or both.

THINK

a 1 Determine if the compound event is inclusive or exclusive by writing out the outcomes of each individual event.

2 Calculate the probabilities of the individual events

3 As the individual events are mutually exclusive, we can calculate the probabilities of the compound event by adding their probabilities.

b 1 Determine if the compound event is inclusive or exclusive by writing out the outcomes of each individual event.

2 Calculate the probabilities of the individual events

3 As the events are not mutually exclusive, we also need to calculate the probability that both individual events occur.

WRITE

a 1 Rolling a 4: 4

Rolling an odd number: 1, 3, 5

There are no outcomes that appear in both individual events, so the compound event is exclusive.

$$\begin{aligned} \mathbf{2} \quad P(\text{rolling a 4}) &= \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}} \\ &= \frac{1}{6} \end{aligned}$$

$$\begin{aligned} P(\text{rolling an odd number}) &= \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}} \\ &= \frac{3}{6} \end{aligned}$$

$$\begin{aligned} \mathbf{3} \quad P(\text{rolling an odd number or rolling a 4}) &= \frac{1}{6} + \frac{3}{6} \\ &= \frac{4}{6} \\ &= \frac{2}{3} \end{aligned}$$

b 1 Selecting a heart:

2♥, 3♥, 4♥, 5♥, 6♥, 7♥, 8♥, 9♥, 10♥, J♥, Q♥, K♥, A♥

Selecting a jack: J♣, J♦, J♥, J♠

There is one outcome that appears in both individual events (J♥), so the compound event is inclusive.

$$\begin{aligned} \mathbf{2} \quad P(\text{selecting a heart}) &= \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}} \\ &= \frac{13}{52} \end{aligned}$$

$$\begin{aligned} P(\text{selecting a jack}) &= \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}} \\ &= \frac{4}{52} \end{aligned}$$

3 P(selecting a heart and selecting a jack)

$$\begin{aligned} &= \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}} \\ &= \frac{1}{52} \end{aligned}$$

- 4 Calculate the probability of the compound event by adding the individual probabilities and subtracting the probability that they both occur.

$$\begin{aligned} 4 \quad P(\text{selecting a heart or selecting a jack}) &= \frac{13}{52} + \frac{4}{52} - \frac{1}{52} \\ &= \frac{16}{52} \\ &= \frac{4}{13} \end{aligned}$$

ACTIVITY: CREATING YOUR OWN COMPOUND EVENT PROBLEM

Using the skills that you have learnt in this subtopic, pose a problem that involves the use of some of these terms. Test your problem on a classmate and check that you both calculate the same answer.

Exercise 12.5 Mutually exclusive and compound events

Individual pathways

PRACTISE

Questions:
1–7, 9, 11

CONSOLIDATE

Questions:
1–7, 9–11, 13, 14

MASTER

Questions:
1–17

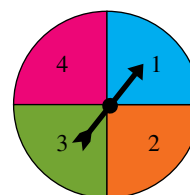
Individual pathway interactivity: int-7049

learnON ONLINE ONLY

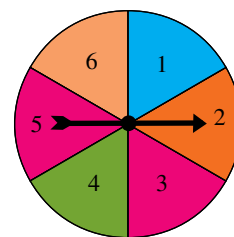
To answer questions online and to receive **immediate feedback** and **fully worked solutions** for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Understanding and fluency

- WEB** Determine whether the following events are mutually exclusive or not.
 - Rolling an even number and rolling a multiple of 5 on a twelve-sided die.
 - Selecting a picture card and selecting a heart from a deck of cards.
 - Landing on a blue section and landing on a 4 when spinning the following spinner.
- MC** Which of the following is a mutually exclusive event to selecting a black card from a pack of cards?
 - Selecting the king of spades
 - Selecting a 2, 3, 4 or 5
 - Selecting a queen
 - Selecting the 5 of diamonds
 - Selecting the queen of clubs
- WE9** Describe the following using the language of compound events.
 - Rolling a 1, 2, 3 or 4 on a ten-sided die.
 - Selecting a club from a pack of cards.
 - Rolling a 9 or 10 on a ten-sided die.
- MC** Which of the following does not represent a compound event?
 - Selecting a heart or club from a pack of cards
 - Rolling at least a 6 on a six-sided die
 - Rolling at most a 2 on a twelve-sided die
 - Not selecting a diamond from a pack of cards
 - Selecting a black card and a king from a pack of cards



5. **WE10** Determine whether the following compound events are inclusive or exclusive.
- Rolling a multiple of a 3 or a 5 on a six-sided die
 - Selecting a heart or a 7 from a pack of cards
 - Landing on a red section or a 3 but not both when spinning the following spinner



6. **WE11** Determine the probability of the following compound events.

- Selecting a queen or selecting a king from a pack of cards
 - Selecting a red card or a queen or both from a pack of cards
7. A ten sided-die is rolled. Calculate the following.
- P(rolling a 3 or a 6)
 - P(rolling at least a 5)
 - P(rolling at most a 5)
 - P(rolling a multiple of 2 or a multiple of 3 or both)
8. Keira was selecting a marble from a bag containing 5 red marbles, 9 blue marbles and 7 green marbles. Calculate P(selecting a red marble or a blue marble).
9. Jack was playing at the local carnival with the chance to win a cuddly toy.

If there was a 0.2 chance that he would win a toy lion, and a $\frac{1}{8}$ chance he would win a toy bear, what is the probability that he would win one of these prizes? Give your answer as a decimal.



10. The local weather forecaster predicted that there was a 50% chance of it being sunny tomorrow, a 20% chance of it being rainy and a 5% chance of it being both sunny and rainy.
- What was the chance of there being either sun or rain or both?
11. Will likes to play both soccer and basketball. On any given evening there is a 30% chance that he will play soccer and a $\frac{1}{4}$ chance he will play basketball. Will never plays both soccer and basketball on the same evening.
- Is the compound event 'Will plays soccer or basketball' inclusive or exclusive?
 - Calculate P(Will plays soccer or basketball). Give your answer as a percentage.

Communicating, reasoning and problem solving

12. In your own words explain mutually exclusive events.
13. Are the events rolling a 1 and rolling a 5 mutually exclusive events? Explain your answer.
14. On a given Saturday the probability of Sasha going to her friend Jess's house was 65% and the probability of Sasha going to the swimming pool was 40%.
- Explain why the probability of Sasha either going to Jess's house or the swimming pool is not 105%.
 - From the given information are we able to determine the probability of Sasha either going to Jess's house or the swimming pool? Explain your answer.
15. The age and gender of 90 teenagers in NSW taking their driving tests have been recorded in the following table.

Age	17	18	19	Total
Boys	27	9	8	44
Girls	26	13	7	46
Total	53	22	15	90

A teenager from the group is selected at random. Find:

- P(selecting a 17 year-old boy)
- P(selecting a 17 or 18 year-old girl)
- P(selecting a boy or a girl)
- P(selecting a boy and a 19-year old)
- P(selecting an 18-year or a girl or both).

16. Explain why the language ‘both’ or ‘not both’ can help to determine whether a compound event is inclusive or exclusive.
17. Does the sum of mutually exclusive events always total 1? Explain your answer.

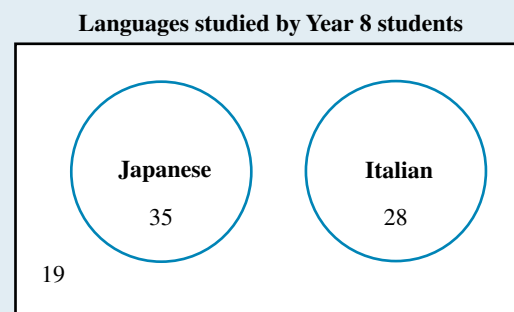
12.6 Venn diagrams and two-way tables

12.6.1 Exploring probability using Venn diagrams

- An **attribute** is a feature or characteristic of someone or something.
- A **Venn diagram** is made up of a rectangle and one or more circles. A Venn diagram is used to show the relationships between attributes.
- The circles in a Venn diagram indicate the different attributes, and the numbers in a Venn diagram indicate the number that have that attribute.
- The circles in a Venn diagram can overlap, with an overlapping region indicating the number that have both attributes.
- If two circles do not overlap then those attributes are mutually exclusive.

WORKED EXAMPLE 12

The following Venn diagram represents the languages studied by Year 8 students at a school.



- Are there any students who studied both Japanese and Italian?
- How many students were there overall?
- How many students studied Japanese?
- How many students studied Italian?
- How many students studied neither Japanese nor Italian?

THINK

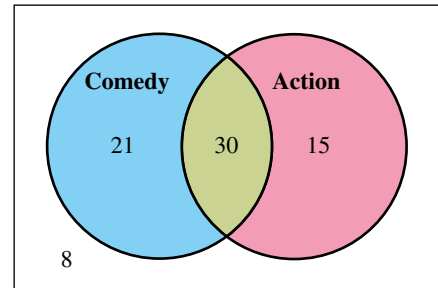
- The circles representing Japanese and Italian do not overlap, so these attributes are mutually exclusive.
- Add all of the numbers in the diagram together. This represents all of the students.
- Look at the region represented by Japanese.
- Look at the region represented by Italian.
- Look at the region outside of the circle.

WRITE/DRAW

- No, there are no students who studied both Japanese and Italian.
- $19 + 35 + 28 = 82$
There were 82 students overall.
- There were 35 students who studied Japanese.
- There were 28 students who studied Italian.
- There were 19 students who studied neither Japanese nor Italian.

- In Worked example 12 the regions representing the Japanese and Italian languages did not overlap, so these attributes were mutually exclusive and there were no students who studied both subjects.
- When two regions in a Venn diagram overlap the attributes are not mutually exclusive, and the number in the overlapping region represents the members that have both attributes.
- For example, the following Venn diagram represents the favourite film genres of Year 8 students.
- Combined, the blue and green regions represent all the students who selected comedy films.
- Combined, the pink and green regions represent all the students who selected action films.
- The blue region represents those students who selected comedy films, but who did *not* select action films.
- The pink region represents those students who selected action films, but who did *not* select comedy films.
- The green section represents those students who selected both comedy *and* action films together.
- The white region represents those students who selected *neither* comedy *nor* action films.
- Combined, the blue and pink regions represent those students who selected either comedy *or* action films *but not both*.
- Combined, the blue, pink and green regions represent those students who selected either comedy *or* action films *or both*.

Favourite film genres of Year 8 students



WORKED EXAMPLE 13

Describe the following regions in this Venn diagram.

Favourite sport(s) of Year 8 students



- The green region
- The purple region
- The orange region
- The white region
- The green and purple regions combined
- The green, purple and orange regions combined

THINK

- Look at this region. It represents NRL without AFL
- Look at this region. It represents AFL without NRL.
- Look at this region. It represents both AFL and NFL together.

WRITE/DRAW

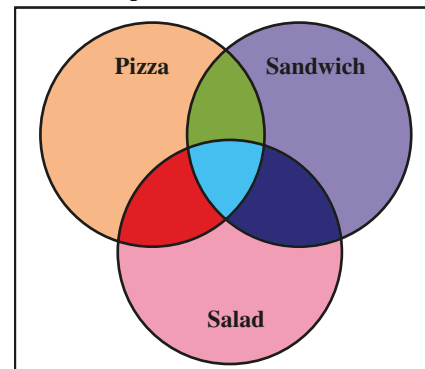
- This region represents the students whose favourite sport is NRL, but not AFL.
- This region represents the students whose favourite sport is AFL, but not NRL.
- This region represents the students whose favourite sports are both AFL and NRL together.

- d** Look at this region. It represents neither NRL nor AFL.
- e** Look at this region. It represents NRL and AFL without those who selected both together
- f** Look at this region. It represents NRL and AFL or both together

- d** This region represents the students whose favourite sport is neither NRL nor AFL.
- e** This region represents the students whose favourite sport is either NRL or AFL, but not both together.
- f** This region represents the students whose favourite sport is either NRL or AFL or both together.

- Venn diagrams can also cover 3 attributes by including 3 circles.
- The diagram at right shows the lunchtime preferences of 50 Year 8 students.
- The light blue section represents those students who like pizzas, sandwiches and salad, while the red section represents those students who like pizza and salad, but not sandwiches.
- Try to work out what the other colours represent!

Lunchtime preferences of Year 8 students



12.6.2 Constructing Venn diagrams

- Given data we can construct Venn diagrams to represent all possible combinations of two attributes, and then use this data to calculate missing values.

WORKED EXAMPLE 14

An ice-creamery conducted a survey of 60 customers on a Monday and obtained the following results on two new ice-cream flavours. The results showed that 35 customers liked Product A, 40 liked Product B, and 24 liked both flavours.

- a** Draw a Venn diagram to illustrate the above information.
- b** Use the Venn diagram to answer the following questions.
 - i** How many customers liked Product A only?
 - ii** How many customers liked Product B only?
 - iii** How many customers liked neither product?

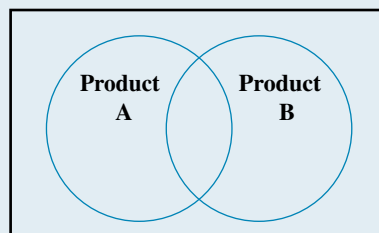


THINK

- a 1** Draw and label two overlapping circles within a rectangle to represent Product A and Product B.
Note: The circles for products A and B overlap because 24 customers liked both products.

WRITE

a



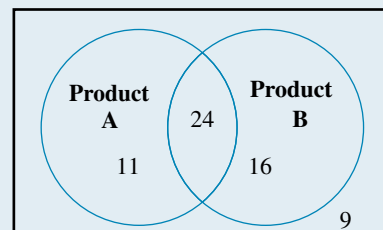
- 2 Working from the overlapping area outwards, determine the number of customers in each region.
Note: The total must equal the number of customers surveyed; that is, 60.

There are 24 customers who liked both products.

Product A contains 11 customers (i.e. $35 - 24$) who liked Product A, but not Product B.

Product B contains 16 customers (i.e. $40 - 24$) who liked Product B, but not Product A.

The remaining 9 customers (i.e. $[60 - (11 + 24 + 16)]$) liked neither product.



- b** **i** Refer to the Venn diagram and answer the question.
Note: The non-overlapping part of Product A's circle refers to the customers that liked Product A only.
- ii** Refer to the Venn diagram and answer the question.
Note: The non-overlapping part of Product B's circle refers to the customers that liked Product B only.
- iii** Refer to the Venn diagram and answer the question.
Note: The region outside the circles refers to the customers who liked neither product.
- b** **i** Eleven customers liked Product A only.
- ii** Sixteen customers liked Product B only.
- iii** Nine customers liked neither product.

ACTIVITY: EYE AND HAIR COLOUR PROBABILITY

Equipment: Post-it notes, marker pens

- On the board, create a large Venn diagram with three overlapping circles. Label the circles 'Blue eyes', 'Brown hair' and 'Black hair'.
- Each student writes their initials on a Post-it note using a marker pen and places the Post-it note in the appropriate place in the Venn diagram.
- Count the number of Post-it notes in each section. Remove the Post-it notes and write the number of Post-it notes for the section in the section on the board.
- As a class, discuss the following. If a student is chosen randomly to receive a prize, what is the probability that he or she has:
 - blue eyes
 - neither blue eyes nor brown hair
 - brown hair
 - black hair
 - not got blue eyes and brown hair
 - blue eyes and black hair
 - not got blue eyes
 - not got black hair?

12.6.3 Two-way tables

- **Two-way tables** can also be used to represent the relationship between non-mutually exclusive attributes.
- In a two-way table one of the attributes is listed on the left-hand side of the table and one of the attributes is listed on the top of the table.

WORKED EXAMPLE 15

The following two-way table shows the relationship between gender and height of Year 8 students.

Gender compared to height of Year 8 students

	Below 160 cm	Above 160 cm	
Female	25	11	36
Male	9	24	33
	34	35	69

- How many females are there overall?
- How many males are above 160 cm?
- How many females are above 160 cm?
- Are there more females below 160 cm than males above 160 cm?
- How many students are either male or above 160 cm?

THINK

- Look at the row represented by the female attribute. The number at the end of this row contains the sum of this row's data.
- Look at the region represented by the intersection between the male and above 160 cm attributes.
- Look at the region represented by the intersection between the female and above 160 cm attributes.
- Compare the two required regions.
- Identify all of the required regions; that is, the row represented by the male attribute and the intersection of the female and above 160 cm attributes.

WRITE/DRAW

- There are 36 females overall.
- There are 24 males above 160 cm.
- There are 11 females above 160 cm.
- Yes, there are more females below 160 cm than males above 160 cm (25 compared to 24).
- $33 + 11 = 44$
There are 44 students who are either male or above 160 cm.

Note: In part e of Worked example 13 we only need to add the number of females above 160 cm as the males above 160 cm have already been included in the male attribute.

WORKED EXAMPLE 16

There are 1400 penguins in an Australian penguin colony. 675 of the penguins are male, of which 370 are up to 5 years old. 410 of the female penguins are over 5 years old.

- a Draw a two-way table to illustrate the above information.
- b Use the two-way table to answer the following questions.
 - i How many of the penguins are up to 5 years old?
 - ii How many of the penguins are over 5 years old?
 - iii How many of the male penguins are over 5 years old?



THINK

- a 1 Draw the shell for your two-way table, listing one attribute on the left-hand side and one on the top.

- 2 Enter in the information given in the question.

- 3 Calculate the missing information.

WRITE/DRAW

a

	Up to 5	Over 5	
Male			
Female			

There are 1400 penguins overall.
 There are 675 male penguins, of which 370 are up to 5 years old.
 There are 410 female penguins over 5 years old.

	Up to 5	Over 5	
Male	370		675
Female		410	
			1400

There are 305 male penguins who are over 5 years old ($675 - 370 = 305$).
 There are 725 female penguins ($1400 - 675 = 725$).
 There are 315 female penguins who are up to 5 years old ($725 - 410 = 315$).
 There are 685 penguins who are up to 5 years old ($370 + 315 = 685$).
 There are 715 penguins who are over 5 years old ($305 + 410 = 715$).

	Up to 5	Over 5	
Male	370	305	675
Female	315	410	725
	685	715	1400

- b i** Look at the column represented by the up to 5 attribute. The number at the end of this column contains the sum of this column's data.
- b ii** Look at the column represented by the over 5 attribute. The number at the end of this column contains the sum of this column's data.
- b iii** Look at the cell represented by the intersection between the male and over 5 attributes.
- b i** 685 penguins are up to 5 years old.
- b ii** 715 penguins are over 5 years old.
- b iii** 305 of the male penguins are over 5 years old.

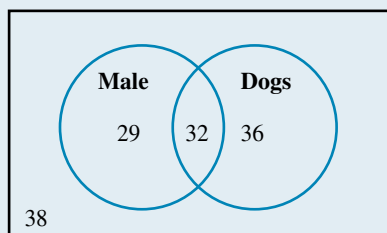
12.6.4 Converting between Venn diagrams and two-way tables

- When representing the relationships between two attributes, we can convert from Venn diagrams to two-way tables and vice versa.

WORKED EXAMPLE 17

The following Venn diagram shows the relationships between males and females and whether they prefer cats or dogs.

The relationship between gender and cats/dogs preference



Convert this information into a two-way table.

THINK

- Draw the shell for your two-way table, listing one attribute on the left-hand side and one on the top.
- Calculate the total number of people.
- Interpret the information in the Venn diagram.

WRITE

	Cats	Dogs	
Male			
Female			

$$29 + 32 + 36 + 38 = 135$$

There are 32 males who prefer dogs.
 There are 36 non-males (females) who prefer dogs.
 There are 29 males who prefer non-dogs (cats)
 There are 38 non-males (females) who prefer non-dogs (cats).

- 4 Enter in the information into the two-way table.

	Cats	Dogs	
Male	29	32	
Female	38	36	
			135

- 5 Calculate the missing values by summing the rows and columns, then enter this information into the two-way table.

$$29 + 32 = 61$$

$$38 + 36 = 74$$

$$29 + 38 = 67$$

$$32 + 36 = 68$$

	Cats	Dogs	
Male	29	32	61
Female	38	36	74
	67	68	135

Exercise 12.6 Venn diagrams and two-way tables

Individual pathways

PRACTISE

Questions:
1–5, 7–9, 11, 13, 15

CONSOLIDATE

Questions:
1–11, 13–15

MASTER

Questions:
1–18

Individual pathway interactivity: int-7050

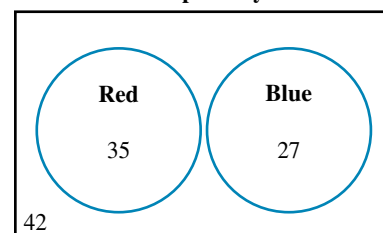
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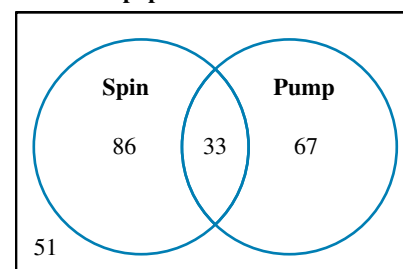
Understanding and fluency

- WE12** The following Venn diagram represents the favourite colours of primary school children.
 - Are there any children who chose both red and blue?
 - How many children were there overall?
 - How many children preferred red?
 - How many children preferred blue?
 - How many children preferred neither red nor blue?
- The following Venn diagram represents a study of the most popular exercise classes at a gym.
 - How many people were surveyed overall?
 - How many people went to spin classes?
 - How many people went to pump classes?
 - How many people went to neither spin nor pump classes?
 - How many people went to either the spin or pump class, but not both?

Favourite colours of primary school children



Most popular exercise classes



3. **WE13** Describe the following regions in this Venn diagram.

- The combined pink and orange regions
- The purple region
- The white region
- The combined pink, purple and orange regions
- The combined pink and purple regions

4. A survey of a Year 8 class found the numbers of class members who play basketball, cricket and soccer. Use the following Venn diagram to calculate the number of students who:

- were in the class
- play basketball
- play cricket and basketball
- play cricket and basketball but not soccer
- play soccer but not cricket
- play all three sports
- do not play cricket, basketball or soccer
- do not play cricket
- play cricket or basketball or both
- play at least one of basketball or cricket or soccer.

5. **WE14** A cosmetics shop surveyed 100 of their customers about two new bath bomb products. The results showed that 63 customers liked Product A, 28 liked Product B, and 11 liked both.

- Draw a Venn diagram to illustrate the above information.
- Use the Venn diagram to answer the following questions.
 - How many customers liked Product A only?
 - How many customers liked Product B only?
 - How many customers liked neither product?

6. A tyre manufacturer conducting a survey of 2200 customers obtained the following results on two tyres: 1390 customers preferred Tyre A, 1084 preferred Tyre B, and 496 preferred both equally.

- Draw a Venn diagram to illustrate the above information.
- Use the Venn diagram to answer the following questions.
 - How many customers preferred Tyre A only?
 - How many customers preferred Tyre B only?
 - How many customers preferred neither tyre?

7. The favourite cruise destinations of 120 Australian holiday makers were as follows. 55 people chose Fiji, 37 chose Tahiti and 42 chose another destination.

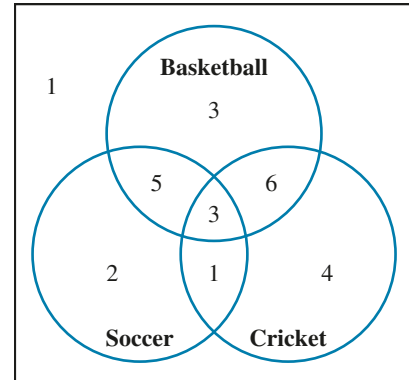
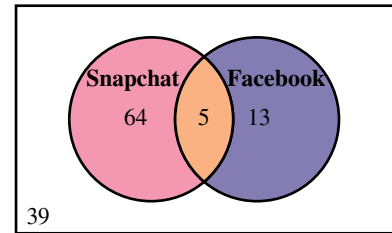
- Draw a Venn diagram to illustrate the above information.
- How many of the holiday makers chose both Fiji and Tahiti?

8. **WE15** The following two-way table shows the relationship between home ownership and household income for 100 individuals.

Home ownership compared to household income

	Under \$100k	Over \$100k	
Home ownership	11	33	44
Renting	38	18	56
	49	51	100

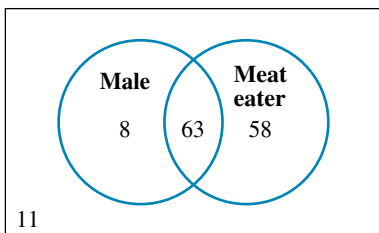
Favourite social media of teenagers



- a. How many home owners are there overall?
 - b. How many home owners earn under \$100k?
 - c. How many renters earn over \$100k?
 - d. Are there more home owners earning over \$100k than renters earning under \$100k?
 - e. How many people are renters or earning above \$100k?
9. **WE16** There are 150 pop and rap songs in rotation on a popular radio channel. This includes 92 songs from this year, of which 63 are pop songs. Of the older songs, 21 can be classified as rap.
- a. Draw a two-way table to illustrate the above information.
 - b. Use the two-way table to answer the following questions.
 - i. How many of the older songs are pop songs?
 - ii. How many of the songs from this year are rap songs?
 - iii. How many of the songs are pop songs?
10. There are 180 pigs and cattle on Farmer Smith's farm. 85 of the animals are male, of which 59 are pigs. 31 of the cattle are female.
- a. Draw a two-way table to illustrate the above information.
 - b. Use the two-way table to answer the following questions.
 - i. How many of the cattle are male?
 - ii. How many of the females are pigs?
11. The following Venn diagram shows the relationship between males and females and whether they are meat eaters or vegetarians.



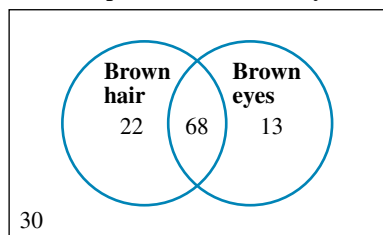
Relationship between gender and diet



Convert this information into a two-way table.

12. The following Venn diagram shows the relationship between hair and eye colour.

Relationship between hair and eye colour



Convert this information into a two-way table.

13. The following two-way table shows the relationships between male and female gamers and whether they prefer PlayStation or Xbox.

Gender compared to console preference

	PlayStation	Xbox	
Male	24	19	43
Female	21	17	38
	45	36	81

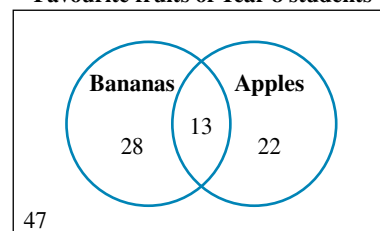
Convert this information into a Venn diagram.

Communicating, reasoning and problem solving

14. The favourite fruits of Year 8 students have been recorded in the following Venn diagram.

- Convert this information into a two-way table.
- Calculate the probability that a student's favourite fruits include apples.
- Calculate the probability that a student's favourite fruits include neither apples nor bananas.
- Calculate the probability that a student's favourite fruits include apples or bananas, but not both.

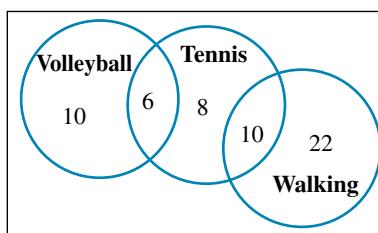
Favourite fruits of Year 8 students



15. The following two-way table shows the relationship between nations where French is an official language and the nations where English is an official language.

	English	Not English	
French	5	24	29
Not French	49	117	166
	54	141	195

- Calculate the probability that a nation has neither French nor English as an official language.
 - Calculate the probability that a nation has French or English or both as an official language.
 - Calculate the probability that a nation has French or English but not both as an official language.
16. What is the minimum number of pieces of information that is required to complete a two-way table? Explain your answer.
17. A sporting club has its members playing different sports, as shown in the Venn diagram.
- Copy the given Venn diagram and shade the areas that represent:
 - members playing tennis only
 - members walking only
 - members both playing tennis and walking.



- How many members:
 - play volleyball
 - are involved in all three activities?
- How many members belong to the sporting club?
- How many members do not:
 - play tennis
 - walk?
- Calculate the probability that a member likes playing volleyball or tennis but does not like walking.
- Calculate the probability that a member likes playing volleyball and tennis but does not like walking.

18. A survey of 140 fifteen-year-old girls investigated how many read magazines (M), crime novels (C) and science fiction (S). It found:
- 23 read both magazines and science fiction
 - 21 read both magazines and crime novels
 - 25 read both crime novels and science fiction
 - 15 read all three
 - 40 read magazines only
 - 38 read crime novels only
 - 10 read science fiction only.
- a. Show this information on a fully labelled Venn diagram.
- b. How many girls read magazines?
- c. How many girls read only crime?
- d. How many girls read science fiction?
- e. How many girls read none of these three?

12.7 Review

Investigation | Rich task

In a spin

When dealing with events involving chance, we try to predict what the outcome will be. Some events have an even chance of occurring, whereas others have little or no chance of occurring. If the outcomes for events are equally likely, we can predict how often each outcome will appear. But, will our predictions always be exact?

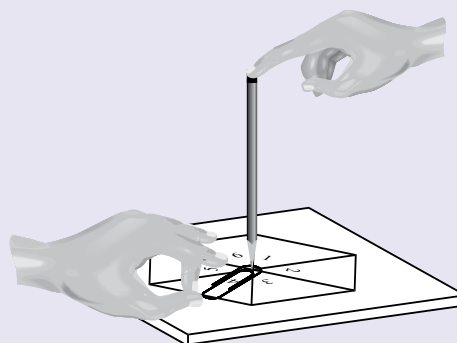
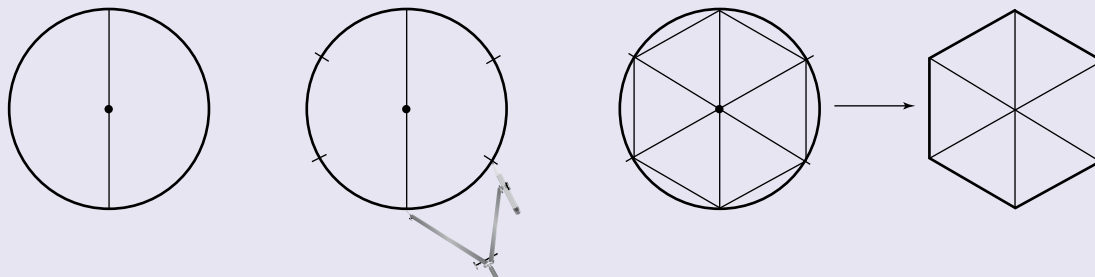
Spinners are often used to help calculate the chance of an event occurring. There are many different types of spinners, and one with 6 equal sections is shown below. A paperclip is flicked around the pencil placed at the centre. The instructions below will enable you to construct a spinner to use in a probability exercise.

Draw a circle with a radius of 8 cm onto a piece of cardboard using a pair of compasses. Using a ruler, draw a line to indicate the diameter of the circle.

Keep the compasses open at the same width. Place marks on the circumference on both sides of the points where the diameter meets the circle.

Join the marks around the circumference to produce your hexagon. Draw a line with a ruler to join the opposite corners and cut out the hexagon.

Number the sections 1 to 6 or colour each section in a different colour.



1. Flick the paperclip around the pencil 120 times. Record each outcome in a copy of the table below.

Score	Tally	Frequency
1		
2		
3		
4		
5		
6		
	Total	

Note: If the paperclip lands on a line, re-spin the spinner.

2. Are the outcomes of each number in your spinner equally likely? Explain why.
3. What would be the theoretical probability of spinning a 6?
4. Based on the results you obtained from your spinner, list the relative frequencies of each outcome on the spinner.
5. How many times would you expect the paperclip to land on each number when you perform 120 spins?
6. How close were your results to the expected results?

Combine your results with the results obtained from your classmates.

7. Design a new frequency table for the class results.
8. How do the relative frequencies of the pooled class results compare with your results? Are they closer to the results you expected?
9. If time permits, continue spinning the spinner and pooling your results with the class. Investigate the results obtained as you increase the number of trials for the experiment.

learnON RESOURCES — ONLINE ONLY

Digital doc: Investigation — In a spin (doc-13945)

Digital doc: Code puzzle — What did the cannibal have for breakfast? (doc-13946)

Digital doc: Summary Topic 12 (doc-23248)

Interactivity: Word search Topic 12 (int-2637)

Interactivity: Crossword Topic 12 (int-2638)

Interactivity: Sudoku Topic 12 (int-3192)

Exercise 12.7 Review questions

To answer questions online and to receive **immediate feedback** and **fully worked solutions** for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Understanding and fluency

1. **MC** The chance of getting a 5 with the spinner at right is:

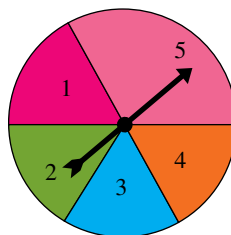
A. $\frac{1}{3}$

B. $\frac{1}{5}$

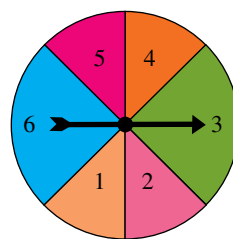
C. $\frac{1}{6}$

D. $\frac{2}{5}$

E. none of these



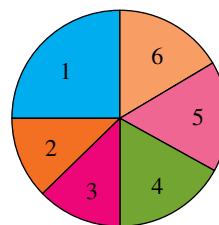
2. Study the spinner shown at right.
- What is the sample space?
 - Are all outcomes equally likely? Explain your answer.
 - In theory, what are the chances of spinning:
 - a 2
 - a 6?



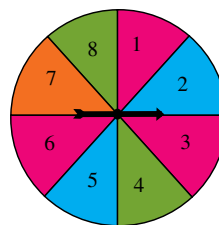
3. One thousand tickets were sold for a raffle. If you purchased five, what are your chances of winning the raffle?
4. A cube has 2 red faces, 2 white faces, 1 green face and 1 yellow face. If rolled, what is the probability of the top face showing:
- red
 - yellow
 - not red
 - green or white?
5. On each of the 5 days, Monday to Friday, garbage is collected. What are the chances that garbage is collected at an address chosen at random on:
- Wednesday
 - Thursday or Friday
 - Sunday
 - a day other than Monday?
6. A multiple-choice question has as alternatives A, B, C, D and E. Only one is correct.
- What is the probability of guessing:
 - the right answer
 - the wrong answer?
 - What is the total of the probabilities?
7. a. Design a circular spinner with the numbers 1 to 4 so that:

$$P(1) = \frac{1}{2}, P(2) = \frac{1}{4}, P(3) = \frac{1}{8}, P(4) = \frac{1}{8}.$$

- What will the size of the angle in each sector be?
8. Give the probability of each of the numbers in this circular spinner.
9. State the complement of each of the following events.
- Tossing a coin and it landing Tails
 - Winning a race in which you run
 - Answering a question correctly
 - Selecting a black marble from a bag of marbles
 - Selecting a number less than 20
10. State whether each of the following pairs of events are complementary or not. Explain your answer.
- Having milk in your coffee and not having milk in your coffee.
 - Going overseas for your holiday or holidaying in Australia.
 - Catching a train to work or catching a bus to work.



11. The probability that a traffic light is green is $\frac{3}{7}$. Find the probability that the traffic light is not green.
12. Determine if the following events are mutually exclusive or not.
- Rolling a multiple of 4 and rolling a 10 on a 10-sided die.
 - Selecting an even number and selecting a multiple of 7 from 1–15.
 - Landing on a blue segment and landing on a 5 when spinning this spinner.
13. Determine the probability of the following compound events.
- Not choosing a multiple of 3 from 1–20.
 - Rolling a multiple of 3 or a multiple of 4 but not both when rolling a 12-sided die.



14. Jacqueline is a keen surfer and skateboarder. On a Sunday there is a $\frac{2}{5}$ chance she will go surfing, a $\frac{1}{4}$ chance she will go skateboarding and a $\frac{1}{10}$ chance she will go both surfing and skateboarding. Calculate the probability Jacqueline will go surfing, skateboarding or both on a Sunday.



15. Shoppers in a supermarket were given a taste test of two varieties of jellybeans — home brand (H) and brand X . Of the 168 shoppers to undertake the test, 111 liked the home brand, 84 liked brand X and 39 liked both.
- Represent these data as a Venn diagram.
 - What is the probability that one of those shoppers did not like either of the two varieties?
 - What is the probability that one of the shoppers liked brand H or brand X ?
 - What is the probability that one of the shoppers liked brand X given that they liked brand H ?
16. Draw a two-way table to display the situation in question 15.

Communicating, reasoning and problem solving

17. During her latest shopping trip, Karen's mother stocked up on a supply of icy poles for the summer. In one box there are 3 icy poles of each of the flavours: orange, raspberry, lime and cola. Karen likes everything but lime.



- What is the probability that when Karen reaches into the box randomly, she will select an icy pole that she likes?
 - Two days later, 2 raspberry, 1 cola, and 1 lime have been eaten by various members of the family. Have Karen's chances of randomly choosing a lime icy pole gone up, down or stayed the same? Explain your answer.
18. In a certain school band there are 6 girls and some boys. A student is selected at random from this group. Find the number of boys in the group if the probability that a girl is selected is $\frac{1}{4}$.
19. Jar A contains 5 black, 3 white and 2 coloured marbles. Jar B contains 6 black, 4 white and 4 coloured marbles. From which jar are you more likely to draw a black marble?
20. A fair 4-sided die in the shape of a regular tetrahedron is tossed twice.
- Draw the sample space.
 - What is the probability that the two numbers that are thrown are both even?
21. There are only three swimmers in the 100-m freestyle. Swimmer A is twice as likely to win as B and three times as likely to win as C. Find the probability that B or C wins.
22. James has two dice. One is a regular die. James has altered the other by replacing the 1 with a 6 (so the altered die has two 6's, and no 1). He rolls the two dice together. Determine the probability that a double is rolled when the two dice are thrown.

23. In a club of 60 members, half support the sausage sizzles, one-third support the raffles and some of these support both. One-fifth of the members support neither of these fundraisers. Determine the probability that a club member, chosen at random, supports both.



24. There are four differently coloured marbles in a bag: red, blue, green and yellow. There are 8 blue marbles and 4 yellow marbles. The probability of drawing a blue or yellow marble is 0.6. If there are 3 green marbles in the bag, how many red marbles are there?
25. The following two-way table shows whether coffee and tea drinkers like milk in their hot drink.

	Milk	No milk	
Tea	38	22	60
Coffee	31	64	95
	69	86	155

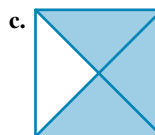
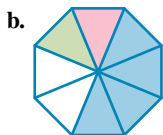
- Convert this information into a Venn diagram.
 - Calculate the probability that someone likes milk in their hot drink.
 - Calculate the probability that someone is either a tea drinker without milk or a coffee drinker with milk.
 - Calculate the probability that someone is either a coffee drinker or likes milk with their hot drink but not both.
26. Margaret is in charge of distributing team uniforms for students representing the school in music, athletics and debating. Margaret knows that representing the school are 43 students, some of whom are involved in more than one activity. They must purchase a uniform for each activity in which they participate.
- Margaret has the following information: 36 students are in the concert band, 31 students are in the athletics team, 12 students are in the debating team, 6 students are involved in all three. Nine students are involved in music and debating, 7 in athletics and debating and 26 students are in music and athletics.
- Show this information on a Venn diagram.
 - Calculate the probability that a student will be required to purchase only the music uniform.
 - Calculate the probability that a student will be required to purchase only the athletics uniform.
 - Calculate the probability that a student will be required to purchase only the debating uniform.
 - Calculate the probability that a student will be required to purchase music and debating uniforms but not an athletics uniform.
 - Calculate the probability that a student will be required to purchase a music or athletics uniform but not a debating uniform.

Answers

Topic 12 Probability

Exercise 12.2 Probability scale

1. a. Certain b. Highly unlikely c. Discuss d. Discuss
 e. Highly unlikely f. Highly unlikely g. Certain h. Impossible
 i. Highly unlikely j. Highly likely k. Even chance l. Highly unlikely
 m. Even chance n. Better than even chance o. Highly likely p. Highly unlikely
2. a–e. Class discussion
3. a. $\frac{1}{2}$ b. $\frac{9}{10}$ c. 0 d. 1 e. $\frac{1}{2}$ f. $\frac{1}{3}$ g. $\frac{1}{52}$
 h. $\frac{11}{20}$ i. $\frac{5}{6}$ j. $\frac{1}{13}$ k. $\frac{2}{13}$ l. $\frac{4}{7}$ m. 1
4. C
5. E
6. a. $\frac{4}{11}$ b. $\frac{7}{11}$ c. $\frac{2}{11}$ d. $\frac{1}{11}$
7. a. i. No
 ii. {blue, green, pink, orange}
 iii. Blue: $\frac{1}{3}$; green: $\frac{1}{6}$; pink: $\frac{1}{3}$; orange: $\frac{1}{6}$
 b. i. Yes
 ii. {blue, green, purple, orange, pink}
 iii. Blue: $\frac{1}{5}$; green: $\frac{1}{5}$; purple: $\frac{1}{5}$; orange: $\frac{1}{5}$; pink: $\frac{1}{5}$
 c. i. No
 ii. {orange, green}
 iii. Orange: $\frac{2}{3}$; green: $\frac{1}{3}$
8. a. Blue. There are more blue jelly beans than any other colour.
 b. Yellow. There are fewer yellow jelly beans than any other colour.
 c. Blue: $\frac{6}{17}$; green: $\frac{5}{17}$; red: $\frac{4}{17}$; yellow: $\frac{2}{17}$
9. Answers will vary. Some examples are shown.



10. Answers will vary.
11. The probability of a green card is 0.15. There are 3 green cards in the bag.

Exercise 12.3 Sample spaces and theoretical probability

1. a. {Heads, Tails}
 b. {a, a, o, u}
 c. {Monday, Tuesday, Wednesday, Thursday, Friday, Saturday, Sunday}
 d. {R, R, R, W, W, B}
 e. {1, 2, 3, 4, 5, 6}
 f. {king of hearts, diamonds, clubs, spades; queen of hearts, diamonds, clubs, spades; jack of hearts, diamonds, clubs, spades}
 g. {1, 2, 3, 4, 5, 6, 7, 8}
 h. {2, 4, 6, 8, 10, 12, 14, 16, 18, 20}
 i. {apple, apple, pear, pear, pear, pear, orange, orange, orange, orange, banana, banana, banana, banana}
 j. {Dolly, Dolly, Dolly, Girlfriend, Girlfriend, Smash Hits, Mathsmag, Mathsmag}
 k. {A, B, C, D, E}
 l. {gold, silver, bronze}

2. a. {1, 2, 3, 4, 5, 6}

b. i. $\frac{1}{6}$

ii. $\frac{1}{2}$

iii. $\frac{2}{3}$

iv. $\frac{1}{3}$

v. $\frac{1}{2}$

vi. $\frac{1}{3}$

vii. 0

viii. 1

3. a. $\frac{1}{52}$, 1.9%

b. $\frac{1}{13}$, 7.7%

c. $\frac{2}{13}$, 15.4%

d. $\frac{1}{4}$, 25%

e. $\frac{1}{2}$, 50%

f. $\frac{4}{13}$, 30.8%

g. $\frac{1}{13}$, 7.7%

4. a. $\frac{2}{45}$

b. $\frac{8}{9}$

c. $\frac{1}{9}$

d. $\frac{1}{9}$

5. a. $\frac{3}{14}$

b. $\frac{1}{7}$

c. $\frac{1}{7}$

d. $\frac{5}{14}$

e. $\frac{11}{14}$

f. $\frac{4}{7}$

g. 0

h. $\frac{13}{14}$

6. a. $\frac{1}{2}$

b. $\frac{1}{2}$

c. 0

d. 1

7. a. $\frac{1}{365}$ or $\left(\frac{1}{366}\right)$

b. $\frac{7}{365}$ (or $\frac{7}{366}$)

c. $\frac{30}{365} = \frac{6}{73}$ (or $\frac{30}{366} = \frac{5}{61}$)

d. $\frac{1}{365}$ (or $\frac{1}{366}$)

Note: In a leap year there are 366 days.

8. a. i Yes, equal sectors

ii. $\frac{1}{2}$

b. i Yes, equal sectors

ii. $\frac{1}{4}$

c. i No, sector 1 occupies a larger area.

ii. $\frac{2}{3}$

d. i No, sector 1 occupies the smallest area.

ii. $\frac{1}{8}$

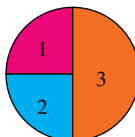
9. $\frac{1}{2}$

10. a.



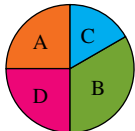
b. Each sector has an angle of 72° at the centre of the spinner.

11. a.



b. Sectors 1 and 2 have angles of 90° and sector 3 has an angle of 180° at the centre of the spinner.

12. a.



b. Sectors A and D: 90° , sector B: 120° and sector C: 60° .

13. a. 1

b. 360°

14. a. Heads, $\frac{1}{2}$; Tails, $\frac{1}{2}$

b. 1

15. a. H H, $\frac{1}{4}$ H T, $\frac{1}{4}$ T H, $\frac{1}{4}$ T T, $\frac{1}{4}$

b. 1

16. a. $\frac{1}{6}$

b. 1

c. Yes

d. The total of probabilities for all elements in a sample space is always 1.

17. C

18. A

19. a. {1, 2, 3, 4, 5, 6}

b. 36

c.

		Die 2					
		1	2	3	4	5	6
Die 1	1	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
	2	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
	3	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
	4	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
	5	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
	6	(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

d.

Sum	2	3	4	5	6	7	8	9	10	11	12
Probability	$\frac{1}{36}$	$\frac{2}{36} = \frac{1}{18}$	$\frac{3}{36} = \frac{1}{12}$	$\frac{4}{36} = \frac{1}{9}$	$\frac{5}{36}$	$\frac{6}{36} = \frac{1}{6}$	$\frac{5}{36}$	$\frac{4}{36} = \frac{1}{9}$	$\frac{3}{36} = \frac{1}{12}$	$\frac{2}{36} = \frac{1}{18}$	$\frac{1}{36}$

e. The probabilities are symmetric.

f. {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}

g.

Number of Heads	0	1	2	3
Probability	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

h. 3 Heads

i. Heads once and Tails twice

20. a. $\frac{1}{4}$ b. $\frac{5}{9}$ c. $\frac{1}{2}$
21. a. 1 b. $\frac{3}{8}$
22. a. $\frac{1}{4}$ b. $\frac{1}{6}$ c. $\frac{35}{36}$ d. 1
23. a. $\frac{1}{4}$ b. $\frac{1}{2}$ c. $\frac{1}{2}$ d. $\frac{1}{3}$

Exercise 12.4 Complementary events

- Selecting an odd number
 - Selecting a consonant
 - The coin landing Tails
 - Getting a number greater than 2
 - Getting a total of 12
 - Not selecting a diamond
 - Not selecting an E
 - Not selecting a blue marble
- B and C
- Not complementary, as there are other things that you could have for breakfast.
 - Not complementary, as there are other ways of travelling to your friend's house.
 - Not complementary, as there are other things that you could be doing.
 - Complementary, as this covers all possible outcomes.
 - Not complementary, as neither case covers the possibility of rolling a 4.
- $\frac{12}{13}$
- A
- $\frac{215}{216}$
- $\frac{4}{5}$
- $\frac{1}{4}$
 - $\frac{3}{4}$
 - $\frac{3}{4}$
 - $\frac{1}{4}$
- No, the two events are not complementary, as the sum of their probabilities does not equal one. Getting one Head is also an outcome.
- $\frac{17}{70}$
 - $\frac{53}{70}$
 - $\frac{9}{70}$
 - $\frac{61}{70}$
- 0
 - $\frac{7}{11}$
 - $\frac{4}{11}$

12. $\frac{3}{10}$

13. a. $\frac{n-r}{n}$

b. $\frac{r}{n}$

14. a. $\frac{3}{11}$

b. $\frac{5}{11}$

15. If an event has only 2 possible outcomes, which have nothing in common, they are complementary events. If the probability of the event occurring (P) is added to the probability of event not occurring (P' , i.e. the complement), the result will be a probability of 1. The event will either occur or it won't occur.

Exercise 12.5 Mutually exclusive and compound events

1. a. Not mutually exclusive b. Not mutually exclusive c. Mutually exclusive

2. D

3. a. Rolling at most a 4 on a ten-sided die

b. Not selecting a heart, diamond or spade from a pack of cards

c. Rolling at least a 9 on a ten-sided die

4. B

5. a. Exclusive

b. Inclusive

c. Exclusive

6. a. $\frac{8}{52} = \frac{2}{13}$

b. $\frac{28}{52} = \frac{7}{13}$

7. a. $\frac{2}{10} = \frac{1}{5}$

b. $\frac{6}{10} = \frac{3}{5}$

c. $\frac{5}{10} = \frac{1}{2}$

d. $\frac{7}{10}$

8. $\frac{14}{21} = \frac{2}{3}$

9. 0.325

10. 65%

11. a. Exclusive b. 55%

12. Two events are mutually exclusive if one event happening excludes the other from happening.

13. Yes, because there are no common outcomes.

14. a. We cannot have a probability greater than 100%, so the two events must not be mutually exclusive. This means that there is a probability that Sasha goes to Jess's house and to the swimming pool.

b. No, we would also need to know the probability of Sasha going to Jess's house and the swimming pool.

15. a. $\frac{27}{90} = \frac{3}{10}$

b. $\frac{39}{90} = \frac{13}{30}$

c. 1

d. $\frac{8}{90} = \frac{4}{45}$

e. $\frac{55}{90} = \frac{11}{18}$

16. By using either 'both' or 'not both' we are indicating whether an outcome where both individual events occur is in the sample space or not. If it is in the sample space then the event is inclusive, if it is not in the sample space then the event is exclusive.

17. No, mutually exclusive events cannot both occur at the same time, but they do not need to be complementary events.

Exercise 12.6 Venn diagrams

1. a. No b. 104 c. 35 d. 27 e. 42

2. a. 237 b. 119 c. 100 d. 51 e. 153

3. a. Teenagers who like Snapchat

b. Teenagers who like Facebook, but not Snapchat

c. Teenagers who like neither Facebook nor Snapchat

d. Teenagers who like Facebook or Snapchat or both

e. Teenagers who like Facebook or Snapchat, but not both

4. a. 25

b. 17

c. 9

d. 6

e. 7

f. 3

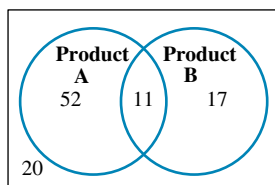
g. 1

h. 11

i. 22

j. 24

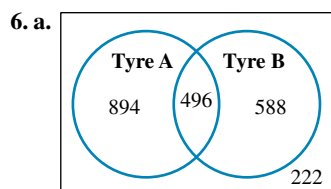
5. a.



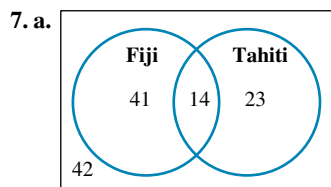
b. i. 52

ii. 17

iii. 20



- b. i. 894 ii. 588 iii. 222



- b. 14

8. a. 44 b. 11 c. 18 d. No e. 89

9. a.

	This year	Older	
Rap	29	21	50
Pop	63	37	100
	92	58	150

- b. i. 37 ii. 29 iii. 100

10. a.

	Male	Female	
Pigs	59	64	123
Cattle	26	31	57
	85	95	180

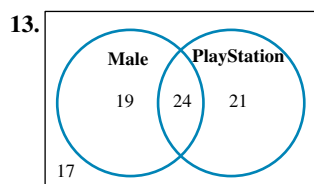
- b. i. 26 ii. 64

11.

	Meat eater	Vegetarian	
Male	63	8	71
Female	58	11	69
	121	19	140

12.

	Brown eyes	Not brown eyes	
Brown hair	68	22	90
Not brown hair	13	30	43
	81	52	133



14.

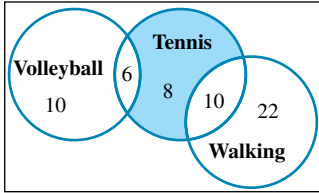
	Apples	Not apples	
Bananas	13	28	41
Not bananas	22	47	69
	35	75	110

- b. $\frac{35}{110} = \frac{7}{22}$ c. $\frac{47}{110}$ d. $\frac{50}{110} = \frac{5}{11}$

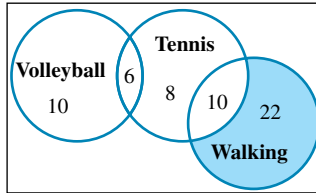
15. a. $\frac{117}{195} = \frac{39}{65}$ b. $\frac{78}{195} = \frac{26}{65}$ c. $\frac{73}{195}$

16. At least 4 pieces of information are required to complete a two-way table. Without 4 pieces of information some of the cells cannot be completed.

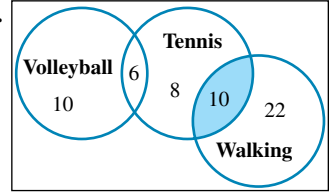
17. a. i.



ii.



iii.



b. i. 16

ii. 0

c. 56

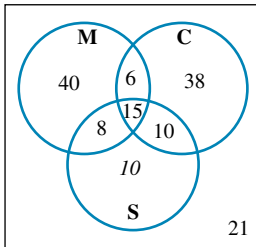
d. i. 32

ii. 24

e. $\frac{24}{56} = \frac{3}{7}$

f. $\frac{6}{56} = \frac{3}{28}$

18. a.



b. 69

c. 38

d. 43

e. 21

Investigation | Rich task

1. Check with your teacher.
2. They are equally likely because each section is the same size.
3. $\frac{1}{6}$
4. Check with your teacher.
5. 20
6. to 9. Check with your teacher.

Exercise 12.7 Review questions

1. A

2. a. {1, 2, 3, 4, 5, 6}

b. No, because the sectors are of varying angle size. 3 and 6 have sectors that are double the size of others; therefore, they will have a larger probability.

c. i. $\frac{1}{8}$

ii. $\frac{1}{4}$

3. $\frac{1}{200}$

4. a. $\frac{1}{3}$

b. $\frac{1}{6}$

c. $\frac{2}{3}$

d. $\frac{1}{2}$

5. a. $\frac{1}{5}$

b. $\frac{2}{5}$

c. 0

d. $\frac{4}{5}$

6. a. i. $\frac{1}{5}$

ii. $\frac{4}{5}$

b. 1

7. a.



b. Sector 1: 180°

Sector 2: 90°

c. Sector 3: 45°

Sector 4: 45°

8. Sector 1: $\frac{1}{4}$ Sector 2: $\frac{1}{8}$ Sector 3: $\frac{1}{8}$

Sector 4: $\frac{1}{6}$ Sector 5: $\frac{1}{6}$ Sector 6: $\frac{1}{6}$

9. a. The coin lands Heads
 c. Answering the question incorrectly
 e. Selecting a number greater than 19
- b. Losing the race
 d. Not selecting a black marble

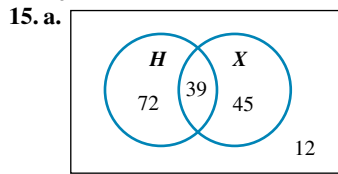
10. a. Complementary, as all possible outcomes are covered
 b. Complementary, as all possible outcomes are covered
 c. Not complementary, as there are other means to travel to work

11. $\frac{4}{7}$

12. a. Mutually exclusive
 b. Not mutually exclusive
 c. Not mutually exclusive

13. a. $\frac{14}{20} = \frac{7}{10}$
 b. $\frac{5}{12}$

14. $\frac{11}{20}$



b. $\frac{12}{168} = \frac{1}{14}$
 c. $\frac{156}{168} = \frac{13}{14}$
 d. $\frac{39}{111} = \frac{13}{37}$

16.

		Brand X		
		Yes	No	
Brand H	Yes	39	72	111
	No	45	12	57
		84	84	168

17. a. $\frac{3}{4}$

- b. Stayed the same. Recalculating the probability gives $\frac{3}{4}$ again.

18. 18 boys

19. Jar A

20. a. $\{(1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (2, 3), (2, 4), (3, 1), (3, 2), (3, 3), (3, 4), (4, 1), (4, 2), (4, 3), (4, 4)\}$

b. $\frac{1}{4}$

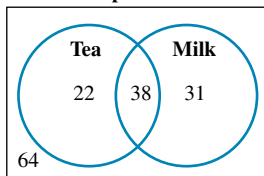
21. $\frac{5}{11}$

22. $\frac{1}{6}$

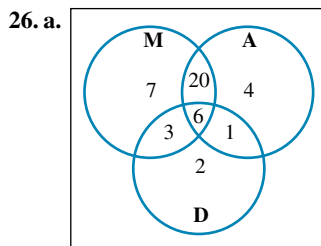
23. $\frac{1}{30}$

24. 5

25. a. Hot drinks compared to milk preference



b. $\frac{69}{155}$
 c. $\frac{53}{155}$
 d. $\frac{102}{155}$



b. $\frac{7}{43}$
 c. $\frac{4}{43}$
 d. $\frac{2}{43}$
 e. $\frac{3}{43}$
 f. $\frac{31}{43}$

TOPIC 13

Data analysis

13.1 Overview

Numerous **videos** and **interactivities** are embedded just where you need them, at the point of learning, in your learnON title at www.jacplus.com.au. They will help you to learn the concepts covered in this topic.

13.1.1 Why learn this?

Every day we are bombarded with statistics in the media. The statistics are often presented as diagrams — graphs, pie charts, bar charts — rather than as tables or lists. This topic shows you how to make sense of such statistical data and graphs.



DISCUSSION

Is there an activity, other than your study, in which you are interested or involved where data plays an important part in any analysis of performance? Do you think this is a fair use of data?

LEARNING SEQUENCE

- 13.1** Overview
- 13.2** Samples and populations
- 13.3** Primary and secondary data
- 13.4** Organising and displaying data
- 13.5** Measures of centre and spread
- 13.6** Analysing data
- 13.7** Review

LEARNING OUTCOMES

A student:

- communicates and connects mathematical ideas using appropriate terminology, diagrams and symbols MA4-1WM
- recognises and explains mathematical relationships using reasoning MA4-3WM
- collects, represents and interprets single sets of data, using appropriate statistical displays MA4-19SP

CONTENT DESCRIPTIONS

Students:

Investigate techniques for collecting data, including census, sampling and observation (ACMSP284)

Explore the practicalities and implications of obtaining data through sampling using a variety of investigative processes (ACMSP206)

Identify and investigate issues involving numerical data collected from primary and secondary sources (ACMSP169)

Construct and compare a range of data displays, including stem-and-leaf plots and dot plots (ACMSP170)

Calculate mean, median, mode and range for sets of data and interpret these statistics in the context of data (ACMSP171)

Explore the variation of means and proportions of random samples drawn from the same population (ACMSP293)

Investigate the effect of individual data values, including outliers, on the mean and median (ACMSP207)

Source: NSW Syllabus for the Australian curriculum

Note: Your teacher may now set you a pre-test to determine how familiar you are with the content in this topic.

13.2 Samples and populations

13.2.1 Collecting data

- Information or **data** is constantly being collected.
- Different organisations collect different types of data. For example, at a cricket match, some of the **statistics** gathered for a batsman are: time spent batting, the number of balls faced, the runs off a particular delivery, where the ball was hit, the number of 4s or 6s hit, and so on.
- Once the data are collected, they can be organised, analysed and interpreted.
- A **survey** is the process of collecting data. If every member of a target **population** is surveyed, the process is called a **census**.
- A census is conducted in Australia every 5 years to obtain an accurate profile of Australians. On census night each person in Australia is required to complete a detailed online questionnaire or booklet containing a series of questions relating to age, marital status, employment, income, housing, education, modes of transport and so on. This allows the government to analyse the population and make decisions on how to improve services.
- Due to limitations in time, cost and practicality, instead of a census in many cases a **sample** of the population is selected at **random** (not in any particular order or pattern) to prevent **biased** (leaning in a favoured direction) results.



DISCUSSION

Discuss the practicalities of collecting data through a census as compared with collecting data of a sample of the population. Consider the size of the population when taking a census, particularly in countries with large populations such as India or China.

WORKED EXAMPLE 1

In each of the following, state if the information was obtained by census or survey.

- A school uses a roll to count the number of students absent each day.
- The television ratings, in which 2000 families complete a questionnaire on the programs they watch over a one-week period.
- A battery manufacturer tests every hundredth battery off the production line.
- A teacher records the examination marks of her class.

THINK

- Every student is counted at roll call each morning.
- Not every family is asked to complete a ratings questionnaire.
- Not every battery is tested.
- The marks of every student are recorded.

WRITE

- Census
- Survey
- Survey
- Census

- Data can be collected using a variety of techniques. Three common methods are:
 - observation
 - survey
 - experiment.

13.2.2 Surveys

- Collecting data by survey is the form most frequently used. The survey is administered with the aid of a questionnaire. The success of obtaining meaningful and relevant data from a questionnaire depends largely on the care taken in designing the questions. Methods used to collect data include:
 - personal interviews, where the interviewer usually asks prepared questions, then records the replies from the respondent
 - telephone interviews, where the interview is conducted over the telephone
 - self-administered questionnaires, which are usually mailed to individuals who then complete the questionnaire and return it in a pre-paid envelope or hold it for collection.

13.2.3 Questionnaires

- When a **questionnaire** is designed, we must keep in mind that the collected data should be in a form that is easy to analyse. Questions can be **open** or **closed**.
- Open questions are those where the respondent has no guided boundaries within which to answer. Questions that belong to this class include:
 - ‘Who is your favourite singer?’
 - ‘What is your favourite food?’
 The main problem with open questions is that their answers are often difficult to classify and analyse.
- Closed questions are of the type where the respondent must answer within a category. The question about food above could be rewritten as:
 - ‘Which one of the following foods do you prefer most?’
 - Meat
 - Seafood
 - Poultry
 - Vegetables
 - Fruit
 These types of answers are easier to analyse than answers to open questions. It must be noted that options such as:
 - None of the above or
 - Don’t know
 should be avoided, if possible, as they provide the respondent with a ‘way out’.

WORKED EXAMPLE 2

Ten students were asked the open question:

‘What do you dislike about your school uniform?’

These were their responses.

1. It’s too old-fashioned.	2. It costs too much.
3. It’s just yucky.	4. It’s a dreadful colour.
5. The girls aren’t allowed to wear slacks.	6. The colour is drab.
7. The skirt is too long.	8. It would be better to have it all one colour rather than three different colours.
9. The shoes are too uncomfortable.	10. We should be allowed to wear leggings in winter.

Classify the responses into categories to identify the two main reasons given.

THINK

- 1 Since two main reasons have to be identified, look for, say, 3 categories which could be used to classify the data.
- 2 Classify the responses.
- 3 Identify the main reasons.

WRITE

Two main categories are apparent:

- 1 Style
- 2 Colour

A third category is not apparent.

Style
Responses 1, 5, 7, 9, 10

Colour
Responses 4, 6, 8

Responses 2 and 3 can be in a miscellaneous category.

The main reasons students dislike the uniform are its style and colour.

- It can be seen from Worked example 2 that it is probably worth taking the time to write closed questions for a questionnaire, as this reduces the incidence of answers that are difficult to classify.

13.2.4 Planning a questionnaire

- To prepare good questions you should keep the following points in mind.
 1. The questions should flow smoothly from one to the next.
 2. Introductory remarks should be included, outlining the aim and purpose of the questionnaire, along with any necessary instructions.
 3. Jargon, slang and abbreviations should be avoided.
 4. Do not ask questions that are vague or ambiguous.
 5. Avoid bias and emotional language.
 6. Avoid double-barrelled questions (i.e. questions that have two purposes or meanings).
 7. Do not ask leading questions (those that lead to an expected response).
 8. Make sure your questions are capable of being answered by your respondents.
 9. Avoid questions with double negatives.
 10. At the conclusion, thank the respondent for answering.

WORKED EXAMPLE 3

Identify the areas of concern with the following questions, then rewrite each so that the meaning is clear, unbiased and understandable.

- a Don't you agree that the school uniform should be changed?
- b Didn't you know that students are not allowed to go to the school canteen at morning break?
- c Those poor, distraught animals in the zoo should be running wild and not restricted in cages. Don't you agree?
- d How old are you?



THINK

- a This is a leading question — leading the respondent to agree. Avoid language like 'don't you know' when writing questions.
- b This question has two negative parts (didn't and not) causing confusion for the respondent.

WRITE

- a Are you in favour of a change in the school uniform?
- b Are you aware of the rule banning students from the school canteen at morning tea?

c This question plays on the emotions of the respondent by using language such as ‘poor’ and ‘distracted’. d People (particularly older people) often don’t like disclosing personal details.	c What’s your opinion about keeping animals in cages at zoos? d To which age group do you belong? • Less than 10 years • 10–20 years • 21–50 years • Over 50 years of age

13.2.5 Selecting samples

- When a sample is selected from a population, care must be taken to ensure that the composition of the sample is as close as possible to that of the population.
- One of the simplest ways to select a reliable sample is to using a random sampling technique.
- A way in which this can be achieved is to number each individual item (element) of the population, then randomly select items for the sample by using random digit tables, the random function on a calculator or (without looking) drawing numbers from a container.

WORKED EXAMPLE 4

As part of their quality control program, staff members at Perplexing Puzzles select 5 puzzles for testing from a batch of 50 coming off a production line. Use the following table of random numbers to select the 5 puzzles to be tested.

94, 01, 54, 68, 74	32, 44, 44, 82, 77	59, 82, 09, 61, 63	64, 65, 42, 58, 43
71, 10, 88, 82, 22	88, 57, 07, 40, 15	25, 70, 49, 10, 35	01, 75, 51, 47, 50
62, 88, 08, 78, 73	95, 16, 05, 92, 21	22, 30, 49, 03, 14	75, 87, 71, 73, 34
11, 74, 81, 21, 02	80, 58, 01, 18, 67	18, 71, 05, 96, 21	06, 55, 40, 78, 50
17, 94, 40, 56, 00	60, 47, 80, 33, 43	25, 85, 02, 89, 05	57, 21, 63, 96, 18

THINK

- 1 Assign each puzzle on the production line a number, according to its order, ranging from 1 to 50.
Note: 01 will represent the first puzzle on the production line.
- 2 Randomly select a starting point and move in a systematic order, jotting down 5 pairs of digits that range from 01 to 50.
Note: Reject any values that equal 00 or are greater than 50. Reject any repeated values.

WRITE

01 — First puzzle on the production line.
 02 — Second puzzle on the production line.
 03 — Third puzzle on the production line.
 ...
 ...
 50 — Fiftieth puzzle on the production line.

Starting position: fourth row, first block, fourth pair of digits; that is, 21.

21 accept
 02 accept
 80 reject since value is greater than 50
 58 reject since value is greater than 50
 01 accept
 18 accept
 67 reject since value is greater than 50
 18 reject since value is repeated
 71 reject since value is greater than 50
 05 accept

The puzzles numbered 21, 02, 01, 18 and 05 will be selected for testing.

13.2.6 Biased samples

- If a sample is not representative of the population, it is said to be biased.
- To avoid bias, the size of the sample should be about $\sqrt{\text{population size}}$.

WORKED EXAMPLE 5

Comment on the selection of the sample in the following cases.

- a** Members of the men's basketball team were interviewed regarding the height of the mirrors in the gym changing room.
- b** Spectators at a music concert were interviewed regarding the sporting facilities in the town.
- c** Of the 100 people in the bush walking group, 5 were interviewed regarding increasing the club's fees.

THINK

- a** Are those in the men's basketball team of typical height compared with those who use the gym?
- b** Those at a music concert probably would not use the sporting facilities very frequently.
- c** This sample is too small.

WRITE

- a** Those in the men's basketball team would be taller than most of the people who use the gym change room. They would prefer the mirrors to be higher. This is a biased sample.
- b** This is a biased sample as it does not include those who would make greatest use of the sporting facilities.
- c** This sample of 5 from the group of 100 is too small to represent the views of the whole group.

ACTIVITY: TAKING RANDOM SAMPLES FROM A POPULATION

Investigate the different ways in which random samples may be drawn from a census. For example, random samples could be drawn by gender, postcode or state.

Prepare a report on the different ways you have found, determining if any of them may produce biased samples.

learnon

RESOURCES — ONLINE ONLY

Interactivity: Random numbers (int-0089)

Interactivity: Collecting data (int-3807)

Interactivity: Questionnaires (int-3809)

Interactivity: Planning a questionnaire (int-3810)

Interactivity: Selecting samples (int-3811)

Interactivity: Biased samples (int-3812)

Digital doc: Investigation — Bias (doc-2473)

Digital doc: Investigation — Data and the Olympics (doc-7013)

Digital doc: Investigation — Collecting data for surveys and questionnaires (doc-2167)

Digital doc: SpreadSHEET Data and the Olympics (doc-7016)

Exercise 13.2 Samples and populations

Individual pathways

PRACTISE

Questions:
1–10, 15, 17

CONSOLIDATE

Questions:
1–9, 11, 12, 13, 16

MASTER

Questions:
1–19

Individual pathway interactivity: int-4451

learnON ONLINE ONLY

To answer questions online and to receive **immediate feedback** and **fully worked solutions** for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Understanding and fluency

- Copy and complete the following.
When we obtain data from the whole population, we conduct a _____; however, a survey obtains data from a _____ of the population.
- A school conducts an election for a new school captain. Every teacher and student in the school casts a vote. Is this an example of a census or a survey? Explain your answer.
- A questionnaire is conducted by the school council to see what library facilities the community needs. If 500 people who live in the community participate, is this an example of a census or a survey?
- WE1** For each of the following, state whether a census or a survey has been used.
 - Fifty people at a shopping centre are asked to nominate the supermarket where they do most of their grocery shopping.
 - To find the most popular new car on the road, 300 new-car buyers are asked what make and model they purchased.
 - To find the most popular new car on the road, the make and model of every new car registered are recorded.
 - To test the life of a light bulb, every 100th bulb is tested.
- For each of the following, recommend whether you would use a census or a survey to find:
 - the most popular TV program on Sunday night at 8.30 pm
 - the number of 4-wheel-drive cars sold in a year
 - the number of cars travelling on a toll road each day
 - the percentage of defective SIM cards produced by a mobile phone manufacturing company.
- Explain what you understand by the terms ‘open’ and ‘closed’ questions. Give an example of an open question, then rewrite your question in a closed format.
- WE2** Ten students were asked their opinions about the cause of congestion at the school’s front gate. Analyse their responses below, suggest categories into which they could be classified and identify the most commonly stated reasons for the congestion.
 - The cars shouldn’t come up the front driveway.
 - The front entrance is too small.
 - There should be another entrance.
 - The buses are the problem.
 - Bike riders should have a separate entrance.
 - Parents don’t care where they park.
 - Kids just sit around there talking.
 - The buses all arrive at the same time.
 - The road is not wide enough.
 - There should be a traffic control officer there to direct the traffic.



8. Fifteen students were asked to identify the one thing in their Maths class that they particularly didn't like. Classify the responses below into appropriate categories, then identify the main reasons.
1. It's too hard.
 2. There's too much homework.
 3. I can't understand the teacher.
 4. It's boring.
 5. The boys are too distracting.
 6. The teacher favours the boys.
 7. It's too much work.
 8. I can't concentrate for that length of time.
 9. We do something new every lesson.
 10. The teacher expects too much.
 11. I don't like working in groups.
 12. Our teacher is too strict.
 13. Our class is too big.
 14. The work is not interesting.
 15. I don't like doing presentations to the class.
9. **WE3** Identify the areas of concern with the following questions, then rewrite each so that the meaning is clear, unbiased and understandable.
- a. How much money do you earn?
 - b. Do you exercise regularly?
 - c. Which venue do you prefer — the SCG, the MCG, the Gabba or the WACA?
 - d. Do you support the Premier's policy on wildlife preservation?
 - e. How tall are you in feet and inches?
 - f. Did you buy your sneakers for comfort and quality?
10. Rewrite the following open questions in closed format.
- a. How much pocket money do you get each week?
 - b. How do you travel to school?
 - c. What type of destination do you prefer for a holiday?

Communicating, reasoning and problem solving

11. What issues do you think may occur if a sample size was very small, for example if only 5 people were surveyed out of a population of 500?
12. **WE5** Comment on the selection of the sample in the following cases.
- a. The choir members were interviewed regarding the opening hours of the school swimming pool.
 - b. Spectators at a football match were interviewed after the match to get their views on the Prime Minister's new health package.
 - c. Visitors arriving at Sydney's international airport were interviewed regarding the transport system from there into the city.
 - d. Twenty spectators at the Australian Tennis Open championships were interviewed regarding an appropriate time of the year to hold the event.
13. Use technology to select at random the following samples:
- a. 6 shoppers from 75 in a store
 - b. 4 babies from 30 born on a particular day
 - c. 6 products from 100 coming off a production line.
14. **WE4** As part of their quality control program, staff members at Perplexing Puzzles select 5 puzzles from a batch of 50 coming off a production line, for testing. Use the following table of random numbers to select the 5 puzzles to be tested.
- | | | | |
|--------------------|--------------------|--------------------|--------------------|
| 94, 01, 54, 68, 74 | 32, 44, 44, 82, 77 | 59, 82, 09, 61, 63 | 64, 65, 42, 58, 43 |
| 71, 10, 88, 82, 22 | 88, 57, 07, 40, 15 | 25, 70, 49, 10, 35 | 01, 75, 51, 47, 50 |
| 62, 88, 08, 78, 73 | 95, 16, 05, 92, 21 | 22, 30, 49, 03, 14 | 75, 87, 71, 73, 34 |
| 11, 74, 81, 21, 02 | 80, 58, 01, 18, 67 | 18, 71, 05, 96, 21 | 06, 55, 40, 78, 50 |
| 17, 94, 40, 56, 00 | 60, 47, 80, 33, 43 | 25, 85, 02, 89, 05 | 57, 21, 63, 96, 18 |



15. Use the table of random numbers given in question 14 to select at random the following samples:
- 3 people from a population of 99 (work horizontally from top left)
 - 4 students from a class of 60 (work horizontally from the bottom row of the second block)
 - 3 raffle tickets from a book of 50 (diagonally in the third block, starting at bottom left).
16. **MC** Using the table of random numbers from question 14, the sample '5 students from a group of 50' (working horizontally and beginning from the first pair of digits in row 2, block 3) would correspond to:
- A.** 25, 70, 49, 10, 35 **B.** 95, 16, 05, 92, 21 **C.** 16, 05, 21, 22, 30
D. 25, 49, 10, 35, 01 **E.** 09, 42, 43, 10, 22
17. Describe how you could use a standard pack of cards to select a random sample from a small population.



18. Are the random numbers generated by a computer or calculator truly random?
19. A survey of students was conducted to see whether they preferred watching sports or playing sports. Eighteen people preferred watching sports and 16 preferred playing sports. If 30 people completed the survey, and 2 people don't like to watch or play sports, how many like to do both?



13.3 Primary and secondary data

13.3.1 Primary data

- **Primary data** are data that you have collected yourself.
- This means that you have ownership of the data, and no one else has access to the data until it is released or published.
- A variety of methods of collecting primary data is available. These include observation, measurement, survey, experiment or simulation.

Observation

- Observation involves recording the behavioural patterns of people, objects and events in a systematic manner.
- The data can be collected as a disguised observation (respondents are unaware they are being observed) or undisguised observation (the respondents are aware). CCT cameras are an example of people knowing that their movements are being recorded, but not always being aware of where the recording takes place.
- Observations can be in a natural environment (e.g. in a food hall), or a contrived environment (a food-tasting session for a food company).
- Mechanical devices (video cameras, closed circuit television, counting devices across a road) can also be used.

Measurement

- Measurement involves using some measuring device to collect data.
- This generally involves conducting an experiment of some type.
 - The height of everyone in your class can be measured.
 - The mass of all newborn babies can be collected.
 - A pedometer measures the number of paces the wearer takes.

Surveys

- Surveys involve designing a questionnaire to interview people. Often the questionnaire requires many rewrites to obtain one which is clear and unbiased.
- The interview can be in person — face to face or by telephone. The advantage of this method is that you are able to see the reactions of those you are interviewing, and explain particular questions, if necessary.
- Most frequently these days, email is used; however, there are advantages and disadvantages to using this type of survey.
- Advantages:
 - Can cover a large number of people or organisations
 - A wide geographic coverage is possible
 - It avoids embarrassment on the part of the respondent
 - There is no interviewer bias
 - The respondent has time to consider responses
 - It's relatively cheap
- Disadvantages:
 - The questions have to be relatively simple
 - The response rate is often quite low (inducements are often given as an incentive to return)
 - The reliability of the answers is questionable
 - No control over who actually completes the questionnaire
 - Problems with incomplete questionnaires

ACTIVITY: CONSTRUCTING AND RUNNING A SURVEY

Design a survey to collect information about a topic in which you are interested. The survey should contain questions that collect both numerical and categorical data.

Once your survey has been designed carry out a small trial. If necessary, refine your survey questions after they have been trialled.

Finally decide whether a census or a sample is most appropriate to collect the required information to investigate your chosen topic.

Experiment

- Generally, when conducting an experiment the data collected are quantitative.
- Particular care should be taken to ensure the experiment is conducted in a manner that would produce similar results if repeated.
- Care must be taken with the recording of results.
- The results must be in a form that can readily be analysed.
- All results need to be recorded, including the weird or unexpected outcomes.

Simulation

- Experiments such as rolling a die, tossing a coin or drawing a card from a deck may be conducted to model some real-life situation.
- Simulations occur in areas such as business, engineering, medical and scientific research.
- Simulations are often used to imitate real-life situations, which may be dangerous, impractical or too expensive to explore by other means.

WORKED EXAMPLE 6

It is widely believed that there is equal chance of having a boy or girl with each birth. Genetics and the history of births in a family sometimes have a great influence on the sex of the child as well. Ignore those factors in this question.

- a Design an experiment to simulate the chance of giving birth to a boy or a girl.**
b Describe how your experiment could be conducted to determine the number of children a couple should have, on average, to ensure they have offspring of both sexes.

THINK

- a** Use a device that can simulate two outcomes which are equally likely. This could be a random number generator to generate two integers, say a 0 (representing a boy) and 1 (representing a girl). A fair coin could be tossed such that a Head represents a boy and a Tail represents a girl.

- b 1** Describe how the experiment will be conducted.

WRITE

- a** A fair coin will be tossed with a Head representing a boy (B) and a Tail representing a girl (G).

- b** The experiment will be conducted 50 times, and a record kept of each experiment. For each experiment, the coin will be tossed until both sexes result. This may mean that there could be, for example, 7 trials in an experiment (GGGGGB) before both sexes are represented.

2 Display the table of results.

The table below shows the results of the 50 experiments.

Exp. no.	Results	No. of trials	Exp. no.	Results	No. of trials
1	BG	2	26	GGGB	4
2	GGB	3	27	GGGGB	5
3	BG	2	28	GGGB	4
4	GGGGB	5	29	BG	2
5	BBBBBBG	7	30	BBBG	4
6	GGGB	4	31	BG	2
7	BBG	3	32	GB	2
8	BBG	3	33	GGGB	4
9	BBBBG	5	34	BG	2
10	GB	2	35	GGGGGGB	7
11	BG	2	36	BBBBBBG	7
12	GGGB	4	37	GB	2
13	BBG	3	38	BG	2
14	BBG	3	39	GGB	3
15	GB	2	40	GGGGB	5
16	BG	2	41	BBG	3
17	GGB	3	42	BBBBBBG	6
18	GB	2	43	GGB	3
19	GGB	3	44	GGB	3
20	BBBG	4	45	BBBG	4
21	BG	2	46	BBG	3
22	GB	2	47	GGGGGGB	7
23	GGGGB	5	48	BG	2
24	BG	2	49	BBG	3
25	GGGGB	5	50	GGGGB	6
				Total	175

This table shows that 175 trials were undertaken in 50 experiments where each experiment resulted in both sexes.

3 Determine the average number of children required to produce offspring of both sexes.

$$\begin{aligned}\text{Average number of children} &= \frac{175}{50} \\ &= 3.5\end{aligned}$$

4 Write a conclusion.

The average number of children a couple should have to reach the goal of having both sexes is 4.

- Before collecting any primary data, it must be clear what data are to be collected.
- A decision must be made as to the method of collection.
- The advantages and disadvantages of the collection method must be acknowledged.
- The reason for the data collection should be clear from the outset.

WORKED EXAMPLE 7

You have been asked to obtain primary data to determine the methods of transport the students at your school use to travel to school. The data collected are to provide support for the Student Council's proposal for a school bus.

- a What data should be collected?
- b Outline possible methods which could be used to collect these data.
- c Decide which method you consider to be the best option, and discuss its advantages and disadvantages.

THINK

a Outline the various forms of transport available to the students.

b Consider all the alternatives for collecting the data.

c 1 Decide on the best option.

2 Discuss the advantages and disadvantages.

WRITE

a The modes of transport available to students at the school are: car, bus, train, bicycle and walking

b Several methods could be used to collect the data.

- Could stand at the school gate one morning and ask students as they arrive.
- A questionnaire could be designed.
- Students could be asked to write their mode of transport on a piece of paper and place in a collection tin.

c The first option of standing at the school gate is very time-consuming, and students could arrive at the back gate. The third option does not seem reliable, as some students may not comply, and other students may place multiple pieces of paper in the collection tin. The second option seems the best of the three.

The advantages of a questionnaire include:

- There is a permanent record on paper.
- It is not time-consuming to distribute or collect.
- Students can complete it at their leisure.

Disadvantages include:

- Students may not return it.
- Expense involved in producing copies.

Note: This example does not represent the views of all those collecting such data. It merely serves to challenge students to explore and discuss available options.

- Sometimes the primary data required are not obvious at the outset of the investigation.
- For example, you are asked to investigate the claim:
Most students do not eat a proper breakfast before school.
What questions would you ask to prove or refute this claim?

13.3.2 Secondary data

- **Secondary data** are data that have already been collected by someone else.
- The data can come from a variety of sources:
 - Paper — books, journals, magazines, company reports
 - Electronic — online databases, internet, broadcasts, videos
 - Government sources — the Australian Bureau of Statistics (ABS) provides a wealth of statistical data

- General business sources — academic institutions, stockbroking firms, sporting clubs
- Media — newspapers, TV reports.
- Secondary data sources often provide data which would not be possible for an individual to collect.
- The data can be qualitative or quantitative.
- The accuracy and reliability of the data sometimes needs to be questioned, depending on its source.
- The age of the data should always be considered.
- Often the data which surrounds us passes by unnoticed.
- It is important to learn the skills to be able to critically analyse secondary data.

ACTIVITY: COLLECTING AND ANALYSING SECONDARY DATA

Collect a variety of pieces of secondary data in the forms of tables and graphs about topics that are of interest to you. The Australian Bureau of Statistics (ABS) website contains a lot of information that you might find useful for this activity.

Look at the scales used on the graphs. Are all the measurements written in full, or are abbreviations used?

Try to identify any features on the graphical displays that may be open to misinterpretation. These may be the labelling of axes or misleading units of measurement.

WORKED EXAMPLE 8

Bigbite advertise the energy and fat content of some of their rollaways on their placemats.

- What information can you gain from this data?
- Bigbite advertise that they have a range of rollaways with less than 6 grams of fat. Comment on this claim.
- This could be the starting point of a statistical investigation. How could you proceed from here?
- Investigations are not conducted simply for the sake of investigating. Suggest some aims for investigating further.



Bigbite fresh as rollaways	Energy (kJ)	Fat (g)	Sat. fat (g)
Roasted vegetable	900	3.0	1.0
HAM	1100	6.0	1.4
TURKEY	1140	4.8	1.7
BBQ BEEF	1150	5.0	1.5
Bigbite ribbon	1130	4.8	1.3
TURKEY AND HAM	1250	4.5	1.5
BBQ CHICKEN	1460	4.7	1.2
CHICKEN Tandoori	1110	4.0	1.0
Fresh as Dessert			
Fruit Slices	200	<1	<1

Bigbite rollaways
Regular rollaways include white and/or wholemeal bread, salads and meat.

Nutritional value is changed by adding cheese or sauces.



THINK

- Look at the data on the placemat to gain as much information as possible.

WRITE

- The placemat reveals the following information:
 - A higher energy content of a rollaway does not necessarily mean that its fat content is higher.
 - As the fat content of a rollaway increases, generally the saturated fat content also increases.
 - The addition of some types of protein (ham, turkey, beef, chicken) increases the energy content of the rollaway.
 - These data are only for those rollaways on white or wholemeal bread with salads and meat.
 - The addition of condiments (sauces) or cheese will alter these figures.
 - A fruit slice has much less energy and fat than a rollaway.

b Examine the data to discover if there is evidence to support the claim. Make further comment.	b All the rollaways displayed have less than 6 grams of fat, so Bigbite's claim is true. It must be remembered that the addition of cheese and sauce to these rollaways would increase their fat content. Also, if the rollaway was on any bread other than white or wholemeal, the fat content could go beyond 6 grams.
c What would be the next step in the investigation?	c The placemat displays a toll-free phone number for further information. Bigbite's web site also contains additional detailed information.
d What are some interesting facts which could be revealed through a deeper investigation?	d Suggested aims for investigating further could be: • How much extra fat is added to a rollaway by the addition of cheese and/or sauce? • What difference does a different type of bread make to the fat content of the rollaway? • Which rollaway contains the highest fat content? • What is the sugar content of the rollaways?

DISCUSSION

Discuss some of the difficulties that you may come across with obtaining data from either primary or secondary sources.

Consider the location of where the data would need to be collected from, the reliability of the data, what digital technologies might be needed and anything else that may influence the results.

learnon

RESOURCES — ONLINE ONLY

Interactivity: Primary and secondary data (int-3814)

Exercise 13.3 Primary and secondary data

Individual pathways

PRACTISE

Questions:
1–4, 6, 8, 14

CONSOLIDATE

Questions:
1–5, 7, 9–11, 13, 14

MASTER

Questions:
1a, d, g, 2, 3, 4–17

Individual pathway interactivity: int-4452

learnon ONLINE ONLY

To answer questions online and to receive **immediate feedback** and **fully worked solutions** for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Understanding and fluency

- 1. WE6** Devise an experiment to simulate each of the following situations and specify the device used to represent the outcomes.
 - a.** A true/false test in which answers are randomly distributed.
 - b.** A casino game with outcomes grouped in colours of either red or black.
 - c.** Breakfast cereal boxes containing 4 different types of plastic toys.

- d. In a group of six people, one person is to be chosen as the leader.
 - e. A choice of three main meals on a restaurant's menu, all of which are equally popular.
 - f. Five possible holiday destinations offered by a travel agent, such that all destinations are equally available and equally priced.
 - g. Five types of takeaway fast foods available in one area, where one pizza is twice as popular as each of the other types of takeaway food (the other 4 are equally popular).
2. **WE7** You have been asked to obtain primary data from students at your school to determine what internet access students have at home. The data collected are to provide support for opening the computer room for student use at night.
- a. What data should be collected?
 - b. Outline possible methods which could be used to collect this data.
 - c. Decide which method you consider to be the best option, and discuss its advantages and disadvantages.
3. **WE8** This label shows the nutritional information of Brand X rolled oats.

Nutrition Information			
Servings Per Package: 25		Serving Size 30g	
	Per Serving 30g	%DI* Per Serving	Per 100g
Energy	486kJ	6%	1620kJ
Protein	4.3g	9%	14.3g
Fat - Total	2.8g	4%	9.3g
- Saturated	0.5g	2%	1.7g
- Trans	Less than 0.1g	-	Less than 0.1g
- Polyunsaturated	1.0g	-	3.2g
- Monounsaturated	1.3g	-	4.4g
Carbohydrate	16.8g	5%	56.0
- Sugars	0.9g	1%	3.0g
Dietary Fibre	3.1g	10%	10.4g
Sodium	0.7mg	0.1%	2mg

* % DI = Percentage daily intake

- a. What information can you gain from this data?
 - b. This could be the starting point of a statistical investigation. How could you proceed from here?
 - c. Suggest some aims for investigating further.
4. a. Provide a list of methods you could use to collect primary data.
- b. Describe which method you would use to collect the following primary data.
- i. Heights of trees along the footpaths of a tree-lined street
 - ii. Number of buses that transport students to your school in the morning
 - iii. Sunrise times during summer
 - iv. Student opinion regarding length of lessons

For questions 5 and 6, design an experiment to simulate the situation, carry out the experiment and give the results of the experiment.

5. A mouse in a maze can make left or right turns at each junction. Assuming each turn is equally likely, how many junctions on average must the mouse go through before each type of turn will have been made?
6. A restaurant menu features 4 desserts which are assumed to be equally popular. How many dessert orders must be filled (on average) before the owner can be sure all types will have been ordered?



7. This label shows the nutritional information of Brand Y rolled oats.

Nutrition information		
Servings per package: 30		
Serving size: 30g		
	Avg. Quantity Per serving 30g	Avg. Quantity Per 100g
Energy	480kJ (115Cal)	1600kJ (383Cal)
Protein	3.2g	10.5g
Fat, total	2.4g	8.0g
- saturated	LESS THAN 1g	1.5g
Carbohydrate	18.3g	61.0g
- sugars	0.0g	0.0g
Dietary Fibre, total	3.3g	10.0g
Sodium	LESS THAN 5mg	LESS THAN 5mg
Ingredients		
Oats (100%)		
Attention		
THIS PRODUCT CONTAINS GLUTEN.		
Storage		
Store in a cool, dry place.		

- Compare the nutritional information with that on the Brand X label in question 3.
8. Comment on this claim.

We did a survey on 100 people regarding eating chocolate.
60 of these people said they regularly ate chocolate.
We then measured the heights of all 100 people.
***** The result *****
Eating chocolate makes you taller!!

9. Russel operates a computer software sales outlet. He keeps a log of all complaints from customers. Suggest how he could organise his log.
10. The following claim has been made regarding secondary data.
There's a lot more secondary data than primary data, it's a lot cheaper and it's easier to acquire.
Comment on this statement.

Communicating, reasoning and problem solving

11. Pizza King conducted a survey of their customers comparing 10 of their pizza varieties with their nearby competitors. After receiving and analysing the data they released an advertising campaign with the headline 'Customers rate our pizzas 25% better than the rest!'. The details in the small print revealed that this was based on the survey on their Hawaiian pizzas.
Was there anything wrong with Pizza King's claim?

12. The local Bed Barn was having a sale on selected beds by Sealy and SleepMaker. Four of the beds on sale were:

Sealy Posturepremier	on sale for \$1499	a saving of \$1000
Sealy Posturepedic	on sale for \$2299	a saving of \$1600
SleepMaker Casablanca	on sale for \$1199	a saving of \$800
SleepMaker Umbria	on sale for \$2499	a saving of \$1800

The store claimed that all these beds had been discounted by at least 40%. Comment on whether this statement is true, supporting your comments with sound mathematical reasoning.

13. Hamish is planning on running a stall as a fund-raiser selling ice-cream. There are 1000 students in his school ranging from Year 7 to Year 12. There are 5 Year 8 classes each of 25 students (boys and girls).

Hamish intends to ask a group of 10 students chosen at random from each of these 5 classes to select their favourite three ice-cream flavours. Hamish is confident that this random sampling method encompassing a total of 50 students should give him an accurate picture of the ice-cream preferences for the school. Is Hamish correct or is he facing a financial disaster?



14. Addison, a prospective home buyer, wishes to get up to date with the cost of a mortgage from financial institutions. She realises that there are a lot of lenders in the marketplace. Explain how she would collect the necessary information in the form of:
- primary data
 - secondary data.
15. Kirsty, chief marketing manager of FARMCO Cheeses, has decided to run a major TV advertising campaign.
- How should she choose a TV channel and time slot to run her advertisements?
 - How should she decide which demographic/age groups to target?
 - Does the answer to part **b** have any bearing on the answer to part **a**?
16. Can you think of any ethical issues that may arise from collecting and representing data from various sources?
17. When using secondary data from other countries, what different unit classifications could you encounter?

13.4 Organising and displaying data

13.4.1 Examining data

- Once collected, the data must be organised.
- When this has been done, any anomalies in the data will be highlighted.
- Before we rush into doing calculations, we must consider how to treat any anomalies. These could have occurred because of:
 - recording errors
 - unusual responses.
- Sometimes a decision is made just to disregard these anomalies, which are regarded as outliers.
- Outliers can greatly affect the results of calculations, as we shall see later.



13.4.2 Frequency tables

- Organising the raw data into a frequency table is the first step in allowing us to see trends in the data.

WORKED EXAMPLE 9

In a suburb of 350 houses, a sample of 20 households was surveyed to find the number of children in them. The data were collected and recorded as follows:

0, 2, 3, 2, 1, 3, 5, 2, 0, 1, 2, 0, 2, 1, 2, 1, 2, 3, 1, 0.

- Organise the data into a frequency table.
- Comment on the distribution of the data.
- What could you say about the number of children per household in the suburb?

THINK

- Draw up a frequency table and complete the entries.

WRITE

Children per household	Frequency
0	4
1	5
2	7
3	3
4	0
5	1

- Look at how the data are distributed.
- The data value of 5 appears to be an outlier. This is probably not a recording error, but it is not typical of the number of children per household. Most households seem to have 1 or 2 children.
- Does this sample seem to reflect the population characteristics?
- The sample is an appropriate size, and would probably reflect the characteristics of the population. It would be reasonably safe to say that most houses in the suburb contained 1 or 2 children.

- Sometimes there are too many data to treat as single entries, and it is necessary to group the data into **class intervals**.
- The choice for the size of class intervals should lead to between 5 and 10 groups being formed.
- Class intervals are set so that each score belongs to one group only.
- This means that it is not possible to have intervals such as 5–10, 10–20, ..., because an entry of 10 would then be in two class intervals.

WORKED EXAMPLE 10

A sample of 40 people was surveyed about the number of hours per week they spent watching TV. The results, rounded to the nearest hour, are listed below.

12, 18, 9, 17, 20, 7, 24, 16, 9, 27, 7, 16, 26, 15, 7, 28, 11, 20, 9, 11, 23, 19, 29, 12, 19, 12, 16, 21, 8, 4, 16, 20, 17, 10, 24, 21, 5, 13, 29, 26

- Organise the data into a frequency table using class intervals of 5–<10, 10–<15 etc. Show the midpoint of each class interval.
- Comment on the distribution of the data.



THINK

- a**
- 1 Draw up a frequency table with three columns: class interval (hours of TV), midpoint and frequency.
 - 2 The midpoint is calculated by adding the two extremes of the class interval and dividing by 2. For example, the midpoint of the first class interval is $\frac{5 + 10}{2} = 7.5$.
 - 3 Systematically go through the list, determine how many times each score occurs and enter the information into the frequency column.
- b** Look at how the data are distributed.

WRITE

- a**
- | Hours of TV | Midpoint | Frequency |
|-------------|----------|-----------|
| 5–<10 | 7.5 | 9 |
| 10–<15 | 12.5 | 7 |
| 15–<20 | 17.5 | 10 |
| 20–<25 | 22.5 | 8 |
| 25–<30 | 27.5 | 6 |
- b** The TV viewing times are fairly evenly distributed, but the most-frequent number of hours per week viewing was about 15 to 20 hours.

13.4.3 Column graphs

- Once a frequency table has been constructed from the data, it can be displayed in graphical form. The most important statistical displays are column graphs. A special type of column graph is called a **histogram**. It has the following characteristics.
 1. All columns are of equal width.
 2. No gaps are left between columns.
 3. Each column ‘straddles’ an x -axis score; that is, the column starts and finishes halfway between scores.
 4. Usually a half-interval is left at the beginning and end of the graph. That is, the first score is one unit in from the frequency (y)-axis.
- The half-interval is left at the beginning and end of the graph so there is space to draw a **frequency polygon**, as shown in Worked example 13.
- When drawing a histogram be careful to pick an appropriate scale for both the horizontal and vertical axes. Before you start to construct your histogram identify the smallest and largest values for both axes to help you pick an appropriate scale.
- Both axes must also be labelled on all histograms that you create. The vertical axis will usually be labelled ‘Frequency’.

WORKED EXAMPLE 11

Consider the frequency table created in Worked example 9.

- a** Display the data as a histogram.
b Comment on the shape of the graph.
c Are these comments consistent with those in Worked example 9?

Children per household	Frequency
0	4
1	5
2	7
3	3
4	0
5	1

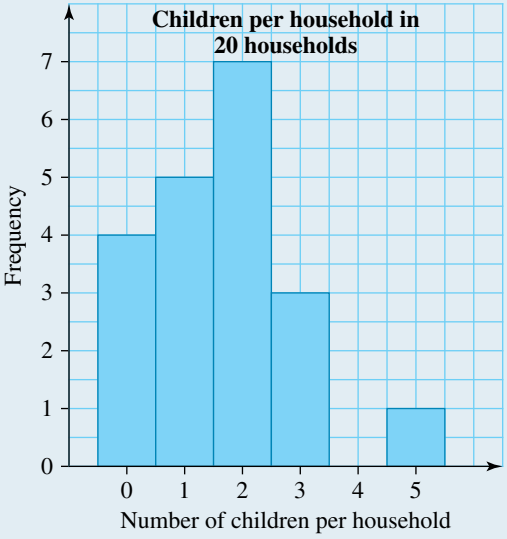
THINK

- a 1** Rule a set of axes on graph paper.
Give the graph a title. Label the horizontal axis 'Number of children per household' and the vertical axis 'Frequency'.

- 2** Leave a $\frac{1}{2}$ unit space at the beginning.
Draw the first column so that it starts and finishes halfway between each household, and reaches a vertical height of 4 units. Repeat this technique for the other scores.

- b** Look at how the data are distributed.
- c** Look for any similarities or differences between the observations in Worked example 9 and those made here.

WRITE/DRAW

- a**
- 
- b** The data show that the score of '5 children per household' seems to be an outlier, and that the majority of households seem to have 1 or 2 children.
- c** These comments are consistent with those made in Worked example 9.

- When presenting grouped data graphically, we label the horizontal axis (score) with either:
 - the class interval, or
 - the midpoint of each class interval.

WORKED EXAMPLE 12

Consider the grouped frequency table created in Worked example 10.

Hours of TV	Midpoint	Frequency
5–<10	7.5	9
10–<15	12.5	7
15–<20	17.5	10
20–<25	22.5	8
25–<30	27.5	6

- a** Display the data as a histogram.
- b** Comment on the shape of the graph.

THINK

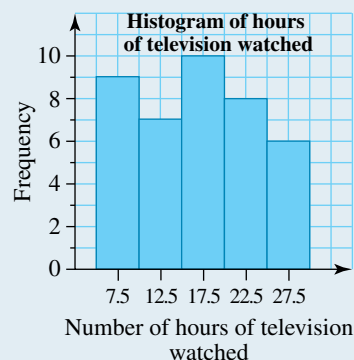
- a 1** Rule a set of axes on graph paper. Give the graph a title. Label the horizontal axis 'Number of hours of television watched' and the vertical axis 'Frequency'.

- 2** Leaving a $\frac{1}{2}$ unit space at the beginning, draw the first column so that it starts and finishes halfway between class intervals, and reaches a vertical height of 9 units. Repeat this technique for the other scores.

- b** Look at how the data are distributed.

WRITE/DRAW

a



- b** The hours of TV watching is fairly consistent throughout the week. A maximum number of people watch about 15 to 20 hours per week.

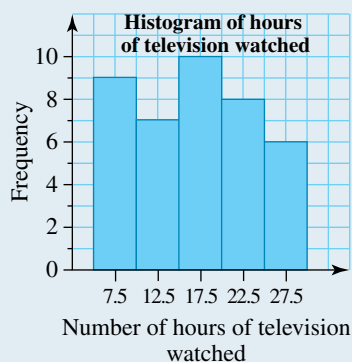
- If we join the midpoints of the tops of the columns of a histogram, then extend the ends to the x -axis, we form what is called a frequency polygon.

WORKED EXAMPLE 13

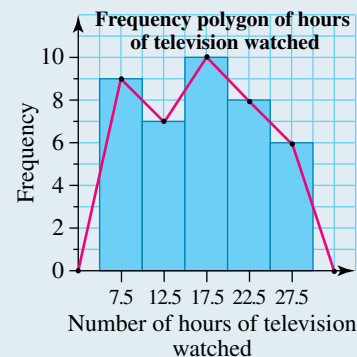
Use the histogram created in Worked example 12 to draw a frequency polygon of the data.

THINK

- 1 Mark the midpoints of the tops of the columns of the histogram.
- 2 Join the midpoints with straight lines.
- 3 Close the polygon by drawing lines at each end down to the x -axis.



WRITE/DRAW



- The procedure for dealing with a large amount of data, organising it into class intervals, then producing a histogram and a frequency polygon can be seen in this next example.

WORKED EXAMPLE 14

The following data are the results of testing the lives (in hours) of 100 torch batteries.

20, 31, 42, 49, 46, 36, 42, 25, 28, 37, 48, 49, 45, 35, 25, 42, 30, 23, 25, 26,
29, 31, 46, 25, 40, 30, 31, 49, 38, 41, 23, 46, 29, 38, 22, 26, 31, 33, 34, 32,
41, 23, 29, 30, 29, 28, 48, 49, 31, 49, 48, 37, 38, 47, 25, 43, 38, 48, 37, 20,
38, 22, 21, 33, 35, 27, 38, 31, 22, 28, 20, 30, 41, 49, 41, 32, 43, 28, 21, 27,
20, 39, 40, 27, 26, 36, 36, 41, 46, 28, 32, 33, 25, 31, 33, 25, 36, 41, 28, 33

- Choose a suitable class interval for the given data and present the results in a frequency distribution table.
- Draw a histogram of the data.
- Add a polygon to the histogram.

THINK

- Choose a suitable size for the class interval.
- Obtain the range for the given data; that is, subtract the smallest value from the largest.
- Divide the results obtained for the range by 5 and round to the nearest whole number.
Note: A class interval of 5 hours will result in 6 groups.
- Draw a frequency table and list the class intervals in the first column, beginning with the smallest value.
Note: The class interval 20–<25 includes hours ranging from and including 20 to less than 25.
- Systematically go through the data and determine the frequency of each class interval.
- Calculate the total of the frequency column.

WRITE

a

$$\begin{aligned}\text{Range} &= \text{largest value} - \text{smallest value} \\ &= 49 - 20 \\ &= 29\end{aligned}$$

$$\begin{aligned}\text{Number of class intervals: } \frac{29}{5} &= 5.8 \\ &= 6\end{aligned}$$

Lifetime (hours)	Tally	Frequency (f)
20–<25		12
25–<30		23
30–<35		20
35–<40		16
40–<45		13
45–<50		16
	Total	100

- b 1** Rule and label a set of axes on graph paper.
Give the graph a title.

- 2** Add scales to the horizontal and vertical axes.
Note: Leave a half interval at the beginning and end of the vertical axis.

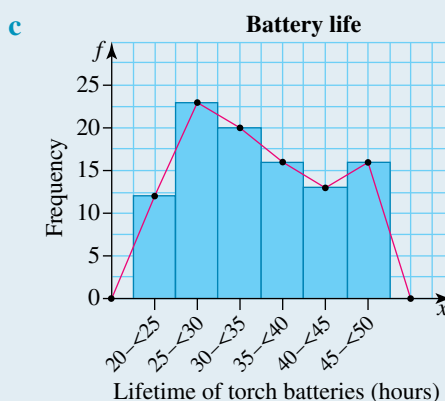
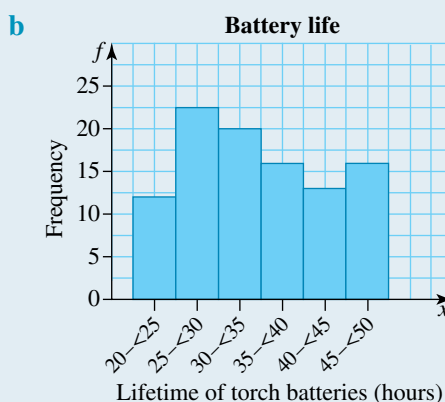
- 3** Draw in the first column so that it starts and finishes halfway between class intervals and reaches a vertical height of 12 units.

- 4** Repeat step 3 for each of the other scores.

- c 1** Mark the midpoints of the tops of the columns obtained in the histogram from part **b**.

- 2** Join the midpoints by straight line intervals.

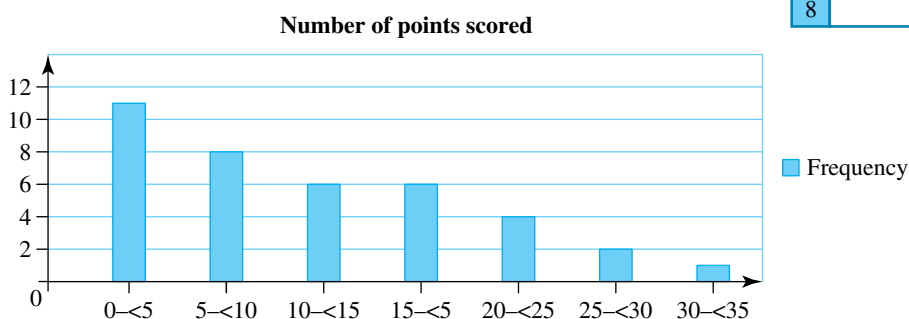
- 3** Close the polygon by drawing lines at each end down to the class interval (x -) axis.



13.4.4 Using a spreadsheet to tabulate and graph data

- Spreadsheets such as Excel can also be used to easily tabulate and graph data.
- Enter the data values or class intervals into the first column and frequencies into the second column, as shown.
- Highlight all of the cells containing data and click on 'Insert' to choose a data display of your choice. You can change the title of your graph as well as the font and the colours.

	A	B
1	Number of points scored	Frequency
2	0-5	11
3	5-10	8
4	10-15	6
5	15-20	6
6	20-25	4
7	25-30	2
8	30-35	1



- This will draw a simple bar chart. There is also a plug-in available which can help you to create histograms. Ask your teacher for help or search online for an appropriate tutorial.

Interactivity: Frequency tables (int-3816)

Interactivity: Column graphs (int-3817)

Digital doc: SkillsHEET Presenting data in a frequency table (doc-6989)

Digital doc: SpreadSHEET Histograms and frequency polygons (doc-2183)

Digital doc: WorkSHEET Representing and interpreting data I (doc-7005)

Exercise 13.4 Organising and displaying data

Individual pathways

PRACTISE

Questions:
1–7, 9, 12

CONSOLIDATE

Questions:
1, 2, 4, 6, 8, 10–12

MASTER

Questions:
1, 2c, 3, 5, 7, 10–13

Individual pathway interactivity: int-4453

learnon ONLINE ONLY

To answer questions online and to receive **immediate feedback** and **fully worked solutions** for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Understanding and fluency

- WE9,11,13** In a suburb of roughly 1500 houses, a random sample of 40 households were chosen, and surveyed to find the number of children in each of them. The data were collected and recorded as follows.
0, 3, 2, 4, 1, 2, 3, 2, 2, 2, 2, 1, 3, 4, 5, 2, 3, 1, 1, 1,
0, 0, 2, 3, 4, 1, 3, 4, 2, 2, 0, 1, 2, 3, 2, 0, 2, 4, 5, 1
 - Comment on the sample.
 - Organise the data into a frequency table.
 - Display the data as a combined histogram and frequency polygon.
 - Comment on the distribution of the data and the shape of the graph.
 - What could you say about the number of children per household in the suburb?
- Consider the following frequency tables.

a.

x	f
1	4
2	3
3	5
4	0
5	2
6	6
Total	

b.

x	f
150	2
151	6
152	9
153	18
154	11
155	3
Total	

c.

x	f
50	3
52	3
54	2
56	2
58	1
60	5
Total	

For each table:

- draw a combined histogram and frequency polygon to suit the data
 - comment on the shape of the graph.
- A quality control officer selected 25 boxes of MP3 players at random from a production line. She tested every MP3 player in each box and displayed the number of defective MP3 players in each box as follows:
1, 3, 2, 5, 2, 2, 1, 5, 2, 1, 2, 4, 3, 0, 5, 3, 2, 1, 3, 2, 1, 3, 4, 2, 1.



- Comment on the sample.
 - Organise the data into a frequency table.
 - Comment on the distribution of the data.
 - What could you say about the population of MP3 players?
4. This table shows the number of hours of sport played per week by a group of Year 8 students.

Score (hours of sport played)	Frequency (f)
1	3
2	8
3	10
4	12
5	16
6	8
7	7
Total	64



- Draw a histogram and frequency polygon to display the data.
 - Comment on the shape of the graph.
 - Discuss whether you feel this sample reflects the sporting habits of Year 8 students generally.
5. **WE10,12,13** A block of houses in a suburb was surveyed to find the size of each house (in m^2). The results are shown in the following table.

Size of house (m^2)	Frequency
100–<150	13
150–<200	18
200–<250	19
250–<300	17
300–<350	14
350–<400	11
Total	92

- Draw a histogram and frequency polygon to display the data.
 - Comment on the shape of the graph.
 - Discuss whether you feel this sample reflects the size of the houses in the suburb.
6. Forty people joined a weight-loss program. Their mass (in kg) was recorded at the beginning of the program and is shown in the frequency table at right.
- Draw a histogram and frequency polygon to display the data.
 - Comment on the shape of the graph.
 - Discuss whether you feel this sample reflects the mass of people in the community.

Class interval	Frequency
60–<70	2
70–<80	5
80–<90	9
90–<100	12
100–<110	7
110–<120	3
120–<130	2
Total	40

7. **WE14** Forty people in a shopping centre were asked about the number of hours per week they spent watching TV. The result of the survey is shown below.

10, 13, 7, 12, 16, 11, 6, 14, 6, 11, 5, 14, 12, 8, 27, 17, 13, 8, 14, 10,
13, 7, 15, 10, 16, 8, 18, 14, 21, 28, 9, 12, 11, 13, 9, 13, 29, 5, 24, 11

- Organise the data into class intervals of 5–<10 hours etc., and draw up a frequency table.
 - Draw a histogram and frequency polygon to display the data.
 - Comment on the shape of the graph.
 - Discuss whether you feel this sample reflects the TV viewing habits of the community.
8. A Year 8 class sat for a 10-question multiple choice test. These are their results.
2, 6, 5, 9, 8, 7, 3, 6, 9, 4, 8, 8, 6, 7, 6, 4, 7, 8, 7, 8, 6, 7, 8, 5, 3, 9, 2, 6, 5, 8
- Organise the data into a frequency table.
 - Display the data as a combined histogram and frequency polygon.
 - Comment on the performance of the class in the test.
9. The number of hours of sleep during school week nights for a Year 8 class are recorded below.
6, 9, 7, 8, 7, $8\frac{1}{2}$, $6\frac{1}{2}$, 8, $7\frac{1}{2}$, $7\frac{1}{2}$, 8, $8\frac{1}{2}$, $6\frac{1}{2}$, 8, 8, 7, $7\frac{1}{2}$, 8, 9, 8
- Organise the data into a frequency table.
 - Display the data as a histogram.
 - Comment on the sleeping habits of the Year 8 students.
 - Discuss whether you feel these sample results reflect those of Year 8 students generally.

Communicating, reasoning and problem solving

10. The amount of pocket money (in dollars) available to a random sample of 13-year-olds each week was found to be as shown below.

10, 15, 5, 4, 8, 10, 4, 15, 5, 6, 10, 6, 5, 10, 8, 10, 5, 10, 10, 6

- Organise the data into a frequency table.
 - Display the data as a histogram.
 - Comment on the shape of the histogram.
 - Discuss whether you feel these sample results reflect those of 13-year-olds generally.
11. Data were collected on the number of times people go to the cinema in a month.
- Put the data in a frequency table.
 - Draw a histogram to represent the data.
 - How many people go to the cinema less than three times a month?
 - How many people go to the cinema at least three times a month?
 - Is it reasonable to draw conclusions about the whole population based on this sample? Give reasons for your answer.

4, 5, 7, 9, 1, 2, 5, 2, 4, 8, 3, 6, 2, 3, 8, 1, 1, 4,
5, 3, 3, 6, 1, 2, 7, 1, 3, 2, 2, 4, 10, 0, 1, 3, 4, 6



Problem solving

12. The following data give the results of testing the lives (in hours) of 100 torch batteries.

25, 36, 30, 34, 21, 40, 36, 46, 29, 38, 20, 41, 34, 45, 25,
40, 31, 39, 24, 45, 27, 44, 23, 35, 47, 49, 20, 37, 43, 26,
35, 28, 48, 30, 20, 36, 41, 26, 32, 42, 21, 31, 45, 42, 26,
37, 33, 24, 45, 38, 36, 43, 21, 34, 38, 35, 28, 41, 30, 22,
29, 32, 39, 25, 44, 21, 35, 38, 41, 35, 30, 23, 37, 43, 33,
34, 28, 39, 22, 31, 35, 42, 38, 27, 36, 46, 28, 34, 37, 29,
24, 30, 39, 44, 31, 24, 36, 28, 47, 21



- a. Choose a suitable class interval for the given data, and present the results in a frequency distribution table.
 - b. Draw a histogram and frequency polygon of the data.
 - c. Comment on the trends shown by the histogram.
 - d. Discuss whether you feel these results reflect those of the battery population.
13. A random sample of 30 students in Year 9 undertook a survey to investigate the height of Year 9 students. These were their measured heights (in cm).
- 146, 163, 156, 168, 159, 170, 152, 174, 156, 163, 157, 161, 178, 151, 148,
167, 162, 157, 166, 154, 150, 166, 160, 155, 164, 157, 171, 168, 158, 162
- a. Organise the data into class intervals $145\text{--}<150$ cm etc., and draw up a frequency distribution table.
 - b. Draw a histogram displaying the data.
 - c. Reorganise the class intervals to be $145\text{--}<148$ cm etc., and construct a new frequency distribution table.
 - d. Draw a new histogram displaying the data in part c.
 - e. Comment on the similarities and differences between the two histograms.

13.5 Measures of centre and spread

13.5.1 Measures of central tendency

- Simple calculations based on collected data can help give us typical values, or values that show how the data **cluster**.
- These typical values are commonly referred to as averages. They are called **measures of centre** or measures of location.
- We will look at 3 different types of averages used in interpreting data: **mean**, **median** and **mode**.

13.5.2 Mean

- The mean or average of a set of scores is the sum of all the data values divided by the number of data values.
- It is denoted by the symbol $\bar{x}$ (pronounced x bar).

WORKED EXAMPLE 15

Jan's basketball scores were 18, 24, 20, 22, 14 and 12. What was his mean score? Calculate your answer, correct to 1 decimal place.



THINK

- Calculate the sum of the basketball scores.
- Count the number of basketball scores.

WRITE

$$\begin{aligned}\text{Total score} &= 18 + 24 + 20 + 22 + 14 + 12 \\ &= 110\end{aligned}$$

$$\text{Number of scores} = 6$$

3 Define the rule for the mean.

$$\text{Mean} = \frac{\text{sum of data values}}{\text{number of data values}}$$

4 Substitute the known values into the rule.

$$\bar{x} = \frac{110}{6}$$

5 Evaluate, rounding to 1 decimal place.

$$= 18.333\ 33\dots$$

Note: The mean is often not one of the given scores.

$$= 18.3$$

- Sometimes calculations need to be performed from a frequency distribution table.

WORKED EXAMPLE 16

Calculate the mean of the frequency distribution data given below.

Score (x)	1	2	3	4	5
Frequency (f)	3	2	4	0	5

THINK

1 Rearrange the rows as columns and include an extra column headed: 'Score $\times$ frequency ($x \times f$)'.

2 Enter the information into the third column; that is, the score of 1 occurred 3 times.

Therefore, $x \times f = 1 \times 3 = 3$.

The score of 2 occurred 2 times.

Therefore, $x \times f = 2 \times 2 = 4$.

Continue this process for each pair of data.

3 Determine the total of the 'Frequency' column. This shows how many scores there are altogether.

4 Determine the total of the 'Score $\times$ frequency' column. This shows the overall value of all the scores.

5 Define the rule for the mean.

$$\text{Mean} = \frac{\text{total of score} \times \text{frequency values}}{\text{total frequency values}}$$

6 Substitute the known values into the rule.

$$\bar{x} = \frac{54}{15}$$

7 Evaluate the answer to 1 decimal place.

$$= 3.6$$

Note: The mean is often not one of the given scores.

WRITE

Score (x)	Frequency (f)	Score $\times$ frequency ($x \times f$)
1	3	$1 \times 3 = 3$
2	2	$2 \times 2 = 4$
3	4	$3 \times 4 = 12$
4	0	$4 \times 0 = 0$
5	1	$5 \times 1 = 5$
6	5	$6 \times 5 = 30$
Total	15	54

13.5.3 Median

- The median is the middle score for an odd number of scores and the average of the two middle scores for an even number of scores.
- Alternatively, if a set of data contains n scores, the median is given by the $\frac{n+1}{2}$ th score.
- To obtain the median, the scores must be arranged in numerical order.

WORKED EXAMPLE 17

Find the median of each of the following sets of scores.

a 10, 8, 11, 5, 17

b 9, 3, 2, 6, 3, 5, 9, 8

THINK

a 1 Arrange the values in ascending order.

2 Select the middle value.

Note: There is an odd number of scores; that is, 5.

Hence, the third value is the middle number or median.

Alternatively, the rule $\frac{n+1}{2}$, where $n = 5$, gives the position of the median. The location of the median is $\left(\frac{5+1}{2} = 3\right)$; that is, the 3rd score.

3 Answer the question.

b 1 Arrange the values in ascending order.

2 Select the middle values.

Note: There is an even number of scores; that is, 8.

Hence, the fourth and fifth values are the middle numbers, or median.

Again the rule $\frac{n+1}{2}$ could be used to locate the position of the median.

3 Obtain the average of the two middle values.

4 Answer the question.

Note: The median in this case is not one of the actual scores.

WRITE

a 5, 8, 10, 11, 17

The median of the scores is 10.

b 2, 3, 3, 5, 6, 8, 9, 9

$$\begin{aligned}\text{Median} &= \frac{5+6}{2} \\ &= \frac{11}{2} \\ &= 5\frac{1}{2} \text{ (or } 5.5\text{)}\end{aligned}$$

The median of the scores is $5\frac{1}{2}$ or 5.5.

Note: For sets of data containing an odd number of scores, the median will be one of the actual scores, while for the sets with an even number of scores, the median will be positioned halfway between the two scores.

13.5.4 Mode

- The mode is the most common score in a set of data.
- It is the score with the highest frequency. It measures clustering of scores.
- Some sets of scores have more than one mode or no mode at all. There is no mode when all values occur an equal number of times.

WORKED EXAMPLE 18

Find the mode of each of the following sets of scores.

a 5, 7, 9, 8, 5, 8, 5, 6

b 10, 8, 11, 5, 17

c 9, 3, 2, 6, 3, 5, 9, 8

THINK

- a** 1 Look at the set of data and circle any values that have been repeated.
 2 Choose the values that have been repeated the most.
 3 Answer the question.
- b** 1 Look at the set of data and circle any values that have been repeated.
 2 Answer the question.
Note: No mode is not the same as having a mode which equals 0.
- c** 1 Look at the set of data and circle any values that have been repeated.
 2 Choose the values that have been repeated the most.
 3 Answer the question.

WRITE

- a** ⑤, 7, 9, ⑧, ⑤, ⑧, ⑤, 6
 The number 5 occurs 3 times.
 The mode for the given set of values is 5.
- b** 10, 8, 11, 5, 17
 No values have been repeated.
 The following set of data has no mode, since none of the scores correspond to a highest frequency. Each number occurs only once.
- c** ⑨, ③, 2, 6, ③, 5, ⑨, 8
 The number 3 occurs twice. The number 9 occurs twice.
 The modes for the given set of values are 3 and 9.

DISCUSSION

When someone quotes an average, how do you know which measure of spread they used?

WORKED EXAMPLE 19

The data from a survey asking people how often per week they purchased takeaway coffee from a café are shown below.

2, 9, 11, 8, 5, 5, 5, 8, 7, 4, 5, 3

Use the data to calculate each of the following.

- a** The mean number of coffees purchased per week
b The median number of coffees purchased per week
c The modal number of coffees purchased per week

THINK

- a** 1 To calculate the mean, add all the values in the data set and divide by how many data values there are.
 There are 12 values in the data set.

WRITE

$$\begin{aligned}\bar{x} &= \frac{\text{sum of all the values}}{\text{number of values}} \\ &= \frac{2 + 9 + 11 + 8 + 5 + 5 + 5 + 8 + 7 + 4 + 5 + 3}{12} \\ &= \frac{72}{12} \\ &= 6\end{aligned}$$

- 2 Answer the question.

The mean number of coffees purchased per week is 6.

- b 1** The median is the middle value by position. There are 12 values in the data set, so the middle position is between the 6th and 7th values.

$$\begin{aligned}\text{Median position} &= \frac{n+1}{2} \\ &= \frac{12+1}{2} \\ &= \frac{13}{2} \\ &= 6.5\end{aligned}$$

- 2** Arrange the data set in order from lowest to highest.
The 6th value is 5.
The 7th value is 5.

$$\begin{aligned}2, 3, 4, 5, 5, 5, 5, 7, 8, 8, 9, 11 \\ \text{Median} &= \frac{5+5}{2} \\ &= \frac{10}{2} \\ &= 5\end{aligned}$$

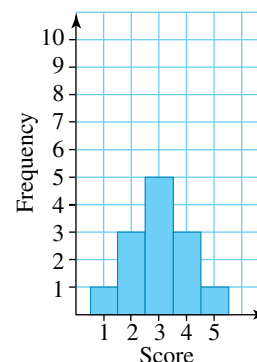
- 3** Answer the question.

The median number of coffees purchased per week is 5.

- c** The mode is the most common value in the data set. The most common value is 5.

The modal number of coffees purchased per week is 5.

- If a histogram is symmetrical then we are able to identify the mean and median directly from the histogram, as they will be equal to the middle value of the histogram.
- For example, in the histogram at right the mean and the median are both equal to 3. We can also see that the mode is 3.



13.5.5 Measures of spread

- In analysing a set of scores, it is helpful to see not only how the scores tend to cluster, or how the middle of the set looks, but also how they spread or scatter.
- Two classes may have the same average mark, but the spread of scores may differ considerably.
- The **range** of a set of scores is the difference between the highest and lowest scores.
- The range is a **measure of spread**.

WORKED EXAMPLE 20

Find the range of the following sets of data.

- a** 7, 3, 5, 2, 1, 6, 9, 8

THINK

- a 1** Obtain the highest and lowest values.
- 2** Define the range.
- 3** Substitute the known values into the rule.
- 4** Evaluate.
- 5** Answer the question.

b

<i>x</i>	7	8	9	10
<i>f</i>	1	3	5	2

WRITE

- a** Highest value = 9
Lowest value = 1
- Range = highest value – lowest value
- = 9 – 1
- = 8
- The set of values has a range of 8.

- b 1** Obtain the highest and lowest values.
Note: Consider the values only, not the frequencies.
- 2** Define the range.
- 3** Substitute the known values into the rule.
- 4** Evaluate.
- 5** Answer the question.

b Highest value = 10
 Lowest value = 7
 $\text{Range} = \text{highest value} - \text{lowest value}$
 $= 10 - 7$
 $= 3$
 The frequency distribution table data have a range of 3.

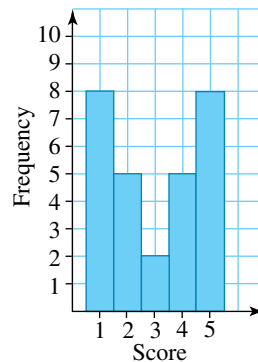
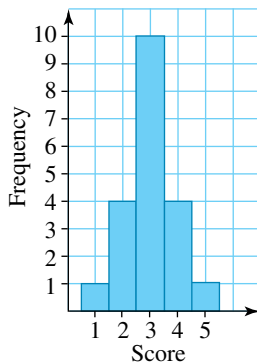
- Although the range identifies both the lowest and highest scores, the spread of the scores within the range is frequently not uniform.

ACTIVITY: CLASS SURVEY

- In groups, survey the class on one of the following areas: hours of sleep per night, number of people in the family, length of right foot, arm span and number of pieces of fruit eaten in a week.
- Collect the class data in frequency tables. Use class intervals if necessary.
- Calculate the following statistics:
 - mean
 - median
 - mode
 - range.
- What is the difference between the mean, the median and the mode?
- Discuss your results with the class. What does the range tell us?
- Construct a histogram to represent your data and collate the class data to display in the classroom.

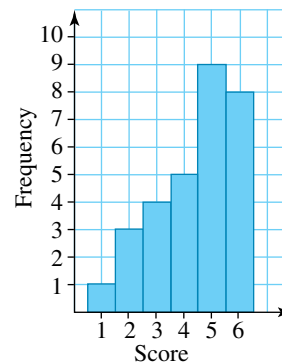
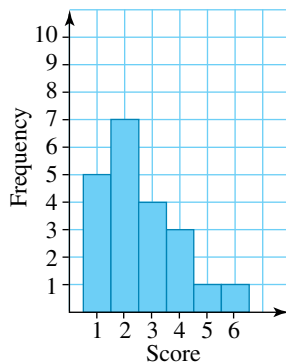
13.5.6 Clusters, gaps and outliers

- By looking at a visual representation of a data set we can identify any clusters, gaps and **outliers** in the data set.
- A cluster is a grouping of data points that are close together.
- Consider the following two histograms.

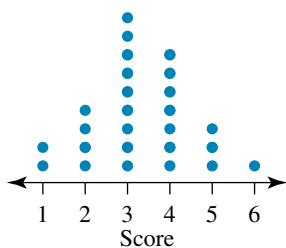
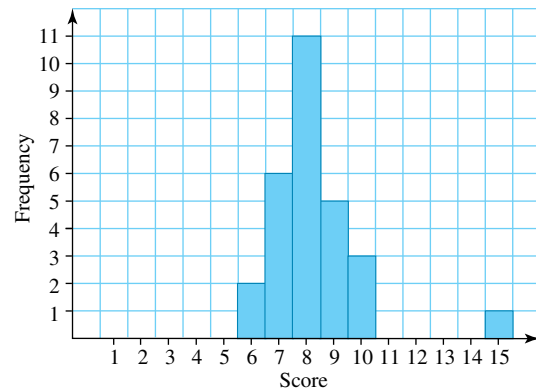


- In the first histogram the mean, median and mode of the data set is 3 and the data is clustered around the mean.
- In the second histogram the data is not clustered and there are two modes — one at either end of the distribution. The mean and median of this data set are also 3, but the modes are now 1 and 5.

- Data can also be clustered at either the lower end or the upper end of a distribution, as shown in the following histograms.



- If there is a gap in the data set and a data point appears on its own then we consider this data point to be an outlier.
- An outlier is a data value that is considerably different from the rest of the values in a set of data.
- Looking at the histogram at right, we can see that the data value 15 is an outlier.
- Clusters, gaps and outliers can also be identified in other data representations, including stem and leaf diagrams and dot plots.



Key: 1 | 1 = 11

Stem	Leaf
1	2 4 6
2	0 1 3 7
3	2 3 4 4 6 7 8
4	0 1 5
5	
6	
7	8

13.5.7 The effect of outliers on measures of centre and spread

- The presence of one or more outliers may have a considerable effect on the measures of centre and spread of a particular set of data.

WORKED EXAMPLE 21

A netball team scored the following amount of points in 10 games:

17, 23, 31, 19, 50, 29, 16, 23, 30, 32.

- Calculate the mean, median, mode and range of the team scores.
- The following Saturday, the regular goal shooter was ill, and Lauren, who plays in a higher division, was asked to play. The team's score for that game was 200.
Recalculate the mean, median, mode and range after 11 games.
- Comment on the similarities and differences between the two sets of summary statistics.

THINK	WRITE
a 1 Rearrange the scores in order.	a 16, 17, 19, 23, 23, 29, 30, 31, 32, 50
2 Calculate the mean.	$\frac{16 + 17 + 19 + 23 + 23 + 29 + 30 + 31 + 32 + 50}{10} = 27$
3 Calculate the median	$\frac{23 + 29}{2} = 26$
4 Calculate the mode.	23 appears the most times.
5 Calculate the range.	$50 - 16 = 34$
6 Write the answer.	Mean = 27 Median = 26 Mode = 23 Range = 34
b 1 Rearrange the scores in order.	b 16, 17, 19, 23, 23, 29, 30, 31, 32, 50, 200
2 Calculate the mean.	$\frac{16 + 17 + 19 + 23 + 23 + 29 + 30 + 31 + 32 + 50 + 200}{11} = 43$
3 Calculate the median	The middle value of the data set is 29.
4 Calculate the mode.	23 appears the most times.
5 Calculate the range.	$200 - 16 = 184$
6 Write the answer.	Mean = 43 Median = 29 Mode = 23 Range = 184
c Compare the two sets of summary statistics.	The inclusion of an extreme value or outlier has dramatically increased the mean and the range of the data, marginally increased the median and left the mode unchanged.

Note: The important point to learn from Worked example 21 is that when a set of data includes extreme values, the mean may not be truly representative of the data.

13.5.8 Which measure of centre is most useful?

- It is important to know which measure of centre will be most useful in a given situation.
 - The mean is appropriate when no extreme values or outliers distort the picture.
 - The median is appropriate when outliers are present.
 - The mode is appropriate when the most common result is significant.

Summary of measures of centre		
Mean	Median	Mode
Uses all the scores as a total, divided by the number of scores. It is affected by exceptionally large or small scores.	The score in the exact middle of the values placed in numerical order. It tells us nothing about other values. It is unaffected by exceptionally large or small values.	The most common value or category. It tells us nothing about the rest of the data. Data may have no mode, one mode or more than one mode.

Interactivity: Measures of centre (int-2362)

Interactivity: Mean (int-3818)

Interactivity: Median (int-3819)

Interactivity: Mode (int-3820)

Interactivity: Range (int-3822)

Interactivity: Outliers (int-3821)

eLesson: Mean and median (eles-1905)

Digital doc: SkillsHEET Finding the mean of ungrouped data (doc-6991)

Digital doc: SkillsHEET Arranging a set of data in ascending order (doc-6992)

Digital doc: SkillsHEET Finding the median (doc-6993)

Digital doc: SkillsHEET Finding the score in a data set that occurs most frequently (doc-6995)

Digital doc: SkillsHEET Finding the middle score for data arranged in a dot plot (doc-6994)

Digital doc: Investigation — How many red M&Ms? (doc-2166)

Digital doc: SpreadSHEET Mean (doc-2184)

Digital doc: SpreadSHEET Median (doc-2185)

Digital doc: SpreadSHEET Mode (doc-2187)

Exercise 13.5 Measures of centre and spread

Individual pathways

PRACTISE

Questions:

1–17, 19, 20, 22–26, 33, 34, 36

CONSOLIDATE

Questions:

1–26, 29, 33–36

MASTER

Questions:

1–28, 30–34, 36, 38–41

Individual pathway interactivity: int-4454

learnon ONLINE ONLY

To answer questions online and to receive **immediate feedback** and **fully worked solutions** for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Understanding and fluency

- WE15** Caroline's basketball scores were 28, 25, 29, 30, 27 and 22. What was her mean score? Give the answer correct to 1 decimal place.
- Find the mean (average) of each set of the following scores. Give the answers correct to 2 decimal places.
 - 1, 2, 3, 4, 7, 9
 - 2, 7, 8, 10, 6, 9, 11, 4, 9
 - 3, 27, 14, 0, 2, 104, 36, 19, 77, 81
 - 4, 8.4, 6.6, 7.0, 7.5, 8.0, 6.9
- Francesca's soccer team has the following goals record this season:
2, 0, 1, 3, 1, 2, 4, 0, 2, 3.
 - What total number of goals have they scored?
 - How many games have they played?
 - Find the team's average score.
- MC** Frisco's athletics coach timed 5 consecutive 200-m training runs. He recorded times of 25.1, 23.9, 24.8, 24.5 and 27.3 seconds. His mean 200-m time (in seconds) is:
A. 24.60 **B.** 25.20 **C.** 25.12 **D.** 25.42 **E.** 26.12



5. Two Year 8 groups did the same Mathematics test. Their results out of 10 were:

Group A: 5, 8, 7, 9, 6, 7, 8, 5, 4, 2

Group B: 5, 6, 4, 5, 9, 7, 8, 8, 9, 7.

Which group had the highest mean?

6. **WE16** Calculate the mean of this frequency distribution, correct to 2 decimal places.

Score (x)	1	2	3	4	5
Frequency (f)	4	3	6	1	0

7. Calculate the mean of this frequency distribution, correct to 2 decimal places.

Score (x)	6	7	8	9	10
Frequency (f)	2	8	3	6	2

8. **WE17a** Find the median of the following scores.

a. 5, 5, 7, 12, 13

b. 28, 13, 17, 21, 18, 17, 14

9. **WE17b** Find the median of each of the following sets of scores.

a. 2, 52, 46, 52, 48, 52, 48

b. 4, 1.5, 1.7, 2.0, 1.8, 1.5, 1.7, 1.8, 1.9

10. **WE18** For each set of scores in questions 8 and 9, find the mode.

Questions 11 and 12 refer to the following set of scores.

1, 1, 1, 4, 4, 5, 5, 6, 3, 3, 7, 6, 5, 4, 6, 2, 1, 8

11. **MC** The median of the given scores is:

A. 1

B. 4.5

C. 4

D. 5

E. 8

12. **MC** The mode of the given scores is:

A. 5

B. 6

C. 4

D. 3

E. 1

13. A third Year 8 group had the following results in the same test as in question 5:

5, 7, 8, 4, 6, 8, 5, 9, 8.

a. What is the average score of this group, correct to 1 decimal place?

b. What must a tenth student (who was originally absent) score to bring this group's average to 7?

14. A survey of the number of occupants in each house in a street gave the following data:

2, 5, 1, 6, 2, 3, 2, 4, 1, 2, 0, 2, 3, 2, 4, 5, 4, 2, 3, 4.

Prepare a frequency distribution table with an $x \times f$ column and use it to find the mean number of people per household.

15. The mean of 5 scores is 7.2.

a. What is the sum of the scores?

b. If four of the scores are 9, 8, 7 and 5, what is the fifth?

16. **WE19** Over 10 matches, a soccer team scored the following number of goals: 2, 3, 1, 0, 4, 5, 2, 3, 3, 4.

a. What was the most common number of goals scored?

b. What was the median number of goals scored?

c. In this case, does the mode or the median give a score that shows a typical performance?

17. The following scores represent the number of muesli bars sold in a school canteen each day over two weeks:

54, 64, 51, 58, 56, 59, 10, 34, 48, 56.

a. Calculate the mean.

b. Calculate the median.

c. Calculate the mode.

d. Which of the mean, median or mode best represents a typical day's sales in the school canteen? Explain.

18. A small business pays the following annual wages (in thousands of dollars) to its employees:

18, 18, 18, 18, 26, 26, 40, 80.

a. What is the mode of the distribution?

b. What is the median wage?



- c. What is the mean wage?
 d. Which measure would you expect the employees' union to use in wage negotiations?
 e. Which might the boss use in such negotiations?

19. **WE20a** Find the range of the following scores.

a. 5, 5, 7, 12, 13

b. 28, 13, 17, 21, 18, 17, 14

c. 2, 52, 46, 52, 48, 52, 48

d. 4, 1.5, 1.7, 2.0, 1.8, 1.5, 1.7, 1.8, 1.9

20. **WE20b** Find the range of the following sets of data.

a.

<i>x</i>	6	7	8	9	10
<i>f</i>	1	5	10	7	3

b.

<i>x</i>	1	2	3	4	5	6
<i>f</i>	7	9	6	8	10	10

c.

<i>x</i>	5	10	15	20
<i>f</i>	1	5	10	7

d.

<i>x</i>	110	111	112	113	114
<i>f</i>	2	2	2	3	3

21. State the range of each of the following data sets.

a. Key: 1 | 5 = 15

Stem	Leaf
1	1 2 7 8 9
2	2 8
3	1 3 7 9
4	0 1 2 6

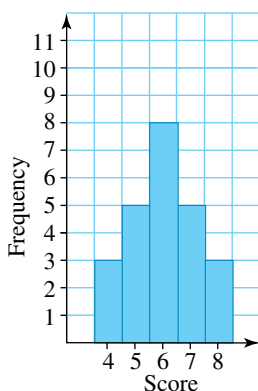
b. Key: 24 | 7 = 247

Stem	Leaf
24	2 7
25	2 4 6 6 8
26	0 1 3 5 9
28	5 6 6 8

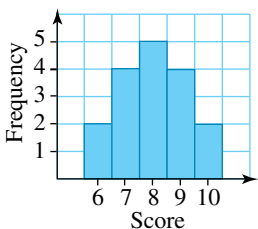
c. Key: 17 | 4 = 174

Stem	Leaf
15	6 2 4
16	8 6 1 3 9
17	0 2 1 8 6 7 3 4
18	4 1 5 2 7 1

22. Calculate the mean, median, mode and range of the data shown in the following histogram.

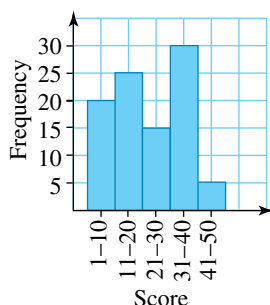


23. The figure below shows the distribution of a set of scores.

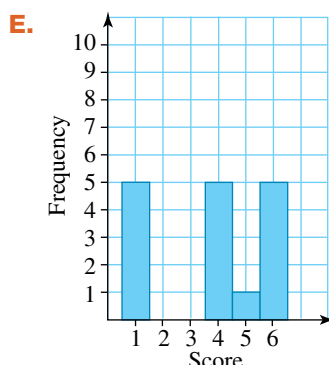
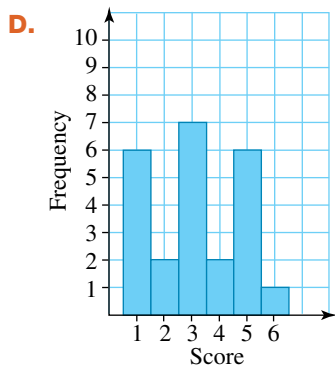
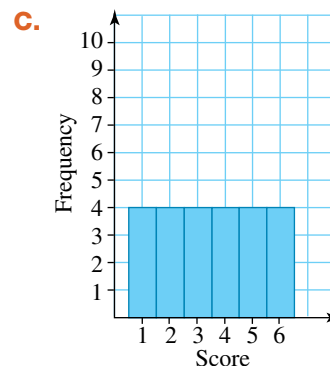
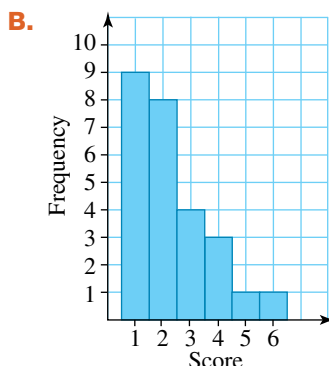
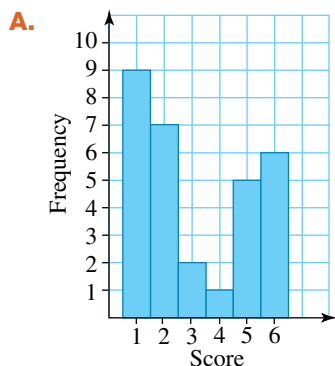


- a. What is the mode?
 b. Are there any outliers in the data set? If so state their value(s).

24. Consider the following distribution. What is the modal class?



25. **MC** In which of the following data sets is the data clustered?



26. **WE21** A rugby team scored the following amount of points in 10 games:

24, 18, 33, 29, 22, 16, 38, 30, 26, 30

- Calculate the mean, median, mode and range of the team scores.
- The following Saturday the team played the bottom team on the ladder, who had lost their previous 35 games. The team's score for that game was 86. Recalculate the mean, median, mode and range after 11 games.

c. Comment on the similarities and differences between the two sets of summary statistics.



Communicating, reasoning and problem solving

27. Determine which measure of centre is most appropriate to use in the following situations. Explain your answers.

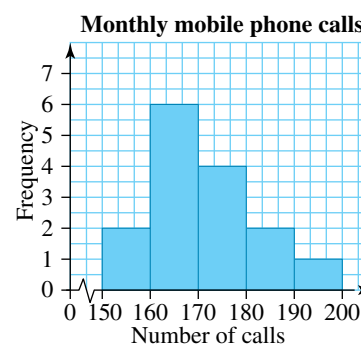
- Analysing property values in different suburbs of Sydney.
- Determining the average shoe size sold at a department store.
- Determining the average number of tries scored over a season of rugby league.

28. Find five numbers that have a mean of 10 and a median of 12.
29. The mean of 5 different test scores is 15. What are the largest and smallest possible test scores, given that the median is 12? All test scores are whole numbers. Justify your answer.
30. The mean of 5 different test scores is 10. What are the largest and smallest possible values for the median? All test scores are whole numbers. Justify your answer.
31. The mean of 9 different test scores that are whole-number percentages is 85. The median is 80. What is the greatest possible range between the highest and lowest possible test scores? Justify your answer.
32. Consider a set of integer values with a mean of 15 and a median of 15.5.
- If another number is added that has the effect of lowering the mean to 14.9, what happens to the median? Give reasons for your answer.
 - If another number is added that has the effect of raising the mean to 15.1, what happens to the median? Give reasons for your answer.
33. The following scores represent data from an online survey asking about the average number of hours students spent exercising each week:
3, 5, 1, 4, 0, 8, 23, 4, 2, 0, 2, 6.
- Identify the potential error in the data set.
 - Could this data value have been a genuine outlier? Explain your answer.
34. The temperatures of Sydney during the first 20 days of July were as follows:
17, 18, 18, 15, 19, 18, 17, 16, 18, 18, 19, 20, 14, 17, 18, 36, 19, 24, 25, 17.
- Identify the potential error in the data set.
 - Could this data value have been a genuine outlier? Explain your answer.
35. The data above right show the number of hours Year 8 students used a computer in a particular week.
- Calculate the mean, median and mode.
 - What do you notice about the value of the mean compared with the median?
 - What do you notice about the value of the mean compared with the mode?
 - Which is the best indicator of the centre of the data set?
36. Identify the mean, median and mode in the following paragraph:
It was an amazing game of cricket today. The winning team hit more sixes than any other number of runs. This meant that, even though the middle value of runs per over was 3, the sixes brought the average up to about five runs per over. What an incredible game!
37. Calculate the mean number of DVDs presented in this frequency table, using the median of each interval as the x -value for the interval.

Number of DVDs (x)	Frequency (f)
1–15	3
16–30	9
31–45	8
46–60	11
61–75	10
76–90	14
91–105	15
106–120	18

38. Calculate the mean number of calls made on mobile phones in the month shown in the graph at right.

5, 3, 6, 7, 3, 5, 2, 5, 2, 3, 6, 7



39. Draw an example of a dot plot that is clustered with an outlier.
40. Why is it more appropriate to use the median than the mean when a data set contains one or more outliers?
41. If the mean of six items of data is 24 and five of the items are 19, 26, 22, 30 and 27, find the sixth item.

13.6 Analysing data

13.6.1 Analysing data sets

- You are now armed with the skills to calculate measures of centre and spread for a data set, and to construct and analyse various forms of graphs.
- It must be remembered that most of the data we analyse are the results of surveys conducted on samples of the population.
- It is appropriate, then, that we consider data collected from a survey in light of the population from which the sample originated.
- To understand what information the data give, and perhaps to draw conclusions from it, we must appreciate what each statistical measure does.

Statistical measures	Definition and purpose
Mode	The most common score or category. It tells us nothing about the rest of the data. Data may have no mode, one mode or more than one mode.
Median	The score in the exact middle of the values placed in numerical order. It tells us nothing about the rest of the data. It is unaffected by exceptionally large or small scores.
Mean	The sum of all the scores divided by the number of scores. It is affected by exceptionally large or small scores.
Range	The difference between the highest score and the lowest score. It shows how far the scores are spread apart. It is particularly useful when combined with the mean or the median.

ACTIVITY: ANALYSING DATA FROM OTHER SUBJECTS

Look for data from one of your other school texts. For example, your geography text or atlas may contain information about sustainable food production or population statistics. Calculate and compare sample statistics for the data you have found.

WORKED EXAMPLE 22

Explain which statistical measure is referred to in these statements.

- The majority of people surveyed prefer Activ-8 sports drink.
- The ages of fans at the Rolling Stones concert varied from 8 to 80.
- The average Australian family has 2.5 children.

THINK

- Write the statement and highlight the key word(s).
- Relate the highlighted word to one of the statistical measures.

WRITE

- The majority of people surveyed prefer Activ-8 sports drink.
Majority implies most, which refers to the mode.

3 Answer the question. b 1 Write the statement and highlight the key word(s). 2 Relate the highlighted word to one of the statistical measures. 3 Answer the question. c 1 Write the statement and highlight the key word(s). 2 Relate the highlighted word to one of the statistical measures. 3 Answer the question.	This statement refers to the mode. b The ages of fans at the Rolling Stones concert varied from 8 to 80. The statement refers to the range of fans' ages at the concert. This statement refers to the range. c The average Australian family has 2.5 children. The statement deals with surveying the population (census) and finding out how many children are in each family. This statement refers to the mean.

13.6.2 Using sample properties to predict those of the population

- Once survey data has been collated and analysed, we can use the data set to predict the characteristics of the population from which it was taken. Consider this example.

WORKED EXAMPLE 23

The 153 students in Year 8 all sat for a 10-question multiple choice test as a practice test for an upcoming exam. A random sample of the results of 42 of the students gave this distribution.

Score (x)	1	2	3	4	5	6	7	8	9	10
Frequency (f)	2	3	6	7	11	8	4	0	0	1

- Calculate the mean mark.
- Determine the median mark.
- Give the modal mark.
- Which measure of centre best represents the data?
- Comment on any prediction about the properties of the population from this sample.

THINK

- Add a third column to the table to calculate the total of all the marks. Use the total to then calculate the mean.

WRITE

a

Mark (x)	Frequency (f)	$f \times x$
1	2	2
2	3	6
3	6	18
4	7	28
5	11	55
6	8	48
7	4	28
8	0	0
9	0	0
10	1	10
Total	42	195

$$\begin{aligned}\text{Mean} &= \frac{\text{Total of all scores}}{\text{Number of scores}} \\ &= \frac{195}{42} \\ &= 4.6\end{aligned}$$

- b** The median is the score in the middle; that is, the $\frac{42 + 1}{2}$ th score — the average of the 21st and 22nd score. Add a cumulative frequency column to the original frequency distribution. (Find the total frequency at that point for each mark.) Look in the cumulative frequency column to see where the 21st and 22nd scores lie.

b

Mark (x)	Frequency (f)	Cumulative frequency
1	2	2
2	3	$2 + 3 = 5$
3	6	$5 + 6 = 11$
4	7	$11 + 7 = 18$
5	11	$18 + 11 = 29$
6	8	$29 + 8 = 37$
7	4	$37 + 4 = 41$
8	0	$41 + 0 = 41$
9	0	$41 + 0 = 41$
10	1	$41 + 1 = 42$
Total	42	42

The 21st and 22nd scores are both 5.

The median is 5.

- c** The modal mark is the one that occurs most frequently. Look for the one with the highest frequency.
- d** Compare the results for the mean, median and mode. Look for similarities and differences.
- e** Consider whether these results from the sample would reflect those of the population.
- c** The mode is 5.
- d** The mean is 4.6, the median is 5 and the mode is 5. It seems that any of these measures of centre would be appropriate to use as a measure of centre of the data. However, check the outlier of 10. When the outlier is disregarded, the mean is calculated to be $185 \div 41 = 4.5$. It seems that the outlier in this case does not have a great effect on the mean. So, the mean, median or mode could be used as a measure of the centre of the data.
- e** It seems likely that these results would reflect those of the whole population. The sample is random, and of sufficient size. The outlier of 10 indicates that there would be a few students with full marks, and at least half the students passed the test.

- If we take more than one sample drawn from the same population then each sample we take will vary in different ways. This means that the summary statistics we derive can vary from sample to sample.
- There may be significant differences between two samples from a population if they are chosen from different subsets of a population. For example, if you were collecting data on the heights of people and one sample consisted only of boys and a second sample consisted only of girls, the summary statistics would vary significantly.

13.6.3 Using a spreadsheet to tabulate and graph data

- We can use spreadsheets such as Excel to calculate summary statistics.
- Enter the data values into a column as shown in column B, rows 2 to 9, at right.
- To calculate the mean, use the formula ‘=AVERAGE(’ and then select all of the cells containing your data. Close the brackets and the mean will be calculated.
- To calculate the median, use the formula ‘=MEDIAN(’ and then select all of the cells containing your data. Close the brackets and the median will be calculated.
- To calculate the mode, use the formula ‘=MODE(’ and select all of the cells containing your data. Close the brackets and the mode will be calculated.

Note: If there is more than one mode, this method will only display one of the modes, so it is always a good idea to double check to see if there is more than one mode in your data set.

- To calculate the range, use the formula ‘=MAX(’ and then select all of the cells containing your data. Close the brackets and enter ‘-MIN(’ and again select all of the cells containing your data. Close the brackets and the range will be calculated.

	A	B
1		Scores
2		5
3		10
4		7
5		5
6		8
7		12
8		6
9		11
10		
11	Mean	8
12	Median	7.5
13	Mode	5
14	Range	7

ACTIVITY: USING A SPREADSHEET TO CALCULATE AND COMPARE SUMMARY STATISTICS

Pick a topic to investigate from the population of students at your school; for example, the ages of their parents. Take three different samples from your population.

Enter your collected data into a spreadsheet and calculate the summary statistics for each of your different samples.

Compare the summary statistics for your three samples and comment on the similarities and differences.

learnon RESOURCES — ONLINE ONLY

Digital doc: WorkSHEET Representing and interpreting data II (doc-7012)

Exercise 13.6 Analysing data

Individual pathways

PRACTISE

Questions:
1–7, 11, 13, 15

CONSOLIDATE

Questions:
1–7, 13–17, 19

MASTER

Questions:
1–21

 Individual pathway interactivity: int-4456

learnon ONLINE ONLY

To answer questions online and to receive **immediate feedback** and **fully worked solutions** for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Understanding and fluency

1. **WE22** Explain which statistical measure is referred to in these statements.
 - a. There was a 15°C temperature variation during the day.
 - b. Children at this school are absent 3.4 days per semester, on average.
 - c. Most often you have to pay \$79.95 for those sports shoes.
 - d. The average Australian worker earns about \$470 per week.
 - e. A middle-income family earns about \$35 000 per annum.



2. **WE23** This frequency table shows the results of a random sample of 15 students (from a class of 30) who sat for a 10-question multiple choice test.
 - a. Calculate the mean mark.
 - b. Determine the median mark.
 - c. Give the modal mark.
 - d. Which measure of centre best represents the data?
 - e. Comment on any prediction of properties of the population from this sample.

Score (x)	Frequency (f)
4	1
5	2
6	5
7	4
8	3
Total	15

3. Consider the following frequency distribution tables.

a.

x	1	2	3	4	5
f	4	3	2	1	0

b.

x	6	7	8	9	10
f	2	8	3	4	2

For each one:

- i. calculate the mean score to 1 decimal place
- ii. determine the median score
- iii. give the modal score
- iv. indicate which measure of centre best describes the distribution.

4. Consider the following frequency stem plots.

- a. Key: 1 | 0 = 10

Stem	Leaf
1	0 2
2	1 3 3 5
3	
4	4

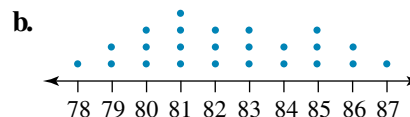
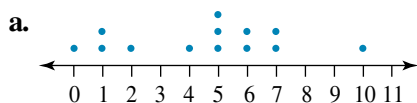
- b. Key: 10 | 0 = 100

Stem	Leaf
10	0
11	0 2 2 2
12	0 4 6 6
13	3

For each one:

- i. calculate the mean score to 1 decimal place
- ii. determine the median score
- iii. give the modal score
- iv. indicate which measure of centre best describes the distribution.

5. Consider the following dot plots.



For each one:

- i. calculate the mean score to 1 decimal place
- ii. determine the median score
- iii. give the modal score
- iv. indicate which measure of centre best describes the distribution.

6. Three different samples looked at the average yearly incomes of NSW households. The figures given are in \$000s and have been rounded to the nearest whole number.

Sample 1: 97, 135, 52, 106, 189, 158, 70, 81, 122, 69

Sample 2: 102, 131, 85, 204, 77, 85, 114, 90, 111, 126

Sample 3: 66, 89, 110, 90, 173, 77, 129, 166, 256, 98

- Calculate the mean, median, mode and range for each sample.
- Compare the summary statistics for the three different samples.

7. A survey of the number of people in each house in a street produced these data:
2, 5, 1, 6, 2, 3, 2, 1, 4, 3, 4, 3, 1, 2, 2, 0, 2, 4.

- Prepare a frequency distribution table with an $f \times x$ column and use it to find the average (mean) number of people per household, correct to 1 decimal place.
- Draw a dot plot of the data and use it to find the median number per household.
- Find the modal number per household.
- Which of the measures would be most useful to:
 - a real estate agent renting out houses
 - a government population survey
 - an ice-cream mobile vendor?



8. The contents of 20 packets of matches were counted after random selection and the following numbers obtained: 138, 139, 139, 141, 137, 140, 137, 141, 139, 142, 140, 141, 141, 139, 141, 138, 139, 140, 141, 138.

- Compile a frequency distribution table, including an $f \times x$ column.
- Find the mode, median and mean of the distribution. Give answers correct to 1 decimal place where necessary.
- Which of the 3 measures best supports the manufacturer's claim that there are 140 matches per box?
- Display the data as a combined histogram and frequency polygon.



9. A class of 26 students had a median mark of 54 in Mathematics; however, no-one actually obtained this result.

- Explain how this is possible.
- Explain how many must have scored below 54.

10. A soccer team had averaged 2.6 goals per match after 5 matches. After their sixth match, the average had dropped to 2.5. How many goals did they score in that latest match?



11. A tyre manufacturer selects 48 tyres at random from the production line for testing. The total distance travelled during the safe life of each tyre is shown in the table.

Distance in km ('000)	82	78	56	52	50	46
Number of tyres	2	4	10	16	12	4

- Calculate the mean, median and mode.
- Recalculate the mean with the 6 longest-lasting tyres removed. By how much is it lowered?
- If you selected a tyre at random, what tyre life would it most likely have?
- In a production run of 10 000 tyres, how many could be expected to last for a maximum of 50 000 km?
- As the manufacturer, for what distance would you be prepared to guarantee your tyres? Why?

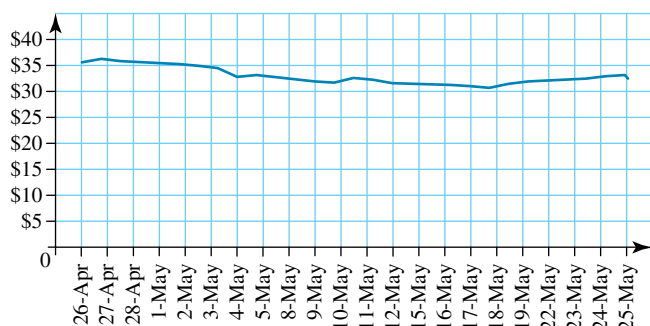
Communicating, reasoning and problem solving

12. The following graph displays the movement of the price of BankSave shares over a 30-day period.

The close price of the shares on the 22 days of trading, rounded to the nearest dollar, were:

35, 36, 36, 35, 35, 34, 33, 33, 32, 32, 33, 32, 32, 31, 31, 31, 32, 32, 33, 33, 33, 33

Calculate the mean (correct to 2 decimal places), median, mode and range of the share prices.



13. At a preview cinema session, the ages of the viewers were recorded and displayed in a stem-and-leaf plot at right. For these data, find the:

- a. range c. median
b. mean d. mode.

Key: 1 | 6 = 16 years

Stem	Leaf
1	5 6 7 7 8 9 9
2	1 2 4 8 8
3	0 1 1 1 5
4	2 3
5	3

14. Create a data set that fits each of the following descriptions.

- a. Five data values with a mean of 3 and a mode of 3
b. Five data values with a mean of 3 and a mode of 4
c. Five data values with a mean of 3 and a median of 2

15. Read the following paragraph and explain what statistics are represented and what they mean.

It's been an exciting day at the races today. There was a record fast time of 38 seconds, and also a record slow time of 4 minutes and 52 seconds. We had an unbelievable number of people who ran the race in exactly 1 minute. Despite this, the average time was well over 2 minutes, due to the injury of a few runners.

16. If you take more than one sample from the same population, why will the summary statistics vary from sample to sample?
17. Give three reasons why different samples from the same population may have different summary statistics.
18. When taking summary statistics of a sample to represent a population, why is it important that the sample is representative of the whole population? Use an example in your explanation.
19. Write a story to represent the following statistics: mean = 45.6, median = 34, mode = 24 and range = 58.
20. The number of goals a netballer scored in the 12 games of a season was as follows:
1, 1, 1, 1, 2, 2, 2, 3, 3, 3, 8, 12.
A local newspaper reporter asked the netballer what his average was for the season.
- a. Which measure of centre (mean, median or mode) should the netballer give the reporter as his 'average' so that the value of the average is as high as possible?
- b. Which measure of centre would you choose to best describe the 'average' number of goals the netballer scored each game? Why?
21. Why are measures of centre important statistical measures for businesses?

13.7 Review

Investigation | Rich task

Analysing the English language



In the English language, our alphabet contains 26 letters that combine to form words. Have you ever wondered why some letters appear more often than others in many of the words we read? Scrabble® is a game that allows players to form interlocking words in a crossword style. The words are formed using lettered tiles that carry a numerical value. Players compete against each other to form words of the highest score until all the tiles have been used.

The following table displays the letter distribution and the value of each letter tile in the game of Scrabble.

Letter	Number of tiles	Letter score	Letter	Number of tiles	Letter score	Letter	Number of tiles	Letter score	Letter	Number of tiles	Letter score
A	9	1	B	2	3	C	2	3	D	4	2
E	12	1	F	2	4	G	3	2	H	2	4
I	9	1	J	1	8	K	1	5	L	4	1
M	2	3	N	6	1	O	8	1	P	2	3
Q	1	10	R	6	1	S	4	1	T	6	1
U	4	1	V	2	4	W	2	4	X	1	8
Y	2	4	Z	1	10	Blank	2	0			

1. Which letters are most common in Scrabble and which are least common?
2. What do you notice about the relationship between the number of tiles used for each letter and their letter score?

This passage about newspapers and magazines is taken from an English textbook.

3. Complete a frequency table for the distribution of letters in this passage.
4. How do the results from this frequency table compare with the results from the Scrabble game's frequency table?

Who reads news?

Newspapers and magazines are produced for different categories of readers. These are known as the 'target audience'. Particular groups of readers buy certain types of newspapers and magazines because these publications have content which interests them as a reader. Magazines and newspapers cover different types of news; for example, a local paper contains information on local issues whereas a magazine like *Movie* has all the latest news on films and film stars. General newspapers like *The Age* and *The Australian* try to appeal to a broader audience by including sections on various topics such as sport, business and property.

Knowing which letters in the English language are most common can be very helpful when we are trying to solve coded messages. The following paragraph is written in code.

Tnlmktebt bl ehvtmxw bg max lhnmaxkg axfbliaxkx tgw bl
ftwx ni hy lbq lmtmxl tgw mph mxkkbmhkbxl. Max vtibmte hy
Tnlmktebt bl Vtguxkkt.

5. Study the coded message carefully. Use an appropriate method to decode the message.
6. Explain the strategies you used to decode the message.
7. Using different codes, write your own messages on a separate page and test them on your classmates.

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Digital doc: Investigation — Analysing the English language (doc-13943)

Digital doc: Code puzzle — An important discovery in 1921 (doc-13944)

Digital doc: Summary Topic 13 (doc-23249)

Interactivity: Word search Topic 13 (int-2759)

Interactivity: Crossword Topic 13 (int-2760)

Interactivity: Sudoku Topic 13 (int-3191)

Exercise 13.7 Review questions

To answer questions online and to receive **immediate feedback** and **fully worked solutions** for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Understanding and fluency

1. For each of the following statistical investigations, state whether a census or a survey has been used.
 - a. The average price of petrol in Sydney was estimated by averaging the price at 40 petrol stations.
 - b. The Australian Bureau of Statistics has every household in Australia complete an online questionnaire or information form every five years.
 - c. The performance of a cricketer is measured by looking at his performance in every match he has played.
 - d. Public opinion on an issue is sought by a telephone poll of 2000 homes.
2. **MC** Which of the following is an example of a census?
 - A. A newspaper conducts an opinion poll of 2000 people.
 - B. A product survey of 1000 homes to determine what brand of washing powder is used.
 - C. Every 200th jar of Vegemite is tested to see if it is the correct mass.
 - D. A federal election is held.
 - E. At a shopping centre, 500 people are questioned regarding the parking facilities at the centre.
3. **MC** Which of the following is an example of a random sample?
 - A. The first 50 students who arrive at school take a survey.
 - B. Fifty students' names are drawn from a hat, and those drawn take the survey.
 - C. Ten students from each year level in the school are asked to complete a survey.
 - D. One class in the school is asked to complete the survey.
 - E. Those who catch the bus to school are asked to complete the survey.
4. Discuss how bias can be introduced into statistics through:
 - a. questionnaire design
 - b. sample selection.
5. How can you determine an appropriate sample size from a population of known size?

6. A number of people were asked to rate a video on a scale of 0 to 5. Here are their scores:
1, 0, 2, 1, 0, 0, 1, 0, 2, 3, 0, 0, 1, 0, 1, 2, 5, 3, 1, 0.
- Sort the data into a frequency distribution table.
 - Display the data as a histogram, with a frequency polygon overlay.
 - Determine the mode.
 - Find the median.
 - Calculate the range.
7. Weekly earnings from casual work performed by a sample of 50 high school students were rounded to the nearest dollar, as follows:
35, 19, 46, 54, 57, 24, 62, 08, 58, 28,
57, 53, 65, 51, 24, 60, 43, 56, 71, 50,
09, 65, 16, 38, 24, 38, 53, 68, 56, 64,
49, 49, 27, 55, 46, 79, 58, 16, 59, 62,
47, 27, 38, 47, 61, 64, 44, 34, 58, 44.
- Organise the data into a frequency distribution with class intervals 0–9, 10–19, etc.
 - Display the data as a histogram and frequency polygon.
8. Calculate the mean of the following scores 1, 2, 2, 2, 3, 3, 5, 4 and 6.
9. The mean of 10 scores was 5.5; nine of the scores were: 4, 5, 6, 8, 2, 3, 4, 6 and 9; what was the tenth score?

Questions 10 and 11 refer to the following distribution table.

<i>x</i>	2	3	5	6
<i>f</i>	3	2	8	2

10. Calculate the mean of the given frequency distribution table.
11. For the given frequency distribution table, determine:
- the mode
 - the median
 - the range.
12. **a.** Determine the mode of the following values: 3, 2, 6, 5, 9, 8, 1, 7. Explain your answer.
b. Determine the median of the following values: 10, 6, 1, 9, 8, 5, 17, 3.
c. Calculate the range of the following values: 1, 6, 15, 7, 21, 8, 41, 7.
13. The following table shows the number of cars that are garaged at each house in a certain street each night.

Number of cars	Frequency
1	9
2	6
3	2
4	1
5	1

Show these data in a frequency histogram.

- 14.
<i>x</i>	1.5	2.0	2.5	3.0	3.5
<i>f</i>	10	20	8	5	6

For this frequency distribution:

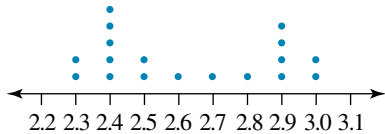
- calculate the mean score
- determine the median score
- give the modal score
- indicate which measure of centre best describes the distribution.

15. Key: $6.1 \mid 8 = 6.18$

Stem	Leaf
6.1	8 8 9
6.2	0 5 6 8
6.3	0 1 2 4 4 4

For this stem plot:

- calculate the mean score
 - determine the median score
 - give the modal score
 - indicate which measure of centre best describes the distribution.
16. For this dot plot:



- calculate the mean score (correct to 2 decimal places)
- determine the median score
- give the modal score
- indicate which measure of centre best describes the distribution.

Communicating, reasoning and problem solving

17. A frozen goods section manager recorded the following sales of chickens by size during a sample week.
 16, 14, 13, 12, 15, 14, 13, 11, 12, 14, 14, 16, 15, 13, 11, 12, 14, 13, 15, 17, 13, 12, 14, 16, 13,
 11, 15, 14, 12, 11, 15, 12, 13, 12, 12, 15, 13, 11, 11, 13, 16, 13, 12, 15, 17, 13, 14, 16, 12, 15
- Construct a frequency distribution table showing x , f and $x \times f$ columns. You may include a tally column if you wish.
 - Draw a histogram to display the data.
 - Identify the mode of the distribution.
 - Calculate the mean and median sizes of the chickens sold.
 - Of which size should the manager order most? Explain.
 - What is the range of sizes?
 - What percentage of total sales are in the size 12 to 14 group?
 - Is the mean a useful measure to the manager? Explain.
18. The following table displays the results of the number of pieces of mail delivered in a week to a number of homes.

Number of pieces of mail	0	1	2	3	4	5	6	7	8	9
Frequency	7	25	34	11	8	2	4	5	3	1

- What is the most common number of pieces of mail delivered?
- What is the mean number of pieces of mail delivered?
- Calculate the range.
- Explain what this shows about the mail delivery service to these homes.

Answers

Topic 13 Data analysis

Exercise 13.2 Samples and populations

1. Census, sample
2. Census — every member of the population participates.
3. Survey
4. a. Survey b. Survey c. Census d. Survey
5. a. Survey b. Census c. Census d. Survey
6. Open questions have no boundaries for response. Closed questions require answers to fall within a category. A variety of examples is possible.
7. A variety of categorical responses is suitable; check with your teacher.
8. A variety of categorical responses is suitable; check with your teacher.
9. a. This is a personal question.
b. This question is vague. Define what is meant by ‘exercise’ and ‘regularly’.
c. What do the abbreviations stand for?
d. What is the Premier’s policy on wildlife preservation?
e. Is this question capable of being answered? What is a ‘foot’?
f. This is a double-barrelled question.
Check with your teacher for alternative wording of questions.
10. Check with your teacher.
11. A small sample size is more likely to produce data that is not representative of the population. By having a larger sample size we can limit the effect of unusual data points that may only occur in the sample by chance.
12. A variety of answers is available here.
a. Typically, the choir members would not be the ones using the pool. The sample is biased.
b. These people may or may not know about the PM’s health-care package. In any case, the sample is biased as it is not truly representative of the population.
c. If the visitors had just arrived at the airport, they would not have had time to experience the transport system into the city. The sample is biased.
d. This sample size is far too small.
13. A variety of answers is possible.
14. 22, 30, 49, 3, 14 is one possible solution.
15. a. 94, 1, 54 b. 60, 47, 33, 43 c. 25, 49, 10
16. D
17. Discuss this question as a class.
18. Most computers and calculators generate ‘pseudo random’ numbers by using an algorithm to generate numbers that appear to be random, but are actually predictable. This is an important thing to be aware of if you are using encryption to secure data.
19. 6

Exercise 13.3 Primary and secondary data

1. These are simply examples of simulations that could be conducted.
a. A coin could be flipped (Heads representing ‘True’ and Tails representing ‘False’).
b. A coin could be flipped (Heads representing ‘red’ and Tails representing ‘black’).
c. Spinner with 4 equal sectors (each sector representing a different toy)
d. Roll a die (each face represents a particular person).
e. Spinner with 3 equal sectors (each one representing a particular meal)
f. Spinner with 5 equal sectors (each one representing a particular destination)
g. Spinner with 5 sectors, one with an angle size of 120° and four with an angle size of 60° (each one representing a particular fast food)
2. a. Answers will vary, but some possible suggestions include:
Which students have internet access at home?

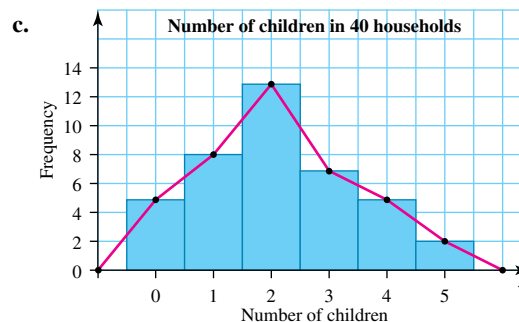
- Do the students need access at night?
 What hours would be suitable?
 How many would make use of this facility?
- Answers will vary, but could include a survey or online questionnaire.
 - Answers will vary.
- Check with your teacher.
 - Answers will vary, but some possible suggestions include:
 - Census, survey, questionnaire, interview, observation, experiment, online response
 - Measurement
 - Observation
 - Newspaper recordings
 - Survey
 - to 7. Check with your teacher.
 - The claim is false. It is not a logical deduction.
 - Check with your teacher.
 - Check with your teacher.
 - Pizza King's advertising campaign was misleading as it sounds as though all of their pizzas were rated 25% better than their competitors, when only one of the 10 varieties was rated 25% better.
 - Sealy Posturepremier: $40\% \text{ off } \left(\frac{1000}{2499} \times 100\% \right)$
 Sealy Posturepedic: $41\% \text{ off } \left(\frac{1600}{3899} \times 100\% \right)$
 SleepMaker Casablanca: $40\% \text{ off } \left(\frac{800}{1999} \times 100\% \right)$
 SleepMaker Umbria: $42\% \text{ off } \left(\frac{1800}{4299} \times 100\% \right)$
 True; the discount is at least 40% off these beds.
 - A sample from one Year group only will not give an indication of the preferences of the whole school. Hamish should ask students from all Year levels.
 - Answers will vary, but some possible suggestions include:
 Primary data: Addison could visit each of the institutions in turn either in person or online.
 Secondary data: She could seek advice from friends and colleagues. She could enlist the help of a mortgage broker or similar professional.
 - Answers will vary, but some possible suggestions include:
 - The most watched channel and the time slot with highest viewing ratings
 - Conduct a survey in a supermarket to establish the typical customer for her product
 - Yes. If the typical customer was a five-year-old child, there would be little point in advertising during the 6 o'clock news, even though that time slot may have the highest rating on the most watched channel.
 - Answers will vary. For example, when collecting primary data you should be aware of any cultural or religious sensitivities before you collect that data, and consider privacy issues when deciding how to present the data.
 - Answers will vary. For example, the United States uses the Imperial measurement system, so instead of centimetres, kilometres and litres, the data may have been collected in inches, miles and gallons.

Exercise 13.4 Organising and displaying data

- The sample is random and is of sufficient size, so calculations from the data could be considered reliable.

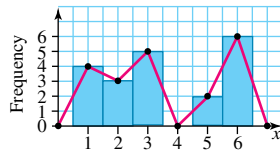
b.

x	Frequency
0	5
1	8
2	13
3	7
4	5
5	2
Total	40



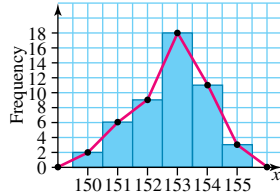
- b. The data are distributed fairly evenly around 2 children per household, and there appear to be no outliers. The graph clearly shows a maximum at 2 children per household.
- c. The sample is a random one, and of sufficient size; we can be confident that the suburb also exhibits these same properties.

2. a. i.



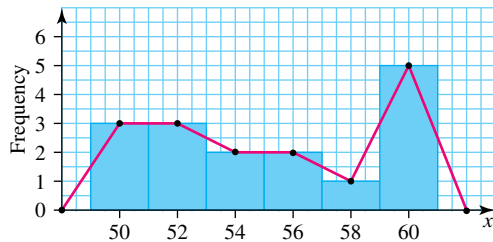
ii. This graph is quite inconsistent, showing no particular trend.

b. i.



ii. This graph is almost symmetrical, rising to a maximum at 153, then decreasing.

c. i.



ii. This graph is quite unusual. It shows a group of smaller values, then a maximum at the upper limit of the data.

3. a. The sample is a random one, so it seems to be a reliable reflection of the population of the portable MP3 players.

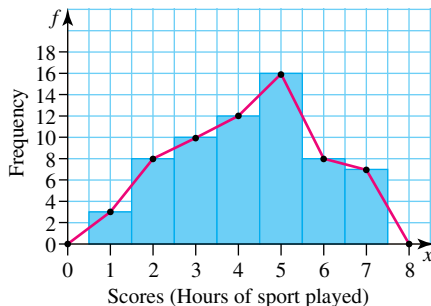
b.

x	Frequency
0	1
1	6
2	8
3	5
4	2
5	3
Total	25

c. There was only 1 box with no defective MP3 players. Most boxes had only 1, 2 or 3 defective MP3 players, while 3 boxes were found to have 5 defective MP3 players.

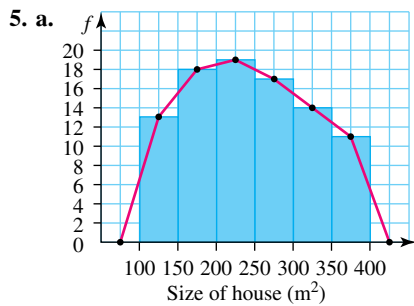
d. Since the sample was randomly selected, it seems to be a reliable reflection of the characteristics of the population. It would be reasonably safe to say that most boxes would have only 1, 2 or 3 defective MP3 players.

4. a. **Hours of sport played by Year 8 students**



b. The graph rises steadily to a maximum, then falls away sharply at the upper end of the data.

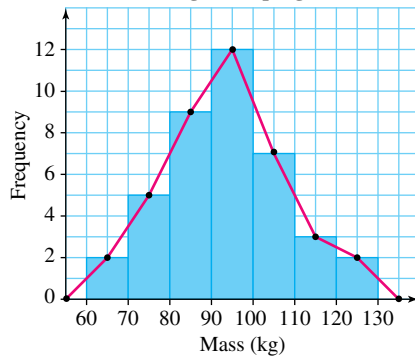
c. This is likely to be a true reflection of the sporting habits of Year 8 students. Some do a minimum of only 1 hour per week, quite a few do 2, 3 or 4 hours per week, with the maximum number doing 5 hours per week. The committed sports players would put in 6 or 7 hours per week.



b. The graph is roughly symmetrical, rising to a maximum at the 200-m² to 250-m² size, then decreasing slowly.

c. Since this is one block of houses in the suburb, it is not a random sample. We often find all the houses in a block of a suburb are of similar style. For this reason, we could not say it reflects the size of houses in the whole suburb.

6. a. **Mass of people joining a weight loss program**

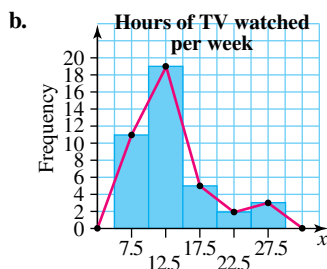


b. The graph is quite symmetrical, rising to a maximum at the 90-kg to 100-kg mass, then decreasing more rapidly to the 130-kg mass.

c. This would most likely not reflect the masses of people in the community because these are people who have enrolled in a program to lose weight.

7. a.

Class interval	Midpoint (class centre)	Tally	Frequency
5–<10	7.5		11
10–<15	12.5		19
15–<20	17.5		5
20–<25	22.5		2
25–<30	27.5		3
		Total	40

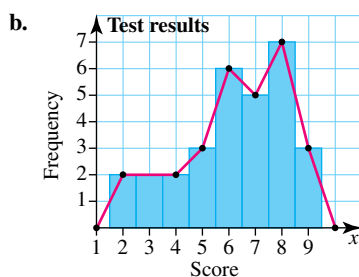


c. The graph is heavily weighted towards the lower end of the scale, with most people watching fewer than 15 hours of TV per week. There were 3 people who watched almost 30 hours of TV per week.

d. Since these people were interviewed in a shopping centre, the sample is not a random one. It could not therefore be taken to reflect the viewing habits of the community.

8. a.

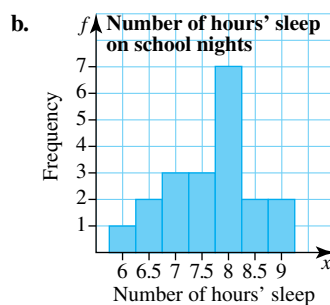
Test results	Frequency
2	2
3	2
4	2
5	3
6	6
7	5
8	7
9	3
Total	30



c. Most Year 8 students achieved a score of about 5 to 9. There were 6 students who received less than 5. This is probably a typical spread of marks for a multiple choice test.

9. a.

Hours of sleep	Frequency
6	1
$6\frac{1}{2}$	2
7	3
$7\frac{1}{2}$	3
8	7
$8\frac{1}{2}$	2
9	2
Total	20

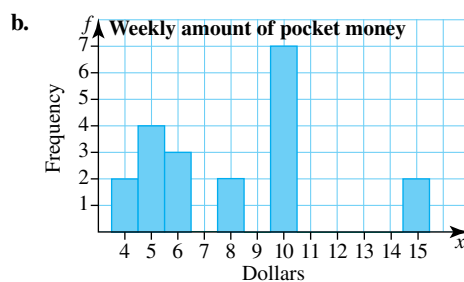


c. The histogram peaks sharply at the 8-hours' sleep mark, indicating that generally, Year 8 students get 8 hours of sleep per night during the week. Some get less, and a few get more.

d. It seems likely that these sample results would reflect the sleeping habits of Year 8 students generally.

10. a.

Pocket money (\$)	Frequency
4	2
5	4
6	3
7	0
8	2
9	0
10	7
11	0
12	0
13	0
14	0
15	2
Total	20

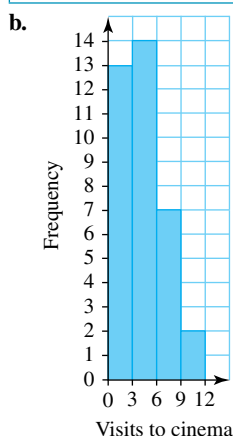


c. The histogram shows no general trend. The maximum is at \$10, indicating that a popular amount of pocket money is \$10 per week. Quite a few receive less than this, with only 2 receiving more.

d. Since this is a random sample, it is quite likely that these results reflect the general population of 13-year-olds when it comes to pocket money.

11. a. Answers will depend on class intervals chosen. An example is given.

Number of visits to the cinema	Tally	Frequency (f)
0–2		13
3–5		14
6–8		7
9–11		2
	Total	36



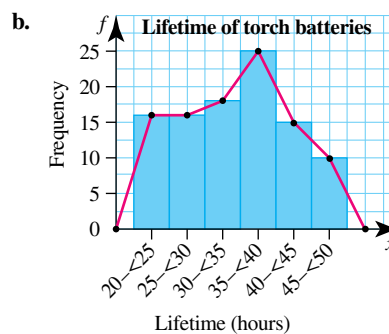
c. 13

d. 23

e. No. Check with your teacher.

12. a.

Lifetime (hours)	Frequency
20–<25	16
25–<30	16
30–<35	18
35–<40	25
40–<45	15
45–<50	10
Total	100

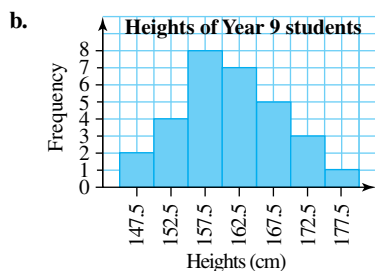


c. The histogram shows that the majority of torch batteries last for about 40 hours. A few last longer than this.

d. It seems reasonable that the torch battery population would display a similar trend.

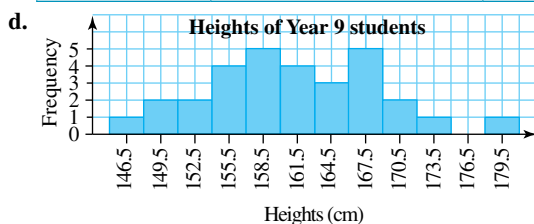
13. a.

Class interval	Midpoint (class centre)	Tally	Frequency
145–<150	147.5		2
150–<155	152.5		4
155–<160	157.5		8
160–<165	162.5		7
165–<170	167.5		5
170–<175	172.5		3
175–<180	177.5		1
		Total	30



c.

Class interval	Midpoint (class centre)	Tally	Frequency
145–<148	146.5		1
148–<151	149.5		2
151–<154	152.5		2
154–<157	155.5		4
157–<160	158.5		5
160–<163	161.5		4
163–<166	164.5		3
166–<169	167.5		5
169–<172	170.5		2
172–<175	173.5		1
175–<178	176.5	—	0
178–<181	179.5		1
		Total	30



- e. The two histograms represent the same data set, but appear to be quite different. The first histogram appears to be roughly symmetrical, with a maximum number of students having a height about 155 cm to 160 cm. The second histogram has two modes, with the most common height for students being about 157 cm to 160 cm, or 166 cm to 169 cm. It illustrates the fact that the interpretation of a histogram displaying grouped data is dependent on the class interval used.

Exercise 13.5 Measures of centre and spread

- 26.8
- a. 4.33 b. 7.33 c. 36.30 d. 6.91
- a. 18 b. 10 c. 1.8 goals
- C
- Group A: 6.1; Group B: 6.8. Group B has a higher mean.
- 2.29 7. 7.90
- a. 7 b. 17
- a. 48 b. 1.8
- Question 8:
 - 5
 - 17
 Question 9:
 - 52
 - 1.5, 1.7, 1.8
- C
- E
- a. 6.7 b. 10

14.

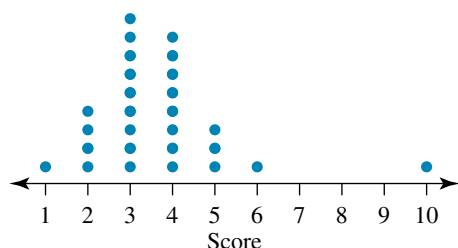
x	f	xf
0	1	0
1	2	2
2	7	14
3	3	9
4	4	16
5	2	10
6	1	6
Total	20	57

Mean = 2.85

15. a. 36 b. 7
16. a. 3 b. 3 c. Both
17. a. 49 b. 55 c. 56 d. Median
18. a. \$18 000 b. \$26 000 c. \$30 000
- d. Mean (the highest value) e. Mode (the lowest value)
19. a. 8 b. 15 c. 50 d. 2.5
20. a. 4 b. 5 c. 15 d. 4
21. a. 35 b. 46 c. 35
22. Mean = 6, Median = 6, Mode = 6, Range = 4
23. a. 8 b. No
24. 31–40
25. B
26. a. Mean = 26.6, Median = 27.5, Mode = 30, Range = 22
- b. Mean = 32, Median = 29, Mode = 30, Range = 70
- c. The inclusion of an outlier dramatically increased the range and significantly increased the mean of the data. The median was marginally increased while the mode was unchanged.
27. a. The median, as there are likely to be some outliers which will significantly affect the mean
- b. The mode, as this will be the most popular shoe size and will be a number which is easy to interpret
- c. The mean, as this will be the average scored in a game
28. One possible answer is 5, 6, 12, 13, 14. If the five numbers are in ascending order, the third number must be 12 and the other 4 numbers must total 38.
29. Highest score is 49 and lowest score is 0.
- The scores would be 0, 1, 12, 13, 49.
30. Largest value for median is 15. Scores would be 0, 2, 15, 16, 17. Smallest value for median is 2. Scores would be 0, 1, 2, a , b , where $a + b = 47$.
31. 43; highest score: 100; lowest score: 57. The scores would be 57, 77, 78, 79, 80, 97, 98, 99, 100.
32. a. Lowered, as the new number must be less than 15.
- b. Raised, as the new number must be 16 or more.
33. a. The potential error is the data value of 23.
- b. This value could have been a genuine outlier if the particular student was a multi-sport athlete who trained for a number of hours each day.
34. a. The potential error is the data value of 36.
- b. This outlier is likely to have been a recording error as temperatures in Sydney do not reach that high in July.
35. a. Mean = 4.5; median = 5; modes = 3 and 5
- b. The median is higher than the mean.
- c. The mean is between the two modal values.
- d. As there are no outliers, the mean is probably the best indicator of the centre of the data set.
36. Mean = 5; median = 3; mode = 6
37. 73.45

38. 171

39. Answers will vary. A potential dot plot is:



40. The median is the middle score and so is less likely to be affected by outliers. As the mean is calculated using all the data in the set, an outlier can significantly affect the mean.

41. 20

Exercise 13.6 Analysing data

1. a. Range b. Mean c. Mode d. Mean e. Median

2. a. 6.4 b. 6 c. 6

d. They are all quite close, so any would do.

e. Since these were the results of half the class, and the sample was random, it seems likely that the population results would be similar.

3. a. i. 2 ii. 2 iii. 1 iv. Mean or median

b. i. 7.8 ii. 7 iii. 7 iv. Any of the three

4. a. i. 22.6 ii. 23 iii. 23 iv. Any of the three

b. i. 117.5 ii. 116 iii. 112 iv. Mean

5. a. i. 4.5 ii. 5 iii. 5 iv. Any of the three

b. i. 82.4 ii. 82 iii. 81 iv. Mean or median

6. a. Sample 1: Mean = 107.9, Median = 101.5, No mode, Range = 137

b. Sample 2: Mean = 112.5, Median = 106.5, Mode = 85, Range = 127

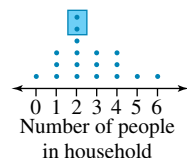
c. Sample 3: Mean = 125.4, Median = 104, No mode, Range = 190

d. All three median values lie close together. Sample 3 has a significantly larger mean and range than the other two samples, possibly caused by the data value 256 which appears to be an outlier. Sample 2 is the only sample which has a mode, so modes cannot be compared.

7. a.

Score (x)	Frequency (f)	Freq. $\times$ score ($f \times x$)
0	1	0
1	3	3
2	6	12
3	3	9
4	3	12
5	1	5
6	1	6
	$n = 18$	$\sum fx = 47$

b. $\bar{x} = 2.6$



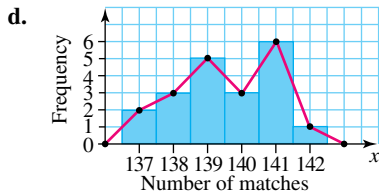
c. 2

d. i. Median ii. Mean iii. Mode

8. a.

Score (x)	Frequency (f)	Freq $\times$ score ($f \times x$)
137	2	274
138	3	414
139	5	695
140	3	420
141	6	846
142	1	142
$n = 20$		$\sum fx = 2791$

b. 141, 139.5, 139.6 c. Mean



9. a. The median was calculated by taking the average of the 2 middle scores.

b. 13

10. 2

11. a. 55 250 km, 52 000 km, 52 000 km

b. 51 810 km; it is reduced by 3440 km.

c. 52 000 km

d. 3333

e. 50 000 km; 92% last that distance or more.

12. Mean = 33.05, Median = 33, Mode = 33, Range = 5

13. a. 38 years

b. 27 years

c. 26 years

d. 31 years

14. a. Answers will vary. An example is 0, 1, 3, 3, 8.

b. Answers will vary. An example is 0, 1, 4, 4, 6.

c. Answers will vary. An example is 1, 2, 2, 4, 6.

15. The range was 4 minutes 14 seconds (fastest time of 38 seconds; slowest time of 4 minutes 52 seconds). The most common time (mode) was 1 minute. The average (mean) time was more than 2 minutes.

16. The data values in each sample will vary, so the sample statistics taken from these samples will also vary.

17. Samples from the same population may have different summary statistics due to natural variation within the population, the presence of outliers within samples and some subsets of the population not being represented in all samples. By taking sufficiently large random samples these variations can be reduced.

18. If a sample is not representative of the whole population then it is likely that some key features of the population will be missed. For example, if the population was the whole of NSW, and all the sample data points were taken from Sydney suburbs, then the rest of NSW would not be represented in the sample. Some statistics, for example median house prices, are likely to be much higher in Sydney than in most of the rest of NSW.

19. Check with your teacher.

20. a. The mean (3.25). The median is 2 and the mode is 1.

b. Answers will vary.

21. Measures of centre are important because they give us an indication what the average result is across a population (mean), what the most frequent result is (mode), and the middle of the range result (median). This information can allow businesses to plan, such as how many products to produce or when to provide a service.

Investigation | Rich task

1. The letters E, A, I and O are the most common; the letters J, K, Q, X and Z are least common.

2. The more tiles for a letter results in a lower tile score. The less tiles for a letter results in a higher tile score.

3. The frequency distribution table includes the letters in the passage heading.

Letter	Frequency	Letter	Frequency
A	61	O	31
B	6	P	23
C	18	Q	0
D	18	R	39
E	73	S	50
F	13	T	34
G	10	U	14
H	14	V	4
I	31	W	11
J	0	X	1
K	3	Y	6
L	18	Z	4
M	10		
N	41		

4. Results from both frequency tables show letters J, K, Q, X and Z to be the least common. However, while results from the scrabble frequency table show letters A, E, I and O to be the most common; results from the textbook frequency table show letters A, E, I, N, O, R, S and T to be most common.
5. Australia is located in the southern hemisphere and is made up of six states and two territories. The capital of Australia is Canberra.
6. Teacher to check. Common strategies involve looking for common letters and frequently used two- and three-lettered words.
7. Teacher to check.

Exercise 13.7 Review questions

1. a. Survey b. Census c. Census d. Survey
2. D
3. B
4. Discuss as a class.
5. The appropriate sample size is $\sqrt{\text{population size}}$.

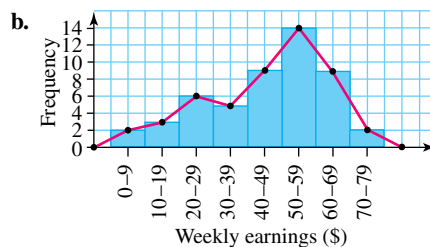
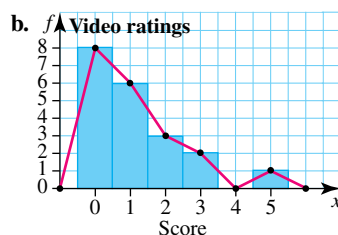
6. a.

Video rating	Frequency
0	8
1	6
2	3
3	2
4	0
5	1
Total	20

- c. 0 d. 1 e. 5

7. a.

Class interval	Frequency (f)
0–9	2
10–19	3
20–29	6
30–39	5
40–49	9
50–59	14
60–69	9
70–79	2
	$n = 50$



8. 3.1

9. 8

10. 4.3

11. a. 5

b. 5

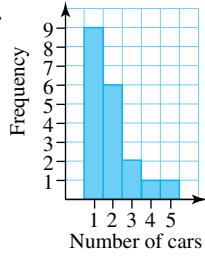
c. 4

12. a. There is no mode since none of the values occurs more than once.

b. 7

c. 40

13.



14. a. 2.3

b. 2.0

c. 2.0

d. Mode

15. a. 6.27

b. 6.28

c. 6.34

d. Mean or median

16. a. 2.63

b. 2.55

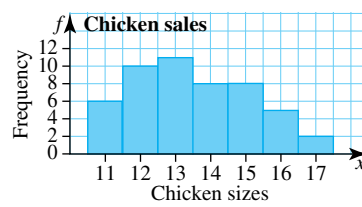
c. 2.4

d. Mean

17. a.

x	f	xf
11	6	66
12	10	120
13	11	143
14	8	112
15	8	120
16	5	80
17	2	34
Total	50	675

b.



c. 13

d. 13.5, 13

e. 13; as this is the most frequently sold size.

f. 6

g. 58%

h. Discuss

18. a. 2

b. 2.6

c. 9

d. Most homes get up to about 3 pieces of mail. Some do get more.

TOPIC 14

Pythagoras' theorem

14.1 Overview

Numerous **videos** and **interactivities** are embedded just where you need them, at the point of learning, in your learnON title at www.jacplus.com.au. They will help you to learn the concepts covered in this topic.

14.1.1 Why learn this?

Pythagoras was a famous mathematician who lived about 2500 years ago. He is credited with being the first person to prove that in any right-angled triangle there is a special relationship between the squares of the three sides. The theorem has an important role to play in everyday life because right-angled triangles occur in construction, navigation, planning, design and packaging. Pythagoras' theorem is one of the great geometrical theorems.



LEARNING SEQUENCE

- 14.1** Overview
- 14.2** Right-angled triangles
- 14.3** Finding the hypotenuse
- 14.4** Finding a shorter side
- 14.5** Working with different units
- 14.6** Composite shapes
- 14.7** Pythagorean triads
- 14.8** Review

LEARNING OUTCOMES

A student:

- communicates and connects mathematical ideas using appropriate terminology, diagrams and symbols MA4-1WM
- applies appropriate mathematical techniques to solve problems MA4-2WM
- applies Pythagoras' theorem to calculate side lengths in right-angled triangles, and solve related problems MA4-16MG

CONTENT

Students:

Investigate the concept of irrational numbers, including π (ACMNA186)

Investigate Pythagoras' theorem and its application to solving simple problems involving right-angled triangles (ACMMG222)

Source: NSW Syllabus for the Australian curriculum

learnon RESOURCES — ONLINE ONLY

eLesson: The story of mathematics — Hypatia (eles-1844)

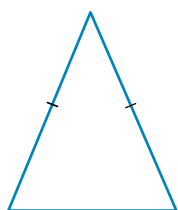
Note: Your teacher may now set you a pre-test to determine how familiar you are with the content in this topic.

14.2 Right-angled triangles

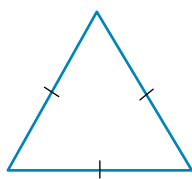
14.2.1 Right-angled triangles

- This topic investigates one of the most important ideas in geometry related to **right-angled triangles**. Triangles can be classified according to the length of their sides. They can be classified as equilateral, isosceles or scalene. Triangles can be classified by their angles as well. A triangle with one right angle is said to be a right-angled triangle.

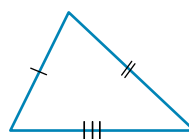
Types of triangles



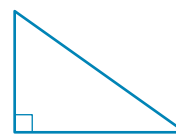
Isosceles



Equilateral



Scalene



Right-angled

- In any triangle, the longest side is opposite the largest angle; so in a right-angled triangle, the longest side is opposite the right angle. This side has a special name. It is called the **hypotenuse**.
- The theorem, or rule, that you will learn more about was named after an ancient Greek mathematician, called Pythagoras (580–501 BC). It will enable you to solve all kinds of practical problems related to right-angled triangles.



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RESOURCES — ONLINE ONLY

Interactivity: Triangles (int-3843)

Digital doc: SkillSHEET Measuring lengths and angles (doc-11425)

Digital doc: SkillSHEET Finding the square of a number (doc-11426)

Exercise 14.2 Right-angled triangles

Individual pathways

PRACTISE

Questions:
1–6, 8, 10

CONSOLIDATE

Questions:
1–5, 7, 8, 10, 11

MASTER

Questions:
1–5, 7–11

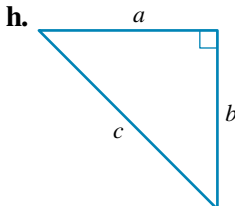
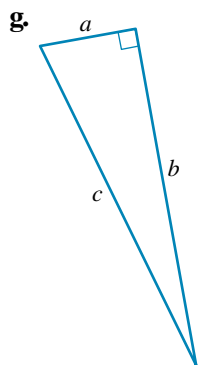
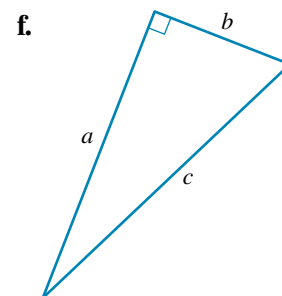
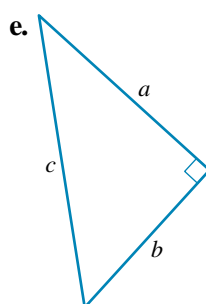
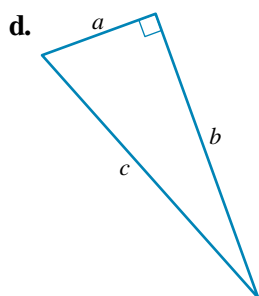
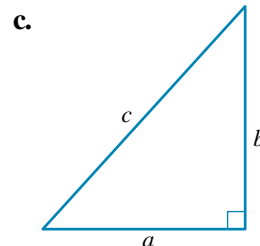
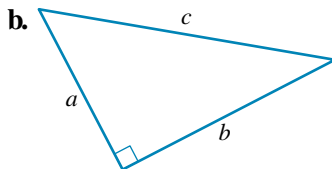
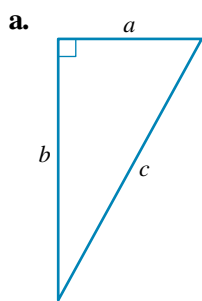
Individual pathway interactivity: int-4469

learnon ONLINE ONLY

To answer questions online and to receive **immediate feedback** and **fully worked solutions** for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Understanding and fluency

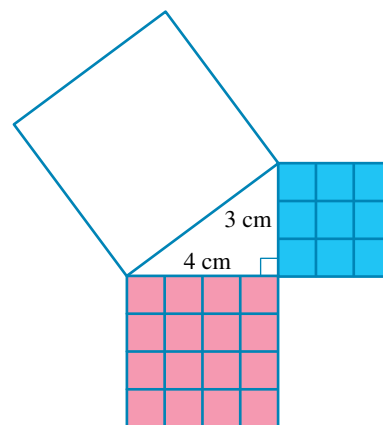
1. For each of the following triangles, the lengths of the sides are recorded in the table that follows. Note that the hypotenuse is always marked as c . Complete the table.



Lengths (mm)	a	b	c	d	e	f	g	h
a	19	24	27	16	31	41	13	24
b	35	32	30	40	25	19	45	24
c	40	40	40	43	40	45	47	34
a^2								
b^2								
$a^2 + b^2$								
c^2								

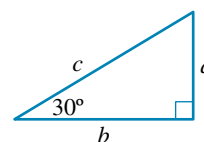
2. What do you notice about the results in the table in question 1?
3. On a sheet of paper, carefully draw 6 right-angled triangles of different sizes. You will need to use a protractor, set square or template to make sure the triangles are right-angled. Carefully measure the sides of each triangle, and complete a table like the one in question 1. What do you notice about these results?
4. Now draw some triangles that are not right-angled, measure their sides and complete the same kind of table. Remember to label the longest side as c . What do you notice this time?

5. Explain why it is not possible to have a triangle with two right angles in it.
6. For this activity you will need graph paper, coloured pencils, glue and scissors.
 - a. On a sheet of graph paper, draw a right-angled triangle with a base of 4 cm and a height of 3 cm.
 - b. Carefully draw a square on each of the three sides of the triangle and mark a grid on each so that the square on the base is divided into 16 small squares, while the square on the height is divided into 9 small squares.
 - c. Colour the square on the base, and the square on the height in different colours as shown at right so that you can still see the grid lines.
 - d. Carefully cut out the two coloured squares from the triangle.
 - e. Now stick the larger of the coloured squares on the uncoloured square of the triangle (the square on the hypotenuse).
 - f. Using the grid lines as a guide, see if you can cut the smaller square up and fit it on the remaining space. The two coloured squares should have exactly covered the third square.
 - g. What do you notice about the hypotenuse and the other two sides of a right-angled triangle?
7. A right-angled triangle is a special triangle, but it has some properties that are common to all triangles. One of these common properties is that the sum of the lengths of any two sides must be greater than the length of the third side.
Using a ruler and compass, attempt to construct a triangle whose side lengths are 7 cm, 4 cm and 12 cm. What do you find?



Communicating, reasoning and problem solving

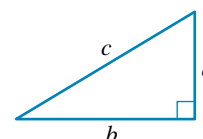
8. What is the sum of the two smaller angles in a right-angled triangle? Justify your answer.
9. If the two smaller angles in a right-angled triangle are equal in size, make a statement about the triangle. What size is each of the smaller angles?
10. Can you draw an equilateral triangle that has a right angle? Explain.
11. a. Draw four triangles with a 30° angle like the one shown at right, ensuring that the length of a is different for each.
b. For each triangle you have drawn in part a, find the value of the following.
 - i. $\frac{b}{a}$
 - ii. $\frac{b}{c}$
 - iii. $\frac{a}{c}$
- c. What do you notice about your answers to part b?
- d. Repeat parts a to c for another four right-angled triangles using an angle other than 30° .



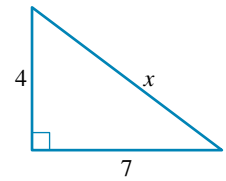
14.3 Finding the hypotenuse

14.3.1 Finding the hypotenuse

- Pythagoras was perhaps the first mathematician to recognise and investigate a very important property of right-angled triangles. The theorem, or rule, named after him states that:
In any right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides. The rule is written as $c^2 = a^2 + b^2$ where a and b are the two shorter sides and c is the hypotenuse.
- The hypotenuse is the longest side of a right-angled triangle and is always the side that is opposite the right angle.

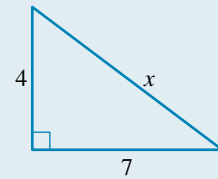


- Pythagoras' theorem gives us a way of finding the length of the third side in a triangle, if we know the lengths of the two other sides.
- We are able to find the length of the hypotenuse when we are given the length of the two shorter sides by substituting into the formula $c^2 = a^2 + b^2$ to calculate the hypotenuse c .
- Calculations with Pythagoras' theorem often result in a number under the root sign ($\sqrt{\quad}$) that is not a square number. In such cases answers may be left unsimplified, and this is called exact (surd) form. A calculator may be used to find an approximate answer, which is usually then given to a specified number of decimal places.



WORKED EXAMPLE 1

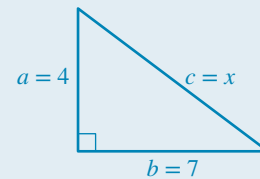
For the triangle at right, find the length of the hypotenuse, x , correct to 1 decimal place.



THINK

- Copy the diagram and label the sides a , b and c . Remember to label the hypotenuse as c .

WRITE/DRAW



- Write Pythagoras' theorem.
- Substitute the values of a , b and c into this rule and simplify.
- Calculate x by taking the square root of 65. As 65 is not a square number, a calculator can be used to find an approximate value for x . Round your answer correct to 1 decimal place.

$$c^2 = a^2 + b^2$$

$$\begin{aligned} x^2 &= 4^2 + 7^2 \\ &= 16 + 49 \\ &= 65 \end{aligned}$$

$$x = \sqrt{65}$$

This is the exact (surd) form.

$$x \approx 8.1$$

- The $\approx$ symbol means 'is approximately equal to'. An alternative symbol, $\doteq$, can also be used.
- In many cases we are able to use Pythagoras' theorem to solve practical problems. We can model the problem by drawing a diagram, and use Pythagoras' theorem to solve the right-angled triangle. We then use the result to give a worded answer.

WORKED EXAMPLE 2

A fire is on the twelfth floor of a building. A child needs to be rescued from a window that is 20 metres above ground level. If the rescue ladder can be placed no closer than 6 m from the foot of the building, what is the minimum length ladder needed to make the rescue? Leave your answer in exact form.

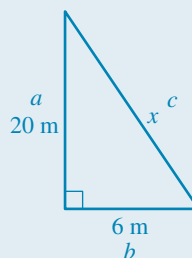


THINK

- 1 Draw a diagram and label the sides a , b and c .
Remember to label the hypotenuse as c .

- 2 Write Pythagoras' theorem.
- 3 Substitute the values of a , b and c into this rule and simplify.
- 4 Calculate x by taking the square root of 436.
- 5 Give a worded answer.

WRITE/DRAW



$$c^2 = a^2 + b^2$$

$$\begin{aligned}x^2 &= 20^2 + 6^2 \\&= 400 + 36 \\&= 436\end{aligned}$$

$$x = \sqrt{436}$$

The ladder needs to be $\sqrt{436}$ metres
(≈ 21 m) long.

ACTIVITY: EXPLORING SURDS

Use a computer to explore decimal approximations of surds. Can all surds be represented by either terminating or recurring decimals?

learnon RESOURCES — ONLINE ONLY

Interactivity: Finding the hypotenuse (int-3844)

Digital doc: SkillsHEET Finding the square root of a number (doc-11427)

Digital doc: SkillsHEET Rounding to a given number of decimal places (doc-11428)

Exercise 14.3 Finding the hypotenuse

Individual pathways

PRACTISE

Questions:
1–5, 9, 14

CONSOLIDATE

Questions:
1–6, 9, 10, 12, 14

MASTER

Questions:
1–15

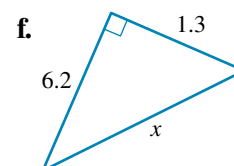
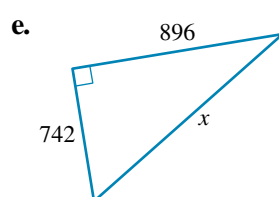
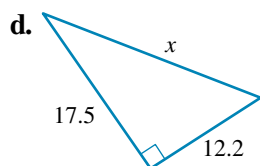
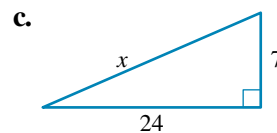
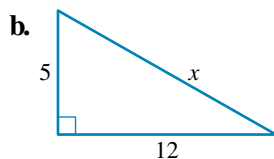
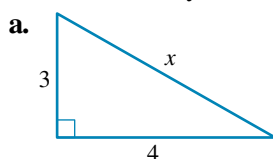
Individual pathway interactivity: int-4470

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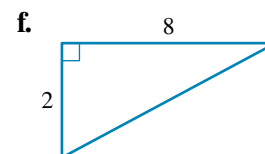
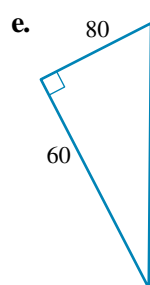
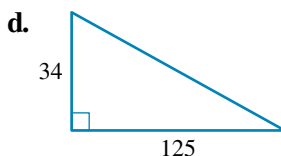
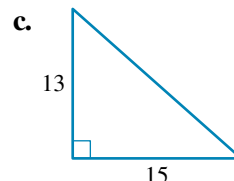
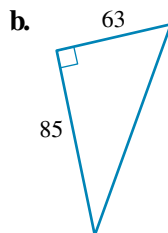
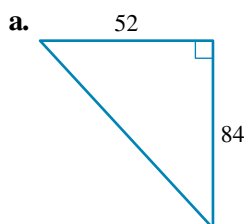
To answer questions online and to receive **immediate feedback** and **fully worked solutions** for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Understanding and fluency

1. **WE1** For the following triangles, find the length of the hypotenuse, x , correct to 1 decimal place where necessary.



2. For each of the following triangles, find the length of the hypotenuse. Leave your answers in exact form.

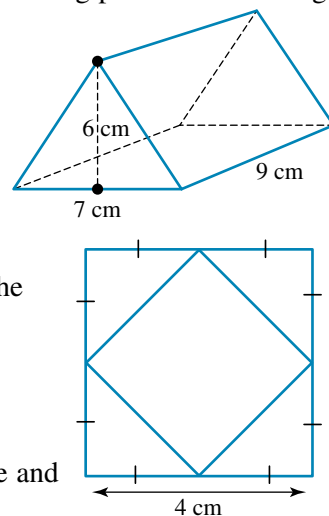


3. A right-angled triangle has a base of 5 cm and a perpendicular height of 11 cm. Find the length of the hypotenuse. Leave your answer in exact form.
4. Find the lengths, correct to 2 decimal places, of the diagonals of squares that have side lengths of:
- 12 cm
 - 20 mm
 - 4.9 cm.
5. What is the length, correct to 2 decimal places, of the diagonal of a rectangle whose sides are:
- 10 cm and 8 cm
 - 620 cm and 400 cm
 - 17 cm and 3 cm?
6. An isosceles triangle has a base of 30 cm and a perpendicular height of 10 cm. Find the length of the two equal sides of the isosceles triangle. Give your answer correct to 2 decimal places.

Communicating, reasoning and problem solving

7. A right-angled triangle has a perpendicular height of 17.2 cm, and a base that is half the height. Find the length of the hypotenuse, correct to 2 decimal places.
8. **WE2** A ladder leans against a vertical wall. The foot of the ladder is 1.2 m from the wall, and the top of the ladder reaches 4.5 m up the wall. How long is the ladder? Give your answer correct to 2 decimal places.
9. Wally is installing a watering system in his garden. The pipe is to go all around the edge of the rectangular garden, and have a branch diagonally across the garden. The garden measures 5 m by 7.2 m. If the pipe costs \$2.40 per metre (or part thereof), what will be the total cost of the pipe? Show your working.

10. When taking the square root of a positive number (e.g. $\sqrt{64}$), there is both a negative and positive solution (e.g. ± 8). Explain why the negative solution is not feasible when solving problems involving Pythagoras' theorem.
11. a. Calculate the volume of the prism shown at right.
b. Find the length of the sloping edge of the cross-section.
12. A ladder rests against a vertical wall with the top of the ladder being 7 m above the ground. If the bottom of the ladder was moved 1 m further away from the foot of the wall, the top of the ladder would rest against the foot of the wall. What is the length of the ladder?
13. A right-angled isosceles triangle has an area of 200 cm^2 . Find the area of the semi-circle which sits on the hypotenuse.
14. Use words to describe the relationship between the sides of a right-angled triangle.
15. A smaller square is drawn inside a larger square as shown in the diagram. Use Pythagoras' theorem to determine the side length of the smaller square and hence the area of the smaller square.



14.4 Finding a shorter side

14.4.1 Finding a shorter side

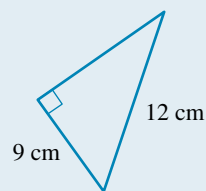
- Sometimes a question will give you the length of the hypotenuse and ask you to find one of the shorter sides. In such examples, we need to rearrange Pythagoras' formula.
- Given that $c^2 = a^2 + b^2$, we can rewrite this as:

$$a^2 = c^2 - b^2$$

$$\text{or } b^2 = c^2 - a^2$$

WORKED EXAMPLE 3

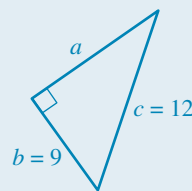
Find the length of the unmarked side of the triangle at right.
Leave your answer in exact form.



THINK

- Copy the diagram and label the sides a , b and c . Remember to label the hypotenuse as c .
- Write Pythagoras' theorem for a shorter side.
- Substitute the values of a , b and c into this rule and simplify.
- Find a by taking the square root of 63.

WRITE/DRAW



$$a^2 = c^2 - b^2$$

$$a^2 = 12^2 - 9^2$$

$$= 144 - 81$$

$$= 63$$

$$a = \sqrt{63} \text{ cm}$$

- Practical problems may also involve you being required to find the shorter side of a right-angled triangle.

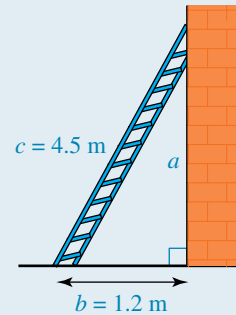
WORKED EXAMPLE 4

A ladder that is 4.5 m long leans up against a vertical wall. The foot of the ladder is 1.2 m from the wall. How far up the wall does the ladder reach? Give your answer correct to 2 decimal places.

THINK

- 1 Draw a diagram and label the sides a , b and c . Remember to label the hypotenuse as c .
- 2 Write Pythagoras' theorem for a shorter side.
- 3 Substitute the values of a , b and c into this rule and simplify.
- 4 Find a by taking the square root of 18.81. Round to 2 decimal places.
- 5 Give a written answer.

WRITE/DRAW



$$a^2 = c^2 - b^2$$

$$\begin{aligned} a^2 &= 4.5^2 - 1.2^2 \\ &= 20.25 - 1.44 \\ &= 18.81 \end{aligned}$$

$$\begin{aligned} a &= \sqrt{18.81} \\ &\approx 4.34 \text{ m} \end{aligned}$$

The ladder will reach a height of 4.34 m up the wall.

- Some questions will require you to decide which method is needed to solve the problem. A diagram will help you decide whether you are finding the hypotenuse or one of the shorter sides.

learnon RESOURCES — ONLINE ONLY

Interactivity: Finding the shorter side (int-3845)

Digital doc: SkillsHEET Rearranging formulas (doc-11429)

Digital doc: WorkSHEET Finding the length of sides (doc-11431)

Exercise 14.4 Finding a shorter side

Individual pathways

PRACTISE

Questions:
1–6, 8, 10, 14

CONSOLIDATE

Questions:
1–10, 13, 14

MASTER

Questions:
1–15

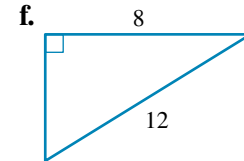
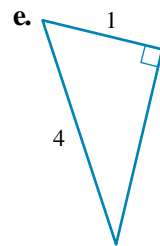
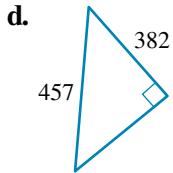
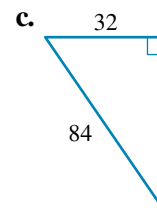
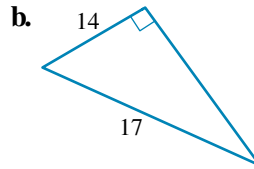
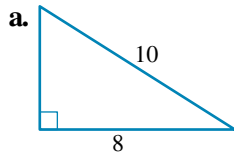
Individual pathway interactivity: int-4471

learnon ONLINE ONLY

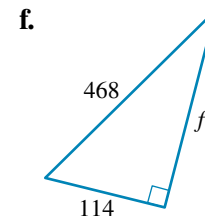
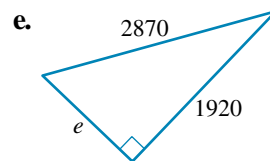
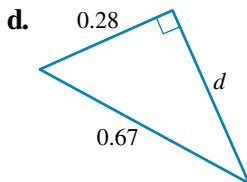
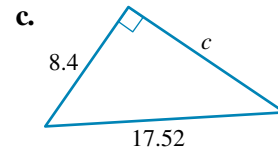
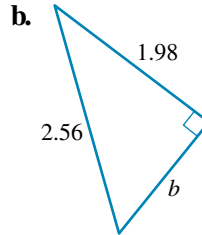
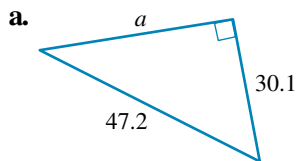
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Understanding and fluency

1. **WE3** Find the length of the unmarked side in each of the following triangles. Leave your answer in exact form.

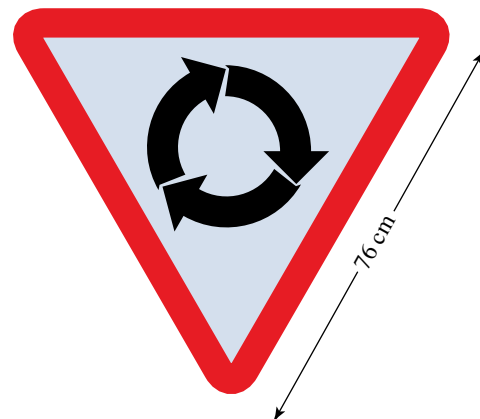
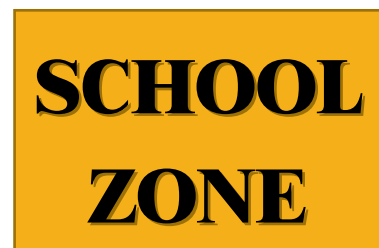


2. Find the value of the pronumeral, correct to 2 decimal places.



For questions 3 to 13, give your answers correct to 2 decimal places.

3. The diagonal of the rectangular sign at right is 34 cm. If the height of this sign is 25 cm, find the width.
4. The diagonal of a rectangle is 120 cm. One side has a length of 70 cm. Find:
- the length of the other side
 - the perimeter of the rectangle
 - the area of the rectangle.
5. An equilateral triangle has sides of length 20 cm. Find the height of the triangle.
6. The roundabout sign shown at right is in the form of an equilateral triangle. Find the height of the sign and, hence, find its area.
7. **WE4** A ladder that is 7 metres long leans up against a vertical wall. The top of the ladder reaches 6.5 m up the wall. How far from the wall is the foot of the ladder?

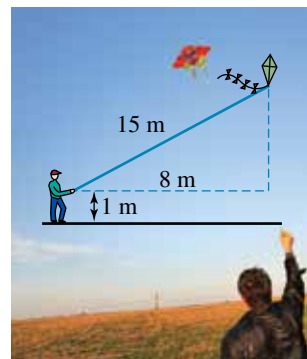
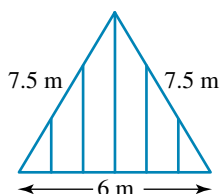


8. A tent pole that is 1.5 m high is to be supported by ropes attached to the top. Each rope is 2 m long. How far from the base of the pole can each rope be pegged?
9. Ben's dog 'Macca' has wandered onto a frozen pond, and is too frightened to walk back. Ben estimates that the dog is 3.5 m from the edge of the pond. He finds a plank, 4 m long, and thinks he can use it to rescue Macca. The pond is surrounded by a bank that is 1 m high. Ben uses the plank to make a ramp for Macca to walk up. Will he be able to rescue his dog?



Communicating, reasoning and problem solving

10. A kite is attached to a string 15 m long. Sam holds the end of the string 1 m above the ground, and the horizontal distance of the kite from Sam is 8 m as shown at right. How far above the ground is the kite? Show your working.
11. Penny, the carpenter, is building a roof for a new house. The roof has a gable end in the form of an isosceles triangle, with a base of 6 m and sloping sides of 7.5 m. She decides to put 5 evenly spaced vertical strips of wood as decoration on the gable as shown below. How many metres of this decorative wood does she need? Show your working.



12. The size of a rectangular television screen is given by the length of its diagonal. The television at right has a 92-cm screen. What is the length of the shorter side? Show your working.
13. An art student is trying to hang her newest painting on an existing hook in a wall. She leans a 1.2-m ladder against the wall so that the distance between the foot of the ladder and the wall is 80 cm.
 - a. Draw a sketch showing the ladder leaning on the wall.
 - b. How far up the wall does the ladder reach?
 - c. The student climbs the ladder to check whether she can reach the hook from the step at the very top of the ladder. Once she extends her arm, the distance from her feet to her fingertips is 1.7 m. If the hook is 2.5 m above the floor, will the student reach it from the top step?
14. a. A rectangle has two sides of length 3 cm and diagonals of length 5 cm. Calculate the length of the other two sides.
 b. It is known that the diagonals of this rectangle bisect each other at the point of intersection. Using this fact, check (using appropriate calculations) whether the diagonals are perpendicular to each other.
15. What should you look out for when using Pythagoras' theorem to calculate the value of missing sides?



14.5 Working with different units

14.5.1 Unit conversion

- When we use Pythagoras' theorem, we are usually working with a practical situation where measurements have been given. In any calculation, it is essential that all of the measurements are in the same units (e.g. cm).

- Do you remember the relationship between the units of length? Here is a reminder:

$$10 \text{ mm} = 1 \text{ cm}$$

$$100 \text{ cm} = 1 \text{ m}$$

$$1000 \text{ m} = 1 \text{ km}$$

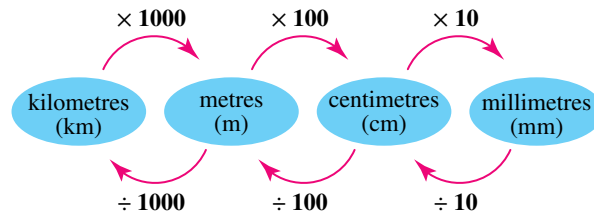
To change to a larger unit, divide by the conversion factor.

To change to a smaller unit, multiply by the conversion factor.

For example, to change kilometres to metres, multiply by 1000.

To change centimetres to metres, divide by 100.

The following chart shows all the conversions of length that you are likely to need.



- When using Pythagoras' theorem, always check the units given for each measurement. If necessary, convert all measurements to the same units before using the rule.

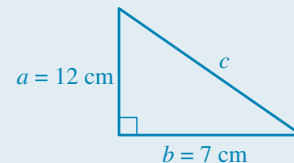
WORKED EXAMPLE 5

Find the length, in mm, of the hypotenuse of a right-angled triangle if the 2 shorter sides are 7 cm and 12 cm. Give your answer correct to 1 decimal place.

THINK

- Draw a diagram and label the sides a , b and c . Remember to label the hypotenuse as c .
- Check that all measurements are in the same units. They are.
- Write Pythagoras' theorem for the hypotenuse.
- Substitute the values of a and b into this rule and simplify.
- Find c by taking the square root. Give the units in the answer.
- Check the units required in the answer and convert if necessary.

WRITE/DRAW



$$c^2 = a^2 + b^2$$

$$\begin{aligned} c^2 &= 12^2 + 7^2 \\ &= 144 + 49 \\ &= 193 \end{aligned}$$

$$\begin{aligned} c &= \sqrt{193} \\ &\approx 13.89 \text{ cm} \end{aligned}$$

$$\begin{aligned} &= 13.89 \times 10 \text{ mm} \\ &= 138.9 \text{ mm} \end{aligned}$$

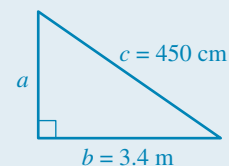
WORKED EXAMPLE 6

The hypotenuse and one other side of a right-angled triangle are 450 cm and 3.4 m respectively. Find the length of the third side, in cm, correct to the nearest whole number.

THINK

- Draw a diagram and label the sides a , b and c . Remember to label the hypotenuse as c .

WRITE/DRAW



2 Check that all measurements are in the same units. They are different, so convert 3.4 m into cm.	$3.4 \text{ m} = 3.4 \times 100 \text{ cm}$ $= 340 \text{ cm}$
3 Write Pythagoras' theorem for a shorter side.	$a^2 = c^2 - b^2$
4 Substitute the values of b and c into this rule and simplify.	$a^2 = 450^2 - 340^2$ $= 202\,500 - 115\,600$ $= 86\,900$
5 Find a by taking the square root of 86 900. Round to the nearest whole number.	$a = \sqrt{86\,900}$ ≈ 295
6 Give a worded answer.	The third side will be approximately 295 cm long.



RESOURCES — ONLINE ONLY

Digital doc: SkillsHEET Converting units of length (doc-11430)

Exercise 14.5 Working with different units

Individual pathways

PRACTISE

Questions:
1–8, 10, 11, 14, 17

CONSOLIDATE

Questions:
1, 2a–e, 3a–e, 4a–d, 5–12, 14, 17

MASTER

Questions:
1, 2e–h, 3e–h, 4c–f, 5–19

Individual pathway interactivity: int-4472

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Understanding and fluency

Where appropriate, give answers correct to 2 decimal places.

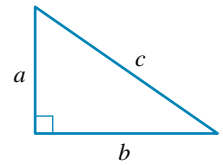
- WE5** Find the length, in mm, of the hypotenuse of a right-angled triangle, if the two shorter sides are 5 cm and 12 cm.
- Find the length of the hypotenuse of the following right-angled triangles, giving the answer in the units specified.
 - Sides 456 mm and 320 mm, hypotenuse in cm.
 - Sides 12.4 mm and 2.7 cm, hypotenuse in mm.
 - Sides 32 m and 4750 cm, hypotenuse in m.
 - Sides 2590 mm and 1.7 m, hypotenuse in mm.
 - Sides 604 cm and 249 cm, hypotenuse in m.
 - Sides 4.06 km and 4060 m, hypotenuse in km.
 - Sides 8364 mm and 577 cm, hypotenuse in m.
 - Sides 1.5 km and 2780 m, hypotenuse in km.
- WE6** The hypotenuse and one other side of a right-angled triangle are given for each case below. Find the length of the third side in the units specified.
 - Sides 46 cm and 25 cm, third side in mm.
 - Sides 843 mm and 1047 mm, third side in cm.
 - Sides 4500 m and 3850 m, third side in km.
 - Sides 20.3 cm and 123 mm, third side in cm.

- e. Sides 6420 mm and 8.4 m, third side in cm.
- f. Sides 0.358 km and 2640 m, third side in m.
- g. Sides 491 mm and 10.8 cm, third side in mm.
- h. Sides 379 000 m and 82 700 m, third side in km.

4. Two sides of a right-angled triangle are given. Find the third side in the units specified. The diagram shows how each triangle is to be labelled.

Remember: c is always the hypotenuse.

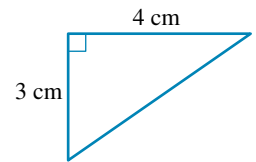
- a. $a = 37$ cm, $c = 180$ cm, find b in cm.
- b. $a = 856$ mm, $b = 1200$ mm, find c in cm.
- c. $b = 4950$ m, $c = 5.6$ km, find a in km.
- d. $a = 125\,600$ mm, $c = 450$ m, find b in m.
- e. $a = 0.0641$ km, $b = 0.153$ km, find c in m.
- f. $a = 639\,700$ cm, $b = 2.34$ km, find c in m.



5. **MC** Note: There may be more than one correct answer.

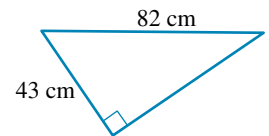
- a. What is the length of the hypotenuse in this triangle?

- A. 25 cm
- B. 50 cm
- C. 50 mm
- D. 500 mm



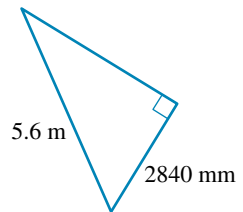
- b. What is the length of the third side in this triangle?

- A. 0.698 m
- B. 69.8 cm
- C. 6.98 cm
- D. 69.8 mm



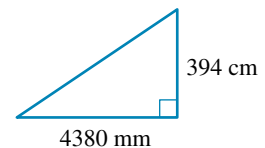
- c. The most accurate measure for the length of the third side in the triangle at right is:

- A. 4.83 m
- B. 23.3 cm
- C. 2330 mm
- D. 4826 mm

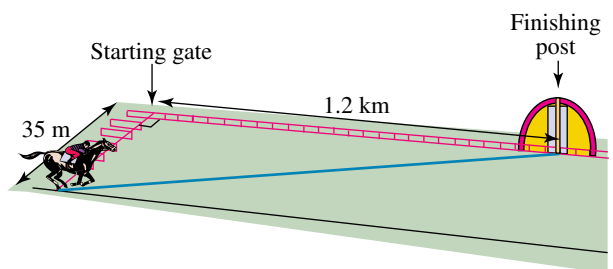


- d. What is the length of the third side in this triangle?

- A. 34.71 m
- B. 5.89 m
- C. 1722 cm
- D. 4.4 m



6. A rectangle measures 35 mm by 4.2 cm. Find the length of the diagonal in mm.
7. A sheet of A4 paper measures 210 mm by 297 mm. Find the length of the diagonal in centimetres.
8. A rectangular envelope has a length of 21 cm and a diagonal measuring 350 mm. Find:
- a. the width of the envelope
 - b. the area of the envelope.
9. A right-angled triangle has a hypotenuse of 47.3 cm and one other side of 0.308 m. Find the area of the triangle.
10. A horse race is 1.2 km. The track is straight, and 35 m wide. How much further than 1.2 km will a horse run if it starts on the outside and finishes on the inside as shown?
11. A ramp is 9 metres long, and rises to a height of 250 cm. What is the horizontal distance, in metres, between the bottom and the top of the ramp?



12. Sarah is making a gate, which has to be 1200 mm wide. It must be braced with a diagonal strut made of a different type of timber. She has only 2 m of this kind of timber available. What is the maximum height of the gate that she can make?

Communicating, reasoning and problem solving

13. A rectangular park is 260 m by 480 m. Danny usually trains by running around the edge of the park. After heavy rain, two adjacent sides are too muddy to run along, so he runs a triangular path along the other two sides and the diagonal. Danny does 5 circuits of this path for training. How far does he run? Give your answer in km.
14. A swimming pool is 50 m by 25 m. Peter is bored by his usual training routine, and decides to swim the diagonal of the pool. How many diagonals must he swim to complete his normal distance of 1.2 km?
15. A hiker walks 4.5 km west, then 3.8 km south. How far in metres is she from her starting point?
16. A square has a diagonal of 10 cm. What is the length of each side in mm?
17. The triangular sides of a wheat field measure 2.4 km, 2500 m and 700 000 mm. Is the triangle right-angled? Show your working.
18. The sides of a rectangle measure a cm by $5a$ mm. The diagonal measures $\frac{b}{10}$ m. Find the relationship between a and b .
19. How do you remember whether to multiply or divide when converting units?

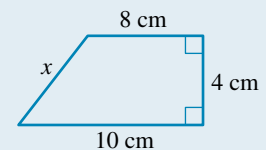
14.6 Composite shapes

14.6.1 Composite shapes

- In the exercises so far, we have been working with only one right-angled triangle. Many situations involve more complex diagrams, where the right-angled triangle is not as obvious. A neat diagram is essential for these questions.

WORKED EXAMPLE 7

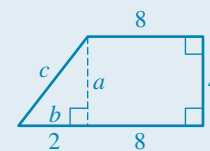
Find the length of the side, x . Give your answer correct to 2 decimal places.



THINK

- Copy the diagram. On the diagram, create a right-angled triangle and use the given measurements to work out the lengths of 2 sides.
- Label the sides of your right-angled triangle as a , b and c . Remember to label the hypotenuse as c .
- Check that all measurements are in the same units. They are the same.
- Write Pythagoras' theorem for the hypotenuse.
- Substitute the values of a , b and c into this rule and simplify.
- Find x by taking the square root of 20. Round your answer correct to 2 decimal places.

WRITE/DRAW



$$c^2 = a^2 + b^2$$

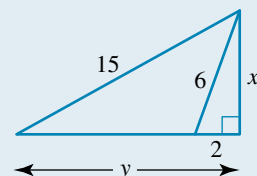
$$\begin{aligned} x^2 &= 4^2 + 2^2 \\ &= 16 + 4 \\ &= 20 \end{aligned}$$

$$\begin{aligned} x &= \sqrt{20} \\ &\approx 4.47 \text{ cm} \end{aligned}$$

- Some situations will involve shapes that contain more than one triangle. In this case it is a good idea to split the diagram into separate right-angled triangles first.

WORKED EXAMPLE 8

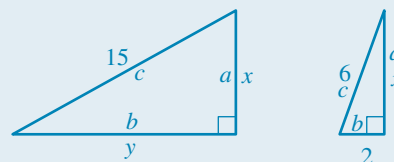
For the diagram at right, find the length of the sides marked x and y to 2 decimal places.



THINK

- Copy the diagram.
- Find and draw any right-angled triangles contained in the diagram. Label their sides.
- To find an unknown side in a right-angled triangle, we need to know 2 sides, so find x first.
- For the triangle containing x , write down Pythagoras' theorem for a shorter side. Use it to find x .
- We now know 2 sides for the other triangle because we can substitute $x = 5.66$.
- For the triangle containing y , write down Pythagoras' theorem for a shorter side and use it to find y .

WRITE/DRAW



$$\begin{aligned} a^2 &= c^2 - b^2 \\ x^2 &= 6^2 - 2^2 \\ &= 36 - 4 \\ &= 32 \\ x &= \sqrt{32} \\ &\approx 5.66 \end{aligned}$$

$$\begin{aligned} b^2 &= c^2 - a^2 \\ y^2 &= 15^2 - 5.66^2 \\ &= 225 - 32 \\ &= 193 \\ y &= \sqrt{193} \\ &\approx 13.89 \end{aligned}$$

learnON RESOURCES — ONLINE ONLY

Interactivity: Composite shapes (int-3847)

Exercise 14.6 Composite shapes

Individual pathways

PRACTISE

Questions:
1–8, 13

CONSOLIDATE

Questions:
1–10, 13, 14

MASTER

Questions:
1–16

Individual pathway interactivity: int-4473

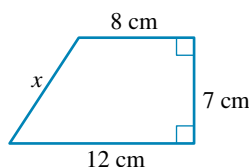
learnON ONLINE ONLY

To answer questions online and to receive **immediate feedback** and **fully worked solutions** for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

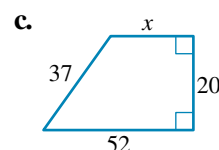
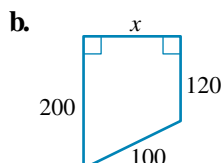
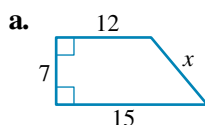
Where appropriate, give answers correct to 2 decimal places.

Understanding and fluency

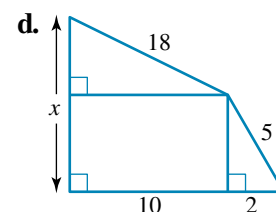
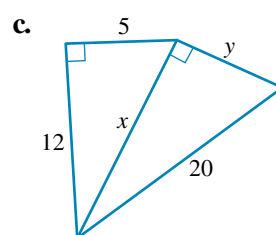
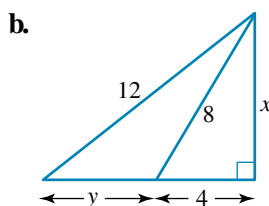
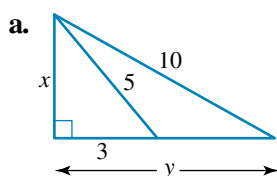
1. **WE7** Find the length of the side x in the figure below.



2. For the following diagrams, find the length of the sides marked x .



3. **WE8** For each of the following diagrams, find the length of the sides marked x and y .



4. **MC** a. The length of the diagonal of the rectangle at right is:

- A. 9.7
- B. 19
- C. 13.9
- D. 12.2

- b. The area of the rectangle at right is:

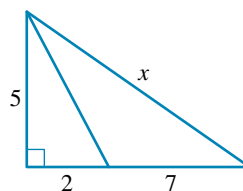
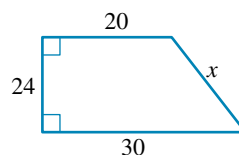
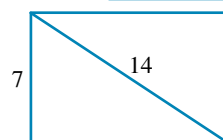
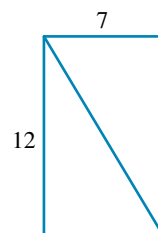
- A. 12.1
- B. 84.9
- C. 98
- D. 169.7

- c. The value of x in this shape is:

- A. 24
- B. 26
- C. 38.4
- D. 10

- d. What is the value of x in this figure?

- A. 5.4
- B. 8.6
- C. 10.1
- D. 10.3

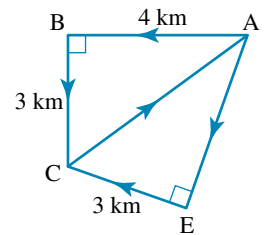


5. A garden bed is in the shape of a right-angled trapezium with the sloping edge 2.0 m, and parallel sides of 3.2 m and 4.8 m. Find the width of the garden and, hence, the area.
6. Jess paddles a canoe 1700 m to the west, then 450 m south, and then 900 m to the east. She then stops for a rest. How far is she from her starting point?

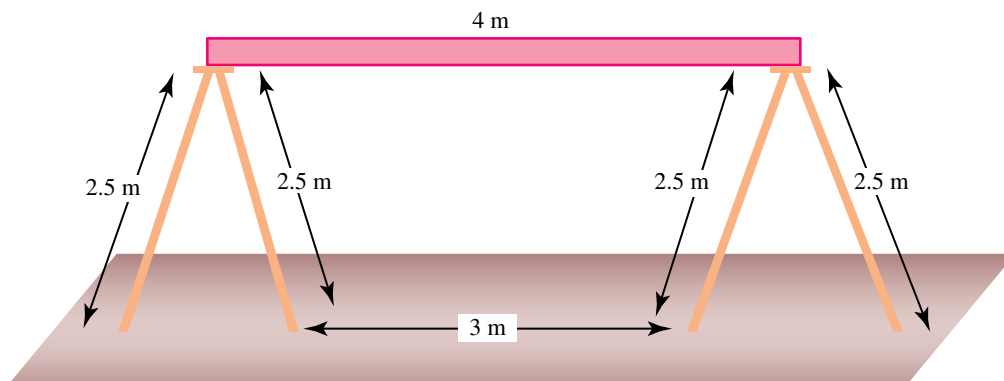
7. Two buildings, 10 and 18 m high, are directly opposite each other on either side of a European street that is 6 m wide (as shown in the following figure). What is the distance between the top of the two buildings?



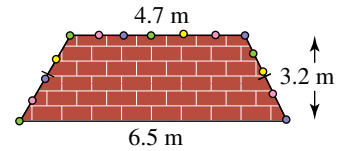
8. A yacht race starts and finishes at A and consists of 6 legs; AB, BC, CA, AE, EC, CA, in that order as shown in the figure at right. If $AB = 4$ km, $BC = 3$ km and $CE = 3$ km, find:
- AC
 - AE
 - the total length of the race.



9. A painter uses a trestle to stand on in order to paint a ceiling. It consists of 2 stepladders connected by a 4 m long plank. The inner feet of the 2 stepladders are 3 m apart, and each ladder has sloping sides of 2.5 m. How high off the ground is the plank?

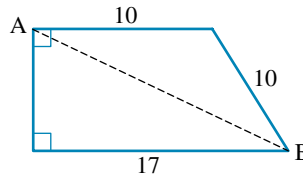
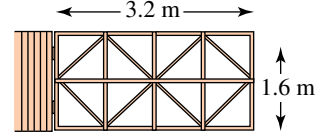


10. A feature wall in a garden is in the shape of a trapezium, with parallel sides of 6.5 m and 4.7 m. The wall is 3.2 m high. It is to have fairy lights around the perimeter (except for the base). How many metres of lighting are required?

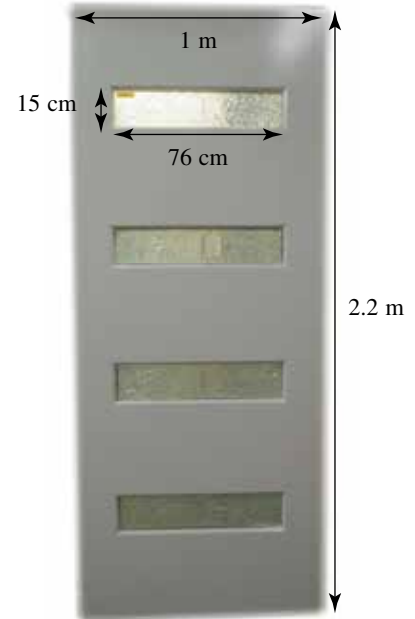


Communicating, reasoning and problem solving

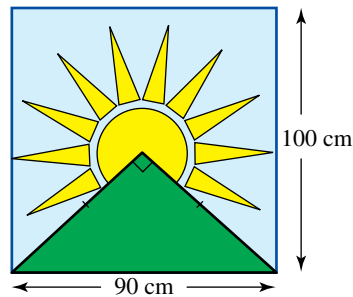
11. A rectangular gate is 3.2 m long and 1.6 m high, and consists of three horizontal beams, and five vertical beams as shown in the diagram at right. Each section is braced with diagonals. How much timber is needed for the gate?
12. Find the distance AB in the following plan of the paddock. Distances are in metres.



13. The front door shown is 1 m wide and 2.2 m high and has four identical glass panels, each 76 cm long and 15 cm wide.
- Calculate the total area of the glass panels.
 - The door is to be painted inside and outside. Calculate the total area to be painted.
 - Two coats of paint are needed on each side of the door. If the paint is sold in 1-L tins at \$24.95 per litre and each litre covers 8 m^2 of the surface, calculate the total cost of painting the door.
14. The leadlight panel shown depicts a sunrise over the mountains. The mountain is represented by a green triangle 45 cm high. The yellow sun is represented by a section of a circle with an 18-cm radius. There are 10 yellow sunrays in the shape of isosceles triangles with a base of 3 cm and a height of 12 cm. The sky is blue. Calculate the area of the leadlight panel made of:



- green glass
- yellow glass
- blue glass.



15. On each of the four sides of a square an equilateral triangle is drawn, the final figure resembling a four-pointed star. If the sides of the square are $\frac{a}{b}$ cm long, what is the area of the complete figure?
16. What do you look for when dividing composite shapes?

14.7 Pythagorean triads

14.7.1 Pythagorean triads

- Pythagorean triads, or triads for short, are 3 integers (whole numbers) that satisfy Pythagoras' theorem. This means that the 3 numbers could be the sides of a right-angled triangle.
- We test Pythagorean triads by first calculating c^2 , then calculating $a^2 + b^2$. If both calculations yield the same result, the original 3 numbers form a Pythagorean triad.
- The numbers 3, 4 and 5 form a Pythagorean triad because $3^2 + 4^2 = 25$ and $5^2 = 25$. If we multiply each of these 3 numbers by another number, the resulting numbers will also form a Pythagorean triad. For example, if we multiply 3, 4 and 5 by 5, we get 15, 20 and 25. These 3 numbers satisfy Pythagoras' theorem.

Check: $c^2 = 25^2 = 625$
 $a^2 + b^2 = 15^2 + 20^2 = 625$
so $c^2 = a^2 + b^2$

- Algebra can be used to find sets of numbers that are Pythagorean triads. There are two ways in which this can be done.

Method 1

- Start with any odd number, and make this the shortest length of the triangle.
- Use the formula $M = \frac{S^2 - 1}{2}$, where S = shortest side and M = middle side, to calculate the middle side of the triangle.
- You now have two sides of the triangle and can use $c^2 = a^2 + b^2$ to calculate the third side (hypotenuse, c).

WORKED EXAMPLE 9

If the smallest number of a Pythagorean triad is 7, find the middle number and, hence, find the third number.

THINK

- Write down the given information, and the formula to find the middle side, M .
- Substitute $S = 7$ into the formula and evaluate.

WRITE

$$\begin{aligned} S &= 7 \\ M &= \frac{S^2 - 1}{2} \\ &= \frac{7^2 - 1}{2} \\ &= \frac{49 - 1}{2} \\ &= \frac{48}{2} \\ &= 24 \end{aligned}$$

The middle number is 24.

- Use Pythagoras' theorem to find the third number.

$$\begin{aligned} c^2 &= a^2 + b^2 \\ &= 7^2 + 24^2 \\ &= 49 + 576 \\ &= 625 \\ c &= \sqrt{625} \\ &= 25 \end{aligned}$$

The third number is 25.

- State the solution.

The Pythagorean triad required is 7, 24, 25.

- Why do you think this rule works only when the smallest side is an odd number?

Method 2

- This method takes any 2 numbers, and applies a set of rules to them, in order to generate Pythagorean triads. Select 2 numbers, x and y , with the following rules:
Rule 1 The number chosen for x must be larger than y ; that is $x > y$.
Rule 2 One number must be odd and the other even, to find the first triad.
Rule 3 The numbers chosen for x and y must have no common factors. For example, we could not choose 6 and 9 since they have a common factor of 3.
- The Pythagorean triad is given by $2xy, x^2 - y^2, x^2 + y^2$. Additional triads can be found by multiplying each of these side lengths by a common multiple.

DISCUSSION

What strategies could you use to memorise as many common triads as possible?

WORKED EXAMPLE 10

Find a Pythagorean triad using the values $x = 7$ and $y = 2$.

THINK

- 1 Write down the values given in the question.
- 2 Substitute these values into the expression for each term.
- 3 Check that the numbers obtained satisfy Pythagoras' theorem.
Does $c^2 = a^2 + b^2$?
- 4 State the solution.

WRITE

$$x = 7, y = 2$$

$$2xy = 2 \times 7 \times 2 \\ = 28$$

$$x^2 - y^2 = 7^2 - 2^2 \\ = 49 - 4 \\ = 45$$

$$x^2 + y^2 = 7^2 + 2^2 \\ = 49 + 4 \\ = 53$$

$$c^2 = 53^2 \\ = 2809$$

$$a^2 + b^2 = 28^2 + 45^2 \\ = 784 + 2025 \\ = 2809$$

The Pythagorean triad required is 28, 45, 53.



RESOURCES — ONLINE ONLY

Interactivity: Pythagorean triples (int-2765)

Interactivity: Pythagorean triads (int-3848)

Digital doc: WorkSHEET Pythagoras' theorem (doc-11433)

Exercise 14.7 Pythagorean triads

Individual pathways

PRACTISE

Questions:
1–9

CONSOLIDATE

Questions:
1–10, 12

MASTER

Questions:
1–15

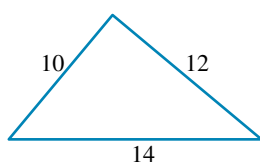
Individual pathway interactivity: int-4474

learnon ONLINE ONLY

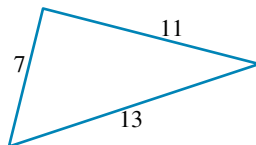
Understanding and fluency

- Use Pythagoras' theorem to decide which of these triangles are right-angled.
 - 6, 8, 10
 - 5, 12, 13
 - 4, 5, 6
 - 24, 7, 25
 - 16, 20, 12
 - 14, 16, 30
- Use the triads below to create three other triads, and check that they satisfy Pythagoras' theorem.
 - 3, 4, 5
 - 5, 12, 13
- WE9** The smallest numbers of four Pythagorean triads are given below. Find the middle number and, hence, find the third number.
 - 9
 - 11
 - 13
 - 29
- What do you notice about the triads in question 3?
- WE10** For each of the following, find a Pythagorean triad using the given values of x and y . (In each case check that the 3 numbers found satisfy Pythagoras' theorem.)
 - $x = 6$ and $y = 1$
 - $x = 7$ and $y = 4$
 - $x = 8$ and $y = 3$
 - $x = 11$ and $y = 6$
 - $x = 14$ and $y = 9$
 - $x = 15$ and $y = 2$
- Are the following triangles right-angled? Use Pythagoras' theorem to check.

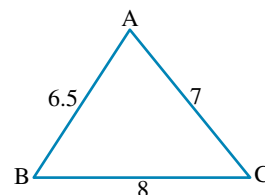
a.



b.



- Use Pythagoras' theorem to prove that the triangle at right is not right-angled.
- Does the method used in question 5 work if both x and y are even?
 - Does it work if both x and y are odd?Try some examples of your own.



Communicating, reasoning and problem solving

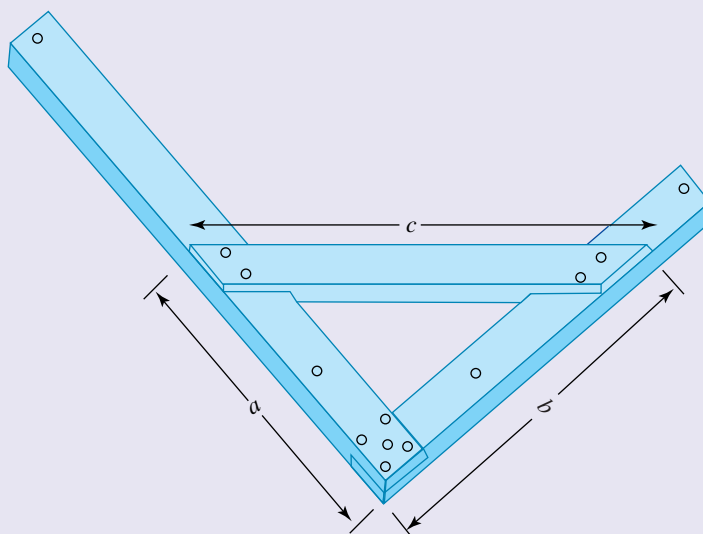
- If 32, x , 68 is a Pythagorean triad, what is the value of x ? Show your working.
- Find a Pythagorean triad in which the smallest number is 33. Explain how you reached the solution.
- Prove that $(a^2 - b^2)$, $2ab$, $(a^2 + b^2)$ form a Pythagorean triad.
- The lengths of the sides of a particular triangle are a cm, $2a$ cm and $3a$ cm. Prove that the triad formed is not a Pythagorean triad.
- If $(p - q)$, p , $(p + q)$ form a Pythagorean triad, find a relationship between p and q .
 - If $p = 8$, find the Pythagorean triad.
- What strategies could you use to memorise as many common triads as possible?
- In a right-angled triangle the smallest angle is two-thirds the size of the other angle, which is not a right angle. What is the size of the smallest angle?

14.8 Review

Investigation | Rich task

Are these walls at right angles?

Builders often use what is called a ‘builder’s square’ when pegging out the foundations of a building to ensure that adjacent walls are at right angles. Even an error of just 1 or 2 degrees could mean that walls at the opposite end of a building may not end up in line. A builder’s square uses the properties of Pythagoras’ theorem. The diagram below shows how a builder’s square is constructed. The hypotenuse side, c , acts as a brace to keep the two adjacent sides, a and b , in the correct position.



We will investigate the use of the builder’s square. Cut two thin strips of paper to represent the arms a and b as shown in the diagram above. Join these two strips at one point with a pin to make the shape of the builder’s square. The length of c can be obtained by measuring the distance from the end of a to the end of b . Complete questions 1 to 4, and record your results in the table on the following page.

1. Open the arms so that they make an angle of 90° . Use a protractor to measure the angle. Carefully measure the strips to obtain the values for a , b and c in millimetres. Record your results in the first row of the table.
2. Open the arms so that they make an angle less than 90° . Use a protractor to measure the angle. Carefully measure the strips to obtain the values for a , b and c in millimetres. Complete the second row of the table.
3. Repeat question 2 with the arms opened up to an angle greater than 90° . Complete the third row of the table.

4. Change the lengths of a and b by constructing a new builder's square. Repeat questions 1 to 3 and complete the last three rows of the table.

Length of a	Length of b	Length of c	$a^2 + b^2$	c^2	Angle

5. Consider the last three columns of the table. Using your results, what conclusions can you draw?
6. Construct a new builder's square and open it to an angle that you estimate to be 90° . Take measurements of a , b and c and record them below. What conclusions would you draw regarding your angle estimate? Check by measuring your angle with a protractor.
7. From your investigations, write a paragraph outlining how the measurements of the lengths of a , b and c on a builder's square can be used by the builder to determine whether the angle between adjacent walls would be equal to, greater than or less than a right angle.

learnon RESOURCES — ONLINE ONLY

Digital doc: Investigation — Are these walls at right angles? (doc-13949)

Digital doc: Code puzzle — What is Bagheera in *The Jungle Book*? (doc-13950)

Digital doc: Summary Topic 14 (doc-23250)

Interactivity: Word search Topic 14 (int-3385)

Interactivity: Crossword Topic 14 (int-3386)

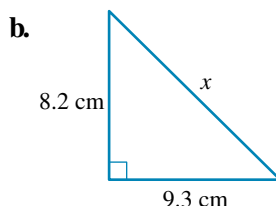
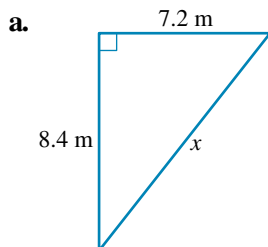
Interactivity: Sudoku Topic 14 (int-3194)

Exercise 14.8 Review questions

To answer questions online and to receive **immediate feedback** and **fully worked solutions** for every question, go to your learnON title at www.jacplus.com.au. *Note:* Question numbers may vary slightly.

Understanding and fluency

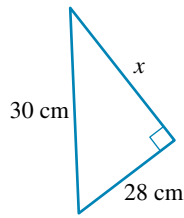
- What is the relationship of the hypotenuse, c , and the two shorter sides, a and b , of a right-angled triangle?
- Calculate x to 2 decimal places.



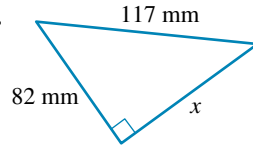
- The top of a kitchen table measures 160 cm by 90 cm. A beetle walks diagonally across the table. How far does it walk?
- A broomstick leans against a wall. The stick is 1.5 m long and reaches 1.2 m up the wall. How far from the base of the wall is the bottom of the broom?

5. Calculate x . Leave your answers in exact form.

a.



b.



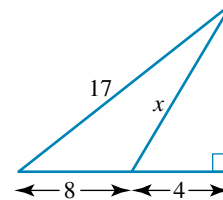
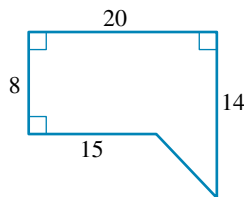
6. How high up a wall can a 20 m ladder reach when it is placed 2 m from the foot of a wall? Give your answer correct to 1 decimal place.

7. A rectangular garden bed measures 3.7 m by 50 cm. Find the length of the diagonal, in cm.

8. A road sign is in the shape of an equilateral triangle with sides of 600 mm. Find the area of the sign, in cm^2 .

9. Calculate x in the figure at right.

10. Find the perimeter of the shape below.



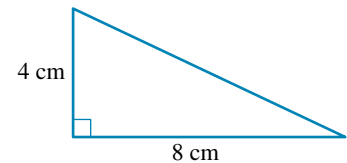
Communicating, reasoning and problem solving

11. Do the sides 4 cm, 5 cm and 6 cm form a right-angled triangle?

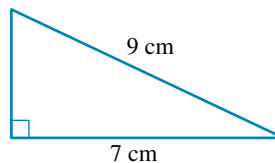
12. Calculate the length of the hypotenuse of a right-angled triangle with sides of 10 cm and 24 cm.

13. Calculate the length of the shortest side of a right-angled triangle with sides of 15 cm and 12 cm.

14. The length of the hypotenuse in the triangle at right is closest to which integer?



15. The length of the shortest side in this triangle is closest to which integer?



16. Calculate the perimeter of a right-angled triangle with a hypotenuse of 6.5 cm and another side of 2.5 cm.

17. An ironwoman race involves 3 swim legs and a beach run as shown in the figure at right.

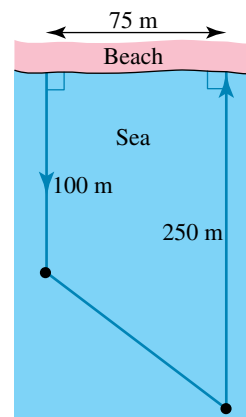
What is the total distance covered in the race?

18. Which of the following are Pythagorean triads?

a. 15, 36, 39

b. 50, 51, 10

c. 50, 48, 14



Answers

Topic 14 Pythagoras' theorem

Exercise 14.2 Right-angled triangles

1.	a	b	c	d	e	f	g	h
a	19	24	27	16	31	41	13	24
b	35	32	30	40	25	19	45	24
c	40	40	40	43	40	45	47	34
a^2	361	576	729	256	961	1681	169	576
b^2	1225	1024	900	1600	625	361	2025	576
$a^2 + b^2$	1586	1600	1629	1856	1586	2042	2194	1152
c^2	1600	1600	1600	1849	1600	2025	2209	1156

2. $a^2 + b^2 \approx c^2$

3. $a^2 + b^2 = c^2$

4. $a^2 + b^2 \neq c^2$

5. The angle sum of a triangle is 180° . If a triangle contained two right angles, the third angle would be 0° , which would mean it was a straight line rather than a triangle.

6. g. The square on the hypotenuse is equal to the sum of the squares on the two other sides.

7. It is not a possible triangle because $7 + 4 < 12$.

8. 90° . The right angle is 90° and the angle sum of a triangle is 180° . Therefore the sum of the other two angles must equal 90° .

9. The triangle is isosceles; the smaller angles are 45° .

10. It is not possible as all angles are 60° in an equilateral triangle.

11. a. Answers will vary.

b. i. 1.732 ii. 0.866 iii. 0.5

c. The values are the same for all the triangles.

d. Answers will vary.

Exercise 14.3 Finding the hypotenuse

1. a. $x = 5$ b. $x = 13$ c. $x = 25$ d. $x = 21.3$ e. $x = 1163.3$ f. $x = 6.3$

2. a. $\sqrt{9760}$ b. $\sqrt{11\,194}$ c. $\sqrt{394}$ d. $\sqrt{16\,781}$ e. 100 f. $\sqrt{68}$

3. $\sqrt{146}$ cm

4. a. 16.97 cm b. 28.28 mm c. 6.93 cm

5. a. 12.81 cm b. 737.83 cm c. 17.26 cm

6. 18.03 cm 7. 19.23 cm 8. 4.66 m 9. \$81.60

10. The negative solution is not feasible as lengths must be positive.

11. a. 189 cm^3 b. 6.95 cm

12. 25 m 13. 314.16 cm^2

14. The square of the hypotenuse (the side opposite the right-angle) is equal to the sum of the squares of the other two sides.

15. Side length ≈ 2.83 cm; area = 8 cm^2

Exercise 14.4 Finding a shorter side

1. a. 6 b. $\sqrt{93}$ c. $\sqrt{6032}$ d. $\sqrt{62\,925}$ e. $\sqrt{15}$ f. $\sqrt{80}$

2. a. 36.36 b. 1.62 c. 15.37 d. 0.61 e. 2133.19 f. 453.90

3. 23.04 cm

4. a. 97.47 cm b. 334.94 cm c. 6822.90 cm^2

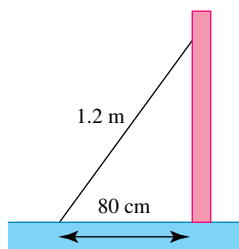
5. 17.32 cm

6. 65.82 cm; 2501.16 cm^2

7. 2.60 m 8. 1.32 m 9. Yes

10. 13.69 m 11. 20.61 m 12. 45.43 cm

13. a.



b. 89.44 cm

c. Yes, she will reach the hook from the top step.

14. a. 4 cm

b. The diagonals are perpendicular to each other because they create right-angled triangles.

15. Identify which side is the hypotenuse before calculating the length of the missing side. Pythagoras' theorem needs to be rearranged if the missing length is a shorter side of the triangle.

Exercise 14.5 Working with different units

1. 130 mm

2. a. 55.71 cm

b. 29.71 mm

c. 57.27 m

d. 3098.08 mm

e. 6.53 m

f. 5.74 km

g. 10.16 m

h. 3.16 km

3. a. 386.13 mm

b. 62.09 cm

c. 2.33 km

d. 16.15 cm

e. 541.70 cm

f. 2615.61 m

g. 478.97 mm

h. 369.87 km

4. a. 176.16 cm

b. 147.40 cm

c. 2.62 km

d. 432.12 m

e. 165.88 m

f. 6811.55 m

5. a. C

b. A, B

c. D

d. B

6. 54.67 mm

7. 36.37 cm

8. a. 28 cm

b. 588 cm²

9. 552.86 cm²

10. 0.51 m

11. 8.65 m

12. 1600 mm

13. 6.43 km

14. 21.46 diagonals, so would need to complete 22

15. 5889.82 m

16. 70.71 mm

17. Yes

18. $a^2 = 80b^2$

19. Divide when you are converting to a larger unit and multiply when you are converting to a smaller unit.

Exercise 14.6 Composite shapes

1. 8.06 cm

2. a. $x = 7.62$

b. $x = 60$

c. $x = 20.87$

3. a. $x = 4, y = 9.17$

b. $x = 6.93, y = 5.80$

c. $x = 13, y = 15.20$

d. $x = 19.55$

4. a. C

b. B

c. B

d. D

5. 1.2 m; 4.8 m²

6. 917.88 m

7. 10 m

8. a. 5 km

b. 4 km

c. 24 m

9. 2.45 m

10. 11.35 m

11. 26.65 m

12. 18.44 m

13. a. 4560 cm² or 0.456 m²

b. 34 880 cm² or 3.488 m²

c. The paint job would need 1 tin of paint, costing \$24.95. The cost of the paint used would be \$21.76.

14. a. 2025 cm²

b. 943.41 cm²

c. 6031.59 cm²

15. $\left(1 + \sqrt{3}\right) \frac{a^2}{b^2}$

16. Look for places where you can make right-angled triangles.

Exercise 14.7 Pythagorean triads

1. Yes: a, b, d, e No: c, f

2. Answers will vary. Possible answers:

a. 6, 8, 10; 9, 12, 15; 12, 16, 20

b. 10, 24, 26; 15, 36, 39; 20, 48, 52

3. a. 40, 41

b. 60, 61

c. 84, 85

d. 420, 421

4. The two larger numbers are consecutive numbers.
5. **a.** 12, 35, 37 **b.** 33, 56, 65 **c.** 48, 55, 73 **d.** 85, 132, 157 **e.** 115, 252, 277 **f.** 60, 221, 229
6. **a.** $10^2 + 12^2 \neq 14^2$ No **b.** $7^2 + 11^2 \neq 13^2$ No
7. $6.5^2 + 7^2 = 91.25$
 $8^2 = 64$
 Therefore, the triangle is not right-angled.
8. **a.** Yes **b.** Yes
9. 60
10. The following Pythagorean triads can be found: 33, 44, 55; 33, 56, 65; 33, 180, 183; 33, 544, 545. Explanations will vary.
11. Check with your teacher.
12. Check with your teacher.
13. **a.** $p = 4q$ **b.** 6, 8, 10
14. Discuss with your teacher.
15. 36°

Investigation | Rich task

- 1 to 4. Answers will vary. Teacher to check.
5. When the angle is equal to 90° , $c^2 = a^2 + b^2$.
 For an angle greater than 90° , $c^2 > a^2 + b^2$.
 For an angle smaller than 90° , $c^2 < a^2 + b^2$.
6. If $c^2 = a^2 + b^2$, then the angle is equal to 90° . Check by measuring the angle.
7. Paragraph explanation of the results in question 5; teacher to check.

Exercise 14.8 Review questions

1. $c^2 = a^2 + b^2$	2. a. 11.06 m	b. 12.40 cm	3. 183.58 cm
4. 0.9 m	5. a. $\sqrt{116}$	b. $\sqrt{6965}$	6. 19.9 metres
7. 373.36 cm	8. 1558.85 cm ²	9. 12.69	10. 64.81
11. No	12. 26 cm	13. 9 cm	14. 9 cm
15. 6 cm	16. 15 cm	17. 592.71 m	
18. a. Yes	b. No	c. Yes	

GLOSSARY

- Alternate angles:** angles on opposite sides of the transversal positioned between the parallel lines. These angles are equal in size.
- Arc:** a section of the circumference of a circle
- Associative Law:** a number law that refers to the order in which three numbers may be added, subtracted, multiplied or divided, taking two at a time
- Attribute:** a feature or characteristic of someone or something
- Base:** the number or variable that is being multiplied when an expression is written in index form
- Biased:** leaning in a favoured direction
- Cartesian plane:** a coordinate grid formed by an x - and y -axis
- Census:** the collection of data in which every member of a target population is surveyed
- Centre of rotation:** the point that a point or shape is rotated about
- Certain:** used in probability to describe an event that will definitely occur
- Chance experiments:** an experiment in which the outcomes are open to chance
- Chord:** a straight line joining any two points on the circumference of a circle
- Circumference:** the distance around the outside of a circle. It is equal to $2\pi r$ or πD , where r = radius and D = diameter of the circle.
- Class interval:** an equal-sized group of scores
- Closed question:** a question with one possible solution
- Cluster:** a grouping of data values in close proximity to each other
- Co-interior angles (allied angles):** angles on the same side of the transversal positioned between the parallel lines. These angles are supplementary.
- Commutative Law:** a number law that refers to the order in which two numbers may be added, subtracted, multiplied or divided
- Complementary events:** events that together form a sample space
- Compound event:** an event that is made up of more than one individual event
- Congruent figures:** two figures that are identical in size and shape. The orientation of the figures does not have to be the same.
- Corresponding angles:** angles on the same side of the transversal that are both either above or below the parallel lines. These angles are equal in size.
- Data:** various forms of information
- Denominator:** the bottom term of a fraction. It shows the total number of parts the whole has been divided into.
- Diagonal:** a line that runs from one corner of a closed figure to an opposite corner
- Diameter:** the straight-line distance across a circle through its centre
- Directed number:** a number that has both size and direction; for example, $+3$ and -7
- Distributive Law:** a rule that states that each term inside a pair of brackets is to be multiplied by the term outside the brackets
- Equivalent fractions:** fractions that are equal in value; for example, $\frac{1}{2} = \frac{3}{6}$
- Estimate:** an approximate answer when an accurate answer is not required
- Evaluate:** to obtain an answer
- Event:** a set of favourable outcomes in each trial of a probability experiment
- Exclusive (event):** a compound event where an outcome cannot satisfy both of the qualities being described
- Experiment:** the process of performing repeated trials of an activity in probability for the purpose of obtaining data in order to be able to predict the chances of certain things happening

Exponent: *see* **Power**

Factorising: the process of transforming an expanded expression into factors which are multiplied together

Favourable outcome: the desired result in a probability experiment

Frequency polygon: the line graph created when the midpoints of the tops of the columns of a histogram are joined by straight lines. The polygon is closed by drawing lines at each end down to the x -axis (score).

Gradient: a measure of how steep something is; that is, its slope. The gradient of a straight line is given by:

$$m = \frac{\text{vertical distance}}{\text{horizontal distance}}.$$

Histogram: a type of column graph in which no gaps are left between columns and each column ‘straddles’ an x -axis score, such that the column starts and finishes halfway between scores. The x -axis scale is continuous and usually a half-interval is left before the first column and after the last column.

Hypotenuse: the longest edge of a right-angled triangle

Identity law for addition: when 0 is added to any number, the original number remains unchanged

Identity law for multiplication: when any number is multiplied by 1, the original number remains unchanged

Image: a point or shape after it has been transformed

Impossible: used in probability to describe an event that will not occur

Improper fraction: a fraction whose numerator is larger than its denominator; for example, $\frac{5}{4}$

Included angle: the angle formed between two lines that meet at a point

Inclusive (event): a compound event where an outcome can satisfy both of the qualities being described

Index: *see* **Power**

Index form: the short way of writing a number or variable when it is multiplied by itself repeatedly

Infinite: not finite; never ending; unlimited

Integers: positive whole numbers, negative whole numbers and zero

Inverse law for addition: when a number is added to its additive inverse, the result is 0

Inverse law for multiplication: when a number is multiplied by its multiplicative inverse, the result is 1

Irrational numbers: numbers that can not be written as fractions

Isometric: having the exact same size and shape

Kite: a quadrilateral in which two pairs of adjacent sides are equal in length and one pair of opposite angles (those between the sides of unequal length) are equal

Like terms: terms that contain exactly the same variables (letters); for example, $3ab$ and $7ab$ are like terms, but $5a$ and $6ab$ are not

Linear equation: an equation in which the dependent variable has an index (power) of 1

Linear graph: a graph in which all of the points fall on a straight line

Linear pattern: a pattern of points which when plotted can be joined to form a straight line

Mean: in summary statistics, the sum of all the scores divided by the number of scores. It is also called the average.

Measures of centre: any of a number of terms used to describe a central value in a data set

Measures of spread: any of a number of terms used to describe how the values in a data set are spread or scattered

Median: in summary statistics, the middle value if the number of data is odd, or the average of the two middle values if the number of data is even. Data must first be arranged in numerical order.

Mixed number: a number made up of a whole number and a fraction; for example, $2\frac{3}{4}$

Mode: in summary statistics, the most common value (score) or values in a set of data

Mutually exclusive: two events that cannot both occur at the same time

Numerator: the top term of a fraction. It shows how many parts there are.

Open question: a question with more than one possible answer

Opposite angles: angles in a polygon that are opposite

Opposite sides: sides in a polygon that are opposite

Ordered pair: a pair of coordinates, with the x -coordinate appearing before the y -coordinate

Origin: the centre of a Cartesian plane, $(0, 0)$, where the x - and y -axes intersect

Outcome: the particular result of a trial in a probability experiment

Outlier: a piece of data that is much larger or smaller than the rest of the data

Parallelogram: a quadrilateral with both pairs of opposite sides parallel to each other. Rectangles, squares and rhombuses are parallelograms.

Per cent: out of 100

Perimeter: the distance around the outside (border) of a shape

Population: every member or data point under consideration

Power: the number that indicates how many times the base is being multiplied by itself when an expression is written in index form. *Also known as* an exponent or index.

Primary data: data that has been collected first-hand (by yourself)

Probability: the likelihood or chance of a particular event (result) occurring.

$$P(\text{event}) = \frac{\text{number of favourable outcomes}}{\text{number of possible outcomes}}$$

The probability of an event occurring ranges from 0 (impossible — will not occur) to 1 (certain — will definitely occur).

Pronumeral: a letter used in place of a number; another name for a variable

Proper fraction: a fraction whose numerator is smaller than its denominator; for example, $\frac{3}{4}$

Proportion: equality of two or more ratios

Quadrant: one of the four sections of a Cartesian plane

Quadratic equation: an equation in which the highest power of the pronumeral is 2

Quadrilateral: a 2-dimensional, closed shape formed by four straight sides

Questionnaire: a set of questions used in a survey

Radius: a straight line from a circle's centre to any point on its circumference

Random: following no particular order or pattern. To ensure that they are free from bias, surveys should be as random as possible.

Range: in summary statistics, the difference between the highest and lowest values (scores)

Ratio: comparison of two or more quantities of the same kind

Rational numbers: numbers that can be expressed as fractions

Real numbers: the set of all rational and irrational numbers

Rectangle: a parallelogram in which the opposite sides are equal in length and all angles are equal to 90°

Recurring decimal: a number with an infinitely repeating pattern of decimal places

Reflections: transformations whereby a point or object is reflected in a mirror line

Rhombus: a parallelogram in which all sides are equal and opposite angles are equal

Right-angled triangle: a triangle that has one of its angles equal to 90° (a right angle)

Rotations: transformations whereby a point or object is rotated around a point (the centre of rotation)

Rounding: expressing a number with a certain number of decimal places

Rounding down: A number ending in 0, 1, 2, 3 or 4 is rounded down.

Rounding up: A number ending in 5, 6, 7, 8 or 9 is rounded up.

Sample: part of a whole population

Sample space: in probability, the complete set of outcomes or results obtained from an experiment. It is shown as a list enclosed in a pair of braces, $\{ \}$.

Secondary data: data that has been collected second-hand (by someone else)

Sectors: a region of a circle bounded by two radii and the arc joining them

Segment: a section of a circle bounded by a chord and an arc

Sequence: a set of numbers that follow a pattern

Square: a parallelogram in which all sides are equal and all angles are equal to 90°

Statistics: the branch of mathematics that deals with the collection, organisation, display, analysis and interpretation of data, which are usually presented in numerical form

Substitution: the process by which a number replaces a variable in a formula

Tangent: a straight line that touches the circumference of a circle only at one given point

Terms: a group of numbers and variables connected by the operations of multiplication or division

Terminating decimal: a number with a set number of decimal places

Theoretical probability: the probability of an event based on the number of possible favourable outcomes and the total number of possible outcomes

Translations: transformations whereby a point or object is moved without flipping, turning or changing size

Transversal: a line that intersects a pair (a set) of parallel lines

Trapezium: quadrilateral in which one pair of opposite sides is parallel

Two-way table: a diagram that represents the relationship between two non-mutually exclusive attributes

Undefined: a numeric value that cannot be calculated

Unit price: the price per unit of an item; for example, the price per 100 mL of beverage

Unitary method: a way of working out the best buy by comparing the unit price of items

Unknown: a variable in an algebraic equation or expression

Variables: letters or symbols in an equation or expression that may take many different values

Venn diagram: a diagram representing different attributes represented by circles, inside a rectangular frame that represents the population

Vertically opposite angles: special angles formed when two straight lines intersect. The two non-adjacent angles are called vertically opposite angles. These angles are equal in size.

x-axis: the horizontal axis in a Cartesian plane

y-axis: the vertical axis in a Cartesian plane

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