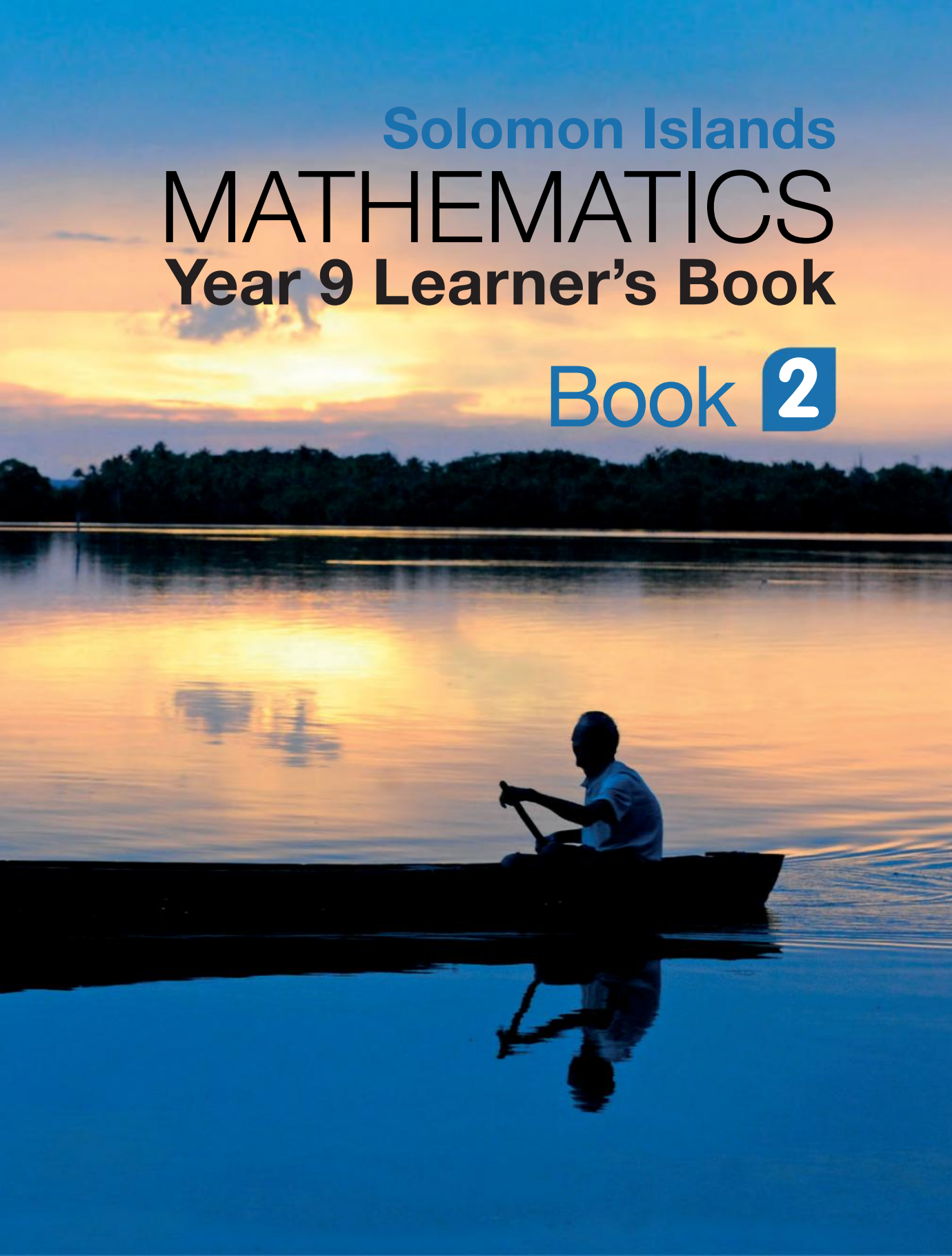
Solomon Islands

# MATHEMATICS

## Year 9 Learner's Book

Book 2



ALWAYS LEARNING

PEARSON

Solomon Islands

# MATHEMATICS

## Year 9 Learner's Book

Book 2

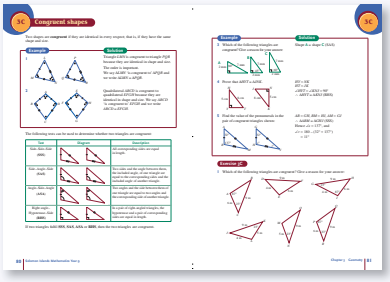
# CONTENTS

# HOW TO USE THIS BOOK

The **Solomon Islands Mathematics** series has been written to cover the General Learning Outcomes of the Solomon Islands Secondary Mathematics Syllabus Years 7 to 9.

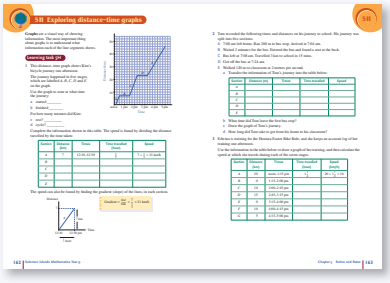

## Chapter opening pages

Chapter opening pages include a contemporary or historical context for the content and provide learners with a list of the skills that are covered in the chapter.




## Theory and exercise sections

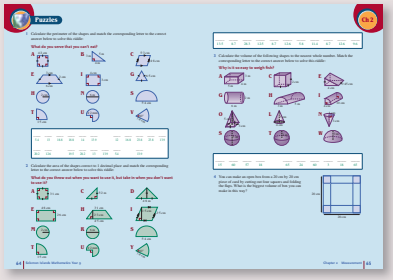
Theory and exercise sections contain explanations, examples and exercises designed to develop understanding of concepts and provide opportunities for students to practise new skills.




## Explorations

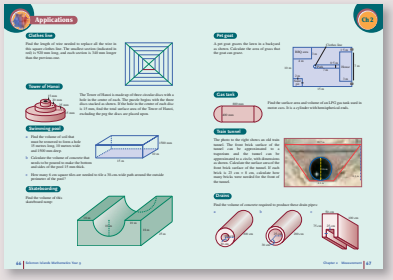
Explorations are scattered throughout the chapters, allowing students to work independently on non-standard problems and construct their own understandings.

# These features are found at the end of each chapter



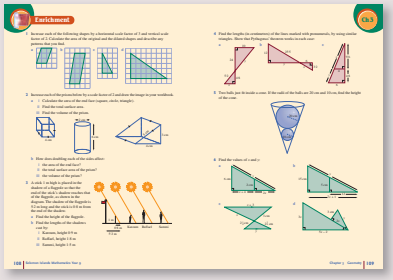

## Puzzles

Puzzles are included for extra skills practice.



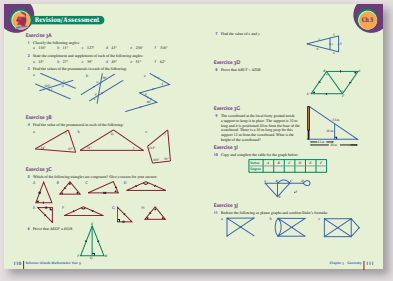

## Application

Application sections investigate and apply mathematical ideas in a creative way and provide activities for a range of learner abilities.




## Enrichment

Enrichment sections contain challenging tasks for learners to apply and extend their understanding of concepts.




## Revision/Assessment

Revision/Assessment sections provide opportunities for learners to consolidate their understanding of concepts.

# Solomon Islands Mathematics Year 9 Learner's Book

## Introduction

This book is written to help you learn Mathematics by actively participating in a variety of activities. The book has a total of 12 chapters. Each chapter focuses on a particular topic from one of the strands in the Solomon Islands Junior Secondary Mathematics Syllabus. The strands are *Number*, *Measurement*, *Algebra*, *Geometry*, *Trigonometry*, *Statistics and Probability*. We hope that the activities in this book will encourage you to learn Mathematics effectively, and gain enjoyment and enrichment from the topics and contexts involved.

## Chapter organisation

The chapter order provides opportunities to revise topics studied in earlier years, to learn new knowledge and skills, and to review and develop your understanding throughout the school year.

## The Number strand

The chapters that will further develop your number skills include *Number Systems*, *Ratios and Rates*, *Consumer Maths* and *Indices*. You will continue your understanding of numbers, the ways they are represented, and the quantities for which they stand. You will develop accuracy, efficiency, and confidence in calculating, both mentally, and on paper. You will refine your ability to estimate and to make approximations, and to be alert to the reasonableness of results and measurements. It is important to maintain your competency in the four basic operations, and apply them confidently with whole numbers, directed numbers, decimals and fractions. By extending these skills to the real numbers, ratios and rates, and indices, you will be able to apply mathematics to solving problems in real-life contexts, and especially to Consumer Maths.

## The Measurement strand

There is just one chapter to consolidate and build on your measurement skills from earlier years in Year 9. New applications explore curved surfaces and composite shapes and you will develop these skills to apply different formulas to surface area and volume problems. You will continue to use and convert metric units, make sensible estimates and round measurements to appropriate degrees of accuracy. Above all, you will need to be able to use measurements to solve problems in practical contexts.

## The Algebra strand

The chapters that will develop your algebra skills in Year 9 are *Algebra Skills*, *Linear Equations and Formulas*, and *Linear Graphs*. You will continue to learn to recognise patterns and relationships in mathematics and the real world, and be able to generalise from them. More importantly, you will develop your ability to think abstractly and to use symbols, graphs and diagrams to represent and communicate mathematical relationships, concepts, and generalisations. You will need to manipulate algebraic expressions confidently to solve practical problems.

### The Geometry strand

The Geometry chapter at Year 9 draws on much of the work from earlier years concerned with size, shape, position, and the properties of space, and introduces the key concepts of congruent and similar figures. You will need to visualise shapes in two and three dimensions, recognise and appreciate their occurrence in the environment, and to make use of the geometrical properties of everyday objects. You will also appreciate how to use geometric models as aids to solve practical problems in time and space with an introduction to networks.

### The Trigonometry strand

Trigonometry is a new strand that is introduced at Year 9 with a chapter of its own. It draws on key skills from other strands, in particular, the geometry and algebra strands. Trigonometry has many applications, especially those of a practical nature such as engineering, architecture, navigation and surveying where problems can be modelled with triangles. By expanding your knowledge and understanding to finding unknown angles and lengths for triangles in two and three dimensions in Year 9, you will provide yourself with a firm foundation for future Mathematical studies.

### The Statistics and Probability strand

There is one chapter for your study of *Statistics* and another for *Probability*. Newspapers are full of statistical information and it is important that you understand statements and graphs to check the accuracy of the conclusions. Your study of Probability will help you describe the chance of various events occurring and the decision-making skills can be applied to many real-life situations.

### How to learn Mathematics

As you work through the chapters you will be asked to work on your own, work with a partner or in a group, and sometimes with the whole class. Therefore, you must be willing to participate actively in all the tasks and not rely on the teacher or friends for answers. When you actively participate you will learn a great deal as well.

### Making mistakes

Learning Mathematics is a skill, like riding a bicycle. You cannot learn to ride a bicycle by just listening to the teacher telling you how to ride, you can only learn by doing it. Nobody has learnt to ride a bicycle without falling off many times. Making mistakes is part of the learning process and this is also true for Mathematics. The more familiar you are with the topic, the fewer mistakes you are likely to make. Like bicycle riding, Mathematics learning needs lots of practice and the exercises in this book are designed to help you practice until you become confident with each new skill. Homework is a chance to further practice the skills learnt in class, and what you can't do on your own, you can ask your teacher or a friend the next day.

### Developing skills

Mathematics is more than a series of facts and rules. It involves understandings and skills that can be applied to new situations. After each lesson it is useful to reflect on your learning and in particular about the problem solving strategies that you used that day. Those same strategies may be useful for other problems in the future. And if you discover a new skill, show it to a friend. Not only will your friend benefit, but it will help you remember it too!

# Suggested teaching plan for the Year 9 Learner's Book

## Semester 1

| Weeks | Sub-strands | Allocated Times |
|---|---|---|
| | **Number** | |
| **1** | Chapter 1: Number Systems | 3 Weeks |
| **2** | | |
| **3** | | |
| | **Measurement** | |
| **4** | Chapter 2: Measurement | 3 Weeks |
| **5** | | |
| **6** | | |
| | **Geometry** | |
| **7** | Chapter 3: Geometry | 2 Weeks |
| **8** | | |
| | **Algebra** | |
| **9** | Chapter 4: Algebra Skills | 2 Weeks |
| **10** | | |
| | **Number** | |
| **11** | Chapter 5: Ratios and Rates | 3 Weeks |
| **12** | | |
| **13** | | |
| **14** | Chapter 6: Consumer Maths | 3 Weeks |
| **15** | | |
| **16** | | |
| | **Statistics and Probability** | |
| **17** | Chapter 7: Statistics | 3 Weeks |
| **18** | | |
| **19** | | |
| **20** | *Mid-year Examinations* | 1 Week |
| Mid-year Holidays | | |

## Semester 2

| Weeks | Sub-strands | Allocated Times |
|---|---|---|
| | **Trigonometry** | |
| **21** | Chapter 8: Trigonometry | 3 Weeks |
| **22** | | |
| **23** | | |
| | **Algebra** | |
| **24** | Chapter 9: Linear Equations and Formulas | 2 Weeks |
| **25** | | |
| **26** | Chapter 10: Linear Graphs | 3 Weeks |
| **27** | | |
| | **Statistics and Probability** | |
| **28** | Chapter 11: Probability | 3 Weeks |
| **29** | | |
| **30** | | |
| | **Number** | |
| **31** | Chapter 12: Indices | 3 Weeks |
| **32** | | |
| **33** | *Final Examinations* | 1 Week |
| End-of-year Holidays | | |

CHAPTER

# 7

# Statistics

In the last 20 years there has been a lot of statistical research into the prevention of infectious diseases. The World Health Organization and UNICEF monitors the number of births and the infant mortality rates for 194 countries and territories. From 1990 to 2011, the infant mortality rate (deaths of babies under 1 year of age) for Solomon Islands improved from 34 to 18 per 1000 live births. Governments find it useful to compare their statistics with those of other countries. For example, the 2011 infant mortality rate for Papua New Guinea was 45 per 1000 live births, Samoa (16), USA (6), New Zealand (5) and Australia (4). Such statistics are used to justify the need for aid money to support health initiatives in developing countries.

## This chapter covers the following skills:

- Drawing histograms with relative or percentage frequency
- Using double column bar graphs
- Calculating relative frequency

  $$\text{Relative frequency} = \frac{\text{frequency}}{\text{total frequency}}$$

- Calculating percentage frequency

  $$\text{Percentage frequency} = \frac{\text{frequency}}{\text{total frequency}} \times 100$$

- Drawing cumulative frequency curves and percentiles
- Drawing and analysing boxplots
- Calculating mean and range

  $$\text{Mean } \bar{x} = \frac{\Sigma x}{n}$$

- Drawing back-to-back stemplots and parallel boxplots
- Taking a random sample for a survey

## Specific Learning Outcome (SLO)

Learners should be able to:

**9.7.1.1** Define the term 'numerical data'.

**9.7.1.2** Name and differentiate between discrete and continuous numerical data.

**9.7.2.1** Define the term 'histogram'.

**9.7.2.2** Identify the features and properties of histograms and construct graphs to display continuous numerical data.

**9.7.3.1** Calculate the *relative frequency* and *percentage frequency* for given data.

**9.7.4.2** Compare the frequencies of data in given classes using percentages.

**9.7.4.3** Identify the modal class interval for a set of scores from a histogram.

**9.7.5.1** Distinguish between *positively* or *negatively skewed* histograms

**9.7.6.1** Identify the features of a double column graph.

**9.7.7.1** Display data with a double column bar graph.

**9.7.7.2** Read and interpret information from a double column graphs.

**9.7.8.1** Define the term 'mean' and identify its symbol.

**9.7.9.1** Calculate the total sum of a set of scores in a frequency table.

**9.7.10.1** Use the formula to calculate the mean of a set of scores for single data.

**9.7.10.2** Identify the middle number for each class for scores given as grouped data.

**9.7.10.3** Use the formula to calculate the mean of a set of scores for group data.

**9.7.11.1** Define the term 'range' of a set of scores and calculate its value for single and grouped data.

**9.7.12.1** Construct a frequency table to represent given data.

**9.7.13.1** Define the term 'cumulative frequency'.

**9.7.14.1** Convert cumulative frequency scores to percentages to become cumulative percentage frequency values.

**9.7.15.1** Graph cumulative percentage frequency data in the form of an ogive curve.

**9.7.16.1** Interpret features from a given cumulative frequency graph.

**9.7.17.1** Identify the three main percentiles for given data, upper quartile, median and lower quartile.

**9.7.18.1** Use the given curve (ogive) to find the percentiles and quartiles of given scores.

**9.7.18.2** Calculate the median and mode for a set of scores using percentiles.

**9.7.19.1** Identify a stemplot graph and explain its properties.

**9.7.20.1** Display given numerical data as an ordered stem-and-leaf plot.

**9.7.21.1** Calculate the median and quartiles for a set of scores from stemplot graph.

**9.7.22.1** Define the term 'boxplot' and explain the four steps in its construction.

**9.7.23.1** Calculate the values for the median and quartiles (Q1, Q2, Q3) for given data.

**9.7.24.1** Draw boxplots using information from cumulative frequency curves using the five number summary.

**9.7.25.1** Construct parallel boxplots use them to make comparisons between two sets of data.

**9.7.26.1** Define and identify when a sample population is necessary to deal with large sets of data.

**9.7.27.1** Define the following terms 'random' and 'stratified sampling', and explain their use to choose unbiased data.

**9.7.28.1** Collate sample data using a random number generator on a calculator.

**9.7.28.2** Collect sample data using stratified sampling where populations are given in ratios.

# 7A Histograms

Numerical data are data that involve counting or measurement.

- **Discrete numerical** data involve distinct values and these are often, but not always, whole numbers. If collecting data involves counting, then the data will be discrete numerical data.
- **Continuous numerical** data are data that can have a value anywhere on a particular scale. If collecting data involves measuring, then the data are probably continuous numerical data.

When we analyse continuous numerical data or discrete numerical data where there is a large variation in the numbers, we need to group the data into classes. Ideally these classes should be the same size.

A histogram is a special kind of bar chart. Histograms are used to display continuous numerical data.

When drawing histograms, you should remember that:

- All the bars are the same width.
- There should be no gaps between the bars.
- The horizontal axis should be evenly numbered.
- The vertical axis is labelled as frequency, relative frequency or percentage frequency.

The highest bar in a histogram shows the modal class interval.

**Example**

The heights, in centimetres, of a group of 25 Year 9 learners are:

144, 153, 167, 178, 171,
149, 153, 162, 167, 166,
156, 154, 161, 156, 166,
162, 160, 164, 148, 157,
170, 165, 163, 174, 168

Collect this information in a frequency table and display it as histogram.

**Solution**

These are continuous data and can be grouped into classes such as 140–, 150–, 160–, etc.

| Height | Frequency |
|---|---|
| 140– | 3 |
| 150– | 6 |
| 160– | 12 |
| 170– | 4 |
| Total | 25 |

There are 12 people with heights between 160 and 170 cm and we call this 'the modal class' because it has the highest frequency.

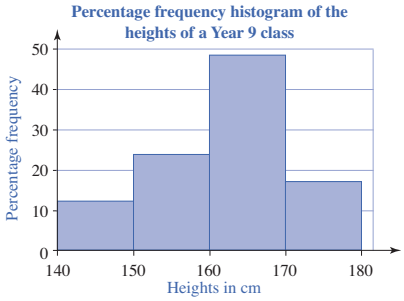
When comparing samples of different sizes, we use relative frequency or percentage frequency:

- Relative frequency $= \dfrac{\text{frequency}}{\text{total frequency}}$
- Percentage frequency $= \dfrac{\text{frequency}}{\text{total frequency}} \times 100$

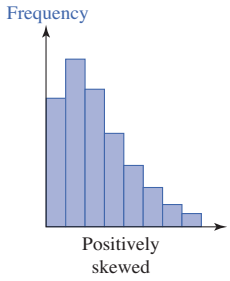
| Height | Frequency | Relative frequency | Percentage frequency |
|---|---|---|---|
| 140– | 3 | 0·12 | 12 |
| 150– | 6 | 0·24 | 24 |
| 160– | 12 | 0·48 | 48 |
| 170– | 4 | 0·16 | 16 |
| Total | 25 | 1·00 | 100 |

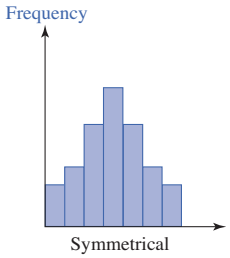
From the table we can see that 48% of the class was between 160 and 170 cm tall.

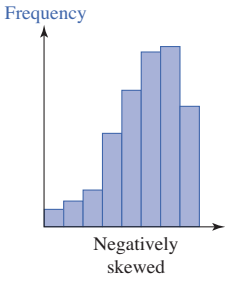
The percentage histogram for height looks like this:




When we are describing a histogram we consider the shape, the centre and the spread of the data. Histograms that tail off are described as **skewed**.




## Exercise 7A

**1** The times, in minutes, 50 learners take to travel to school are:
85, 45, 20, 7, 36, 28, 45, 69, 72, 56, 32, 15, 38, 62, 44, 36, 29, 46, 21, 45, 52, 34, 27, 24, 16, 18, 24, 23, 17, 12, 5, 43, 28, 36, 15, 12, 26, 28, 17, 13, 14, 9, 24, 28, 27, 29, 56, 38, 29, 34

**a** Group the data in classes as 0–, 20–, 40– … **b** Find the relative frequency for each group.

**c** Display this as a histogram. **d** Comment on what the graph shows.

**e** What is the probability that a learner takes less than 20 minutes to get to school?

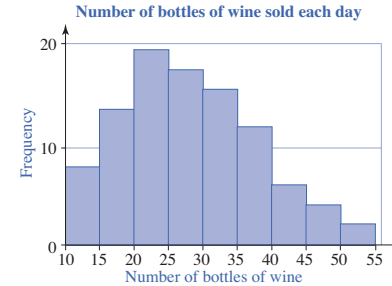
**2** For this question use the data from Question 1 (time taken to travel to school):

**a** Group the data in the groups 0–, 10–, 20– … **b** Find the relative frequency for each group.

**c** Display this as a histogram. **d** Comment on what the graph shows.

**e** Do you consider this graph to be more informative than the graph in Question 1. Why?

**3** The graph shows the number of bottles of wine sold in a restaurant each day:




- **a** On how many days did they sell more than 45 bottles of wine?
- **b** Describe the shape of the graph.
- **c** What is the modal class for the data?
- **d** Estimate the average number of bottles sold each day.

**4** The time, in minutes, that 120 learners spent connected to the Internet over the course of a month is displayed in the table:

| Time on the Internet (min) | Frequency |
|---|---|
| 0– | 5 |
| 100– | 17 |
| 200– | 63 |
| 300– | 27 |
| 400– | 8 |

- **a** Display this information as a frequency histogram.
- **b** Describe the shape of the histogram.
- **c** Find the relative frequency for each group.
- **d** Display this as a histogram.
- **e** Comment on what the graph shows.

**5** The height, in metres, of the pine trees in a reserve were recorded as:

10·6, 7·5, 8·3, 8·5, 10·1, 7·3, 10·6, 10·2, 8·3, 9·4, 9·6, 7·6, 9·4, 8·1, 6·9, 8·4, 9·3, 9·2, 8·5, 9·5, 8·4, 7·5, 6·1, 7·9, 8·6, 9·4, 8·3, 8·9, 8·8, 7·6

- **a** Group the data in the classes 6·0– , 7·0– … and display it as a frequency table.
- **b** Find the percentage frequency for each group.
- **c** Display the data as a percentage frequency histogram.
- **d** Describe the shape of the histogram.
- **e** What percentage of trees were more than 10 m tall?
- **f** What percentage of trees were between 8 m and 10 m tall?

**6** In a government review of gambling, a survey was done to find how much people lost in a single night playing the 'pokies'. The results of the 50 people surveyed were:

$72, $23, $78, $12, $26, $36, $27, $89, $95, $82, $35, $21,$27, $43, $53, $18, $15, $21, $27, $38, $62, $23, $24, $32, $37, $32, $26, $39, $45, $23, $47, $35, $28, $22, $56, $40, $47, $35, $40, $32, $56, $37, $36, $52, $23, $36, $27, $28, $42, $35

- **a** Choose an appropriate method of grouping the data and display this information in a percentage frequency table.
- **b** Display this information as a percentage histogram.

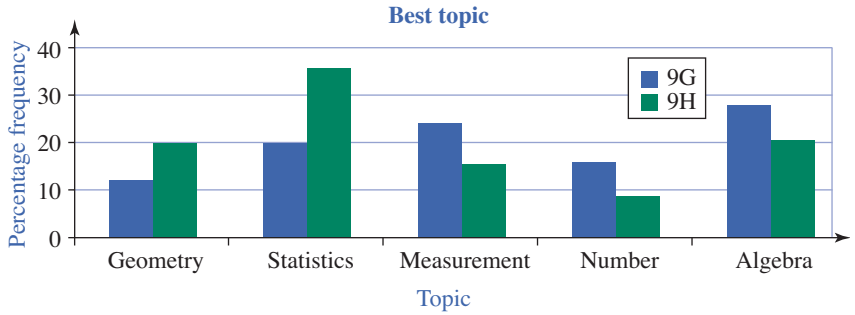
# Double column bar graphs 7B

The Maths teachers of 9G and 9H were keen to see which topics they taught well. They decided to do an analysis to see in which topic their learners scored highest. There were five topics, each worth 20 marks in the exam. The topics were Geometry, Statistics, Measurement, Number and Algebra.

The table below shows the number of learners who got their best result in each of the topics:

| Topic | Percentage of learners in 9G for whom this was their best result | Percentage of learners in 9H for whom this was their best result |
|---|---|---|
| Geometry | 12 | 20 |
| Statistics | 20 | 36 |
| Measurement | 24 | 16 |
| Number | 16 | 8 |
| Algebra | 28 | 20 |

We could display this information as two separate bar graphs or as a **double column bar graph**.




From the graph we can see that:

- Algebra was the best topic for 9G as 28% of the class got their best result in Algebra.
- Statistics was the best topic for 9H as 36% of the class got their best result in it.
- Geometry was the worst topic for 9G, while Number was the worst topic for 9H.

## Exercise 7B

**1** Year 9 learners were asked their preferred way of keeping in contact with friends while away on holiday.

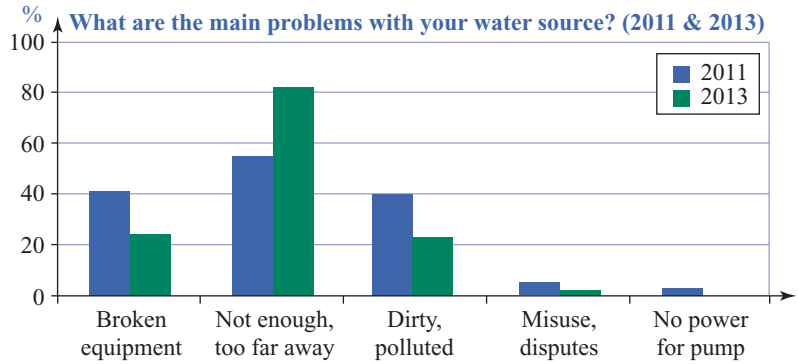
The results are given in the table:

| | Male | Female |
|---|---|---|
| Phone calls | 9 | 20 |
| SMS messages | 13 | 8 |
| Emails | 25 | 12 |
| Writing letters | 3 | 10 |
| Total | 50 | 50 |

**a** Display the data as a double column bar graph.

**b** What does the graph tell us?

**2** The graph compares the answers to a RAMSI People's survey question about problems with peoples' water sources in 2011 and 2013.




**a** What percentage of people reported broken equipment as the main problem in 2013?

**b** What was the percentage difference for dirty–polluted water from 2011 to 2013?

**c** Which water problem got worse over the period, and by how much?

**3** The table shows the responses to a question in the 2013 RAMSI People's Survey about where people sell their produce.

**Where do you sell? (by residence, 2013)**

| | Rural % | Honiara % | All % |
|---|---|---|---|
| Market | 75·8 | 44·0 | 68·7 |
| Roadside | 27·7 | 54·4 | 33·7 |
| House | 30·5 | 33·4 | 31·2 |
| Sell to other business | 16·6 | 2·7 | 13·5 |
| Canteen/small store | 1·1 | 19·3 | 11·5 |
| Outside other business | 4·7 | 3·2 | 4·3 |
| Wharf | 4·0 | 0·5 | 3·2 |
| Big shop | 1·1 | 0·6 | 1·0 |
| No details | 0·4 | 0·3 | 0·4 |
| Number of respondents | 2161 | 621 | 2782 |

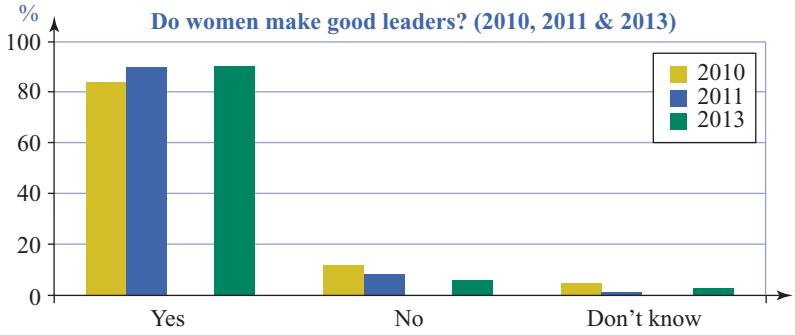
Multiple response table. Percentages are totals of up to three responses per respondent.
No answer—1 case, Not asked—622 cases

**a** Display the data as a double column bar graph comparing responses from people who live in rural areas with responses with people who live in Honiara.

**b** Identify the main differences of produce selling sites for people who live in rural areas compared with those who live in Honiara according to your graph.

**4** The graph compares the answers to a RAMSI People's survey question about whether women make good leaders.

- **a** In which years was this question surveyed?
- **b** Why is there a gap between the blue and green bars on the graph?
- **c** What percentage of people surveyed in 2010 said that women do not make good leaders?
- **d** Is there any evidence that the number of people who think women do make good leaders has increased from 2010 to 2013? Provide some data from the graph to support your opinion.




# 7C The mean and the range

A commonly used measure of the centre of data is called the mean. The **mean** is found by adding up all the values and then dividing the total by the number of values.

We use the notation $\bar{x}$ to represent the mean.

A commonly used measure of the spread of data is the range. The **range** of the data is the difference between the highest and lowest scores.

**Example**

**1** The heights, in centimetres, of 25 Year 9 learners were:

144, 153, 167, 178, 171,
149, 153, 162, 167, 166,
156, 154, 161, 156, 166,
162, 160, 164, 148, 157,
170, 165, 163, 174, 168

Find the mean height and range for the group.

**Solution**

$$\bar{x} = \frac{\Sigma x}{n}$$

where $\Sigma x$ is the total and $n$ is the number of values.

$$\bar{x} = \frac{4034}{25}$$
$$= 161{\cdot}36$$

The mean height is approximately 161 cm.

Highest score = 178 cm
Lowest score = 144 cm
Range = 178 – 144 = 34 cm

When we have information in a frequency table, we can use multiplication instead of adding up all the values individually.

**Example**

**2** The number of children per family in a group of families is displayed in a frequency table.

| No. of children | Frequency |
|---|---|
| 1 | 4 |
| 2 | 10 |
| 3 | 6 |
| 4 | 3 |
| 5 | 2 |
| Total | 25 |

From the table we know there are 4 families with only 1 child, 10 families with 2 children, 6 families with 3 children and so on.

Find the mean number of children per family in the group.

**Solution**

To find the total number of children we multiply the number of children by the number of families.

| No. of children $x$ | Frequency $f$ | $xf$ |
|---|---|---|
| 1 | 4 | 4 |
| 2 | 10 | 20 |
| 3 | 6 | 18 |
| 4 | 3 | 12 |
| 5 | 2 | 10 |
| | $\Sigma f = 25$ | $\Sigma xf = 64$ |

$$\bar{x} = \frac{\Sigma xf}{\Sigma f} = \frac{\Sigma xf}{n}$$
$$= \frac{64}{25} = 2{\cdot}56$$

The mean number of children per family in the group is 2.56.

If we are only given grouped data in a frequency table, we can estimate the mean by using the **middle class value** or middle value of the group.

## Example

**3** The heights, in centimetres, of 25 Year 9 learners are shown in the grouped frequency table below. Find an estimate for the mean using the data in the frequency table:

| Height | Frequency |
|---|---|
| 140– | 3 |
| 150– | 6 |
| 160– | 12 |
| 170– | 4 |
| Total | 25 |

## Solution

The middle value for the first class is halfway between 140 and 149·9… or 150.

| Middle value of the heights $m$ | Frequency $f$ | $mf$ |
|---|---|---|
| 145 | 3 | 435 |
| 155 | 6 | 930 |
| 165 | 12 | 1980 |
| 175 | 4 | 700 |
| Total | 25 | 4045 |

$$\text{Middle value} = \frac{140 + 150}{2}$$
$$= 145$$

We don't know the exact heights of the three people whose heights were between 140 and 150 so we assign each of them the middle value of 145 cm.

$$\bar{x} = \frac{\Sigma mf}{\Sigma f}$$
$$= \frac{\Sigma mf}{n}$$
$$= \frac{4045}{25}$$
$$= 161{\cdot}8$$

In Example 1, the actual mean height of 161·36 was calculated, so we can compare the answers.

There is only a 0·44 difference between this approximate value and the exact answer.

7C

## Exercise 7C

**1** The times taken, in minutes, by 50 learners to travel to school are:

85, 45, 20, 7, 36, 28, 45, 69, 72, 56, 45, 32, 15, 38, 62, 44, 36,
29, 46, 21, 52, 34, 27, 24, 16, 18, 24, 23, 17, 12, 5, 43, 28, 36,
15, 12, 26, 28, 17, 13, 14, 9, 24, 28, 27, 29, 56, 38, 29, 34

**a** What was the mean time taken by the learners to travel to school?

**b** What was the range of times taken by learners to travel to school?

**2** Some of the learners in Question 1 travel by bus, others by car.

| | |
|---|---|
| **Bus** | 85, 45, 20, 28, 69, 72, 56, 45, 32, 38, 62, 44, 36, 29, 46, 21, 52, 34, 27, 18, 24, 23, 38, 29 |
| **Car** | 7, 36, 45, 15, 24, 16, 17, 12, 5, 43, 28, 36, 15, 12, 26, 28, 17, 13, 14, 9, 24, 28, 27, 29, 56, 34 |

**a** Find the mean time and the range of times taken for the learners to travel to school by bus.

**b** Find the mean time and the range of times taken for the learners to travel to school by car.

**c** Comment on the results.

**3** In the survey carried out to find how much people lost in a single night playing the 'pokies', the results of the 50 people surveyed were:

$72, $23, $78, $12, $26, $36, $27, $89, $95, $82, $35, $21, $27, $43, $53, $18, $15, $21, $27, $38, $62, $23, $24, $32, $37, $32, $26, $39, $45, $23, $47, $35, $28, $22, $56, $40, $47, $35, $40, $32, $56, $37, $36, $52, $23, $36, $27, $28, $42, $35.

**a** What was the mean amount lost in a night?

**b** What was the range in the amounts lost in a night?

**4** If we divide the results in Question 3 into male and female responses, we get:

| | |
|---|---|
| **Males** | $32, $26, $39, $45, $23, $47, $35, $28, $22, $56, $40, $47, $35, $40, $32, $56, $37, $36, $52, $23, $36, $27, $28, $42, $35 |
| **Females** | $72, $23, $78, $12, $26, $36, $27, $89, $95, $82, $35, $21, $27, $43, $53, $18, $15, $21, $27, $38, $62, $23, $24, $32, $37 |

**a** What was the mean amount lost by males in a night?

**b** What was the mean amount lost by females in a night?

**c** Find the range of the data for each group.

**5** The height, in metres, of pine trees in a reserve is given by the frequency table:

| Height (m) | Frequency |
|---|---|
| 6– | 2 |
| 7– | 6 |
| 8– | 11 |
| 9– | 7 |
| 10– | 4 |

**a** Calculate an estimate for the mean height from the data in the frequency table.

**b** Calculate the mean height from the raw data:
10·6, 7·5, 8·3, 8·5, 10·1, 7·3, 10·6, 10·2, 8·3, 9·4,
9·6, 7·6, 9·4, 8·1, 6·9, 8·4, 9·3, 9·2, 8·5, 9·5,
8·4, 7·5, 6·1, 7·9, 8·6, 9·4, 8·3, 8·9, 8·8, 7·6

**c** What was the difference between the estimate of the mean and the actual mean?

**6** The time, in minutes, that 120 learners spent connected to the Internet over the course of a month is given in the table below.

| Time on the Internet (min) | Frequency |
|---|---|
| 0– | 5 |
| 100– | 17 |
| 200– | 63 |
| 300– | 27 |
| 400– | 8 |

Find the approximate average time spent on the Internet by these learners.

**7** This frequency table shows the number of bottles of wine sold in a restaurant each day.

| No. bottles of wine sold | Frequency |
|---|---|
| 10– | 4 |
| 15– | 7 |
| 20– | 10 |
| 25– | 9 |
| 30– | 8 |
| 35– | 6 |
| 40– | 3 |
| 45– | 2 |
| 50– | 1 |

**a** Display the data as a histogram. Comment on its shape.

**b** From the histogram, estimate the average number of bottles sold each day.

**c** From the frequency table, calculate an approximation for the mean number of bottles sold each day.

**8** In a game of chance, two dice are tossed. If the total of the two numbers shown on the dice is 9 or more, the player wins that number in dollars. If the total of the two numbers is 8 or less, the player loses that number in dollars. One player tosses the dice 36 times and obtains these scores:

10, 8, 9, 5, 6, 11, 7, 8, 5, 7, 2, 6, 4, 4, 3, 9, 3, 12,
5, 5, 8, 9, 6, 7, 4, 7, 10, 6, 11, 8, 7, 10, 6, 9, 7, 8

**a** Construct a frequency table to represent the data.

**b** Add in a column to represent the amount won or lost.

**c** Calculate the mean amount won or lost in this game.

**d** Obtain two dice and toss them 36 times. Did you win or lose on the game?

**e** Use a grid to calculate the theoretical probability of obtaining each score.

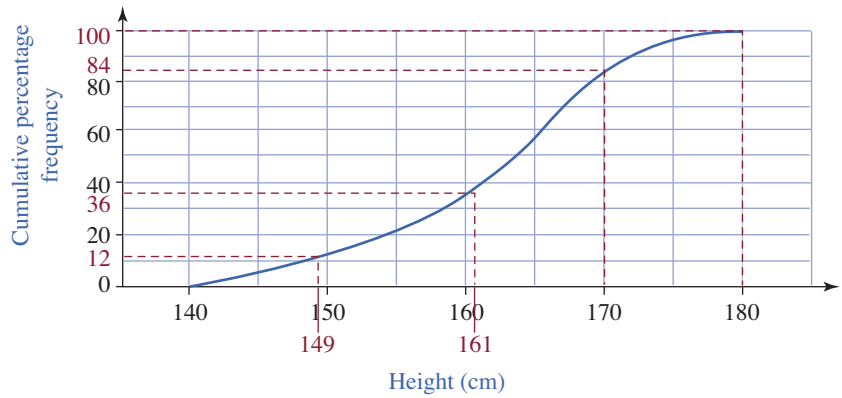
# 7D Cumulative frequency

**Cumulative frequency** is the sum of the frequency up to and including a certain score. The cumulative percentage frequency is the percentage of the sample that is above or below a certain value. We display the cumulative frequency and the percentage cumulative frequency as a curve or ogive.

When we display this information as an ogive, we plot the cumulative frequency against the **upper class value** or the **upper extreme of the group**. So for the height example, we would start at 0 against 140, then 12% would be plotted against 150, 36% against 160 and so on.

| Height | Frequency | Cumulative frequency | Cumulative percentage frequency |
|---|---|---|---|
| 130– | 0 | 0 | $\frac{0}{25} \times 100 = 0\%$ |
| 140– | 3 | 3 | $\frac{3}{25} \times 100 = 12\%$ |
| 150– | 6 | 9 | $\frac{9}{25} \times 100 = 36\%$ |
| 160– | 12 | 21 | $\frac{21}{25} \times 100 = 84\%$ |
| 170– | 4 | 25 | $\frac{25}{25} \times 100 = 100\%$ |

| Upper class value | Cumulative percentage frequency |
|---|---|
| 140 | 0 |
| 150 | 12 |
| 160 | 36 |
| 170 | 84 |
| 180 | 100 |




Cumulative percentage frequency curves can be used to estimate percentiles. From the graph we can read off the 80% of our class with heights of less than 168 cm. We call this the 80th percentile.

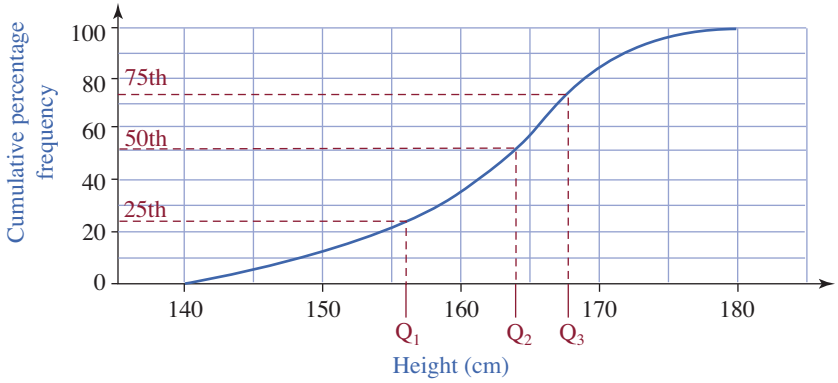
**Example** **Solution**

From the graph estimate the percentage of learners with heights less than:

| | | |
|---|---|---|
| a | 149 cm | Approximately 11% |
| b | 161 cm | Approximately 38% |
| c | 170 cm | Approximately 84% |
| d | 180 cm | 100% |

Three of the percentiles have special names:

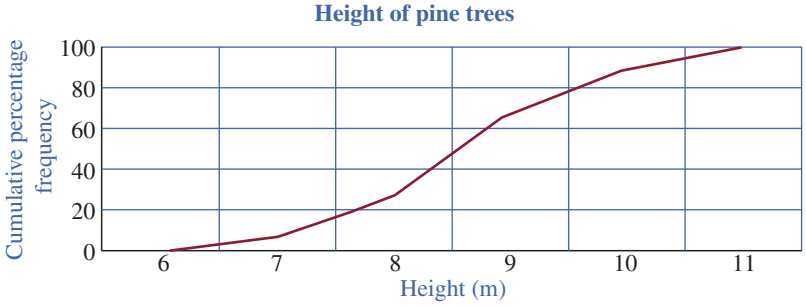
| Percentile | Name | |
|---|---|---|
| 25th | Lower quartile or $Q_1$ | 25% or $\frac{1}{4}$ of the sample lies below this value |
| 50th | Median or $Q_2$ | 50% or $\frac{1}{2}$ of the sample lies below this value |
| 75th | Upper quartile or $Q_3$ | 75% or $\frac{3}{4}$ of the sample lies below this value |




From our graph we can estimate that the median is 164 cm, the lower quartile is 156 cm and the upper quartile is 168 cm.

## Exercise 7D

**1** The cumulative percentage frequency for the heights, in metres, of pine trees is displayed as a cumulative frequency curve below:




**a** Complete the following:

**i** Approximately 60% of the trees were less than _____ metres tall.

**ii** Approximately _____ % of the trees were less than 10 metres tall.

**iii** Approximately 80% of the trees were taller than _____ metres tall.

**b** Find the 25th, 50th and 75th percentiles for this data.

**2** The time to pack cans of Taiyo on an assembly line are given in this frequency table:

| Time (min) | Cumulative % frequency |
|---|---|
| <2 | 2 |
| <3 | 6 |
| <4 | 11 |
| <5 | 8 |
| <6 | 2 |
| <7 | 1 |


**a** Find the cumulative percentage frequency.

**b** Display this information as a cumulative percentage curve.

**c** Complete the following:

**i** Approximately 80% of the cans were packed in less than _____ minutes.

**ii** Approximately _____ of the cans were packed in less than 3·5 minutes.

**iii** Approximately 40% of the cans took longer than _____ minutes to pack.

**3** The time, in minutes, that 120 learners spent connected to the Internet over the course of a month is given in the table.

| Time on the Internet (min) | Cumulative % frequency |
|---|---|
| 0– | 5 |
| 100– | 17 |
| 200– | 63 |
| 300– | 27 |
| 400– | 8 |

**a** Find the cumulative percentage frequency for this data.

**b** Display this data as a cumulative percentage frequency curve.

**c** Learners are charged an excess computer-use fee if they are on the computer for more than 4 hours. What percentage of learners will be charged this fee?

**d** How many learners will be charged the fee?

**e** Use the graph to find:

**i** the lowest value ($x_L$) **ii** the lower quartile ($Q_1$) **iii** the median ($Q_2$)

**iv** the upper quartile ($Q_3$) **v** the highest value ($x_H$)

**4** This entry gives the number of deaths of infants under one year old in a given year per 1000 live births in the same year. This rate is often used as an indicator of the level of health in a country. Find the cumulative percentage frequency and display this information as a cumulative percentage curve. Comment on what the graph shows.

| Country | 2000 | 2001 | 2002 | 2003 | 2004 | 2005 | 2006 | 2007 | 2008 | 2009 | 2010 | 2011 | 2012 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Solomon Islands | 25·3 | 24·5 | 23·7 | 22·9 | 22·1 | 21·3 | 20·6 | 20·0 | 19·77 | 19·0 | 18·4 | 17·8 | 17·35 |

**5** The time taken, in minutes, by 50 learners to travel to school is summarised in this frequency table:

| Time (min) | Cumulative % frequency |
|---|---|
| 0– | 3 |
| 10– | 10 |
| 20– | 16 |
| 30– | 8 |
| 40– | 6 |
| 50– | 3 |
| 60– | 2 |
| 70– | 1 |
| 80– | 1 |

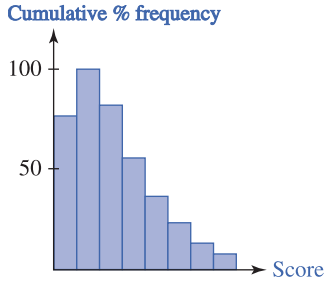
**a** Find the cumulative percentage frequency for this data.

**b** Display the data as a cumulative percentage frequency curve.

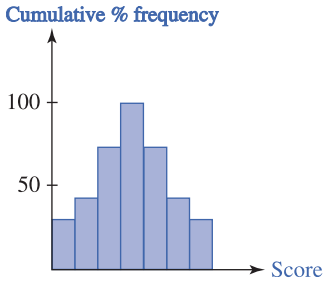
**c** Find the median and the lower and upper quartiles

**6** Draw cumulative percentage frequency curves to represent the following histograms:

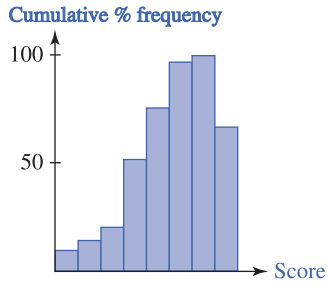
**a**




**b**




**c**



# 7E Stemplots

Stem-and-leaf plots or stemplots are a clever way of displaying numerical data. The stemplot must always include a legend to show what each stem and leaf represents. The leaves must be evenly spread, and when this is done carefully the stemplot looks a little like a histogram on its side.

For two-digit numbers we divide the numbers so that the tens are the stem and the units are the leaves. So 49 is split as 4|9. For three-digit numbers we could divide the numbers so that the tens are the stem and the units are leaves, or we could divide the numbers so that the hundreds form the stem and the tens and units form two-digit leaves. So 149 could be split as 14|9 or 1|49.

In an **ordered** stem-and-leaf plot we write the leaves in numerical order and this can be very useful for finding the median and the quartiles.

The **median value** is the middle value when the data are listed in order.

If there are $n$ pieces of data then the median would be the $\left(\frac{n+1}{2}\right)$th value.

For an odd number of pieces of data this would be an actual value. For an even number of pieces of data this would be midway between the middle two values.

The lower quartile, $Q_1$, is the median of the lower half of the sample and the upper quartile, $Q_3$, is the median of the upper half of the sample. If the median is an actual value it is not included when finding $Q_1$ or $Q_3$.

**Example** **Solution**

**1** The heights, in centimetres, of a group of 25 Year 9 learners were:

144, 153, 167, 178, 171, 149, 153, 162, 167, 166, 156, 154, 161, 156, 166, 162, 160, 164, 148, 157, 170, 165, 163, 174, 168

**a** Display the data as a stemplot.

If we display these heights as an *ordered* stem-and-leaf plot, we get:

| | |
|---|---|
| 14 | 4 8 9 |
| 15 | 3 3 4 6 6 7 |
| 16 | 0 1 2 2 3 4 5 6 6 7 7 8 |
| 17 | 0 1 4 8 |

**b** Find the median value.

There are 25 values listed, so the median is the $\left(\frac{25+1}{2}\right)$th or 13th value.

| | |
|---|---|
| 14 | 4 8 9 |
| 15 | 3 3 4 6 6 7 |
| 16 | 0 1 2 (2) 3 4 5 6 6 7 7 8 |
| 17 | 0 1 4 8 |

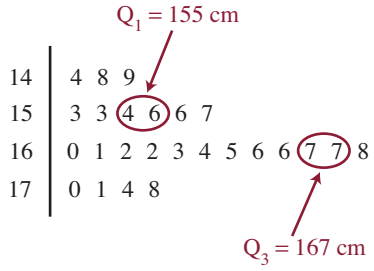
median is 162 cm

**Example**

**c** Find the upper and lower quartiles.

**Solution**

There are 12 values above and below the median, and the median of each of these is between the 6th and 7th values.




## Back-to-back stemplots

One way of comparing two sets of data is to display them in an ordered back-to-back stemplot. The first set of data is recorded going out from the centre of the stemplot to the right and the other set is recorded going out from the centre of the stemplot to the left.

**Example**

**2** The test results for 9G and 9H are shown below. Display the results as an ordered back-to-back stemplot.

- Results for 9G were:
  59, 60, 66, 67, 76, 74, 61, 60, 68, 71, 76, 72, 58, 57, 66, 65, 72, 67, 62, 55, 56, 71, 64, 69, 70
- Results for 9H were:
  56, 42, 76, 71, 47, 53, 52, 49, 60, 61, 54, 52, 57, 63, 65, 46, 57, 55, 48, 43, 52, 64, 78, 50, 64

| 9G | Stem | 9H |
|---|---|---|
| | 4 | 2 3 6 7 8 9 |
| 9 8 5 | 5 | 0 2 2 2 3 4 5 6 7 7 |
| 9 8 7 7 6 6 5 4 2 1 0 0 | 6 | 0 1 3 4 4 5 |
| 6 6 4 2 2 1 1 0 | 7 | 1 6 8 |
| | 8 | |
| | 9 | |

From the stemplots we can see that:

- the classes are the same size
- the results for 9H are more positively skewed
- the highest and lowest results were in class 9H and it has a greater range
- the results for 9G were generally higher than the results for 9H
- there was less variation in the results for 9G.

# 7E

## Exercise 7E


**1** The height, in metres, of the pine trees in a reserve were recorded as:

10.6, 7.5, 8.3, 8.5, 10.1, 7.3, 10.6, 10.2, 8.3, 9.4, 9.6, 7.6, 9.4, 8.1, 6.9, 8.4, 9.3, 9.2, 8.5,9.5, 8.4, 7.5, 6.1, 7.9, 8.6, 9.4, 8.3, 8.9, 8.8, 7.6

**a** Display the data as a stemplot.

**b** Find the median and the quartiles.

**c** Find the range of the data.

**2** The ordered back-to-back stemplot below shows the data collected for two classes *A* and *B*:

| *A* | | | | | | | Stem | *B* | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 9 | 9 | 8 | 7 | 6 | 3 | 2 | 4 | 1 | 2 | | | | | | |
| | | 7 | 5 | 3 | 2 | 0 | 5 | 5 | 6 | 7 | 8 | | | | |
| | | | | 3 | 1 | 0 | 6 | 0 | 0 | 2 | 2 | 8 | | | |
| | | | | | 2 | 1 | 7 | 0 | 1 | 1 | 2 | 2 | 4 | 6 | 6 |
| | | | | | 5 | 0 | 8 | 3 | 6 | 6 | 7 | 7 | | | |
| | | | | | | 7 | 9 | 0 | 4 | | | | | | |

**a** Comment on the differences, and describe the shape of the two data sets by using terms such as positively or negatively skewed, and symmetrical.

**b** Find the median result for each class and compare them.

**c** Find the range of the results for each class and compare them.

**3** The back-to-back stemplot below shows the data collected from two classes *C* and *D*:

| *C* | | | | | | | Stem | *D* | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | 2 | 2 | 1 | 4 | 1 | 2 | 6 | 7 | 8 | | |
| | | 4 | 4 | 2 | 2 | 0 | 5 | 0 | 0 | 2 | 2 | | | |
| 9 | 9 | 8 | 8 | 3 | 1 | 0 | 6 | 6 | | | | | | |
| | | | 5 | 4 | 2 | 1 | 7 | 5 | | | | | | |
| | | | | | | 1 | 8 | 3 | 6 | 6 | 7 | | | |
| | | | | | | 5 | 9 | 0 | 2 | 2 | 3 | 8 | 8 | 9 |

**a** Comment on the differences and describe the shape of the two data sets.

**b** Find the median score for each class and compare them.

**c** Find the range of the results for each class and compare.

**4** The times taken, in minutes, for 50 learners to travel to school are:

85, 45, 20, 7, 36, 28, 45, 69, 72, 56, 45, 32,
15, 38, 62, 44, 36, 29, 46, 21, 52, 34, 27, 24, 16,
18, 24, 23, 17, 12, 5, 43, 28, 36, 15, 12, 26, 28, 17,
13, 14, 9, 24, 28, 27, 29, 56, 38, 29, 34

**a** Display the data as a stemplot and hence find the median and quartiles.

Some of the learners travelled by bus and the others travelled by car.

| | |
|---|---|
| **Bus** | 85, 45, 20, 28, 69, 72, 56, 45, 32, 38, 62, 44, 36, 29, 46, 21, 52, 34, 27, 18, 24, 23, 38, 29 |
| **Car** | 7, 36, 45, 15, 24, 16, 17, 12, 5, 43, 28, 36, 15, 12, 26, 28, 17, 13, 14, 9, 24, 28, 27, 29, 56, 34 |

**b** Find the median travel time for learners travelling to school by:

**i** bus **ii** car

**c** Comment on what the medians show about the difference between the times taken for learners travelling by bus and those travelling by car.

**5** The number of bottles of wine sold in a restaurant each day was divided into white wine and red wine sales.

- Number of bottles of white wine sold:
  9, 16, 15, 20, 32, 20, 23, 8, 9, 35, 19, 20, 24, 19, 15, 18, 12, 18,
  15, 12, 14, 15, 14, 20, 24, 16, 12, 28, 26, 16, 12, 14, 25, 28, 28,
  20, 18, 18, 15, 8, 13, 18, 14, 21, 27, 23, 28, 11, 14, 22
- Number of bottles of red wine sold:
  9, 9, 8, 12, 14, 8, 10, 6, 6, 18, 8, 11, 11, 10, 8, 6, 3, 10, 8, 2, 9, 10, 8, 12, 14, 8,
  6, 14, 18, 8, 4, 5, 13, 17, 18, 12, 16, 10, 9, 4, 3, 5, 8, 10, 11, 18, 18, 2,12, 10

**a** Display this information as an ordered back-to-back stemplot.

**b** Comment on what the graph shows.

**6** The results below show the amounts lost at the 'pokies' one night.

| | |
|---|---|
| **Males** | $32, $26, $39, $45, $23, $47, $35, $28, $22, $56, $40, $47, $35, $40, $32, $56, $37, $36, $52, $23, $36, $27, $28, $42, $35 |
| **Females** | $72, $23, $78, $12, $26, $36, $27, $89, $95, $82, $35, $21, $27, $43, $53, $18, $15, $21, $27, $38, $62, $23, $24, $32, $37 |

**a** Display this information as an ordered back-to-back stemplot.

**b** Comment on what the graph shows.

**7** Draw a stemplot with 10 scores that could represent each of the following situations:

**a** Lowest score = 2·5, $Q_1$ = 3·0, median = 4·2, $Q_3$ = 6·5, highest score = 9·8

**b** Lowest score = 42, $Q_1$ = 45, median = 50, $Q_3$ = 67, highest score = 76

**c** Lowest score = 102, $Q_1$ = 118, median = 123, $Q_3$ = 130, highest score = 154

# 7F Boxplots and measures of spread

Box-and-whisker plots or boxplots are an excellent way of displaying the medians and quartiles. The five-number summary can be used to measure the spread of data.

Step 1: Draw a horizontal axis similar to the horizontal axis of a histogram.

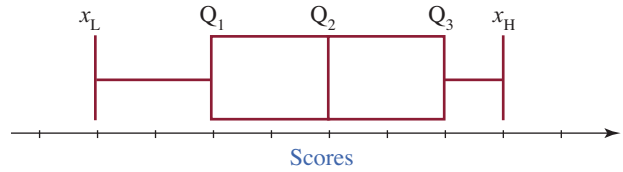
Step 2: Find the following five values:

- lowest value ($x_L$)
- lower quartile ($Q_1$)
- median ($Q_2$)
- upper quartile ($Q_3$)
- highest value ($x_H$)

These figures are known as the **five-number summary**.

Step 3: Mark these above the axis.

Step 4: The middle three marks—$Q_1$, $Q_2$ and $Q_3$—are used to form a box, and the 'whiskers' go out to the outer marks—$x_L$ and $x_H$.




However, to get really accurate results for these points we need to use the original data.

**Example**

Find the exact values for the median and the quartiles for the heights of the group of 25 Year 9 learners, whose heights, in centimetres, are:

144, 153, 167, 178, 171, 149, 153, 162, 167, 166, 156, 154, 161,
156, 166, 162, 160, 164, 148, 157, 170, 165, 163, 174, 168

Hence, draw a boxplot for the heights of the group of Year 9 learners.

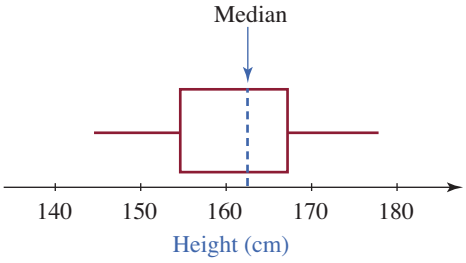
**Solution**

The five-figure summary for the heights of the learners (previously calculated in Exercise 7E) is:

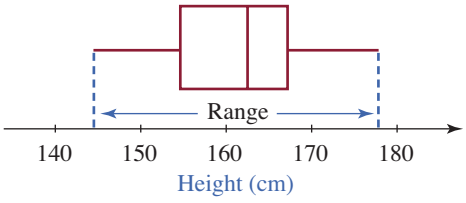
$x_L = 144$
$Q_1 = 155$
Median = 162
$Q_3 = 167$
$x_H = 178$

**Solution**

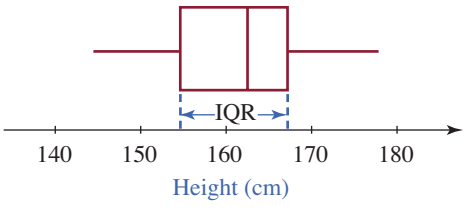
The boxplot for the heights is shown below:



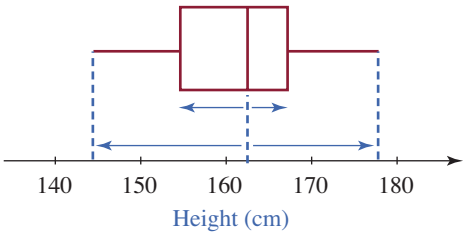

The data is spread around a median of 162 cm.




The range is 34 cm.

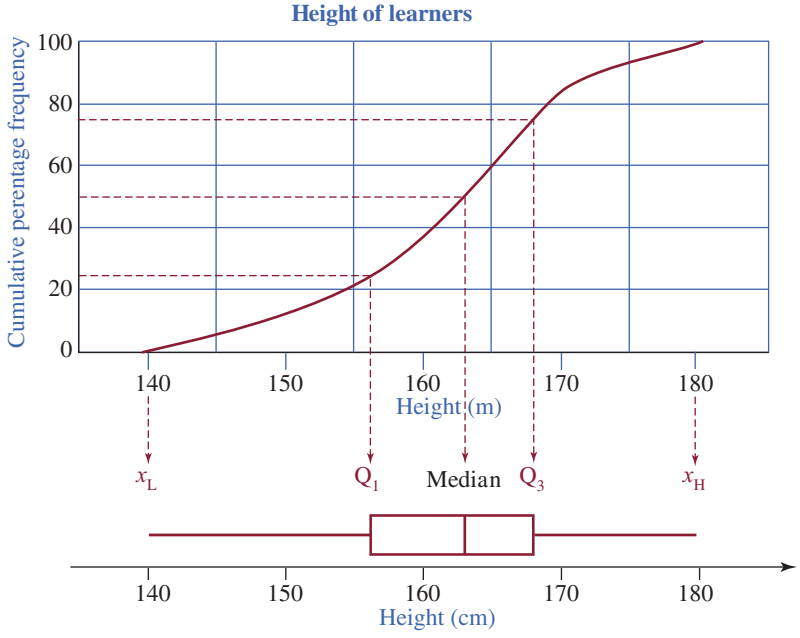



The interquartile range (IQR = $Q_3 - Q_1$) is 12.




The data spread is almost symmetrical.

Boxplots can also be drawn using the cumulative percentage frequency curve to find the five-number summary.

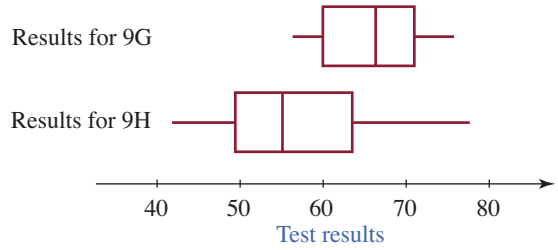


# 7F

Parallel boxplots can also be used to compare different data sets. When comparing two or more samples, we draw all the boxplots above the same horizontal axis.

## Example

Here are the boxplots of the exam results for 9G and 9H.




From the boxplots we can obtain the following information:

- There was more variation for the results for 9H, as the range is much greater than for the results for 9G.
- The median result was much higher for class 9G, and their results are generally higher.
- The results for 9H were slightly positively skewed.
- The interquartile ranges for the classes were about the same.

## Exercise 7F

**1** Display the following information as a boxplot:

**a** $x_L = 12$
$Q_1 = 23$
Median $= 26{\cdot}5$
$Q_3 = 34$
$x_H = 53$

**b** $x_L = 5$
$Q_1 = 18$
Median $= 28$
$Q_3 = 43$
$x_H = 85$

**c** $x_L = 12$
$Q_1 = 26$
Median $= 35$
$Q_3 = 45$
$x_H = 95$

**2** The heights, in metres, of pine trees are shown below:

10·6, 7·5, 8·3, 8·5, 10·1, 7·3, 10·6, 10·2, 8·3, 9·4, 9·6, 7·6, 9·4, 8·1, 6·9, 8·4, 9·3, 9·2, 8·5, 9·5, 8·4, 7·5, 6·1, 7·9, 8·6, 9·4, 8·3, 8·9, 8·8, 7·6

**a** Find the five-number summary for the heights.

**b** Display this information as a boxplot.

**c** Complete the following:

**i** 25% of the trees were less than _____ m tall.

**ii** 50% of the trees were less than _____ m tall.

**iii** 75% of the trees were less than _____ m tall.

**iv** 25% of the trees were more than _____ m tall.

**v** 75% of the trees were more than _____ m tall.

**d** Complete the following:
The height of the middle 50% of the sample is between _____ and _____, so the interquartile range for the heights is _____ .

**3** The number of songs stored on the MP3 players of 150 learners is listed in the table:

| Number of songs | Cumulative % frequency |
|---|---|
| 0– | 12 |
| 200– | 24 |
| 400– | 35 |
| 600– | 51 |
| 800– | 18 |
| 1000– | 10 |

**a** Display this as a histogram and describe the shape.

**b** Calculate the percentage cumulative frequency for this data.

**c** Draw a cumulative frequency curve.

**d** Assume the upper value is 1200. From the curve find the five-number summary, and draw the boxplot for the data.

**e** Complete the following:

**i** 25% of the learners have fewer than _____ songs on their MP3 players.

**ii** 50% of the learners have fewer than _____ songs on their MP3 players.

**iii** 75% of the learners have fewer than _____ songs on their MP3 players.

**iv** 25% of the learners have more than _____ songs on their MP3 players.

**v** 75% of the learners have more than _____ songs on their MP3 players.

**f** Complete the following:
The learners in the middle 50% of the sample have between _____ and _____ songs on their MP3 players, so the interquartile range is _____ .

**4** The five-number summaries for the sales of white and red wine are given in the table.

| | White | Red |
|---|---|---|
| $x_L$ | 8 | 2 |
| $Q_1$ | 14 | 5 |
| Median | 18 | 9·5 |
| $Q_3$ | 23 | 12 |
| $x_H$ | 35 | 18 |

Display this information as parallel boxplots and describe what the graphs show.

**5** The intelligence quotient or IQ scores of a group of 30 learners were recorded. There were 15 girls and 15 boys in the group.

Female: 100, 83, 94, 107, 115, 120, 97, 82, 98, 111, 104, 103, 94, 85, 73

Male: 99, 91, 103, 99, 83, 126, 102, 94, 86, 105, 100, 92, 105, 93, 76

**a** Display the data as parallel boxplots.

**b** Comment on the difference in IQ between the boys and girls in this group.

**6** The table shows the number of deaths, in thousands, per year for Australians under the age of 60.

| Age | Number of deaths per year (male) | Number of deaths per year (female) |
|---|---|---|
| 0– | 30 | 24 |
| 10– | 15 | 7 |
| 20– | 27 | 8 |
| 30– | 31 | 15 |
| 40– | 50 | 32 |
| 50– | 136 | 80 |

**a** Draw a cumulative percentage frequency curve for the number of male deaths.

**b** Draw a cumulative percentage frequency curve for the number of female deaths.

**c** Use the graphs to find the five-number summaries for each sex.

**d** Display this information as parallel boxplots.

**e** Comment on the difference between the number of premature deaths for males and females.

# 7G Exploring sampling

When we carry out a statistical survey, it is often impossible to survey everybody involved.

It is usually better to survey a **sample** or smaller group of people. A good sample size, $n$, is $\sqrt{N}$, where $N$ is the number in the population. It is also important not to have a sample that is biased in any way. Two ways of choosing a fair sample are *stratified sampling* and *random sampling*.

**Example**

Imagine you are the year level coordinator for a school and you wanted to randomly select a group of 12 from a year level with 140 learners. How can you select learners to survey?

## Stratified sampling

In this year level there are 80 males and 60 females. When selecting a sample we should try to ensure that it is in the same ratio of sexes as the population.

The ratio of males to females in the population is 80:60 or 4:3.

The number of males in the sample should be $\frac{4}{7} \times 12 = 6{\cdot}857$.

So the best breakdown of learners is 7:5—that is, seven boys and five girls.

## Random sampling with a calculator

Calculators have a random number function. These produce numbers from 0.0000 to 0.9999 that can then be used to get whole numbers from 1 up to a required number. The coordinator could use the random number function to choose the sample by following these steps.

| | Example |
|---|---|
| Number the learners from 1 to 140. | |
| Use the calculator to generate a random number (0.0000 to 0.9999). | 0.7123 |
| Multiply the number by the number in the population, in this case 140. | 99.722 |
| Add 1 to the number. | 100.722 |
| Ignore the decimal places. | 100<br>Learner 100 is chosen. |

This process is repreated until there are 12 different numbers.

## Random sampling using a table of random numbers

An alternative to generating random numbers with a calculator is to use a table of random numbers.

### Random numbers

| | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 2387 | 9003 | 3951 | 5695 | 1284 | 4761 | 7118 | 1196 | 1741 | 3791 | 3405 | 3132 | 6682 | 9493 | 9864 |
| 7359 | 1250 | 7036 | 2916 | 7562 | 9299 | 8910 | 6713 | 5173 | 8617 | 4222 | 0244 | 3045 | 4923 | 1740 |
| 3200 | 2876 | 4591 | 6695 | 0574 | 6376 | 6141 | 1322 | 6031 | 0193 | 0469 | 9160 | 4010 | 4583 | 9834 |
| 3100 | 0586 | 5833 | 5593 | 7181 | 3078 | 2430 | 4682 | 2133 | 8413 | 4987 | 4854 | 4680 | 0229 | 9913 |
| 8661 | 0584 | 4736 | 6834 | 6189 | 1441 | 5331 | 9766 | 6544 | 5230 | 5949 | 2036 | 8082 | 0606 | 3178 |
| 4954 | 7400 | 4954 | 7117 | 0237 | 7064 | 6177 | 0984 | 6178 | 2096 | 8250 | 9478 | 7569 | 5211 | 0513 |
| 5885 | 9373 | 0797 | 0721 | 7942 | 4146 | 7649 | 1836 | 2440 | 3209 | 0461 | 2579 | 8590 | 1568 | 0050 |
| 1725 | 0128 | 6589 | 2347 | 6147 | 6176 | 0417 | 6131 | 2618 | 0851 | 6151 | 9509 | 3355 | 1625 | 7762 |
| 5157 | 5315 | 6433 | 3768 | 4298 | 3244 | 9535 | 4655 | 9440 | 6473 | 6120 | 4048 | 5861 | 9464 | 5673 |
| 3633 | 1840 | 6108 | 4079 | 1830 | 7645 | 7773 | 4482 | 6216 | 3263 | 5098 | 6899 | 9692 | 4889 | 3542 |
| 6574 | 0410 | 0160 | 2679 | 1092 | 5442 | 2134 | 9247 | 6234 | 8205 | 2031 | 3945 | 0590 | 9626 | 0714 |
| 1976 | 8918 | 3074 | 8923 | 7351 | 4249 | 3023 | 6261 | 0927 | 8754 | 7477 | 8929 | 2211 | 3346 | 4145 |
| 6206 | 4905 | 6043 | 0356 | 5689 | 6199 | 9079 | 6739 | 7273 | 9824 | 4758 | 4739 | 5890 | 2846 | 0069 |
| 7289 | 2986 | 6212 | 0041 | 4736 | 6766 | 9046 | 9858 | 2510 | 1166 | 6701 | 5710 | 4045 | 3597 | 2550 |
| 4411 | 6853 | 1427 | 3536 | 1752 | 2445 | 7212 | 9468 | 9614 | 0003 | 2685 | 6069 | 1826 | 3801 | 1485 |
| 1771 | 8774 | 4379 | 0649 | 8263 | 0764 | 3463 | 0838 | 4789 | 0123 | 2636 | 5327 | 6190 | 2069 | 5763 |
| 7737 | 4709 | 4312 | 5161 | 8522 | 5531 | 5418 | 1520 | 0200 | 6081 | 1915 | 7577 | 1337 | 9076 | 4156 |
| 5535 | 9287 | 9313 | 0926 | 8240 | 1669 | 4225 | 2803 | 4097 | 6608 | 8171 | 4265 | 5743 | 1713 | 0488 |
| 4550 | 2475 | 1818 | 7282 | 7269 | 6411 | 5460 | 6982 | 8760 | 2424 | 0120 | 9632 | 5201 | 1804 | 6200 |
| 9937 | 8467 | 9686 | 9763 | 4066 | 0743 | 6217 | 4017 | 0190 | 7878 | 1483 | 9544 | 2788 | 1500 | 4177 |
| 3293 | 0687 | 9872 | 4024 | 6307 | 8863 | 7500 | 7949 | 0209 | 3984 | 3498 | 0002 | 2441 | 3713 | 8691 |
| 5030 | 0240 | 9748 | 1479 | 6095 | 3383 | 4622 | 2443 | 5652 | 5660 | 2252 | 7708 | 0464 | 5117 | 2232 |
| 1049 | 0281 | 3037 | 7882 | 4752 | 0255 | 1674 | 8044 | 9224 | 7117 | 0739 | 2631 | 1349 | 9098 | 2631 |
| 1919 | 1603 | 9597 | 6428 | 9601 | 9909 | 4608 | 0614 | 9578 | 0245 | 3889 | 4567 | 6506 | 4011 | 5885 |
| 8793 | 0099 | 2130 | 5397 | 3912 | 3167 | 5911 | 1011 | 8387 | 3421 | 6660 | 1260 | 5016 | 9015 | 4034 |
| 4555 | 6161 | 7844 | 3157 | 7850 | 7732 | 0591 | 0476 | 5068 | 1927 | 7525 | 8369 | 8273 | 8708 | 9045 |
| 8507 | 2588 | 6043 | 9980 | 3962 | 4872 | 8887 | 7635 | 0067 | 6136 | 6916 | 3587 | 4026 | 7721 | 1012 |
| 9231 | 3270 | 1216 | 8498 | 5081 | 2571 | 2131 | 7121 | 3811 | 9900 | 7319 | 7788 | 7057 | 1032 | 6147 |
| 6050 | 5962 | 2913 | 5418 | 2334 | 5471 | 5763 | 8046 | 0931 | 3381 | 3939 | 5473 | 7728 | 7135 | 5541 |
| 3844 | 6694 | 9373 | 0776 | 3926 | 5111 | 7224 | 9064 | 4785 | 7563 | 0355 | 4898 | 1204 | 8669 | 6770 |

This table gives 1800 random digits, grouped for convenience, in four-digit numbers. To select a group of 9 from a year level with 85 learners using the data from the table, read off successive groups of two digits, rejecting those greater than 85. The result is:

23, (87), (90), 03, 39, 51, 56, (95), 12, (98), 44, 76, 17

The numbers in the brackets are greater than 85 and are rejected. The 9 random numbers less than 86 that are selected are:

23, 3, 39, 51, 56, 12, 44, 76, 17

You do not need to always begin reading random numbers from the top of the table. Random numbers can be started at any point in the table and read horizontally or vertically. Ideally the starting point should also be selected at random, so a row or column could be identified by rolling a die, or even by using today's date. For example on 17 July, you could begin with the 17th number in row 7.

## Learning task 7G

**1** The Betikama Adventist College (BAC) Year 9 learners are trying to decide on a theme for their social night program. The learners' committee decided to survey a sample of learners for their opinion. They considered the following options:

- **a** Ask the learners at Year 9 soccer training (boys and girls).
- **b** Ask the Year 9 learners in the library at lunchtime.
- **c** Survey the Year 9 learners who board at the school (approximately three quarters of the learners are boarders).
- **d** Survey all the people in the school list with a surname beginning with S.
- **e** Survey the Year 9 learners who work in the school garden.
- **f** Ask the Year 9 learners who are studying Business Studies as their elective.

Explain why each of these options may not give results that reflect the opinions of the whole year level.

**2** Use the random number function of the calculator or the random number table on page 27 to produce 10 random numbers from:

**a** 1 to 200 **b** 1 to 500 **c** 1 to 50

**3** Use the random number function of the calculator or the random number table on page 27 to produce 10 random numbers from:

**a** 100 to 200 **b** 40 to 80 **c** 101 to 150

(Hint: How many numbers are there? What should be added to the random number?)

**4** A teacher needs to randomly select two learners from a class of 27. Explain how she could do this using random numbers.

**5** There are 14 boys and 13 girls. How could she randomly select a boy and a girl?

**6** The breakdown by sex and boarding status of the year level in the example is:

| | Boarders | Day learners | Total |
|---|---|---|---|
| Male | 25 | 55 | 80 |
| Female | 10 | 50 | 60 |
| Total | 35 | 115 | 140 |

The year level coordinator would like to survey 40 learners. Explain how the coordinator could select learners using stratified sampling.

**7** A gymnastics club has 400 members in the following divisions:

| | Juniors | Adults | Seniors |
|---|---|---|---|
| Male | 90 | 50 | 40 |
| Female | 110 | 40 | 70 |

The committee wants to survey 60 members. Explain how they could use stratified sampling to select members.

**8** The 2013 RAMSI People's Survey collected 3405 responses, a quarter from each of four age/gender groups: Men and Women aged 30 years and over and Young Men and Young Women aged 18 to 29. As in previous years, interviews were conducted on Guadalcanal, Honiara, Malaita and Western Province, plus a sample of other provinces surveyed in alternate years.

**a** What type of sampling is used for the RAMSI People's Survey?

**b** Do the survey results fairly represent the opinions of all Solomon Islanders? Give reasons for your opinion.

**c** One of the RAMSI survey questions is "What do you use a phone for?" If you were hired to interview a random sample of people in your village to record their answers to this question, describe how you would select the people according to the RAMSI criteria.

**9** Which of the following sampling methods would be unlikely to produce a random sample? Give a reason why any sample might be biased.

**a** To choose a sample of five learners from a class, the teacher selects the first five listed in the class roll book.

**b** Each learner's name is written on a card. The cards are put into a box and the box is shaken. The principal is invited to draw out 5 cards without looking in the box. The first five names drawn are used as the sample.

**c** A needle is pushed through a telephone directory. The first 25 numbers with needle holes are used for the sample.

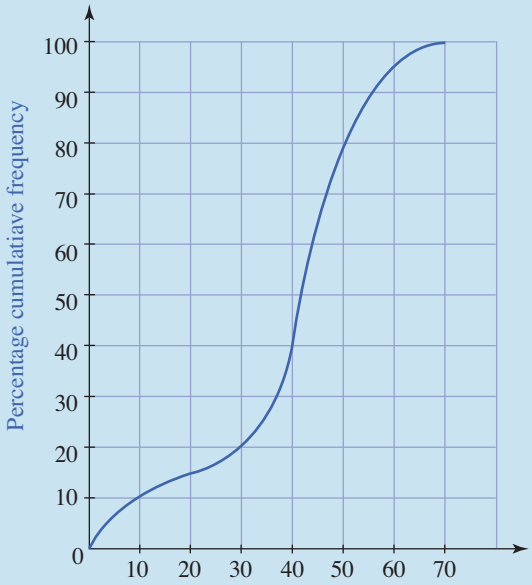
**d** A police officer stops every fifth car that passes the market to check the car is properly registered. He checks a sample of 32 cars in one hour.

**e** A supermarket owner checks the top tray of 20 trays of eggs to see whether they are all fresh and can be sold in her shop. The test is to see whether each egg will sink or float in a pot of water.

# Puzzles

**1** Complete the table by using the data in the table and the cumulative percentage frequency curve. Match the letters to the correct numbers below to find the name of the bacterium that causes typhoid fever:

| Interval | Frequency | Cumulative percentage frequency |
|---|---|---|
| 0– | 10 | **A** |
| 10– | **B** | 16 |
| 20– | **U** | **C** |
| **S** | 20 | **E** |
| 40– | 40 | **I** |
| 50– | **L** | **M** |
| 60– | **N** | **O** |




| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 30 | 10 | 15 | 95 | 100 | 5 | 40 | 16 | 16 | 10 |
| 6 | 10 | 20 | 80 | 15 | 15 | 4 | 30 | | |

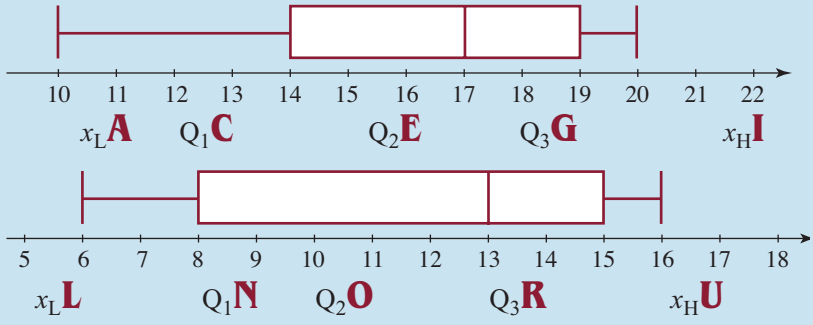
**2** Find the five-number summary for the two data sets. Match the letter to the correct answer below to find the name of the person born in September 1836 in England who supposedly invented the first toilet:

| 7, 5, 8, 9, 2, 1, 3, 7, 5, 2, 9, 8, 7, 7, 3, 1 | | 15, 14, 9, 2, 2, 9, 15,16, 4, 5, 8, 9, 4, 3, 12, 5 | |
|---|---|---|---|
| $x_L$ | **A** | $x_L$ | **O** |
| $Q_1$ | **C** | $Q_1$ | **P** |
| $Q_2$ | **E** | $Q_2$ | **R** |
| $Q_3$ | **H** | $Q_3$ | **S** |
| $x_H$ | **M** | $x_H$ | **T** |

| | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 16 | 7.5 | 2 | 9 | 1 | 13 | | 2.5 | 8.5 | 1 | 4 | 4 | 6 | 8.5 |

**3** During the Roman Empire *plumberi* (plumbers) built sophisticated sewerage systems made from lead. Some historians think that lead poisoning may have contributed to the downfall of the Roman Empire. Use the boxplots to find the five-number summaries, then match the letters to the correct numbers below to find the answer to the question:

**What are the names of two famous Roman emperors?**




| | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ___ | ___ | ___ | ___ | ___ | ___ | ___ | ___, | | ___ | ___ | ___ | ___ |
| 14 | 10 | 6 | 20 | 19 | 16 | 6 | 10 | | 8 | 17 | 15 | 13 |

**4** Use the data sets for 1 and 2 to complete the back-to-back stemplots. Match the letter to the missing numbers below to find the answer to the question:

**What is the name given to a scientist who studies plagues and epidemics?**

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 27 | 14 | 36 | 28 | 29 | 15 | **G** | 42 | 56 |
| | 10 | 15 | 8 | 33 | 52 | 65 | 5 | 47 | 47 |
| 2 | **L** | 51 | 60 | 18 | 26 | 53 | 61 | 8 | 66 |
| | 9 | 58 | 69 | 25 | 57 | 64 | 12 | 28 | 59 |

| 2 | | | | | Stem | 1 | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | 9 | 8 | 0 | 5 | 8 | | | |
| | | | **M** | 2 | 1 | 0 | 4 | **E** | 5 | 8 |
| | 8 | 6 | 5 | 1 | 2 | 7 | 8 | 9 | | |
| | | | | | 3 | 3 | **S** | | | |
| | | | | | **D** | 2 | 7 | 7 | | |
| **O** | 8 | 7 | **I** | 1 | 5 | **T** | 6 | | | |
| 9 | 6 | 4 | **P** | 0 | 6 | 5 | | | | |

| | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ___ | ___ | ___ | ___ | ___ | ___ | ___ | ___ | ___ | ___ | ___ | ___ | ___ | ___ | ___ |
| 5 | 1 | 3 | 4 | 5 | 8 | 3 | 9 | 21 | 9 | 18 | 3 | 6 | 2 |

# Applications

## Samples and surveys

Imagine that a large school of more than 1000 learners wanted to make major changes to its timetable, to introduce new subjects, or to change the policy on uniform or on how the school is run. The School Council members would need to know whether the staff, learners and parents would support its decisions.

One way of collecting information is to conduct a survey. The process of conducting a manageable survey that gives results that are easy to analyse is not easy.

### WHY?

- There should be a clear reason for doing a survey.
- It is important that the survey is limited to a few specific issues.
- Think about the type of data you wish to collect.
- Ensure that the survey is not too open-ended, or the data will be difficult to analyse.

### WHO?

- It is usually not appropriate to survey all the population.
- Ensure that individuals are randomly selected so that there is no bias.
- Include a cross-section with members from each of the relevant interest groups.
- The sample needs to be large enough to be representative of the whole population.

### HOW?

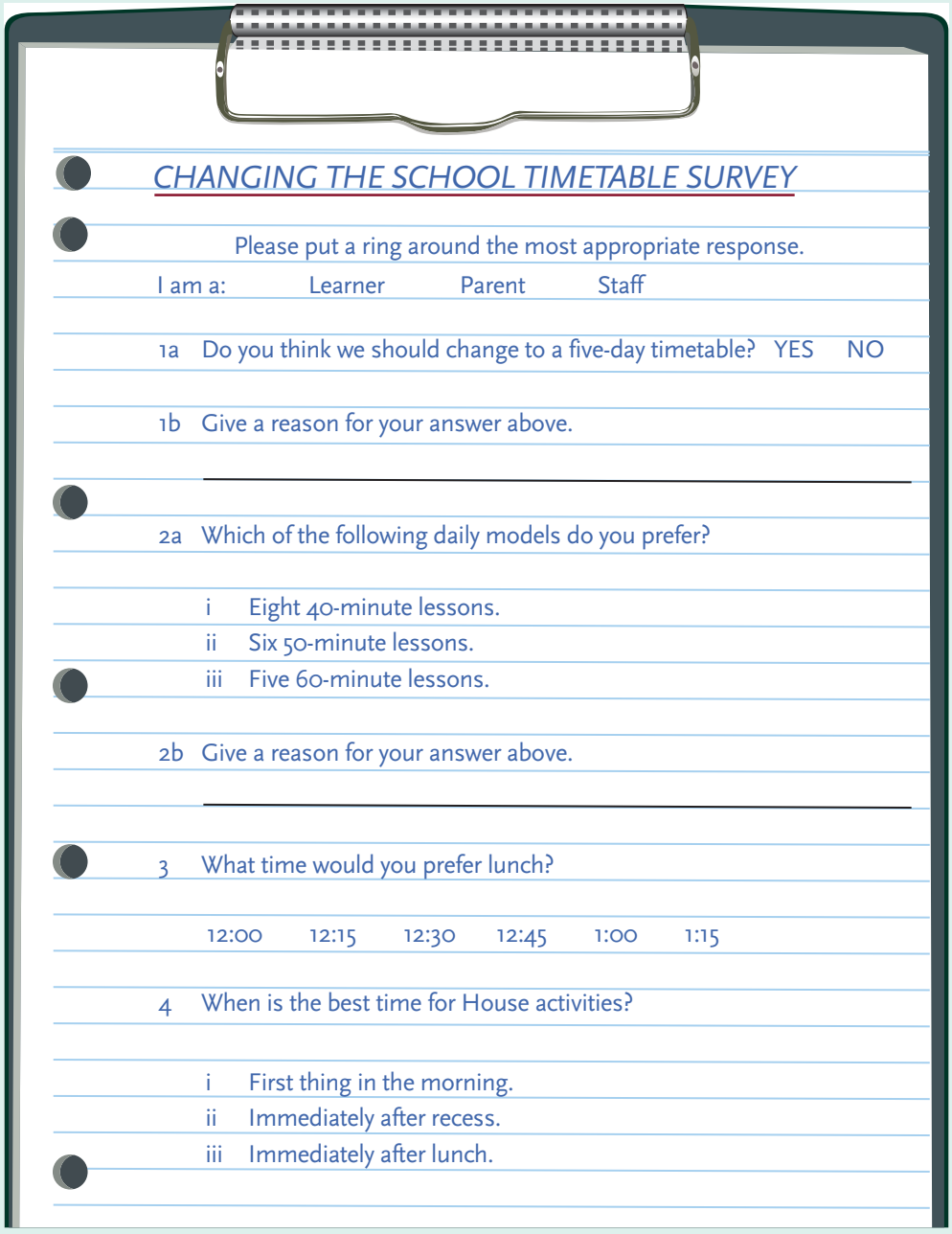
- Limit the survey to only as many questions as are really necessary.
- Keep the questions brief and clear.
- Limit the responses to ensure that the answer format is as simple as possible.
- Ask for comments only for clarification of an earlier limited-response question.

**Example**

### Changing the timetable

**Why?** A few years ago a large school wanted to change its timetable. The timetable was a revolving 6-day timetable with eight 40-minute lessons a day. Each day also had to include a recess, lunchtime and one 20-minute period for House activities. The curriculum coordinator decided to conduct a survey before making any changes. Should the school change to a 5-day, Monday-to-Friday timetable, or stay with a 6-day model? Why did people want to change or stay the same? What was the best length of time for a lesson and the ideal number of lessons per day? Where should lunch, recess and House activities fall in the day?

**Who?** The school had 1200 learners: 400 in the primary school (Prep–Year 6) and 800 in the senior school. There were 720 families at the school and 80 teaching staff. The coordinator decided to survey 100 people, and used learner/staff identification numbers to randomly select 20 learners from the primary school, 40 learners from the senior school, 36 parents and 4 staff members.



## CHANGING THE SCHOOL TIMETABLE SURVEY

Please put a ring around the most appropriate response.

I am a: Learner Parent Staff

1a Do you think we should change to a five-day timetable? YES NO

1b Give a reason for your answer above.

2a Which of the following daily models do you prefer?

i Eight 40-minute lessons.
ii Six 50-minute lessons.
iii Five 60-minute lessons.

2b Give a reason for your answer above.

3 What time would you prefer lunch?

12:00 12:15 12:30 12:45 1:00 1:15

4 When is the best time for House activities?

i First thing in the morning.
ii Immediately after recess.
iii Immediately after lunch.

1. Choose an issue that is relevant for your community, school or year level.
2. Decide on the target group to be surveyed and select a sample of 50.
3. Write a survey with no more than eight questions.
4. Analyse the data using skills learnt in this chapter or in previous years.
5. Compare the results of two different groups, e.g. learners and teachers.
6. Use these results to produce a report and present this report to the rest of your group.

# Enrichment

## Boxplots with outliers

An **outlier** is a value that is extremely unusual and possibly should not have been included in the data. We define outliers as values that lie outside the limits of:

$$Q_1 - 1{\cdot}5\text{IQR},\ Q_3 + 1{\cdot}5\text{IQR}$$

### Example

The times, in minutes, taken by 50 learners to travel to school are:

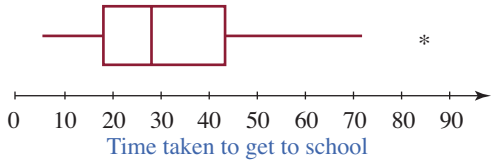
85, 45, 20, 7, 36, 28, 45, 69, 72, 56, 45, 32, 15, 38, 62, 44, 36,
29, 46, 21, 52, 34, 27, 24, 16, 18, 24, 23, 17, 12, 5, 43, 28, 36,
15, 12, 26, 28, 17, 13, 14, 9, 24, 28, 27, 29, 56, 38, 29, 34

$Q_1 = 18$ and $Q_3 = 43$

To find the outliers we calculate:

$Q_1 - 1{\cdot}5\text{IQR}$
$= 18 - (1{\cdot}5)(43 - 18)$
$= -19.5$

$Q_3 + 1{\cdot}5\text{IQR}$
$= 43 + (1{\cdot}5)(43 - 18)$
$= 80{\cdot}5$




An outlier is any value less than –19·5 (which is impossible) or greater than 80·5.

Therefore, 85 is an outlier, and when we draw the boxplot we should put a star at the outlier and the whisker should finish at the next value within the limits, in this case 72. The boxplot is shown with the outlier marked.

**1** In the survey carried out to find how much people lost in a single night playing the 'pokies', the results of the 50 people surveyed are shown below:

$72, $23, $121, $12, $26, $36, $27, $89, $139, $82, $35, $21, $27, $43, $53, $18, $15, $21, $27, $38, $160, $23, $24, $32, $37, $32, $26, $39, $45, $23, $47, $35, $28, $22, $56, $40, $47, $35, $40, $32, $159, $37, $36, $52, $23, $36, $27, $28, $42, $35

Find the outliers and hence draw the boxplot.

**2** The number of bottles of wine sold in a restaurant each day is shown below:

9, 16, 15, 49, 32, 20, 23, 8, 9, 5, 19, 20, 24, 19,
15, 18, 12, 18, 15, 12, 14, 60, 14, 20, 24, 16, 12, 68, 26,
16, 12, 4, 25, 28, 28, 20, 18, 18, 15, 8

Find the outliers and hence draw the boxplot.

**3** The meat content, in grams, in a sample of hamburgers is shown below:

82, 123, 123, 109, 103, 125, 120, 118, 98, 126,
149, 122, 124, 120, 145, 117, 113, 111, 121, 101

Find the outliers and hence draw the boxplot.

## Density histograms

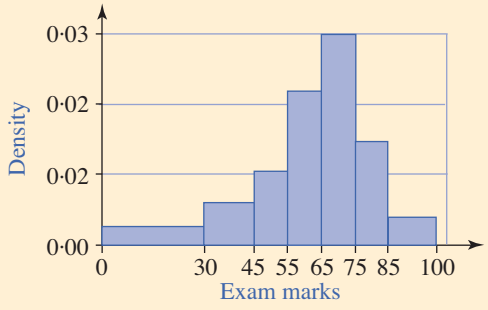
When drawing histograms we ensured that the intervals had the same width. This is not always how data are recorded. It is not appropriate to use frequency when intervals have different widths. When we look at a histogram we register the area a bar covers rather than its height. If we have a class that is twice as wide as the others we must compensate by making the bar half as high. We use **density** on the vertical axis.

$$\text{Density} = \frac{\text{relative frequency}}{\text{class width}}$$

The Year 9 Maths exam marks are shown below:

| Exam marks | Frequency | Relative frequency | Density |
|---|---|---|---|
| 0– | 15 | 0·075 | 0·0025 |
| 30– | 18 | 0·09 | 0·006 |
| 45– | 20 | 0·1 | 0·01 |
| 55– | 45 | 0·225 | 0·0225 |
| 65– | 58 | 0·29 | 0·029 |
| 75– | 32 | 0·16 | 0·016 |
| 85– | 12 | 0·06 | 0·004 |

The density histogram is shown below:




**4** Draw density histograms for the following car accident insurance data:

**a**

| Age of driver | Frequency of accidents |
|---|---|
| 18– | 610 |
| 25– | 440 |
| 35– | 380 |
| 45–65 | 570 |

**b**

| Number of years driving experience | Frequency of accidents |
|---|---|
| 0 | 60 |
| 2– | 240 |
| 10– | 150 |
| 20– | 60 |
| 30– | 20 |
| 40–50 | 330 |

# Revision/Assessment

## Exercise 7A

**1** The meat in a quarter-pound burger should weigh approximately 114 g. In a quality control survey a company weighed the meat in 30 quarter-pound burgers to check that they were not misleading their customers.

The weights, in grams, rounded to the nearest whole number were:

109, 113, 118, 123, 126, 122, 124, 127, 107, 105, 114, 112, 115, 127, 124, 118, 120, 122, 111, 102, 115, 107, 116, 118, 120, 127, 120, 110, 112, 127

Summarise the data in a percentage frequency table with a class width of 5.

**2** The table shows the number of people below the age of 60 who died in one year in Australia.

**a** Display the data as a percentage histogram.

**b** Describe the shape of the graph. Is this what you would expect?

**c** What percentage were between the ages of 20 and 30?

| Age | Number of deaths per year (in thousands) |
|---|---|
| 0– | 54 |
| 10– | 22 |
| 20– | 35 |
| 30– | 46 |
| 40– | 82 |
| 50– | 216 |

## Exercise 7B

**3** In a survey to find the different eating habits of a group of adults and children, data were collected and displayed in this table.

Display the data as a double column bar graph and write three comments on what the graph shows.

| | Adults | Children |
|---|---|---|
| Fries | 45 | 78 |
| Hamburger | 21 | 67 |
| Cheeseburger | 63 | 89 |
| Chicken burger | 87 | 32 |
| Nuggets | 74 | 12 |

**4** The table shows the percentage of six-year-olds who were fully immunised before and after a campaign to encourage immunisation:

| | Before the campaign | After the campaign |
|---|---|---|
| Diphtheria and tetanus | 93 | 93 |
| Whooping cough | 93 | 85 |
| Polio | 59 | 80 |
| Measles | 65 | 92 |
| Mumps | 14 | 87 |

**a** Display this information as a double column bar graph.

**b** For which diseases was there an increase in immunisation rates?

**c** For which diseases was there an decrease in immunisation rates?

**d** Do you think the campaign was successful? Explain your answer.

## Exercise 7C

**5** Find the mean for the weight of meat in 25 quarter-pound burgers. Their weights, in grams, were:

109, 113, 118, 123, 126, 122, 124, 127, 107, 105, 114, 112, 115,
127, 124, 118, 120, 122, 111, 102, 115, 107, 116, 118, 120

**6** The results of a larger survey of hamburger weights are displayed in this table:

| Weight (g) | Frequency |
|---|---|
| 100– | 216 |
| 105– | 452 |
| 110– | 538 |
| 115– | 640 |
| 120– | 541 |
| 125– | 113 |

Estimate the mean weight for this larger sample.

## Exercise 7D

**7** **a** Display the data from Question 5 as a percentage cumulative frequency curve.

**b** Find the quartiles for the weights.

## Exercise 7E

**8** The data below show the costs of haircuts at Dr Follicles:

| | |
|---|---|
| Males | \$52, \$26, \$39, \$45, \$23, \$47, \$35, \$28, \$22, \$56, \$40, \$15, \$35, \$40, \$32, \$56, \$37, \$36, \$52, \$23, \$36, \$27, \$28, \$42, \$35 |
| Females | \$75, \$25, \$76, \$32, \$26, \$36, \$27, \$89, \$95, \$82, \$35, \$21, \$27, \$43, \$53, \$18, \$35, \$21, \$27, \$38, \$62, \$23, \$34, \$32, \$37 |

**a** Display this information as an ordered back-to-back stemplot.

**b** Comment on what the graph shows.

## Exercise 7F

**9** A competitor claims to have more meat in its quarter-pound burgers and publishes the following data results for its survey of 25 burgers.

The weights, in grams, were:

112, 123, 113, 109, 123, 125, 120, 118, 118, 116, 114, 122, 124,
120, 115, 117, 113, 111, 121, 117, 126, 120, 124, 124, 110

**a** Find the five-number summary for the data set.

**b** Draw a boxplot.

**c** Comment on what the graph shows.

**10** Sue is an asthmatic. In an attempt to improve the treatment of her asthma, her doctor asked her to record her 'expiratory peak flow rate' each day for three weeks. The results were:

245, 250, 340, 320, 280, 310, 275, 290, 315, 375, 400,
320, 285, 230, 240, 265, 280, 280, 325, 360, 380

**a** Rewrite the flow rates in order.

**b** Find the five-number summary for the flow rates.

**c** Display this information as a boxplot.

CHAPTER

# 8

# Trigonometry

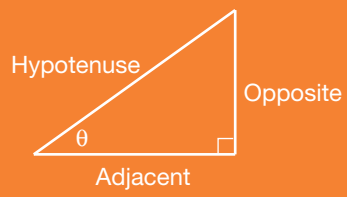
The study of trigonometry began in 140 BC when ancient Greek mathematicians investigated the chords in a circle. The name trigonometry was first published in 1595 by Pitiscus and comes from the word *trigons*, which is Latin for triangle.

Vavaya Ridge, just above Point Cruz main market in Honiara, is one of the hills that was used by the Japanese to shoot Allied warships that attempted to land on the Island of Guadalcanal during the Second World War. It was a long and fierce battle and the wrecks of many Japanese and Allied ships remain in Iron Bottom Sound between the islands of Savo, Guadalcanal and Florida (Gela). In order for guns to sink a ship from Vavaya Ridge, trigonometry was used to calculate the necessary range and angles of elevation.

## This chapter covers the following skills:

- Calculating unknown sides of a right-angled triangle using the trigonometry ratios

$\sin\theta = \frac{O}{H}$ $\cos\theta = \frac{A}{H}$ $\tan\theta = \frac{O}{A}$




- Finding unknown angles
  If $\sin\theta = a$, $a = \sin^{-1}\theta$
  $\cos\theta = b$, $b = \cos^{-1}\theta$
  $\tan\theta = c$, $c = \tan^{-1}\theta$
- Solving practical problems
- Using bearings and angles of elevation and depression
  - True bearing is an angle drawn clockwise from north.
  - Conventional or compass bearings start from north or south then give the amount of turn east or west.
  - Angles of elevation and depression are measured from the horizontal.
- Exploring the unit circle and graphs of trigonometric functions

## Specific Learning Outcome (SLO)

Learners should be able to:

**9.8.1.1** Identify a right-angled triangle and name the sides to a given angle: *hypotenuse*, *adjacent*, *opposite*.

**9.8.2.1** Identify the sine ratio:

$$\text{sine ratio} = \frac{\text{opposite side}}{\text{hypotenuse}}$$

**9.8.3.1** Calculate the length of the side opposite to a given angle using the sine ratio.

**9.8.4.1** Calculate and solve word problems using the sine ratio.

**9.8.5.1** Identify the cosine ratio:

$$\text{cosine ratio} = \frac{\text{adjacent side}}{\text{hypotenuse}}$$

**9.8.6.1** Calculate the length of the side adjacent to a given angle using the cosine ratio.

**9.8.7.1** Calculate and solve word problems using the cosine ratio.

**9.8.8.1** Identify the hypotenuse of the right-angle triangle.

**9.8.9.1** Calculate the length of the hypotenuse of a right-angled triangle using the sine or cosine ratio.

**9.8.10.1** Solve word problems that involve the hypotenuse side of the right-angle triangle.

**9.8.11.1** Calculate a missing angle of a right-angled triangle using the sine or cosine ratio.

**9.8.11.2** Solve practical word problems using the sine or cosine ratio.

**9.8.12.1** Identify the tangent ratio:

$$\text{tangent ratio} = \frac{\text{opposite side}}{\text{adjacent side}}$$

**9.8.13.1** Calculate the length of the opposite and adjacent sides of the right angle triangle using the tangent ratio.

**9.8.14.1** Find missing angles of right-angled triangles using the tangent ratio.

**9.8.15.1** Solve practical word problems using the tangent ratio.

**9.8.16.1** Define angles of elevation and depression.

**9.8.17.1** Solve various practical word problems dealing with angles of elevation and depression.

**9.8.18.1** Define the term ‘bearings’.

**9.8.19.1** Differentiate between *conventional* and *true bearings*.

**9.8.20.1** Calculate bearings problems with diagrams.

**9.8.20.2** Solve practical word problems using conventional and true bearings.

**9.8.21.1** Use the scales of the unit circle find trigonometrical ratios and angles.

**9.8.22.1** Calculate values of given sine and cosine angles using the unit circle graph.

**9.8.23.1** Estimate to the nearest degree the corresponding angles for sine and cosine ratios decimal values.

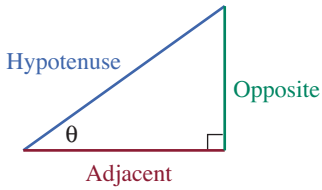
**9.8.24.1** Use a calculator or trigonometric tables to plot sine values on a graph.

**9.8.25.1** Use a calculator or trigonometric tables to plot cosine values on a graph.

**9.8.25.2** Identify the key features of sine and cosine graphs.

# 8A Labelling a right-angled triangle

The sides of a right-angled triangle can be identified as opposite to and adjacent (next) to a given angle. The hypotenuse is the longest side of a right-angled triangle because it is opposite the right angle—the largest angle in the triangle.

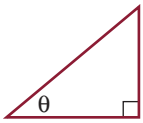


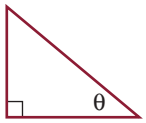

The symbols θ and α are often used to represent unknown angles in a triangle.

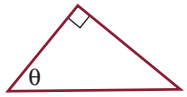
Theta (θ) and Alpha (α) are both letters in the Greek alphabet.

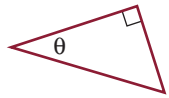
## Exercise 8A

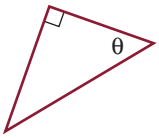
**1** Label the sides of each triangle relative to the angle θ, using the letters H (hypotenuse), O (opposite) and A (adjacent):

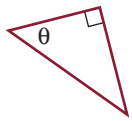
**a**


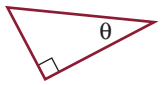

**b**


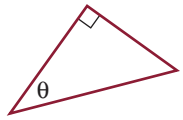

**c**


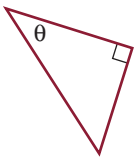

**d**


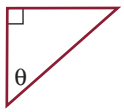

**e**


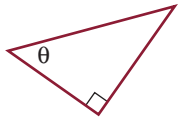

**f**


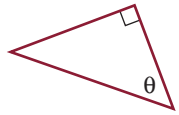

**g**



**h**


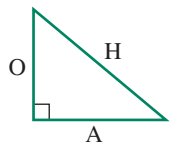

**i**


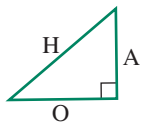

**j**


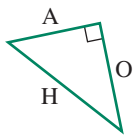

**k**


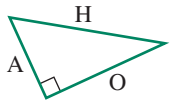

**l**


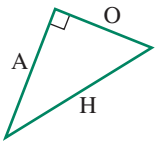

**2** Place the angle α in the following triangles to match the labels:

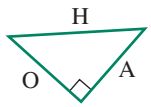
**a**


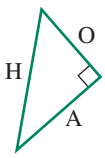

**b**


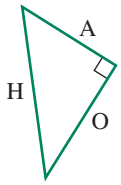

**c**


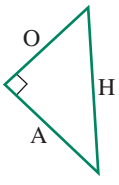

**d**


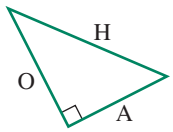

**e**


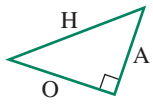

**f**


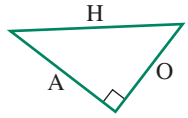

**g**



**h**



**i**


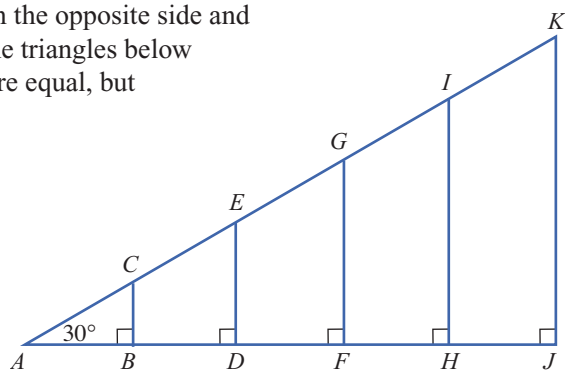

**j**



**k**



**l**


# Exploring the sine ratio 8B

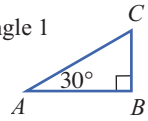
This task examines the relationship between the opposite side and the hypotenuse in right-angled triangles. The triangles below are all similar as all corresponding angles are equal, but their side lengths are different.



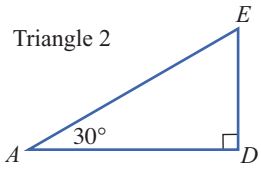

## Learning task 8B

**1** For each triangle shown below, measure the lengths of the opposite side and the hypotenuse to the nearest millimetre:

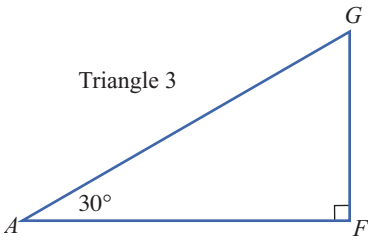
Triangle 1



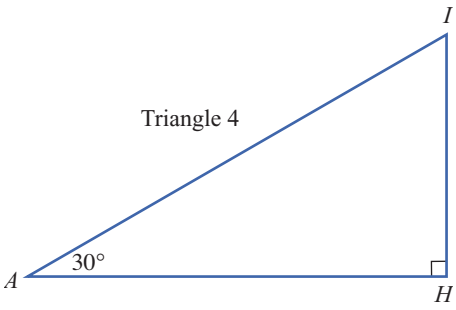

Triangle 2



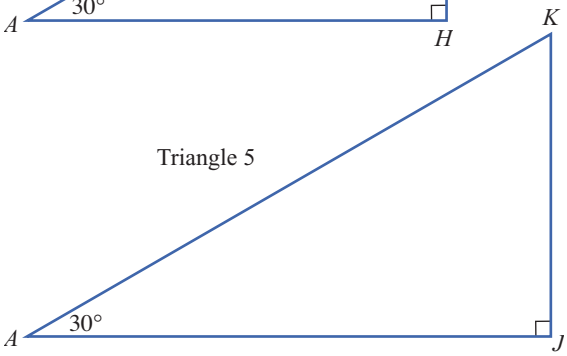

Triangle 3




Triangle 4




Triangle 5




**2** Copy and complete the table below, then use a calculator to find the ratio of the length of the opposite side to the length of the hypotenuse, correct to 2 decimal places:

| Triangle | Length of opposite side (mm) | Length of hypotenuse (mm) | Ratio: $\frac{\text{opposite side}}{\text{hypotenuse}}$ |
|---|---|---|---|
| 1 | | | |
| 2 | | | |
| 3 | | | |
| 4 | | | |
| 5 | | | |

**3** What conclusion can you reach about the ratio of the length of the opposite side to the length of the hypotenuse in each of the triangles?

**4** This ratio is called **sine**.

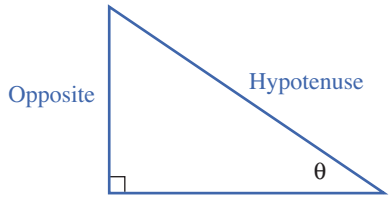
$\sin 30° =$ _____

# 8C Finding the length of the opposite side

Trigonometry can be used to find the lengths of the sides of right-angled triangles by using algebra to solve the appropriate equation. The letters **SOH CAH TOA** (pronounced *soh car towa*) help us to get the order of the equation correct. The first of three trigonometric relations, sine, uses the opposite side to a given angle and the hypotenuse. It is abbreviated to $\sin\theta$, where $\theta$ is the angle in the triangle.

SOH CAH TOA

$$\text{Sine}\,\theta = \frac{\text{Opposite}}{\text{Hypotenuse}}$$

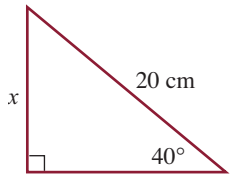



Sine uses the opposite side and the hypotenuse. When finding the sine of an angle, make sure that your calculator is set in degree mode.

**Example**

Use the three figure tables to find the length of side marked $x$ in the following triangles, expressed correct to 2 decimal places:

**a**




**Solution**

SOH CAH TOA

$$\sin\theta = \frac{O}{H}$$

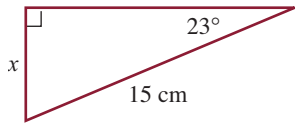
$$\sin 40° = \frac{x}{20}$$

$$20 \times \sin 40° = \frac{x}{20} \times 20$$

$$\therefore x = 20\sin 40°$$

$$x = 12{\cdot}86 \text{ cm}$$

**b**



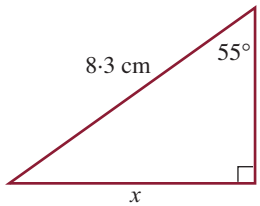

$$\sin\theta = \frac{O}{H}$$

$$\sin 23° = \frac{x}{15}$$

$$\therefore x = 15\sin 23°$$

$$x = 5{\cdot}86 \text{ cm}$$

**c**




$$\sin\theta = \frac{O}{H}$$

$$\sin 55° = \frac{x}{8{\cdot}3}$$

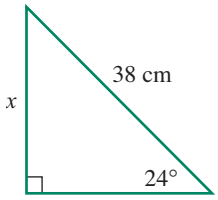
$$\therefore x = 8{\cdot}3\sin 55°$$

$$x = 6{\cdot}80 \text{ cm}$$

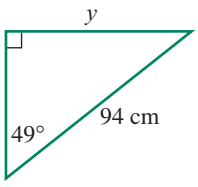
## Exercise 8C

**1** Find the length of the side marked with a pronumeral. Using the three figure tables, express your answers to 2 decimal places:

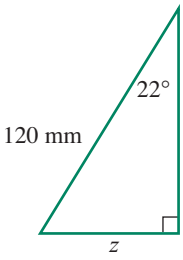
**a**



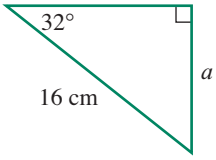

**b**



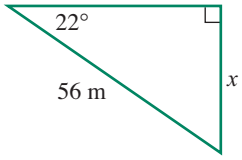

**c**



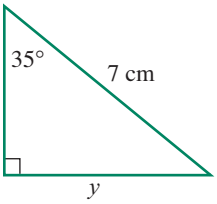

**d**



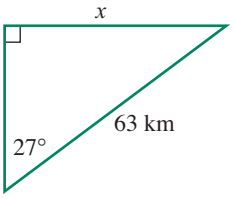

**e**



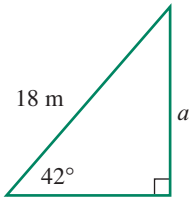

**f**



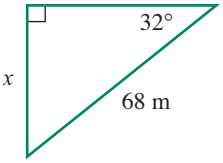

**g**




**h**

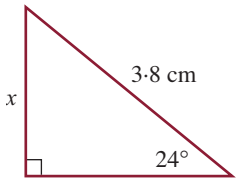



**i**

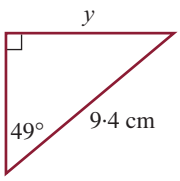



**2** Find the length of the side marked with a pronumeral. Using the three figure tables, express your answers to 2 decimal places:

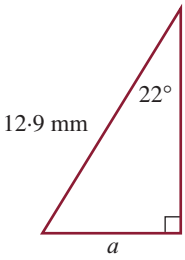
**a**



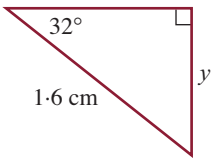

**b**



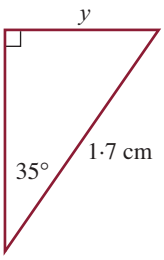

**c**



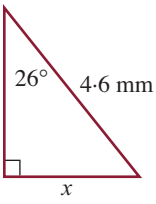

**d**



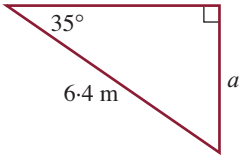

**e**



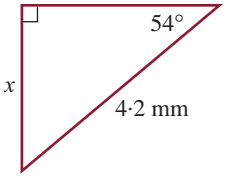

**f**



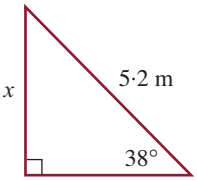

**g**



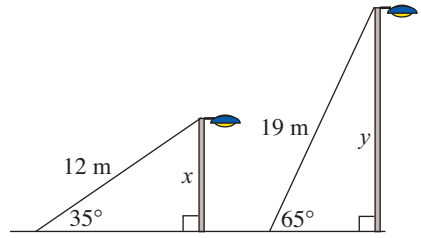

**h**




**i**

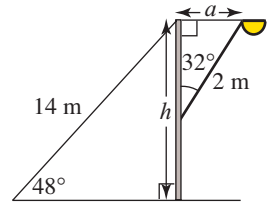


**3** Two street lights are mounted on poles. Both poles are kept vertical by wires, as shown in the diagram. One wire is 12 m long and makes an angle of 35° with the ground, the other wire is 19 m long and makes an angle of 65º with the ground. Find the height of each light pole.




**4** **a** A light is attached to a pole by a 2-metre-long strut which makes an angle of 32° with the pole. Find the horizontal distance of the light from the pole ($a$).

**b** The top of the pole has a 14-metre-long wire connecting it to the ground. If this wire makes an angle of 48° with the ground, find the height of the pole ($h$).

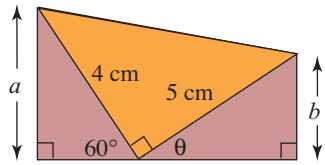



**5** This painting in the shape of a trapezium has some measurements marked on it. Find:

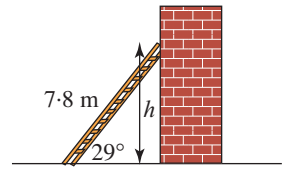
**a** height $a$

**b** the size of the angle marked θ

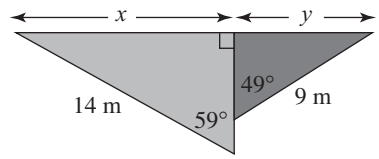
**c** height $b$



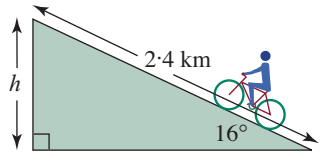

**6** A 7·8-metre-long ladder is placed against a wall. Find the height the ladder reaches up the wall if it makes an angle of 29° with the ground.



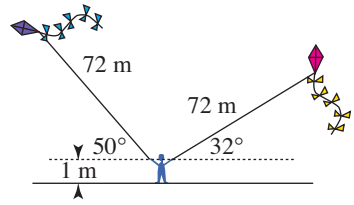

**7** These two sections of concrete are to be used as part of a bridge construction. Find the lengths marked $x$ and $y$ and so find the total length of the horizontal section.




**8** William rides up a hill for 2·4 kilometres. If the hill is angled at 16° to the horizontal find the height of the hill ($h$).

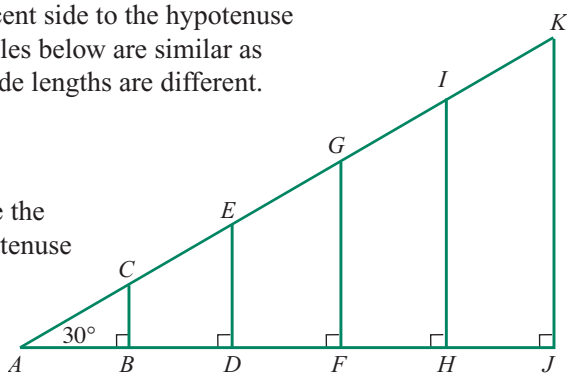



**9** Una flies two stunt kites at the same time, using a 72-metre-long line for each one. She holds each line 1 metre above the ground. If the angles to the horizontal are 50° and 32° respectively find the height of each kite above the ground.



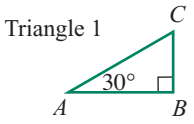
# Exploring the cosine ratio 8D

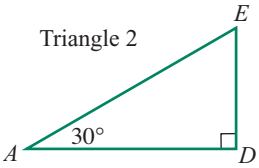
This section examines the ratio of the adjacent side to the hypotenuse in similar right-angled triangles. The triangles below are similar as corresponding angles are equal, but their side lengths are different.

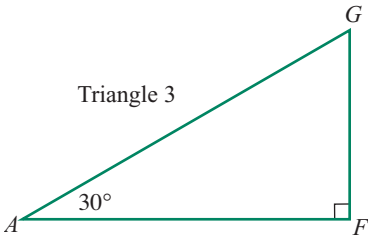


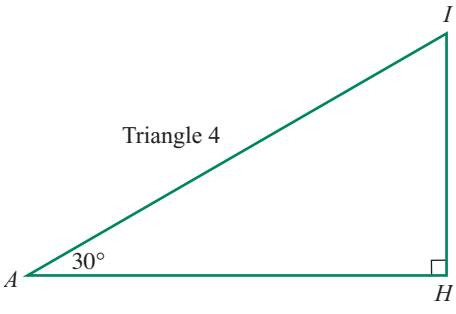

## Learning task 8D

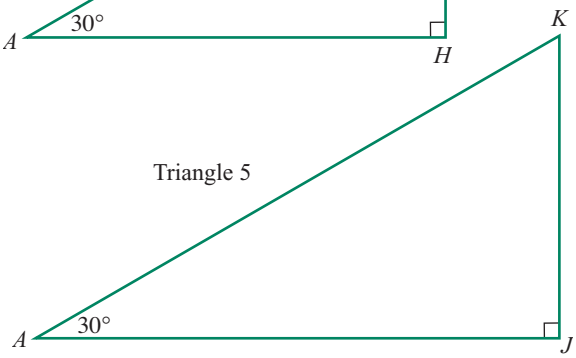
**1** For each triangle shown below, measure the length of the adjacent side and the hypotenuse to the nearest millimetre:




**2** Copy and complete the table below, then use three figure tables to find the ratio of the length of the adjacent side to the length of the hypotenuse, correct to 2 decimal places:

| Triangle | Length of adjacent side (mm) | Length of hypotenuse (mm) | Ratio: $\frac{\text{adjacent side}}{\text{hypotenuse}}$ |
|---|---|---|---|
| 1 | | | |
| 2 | | | |
| 3 | | | |
| 4 | | | |
| 5 | | | |

**3** What conclusion can be reached about the ratio of the length of the adjacent side to the length of the hypotenuse in each of the triangles?

**4** This ratio is called **cosine**.

$\cos 30° =$ _____

# 8E Finding the length of the adjacent side

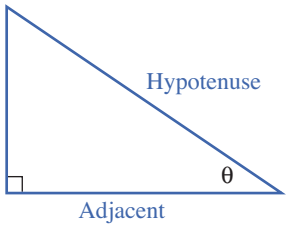
Trigonometry can be used to find the length of the sides of right-angled triangles.

The letters **SOH CAH TOA** (pronounced *soh car towa*) help us to get the order of the equation correct. The trigonometric relation called cosine uses the adjacent side to a given angle and the hypotenuse. It is abbreviated to $\cos\theta$, where $\theta$ is the angle in the triangle.

SOH CAH TOA

$$\text{Cosine}\,\theta = \frac{\text{Adjacent}}{\text{Hypotenuse}}$$

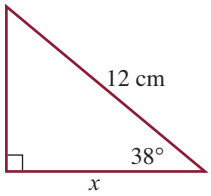
Cosine uses the adjacent side and the hypotenuse.



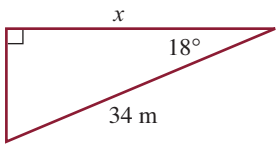

## Example

Use three figure tables to find the length of the side marked $x$ in the following diagrams:

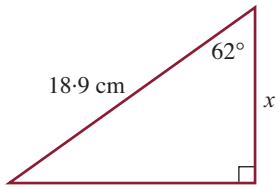
**a**




**b**




**c**




## Solution

SOH CAH TOA

$\cos\theta = \frac{A}{H}$

$\cos 38° = \frac{x}{12}$

$\therefore x = 12\cos 38°$

$x = 9{\cdot}46$ cm

$\cos\theta = \frac{A}{H}$

$\cos 18° = \frac{x}{34}$

$\therefore x = 34\cos 18°$

$x = 32{\cdot}34$ m

$\cos\theta = \frac{A}{H}$

$\cos 62° = \frac{x}{18{\cdot}9}$

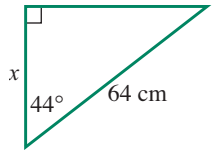
$\therefore x = 18{\cdot}9\cos 62°$

$x = 8{\cdot}87$ cm

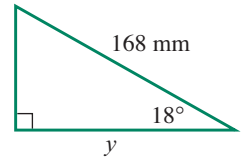
## Exercise 8E

**1** Find the length of the side marked with the pronumeral. Use three figure tables to express the answers correct to 2 decimal places:

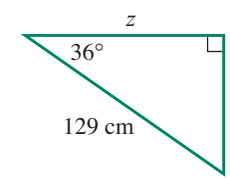
**a**



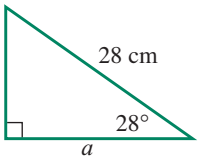

**b**



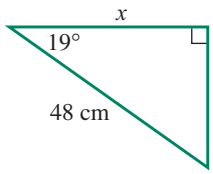

**c**



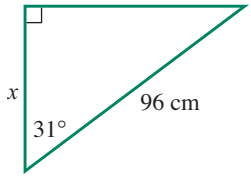
**d**



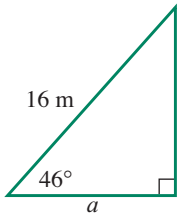

**e**



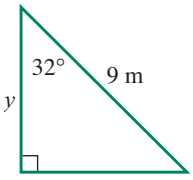

**f**



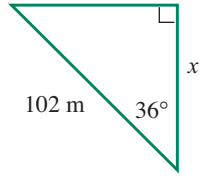

**g**




**h**

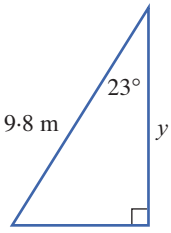



**i**

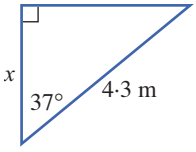



**2** Find the length of the side marked with the pronumeral. Use three figure tables to express the answers correct to 2 decimal places:

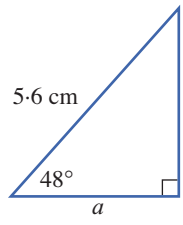
**a**



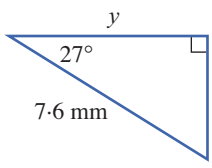

**b**



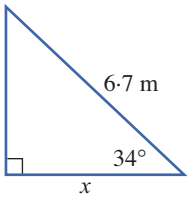

**c**



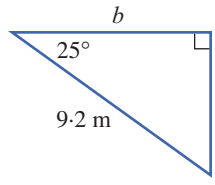

**d**



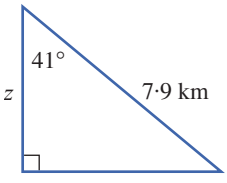

**e**



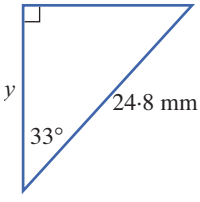

**f**



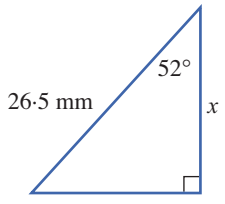

**g**



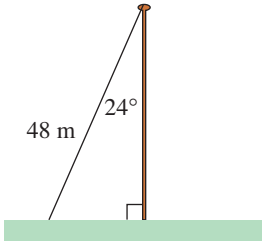

**h**




**i**

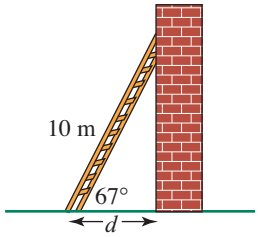



**3** A wire of length 48 metres is attached to the top of a pole. If the angle between the wire and the pole is 24°, find the height of the pole to the nearest metre.

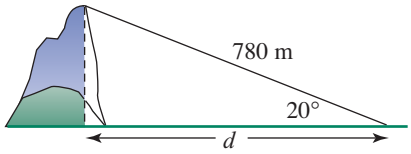


# 8E

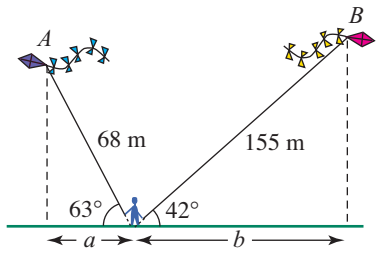
**4** A 10-metre-long ladder makes an angle of 67° with the ground. Find the distance from the foot of the ladder to the wall ($d$).




**5** Bushwalkers sight the top of a mountain. If the distance to the top of the mountain is 780 metres and the angle of elevation is 20°, find the distance ($d$) they need to walk to reach its base.



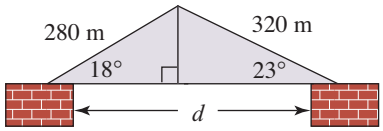

**6** Percy flies two stunt kites at once. Kite $A$ makes an angle of 63° with the ground, while kite $B$ makes an angle of 42° with the ground. Use the information on the diagram to find the horizontal distance from each kite to Percy ($a$ and $b$).



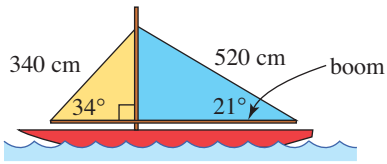

**7** Two triangular concrete sections are placed on top of brick piers and joined together.

**a** Find the base length of each triangular section.

**b** Find the maximum distance ($d$) that they can span if each section must sit on at least 10 metres of brick pier.




**8** This yacht has a yellow and a blue sail. Find the bottom length of each sail and so find the length of the boom.




**9** Three blocks are stacked together as shown. $A$ and $B$ are prisms with right-angled triangular ends.

Find:

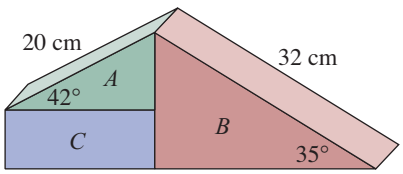
**a** the height of block $A$

**b** the height of block $B$

**c** the height of block $C$

**d** the base length of block $B$

**e** the base length of block $C$



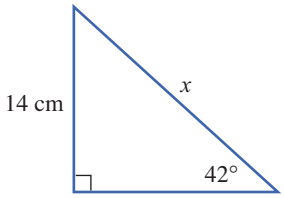
# Finding the hypotenuse 8F

When the hypotenuse is the unknown side, the equations for sine and cosine require an extra step in the solution. Good equation-solving skills are needed, as the unknown appears in the denominator of the equations. The same initial equations obtained from **SOH CAH TOA** apply. The most important decision to make is whether to use sine or cosine to answer the question.

## Example — Solution

Find the length of the side marked $x$ with the pronumeral.
Use three figure tables to express the answers correct to 2 decimal places:

**a**




SOH CAH TOA

$\sin\theta = \frac{O}{H}$

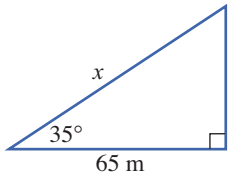
$\sin 42° = \frac{14}{x}$

$\therefore x\sin 42° = 14$

$\therefore x = \frac{14}{\sin 42} = \frac{14}{0{\cdot}669} = 20{\cdot}93$ using three figure tables or

$\therefore x = \frac{14}{\sin 42} = 20{\cdot}92$ cm using a calculator

**b**




SOH CAH TOA

$\cos\theta = \frac{A}{H}$

$\cos 35° = \frac{65}{x}$

$\therefore x\cos 35° = 65$

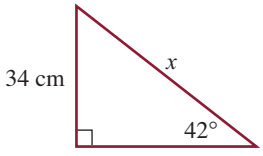
$\therefore x = \frac{65}{\cos 35} = \frac{65}{0{\cdot}819} = 79{\cdot}37$ using three figure tables or

$\therefore x = \frac{65}{\cos 35} = 79{\cdot}35$ m using a calculator

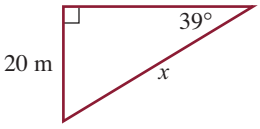
## Exercise 8F

**1** Find the length of the sides labelled $x$. Use three figure tables and express all answers correct to 2 decimal places:

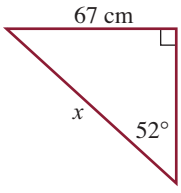
**a**



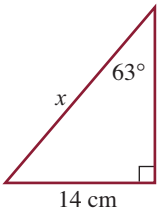

**b**



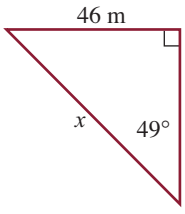

**c**



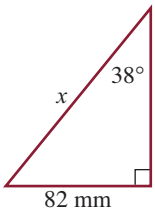

**d**




**e**



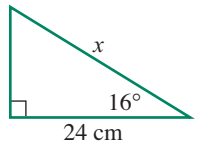

**f**



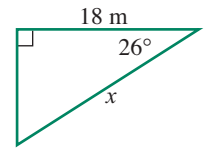
# 8F

**2** Find the length of the sides labelled $x$. Use three figure tables and express all answers correct to 2 decimal places:

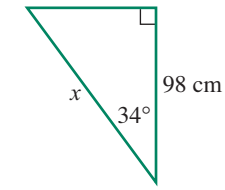
**a**



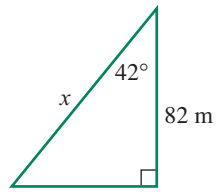

**b**



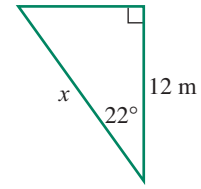

**c**



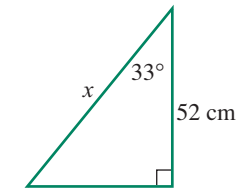

**d**



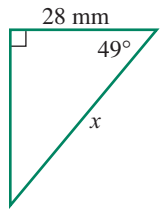

**e**



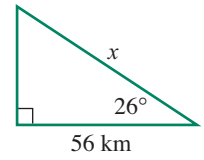

**f**



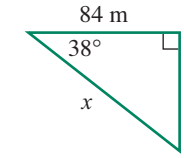

**g**




**h**

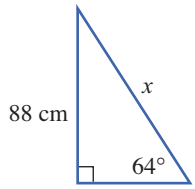



**i**

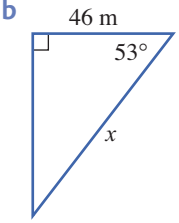



**3** Find the length of the sides marked $x$. You will have to use sine or cosine—the choice is yours. Using three figure tables, give your answers correct to 2 decimal places:

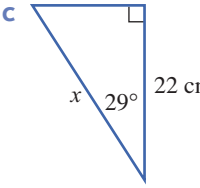
**a**



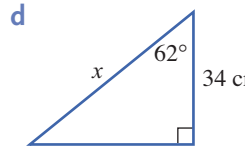

**b**



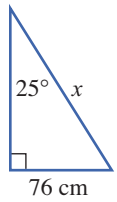

**c**



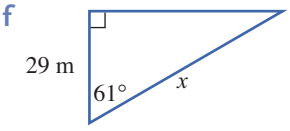

**d**



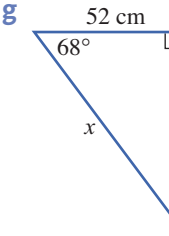

**e**



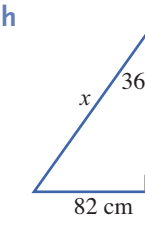

**f**



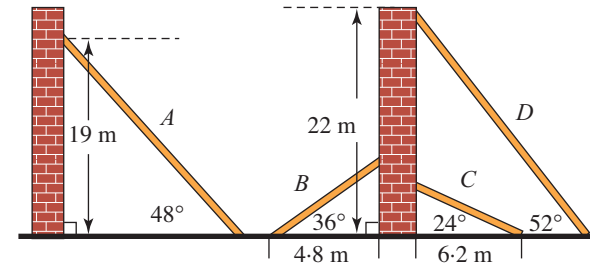

**g**



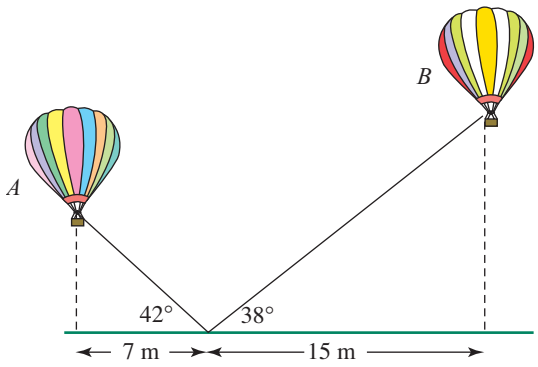

**h**



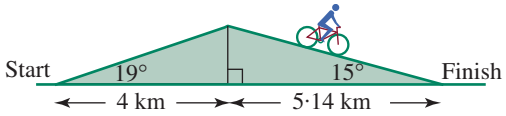

**4** Ladders are placed against walls. Use the information on the diagram to find the length of each ladder.



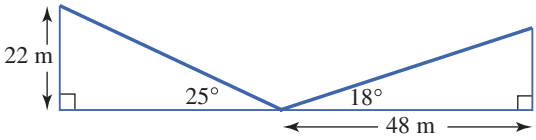
**5** Two hot-air balloons are tethered tightly to the ground by ropes as shown. Find the lengths of the ropes.



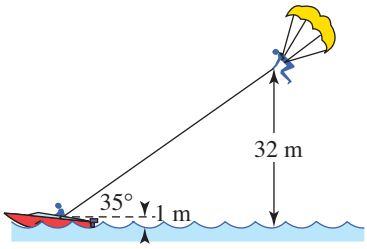

**6** A cross-country trail bike ride is planned so that competitors climb up one hill and then race down the other side, as shown here. Find the length of the race.



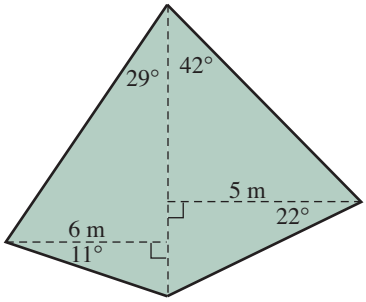

**7** This is a sectional sketch for the new Koloale drainage channel, which has been approved by the Honiara City Council for China Town. Find the total distance from the top of one side of the channel to the bottom of the channel and back up to the top of the other side.



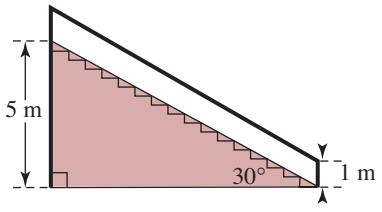

**8** The man in a parasuit is 32 metres above the sea. If his rope is attached to the boat at a point 1 metre above the waterline and it makes an angle of 35° to the horizontal, what is the length of rope?




**9** Find the perimeter of this shape to the nearest centimetre.




**10** The diagram is a design for a spectator stand for the Pacific Arts Festival. Calculate the total length of the handrail on the edge of the stand to the nearest centimetre.



# 8G Finding angles using sine and cosine

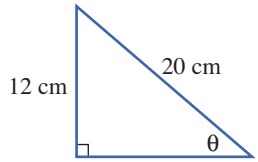
Trigonometry can be used to find angles in right-angled triangles when the side lengths are known. A calculator must be used to 'undo' the given trigonometric function, using the inverse function, so if $\sin\theta = a$, then $\theta = \sin^{-1} a$.

Alternatively, the sine or cosine three figure tables can also be used to look up the angle that corresponds to the decimal ratio value.

**Example**

Using the three figure table, find the angles in the following triangles correct to 2 decimal places:

**a**




**Solution**

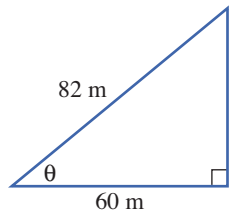
SOH CAH TOA

$\sin\theta = \frac{O}{H} = \frac{12}{20}$

$\theta = \sin^{-1}(0{\cdot}6)$

$\therefore \theta = 36{\cdot}87°$

**b**




SOH CAH TOA

$\cos\theta = \frac{A}{H} = \frac{60}{82}$

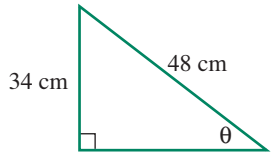
$\theta = \cos^{-1}\left(\frac{60}{82}\right)$

$\therefore \theta = 42{\cdot}97°$

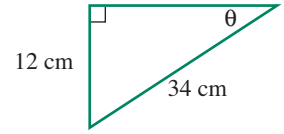
## Exercise 8G

**1** Use a calculator or the three figure tables to find the unknown angles correct to 2 decimal places:

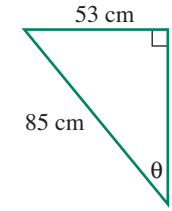
**a**



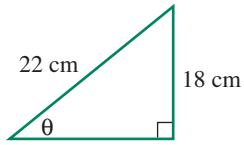

**b**



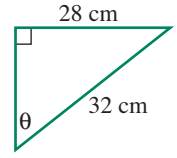

**c**



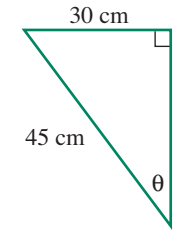

**d**



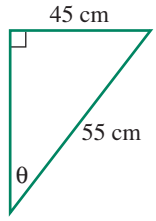

**e**



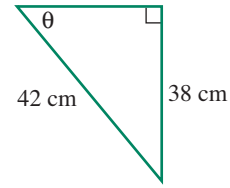

**f**



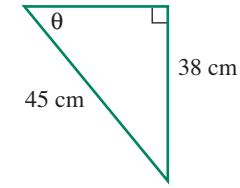

**g**




**h**

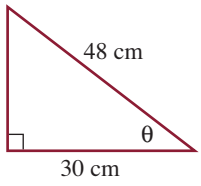



**i**

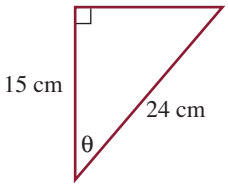


**2** Find the unknown angles correct to 2 decimal places:

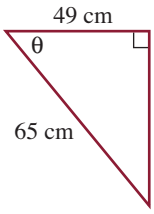
**a**



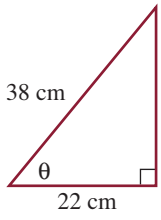

**b**



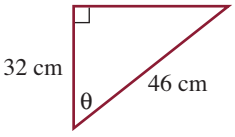

**c**



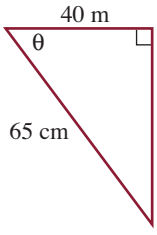

**d**



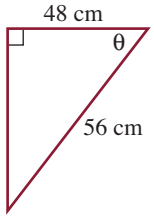

**e**



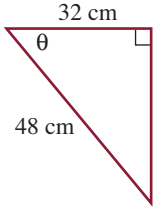

**f**



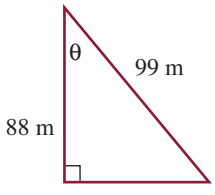

**g**




**h**

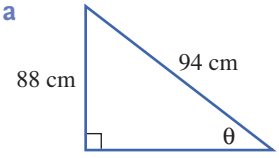



**i**

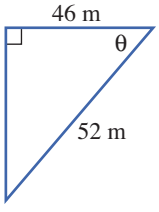



**3** Using the sine and cosine functions, find the unknown angles correct to 2 decimal places:

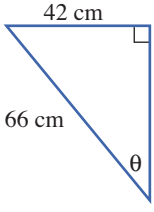
**a**



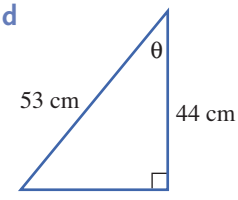

**b**



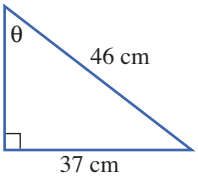

**c**



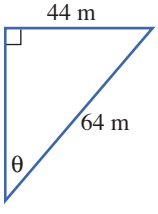

**d**



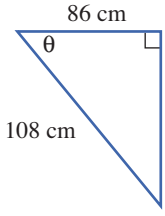

**e**



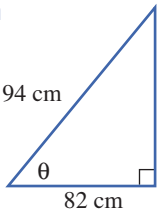

**f**



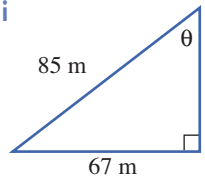

**g**



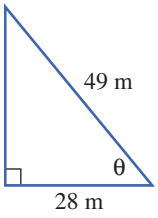

**h**



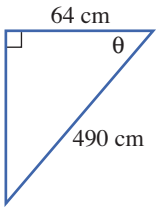

**i**



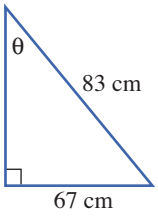

**j**



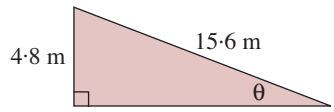

**k**




**l**

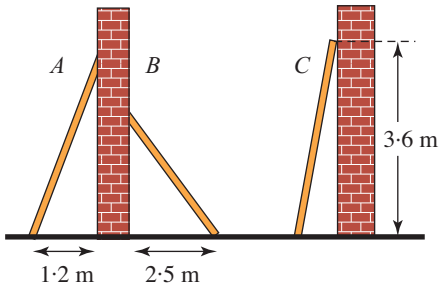


**4** An escalator which is 15·6 metres long rises 4·8 metres. Find the angle that it makes with the horizontal.

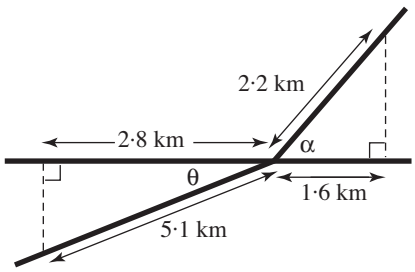



**5** Three ladders each 4·8 metres long are leant against walls as shown. For each ladder find the angle that it makes:

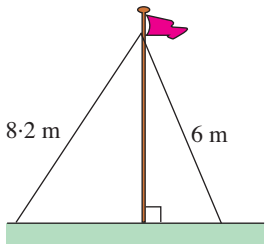
**a** with the ground **b** with the wall




**6** This road map shows an intersection. Use the information in the diagram to find the angles θ and α to the nearest degree.




**7** A flagpole 6·8 m tall is supported by two wires attached to a point 1·5 metres below the top of the pole. If the wires are 8·2 m and 6 m long, find the angle that each wire makes with the ground.

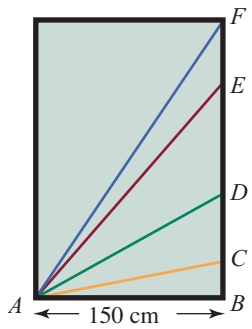



**8** There are four lines in this abstract painting. The lengths of the lines are given:

$AC = 165$ cm, $AD = 190$ cm, $AE = 220$ cm, $AF = 240$ cm

**a** Find the angle that each line makes with the base of the painting (i.e. with line $AB$).

**b** Find the angle that each line makes with the right edge of the painting (i.e. with line $BF$).



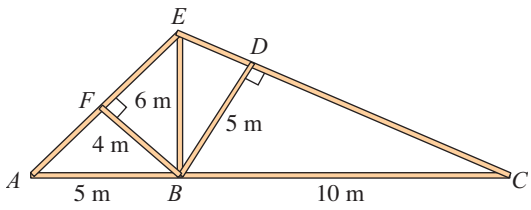

**9** Find the angles indicated on this building truss:

**a** $\angle BAF$ **b** $\angle ABF$

**c** $\angle FEB$ **d** $\angle EBF$

**e** $\angle BED$ **f** $\angle DBE$

**g** $\angle DBC$ **h** $\angle DCB$



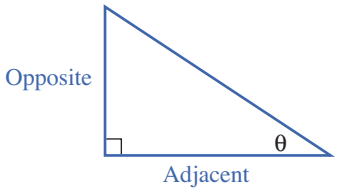
# Using the tangent ratio 8H

As we have seen, trigonometry can be used to find the length of the sides and the size of the angles inside right-angled triangles.

The letters **SOH CAH TOA** (pronounced *soh car towa*) help us to get the order of the equation correct. The side opposite a given angle and the side adjacent to it use the trigonometric relation called tangent.

SOH CAH TOA

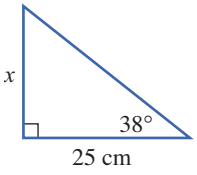
$\text{Tangent}\,\theta = \dfrac{\text{Opposite}}{\text{Adjacent}}$



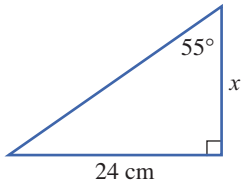

Tangent uses the opposite and adjacent sides. The solution process is the same as for the calculations for sine and cosine.

## Example

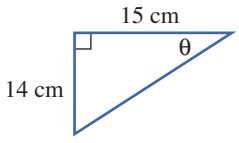
**1** Using three figure tables find the opposite sides:




**2** Find the adjacent side using three figure tables:




**3** Use three figure tables to find the angle θ:




## Solution

SOH CAH TOA

$\tan\theta = \dfrac{O}{A}$

$\tan 38° = \dfrac{x}{25}$

$\therefore x = 25\tan 38°$

$x = 19{\cdot}53 \text{ cm}$

$\tan\theta = \dfrac{O}{A}$

$\tan 55° = \dfrac{24}{x}$

$\therefore x = \dfrac{24}{\tan 55°}$

$x = 16{\cdot}80 \text{ cm}$

$\tan\theta = \dfrac{O}{A}$

$\tan\theta = \dfrac{14}{15}$

$\therefore \theta = \tan^{-1}\left(\dfrac{14}{15}\right)$

$\theta = 43°$

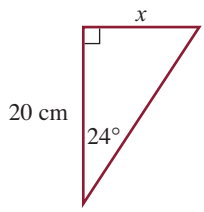
correct to the nearest degree

# 8H

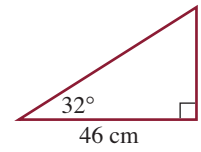
## Exercise 8H

**1** Find the length of the sides marked $x$ in these triangles. Using the three figure tables, give your answers correct to 2 decimal places:

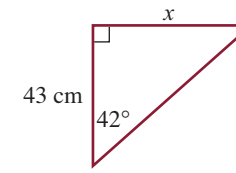
**a**



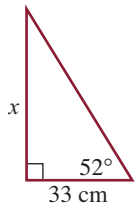

**b**



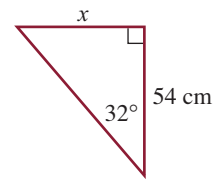

**c**



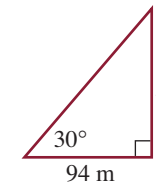

**d**



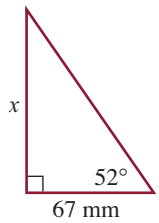

**e**



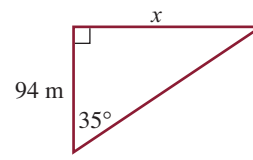

**f**



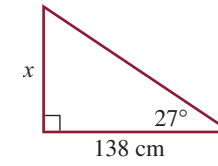

**g**




**h**

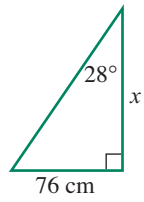



**i**

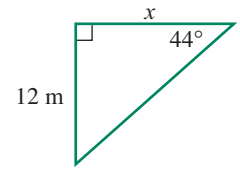



**2** Find the length of the sides marked $x$ in these triangles, correct to 2 decimal places:

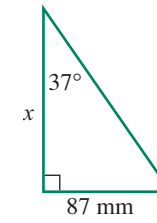
**a**



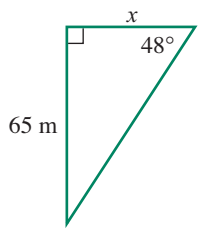

**b**



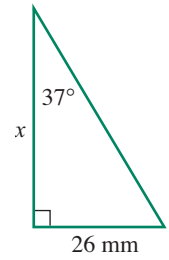

**c**



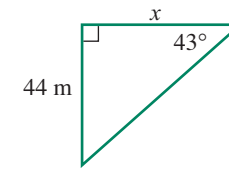

**d**



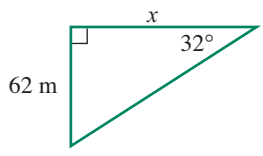

**e**



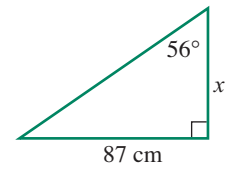

**f**



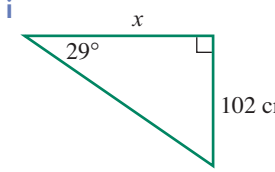

**g**




**h**

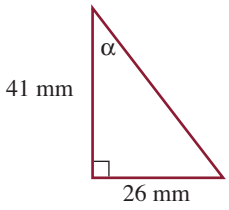



**i**

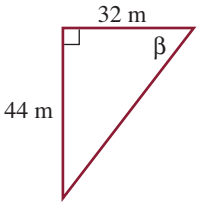


**3** Find the size of the marked angles in degrees, correct to 2 decimal places:

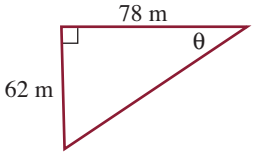
**a**



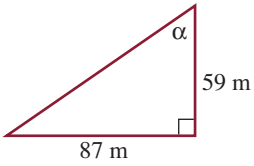

**b**



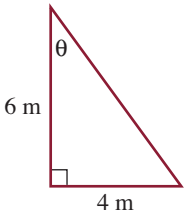

**c**



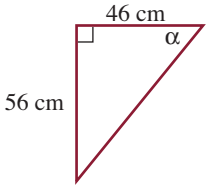

**d**



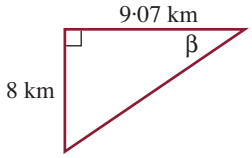

**e**



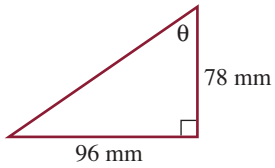

**f**



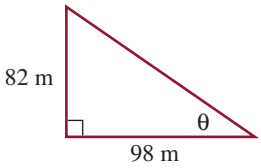

**g**



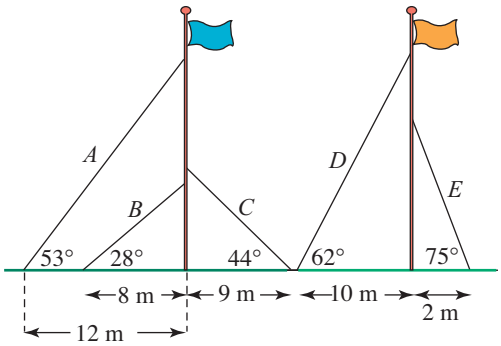

**h**



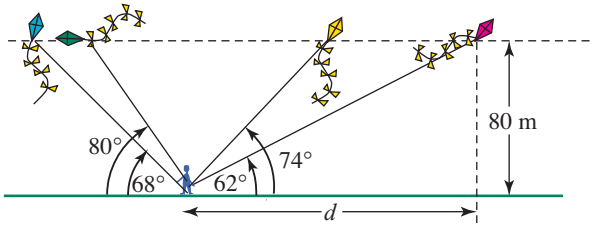

**i**




**4** Wires are attached to two flagpoles as shown. Using the information on the diagram, find the height from the bottom of the pole to where the wires *A*, *B*, *C*, *D* and *E* are attached.



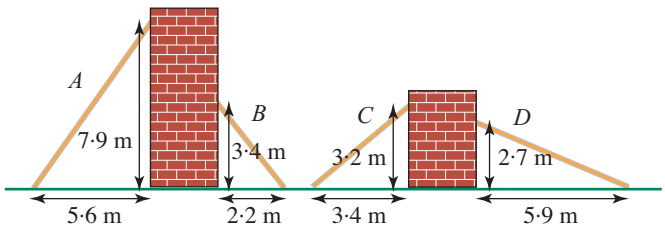

**5** Luma's four stunt kites are all 80 metres above the ground. Assuming he holds the strings 1 m above the ground, find the horizontal distance from Luma to each kite. The horizontal distance from Luma to the red kite is marked *d*.




**6** Four ladders lean against the walls as shown. Find the angle, to the nearest degree, that each ladder makes with:

**a** the ground

**b** the wall

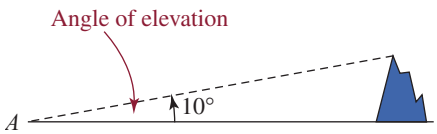


# 8I Angles of elevation and depression

When objects are sighted over a long distance, the angle used can be that of elevation or depression.

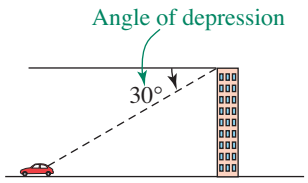
When an observer has to look up to observe an object, then the angle measured from the horizontal is called the **angle of elevation**.

This angle of elevation is 10°.

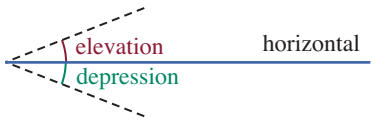



When an observer has to look down to observe an object, then the angle measured from the horizontal is called the **angle of depression**.

This angle of depression is 30°.

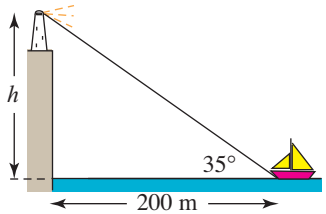



Angles of elevation and depression are measured from the horizontal.



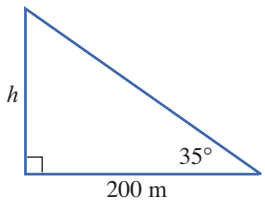

## Example

**1** The angle of elevation to the top of a lighthouse from a boat that is positioned 200 m from the base of a cliff is 35°. Find the height above sea level of the top of the lighthouse.




## Solution

Draw the triangle and mark the information on it.



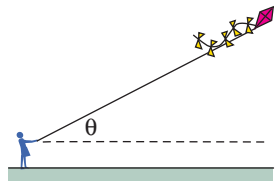

$\tan 35° = \frac{h}{200}$

$h = 200 \tan 35°$

$= 140$

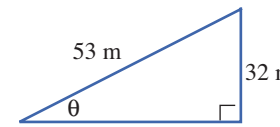
The top of the lighthouse is 140 metres above sea level.

**2** A girl flies a kite on a 53-metre-long string. The kite is 33 m above the ground. Find the angle of elevation, θ, of the kite, to the nearest degree, assuming the girl holds the string 1 m above the ground.




Draw the triangle and mark the information on it.

$33 - 1 = 32$



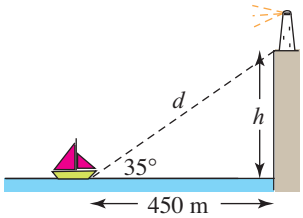

$\sin\theta = \frac{32}{53}$

$\theta = \sin^{-1}\left(\frac{32}{53}\right)$

$= 37°$

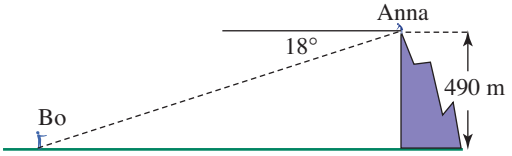
## Exercise 8I

**1** A sailor on a boat 450 metres from the base of a cliff sights the top of the cliff. If the angle of elevation is 35° from the surface of the ocean to the top of the cliff, find:

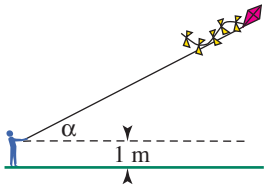



- **a** the height of the cliff ($h$)
- **b** the straight-line distance from the boat to the top of the cliff ($d$)

**2** The angle of depression from Anna on the top of a cliff to Bo her brother, who is standing on the ground, is 18°. If the height of the cliff is 490 m, find the distance from Bo to the foot of the cliff.

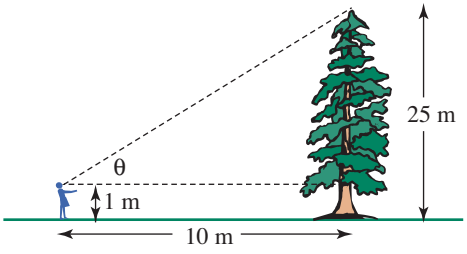



**3** A kite is flown on a string that is 230 metres long. What angle of elevation ($\alpha$) does the string make with the horizontal when the height of the kite above the ground is:

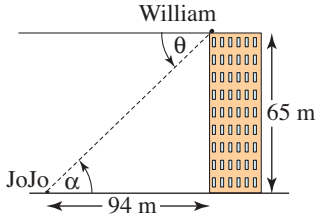



- **a** 20 m?
- **b** 40 m?
- **c** 60 m?
- **d** 100 m?
- **e** 180 m?

**4** Lucinda sights the top of a tree that is 25 m tall. If she is standing 10 metres from the base of the tree and her eyes are 1 metre above the ground, find the angle of elevation ($\theta$).

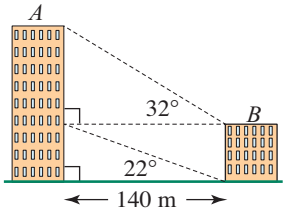



**5** William is positioned on the top of a 65-metre-tall building and his dog JoJo is standing 94 m from the base of the building.




- **a** Find the angle of elevation from JoJo to William.
- **b** Find the angle of depression to JoJo from William.

**6** Two angles of elevation to building $A$ were taken from positions on building $B$ as shown.




- **a** Find the height of building $A$.
- **b** Find the angle of depression from the top of building A to the bottom of building $B$.
- **c** Find the angle of elevation of the bottom of building $B$ to the top of building $A$.

# 8J Trigonometry and bearings

**Bearings** are angle measurements which are used to track the direction of objects for navigation purposes. When working with bearings, always draw a diagram with the centre of the compass on the point from where the bearing is given.

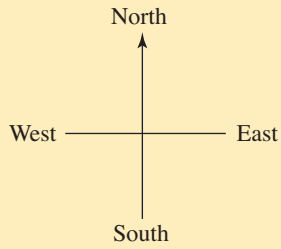
There are two types of bearings: magnetic or conventional bearings and true bearings.

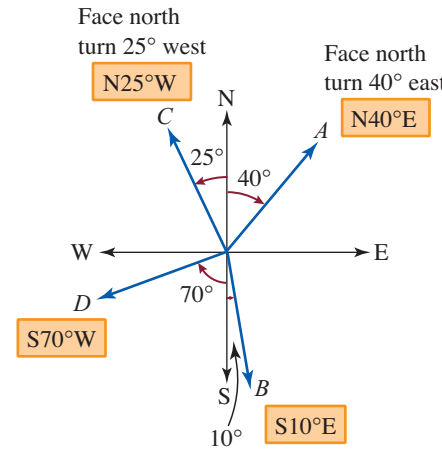
### Conventional bearings

North originally was magnetic north and the other directions are drawn relative to it.

Two directions and an angle are used to specify a bearing. The first direction is the way in which you need to face. The angle is the amount that you need to turn in the second direction.

Conventional bearings usually start from north or south, then give the amount of turn to the east or west.

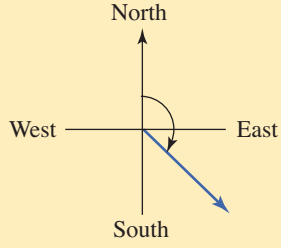


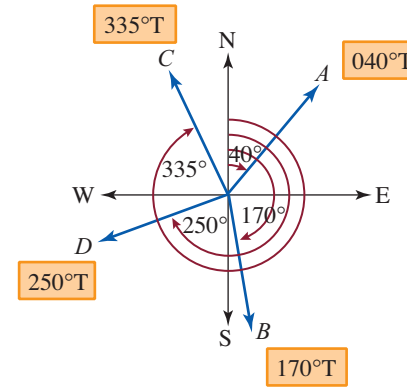




### True bearings

These bearings are measured clockwise from north.

Note that true bearings all have three digits. This is to ensure bearings such as 020°T and 200°T are never confused in radio communications.




## Example

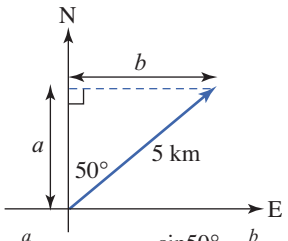
**1** A bushwalker set out on the bearing N50°E. If he travels 5 km on this bearing find the distance that he has travelled:

**a** north

**b** east

## Solution

Mark the bearing and the distance travelled on a diagram.




$\cos 50° = \frac{a}{5}$ $\quad$ $\sin 50° = \frac{b}{5}$

$a = 5\cos 50°$ $\quad$ $b = 5\sin 50°$

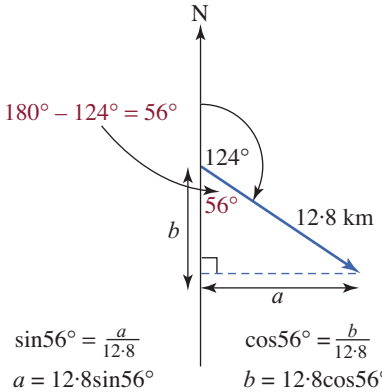
$a = 3{\cdot}21$ km north $\quad$ $b = 3{\cdot}83$ km east

**2** A yacht tacks on the bearing 124°T and travels for 12·8 km. Show this on a diagram and find the distance it has travelled:

**a** east

**b** south

Draw the bearing and distance travelled on the diagram.

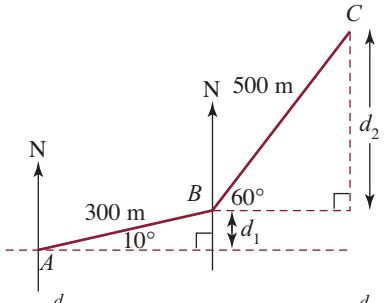



$\sin 56° = \frac{a}{12{\cdot}8}$ $\quad$ $\cos 56° = \frac{b}{12{\cdot}8}$

$a = 12{\cdot}8\sin 56°$ $\quad$ $b = 12{\cdot}8\cos 56°$

$a = 10{\cdot}6$ km east $\quad$ $b = 7{\cdot}2$ km south

**3** Rob travels 300 m from $A$ on bearing 080°T to $B$, then 500 m on bearing 030°T to finish at $C$. Find the total distance, to the nearest metre, he travels north.




$\sin 10° = \frac{d_1}{300}$ $\quad$ $\sin 60° = \frac{d_2}{500}$

$d_1 = 300\sin 10°$ $\quad$ $d_2 = 500\sin 60°$

$d_1 = 52$ m $\quad$ $d_2 = 433$ m

Total distance $= d_1 + d_2 = 52 + 433 = 485$ m

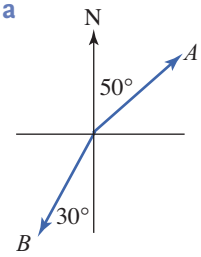
Total distance north is 485 m.

# 8J

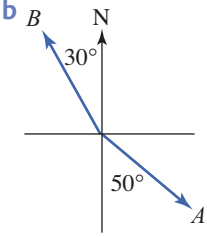
## Exercise 8J

**1** Write the magnetic and true bearings shown in the following diagrams:

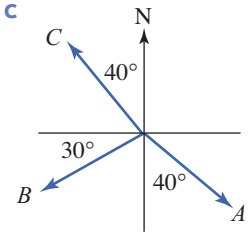
**a**



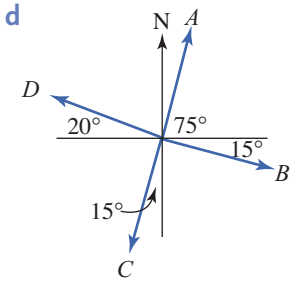

**b**




**c**




**d**




**2** Draw diagrams showing the following bearings:

**a** **i** N45°E **ii** S30°W **b** **i** S70°W **ii** N55°E **c** **i** 130°T **ii** 240°T **d** **i** 250°T **ii** 335°T

**3** Give the true bearings of the following magnetic bearings:

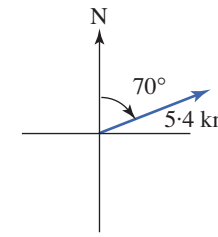
**a** S20°E **b** N30°W **c** S40°E **d** N80°W **e** S25°W **f** S32°E

**4** Give the magnetic bearings of the following true bearings:

**a** 047°T **b** 095°T **c** 126°T **d** 225°T **e** 325°T **f** 231°T

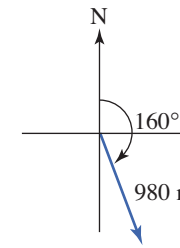
**5** An aeroplane flies for 5·4 km on the bearing 070°T.
Use the diagram to find the distance that it has travelled:

**a** north **b** east



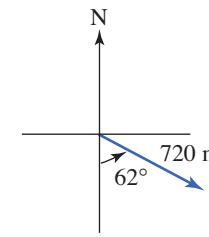

**6** A bushwalker walks on the bearing 160°T for 980 m.
Use the diagram to find the distance that he has travelled:

**a** south **b** east

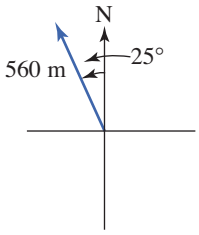



**7** A canoe is paddled for 720 metres on the bearing S62°E.
Use the diagram to find the distance that it has travelled:

**a** south **b** east




**8** A rally car is driven for 560 metres on the bearing N25°W. Use the diagram to find the distance that it has travelled:




**a** north **b** west

**9** Draw diagrams for the following questions, clearly showing all calculations.

**a** A boat travels 20 km from port on the bearing 115°T. Find the distance from port that it travelled:

**i** south **ii** east

**b** Ha'apio walks on the bearing N70°W for 800 metres. Find the distance that he has walked:

**i** west **ii** north

**c** The position of Honiara from Auki is 114 km on the bearing of 220°T. Find how far Honiara is from Auki in the following directions:

**i** south **ii** west

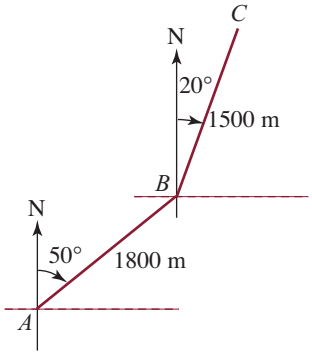
**d** A bird flies on the bearing S28°W for 1500 metres. Find the distance from its starting point that the bird has travelled:

**i** south **ii** west

**e** A ship travelled 40 km from port on the bearing of 050°T. Find the distance from port that the ship has travelled:

**i** north **ii** east

**10** Fred starts from point $A$ and travels through point $B$ to point $C$, as shown on the diagram. How far south and then west will he need to travel to get back to the start at point $A$?



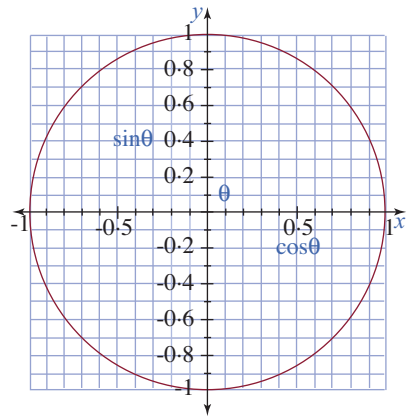

**11** Draw a diagram and then find the total distance travelled east when Rodrick walks 900 m on bearing S70°E, and then 100 m on bearing N80°E.

# 8K Exploring the unit circle

A unit circle has a radius of 1 unit, and can be used to find the trigonometric ratios.

The value of $\sin\theta$ can be read off on the $y$-axis.

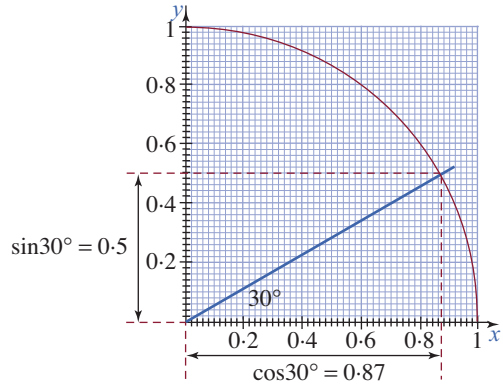
The value of $\cos\theta$ can be read off on the $x$-axis.




**Example**

Use this part of the unit circle to read off the values of $\sin 30°$ and $\cos 30°$ and check your answers using a calculator or three figure tables.

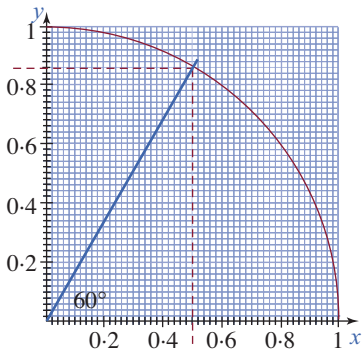
**Solution**



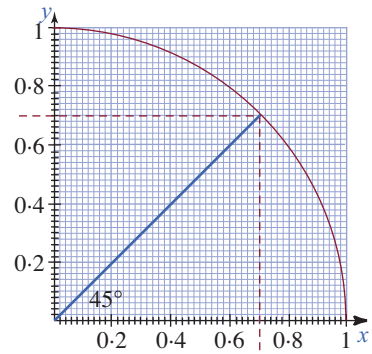

$\sin 30° = 0{\cdot}5$
$\cos 30° = 0{\cdot}87$

## Learning task 8K

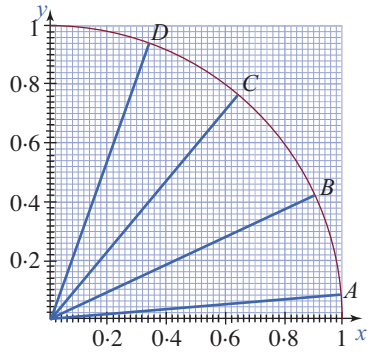
**1** Use this part of the unit circle to read off the values of $\sin 60°$ and $\cos 60°$.




**2** Use this part of the unit circle to read off the values of $\sin 45°$ and $\cos 45°$.



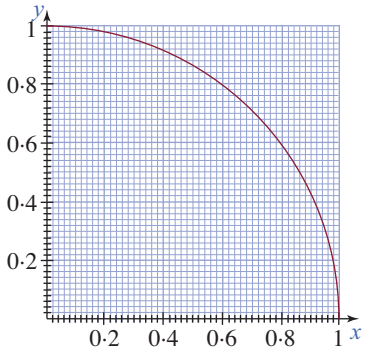
**3** Measure the angles and determine their sine and cosine values. Check the values with a calculator or three figure table.




**4** Draw the following angles on the diagram and use it to find their sine and cosine values correct to 2 decimal places:

**a** 20° **b** 40° **c** 55°

**d** 75° **e** 80°

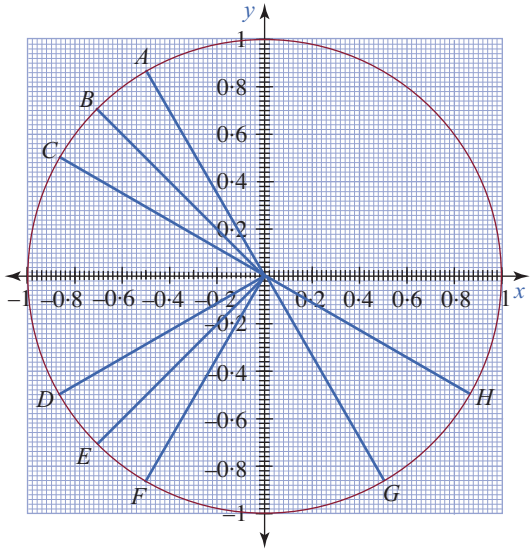



**5** Draw a diagram of the first quadrant of a unit circle and use it to estimate to the nearest degree the angles with the following sine and cosine values:

0·1, 0·2, 0·3, 0·4, 0·5, 0·6, 0·7, 0·8, 0·9

## Does it work for angles greater than 90°?

**6** The sine and cosine of all angles can be read off the unit circle. Read the values of sine and cosine for the angles using this circle. Copy and complete the table to record your answers. Check your answers with your calculator.




| Angle | Sine (*y*-axis) | Cosine (*x*-axis) |
|---|---|---|
| *A*: 120° | | |
| *B*: 135° | | |
| *C*: 150° | | |
| *D*: 210° | | |
| *E*: 225° | | |
| *F*: 240° | | |
| *G*: 300° | | |
| *H*: 330° | | |

# 8L Exploring trigonometric graphs

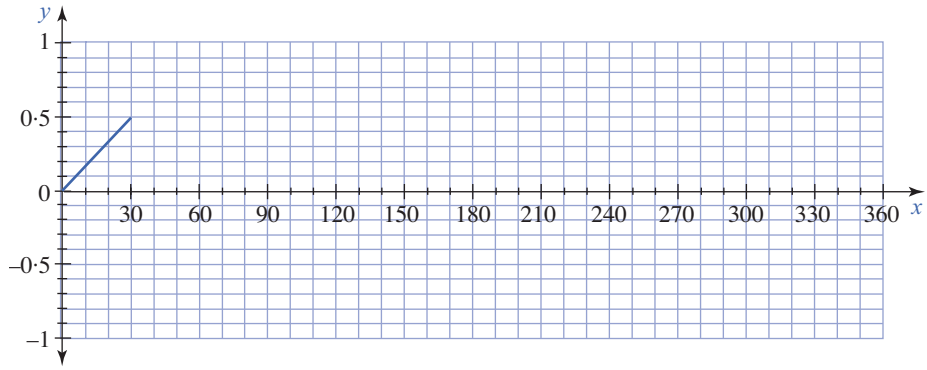
## Learning task 8L

### Plotting the graph of $y = \sin x$

**1 a** Complete the table of sine values by using a three figure table. Give the values correct to 2 decimal places.

| $x$ | 0° | 15° | 30° | 45° | 60° | 75° | 90° | 105° | 120° | 135° | 150° | 165° | 180° |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $y = \sin x$ | 0 | 0·26 | 0·5 | | | | | | | | | | |
| $x$ | 195° | 210° | 225° | 240° | 255° | 270° | 285° | 300° | 315° | 330° | 345° | 360° | |
| $y = \sin x$ | | | | | | | | | | | | | |

**b** Copy and complete the graph below using the points in the table. Join the points to make a smooth curve.

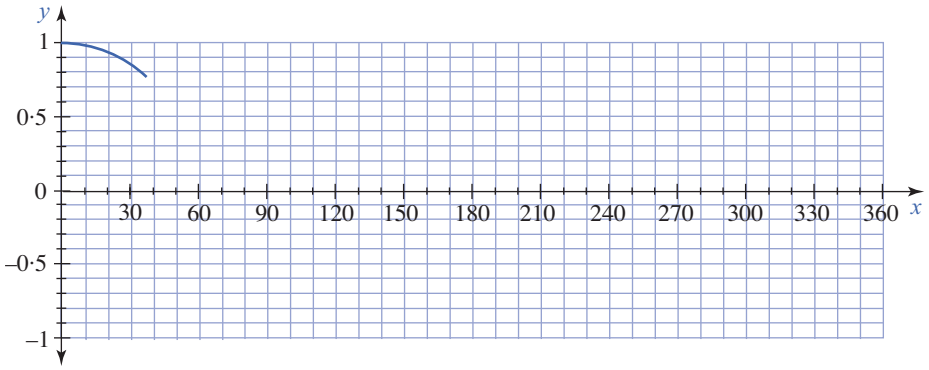



### Plotting the graph of $y = \cos x$

**2 a** Complete the table of cosine values by using a three figure table. Give the values correct to 2 decimal places.

| $x$ | 0° | 15° | 30° | 45° | 60° | 75° | 90° | 105° | 120° | 135° | 150° | 165° | 180° |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $y = \cos x$ | 1 | 0·97 | 0·87 | | | | | | | | | | |
| $x$ | 195° | 210° | 225° | 240° | 255° | 270° | 285° | 300° | 315° | 330° | 345° | 360° | |
| $y = \cos x$ | | | | | | | | | | | | | |

**b** Copy and complete the graph below by using the points in the table. Join the points to make a smooth curve.



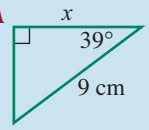

**c** State the similarities between the graphs $y = \sin x$ and $y = \cos x$.

# Puzzles

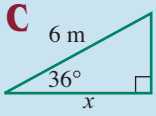
**1** Find the length of the sides marked $x$ in each of the following triangles and round to the nearest whole number. Match the letter to the correct answer below to solve the riddle:

**What keeps a square from moving?**

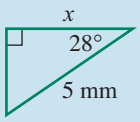
**A**



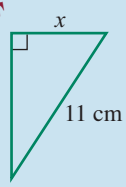

**C**



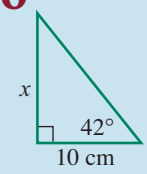

**E**



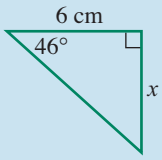

**F**



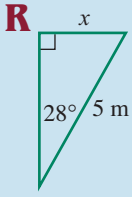

**O**



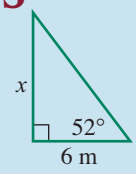

**Q**



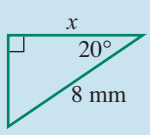

**R**



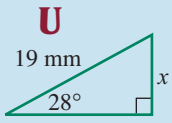

**S**




**T**




**U**

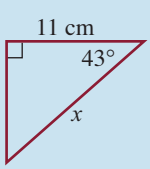



| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 8 m | 6 cm | 9 mm | 7 cm | 2 m | 4 mm | | 2 m | 9 cm | 9 cm | 8 mm |
| 9 cm | 4 cm | | 5 m | 9 cm | 9 mm | 2 m | 8 m | 4 mm | | |

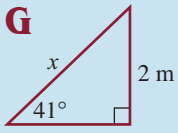
**2** Find the length of the hypotenuse in each of the following triangles and round to the nearest whole number. Match the letter to the correct answer below to solve the riddle:

**What do you call a teapot of boiling water on top of Mount Everest?**

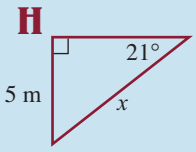
**E**



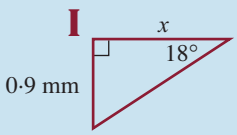

**G**



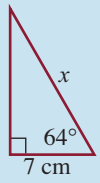

**H**



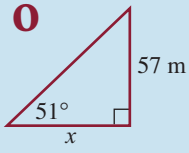

**I**



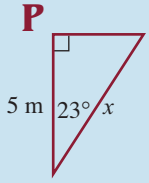

**N**



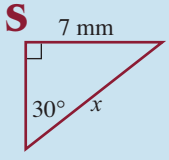

**O**



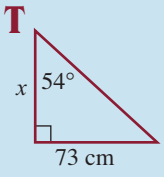

**P**



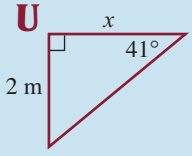

**S**




**T**



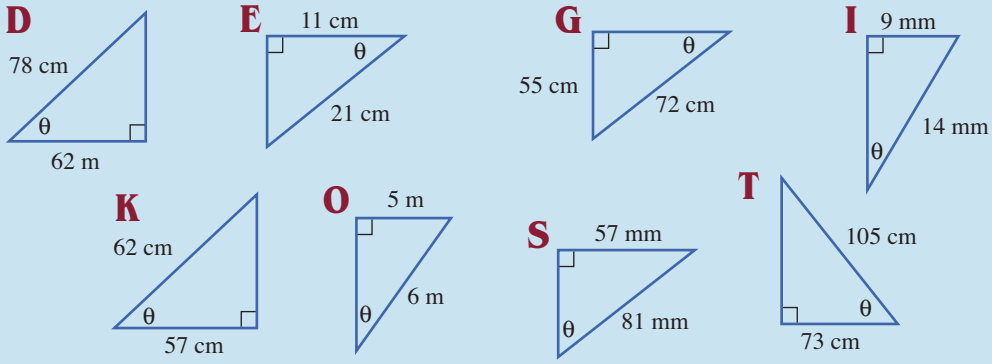

**U**




| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 14 m | 3 mm | 3 m | 14 m | | 5 m | 46 m | 53 cm | | 3 mm | 16 cm |
| 2 m | 14 mm | 15 cm | | | | | | | | |

**3** Find the angle θ to the nearest degree in each of the triangles, and then match the letter to the correct answer below to solve the riddle:

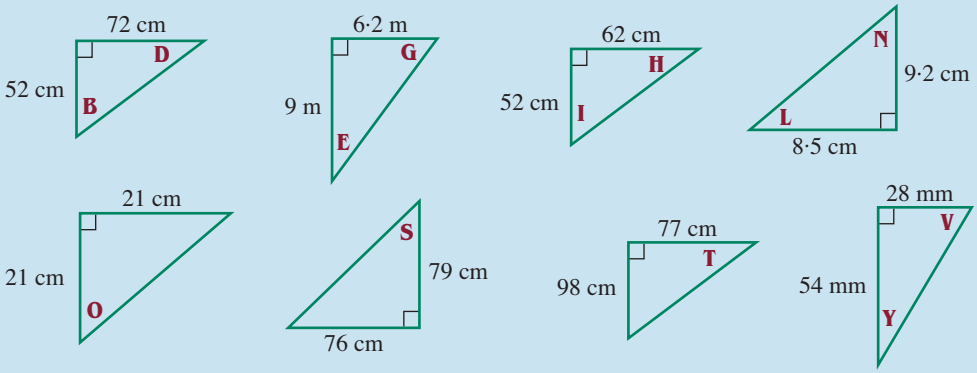
**What do you give a dog that loves computers?**




| | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | | | |
| 37° | 56° | 50° | | 37° | 40° | 45° | 23° | 58° | 46° | 46° | 58° | 45° | | |

**4** Find the missing angles to the nearest degree in each of the triangles. Match the letter to the correct answer below to solve the riddle:

**What are dyslexic atheists?**

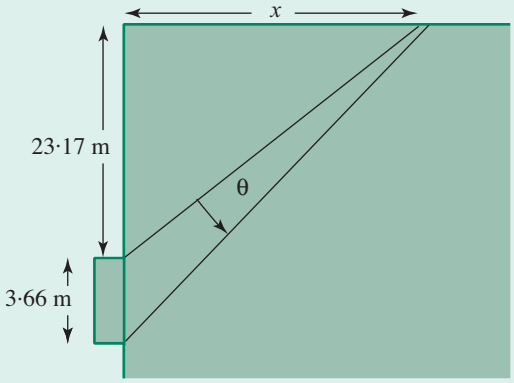


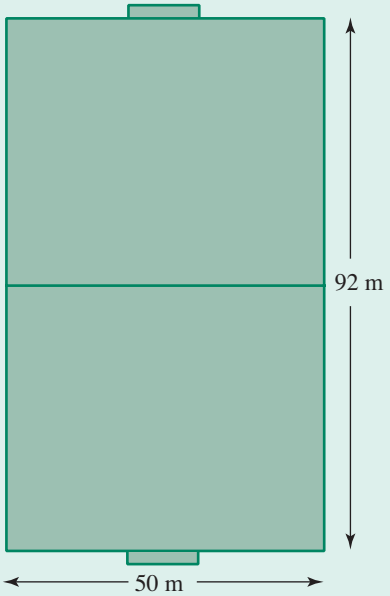

| | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | | | |
| 52° | 40° | 35° | 27° | | 36° | 45° | 43° | 52° | | | | | | |
| | | | | | | | | | | | | | | |
| 54° | 35° | 47° | 50° | 35° | 63° | 35° | | 50° | 43° | | | | | |
| | | | | | | | | | | | | | | |
| 36° | 45° | 55° | 44° | | | | | | | | | | | |

# Applications

## Shooting for goal

**a** A hockey field is a rectangle 50 m wide and 92 m long. The goals are 3·66 m wide. As a player moves back along the side line the angle (θ) gets bigger.

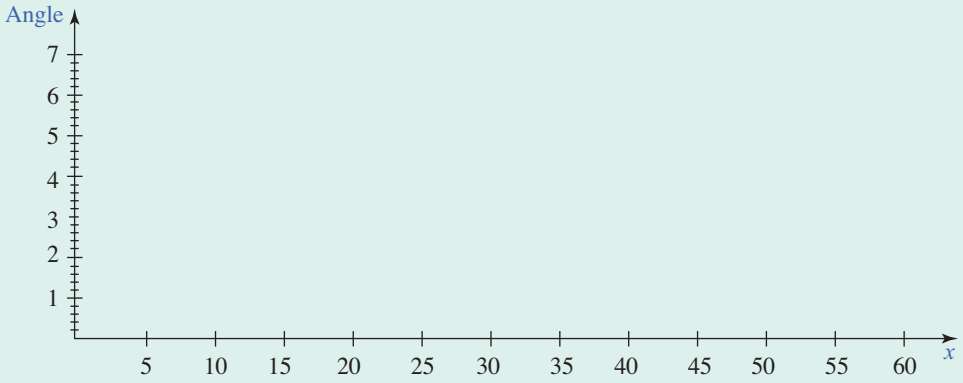



Using trigonometry, find the angle θ to the nearest tenth of a degree for the following values of $x$. Copy and complete the table.

| **Distance $x$** | 5 m | 10 m | 15 m | 20 m | 25 m | 30 m |
|---|---|---|---|---|---|---|
| **Angle θ** | | | | | | |

Copy the axes below to plot your results.

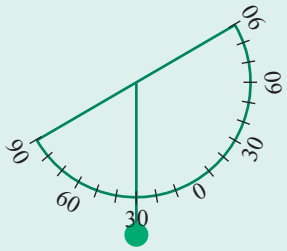


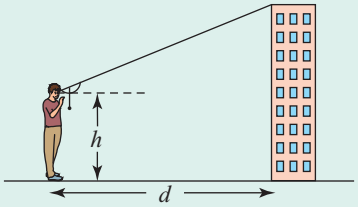

From the graph and by further calculation, find the distance $x$ that results in the largest goal angle. Give your answer correct to 1 decimal place.

**b** Repeat the above investigation to find the best position to make the largest goal angle for soccer. A soccer pitch is a rectangle 100 m long by 64 m wide. The goals are 7·32 m apart.

## Inaccessible heights

An inclinometer can be made by attaching a piece of string with a weight (plumb line) to the centre of a large protractor. The angles are numbered from zero from the plumb line as shown below. The angle of elevation is found by reading off the angle shown by the plumb line.

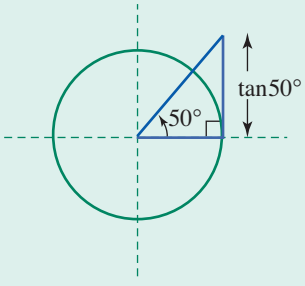



Use an inclinometer to sight the top of an object such as a tree or tall building. Measure the angle of elevation and the distance to the base of the object ($d$), as well as the height of the inclinometer above the ground ($h$). Use this information to find the height of your chosen object.

## Measuring the tangent of an angle

Draw a circle with a radius of 1 cm. Construct a right-angled triangle from the centre of the circle. Measure the height of the triangle to the nearest mm. This length is a measure of the tangent of the angle. Copy and complete the table below to record your results, and then check your measurements using a three figure table.

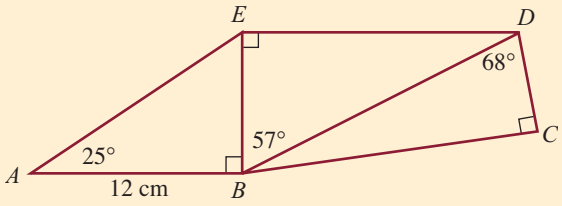



| Triangle | Angle | Height | Tangent of the angle |
|---|---|---|---|
| 1 | 45° | | |
| 2 | 50° | | |
| 3 | 55° | | |
| 4 | 60° | | |
| 5 | 65° | | |
| 6 | 70° | | |
| 7 | 75° | | |
| 8 | 80° | | |
| 9 | 85° | | |
| 10 | 90° | | |

# Enrichment

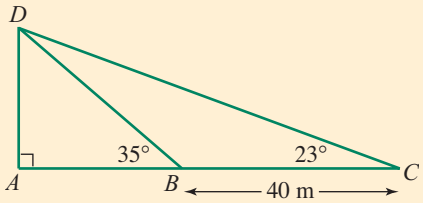
**1** Find the length of the following lines expressed to 2 decimal places, keeping your answers as accurate as possible (don't use rounded answers to find related lengths):

**a** *AE* **b** *BE* **c** *ED* **d** *BD* **e** *BC* **f** *CD*

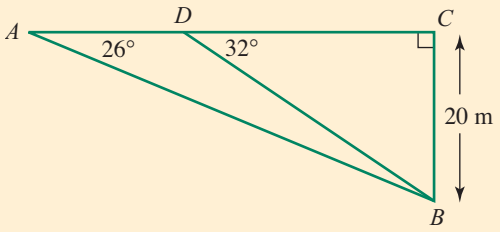



**2** Find the length of the line *AD* to the nearest metre:

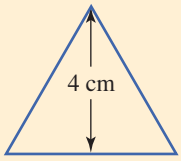
**a**




**b**

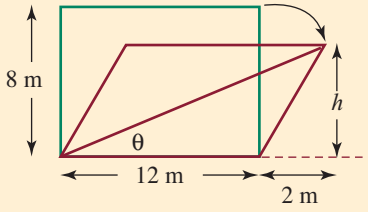



**3** What are the side lengths of an equilateral triangle that has a height of 4 cm?

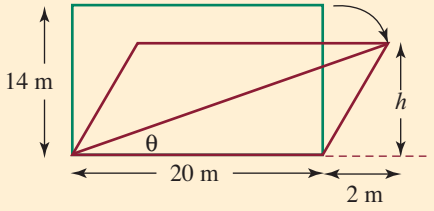



**4** The two rectangular gates shown below are pushed over from the side during a fierce storm:

**a**



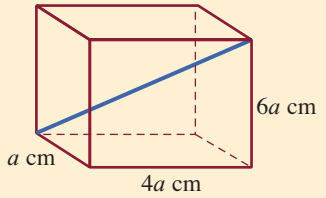

**b**




**i** Find the new height of each gate ($h$).

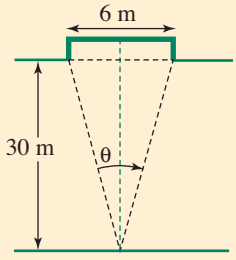
**ii** Find the angle ($\theta$) that the diagonal strut now makes with the bottom of the fence.

**5** Find the angle that the blue line makes with the base of the cuboid, to the nearest tenth of a degree.

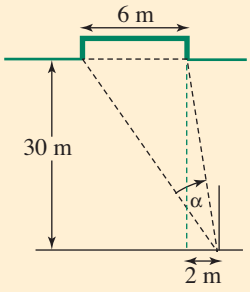


**6** During a game of soccer, a player takes a shot for goal from 30 metres out.

**a** Find the angle that the goal face is open.



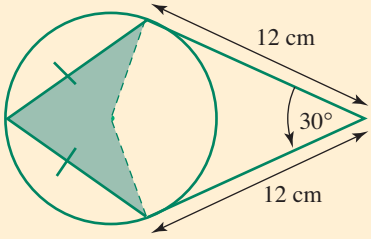

**b** Another shot is taken from a different position. Find the new angle $\alpha$ of the goal face.



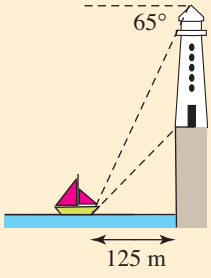

**7** In the diagram shown, find the area of:

**a** the circle

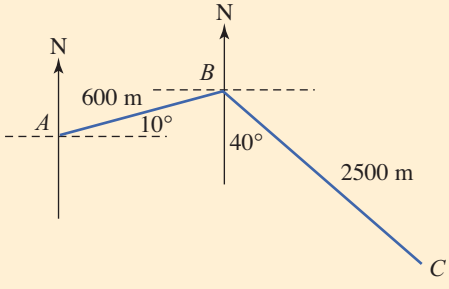
**b** the shaded section




**8** A sailor on a yacht at sea sights the bottom of a lighthouse on a cliff at an angle of elevation of 43°, while the angle of depression from the top of the lighthouse to the yacht is 65°. If the yacht is 125 metres from the base of the cliff, find the height of the lighthouse.




**9** A bushwalker walks from point $A$ to $B$ to $C$ according to this diagram. Find the distance and bearing of $A$ from $C$.




**10** A ship travels 1500 m from port on bearing 070°T, and then travels on bearing 120°T for 300 m to a marker buoy.

**a** Find the distance from the port to the marker buoy.

**b** Find the bearing:

**i** from the port to the marker buoy

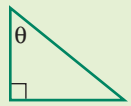
**ii** from the marker buoy to the port

# Revision/Assessment

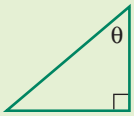
## Exercise 8A

**1** Label the sides of the following triangles with the letters H (hypotenuse), O (opposite) and A (adjacent):

**a**




**b**

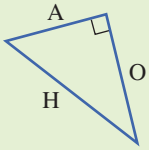



**c**

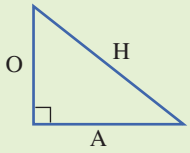

**d**


**2** Place the angle θ in these triangles to match the labels:

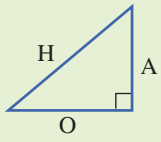
**a**



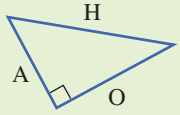

**b**




**c**



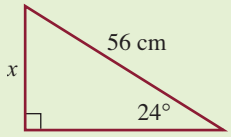

**d**



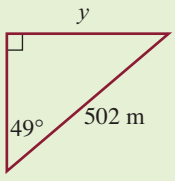

## Exercise 8C

**3** Find the length of the side marked with a pronumeral, expressing answers correct to 2 decimal places:

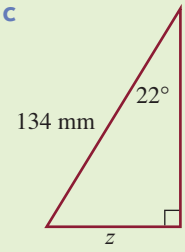
**a**



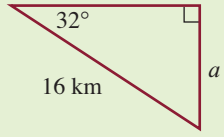

**b**



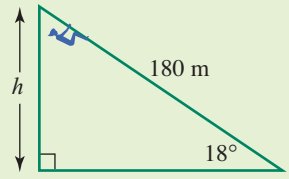

**c**




**d**



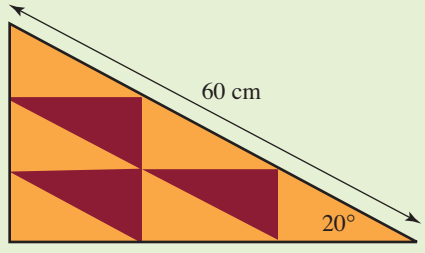

**4** Nellie slides down the 180-metre-long wire shown here. Find the vertical drop ($h$).




**5** This abstract painting is made up of nine identical triangles. Use the diagram to find:

**a** the height of the painting

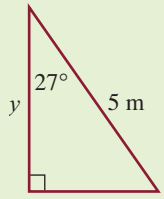
**b** the height of each triangle



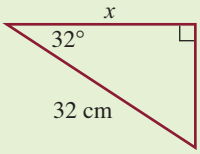
## Exercise 8E

**6** Find the length of the sides marked with a pronumeral, expressing the answers correct to 2 decimal places:

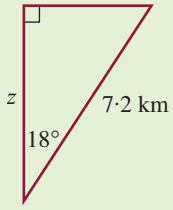
**a**



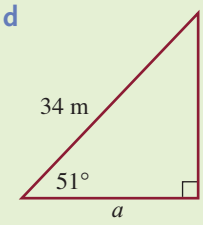

**b**



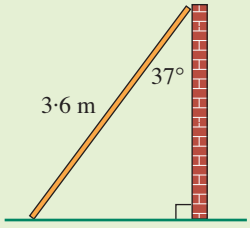

**c**



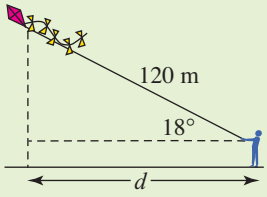

**d**




**7** A piece of timber 3·6 metres long leans against a wall. If the angle between the wall and the timber is 37°, find the height of the wall to the nearest centimetre.



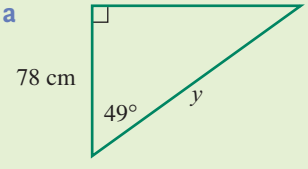

**8** Tim is flying a kite using a string of length 120 metres. Find the horizontal distance ($d$) from Tim to the kite.



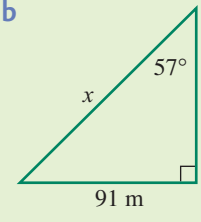

## Exercise 8F

**9** Find the length of the sides marked with a pronumeral, expressing the answers correct to 2 decimal places:

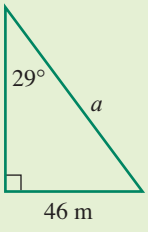
**a**



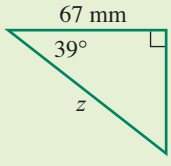

**b**



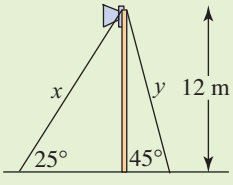

**c**



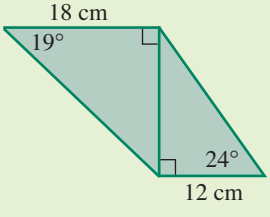

**d**




**10** A loudspeaker is mounted on the top of a pole that is 12 metres tall. The pole is kept vertical by two wires. One wire ($x$) makes an angle of 25° with the ground and the other one ($y$) makes an angle of 45° with the ground. Find the length of each wire.



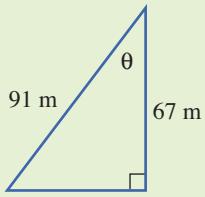

**11** Find the perimeter of this shape to the nearest millimetre.



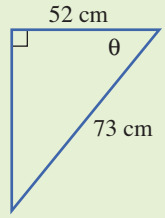
## Exercise 8G

**12** Find the angle marked θ expressed to 1 decimal place:

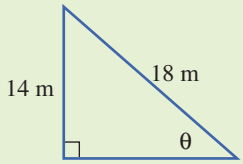
**a**



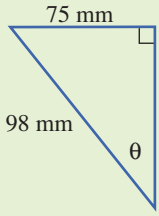

**b**



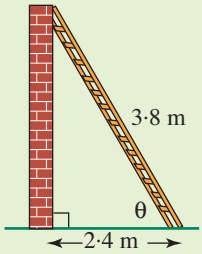

**c**



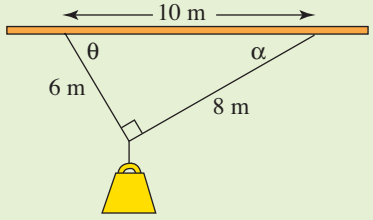

**d**




**13** A ladder which is 3·8 metres long leans against a wall. If its foot is 2·4 metres from the base of the wall, find the angle (θ) that the ladder makes with the ground.



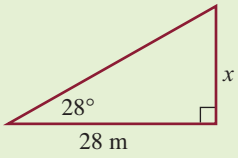

**14** A weight is supported by two ropes which are tied 10 metres apart. Find the angles θ and α shown in the diagram.



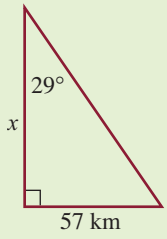

## Exercise 8H

**15** Find the missing lengths or angles in the following:

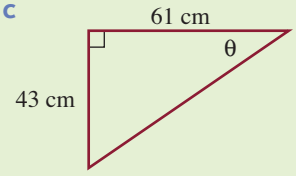
**a**



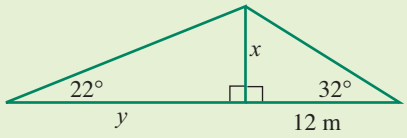

**b**



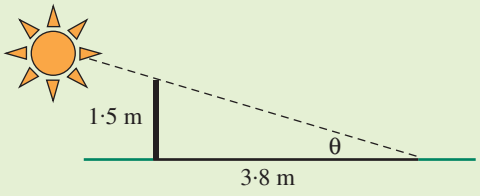

**c**




**16** Find the length marked *x* in the diagram, then use it to find the length marked *y*.

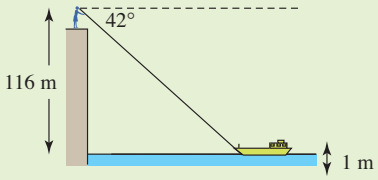



**17** A stick 1·5 metres tall casts a shadow 3·8 metres in length. Find the angle of the Sun θ, to the nearest tenth of a degree.



## Exercise 8I

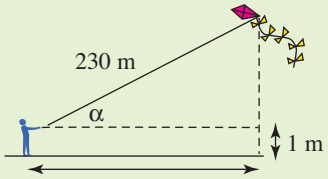
**18** An observer on a cliff sights a ship 116 m below. If the angle of depression is 42°, how far is the ship from the base of the cliff?




**19** A kite is flown using a string which is 230 metres long. Find the angle of elevation ($\alpha$) shown in the diagram when the kite's height above the ground is:

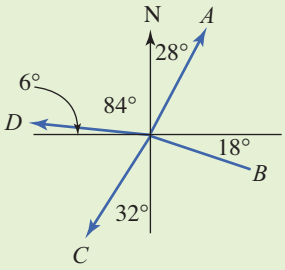
**a** 30 m **b** 50 m **c** 80 m

**d** 150 m **e** 190 m



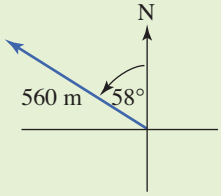

## Exercise 8J

**20** Write the magnetic and true bearings of the four points shown in this diagram.




**21** A yacht sails for 560 metres on the bearing N58°W. Use the diagram to find the distance that it has travelled:

**a** north **b** west




**22** A ship travelled 53 km from port on the bearing of 120°T. Draw a diagram and find the distance from port that the ship has travelled:

**a** south **b** east

CHAPTER

# 9

# Linear Equations and Formulas

Humans have longed to glide and soar like a bird. How do birds fly? Flight is governed by the rules and formulas of Biomechanics, a branch of Physics, involving three forces: drag, lift and thrust.

Birds are streamlined in a shape to minimise drag and they maintain a delicate balance between body weight and muscle strength. They need a low body weight to maximise lift and powerful flight muscles to produce thrust. Feathers have evolved as a lightweight covering for birds that control body temperature and attracts mates.

## This chapter covers the following skills:

- Developing and solving linear equations
- Developing and solving linear inequations
- Applying and rearranging formulas
- Formulating and solving linear equations
- Modelling real-life problems with linear equations
- Managing equations that use the distributive law $a(b + c) = ab + ac$
- Managing equations that involve simple algebraic fractions
- Formulating and solving linear inequations
- Modelling real-life problems with linear inequations
- Solving literal equations
- Applying formulas to solve problems
- Transposing formulas

## Specific Learning Outcomes (SLO)

Learners should be able to:

**9.9.1.1** Find value of pronumerals for simple equations using one-step inverse operations.

**9.9.1.2** Write and solve equations for simple word problems.

**9.9.2.1** Solve complex equations requiring two or three steps using inverse operations.

**9.9.3.1** Write and solve equations for complex word problems.

**9.9.4.1** Solve equations involving brackets.

**9.9.5.1** Solve complex equations involving fractions.

**9.9.6.1** Write and solve equations involving fractions for given maths statements.

**9.9.7.1** Define the term 'inequation' and identify the four inequation signs:
$\leq$: *less than* or *equal to*
$\geq$: *greater than* or *equal to*
$<$: *less than*
$>$: *greater than*

**9.9.8.1** Write simple statements as inequations and solve them.

**9.9.9.1** Solve inequations that are divided by 'negative' numbers.
Note: When we solve an inequation by dividing it by a negative number, reverse the inequality sign.

**9.9.10.1** Complete sentences as true inequalities.

**9.9.11.1** Write inequations statements and display the solutions on a number line.

**9.9.12.1** Solve complex inequations algebraically and display the solutions on a number line.

**9.9.13.1** Define the term 'literal equation' and write equations for given statements that involve several pronumerals.

**9.9.14.1** Solve literal equations.

**9.9.15.1** Define the term 'formula' and identify different examples of formulas.

**9.9.16.1** Solve problems by substituting values into given formulas.

**9.9.17.1** Define the term 'transpose' and apply it to different formulas.

**9.9.18.1** Re-arrange formulas to make a given pronumeral the subject.

**9.9.18.2** Solve word problems by transposing or substituting values into formulas.

# 9A One-step equations: A review

Inverse operations are best used to solve equations.

**Example** **Solution**

**1** Solve the equations:

**a** $x + 6 = -2$

Subtract 6 from *both sides* to keep the 'balance':

$$x + 6 = -2$$
$$x + 6 - 6 = -2 - 6$$

Solution is $x = -8$.

**b** $n - 8 = -5$

Add 8 to both sides:

$$n - 8 = -5$$
$$n - 8 + 8 = -5$$

Solution is $n = 3$.

**2** A number $y$ is multiplied by four to get a result of negative thirty-two. Write an equation for $y$ and solve it.

The equation is $4y = -32$.

Divide both sides by 4:

$$\frac{4y}{4} = \frac{-32}{4}$$

Solution is $y = -8$.

**3** A certain number $q$ divided by negative eight gives a result of nine. Write an equation for $q$ and solve it.

The equation is $\frac{q}{-8} = 9$.

Multiply both sides by $-8$:

$$\frac{q}{-8} \times -8 = 9 \times -8$$

Solution is $q = -72$.

## Exercise 9A

**1** Solve the following equations:

**a** $x + 3 = 7$ **b** $y + 12 = 27$ **c** $z + 9 = 6$

**d** $m - 7 = -9$ **e** $n - 18 = -6$ **f** $p - 5 = -7$

**g** $p - \frac{1}{4} = -\frac{1}{2}$ **h** $q - 3{\cdot}5 = 1{\cdot}75$ **i** $t - 5{\cdot}25 = -1{\cdot}75$

**2** Solve the following equations:

**a** $-5x = 20$ **b** $12y = -72$ **c** $-5n = -45$

**d** $-9g = 27$ **e** $5p = -55$ **f** $-12m = -60$

**g** $7r = -21$ **h** $-3s = 18$ **i** $-7t = -42$

**j** $4z = -15$ **k** $-8q = 43$ **l** $-6h = -25$

**m** $10x = -5$ **n** $-12y = 8$ **o** $-24z = -16$

**p** $8x = -2$ **q** $-9x = 12$ **r** $-75y = 15$

**s** $-24z = -10$ **t** $-39y = -13$ **u** $30x = -20$

**3** Solve the following equations:

**a** $\frac{x}{8} = 10$ **b** $\frac{x}{-4} = 5$ **c** $\frac{x}{-5} = -12$

**d** $\frac{x}{12} = -8$ **e** $\frac{x}{-8} = 6$ **f** $\frac{x}{-6} = -11$

**g** $\frac{y}{7} = 9$ **h** $\frac{y}{-9} = 12$ **i** $\frac{y}{-12} = -9$

**j** $\frac{y}{10} = -7$ **k** $\frac{y}{-6} = -16$ **l** $\frac{y}{-8} = -15$

**4** Write equations for the following statements and solve them:

**a** Ian has $p$ marbles and when he wins six more he has a total of eleven.

**b** Ten plus a certain number $y$ equals negative three.

**c** A particular number $x$ minus five equals negative twelve.

**d** A number $y$ minus twelve equals negative three.

**e** Seven times a certain number $q$ equals negative eighty-four.

**f** Negative six times the number $m$ gives a result of negative thirty.

**g** A number $n$ when multiplied by negative eight gives a result of eighty.

**h** When a number $z$ is divided by seven, the result is negative six.

**i** When $r$ is divided by negative nine, the result is twenty.

**j** When $s$ is divided by negative four, the result is negative sixteen.

**5** My bank account has an unknown balance $b$, and when I withdraw \$50 the balance is –\$80; that is, I am \$80 overdrawn. Write an equation for $b$ and then solve it.

**6** When my pay \$$d$ is deposited into my bank account, the balance rises from –\$60 to \$150. Write an equation for $d$ and solve it.

**7** A Gavutu dolphin pen had an unknown number of dolphins $d$ and when 35 new dolphins were introduced the numbers grew to 81. Write an equation for $d$ and then solve it.

**8** A wildlife park on the island of Tetepare in the Western Province of Solomon Islands introduced a number of crocodiles $c$ into a community of 27. When three new baby crocodiles were born, the total number grew to 46. Write an equation for $c$ and then solve it.

**9** The cost \$$C$ of having a DJ for the school disco was shared by 120 learners. If the learners each paid \$3·50, write an equation for $C$ and solve it.

**10** A class of 25 learners gained entry to a special exhibition at the National Museum for a group price of \$146·25. Write an equation and solve it to find the entry price $p$ paid by each learner.

# 9B Solving two- and three-step equations

To solve more complex equations, undo the equations using inverse operations.

**Example** **Solution**

**1** Solve the equations:

**a** $4x + 7 = 23$

$$4x + 7 = 23$$
$$4x + 7 - 7 = 23 - 7$$
$$4x = 16$$
$$\frac{4x}{4} = \frac{16}{4}$$
$$x = 4$$

**b** $\frac{3y}{-4} = 12$

$$\frac{3y}{-4} = 12$$
$$\frac{3y}{-4} \times -4 = 12 \times -4$$
$$3y = -48$$
$$\frac{3y}{3} = \frac{-48}{3}$$
$$y = -16$$

**2** A certain number $x$ is multiplied by two and four is added. The result is divided by seven and an answer of negative two is obtained. Write the equation and solve for $x$.

The equation is $\frac{2x + 4}{7} = -2$.

$$\frac{2x + 4}{7} \times 7 = -2 \times 7$$
$$2x + 4 = -14$$
$$-4 \quad -4$$
$$\frac{2x}{2} = \frac{-18}{2}$$
$$x = -9$$

**3** I think of a number $x$, multiply it by five, divide the result by four and finally subtract twelve to get an answer of thirteen. Write the equation and solve for $x$.

The equation is $\frac{5x}{4} - 12 = 13$.

$$\frac{5x}{4} - 12 = 13$$
$$\frac{5x}{4} - 12 + 12 = 13 + 12$$
$$5\frac{x}{4} = 25$$
$$\frac{5x}{4} \times 4 = 25 \times 4$$
$$5x = 100$$
$$\frac{5x}{5} = \frac{100}{5}$$
$$x = 20$$

## Exercise 9B

**1** Solve these equations by following the setting out given in the worked examples:

**a** $3x + 7 = 16$ **b** $3y - 16 = 8$ **c** $9m - 3 = -39$ **d** $7 + 3n = -5$

**e** $10 + 5q = -5$ **f** $-8a - 19 = 13$ **g** $-6b + 48 = 6$ **h** $45 - 5c = 15$

**i** $24 - 4m = 12$ **j** $9 - 7a = 37$ **k** $-15 - 4r = 9$ **l** $-6s + 5 = 14$

**2** Solve these two-step equations by following the setting out given in the worked examples:

**a** $\frac{5x}{-4} = 15$ **b** $\frac{3x}{5} = -12$ **c** $\frac{-2x}{3} = 6$

**d** $\frac{3}{-5}a = 9$ **e** $\frac{-2}{9}b = 2$ **f** $\frac{5}{2}c = -15$

**g** $\frac{-2m}{3} = 7$ **h** $\frac{3p}{-2} = 5$ **i** $\frac{4}{3}c = -5$

**3** Solve the following equations by using the setting out given in the worked examples:

**a** $\frac{p}{6} + 5 = 2$ **b** $\frac{y}{9} + 7 = 3$ **c** $\frac{z}{2} - 6 = -3$

**d** $\frac{1}{8}m + 7 = 3$ **e** $\frac{1}{4}n - 6 = -2$ **f** $\frac{1}{12}q - 3 = -7$

**g** $\frac{t}{2} + \frac{3}{4} = -\frac{1}{4}$ **h** $\frac{1}{4}r - \frac{1}{2} = \frac{1}{4}$ **i** $\frac{1}{3}s - 2 = 1\frac{1}{3}$

**4** Solve the following equations:

**a** $\frac{x+3}{2} = 6$ **b** $\frac{x+9}{3} = 2$ **c** $\frac{x-7}{8} = -2$

**d** $\frac{a-5}{6} = -3$ **e** $\frac{2x+1}{5} = 3$ **f** $\frac{3y+7}{2} = -4$

**g** $\frac{3z-4}{-2} = 5$ **h** $\frac{2-3y}{7} = 2$ **i** $\frac{3-5x}{4} = -3$

**5** Solve the following equations:

**a** $\frac{2x}{3} - 5 = 3$ **b** $\frac{3a}{4} + 8 = 5$ **c** $\frac{4n}{3} - 17 = 7$

**d** $\frac{5n}{3} + 9 = -11$ **e** $\frac{4p}{5} - 9 = 7$ **f** $\frac{2q}{7} - 1 = 7$

**g** $\frac{5}{6}m - 3 = -18$ **h** $\frac{2}{5}z - 4 = 12$ **i** $\frac{3}{8}t - 2 = -11$

**6** Write an equation for each of the following statements and then solve it:

**a** I think of a number $x$, multiply it by five, then subtract seventeen to get a result of negative two.

**b** Four times a certain number $x$ divided by negative five gives a result of two.

**c** A certain number $y$ divided by five, plus seven gives a result of three.

**d** Ten is added to a certain number $x$ and the result is divided by eleven to give an answer of negative two.

**e** The result of three times a certain number $q$, minus seven, is divided by two and the result is four.

**f** Three times a certain number $y$ divided by four from which two is subtracted gives a result of negative fourteen.

**g** Two-fifths of a certain number $z$, minus three, equals thirteen.

**h** Twelve minus three-quarters of a certain number $n$ gives a result of negative nine.

# 9C Equations involving brackets

A number multiplying a bracket can be divided out, or the bracket can be removed, depending on the equation.

**Example** **Solution**

**1** Four is subtracted from a certain number $x$ and the result is multiplied by two to give an answer of negative twelve.

Write an equation and solve for $x$.

The equation is $2(x-4) = -12$.

As the 2 in front of the bracket divides into the –12 on the right, we have:

$$\frac{2(x-4)}{2} = \frac{-12}{2} \quad \text{dividing out by two}$$

$$x - 4 = -6$$

$$+4 \quad +4 \quad \text{adding four to both sides}$$

$$x - 4 + 4 = -6 + 4$$

$$x = -2$$

**2** Solve $4(x+5) = 17$.

It is best to remove the bracket as 4 does not divide exactly into 17.

$$4(x+5) = 17$$

$$4x + 20 = 17$$

$$-20 \quad -20$$

$$4x = -3$$

$$\frac{4x}{4} = \frac{-3}{4}$$

$$x = -\frac{3}{4}$$

**3** Three is subtracted from a certain number $x$ and the result is multiplied by two. This answer is divided by three and then four is added to get a final answer of eight.

Write an equation and solve for $x$.

The equation is:

$$\frac{2(x-3)}{3} + 4 = 8$$

$$-4 \quad -4$$

$$\frac{2(x-3)}{3} = 4$$

$$\frac{2(x-3)}{3} + 4 - 4 = 8 - 4$$

$$\frac{2(x-3)}{3} \times 3 = 4 \times 3$$

$$2(x-3) = 12$$

$$\frac{2(x-3)}{2} = \frac{12}{2}$$

$$x - 3 = 6$$

$$x = 9$$

## Exercise 9C

**1** Solve the following equations by first dividing by the number multiplying the bracket:

**a** $2(x+3) = 8$ **b** $3(x+2) = 12$ **c** $4(m+1) = 16$ **d** $5(x-8) = 10$

**e** $2(y-6) = 14$ **f** $6(n-5) = 66$ **g** $3(z-5) = -18$ **h** $4(s+3) = -12$

**i** $7(p-6) = -21$ **j** $-2(q-3) = 10$ **k** $-6(t-5) = 12$ **l** $-5(h+2) = 20$

**m** $-9(x-8) = -18$ **n** $-4(y+4) = -16$ **o** $-8(p-7) = -32$ **p** $-5(x-7) = 7.5$

**q** $-2(x-9) = -1.4$ **r** $-3(y-5) = -1.5$

**2** Solve the following equations by first removing the brackets:

**a** $2(x + 4) = 12$ **b** $3(x + 6) = 15$ **c** $5(x + 6) = 25$

**d** $6(z - 9) = 18$ **e** $9(q - 8) = 36$ **f** $7(t - 3) = 21$

**g** $2(x + 5) = 15$ **h** $4(p + 6) = 27$ **i** $3(m + 2) = 14$

**j** $5(y - 3) = -12$ **k** $7(z - 2) = -5$ **l** $4(p - 3) = -7$

**m** $2(s - 3) = -7$ **n** $8(t - 9) = -35$ **o** $9(r - 2) = -16$

**3** Solve the following equations by using whichever approach you wish:

**a** $3(2x + 5) = 27$ **b** $6(3y - 17) = 18$ **c** $4(3z - 11) = 12$

**d** $2(4m + 7) = 8$ **e** $5(2n + 11) = 6$ **f** $7(4q + 14) = 21$

**g** $4(5 + 2q) = 42$ **h** $4(9 + 3p) = 55$ **i** $8(2 + 3q) = 60$

**j** $3(8 - 4y) = 12$ **k** $2(8 - 3y) = 5$ **l** $5(1 - 4z) = 18$

**m** $4(9 - 3r) = 12$ **n** $12(2 - 3s) = -48$ **o** $7(3 - 4t) = 22$

**4** Solve the following equations:

**a** $\frac{2(x-3)}{7} + 4 = 6$ **b** $\frac{4(x-5)}{5} - 1 = 7$ **c** $\frac{9(x-5)}{5} - 6 = 3$

**d** $\frac{2(x-4)}{7} + 5 = 9$ **e** $\frac{11(x-5)}{3} - 25 = 30$ **f** $\frac{5(x-4)}{3} + 6 = 21$

**5** Write an equation for each of the following statements and then solve it:

**a** Six is subtracted from a certain number $x$ and the result is multiplied by five to give an answer of negative twenty.

**b** Ten is added to a certain number $y$ and the result is multiplied by three to give an answer of forty-five.

**c** A certain number $y$ is subtracted from eight and the result is multiplied by four to give an answer of twenty-four.

**d** A certain number $x$ is multiplied by three and the result is added to four. This answer is then multiplied by two to give a final answer of thirty-eight.

**e** A certain number $z$ is multiplied by five and the result is subtracted from eight. This answer is then multiplied by four to give a final answer of seventy-two.

**f** Nine is subtracted from a certain number $y$ and the result is multiplied by three. This answer is divided by four and then seven is added to give a final answer of sixteen.

**g** Seven is added to a certain number $z$ and the result is multiplied by five. This answer is divided by eight, then four is subtracted to give a final answer of six.

**h** The price $P$ dollars of a scuba-diving lesson is discounted by ten dollars. If twenty learners pay the discounted price and the total is $340, write an equation involving $P$ and solve it.

# 9D Further equations

More complicated equations often have the pronumeral appearing more than once.

**Example** **Solution**

**1** A certain number $x$ is multiplied by two and four is subtracted. This result is multiplied by three and the answer is equal to twice the sum of $x$ and four.

Write an equation and solve for $x$.

The equation is $3(2x - 4) = 2(x + 4)$.

Removing brackets:

$$\begin{aligned} 6x - 12 &= 2x + 8 \\ 6x - 12 - 2x &= 2x + 8 - 2x \\ 4x - 12 &= 8 \\ 4x - 12 + 12 &= 8 + 12 \\ 4x &= 20 \\ x &= 5 \end{aligned}$$

**2** Solve $\frac{5x}{2} - \frac{4x}{3} = 14$.

Clear the fractions by multiplying through by 6:

$$\begin{aligned} \frac{5x}{2} \times 6 - \frac{4x}{3} \times 6 &= 14 \times 6 \\ 15x - 8x &= 84 \\ 7x &= 84 \\ x &= 12 \end{aligned}$$

**3** Solve $\frac{2x+1}{5} - \frac{x-2}{3} = 1$.

Clear the fractions by multiplying through by 15. Also use brackets around the terms in each numerator to ensure correctness.

$$\begin{aligned} \frac{(2x+1)}{5} \times 15 - \frac{(x-2)}{3} \times 15 &= 1 \times 15 \\ (2x+1) \times 3 - (x-2) \times 5 &= 15 \\ 6x + 3 - 5x + 10 &= 15 \\ x + 13 &= 15 \\ x &= 2 \end{aligned}$$

**4** A mother is five years older than double her daughter's age. The total of their ages is 50 years.

Set up an equation and solve it to find their ages.

Let $x$ represent the daughter's age.

$\therefore$ the mother's age is represented by $2x + 5$.

The equation is then

$$\begin{aligned} x + 2x + 5 &= 50 \\ 3x + 5 &= 50 \\ 3x + 5 - 5 &= 50 - 5 \\ 3x &= 45 \\ x &= 15 \end{aligned}$$

Statement of answers:

- The daughter is 15 years old.
- The mother is $2 \times 15 + 5 = 35$ years old.
- Checking: $15 + 35 = 50$

## Exercise 9D

**1** Remove the brackets and combine like terms to solve the following equations:

**a** $5(x-3)-3x=1$ **b** $3(x-1)+2x=12$

**c** $2(x-4)+3(x+1)=15$ **d** $2(x+1)+3(x-2)=6$

**e** $5(x-2)-2(x-1)=1$ **f** $6(x-1)-(x-1)=20$

**2** Solve the following equations:

**a** $6x=2x+8$ **b** $5y=2y-6$ **c** $3z=-2z-15$

**d** $5m+6=2m$ **e** $7n-8=3n$ **f** $3p+20=-7p$

**g** $5x-8=4x+7$ **h** $9y+4=6y-8$ **i** $3z+5=13-z$

**3** Solve the following equations by first removing the brackets:

**a** $3(2x+5)=5x+18$ **b** $11y-10=3(3y-2)$

**c** $4(3z-11)=14z-48$ **d** $2(2x-3)=3(x+4)$

**e** $2(3z-2)=2(z-4)$ **f** $3(x-5)=2(x+4)$

**g** $4(x+1)=-2(x+4)$ **h** $3(q-7)=-4(6-q)$

**i** $-4(3x-1)=5(6-2x)$ **j** $5(2x-1)=-4(1-2x)$

**k** $-3(2-q)=5(q+4)$ **l** $3(1-4z)=-2(2-z)$

**4** Solve the following equations:

**a** $\frac{x}{2}+\frac{x}{5}=7$ **b** $\frac{x}{2}+\frac{x}{3}=5$ **c** $\frac{z}{5}-\frac{z}{9}=4$

**d** $\frac{x}{2}-\frac{x}{3}=2$ **e** $\frac{x}{2}-\frac{x}{5}=6$ **f** $\frac{x}{2}+\frac{x}{4}=3$

**g** $\frac{3x}{2}+\frac{x}{5}=17$ **h** $\frac{3x}{4}+\frac{2x}{5}=23$ **i** $\frac{5x}{8}+\frac{3x}{4}=11$

**j** $\frac{3x}{2}-\frac{5x}{8}=7$ **k** $\frac{3x}{4}-\frac{2x}{3}=2$ **l** $\frac{3x}{2}-\frac{2x}{5}=11$

**5** Solve the following equations:

**a** $\frac{2x+3}{5}+\frac{x+3}{3}=6$ **b** $\frac{3x-1}{4}+\frac{x+2}{3}=8$ **c** $\frac{2x+1}{3}+\frac{3x-2}{2}=8$

**d** $\frac{4x+3}{2}-\frac{x+3}{4}=6$ **e** $\frac{3x-1}{2}-\frac{2x+3}{5}=11$ **f** $\frac{5x-3}{3}-\frac{3x+1}{2}=-2$

**g** $\frac{3x-1}{4}-\frac{x-2}{2}=2$ **h** $\frac{2x-1}{5}-\frac{2x-3}{3}=4$ **i** $\frac{7x+2}{3}-\frac{5x-4}{9}=10$

**6** For each of the following, write an equation that models the problem, and define the pronumeral used. Solve the equation and state clearly the solution(s) at the end.

**a** A father is three times the age of his son, and together their ages total 48 years. How old is each person?

**b** Ian is six years older than Liz and their combined ages total sixty years. How old is each person?

**c** At the school bazaar, my friend spent eight dollars less than I, and together we spent a total of $22. How much did we each spend at the bazaar?

**d** At the same school bazaar, three large green coconuts and eight half-price coconuts cost $21. How much was the large coconut?

**e** In a 'relay for life' event with four competitors, the first competitor runs a certain distance, the second runs this same distance plus an extra 100 metres, the third competitor runs the same distance as the second runner plus an extra 100 metres, and the fourth competitor runs the same distance as the third runner plus an extra 100 metres. If the total distance covered by all four competitors is 3000 metres, how far does each competitor run?


**f** On route to the Ruavtu School, the bus travelled at 80 km/h for a certain period of time $t$ and at 100 km/h for twice this period of time. If the total distance covered was 140 km, for how long did the bus travel at each speed? (Hint: Distance = speed × time)

**g** At the movies a discount ticket sells for $5 dollars off the normal price. If four discount tickets and three full price tickets cost $92, how much does each type of ticket cost?

**h** At the Agricultural Show, a certain ride offers a discount of $1 on the full price of the ride when 10 or more tickets are purchased. If 20 discount tickets cost the same as fifteen full price tickets, find the cost of each type of ticket.

**i** On the RAMSI Ride, Peter leaves for the day's ride at a speed of 18 km/h and 1 hour later Bob follows him at a speed of 24 km/h. How long does it take Bob to catch Peter, and how far has each travelled when this occurs?

**j** The sum of five consecutive whole numbers is equal to six times the smallest number. Find the five numbers.

**k** Two numbers are such that one number is six times the other. When the two are added the result is equal to three times the smaller number plus 20. What are the two numbers?

**l** Six friends booked a table at the local pizza restaurant and planned to share the final bill six ways. If only four of the friends turned up and had to pay an *extra* $5 each to cover the bill, find the cost of the final bill.

# Exploring inequalities 9E

Inequalities can be readily found around us. For example, the combined weights of two people crossing the Mataniko River on floating boards must not exceed 220 kg. Mathematically we would write this as $W \leq 220$ (kg).

The four inequality signs used are:

| | |
|---|---|
| $<$ meaning 'less than' | $\leq$ meaning 'less than or equal to' |
| $>$ meaning 'greater than' | $\geq$ meaning 'greater than or equal to' |

## Learning task 9E

**1** Complete the following and include the correct inequality sign in each box:

**a** $8 > 6$

$\times 3$ $\times 3$

$\_\_ \square \_\_$

**b** $-5 < 12$

$\times 2$ $\times 2$

$\_\_ \square \_\_$

**c** $16 > 8$

$\div 4$ $\div 4$

$\_\_ \square \_\_$

**d** $-20 < -10$

$\div 5$ $\div 5$

$\_\_ \square \_\_$

**2** Complete the following conclusion:
When an inequality is multiplied or divided by a positive number, the inequality sign ____ (choose from 'keeps' or 'reverses') its direction.

**3** Complete the following and include the correct inequality sign in each box:

**a** $7 > 5$

$\times -2$ $\times -2$

$\_\_ \square \_\_$

**b** $4 > -2$

$\times -3$ $\times -3$

$\_\_ \square \_\_$

**c** $5 < 8$

$\times -4$ $\times -4$

$\_\_ \square \_\_$

**d** $-4 < 2$

$\times -2$ $\times -2$

$\_\_ \square \_\_$

**4** Complete the following conclusion:
When an inequality is multiplied or divided by a negative number, the inequality sign ____ (choose from 'keeps' or 'reverses') its direction.

**5** Complete the following:

**a** $7x > 14$

$\div 7$ $\div 7$

$x \square \_\_$

**b** $5x < -15$

$\div 5$ $\div 5$

$x \square \_\_$

**c** $-2x > -12$

$\div -2$ $\div 2$

$x \square \_\_$

**d** $-4x > 8$

$\div -4$ $\div -4$

$x \square \_\_$

# 9F Inequations

This section gives further practise of the solution methods for inequations covered in the previous section.

An **inequation** occurs when the equals (=) sign in an equation is replaced by one of the following four **inequality signs**.

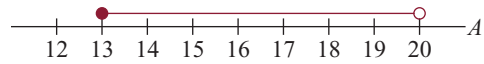
| | |
|---|---|
| $<$ meaning 'less than' | $\leq$ meaning 'less than or equal to' |
| $>$ meaning 'greater than' | $\geq$ meaning 'greater than or equal to' |

**Example** **Solution**

**1** If $A$ is the age at which you are called a teenager, write an inequation for $A$ and show the solution on a number line.

$A$ has to be thirteen or more but less than twenty. Reading from the centre to the left and then to the right, we have:

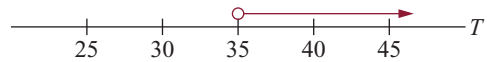
$13 \leq A < 20$




Note that an 'empty' or 'open' end circle excludes the end number, but a closed or filled circle includes it.

**2** Solve the inequation $T - 5 > 30$.

$$T - 5 > 30$$
$$T - 5 + 5 > 30 + 5$$
$$T > 35$$




Note that an 'empty' or 'open' circle excludes the end number 35.

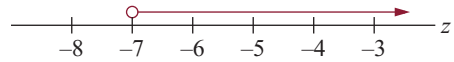
**3** A number $z$ is multiplied by negative two and four is added to get a result less than 18.

Write an inequation and solve it, showing the solution on a number line.

We apply the algebraic steps we have used before.

Start with $-2z + 4 < 18$.

$$-2z + 4 - 4 < 18 - 4$$
$$\frac{-2z}{-2} < \frac{-14}{-2}$$
$$z > -7$$




When we solve an inequation, multiplying or dividing both sides by a negative number *reverses* the inequality sign.

For example: $-8 < -5$

$\times -2 \quad \times -2$

$+16 > +10$

## Exercise 9F

**1** Use the symbols <, > or = to correctly complete the following statements:

**a** $15 + 12 \_\_\_\_ 4 \times 6$

**b** $3 \times 5 \_\_\_\_ 20 \div 2$

**c** $6 \times -2 \_\_\_\_ -4 + 9$

**d** $16 - 4 \_\_\_\_ 24 \div -3$

**e** $-4 \times -0{\cdot}5 \_\_\_\_ -20 \div -10$

**f** $-36 + 25 \_\_\_\_ 6 \times -7$

**g** $9 \times -12 \_\_\_\_ 216 \div -2$

**h** $4^3 + 5 \_\_\_\_ 40 - 22$

**2** Write each of the following statements as an inequation and show the solution on a number line. Use a number line that displays both negative and positive numbers. Be careful to use the appropriate open or closed circle at each endpoint:

**a** $x$ is less than ten.

**b** $y$ is more than negative five.

**c** $z$ is negative three or less.

**d** $m$ is negative one or more.

**e** $x$ is greater than $-2$ but less than 2.

**f** $x$ is between $-4$ and $-1$.

**g** $x$ is $-2$ or more but less than 3.

**h** $y$ is greater than $-2$ but less than or equal to 4.

**i** The temperature during a day at school ranged from 15°C to 25°C inclusive.

**3** Harry's height is given by $H$ (cm). The required height for a person to be recruited to the Solomon Islands Royal Police Force is 170 cm or greater. Write an inequality to represent the restriction on Harry's height for the Police Force.

**4** Solve the following inequations by using the setting out of earlier sections:

**a** $x + 5 < 3$

**b** $y - 8 < -12$

**c** $z + 18 \geq 10$

**d** $4x \geq -24$

**e** $9y > -72$

**f** $5z < -30$

**g** $-5x \geq 15$

**h** $-3y > -27$

**i** $-7z < -35$

**j** $\frac{x}{7} \geq -3$

**k** $\frac{y}{13} < -4$

**l** $\frac{z}{15} < -2$

**m** $\frac{x}{-4} \geq 5$

**n** $\frac{y}{-2} < -8$

**o** $\frac{z}{-15} < 4$

**p** $\frac{1}{4}m < -5$

**q** $\frac{1}{-2}n \geq 3$

**r** $\frac{1}{-3}p < -5$

**s** $\frac{2x}{-3} \leq -6$

**t** $\frac{3y}{-5} > -12$

**u** $\frac{4z}{-3} < -8$

**v** $\frac{-2}{5}a \geq -6$

**w** $\frac{-7}{6}b \leq -21$

**x** $\frac{-3}{4}c \leq -12$

# 9F

**5** Solve the following inequations by using the setting out of earlier sections:

**a** $4x - 8 \geq -20$ **b** $15 + 2y < 11$ **c** $6z - 2 \leq -26$

**d** $-2a + 15 < 11$ **e** $-10b + 9 \geq -11$ **f** $5c - 8 < -18$

**g** $\frac{m}{7} + 10 \geq 6$ **h** $\frac{n}{-6} - 9 < -2$ **i** $\frac{p}{-5} - 1 \geq -2$

**j** $\frac{1}{-4}x + 7 < 3$ **k** $\frac{1}{5}y - 9 \geq -6$ **l** $\frac{1}{-3}z - 5 \leq -2$

**m** $\frac{2a}{-5} + 3 \leq -5$ **n** $\frac{-7b}{6} - 2 \leq -9$ **o** $\frac{4z}{-3} - 7 \leq -15$

**p** $\frac{2}{-3}x + 5 \leq -7$ **q** $\frac{-3}{5}y - 2 \leq -14$ **r** $\frac{2}{-5}p - 6 \leq -16$

**6** Solve the following inequations:

**a** $-2(x + 4) < 10$ **b** $-3(x + 6) > -15$ **c** $-5(x + 8) \geq 30$

**d** $-6(z - 9) \leq -18$ **e** $-9(q + 8) < 36$ **f** $-7(t - 3) \geq -21$

**7** Solve the following inequations:

**a** $\frac{3x + 8}{5} > 4$ **b** $\frac{2x + 8}{4} \leq -6$ **c** $\frac{3x - 9}{10} \geq -3$

**d** $\frac{-2x - 5}{5} \leq 1$ **e** $\frac{4x - 3}{-5} > 3$ **f** $\frac{-5x - 10}{2} \leq 5$

**8** Solve the following inequations which involve brackets:

**a** $5(x - 5) > 2(x + 1)$ **b** $3(x + 2) > 2(x - 1)$ **c** $7(x - 2) \leq 3(x + 6)$

**d** $2(x + 5) \geq 3(x - 1)$ **e** $5(z - 3) < 7(z - 1)$ **f** $-3(y - 2) > 2(y - 2)$

**9** Solve the following inequations:

**a** $\frac{3(x - 4)}{2} - 3 \geq 3$ **b** $\frac{5(9 - x)}{3} + 2 < 12$ **c** $\frac{7(x - 4)}{6} - 5 \leq 2$

**d** $\frac{2(x - 3)}{3} + 7 > 15$ **e** $\frac{3(2x - 5)}{13} - 8 \geq -5$ **f** $\frac{5(3x + 10)}{2} - 8 \leq 2$

**10** Solve the following inequations:

**a** $\frac{2x + 1}{5} + \frac{x - 4}{3} > -7$ **b** $\frac{3x + 1}{4} + \frac{2x - 3}{7} \leq 5$ **c** $\frac{2x + 1}{5} + \frac{4x + 3}{3} < -4$

**d** $\frac{3x - 2}{5} - \frac{2x - 5}{3} \geq 1$ **e** $\frac{5x - 3}{2} - \frac{4x - 3}{7} > -3$ **f** $\frac{3 - 2x}{9} - \frac{5x + 8}{7} \geq 2$

**11** A daughter is half the age of her mother, and the total of their ages is less than 60 years. Solve an inequation to find the restriction on the age of the daughter. Is there any restriction on the mother's age, and if so what is it?

**12** Initially I had \$1000 in my bank account. Yesterday I withdrew \$$d$ and today I withdrew \$20 more than yesterday's withdrawal. If the balance of my account is still at least \$600, what restrictions applied to my withdrawals each day?

# Literal equations 9G

Literal equations involve several pronumerals (letters); however, the methods of solution follow the same processes as those of ordinary equations.

**Example** | **Solution**

**1** I think of a number $x$ and add $a$ to get a result of $b$. Write the equation and solve for $x$.

The equation is $x + a = b$.

Subtract $a$ from both sides:

$$\begin{aligned} x + a &= b \\ x + a - a &= b - a \\ x &= b - a \end{aligned}$$

**2** A certain number $y$ is multiplied by $-a$ to get a result of $b$. Write the equation and solve for $y$.

The equation is $-ay = b$.

Divide both sides by $-a$:

$$\frac{-ay}{-a} = \frac{b}{-a}$$

$$y = \frac{b}{-a} = -\frac{b}{a}$$

**3** A certain number $q$ divided by $-a$ gives a result of $-b$. Write the equation and solve for $q$.

The equation is $\frac{q}{-a} = -b$.

Multiply both sides by $-a$:

$$\frac{q}{-a} \times -a = -b \times -a$$

$$q = ba \text{ or } ab$$

**4** Solve the following literal equations for $x$:

**a** $\frac{ax}{b} - c = d$

$$\frac{ax}{b} - c = d$$

$$\frac{ax}{b} = d + c$$

$$ax = b(d + c)$$

$$\frac{ax}{a} = \frac{b(d + c)}{a}$$

$$x = \frac{b(c + d)}{a}$$

**b** $a(x - b) = c$

$$a(x - b) = c$$

$$ax - ab = c$$

$$ax = c + ab$$

$$x = \frac{c + ab}{a} \text{ or } x = \frac{c}{a} + b$$

## Exercise 9G

**1** Solve the following equations for $x$:

**a** $x + m = n$ **b** $x + a = b$ **c** $x + p = -q$

**d** $x - a = c$ **e** $x - r = -s$ **f** $x - m = -n$

**g** $b - x = a$ **h** $m - x = h$ **i** $e - x = -f$

**2** Solve the following equations for $x$:

**a** $px = q$ **b** $rx = s$ **c** $px = q$

**d** $cx = -d$ **e** $-mx = n$ **f** $-fx = -p$

**g** $ax + b = 0$ **h** $cx - d = 0$ **i** $-kx + h = 0$

**3** Solve the following equations for $x$:

**a** $\frac{x}{m} = n$ **b** $\frac{x}{a} = c$ **c** $\frac{x}{f} = f$ **d** $\frac{x}{-c} = d$

**e** $\frac{x}{-r} = z$ **f** $\frac{x}{-v} = v$ **g** $\frac{x}{g} = -h$ **h** $\frac{x}{j} = -k$

**i** $\frac{x}{w} = -w$ **j** $\frac{x}{-m} = -p$ **k** $\frac{x}{-q} = -p$ **l** $\frac{x}{-z} = -z$

**4** Solve the following equations for $y$:

**a** $ay + b = c$ **b** $my + c = d$ **c** $zy - x = q$

**d** $py - q = k$ **e** $by - c = -q$ **f** $p - hy = b$

**g** $m = ky - n$ **h** $d = c + ay$ **i** $f = g - hy$

**5** Solve the following equations for $y$:

**a** $\frac{y}{c} + a = b$ **b** $\frac{y}{b} + k = h$ **c** $\frac{y}{a} - c = b$

**d** $\frac{y}{m} + n = -b$ **e** $\frac{y}{h} - h = h$ **f** $\frac{y}{p} + p = 2p$

**g** $b + \frac{y}{a} = c$ **h** $m - \frac{y}{h} = k$ **i** $-p - \frac{y}{q} = r$

**6** Solve the following equations for $x$:

**a** $\frac{ax}{c} + b = d$ **b** $\frac{cx}{a} - c = b$ **c** $\frac{dx}{b} - c = d$

**d** $\frac{ax}{b} - f = -g$ **e** $\frac{mx}{n} + p = -q$ **f** $\frac{ax}{-b} + c = d$

**g** $b + \frac{cx}{d} = e$ **h** $r - \frac{tx}{s} = u$ **i** $-k - \frac{fx}{q} = h$

**7** Solve the following equations for $x$:

**a** $b(x - c) = a$ **b** $m(x - n) = k$ **c** $p(x - q) = r$

**d** $h(x + k) = m$ **e** $z(x + y) = w$ **f** $t(x + r) = s$

**g** $u(x - d) = -f$ **h** $n(x - r) = -s$ **i** $p(x - t) = -q$

**j** $y(x + z) = -w$ **k** $k(g - x) = h$ **l** $p(m - x) = -n$

**m** $m(x - n) = 3m$ **n** $p(q - x) = r$ **o** $d(b - x) = a^2$

**p** $m\left(\frac{2}{m} - x\right) = n$ **q** $a\left(x + \frac{2}{a}\right) = b$ **r** $2z\left(\frac{3}{4z} + x\right) = 2{\cdot}5$

# Applying formulas 9H

A formula is an equation that relates two or more variables that are represented by pronumerals. We substitute into formulas to find values of required variables. This sometimes requires us to solve an equation or to rearrange a formula.

## Example

**1** The formula which finds the surface area ($A$) of a closed cylinder of radius $r$ and height $h$ is:

$A = 2\pi r(r + h)$

Find $A$ in cm$^2$ when $r = 4$ cm and $h = 6$ cm.

## Solution

$A = 2\pi r(r + h)$

Substitute values:

$r = 4$ cm, $h = 6$ cm

$A = 2 \times \pi \times 4 \times (4 + 6)$

$A = 2 \times \pi \times 4 \times 10$

$A = 80\pi$ cm$^2$ exact form

$A = 251{\cdot}33$ cm$^2$ to 2 decimal places

**2** The perimeter ($P$) of a rectangle is given by the formula:

$P = 2L + 2W$

where $L$ is the length and $W$ is the width.

Solve an equation to find $L$ in cm when $P = 70$ cm and $W = 20$ cm.

$P = 2L + 2W$

Substituting values:

$P = 70$ cm, $W = 20$ cm

$70 = 2L + 2 \times 20$

$70 = 2L + 40$

$70 - 40 = 2L + 40 - 40$

$30 = 2L$

$L = 15$ cm

**3** The kinetic energy ($E$) of a mass of $m$ kilograms moving at $v$ metres per second is given by:

$E = \frac{1}{2}mv^2$

By solving an equation, find $v$ when $E = 200$ and $m = 4$.

$E = \frac{1}{2}mv^2$

Substituting values we have:

$E = 200$, $m = 4$

$200 = \frac{1}{2} \times 4 \times v^2$

$200 = 2v^2$

$\frac{200}{2} = \frac{2v^2}{2}$

$v^2 = 100$

$v = \sqrt{100}$

$v = 10$ m/s

## Exercise 9H

**1** Find the value of the 'subject' of each formula by substituting values and simplifying:

**a** $E = IR$ where $I = 20$, $R = 60$

**b** $A = \frac{1}{2}bh$ where $b = 8$, $h = 10$

**c** $v = u + at$ where $u = 6$, $a = 10$, $t = 5$

**d** $a = \frac{F}{m}$ where $F = 600$, $m = 15$

**e** $E = \frac{1}{2}mv^2$ where $m = 5$, $v = 4$

**f** $s = \frac{1}{2}at^2$ where $a = 4$, $t = 5$

**g** $C = \frac{5(F-32)}{9}$ where $F = 68$

**h** $A = 4\pi r^2$ where $r = 5$

**i** $m = \sqrt{a \times b}$ where $a = 50, b = 8$

**j** $A = 2\pi r(r + h)$ where $r = 9, h = 11$

**k** $s = ut + 5t^2$ where $u = 4, t = 6$

**l** $c = \sqrt{a^2 + b^2}$ where $a = 5, b = 12$

**m** $V = \frac{4\pi r^3}{3}$ where $r = 6$

**n** $S = \frac{a}{(1-r)}$ where $a = 5, r = \frac{3}{4}$

**o** $A = \frac{(a+b)h}{2}$ where $a = 4, b = 6, h = 8$

**p** $V = \frac{\pi r^2 h}{3}$ where $r = 9, h = 10$

**q** $A = P(1+r)^n$ where $P = 500, r = 0{\cdot}1, n = 3$

**r** $I = \frac{PRT}{100}$ where $P = 400, R = 6, T = 8$

**s** $R = \frac{100I}{PT}$ where $I = 300, P = 1000, T = 2$

**t** $a = \sqrt{c^2 - b^2}$ where $c = 10, b = 8$

**u** $s = \frac{v^2 - u^2}{2a}$ where $v = 13, u = 12, a = 2{\cdot}5$

**v** $S = \frac{a(r^n - 1)}{(r-1)}$ where $a = 4, r = 2, n = 3$

**2** Substitute the given values into each formula and then solve the resultant equation for the remaining pronumeral:

**a** $d = vt$ where $d = 80, v = 16$

**b** $d = vt$ where $d = 150, t = 5$

**c** $E = IR$ where $E = 15, I = 3$

**d** $E = IR$ where $E = 16, R = 8$

**e** $v = u + at$ where $v = 15, a = 5, t = 2$

**f** $v = u + at$ where $v = 20, a = 6, u = 8$

**g** $v = u + at$ where $v = 16, t = 5, u = 6$

**h** $E = \frac{1}{2}mv^2$ where $E = 2000, v = 20$

**i** $E = \frac{1}{2}mv^2$ where $E = 1280, m = 10$

**j** $C = \frac{F-32}{1{\cdot}8}$ where $C = 95$

**k** $C = 2\pi r$ where $C = 16\pi$

**l** $A = \frac{1}{2}bh$ where $A = 75, b = 10$

**m** $A = \frac{1}{2}bh$ where $A = 24, h = 8$

**n** $A = \pi r^2$ where $A = 36\pi$

**o** $A = \frac{(a+b)h}{2}$ where $A = 63, a = 6, b = 8$

**p** $A = \frac{(a+b)h}{2}$ where $A = 48, a = 9, h = 6$

**q** $V = \pi r^2 h$ where $V = 64\pi, r = 2$

**r** $V = \pi r^2 h$ where $V = 27\pi, h = 3$

**s** $c^2 = a^2 + b^2$ where $a = 5, b = 12$

**t** $c^2 = a^2 + b^2$ where $c = 25, b = 24$

**3** The speed $v$ (m/s) that a skateboarder achieves when accelerating down a ramp is given by $v^2 = u^2 + 2as$, where $u$ is the initial launching velocity (in m/s), $a$ is the acceleration (in m/s$^2$) and $s$ is the distance covered down the slope (in metres).

**a** Find $v^2$ when $u = 3, a = 2, s = 4$ and hence find $v$.

**b** Find $s$ when $v = 6, u = 4, a = 2$.

**c** Find $u^2$ and then $u$, when $v = 5, a = 0{\cdot}5, s = 9$.

**4** The distance covered by the skateboarder in Question 3 after $t$ seconds is given by $s = ut + \frac{1}{2}at^2$.

**a** Find $s$ when $u = 2$, $a = 0.25$, $t = 10$.

**b** Find $u$ when $s = 50$, $t = 20$, $a = 0.1$.

**5** The formula $F = 1{\cdot}8C + 32$ converts temperatures in degrees Celsius ($C$) to degrees Fahrenheit ($F$).

**a** Convert the following Celsius temperatures to Fahrenheit:

**i** 0°C **ii** 40°C **iii** –10°C

**b** Solve equations to find the Celsius equivalent of the following Fahrenheit temperatures:

**i** 59°F **ii** 86°F **iii** 23°F

**6** A scoop of ice cream in a cone is assumed to be in the shape of a sphere. The volume of the ice cream is given by the formula $V = \frac{4}{3}\pi r^3$, where $r$ is the radius of the sphere.

**a** Find the volume in terms of $\pi$ of a scoop of ice cream with radius $r = 2$ cm.

**b** Express this result as an answer correct to 2 decimal places.

**c** Find $r$ when $V = 36\pi$ cm$^3$.

**7** The volume of a cone is given by the formula $V = \frac{\pi r^2 h}{3}$, where $r$ is the radius of the circular base and $h$ is the height.

**a** Find the volume of a cone with radius $r = 6$ cm and height $h = 8$ cm in *exact form*. Express this result as an answer correct to 2 decimal places.

**b** Find $h$ when $V = 960\pi$ (cm$^3$) and $r = 12$ cm.

**c** Find $r$ when $V = 75\pi$ (cm$^3$) and $h = 9$ cm.

**8** The simple interest, $I$, earned when a principal of $P$ is invested for $T$ years at $R$(%) p.a. is given by $I = \frac{PRT}{100}$.

**a** Find $I$ when $P = 1500$, $R = 5$(%), $T = 4$.

**b** Find $T$ when $I = 300$, $P = 1000$, $R = 6$(%).

**9** When money \$$P$ (the principal) is invested in an interest-bearing account paying $r$% interest compounded annually, the amount \$$A$ to which it grows is given by $A = P\left(1 + \frac{r}{100}\right)^n$, where $n$ is the number of years.

**a** Find $A$ if \$2000 is invested at 8% ($r = 8$) p.a. for 5 years.

**b** What principal amount was invested at 6·5% p.a. to produce an amount of \$8000 over 10 years?

**c** If the interest rate is 7% p.a., use trial and error to find the time it takes (to the nearest year) for the principal \$$P$ to double in value.

# 9I Transposing formulas

Often we wish to switch a formula around to make another variable the subject. This enables us to perform repeated calculations of the variable of interest.

**Example**

**1** The distance $d$ km covered by a car travelling at $v$ km/h for $t$ hours is given by the formula $d = vt$.

Transpose the formula to make $v$, and then $t$, the subject.

**Solution**

Solving for $v$:

$$d = vt$$

$$\frac{d}{t} = \frac{vt}{t} \quad \text{dividing both sides by } t$$

$$v = \frac{d}{t}$$

Solving for $t$:

$$d = vt$$

$$\frac{d}{v} = \frac{vt}{v} \quad \text{dividing both sides by } v$$

$$t = \frac{d}{v}$$

**2** The formula $v = u + at$ is used to find the speed of an accelerating body.

Transpose the formula to make $u$, then $t$, the subject.

Solving for $u$:

$$v = u + at$$

$$v - at = u + at - at \quad -at \text{ from both sides}$$

$$v - at = u$$

i.e. $u = v - at$

Solving for $t$:

$$v - u = at \quad -u \text{ from both sides}$$

$$\frac{at}{a} = \frac{v - u}{a} \quad \div \text{ both sides by } a$$

$$t = \frac{v - u}{a}$$

**3** The volume of a cone of radius $r$ and height $h$ is given by the formula $V = \frac{1}{3}\pi r^2 h$.

Transpose this formula to make $h$, then $r$, the subject.

Solving for $h$:

$$V = \frac{1}{3}\pi r^2 h$$

$$3V = \pi r^2 h \quad \times \text{ both sides by } 3$$

$$h = \frac{3V}{\pi r^2} \quad \div \text{ both sides by } \pi r^2$$

Solving for $r$:

$$3V = \pi r^2 h \quad \times \text{ both sides by } 3$$

$$r^2 = \frac{3V}{\pi h} \quad \div \text{ both sides by } \pi h$$

$$r = \sqrt{\frac{3v}{\pi h}}$$

## Exercise 9I

**1** In each case, transpose the given formula to make the required variable the subject:

**a** $F = ma$, transpose for $m$ first, and then for $a$

**b** $E = IR$, transpose for $R$ first, and then for $I$

**c** $A = LW$, transpose for $L$ first, and then for $W$

**d** $C = \pi D$, transpose for $D$

**e** $V = LWH$, transpose in turn to make each of $L$, $W$, $H$ the subject

**f** $A = 2\pi rh$, transpose for $r$ and then for $h$

**g** $100I = PRT$, transpose in turn to make each of $P$, $R$, $T$ the subject

**h** $V = U + AT$, transpose in turn to make each of $U$, $A$, $T$ the subject

**i** $V = L^2H$, transpose in turn to make $H$ and then $L$ the subject

**j** $V = \pi r^2 h$, transpose to make $h$ and then $r$ the subject

**k** $s = \dfrac{(u+v)t}{2}$, transpose to make $t$, then each of $u$ and $v$, the subject

**l** $v^2 = u^2 + 2as$, transpose to make $a$, then $s$, the subject

**m** $v^2 = u^2 + 2as$, transpose to make $u^2$ the subject; from this make $u$ the subject

**n** $c^2 = a^2 + b^2$, transpose to make $c$, then each of $a$ and $b$, the subject

**o** $A = 2\pi r^2 + 2\pi rh$, transpose to make $h$ the subject

**p** $T^2 = 2\pi \times \dfrac{l}{g}$, transpose to make $l$ the subject, then transpose to make $g$ the subject

**q** $F = \dfrac{GMm}{d^2}$, transpose to make $M$ the subject, then transpose to make $m$ the subject

**r** $F = \dfrac{GMm}{d^2}$, transpose to make $d^2$, then $d$, the subject

**s** $V = \dfrac{4\pi r^3}{3}$, transpose to make $r^3$, then $r$, the subject (you will need to use the cube root operator $\sqrt[3]{\ }$ for this last step)

**t** $A = P(1 + r)^3$, transpose to make $P$, then $r$, the subject

**2** Temperatures measured in Fahrenheit ($F$) are related to those measured in Celsius ($C$) by the formula $F = 1{\cdot}8C + 32$.

**a** Transpose the formula to make $C$ the subject.

**b** Convert the following Fahrenheit temperatures to Celsius:

**i** 32 **ii** 20 **iii** 50

**c** Does doubling any temperature in Fahrenheit mean that the corresponding Celsius temperature doubles? Explain your answer with an example.

**3** The kinetic energy $E$ of a skateboarder travelling at $v$ (metres per second), is given by the formula $E = \dfrac{1}{2}mv^2$.

**a** Transpose this formula to make $m$ the subject and so find $m$ if $E = 750$ and $v = 5$.

**b** Transpose the formula to make $v$ the subject and so find $v$ if $E = 560$ and $m = 70$.

# Puzzles

**1** Solve the simple one-step equations and match the letter in the equation to the correct solution below to answer the riddle:

**Why did Dracula have his computer repaired?**

| | | | |
|---|---|---|---|
| $A - 3 = 7$ | $B + 12 = 28$ | $D - 5 = 1$ | $E + 7 = 8$ |
| $H + \frac{1}{4} = -\frac{7}{4}$ | $I - 3{\cdot}5 = 1{\cdot}5$ | $-5M = 20$ | $12N = -72$ |
| $-5O = 45$ | $-9P = -27$ | $-3R = -12$ | $-7S = -49$ |
| $\frac{T}{1} = 1$ | $\frac{V}{-4} = 2$ | $\frac{W}{-5} = -3$ | $\frac{Y}{-6} = -3$ |

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| ___ | ___ | | ___ | ___ | ___ | ___ | ___ | ___ | |
| –2 | 1 | | 15 | 10 | –6 | 8 | 1 | 6 | |
| ___ | ___ | | ___ | ___ | ___ | ___ | ___ | ___ | ___ |
| 8 | –9 | | 5 | –4 | 3 | 4 | –9 | –8 | 1 |
| ___ | ___ | ___ | | ___ | ___ | ___ | ___ | ___ | |
| –2 | 5 | 7 | | 16 | 18 | 8 | 1 | 7 | |

**2** Solve the following equations and match the letter in the equation to the correct solution below to answer the riddle:

**What is 20 – 20 – 20 – 20 – 20 – 20?**

| | | |
|---|---|---|
| $-2(C - 3) = 22$ | $-6(E - 5) = -12$ | $-5(F + 2) = 50$ |
| $2(I + 4) = 12$ | $7(M - 1) = -14$ | $3(2N + 5) = 21$ |
| $2(9 - 3O) = 30$ | $2(2 - 3P) = -14$ | $3(3 - 4R) = 45$ |
| $\frac{2(S - 3)}{7} + 4 = 6$ | $\frac{4(T - 5)}{5} - 1 = 7$ | $\frac{9(V - 5)}{2} - 6 = 12$ |

| | | | | | | |
|---|---|---|---|---|---|---|
| ___ | ___ | ___ | ___ | ___ | ___ | ___ |
| 3 | 7 | –3 | –12 | 7 | –8 | 15 |
| ___ | ___ | ___ | ___ | ___ | ___ | ___ |
| –1 | –2 | 1 | 10 | 15 | 7 | –3 |
| ___ | ___ | ___ | ___ | ___ | ___ | |
| 9 | 2 | 10 | 2 | –2 | 1 | |

**3** Solve the following inequations and match the letter in the equation to the correct solution below to answer the riddle:

**What did the girl zombie do when the boy zombie rolled his eyes at her?**

**A** $4x \geq -24$

**B** $\frac{x}{7} \geq -3$

**C** $-\frac{3}{4}x \leq -12$

**D** $-4x - 8 \geq -20$

**E** $\frac{2x}{5} + 3 < 5$

**H** $-\frac{7}{2}x - 2 \leq -9$

**I** $\frac{3x+8}{5} < 4$

**K** $\frac{2x+8}{3} > 6$

**L** $\frac{2(x-3)}{3} + 7 > 15$

**M** $2(x+4) < 10$

**N** $-3(x+6) > -15$

**O** $5(x-5) > 2(x+1)$

**P** $3(x+2) > 2(x+4)$

**R** $\frac{3(2x-5)}{13} - 8 \leq -5$

**T** $\frac{5(3x+1)}{2} - 8 > 2$

**U** $\frac{2x+1}{5} + \frac{x-4}{3} > 4$

| | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $x > 2$ | $x < 4$ | $x \geq 16$ | $x > 5$ | $x < 5$ | $x \leq 3$ | | $x > 1$ | $x \geq 2$ | $x < 5$ | $x < 1$ | | | | | | | |
| $x > 7$ | $x > 2$ | | $x \geq -6$ | $x < -1$ | $x \leq 3$ | | $x \leq 9$ | $x > 9$ | $x > 15$ | $x > 15$ | $x < 5$ | $x \leq 3$ | | | | | |
| $x > 1$ | $x \geq 2$ | $x < 5$ | $x < 1$ | | $x \geq -21$ | $x \geq -6$ | $x \geq 16$ | $x > 5$ | | | | | | | | | |

# Applications

## Car distances

The distance $d$ km covered by a car travelling at velocity $v$ km/h when it travels for $t$ hours is given by the formula $d = vt$.


**a** Write down the formula that gives the distance travelled by a car travelling at $v = 60$ km/h. Using this formula, copy and complete the following table:

| $t$ (time taken) | 0 | 2 | 4 | 6 | 8 | 10 |
|---|---|---|---|---|---|---|
| $d$ (km travelled) | | | | | | |

**b** Transpose the formula in part a to make $t$ the subject.

**c** Using the transposed formula in part b, copy and complete the following table:

| $d$ (km travelled at 60 km/h) | 0 | 30 | 60 | 180 | 420 | 540 |
|---|---|---|---|---|---|---|
| $t$ (time taken) | | | | | | |

**d** Return to the formula $d = vt$ and let the distance of a certain journey $d = 300$ km. Substitute $d = 300$ into this formula and transpose the formula to make $v$ the subject.

**e** Using the transposed formula in part d, copy and complete the following table:

| $t$ (time to travel 300 km) | 1 | 2 | 3 | 4 | 5 | 6 | 8 | 10 |
|---|---|---|---|---|---|---|---|---|
| $v$ (speed needed) | | | | | | | | |

**f** Use this diagram 

to help you complete these important formulas.

**i** $d =$_____ **ii** $v =$ _____ **iii** $t =$ _____

## Distance out to sea

The distance $d$ km a person can see out to sea when their eyes are $h$ metres above sea level is recorded in the table below.

| $h$ | 0 | 5 | 10 | 15 | 20 | 25 | 30 |
|---|---|---|---|---|---|---|---|
| $d$ | 0 | 8 | 11·4 | 14 | 16 | 18 | 19·7 |

**a** Take the square root of the $h$ values and produce a new table of $\sqrt{h}$ values versus $d$.

**b** By dividing each $d$ value by its corresponding $\sqrt{h}$ value, and averaging results as before, develop a formula that relates $d$ to $h$.

# Enrichment

**Example** **Solution**

**1** Solve the following equations for $x$:

**a** $\frac{3}{2x} = \frac{4}{5}$

We use the method of 'cross multiplication':

$$2x \times 4 = 5 \times 3$$
$$8x = 15$$
$$x = \frac{15}{8} = 1\frac{7}{8}$$

This is equivalent to first multiplying both sides by $2x$ and then by 5.

**b** $\frac{3}{x+1} = \frac{2}{x+5}$

Using cross multiplication:

$$3 \times (x+5) = 2 \times (x+1)$$
$$3x + 15 = 2x + 2$$
$$x + 15 = 2$$
$$x = -13$$

**2** Solve the equation $\frac{4}{5x} - \frac{3}{2x} = \frac{1}{2}$.

Multiply through by $10x$ to clear the fractions:

$$\frac{4}{5x} \times 10x - \frac{3}{2x} \times 10x = \frac{1}{2} \times 10x$$
$$8 - 15 = 5x$$
$$5x = -7$$
$$x = -\frac{7}{5} = -1\frac{2}{5}$$

**1** Solve each of the following equations for $x$:

**a** $\frac{7}{3x} = \frac{1}{5}$ **b** $-\frac{2}{3} = \frac{9}{4x}$ **c** $\frac{8}{5x} = -3$

**d** $\frac{4}{x-6} = \frac{3}{x-5}$ **e** $\frac{1}{2x-1} = \frac{4}{3x+5}$ **f** $\frac{3}{4-3x} = \frac{-2}{4x-3}$

**g** $\frac{x-4}{x+1} = \frac{2}{5}$ **h** $\frac{2x-3}{5x+1} = \frac{-3}{4}$ **i** $\frac{2-5x}{4x+1} = -3$

**j** $\frac{3}{7x} - \frac{2}{x} = 1$ **k** $\frac{5}{3x} - \frac{1}{6x} = \frac{1}{2}$ **l** $\frac{-3}{4x} + \frac{5}{8x} = \frac{1}{4}$

**m** $\frac{2}{3x} + \frac{3}{4x} = -\frac{5}{6}$ **n** $\frac{1}{x} + \frac{2}{x+1} = \frac{1}{2x}$ **o** $\frac{1}{x} - \frac{2}{x+1} = \frac{-1}{x(x+1)}$

**p** $\frac{1}{x+1} + \frac{2}{x-2} = \frac{9}{(x+1)(x-2)}$ **q** $\frac{2}{x-1} - \frac{1}{x+3} = \frac{11}{(x-1)(x+3)}$

**r** $\frac{2}{x-1} + \frac{3}{x+2} = \frac{2x+7}{(x-1)(x+2)}$ **s** $\frac{2x}{x+1} + \frac{4}{x-4} = \frac{2x^2+8}{(x+1)(x-4)}$

**t** $\frac{3x}{x-2} + \frac{2x}{x+1} = \frac{5x^2+3x+12}{(x-2)(x+1)}$ **u** $\frac{4x}{x-1} - \frac{3x}{x+1} = \frac{x^2+7}{(x-1)(x+1)}$

**Example** **Solution**

**3** Solve the following equations for $x$:

**a** $\frac{a}{x} - b = c$

$$\frac{a}{x} - b = c$$
$$\frac{a}{x} = \frac{c+b}{1} \quad \text{add } b \text{ to both sides}$$
$$x(c+b) = a \quad \text{cross multiply}$$
$$x = \frac{a}{c+b}$$

**b** $\frac{x}{a} + \frac{b}{c} = \frac{d}{e}$

$$\frac{x}{a} \times ace + \frac{b}{c} \times ace$$
$$= \frac{d}{e} \times ace \quad \text{clearing all fractions}$$
$$xce + bae = dac$$
$$xce = dac - bae$$
$$x = \frac{dac - bae}{ce}$$

**c** $\frac{ax+b}{cx+d} = \frac{2}{3}$

$$2(cx+d) = 3(ax+b) \quad \text{cross multiplying}$$
$$2cx + 2d = 3ax + 3b$$
$$2cx - 3ax = 3b - 2d$$
$$x(2c - 3a) = 3b - 2d$$
$$x = \frac{3b-2d}{2c-3a}$$

**2** Solve the following equations for $x$:

**a** $\frac{m}{x} + n = k$

**b** $\frac{r}{2x} - s = t$

**c** $h - \frac{k}{4x} = -n$

**d** $\frac{x}{b} - \frac{a}{c} = f$

**e** $\frac{a}{b} - \frac{x}{d} = \frac{c}{e}$

**f** $\frac{3x}{2m} + \frac{p}{3n} = q$

**g** $ax - c = bx + d$

**h** $f - gx = hx + k$

**i** $\frac{a}{bx+c} = \frac{d}{ex+f}$

**j** $\frac{px-q}{mx+c} = \frac{1}{2}$

**k** $\frac{f-gx}{kx-v} = \frac{a}{b}$

**l** $\frac{2x-a}{b-3x} = \frac{b}{a}$

**m** $\frac{a}{bx} + \frac{c}{x} = d$

**n** $\frac{m}{px} - \frac{q}{sx} = e$

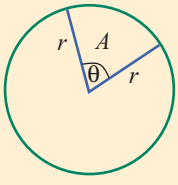
**o** $\frac{d}{fx} + \frac{g}{mx} = \frac{a}{b}$

**p** $\frac{a}{x} + \frac{b}{x+1} = \frac{c}{x+1}$

**q** $\frac{p}{x+2} + \frac{q}{x} = \frac{r}{x}$

**r** $\frac{m}{x-3} - \frac{n}{x+1} = \frac{p}{x-3}$

**3** The area $A$ cm$^2$ of a sector of a circle of radius $r$ cm, with an angle $\theta$ subtended at the centre, is given by $A = \frac{\theta}{360} \times \pi r^2$.




**a** Rearrange the formula to make $\theta$ the subject and so find $\theta$ when $A = \pi$ and $r = 6$.

**b** Rearrange the formula to make $r$ the subject and so find $r$ when $\theta = 90^\circ$ and $A = 49\pi$.

# Revision/Assessment

## Exercise 9A

**1** Solve the following equations:

**a** $p - \frac{3}{4} = -\frac{1}{2}$ **b** $q - 2{\cdot}5 = 5{\cdot}5$ **c** $t - 7{\cdot}25 = -3{\cdot}75$

**d** $3z = -15$ **e** $-9q = 72$ **f** $-6h = -35$

**g** $\frac{y}{10} = -8$ **h** $\frac{y}{-6} = 9$ **i** $\frac{y}{-8} = -12$

## Exercise 9B

**2** Solve the following equations:

**a** $-7a - 19 = 16$ **b** $-5b + 9 = 29$ **c** $7 - 5c = -18$

**d** $\frac{a-5}{6} = -2$ **e** $\frac{2x+1}{3} = 5$ **f** $\frac{4-3y}{4} = -2$

**g** $\frac{5m}{6} + 2 = -3$ **h** $\frac{2z}{5} - 4 = -6$ **i** $\frac{3}{8}t - 5 = -14$

## Exercise 9C

**3** Solve the following equations:

**a** $2(4m - 7) = 10$ **b** $5(2n - 1) = 7$ **c** $7(12 - 4q) = 28$

**d** $\frac{2(x-4)}{7} + 7 = 11$ **e** $\frac{7(x-5)}{3} - 4 = 10$ **f** $\frac{5(4-x)}{3} + 7 = 22$

## Exercise 9D

**4** Solve the following equations:

**a** $4(2 - x) = -3(x + 4)$ **b** $-3(q - 7) = 4(5 - q)$ **c** $4(3x - 1) = -5(6 - 2x)$

**d** $\frac{3x}{2} + \frac{x}{8} = 13$ **e** $\frac{2x}{3} - \frac{3x}{4} = 2$ **f** $\frac{3x}{2} - \frac{2x}{5} = 22$

**g** $\frac{x-2}{2} - \frac{3x-1}{4} = 2$ **h** $\frac{2x-1}{5} + \frac{3-2x}{3} = 4$ **i** $\frac{5x-4}{9} - \frac{7x+2}{3} = -10$

## Exercise 9F

**5** Solve the following inequations:

**a** $4x - 8 \geq -24$ **b** $9 + 2y < 15$ **c** $6z - 2 \leq -32$

**d** $-2a + 13 < 9$ **e** $-5b + 4 \geq -11$ **f** $8 - 5c < 18$

**g** $\frac{2a}{-5} + 5 \leq -7$ **h** $\frac{-7b}{6} - 1 \leq -8$ **i** $\frac{4z}{-3} - 5 \leq -13$

**j** $-2(x + 4) < 12$ **k** $-3(x + 6) > -18$ **l** $-5(x + 8) \geq 35$

**m** $\frac{3x+2}{5} > 4$ **n** $\frac{2x+8}{4} \leq -3$ **o** $\frac{3x-9}{7} \geq -3$

**p** $3(x + 5) \geq 2(x - 1)$ **q** $7(z + 3) < 3(z - 1)$ **r** $-3(y - 4) > 2(y - 3)$

**s** $\frac{3(x-4)}{2} - 1 \geq 2$ **t** $\frac{5(9-x)}{3} + 1 < 11$ **u** $\frac{7(x-4)}{6} - 3 \leq 4$

**v** $\frac{2x-5}{3} - \frac{3x-2}{5} \geq 1$ **w** $\frac{4x-3}{7} - \frac{5x-3}{2} > -3$ **x** $\frac{5x+8}{7} - \frac{3-2x}{9} \geq 2$

## Exercise 9G

**6** Solve the following literal equations for $x$:

**a** $\frac{ax}{b} + c = d$ **b** $\frac{nx}{m} - p = -q$ **c** $\frac{qx}{-p} - r = t$

**d** $a(x-b) = -d$ **e** $n(r-x) = -s$ **f** $-p(x-t) = q$

**g** $\frac{x}{m} + n = -n$ **h** $\frac{x}{h} - h = -2h$ **i** $\frac{x}{p} + p = \frac{1}{2}p$

## Exercise 9H

**7** Substitute the given values and solve for the remaining variable in each formula:

**a** $A = 2\pi r(r+h)$ where $r = 7$, $h = 9$ **b** $s = ut + 5t^2$ where $u = 3$, $t = 4$

**c** $c = \sqrt{a^2 + b^2}$ where $a = 7$, $b = 24$ **d** $V = \frac{4\pi r^3}{3}$ where $r = 9$

**e** $C = \frac{F-32}{1{\cdot}8}$ where $C = 30$ **f** $C = 2\pi r$ where $C = 12\pi$

**g** $A = \frac{1}{2}bh$ where $A = 80$, $b = 10$ **h** $A = \pi r^2$ where $A = 49\pi$

## Exercise 9I

**8** Transpose each of the following formulas to make the required variable the subject:

**a** $A = 2\pi rh$, transpose to make $r$ the subject

**b** $100I = PRT$, transpose to make $T$ the subject

**c** $v = u + at$, transpose make $a$ the subject

**d** $V = L^2H$, transpose to make $L$ the subject

**e** $V = \pi r^2 h$, transpose to make $r$ the subject

**f** $s = \frac{(u+v)t}{2}$, transpose to make $u$ the subject

**g** $v^2 = u^2 + 2as$, transpose to make $u$ the subject

**h** $F = \frac{k}{d^2}$, transpose to make $d$ the subject

**i** $V = \frac{4\pi r^3}{3}$, transpose to make $r$ the subject

**j** $A = P(1+r)^3$, transpose to make $r$ the subject

**9** When an amount of money $P$ (the principal) is invested in an account that pays $R$ per cent per annum compound interest, the amount $A$ that it grows to after $n$ years is given by the formula $A = P\left(1 + \frac{R}{100}\right)^n$. If \$1000 ($P$) grows to \$1210 ($A$) after 2 years ($n$), find the compound interest rate $R$.

CHAPTER

# 10

# Linear Graphs

Linear programming was developed in the US during World War II to optimise military-related scheduling and planning tasks. George Dantzig, who was a professor at Stanford University at the time, is credited with the development of this field of mathematics, and was the inventor of the simplex method of solving linear programming problems. The petroleum industry was the first civilian industry to use linear programming.

The mining industry in Solomon Islands makes extensive use of linear programming to ensure management decisions on the supply, use and maintenance of equipment, and the extraction, refining and export of minerals can be achieved economically and according to shipping deadlines.

## This chapter covers the following skills:

- Plotting points on a set of Cartesian axes
- Reflecting sets of points about a given line
- Plotting lines using a table of values
- Graphing lines using $x$- and $y$-intercepts
- Finding gradients of lines
- Graphing lines using the gradient and $y$-intercept
- Finding the equation of a line through two given points
- Graphing regions in the Cartesian plane

## Specific Learning Outcomes (SLO)

Learners should be able to:

**9.10.1.1** Name and identify parts of the Cartesian plane: $x$- and $y$-axes, the origin (0, 0), and coordinates $(x, y)$.

**9.10.2.1** Draw Cartesian planes, and plot and identify the coordinates of points on the plane.

**9.10.3.1** Plot straight line graphs using equations and tables of linear equations.

**9.10.4.1** Draw linear graphs representing practical problems using $x$- and $y$-values.

**9.10.5.1** Calculate the values for $x$- and $y$-intercepts for linear equations and draw the graphs.

**9.10.6.1** Find the equations for given vertical and horizontal lines on a graph.

**9.10.6.2** Sketch and graph equations that are given in the form of $y = c$ and $x = c$.

**9.10.7.1** Define the term 'gradient' for a linear graph as the rise over the run.

**9.10.8.1** Calculate the gradient of a linear graph given two points on the graph.

**9.10.8.2** Find the gradients of graphs using the formula:

$$m = \frac{y^2 - y^1}{x^2 - x^1}$$

**9.10.9.1** Sketch linear graphs for given equations in the form of: $y = mx + c$ by finding: the $y$-intercept ($c$) and the gradient ($m$).

**9.10.10.1** Calculate the gradients for given linear graphs.

**9.10.10.2** Find the gradients and $y$-intercepts for given linear equations.

**9.10.10.3** Find the equations when gradients and $y$-intercepts are given.

**9.10.10.4** Re-arrange equations into the form $y = mx + c$ then find the gradient, $y$-intercept and graph the equations.

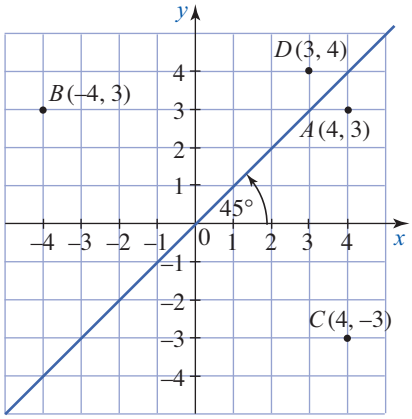
**9.10.11.1** Find the equation of given linear graphs by finding the $y$-intercepts and the gradient.

**9.10.11.2** Finding the equation of lines that pass through given points.

# 10A Plotting points on the Cartesian plane

A **Cartesian plane** is made up of two number lines intersecting at right angles. We use ***x*** and ***y*-coordinates** to locate the positions of points in the plane, and label them with coordinates $(x, y)$. The **origin** $O$ is the point $(0, 0)$ where the lines intersect.

**Example**




**Solution**

Point $A$ has the coordinates $(4, 3)$, as its position is 4 on the $x$-axis and 3 on the $y$-axis.

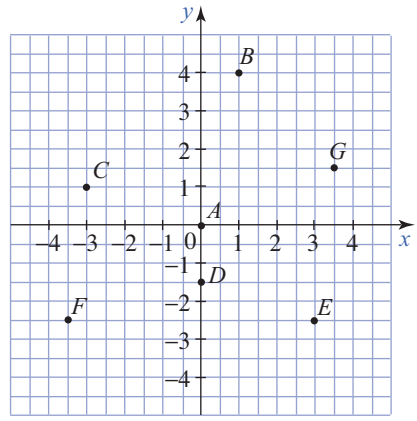
Point $B$ is the reflection of point $A$ across the $y$-axis and has coordinates $(–4, 3)$.

Point $C$ is the reflection of point $A$ across the $x$-axis and has coordinates $(4, –3)$.

Point $D$ is the reflection of point $A$ across the 45° line $y = x$ and has coordinates $(3, 4)$.

## Exercise 10A

**1** Write down the coordinates of each of the points shown on the grid:




**a** $A$ _____

**b** $B$ _____

**c** $C$ _____

**d** $D$ _____

**e** $E$ _____

**f** $F$ _____

**g** $G$ _____

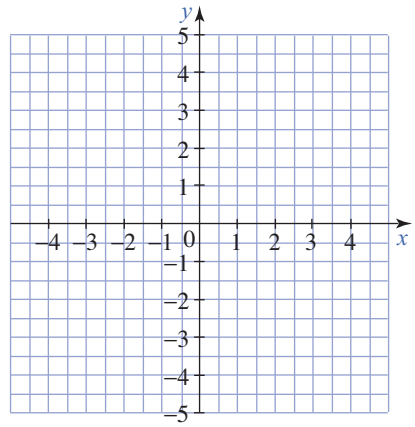
**2** Copy the grid and plot the points given below. Join the points in the order given to produce a shape:

START $(–4, 0)$, $(–2, 4)$, $(2, 4)$, $(4, 0)$, $(2, –4)$, $(–2, –4)$, $(–4, 0)$, STOP.

START $(–3, –2)$, $(3, 2)$, $(3, –2)$, $(–3, 2)$, $(–3, –2)$, $(0, –4)$, $(3, –2)$, $(–3, –2)$, STOP.

START $(3, 2)$, $(0, 4)$, $(–3, 2)$, $(3, 2)$, STOP.

Colour it in to produce different designs.



**3** Draw up a set of Cartesian axes on graph paper. Let 1 centimetre represent 1 unit, and allow the values on both the $x$- and $y$-axes to run from –6 to +6.

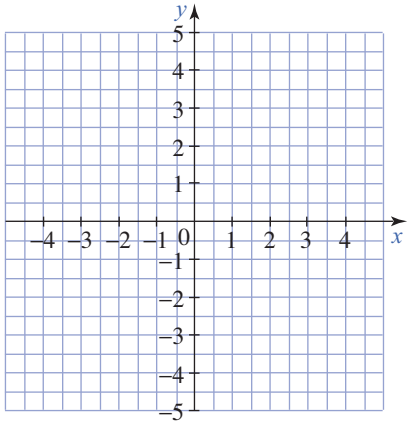
- **a** Plot the points (–5, –5), (–2, –2), (0, 0), (2, 2), (5, 5) and join these points with a straight line. The equation for this line is $y = x$ (where the $y$ value is equal to the $x$ value).
- **b** Plot the points (2, 0), (3, 3), (5, 4) and connect these points to form a triangle. What type of triangle have you drawn?
- **c** Plot the points (–2, 0), (–3, 3), (–5, 4) and connect these points to form a triangle. About which axis has the triangle in part a been reflected to obtain this triangle (the image)?
- **d** Next plot the points (0, 2), (3, 3), (4, 5) and connect these points to form a triangle. In terms of reflections, relate this triangle to the one in part b.
- **e** Finally plot the points (2, 0), (3, –3), (5, –4) and connect these points to form a triangle. In terms of reflections, relate this triangle to the one in part b.

**4** Draw another set of Cartesian axes on graph paper. Let 1 centimetre represent 1 unit and allow the values on both the $x$- and $y$-axes to run from –6 to +6.

- **a** Connect the points (0, 0), (4, –2), (5, –5), (2, –4) to form a kite.
- **b** Connect the points (0, 0), (4, 2), (5, 5), (2, 4) and, in terms of reflections, relate this image to the shape in part a.
- **c** Now draw the reflection of the kite in part a about the $y$-axis and label the coordinates of its vertices.
- **d** Finally, draw the reflection of the kite in part a about the line $y = x$ and label the coordinates of the vertices.

**5** **a** Draw a set of Cartesian axes as shown and plot the following sets of points:
(3, –5), (3, –4), (3, –2), (3, –1), (3, 0), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5)
Join the plots with a straight line to form a *line segment*. Because the $x$-coordinate of all points is 3, the equation of the line is $x = 3$ (where $-5 \leq y \leq 5$ ).

- **b** Reflect the points in part a about the $y$-axis and draw in the image of the line segment. What is the equation of this new line segment?
- **c** Reflect the points in part a about the line $y = x$ and draw the image of the line segment. What is the equation of this new line segment?
- **d** Reflect the points in part b about the line $y = x$ and draw in the image of the line segment. What is the equation of this new line segment?
- **e** What shape do the four line segments form?



# 10B Plotting straight line graphs

A **linear relation** is a set of points which when plotted fall on a straight line.

Often the graph of a linear relation is drawn starting from the rule or equation that relates the $x$ and $y$ values in each coordinate pair. Usually $x$ is called the **independent variable** and $y$ the **dependent variable** as $y$ is generally calculated from a rule which depends on $x$.

## Example

Plot the graph of $y = -2x + 6$.

## Solution

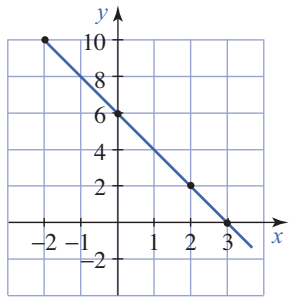
We work out the coordinates of a sample of points:

| $x$ | –2 | 0 | 2 | 3 |
|---|---|---|---|---|
| $y$ | 10 | 6 | 2 | 0 |

The $y$ values have been worked out from the equation; for example, when $x = -2$

$y = -2 \times -2 + 6 = 4 + 6 = 10$

Plot the points and notice they can be joined to form a straight line.




Note that the $y$-intercept is 6. This is the value of $y$ when $x = 0$.

The $x$-intercept is 3. This is the value of $x$ when $y = 0$.

## Exercise 10B

**1** Complete the following tables, using the given rules, then plot the points on separate sets of Cartesian axes. Join the points with a straight line, extending the line in each direction.

State the $x$-intercept and the $y$-intercept in each case:

**a** $y = 3x$

| $x$ | –1 | 0 | 1 | 2 |
|---|---|---|---|---|
| $y$ | | | | |

**b** $y = 2x - 4$

| $x$ | –1 | 0 | 1 | 2 |
|---|---|---|---|---|
| $y$ | | | | |

**c** $y = -x + 4$

| $x$ | –2 | –1 | 0 | 4 |
|---|---|---|---|---|
| $y$ | | | | |

**d** $y = -2x + 2$

| $x$ | –1 | 0 | 1 | 2 |
|---|---|---|---|---|
| $y$ | | | | |

**e** $y = 2x + 4$

| $x$ | –2 | 0 | 1 | 2 |
|---|---|---|---|---|
| $y$ | | | | |

**f** $y = 3x - 9$

| $x$ | –1 | 0 | 1 | 3 |
|---|---|---|---|---|
| $y$ | | | | |

**g** $y = x + 3$

| $x$ | –3 | –1 | 0 | 1 |
|---|---|---|---|---|
| $y$ | | | | |

**h** $y = -2x - 4$

| $x$ | –2 | 0 | 1 | 2 |
|---|---|---|---|---|
| $y$ | | | | |

**2** Find the $y$ values in each ordered pair, then graph the straight lines that represent the following rules. Give the $x$-intercept and the $y$-intercept in each case:

**a** $y = x + 2$, (–2, __), (0, __), (2, __)
**b** $y = x - 2$, (–2, __), (0, __), (2, __)
**c** $y = -x + 3$, (–2, __), (0, __), (3, __)
**d** $y = -x - 1$, (–1, __), (0, __), (3, __)
**e** $y = 4x$, (–2, __), (0, __), (2, __)
**f** $y = -0{\cdot}5x$, (–2, __), (0, __), (2, __)
**g** $y = 2x - 8$, (–2, __), (0, __), (4, __)
**h** $y = -2x + 3$ (–2, __), (0, __), (1·5, __)

**3** The monthly cost $C$ (dollars) of running my mobile phone is given by the rule $C = 0{\cdot}40t + 10$, where $t$ is the number of minutes spent making calls and \$10 is the monthly rental cost, which includes insurance.

**a** Copy and complete the following table, which gives the cost for various times spent making calls.

| $t$ | 60 | 80 | 100 |
|---|---|---|---|
| $C$ | | | |

**b** On a set of axes of your own, plot the points whose coordinates are given by the values in the table. Don't forget to correctly scale and label your axes.

**c** Join the plots with a straight line. Extend the line back to the vertical axis (where $t = 0$), and continue it beyond $t = 100$.

**d** Use the rule to find the cost of making calls totalling 90 minutes.

**e** How many minutes worth of calls were made if the cost was \$56?

**4** The temperature $T$°C at a certain beach is given by $T = 18 + 2t$, where $t$ is the number of hours after 9 am on the given day.

**a** Copy and complete the following table, which gives the temperature up to 2 pm:

| $t$ | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| $T$ | | | | | | |

**b** Plot the points whose coordinates are given by the values in the table above on a set of axes of your own. Don't forget to correctly scale and label your axes.

**c** Join the plotted points with a straight line. Do not extend the line this time.

**d** From your graph, read off the temperature at 1:30 pm.

**e** Use the rule (formula) that relates $T$ to $t$ to find the temperature at 1:30 pm.

**f** Solve the equation to find when the temperature is 23°C.

**5** When travelling to my holiday destination, the distance $d$ km remaining after travelling for $t$ hours is given by the equation $d = 200 - 80t$.

**a** Find the $t$- and $d$-intercepts of this linear relation, and use them to sketch the graph of $d$ against $t$. Label your axes and show appropriate scales.

**b** How far is the holiday destination from home, and how long does it take me to reach it?

**c** How far do I still have to go after an hour and a half?

**d** When I have travelled 50 km, how long will it take to reach my destination?

**e** How far have I travelled after an hour and a half?

**f** At what speed am I travelling to my holiday destination?

# 10C Using the *x*- and *y*-intercepts

The two most useful points to plot when graphing a straight line of the form $y = mx + c$ are the $x$-intercept and the $y$-intercept. This method does not work for equations of the form $y = mx$, $y = c$ and $x = c$, which are best dealt with by other methods.

## Example

Sketch the line $y - 2x + 4 = 0$ by finding the intercepts.

## Solution

Substitute $x = 0$ into the equation and solve to find the $y$-intercept:

$y - 2 \times 0 + 4 = 0$

$y + 4 = 0$

$y = -4$

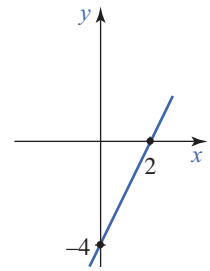
Substitute $y = -4$ and solve to find the $x$-intercept:

$-2x + 4 = 0$

$-2x = -4$

$x = 2$

Plot the two intercepts and draw a straight line through them.




## Exercise 10C

**1** On a set of axes of your own, sketch the following straight lines using the intercept method:

**a** $y = x + 1$ **b** $y = x + 2$ **c** $y = x + 4$ **d** $y = x - 5$

**e** $y = 2x - 8$ **f** $y = 3x - 6$ **g** $y = \frac{1}{2}x + 1$ **h** $y = 2x + 2$

**2** On a separate set of axes, sketch the following straight lines using the intercept method:

**a** $y = -x + 5$ **b** $y = -2x + 4$ **c** $y = -3x + 9$ **d** $y = -4x + 8$

**e** $y = -5x + 5$ **f** $y = -6x - 12$ **g** $y = -\frac{1}{2}x + 1$ **h** $y = -\frac{1}{4}x - 2$

**3** On a new set of axes, sketch the following straight lines using the intercept method:

**a** $3y + 4x = 12$ **b** $2y - 3x = 6$ **c** $3x - 2y = 6$ **d** $4y - 3x = 12$

**e** $x + 2y - 4 = 0$ **f** $2y - x + 4 = 0$ **g** $2x + y - 6 = 0$ **h** $2x - y - 6 = 0$

**i** $2x - y - 3 = 0$ **j** $x - 2y - 3 = 0$ **k** $3x + 4y - 8 = 0$ **l** $4x - 3y + 8 = 0$

**4** The temperature $T$°C on the ski slopes $t$ hours after midday is given by $T = 2 - 0{\cdot}5t$.

**a** Find the $T$- and $t$-intercepts of this linear relation, and use them to sketch the graph of $T$ against $t$. Label your axes and show appropriate scales.

**b** What is the temperature at midday and what is the temperature at 4 pm?

**c** Find the temperature at 6 pm.

**d** According to this model, when will the temperature drop to –3°C?

**e** Comment on how realistic the model is.

**5** The temperature $T$°C on the beach $t$ hours after midday is given by $T = 20 + 2t$.

**a** Find the $T$- and $t$-intercepts of this linear relation, and use them to sketch the graph of $T$ against $t$. Label your axes and show appropriate scales.

**b** What is the temperature at midday and at 4 pm?

**c** According to this model, when will the temperature rise to 30°C?

**d** Comment on how realistic the model is.

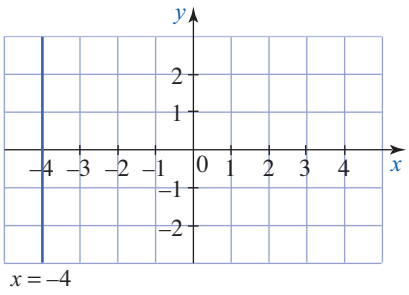
# Horizontal and vertical lines 10D

## Lines of the form $x = c$ and $y = c$

**Example** **Solution**

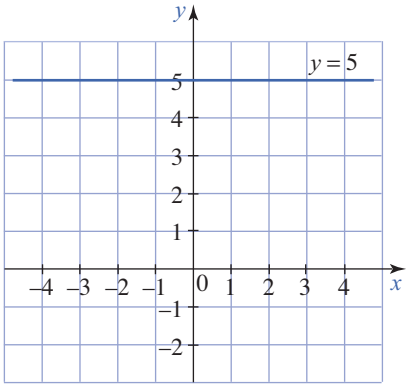
**1** A vertical line passing through –4 on the $x$-axis has the equation $x = -4$.

- All points on this line have an $x$-coordinate of –4.
- The line $x = -4$ is parallel to the $y$-axis and perpendicular to the $x$-axis.




**2** A horizontal line passing through 5 on the $y$-axis has the equation $y = 5$.

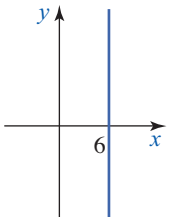
- All points on this line have a $y$-coordinate of 5.
- The line $y = 5$ is parallel to the $x$-axis and perpendicular to the $y$-axis.



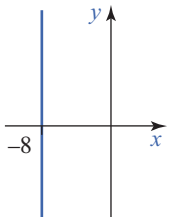

## Exercise 10D

**1** Write down the equations for each of the following lines:

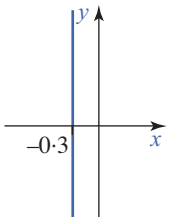
**a**



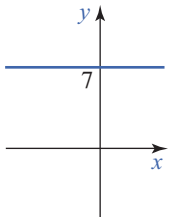

**b**




**c**




**d**




**2** On a set of axes of your own, sketch the graphs of the straight lines with equations:

**a** $x = 9$ **b** $x = -\frac{3}{4}$ **c** $x = \frac{1}{4}$ **d** $x = 0$

**3** On a set of axes of your own, sketch the graphs of the straight lines with equations:

**a** $y = 7$ **b** $y = -3$ **c** $y = -\frac{1}{4}$ **d** $y = 0$

**4** Write down the equation of the line which is:

**a** parallel to the $x$-axis and passes through the point (2, 3)

**b** parallel to the $y$-axis and passes through the point (6, –1)

**c** perpendicular to the line $x = 2$ and passes through the point (–2, 4)

**d** perpendicular to the line $y = -3$ and passes through the point (–4, 1)

# 10E Gradients of straight lines

The **gradient** or steepness of a straight line is the rate at which the line rises (or falls). The gradient is worked out by dividing the vertical **rise** by the horizontal **run** between any two convenient points.

- The gradient of any horizontal line is zero.
- The gradient of any vertical line is undefined.

## Example

Find the gradient of the line passing through the points:

**a** (1, 2) and (4, 8) **b** (1, 8) and (3, 4) **c** (0, 4) and (3, 0)

## Solution

**a** $m = \frac{\text{rise}}{\text{run}}$

$= \frac{8-2}{4-1}$

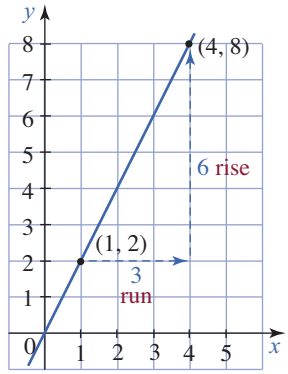
$= \frac{6}{3}$

$= 2$

Note that the gradient is positive as the line has an *uphill* slope to the right.

For every 3 units run to the right, the line rises 6 units.

$\Rightarrow$ For every 1 unit run to the right, the line rises 2 units.




**b** $m = \frac{4-8}{3-1}$

$= \frac{-4}{2}$

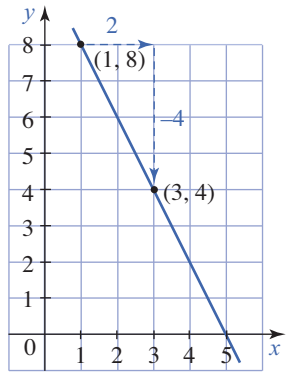
$= -2$

Note there is a *fall* or *negative rise.*

The gradient is negative as the line has a *downhill* slope to the right.

For every 2 units run to the right, the line falls 4 units.

$\Rightarrow$ For every 1 unit run to the right, the line falls 2 units.



## Solution

**c** $m = \dfrac{0-4}{3-0}$

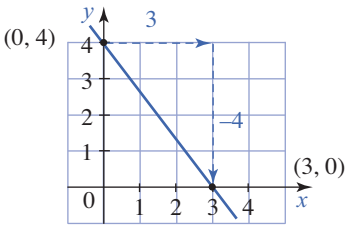
$= \dfrac{-4}{3}$ or $-1\dfrac{1}{3}$

Note there is a fall or *negative rise.*

The gradient is negative, as the line is has a *downhill* slope to the right.

For every 3 units run to the right, the line falls 4 units.

$\Rightarrow$ For every 1 unit run to the right, the line falls $1\frac{1}{3}$ units.




Where two points on a line have coordinates $(x_1, y_1)$ and $(x_2, y_2)$, the vertical rise is the difference between the $y$ values, and the horizontal run is the difference between the $x$ values.

The gradient can be quickly worked out by using the formula gradient $= \dfrac{y_2 - y_1}{x_2 - x_1}$.

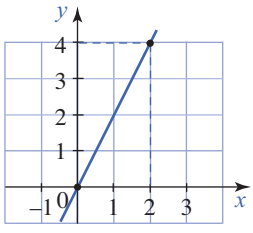
The gradient of the line joining the points (–4, 8 ) and (–1, 2) would be:

$$m = \frac{2-8}{-1--4}$$
$$= \frac{-6}{3}$$
$$= -2$$

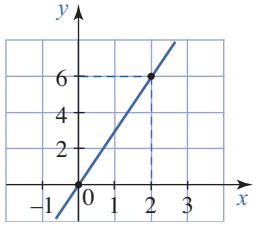
## Exercise 10E

**1** Find the gradients of the following lines:

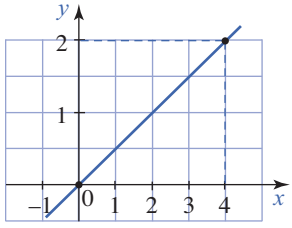
**a**



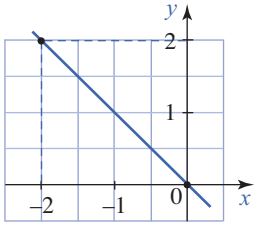

**b**



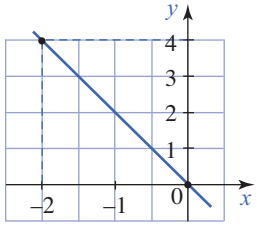

**c**



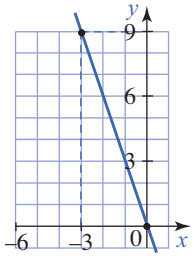

**d**




**e**



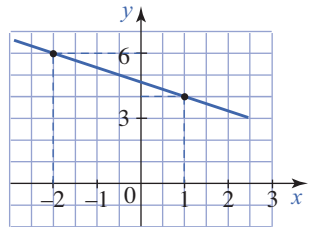

**f**



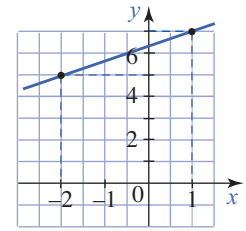
## 10E

**2** Find the gradients of the following lines:

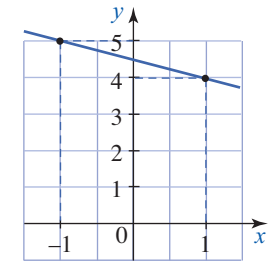
**a**




**b**



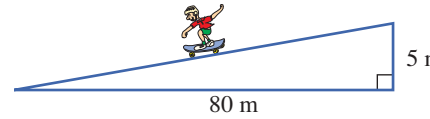

**c**




**3** Use the formula for gradient to find the gradient of the line joining each pair of points:

| | | | |
|---|---|---|---|
| **a** (1, 3), (2, 5) | **b** (2, 2), (5, 8) | **c** (–2, –1), (2, 7) | **d** (–1, 3), (2, 0) |
| **e** (–1, –3), (3, –7) | **f** (–1, 1), (3, –5) | **g** (–2, 0), (0, 4) | **h** (0, 6), (2, 0) |
| **i** (0, –5), (3, 0) | **j** (–4, 3), (2, 3) | **k** (4, 5), (4, 7) | **l** (–1, –4), ( 3, –4) |

**4** A skateboarder descends a slope that shows a rise of 5 metres for a run of 80 metres.




**a** Find the gradient of the incline expressed as a simple fraction.

**b** What distance forward would the skateboarder have travelled after dropping a vertical distance of 1 metre?

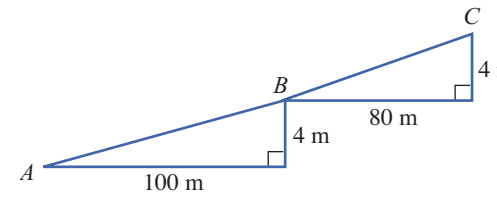
**5** Part of a hiking track up the side of a mountain has two sections, as shown in the diagram.

Expressing your answers as simple fractions, find:

**a** the gradient of $\overline{AB}$

**b** the gradient of $\overline{BC}$

**c** the average gradient when hiking from $A$ to $C$




# The gradient and *y*-intercept 10F

Linear equations of the form $y = mx + c$ can be graphed using the gradient ($m$) and $y$-intercept ($c$).

## Example

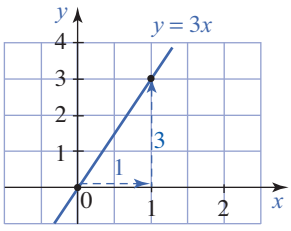
**1** Sketch:

**a** $y = 3x$ **b** $y = \frac{3}{4}x - 1$ **c** $y = -2x$ **d** $y = -3x + 4$

## Solution

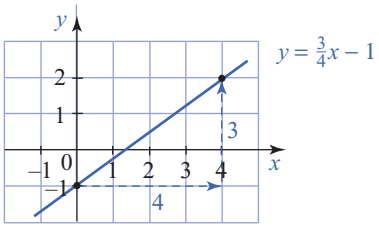
**1 a** The line $y = 3x$ has a gradient of 3 and a $y$-intercept of zero because, by substitution, when $x = 0$, $y = 0$, and when $x = 1$, $y = 3$.

To sketch the line, start at zero on the $y$-axis, run forward (positively) 1 unit, then rise 3 units. Draw a line through the two points (0, 0) and (1, 3).



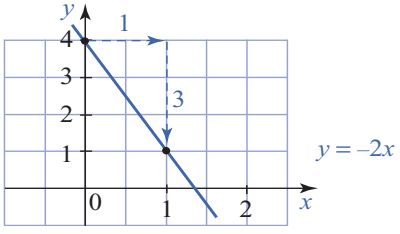

**b** The line $y = \frac{3}{4}x - 1$ has a gradient of $\frac{3}{4}$ and a $y$-intercept of $-1$ because, by substitution, when $x = 0$, $y = -1$, and when $x = 4$, $y = 2$.

To sketch the line, start at $-1$ on the $y$-axis, run forward (positively) 4 units, then rise 3 units. Draw a line through the two points (0, $-1$) and (4, 2).



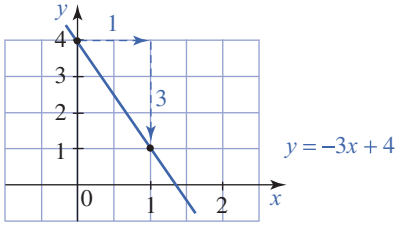

**c** The line $y = -2x$ has a gradient of $-2$ and a $y$-intercept of zero because, by substitution, when $x = 0$, $y = 0$, and when $x = 1$, $y = -2$.

To sketch the line, start at zero on the $y$-axis, run forward (positively) 1 unit, then drop 2 units, and draw a line through the two points (0, 0) and (1, $-2$).




**d** The line $y = -3x + 4$ has a gradient of $-3$ and a $y$-intercept of 4 because, by substitution, when $x = 0$, $y = 4$, and when $x = 1$, $y = 1$.

To sketch the line, start at 4 on the $y$-axis, run forward (positively) 1 unit, then drop 3 units. Draw a line through the two points (0, 4) and (1, 1).



# 10F

## Example

**2** A line is given by the equation $2y + x - 4 = 0$.

Rearrange this equation and read off the gradient and the $y$-intercept.

Use this information to sketch the line.

## Solution

**2** $2y + x - 4 = 0$

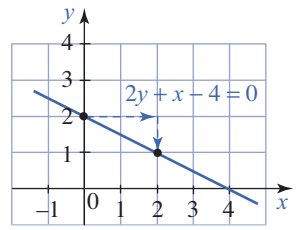
$+4 \quad +4$ add 4 to both sides

$2y + x - 4 + 4 = 0 + 4$ adding to both sides

$2y + x = 4$

$2y + x - x = 4 - x$ take $x$ from both sides

$y = -\frac{1}{2}x + 2$




Gradient is $-\frac{1}{2}$, the $y$-intercept is 2.

## Exercise 10F

**1** On a set of axes of your own, sketch the following straight lines by using the gradient and $y$-intercept method. For those involving fractions, use a run (forward) which is the denominator:

**a** $y = 3x$ **b** $y = 2x$ **c** $y = x$ **d** $y = \frac{1}{2}x$

**e** $y = \frac{1}{4}x$ **f** $y = \frac{4}{3}x$ **g** $y = \frac{5}{2}x$ **h** $y = \frac{2}{5}x$

**i** Complete this statement. The positive number in front of $x$, called the *coefficient* of $x$, specifies the $g$_____.

**2** On a separate set of axes, sketch the following straight lines by using the gradient and $y$-intercept method. For those involving fractions, use a run (forward) which is the denominator:

**a** $y = -2x$ **b** $y = -3x$ **c** $y = -x$ **d** $y = -\frac{1}{4}x$

**e** $y = -\frac{1}{2}x$ **f** $y = -\frac{3}{4}x$ **g** $y = -\frac{2}{3}x$ **h** $y = -\frac{3}{2}x$

**i** Complete this statement. The negative number in front of $x$, called the *coefficient* of $x$, specifies the $g$_____.

**3** **a** On a new set of axes, sketch the following straight lines by using the gradient and $y$-intercept method:

**i** $y = 3x$ **ii** $y = 3x + 1$ **iii** $y = 3x - 4$ **iv** $y = 3x + 5$

**b** What happens to the basic graph of $y = 3x$ when a number is added or subtracted?

**c** What is the gradient and the $y$-intercept for each of the above graphs?

**4** **a** On a separate set of axes of your own, sketch the following straight lines by using the gradient and $y$-intercept method:

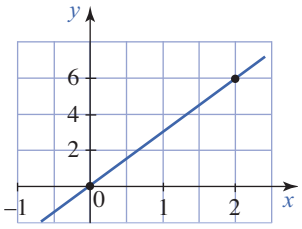
**i** $y = -2x$ **ii** $y = -2x + 2$ **iii** $y = -2x - 1$ **iv** $y = -2x - 3$

**b** What happens to the basic graph of $y = -2x$ when a number is added or subtracted?

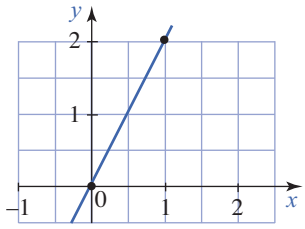
**c** What is the gradient and the $y$-intercept for each of the above graphs?

**5** Read off the gradient and the $y$-intercept from the graph and then write down the equations of the following lines:

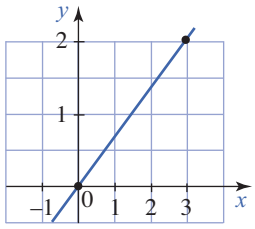
**a**



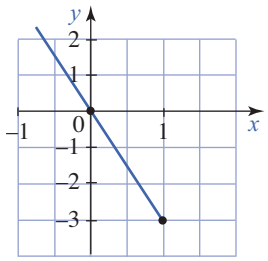

**b**



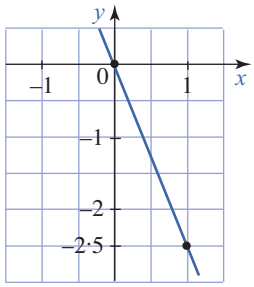

**c**



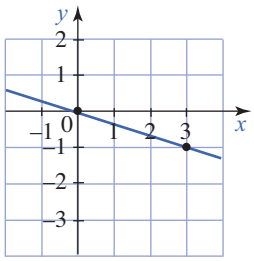

**d**




**e**

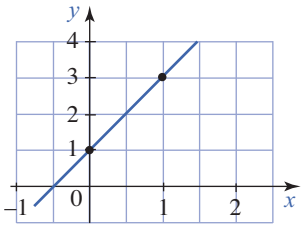



**f**

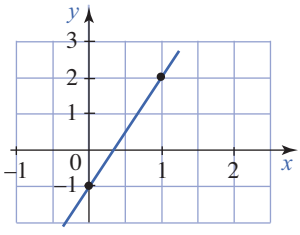



**6** Read off the gradient and the $y$-intercept from the graph and then write down the equations of the following lines:

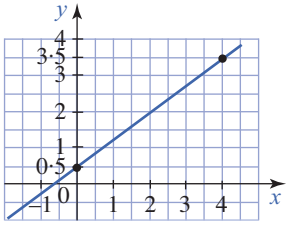
**a**



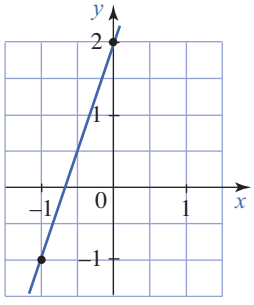

**b**



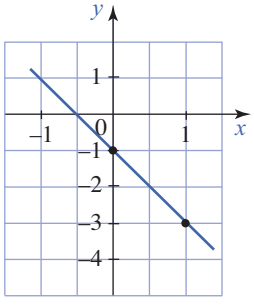

**c**



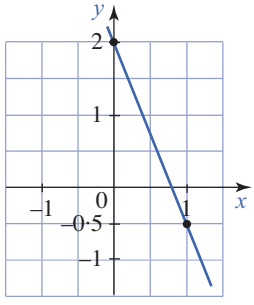

**d**



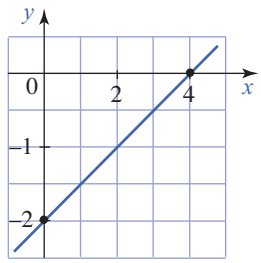

**e**



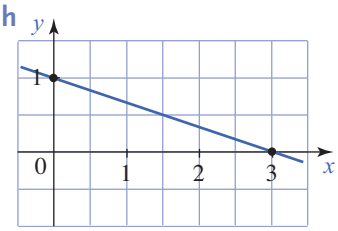

**f**



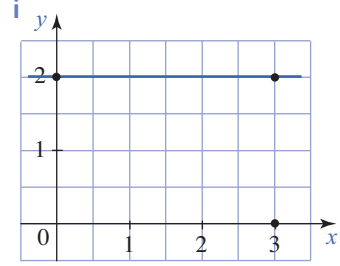
**g**




**h**




**i**




**7** Complete the following statement:

When the equation of a line appears as $y = mx + c$, then _____ (the coefficient of $x$) specifies the gradient and _____ gives the $y$-intercept.

**8** Read off the gradient and $y$-intercept for each of the following lines:

**a** $y = 2x - 7$ **b** $y = 5x + 9$ **c** $y = x - 1$

**d** $y = x$ **e** $y = -3x - 5$ **f** $y = 5 - 2x$

**g** $y = 2 - 8x$ **h** $y = 5 + 3x$ **i** $y = -x$

**j** $y = 8 - x$ **k** $y = \frac{2}{3}x + 2$ **l** $y = -\frac{1}{2}x + 1$

**m** $y = 5 - \frac{1}{4}x$ **n** $y = \frac{3x}{2} - 2$ **o** $y = \frac{x}{4} + 1$

**p** $y = \frac{x}{2} - 4$ **q** $y = \frac{x}{3} - 1$ **r** $y = 6 - \frac{2x}{3}$

**s** $y = 3 - \frac{3}{4}x$ **t** $y = -2 - \frac{2x}{5}$ **u** $y = \frac{1}{2} - \frac{3x}{4}$

**9** Write down the equation of a line that has:

**a** a gradient of 3 and a $y$-intercept of $-2$

**b** a gradient of $-1$ and a $y$-intercept of 5

**c** a gradient of 7 and a $y$-intercept of zero

**d** a gradient of $-6$ and a $y$-intercept of $-1$

**e** a gradient of $a$ and a $y$-intercept of $b$

**f** a gradient of $a$ and a $y$-intercept of $-b$

**g** a gradient of $-a$ and a $y$-intercept of $b$

**h** a gradient of $-a$ and a $y$-intercept of $-b$

**10** Rearrange the following equations to make $y$ the subject, then state the gradient and $y$-intercept in each case. Use this information to sketch the graph of each:

**a** $y - 2x - 3 = 0$ **b** $y - x + 1 = 0$ **c** $y + 2x - 1 = 0$

**d** $2y + x = 6$ **e** $2y - x = 4$ **f** $3y + 6x - 12 = 0$

**g** $3x + y - 4 = 0$ **h** $-2x + y + 5 = 0$ **i** $4x + 2y - 7 = 0$

**j** $4x - 2y + 3 = 0$ **k** $2x + 5y = 10$ **l** $9x - 3y - 2 = 0$

**m** $2y - 6x + 1 = 0$ **n** $5x - 2y - 1 = 0$ **o** $3x - 4y - 2 = 0$

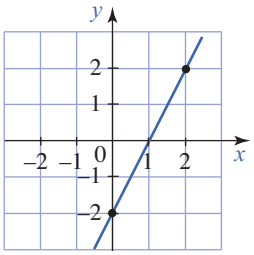
# Finding the equation of a line 10G

The gradient of a line between the two points $(x^1, y^1)$ and $(x^2, y^2)$ is: $\frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$.

To find the equation of a given line, work out the gradient and the $y$-intercept, then write down the equation in the form $y = mx + c$.

## Example

**1** Write down the equation of the line that passes through the points (0, –2) and (2, 2).




## Solution

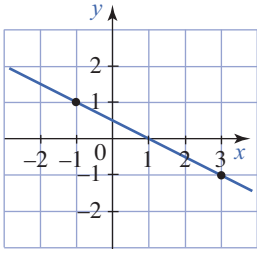
From the diagram, or by using the coordinates:

$$\text{gradient} = \frac{2 - -2}{2 - 0} = \frac{4}{2} = 2$$

The $y$-intercept is –2.

The equation of the line is $y = 2x - 2$.

**2** Find the equation of the line that passes through the points (–1, 1) and (3, –1).




From the diagram, or by using the coordinates:

$$\text{gradient} = \frac{-1 - 1}{3 - -1} = \frac{-2}{4} = -\frac{1}{2}$$

The $y$-intercept is not given this time, so we must substitute a point to find $c$.

Start with $y = -\frac{1}{2}x + c$ and substitute (–1, 1):

$$1 = -\frac{1}{2} \times -1 + c$$

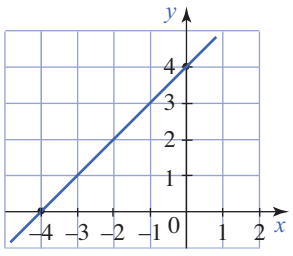
$$1 = \frac{1}{2} + c \quad \text{so } c = \frac{1}{2}$$

The equation of the line is $y = -\frac{1}{2}x + \frac{1}{2}$.

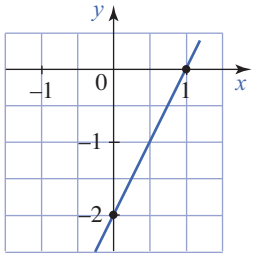
## Exercise 10G

**1** Write down the equations of the following lines:

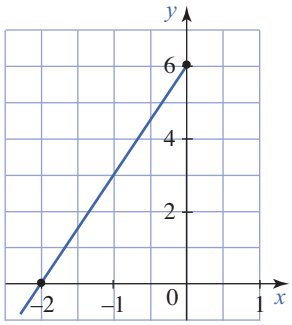
**a**



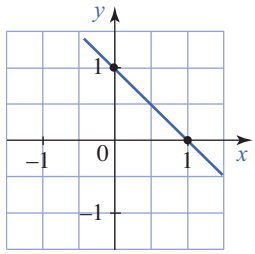

**b**



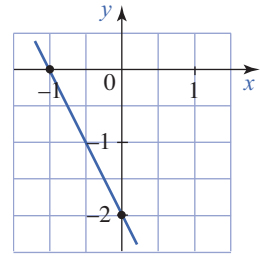

**c**



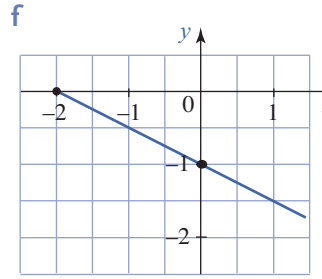
**d**




**e**




**f**




**2** Complete the following line equations, by substituting the given point to find the value of the $y$-intercept, $c$, in each case:

**a** $y = 2x + c$ and the line passes through the point (1, 1)

**b** $y = x + c$ and the line passes through the point (2, 9)

**c** $y = 3x + c$ and the line passes through the point (2, 4)

**d** $y = 4x + c$ and the line passes through the point (1, –1)

**e** $y = 5x + c$ and the line passes through the point (3, 14)

**f** $y = -x + c$ and the line passes through the point (5, 3)

**g** $y = -2x + c$ and the line passes through the point (3, –2)

**h** $y = -4x + c$ and the line passes through the point (–2, 9)

**i** $y = -0{\cdot}5x + c$ and the line passes through the point (4, –1)

**j** $y = -\frac{3}{2}x + c$ and the line passes through the point (3, –4)

**3** Find the equation of each of the lines with the following properties:

**a** has a gradient of 2 and passes through the point (2, 6)

**b** has a gradient of 1 and passes through the point (1, 3)

**c** has a gradient of 4 and passes through the point (3, 14)

**d** has a gradient of –1 and passes through the point (2, 1)

**e** has a gradient of –3 and passes through the point (–2, 13)

**f** has a gradient of –5 and passes through the point (2, –12)

**4** Find the equations of the lines which pass through points:

**a** (–1, 3) and (0, 4)

**b** (–2, –3) and (0, 1)

**c** (2, 4) and (0, 3)

**d** (4, 5) and (0, 2)

**e** (1, 1) and (0, 2)

**f** (2, 1) and (0, 5)

**g** (1, –2) and (0, –1)

**h** (2, –8) and (0, –2)

**i** (1, 1) and (0, –2)

**j** (–2, 0) and (0, –1)

**k** (0, $a$) and ($a$, 7$a$)

**l** (0, –$b$) and ($b$, 3$b$)

**5** Find the equation of the lines which pass through each of the following pairs of points:

**a** (1, 8) and (3, 12)

**b** (3, 13) and (5, 15)

**c** (4, 6) and (6, 12)

**d** (–1, 3) and (0, 4)

**e** (1, 1) and (4, 22)

**f** (1, 5) and (5, 1)

**g** (–1, 3) and (0, 5)

**h** (–2, 7) and (2, –1)

**i** (2, –9) and (4, –17)

**j** (–1, –3) and (3, –11)

**k** (–2, 4) and (1, 1)

**l** (–3, –1) and (2, 1)

**m** $\left(-\frac{1}{2}, 1\right)$ and $\left(1, -\frac{1}{2}\right)$

**n** $\left(\frac{1}{4}, \frac{3}{4}\right)$ and $\left(\frac{3}{4}, \frac{1}{4}\right)$

**o** ($a$, $b$) and ($b$, $a$)

# Exploring gradients and intercepts 10H

For the equation $y = 4x - 7$, the coefficient of $x$ is 4, the coefficient of $y$ is 3 and the constant is $-7$.

## Learning task 10H

**1** Prepare a graph with the scales on both the $x$-axis and $y$-axis from $-10$ to 10.

- **a** Plot and label the graphs of the lines $y = x$, $y = 2x$, $y = 3x$, $y = 4x$ and $y = 5x$.
- **b** Plot and label the graphs of the lines $y = \frac{1}{2}x$, $y = \frac{1}{4}x$ and $y = \frac{1}{5}x$ on the same graph.
- **c** Which coordinate is common for each of these lines?
- **d** Sketch and label your best guess for where you would expect the graphs of the lines $y = 2\frac{1}{2}x$, $y = \frac{3}{4}x$ and $y = 10x$ would be plotted.
- **e** Write in your own words what a graph with a positive $x$-coefficient looks like.

**2** Prepare a graph with the scales on both the $x$-axis and $y$-axis from $-10$ to 10.

- **a** Plot and label the graphs of the lines $y = -x$, $y = -2x$, $y = -3x$, and $y = -5x$.
- **b** Plot and label the graphs of the lines $y = -\frac{1}{2}x$, $y = -\frac{1}{4}x$ and $y = -\frac{1}{5}x$ on the same graph.
- **c** Which coordinate is common for each of these lines?
- **d** Sketch and label your best guess for where you would expect the graphs of the lines $y = -2\frac{1}{2}x$, $y = -\frac{2}{3}x$ and $y = -20x$ would be plotted.
- **e** Write in your own words what a graph with a negative $x$-coefficient looks like.

**3** Write in your own words what a graph of $x = 0$ looks like.

**4** Write in your own words what a graph of $y = 0$ looks like.

**5** Prepare a graph with the scales on both the $x$-axis and $y$-axis from $-10$ to 10.

- **a** Plot and label the graphs of the lines $y = x$, $y = x + 1$, $y = x + 2$, $y = x + 3$ and $y = x + 5$.
- **b** Plot and label the graphs of the lines $y = x - 1$, $y = x - 4$, $y = x - 6$ on the same graph.
- **c** What geometric property is common for each of these lines?
- **d** Sketch and label your best guess for where you would expect the graphs of the lines $y = x + 2\frac{1}{2}$ and $y = x - 3\frac{1}{3}x$ would be plotted.
- **e** Write in your own words what a graph of $y = x \pm c$ looks like for different values for the constant $c$.

**6** **a** On a new set of axes, plot and label the graphs of $y = 3x$, $y = 3x + 2$, and $y = 3x + 4$.

- **b** Describe in your own words how the line $y = 3x$ might move to become each of the lines: $y = 3x + 2$, $y = 3x + 4$ and $y = 3x - 4$.

**7** By now you will have experimented with straight line equations of the form $y = mx + c$. Explain the following in your own words:

**a** How does the coefficient of $x$ ($m$) affect the straight line graph?

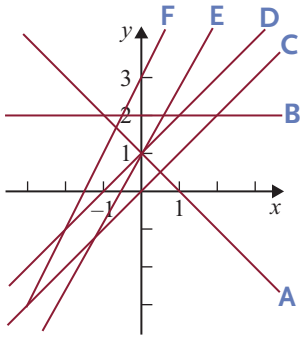
**b** What does the term $c$ in the equation tell you about the graph?

**8** Match each of the following equations with its graph. Each graph has been used only once.

**a** $y = 2x + 1$ **b** $y = x + 1$

**c** $y = 2x + 3$ **d** $y = -x + 1$

**e** $y = x$ **f** $y = 2$




**9** Prepare a graph with the scales on both the $x$-axis and $y$-axis from –10 to 10.

**a** Draw the line joining the coordinates (0, 10) and (1, 0). Label it with its equation.

**b** Draw the line joining the coordinates (0, 9) and (2, 0). Label it with its equation.

**c** Draw the line joining the coordinates (0, 8) and (3, 0). Label it with its equation.

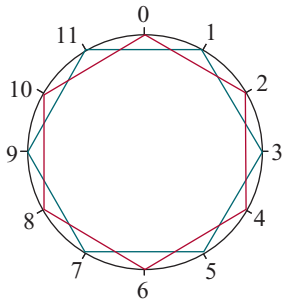
**d** Continue the pattern of lines as far as the one joining (0, 1) and (10, 0).

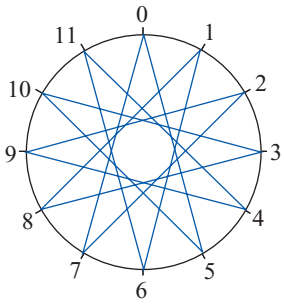
**e** Reflect each of the lines drawn in the first quadrant of the graph in the $x$-axis. Label each line with its equation and note any patterns in the pairs of equations of each line and its mirror image.

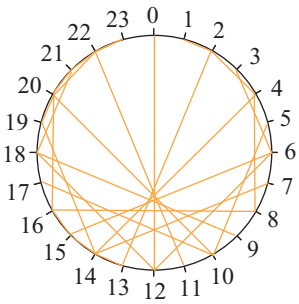
**f** Reflect each of the lines drawn in the first quadrant of the graph in the $y$-axis. Label each line with its equation and note any patterns in the pairs of equations of each line and its mirror image.

**g** Finish the design by completing the graph pattern of lines in the third quadrant. Again label each line with its equation and note any patterns with the equations of its mirror images.

**10** Experiment with your own graph string designs following the pattern of Question 9. For example, you might start with joining the pairs of points (0, 10) and (3, 0), (0, 9) and (4, 0), etc. Alternatively, start with a different set of axes that meet at 30° instead of 90°. Use different colours and display your design on a class noticeboard.

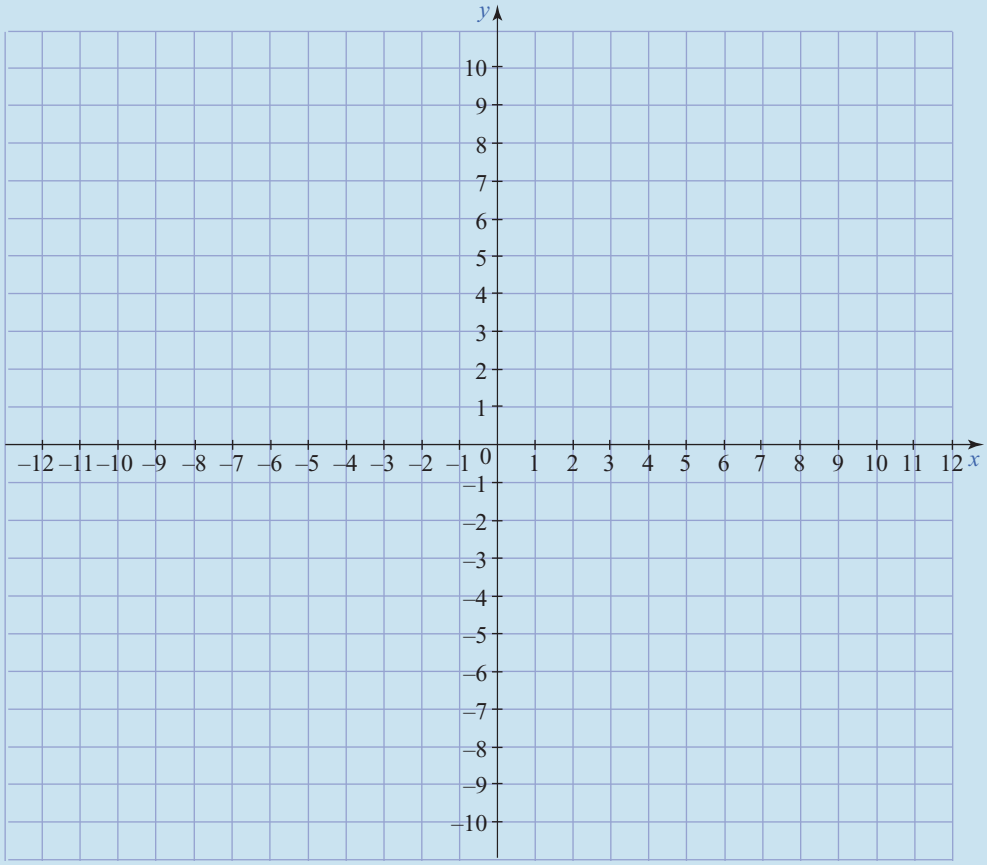


# Puzzles

1 Copy the grid below. Plot the coordinate points in the order given to create a picture, then colour it in:




JOIN (–6, 10), (–2, 7), (–2, –1), (–1, –7), (–4, –3), (–6, –7 ), (–8, –3), (–11, –7), (–10, –1), (–10, 7), (–6, 10) STOP

JOIN (–6, 7), (–4, 6), (–4, 4), (–6, 3), (–8, 4), (–8, 6), (–6, 7) STOP

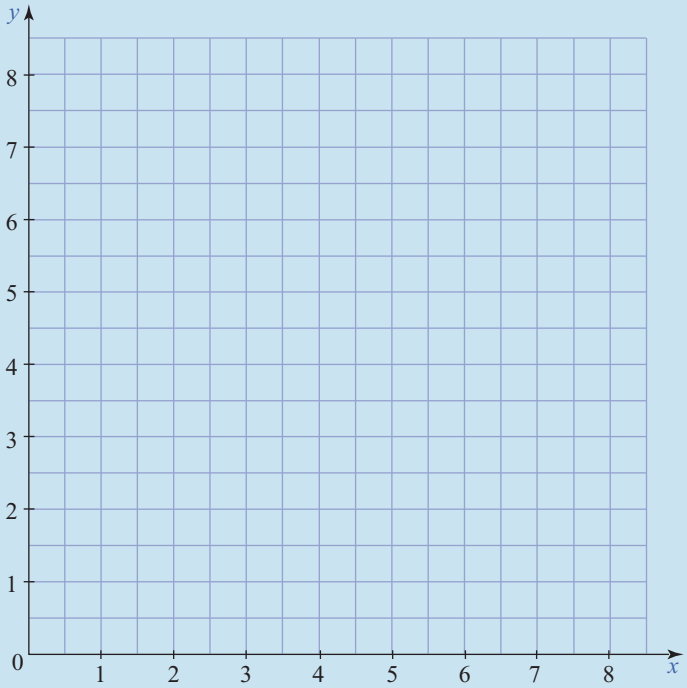
JOIN (–6, 2), (–4, 1), (–4, –1), (–6, –2), (–8, –1), (–8, 1), (–6, 2) STOP

JOIN (1, 0), (2, 0), (2, –1), (4, –1), (4, 1), (3, 2), (3, 4), (6, 4), (6, 2), (5, 1), (5, –1), (8, –1), (8, –3), (7, –3), (7, –2), (5, –2), (5, –4), (6, –4), (6, –6), (7, –6), (7, –7), (5, –7), (5, –5), (4, –5), (4, –7), (2, –7), (2, –6), (3, –6), (3, –4), (4, –4), (4, –2), (1, –2), (1, 0) STOP

JOIN (3, 2), (2, $2\frac{1}{2}$), (3, 3), (2, $3\frac{1}{2}$), (3, 4), (4, 5), ($4\frac{1}{2}$, 4), (5, 5), (6, 4), (7, $3\frac{1}{2}$), (6, 3), (7, $2\frac{1}{2}$), (6, 2) STOP

**2** Copy the grid below, then sketch the following lines:

$y = 1$ $y = 5$ $y = 2x + 1$ $y = -2x + 15$




Name the shape made by the borders of the four lines:

**3** Copy the grid below, then sketch the following lines:

$y = x$

$y = -x$

$y = \frac{1}{3}x + 4$

$y = -\frac{1}{3}x + 4$

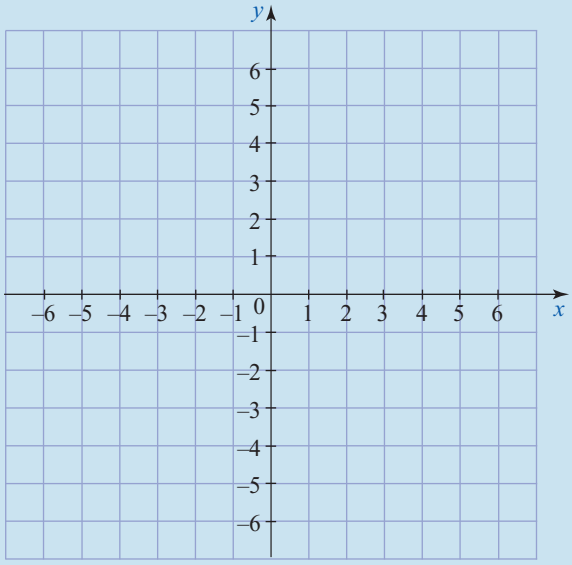
Shade in the shape made by the borders of the four lines.

Draw the reflection of the shape in the line $y = x$ and shade.

Draw the reflection of the shape in the line $y = -x$ and shade.

Draw the reflection of the shape about the $x$-axis.

Name the shape that is created:



# Applications

## Electrician

To qualify as an electrician at SICHE, a learner has to pay \$180 to complete the required course. After qualifying, an electrician can expect to earn \$12·50 per hour.

**a** Copy and complete the following table, which gives the learner's net gain $A$ when various numbers of hours $n$ are worked:

| $n$ | 0 | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 |
|---|---|---|---|---|---|---|---|---|---|---|
| $A$ | | | | | | | | | | |

**b** Draw up a set of axes with $n$ represented horizontally and $A$ vertically. Plot the points given in the table.

**c** Connect the plotted points with a straight line, and extend the line from $n = 18$.

**d** What is the 'break-even' whole number of hours needed to cover the cost of qualifying?

**e** Write down the equation relating $A$ to $n$.

**f** Use the equation in part e to find $A$ when $n$ is 40 hours.

**g** How many (whole) hours need to be worked for the net gain $A$ to be \$300?

## Pulse rates

While resting, measure your pulse by counting how many beats there are over a 1-minute interval. Record your resting pulse rate in beats per minute.

Now raise your pulse rate by running one lap of the school oval or another suitable circuit. When you stop, wait 30 seconds and then measure your pulse over a minute. The number of beats you count will estimate your pulse rate at the 1-minute mark after you stopped running. After another 2 minutes, measure your pulse again over a full minute, and this result will estimate your pulse rate at the 4-minute mark. Copy and complete this table:

| $t$ (minutes) | 1 | 4 | 7 | 10 | 13 | 16 |
|---|---|---|---|---|---|---|
| $P$ (beats/min) | | | | | | |

**a** Draw up a set of axes with $t$ marked horizontally and $P$ marked vertically. Plot the points given in the table above.

**b** The plots will not lie on a perfect straight line. However, draw in a 'trend line' among the plots so that half the plots are above and half are below the line.

**c** Read off the $P$-intercept of your line as accurately as you can.

**d** Now from the line, read the pulse rate when $t = 10$. Label this point on the line (10, __).

**e** Using the point in part d above and the $y$-intercept, find the gradient of the trend line. Write down its equation in the form $P =$ ____$t +$ ____.

**f** Use the formula obtained in part e to work out the pulse rate when $t = 15$ minutes.

**g** Use the formula obtained in part e to find how long it would (theoretically) take for the pulse to return to its resting rate.

## Cycling

The aim is to measure how far you travel with various numbers of turns of the pedals in different gears.

Select a gear, say first gear, and put a chalk mark on the ground below the position of the 'V' made by the frame in the centre of the bike. Turn the pedals through one rotation and measure the distance travelled in metres.

**a** Record the distance for several rotations, then complete a table of the form:

| Turns $T$ | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| Distance $D$ (metres) | 0 | | | | | |

**b** Draw up a set of axes and plot the above points. Join the points with a straight line. Don't forget to label and scale your axes.

**c** Find the gradient of the line and so write the equation relating $D$ to $T$.

**d** Use your equation in part c to find how many turns are required to cover 1 kilometre.

**e** How many times would the pedal turn on a bicycle ride, which is 560 kilometres long?

**f** Repeat the steps in parts a–d for different gears on your bike.

**g** Another learner recorded these incomplete results for his bike. Copy and complete the table:

| Turns $T$ | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| Distance $D$ (metres) | 0 | 4·5 | | | | |

**h** Graph the results on a set of axes of your own.

**i** How far would the learner travel with 10 turns of the pedals?

**j** Write a formula that relates $D$ to $T$.

**k** How many turns are required to cover 1 kilometre?

**l** The learner rides to school each day, a return trip of 8·5 kilometres. How many times will he turn the pedals over the course of a week?

# Enrichment

## Inequations in the Cartesian plane

**1** **a** Draw up a set of axes and using any method you wish draw in the line $y = 2x - 1$ in pencil.

**b** Plot the points (1, 1), (2, 3), (3, 5) and check whether or not they lie on the line.

Points on this line have the $y$ value *equal* to twice the $x$ value minus 1, as per the relation $y = 2x - 1$.

**c** Now plot the points (1, 2), (2, 4), (3, 6). Where are these points in relation to the line? The $y$ values of these points are *greater* than twice the $x$ value minus 1. Using a different-coloured pencil, shade in the region *above* the line. In this region $y > 2x - 1$.

**d** Now rule over the line $y = 2x - 1$, with the same colour as used to shade the region above the line. The region of points both *on* and *above* the line is described by the relation $y \geq 2x - 1$.

**e** Plot the points (1, 0), (2, 2), (3, 4). Where are these points in relation to the line? The $y$ values of these points are *less* than twice the $x$ value minus 1. Shade the region given by $y < 2x - 1$ in a different colour.

**2** **a** On a new set of axes draw the line with equation $y = -x + 3$.

**b** Shade the region where $y \leq -x + 3$.

**c** Shade the region where $y > -x + 3$.

**3** **a** On a new set of axes draw the line with equation $x = 5$.

**b** Shade the region where points have $x$-coordinates greater than 5.

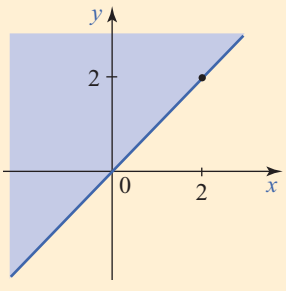
**c** Shade the region where $x < 5$.

**4** **a** On a new set of axes draw in the line $y = -0{\cdot}5x + 6$, using a broken line (---).

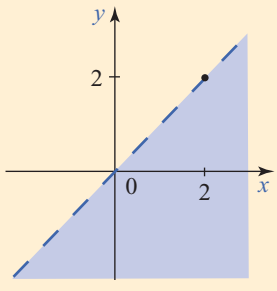
**b** Shade the region $y > -0{\cdot}5x + 6$. **c** Shade the region $y < -0{\cdot}5x + 6$.

**5** Write down inequations that describe the following regions:

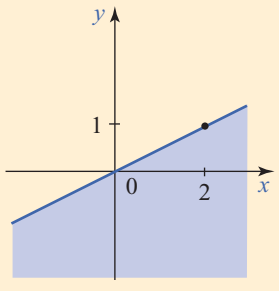
**a**



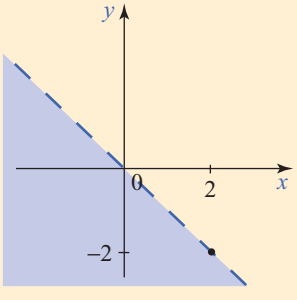

**b**



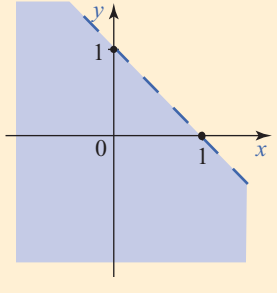

**c**



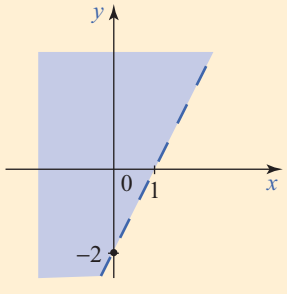

**d**



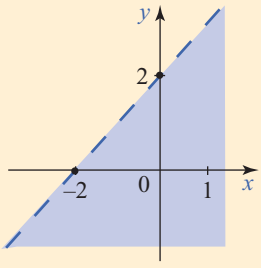

**e**



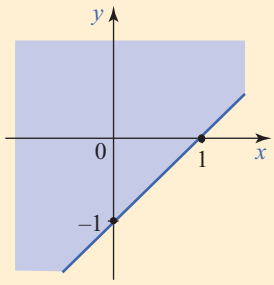

**f**



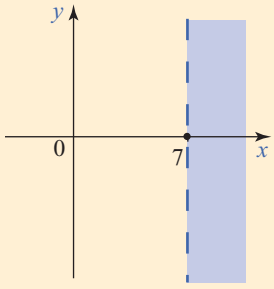
g



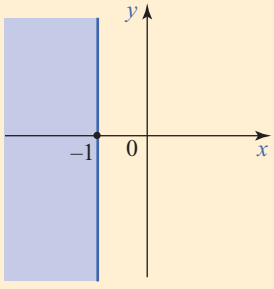

h



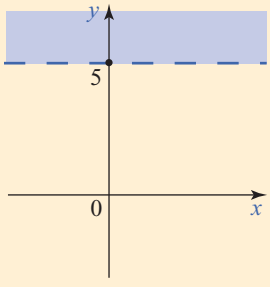

i



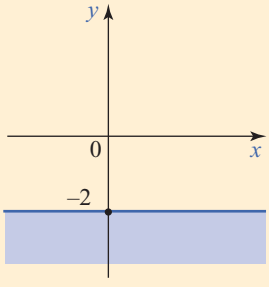

j




k




l




**6** **a** On a set of Cartesian axes of your own, shade the common overlapping region of the following relations:

**i** $y \leq -x + 4$ **ii** $y \geq x - 4$ **iii** $x \geq -1$

**b** Identify the shape you have shaded.

**c** What is the area of the shape?

**7** **a** On a set of Cartesian axes of your own, shade the overlapping region of the following relations:

**i** $y \leq x + 3$ **ii** $y \geq -2$ **iii** $x \leq 2$ **iv** $x \geq -3$

**b** Identify the shape you have shaded.

**c** Label the coordinates of the vertices of the shaded region.

**d** Find the area of the shaded region.

**8** Uepi Resort down at Marovo Lagoon hires its outboard motor (OBM) engine and ray-boat for \$30 and \$60 respectively per day.

At a certain beach the company has three OBM and four ray-boats, in all up to seven items that can be hired out.

**a** Draw up a Cartesian set of axes. Let $x$ represent the number of ray-boats and $y$ the number of OBM hired out for the day.

**b** On your grid, plot as points the possible hire combinations.

**c** Experience has shown that the number of hires never exceeds five on any day. With this information in mind, identify the feasible hire combinations. Shade the region occupied by these points.

**d** Beside each plotted point determined in part c, write the income received.

**e** Which hire combination produces the highest daily income?

**f** What is the maximum daily income that can be achieved?

**g** What is the next maximum daily income that can be achieved?

# Revision/Assessment

## Exercise 10A

**1** Draw a set of Cartesian axes on graph paper. Let 1 centimetre represent 1 unit, and allow the values on both the $x$- and $y$-axes to run from –6 to +6.

**a** Plot the points (0, 2), (3, 3), (4, 5) and connect these points to form a triangle.

**b** Reflect the points in part a across the $x$-axis and draw the image of the triangle. Clearly label the coordinates of the vertices.

**c** Next draw the reflection of the triangle in part a about the line $y = x$ and label the coordinates of the vertices.

**d** Finally, draw the reflection of the triangle in part a about the $y$-axis and label the coordinates of the vertices.

## Exercise 10B

**2** Complete the following tables, then plot the points on a set of Cartesian axes and join them with a straight line. State the $x$- and the $y$-intercepts in each case:

**a** $y = 2x - 4$

| $x$ | –1 | 0 | 1 | 2 |
|---|---|---|---|---|
| $y$ | | | | |

**b** $y = -3x + 6$

| $x$ | –2 | 0 | 1 | 2 |
|---|---|---|---|---|
| $y$ | | | | |

## Exercise 10D

**3** On a set of axes, sketch the following straight lines by using the intercept method:

**a** $y = x + 2$ **b** $y = -2x + 4$ **c** $3y + 4x = 12$

**d** $2x - 3y = 6$ **e** $5x + 2y - 10 = 0$ **f** $3x - 5y - 30 = 0$

## Exercise 10E

**4** On a set of axes, sketch the graphs of the straight lines with equations:

**a** $x = 4$ **b** $x = -\frac{3}{4}$ **c** $y = \frac{1}{4}$ **d** $y = 0$

**e** $y = -2$ **f** $x = 0$ **g** $x = 2{\cdot}5$ **h** $y = -3{\cdot}5$

## Exercise 10F

**5** Find the gradient of the line joining each pair of points:

**a** (1, 3), (2, 6) **b** (2, 2), (5, 11) **c** (–2, 8), (1, 2)

## Exercise 10G

**6** On a set of axes, sketch the following straight lines by using the gradient and $y$-intercept method:

**a** $y = 4x$ **b** $y = -2x + 1$ **c** $y = -x + 2$ **d** $y = \frac{1}{2}x - 1$

**e** $y = \frac{1}{2}x$ **f** $y = \frac{1}{2}x - 1$ **g** $y = \frac{2}{3}x$ **h** $y = -\frac{3}{4}x + 1$

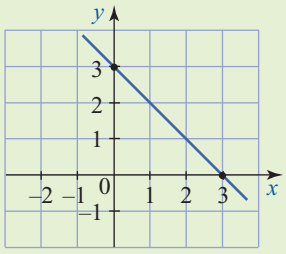
**i** $y = -\frac{1}{2}x + 1$ **j** $y = 2 - \frac{1}{4}x$ **k** $y = \frac{1}{4}x + \frac{1}{2}$ **l** $y = \frac{3}{4} - \frac{1}{2}x$

**m** $y = \frac{2}{3}x + 1$ **n** $y = 1 - \frac{2}{3}x$ **o** $y = \frac{4}{5}x + 2$ **p** $y = 2 - 1\frac{4}{5}x$

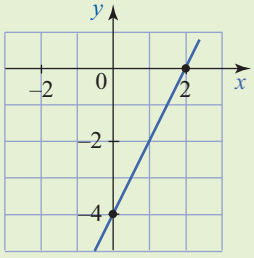
## Exercise 10H

**7** Read off the gradient and the $y$-intercept and then write down the equation of each of the following lines:

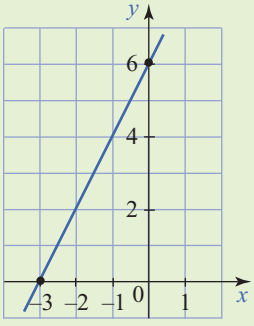
**a**



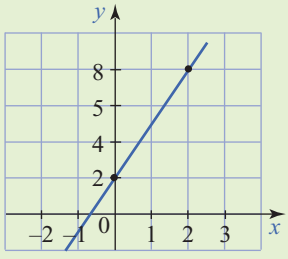

**b**



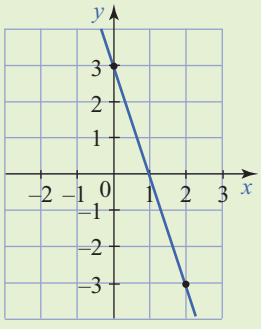

**c**



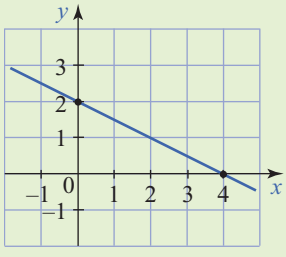

**d**




**e**




**f**




**8** Find the equation of the line that passes through:

**a** (–1, 2) and (0, 4)

**b** (–2, –3) and (0, 3)

**c** (2, 7) and (0, 3)

**d** (2, 1) and (0, 2)

**9** Find the equation of the line with the following properties:

**a** a gradient of 2 and passes through the point (2, 8)

**b** a gradient of 1 and passes through the point (–1, 3)

**c** a gradient of –3 and passes through the point (2, –1)

**10** Find the equation of the lines that pass through each of the following pairs of points:

**a** (1, 6) and (3, 10)

**b** (3, 3) and (6, 12)

**c** (4, 6) and (5, 1)

**d** (–1, 6) and (1, 2)

**11** A tank which initially has an unknown volume of water in it is being filled, and after 5 minutes it contains 4000 litres. After another 10 minutes the volume has increased to 6000 litres.

**a** Find a formula which gives the volume, $V$ litres, in the tank after $t$ minutes, assuming a linear relation between volume and time.

**b** What was the initial volume of water in the tank?

**c** At what rate (in litres per minute) is the tank being filled?

**d** How long does it take the volume to reach 5500 litres?

CHAPTER

# 11

# Probability

Tides have long been known as an important factor for success when saltwater fishing. Our ancestors understood the fact that the moon determines the type of tide and used it to predict the best days and times for fishing. Using weather instruments and computers, the Meteorological Department has been keeping records of all low tides and high tides over past years. Using this data the likelihood of particular tidal patterns can be predicted.

## This chapter covers the following skills:

- Finding sample spaces and calculating probabilities
  The probability of an event
  $= \frac{\text{number of favourable outcomes}}{\text{total number of possible outcomes}}$
- Calculating complementary events
  $\Pr(A') = 1 - \Pr(A)$
- Calculating mutually exclusive events
  If two events $A$ and $B$ are mutually exclusive then
  $\Pr(A \cap B) = 0$
- Using the addition law for combined events
  $\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$
- Using the multiplication principle
  If two events are independent then
  $\Pr(A \cap B) = \Pr(A) \times \Pr(B)$
- Calculating odds in horseracing
  Odds are written as the ratio
  ways to lose : ways to win

## Specific Learning Outcomes (SLO)

Learners should be able to:

**9.11.1.1** Define the terms 'probability' and 'event space'.

**9.11.1.2** Apply the formula for calculating the probability of an event:
probability of an event
$= \frac{\text{no. of favourable outcomes}}{\text{total no. of possible outcomes}}$

**9.11.2.1** Construct lattice and tree diagrams to display the total possible outcomes for one or more events.

**9.11.2.2** Use the probability formula to calculate the probability of events.

**9.11.3.1** Define the term 'complementary events'.

**9.11.4.1** Calculate the probability of complementary events using $\Pr(A') = 1 - \Pr(A)$.

**9.11.5.1** Use Venn Diagrams to display complement events.

**9.11.6.1** Identify the symbols that are used for union: $\cup$ and intersection: $\cap$.

**9.11.6.2** Use the additional law with the term 'or' to combine probabilities using the formula:
$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$

**9.11.7.1** Define the term 'mutually exclusive events' and identify such events.

**9.11.8.1** Use Venn diagrams to calculate probabilities of events that are mutually exclusive.

**9.11.9.1** Define and identify Independent Events.

**9.11.9.2** Apply the general equation for two events that occur Independently:
$\Pr(A \text{ and } B) = \Pr(A \cap B) = \Pr(A) \times \Pr(B)$

**9.11.10.1** Calculate the probability of independent events.

**9.11.11.1** Define the term 'odds' to probability and that high odds means that the event is less likely to occur.

**9.11.12.1** Express the odds in ratio form as: ways to lose : ways to win.

**9.11.13.1** Convert probabilities to odds and vice versa using the formula:
$\Pr(\text{win}) = \frac{\text{ways to win}}{\text{ways to win} = \text{ways to lose}}$

**9.11.14.1** Use formula to calculate the unit payout:
$\text{unit payout} = \frac{\text{payout}}{\text{original bet}}$

**9.11.14.2** Calculate the expected return using the formula: payout = bet $\times$ odds + bet.

# 11A Event spaces

To calculate the probability of an event we need to consider all the possible outcomes. Usually we write these probabilities as fractions in their simplest form.

$$\text{The probability of an event} = \frac{\text{number of favourable outcomes}}{\text{total number of possible outcomes}}$$

The **event space** is a list of all the possible outcomes.

A **lattice diagram** or grid can be used to display the sample space for two events.

## Example

**1** **a** Show the event space when two fair six-sided dice are rolled.

**b** Find Pr(two numbers the same).

## Solution

**a** The event space could be displayed as a grid:

| | | 1st die | | | | | |
|---|---|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 | 5 | 6 |
| 2nd die | 1 | 1, 1 | 1, 2 | 1, 3 | 1, 4 | 1, 5 | 1, 6 |
| | 2 | 2, 1 | 2, 2 | 2, 3 | 2, 4 | 2, 5 | 2, 6 |
| | 3 | 3, 1 | 3, 2 | 3, 3 | 3, 4 | 3, 5 | 3, 6 |
| | 4 | 4, 1 | 4, 2 | 4, 3 | 4, 4 | 4, 5 | 4, 6 |
| | 5 | 5, 1 | 5, 2 | 5, 3 | 5, 4 | 5, 5 | 5, 6 |
| | 6 | 6, 1 | 6, 2 | 6, 3 | 6, 4 | 6, 5 | 6, 6 |

The 36 possible outcomes could be listed as a set:

{(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}

**b** {(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)}

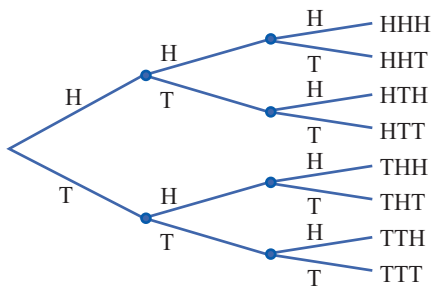
$$\text{Pr(two numbers the same)} = \frac{6}{36} = \frac{1}{6}$$

A **tree diagram** could be used to display three or more events.

## Example

**2** Display the event space as a tree diagram when three coins are tossed.

## Solution




The eight outcomes could be listed as a set:

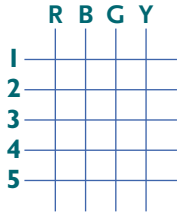
{(H, H, H), (H, H, T), (H, T, H), (H, T, T), (T, H, H), (T, H, T), (T, T, H), (T, T, T)}

## Exercise 11A

**1** List the event spaces for:

- **a** possible answers for a 5-response multiple-choice question
- **b** star signs
- **c** scores on a fair six-sided die
- **d** suits of a playing card

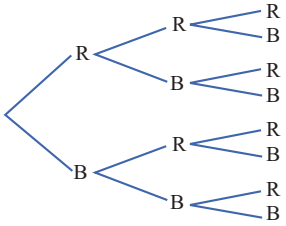
**2** One ball is chosen at random from a red, a blue, a green and a yellow ball, then a spinner numbered 1 to 5 is spun. The event space is shown here in a grid. List all possible outcomes as a set.




**3** Show in a grid or a lattice diagram the event space for:

- **a** sex of children in a two-child family
- **b** suits of two playing cards selected at random from a deck of 52 cards

**4** One card is dealt from a pack of 52 playing cards, its colour is noted and the card is replaced. This is done three times. The event space is shown in this tree diagram. List all possible outcomes as a set.




**5** What is the probability that a letter chosen at random from the alphabet is in the word:

- **a** geometry?
- **b** trigonometry?
- **c** mathematics?

**6** What is the probability that a card chosen at random from a pack of cards is

- **a** a heart?
- **b** red?
- **c** an ace?

**7** What is the probability that a person chosen at random has their birthday in May?

**8** When a die is thrown, what is the probability of getting:

- **a** an even number?
- **b** a prime number?
- **c** a number less than 5?

**9** What is the probability that a person guessing a multiple-choice question with five possible answers *A*–*E* will choose the letter *B*?

**10** In a two-child family, what is the probability that the children will be:

- **a** two boys?
- **b** a boy and a girl?

**11** A coin and a die are thrown, one after the other.

- **a** Complete a grid to show all possible outcomes in the experiment.
- **b** How many different outcomes are possible?
- **c** Are all the outcomes equally likely?
- **d** Calculate the probability of obtaining:
  - **i** a head and a 2
  - **ii** a head and an even number
  - **iii** a tail and a prime
  - **iv** a tail and a multiple of 3
  - **v** a head and a number greater than 1
  - **vi** a tail and a number less then 3

# 11B Complementary events

When all the possible outcomes for a situation are equally likely, the probability of an event can be calculated by dividing the number of possible outcomes in an event, by the total number of outcomes in the event space.

$$\Pr(A) = \frac{n(A)}{n(\xi)}$$

**Example**

**1** A teacher randomly selects a learner to answer a question. If there are 8 girls and 12 boys in the class, what is the probability that a girl is selected?

**Solution**

Let $A = \{\text{girls}\}$
$n(A) = 8$ 8 girls
$n(\xi) = 20$ 20 learners in total

$$\Pr(A) = \frac{n(A)}{n(\xi)}$$
$$= \frac{8}{20}$$
$$= \frac{2}{5} \text{ or } 0{\cdot}4$$

In many situations there are only two possibilities: male or female, win or lose, and so on.

These two events are the **complements** of each other. We call these events **complementary events**. In a trial either the event or its complement must occur.

Probability of an event + Probability of its complement $= 1$

This can be rewritten as:

Probability of an event $= 1 -$ Probability of its complement

Probability of an event *not* occurring $= 1 -$ Probability of an event occurring

The complement of $A$ is $A'$ so $\mathbf{Pr(A') = 1 - Pr(A)}$

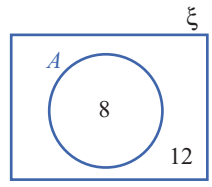
**Example**

**2** In the example above, the probability that the selected learner is a girl is $\frac{2}{5}$.
What is the probability that the selected learner is *not* a girl?

**Solution**

$\Pr(A') = 1 - \Pr(A)$
$= 1 - \frac{2}{5}$
$= \frac{3}{5}$ or $0{\cdot}6$

A **Venn diagram** can be used to display the number of outcomes in each set. The sample space or **universal set** $\xi$ is contained within the box and the circles contain the elements or the number of elements in each set.




For the example above:
$n(A) = 8$
$n(A') = 12$
$n(\xi) = 20$

## Exercise 11B

**1** Which of the following are complementary events?

- **A** Passing or failing a test
- **B** Hitting a six or being bowled out in a cricket over
- **C** Going out or staying in on a Friday night
- **D** A newborn baby is male or female
- **E** It rains or is sunny on Sunday

**2** The probability of throwing a double when you throw two dice is $\frac{1}{6}$. What is the probability that you don't throw a double?

**3** The probability of a learner winning a prize in the school raffle is $\frac{3}{250}$. What is the probability that the learner doesn't win a prize?

**4** What is the probability that a face card (a king, queen or jack) is not drawn when a card is randomly selected from a 52-card pack?

**5** What is the probability that a month has less than 31 days?

**6** There are four possible options for a quiz question and only one is correct. What is the probability that someone who is just guessing gets the wrong answer?

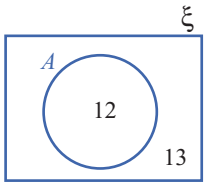
**7** A multiple-choice test has three questions and four possible options, but only one answer to each question. The probability of getting the answers correct are given in the table:

| No. of correct answers | Probability |
|---|---|
| 1 | $\frac{27}{64}$ |
| 2 | $\frac{9}{64}$ |
| 3 | $\frac{1}{64}$ |

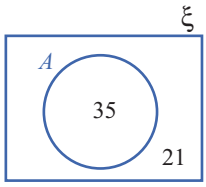
- **a** What is the probability of getting two correct answers?
- **b** What is the probability of getting at least one correct answer?
- **c** What is the probability of getting no correct answers?
- **d** What is the probability of getting less than three correct answers?

**8** For each Venn diagram shown below, calculate Pr($A$) and Pr($A'$) and show that they add to 1:

**a**




**b**



# 11C The addition law of probability

The combination of two sets is called the **union** and is denoted by the symbol $\cup$. $A$ union $B$ means $A$ *or* $B$ *or* both. $\Pr(A \cup B)$ means the probability that $A$ *or* $B$ *or* both occur.

The overlap of two sets is called the **intersection** and is denoted by the symbol $\cap$.
$A$ intersection $B$ means $A$ *and* $B$. $\Pr(A \cap B)$ means the probability that both $A$ *and* $B$ occur.

**Example**

**1** In a class of 24 music learners, 12 play the piano, 10 play the guitar and 3 play both.

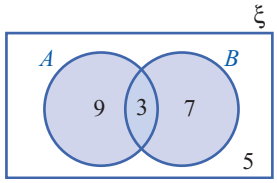
Let $A$ be the event that a learner plays the piano.

Let $B$ be the event that a learner plays the guitar.

Find the probability that a learner randomly selected from this music class plays the piano *or* the guitar.

**Solution**

If 3 learners play both instruments, then 9 play the piano only and 7 play the guitar only. So the Venn diagram looks like this:




Playing the piano *or* the guitar is written as $A \cup B$.

Adding the numbers in the union:

$9 + 3 + 7 = 19$

$\Pr(\text{plays piano } or \text{ guitar}) = \frac{19}{24}$

The probability of one event *or* another event could also be calculated by using the **addition law of probability**, which states:

$$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$$

This rule allows for the fact that when we add $\Pr(A)$ and $\Pr(B)$, we include the intersection twice.

**Example**

**2** Without drawing a Venn diagram, show that the probability that a randomly selected learner from the music class above plays the piano *or* the guitar is $\frac{19}{24}$.

**Solution**

$\Pr(\text{plays piano or guitar})$

$= \Pr(A \cup B)$

$= \Pr(A) + \Pr(B) - \Pr(A \cap B)$

$= \frac{12}{24} + \frac{10}{24} - \frac{3}{24}$

$= \frac{19}{24}$

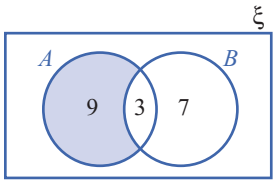
When we want to solve a probability question involving *not*, we could also use an appropriate Venn diagram.

**Example**

**3** Find the probability that a randomly selected learner from the music class above plays the piano but *not* the guitar.

**Solution**

The shaded region in the Venn diagram represents this:



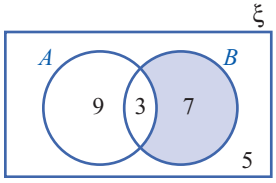

For Pr(plays the piano *not* the guitar), we use the notation $\Pr(A \cap B')$.

$\Pr(A \text{ not } B) = \Pr(A \cap B')$

$= \frac{9}{24}$

**4** Find the probability that a randomly selected learner from the music class above plays the guitar but *not* the piano?

The shaded region in the Venn diagram represents this:



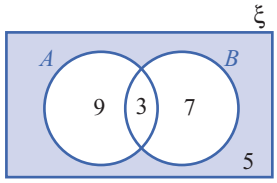

Pr(plays the guitar *not* the piano)

$= \Pr(A' \cap B)$

$= \frac{7}{24}$

**5** Find the probability that a randomly selected learner from the music class above plays *neither* the piano *nor* the guitar.

The shaded region in the Venn diagram represents this:




To calculate Pr(plays *neither* piano *nor* guitar), we use $\Pr(A' \cap B')$.

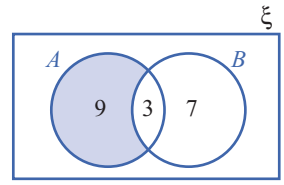
$\Pr(\textit{not } A \textit{ not } B) = \Pr(A' \cap B')$

$= \frac{5}{24}$

# 11C

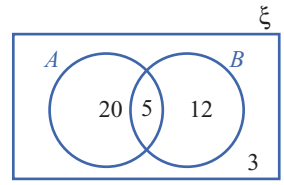
## Exercise 11C

**1** Copy the diagram and shade the area that corresponds to each of the following:




**a** $A$ **b** $B$ **c** $A \cap B$ **d** $A \cup B$
**e** $A'$ **f** $B'$ **g** $A \cap B'$ **h** $A' \cap B$
**i** $A' \cup B$ **j** $A \cup B'$ **k** $(A \cup B)'$ **l** $(A \cap B)'$

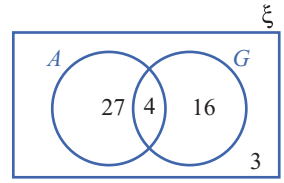
**2** Consider the Venn diagram shown with two events $A$ and $B$.




Calculate the following:

**a** $\Pr(A \cap B)$ **b** $\Pr(A \cup B)$ **c** $\Pr(A \cap B')$
**d** $\Pr(A' \cap B)$ **e** $\Pr(A' \cap B')$ **f** $\Pr(A \cap B)'$

**3** The results of a survey of 50 teachers in Honiara who were asked whether they went to Auki or Gizo for their holidays are shown in the Venn diagram.




Calculate the probability that a randomly selected teacher went to:

**a** Auki as well as Gizo **b** Auki but not Gizo
**c** Gizo but not Auki **d** either Auki or Gizo
**e** Auki **f** Gizo
**g** neither Auki nor Gizo

**4** In a 52-card pack of playing cards, there are 26 red cards, 12 face cards (king, queen or jack) and 6 red face cards.

**a** Display this information in a Venn diagram.
**b** Find the probability that a randomly selected card is:
**i** a red card **ii** a face card
**iii** a red card that is not a face card **iv** a black face card
**v** a red face card **vi** a black number card

**5** Find $\Pr(A \cup B)$ if:

**a** $\Pr(A) = 0{\cdot}5$, $\Pr(B) = 0{\cdot}2$ and $\Pr(A \cap B) = 0{\cdot}1$
**b** $\Pr(A) = 0{\cdot}7$, $\Pr(B) = 0{\cdot}2$ and $\Pr(A \cap B) = 0{\cdot}1$
**c** $\Pr(A) = 0{\cdot}4$, $\Pr(B) = 0{\cdot}3$ and $\Pr(A \cap B) = 0{\cdot}2$

**6** Find $\Pr(A \cap B)$ if:

**a** $\Pr(A) = 0{\cdot}5$, $\Pr(B) = 0{\cdot}4$ and $\Pr(A \cup B) = 0{\cdot}7$
**b** $\Pr(A) = 0{\cdot}7$, $\Pr(B) = 0{\cdot}3$ and $\Pr(A \cup B) = 0{\cdot}8$
**c** $\Pr(A) = 0{\cdot}4$, $\Pr(B) = 0{\cdot}5$ and $\Pr(A \cup B) = 0{\cdot}6$

**7** Given the probabilities, find:

**i** $\Pr(A' \cap B)$ **ii** $\Pr(A \cap B)'$ **iii** $\Pr(A' \cap B')$

**a** $\Pr(A) = 0{\cdot}5$, $\Pr(B) = 0{\cdot}2$ and $\Pr(A \cap B) = 0{\cdot}1$
**b** $\Pr(A) = 0{\cdot}5$, $\Pr(B) = 0{\cdot}4$ and $\Pr(A \cup B) = 0{\cdot}7$

# Mutually exclusive events 11D

Two events are **mutually exclusive** if it is impossible for them both to happen. When we use a Venn diagram to represent mutually exclusive events there is no overlapping of the sets.

If the two events $A$ and $B$ are mutually exclusive, then:

$$\Pr(A \cap B) = 0 \text{ and } \Pr(A \cup B) = \Pr(A) + \Pr(B)$$

When the two events are *not* mutually exclusive there will be an overlap.

## Example

**1** In a class of 24 music learners, 12 play the piano, 10 play the guitar and 3 play both. Let $A$ be the event that a learner plays the piano and let $B$ be the event that a learner plays the guitar. Display this as a Venn diagram.

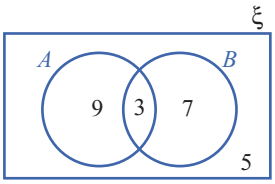
Are the events mutually exclusive?

**2** What is the probability that a randomly selected learner from the music class above plays the piano *and* the guitar?

## Solution

If 3 learners play both instruments, then 9 play the piano only and 7 play the guitar only, while 5 learners play neither instrument.

The Venn diagram shows the information:

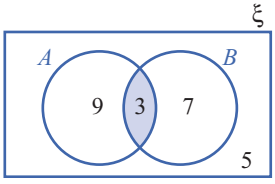



The events 'playing the guitar' and 'playing the piano' are *not* mutually exclusive as they have an overlap. Three learners play both the piano and the guitar.

The learners who play the piano *and* the guitar are shown in the intersection:

$$\Pr(\text{plays piano and guitar}) = \frac{3}{24}$$
$$= \frac{1}{8}$$

The shaded area on the Venn diagram represents $A \cap B$.




$$\Pr(A \cap B) = \frac{1}{8}$$
$$\neq 0$$

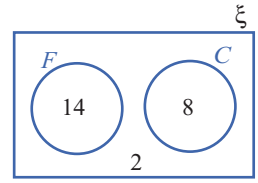
So the events are *not* mutually exclusive.

**Example**

**3** In a class of 24 Year 9 learners, 14 study French, 8 study Chinese and 2 don't study a language. Display this as a Venn diagram and show that studying Chinese and French are mutually exclusive events.

**Solution**

There are 24 learners in the group:
$14 + 8 + 2 = 24$
This means that there is no overlap.




$\Pr(C \cap F) = 0$

$\Pr(C) + \Pr(F) = \frac{8}{24} + \frac{14}{24} = \frac{22}{24} = \Pr(C \cup F)$

## Exercise 11D

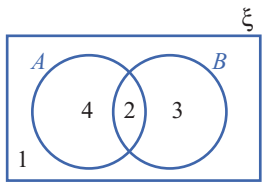
**1** Which of the following are mutually exclusive events?

**A** It will rain or it will be sunny tomorrow
**B** Throwing a six or throwing a five on a fair six-sided die
**C** Giving birth to a boy or a girl
**D** A learner plays football or plays hockey
**E** Winning or losing a football game
**F** Hitting a six or a four in a game of cricket

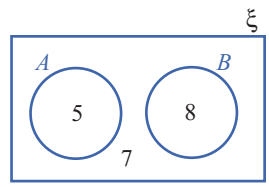
**2** For each of the Venn diagrams shown, find:

**i** $\Pr(A)$ **ii** $\Pr(B)$ **iii** $\Pr(A \cap B)$ **iv** $\Pr(A \cup B)$
**v** if $A$ and $B$ mutually exclusive

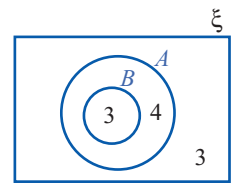
**a**



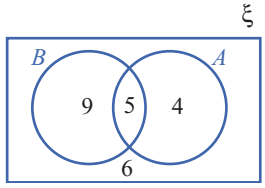

**b**



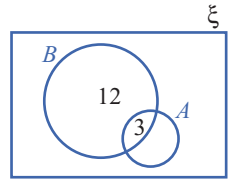

**c**



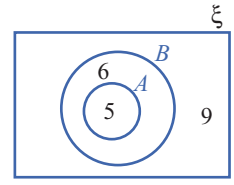

**d**




**e**




**f**




**3** **a** What is the probability of randomly selecting the following card from a pack of 52 playing cards?

**i** a face card **ii** a red card **iii** a spade
**iv** a red spade **v** a red face card **vi** a black card

**b** Which of the events above are mutually exclusive?

**4** In a group of 10 learners, the following characteristics were noted:

| Learner | Sex | Eye colour | Hair colour | Right- or left-handed | Siblings |
|---|---|---|---|---|---|
| Emma | Female | Blue | Blond | Left | 2 |
| Thomas | Male | Brown | Brown | Right | 1 |
| Joanna | Female | Brown | Brown | Right | 1 |
| Kade | Male | Blue | Blond | Right | 2 |
| Amy | Female | Green | Red | Right | 2 |
| Rowan | Male | Brown | Brown | Right | 1 |
| Daniel | Male | Brown | Brown | Right | 1 |
| Myron | Male | Blue | Blond | Right | 2 |
| Poornima | Female | Brown | Brown | Left | 2 |
| Kent | Male | Blue | Brown | Right | 2 |

**a** What is the probability that a randomly selected learner:
**i** is male? **ii** is blue-eyed? **iii** has brown hair? **iv** is left-handed?

**b** What is the probability that a randomly selected learner:
**i** has brown eyes and brown hair? **ii** has green eyes and red hair?
**iii** has blue eyes and is female? **iv** is left-handed and is male?
**v** has two siblings and is right-handed? **vi** has only one sibling and is blonde?

**c** What is the probability that a randomly selected learner:
**i** has brown hair or blue eyes? **ii** is male or left-handed?
**iii** is female or has blond hair? **iv** is left-handed or has only one sibling?
**v** has green eyes or is right-handed? **vi** has blond or brown hair?

**d** Which of the situations in part b are mutually exclusive? Explain your answer.

**5** Learners selecting their electives could choose from Sports, Home Economics, Art, Religious Studies, Agriculture and Technology. There are 120 learners in the year level and the level coordinator had to sort the electives into three blocks, given the information in the table.

| Elective | Number of learners |
|---|---|
| Sports | 52 |
| Home Economics | 46 |
| Art | 86 |
| Religious Studies | 64 |
| Agriculture | 58 |
| Technology | 27 |

**a** List the combinations of two electives that cannot be mutually exclusive.

**b** The number of learners who select not to do either Sports or Home Economics is 32. Are these two elective choices mutually exclusive? Explain your answer.

**c** The number of learners who select to do either Agriculture or Technology is 65. Are these two elective choices mutually exclusive? Explain your answer.

# 11E Multiplication law

Two or more events are **independent** if the result of one event does not have an effect on the other events. For example, the probability of getting a tail when you toss a coin is $\frac{1}{2}$. The second time you toss the coin, the probability of getting a tail is still $\frac{1}{2}$. Getting a tail on the first toss of the coin and getting a tail on the second toss of the coin are independent events.

When two events are independent of each other:

$$\Pr(A \text{ and } B) = \Pr(A \cap B) = \Pr(A) \times \Pr(B)$$

It follows that $\Pr(A \cap B) \neq \Pr(A) \times \Pr(B)$ means that events are not independent and that the result of the first event could have an effect on the result of the other event.

## Example

**1** Assuming that the chance of being born with a certain star sign is $\frac{1}{12}$, what is the probability that two randomly selected people are both Librans?

## Solution

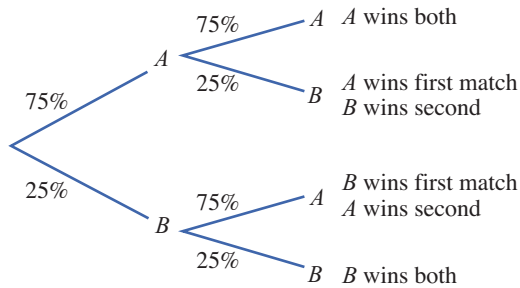
The probability that a person is a Libran is independent of another person's star sign, so we can use the multiplication principle:

Pr(two people are Librans)
= Pr(first Libran) × Pr(second Libran)

$$= \frac{1}{12} \times \frac{1}{12}$$

$$= \frac{1}{144}$$

**2** Experience has shown that, on average, Ann beats Bern at tennis about 75% of the time. Assume that winning each game is an independent event. They play two games.




**a** What is the probability that Ann wins both games?

Pr($A$ wins both)
= Pr($A$ wins first *and* $A$ wins second)
= Pr($A \cap A$)
= Pr($A$) × Pr($A$)
= 0·75 × 0·75
= 0·5625

Probability Ann wins both is about 56%.

**b** What is the probability that Bern wins both games?

Pr($B$ wins both)
= Pr($B$ wins first *and* $B$ wins second)
= Pr($B \cap B$)
= Pr($B$) × Pr($B$)
= 0·25 × 0·25
= 0·0625

Probability Bern wins both is about 6%.

**Example**

**c** What is the probability that they each win one game?

**Solution**

Pr(each wins a game)
= Pr($A$ wins first *and* $B$ wins second) *or* ($B$ wins first *and* $A$ wins second)
= $\Pr(A \cap B) + \Pr(B \cap A)$
= $\Pr(A) \times \Pr(B) + \Pr(B) \times \Pr(A)$
= $0{\cdot}75 \times 0{\cdot}25 + 0{\cdot}25 \times 0{\cdot}75$
= $0{\cdot}375$

Probability each wins a game is 37·5%.

## Exercise 11E

**1** What is the probability of throwing a double six when you throw two dice?

**2** What is the probability of throwing five heads in a row when you toss a coin?

**3** On a dinner party menu there are:
three soups: tomato, French onion and pumpkin
four roasts: beef, fish, pork and chicken
two desserts: ice cream and sweet biscuits.

If the food is served randomly, find the probability of being served:

**a** tomato soup

**b** sweet biscuits

**c** pumpkin soup and roast pork

**d** tomato soup, fish and ice cream

**e** roast beef and sweet biscuits

**4** Peter has:
9 shirts: 4 white shirts, 3 black shirts, 2 light blue shirts
6 ties: 3 navy ties, 2 red ties, 1 white and black striped tie
3 suits: 2 navy suits, 1 black suit

Peter is always a bit sleepy when he gets dressed, and one morning he doesn't pay much attention to what he is putting on. Find the probability that he wears:

**a** a white shirt

**b** a black suit

**c** a navy tie

**d** a navy suit and a light blue shirt

**e** a black shirt and a red tie

**f** a black suit, a white shirt and a white and black striped tie

**5** Kevin has a sock drawer containing 10 black, 7 blue and 5 red socks.

**a** If he pulls out a pair of socks at random, find the probability he selects:

**i** two black socks

**ii** two red socks

**iii** one red and one blue sock

**iv** a red and a black sock

**b** What is the minimum number of socks he must select at random to ensure he has a matching pair?

# 11F Odds

In gambling, especially in horseracing, we often express the likelihood of an event as 'odds'. These odds are estimates of probability based on the past experiences of the horse under similar conditions. However, odds can also be used where the probabilities are known, such as in the games at a casino.

Odds are written in the form of a ratio: ways to lose : ways to win

If the odds are high then the event is less likely to occur than an event with lower odds. Where there is a fifty-fifty chance of an event then the odds are 1 : 1.

**Example**

**1** What are the odds of getting a queen when you randomly select a card from a 52-card pack?

**Solution**

There are 52 cards in the pack.

There are 4 queens and 48 other cards.

The odds of getting a queen are 48 : 4 or 12 : 1.

To convert from odds to probability we use $\Pr(\text{win}) = \dfrac{\text{ways to win}}{\text{ways to win} + \text{ways to lose}}$

**Example**

**2** If the odds against an event occurring are 8 : 1, what is the probability of the event occurring?

**Solution**

$$\text{Probability} = \frac{1}{1+8} = \frac{1}{9}$$

**3** If the probability of a win is $\frac{5}{6}$, what are the odds?

$$\text{Odds} = \frac{5}{6} \div \frac{1}{6} = 5:1$$

Bookmakers at racecourses commonly use odds, but at the TAB or in the newspapers the odds are generally written as **unit payouts**. The unit payout is the amount a punter can expect to get in return for a $1 bet.

$$\text{Unit payout} = \frac{\text{payout}}{\text{original bet}}$$

For comparison the unit payout can be calculated from the odds. If a bet is placed with a bookmaker then the payout is calculated by multiplying the bet by the odds and adding the original bet.

$$\text{Payout} = \text{bet} \times \text{odds} + \text{bet}$$

**Example**

**4** Jim bets $5 on a horse when the odds are 7 : 2.

**a** If the horse wins, how much should he expect as a payout?

**Solution**

The odds of 7 : 2 are equivalent to 3·5 : 1

$$\text{Payout} = 5 \times 3{\cdot}5 + 5 = 17{\cdot}5 + 5 = \$22{\cdot}50$$

**Example**

b What is the unit payout?

**Solution**

Unit payout $= \frac{22{\cdot}50}{5}$

Unit payout is 4·5

## Exercise 11F

**1** Convert these probabilities to odds:

a $\frac{1}{2}$ b $\frac{1}{5}$ c $\frac{2}{13}$ d $\frac{3}{4}$

e $\frac{5}{8}$ f $\frac{3}{10}$ g $\frac{4}{15}$ h $\frac{7}{12}$

**2** Convert these odds to probabilities:

a 3:1 b 7:1 c 6:2 d 11:4

e 33:1 f 9:4 g 15:7 h 13:8

**3** In a roulette game at a casino there are 37 possible outcomes numbered 0–36. The even numbers are red, the odd numbers are black and the zero is green. The casino pays out 2:1 if a ball lands on an even number.

a What is the probability of an even number?

b What are the odds of getting a red?

c Are the odds more or less than the actual payout?

**4** Caroline bet $20 'on the nose' on a horse that had odds of 3:1. What should she expect to get back if the horse wins its race? (Note: 'On the nose' means to win.)

**5** Calculate how much Caroline would expect to win on a $20 bet if the odds were:

a 10:1 b 5:1 c 7:4 d 3:2

**6** Find the unit payout if the odds are:

a 7:1 b 9:1 c 11:2 d 7:4

**7** State the odds for the unit payouts given in the race as shown (right):

**Golfhouse Hotel Members Plate 12.00**

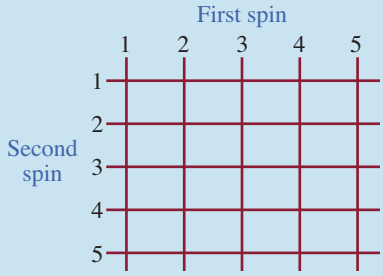
**2200 m** $11,000. Maiden. 3YO&UP. Set weights. Apprentices can claim.

| | | | | |
|---|---|---|---|---|
| **NOLAN LANE, 6** | J R Smith | **N Howes** | **57** | **5.50** |
| **HOT PURSUIT, 9** | P A Holt | **V Bull** | **57** | **6.00** |
| **REIGNITE, 4** | P A Holt | **G Cross** | **57** | **★3.50** |
| **SUGAR DAD, 1** | D A Ryan | **M Ross** | **57** | **11.00** |
| **DALTON'S BEST, 2** | D C Harris | **S Noonan** | **54.5** | **15.00** |
| **MARGUEETA, 2** | D R Lewis | **B Chu** | **54.5** | **7.00** |
| **ON THE KNOCKER, 3** | D R Lewis | **M Phillips** | **54.5** | **15.00** |
| **RISING RIVER, 8** | D R Lewis | **R Hinton** | **54.5** | **15.00** |
| **MAGICAL MOON, 7** | S Trioli | **N Stock** | **52** | **5.00** |

**8** In a 'daily double' a punter correctly picks the winners of two specified races in a day. The winnings of the first race are placed on the second race. Mick chooses a horse with odds of 12:1 in the first race and 7:2 in the second race. He bets $5. What would he expect to get back if both horses win?

# Puzzles

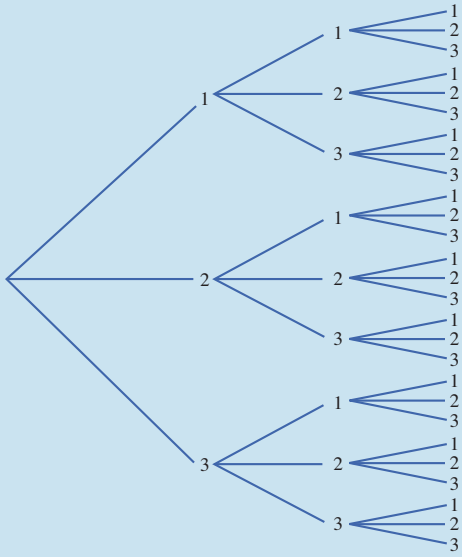
**1** A spinner divided into five equal sectors numbered 1–5 was spun twice.The grid shows the event space. Match the letters below to the correct probability of each event to find the name of the horse and jockey who won the Melbourne Cup in 1954:


**A** Pr(both the same)
**E** Pr(add to 3)
**F** Pr(different by 1)
**G** Pr(not the same)
**I** Pr(add to 2)
**L** Pr(product is >12)
**N** Pr(product of 4)
**P** Pr(add to >5)
**R** Pr(both odd)
**S** Pr(both even)
**T** Pr(add to <10)
**U** Pr(first number is even)

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $\frac{9}{25}$ | $\frac{1}{25}$ | $\frac{4}{25}$ | $\frac{1}{25}$ | $\frac{3}{25}$ | $\frac{4}{5}$ | | $\frac{2}{25}$ | $\frac{1}{5}$ | $\frac{4}{25}$ | $\frac{24}{25}$, |
| $\frac{3}{5}$ | $\frac{2}{5}$ | $\frac{9}{25}$ | $\frac{24}{25}$ | $\frac{2}{25}$ | $\frac{6}{25}$ | $\frac{6}{25}$ | | | | |

**2** A spinner divided equally into three sectors and numbered 1–3 was spun three times. The event space is shown as a tree diagram. Find the probabilities of the outcomes and match the letters to the correct probability below to find the name of the horse who won the Melbourne Cup in 1970:




**A** Pr(add to 3)
**B** Pr(all 3 are the same)
**D** Pr(1, 2 and 3 in any order)
**E** Pr(2 or more numbers are the same)
**G** Pr(multiply to 3)
**H** Pr(add to >6)
**N** Pr(first 2 numbers are the same)
**O** Pr(last number is not 1)
**T** Pr(do not add to 3)

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $\frac{1}{9}$ | $\frac{1}{27}$ | $\frac{4}{27}$ | $\frac{10}{27}$ | $\frac{2}{9}$ | $\frac{1}{27}$ | $\frac{2}{9}$ | | $\frac{1}{3}$ | $\frac{2}{3}$ | $\frac{26}{27}$ | $\frac{7}{9}$ |

**3** Find the missing probabilities in the table below by using the rules:
$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$ and $\Pr(A') = 1 - \Pr(A)$
Match the letters to the correct probabilities to find the name of the horse, jockey and trainer of the 1993 Melbourne Cup winner:

| $\Pr(A)$ | $\Pr(B)$ | $\Pr(A \cap B)$ | $\Pr(A \cup B)$ | $\Pr(A')$ | $\Pr(B')$ |
|---|---|---|---|---|---|
| 0·5 | 0·3 | 0·2 | **A** | **C** | **D** |
| **E** | 0·6 | **G** | 0·8 | 0·7 | **I** |
| 0·8 | **K** | 0·7 | **L** | **N** | 0·2 |
| 0·62 | 0·55 | O | 0·92 | **P** | **R** |
| **T** | **V** | 0·56 | **W** | 0·25 | 0·48 |

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| ____ | ____ | ____ | ____ | ____ | ____ | ____ | | ____ | ____ | ____ | ____, |
| 0·52 | 0·4 | 0·2 | 0·75 | 0·6 | 0·1 | 0·3 | | 0·5 | 0·45 | 0·25 | 0·38 |
| ____ | ____ | ____ | ____ | ____ | ____, | | ____ | ____ | ____ | ____ | |
| 0·8 | 0·4 | 0·2 | 0·6 | 0·2 | 0·3 | | 0·71 | 0·3 | 0·9 | 0·7 | |

**4** Convert the following odds to probabilities. Match the letters to the correct probabilities below to find the name of the horse and rider who won the 1985 Melbourne Cup:

| | | | |
|---|---|---|---|
| **A** 1:1 | **C** 2:1 | **D** 3:1 | **E** 1:2 |
| **G** 5:4 | **H** 1:7 | **I** 1:5 | **L** 1:3 |
| **M** 13:2 | **N** 2:5 | **R** 5:7 | **S** 12:3 |
| **T** 3:5 | **U** 7:6 | **W** 7:10 | **Y** 5:8 |

| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ____ | ____ | ____ | ____ | | ____ | | ____ | ____ | ____ | ____ | ____ | ____ | ____ | ____ | ____, |
| $\frac{10}{17}$ | $\frac{7}{8}$ | $\frac{1}{2}$ | $\frac{5}{8}$ | | $\frac{1}{2}$ | | $\frac{5}{7}$ | $\frac{6}{13}$ | $\frac{5}{6}$ | $\frac{3}{15}$ | $\frac{1}{2}$ | $\frac{5}{7}$ | $\frac{1}{3}$ | $\frac{2}{3}$ | |
| ____ | ____ | ____ | ____ | ____ | ____ | | ____ | ____ | ____ | ____ | ____ | ____ | ____ | | |
| $\frac{7}{8}$ | $\frac{8}{13}$ | $\frac{3}{4}$ | $\frac{1}{2}$ | $\frac{5}{7}$ | $\frac{1}{4}$ | | $\frac{2}{15}$ | $\frac{2}{3}$ | $\frac{1}{2}$ | $\frac{4}{9}$ | $\frac{7}{8}$ | $\frac{2}{3}$ | $\frac{7}{12}$ | | |

# Applications

## A day at the races

The first rule of gambling is 'never bet more than you can afford to lose'.

Imagine you have put away $50 to bet on a particular racing event.

**a** Check the racing pages of the paper for that event and calculate how you would place your bets for the day if you were at the racecourse. No each-way betting is allowed.

**b** The next day check the results in the newspaper.

**c** Calculate how much you would have won or lost.

**d** Compare your results with those of other learners in the class.

## Star signs

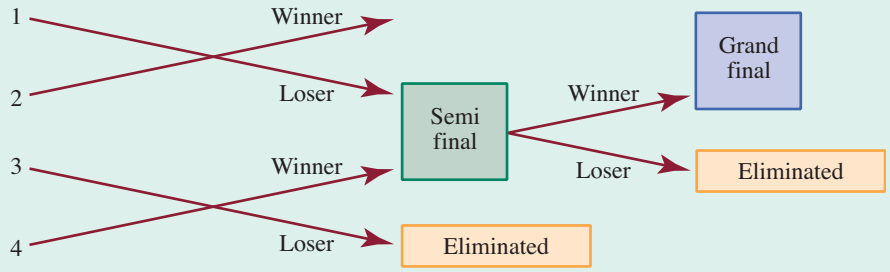
Is there any truth in the theory that people born under certain star signs are attracted to other people born under the same sign? If there is no truth to the theory you would expect that the star signs of partners would be random.

**a** Find the star signs for both partners in successful relationships among your friends and family, or of famous people.

**b** Does the selection of partners seem random?

## Final eight

The playoff format between the top eight teams in the Solomon Islands National Soccer League (NSL) championship is as follows:



The current NSL final series involves the top eight teams at the end of the home-and-away game series.

| Week 1 | Week 2 | Week 3 | Week 4 |
|---|---|---|---|
| *Qualifying Finals*<br>(A) Team 1 v Team 4<br>(B) Team 2 v Team 3<br>Teams 1 and 2 will host the matches.<br>All teams have double chance.<br>*Elimination Finals*<br>(C) Team 5 v Team 8<br>(D) Team 6 v Team 7<br>Teams 5 and 6 will host the matches.<br>Losers are eliminated. | *Semi Finals*<br>(E) Loser (A) v Winner (C)<br>(F) Loser (B) v Winner (D)<br>Losers from qualifying finals will host the matches.<br>Losers eliminated. | *Preliminary Finals*<br>(G) Winner (A) v Winner (F)<br>(H) Winner (B) v Winner (E)<br>Winners from qualifying finals host preliminary finals.<br>The crossover of matches ensures there is no repeat of earlier finals.<br>Winners proceed to Grand Final.<br>Losers eliminated. | *Grand Final*<br>(I) Winner (G) v Winner (H) |

**a** Use a tree diagram similar to the one above to represent the outcomes of the finals series of games.

**b** Assuming that the probability of each team winning a match is 0·5 and that they are independent events, calculate the theoretical probability of each team winning the grand final, by using:

- **i** the multiplication principle to give a theoretical answer
- **ii** a coin or random number generator as a simulation

**c** Compare the chances for each team of winning if it is in:

- **i** the final four
- **ii** the final eight

# Enrichment

**1** True, false or unsure?
For each of these statements, decide whether you think it is true, false or you are unsure. Share your opinion with a classmate, and justify your opinion using strategies you learnt earlier in this chapter.

**a** When you roll a fair six-sided die, it is harder to roll a six than a four.

**b** In a lottery, the six numbers 3, 12, 326, 37, 38, 40 are more likely to come up than the numbers 1, 2, 3, 4, 5, 6.

**c** If a family has already got four boys, then the next baby is more likely to be a girl than a boy.

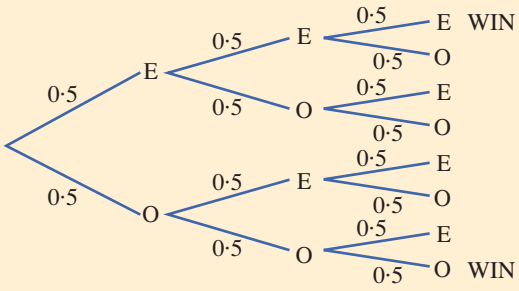
**d** Scoring a total of three with two dice is twice as likely as scoring a total of two.

**e** There are three outcomes in a football match, win, lose or draw. The probability of winning is therefore $\frac{1}{3}$.

**f** In a 'true or false' quiz with 10 questions, you are certain to get five right if you just guess.

**2** In a game of chance 10 balls numbered 1–10 were placed in a bag. A ball was drawn at random, its number was noted and the ball was replaced. This was repeated three times. Players won a jackpot if the three numbers were all even or all odd.

The tree diagram below shows the possible ways of winning:




**a** Use the multiplication principle to calculate the probability of a win.

The game was changed and the balls were not replaced.

**b** Redraw the tree diagram to show the possible ways of winning now.

**c** Use the multiplication principle to calculate the probability of winning now.

**d** Compare the two probabilities. Which has the better chance of winning? Explain why.

**3** Vehicle registration plates have three letters followed by three digits.

**a** If we allow digits and letters to be used more than once, there are $26 \times 26 \times 26 \times 10 \times 10 \times 10 = 17\,576\,000$ different arrangements.
What is the probability that a randomly selected registration plate:

**i** starts with a Q?
**ii** ends with a 7?
**iii** has two letters the same?
**iv** has three digits the same?
**v** is an even number?

**b** If we do not allow digits and letters to be used more than once, there are $26 \times 25 \times 24 \times 10 \times 9 \times 8 = 11\,232\,000$ different arrangements.
What is the probability that a randomly selected registration plate:

**i** starts with a Q?
**ii** ends with a 7?
**iii** has an even digit?
**iv** ends with 0 or 8?

**4** 80 families were surveyed and asked which newspaper they bought:
60 bought the *Solomon Star*, 30 bought *Solomon Times* and 10 bought no newspaper.

**a** Draw a Venn diagram to represent the information.

**b** What is the probability that a family buys:

**i** the *Solomon Star*?

**ii** *Solomon Times*?

**iii** neither newspaper?

**iv** both newspapers?

**v** the *Solomon Star* but not the *Solomon Times*?

**vi** *Solomon Times* but not the *Solomon Star*?

**5** In a 52-card pack of playing cards, there are 13 hearts, 13 clubs and 12 face cards.

**a** Display this information in a Venn diagram.

**b** Find the probability that a randomly drawn card is:

**i** a heart

**ii** a face card

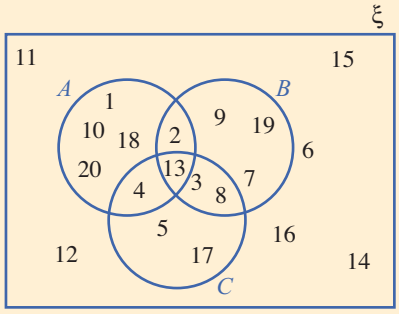
**iii** a club that is not a face card

**iv** a heart and a face card

**v** not a club or a heart

**vi** a face card that is not a club or a heart

**6** The Venn diagram shows the numbers 1–20 and the sets $A$, $B$ and $C$. Use the Venn diagram to list the elements in each of the sets below:




**a** $A$

**b** $B$

**c** $C$

**d** $A'$

**e** $A \cap B$

**f** $B \cap C$

**g** $A \cap B \cap C$

**h** $A \cap B'$

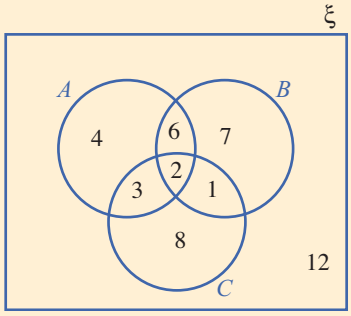
**i** $B' \cup C'$

**j** $(A \cup B \cup C)'$

**k** $A \cap B \cap C'$

**l** $A' \cap B' \cap C'$

**7** The Venn diagram shows the number of elements in each of the sets $A$, $B$ and $C$. Use the Venn diagram to find the probabilities of the following:




**a** $\Pr(A)$

**b** $\Pr(A \cap B)$

**c** $\Pr(A \cap B \cap C)$

**d** $\Pr(A \cap B')$

**e** $\Pr(A')$

**f** $\Pr(B \cup C)$

**g** $\Pr(A \cup B \cup C)$

**h** $\Pr(B \cup C')$

**i** $\Pr(B)$

**j** $\Pr(C)$

**k** $\Pr(B')$

**l** $\Pr(B' \cap C')$

# Revision/Assessment

## Exercise 11A

**1** When two dice are thrown, what is the probability that:

- **a** they both show a 6?
- **b** they show the same number?
- **c** the sum of the numbers shown is greater than 8?
- **d** the difference between the numbers is less than 2?

**2** What is the probability that the answers to two multiple-choice questions with five possible answers $A$–$E$:

- **a** are both $C$?
- **b** are the same?
- **c** are different?

**3** What is the probability that when two people are selected at random:

- **a** they are both Taurean?
- **b** they are the same star sign?
- **c** only one is a Taurean?
- **d** they are different star signs?

**4** What is the probability that when two people randomly select a playing card:

- **a** they both select a red card?
- **b** they both select a heart?
- **c** only one selects a heart?
- **d** only one selects a black card?
- **e** they select cards of the same suit?
- **f** they select cards of a different suit?

**5** When three coins are tossed, what is the probability of:

- **a** three heads?
- **b** no heads?
- **c** at least one head?

**6** In a three-child family, what is the probability of:

- **a** three girls?
- **b** two boys?
- **c** a boy and a girl?

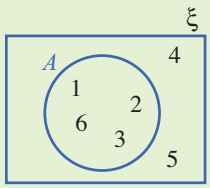
## Exercise 11B

**7** A bag contains 36 lollies, 15 are red, 12 are green and the rest are black. When a lolly is picked at random from the bag, what is the probability of getting:

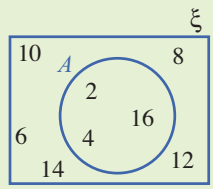
- **a** a red lolly?
- **b** a green lolly?
- **c** a black lolly?
- **d** not a red lolly?
- **e** not a green lolly?
- **f** not a black lolly?

**8** For each Venn diagram shown below, list Pr($A$) and Pr($A'$) and show that they add to 1:

**a**




**b**




## Exercise 11C

**9** Draw a Venn diagram to find the probability that a randomly selected playing card is:

- **a** a black card
- **b** an even-numbered card
- **c** a black, even-numbered card
- **d** a black card or an even-numbered card

**10** Find $\Pr(A \cap B)$ if:

**a** $\Pr(A) = 0{\cdot}4$, $\Pr(B) = 0{\cdot}5$ and $\Pr(A \cup B) = 0{\cdot}6$

**b** $\Pr(A) = 0{\cdot}2$, $\Pr(B) = 0{\cdot}3$ and $\Pr(A \cup B) = 0{\cdot}4$

**c** $\Pr(A) = 0{\cdot}4$, $\Pr(B) = 0{\cdot}5$ and $\Pr(A \cup B) = 0{\cdot}5$

## Exercise 11D

**11** A learner is asked to choose a number between 10 and 30 inclusive.

**a** What is the probability that the number is:

**i** a prime number? **ii** an even number? **iii** an odd number?

**iv** divisible by 3? **v** divisible by 5? **vi** divisible by 7?

**b** What is the probability that the number is:

**i** a prime or an odd number? **ii** an even number or divisible by 7?

**iii** an even or a prime number? **iv** divisible by 3 or 7?

**v** divisible by 5 or 7? **vi** divisible by 3 or 5?

**c** Which of the situations in part b are mutually exclusive? Explain your answer.

## Exercise 11E

**12** Experience has shown that, on average, Amy beats Bev at tennis about 80% of the time. If they play each other three times in one week, what is the probability that:

**a** Amy wins all the games?

**b** Bev wins all the games?

**c** Amy wins only one game?

**d** Bev wins at least one game?

**13** In a quiz show, a person wins \$1 000 000 if they correctly answer 15 questions in a row. They are given 4 possible answers. If they guess the answer to each question, what is the probability that they win \$1 000 000?

**14** **a** What is the probability of correctly guessing the answers of all six questions in a multiple-choice test, given that there are five options for each question?

**b** What is the probability of correctly guessing only one of the answers to the six questions?

## Exercise 11F

**15** Convert these probabilities to odds:

**a** $\frac{1}{2}$ **b** $\frac{2}{5}$ **c** $\frac{2}{3}$ **d** $\frac{3}{4}$

**e** $\frac{5}{12}$ **f** $\frac{1}{10}$ **g** $\frac{4}{15}$ **h** $\frac{7}{15}$

**16** Convert these odds to probabilities:

**a** 2:1 **b** 5:1 **c** 7:2 **d** 15:4

**e** 30:1 **f** 9:2 **g** 18:7 **h** 25:8

CHAPTER

# 12

# Indices

Edwin Hubble, an American astronomer in the 1920s, discovered that the universe was expanding and that the more-distant galaxies were receding faster than the closer stars. From the value of the Hubble constant, currently estimated to be 22·46 km/s per million light-years (an extremely small number), the age of the universe is thought to be about 15 billion years. Scientists manage these large numbers in their calculations by writing them as $2{\cdot}246 \times 10^{-5}$ and $1{\cdot}5 \times 10^{7}$. These are called standard form numbers and such numbers make ready use of indices and powers of ten.

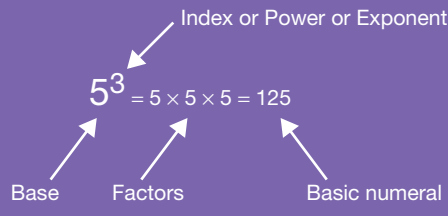
## This chapter covers the following skills:

- Expressing algebraic and numerical quantities in index form
- Applying the index laws to simplify numerical and algebraic expressions
- Working with zero and negative powers
- Working with simple fractional indices
- Graphing simple exponential curves in modelling situations

## Specific Learning Outcomes (SLO)

Learners should be able to:

**9.12.1.1** Define and identify the following terms: *index numbers*, *exponential numbers*, *factor*




**9.12.2.1** Convert index numbers into factor form and evaluate as basic numerals.

**9.12.3.1** Apply index laws that deal with multiplication and division of index numbers:
Index law 1: $b^x \times b^y = b^{x+y}$
*for multiplication: ADD the power*
Index law 2: $b^x \div b^y = b^{x-y}$
*for division: SUBTRACT powers*

**9.12.4.1** Identify and apply index laws that expand (remove) brackets:
Index law 3: $(x^P)^q = x^{p \times q}$
Index law 4: $(xy)^P = x^p y^p$
Index law 5: $\left(\frac{x}{y}\right)^q = \frac{x^P}{y^P}$

**9.12.5.1** Apply the index law for any base that has a powers equal to zero:
*Any base to the power of zero equals one*: $b^0 = 1$

**9.12.6.1** Apply index laws that relate to negative Indices:
In general: $x^{-3} = \frac{1}{x^3}$ so $b^{-P} = \frac{1}{b^P}$
and therefore: $b^{-P} = \frac{1}{b^P}$ and $\frac{1}{b^{-P}} = b^P$

**9.12.7.1** Simplify negative indices by removing brackets to basic numerals with positive powers.

**9.12.8.1** Identify a Base with a power of $\frac{1}{2}$ as the square root of that base.
Example: $49^{\frac{1}{2}} = \sqrt{49}$
In general: $x^{\frac{1}{2}} = \sqrt{x}$ for any non-negative $x$

**9.12.9.1** Simplify indices with fractional powers as basic numerals.

**9.12.10.1** Express large and small numbers in standard form and as basic numerals.

**9.12.11.1** Apply standard form to word problems.

# 12A Index numbers

$5^3$ is an example of an **index number** in which 5 is the base and 3 is the index or **power** or **exponent**.

$5^3$ means $5 \times 5 \times 5$; that is, five multiplied three times, which equals 125 and is called the **basic numeral**.

$5^3$ is a number in **index** form, $5 \times 5 \times 5$ is the same number in **factor** form.

Note that $5^1$ means 5.

Note also that $(-5)^2$ means $-5 \times -5 = +25$.

However, $-5^2$ means the *negative of* $5^2$, so that $-5^2 = -(5 \times 5) = -25$.

**Example** **Solution**

**1** Write each of the following in factor form and then evaluate to a basic numeral:

**a** $3^5$

$3^5 = 3 \times 3 \times 3 \times 3 \times 3$
$= 243$

**b** $(-2)^4$

$(-2)^4 = -2 \times -2 \times -2 \times -2$
$= 16$

**c** $-7^2$

$-7^2 = -(7 \times 7)$
$= -49$

**d** $\left(\frac{2}{3}\right)^3$

$\left(\frac{2}{3}\right)^3 = \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3}$
$= \frac{8}{27}$

**2** Write in index form, simplifying where possible:

**a** $x \times x \times x \times x \times x \times x \times x$

$x \times x \times x \times x \times x \times x \times x = x^6$
There are six $x$s multiplied together.

**b** $x \times y \times x \times y \times y \times x \times x \times y$

$x \times y \times x \times y \times y \times x \times x \times y$
$= x^4 \times y^4$ or $x^4y^4$

**c** $3ab \times 2ba \times 5c \times 4bc$

$3ab \times 2ba \times 5c \times 4bc$
$= 3 \times 2 \times 5 \times 4 \times a \times a \times b \times b \times b \times c \times c$
$= 120a^2b^3c^2$

**d** $\frac{a}{b} \times \frac{a}{b} \times \frac{1}{b}$

$\frac{a}{b} \times \frac{a}{b} \times \frac{1}{b} = \frac{a^2}{b^3}$

## Exercise 12A

**1** Write each of the following in factor form and then evaluate:

| | | | | |
|---|---|---|---|---|
| **a** $5^2$ | **b** $3^2$ | **c** $6^3$ | **d** $2^4$ | **e** $10^3$ |
| **f** $6^1$ | **g** $1^6$ | **h** $0^2$ | **i** $9^2$ | **j** $2^6$ |
| **k** $10^2$ | **l** $10^6$ | **m** $4^3$ | **n** $3^4$ | **o** $100^3$ |

**2** Write in factor form and then evaluate:

**a** $(-1)^2$ **b** $(-3)^2$ **c** $(-4)^2$ **d** $(-5)^1$

**e** $(-10)^2$ **f** $(-1)^4$ **g** $(-9)^2$ **h** $(-6)^3$

**i** $(-2)^1$ **j** $(-2)^2$ **k** $(-2)^3$ **l** $(-2)^4$

**3** Evaluate the following to a basic numeral:

**a** $-3^1$ **b** $-3^2$ **c** $-5^3$ **d** $-10^4$

**e** $-(-4)^2$ **f** $-(-3)^1$ **g** $-(-5)^4$ **h** $-(-1)^3$

**i** $-(-2)^3$ **j** $-(-2)^4$ **k** $-(-2)^5$ **l** $-(-2)^6$

**4** Write in factor form and then evaluate:

**a** $\left(\frac{2}{5}\right)^2$ **b** $\left(\frac{1}{3}\right)^3$ **c** $\left(\frac{2}{5}\right)^4$ **d** $\left(\frac{3}{4}\right)^3$

**e** $\left(-\frac{1}{5}\right)^3$ **f** $\left(-\frac{2}{3}\right)^2$ **g** $\left(-\frac{1}{10}\right)^2$ **h** $\left(-\frac{5}{7}\right)^2$

**i** $\left(-\frac{4}{9}\right)^2$ **j** $-\left(-\frac{3}{4}\right)^3$ **k** $-\left(-\frac{1}{2}\right)^5$ **l** $-\left(-\frac{5}{6}\right)^2$

**5** Write down the index numbers represented by the following statements, and evaluate them to a basic numeral:

**a** base 4, power 3 **b** base 3, index 2 **c** base 5, exponent 1

**d** base –5, index 2 **e** base zero, power 3 **f** base 1, index 6

**g** base –2, index 4 **h** base 6, power 5 **i** base 10, index 3

**6** Write the following expressions in index form:

**a** $x \times x \times x$ **b** $y \times y \times y \times y \times y \times y$ **c** $a \times a \times a \times a$

**d** $x \times x \times y \times y \times y$ **e** $a \times a \times a \times b \times b$ **f** $x \times x \times y \times y \times z \times z$

**g** $x \times y \times x \times y \times y \times x$ **h** $a \times b \times a \times b \times a \times a$ **i** $a \times b \times c \times a \times c \times b$

**j** $-a \times b \times a \times -b \times c \times c$ **k** $m \times -n \times p \times -m \times -p$ **l** $-a \times -c \times b \times -b \times c \times -a$

**7** Write in index form and simplify where possible:

**a** $2ab \times 4ba \times 3c \times bc$ **b** $6xy \times 2yz \times 5yz \times xy$ **c** $pq \times 7qr \times 2pr \times 4qp$

**d** $-2yx \times 3zx \times 4zy$ **e** $2ab \times -4bc \times -3ac$ **f** $-2st \times 5tr \times -3srt \times 2rs$

**g** $-5x \times -6zy \times -3xzy$ **h** $-10ab \times -2ab \times -5bc$ **i** $6rst \times -str \times 5rts \times -s$

**8** Write in index form and simplify where possible:

**a** $\frac{x}{y} \times \frac{x}{y} \times \frac{1}{y}$ **b** $\frac{a}{c} \times \frac{a}{c} \times \frac{1}{c} \times \frac{a}{1}$ **c** $\frac{2}{t} \times \frac{3}{t} \times \frac{2s}{t} \times \frac{s}{5}$

**d** $\frac{3p}{q} \times \frac{2r}{5s} \times \frac{p}{q} \times \frac{r}{s}$ **e** $\frac{x}{y} \times \frac{5z}{7y} \times \frac{3x}{t} \times \frac{z}{4t}$ **f** $\frac{-2a}{b} \times \frac{c}{d} \times \frac{-3c}{5b} \times \frac{ac}{-7d}$

**g** $\frac{2x}{y} \times \frac{xz}{8xy} \times \frac{4x}{t} \times \frac{zt}{x}$ **h** $\frac{-3a}{bc} \times \frac{-2b}{ad} \times \frac{ac}{bd} \times \frac{cd}{6a}$ **i** $\frac{-2m}{w} \times \frac{3m}{p} \times \frac{q}{-4p} \times \frac{rt}{w}$

# 12B Exploring the index laws

## Learning task 12B

### Multiplication

**1** Complete the following:

**a** $3^2 \times 3^3$

$= 3 \times 3 \times \_\_ \times \_\_ \times \_\_$

$= 3^{\_\_}$ the index form of the answer

$=$ _____ worked out as a *basic numeral*

**b** $x^4 \times x^2$

$= x \times x \times \_\_ \times \_\_ \times \_\_ \times \_\_$

$= x^{\_\_}$ the index form of the answer

**c** Generalising: $b^x \times b^y = b^{\_\_}$

**d** The quicker way is to retain the base, $b$, and to _____ the powers, but only where the base is the same.

This is often referred to as the **first index law**.

### Division

**2** Complete the following:

**a** $5^6 \div 5^4$

$= \dfrac{5 \times 5 \times 5 \times \_\_ \times \_\_ \times \_\_}{5 \times \_\_ \times \_\_ \times \_\_}$

$= 5^{\_\_}$ (count up the number of remaining fives) the index form of the answer

$=$ ___ worked out as a *basic numeral*.

**b** $b^7 \div b^3$

$= \dfrac{b \times b \times \_\_ \times \_\_ \times \_\_ \times \_\_ \times \_\_}{b \times \_\_ \times \_\_}$

$= b^{\_\_}$ (count up the number of remaining $b$s) the index form of the answer

**c** Generalising: $b^x \div b^y = b^{\_\_}$

**d** The quicker way is to retain the base, $b$, and to _____ the powers, but only where the base is the same.

This is often referred to as the **second index law**.

**3** Simplify the following using index laws and write the answer as a basic numeral:

**a** $3^3 \times 3^2$ **b** $5^2 \times 5^1$ **c** $2^3 \times 2^4$

**d** $6^2 \times 6$ **e** $10^2 \times 10^3$ **f** $(-7)^2 \times (-7)^1$

**g** $(-4)^2 \times (-4)^2$ **h** $(-9) \times (-9)^2$ **i** $2^3 \times 2^2 \times 2^1$

**j** $3^3 \times 3 \times 3^2$ **k** $(-5)^2 \times (-5)^1 \times (-5)$ **l** $(-10)^2 \times (-10) \times (-10)^3$

**4** Simplify the following to an answer in index form:

**a** $x^4 \times x^5$ **b** $y^3 \times y^1$ **c** $m^4 \times m^2$ **d** $x^1 \times x^7$

**e** $a^1 \times a^3$ **f** $b \times b^5$ **g** $z^2 \times z^3$ **h** $b \times b^9$

**i** $x^2 \times x^3 \times x^4$ **j** $n \times n^2 \times n^4$ **k** $n \times n^1 \times n^3$ **l** $p^5 \times p^2 \times p^4$

**5** Simplify the following using index laws and write the answer as a basic numeral:

**a** $5^4 \div 5^3$ **b** $3^7 \div 3^2$ **c** $6^4 \div 6^2$ **d** $8^2 \div 8$

**e** $(-10)^6 \div (-10)^3$ **f** $(-4)^2 \div -4$ **g** $(-9)^5 \div (-9)^2$ **h** $(-5)^7 \div (-5)^5$

**6** Simplify the following:

**a** $x^{10} \div x^7$ **b** $y^9 \div y^6$ **c** $z^5 \div z^2$ **d** $b^6 \div b^5$

**e** $x^2 \div x^1$ **f** $p^3 \div p$ **g** $q^3 \div q^1$ **h** $z^7 \div z$

# Applying the index laws 12C

**Index law 1**: $b^x \times b^y = b^{x+y}$

**Index law 2**: $b^x \div b^y = b^{x-y}$

The base terms, $b$, must be the same in order to add or subtract indices in the above rules. Ordinary numbers and values multiply, divide and cancel in the usual way.

**Example** **Solution**

**1** Simplify $3x^7 \times 5x^3$ by using the index laws.

$3x^7 \times 5x^3 = 3 \times 5 \times x^7 \times x^3$
$= 15x^{10}$ retain base, add the powers

**2** Simplify $4xy^4 \times 6x^3y^6$ by using the index laws.

Working with each group separately:
$4xy^4 \times 6x^3y^6 = 4 \times 6 \times x^1 \times x^3 \times y^4 \times y^6$
$= 24 \times x^4 \times y^{10}$
$= 24x^4y^{10}$

**3** Simplify $20x^{12} \div 5x^7$ by using the index laws.

$$20x^{12} \div 5x^7 = \frac{20 \times x^{12}}{5 \times x^7}$$
$$= \frac{20}{5} \times \frac{x^{12}}{x^7}$$
$$= 4x^5$$

**4** Simplify $-\dfrac{4x^4y^7}{12x^3y^5}$ by using the index laws.

Treating each group separately:
$$-\frac{4x^4y^7}{12x^3y^5} = -\frac{4 \times x^4 \times y^7}{12 \times x^3 \times y^5}$$
$$= -\frac{4}{12} \times \frac{x^4}{x^3} \times \frac{y^7}{y^5}$$
$$= -\frac{1}{3}xy^2$$

**5** Simplify $\dfrac{24x^6y^5}{5x^2y} \div \dfrac{8xy^2}{15x^2y^4}$.

Treating each like group of terms separately:
$$\frac{24x^6y^5}{5x^2y} \div \frac{8xy^2}{15x^2y^4}$$
$$= \frac{24x^6y^5}{5x^2y} \times \frac{15x^2y^4}{8xy^2}$$
$$= \frac{\cancel{24}^{3}}{\cancel{8}_{1}} \times \frac{\cancel{15}^{3}}{\cancel{5}_{1}} \times \frac{x^6 \times x^2}{x^2 \times x^1} \times \frac{y^5 \times y^4}{y^1 \times y^2}$$
$$= 3 \times 3 \times \frac{x^8}{x^3} \times \frac{y^9}{y^3}$$
$$= 9x^5y^6$$

# 12C

## Exercise 12C

**1** Simplify the following:

**a** $3x^4 \times 7x^2$ **b** $4x^2 \times 5x^5$ **c** $8x^4 \times 5x^3$

**d** $4x^2 \times 2x^5$ **e** $-3y^2 \times 7y^1$ **f** $8z \times -5z^5$

**g** $-7a \times -a^3$ **h** $b^3 \times -8b^2$ **i** $-2x^3 \times 5x^2 \times x^3$

**j** $-3b \times b^2 \times -5b^1$ **k** $-7y^2 \times 4y^3 \times 2y^4$ **l** $-2x \times x^1 \times -x^5$

**2** Simplify the following:

**a** $2x^2y^3 \times 5x^3y^2$ **b** $3x^1y^3 \times 8x^2y^4$ **c** $4x^2y^2 \times 5x^2y^2$

**d** $3a^3b^3 \times 6a^2b^2$ **e** $-5m^1n^2 \times m^3n^5$ **f** $p^1q^3 \times -9p^2q^4$

**g** $4xy^3 \times 2x^2y^4 \times 10x^4y$ **h** $3x^2y^3 \times 4x^3y^2 \times 2x^2y^3$ **i** $x^2y^3 \times -4xy \times -2x^1y^3$

**j** $xy \times -3x^3y^1 \times -9xy$ **k** $2x^2y \times -3xyz^2 \times 4xyz$ **l** $3a^3b^2c \times -5abc^2 \times a^2bc$

**m** $-5m^2n \times -2mn^2p \times -mnp$ **n** $6mnp^2 \times -4mnp \times -2m^2np$

**3** Simplify the following as far as possible:

**a** $\dfrac{12x^5y^6}{3x^3y^5}$ **b** $\dfrac{24x^7y^5}{12x^5y^3}$ **c** $\dfrac{10x^6y^{10}}{-2x^3y^7}$ **d** $\dfrac{-15x^8y^6}{3x^4y^2}$

**e** $\dfrac{7r^3s^4}{14rs^3}$ **f** $\dfrac{4m^3n^4}{12mn^2}$ **g** $\dfrac{-9p^5q^4}{36p^3q^2}$ **h** $\dfrac{16a^3b^4}{-24a^2b^3}$

**i** $\dfrac{-20s^5t^3}{-25s^2t^2}$ **j** $\dfrac{-28x^6y^3}{35x^4y}$ **k** $\dfrac{-8z^5x^4}{6z^2x^3}$ **l** $\dfrac{-30x^4y}{25x^2y}$

**m** $\dfrac{12x^4y^6z^3}{8x^2y^5z}$ **n** $\dfrac{125a^3b^2c^4}{75a^2bc}$ **o** $\dfrac{60m^5n^6p^7}{45m^2n^3p^4}$

**4** Simplify the following as far as possible:

**a** $\dfrac{10x^2}{3y} \times \dfrac{6y^4}{5x}$ **b** $\dfrac{12y^5}{5x^2} \times \dfrac{10x^3}{6y^2}$

**c** $\dfrac{4y^4x}{3y} \times \dfrac{9x^3y}{2x^2y^2}$ **d** $\dfrac{-18b^7a^5}{3a^2b^2} \times \dfrac{8a^9b^5}{-4b^3a^6}$

**e** $\dfrac{14x^5}{3y} \div \dfrac{7y}{18x}$ **f** $\dfrac{20y^4}{3x^2} \div \dfrac{5y}{18x^4}$

**g** $\dfrac{32x^3y^4}{7x^2} \div \dfrac{8y^2x}{-21x^4y}$ **h** $\dfrac{-25m^3n^4}{4m^2n^2} \div \dfrac{10m^2n}{-14n^4m}$

**i** $\dfrac{8y^5}{3x^2} \times \dfrac{9x^3}{5y^2} \div \dfrac{12y^2}{30x}$ **j** $\dfrac{-4y^3}{3x^5} \times \dfrac{6x^3}{10y^2} \div \dfrac{12y}{-30x^2}$

**k** $\dfrac{12x^2y}{7xy^2} \times \dfrac{2x^2y}{3xy^3} \div \dfrac{48xy}{42x^3y^6}$ **l** $\dfrac{6m^4n^2p^5}{5m^2p} \div \left(\dfrac{p^2}{10mn} \times \dfrac{3m^2p}{n}\right)$

# Managing brackets and the zero power

12D

The index quantity $(x^2)^3$ means $x^2 \times x^2 \times x^2$, which works out to $x^6$, when we add the powers (of the same base). When we expand brackets, we multiply the powers.

**Index law 3**: $(x^p)^q = x^{p \times q}$

The index quantity $(xy)^4$ means $xy \times xy \times xy \times xy = x \times x \times x \times x \times y \times y \times y \times y = x^4y^4$

**Index law 4**: $(xy)^p = x^p y^p$

The index quantity $\left(\frac{x}{y}\right)^3$ means $\frac{x}{y} \times \frac{x}{y} \times \frac{x}{y} = \frac{x^3}{y^3}$

**Index law 5**: $\left(\frac{x}{y}\right)^p = \frac{x^p}{y^p}$

## The zero power

Consider $3^4 \div 3^4$
$= 3^{4-4}$
$= 3^0$ worked out by the second index law

Also $3^4 \div 3^4$
$= 81 \div 81$
$= 1$ worked out normally

So: $3^0 = 1$

This would be so for any base number $b$, not just 3.
Any base to the power of zero equals 1.

Hence: $b^0 = 1$

**Example**

**1** Remove brackets from the following expressions and, where possible, evaluate to a basic numeral:

**a** $(2^3)^2$

**Solution**

$(2^3)^2 = 2^{3 \times 2} = 2^6$
$= 2 \times 2 \times 2 \times 2 \times 2 \times 2$
$= 64$

**b** $(-3xy^2)^3$

$(-3xy^2)^3 = (-3)^3 \times x^{1 \times 3} \times y^{2 \times 3}$
$= -3 \times -3 \times -3 \times x^3 \times y^6$
$= -27x^3y^6$

**c** $\left(\frac{x^2y}{2z^3}\right)^5$

$\left(\frac{x^2y}{2z^3}\right)^5 = \frac{x^{2 \times 5} \times y^{1 \times 5}}{2^5 \times z^{3 \times 5}}$
$= \frac{x^{10}y^5}{32z^{15}}$

**Example** | **Solution**

**d** $(5^0)^8$

$(5^0)^8 = 5^{0\times 8}$
$= 5^0$
$= 1$

**e** $6(x^2)^0$

$6(x^2)^0 = 6 \times x^{2\times 0}$
$= 6 \times x^0$
$= 6 \times 1$
$= 6$

**f** $(-10x^5y^2)^0$

$(-10x^5y^2)^0 = 1$ any quantity to the power of zero is 1

**g** $4(7a^2)^0 - 6y^0$

$4(7a^2)^0 - 6y^0 = 4 \times 1 - 6 \times 1$
$= 4 - 6$
$= -2$

**2** Simplify the following expressions:

**a** $\left(\frac{2x^2}{y^3}\right)^2 \times \left(\frac{3y^2}{4x}\right)^3$

$\left(\frac{2x^2}{y^3}\right)^2 \times \left(\frac{3y^2}{4x}\right)^3$
$= \frac{2^2x^4}{y^6} \times \frac{3^3y^6}{4^3x^3}$
$= \frac{4}{64} \times \frac{27}{1} \times \frac{x^4}{x^3} \times \frac{y^6}{y^6}$
$= \frac{1}{16} \times \frac{27}{1} \times x^1 \times y^0$
$= \frac{27}{16}x$

**b** $\left(\frac{(-2)x^3z}{3y}\right)^4 \div \frac{(4z^2y^2x^5)^2}{(9y^5)^2}$

$\left(\frac{(-2)x^3z}{3y}\right)^4 \div \frac{(4z^2y^2x^5)^2}{(9y^5)^2}$
$= \frac{(-2)^4x^{12}z^4}{3^4y^4} \times \frac{9^2y^{10}}{4^2z^4y^4x^{10}}$
$= \frac{16}{81} \times \frac{81}{16} \times \frac{x^{12}}{x^{10}} \times \frac{y^{10}}{y^8} \times \frac{z^4}{z^4}$
$= x^2 \times y^2 \times z^0$
$= x^2 \times y^2 \times 1$
$= x^2y^2$

## Exercise 12D

**1** Simplify the following expressions and evaluate to a basic numeral:

**a** $8^0$ **b** $1000^0$ **c** $(-28)^0$ **d** $-50^0$

**e** $\left(\frac{3}{4}\right)^0$ **f** $\left(-\frac{5}{6}\right)^0$ **g** $-\left(1\frac{3}{5}\right)^0$ **h** $(0{\cdot}5)^0$

**i** $(4^0)^2$ **j** $(10^6)^0$ **k** $((-3)^2)^0$ **l** $((-3)^0)^2$

**m** $(5y)^0$ **n** $8x^0$ **o** $7(x^4)^0$ **p** $9(-a^3)^0$

**q** $(-9)^0 \times x^0 \times -y^0$ **r** $-8(a^0b^3)^0$ **s** $3^0 \times (2x)^0$ **t** $2x^0 + (3x)^0$

**2** Simplify the following expressions:

**a** $(8x^2y^3)^0$ **b** $(10m^3n^2)^0$ **c** $10(-3x^2y)^0$ **d** $(-p^2q^3r^4)^0$

**e** $3b^0 - 2y^0$ **f** $(4a^7)^0 - 3p^0$ **g** $(5x)^0 + 5x^0$ **h** $(3y)^0 \times 3^0y$

**3** Simplify the following expressions by removing brackets and evaluating to a basic numeral:

**a** $(2^2)^3$ **b** $(3^4)^2$ **c** $(10^2)^2$ **d** $((-2)^3)^2$

**e** $(5^1)^3$ **f** $((-3)^2)^2$ **g** $((-10)^3)^2$ **h** $-((-2)^2)^3$

**4** Simplify the following expressions by removing brackets:

**a** $(x^4y^3)^2$ **b** $(xy^3)^3$ **c** $(x^4y)^5$ **d** $(2a^3)^6$

**e** $(3xy^3)^2$ **f** $(2mn^2)^4$ **g** $(-4x^2y^3)^2$ **h** $(-5a^3b^4)^2$

**i** $(2m^4n^2p^3)^3$ **j** $(3a^3b^0c^2)^4$ **k** $(-2xy^3z^5)^3$ **l** $(-10m^0n^1p)^3$

**5** Simplify the following expressions by removing brackets:

**a** $\left(\frac{a^3}{c^2}\right)^2$ **b** $\left(\frac{y^3}{3z^4}\right)^3$ **c** $\left(\frac{2m^2}{p^4}\right)^3$ **d** $\left(\frac{3r^4s^2}{2t^3}\right)^4$

**e** $\left(\frac{-2uv^2}{5w^3z^2}\right)^2$ **f** $\left(\frac{4x^2y^3}{-3t^5z^3}\right)^3$ **g** $\left(\frac{10x^3y^2z}{u^2v}\right)^3$ **h** $\left(\frac{-3r^2s^1t}{u^2v^4w}\right)^3$

**6** Simplify the following products by first removing brackets:

**a** $(x^3)^2 \times (x^2)^5$ **b** $(a^3)^4 \times (a^2)^3$ **c** $(m^2)^4 \times (n^3)^2$

**d** $(2r^2)^4 \times (3s^2)^2$ **e** $(3m^2)^2 \times (2n^2)^3$ **f** $(5x^3)^2 \times (2z^5)^2$

**g** $(x^2y^4)^2 \times (x^3y^2)^3$ **h** $(ab^3)^2 \times (a^3b)^3$ **i** $(xy^2)^2 \times (x^3y^0)^3$

**j** $(3x^2y^4)^2 \times (5x^3y^2)^3$ **k** $(2mn^2)^3 \times (4m^2n^1)^2$ **l** $(10rs^3)^4 \times (r^3s^2)^2$

**7** Simplify the following quotients by first removing brackets:

**a** $x^7 \div (x^2)^3$ **b** $(y^3)^2 \div y^5$ **c** $(m^3)^4 \div (m^2)^5$

**d** $4(p^2)^3 \div 2p^5$ **e** $^-50(q^4)^2 \div (5q^3)^2$ **f** $(3s^4)^5 \div (3s^3)^2$

**g** $\frac{(x^2y^3)^4}{(x^1y^2)^5}$ **h** $\frac{(9m^3n^2)^3}{(3m^2n^2)^2}$ **i** $\frac{(16p^2q^4)^3}{(2p^3q^6)^2}$

**8** Simplify the following expressions:

**a** $\left(\frac{3x^3}{y}\right)^3 \times \left(\frac{2y}{x}\right)^3$ **b** $\left(\frac{4p^3}{q^4}\right)^2 \times \left(\frac{3q^3}{2p}\right)^3$ **c** $\left(\frac{m^3}{5n^2}\right)^2 \times \left(\frac{10n^3}{m^2}\right)^3$

# 12E Negative indices

Consider $x^4 \div x^7$.

Worked out by the second index law:

$x^4 \div x^7$
$= x^{4-7}$
$= x^{-3}$

When worked using factor form:

$$x^4 \div x^7$$
$$= \frac{\cancel{x} \times \cancel{x} \times \cancel{x} \times \cancel{x} \times \cancel{x}}{x \times x \times x \times x \times \cancel{x} \times \cancel{x} \times \cancel{x} \times \cancel{x} \times \cancel{x}}$$
$$= \frac{1}{x \times x \times x}$$
$$= \frac{1}{x^3}$$

So: $x^{-3} = \frac{1}{x^3}$

In general: $b^{-p} = \frac{1}{b^p}$ which gives meaning to a negative power.

If $p$ is replaced by $-p$, the above statement becomes $b^{-(-p)} = \frac{1}{b^{-p}}$, which gives $\frac{1}{b^{-p}} = b^p$.

Key concepts to know then are: $b^{-p} = \frac{1}{b^p}$ and $\frac{1}{b^{-p}} = b^p$

**Example** **Solution**

**1** Simplify the following by writing as basic numerals:

**a** $4^{-1}$

$4^{-1} = \frac{1}{4^1} = \frac{1}{4}$

**b** $3^{-2}$

$3^{-2} = \frac{1}{3^2} = \frac{1}{9}$

**c** $\frac{1}{2^{-1}}$

$\frac{1}{2^{-1}} = 2^1 = 2$

**d** $\frac{1}{3^{-4}}$

$\frac{1}{3^{-4}} = 3^4 = 81$

**e** $\left(\frac{2}{3}\right)^{-1}$

$$\left(\frac{2}{3}\right)^{-1} = \frac{2^{-1}}{3^{-1}}$$
$$= \frac{1}{2^1} \times \frac{3^1}{1}$$
$$= \frac{3}{2}$$

**Example** **Solution**

**2** Simplify the following, writing the answers with positive powers:

**a** $4x^{-1}$

$$4x^{-1} = \frac{4}{x^1} = \frac{4}{x}$$

**b** $xy^{-2}$

$$xy^{-2} = \frac{x}{y^2}$$

**c** $\dfrac{2}{m^{-1}}$

$$\frac{2}{m^{-1}} = 2m^1 = 2m$$

**d** $\dfrac{5}{z^{-3}}$

$$\frac{5}{z^{-3}} = 5z^3$$

**e** $\dfrac{x^{-1}}{y^{-1}}$

$$\frac{x^{-1}}{y^{-1}} = \frac{x^{-1}}{1} \times \frac{1}{y^{-1}}$$
$$= \frac{1}{x^1} \times \frac{y^1}{1}$$
$$= \frac{y}{x}$$

**f** $6a^{-3}b^2$

$$6a^{-3}b^2 = \frac{6}{1} \times \frac{1}{a^3} \times \frac{b^2}{1}$$
$$= \frac{6b^2}{a^3}$$

**g** $\left(\dfrac{2x^2}{y^{-3}}\right)^{-2} \times \left(\dfrac{3y^{-2}}{4x}\right)^3$

$$\left(\frac{2x^2}{y^{-3}}\right)^{-2} \times \left(\frac{3y^{-2}}{4x}\right)^3$$
$$= \frac{2^{1\times-2}x^{2\times-2}}{y^{-3\times-2}} \times \frac{3^{1\times3}y^{-2\times3}}{4^{1\times3}x^{1\times3}}$$
$$= \frac{2^{-2}x^{-4}}{y^6} \times \frac{3^3y^{-6}}{4^3x^3}$$
$$= \frac{2^{-2}\times3^3}{4^3} \times \frac{x^{-4}}{x^3} \times \frac{y^{-6}}{y^6}$$
$$= \frac{3^3}{2^2\times4^3} \times x^{-7} \times y^{-12}$$
$$= \frac{27}{256x^7y^{12}}$$

## Exercise 12E

**1** Simplify the following by writing as basic numerals:

**a** $5^{-1}$ **b** $10^{-1}$ **c** $10^{-2}$ **d** $10^{-3}$ **e** $3^{-3}$

**f** $1^{-1}$ **g** $-(1)^{-1}$ **h** $(-1)^{-1}$ **i** $2^{-1}$ **j** $6^{-2}$

**2** Simplify the following by writing as basic numerals:

**a** $\frac{1}{3^{-1}}$ **b** $\frac{1}{4^{-1}}$ **c** $\frac{1}{5^{-1}}$ **d** $\frac{1}{8^{-1}}$

**e** $\frac{1}{2^{-3}}$ **f** $\frac{1}{4^{-2}}$ **g** $\frac{2}{5^{-2}}$ **h** $\frac{5}{10^{-4}}$

**3** Remove brackets from the following and simplify:

**a** $\left(\frac{3}{4}\right)^{-1}$ **b** $\left(\frac{6}{5}\right)^{-1}$ **c** $\left(\frac{2}{7}\right)^{-1}$ **d** $\left(\frac{9}{10}\right)^{-1}$

**e** $\left(\frac{1}{10}\right)^{-1}$ **f** $\left(\frac{1}{6}\right)^{-1}$ **g** $\left(\frac{5}{1}\right)^{-1}$ **h** $\left(\frac{1}{100}\right)^{-1}$

**4** Simplify the following, writing the answers with positive powers:

**a** $5y^{-1}$ **b** $10x^{-1}$ **c** $7x^{-2}$ **d** $9w^{-3}$

**e** $10^{-1} \times z$ **f** $5^{-1} \times p$ **g** $4^{-2} \times t$ **h** $10^{-3} \times z$

**i** $4^{-1} \times p^{-1}$ **j** $8^{-1} \times p^{-1}$ **k** $2^{-2} \times z^{-3}$ **l** $3^{-2} \times t^{-4}$

**5** Simplify the following, writing the answers with positive powers:

**a** $xy^{-1}$ **b** $xy^{-3}$ **c** $a^2b^{-4}$ **d** $p^5q^{-5}$

**e** $m^{-1}n$ **f** $s^{-2}r^2$ **g** $u^{-6}v^2$ **h** $b^{-2}c$

**i** $u^{-1}v^{-1}$ **j** $x^{-2}y^{-1}$ **k** $m^{-2}n^{-3}$ **l** $p^{-4}q^{-3}$

**6** Simplify the following as far as possible by using the index laws:

**a** $\frac{3}{x^{-1}}$ **b** $\frac{5}{y^{-1}}$ **c** $\frac{10}{z^{-2}}$ **d** $\frac{8}{m^{-3}}$

**e** $\frac{x}{3^{-1}}$ **f** $\frac{y}{7^{-1}}$ **g** $\frac{z}{9^{-2}}$ **h** $\frac{s}{2^{-2}}$

**i** $\frac{3^{-1}}{x}$ **j** $\frac{2^{-1}}{r}$ **k** $\frac{10^{-1}}{x^2}$ **l** $\frac{5^{-2}}{z^3}$

**7** Simplify the following, writing the answers with positive powers:

**a** $\frac{y^{-1}}{x^{-1}}$ **b** $\frac{p^{-1}}{q^{-2}}$ **c** $\frac{r^{-2}}{s^{-1}}$ **d** $\frac{m^{-2}}{n^{-2}}$

**e** $\frac{z^2x^{-1}}{y^{-2}}$ **f** $\frac{c^{-2}}{ab^{-3}}$ **g** $\frac{p^{-2}q}{r^2s^{-1}}$ **h** $\frac{t^{-1}u^{-2}}{v^{-3}w^{-3}}$

**8** Simplify the following, writing the answers with positive powers:

**a** $\left(\frac{y}{x^{-2}}\right)^2 \times \left(\frac{x^{-2}}{y^{-1}}\right)^{-1}$ **b** $\left(\frac{2p^{-1}}{q}\right)^{-3} \times \left(\frac{q^2}{4p^{-1}}\right)^{-1}$ **c** $\left(\frac{5m^{-2}}{n^{-1}}\right)^{-3} \times \left(\frac{10n^{-1}}{m}\right)^2$

**d** $\left(\frac{r^{-1}}{4s^2}\right)^{-3} \times \left(\frac{s^{-3}}{8r}\right)^2$ **e** $\left(\frac{r^{-1}}{3s^2}\right)^3 \div \left(\frac{r^2}{12s^2}\right)^2$ **f** $\left(\frac{u^2v^{-1}}{w^{-1}x^2}\right)^{-1} \div \left(\frac{u^{-1}v^2}{w^2x^{-1}}\right)^2$

# Simple fractional indices 12F

**Power of one-half**

Consider $7^2 = 49 \Rightarrow 7 = \sqrt{49}$
(square root of 49)

Also $(7^2)^{\frac{1}{2}} = 49^{\frac{1}{2}} \Rightarrow 7^1 = 49^{\frac{1}{2}}$
(second root of 49)

So $49^{\frac{1}{2}} = \sqrt{49}$

In general, $x^{\frac{1}{2}} = \sqrt{x}$ for any non-negative $x$.

**One-third power**

Consider $5^3 = 125 \Rightarrow 5 = \sqrt[3]{125}$
(cube root of 125)

Also $(5^3)^{\frac{1}{3}} = 125^{\frac{1}{3}} \Rightarrow 5^1 = 125^{\frac{1}{3}}$
(third root of 125)

So $125^{\frac{1}{3}} = \sqrt[3]{125}$

In general, $x^{\frac{1}{3}} = \sqrt[3]{x}$

**One-fourth power**

Consider $3^4 = 81 \Rightarrow 3 = \sqrt[4]{81}$
(fourth root of 81)

Also $(3^4)^{\frac{1}{4}} = 81^{\frac{1}{4}} \Rightarrow 3^1 = 81^{\frac{1}{4}}$
(fourth root of 81)

So $81^{\frac{1}{4}} = \sqrt[4]{81}$

In general, $x^{\frac{1}{4}} = \sqrt[4]{x}$ for any non-negative $x$.

**One-fifth power**

Consider $2^5 = 32 \Rightarrow 2 = \sqrt[5]{32}$
(fifth root of 32)

Also $(2^5)^{\frac{1}{5}} = 32^{\frac{1}{5}} \Rightarrow 2^1 = 32^{\frac{1}{5}}$
(fifth root of 32)

So $32^{\frac{1}{5}} = \sqrt[5]{32}$

In general, $x^{\frac{1}{5}} = \sqrt[5]{x}$

In general then, $x^{\frac{1}{n}} = \sqrt[n]{x}$, where $x$ must only be a positive number if $n$ is even.

Note that fractional powers can be found on scientific calculators by using the $x^{\frac{1}{y}}$, $x^{\frac{p}{q}}$ or associated keys.

**Example** **Solution**

**1** Simplify the following by writing as basic numerals:

**a** $36^{\frac{1}{2}}$

$36^{\frac{1}{2}} = \sqrt{36} = 6$

**b** $64^{-\frac{1}{2}}$

$64^{-\frac{1}{2}} = \dfrac{1}{64^{\frac{1}{2}}} = \dfrac{1}{\sqrt{64}} = \dfrac{1}{8}$

**c** $8^{\frac{1}{3}}$

$8^{\frac{1}{3}} = \sqrt[3]{8} = 2$

| Example | Solution |
|---|---|
| **d** $64^{-\frac{1}{3}}$ | $64^{-\frac{1}{3}} = \dfrac{1}{64^{\frac{1}{3}}} = \dfrac{1}{4}$ |
| **e** $16^{\frac{1}{4}}$ | $16^{\frac{1}{4}} = 2$ (as $2^4 = 16$) |
| **f** $243^{-\frac{1}{5}}$ | $243^{-\frac{1}{5}} = \dfrac{1}{243^{\frac{1}{5}}} = \dfrac{1}{3}$ |

**2** Simplify the following expressions, writing the answers with positive powers:

**a** $(8m)^{\frac{1}{3}}$

$(8m)^{\frac{1}{3}} = 8^{\frac{1}{3}} \times m^{\frac{1}{3}} = 2m^{\frac{1}{3}}$

**b** $(16y^{-4})^{-\frac{1}{4}}$

$(16y^{-4})^{-\frac{1}{4}} = 16^{-\frac{1}{4}} \times y^{-4 \times -\frac{1}{4}}$

$= \dfrac{1}{16^{\frac{1}{4}}} \times y^1$

$= \dfrac{1}{2} \times y = \dfrac{y}{2}$

**c** $\left(\dfrac{36z^{-2}}{25}\right)^{\frac{1}{2}}$

$\left(\dfrac{36z^{-2}}{25}\right)^{\frac{1}{2}} = \left(\dfrac{36^{\frac{1}{2}} z^{-2 \times \frac{1}{2}}}{25^{\frac{1}{2}}}\right)$

$= \dfrac{6z^{-1}}{5} = \dfrac{6}{5z}$

**Summary of index laws**

1. $x^m \times x^n = x^{m+n}$
2. $x^m \div x^n = x^{m-n}$
3. $(x^m)^n = x^{m \times n}$
4. $(xy)^m = x^m y^m$
5. $\left(\dfrac{x}{y}\right)^m = \dfrac{x^m}{y^m}$

Meaning of the zero power: $x^0 = 1$

Meaning of the negative power: $x^{-p} = \dfrac{1}{x^p}$

Reciprocal of a negative power: $\dfrac{1}{x^{-p}} = x^p$

The $p$th root of $x$: $x^{\frac{1}{p}} = \sqrt[p]{x}$

## Exercise 12F

**1** Simplify the following by writing as basic numerals:

**a** $4^{\frac{1}{2}}$ **b** $16^{\frac{1}{2}}$ **c** $49^{\frac{1}{2}}$ **d** $100^{\frac{1}{2}}$

**e** $27^{\frac{1}{3}}$ **f** $64^{\frac{1}{3}}$ **g** $125^{\frac{1}{3}}$ **h** $1000^{\frac{1}{3}}$

**i** $81^{\frac{1}{4}}$ **j** $256^{\frac{1}{4}}$ **k** $32^{\frac{1}{5}}$ **l** $10\,000^{\frac{1}{4}}$

**2** Simplify the following by writing as basic numerals in fraction form:

**a** $9^{-\frac{1}{2}}$ **b** $25^{-\frac{1}{2}}$ **c** $81^{-\frac{1}{2}}$ **d** $36^{-\frac{1}{2}}$

**e** $8^{-\frac{1}{3}}$ **f** $27^{-\frac{1}{3}}$ **g** $64^{-\frac{1}{3}}$ **h** $216^{-\frac{1}{3}}$

**i** $16^{-\frac{1}{4}}$ **j** $81^{-\frac{1}{4}}$ **k** $32^{-\frac{1}{5}}$ **l** $100\,000^{-\frac{1}{5}}$

**3** Simplify the following expressions:

**a** $(9x)^{\frac{1}{2}}$ **b** $(16x^2)^{\frac{1}{2}}$ **c** $(49y^4)^{\frac{1}{2}}$ **d** $(25z^6)^{\frac{1}{2}}$

**e** $(8t)^{\frac{1}{3}}$ **f** $(27x^3)^{\frac{1}{3}}$ **g** $(64z^6)^{\frac{1}{3}}$ **h** $(125x^9)^{\frac{1}{3}}$

**i** $(16p)^{\frac{1}{4}}$ **j** $(81q^4)^{\frac{1}{4}}$ **k** $(32s^5)^{\frac{1}{5}}$ **l** $(243y^{10})^{\frac{1}{5}}$

**4** Simplify the following, writing the answers with positive powers:

**a** $(36x)^{-\frac{1}{2}}$ **b** $(36y^{-2})^{-\frac{1}{2}}$ **c** $(121y^{-4})^{-\frac{1}{2}}$ **d** $(4^{-1}z^6)^{-\frac{1}{2}}$

**e** $(8x)^{-\frac{1}{3}}$ **f** $(27x^{-3})^{-\frac{1}{3}}$ **g** $(64z^{-6})^{-\frac{1}{3}}$ **h** $(125^{-1}p^3)^{-\frac{1}{3}}$

**i** $(16x)^{-\frac{1}{4}}$ **j** $(81^{-1}m^4)^{-\frac{1}{4}}$ **k** $(32y^{-5})^{-\frac{1}{5}}$ **l** $(243^{-1}p^5)^{-\frac{1}{5}}$

**5** Simplify the following, writing the answers with positive powers:

**a** $\left(\frac{25t^{-2}}{49}\right)^{\frac{1}{2}}$ **b** $\left(\frac{4s^{-4}}{9}\right)^{\frac{1}{2}}$ **c** $\left(\frac{36m^{-6}}{25}\right)^{\frac{1}{2}}$ **d** $\left(\frac{121q^{-1}}{144}\right)^{\frac{1}{2}}$

**e** $\left(\frac{9z^{-2}}{25}\right)^{-\frac{1}{2}}$ **f** $\left(\frac{81u^{-8}}{100}\right)^{-\frac{1}{2}}$ **g** $\left(\frac{27u^{-3}}{8}\right)^{\frac{1}{3}}$ **h** $\left(\frac{64w^{-9}}{125}\right)^{\frac{1}{3}}$

**i** $\left(\frac{u^{-4}}{16}\right)^{\frac{1}{4}}$ **j** $\left(\frac{81x^{-8}}{y^4}\right)^{-\frac{1}{4}}$ **k** $\left(\frac{x^{-5}}{32y^{10}}\right)^{\frac{1}{5}}$ **l** $\left(\frac{243m^{-10}}{n^{-5}}\right)^{\frac{1}{5}}$

# 12G Standard form

Scientists use **standard form** or **scientific notation** to write both large and small numbers.

To write a large number in standard form:

- Shift the decimal point to the left to make a number between 1 and 10.
- Multiply by a positive power of 10; the power is the number of places you moved the decimal point.
- The first part of the number is always between 1 and 10 and the second is a positive power of 10.

**Example** **Solution**

**1** The speed of light is 300 000 km/s. Write it in standard form.

$300\,000 \text{ km/s} = 3{\cdot}0 \times 10^5 \text{ km/s}$

**2** The distance of the planet Pluto from the Sun is $5{\cdot}9 \times 10^9$ km. Write this as a basic numeral.

$5{\cdot}9 \times 10^9 \text{ km} = 5\,900\,000\,000 \text{ km}$

To write very small numbers in standard form:

- Shift the decimal point to the right to make a number between 1 and 10.
- Multiply by a negative power of 10; the negative power is the number of places you moved the decimal point.
- The first number is always between 1 and 10 and the second is a negative power of 10.

**Example** **Solution**

**3** The diameter of an atom is 0·000 000 1 millimetres. Write it in standard form.

$$0{\cdot}000\,000\,1 = 1{\cdot}0 \div 10^7$$
$$= 1{\cdot}0 \times \frac{1}{10^7}$$
$$= 1{\cdot}0 \times 10^{-7} \text{ mm}$$

**4** Write $1{\cdot}09 \times 10^{-4}$ as a basic numeral.

$1{\cdot}09 \times 10^{-4} = 0{\cdot}000\,109$

**5** Use index laws to perform the following calculations with standard form numbers:

**a** $(3{\cdot}0 \times 10^3) \times (5{\cdot}0 \times 10^{-2})$

$$(3{\cdot}0 \times 10^3) \times (5{\cdot}0 \times 10^{-2})$$
$$= (3{\cdot}0 \times 5{\cdot}0) \times (10^3 \times 10^{-2})$$
$$= 15{\cdot}0 \times 10^{3\,+\,-2}$$
$$= 15{\cdot}0 \times 10^1$$
$$= 1{\cdot}5 \times 10^2$$

**b** $(8{\cdot}0 \times 10^2) \div (4{\cdot}0 \times 10^{-2})$

$$(8{\cdot}0 \times 10^2) \div (4{\cdot}0 \times 10^{-2})$$
$$= \frac{8{\cdot}0}{4{\cdot}0} \times \frac{10^2}{10^{-2}}$$
$$= 2 \times 10^4$$

## Exercise 12G

**1** Write the following basic numerals in standard form:

| | | | |
|---|---|---|---|
| **a** 50 000 | **b** 120 500 | **c** 580 | **d** 52 000 000 |
| **e** 360 000 | **f** 250 | **g** 375·2 | **h** 1024·6 |
| **i** 250 000 000 | **j** 100 | **k** 1000 | **l** 200 000 |

**2** Write the following standard form numbers as basic numerals:

| | | | |
|---|---|---|---|
| **a** $1{\cdot}5 \times 10^3$ | **b** $4{\cdot}05 \times 10^6$ | **c** $3{\cdot}621 \times 10^5$ | **d** $7{\cdot}205 \times 10^6$ |
| **e** $5{\cdot}9 \times 10^8$ | **f** $4{\cdot}5 \times 10^2$ | **g** $2{\cdot}0 \times 10^1$ | **h** $9{\cdot}8 \times 10^0$ |
| **i** $1{\cdot}006 \times 10^3$ | **j** $3{\cdot}125\,63 \times 10^3$ | **k** $2{\cdot}25 \times 10^0$ | **l** $9{\cdot}99 \times 10^3$ |

**3** Write the following basic numerals in standard form:

| | | |
|---|---|---|
| **a** 0·005 | **b** 0·001 07 | **c** 0·25 |
| **d** 0·000 375 | **e** 0·0001 | **f** 0·000 653 2 |
| **g** 0·105 | **h** 0·7 | **i** 0·9 |
| **j** 0·000 000 000 2 | **k** 0·000 000 005 5 | **l** 0·000 000 006 1 |

**4** Write the following standard form numbers as basic numerals:

| | | | |
|---|---|---|---|
| **a** $4{\cdot}3 \times 10^{-3}$ | **b** $6{\cdot}0 \times 10^{-5}$ | **c** $3{\cdot}5 \times 10^{-1}$ | **d** $1{\cdot}5 \times 10^{-3}$ |
| **e** $1{\cdot}06 \times 10^{-2}$ | **f** $6{\cdot}25 \times 10^{-1}$ | **g** $9{\cdot}99 \times 10^{-6}$ | **h** $4{\cdot}3 \times 10^{-3}$ |
| **i** $4{\cdot}75 \times 10^{-6}$ | **j** $1{\cdot}025 \times 10^{-7}$ | **k** $8{\cdot}3 \times 10^{-2}$ | **l** $6{\cdot}25 \times 10^{-9}$ |

**5** Complete the following standard form calculations:

| | |
|---|---|
| **a** $(6{\cdot}0 \times 10^6) \times (4{\cdot}0 \times 10^{-3})$ | **b** $(9{\cdot}0 \times 10^4) \div (3{\cdot}0 \times 10^3)$ |
| **c** $(3{\cdot}0 \times 10^5) \times (2{\cdot}0 \times 10^{-3})$ | **d** $(1{\cdot}5 \times 10^{-2}) \times (2{\cdot}0 \times 10^4)$ |
| **e** $(4{\cdot}0 \times 10^{-4}) \times (3{\cdot}0 \times 10^9)$ | **f** $(4{\cdot}0 \times 10^{-2}) \times (6{\cdot}0 \times 10^{-4})$ |
| **g** $(8{\cdot}0 \times 10^3) \div (4{\cdot}0 \times 10^2)$ | **h** $(9{\cdot}3 \times 10^{-1}) \div (3{\cdot}1 \times 10^{-3})$ |

**6** **a** A light-year is the distance that light will travel in 1 year. The speed of light is normally given as 300 000 kilometres per second.

**i** Calculate the number of seconds in one day.

**ii** Calculate the number of seconds in a year (take 365 days = 1 year).

**iii** Determine the distance (in km) that light travels in a year, expressing your answer in standard form.

**b** The distance from the Earth to the Moon is 365 000 kilometres. How long does it take light reflected from the Moon to reach the Earth?

**c** Given that the Sun is 150 million kilometres from the Earth, how long does light from the Sun take to reach the Earth?

# Puzzles

**1** Simplify the index expressions and match the letter to the correct answer below to find the title of this puzzle:

**SKATING**

**ICE**

**A** $2x^2 \times 3x^4$

**C** $^-4x \times {}^-3x^3$

**E** $x \times x^2 \times x^3$

**G** $xy^2 \times x^2y$

**H** $^-3x^2y \times {}^-2xy$

**I** $xy \times yx$

**K** $\dfrac{12x^2y}{6xy}$

**N** $\dfrac{10x^5y^3}{2x^4y^2}$

**O** $\dfrac{16x^7y}{2x^6y}$

**S** $\dfrac{5x^2y}{2x^3} \times \dfrac{4x^2}{10y}$

**T** $\dfrac{8y^3}{3xy} \times \dfrac{9xy}{4y}$

**W** $\dfrac{5xy}{4y^4} \times \dfrac{8y^5}{2x}$

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| $x$ | $2x$ | $6x^6$ | $6y^2$ | $x^2y^2$ | $5xy$ | $x^3y^3$ | $8x$ | $5xy$ |
| $6y^2$ | $6x^3y^2$ | $x^2y^2$ | $5xy$ | $x^2y^2$ | $12x^4$ | $x^6$ | | |

**2** Simplify the index expressions and match the letter to the correct answer below to find the title of this puzzle:

**LO** HEAD / HEELS **VE**

**A** $(x^4)^3$

**D** $(2x^3)^2$

**E** $2(x^2)^3$

**H** $(4x^2y)^2$

**I** $(^-3xy)^2$

**L** $3(^-2x^2y)^2$

**N** $2(x^2)^3 \times (x^3)^2$

**O** $(2x^3)^2 \times 3x^2$

**R** $(^-4xy)^2 \times xy$

**S** $\dfrac{(x^4y^2)^3}{(x^5y)^2}$

**V** $\dfrac{(4xy^3)^2}{(2y)^4}$

**W** $\dfrac{(10x^2y)^2}{(5x)^2}$

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $16x^4y^2$ | $2x^6$ | $x^{12}$ | $4x^6$ | | $12x^8$ | $x^2y^2$ | $2x^6$ | $16x^3y^3$ | | | | |
| $16x^4y^2$ | $2x^6$ | $2x^6$ | $12x^4y^2$ | $x^2y^4$ | | $9x^2y^2$ | $2x^{12}$ | | $12x^4y^2$ | $12x^8$ | $x^2y^2$ | $2x^6$ |

**3** Simplify the index expressions to those using only positive indices and match the letter to the correct answer below to find the title of this puzzle:

| | | | |
|---|---|---|---|
| **B** $x^{-1}$ | **D** $(2x)^{-1}$ | **E** $2x^{-1}$ | **H** $\frac{2}{x^{-1}}$ |
| **N** $2(x^0)^{-1}$ | **M** $\left(\frac{2}{x}\right)^{-1}$ | **O** $3x^{-1}y$ | **R** $3(xy)^{-1}$ |
| **T** $3x^{-2}$ | **U** $\frac{x^{-1}}{y^{-1}}$ | **W** $\frac{2x^{-2}}{(2y)^{-1}}$ | **X** $\frac{3x^{-2}}{(3y)^{-1}}$ |

| | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\frac{3y}{x}$ | $\frac{y}{x}$ | $\frac{3}{x^2}$ | 2 | $\frac{y}{x}$ | $\frac{x}{2}$ | $\frac{1}{x}$ | $\frac{2}{x}$ | $\frac{3}{xy}$ | $\frac{2}{x}$ | $\frac{1}{2x}$ | | | | |
| $\frac{3}{x^2}$ | $2x$ | $\frac{3}{xy}$ | $\frac{2}{x}$ | $\frac{2}{x}$ | | $\frac{3}{x^2}$ | $\frac{4y}{x^2}$ | $\frac{3y}{x}$ | | $\frac{3y}{x}$ | 2 | $\frac{2}{x}$ | | |

**4** Simplify the index expressions and match the letter to the correct answer below to find the title of this puzzle:

| | | |
|---|---|---|
| **A** $4^{\frac{1}{2}}$ | **B** $9^{\frac{1}{2}} \times 2^{-1}$ | |
| **C** $25^{\frac{1}{2}}$ | **D** $27^{\frac{1}{3}}$ | |
| **E** $256^{\frac{1}{4}}$ | **K** $1000^{\frac{1}{3}}$ | **N** $81^{-\frac{1}{4}}$ |
| **O** $32^{-\frac{1}{5}}$ | **P** $36^{-\frac{1}{2}}$ | **U** $(4^0)^{\frac{1}{2}}$ |
| **R** $(6^4)^{\frac{1}{2}}$ | **S** $\left(-64^{\frac{1}{3}}\right)^2$ | **T** $36^{\frac{1}{2}} \times 5^0$ |
| **W** $100^{-\frac{1}{2}} \times 3^2$ | **Y** $4^{-1} \times 243^{-\frac{1}{5}}$ | |

| | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\frac{1}{12}$ | $\frac{1}{2}$ | 1 | 36 | | $\frac{1}{6}$ | 2 | $\frac{1}{3}$ | 6 | 16 | | 2 | 36 | 4 | |
| $\frac{1}{2}$ | $\frac{1}{3}$ | | $\frac{3}{2}$ | 2 | 5 | 10 | $\frac{9}{10}$ | 2 | 36 | 3 | 16 | | | |

Your pants are no

# Applications

## Red blood cells

The human body contains about $8 \times 10^4$ μL (microlitres) of blood for each kilogram of body weight. One microlitre (μL) is one-millionth of a litre.

Each microlitre of blood contains about $5 \times 10^6$ red blood cells.

**a** Find the approximate number of red blood cells in a person who weighs 56 kg.

**b** Find your weight in kilograms and estimate the number of red blood cells in your body.

**c** Assuming the average blood donor gives about 470 mL, estimate the number of red blood cells in one donation.

**d** Carry out some research at your local blood bank and find out how many litres of each blood type are donated and used on a daily or weekly basis. Convert the volume of blood to the number of red blood cells and present the information in standard form. Write a report on your findings and include information such as the different uses of donated blood.

## Hospital costs

The estimated average daily cost of patient care in a hospital in 1995 was $1050.

These costs are thought to be rising steadily at about 8·5% per year.

**a** Use this information to estimate what the average cost per day was in 1996.

The formula $C = 1050 \times (1{\cdot}085)^n$ can be used to estimate costs, where $C$ = average daily cost of patient care in a hospital and $n$ = number of years after 1995.

**b** Using this formula, copy and complete the table below:

| | 1995 | 2000 | 2005 | 2010 | 2015 | 2020 |
|---|---|---|---|---|---|---|
| $n$ | | | | | | |
| $C$ | | | | | | |

**c** Draw a graph of the data and describe its shape. Comment on it.

**d** Carry out some research at your local hospital and find out the estimated current cost per day of patient care. What are the issues affecting hospital costs and patient care?

## Vehicle depreciation

A second-hand car cost \$30 000 and depreciates at 15% of its current value each year. Its value $V$ (\$) after $t$ (years) can be worked out by using the formula $V = 30\,000\ (0{\cdot}85)^t$.

**a** Use the formula above to calculate the value of the car each year after depreciation is taken then tabulate them for the first 10 years.

**b** Plot these values on a suitably scaled set of axes, with $t$ plotted horizontally. Join your points with a smoothly falling curve.

**c** From your graph, read as accurately as you can the time required for the second-hand car to halve in value.

**d** Using trial and error, find the answer required in part c correct to 1 decimal place.

**e** Write down the rule you would work with if the second-hand car cost \$40 000 and it depreciated at 20% per year.

# Enrichment

## More on fractional indices

The value of $32^{\frac{3}{5}}$ can be found by writing it as $\left(32^{\frac{1}{5}}\right)^3$, giving $2^3$ which is 8.

It is also equivalent to $(32^3)^{\frac{1}{5}}$, but this form is less manageable than the one above.

$32^{\frac{3}{5}}$ can also be evaluated using the $x^y$, $\sqrt[x]{y}$, or $x\hat{}$ keys on a calculator.

$32^{(3/5)}$, $\sqrt[5]{32^3}$, $(\sqrt[5]{32})^3$, $32\hat{}(3/5)$ would each produce the required answer.

**1** Evaluate the following:

**a** $4^{\frac{3}{2}}$ **b** $9^{\frac{3}{2}}$ **c** $25^{-\frac{3}{2}}$ **d** $100^{-\frac{5}{2}}$

**e** $8^{\frac{2}{3}}$ **f** $27^{-\frac{4}{3}}$ **g** $64^{-\frac{2}{3}}$ **h** $125^{\frac{4}{3}}$

**i** $16^{\frac{3}{4}}$ **j** $81^{-\frac{5}{4}}$ **k** $32^{-\frac{6}{5}}$ **l** $243^{\frac{2}{5}}$

**m** $\left(\frac{125}{8}\right)^{\frac{2}{3}}$ **n** $\left(\frac{64}{121}\right)^{-\frac{3}{2}}$ **o** $\left(\frac{81}{16}\right)^{-\frac{3}{4}}$ **p** $\left(\frac{32}{243}\right)^{-\frac{4}{5}}$

**2** Use the index laws to simplify the following and express the answers with positive indices:

**a** $(16x^2)^{-\frac{1}{4}}$ **b** $(25y^3)^{-\frac{1}{2}}$ **c** $(27x^2y^3)^{-\frac{2}{3}}$ **d** $\left(5^{\frac{1}{2}}x^{-\frac{3}{4}}y^{-2}\right)^4$

**e** $\left(4^{\frac{2}{3}}a^{-\frac{1}{6}}b^{\frac{2}{9}}\right)^{-3}$ **f** $\left(x^{\frac{1}{5}}\right)^2 \times (x^{-2})^{\frac{1}{5}}$ **g** $\left(\frac{x^{\frac{3}{2}}}{2y^{-2}}\right)^{-2} \times \left(\frac{2^{-1}y}{x^{\frac{1}{3}}}\right)^6$

Calculators can be used to work out powers of numbers via the $x^y$, $x^{\frac{1}{y}}$, $x^{\frac{p}{q}}$, $x\hat{}$ or related keys, depending on the type of calculator.

**3** Evaluate the following:

**a** $10 \cdot 5^2$ **b** $5 \cdot 3^3$ **c** $7 \cdot 25^4$ **d** $4 \cdot 125^5$ **e** $(-1 \cdot 5)^2$ **f** $(-2 \cdot 5)^3$

**g** $(-5 \cdot 1)^4$ **h** $(-6 \cdot 8)^5$ **i** $(-2 \cdot 4)^6$ **j** $(-1 \cdot 2)^7$ **k** $(-2 \cdot 2)^8$ **l** $(-1 \cdot 1)^9$

How can you predict the sign of the answer from the power used?

**4** Find the following and check your answers using a calculator:

**a** $\left(\frac{1}{2}\right)^{-1}$ **b** $\left(2\frac{1}{2}\right)^{-1}$ **c** $\left(1\frac{3}{4}\right)^{-1}$ **d** $\left(2\frac{4}{5}\right)^{-2}$

**e** $\left(-\frac{1}{2}\right)^{-1}$ **f** $\left(-1\frac{1}{4}\right)^{-2}$ **g** $\left(-\frac{2}{3}\right)^{-3}$ **h** $\left(-1\frac{1}{2}\right)^{-4}$

**5** Evaluate the following:

**a** $\left(\frac{1}{4}\right)^{-\frac{1}{2}}$ **b** $\left(\frac{1}{8}\right)^{-\frac{1}{3}}$ **c** $\left(\frac{1}{81}\right)^{-\frac{1}{4}}$ **d** $\left(\frac{1}{32}\right)^{-\frac{1}{5}}$

**e** $\left(\frac{16}{9}\right)^{-\frac{1}{2}}$ **f** $\left(\frac{8}{27}\right)^{-\frac{1}{3}}$ **g** $\left(\frac{81}{16}\right)^{-\frac{1}{4}}$ **h** $\left(\frac{32}{243}\right)^{-\frac{1}{5}}$

**6** Calculate the following, expressing your answers for question e onwards as simple fractions:

**a** $(^{-}3{\cdot}4)^6$ **b** $(^{-}1{\cdot}5)^7$ **c** $(1{\cdot}05)^{20}$ **d** $(^{-}1{\cdot}1)^6$

**e** $\left(-\frac{1}{4}\right)^{-1}$ **f** $\left(1\frac{1}{4}\right)^{-2}$ **g** $\left(-\frac{2}{5}\right)^{-3}$ **h** $\left(-1\frac{1}{2}\right)^{-4}$

**i** $\left(\frac{9}{25}\right)^{-\frac{1}{2}}$ **j** $\left(\frac{27}{8}\right)^{-\frac{1}{3}}$ **k** $\left(\frac{16}{81}\right)^{-\frac{1}{4}}$ **l** $\left(\frac{243}{32}\right)^{-\frac{1}{5}}$

**7** Given that the speed of light is 300 000 kilometres per second, and that the closest star to the Earth, Proxima Centauri, is 4·3 light-years away, determine how far this is in kilometres.

**8** The radius of the Earth is approximately 6400 kilometres and the Earth is considered to be a sphere. What is its volume in cubic metres?

(The formula $V = \frac{4}{3}\pi r^3$ may help.)

**9** What is the circumference of the Earth at the equator? How long would it take a signal of light to travel around the Earth at the equator, if light could bend to this extent?

The radius $R$ metres (to the 'event horizon') of a black hole of mass $M$ (kilograms) is given by the relation:

$$R = \frac{2GM}{c^2}$$

where $G$ is the universal constant of gravitation ($6{\cdot}67 \times 10^{-11}$ N m$^2$/kg$^2$), and $c$ is the speed of light ($3 \times 10^8$ m/s).

**10** If the Sun, whose mass is $2 \times 10^{30}$ kg, were to collapse to become a black hole, what would be its radius in kilometres?

**11** If the Earth, whose mass is $6{\cdot}006 \times 10^{24}$ kg, were to collapse to become a black hole, what would be its radius in millimetres?

# Revision/Assessment

## Exercise 12A

**1** Evaluate the following to basic numerals:

**a** $9^2$ **b** $(^-1)^3$ **c** $(^-4)^3$ **d** $^-(^-11)^2$

**e** $\left(-\frac{1}{4}\right)^3$ **f** $\left(\frac{2}{5}\right)^2$ **g** $\left(-\frac{1}{10}\right)^3$ **h** $\left(\frac{3}{4}\right)^4$

**2** Write in index form, simplifying where possible:

**a** $2ab \times 3ba \times 5c \times bc$ **b** $7xy \times 3yz \times yz \times 4xy$

**c** $3pq \times 6qr \times 3pr \times 2qp$ **d** $-4yx \times 2zx \times 5zy$

**e** $3ab \times -bc \times -2ac$ **f** $-st \times 2tr \times -4srt \times 3rs$

## Exercise 12C

**3** Simplify the following:

**a** $5x^2y^3 \times 6x^2y^2$ **b** $2x^1y^3 \times 7x^2y^4$ **c** $3x^2y^2 \times 5x^2y^2$

**d** $-2a^3b^3 \times 3a^2b^2$ **e** $6m^1n^2 \times -2m^3n^5$ **f** $-3p^1q^3 \times -9p^2q^4$

**g** $\frac{12x^6y^5}{4x^3y^4}$ **h** $\frac{10x^4y^9}{-5x^3y^7}$ **i** $\frac{-15x^5y^4}{5x^4y^2}$

**j** $\frac{20x^7y^4}{4x^2y^3}$ **k** $\frac{16x^2y^3}{-4xy}$ **l** $\frac{-25m^3n^2}{-5m^2n}$

**4** Simplify the following as far as possible:

**a** $\frac{12x^3}{2y} \times \frac{8y^2}{4x}$ **b** $\frac{-16b^7a^5}{3a^4b^2} \times \frac{9a^9b^4}{-4b^3a^6}$ **c** $\frac{21x^5}{4y} \div \frac{7y}{20x}$

**d** $\frac{32x^3y^4}{7x^2} \div \frac{16y^2x}{-28x^2y}$ **e** $\frac{-25m^3n^4}{4m^2n^2} \div \frac{5m^2n}{-16n^2m^2}$ **f** $\frac{-2xy^2}{14x^2yw^3} \div \frac{x^5y}{7x^7w^4}$

## Exercise 12D

**5** Simplify the following:

**a** $(8y)^0$ **b** $8x^0$ **c** $9(x^4)^0$ **d** $6(-a^3)^0$

**e** $7b^0 - 4y^0$ **f** $(3a^7)^0 - 2p^0$ **g** $(4x)^0 + 4x^0$ **h** $(6y)^0 \times 6^0y$

**i** $(4m^4n^1p^2)^3$ **j** $(2a^2b^0c^1)^4$ **k** $(-4xy^2z^4)^3$ **l** $(-5m^0n^1p)^3$

**m** $\left(\frac{-3uv^2}{5w^2z^1}\right)^2$ **n** $\left(\frac{2x^2y^4}{-3t^5z^3}\right)^3$ **o** $\left(\frac{5x^3y^1z}{u^2v}\right)^3$ **p** $\left(\frac{-4r^2s^1t}{u^2v^3w}\right)^3$

**6** Simplify the following expressions:

**a** $\left(\frac{m^4}{8n^2}\right)^2 \times \left(\frac{4n^1}{m^2}\right)^3$ **b** $\left(\frac{m^3}{5n^2}\right)^2 \times \left(\frac{10n^3}{m^2}\right)^2$

**c** $\left(\frac{2x^2}{y}\right)^4 \div \frac{(4x^3)^2}{(5y^1)^2}$ **d** $\left(\frac{9x^3}{y^2}\right)^3 \div \frac{(9x^4)^2}{7y^6}$

## Exercise 12E

**7** Simplify the following, writing the answers as basic numerals:

**a** $4^{-1}$ **b** $5^{-2}$ **c** $10^{-4}$ **d** $2^{-2}$

**e** $\frac{1}{4^{-1}}$ **f** $\frac{1}{3^{-3}}$ **g** $\frac{1}{5^{-2}}$ **h** $\frac{2}{4^{-2}}$

**i** $\left(\frac{4}{5}\right)^{-1}$ **j** $\left(\frac{2}{3}\right)^{-2}$ **k** $\left(\frac{3}{4}\right)^{-3}$ **l** $\left(\frac{3}{2}\right)^{-4}$

**8** Simplify the following, writing the answers with positive powers:

**a** $\frac{z^3x^{-1}}{y^{-4}}$ **b** $\frac{c^{-3}}{ab^{-4}}$ **c** $\frac{p^{-3}q}{r^2s^{-2}}$

**d** $\frac{t^{-2}u^{-1}}{v^{-2}w^{-4}}$ **e** $\left(\frac{4m^{-1}}{n^{-2}}\right)^{-3} \times \left(\frac{8n^{-1}}{m}\right)^{2}$ **f** $\left(\frac{r^{-2}}{6s^2}\right)^{3} \div \left(\frac{3r^2}{s^2}\right)^{-2}$

## Exercise 12F

**9** Simplify the following, writing the answers as basic numerals:

**a** $81^{\frac{1}{2}}$ **b** $64^{\frac{1}{3}}$ **c** $16^{\frac{1}{4}}$ **d** $243^{\frac{1}{5}}$

**e** $36^{-\frac{1}{2}}$ **f** $8^{-\frac{1}{3}}$ **g** $81^{-\frac{1}{4}}$ **h** $32^{-\frac{1}{5}}$

**10** Simplify the following, writing the answers with positive powers:

**a** $(16t)^{\frac{1}{2}}$ **b** $(64x^6)^{\frac{1}{3}}$ **c** $(16z^4)^{\frac{1}{4}}$ **d** $(243x^{10})^{\frac{1}{5}}$

**e** $(64z^2)^{-\frac{1}{2}}$ **f** $(125^{-1}x^3)^{-\frac{1}{3}}$ **g** $(16x^{-8})^{-\frac{1}{4}}$ **h** $(32^{-1}y^5)^{-\frac{1}{5}}$

**i** $\left(\frac{u^{-4}}{81}\right)^{\frac{1}{4}}$ **j** $\left(\frac{81x^{-8}}{y^4}\right)^{-\frac{1}{2}}$ **k** $\left(\frac{x^{-5}}{32y^{10}}\right)^{-\frac{1}{5}}$ **l** $\left(\frac{27m^{-9}}{n^{-6}}\right)^{\frac{1}{3}}$

## Exercise 12G

**11** Write the following basic numerals in standard form:

**a** 500 000 **b** 120 000 000 **c** 80 000 **d** 22 000 000

**12** Write the following standard form numbers as basic numerals:

**a** $1{\cdot}7 \times 10^4$ **b** $4{\cdot}07 \times 10^3$ **c** $3{\cdot}6 \times 10^6$ **d** $7{\cdot}25 \times 10^8$

**13** Write the following basic numerals in standard form:

**a** 0·0015 **b** 0·000 007 **c** 0·0250 **d** 0·000 000 03

**14** Write the following standard form numbers as basic numerals:

**a** $4{\cdot}8 \times 10^{-3}$ **b** $6{\cdot}1 \times 10^{-6}$ **c** $3{\cdot}05 \times 10^{-1}$ **d** $1{\cdot}25 \times 10^{-8}$

# Answers

These are selected answers only.

## Chapter 7

### Exercise 7A

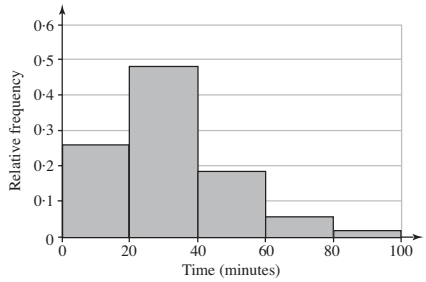
**1 a**

| Travel time | Frequency |
|---|---|
| 0– | 13 |
| 20– | 24 |
| 40– | 9 |
| 60– | 3 |
| 80– | 1 |

**b**

| Travel time | Frequency | Rel. frequency |
|---|---|---|
| 0– | 13 | 0·26 |
| 20– | 24 | 0·48 |
| 40– | 9 | 0·18 |
| 60– | 3 | 0·06 |
| 80– | 1 | 0·02 |

**c**



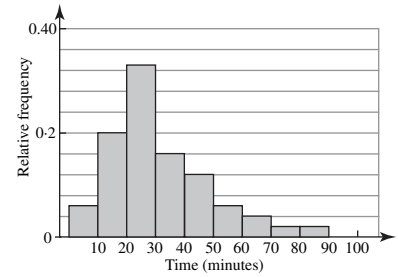

**d** The data is positively skewed. The majority of learners take less than 40 minutes to get to school.

**e** 26% of the learners take less than 20 minutes to get to school.

**2 a, b**

| Travel time | Frequency | Rel. frequency |
|---|---|---|
| 0– | 3 | 0·06 |
| 10– | 10 | 0·20 |
| 20– | 16 | 0·32 |
| 30– | 8 | 0·16 |
| 40– | 6 | 0·12 |
| 50– | 3 | 0·06 |
| 60– | 2 | 0·04 |
| 70– | 1 | 0·02 |
| 80– | 1 | 0·02 |
| 90– | 0 | 0 |

**c**




**d** The data is positively skewed.

**e** Yes, because the classes are smaller.

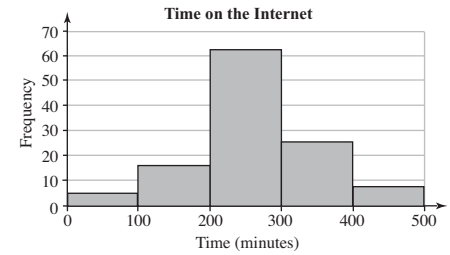
**3 a** On 8 days.

**b** The histogram is positively skewed.

**c** The modal group for this histogram is 20–25 bottles.

**d** 25 bottles

**4 a**

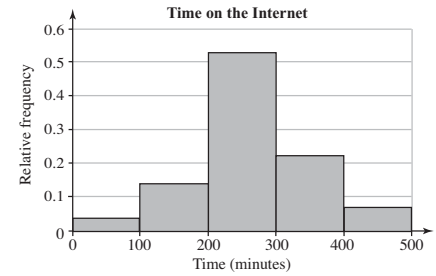



**b** The shape of the histogram is basically symmetrical.

**c**

| Time on the Internet (min) | Relative frequency |
|---|---|
| 0– | 0·04 |
| 100– | 0·14 |
| 200– | 0·53 |
| 300– | 0·22 |
| 400– | 0·07 |

**d**

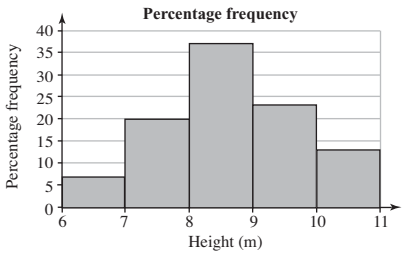



**e** The graph shows that 53% of the learners, a majority, spent between 200 and 300 minutes on the Internet over the month. The graph is roughly symmetrical, with a slight negative skew.

**5** a, b

| Height | Frequency | Percentage frequency |
|---|---|---|
| 6·0– | 2 | 7 |
| 7·0– | 6 | 20 |
| 8·0– | 11 | 37 |
| 9·0– | 7 | 23 |
| 10·0– | 4 | 13 |
| Total | 30 | 100% |

c

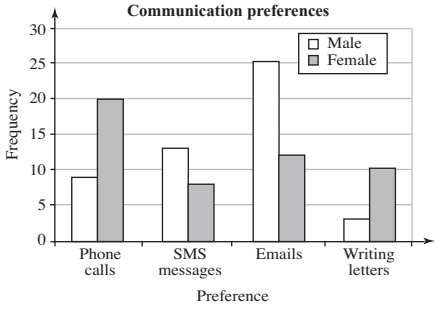



d The histogram is roughly symmetrical.

e 13% f 60%

## Exercise 7B

**1** a

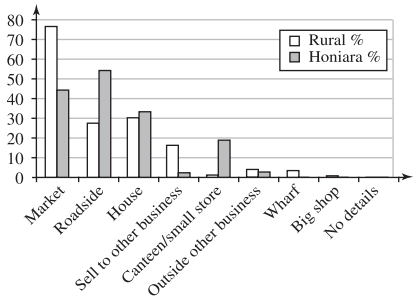



b The graph indicates that males prefer emails whereas females prefer phone calls.

**2** a 22% b 19%

c Not enough, too far away, by 23%

**3** a




b Approximately 30% more sellers use the markets in rural areas than in Honiara, 25% more sell at the roadside in Honiara compared with rural sellers, 15% more rural sellers sell to other businesses than those in Honiara, and 19% more Honiara sellers use Canteens/small stores compared to rural sellers.

**4** a 2010, 2011 & 2013

b No data for 2012

c 10%

d Yes, rise of 7% from 2010 to 2011, though no apparent change between 2011 and 2013, so increase from 2010 was maintained.

## Exercise 7C

**1** a 31·78 min b 80 min

**2** a 40·54 min, 67 min

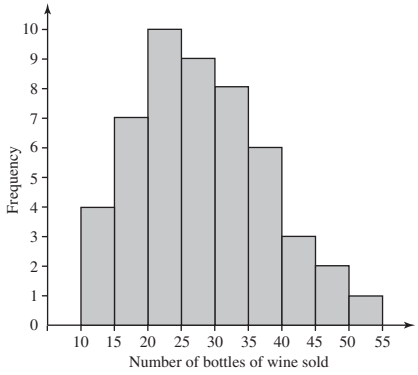
**3** a $38·70 b $83

**4** a $36·76 b $40·64 c $34, $83

**5** a 8·67 m b 8·63 m c 0·04 m

**6** 263·3 min

**7** a




b Histogram is positively skewed.

c 28·1 bottles

**8** a, b

| Total | Frequency | Winnings |
|---|---|---|
| 2 | 1 | –2 |
| 3 | 2 | –3 |
| 4 | 3 | –4 |
| 5 | 4 | –5 |
| 6 | 5 | –6 |
| 7 | 5 | –7 |
| 8 | 5 | –8 |
| 9 | 4 | 9 |
| 10 | 3 | 10 |
| 11 | 2 | 11 |
| 12 | 1 | 12 |

c Loss of $1·29

**e**

| Total | Probability |
|---|---|
| 2 | $\frac{1}{36}$ |
| 3 | $\frac{2}{36}$ |
| 4 | $\frac{3}{36}$ |
| 5 | $\frac{4}{36}$ |
| 6 | $\frac{5}{36}$ |
| 7 | $\frac{6}{36}$ |
| 8 | $\frac{5}{36}$ |
| 9 | $\frac{4}{36}$ |
| 10 | $\frac{3}{36}$ |
| 11 | $\frac{2}{36}$ |
| 12 | $\frac{1}{36}$ |

## Exercise 7D

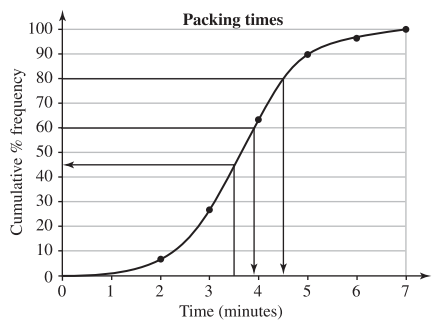
**1** **a** **i** 9 m **ii** 90% **iii** 7·8 m

**b** ≈ 8 m, 8·5 m, 9·4 m

**2** **a**

| Time (min) | Cumulative % frequency |
|---|---|
| <2 | 6·67 |
| <3 | 26·67 |
| <4 | 63·33 |
| <5 | 90 |
| <6 | 96·67 |
| <7 | 100 |

**b**

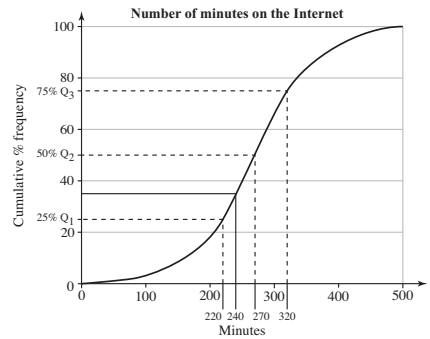



**c** **i** 4·5 mins **ii** 45% **iii** 3·9 mins

**3** **a**

| No. of minutes on the Internet | Cumulative % frequency |
|---|---|
| <100 | 4·2 |
| <200 | 18·3 |
| <300 | 70·8 |
| <400 | 93·3 |
| <500 | 100 |

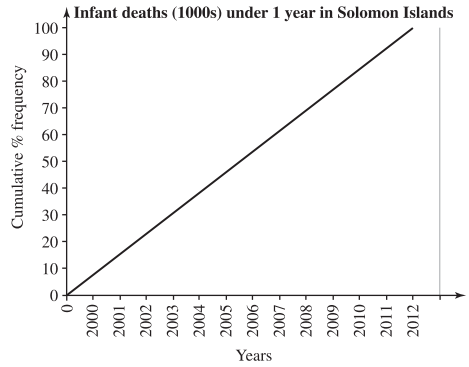
**b**




**c** 65% **d** 78

**e** **i** 0 **ii** 220 **iii** 270 **iv** 320 **v** 500

**4**

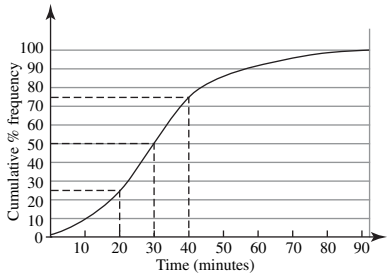



Graph shows a gradual but steady reduction in infant deaths from 200 to 2012.

**5** **a**

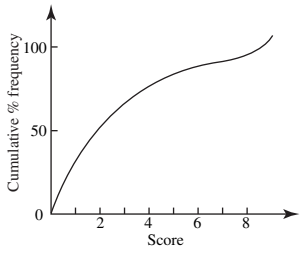
| Time (min) | Cumulative % frequency |
|---|---|
| <10 | 6 |
| <20 | 26 |
| <30 | 58 |
| <40 | 74 |
| <50 | 86 |
| <60 | 92 |
| <70 | 96 |
| <80 | 98 |
| <90 | 100 |

**b**

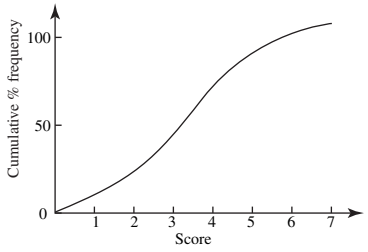



**c** 15, 45

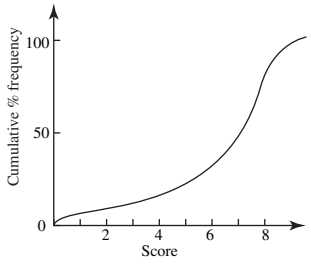
**6 a**




**b**




**c**




## Exercise 7E

**1 a**

| Stem | Height |
|---|---|
| 6 | 1 9 |
| 7 | 3 5 5 6 6 9 |
| 8 | 1 3 3 3 4 4 5 5 6 8 9 |
| 9 | 2 3 4 4 4 5 6 |
| 10 | 1 2 6 6 |

6|1 means 6·1

**b** Median = 8·5, $Q_1$ = 6·75, $Q_3$ = 9·4

**c** 45

**2 a** Class *A* has more low scores between 40 and 70, whereas Class *B* has scores closely centred around 60.

Class *A*: Positively skewed
Class *B*: Symmetrical

**b** Class *A*: 54 Class *B*: 71

**c** Class *A*: 55 Class *B*: 53

**3 a** Class *C* scores are roughly symmetrical and centred around 60, whereas Class *D* has two peaks at the extremities.

**b** Class *C*: 63 Class *D*: 79
Class *D* has a higher median.

**c** Class *C*: 54 Class *D*: 58

Class *D* has more variation.

**4 a**

| Stem | Time |
|---|---|
| 0 | 5 7 9 |
| 1 | 2 2 3 4 5 5 6 7 7 8 |
| 2 | 0 1 3 4 4 4 6 7 7 8 8 8 8 9 9 9 |
| 3 | 2 4 4 6 6 6 8 8 |
| 4 | 3 4 5 5 5 6 |
| 5 | 2 6 6 |
| 6 | 2 9 |
| 7 | 2 |
| 8 | 5 |

$Q_1$ = 18, Median = 28, $Q_3$ = 43

**b**

| Bus | Stem | Car |
|---|---|---|
| | 0 | 5 7 9 |
| 8 | 1 | 2 2 3 4 5 5 6 7 7 |
| 9 9 8 7 4 3 1 0 | 2 | 4 4 6 7 8 8 8 9 9 |
| 8 8 6 4 4 2 | 3 | 4 6 6 |
| 6 5 5 4 | 4 | 3 5 |
| 6 2 | 5 | 6 |
| 6 2 | 6 | |
| 2 | 7 | |
| 5 | 8 | |

1|2 = 12

**i** 35 minutes **ii** 24 minutes

**c** It is faster to travel by car.

**5 b** The stemplot indicates higher white wine sales with more variation.

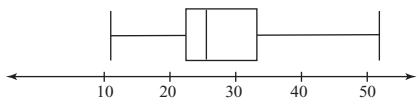
**6 a**

| Males | Stem | Females |
|---|---|---|
| | 1 | 2 5 8 |
| 8 8 7 6 3 3 2 | 2 | 1 1 3 3 4 6 7 7 7 |
| 9 7 6 6 5 5 5 2 2 | 3 | 2 5 6 7 8 |
| 7 7 5 2 0 0 | 4 | 3 |
| 6 6 2 | 5 | 3 |
| | 6 | 2 |
| | 7 | 2 8 |
| | 8 | 2 9 |
| | 9 | 5 |

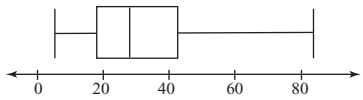
**b** The graph indicates that females have a greater variation in the amounts of money lost at the pokies. The amounts lost by males are closely centred around \$20 to \$40. There are more extreme amounts lost by the females.

## Exercise 7F

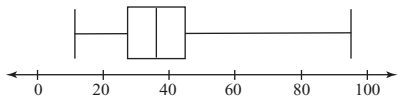
**1 a**




**b**

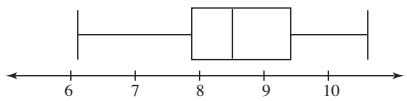



**c**




**2 a** $x_L = 6{\cdot}1$
$Q_1 = 7{\cdot}9$
Median $= 8{\cdot}5$
$Q_3 = 9{\cdot}4$
$x_H = 10{\cdot}6$

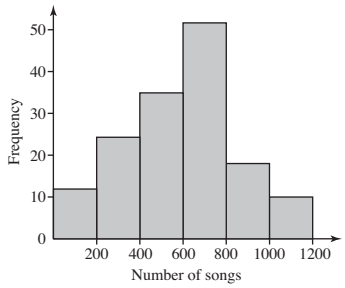
**b**




**c** **i** 7·9 **ii** 8·5 **iii** 9·4
**iv** 9·4 **v** 7·9

**d** 7·9 m, 9·4 m, 1·5 m

**3 a**

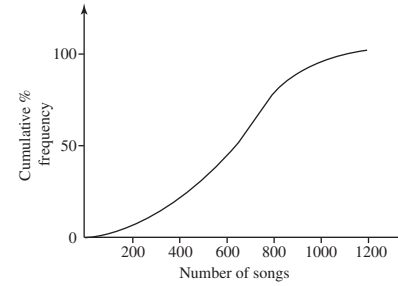



Close to symmetrical

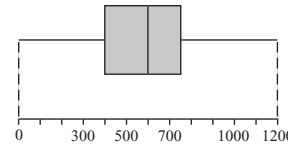
**b**

| Number of songs | Cumulative % frequency |
|---|---|
| <200 | 8 |
| <400 | 24 |
| <600 | $47\frac{1}{3}$ |
| <800 | $81\frac{1}{3}$ |
| <1000 | $93\frac{1}{3}$ |
| <1200 | 100 |

**c**



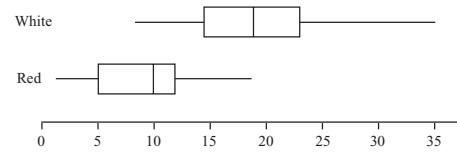

**d** $x_L = 0$
$Q_1 = 400$
Median $= 600$
$Q_3 = 750$
$x_H = 1200$




**e** **i** 400 **ii** 600 **iii** 750
**iv** 750 **v** 400

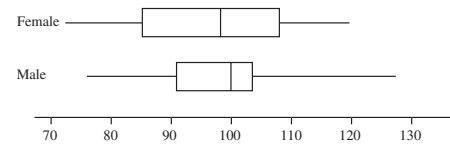
**f** 400, 750, 350 songs

**4**



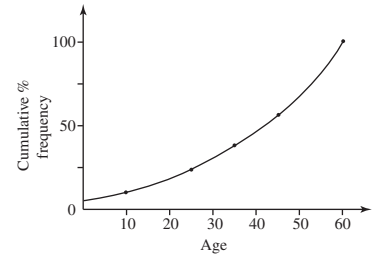

Sales of white wine are generally higher and there is more variation than for sales of red wine.

**5 a**

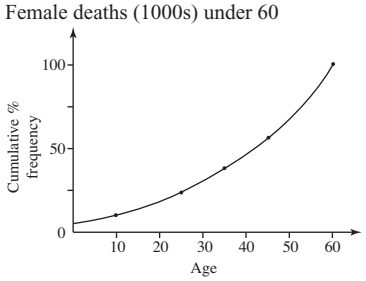



**b** Girls' IQs have a larger spread but boys' IQs are more concentrated about the central value.

**6 a** Male deaths (1000s) under 60



**b** Female deaths (1000s) under 60




**c** Male: $x_L = 0$ Female: $x_L = 0$

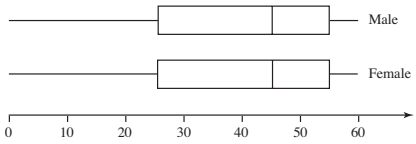
$Q_1 = 25$ $Q_1 = 25$

Median = 45 Median = 45

$Q_3 = 55$ $Q_3 = 55$

$x_H = 60$ $x_H = 60$

**d**




**e** Very similar

## Learning task 7G

**4** Number the students from 1 to 27.
Use the calculator to get a random number (0·0000 to 0·9999). If using the random number table, the teacher could follow the method provided in the Examples.
Multiply this number by 27.
Add 1 to the number.
Ignore the decimal places. Select the student with this number.
Repeat until two different students are selected.

**5** Number the boys from 1 to 14, and the girls from 1 to 13.
Perform each calculation as in Q4.

**6** Ratio of males : females is 80 : 60 = 4 : 3
Ratio of boarders : day students is 35 : 115 = 7 : 23

Number of males: $\frac{4}{7} \times 40 = 22{\cdot}857 = 23$, 17 females.

Number of boarders: $\frac{7}{30} \times 40 = 9{\cdot}33 = 9$, 31 day students.

$\frac{7}{30} \times 23 = 5{\cdot}36 = 5$ male boarders

18 male day students
4 female boarders
13 female day students

**7** Ratio of males: females is 180 : 220 = 9 : 11

Number of males: $\frac{9}{20} \times 60 = 27$ males, 33 females

Ratio of juniors : adult : seniors = 200 : 90 : 110 = 20 : 9 : 11

$\frac{20}{40} \times 27 = 14$ junior males, 17 junior females

$\frac{9}{40} \times 27 = 6$ adult males, 7 adult females

$\frac{11}{40} \times 27 = 7$ senior males, 9 senior females

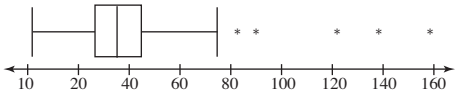
**8** **a** Stratified sampling

**b** Learners' ideas

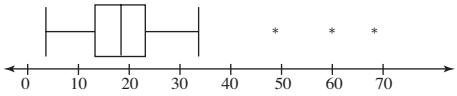
**c** Learners' ideas

## Enrichment

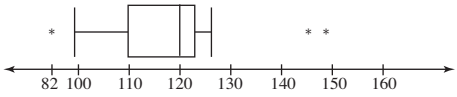
**1** $Q_1 = 26$, $Q_2 = 35$, $Q_3 = 45$, IQR = 19
1·5IQR = 1·5(19) = 28·5
Outliers are outside the range:
[26 – 28·5, 45 + 28·5] or [0, 73·5]



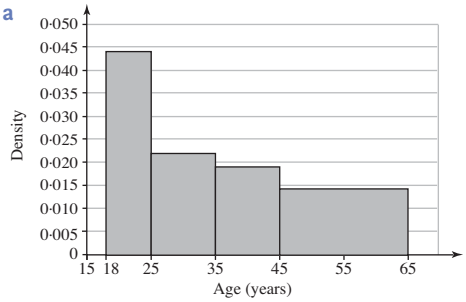

**2** $Q_1 = 13$, $Q_3 = 23{\cdot}5$, IQR = 10·5
1·5IQR = 1·5(10·5) = 15·75
Outliers are outside the range:
[13 – 15·75, 23·5 + 15·75] or [0, 40]



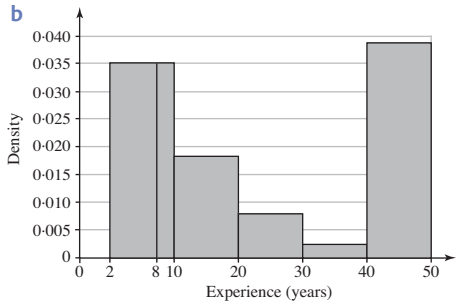

**3** $Q_1 = 110$, $Q_3 = 123{\cdot}5$, IQR = 13·5
1·5IQR = 1·5(13·5) = 20·25
Outliers are outside the range:
[110 – 20·25, 123·5 + 20·25] or [89, 144]




**4** **a**



**b**

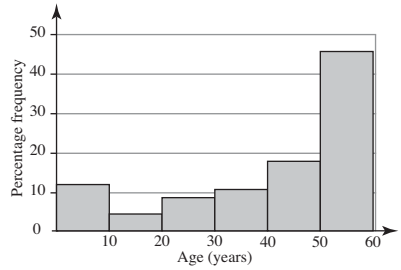



## Revision/Assessment

**1**

| Weight (grams) | Frequency | Percentage frequency |
|---|---|---|
| 100– | 1 | 3 |
| 105– | 4 | 13 |
| 110– | 6 | 20 |
| 115– | 6 | 20 |
| 120– | 8 | 27 |
| 125– | 5 | 17 |
| Total | 29 | 100 |

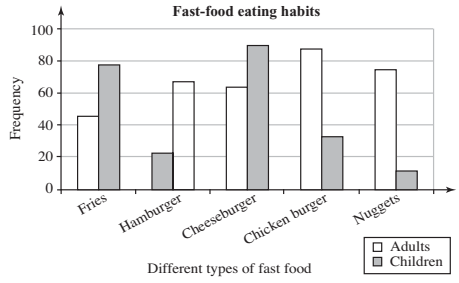
**2 a**




**b** The graph dips in the middle as expected. Early childhood deaths occur between 0 and 10 years of age and rates of death increase significantly after age 50.

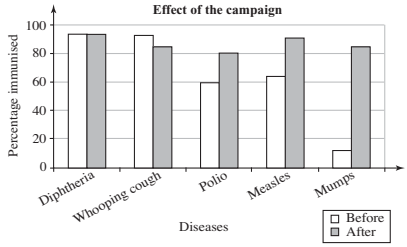
**c** 8%

**3**




The adults prefer chicken products and the children like fries and burgers.

**4 a**




**b** There was an increase in immunisation rates for polio, measles and mumps.

**c** There was a decrease in immunisation rates for whooping cough.

**d** Yes, as there was an increase in immunisation rates for most diseases.

**5** $\bar{x} = 116{\cdot}60$ g

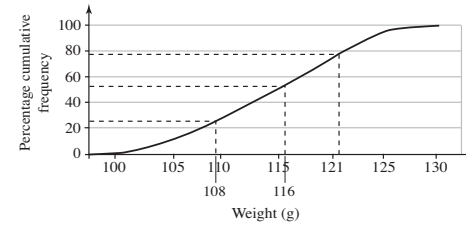
**6**

| Weight | Frequency | Mid value | Frequency × mid value |
|---|---|---|---|
| 100– | 216 | 102·5 | 22 140 |
| 105– | 452 | 107·5 | 48 590 |
| 110– | 538 | 112·5 | 60 525 |
| 115– | 640 | 117·5 | 75 200 |
| 120– | 541 | 122·5 | 66 272·5 |
| 125– | 113 | 127·5 | 14 407·5 |
| Total | 2500 | | 287 135 |

$\bar{x} = 114{\cdot}9$ g

**7 a**

| Weight (g) | Percentage frequency | Upper value | Cumulative percentage frequency |
|---|---|---|---|
| 95– | 0 | <100 | 0 |
| 100– | 8·64 | <105 | 8·64 |
| 105– | 18·08 | <110 | 26·72 |
| 110– | 21·52 | <115 | 48·24 |
| 115– | 25·6 | <120 | 73·84 |
| 120– | 21·64 | <125 | 95·48 |
| 125– | 4·52 | <130 | 100 |




**b** $Q_1 = 108$
Median = 116
$Q_3 = 121$

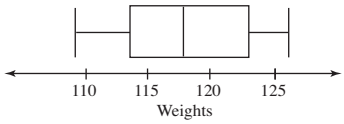
**8 a**

| Females | | Males |
|---|---|---|
| 8 | 1 | 5 |
| 7 1 7 1 7 6 5 | 2 | 6 3 8 2 3 7 8 |
| 3 5 5 6 2 | 3 | 9 5 5 2 7 6 6 5 |
| 7 2 4 8 3 | 4 | 5 7 0 0 2 |
| 3 | 5 | 2 6 6 2 |
| 2 | 6 | |
| 6 5 | 7 | |
| 2 9 | 8 | |
| 5 | 9 | |

**b** Male haircuts are generally cheaper than female haircuts and there is less variation in prices.

**9 a** $x_L = 109$
$Q_1 = 113{\cdot}5$
Median = 118
$Q_3 = 123$
$x_H = 126$

**b**

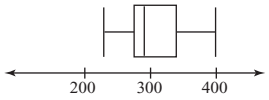



**c** The weights are basically symmetrically spread around a median of 118.

**10 a** 230, 240, 245, 250, 265, 275, 280, 280, 280, 285, 290, 310, 315, 320, 320, 325, 340, 360, 375, 380, 400

**b** $x_L = 230$
$Q_1 = 270$
Median = 290
$Q_3 = 332{\cdot}5$
$x_H = 400$

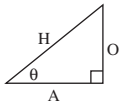
**c**



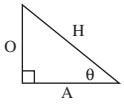

# Chapter 8

## Exercise 8A

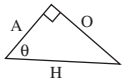
**1 a**



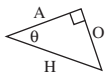

**b**



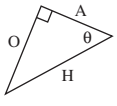

**c**



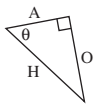

**d**



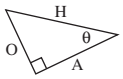

**e**



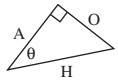

**f**



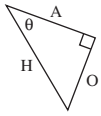

**g**



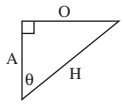

**h**



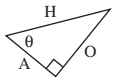

**i**



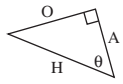

**j**



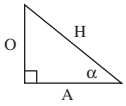

**k**



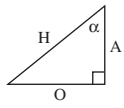

**l**



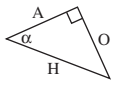

**2 a**



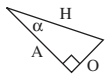

**b**



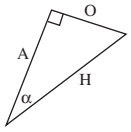

**c**



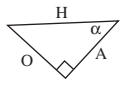

**d**



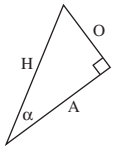

**e**



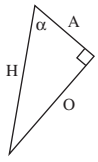

**f**



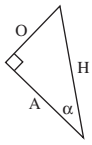

**g**



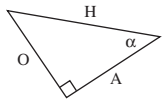

**h**



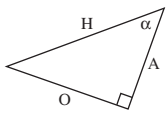

**i**



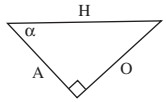

**j**




**k**




**l**




## Learning task 8B

**2**

| Triangle | Length of opposite side (mm) | Length of hypotenuse (mm) | Ratio: $\frac{\text{opposite side}}{\text{hypotenuse}}$ |
|---|---|---|---|
| 1 | 8 | 16 | 0·50 |
| 2 | 16 | 32 | 0·50 |
| 3 | 25 | 50 | 0·50 |
| 4 | 32 | 64 | 0·50 |
| 5 | 40 | 80 | 0·50 |

**3** The ratio of the opposite side to the hypotenuse is the same for all these triangles and equal to 0·5.

**4** $\sin 30° = 0{\cdot}5$

## Exercise 8C

**1** **a** 15·46 cm **b** 70·94 cm **c** 44·95 mm
**d** 8·48 cm **e** 20·98 m **f** 4·02 cm
**g** 28·60 km **h** 12·04 m **i** 36·03 m

**2** **a** 1·55 cm **b** 7·09 cm **c** 4·83 mm
**d** 0·85 cm **e** 0·98 cm **f** 2·02 mm
**g** 3·67 m **h** 3·40 mm **i** 3·20 m

**3** $x = 6{\cdot}88$ m, $y = 17{\cdot}22$ m

**4** **a** 1·06 m **b** 10·40 m

**5** **a** 3·46 cm **b** 30° **c** 2·5 cm

**6** 3·78 m

**7** $x = 12$ m $y = 6{\cdot}79$ m
Total length = 18·79 m

**8** 0·66 km

**9** 56·16 m 39·15 m

## Learning task 8D

**2**

| Triangle | Length of adjacent side (mm) | Length of hypotenuse (mm) | Ratio: $\frac{\text{adjacent side}}{\text{hypotenuse}}$ |
|---|---|---|---|
| 1 | 14 | 16 | 0·88 |
| 2 | 28 | 32 | 0·88 |
| 3 | 44 | 50 | 0·88 |
| 4 | 56 | 64 | 0·88 |
| 5 | 70 | 80 | 0·88 |

**3** The ratio of the adjacent side to the hypotenuse is approximately the same for these triangles and the average is equal to 0·88, correct to 2 decimal places.

**4** $\cos 30° = 0{\cdot}88$

## Exercise 8E

**1** **a** $x = 46{\cdot}04$ cm **b** $y = 159{\cdot}78$ mm
**c** $z = 104{\cdot}36$ cm **d** $a = 24{\cdot}72$ cm
**e** $x = 45{\cdot}38$ cm **f** $x = 82{\cdot}29$ cm
**g** $a = 11{\cdot}11$ m **h** $y = 7{\cdot}63$ m
**i** $x = 82{\cdot}52$ m

**2** **a** $y = 9{\cdot}02$ m **b** $x = 3{\cdot}43$ m
**c** $a = 3{\cdot}75$ cm **d** $y = 6{\cdot}77$ mm
**e** $x = 5{\cdot}55$ m **f** $b = 8{\cdot}34$ m
**g** $z = 5{\cdot}96$ km **h** $y = 20{\cdot}80$ mm
**i** $x = 16{\cdot}32$ mm

**3** 44 m **4** 3·91 m **5** 732·96 m

**6** $a = 30{\cdot}87$ m, $b = 115{\cdot}19$ m

**7** **a** 266·30 m, 294·56 m **b** 540·86 m

**8** 767·33 cm

**9** **a** 13·38 cm **b** 18·35 cm **c** 5·03 cm
**d** 26·21 cm **e** 14·86 cm

## Exercise 8F

**1** **a** 50·81 cm **b** 31·78 m
**c** 85·02 cm **d** 15·71 cm
**e** 60·95 m **f** 133·19 mm

**2** **a** 24·97 cm **b** 20·03 m
**c** 118·21 cm **d** 110·34 m
**e** 12·94 m **f** 62·00 cm
**g** 42·68 mm **h** 62·31 km
**i** 106·60 m

**3** **a** 97·91 cm **b** 76·44 m
**c** 25·15 cm **d** 72·42 cm
**e** 179·83 cm **f** 59·82 m
**g** 138·81 cm **h** 139·51 cm

**4** Ladder *A*: 25·57 m Ladder *B*: 5·93 m
Ladder *C*: 6·79 m Ladder *D*: 27·92 m

**5** Balloon *A*: 9·42 m Balloon *B*: 19·04 m

**6** 9·55 km **7** 102·53 m **8** 54·05 m

**9** 31·35 m **10** 10·00 m

## Exercise 8G

**1** **a** 45·10° **b** 20·67° **c** 38·57°
**d** 54·90° **e** 61·04° **f** 41·81°
**g** 54·90° **h** 64·79° **i** 57·61°

**2** **a** 51·32° **b** 51·32° **c** 41·08°
**d** 54·62° **e** 45·92° **f** 52·02°
**g** 31·00° **h** 48·19° **i** 27·27°

**3** **a** 69·42° **b** 27·80° **c** 39·52°
**d** 33·88° **e** 53·55° **f** 43·43°
**g** 37·22° **h** 29·27° **i** 52·02°
**j** 55·15° **k** 82·50° **l** 53·83°

**4** 17·92°

**5** **a** A: 75·52° B: 58·61° C: 48·59°
**b** A: 14·48° B: 31·39° C: 41·41°

**6** $\theta = 57°$, $\alpha = 43°$

**7** 40·27°, 62·05°

**8** **a** *AC*: 24·62° *AD*: 37·86° *AE*: 47·01° *AF*: 51·32°
**b** *AC*: 65·38° *AD*: 52·14° *AE*: 42·99° *AF*: 38·68°

**9** **a** 53·13° **b** 36·87° **c** 41·81° **d** 48·19°
**e** 56·44° **f** 33·56° **g** 60° **h** 30°

## Exercise 8H

**1** **a** 8·90 cm **b** 28·74 cm **c** 38·72 cm
**d** 42·24 cm **e** 33·74 cm **f** 54·27 m
**g** 85·76 mm **h** 65·82 m **i** 70·31 cm

**2** **a** 142·94 cm **b** 12·43 m **c** 115·45 mm
**d** 58·53 m **e** 34·50 mm **f** 47·18 m
**g** 99·22 m **h** 58·68 cm **i** 184·01 cm

**3** a 32·38° b 53·97° c 38·48°
d 55·86° e 33·69° f 50·60°
g 41·41° h 50·91° i 39·92°

**4** A: 15·92 m B: 4·25 m C: 8·69 m
D: 18·81 m E: 7·46 m

**5** Blue kite 31·92 m Green kite 13·93 m
Yellow kite 22·65 m Red kite 42·01 m

**6** a Ladder *A*: 55° Ladder *B*: 57°
Ladder *C*: 43° Ladder *D*: 25°
b Ladder *A*: 35° Ladder *B*: 33°
Ladder *C*: 47° Ladder *D*: 65°

## Exercise 8I

**1** a 315·09 m b 549·35 m

**2** 1508·06 m

**3** a 4·74° b 9·76° c 14·86°
d 25·50° e 51·10°

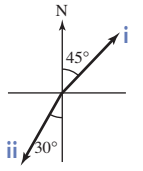
**4** 67·38°

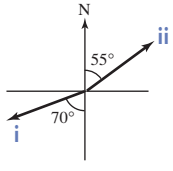
**5** a 34·66° b 34·66°

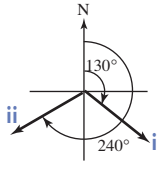
**6** a 144·05 m b 45·82° c 45·82°

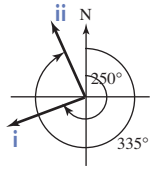
## Exercise 8J

**1** a *A*: N50°E or 50°T *B*: S30°W or 210°T
b *A*: S50°E or 130°T *B*: N30°W or 330°T
c *A*: S40°E or 140°T *B*: S60°W or 240°T
*C*: N40°W or 320°T
d *A*: N15°E or 15°T *B*: S75°E or 105°T
*C*: S15°W or 195°T *D*: N70°W or 290°T

**2** a


b


c


d



**3** a 160°T b 330°T c 140°T
d 280°T e 205°T f 148°T

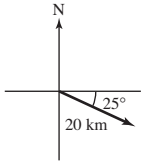
**4** a N47°E b S85°E c S54°E
d S45°W e N35°W f S51°W

**5** a 1·85 km north b 5·07 km east

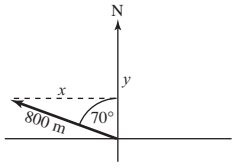
**6** a 338·02 m south b 635·72 m east

**7** a 920·90 m south b 335·18 m east

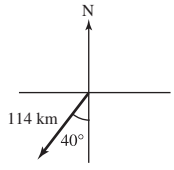
**8** a 507·53 m north b 236·67 m west

**9** a


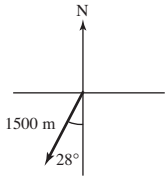
i 8·45 km ii 18·13 km

b


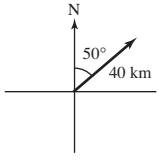
i 751·75 m ii 273·62 m

c


i 87·33 km south ii 73·28 km west

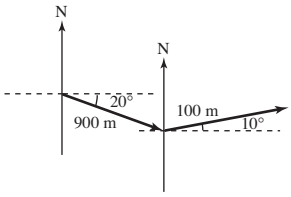
d


i 1324·42 m ii 704·21 m

e


i 25·71 km ii 30·64 km

**10** 2566·56 m south, 1891·91 m west

**11**


944·20 m east

## Learning task 8K

**1** $\sin 60° = 0{\cdot}87$, $\cos 60° = 0{\cdot}50$

**2** $\sin 45° = 0{\cdot}71$, $\cos 45° = 0{\cdot}71$

**3** *A*: $\sin 5° = 0{\cdot}09$, $\cos 5° = 0{\cdot}99$
*B*: $\sin 25° = 0{\cdot}42$, $\cos 25° = 0{\cdot}91$
*C*: $\sin 50° = 0{\cdot}77$, $\cos 50° = 0{\cdot}64$
*D*: $\sin 70° = 0{\cdot}94$, $\cos 70° = 0{\cdot}34$

**4** **a** $\sin 20° = 0{\cdot}3$, $\cos 20° = 0{\cdot}9$

**b** $\sin 40° = 0{\cdot}6$, $\cos 40° = 0{\cdot}8$

**c** $\sin 55° = 0{\cdot}82$, $\cos 55° = 0{\cdot}57$

**d** $\sin 75° = 0{\cdot}97$, $\cos 75° = 0{\cdot}26$

**e** $\sin 80° = 1{\cdot}0$, $\cos 80° = 0{\cdot}2$

**5** $\sin 6° = 0{\cdot}1$, $\cos 84° = 0{\cdot}1$
$\sin 12° = 0{\cdot}2$, $\cos 78° = 0{\cdot}2$
$\sin 17° = 0{\cdot}3$, $\cos 73° = 0{\cdot}3$
$\sin 24° = 0{\cdot}4$, $\cos 66° = 0{\cdot}4$
$\sin 30° = 0{\cdot}5$, $\cos 60° = 0{\cdot}5$
$\sin 37° = 0{\cdot}6$, $\cos 53° = 0{\cdot}6$
$\sin 44° = 0{\cdot}7$, $\cos 46° = 0{\cdot}7$
$\sin 53° = 0{\cdot}8$, $\cos 37° = 0{\cdot}8$
$\sin 64° = 0{\cdot}9$, $\cos 26° = 0{\cdot}9$

**6**

| Angle | Sine (*y*-axis) | Cosine (*x*-axis) |
|---|---|---|
| 120° | 0·9 | −0·5 |
| 135° | 0·7 | −0·7 |
| 150° | 0·5 | −0·9 |
| 210° | −0·5 | −0·9 |
| 225° | −0·7 | −0·7 |
| 240° | −0·9 | −0·5 |
| 300° | −0·9 | 0·5 |
| 330° | −0·5 | 0·9 |

## Exercise 8L

**1** **a** 0, 0·26, 0·5, 0·71, 0·87, 0·97, 1, 0·97, 0·87, 0·71, 0·5, 0·26, 0, –0·26, –0·5, –0·71, –0·87, –0·97, –1, –0·87, –0·71, –0·5, –0·26, 0

**b** See graph below.

**2** **a** 1, 0·97, 0·87, 0·71, 0·5, 0·26, 0, –0·26, –0·5, –0·71, –0·87, –0·97, –1, –0·97, –0·87, –0·71, –0·5, –0·26, 0, 0·26, 0·5, 0·71, 0·87, 0·97, 1

**b** See graph below.

**c** They have the same shape—are both periodic over 360°. The height is bounded between 1 and –1. If either graph was shifted 90° right or left then it would be in the same position as the other.

## Applications

### Shooting for goal

**a**

| $x$ | 5 m | 10 m | 15 m | 20 m | 25 m | 30 m |
|---|---|---|---|---|---|---|
| θ | 1·6° | 2·9° | 3·7° | 4·1° | 4·2° | 4·1° |

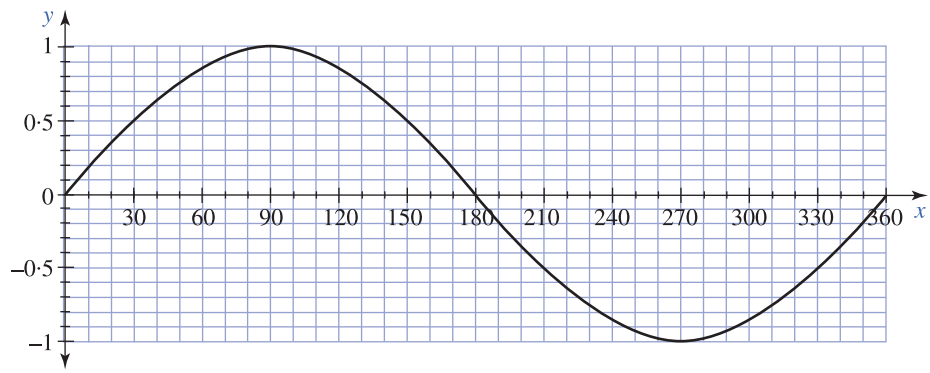
The largest goal angle is 4·2° when $x = 24{\cdot}9$ m.

**b**

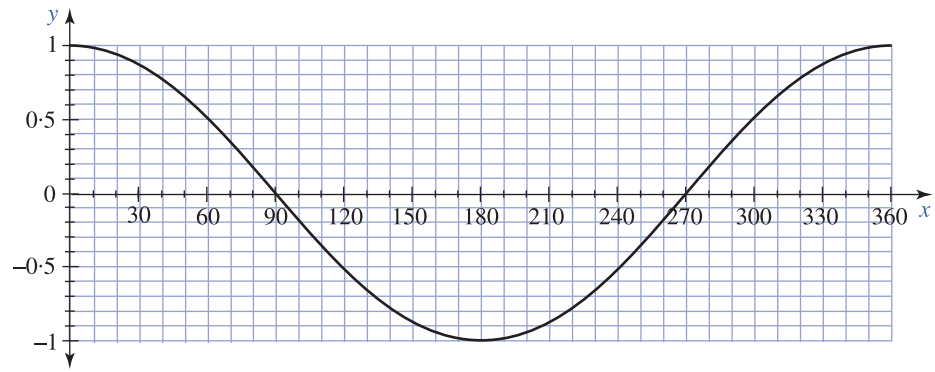
| $x$ | 15 m | 20 m | 25 m | 30 m | 35 m | 40 m |
|---|---|---|---|---|---|---|
| θ | 5·1° | 5·9° | 6·4° | 6·6° | 6·5° | 6·4° |

The largest angle is 6·6° when $x = 31{\cdot}8$ m.

**1** **b**




**2** **b**



## Measuring the tangent of an angle

| Triangle | Angle | Height | Tangent of the angle |
|---|---|---|---|
| 1 | 45° | 1·0 cm | 1·00 |
| 2 | 50° | 1·2 cm | 1·19 |
| 3 | 55° | 1·4 cm | 1·43 |
| 4 | 60° | 1·7 cm | 1·73 |
| 5 | 65° | 2·1 cm | 2·14 |
| 6 | 70° | 2·7 cm | 2·75 |
| 7 | 75° | 3·7 cm | 3·73 |
| 8 | 80° | 5·7 cm | 5·67 |
| 9 | 85° | 11·4 cm | 11·43 |
| 10 | 90° | Can't do | Not possible |

## Enrichment

**1** **a** 13·24 cm **b** 5·60 cm **c** 8·62 cm
**d** 10·27 cm **e** 9·53 cm **f** 3·85 cm

**2** **a** 43 m **b** 9 m

**3** 4·62 cm

**4** **a** **i** 7·75 m **ii** 28·97°
**b** **i** 13·86 m **ii** 32·2°

**5** 55·5°

**6** **a** 11·42° **b** 11·12°

**7** **a** 32·17 cm$^2$ **b** 9·6 cm$^2$

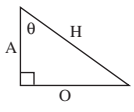
**8** 151·50 m

**9** 2847·80 m, 309·49°T

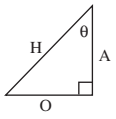
**10** **a** 1708·37 m
**b** **i** 12·3°T **ii** 257·7°T

## Revision/Assessment

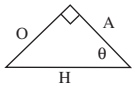
**1** **a**



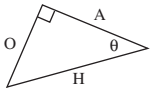

**b**



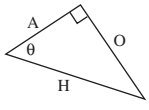

**c**



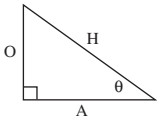

**d**



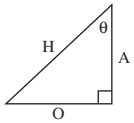

**2** **a**



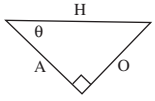

**b**




**c**




**d**




**3** **a** 22·78 cm **b** 378·86 m
**c** 50·20 mm **d** 8·48 km

**4** 55·62 m

**5** **a** 20·52 cm **b** 6·84 cm

**6** **a** 4·46 m **b** 27·14 cm
**c** 6·85 km **d** 21·40 m

**7** 2·88 m **8** 114·13 m

**9** **a** 118·89 cm **b** 108·51 m
**c** 94·88 m **d** 86·21 mm

**10** $x = 28{\cdot}39$ m, $y = 16{\cdot}97$ m

**11** 62·2 cm

**12** **a** 42·6° **b** 44·6° **c** 51·1° **d** 49·9°

**13** 50·83°

**14** $\theta = 53{\cdot}13°$, $\alpha = 36{\cdot}87°$

**15** **a** 14·89 m **b** 102·83 km **c** 35·18°

**16** $x = 7{\cdot}50$ m, $y = 18{\cdot}56$ m

**17** 21·5°

**18** 128·83 m

**19** **a** 7·24° **b** 12·30° **c** 20·09°
**d** 40·38° **e** 55·26°

**20** *A*: N28°E or 28°T *B*: S72°E or 108°T
*C*: S32°W or 212°T *D*: N84°W or 276°T

**21** **a** 296·75 m **b** 474·91 m

**22** **a** 26·5 km **b** 45·90 km

# Chapter 9

## Exercise 9A

**1** **a** $x = 4$ **b** $y = 15$ **c** $z = -3$
**d** $m = -2$ **e** $n = 12$ **f** $p = -2$
**g** $p = -\frac{1}{4}$ **h** $q = 5{\cdot}25$ **i** $t = 3{\cdot}5$

**2** **a** $x = -4$ **b** $y = -6$ **c** $n = 9$
**d** $g = -3$ **e** $p = -11$ **f** $m = 5$
**g** $r = -3$ **h** $s = -6$ **i** $t = 6$
**j** $z = -3{\cdot}75$ **k** $q = -5{\cdot}375$ **l** $h = 4\frac{1}{6}$
**m** $x = -\frac{1}{2}$ **n** $y = -\frac{2}{3}$ **o** $z = \frac{2}{3}$
**p** $x = -\frac{1}{4}$ **q** $x = -1\frac{1}{3}$ **r** $y = -\frac{1}{5}$
**s** $z = \frac{5}{12}$ **t** $y = \frac{1}{3}$ **u** $x = -\frac{2}{3}$

**3** **a** $x = 80$ **b** $x = -20$ **c** $x = 60$
**d** $x = -96$ **e** $x = -48$ **f** $x = 66$
**g** $y = 63$ **h** $y = -108$ **i** $y = 108$
**j** $y = -70$ **k** $y = 96$ **l** $y = 120$

**4** **a** $p = 5$ **b** $y = -13$ **c** $x = -7$
**d** $y = 9$ **e** $q = -12$ **f** $m = 5$

g $n = -10$ h $z = -42$ i $r = -180$

j $s = 64$

**5** $b = -\$30$ **6** $d = \$210$ **7** $d = 46$

**8** $c = 16$ **9** $\frac{C}{120} = 3{\cdot}5$, $C = \$420$

**10** $25p = 146{\cdot}25$, $p = \$5{\cdot}85$

## Exercise 9B

**1** a $x = 3$ b $y = 8$ c $m = -4$

d $n = -4$ e $q = -3$ f $a = -4$

g $b = 7$ h $c = 6$ i $m = 3$

j $a = -4$ k $r = -6$ l $s = -1\frac{1}{2}$

**2** a $x = -12$ b $x = -20$ c $x = -9$

d $a = -15$ e $b = -9$ f $c = -6$

g $m = -10\frac{1}{2}$ h $p = -3\frac{1}{3}$ i $c = -3\frac{3}{4}$

**3** a $p = -18$ b $y = -36$ c $z = 6$

d $m = -32$ e $n = 16$ f $q = -48$

g $t = -2$ h $r = 3$ i $s = 10$

**4** a $x = 9$ b $x = -3$ c $x = -9$

d $a = -13$ e $x = 7$ f $y = -5$

g $z = -2$ h $y = -4$ i $x = 3$

**5** a $x = 12$ b $a = -4$ c $n = 18$

d $n = -12$ e $p = 20$ f $q = 28$

g $m = -18$ h $z = 40$ i $t = -24$

**6** a $x = 3$ b $x = -2\frac{1}{2}$ c $y = -20$

d $x = -32$ e $q = 5$ f $y = -16$

g $z = 40$ h $n = 28$

## Exercise 9C

**1** a $x = 1$ b $x = 2$ c $m = 3$

d $x = 10$ e $y = 13$ f $n = 16$

g $z = -1$ h $s = -6$ i $p = 3$

j $q = -2$ k $t = 3$ l $h = -6$

m $x = 10$ n $y = 0$ o $p = 11$

p $x = 5{\cdot}5$ q $x = 9{\cdot}7$ r $y = 5{\cdot}5$

**2** a $x = 2$ b $x = -1$ c $x = -1$

d $z = 12$ e $q = 12$ f $t = 6$

g $x = 2\frac{1}{2}$ h $p = \frac{3}{4}$ i $m = 2\frac{2}{3}$

j $y = \frac{3}{5}$ k $z = 1\frac{2}{7}$ l $p = 1\frac{1}{4}$

m $s = -\frac{1}{2}$ n $t = 4\frac{5}{8}$ o $r = \frac{2}{9}$

**3** a $x = 2$ b $y = 6\frac{2}{3}$ c $z = 4\frac{2}{3}$

d $m = -\frac{3}{4}$ e $n = -4\frac{9}{10}$ f $q = -2\frac{3}{4}$

g $q = 2\frac{3}{4}$ h $p = 1\frac{7}{12}$ i $q = 1\frac{5}{6}$

j $y = 1$ k $y = \frac{11}{6}$ l $z = -\frac{13}{20}$

m $r = 2$ n $s = 2$ o $t = -\frac{1}{28}$

**4** a $x = 10$ b $x = 15$ c $x = 10$

d $x = 18$ e $x = 20$ f $x = 13$

**5** a $x = 2$ b $y = 5$ c $y = 2$

d $x = 5$ e $z = -2$ f $y = 21$

g $z = 9$ h $P = \$27$

## Exercise 9D

**1** a $x = 8$ b $x = 3$ c $x = 4$

d $x = 2$ e $x = 3$ f $x = 5$

**2** a $x = 2$ b $y = -2$ c $z = -3$

d $m = -2$ e $n = 2$ f $p = -2$

g $x = 15$ h $y = -4$ i $z = 2$

**3** a $x = 3$ b $y = 2$ c $z = 2$

d $x = 18$ e $z = -1$ f $x = 23$

g $x = -2$ h $q = 3$ i $x = -13$

j $x = \frac{1}{2}$ k $q = -13$ l $z = \frac{1}{2}$

**4** a $x = 10$ b $x = 6$ c $z = 45$

d $x = 12$ e $x = 20$ f $x = 4$

g $x = 10$ h $x = 20$ i $x = 8$

j $x = 8$ k $x = 24$ l $x = 10$

**5** a $x = 6$ b $x = 7$ c $x = 4$

d $x = 3$ e $x = 11$ f $x = -3$

g $x = 5$ h $x = -12$ i $x = 5$

**6** a The son is 12 years old, the father is 36 years old.

b Liz is 30 years old and Ian is 36.

c I spent \$15 and my friend spent \$7.

d An adult ride costs \$3.

e The first competitor runs 600 m, the second 700 m, the third 800 m and the fourth 900 m.

f The bus travelled 80 km/h for half an hour, and at 100 km/h for 1 hour.

g Full price tickets are \$16, discount tickets are \$11.

h The cost of a full price ticket is \$4 and a discount ticket is \$3.

i It takes 3 hours for Bob to catch Peter. At this time they have travelled 72 km

j 10, 11, 12, 13, and 14

**k** 5 and 30

**l** The cost of the final bill was $60.

## Learning task 9E

**2** When an inequality is multiplied/divided by a positive number, the inequality sign keeps its direction.

**4** When an inequality is multiplied/divided by a negative number, the inequality sign reverses its direction.

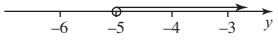
**5** **a** $x > 2$ **b** $x < -3$ **c** $x < 6$ **d** $x < -2$

## Exercise 9F

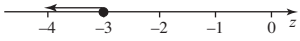
**1** **a** > **b** > **c** < **d** >
**e** = **f** > **g** = **h** >

**2** **a** $x < 10$


**b** $y > -5$

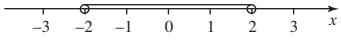



**c** $z \leq -3$



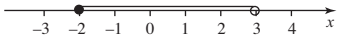

**d** $m \geq -1$


**e** $-2 < x < 2$

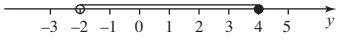



**f** $-4 < x < -1$


**g** $-2 \leq x < 3$




**h** $-2 < y \leq 4$




**i** $15 \leq T \leq 25$


**3** $H \geq 170$ cm

**4** **a** $x < -2$ **b** $y < -4$ **c** $z \geq -8$
**d** $x \geq -6$ **e** $y > -8$ **f** $z < -6$
**g** $x \leq -3$ **h** $y < 9$ **i** $z > 5$
**j** $x \geq -21$ **k** $y < -52$ **l** $z < -30$
**m** $x \leq -20$ **n** $y > 16$ **o** $z > -60$
**p** $m < -20$ **q** $n \leq -6$ **r** $p > 15$
**s** $x \geq 9$ **t** $y < 20$ **u** $z > 6$
**v** $a \leq 15$ **w** $b \geq 18$ **x** $c \geq 16$

**5** **a** $x \geq -3$ **b** $y < -2$ **c** $z \leq -4$
**d** $a > 2$ **e** $b \leq 2$ **f** $c < -2$
**g** $m \geq -28$ **h** $n > -42$ **i** $p \leq 5$
**j** $x > 16$ **k** $y \geq 15$ **l** $z \geq -9$
**m** $a \geq 20$ **n** $b \geq 6$ **o** $z \geq 6$
**p** $x \geq 18$ **q** $y \geq 20$ **r** $p \geq 25$

**6** **a** $x > -9$ **b** $x < -1$ **c** $x \leq -14$
**d** $z \geq -12$ **e** $q > -12$ **f** $t \leq 6$

**7** **a** $x > 4$ **b** $x \leq -16$ **c** $x \geq -7$
**d** $x \geq -5$ **e** $x < -3$ **f** $x \geq -4$

**8** **a** $x > 9$ **b** $x > -8$ **c** $x \leq 8$
**d** $x \leq 13$ **e** $z > -4$ **f** $y < 2$

**9** **a** $x \geq 8$ **b** $x > 3$ **c** $x \leq 10$
**d** $x > 15$ **e** $x \geq 9$ **f** $x \leq -2$

**10** **a** $x > -8$ **b** $x \leq 5$ **c** $x < -3$
**d** $x \leq 4$ **e** $x > -1$ **f** $x \leq -3$

**11** The daughter must be <20 and the mother <40 years of age.

**12** Yesterday's withdrawal was less than or equal to $190 and today's was less than or equal to $380.

## Exercise 9G

**1** **a** $x = n - m$ **b** $x = b - a$ **c** $x = -q - p$
**d** $x = c + a$ **e** $x = r - s$ **f** $x = m - n$
**g** $x = b - a$ **h** $x = m - h$ **i** $x = e + f$

**2** **a** $x = \frac{q}{p}$ **b** $x = \frac{s}{r}$ **c** $x = \frac{q}{p}$
**d** $x = \frac{-d}{c}$ **e** $x = \frac{n}{-m}$ **f** $x = \frac{p}{f}$
**g** $x = \frac{-b}{a}$ **h** $x = \frac{d}{c}$ **i** $x = \frac{h}{k}$

**3** **a** $x = nm$ **b** $x = ca$ **c** $x = f^2$
**d** $x = -dc$ **e** $x = -rz$ **f** $x = -v^2$
**g** $x = -hg$ **h** $x = -kj$ **i** $x = -w^2$
**j** $x = pm$ **k** $x = pq$ **l** $x = z^2$

**4** **a** $y = \frac{c - b}{a}$ **b** $y = \frac{d - c}{m}$ **c** $y = \frac{q + x}{z}$
**d** $y = \frac{k + q}{p}$ **e** $y = \frac{c - q}{b}$ **f** $y = \frac{p - b}{h}$
**g** $y = \frac{m + n}{k}$ **h** $y = \frac{d - c}{a}$ **i** $y = \frac{g - f}{h}$

**5** **a** $y = c(b - a)$ **b** $y = b(h - k)$
**c** $y = a(b + c)$ **d** $y = -m(b + n)$
**e** $y = 2h^2$ **f** $y = p^2$
**g** $y = a(c - b)$ **h** $y = h(m - k)$
**i** $y = -q(r + p)$

**6** **a** $x = \frac{c(d - b)}{a}$ **b** $x = \frac{a(b + c)}{c}$

c $x = \frac{b(d+c)}{d}$ d $x = \frac{b(f-g)}{a}$

e $x = \frac{-n(q+p)}{m}$ f $x = \frac{-b(d-c)}{a}$

g $x = \frac{d(e-b)}{c}$ h $x = \frac{s(r-u)}{t}$

i $x = \frac{-g(h+k)}{f}$

**7** a $x = \frac{a+bc}{b}$ b $x = \frac{k+mn}{m}$

c $x = \frac{r+pq}{p}$ d $x = \frac{m-hk}{h}$

e $x = \frac{w-yz}{z}$ f $x = \frac{s-rt}{t}$

g $x = \frac{du-f}{u}$ h $x = \frac{nr-s}{n}$

i $x = \frac{pt-q}{p}$ j $x = \frac{-(w+yz)}{y}$

k $x = \frac{gk-h}{k}$ l $x = \frac{n+mp}{p}$

m $x = 3+n$ n $x = \frac{pq-r}{p}$

o $x = \frac{db-a^2}{d}$ p $x = \frac{2-n}{m}$

q $x = \frac{b-2}{a}$ r $x = \frac{1}{2z}$

## Exercise 9H

**1** a $E = 1200$ b $A = 40$

c $v = 56$ d $a = 40$

e $E = 40$ f $s = 50$

g $C = 20$ h $A = 314{\cdot}16$

i $m = 20$ j $A = 360\pi \approx 1130{\cdot}97$

k $s = 204$ l $c = 13$

m $V = 360\pi \approx 904{\cdot}78$ n $S = 20$

o $A = 40$ p $V = 270\pi \approx 848{\cdot}23$

q $A = 665{\cdot}5$ r $I = 192$

s $R = 15$ t $a = 6$

u $s = 5$ v $S = 28$

**2** a $t = 5$ b $v = 30$ c $R = 5$

d $I = 2$ e $u = 5$ f $t = 2$

g $a = 2$ h $m = 10$ i $v = 16$

j $F = 203$ k $r = 8$ l $h = 15$

m $b = 6$ n $r = 6$ o $h = 9$

p $b = 7$ q $h = 16$ r $r = 3$

s $c = 13$ t $a = 7$

**3** a $v = 5$ m/s b $s = 5$ m c $u^2 = 16$, $u = 4$ m/s

**4** a $s = 32{\cdot}5$ b $u = \frac{3}{2} = 1{\cdot}5$

**5** a i 32° ii 104° iii 14°

b i 15° ii 30° iii $^{-}5°$

**6** a $V = \frac{32}{3}\pi$ cm$^3$ b 33·51 cm$^3$ c $r = 3$ cm

**7** a $V = \frac{288\pi}{3} = 301{\cdot}59$ cm$^3$

b $h = 20$ cm c $r = 5$ cm

**8** a $I = 300$ b $T = 5$

**9** a $A = \$2938{\cdot}66$ b \$4261·80 c 10 years

## Exercise 9I

**1** a $m = \frac{F}{a}$, $a = \frac{F}{m}$ b $R = \frac{E}{I}$, $I = \frac{E}{R}$

c $L = \frac{A}{W}$, $W = \frac{A}{L}$ d $D = \frac{C}{\pi}$

e $L = \frac{V}{WH}$ f $r = \frac{A}{2\pi h}$

$W = \frac{V}{LH}$ $h = \frac{A}{2\pi r}$

$H = \frac{V}{LW}$

g $P = \frac{100I}{RT}$ h $U = V - AT$

$R = \frac{100I}{PT}$ $A = \frac{V-U}{T}$

$T = \frac{100I}{PR}$ $T = \frac{V-U}{A}$

i $H = \frac{V}{L^2}$ j $h = \frac{V}{\pi r^2}$

$L = \sqrt{\frac{V}{H}}$ $r = \sqrt{\frac{V}{\pi h}}$

k $t = \frac{2s}{(u+v)}$ l $a = \frac{v^2-u^2}{2s}$

$u = \frac{2s}{t} - v$ $s = \frac{v^2-u^2}{2a}$

$v = \frac{2s}{t} - u$

m $u^2 = v^2 - 2as$ n $c = \sqrt{a^2+b^2}$

$u = \sqrt{v^2-2as}$ $a = \sqrt{c^2-b^2}$

$b = \sqrt{c^2-a^2}$

o $h = \frac{A-2\pi r^2}{2\pi r}$ p $l = \frac{T^2 g}{2\pi}$

$g = \frac{2\pi l}{T^2}$

q $M = \frac{Fd^2}{Gm}$ $m = \frac{Fd^2}{GM}$ r $d^2 = \frac{GMm}{F}$ $d = \sqrt{\frac{GMm}{F}}$

s $r^3 = \frac{3V}{4\pi}$ $r = \sqrt[3]{\frac{3V}{4\pi}}$ t $P = \frac{A}{(1+r)^3}$ $r = \sqrt[3]{\frac{A}{P}} - 1$

**2** a $C = \frac{F-32}{1{\cdot}8}$

b i 0 ii $-6\frac{2}{3}$ iii 10

c No

**3** a $m = \frac{2E}{v^2} = 60$ kg b $v = \sqrt{\frac{2E}{m}} = 4$ m/s

## Applications

### Car distances

a $d = 60t$

| $t$ | 0 | 2 | 4 | 6 | 8 | 10 |
|---|---|---|---|---|---|---|
| $d$ | 0 | 120 | 240 | 360 | 480 | 600 |

b $t = \frac{d}{60}$

c

| $d$ | 0 | 30 | 60 | 180 | 420 | 540 |
|---|---|---|---|---|---|---|
| $t$ | 0 | $\frac{1}{2}$ | 1 | 3 | 7 | 9 |

d $v = \frac{300}{t}$

e

| $t$ | 1 | 2 | 3 | 4 | 5 | 6 | 8 | 10 |
|---|---|---|---|---|---|---|---|---|
| $v$ | 300 | 150 | 100 | 75 | 60 | 50 | 37·5 | 30 |

f i $d = vt$ ii $v = \frac{d}{t}$ iii $t = \frac{d}{v}$

### Distance out to sea

b $d \approx 0{\cdot}59\sqrt{h}$

## Enrichment

**1** a $x = 11\frac{2}{3}$ b $x = -3\frac{3}{8}$ c $x = -\frac{8}{15}$

d $x = 2$ e $x = 1\frac{4}{5}$ f $x = \frac{1}{6}$

g $x = 7\frac{1}{3}$ h $x = \frac{9}{23}$ i $x = -\frac{5}{7}$

j $x = -1\frac{4}{7}$ k $x = 3$ l $x = -\frac{1}{2}$

m $x = -1\frac{7}{10}$ n $x = -\frac{1}{5}$ o $x = 2$

p $x = 3$ q $x = 4$ r $x = 2$

s $x = -1$ t $x = -3$ u $x = 1$

**2** a $x = \frac{m}{k-n}$ b $x = \frac{r}{2(t+s)}$

c $x = \frac{k}{4(n+h)}$ d $x = b\left(f + \frac{a}{c}\right)$

e $x = d\left(\frac{a}{b} - \frac{c}{e}\right)$ f $x = \frac{2}{3}m\left(q - \frac{p}{3n}\right)$

g $x = \frac{d+c}{a-b}$ h $x = \frac{f-k}{g+h}$

i $x = \frac{dc-af}{ae-db}$ j $x = \frac{c+2q}{2p-m}$

k $x = \frac{bf+av}{ak+bg}$ l $x = \frac{b^2+a^2}{2a+3b}$

m $x = \frac{a+bc}{db}$ n $x = \frac{sm-pq}{pse}$

o $x = \frac{b(md+fg)}{afm}$ p $x = \frac{-a}{a+b-c}$

q $x = \frac{2r-2q}{p+q-r}$ r $x = \frac{p-m-3n}{m-n-p}$

**3** a $\theta = 10°$ b $r = 14$ cm

## Revision/Assessment

**1** a $p = \frac{1}{4}$ b $q = 8$ c $t = 3{\cdot}5$

d $z = -5$ e $q = -8$ f $h = \frac{35}{6} = 5\frac{5}{6}$

g $y = -80$ h $y = -54$ i $y = 96$

**2** a $a = -5$ b $b = -4$ c $c = 5$

d $a = -7$ e $x = 7$ f $y = 4$

g $m = -6$ h $z = -5$ i $t = -24$

**3** a $m = 3$ b $n = 1{\cdot}2$ c $q = 2$

d $x = 18$ e $x = 11$ f $x = -5$

**4** a $x = 20$ b $q = -1$ c $x = -13$

d $x = 8$ e $x = -24$ f $x = 20$

g $x = -11$ h $x = -12$ i $x = 5$

**5** a $x \geq -4$ b $y < 3$ c $z \leq -5$

d $a > 2$ e $b \leq 3$ f $c > -2$

g $a \geq 30$ h $b \geq 6$ i $z \geq 6$

j $x > -10$ k $x < 0$ l $x \leq -15$

m $x > 6$ n $x \leq -10$ o $x \geq -4$

p $x \geq -17$ q $z < -6$ r $y < 3\frac{3}{5}$

s $x \geq 6$ t $x > 3$ u $x \leq 10 x \leq 10$

**v** $x \geq 34$ **w** $x < 2\frac{1}{9}$ **x** $x \geq 1\frac{16}{59}$

**6** **a** $x = \frac{b(d-c)}{a}$ **b** $x = \frac{m(p-q)}{n}$

**c** $x = \frac{-p(t+r)}{q}$ **d** $x = \frac{ab-d}{a}$

**e** $x = \frac{s+nr}{n}$ **f** $x = \frac{pt-q}{p}$

**g** $x = -2mn$ **h** $x = -h^2$

**i** $x = -\frac{1}{2}p^2$

**7** **a** $A = 224\pi \approx 703{\cdot}72$

**b** $s = 92$ **c** $c = 25$ **d** $V = 972\pi \approx 3053{\cdot}63$

**e** $F = 86$ **f** $r = 6$ **g** $h = 16$ **h** $r = 7$

**8** **a** $r = \frac{A}{2\pi h}$ **b** $T = \frac{100I}{PR}$

**c** $a = \frac{v-u}{t}$ **d** $L = \sqrt{\frac{V}{H}}$

**e** $r = \sqrt{\frac{V}{\pi h}}$ **f** $u = \frac{2s}{t} - v$

**g** $u = \sqrt{v^2 - 2as}$ **h** $d = \sqrt{\frac{K}{F}}$

**i** $r = \sqrt[3]{\frac{3V}{4\pi}}$ **j** $r = \sqrt[3]{\frac{A}{P}} - 1$

**9** 10% per annum

## Chapter 10

### Exercise 10A

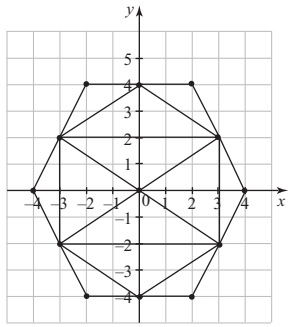
**1** **a** $A = (0, 0)$ **b** $B = (1, 4)$

**c** $C = (-3, 1)$ **d** $D = (0, -1{\cdot}5)$

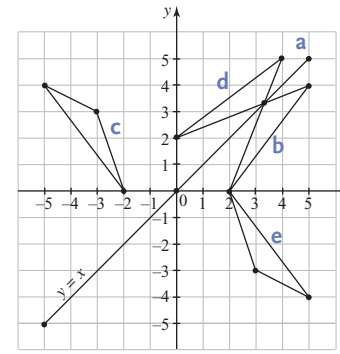
**e** $E = (3, -2{\cdot}5)$ **f** $F = (-3{\cdot}5, -2{\cdot}5)$

**g** $G = (3{\cdot}5, 1{\cdot}5)$

**2**



**3**



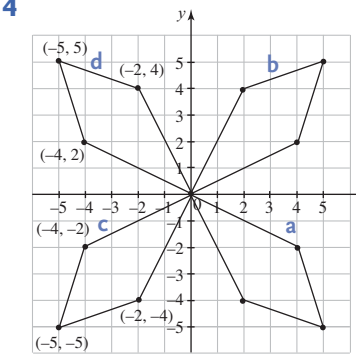

**b** Scalene triangle

**c** Reflected about the $y$-axis

**d** Reflected about the line $y = x$

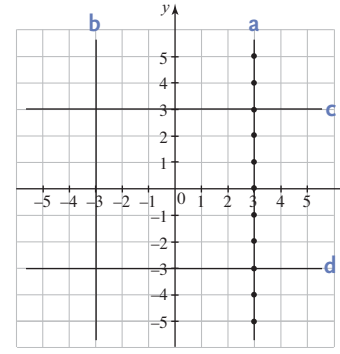
**e** Reflected about the $x$-axis

**4**




**b** Reflection about the $x$-axis

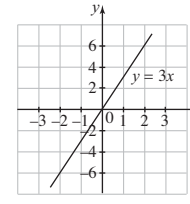
**5**




**b** $x = -3$ **c** $y = 3$ **d** $y = -3$ **e** Square

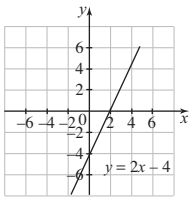
### Exercise 10B

**1** **a**



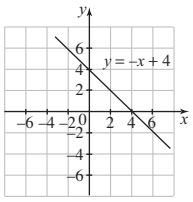

$x$-intercept is 0
$y$-intercept is 0

**b**



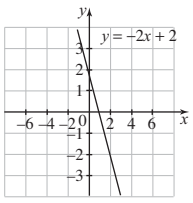

*x*-intercept is 2
*y*-intercept is –4

**c**



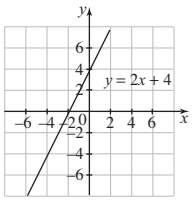

*x*-intercept is 4
*y*-intercept is 4

**d**



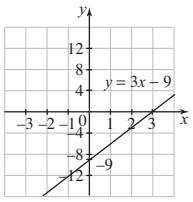

*x*-intercept is 1
*y*-intercept is 2

**e**



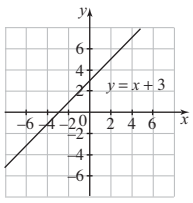

*x*-intercept is –2
*y*-intercept is 4

**f**



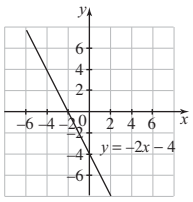

*x*-intercept is 3
*y*-intercept is –9

**g**



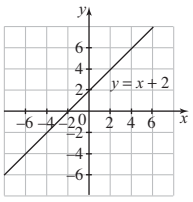

*x*-intercept is –3
*y*-intercept is 3

**h**



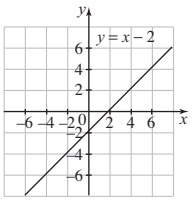

*x*-intercept is –2
*y*-intercept is –4

**2 a**



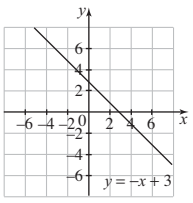

*x*-intercept is –2
*y*-intercept is 2

**b**



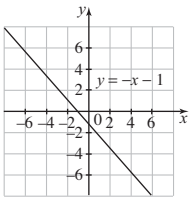

*x*-intercept is 2
*y*-intercept is –2

**c**



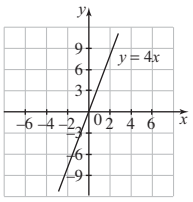

*x*-intercept is 3
*y*-intercept is 3

**d**



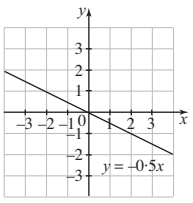

*x*-intercept is –1
*y*-intercept is –1

**e**



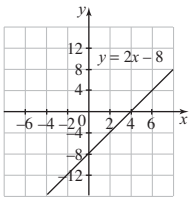

*x*-intercept is 0
*y*-intercept is 0

**f**



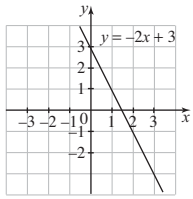

*x*-intercept is 0
*y*-intercept is 0

**g**




*x*-intercept is 4
*y*-intercept is –8

**h**

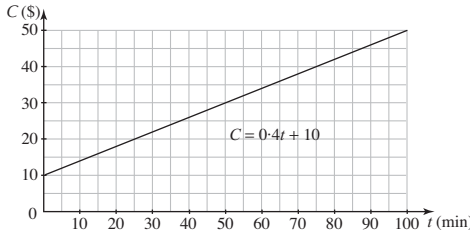



$x$-intercept is 1·5
$y$-intercept is 3

**3 a**

| $t$ | 60 | 80 | 100 |
|---|---|---|---|
| $C$ | 34 | 42 | 50 |

**b**

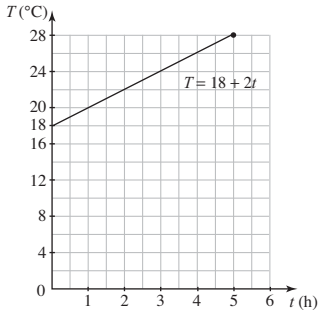



**d** \$46 **e** 115 min

**4 a**

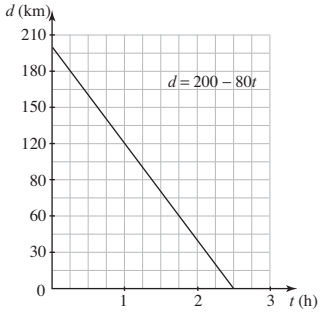
| $t$ | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| $T$ | 18 | 20 | 22 | 24 | 26 | 28 |

**b**




**d** 27°C **e** 27°C **f** 11:30 am

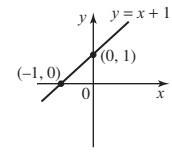
**5 a**



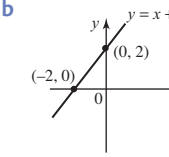

**b** 200 km, 2·5 hours
**c** 80 km **d** 1·875 hours
**e** 120 km **f** 80 km

## Exercise 10C

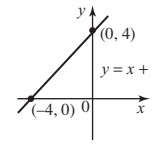
**1 a**



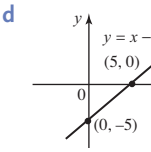

**b**



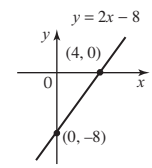

**c**



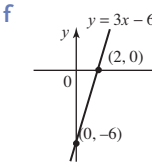

**d**



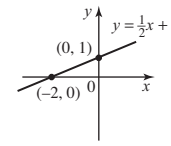

**e**



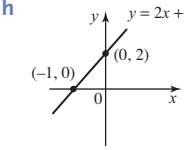

**f**



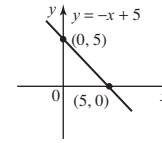

**g**



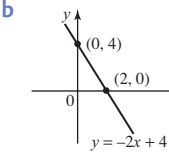

**h**



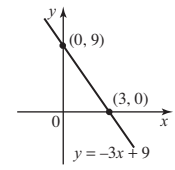

**2 a**



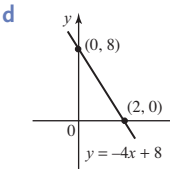

**b**



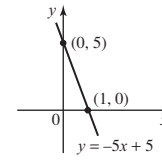

**c**



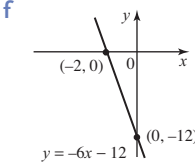

**d**



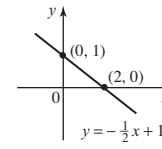

**e**



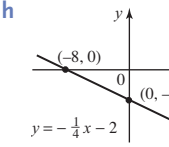

**f**



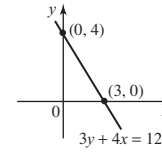

**g**



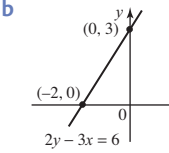

**h**



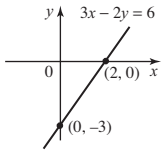

**3 a**



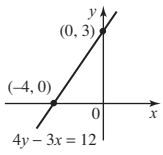

**b**



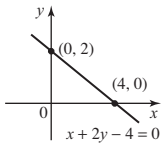
**c**



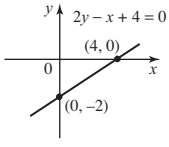

**d**



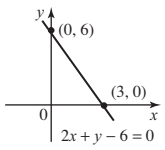

**e**



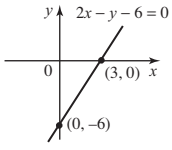

**f**



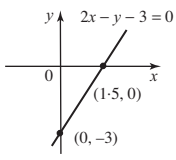

**g**



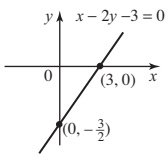

**h**



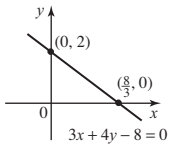

**i**



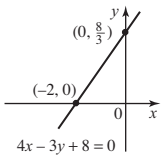

**j**



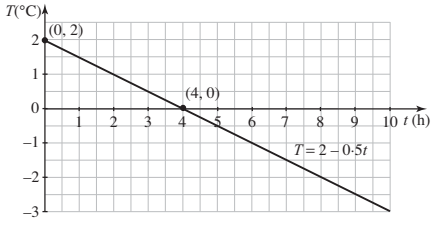

**k**




**l**




**4 a**

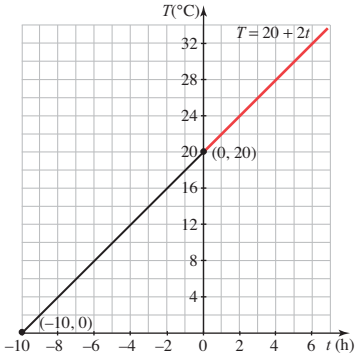



**b** 2°C ($t = 0$), 0°C ($t = 4$)

**c** −1°C **d** 10 pm

**e** It is not realistic due to the linear relationship, which would indicate a constant drop in temperature over an indefinite period of time.

**5 a**




**b** 20°C ($t = 0$), 28°C ($t = 4$)

**c** 5 pm

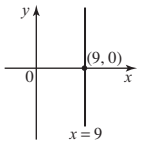
**d** It is not realistic due to the linear relationship, which would indicate a constant increase in temperature over an indefinite period of time.

## Exercise 10D

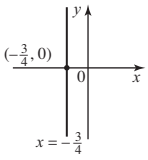
**1 a** $x = 6$ **b** $x = -8$

**c** $x = -0{\cdot}3$ **d** $y = 7$

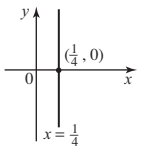
**2 a**



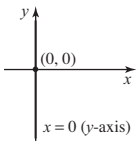

**b**



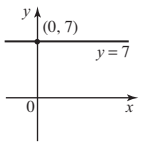

**c**



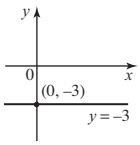

**d**



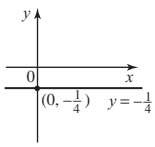

**3 a**



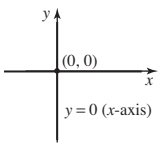

**b**




**c**




**d**




**4 a** $y = 3$ **b** $x = 6$

**c** $y = 4$ **d** $x = -4$

## Exercise 10E

**1 a** 2 **b** 3 **c** $\frac{1}{2}$ **d** −1 **e** −2 **f** −3

**2 a** $-\frac{2}{3}$ **b** $\frac{2}{3}$ **c** $-\frac{1}{2}$

**3 a** 2 **b** 2 **c** 2 **d** −1 **e** −1

**f** $-\frac{3}{2}$ **g** 2 **h** −3 **i** $\frac{5}{3}$ **j** 0

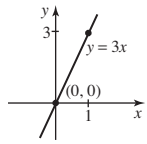
**k** undefined **l** 0

**4 a** $\frac{1}{16}$ **b** 16 metres

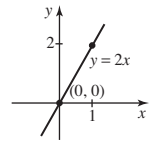
**5 a** $\frac{1}{25}$ **b** $\frac{1}{20}$ **c** $\frac{2}{45}$

## Exercise 10F

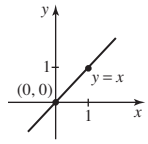
**1** **a**



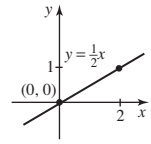

**b**



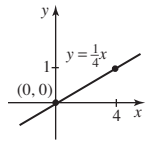

**c**



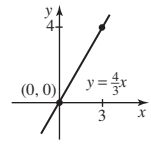

**d**



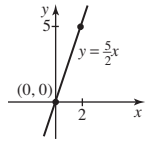

**e**



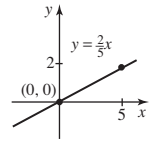

**f**




**g**

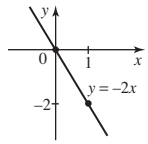



**h**

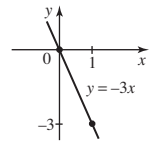



**i** The positive number in front of $x$, called the *coefficient* of $x$, specifies the *gradient*.

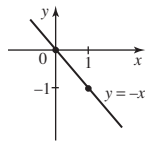
**2** **a**



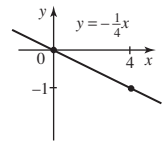

**b**



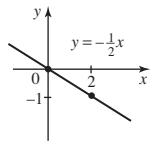

**c**



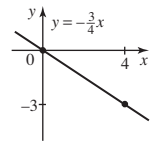

**d**



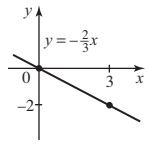

**e**



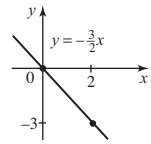

**f**




**g**

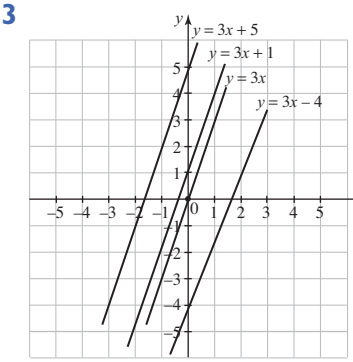



**h**




**i** The negative number in front of $x$, called the *coefficient* of $x$, specifies the *gradient*.

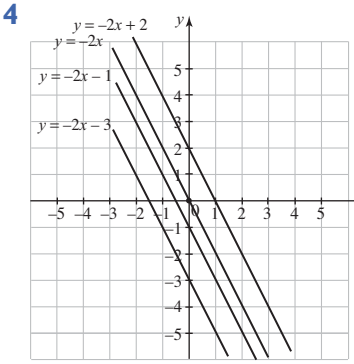
**3**




**b** When a number is added or subtracted the graph is translated up or down.

**c** Gradient is 3 for all graphs, $y$-intercepts are (0, 0), (0, 1), (0, –4), (0, 5).

**4**




**b** The graph is shifted up or down.

**c** Gradient is –2 for all lines, $y$-intercepts are (0, 0), (0, 2), (0, –1), (0, –3).

**5** **a** $y = 3x$ **b** $y = 2x$ **c** $y = \frac{2}{3}x$

**d** $y = -3x$ **e** $y = -\frac{5}{2}x$ **f** $y = -\frac{1}{3}$

**6** **a** $y = 2x + 1$ **b** $y = 3x - 1$

**c** $y = \frac{3}{4}x + \frac{1}{2}$ **d** $y = 3x + 2$

**e** $y = -2x - 1$ **f** $y = -\frac{5}{2}x + 2$

**g** $y = \frac{1}{2}x - 2$ **h** $y = -\frac{1}{3}x + 1$

**i** $y = 2$

**7** When the equation of a line appears as $y = mx + c$, then $m$ (the coefficient of $x$) specifies the gradient and $c$ gives the $y$-intercept.

**8** **a** $m = 2, c = -7$ **b** $m = 5, c = 9$

**c** $m = 1, c = -1$ **d** $m = 1, c = 0$

**e** $m = -3, c = -5$ **f** $m = -2, c = 5$

**g** $m = -8, c = 2$ **h** $m = 3, c = 5$

**i** $m = -1, c = 0$ **j** $m = -1, c = 8$

**k** $m = \frac{2}{3}, c = 2$ **l** $m = -\frac{1}{2}, c = 1$

**m** $m = -\frac{1}{4}, c = 5$ **n** $m = \frac{3}{2}, c = -2$

**o** $m = \frac{1}{4}, c = 1$ **p** $m = \frac{1}{2}, c = -4$

**q** $m = \frac{1}{3}, c = -1$ **r** $m = -\frac{2}{3}, c = 6$

**s** $m = -\frac{3}{4}, c = 3$ **t** $m = -\frac{2}{5}, c = -2$

**u** $m = -\frac{3}{4}, c = \frac{1}{2}$

**9** **a** $y = 3x - 2$ **b** $y = -x + 5$

**c** $y = 7x$ **d** $y = -6x - 1$

**e** $y = ax + b$ **f** $y = ax - b$

**g** $y = -ax + b$ **h** $y = -ax - b$

**10** **a** $y - 2x - 3 = 0$
$y = 2x + 3$
$m = 2, c = 3$

**b** $y - x + 1 = 0$
$y = x - 1$
$m = 1, c = -1$

**c** $y + 2x - 1 = 0$
$y = -2x + 1$
$m = -2, c = 1$

**d** $2y + x = 6$
$y = 3 - \frac{x}{2}$
$m = -\frac{1}{2}, c = 3$

**e** $2y - x = 4$
$y = 2 + \frac{x}{2}$
$m = \frac{1}{2}, c = 2$

**f** $3y + 6x - 12 = 0$
$y = 4 - 2x$
$m = -2, c = 4$

**g** $3x + y - 4 = 0$
$y = 4 - 3x$
$m = -3, c = 4$

**h** $-2x + y + 5 = 0$
$y = 2x - 5$
$m = 2, c = -5$

**i** $4x + 2y - 7 = 0$
$y = \frac{7}{2} - 2x$
$m = -2, c = \frac{7}{2}$

**j** $4x - 2y + 3 = 0$
$y = 2x + \frac{3}{2}$
$m = 2, c = \frac{3}{2}$

**k** $2x + 5y = 10$
$y = 2 - \frac{2}{5}x$
$m = -\frac{2}{5}, c = 2$

**l** $9x - 3y - 2 = 0$
$y = 3x - \frac{2}{3}$
$m = 3, c = -\frac{2}{3}$

**m** $2y - 6x + 1 = 0$
$y = 3x - \frac{1}{2}$
$m = 3, c = -\frac{1}{2}$

**n** $5x - 2y - 1 = 0$
$y = \frac{5}{2}x - \frac{1}{2}$
$m = \frac{5}{2}, c = -\frac{1}{2}$

**o** $3x - 4y - 2 = 0$
$y = \frac{3}{4}x - \frac{1}{2}$
$m = \frac{3}{4}, c = -\frac{1}{2}$

## Exercise 10G

**1** **a** $y = x + 4$ **b** $y = 2x - 2$ **c** $y = 3x + 6$

**d** $y = -x + 1$ **e** $y = -2x - 2$ **f** $y = -\frac{1}{2}x - 1$

**2** **a** $y = 2x - 1$ **b** $y = x + 7$

**c** $y = 3x - 2$ **d** $y = 4x - 5$

**e** $y = 5x - 1$ **f** $y = -x + 8$

**g** $y = -2x + 4$ **h** $y = -4x + 1$

**i** $y = -0{\cdot}5x + 1$ **j** $y = -\frac{3}{2}x + \frac{1}{2}$

**3** **a** $y = 2x + 2$ **b** $y = x + 2$

**c** $y = 4x + 2$ **d** $y = -x + 3$

**e** $y = -3x + 7$ **f** $y = -5x - 2$

**4** **a** $y = x + 4$ **b** $y = 2x + 1$

**c** $y = \frac{1}{2}x + 3$ **d** $y = \frac{3}{4}x + 2$

**e** $y = -x + 2$ **f** $y = -2x + 5$

**g** $y = -x - 1$ **h** $y = -3x - 2$

**i** $y = 3x - 2$ **j** $y = -\frac{1}{2}x - 1$

**k** $y = 6x + a$ **l** $y = 4x - b$

**5** **a** $y = 2x + 6$ **b** $y = x + 10$

**c** $y = 3x - 6$ **d** $y = x + 4$

**e** $y = 7x - 6$ **f** $y = -x + 6$

**g** $y = 2x + 5$ **h** $y = -2x + 3$

**i** $y = -4x - 1$ **j** $y = -2x - 5$

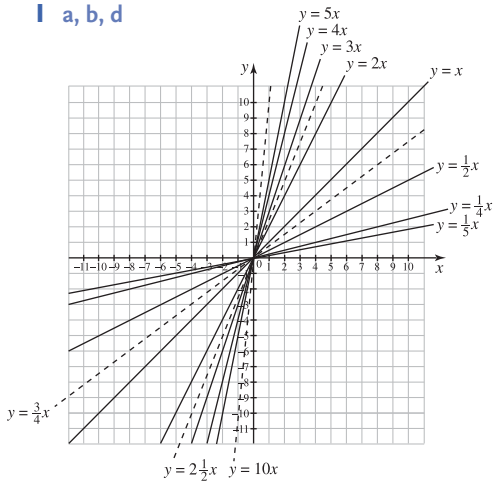
**k** $y = -x + 2$ **l** $y = 0{\cdot}4x + 0{\cdot}2$

**m** $y = -x + 0{\cdot}5$ **n** $y = -x + 1$

**o** $y = -x + (a + b)$

## Exercise 10H

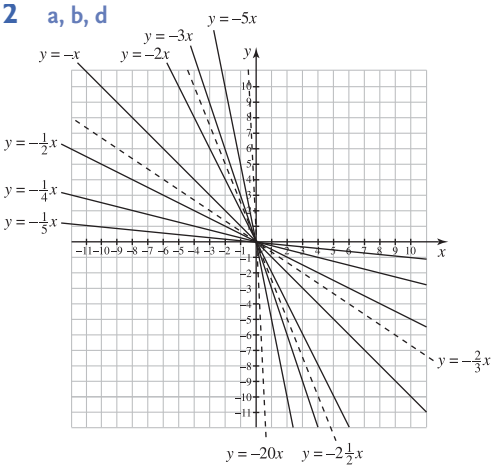
**1** a, b, d




c (0, 0)

e a straight line passing through the origin (0, 0) with a positive gradient

**2** a, b, d



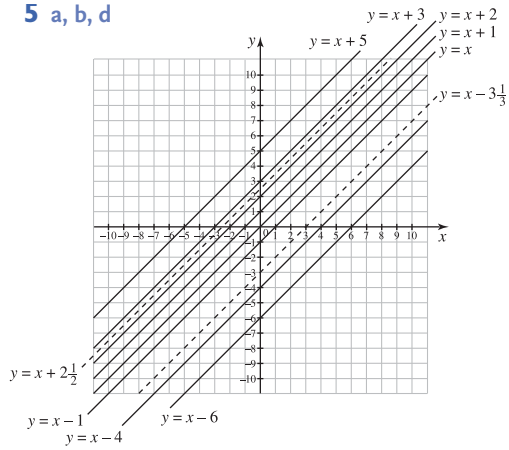

c (0, 0)

e a straight line passing through the origin (0, 0) with a negative gradient

**3** The graph of $x = 0$ looks like a vertical line along the $y$-axis.

**4** The graph of $y = 0$ looks like a horizontal line along the $x$-axis.

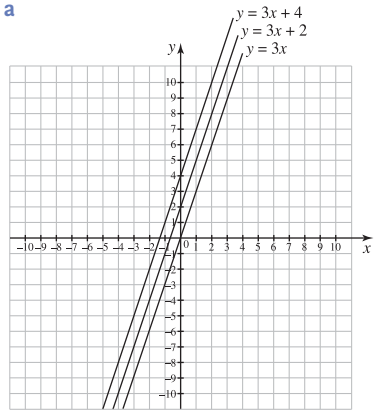
**5** a, b, d




c all lines are parallel

e $y = x \pm c$ will be parallel to $y = x$. For $y = x + c$, the line moves up $c$ units, and for $y = x - c$ it moves down $c$ units.

**6** a



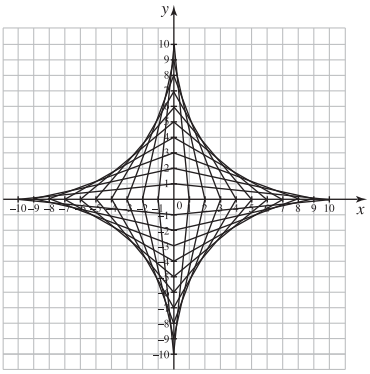

b $y = 3x$ becomes $y = 3x + 2$ by moving up 2 units (translation).
$y = 3x$ becomes $y = 3x + 4$ by moving up 4 units.
$y = 3x$ becomes $y = 3x - 4$ by moving down 4 units.

**7** a $m$ affects the slope or gradient of the line.

b $c$ indicates the position the line crosses the $y$-axis.

**8** a E b D c F d A e C f B

**9**



**a** $y = -10x + 10$

**b** $y = -\frac{9}{2}x + 9$

**c** $y = -\frac{8}{2}x + 8$

**e** reflection of $y = -10x + 10$ in $x$-axis is $y = 10x - 10$

reflection of $y = -\frac{9}{2}x + 9$ in $x$-axis is $y = \frac{9}{2}x - 9$

reflection of $y = -\frac{8}{3}x + 8$ in $x$-axis is $y = \frac{8}{3}x - 8$

etc.

**f** reflection of $y = -10x + 10$ in $y$-axis is $y = +10x + 10$

reflection of $y = -\frac{9}{2}x + 9$ in $y$-axis is $y = \frac{9}{2}x + 9$

reflection of $y = -\frac{8}{3}x + 8$ in $y$-axis is $y = \frac{8}{3}x + 8$

etc.

**g** image of $y = -10x + 10$ in third quadrant is $y = 10x + 10$

image of $y = -\frac{9}{2}x + 9$ in third quadrant is $y = \frac{9}{2}x + 9$

image of $y = -\frac{8}{3}x + 8$ in third quadrant is $y = \frac{8}{3}x + 8$ etc.

**10** Learner's own work.

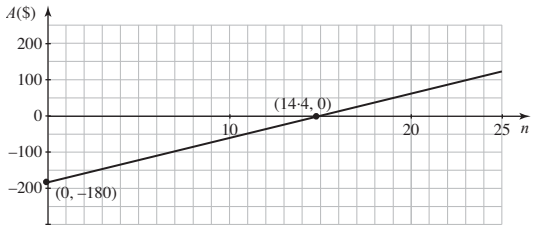
# Applications

## Electrician

**a**

| $n$ | $A$ |
|---|---|
| 0 | –180 |
| 2 | –155 |
| 4 | –130 |
| 6 | –105 |
| 8 | –80 |
| 10 | –55 |
| 12 | –30 |
| 14 | –5 |
| 16 | 20 |
| 18 | 45 |

**b, c**




**d** From the graph, the 'break-even' point is after 15 hours, to the nearest whole number of hours.

**e** $A = 12{\cdot}5n - 180$

**f** $A = 12{\cdot}5 \times 40 - 180 = \$320$ **g** 39 hours

## Cycling

**g**

| **Turns $T$** | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| **Distance $D$ (metres)** | 0 | 4·5 | 9 | 13·5 | 18 | 22·5 |

**h**


**i** 45 metres

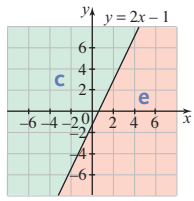
**j** $D = 4{\cdot}5T$

**k** $1000 = 4{\cdot}5T$, $\therefore$ $T = 223$ full turns

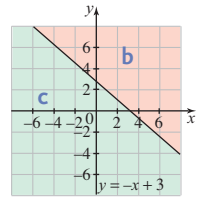
**l** $9444{\cdot}4 \approx 9444$ full turns

## Enrichment

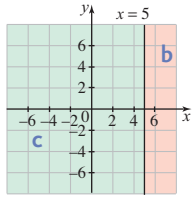
**1**



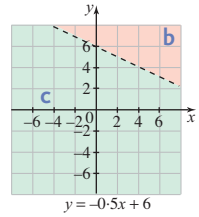

**2**




**3**




**4**




**5** **a** $y \geq x$ **b** $y < x$ **c** $y \leq 0{\cdot}5x$

**d** $y < -x$ **e** $y < -x + 1$ **f** $y > 2x - 2$

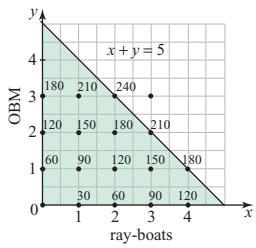
**g** $y < x + 2$ **h** $y \geq x - 1$ **i** $x > 7$

**j** $x \leq -1$ **k** $y > 5$ **l** $y \leq -2$

**6** **b** Isosceles triangle **c** 25 square units

**7** **b** Trapezium **d** 22·5 square units

**8**




**d**

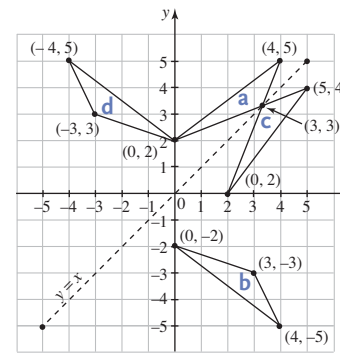
| | |
|---|---|
| (0, 1) = \$60 | (2, 1) = \$120 |
| (0, 2) = \$120 | (2, 2) = \$180 |
| (0, 3) = \$180 | (2, 3) = \$240 |
| (1, 0) = \$30 | (3, 0) = \$90 |
| (1, 1) = \$90 | (3, 1) = \$150 |
| (1, 2) = \$150 | (3, 2) = \$210 |
| (1, 3) = \$210 | (4, 0) = \$120 |
| (2, 0) = \$60 | (4, 1) = \$180 |

**e** 3 OBM and 2 ray-boats

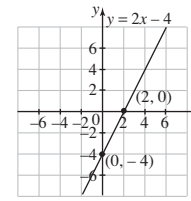
**f** \$240 **g** \$120

## Revision/Assessment

**1**

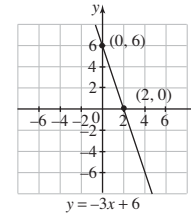



**2** **a**



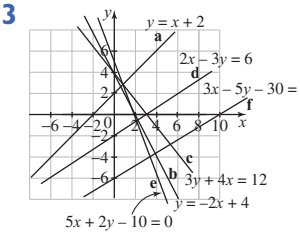

*x*-intercept is 2
*y*-intercept is –4

**b**

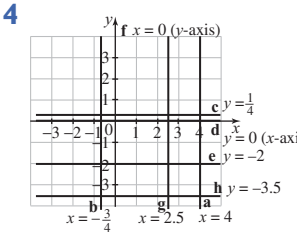



*x*-intercept is 2
*y*-intercept is 6

**3**

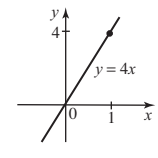



**4**

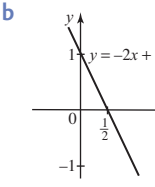


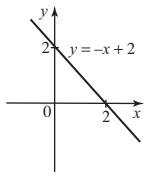

**5** **a** 3 **b** 3 **c** –2

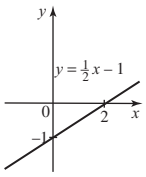
**6** **a**

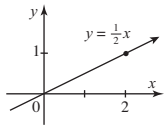


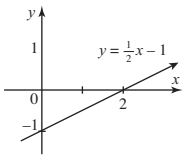

**b**

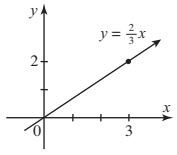


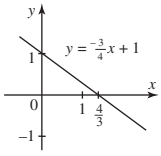
c 

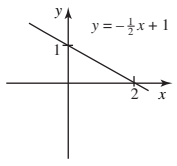

d 

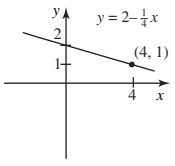

e 

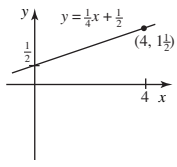

f 

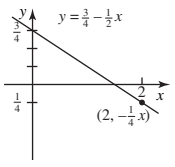

g 

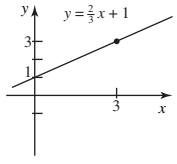

h 

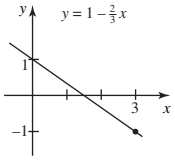

i 

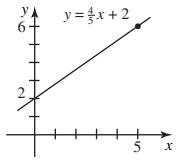

j 

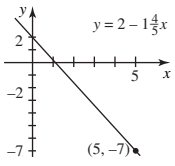

k 


l 


m 


n 


o 


p 


**7** a $y = -x + 3$ b $y = 2x - 4$
c $y = 2x + 6$ d $y = 3x + 2$
e $y = -3x + 3$ f $y = -0{\cdot}5x + 2$

**8** a $y = 2x + 4$ b $y = 3x + 3$
c $y = 2x + 3$ d $y = -0{\cdot}5x + 2$

**9** a $y = 2x + 4$ b $y = x + 4$ c $y = -3x + 5$

**10** a $y = 2x + 4$ b $y = 3x - 6$
c $y = -5x + 26$ d $y = -2x + 4$

**11** a $V = 200t + 3000$ b 3000 litres
c 200 litres per minute d 12·5 min

# Chapter 11

## Exercise 11A

**1** a {A, B, C, D, E}
b {Taurus, Aries, Pisces, Aquarius, Capricorn, Sagittarius, Scorpio, Libra, Virgo, Leo, Cancer, Gemini}
c {1, 2, 3, 4, 5, 6}
d {hearts, diamonds, clubs, spades}

**2** {(red, 1), (red, 2), (red, 3), (red, 4), (red, 5), (blue, 1), (blue, 2), (blue, 3), (blue, 4), (blue, 5), (green, 1), (green, 2), (green, 3), (green, 4), (green, 5), (yellow, 1), (yellow, 2), (yellow, 3), (yellow, 4), (yellow, 5)}

**3** a

| Second child \ First child | B | G |
|---|---|---|
| B | | |
| G | | |

b

| Second card \ First card | ♣ | ♠ | ♦ | ♥ |
|---|---|---|---|---|
| ♣ | | | | |
| ♠ | | | | |
| ♦ | | | | |
| ♥ | | | | |

**4** {(red, red, red), (red, red, black), (red, black, red), (red, black, black), (black, red, red), (black, red, black), (black, black, red), (black, black, black)}

**5** a $\frac{7}{26}$ b $\frac{9}{26}$ c $\frac{8}{26}$

**6** a $\frac{1}{4}$ b $\frac{1}{2}$ c $\frac{1}{13}$

**7** $\frac{1}{12}$

**8** a $\frac{1}{2}$ b $\frac{1}{2}$ c $\frac{2}{3}$

**9** $\frac{1}{5}$

**10** a $\frac{1}{4}$ b $\frac{1}{2}$

**11** a

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| H | | | | | | |
| T | | | | | | |

b 12 c Yes

d i $\frac{1}{12}$ ii $\frac{1}{4}$ iii $\frac{1}{4}$
iv $\frac{1}{6}$ v $\frac{5}{12}$ vi $\frac{1}{3}$

## Exercise 11B

**1** A, C, D

**2** $\frac{5}{6}$

**3** $\frac{247}{250}$

**4** $\frac{10}{13}$

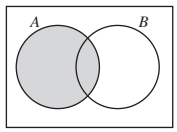
**5** $\frac{5}{12}$

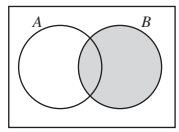
**6** $\frac{3}{4}$

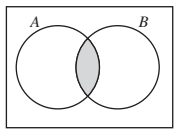
**7** a $\frac{9}{64}$ b $\frac{37}{64}$ c $\frac{27}{64}$ d $\frac{63}{64}$

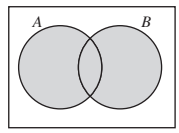
**8** a $\frac{12}{25}+\frac{13}{25}=1$ b $\frac{35}{56}+\frac{21}{56}=1$

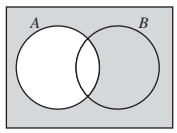
## Exercise 11C

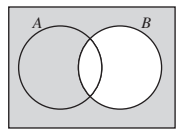
**1** a 

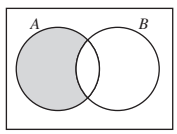

b 

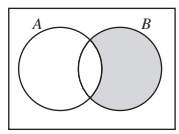

c 

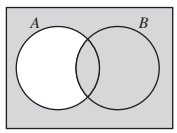

d 

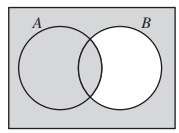

e 

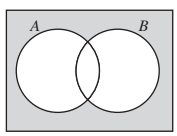

f 

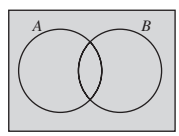

g 


h 


i 


j 

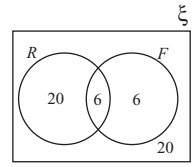

k 


l 


**2** a $\frac{1}{8}$ b $\frac{37}{40}$ c $\frac{1}{2}$ d $\frac{3}{10}$ e $\frac{3}{40}$ f $\frac{7}{8}$

**3** a $\frac{2}{25}$ b $\frac{2}{25}$ c $\frac{8}{25}$ d $\frac{47}{50}$

e $\frac{31}{50}$ f $\frac{2}{5}$ g $\frac{3}{50}$

**4** a 


b i $\frac{1}{2}$ ii $\frac{3}{13}$ iii $\frac{5}{13}$

iv $\frac{3}{26}$ v $\frac{3}{26}$ vi $\frac{5}{13}$

**5** a 0·6 b 0·8 c 0·5

**6** a 0·2 b 0·2 c 0·3

**7** a i 0·1 ii 0·9 iii 0·4

b i 0·2 ii 0·8 iii 0·3

## Exercise 11D

**1** B, C, E

**2** a i $\frac{3}{5}$ ii $\frac{1}{2}$ iii $\frac{1}{5}$ iv $\frac{9}{10}$

v $A$ and $B$ are not mutually exclusive as $\Pr(A \cap B) \neq 0$

b i $\frac{1}{4}$ ii $\frac{2}{5}$ iii 0 iv $\frac{13}{20}$

v $A$ and $B$ are mutually exclusive as $\Pr(A \cap B) = 0$

c i $\frac{7}{10}$ ii $\frac{3}{10}$ iii $\frac{3}{10}$ iv $\frac{7}{10}$

v $A$ and $B$ are not mutually exclusive as $\Pr(A \cap B) \neq 0$

d i $\frac{3}{8}$ ii $\frac{7}{12}$ iii $\frac{5}{24}$ iv $\frac{3}{4}$

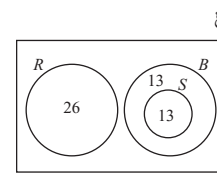
v $A$ and $B$ are not mutually exclusive as $\Pr(A \cap B) \neq 0$

e i $\frac{1}{5}$ ii 1 iii $\frac{1}{5}$ iv 1

v $A$ and $B$ are not mutually exclusive as $\Pr(A \cap B) \neq 0$

f i $\frac{1}{4}$ ii $\frac{11}{20}$ iii $\frac{1}{4}$ iv $\frac{11}{20}$

v $A$ and $B$ are not mutually exclusive as $\Pr(A \cap B) \neq 0$

**3** 


a i $\frac{3}{13}$ ii $\frac{1}{2}$ iii $\frac{1}{4}$ iv 0

v $\frac{3}{26}$ vi $\frac{1}{2}$

b Red and black cards are mutually exclusive, as are red cards and spades.

**4** a i $\frac{3}{5}$ ii $\frac{2}{5}$ iii $\frac{3}{5}$ iv $\frac{1}{5}$

b i $\frac{1}{2}$ ii $\frac{1}{10}$ iii $\frac{1}{10}$ iv 0

v $\frac{2}{5}$ vi 0

c i $\frac{9}{10}$ ii $\frac{4}{5}$ iii $\frac{3}{5}$ iv $\frac{3}{5}$

v $\frac{4}{5}$ vi $\frac{9}{10}$

d Mutually exclusive events in part b are i being male and being left-handed, ii having only one sibling and being blonde, as there are no people who have these pairs of characteristics.

**5** a (Sports, Art)
(Home Economics, Art)
(Art, Religious Studies)
(Art, Agriculture)
(Religious Studies, Agriculture)

b Not mutually exclusive

c Not mutually exclusive

## Exercise 11E

**1** $\frac{1}{36}$ **2** $\frac{1}{32}$

**3** a $\frac{1}{3}$ b $\frac{1}{2}$ c $\frac{1}{12}$ d $\frac{1}{24}$ e $\frac{1}{8}$

**4** a $\frac{4}{9}$ b $\frac{1}{3}$ c $\frac{1}{2}$ d $\frac{4}{27}$ e $\frac{1}{9}$ f $\frac{2}{81}$

**5** a i $\frac{15}{77}$ ii $\frac{10}{231}$ iii $\frac{10}{77}$ iv $\frac{50}{231}$

b 4

## Exercise 11F

**1** a 1:1 b 4:1 c 11:2 d 1:3
e 3:5 f 7:3 g 11:4 h 5:7

**2** a $\frac{1}{4}$ b $\frac{1}{8}$ c $\frac{1}{4}$ d $\frac{4}{15}$
e $\frac{1}{34}$ f $\frac{4}{13}$ g $\frac{7}{22}$ h $\frac{8}{21}$

**3** a $\frac{18}{37}$ b $\frac{18}{37}$ c Less

**4** \$80

**5** a \$220 b \$120 c \$55 d \$50

**6** a 8 b 10 c 6·5 d 2·75

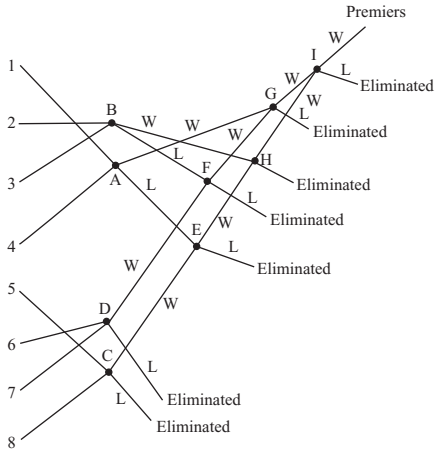
**7** 9:2, 5:1, 5:2, 10:1, 14:1, 6:1, 14:1, 14:1, 4:1

**8** \$292·50

## Applications

### Final eight

a




b Teams 1–4:
$\Pr(\text{win Grand Final}) = 0{\cdot}5^3 + 0{\cdot}5^4 = 0{\cdot}1875$

Teams 5–8:
$\Pr(\text{win Grand Final}) = 0{\cdot}5^4 = 0{\cdot}0625$

c Final four:
Teams 1 and 2:
$\Pr(\text{win Grand Final}) = 0{\cdot}5^2 + 0{\cdot}5^3 = 0{\cdot}375$

Teams 3 and 4:
$\Pr(\text{win Grand Final}) = 0{\cdot}5^3 = 0{\cdot}125$

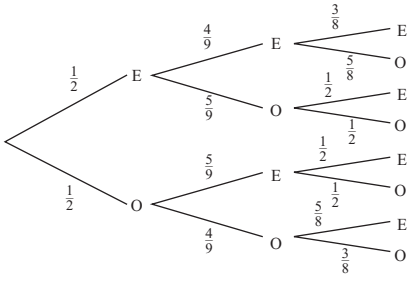
Therefore being in the final four increases the chances of the top two teams winning but decreases the chances of the 3rd and 4th team winning, while the chances of the 5th to 8th teams are obviously increased.

## Enrichment

**1** a false b false c false d true
e false f false

**2** a 0·25

b



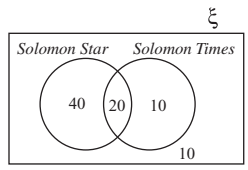

c $\frac{1}{6}$

**d** When you replace the balls you have a better chance of winning, as when they are not replaced the probability of choosing another ball the same decreases each time.

**3** **a** i $\frac{1}{26}$ ii $\frac{1}{10}$ iii $\frac{75}{676}$ iv $\frac{1}{100}$ v $\frac{1}{2}$

**b** i $\frac{1}{26}$ ii $\frac{1}{10}$ iii $\frac{11}{12}$ iv $\frac{1}{5}$

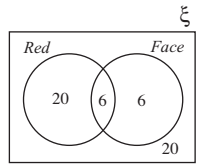
**4** **a**




**b** i $\frac{3}{4}$ ii $\frac{3}{8}$ iii $\frac{1}{8}$

iv $\frac{1}{4}$ v $\frac{1}{2}$ vi $\frac{1}{8}$

**5** **a**




**b** i $\frac{1}{2}$ ii $\frac{3}{13}$ iii $\frac{5}{13}$

iv $\frac{3}{26}$ v $\frac{3}{26}$ vi $\frac{5}{13}$

**6** **a** $A = \{1, 2, 4, 10, 13, 18, 20\}$

**b** $B = \{2, 3, 7, 8, 9, 13, 19\}$

**c** $C = \{3, 4, 5, 8, 13, 17\}$

**d** $A' = \{3, 5, 6, 7, 8, 9, 11, 12, 14, 15, 16, 17, 19\}$

**e** $A \cap B = \{2, 13\}$

**f** $B \cap C = \{3, 8, 13\}$

**g** $A \cap B \cap C = \{13\}$

**h** $A \cap B' = \{1, 4, 10, 18, 20\}$

**i** $B' \cup C' = \{1, 2, 4, 5, 6, 7, 9, 10, 11, 12, 14, 15, 16, 17, 18, 19, 20\}$

**j** $(A \cup B \cup C)' = \{6, 11, 12, 14, 15, 16\}$

**k** $A \cap B \cap C' = \{2\}$

**l** $A' \cap B' \cap C' = \{6, 11, 12, 14, 15, 16\}$

**7** **a** $\frac{15}{43}$ **b** $\frac{8}{43}$ **c** $\frac{2}{43}$ **d** $\frac{7}{43}$

**e** $\frac{28}{43}$ **f** $\frac{27}{43}$ **g** $\frac{31}{43}$ **h** $\frac{32}{43}$

**i** $\frac{16}{43}$ **j** $\frac{14}{43}$ **k** $\frac{27}{43}$ **l** $\frac{16}{43}$

## Revision/Assessment

**1** **a** $\frac{1}{36}$ **b** $\frac{1}{6}$ **c** $\frac{5}{18}$ **d** $\frac{4}{9}$

**2** **a** $\frac{1}{25}$ **b** $\frac{1}{5}$ **c** $\frac{4}{5}$

**3** **a** $\frac{1}{144}$ **b** $\frac{1}{12}$ **c** $\frac{11}{72}$ **d** $\frac{11}{12}$

**4** **a** $\frac{1}{4}$ **b** $\frac{1}{16}$ **c** $\frac{3}{8}$

**d** $\frac{1}{2}$ **e** $\frac{1}{16}$ **f** $\frac{15}{16}$

**5** **a** $\frac{1}{8}$ **b** $\frac{1}{8}$ **c** $\frac{7}{8}$

**6** **a** $\frac{1}{8}$ **b** $\frac{1}{4}$ **c** $\frac{3}{4}$

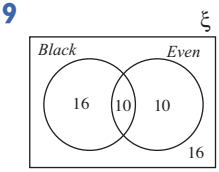
**7** **a** $\frac{5}{12}$ **b** $\frac{1}{3}$ **c** $\frac{1}{4}$

**d** $\frac{7}{12}$ **e** $\frac{2}{3}$ **f** $\frac{3}{4}$

**8** **a** $\Pr(A) + \Pr(A') = \frac{2}{3} + \frac{1}{3} = 1$

**b** $\Pr(A) + \Pr(A') = \frac{3}{8} + \frac{5}{8} = 1$

**9**




**a** $\frac{1}{2}$ **b** $\frac{5}{13}$ **c** $\frac{5}{26}$ **d** $\frac{9}{13}$

**10** **a** 0·3 **b** 0·1 **c** 0·4

**11** **a** i $\frac{2}{7}$ ii $\frac{11}{21}$ iii $\frac{10}{21}$

iv $\frac{1}{3}$ v $\frac{5}{21}$ vi $\frac{1}{7}$

**b** i $\frac{10}{21}$ ii $\frac{4}{7}$ iii $\frac{17}{21}$ iv $\frac{3}{7}$

v $\frac{8}{21}$ vi $\frac{10}{21}$

**c** Even number and a prime number
Number divisible by 5 and 7

**12** **a** 0·512 **b** 0·008 **c** 0·096 **d** 0·488

**13** $\frac{1}{1\,073\,741\,824}$

**14** **a** $\frac{1}{15\,625}$ **b** $\frac{6144}{15\,625}$

**15** a 1:1 b 3:2 c 1:2 d 1:3
e 7:5 f 9:1 g 11:4 h 8:7

**16** a $\frac{1}{3}$ b $\frac{1}{6}$ c $\frac{2}{9}$ d $\frac{4}{19}$
e $\frac{1}{31}$ f $\frac{2}{11}$ g $\frac{7}{25}$ h $\frac{8}{33}$

# Chapter 12

## Exercise 12A

**1** a 25 b 9 c 216 d 16
e 1000 f 6 g 1 h 0
i 81 j 64 k 100 l 1 000 000
m 64 n 81 o 1 000 000

**2** a 1 b 9 c 16 d –5
e 100 f 1 g 81 h –216
i –2 j 4 k –8 l 16

**3** a –3 b –9 c –125 d –10 000
e –16 f 3 g –625 h 1
i 8 j –16 k 32 l –64

**4** a $\frac{4}{25}$ b $\frac{1}{27}$ c $\frac{16}{625}$ d $\frac{27}{64}$
e $-\frac{1}{125}$ f $\frac{4}{9}$ g $\frac{1}{100}$ h $\frac{25}{49}$
i $\frac{16}{81}$ j $\frac{27}{64}$ k $\frac{1}{32}$ l $-\frac{25}{36}$

**5** a 64 b 9 c 5
d 25 e 0 f 1
g 16 h 7776 i 1000

**6** a $x^3$ b $y^6$ c $a^4$
d $x^2y^3$ e $a^3b^2$ f $x^2y^2z^2$
g $x^3y^3$ h $a^4b^2$ i $a^2b^2c^2$
j $a^2b^2c^2$ k $-m^2np^2$ l $a^2b^2c^2$

**7** a $24a^2b^3c^2$ b $60x^2y^4z^2$ c $56p^3q^3r^2$
d $-24x^2y^2z^2$ e $24a^2b^2c^2$ f $60r^3s^3t^3$
g $-90x^2y^2z^2$ h $-100a^2b^3c$ i $30r^3s^4t^3$

**8** a $\frac{x^2}{y^3}$ b $\frac{a^3}{c^3}$ c $\frac{12s^2}{5t^3}$
d $\frac{6p^2r^2}{5q^2s^2}$ e $\frac{15x^2z^2}{28t^2y^2}$ f $\frac{6a^2c^3}{-35)b^2d^2}$
g $\frac{xz^2}{y^2}$ h $\frac{c}{bd}$ i $\frac{3m^2qrt}{2p^2w^2}$

## Learning task 12B

**1** a 243 b $x^6$
c $b^x \times b^y = b^{x+y}$ d **Add** the powers.

**2** a 25 b $b^4$
c $b^x \div b^y = b^{x-y}$ d **Subtract** the powers.

**3** a 243 b 125
c 128 d 216
e 100 000 f $^-343$
g 256 h $^-729$
i 64 j 729
k 625 l 1 000 000

**4** a $x^9$ b $y^4$ c $m^6$ d $x^8$
e $a^4$ f $b^6$ g $z^5$ h $b^{10}$
i $x^9$ j $n^7$ k $n^5$ l $p^{11}$

**5** a $5^1 = 5$ b $3^5 = 243$
c $6^2 = 36$ d $8^1 = 8$
e $(^-10)^3 = ^-1000$ f $(^-4)^1 = ^-4$
g $(^-9)^3 = ^-729$ h $(^-5)^2 = 25$

**6** a $x^3$ b $y^3$ c $z^3$ d $b^1$
e $x^1$ f $p^2$ g $q^2$ h $z^6$

## Exercise 12C

**1** a $21x^6$ b $20x^7$ c $40x^7$
d $8x^7$ e $-21y^3$ f $-40z^6$
g $7a^4$ h $-8b^5$ i $-10x^8$
j $15b^4$ k $-56y^9$ l $2x^7$

**2** a $10x^5y^5$ b $24x^3y^7$ c $20x^4y^4$
d $18a^5b^5$ e $-5m^4n^7$ f $-9p^3q^7$
g $80x^7y^8$ h $24x^7y^8$ i $8x^4y^7$
j $27x^5y^3$ k $-24x^4y^3z^3$ l $-15a^6b^4c^4$
m $-10m^4n^4p^2$ n $48m^4n^3p^4$

**3** a $4x^2y$ b $2x^2y^2$ c $-5x^3y^3$
d $-5x^4y^4$ e $\frac{1}{2}r^2s$ f $\frac{1}{3}m^2n^2$
g $-\frac{1}{4}p^2q^2$ h $-\frac{2}{3}ab$ i $\frac{4}{5}s^3t$
j $-\frac{4}{5}x^2y^2$ k $-\frac{4}{3}xz^3$ l $-\frac{6}{5}x^2$
m $\frac{3x^2yz^2}{2}$ n $\frac{5abc^3}{3}$ o $\frac{4m^3n^3p^3}{3}$

**4** a $4xy^3$ b $4xy^3$ c $6x^2y^2$
d $12a^6b^7$ e $12\frac{x^6}{y^2}$ f $24x^2y^3$
g $^-12x^4y^3$ h $\frac{35n^5}{4}$ i $12x^2y$
j 2 k $x^4y^2$ l $4mn^4p$

## Exercise 12D

**1** a 1 b 1 c 1 d 1 e 1 f 1
g 1 h 1 i 1 j 1 k 1 l 1

**m** 1 **n** 8 **o** 7 **p** 9 **q** $-1$ **r** $-8$
**s** 1 **t** 3

**2** **a** 1 **b** 1 **c** 10 **d** 1 **e** 1 **f** $-2$
**g** 6 **h** $y$

**3** **a** 64 **b** 6561 **c** 10 000
**d** 64 **e** $5^3 = 125$ **f** 81
**g** 1 000 000 **h** $-64$

**4** **a** $x^8y^6$ **b** $x^3y^9$ **c** $x^{20}y^5$
**d** $64a^{18}$ **e** $9x^2y^6$ **f** $16m^4n^8$
**g** $16x^4y^6$ **h** $25a^6b^8$ **i** $8m^{12}n^6p^9$
**j** $81a^{12}c^8$ **k** $-8x^3y^9z^{15}$ **l** $-1000n^3p^3$

**5** **a** $\frac{a^6}{c^4}$ **b** $\frac{y^9}{27z^{12}}$ **c** $\frac{8m^6}{p^{12}}$
**d** $\frac{81r^{16}s^8}{16t^{12}}$ **e** $\frac{4u^2v^4}{25w^6z^4}$ **f** $\frac{64x^6y^9}{-27t^{15}z^9}$
**g** $\frac{1000x^9y^6z^3}{u^6v^3}$ **h** $\frac{-27r^6s^3t^3}{u^6v^{12}w^3}$

**6** **a** $x^{16}$ **b** $a^{18}$ **c** $m^8n^6$
**d** $144r^8s^4$ **e** $72m^4n^6$ **f** $100x^6z^{10}$
**g** $x^{13}y^{14}$ **h** $a^{11}b^9$ **i** $x^{11}y^4$
**j** $1125x^{13}y^{14}$ **k** $128m^7n^8$ **l** $10\,000r^{10}s^{16}$

**7** **a** $x$ **b** $y$ **c** $m^2$
**d** $2p$ **e** $-2q^2$ **f** $27s^{14}$
**g** $x^3y^2$ **h** $81m^5n^2$ **i** 1024

**8** **a** $216x^6$ **b** $54p^3q$ **c** $40n^5$

## Exercise 12E

**1** **a** $\frac{1}{5}$ **b** $\frac{1}{10}$ **c** $\frac{1}{100}$ **d** $\frac{1}{1000}$
**e** $\frac{1}{27}$ **f** 1 **g** $-1$ **h** $-1$
**i** $\frac{1}{2}$ **j** $\frac{1}{36}$

**2** **a** 3 **b** 4 **c** 5 **d** 8
**e** 8 **f** 16 **g** 50 **h** 50 000

**3** **a** $1\frac{1}{3}$ **b** $\frac{5}{6}$ **c** $3\frac{1}{2}$ **d** $1\frac{1}{9}$
**e** 10 **f** 6 **g** $\frac{1}{5}$ **h** 100

**4** **a** $\frac{5}{y}$ **b** $\frac{10}{x}$ **c** $\frac{7}{x^2}$ **d** $\frac{9}{w^3}$
**e** $\frac{z}{10}$ **f** $\frac{p}{5}$ **g** $\frac{t}{16}$ **h** $\frac{z}{1000}$
**i** $\frac{1}{4p}$ **j** $\frac{1}{8p}$ **k** $\frac{1}{4z^3}$ **l** $\frac{1}{9t^4}$

**5** **a** $\frac{x}{y}$ **b** $\frac{x}{y^3}$ **c** $\frac{a^2}{b^4}$ **d** $\frac{p^5}{q^5}$
**e** $\frac{n}{m}$ **f** $\frac{r^2}{s^2}$ **g** $\frac{v^2}{u^6}$ **h** $\frac{c}{b^2}$
**i** $\frac{1}{uv}$ **j** $\frac{1}{x^2y}$ **k** $\frac{1}{m^2n^3}$ **l** $\frac{1}{p^4q^3}$

**6** **a** $3x$ **b** $5y$ **c** $10z^2$ **d** $8m^3$
**e** $3x$ **f** $7y$ **g** $81z$ **h** $4s$
**i** $\frac{1}{3x}$ **j** $\frac{1}{2r}$ **k** $\frac{1}{10x^2}$ **l** $\frac{1}{25z^3}$

**7** **a** $\frac{x}{y}$ **b** $\frac{q^2}{p}$ **c** $\frac{s}{r^2}$ **d** $\left(\frac{n}{m}\right)^2$
**e** $\frac{y^2z^2}{x}$ **f** $\frac{b^3}{ac^2}$ **g** $\frac{qs}{p^2r^2}$ **h** $\frac{v^3w^3}{tu^2}$

**8** **a** $x^6y$ **b** $\frac{p^2q}{2}$ **c** $\frac{4m^4}{5n^5}$ **d** $r$
**e** $\frac{16}{3r^7s^2}$ **f** $\left(\frac{w}{v}\right)^3$

## Exercise 12F

**1** **a** 2 **b** 4 **c** 7 **d** 10 **e** 3 **f** 4
**g** 5 **h** 10 **i** 3 **j** 4 **k** 2 **l** 10

**2** **a** $\frac{1}{3}$ **b** $\frac{1}{5}$ **c** $\frac{1}{9}$ **d** $\frac{1}{6}$
**e** $\frac{1}{2}$ **f** $\frac{1}{3}$ **g** $\frac{1}{4}$ **h** $\frac{1}{6}$
**i** $\frac{1}{2}$ **j** $\frac{1}{3}$ **k** $\frac{1}{2}$ **l** $\frac{1}{10}$

**3** **a** $3x^{\frac{1}{2}}$ **b** $4x$ **c** $7y^2$ **d** $5z^3$
**e** $2t^{\frac{1}{3}}$ **f** $3x$ **g** $4z^2$ **h** $5x^3$
**i** $2p^{\frac{1}{4}}$ **j** $3q$ **k** $2s$ **l** $3y^2$

**4** **a** $\frac{1}{6x^{\frac{1}{2}}}$ **b** $\frac{y}{6}$ **c** $\frac{y^2}{11}$ **d** $\frac{2}{z^3}$
**e** $\frac{1}{2x^{\frac{1}{3}}}$ **f** $\frac{x}{3}$ **g** $\frac{z^2}{4}$ **h** $\frac{5}{p}$
**i** $\frac{1}{2x^{\frac{1}{4}}}$ **j** $\frac{3}{m}$ **k** $\frac{y}{2}$ **l** $\frac{3}{p}$

**5** **a** $\frac{5}{7t}$ **b** $\frac{2}{3s^2}$ **c** $\frac{6}{5m^3}$ **d** $\frac{11}{12q^{\frac{1}{2}}}$

e $\frac{5z}{3}$ f $\frac{10u^4}{9}$ g $\frac{3}{2u}$ h $\frac{4}{5w^3}$

i $\frac{1}{2u}$ j $\frac{x^2y}{3}$ k $\frac{1}{2xy^2}$ l $\frac{3n}{m^2}$

## Exercise 12G

**1** a $5{\cdot}0 \times 10^4$ b $1{\cdot}205 \times 10^5$
c $5{\cdot}8 \times 10^2$ d $5{\cdot}2 \times 10^7$
e $3{\cdot}6 \times 10^5$ f $2{\cdot}5 \times 10^2$
g $3{\cdot}752 \times 10^2$ h $1{\cdot}0246 \times 10^3$
i $2{\cdot}5 \times 10^8$ j $1{\cdot}0 \times 10^2$
k $1{\cdot}0 \times 10^3$ l $2{\cdot}0 \times 10^5$

**2** a 1500 b 4 050 000
c 362 100 d 7 205 000
e 590 000 000 f 450
g 20 h 9·8
i 1006 j 3125·63
k 2·25 l 9990

**3** a $5{\cdot}0 \times 10^{-3}$ b $1{\cdot}07 \times 10^{-3}$
c $2{\cdot}5 \times 10^{-1}$ d $3{\cdot}75 \times 10^{-4}$
e $1{\cdot}0 \times 10^{-4}$ f $6{\cdot}532 \times 10^{-4}$
g $1{\cdot}05 \times 10^{-1}$ h $7{\cdot}0 \times 10^{-1}$
i $9{\cdot}0 \times 10^{-1}$ j $2{\cdot}0 \times 10^{-10}$
k $5{\cdot}5 \times 10^{-9}$ l $6{\cdot}1 \times 10^{-9}$

**4** a 0·0043 b 0·000 06
c 0·35 d 0·0015
e 0·0106 f 0·625
g 0·000 009 99 h 0·0043
i 0·000 004 75 j 0·000 000 102 5
k 0·083 l 0·000 000 006 25

**5** a $2{\cdot}4 \times 10^4$ b $3 \times 10^1$
c $6{\cdot}0 \times 10^2$ d $3{\cdot}0 \times 10^2$
e $1{\cdot}2 \times 10^6$ f $2{\cdot}4 \times 10^{-5}$
g $2{\cdot}0 \times 10^1$ h $3{\cdot}0 \times 10^2$

**6** a i $8{\cdot}64 \times 10^4$ seconds per day
ii $3{\cdot}1536 \times 10^7$ seconds per year
iii $9{\cdot}4608 \times 10^{12}$ km
b 1·22 seconds
c $8\frac{1}{3}$ min or 8 min 20 s

## Applications

### Red blood cells

a $2{\cdot}24 \times 10^{13}$ cells
c $2{\cdot}35 \times 10^{12}$ cells

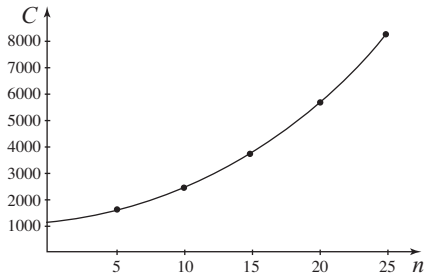
### Hospital costs

a $1139 for 1996

b

| | 1995 | 2000 | 2005 | 2010 | 2015 | 2020 |
|---|---|---|---|---|---|---|
| $n$ | 0 | 5 | 10 | 15 | 20 | 25 |
| $C$ | 1050 | 1579 | 2374 | 3570 | 5368 | 8071 |

c Rising exponential curve, $C$ intercept is 1050.

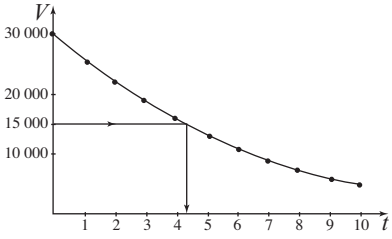



### Vehicle depreciation

a

| $t$ | $V$ | $t$ | $V$ |
|---|---|---|---|
| 0 | 30 000 | 6 | 11 314 |
| 1 | 25 500 | 7 | 9 617 |
| 2 | 21 675 | 8 | 8 174 |
| 3 | 18 424 | 9 | 6 948 |
| 4 | 15 660 | 10 | 5 906 |
| 5 | 13 311 | | |

b




c 4·2 years to halve d 4·3 (4·265 years)

e $V = 40\,000 \times 0{\cdot}08^t$

## Enrichment

**1** a 8 b 27 c $\frac{1}{125}$ d $\frac{1}{100\,000}$

e 4 f $\frac{1}{81}$ g $\frac{1}{16}$ h 625

i 8 j $\frac{1}{243}$ k $\frac{1}{64}$ l 9

m $6\frac{1}{4}$ n $\frac{1331}{512}$ o $\frac{8}{27}$ p $\frac{81}{16}$

**2** a $\frac{1}{2x^{\frac{1}{2}}}$ b $\frac{1}{5y^{\frac{3}{2}}}$ c $\frac{1}{9x^{\frac{4}{3}}y^2}$ d $\frac{25}{x^3y^8}$

e $\frac{a^{\frac{1}{2}}}{16b^{\frac{2}{3}}}$ f $x^0 = 1$ g $\frac{y^2}{16x^5}$

**3** a 110·25 b 148·877 c 2762·8
d 1194·32 e 2·25 f $^{-}$15·625
g 676·52 h $^{-}$14 539·3 i 191·10
j –3·583 k 548·76 l –2·358

An even power gives a positive result, an odd power gives a negative result.

**4** a 2 b $\frac{2}{5}$ c $\frac{4}{7}$ d $\frac{25}{196}$
e –2 f $\frac{16}{25}$ g $\frac{^{-}27}{8}$ h $\frac{16}{81}$

**5** a 2 b 2 c 3 d 2
e $\frac{3}{4}$ f $\frac{3}{2}$ g $\frac{2}{3}$ h $\frac{3}{2}$

**6** a 11·56 b –17·089 c 2·653
d 1·772 e –4 f $\frac{16}{25}$
g $\frac{-125}{8}$ h $\frac{16}{81}$ i $\frac{5}{3}$
j $\frac{2}{3}$ k $\frac{3}{2}$ l $\frac{2}{3}$

**7** $4{\cdot}068 \times 10^{13}$ km **8** $1{\cdot}0986 \times 10^{21}$ m$^3$
**9** 0·13 seconds **10** 2964 m ≈ 3·0 km
**11** 0·0089 m ≈ 8·9 mm

## Revision/Assessment

**1** a 81 b –1 c –64 d –121
e $-\frac{1}{64}$ f $\frac{4}{25}$ g $-\frac{1}{1000}$ h $\frac{81}{256}$

**2** a $30a^2b^3c^2$ b $84x^2y^4z^2$ c $108p^3q^3r^2$
d $-40x^2y^2z^2$ e $6a^2b^2c^2$ f $24r^3s^3t^3$

**3** a $30x^4y^5$ b $14x^3y^7$ c $15x^4y^4$
d $-6a^5b^5$ e $-12m^4n^7$ f $27p^3q^7$
g $3x^3y$ h $-2xy^2$ i $-3xy^2$
j $5x^5y$ k $-4xy^2$ l $5mn$

**4** a $12x^2y$ b $12a^4b^6$ c $15\frac{x^6}{y^2}$
d $-8x^2y^3$ e $20mn^3$ f $-xw$

**5** a 1 b 8 c 9
d 6 e 3 f –1
g 5 h $y$ i $64m^{12}n^3p^6$
j $16a^8c^4$ k $-64x^3y^6z^{12}$ l $-125n^3p^3$
m $\frac{9u^2v^4}{25w^4z^2}$ n $-\frac{8x^6y^{12}}{27t^{15}z^9}$
o $\frac{125x^9y^3z^3}{u^6v^3}$ p $\frac{-64r^6s^3t^3}{u^6v^9w^3}$

**6** a $\frac{m^2}{n}$ b $4m^2n^2$ c $\frac{25x^2}{y^2}$ d $63x$

**7** a $\frac{1}{4}$ b $\frac{1}{25}$ c $\frac{1}{10\,000}$
d $\frac{1}{4}$ e 4 f 27
g 25 h 32 i $\frac{5}{4}$
j $\frac{9}{4}$ k $\frac{64}{27}$ l $\frac{16}{81}$

**8** a $\frac{z^3y^4}{x}$ b $\frac{b^4}{ac^3}$ c $\frac{qs^2}{r^2p^3}$
d $\frac{v^2w^4}{t^2u}$ e $\frac{m}{n^8}$ f $\frac{1}{24s^{10}r^2}$

**9** a 9 b 4 c 2 d 3
e $\frac{1}{6}$ f $\frac{1}{2}$ g $\frac{1}{3}$ h $\frac{1}{2}$

**10** a $4t^{\frac{1}{2}}$ b $4x^2$ c $2z$ d $3x^2$
e $\frac{1}{8z}$ f $\frac{5}{x}$ g $\frac{x^2}{2}$ h $\frac{2}{y}$
i $\frac{1}{3u}$ j $\frac{x^4y^2}{9}$ k $2xy^2$ l $\frac{3n^2}{m^3}$

**11** a $5{\cdot}0 \times 10^5$ b $1{\cdot}2 \times 10^8$
c $8{\cdot}0 \times 10^4$ d $2{\cdot}2 \times 10^7$

**12** a 17 000 b 4070
c 3 600 000 d 725 000 000

**13** a $1{\cdot}5 \times 10^{-3}$ b $7{\cdot}0 \times 10^{-6}$
c $2{\cdot}5 \times 10^{-2}$ d $3{\cdot}0 \times 10^{-8}$

**14** a 0·0048 b 0·000 006 1
c 0·305 d 0·000 000 012 5

# Solomon Islands
# MATHEMATICS
## Year 9 Learner's Book

## Book 2

Mathematical knowledge is essential for full participation in Solomon Islands life, both at school as learners and in the future as adults.

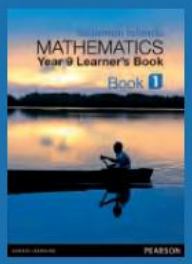
Mathematics is the exploration and use of patterns, relationships and variations in quantity, space, and time, as well as the interpretation of statistical data. ***Solomon Islands Mathematics Year 9 Learner's Book*** integrates these aspects of mathematics into a wide range of social, cultural, scientific, technological, environmental, health and economic contexts, representing both real-life and hypothetical situations.

Learners in secondary schools will build on their existing knowledge and skills in five main strands:

- Number
- Measurement
- Algebra
- Geometry
- Probability and Statistics.

Learners will acquire effective strategies for investigating, interpreting, explaining and making sense of the world, using numbers, symbols and graphs. They will develop the ability to think creatively, critically, strategically and logically. These skills and approaches will have long-term applications throughout the learners' lives.

solomon islands
CURRICULUM DEVELOPMENT DIVISION




ISBN: 978-1-4860-1397-5
9 781486 013975