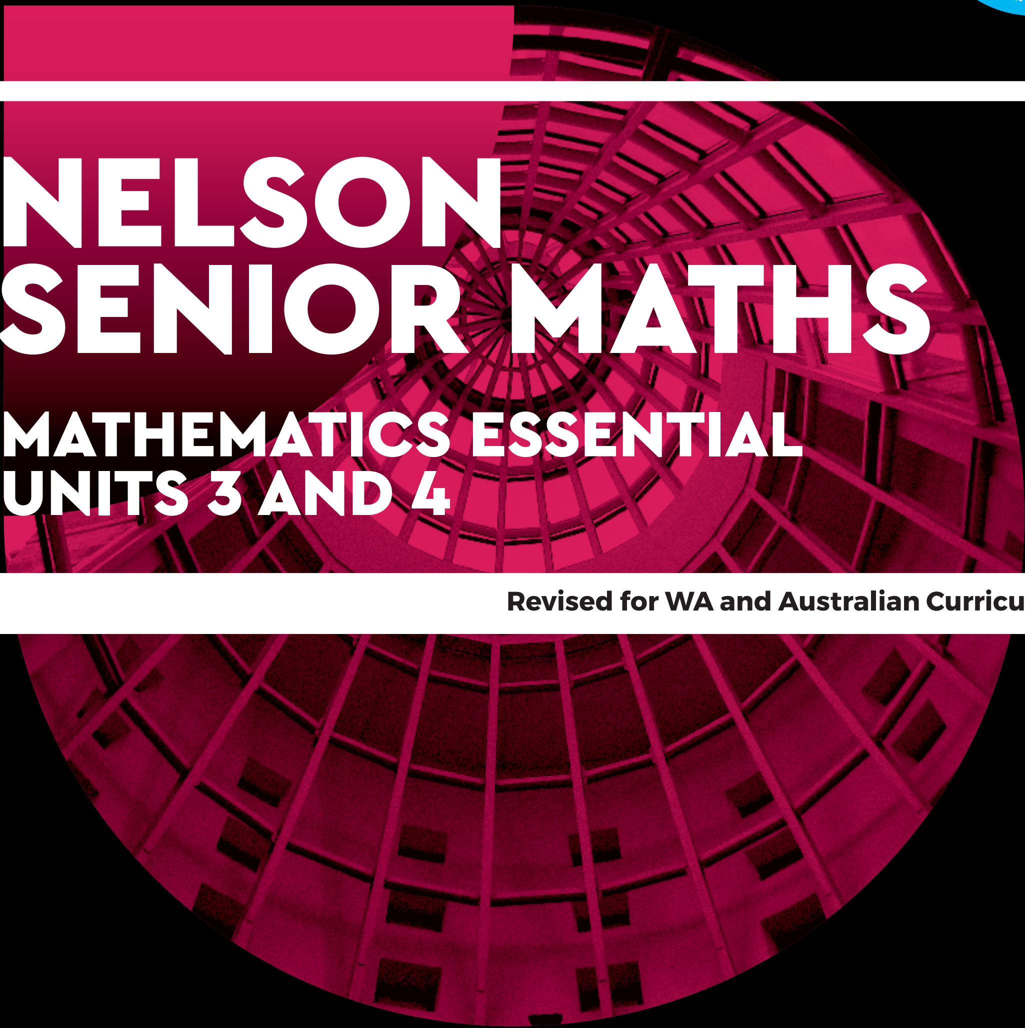

NELSON SENIOR MATHS

MATHEMATICS ESSENTIAL UNITS 3 AND 4

Revised for WA and Australian Curriculum

YEAR 12

Sue Thomson
Judy Binns

2ND EDITION

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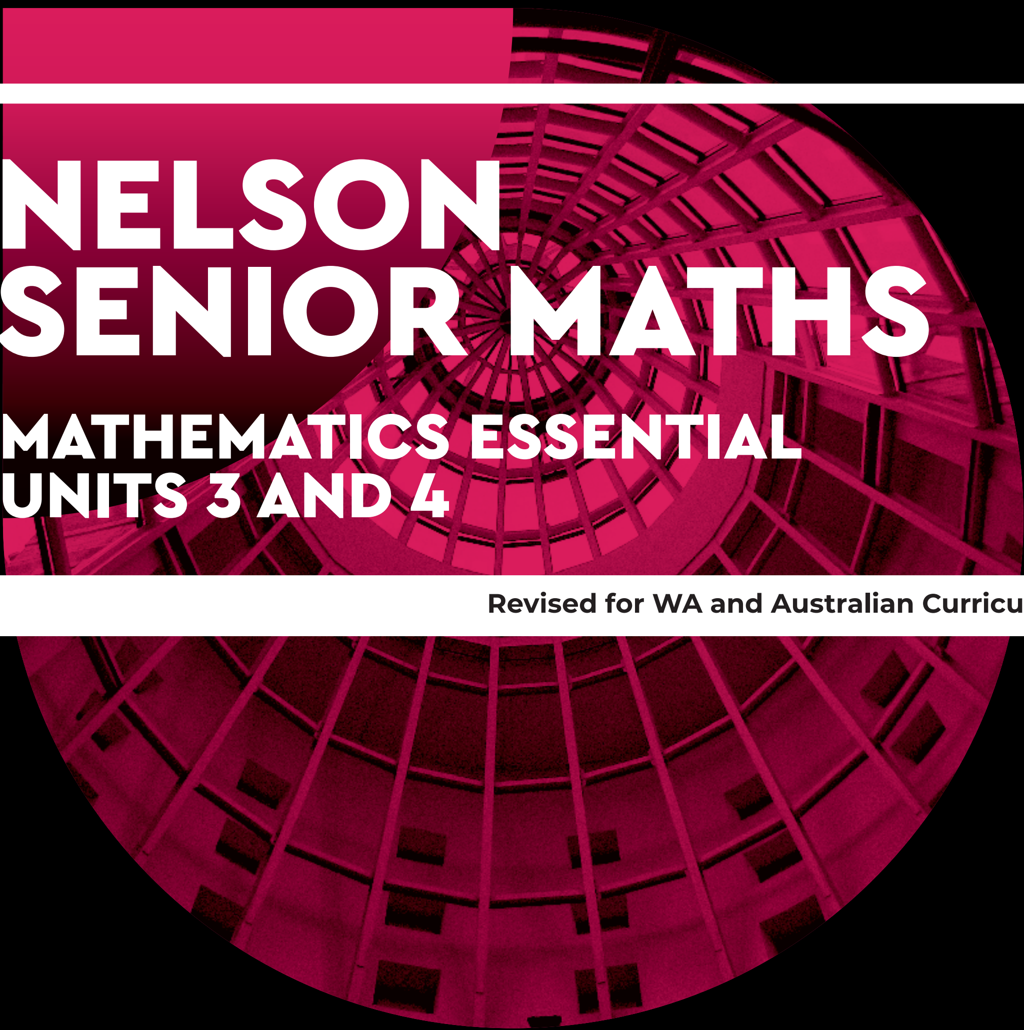
NELSON SENIOR MATHS

MATHEMATICS ESSENTIAL UNITS 3 AND 4



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Judy Binns
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CONTENTS

ISBN 9780170443975

*Australian curriculum only, not WA syllabus

ISBN 9780170443975

ISBN 9780170443975

14

BORROWING MONEY 388

15

WHERE ON EARTH ARE YOU? 414

16

TAKING CHANCES 436

ISBN 9780170443975

PREFACE

Nelson Senior Maths 12: Mathematics Essential Units 3 and 4 has been revised and updated for the WA and Australian curriculum. In this book, students and teachers will find familiar features such as clear worked examples, graded exercises, strong syllabus coverage, Investigations, Technology and a glossary/index. You will also notice new features including *Test yourself* chapter reviews and *Practice set* mixed reviews. We have also introduced a *NelsonNet* student website of worksheets, video tutorials and *ExamView* quizzes, while the *NelsonNet* teacher website contains topic tests and the entire *ExamView* questionbank and software.

The Mathematics Essential course is designed for students heading towards the workforce or further training after school. The course demonstrates interesting and relevant applications of mathematics and teaches useful maths skills for life.

The Year 11 and 12 books each cover the syllabus with 16 short chapters that are written in plain English, with an emphasis on numeracy, literacy and real-life applications. Themes include sport, health, travelling, personal finance, art, building and measurement. We wish all teachers and students using this book every success in embracing the Mathematics Essential course.

ABOUT THE AUTHORS

Sue Thomson is an experienced teacher and educational leader. She was an examination writer, assessor, marker and curriculum writer. Sue is a prolific and successful author with an interest in language development, financial literacy and making mathematics accessible to all. With her husband, Ian Forster, she wrote the successful *Workable Maths* and *Hands-On Maths in Practice* series for WA.

Judy Binns is a mathematics coordinator and experienced author who has taught in urban and rural schools. She has an interest in motivating students with learning difficulties, and wide experience in teaching senior practical mathematics courses. Judy co-wrote a successful mathematics series for Years 7–8 and often presents at local and state conferences.

CONTRIBUTING AUTHORS

Deborah Van Hoek wrote many of the *NelsonNet* worksheets.

John Drake, Katie Jackson and **Joanne Magner** created the video tutorials.

Roger Walter wrote the *ExamView* questions.

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SYLLABUS REFERENCE GRID

Topics and subtopics	*Nelson Senior Maths 12 Mathematics Essential chapter*
UNIT 3	
3.1 Measurement	
Linear measure	**7** On the surface
Area measure	**7** On the surface
Volume and capacity	**10** Fill it up
3.2 Scales, plans and models	
Geometry	**2** The shape of our world
Interpret scale drawings	**8** From paper to reality
Creating scale drawings	**8** From paper to reality
Three dimensional objects	**2** The shape of our world **8** From paper to reality
Right-angled triangles	**3** So you've got a right angle
3.3 Graphs in practical situations	
Cartesian plane	**5** Practical graphs
Using graphs	**5** Practical graphs
3.4 Data collection	
Census	**1** Census and surveys
Surveys	**1** Census and surveys
Simple survey procedure	**4** That's biased
Sources of bias	**4** That's biased
Bivariate scatterplots	**6** Scattering the data
Trendlines	**9** Fitting the data
UNIT 4	
4.1 Probability and relative frequencies	
Probability expressions, Simulations	**12** Will it happen? **16** Taking chances
Simple probabilities in practical situations	**16** Taking chances
Probability applications	**16** Taking chances
4.2 Earth geometry and time zones	
Location	**15** Where on Earth are you?
Time	**13** Time travelling **15** Where on Earth are you?
4.3 Loans and compound interest	
Compound interest	**11** Investing money
Reducing balance loans	**14** Borrowing money

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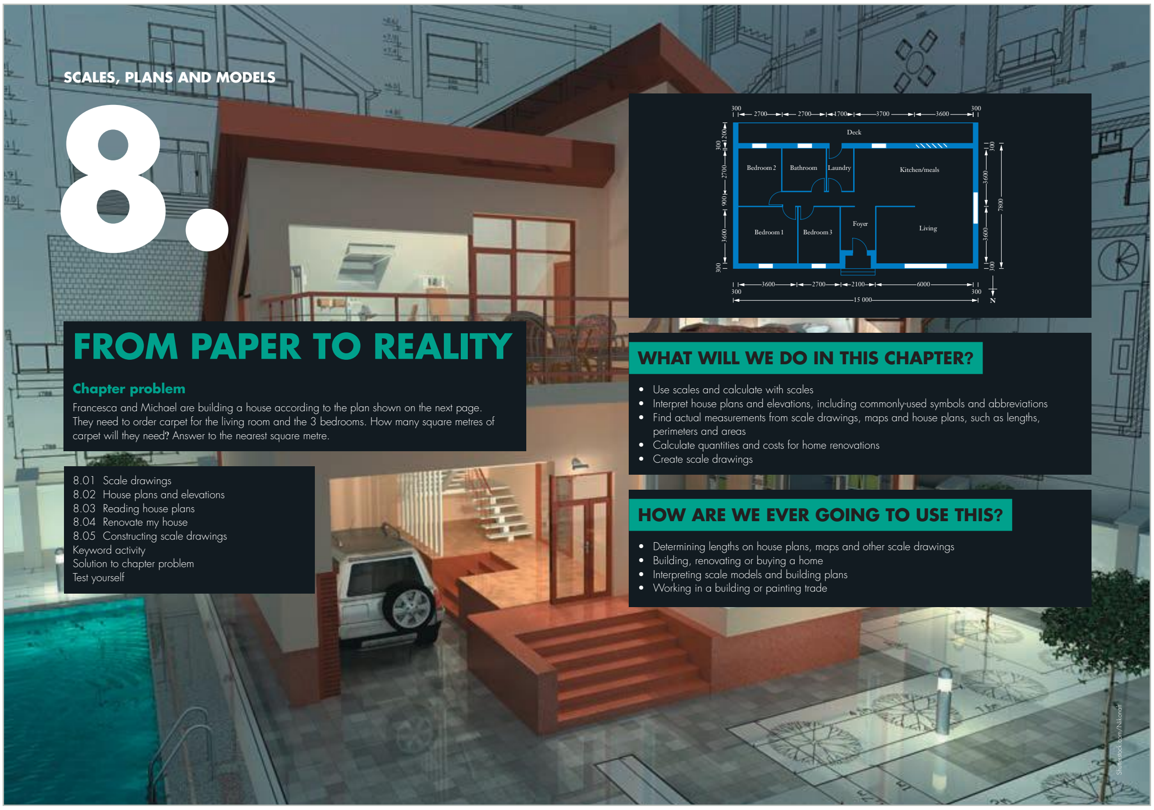
ABOUT THIS BOOK

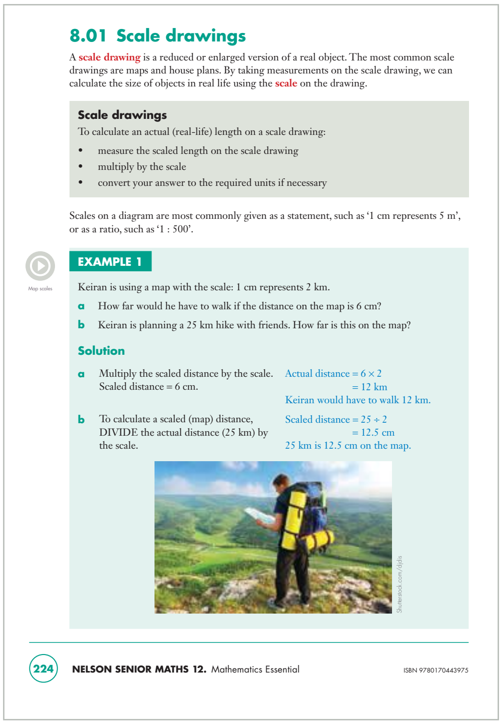
AT THE BEGINNING OF EACH CHAPTER

- Each chapter begins on a double-page spread showing a **Chapter Problem** to be solved, a chapter table of contents, a **What we will do in this chapter?** list of outcomes, and a **How are we ever going to use this?** list of applications.



IN EACH CHAPTER

- Worked examples are explained clearly step-by-step, with the mathematical working shown on the right-hand-side.
- Important facts and formulas are highlighted in a shaded box.
- Important words and phrases are printed in **red** and listed in the glossary at the back of the book.
- Graded exercises are linked to the worked examples and include problems and realistic applications.
- **Investigations** and **practical activities** explore the syllabus in more detail, through group work, discovery and modelling activities.
- **Technology** promotes ICT in the classroom, using spreadsheets and the Internet.



8.01 Scale drawings

A scale drawing is a reduced or enlarged version of a real object. The most common scale drawings are maps and house plans. By taking measurements on the scale drawing, we can calculate the size of objects in real life using the scale on the drawing.

Scale drawings
To calculate an actual (real-life) length on a scale drawing:
- measure the scaled length on the scale drawing
- multiply by the scale
- convert your answer to the required units if necessary

Scales on a diagram are most commonly given as a statement, such as '1 cm represents 5 m', or as a ratio, such as '1 : 500'.

Map scales

EXAMPLE 1

Keiran is using a map with the scale: 1 cm represents 2 km.
a How far would he have to walk if the distance on the map is 6 cm?
b Keiran is planning a 25 km hike with friends. How far is this on the map?

Solution

a Multiply the scaled distance by the scale. Scaled distance = 6 cm. Actual distance = 6 × 2 = 12 km. Keiran would have to walk 12 km.
b To calculate a scaled (map) distance, DIVIDE the actual distance (25 km) by the scale. Scaled distance = 25 ÷ 2 = 12.5 cm. 25 km is 12.5 cm on the map.

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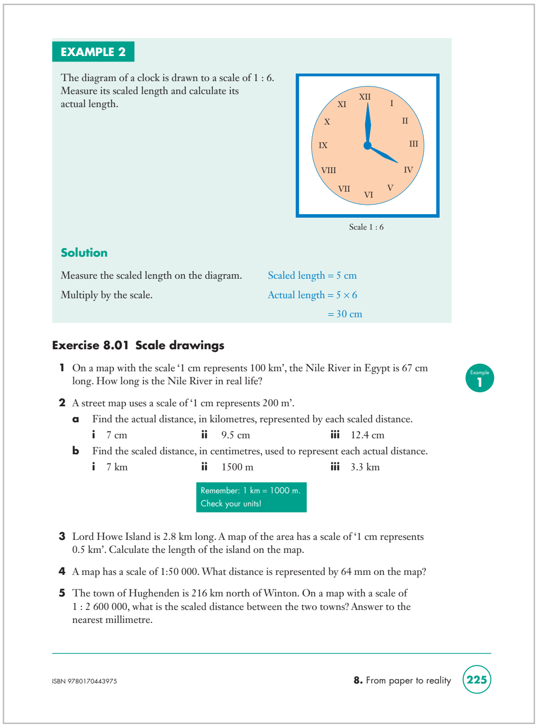
224 NELSON SENIOR MATHS 12. Mathematics Essential ISBN 9780170443975



EXAMPLE 2

The diagram of a clock is drawn to a scale of 1 : 6. Measure its scaled length and calculate its actual length.

Scale 1 : 6

Solution

Measure the scaled length on the diagram. Scaled length = 5 cm
Multiply by the scale. Actual length = 5 × 6 = 30 cm

Exercise 8.01 Scale drawings

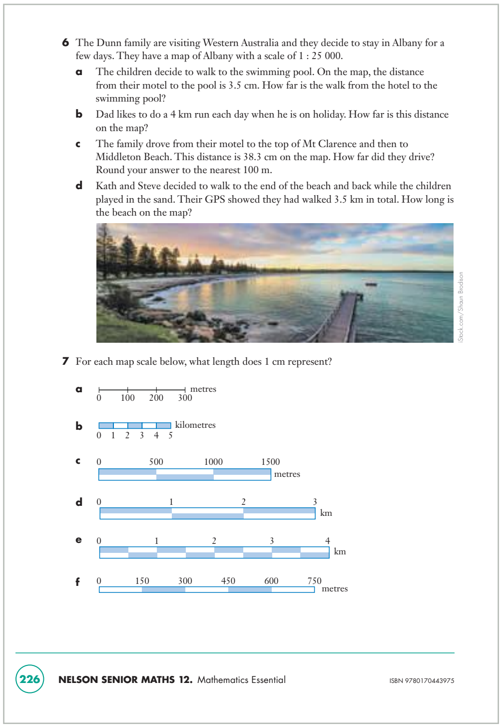
Example 1

1 On a map with the scale '1 cm represents 100 km', the Nile River in Egypt is 67 cm long. How long is the Nile River in real life?
2 A street map uses a scale of '1 cm represents 200 m'.
a Find the actual distance, in kilometres, represented by each scaled distance.
i 7 cm ii 9.5 cm iii 12.4 cm
b Find the scaled distance, in centimetres, used to represent each actual distance.
i 7 km ii 1500 m iii 3.3 km

Remember: 1 km = 1000 m. Check your units!

3 Lord Howe Island is 2.8 km long. A map of the area has a scale of '1 cm represents 0.5 km'. Calculate the length of the island on the map.
4 A map has a scale of 1:50 000. What distance is represented by 64 mm on the map?
5 The town of Hughenden is 216 km north of Winton. On a map with a scale of 1 : 2 600 000, what is the scaled distance between the two towns? Answer to the nearest millimetre.

ISBN 9780170443975 8. From paper to reality 225



6 The Dunn family are visiting Western Australia and they decide to stay in Albany for a few days. They have a map of Albany with a scale of 1 : 25 000.
a The children decide to walk to the swimming pool. On the map, the distance from their motel to the pool is 3.5 cm. How far is the walk from the hotel to the swimming pool?
b Dad likes to do a 4 km run each day when he is on holiday. How far is this distance on the map?
c The family drove from their motel to the top of Mt Clarence and then to Middleton Beach. This distance is 38.3 cm on the map. How far did they drive? Round your answer to the nearest 100 m.
d Kath and Steve decided to walk to the end of the beach and back while the children played in the sand. Their GPS showed they had walked 3.5 km in total. How long is the beach on the map?

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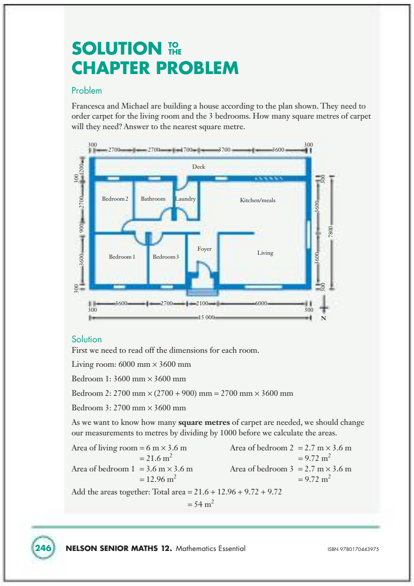
7 For each map scale below, what length does 1 cm represent?
a 0 100 200 300 metres
b 0 1 2 3 4 5 kilometres
c 0 500 1000 1500 metres
d 0 1 2 3 km
e 0 1 2 3 4 km
f 0 150 300 450 600 750 metres

226 NELSON SENIOR MATHS 12. Mathematics Essential ISBN 9780170443975

 ISBN 9780170443975

AT THE END OF EACH CHAPTER

- **Keyword activity** focuses on the mathematical language and terminology learned in the chapter.
- **Solution to the chapter problem** revisits the problem introduced at the start of the chapter and presents a solution to the problem.
- **Test yourself** contains revision linked to the relevant exercise set.
- **Practice sets** revise the skills and knowledge of previous chapters.



SOLUTION TO THE CHAPTER PROBLEM

Problem

Francesca and Michael are building a house according to the plan shown. They need to order carpet for the living room and the 3 bedrooms. How many square metres of carpet will they need? Answer to the nearest square metre.

Solution

First we need to read off the dimensions for each room.

Living room: 6000 mm × 3600 mm

Bedroom 1: 3600 mm × 3600 mm

Bedroom 2: 2700 mm × (2700 + 900) mm = 2700 mm × 3600 mm

Bedroom 3: 2700 mm × 3600 mm

As we want to know how many **square metres** of carpet are needed, we should change our measurements to metres by dividing by 1000 before we calculate the areas.

Area of living room = 6 m × 3.6 m = 21.6 m²

Area of bedroom 2 = 2.7 m × 3.6 m = 9.72 m²

Area of bedroom 1 = 3.6 m × 3.6 m = 12.96 m²

Area of bedroom 3 = 2.7 m × 3.6 m = 9.72 m²

Add the areas together: Total area = 21.6 + 12.96 + 9.72 + 9.72 = 54 m²

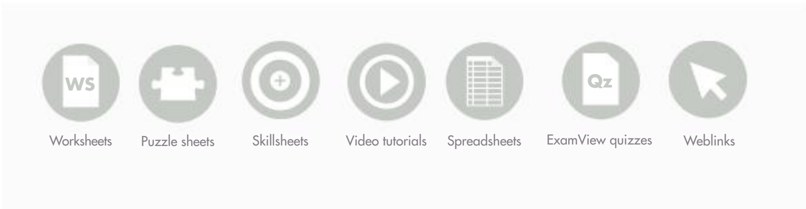
246 NELSON SENIOR MATHS 12. Mathematics Essential ISBN 9780170443975

AT THE END OF THE BOOK

- **Glossary/Index** is a comprehensive dictionary of course terminology.
- **Answers**.

NELSONNET STUDENT WEBSITE

Margin icons link to print (PDF) and multimedia resources found on the *NelsonNet* student website, **www.nelsonnet.com.au**. These include:



- **Worksheets** and **puzzle sheets** that are write-in enabled PDFs
- **Skillsheets** of examples and exercises of prerequisite skills and knowledge
- **Video tutorials:** worked examples explained by ‘flipped classroom’ teachers
- **Spreadsheets:** *Excel* files
- ***ExamView* quizzes:** interactive and self-marking
- **Weblinks**

NELSONNET TEACHER WEBSITE

The *NelsonNet* teacher website, also at **www.nelsonnet.com.au**, contains:

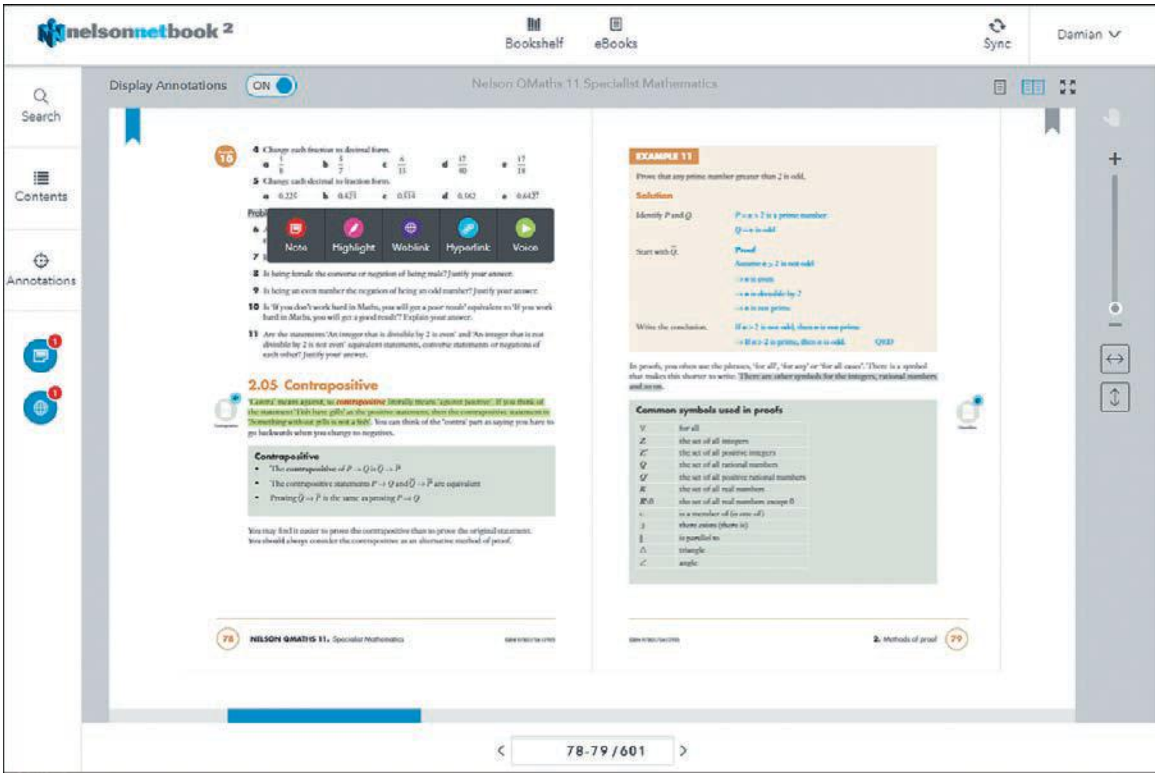
- ***ExamView*** exam-writing software and questionbanks
- **Topic tests**, in Microsoft Word and PDF formats
- A **teaching program**, in Microsoft Word and PDF formats
- **Chapter PDFs** of the textbook
- **Resource Finder:** search engine for *NelsonNet* resources

Note: Complimentary access to these resources is only available to teachers who use this book as a core educational resource in their classroom. Contact your Cengage Education Consultant for information about access codes and conditions.

NELSONNETBOOK

NelsonNetBook is the web-based interactive version of this book found on *NelsonNet*.

- To each page you can add notes, voice and sound bites, highlighting, weblinks and bookmarks
- **Zoom** and **Search** functions
- Chapters can be customised for different groups of students



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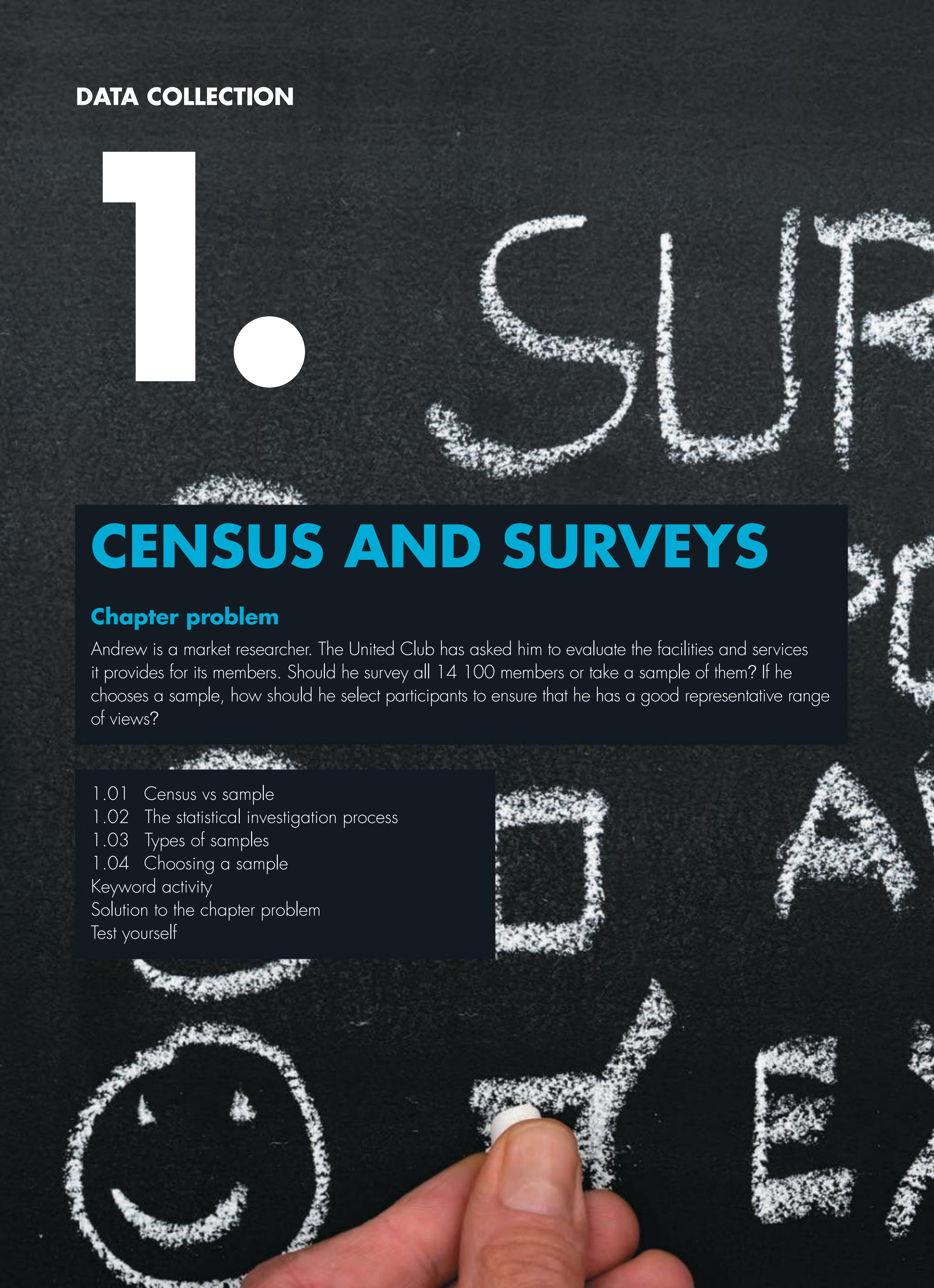
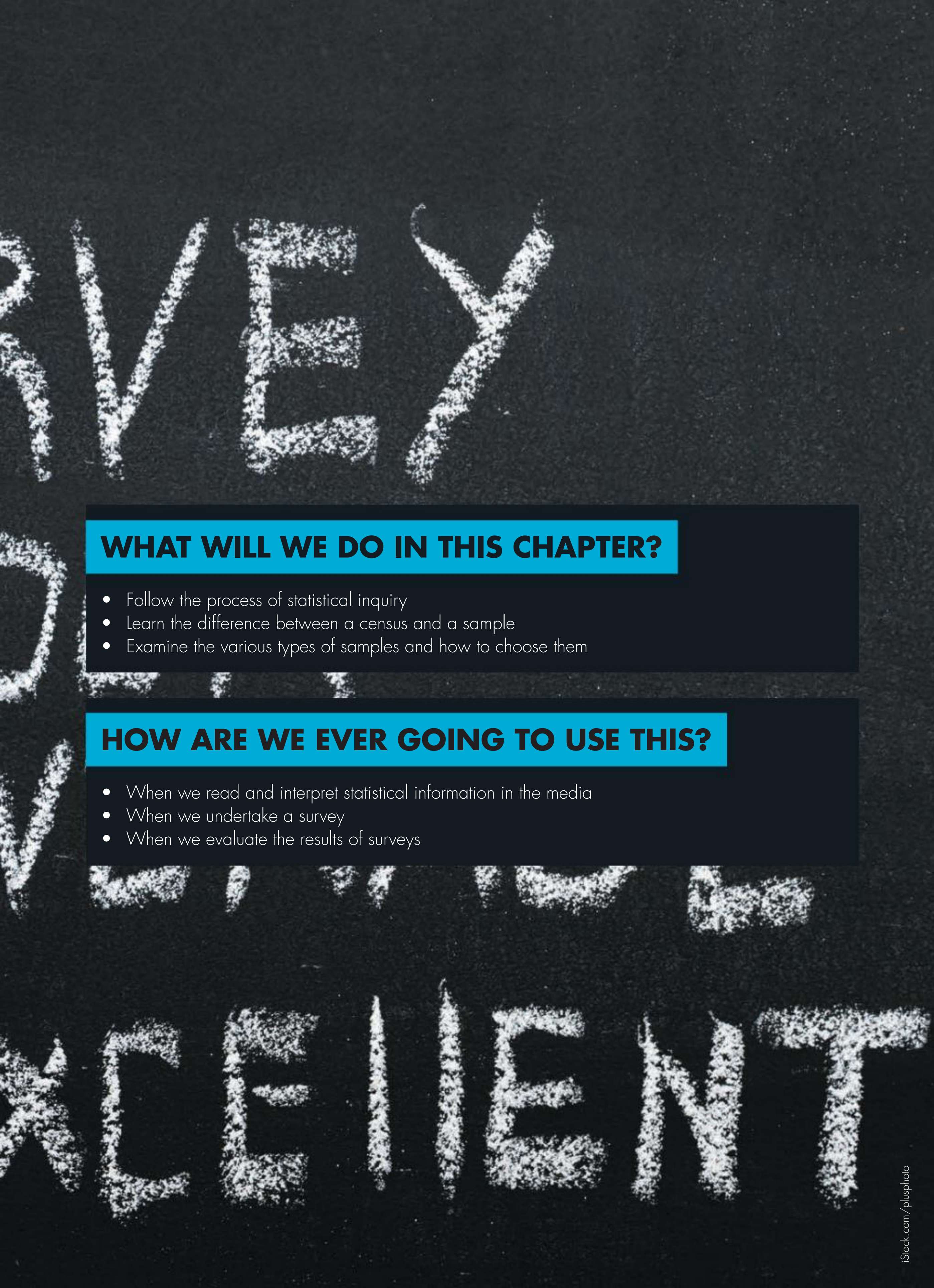
DATA COLLECTION

1. CENSUS AND SURVEYS

Chapter problem

Andrew is a market researcher. The United Club has asked him to evaluate the facilities and services it provides for its members. Should he survey all 14 100 members or take a sample of them? If he chooses a sample, how should he select participants to ensure that he has a good representative range of views?





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WHAT WILL WE DO IN THIS CHAPTER?

- Follow the process of statistical inquiry
- Learn the difference between a census and a sample
- Examine the various types of samples and how to choose them

HOW ARE WE EVER GOING TO USE THIS?

- When we read and interpret statistical information in the media
- When we undertake a survey
- When we evaluate the results of surveys

WS
Student survey form

1.01 Census vs sample

Statistics involves the collecting of information, which is then analysed and used to make decisions. To collect information, we usually survey a representative group of the target **population**. This process is called taking a **sample**.

To collect information about a whole population, *all* people or items must be surveyed. This is called taking a **census**. People who are members of small groups can be missed in a sample. We always use a census when we want to make sure that the views of small groups are included.

The Australian Bureau of Statistics conducts a national census every 5 years. This census is held in years ending in a 1 or a 6, such as 2021.

Sample	Census
• Surveys a selected group of people or items	• Surveys all people or items in the population
• Gives approximate information about the population	• Gives exact information about the population
• Simple and inexpensive	• Complex and expensive
• Can be done quickly	• It takes a lot of time to collect and process the information

EXAMPLE 1

The United Club asked Andrew to evaluate the facilities and services it provides to its 14 100 members. Should he use a census or a sample to gather information?

Solution

A census would be expensive and it would take a long time to process the information.

A sample should be used in this situation.

EXAMPLE 2

From what target population should Andrew take a sample to investigate the reputation of the United Club in the local community?

Solution

Target population means the people represented by the sample.

The target population would be the residents of the community where the United Club is located.

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Exercise 1.01 Census vs sample

1 Write a paragraph in your own words describing the advantages and disadvantages of using a census to collect information.

2 For each investigation, should a census or a sample be used? Give a reason for your answer.

Example 1

a The most popular car colour in Australia
b The number of retired people living in Darwin
c The number of Australians who watch the AFL Grand Final
d The use of soap versus body wash
e Testing coffee for taste
f The population of Hobart
g The number of people using the emergency department at Broome Hospital on Saturday night
h Length of time a certain type of car battery lasts
i The political party supported by people in South Australia
j The favourite TV show of your 10 best friends
k Steve Smith's batting average
l Internet usage by Year 8 students at your school
m Hours of paid work per week completed by students in your school
n Nelson Pay TV wants to know how satisfied its customers are with its service

3 What is the target population for each investigation?

Example 2

a Girls are better than boys at Maths
b Voting intentions for the next State election
c The best song of the last decade
d Student attitudes to school uniform at your school
e Favourite make of car in Queensland
f The amount of money people are prepared to spend on going to the gym each week
g The venue Year 12 should use for their formal
h Donations to charities from wealthy individuals
i The factors influencing the choice of supermarket
j The batting performance of the Australian cricket team
k NSM Bank wants to know if its customers find Internet banking easy to use
l The time taken to deal with complaints to a phone company
m The local council wants to know what recreational equipment should be added to the town's parks

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Australian Bureau of Statistics

Australian statistics

Census questions

INVESTIGATION

THE AUSTRALIAN CENSUS

The Australian census is conducted by the **Australian Bureau of Statistics**. Visit its website and search for the answers to these questions.

1. When was the first national census conducted?
2. When was the last census?
3. List 5 questions that were asked in the last census.
4. Find 3 questions that have been asked in the past but were not asked in the most recent census.
5. Were there any questions in the most recent census that have not been asked before? If so, what were they?
6. Are all questions in the Census compulsory?
7. Give 3 examples of how census information is used.
8. What was different about the Census in 2016? What challenges did this present?
9. Who is NOT required to complete the Census?

1.02 The statistical investigation process

Information is collected by a variety of groups to help people make informed decisions. This process is called **statistical investigation** and it involves the following steps:

- **Posing questions** – identifying the problem and deciding what information is needed and in what form
- **Collecting** data – identifying the target population, choosing between using a **census** or a **sample** of the population, deciding how we will collect information and then collecting the information
- **Organising data** – organising the collected information for clearer analysis, such as using a frequency table or spreadsheet, with either grouped or ungrouped data
- **Displaying data** – presenting the information in a way that makes it easy to follow and understand, such as with graphs or tables
- **Analysing data** – calculating summary statistics such as mean, median and mode, then looking for patterns and relationships in the data
- **Writing a report** – communicating the results of the investigation in a clear way, with the conclusions supported by the statistics

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Exercise 1.02 The statistical investigation process

1 In your own words, summarise the steps in the process of statistical investigation.

2 Classify each activity below as one of these steps:

- posing questions (PQ)
- collecting data (CD)
- organising data (OD)
- displaying data (DD)
- analysing data and drawing conclusions (AD)
- writing a report (WR)

a Liong asks fellow students in Year 12 how they travel to school

b Jane draws a frequency histogram of her data

c Anna finds the mean house price for her suburb

d Managing Director Theo decides he needs to know customers' favourite car colour

e Kieran makes recommendations in his final report

f Emilia puts her information on favourite holiday destinations into a frequency table

g Lee concludes that coffee is her friends' favourite daytime drink

h Simon visits the residents of Lawson Street to collect their Census forms

i Madeline decides to research the drinking habits of 18-year-olds

j Will displays data about pocket money received by Year 11 students in a sector graph

k Phoebe writes a report on her data about popular sports

l Kim goes through a pile of surveys and records the responses to a specific question

Alamy Stock Photo/Richard G. Bingham II

3 Choose a topic to research and describe how you would implement each of the 6 steps of a statistical investigation.

Australian Bureau of Statistics

4 Some organisations collect information on a large scale. Visit the websites of each organisation listed and write at least 5 things they collect information about.

a the Australian Bureau of Statistics (ABS)

b the United Nations (UN)

c the World Health Organization (WHO)

United Nations

World Health Organisation

5 The privacy of collected information can be important. Visit the ABS website and search for the **Census** page. Scroll down and click on **Privacy, Confidentiality & Security**. Describe in your own words how the ABS ensures the privacy of the information we provide in the Census.

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1.03 Types of samples

If we decide to use a sample for a survey, there are 4 types of samples we can use.

In a **random sample**, every member of the population has an equal chance of being included in the sample. For example, when a computer selects a customer of a phone company at random to be surveyed about the company's performance, every customer of that company has an equal chance of being selected.

In a **stratified sample**, different categories in a population are represented according to their proportion size of the population and then members of each category are selected randomly.

For example, a school's population may be made up of 72% junior students and 28% senior students. A stratified sample of school students must also be made up of 72% junior students and 28% senior students.

In a **systematic sample**, selections are made on a regular basis. For example, testing every 500th battery produced in a factory to check that the machines producing the batteries are working properly.

In a **self-selected sample**, whoever wishes to participate answers the questions asked. For example, when a current affairs TV program asks people to vote online about who should be Australia's next prime minister, anyone can participate.

EXAMPLE 3

The United Club has 14 100 members. This table gives a breakdown of members by age group.

Age group	Number of members
Less than 30 years	2100
30 to 39 years	2700
40 to 49 years	5100
50 years and over	4200

Andrew is considering 4 different of ways of choosing a sample for his market research. Which type of sample is each one?

a Every 100th member on the alphabetical membership list

b Names selected randomly by the computer

c Putting up a sign at the entrance of the club asking members to volunteer

d 21 members who are less than 30 years old, 27 members who are 30 to 39 years old, 51 members who are 40 to 49 years old and 42 members aged 50 years or more.

Solution

a Selections are made on a regular basis, in this case every 100th member. — Systematic sample

b Every member of the population has an equal chance of being included and the computer chooses randomly. — Random sample

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c	Any member who volunteers can participate.	Self-selected sample
d	A predetermined number of members are chosen based on the number of members in each age group.	Stratified sample

Exercise 1.03 Types of samples

Example 3

1 Which type of sampling is described in each case?

- **a** Selecting every 105th name from a customer database
- **b** The names of all Years 11-12 students are placed in a hat and 2 are drawn out to represent the school at a council function
- **c** A television program asks viewers to respond to a Yes/No question
- **d** Selecting an appropriate number of students from each year at your local high school
- **e** Subscribers are sent an email asking for their opinion of the play they have just seen
- **f** Names of employees at a bank are drawn out of a barrel
- **g** The audience at a concert finds prize tickets under every 40th seat in each row
- **h** A company sends out an email to customers asking for their opinions
- **i** Employees are sorted from tallest to shortest and every 5th employee completes a questionnaire
- **j** 20 females and 28 males were surveyed out of a group of athletes with 100 females and 140 males
- **k** An airport customs officer searching every 10th airline passenger
- **l** A medical researcher advertises for people to participate in a health survey
- **m** 5 cards are selected from a pack of cards without looking
- **n** An import/export business employs 125 women and 250 men, and 17 women and 34 men are surveyed about their work hours
- **o** A computer selects every 1000th name from the electoral (voters') roll
- **p** A bank surveys 5% of its customers in each age group

2 Children from 400 families attend the local primary school. The P&C has raised money for new play equipment for the playground and wants to interview parents about their ideas. The committee would like to survey 40 families. Suggest how they might select the 40 families using a:

- **a** random sample
- **b** stratified sample
- **c** systematic sample
- **d** self-selected sample

3 For the survey above, which method of sampling is the best to use? Explain your answer.

4 What are the possible disadvantages of using:

- **a** a self-selected sample?
- **b** a random sample?
- **c** a systematic sample?
- **d** a stratified sample?

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Chapter problem

You've covered the skills required to solve the chapter problem. Can you solve it now?

INVESTIGATION

SAMPLE SIZES IN POLLS

In this investigation, we will look at the sizes of samples used by polling organisations.

Roy Morgan

1 Visit the **Roy Morgan** research website.

2 Click on the **Findings** tab and select one of the polls in Australia that interests you.

3 Write down what the poll is about and the size of the sample.

4 Use the Internet to find the population of Australia.

5 What percentage is the sample of Australia's population?

Remember: $\frac{\text{sample}}{\text{population}} \times 100$

YouGov

6 Visit the **YouGov Australia** website.

7 Select a recent national or state poll and write down the sample size they used.

8 Use the Internet to find the population of Australia or the state for the poll you have chosen.

9 What percentage is the sample of the population?

10 Write 2 or 3 sentences describing what you have found.

11 How well do you think the samples reflect the opinions of the population?

Alamy Stock Photo/Jeff Greenberg

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1.04 Choosing a sample

When we have decided what type of sample to use, we need to know how to choose the sample.

EXAMPLE 4

Selina decided to use a stratified sample to survey members at her local gym. There are 970 gym members, of which 590 are female and 380 are male. She plans to survey 10% of the members.

a How many members will Selina survey?

b How many female members should she survey?

c How many male members should she survey?

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Solution

a Calculate 10% of the membership.

10% of 970 = 97

Selina should survey 97 members.

b Female members make up 590 out of 970 members.

Write this as a fraction and find this fraction of the 97 members to be surveyed.

$\frac{590}{970} \times 97 = 59$

Selina should survey 59 female members

c Male members make up 380 out of 970 members.

$\frac{380}{970} \times 97 = 38$

OR, from part **b**.

97 members – 59 females = 38 males

Selina should survey 38 male members.

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Exercise 1.04 Choosing a sample

1 Maria wishes to survey the members of the fan club she belongs to. Suggest any method she could use to find a random sample of members to survey.

2 Mr Carrozza, the school principal, wants to survey a sample of students' parents at Nelson Valley College. Suggest how he could find a systematic sample of parents.

3 In a self-selected sample, people volunteer to complete a survey. Suppose you need to collect data on usage of the school canteen. Suggest 3 places in your school where you might make questionnaires available for those who wanted to fill them in.

Example 4

4 Amanda is going to use a stratified sample to survey the parents of her local netball club. There are 540 children playing netball, made up of 380 primary school students and 160 high school students. Amanda is going to survey 15% of parents.

- **a** How many parents should complete the survey?
- **b** How many parents of primary students should complete the survey?
- **c** How many parents of high school students should complete the survey?

5 Kieran is going to survey Year 8 students about their opinions of the school. There are 96 boys and 69 girls in Year 8. He aims to survey 20% of students.

- **a** How many students should complete the survey?
- **b** How many boys should complete the survey?

6 Jamiela is going to ask a sample of her Facebook friends about their favourite sport. She has 376 single friends and 416 friends in a relationship. She is going to survey 12.5% of her friends.

- **a** How many friends will she survey?
- **b** How many single friends will be in the survey?
- **c** How many friends in a relationship will be in the survey?

7 Global Communications employs 750 people made up of 479 males and 271 females. The company intends to survey 75 employees about their working conditions.

- **a** How many males should be surveyed?
- **b** How many females should be surveyed?

8 Darren is researching political views in Western Australia. He stands on Wellington Street in central Perth and interviews every 10th person who walks past.

- **a** Is this a representative sample of the population of WA? Why or why not?
- **b** If not, describe how Darren could achieve a representative sample.

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KEYWORD ACTIVITY

Census and surveys find-a-word

WORD MATCH

Match each word with its definition.

1	census	**A**	A representative group in a survey
2	population	**B**	Every member of the population has an equal chance of being selected
3	privacy	**C**	Different categories in a population are represented according to the size of each category
4	questionnaire	**D**	Your right not to share personal information
5	random	**E**	The process of collecting information to enable people to make informed decisions
6	sample	**F**	One of the most common ways to collect information
7	self-selected	**G**	Sample where selections are made on a regular basis, such as every 100th item
8	stratified	**H**	The group of people from whom a sample is chosen
9	statistical investigation	**I**	Whoever wishes to participate answers the questions asked
10	systematic	**J**	All people or items are surveyed

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SOLUTION TO THE CHAPTER PROBLEM

Problem

Andrew is a market researcher. The United Club has asked him to evaluate the facilities and services it provides for its members. Should he survey all 14 100 members or take a sample of them? If he chooses a sample, how should he select participants to ensure that he gets a good representative range of views?

Solution

It would be too expensive to survey all 14 100 members and it would take too long to analyse the results. A sample is the better option.

To ensure a representative sample, Andrew should take a stratified sample of the different age groups. If he is using a sample size of 10%, then he should survey 10% of each age group.

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1. TEST YOURSELF

Census and surveys

Practice quiz

Exercise 1.01

1 Should you use a census or a sample to find the most popular TV program in Australia? Give a reason for your answer.

Exercise 1.02

2 For each investigation listed below:

   i what is the target population?

   ii should a census or sample be used?

   a The number of children in each family in your street

   b Make of car in car fleets of companies

   c Voting intentions at the next federal election

   d Whether boys complete more homework than girls in Northern Territory schools

Exercise 1.02

3 List the steps in a statistical investigation in your own words.

Exercise 1.03

4 Cengage Engineering has 430 employees. The CEO wishes to survey the employees about working conditions at the company. He would like to survey 86 employees. Suggest how the firm contracted to do the research might select the 86 employees using a:

   a random sample

   b stratified sample

   c systematic sample

   d self-selected sample

Exercise 1.04

5 Lindsay is going to use a stratified sample to survey the parents of all students in her dance classes. There are 375 children in the dance classes: 295 girls and 80 boys. Lindsay is going to survey 20% of parents.

   a How many parents should complete the survey?

   b How many parents of girls should complete the survey?

   c How many parents of boys should complete the survey?

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MEASUREMENT

2. THE SHAPE OF OUR WORLD

Chapter problem

Andrew and Robyn are renovating their bathroom. They want to have one feature wall with tiles that are NOT square. What shapes could the tiles be? Create some designs for their feature wall.

WHAT WILL WE DO IN THIS CHAPTER?

- Use the symbols and language of geometry
- Draw and recognise the properties of 2D shapes and 3D solids
- Interpret 2D representations of 3D objects
- Interpret diagrams of 3D objects

HOW ARE WE EVER GOING TO USE THIS?

- Whenever we use shapes or solids in design
- To make models of 3D shapes
- To sketch 3D solids from different views
- In a career where design is an important element, for example, graphic design or home renovations

iStock.com/Barrirret

Polygon puzzle

2.01 2D shapes

2D shapes are flat and have 2 **dimensions**: length and width.

3D shapes are solids and have 3 dimensions: length, width and depth.

2D shapes surround us every day. The most common 2D shapes are **triangles** and **quadrilaterals**.

WS 2D shapes

Exercise 2.01 2D shapes

This exercise can be printed from NelsonNet as the worksheet ‘2D shapes’.

1 Triangles

Use the word list to complete the following paragraphs. You can use words more than once. You may like to cut out some triangles and use them to help you fill in the blanks.

Alamy/Mediablitzimages

Alamy/DIZ München GmbH

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Word list: angle, angles, equilateral, scalene, sides, two.

We name triangles according to the lengths of their ______. A triangle with three sides the same length is called an ________________ triangle. This triangle also has three ________ equal. Each ______ is equal to 60°. An isosceles triangle has _____ sides and _____ angles equal. A ________ triangle has no sides the same length.

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The surface of this side of the pyramid is an equilateral triangle.

Word list: acute , obtuse, right.

We can also name triangles according to the types of angles in them. A triangle with three angles less than 90° is called an ______-angled triangle. A triangle with one angle between 90° and 180° is called an ______-angled triangle. A right-angled triangle has one ______ angle.

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2 The parallelogram and rhombus

Use the word list to complete the following paragraphs. You can use words more than once. You may like to cut out a parallelogram and a rhombus and use them to help you fill in the blanks.

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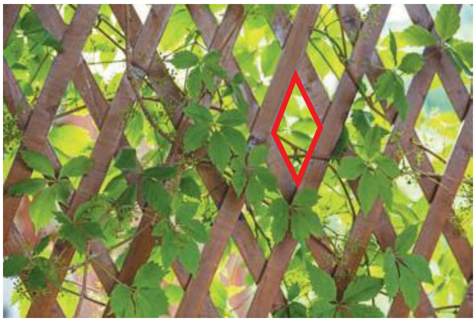

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Word list: bisect, equal, four, opposite, parallel, 90°, rectangle, square.

Each of these shapes has ____ sides. A parallelogram has both pairs of ________ sides equal. Each pair of opposite sides is also _________. The opposite pairs of angles are _______. The diagonals of a parallelogram cut each other in half. We say they _______ each other.

A rhombus has all of the features of a parallelogram. In addition, all four sides of the rhombus are _______. This means the diagonals bisect each other at an angle of _______.

A parallelogram looks like a ______________ pushed over and the rhombus looks like a ___________ pushed over.

3 The rectangle and square

Use the word list to complete the following paragraphs. You can use words more than once. You may like to cut out a rectangle and a square and use them to help you fill in the blanks.

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Word list: diagonals, equal, parallelogram, rhombus, 90°

A rectangle has all the features of a _________. However, in a rectangle all the angles are _____ and the diagonals are _____ lengths.

A square has all the features of a _________. Also, the sides are all _________. The angles are all _____. The __________ are the same length.

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4 Other quadrilaterals

Use the word list to complete the following paragraphs. You can use words more than once.

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Word list: diagonals, equal, parallel, quadrilateral, 90°.

Any shape with four sides is called a ______________. The trapezium is a quadrilateral with one pair of opposite sides ________. A kite is a bit different. The 'top' 2 sides are ______ and the 'bottom' 2 sides are also _______. The angles at the 'sides' are _____. The __________ intersect at an angle of _______.

5 More polygons

Use the word list to complete the following paragraphs. You can use words more than once.

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Shutterstock.com/koosen

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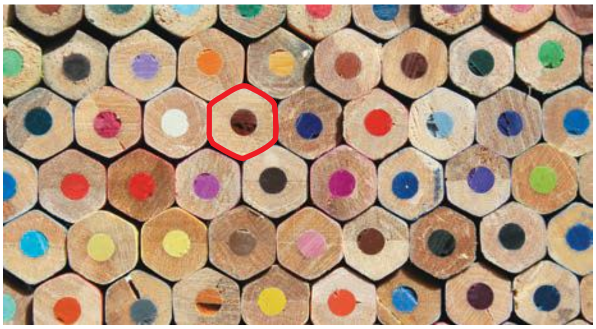

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Word list: decagon, hexagon, number, octagon, pentagon, quadrilaterals, regular.

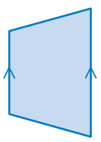
A **polygon** is a shape with straight sides. We use the __________ of sides they have to name each polygon. We have already looked at triangles and ____________________ in detail. We have the following names for polygons with more than 4 sides:

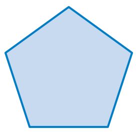
Name	Number of sides
	5
	6
Heptagon	7
	8
Nonagon	9
	10
Undecagon	11
Dodecagon	12

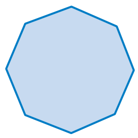
A **rhombus** has 4 equal sides but **not** 4 equal angles, so it is **not** a **regular polygon**.

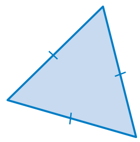
When all the sides and angles of a polygon are equal, it is called a __________ polygon. The equilateral triangle and the square are ___________ polygons.

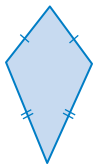
6 Name each shape.

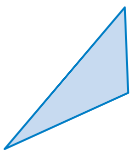
a 

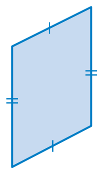
b 

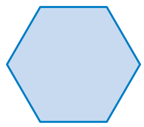
c 

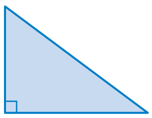
d 

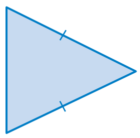
e 

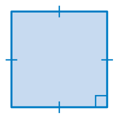
f 

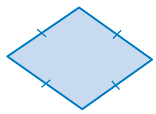
g 

h 

i 

j 

k 

l 

7 Is each statement TRUE or FALSE?

a An isosceles triangle must be acute-angled.

b A square is a regular quadrilateral.

c An obtuse-angled triangle must also have acute angles.

d A rhombus is a special parallelogram.

e An acute-angled triangle cannot be isosceles.

f A rectangle is a special trapezium.

g An equilateral triangle must be acute-angled.

h A trapezium must have 2 equal sides.

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INVESTIGATION

ROAD SIGNS

Road signs in Australia are based on a number of geometrical shapes. Use the Internet to research the following questions. Google Images may be helpful.

1 The regular octagon is used for only one sign. What sign is it? Draw or copy an example.

2 What signs use the equilateral triangle? Draw or copy an example.

3 Many signs are rectangular. Name some of the signs that use rectangles and draw or copy some examples.

4 Warning signs are usually black on yellow. Why are those colours used? What shape are they? Draw or copy an example.

5 Some signs have this shape. What sort of shape is it? Draw or copy an example.

Present your findings using a video or presentation software.

Chapter problem

You've covered the skills required to solve the chapter problem. Can you solve it now?

WS Sorting solids

WS 3D solids

2.02 3D solids

We live in a 3D world. Everywhere around us we see 3D solids – buildings, cars, supermarket items.

Exercise 2.02 3D solids

This exercise can be printed from NelsonNet as the worksheet '3D solids'.

Use the word list to complete each paragraph.

1 Polyhedrons

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Getty/Dorling Kindersley/Steve Gorton

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Word list

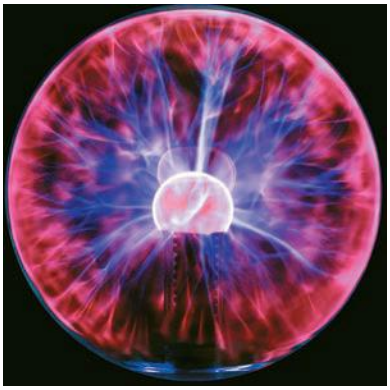
edge	flat	hexahedron
pentahedron	prism	pyramid
rectangular	vertex	

Solids can have flat faces and curved faces. Solids that only have ________ faces are called polyhedrons. Some of the common polyhedrons are the ________ prism, the triangular ________ and the square ________. Polyhedrons have similar names to polygons. We call a solid with 5 faces a ________; a solid with 6 faces is called a ________. When the faces of a polyhedron meet, they form an ________. When 3 or more edges meet, they form a ________.

2 Solids with curved surfaces

iStock.com/enviromantic


Alamy Stock Photo/David Moore


Shutterstock.com/WhiteRabbit83

Word list

circle	cone	curved
flat	rectangle	sphere

Some solids are not polyhedrons because they have ________ faces. A solid that has only one curved face is the ________. The cylinder has 2 ________ faces and one ________ face. When we flatten the curved face, it is a ________. A ________ has one flat face and one curved face. When we flatten the curved section, it is a sector of a ________.

3 Copy and complete this table.

Solid	Number of faces	Shapes of faces	Number of identical faces
Cube			
Cylinder			
Square pyramid			
Triangular prism			
Rectangular prism			
Cone			
Triangular pyramid			

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4 Prisms and pyramids

Prisms and pyramids are special types of polyhedrons.

Dreamstime.com/Paop

iStock.com/falun

Shutterstock.com/Tobias Arhelger

Word list

bottom	cross-section	end
pointed	rectangle	shape
size	square	triangle

A prism has the same __________ from one end to the other. Both ends are the same __________. Prisms take their names from the shape at each __________. A rectangular prism has a __________ at each end and a triangular prism has a __________ at each end.

A pyramid has a __________ top, called the apex. The shape at the __________ of the pyramid gives the pyramid its name. A square pyramid has a __________ at the bottom. A pyramid's __________ is NOT the same from the bottom to the top.
Each cross-section is the same shape but not the same __________.

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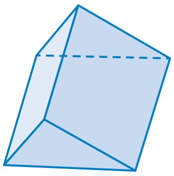
5 Name each solid.

a



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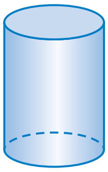

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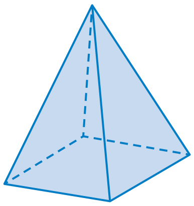

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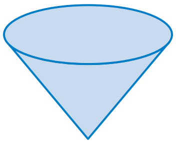

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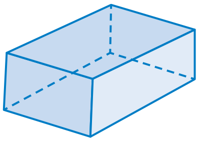

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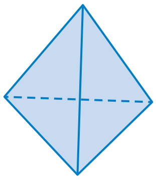

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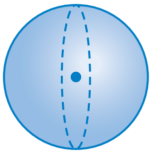

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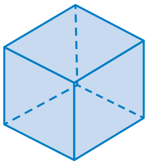


6 Name each solid shown in these pictures.

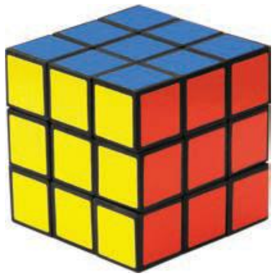
a

Alamy Stock Photo/Martin Lee

b

Shutterstock.com/M. Unal Ozmen

c



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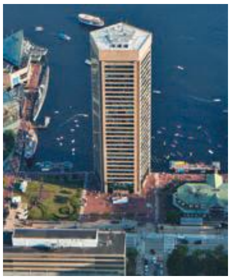

Getty Images/Greg Pease

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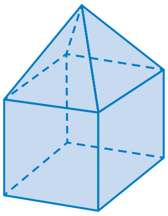
7 Each composite solid below is made up of 2 basic solids. Name the 2 solids that have been combined.

a



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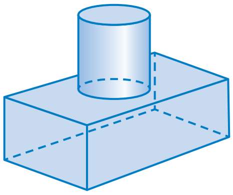

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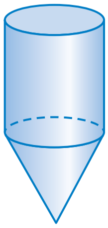

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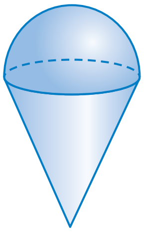

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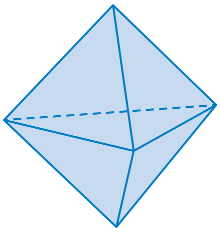

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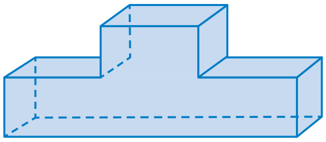


8 Is each statement TRUE or FALSE?

a A triangular prism has a triangle for its cross-section.

b A cylinder is a polyhedron.

c A rectangular prism is a hexahedron.

d A cone has 2 faces.

e A cube and a square pyramid each has 6 faces.

f A pyramid has the same cross-section from bottom to top.

INVESTIGATION

LOGOS AND BUILDINGS

1 Find 5 examples of company logos.

- Present each logo and state what shapes have been used to create the logo.

2 Find 5 examples of buildings or bridges or sculptures.

- Present each example and state what solids have been used.
- Identify what shapes are on the surface of the solids you have chosen.

Present your findings as a poster or using presentation software.

ISBN 9780170443975

INVESTIGATION

STACKING AND PACKAGING

1 Look around your classroom. Why are bricks the shape of rectangular prisms? Why is the room a rectangular prism? What shape are most books? Why do you think this is?

2 Imagine you are in a supermarket. What solids are used for packaging? Why do you think manufacturers use different shapes? How are items stacked on the shelves? What part does packaging play in this?

3 Research the solid shape bees use to make a beehive and why they use it.

Shutterstock.com/StudioSmart

4 Find other 3D structures in your environment. What solid shapes have been used? Why might this be so? If you can't think of any, use the Internet to find some examples of different structures.

ISBN 9780170443975

2.03 Designing a logo

WS Enlarging a logo

Geometric designs are all around us. They are used in creating logos for businesses and organisations. They are part of the design of products we see in the supermarket and in other graphic design work.

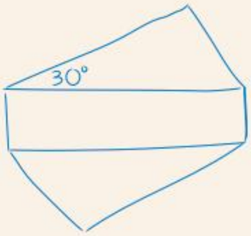
Alamy/Peter Lane

Alamy/Mark Richardson

Alamy/Kevin Foy

EXAMPLE 1

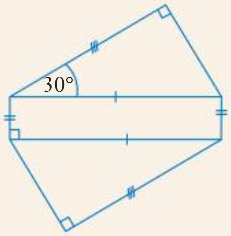
Chloe works for a design company. When she produces a design for a logo she does a rough sketch first. Draw this design accurately.



Solution

This logo is made up of a rectangle and 2 identical right-angled triangles.
Draw the rectangle first.

Draw 2 triangles with a 30° angle and a right angle.



Sometimes, Chloe writes down a description of the design and other people are responsible for drawing it.

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EXAMPLE 2

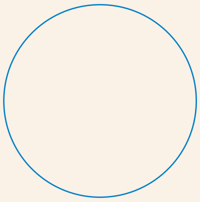
Chloe described the design for a new logo as follows: 'The logo is a circle overlaid by an equilateral triangle. The vertices of the triangle are outside the circle and the segments of the circle outside the triangle are shaded.'

Draw this logo.

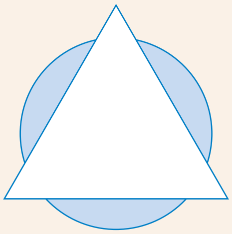
'overlaid' means the triangle is 'on top' of the circle.

Solution

Draw the circle first



Add the equilateral triangle and shade the segments of the circle outside the triangle.



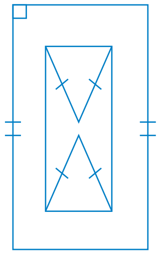
Exercise 2.03 Designing a logo

Example 1

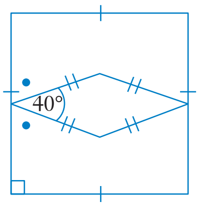
1 Draw each design accurately, using a ruler and protractor or using technology.

a In the large rectangle the length is twice the width.

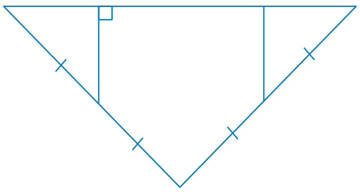
In the small rectangle the width is half the width of the large rectangle.



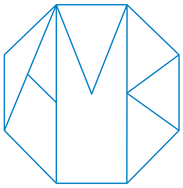
b



c



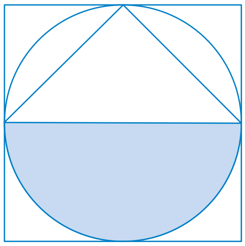
d



All sides of the octagon are equal.

2 Design your own logo. Make a rough sketch first, then an accurate sketch. Use your own initials somewhere in the logo.

3 Use the word list to complete the description of the logo.



Word list:

circle	circumference	diameter
four	isosceles	semicircle
square	vertex	

The perimeter of the logo is a square. Inside the ________ there is a large circle whose ________ touches each of the ________ sides of the square. The lower ________ is shaded. There is an ________ triangle in the upper semicircle. Two vertices of the triangle are on either ends of the ________ and the third ________ is on the circumference of the ________.

Example 2

4 Read each description of a logo, then draw the logo.

- **a** The vertices of an equilateral triangle are on the circumference of a circle. The segments between the triangle and the circle are shaded.
- **b** The four vertices of a square are on the circumference of a circle. Both diagonals of the square have been drawn. Two of the four identical triangles have been shaded.
- **c** A regular hexagon has two equilateral triangles inside it. One vertex of each triangle is at the centre of the hexagon and the triangles are opposite each other. One triangle has the letter R in it and the other has the letter M in it.
- **d** A rectangle contains the letters B and B, back-to-back.

5 Choose three logos of well-known companies or products. What shapes are they based on? Write a description of them. Share you descriptions with your friends and see if they can guess what logos you have chosen.

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2.04 Drawing solids

Solid shapes can be difficult to draw. This exercise will give you some ideas on how to sketch some common solids.

WS
Sketching solids

Exercise 2.04 Drawing solids

WS
Isometric dot paper

1 Copy each diagram on isometric dot paper (which can be downloaded from NelsonNet).

a


b


c

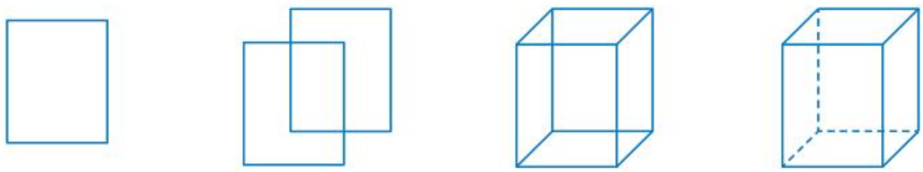

d


e


f


This T-shape is correctly drawn.

2 Sketch a rectangular prism using these diagrams and instructions.



a Draw a rectangle that will be the front face of the prism.

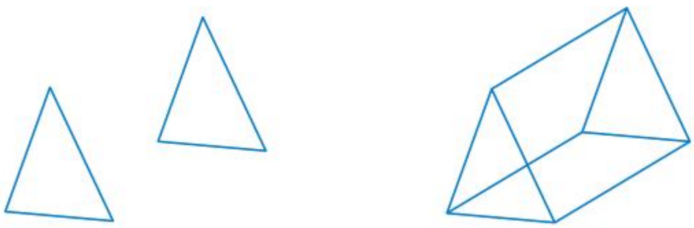
b Draw another rectangle the same size, but position it slightly above and to the right of the first rectangle.

c Join the matching vertices (corners).

d Make the outside edges of the prism darker and use lighter or dotted lines for the inside edges.

3 Draw each solid using the method shown in question **2**.

a A triangular prism



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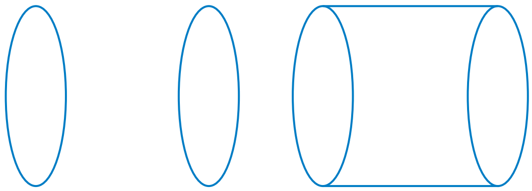
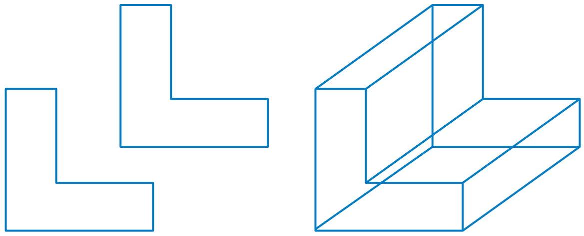
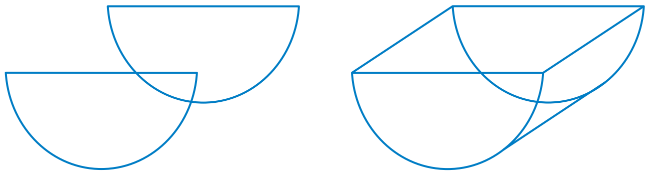
b A cylinder



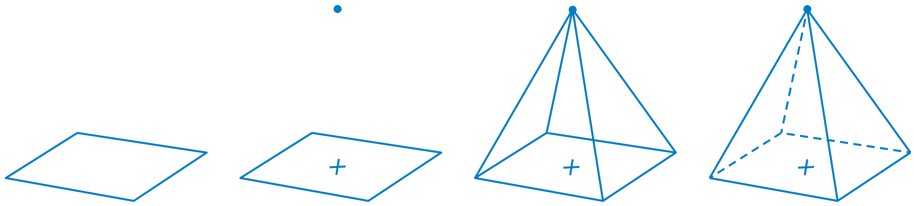
c An L-shaped prism



d A half-cylinder



4 Sketch a rectangular pyramid using these diagrams and instructions.



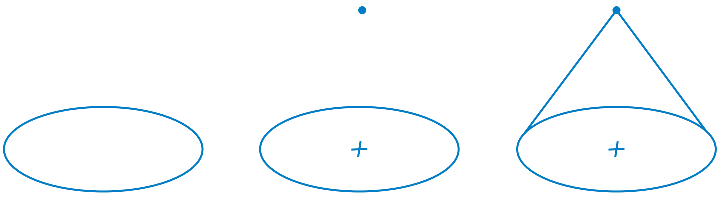
a Draw a parallelogram for the rectangular base.

b Determine the centre of the parallelogram (where the diagonals cross) and place a dot above this position: this dot will be the top of the pyramid.

c Join the 4 vertices of the parallelogram to the dot.

d Make the outside edges of the pyramid darker and the inside edges lighter or dotted.

5 Sketch a cone using these diagrams and instructions.



a Draw an oval for the circular base.

b Draw a dot above the centre of the oval.

c Join the dot to the oval.

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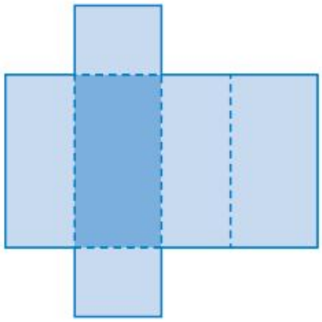
2.05 From 3D to 2D

Nets of solids

WS
Making solids

A cardboard box starts out as a flat piece of cardboard. We cut out a particular shape and fold it to make the box. The 2D shape we cut out is called the **net** of a solid. For the box, the net would look like this:

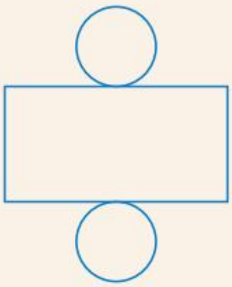


We have not included the tabs we would need to glue it together.

Shutterstock.com/Anastasiya 99

EXAMPLE 3

What solid would this net make?



Solution

The rectangle would curve around to match the circles.

This is the net of a cylinder.

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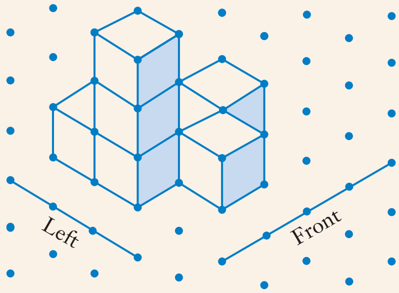
Different views of solids

We can also draw a solid from different perspectives (points of view). We can draw what we would see when looking from the front or the top or one of the sides.

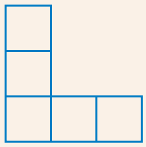
EXAMPLE 4

For this solid, draw the front, left and top views.



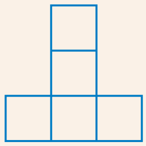
Solution

Imagine you are standing in front of the solid – this is what you would see.



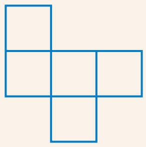
Front view

Imagine you are standing on the left of the solid – this is what you would see.



Left view

Looking down at the solid from above – this is what you would see.



Top view

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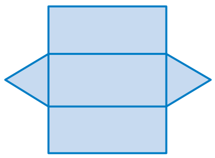
Exercise 2.05 From 3D to 2D

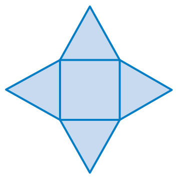
Example 3

1 Match each net in parts **a** to **f** with the solid it makes in parts **A** to **F**. Write the name of each solid.

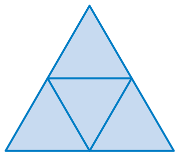
a



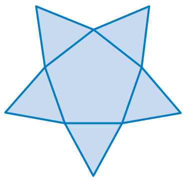
b



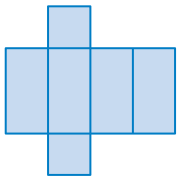
c



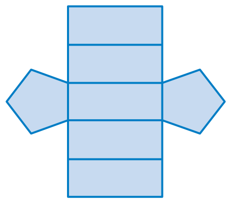
d



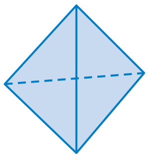
e



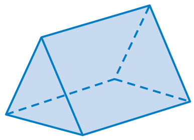
f



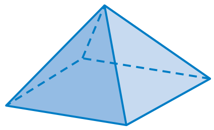
A



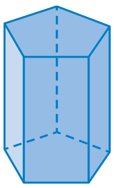
B



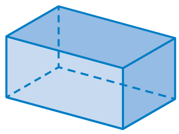
C



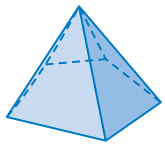
D



E

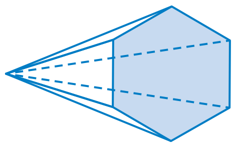


F

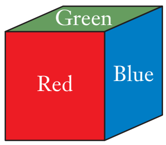


2 **a** What is the name of this solid?

b Draw the net of this solid.



3 Draw the net of this cube, showing the correct positions of the 3 coloured faces shown.



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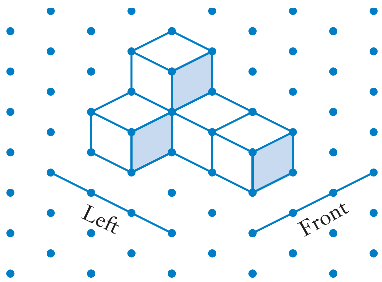
Example 4

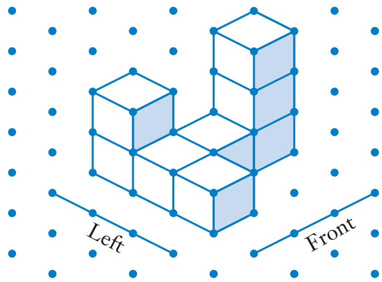
4 For each solid, draw each view.

a



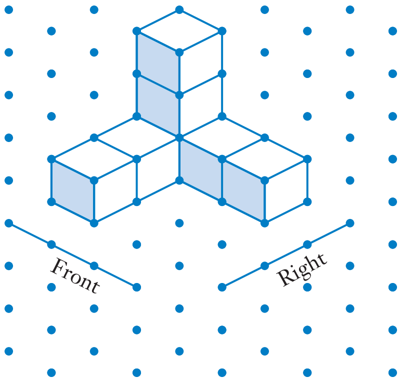
i front view
ii left view
iii top view

b



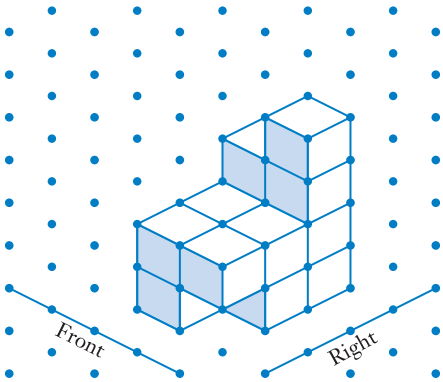
i left view
ii back view
iii top view

c



i front view
ii right view
iii top view

d



i back view
ii right view
iii top view

5 Draw the top and front view of each object.

a

Shutterstock.com/Maxx-Studio

b

Shutterstock.com/Vereshchagin Dmitry

c

Shutterstock.com/Olga Kovalenko

d

Shutterstock.com/Africa Studio

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6 Sketch the front, left and right view of this children's cubby house.

Photos courtesy Sue Thomson

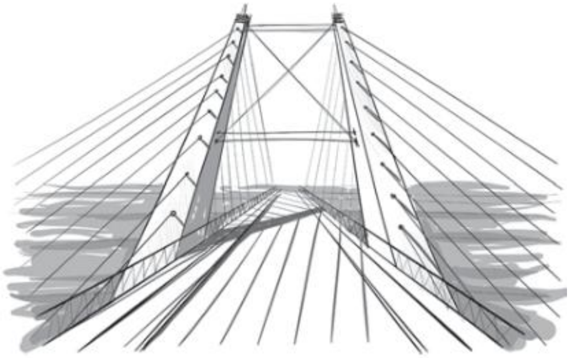
7 Sketch the front, left and 'bird's eye' view of this small country church.

'Bird's eye view' means the top view.

Photos courtesy Sue Thomson

2.06 Perspective drawings

In **perspective drawings**, objects that are further away are drawn smaller than objects that are up close. If a solid shape is drawn in perspective, it will have a vanishing point. Sometimes pictures can have 2 or 3 vanishing points.



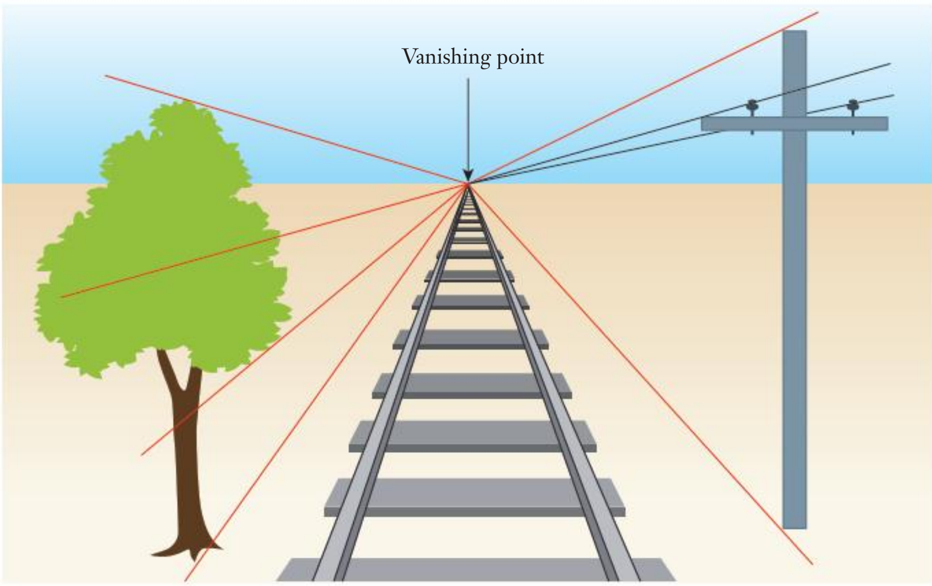
Exercise 2.06 Perspective drawings

This exercise can also be downloaded as a worksheet from NelsonNet.

WS Perspective drawings

1 This straight railway track is drawn in perspective. The point on the horizon where the lines meet and the railway seems to disappear is called the 'vanishing point'. Notice that the sleepers (rectangular planks) appear to be smaller and closer together near the vanishing point.

Complete a row of trees on the left of the track and a row of telegraph poles on the right.



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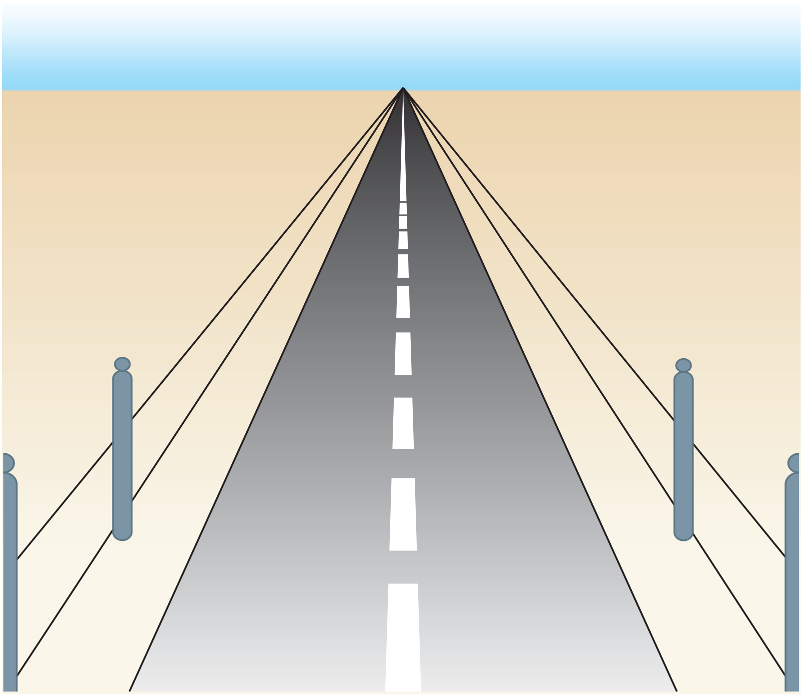
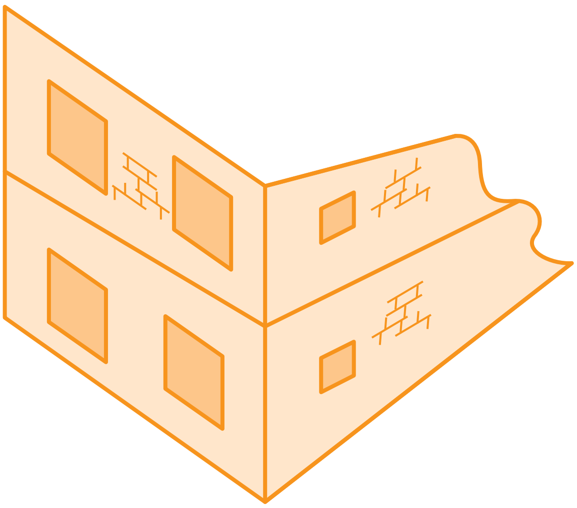
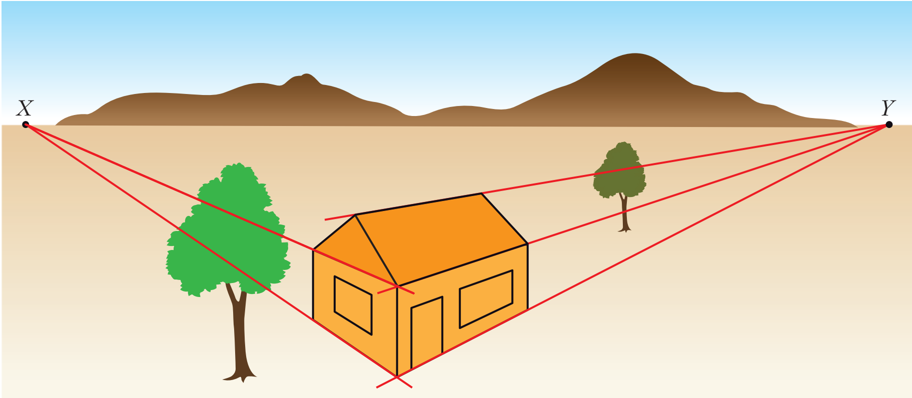
2 Copy this diagram and complete the drawing for the fences.



3 Copy and complete this perspective drawing of a building by drawing lines to the vanishing point.

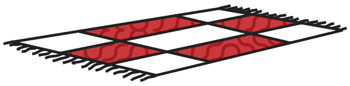


4 This perspective drawing has two vanishing points X and Y.

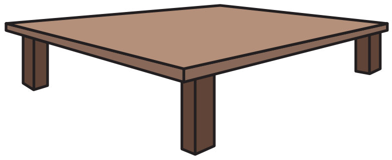


Copy each perspective drawing and find the two vanishing points for each one.

a

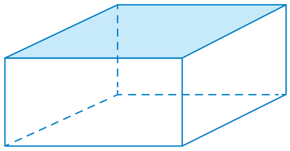


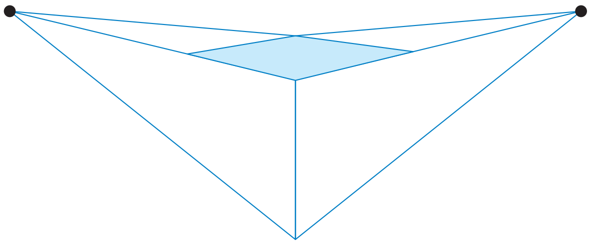
b



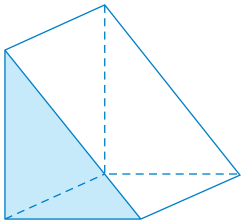
5 Copy and complete the perspective drawing of each prism using the lines and vanishing points shown.

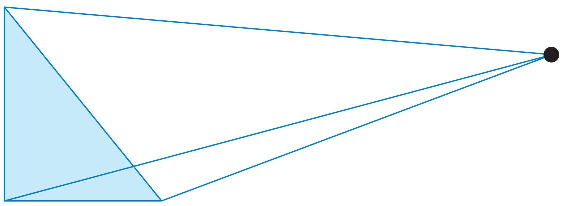
a



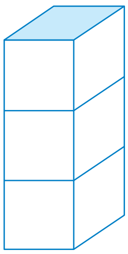


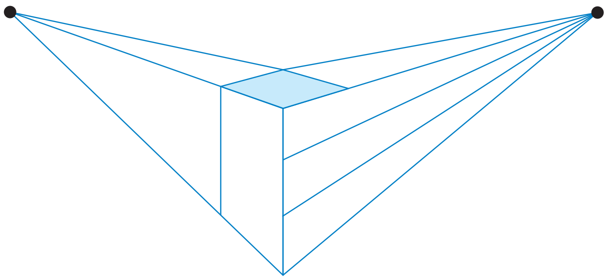
b





c





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INVESTIGATION

PERSPECTIVE DRAWING IN ART

1 Search the Internet for examples of perspective drawing in art. Find examples that have one vanishing point and two vanishing points.

2 Print copies of two pictures with:

a one vanishing point **b** two vanishing points

3 On each example, draw in the lines to the vanishing points.

4 If you are artistic, draw your own picture with vanishing point(s).

iStock.com/KreangchaiRungfamai

Anamorphic street art at a shopping mall

KEYWORD ACTIVITY

DEFINITION MATCH

Match each word with its definition

	Words		**Definitions**
1	net	**A**	A solid with all flat faces
2	polyhedron	**B**	Describes an object that has length, width and height
3	prism	**C**	A solid that comes to a point
4	pyramid	**D**	Describes an object that has length and width only
5	quadrilateral	**E**	A solid whose cross-section is the same from end to end
6	3D	**F**	A 4-sided shape
7	triangle	**G**	What you get when you “flatten” a solid into its faces
8	2D	**H**	A 3-sided shape

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SOLUTION TO THE CHAPTER PROBLEM

Problem

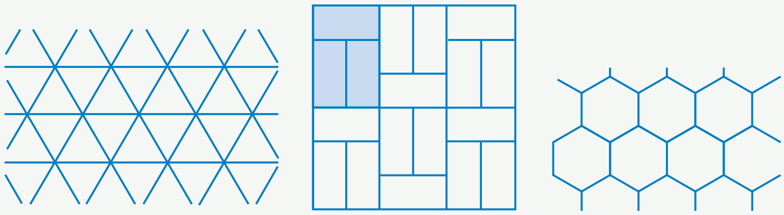
Andrew and Robyn are renovating their bathroom. They want to have one feature wall with tiles that are NOT square. What shapes could the tiles be? Create some designs for their feature wall.

Solution

Work out what shapes can be used:

The shapes will need to fit together to cover the wall. We could use equilateral triangles, rectangles, parallelograms, other quadrilaterals or even regular hexagons. However tiles usually come as squares, rectangles or hexagons. So we would probably use a triangular, rectangular or hexagonal tile for the feature wall.

Designs



Many other designs are possible if we mix the shapes together.

Andrew and Robyn could search online to find a design they like.

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2. TEST YOURSELF

The shape of our world

Practice quiz

Exercise 2.01

1 Draw a neat sketch of each 2D shape.

a A triangle that is right-angled and isosceles

b A hexagon

c A quadrilateral with 4 right angles

d A triangle that is scalene and obtuse-angled

e A quadrilateral whose diagonals cross at right angles

f An octagon

Exercise 2.01

2 What polygon am I? There may be more than one answer.

a I have 3 sides and all of my angles are equal.

b I am a quadrilateral with both pairs of opposite sides parallel.

c I have 5 sides.

d I have 4 sides and my diagonals bisect one another.

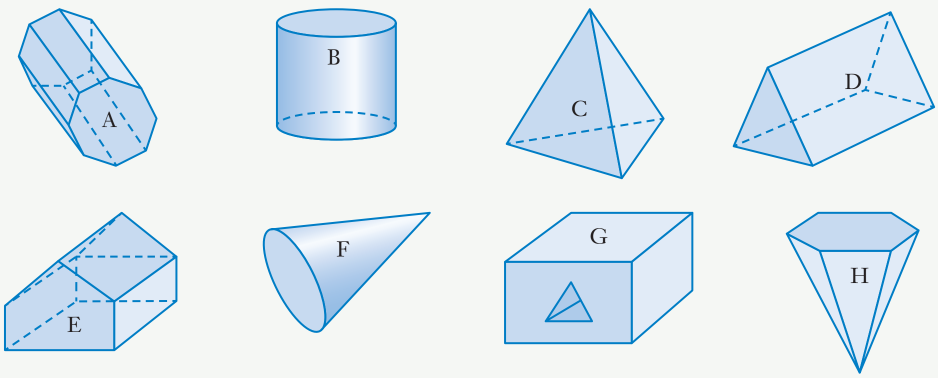
e I am a quadrilateral with one pair of parallel sides.

f I have 3 sides. My angles are 60°, 80° and 40°.

Exercise 2.02

3 Which of these shapes are:

a prisms? **b** pyramids? **c** solids with curved faces?



Exercise 2.03

4 Draw the logo described as follows.

The logo is a square. One diagonal is drawn. From one end of the diagonal, lines are drawn to the midpoints of the 2 opposite sides. Write your initials in 2 of the triangles formed.

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Exercise 2.04

5 Copy each diagram on isometric dot paper.

WS

Isometric dot paper

a

b

Exercise 2.04

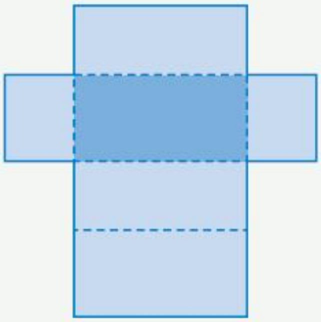
6 Draw each solid using the method shown in Exercise 2.04.

a rectangular prism **b** cylinder

Exercise 2.05

7 Name the solid that can be made from each net.

a



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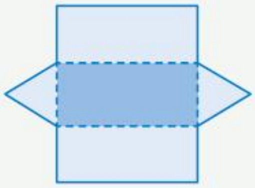

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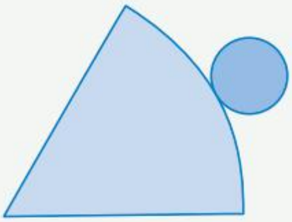

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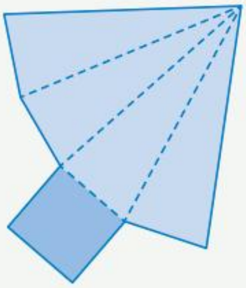

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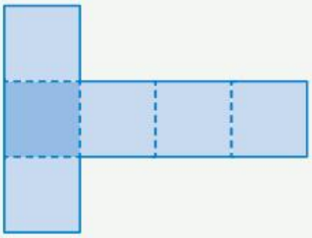

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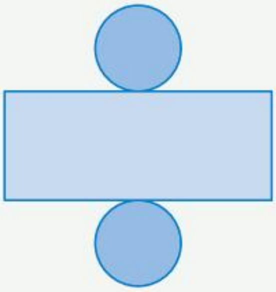

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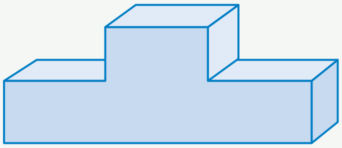
8 This diagram shows a podium for awarding medals to competitors at a sports tournament.

Exercise 2.05



For the podium, draw:

a the right view **b** the front view **c** the top view

9 Copy this diagram and complete the stairs and handrail.

Exercise 2.06

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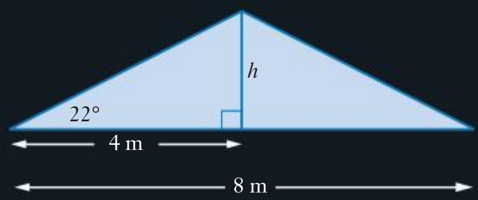
3.

SO YOU'VE GOT A RIGHT ANGLE

Chapter problem

Denis is building a garage. The garage is 8 m wide and the pitch of the roof is 22°. How high does he need to make the truss, h, correct to 2 decimal places?



iStockphoto/jhorrocks

WHAT WILL WE DO IN THIS CHAPTER?

- Solve practical problems using Pythagoras' theorem
- Use trigonometry to calculate the lengths of sides in right-angled triangles, including the hypotenuse
- Use trigonometry to calculate the sizes of angles in right-angled triangles
- Use trigonometry to solve practical problems, including those involving angles of elevation and depression
- Describe direction using compass and true bearings, and use trigonometry to solve problems involving bearings*

HOW ARE WE EVER GOING TO USE THIS?

- Determine a length when we can't measure it
- Many people who work in trades, for example, carpenters, builders and land surveyors, use right-angled triangle calculations in their work

3.01 Pythagoras' theorem

Pythagoras' theorem

Pythagoras' puzzle

WS Pythagoras' leopard

WS Pythagoras' problems

WS Pythagoras' theorem time trial

WS Pythagorean two-step problems

WS Applications of Pythagoras' theorem

Pythagoras was an ancient Greek mathematician who lived from 580 to 500 BCE. **Pythagoras' theorem** is named after him, even though no-one is sure whether it was Pythagoras himself or one of his followers who proved the theorem. What we do know is that mathematicians from different ancient civilisations knew about the theorem well before Pythagoras himself.

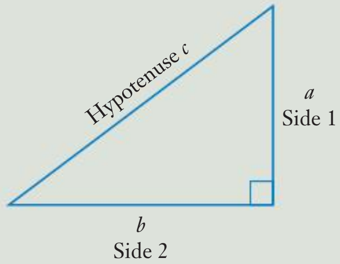
Pythagoras' theorem

In a right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

$$(\text{hypotenuse})^2 = (\text{side } 1)^2 + (\text{side } 2)^2$$

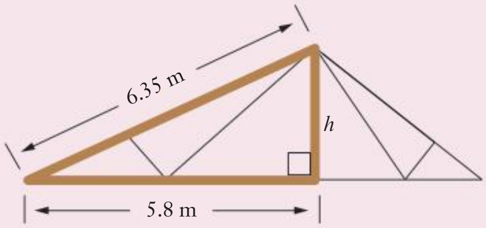
$$c^2 = a^2 + b^2$$

The **hypotenuse** is the longest side of a right-angled triangle.



EXAMPLE 1

Use Pythagoras' theorem to find the height (h) of this roof truss, correct to one decimal place.



Solution

The hypotenuse is 6.35 and one side is 5.8.

Pythagoras' theorem is $c^2 = a^2 + b^2$.

Solve the equation for h.

$$\begin{aligned} 6.35^2 &= 5.8^2 + h^2 \\ 6.35^2 - 5.8^2 &= h^2 \\ h^2 &= 6.6825 \\ h &= \sqrt{6.6825} \\ &= 2.585 \\ &\approx 2.6 \text{ m} \end{aligned}$$

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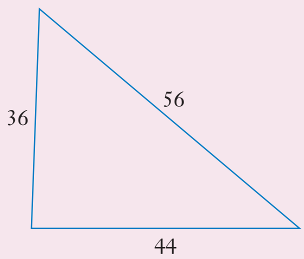
We can also use Pythagoras' theorem to test whether a triangle contains a right angle. If the theorem works, there's a right angle. If the theorem doesn't work, the triangle isn't right-angled.

EXAMPLE 2

Is this triangle right-angled?



Solution

We need to check whether $56^2 = 36^2 + 44^2$.

If both sides of the equation are equal, then the triangle is right-angled.

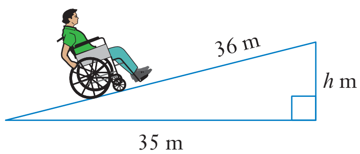
$56^2 = 3136$

$36^2 + 44^2 = 3232 \neq 3136$

Both sides are not equal.
Pythagoras' theorem doesn't work, so the triangle is not right-angled.

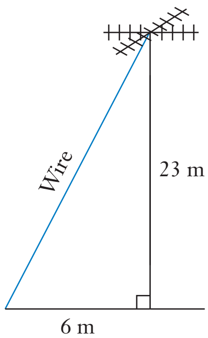
Exercise 3.01 Pythagoras' theorem

1 This wheelchair ramp is 36 m long and covers a horizontal distance of 35 m. Calculate the rise, h m, of the ramp, correct to 1 decimal place.



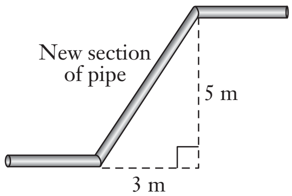
Example 1

2 What length of wire is required to connect the top of a 23 m TV antenna to a hook 6 m from the base of the antenna? Answer correct to one decimal place.



3 Tomasz installs a new section of pipe to join 2 existing pipes. Calculate the length of the new section of pipe. Express your answer in metres, correct to the nearest millimetre.

When a measurement is in metres, the nearest mm means 3 decimal places. The nearest cm is 2 decimal places.



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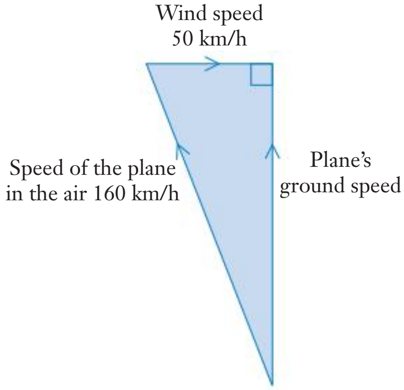
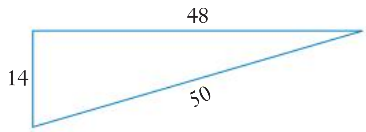
4 Ellie is flying a small plane at 160 km/h against a 50 km/h wind, as shown in the diagram. Calculate the plane's ground speed, correct to the nearest km/h.



Example 2

5 Use Pythagoras' theorem to test whether each triangle is right-angled.

a



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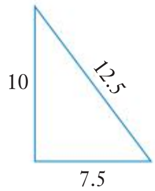

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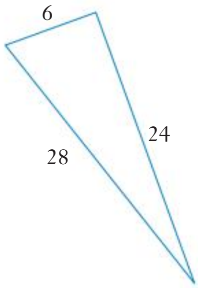

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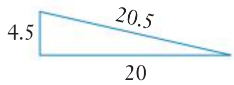
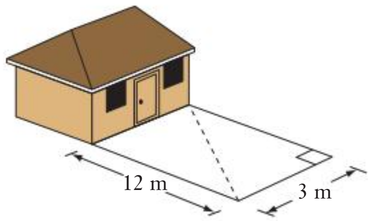
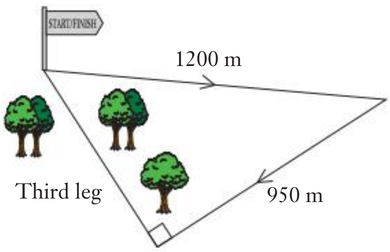


6 Mohammed is laying a concrete slab 12 m by 3 m in front of his shed. He uses Pythagoras' theorem to check that the corners of the slab are right angles. How long should the diagonal be? Express your answer in metres, correct to the nearest centimetre.



7 The school cross-country course is in the shape of a right-angled triangle. The first leg is 1200 m and the second leg is 950 m. The third leg, through thick scrub, is difficult to measure. Calculate its length, correct to the nearest metre.



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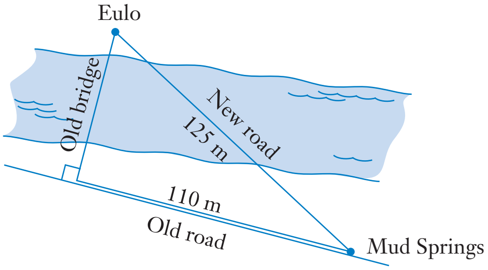
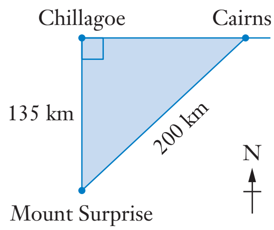
8 Bella had to cross the river to get from Mud Springs to Eulo. The new road across the river was closed for repairs so she had to use the old bridge. How much further did she have to travel using the old road and old bridge compared to the direct route across the new bridge? Answer correct to one decimal place.



9 Joe is the pilot of a small plane. He planned to fly 200 km from Mount Surprise to Cairns. Because of poor weather conditions between Mount Surprise and Cairns, Joe flew 135 km due north to Chillagoe then turned due east and flew to Cairns. Calculate the distance from Chillagoe to Cairns, correct to the nearest kilometre.

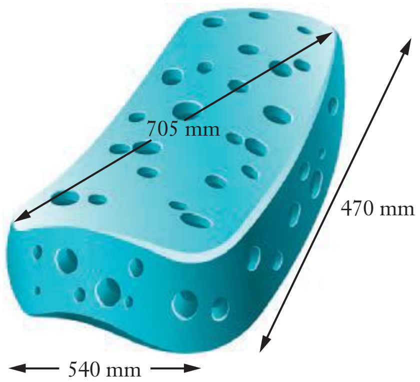


10 The hypotenuse of a right-angled triangle is 24 m. Draw 2 possible triangles, showing the lengths of the other 2 sides, correct to 2 decimal places.

INVESTIGATION

CUTTING RECTANGLES

Renée works in an upholstery business. One of her jobs is cutting out rectangular pieces of foam to make seat cushions. She always has a problem judging whether the cut foam is square (square means 'at right angles').



- Renée thinks the foam in the diagram is square. Is she right?
- Describe a process Renée could use to check whether her foam blocks are square.

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3.02 The sides of a right-angled triangle

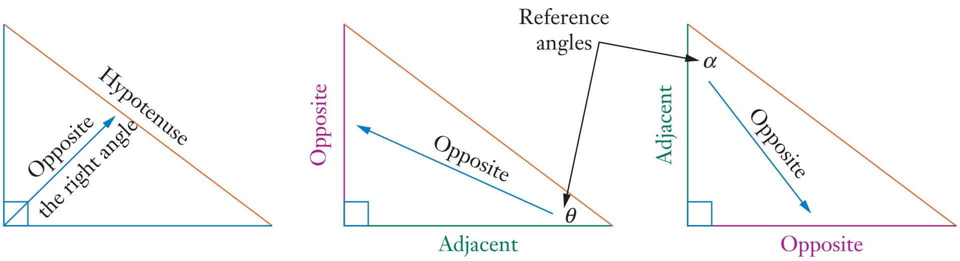
The word 'trigonometry' comes from two Greek words that mean 'the measurement of triangles'. **Trigonometry** is the branch of mathematics used to solve problems involving triangles.

In a right-angled triangle, the longest side is the side opposite the right angle and it is called the **hypotenuse**. The names of the other 2 sides are determined by the reference angle.

The **opposite side** is the side facing the reference angle.

The **adjacent side** is next to the reference angle.

Adjacent means 'next to'.

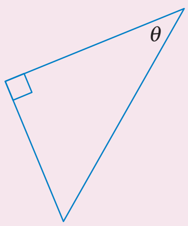


When naming the sides of a right-angled triangle, always start with the hypotenuse, because its position never changes. Then determine the opposite and adjacent sides.

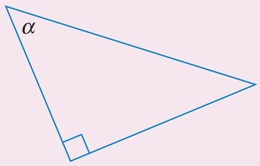
EXAMPLE 3

For each triangle, name its sides according to the marked angle.

a

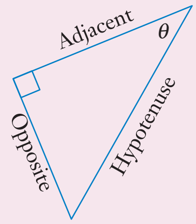


b

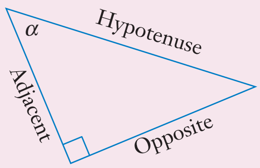


Solution

a The **hypotenuse** is always the longest side, opposite the right angle. The **opposite** side is on the other side of the triangle from θ. The **adjacent** side joins θ to the right angle.



b The **hypotenuse** is opposite the right angle. The **opposite** side is opposite α. The **adjacent** side joins α to the right angle.



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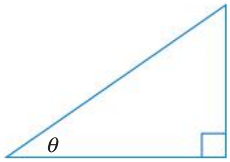
Exercise 3.02 The sides of a right-angled triangle

Example 3

1 Copy each triangle and name its sides according to the marked angle.

a



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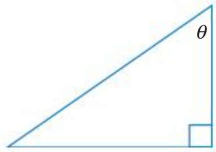

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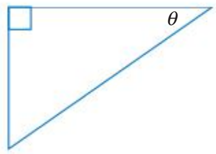
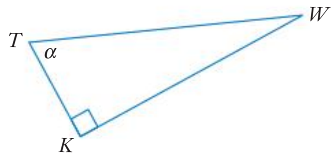


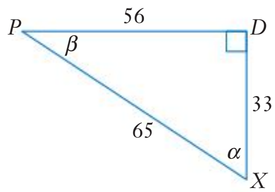
2 The sides of $\triangle TWK$ are TW, TK and WK.
Which side is opposite angle α?



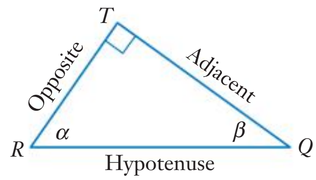
3 **a** How long is the hypotenuse in this triangle?

b How long is the opposite side to angle α?

c How long is the adjacent side to angle β?

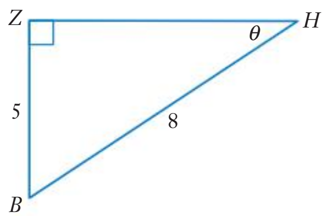


4 Which angle, α or β, was the reference angle for naming the sides of $\triangle RTQ$?

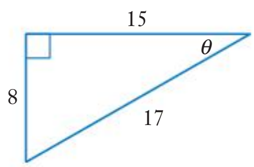


5 In $\triangle HZB$, the fraction $\dfrac{\text{opposite}}{\text{hypotenuse}}$ for θ is equal to $\dfrac{5}{8}$ because opposite = 5 and hypotenuse = 8.

Determine the fraction $\dfrac{\text{opposite}}{\text{hypotenuse}}$ for the marked angle in each triangle.



a



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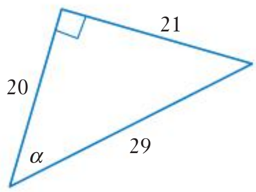

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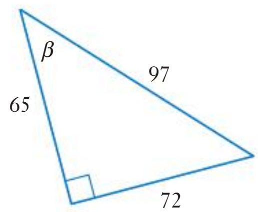

6 The side opposite angle β in a right-angled triangle is 3 cm long and the adjacent side is 5 cm long.

a Draw the triangle as accurately as possible.

b Measure the size of angle β, correct to the nearest degree.

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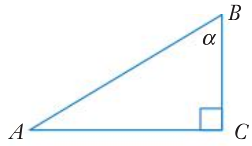
7 **a** Which side is the hypotenuse in this triangle: AB, AC or BC?

b What is the opposite side to angle α?

c What is the adjacent side to angle α?



3.03 The tangent ratio

The **tangent** ratio, abbreviated **tan**, is the ratio of the length of the **opposite side** to the length of the **adjacent side** for a reference angle in a right-angled triangle. It is the fraction $\frac{\text{opposite}}{\text{adjacent}}$.

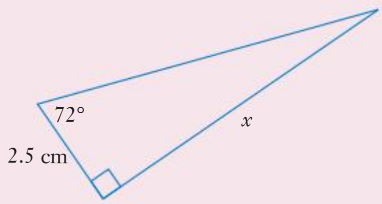
The tangent ratio

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

The tangent ratio can be used to calculate the length of an unknown side in a right-angled triangle.

EXAMPLE 4

Use the tangent ratio to find the length of x in the triangle. Express your answer correct to 2 decimal places.



Solution

For 72° in the triangle, x is the opposite side and 2.5 cm is the adjacent side.

Use the formula $\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$.

$$\tan 72° = \frac{x}{2.5}$$

Solve the equation.
First, swap sides, then multiply both sides by 2.5.

$$\frac{x}{2.5} = \tan 72°$$

$$\cancel{2.5} \times \frac{x}{\cancel{2.5}} = \tan 72° \times 2.5$$

$$x = 2.5 \tan 72°$$

Enter 2.5 tan 72 = on your calculator.

$$x = 7.69\,420\ldots$$
$$\approx 7.69 \text{ cm}$$

From the diagram, $x \approx 7.69$ cm seems a reasonable answer.

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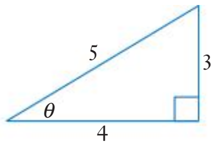
Make sure that your calculator is set in degrees mode DEG or D. If it is set to RAD or GRAD, your calculator will give you the wrong answer. Ask your teacher for help if needed.

Exercise 3.03 The tangent ratio

1 For each triangle, write tan θ as a fraction.

a



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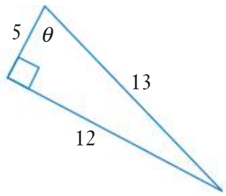
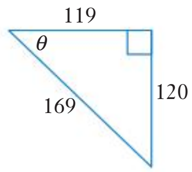
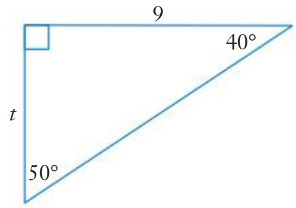


c



2 Accurately construct a triangle in which $\tan \theta = \frac{2}{3}$.

3 Copy and complete the working to calculate the value of t in the diagram. Express your answer correct to 1 decimal place.



$$\frac{t}{9} = \tan \square$$

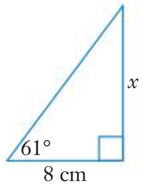
$$\square \times \frac{t}{9} = \tan \square \times \square$$

$$t \approx \square$$

4 Find the value of each variable, correct to 2 decimal places.

Example 4

a



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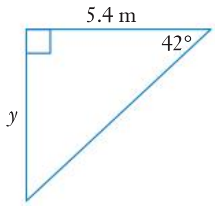

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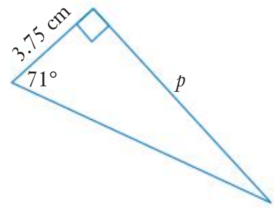

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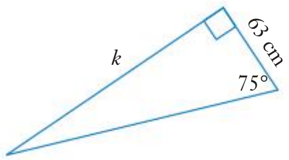
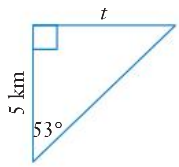
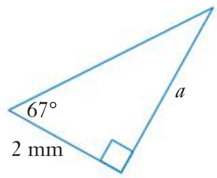


e



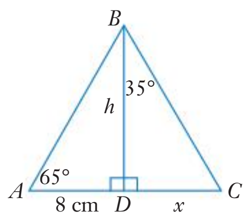
f



5 These 2 right-angled triangles share a side BD.

a Use $\triangle ABD$ to find h, correct to 2 decimal places.

b Use $\triangle BDC$ to find x, correct to one decimal place.



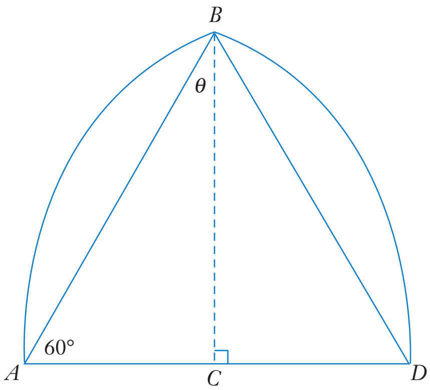
6 Gothic arches are a common feature in the design of Gothic cathedrals. Equilateral triangles are the design basis for most Gothic arches.

Shutterstock.com/Maigi

a What must be the size of angle θ in the diagram?

b One Gothic arch is 1.2 m high, so $BC = 1.2$ m. Calculate correct to 1 decimal place the length of AC and so find the width of arch AD.

Remember, the angles in ΔABC add to 180°. Also, ΔABD is equilateral.



Chapter problem

You've covered the skills required to solve the chapter problem. Can you solve it now?

ISBN 9780170443975

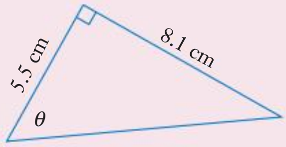
3.04 Using tan to find an angle

We can use the formula $\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}}$ to calculate angles as well as sides in a right-angled triangle.

WS Using the tan ratio to calculate an angle

EXAMPLE 5

Calculate the size of angle θ, correct to the nearest degree.



Solution

The opposite side is 8.1 cm and the adjacent side is 5.5 cm.

Use $\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}}$ to find the angle θ. $\tan\theta = \dfrac{8.1}{5.5}$

To 'undo the tan' we need to press SHIFT tan on our calculator.

Enter SHIFT tan 8.1 ÷ 5.5 =. $\theta = 55.8230\ldots$

Round to the nearest degree. $= 56°$

From the diagram, $\theta = 56°$ seems a reasonable answer.

Exercise 3.04 Using tan to find an angle

Example 5

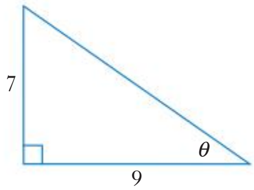
1 Use your calculator to find the value of θ, correct to the nearest degree.

a $\tan\theta = 0.86$ **b** $\tan\theta = 1.07$ **c** $\tan\theta = \frac{3}{4}$

d $\tan\theta = \frac{3}{5}$ **e** $\tan\theta = \frac{11}{8}$ **f** $\tan\theta = 1\frac{1}{3}$

2 Use the tan ratio to find the size of each marked angle, correct to the nearest degree.

a



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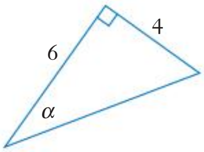

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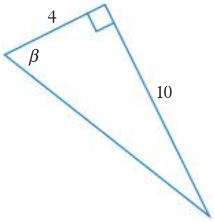

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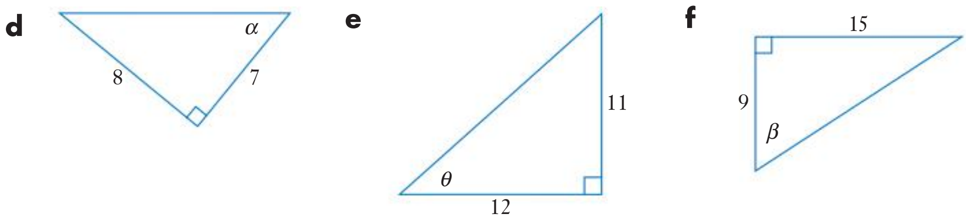


3 The shorter (non-hypotenuse) sides of a right-angled triangle are 4 cm and 6 cm long. Calculate the sizes of all the angles in the triangle, correct to the nearest degree.

4 The shorter sides in a right-angled triangle are both 5 cm long.

a Find the sizes of the angles in the triangle.

b Explain why $\tan 45° = 1$.

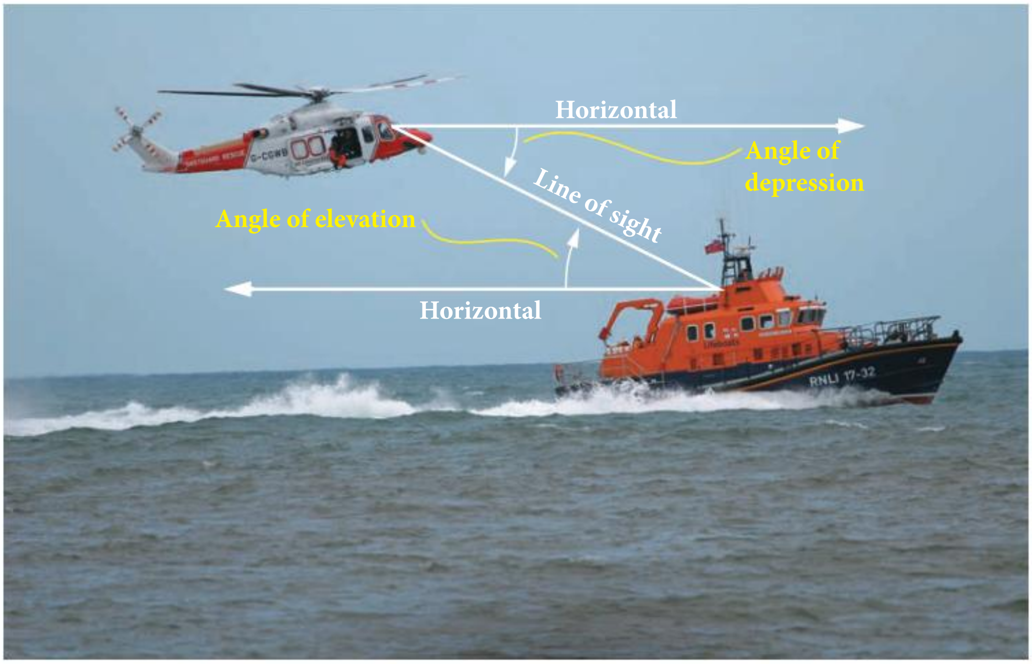
3.05 Angles of elevation and depression

WS Angles of elevation and depression

Angles of elevation and depression

Angle of depression

Angles of elevation and depression are used regularly to solve practical measurement problems.



Alamy Stock Photo/stephen How

The **angle of elevation** is the angle the eye turns **up** from the horizontal to look at an object in a higher position.

The **angle of depression** is the angle the eye turns **down** from the horizontal to look at an object in a lower position.

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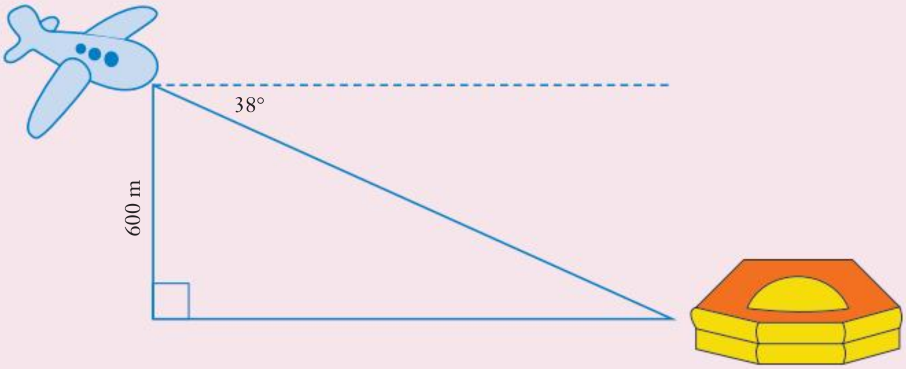
EXAMPLE 6

A pilot flying at a height of 600 m saw a life raft in the sea at an angle of depression of 38°. Calculate the horizontal distance from the plane to the life raft, correct to the nearest metre.

Watch out! It's a common error to write the angle of depression as the top angle in the triangle. The angle of depression is outside the triangle.

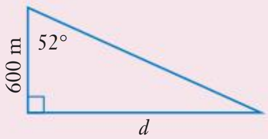


Solution

Let d be the horizontal distance from the plane to the life raft.

The angle of depression and the angle at the top of the triangle add to 90°.

The top angle $= 90° - 38°$
$= 52°$



Use $\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$ because the opposite and the adjacent sides are involved.

$$\tan 52° = \frac{d}{600}$$
$$d = 600 \times \tan 52°$$
$$= 767.9649\ldots$$
$$\approx 768 \text{ m}$$

Write the answer.

The horizontal distance from the plane to the life raft is 768 m.

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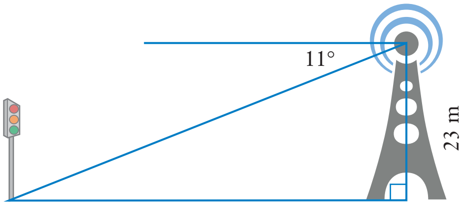
Exercise 3.05 Angles of elevation and depression

In this exercise, express your answers correct to one decimal place (for lengths) or the nearest degree (for angles), unless instructed otherwise.

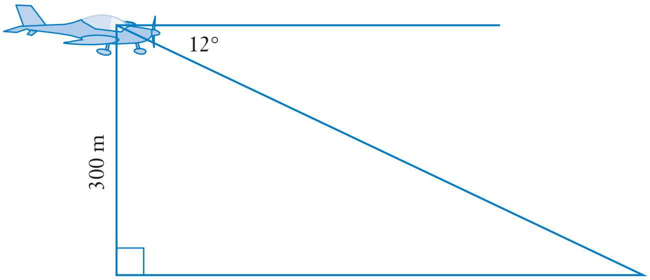
Example 6

1 From the top of a mobile phone tower 23 m high, the angle of depression to the bottom of a set of traffic lights is 11°. How far is the set of traffic lights from the base of the mobile phone tower?

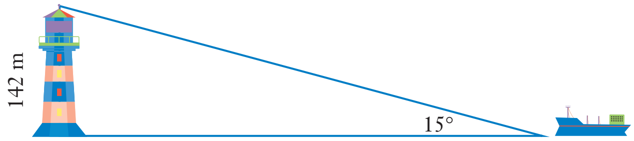


Usually the angle of depression is NOT the top angle in the triangle. Subtract the angle of depression from 90° to determine the size of the top angle.

2 Samantha is flying an ultra-light aircraft at a height of 300 m. Her angle of depression to the landing strip is 12°. Calculate the horizontal distance from the plane to the landing strip.



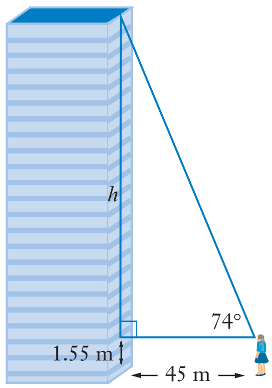
3 A navigation chart shows that the top of a lighthouse is 142 m above sea level. A ship's navigator measured the angle of elevation to the top of the lighthouse as 15°. How far is the ship from the base of the lighthouse?



4 Sabira is 45 m away from a tall office block. She measured the angle of elevation to the top of the office block as 74°.

a Calculate the height, h, correct to 3 decimal places.

b Sabira's eyes are 1.55 m above the ground. Calculate the height of the building.



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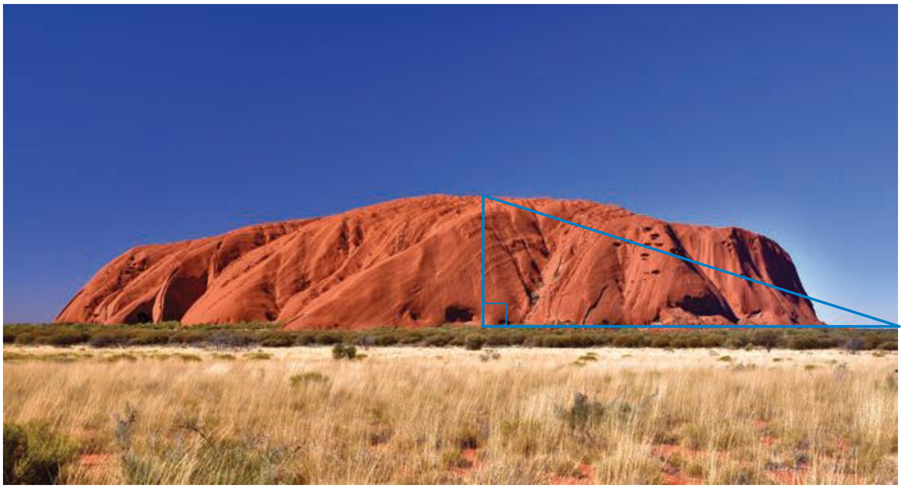
5 When Simon was 810 m away from Uluru, he measured the angle of elevation to the top of the rock as 20°.



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a Copy and complete the diagram, showing Simon's measurements on it.

b Calculate, correct to the nearest metre, the height of Uluru.

6 The third level of the Eiffel Tower in Paris is 276 m above the ground. When Elyse was standing on the third level of the tower, she measured the angle of depression to a group of her friends on the ground as 19°. How far were Elyse's friends from the base of the Eiffel Tower below her?

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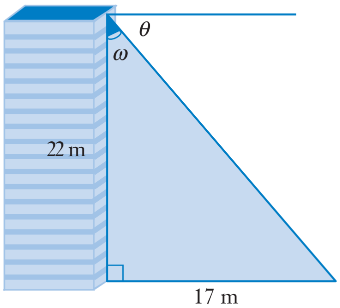
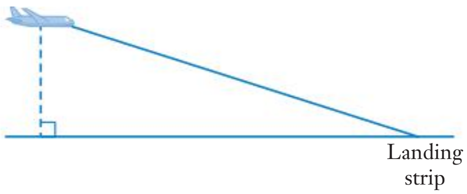
7 The front of an office block has security lights that come on when they detect movement. The lights are 22 m above the ground, and they light up a 17 m wide strip in front of the building.

a Calculate the size of angle ω.

b Determine the size of angle θ, the light's angle of depression.



8 From a small plane flying at a height of 800 m, the angle of depression to the landing strip is 14°.

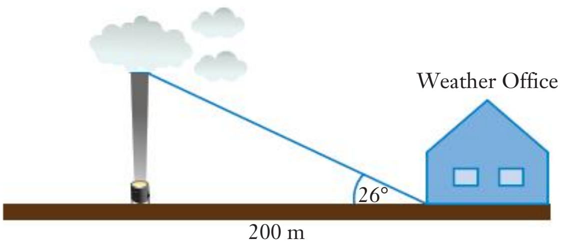


a Copy the diagram and show the position of the angle of depression.

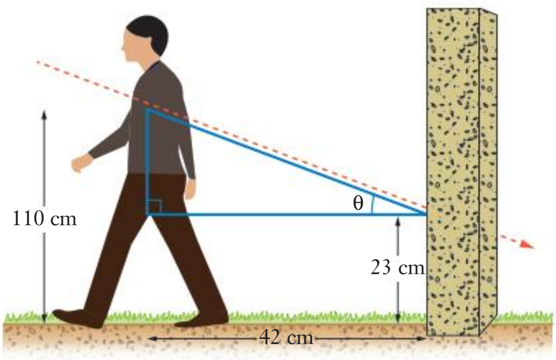
b Calculate, correct to the nearest metre, the horizontal distance from the plane to the landing strip.

9 During World War II, searchlights were used with trigonometry to calculate the height of clouds.

Calculate the height of the clouds in the diagram below when the angle of elevation to the clouds is 26°.



10 Forensic experts are investigating a murder scene. A man was standing up when he was shot and the bullet went through him 110 cm above the ground. The bullet then lodged in a wall 42 cm behind the man and 23 cm above the ground.



a Calculate the length of the opposite side by subtracting 23 from 110.

b Calculate the angle of elevation from the bullet to where the gun was fired.

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3.06 The sine and cosine ratios

We use the **sine** and **cosine** ratios when calculations involve the hypotenuse.
The abbreviation for sine is **sin** and it is pronounced 'sign'.
The abbreviation for cosine is **cos**.

WS Trigonometric ratios

WS Trigonometric calculations

WS Identifying the correct trigonometric ratio

WS Finding an unknown angle

Finding an unknown side

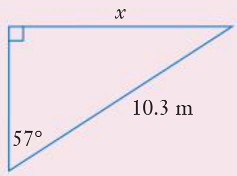
The sine and cosine ratios

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

EXAMPLE 7

Find the value of x in this triangle, correct to 1 decimal place.



Solution

x is the opposite side and 10.3 m is the hypotenuse.

$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$

$$\sin 57° = \frac{x}{10.3}$$

$$\frac{x}{10.3} = \sin 57°$$

Put the values in the formula, then solve the equation.

Multiply both sides by 10.3.

$$\cancel{10.3} \times \frac{x}{\cancel{10.3}} = \sin 57° \times 10.3$$

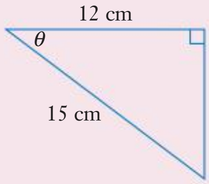
Evaluate x.

$$\begin{aligned} x &= 10.3 \sin 57° \\ &= 8.6383 \ldots \\ &\approx 8.6 \text{ m} \end{aligned}$$

From the diagram, $x \approx 8.6$ m seems a reasonable answer.

EXAMPLE 8

Calculate the size of θ, correct to the nearest degree.



Solution

For angle θ, 12 cm is the adjacent side. 15 cm is the hypotenuse.

$$\cos \theta = \frac{12}{15}$$

$$\cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

Use the 'undo the cos' keys on a calculator:

SHIFT COS 12 ÷ 15 =.

$$\theta = 36.8698 \ldots$$
$$\approx 37°$$

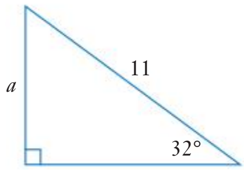
From the diagram, $\theta = 37°$ seems a reasonable answer.

Exercise 3.06 The sine and cosine ratios

Example 7

1 Use the sin ratio to calculate the value of each variable, correct to one decimal place.

a



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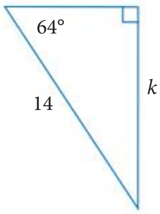
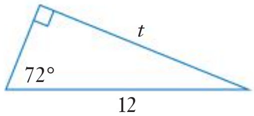
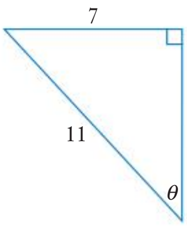


c



2 Use the sin ratio to determine the size of each marked angle, correct to the nearest degree.

a



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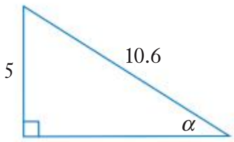
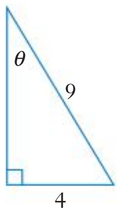


c



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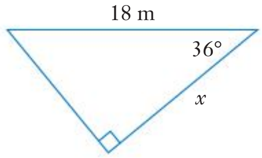
3 Use the cos ratio to determine the values of each variable, correct to the nearest metre. Example 8

a



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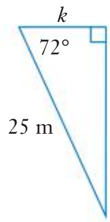

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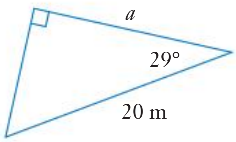
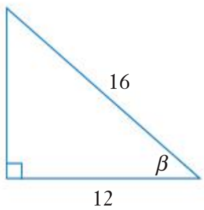


4 Calculate the value of each marked angle using the cos ratio, correct to the nearest degree.

a



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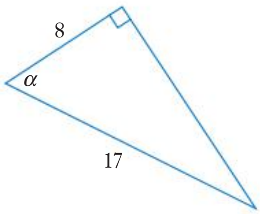

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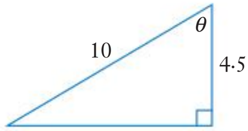
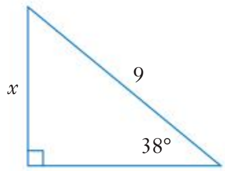


5 For each triangle, choose the sin or cos ratio, then calculate the value of x correct to 1 decimal place.

When the side you know is the hypotenuse, you will need to use sin if the side with the variable is opposite and cos if it is adjacent.

a



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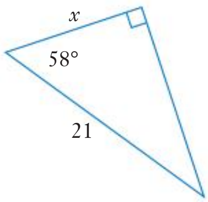

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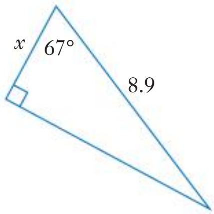

3.07 Sine, cos or tan?

One way to remember the trigonometry ratio formulas is with the phrase 'Only half an hour of algebra' and the trigonometry keys on your calculator in order:

Only Half	An Hour	Of Algebra
sin $= \dfrac{\text{opposite}}{\text{hypotenuse}}$	cos $= \dfrac{\text{adjacent}}{\text{hypotenuse}}$	tan $= \dfrac{\text{opposite}}{\text{adjacent}}$

Some students like to say SOH-CAH-TOA to help them remember the formulas.

Choose the memory method that works for you.

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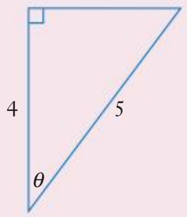
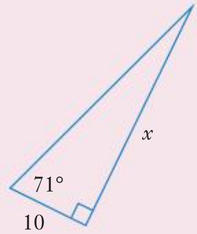
EXAMPLE 9

Which trigonometry ratio can we use to find the value of the variable in each triangle?

a



b



Solution

a The marked sides are the adjacent, **a**, and the hypotenuse, **h**. '**A**n **h**our' is the middle section of the phrase, and the middle trig key on the calculator is cos.

The cos ratio can be used.

b The marked sides are the opposite, **o**, and the adjacent, **a**. '**O**f **a**lgebra' is the last section of the phrase, and the last trig key on the calculator is tan.

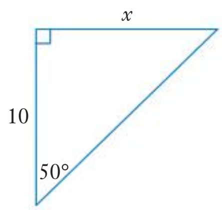
The tan ratio can be used.

Exercise 3.07 Sine, cos or tan?

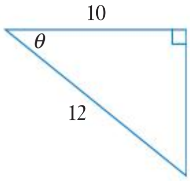
Example 9

1 In each triangle, which trigonometry ratio could be used to find the value of the variable?

a



b



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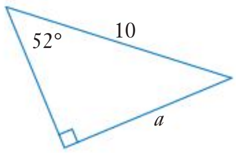
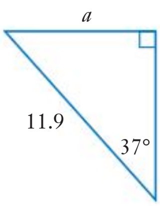
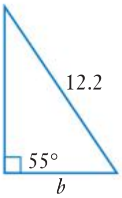


2 Find the value of each variable, correct to one decimal place.

a



b



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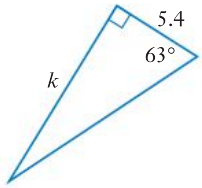

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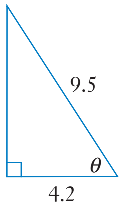
3 Determine the size of each marked angle, correct to the nearest degree.

a



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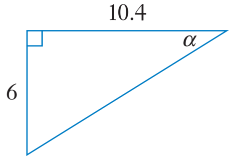
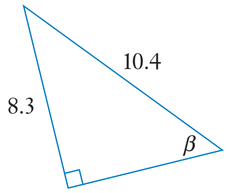
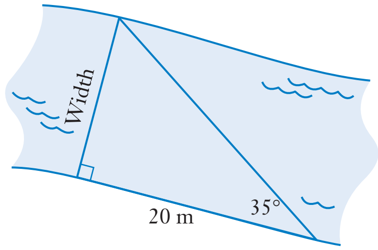


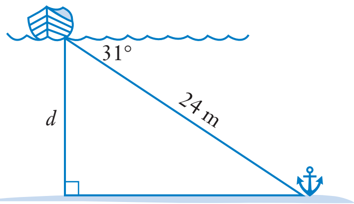
c



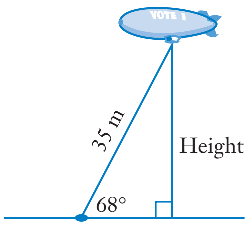
4 During a flood, Grant needed to work out the width of the river. The diagram shows the measurements he took. Use Grant's measurements to determine the width of the flooded river, correct to the nearest metre.



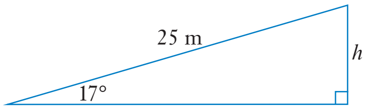
5 A boat's anchor chain is 24 m long, and it is making an angle of 31° with the top of the water. How deep, d m, is the water, correct to 1 decimal place?



6 An airship is anchored to the ground by a 35 m long rope. The angle between the rope and the ground is 68°. How high is the airship above the ground, correct to one decimal place?

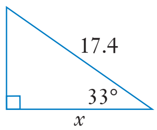


7 A skateboard ramp 25 m long leans at 17° to the horizontal. How much higher is one end of the ramp than the other, correct to one decimal place?



8 Find the value of each variable, correct to one decimal place.

a



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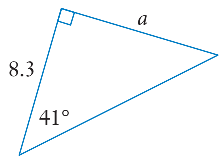

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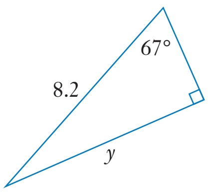

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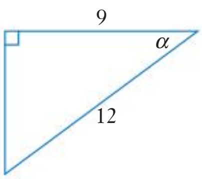
9 Determine the size of α, correct to the nearest degree.

a



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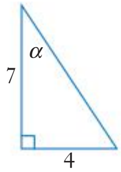

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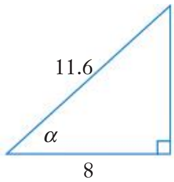


3.08 Finding the hypotenuse

A triggy riddle

Finding the hypotenuse

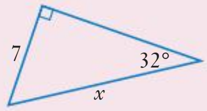
WS Finding an unknown side

WS Calculating lengths and angles

WS Mixed trig questions

EXAMPLE 10

Calculate the length of the hypotenuse, correct to one decimal place.



Solution

The sides involved are the opposite and the hypotenuse.

$\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}}$ $\qquad \sin 32° = \dfrac{7}{x}$

Take the reciprocal of both sides to put x on top. $\qquad \dfrac{1}{\sin 32°} = \dfrac{x}{7}$

Swap sides to put x on the left. $\qquad \dfrac{x}{7} = \dfrac{1}{\sin 32°}$

Multiply both sides of the equation by 7. $\qquad \cancel{7} \times \dfrac{x}{\cancel{7}} = \dfrac{1}{\sin 32°} \times 7$

$$x = \frac{7}{\sin 32°}$$

The calculator steps to find x are: $\qquad = 13.2095...$

7 [÷] [sin] 32 [=] $\qquad \approx 13.2$

From the diagram, $x \approx 13.2$ cm seems a reasonable answer.

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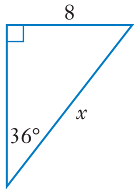
Exercise 3.08 Finding the hypotenuse

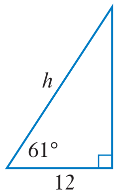
Example 10

1 Calculate the length of the hypotenuse in each triangle, correct to 1 decimal place.

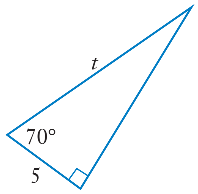
a



b

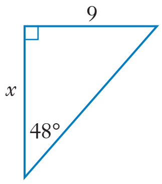


c



2 Determine, correct to one decimal place, the value of x in each triangle.

a



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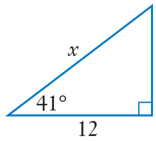

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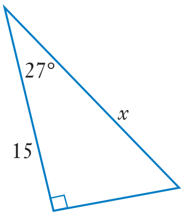
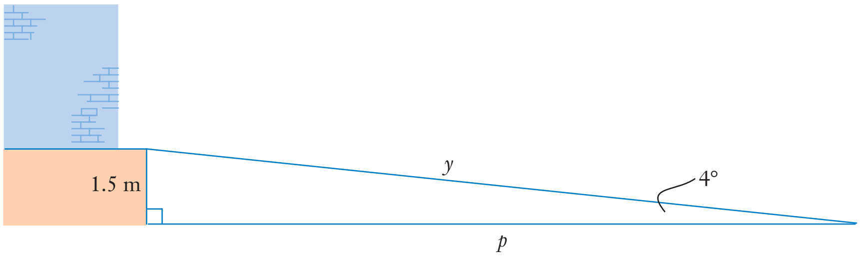


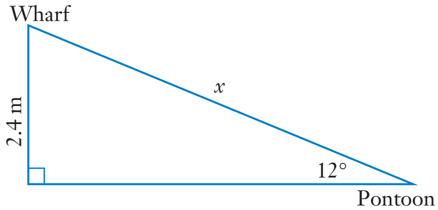
3 Scott is designing a wheelchair access ramp. The ramp will connect the car park with the building. The building is 1.5 m higher than the car park. The angle of inclination must not exceed 4°.



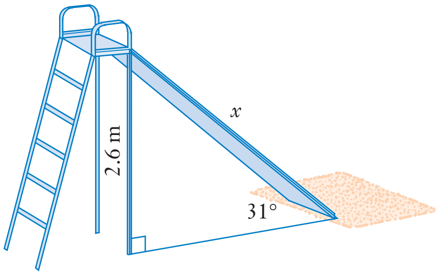
a Calculate the horizontal distance, p, required for the ramp, correct to 2 decimal places.

b How long is the ramp, y, correct to one decimal place?

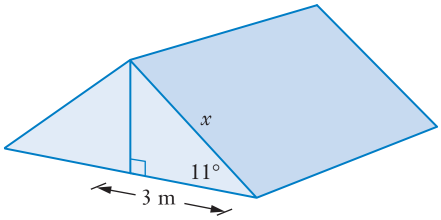
4 At low tide, a wharf is 2.4 m above the water. The ramp joining the wharf to a floating pontoon makes an angle of 12° to the horizontal. Calculate the length of the ramp, x, correct to 1 decimal place.



5 The council installs a children's slide in the park. The top of the side is 2.6 m high, and the slide makes an angle of 31° with the ground. Calculate the length of the slide, x, correct to 2 decimal places.

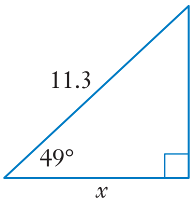


6 This diagram shows a shed roof. Calculate x, the length of each piece of iron sheeting required. Round your answer up to the nearest 10 cm.



7 Find the value of each variable, correct to one decimal place.

a



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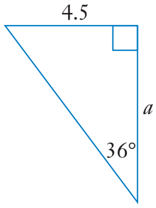

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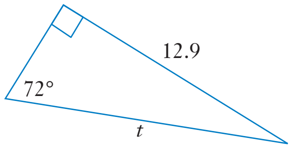

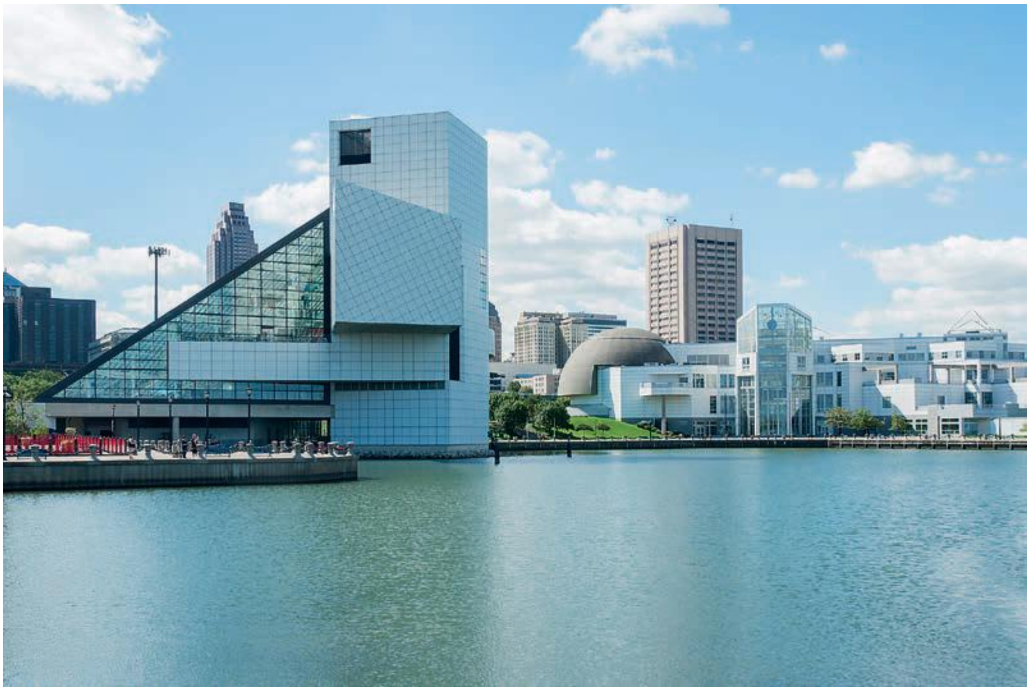

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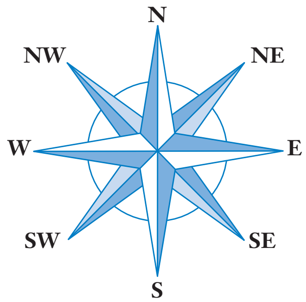
3.09 Compass bearings*

*Australian curriculum only, not in WA syllabus

The 4 major compass directions are north (N), east (E), south (S) and west (W). The directions halfway between these are northeast (NE), southeast (SE), southwest (SW) and northwest (NW). We use these 8 compass directions to describe wind direction, position of the Sun, shadows, and the path of a plane or ship.



DID YOU KNOW?

It's a common myth that our word 'news' was derived from a combination of the first letters of the directions **n**orth, **e**ast, **w**est and **s**outh. 'News' comes from the word 'new', short for 'new information'. 'New' became 'news' when there was more than one piece of new information.

EXAMPLE 11

iStock.com/pop_jop

a What compass direction is it from Coober Pedy to Port Augusta?

b In what direction is Coober Pedy from Port Augusta?

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Solution

a Position the centre of a compass rose on the location the direction is *from*. In this case, the bearing is from Cooper Pedy.

The direction to Port Augusta is in the middle of east and south. The direction from Cooper Pedy to Port Augusta is southeast.

b Now the bearing is from Port Augusta. Position the compass rose on Port Augusta.

The direction to Cooper Pedy is between north and west. Cooper Pedy is northwest of Port Augusta.

There are many directions that lie between north, south, east and west. If we want to specify a precise direction using a **compass bearing**, we write something like S 15° E, which means 'look south, then turn 15° towards the east'.

When we're using compass bearings, we always start by heading north or south, then give the number of degrees we have to turn towards the east or west.

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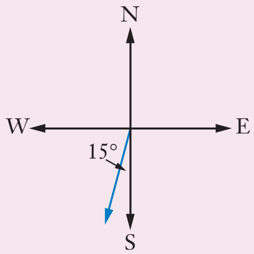
Compass bearings

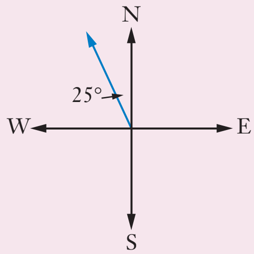
EXAMPLE 12

Write each direction as a compass bearing.

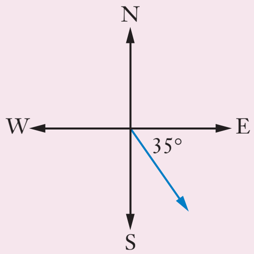
a



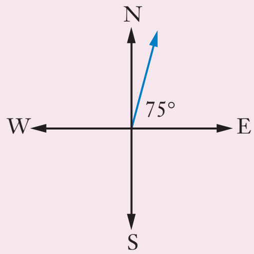
b



c



d



Solution

a This direction is towards the south, so it starts at S. It's 15° towards the west. The bearing ends with 15°W. — S 15° W

b This direction is initially towards the north with a turn of 25° towards the west. — N 25° W

c Calculate the angle between south and the direction. — S 55° E

$$\text{Angle} = 90^\circ - 35^\circ$$
$$= 55^\circ$$

The direction is towards south with a turn of 55° towards the east.

d The angle between north and the direction $= 90^\circ - 75^\circ$ — N 15° E

$$= 15^\circ$$

The direction is towards north with a turn of 15° towards the east.

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Exercise 3.09 Compass bearings

Example 11

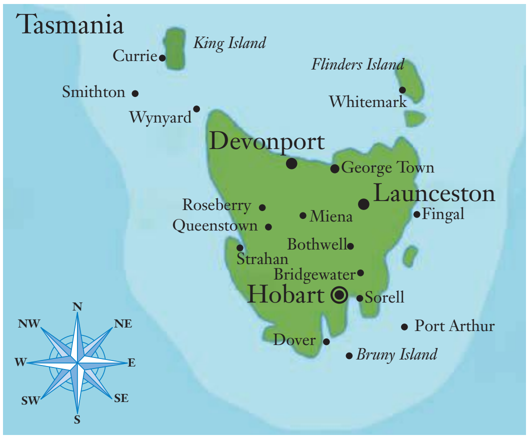
1 What is the compass direction:

a from Hobart to Roseberry?

b from Launceston to Bothwell?

c to Roseberry from Queenstown?

d to Launceston from George Town?



iStock.com/pop_jop

2 In 1986, following an explosion and fire at the Chernobyl nuclear power station, winds carried the radiation cloud over a vast area.

iStock.com/Kosmozoo

a What compass direction, NW, NE, SE or SW is it from Chernobyl to Austria?

b Easterly winds are winds that come from an easterly direction.

What countries experienced radiation clouds as a result of easterly winds?

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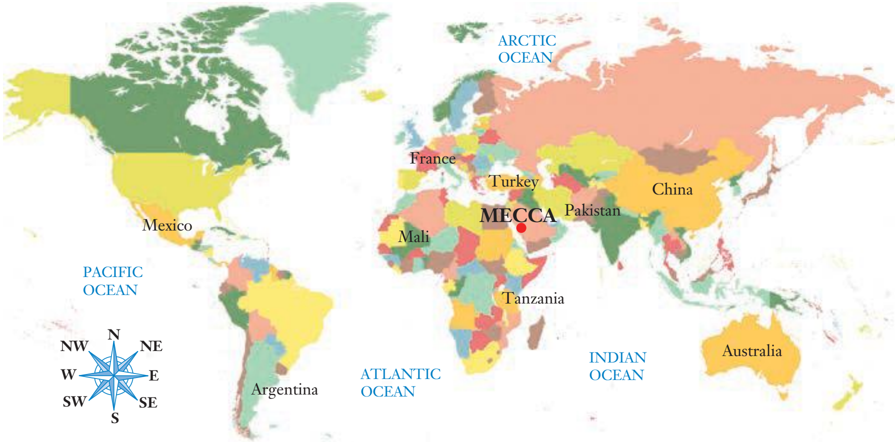
3 When Muslims pray, they face the central shrine in Mecca's Great Mosque, Islam's holiest place. What is the approximate compass direction (N, S, E, W, NE, SE, SW or NW) from each of the following places to Mecca?



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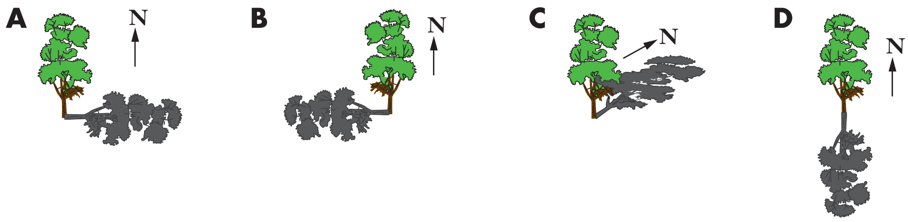
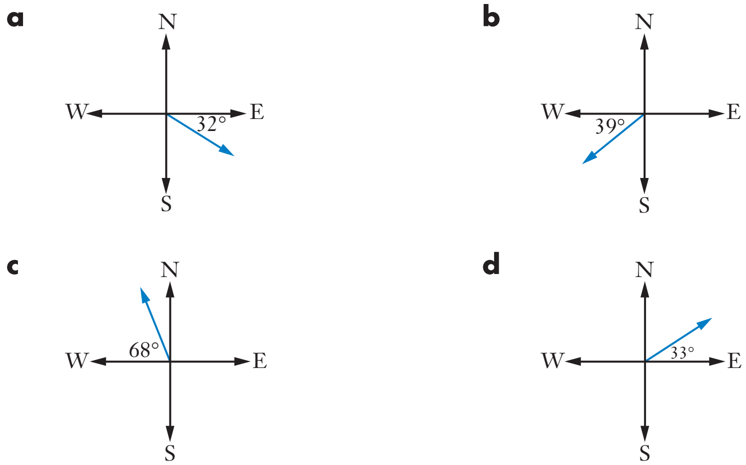
a Australia	**b** China	**c** Turkey
d Pakistan	**e** Mali	**f** Argentina
g France	**h** Tanzania	**i** Mexico

4 Late in the afternoon, the Sun is in the west. Which one of the diagrams shows the correct position of the tree's shadow?

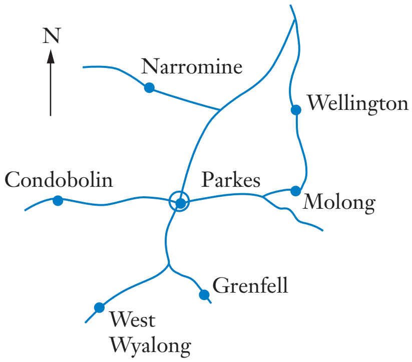


5 Use compass bearings to describe the direction in each diagram.

Example 12



6 A westerly wind brought smoke from a bushfire to Parkes.



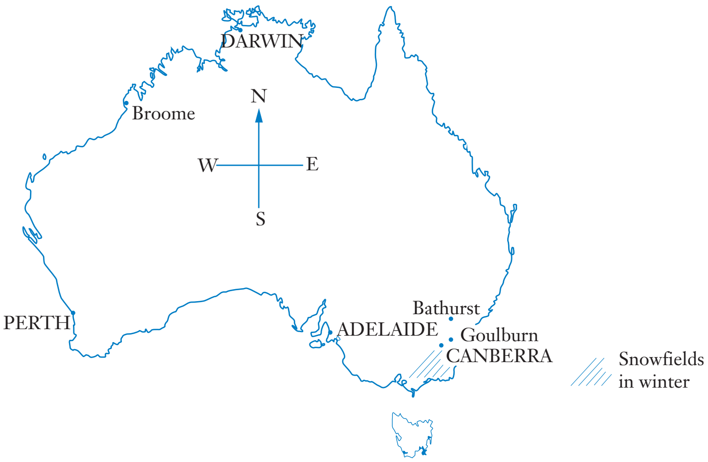
A westerly wind comes from the west.

a Near which town on the map could the bushfire be located?

b If the wind keeps blowing in the same direction, which other town will receive smoke from the fire?

c The wind changed direction and smoke went in the direction of S 10° E. Which town experienced smoke?

7 Winds from over the ocean carry moisture and are likely to bring rain, but winds from over a land mass are likely to be dry. The temperature of the land the winds blow over determines whether the winds will be hot or cold. In winter, winds from over the snowfields will be cold, but summer winds from Central Australia are likely to be dry and hot.



Are the following winds likely to be hot, cold, bring rain or fine weather?

Remember! A wind's direction is the direction it has come *from*.

a westerly winds in Perth

b westerly winds in Bathurst in summer

c northeasterly wind in Goulburn

d southerly winds in Canberra in winter

e southerly winds in Darwin in summer

f southerly winds in Adelaide

g northwesterly winds in Broome

h southeasterly winds in Broome in summer

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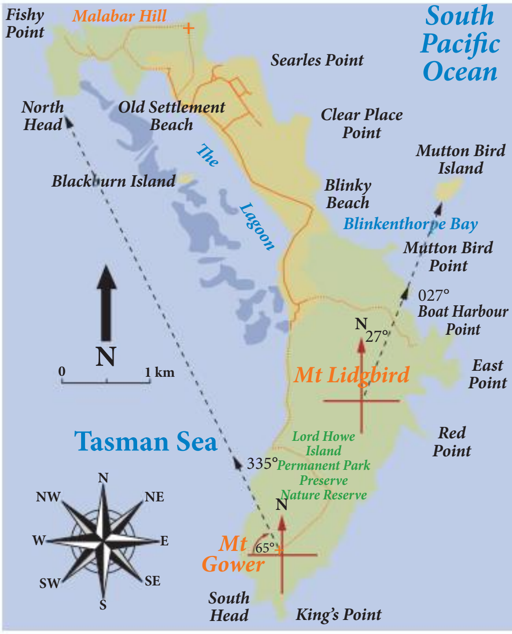
3.10 True bearings*

*Australian curriculum only, not in WA syllabus

Aircraft and ship navigators are responsible for getting their vessels to their destination. They use **true bearings** or **three-figure bearings** (three-digit angles) to specify directions accurately. Three-figure bearings measure angles from true north in a clockwise direction, from 000° to 360°.

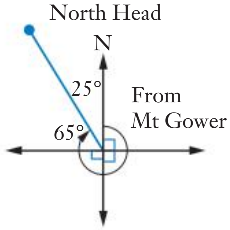
On this map of Lord Howe Island, to find the bearing *from* Mt Gower to North Head, we draw a compass rose on Mt Gower, draw a line from Mt Gower to North Head, then measure the angles.



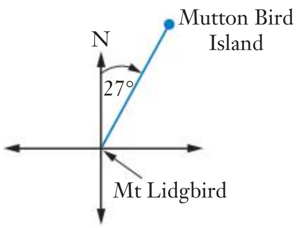
We add all the angles in a clockwise direction from north around to the direction line.

Bearing from Mt Gower to North Head = 90° + 90° + 90° + 65°

= 335°



To find the bearing of Mutton Bird Island *from* Mt Lidgbird, position the compass rose on Mt Lidgbird and draw a line from Mt Lidgbird to Mutton Bird Island. The angle between north and the direction line to Mutton Bird Island is 27°, but we write the bearing as 027° because we must use 3 figures.



WS Bearings

WS A page of bearings

WS Identifying bearings

WS 16 points of the compass

WS NSW map bearings

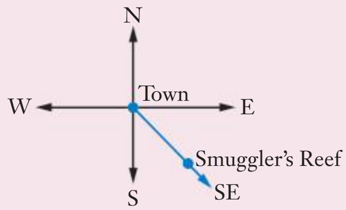
WS Bearings match-up

Every which way

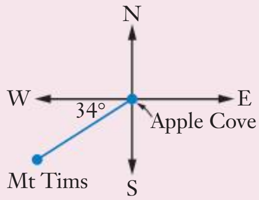
WS Elevations and bearings

EXAMPLE 13

a What is the bearing of Smuggler's Reef from Town?

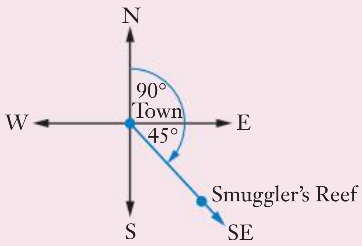


b What is the bearing from Apple Cove to Mt Tims?



Solution

a The bearing is *from* Town. Position a compass rose on Town and rule the southeast direction.

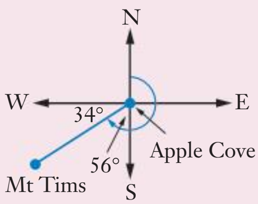


Southeast is exactly in the middle of south and east. $\frac{1}{2}$ of 90° is 45°.

Add the angles from north, clockwise to the direction of Smuggler's Reef.

The bearing from Town to Smuggler's Reef = 90° + 45°
= 135°

b The bearing is *from* Apple Cove. Position a compass rose on Apple Cove and the southwest direction. The angle between Mt Tims and south is 90° – 34° = 56°



Bearing from Apple Cove to Mt Tims = 90° + 90° + 56°
= 236°

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EXAMPLE 14

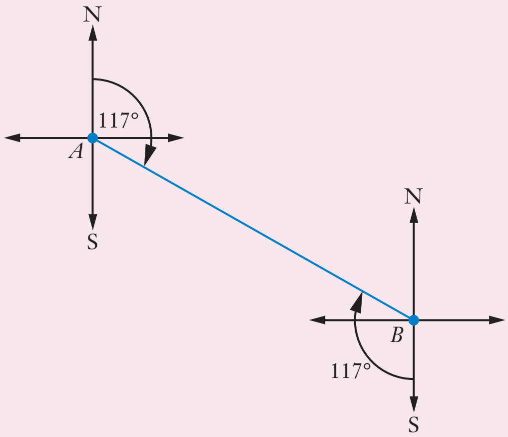
Kevin left camp and walked on a bearing of 117° to the creek.

What bearing should Kevin follow to return to his camp from the creek?

Solution

Sketch the bearing of the creek, B, from camp, A. On the same diagram, draw a compass rose at the creek, B.

Use alternate angles on parallel lines to find $\angle ABS$, where S is south.



$\angle ABS = 117°$ (alternate angles, NA || NB)

Add the angles from north to find the bearing back to Kevin's camp from the creek.

$\therefore$ Bearing of camp from creek $= 180° + 117°$

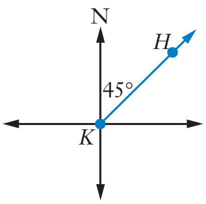
$= 297°$

Exercise 3.10 True bearings

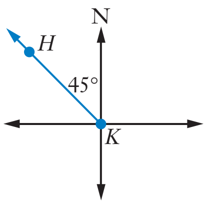
1 What is the three-figure (true) bearing and compass bearing from K to H in each diagram?

Example 13

a



b



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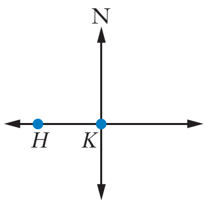
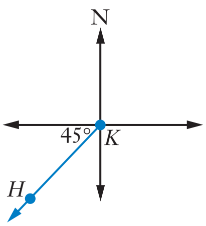


d



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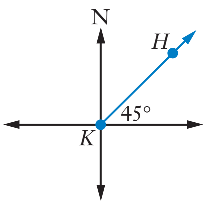

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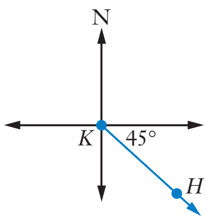
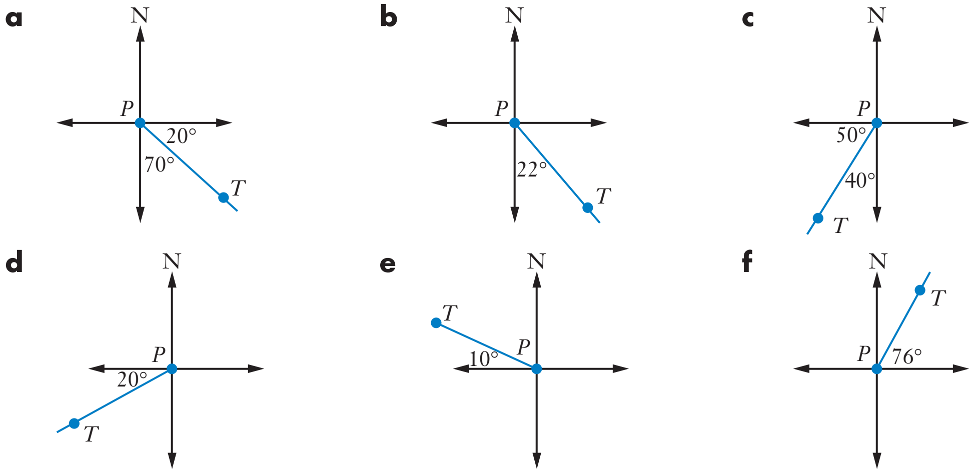


2 What bearing is required to travel from P to T in each diagram?
Answer as a compass bearing and as a true bearing.



Example 14

3 Copy and complete the bearings for each trip.

	Forward trip bearing	Bearing for the return trip
a	134°	
b	092°	
c		316°
d		065°

4 The bearing of Mecca, Saudi Arabia from Brisbane is 280°.
a What is the true bearing from Mecca to Brisbane?
b Express your answer to part **a** as a compass bearing.

5 The direction from Melbourne, Victoria, to Mecca is 279°.
a What is the true bearing from Mecca to Melbourne?
b Express your answer to part **a** as a compass bearing.

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INVESTIGATION

WHERE IS MECCA?

In this investigation, you are going to use a website to find the distance and the compass direction from your home to Mecca.

1 Go to the **Al Habib** website and select 'Qibla pointer'. Enter the name of your town or city. Record the distance and the bearing from true north.

For example, from Melbourne, Mecca is 12 734 km and 279°.

Al Habib

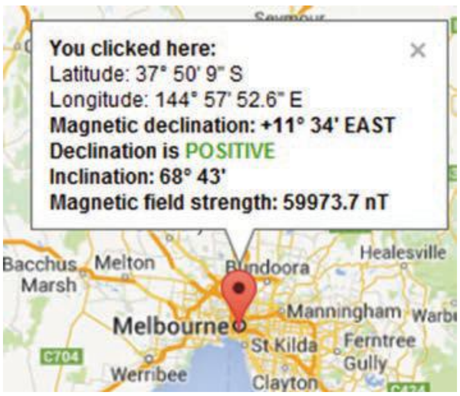
Powered by Google

When we draw straight line compass directions on a map they look curved because a map is a flat representation of the Earth's curved surface.

2 A compass gives directions from magnetic north, not true north. The difference between magnetic north and true north is called **magnetic declination**. Magnetic declination depends on where you are. Go to the **Magnetic declination** website and enter the name of your town or city followed by Australia and click **Search map**. Click the link to your town or city and record the magnetic variation and whether the variation is positive or negative.

Magnetic declination

For example, for Melbourne, the declination is +11°.



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3 The final step is to subtract the size of the magnetic declination from the bearing to Mecca.

True north bearing from Melbourne to Mecca is 279° – 11° = 268°.

3.11 Bearings problems*

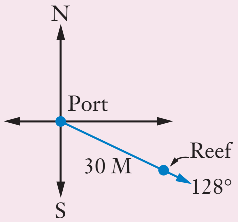
Bearings

*Australian curriculum only, not in WA syllabus

EXAMPLE 15

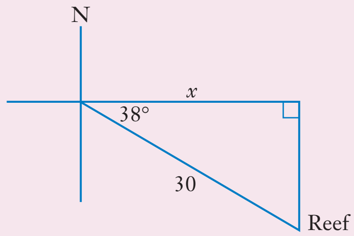
True bearings

A fishing boat left port and sailed on a bearing of 128° for 30 M (nautical miles) to a fishing reef. How far is the reef east of port? Answer to the nearest nautical mile.



Solution

Construct a right-angled triangle by drawing a line from the reef straight up to the east direction line.



The angle in the triangle = 128° – 90°

= 38°

Let x be how far the reef is east of port.

Then use trigonometry to find x.

$$\cos 38° = \frac{x}{30}$$

$$x = 30 \cos 38°$$

$$= 23.6403\ldots$$

$$\approx 24 \text{ M}$$

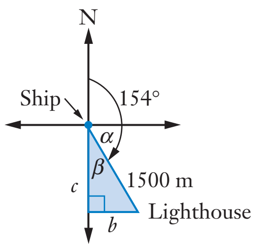
Write your answer. The reef is 24 M east of port.

Exercise 3.11 Bearings problems

Example 15

1 A lighthouse is 1500 m from a ship on a bearing of 154°.

- **a** Explain why $\alpha = 64°$ and $\beta = 26°$ in the diagram.
- **b** How far is the ship west of the lighthouse?
- **c** Calculate the distance that the lighthouse is south of the ship.



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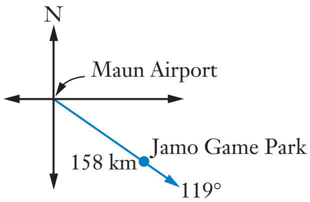
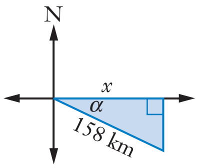
2 A light aircraft flew 158 km in a straight line from Maun Airport to Jamo Game Park on a bearing of 119°.

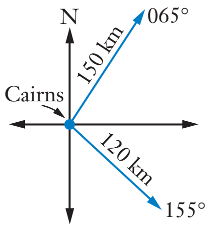


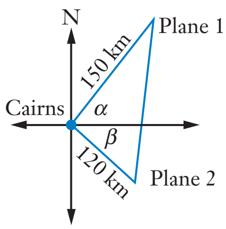


a Calculate the size of the angle α.

b How far is Jamo Game Park east of Maun Airport?

3 Two planes flew out of Cairns to search for a missing yacht. The first plane flew 150 km on a bearing of 065° and the second plane flew on a bearing of 155° for 120 km.



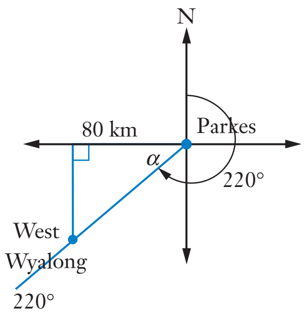


a Calculate the sizes of angles α and β.

b What is $\alpha + \beta$?

c Use Pythagoras' theorem to calculate how far the planes are apart.

4 Ben drove along the highway from Parkes to West Wyalong on a bearing of 220°. West Wyalong is 80 km west of Parkes.



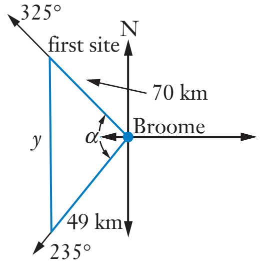
a Explain why $\alpha = 50°$.

b How far, correct to the nearest kilometre, is it along the highway from Parkes to West Wyalong?

5 Two dive boats sailed out of Broome heading for popular dive sites. The first sailed on a bearing of 325° for 70 nautical miles. The second boat sailed 49 km on a bearing of 235° until it was due south of the first dive boat.

a Explain why $\alpha = 90°$.

b How far, y km, are the two dive sites apart?



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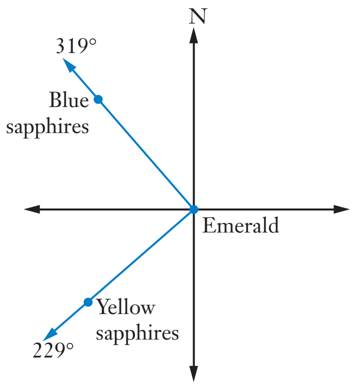
6 Caleb was looking for sapphires near the city of Emerald in Queensland. He found blue sapphires 60 km on a bearing of 319° from Emerald and yellow sapphires 36 km on a bearing of 229° from Emerald. How far is it between the two locations where Caleb found sapphires?



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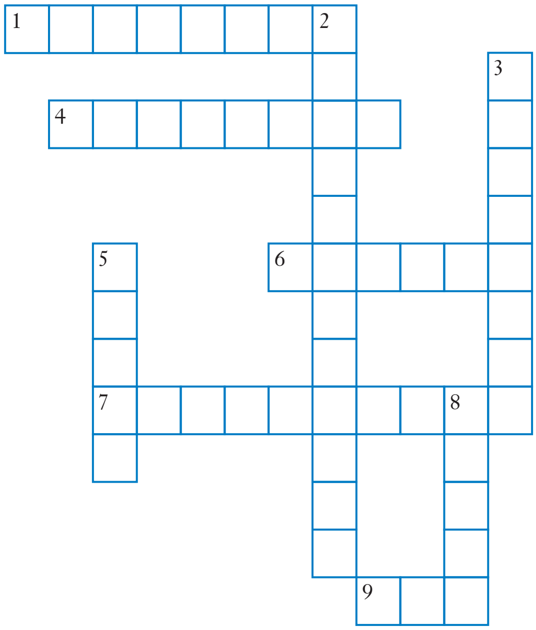
KEYWORD ACTIVITY

That's the right angle!

TRIGONOMETRY CROSSWORD

Copy the crossword and complete the trigonometry summary below to complete the crossword puzzle.



The word 'trigonometry' comes from two Greek words that combine to mean 'the measurement of triangles'. In modern times **2**__________ has many applications, including determining the size of **6**_____ and **8**_____ in right-angled triangles.

In a **5**____ angled triangle the side opposite the right angle is called the **7**________, but the names of the other sides are determined by the reference angle. The **3**______ side is opposite the reference angle and the **1**_______ side is next to the reference angle.

The ratio of the opposite side to the adjacent side is called the **4**______ _______ (2 words). We use the sin or the **9**_______ ratios when we do calculations involving the hypotenuse.

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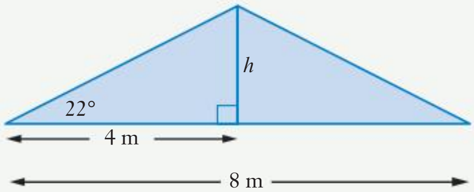
SOLUTION TO THE CHAPTER PROBLEM

Problem



Denis is building a garage. The garage is 8 m wide and the pitch of the roof is 22°. How high does he need to make the truss, h, correct to 2 decimal places?

Solution

h is the opposite side to 22°, 4 m is the adjacent side.

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan 22° = \frac{h}{4}$$

$$\frac{h}{4} = \tan 22°$$

$$\begin{aligned} h &= \tan 22° \times 4 \\ &= 1.6161\ldots \\ &\approx 1.62 \text{ m} \end{aligned}$$

Denis will need to make the truss 1.62 m high.

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3. TEST YOURSELF

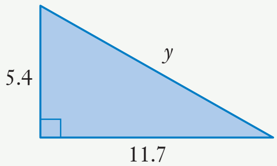
So you’ve got a right angle

Practice quiz

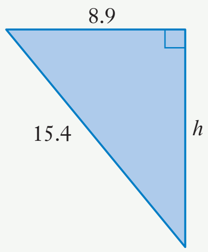
Exercise 3.01

1 Find the value of each variable correct to one decimal place.

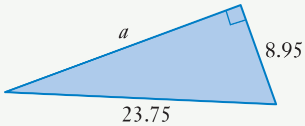
a



b

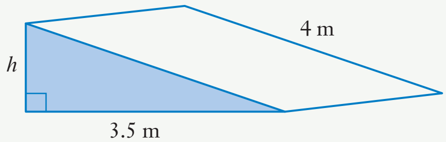


c



Exercise 3.01

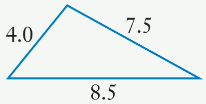
2 Gianni works in a factory that makes solar panels. He is making a frame to hold a large panel on a roof and angle it correctly for the sun. The panel is 4 m long and the base of the frame is 3.5 m long. How high, h, does Gianni need to make the frame (correct to 1 decimal place)?



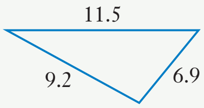
Exercise 3.01

3 Use Pythagoras’ theorem to test whether each triangle is right-angled.

a



b



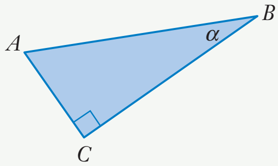
Exercise 3.02

4 This triangle has sides AB, AC and CB. Name the side that is:

a the hypotenuse

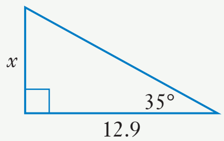
b opposite to angle α

c adjacent to angle α



Exercise 3.03

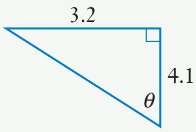
5 Use the tan ratio to find the value of x, correct to one decimal place.



Exercise 3.04

6 Determine, correct to the nearest degree, the size of each angle θ.

a



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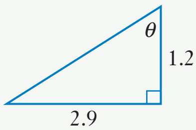

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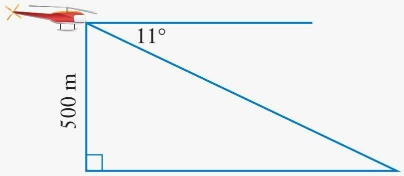
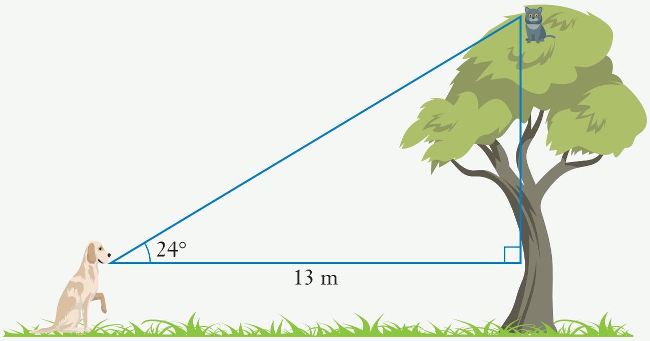
Exercise 3.05

7 State emergency workers are flying at a height of 500 m towards a mine disaster. The pilot can see the disaster site at an angle of depression of 11°. Calculate the horizontal distance from the helicopter to the disaster, correct to the nearest 10 metres.



Exercise 3.05

8 Ginger the dog chased Oscar the cat up a tree. Ginger is 13 m from the base of the tree and her angle of elevation to Oscar is 24°. Ginger's eyes are 0.6 m above the ground. How high above the ground is Oscar, correct to one decimal place?



Exercise 3.06

9 When Sharyn skied 50 m down a slope, her height above sea level changed by 10 m. Calculate the size of angle θ, correct to the nearest degree.

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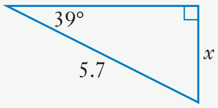
Exercise 3.07

10 Which trigonometry ratio (sin, cos or tan) can be used to calculate the value of each variable? Write the ratio, then calculate the value of the variable.

a



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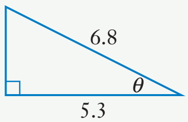
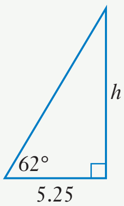
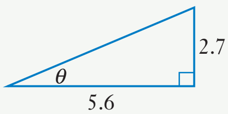


c



d



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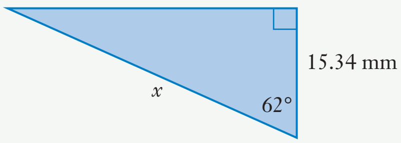
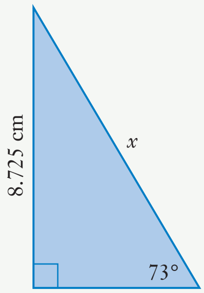
11 Determine the value of x, correct to 2 decimal places.

Exercise **3.08**

a



b

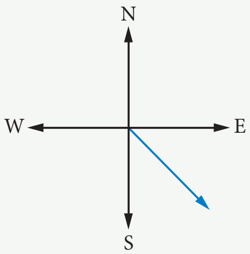


12*Which diagram shows the direction south-west?

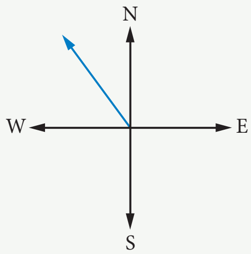
*Australian curriculum only, not in WA syllabus

Exercise **3.09**

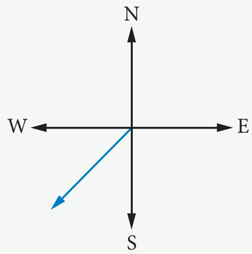
A



B



C

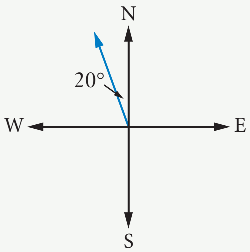


13*Match each bearing to its diagram **A**, **B** or **C**.

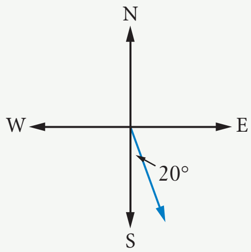
Exercise **3.10**

a S 20° W **b** 160° **c** 340°

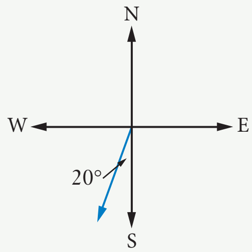
A



B

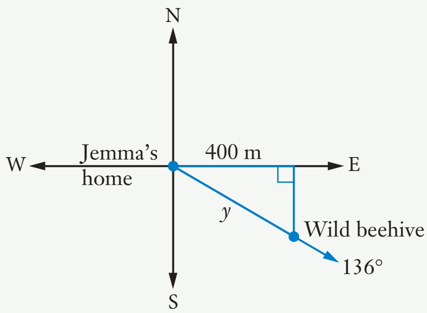


C



14*Jemma found a wild beehive 400 m east on a bearing of 136° from her home. Calculate the straight-line distance, y, from Jemma's home to the beehive. Answer to the nearest metre.

Exercise **3.11**



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DATA COLLECTION

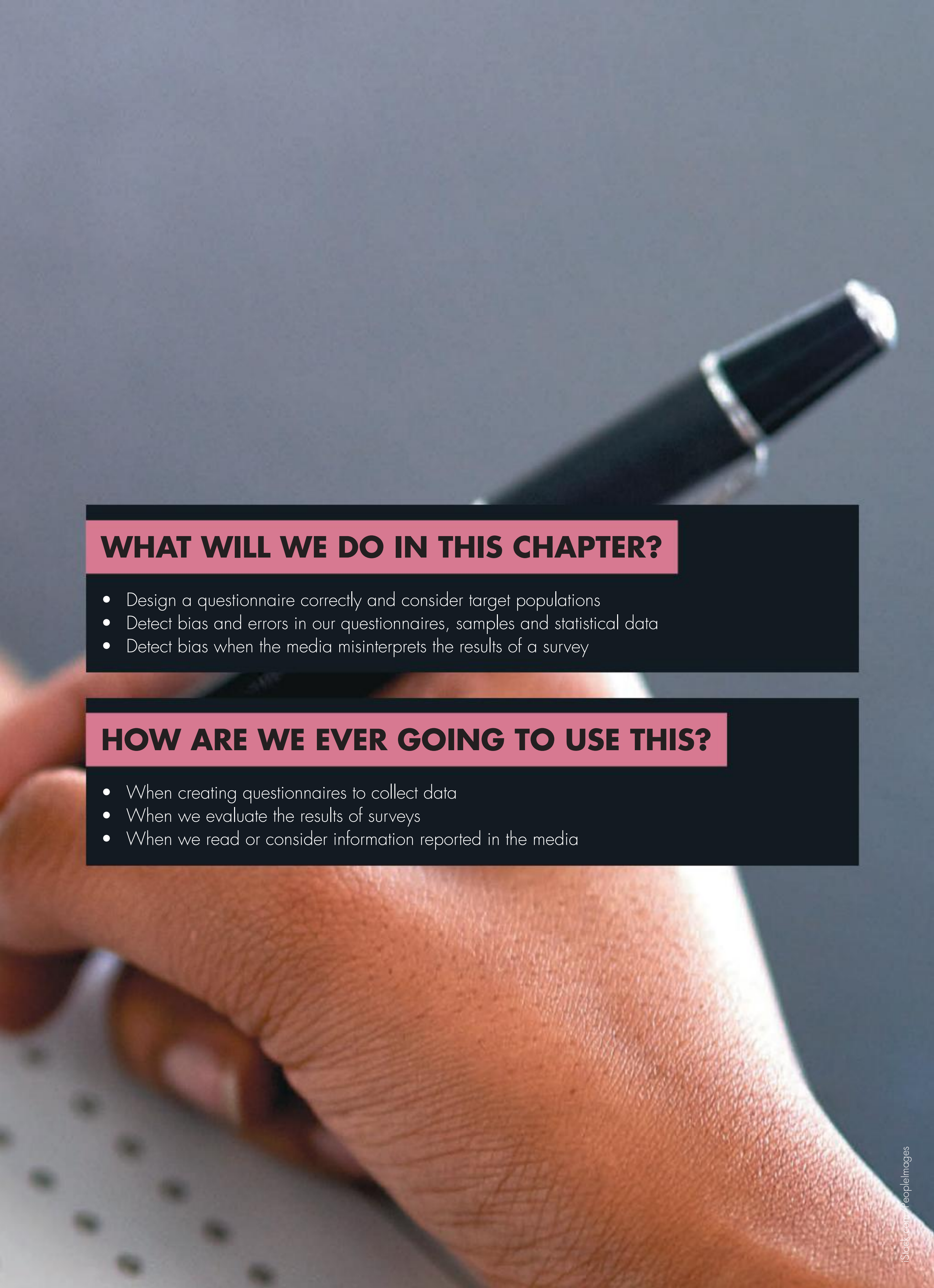
4.

THAT'S BIASED

Chapter problem

Juanita is researching political views in her community. One Wednesday morning she stands at an intersection on the main street of town. She asks people passing, 'Are you going to vote Liberal or Labor in the next election?'.

Will Juanita's data be reliable and unbiased? If not, how can she improve her methods?



WHAT WILL WE DO IN THIS CHAPTER?

- Design a questionnaire correctly and consider target populations
- Detect bias and errors in our questionnaires, samples and statistical data
- Detect bias when the media misinterprets the results of a survey

HOW ARE WE EVER GOING TO USE THIS?

- When creating questionnaires to collect data
- When we evaluate the results of surveys
- When we read or consider information reported in the media

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WS
Student survey form

WS
Census questions

4.01 Questionnaires

Using **questionnaires** is one of the most common ways to collect information.
A good questionnaire:

- uses simple language
- has clear and precise questions
- meets privacy requirements: unauthorised people cannot access your information
- is free from bias

Bias is an unwanted influence in sampling or questionnaires that favours a particular section of the population unfairly. Bias produces unreliable results because the sample or questionnaire answers are not truly representative of the population.

Exercise 4.01 Questionnaires

Andrew has designed the first draft of a questionnaire to survey club members about the entertainment that the United Club provides, including trips away. Andrew's draft is given below. Read the questionnaire and answer the questions that follow.

United Club **Help us help you!**

Please answer this short questionnaire to help the United Club better serve you.

1 How old are you?
- ☐ Less than 30 years
- ☐ Between 31 and 40 years
- ☐ Between 41 and 50 years
- ☐ Over 50 years

2 Are you male or female?
- ☐ Male
- ☐ Female

3 Which of the Club's entertainments do you attend?

4 How often do you attend the following Club activities?
Concerts Films Dance nights Trips away

5 Do you think the entertainment offered is reasonably priced?

6 Should the club offer more types of entertainment and, if so, what should it offer?

7 Are there any other comments you would like to make about the Club's entertainment program?

Thank you for taking the time to complete this questionnaire. Please hand it in at the bar.

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1 **Closed questions** give a number of options for the answer. How many questions in the questionnaire are closed?

2 **Open-ended questions** provide no options and require the person to write their own answer. How many questions in the questionnaire are open-ended?

3 The first question about age has a problem in the options given. What error has Andrew made? (Hint: where would a person aged 30 tick?). Rewrite the options to this question to fix this error.

4 When Andrew asks 'Which of the Club's entertainments do you attend?' it is not clear exactly what he means. Club members could be unsure which of the Club's activities is classified as entertainment. Rewrite the question to make it clear.

5 When Andrew asks 'How often do you attend the following Club activities?' he will get a range of answers. Analysing the data will be easier if Andrew provides some options. Suggest a series of tick boxes that Andrew could provide for people to record their answers.

6 Would all Club members agree on what 'reasonably priced' means? Suggest how Andrew could change this question to make it clearly understood.

7 Write another question about the cost of entertainment for this questionnaire.

8 There is a risk that answers won't remain private if the questionnaires are handed in at the bar. Suggest a more secure way to collect the questionnaire forms.

9 Do you think Andrew has missed anything in his questionnaire? (Hint: how would people who don't attend the entertainment activities express their views?)
Write 3 additional questions Andrew could use.

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4.02 Bias in questionnaires

The way a question is worded in a questionnaire can introduce bias. The wording of the questions can lead people towards a particular view. This can unfairly influence the way people answer the questions.

EXAMPLE 1

Andrew has written some questions to ask United Club members about its restaurant.

a How often do you eat at our fabulous restaurant?

b Rate your last meal: Great Yummy Quite nice

In what way are these questions biased? Rewrite them so they are not biased.

Solution

a Using the word 'Fabulous' encourages the person answering the question to give a positive answer.
It would be better to ask: 'How often do you eat at the restaurant?'

b This question doesn't give any negative options. The question could be:
'Rate your last meal:
Poor Average Good'

Exercise 4.02 Bias in questionnaires

Example 1

1 For each questionnaire question below,
- **i** state how it is biased
- **ii** rewrite it so that it is not biased.

a Do you prefer to holiday in fascinating Sydney or in wet Melbourne?
b Do you agree that we should increase the tax on those disgusting cigarettes?
c Rate the last movie you saw: OK Good Fantastic
d Do you prefer exciting soccer or boring old football?
e Do you think the Club is open long enough or should we be able to rage on into the early morning?
f Rate your health fund: useless not much good reasonable
g Are there enough events for young people in this boring town?
h Isn't this just the greatest movie you've ever seen?
i Are you one of those dumb people who walk to school?
j Rate the gym you attend: best ever good Ok

2 You have decided to survey your fellow students about the sports offered at your school. Make up 3 examples of biased questions. Then give 3 unbiased versions of these questions.

ISBN 9780170443975

4.03 Bias in sampling

To be fair and reliable, a sample being surveyed should represent the population accurately. Bias can happen if a sample is not selected properly.

EXAMPLE 2

For his survey, Andrew is considering finding a fair sample of United Club members by asking 100 people:

a as they come through the door on a Friday night

b playing Bingo on a Wednesday morning

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In what way is each sample biased? How could you ensure that an accurate sample is found?

Solution

a The people at the club on a Friday night are more likely to be younger people.

All age groups would not be represented fairly.

b The people at the club on a Wednesday morning are more likely to be retired or not working.

Working members would be under-represented in the survey.

For a more accurate sample, Andrew could find a list of all Uniting Club members and choose every, say, 10th member (other answers are possible).

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Exercise 4.03 Bias in sampling

Example 2

1 For each sample given below, state how:

- **i** the sample is biased
- **ii** you could ensure that an accurate sample is found.

- **a** Asking people in the crowd at an AFL match for their favourite sport
- **b** Calling every 6th phone number on a page in the phone book to ask about mobile phone usage
- **c** Surveying people about their preferred supermarket in a shopping centre with only one supermarket
- **d** Asking people in the inner-city their view on drought support by the Government
- **e** Asking teachers what music should be played at the Year 12 formal
- **f** Surveying the Maths staff on what books should be bought for the school library
- **g** Finding out what people think of a new design for a football club jersey by asking people who walk past the office door
- **h** Surveying taxi drivers on who they will vote for in the next election
- **i** Emailing a survey to club members to complete
- **j** Asking McDonalds customers to name their favourite coffee shop

2 Ms Benedict's class is completing a statistical investigation project as part of their assessment. The students need to decide who they will ask to answer their questions. For each investigation, suggest:

- **i** what kind of group should be surveyed to get useful results
- **ii** what kind of group is likely to give less useful or biased results

- **a** New tourist attractions that could be built in Adelaide
- **b** Children's favourite holiday destinations
- **c** Most popular teacher in Nelson Valley State High
- **d** Most popular sport in Australia
- **e** Changes to the school uniform
- **f** Exercise equipment to be installed in outdoor areas
- **g** Types of stores to include in a new shopping centre
- **h** Entertainment provided in a chain of retirement homes

Chapter problem

You've covered the skills required to solve the chapter problem. Can you solve it now?

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INVESTIGATION

TRAVELLING TO SCHOOL

Conduct a survey of how the students at your school commute to school.

1 Before you start, you will need to make some decisions.

- How large a sample will you use?
- How will you select your sample?
- What questions will you ask?
- What are the possible categories of travel that students at your school might use? List them.
- What will you do if there is a category you haven't thought of?
- What will you do if someone uses more than one method of transport?

2 **Collect** the information.

3 **Answer** the following questions:

- What is the most common method of travel?
- What is the least common method?
- What effect does the location of your school have on the method of travel?

4 **Present** your results in a **report**, either written or as PowerPoint slides.

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4.04 Errors in surveys

There are different ways in which error can be part of the data collection process. Errors can result from:

- The surveyor not asking the question correctly
- The respondent interpreting the question incorrectly
- The respondent answering the questionnaire incorrectly
- The surveyor recording the answers incorrectly

When collecting data, the aim is to avoid or minimise these errors.

Information that is collected can also be interpreted carelessly to favour a particular point of view.

EXAMPLE 3

Alex stood at the school gate one morning and asked 10 students if they liked the school uniform. Five students said something negative about the uniform. Alex submitted a report to the school principal, claiming that half of the students hate the uniform. Describe 3 things that are wrong with this conclusion.

Solution

1 Alex only asked 10 students and it is not a random selection. The sample is unlikely to be representative of the whole school.

2 The question is vague and it's not clear what students don't like. It may not be the whole uniform that students don't like, but some particular item.

3 Alex exaggerates the answers when he says 'hate'. We don't know how strongly students feel about the uniform.

AAP Image/Dan Peled

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Exercise 4.04 Errors in surveys

1 Each situation listed below can affect the way a questionnaire is answered. Explain how it could affect the usefulness of the survey results.

- **a** The questionnaire is very long
- **b** The presentation of the questionnaire is messy and hard to follow
- **c** The choice of categories in some questions don't cover all the options
- **d** The time allowed to answer the questionnaire is limited

2 There are different ways to collect data. For each method listed, describe one possible error to avoid if using that method.

- **a** Sending a questionnaire by post
- **b** Asking people to complete a diary each day recording their behaviour
- **c** Calling up households and asking questions over the phone
- **d** Having an interviewer ask a person the questions and completing the questionnaire
- **e** Direct observation, for example, how many vehicles travel through an intersection at a given time of day

3 Jane asks 5 Year 12 students if they like the food served at the school canteen and 3 students say they don't like it. Jane writes an article for the school newspaper with the headline '60% of students hate canteen food'. Explain 3 things that are wrong with this headline.

Example 3

4 The Hightop Hotel emails its customers to complete a survey about its service. It receives 25 replies, each of which makes some positive comments about the hotel. An advertisement is published saying 'Customers 100% happy with our hotel'. Explain what is wrong with this claim.

5 A newspaper headline reads 'Government program for disadvantaged students a disaster'. In the article, it is revealed that 35 students out of 1000 students receiving government support have dropped out of school. Explain why this headline is misleading.

6 Using the information you collected in the Investigation 'Travelling to school', write a presentation that is deliberately misleading. Perhaps you might like to write about additional student parking, motorbike parking, later school starting times or some other issue, but make sure you distort the true facts!

7 Use the Internet or library to find some newspaper or magazine articles that are based on statistics. Read them carefully to check if they have presented the information properly. Write a report on 3 articles you find, highlighting their use of statistics, either correctly or incorrectly.

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INVESTIGATION

STATISTICS IN THE MEDIA

We live in a world of 24-hour news, through TV, news websites, Facebook, Twitter and blogs. To detect bias, we need to consider where the news comes from and what samples were used. Is the information supplied by a journalist, the police, a company executive, a government official or an opinionated blogger? Are the statistics based on a small sample, a large sample, an unrepresentative sample, or a phone/online poll? Each may have a particular bias that may influence how the story is reported.

Find 3 examples of news items or surveys reported in a newspaper or online and investigate the following questions.

1. Where did it come from? What could influence how the story was reported?
2. Who wrote the story?
3. Does it show any bias?
4. Can it be supported by other news providers?
5. What type of sampling was used? Was it representative?
6. How many people were questioned?
7. When was the survey conducted?

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KEYWORD ACTIVITY

BIAS IN STATISTICS

1 Explain what is meant by the word *bias* in the statistical context.

2 What are the qualities of a good questionnaire that doesn't have bias?

3 Write a paragraph explaining how samples can be biased and how to avoid this.

4 List 5 errors than can occur in a survey that may lead to unreliable results.

Questionnaires word match

Statistical scramble

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SOLUTION TO THE CHAPTER PROBLEM

Problem

Juanita is researching political views in her community. One Wednesday morning she stands at an intersection on the main street of town. She asks people passing, 'Are you going to vote Liberal or Labor in the next election?'

Will Juanita's data be reliable and unbiased? If not, how can she improve her methods?

Solution

The sample:

On a Wednesday morning she will miss people who are working or shopping. She will mainly be asking older people who have retired, and unemployed people. Her sample is NOT representative of her community.

The question:

There are alternatives to voting Liberal or Labor – other political parties or independent candidates. Her question doesn't cover all the options. Also, she has not specified which election – local council, state or federal governments.

Improvements:

She should choose her sample from the electoral roll of the community.

The question could be asked with a tick box option for answering or it could be an open-ended question. It should specify what level of government is meant. For example, 'Who do you intend to vote for in the next federal election?'

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4. TEST YOURSELF

That's biased

Practice quiz

1 Anthony wrote 'How often do you drink?' as part of a questionnaire he was giving to Year 12 students about their alcohol consumption.

Exercise 4.01

- **a** What problem might people have in answering this question?
- **b** Rewrite the question, providing some tickbox options.

2 Tranh is surveying students about the quality of food sold at the school canteen.

Exercise 4.02

- **a** Write an example of a biased question he could use if he wanted to show how great the food is.
- **b** Rewrite the question you wrote in part **a** so that it is NOT biased.

3 Jenni asks a sample of people in her town, 'Which coffee shop makes the best coffee?'

Exercise 4.03

- **a** Give an example of a sample that would be biased.
- **b** State how could you ensure that an accurate sample is found.

4 Kendall interviewed her Year 11 class about changing the times that school starts. 11 of the 20 students surveyed said they wouldn't mind if school started a bit earlier. Kendall presents a report to the student council with the title 'Students support starting school half an hour earlier'. Describe 3 things that are wrong with this claim.

Exercise 4.04

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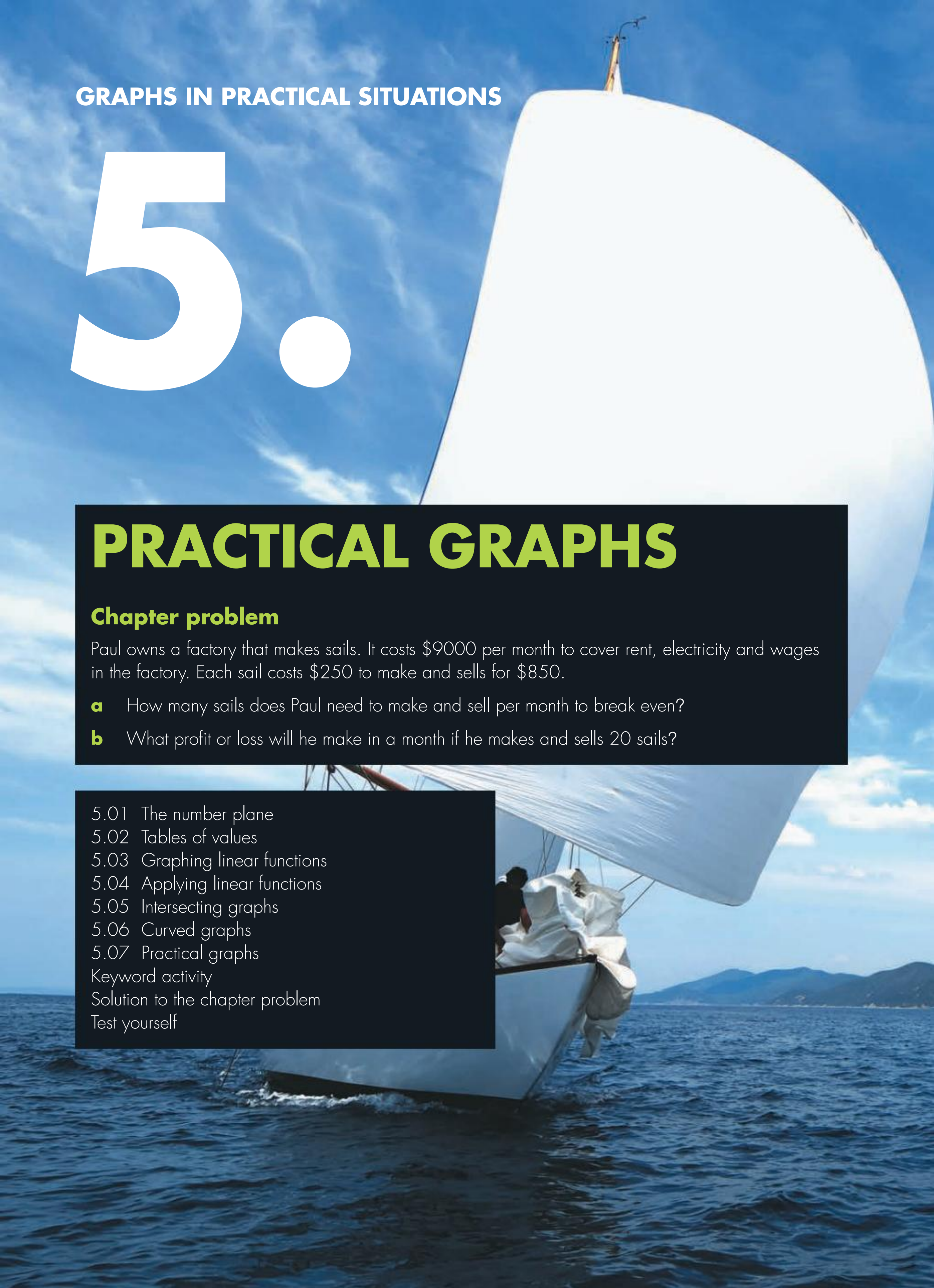


GRAPHS IN PRACTICAL SITUATIONS

5.

PRACTICAL GRAPHS

Chapter problem

Paul owns a factory that makes sails. It costs $9000 per month to cover rent, electricity and wages in the factory. Each sail costs $250 to make and sells for $850.

a How many sails does Paul need to make and sell per month to break even?

b What profit or loss will he make in a month if he makes and sells 20 sails?

WHAT WILL WE DO IN THIS CHAPTER?

- Plot points on the Cartesian plane (number plane)
- Complete and graph a table of values for a linear function
- Graph straight lines and find the gradient and y-intercept
- Apply linear functions in practical situations and interpret the meaning of the gradient and y-intercept
- Solve problems involving 2 intersecting lines and identify break-even points for a business to make a profit
- Graph curves and use curves in practical situations
- Interpret practical graphs, including conversion graphs and distance–time graphs

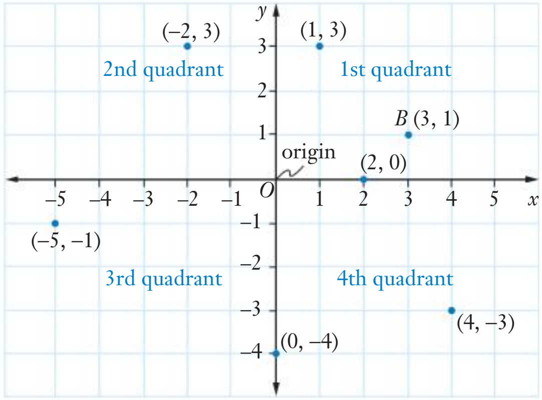
HOW ARE WE EVER GOING TO USE THIS?

- When we want to represent relationships graphically
- When we model real-life situations with algebra and a graph

WS
Number plane grid paper

5.01 The number plane

The **Cartesian plane**, another name for **number plane**, is a grid made from a horizontal number line called the ***x*-axis**, and a vertical number line called the ***y*-axis**, as shown. The centre of the number plane is called the **origin**.



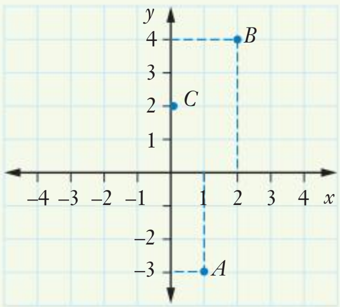
We find points on the number plane by using a pair of numbers called **coordinates**, where the first number goes across from the origin and the second number goes up/down. For example, the point (3, 1) is 3 across and 1 up from the origin, labelled point *B* on the diagram. The origin has coordinates (0, 0).

The number plane is divided into 4 regions called **quadrants**.

EXAMPLE 1

a What are the coordinates of points *A*, *B* and *C* on the graph?

b Plot the points (3, −2), (0, 3) and (−2, 0) on a graph.



Solution

a Find the across number, then the up-and-down number.

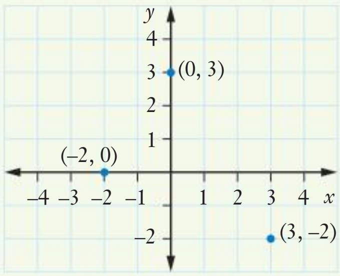
Point *A* has coordinates (1, −3).

B is (2, 4) and *C* is (0, 2).

b To plot (3, −2), find 3 on the across axis, then go down 2.

To plot (0, 3), find 0 on the across axis, then go up 3. The point (0, 3) is on the *y*-axis.

To plot (−2, 0), find −2 on the across axis, then stay on the *x*-axis. The point (−2, 0) is on the *x*-axis.



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A set of points can be written in a table. The coordinates of each point are shown in a table column.

EXAMPLE 2

Plot the points shown in this table of values.

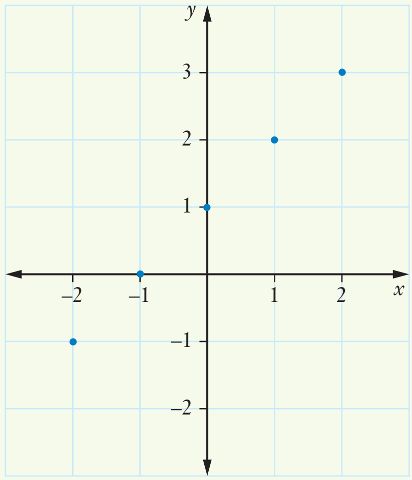
x	–2	–1	0	1	2
y	–1	0	1	2	3

Solution

From the table, the first point is (–2, –1).
The other points are (–1, 0), (0, 1), (1, 2) and (2, 3).

Remember: first number across, second number up/down.

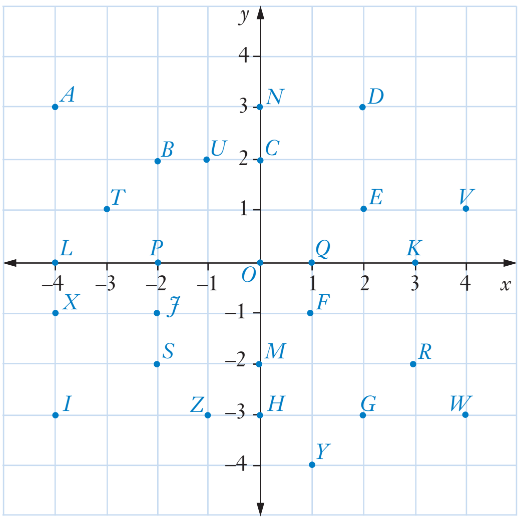
An easy way to remember that x values go across the number plane:
x is 'a cross'



Exercise 5.01 The number plane

Example 1

1 Write the coordinates of each of the points A to Z.



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WS

Number plane grid paper

2 **a** Draw a Cartesian plane with both axes (lines) extending from –6 to 6.

b Plot these points on the number plane.

$A(3, 1)$	$B(-4, 3)$	$C(-3, 4)$	$D(-2, -2)$
$E(0, -2)$	$F(1, -5)$	$G(4, -4)$	$H(-3, 0)$
$I(-6, 0)$	$J(-1, -6)$	$K(-3, -4)$	$L(0, 5)$
$M(6, 2)$	$N(-3, 5)$	$P(5, -5)$	$Q(-1, 6)$

3 List all of the points from question **2** that are:

a in the 1st quadrant **b** in the 2nd quadrant

c in the 3rd quadrant **d** in the 4th quadrant

e on the border of 2 quadrants.

4 In which quadrant would you find each point?

a (3, –5) **b** (–2, –4) **c** (–8, 1)

5 Copy this table and complete it by placing a + or a – sign in the blank spaces.

Points in:	x-coordinate	y-coordinate
1st quadrant	+	
2nd quadrant	–	
3rd quadrant		
4th quadrant		

Example 2

6 Plot the points given in this table on a number plane.

x	0	1	2	3	4
y	–2	–1	0	1	2

You need to decide what numbers to use on the x and y axes. Look at the table you are given. Check what numbers you need on each axis.

7 For each table, plot the points on a number plane.

a

x	–6	–3	0	3	6
y	–2	–1	0	1	2

b

x	–5	–2	1	3	4
y	3	0	–3	–5	–6

c

x	4	2	0	–1	–2
y	0	2	4	5	6

d

x	–1	0	1	2	3
y	–2	0	2	4	6

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INVESTIGATION

CHANGING POINTS OF VIEW

In this investigation, we will see what happens when we change coordinates in different ways.

1 Draw a number plane with an x-axis from –15 to 15 and a y-axis from –20 to 20. Use 0.5 cm = 1 unit.

2 Plot these points and join them in the given order:

(3, 2) (11, 14) (9, 13) (10, 19) (5, 10) (6, 11) (3, 2)

You have drawn a lightning bolt in the first quadrant.

3 What happens to the lightning bolt if you multiply all the x values by –1? Rewrite the points with these new coordinates. Plot these points and join them as before. Describe in words what has happened to the lightning bolt.

4 Now investigate what happens if you multiply all the y values by –1. Rewrite the points, plot them and join them as before. Describe in words what has happened to the lightning bolt.

5 How would you place the lightning bolt in the other quadrant? Show that what you have said will actually work.

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WS Formula frolics

5.02 Tables of values

We can complete a table of values when we are given an equation (**formula**) relating the x and y values.

EXAMPLE 3

Complete each table of values given the equation.

a $y = 4x + 3$

x	–2	–1	0	1	2	3
y						

b $y = 10 - 2x$

x	–2	–1	0	1	2	3
y						

Solution

a Replace x with –2 in the equation, work out the value and write it in the table under $x = -2$.

$$\begin{aligned} y &= 4 \times (-2) + 3 \\ &= -8 + 3 \\ &= -5 \end{aligned}$$

x	–2	–1	0	1	2	3
y	–5					

Repeat with the other 5 values of x in the table.

When $x = -1$, $y = 4 \times (-1) + 3 = -1$.

When $x = 0$, $y = 4 \times (0) + 3 = 3$.

When $x = 1$, $y = 4 \times (1) + 3 = 7$.

When $x = 2$, $y = 4 \times (2) + 3 = 11$.

When $x = 3$, $y = 4 \times (3) + 3 = 15$.

x	–2	–1	0	1	2	3
y	–5	–1	3	7	11	15

b Replace x with –2 in the equation, work out the value and write it in the table under $x = -2$.

$$\begin{aligned} y &= 10 - 2 \times (-2) \\ &= 10 - (-4) \\ &= 10 + 4 \\ &= 14 \end{aligned}$$

x	–2	–1	0	1	2	3
y	14					

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Repeat with the other 5 values of x in the table.

When $x = -1$, $y = 10 - 2 \times (-1) = 12$.

When $x = 0$, $y = 10 - 2 \times 0 = 10$.

When $x = 1$, $y = 10 - 2 \times 1 = 8$.

When $x = 2$, $y = 10 - 2 \times 2 = 6$.

When $x = 3$, $y = 10 - 2 \times 3 = 4$.

x	–2	–1	0	1	2	3
y	14	12	10	8	6	4

Exercise 5.02 Tables of values

Example 3

1 Copy and complete each table of values for the given equation.

a $y = 2x + 3$

x	–5	–2	3	6	11
y					

b $y = 3x - 5$

x	4	2	5	7	–3
y					

c $y = \frac{1}{2}x$

x	–4	–2	3	6	9
y					

d $y = 20 - 2x$

x	–2	–1	4	7	10
y					

2 Complete the table for the relationship $y = 12 - x$.

x	1	2	5	9	10
y					

3 Match each equation to its correct table of values.

a $y = 3x$

b $y = 3 + x$

c $y = x^2$

d $x \times y = 12$

e $y = 6 - x$

f $y = \frac{1}{2}x + 1$

A

x	1	2	3	4	6
y	1	4	9	16	36

B

x	1	2	–3	4	6
y	4	5	0	7	9

C

x	1	2	3	–4	6
y	12	6	4	–3	2

D

x	0	2	8	–4	6
y	1	2	5	–1	4

E

x	1	2	3	4	6
y	5	4	3	2	0

F

x	1	–2	3	4	–6
y	3	–6	9	12	–18

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4 Find an equation for each table of values.

a

x	1	2	3	4	5
y	2	3	4	5	6

b

x	1	2	3	4	5
y	0.1	0.2	0.3	0.4	0.5

c

x	1	2	3	4	5
y	4	8	12	16	20

d

x	1	2	3	4	5
y	1	4	9	16	25

e

x	1	2	3	4	5
y	9	8	7	6	5

f

x	1	2	3	4	6
y	24	12	8	6	4

g

x	1	2	3	4	5
y	6	5	4	3	2

h

x	1	2	3	4	5
y	2	4	6	8	10

5 Construct your own table of values for the equation $x - y = 1$.

5.03 Graphing linear functions

Graphing linear equations

WS Graphing points and lines

WS Graphing linear functions

WS A page of number planes

We can graph algebraic equations on a number plane. Some equations have graphs that are straight lines. The equation of a straight line is called a **linear function**.

Graphing linear functions

Linear means 'of a line'.

- Construct a table of values for the equation
- Plot the points from the table of values on a number plane
- Rule a straight line through the points
- Label the line with its equation.

We can choose any numbers we want for x, but we need to make sure the points will fit on our graph and be easy to calculate. It is easiest to choose whole numbers close to 0.

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EXAMPLE 4

Graphing linear functions

Graph the linear function $y = x - 3$.

Solution

Draw a table and choose some x values.

x	−1	0	1	2	3	4
y						

Calculate the y values to complete the table.

$y = -1 - 3 = -4$

$y = 0 - 3 = -3$

$y = 1 - 3 = -2$

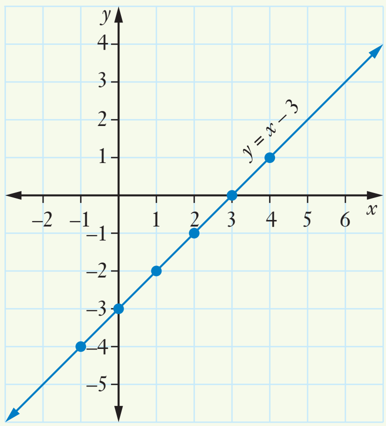
$y = 2 - 3 = -1$

$y = 3 - 3 = 0$

$y = 4 - 3 = 1$

Do you notice a pattern with the y values?

Draw a set of axes and plot the points. Rule a straight line through the points, place arrows at each end and label the line with its equation.



Gradient and *y*-intercept

Gradient and y-intercept of a line

The **gradient** (or **slope**) of a line measures how steeply the line goes up or down.

The ***y*-intercept** of a line is the value where the line crosses the y-axis.

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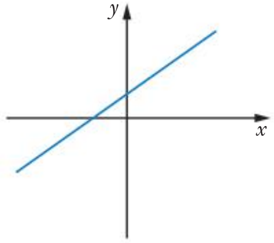
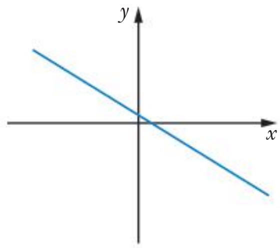
A **positive gradient** means the line goes up from left to right.

A **negative gradient** means the line goes down from left to right.

Positive gradient (increasing)



Negative gradient (decreasing)



Graph gradients

Drawing gradients

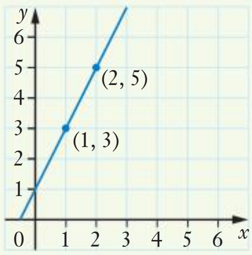
EXAMPLE 5

This line has the equation $y = 2x + 1$.

a What is the gradient of the line?

b What is the value of the y-intercept?

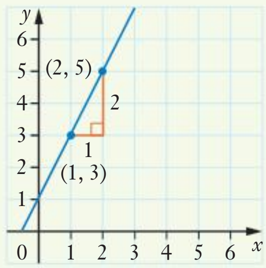
c How do your answers to **a** and **b** relate to the equation of the line?



Solution

a Draw a right-angled triangle using the 2 points, using the line as the hypotenuse.

The line goes up 2 units for every 1 unit across to the right.



Gradient $= \frac{2}{1} = 2$

The gradient is 2.

b The y-intercept is the value where the line crosses the y-axis.

The y-intercept is 1.

c The equation of the line is $y = 2x + 1$.

The gradient 2 is the number in front of the x in the equation.

The y-intercept 1 is the number on its own in the equation.

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EXAMPLE 6

Find the gradient and y-intercept of the line with equation:

a $y = 4x - 1$ **b** $y = -2x$ **c** $y = 8 - 3x$

Solution

a The gradient is the number in front of the x.

$y = 4x - 1$

gradient = 4

The y-intercept is the **constant** or the number on its own.

y-intercept = −1

b The gradient is the number in front of the x.

$y = -2x$

gradient = −2

y-intercept = 0

c $8 - 3x$ can be rewritten as $-3x + 8$.

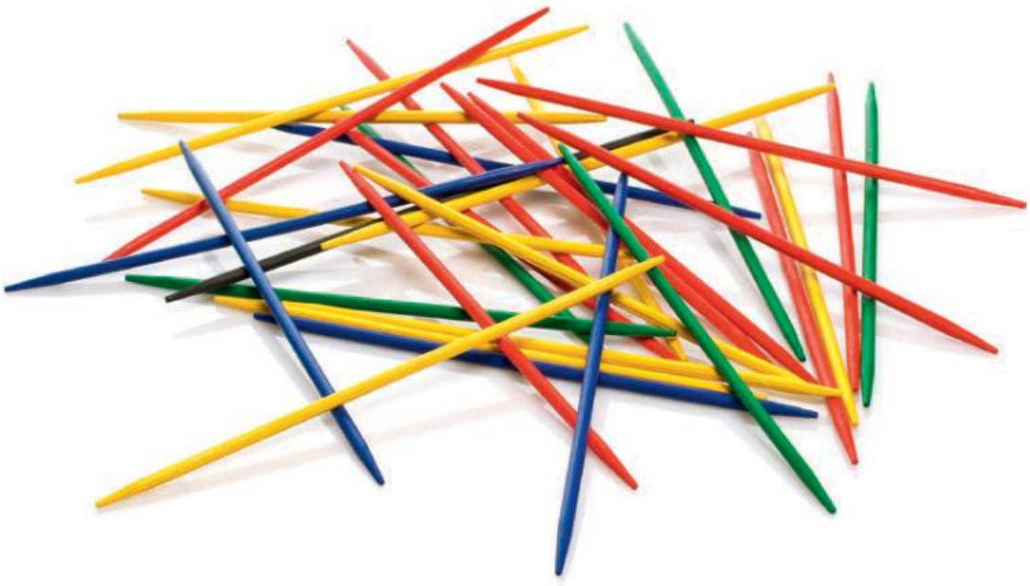
$y = -3x + 8$

gradient = −3

y-intercept = 8

Gradient goat

Gradient and y-intercept



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Exercise 5.03 Graphing linear functions

Example 4

1 Graph each linear function.

a $y = x - 4$ **b** $y = 2x$ **c** $y = x + 3$

d $y = \frac{x}{2}$ **e** $y = -3x$ **f** $y = 2x - 4$

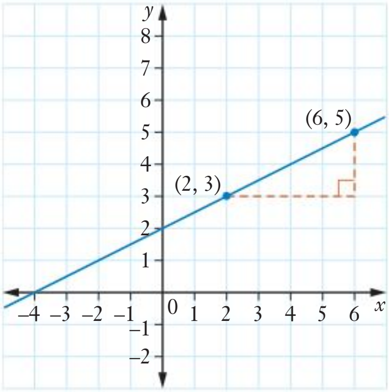
g $y = -x + 2$ **h** $y = 3x$ **i** $y = \frac{x}{4}$

Remember: $\frac{x}{4} = \frac{1}{4}x$

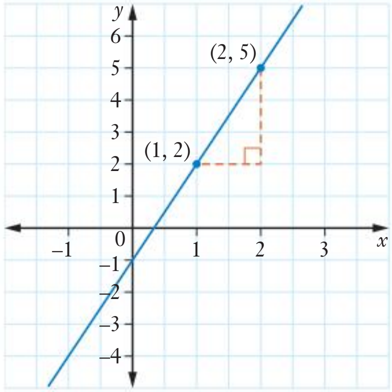
Example 5

2 For each line, find its gradient and y-intercept.

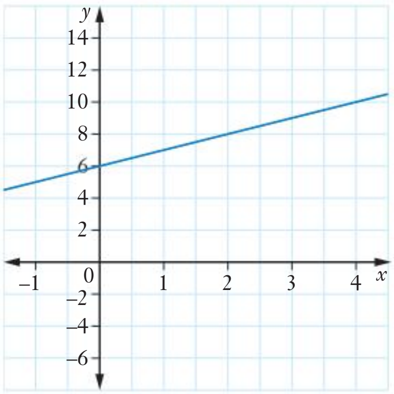
a



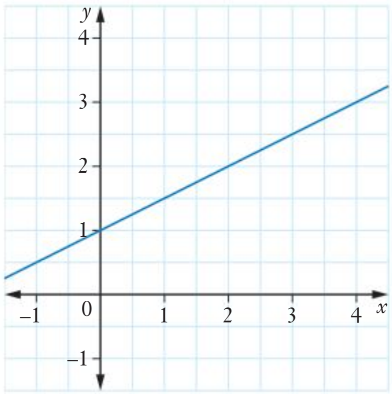
b



c



d



Example 6

3 For each linear function, state:

i the gradient **ii** the y-intercept.

a $y = 3x + 2$ **b** $y = -2x + 3$ **c** $y = 7 - x$

d $y = 4x$ **e** $y = \frac{x}{4} - 2$ **f** $y = -\frac{x}{3} - 4$

4 For each graph you drew in question **1**, state whether the gradient is positive or negative and write the gradient from the equation.

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INVESTIGATION

USING TECHNOLOGY TO GRAPH LINES

We can graph linear functions using technology such as:

- spreadsheets
- graphing/CAS calculators
- graphing software such as GeoGebra or Desmos
- graphing websites

1 Graph the line $y = 2x - 3$ using at least 2 forms of technology.

2 Which technology did you find the easiest to use? Why?

3 Were there any difficulties with any of the technologies? If so, what were they?

4 Use your preferred technology to graph a line that has:

a a positive gradient and a negative y-intercept

b a negative gradient and a negative y-intercept

c a negative gradient and goes through the origin

d a positive gradient and goes through (2, 3)

e a zero gradient and a positive y-intercept

5.04 Applying linear functions

Linear modelling

Linear functions can be used to model many real-life situations.

EXAMPLE 7

The concentration of a particular drug in a person's body decreases as time passes. This is represented by the graph shown, which has equation $C = -25t + 200$. Note that the variables x and y have been replaced by t and C respectively.

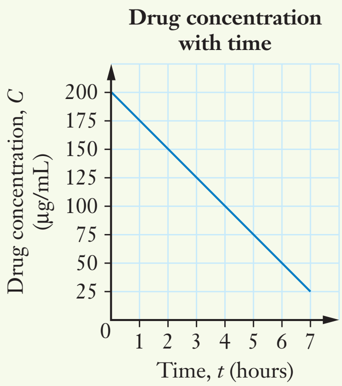
a What is the gradient of the line?

b What does the gradient represent?

c Find the **vertical intercept** of the line.

d What does the vertical intercept represent?

e When will there be no drug remaining in the body?



μg means micrograms or one-millionth of a gram.

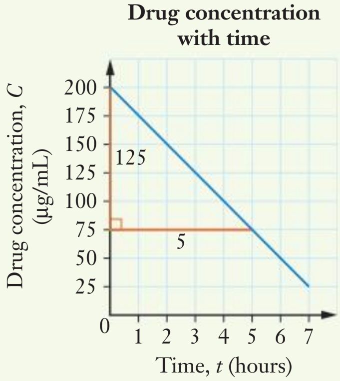
Solution

a Draw a triangle below the line to calculate the gradient.

The line goes down 125 units for every 5 units across. The gradient will be negative because the line goes down rather than up, representing the decreasing drug concentration.

OR

Read the gradient from the equation of the line. It is the number in front of t.



$$\text{Gradient} = -\frac{125}{5}$$
$$= -25$$

b The gradient measures the rate of change of the drug concentration. It is negative because the concentration is decreasing.

The gradient represents the decrease in drug concentration per hour.
It goes down 25 μg/mL every hour.

c The vertical intercept is the general name for the y-intercept. It is the value where the line crosses the vertical axis and is the constant in $C = -25t + 200$.

The vertical intercept is 200.

We don't say 'y-intercept' here because the equation involves t and C, not x and y.

d The vertical intercept is the drug concentration when $t = 0$.

The vertical intercept represents the initial concentration of the drug: 200 μg/mL.

e There is no drug remaining when $C = 0$. We can either extend the line to see where it crosses the horizontal axis, or substitute $C = 0$ into the equation and solve the equation.

When $C = 0$,

$$0 = -25t + 200$$
$$25t = 200$$
$$\frac{25t}{25} = \frac{200}{25}$$
$$t = 8.$$

There will be no drug in the body after 8 hours.

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EXAMPLE 8

Carly has a window cleaning business. Each day it costs her an average of \$75 for fuel and car maintenance and \$8 per job.

a Let C = Carly's daily costs in dollars and n = the number of jobs she does per day.

Write an equation for Carly's daily costs.

b Construct a graph to illustrate Carly's daily costs.

c What is the gradient and vertical intercept of the graph and what do these values represent?

Solution

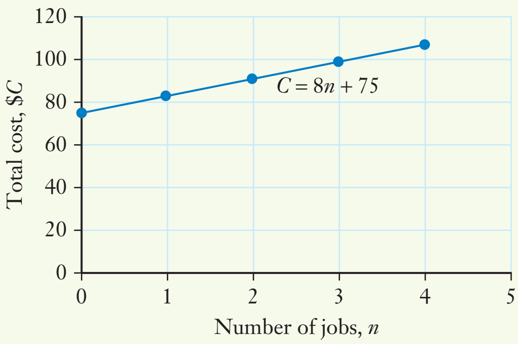
a Carly's daily costs
= \$75 car costs + \$8 per job

$C = 75 + 8 \times n$
$C = 8n + 75$

b Complete a table of values for $C = 8n + 75$.

n	0	1	2	3	4
C	75	83	91	99	107

Use the table of values to graph a line on a number plane.



c The gradient measures the rate of change of Carly's costs as the number of jobs increase by 1. It is positive because the cost is increasing.

For $C = 8n + 75$, the gradient is 8 and the vertical intercept is 75.

The gradient represents Carly's costs per job (\$8). The vertical intercept represents Carly's fixed daily costs (\$75); the cost without doing any jobs.

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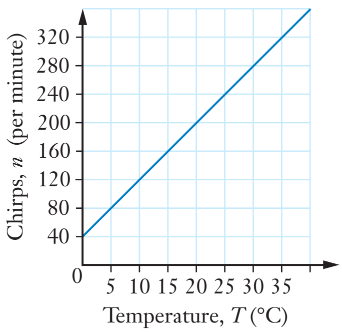
Exercise 5.04 Applying linear functions

Example 7

1 The number of times a cricket chirps per minute is related to the temperature. The relationship is represented by the equation $n = 8T + 40$, where T is the temperature in °C and n is the number of chirps. This is shown on the graph.



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a What is the graph's vertical intercept?

b What does the vertical intercept represent in this situation?

c Calculate the gradient of the line.

d In this context, what does the gradient represent?

e How many times per minute does a cricket chirp when it is 32°C?

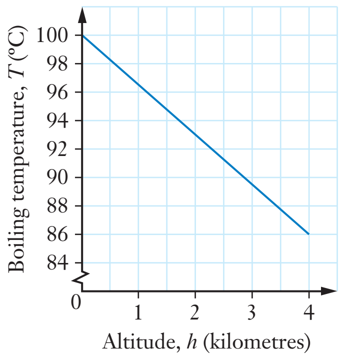
f At what temperature do crickets chirp 160 times per minute?

g Using the formula, calculate how many times per minute a cricket chirps at 100°C. Is this realistic?

2 At sea level, water boils at 100°C. At different altitudes, the boiling temperature changes.

'Altitude' is another word for 'height'.

This graph shows the relationship between altitude (h km) and the temperature (T°C) at which water boils. The equation of the line is $T = -3.5h + 100$.



a Water boils at 100°C at sea level. What altitude is sea level?

b Calculate the gradient of the line.

c What physical quantity does the gradient represent?

d Use the graph to determine the boiling point of water at an altitude of 2.8 km.

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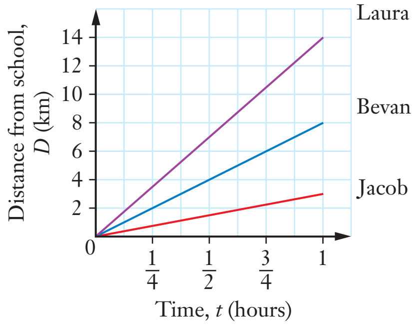
3 After school, 3 students went home by walking, jogging or riding a bike. Each of them travelled at a constant speed and took one hour to reach home. This distance–time graph shows the distance they travelled.



a Calculate the gradient of each line.

b In this context, what does the gradient represent?

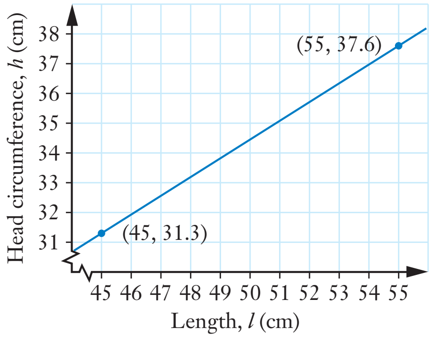
c Which student travelled at the slowest speed? Give a reason for your answer.

d Match Laura, Bevan and Jacob to their method of travelling home: walking, jogging or riding a bike.

4 This graph shows the length and head circumference of average newborn boys.

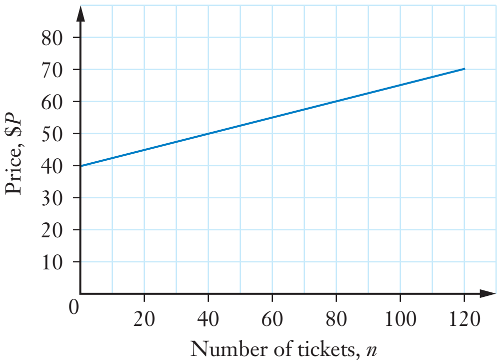
Newborn baby Jason is 51 cm long and his head circumference is 35.1 cm. Do you think Jason's mother should be worried about him? Give a reason for your answer.

Newborn boys' length and head circumference



5 Anastasia operates a printing business. When she quotes a price for printing tickets, she charges an initial fee to cover the cost of design and an additional fee of $0.25 per ticket printed. The price she charges for printing tickets is shown on the graph.

Printing costs



a How much is Anastasia's initial charge for the design?

b What is the gradient of the line?

c What physical quantity does the gradient represent?

d How much does she charge to design and print 500 tickets?

e When Anastasia designs and prints elaborate wedding invitations, she charges an initial fee of $110 and 50c per invitation. Write a formula to determine the price, in D dollars, for designing and printing n elaborate wedding invitations.

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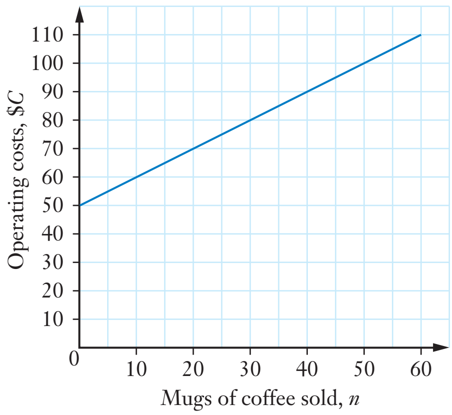
6 Alan runs a coffee stall at sporting events. The graph shows how much it costs Alan to set up the stall and make mugs of coffee.

a How much does it cost Alan to set up the stall?

b How much does it cost Alan to make each mug of coffee?

c If n is the number of mugs sold and C are Alan's costs in dollars, what is the equation of the line on the graph?



Example 8

7 Belinda's hobby is jumping out of planes! This table of values shows her height above the ground t seconds after she opened her parachute.

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Time t seconds	0	60	90	120
Height, h metres	600	300	150	0

a How high above the ground was Belinda when she opened her parachute?

b How many metres is Belinda falling every second?

c Explain why the equation $h = 600 - 5t$ represents this situation.

d Graph this equation.

8 Gabriel is investing $2000 at a simple interest rate of 3.5% p.a. This table shows the interest earned using the formula $I = Prn$.

Number of years (n)	0	1	2	3	4	5
Interest earned (I)	0	$70	$140	$210	$280	$350

a Graph this function.

b Find the gradient of the line and explain what this value represents.

c Find the vertical intercept and explain what this value represents.

d Use the equation to find the interest earned if $2000 is invested at 3.5% p.a. for 7 years.

e Gabriel withdraws his money after 2 years. How much money in total does he receive?

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9 The cost of filling your petrol tank is related to the number of litres it holds, as shown in this table.

Petrol amount (*L* litres)	0	10	20	30	40	50
Cost (\$*C*)	0	14	28	42	56	70

a Graph the relationship between petrol amount and cost.

b What is the gradient of the graph?

c Calculate the cost of buying 75 litres of petrol.

10 Amy has old dressmaking patterns that show material measurements in yards, an imperial measure, which she needs to convert to metres.

a Use this table of values to construct a conversion graph between yards and metres.

Yards	0	1	3	5	10
Metres	0	0.9	2.7	4.5	9.1

b Use your graph to estimate the number of metres of material Amy needs when the pattern states $2\frac{3}{4}$ yards.

c Amy has 4.2 m of material left for making a dress. The pattern says she needs 4 yards of material. Does Amy have enough material to make the dress? Justify your answer.


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WS
Intersection of lines

5.05 Intersecting graphs

We can graph 2 lines on the same number plane to help us solve problems.

EXAMPLE 9

Cherie sells scones for \$3 each. It costs her \$240 for the necessary equipment and \$1 to make each scone.

a Find an equation for C, the cost in dollars for making n scones.

b Find an equation for I, the income in dollars from selling n scones.

c Graph both equations on the same axes for values of n from 0 to 200.

d How many scones does Cherie need to sell to break even?

e How much profit will Cherie make if she sells 390 scones?

Solution

a The cost is \$240 plus \$1 for each scone Cherie makes.

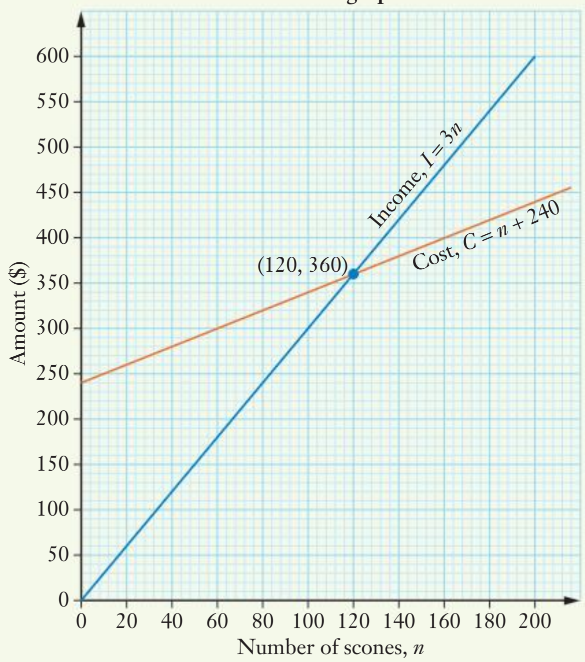
$$\text{Cost} = \$240 + n \times \$1$$
$$C = 240 + n$$
$$C = n + 240$$

b Income is \$3 for each scone.

$$\text{Income} = n \times \$3$$
$$I = 3n$$

c Graph both lines together.

Cost and income graphs for scones



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d 'Break even' means that costs and income are the same. This occurs on the graph where the lines intersect, at (120, 360). At this **break-even point**, 120 represents the number of scones.

Cherie needs to sell 120 scones to break even.

e The profit is how much the income is more than the cost: substitute $n = 390$ into both the Income and Cost equations.

Income: $I = 3n$
$= 3 \times 390$
$= \$1170.$
For 390 scones, the income will be \$1170.
Cost: $C = n + 240$
$= 390 + 240$
$= 630$
For making 390 scones, the cost will be \$630.

Profit = income − cost

Profit = \$1170 − \$630
= \$540
Cherie will make a \$540 profit when she makes and sells 390 scones.

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Break-even points

In the previous example, when $n = 120$, $C = 360$ and $I = 360$. The cost and income are the same. This means that when Cherie makes 120 scones, it costs her \$360 and she receives \$360 from the sales. From the graphs, we can see that if she sells more than 120 scones, she will make a profit because her income is greater than her costs, but if she sells fewer than 120 scones, she will make a loss because her income is less than her costs. The value of 120 is called the **break-even point**.

If we graph the equations of a business' costs and income, then the break-even point is the point where the 2 lines intersect. Below the break-even point, the costs are greater than the income and the business makes a loss. Above the break-even point, the income is greater than the costs and the business makes a profit.

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Exercise 5.05 Intersecting graphs

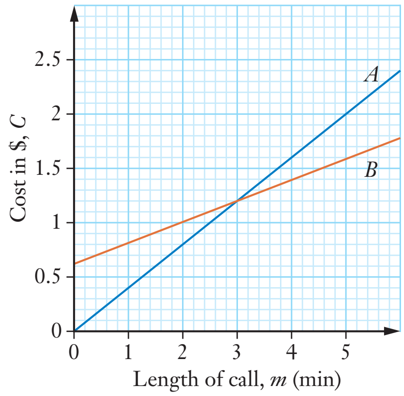
Example 9

1 Khalid bakes and sells muffins. The equipment used to bake the muffins costs \$200 and each muffin he makes costs \$1. Khalid sells the muffins for \$2 each.

Let n = the number of muffins, $\$C$ = the total cost to make n muffins and $\$I$ = the income from selling n muffins.

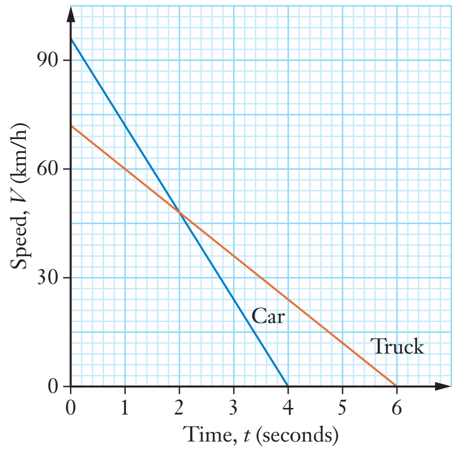
- **a** Explain why $C = n + 200$ and $I = 2n$.
- **b** Graph $C = n + 200$ and $I = 2n$ on the same axes using n from 0 to 400.
- **c** What are the coordinates of the point where the 2 lines intersect?
- **d** What does this point represent?
- **e** Will Khalid make a profit if he sells 120 muffins? Explain your answer.
- **f** How much profit will Khalid make if he sells 230 muffins?

2 Chloe is deciding between 2 mobile phone plans. Plan A offers her no connection fee and charges of 40c per minute for each call. Calls under plan B cost 20c per minute with a 60c connection fee. The graphs show the costs of phone calls under each plan for calls of up to 6 minutes.



- **a** Which plan (A or B) has the equation $C = 0.6 + 0.2m$? Explain the reason for your choice.
- **b** For what length of phone call is the cost the same under both plans?
- **c** Which plan is cheaper for longer calls?

3 A truck and a car were travelling on the freeway. When an accident occurred ahead of them, both vehicles had to stop quickly. The graphs show the speed each vehicle was travelling at t seconds after they applied their brakes.



- **a** How fast were the car and the truck going in km/h before the accident?
- **b** How many seconds after applying their brakes were the vehicles travelling at the same speed?
- **c** How much longer than the car did it take for the truck to stop?
- **d** Write a linear equation for V, the speed in km/h, for the speeds of each vehicle t seconds after applying their brakes.
- **e** Explain why the gradients of both lines are negative.
- **f** Suggest a reason why the truck took longer to stop than the car, even though the car was travelling faster initially.

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4 Allana sells flowers at the market for $10 per bunch.

a Copy and complete this table.

Bunches of flowers sold	0	10	20	30	40	50	60
Income received ($)							

b This graph shows Allana's *costs* when she sells *n* bunches of flowers. Copy the graph and use your answers to part **a** to graph Allana's income on the same axes.

c Explain how you know that Allana will break even when she sells 32 bunches of flowers.

d How many bunches of flowers will Allana need to sell to make a profit?

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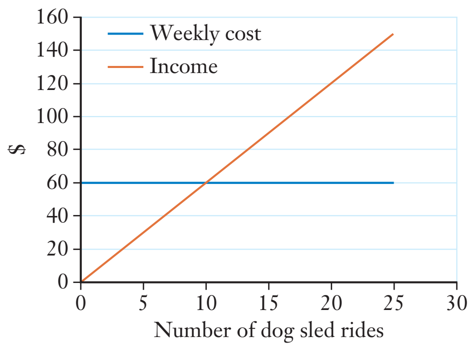
5 Each week, a toy factory's fixed costs total \$1500. Its costs increase by \$15 for every toy it produces. Each item produced sells for \$35.

Let n = the number of items the factory produces each week, $\$C$ = the total costs to produce n items, and $\$I$ = the income the factory receives from selling n items.

- **a** Does the expression $35n$ represent the income from selling n items or the cost to produce n items?
- **b** What does the expression $15n + 1500$ represent?
- **c** Graph both equations $I = 35n$ and $C = 15n + 1500$ on the same axes, using n from 0 to 200.
- **d** How many toys does the factory need to produce each week to break even?
- **e** How much profit will the company make if it sells 100 toys in a week?

6 Jon earns money taking tourists on dog sled rides in the snow. Jon has displayed his expenses and income on a graph.



Number of dog sled rides	0	5	10	15	20	25
Weekly cost	60	60	60	60	60	60
Income	0	30	60	90	120	150

- **a** How much are Jon's weekly costs?
- **b** How many dog sled rides does Jon require each week to break even?
- **c** How much does Jon charge for a dog sled ride?
- **d** Suggest a reason why Jon's cost line is horizontal.

7 To advertise the school musical, the drama class is making and selling promotional T-shirts. The set-up costs to make the T-shirts is \$160 and each shirt will cost \$10 to make. The class will sell the T-shirts for \$20 each.

- **a** Find an equation for C, the cost in dollars for making n T-shirts.
- **b** Find an equation for I, the income in dollars from selling n T-shirts.
- **c** Graph both equations on the same axes, for values of n from 0 to 50.
- **d** How many T-shirts need to be sold to break even?
- **e** The class estimates it will sell 90 T-shirts. How much profit can they expect to make?

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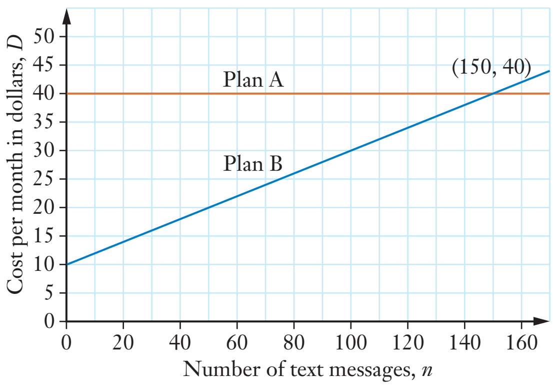
8 Grant is hard of hearing and uses his mobile phone for texting only. This graph shows the monthly cost, $\$D$ dollars for n texts on 2 phone plans. Plan A is a fixed price per month with unlimited texts while Plan B has a monthly charge and a cost per text.



a What is the monthly cost for Plan A?

b What is the vertical intercept of the graph of Plan B?

c What does the vertical intercept in Plan B represent?

d How much does each text cost to send on Plan B?

e What is the equation of the line representing Plan B?

f For what number of calls per month is Plan A the same cost as Plan B?

g Grant sends an average of 5 or 6 texts per day. Which plan would you recommend he chooses? Use calculations to justify your recommendation.

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Chapter problem

You've covered the skills required to solve the chapter problem. Can you solve it now?

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INVESTIGATION

COSTING THE YEAR 12 FORMAL

In this investigation you will model the costs and income for running the Year 12 formal.

1 You will need to collect the following information for where you live:

Costs	Hire of venue	______________________
	Decorations	______________________
	Band/DJ	______________________
	Other costs	______________________
	Total costs	______________________

These are the fixed costs for the formal.

The committee decides to charge $15 per person to cover the fixed costs.

2 On the same number plane:

- draw a graph for $I = 15n$ for n from 0 to 200.
- draw a graph for $F =$ _______ for the fixed costs. This will be a horizontal line.

3 **a** Where do the 2 lines intersect? How many people need to attend the formal to cover the fixed costs?

b How many people do you expect to attend the formal? Will you cover the fixed costs if you charge $15 per person?

c Depending on what you have found, adjust the amount you need to charge per person to cover the fixed costs. This could be more or less than $15. Draw the income graph for this new amount.

d How many people now need to attend the formal to cover the fixed costs?

4 Find out from the venue how much they charge per person for catering and calculate what you need to charge for your formal tickets.

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5.06 Curved graphs

Not all practical situations give straight-line graphs. Now we will look at some situations that when graphed give us curves rather than straight lines.

Quadratic functions

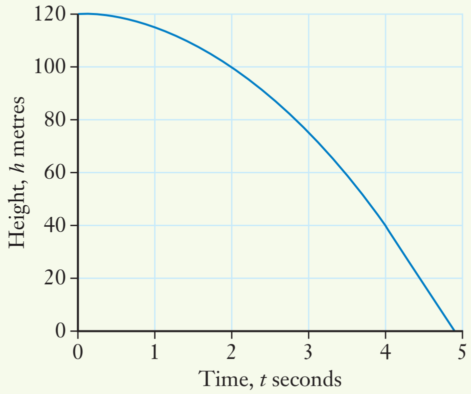
EXAMPLE 10

A rock fell from the top of a cliff onto a road. The graph shows the height of the rock above the road t seconds after it fell.

a How high was the rock above the road after 2 seconds?

b How far did the rock fall during those first 2 seconds?

c Approximately after how many seconds did the rock hit the road?



Solution

a Reading from the graph at $t = 2$ gives $h = 100$.

The rock was 100 m above the road after 2 seconds.

b The rock was 120 m high before it fell and 100 m high after 2 seconds.

During the first 2 seconds it fell:
120 m – 100 m = 20 m

c Read from the graph.

The rock hit the road at about 4.9 s.

EXAMPLE 11

Isobel plays basketball. When she takes a shot, the ball follows a curve. This table gives the height, h metres, of the ball t seconds after it leaves her hands.

Time, t (seconds)	0	0.5	1	1.5	2	2.5
Height, h (metres)	2.4	3.3	3.7	4.0	3.6	2.8

Graph this data and join the points with a smooth curve.

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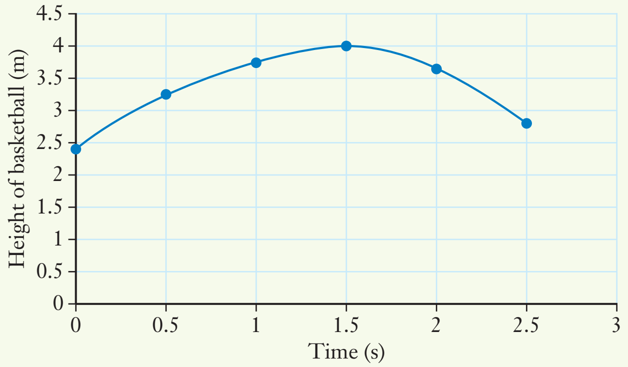
Solution

Draw a set of axes with Time on the horizontal axis and Height on the vertical axis.

Plot the points.

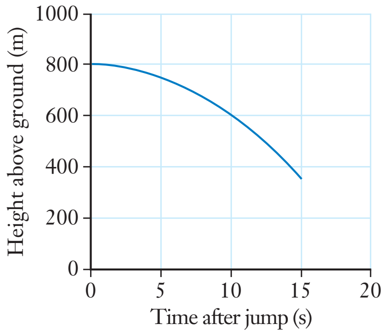
Join them with a smooth curve.



Exercise 5.06 Curved graphs

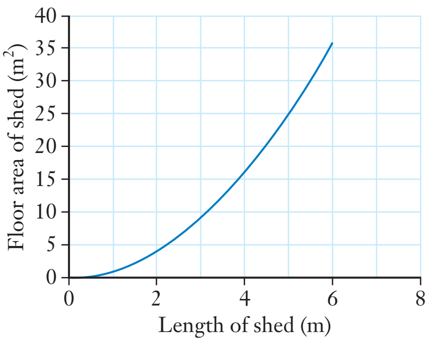
Example 10

1 When Jose jumps out of a plane, he free-falls for 15 seconds before pulling the parachute cord. The graph shows his height during the free-fall.



- **a** At what height did Jose jump out?
- **b** How high was Jose above the ground 10 seconds after he jumped?
- **c** How far did Jose fall in the first 10 seconds?
- **d** Approximately how high above the ground was Jose when he pulled the cord?

2 Adriana is planning to make a square chicken shed on her farm. The graph shows the relationship between the length and area of the shed.



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Use the graph to estimate the length of the chicken shed with an area of 30 m^2.

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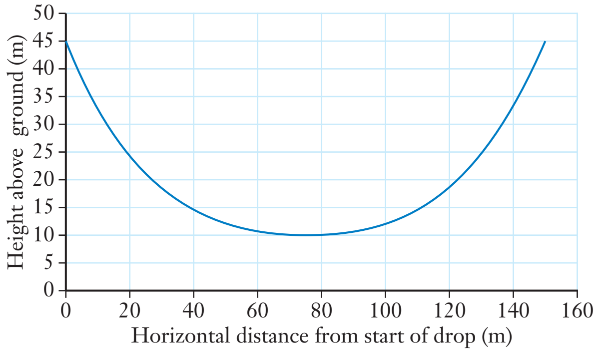
3 This graph represents part of a roller coaster track at a theme park.



a At what height does this section of the ride start?

b How high above the ground is the ride when it has travelled 50 m horizontally?

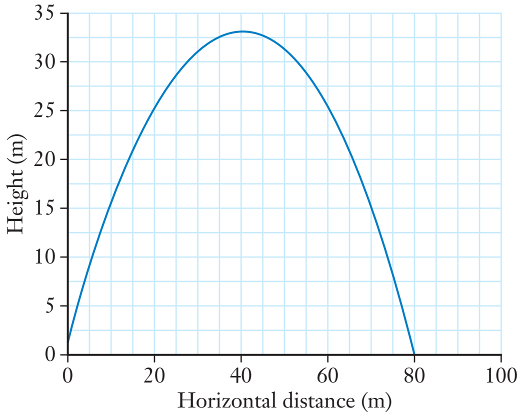
c How far has the ride travelled horizontally when it reaches this height again?

4 Naresh hit a cricket ball out of the oval. The graph shows the path of the ball.

a Estimate the maximum height of the ball.

b Approximately how far from Naresh did the ball land?

c While falling, the ball just cleared the top of the scoreboard, which is 12 m high. How far is the scoreboard (horizontally) from Naresh?



5 This table shows the height of tides in Cairns on 3 consecutive days.

Example 11

Day	Wednesday				Thursday				Friday			
Time	1 a.m.	7:30 a.m.	1:15 p.m.	7:45 p.m.	1:45 a.m.	8 a.m.	2 p.m.	8:15 p.m.	2:15 a.m.	8:45 a.m.	2:30 p.m.	8:45 p.m.
Height of tide (m)	0.5	2.6	0.6	2.8	0.4	2.7	0.6	2.7	0.4	2.8	0.8	2.6

a Graph this data with Time on the horizontal axis and Height on the vertical axis. Join the points with a smooth curve.

b Use your graph to estimate the height of the tide at midnight Wednesday.

c Joe needs a tide height of 1 m to go fishing. Use your graph to estimate when he can go fishing on Friday afternoon.

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6 For a Science project, Ellie measures the temperature of a cup of coffee at different times after she makes it. Her data is given in the table below.

Time (min)	0	4	8	12	16
Temperature (°C)	80	52	47	43	40

a When you graph this table, should Time or Temperature be on the horizontal axis?

b Graph this table and join the points with a smooth curve.

c Use your graph to estimate the temperature of the coffee after 2 minutes.

d Ellie likes to drink her coffee at approximately 53°C. After she makes her coffee how long should she wait to drink it?

7 These tables show the hours and minutes of daylight in 2 cities on the 21st day of the month over one year. For example, 6:52 means 6 hours 52 minutes of daylight.

Anchorage, Alaska											
Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
6:52	9:39	12:20	15:18	17:59	19:22	18:00	15:16	12:21	9:31	6:47	5:27

Perth, Australia											
Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
14:39	13:25	12:07	10:43	9:37	9:09	9:36	10:42	12:05	13:27	14:40	15:11

a On the same set of axes, graph the data for each city and join the points with a smooth curve. Use a different colour for each city.

b What are the similarities between the 2 graphs?

c What are the differences between the 2 graphs?

d In which months do the graphs cross?

e Why is the maximum hours of daylight in Anchorage in June greater than the maximum hours of daylight in Perth in December?

8 Toby is organising a bus trip to see a concert. The cost of the bus is $840 and the maximum number of people it carries is 40.

a Copy and complete this table to show the cost per person for different numbers of people.

Cost per person = 840 ÷ the number of people

Number of people	1	5	10	20	30	40
Cost per person ($)	840	168				

b Graph this data, joining the points with a smooth curve.

c From your graph estimate the cost per person if 28 people attend.

d Calculate the cost using the formula. How close was your estimate from the graph?

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5.07 Practical graphs

WS Currency conversion graph

EXAMPLE 12

Karen discovered some recipes in an old cookbook, but the measurements are in pounds. She found a table showing some conversions from pounds to grams.

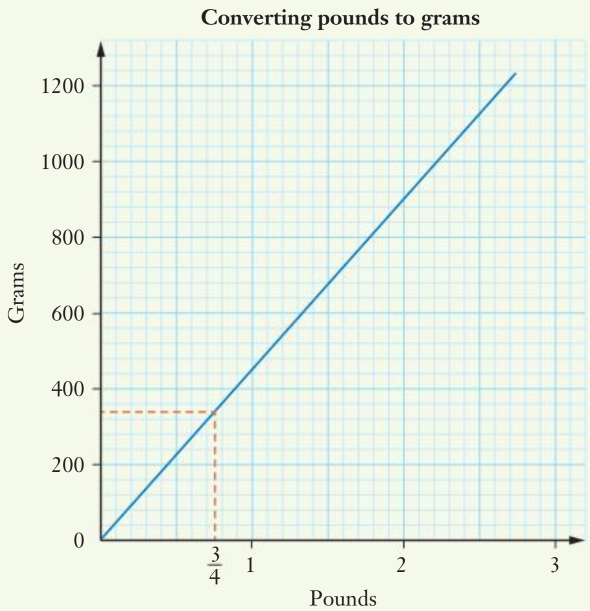
Pounds	0	1	1.5	2
Grams	0	454	680	907

a Construct a **conversion graph** that Karen can use to convert pounds to grams.

b Karen needs $\frac{3}{4}$ pound of meat to make pizzas. How many grams of meat is this?

Solution

a Draw a set of axes and label the horizontal axis 'Pounds' and the vertical axis 'Grams'. Graph the straight line given by the table of values.



b Find $\frac{3}{4}$ or 0.75 pounds on the horizontal axis. Read up to the line, then across to the vertical axis.

Karen needs approximately 340 g of meat for the pizzas.

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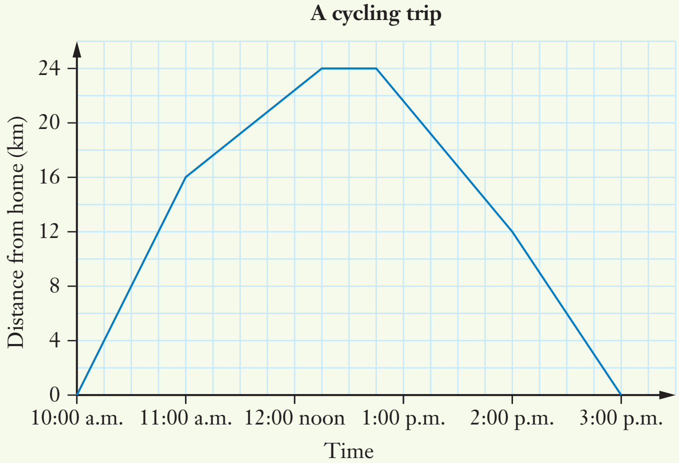
EXAMPLE 13

This **distance–time graph** shows Thuy's cycling trip.



a At what time did Thuy leave home?

b When did Thuy stop? How far from home was he?

c Find his speed in km/h:

  i in the first hour **ii** in the second hour

d How is a change in speed shown on the graph?

e When did Thuy start returning home?

f How long did it take him to return home?

Solution

a For leaving home time, look at the start of the graph, when distance = 0.

Thuy left home at 10 a.m.

b For stopping, look for where the graph is flat.

He stopped at 12:15 p.m., 24 km from home.

Where the graph is flat, the distance from home does not change, which means that Thuy has stopped.

c **i** In the first hour, Thuy travelled from 0 km to 16 km.

Thuy's speed in the first hour was 16 km/h.

ii In the second hour, Thuy travelled from 16 km to 24 km, a distance of 24 – 16 = 8 km.

Thuy's speed in the second hour was 8 km/h.

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d	Where the graph is steeper, the speed is greater: more distance travelled in less time.	A change in speed is shown by a change in the steepness of the graph. The graph is steeper in the first hour than in the second hour because the speed is greater in the first hour.
e	Thuy is heading home when the graph goes downward, with a negative gradient.	Thuy started returning home at 12:45 p.m.
f	He reached home at 3 p.m.	Time to return home = 3 p.m. – 12:45 p.m. = 2 h 15 min

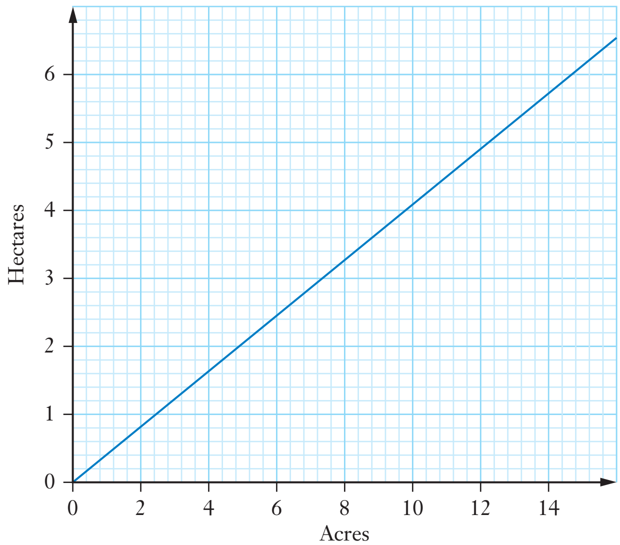
On a distance-time graph:

- a horizontal (flat) section on the graph indicates a stop
- the steeper the line, the greater the speed (more distance covered in less time)
- a section going down, towards the right, indicates a change in direction or that the traveller is returning towards the start

Exercise 5.07 Practical graphs

Example 12

1 This graph is used to convert from acres to hectares. The acre is an old measure of land area.



a What is the size of one small unit on the horizontal axis?

b What is the size of one small unit on the vertical axis?

c What is the gradient of this line, correct to one decimal place?

d Use the graph to convert 12 acres to hectares.

e Jane's garden has an area of 5 acres. What is this area in hectares?

f Use the graph to convert 4.4 hectares to acres.

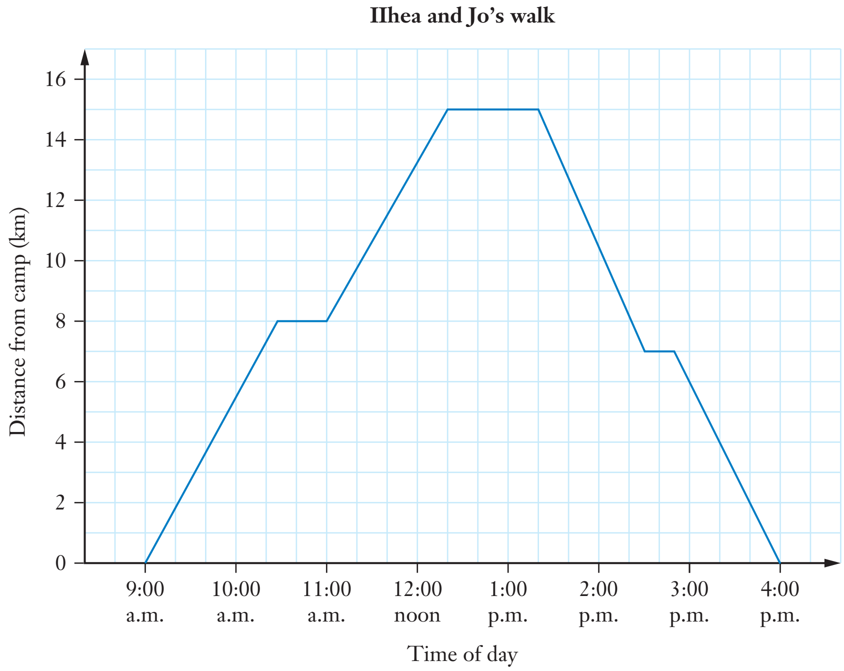
g Ciaran has a property with an area of 6 hectares. How big is this in acres?

h Nelson Park is rectangular with dimensions 250 m by 128 m.

- **i** What is the area of the park in square metres?
- **ii** What is the area of the field in hectares? Remember: 1 ha = 10 000 m^2.
- **iii** What is the area of the field in acres?

Example 13

2 Ilhea and Jo decided to walk from their campsite to the beach. This travel graph shows their distance from camp during their walk.



a What is the scale on the horizontal axis?

b What is the scale on the vertical axis?

c How far from camp is the beach?

d At what times and for how long did Ilhea and Jo stop each time?

e What is the gradient of the section of the graph from 9 a.m. to 10.30 a.m.? What does this gradient represent?

f How far did Ilhea and Jo walk that day?

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3 Adele is backpacking in Europe. She doesn't have much money, so she needs to know how much everything costs. This table shows the conversion between Australian dollars (AUD) and euros (EUR), the common currency in Europe.

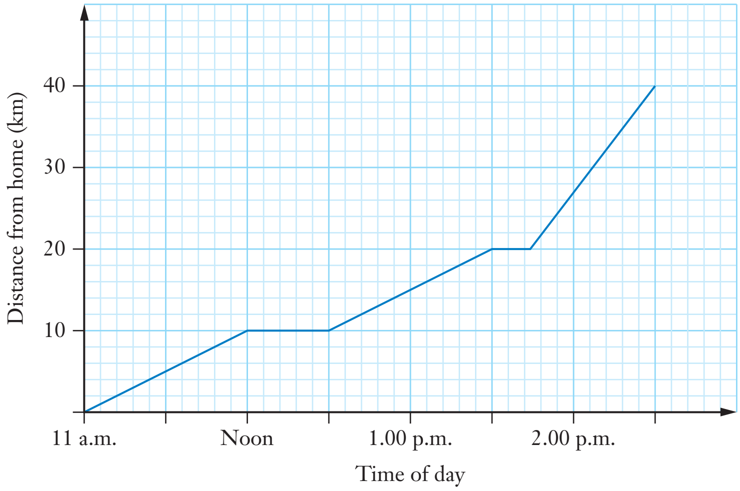
Australian dollars	0	10	20	50
Euros	0	6.63	13.26	33.15

a Construct a conversion graph for Australian dollars to euros.

b Approximately how many euros should Adele receive for 30 AUD?

c Adele's lunch cost 9 EUR. How much is this in AUD?

4 Tara rode her bicycle to her friend Alana's place. This graph describes her trip.



a At what time did Tara leave home?

b How far did she travel altogether?

c Calculate her speed between 12:30 p.m. and 1:30 p.m.

d When did Tara rest?

e At what time is Tara travelling the fastest? How can you tell?

5 One Australian dollar (AUD) buys $0.76 US dollars (USD).

a Copy and complete the table of values.

Australian dollars (AUD)	0	10	100	200
US dollars (USD)				

b Construct a conversion graph between AUD and USD.

c Tahira ordered headphones online for $25 USD. How much did the headphones cost in AUD? Round to the nearest cent.

d Rafi is going to change $400 AUD into USD before he flies to Hawaii. How many USD will he receive?

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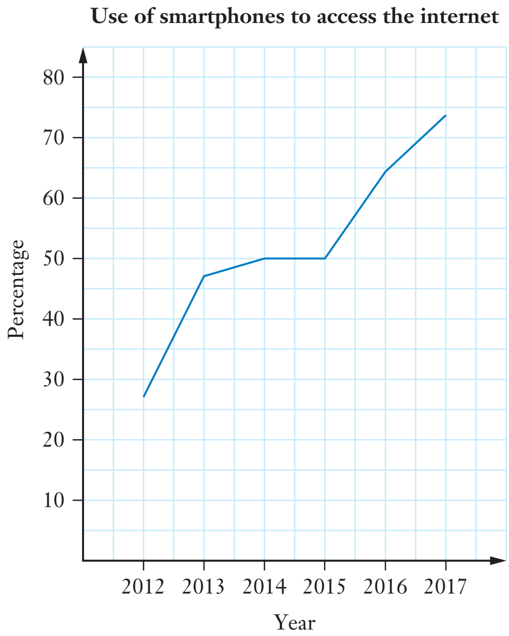
6 This graph shows the percentage of people with smartphones who use them to access the internet.

a Estimate from the graph what percentage of people used their phones to access the internet in 2013.

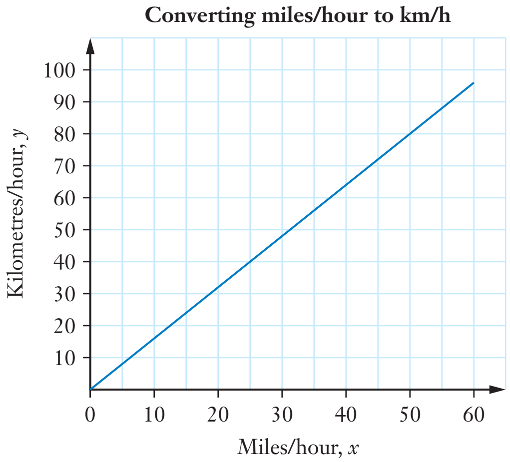
b Estimate from the graph what percentage of people used their phones to access the internet halfway through 2017.

c Describe how internet usage via phone is changing over time.

d What do you think the percentage would be now? Why?



7 This conversion graph converts between miles per hour (x) and kilometres per hour (y).



a What speed in km/h is equivalent to 30 miles/h?

b What is the gradient of the conversion graph?

c Write an equation for y involving x.

d The speed limit on a British motorway is 100 miles per hour. What is this speed in km/h?

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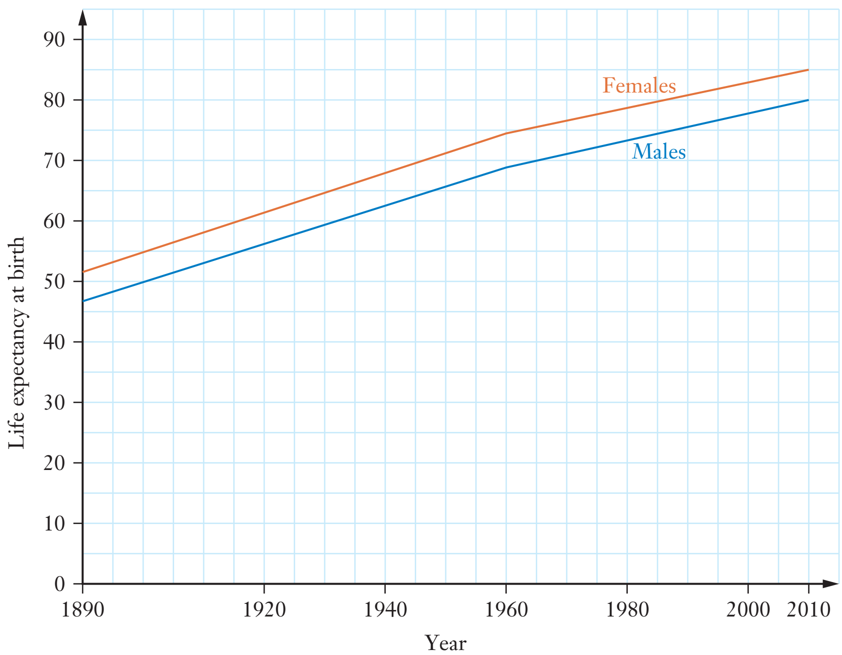
8 The graphs in this question show how life expectancy has changed over time.

Life expectancy at birth



a What was the life expectancy of a newborn male in 2010?

b Whose life expectancy (males or females) has increased the most between 1960 and 2010 and by how much?

c Which part of the graphs is the steepest? What does this mean?

d What was the life expectancy of a female in 1890?

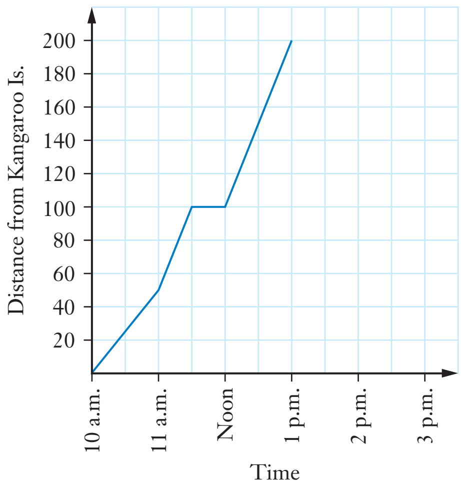
e What is the difference between men and women's life expectancy in 1960?

f Describe the trend in life expectancy as shown in both graphs.

9 This incomplete graph shows Hong's trip from Kangaroo Island to Adelaide and back.

a Describe the section of the journey shown on the graph.

b Hong stays in Adelaide for 2 hours and then returns to Kangaroo Island without stopping, arriving at 5 p.m. Copy the graph, extending the Time axis to 5 p.m., and complete the graph to show the return section of Hong's trip.



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10 Jack goes on a trip to visit his grandparents.

a Draw a travel graph to represent Jack's journey:

- Jack starts at 9 a.m.
- He travels 180 km in 2 hours.
- He stops for $\frac{1}{2}$ an hour.
- He travels 120 km in $1\frac{1}{2}$ hours.
- He stops at his grandparents' house for 6 hours.
- He travels back home at 100 km/h.

b How far from Jack's home do his grandparents live?

c At what time does Jack arrive there?

d In which part of the journey is Jack travelling the fastest?

e How long does the journey home take?

f What is the total time taken for the trip?

g How far from home is Jack at 9 p.m.?

KEYWORD ACTIVITY

CHAPTER SUMMARY

Use the listed words to copy and complete the summary of this chapter below.

break-even	costs	curved	expenses	gradient
income	intersecting	linear	lines	loss
model	point	profit	*y*-intercept	

In this chapter, we learned about [1]__________ functions that give us straight[2]__________ when we graph them. The [3]__________ of the line tells us how steep the line is. The [4] __________ tells us where the line crosses the *y*-axis.

We used linear functions to [5]________ real-life situations.

For 2 [6]____________ lines, we examined the [7]_______ where these lines crossed. We applied this idea to practical problems such as business, travelling and comparing mobile phone plans.

In business, we drew a graph for the money coming in, called [8]________, and the money being spent, called [9]________ or [10]____________. The point of intersection of these 2 lines is called the [11]______________ point. At this point, the business owner does not make a [12]________ or a [13]____.

We also learned about practical situations that don't have a straight-line graph. Instead, many graphs are [14]_________.

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SOLUTION TO THE CHAPTER PROBLEM

Problem

Paul owns a factory that makes sails. It costs $9000 per month to cover rent, electricity and wages in the factory. Each sail costs $250 to make and sells for $850.

a How many sails does Paul need to make and sell per month to break even?

b What profit or loss will he make in a month when he makes and sells 20 sails?

Solution

a Let n = the number of sails

C = the cost to produce n sails per month

I = the income from selling n sails.

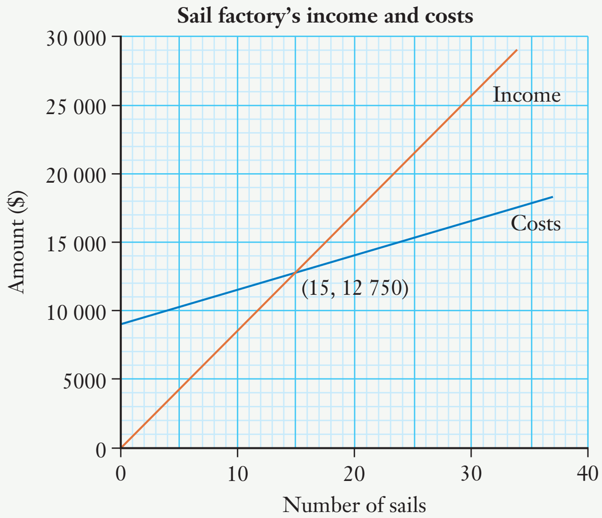
The linear functions are:

$C = 250n + 9000$

$I = 850n$

The lines intersect at (15, 12 750), which means that the break-even point is $n = 15$.

Paul needs to make and sell 15 sails per month to break even.



b When $n = 20$:

$\text{Cost} = \$250 \times 20 + \$9000$

$= \$14\ 000$

$\text{Income} = \$850 \times 20$

$= \$17\ 000$

Income is greater than cost, so a profit will be made.

$\text{Profit} = \$17\ 000 - \$14\ 000$

$= \$3000$

Paul will make a profit of $3000 when he makes and sells 20 sails in a month.

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5. TEST YOURSELF

Practical graphs

Qz Practice quiz

Exercise 5.01

1 Plot each point on a number plane.

a $P(1, -4)$ **b** $Q(-3, -1)$ **c** $R(-1, 2)$ **d** $S(1, 3)$

e $T(0, 4)$ **f** $U(2.5, -2)$ **g** $W(-2, 0)$ **h** $Y(-3, 1.5)$

Exercise 5.01

2 Which points from question **1** are:

a on the y-axis? **b** in the 2nd quadrant?

c on the x-axis? **d** in the 4th quadrant?

Exercise 5.02

3 Copy and complete each table of values.

a $y = x + 3$

x	−1	0	1	2	3
y					

b $y = 2x - 7$

x	−2	−1	2	4	5
y					

c $y = 5 - x$

x	−3	−1	0	2	4
y					

Exercise 5.03

4 Graph each linear function on a number plane.

a $y = 2x - 2$ **b** $y = -x + 3$ **c** $y = \frac{x}{2} - 2$

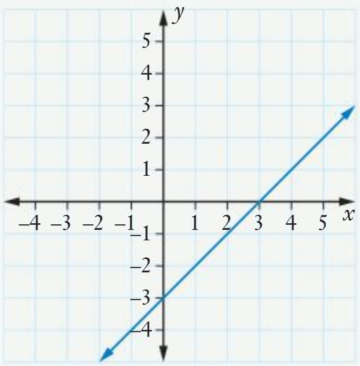
Exercise 5.03

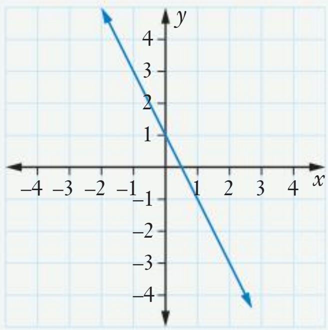
5 Find the gradient and y-intercept of each linear function in question **4**.

Exercise 5.03

6 For each graph find:

i the gradient **ii** the y-intercept





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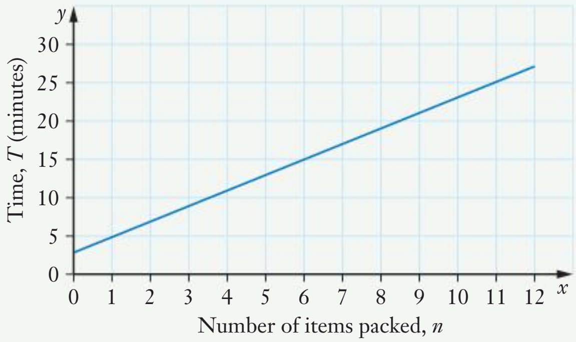
7 For each linear function, state:

i the gradient **ii** the y-intercept

a $y = 4x - 3$ **b** $y = -x + 2$ **c** $y = \frac{x}{3}$

Exercise 5.03

8 Goran packs glass items in boxes. He has to assemble the box before he can pack it. The graph shows the time, T minutes, it takes him to assemble one box and pack n glass items in it.



Exercise 5.04

a How long does it take Goran to assemble a box before he starts to pack it?

b Calculate the gradient of the line.

c What physical amount does the gradient represent?

d Use the graph to find how many items can be packed in 15 minutes.

e Calculate the time it takes Goran to assemble a box and pack 15 items in it.

9 Adrian's Awesome Appetisers charges \$10 per person plus an \$80 fixed charge. This table shows Adrian's charges.

Exercise 5.04

Number of people, n	0	50	100	150	200	250
Charge, \$$C$	80	580	1080	1580	2080	2580

a Construct a graph showing Adrian's charges for n people.

b Use the graph to find the cost for 170 people.

c Adrian charged NSM Bank \$830 for catering for a party. How many people were catered for?

d Find the vertical intercept of the line and explain what this value represents.

e Find the gradient and explain what this value represents.

10 The Year 12 fundraising committee plans to sell school souvenir USB drives. The manufacturer quoted an 'initial charge of \$126 plus \$5 per unit' and the committee thinks that it will sell the units for \$12 each.

Exercise 5.05

a Find an equation for \$$C$, the cost of making n USB drives.

b Find an equation for \$$I$, the income from selling n USB drives.

c Graph both equations on the same axes for values of n from 0 to 20.

d How many USB drives does the committee need to sell to break even?

e There are 140 students in Year 12. If half of them buy a USB drive each, how much profit can the committee expect to make?

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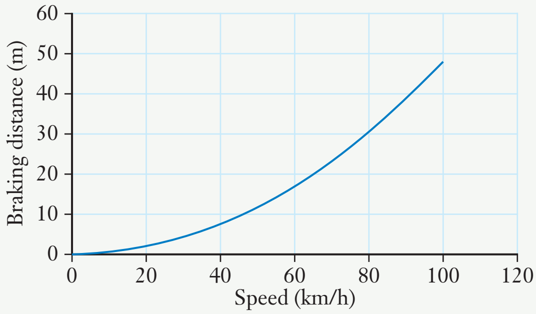
Exercise 5.06

11 This graph shows the braking distance for a car at different speeds.



a What is the braking distance at 100 km/h?

b At what speed is the car travelling if the braking distance is 30 m?

c Bridie is travelling at 60 km/h. She sees an accident ahead. She travels 25 m before she applies the brakes. What is Bridie's total stopping distance from when she sees the accident?

Exercise 5.06

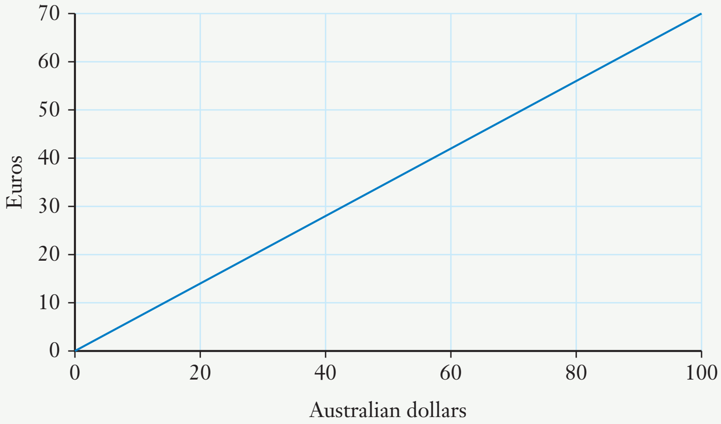
12 Susie planted a new shrub in her garden. She measured its height each week.

Week	1	2	3	4	5	6
Height (cm)	3	7	10.5	13.5	16	18

Draw a graph of this data and join the points with a smooth curve.

Exercise 5.07

13 This graph can be used to convert Australian dollars (AUD) to euros (€).



a Convert \$15 to euros.

b Convert €50 to Australian dollars.

c A meal in Paris costs €35. What is this in Australian dollars?

d Gustav is in Australia. He has €25 left on his debit card. Is this enough to pay for a \$25 meal at the local club?

e Calculate how many euros you would get for \$220.

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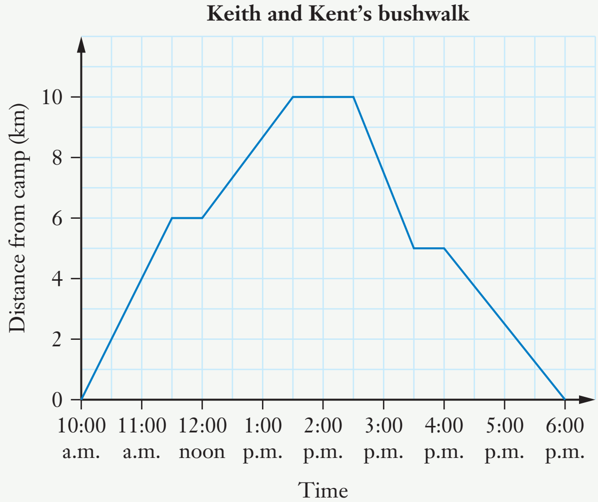
14 Keith and Kent went bushwalking one day. This distance-time graph shows their journey.

Exercise 5.07



a How far did Keith and Kent walk?

b How many stops did they make?

c At what time did they start back?

d Between what times were Keith and Kent walking the fastest?

e Calculate their speed in the last 2 hours.

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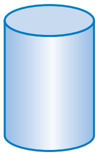
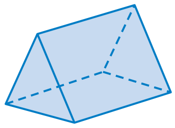
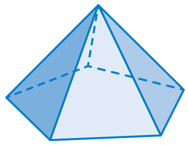
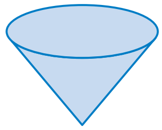
Practice set 1

Section A Multiple-choice questions

For each question select the correct answer **A**, **B**, **C** or **D**.

Exercise 2.02

1 Which solid shown is a prism?

A  **B**  **C**  **D** 

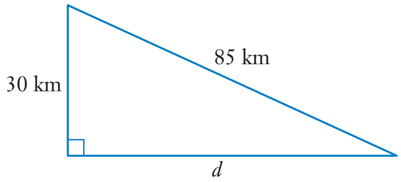
Exercise 1.02

2 Joshua creates 3 graphs showing the information he found in his survey. What step is this in the statistical investigation process?

A collecting data **B** organising data
C displaying data **D** analysing data

Exercise 3.01

3 Find the length of the side marked *d*, rounded to the nearest whole number.



A 55 km **B** 80 km
C 90 km **D** 115 km

Exercise 1.01

4 For which of the following would you use a census rather than a sample to investigate?

A Testing coffee for the best taste
B Finding the number of migrants from Japan to Australia
C Obtaining general views on road safety in Victoria
D Finding out people's views on whether Australia should become a republic

Exercise 5.03

5 Which equation below has a graph that is a line with a gradient of 3?

A $y = 3x$ **B** $y = 3$ **C** $y = x + 3$ **D** $y = 3 - x$

Exercise 3.04

6 Find the size of the angle θ, given tan θ = 0.859.

A 31° **B** 40° **C** 41° **D** 59°

Exercise 1.03

7 A radio station asks listeners to complete an online survey about their favourite music. What type of sample is this?

A random **B** self-selected **C** stratified **D** systematic

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8 I am a quadrilateral with opposite sides equal. My diagonals bisect each other and my corners are right angles. What am I?

A parallelogram **B** trapezium **C** rectangle **D** rhombus

Exercise 2.01

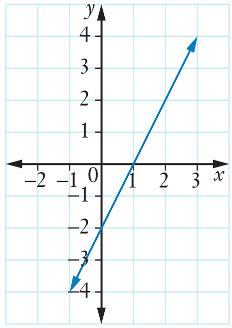
9 What is the equation of this line?

A $y = -x + 2$

B $y = x - 2$

C $y = 2 - 2x$

D $y = 2x - 2$

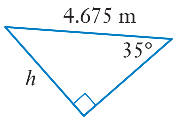


Exercise 5.03

10 Find the length of the side marked h.

A 2.68 m **B** 3.27 m

C 3.83 m **D** 4.68 m



Exercise 3.06

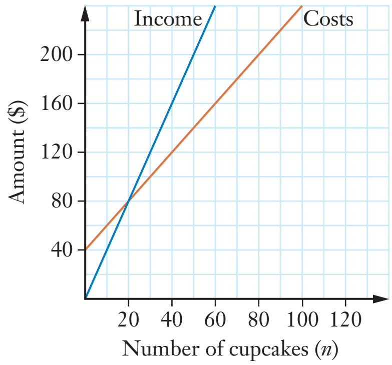
11 This graph shows the income and expenditure for Catriona's Cupcakes. What happens if Catriona sells 40 cupcakes?

A She makes a profit

B She makes a loss

C She breaks even

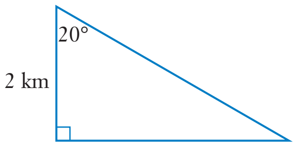
D You can't tell from the graph



Exercise 5.05

12 Mala is flying a glider at a height of 2 km. To land, she descends at an angle of depression of 20°. Which diagram illustrates this correctly?

A



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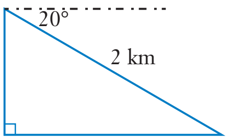

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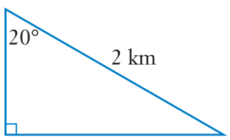

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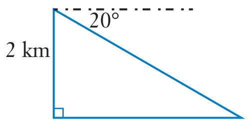

Exercise 3.05

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Exercise 5.03

Exercise 1.01

Exercise 2.02

Exercise 3.03

Section B Short-answer questions

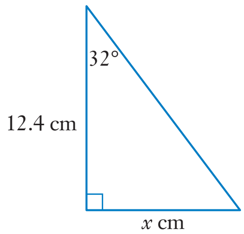
1 Graph each linear function.

a $y = x - 3$ **b** $y = 3 - x$

2 The Australian Bureau of Statistics conducts a census every 5 years. Give an example of how they summarise and display the data they collect.

3 Sketch a neat diagram of a cylinder.

4 Find the value of x, correct to 2 decimal places.



Exercise 1.03

5 Melinda wants to survey student views on the school uniform. Describe how she could choose each type of sample.

a random sample **b** stratified sample

c self-selected sample **d** systematic sample

Exercise 1.03

6 Melinda decides to use a stratified sample of 10% of students. She has the following data from the school office.

Male	390
Female	450
Total students	840

Year 7	165
Year 8	128
Year 9	171
Year 10	155
Year 11	127
Year 12	94

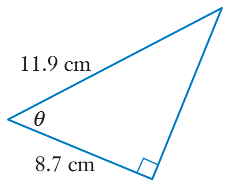
a How many students should complete the survey?

b How many female students should be part of the survey?

c How many Year 8 students will be surveyed?

Exercise 3.06

7 Find the value of θ, correct to the nearest degree.



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8 Draw a neat diagram of each shape.

a An isosceles, obtuse-angled triangle

b A parallelogram with side lengths 4 cm and 2 cm

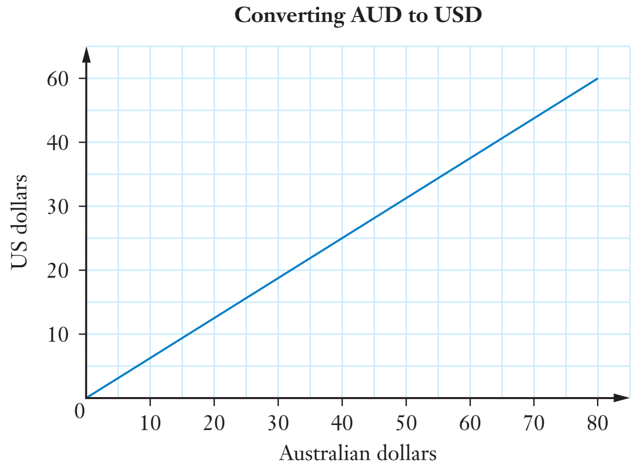
c A right-angled, scalene triangle

d A kite

Exercise 2.01

9 This conversion graph converts between Australian dollars (AUD) and US dollars (USD).

Exercise 5.07



a Convert \$50 AUD to US dollars.

b Convert \$20 USD to Australian dollars.

c Kelsey buys a bus tour ticket in New York for \$55 USD. How much is this in Australian dollars?

d Calculate how many Australian dollars you would get for \$300 USD.

10 **a** Give an example of an investigation where you would use a census to collect the information.

b Who would be the target population of your investigation?

Exercise 1.01

11 Linda is standing 750 m from the base of a tree. The angle of elevation to the top of the tree is 7°.

Exercise 3.05

a Draw a diagram showing this information.

b Find the height of the tree, correct to one decimal place.

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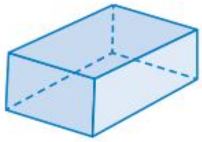
Exercise 2.05

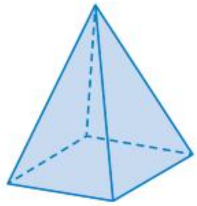
12 Draw the net of each solid.

a



b

c



Exercise 4.01

Exercise 4.02

13 **a** Jim's survey asks 'How often do you go out to dinner?' Suggest a series of tick boxes that Jim could provide for people to record their answers.

b Jim's survey also asked the question: 'Rate your meal at the fabulous Diggers' Steak House:

Great Yummy Quite nice'

i In what way is this question biased?

ii Rewrite it so it is not biased.

Exercise 5.04

14 Simon and Maddy are organising the catering for an outdoor wedding reception. The caterer charges \$400 plus \$55 per person.

a Copy and complete the table.

Number of wedding guests, *n*	0	50	100	150	200
Cost of catering, *C* dollars					

b Draw a graph from the table in part **a**.

c Use your graph to calculate the:

i cost of catering for 125 wedding guests

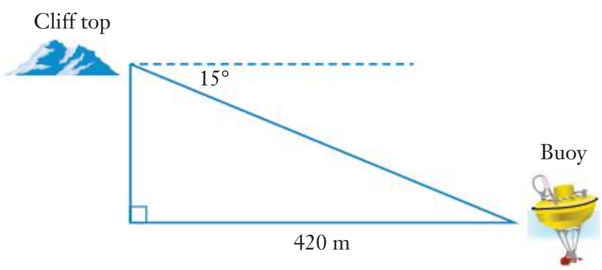
ii the number of wedding guests when the cost was \$9200.

d What is the vertical intercept of the graph and what does it represent?

e What is the gradient of the line and what does it represent?

Exercise 3.05

15 From the top of a cliff the angle of depression to a weather buoy is 15°. The buoy is moored 420 m from the cliff. Find the height of the cliff, correct to one decimal place.



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16 Year 12 students run a fundraising sausage sizzle at the athletics carnival. It will cost them $32 for the gas bottle for the barbecue and each sausage in bread will cost $0.90 to make.

Exercise 5.04

a Copy and complete this table.

Number of sausages in bread, n	0	10	20	30	40	50	100	150	200
Cost, $\$C$	32	41							

b Write an equation to calculate the cost, $\$C$, of making n sausages in bread.

c Use the equation to calculate the cost of making 170 sausages in bread.

d Construct a graph showing the cost of making n sausages in bread.

e Find the vertical intercept and explain what this value represents.

f Find the gradient and explain what this value represents.

17 The students decide to *sell* the sausages in bread for $2 each.

Exercise 5.05

Number of sausages in bread, n	0	10	20	30	40	50	100	150	200
Income, $\$I$	0	20							

a Copy and complete this table for the income from selling the sausages in bread.

b Write an equation to calculate the income from selling n sausages in bread and graph it on the same axes you used in question **16**.

c What are the coordinates of the point where the 2 lines intersect?

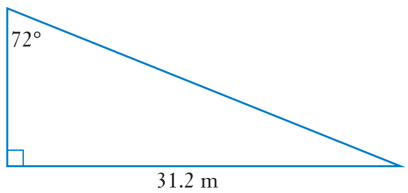
d What does this point represent?

e Will Year 12 make a profit if they sell 70 sausages in bread? Explain your answer.

f How much profit will the Year 12 students make if they sell all 200 sausages in bread?

18 Calculate the length of the hypotenuse of this triangle, correct to one decimal place.

Exercise 3.08



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DATA COLLECTION

6.

SCATTERING THE DATA

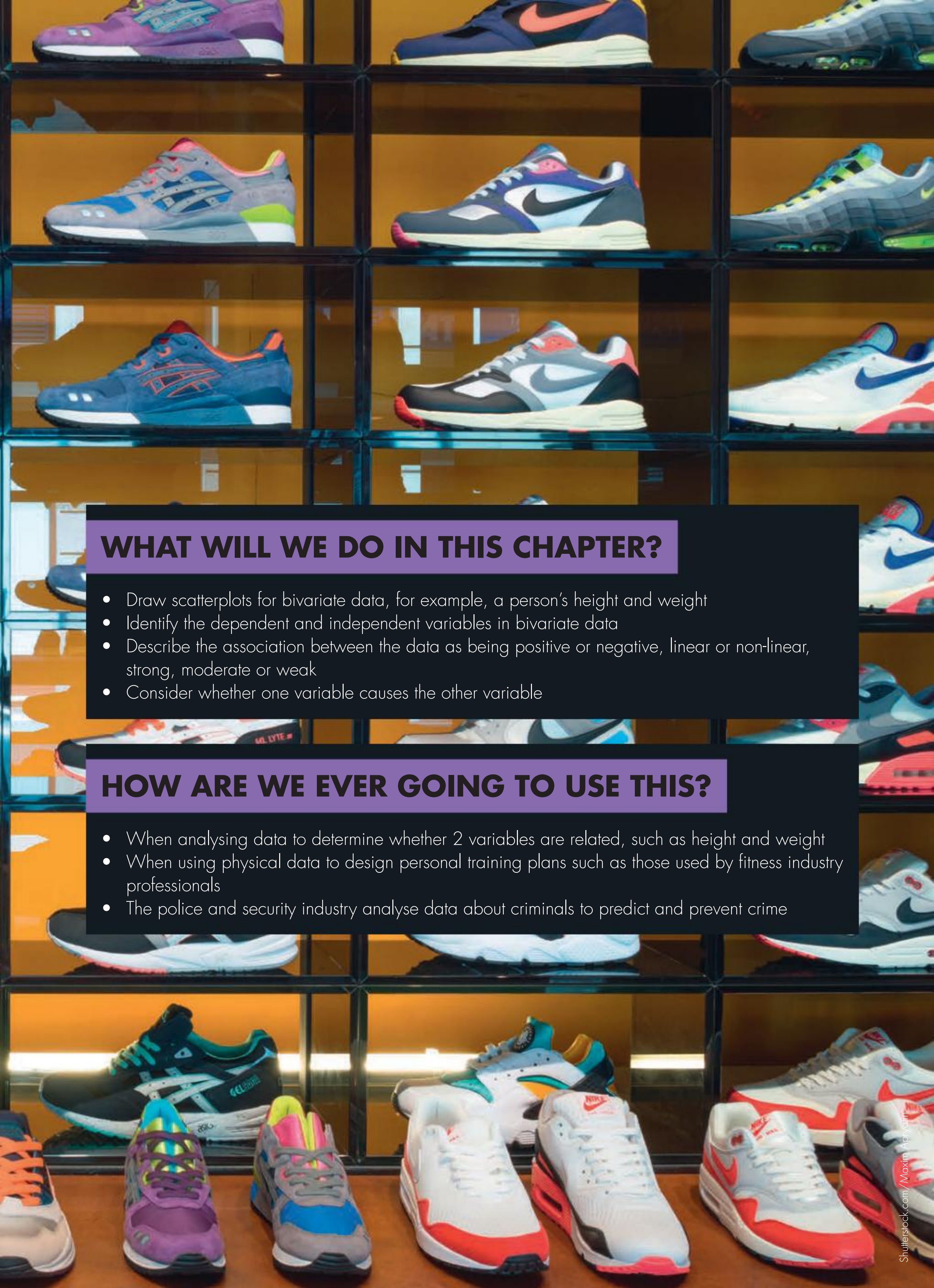
Chapter problem

A consumer association investigated the quality and price of 8 pairs of gym shoes labelled A to H. The investigation team gave each pair of shoes a quality rating out of 10.

This table shows the results.

	A	B	C	D	E	F	G	H
Price	\$320	\$280	\$260	\$240	\$180	\$180	\$80	\$40
Quality	8	10	9	6	6	4	1	3

Is there a relationship between the price of shoes and the quality rating assigned by the team? If so, what is the relationship?



WHAT WILL WE DO IN THIS CHAPTER?

- Draw scatterplots for bivariate data, for example, a person's height and weight
- Identify the dependent and independent variables in bivariate data
- Describe the association between the data as being positive or negative, linear or non-linear, strong, moderate or weak
- Consider whether one variable causes the other variable

HOW ARE WE EVER GOING TO USE THIS?

- When analysing data to determine whether 2 variables are related, such as height and weight
- When using physical data to design personal training plans such as those used by fitness industry professionals
- The police and security industry analyse data about criminals to predict and prevent crime

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6.01 Scatterplots

WS A page of scatterplots

WS Height vs shoe size

WS Body measurements

Bivariate data is data with 2 variables. For example, you might collect data on the height and weight of people. Bivariate data can be graphed on a **scatterplot**.

The first variable is called the **independent variable** and is graphed on the **horizontal axis**.

The second variable is called the **dependent variable** and is graphed on the **vertical axis**.

We can look at the scatterplot to see if it has any of the following features:

- there is a pattern to the points
- as one variable increases, the other variable increases (or decreases)
- there are groups of points
- most of the points are together but a few are out on their own

EXAMPLE 1

10 students had their heights and waist heights (above the ground) measured.

Total height, *H* cm	175	177	178	184	162	172	173	191	163	175
Waist height, *W* cm	101	106	107	115	100	87	106	120	90	108

a Graph this bivariate data in a scatterplot.

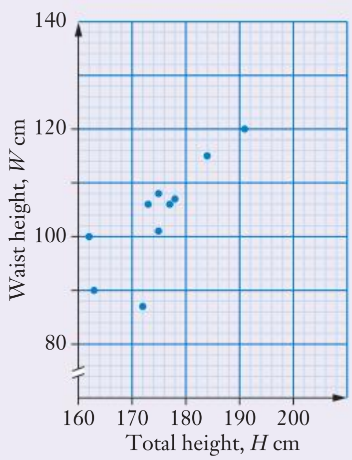
b Which is the independent variable and which is the dependent variable?

c Comment on the features of the scatterplot.

Solution

a The total height will go on the horizontal axis.

The waist height will go on the vertical axis.



b The independent variable is on the horizontal axis.

The independent variable is height.
The dependent variable is waist height.

c Describe any patterns.

As total height increases, waist height increases.

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Exercise 6.01 Scatterplots

Graph paper is required for this exercise. Keep your scatterplots to use in later exercises and in Chapter 9, *Fitting the data*.

WS

2mm grid paper

Example 1

1 This table shows the heights of a sample of girls when they were $2\frac{1}{2}$ years old and when they were 18 years old.

Height at $2\frac{1}{2}$ years (cm)	87	83	88	84	82	81	89	86	85	80
Height at 18 years (cm)	171	168	171	167	161	160	174	170	172	158

a Graph this bivariate data in a scatterplot.

b Which is the independent variable and which is the dependent variable?

c Comment on the features of the scatterplot.

2 Matthew asked a group of 11 students how many hours per week they spent playing sport and playing video games.

Sport	6	2	10	4	7	6	10	4	7	5	3
Video games	10	1	0	5	2	12	0	1	3	2	4

a Present this bivariate data in a scatterplot.

b Which is the dependent variable and which is the independent variable?

c Comment on the features of the scatterplot.

3 Simone measured the heights and arm spans of a group of 10 senior students.

Height, *H* cm	170	195	181	181	166	200	163	162	183	167
Arm span, *A* cm	171	186	187	178	165	160	147	143	115	169

a Construct a scatterplot to show this bivariate data.

b Which is the independent variable and which is the dependent variable?

c What does the scatterplot show?

4 This table shows the normal resting pulse of a sample of students and the time it takes each of them to swim 50 m.

Resting pulse (beats/min)	42	70	64	62	55	60	50	72	80
Swimming time (s)	30	48	50	43	40	45	36	49	59

a Construct a scatterplot for this set of data.

b Comment on the features of the scatterplot.

5 This table shows the sales of CD albums and digital albums over 9 years. The data is in thousands.

CD albums	46 174	49 818	44 045	38 659	39 529	33 114	30 223	27 356	14 226
Digital albums	91	418	788	1322	2279	3301	4818	6838	7377

a Present this bivariate data in a scatterplot.

b Describe what the scatterplot shows.

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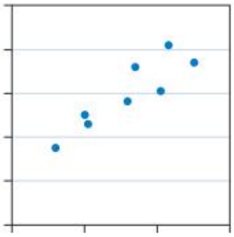
WS
Height vs shoe size

WS
Body measurements

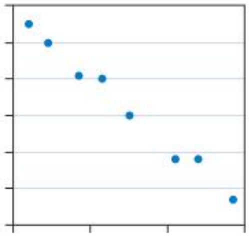
6.02 What is the relationship?

When bivariate data is graphed on a scatterplot we can use it to see if there is a relationship between the 2 variables. The **association** between 2 variables should be considered in terms of **direction**, **shape** and **strength**.

The **direction** of the association can be **positive** or **negative**.

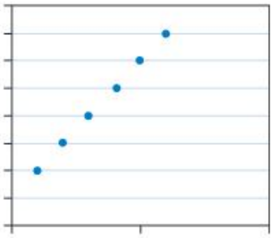


Positive (going up)

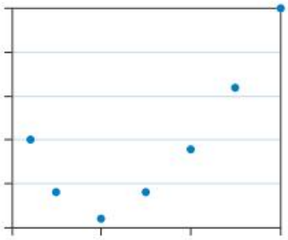


Negative (going down)

The **shape** of the association can be **linear** or **non-linear**.

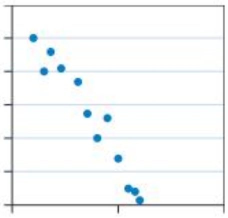


Linear pattern

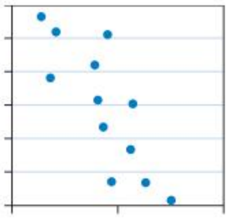


Non-linear pattern

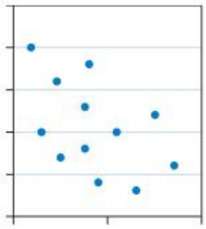
The **strength** of the association can be **strong**, **moderate** or **weak**.



Strong

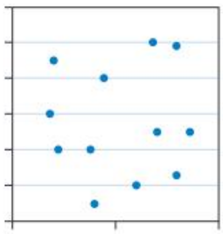


Moderate



Weak

Sometimes, the points have **no association**.



No association

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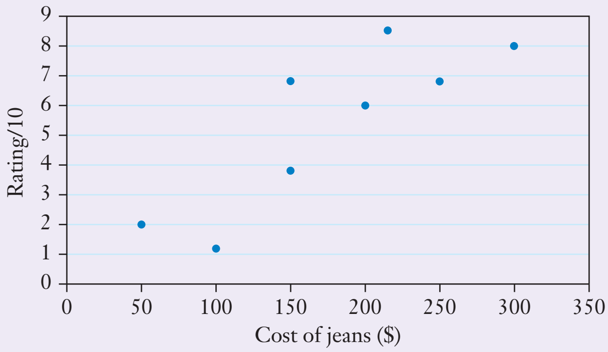
EXAMPLE 2

Describe the association between the variables shown in this bivariate scatterplot.



Solution

The dots go up from left to right.	Positive
The dots are close to forming a straight line.	Linear
The points are spread out.	Moderate
Write the answer.	This association is positive, linear and moderate.

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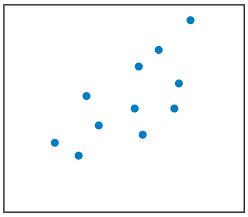
Exercise 6.02 What is the relationship?

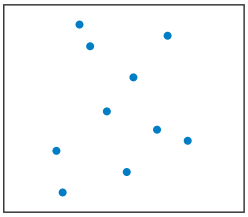
Example 2

1 Describe the association (direction, shape, strength) between the 2 variables shown in each scatterplot.

a



b



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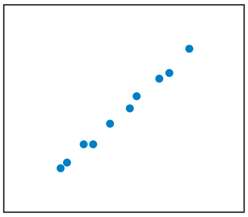
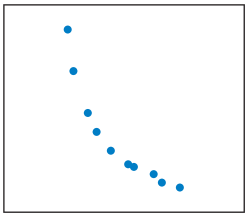
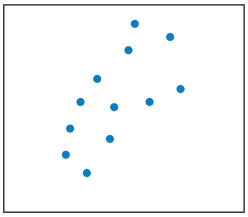


d



e



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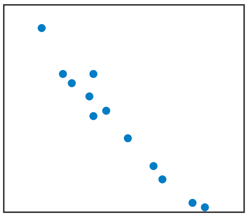

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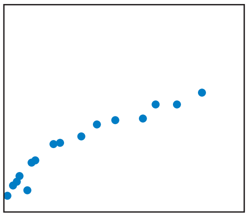

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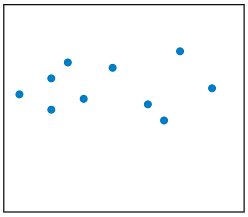

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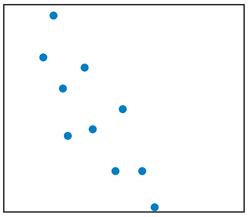

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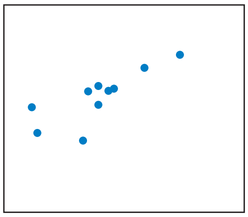

2 Look at the scatterplots you drew in Exercise 6.01. Describe the association of the variables in:

a question **1** on heights

b question **4** on pulse and time

c question **5** on album sales

3 The table shows the ages of 10 boys and the time it took them to swim 50 metres.

Age (years)	6	15	7	8	7	9	14	12	10	11
Swim time (s)	76	32	56	49	54	45	36	41	38	36

a Construct a scatterplot to display the information.

b Is there a relationship between the boy's ages and the time it takes them to swim 50 m? Describe the association.

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4 **a** For each student in your class, record the number of letters in their family name and the number of minutes they spend travelling to school (rounded to the nearest 10 minutes).

b Draw a scatterplot of the data with number of letters on the horizontal axis and travelling time on the vertical axis.

c Would you expect these 2 variables to have any association? Why or why not?

d Is there an association between these 2 variables? If so, describe it.

5 **a** Collect the following data for each student in your class.

- circumference of the student's head
- the student's height

b Draw a scatterplot of the data with the head circumference on the horizontal axis and height on the vertical axis.

c Is there a relationship between these 2 variables? If so, describe it.

6 Sketch a scatterplot to illustrate a set of data that has each type of association.

a Weak, positive linear association

b Strong, negative linear association

c Strong, positive, non-linear association

d No association

INVESTIGATION

BODY CIRCUMFERENCES

For each student in your class, record the following measurements correct to the nearest cm:

- wrist circumference
- neck circumference
- waist circumference

Construct scatterplots for:

- wrist circumference and neck circumference
- wrist circumference and waist circumference

Describe any relationships you can see in each scatterplot. Keep these scatterplots for future investigations.

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6.03 Does one variable cause the other?

Just because 2 variables have an association, it doesn't necessarily mean that one causes the other. At one time twenty years ago, there was a strong positive association between the price of petrol and the Australian cricket team's run rate! The price of petrol and cricket run rates are unrelated; one couldn't possibly cause the other.

Where we find an association, we need to examine the variables and decide whether there is a **causal relationship** or not.

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EXAMPLE 3

Each pair of variables below have a strong association. For each pair:

i decide if a change in one variable *causes* a change in the other variable

ii if there is no causal relationship, suggest other factors that might make the variables have an association.

a the height and weight of a person

b the price of petrol and the amount of petrol sold

Solution

a A change in height does NOT cause a change in weight. — Not causal. Both height and weight are affected by age, diet and body shape.

b Yes, a change in petrol price causes a change in amount of petrol purchased. — Causal. As the petrol price increases, petrol sales will decrease, because people wait for the price to come down again.

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Exercise 6.03 Does one variable cause the other?

Example 3

1 Each pair of variables below have a strong association. For each pair:

  i determine if a change in one variable causes a change in the other variable

  ii if there is no causal relationship, suggest other factors that might make the variables have an association.

a the driving speed of a car and the amount of petrol used

b the length of the right foot and the length of the left foot of the same person

c the price of a particular brand of car and the number sold

d the sale of hot chips and soft drinks at a football game

e the height and arm span of students

f kilojoules of energy consumed by a person and the weight gained

g number of rainy days in a month and the sales of umbrellas in that month

Dreamstime/Ross Tomei

2 Write down 2 variables you would expect to have a positive linear relationship, where a change in one variable would cause a change in the other variable.

3 Write down 2 variables you would expect to have a positive linear relationship, where a change in one variable would NOT cause a change in the other variable.

4 Write down 2 variables you would expect to have a negative linear relationship, where a change in one variable would cause a change in the other variable.

Chapter problem

You've covered the skills required to solve the chapter problem. Can you solve it now?

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Data crossword

6.04 Analysing data

Let's apply what we have learned about bivariate data to analyse data about a sample of Year 12 students.

Exercise 6.04 Analysing data

This table gives information collected from 15 Year 12 students. You can download this table as a spreadsheet ('Year 12 data') from NelsonNet. You can also use the spreadsheet to draw scatterplots.

Year 12 data

Student	Height (cm)	Arm span (cm)	Right foot length (cm)	Time to travel to school (mins)	Hours of homework per week	Resting pulse (beats per minute)	Hours watching TV per week	Number of siblings
Amy	176	175	26	7	6	64	2	4
Joe	178	175	23	40	6	66	10	2
Annika	151	150	30	10	23	95	4	1
Janine	168	175	25	15	7	76	14	2
Stephen	186	181	37	30	23	60	9	3
Thanh	187	187	28	5	2	74	5	2
Gillian	149	149	22	50	17	70	4	1
Vamsee	174	172	28	4	1	62	3	1
Lalaja	172	178	26	20	16	76	20	4
Darryl	177	177	23	15	10	64	10	3
Lyn	169	160	22	7	15	60	0	3
Jeremy	159	155	24	25	9	83	4	2
Ben	169	184	26	34	0	68	17	1
Abdul	163	159	24	20	3	77	9	2
Miriam	163	165	25	20	4	75	1	3

1 Use the data for arm span and the length of the right foot.

a Draw a scatterplot for this data.

b Name the independent and dependent variables.

c Describe any features of the scatterplot.

d Is there an association between these 2 variables? Describe it.

e If there is an association, is it a causal relationship? If not, suggest other factors that might result in an association between these two variables.

2 Use the data for number of siblings and time to travel to school.

a Would you expect there to be an association between these 2 variables? Why or why not?

b Construct a scatterplot for this data.

c Describe any features of the scatterplot.

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3 Choose 2 variables from the table that you would expect to have an association.

- **a** Draw a scatterplot for this data.
- **b** Name the independent and dependent variables.
- **c** What features does the scatterplot have?
- **d** Does the scatterplot show a relationship between these 2 variables? Describe it.
- **e** If there is a relationship, is it a causal relationship? If not, suggest other factors that might result in an association between these two variables.

4 Choose two variables from the table that you would NOT expect to have an association.

- **a** Draw a scatterplot for this data.
- **b** Describe any features of the scatterplot.
- **c** Is there an association between these 2 variables? Describe it.

KEYWORD ACTIVITY

Scattered find-a-word

CHAPTER SUMMARY

Use the list of words below to copy and complete the summary of the chapter.

association	bivariate	causes
dependent variable	independent variable	linear
moderate	negative	non-linear
positive	scatterplot	strong
weak		

In this chapter, we studied [1]___________ data, which is data with 2 variables. We learned how to graph this data on a [2]______________. On the graph, the variable on the horizontal axis is called the [3]_____________ ___________ and the variable on the vertical axis is called the [4]_________ __________.

We can use the graph to decide whether there is an [5]_______________ between the variables or not. We consider this relationship in terms of its:

Direction: whether it is [6]________________ or [7]______________.

Shape: is it [8]_______________ or [9]______________?

Strength: is the relationship [10]__________, [11]____________ or [12]___________?

When we see a relationship, we considered whether one variable [13]____________ the other.

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SOLUTION TO THE CHAPTER PROBLEM

Problem

A consumer association investigated the quality and price of 8 pairs of gym shoes, labelled A to H. The investigation team gave each pair of shoes a quality rating out of 10.

This table shows the results.

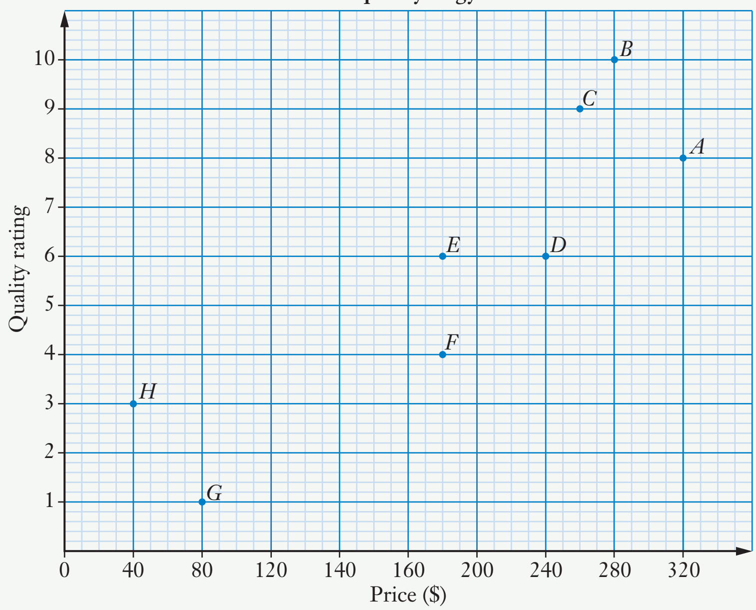
	A	B	C	D	E	F	G	H
Price	\$320	\$280	\$260	\$240	\$180	\$180	\$80	\$40
Quality	8	10	9	6	6	4	1	3

Is there a relationship between the price of shoes and the quality rating assigned by the team? If so, what is the relationship?

Solution

Draw a scatterplot to see if there is a relationship.

Price and quality of gym shoes



The scatterplot shows there is relationship between the 2 variables: the more expensive the gym shoe, the better the quality.

The relationship is strong, positive and linear. This would be a causal relationship. You would expect that the more expensive shoes use better materials and are better constructed than the cheap shoes.

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6. TEST YOURSELF

Scattering the data

Qz Practice quiz

Exercise 6.01

1 Eliza works in a coffee shop. She thinks there is a relationship between the daily average temperature and their hot chocolate drink sales.

Temperature (°C)	8	36	12	28	14	22	16	10	24	18	7	31
Hot chocolate sold	42	15	37	24	9	20	35	20	10	30	30	4

a Graph Eliza's bivariate data in a scatterplot.

b Which is the independent variable and which is the dependent variable?

c Comment on the features of the scatterplot.

Exercise 6.02

2 Describe the association (direction, shape and strength) between the 2 variables shown in the scatterplot you drew in question **1**.

Exercise 6.02

3 Sketch a scatterplot to illustrate bivariate data that have each type of association:

a Weak, negative, linear association

b Strong, positive linear association

c No association

Exercise 6.03

4 Each pair of variables below have a strong association. Decide if the relationship is causal or suggest other factors that might link the variables.

a The leg and arm lengths of the same person

b The number of wins for a football team in a season and the sale of its merchandise

Exercise 6.04

5 This table comes from the larger table in Exercise 6.04 on page 164.

a Do you expect to find a strong association between these 2 variables? Why or why not?

b Draw a scatterplot for this data.

c Name the independent and dependent variables.

d Is there an association between these 2 variables? If so, describe it.

Student	Hours of homework per week	Hours watching TV per week
Amy	6	2
Joe	6	10
Annika	23	4
Janine	7	14
Stephen	23	9
Thanh	2	5
Gillian	17	4
Vamsee	1	3
Lalaja	16	20
Darryl	10	10
Lyn	15	0
Jeremy	9	4
Ben	0	17
Abdul	3	9
Miriam	4	1

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MEASUREMENT

7. ON THE SURFACE

Chapter problem

Jess is making gift boxes to sell. She plans to make three types of boxes:

Cube: 15 cm long

Rectangular prism: 20 cm by 10 cm by 8 cm

Cylinder: radius 4.5 cm and height 34 cm

Should she charge the same for each gift box?

Give reasons for your answer.


iStock.com/Milkos

WHAT WILL WE DO IN THIS CHAPTER?

- Calculate perimeters of shapes, including circular and composite shapes
- Use metric units for area and convert between them
- Use formulas to calculate area, including composite shapes
- Calculate lengths of arcs and areas of sectors*
- Calculate the surface area of prisms, pyramids*, cylinders, spheres and composite solids

HOW ARE WE EVER GOING TO USE THIS?

- When we are doing craft and need to work out the amount of materials required
- If we are covering furniture, cushions or making quilts
- When we calculate the quantities of tiles, paint and carpet for building or renovating a home or office
- Many trades require knowledge of area and surface area

Metric match-up

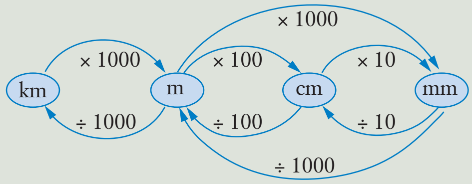
7.01 Length and perimeter

Unit	Relationships	How we use it
kilometre (km)	1 km = 1000 m	Used to measure long distances For example, the distance between cities or countries
metre (m)	1 m = 100 cm 1 m = 1000 mm	Used to measure medium lengths For example, the length of a sporting field or a room's width
centimetre (cm)	1 cm = 10 mm	Used to measure small lengths For example, the dimensions of a brick
millimetre (mm)		Used to measure very small lengths For example, the length of an insect or the size of your toenail

Metric units of length

To change from a small unit to a bigger unit, **divide** by the conversion factor.

To change from a big unit to a smaller unit, **multiply** by the conversion factor.



EXAMPLE 1

Convert:

a 30 m to cm **b** 780 m to km **c** 65 cm to mm.

Solution

a There are 100 cm in 1 metre. The conversion factor is 100. Changing from m to cm is changing to a smaller unit. We need to multiply by the conversion factor.

$30 \text{ m} = 30 \times 100 \text{ cm}$
$= 3000 \text{ cm}$

b There are 1000 m in one km. Changing from m into km is changing into a bigger unit. We need to divide by the conversion factor.

$780 \text{ m} = 780 \div 1000 \text{ km}$
$= 0.78 \text{ km}$

c There are 10 mm in one cm. Changing from cm into mm is changing into a smaller unit. We need to multiply by the conversion factor.

$65 \text{ cm} = 65 \times 10 \text{ mm}$
$= 650 \text{ mm}$

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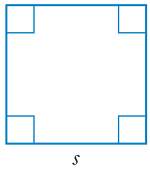
Perimeters of squares, rectangles and regular polygons

Perimeter code puzzle

The **perimeter** of a shape is the distance around the outside of the shape. It is a measure of **length**.

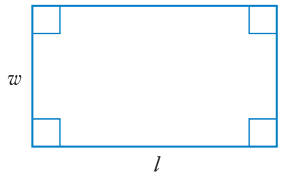


The **perimeter of a square** can be found by adding all of the sides:

Perimeter $= s + s + s + s$

$= 4s$

The **perimeter of a rectangle** can be found by adding all of the sides:



Perimeter $= l + w + l + w$

$= 2l + 2w$ or $2(l + w)$.

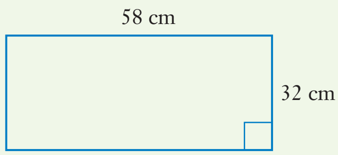
Perimeters of squares and rectangles

The **perimeter of a square** is $P = 4s$, where s is the length of one side.

The **perimeter of a rectangle** is $P = 2l + 2w$ or $P = 2(l + w)$, where l is the length and w is the width.

EXAMPLE 2

Use a formula to calculate the perimeter of this rectangle.



Solution

Using $P = 2l + 2w$, where $l = 58$ and $w = 32$

$P = 2 \times 58 + 2 \times 32$
$= 116 + 64$
$= 180$

OR using the formula $P = 2(l + w)$

$P = 2 \times (58 + 32)$
$= 2 \times 90$
$= 180$

Write the answer, including units.

The perimeter is 180 cm.

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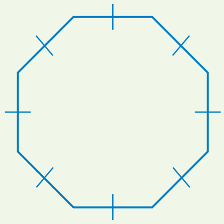
EXAMPLE 3

Calculate the perimeter of this octagon.



Solution

As all sides of the regular octagon are the same length, we can simply multiply 23 mm by 8.

$$\text{Perimeter} = 8 \times 23$$
$$= 184$$

Write the answer, including units.

The perimeter is 184 mm.

Exercise 7.01 Length and perimeter

Example 1

1 In each part, state the conversion factor and whether you have to multiply or divide.

a km to m **b** mm to cm **c** cm to m **d** m to km
e m to cm **f** m to km **g** cm to mm

2 Copy and complete each conversion.

a 3 cm = ___ mm **b** 5 m = ___ cm **c** 400 m = ___ km
d 2 km = ___ m **e** 30 mm = ___ cm **f** 200 cm = ___ m
g 500 mm = ___ m **h** 250 m = ___ km **i** 60 cm = ___ m
j 60 cm = ___ mm **k** 4500 m = ___ km **l** 0.8 km = ___ m
m 8 mm = ___ cm **n** 90 m = ___ cm **o** 90 m = ___ km
p 6.5 m = ___ mm

3 Follow each step to convert 0.64 km into cm.

a Change 0.64 km into m.
b Change your answer to part **a** into cm.

4 Follow each step to convert 85 000 cm into km.

a Convert 85 000 cm into m.
b Change your answer to part **a** into km.

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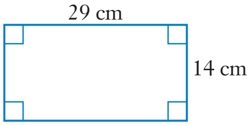
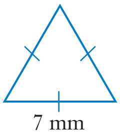
5 Calculate the perimeter of each shape.

Example 2

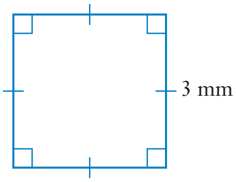
a



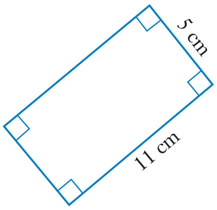
b



c



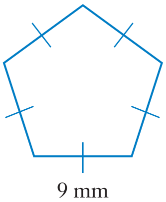
d



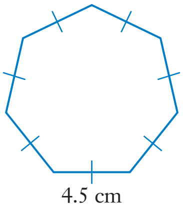
6 Calculate the perimeter of each regular polygon.

Example 3

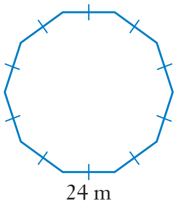
a



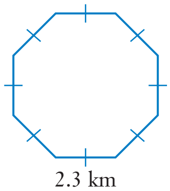
b



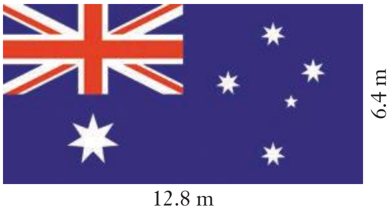
c



d



7 The Australian flag that flies above Parliament House in Canberra is 12.8 m long and 6.4 m high. Calculate the perimeter of this flag.



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8 Our 50c coin is in the shape of a regular dodecagon, a 12-sided polygon. Each side is 8.2 mm long. Calculate the perimeter of a 50c coin.

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9 Calculate the perimeter of each regular polygon.

a

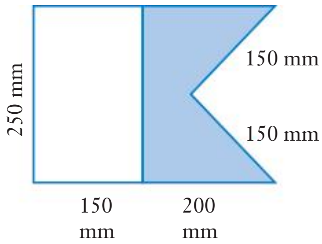
Shutterstock.com/Zoart Studio

b

Shutterstock.com/Starsphinx

10 The perimeter of a 10-sided regular polygon is 60 m. How long is each side?

11 Maritime rules require dive boats to fly a blue and white flag when there are divers in the water. The flag tells other boat drivers to go slowly and stay away from the divers.



a What geometrical shape is the white section of the flag?
b What is the perimeter of the white section of the flag?
c Calculate the perimeter of the blue section of the flag.
d What is the perimeter of the whole flag?
e Why isn't the perimeter of the whole flag the same amount as adding the perimeter of the white section and the blue section together?

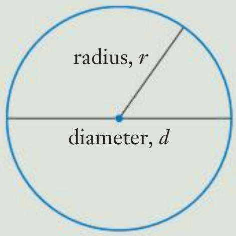
WS
Circles and arcs

7.02 Circumference and arc length of a circle

The perimeter of a circle is called its **circumference**.

Circumference of a circle

- $C = \pi d$, where d is the **diameter** of the circle
- $C = 2\pi r$, where r is the **radius** of the circle

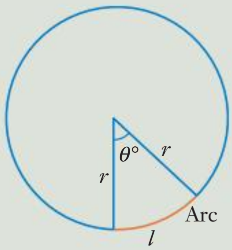


Arc length of a circle*

$$l = \frac{\theta}{360} \times 2\pi r$$

where θ is the size of the angle at the centre of the circle.

*Australian curriculum only, not in WA syllabus



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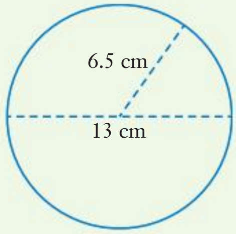
The arc length is a fraction of the circumference of the circle. The fraction is $\frac{\theta}{360}$ because there are 360° in a circle.

EXAMPLE 4

Circumference and arc length of a circle

Calculate the circumference of this circle.



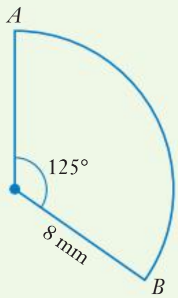
Solution

Use the formula $C = \pi d$, where $d = 13$ cm	$C = \pi \times 13$ $= 40.8407\ldots$
Or use the formula $C = 2\pi r$, where $r = 6.5$ cm	$C = 2 \times \pi \times 6.5$ $= 40.8407\ldots$
Write the answer. The values in the question had one decimal place so use one decimal place in your answer.	The circumference of the circle is 40.8 cm.

EXAMPLE 5 *

Circumference and arc length of a circle

Calculate the length of arc AB, correct to the nearest mm.



Solution

Arc length: $l = \frac{\theta}{360} \times 2\pi r$	Arc $AB = \frac{125}{360} \times 2\pi \times 8$
where $\theta = 125°$ and $r = 8$ mm.	$= 17.453\ldots$
Write the answer.	The length of arc AB is 17 mm.

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Exercise 7.02 Circumference and arc length of a circle

Example 4

1 Calculate, correct to one decimal place, the circumference of a circle with:

a radius 15 cm **b** diameter 18 mm

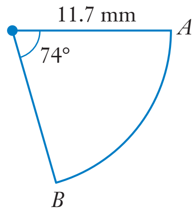
c radius 3.1 km **d** diameter 17.5 m

Example 5

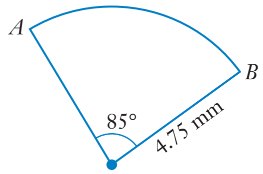
2* Calculate, correct to one decimal place, the length of the arc AB in each diagram.

*Australian curriculum only, not in WA syllabus

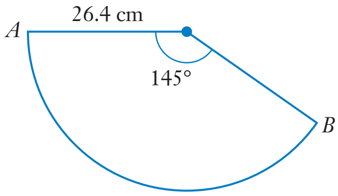
a



b

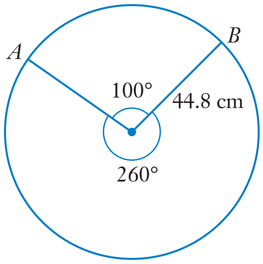


c

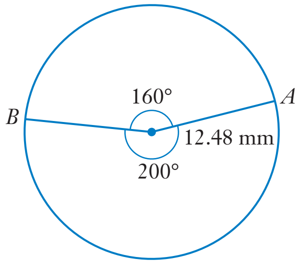


3* Calculate the length of the larger arc AB in each diagram, correct to the nearest whole number.

a



b



4 The radius of the Earth is 6400 km. The equator is a circumference of the Earth. Calculate the length of the equator, correct to the nearest 100 km.

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5 In Exercise 7.01, Question 8, you calculated the perimeter of a 50c coin. Now imagine that there is a circle around the coin.

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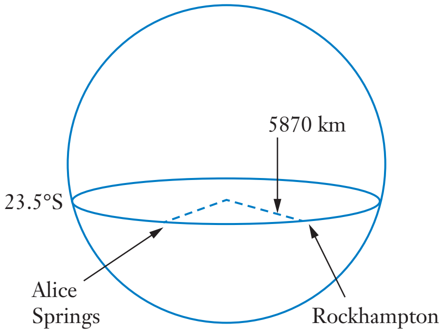
a Which do you think will be larger; the circumference of the circle or the perimeter of the coin?

b The diameter of the circle is 31.65 mm. Calculate the circumference, correct to one decimal place.

c Each side of the coin is 8.2 mm long. Was your guess in part **a** correct? Why is one of these measures bigger than the other?

6 Sam has a circular garden, diameter 2.4 m, in her front yard. She is going to plant flowers every 30 cm around the circumference. How many plants will she need?

7 Rockhampton and Alice Springs both lie on the 23.5°S parallel of latitude. The radius of the 23.5°S parallel is 5870 km.



a Calculate the entire length of the 23.5° south parallel of latitude, correct to the nearest 10 km.

b* The angle at the centre of the 23.5° circle made by joining the positions for Alice Springs and Rockhampton is 17°. Calculate the distance along the 23.5°S parallel from Rockhampton to Alice Springs. Answer correct to the nearest 10 km.

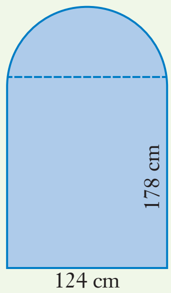
c* Use Google Maps or a similar website to find the distance from Rockhampton to Alice Springs by car. Suggest a reason why this distance is more than the answer to part **b**.

7.03 Perimeters of composite shapes

Most things in real life aren't just one shape. Often, items are made from a **composite** shapes.

EXAMPLE 6

The window in Bella's lounge room is in the shape of a semicircle on top of a rectangle, as shown. Calculate the perimeter of Bella's window in metres, correct to 2 decimal places.



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Solution

The perimeter is made up of half the circumference of a circle and 3 sides of the rectangle.

First calculate half the circumference: $\frac{1}{2}\pi d$.

Don't round until the last step.

$$\text{Half the circumference} = \frac{1}{2} \times \pi \times 124$$
$$= 194.7787\ldots$$

Three sides of the rectangle.

$$\text{Rectangle} = 178 + 124 + 178$$
$$= 480$$

Calculate the total perimeter.

$$\text{Total perimeter} = 194.7787\ldots + 480$$
$$= 674.7787\ldots \text{ cm}$$

Convert to metres.

$$= 6.747787\ldots \text{ m}$$

Write the answer.

The perimeter of Bella's window is 6.75 m.

EXAMPLE 7

This logo's shape is $\frac{3}{4}$ of a circle with a radius of 8 mm. Calculate its perimeter, correct to one decimal place.

Solution

The perimeter is $\frac{3}{4}$ of the circumference + 2 radii.

$$\text{Perimeter} = \left(\frac{3}{4} \times 2 \times \pi \times 8\right) + (2 \times 8)$$
$$= 53.6991\ldots$$

Radii is the plural form of radius. One radius, two radii.

Write the answer.

The logo's perimeter is 53.7 mm.

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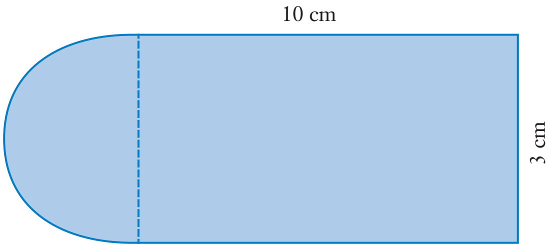
Exercise 7.03 Perimeters of composite shapes

Example 6

1 This diagram shows the shape of a tool that scrapes paint off glass. Calculate the perimeter of the tool, correct to 1 decimal place.



2 The South Coast Lions run around the outside of a circular field as part of their training. The diameter of the field is 180 m.

a How far do they run (to the nearest metre) when they complete 8 laps of the field?

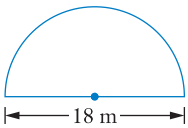
b The coach wants the team to run 2 km. How many laps of the field will the team have to run to complete 2 km? Answer to one decimal place.


Dreamstime/Vadymvdrobot

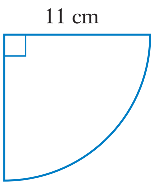
Example 7

3 Calculate, correct to one decimal place, the perimeter of each shape.

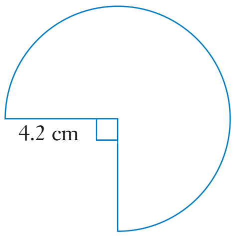
a



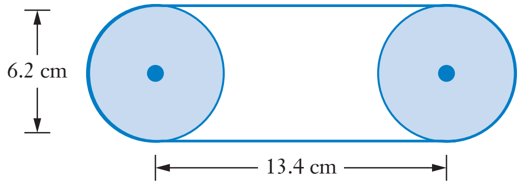
b



c

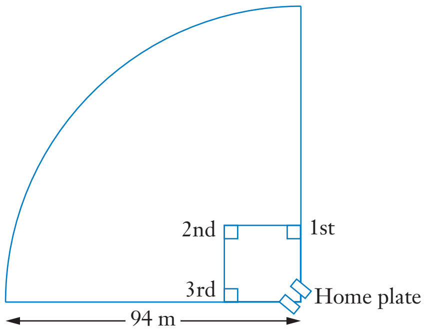


4 The diagram shows a belt around a pair of pulleys. Calculate the length of the belt, correct to 3 significant figures.

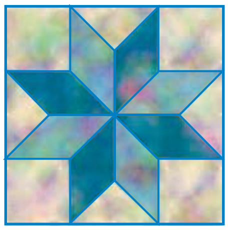


5 A traditional softball field is in the shape of a circle quadrant. What is the perimeter of the field? Answer to the nearest metre.

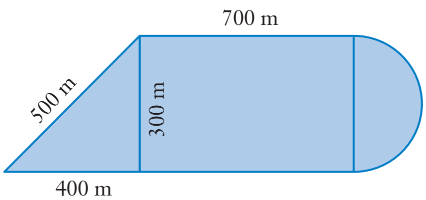
A **quadrant** is a quarter of a circle.



6 Rachel is making a quilt with a star pattern as shown. Each edge of the star is 6 cm long. She is going to put a trim around the outside of all 12 stars in the quilt. How many metres of trim will she use?



7 The diagram shows a walking circuit in a national park. The paths surround a shape that includes a triangle, a rectangle and a semicircle. Calculate the length of the circuit, correct to the nearest metre.



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7.04 Area

The **area** of a shape is the amount of surface enclosed by the shape. It is measured in square units, which are based on the length units.

WS
Australian areas

Area unit	The size of a square of length:	Approximately the size of:
square millimetre (mm^2)	1 mm (Actual size:)	
square centimetre (cm^2)	1 cm (Actual size:)	a fingernail
square metre (m^2)	1 m	the floor of a large shower recess
hectare (ha)	100 m	the area bounded by an athletics track, or an international rugby pitch
square kilometre (km^2)	1 km	a theme park


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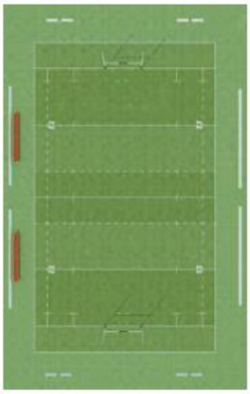
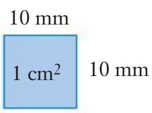
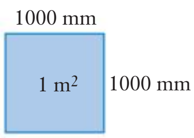

Alamy Stock Photo/Archideaphoto

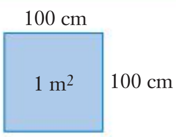
$1 \text{ cm} = 10 \text{ mm}$
$1 \text{ cm}^2 = 10 \times 10 \text{ mm}^2$
$= 100 \text{ mm}^2$



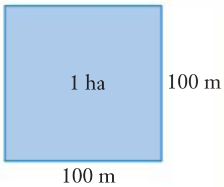
$1 \text{ m} = 1000 \text{ mm}$
$1 \text{ m}^2 = 1000 \times 1000 \text{ mm}$
$= 1\,000\,000 \text{ mm}^2$



$1 \text{ m} = 100 \text{ cm}$
$1 \text{ m}^2 = 100 \times 100 \text{ cm}^2$
$= 10\,000 \text{ cm}^2$



$1 \text{ ha} = 100 \times 100 \text{ m}$
$= 10\,000 \text{ m}^2$



Similarly, $1 \text{ km}^2 = 1 \text{ km} \times 1 \text{ km} = 1000 \text{ m} \times 1000 \text{ m} = 1\,000\,000 \text{ m}^2$

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When converting area units, we have to convert the length unit twice. One conversion is for the length and the other is for the width. We *square* the simple linear conversion factor to get the area conversion factor.

For example,	to change from m to cm,	multiply by 100,
	but to change from m^2 to cm^2,	multiply by $100^2 = 10\ 000$.

Metric units of area

$1\ cm^2 = 10^2\ mm^2 = 100\ mm^2$

$1\ m^2 = 100^2\ cm^2 = 10\ 000\ cm^2$

$1\ m^2 = 1000^2\ mm^2 = 1\ 000\ 000\ mm^2$

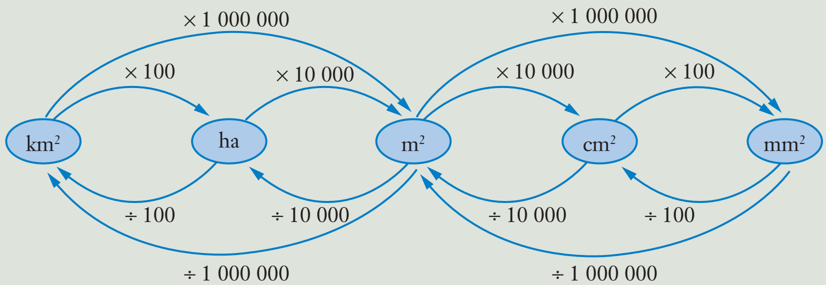
$1\ ha = 100^2\ m^2 = 10\ 000\ m^2$

$1\ km^2 = 1000^2\ m^2 = 1\ 000\ 000\ m^2$

To change from a small unit to a bigger unit, **divide** by the conversion factor.

To change from a big unit to a smaller unit, **multiply** by the conversion factor.

This diagram shows how to convert between units of area.



EXAMPLE 8

Convert

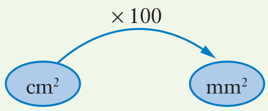
a $3\ cm^2$ to mm^2 **b** $4000\ mm^2$ to m^2 **c** $81\ 000\ m^2$ to ha

Solution

a cm^2 to mm^2, large to small unit: $\times 100$.

$3\ cm^2 = 3 \times 100\ mm^2$

$= 300\ mm^2$



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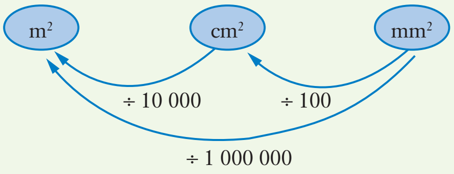
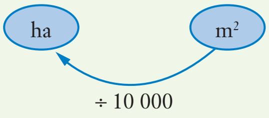
b mm^2 to m^2, small to large unit: ÷ 1 000 000.



$4000\text{ mm}^2 = 4000 \div 1\ 000\ 000\text{ m}^2$
$= 0.004\text{ m}^2$

c m^2 to ha, small to large unit: ÷ 10 000.

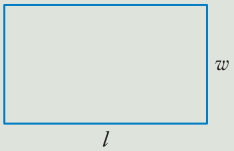


$81\ 000\text{ m}^2 = 81\ 000 \div 10\ 000\text{ ha}$
$= 8.1\text{ ha}$

Area of a rectangle

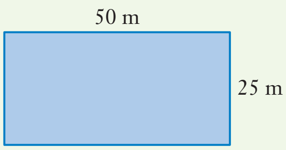
Area = length × width

$A = l \times w$

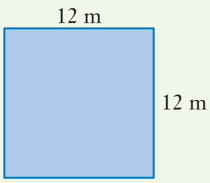


EXAMPLE 9

a The floor of an Olympic swimming pool is 50 m long and 25 m wide. Calculate the area of the floor.



b Gymnastic floor competitions are held in a square with sides 12 m long. What is the area of a square with sides of 12 m?



Solution

a Multiply the length by the width.

The units in the question are metres, so the answer for area is in square metres (m^2)

$\text{Area} = l \times w$
$= 50 \times 25$
$= 1250\text{ m}^2$

b The length and the width are both 12 m.

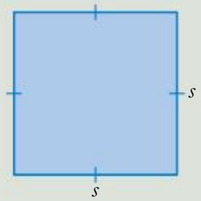
To calculate the area, multiply 12 by 12.

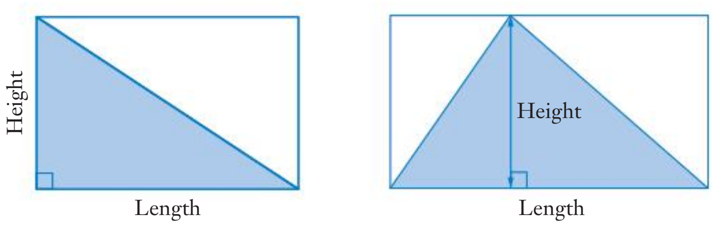
$$\begin{aligned} \text{Area} &= l \times w \\ &= 12 \times 12 \\ &= 144\ \text{m}^2 \end{aligned}$$

Area of a square

Area = side × side

$A = s^2$



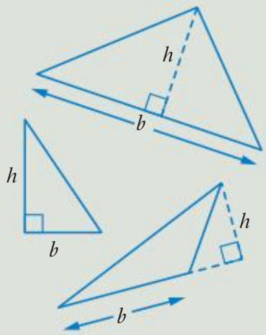


The above diagrams show that the area of the triangle is half the area of the rectangle that encloses it. For a triangle, the length is called the **base** and the height is called the **perpendicular height** because it is at a right angle (90°) to the base.

Area of a triangle

$\text{Area} = \frac{1}{2} \times \text{base} \times \text{perpendicular height}$

$A = \frac{1}{2}bh$



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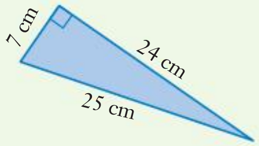
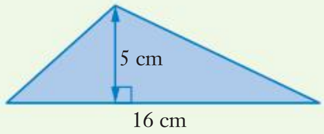
EXAMPLE 10

Calculate the area of each triangle.

a



b



Solution

a The area of a triangle is $A = \frac{1}{2}bh$.

The base and the height must be at 90° to each other. Base = 24 and height = 7.

The 25 cm length is not used in this calculation.

$\text{Area} = \frac{1}{2} \times 24 \times 7$

$= 84 \text{ cm}^2$

b Base = 16 cm and height = 5.

$\text{Area} = \frac{1}{2} \times 16 \times 5$

$= 40 \text{ cm}^2$

Exercise 7.04 Area

1 Select the best unit (km², ha, m², cm² or mm²) for measuring each area.

- **a** a farm
- **b** the floor of a classroom
- **c** a shirt
- **d** a football field
- **e** a sheet of paper
- **f** your eardrum
- **g** Australia
- **h** a butterfly
- **i** Garden Island

2 Copy and complete each conversion.

Example 8

- **a** $7.9 \text{ cm}^2 = \_\_\_\_\_ \text{ mm}^2$
- **b** $1\,500\,000 \text{ mm}^2 = \_\_\_\_\_ \text{ m}^2$
- **c** $690 \text{ mm}^2 = \_\_\_\_\_ \text{ cm}^2$
- **d** $76\,000\,000 \text{ m}^2 = \_\_\_\_\_ \text{ km}^2$
- **e** $865\,000 \text{ cm}^2 = \_\_\_\_\_ \text{ m}^2$
- **f** $12 \text{ ha} = \_\_\_\_\_ \text{ m}^2$
- **g** $0.32 \text{ km}^2 = \_\_\_\_\_ \text{ m}^2$
- **h** $4.5 \text{ m}^2 = \_\_\_\_\_ \text{ cm}^2$
- **i** $0.75 \text{ m}^2 = \_\_\_\_\_ \text{ mm}^2$
- **j** $19\,000 \text{ m}^2 = \_\_\_\_\_ \text{ ha}$

3 Arrange these areas from smallest to largest: 6.5 m^2, $25\,050\,000 \text{ mm}^2$ and $114\,000 \text{ cm}^2$.

To compare sizes, the measurements need to be in the same units.

4 Arrange these areas from largest to smallest: 990 mm^2, 54 cm^2 and $0.000\,032 \text{ m}^2$.

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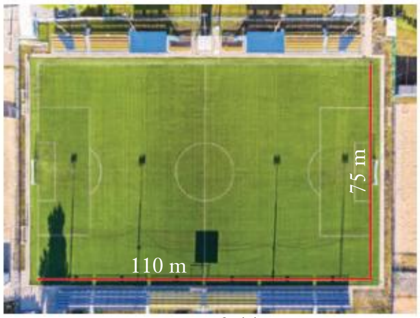
5 The centre of Adelaide is approximately a square bounded by streets called North, South, East and West Terraces. Its area is about 3 521 000 m^2. Express the area of this square in:

a hectares **b** square kilometres

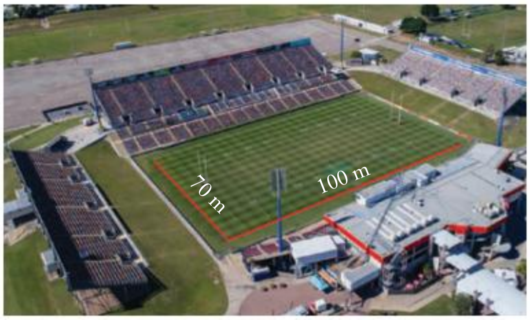
Example 9

6 Which field below has an area closer to 1 ha? Justify your answer.



iStock.com/Mr_Twister

Soccer field



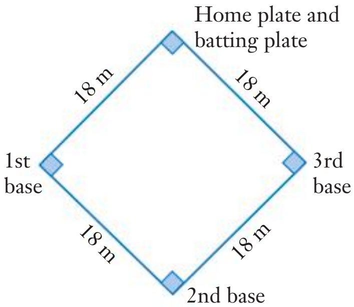
iStock.com/Cameron Laird Photography

Rugby union field

7 The infield of a junior softball ground is in the shape of a square with sides 18 m long.

a What is the area of the infield?

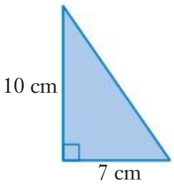
b A batter can score a home run if she hits the ball a long way and then runs through all the bases and back to the home plate. Kate hit a home run. How far did she run?



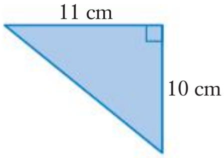
Example 10

8 Calculate the area of each triangle.

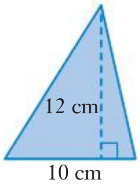
a



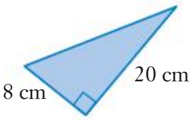
b



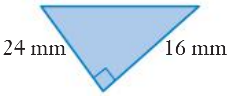
c



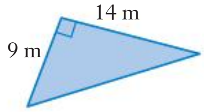
d



e

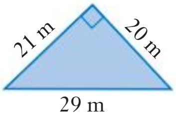


f



9 **a** Which 2 sides of the triangle on the right do you use in the formula $A = \frac{1}{2}bh$ to calculate the area of the triangle?

b What is the area of the triangle?



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10 Nabil’s driveway is in the shape of a rectangle, 15 m long with area 45 m^2. How wide is the driveway?

11 A large bushfire is burning out of control. Overnight it destroyed 2400 ha of bush.

a Convert 2400 ha to m^2.

b Convert your answer from part **a** to km^2.

c The burnt bush is in the shape of a rectangle. What could the **dimensions** of the burnt area be?

12 Ross is going to tile his verandah. The square tiles are 30 cm long.

a Calculate the area of one tile in cm^2.

b How many tiles cover 1 m^2?

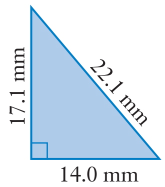
c Ross’ verandah is a rectangle 3 m wide by 8.4 m long. How many tiles will he need to cover the verandah?

d To allow for cutting and breakage, Ross is going to order 5% more than the minimum number of tiles he requires. How many tiles should he order?

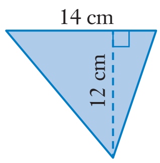
13 Calculate the area of each triangle.

Make sure that you use the two dimensions that are at right angles to each other.

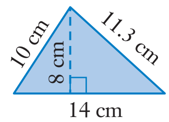
a



b



c



14 The area of a rectangle is 36 cm^2.

a What could the dimensions of the rectangle be? Give two possible sets of values.

b Calculate the perimeter of the rectangle for your suggested values in part **a**.

c What is the smallest the perimeter could be?

15 **a** Estimate the area of this stamp.

Designer: Jason Watts, Australia Post Design Studio; © Australian Postal Corporation 2018

b Use a ruler to measure the stamp and then calculate the stamp’s area. How close was your approximation?

16 Draw two possible right-angled triangles that each has an area of 24 cm^2, showing values for the base and height of the triangle.

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PRACTICAL ACTIVITY

ESTIMATING AREA

It takes practice to become good at estimating. Complete these practical estimation activities to develop your group's estimation skills.

To complete these activities, each group will need:

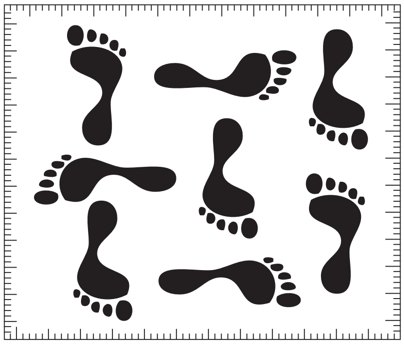
- 4 metre rulers or measuring tapes
- Paper, pencils and scissors

Make a copy of this table:

Estimating area					
Names	**Group members' estimates**				**Real measurement**
Footprints in a square metre					
Handprints in a square metre					
Area of a car number plate in square centimetres					

Activity 1: Footprints in a square metre

Imagine you are going to cover a square metre with your footprints. How many footprints will fit inside 1 m^2 without any overlaps? Record your group's estimates on the record sheet.



How good was your estimate?

- Make a template of your foot. Put a piece of paper under your foot and trace around the outside. Use scissors to cut out the template. Make several templates.
- Place four 1-metre rulers on the floor to outline 1 m^2.
- Systematically place your foot template in the 1 m^2 and count the number required to cover the square.

Activity 2: Handprints in a square metre

Now imagine that you are going to cover a square metre with your handprints without any overlaps. How many handprints will you need?

Use a similar method to that for checking footprints in a square metre to check the accuracy of your group's handprint estimates.

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Activity 3: The area of a car number plate

Estimate the area covered by a standard car number plate.

To check your estimate, measure the length and the height of an appropriate number plate, then use the formula Area = $l \times w$ to calculate the area.

Alamy Stock Photo/ Ian Dagnall

Activity 4: Laying grass

Keira is planning to use rolls of grass to cover her backyard that is 15.4 m wide by 12 m long. Each roll of grass is 50 cm wide and 2 m long.

a Draw a diagram to represent Keira's backyard.

b Calculate the area of Keira's backyard and the area covered by one roll of grass.

c How many rolls of grass will Keira need to buy to cover her backyard?

We can only buy whole rolls of grass.

d On the diagram you drew in part **a**, decide whether it will be better to lay the grass in rows or columns or both. Find the best way.

e Describe the size and shape of the pieces of grass that will be left over after Keira has finished covering her backyard.

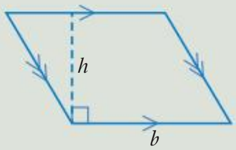
7.05 Areas of quadrilaterals, circles and sectors

Areas of quadrilaterals, circles and sectors

Area of a parallelogram

Area = base × perpendicular height

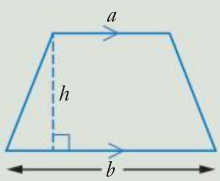
$A = bh$



Area of a trapezium

Area = $\frac{1}{2}$ × sum of parallel sides × perpendicular height

$A = \frac{1}{2}(a + b)h$



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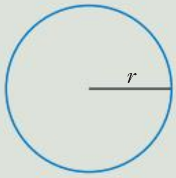
Area of a circle

Area = $\pi \times (\text{radius})^2$

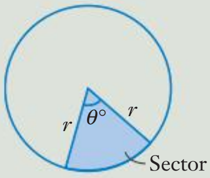
$A = \pi r^2$



Area of a sector*

$$\text{Area} = \frac{\text{sector angle}}{360} \times \pi \times (\text{radius})^2$$

$$A = \frac{\theta}{360} \times \pi r^2$$



*Australian curriculum only, not in WA syllabus

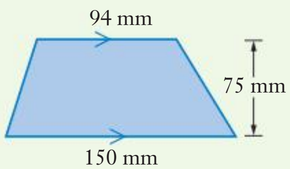
The sector area is a fraction of the area of the circle, and $\frac{\theta}{360}$ is the fraction because there are 360° in a circle.

Areas of trapeziums, circles and sectors

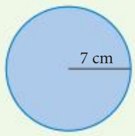
EXAMPLE 11

Find the area of each shape, correct to 2 decimal places where necessary.

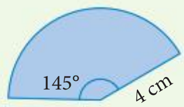
a



b



c*



Solution

a For a trapezium, $A = \frac{1}{2}(a + b)h$.

$a = 94$, $b = 150$, $h = 75$.

$$\text{Area} = \frac{1}{2} \times (94 + 150) \times 75.$$
$$= 9150 \text{ mm}^2$$

b For a circle, $A = \pi r^2$.

$r = 7$.

$$\text{Area} = \pi \times 7^2$$
$$= 153.9380\ldots$$
$$\approx 153.94 \text{ cm}^2$$

c For a sector, $A = \frac{\theta}{360} \times \pi r^2$.

$\theta = 145$, $r = 4$.

$$\text{Area} = \frac{145}{360} \times \pi \times 4^2$$
$$= 20.2458\ldots$$
$$\approx 20.25 \text{ cm}^2$$

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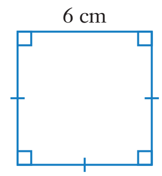
Exercise 7.05 Areas of quadrilaterals, circles and sectors

Example 11

1 Calculate the area of each quadrilateral.

a



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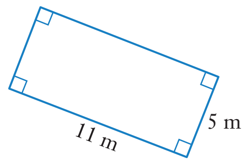
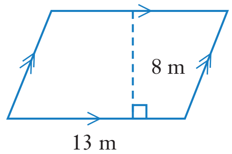
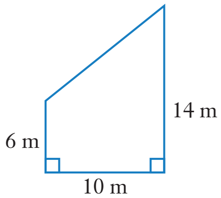


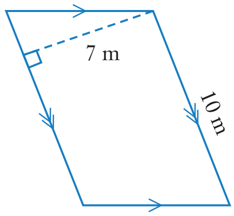
c



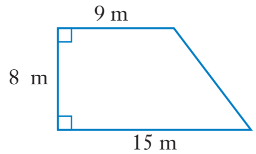
d



e

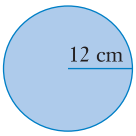


f



2 Find the area of each circle or sector*, correct to 2 decimal places.

a



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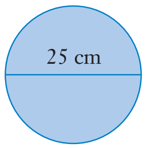

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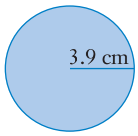

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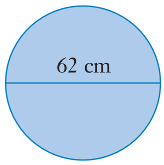

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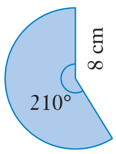
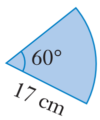
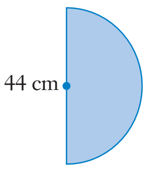


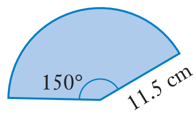
f



g



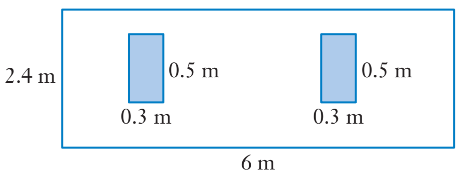
h



3 Mark is a bricklayer. He uses 65 bricks to build 1 m² of wall.

a How many square metres of wall can Mark build with 715 bricks?

b Mark is going to build this brick fence leaving 2 holes as requested by the customer. How many square metres of brickwork are in the fence?



c How many bricks will Mark need to build the fence?

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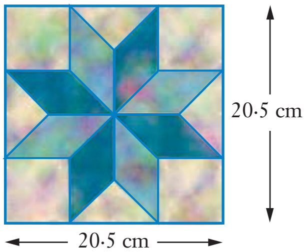
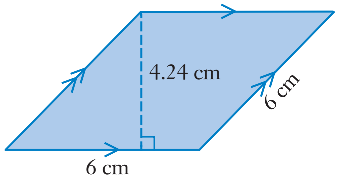
4 Rachel uses parallelograms to make a star pattern on a quilt. The star pattern is shown on the left, and one of the parallelograms shown on the right.





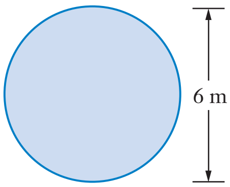
a Calculate the area of one parallelogram.

b What is the area covered by one star?

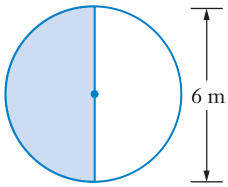
c How much of the area of the square is **not** covered by the star?

5 Calculate the area of each shaded region, correct to one decimal place.

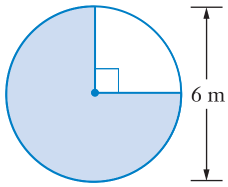
a



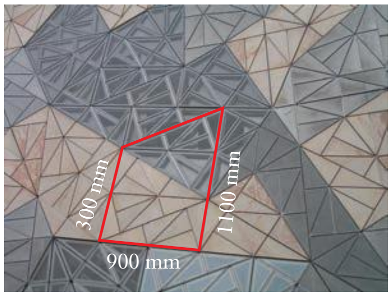
b



c



6 Find the area of the trapezoidal shape in the photo of the exterior of Federation Square, in central Melbourne.



7 A pizza is 30 cm in diameter.

a What is the area of the pizza, correct to 2 decimal places?

b The pizza is cut into 8 equal pieces. What is the area of each piece?

c Corey eats 3 slices and Anya eats 5 slices. How many more square centimetres of pizza does Anya eat?

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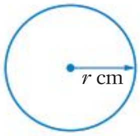
8 During World War II in the 1940s, army barracks were built with a semicircular end with diameter 7 m. Find the area of tin used for one end of the building, correct to 2 decimal places.

Shutterstock.com/Paul Briden

9 This circle has an area of 36 cm^2. Find its radius, r cm, correct to 2 decimal places.



7.06 Areas of composite shapes

Composite shapes are made up of smaller simpler shapes. We can find the areas of composite shapes by adding or subtracting the areas of simpler shapes.

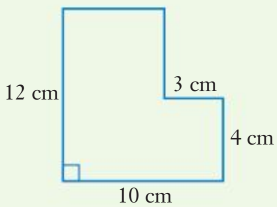
Area code puzzle

WS Composite areas

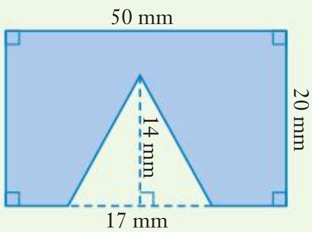
EXAMPLE 12

Find the area of each shape.

a



b



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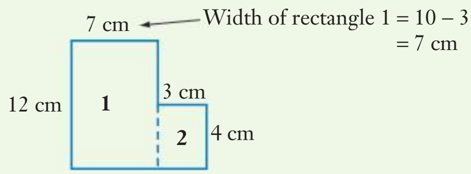
Solution

a Divide the shape into 2 rectangles as shown.



Calculate the area of each rectangle and add them together.

There is more than one way to divide this shape. All methods result in the same answer.

$$\text{Area of rectangle 1} = 7 \times 12$$
$$= 84 \text{ cm}^2$$
$$\text{Area of rectangle 2} = 3 \times 4$$
$$= 12 \text{ cm}^2$$
$$\text{Total area} = 84 + 12$$
$$= 96 \text{ cm}^2$$

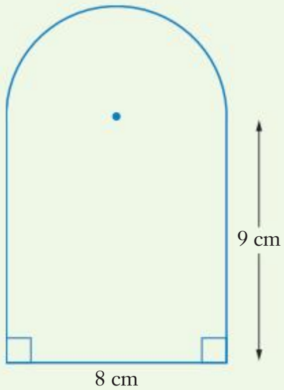
b Find the area of the rectangle and subtract the area of the triangle.

Sometimes, it is easier to find the area by subtraction.

$$\text{Area} = 50 \times 20 - \frac{1}{2} \times 17 \times 14$$
$$= 881 \text{ mm}^2$$

EXAMPLE 13

Find the area of this shape, correct to 2 decimal places.



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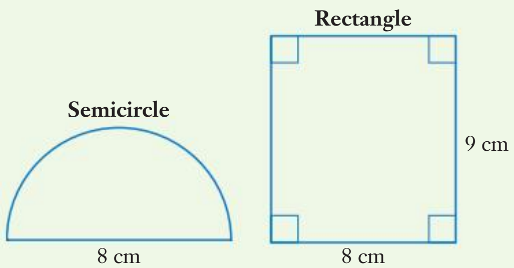
Solution

This shape is a semicircle and a rectangle.



Find the area of the semicircle first.

Radius, $r = \frac{1}{2} \times 8 = 4$

Area of semicircle $= \frac{1}{2} \times \pi r^2$

$= \frac{1}{2} \times \pi \times 4^2$

$= 25.1327 \ldots$

Find the area of the rectangle.

Area of rectangle $= lw$

$= 8 \times 9$

$= 72$

Add both areas together.

Total area $= 25.1327\ldots + 72$

$= 97.1327\ldots$

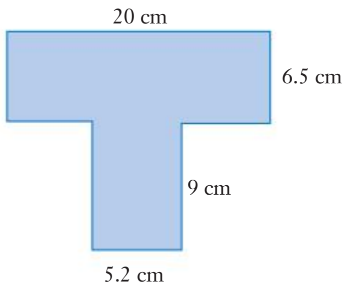
$\approx 97.13 \text{ cm}^2$

Exercise 7.06 Areas of composite shapes

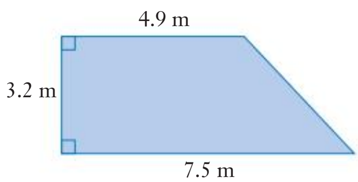
Example 12

1 Find the area of each shape, correct to 2 decimal places where necessary.

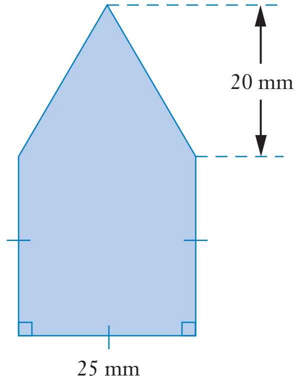
a



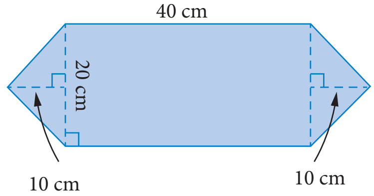
b



c

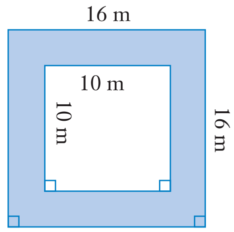


d

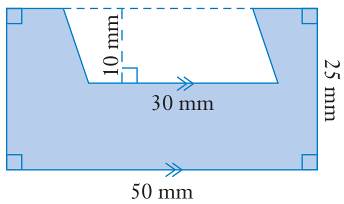


2 Calculate the area of each shaded region.

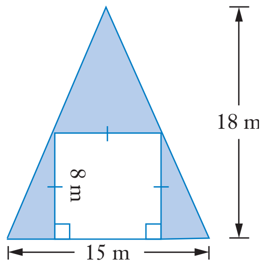
a



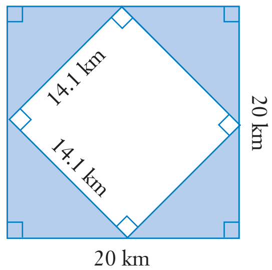
b



c



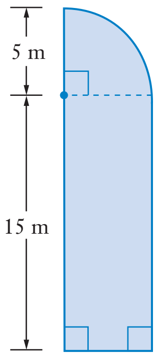
d



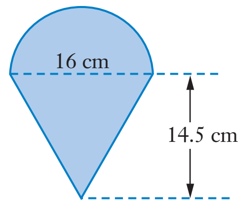
Example 13

3 Calculate the area of each shaded region, correct to 2 decimal places.

a



b



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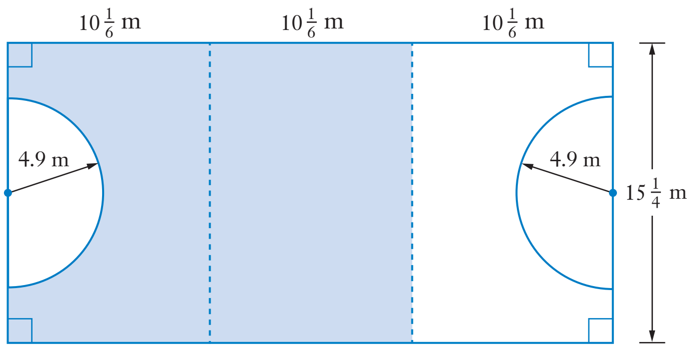
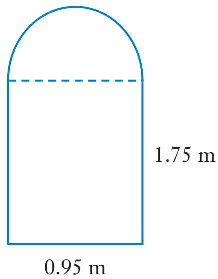
4 The diagram shows a netball court.



a Calculate, correct to 2 decimal places, the area of one of the goal semicircles.

b Calculate, correct to one decimal place, the size of the shaded region where the 'wing attack' player can play.

5 Susan and Ian are putting 3 arch windows in the front room of their house. The diagram shows one of the windows with its dimensions.

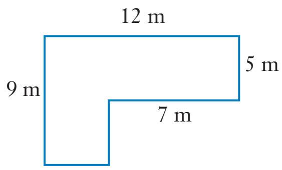


a Find the area of one arch window, correct to 2 decimal places.

b Find the area of all 3 arch windows.

c 10% of the area is NOT glass. Find the area of glass used.

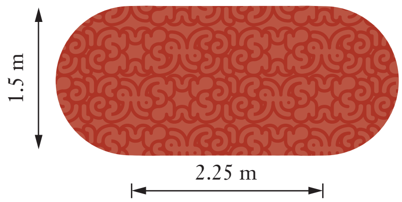
6 A children's playground has an L-shape as shown.



a What is the area of the playground?

b The playground is to be covered with woodchips for safety. How much will the woodchips cost if they are sold for \$7.20 per square metre?

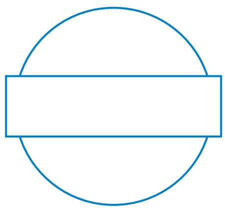
7 Angelina buys a new rug for her living room. The rug is composed of one rectangle and two semicircles as shown.



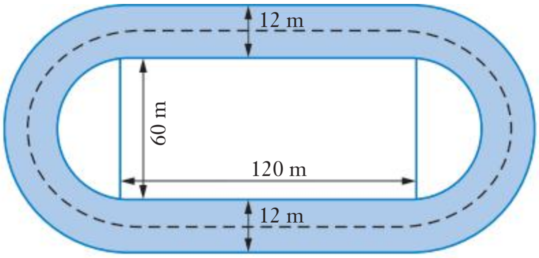
a Calculate the area of the rug, correct to 2 decimal places.

b Angelina's living room floor is a rectangle 3 m by 2.5 m. What area of the floor will NOT be covered by the rug?

8 This badge is made up of a metal disc overlaid with a rectangle. The metal disc has a radius of 3.5 cm. The rectangle is 1 cm longer than the diameter of the circle and 1.5 cm wide. Calculate the total area of the badge, correct to 2 decimal places.



9 Gotham City has a multi-purpose football field surrounded by an athletics track as shown. Calculate the area of the athletics track, correct to 2 decimal places.

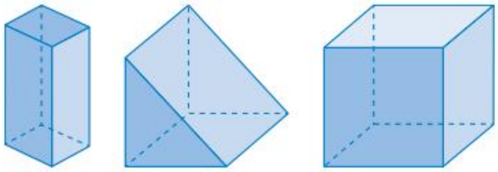


Nets of solids


Surface area of a prism

7.07 Surface areas of prisms

A **prism** is a solid shape that has identical ends. Some examples are shown: a rectangular prism, **triangular prism** and cube. All side faces are rectangles.

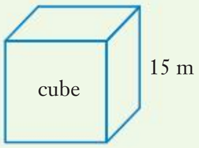


When we want to make boxes for packaging, we need to know the area of the faces of the box. This is called **surface area** because it is the sum of the areas of all the surfaces of the box. The easiest way to calculate the surface area of an object is to draw its **net**. We learned about the nets of solids in Chapter 2, *The shape of our world*. We calculate the area of each shape on the net individually and then add them together to find the total surface area.

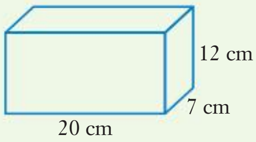
EXAMPLE 14

Find the surface area of each prism.

a



b



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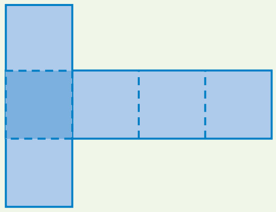
Solution

a A cube has 6 identical faces, all squares.

This is the net.

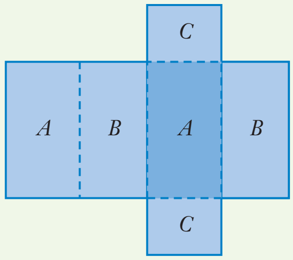


The surface area is 6 times the area of each square.

Total surface area $= 6 \times 15^2$

$= 1350 \text{ m}^2$

b This is the net of a rectangular prism. Notice that the rectangles are in matching pairs.



The ends (C) are the same rectangles.

Area of ends $= 2 \times (7 \times 12)$

$= 168 \text{ cm}^2$

The top and bottom faces (A) are the same size.

Area of top/bottom $= 2 \times (20 \times 7)$

$= 280 \text{ cm}^2$

The front and back faces (B) are the same size.

Area of front/back $= 2 \times (20 \times 12)$

$= 480 \text{ cm}^2$

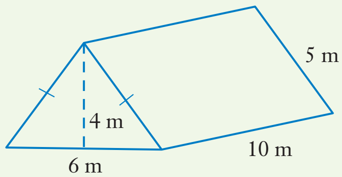
Add the areas together.

Total surface area $= 168 + 280 + 480$

$= 928 \text{ cm}^2$

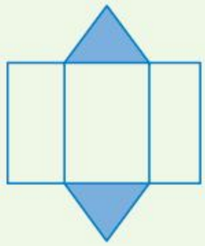
EXAMPLE 15

Find the surface area of this triangular prism.



Solution

This is the net of the prism.
There are 3 rectangles and 2 identical triangles.



Find the area of the base rectangle.

$$\text{Area of base} = 6 \times 10 = 60 \text{ m}^2$$

Find the area of the two side rectangles.

$$\text{Area of side faces} = 2 \times (5 \times 10) = 100 \text{ m}^2$$

Find the area of the two triangles.

$$\text{Area of 2 triangles} = 2 \times \left(\frac{1}{2} \times 6 \times 4\right) = 24 \text{ m}^2$$

Add the areas together.

$$\text{Total surface area} = 60 + 100 + 24 = 184 \text{ m}^2$$

Exercise 7.07 Surface areas of prisms

Example 14

1 Find the surface area of each cube.

a

b

2 Charlotte has a plastic storage cube that is 35 cm long with no top. Calculate its external surface area. (Ignore the holes for the handles.)

Alamy Stock Photo/Stephanie Jackson

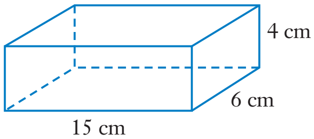
ISBN 9780170443975

3 This metal cube sculpture has a side length of 450 cm. The surface is to be covered with a weather-resistant finish. How many square metres need to be covered?

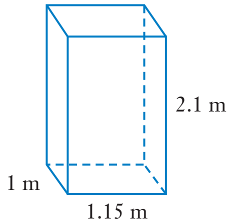
Shutterstock.com/Holly Vegter

4 Find the surface area of each prism.

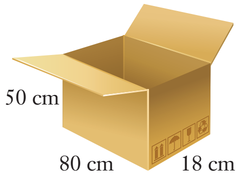
a



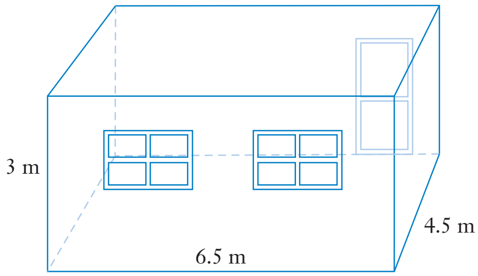
b



5 James' new TV was packed in this box. Find the surface area of the box.



6 Alexa and Jai built a new living room.



a Calculate the area of the walls and ceiling to be painted. Subtract 6 m^2 for the windows and door.

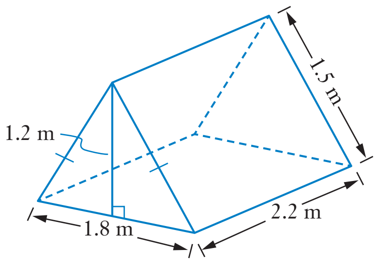
b If paint costs \$23 for a 4-litre can that covers 10 m^2, calculate how much it would cost to paint the room with 2 coats.

c Calculate the area of the floor to be covered with wood parquet.

d If wood parquet costs \$15.70 per square metre, how much would it cost to cover the floor? Answer to the nearest 10 dollars.

7 Tom's company makes camping equipment, including this tent. What amount of material is required to make the tent (including the floor)?

Example 15



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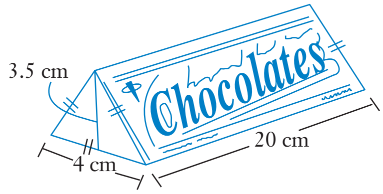
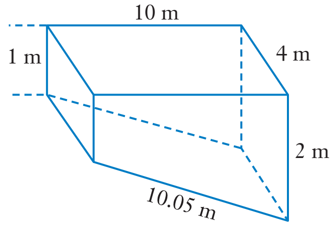
8 The diagram shows a chocolate box in the shape of a prism. The ends of the box are equilateral triangles.



a Calculate the surface area of the box.

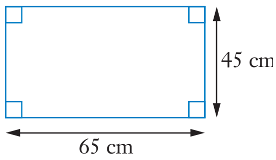
b The manufacturer allows an extra 10% of the surface area for tabs and wastage. How much cardboard is required for one box?

9 Harish and Rachna have a backyard pool shaped as shown. It needs to be repainted. There are tiles around the top edge to a depth of 40 cm. Calculate the area to be painted.



Remember to convert 40 cm to m.

10 A metal tray is made by taking a rectangular piece of metal and cutting squares from the corners. The edges are then bent upwards and welded at the corners to form the tray.



a This piece of metal has squares of side length 10 cm cut out of each corner. What are the dimensions of the tray that can be formed?

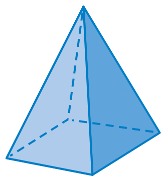
b The bottom of the tray is to be lined with material. What area of material is required?

c The sides and the outside are to be enamelled. What area is to be enamelled?

7.08 Surface areas of pyramids*

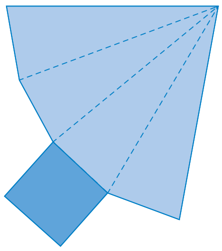
*Australian curriculum only, not in WA syllabus

A **pyramid** has a triangle, quadrilateral or other polygon as its base, and all side faces are triangles that meet at a point called the **apex**. We name pyramids by the shape of their base. For example, this is a square pyramid.

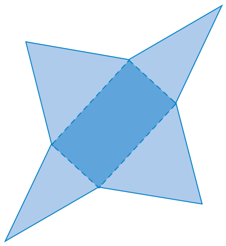


The nets of some pyramids are shown below: these will help us find their surface areas.

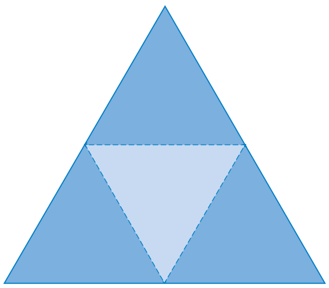
Square pyramid



Rectangular pyramid



Triangular pyramid



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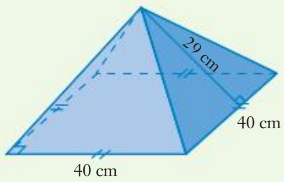
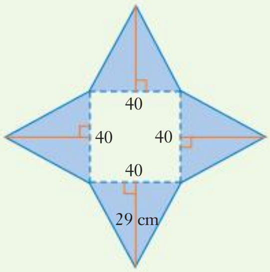
Surface area of a pyramid

Surface area of a pyramid = area of base + area of all triangular side faces

EXAMPLE 16*

A square pyramid and its net are shown. Calculate the surface area of the pyramid.





Solution

Calculate the area of the square base.

$$\text{Area} = 40^2$$
$$= 1600\ \text{cm}^2$$

Calculate the area of the 4 triangles. They are all the same because the base is a square.

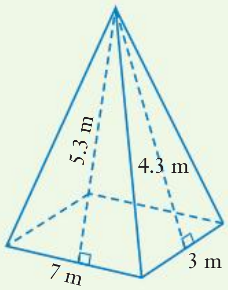
$$\text{Area} = 4 \times \left(\frac{1}{2} \times 40 \times 29\right)$$
$$= 2320\ \text{cm}^2$$

Add the areas together.

$$\text{Total surface area} = 1600 + 2320$$
$$= 3920\ \text{cm}^2$$

EXAMPLE 17*

Find the surface area of this rectangular pyramid.



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Solution

Find the area of the rectangular base.

$$\text{Area} = 7 \times 3$$
$$= 21 \text{ m}^2$$

Opposite triangles are identical.

Find the area of the left/right triangles.

$$\text{Area} = 2 \times \left(\frac{1}{2} \times 3 \times 4.3\right)$$
$$= 12.9 \text{ m}^2$$

Find the area of the front/back triangles.

$$\text{Area} = 2 \times \left(\frac{1}{2} \times 7 \times 5.3\right)$$
$$= 37.1 \text{ m}^2$$

Add the areas together.

$$\text{Total surface area} = 21 + 12.9 + 37.1$$
$$= 71 \text{ m}^2$$

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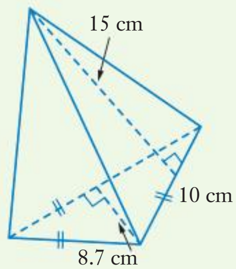
EXAMPLE 18*

Find the surface area of this triangular pyramid.

Note that its base is an equilateral triangle.



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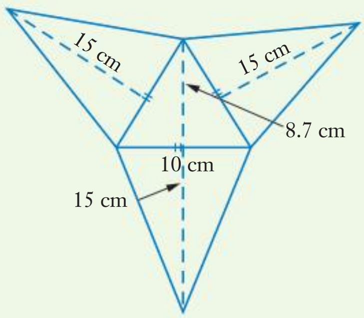
Solution

The net of this pyramid is shown: the base is an equilateral triangle and the 3 side faces are identical triangles.



Find the area of the 3 side faces. $\text{Area} = 3 \times \left(\frac{1}{2} \times 10 \times 15\right)$

$= 225 \text{ cm}^2$

Find the area of the base triangle. $\text{Area} = \frac{1}{2} \times 10 \times 8.7$

$= 43.5 \text{ cm}^2$

Add the areas together. $\text{Total surface area} = 225 + 43.5$

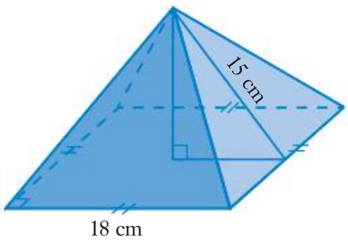
$= 268.5 \text{ cm}^2$

Exercise 7.08 Surface areas of pyramids*

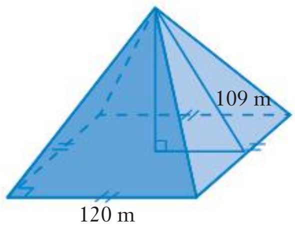
Example 16

1 Find the surface area of each square pyramid.

a



b



2 Andrea is making a stained glass lampshade in the shape of a square pyramid. The bottom edges are 20 cm long and the slant height of the triangles is 24 cm. Find the total area of glass that Andrea will need to make the lampshade.

Note: A lampshade does not have a base!

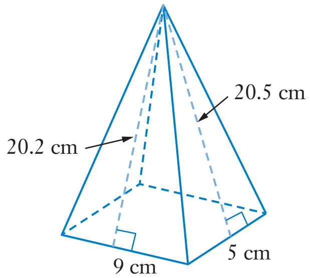
Photo courtesy of Sue Thomson

3 The Great Pyramid in Egypt is a square pyramid with the sides of the base being 230 m and the slant height of the sides 300 m. Calculate the surface area of the Great Pyramid, including the base.

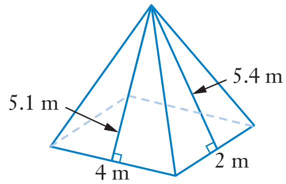
Example 17

4 Calculate the surface area of each rectangular pyramid.

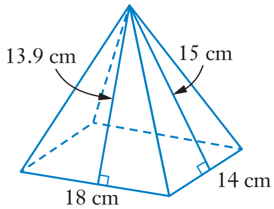
a



b



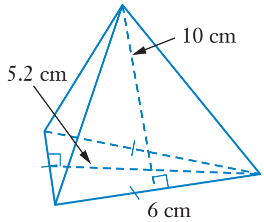
5 Harry believes his seedlings grow best when they are planted in glass pyramids. He uses a glass cover in the shape of a rectangular pyramid that fits over a glass base 18 cm by 14 cm. Calculate the total surface area of the pyramid and the glass base.



Example 18

6 Calculate the surface area of each triangular pyramid.

a



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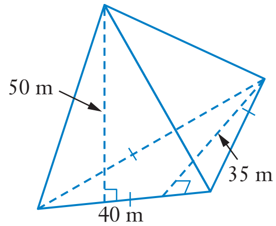

7 Edward wins the state public speaking competition and receives a crystal glass trophy in the shape of a triangular pyramid. The base is an equilateral triangle with sides 4 cm and perpendicular height 3.5 cm. The slant height of the sides of the trophy is 22 cm. Calculate the surface area of the glass.

8 The entrance to the Louvre museum in Paris is a large square pyramid made of glass. The base of the pyramid is 35 m long. The slant height of the sides of the pyramid is 27.8 m.

a Calculate the area of the 4 side faces of the pyramid.

b The side faces are made out of glass pieces in the shape of a rhombus. There are approximately 640 of these on the 4 sides. What is the approximate area of one glass piece?

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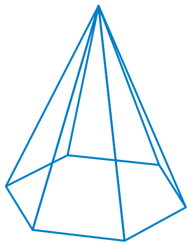
9 This is a hexagonal pyramid box made to hold chocolates. The base has sides of 8 cm and its area is 83 cm^2. The slant height of the triangles is 15 cm. Calculate the surface area of this chocolate box.



Hexagonal pyramid

10 The roof of one section of Sue's house is a square pyramid. The base of each section of roof is 10 m and the slant height is 5.3 m. At the top, a square pyramid has been sliced off. The removed section has a square base 1.8 m long and a slant height of 1.9 m.

Photo courtesy of Sue Thomson

a What was the surface area of the roof *before* the top section was removed?

b What is the surface area of the section of roof that has been removed?

c What is the surface area of the roof that Sue has to cover with roof tiles?

INVESTIGATION

FINDING PYRAMID BUILDINGS

Search the Internet for buildings or structures that are pyramids.

- Write down the locations of at least 10 pyramid buildings or structures. Include pictures where possible.
- Choose one of the buildings or structures you have found and find its dimensions.
- Calculate the surface area of the building or structure.

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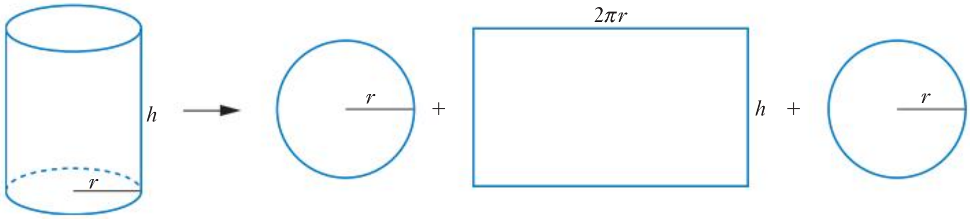
7.09 Surface areas of cylinders and spheres

If we flatten out a **cylinder**, we get this net of 2 circles and a rectangle.

WS Solid shapes

WS Nets of solids



Note: The length of the rectangle is the circumference of the circle ($2\pi r$).

Surface area of a cylinder = area of 2 circles + area of rectangle

$$= 2 \times \pi r^2 + 2\pi r \times h \qquad \text{Circumference} = 2\pi r$$

$$= 2\pi r^2 + 2\pi rh$$

There is a simple formula for the surface area of a **sphere**.

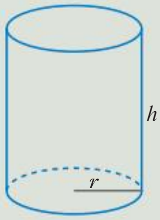
Surface area of a sphere = $4 \times \pi \times (\text{radius})^2$ Like 4 times the area of a circle

$$= 4\pi r^2$$

Surface area of a cylinder

Surface area = area of 2 circles + area of rectangle

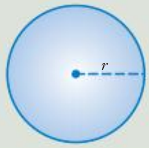
$$SA = 2\pi r^2 + 2\pi rh$$



Surface area of a sphere

Surface area = $4 \times \pi \times (\text{radius})^2$

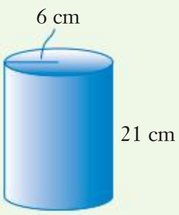
$$SA = 4\pi r^2$$



EXAMPLE 19

Find the surface area of each solid, correct to 2 decimal places.

a



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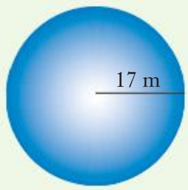

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Solution

a Use the formula for the surface area of a closed cylinder with $r = 6$ and $h = 21$.

$$\begin{aligned} SA &= 2\pi r^2 + 2\pi rh \\ &= 2 \times \pi \times 6^2 + 2 \times \pi \times 6 \times 21 \\ &= 1017.8760\ldots \\ &= 1017.88 \text{ cm}^2 \end{aligned}$$

b Use the formula for the surface area of a sphere with $r = 17$.

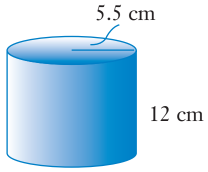
$$\begin{aligned} SA &= 4 \times \pi \times r^2 \\ &= 4 \times \pi \times 17^2 \\ &= 3631.6811\ldots \\ &\approx 3631.68 \text{ m}^2 \end{aligned}$$

Exercise 7.09 Surface areas of cylinders and spheres

Example 19

1 Find the surface area of each solid, correct to 2 decimal places.

a



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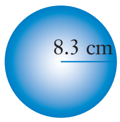
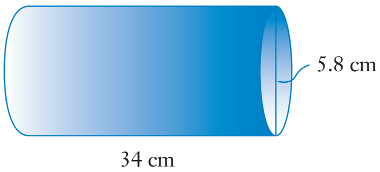
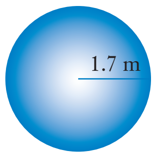


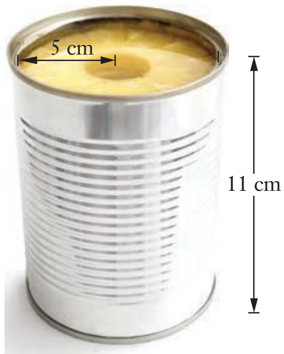
c



d



2 A can of pineapple rings has height 11 cm and radius 5 cm. A label is wrapped around the curved surface. Calculate the area of the label, correct to 2 decimal places.



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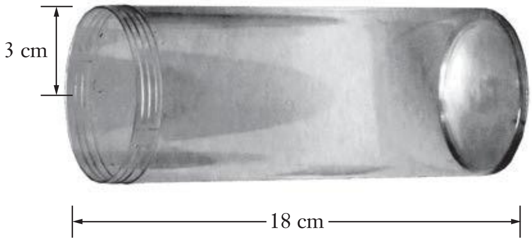
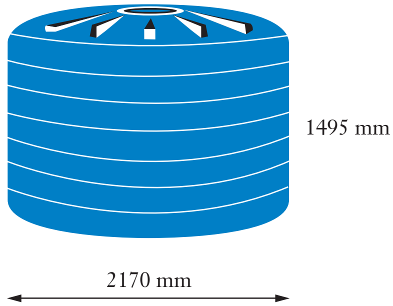
3 Calculate, correct to one decimal place, the area of recycled plastic used to make this food container. Note that it has an open end.



4 A tennis ball has a diameter of 7 cm. Calculate the surface area of the tennis ball, correct to the nearest cm^2.

5 How much plastic would be needed to make this water tank? Answer correct to the nearest square metre.



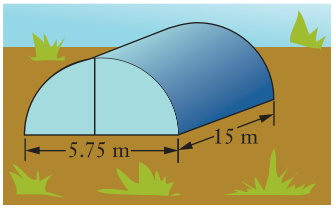
6 The Earth is approximately a sphere with a radius of 6400 km.

a Calculate the surface area of the Earth, correct to the nearest 1000 square km.

b Approximately 70% of the Earth's surface is water. What area of the Earth's surface is covered with water?

c What area of the Earth's surface is land?

7 A hydroponics shed is made in the shape of half of a cylinder. Find the area of sheet metal needed for the shed, correct to the nearest square metre. Do not include the floor.



8 A disco mirror ball is 33 cm in diameter. It is covered with small mirror tiles that have an area of 1.44 cm^2 each.

a Find the surface area of the sphere, correct to 2 decimal places.

b How many tiles are used to cover the mirror ball?

ISBN 9780170443975

9 Elise bought a tube for sending posters through the mail. It is a cardboard cylinder with plastic plugs at each end. The tube is 1.1 m long and has a radius of 2.5 cm.

a Calculate the amount of cardboard used in creating the tube, correct to the nearest square centimetre.

b Calculate the total area of the plastic ends, correct to 2 decimal places.

10 Abhishek has a set of 3 steel bowls he uses in cooking. Each bowl is the shape of a hemisphere (half a sphere).

a What is the radius of each hemisphere?

b Calculate, correct to the nearest cm^2, the internal surface area of each bowl.

11 Charlotte has a large bowl that she is going to use for a water feature in her garden. She needs to paint the inside surface with a waterproofing liquid.

a Calculate the surface area that Charlotte needs to cover with the waterproofing liquid.

b A 1-litre bottle of the liquid covers 10 m^2. How many bottles will Charlotte need to buy?

Chapter problem

You've covered the skills required to solve the chapter problem. Can you solve it now?

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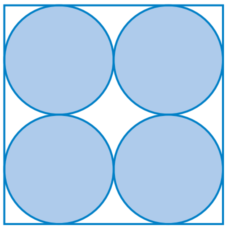
INVESTIGATION

CHRISTMAS BAUBLE BOXES

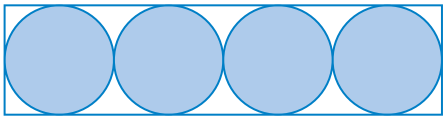
Irena has been asked to design a new cardboard box for a set of 4 spherical Christmas baubles with a diameter of 6.5 cm. She presents 2 designs:

An open square box



An open box has no top.

An open rectangular box



1 Calculate the area of cardboard required for the open square box. Allow for an extra centimetre on each dimension (length, width, height) to have space for interior packaging. Add 10% extra area for overlapping sections to glue the box together.

2 Calculate the area of cardboard required for the open rectangular box. Allow an extra centimetre on each dimension for interior packaging space. Add 10% extra area for overlapping sections.

3 Is there another possible design for holding the 4 baubles? If so, provide a sketch and calculate the area of cardboard required for it. Allow an extra centimetre on each dimension for interior packaging space. Add 10% extra area for overlapping sections.

4 Which design would you recommend? Justify your decision.

iStock.com/yasinguneysu

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7.10 Surface areas of composite solids

When we were calculating the area of some shapes, we had to separate the shape into several smaller shapes. We can use the same technique when we are calculating the surface areas of composite solids.

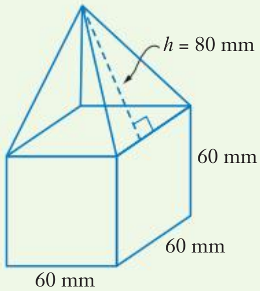
Surface area

To find the surface area of a solid:

- calculate the area of each face
- add the areas together to calculate the total surface area

EXAMPLE 20

Calculate the surface area of this composite solid made from a square pyramid and a cube.



Solution

This solid has 4 triangular faces and 5 square faces.

Find the area of the 4 triangles. Area of triangular faces $= 4 \times \frac{1}{2} \times 60 \times 80$

$= 9600\ \text{mm}^2$

Find the area of the 5 squares. Area of square faces $= 5 \times 60 \times 60$

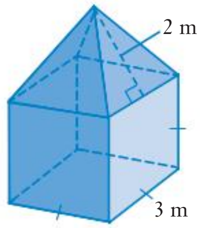
$= 18\ 000\ \text{mm}^2$

Add the areas together. Total surface area $= 9600 + 18\ 000$

$= 27\ 600\ \text{mm}^2$

Exercise 7.10 Surface areas of composite solids

1 Manuel is building a greenhouse, as shown in the diagram. Calculate the surface area of his greenhouse, *not including the floor*.

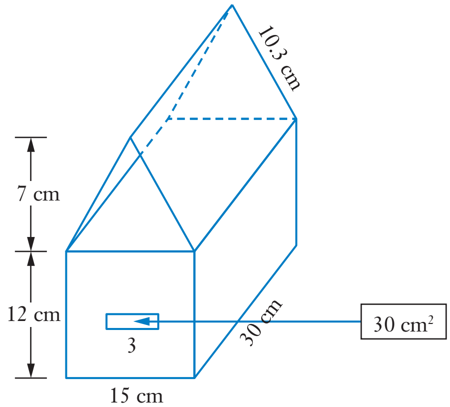


Example 20

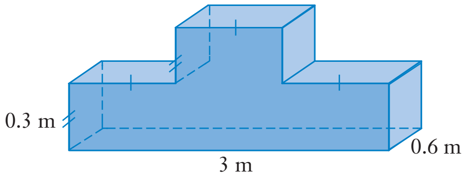
2 This metal water tank supplies water to steam trains. It has the shape of a square prism, where each side face is a rectangle 3.2 m wide by 1.4 m high. Find the area of metal in the water tank, including the base and the lid.

Shutterstock.com/Desmond Kean

3 The lower section of Tom's metal letterbox is a rectangular prism and the upper section is a triangular prism. The upper section is open at both triangular ends to hold long, rolled-up articles. The rectangular slot is 30 cm². Calculate the amount of sheet metal in Tom's letterbox.



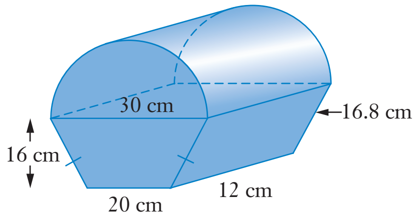
4 The local athletics club has a portable podium for medal presentations. Every surface, excluding the base, needs repainting. Calculate the surface area to be repainted.



5 Lisa is making a jewellery box shaped as shown. She is going to cover it with special material.

a Find the surface area of the jewellery box, correct to the nearest square centimetre.

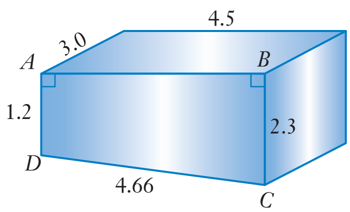
b Lisa allows an extra 10% of material for the edges and wastage. How much material should Lisa buy, correct to the nearest square centimetre?



6 Bruno needs to resurface the interior of his pool. (All measurements are in metres.)

a Calculate the area of the trapezium *ABCD*.

b Calculate the total surface area of the 5 faces that Bruno needs to resurface, correct to 2 decimal places.



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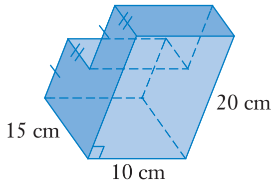
7 Ginny has made a pair of wooden bookends in an L-shape. She is going to stain the bookends before she uses them. Find the total surface area of both bookends.



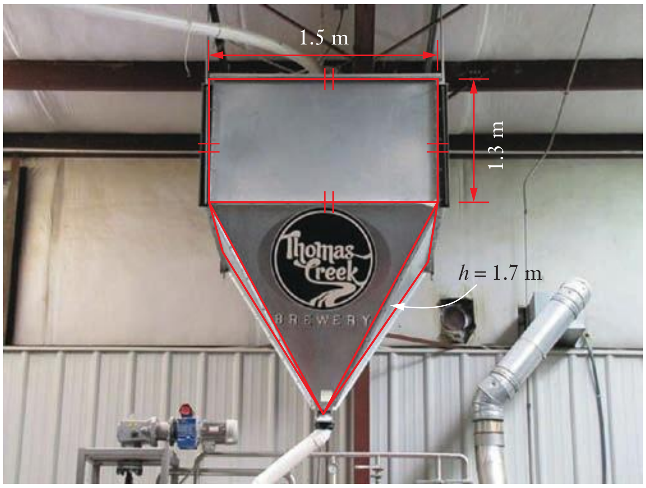
8 This truck delivers LPG to homes for use in cooking. The tank is comprised of a cylinder with a hemisphere at each end.

Alamy Stock Photo/Justin Kaseznnez

a Calculate the surface area of the LPG tank, correct to 1 decimal place.

b 1 L of paint covers 14 m^2 of the tank. How much paint will Bill need to cover the tank with 2 coats? Answer in litres, correct to 1 decimal place.

9 Jeff uses sheet metal to make this hopper to use in the beer making process. The hopper is in the shape of a square pyramid under a square prism, without a top. How much sheet metal did Jeff use to make the hopper?



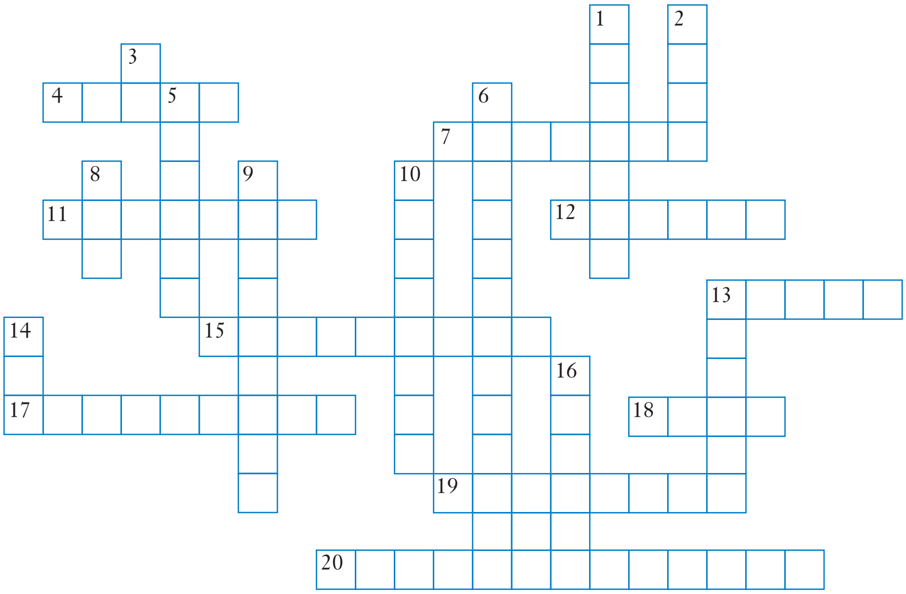
GreerToday.com Photo

KEYWORD ACTIVITY

CLUELESS CROSSWORD

Copy the crossword below and fit the words into it. The number of letters in each word is shown in brackets.

pi [2]	arc [3]	net [3]	area [4]
cube [4]	prism [5]	solid [5]	circle [6]
sector [6]	sphere [6]	square [6]	hectare [7]
pyramid [7]	surface [7]	cylinder [8]	triangle [8]
composite [9]	rectangle [9]	trapezium [9]	parallelogram [13]
quadrilateral [13]			



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SOLUTION TO THE CHAPTER PROBLEM

Problem

Jess is making gift boxes to sell. She plans to make three types of boxes:

Cube: 15 cm long

Rectangular prism: 20 cm by 10 cm by 8 cm

Cylinder: radius 4.5 cm and height 34 cm

Should she charge the same for each gift box?

Give reasons for your answer.

Solution

We have to compare the surface areas of the boxes to see if they are equal.

Cube:

$$\begin{aligned}\text{Surface area} &= 6 \times (\text{area of a square face}) \\ &= 6 \times 15^2 \\ &= 1350 \text{ cm}^2\end{aligned}$$

Rectangular prism:

$$\begin{aligned}\text{Surface area} &= 2 \times (\text{base area}) + 2 \times (\text{front face area}) + 2 \times (\text{side face area}) \\ &= 2 \times (20 \times 10) + 2 \times (20 \times 8) + 2 \times (10 \times 8) \\ &= 880 \text{ cm}^2\end{aligned}$$

Cylinder:

$$\begin{aligned}\text{Surface area} &= 2\pi r^2 + 2\pi rh \\ &= 2 \times \pi \times (4.5)^2 + 2 \times \pi \times 4.5 \times 34 \\ &\approx 1089 \text{ cm}^2\end{aligned}$$

The boxes have different surface areas.

Jess should not charge the same amount for each box because they use different amounts of materials. Compared to the price of the rectangular prism, she should charge more for the cylinder and even more for the cube.

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7. TEST YOURSELF

On the surface

Practice quiz

Exercise 7.01

1 Copy and complete each conversion.

a 5 cm = ____ mm

b 3 m = _____ cm

c 3600 m = ____ km

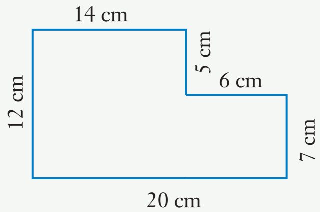
d 4.2 m = ____ mm

e 80 m = _____ km

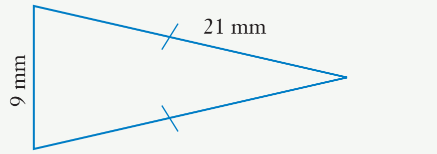
Exercise 7.01

2 Calculate the perimeter of each shape.

a



b



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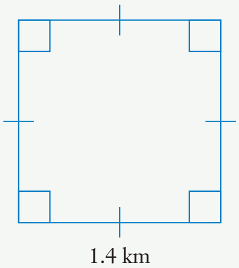
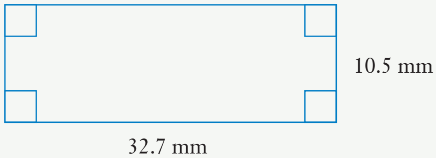
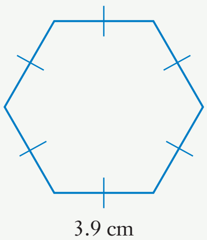


d



e

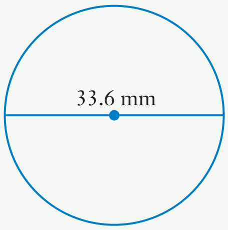


Exercise 7.02

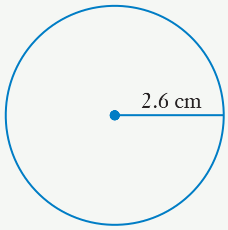
3 Calculate the circumference of each circle, correct to 1 decimal place.

a

b



c



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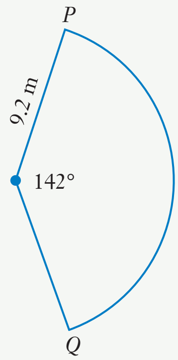
4 Calculate the length of the arc PQ, correct to 1 decimal place.

Exercise 7.02

a*



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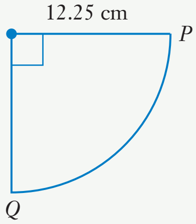

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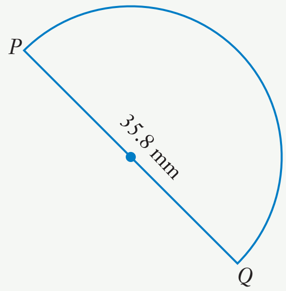
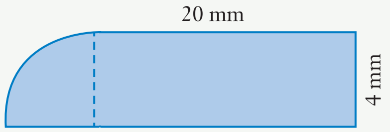


*Australian curriculum only, not in WA syllabus.

5 What is the perimeter of this shape made from a quarter of a circle and a rectangle? Answer correct to one decimal place.

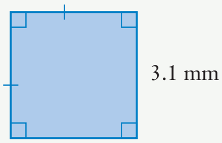
Exercise 7.03



6 Calculate the area of each shape, correct to one decimal place.

Exercise 7.04

a



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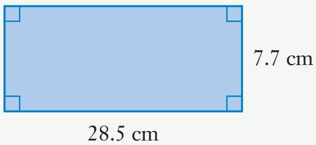

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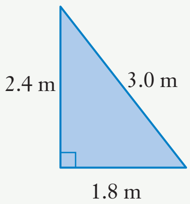
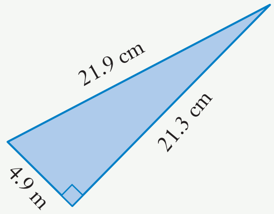


d



7 Grant is going to buy tiles to cover his rectangular verandah, which is 11 m long by 2.5 m wide. How many square metres of tiles should he order, allowing an additional 10% for cutting and breakages? Answer correct to the nearest square metre.

Exercise 7.04

8 Copy and complete each conversion.

Exercise 7.04

a 400 mm^2 = ________ cm^2

b 5.4 cm^2 = _____ mm^2

c 25 000 m^2 = _____ ha

d 5500 cm^2 = _______ m^2

e 70 cm^2 = _____ mm^2

f 6 m^2 = _____ cm^2

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Exercise 7.05

9 Match each shape in **a** to **g** with the formula for its area in **A** to **G**.

a square **b** rectangle **c** circle **d** parallelogram
e trapezium **f** sector **g** triangle

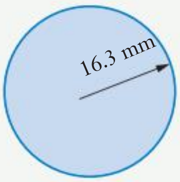
A $A = \frac{1}{2}(a + b)h$ **B** $A = \pi r^2$ **C** $A = s^2$ **D** $A = \frac{\theta}{360}\pi r^2$

E $A = bh$ **F** $A = lw$ **G** $A = \frac{1}{2}bh$

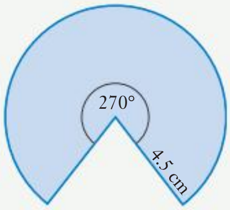
Exercise 7.05

10 Calculate the area of each shape, correct to one decimal place.

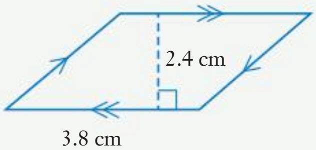
a



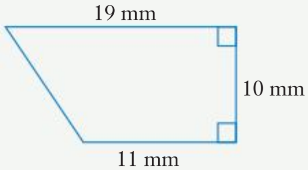
b*



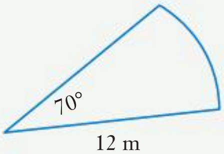
c



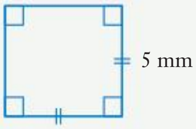
d



e*



f

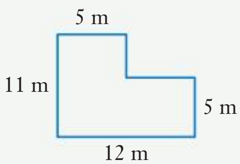


*Australian curriculum only, not in WA syllabus.

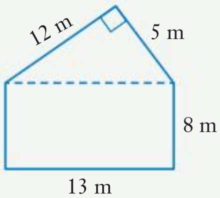
Exercise 7.06

11 Find the area of each shape.

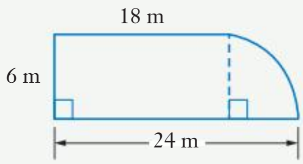
a



b



c



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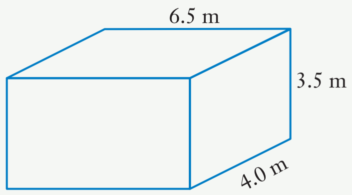
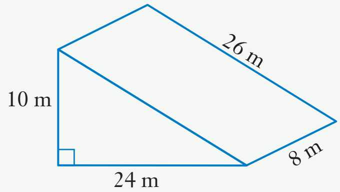
12 Find the surface area of each prism.

Exercise 7.07

a



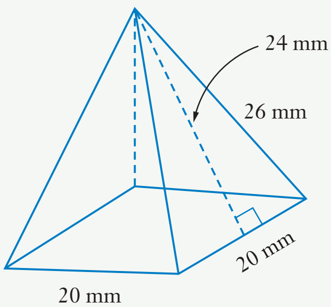
b



13* Find the surface area of this square pyramid.

Exercise 7.08

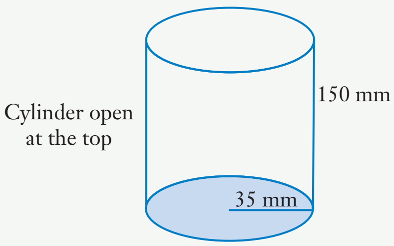
*Australian curriculum only, not in WA syllabus.



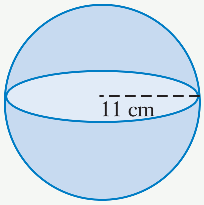
14 Calculate the surface area of each solid, correct to the nearest whole number.

Exercise 7.09

a



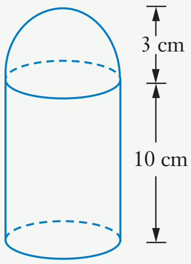
b



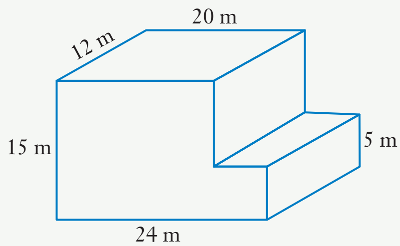
15 Calculate the surface area of each solid.

Exercise 7.10

a



b



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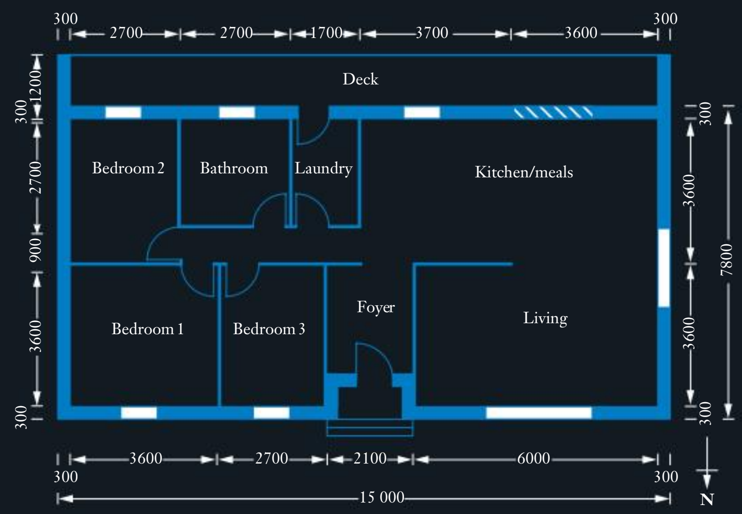
SCALES, PLANS AND MODELS

8. FROM PAPER TO REALITY

Chapter problem

Francesca and Michael are building a house according to the plan shown on the next page. They need to order carpet for the living room and the 3 bedrooms. How many square metres of carpet will they need? Answer to the nearest square metre.



WHAT WILL WE DO IN THIS CHAPTER?

- Use scales and calculate with scales
- Interpret house plans and elevations, including commonly-used symbols and abbreviations
- Find actual measurements from scale drawings, maps and house plans, such as lengths, perimeters and areas
- Calculate quantities and costs for home renovations
- Create scale drawings

HOW ARE WE EVER GOING TO USE THIS?

- Determining lengths on house plans, maps and other scale drawings
- Building, renovating or buying a home
- Interpreting scale models and building plans
- Working in a building or painting trade

8.01 Scale drawings

A **scale drawing** is a reduced or enlarged version of a real object. The most common scale drawings are maps and house plans. By taking measurements on the scale drawing, we can calculate the size of objects in real life using the **scale** on the drawing.

Scale drawings

To calculate an actual (real-life) length on a scale drawing:

- measure the scaled length on the scale drawing
- multiply by the scale
- convert your answer to the required units if necessary

Scales on a diagram are most commonly given as a statement, such as '1 cm represents 5 m', or as a ratio, such as '1 : 500'.

Map scales

EXAMPLE 1

Keiran is using a map with the scale: 1 cm represents 2 km.

a How far would he have to walk if the distance on the map is 6 cm?

b Keiran is planning a 25 km hike with friends. How far is this on the map?

Solution

a Multiply the scaled distance by the scale. Scaled distance = 6 cm.

Actual distance $= 6 \times 2$
$= 12$ km
Keiran would have to walk 12 km.

b To calculate a scaled (map) distance, DIVIDE the actual distance (25 km) by the scale.

Scaled distance $= 25 \div 2$
$= 12.5$ cm
25 km is 12.5 cm on the map.

Shutterstock.com/djdis

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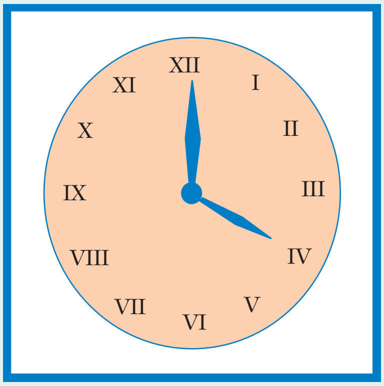
EXAMPLE 2

The diagram of a clock is drawn to a scale of 1 : 6. Measure its scaled length and calculate its actual length.



Scale 1 : 6

Solution

Measure the scaled length on the diagram. Scaled length = 5 cm

Multiply by the scale. Actual length = 5×6

= 30 cm

Exercise 8.01 Scale drawings

Example 1

1 On a map with the scale '1 cm represents 100 km', the Nile River in Egypt is 67 cm long. How long is the Nile River in real life?

2 A street map uses a scale of '1 cm represents 200 m'.

a Find the actual distance, in kilometres, represented by each scaled distance.

i 7 cm **ii** 9.5 cm **iii** 12.4 cm

b Find the scaled distance, in centimetres, used to represent each actual distance.

i 7 km **ii** 1500 m **iii** 3.3 km

Remember: 1 km = 1000 m.
Check your units!

3 Lord Howe Island is 2.8 km long. A map of the area has a scale of '1 cm represents 0.5 km'. Calculate the length of the island on the map.

4 A map has a scale of 1:50 000. What distance is represented by 64 mm on the map?

5 The town of Hughenden is 216 km north of Winton. On a map with a scale of 1 : 2 600 000, what is the scaled distance between the two towns? Answer to the nearest millimetre.

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6 The Dunn family are visiting Western Australia and they decide to stay in Albany for a few days. They have a map of Albany with a scale of 1 : 25 000.

a The children decide to walk to the swimming pool. On the map, the distance from their motel to the pool is 3.5 cm. How far is the walk from the hotel to the swimming pool?

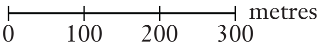
b Dad likes to do a 4 km run each day when he is on holiday. How far is this distance on the map?

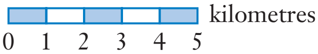
c The family drove from their motel to the top of Mt Clarence and then to Middleton Beach. This distance is 38.3 cm on the map. How far did they drive? Round your answer to the nearest 100 m.

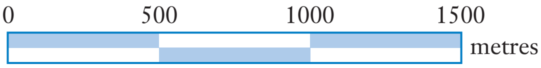
d Kath and Steve decided to walk to the end of the beach and back while the children played in the sand. Their GPS showed they had walked 3.5 km in total. How long is the beach on the map?

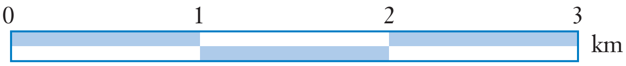
iStock.com/Shaun Brodison

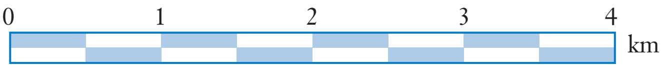
7 For each map scale below, what length does 1 cm represent?

a


b


c


d


e


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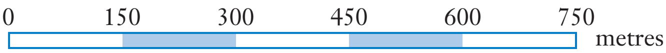

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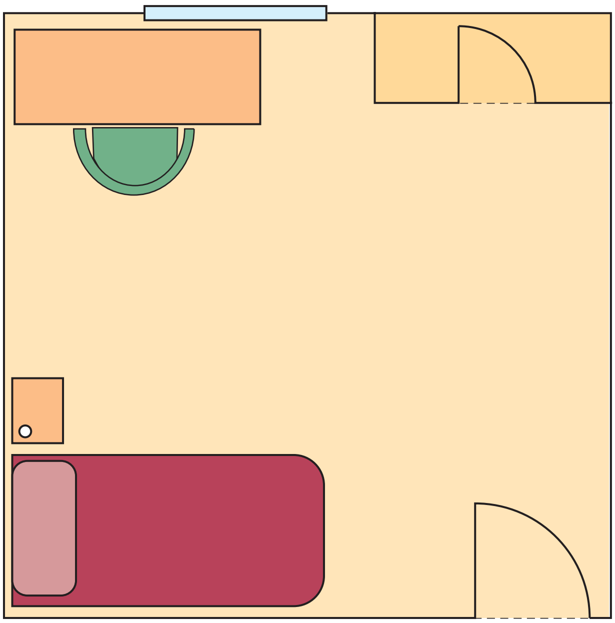
8 A scale plan of a bedroom is shown below.

Example 2



Scale 1 : 50

By measurement and calculation, find the actual:

a length of the bedroom
b width of the doorway
c length of the bed
d length of the window
e length of the table
f width of the bed

9 Measure the length of each scale drawing, then use the scale to calculate its actual length.

a Scale: 1 cm represents 0.6 m

Shutterstock.com/tamaguramo

b Scale: 1 : 24

Shutterstock.com/AlexMaster

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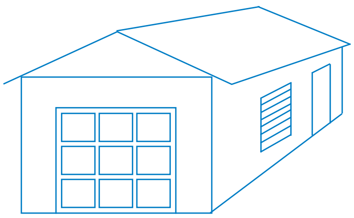
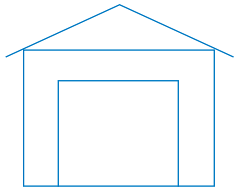
8.02 House plans and elevations

A draftsperson drawing a design for a new building includes different views of the building.

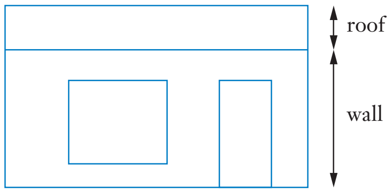
- a **plan**: a diagram of the floor
- the **elevation views**: views of the front, back, left side and right side of the building.

These diagrams show a garage with the front and right-side elevation and the view from above (plan):

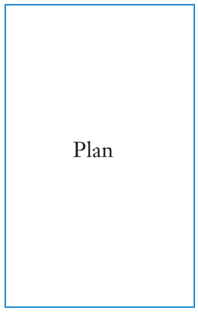




Front elevation



Right elevation



Shutterstock.com/Romakoma

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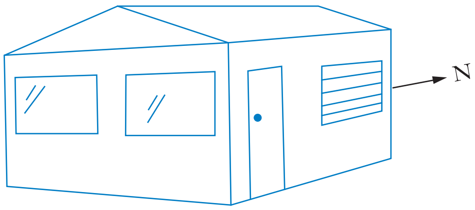
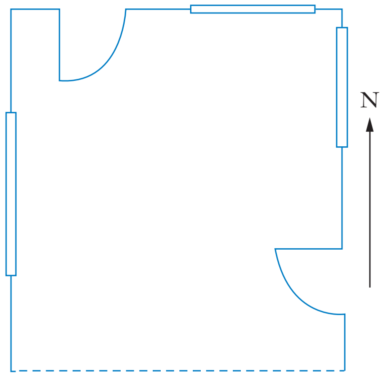
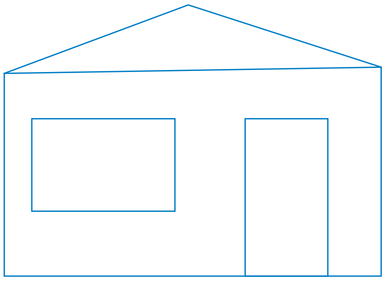
Exercise 8.02 House plans and elevations

1 The plan and 2 elevations for Erik's workshed are shown below.

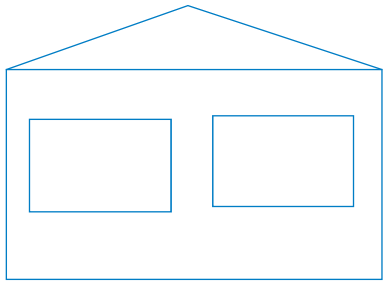




Plan



Northern elevation



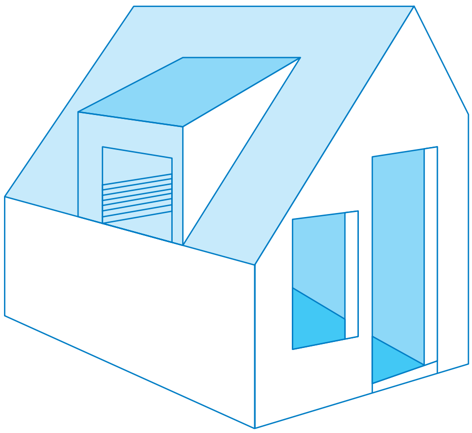
Southern elevation

a The dotted line shows that the plan is incomplete. Copy and complete the plan.

b Draw the eastern and western elevations.

Remember: the eastern elevation is the view from the eastern side.

2 For the house shown, sketch the front and side elevations. You do not need to draw the elevations to scale, but use a ruler and be as accurate as you can.



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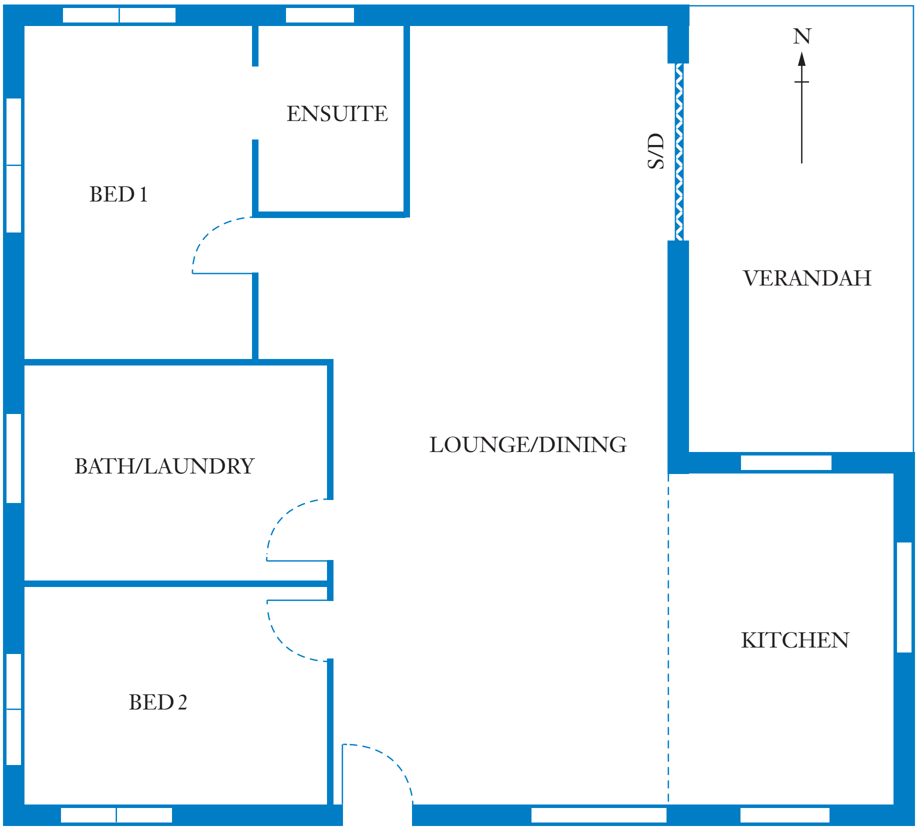
3 For the house shown, sketch the front and possible side elevations. You do not need to draw them to scale but use a ruler and be as accurate as you can.

Shutterstock.com/Ewelina Wachala

4 This is the plan of Grigor and Silvana's holiday house.



Sketch the northern and western elevations. You do not need to draw them to scale but use a ruler and be as accurate as you can.

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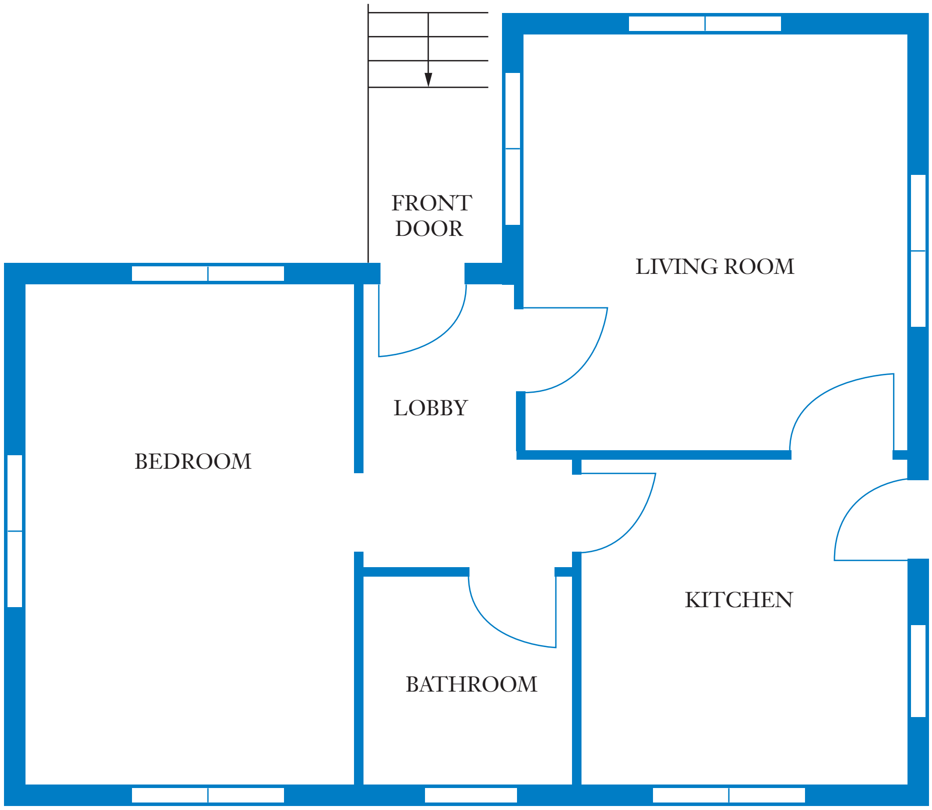
5 This is the plan for a one-bedroom apartment, the front door of which faces north. Draw elevation views of each side of the house.



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WS

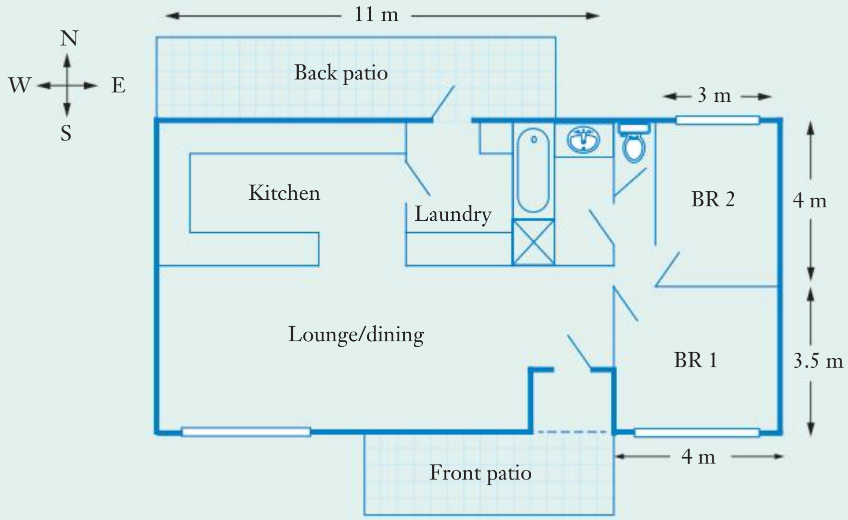
Interpreting an office plan

8.03 Reading house plans

Before any house or building can be built, a plan must be drawn up. House plans use many different symbols and abbreviations. They are either drawn to scale or have measurements written on them. Often, measurements are shown in **millimetres** to avoid the use of decimal points, which can lead to errors in printing and reading.

EXAMPLE 3

This is the floor plan for Grant's house.



a What are the dimensions of Bedroom 1?

b What is the area of Bedroom 1?

c What is the length across the front of the house?

d What is the width of the house excluding the patios?

Solution

a Find bedroom 1 on the plan. The dimensions are the length and the width.

The dimensions of bedroom 1 are 4 m by 3.5 m

b Area = lw

$$\begin{aligned} \text{Area} &= 4 \times 3.5 \\ &= 14 \text{ m}^2 \end{aligned}$$

c Find the full length from the plan.

$$\begin{aligned} \text{Length} &= 11 + 4 \\ &= 15 \text{ m} \end{aligned}$$

d Find the full width from the plan.

$$\begin{aligned} \text{Width} &= 4 + 3.5 \\ &= 7.5 \text{ m} \end{aligned}$$

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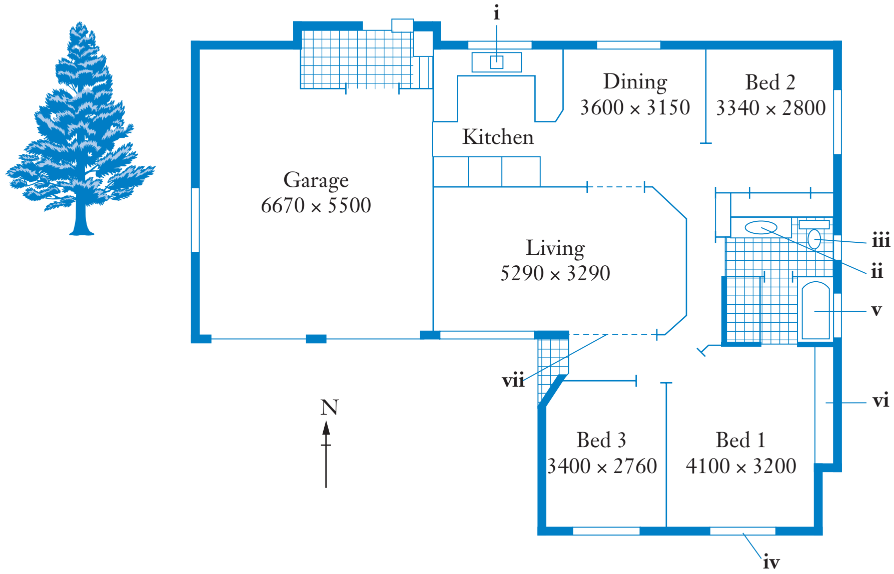
Exercise 8.03 House plans

1 This is the plan for Menhal's new house.



a What does each symbol used on the plan represent?

i  **ii**  **iii**  **iv** 

v  **vi**  **vii**

b Three areas on the plan are shaded with this pattern.  What does this mean?

c How many bedrooms does the house have?

d Which bedrooms have built-in wardrobes?

e What are the dimensions of bedroom 1?

f Where is the laundry?

g Where is the linen press?

h How many bathrooms does the house have?

i How many toilets are there?

j How many doors lead into the bathroom?

k Which room has dimensions 5.29 m by 3.29 m?

l What are the dimensions of the garage?

m If Menhal looks out of the window of each room mentioned below, in which direction is she facing?

i living room **ii** dining room **iii** bedroom 1 **iv** bedroom 2 **v** garage

n In which direction do the garage doors face?

o If Menhal stands on the front door step, can she see into the bathroom?

p If Menhal stands inside the front doorway, list the 4 rooms she can see into.

q If Menhal is working in the kitchen, list the 3 rooms she can see into.

2 a How many bedrooms does this house have?

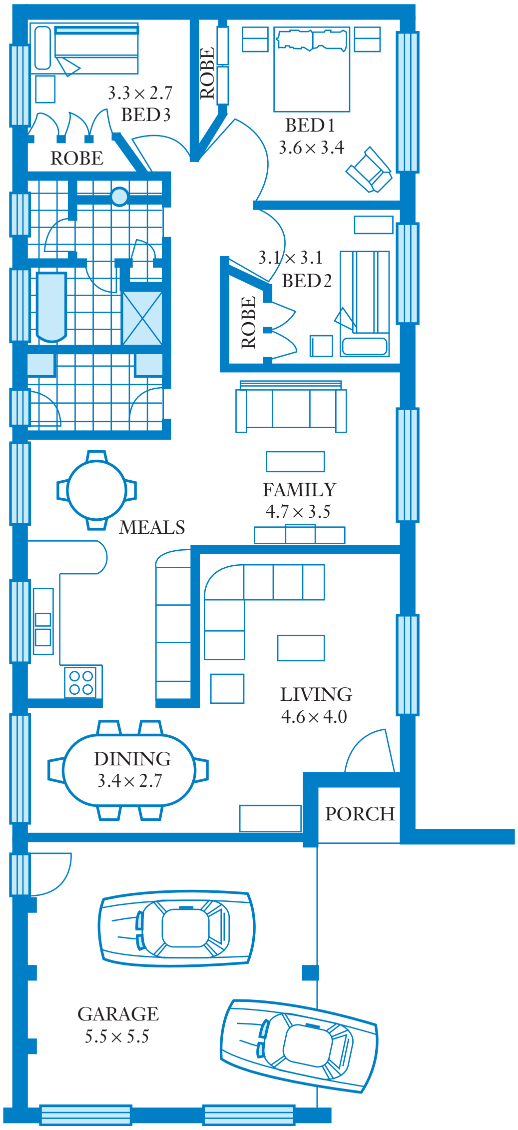
b How many of the bedrooms have built-in wardrobes?

c Is there an ensuite bathroom?

d What are the dimensions of the family room?

e Which bedroom has the largest floor area?

f The floor area of the house is 128 square metres. Calculate the cost of building this house at the rate of $672 per square metre.



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3 This house plan has a scale of 1 : 100.

Measure and calculate:

a the length of the main bedroom

b the length of the window in that room

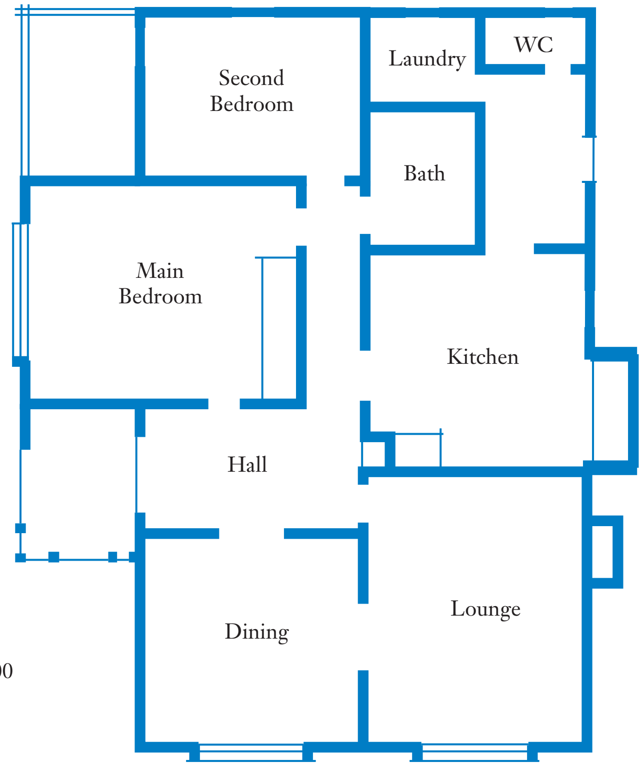
c the length of the laundry

d the area of the bathroom

e the longer side of the lounge room

f the area of the dining room

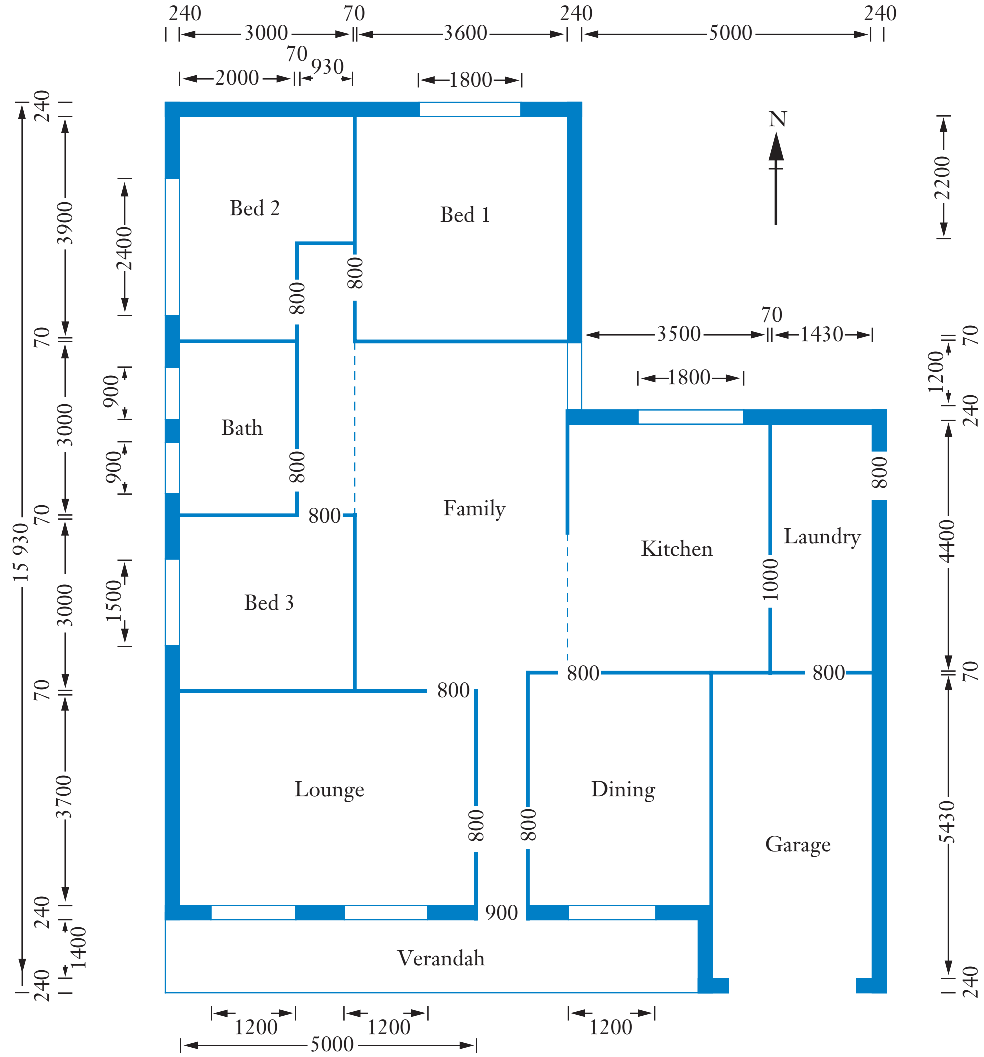
Example 3



Scale 1 : 100

Alamy Stock Photo/Fir Mamat

4 Judy and Keith are buying the house with this floor plan.



a What is the thickness of the external walls of this house in mm?

b What is the thickness of the internal walls?

c What is the front width of the house in metres?

d What is the length of the left side of the house?

e What are the dimensions of the family room?

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5 **a** All measurements on this plan are in metres. What are the dimensions of the family room? Express your answer in mm.

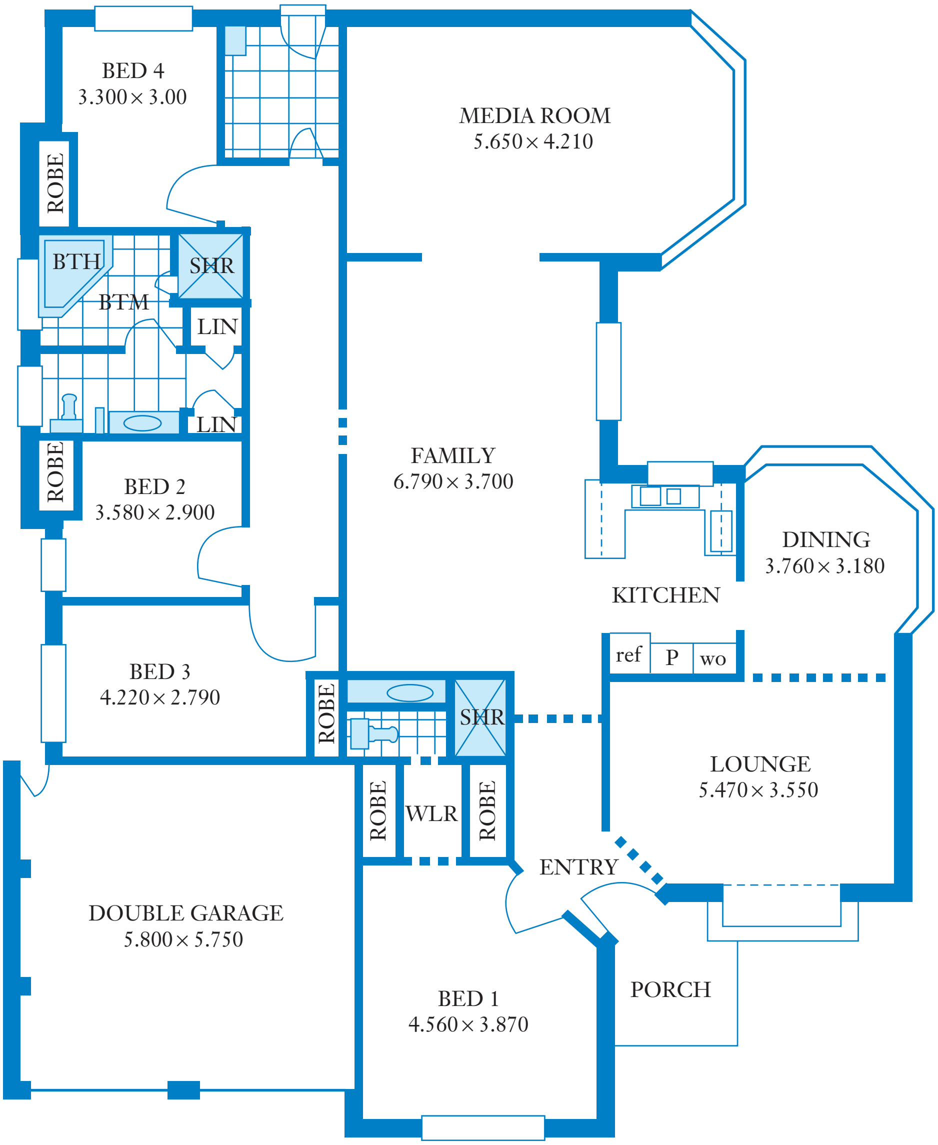
b How many toilets are there?

c Where are the 2 linen presses?

d What does 'SHR' stand for?

e Which bedroom has the smallest floor area?

f Which is bigger in area: the family room or the media room? By how many square metres?



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INVESTIGATION

MY HOME

There are lots of websites that contain house floor plans. Search for 'project homes' on the Internet and choose some floor plans for houses that you like.

For each plan:

- calculate the floor area of the whole house
- if possible, obtain an estimate for the cost of building the house
- calculate the cost per square metre

Shutterstock.com/Lev Kropotov

Compare and contrast the features of each house.

Decide which house you prefer based on the information you have found above and present your findings.

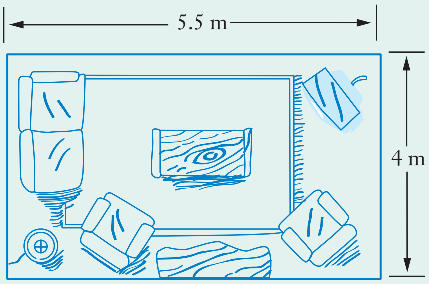
8.04 Renovate my house

We use house plans to calculate the materials required for a job and to estimate costs.

EXAMPLE 4

This is a diagram of Jackie's family room.

Jackie is laying cork tiles on the family room floor. The tiles cost $37.50 per square metre. How much will the cork tiles cost?



Solution

First find the area of the floor.

$$\text{Area} = 5.5 \times 4$$
$$= 22 \text{ m}^2$$

Multiply the area by the price.

$$\text{Cost} = 22 \times \$37.50$$
$$= \$825$$

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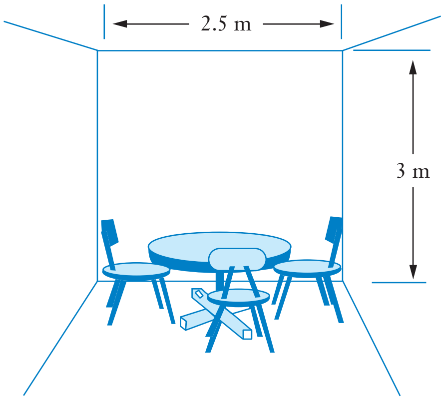
Exercise 8.04 Renovate my house

Example 4

1 The dining room in Nicole's home unit is very small. To make it look bigger, she plans to cover one wall with mirror tiles. The wall she plans to cover is 3 metres high and 2.5 metres wide.

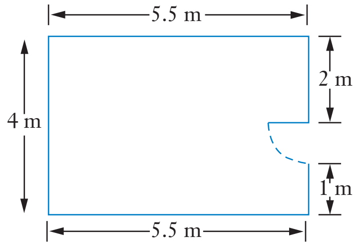


a What is the area of the wall?

b The mirror tiles Nicole has chosen cost \$36.50 per square metre. What will be the total cost of the mirror tiles for the wall in Nicole's dining room?

2 Luis is replacing the skirting boards in his lounge room.

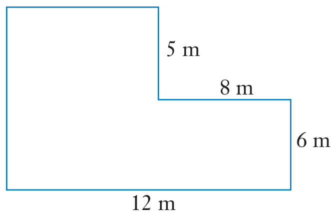
Skirting boards are the wooden boards that run around a room along the base of the walls.



a How many metres of skirting boards will he need?

b The skirting boards cost \$11 per metre. Calculate the total cost of the skirting boards for the job.

3 The floor area of Jesse's new house is shown.



a What is the floor area of this house?

b Using a cost of \$840 per square metre, calculate the amount Affordable Homes charges to build this house.

Use the floor plan for Grant's house on page 232 to answer questions **4** to **7**.

4 Grant is renovating Bedroom 2. The skirting boards need replacing.

a Calculate the perimeter of Bedroom 2.

b The door is 1 m wide. How many metres of skirting board are needed?

c Skirting boards cost approximately \$15 per metre. What will be the cost of the skirting boards?

5 Grant is also replacing the carpet in Bedroom 2.

a How many square metres of carpet will he need?

b Carpet comes in rolls that are 3.6 m wide. What length of carpet will Grant need, rounded up to the nearest metre?

c The carpet he likes costs \$79.45 a metre. How much will the carpet cost in total?

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6 **a** Grant also needs to paint the ceiling. What is the area of the ceiling?

b The ceiling requires 2 coats of paint. One litre of paint covers approximately 12 m^2 of the ceiling.

i Calculate the number of litres of paint that Grant needs.

ii The ceiling paint costs $34.90 for a 4-litre can. Calculate the cost of paint for the ceiling.

7 **a** In what order should Grant do the renovations from questions **4**, **5** and **6**? Give reasons for your answer.

b What is the approximate total cost of these renovations?

Use the floor plan for Judy and Keith's house on page 236 to answer questions **8** to **11**.

8 Judy and Keith have decided to replace all the floor coverings before they move into the house. They are going to lay wood parquet flooring in the lounge room. The flooring costs $68 per square metre. How much will the flooring cost?

9 Other rooms need new carpet. Judy and Keith have chosen a carpet that is 3.66 m wide. How many metres of carpet are required for:

a bedroom 1? **b** bedroom 2? **c** bedroom 3?

10 The kitchen and laundry are to have new tiled floors. The tiles cost $41.10 per square metre, allowing for breakage and wastage. How much will it cost to buy tiles for the floor? Answer correct to the nearest dollar.

11 Judy and Keith are expecting a baby and they are going to make bedroom 2 the baby's room. Judy wants to decorate the room with a frieze pattern around the walls. It will go above the door and the windows.

a How many metres long will the \frieze be?

b Each roll of frieze is 5 m long. How many rolls of frieze will they need to buy?

12 Judy and Keith were surprised by the estimated cost of their renovations. Before signing off on the purchase of this house, they checked out the cost of building a project home of similar style, with the same floor area as this house. The cost of the project home was approximately $594 per square metre. Calculate the estimated cost of building this project home.

Chapter problem

You've covered the skills required to solve the chapter problem. Can you solve it now?

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8.05 Constructing scale drawings

To make a scale drawing, start with a rough sketch, including the required measurements. Then choose a scale and draw the diagram accurately.

WS
Scale drawings

Lengths on scale drawings

To calculate a scaled length, divide the actual length by the scale.

EXAMPLE 5

This is a sketch of Farmer Freda's field.

a Construct a scale drawing of the field.

b What is the length of the unknown side of the field?

Solution

a Choose a suitable scale – we want it to fit easily on the page.

Use the scale: 1 cm represents 200 m.

Calculate the scaled length for each measurement by dividing by the scale.

$900 \div 200 = 4.5$ cm

$700 \div 200 = 3.5$ cm

$1200 \div 200 = 6$ cm

$350 \div 200 = 1.75$ cm

Use these measurements to draw your scale drawing.

b Measure the unknown length on the scale drawing and multiply by the scale.

Measured length = 2.3 cm

$2.3 \times 200 = 460$ m

The unknown side is 460 m.

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EXAMPLE 6

Sue is going on a bushwalk from her camping site. She walks 2.5 km due east and then 1.9 km due northwest.

a Make a scale drawing of Sue's walk.

b Use the drawing to determine how far Sue is from her campsite.

iStock.com/pixdeluxe

Solution

a Choose a suitable scale.

Use the scale: '1 cm represents 0.5 km'

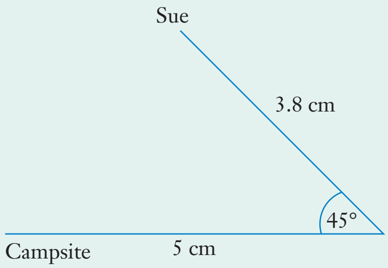
Find the scaled measurements by dividing by the scale.

$2.5 \div 0.5 = 5$ cm

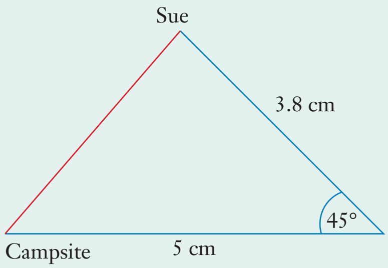
$1.9 \div 0.5 = 3.8$ cm

Use a ruler and a protractor to construct the scale drawing.

Northwest is at an angle of 45° (halfway) between west and north.



b Join the starting point to the finishing point and measure this line.



The line is 3.5 cm long.

Find the actual distance by multiplying by the scale.

$$\begin{aligned}\text{Actual distance} &= 3.5 \times 0.5\\ &= 1.75 \text{ km}\end{aligned}$$

Sue is 1.75 km from her campsite.

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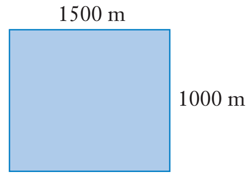
Exercise 8.05 Constructing scale drawings

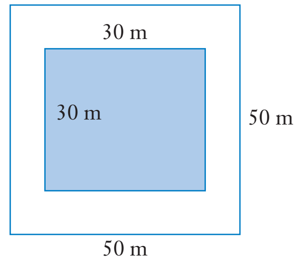
You will need a ruler and a protractor to complete this exercise.

1 Make a scale drawing of this field.

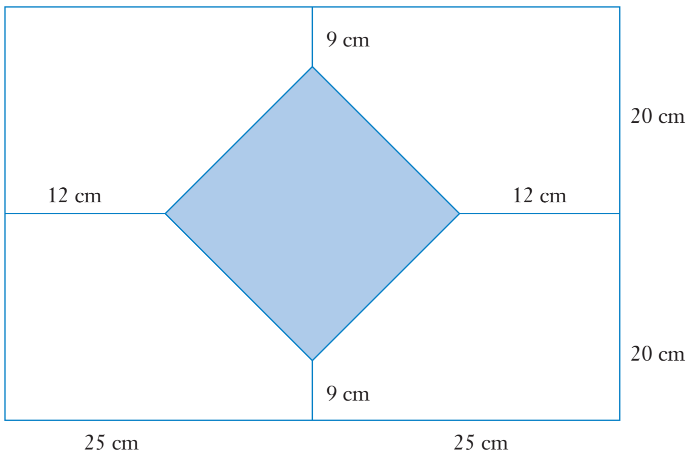


Example 5

2 Antje drew a diagram of a courtyard.
Make a scale drawing of the courtyard.



3 Samantha designed this cutting plate. Make a scale drawing of her cutting plate.



4 Hamish and Jacob are setting off on a hike. They walk 5 km due west of their starting point and then turn to walk 7 km south. They stop for lunch and then walk another 6 km northeast before stopping for afternoon tea.

Example 6

- **a** Make a scale drawing of Hamish and Jacob's walk.
- **b** How far are they from their starting point?

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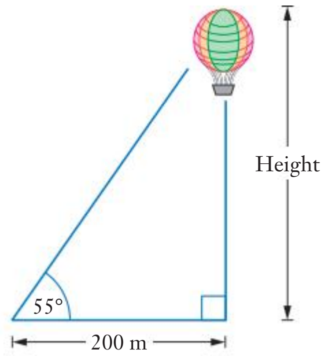
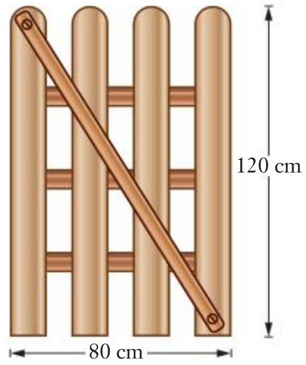
5 Construct a scale drawing to calculate the actual height of the hot air balloon above the ground.



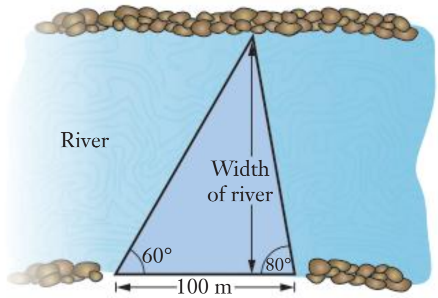
6 An old wooden gate 80 cm wide by 120 cm high needs a diagonal brace for support.

a Construct a scale drawing of the gate and find the actual length of the brace.

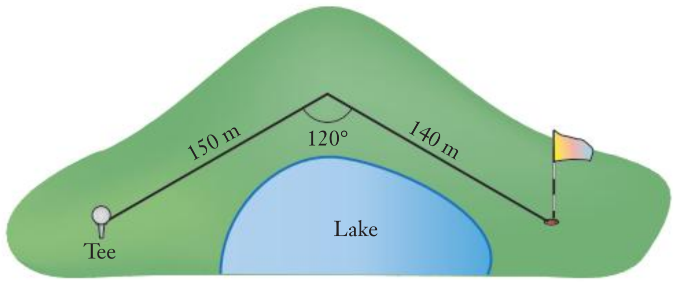
b Use Pythagoras' theorem to check the answer you obtained from the scale drawing. Answer correct to one decimal place.



7 Jodie wants to swim across the river. Draw this diagram to scale and calculate the width of the river.



8 A golf course has a large lake as an obstacle. Most golfers follow the dog leg around the lake. How far is it straight across the lake, from the tee to the hole? Use a scale drawing to answer this question.



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INVESTIGATION

MAKE MY OWN SCALE DRAWING

1 Choose something that is a large rectangle, for example, a paved area, a quadrangle, a brick wall, a whiteboard or the classroom floor.

2 Measure the length and width of the rectangle.

3 Make a scale drawing of your rectangle. Be sure to include the scale you used.

4 Measure the length and width of the school reception area or foyer.

5 There are plans to enlarge the length and width of this area by 50%. What will be the new dimensions of this area?

6 Draw a scale diagram of the enlarged area. Add the position of furniture and any other items in the school entrance area. Show the scale you used.

KEYWORD ACTIVITY

WORD MATCH

Match each word in the left column to its correct meaning in the right column.

	Word		Meaning
1	centimetre	**A**	A unit used to measure distances between towns
2	dimensions	**B**	Represents real objects that are too big to draw with the actual measurements
3	elevation	**C**	The relationship between an actual object and its diagram
4	enlarge	**D**	One-thousandth of a kilometre
5	house plan	**E**	To make a drawing or object larger
6	kilometre	**F**	Icons that illustrate features on a house plan
7	metre	**G**	One-hundredth of a metre
8	millimetre	**H**	A diagram showing rooms and measurements of a house
9	reduce	**I**	The unit of length used in house plans
10	scale	**J**	The length and width of a room are called its _________
11	scale drawing	**K**	The view of a building from the side
12	symbols	**L**	To make a drawing or object smaller

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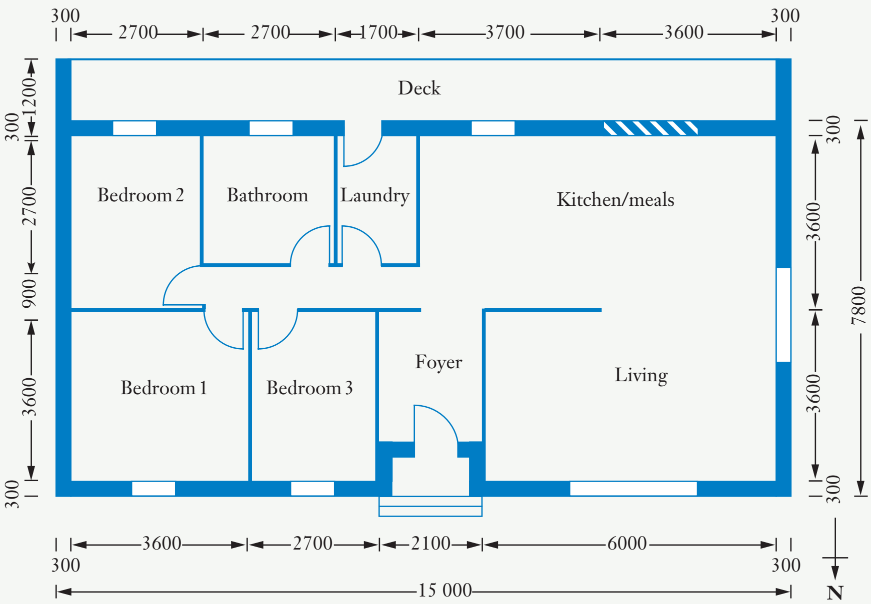
SOLUTION TO THE CHAPTER PROBLEM

Problem

Francesca and Michael are building a house according to the plan shown. They need to order carpet for the living room and the 3 bedrooms. How many square metres of carpet will they need? Answer to the nearest square metre.



Solution

First we need to read off the dimensions for each room.

Living room: 6000 mm × 3600 mm

Bedroom 1: 3600 mm × 3600 mm

Bedroom 2: 2700 mm × (2700 + 900) mm = 2700 mm × 3600 mm

Bedroom 3: 2700 mm × 3600 mm

As we want to know how many **square metres** of carpet are needed, we should change our measurements to metres by dividing by 1000 before we calculate the areas.

Area of living room $= 6 \text{ m} \times 3.6 \text{ m}$
$= 21.6 \text{ m}^2$

Area of bedroom 1 $= 3.6 \text{ m} \times 3.6 \text{ m}$
$= 12.96 \text{ m}^2$

Area of bedroom 2 $= 2.7 \text{ m} \times 3.6 \text{ m}$
$= 9.72 \text{ m}^2$

Area of bedroom 3 $= 2.7 \text{ m} \times 3.6 \text{ m}$
$= 9.72 \text{ m}^2$

Add the areas together: Total area $= 21.6 + 12.96 + 9.72 + 9.72$
$= 54 \text{ m}^2$

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8. TEST YOURSELF

From paper to reality

Practice quiz

Exercise 8.01

1 Measure the marked height of the house in the photograph below and work out the actual height of the house. Scale: 1 cm represents 2.5 m.

Shutterstock.com/karamysh

Exercise 8.01

2 On a tourist map of Perth, the scale is given by:

a Write this scale as a simplified ratio.

b Find the actual distance between the following places given the scaled distance.
- **i** Parliament House to Perth Arena (15.2 cm)
- **ii** Perth Station to the State Library (4.6 cm)

c Find the scaled distance between the following places given the actual distance:
- **i** Stirling Gardens to Barrack Street Jetty (380 m)
- **ii** St Marys Cathedral to the Supreme Court Gardens (725 m)

d A section of the Graham Farmer Freeway on the map measures 22.6 cm. How long is this section?

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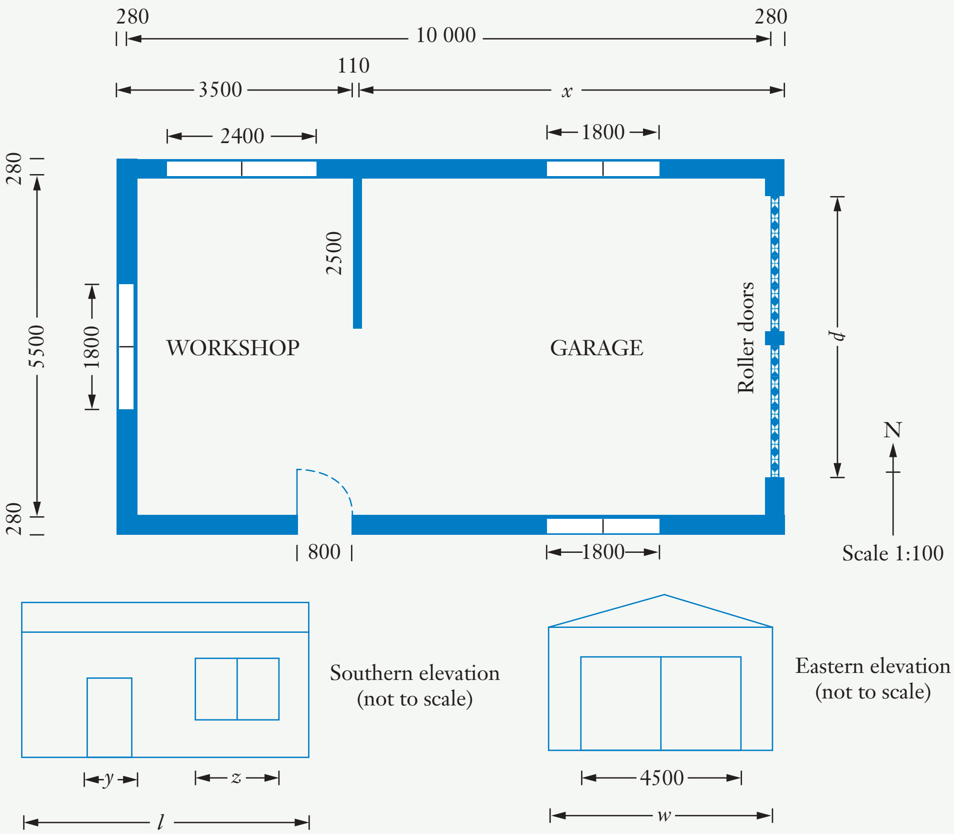
Exercise 8.02

3 **a** Draw elevation views of the northern and western sides of this building.

You do not need to draw them to scale, but use a ruler and be as accurate as possible.



b What is the length of x on the plan?

c Find the lengths of y, z, l, w and d.

Exercise 8.03

4 Use the plan in Question **3** above to answer the following questions.

a What is the width of the external walls of the building?

b What is the width of the wall between the workshop and the garage?

c Find the internal width of the building.

d What are the dimensions of the workshop?

e What are the dimensions of the garage?

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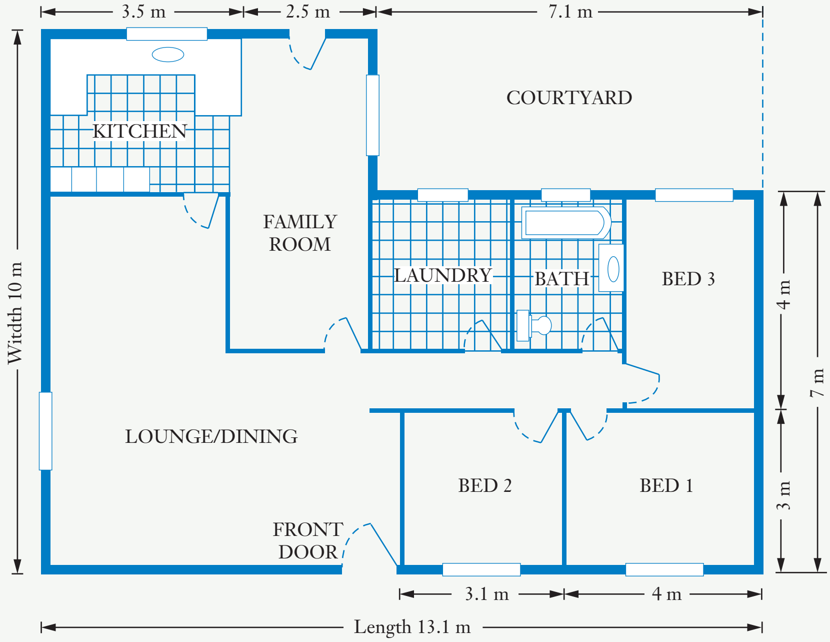
Exercise 8.03

Exercise 8.04

5 This is the plan for the house that Sam and Nic are buying.



a How many doors are shown on this plan?

b What are the dimensions of the house?

c Calculate the width of the kitchen.

d **i** Find the floor area of the house.

ii An estimate for the cost of building a house is $672/m^2. Estimate the cost of building this house, correct to the nearest dollar.

e What percentage (correct to one decimal place) of the house's floor area is taken up by bedroom 1?

f A builder is going to tile the courtyard. What is the area of the courtyard?

g To allow for cutting and fitting, a builder always buys 10% more tiles than required to cover a floor area.

i How many square metres of tiles should the builder buy for the courtyard, including the extra 10%?

ii If each tile measures 32 cm by 32 cm, how many tiles are needed in total?

h The guttering across the front of the house needs replacing.

i How much guttering is needed?

ii The guttering company charges a fee of $50 plus $24.75 per metre to supply and install the guttering. How much will they charge to gutter the front of the house?

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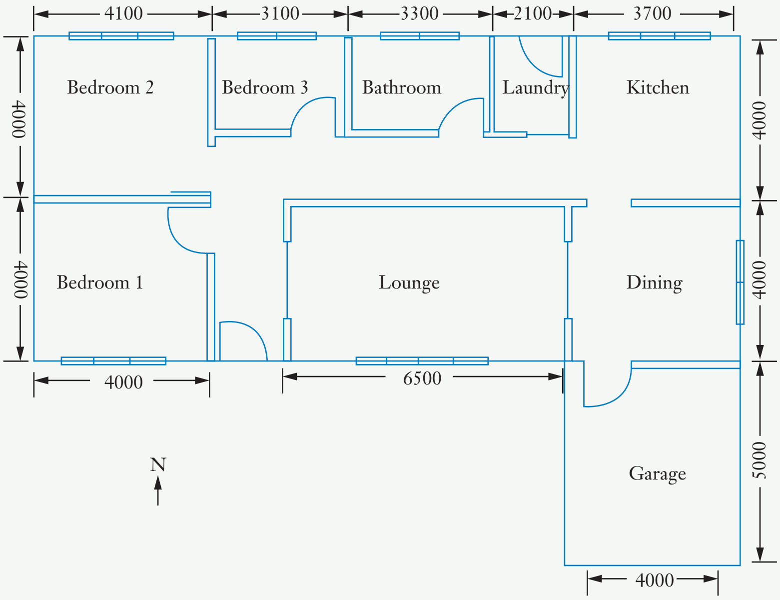
Exercise 8.04

6 This is the floor plan for Harry and Meghan's house. They have decided to renovate some areas of the house.



a Harry and Meghan are going to lay wood parquet flooring in the lounge room. The flooring costs \$68 per square metre. How much will the flooring cost?

b They are going to replace the skirting boards in Bedroom 1. Assume the door is 820 mm wide.

  i How many metres of skirting board are needed?

  ii Skirting boards cost approximately \$15 per metre. What will be the cost of the skirting boards?

c The tiles on the floor of the bathroom and laundry need replacing. The width of these rooms is 2300 mm.

  i Calculate the total floor area of the bathroom and laundry.

  ii Each tile is 200 mm by 200 mm. How many tiles are needed?

  iii The tiles cost \$41.10 per square metre, allowing for wastage. How much will the new tiles cost?

d The ceiling of the lounge room needs to be repainted.

  i What is the area of the ceiling?

  ii The ceiling requires 2 coats of paint. One litre of paint covers approximately 12 m^2. How many litres of paint will be required to do the ceiling?

  iii Ceiling paint costs \$34.90 for a 4-litre can. Estimate the cost of painting the ceiling.

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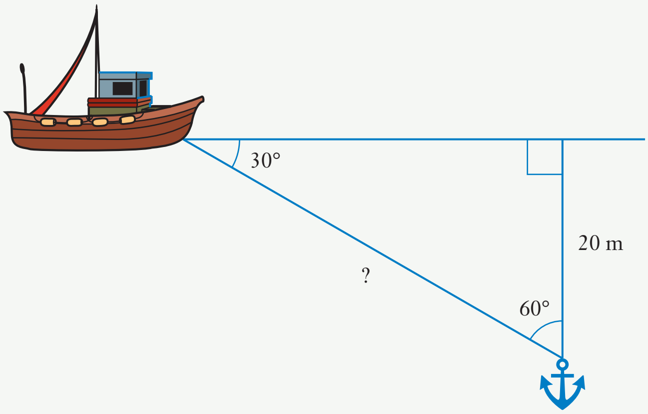
Exercise 8.05

7 Neil and Ted are keen scuba divers. When they moor their boat, the angle between the anchor rope and the top of the water is 30°. Construct a scale drawing to calculate what length anchor rope they need if the water is 20 m deep.



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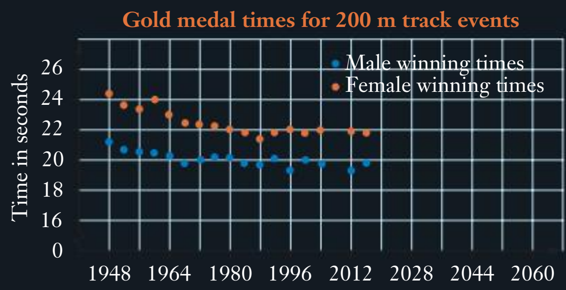
BIVARIATE GRAPHS

9.

FITTING THE DATA

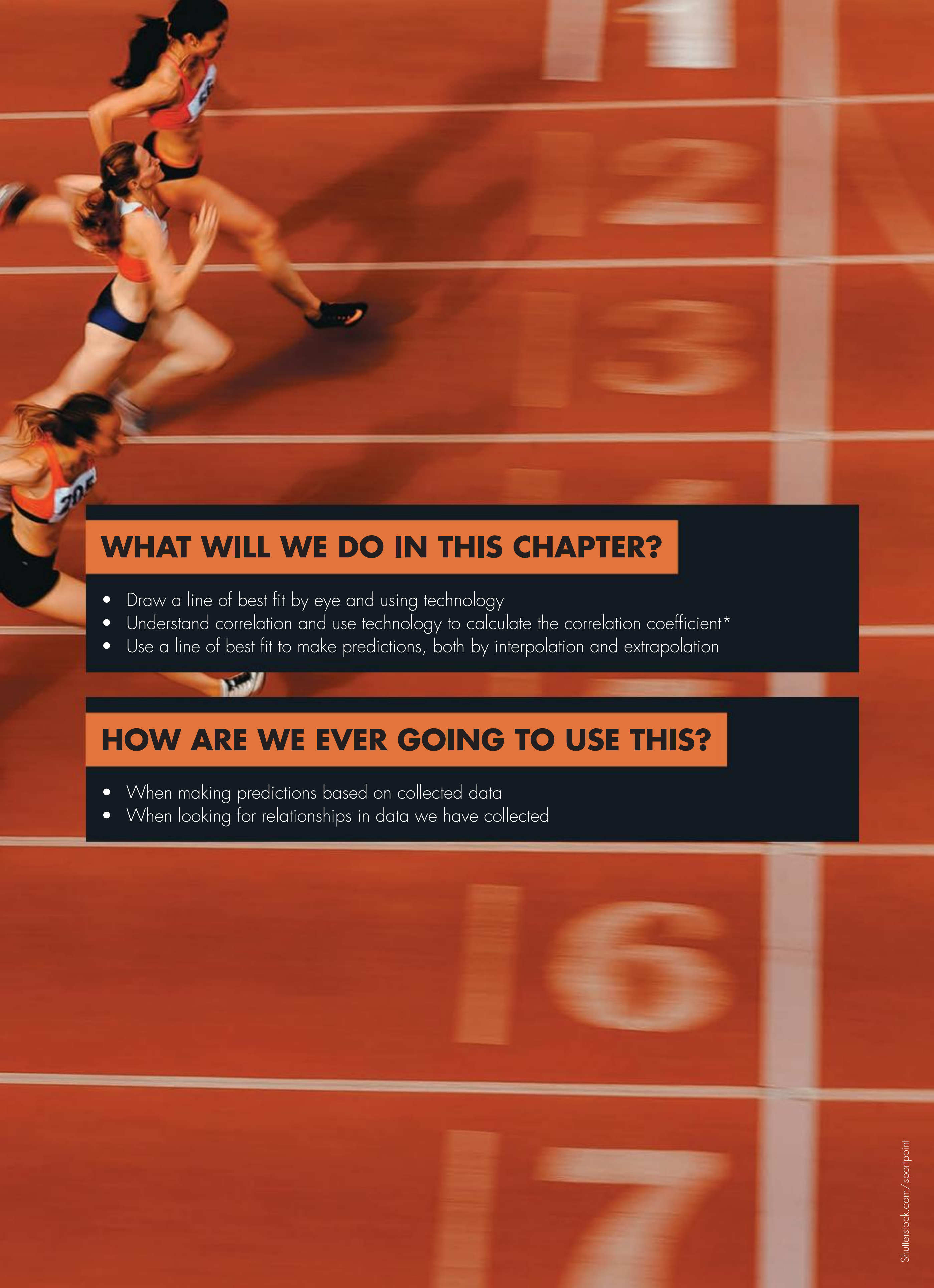
Chapter problem

Kaylene is a competitive runner. She studies the past winning times for the women's Olympic 200 m race, and is surprised by the rate at which the times have improved. The women's winning times are slower than those of the men in their 200 m event, but the women seem to be catching up. She wondered whether women would ever run the event in the same or faster time than the men.



This graph shows the gold medal times for men and women's 200 m track events at the Olympic Games since 1948.

Will the women ever run the 200 m race faster than the men?



WHAT WILL WE DO IN THIS CHAPTER?

- Draw a line of best fit by eye and using technology
- Understand correlation and use technology to calculate the correlation coefficient*
- Use a line of best fit to make predictions, both by interpolation and extrapolation

HOW ARE WE EVER GOING TO USE THIS?

- When making predictions based on collected data
- When looking for relationships in data we have collected

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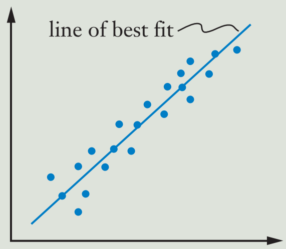
9.01 Drawing a line of best fit

In Chapter 6, *Scattering the data*, we drew scatterplots for bivariate data. If the data shows a strong linear association, we can approximate the linear relationship by drawing a **line of best fit** through the points.

A line of best fit:

- Represents most or all of the points as closely as possible
- Goes through as many points as possible
- Has roughly the same number of points above and below it
- Is drawn so that the distances of points from the line are as small as possible.



EXAMPLE 1

This table from Exercise 6.01, question **1** on page 157 shows the heights of a sample of girls when they were $2\frac{1}{2}$ years old and when they were 18 years old.

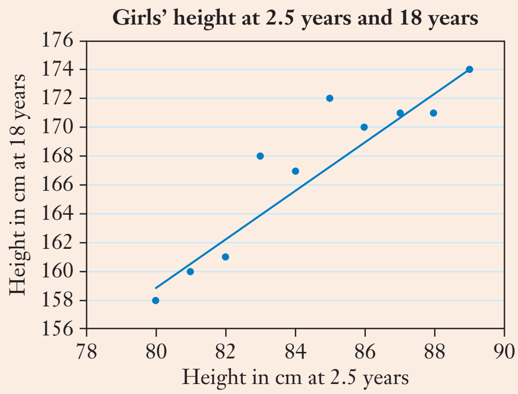
Height at $2\frac{1}{2}$ years (cm)	87	83	88	84	82	81	89	86	85	80
Height at 18 years (cm)	171	168	171	167	161	160	174	170	172	158

Graph this bivariate data on a scatterplot and draw a line of best fit for it.

Solution

Graph the data first.

Then draw a line through the middle of the points with roughly the same number of points above and below the line.



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We can find the equation of the line of best fit using technology.

EXAMPLE 2

For the height data in Example **1**, find the equation of the line of best fit using:

a a scientific calculator

b a spreadsheet.

Solution

a Enter this table of values into the calculator by following the instructions below.

Height at $2\frac{1}{2}$ years (cm)	87	83	88	84	82	81	89	86	85	80
Height at 18 years (cm)	171	168	171	167	161	160	174	170	172	158

Operation	Casio scientific	Sharp scientific
Start statistics mode.	MODE STAT A+BX	MODE STAT LINE
Clear the statistical memory.	SHIFT 1 Edit, Del-A	2ndF DEL
Enter data.	SHIFT 1 Data to get table Enter in X column 87 = 83 =, etc. Enter in Y column 171 = 168 =, etc. AC to leave table	87 2ndF STO 83 M+ 171 2ndF STO 168 M+ etc.
On these calculators, the gradient is b and the y-intercept is a (b = 1.709 ..., a = 22.781 …)	SHIFT 1 Reg b = SHIFT 1 Reg a =	ALPHA b = ALPHA a =

Another name for line of best fit is ‘regression line’, and ‘Reg’ on the calculator stands for regression. The calculator expresses the equation of the line in the form $y = bx + a$.

So the line of best fit using the calculator has equation $y = 1.709x + 22.781$.

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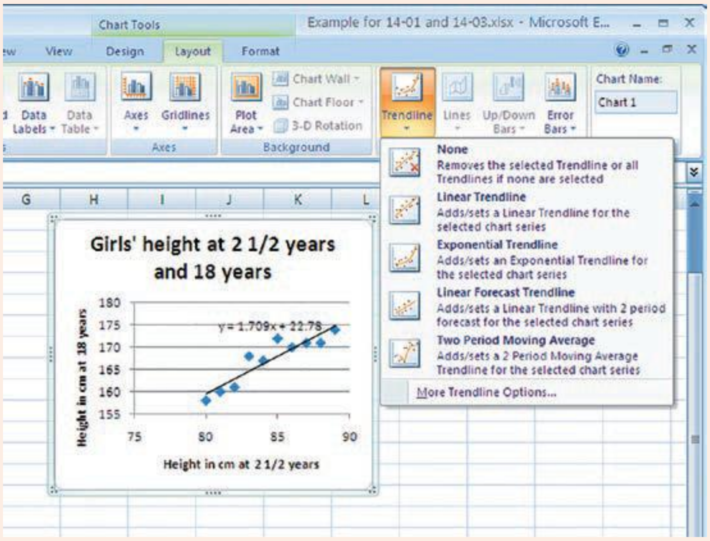
b Enter the table of values into a spreadsheet, then follow these instructions:

1. Graph the data on a scatterplot.
2. Select the graph and select **Trendline** from the **Layout** menu.



3. Select **Linear Trendline** and the line will appear on the graph.
4. Select **More Trendline Options** from the **Trendline** menu, then select **Display equation on the chart**.

The spreadsheet equation for the line of best fit is $y = 1.709x + 22.78$.

Note: The calculator and spreadsheet use a formula to calculate the line of best fit, but when we draw a line of fit by hand, we have to position the line 'by eye'. It's unlikely that we will position the line in exactly the same place that the technology does. This means that our line will be a little different from the line drawn using technology.

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Exercise 9.01 Drawing a line of best fit

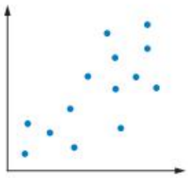
Keep your answers to this set of exercises. You will be using them again in Exercise 9.03.

1 Copy or print each scatterplot and draw a line of best fit. You can print a copy of the graphs from the worksheet 'Scatterplots' from NelsonNet.

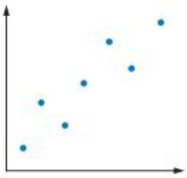
Example 1

WS Scatterplots

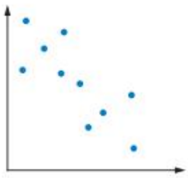
a



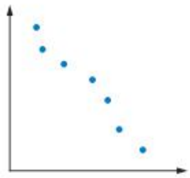
b



c



d



2 This table shows the birth rate and female life expectancy for a number of countries. The birth rate is the number of births per year per 1000 of the population. Life expectancy is measured in years.

a Draw a scatterplot of birth rate against life expectancy.

b Draw in a line of best fit for this data.

Country	Birth rate	Female life expectancy
Australia	13.3	85.5
Brazil	14.7	79.8
Canada	11.2	84.6
Fiji	21.4	76
Germany	8.1	83.5
Iraq	31.0	72
Kenya	36.1	61
Laos	28.0	69.5
Nepal	24.3	70
Niger	50.0	57
Rwanda	42.1	61

3 This table shows the height above sea level and the average annual rainfall for some places in Australia.

City	Height above sea level (m)	Mean annual rainfall (mm)
Alice Springs	581	282
Ballarat	432	694
Hobart	24	616
Kalgoorlie	387	266
Mount Isa	365	463
Norfolk Island	73	1017
Perth	15	736
Winton	188	382

a Draw a scatterplot of height above sea level against mean annual rainfall.

b Draw a line of best fit for this data.

4 Use technology to find the equation of the line of best fit for the birth rate and life expectancy data from question **2**.

Example 2

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5 Use technology to find the equation of the line of best fit for the height and rainfall data from question **3**.


Fairfaxphotos/Wayne Taylor

6 This table shows information about the amounts of energy, carbohydrate and fat contained in 100 g of some takeaway foods.

Food	Energy (kilojoules)	Carbohydrate (grams)	Fat (grams)
Hamburger	1030	26.6	9.1
Cheeseburger	1070	23.9	11.2
Chicken burger	921	20.2	10.1
Fish burger	988	24.5	10.2
Grilled chicken wrap	771	13.4	10.2
Chicken salad	325	5.5	3.4
Egg and bacon wrap	767	15.3	9.1
Hash browns	1150	26.4	17.2
Chips	1480	39.7	19.0

a Draw a scatterplot of energy against carbohydrate.

b Draw a line of best fit for this data.

c Use technology to find the equation of the line of best fit.

d Draw a scatterplot of energy against fat and a line of best fit for this data.

e Use technology to find the equation of the line of best fit.

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9.02 Using a line of best fit

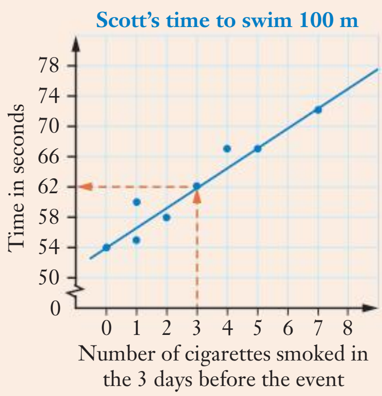
WS Lines of fit

Line of best fit

Lines of best fit show the association between variables and we can use them to make predictions within the range of the data. This is called **interpolation**.

EXAMPLE 3

Scott is a competitive swimmer. He noticed that his times for the 100 m freestyle event are slower when he smokes cigarettes in the days before the event. He displayed his times for his last 8 events and the numbers of cigarettes he smoked in the 3 days before on a scatterplot. He drew a line of best fit through the data.



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a What is the independent variable?

b What is the dependent variable?

c What does the line of best fit show about the relationship between these 2 variables?

d Use the line of best fit to predict Scott's time for the 100 m freestyle when he has smoked 3 cigarettes in the 3 days before the event.

Solution

a Independent variable is on the horizontal axis. — The independent variable is the number of cigarettes smoked.

b Dependent variable is on the vertical axis. — The dependent variable is the swimming time.

c Describe the relationship. — As the number of cigarettes smoked increases, Scott's time for the 100 m increases.

d Find 3 on the horizontal axis, go up to the line and then across to the time. — Scott's time will be approximately 62 seconds.

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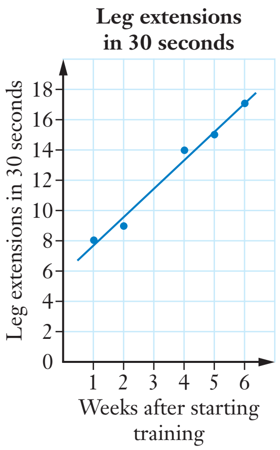
Exercise 9.02 Using the line of best fit

Example 3

1 Anita is trying to strengthen her quadriceps muscles. This graph shows the number of leg extensions she was able to complete in 30 seconds each week after she started training.



Shutterstock.com/antonio diaz

a What is the independent variable?

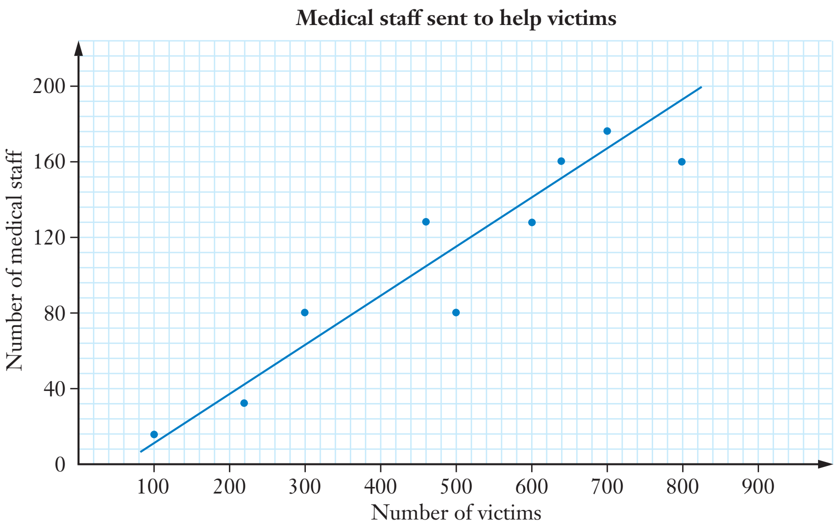
b What is the dependent variable?

c What does the line of best fit show about the relationship between these 2 variables?

d Anita forgot to record the number of leg extensions she could do in week 3. Use the line of best fit to predict this number.

2 Humanitarian agencies send people to assist when a natural disaster occurs.

This scatterplot shows the number of medical staff one small agency sent to assist after earthquakes and floods occurred in different parts of the world, and the estimated number of victims in each disaster.



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a What is the independent variable?

b What is the dependent variable?

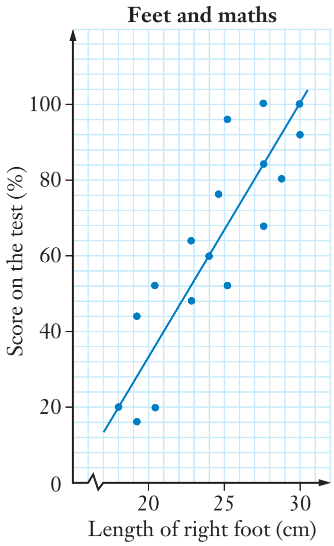
c Use the line of best fit to estimate the number of medical staff the agency will send to a disaster with an estimated 200 victims.

d The agency sent 180 medical staff to assist in a disaster. Approximately how many victims were affected by the disaster?

e Describe the relationship between the number of medical staff and the number of victims.

f Kyle noted that the greater the number of medical staff, the more victims there were requiring help. He argued that if the agency decreased the number of medical staff, there would be less people needing help. What is wrong with Kyle's argument?

3 Billy announced, 'People with big feet are better at maths than people with small feet'. He had given a maths test to a large sample of students in the school library and measured the length of each person's right foot. He displayed the results on this scatterplot.



a According to Billy's line of best fit, what is the right foot length of a person who scored 58 on the maths test?

b Describe the relationship between the length of the right foot and the score on the test as shown by this line of best fit.

c Big feet don't cause high maths scores. How can Billy's results be explained?

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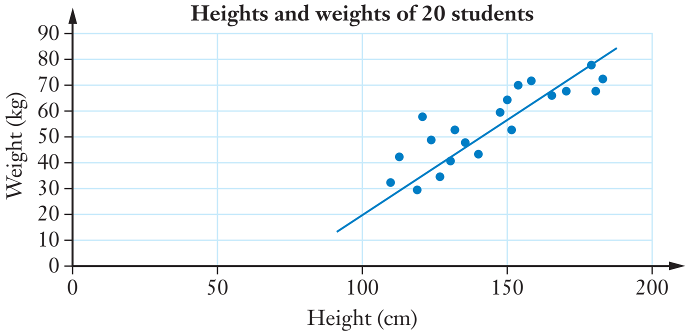
4 This graph shows the scatterplot for the heights and weights of 20 students with a line of best fit drawn on it.

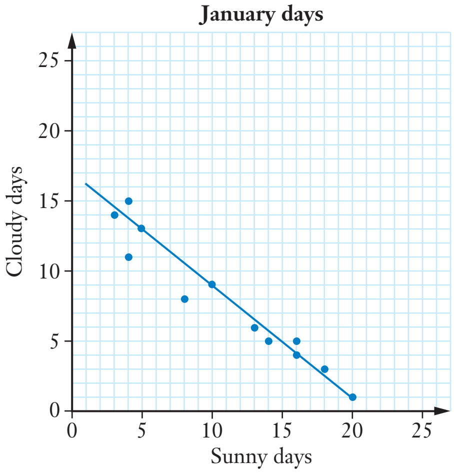


a Describe the relationship between the height and weight of students as shown by this line of best fit.

b Use the line of best fit to estimate the weight of a student who is 150 cm tall.

c Does an increase in height *cause* an increase in weight? Why or why not?

5 Yasmina graphed the number of sunny days in January against the number of dry cloudy days in January for 12 different Australian places. She then drew a line of best fit on her scatterplot.



a Use the line of best fit to estimate how many dry cloudy days there would be if there were 7 sunny days.

b Describe the relationship between the number of sunny days and the number of cloudy days in January.

c Suggest a reason for this relationship.

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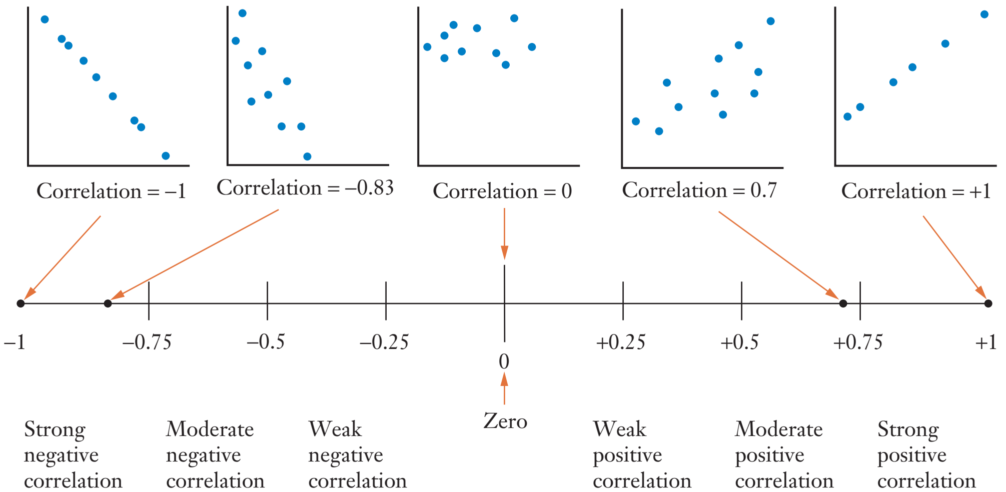
9.03 Correlation*

*Australian curriculum only, not in WA syllabus

We learned about the **association** between 2 variables in Chapter 6, *Scattering the data*. The association can be positive or negative, linear or non-linear, strong, moderate or weak.

Correlation is a measure of how strongly 2 variables are related to each other.

We can measure the strength of the relationship by calculating a value called the **correlation coefficient**. The correlation coefficient can have values from –1 to 1.



- The more the data points lie in a straight line and the stronger the association, the closer the correlation is to 1 or –1.
- The more spread out the data and the weaker the association, the closer the correlation is to 0.
- A positive association gives a positive value for the correlation coefficient.
- A negative association gives a negative value for the correlation coefficient.

You can calculate the correlation coefficient using an online calculator, scientific calculator or spreadsheet. As for standard deviation, there is a complex formula for calculating the correlation coefficient, but you are not required to use it.

EXAMPLE 4

This table shows the number of stuffed toys owned by a group of children of different ages. Calculate the correlation coefficient for the data.

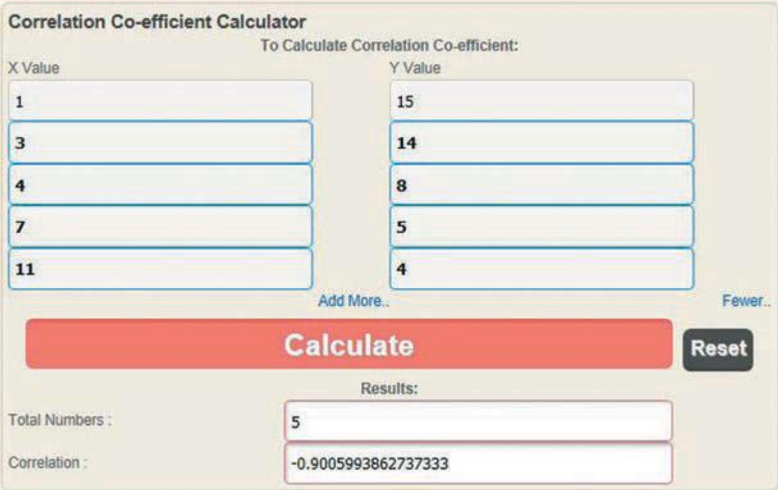
Age	1	3	4	7	11
Number of stuffed toys owned	15	14	8	5	4

Solution

Method 1: Online calculator

Easy calculation

1 Search the Internet for ‘correlation coefficient calculator’ or find it on the Easy Calculation website.



Source: easycalculation.com

2 Enter each pair of values, then click Calculate to display the correlation coefficient as –0.9005...

Method 2: Scientific calculator

Operation	Casio scientific	Sharp scientific
Start statistics mode.	MODE STAT A+BX	MODE STAT LINE
Clear the statistical memory.	SHIFT 1 Edit, Del-A	2ndF DEL
Enter data.	SHIFT 1 Data to get table Enter in X column 1 = 3 =, etc. Enter in Y column 15 = 14 =, etc. AC to leave table	1 2ndF STO 3 M+ 15 2ndF STO 14 M+ etc.
Calculate the correlation coefficient (r = –0.9005 …)	SHIFT 1 Reg r =	ALPHA r =

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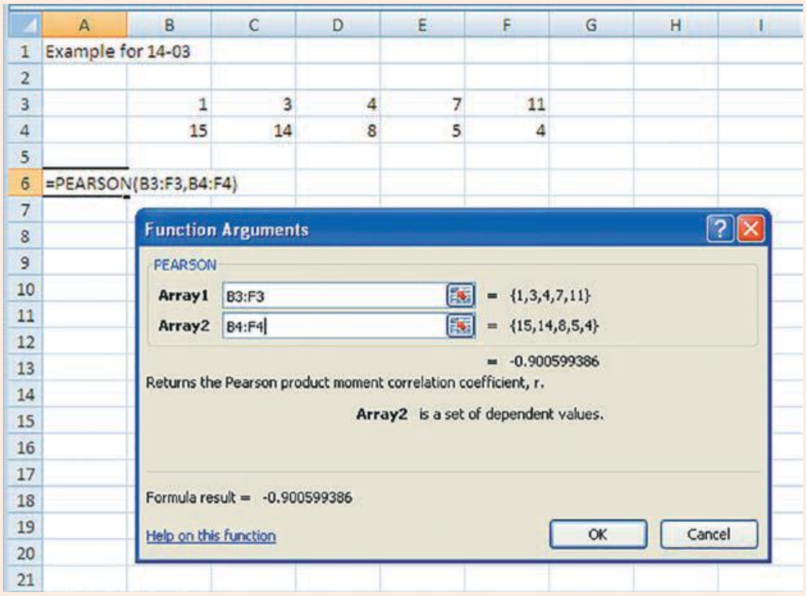
Method 3: Spreadsheet

1 Type the data into the spreadsheet. The example shows the first row of data in cells B3 to F3 and the second set in B4 to F4.



2 Select the cell where you want to display the correlation coefficient, and select Insert function from the tool bar. In the Search for a function box, type PEARSON, then let the spreadsheet help you complete the formula to calculate the correlation coefficient –0.9005…

The correlation coefficient is also called the Pearson correlation coefficient, which is why the spreadsheet calls the formula PEARSON.

EXAMPLE 5

A sample of 10 people had their left foot length and left lower arm length (from their elbow to the tip of the middle finger) measured.

Left foot length (cm)	27	22	23	24	25	25.5	28	26.5	28	29
Left lower arm length (cm)	48.5	43	44.5	43.5	45	46	47	48	49	50

a Construct a scatterplot of this data.

b Calculate, correct to 2 decimal places, the correlation coefficient and describe the correlation in words.

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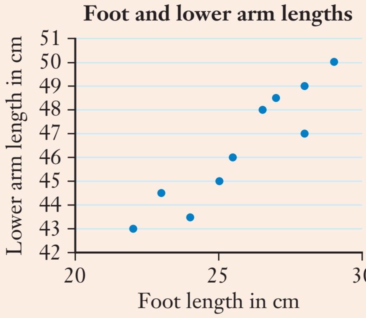
Solution

a Foot length is on the horizontal axis and arm length is on the vertical axis.



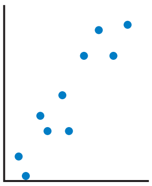
b Calculate the correlation coefficient using one of the three methods shown in Example 4.

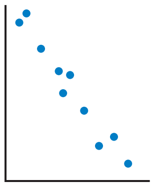
The correlation coefficient is 0.93.

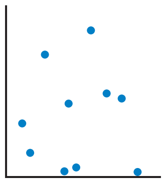
The data shows a strong, positive linear relationship between the length of the left foot and the length of the lower left arm.

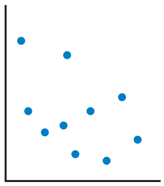
Exercise 9.03 Correlation*

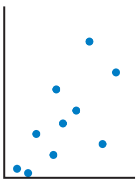
1 Match each scatterplot to its correlation coefficient.

a


b


c


d


e


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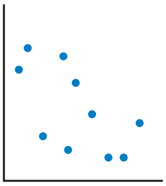
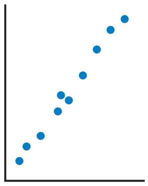
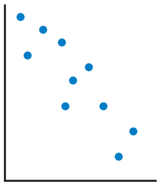


g


h


Correlation coefficients

A	–0.623	**B**	0.008	**C**	–0.977	**D**	–0.870
E	0.923	**F**	0.995	**G**	–0.441	**H**	0.681

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2 Eliza owns a cafe. She thinks there is a relationship between the daily temperature and hot chocolate drink sales.

Example 4

Temperature (°C)	8	36	12	28	14	22	16	10	24	18	7	31
Hot chocolate drinks sold	42	15	37	24	9	20	35	20	10	30	30	4

a Construct a scatterplot to illustrate Eliza's information.

b Calculate, correct to 2 decimal places, the correlation between temperature and the number of hot chocolate drinks sold.

c Is Eliza correct in thinking there is a relationship between temperature and high chocolate sales? Justify your answer from your answers to parts **a** and **b**.

d Is this a causal relationship? Does a change in one variable cause the change in the other variable?

3 Calculate the correlation coefficient for the following data in Exercise 9.01 on pages 257–258:

a Question **2** between birth rate and female life expectancy

b Question **3** between height above sea level and mean annual rainfall

c Question **6** between energy and carbohydrate content

d Question **6** between energy and fat content

e Question **6** between carbohydrate content and fat content

4 Vilas wanted to investigate whether the latitude of a city is related to its temperature.

Example 5

City	Latitude (°S)	Mean January temperature (°C)
Adelaide	35	30
Alice Springs	23	40
Brisbane	27	31
Darwin	12	33
Hobart	43	24
Melbourne	38	27
Norfolk Island	29	25
Perth	32	32
Townsville	19	33

a Draw a scatterplot showing Vilas' data.

b Calculate the correlation coefficient for this data.

c Do you think these 2 variables are closely related? Why or why not?

d What other factors would influence the mean January temperature in a city?

5 In Exercise 6.01, question **5** on page 157 looked at the sales of CD and digital albums over 9 years.

a Calculate the correlation coefficient for this data.

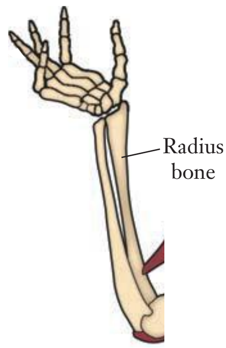
b What does this tell you about the relationship between these 2 variables?

c Is this a causal relationship – does a change in one variable cause the change in the other variable?

6 Forensic scientists use formulas to predict a person's height from the length of the radius bone in the lower arm. All measurements are in centimetres.

Male height = $6.650r + 80.405$

Female height = $3.876r + 73.502$



a Copy and complete this table after measuring the radius bone of the members in your group.

Name of person							
Length of radius bone (cm)							
Predicted height using the formula (cm)							
Measured height (cm)							

b Construct separate scatterplots for males and females, comparing predicted heights to measured heights.

c Calculate the correlation coefficient.

d Describe the correlation.

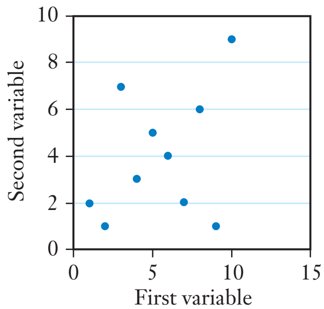
e Suggest a reason the predicted heights may not be very accurate.

INVESTIGATION

CHANGING CORRELATIONS

Changing correlations

For this table of values and scatterplot, the correlation coefficient for the variables is 0.366. Download the spreadsheet 'Changing correlations' from NelsonNet.



First variable	1	2	3	4	5	6	7	8	9	10
Second variable	2	1	7	3	5	4	2	6	1	9

The spreadsheet contains the data in the above table and calculates the correlation coefficient. Change the values in the Second Variable (light blue) row to produce a set of data whose correlation coefficient is:

a 0.9 **b** –0.3 **c** close to 0.

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9.04 Interpolation and extrapolation

A line of best fit helps us to make **predictions** as we did in some of the questions in Exercise 9.02. If we make a prediction on information that lies *within* the data, this is called **interpolation**. If we make a prediction on information that lies *outside* the data, this is called **extrapolation**.

However, we need to be careful when we are making predictions from sets of data.

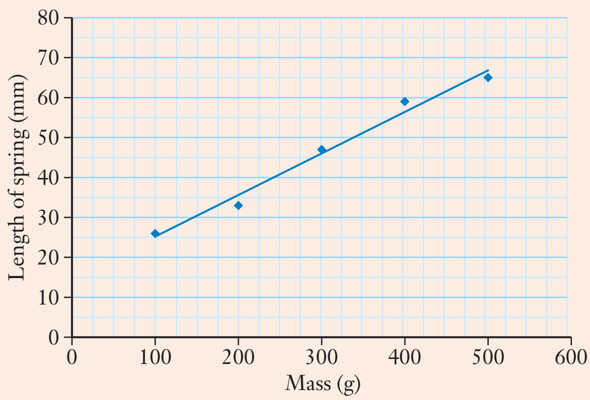
- Predictions within the data can be unreliable if the association is weak. Predictions are more reliable if the association is strong and linear.
- Predictions that go outside the data can be very unreliable because we can't be sure that the linear relationship continues beyond the data.

EXAMPLE 6

In a science experiment, Clare and Dilani attached different weights to a spring. They recorded the length of the spring for each weight.

Mass (g)	100	200	300	400	500
Length of spring (mm)	26	33	47	59	65

They graphed the results on the scatterplot below and drew a line of best fit.



a Use the line to find:

  i the length of the spring for a mass of 250 g

  ii the mass when the length of the spring is 30 mm

b How reliable are these predictions?

c The equation of the line is $L = 0.104m + 14.8$. Use the equation to predict the length of the spring when the attached mass is 800 g.

d How reliable is this prediction?

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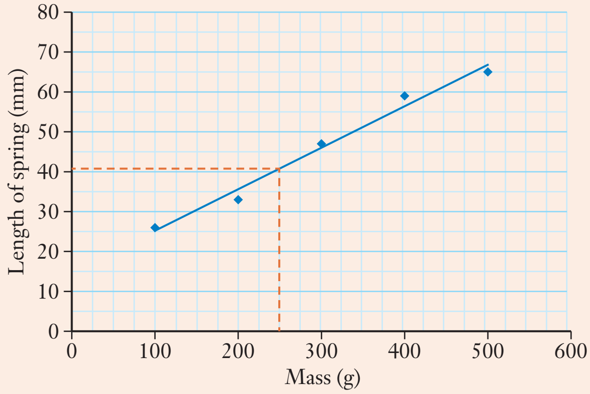
Solution

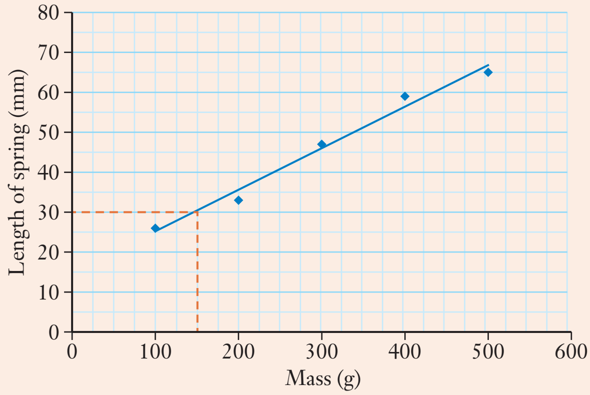
a **i** Find 250 g on the horizontal axis, go up to the graph and across to length.

The length of the spring is approximately 41 mm.



ii Find 30 mm on the vertical axis, go across to the line and down to mass.

The mass is approximately 150 g.



b These predictions are within the given data.

These predictions are reasonably reliable as the association is strong and linear.

c Substitute m = 800 into the formula

$$L = 0.104m + 14.8$$
$$= 0.104 \times 800 + 14.8$$
$$= 98$$

The length of the spring with 800 g attached is approximately 98 mm.

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d A mass of 800 g is outside the data shown on the graph, so it is an example of **extrapolation**.

The prediction is not necessarily reliable as we don't know if the relationship continues to be linear. It is also possible that at some point the spring might break.

Strength of the correlation*

*Australian curriculum only, not in WA syllabus

The strength of the correlation also indicates whether predictions on the data will be reliable. The closer the coefficient is to –1 or 1, the more reliable the prediction will be.

EXAMPLE 7*

This student data comes from Chapter 6, *Scattering the data*, Example **1** on page 156.

Total height, H cm	175	177	178	184	162	172	173	191	163	175
Waist height, W cm	101	106	107	115	100	87	106	120	90	108

a Calculate the correlation coefficient for this data.

b If we used this data to predict the waist height of a person 170 cm tall, how reliable would it be?

c If we used this data to predict the waist height of a person 130 cm tall, how reliable would it be?

Solution

a Calculate the correlation coefficient using technology as shown on page 264.

$r = 0.81$

b The heights in the table range from 162 to 191 cm, so 170 cm is within the data range: this is interpolation.

As the correlation coefficient is quite close to 1 and 170 cm is within the data range, this prediction would be reasonably reliable.

c 130 cm is outside this data range: this is extrapolation.

The prediction may not be reliable as 130 cm is well outside the given data.

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Exercise 9.04 Interpolation and extrapolation

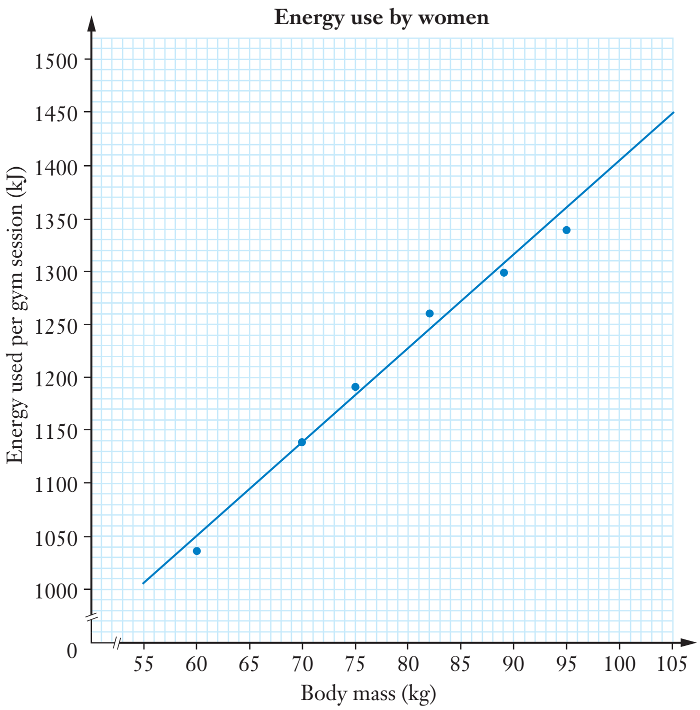
This exercise requires some of your answers from Exercise 9.01.

Example 6

1 Jasmine recorded the body mass of a sample of women at the gym and the amount of energy they use per day.

Body mass (kg)	60	70	75	82	89	95
Energy used (kJ)	1040	1140	1190	1260	1300	1340

She graphed the data on a scatterplot and drew a line of best fit.



a Use the line of best fit to find:

i the amount of energy used by a woman of mass 65 kg

ii the body mass of a woman who uses 1450 kJ of energy per day

b How reliable are these predictions? Justify your answer.

c The equation of the line is $E = 16.9B - 143$, where B is the body mass and E is the energy used. Use it to predict the energy used by a woman of mass 120 kg.

d Would you expect this situation to continue to be a linear relationship beyond the data points? Why or why not?

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2 In Exercise 9.01, question **2**, you drew a scatterplot on the birth rate (B) and female life expectancy (E) in a number of countries. A line of best fit for this data has equation $E = -0.725B + 91.2$.

a Predict the female life expectancy in a country where the birth rate is 40.

b Determine the birth rate in a country where female life expectancy is 65 years.

c How reliable are these predictions? Justify your answer.

d What would be the female life expectancy in a country where the birth rate is 60?

e How reliable do you think estimates outside this data would be? Justify your answer.

3 In Exercise 9.01, question **3**, you drew a scatterplot for the height above sea level (H) and the mean annual rainfall (R) for a number of cities. A line of best fit for this data has equation $R = -0.769H + 755.5$.

a Predict the mean annual rainfall for a city 300 m above sea level.

b Determine the height above sea level for a city that receives 400 mm of rain per year.

c How reliable are these predictions? Justify your answer.

d Calculate the mean annual rainfall for a city 800 m above sea level.

e Predict the mean annual rainfall for a city 1000 m above sea level. Is this possible? What is wrong with this prediction?

4 Mr Armstrong, the Science teacher, sets up an experiment to measure the pressure of a gas at different temperatures. This table show the results:

Example 7

Temperature (°C)	10	20	30	40	50	60	70	80	90
Pressure (g/cm³)	27.9	30	29.8	32.1	31.9	34.1	33.8	34.8	36.6

a* Calculate the correlation coefficient for this data.
*Australian curriculum only, not in WA syllabus

b Mr Armstrong used this data to estimate the pressure at 56°C. How reliable is this estimate?

c Mr Armstrong used this data to predict the pressure at –10°C. How reliable is this estimate?

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5 Ms Cranston has a set of results for her class on two Maths tests.

Student	Bill	Ruth	Mary	Ella	Greg	Jim	Meg	Tara	Bob	Clem	Bree	Amy
Algebra test	60	38	65	??	75	48	67	23	82	16	92	80
Data test	35	21	47	31	56	40	54	11	59	20	62	??

Ella and Amy missed a test and Ms Cranston wants to give them an estimate.

a Draw a scatterplot for this data (except for Ella and Amy) and draw a line of best fit.

b Predict the results that Ella and Amy might have received in the test they missed.

c* Calculate the correlation coefficient for this data.

d* Given this correlation coefficient, how reliable do you think these predictions are? Justify your answer.

6 Ryan investigated the heights (x cm) of a group of his friends and their hand spans (y cm, the maximum distance y between the tips of their thumb and little finger).

Height (x cm)	170	178	160	183	168	145	155
Hand span (y cm)	20	21	19	22	20	17	19

a Construct a scatterplot for the data and draw a line of best fit on the plot.

b A line of best fit has equation $y = 0.12x - 0.02$. Use the equation to predict the hand span of Izak, who is 165 cm tall. Answer correct to the nearest centimetre.

c Robert Wadlow was the tallest man in the world. His height was 272 cm (see photo below of his statue at a London museum). Use the equation of the line of fit to predict his hand span, correct to the nearest centimetre, and explain why this measurement is unlikely to be correct.

Alamy Stock Photo/Gary Wilkinson

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Chapter problem

You've covered the skills required to solve the chapter problem. Can you solve it now?

KEYWORD ACTIVITY

CHAPTER SUMMARY

Use this list of words to complete this summary of the chapter below.

bivariate	correlation	correlation coefficient
equation	extrapolation	interpolation
line of best fit	predictions	scatterplot
technology		

In this chapter, we have continued our study of **1**__________ data. We graphed such data on a **2**_____________ and drew a **3**_____________ _____________ _____________ _____________ through them 'by eye'. We can also use **4**_____________ to graph the line and find its **5**_____________.

Two variables can have a strong association between them, also called the **6**____________. We can also enter a table of values into a calculator or spreadsheet to calculate the **7**__________ ____________.

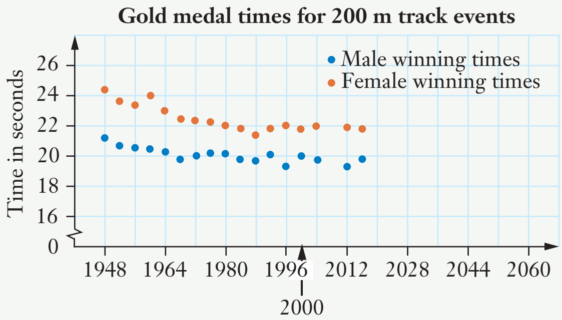
We can use both the lines of best fit and the correlation coefficient to help us make **8**____________ from the data. When we make estimates from within the data range, this is called **9**____________ and it is reliable if there is a high correlation between the data. When we make estimates outside the range of the data, this is called **10**____________ and can often be unreliable or impossible.

SOLUTION TO THE CHAPTER PROBLEM

Problem

Kaylene is a competitive runner. She studies the past winning times for the women's Olympic 200 m race, and is surprised by the rate at which the times have improved. The women's winning times are slower than those of the men in their 200 m event, but the women seem to be catching up. She wondered whether women would ever run the event in the same or faster time than the men.

This graph shows the gold medal times for men and women's 200 m track events at the Olympic Games since 1948.



Will the women ever run the 200 m race faster than the men?

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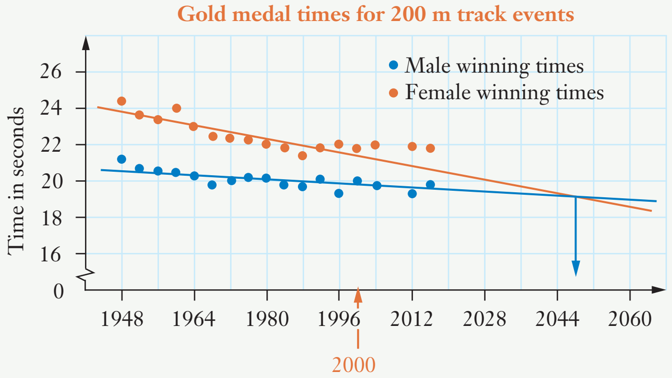
Solution

Add a line of best fit for men and for women. These lines intersect at around 2048.



If the linear model is valid, men and women will run the 200 m event in the same time in 2048 and after that women will be faster.

However, this is unlikely. The year 2048 is a long way beyond the data, and the data cannot be linear forever. If the linear graph continues, there will come a year when the gold medal time is 0 or negative! It is more likely that both sets of times will level off.

9. TEST YOURSELF

Fitting the data

Qz Practice quiz

Exercise 9.01

1 In Exercise 9.01, Question **2**, Eliza studied the relationship between the daily average temperature and her cafe's hot chocolate drink sales.

Temperature (°C)	8	36	12	28	14	22	16	10	24	18	7	31
Hot chocolate sold	42	15	37	24	9	20	35	20	10	30	30	4

a Graph Eliza's bivariate data in a scatterplot and draw a line of best fit for this data.

b Use technology to find the equation of the line of best fit.

Exercise 9.02

2 For the hot chocolate data from Question **1**:

a what is the independent variable?

b what is the dependent variable?

c how many hot chocolate drinks could Eliza expect to sell on a day with a temperature of 20°C?

d predict the temperature on a day when Eliza sells 25 hot chocolate drinks

e describe the relationship between the temperature and the number of hot chocolate drinks sold

Exercise 9.03

3* Sketch a scatterplot to illustrate a set of data that has:

a a weak, positive correlation

b a strong, negative correlation

c no correlation

*Australian curriculum only, not in WA syllabus

Exercise 9.03

4* This data from Chapter 6, *Scattering the data*, compares the hours of homework and the hours of TV watching per week for a group of 15 students.

a Calculate the correlation coefficient for this data.

b Do you think these 2 variables are closely related? Why or why not?

Student	Hours of homework per week	Hours watching TV per week
Amy	6	2
Joe	6	10
Annika	23	4
Janine	7	14
Stephen	23	9
Thanh	2	5
Gillian	17	4
Vamsee	1	3
Lalaja	16	20
Darryl	10	10
Lyn	15	0
Jeremy	9	4
Ben	0	17
Abdul	3	9
Miriam	4	1

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Exercise 9.04

5 A group of 12 students measured their heights and length of stride in centimetres. A stride is the biggest step a person can take from a standing position, measured from toe to toe.

Height	165	140	180	176	160	164	178	170	148	157	150	162
Length of stride	105	85	111	104	98	95	108	102	92	97	88	100

a Draw a scatterplot for this data and draw a line of best fit.

b* Calculate the correlation coefficient for this data.

c Use your line of best fit to predict:

- **i** the length of stride of Natalie who is 155 cm tall
- **ii** the height of Emir who has a stride length of 90 cm

d How reliable are these predictions? Justify your answer.

e Use technology to find the equation of the line of best fit.

f Use the equation to predict:

- **i** the length of stride of Zhang, who is 200 cm tall
- **ii** the height of Martika, who has a stride length of 80 cm

g How reliable are these predictions? Justify your answer.

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MEASUREMENT

10.

FILL IT UP

Chapter problem

Stuart has to supply and install a septic tank with a capacity of between 5000 and 6000 L on a farm. The diagram shows the top view of a septic tank that Stuart is considering for the job. The tank is in the shape of a rectangle with a semicircle at each end. The tank is 1.7 m deep. Is this septic tank suitable for the job?

WHAT WILL WE DO IN THIS CHAPTER?

- Choose appropriate metric units of mass, volume and capacity and convert between them
- Calculate the volume and capacity of prisms, cylinders, spheres and pyramids

HOW ARE WE EVER GOING TO USE THIS?

- When comparing the quantities inside different-sized food containers
- When calculating the quantity of materials for a job, for example, the amount of soil or mulch needed in our garden
- Volume is a key component of important trades such as plumbing and landscape gardening

iStockphoto/Sylvia Schug

10.01 Mass

WS
Mass time trial

The **gram** is the basic unit for **mass** and all other mass units are based on the gram.

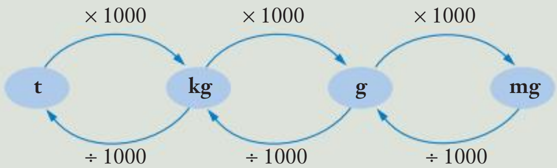
The **milligram** is often used to measure the mass of medicine, vitamins, food and jewellery.

Unit	Relationships
milligram (mg)	
gram (g)	1 g = 1000 mg
kilogram (kg)	1 kg = 1000 g
tonne (t)	1 t = 1000 kg

Metric units of mass

To change from a small unit to a bigger unit, **divide** by the conversion factor.

To change from a big unit to a smaller unit, **multiply** by the conversion factor.



EXAMPLE 1

Convert 38 g to

a mg **b** kg

Solution

a There are 1000 mg in 1 g. Changing from g to mg is changing to a smaller unit, so multiply by the conversion factor.

$38\text{ g} = 38 \times 1000\text{ mg}$
$= 38\,000\text{ mg}$

b There are 1000 g in 1 kg. Changing from g to kg is changing to a larger unit, so divide by the conversion factor.

$38\text{ g} = 38 \div 1000\text{ kg}$
$= 0.038\text{ kg}$

Exercise 10.01 Measuring mass

Example 1

1 Copy and complete each conversion.

a 3 kg = ___ g
b 12 t = ____ kg
c 1500 g = ___ kg
d 2400 kg = ___ t
e 850 kg = ____ g
f 900 g = ____ kg
g 2.5 g = _____ mg
h 500 mg = ___ g

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2 A hospital pharmacist ordered 2000 tablets. Each tablet has a mass of 5 mg.

- **a** Calculate the total mass of the tablets in mg.
- **b** What is the total mass in grams?

3 Vitamin C powder contains 90% ascorbic acid and 10% calcium.

- **a** What mass of calcium is in 40 milligrams of vitamin C?
- **b** What mass of ascorbic acid is in 60 milligrams of vitamin C?
- **c** Calculate the number of milligrams of ascorbic acid in 2.4 grams of vitamin C.

4 The gross mass of a bottle of 500 tablets is 155 g. The mass of the bottle only is 20 g.

gross mass = total mass including bottle
net mass = mass of tablets only

- **a** Calculate the net mass of the tablets.
- **b** What is the net mass of the tablets in mg?
- **c** What is the mass of one tablet in mg?

5 How many 50 mg injections can a nurse make from a 1 g container of streptomycin medicine?

6 List 3 items whose mass you would measure in

a tonnes **b** kilograms **c** grams **d** milligrams

7 We measure the size of precious stones in carats. Erin's engagement ring contains a 1.8 carat diamond. What is the mass of the diamond in mg? (1 carat = 200 mg).

8 Nelsonlink Airlines has a carry-on luggage limit of 12 pounds. Karen's bag is 5 kg.

- **a** Calculate the mass of Karen's bag in pounds. 1 kg = 2.2 pounds.
- **b** Is Karen's bag light enough to take on the flight? Justify your answer.

9 Jettison Air has two sets of restrictions on the size of bags it allows on flights.

- The mass of the bag must be 50 pounds or less.
- The sum of the bag's dimensions (length + width + height) must be less than 62 inches.

Orlando's bag is 50 cm long, 19 cm high, 32 cm wide and has a mass of 24 kg.

Is Orlando's bag allowed on the flight? Justify your answer.

(Note: 1 kg = 2.2 pounds and 1 inch = 2.5 cm.)

10 A standard house brick has a mass of 2.7 kg.

- **a** A pallet of bricks contains 500 bricks. Calculate the mass of one pallet of bricks.
- **b** A truck carries 8 pallets of bricks. Calculate the weight of the bricks in tonnes.

11 In China, the mass of tea leaves is measured in 'jins'. One jin = 500 g. Calculate in grams the mass of a packet of tea that is 3.2 jin.

12 We measure the mass of precious metals in troy ounces (1 troy ounce = 31.103 g). Gazi bought a 1 kg gold bar as an investment. How much was Gazi's gold bar worth on the day when gold was valued at $1331 per troy ounce?

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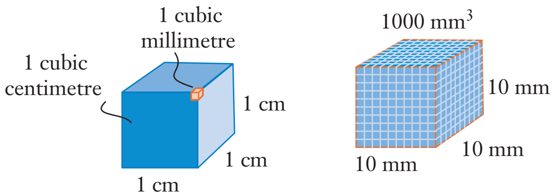
10.02 Volume

The **volume** of a solid is the amount of space occupied by the solid. Volume is measured in cubic units.

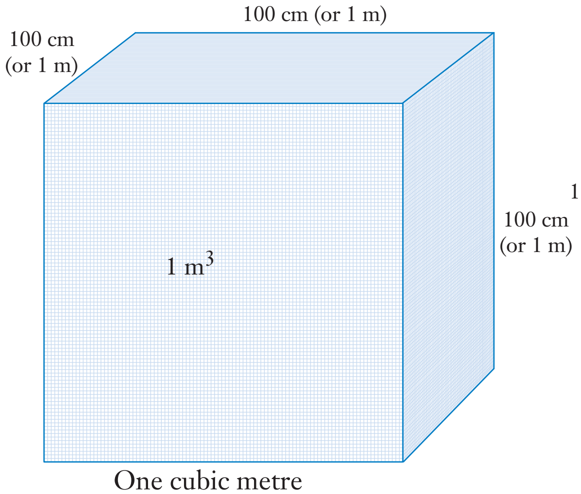
Volume unit	The size of a cube of length:	Approximately the size of:
cubic millimetre (mm^3)	1 mm	a grain of raw sugar or rock salt
cubic centimetre (cm^3)	1 cm	a tooth or a pain relief tablet
cubic metre (m^3)	1 m	2 washing machines

These diagrams compare a cubic millimetre with a cubic centimetre.



$$1\text{ cm}^3 = 10\text{ mm} \times 10\text{ mm} \times 10\text{ mm}$$
$$= 1000\text{ mm}^3$$

This diagram compares a cubic centimetre with a **cubic metre**.



$$1\text{ m}^3 = 100\text{ cm} \times 100\text{ cm} \times 100\text{ cm}$$
$$= 1\,000\,000\text{ cm}^3$$

One cubic metre

When converting area units in Chapter 7, *On the surface*, we had to convert the length unit twice. When converting volume units, we have to convert the length unit 3 times. We *cube* the simple linear conversion factor to get the volume conversion factor.

For example,	to change from m to cm,	multiply by 100,
but	to change from m^3 to cm^3,	multiply by $100^3 = 1\,000\,000$.

 ISBN 9780170443975

Metric units of volume

$1 \text{ cm}^3 = 1000 \text{ mm}^3$

$1 \text{ m}^3 = 1\,000\,000 \text{ cm}^3$

To change from a small unit to a bigger unit, **divide** by the conversion factor.

To change from a big unit to a smaller unit, **multiply** by the conversion factor.

EXAMPLE 2

Convert:

a 36 cm^3 to mm^3 **b** $84\,000\,000 \text{ cm}^3$ to m^3

Solution

a cm^3 to mm^3, large to small unit: $\times 10^3 = 1000$.

$$36 \text{ cm}^3 = 36 \times 1000 \text{ mm}^3$$
$$= 36\,000 \text{ mm}^3$$

b cm^3 to m^3, small to large unit: $\div 100^3 = 1\,000\,000$.

$$84\,000\,000 \text{ cm}^3 = 84\,000\,000 \div 1\,000\,000 \text{ m}^3$$
$$= 84 \text{ m}^3$$

Exercise 10.02 Volume

1 What units would you use (m^3, cm^3 or mm^3) to measure the volume of each object?

a A bedroom **b** A backpack **c** A mobile phone
d A matchbox **e** A concert hall **f** A swimming pool
g Your calculator **h** A car **i** A driver's licence
j A glass of water **k** A USB drive **l** A box of laundry powder

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2 Match the correct volume (**A** to **G**) with each of the items (**a** to **g**) listed.

a	bottle of nail polish	**A**	200 m^3
b	box of tissues	**B**	3980 m^3
c	glass of fruit juice	**C**	1250 cm^3
d	bottle of lemonade	**D**	5000 cm^3
e	classroom	**E**	20 000 mm^3
f	school hall	**F**	250 cm^3
g	box of cereal	**G**	2200 cm^3

Hint: You may find it helpful to put the items in descending order of volume and then choose the measurements.

Example 2

3 Copy and complete each conversion.

a 5000 cm^3 = _____ mm^3 **b** 1.6 m^3 = _____ cm^3

c 6000 cm^3 = _____ m^3 **d** 4000 mm^3 = _____ cm^3

e 160 000 cm^3 = _____ m^3 **f** 250 mm^3 = _____ cm^3

g 0.18 m^3 = _____ cm^3 **h** 0.12 cm^3 = _____ mm^3

i 4 m^3 = _____ mm^3 **j** 9 600 000 mm^3 = _____ m^3

4 Arrange in ascending order: 42 cm^3, 4210 mm^3, 0.0042 m^3

5 Arrange in descending order: 65 000 cm^3, 0.6 m^3, 7 000 000 mm^3

6 The volume of Marty's chest of drawers is 306 000 cm^3. What is this in cubic metres?

7 Ruchi's lunchbox has a volume of 2520 cm^3. What is the volume of her lunchbox in cubic millimetres?

8 Hans ordered 1 m^3 of pine bark to mulch his vegetable garden. After he finished putting the mulch on his garden, he had 20% of the original amount left. How many cubic centimetres of mulch were left over?

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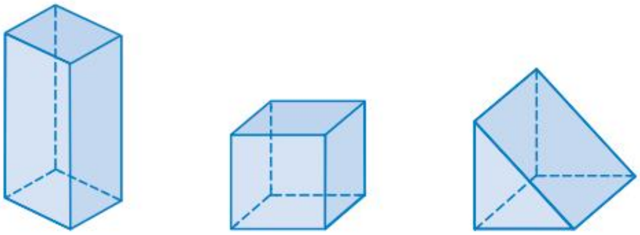
10.03 Volumes of prisms

WS Converting volume units

A solid with identical ends and flat sides is called a **prism**. The 3 prisms shown below are a rectangular prism, cube and triangular prism. The ends of a prism are shapes with straight sides (rectangles, triangles or any other polygon) and is part of the name of the prism, for example, rectangular prism, triangular prism.



Volume of a prism or other solid with identical ends

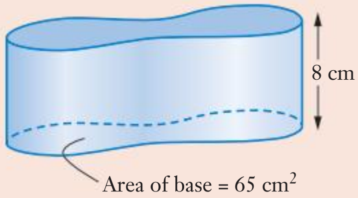
$$V = A \times h$$

where A is the area of the end or **base** and h is the height.

EXAMPLE 3

The area of the base of this solid is 65 cm^2.
What is the volume of the solid?

This is not a prism but it does have identical ends.



Solution

Both ends of the solid are identical, so we can use the formula $V = A \times h$.

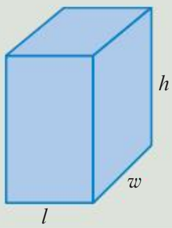
$A = 65$ and $h = 8$.

$$\text{Volume} = 65 \times 8$$
$$= 520 \text{ cm}^3$$

Volume of a rectangular prism

$$V = lwh$$

where l = length, w = width, h = height



Volume of a cube

$$V = s^3$$

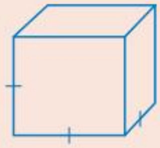
where s = side length.

EXAMPLE 4

Find the volume of each prism.

a

b



Solution

a Use the formula $V = lwh$

$V = 40 \times 9 \times 12$

$= 4320 \text{ mm}^3$

b This is a cube, a special type of rectangular prism. $V = s^3$.

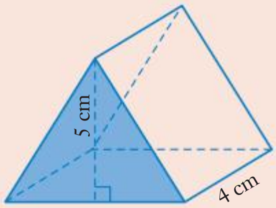
$V = 6.5 \times 6.5 \times 6.5$

$= 6.5^3$

$= 274.625 \text{ cm}^3$

EXAMPLE 5

Find the volume of this triangular prism.



Solution

Use the formula $V = A \times h$.

For a triangular prism, $A = \frac{1}{2}bh$.

$b = 6, h = 5$.

$V = (\frac{1}{2} \times 6 \times 5) \times 4$

$= 60 \text{ cm}^3$

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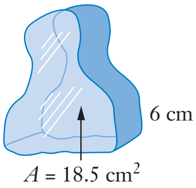
Exercise 10.03 Volumes of prisms

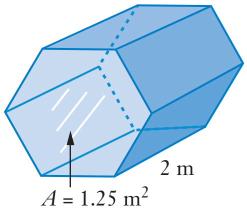
1 Find the volume of each solid.

Example 3

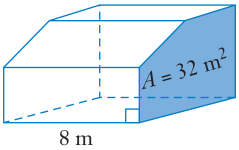
a



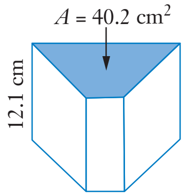
b



c



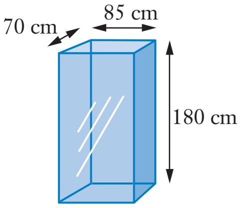
d



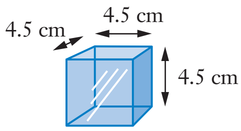
2 Find the volume of each prism.

Example 4

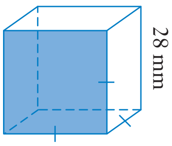
a



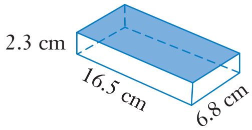
b



c



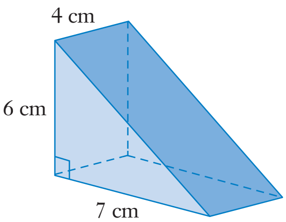
d



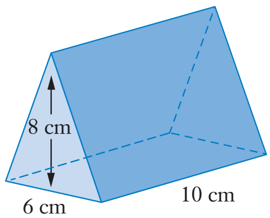
3 Find the volume of each triangular prism.

Example 5

a



b



4 Concrete blocks with the shape of rectangular prisms are used to build houses.

a If each concrete block measures 45 cm by 17 cm by 21 cm, calculate the volume of one block.

b A wall is made from 80 concrete blocks. Calculate the volume of the wall.

5 These storage cubes have a side length of 35 cm.

a Calculate the volume of one cube.

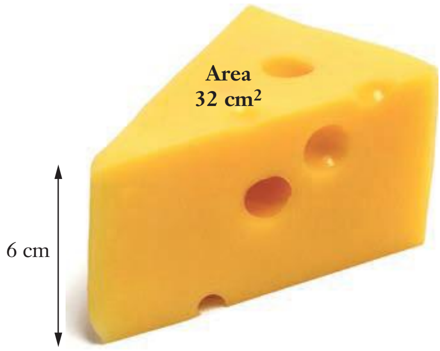
b Inga has a stack of 11 cubes in her bedroom to store books. What volume of books can she store?

c Inga's books have an average volume of 1425 cm^3 each. Approximately how many books can she store?


Shutterstock.com/Leszek Glasner

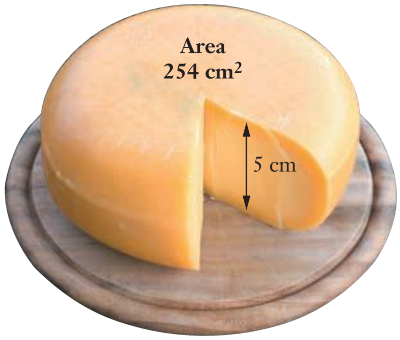
6 Calculate the volume of each food item.

a



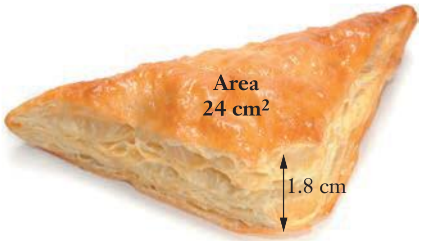
iStockphoto/PIKSEL

b



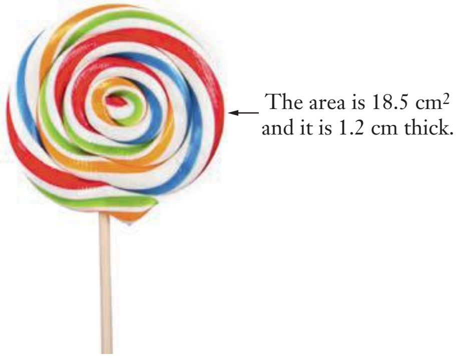
Dreamstime.com/Emprize

c



Thinkstock/Vitali Dyatchenko

d



Thinkstock/ekinyalgin

7 Michael is laying pavers for a new rectangular outdoor area, 4.2 m by 2.7 m. Each paver is 300 mm by 300 mm and 50 mm thick.

a How many pavers will he need?

b Calculate the volume of one paver.

c Calculate the volume for the total number of pavers required for this job.

d Pavers come in boxes of 10. How many boxes will Michael need to purchase?

e Each box costs \$56.70. How much will the pavers cost?

Remember to have all your measurements in the same units!

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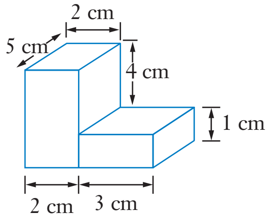
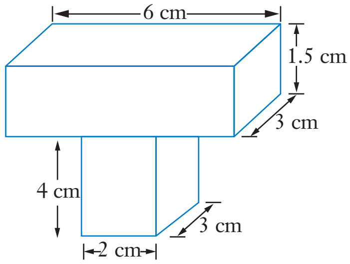
8 Calculate the volume of each prism.

a



b



9 Nazneen plans to install air-conditioning inside her house. The air conditioner is available in 4 sizes.

Air conditioner size	Volume of air in house
Small	210 m^3
Medium	350 m^3
Large	500 m^3
Extra large	720 m^3

Nazneen's house is in the shape of a rectangular prism, with the dimensions shown on the photograph.

Photo courtesy of Sue Thomson

What size air conditioner will Nazneen need?

10 a The volume of this 1 L carton of milk is 1000 cm^3. The height of this container is 20.4 cm. What is the area of the base, correct to the nearest cm^2?

b The base is a square. How long are the sides of the square?

Newspix/Jessica Lee

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Mass and capacity match

Mass, volume and capacity time trial

10.04 Volume and capacity

Volume measures the amount of space inside a container, while **capacity** measures the amount of liquid or gas a container will hold.

Capacity unit	Approximately the size of:
millilitre (mL)	a large drop of water
litre (L)	a tall carton of milk
kilolitre (kL)	a small rainwater tank
megalitre (ML)	half an Olympic-sized swimming pool

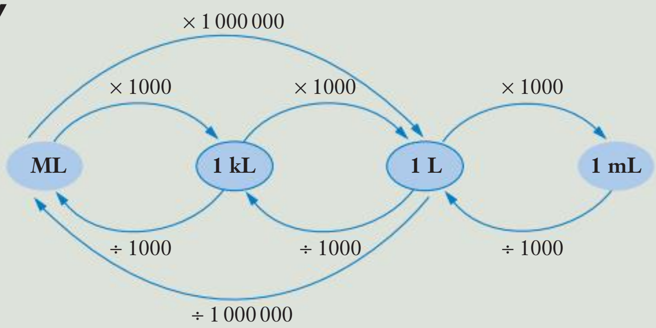
Metric units of capacity

1 L = 1000 mL

1 kL = 1000 L

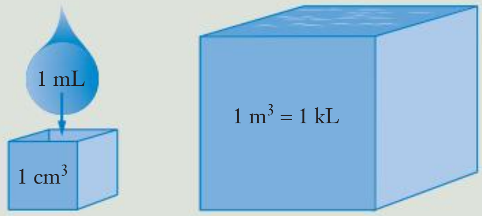
1 ML = 1000 kL

= 1 000 000 L



1 cm^3 holds 1 mL

1 m^3 holds 1000 L or 1 kL



Volume and capacity

EXAMPLE 6

Convert:

a 5 cm^3 to mL **b** 1850 mL to litres

Solution

a 1 cm^3 holds 1 mL. The number of cm^3 and mL are always the same. 5 cm^3 holds 5 mL

b mL to L: small to large unit: ÷ 1 000. 1850 mL = 1850 ÷ 1000 L

= 1.85 L

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EXAMPLE 7

Volume and capacity

The volume of a large fishpond is 3.4 m^3. How many litres of water does it hold?

Shutterstock.com/Ron Zmiri

Solution

1 m^3 holds 1000 L. Multiply the number of m^3 by 1000.	3.4 m^3 holds 3.4×1000 L = 3400 L
Write your answer.	The fishpond holds 3400 L of water.

Exercise 10.04 Volume and capacity

1 State what unit of capacity you would use to measure the size of:

- **a** a glass of milk
- **b** a dam
- **c** a car's petrol tank
- **d** a bottle of medicine
- **e** an office water cooler
- **f** a swimming pool

2 Match the correct capacity (**A** to **J**) with the items (**a** to **j**) listed.

Hint: You may find it helpful to put the items in descending order of capacity and then choose the measurements.

a	car petrol tank	**A**	200 mL
b	a cup of flour	**B**	23 kL
c	bathtub	**C**	5 mL
d	bucket of water	**D**	70 L
e	can of drink	**E**	1250 mL
f	glass of water	**F**	1.875 ML
g	Olympic swimming pool	**G**	250 mL
h	bottle of lemonade	**H**	9 L
i	teaspoon	**I**	375 mL
j	water storage tank	**J**	180 L

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Example 6

3 Convert each measurement to mL.

a 8 cm^3 **b** 1500 cm^3 **c** 425 cm^3

4 Convert each measurement to litres.

a 2000 mL **b** 3500 mL **c** 250 mL

Example 7

5 **a** The volume of a large container is 5000 cm^3. How many millilitres does the container hold?

b How many litres will a container with a volume of 5000 cm^3 hold?

6 How many litres can a 2 m^3 container hold?

7 What is the volume in cubic centimetres of a 1.25-litre soft drink bottle?

8 Each can in a box of 24 cans of soft drink holds 375 mL. How many litres of soft drink are contained in the box?

9 This inflatable children's pool contains water 20 cm deep. How many litres of water are in the pool?

Make sure you convert 20 cm to metres before you start the calculations!

123RF Stock Photo/smikeymikey1

10 A tap leaks 10 mL of water every 50 seconds. How much water will the tap lose in:

a 1 second? **b** 1 minute? **c** 3 hours? **d** 1 day?

11 Zina's swimming pool is 5.8 m long and 3.2 m wide. Hot, dry winds from Central Australia caused 11 cm of water in the pool to evaporate.

a What solid shape could be used to represent the volume of water that evaporated from Zina's pool?

b How many cubic metres of water evaporated from the pool?

c How many litres of water are required to top up Zina's pool?

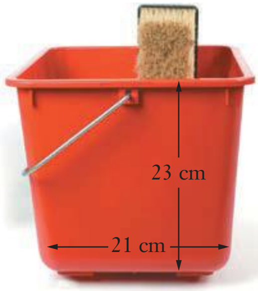
d The pump on Zina's water tank delivers 105 L per minute. For how long will she need to pump water from the tank into her pool to replace the evaporated water? Express your answer correct to the nearest minute.

12 This bucket has a square base with sides of 21 cm.

a The sides of the bucket are 23 cm high. Calculate the volume of the bucket in cm^3.

b How many whole litres of water can the bucket hold?

c One litre of water weighs 1 kg. Approximately how much will the bucket weigh when it is half-full of water?



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13 a How many adult doses of cough medicine are contained in a full bottle?

b Young's rule is a formula for calculating a child's dose of medicine.

$$\text{Child dose} = \frac{\text{age of the child in years} \times \text{adult dose}}{\text{Age of the child in years} + 12}$$

Use Young's rule to calculate the amount of cough medicine that a 7-year-old child can have. Answer in mL correct to 1 decimal place.

Alamy Stock Photo/Aleksandra Gigowska

14 Charlotte has a bad cold and she wants to take some of this medicine.

Who can use this product		
✓ Adults and Children OVER 12 years ✓ Suitable for diabetics ✓ Gluten free		
Age	**How much**	**How often**
Over 12 years	20 mL	1 dose every 4 hours as necessary. No more than 6 doses in 24 hrs.

a Charlotte is 16 years old. What dose can Charlotte take?

b How often can she take the medicine?

c What is the maximum amount of medicine that Charlotte can take in 24 hours?

d Charlotte is concerned about taking the full adult dose because she only weighs 40 kg. Clark's rule is a formula for calculating a child's dose of medicine based on weight.

$$\text{Child dose} = \frac{\text{weight of the child in kilograms} \times \text{adult dose}}{70}$$

According to Clark's rule, what dose should Charlotte take? Answer correct to the nearest mL.

e Charlotte and her 11-year-old brother, Noah, are the same weight. Can Noah take the dose you calculated for Charlotte in part **d**? Explain.

15 Fried's rule is a different formula for calculating a child's dose of medicine.

$$\text{Child dose} = \frac{\text{age of the child in months}}{150} \times \text{adult dose}$$

The weekly adult dose of a dietary supplement is 1.25 L. According to Fried's rule, what amount of the dietary supplement can a 5-year-old child have per week? Express your answer in mL.

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INVESTIGATION

PACKAGING CASKS

Garry works in the delivery section of a supermarket. He has a 1 m^3 package of 5-litre casks of fruit juice. Your group's task is to determine the number of casks in the package.

Volume code puzzle

Officer Cubic

Measurement formulas chart

Formula matching game

A page of solid shapes

Sweet areas and volumes

10.05 Volumes of cylinders, spheres and pyramids

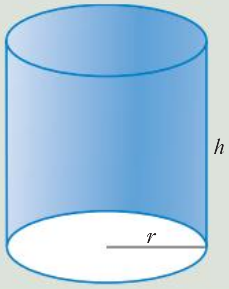
Volume of a cylinder

We can also use the formula $V = Ah$ to find the volume of a cylinder, because a cylinder has identical ends. For a cylinder, the base is a circle, with area $A = \pi r^2$.

$$\begin{aligned} V &= Ah \\ &= \pi r^2 \times h \\ &= \pi r^2 h \end{aligned}$$

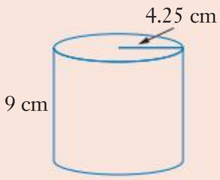
Volume of a cylinder

$$V = \pi r^2 h$$



EXAMPLE 8

Find the volume of this cylinder, correct to 2 decimal places.



Solution

$V = \pi r^2 h$

$r = 4.25$ and $h = 9$.

$$\begin{aligned} V &= \pi \times 4.25^2 \times 9 \\ &= 510.7051 \ldots. \\ &\approx 510.71 \text{ cm}^3 \end{aligned}$$

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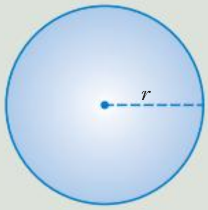
Volume of a sphere

There is a special formula for the volume of a sphere.

Volume of a sphere

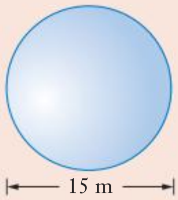
$$\text{Volume} = \frac{4}{3} \times \pi \times \text{radius}^3$$

$$V = \frac{4}{3}\pi r^3$$



EXAMPLE 9

Find the volume of this sphere, correct to the nearest cubic metre.



Solution

The diameter is 15 m.

So the radius r is half of 15.

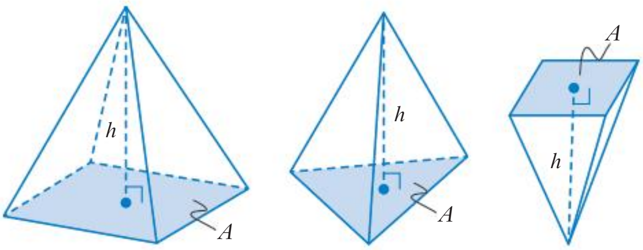
$V = \frac{4}{3}\pi r^3$.

$$r = \frac{1}{2} \times 15 = 7.5$$

$$\begin{aligned} V &= \frac{4}{3} \times \pi \times 7.5^3 \\ &= 17673.1458\ldots \\ &\approx 1767\text{ m}^3 \end{aligned}$$

Volume of a pyramid

The volume of a pyramid is $\frac{1}{3}$ of the volume of a prism with the same base and height.



Volume of a pyramid

$$V = \frac{1}{3}Ah$$

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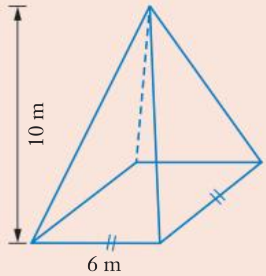
EXAMPLE 10

Calculate the volume of this square pyramid.



Solution

$$V = \frac{1}{3}Ah.$$

The base, A, is a square.

$A = s^2$, where $s = 6$.

$$A = 6^2$$
$$= 36 \text{ m}^2$$

$$V = \frac{1}{3}Ah.$$

$h = 10.$

$$V = \frac{1}{3} \times 36 \times 10$$
$$= 120 \text{ m}^3$$

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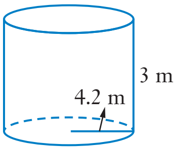
Exercise 10.05 Volumes of cylinders, spheres and pyramids

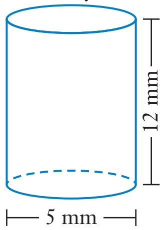
1 Calculate, correct to 1 decimal place, the volume of each cylinder.

Example 8

a



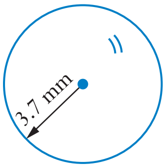
b



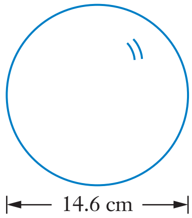
2 Calculate, correct to 1 decimal place, the volume of each sphere.

Example 9

a



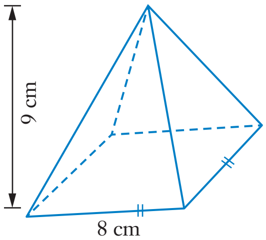
b



3 Calculate the volume of each pyramid.

Example 10

a



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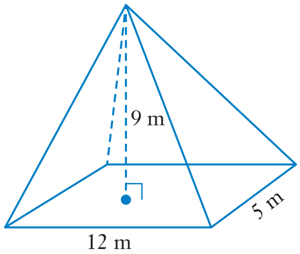

4 Large spherical tanks are used to hold gas. Calculate the volume of this tank, correct to the nearest m^3.

iStock.com/HAYKIRDI

5 Jo uses a cylindrical tank to store water on her property. What is the volume of Jo's tank? Answer correct to 2 decimal places.

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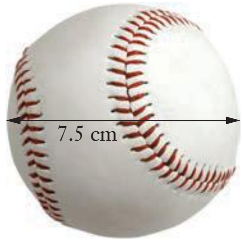
6 The Pyramid of Khafre in Egypt is famous because the smooth surface covering the top is still there. The pyramid has a square base that is 215.3 m long and its height is 136 m. Calculate the volume of the Khafre pyramid. Express your answer correct to the nearest 100 m^3.

Shutterstock.com/Kirsty Bisset

7 The diameter of a leather baseball is 7.5 cm. Calculate its volume correct to the nearest cm^3.



Shutterstock.com/Alex Staroseltsev

8 One of the old fuel tanks on the Space Shuttle contained liquid hydrogen. The tank is a cylinder, 21.2 m long and 8.4 m wide, with a hemisphere of radius 4.2 m at each end. Calculate, correct to 1 decimal place, the volume of the tank in cubic metres.

Alamy Stock Photo/ZUMA Press, Inc.

9 A small cylindrical water tank has a diameter of 720 mm and a height of 970 mm.

a Calculate the volume of the water tank in m^3, correct to 2 decimal places.

b What is the capacity of the water tank in litres? Answer correct to the nearest 10 litres.

Hint: Change the measurements to metres before you start.

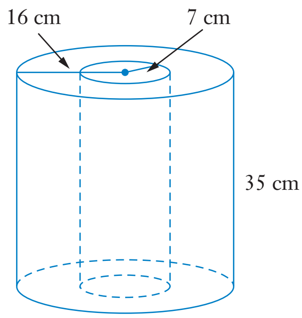
Remember: $1 m^3 = 1000$ L

10 A hollow cylinder is shown.

a Find the volume of a cylinder with radius 16 cm, correct to 2 decimal places.

b Find the volume of the 'hole' with radius 7 cm, correct to 2 decimal places.

c Calculate the volume of the hollow cylinder by subtracting your answer to part **b** from your answer to part **a**.



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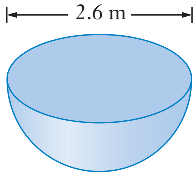
11 Find, correct to one decimal place, the volume of this hemisphere ('half sphere').

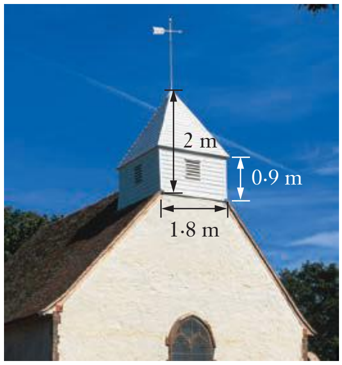


12 The shape of this church tower is a square pyramid on top of a square prism.

a Find the height of the square pyramid.

b Calculate the volumes of the pyramid and the prism.

c How much space is contained inside this tower?



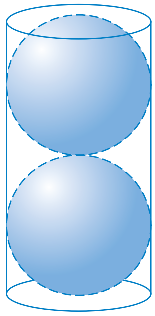
Alamy/Nick Hawkes

13 The radius of each sphere in the diagram is 6 cm.

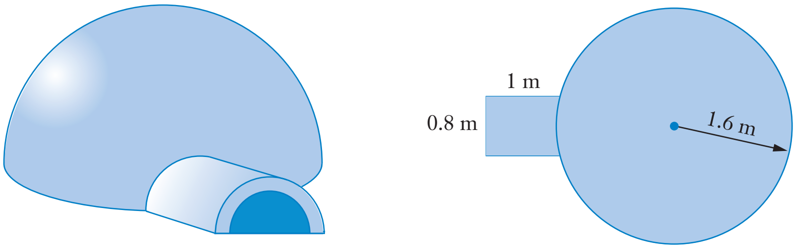
a Calculate the total volume of the 2 spheres.

b What is the radius and height of the cylinder?

c Calculate the volume of the cylinder.

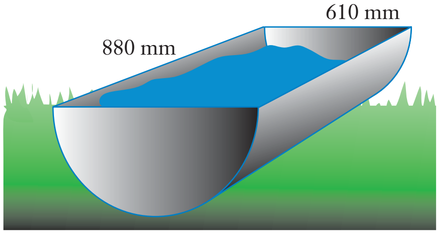


14 The Canadian Inuit people used igloos as temporary shelters. The main section was a hemisphere, and the entrance was half a cylinder. The diagram on the right below shows the internal dimensions of an igloo. What is the internal volume of the igloo? Express your answer correct to 1 decimal place.



Front view Top view

15 Silvana uses half-cylinder water troughs as shown for the animals on a farm.



a Calculate the volume of one water trough in cubic metres, correct to 2 decimal places.

b How much water is needed to fill one water trough?

c There 25 water troughs on the farm. How much water is needed to fill all troughs?

d Silvana uses a cylindrical tank on a trailer to fill the water troughs. Calculate the volume of the tank, correct to 2 decimal places, if its diameter is 1560 mm and its height is 1210 mm.

e Can this tank hold enough water to fill all the troughs? If not, how many trips will Silvana have to make?

Chapter problem

You've covered the skills required to solve the chapter problem. Can you solve it now?

INVESTIGATION

DESIGNING A SMALLER CAN

Dlibrary/Mars Petcare Australia

Changing the dimensions of a package is one method that manufacturers use to disguise price rises. Keeping the price the same but reducing the size of the contents is equivalent to increasing the price.

This can has a base radius of 5 cm and a height of 10 cm and it contains 785 cm^3.

Design a can that looks almost the same size, but contains about 20% less volume.

Hints

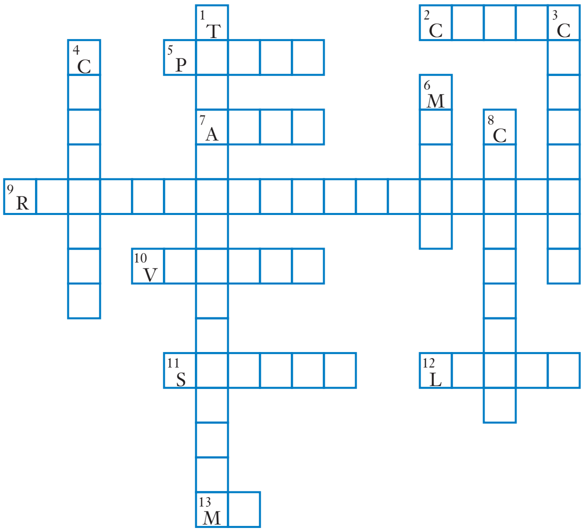
- How much is 20% less than 785 cm^3?
- Reduce the radius or the height, or both, by a small amount and calculate the volume. Is it in the range you want? If not, try some other values for the radius and height.

ISBN 9780170443975

KEYWORD ACTIVITY

CROSSWORD

Copy and complete this crossword, using the clues below.



ACROSS

2 Volume can be measured in _______ metres.

5 A solid that has flat faces and identical cross-sections.

7 The amount of surface a flat shape covers.

9 A solid shape that has a rectangle for the base and comes to a point at the top (2 words).

10 The amount of space a solid takes up.

11 The shape of a ball.

12 A unit of capacity equal to 1000 cm^3.

13 Millimetre (abbreviation).

DOWN

1 A solid shape with identical triangles at each end. The other faces are rectangles. (2 words)

3 A solid shape with circular ends, the shape of a can.

4 The amount of liquid or gas a container can hold.

6 A standard unit of length.

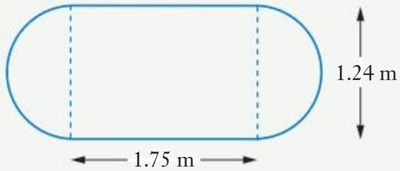
8 A solid made from 2 or more solids.

ISBN 9780170443975

SOLUTION TO THE CHAPTER PROBLEM

Problem

Stuart has to supply and install a septic tank with a capacity of between 5000 and 6000 L on a farm. The diagram shows the top view of a septic tank which Stuart is considering for the job. The tank is in the shape of a rectangle with a semicircle at each end. The tank is 1.7 m deep. Is this septic tank suitable for the job?



Solution

$$\begin{aligned}\text{Area of the top of the tank} &= 2 \times \text{area of the semicircles} + \text{area of the rectangle}\\ &= 2 \times \frac{1}{2} \times \pi \times (0.62)^2 + 1.75 \times 1.24\\ &= 3.3776 \ldots \text{ m}^2\end{aligned}$$

$$\begin{aligned}\text{Volume of the tank} &= \text{area of the top} \times \text{depth}\\ &= 3.3776 \ldots \times 1.7\\ &\approx 5.7419 \ldots \text{ m}^3\\ &= 5.7419 \ldots \times 1000 \text{ L} \leftarrow \boxed{1\,\text{m}^3 = 1\,\text{kL}}\\ &= 5741.9 \ldots \text{ L}\\ &\approx 5742 \text{ L}\end{aligned}$$

5742 L is between 5000 L and 6000 L. The tank is suitable for the job.

iStockphoto/Sylvia Schug

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10. TEST YOURSELF

Fill it up

Practice quiz

1 Copy and complete each conversion. Exercise 10.01

a 5 kg = _____ g **b** 200 g = _____ kg

c 1.4 t = _____ kg **d** 3.5 g = _____ mg

e 7500 kg = _____ t

2 Copy and complete each conversion. Exercise 10.02

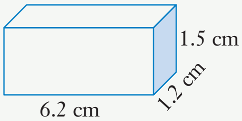
a 5 cm^3 = ____ mm^3 **b** 2 m^3 = _____ cm^3

c 500 mm^3 = ___ cm^3 **d** 0.25 m^3 = ____ cm^3

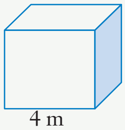
e 24 000 cm^3 = ___ m^3 **f** 36 000 mm^3 = ___ cm^3

3 Calculate the volume of each solid. Exercise 10.03

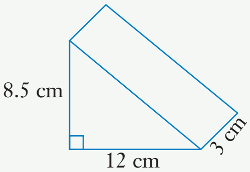
a



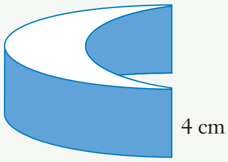
b



c



d



Area of base = 12 cm^2

4 Convert 2600 mL to litres. Exercise 10.04

5 How many 120 mL baby's bottles can be filled from a 2-litre container of milk? Exercise 10.04

6 Complete each statement. Exercise 10.04

a A container with a volume of 24 cm^3 holds ____ mL.

b The volume of a container with a capacity of 6 L is _______.

7 The volume of a small wine barrel is 0.7 m^3. How many litres does the barrel hold? Exercise 10.04

Dreamstime/Eltoro69

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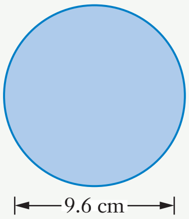
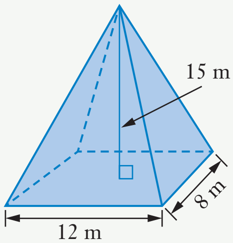
Exercise 10.05

8 Calculate correct to one decimal place the volume of each solid.

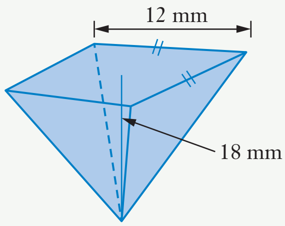
a



b



c

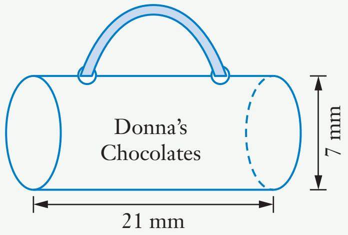


Exercise 10.05

9 This is the container Donna designed to hold the chocolates she makes and sells.

a Calculate correct to the nearest cubic centimetre the volume of the container.

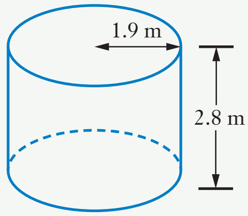
b Express the capacity of the container in litres.



Exercise 10.05

10 **a** Calculate the volume of the water tank, correct to the nearest m^3.

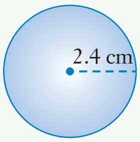
b What is the capacity of the tank in litres?



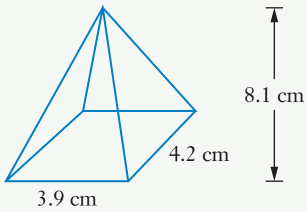
Exercise 10.05

11 Calculate the volume of each solid, correct to the nearest cm^3.

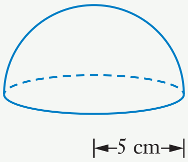
a



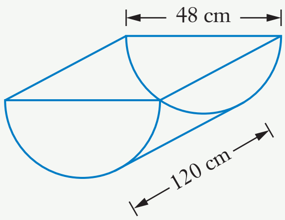
b



c



d



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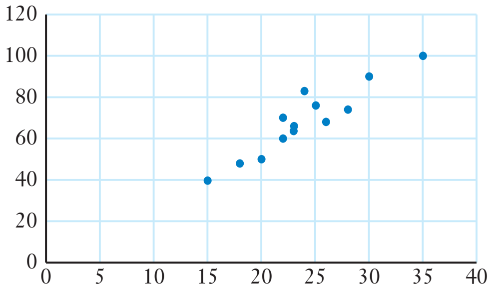
Practice set 2

Section A Multiple-choice questions

For each question, select the correct answer **A**, **B**, **C** or **D**.

1 Which statement is true for this scatterplot?

Exercise 6.01



A As one variable increases, the other variable increases

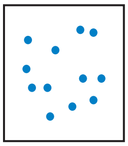
B As one variable increases, the other variable decreases

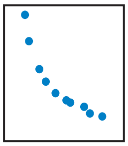
C There are groups of points

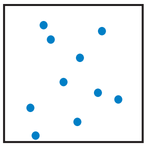
D Most of the points are together but a few are out on their own

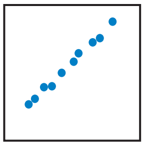
2 On which diagram would we be able to draw a line of best fit?

Exercise 9.01

A 

B 

C 

D 

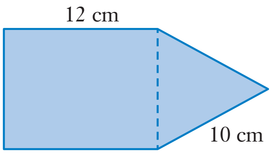
3 Evan is using a map with a scale of 1 : 20 000. The distance from home to the shops on the map is 3.4 cm. What is the actual distance?

Exercise 8.01

A 6800 m **B** 680 m **C** 571 m **D** 57 m

4 This shape is made up of a rectangle and an equilateral triangle. Find its perimeter.

Exercise 7.03

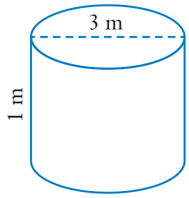


A 22 cm **B** 44 cm

C 54 cm **D** 64 cm

5 Which calculation could be used to find the volume of this cylinder?

Exercise 10.04



A $\pi \times 3 \times 1$ **B** $\pi \times 3^2 \times 1$

C $\pi \times 1.5 \times 1$ **D** $\pi \times 1.5^2 \times 1$

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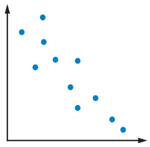
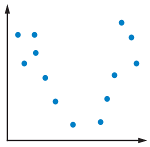
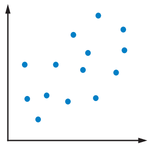
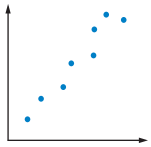
Exercise 6.02

6 Which scatterplot shows an association that is linear and positive?

A 

B 

C 

D 

Exercise 8.04

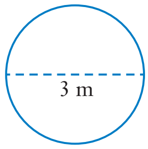
7 John and Julie decide to repaint their ceilings. The total area of their ceilings is 90 m^2 and they require 2 coats of paint. 1 L of paint covers 12 m^2. Paint costs $34.90 for a 4 L can.

How much does the paint cost to repaint the ceilings?

A $69.80 **B** $139.60 **C** $261.75 **D** $523.50

Exercise 7.05

8 Which calculation could be used to find the area of this circle?

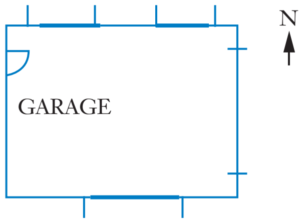


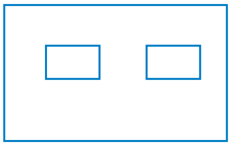
A $\pi \times 3$ **B** $\pi \times 3^2$

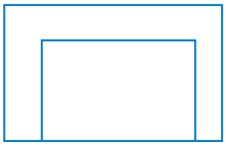
C $2 \times \pi \times 1.5$ **D** $\pi \times 1.5^2$

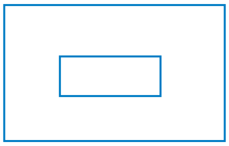
Exercise 8.02

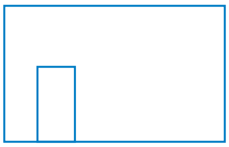
9 Which diagram shows the northern elevation for the garage shown on the plan?



A 

B 

C 

D 

Exercise 10.03

10 The internal dimensions of a refrigerator are 150 cm (height), 60 cm (width) and 40 cm (depth). What is the capacity of the refrigerator in litres?

A 250 L **B** 300 L **C** 360 L **D** 430 L

Exercise 7.04

11 Find the area of a window that is 2 metres long and 90 cm wide.

A 0.18 m^2 **B** 1.8 m^2 **C** 18 m^2 **D** 180 m^2

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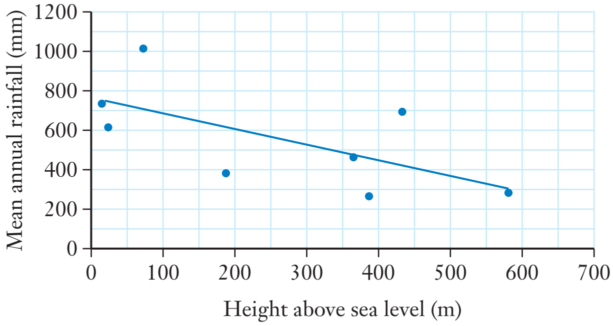
12 This scatterplot and line of best fit shows the relationship between the height above sea level and the mean annual rainfall for some Australian towns.

Exercise 9.02



Use the line of best fit to predict the mean annual rainfall of Avatown, which is 400 m above sea level, and the height above sea level of Clareville, which has a mean annual rainfall of 600 mm.

	Avatown's rainfall	Clareville's height
A	450 mm	200 m
B	450 mm	250 m
C	490 mm	250 m
D	490 mm	200 m

Section B Short-answer questions

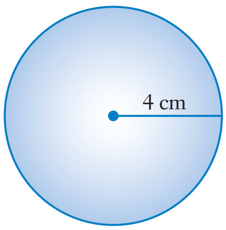
1 The front yard of a house is rectangular and measures 20 m by 8 m. The backyard is also rectangular and measures 35 m by 7.5 m. Calculate the total cost of covering both yards with turf that costs \$18.60/m^2.

Exercise 7.04

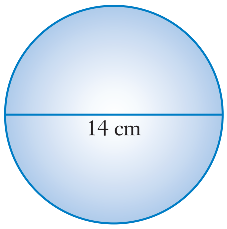
2 Calculate the volume of each sphere, correct to 2 decimal places.

Exercise 10.04

a



b



3 Calculate the surface area of each sphere above, correct to 2 decimal places.

Exercise 7.09

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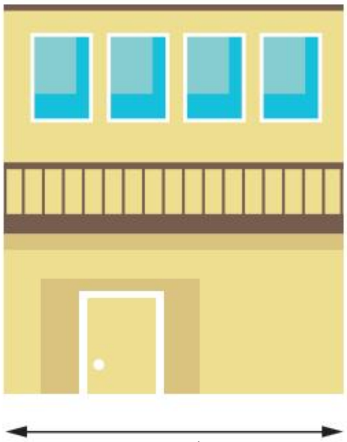
Exercise 8.01

4 Measure the length of each scale drawing, then use the scale ratio to calculate its actual length.

a

b



Exercise 6.04

5 This table compares the heights and resting pulses of a sample of 15 students.

Height (cm)	Resting pulse (beats per minute)
176	64
178	66
151	95
168	76
186	60
187	74
149	70
174	62

Height (cm)	Resting pulse (beats per minute)
172	76
177	64
169	60
159	83
169	68
163	77
163	75

a Would you expect there to be an association between these 2 variables? Why or why not?

b Construct a scatterplot for this data.

c Describe any features of the scatterplot.

d Is there an association between these 2 variables? If so, describe it.

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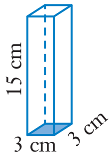
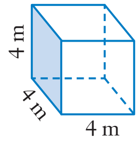
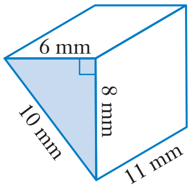
6 Calculate the volume of each prism. Exercise 10.02

a 

b 

c 

7 Find the surface area of each prism above. Exercise 7.07

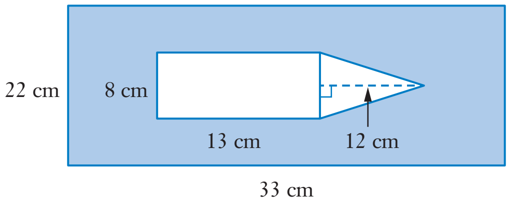
8 A sample of 10 people had their left foot length and left lower arm length (from their elbow to the tip of their middle finger) measured. Exercise 9.01

Left foot length (cm)	22	23	24	25	25.5	26.5	27	28	28	29
Left lower arm length (cm)	43	44.5	43.5	45	46	48	48.5	47	49	50

a Plot this data on a scatterplot.

b Draw a line of best fit for this data.

9 A metal-cutting template is shown below. Exercise 7.06



a Calculate the area of metal removed from the plate.

b Calculate the area of the remaining metal plate.

10 Tim and Kim bought a house with the floor plan shown on page 234 in Exercise 8.03, question **2**. Exercise 8.03

a What are the dimensions of the garage?

b Tim and Kim decide to concrete the garage floor.

i How many square metres of concrete will they need?

ii Concrete costs \$75/square metre for a depth of 100 mm. How much will the concrete for the garage floor cost?

c How many windows are there in this house?

d The carpet in Bedroom 1 needs to be replaced.

i How many square metres of carpet will they need?

ii Carpet costs \$85/m^2. How much will it cost to replace the carpet?

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Exercise 6.03

11 Each pair of variables below have a strong association. For each pair:

- **i** decide if a change in one variable causes a change in the other variable
- **ii** if there is no causal relationship, suggest other factors that might result in the variables having an association.

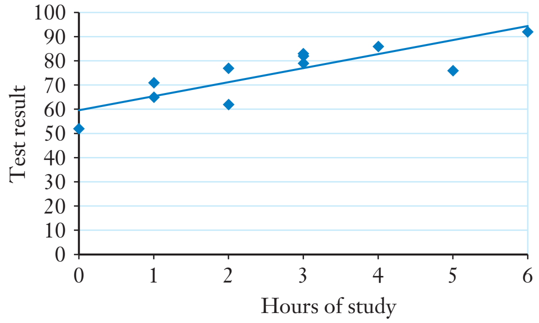
a The length of the right arm and the length of the left arm of the same person.

b The price of a brand of sports shoes and the number of pairs sold.

c The hours of exercise per week for a person and the person's fitness level.

Exercise 9.04

12 Harry asked 11 students how many hours they studied and their test result. This scatterplot shows the results of his survey with a line of best fit for the data.



a Use the line of best fit to predict:

- **i** the test result for a student who studied for 3.5 hours
- **ii** the hours studied by a student who scored 75 in the test

b How reliable are these predictions?

c Would you expect this situation to continue to be a linear relationship beyond the data points? Why or why not?

Exercise 10.04

13 The Pyramid of Cestius in Rome is a square pyramid of solid stone. Its base is 29.6 m square and its height is 37 m. Calculate the volume of stone in this pyramid, correct to the nearest cubic metre.

Exercise 6.01

14 Suravi asked 11 students at her school about how many hours per week they spent playing sport and how many hours per week they spent watching TV.

Sport (hours)	5	6	17	9	3	13	17	5	4	0	10
TV (hours)	8	6	1	2	10	0	2	9	9	10	2

a Present this bivariate data in a scatterplot.

b Comment on the features of the scatterplot.

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15 a Draw a line of best fit for the data on your scatterplot from question **14**.

Exercise 9.04

b What does the line of best fit show about the relationship between these 2 variables?

c Use the line of best fit to predict how many hours of TV a person who plays 8 hours of sport would watch.

d Use the line of best fit to predict how many hours of sport a person who watches 3 hours of TV would play.

e How reliable are these predictions? Justify your answer.

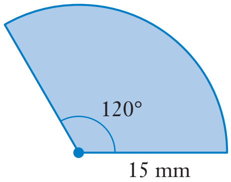
16 *Calculate correct to 2 decimal places:

Exercise 7.02, 7.05

a the area of this sector

b the perimeter of this sector,

*Australian curriculum only, not in WA syllabus



17 Stuart installed a large pipe to transport water from a dam to a nearby community. The area of the circular cross-section of the pipe is 0.95 m^2 and the pipe is 8.7 km long.

Exercise 10.04

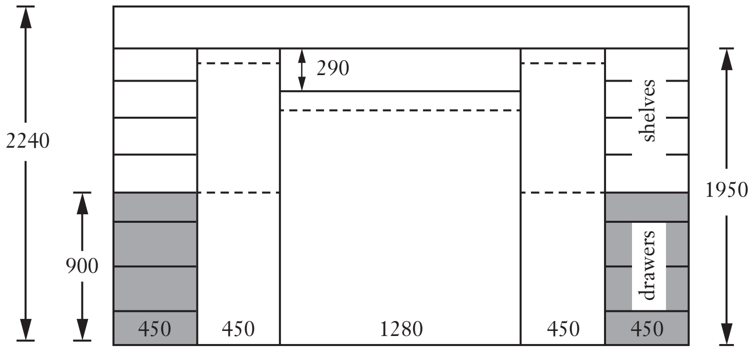
a What is the length of the pipe in metres?

b Calculate the volume of water that the pipe can hold. Answer in cubic metres.

c How many litres of water are in the pipe when it is full?

18 This is the design for a built-in wardrobe. All measurements are in millimetres. Make a scale drawing of this design using a scale of 1 : 20.

Exercise 8.05



Rails for hanging clothes (---). Top rails are 150 mm below the shelves above. Design is symmetrical.

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11.

INVESTING MONEY

Chapter problem

Ryan invested \$25 000 for 3 years, earning compound interest at 4% per year.
Melanie also invested \$25 000 for 3 years, earning simple interest at 4.2% per year.
Whose investment will earn more interest?

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WHAT WILL WE DO IN THIS CHAPTER?

- Calculate and compare simple and compound interest
- Use technology to calculate the future value of a compound interest investment
- Use technology to compare the progress of loans and investments, and the effects of changing interest rates and compounding periods
- Solve problems involving inflation, appreciation and other situations that increase in a similar way to compound interest

HOW ARE WE EVER GOING TO USE THIS?

- Determine the chance that an investment is safe
- Choose a safe investment that will give us reasonable return
- Make calculations to check that we have received the correct investment returns
- Avoid falling for financial scams

11.01 Simple interest

Interesting puzzle

WS Simple interest

WS What's the interest?

WS Simple interest riddle

WS Applications of simple interest

Simple interest

When we invest money with **simple interest** or **flat rate interest**, the amount of interest earned is the same every year. The interest is a percentage of the **principal**, the original amount invested.

Simple interest formula

$$I = Prn$$

where P = principal (what you invest or borrow)

r = **interest rate** per time **period**, as a decimal

n = number of time periods

EXAMPLE 1

Chloe invested \$6000 at 3.25% p.a. simple interest for 4 years.

p.a. = **per annum** = per year

a How much interest will she earn?

b How much will be in her account at the end of 4 years?

Solution

a In the simple interest formula $I = Prn$, $P = 6000$, $r = 3.25\% = 0.0325$ and $n = 4$.

$I = 6000 \times 0.0325 \times 4$

$= 780$

Write the answer.

Chloe will earn \$780 in simple interest.

b Add the interest to the principal.

Account total = \$6000 + \$780

= \$6780

Write the answer.

Chloe will have \$6780 in her account.

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Sometimes money is not invested or borrowed for a whole number of years. We need to make sure the interest rate and the time period match.

EXAMPLE 2

Rosie invested \$9500 at 4.2% p.a. for 30 months. How much interest did she earn?

Solution

The interest rate is per year, but the time is in *months*. We must change 30 months to years by dividing by 12.	Time $= \frac{30}{12}$ years
$P = 9500$, $r = 0.042$, $n = \frac{30}{12}$	$I = 9500 \times 0.042 \times \frac{30}{12}$ $= 997.5$
Write the answer.	Rosie earned \$997.50 in interest.

Converting months to years

- When the interest rate is p.a., the time must be in years.
- Divide the number of months by 12 to change the time into years.

EXAMPLE 3

What percentage interest rate per month is equivalent to 5.52% p.a.?

Solution

Divide an annual interest rate by 12 to change it to a monthly interest rate.	$5.52\% \div 12 = 0.46\%$
Write the answer.	5.52% p.a. is equivalent to 0.46% per month.

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Exercise 11.01 Simple interest

Example 1

1 Calculate the simple interest on each principal.

a \$900 at 3.2% p.a. for 4 years

b \$1560 at 3.9% p.a. for 4 years

c \$4500 at 5.5% p.a. for $3\frac{1}{2}$ years

d \$2750 at 5.1% p.a. for $2\frac{1}{2}$ years

Example 2

2 What is the simple interest on each investment?

a \$840 at 3.5% p.a. for 18 months

b \$12 800 at 6.05% p.a. for 3 months

c \$2960 at $5\frac{1}{2}$% p.a. for 4 months

d \$880 at $6\frac{1}{4}$% p.a. for 1 month.

Example 3

3 What percentage interest per month is equivalent to 4.56% p.a.?

4 NSM Credit Union offers investors 3.12% p.a. simple interest. Using 4 decimal places when required, express this rate of interest as a:

a monthly rate

b weekly rate

c 6-monthly rate

d daily rate

e fortnightly rate

f quarterly rate (3-monthly rate)

Divide by 52 for the weekly rate, divide by 2 for the 6-monthly rate and divide by 4 for the quarterly rate.

5 Nina borrowed \$3200 from a finance company for 2 years at 17% p.a. interest to buy some furniture for her home.

a How much interest did she have to pay?

b How much, including interest, did Nina have to repay the finance company?

6 Mark owed \$865 on his credit card. The credit card company charged him one month's interest at 22% p.a.

a How much interest was he charged?

b Calculate the total amount he had to repay the credit card company.

7 Eddie borrowed \$55 000 from a finance company to set up an online business. He borrowed the money for 6 months and was charged 17.25% p.a. interest. How much did he have to repay the finance company, including interest?

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8 Kelly made the mistake of signing a contract to buy a new house before she sold her old house. When it was time to pay for the new property, Kelly had to borrow \$960 000 in bridging finance at 1.44% monthly.

a What daily interest rate is equivalent to 1.44% per month? Assume a month has 30 days.

b Kelly borrowed the money for 45 days. Calculate the amount of interest she had to pay.

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9 Last January, Jude received a text message from Darren, someone she knew briefly years ago. He told her that he was now working for an investment company and knew of a investment opportunity that was paying 15% p.a. interest, paid every 6 months. Jude decided to invest \$20 000 for 6 months in this business.

a How much interest will Jude's investment earn at 15% p.a. for one month?

b How much, including interest, should Jude receive when she withdraws her investment after 6 months?

c In July, Jude couldn't contact Darren. He had disappeared with her money. Why should Jude have been suspicious of the **investment opportunity**?

10 This graph shows the total value of a \$1000 simple interest investment at 15% p.a. and 20% p.a. for 5 years.

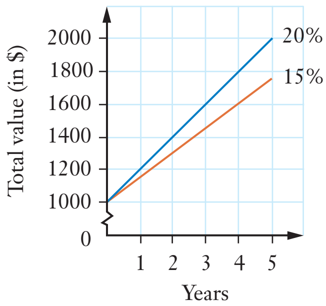
a How much is the investment worth after 4 years at 15% p.a.?

b How much simple interest is earned in 3 years at 20% p.a.?

c How much more interest is earned in 5 years at 20% p.a. than at 15% p.a.?

d Both graphs start at 1000 on the vertical axis. What is the meaning of the 1000?

Total value of \$1000 simple interest investment



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MoneySmart

INVESTIGATION

IS MY INVESTMENT SAFE?

In this investigation, we will visit the MoneySmart website to find out about checking whether an investment is safe.

What you have to do

1. Go to 'Scams' and 'Companies you should not deal with'.
2. Should you consider investing with the company Thomas Moore Global? Explain.
3. In which country is Thomas Moore Global based?
4. List 4 different companies, each in a different country, with which you shouldn't invest.
5. Randomly choose 8 letters from the alphabet, for example, B, F, G, K, N, P, Q and Z.
6. Categorise each company that starts with the 8 letters according to the company's location, using this frequency table.

Geographical region	Tally	Frequency
Africa		
Asia		
Australia		
Europe		
Middle East, including United Arab Emirates		
South America		
United States		
Total		

7. What conclusion can you draw about the locations of these companies?
8. Read the section on the website about why overseas scammers target Australians, then write a paragraph describing your findings in this investigation. Include a sector graph to display your data visually.

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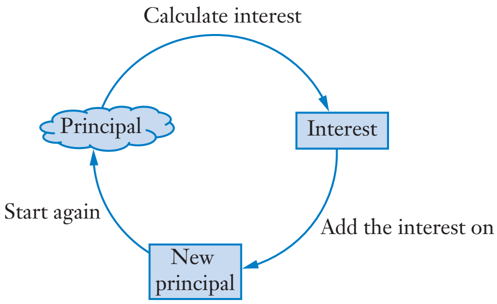
11.02 Compound interest

There's a well-known claim often quoted by financial investors:

'Compound interest is the eighth wonder of the world.

Those who understand it, earn it. Those who don't, pay it.'

With **compound interest,** you receive 'interest on your interest'. After interest is added to the investment, the next time interest is calculated, it will be based on this larger amount. The amount of interest grows or 'is **compounded**' as the size of the investment grows.



EXAMPLE 4

Sarina invested \$2000 at 5% p.a. interest compounded annually.

a Calculate the future value of Sarina's investment at the end of three years.

b Calculate the total interest she earned.

Solution

a We need to calculate the interest for the first year and add it to the principal before we calculate the following year's interest.

$I = Prn$, where $r = 5\% = 0.05$, $n = 1$ and P changes each year, but its initial value is 2000.

	Interest	Balance
End of the 1st year	$I = \$2000 \times 0.05 \times 1$ $= \$100$	$\$2000 + \$100 = \$2100$
End of the 2nd year	$I = \$2100 \times 0.05 \times 1$ $= \$105$	$\$2100 + \$105 = \$2205$
End of the 3rd year	$I = \$2205 \times 0.05 \times 1$ $= \$110.25$	$\$2205 + \$110.25 = \$2315.25$

The future value of Sarina's investment is worth \$2315.25.

Alternatively, calculate the value of 5% interest added to the principal by multiplying by 1.05, for example, \$2000 × 1.05 = \$2100, giving you the answers in the Balance column straight away.

b	Total interest = final balance – original principal	Total interest = \$2315.25 – \$2000 = \$315.25
	OR add the 3 interest amounts in the middle column of the table.	Total interest = \$100 + \$105 + \$110.25 = \$315.25
	Write the answer.	The total interest earned over the 3 years is \$315.25.

Exercise 11.02 Compound interest

Example 4

1 Yumi invested \$8000 at 6% p.a. interest compounded annually for 3 years. Copy and complete this table to calculate the future value of her investment at the end of 3 years.

	Interest	Balance
End of the 1st year	$I = Prn$ $= \$8000 \times 0.06 \times 1$ = \$______	\$8000 + \$____ = \$______
End of the 2nd year	$I = Prn$ = \$______ × 0.___ × 1 = \$____	\$_____ + \$_____ = \$_____
End of the 3rd year	$I = Prn$ = \$____ × 0.____ × 1 = \$_____	\$_____ + \$_____ = \$______

2 Jayden invested \$12 000 for 3 years at 4% p.a. **compounding** yearly.

a How much interest did he earn in the first year?
b How much was in his account at the end of the first year?
c How much was in his account at the end of the second year?
d Calculate the future value of his investment at the end of 3 years.
e How much interest will Jayden earn during his 3-year investment?

3 Nhi invested \$4000 at 3.2% p.a. for 2 years compounding annually.

a Calculate the future value of her investment at the end of the 2 years.
b How much interest will she earn during the 2 years?
c How much less interest would Nhi earn had it been simple interest rather than compound interest?

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4 Suresh has saved \$14 000 from his after-school job that he plans to spend on a car. He is going to invest the money for 3 months until he has his P-plates. Suresh's investment is going to pay 0.7% per month, interest compounded monthly.

a Copy and complete the table.

	Interest	Balance
End of the 1st month	$I = Prn$ $= \$14\,000 \times 0.007 \times 1$ $= \$\_\_\_\_\_\_$	$\$14\,000 + \$\_\_\_\_\_ = \$\_\_\_\_$
End of the 2nd month	$I = Prn$ $= \$\_\_\_\_ \times 0.\_\_\_ \times 1$ $= \$\_\_\_$	$\$\_\_\_\_\_ + \$\_\_\_\_\_ = \$\_\_\_\_\_$
End of the 3rd month	$I = Prn$ $= \$\_\_\_\_\_ \times 0.\_\_\_ \times 1$ $= \$\_\_\_\_\_\_$	$\$\_\_\_\_\_ + \$\_\_\_\_\_\_ = \$\_\_\_\_\_\_\_$

b How much interest will Suresh make during the 3-month investment?

5 This table shows the interest and balance calculations for Keira's investment.

	Interest	Balance
End of the 1st year	$I = Prn$ $= \$9600 \times 0.08 \times 1$ $= \$768.00$	$\$9600 + \$768.0$ $= \$10\,368$
End of the 2nd year	$I = Prn$ $= \$10\,368 \times 0.08 \times 1$ $= \$829.44$	$\$10\,368 + \$829.44$ $= \$11\,197.44$

a How much money did Keira invest?

b What was the annual rate of compound interest?

c How much interest did Keira's investment earn?

6 Last night Zac received a phone call from an investment advisor he doesn't know telling him about an investment opportunity that is virtually risk-free. The offer is likely to be fully subscribed quickly, so he has to agree straight away. The minimum investment is \$10 000 and the interest rate is 15% p.a. compounding annually for 3 years.

a Calculate the amount of interest the advisor claims \$10 000 will receive in the next 3 years.

b What signs show that this opportunity is most probably a scam?

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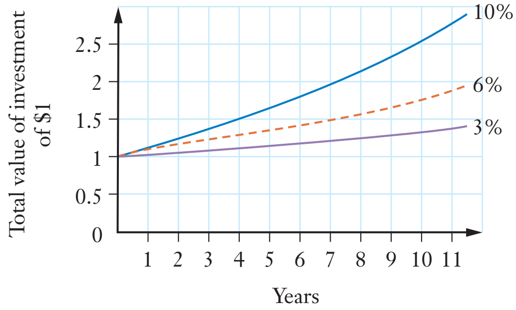
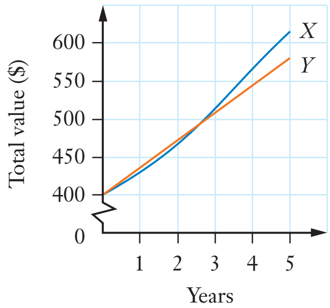
7 This graph shows the future value of a \$1 compound investment at 3% p.a., 6% p.a. and 10% p.a.



a Describe how the shape of a compound interest graph is different to the shape of a simple interest graph.

b Approximately how long does it take for a \$1 investment to double its value at 10% p.a. compound interest?

8 Nick and Adam invested the same amount of money at 9% p.a., but Nick's investment was 9% p.a. compounded annually while Adam's was 9% p.a. simple. The graphs show the future value of their investments after 5 years.



a How much was Nick and Adam's original investment?

b Match the graphs X and Y to Adam's and Nick's investment.

c After 5 years, state whose investment is higher and by how much.

INVESTIGATION

IS IT A SCAM?

Technology and the Internet have changed the way we do things and made life easier for us. But technology has also made life easier for criminals to use scams and financial fraud to rob us of our money.

1 Visit the **Scamwatch** website.

2 Go to **Types of scams** and investigate **Unexpected money** scams. List 3 different ways people fall for scams.

3 Prepare a list of answers you could give to possible scammers when they are trying to convince you to do something you don't want to do.

4 Investigate **Get help**. Describe what you should do if you have sent money to someone you think is a scammer.

Chapter problem

You've covered the skills required to solve the chapter problem. Can you solve it now?

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11.03 Inflation and appreciation

We can use the same process that we use to calculate the future value of a compound interest investment to solve problems involving inflation, appreciation, population growth and other types of growth.

Calculating compound interest can be tedious and time-consuming. It's time to learn a calculator shortcut for increasing an amount by a percentage. For example, to increase a quantity by 7.5%, all we have to do is multiply by (100% + 7.5%) = 107.5% = 1.075.

EXAMPLE 5

Compound interest

Calculate the future value of a principal of $1000 in 3 years at a compound interest rate of 4.5% p.a.

Solution

First, calculate the future value after the first year by increasing $1000 by 4.5%.

The shortcut is to multiply $1000 by 100% + 4.5% = 104.5% = 1.045.	Amount at the end of the 1st year = $1000 × 1.045 = $1045
Repeat: increase $1045 by 4.5%.	Amount at the end of the 2nd year = $1045 × 1.045 = $1092.025 ≈ $1092.03
Repeat: increase $1092.03 by 4.5%.	Amount at the end of the 3rd year = $1092.03 × 1.045 = $1141.17135 ≈ $1141.17
Write the answer.	The future value of the investment is $1141.17.

Inflation is an increase in the prices of goods and services.

Appreciation is an increase in the value of an item over time, such as artwork, gold, a prestige car, land or a house.

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EXAMPLE 6

Emma's weekly groceries cost \$180. If the rate of inflation of 2.5% p.a. continues for the next 3 years, how much more will Emma have to pay for the same groceries then?

Solution

Increase \$180 by 2.5% for 3 years.	Future cost of the groceries
To increase by 2.5% three times, multiply by (100% + 2.5%) = 102.5% = 1.025 three times.	$= \$180 \times 1.025 \times 1.025 \times 1.025$ $= \$193.840\ 312\ 5$ $= \$193.84$
To calculate 'how much more', subtract the original cost from the future cost.	Extra amount = \$193.84 – \$180 = \$13.84
Write the answer.	Emma's groceries will cost \$13.84 more in 3 years.

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Exercise 11.03 Inflation and appreciation

Example 5

1 Write the decimal beginning with '1.' (such as 1.09) that I have to multiply an amount by to increase the amount by:

a 8% **b** 6% **c** 3.5% **d** 2.8% **e** 1.75%

2 Write the percentage that an amount increases by when I multiply the amount by:

a 1.07 **b** 1.065 **c** 1.015 **d** 1.12 **e** 1.1

Example 6

3 Dianna invested $1000 at 3.6% p.a. compound interest. Which expression is the calculation for the future value of Dianna's investment in 2 years' time?

A $\$1000 \times 1.036$ **B** $\$1000 \times 1.036 \times 1.036$ **C** $\$1000 \times 1.036 \times 2$

4 If inflation is 3.1% p.a., calculate the cost of a pair of jeans in 4 years if it costs $45 today.

5 Annabel is saving to buy a new car. Prices are increasing at a constant rate of 4% p.a. The car is priced at $18 000 today. How much will a similar new car cost in 3 years' time? Answer to the nearest dollar.

6 An antique table is valued at $560 and its value is increasing at 7% p.a. How much will the table be worth in 6 years' time, to the nearest dollar?

7 The average rate of Australian wage increases has been 3.38% p.a. since the year 2010.

a In 2010, Joe earned $13 per hour. 10 years later, Joe was still doing exactly the same job. How much did he earn per hour in 2020?

b In 2014, Maggie earned $23/hour. What was her wage in 2016?

8 From 1951 to 2016, Australia's average rate of inflation was 5.09% per year. Imagine a soccer ball cost $1 in 1951. How much would a similar soccer ball cost in 2016, 65 years later?

9 At the end of World War II, Hungary experienced hyperinflation. Hyperinflation is when prices rise by 50% or more per month. The currency in Hungary at the time was called a pengo, and prices rose by 207% each day!

Complete this table to show how the cost of a basket of essential food supplies changed during the Hungarian time of hyperinflation. Write values correct to 2 decimal places.

	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
Cost of basket of food	8 pengo					

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10 Wars are a common cause of hyperinflation. In 1921, after World War I, Austria's inflation was 134% p.a. or 11.17% per month.

a Use a calculation to show that you can increase a price by 11.17% by multiplying by 1.1117.

b At the beginning of 1921, a meal in Austria cost 9 Austrian shillings. What did the same meal cost 5 months later? Answer to the nearest shilling.

Getty Images/Universal History Archive

11 Jono collects mint condition *Donald Duck* comics. One of his mint comics is signed by the creator, Carl Barks. Currently it's valued at $1200 and it is appreciating (increasing in value) by 9% p.a. How much will this comic be worth in 4 years' time? Answer correct to the nearest $100.

12 The population of a small rural town is increasing by 3% p.a. Today the population is 2450. If the annual growth continues at 3% per year, what will be the size of the population in 5 years?

13 In 2019, Australia's population was 25 500 000. If the population increased at approximately 3.2% p.a.:

a estimate Australia's population, correct to the nearest thousand, in 2025

b when is Australia's population expected to reach 50 million?

Use a guess-and-check method.

14 This table shows the value of some prize collectables in 2018.

	Item	Value
a	Certified American baseball cards	$150 000
b	Mint condition *Spiderman* comic book	$20 000
c	Vintage *Star Wars* costume in original packet	$18 000
d	Original Apple computer in unopened box	$670 000
e	Limited edition *Frozen* doll in unopened box	$2500
f	Original copy of one of Da Vinci's notebooks	$50 000

Assuming an annual rate of appreciation of 5%, calculate the 2021 value of each collectable, correct to the nearest dollar.

No one knows what future rates of appreciation will be nor what particular items will become valuable collectables. If you own any collectables, keep them in their unopened, original box to keep their value as high as possible.

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INVESTIGATION

COLLECTABLES

Do an online search for collectables.

- Write down a list of items that are currently valuable.
- Which collectables increase in value more quickly than others?

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11.04 Interest calculators

We can use an online interest calculator to calculate compound interest, such as the one on **The Calculator Site** or **Money Smart.**

MoneySmart

WS Compounding periods: spreadsheet

WS Compound interest table

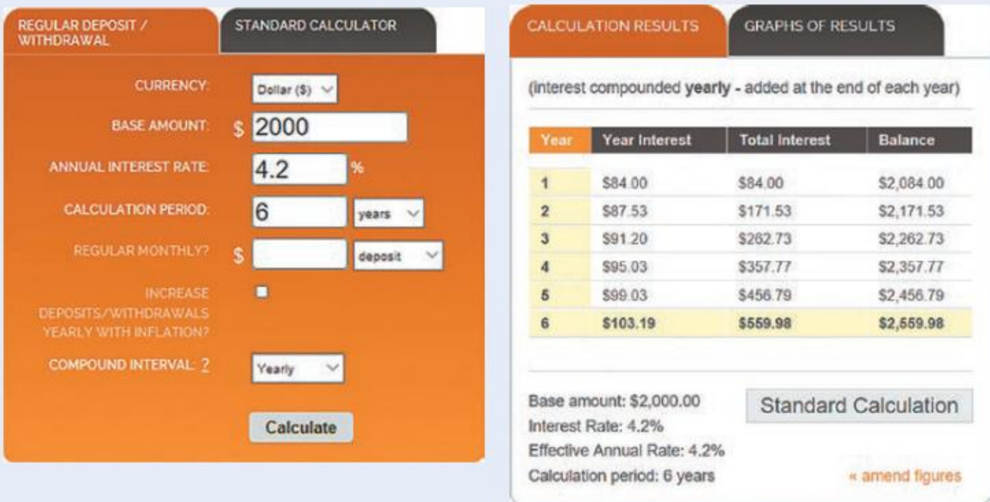
Interest calculator

EXAMPLE 7

Jana invested \$2000 in an account that earned 4.2% p.a. interest, compounded yearly for 6 years. Calculate the final value of her investment and the amount of interest she earned.

Solution

Entering the principal, interest rate, period (6 years) and compound interval (yearly) into the compound interest calculator on The Calculator Site and clicking 'Calculate':



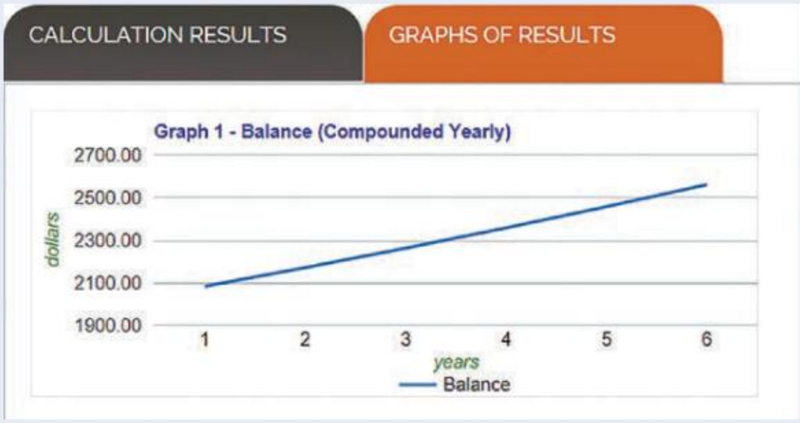
Year	Year Interest	Total Interest	Balance
1	\$84.00	\$84.00	\$2,084.00
2	\$87.53	\$171.53	\$2,171.53
3	\$91.20	\$262.73	\$2,262.73
4	\$95.03	\$357.77	\$2,357.77
5	\$99.03	\$456.79	\$2,456.79
6	\$103.19	\$559.98	\$2,559.98

The Calculator Site (thecalculatorsite.com)

Final value = \$2559.98

Total interest = \$559.98

Click on the 'Graphs of results' tab to see a graph of the future value over the 6 years.



The Calculator Site (thecalculatorsite.com)

Exercise 11.04 Interest calculators

Use an online compound interest calculator for each question.

Example 7

1 Jayne invested \$3000 for 7 years in annually compounding interest at 4.1% p.a.

- **a** Calculate the future value of her investment at the end of 7 years.
- **b** How much interest will the investment earn?
- **c** Use the online calculator to construct a graph showing the value of her investment during the 7 years.

2 Determine the future value of each investment.

	Principal	Interest rate p.a.	Term in years
a	\$5500	2.9%	6
b	\$1260	1.8%	4
c	\$12 000	3.15%	3
d	\$36 500	3.2%	8

3 Taj invested \$46 000 at 2.85% p.a. annually compounding interest for 3 years. Determine the interest his investment earned each year and the total amount of interest.

4 When you lend someone money at compound interest, the calculations involved are exactly the same as when you invest in compound interest. Mr Kahill lent the soccer club \$50 000 to use in upgrading club facilities. The loan was for 8 years at 5.6% p.a. annually compounding interest. The club agreed to repay the loan plus interest in one lump sum at the end of 8 years.

- **a** How much will the club have to repay at the end of the loan?
- **b** How much interest is the club being charged?
- **c** Construct a graph to display the amount the club owes Mr Kahill for the term of the loan.

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5 Determine the future value of a $6400 loan annually compounding at 3.67% p.a. for 10 years, assuming no repayments are made.

6 Sandra has $20 000 to invest for 3 years. She is considering 2 different investments.

Option 1	Simple interest at 5.5% p.a.
Option 2	Compound interest at 4.5% p.a.

Which option do you recommend she take? Use figures to justify your answer.

11.05 Compound interest spreadsheets

Spreadsheets are powerful tools to use for financial calculation. In this section, we will use a prepared spreadsheet as well as make one of our own.

Interest calculator

Compound interest graph

Compound interest

EXAMPLE 8

Download the 'Compound interest' spreadsheet from NelsonNet and use it to determine the amount of interest that a $1200 investment will earn at 12% p.a. interest compounded yearly for 8 years.

Solution

Insert $1200 for the principal and 12% for the interest.

	A	B	C	D
1	**Compound interest spreadsheet**			
2	Only enter data in cells shaded blue.			
3				
4	Principal	$1,200.00	Annual rate of interest as a percentage	12%
5				
6				
7		**Account balance at the beginning of the year**	**Interest earned during the year**	**Account balance at the end of the year**
8	**Year 1**	$1,200.00	$144.00	$1,344.00
9	**Year 2**	$1,344.00	$161.28	$1,505.28
10	**Year 3**	$1,505.28	$180.63	$1,685.91
11	**Year 4**	$1,685.91	$202.31	$1,888.22
12	**Year 5**	$1,888.22	$226.59	$2,114.81
13	**Year 6**	$2,114.81	$253.78	$2,368.59
14	**Year 7**	$2,368.59	$284.23	$2,652.82
15	**Year 8**	$2,652.82	$318.34	$2,971.16
16				

Future value after 8 years = $2971.16

Interest = Future value – Principal

= $2971.16 – $1200

= $1771.16

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EXAMPLE 9

David often lends money to members of his family and he wants the principal and the interest repaid all at the end. Create a spreadsheet that he can use to calculate the total amount to be repaid at the end of the loan.

Solution

The spreadsheet needs to have places to enter the principal (C3) and the interest rate (C4).

We'll enter the interest rate as a decimal to simplify our formulas.

In B7 we want the amount loaned.
Formula for B7: =C3

D12

	A	B	C	D
1	David's spreadsheet			
2	Only enter information in the			
3		Amount loaned		
4		Interest rate as a decimal		
5				
6	Time in Years	Amount owed at the beginning of the year	Interest	Amount owed at the end of the year
7				
8				

We need 1 here as the starting year. The next year A8 = A7 + 1.

The amount owing at the beginning of the second year is the same as the amount owing at the end of the first year.
Formula for B8: =D7

Use *Prn* for the interest.
P = B7, r = C4, n = 1, so we can leave n out.
But we don't want the interest rate to change when we 'copy down', so we put \$ signs in the formula for C7: =B7*\$C\$4

This is the amount at the beginning of the year, plus interest.
Formula for D7: =B7 + C7

Here is the spreadsheet with the formulas included.

D12

	A	B	C	D
1	David's spreadsheet			
2	Only enter information in the			
3		Amount loaned		
4		Interest rate as a decimal		
5				
6	Time in Years	Amount owed at the beginning of the year	Interest	Amount owed at the end of the year
7	1	=C3	=B7*\$C\$4	=B7+C7
8	=A7+1	=D7		

Enter 5000 in C3 and 0.06 in C4 for a principal of \$5000 and an interest rate of 6% p.a., and use 'currency' format for all the cells to display money values. The final step is to highlight the last cell containing a formula in each column and fill it down.

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You'll get something like this:

	A	B	C	D
1	David's spreadsheet			
2	Only enter information in the cells shaded green			
3		Amount loaned	$5,000.00	
4		Interest rate as a decimal	0.06	
5				
6	Time in Years	Amount owed at the beginning of the year	Interest	Amount owed at the end of the year
7	1	$5,000.00	$300.00	$5,300.00
8	2	$5,300.00	$318.00	$5,618.00
9	3	$5,618.00	$337.08	$5,955.08
10	4	$5,955.08	$357.30	$6,312.38

Now the spreadsheet is ready for David to use.

Shutterstock.com/Koldunova Anna

Exercise 11.05 Compound interest spreadsheets

Use the 'Compound interest' spreadsheet from NelsonNet for questions **1** to **3**.

Compound interest

Example 8

1 Use the spreadsheet to determine the future value of each investment.

	Principal	Interest rate p.a.	Term in years
a	$1000	5%	4
b	$2500	4.3%	6
c	$32 500	3.75%	8
d	$50 000	2.1%	5

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2 Calculate the interest earned on each investment.

	Principal	Interest rate p.a.	Term in years
a	\$3000	4%	5
b	\$5250	3.25%	4
c	\$9600	2.9%	8
d	\$12 000	1.3%	6

3 Goran is saving to buy a car. He now has \$5460 saved from his part-time job that he plans to invest at 2.9% p.a. for 3 years.

a Calculate the future value of this investment.

b Goran hopes to have saved another \$4000 when the investment matures. Calculate the total amount he will have to spend on the car.

Example 9

Peta's spreadsheet

4 Peta is creating a spreadsheet to show the future value of a compound interest investment. Ask your teacher to download 'Peta's spreadsheet' from NelsonNet.

	A	B	C	D
1	**Peta's spreadsheet**			
2	Only enter data in the cells shaded yellow.			
3				
4	Present value	\$6,000.00		
5	Annual interest rate as a percentage	0.45		
6				
7	Year	Value at the beginning of the year	Interest	Value at the end of the year
8	1			
9				

a Write the formulas for cells B8, C8, D8 and A9, enter the formulas into the spreadsheet, then copy the formulas down to row 25.

b Peta meant to enter the decimal for 4.5% in B5 but she made a mistake. Correct her mistake.

c Use Peta's spreadsheet to determine the future value of a \$24 000 investment at 4.5% annually compounding interest for 16 years.

d How much interest will this investment earn?

5 **a** Design your own spreadsheet to show the future value of a compound interest investment. Can you arrange it differently to the examples in this chapter? Add colour to make your spreadsheet interesting.

b Use your spreadsheet to determine the amount of time it takes for a \$3000 investment to double in size (that is, become \$6000) when the interest rate is 4% p.a.

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11.06 Different compounding periods

So far in this chapter we've only considered investments with interest compounding yearly, but interest can compound monthly, quarterly, daily or any time period. However, the calculations can be very repetitive and time-consuming. Fortunately, we can use technology to do the long calculations for us.

In this section you will need access to an online calculator or ask your teacher to download the 'Different compounding periods' spreadsheet from NelsonNet.

Different compounding periods

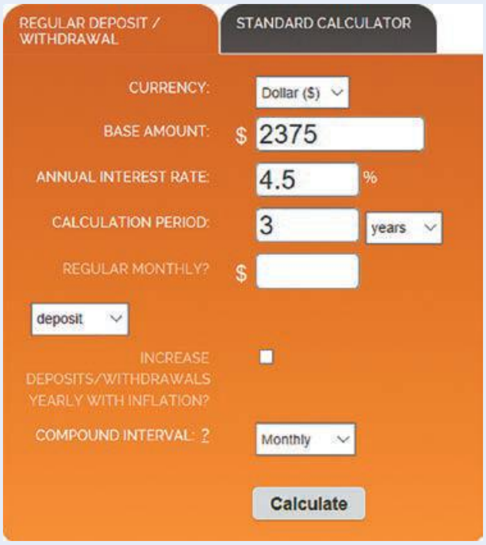
EXAMPLE 10

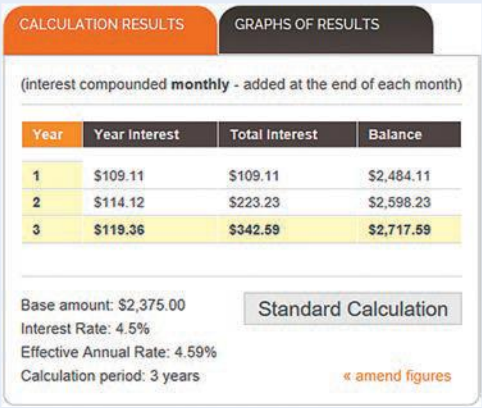
Zack invested \$2375 for 3 years in an account that earned 4.5% p.a. interest, compounded monthly. Use technology to calculate the future value of his investment and the amount of interest he will earn.

Solution

Principal \$2375, interest rate 4.5% p.a., period 3 years, compounded monthly.

Online calculator





The Calculator Site (thecalculatorsite.com)

Future value = \$2717.59

Total interest = \$342.59

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‘Different compounding periods’ spreadsheet

As interest is compounded monthly, the number of periods per year = 12.

	A	B	C	D	E
1	Different compounding periods				
2					
3	How much compound interest will I earn?				
4	Only enter data in cells shaded in blue.				
5					
6	Principal	$2,375.00			
7	Annual rate of interest as a percentage	4.50%		Interest rate per compounding period, as a decimal	0.00375
8	Number of compounding periods per year	12		Number of compounding periods	36
9	Length of the investment in years	3		Final value of the investment	$2,717.59
10				Interest earned during the investment	$342.59
11					

Future value = $2717.59

Total interest = $342.59

Exercise 11.06 Different compounding periods

Use the technology of your choice to answer these questions.

Example 10

1 Determine the amount of interest that each investment will earn.

	Principal	Interest rate p.a.	Term	Compounding period
a	$10 000	6%	3 years	monthly
b	$2500	4.2%	4.5 years	quarterly
c	$1840	3.6%	5 years	half yearly
d	$3800	4.6%	2 years	daily

‘Quarterly’ means 4 times per year, that is, every 3 months

2 Marissa won $5800 in lotto and invested it at 7.2% p.a. monthly compounding interest for 3 years.

a How much will Marissa’s win grow to in 3 years?

b How much interest will Marissa’s investment earn?

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3 Brody saved $1660 for a holiday after he finishes Year 12. He invested it at 5% p.a. interest compounded monthly for one year. How much will be in his holiday account at the end of the year?

4 Jackie received a $2500 bonus as a result of increased company profits. She is going to invest it for one year at 6% p.a. How much *more* interest will she earn in monthly compounding compared to simple interest?

5 When Kristy was made redundant at work, she was given a lump sum payment of $40 000, which she invested for 5 years. How much more interest will she earn from *daily* compounding than *annually* compounding interest at 6% p.a.?

6 On his retirement, Yo-han received a lump sum superannuation payment of $450 000 and he is going to invest it for 2 years. He can invest it at 6.4% p.a. monthly compounding or 6.55% p.a. annually compounding interest. Which investment will give him the better return? Justify your answer.

iStock.com/XiXinXing

7 Lucy has $200 000 to invest to provide for her retirement. She considers 2 different investment options.

	Investment term	Fees
Finance company	5% p.a. compounding monthly	$10 per month
Managed funds	6% p.a. compounding daily	6.5% of the interest earned

Which option will give Lucy the higher return in 12 months? Explain your answer.

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INVESTIGATION

DIFFERENT COMPOUNDING PERIODS

In this investigation, you are going to determine how changing the compounding period (how often interest is compounded) affects the amount of interest earned.

What you have to do

1 Use either an online calculator or a spreadsheet to complete the missing values in the table.

Invest \$10 000 for 4 years at 8% p.a.

Compounding period	Total interest earned
Annually	
Every 6 months	
Monthly	
Weekly	
Daily	

2 Write a sentence to describe the observations you've made.

3 Check the correctness of your observation by completing a table for another investment.

Invest \$50 000 for 7 years at 12% p.a. compounding interest

Compounding period	Total interest earned
Annually	
Every 6 months	
Monthly	
Weekly	
Daily	

4 Chris is going to invest \$2000 for 3 years at 6% p.a. He can select either monthly compounding or fortnightly compounding interest. Which compounding period will give him the better return? Give a reason for your answer.

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INVESTIGATION

IS COMPOUND INTEREST ALWAYS BETTER THAN SIMPLE INTEREST?

In this investigation, you are going to make some calculations to help you choose the investment with the better return.

What you have to do

1 Copy the tables below.

2 Use the technology of your choice to complete the tables. To make it easier, make the principal $1000 in every calculation.

3 Complete the class discussion questions after you have finished the calculations.

WS Simple vs compound interest: spreadsheet

WS Comparing interest rates

Comparing interest rates

Which is better: simple or compound interest?

Part A

The interest rates and the terms are the same.

Simple interest	Interest compounded annually	Summary
Term: 4 years Interest rate: 5% p.a.	Term: 4 years Interest rate: 5% p.a.	Simple interest = Compound interest = Which investment is better?
Term: 6 years Interest rate: 3% p.a.	Term: 6 years Interest rate: 3% p.a.	Simple interest = Compound interest = Which investment is better?
Term: 20 years Interest rate: 7.5% p.a.	Term: 20 years Interest rate: 7.5% p.a.	Simple interest = Compound interest = Which investment is better?

Part B

The terms are the same, but the compound interest rate is higher than the simple interest rate.

Simple interest	Interest compounded annually	Summary
Term: 4 years Interest rate: 5% p.a.	Term: 4 years Interest rate: 5.1% p.a.	Simple interest = Compound interest = Which investment is better?

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Simple interest	Interest compounded annually	Summary
Term: 6 years Interest rate: 3.75% p.a.	Term: 6 years Interest rate: 4% p.a.	Simple interest = Compound interest = Which investment is better?
Term: 20 years Interest rate: 7.5% p.a.	Term: 20 years Interest rate: 8.1% p.a.	Simple interest = Compound interest = Which investment is better?

Part C

The terms are the same, but the simple interest rate is higher than the compound interest rate.

Simple interest	Interested compounded annually	Simple interest
Term: 4 years Interest rate: 5% p.a.	Term: 4 years Interest rate: 4.8% p.a.	Simple interest = Compound interest = Which investment is better?
Term: 6 years Interest rate: 3% p.a.	Term: 6 years Interest rate: 2.75% p.a.	Simple interest = Compound interest = Which investment is better?
Term: 20 years Interest rate: 12% p.a.	Term: 20 years Interest rate: 7.5% p.a.	Simple interest = Compound interest = Which investment is better?
Term: 3 years Interest rate: 12% p.a.	Term: 3 years Interest rate: 7.5% p.a.	Simple interest = Compound interest = Which investment is better?

Class discussion questions

- When the interest rates and the terms are the same, does simple or compound interest give the better return?
- When the compound interest rate is higher than the simple interest rate, and the terms are the same, which type of investment produces the better return?
- When the simple interest rate is bigger than the compound rate, and the terms are the same, will one or the other type of interest always give the better return?

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KEYWORD ACTIVITY

DEFINITIONS MATCH

Match the terms in the left column with their correct meanings in the right column.

	Word		Meaning
1	future value	**A**	An illegal method used to trick people out of their money
2	compound interest	**B**	Occurring every year
3	compounding period	**C**	The rate of return on an investment, usually expressed as a percentage per year.
4	scam	**D**	Interest that is calculated only on the original principal.
5	annual	**E**	The amount of time between compound interest calculations for an investment
6	appreciate	**F**	The original amount invested or borrowed
7	inflation	**G**	To increase in value, such as the value of an antique.
8	interest rate	**H**	The value of an investment over time.
9	principal	**I**	An increase in the price of an item, such as the cost of bread.
10	superannuation	**J**	Interest that is calculated on the current value of an investment, including interest previously added to the principal.
11	simple interest	**K**	A regular saving fund to provide an income when you are retired

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SOLUTION TO THE CHAPTER PROBLEM

Problem

Ryan invested \$25 000 for 3 years, earning compound interest at 4% per year.

Melanie also invested \$25 000 for 3 years, earning simple interest at 4.2% per year.

Whose investment will earn more interest?

Solution

Use an online calculator or spreadsheet to calculate the future value of Ryan's investment.

Principal is \$25 000, interest rate 4% p.a. and the time is 3 years.

An online calculator shows that Ryan's investment will be worth \$28 121.60 at the end of 3 years.

$$\begin{aligned}\text{Ryan's interest} &= \$28\ 121.60 - \$25\ 000 \\ &= \$3121.60\end{aligned}$$

Use the simple interest formula to calculate Melanie's interest.

Interest = Prn

$P = \$25\ 000$, $r = 0.042$ and $n = 3$

$$\begin{aligned}\text{Melanie's interest} &= \$25\ 000 \times 0.042 \times 3 \\ &= \$3150\end{aligned}$$

Compare the interest:

Melanie's investment will earn \$28.40 more interest than Ryan's.

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11. TEST YOURSELF

Investing money

Practice quiz

1 Calculate the simple interest earned on a \$4000 investment at 4.1% p.a. for 8 years. Exercise 11.01

2 Josephine invested \$8400 at 4% p.a. compounded yearly. Exercise 11.02

a Copy and complete this table to determine the future value of her investment over 3 years.

	Interest	Balance
End of the 1st year	$I = Prn$ $= \$8400 \times 0.04 \times 1$ $= \$\_\_\_\_\_\_$	$\$8000 + \$\_\_\_\_\_ = \$\_\_\_\_$
End of the 2nd year	$I = Prn$ $= \$\_\_\_\_ \times 0.\_\_\_ \times 1$ $= \$\_\_\_$	$\$\_\_\_\_\_ + \$\_\_\_\_\_ = \$\_\_\_\_\_$
End of the 3rd year	$I = Prn$ $= \$\_\_\_\_\_ \times 0.\_\_\_ \times 1$ $= \$\_\_\_\_\_\_$	$\$\_\_\_\_\_ + \$\_\_\_\_\_\_ = \$\_\_\_\_\_\_\_$

b How much interest did the investment earn?

3 By what number do I multiply an amount to increase the amount by 7%? Exercise 11.03

4 Today, a box of groceries costs \$56. If inflation continues at 2% p.a., how much will the box of groceries cost in 4 years time? Exercise 11.03

5 Tim owns some collectable baseball cards that are appreciating at 5% p.a. Today the cards are worth \$720. Calculate their value in 3 years time. Exercise 11.03

6 A tablet device is currently priced at \$340. Predict the price of the device in 5 years time, assuming an average 3% p.a. inflation rate. Exercise 11.03

7 Use an online calculator to determine the total amount of interest Pooja will earn when she invests \$26 500 at 4.1% p.a. annually compounding interest for 7 years. Exercise 11.04

8 Which investment has the better return for an \$800 principal for a 6-year term? Exercise 11.04

Investment A: 3.4% p.a. simple interest

Investment B: 3.3% p.a. interest compounded yearly

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Exercise 11.05

9 Jill used this spreadsheet to calculate the future value and interest when she invested \$60 000 at 8% p.a. with monthly compounding interest.

G34

	A	B	C	D
1	**Jill's spreadsheet**			
2				
3				
4		**Initial investment C4**	\$60,000.00	
5		**Annual interest rate as a decimal C5**	0.08	
6		**Monthly interest rate C6**	0.006666667	
7				
8				
9	**Number of months**	**Value of the investment at the beginning of the month**	**Monthly interest**	**Value of the investment at the end of the month**
10	1	\$60,000.00	\$400.00	\$60,400.00
11	2	\$60,400.00	\$402.67	\$60,802.67
12	3	\$60,802.67	\$405.35	\$61,208.02
13	4	\$61,208.02	\$408.05	\$61,616.07
14	5	\$61,616.07	\$410.77	\$62,026.85
15	6	\$62,026.85	\$413.51	\$62,440.36
16	7	\$62,440.36	\$416.27	\$62,856.63
17	8	\$62,856.63	\$419.04	\$63,275.67
18	9	\$63,275.67	\$421.84	\$63,697.51
19	10	\$63,697.51	\$424.65	\$64,122.16
20	11	\$64,122.16	\$427.48	\$64,549.64
21	12	\$64,549.64	\$430.33	\$64,979.97
22	13	\$64,979.97	\$433.20	\$65,413.17
23	14	\$65,413.17	\$436.09	\$65,849.26
24	15	\$65,849.26	\$439.00	\$66,288.25
25	16	\$66,288.25	\$441.92	\$66,730.17
26	17	\$66,730.17	\$444.87	\$67,175.04
27	18	\$67,175.04	\$447.83	\$67,622.88
28				

a One of the formulas Jill used is =B10*\$C\$6. What does this formula calculate?

b Why did Jill use \$ signs in the formula?

c How much interest did Jill's investment earn in the 14th month?

d What formula could Jill use to make the spreadsheet calculate the total amount of interest earned in the first 12 months?

e What was Jill's investment worth at the end of 18 months?

f Construct a spreadsheet similar to Jill's if instead of compounding monthly, her investment compounds weekly.

g Use your spreadsheet to determine the extra interest Jill will earn in 6 months if the investment compounds weekly instead of monthly.

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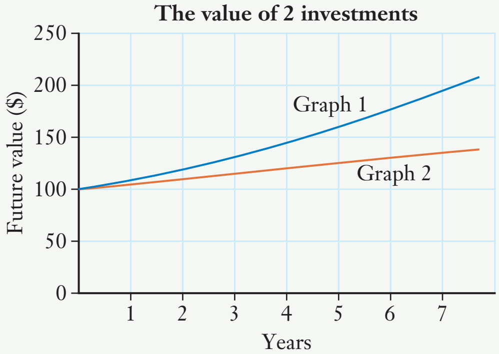
10 Jason drew a graph of 2 investments. One investment is paying simple interest and the other compound interest. Which graph is which?

Exercise 11.02



11 Sam invested \$3000 at 4.8% p.a. monthly compounding interest for 3 years.

Exercise 11.06

a What was the monthly interest rate?

b Use the technology of your choice to determine how much interest Sam will earn.

12 Pedro has \$10 000 to invest for 6 years. Use the technology of your choice to determine which is the best investment:

Exercise 11.06

- 4% p.a. monthly compounding
- 4.1% p.a. annually compounding
- 4.4% p.a. simple interest

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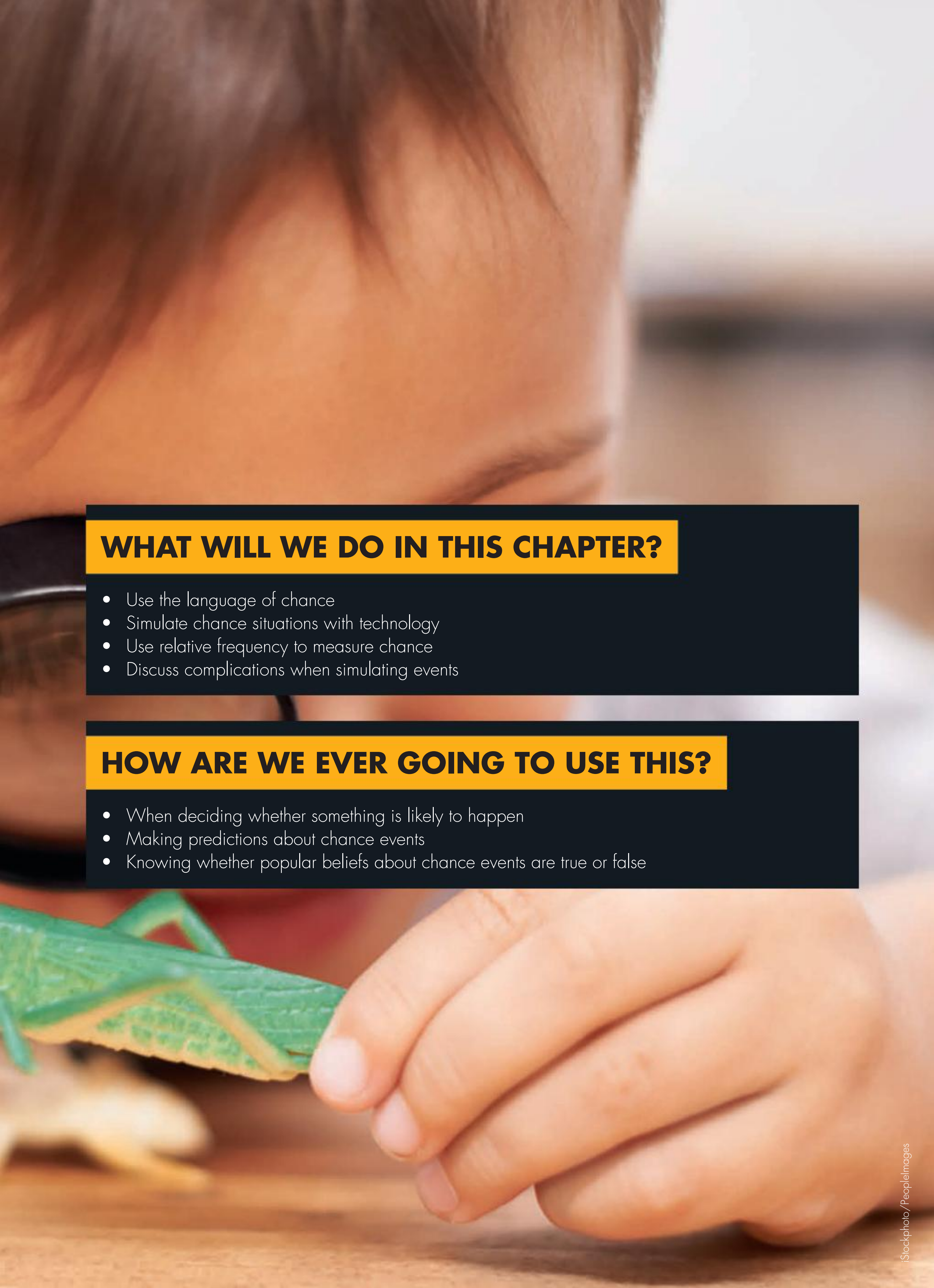
PROBABILITY AND RELATIVE FREQUENCIES

12.

WILL IT HAPPEN?

Chapter problem

Currently, each packet of *Ozbix* breakfast cereal comes with a free little toy. There are 12 different toys available but there's only one toy in each packet, each equally likely. Jordan wants to collect every toy. How many packets of cereal should he buy to get all 12 toys?



WHAT WILL WE DO IN THIS CHAPTER?

- Use the language of chance
- Simulate chance situations with technology
- Use relative frequency to measure chance
- Discuss complications when simulating events

HOW ARE WE EVER GOING TO USE THIS?

- When deciding whether something is likely to happen
- Making predictions about chance events
- Knowing whether popular beliefs about chance events are true or false

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12.01 Heads or tails?

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Anyone who's been involved in playing or watching sport is familiar with tossing a coin. We may toss a coin to decide things like:

- which team will kick off
- who will bat or serve first
- which goal we will attack

We toss a coin because we believe it's a fair way to decide. Both players or teams have the same chance of winning, but sometimes we hear claims that one team 'always loses the toss' or that the coin 'always comes up heads'. In this section we are going to investigate such claims. Fortunately, we don't have to spend a lot of time tossing coins and writing down the results. We can use a spreadsheet to simulate tossing a coin. Simulate means to pretend, imitate or model a real situation.

Relative frequency

We can estimate the **probability** that an **event** will happen by using the results of a **simulation**. We count the number of times the event happened and write it as a fraction over the total number of trials.

$$\text{Relative frequency} = \frac{\text{number of times the event happens}}{\text{total number of trials}}$$

Relative frequency is sometimes called **experimental probability** because it is used to estimate the actual probability or **theoretical probability**.

EXAMPLE 1

Toni tossed a coin 10 times and got 4 heads and 6 tails. Calculate the relative frequency of getting a head.

Solution

There were 4 heads in 10 trials.

Relative frequency = $\dfrac{\text{number of times the event happens}}{\text{total number of trials}}$

$$P(\text{head}) = \frac{4}{10}$$
$$= \frac{2}{5}$$

P(head) means 'probability of a head'.

Write the answer.

In Toni's experiment, the relative frequency of getting a head is $\frac{2}{5}$.

Note: This is different to the theoretical probability that we will consider in Chapter 16, *Taking chances*.

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Exercise 12.01 Heads or tails?

Download the 'Heads and tails' spreadsheet to simulate tossing a coin repeatedly.

Heads and tails

1 Run the simulation 10 times, concentrating only on the first 10 results highlighted in yellow.

2 Record the number of heads and tails each time.

3 Record the biggest number of the same (repeated) result, for example, 5 heads in a row or 3 tails in a row.

4 Make a note of any times the results alternate between heads and tails, that is, HTHTHTHTHT or THTHTHTHTH.

5 Do heads always show exactly half the time?

6 In what percentage of the 10 trials were there exactly 5 heads and 5 tails?

7 What was the largest number of the same result in a row?

8 On how many occasions were the results alternating between heads and tails?

9 Run the spreadsheet simulation again. This time, concentrate on the bottom table that shows the percentage of heads. For how many coin tosses is the percentage of heads closest to 50%?

10 Which percentage value changes the most? Can you explain why?

11 Is it possible to tamper with a coin so that it doesn't show heads 50% of the time?

12 In recent years, it was discovered that some British coins were biased, that is, didn't show heads 50% of the time. Search the Internet to find out which coin it was.

13 Try this experiment. Stick a small amount of Blutac to one side of a coin. Toss the coin 20 times, recording the result each time. Which side showed more often? The side with or without the Blutac?

14 Run the spreadsheet simulation 3 more times, noting the number of heads that occurred in the first 10 tosses (the cells highlighted in yellow). Calculate the relative frequency involved each time.

Example 1

15 Is the value of the relative frequency the same each time?

GROUP ACTIVITY

THE LANGUAGE OF CHANCE

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Probability is the chance that something will happen. In everyday conversation we use words that indicate the chance or probability that something will happen.

Here are some words that Australians use to describe the chance that something will happen:

50-50	Buckley's chance	certainty	fair chance
even chance	fat chance	impossible	never
no way	not in 100 years	odds on	one in a million
outside chance	probably	slim chance	unlikely

In your group:

1 Discuss the meaning of each term.

2 Place each term in one of the categories:

- certain or very likely to happen
- could happen
- can't or is very unlikely to happen.

3 Write a list of other words or expressions that the members of your group use to describe chance. Include the meaning with each one.

4 When you watch the weather forecast on TV, you will hear some 'chance' words used to describe what the future weather could be like. In your group, write a weather forecast for tonight.

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12.02 Rolling a die

WS
Greedy pig game

'Dice' is the plural form of the word 'die'. When we have 2 or more, they are called dice, but if we only have one of them, it's a 'die'.

We are going to investigate what happens when we roll a normal, 6-sided die. We will use a spreadsheet simulation rather than do lots of die-rolling.

Exercise 12.02 Rolling a die

Rolling a die

Download the 'Rolling a die' spreadsheet to complete this group activity. The spreadsheet simulates rolling a die 24 times.

1 When we roll a normal 6-sided die, any one of the numbers from 1 to 6 can show.

Run the simulation once. Did each of the numbers from 1 to 6 occur the same number of times? Is this what you expected would happen? Why or why not?

2 Calculate the relative frequency for each number based on your simulation.

3 Will other groups in your class have the same probabilities? Give a reason for your answer.

4 Run the spreadsheet 4 times to simulate rolling a die 96 times. Calculate the total frequency for each of the numbers from 1 to 6.

5 Theoretically, in 96 rolls each number should show 16 times. How close did your results come to 16?

6 If you were rolling a real die and you put a small piece of Blu Tack on one side to make that side a little bit heavier, do you think that all the numbers would still show the same number of times? If not, which number would occur more frequently and which less frequently than the others?

7 Imagine you had a wooden die and you were able to use sandpaper to round one of the edges to make it smoother. What effect would this have on the frequency of the different numbers?

Chapter problem

You've covered the skills required to solve the chapter problem. Can you solve it now?

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12.03 Relative frequency

WS A page of spinners

WS Coins probability

WS Dice probability

Quite often, it is impossible to calculate the actual probability of an event. When we can't calculate a theoretical probability, we can use past records or perform an experiment to calculate the relative frequency.

$$\text{Relative frequency of an event} = \frac{\text{number of times the event happens}}{\text{total number of trials}}$$

The **relative frequency** of an event can be written as a fraction, decimal or percentage.

EXAMPLE 2

Emily is a park ranger. This table shows the data she recorded about kangaroo deaths in the park.

Cause of death	No. of deaths
Hit by a motor vehicle	78
Shot	12
Caught in a fence or trap	11
Old age	25
Starvation	3
Other	7
No known cause	4
Total	140

a Calculate the probability that the death of a kangaroo was caused by a motor vehicle. Express your answer as a decimal, correct to 3 decimal places.

b Use Emily's data to estimate the probability that a kangaroo in her area will die from old age. Express your answer correct to the nearest percentage.

Solution

a Number of deaths due to motor vehicles = 78

Total = 140

$$P(\text{killed by motor vehicle}) = \frac{78}{140} = 0.557\ 14\ldots \approx 0.557$$

b Number of deaths due to old age = 25

Total = 140

$$P(\text{death from old age}) = \frac{25}{140} \times 100\% = 17.8571\ldots\% \approx 18\%$$

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EXAMPLE 3

Souraya counted the contents of 10 boxes of matches that were each labelled as containing 50 matches. Her results were:

53 49 50 48 52

51 50 49 50 51

iStock.com/Antagain

a What is the relative frequency of a box containing exactly 50 matches?

b What is the probability that a box contains more than 50 matches?

Solution

a Number containing 50 matches = 3

Total = 10

$P(50 \text{ matches}) = \dfrac{3}{10}$

b Number containing over 50 matches = 4

Total = 10

$P(\text{more than } 50 \text{ matches}) = \dfrac{4}{10}$

$= \dfrac{2}{5}$

The probability that a box contains more than 50 matches is $\dfrac{2}{5}$.

Exercise 12.03 Relative frequency

1 Renee measured the tail lengths of a sample of adult quokkas on Rottnest Island, WA. Example 2

Tail length (cm)	Frequency
24	2
25	7
26	9
27	10
28	8
29	7
30	4
31	3
Total	50

Shutterstock.com/Jessika Knaupe

a What is the probability that an adult quokka will have a 27 cm long tail?

b Renee's wildlife manual says that adult quokka tails range from 25 to 30 cm long. What is the probability that an adult quokka will have a tail that is outside this range?

c What is the most **likely** length for an adult quokka's tail?

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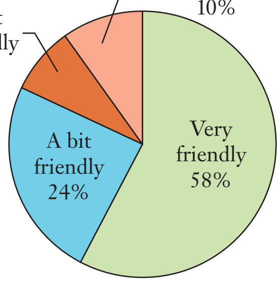
2 Renee was surprised by how friendly the 50 quokkas were. She gave each animal a friendliness rating and displayed the data in a pie chart. Calculate the experimental probability that a randomly-selected quokka will be:

a not friendly (give your answer as a decimal)

b very friendly or too friendly (give your answer as a fraction)



Example 3

3 In the past 12 months, when Jason visited his favourite restaurant, he noted that he had to wait for a table 8 times and he got a table straight away 16 times.

a How many times did Jason go to the restaurant?

b What is the relative frequency of 'not having to wait'?

c What is the experimental probability that next time Jason goes to the restaurant he will have to wait for a table?

4 Murphy's Law states that if anything can go wrong, then it will! Libby decided to test this theory by dropping a piece of toast and seeing whether it landed buttered-side up (good) or buttered-side down (bad). She performed 40 trials of her experiment, and her results were:

Buttered-side up: 5

Buttered-side down: 35

a What is the relative frequency of the toast landing buttered-side up?

b Use Libby's data to determine the percentage probability that a dropped piece of toast will land buttered-side down.

c Repeat Libby's experiment to determine the probability that the bread will land buttered-side up if it is knocked off the edge of a table.

Watch out! Make sure that you first cover the floor to avoid staining.

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5 Mitchell visited the old whaling station in Albany. He asked a random selection of people some questions about whaling and presented his results below.

Question	YES	NO
Do you agree with the international ban on killing whales?	19	1
Is it OK to make and sell souvenirs made from whale bones?	13	7
Are there too many whales in the oceans around WA?	2	18

a What is the probability that a person selected at random agrees with the international ban on killing whales?

b Calculate the probability that a person selected at random does not agree with making and selling souvenirs from whale bones. Express your answer as a decimal.

c What is the probability that a randomly-selected person thinks there are too many whales in the oceans around WA? Express your answer as a percentage.

d Predict the answers that a person who is a member of the 'Save our whales' group would give to each of Mitchell's questions.

e Suggest a factor that could create bias in Mitchell's results.

6 Latu rolled a die 75 times and displayed the results in a table.

Number	Frequency
1	21
2	12
3	10
4	11
5	12
6	9

a What is the experimental probability of rolling 3 with this die?

b Copy and complete the table to show the experimental probability of rolling each number as a decimal, correct to 2 places.

Number on the die	1	2	3	4	5	6
Experimental probability			0.13			

c To 2 decimal places, the theoretical probability of tossing each number on a normal die is 0.17. Latu believes his die is biased. Is he correct? Justify your answer.

7 Lauren loves to collect Sports Heroes cards. There are 10 different cards in the set and one of them is placed randomly in each packet of bubble gum.

Sports heroes

a Predict the number of packets of gum that Lauren will need to buy to get all 10 cards.

b Download the 'Sports heroes' spreadsheet.

c Run the spreadsheet 20 times to simulate buying 20 packets of gum. Record the number of packets required to get all 10 cards.

d Find the relative frequency of getting a full set of cards from fewer than 15 packets of gum.

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INVESTIGATION

VISITORS TO WESTERN AUSTRALIA

Tourism WA

For this activity, use the link to visit the Tourism Western Australia website www.tourism.wa.gov.au.

In 2017, 600 000 people from overseas visited Western Australia for a holiday or other leisure activities. Of these visitors, 42 800 came from China. This means that the relative frequency that an overseas visitor came from China was 42 800 ÷ 600 000 ≈ 0.071.

1 Research the *most recent data* available about the country of origin for visitors to WA and complete a table like the one below to display the information. The first row has been completed as an example.

Country of origin	Number of visitors	Relative frequency (to 3 decimal places)
China	42 800	0.071

2 Based on your data, what is the most likely language a visitor from overseas would speak?

3 If you were employing tour guides to work in Western Australia, what languages would you like them to speak?

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PRACTICAL ACTIVITY

THE GAME SHOW PROBLEM

For this activity, each pair of students will need 3 cards from a deck of playing cards.

You are the contestant in a TV game show. Behind one of the 3 doors there is a car that you could win. The host asks you to choose a door and you choose Door 1. The host then opens *Door 3* to show you that the car isn't behind it. This means that the car is behind either Door 1 or Door 2. Should you stick with Door 1 or switch to Door 2?

To simulate this problem, decide which of your three cards will be the 'CAR' card. Then you are going to work out the relative frequency that the car is behind the door that wasn't your first choice.

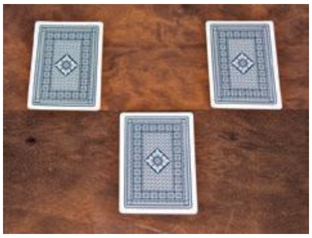
1 Decide who will be the game show host and who will be the contestant.

2 Copy this frequency table.

Outcome	Tally	Frequency
Car was first choice		
Car wasn't first choice		

3 The host shuffles the 3 cards, looks at them, then places them face down on the table.

4 The contestant chooses one card and moves it down, as shown in the photo.



5 The host displays one of the 2 remaining cards that is *not* the car.

6 Turn over both the remaining cards. Record in your frequency table whether the car was the card first chosen by the contestant or whether it was the other card.

7 Perform the simulation at least 24 times.

8 Determine the relative frequency that the car is behind the door which wasn't the contestant's first choice.

9 Decide on the best strategy. Should the contestant stick with their original choice or switch?

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12.04 Does bad luck happen often?

People often have the wrong idea about probability and chance. We tend to overestimate the chance of something good happening to us, for example, we think it's easier to win lotto than it really is. We also greatly underestimate the chance of something bad happening to us, such as being struck by lightning. The truth is that we are much more likely to be struck by lightning than to win a major prize in lotto.

Exercise 12.04 Does bad luck happen often?

Work in groups to complete this exercise.

50-year flood

1 The 1-in-50-year flood

Download the '50-year flood' spreadsheet.

Imagine you are buying a block of land that is in a 1-in-50-year flood zone. Will it flood very often?

A 1-in-50-year flood zone means that in every year, there is a probability of $\frac{1}{50}$ that there will be a major flood. The **law of averages** or **law of large numbers** says that over the long run, this flood will happen once in every 50 years, but in the short run, there is no pattern to the frequency of floods.

Newspix/Stuart Quinn

a Run the flood simulation several times and make a note of the number of times there is a flood.

b How often does a flood happen? Run the simulation 5 times and make a note of the number of years between floods. Is there a pattern?

c Run the simulation again, looking for floods in 2 consecutive years (2 years in a row). How many simulations were required to get one with floods 2 years in a row?

d Do 1-in-50-year floods happen every 50 years?

e The real estate agent says to you: 'I know it's in a 1-in-50-year flood zone, but you don't have to worry. It flooded last year, so it will be 49 years before another flood.' How would you answer him?

f What other factors are there, apart from rainfall, that contribute to flooding?

g Can a location have more than one flood in a year? Research floods in Gympie in 1898.

h If a town is in a 1-in-50-year flood zone and it experiences a large flood, do you think the chance of experiencing another large flood a few weeks later if there is more heavy rain is greater or smaller than $\frac{1}{50}$? Give some real-world reasons to justify your answer.

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2 The medical operation

Download the 'Medical treatment' spreadsheet.

Medical treatment

Imagine you have a serious medical condition. It isn't life-threatening, but it's very painful and restricts a lot of what you can do each day. Your doctor says that there is an operation that will completely cure the condition, but there is a 1-in-20 (5%) chance that you could die during the operation. What would you do? Would you still have the operation?

- **a** Suppose you say YES and decide to go through with the operation. Choose a number at random from 1 to 65 to represent your patient number and write it down.
- **b** Run the spreadsheet several times. It simulates 65 patients having the operation. Record what happens to the patient with your number.
- **c** Did you die in any of the simulations?
- **d** Did anyone in your group die?
- **e** A common belief is that something with a small probability isn't going to happen on the first trial. Run the simulation until the first patient dies. How many simulations did it take?
- **f** Can something with a small probability happen in the first trial?
- **g** What other factors might influence the result of an operation?

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4-child families

3 Number of girls in a family

Work in pairs and download a copy of the spreadsheet '4-child families' to complete this activity.

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This spreadsheet simulates the sex of the children in 4-child families.

a Run the simulation 12 times, recording the number of girls in the family each time.

b What is the most common number of girls in a 4-child family?

c What is the relative frequency of the most common number of girls in a 4-child family?

d Melissa is the mother of 3 girls and she would like to have a son. She thinks that if she has another baby, it's more likely to be a boy because she has 3 girls already. Run the simulation again, looking for families with girls for the first 3 children and record the sex of the 4th child. Continue until you've got 12 results. Were there more boys than girls for the 4th child?

e Repeat the simulation, this time concentrating on families with boys for the first 3 children. Was a boy or a girl more common as the 4th child?

f Are the same numbers of baby boys and girls born in Australia? Research the question and determine an experimental probability that an Australian baby will be a girl.

g Is it just a chance event that makes the numbers of boy and girl babies different? Research the reasons why different numbers of boys and girls are born in Australia.

Sum of 2 dice

4 The total rolled on 2 dice

You will need to work in pairs and download the 'Sum of 2 dice' spreadsheet to complete this activity.

a If we roll a pair of dice and add the 2 numbers, the most common sum is 7. Run the simulation and check that this statement is true.

b Think about this challenge: Roll a sum of 7 twice before you roll a sum of 6 and a sum of 8 in any order. Which is more likely: 7 twice or a 6 and an 8? Write down your prediction.

c Run the simulation at least 12 times to see which occurs first.

d Were you right? How good is your prediction?

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INVESTIGATION

FREEWAY ACCIDENTS

Freeway accidents

Accidents are a serious problem on freeways, especially for heavy vehicles. The probability that a severe accident (where a person is killed or seriously injured) on a particular freeway involves a truck or other heavy vehicle is 0.12.

To complete this activity, download the **Freeway accidents** spreadsheet. This spreadsheet simulates 50 accidents on a freeway and counts the number of accidents involving trucks and other heavy vehicles.

1. Over 50 accidents, what is the expected number of times that a truck is involved?
2. Run 40 simulations of 50 accidents using the spreadsheet and record the number of times a truck is involved.
3. Calculate the percentage of simulations in which 6 trucks were involved in accidents.
4. Calculate the percentage of simulations in which the number of accidents involving trucks was:
   - **a** 5, 6 or 7
   - **b** within the range 3 to 9
5. What conclusion could you make from this simulation?

KEYWORD ACTIVITY

DEFINITIONS

Write a sentence explaining the meaning of each of the following terms in this chapter.

die	dice	prediction
probability	relative frequency	simulation

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SOLUTION TO THE CHAPTER PROBLEM

Problem

Currently, each packet of *Ozbix* breakfast cereal comes with a free little toy. There are 12 different toys available but there's only one toy in each packet, each equally likely. Jordan wants to collect every toy. How many packets of cereal should he buy to get all 12 toys?

Solution

Breakfast cereals

We can simulate this problem. Download the 'Breakfast cereals' spreadsheet.

Run the simulation several times and record the smallest and largest number of packets Jordan needs to buy to collect all 12 toys. The answers will depend on the simulation.

For example:

Smallest number obtained in the simulation: 18 packets.

Largest number: 51.

Because it's a chance event, we can't say exactly how many boxes Jordan will have to buy, but it will be somewhere between 18 and 51.

Note: This simulation assumes that the toys are placed in the boxes at random and in equal numbers. This may not be the case. The cereal company might make 1 or 2 toys more 'rare' and harder to get.

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12. TEST YOURSELF

Will it happen?

Qz
Practice quiz

1 Matt wanted to determine the probability that a dropped thumbtack would land with its sharp point facing up. He dropped 30 thumbtacks.

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a What is the relative frequency as a decimal that a thumbtack will land with the point facing up?

b Is the dropped thumbtack more likely to land with the point facing up or down? Give a reason for your answer.

2 Saskia tossed a coin 6 times and it showed heads 4 times. She claims that the coin is biased. Write a sentence to explain why Saskia's conclusion may be wrong.

3 Describe a real-world event that has a probability of 50%.

4 Decide whether each statement is true or false. If they are false, correct them.

a If you toss a coin and get 5 heads in a row, then most likely the next toss will be a tail.

b The first 4 children in a family are all girls. If the mother has another baby, there's a 50% chance that it will be another girl.

5 Some children in Lara's year are having trouble with their maths homework. Lara recorded the following information.

Outcome	Frequency	Relative frequency
No trouble	43	
Some trouble	51	
Lot of trouble	36	
Total	120	

a Copy and complete the relative frequency column as a decimal correct to 3 decimal places.

b Assuming that the values in Lara's information are correct, what is the probability that a student selected at random from Lara's year will be having some or a lot of trouble with their maths homework?

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EARTH GEOMETRY AND TIME ZONES

13. TIME TRAVELLING

Chapter problem

The FA Cup soccer final is played at Wembley Stadium in London in May. At this time of year, England is on daylight saving time. The match starts at 5 p.m. local time. At what time should TV viewers in Australia tune in to watch the final?

WHAT WILL WE DO IN THIS CHAPTER?

- Learn about time zones and daylight saving in Australia and around the world
- Calculate time differences between 2 places in the world
- Solve practical problems involving communicating and travelling between time zones, including across the International Date Line

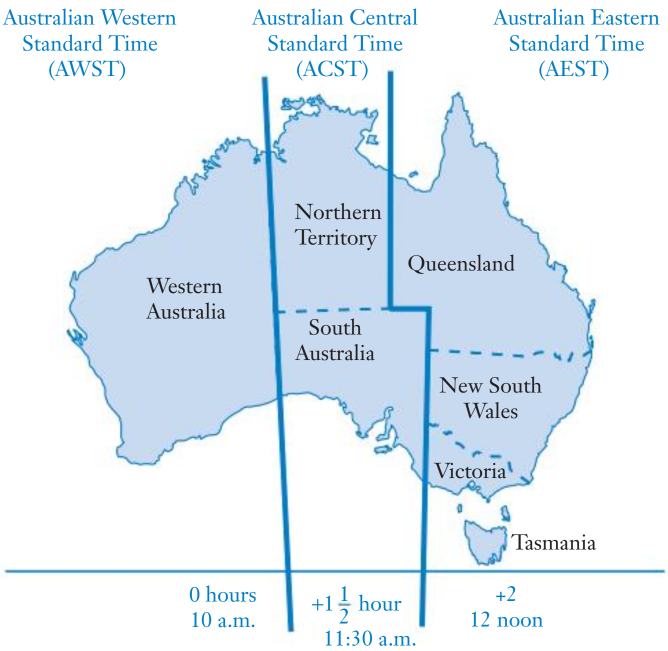
HOW ARE WE EVER GOING TO USE THIS?

- Planning an Australian or overseas trip
- When making phone or Internet contact with people in other states or countries
- When watching live sport from other states and countries
- Adjusting the time for daylight saving

13.01 Australian time zones

Australia has 3 **time zones**:

- **Australian Western Standard Time (AWST)**, covering Western Australia.
- **Australian Central Standard Time (ACST)**, covering South Australia and Northern Territory and $1\frac{1}{2}$ hours ahead of AWST.
- **Australian Eastern Standard Time (AEST)**, covering Queensland, New South Wales, Australian Capital Territory, Victoria and Tasmania, and 2 hours ahead of AWST.

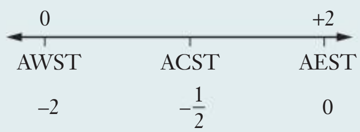


EXAMPLE 1

Show the Australian time zones on a timeline with NSW as zero hours.

Solution

If NSW (AEST) is 0, then Western Australia is –2 hours and Central Australia is $-\frac{1}{2}$ hours.



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Calculating times using time zones

When moving east, add on the time difference.

When moving west, subtract the time difference.

EXAMPLE 2

a It is 9 a.m. in Perth. What time is it in Canberra?

b It is 2 p.m. in Melbourne. What time is it in Adelaide?

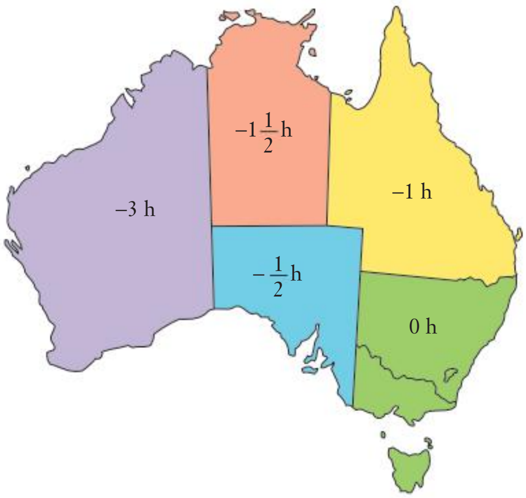
Solution

a Moving east from Perth to Canberra: add the time difference. — 9 a.m. + 2 hours = 11 a.m.

b Moving west from Melbourne to Adelaide: subtract the time difference. — 2 p.m. $- \frac{1}{2}$ hour = 1:30 p.m.

Daylight saving

Many countries adopt **daylight saving** during the warmer half of the year to take advantage of the extra hours of sunlight. In Australia, this involves advancing clocks by one hour at the start of October and adjusting back one hour at the start of April ('spring' forward, 'fall' back). During daylight saving, our time zones are an extra hour ahead of the standard time zones, except in Queensland, the Northern Territory and Western Australia, the states that do not participate in daylight saving. The map below shows the 5 time zones in Australia during daylight saving.



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EXAMPLE 3

When it is 10:20 a.m. in December in Adelaide, what time is it in Perth?

Solution

In December, Adelaide is on daylight saving time. This means it is $2\frac{1}{2}$ hours ahead of Perth instead of the usual $1\frac{1}{2}$ hours.

Time in Perth = 10:20 a.m. – $2\frac{1}{2}$ hours
= 7:50 a.m.

Exercise 13.01 Australian time zones

Example 1

1 Show the time zones on a timeline with the Northern Territory as zero hours.

2 State whether each location is ahead of, behind, or has the same time as Adelaide.

a Sydney **b** Melbourne
c Darwin **d** Canberra
e Geraldton (WA) **f** Mt Isa (Qld)
g Ballarat (Vic) **h** Ceduna (SA)
i Hobart **j** Broome (WA)

Example 2

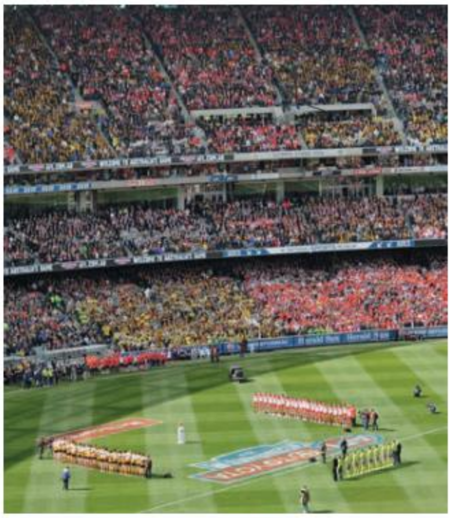
3 It is 10:30 a.m. in Adelaide. Find the time in:

a Cairns (Qld) **b** Freemantle (WA) **c** Alice Springs (NT)
d Launceston (Tas) **e** Bendigo (Vic) **f** Broome (WA)

4 What is the time in each city when it is 11 p.m. in NSW?

a Brisbane **b** Adelaide **c** Perth
d Darwin **e** Hobart **f** Canberra

5 The AFL Grand Final started in Melbourne at 2:15 p.m. and was shown live in Adelaide. What time did the game start on Adelaide TV?



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6 **a** Jonah's flight from Sydney to Perth will take 4 hours. He leaves Sydney at 2 p.m. What time does he land in Perth? Give your answer as Perth local time.

b When Jonah flies home, he leaves Perth at 11:30 a.m. What time does he land in Sydney? Give your answer as Sydney local time.

7 When Kris was holidaying at Margaret River in WA, she phoned her parents in Canberra. She rang at 6:30 p.m. Western Standard Time. What time was it in Canberra?

8 Zoe, in Rockhampton, Qld, uses the Internet to talk with her cousins in Broome, WA. At what time should Zoe log on to catch her cousins at 8:00 p.m. Broome time?

9 **a** Use the Internet to find out when daylight saving begins and ends in NSW and Victoria.

b Why do some states have daylight saving?

c How does daylight saving affect the different time zones?

d When it is 12:30 p.m. in Western Australia (not on daylight saving), what time is it in NSW on Eastern Standard Daylight Saving Time?

10 Broken Hill in NSW operates on Central Standard Time rather than Eastern Standard Time like the rest of NSW. Suggest a reason why the people of Broken Hill might have chosen to do this. (Hint: look at a map.)

11 For this question, assume daylight saving is operating.

Example 3

a When it is 8:30 p.m. in Darwin, what time is it in Hobart?

b What time is it in Adelaide when it is 11:45 a.m. in Melbourne?

c When it is 1:05 p.m. in Sydney, what time is it in Perth?

d What time is it in Brisbane when it is 8:20 a.m. in Darwin?

12 Ben, working in the mines in Western Australia, rings his wife in Victoria during summer. He knows she will have the children in bed by 8 p.m. After what time in Western Australia can he ring so that he can talk to her after the children are in bed?

13 Janine, who lives in Queensland, wants to ring her daughter Alexa in Adelaide before she goes to work to start a new job. She knows Alexa intends to leave for work at 7:45 a.m., so wants to talk to her at 7:15 a.m. At what time in Queensland should Janine ring her?

International time zones

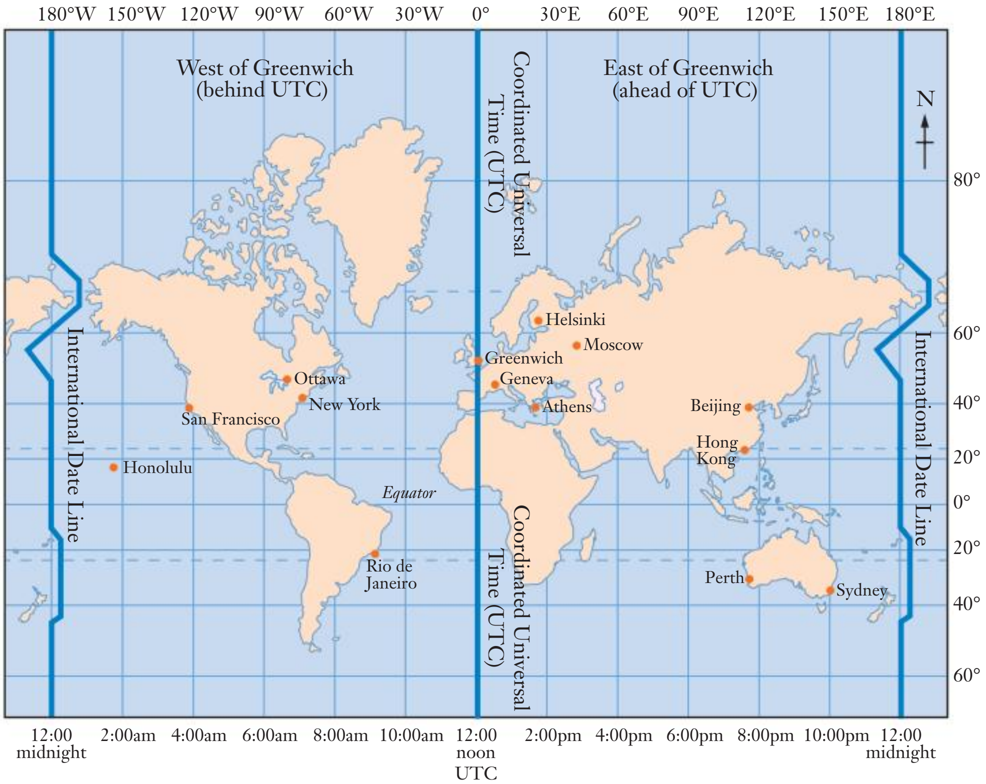
WS

Map of the world

13.02 World time zones

The rest of the world also operates on different time zones, which are measured relative to the Greenwich Observatory near London, England. This map shows international time zones, related to the time zone called **Coordinated Universal Time (UTC)** or **Greenwich Mean Time (GMT)**.

Greenwich is pronounced 'gren-itch'.



Places west of England (such as the USA) are behind UTC and places east of England (such as Asia) are ahead of UTC.

West (–) UTC East (+)

Australian Western Standard Time is UTC + 8 hours.
Notice on the map that because the world is round, when you cross the International Date Line on the right (heading east), you end up on the left side of the map and 'go back' one day!

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EXAMPLE 4

State whether each country is ahead of or behind UTC.

a Australia **b** Canada **c** Chile **d** China

Solution

a On the map, Australia is east (to the right) of England. Australia is ahead of UTC.

b USA is west (to the left) of England. USA is behind UTC.

c Chile is in South America, west of England. Chile is behind UTC.

d China is in Asia, east of England. China is ahead of UTC.

This table shows the time in various places in relation to UTC.

WS Table of time zones

Location	Hours from UTC	Location	Hours from UTC
Hawaii	–10	Lebanon, Finland, Egypt, South Africa	+2
Anchorage (Alaska)	–9	Moscow, Kenya, Saudi Arabia, Iraq	+3
Las Vegas, Los Angeles, Seattle, Vancouver	–8	Dubai, Mauritius, Azerbaijan	+4
Banff (Canada), Bolivia	–7	Pakistan	+5
Chicago, Mexico City	–6	East Kazakhstan, Bangladesh	+6
Atlanta, New York, Peru	–5	Jakarta, Vietnam, Thailand	+7
Barbados, Chile	–4	Perth, Bali, Hong Kong, Singapore	+8
Rio de Janeiro, Argentina	–3	Japan, Korea, East Timor	+9
South Sandwich Islands	–2	Sydney, Papua New Guinea, Guam	+10
Azores (Portugal)	–1	Vanuatu, Solomon Islands	+11
United Kingdom, Ireland, Iceland, Burkina Faso	0	New Zealand, Fiji	+12
Italy, France, Germany, Algeria	+1	Tonga	+13

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EXAMPLE 5

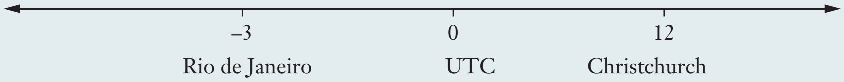
a What is the time difference between Rio de Janeiro (Brazil) and Christchurch (New Zealand)?

b When it is 2 a.m. in Rio de Janeiro, what time is it in Christchurch?

c What time is it in Rio de Janeiro when it is 7 a.m. in Christchurch?

Solution

a



Rio de Janeiro is –3 from UTC.

Christchurch is +12 from UTC.

Time difference = 3 + 12

= 15 hours

b Christchurch is 15 hours ahead of Rio de Janeiro.

2 a.m. + 15 hours = 5 p.m.

c Rio de Janeiro is 15 hours behind Christchurch.

7 a.m. – 15 hours = 4 p.m. the day before

In your answer, remember to include which day it is.

Exercise 13.02 World time zones

Use the map of time zones and the table of places for this exercise.

Example 4

1 For each country, state whether it is ahead of or behind UTC.

a Alaska **b** Japan **c** India

d Argentina **e** Hawaii **f** Saudi Arabia

Example 5

2 What is the time difference between the following cities?

a Mecca (Saudi Arabia) and Las Vegas (USA)

b Lima (Peru) and Melbourne

c Honolulu (Hawaii) and Buenos Aires (Argentina)

d Helsinki (Finland) and Suva (Fiji)

e Banff (Canada) and Florence (Italy)

Remember a line diagram can help you work it out!

3 Use your answers to Question **2** to answer the following.

a When it is 9 a.m. in Las Vegas, what time is it in Saudi Arabia?

b When it is 4 p.m. in Saudi Arabia, what time is it in Las Vegas?

c What time is it in Sydney when it is 3 a.m. in Lima?

d When it is 10 a.m. in Sydney, what time is it in Lima?

e What time is it in Honolulu when it is 2 p.m. in Buenos Aires?

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f What time is it in Buenos Aires when it is 10 p.m. in Honolulu?
g When it is 10 a.m. in Suva, what time is it in Helsinki?
h When it is 10 p.m. in Helsinki, what time is it in Suva?
i When it is 8 p.m. in Florence, what time is it in Banff?
j What time is it in Florence when it is 11 a.m. in Banff?

4 a When it is 5 a.m. in Mexico City, what time is it in Helsinki?
b When it is 4 p.m. in Hong Kong, what time is it in Vancouver (Canada)?
c What time is it in Algiers (Algeria) when it is 10 a.m. in Honolulu?
d When it is 1 p.m. in La Paz (Bolivia), what time is it in Reykjavik (Iceland)?
e If it is 9 p.m. in Atlanta (USA), what time is it in Dhaka (Bangladesh)?
f When it is 11 a.m. in Chicago (USA), what time is it in Port Vila (Vanuatu)?

5 At 9:00 a.m. in Bridgetown (Barbados), Wesley phones his cousin in Cairo (Egypt). What time is it in Cairo?

6 James is playing an online game at 10 p.m. in Albany, WA. He is multi-playing with Juan in Santiago (Chile). What time is it in Santiago?

7 Michelle caught a flight from Melbourne to Christchurch (New Zealand). This flight left Melbourne at 1030.
a What time was it in Christchurch when Michelle's flight left Melbourne?
b The flight arrived in Christchurch at 1800 (New Zealand time). How long did Michelle's flight take?

8 Carol flew from New York to Honolulu. Her flight left New York at 1520 on Saturday and took 11 hours to get to Honolulu. At what local time did she arrive in Honolulu?

9 Simon flies from Brisbane to Los Angeles. He leaves Brisbane at 1030 on Saturday. He arrives in Los Angeles at 0900 on the same day (Saturday).
a How is this possible?
b How long is the flight from Brisbane to Los Angeles?

10 Michael is conducting business in Melbourne with a company in Jakarta, Indonesia. Melbourne is on daylight saving, while Indonesia is not. Michael needs to organise a video conference for 2 p.m. Indonesian time. What time will it be in Melbourne?

11 Joe is visiting Ouagadougou in Burkina Faso in West Africa, where the time zone is UTC 0. He wants to connect to his friend Steve in Sydney via the Internet. He logs on at 6 p.m. local time.
a What time will it be in Sydney?
b Is this a good time for Joe to try to talk to his friend? Why or why not?

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12 Alycia is studying in Adelaide. Her brother is working in Baku in the former Russian region of Azerbaijan, whose time zone is UTC+4. It is February and daylight saving is operating in Adelaide. Between what times in Adelaide can Alycia ring him so that she rings between 6 p.m. and 10 p.m. in Baku?

Chapter problem

You've covered the skills required to solve the chapter problem. Can you solve it now?

INVESTIGATION

THE INTERNATIONAL DATE LINE

Research the following questions.

1 What is the International Date Line?

2 How is it related to the prime meridian through Greenwich?

3 What happens when you cross the International Date Line travelling west; for example, flying from the USA to Australia?

4 What happens when you cross the International Date Line travelling east; for example, flying from Australia to the USA?

5 Name an island on each side of the International Date Line.

6 In 2011, one Pacific island changed its time zone in relation to the International Date Line. What island was this and why did it make the change?

7 Find and copy a map showing the International Date Line.

INVESTIGATION

HOW DID WE GET TIME ZONES?

Research the history of time zones, including:

- how people kept time before clocks were widely used and why this worked
- why Greenwich was established as 'zero' time
- what developments led to the need for common time zones
- what people were involved in the development of time zones
- when time zones became established

Add anything else you find interesting.

Prepare a presentation showing what you have found.

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13.03 Happy New Year!

As the time approaches midnight on the night of 31 December, every place in the world celebrates the arrival of the New Year but we don't all do it together. Australian and the Pacific region are the first to celebrate the new year compared to the rest of the world. Furthermore, some countries in the southern hemisphere are on daylight saving time but countries in the northern hemisphere are not.

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EXAMPLE 6

What time is it in the United Kingdom when Tasmania is celebrating midnight on New Year's Day?

Solution

Tasmania's time zone is UTC+10, but because it observes daylight saving in January, Tasmania is UTC+11.

At this time of the year, Hobart is 11 hours ahead of UTC.

Midnight – 11 hours = 1 p.m.

The time in London is 1 p.m. on 31 December.

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EXAMPLE 7

How many hours later than the New Year in Perth is the New Year in Las Vegas?

Solution

Find the time difference between Perth and Las Vegas. There is no daylight saving in Western Australia and none in Las Vegas as it is winter there.

Perth is GMT +8.

Las Vegas is GMT −8.

$8 + 8 = 16$

The New Year in Las Vegas is 16 hours after the New Year in Perth.

Exercise 13.03 Happy New Year!

Example 6

1 What time is it in Dubai when the residents of Perth are celebrating at midnight for the new year?

2 What time is it in Hawaii when people in Darwin are celebrating the arrival of the New Year?

3 The pyramids in Egypt have a special light show at the start of the New Year. At what time in Melbourne should Jack switch on the TV to see the light show on the pyramids?

4 Kit and Jennifer want to watch the spectacular fireworks at the Space Needle in Seattle online.
At what time should they log on in Broome (WA) to watch the fireworks, which start at 11:45 p.m. in Seattle (UTC−8)?

Example 7

5 How many hours are there between the New Year in Brisbane and the New Year in Germany?

6 How many hours earlier is the New Year in New Zealand than the New Year in Moscow?

7 Use the information in the table on page 371 to answer the following questions.
   a Which is the first place in the world to celebrate New Year?
   b Which is the last place in the world to celebrate New Year?
   c How many hours are there between these 2 celebrations?

8 Use the information in the table on page 371 to answer this question.
   a Choose a place that you would like to visit for the New Year's celebration.
   b Your parents want to ring you when it is the New Year in the place you have chosen. At what time should your parents ring you from home?

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INVESTIGATION

HOW MANY NEW YEARS CAN YOU CELEBRATE?

Is it possible to celebrate the New Year in your city and celebrate it again in another city? Is it possible to do this more than once?

In this investigation, we will assume that you are at the airport and that a plane will take off as soon as midnight passes.

You will need to find the flight times between your city and other cities in the world that have a large time difference with your city.

- Make a list of possible cities.
- Calculate the time difference between your city and each city.
- Use the Internet to find the flight times from your city to each city.
- By comparing the times, decide if it's possible to celebrate the New Year in more than one city.

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13.04 Watching international sport

Major sporting events such as the tennis at Wimbledon, the World Cup soccer, the cricket Ashes series and Formula 1 motor races are watched by millions of people in many different countries. We can use time zones to work out when different events will be shown live on TV in Australia.

Exercise 13.04 Watching international sport

1 Formula 1 Grand Prix races are held all over the world, 19 races in all. What time do you need to turn on the TV to see the start of each race?

- **a** Sakhir in Bahrain (UTC+3) in April, starting at 6 p.m.
- **b** Shanghai in China (UTC+8) in April, starting at 3 p.m.
- **c** Monza in Italy (GMT+2) in September, starting at 2 p.m.
- **d** Austin, Texas (UTC-5) in the USA in November, starting at 2 p.m.

2 Wimbledon, the oldest tennis tournament in the world, takes place in London (UTC 0) at the end of June and the start of July.

- **a** Each day, matches start at 11 a.m. local time. What time is this in your location?
- **b** The semi-finals start at 2 p.m. local time. What time will you need to switch on the TV to watch them?
- **c** The Men's Final starts at 2:30 p.m. local time. One final lasted 3 hours and 35 minutes. At what time, in your location, did this final finish?

3 The FIFA World Cup was held in Russia in June-July 2018. Australia played 3 other countries in the Group rounds in cities where the time zone was UTC+3.

- **a** The Australia versus France match was scheduled for 1 p.m. in Kazan. What time was this where you live?
- **b** Australia's match against Peru was scheduled for 5 p.m. in Sochi. What time was this where you live?
- **c** The final was played at 4 p.m. in Moscow. The broadcast lasted 2 hours and 45 minutes. What time did it finish where you live?

4 Every 4 years in July, the Australian Cricket team plays the Ashes Series against England in England. At this time of year, England is on daylight saving time and Australia isn't. The daily schedule is:

- Start of play: 10:30 a.m.
- Lunch: 12:30 p.m. to 1:10 p.m.
- Tea: 3:10 p.m. to 3:30 p.m.
- Close of play: 5:30 p.m.

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List the daily schedule in your local time.

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5 The US Masters Golf Tournament is played each year at Augusta, Georgia in the USA in April. Augusta is UTC–5. In April, Augusta is on daylight saving time.

a Practice rounds are broadcast from 12 noon to 7 p.m. local time. Peter lives in Hobart and wants to watch practice. Between what times should he be watching?

b Each of the 4 days of competition are broadcast from 3 p.m. to 7 p.m. Between what times can you watch the daily rounds in your time zone?

c Play finishes at 7 p.m. each day, but, if there is a tie at the end of the competition, extra play-off holes have to be played on the last day. The play-off takes an extra 45 minutes. In your time zone, what time will the play-off finish on the last day?

6 The Tour de France is a cycling race that takes place in July each year. It lasts 3 weeks, covers about 3700 km and is raced mostly in France, whose time zone is UTC+1. There are 20 racing days and 2 rest days.

The race on each day is called a stage. In July, France is on daylight saving.

a Stage 10 starts at approximately 10 a.m. local time. What time is this where you are?

b Live broadcast on Australian television starts at 10.30 p.m. AEST.

i What time is this where you live?

ii What time is this in France?

iii If the race started at 10 a.m. in France, how much of the race would you have missed?

c On one of the days, the riding takes 5.5 hours, starting at 12 noon in France. You want to watch the last half hour of the race. At what time would you need to switch on your TV?

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World Clock

INVESTIGATION

WHERE?

In this investigation, use the World Clock website to answer the following questions.

1 Find 3 locations that have the same time as you.

2 Find 3 locations that are 5 hours behind you.

3 Find 3 locations that are 2 hours ahead of you.

4 What town or city is as far behind you in time as possible?

5 What town or city is as far ahead of you in time as possible?

6 Choose 3 places you would like to visit and find the time in those places when it is 12 noon where you are.

INVESTIGATION

CHOOSE YOUR SPECIAL EVENT

Many special events around the world are broadcast live on Australian television or on the internet. These events include the Olympics, the Commonwealth Games, the Oscars, religious festivals, ANZAC Day services, Remembrance Day services and major sporting events.

John Pyke / Getty Images

- Choose one event that you are interested in that takes place in our summer and one event that takes place in our winter.
- For each of your chosen events, work out the times that you will be able to watch it at home on TV.

For one of the events you have chosen:

- find out how long it would take you to fly from your city to the event
- imagine that you are able to fly to see the event live and that you wish to arrive at least 2 hours before the event commences. By what time must you leave your city?

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KEYWORD ACTIVITY

CHAPTER SUMMARY

Australian Central Standard Time	International Date Line
Daylight saving	UTC
Time zone	

For each term, write a short paragraph to explain its meaning and importance.

Then write a single paragraph including all 5 terms.

SOLUTION TO THE CHAPTER PROBLEM

Problem

The FA Cup soccer final is played at Wembley Stadium in London in May. At this time of year, England is on daylight saving time. The match starts at 5 p.m. local time. At what time should TV viewers in Australia tune in to watch the final?

Solution

We will answer for all 3 Australian time zones. In London, daylight saving is operating, so the local time is UTC + 1.

Australian Western Standard Time is UTC + 8.

The time difference is 8 – 1 = 7 hours.

When it is 1700 in London, the time in Western Australia is 5 pm + 7 hours = 12 midnight.

Australian Central Standard Time is UTC + 9.5, which is 1.5 hours ahead of WA, so their local time will be 1:30 am.

Australian Eastern Standard Time is on UTC + 10, which is 2 hours ahead of WA, so their local time will be 2 am.

TV viewers of the FA Cup soccer final need to tune in at:

- 12 midnight — for Western Australia
- 1:30 am — for South Australia, Northern Territory
- 2 a.m. — for NSW, Queensland, Victoria, ACT and Tasmania

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13. TEST YOURSELF

Practice quiz

1 **a** What time is it in Western Australia when it is 6 a.m. in NSW?

b It is 10:15 a.m. in South Australia. What time is it in Tasmania?

c Determine the time in the Northern Territory when it is 12 noon in WA.

d Sally is talking to her family at home on the phone from her holiday location. At Sally's holiday location it is 2 p.m. and it is 1:30 p.m. at home. Where could Sally's home be?

Exercise 13.01

2 Queensland and Western Australia don't follow daylight saving. Calculate the time in each city when it is midnight in Sydney.

Exercise 13.01

a Melbourne **b** Brisbane **c** Perth **d** Hobart

3 It is 11 a.m. in Canberra. What is the local time in each of the following cities?

Exercise 13.02

a Auckland **b** Bangkok **c** Atlanta

d Cape Town **e** Beirut **f** New Delhi (UTC + 5.5)

4 **a** Calculate the time difference between Atlanta and Beirut (Lebanon).

b What is the time in New Delhi when it is 1340 in Bangkok?

Exercise 13.02

5 **a** When it is 10 a.m. in New York, what time is it in Sydney?

b When it is 7 p.m. in Sydney, what time is it in New York?

c What time is it in Rio de Janeiro when it is 11:30 a.m. in Las Vegas?

d What time is it in Berlin (Germany) when it is 4:30 p.m. in New Zealand?

e When it is 12 noon in Darwin, what time is it in Vanuatu?

Exercise 13.02

6 Amy flies from Darwin to Paris. She departs Adelaide at 11:50 p.m. local time on a Friday night. The flight takes 23 hours 5 minutes. What time is it in Paris when she lands?

Exercise 13.02

7 The director of the Art Gallery of Western Australia in Perth needs to talk to the director of the Museum of Modern Art in New York about an upcoming exhibition. The opening hours of the New York gallery are 10 a.m. to 5 p.m. Between what times in Perth could the WA director call?

Exercise 13.02

8 What time is it in Hong Kong when Fiji is celebrating the New Year at midnight, when Fiji has daylight saving?

Exercise 13.03

9 The French Open for tennis is held in Paris each May/June when daylight saving is operating in France.

Exercise 13.04

a Each day, matches start at 11 a.m. local time. What time is this where you live?

b The Women's Final starts at 2:30 p.m. local time. The match lasted 2 hours and 15 minutes. At what time, in your location, did this final finish?

10 Ella likes to record English soccer matches. The game starts at 6 p.m. Friday local time in the UK. Ella lives in Wollongong and it is January. At what time should she set her TV to record the start of the game in the UK?

Exercise 13.04

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Medical treatment

Rolling a die

Breakfast cereals

Exercise 12.01

Practice set 3

- You will need the *Medical treatment*, *Rolling a die* and *Breakfast cereals* spreadsheets from Chapter 12, which you can download from NelsonNet
- You will need an online compound interest calculator for question **5** of Section B

Section A Multiple-choice questions

For each question, select the correct answer **A**, **B**, **C** or **D**.

1 Which event has a probability of $\frac{1}{2}$?

A Having a sunny day tomorrow

B Throwing a 5 on a die

C Having a baby girl for your first child

D Catching a cold

Use this table for questions **2** and **3**.

Location	Hours from UTC	Location	Hours from UTC
Hawaii	–10	Lebanon, Finland, Egypt, South Africa	+2
Anchorage (Alaska)	–9	Moscow, Kenya, Saudi Arabia, Iraq	+3
Las Vegas, Los Angeles, Seattle, Vancouver	–8	Dubai, Mauritius, Azerbaijan	+4
Banff (Canada), Bolivia	–7	Pakistan	+5
Chicago, Mexico City	–6	East Kazakhstan, Bangladesh	+6
Atlanta, New York, Peru	–5	Jakarta, Vietnam, Thailand	+7
Barbados, Chile	–4	Perth, Bali, Hong Kong, Singapore	+8
Rio de Janeiro, Argentina	–3	Japan, Korea, East Timor	+9
South Sandwich Islands	–2	Sydney, Papua New Guinea, Guam	+10
Azores (Portugal)	–1	Vanuatu, Solomon Islands	+11
United Kingdom, Ireland, Iceland, Burkina Faso	0	New Zealand, Fiji	+12
Italy, France, Germany, Algeria	+1	Tonga	+13

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2 Find the time in Hong Kong when it is 9 p.m. Saturday in Rio de Janeiro.

A 10 a.m. Saturday **B** 4 p.m. Saturday
C 2 a.m. Sunday **D** 8 a.m. Sunday

Exercise 13.02

3 How many hours are there between Vancouver, Canada and Finland?

A 6 h **B** 9 h **C** 10 h **D** 11 h

Exercise 13.03

4 Calculate the simple interest earned on \$800 at 4% p.a. for 3 years.

A \$12 **B** \$24 **C** \$32 **D** \$96

Exercise 11.01

5 Amila rolls a die repeatedly and records the frequency of each number rolled.

Exercise 12.03

Number on the die	1	2	3	4	5	6
Frequency	18	15	15	19	16	13

What is the relative frequency for rolling an even number from this data?

A $\frac{49}{96}$ **B** $\frac{47}{96}$ **C** $\frac{47}{49}$ **D** $\frac{34}{62}$

6 Sungwoo has recently moved to Auckland, New Zealand from Perth. He plans to ring his parents at 8 a.m. on his first day of work. New Zealand is on daylight saving time but Western Australia is not. What time will it be in Perth when Sungwoo rings? Use the table from the previous page.

A 3 a.m. **B** 4 a.m. **C** 12 noon **D** 1 p.m.

Exercise 13.02

7 By which decimal do I multiply to increase an amount by 1.75%?

A 1.0175 **B** 1.075 **C** 1.175 **D** 1.75

Exercise 11.03

8 Jane invests \$4500 at 3.5% p.a. interest compounding annually for 8 years. Calculate the amount of interest she earns.

A \$1260 **B** \$1425.64 **C** 5925.64 **D** \$9000

Exercise 11.04

Section B Short-answer questions

1 Name 3 countries or cities that are:

a ahead of UTC

b behind UTC

Exercise 13.02

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Exercise 12.04

2 For this question, you will need the *Medical treatment* spreadsheet from NelsonNet.

a Select a number at random from 1 to 65 to represent your patient number and write it down.

b Run the spreadsheet 20 times. Record what happens to the patient with your number.

c Did your patient die in any of the simulations?

Exercise 11.02

3 Joanna invested \$11 500 at 2% p.a. compounding annually for 3 years. Copy and complete the table to calculate the future value of her investment at the end of 3 years.

	Interest	**Balance**
End of the 1st year	$I = Prn$ $= \$11\ 500 \times 0.02 \times 1$ $= \$\_\_\_\_\_\_$	$\$11\ 500 + \$\_\_\_\_ = \$\_\_\_\_\_\_$
End of the 2nd year	$I = Prn$ $= \$\_\_\_\_\_\_ \times 0.\_\_\_ \times 1$ $= \$\_\_\_\_$	$\$\_\_\_\_\_ + \$\_\_\_\_ = \$\_\_\_\_\_$
End of the 3rd year	$I = Prn$ $= \$\_\_\_\_ \times 0.\_\_\_\_ \times 1$ $= \$\_\_\_\_\_$	$\$\_\_\_\_\_ + \$\_\_\_\_\_ = \$\_\_\_\_\_\_$

Exercise 12.03

4 For this question, you need the *Rolling a die* spreadsheet.

a Run the spreadsheet 4 times to simulate rolling a die 96 times. Calculate the total frequency for each of the numbers from 1 to 6.

b Theoretically, in 96 rolls each number should show 16 times. How close did your results come to 16?

c Calculate the relative frequency for each number based on your simulations.

Exercise 11.04

5 Rashid invested \$3500 at 5.9% p.a. compounding annually for 7 years.

a Use an online compound interest calculator to calculate the value of Rashid's investment at the end of the 7 years.

b How much interest did Rashid earn?

Use the table from page 384 to answer Questions **6** and **7**.

Exercise 13.02

6 Nina and Phillip fly from Adelaide (UTC+9.5) to Chile. They leave Adelaide at 1815 Friday and the flight takes 18 hours and 45 minutes. At what local time did they arrive in Chile?

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7 Damien is working in Atlanta, USA and needs to conference call with his company's office in Hong Kong. Atlanta is on daylight saving time. If Damien makes the call at 10 p.m. Wednesday in Atlanta, what time is it in Hong Kong? Exercise 13.02

8 Oliver invests \$8400 for 18 months at 2.4% p.a. compounding monthly. Exercise 11.06

a What is the value of the investment at the end of 18 months?

b How much interest did Oliver earn?

9 Use the *Breakfast cereals* spreadsheet from NelsonNet to answer these questions. Exercise 12.03

a How many boxes do you think you would have to buy to get all 12 toys?

b Run the simulation 20 times and record the number of boxes Jayden will have to buy to get all 12 toys.

c What was the lowest number of boxes?

d What was the highest number of boxes?

e How did your results compare with what you expected in part a?

10 Anton visited a wharf from which tourists depart to see the Great Barrier Reef. He asked a random selection of 20 people some questions about the Reef. These are his results. Exercise 12.03

Question	Yes	No
Do you agree the Great Barrier Reef needs environmental protection?	17	3
Are there too many tourists visiting the Reef?	12	8
Is it OK to destroy parts of the Reef for economic benefits?	4	16

a What is the probability that a person selected at random agrees that there are too many tourists visiting the Reef?

b Calculate as a decimal the probability that a person selected at random does not agree with environmental protection for the Reef.

c Suggest a factor that could create bias in Anton's results.

11 Use the table from page 384 to answer this question. The French Open tennis is played in Paris, France at the end of May and early June. At this time of year Paris is on daylight saving time. Exercise 13.04

a Qualifying matches start at 10 a.m. local time. What time is this in Western Australia?

b You are watching the tennis on TV in WA and the semi-finals start at 8 p.m. What time is it in Paris?

c The Women's Final starts at 2.30 p.m. Saturday in Paris. The final lasts 3 hours and 35 minutes. At what time in WA will this final finish?

12 Ethan earns a salary of \$72 300. The salary is increased each year by the inflation rate of 2.1% p.a. Calculate Ethan's salary in 5 years, correct to the nearest dollar. Exercise 11.03

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14. BORROWING MONEY

Chapter problem

Lee borrowed $240 000 to buy an apartment. She is going to repay the loan plus interest in monthly instalments at 7.8% p.a. monthly reducible finance.

If Lee borrows the money over 15 years, the monthly repayments will be $2265.95, and if she borrows the money over 30 years, the monthly repayments will be $1727.70.

a Will Lee pay more money if she takes the loan for 15 years or 30 years? What is the difference in amount paid?

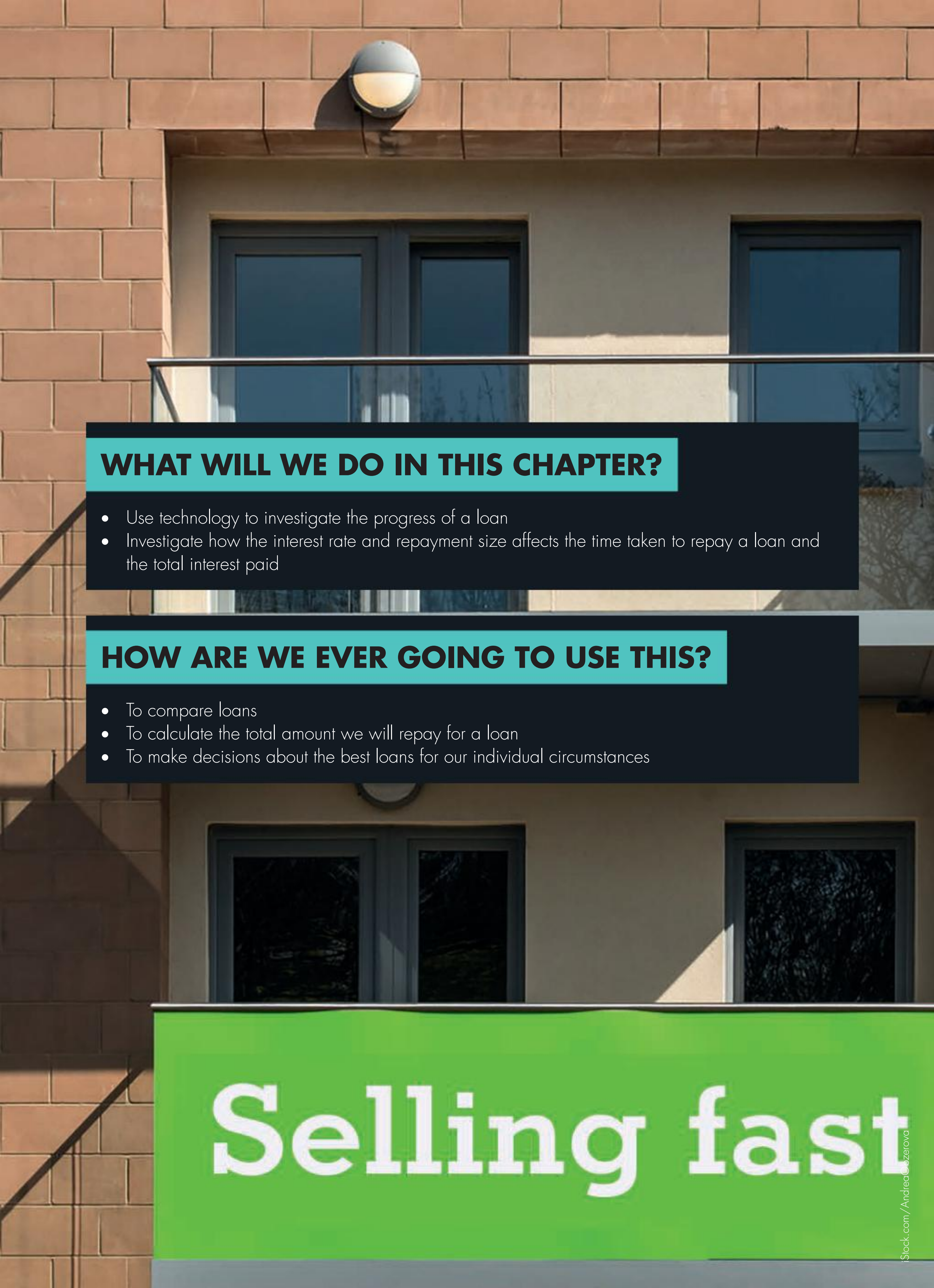
b Suggest a reason Lee might decide to take the loan over 30 years.

WHAT WILL WE DO IN THIS CHAPTER?

- Use technology to investigate the progress of a loan
- Investigate how the interest rate and repayment size affects the time taken to repay a loan and the total interest paid

HOW ARE WE EVER GOING TO USE THIS?

- To compare loans
- To calculate the total amount we will repay for a loan
- To make decisions about the best loans for our individual circumstances



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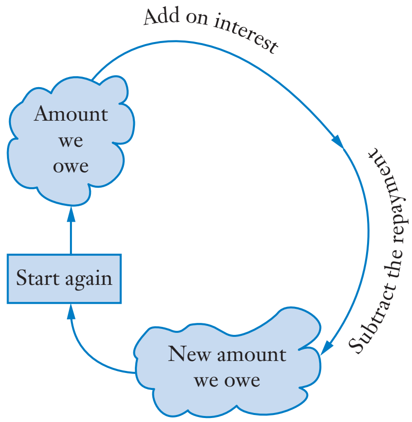
14.01 Reducing balance loans

When we borrow money from a financial institution such as a bank, credit union or finance company, we are charged **interest**. The interest rate depends on the type of financial institution and the reason we are borrowing the money. Wise consumers shop around for the best value before they commit themselves.

There are 2 types of interest on loans.

With **simple interest**, also known as **flat-rate interest**, we pay the same amount of interest throughout the loan, regardless of the amount we owe.

With **reducible interest,** we only pay interest on the amount of money still owing. As the amount we owe decreases, the interest decreases.



This table shows the progress of a **reducing balance loan**. The principal is \$15 000, borrowed at 9% p.a. monthly reducing interest, with monthly **repayments** of \$800. All amounts are rounded to the nearest cent.

1st month's interest
0.09 ÷ 12 × \$15 000

\$15 000 + \$112.50

\$15 112.50 – \$800

Reducing balance loan
Amount borrowed: \$15 000
Interest rate: 9% p.a. monthly reducible
Monthly repayment (*R*): \$800

Month	Principal (*P*)	Interest (*I*)	Principal + Interest (*P*+*I*)	Amount owing (*P*+*I*–*R*)
1st	\$15 000	\$112.50	\$15 112.50	\$14 312.50
2nd	\$14 312.50	\$107.34	\$14 419.84	\$13 619.84
3rd	\$13 619.84	\$102.15	\$13 721.99	\$12 921.99
4th	\$12 921.99	\$96.91	\$13 018.90	\$12 218.90

\$14 419.84 – \$800

The principal at the start of the 2nd row is the same as the amount at the end of the 1st row.

2nd month's interest
0.09 ÷ 12 × \$14 312.50

\$14 312.50 + \$107.34

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EXAMPLE 1

Reducing balance loans

When Ethan borrowed \$10 000 from the bank to buy a small car, the bank charged 9% p.a. reducible interest and his monthly repayments were \$240. Find the values of **a** to **h** in the table, then find how much Ethan will owe after he has made 3 repayments, and the total interest he will pay in the first 3 months. Round all values to the nearest cent.

Ethan's reducing balance car loan				
Amount borrowed: \$10 000				
Interest rate: 9% p.a. monthly reducible				
Monthly repayments: \$240				
Month	**Principal (P)**	**Interest (I)**	**Principal + interest ($P + I$)**	**Amount owing ($P + I - R$)**
1	\$10 000	\$75	\$10 075	\$9835
2	**a**	**b**	**c**	**d**
3	**e**	**f**	**g**	**h**

a The principal for the 2nd month is the same as the amount owing at the end of the 1st month. \$9835

b Calculate 1 month's interest at 9% p.a. on the amount Ethan owes. $\$9835 \times 0.09 \div 12 = \$73.76$

c $P + I$ is the sum of **a** and **b**. $\$9835 + \$73.76 = \$9908.76$

d The amount owing at the end of the 2nd month is **c** minus the monthly repayment of \$240. $\$9908.76 - \$240 = \$9668.76$

e The principal for the 3rd month is the same as the amount owing at the end of the 2nd month. \$9668.76

f Calculate 1 month's interest on \$9668.76. $\$9668.76 \times 0.09 \div 12 = \$72.52$

g $P + I$ is the sum of **e** and **f**. $\$9668.78 + \$72.52 = \$9741.28$

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h The amount owing at the end of the 3rd month is **g** minus $240.

$9741.28 – $240 = $9501.28

Ethan's reducing balance car loan				
Amount borrowed: $10 000				
Interest rate: 9% p.a. monthly reducing				
Monthly repayments: $240				
Month	**Principal (P)**	**Interest (I)**	**Principal + interest ($P + I$)**	**Amount owing ($P + I - R$)**
1st	$10 000	$75	$10 075	$9835
2nd	$9835	$73.76	$9908.76	$9668.76
3rd	$9668.76	$72.52	$9471.28	$9501.28

The amount Ethan still owes after he has made 3 repayments is the amount owing after the 3rd month.

$9501.28

Notice that the interest decreases as the amount owing decreases.

The interest Ethan has paid is the sum of the values in the interest column.

Interest = $75 + $73.76 + $72.52

= $221.28

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Exercise 14.01 Reducing balance loans

Example 1

1 Chantelle borrowed $7000 from the bank at 9% p.a. monthly reducible interest to buy her first car. Her monthly instalments are $320.

a Use a calculation to show that Chantelle will pay $52.50 for interest in the first month.

b Use another calculation to show that Chantelle will owe $7052.50 at the end of the first month immediately before she makes her first repayment.

c Explain how you know that Chantelle will owe $6732.50 immediately after she makes her first repayment.

d Find the values of **A** to **H** in Chantelle's repayment table.

Chantelle's reducing balance car loan				
Amount borrowed: $7000				
Interest rate: 9% p.a. monthly reducible				
Monthly repayments: $320				
Month	**Principal (P)**	**Interest (I)**	**Principal + interest ($P + I$)**	**Amount owing ($P + I - R$)**
1	$7000	$52.50	$7052.50	$6752.50
2	**A**	**B**	**C**	**D**
3	**E**	**F**	**G**	**H**

e How much will Chantelle owe immediately after she makes her 3rd repayment?

f How much interest will Chantelle pay in the first 3 months of the loan?

g How much less interest will she pay in the 4th month than in the 1st month?

h Why is the amount of interest getting smaller each month?

2 **a** Copy and complete the first 4 lines of Ryan's personal loan.

Ryan's reducing balance loan				
Amount borrowed: $16 000				
Interest rate: 7.2% p.a. monthly reducing				
Monthly repayments: $400				
Month	**Principal (P)**	**Interest (I)**	**Principal + interest ($P + I$)**	**Amount owing ($P + I - R$)**
1	$16 000	$96	$16 096	$15 696
2				
3				
4				

b How much will Ryan owe immediately after he has made his 4th repayment?

c What percentage of Ryan's 4th repayment was interest? Give your answer correct to one decimal place.

3 Bianca borrowed \$18 000 at 8% p.a. reducible interest, calculated every 6 months, to expand her business. She agreed to repay the loan over 3 years with half-yearly repayments of \$3434.

a Calculate the amount of interest that Bianca will pay in her first half-year.

b Find the values of **A** to **E** in the table.

Bianca's business loan				
Amount borrowed: \$18 000				
Interest rate: 8% p.a. reducible				
Half-yearly repayments: \$3434				
Half-years	**Principal (P)**	**Interest (I)**	**Principal + interest ($P + I$)**	**Amount owing ($P + I - R$)**
1	\$18 000	\$720	\$18 720	\$15 286
2	**A**	**B**	\$15 897.44	**C**
3	\$12 463.44	**D**	\$12 961.98	\$9527.98
4	\$9527.98	\$381.12	\$9909.10	\$6475.10
5	\$6475.10	\$259.00	**E**	\$3300.10
6	\$3300.10	\$132.00	\$3432.10	0

c After how long will Bianca have repaid more than half the loan?

4 Jackson borrowed \$12 800 at 7.56% p.a. monthly reducible interest to buy a boat.

a Complete the first 3 rows of this table.

Jackson's loan				
Amount borrowed: \$12 800				
Interest rate: 7.56% p.a. monthly reducible				
Monthly repayments: \$900				
Month	**Principal (P)**	**Interest (I)**	**Principal + interest ($P + I$)**	**Amount owing ($P + I - R$)**
1	\$12 800			
2				
3				

b Calculate the total amount Jackson will repay in the first 3 months.

c How much will Jackson repay off the principal in the first 3 months of the loan?

d How much interest will he pay in the first 3 months?

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5 a Create a spreadsheet to calculate the progress of a \$10 000 loan at 6% p.a. monthly reducible interest with repayments of \$260 per month. Use this spreadsheet outline to get started.

	A	B	C	D	E
1	Month	Principal (P)	Interest (I)	Principal + interest ($P + I$)	Amount owing ($P + I - R$)
2	1	\$10 000			
3	2				
4	3				
5	4				
6	5				

b Use your spreadsheet to calculate the amount still owing on the loan immediately after the 5th repayment.

14.02 Loan spreadsheets

WS Reducing balance loan: spreadsheet

Home loans are an example of a reducing balance loan that lasts for a long time, for example, 30 years. Before calculators were available, banks had to calculate loan and interest amounts using pen-and-paper! The table for a 30-year loan would require 360 rows, one for each month.

Today, however, we have technology such as spreadsheets to do the 'number-crunching' involved.

Reducible loans

EXAMPLE 2

To answer this question, download the 'Reducible loans' spreadsheet from NelsonNet.

Jordan is borrowing \$20 000 at 6% p.a. reducible interest and his monthly repayments are \$810.

a How long will it take Jordan to repay the loan?

b How much interest will Jordan pay?

Solution

In the blue cells, enter 20 000 for the loan amount, 0.06 for the interest rate and 810 for his monthly repayment.

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	A	B	C	D	E	F
1	**Reducible loans**					
2	Only enter data in cells shaded blue. Enter the loan and repayment amounts without any spaces, commas or $ sign.					
3						
4	Loan amount	$20,000.00				
5	Interest rate as a decimal	0.06				
6	Monthly repayment	$810.00				
7						
8		Amount owing at the beginning of the month	Interest charge for the month	Amount owing plus interest	Amount owing after the repayment	
9	1st month	$20,000.00	$100.00	$20,100.00	$19,290.00	
10	2nd month	$19,290.00	$96.45	$19,386.45	$18,576.45	
11	3rd month	$18,576.45	$92.88	$18,669.33	$17,859.33	
12	4th month	$17,859.33	$89.30	$17,948.63	$17,138.63	
13	5th month	$17,138.63	$85.69	$17,224.32	$16,414.32	
14	6th month	$16,414.32	$82.07	$16,496.39	$15,686.39	
15	7th month	$15,686.39	$78.43	$15,764.83	$14,954.83	
16	8th month	$14,954.83	$74.77	$15,029.60	$14,219.60	
17	9th month	$14,219.60	$71.10	$14,290.70	$13,480.70	
18	10th month	$13,480.70	$67.40	$13,548.10	$12,738.10	
19	11th month	$12,738.10	$63.69	$12,801.79	$11,991.79	
20	12th month	$11,991.79	$59.96	$12,051.75	$11,241.75	
21	13th month	$11,241.75	$56.21	$11,297.96	$10,487.96	
22	14th month	$10,487.96	$52.44	$10,540.40	$9,730.40	
23	15th month	$9,730.40	$48.65	$9,779.05	$8,969.05	
24	16th month	$8,969.05	$44.85	$9,013.90	$8,203.90	
25	17th month	$8,203.90	$41.02	$8,244.92	$7,434.92	
26	18th month	$7,434.92	$37.17	$7,472.09	$6,662.09	
27	19th month	$6,662.09	$33.31	$6,695.40	$5,885.40	
28	20th month	$5,885.40	$29.43	$5,914.83	$5,104.83	
29	21st month	$5,104.83	$25.52	$5,130.35	$4,320.35	
30	22nd month	$4,320.35	$21.60	$4,341.95	$3,531.95	
31	23rd month	$3,531.95	$17.66	$3,549.61	$2,739.61	
32	24th month	$2,739.61	$13.70	$2,753.31	$1,943.31	
33	25th month	$1,943.31	$9.72	$1,953.03	$1,143.03	
34	26th month	$1,143.03	$5.72	$1,148.74	$338.74	
35	27th month	$338.74	$1.69	$340.44	-$469.56	
36	28th month	-$469.56	-$2.35	-$471.91	-$1,281.91	
37	29th month	-$1,281.91	-$6.41	-$1,288.32	-$2,098.32	
38	30th month	-$2,098.32	-$10.49	-$2,108.81	-$2,918.81	
39	31st month	-$2,918.81	-$14.59	-$2,933.41	-$3,743.41	
40	32nd month	-$3,743.41	-$18.72	-$3,762.12	-$4,572.12	
41	33rd month	-$4,572.12	-$22.86	-$4,594.98	-$5,404.98	
42	34th month	-$5,404.98	-$27.02	-$5,432.01	-$6,242.01	
43	35th month	-$6,242.01	-$31.21	-$6,273.22	-$7,083.22	
44	36th month	-$7,083.22	-$35.42	-$7,118.63	-$7,928.63	

a Look at the values in the last column, E. When the amount owing becomes negative, the loan has been paid off. The first negative amount owing is in cell E35, which corresponds to the 27th payment month.

Jordan will make 26 monthly payments of $990, but his final payment only needs to be $340.44.

Jordan will repay the loan after 27 months (which is 2 years and 3 months).

b Add all the values in the interest column up to the 27th month. Entering a formula into the spreadsheet will be the easiest way.

The formula is = SUM(C9:C35).

Jordan will pay $1400.44 interest.

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Exercise 14.02 Loan spreadsheets

Use the 'Reducible loans' spreadsheet from NelsonNet for this exercise.

Reducible loans

Example 2

1 Samantha borrowed \$5000 at 7% p.a. reducible interest to buy a motor scooter. Her monthly repayments are \$350.

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- **a** How much interest will Samantha pay in the first month?
- **b** How much of Samantha's second repayment will be interest?
- **c** How much will Samantha owe immediately after she has made her 12th repayment?
- **d** How long will it take Samantha to repay the loan?
- **e** What is the value of Samantha's last repayment?
- **f** How much interest will Samantha pay on the loan?
- **g** Calculate the total amount Samantha will repay.

2 Trinh borrowed \$6000 at 4.25% p.a. reducible interest to buy some new computer equipment. Each month Trinh repays \$360.

- **a** How long will it take Trinh to repay the loan?
- **b** How much interest will he pay?
- **c** How much more than he borrowed will Trinh repay?

3 Jacinta's grandmother lends her \$7000 interest-free to buy her first car, provided that she repays her \$400 each month.

- **a** How long will it take Jacinta to repay the interest-free loan?
- **b** If she had to borrow the money from the bank, Jacinta would be charged 9% p.a. reducible interest. How much interest is she saving with her grandmother's interest-free loan?

4 Ashok wants to borrow \$9000. The bank has offered him a reducible interest loan at 7.4% p.a. and suggested that he repay \$400 each month.

- **a** How long will it take Ashok to repay the loan with monthly repayments of \$400?
- **b** Ashok thinks he can afford to repay \$550 per month. How long will it take him to repay the loan at \$550 per month?
- **c** Will Ashok save any money by paying the loan off more quickly? Justify your answer.
- **d** What general conclusion can you make about the advantage of repaying a reducible loan quickly?
- **e** Is the same conclusion true for flat-rate loans? Explain your answer.

5 **a** On the ‘Reducible loans’ spreadsheet, what formulas are in cells B9, C9, D9 and E9?

b Why do the formulas in C9 and E9 contain $ signs, but the formulas in B9 and D9 don’t?

6 Lily made a spreadsheet of her reducing balance loan.

	A	B	C	D	E	F
1	**Reducible loans**					
2	Only enter data in cells shaded blue. Enter the loan and repayment amounts without any spaces, commas or $ sign.					
3						
4	Loan amount	$4,000.00				
5	Interest rate as a decimal	0.06				
6	Monthly repayment	$120.00				
7						
8		Amount owing at the beginning of the month	Interest charge for the month	Amount owing plus interest	Amount owing after the repayment	
9	1st month	$4,000.00	$20.00	$4,020.00	$3,900.00	
10	2nd month	$3,900.00	$19.50	$3,919.50	$3,799.50	
11	3rd month	$3,799.50	$19.00	$3,818.50	$3,698.50	
12	4th month	$3,698.50	$18.49	$3,716.99	$3,596.99	
13	5th month	$3,596.99	$17.98	$3,614.97	$3,494.97	
14	6th month	$3,494.97	$17.47	$3,512.45	$3,392.45	
15	7th month	$3,392.45	$16.96	$3,409.41	$3,289.41	
16	8th month	$3,289.41	$16.45	$3,305.86	$3,185.86	
17	9th month	$3,185.86	$15.93	$3,201.79	$3,081.79	
18	10th month	$3,081.79	$15.41	$3,097.20	$2,977.20	
19	11th month	$2,977.20	$14.89	$2,992.08	$2,872.08	
20	12th month	$2,872.08	$14.36	$2,886.44	$2,766.44	
21	13th month	$2,766.44	$13.83	$2,780.28	$2,660.28	
22	14th month	$2,660.28	$13.30	$2,673.58	$2,553.58	
23	15th month	$2,553.58	$12.77	$2,566.35	$2,446.35	
24	16th month	$2,446.35	$12.23	$2,458.58	$2,338.58	
25	17th month	$2,338.58	$11.69	$2,350.27	$2,230.27	
26	18th month	$2,230.27	$11.15	$2,241.42	$2,121.42	
27	19th month	$2,121.42	$10.61	$2,132.03	$2,012.03	
28	20th month	$2,012.03	$10.06	$2,022.09	$1,902.09	
29	21st month	$1,902.09	$9.51	$1,911.60	$1,791.60	
30	22nd month	$1,791.60	$8.96	$1,800.56	$1,680.56	
31	23rd month	$1,680.56	$8.40	$1,688.96	$1,568.96	
32	24th month	$1,568.96	$7.84	$1,576.80	$1,456.80	
33	25th month	$1,456.80	$7.28	$1,464.09	$1,344.09	
34	26th month	$1,344.09	$6.72	$1,350.81	$1,230.81	
35	27th month	$1,230.81	$6.15	$1,236.96	$1,116.96	
36	28th month	$1,116.96	$5.58	$1,122.55	$1,002.55	
37	29th month	$1,002.55	$5.01	$1,007.56	$887.56	
38	30th month	$887.56	$4.44	$892.00	$772.00	
39	31st month	$772.00	$3.86	$775.86	$655.86	
40	32nd month	$655.86	$3.28	$659.14	$539.14	
41	33rd month	$539.14	$2.70	$541.83	$421.83	
42	34th month	$421.83	$2.11	$423.94	$303.94	
43	35th month	$303.94	$1.52	$305.46	$185.46	
44	36th month	$185.46	$0.93	$186.39	$66.39	

What formulas did Lily enter in cells B9, C9, D9, E9 and B10?

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7 Gabriel borrowed \$30 000 from the bank to buy a car. The bank is charging him 7.5% p.a. monthly reducible finance, and each month Gabriel repays \$950.

Use a spreadsheet to determine the number of monthly repayments Gabriel will make and the total amount of interest he will pay on the loan.

8 Monique borrowed \$6400 at 5.8% p.a. monthly reducing interest to go on a holiday to Japan. Her monthly repayments are \$520.

a How long will it take Monique to repay the loan?

b How much interest will Monique pay?

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9 Use the spreadsheet to investigate how increasing the interest rate affects the time taken to repay a loan, if the monthly repayment stays the same.
Copy and complete this statement:

When interest rates go up, it takes us _______ to repay the loan and we pay _______ interest.

10 Use the spreadsheet to investigate how increasing the monthly repayment amount affects the time taken to repay a loan, if the interest rate stays the same.
Copy and complete this statement:

If we increase our monthly repayments, it takes us _______ to repay the loan and we pay _______ interest.

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MoneySmart

14.03 Online loan calculators

Most people use online calculators to help them investigate and manage reducing balance loans. Find an online loan calculator to use in this section, for example, the mortgage calculator on the **MoneySmart** website.

EXAMPLE 3

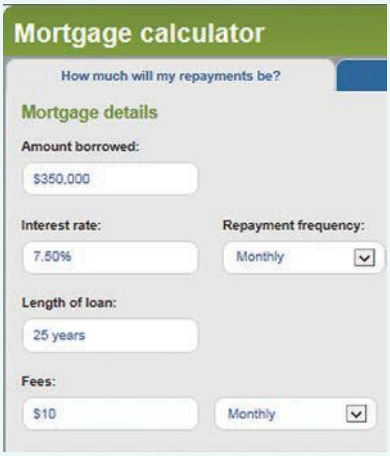
Elle borrows \$350 000 to buy an apartment to live in. She will repay the loan in equal monthly instalments over 25 years at 7.5% p.a. monthly reducible interest. In addition, she will be charged a \$10 monthly account-keeping fee.

a How much are Elle's monthly repayments?

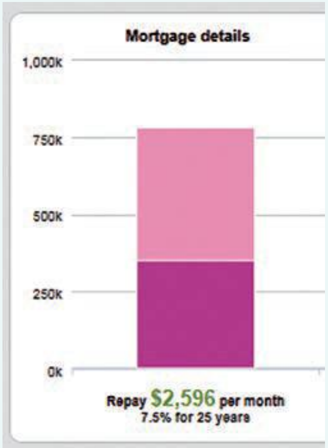
b How much will Elle pay in interest and fees?

Solution

a Enter the values into the mortgage calculator.



ASIC's MoneySmart website Mortage Calculator (https://www.moneysmart.gov.au/tools-and-resources/calculators-and-tools/mortgage-calculator). Date accessed March 2017



Elle's monthly repayments will be \$2596.

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b	She will make monthly repayments for 25 years.	Number of months = 25×12 $= 300$
	Find the total repaid.	Total repaid = repayment × number of months in 25 years = \$2596 × 300 = \$778 800
	Calculate the interest and fees.	Interest and fees = Total repaid – amount borrowed = \$778 800 – \$350 000 = \$428 800
	Write your answer.	Elle will pay \$428 800 in interest and fees.

If the online calculator's answer is a little different to yours, that's because it shows the repayment amount (\$2596) **correct to the nearest dollar**, but when it does its calculations it uses a more precise value.

Alamy Stock Photo/Stephen Dwyer

MoneySmart

EXAMPLE 4

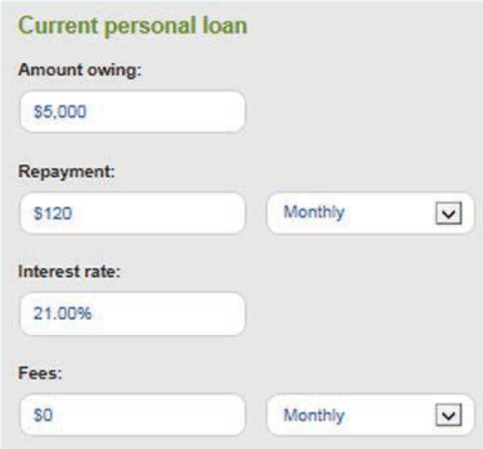
Owen overspent on his credit card and bought \$5000 worth of items with it. His credit card charges 21% p.a. monthly reducible interest, but he can only afford to repay \$120 per month.

a How long will it take Owen to repay the \$5000 credit card bill if he makes \$120 repayments each month?

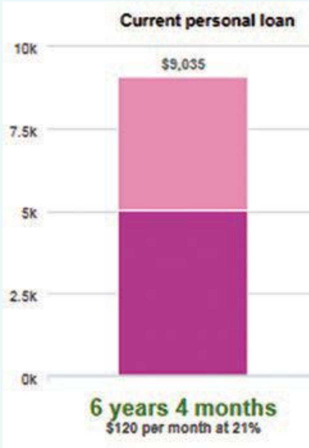
b How much will it cost Owen to pay off his \$5000 credit card bill?

Solution

a Enter the values into the personal loan calculator on MoneySmart. Use the calculator in the tab 'How can I repay my loan sooner'. Set fees to \$0.



ASIC's MoneySmart website Mortgage Calculator (https://www.moneysmart.gov.au/tools-and-resources/calculators-and-tools/personal-loan-calculator). Date accessed March 2017



The calculator says that it will take Owen 6 years and 4 months to pay off the loan.

b Calculate the number of repayments.

$$\text{Number of repayments} = 6 \times 12 + 4$$
$$= 76$$

Calculate the total repaid.

$$\begin{aligned}\text{Total repaid} &= \text{repayment} \\ &\quad \times \text{number of months} \\ &= \$120 \times 76 \\ &= \$9120\end{aligned}$$

Write your answer.

It will cost Owen \$9120 to pay off his \$5000 credit card bill.

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Exercise 14.03 Online loan calculators

You will need access to online calculators for this exercise.

Use the personal loan calculator to answer questions **1** to **3**.

MoneySmart

1 Claire is borrowing $15 000 at 9.8% p.a. monthly reducible interest to start a business. She plans to repay the loan in monthly repayments over 5 years.

Example 3

- **a** How much are her monthly repayments?
- **b** Calculate the total amount Claire will pay in the monthly instalments.
- **c** How much interest will Claire pay during the 5 years?

2 Juan wants to borrow $40 000 to buy an SUV. He can get the money from a finance company at 17% p.a. reducible interest.

- **a** Calculate the monthly repayments if Juan takes the loan over 15 years.
- **b** How much interest will he pay if he takes the loan over 15 years?
- **c** Explain why 15 years isn't a suitable term for a car loan.
- **d** If you were Juan, what would you do?

3 Hoa wants to borrow $10 000 to go on a trip to Canada. Her bank will lend her the money at 8.75% p.a. reducible finance with no monthly account fees. Hoa can afford to repay $650 per month.

Example 4

- **a** Use the 'How can I repay my loan sooner' tab on the personal loan calculator to calculate the number of monthly repayments Hoa will make to repay the loan.
- **b** How much interest will Hoa pay?

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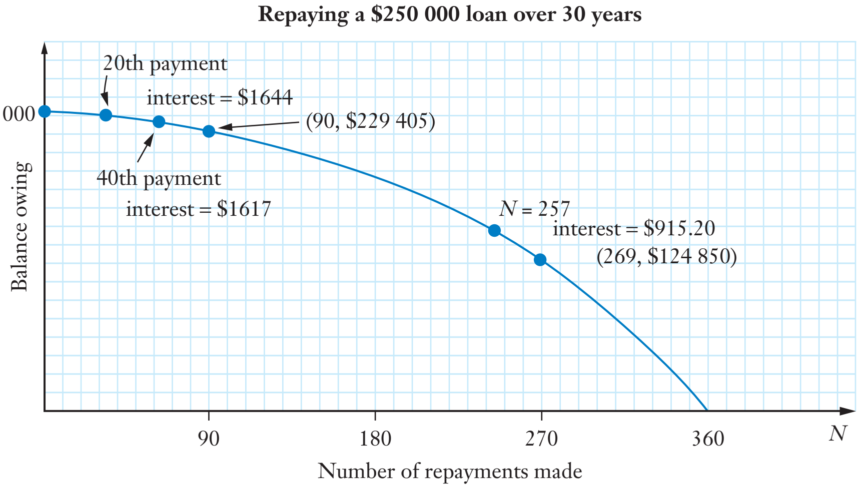
4 When Chris bought his townhouse, he borrowed \$250 000 at 8% p.a. monthly reducing interest over 30 years and his monthly repayments were \$1834.40. The graph shows the balance he owed on the loan after N months.



a The graph shows that in the 20th month, the interest paid was \$1644, but the monthly repayment was \$1834.40. How much of the repayment was not interest but a decrease in the loan?

b The graph shows that in the 40th month, the interest paid was \$1617. For a reducing balance loan, why is the interest in the 40th repayment less than the interest in the 20th repayment?

c Use the graph to determine the amount that Chris still owed on the loan after 90 months.

d Calculate the total monthly repayments Chris made during the 90 months.

e In the 257th month, how much of the repayment:

   i was interest?

   ii was a decrease in the loan?

f What percentage of the loan is still owing after 269 months?

g How many years and months is 269 months?

h It took Chris 269 months to repay the first half of the loan. How long did it take him to repay the second half of the loan?

i For a reducing balance loan, when do we pay the most interest: at the beginning of the loan or at the end of the loan?

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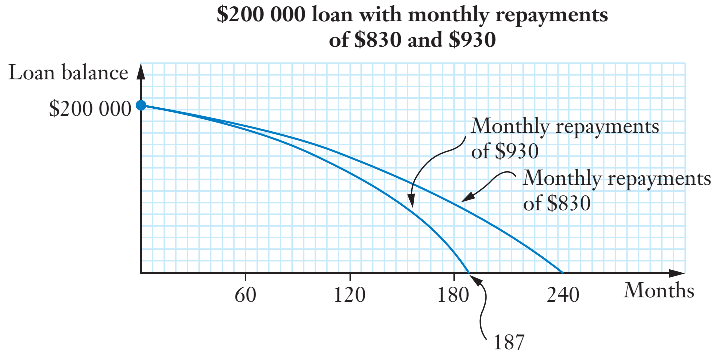
5 When Lara borrowed \$200 000 to buy an apartment, the bank told her the repayments would be \$830 per month. Lara decided that she could afford to repay \$930 per month. The graph shows the balance of a \$200 000 loan with monthly repayments of \$830 and \$930.



a How long will Lara take to repay the loan if she makes monthly repayments of:

i \$830? **ii** \$930?

b How many months will Lara save by making monthly repayments of \$930?

c How much money will Lara save in repayments by making monthly repayments of \$930?

d Why do you think some people may choose to make the smaller monthly repayments even though they will have to pay more in the long run?

Use the mortgage calculator to answer the remaining questions in this exercise.

6 Bree borrowed \$260 000 at 6.75% p.a. over 15 years.

a How much were her monthly repayments?

b Interest rates fell to 6.5% p.a. How much cheaper were Bree's monthly repayments after the fall in interest rates?

7 When interest rates go up, the loan repayments increase. When Jai borrowed \$170 000, the interest rate was 6.75% p.a. and he took the loan over 10 years.

a How much was Jai's monthly repayment when he took out the loan?

b By how much did Jai's monthly repayments increase when the interest rate went up to 7% p.a.?

c How much more did Jai have to repay each year at 7% p.a. compared to the original annual amount?

8 Zhi is buying a house. He is going to borrow \$240 000 at 8% p.a. He is trying to decide whether to take the loan over 20 or 30 years.

a How much more are the monthly repayments over 20 years than over 30 years?

b How much more interest will he pay if he takes the loan over 30 years than over 20 years?

c If you were Zhi, would you take the loan over 20 or 30 years? Give reasons.

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9 During the 1980s, home loan interest rates rose as high as 18% p.a. How much more were the monthly repayments on a $150 000 loan over 20 years in the 1980s compared to the same loan at 5.7% p.a. in the late 2010s?

10 Sarah is deciding between two different home loans. She is borrowing $140 000 over 25 years. Sarah summarised the terms and conditions of the 2 loans in a table.

	Interest rate p.a.	Loan establishment fee	Annual loan fee	Mortgage discharge fee
Big bank	7.1%	$320	$248	$228
Small mortgage company	7.0%	$598	$76	$314

a Calculate the value of the monthly repayments for each loan.

b Copy and complete this table to help Sarah determine the better loan.

	Big bank	Small mortgage company
Loan establishment fee		
Mortgage discharge fee		
Total annual loan fee over 25 years		
Total monthly repayments		
Total cost of the loan		

c Which 2 features of a loan–the interest rate, loan establishment fee, annual loan fee or mortgage discharge fee–most influence the total cost of a loan?

d Which of the 2 loans do you recommend Sarah take? Why?

Chapter problem

You've covered the skills required to solve the chapter problem. Can you solve it now?

14.04 Investigating loans

MoneySmart

When you borrow a large amount of money, does a change in the interest rate affect how long it takes to pay off the loan? Does increasing the monthly repayment amount save any money in the long run? In the following exercise, you are going to investigate these 2 questions yourself. You will need to visit the **MoneySmart** website and search for **Mortgage calculators**. Select **How can I repay my loan sooner**?

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Exercise 14.04 Investigating loans

Work in groups for each investigation.

Changes in interest rates

1 Copy this table and use the online calculator to complete it. The first row has been completed as an example.

Amount borrowed $200 000		**Monthly repayment $1500** **Monthly fee $10**	
Interest rate	**Time to repay the loan**	**Total repaid**	**Interest**
5.00%	16 years 6 months	$295 582	$95 582
5.5%			
5.75%			
6.00%			
7.00%			

2 Change the amount borrowed, then copy and complete this table.

Amount borrowed $		**Monthly repayment $1500** **Monthly fee $10**	
Interest rate	**Time to repay the loan**	**Total repaid**	**Interest**
5.00%			
5.5%			
5.75%			
6.00%			
7.00%			

3 What happens to the amount of interest you pay on a $150 000 loan with monthly repayments of $1000 if the interest rate drops from 4% p.a. to 3.75% p.a.?

4 Write a sentence to describe how changes in interest rates affect the length of time it takes to repay a loan and the total interest involved. Assume the monthly repayment remains the same.

Changes in the monthly repayments

5 Copy this table and use the online calculator to complete it. The first row has been completed as an example.

Amount borrowed $200 000		**Interest rate: 6% pa** **Monthly fee $10**	
Monthly repayment	**Time to repay the loan**	**Total repaid**	**Interest**
$1600	16 years 7 months	$318 030	$118 030
$1400			
$1250			
$2000			
$2400			

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6 How does increasing the monthly repayment amount affect the total interest paid? What effect does decreasing the monthly repayments have?

7 How does increasing the monthly repayment amount affect the time required to pay off the loan? What effect does decreasing the monthly repayments have?

8 Does doubling the monthly repayments halve the time it takes to pay off the loan?

9 Suppose you are going to borrow money to buy a place to live in. What advice can you give yourself about interest rates and repayments?

INVESTIGATION

MAKING SMART REPAYMENTS

MoneySmart

Let's examine the effects of making loan repayments more frequently and increasing the size of each repayment. Visit the **MoneySmart** website and search for **Mortgage calculators**. Select **How can I repay my home sooner?**

1 Declan has a \$250 000 loan at 8% p.a. and he can afford to repay \$3000 per month or \$1500 per fortnight. Does paying off a home loan fortnightly instead of monthly make any difference?

Use the online calculator to help you copy and complete the missing values in the table.

	Monthly repayments	Fortnightly repayments
Value of the repayment	\$3000	\$1500
Time to pay off the loan		
Total repayments		
Total interest		

2 Repeat question **1** for a \$300 000 loan at 12% p.a. with a monthly repayment of \$5000 and a fortnightly repayment of \$2500.

3 Investigate some other loan amounts (principals), but make sure the monthly repayment is twice the value of the fortnightly repayment.

4 What conclusions can you make? Does repaying half the monthly repayment each fortnight make any difference to the loan?

5 Madison borrowed \$250 000 at 8% p.a. with a monthly repayment of \$2000. Select the tab for **How can I repay my home loan sooner?** Use the calculator to find the term (length of time) of Madison's loan. Record the total amount and the interest paid by Madison.

6 Does increasing the monthly repayments by \$20 make any difference? Change the repayment from \$2000 to \$2020. Record the new term of the loan, as well as the new total amount and interest paid.

7 Repeat question **6** for a larger monthly repayment.

8 What conclusions can you make? Does repaying a larger amount make much difference to the loan?

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INVESTIGATION

CAN YOU AFFORD TO BUY A HOME?

Could you buy a home at age 25? Copy the table below or download it from NelsonNet. Complete it by answering the questions on the next page.

WS Can you afford to buy a home?

Name				
1 Future job				
2 Annual gross pay				
3 Monthly gross pay				
4 Maximum monthly repayment				
5 Savings interest rate				
6 Deposit				
7 Mortgage interest rate				
	15 yrs	20 yrs	25 yrs	30 yrs
8 Maximum loan amount				
9 Monthly repayment				
10 Total repayments				
11 Total repayments if paid fortnightly				
12 The amount you can afford to spend on a property				
13 A property you can afford in a suitable location. Paste photo here.	**Address**			
	Price			
14 Net monthly income after deducting income tax and mortgage repayments.				
15 Five more expenses from my net monthly income.				
16 Strategies for making buying property more affordable.				

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1. Write the type of career you plan to have when you are 25, for example, panel beater, vet nurse, childcare worker.
2. Research the annual gross pay you will receive in this occupation. Do not include any overtime.
3. Calculate your monthly gross pay.
4. Calculate 30% of your monthly pay. This is the maximum amount your monthly repayment can be.
5. Research the current savings interest rate.
6. Suppose you are going to save 30% of your income each month for 5 years at the current savings interest rate. Use the MoneySmart **Monthly savings calculator** to determine your savings after 5 years. This amount is your **deposit**.
7. Research the current mortgage interest rate.

MoneySmart

8. Use the MoneySmart **How much can I borrow?** mortgage calculator to determine the maximum amount you can borrow over 15, 20, 25 and 30 years.
9. Record the monthly repayments in row 9 of the table.
10. Record the total you will repay in row 10 of the table.
11. Use the online calculator to determine the total you will repay if you make half the monthly repayment each fortnight. Record these amounts in row 11 of the table.
12. Calculate the amount you can afford to spend on a property by adding the amount you can borrow to the deposit you have saved.
13. Visit a real estate website and search for a property you can afford to buy in a suitable location. Paste a photograph of the property in the table. Record the address of the property and the selling price.
14. Assume that you will pay 25% of your gross pay in income tax. Subtract this tax and your monthly mortgage repayment from your gross pay. This amount represents the net pay you will have left each month. Write it in line 14 of the table.
15. List 5 different things you will have to pay out of the amount remaining in line 15.
16. Buying your first home is financially challenging. In your group, discuss strategies you could use to make it easier. Record the strategies in line 16 of the table.

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KEYWORD ACTIVITY

CHAPTER SUMMARY

Use the terms in this list to copy and complete the chapter summary below.

credit union	decreases	flat	fortnightly	huge	interest
minimum	money	real estate	reducible	repayment	same

When we borrow money from a bank, **1**______________ or finance company, we have to pay **2**______________. Often finance companies charge simple or **3**______________ rate interest. When we borrow money at simple interest, we pay the **4**______________ amount of interest every year, based on the principal borrowed. Reducing balance loans charge **5**______________ interest, where the amount of interest **6**______________ as it is based on the loan amount still owing.

When we borrow a lot of money over a long time, for example, when we buy **7**______________, the total amount we repay can be **8**______________. Even small changes in interest rates can make a big difference to the size of each **9**______________. If we can pay more than the **10**______________ amount required each month, or repay half the monthly repayment **11**______________, we can save a lot of **12**______________ in interest.

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SOLUTION TO THE CHAPTER PROBLEM

Problem

Lee borrowed $240 000 to buy an apartment. She is going to repay the loan plus interest in monthly instalments at 7.8% p.a. monthly reducible finance.

If Lee borrows the money over 15 years, the monthly repayments will be $2265.95, and if she borrows the money over 30 years, the monthly repayments will be $1727.70.

a Will Lee pay more money if she takes the loan for 15 years or 30 years? What is the difference in the amount paid?

b Suggest a reason Lee might decide to take the loan over 30 years.

Solution

a Total amount repaid = monthly repayment × number of months

Number of months = number of years × 12

Loan over 15 years

Total amount repaid = $2265.95 × 15 × 12

= $407 871

Loan over 30 years

Total amount repaid = $1727.70 × 30 × 12

= $621 972

The total amount paid is higher for the 30-year loan.

Difference = $621 972 – $407 871

= $214 101

b Even though Lee will have to repay $214 101 more, she might choose the 30-year loan because she can't afford the monthly repayments of $2265.95 for the 15-year loan.

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14. TEST YOURSELF

Borrowing money

Practice quiz

1 Dee borrowed \$12 000 to buy some equipment for her photography business. The bank charges Dee 7.2% p.a. monthly reducible interest with monthly repayments of \$370.

Exercise 14.01

a Find the values of **A** to **H** in Dee's loan table.

Dee's loan				
Amount borrowed: \$12 000				
Interest rate: 7.2% p.a. monthly reducible				
Monthly repayments: \$370				
Month	**Principal (P)**	**Interest (I)**	**Principal + interest ($P + I$)**	**Amount owing ($P + I - R$)**
1	\$12 000	\$72	\$12 072	\$11 702
2	**A**	**B**	**C**	**D**
3	**E**	**F**	**G**	**H**

b How much will Dee owe immediately after she makes her 3rd repayment?

c How much interest will Dee pay in the first 3 months?

2 Use the 'Reducible loans' spreadsheet from NelsonNet for this question.

Exercise 14.02

Kim is borrowing \$28 000 at 8% p.a. reducible interest to buy a new ute and her monthly repayments are \$950.

Reducible loans

a How long will it take Kim to repay the loan?

b How much interest will she pay?

3 Sam is borrowing \$270 000 to buy an apartment. He is going to repay the loan in equal monthly instalments over 20 years at 7.2% p.a. monthly reducible interest. In addition, he will be charged a \$10 monthly account-keeping fee. Use an online calculator for the following questions.

Exercise 14.03

a How much are Sam's monthly repayments?

b How much will he pay in interest and fees?

c Sam can afford to repay \$2400 per month. How long will it take Sam to repay the loan if he pays \$2400 per month?

d How much will Sam save in interest if he pays \$2400 per month?

4 **a** How does an increase in interest rates affect the value of the repayments on a loan?

Exercise 14.04

b How does an increase in the size of each loan repayment affect the time it takes to pay off the loan and the total amount paid on the loan?

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EARTH GEOMETRY AND TIME ZONES

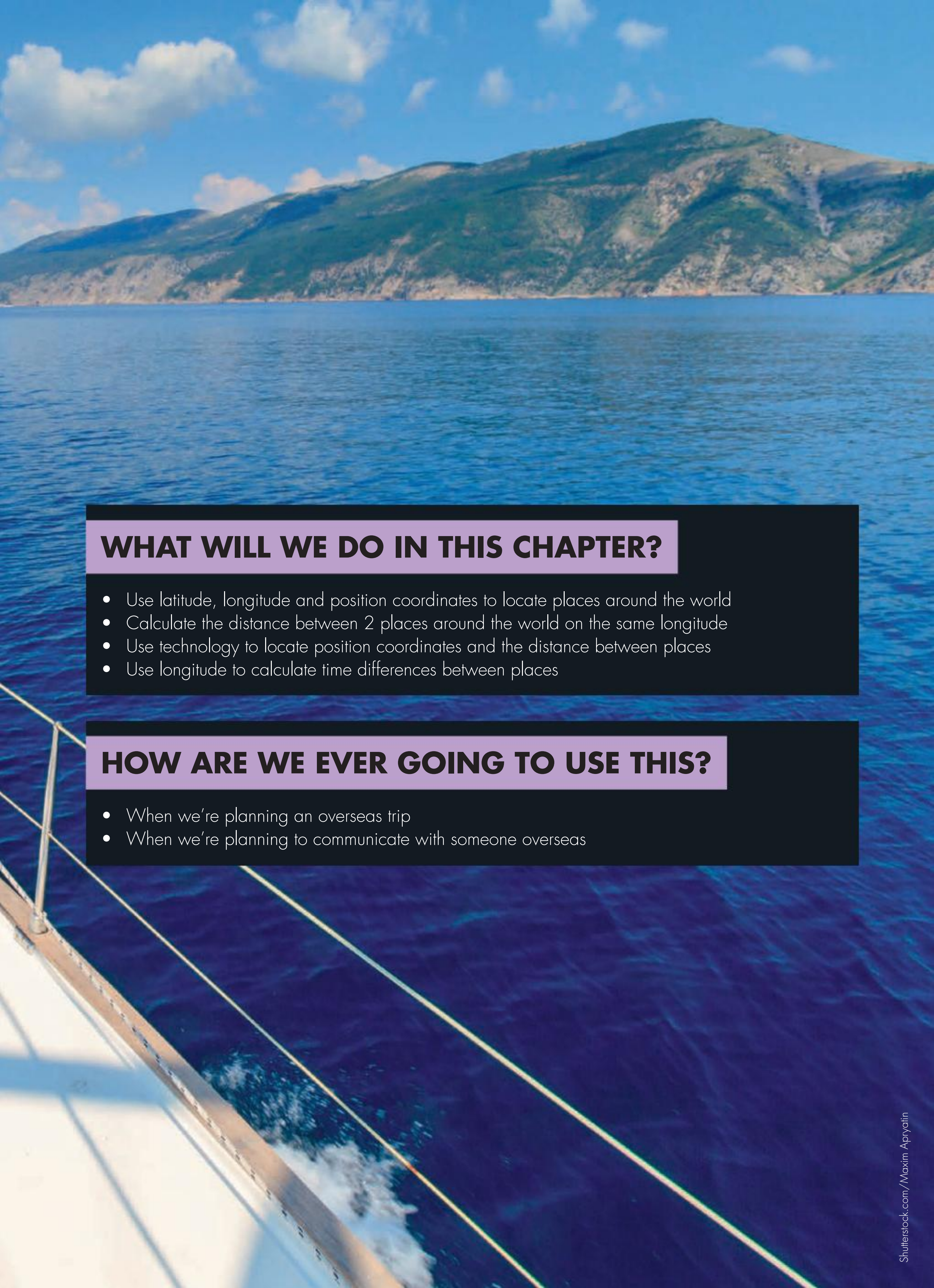
15.

WHERE ON EARTH ARE YOU?

Chapter problem

At 8 a.m. on Monday, Jade left the Cook Islands at 21°S, 160°W to sail up the 160°W meridian of longitude to Hawaii at 21°N, 160°W. Her boat cruises at an average speed of 36 km/h. On what day and at approximately what time can she expect to reach Hawaii?

Chapter outline



WHAT WILL WE DO IN THIS CHAPTER?

- Use latitude, longitude and position coordinates to locate places around the world
- Calculate the distance between 2 places around the world on the same longitude
- Use technology to locate position coordinates and the distance between places
- Use longitude to calculate time differences between places

HOW ARE WE EVER GOING TO USE THIS?

- When we're planning an overseas trip
- When we're planning to communicate with someone overseas

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15.01 Latitude and longitude

WS Longitude and latitude

WS Positions on the globe

WS Map of the world

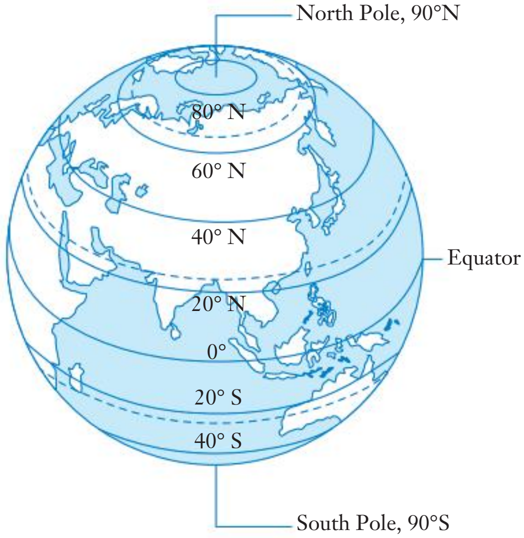
WS Australian coordinates

Street maps use grid references, such as D7, to locate streets, parks and other features. To locate positions on the Earth's surface, we use imaginary latitude and longitude lines on the globe as a grid.

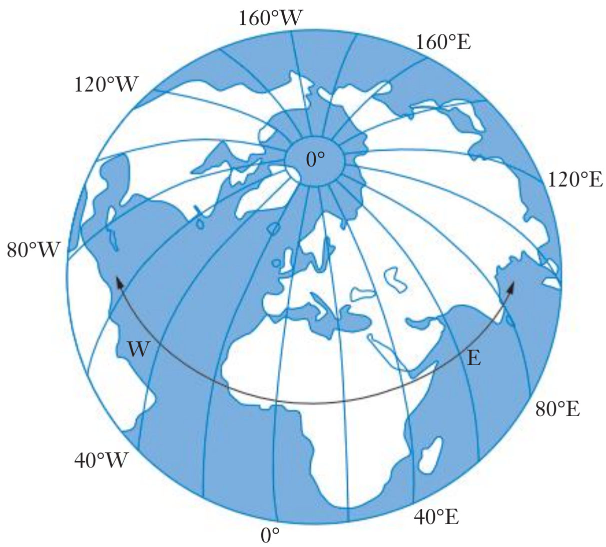
Parallels of **latitude** are imaginary lines that are parallel to the **Equator**. The North Pole has a latitude of 90° north (90°N).

The Equator is at 0° latitude.

The South Pole's latitude is 90°S.



Meridians of **longitude** are imaginary lines running around the Earth from pole to pole. The 0° meridian of longitude runs through the Greenwich Observatory, near London, and is called the **prime meridian** or **Greenwich meridian**. Remember from Chapter 13, *Time travelling*, that time zones are measured from the Greenwich meridian, where the time is UTC 0, also called **Greenwich Mean Time (GMT)**.



Greenwich is pronounced 'gren-itch'.

Longitude is measured in degrees east and west of the prime meridian. The 180°E and 180°W meridians are the same line, on the opposite side of the Earth, where the **International Date Line** is located.

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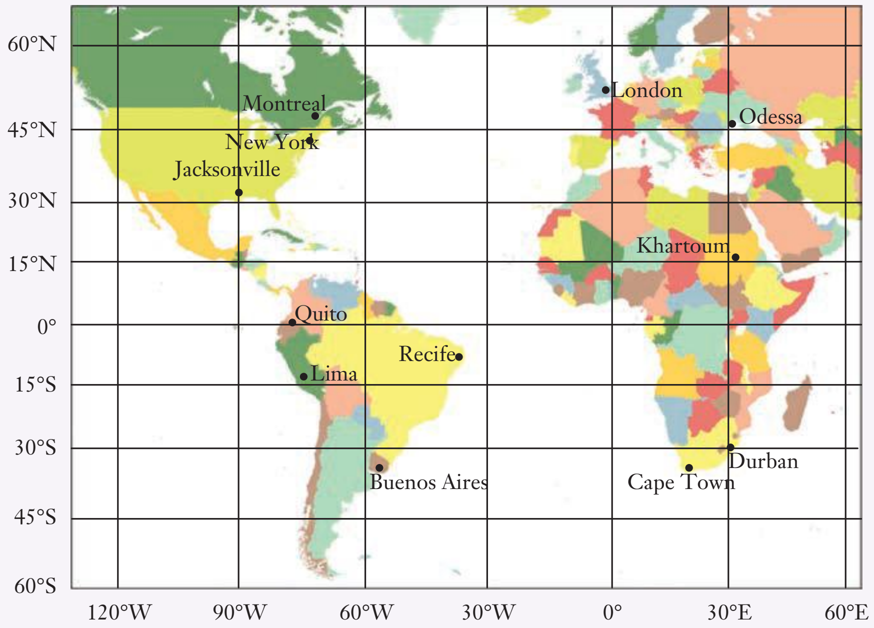
EXAMPLE 1

This map shows the lines of latitude and longitude running across North and South America, Europe and Africa.



a Is the city of Buenos Aires in Argentina north or south of the Equator?

b Is Buenos Aires east or west of Greenwich?

c What are the position coordinates of Odessa, in the Ukraine?

Solution

a Buenos Aires is below the Equator (0° horizontal line), so it is south.

Buenos Aires is south of the Equator.

b Buenos Aires is to the left of the Greenwich meridian (0° vertical line), so it is west.

Buenos Aires is west of Greenwich.

c Odessa is above the Equator at 45°N and right of Greenwich at 30°E.

The coordinates of Odessa are approximately 45°N, 30°E.

Latitude and longitude

When we write position coordinates we always give the **latitude** (**north** or **south**) first and **longitude** (**east** or **west**) second.

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Exercise 15.01 Latitude and longitude

Use the map from the previous page for Questions 1 to 5.

Example 1

1 Which of the cities Cape Town, Montreal, New York and Recife are:

a north of the Equator? **b** south of the Equator?
c east of Greenwich? **d** west of Greenwich?

2 Write 2 cities with approximate latitude 45°N.

3 Write 3 cities with approximate longitude 30°E.

4 Give the approximate latitude and longitude of each city.

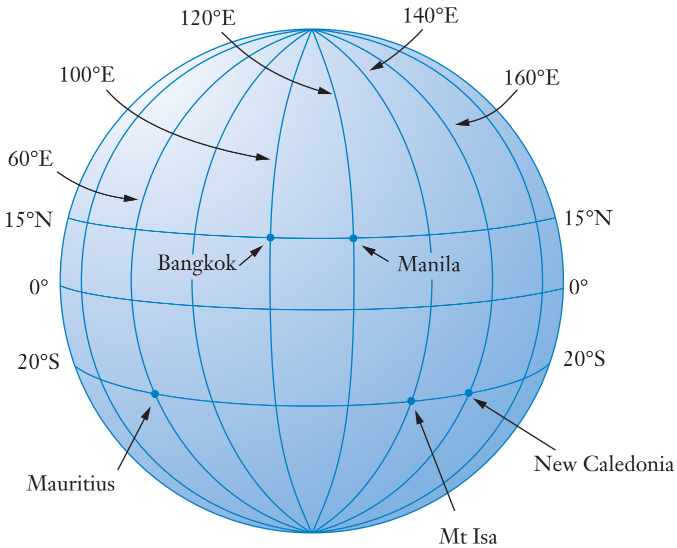
a Jacksonville **b** Buenos Aires **c** Recife
d Khartoum **e** Greenwich (London)

5 Name the city with each pair of coordinates given.

a 45°N, 67°W **b** 40°N, 70°W **c** 32°S, 26°E
d 30°S, 30°E **e** 15°S, 70°W **f** 0°, 75°W

6 Use this globe diagram to write the position coordinates of:

a New Caledonia
b Mt Isa
c Manila
d Mauritius
e Bangkok



7 When Neil was scuba diving, he discovered the wreck of an old Spanish ship at 15°S, 70°E. Is the wreck closest to New Caledonia, Manila, Mauritius or Bangkok?

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INVESTIGATION

WHAT ARE YOUR COORDINATES?

1. Go to the Google Maps website and search for Canberra. Right-click on the map of Canberra and select **What's here?** This will show you that the position coordinates of Canberra are 35°S, 149°E (coordinates are given as North and East so –35°N means 35°S). Now find the coordinates of the capital city of each Australian state and territory.
2. Use Google Maps to find:
   - the position coordinates of your home and school
   - the places with position coordinates 32°S, 115.5°E and 35.7°N, 139.7°W (type the coordinates in the search box without the ° symbols).

Google Maps

15.02 Distances on the Earth's surface

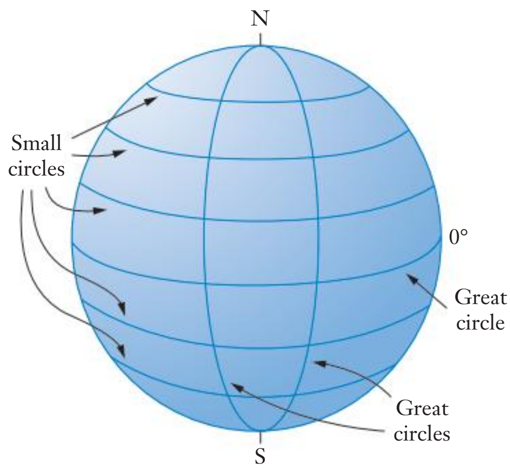
WS Arcs of circles

WS Distances on great circles time trial

We can calculate distances on the Earth by treating it as a **sphere** and using the formula for the arc length of the circle from Chapter 7, *On the surface*. There are 2 types of imaginary circles on the Earth: great circles and small circles.

Great circles are the big circles on the Earth. All the meridians of longitude and the equator are great circles. The radius of a great circle is the same as the radius of the Earth, which is approximately 6400 km.

All the parallels of latitude except the equator are **small circles** of different sizes. The radius of a small circle gets smaller as it moves closer to the North or South poles.



Distance between 2 places on a great circle

To calculate the distance between 2 places on the same great circle, use the formula

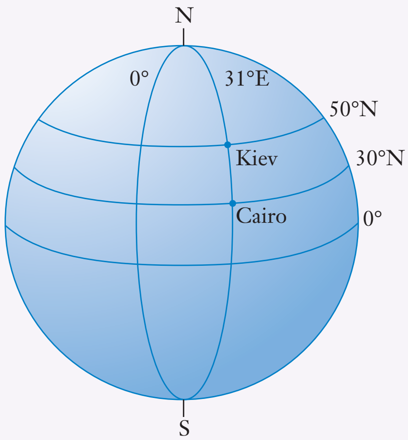
$$\text{Distance} = \frac{\text{Change in the angle}}{360} \times 2\pi R$$

where R = the radius of the Earth (6400 km)

Papa Bravo/Shutterstock.com

EXAMPLE 2

Kiev in the Ukraine (50°N, 31°E) and Cairo in Egypt (30°N, 31°E) are on the same meridian of longitude. Calculate the distance between the 2 cities, correct to the nearest 10 km, using 6400 km as the radius of the Earth.



Solution

Kiev and Cairo have the same longitude. The change in the angle between them is $50° - 30° = 20°$.

$$\text{Distance} = \frac{20}{360} \times 2 \times \pi \times 6400$$
$$\approx 2234.02\ldots \text{ km}$$
$$\approx 2230 \text{ km}$$

Write the answer.

The distance between Kiev and Cairo is approximately 2230 km.

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DID YOU KNOW?

Radius of the Earth

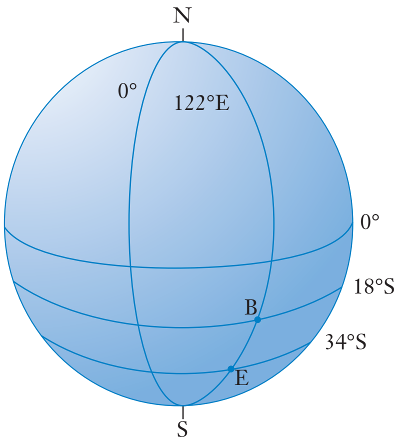
The Earth isn't a perfect sphere because its radius isn't the same everywhere. It bulges at the equator, where the radius is approximately 6378 km. At the poles, it's approximately 6357 km. When we use 6400 km for the radius of the Earth, we have rounded the length to the nearest 100 km. For this reason, we should round off the answers too.

NASA, Rob Simmons

Exercise 15.02 Distances on the Earth's surface

Example 2

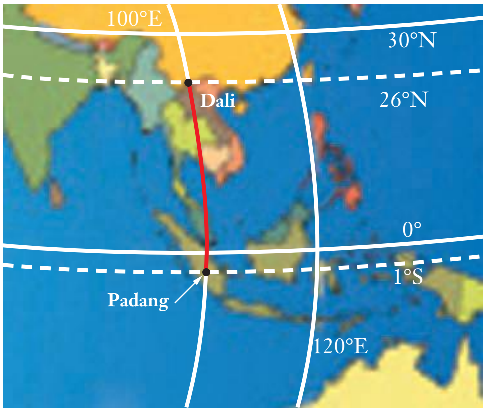
1 Esperance (E) and Broome (B) are on the 122°E meridian of longitude through Western Australia. Esperance is at 34°S, 122°E and Broome is at 18°S, 122°E.



a Use a calculation to show that the change in the angle between Broome and Esperance is 16°.

b Calculate the distance between Broome and Esperance, correct to the nearest 10 km.

2 Angelini chartered a plane in Dali in China to fly to Padang in Sumatra. Dali has position coordinates 26°N, 100°E and Padang's position is 1°S, 100°E.



Ziggymaj/iStock Vectors/Getty Images

a What is the angular difference between Dali and Padang along the 100°E meridian of longitude?

b How far is it from Dali to Padang, correct to the nearest 100 km?

c The flight took 12 hours. Calculate the plane's average speed.

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3 Buenos Aires in Argentina is at 34°S, 58°W and Bridgetown in Barbados is at 13°N, 58°W.

   a Explain why we add 34° and 13° to determine the angular difference between Buenos Aires and Barbados along the 58°W meridian of longitude.

   b Calculate the distance from Bridgetown to Buenos Aires, correct to the nearest 10 km.

4 Durban in South Africa is at 30°S, 30°E and Odessa in the Ukraine is at 45°N, 30°E.

   a What is the angular difference along the 30°E meridian of longitude between Durban and Odessa?

   b Calculate the distance from Durban to Odessa, correct to the nearest 100 km.

5 Port Fairy in Victoria and Weipa in Queensland are both on the 142°E meridian of longitude. Port Fairy is at 38°S and Weipa is 12°S. How far is it from Weipa to Port Fairy?

6 Jonathan was born in Port Hedland, 20°S, 119°E in the north of Western Australia and he works for the council in Sandstone 28°S, 119°E. On long weekends, he catches a light plane home to visit his family in Port Hedland.

   a How far is it from Sandstone to Port Hedland?

   b The light plane flies at an average speed of 120 km/h. How long does Jonathan's flight home take?

7 A mining survey helicopter left Perth, 32°S, 116°E. During an electrical storm, the helicopter was forced to land 1100 km due north of Perth. What were its new coordinates?

Distance calculator

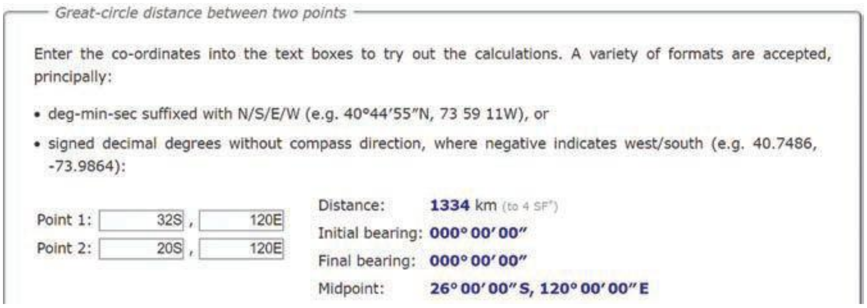
INVESTIGATION

HOW ACCURATE IS IT?

In this activity, you are going to compare your answers to the distance calculations you made in Exercise **15.02** to the distances provided by online calculators.

1 Calculate the distance between the locations 32°S, 120°E and 20°S, 120°E.

2 Use the **distance calculator** on the **Movable Type** website and compare the distance to your calculated distance.



3 Compare your answers to the questions in Exercise **15.02** to those obtained using the website.

4 What conclusions can you make?

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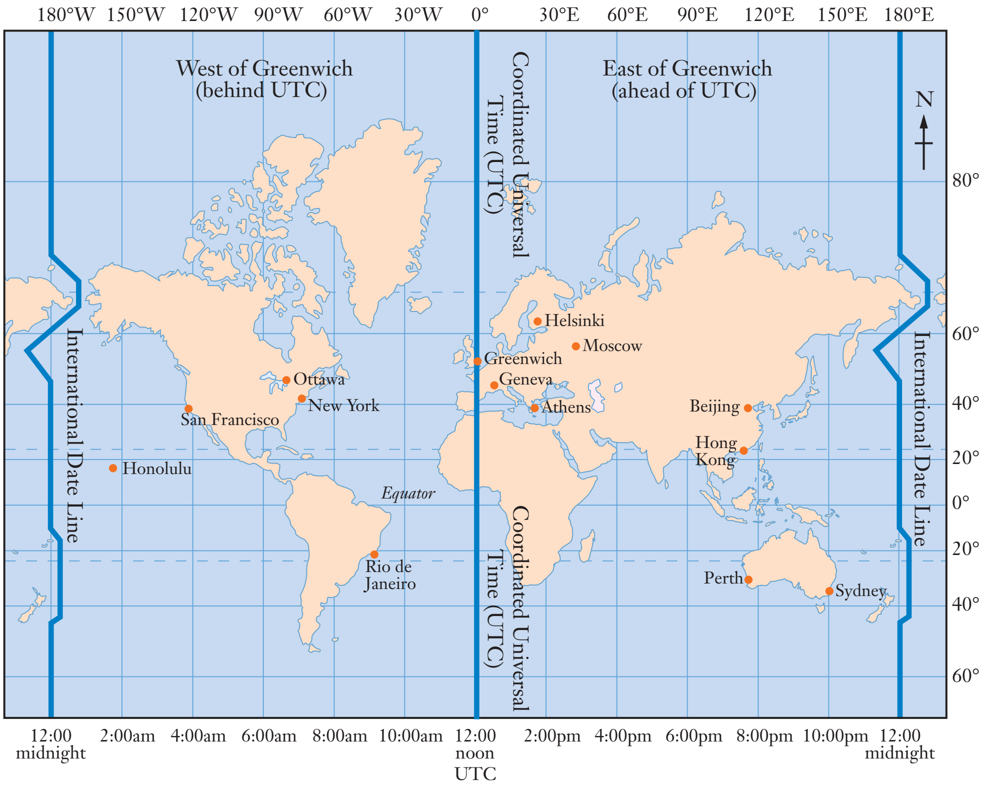
15.03 Longitude and time zones

In Chapter 13, *Time travelling*, we learned about world time zones and UTC time measured from the Greenwich meridian.



Notice from the world map that the meridians of longitude match up with the time zones. The world spans 360 degrees of longitude from 180°W to 180°E, which you would expect as there are 360 degrees in a circle. Also, the world has 24 hourly time zones, which you would expect as there are 24 hours in one day.

So 360° of longitude = 24 hours

Dividing both sides by 24 will give the size of one time zone:

$360° \div 24 = 1$ hour

15° difference in longitude = 1 hour time difference

Notice on the map that 30° difference in longitude = 2 hours' time difference, so this makes sense.

Longitude and time zones

A 15° change in longitude corresponds to a 1 hour time difference.

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EXAMPLE 3

The longitude in Perth is 116°E.

a Explain why the time in Hong Kong, 22°N, 116°E is the same as the time in Perth.

b Bangkok is at 13°N, 101°E. When it is 8 p.m. in Perth, what time is it in Bangkok?

Solution

a Hong Kong and Perth are both on the 116°E meridian of longitude. Places on the same longitude have the same time.

Perth and Hong Kong have the same time because they are on the same longitude.

b First work out the time difference, then determine the local time.

$$\begin{aligned} \text{Difference in longitudes} &= 116° - 101° \\ &= 15° \end{aligned}$$

15° difference in longitude represents a 1 hour time difference.

Perth (116°E) is east of Bangkok (101°E). The time in Perth is 1 hour later than the time in Bangkok.

It is 8 p.m. in Perth.

Write the answer.

It is 7 p.m. in Bangkok.

EXAMPLE 4

Zelko works in an office in Warsaw at 52°N, 20°E. At 11 a.m. local time, he called the office in Adelaide at 35°S, 140°E. What time was it in Adelaide when Zelko phoned?

Solution

Only the longitudes are relevant for time calculations.
First work out the difference in the longitudes, then divide by 15 to calculate the time difference in hours.

$$\begin{aligned} \text{Difference in longitudes} &= 140° - 20° \\ &= 120° \end{aligned}$$

$$\begin{aligned} \text{Time difference} &= 120 \div 15 \\ &= 8 \text{ hours} \end{aligned}$$

Adelaide (140°E) is east of Warsaw (20°E). The time in Adelaide is later than the time in Warsaw.

$$\begin{aligned} \text{Time in Adelaide} &= 11 \text{ a.m.} + 8 \text{ hours} \\ &= 1900 \text{ hours} \\ &= 7 \text{ p.m.} \end{aligned}$$

Write the answer.

When it is 11 a.m. in Warsaw, it is 7 p.m. in Adelaide.

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Exercise 15.03 Longitude and time differences

Example 3

1 Explain why Istanbul, 41°N, 30°E and Cairo 30°N, 30°E have the same time.

2 The difference in longitude between 2 locations is 60°. Use a calculation to show that there is a 4 hour time difference between the locations.

3 What is the time difference between 2 cities that have a 150° difference in longitude?

4 It is 10 a.m. in London, 52°N, 0°. What time is it in each of these locations?

- **a** Ephesus, Turkey, 38°N, 30°E
- **b** Shanghai, China, 31°N, 120°E
- **c** Cairo, Egypt, 30°N, 30°E
- **d** New Orleans, USA, 30°N, 90°W
- **e** Townsville, Queensland, 19°S, 150°E
- **f** Buenos Aires, Argentina 34°S, 60°W
- **g** Texas, USA, 29°N, 90°W

Example 4

5 Kylie is backpacking around India. At 4 p.m. local time at 27°N, 75°E she sent a text message to her mother in Singapore 1°N, 105°E about her visit to the Taj Mahal in Agra. What time was it in Singapore when Kylie sent the text message?

6 Phillipa is working in her company's Pittsburgh office, Pennsylvania, USA at 41°N, 80°W. At 8 a.m. she called Bob in the Las Vegas office at 36°N, 115°W about a legal problem. Suggest a reason why no one answered the phone.

7 Perth is located at 32°S, 116°E and Johannesburg in South Africa is at 26°S, 26°E.

- **a** When it is 8 p.m. on Friday in Perth, what day and time is it in Johannesburg?
- **b** Christine caught a flight that left Perth at 8 p.m. on Friday and flew non-stop to Johannesburg. The flight took 11 hours. What was the day and the time when she landed in Johannesburg?

8 The first cricket test between the West Indies and South Africa started on Thursday at 10 a.m. in Kingston, Jamaica 18°N, 72°W. Albert lives in Cape Town 33°S, 18°E and he is a keen cricket fan. What time should he switch his TV on to watch the start of the test match?

9 Sydney is at 34°S, 151°E and New York is at 41°N, 74°W.

- **a** When it is 7 p.m. on Monday in New York, what day and time is it in Sydney?
- **b** Ryan flew from New York to Sydney. He left New York at 7 p.m. on Monday and his flight took 20 hours. What day and time was it in Sydney when he arrived?

ISBN 9780170443975

Time and date

INVESTIGATION

TIME ZONE MAP

Go to the **Time and date** website and select **Time Zone Map**.

1 **a** Find 5 places that are unfamiliar to you that have the same time as where you are.

b Click on the name of each location to find out where it is and how its time compares to UTC.

c What do all of the places have in common?

2 **a** Find 4 non-Australian places that have a time 2 hours later than where you are.

b What do these places have in common?

Chapter problem

You've covered the skills required to solve the chapter problem. Can you solve it now?

15.04 Pacific islands cruise

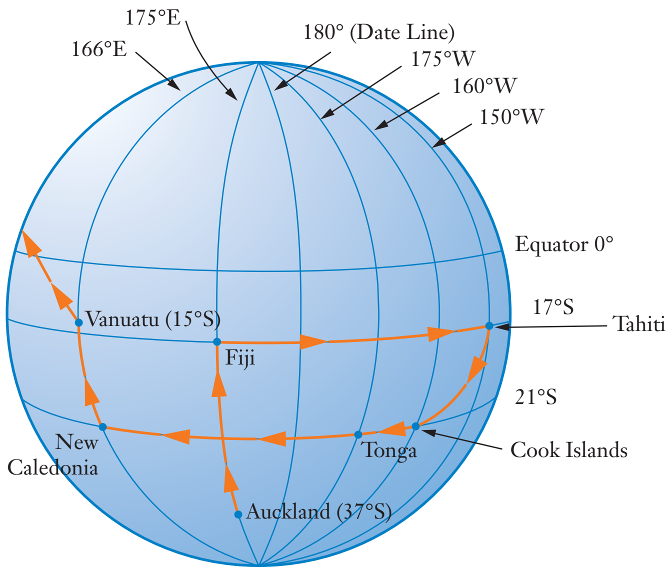
Jade has always loved sailing and when she left school, she decided to make sailing her career. Today she takes charter groups on her ocean-going yacht. The clients select the destinations and Jade does the rest.

Rawpixel/Shutterstock.com

Jade's next charter leaves from Auckland and visits Fiji, Tahiti, the Cook Islands, Tonga, New Caledonia and Vanuatu. Her guests will leave the yacht in Vanuatu, then Jade will sail north-west to Asia to pick up her next charter group.

ISBN 9780170443975

Jade's route is shown in orange in this diagram.



Exercise 15.04 Pacific islands cruise

1 The position coordinates for Auckland and Fiji are 37°S, 175°E and 17°S, 175°E respectively.

Denis Burdin/Shutterstock.com

- **a** Which meridian of longitude goes through Auckland and Fiji?
- **b** Through how many degrees of latitude will Jade sail as she travels from Auckland to Fiji?
- **c** How many kilometres is the journey from Auckland to Fiji? Use 6400 km as the radius of the Earth and express your answer correct to the nearest 10 km.
- **d** How long will it take Jade to sail from Auckland to Fiji at an average speed of 15 km/h? Express your answer correct to the nearest 10 hours.
- **e** Express your answer to part **d** in days and hours.

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2 This table shows daylight saving start and finish times in Auckland and Fiji.

	Daylight savings starts	Daylight savings finishes
Auckland	End of September	Beginning of April
Fiji	Mid November	Mid January

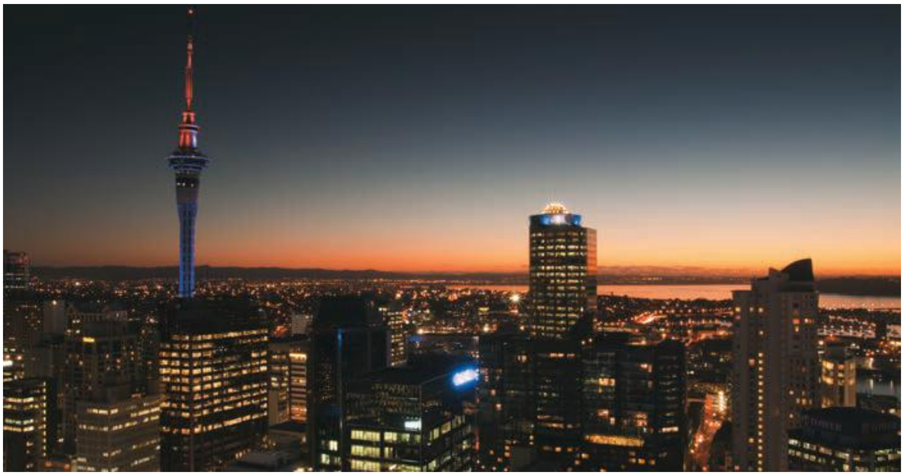
a Use longitude to explain how you know that standard time is the same in Auckland and Fiji.

b Explain why the times are different in the 2 locations during March.

c During which 2 time periods is the time in Auckland ahead of the time in Fiji?

d Is the time in Fiji ever ahead of the time in Auckland?

e Jade's charter will leave Auckland on 4 February on the high tide at 10 a.m. Use your answer to question 1e to calculate the date and approximate time the charter will arrive in Fiji.


Hkomala/Shutterstock.com

Auckland, New Zealand

3 The second leg of Jade's trip is from Fiji (17°S, 175°E) to Tahiti (17°S, 150°W).

a This table shows the UTC time zones for both places.

Fiji	+12
Tahiti	–10

What is the time difference between Tahiti and Fiji?

b How much angular difference is there in the longitude for Tahiti compared to Fiji?

c As we travel around the world, the time changes by 1 hour every 15° the longitude changes. According to this rule, the time difference between Fiji and Tahiti should be a bit more than 2 hours. How can you explain the apparent contradiction between this 2-hour time difference and your answer to part **a**?

d When it is 3 p.m. on Tuesday in Tahiti, what is the time in Fiji?

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Brian Kinney/Shutterstock.com

4 As Jade's yacht sails from Fiji to Tahiti, it will travel 97 km for every 1° change in longitude. Approximately how far is it from Fiji to Tahiti?

5 Jade's guests want to do a 4-wheel drive island experience tour while they're in Tahiti. This tour costs 15 000 French Pacific Francs (CFP) per person. The conversion rate between French Pacific Francs and Australian dollars is 100 CFP = \$1.24. Calculate the cost per person for the 4-wheel drive tour in Australian dollars.

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6 The third leg of the charter is from Tahiti to the Cook Islands. Use the **distance calculator** on the **Time and date** website (under the **Calculators** menu) to determine:

a the distance from Papeete in Tahiti to the Cook Islands

b the compass direction required to sail from Papeete to the Cook Islands

c the time difference between Papeete and Tahiti.

Time and date

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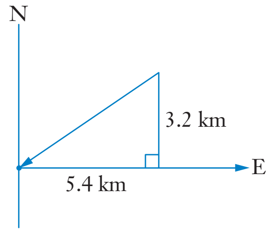
7 When they are in the Cook Islands the charter group wants to go on a fishing trip. The fishing site is 3.2 km north and 5.4 km east of Captain Tam's wharf.

Use Pythagoras' theorem to calculate the straight line distance from the wharf to the fishing site. Express your answer in km, correct to 1 decimal place.



8 When the charter leaves the Cook Islands it will head to New Caledonia via Tonga. Remember Jade's trip is shown on the map on page 427.

a In what compass direction will they be heading?

b Will they cross the International Date Line before or after they call at Tonga?

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Mari Anuhea/Shutterstock.com

9 In New Caledonia, the group will take a day trip to the Isle of Pines. The tour leaves the ferry wharf at 0720 on Saturday and returns the same day at 1635.

a Express the times the tour leaves and returns in a.m. and p.m. times.

b How long is the tour?

c Jade plans to leave New Caledonia on the Saturday night's high tide at 8:15 p.m. How long will the tour group have to get back on board the yacht when they return to the wharf after the tour?

10 **a** In what direction will the yacht sail from New Caledonia to Vanuatu?

b Calculate the distance from New Caledonia to Vanuatu. Use 6400 km as the radius of the Earth and express your answer correct to the nearest km.

c How long, correct to the nearest hour, will the trip take at an average speed of 15 km/h?

d On what day and at what approximate time will the yacht arrive in Vanuatu?

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INVESTIGATION

YOUR SAILING TRIP

Imagine that you can travel on Jade's yacht. Start at any seaside location with a suitable harbour and sail to any location between 20°N and 40°S latitudes.

Your task is to plan the trip of your dreams!

Juancat/Shutterstock.com

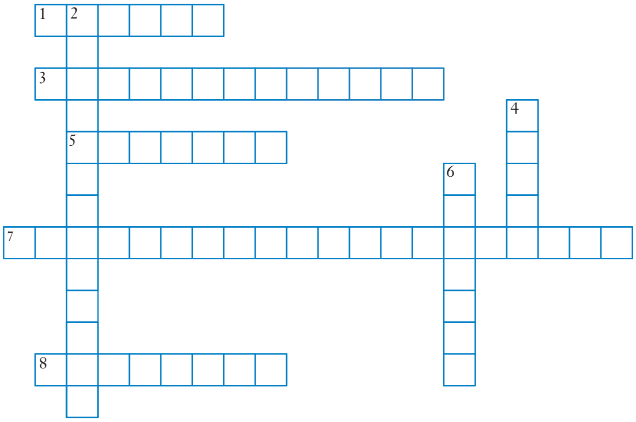
- Select 4 ports of call.
- Use the Internet to determine the position coordinates of each port.
- Position the 4 ports and the harbour from which you will leave on a map and plan a sailing route.
- Research some interesting things to do or places to visit in the ports.
- Use the Internet to determine the distance between each port and any time differences.
- Calculate the times that it will take to sail between the ports at an average speed of 15 km/h.
- Construct a schedule for your trip. Include sailing times and arrival times.

ISBN 9780170443975

KEYWORD ACTIVITY

Where on Earth crossword

Copy and complete the crossword with the missing terms in the chapter summary. You can also print a copy using the 'Where on Earth crossword' link.



In this chapter we learnt about locations, distances and time on the Earth. We specify positions by using[8]________ and longitude. Latitude lines go across the Earth and the [5]________ is the only parallel of latitude that is a [4]_______ circle. [7]______________________ [3 words] go up and down through the North and South poles. The [2]_________________ [2 words], which passes through Greenwich in England is the zero meridian of longitude.

Because the Earth is close to the shape of a [1]________ we can adapt the formula for the [3]______________ of a circle to work out the distance (arc length) between 2 places on a great circle. We work out the [6]_____________difference between the 2 places then use the formula:

$$\text{Distance} = \frac{\text{Change in the angle}}{360} \times 2\pi R$$, where R is the radius of the Earth.

We can use longitude to calculate time differences. Time changes by one hour for every 15° change in longitude.

ISBN 9780170443975

SOLUTION TO THE CHAPTER PROBLEM

Problem

At 8 a.m. on Monday, Jade left the Cook Islands at 21°S, 160°W to sail up the 160°W meridian of longitude to Hawaii at 21°N, 160°W. Her boat cruises at an average speed of 36 km/h. On what day and at approximately what time can she expect to reach Hawaii?

Solution

Jade will be sailing through 42° as she sails from 21°S to 21°N.

$$\begin{aligned}\text{Distance Jade will sail} &= \frac{42}{360} \times 2 \times \pi \times 6400 \text{ km} \\ &= 4691 \text{ km}\end{aligned}$$

$$\begin{aligned}\text{The time the trip will take} &= \text{distance} \div \text{speed} \\ &= 4691 \div 36 \\ &= 130.305\ldots \text{ hours} \\ &= 130.305\ldots \div 24 \text{ days} \\ &= 5.429\ldots \text{ days} \\ &= 5 \text{ days } 10 \text{ hours}\end{aligned}$$

Hawaii and the Cook Islands have the same time because they are on the same meridian of longitude.

Jade will arrive in Hawaii approximately 5 days and 10 hours after 8 a.m. on Monday. She will arrive on Saturday afternoon at approximately 6 p.m.

ISBN 9780170443975

15. TEST YOURSELF

Use the map from Example 1 on page 417 to answer questions **1** and **2**.

Practice quiz

1 **a** Is Jacksonville north or south of the equator?
b Is Cape Town east or west of London?
c What place is located at approximately (45°N, 30°E)?
d What are the approximate position co-ordinates of Lima?

Exercise 15.01

2 **a** What is the difference in latitude between Khartoum and Durban?
b Calculate the distance between Durban and Khartoum, correct to the nearest 100 km.

Exercise 15.02

3 Winnipeg in Canada is at (50°N, 97°W) and Oklahoma City in USA is at (35°N, 97°W).
a How many degrees along the 97°W meridian separate Winnipeg and Oklahoma City?
b Calculate the distance between Winnipeg and Oklahoma, correct to the nearest 100 km.
c What time is it in Oklahoma when it is 11 a.m. in Winnipeg?

Exercise 15.02

Exercise 15.03

4 What are people in other cities doing when it is 10 a.m. on Saturday in Perth (120°E longitude)? Calculate the local time and day, correct to the nearest half hour, in each location **a** to **f**, then match each city with the most likely activity in **A** to **F**.

Exercise 15.03

Location	**Activity**
a Bridgetown, Barbados (60°W)	**A** Leaving a nightclub
b Cook Islands (165°W)	**B** Breakfast
c Denver, USA (105°W)	**C** Afternoon rest
d Nagasaki, Japan (135°E)	**D** Watching late-night movie on TV
e New Delhi, India (75°E)	**E** Morning tea
f London, UK (0°)	**F** Watching TV nightly news

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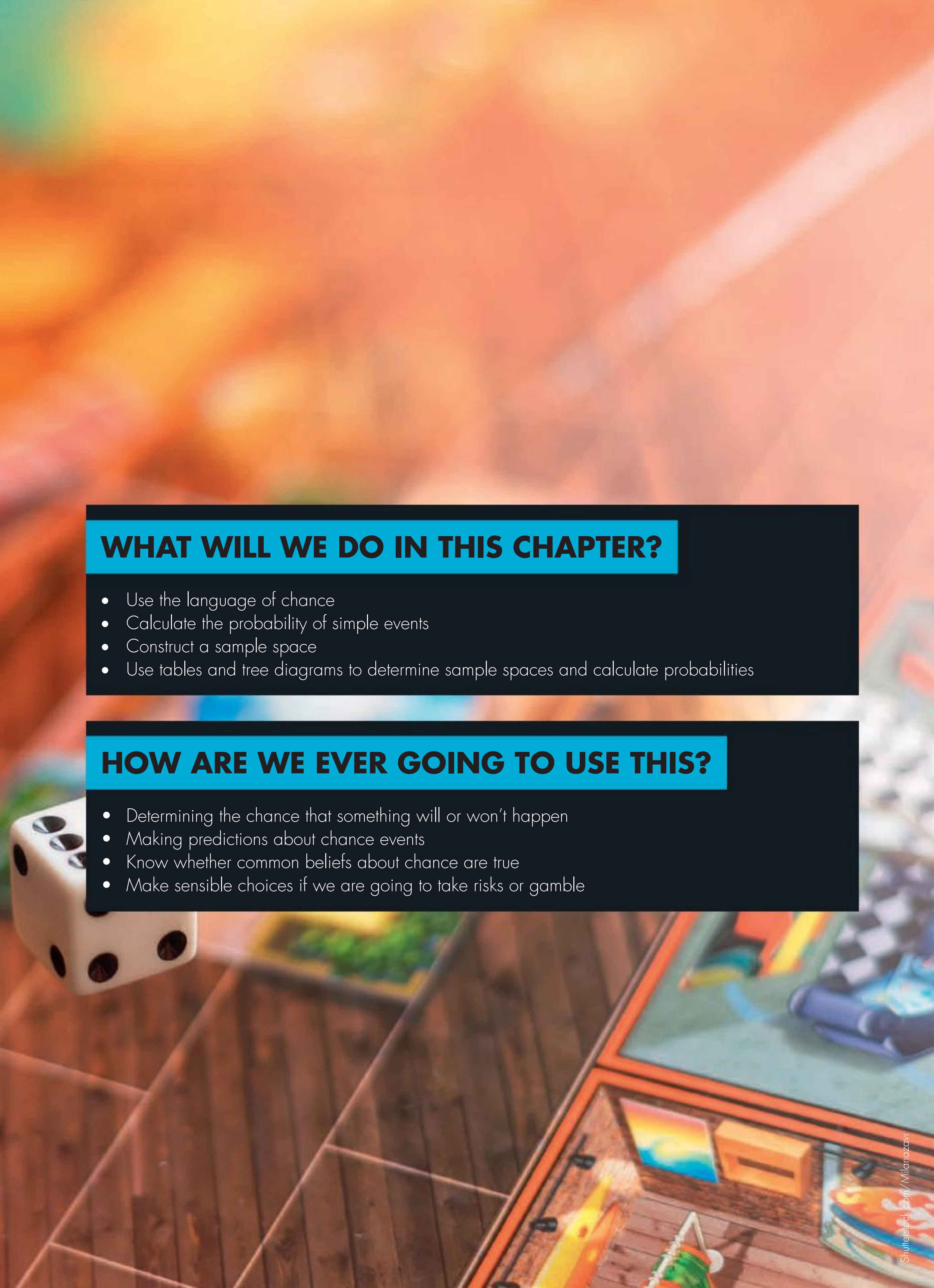
16. TAKING CHANCES

Chapter problem

Tanika thinks that there is something wrong with her pair of dice. When she plays dice games, some numbers seem to come up more often than others on each die. How can she test whether each die is fair and not 'loaded'?



WHAT WILL WE DO IN THIS CHAPTER?

- Use the language of chance
- Calculate the probability of simple events
- Construct a sample space
- Use tables and tree diagrams to determine sample spaces and calculate probabilities

HOW ARE WE EVER GOING TO USE THIS?

- Determining the chance that something will or won't happen
- Making predictions about chance events
- Know whether common beliefs about chance are true
- Make sensible choices if we are going to take risks or gamble

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16.01 Theoretical probability

The language of chance

Sample space

WS Games of chance

Maya is hoping to score a 5 when she rolls a normal 6-sided die. When she rolls the die, the number showing on the top of the die could be 1, 2, 3, 4, 5 or 6, and each number is just as likely to show as any of the others. There is only one 5 from a possible 6 numbers.

The probability or chance that she will roll a 5 is 1 number out of 6 numbers, or $\frac{1}{6}$.

Probability of an event

The probability of an event occurring, where all outcomes are **equally likely**, is given by the formula:

$$P(\text{event}) = \frac{\text{number of ways the event can happen}}{\text{total number of possible outcomes}}$$

This can also be written: $P(\text{event}) = \dfrac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$

Because probability is a fraction, the smallest value for probability is 0 and the largest value is 1. An **impossible event** has a probability of 0, while a **certain event** has a probability of 1. Unlikely events have a probability close to 0 while likely events have a probability close to 1.

A list of all the possible outcomes in a chance situation is called the **sample space**.

EXAMPLE 1

An 8-sided die has the numbers from 1 to 8 written on its faces. Dean rolls this die.

a List the sample space for this situation.

b What is the probability that Dean rolls a number bigger than 5?

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Solution

a The sample space is a list of all the possible outcomes, so list the numbers from 1 to 8.

Sample space = 1, 2, 3, 4, 5, 6, 7, 8

b There are 3 numbers bigger than 5 on the die: 6, 7 and 8. Write 3 on the top of the fraction. There are 8 possible numbers in the sample space. Write 8 in the bottom of the fraction.

$P(\text{bigger than } 5) = \frac{3}{8}$

'3 chances out of 8'

A probability of $\frac{3}{8}$ can also be written as a decimal (0.375), percentage (37.5%) or ratio (3 : 8).

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Exercise 16.01 Theoretical probability

Don't forget to simplify your fraction answer.

Example 1

1 **a** List the sample space for rolling a normal die.

b What is the probability of rolling a number less than 3 on a normal die?

2 **a** List the sample space for tossing a coin.

b What is the probability that a coin will land showing a head?

3 Peter has a paper bag containing 6 red discs and 4 blue discs. When he selects a disc at random from the bag, what is the percentage probability that it will be red?

The word 'random' is very important. If Peter wasn't selecting at random, he would be able to deliberately pick a specific colour.

4 The game of pool is played on a table with 15 coloured balls numbered 1 to 15.

Samir is going to choose a ball at random to hit. What is the probability that he will choose either the 6 or the 8? Write the answer as a ratio.

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5 Daniela is going to choose a coloured chocolate at random from a packet of 50 chocolates. In the packet, there are 14 green, 10 orange, 20 blue and 6 red chocolates.

a What is the decimal probability that Daniela will choose an orange chocolate?

b What is the percentage probability that she will choose a chocolate that is not green?

c What is the most likely colour she will choose?

6 A letter is selected at random from the expression HAPPY HOLIDAY.
What is the probability that the letter is:

a an L? **b** a Y?

c either a P or an H? **d** a consonant?

The letters A, E, I, O and U are called vowels. The other letters of the alphabet are called consonants.

7 **a** What is the probability of randomly choosing E from the letters in the alphabet?

b Aaron is going to choose a letter at random from a page in a novel.

Explain why the probability that the letter will be an E is *not* $\frac{1}{26}$.

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8 The tickets in a raffle are numbered from 1 to 200.

a Calculate the percentage probability that the winning ticket will be number 157.

b What is the decimal probability that the winning ticket will be bigger than 150?

c What is the probability that the winning ticket will be an even number?

d What is the probability that the digit 1 will be part of the winning number?

Alamy Stock Photo/Deborah Harmes

9 In a bag there are 20 balls, coloured either red, blue or green. 6 of the balls are red.

a What is the probability ratio of selecting a red ball at random from the bag?

b The probability of selecting a blue ball from the bag is 2 : 5. How many of the balls in the bag are blue?

c What is the probability ratio of selecting a green ball from the bag?

INVESTIGATION

JOHN KERRICH (1903–1985)

Who was John Kerrich and how did he pass his time in a Nazi prisoner of war camp?

Research John Kerrich and prepare a presentation of your findings.

What contribution did he make to probability? What discoveries did he confirm as part of his experiments in a prisoner of war camp during World War 2?

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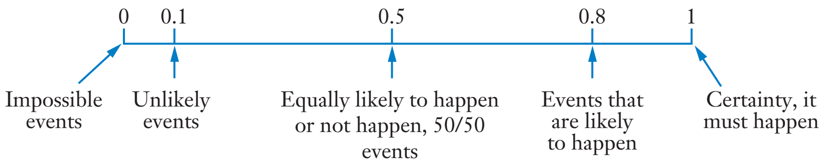
GROUP ACTIVITY

THE PROBABILITY SCALE

The value of a probability ranges from 0 to 1. The closer the probability of an event is to 1, the more likely it is to happen. This probability scale shows some probabilities and their meaning.



In your opinion, how likely is each event below? Discuss and decide where on the probability scale the event should be. Compare your group's opinion with other groups.

1 A coin will land showing a tail.

2 A die will show a 5.

3 It will rain tonight.

4 You will come to school tomorrow.

5 A pink elephant will fly past the window in the next 5 minutes.

6 You will pass your driving test the first time you try.

7 You will roll a total from 2 to 12 on a pair of dice.

8 Prince William will be crowned King of the United Kingdom by the year 2035.

9 All Australian states will switch to daylight saving next summer.

10 There will be a road accident in Brisbane today.

11 The price of petrol will fall to 85c/litre.

12 A baby boy will be born in Armadale Hospital this week.

13 Australia's prime minister will change this year.

14 The Australian cricket team will win the next Ashes series.

15 The West Coast Eagles will win the next AFL competition.

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16.02 Using tables to list outcomes

Listing all possible outcomes in a sample space can be difficult when probability problems become more complex. There are several systematic ways to work out the total number of possibilities. For more complicated situations like rolling 2 dice together, drawing a grid or table is a useful method.

EXAMPLE 2

Zoe tosses a coin and a die together. What is the probability that she tosses:

a a 5 and a head? **b** a 5 or a head?

Solution

For the coin, there are 2 possible outcomes: heads or tails.

For the die, there are 6 possible outcomes: 1 to 6.

The table shows the sample space for a coin and a die tossed together.

		Die					
		1	**2**	**3**	**4**	**5**	**6**
Coin	**Head**	H1	H2	H3	H4	H5	H6
	Tail	T1	T2	T3	T4	T5	T6

a The table shows that there are 12 possibilities.

Only one of the outcomes (H5) shows 5 and a head. $P(5 \text{ and a head}) = \frac{1}{12}$

b Count every outcome in the Head row and 5 column (but don't count H5 twice). $P(5 \text{ or a head}) = \frac{7}{12}$

iStock.com/manfredxy

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Exercise 16.02 Using tables to list outcomes

Example 2

1 Ziad tosses a coin and an 8-sided die numbered 1 to 8.

a Copy and complete the table to show all possible outcomes.

		Die							
		1	2	3	4	5	6	7	8
Coin	Head								
	Tail								

b What is the probability that Ziad tosses:

i a 7 and a tail? **ii** a 7 or a tail?

iii a number less than 5 and a tail? **iv** a head and an odd number?

2 Jemma rolls a pair of dice and adds the 2 numbers that come up.

a Copy and complete this 2-dice grid to list all possible totals.

		2nd die					
	+	1	2	3	4	5	6
1st die	1			4			
	2						
	3				7		
	4						
	5	6				10	
	6						12

b How many outcomes are in the sample space?

c What is the probability of rolling a total of:

i 5? **ii** 8?

iii 10? **iv** 12?

d Which is more likely: a total of 9 or a total greater than 10?

e How many times more likely is a total of 7 than a total of 4?

3 Felicity uses a pair of unusual dice in a board game she is designing. She numbers one die 0, 1, 2, 3, 4 and 5, and the other die 1, 1, 3, 3, 4 and 6. Players roll the dice and add the 2 numbers to determine their score.

a Copy and complete this grid for Felicity's dice.

b What is the probability of rolling the following scores on Felicity's dice?

i 5 **ii** 10 **iii** 3 or 4

c What is the most likely score on Felicity's dice?

+	0	1				
1						
1						
3						
3	3	4				
4						
6						

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4 Stefan uses a normal pair of dice in his board game, but his rules require players to *subtract* the smaller number from the larger number.

a Copy the grid and show all the possible scores using Stefan's rules.

–	1	2	3	4	5	6
1						
2		0				
3					2	
4						
5						
6			3			

b What is the probability of scoring a 3?

c What is the probability of scoring 6?

d What is the most likely score with Stefan's rules?

5 Julianne uses a normal pair of dice in her board game, but her rules require players to use just the *larger* number on the 2 dice as the score.

a Copy and complete the table for Julianne's dice.

	1	2	3	4	5	6
1				4		
2						
3						
4						
5					5	
6	6					

b What is the probability of scoring 3 with Julianne's rules?

c What is the most likely score?

d What score has a probability of $\frac{1}{4}$?

6 Christina made up an interesting special rule for a board game she is designing. Players roll a pair of dice, but they can choose to move:

- the sum of the 2 numbers on the dice, or
- either of the individual numbers showing.

Jayden is playing Christina's game and he needs 6 to win. What is the probability that Jayden can move 6 on his next roll of the dice?

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INVESTIGATION

THE GREAT COCKROACH RACE

A board game uses a pair of dice and features 4 racing cockroaches. The numbers on the faces of the dice are 0, 1, 2, 3, 4 and 5.

Zero to 5  Shutterstock.com/ Zania Studio								
6 to 11 								
12 to 18 								
19 to 25 								

Rules

- Roll the pair of dice and multiply the 2 resulting numbers together.
- The cockroach that has the product of the 2 numbers moves forward 1 square.
- The first cockroach to the end is the winner.

Do you think this game is fair? Does each cockroach have an equal chance of winning?

Give a reason for your opinion.

If you don't think it's fair, which cockroach has an unfair advantage?

Play the game several times.

Have you changed your opinion?

Explain why one cockroach has an unfair advantage.

ISBN 9780170443975

Tables and tree diagrams

Tree diagrams

Matching probabilities

Probability review

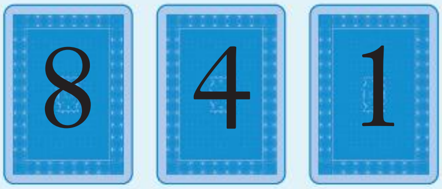
Chance puzzle

16.03 Using tree diagrams

Tree diagrams are another way of listing all possible outcomes systematically. Tree diagrams start from a point on the left side of the page and grow sideways to the right. It's a good idea to leave space above the starting point so that the 'tree' has room to grow!

EXAMPLE 3

Caleb has these 3 cards. He chooses 2 cards at random to make a 2-digit number.

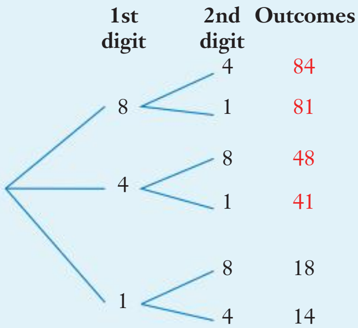


a How many different 2-digit numbers can he make?

b What is the probability that he makes a number greater than 40?

Solution

Draw a tree diagram that lists all possible 2-digit numbers. For the first digit, Caleb can select 8, 4 or 1. For the second digit, he can choose one of the remaining 2 cards (he can't choose the same card twice).



a Count the outcomes.

Caleb can make 6 different 2-digit numbers.

b 4 of the numbers listed (in red) are greater than 40.

$$P(\text{number} > 40) = \frac{4}{6}$$
$$= \frac{2}{3}$$

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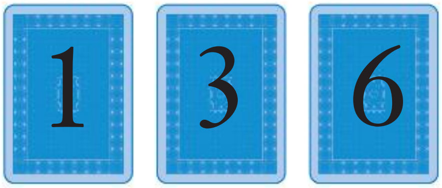
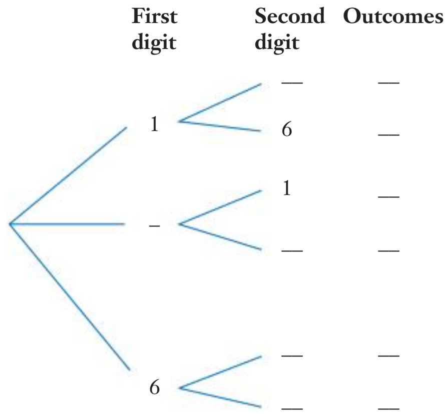
Exercise 16.03 Using tree diagrams

Example 3

1 Dimar selects 2 cards at random from these 3 cards to make a 2-digit number.



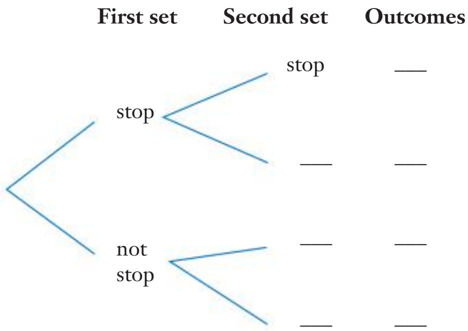
a Copy and complete the tree diagram to show all possible 2-digit numbers.



b What is the probability that Dimar will make a number less than 35?

2 Every morning on her way to school, Gordana drives through 2 sets of traffic lights at which she is equally likely to have to stop or not stop.

a Copy and complete this tree diagram to show the possible light combinations.



b What is the probability that Gordana will have to stop at 2 lights on her way to school tomorrow?

c Calculate the probability that she will have to stop at at least one set of traffic lights tomorrow morning.

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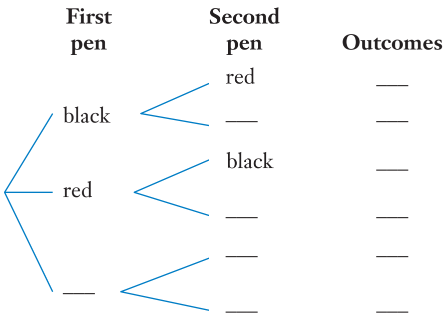
3 Claire has a bag that contains 3 pens: one red, one blue and one black.

a She selects a pen from the bag at random and then another without putting the first one back. Complete the tree diagram to show the possible colour combinations Claire could choose.

> When we choose a second item without putting the first item back, we call it 'without replacement'. If we put the first one back before we choose the second, it's called 'with replacement'.



b What is the probability Claire chooses a red pen followed by a blue pen?

c What is the probability she chooses a black and a blue pen (in any order)?

d Construct another tree diagram to determine the sample space if Claire *replaces* the first pen before she selects the second.

e When Claire replaces the first pen, what's the probability that she will select the same pen twice?

4 A tennis squad has 3 boys, Mark, Wayne and Peter, and 3 girls, Sonia, Delta and Aniela. The coach chooses a boy and a girl at random from the squad to represent the school in a mixed doubles competition.

a Construct a tree diagram to determine all the possible mixed doubles pairs.

b What is the probability that Wayne and Delta will be selected to represent the school?

5 **a** Seth selects 2 cards at random from these 4 cards. Construct a tree diagram to list the sample space.

b How many different 2-card pairs can he select?

c What is the probability that Seth chooses 2 cards of the same colour?

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16.04 Comparing relative frequency and theoretical probability

WS Matching probabilities

WS Probability review

Theoretical probability does not tell you what is definitely going to happen the next time a die is rolled. If the probability of rolling a 5 is $\frac{1}{6}$, it does not mean that in the next 60 rolls, a 5 will come up *exactly* 10 times. Theoretical probability tells you what will happen in the long run. Over many rolls, a 5 should come up in approximately $\frac{1}{6}$ of the rolls.

This is called **the law of averages** or **the law of large numbers**.

In this section, we will compare theoretical probability with relative frequency.

EXAMPLE 4

Keenan rolled an 8-sided die 120 times and he recorded the results in a frequency table.

Number	Frequency
1	17
2	10
3	11
4	9
5	20
6	22
7	13
8	18

a Determine the relative frequency and theoretical probability of rolling a 6 on this die. Express the values as decimals, correct to 3 decimal places.

b Comment on the statement: 'According to the table, the law of averages says that the next roll will probably be a 4 because there haven't been enough 4s rolled so far.'

Solution

a In the experiment, the number 6 occurred 22 times in 120 rolls.

$$\text{Relative frequency} = \frac{22}{120} = 0.1833\ldots \approx 0.183$$

Theoretically, there are 8 possible outcomes and the number 6 is one of them.

$$\text{Theoretical probability} = \frac{1}{8} = 0.125$$

b With each new roll, the probability of each number is the same, so 4 is not more likely.

The statement is incorrect. Each time the die is rolled, there is a $\frac{1}{8}$ chance that a 4 will be rolled.

A common mistake that gamblers make is thinking that if a number hasn't come up much in the past, it's got a higher chance of coming up next. The die doesn't have a memory of the numbers rolled in the past. Each number has the same chance.

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Exercise 16.04 Comparing relative frequency and theoretical probability

Example 4

1 **a** What is the theoretical probability of getting a head when you toss a coin?

b Toss a coin 40 times and record the number of heads you get.

c Use the data you obtained in part **b** to determine the relative frequency of getting a head.

d Calculate the difference between the theoretical probability and the relative frequency.

e Josie has been playing a game that involves tossing a coin. The coin was tossed 10 times and heads came up only 3 times. Josie thinks that there is something wrong with the coin or someone is cheating.

i Download the 'Heads and tails' spreadsheet from NelsonNet.

Heads and tails

ii Run the simulation 40 times and calculate the relative frequency of getting 3 or fewer heads when you toss a coin 10 times.

iii Josie expected to get heads about half of the time. Run the simulation numerous times and concentrate on the percentage of heads. In which group, 10, 50, 100, 200 or 300 tosses, does the percentage of heads change by the biggest amount?

iv Write a sentence to explain how the percentage of heads changes as you increase the number of tosses.

v Are Josie's concerns about the coin, or the people she is playing with, justified? Explain your answer.

2 When you roll a pair of dice, which event do you think is more likely to happen:

- a 1 or a 2 (or both) will show, or
- neither a 1 nor a 2 will show?

a Roll a pair of dice 40 times and record how many times a 1 or a 2 (or both) shows.

b Calculate the relative frequency that a 1 or a 2 (or both) shows.

c Repeat your experiment to check your results.

d This table lists all possible outcomes when a pair of dice is rolled. What is the theoretical probability of rolling 2 numbers that are the same?

e Determine the probability that at least one of the numbers showing will be a 1 or a 2.

f Why is it more likely that a 1 or a 2 will show than neither a 1 nor a 2 will show?

g Write a sentence to compare and contrast the relative frequency and theoretical probability of rolling a 1 or a 2 on a pair of dice.

	Die 2					
Die 1	**1**	**2**	**3**	**4**	**5**	**6**
1	1, 1	1, 2	1, 3	1, 4	1, 5	1, 6
2	2, 1	2, 2	2, 3	2, 4	2, 5	2, 6
3	3, 1	3, 2	3, 3	3, 4	3, 5	3, 6
4	4, 1	4, 2	4, 3	4, 4	4, 5	4, 6
5	5, 1	5, 2	5, 3	5, 4	5, 5	5, 6
6	6, 1	6, 2	6, 3	6, 4	6, 5	6, 6

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Chapter problem

You've covered the skills required to solve the chapter problem. Can you solve it now?

INVESTIGATION

FACT OR FALLACY?

Scott is attempting to swim to Rottnest Island. There are 2 possibilities; either he will swim the distance or he won't. Therefore, the probability that Scott will swim the distance is $\frac{1}{2}$.

This claim is a fallacy (false statement) because the 2 outcomes, 'swimming the distance' and 'not swimming the distance' are NOT equally likely.

As a group activity, discuss each probability statement and decide whether they are fact (true) or fallacy (false). Be ready to justify your group's opinion when other groups disagree!

1 I will either die when I'm 99 or I won't die when I'm 99. That's 1 possibility out of 2. The probability that I will die when I'm 99 is $\frac{1}{2}$.

2 The probability of throwing 2 heads on a pair of coins is $\frac{1}{3}$, because there are 3 possible outcomes: 2 heads, 2 tails, or a head and a tail.

3 There are 10 runners in a race.

a The name of each runner is on a separate piece of paper in a hat. The chance of picking the name of the winner out of the hat at random is $\frac{1}{10}$.

b Each runner in the race has a probability of $\frac{1}{10}$ of winning the race.

4 There is a 60% chance of rain on Saturday and a 40% chance of rain on Sunday. There is a 100% chance of rain on the weekend.

5 There is a mixture of red and blue balls in a bag. The probability of selecting a red ball at random from the bag is $\frac{1}{2}$.

6 Tony plays table tennis. He wins 3 out of 5 matches he plays, making the probability that he will win any match $\frac{3}{5}$. He is playing in a 5-match competition and he has won the first 3 games. It is likely that he will lose the next 2 matches.

7 In Lotto, players select 6 numbers from the numbers 1 to 45. The numbers 3, 11, 15, 16, 25 and 31 are more likely to be the 6 winning numbers than the numbers 1, 2, 3, 4, 5 and 6.

8 Mandy needs an operation to remove a dangerous melanoma from her back. Her doctor said that she had a 98% chance of surviving the surgery. Mandy could die during the surgery.

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WS Probability of $\frac{1}{2}$

WS Probability crossword

KEYWORD ACTIVITY

PROBABILITY OF $\frac{1}{2}$

What does a theoretical probability of $\frac{1}{2}$ really mean?

Group activity

The lines of text in the following paragraph are in the wrong order. Your challenge is to arrange the lines in the correct order. To make the task easier, print a copy of this activity from NelsonNet.

row then the next toss of the coin is more likely to be a head. Coins can't

happen. The theoretical probability of getting a head when we toss a coin

of a head being $\frac{1}{2}$ does mean is that if we toss the coin thousands of times, about

get a head. Neither does it mean that when we toss a coin 100 times we

will get a head 50 times. It also doesn't mean that if we get 8 tails in a

is $\frac{1}{2}$. This doesn't mean that every second time we toss a coin we will

what is going to happen.

half of the time we'll get a head but on no individual future occasion can we know

Theoretical probability is about the long term chance that something will

remember what's happened in the past and the chance of getting a head in the

future doesn't change because we've had lots of tails. What the probability

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SOLUTION TO THE CHAPTER PROBLEM

Problem

Tanika thinks that there is something wrong with her pair of dice. When she plays dice games, some numbers seem to come up more often than others on each die. How can she test whether each die is fair and not 'loaded'?

Solution

On a normal, fair die each number shows approximately $\frac{1}{6}$ of the time. One die at a time, Tanika should roll each die a large number of times, for example, 120, recording the number that it shows each time. Each number should show approximately $\frac{1}{6} \times 120 = 20$ times.

If one number shows a lot more or a lot less than 20 times, for example, 29 or 8, then Tanika should suspect that there is something wrong with the die. In this case, she should check the die again by rolling it another 120 times to see if the large or small amount is repeated.

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16. TEST YOURSELF

Taking chances

Qz
Practice quiz

Exercise 16.01

1 Isabella is rolling a normal 6-sided die. List the sample space.

Exercise 16.01

2 I have an 8-sided die that has the numbers 0, 1, 2, 3, 4, 5, 6, and 7 on it. When I roll this die, what is the probability I will get:

- **a** a 4?
- **b** a number less than 3?
- **c** an odd number more than 2?
- **d** an 8?

Exercise 16.01

3 In a container there are 18 green, 12 orange, 10 yellow, 9 blue, 5 red and 8 brown chocolates. Helen selects a chocolate at random. What is the probability that the chocolate:

- **a** is green?
- **b** is red or blue?
- **c** is not orange or red?

Exercise 16.02

4 Dinesh rolls a 4-sided die numbered 1 to 4, and a normal 6-sided die together. He adds the 2 numbers shown on the dice.

- **a** Construct a table for the sample space and determine the total number of possibilities.
- **b** What is the probability of rolling a sum of 5?

Exercise 16.03

5 Suppose that the weather forecast for each day of the week is sunny, cloudy or raining, each being equally likely.

- **a** Use a tree diagram to show all 9 possible outcomes for the weather for Saturday and Sunday, the 2 days of the weekend.
- **b** Find the probability that:
  - **i** it rains on both days
  - **ii** the weather is the same on both days
  - **iii** it doesn't rain on the weekend
  - **iv** it is sunny on at least one of the days
  - **v** it is cloudy on one of the days and sunny on the other.

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6 **a** Use the spreadsheet 'Heads and tails' from NelsonNet to simulate tossing a coin 25 times. According to this simulation, what is the relative frequency of obtaining a head?

Exercise 16.04

b What is the theoretical probability of obtaining a head when you toss a coin?

c The answers to parts **a** and **b** aren't identical. How can you explain the difference?

Heads and tails

7 In the Davidson family, the probability that any child will be short-sighted is $\frac{1}{4}$.

There are 15 children in the extended Davidson family and 7 of them are short-sighted.

Exercise 16.04

a Using theoretical probability, how many of the 15 children would you expect to be short-sighted?

b How can you explain the difference between the expected frequency and the reality?

c Another baby is due to be born into the Davidson family. The family believes that this baby won't be short-sighted because they already have 7 short-sighted children. Is this thinking correct? Explain your answer.

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Practice set 4

Reducible loans

- You will need the *Reducible loans* spreadsheet from Chapter 14 for Question **1** of Section **B**, which you can download from NelsonNet
- You will need to use the mortgage calculator on the *MoneySmart* website for questions **6** and **11** of Section **B**

Section A Multiple-choice questions

For each question, select the correct answer A, B, C or D.

Exercise 16.01

1 What is the probability that a student randomly chosen has a birthday in a month beginning with the letter J?

A $\frac{1}{3}$ **B** $\frac{1}{4}$ **C** $\frac{1}{6}$ **D** $\frac{1}{12}$

Exercise 15.01

2 Which description is correct about the location of Australia?

A East of Greenwich, south of the Equator
B East of Greenwich, north of the Equator
C West of Greenwich, south of the Equator
D West of Greenwich, north of the Equator

Exercise 14.01

3 Nina is repaying a loan. The progress of her loan is shown below.

Amount borrowed: \$12 000				
Interest rate: 6% p.a. reducible				
Half-yearly repayments: \$1070				
Half-year	Principal (P)	Interest (I)	Principal + interest ($P + I$)	Amount owing ($P + I - R$)
1	\$12 000	\$360	\$12 360	\$11 290
2	\$11 290	\$338.70	\$11 628.70	\$10 558.70
3	\$10 558.70	\$316.76	\$10 875.46	\$9805.46
4	\$9805.46	\$294.16	\$10 099.62	\$9029.62
5	\$9029.62	\$270.89	\$9300.51	\$8230.51
6	\$8230.51	\$246.92	\$8477.43	\$7407.43

How much does Nina owe immediately after her 3rd repayment?

A \$10 875.46 **B** \$10 558.70 **C** \$9805.46 **D** \$9029.62

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4 Liam rolls a die 60 times and records the following results. Exercise 16.04

Number showing on die	1	2	3	4	5	6
Frequency	12	7	13	12	7	9

Based on these results, if he rolls this die 300 times, how many times would he expect to roll a 6?

A 35 **B** 45 **C** 50 **D** 65

5 Tiana has 2 \$5 notes, 4 \$10 notes and 3 \$20 notes in her wallet. She takes out a note without looking. What is the probability that it is a \$20 note? Exercise 16.01

A $\frac{60}{110}$ **B** $\frac{20}{35}$ **C** $\frac{3}{35}$ **D** $\frac{1}{3}$

6 Which calculation would you use to find the distance between Gotland, Sweden (57°N, 18°E) and Capetown, South Africa (34°S, 18°E) in kilometres? Exercise 15.02

A $\frac{16}{360} \times 2 \times \pi \times 6400$ **B** $\frac{23}{360} \times 2 \times \pi \times 6400$

C $\frac{91}{360} \times 2 \times \pi \times 6400$ **D** $\frac{18}{360} \times 2 \times \pi \times 6400$

7 This spreadsheet from Chapter 14 shows the progress of a reducing balance loan of \$20 000. Exercise 14.02

		Amount owing at the beginning of the month	Interest charge for the month	Amount owing plus interest	Amount owing after the repayment
9	1st month	\$20,000.00	\$100.00	\$20,100.00	\$19,290.00
10	2nd month	\$19,290.00	\$96.45	\$19,386.45	\$18,576.45
11	3rd month	\$18,576.45	\$92.88	\$18,669.33	\$17,859.33
12	4th month	\$17,859.33	\$89.30	\$17,948.63	\$17,138.63
13	5th month	\$17,138.63	\$85.69	\$17,224.32	\$16,414.32
14	6th month	\$16,414.32	\$82.07	\$16,496.39	\$15,686.39
15	7th month	\$15,686.39	\$78.43	\$15,764.83	\$14,954.83
16	8th month	\$14,954.83	\$74.77	\$15,029.60	\$14,219.60
17	9th month	\$14,219.60	\$71.10	\$14,290.70	\$13,480.70
18	10th month	\$13,480.70	\$67.40	\$13,548.10	\$12,738.10
19	11th month	\$12,738.10	\$63.69	\$12,801.79	\$11,991.79
20	12th month	\$11,991.79	\$59.96	\$12,051.75	\$11,241.75

How much has been paid off this loan after 12 months?

A \$8758.25 **B** \$11 241.75 **C** \$11 991.79 **D** \$12 051.75

8 Perth is located at (32°S, 116°E) and Mpeketoni, Kenya is located at (2°S, 41°E). When it is 11 a.m. in Perth, what time is it in Mpeketoni? Exercise 15.03

A 4 p.m. **B** 1 p.m. **C** 9 a.m. **D** 6 a.m.

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Section B Short-answer questions

Exercise 14.02

1 Use the *Reducible loans* spreadsheet from NelsonNet for this question.
Anton borrowed $13 000 at 5% p.a. reducible interest to buy a car. His monthly repayments are $470.

a How much interest will Anton pay in the first month?

b How much will he owe immediately after the 15th month?

c How long will it take Anton to repay the loan?

d What is the value of his last repayment?

e How much interest will Anton pay on the loan?

Exercise 16.01

2 Ken has a box of 50 chocolates. In the box there are 11 hard milk chocolates, 22 soft milk chocolates, 7 hard white chocolates and 10 soft white chocolates. Ken chooses a chocolate at random.

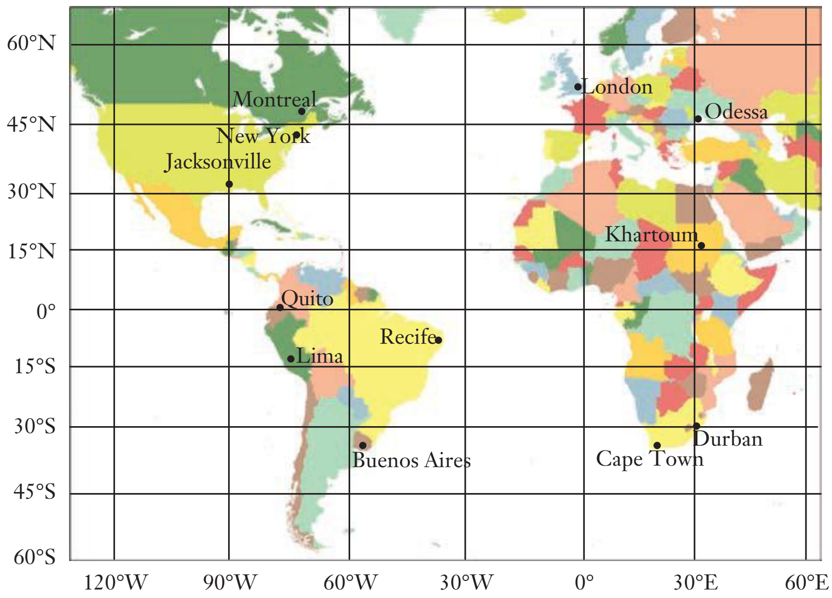
a What is the probability that Ken chooses a hard white chocolate?

b What is the probability he will choose a chocolate that is not hard?

c What is the most likely chocolate he will choose?

Exercise 15.01

3 Use this map to write the position coordinates of:



a Durban **b** Quito **c** New York

Exercise 15.02,15.03

4 a Explain why Helsinki (60°N, 25°E) and Bucharest (44°N, 25°E) have the same standard time.

b Find the distance between these 2 cities correct to the nearest 10 km.

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5 Two dice are rolled and the numbers are multiplied together. Exercise 16.02

a Copy and complete this table to show all possible products.

		1st die					
	×	**1**	**2**	**3**	**4**	**5**	**6**
2nd die	**1**	1	2	3			
	2	2	4				
	3	3	6				
	4			12			
	5						
	6						

b How many different products are possible?

c Which product is the most likely?

d Which product is the least likely?

e What is the probability of a product of:

i 6? **ii** 20? **iii** at least 20?

6 For this question, use an online loan calculator such as the mortgage calculator on the *MoneySmart* website. Exercise 14.03

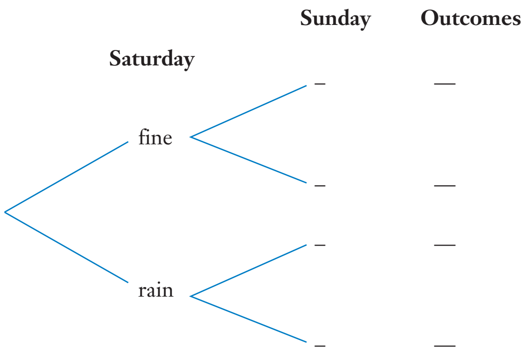
Joshua borrows $450 000 to buy an apartment. He is going to repay the loan in equal monthly instalments over 30 years at 5.5% p.a. reducible interest. He will also be charged a $5 monthly account-keeping fee.

a How much are Joshua's monthly repayments?

b How much will Joshua pay in interest and fees?

7 The weather this weekend has an equal chance of being fine or rainy. Exercise 16.03

a Copy and complete this tree diagram to show the possible outcomes for the weather on Saturday and Sunday.



b What is the probability that it will be fine on both days?

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Exercise 15.02

8 Find the distance between each pair of cities, correct to the nearest 10 km. The radius of the Earth is 6400 km.

a Karratha (21°S, 116°E) and Busselton (34°S, 116°E)

b Lumpungu, Congo (70°S, 25°E) and Kaoma, Zambia (15°S, 25°E)

c Paramaribo, Suriname (6°N, 55°W) and Montevideo, Uruguay (55°S, 55°W)

Exercise 14.01

9 Rose and Ian take out a personal loan to pay for an overseas holiday. They borrow \$18 000 at 8.4% p.a. with monthly repayments of \$475. Copy and complete this table showing the first 4 months of their loan.

Rose and Ian's loan
Amount borrowed: \$18 000
Interest rate: 8.4% p.a. monthly reducible
Monthly repayments: \$475

Month	Principal (P)	Interest (I)	Principal + interest ($P + I$)	Amount owing ($P + I - R$)
1	\$18 000	\$126	\$18 126	\$17 651
2				
3				
4				

Exercise 16.04

10 Anish rolled a die 90 times and recorded the results shown below.

Number on die	1	2	3	4	5	6
Frequency	12	15	9	16	13	25

a What is the theoretical probability of rolling a 6? Answer as a decimal, correct to 3 decimal places.

b From the data in this experiment, what is the relative frequency of rolling a 6 with this die? Answer as a decimal, correct to 3 decimal places.

c Explain why Anish's relative frequencies and theoretical probabilities aren't the same.

d Based on this data, if the die was rolled 200 times, how many times would you expect the number 3 to come up?

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11 For this question, you need to visit the *MoneySmart* website and search for **Mortgage calculators**. Select **How can I repay my loan sooner?**

Exercise **14.04**

a Copy this table and use the online calculator to complete it. The first row has been completed as an example.

Amount borrowed $250 000		Interest rate: 4% pa Monthly fee: $10	
Monthly repayment	**Time to repay the loan**	**Total repaid**	**Interest + Fees**
$1600	18 years 8 months	$357 026	$107 026
$1400			
$1250			
$2000			
$2400			

b Write a short paragraph explaining how changing the monthly repayment affects the time taken to repay the loan and the total repaid.

12 Perth is located at 32°S, 116°E and Johannesburg in South Africa is at 26°S, 26°E.

Exercise **15.03**

a When it is 7 a.m. in Perth, what is the time in Johannesburg?

b Kayla caught a flight that left Perth at 4 p.m. and flew 11 hours non-stop to Johannesburg. What was the time when she landed in Johannesburg?

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ANSWERS

Chapter 1

Exercise 1.01

1 Teacher to check

2 a sample b census c sample
d sample e sample f census
g census h sample i sample
j census k census l sample
m census n sample

3 a students in schools
b people of voting age in the state
c listeners of a radio station, or readers of a magazine, or musicians
d students at my school
e people of Qld
f the general population
g Year 12 students
h rich people
i people who shop at supermarkets
j the Australian cricket team
k internet banking customers of the bank
l customers of the communications company
m the population of the town and surrounding areas

Exercise 1.02

1 Teacher to check

2 a CD b DD c AD d PQ
e WR f OD g AD h CD
i PQ j DD k WR l OD

3–5 Teacher to check

Exercise 1.03

1 a systematic b random
c self-selected d stratified
e self-selected f random
g systematic h self-selected
i random j stratified
k systematic l self-selected
m random n stratified
o systematic p stratified

2–4 Teacher to check

Exercise 1.04

1–3 Teacher to check

4 a 81 b 57 c 24

5 a 33 b 19

6 a 99 b 47 c 52

7 a 48 b 27

8 a No. He only surveys people who work in the centre of Perth. He misses country people and people who work elsewhere.
b Teacher to check

Keyword activity

1 J 2 H 3 D 4 F 5 B
6 A 7 I 8 C 9 E 10 G

Test yourself 1

1 Sample, too expensive and time-consuming to do a census

2 a i families in the street ii census
b i companies ii sample
c i voting population of Australia
ii sample
d i school students of NT ii sample

3–4 Teacher to check

5 a 75 b 59 c 16

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Chapter 2

Exercise 2.01

1 sides, equilateral, angles, two, two, scalene, acute, obtuse, right

2 four, opposite, parallel, equal, bisect, equal, 90°, rectangle, square

3 parallelogram, 90°, equal, rhombus, equal, 90°, diagonals

4 quadrilateral, parallel, equal, equal, equal, diagonals, 90°

5 number, triangles, quadrilaterals, pentagon, hexagon, octagon, decagon, regular, regular

6 **a** trapezium **b** pentagon
c octagon **d** equilateral triangle
e kite **f** scalene triangle
g parallelogram **h** hexagon
i right-angled triangle **j** isosceles triangle
k square **l** rhombus

7 **a** F **b** T **c** T **d** T
e F **f** F **g** T **h** F

Exercise 2.02

1 flat, rectangular, prism (or pyramid), pyramid (or prism), pentahedron, hexadron, edge, vertex

2 curved, sphere, flat, curved, rectangle, cone, circle

3

Solid	Number of faces	Shapes of faces	Number of identical faces
Cube	6	squares	6
Cylinder	3	2 circles and a cylinder	2
Square pyramid	5	square and 4 triangles	4
Triangular prism	5	2 triangles and 3 rectangles	2
Rectangular prism	6	rectangles	3 pairs of 2
Cone	2	circle and a sector	0
Triangular pyramid	4	triangles	0

4 cross-section, shape, end, rectangle, triangle, pointed, bottom, square, cross-section, size

5 **a** triangular prism **b** cylinder
c square pyramid **d** cone
e rectangular prism **f** triangular pyramid
g sphere **h** cube

6 **a** triangular prism **b** cone, sphere
c cube **d** pentagonal prism
e sphere
f square or rectangular pyramid

7 **a** cube, square pyramid
b rectangular prism, cylinder
c cylinder, cone
d cone, hemisphere
e 2 triangular pyramids
f 2 rectangular prisms

8 **a** T **b** F **c** T
d T **e** F **f** F

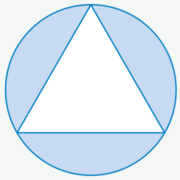
Exercise 2.03

1,2 Teacher to check.

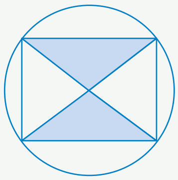
3 square, circumference, four, semicircle, isosceles, diameter, vertex, circle.

4 Other answers possible.

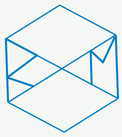
a



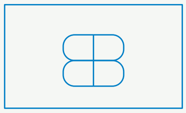
b



c



d



5 Teacher to check.

Exercise 2.04

Teacher to check.

Exercise 2.05

1 a-B, b-C, c-A, d-F, e-E, f-D

2 **a** hexagonal pyramid **b** Teacher to check.

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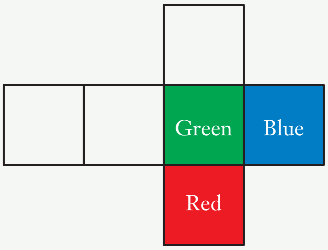
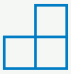
3



Other answers possible.

4 **a** **i**



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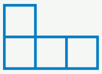

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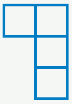
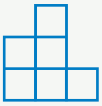


b **i**



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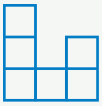

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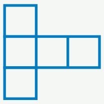
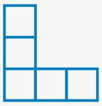


c **i**



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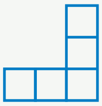

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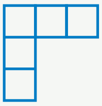
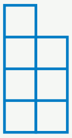


d **i**



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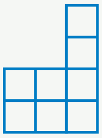

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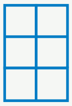
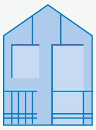


5 Teacher to check.

6



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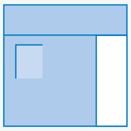
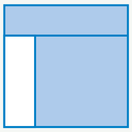
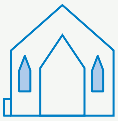


Left side

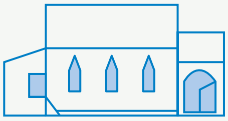


Right side

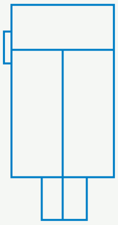
7



Front



Left side



Bird's eye

Keyword activity

1 G **2** A **3** E **4** C
5 F **6** B **7** H **8** D

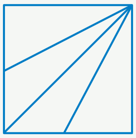
Test yourself 2

1 Teacher to check.

2 **a** equilateral triangle
b parallelogram, rectangle, square, rhombus
c pentagon
d parallelogram, rectangle, square, rhombus
e trapezium
f scalene triangle, acute-angled triangle

3 **a** A, D, E, G **b** C, H **c** B, F

4



5,6 Teacher to check.

7 **a** rectangular prism **b** triangular prism
c cone **d** square pyramid
e cube **f** cylinder

8 Teacher to check.

9 Teacher to check

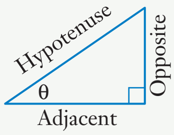
Chapter 3

Exercise 3.01

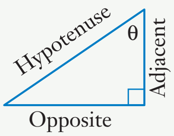
1 8.4 m **2** 23.8 m
3 5.831 m **4** 152 km/h
5 **a** yes **b** yes **c** no **d** yes
6 12.37 m **7** 733 m
8 44.4 m **9** 148 km
10 Teacher to check.

Exercise 3.02

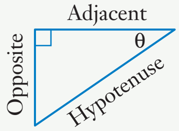
1 **a**



b



c



2 *WK*

3 **a** 65 **b** 56 **c** 56

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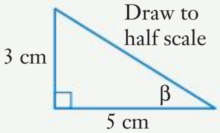
4 β

5 **a** $\frac{8}{15}$ **b** $\frac{21}{20}$ **c** $\frac{72}{65}$

6 **a**



b 31°

7 **a** *AB* **b** *AC* **c** *BC*

Exercise 3.03

1 **a** $\frac{3}{4}$ **b** $\frac{12}{5}$ **c** $\frac{120}{119}$

2 Teacher to check.

3 40°; 9, 40°, 9; 7.6

4 **a** 14.43 cm **b** 4.86 m **c** 10.89 cm
d 235.12 cm **e** 6.64 km **f** 4.71 mm

5 **a** 17.16 cm **b** 12.0 cm

6 **a** 30° **b** $AC = 0.7$ m, $AD = 1.4$ m

Exercise 3.04

1 **a** 41° **b** 47° **c** 37° **d** 31°
e 54° **f** 53°

2 **a** 38° **b** 34° **c** 68° **d** 49°
e 43° **f** 59°

3 34°, 56° and 90°

4 **a** 45°, 45° and 90°
b The opposite and adjacent sides have the same length. When the numerator (top) of a fraction and the denominator (bottom) are the same, the fraction is 1.

Exercise 3.05

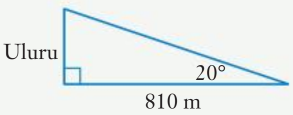
1 118.3 m

2 1411.4 m

3 530.0 m

4 **a** 156.93 m **b** 158.5 m

5 **a**

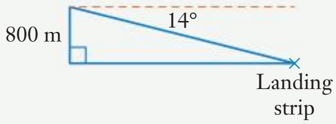


b 295 m

6 801.6 m

7 **a** 38° **b** 52°

8 **a**



b 3209 m

9 97.5 m

10 **a** 87 cm **b** 64°

Exercise 3.06

1 **a** 5.8 **b** 12.6 **c** 11.4

2 **a** 39° **b** 28° **c** 26°

3 **a** 15 m **b** 8 m **c** 17 m

4 **a** 41° **b** 62° **c** 63°

5 **a** 5.5 **b** 11.1 **c** 3.5

Exercise 3.07

1 **a** tan **b** cos **c** sin

2 **a** 7.2 **b** 7.0 **c** 10.6

3 **a** 64° **b** 30° **c** 53°

4 14 m **5** 12.4 m

6 32.5 m **7** 7.3 m

8 **a** 14.6 **b** 7.2 **c** 7.5

9 **a** 41° **b** 30° **c** 46°

Exercise 3.08

1 **a** 13.6 **b** 24.8 **c** 14.6

2 **a** 8.1 **b** 15.9 **c** 16.8

3 **a** 21.45 m **b** 21.5 m

4 11.5 m

5 5.05 m

6 310 cm

7 **a** 7.4 **b** 6.2 **c** 13.6

Exercise 3.09

1 **a** NW **b** S **c** N **d** SE

2 **a** SW **b** Poland, West Germany, UK

3 **a** NW **b** SW **c** S
d W **e** E **f** NE
g SE **h** N **i** E

4 A

5 **a** S 58° E **b** S 51° W **c** N 22° W
d N 57° E

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6 **a** Condobolin **b** Molong
c West Wyalong
7 **a** rain **b** hot, dry **c** rain
d cold **e** hot, dry **f** wet
g wet **h** hot, dry

Exercise 3.10

1 **a** 045°, N 45° E **b** 315°, N 45° W
c 270°, W **d** 225°, S 45° W
e 045°, N 45° E **f** 135°, S 45° E
2 **a** 110°, S 70° E **b** 158°, S 22° E
c 220°, S 40° W **d** 250°, S 70° W
e 280°, N 80° W **f** 014°, N 14° E
3 **a** 314° **b** 272° **c** 136° **d** 245°
4 **a** 100° **b** S 80° E
5 **a** 099° **b** S 81° E

Exercise 3.11

1 **a** Teacher to check **b** 658 m
c 1348 m
2 **a** 29° **b** 138 km
3 **a** $\alpha = 25°, \beta = 65°$ **b** 90°
c 192 km
4 **a** Teacher to check **b** 124 km
5 **a** Teacher to check **b** 85 km
6 70 km

Keyword activity

ACROSS
1 ADJACENT **4** TAN RATIO
6 ANGLES **7** HYPOTENUSE
9 COS

DOWN
2 TRIGONOMETRY **3** OPPOSITE
5 RIGHT **8** SIDES

Test yourself 3

1 **a** 12.9 **b** 12.6 **c** 22.0
2 1.9 m
3 Yes, both contain right angles. Pythagoras' theorem works in both triangles.
4 **a** *AB* **b** *AC* **c** *CB*
5 9.0
6 **a** 38° **b** 68°
7 2570 m
8 6.4 m
9 12°
10 **a** sin, $x = 3.6$ **b** cos, $\theta = 39°$
c tan, $h = 10.0$ **d** tan, $\theta = 25°$
11 **a** 32.68 mm **b** 9.12 cm
12 C
13 **a** C **b** B **c** A
14 576 m

Chapter 4

Exercise 4.01

1 3
2 4
3 Some people would have nowhere to tick, for example, a 30-year-old.
☐ 30 years or under
☐ From 31 to 40 years
☐ From 41 to 50 years
☐ Over 50 years
Other answers possible.
4 Which of the following do you attend?
☐ Concerts ☐ Special events
☐ Films ☐ Trips away
☐ Fundraisers ☐ Other
Other answers possible.
5 ☐ Weekly
☐ Once a month
☐ 2–3 times per month
☐ Over 3 times per month
Other answers possible.
6 For the activities you attend, the cost is:
☐ cheap
☐ OK
☐ expensive
☐ I don't attend activities because they are too expensive
Other answers possible.
7 Teacher to check
8 A box could be provided. Other answers possible.
9 Teacher to check

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Exercise 4.02

1 **a** use of ‘fascinating’ and ‘wet’
b use of ‘disgusting’
c no negative options
d use of ‘exciting’ and ‘boring’
e second half of question shouldn’t be used
f no positive options
g use of ‘boring’
h use of ‘greatest’, yes/no response only
i use of ‘one of those’
j no negative options

2 Teacher to check

Exercise 4.03, 4.04

Teacher to check

Test yourself 4

1 **a** knowing what time frame to consider, other answers possible
b Make the question ‘How often do you drink each week?’

☐ Once a week
☐ 2 to 3 times a week
☐ More than 3 times a week
☐ I don’t drink

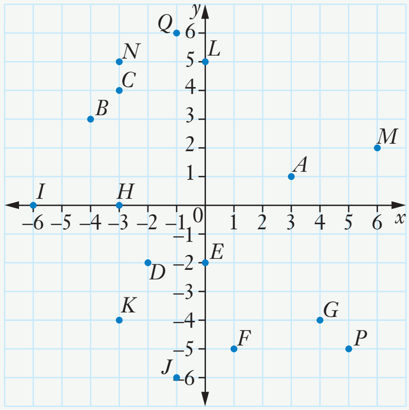
2–4 Teacher to check

Chapter 5

Exercise 5.01

1 $A(-4, 3)$, $B(-2, 2)$, $C(0, 2)$, $D(2, 3)$, $E(2, 1)$, $F(1, -1)$, $G(2, -3)$, $H(0, -3)$, $I(-4, -3)$, $J(-2, -1)$, $K(3, 0)$, $L(-4, 0)$, $M(0, -2)$, $N(0, 3)$, $O(0, 0)$, $P(-2, 0)$, $Q(1, 0)$, $R(3, -2)$, $S(-2, -2)$, $T(-3, 1)$, $U(-1, 2)$, $V(4, 1)$, $W(4, -3)$, $X(-4, -1)$, $Y(1, -4)$, $Z(-1, -3)$

2



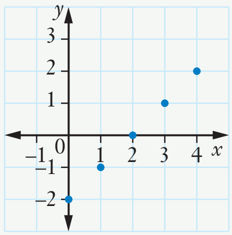
3 **a** A, M **b** B, C, N, Q **c** D, K, J
d F, G, P **e** E, I, H, L

4 **a** 4th **b** 3rd **c** 2nd

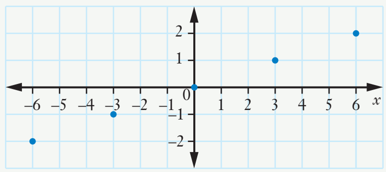
5

Points in:	x-coordinate	y-coordinate
1st quadrant	+	+
2nd quadrant	–	+
3rd quadrant	–	–
4th quadrant	+	–

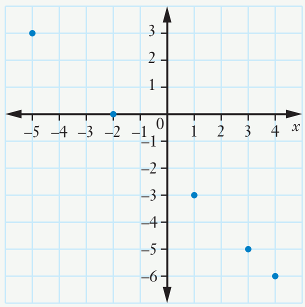
6



7 **a**



b



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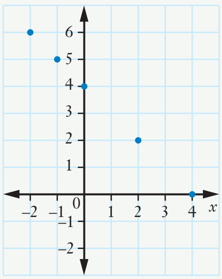
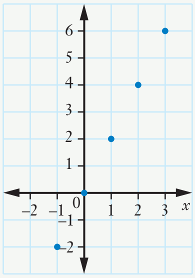
c



d



Exercise 5.02

1 a

x	–5	–2	3	6	11
y	–7	–1	9	15	25

b

x	4	2	5	7	–3
y	7	1	10	16	–14

c

x	–4	–2	3	6	9
y	–2	–1	1.5	3	4.5

d

x	–2	–1	4	7	10
y	24	22	12	6	0

2

x	1	2	5	9	10
y	11	10	7	3	2

3 **a** F **b** B **c** A
d C **e** E **f** D

4 Other answers possible:

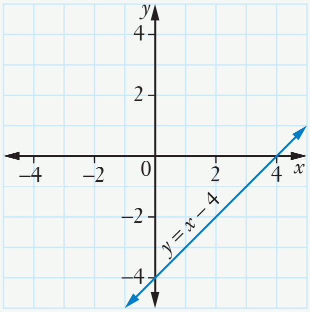
a $y = x + 1$ **b** $y = x \div 10$
c $y = 4x$ **d** $y = x^2$
e $x + y = 10$ **f** $x \times y = 24$
g $x + y = 7$

5 Other answers possible:

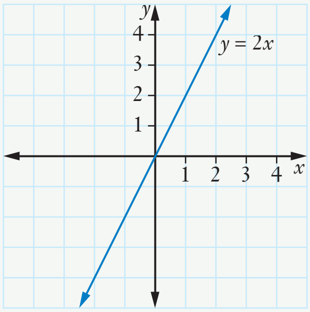
x	5	6	7	8	9
y	4	5	6	7	8

Exercise 5.03

1 a



b



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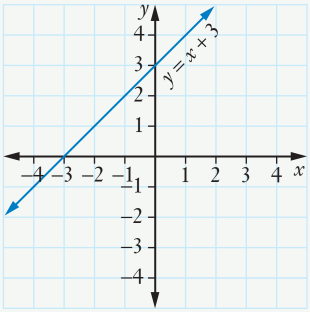

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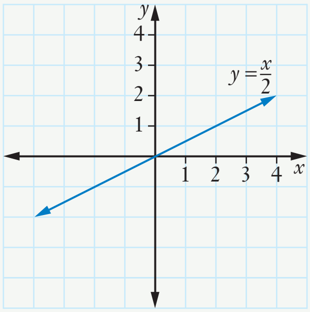

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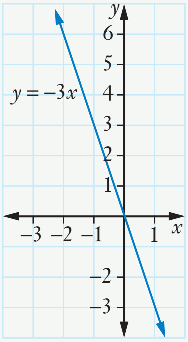
e



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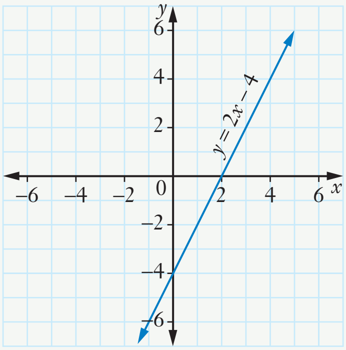

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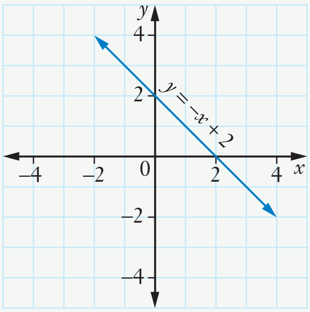

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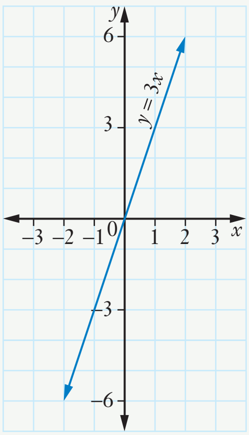
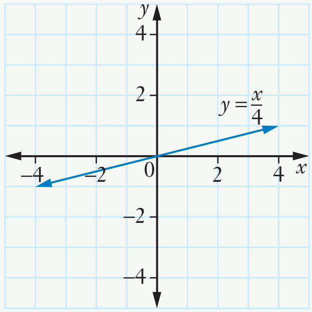


i



2

	Gradient	y-intercept
a	$\frac{1}{2}$	2
b	3	–1
c	1	6
d	$\frac{1}{2}$	1

3 **a** **i** 3 **ii** 2
b **i** –2 **ii** 3
c **i** –1 **ii** 7
d **i** 4 **ii** 0
e **i** $\frac{5}{4(}$ **ii** –2
f **i** $\frac{35}{40}$ **ii** –4

4 **a** positive, 1 **b** positive, 2
c positive, 1 **d** positive, $\frac{1}{2}$
e negative, –3 **f** positive, 2
g negative, –1 **h** positive, 3
i positive, $\frac{1}{4}$

Exercise 5.04

1 **a** 40
b The number of chirps when the temperature is zero.
c 8
d The additional number of chirps per minute when the temperature increases by 1°C.
e 296 **f** 15°C
g 840, not realistic

2 **a** 0 **b** –3.5
c Number of degrees cooler at which water boils for every extra km in altitude.
d about 90°C

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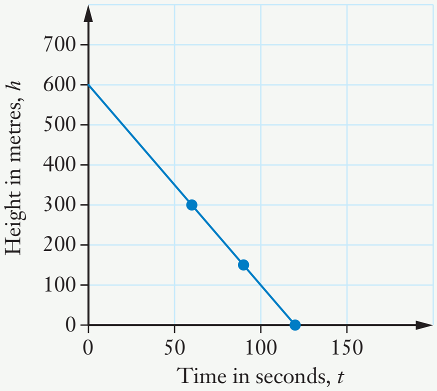
3 **a** Laura 14, Bevan 8, Jacob 3
b speed
c Jacob, he has the smallest gradient, or he didn't travel as far as the others in 1 hour.
d Laura bike, Bevan jog, Jacob walk

4 No, the graph says the head circumference should be about 35 cm, which it is.

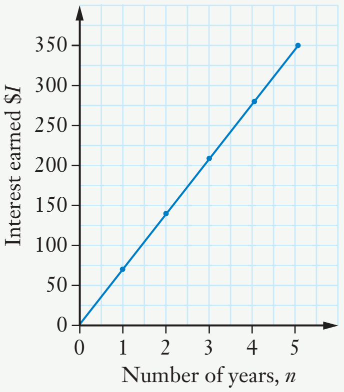
5 **a** \$40 **b** $\frac{1}{4}$
c Additional cost per ticket
d \$165 **e** $D = \frac{1}{2}n + 110$

6 **a** \$50 **b** \$1
c $C = n + 50$

7 **a** 600 m **b** 5
c She starts at 600 and is going down 5 m every second (–5).
d

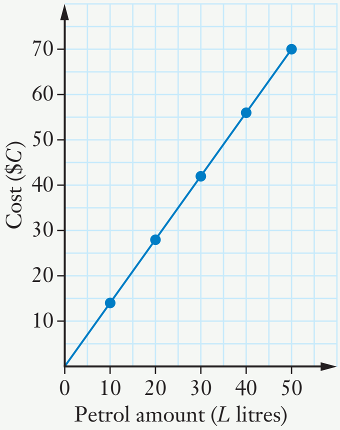


8 **a**



b 70, amount of interest each year
c 0, amount of interest at the start
d \$490
e \$2140

9 **a**

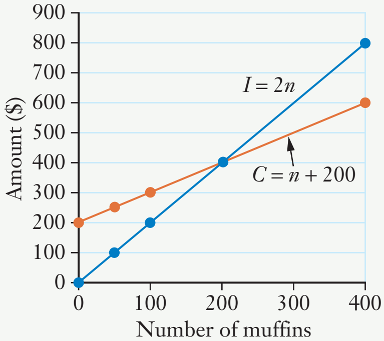


b 1.4 **c** \$105

10 **a** Teacher to check **b** 2.5 m
c Yes, 4.2 m is more than 4 yards.

Exercise 5.05

1 **a** Teacher to check
b



c (200, 400) **d** break-even point
e No, he needs to sell more than 200 to break even.
f \$30

2 **a** B **b** 3 min **c** B

3 **a** truck 72 km/h, car 96 km/h
b 2 **c** 2 seconds
d car: $m = -96 - 24t$, truck: $m = 72 - 12t$
e Both vehicles were slowing down.
f The truck was heavier.

4 **a**

Bunches of flowers sold	0	10	20	30	40	50	60
Income received (\$)	0	100	200	300	400	500	600

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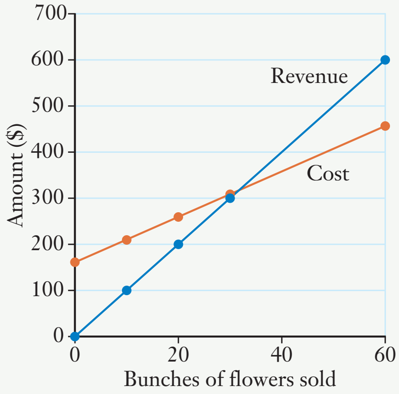
b

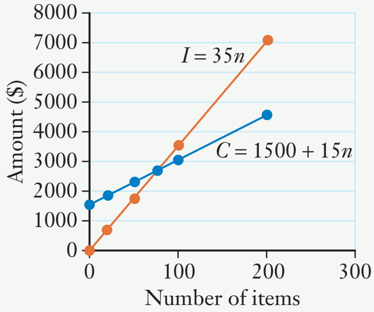


The graphs intersect at (32, 320) which is the breakeven point.

c More than 32 bunches

5 **a** Income **b** Expenses

c



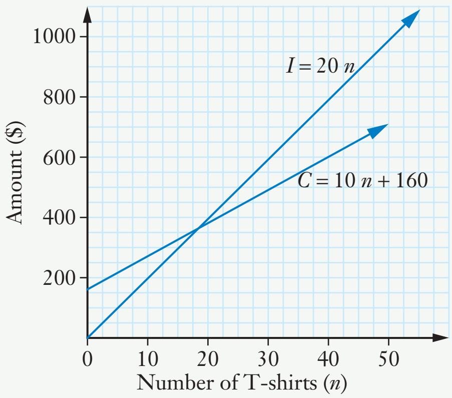
d 75 **e** $500

6 **a** $60 **b** 10 **c** $6

d It's the weekly cost of running the dog sleds, such as feeding the dogs. It doesn't change with the number of dog sled rides.

7 **a** $C = 10n + 160$ **b** $I = 20n$

c



d 16 **e** $740

8 **a** $40 **b** $10

c Monthly service charge

d 20c **e** $D = 0.2n + 10$

f 150 **g** plan A

Exercise 5.06

1 **a** 800 m **b** 600 m

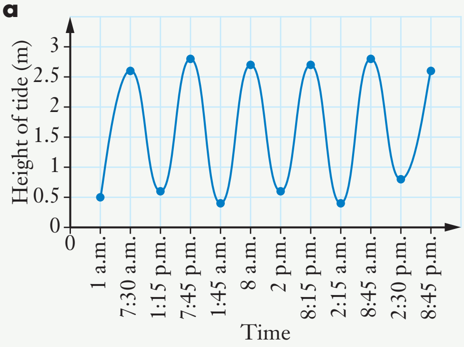
c 200 m **d** 360 m

2 5.5 m

3 **a** 45 m **b** 12 m **c** 100 m

4 **a** 33 m **b** 80 m **c** 72 m

5 **a**

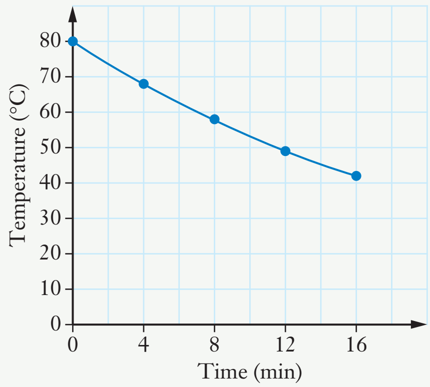


b Approximately 0.9 m

c Approximately 4.30 p.m.

6 **a** Time

b



c approximately 74°C

d approximately 10 minutes

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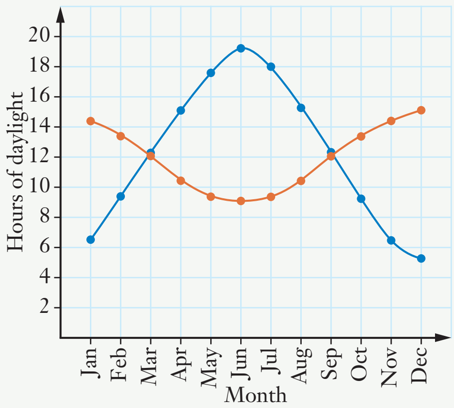
7 a



b They both begin and end at about the same place. Other answers possible.

c Anchorage starts low, goes up and comes down again – Perth is the opposite. Other answers possible.

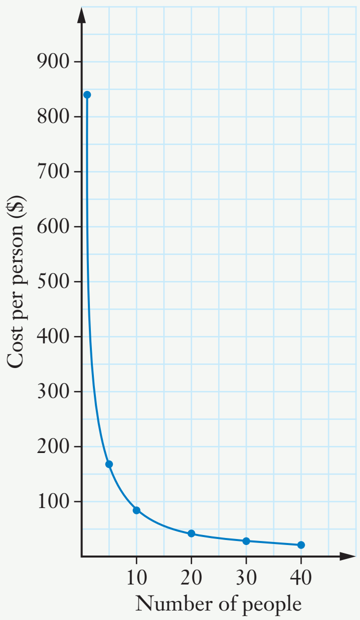
d March, September

e Anchorage is further north of the Equator than Perth, which is south of the Equator.

8 a

Number of people	1	5	10	20	30	40
Cost per person ($)	840	168	84	42	28	21

b



c approximately 30 **d** $30

Exercise 5.07

1 a 0.4 acres **b** 0.2 ha

c gradient = 0.4 **d** 4.9 ha

e 2 ha **f** 10.8 acres

g 14.8 acres

h **i** 32 000 m²

ii 3.2 ha

iii 8 acres

2 a 1 unit = 20 minutes

b 1 unit = 1 km

c 15 km

d 10:30 a.m. for $\frac{1}{2}$ hour, 12:20 p.m. for 1 hr, 2:30 p.m. for 20 mins

e $\frac{16}{3}$ or $5\frac{1}{3}$, this represents the average speed over this time ($5\frac{1}{3}$ km/h)

f 30 km

3 a Teacher to check

b 20 euros

c $13.50

4 a 11 a.m.

b 40 km

c 10 km/h

d At 12 p.m. and 1:30 p.m.

e After 1:45 p.m., graph is steepest

5 a 0, 7.6, 76, 152

b Teacher to check

6 a 47%

b approximately 67%

c Internet usage via phone is increasing over time.

d Teacher to check

7 a 48 km/h **b** 1.6

c $y = 1.6x$ **d** 160 km/h

8 a 80 years

b Male 21 years, female 23 years – female has increased more by 2 years

c 1890 to 1960, life expectancy has increased more rapidly over this time

d 52 years

e 6 years

f In both cases, life expectancy is increasing over time.

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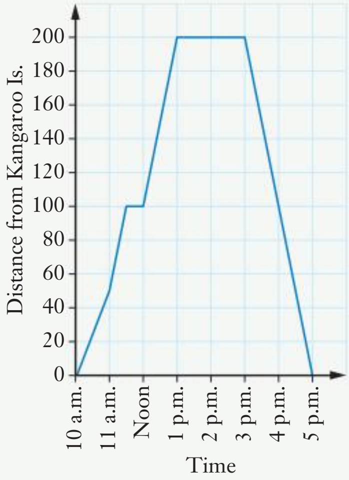
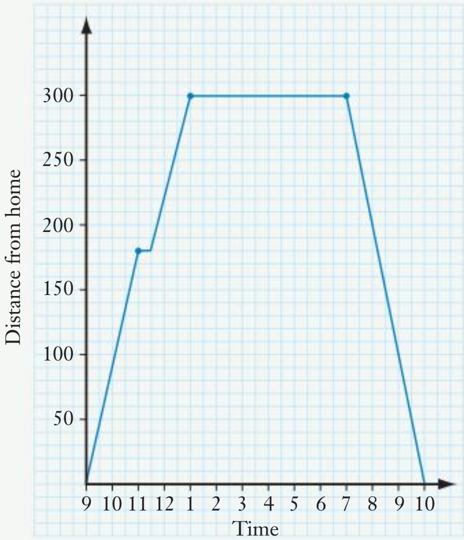
9 **a** Teacher to check

b



10 **a**



b 300 km **c** 1 p.m.

d When he is returning home at 100 km/h from 7 p.m. to 10 p.m.

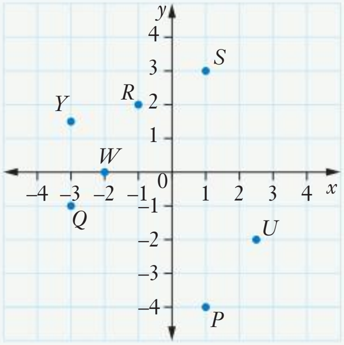
e 3 hours **f** 13 h **g** 100 km

Keyword activity

1 algebraic
2 straight
3 gradient
4 y-intercept
5 model
6 intersecting
7 point
8 income
9 costs
10 expenses
11 break-even
12 profit
13 loss
14 curved

Test yourself 5

1



2 **a** T **b** R, Y **c** W **d** P, U

3 **a**

x	−1	0	1	2	3
y	2	3	4	5	6

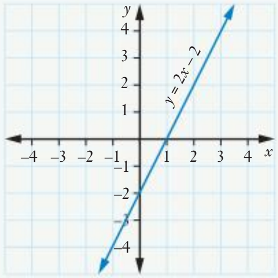
b

x	−2	−1	2	4	5
y	−11	−9	−3	1	3

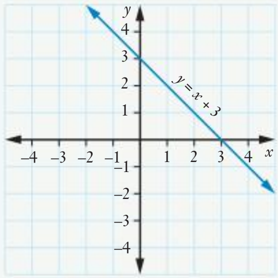
c

x	−3	−1	0	2	4
y	8	6	5	3	1

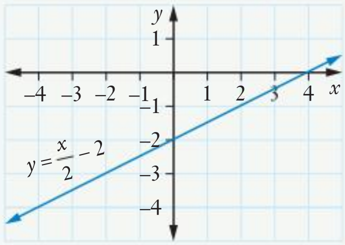
4 **a**



b



c



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5 **a** Gradient 2, y-intercept –1

b Gradient –1, y-intercept 3

c Gradient $\frac{1}{2}$, y-intercept –2

6 **a** **i** 1 **ii** –3

b **i** –2 **ii** 1

7 **a** **i** 4 **ii** –3

b **i** –1 **ii** 2

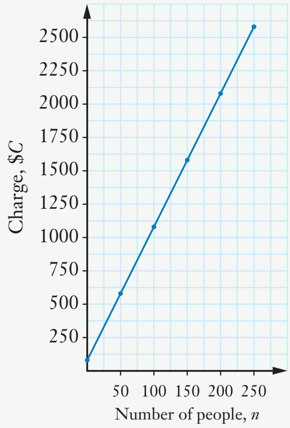
c **i** $\frac{1}{3}$ **ii** 0

8 **a** 3 min **b** 2

c Time to pack an item

d 6 items **e** 33 minutes

9 **a**



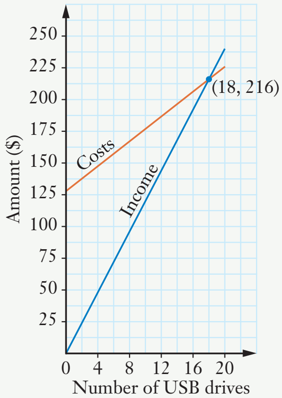
b \$1780 **c** 75

d 80, the fixed charge

e 10, the cost per person

10 **a** $C = 5n + 126$ **b** $I = 12n$

c

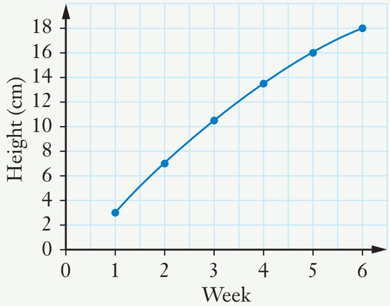


d 18 **e** \$364

11 **a** approximately 48 m **b** 80 km/h

c 43 m

12



13 **a** €10.5 **b** \$70 **c** \$50

d Yes, as €25 ≈ \$35 **e** €154

14 **a** 20 km **b** 3 **c** 2:30 p.m.

d 10:00 a.m. – 11:30 a.m. **e** 2.5 km/h

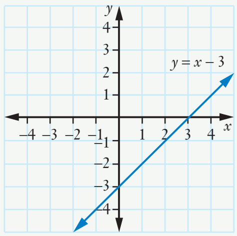
Practice set 1

Section A

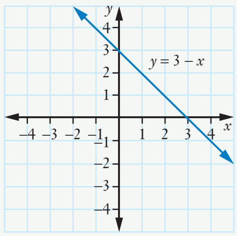
1 B	**2** C	**3** B	**4** B
5 A	**6** C	**7** B	**8** C
9 D	**10** A	**11** A	**12** D

Section B

1 **a**

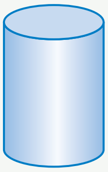


b



2 Teacher to check

3



4 7.75

5 Teacher to check

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6 **a** 84 **b** 45 **c** 13

7 $\theta = 43°$

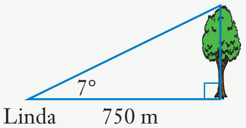
8 Teacher to check

9 **a** \$37.50 USD **b** \$27 AUD

c \$73 AUD **d** \$400 AUD

10 Teacher to check

11 **a**



b 92.1 m

12 Teacher to check

13 **a** Teacher to check

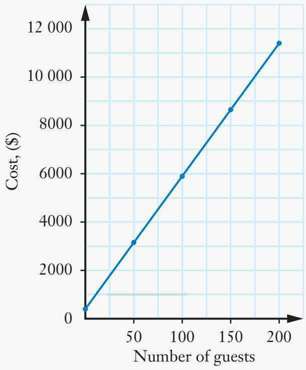
b **i** use of 'fabulous', no negative options

ii Teacher to check

14 **a**

Number of wedding guests, *n*	0	50	100	150	200
Cost of catering, *C* dollars	400	3150	5900	8650	11400

b



c **i** \$7300 **ii** 160

d 400, the initial cost in dollars when there are no guests

e 55, the cost of catering per guest

15 112.5 m

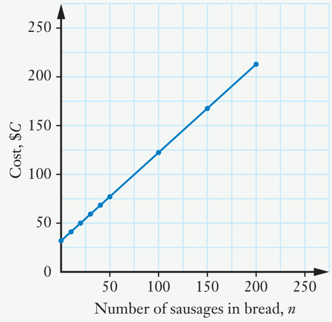
16 **a**

Number of sausages in bread, *n*	0	10	20	30	40	50	100	150	200
Cost, \$*C*	32	41	50	59	68	77	122	167	212

b $C = 32 + 0.9n$

c \$185

d



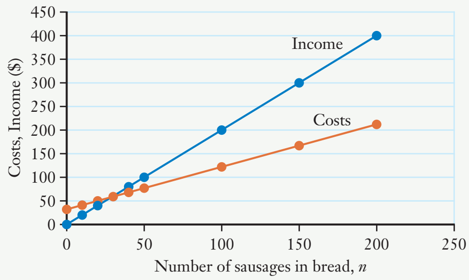
e 32, initial cost of gas bottle in dollars

f 0.9, represents the cost per sausage in bread in dollars

17 **a**

Number of sausages in bread, *n*	0	10	20	30	40	50	100	150	200
Income, \$*I*	0	20	40	60	80	100	200	300	400

b $I = 2n$



Number of sausages in bread, *n*

c Approximately (30, 60)

d Break-even point at which Year 12 doesn't lose or make money.

e Yes, the income line is above the costs line.

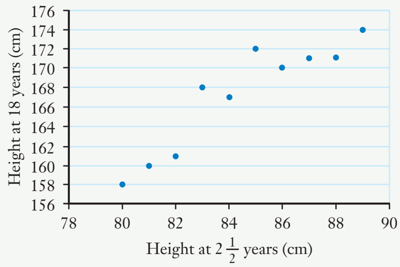
f \$188

18 32.8 m

Chapter 6

Exercise 6.01

1 **a**



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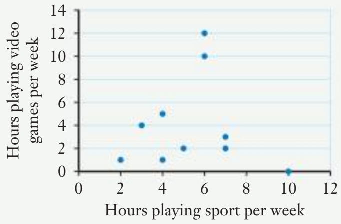
b Height at $2\frac{1}{2}$ years is the independent variable. Height at 18 years is the dependent variable.

c As height at $2\frac{1}{2}$ years increases, the height at 18 years increases.

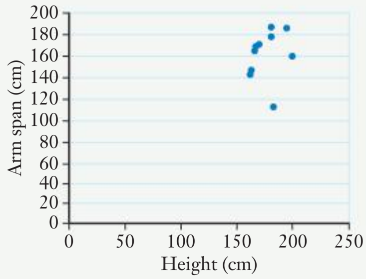
2 a



b 'Hours playing sport per week' is the independent variable and 'Hours playing video games per week' is the dependent variable.

c There is no pattern to this data. It is mostly in a bunch.

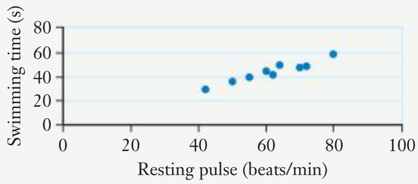
3 a



b 'Height' is the independent variable and 'Arm span' is the dependent variable.

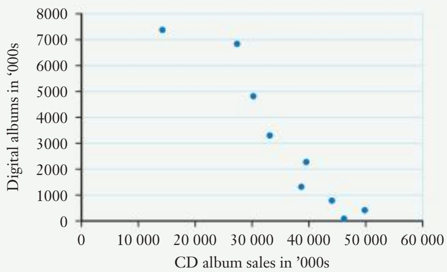
c There is one point out on its own. The other points show that as height increases, the arm span increases.

4 a



b As resting pulse increases, swimming time increases.

5 a



b As CD album sales increase, digital album sales decrease.

Exercise 6.02

1 a positive, linear, moderate

b no association

c positive, linear, strong

d negative, non-linear, strong

e positive, linear, weak

f negative, linear, strong

g positive, non-linear, strong

h no association

i negative, linear, moderate

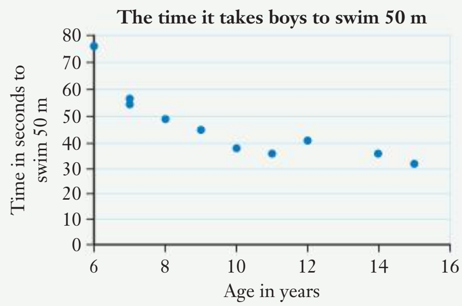
j positive, linear moderate

2 a positive, linear strong

b positive, linear strong

c negative non-linear, moderate

3



b Yes, there is a strong, negative, linear correlation.

4–6 Teacher to check.

Exercise 6.03

1 a causal

b not causal, age causes the length of feet

c causal

d not causal, number of people at the game influences these

e not causal, age causes these two to be linked

f usually causal

g causal

2–4 Teacher to check.

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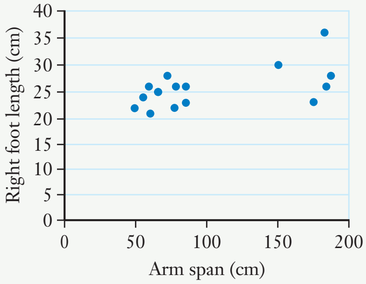
Exercise 6.04

1 **a**



b Independent variable: Arm span; dependent variable: foot length

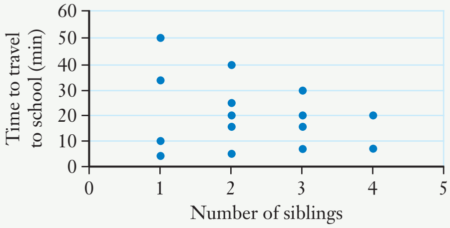
c As arm span increases, foot length increases.

d positive, linear, moderate

e No, both are linked to age and body shape.

2 **a** No, teacher to check reasons.

b



c The scatterplot shows no relationship between the two variables.

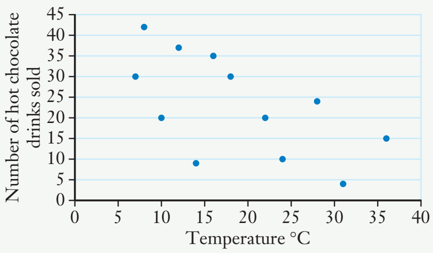
3, 4 Teacher to check.

Keyword activity

1 bivariate
2 scatterplot
3 independent variable
4 dependent variable
5 association
6 positive
7 negative
8 linear
9 non-linear
10 weak
11 moderate
12 strong
13 causes

Test yourself 6

1 **a**



b Independent variable is temperature, dependent variable is hot chocolate sold.

c Teacher to check.

2 The association is negative, linear and moderate.

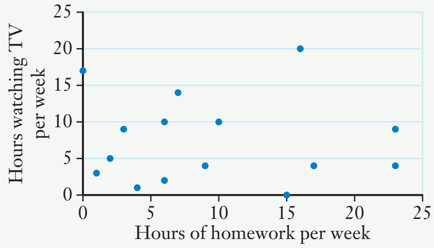
3 Teacher to check.

4 **a** Not causal, these are linked by age and body type.

b Causal, the more the team wins, the more likely fans will buy their merchandise.

5 **a** Teacher to check.

b



c Independent variable is hours of homework per week, dependent variable is hours watching TV per week.

d There is no association between these two variables.

Chapter 7

Exercise 7.01

1 **a** 1000, × **b** 10, ÷ **c** 100, ÷
d 1000, ÷ **e** 100, × **f** 1000, ÷
g 10, ×

2 **a** 30 **b** 500 **c** 0.4 **d** 2000
e 3 **f** 2 **g** 0.5 **h** 0.25
i 0.6 **j** 600 **k** 4.5 **l** 800
m 0.8 **n** 9000 **o** 0.09 **p** 6500

3 **a** 640 **b** 64 000

4 **a** 850 **b** 0.85

5 **a** 86 cm **b** 21 mm
c 12 m **d** 32 cm

6 **a** 45 mm **b** 31.5 cm
c 240 m **d** 18.4 m

7 38.4 m **8** 98.4 mm

9 **a** 520 cm or 5.2 m **b** 83.3 mm

10 6 m

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11 **a** Rectangle **b** 800 mm
c 950 mm **d** 1250 mm
e The length of the join between the blue and white sections isn't part of the flags perimeter, but it is included in the perimeter of each separate section.

Exercise 7.02

1 **a** 94.2 cm **b** 56.5 mm
c 19.5 km **d** 55.0 m
2 **a** 15.1 mm **b** 7.0 mm **c** 33.4 cm
3 **a** 203 cm **b** 22 mm
4 40 200 km
5 **a** Teacher to discuss **b** 94.4 mm
c The shortest distance between any two points is a line, not a curve.
6 25 or 26
7 **a** 36 880 km **b** 1740 km
c 2040 km. The road doesn't follow a straight line.

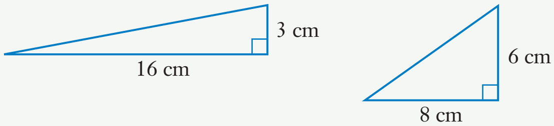
Exercise 7.03

1 27.7 cm
2 **a** 4524 m **b** 3.6 laps
3 **a** 46.3 m **b** 39.3 cm **c** 28.2 cm
4 46.3 cm **5** 336 m
6 1152 cm **7** 2771 m

Exercise 7.04

1 **a** hectares **b** square metres
c square centimetres **d** square metres
e square centimetres **f** square millimetres
g square kilometres **h** square millimetres
i square kilometres
2 **a** 790 **b** 1.5 **c** 6.9
d 76 **e** 86.5 **f** 120 000
g 320 000 **h** 45 000 **i** 750 000
j 1.9
3 6.5 m^2, 114 000 cm^2, 25 050 000 mm^2
4 54 cm^2, 990 mm^2, 0.000 032 m^2
5 **a** 352.1 **b** 3.521
6 Soccer field. It is 1750 m^2 smaller than 1 ha. A rugby field is 3000 m^2 smaller than 1 ha.
7 **a** 324 m^2 **b** 72 m

8 **a** 35 cm^2 **b** 55 cm^2 **c** 60 cm^2
d 80 cm^2 **e** 192 cm^2 **f** 63 cm^2
9 **a** 21 m and 20 m **b** 210 m^2
10 3 m
11 **a** 24 000 000 **b** 24
c Many answers possible, e.g. 6 km by 4 km.
12 **a** 900 cm^2 **b** 11.11
c 280 **d** 294
13 **a** 119.7 mm^2 **b** 84 cm^2 **c** 56 cm^2
14 **a** Any pair of numbers that multiply to give 36, for example 4×9, 3×12, 6×6.
b Teacher to check.
c 24 cm
15 **b** 15 cm^2
16 Many answers are possible, e.g.



Exercise 7.05

1 **a** 36 cm^2 **b** 55 m^2 **c** 104 m^2
d 100 m^2 **e** 70 m^2 **f** 96 m^2
2 **a** 452.39 cm^2 **b** 49.87 cm^2
c 47.78 cm^2 **d** 3019.07 cm^2
e 117.29 cm^2 **f** 151.32 cm^2
g 760.27 cm^2 **h** 173.11 cm^2
3 **a** 11 m^2 **b** 14.1 m^2 **c** 917 bricks
4 **a** 25.44 cm^2 **b** 203.52 cm^2 **c** 216.73 cm^2
5 **a** 113.1 m^2 **b** 56.5 m^2 **c** 84.8 m^2
6 630 000 mm^2
7 **a** 706.89 cm^2 **b** 88.36 cm^2 **c** 176.72 cm^2
8 19.24 m^2 **9** 3.39 cm

Exercise 7.06

1 **a** 176.8 cm^2 **b** 19.84 m^2
c 875 mm^2 **d** 1000 cm^2
2 **a** 156 m^2 **b** 500 mm^2
c 71 m^2 **d** 201.19 km^2
3 **a** 94.63 m^2 **b** 216.53 cm^2
4 **a** 37.71 m^2 **b** 272.4 m^2
5 **a** 2.02 m^2 **b** 6.06 m^2 **c** 5.45 m^2
6 **a** 80 m^2 **b** $576

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7 a $5.14\ m^2$ b $2.36\ m^2$
8 $50\ 48\ cm^2$
9 a $15\ 621.77\ m^2$ b $10\ 027.43\ m^2$
c $5594.34\ m^2$

Exercise 7.07

1 a $29\ 400\ cm^2$ b $600\ mm^2$
2 $6125\ cm^2$
3 $121.5\ m^2$
4 a $348\ cm^2$ b $11.33\ cm^2$
5 $12\ 680\ cm^2$
6 a $89.25\ m^2$ b $414
c $29.25\ m^2$ d $460
7 $12.72\ m^2$
8 a $254\ cm^2$ b $279.4\ cm^2$
9 $71\ m^2$
10 a 45 cm by 25 cm b $1125\ cm^2$
c $4325\ cm^2$

Exercise 7.08

1 a $864\ cm^2$ b $40\ 560\ m^2$
2 $960\ cm^2$
3 $228\ 000\ m^2$
4 a $329.3\ cm^2$ b $39.2\ m^2$
5 $712.2\ cm^2$
6 a $105.6\ cm^2$ b $3700\ m^2$
7 $139\ cm^2$
8 a $1946\ m^2$ b about $3\ m^2$
9 $443\ cm^2$
10 a $106\ m^2$ b $6.84\ m^2$ c $99.16\ m^2$

Exercise 7.09

1 a $604.76\ cm^2$ b $865.70\ cm^2$
c $672.36\ cm^2$ d $36.32\ m^2$
2 $345.58\ cm^2$
3 $367.6\ cm^2$
4 $154\ cm^2$
5 $18\ m^2$
6 a $514\ 719\ 000\ km^2$ b $360\ 303\ 000\ km^2$
c $154\ 416\ 000\ km^2$
7 $161\ m^2$
8 a $3421.19\ cm^2$ b 2376 tiles
9 a $1728\ cm^2$ b $39.27\ cm^2$
10 a 21 cm, 18 cm, 10 cm
b $2771\ cm^2$, $2036\ cm^2$, $628\ cm^2$
11 a $4.02\ m^2$ b 1

Exercise 7.10

1 $185\ cm^2$
2 $38.4\ m^2$
3 $2118\ cm^2$
4 $4.56\ m^2$
5 a $2617\ cm^2$ b $2879\ cm^2$
6 a $7.875\ m^2$ b $40.23\ m^2$
7 $2300\ cm^2$
8 a $31.4\ m^2$ b 4.5 L
9 $13\ m^2$

Keyword activity

Across

4 PRISM
7 SURFACE
11 HECTARE
12 CIRCLE
13 SOLID
15 RECTANGLE
17 COMPOSITE
18 AREA
19 TRIANGLE
20 PARALLELOGRAM

Down

1 PYRAMID
2 CUBE
3 PI
5 SECTOR
6 QUADRILATERAL
8 NET
9 TRAPEZIUM
10 CYLINDER
13 SPHERE
14 ARC
16 SQUARE

Test yourself 7

1 a 50 b 300 c 3.6
d 4200 e 0.08
2 a 64 cm b 51 mm c 5.6 km
d 86.4 mm e 23.4 cm
3 a 179.1 km b 105.6 mm c 16.3 cm
4 a 22.8 m b 19.2 cm c 56.2 mm
5 50.3 mm
6 a $9.6\ mm^2$ b $219.5\ cm^2$
c $2.2\ m^2$ d $52.29\ cm^2$
7 $30\ m^2$
8 a 4 b 540 c 2.5
d 0.55 e 7000 f 60 000
9 a C b F c B d E
e A f D g G

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10 **a** 834.7 mm^2 **b** 47.7 cm^2 **c** 9.12 cm^2
d 150 m^2 **e** 88.0 m^2 **f** 25.0 mm^2

11 **a** 90 m^2 **b** 134 m^2 **c** 136.3 mm^2

12 **a** 12.5 m^2 **b** 720 m^2

13 1360 mm^2

14 **a** $36\,835\text{ mm}^2$ **b** 1521 m^2

15 **a** 273.32 cm^2 **b** 1576 m^2

Chapter 8

Exercise 8.01

1 6700 km

2 **a** **i** 1.4 km **ii** 1.9 km **iii** 2.48 km
b **i** 35 cm **ii** 7.5 cm **iii** 16.5 cm

3 5.6 cm

4 3200 m or 3.2 km **5** 83 mm

6 **a** 875 m **b** 16 cm
c 9600 m or 9.6 km **d** 7 cm

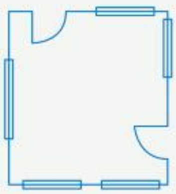
7 **a** 1 : 100 000 **b** 1 : 40 000 **c** 1 : 25 000
d 3 : 160 000 **e** 1 : 50 000 **f** 1 : 10 000

8 **a** 4 m **b** 75 cm **c** 2.05 m
d 1.2 m **e** 1.6 m **f** 1 m

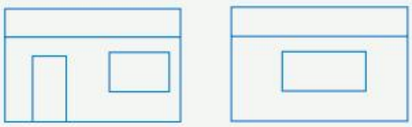
9 **a** 4.02 m **b** 93.6 cm

Exercise 8.02

1 **a**



b



East elevation West elevation

2–5 Teacher to check

Exercise 8.03

1 **a** **i** kitchen sink
ii vanity (or wash basin)
iii toilet
iv window
v bath
vi built-in wardrobe
vii sliding door

b They are all tiled. **c** 3
d 1 and 2 **e** 4100 mm × 3200 mm
f in the back of the garage
g near the bathroom
h 1 **i** 1
j 2 **k** living room
l 6670 mm × 5500 mm
m **i** south **ii** north **iii** south
iv east **v** west
n south **o** no
p Living room, Dining room, Bedroom 1, Bedroom 3
q Dining room, Bedroom 2, Living room

2 **a** 3 **b** 3
c no **d** 4.7 m by 3.5 m
e Bed 1 **f** $86 016

3 **a** 5 m **b** 2.5 m
c 2 m **d** $2\text{ m} \times 2.5\text{ m} = 5\text{ m}^2$
e 5 m
f $4\text{ m} \times 3.8\text{ m} = 15.2\text{ m}^2$

4 **a** 240 mm **b** 70 mm **c** 12.39 m
d 15.69 m **e** 3.6 m by 6.07 m

5 **a** 6790 mm × 3700 mm
b 2
c outside the bathroom
d shower
e bedroom 4
f Family room = 25.123 m^2, media room = 23.787 m^2. Family room is bigger by 1.336 m^2

Exercise 8.04

1 **a** 7.5 m^2 **b** $273.75

2 **a** 18 m **b** $198

3 **a** 92 m^2 **b** $77 280

4 **a** 14 m **b** 13 m **c** $195

5 **a** 12 m^2 **b** 4 m **c** $317.80

6 **a** 14 m^2
b **i** $2\frac{1}{3}\text{ L} \approx 3\text{ L}$ **ii** $34.90

7 **a** painting, skirting boards, carpet
b $562.70

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8 \$1258

9 **a** 3.9 m **b** 3.9 m **c** 3 m

10 \$892

11 **a** 11.53 m **b** 3

12 Approximately \$87 912

Exercise 8.05

1–3 Teacher to check.

4 **a** Teacher to check. **b** 2.9 km

5 285 m

6 **a** Teacher to check. **b** 144.2 cm

7 133 m **8** 251 m

Keyword activity

1 G	**2** J	**3** K	**4** E
5 H	**6** A	**7** D	**8** I
9 L	**10** C	**11** B	**12** F

Test yourself 8

1 10.75 m

2 **a** 1 : 5000

b **i** 760 m **ii** 230 m

c **i** 7.6 cm **ii** 14.5 cm

d 1130 m or 1.13 km

3 **a** Teacher to check

b $x = 6390$ mm

c $y = 800$ mm, $z = 1800$ mm, $l = 10\ 560$ mm, $w = 6060$ mm, $d = 4500$ mm

4 **a** 280 mm

b 110 mm

c 5500 mm

d 3500 mm by 5500 mm

e 6390 mm by 5500 mm

5 **a** 9

b Length 13.1 m, Width 10 m

c 3 m

d **i** 109.7 m^2 **ii** \$73 719 approximately

e 10.9% **f** 21.3 m^2

g **i** 23.43 m^2 ≈ 24 m^2 **ii** 229 tiles

h **i** 13.1 m **ii** \$374.23

6 **a** \$1768

b **i** 15.18 m **ii** \$227.70

c **i** 12.42 m^2 **ii** 310.5 ≈ 311 tiles

iii \$510.46 ≈ \$511

d **i** 26 m^2 **ii** $4\frac{1}{3}$ L

iii \$69.80 (2 cans required)

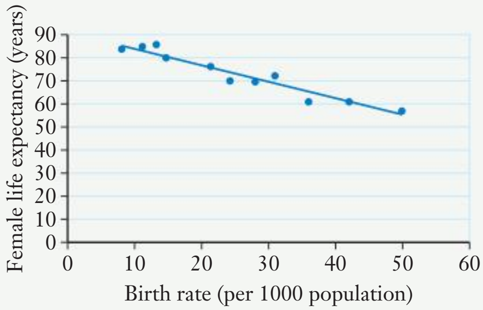
7 Teacher to check diagram, 40 m.

Chapter 9

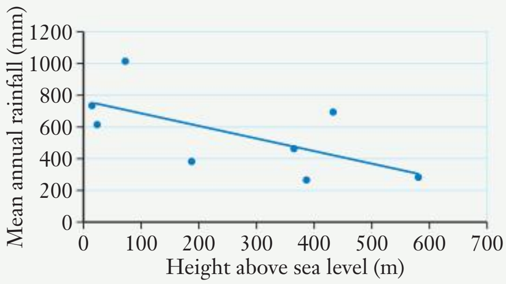
Exercise 9.01

1 Teacher to check.

2



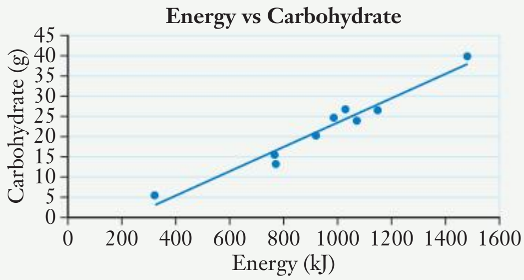
3



4 $y = -0.7248x + 91.18$, other answers are possible

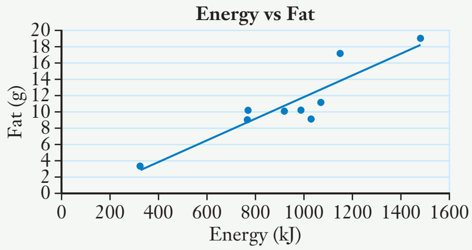
5 $y = -0.7691x + 755.52$, other answers are possible

6 **a, b**



c $y = 0.0301x - 6.7032$, other answers possible

d



e $y = 0.0132x - 1.4528$, other answers possible

Exercise 9.02

1 **a** Weeks after starting training

b Leg extensions in 30 seconds

c The longer Anita has trained, the more leg extensions she can do in 30 seconds.

d 11 or 12

2 **a** Number of victims

b Number of medical staff

c About 40 staff

d About 750 victims

e The greater the number of victims, the greater the number of medical staff sent.

f The agency calculated the number of medical staff required based on the number of victims, not the reverse as Kyle's statement implies.

3 **a** About 23.5 cm

b The longer the right foot, the higher the test score.

c The age of the students is relevant. Younger students who haven't learned as much maths generally have smaller feet and would score lower on the test.

4 **a** As the height of students increases, so does the weight of students.

b About 55kg

c No, teacher to check reasoning

5 **a** About 10 or 11

b As the number of sunny days increases, the number of dry cloudy days decreases.

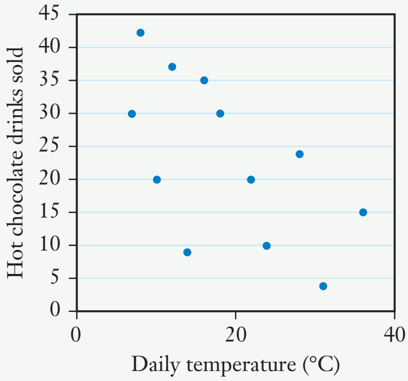
c The total number of sunny, dry cloudy and wet days must be the same as the number of days in the month. As the number of sunny days increases, the number of dry cloudy or wet days must decrease to keep the total the same as the number of days in the month.

Exercise 9.03

1 **a** E **b** C **c** B **d** G

e H **f** A **g** F **h** D

2 **a**



b –0.611

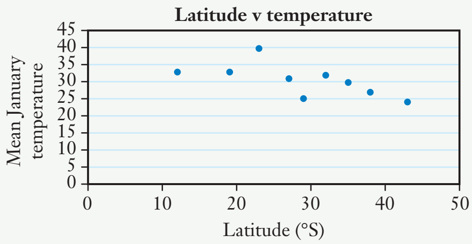
c Yes. The scatterplot shows a moderate negative linear association and the correlation coefficient confirms this.

d Yes, when it is colder, people are more likely to feel like a hot drink.

3 **a** –0.9662 **b** –0.6332 **c** 0.9779

d 0.9095 **e** 0.8520

4 **a**



b –0.65217

c No, the correlation coefficient is only in the moderate range.

d Teacher to check.

5 **a** –0.9484

b They are closely related. As digital sales increase, CD sales decrease.

c Yes, if more people buy digital albums, then fewer people will buy CD albums.

6 Teacher to check.

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Exercise 9.04

1 **a** **i** 1090 kJ **ii** 105 kg

b Reliable, as the correlation is strong.

c 1885 kJ

d Teacher to check.

2 **a** About 62 years

b About 36 births

c Reliable, as the correlation is strong.

d About 48 years

e Teacher to check.

3 **a** About 525 mm

b About 460 m

c Not very reliable, as the correlation is weak.

d About 140 mm

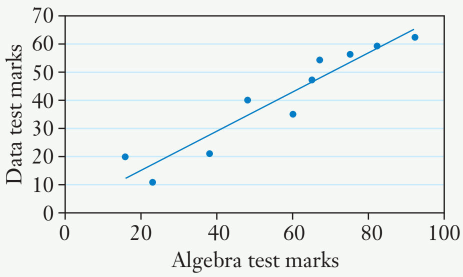
e –14 mm, not possible as you can't have negative rainfall

4 **a** 0.975 989

b Very reliable, as the correlation coefficient is close to 1

c May not be reliable as –10°C is outside the data range.

5 **a**

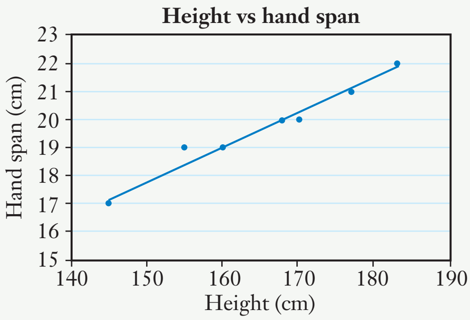


b Ella 43 in Algebra test, Amy 57 in Data test.

c 0.949 98

d Very reliable, as the correlation coefficient is close to 1.

6 **a**



b 20 cm

c 33 cm. The value is very likely to be wrong because Robert Wadlow's height is much greater than the heights used in the scatterplot.

Keyword activity

1 bivariate **2** scatterplot

3 line of best fit **4** technology

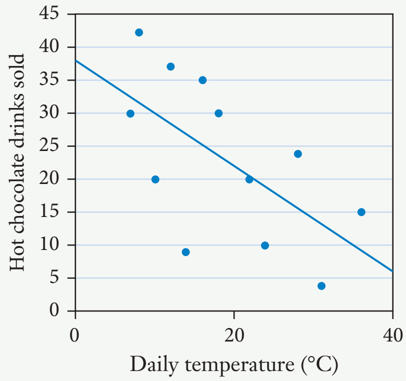
5 equation **6** correlation

7 correlation coefficient **8** predictions

9 interpolation **10** extrapolation

Test yourself 9

1 **a**



b $y = -0.7835x + 37.756$

2 **a** temperature **b** drinks sold

c 22 drinks **d** 16°C

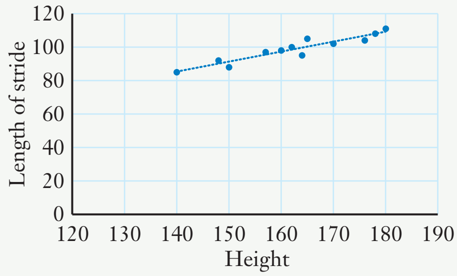
e the higher the temperature, the fewer hot drinks sold

3 Teacher to check.

4 **a** –0.0365

b No, they are not closely related, as the correlation coefficient is close to 0.

5 **a**



b 0.9421

c **i** 88 cm **ii** 150 cm

d Very reliable as the correlation coefficient is close to 1

e $y = 0.5963x + 1.8511$

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f **i** 121 cm **ii** 131 cm

g These predictions are outside the original data, so may be unreliable. We don't know if the linear relationship continues.

Chapter 10

Exercise 10.01

1 **a** 3000 **b** 12 000 **c** 1.5 **d** 2.4
e 850 000 **f** 0.9 **g** 2500 **h** 0.5

2 **a** 10 000 **b** 10

3 **a** 4 **b** 54 **c** 2160

4 **a** 135 g **b** 135 000 **c** 270 mg

5 20

6 Teacher to discuss answers with the class.

7 360 mg

8 **a** 11
b Yes, its mass is less than 12 pounds.

9 Sum of the dimensions = 40.4 inches, mass = 52.8 pounds. The bag is small enough, but it is too heavy.

10 **a** 1350 kg **b** 10.8 t

11 1600 g

12 It weighs 32.15 troy ounces and is worth approximately $42 790.

Exercise 10.02

1 **a** m^3 **b** cm^3 **c** cm^3
d mm^3 **e** m^3 **f** m^3
g cm^3 **h** m^3 **i** mm^3
j cm^3 **k** mm^3 **l** cm^3

2 **a** E **b** G **c** F **d** C
e A **f** B **g** D

3 **a** 5 000 000 **b** 1 600 000
c 0.006 **d** 4
e 0.16 **f** 0.25
g 180 000 **h** 120
i 4 000 000 000 **j** 0.0096

4 4210 mm^3, 42 cm^3, 0.0042 m^3

5 0.6 m^3, 65 000 cm^3, 7 000 000 mm^3

6 0.306 m^3 **7** 2 520 000 mm^3

8 200 000 cm^3

Exercise 10.03

1 **a** 111 cm^3 **b** 2.5 m^3
c 256 m^3 **d** 486.42 cm^3

2 **a** 1 071 000 cm^3 **b** 91.125 cm^3
c 21 952 cm^3 **d** 258.06 cm^3

3 **a** 84 cm^3 **b** 240 cm^3

4 **a** 16 065 cm^3 **b** 1 285 299 cm^3

5 **a** 42 875 cm^3 **b** 471 625 cm^3
c approximately 330 books

6 **a** 192 cm^3 **b** 1270 cm^3
c 43.2 cm^3 **d** 22.2 cm^3

7 **a** 126 pavers **b** 4 500 000 mm^3
c 567 000 000 mm^3 **d** 13
e $737.10

8 **a** 65 cm^3 **b** 51

9 Volume = 420 m^3, so Nazneen requires a large air-conditioner.

10 **a** 49 cm2 **b** 7 cm

Exercise 10.04

1 **a** mL **b** kL **c** L
d mL **e** L **f** kL

2 **a** D **b** G **c** J **d** H **e** I
f A **g** F **h** E **i** C **j** B

3 **a** 8 mL **b** 1500 mL **c** 425 mL

4 **a** 2 L **b** 3.5 L **c** 0.25 L

5 **a** 5000 mL **b** 5

6 2000 **7** 1250 cm^3 **8** 9 L

9 504 L

10 **a** 0.2 mL **b** 12 mL
c 2160 mL **d** 17.28 L

11 **a** rectangular prism **b** 2.042 m^3
c 2042 L **d** 19 minutes

12 **a** 10 143 cm^3 **b** 10 L **c** 5 kg

13 **a** 16 **b** 4.4 mL

14 **a** 20 mL **b** every 4 hours
c 120 mL **d** 11 mL
e No, Noah is too young. The label says for people over 12 years old.

15 500 mL

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Exercise 10.05

1 **a** $166\ mm^3$ **b** $236\ mm^3$
2 **a** $212.2\ mm^3$ **b** $1629.5\ cm^3$
3 **a** $192\ cm^3$ **b** $180\ m^3$
4 $333\ m^3$ **5** $41.56\ m^3$
6 $2\ 101\ 400\ m^3$ **7** $1767\ cm^3$
8 $1485.2\ m^3$
9 **a** $0.39\ m^3$ **b** 390 L
10 **a** $28\ 148.67\ cm^3$ **b** $5387.83\ cm^3$
c $22\ 760.84\ cm^3$
11 $4.6\ m^3$
12 **a** 1.1 m
b Pyramid $1.2\ m^3$, Prism $2.9\ m^3$
c $4.1\ m^3$
13 **a** $1810\ cm^3$
b $r = 6$ cm, $h = 24$ cm **c** $2714\ cm^3$
14 $8.8\ m^3$
15 **a** $0.13\ m^3$ **b** 130 L
c 3250 L **d** $2.31\ m^3$
e No, only holds 2310 L, 2 trips required

Keyword activity

ACROSS

2 CUBIC **5** PRISM
7 AREA
9 RECTANGULAR PYRAMID
10 VOLUME **11** SPHERE
12 LITRE **13** MM

DOWN

1 TRIANGULAR PRISM
3 CYLINDER **4** CAPACITY
6 METRE **8** COMPOSITE

Test yourself 10

1 **a** 5000 **b** 0.2 **c** 1400
d 3500 **e** 7.5
2 **a** 5000 **b** 2 000 000 **c** 0.5
d 250 000 **e** 0.024 **f** 36
3 **a** $11.16\ cm^3$ **b** $64\ m^3$
c $153\ cm^3$ **d** $48\ cm^3$
4 2.6 L **5** 16
6 **a** 24 **b** $6000\ cm^3$
7 700
8 **a** $463.2\ cm^3$ **b** $480.0\ m^3$ **c** $864.0\ mm^3$
9 **a** $1336\ cm^3$ **b** 1.336 L
10 **a** $32\ m^3$ **b** 32 000 L
11 **a** $58\ cm^3$ **b** $44\ cm^3$
c $262\ cm^3$ **d** $108\ 573\ cm^3$

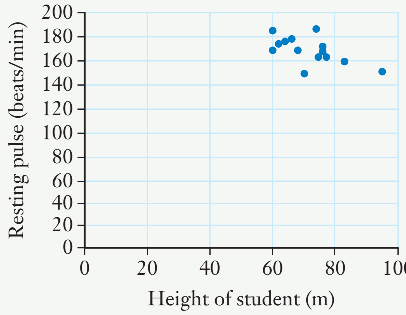
Practice Set 2

Section A

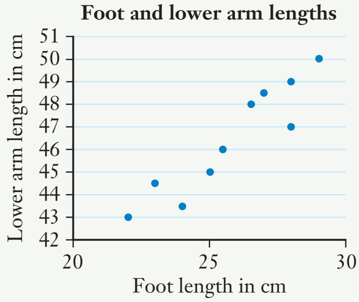
1 A	**2** D	**3** B	**4** C
5 D	**6** C	**7** B	**8** D
9 A	**10** C	**11** B	**12** A

Section B

1 \$7858.50
2 **a** $268.08\ cm^3$ **b** $1436.76\ cm^3$
3 **a** $201.06\ cm^2$ **b** $615.75\ cm^2$
4 **a** 300 cm = 3 m **b** 1125 cm = 11.25 m
5 **a** Teacher to check
b



c Most of the points are together, but a few are out on their own
d There is no association between these variables.
6 **a** $135\ cm^3$ **b** $64\ cm^3$ **c** $264\ mm^3$
7 **a** $198\ cm^2$ **b** $96\ m^2$ **c** $312\ mm^2$
8 **Foot and lower arm lengths**



9 **a** $152\ cm^2$ **b** $574\ cm^2$

10 **a** 5.5 m by 5.5 m
b **i** 30.25 m^2 **ii** $2268.75
c 12
d **i** 12.24 m^2 **ii** $1040.40

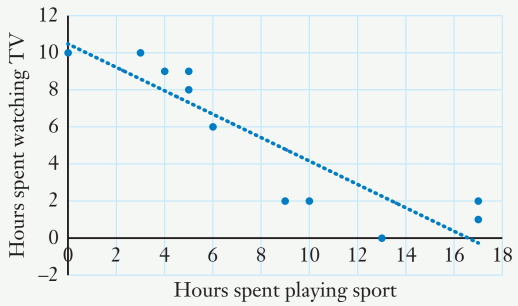
11 **a** **i** not causal
ii age links these 2 measurements
b causal
c usually causal

12 **a** **i** 80 **ii** 2.5 hours
b Reasonably reliable as the points are fairly close to the line
c Teacher to check

13 10 806 m^3

14 **a** See graph at question **15** below (but without the line of best fit).
b Overall as the hours spent playing sport increase, the hours spent watching TV decrease.

15 **a**



b It is negative, linear and moderate.
c Approximately 5.5 hours of watching TV
d Approximately 12 hours playing sport
e Not very reliable as the association is only moderate and not very strong.

16 **a** 235.62 mm^2 **b** 61.42 mm^2

17 **a** 8700 m **b** 8625 m^3
c 8 625 000 L

18 Teacher to check

Chapter 11

Exercise 11.01

1 **a** $115.20 **b** $243.36
c $866.25 **d** $350.63

2 **a** $44.10 **b** $193.60
c $54.27 **d** $4.58

3 0.38%

4 **a** 0.26% **b** 0.06% **c** 1.56%
d 0.0085% **e** 0.12% **f** 0.78%

5 **a** $10 088 **b** $4288

6 **a** $15.86 **b** $880.86

7 $59 743.75

8 **a** 0.048% **b** $20 736

9 **a** $250 **b** $21 500
c The interest rate was too high and she didn't check whether the investment company was known to be a scam risk.

10 **a** $1600 **b** $600 **c** $250
d It's the original investment (principal).

Exercise 11.02

1

	Interest	Balance
End of the 1st year	$I = Prn$ $= \$8000 \times 0.06 \times 1$ $= \$480$	$\$8000 + \$480$ $= \$8480$
End of the 2nd year	$I = Prn$ $= \$8480 \times 0.06 \times 1$ $= \$508.80$	$\$8480 + \$508.80$ $= \$8988.80$
End of the 3rd year	$I = Prn$ $= \$8988.80 \times 0.06 \times 1$ $= \$539.33$	$\$8988.80 + \$539.33$ $= \$9528.13$

2 **a** $1440 **b** $13 440
c $13 977.60 **d** $14 536.70
e $2536.70

3 **a** $4260.10 **b** $260.10 **c** $4

4 **a**

	Interest	Balance
End of the 1st year	$I = Prn$ $= \$14\,000 \times 0.007 \times 1$ $= \$98$	$\$14\,000 + \$98$ $= \$14\,098$
End of the 2nd year	$I = Prn$ $= \$14\,098 \times 0.007 \times 1$ $= \$28.69$	$\$14\,098 + \$28.69$ $= \$14\,126.69$
End of the 3rd year	$I = Prn$ $= \$14\,126.69 \times 0.007 \times 1$ $= \$98.89$	$\$14\,126.69 + \$98.89$ $= \$14\,225.58$

b $224.58

5 **a** $9600 **b** 8% **c** $1597.44

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6 **a** $5208.75

b Scam indicators include:

- cold phone call from someone Zac doesn't know
- push from the 'advisor' to act quickly
- very high interest rate
- claim that the opportunity is 'risk-free'
- minimum investment of $10 000.

7 **a** Because the interest increases each year, a compound graph curves up. Simple interest graphs are straight lines.

b 7 years

8 **a** $400 **b** *X* Nick, *Y* Adam

c Nick's, by $35.45

Exercise 11.03

1 **a** 1.08 **b** 1.06 **c** 1.035

d 1.028 **e** 1.0175

2 **a** 7% **b** 6.5% **c** 1.5%

d 12% **e** 10%

3 B **4** $50.84 **5** $20 248 **6** $840

7 **a** $18.13 **b** $24.58

8 $25.21

9 24.56, 75.40, 231.48, 710.63, 2181.63

10 **a** 100% + 11.17% = 111.17% = 1.1117

b 16 shillings

11 $1700

12 2840

13 **a** 30 805 000 **b** 2041

14 **a** $173 644 **b** $23 153 **c** $20 837

d $775 609 **e** $2894 **f** $57 881

Exercise 11.04

1 **a** $3974.44 **b** $974.44

c Teacher to check.

2 **a** $6529.12 **b** $1353.20

c $13 170.10 **d** $46 960.25

3 Year 1 $1311, Year 2 $1348.36, Year 3 $1386.80, Total $4046.16

4 **a** $77 318.13 **b** $27 318.13

c Teacher to check.

5 $9177.22

6 The future values are: Option 1 $23 300, Option 2 $22 823.32

Option 1 is the better one if it is an investment because she ends up with more money.

Exercise 11.05

1 **a** $1215.61 **b** $3218.44

c $43 630.30 **d** $55 475.18

2 **a** $649.96 **b** $716.50

c $2488.86 **d** $966.95

3 **a** $5948.93 **b** $9948.93

4 **a** B8: = B4 C8: =B8*$B$5 D8: =B8+C8

A9: =D8

b 0.045 **c** $48 536.88

d $24 636.88

5 **a** Teacher to check.

b Approximately 18 years

Exercise 11.06

1 **a** $1966.81 **b** $517.13

c $359.36 **d** $366.16

2 **a** $7193.75 **b** $1393.75

3 $1744.93 **4** $4.19 **5** $464

6 The monthly compounding gives the better return (final values of $511 274.92 compared to $510 880.61).

7 The managed fund gives the better return (final values of $211 56.45 compared to $210 112.38).

Keyword activity

1 H **2** J **3** E **4** A

5 B **6** G **7** I **8** C

9 F **10** K **11** D

Test yourself 11

1 $1312

2 **a** Josephine invested $8400 at 4% p.a. annually compounding interest.

	Interest	Balance
End of the 1st year	$I = Prn$ $= \$8400 \times 0.04 \times 1$ $= \$336$	$8000 + $336 = $8336
End of the 2nd year	$I = Prn$ $= \$8336 \times 0.04 \times 1$ $= \$333.44$	$8336 + $333.44 = $8669.44
End of the 3rd year	$I = Prn$ $= \$8669.44 \times 0.04 \times 1$ $= \$346.78$	$8669.44 + $346.78 = $9016.22

b $1016.22

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3 1.07

4 \$60.62

5 \$833.49

6 \$371.53

7 \$8607.59

8 Investment B is better (\$172.06 compared to \$163.20)

9 **a** Monthly interest for Month 1

b To keep the cell reference the same when she copied down

c \$436.09

d Either =SUM(C10:C21) or =D12-B10

e \$67 622.88

f

G15 fx

	A	B	C	D
1	Jill's spreadsheet			
2				
3				
4		Initial investment C4	\$60,000.00	
5		Annual interest rate as a decimal C5	0.08	
6		Monthly interest rate C6	0.006666667	
7		Weekly interest rate	0.001538462	
8				
9	Number of weeks	Value of the investment at the beginning of the week	Weekly interest	Value of the investment at the end of the week
10	1	\$60,000.00	\$92.31	\$60,092.31
11	2	\$60,092.31	\$92.45	\$60,184.76
12	3	\$60,184.76	\$92.59	\$60,277.35
13	4	\$60,277.35	\$92.73	\$60,370.08
14	5	\$60,370.08	\$92.88	\$60,462.96
15	6	\$60,462.96	\$93.02	\$60,555.98
16	7	\$60,555.98	\$93.16	\$60,649.14
17	8	\$60,649.14	\$93.31	\$60,742.45
18	9	\$60,742.45	\$93.45	\$60,835.90
19	10	\$60,835.90	\$93.59	\$60,929.49
20	11	\$60,929.49	\$93.74	\$61,023.23
21	12	\$61,023.23	\$93.88	\$61,117.11
22	13	\$61,117.11	\$94.03	\$61,211.14
23	14	\$61,211.14	\$94.17	\$61,305.31
24	15	\$61,305.31	\$94.32	\$61,399.63
25	16	\$61,399.63	\$94.46	\$61,494.09
26	17	\$61,494.09	\$94.61	\$61,588.69
27	18	\$61,588.69	\$94.75	\$61,683.45
28	19	\$61,683.45	\$94.90	\$61,778.34
29	20	\$61,778.34	\$95.04	\$61,873.39
30	21	\$61,873.39	\$95.19	\$61,968.58
31	22	\$61,968.58	\$95.34	\$62,063.91
32	23	\$62,063.91	\$95.48	\$62,159.40
33	24	\$62,159.40	\$95.63	\$62,255.03
34	25	\$62,255.03	\$95.78	\$62,350.80
35	26	\$62,350.80	\$95.92	\$62,446.73

g \$6.37

10 Graph 1, Compound interest;
Graph 2, Simple interest

11 **a** 0.4% **b** \$463.66

12 The annual compound interest (\$12 726.37 compared to \$11 272.72 and \$12 640).

Chapter 12

Exercises 12.01 and 12.02

Teacher to discuss with the class.

Exercise 12.03

1 **a** $\frac{1}{5}$ **b** $\frac{1}{10}$ **c** 27 cm

2 **a** 0.08 **b** $\frac{17}{25}$

3 **a** 24 **b** $\frac{16}{24}$ **c** $\frac{1}{3}$

4 **a** $\frac{5}{40}$ **b** $\frac{35}{40} = 87.5\%$

c Teacher to check.

5 **a** $\frac{19}{20}$ **b** 0.35 **c** 10%

d yes, no, no

e Special interest groups who have non-representative views.

6 **a** $\frac{10}{75} = \frac{2}{15}$

b 0.28, 0.16, 0.13, 0.15, 0.16, 0.12

c It could be biased. Number 1 appears to occur nearly twice as often as it should. More trials are required before a conclusion can be reached.

7 Teacher to check.

Exercise 12.04

Teacher to discuss with the class.

Keyword activity

Teacher to check.

Test yourself 12

1 **a** 0.3

b point down, higher relative frequency than pointing up

2 **a** It is quite possible for a normal coin to show heads 4 out of 6 times in the short-run. It is only in the long-run that the percentage of heads approaches 50%.

3 Tossing a coin, birth of boys or girls, guessing the right answer on a true/false question.

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4 **a** False. The coin doesn't have a memory. The next toss could result in either a head or a tail.
b true

5 **a** 0.358, 0.425, 0.3
b $\frac{87}{120} = \frac{29}{40}$

Chapter 13

Exercise 13.01

1

NT		
AWST	ACST	AEST
$-1\frac{1}{2}$	0	$+\frac{1}{2}$

2 **a** ahead **b** ahead
c same **d** ahead
e behind **f** ahead
g ahead **h** same
i ahead **j** behind

3 **a** 11 a.m. **b** 9 a.m. **c** 10:30 a.m.
d 11 a.m. **e** 11 a.m. **f** 9 a.m.

4 **a** 11 p.m. **b** 10:30 p.m. **c** 9 p.m.
d 10:30 p.m. **e** 11 p.m. **f** 11 p.m.

5 1:45 p.m.

6 **a** 4 p.m. **b** 5:30 p.m.

7 8:30 p.m.

8 10 p.m.

9 **a–c** Teacher to check **d** 3:30 p.m.

10 Teacher to check

11 **a** 10 p.m. **b** 11:15 a.m.
c 10:05 a.m. **d** 8:50 a.m.

12 after 5 p.m.

13 6:45 a.m.

Exercise 13.02

1 **a** behind **b** ahead **c** ahead
d behind **e** behind **f** ahead

2 **a** 11 hours **b** 15 hours **c** 7 hours
d 10 hours **e** 8 hours

3 **a** 8 p.m. same day **b** 5 a.m. same day
c 6 p.m. same day **d** 7 p.m. day before
e 7 a.m. same day **f** 5 a.m. next day
g Midnight **h** 8 a.m. next day
i Midday same day **j** 7 p.m. same day

4 **a** 1 p.m. same day **b** Midnight
c 9 p.m. same day **d** 8 p.m. same day
e 8 a.m. next day **f** 4 a.m. next day

5 3 p.m. same day

6 10 a.m. same day

7 **a** 1230 **b** $5\frac{1}{2}$ hours

8 2120

9 **a** Los Angeles is 18 hours behind Brisbane
b 16 hours 30 minutes

10 6 p.m.

11 **a** 5 a.m.
b Probably not; too early

12 11:30 a.m. to 3:30 p.m.

Exercise 13.03

1 8 p.m. 31 December

2 4:30 a.m. 31 December

3 9 a.m. 1 January

4 3:45 p.m. 1 January

5 9 hours

6 9 hours

7 **a** Tonga **b** Hawaii **c** 23 hours

8 Teacher to check.

Exercise 13.04

1–3 Teacher to check

4 **a** 0805 **b** 1535
c 0310 next day

5 Teacher to check

6 **a** 2 a.m. to 9 a.m. the next day
b, c Teacher to check

Keyword activity

Teacher to check

Test yourself 13

1 **a** 4 a.m. **b** 10:45 a.m.
c 1:30 p.m. **d** NT or SA

2 **a** midnight **b** 11 p.m.
c 9 p.m. **d** midnight

3 **a** 1 p.m. **b** 8 a.m.
c 8 p.m. previous day **d** 3 a.m.
e 3 a.m. **f** 6:30 a.m.

4 **a** 7 hours **b** 1210

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5 **a** 1 a.m.
b 8 a.m.
c 4:30 p.m.
d 5:30 a.m.
e 4:30 p.m.
6 2:25 a.m. Sunday
7 Between 10 p.m. and 5 a.m.
8 7 p.m.
9 **a** 5 p.m. AWST
b 10:45 p.m. AWST
10 5 a.m. Saturday

Practice Set 3

Section A

1 C	**2** D	**3** C	**4** D
5 B	**6** A	**7** A	**8** B

Section B

1 Teacher to check
2 Teacher to check
3

	Interest	Balance
End of the 1st year	$I = Prn$ $= \$11500 \times 0.02 \times 1$ $= \$230$	$\$11500 + \$230$ $= \$11\,730$
End of the 2nd year	$I = Prn$ $= \$11730 \times 0.02 \times 1$ $= \$234.60$	$\$11730 + \$234.60$ $= \$11\,964.60$
End of the 3rd year	$I = Prn$ $= \$11964.60 \times 0.02 \times 1$ $= \$239.29$	$\$11964.60 + \$239.29$ $= \$12\,203.89$

4 Teacher to check
5 **a** $5228.05 **b** $728.05
6 2330
7 10 a.m. Thursday
8 Teacher to check
9 **a** $8707.60 **b** $307.60
10 **a** $\frac{12}{20} = \frac{3}{5}$ **b** 0.15
c Teacher to check
11 **a** 4 p.m. **b** 2 p.m.
c 12:05 a.m. Sunday
12 $80 217

Chapter 14

Exercise 14.01

1 **a** $\$7000 \times 0.09 \div 12 = \$52.50$
b $\$7000 + \$52.50 = \$7052.50$
c $\$7052.50 - \$320 = \$6732.50$
d A $6732.50 B $50.49
C $6782.99 D $6462.99
E $6462.99 F $48.47
G $6511.46 H $6196.46
e $6191.46 **f** $151.46
g $4.05 **h** She owes less
2 **a**

$15 696	$94.18	$15 790.18	$15 390.18
$15 390.18	$92.34	$15 482.52	$15 082.52
$15 082.52	$90.50	$15 173.02	$14 773.02

b $14 773.02 **c** 22.6%
3 **a** $720 **b** A $15 286
b $611.44 **c** $12 463.44
d $498.54 **e** $6734
f 4 months
4 **a**

1	$12 800	$80.84	$12 880.64	$11 980.64
2	$11 980.64	$75.48	$12 056.12	$11 156.12
3	$11 156.12	$70.28	$11 226.40	$10 326.40

b $2700 **c** $2473.60 **d** $226.40
5 **a**

	A	B	C	D	E
1	Month	Principal (P)	Interest (I)	Principal + interest ($P + I$)	Amount owing ($P + I - R$)
2	1	10000	=B2*0.06/12	=B2+C2	=D2–260
3	2	=E2	Copy down	Copy down	Copy down
4	3	Copy down			

b $9153.68

Exercise 14.02

1 **a** $29.17 **b** $27.30 **c** $1024.05
d 15 months **e** $335.93 **f** $235.93
g $5235.93

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2 a 18 months b $195.45 c $195.45

3 a 18 months b $532.31

4 a 25 months b 18 months

c Yes, he will save 4202.26.

d The quicker you repay a reducing balance loan, the less interest you will pay.

e No, the interest is a fixed amount.

5 a =B4, =B9*$B$5/12, =B9+C9, =D9–$B$6

b The dollar signs keep the interest rate and the repayment amount the same when we copy down.

6 =B3, =B7*$B$4/14, =B7+V7, =D7-$B$5, =E7

7 The diagram shows the final lines of the spreadsheet.

	A	B	C	D	E
37	31st month	$ 4,925.75	$ 30.79	$ 4,956.54	$ 4,006.54
38	32nd month	$ 4,006.54	$ 25.04	$ 4,031.58	$ 3,081.58
39	33rd month	$ 3,081.58	$ 19.26	$ 3,100.84	$ 2,150.84
40	34th month	$ 2,150.84	$ 13.44	$ 2,164.28	$ 1,214.28
41	35th month	$ 1,214.28	$ 7.59	$ 1,221.87	$ 271.87
42	36th month	$ 271.87	$ 1.70	$ 273.57	-$ 676.43
43					
44					
45		Sum of Interest = SUM(C7:C42)			
46		$	3,523.57		
47					

The spreadsheet shows that it will take 36 months for Gabriel to repay the loan. The interest is in column C. The formula =SUM(C7:C42) will produce the sum of the interest.

Gabriel will make 36 repayments and pay a total of $3523.57 in interest.

8 a 13 months b $214.45

9 longer, more

10 shorter, less

Exercise 14.03

1 a $327 b $19 620 c $4620

2 a $616 b $70 880

c The car will be worn out before it is paid for.

d Save up for the car or buy a cheaper car.

3 a 17 b $1050

4 a $190.40

b He owes less and reducible interest is calculated on the amount owing.

c $229 405 d $165 096

e i $915.20 ii $919.20.

f 49.94% g 22 years 5 months

h 91 months (or 7 years 7 months)

i At the beginning, when the loan amount is highest.

5 a i 240 months ii 187 months

b 53 c $25 290

d They can't afford to pay the higher amount of monthly repayments or they don't want to.

6 a $2301 b $36

7 a $1952 b $22 c $264

8 a $246 b $152 280

c If I could afford the extra $246 per month, I would pay it. If I can't afford the whole $246 / month, I would pay the extra that I can afford.

9 $1266. Payments in the 1980s were more than double the late 2010s value.

10 a $998, $989

b

	Big bank	Small mortgage company
Loan establishment fee	$320	$598
Mortgage discharge fee	$228	$314
Total annual loan fee over 25 years	$6200	$1900
Total monthly repayments	$299 400	$296 700
Total cost of the loan	$306 148	$299 512

c The annual loan fee

d The small mortgage company, because she will save $6636.

Exercise 14.04

Teacher to discuss answers with the class

Keyword activity

1 credit union 2 interest

3 flat 4 same

5 reducible 6 decreases

7 real estate 8 huge

9 repayment 10 minimum

11 fortnightly 12 money

Test yourself 14

1 a A $11 702 B $70.21

C $11 772.21 D $11 402.21

E $11 402.21 F $68.41

G $11 470.62 H $11 100.62

b $11 100.62 c $210.62

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2 a 2 years 9 months b $3350
3 a $2136 b $512 640
c 15 years 10 months d $56 640
4 a Increases the value of the repayments.
b Decreases the time and the total amount.

Chapter 15

Exercise 15.01

1 a Montreal, New York
b Recife, Cape Town
c Cape Town
d Montreal, New York, Recife
2 Montreal, Odessa
3 Odessa, Khartoum, Durban
4 a 32°N, 90°W b 34°S, 55°W
c 8°S, 40°W d 15°N, 30°E
e 50°N, 0°
5 a Montreal b New York
c Cape Town d Durban
e Lima f Quito
6 a 20°S, 160°E b 20°S, 140°E
c 15°N, 120°E d 20°S, 60°E
e 15°N, 100°E
7 Mauritius

Exercise 15.02

1 a $34 - 18 = 16$ b 1790 km
2 a 27° b 3000 km
c 250 km/h
3 a They are on the opposite sides of the equator.
b 5250 km
4 a 75° b 8400 km
5 2900 km
6 a Approximately 900 km
b $7\frac{1}{2}$ hours
7 22°S, 116°E

Exercise 15.03

1 They have the same longitude, 30°E.
2 $60 \div 15 = 4$
3 10 hours
4 a 12 noon b 6 p.m. c 12 noon
d 4 a.m. e 8 p.m. f 6 a.m.
g 4 a.m.
5 6 p.m.
6 It's only 5:00 a.m.! It's too early for anyone to be in the office.
7 a Friday 2 p.m. b 1 a.m. Saturday
8 4 p.m.
9 a 10 a.m. Tuesday b 6 a.m. Wednesday

Exercise 15.04

1 a 175°E b 20
c 2230 km d 150 hours
e 6 days 6 hours
2 a They are on the same longitude. Places on the same longitude have the same time.
b Auckland is on daylight saving time and Fiji isn't.
c Beginning of October until mid-November, mid January until end of March.
d No
e 10 February, 3 p.m. (allowing for daylight saving)
3 a 22 hours b 35°
c The International Date Line is between the 2 locations.
d 1 p.m. Wednesday
4 3395 km
5 $186 per person
6 a 1148 km
b west south west (WSW)
c no time difference
7 6.3 km
8 a West b after Tonga
9 a 7:20 a.m., 4:35 p.m.
b 9 h 15 min c 3 h 40 min
10 a North b 670 km
c 45 hours d Monday 5:15 p.m.

Keyword activity

1 sphere
2 prime meridian
3 circumference
4 great

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5 Equator
6 angular
7 meridians of longitude
8 latitude

Test yourself 15

1 a north b east
c Odessa d (12°S, 75°W)
2 a 45° b 5000 km
3 a 15° b 1700 km c 11 a.m.
4 a 10 p.m. Friday, D
b 2 p.m. Friday, C
c 6 p.m. Friday, F
d 9:30 a.m. Saturday, E
e 7 a.m. Saturday, B
f 1 a.m. Saturday, A

Chapter 16

Exercise 16.01

1 a 1, 2, 3, 4, 5, 6 b $\frac{2}{6} = \frac{1}{3}$

2 a heads, tails b $\frac{1}{2}$

3 $\frac{6}{10} = 60\%$

4 2:15

5 a $\frac{10}{50} = 0.2$ b $\frac{36}{50} = 72\%$ c Blue

6 a $\frac{1}{12}$ b $\frac{2}{12} = \frac{1}{6}$
c $\frac{4}{12} = \frac{1}{3}$ d $\frac{8}{12} = \frac{2}{3}$

7 a $\frac{1}{26}$
b The letter E is used more frequently than other letters, e.g. Z.

8 a 0.5% b $\frac{50}{200} = 0.25$
c $\frac{1}{2}$ d $\frac{118}{200} = \frac{59}{100}$

9 a 6 : 20 = 3 : 10 b 8
c 6 : 20 = 3 : 10

Exercise 16.02

1 a

	1	2	3	4	5	6	7	8
Head	Head, 1	Head, 2	Head, 3	Head, 4	Head, 5	Head, 6	Head, 7	Head, 8
Tail	Tail, 1	Tail, 2	Tail, 3	Tail, 4	Tail, 5	Tail, 6	Tail, 7	Tail, 8

b i $\frac{1}{16}$ ii $\frac{9}{16}$
iii $\frac{4}{16} = \frac{1}{4}$ iv $\frac{4}{16} = \frac{1}{4}$

2 a

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

b 36
c i $\frac{5}{36}$ ii $\frac{5}{36}$
iii $\frac{3}{36} = \frac{1}{12}$ iv $\frac{1}{36}$
d total of 9 e 1.5

3 a

	0	1	2	3	4	5
1	1	2	3	4	5	6
1	1	2	3	4	5	6
3	3	4	5	6	7	8
3	3	4	5	6	7	8
4	4	5	6	7	8	9
6	6	7	8	9	10	11

b i $\frac{5}{36}$ ii $\frac{1}{36}$ iii $\frac{9}{36} = \frac{1}{4}$
c 6

4 a

	1	2	3	4	5	6
1	0	1	2	3	4	5
2	1	0	1	2	3	4
3	2	1	0	1	2	3
4	3	2	1	0	1	2
5	4	3	2	1	0	1
6	5	4	3	2	1	0

b $\frac{6}{36} = \frac{1}{6}$ **c** 0 **d** 1

5 a

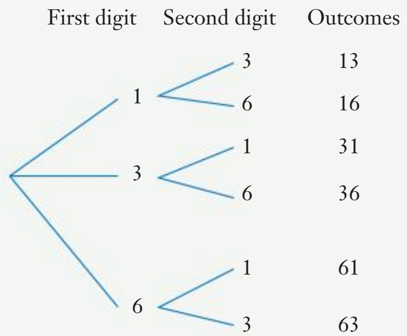
	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	2	3	4	5	6
3	3	3	3	4	5	6
4	4	4	4	4	5	6
5	5	5	5	5	5	6
6	6	6	6	6	6	6

b $\frac{5}{36}$ **c** 6 **d** 5

6 $\frac{16}{36} = \frac{4}{9}$

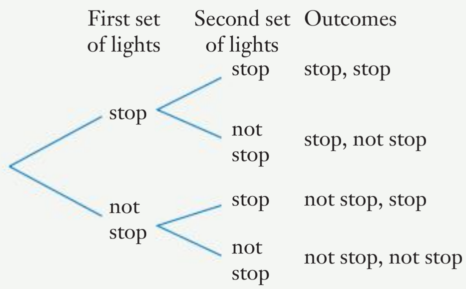
Exercise 16.03

1 a



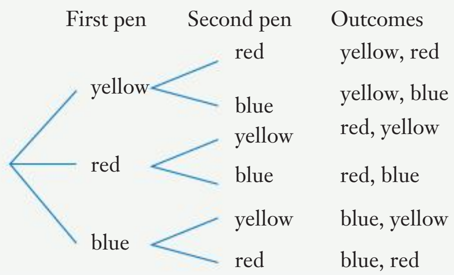
b $\frac{3}{6} = \frac{1}{2}$

2 a



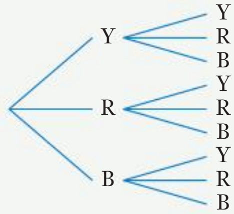
b $\frac{1}{4}$ **c** $\frac{3}{4}$

3 a



b $\frac{1}{6}$ **c** $\frac{2}{6} = \frac{1}{3}$

d



e $\frac{3}{9} = \frac{1}{3}$

4 a Teacher to check. **b** $\frac{1}{9}$

5 a Teacher to check. **b** 6 **c** $\frac{2}{6} = \frac{1}{3}$

Exercise 16.04

Teacher to discuss answers with the class.

Keyword activity

Theoretical probability is about the long term chance that something will happen. The theoretical probability of getting a head when we toss a coin is $\frac{1}{2}$. This doesn't mean that every second time we toss a coin we will get a head. Neither does it mean that when we toss a coin 100 times we will get a head 50 times. It also doesn't mean that if we get 8 tails in a row then the next toss of the coin is more likely to be a head. Coins can't remember what's happened in the past and the chance of getting a head in the future doesn't change because we've had lots of tails. What the probability of a head being $\frac{1}{2}$ does mean is that if we toss the coin thousands of times, about half of the time we'll get a head but on no individual future occasion can we know what is going to happen.

Test yourself 16

1 1, 2, 3, 4, 5, 6

2 a $\frac{1}{8}$ **b** $\frac{3}{8}$ **c** $\frac{1}{2}$ **d** 0

3 a $\frac{9}{31}$ **b** $\frac{7}{31}$ **c** $\frac{45}{62}$

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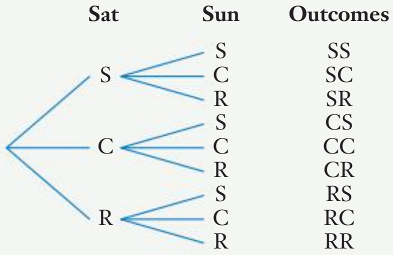
4 a

	Die 2					
Die 1	**1**	**2**	**3**	**4**	**5**	**6**
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10

24

b $\frac{1}{6}$

5 a



b **i** $\frac{1}{9}$ **ii** $\frac{1}{3}$ **iii** $\frac{8}{9}$

iv $\frac{5}{9}$ **v** $\frac{2}{9}$

6 a Teacher to check. **b** $\frac{1}{2}$

c Theoretical probability is only a long-term average. It doesn't predict what happens in each trial of a chance experiment.

7 a About 4

b Probability is only a long-term average. It can't predict accurately for small events.

c No, each child has the same chance.

Practice set 4

Section A

1 B **2** A **3** C **4** B
5 D **6** C **7** A **8** D

Section B

1 a \$54.17 **b** \$6577.25 **c** 30 months
d \$211.15 **e** \$841.15

2 a $\frac{7}{50}$ **b** $\frac{16}{25}$

c soft milk chocolates

3 a 30°S, 30°E **b** 0°, 80°W
c 43°N, 74°W

4 a They are on the same longitude (25°E).
b 1790 km

5 a

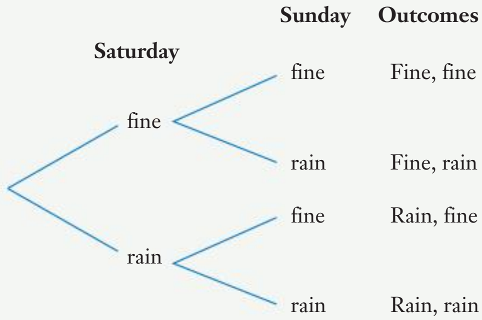
		1st die					
	×	**1**	**2**	**3**	**4**	**5**	**6**
2nd die	**1**	1	2	3	4	5	6
	2	2	4	6	8	10	12
	3	3	6	9	12	15	18
	4	4	8	12	16	20	24
	5	5	10	15	20	25	30
	6	6	12	18	24	30	36

b 18 **c** 6, 12 **d** 1, 36

e **i** $\frac{1}{9}$ **ii** $\frac{1}{18}$ **iii** $\frac{2}{9}$

6 a \$2560 **b** \$471 600

7 a



b $\frac{1}{4}$

8 a 1450 km **b** 6140 km **c** 6810 km

9

2	\$17 651	\$123.56	\$17 774.56	\$17 299.56
3	\$17 299.56	\$121.10	\$17 420.66	\$16 945.66
4	\$16 945.66	\$118.62	\$17 064.28	\$16 589.28

10 a 0.167 **b** 0.278
c Teacher to check **d** 20

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11 a

Amount borrowed $250 000		Interest rate: 4% pa Monthly fee $10	
Monthly repayment	**Time to repay the loan**	**Total repaid**	**Interest + Fees**
$1600	18 years 8 months	$357 026	$107 026
$1400	22 years 11 months	$384 979	$134 979
$1250	28 years	$418 774	$168 774
$2000	13 years 8 months	$326 098	$76 098
$2400	10 years 9 months	$309 212	$59 212

b Teacher to check

12 a 1 a.m. **b** 9 p.m.

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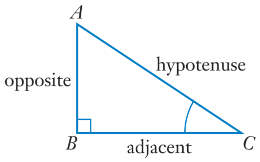
GLOSSARY AND INDEX

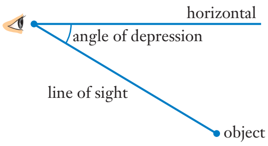
2D shapes: Flat shapes that have 2 dimensions: length and width. (p. 18)

3D shapes: Solid shapes that have 3 dimensions: length, width and depth. (p. 18)

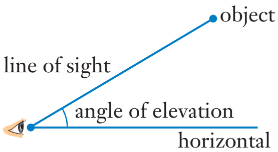
adjacent side: The side next to a given angle in a triangle leading to the right angle. In the diagram, the side *BC* is adjacent to angle *C*. (p. 52)



angle of depression: When an observer looks at an object that is lower, the angle that the eye turns down from the horizontal. (p. 58)



angle of elevation: When an observer looks at an object that is higher, the angle that the eye turns up from the horizontal. (p. 58)



appreciation: Increase in value of an item or asset over time. (p. 325)

area: The amount of surface occupied by a flat shape, measured in square units. (p. 181)

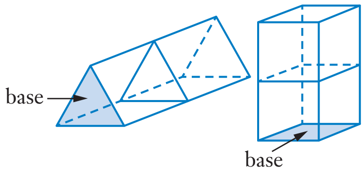
association: A statistical term referring to the relationship between 2 variables. (p. 158)

Australian Central Standard Time (ACST): The time zone for central Australia: the Northern Territory and South Australia, UTC+9.5. (p. 366)

Australian Eastern Standard Time (AEST): The time zone for eastern Australia: Queensland, NSW, the ACT, Victoria and Tasmania, UTC+10. (p. 366)

Australian Western Standard Time (AWST): The time zone for Western Australia, UTC+8. (p. 366)

base (of a prism): One of the parallel end faces of a prism. (p. 287)



bearing: A direction from one point on the Earth's surface to another. There are 2 types of bearings: **compass bearings** and **true (three-figure) bearings.** (p. 72)

bias: In statistics, an unwanted influence that stops the sample or survey from being representative of a population (p. 92)

bivariate data: Data that relates 2 variables measured in the same group, for example, height and weight of students. (p. 156)

break-even point: The point or value of sales at which a business stops making a loss and starts making a profit (p. 125)

capacity: Amount of liquid or gas that can be held by a container, usually measured in millilitres (mL) or litres (L). (p. 292)

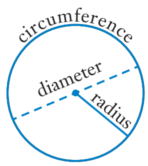
Cartesian plane: Another name for the number plane. (p. 106)

causality: When a change in one variable causes a change in a second variable. (p. 162)

census: A survey of all the items or people in a population. (pp. 4, 6)

centi-: One-hundredth $\left(\frac{1}{100}\right)$.

circumference: The perimeter of a circle. $C = \pi d$ or $C = 2\pi r$, where C is the circumference, π is pi (3.141 59 …), d is the **diameter** and r is the **radius**. (p. 174)



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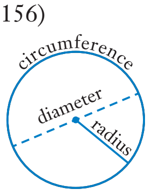
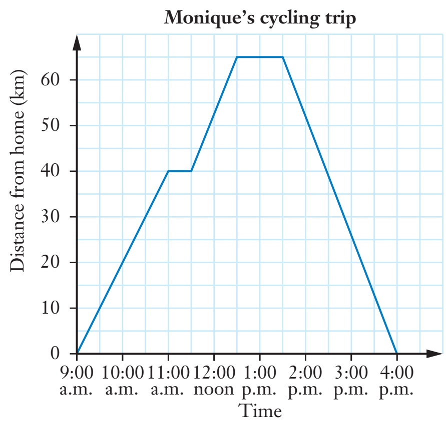
compass bearing: A bearing given as an angle either side of north and south. (p. 72) *See also* **true bearing**.
composite: Made up of more than one thing; for example, a composite shape is made up of 2 or more simpler shapes. (p. 177)
compound interest: Interest paid on the principal invested as well as on any accumulated interest. Differs from **simple interest**. (p. 321)
compounding period: How often interest is calculated when using compound interest, for example, monthly, quarterly or yearly. (p. 322)
conversion graph: A graph that is used to convert between different units, such as between metric and imperial units of measurement, or between currencies in foreign currency exchange. (p. 135)
Coordinated Universal Time (UTC): *See* **UTC.**
coordinates: A pair of numbers that give the location of a point on the number plane. (p. 106)
correlation: The strength of the relationship between 2 variables, can be positive or negative, strong or weak. (p. 263)
correlation coefficient: A value between –1 and 1 that represents the correlation between 2 variables. (p. 263)
cosine (cos): A trigonometry ratio in a right-angled triangle: $\cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}}$, where θ is an angle. (p. 63) *See also* **sine** and **tangent**.
cubic metre (m^3): A unit of volume equal to the volume of a cube of length 1 m (the size of 2 washing machines). 1 m^3 = 1 000 000 cm^3. *See also* **square metre**. (p. 284)

data: Observations or facts which, when collected, organised and evaluated, become information. (p. 6)
daylight saving: The practice of moving the time of day forward one hour during the warmer seasons, over the 6 months from October to March in some states of Australia, to take advantage of the increased hours of daylight. (p. 367)
dependent variable: In statistics, a variable whose value depends on another variable. It is represented on the vertical axis of a scatterplot. (p. 156)
diameter: The length of the interval passing through the centre of a circle and joining 2 points on the **circumference** of the circle. The diameter is double the **radius**. (p. 174)



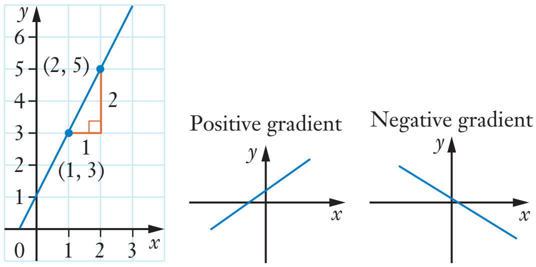
dimensions: The length, width or height of an object. (pp. 18, 187)
distance-time graph: A line graph (see next column) that describes a journey, by comparing distance on the vertical axis with time on the horizontal axis. The slope or steepness of the graph measures speed. (p. 136)



elevation view: On a plan, the view of a building from the sides, front and back. (p. 228)
experimental probability: *See* **relative frequency**.
extrapolation: A modelling situation where predictions are made outside the range of the original data. (p. 269) *See also* **interpolation**.

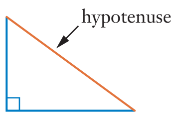
flat-rate interest: *See* **simple interest**.
formula: A rule written as an algebraic equation, using variables. For example, the formula for the area of a triangle is $A = \dfrac{1}{2}bh$. (p. 110)

gradient: The steepness of a line. For the line below, gradient $= \dfrac{2}{1} = 2$. (p. 113)



great circle: A circle on the Earth that has the same radius as the Earth. All meridians of longitude and the Equator are examples of great circles. (p. 419)
Greenwich Mean Time: *See* **UTC.**

hectare: A large unit for measuring area, equal to 10 000 square metres. (p. 181)
hypotenuse: The longest side of a right-angled triangle; the side opposite the right angle. (p. 52)



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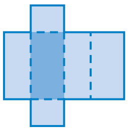
independent variable: In statistics, a variable whose value does not depend on another variable. It is represented on the horizontal axis of a scatterplot. (p. 156)
inflation: The rate at which the overall costs of goods and services are increasing. (p. 325)
instalment: *See* repayment.
International Date Line: The imaginary line that runs through the Pacific Ocean and is approximately the 180° meridian of longitude. A day is either gained or lost when this line is crossed. (p. 416)
interest: Money earned on an investment, or money paid to a financial institution for borrowing. (p. 390)
interest rate: The percentage of the investment or loan on which interest is calculated. (p. 316)
interpolation: A modelling situation where predictions are made within the range of the original data. (pp. 259, 269) *See also* **extrapolation**.

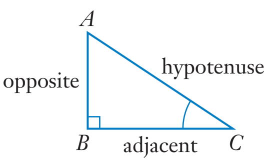
kilo-: One thousand.
kilogram: 1000 grams. (p. 282)

latitude: The angular distance north or south of the Equator of a point on the Earth's surface; parallel imaginary lines of latitude run across the Earth. (p. 416)
line of best fit: A straight line that represents a set of points on a scatter plot, obtained through experiment or observation. (p. 254)
linear function: An equation whose graph is a straight line. (p. 112)
litre: A unit for measuring capacity, equal to 1000 mL. The size of a tall carton of milk. (p. 292)
longitude: The angular distance east or west of the prime meridian of a point on the Earth's surface; lines of longitude run across the between the North and South Poles, down the Earth. (p. 416)

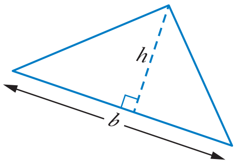
mass: A measure of size or weight in units such as grams, kilograms or tonnes. (p. 282)
mega-: One million.
milli-: One-thousandth $\left(\frac{1}{1000}\right)$.
milligram (mg): One-thousandth $\left(\frac{1}{1000}\right)$ of a gram, a unit of mass. (p. 282)
net (of a solid): The faces of a solid shape laid out flat. For example, this is the net of a rectangular prism. (p. 33)



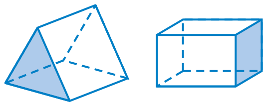
opposite side: The side facing a given angle in a right-angled triangle. In the diagram, the side *AB* is opposite to angle *C*. (p. 52)



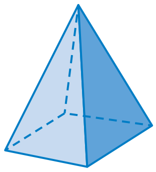
per annum (p.a.): Per year. (p. 316)
perimeter: The distance around the outside of a shape. (p. 171)
perpendicular height: The height of a shape that is at right angles (90°) to the base. For example, *h* is the perpendicular height of this triangle. (p. 184)



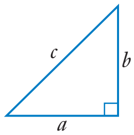
period: Amount of time (for example, month, week, year) used in interest calculations. (p. 316)
plan: Diagram of the floor of a house or building. (p. 228)
polygon: Any flat shape with straight sides, such as a rectangle or octagon. (p. 21)
population: In statistics, all of the items under investigation. (p. 4)
prime meridian or Greenwich meridian: The 0° meridian of longitude. (p. 416)
principal: The original amount of money invested or borrowed, upon which interest is calculated. (p. 316)
prism: A solid shape with flat sides and identical ends. A triangular prism and square prism are shown below. (pp. 198, 287)



probability: Another word for chance or how likely an event is to occur. Its value ranges from 0 to 1. (pp. 348, 350).
pyramid: A solid shape that has a polygon as its base, with side faces that are triangles that meet at a point called the apex. This is a diagram of a square pyramid. (p. 202).



Pythagoras' theorem: In a right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other 2 sides. $c^2 = a^2 + b^2$ (p. 48)

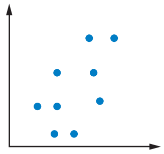


quadrilateral: A 4-sided shape. (p. 18)
questionnaire: A form with questions used to collect information, usually about people's habits or opinions. (p. 92)

radius (plural: radii): The length of the interval joining the centre of a circle to the circumference. The radius is half of the **diameter**. (p. 174)
random sample: A sample for which every member of a population has an equal chance of selection. (p. 8)
reducible interest: Interest that is charged on the amount still owing on the loan, the type of interest charged on a reducing balance loan. (p. 390)

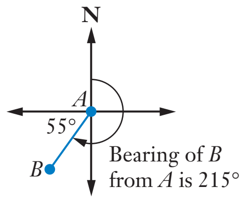
reducing balance loan: A loan where the interest charged is calculated on the balance owing on the loan after each repayment. (p. 390)
regular polygon: A polygon with all sides (and all angles) the same size. For example, a regular pentagon has 5 equal sides. (p. 21)
relative frequency: The number of times an event or score occurs, written as a fraction of the total number of events or scores. (p. 348)
repayment (or instalment): Amount paid regularly to pay off a loan. (p. 390)

sample: A group of items selected from a population to study or survey. (pp. 4, 6)
sample space: A list of all the possible outcomes in a chance situation. For example, when rolling a die, the sample space is 1, 2, 3, 4, 5, 6. (p. 438)
scale (on a map or diagram): The ratio of scaled length to actual length, for example, a scale of 1 : 500 means that lengths represented on the map or diagram are actually 500 times larger in real life. (p. 224)
scale drawing: A drawing of an object, usually smaller, whose lengths are in the same ratio as the actual lengths of the object. (p. 224)
scatterplot: A graph of points on a number plane showing a relationship between 2 variables. (p. 156)



self-selected sample: A sample in which people volunteer to be part of the sample, such as an SMS poll or a website survey, so it is not random. (p. 8)
simple interest (or **flat rate interest**): Interest earned or charged only on the original amount of money (principal) invested or borrowed, different from **compound interest**. (pp. 316, 390)
simulation: An experiment that mimics or imitates the chance behavior of a practical situation. (p. 348)
sine (sin): A trigonometry ratio in a right-angled triangle: $\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}}$, where θ is an angle. (p. 63). *See also* **cosine** and **tangent**.
square metre (m^2): A unit of area equal to the area of a square of length 1 m (the size of a large shower floor). 1 m^2 = 10 000 cm^2. (p. 181) *See also* **cubic metre**.
stratified sample: A sample consisting of a percentage of items from each 'strata' or 'layer' of a population. For example, a stratified sample from a population of 35% children and 65% adults should contain 35% children and 65% adults. (p. 8)
surface area: The total area of all the faces of a solid shape. (p. 198)
systematic sample: A sample chosen by using a set pattern, for example, choosing every tenth number in the phone book. (p. 8)

tangent (tan): A trigonometry ratio in a right-angled triangle: $\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}}$, where θ is an angle. (p.54)
See also **sine** and **cosine**.
time zone: A vertical zone on the Earth where every location within it has the same time of day. For example, central Australia (Northern Territory and South Australia) uses the Australian Central Standard Time zone (ACST). (p. 366)
tonne (t): A unit of mass equal to 1000 kilograms. (p. 282)
tree diagram: A diagram for listing all the possible outcomes of a multi-stage experiment, such as tossing 3 coins together. (p. 446)
triangular prism: A solid shape with a constant, triangular cross-section. (p. 198). *See* **prism** for diagram.
trigonometry: The study of the measurement of sides and angles in triangles. (p. 52)
true bearing (or **three-figure bearing**): Written as a 3-digit angle from north going in a clockwise direction, from 000° to 360°. (p. 77) *See also* **compass bearing**.

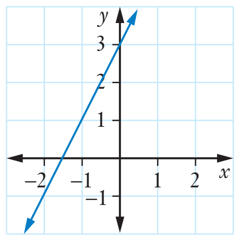


UTC (Coordinated Universal Time, or Greenwich Mean Time, GMT): Local time at the prime meridian (0° longitude) time zone, from which other time zones in the world are measured. (p. 370)

volume: The amount of space occupied by a solid, measured in cubic units. (p. 284)

***x*-axis**: The axis or number line going across (horizontally) on a number plane. (p. 106)

***y*-axis**: The axis or number line going up and down (vertically) on a number plane. (p. 106)
***y*-intercept (or vertical intercept)**: The value at which a graph cuts the *y*-axis on the number plane. For example, the vertical intercept of this graph is 3. (p. 113)



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