FOUNDATION MATHEMATICS

VCE UNITS 1&2

THIRD EDITION

BRYCE GATON
CLAIRE DELANEY
SAMANTHA HORROCKS
TRISH JELBART
MICHAEL O'CONNOR

Suitable for VET and VCAL courses

OXFORD

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Contents

Using Foundation Mathematics

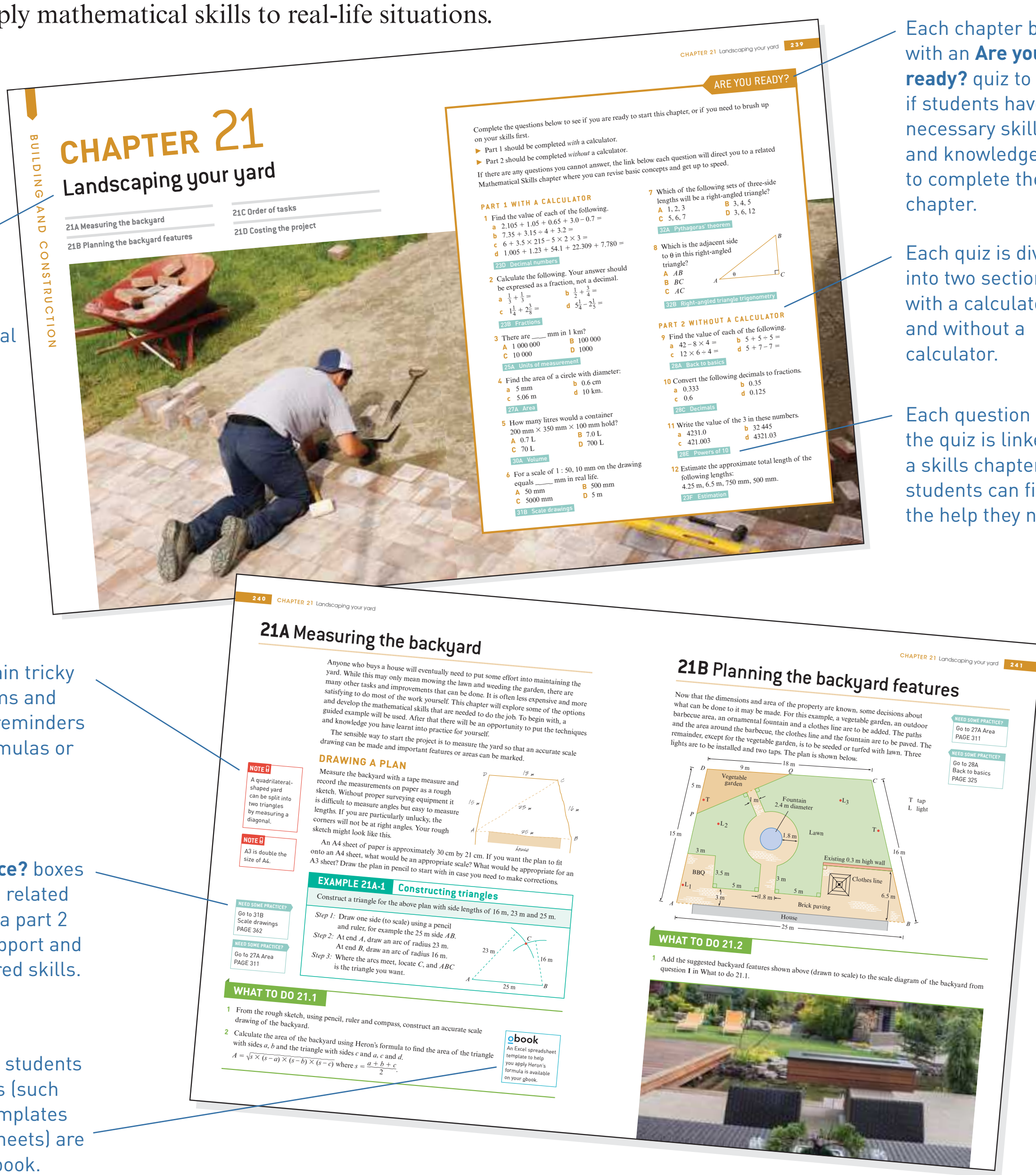
Foundation Mathematics (Third edition) has been completely revised to meet the requirements of the VCE Foundation Mathematics Study Design (2016–18). The series also meets the requirements of VCAL Numeracy and VET courses in Victoria and foundation mathematics courses in other states and territories. This edition offers a range of print, digital and blended resources geared towards practical outcomes.

Student Book

The Student Book is divided into two parts:

PART 1: APPLICATIONS

Part 1 chapters are organised according to key life skills and popular vocational areas, such as building and construction, hospitality, sport sciences, travel and tourism, and health sciences. Topics encourage students to apply mathematical skills to real-life situations.



Chapters cover a wide range of practical, vocational topics designed to engage students.

Each chapter begins with an **Are you ready?** quiz to gauge if students have the necessary skills and knowledge to complete the chapter.

Each quiz is divided into two sections: with a calculator and without a calculator.

Each question in the quiz is linked to a skills chapter so students can find the help they need.

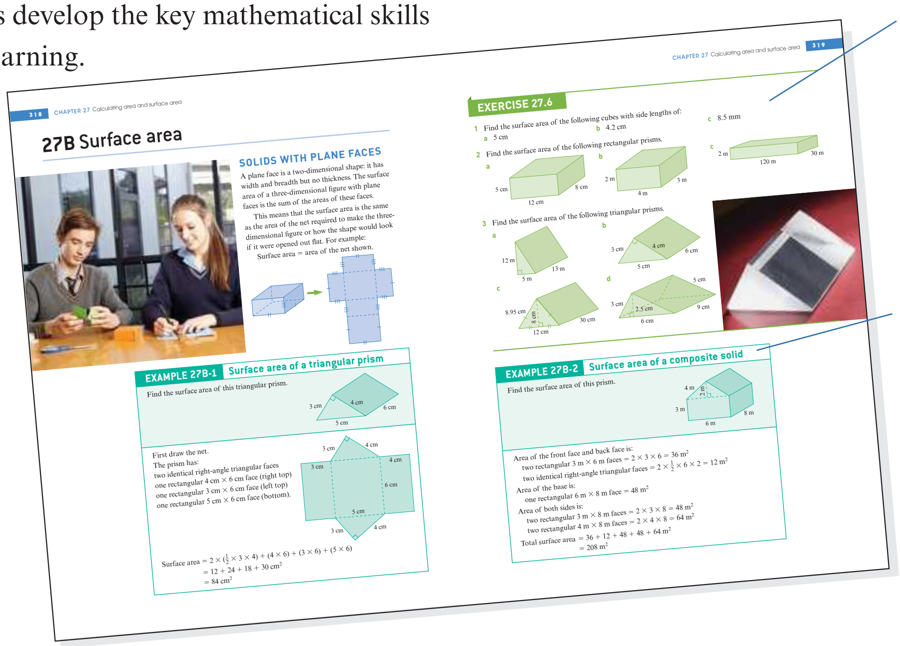
NOTE! boxes explain tricky mathematical terms and concepts or offer reminders about the best formulas or concepts to apply.

Need some practice? boxes refer students to a related skill or concept in a part 2 chapter, to find support and practise the required skills.

obook boxes show students when digital extras (such as spreadsheet templates and project worksheets) are available on the obook.

PART 2: MATHEMATICAL SKILLS

Part 2 chapters develop the key mathematical skills that support learning.



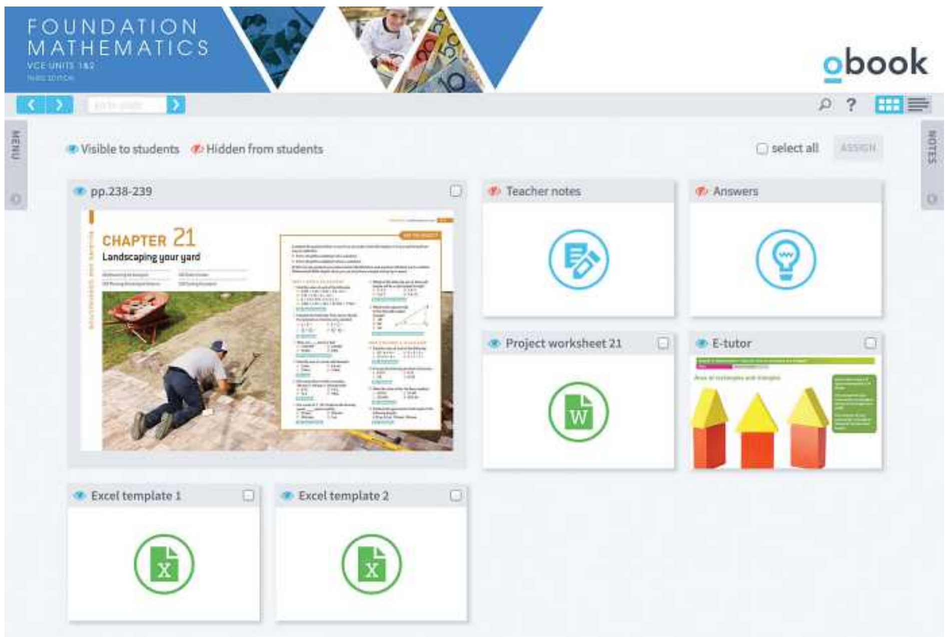
EXERCISES provide opportunities for students to practise key skills before applying them to a real-world context.

WORKED EXAMPLES clearly model the key skill that will be practised in the activity that follows.

obook assess

obook assess enables students to:

- access all of the student book content in digital format (known as obook)
- access a range of extra resources simply and easily via the DASHBOARD view, such as interactive learning modules and e-tutors, write-in project worksheets and Excel spreadsheet templates
- complete hundreds of graded, auto-correcting questions and problems that provide instant feedback (known as assess).



TEACHER obook assess

Teacher obook assess enables teachers to:

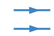
- access all of the Student Book content in digital format
- assign work to students
- create and set class tests
- monitor student progress and manage results
- access answers for all activities and worksheets
- access course planning grids
- access correlation grids for VCAL Numeracy and VET courses.

Mathematical symbols

$=$	equals
$\approx$	is approximately equal to
$\neq$	is not equal to
$+$	add
$-$	subtract
$+4$	positive number
-4	negative number
± 4	plus/minus
$4 > 2$	greater than
$2 < 4$	less than
$\div$	divide
$\times$	multiply
( )	brackets (parentheses)

$\therefore$	therefore
$\frac{2}{4}$	fraction/division
4^2	power, index
2.4	decimal point
2 : 4	ratio of 2 to 4
$\sqrt{\ }$	square root
Σ	sum of
%	per cent
/	per
A	area
l	length
b	breadth; base
w	width

h	height
s	side
P	perimeter
SA	surface area
C	circumference of a circle
r	radius of a circle
d	diameter of a circle
π	pi (approximately 3.14)
m	gradient
b	y-intercept
(x, y)	x- and y-coordinates
r	correlation coefficient
$\angle$	angle

$\triangle$	triangle
sin	sine ratio
cos	cosine ratio
tan	tangent ratio
40°	40 degrees
4′2″	4 minutes 2 seconds
	right angle
	parallel sides
	equal sides
mean	$\bar{x}$ (read 'x bar')
f	frequency
IQR	interquartile range

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PART 1

APPLICATIONS

LIFE SKILLS

CHAPTER 1
Balancing your budget

ARE YOU READY?

Complete the questions below to see if you are ready to start this chapter, or if you need to brush up on your skills first.

- Part 1 should be completed *with* a calculator.
- Part 2 should be completed *without* a calculator.

If there are any questions you cannot answer, the link below each question will direct you to a related Mathematical Skills chapter where you can revise basic concepts and get up to speed.

PART 1 WITH A CALCULATOR

1 Find the value of the following.

a 46 370 − 45 370 =
b 100 − (−10) =
c 11 + 7 + 2 − 3 − 1 − 2 =
d −2 × 5 =

23A Fundamental concepts

2 Find the value of the following.

a 12 ÷ 4 × 65 = **b** 12 ÷ 4 + 65 =
c 12 ÷ (4 + 65) = **d** 12 + 4 ÷ 4 =

23A Fundamental concepts

3 Which of the following is two hundred and forty-five thousand, two hundred and nine?

A 245 209 **B** 20 045 000 209
C 204 5290 **D** 245 290

23A Fundamental concepts

4 Find the value of the following.

a $26 + 1\frac{1}{2} =$ **b** $26 - 1\frac{1}{2} =$

23B Fractions

5 a What is $\frac{1}{6}$ of 360?
b Write the calculator button sequence you would use to find you answer for part **a**.

23C Fractions of quantities

6 Find the value of the '5' in each of the following numbers.

a 125 **b** 12.5
c 12.05 **d** 512

23D Decimal numbers

PART 2 WITHOUT A CALCULATOR

7 Round these numbers to 1 decimal place.

a 37.1485 **b** 37.1513
c 37.1499 **d** 37.1555

23D Decimal numbers

8 Convert the following to percentages.

a 75 out of 100 **b** 0.305 out of 1
c $\frac{5}{8}$ **d** $\frac{2}{3}$

24A Converting to percentages

9 What is 10% of each of these numbers?

a 10 **b** 90
c 555 **d** 1009

24B Percentage of a quantity

10 Determine the following percentages.

a 6% of \$300 **b** 20% of 11 g
c 55% of 3500 m **d** 35% of 620 L

24B Percentage of a quantity

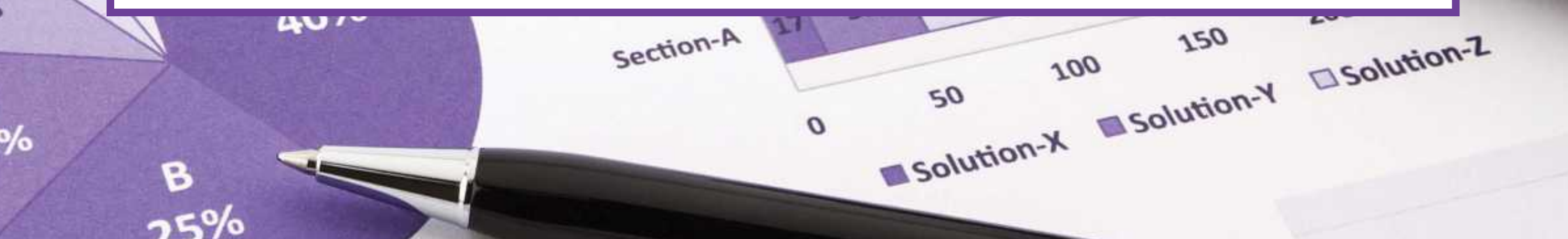
11 a What would be the new weight if 85 kg is increased by 20%?
b How much would \$100 be if it is decreased by 5%?

24C Percentage change

12 A \$150-dollar dress has been reduced to \$75. By what percentage has the price decreased?

A 12% **B** 50%
C 66.6% **D** 75%

24C Percentage change



1A Budgeting

Every year the government releases its 'budget'. Companies and other businesses are always trying to 'meet budget'. Your parents tell you 'it's not in the budget' or 'we can't afford it'. So what exactly is a budget and how are budgets worked out?

The method of preparing a budget is the same for a whole country as it is for a family. In this chapter you will prepare a personal budget to help develop the necessary skills. But first you need to get your definitions right.

NEED SOME PRACTICE?
Go to 23A Fundamental concepts PAGE 263

- A *budget* is a detailed statement of income and expenses.
- Income is also called *revenue* and expenses are also called *expenditures*.
- A budget *deficit* occurs when expenses are greater than income. When a budget is in deficit, the person, business or country goes into debt.
- A budget *surplus* occurs when income is greater than expenses. When a budget is in surplus, a person, business or country can save money for the future.
- A *balanced* budget occurs when income is the same as expenses. The goal when preparing a budget is to make it balance or end up with a surplus.

DECIDING ON A TIME FRAME

Budgets are prepared for different lengths of time depending on what they are being used for. For example, a government budget is a plan for income and spending generally over a whole year. Special events budgets cover the time from the start of collecting revenue to the end of the event. For example, a holiday budget may include the time needed to save the money for the holiday and the length of the holiday itself.

Personal budgets are usually planned around a week, a fortnight or a month. This is because most people are paid weekly, fortnightly or monthly. For the personal budget section of this chapter, a weekly time frame will be used, so all revenue and expenses must be converted into weekly amounts. For the project section of the topic you will need to decide on and justify your own time frame.

NEED SOME PRACTICE?
Go to 23B Fractions PAGE 267

Converting to weekly amounts

Whether income or expenses are being considered, the conversion into a weekly amount follows the same rules.

1 **Fortnightly** (every 2 weeks, for example a pay slip)
  - If the fortnightly amount ends in an even number of cents, divide by 2.
    For example:
    To convert a fortnightly wage of \$432 to an amount per week:
    $\$432 \div 2 = \$216$ a week
  - If the fortnightly amount ends in an odd number of cents, add 1 cent and divide the amount by 2.
    For example:
    To convert a fortnightly wage of \$330.35 to an amount per week, change the amount to \$330.36:
    $\$330.36 \div 2 = \$165.18$ a week

2 Monthly (for example rent, phone bill)

▶ Monthly payments are calendar months, not months of exactly 4 weeks. A calendar month can have between 28 and 31 days. A 4-week month always has 28 days. In order to even out the differences between calendar months, first convert to a yearly amount by multiplying by 12 (the number of months in the year), then to a weekly amount by dividing by 52 (the number of weeks in a year).

For example:

To convert a monthly rent bill of $1278 into an amount per week:

$$\$1278 \times 12 \div 52 = \$294.92 \text{ a week}$$

NOTE

Monthly rent is not the same as four times the weekly rent.

3 Bimonthly (every 2 months, for example gas bills)

▶ To convert from a bimonthly amount to a weekly amount, multiply by 6 (to get the yearly amount), then divide by 52 (the number of weeks in a year).

For example:

To convert a bimonthly gas bill of $233 into an amount per week:

$$\$233 \times 6 \div 52 = \$26.884\,6\ldots = \$26.88 \text{ a week}$$

NEED SOME PRACTICE?

Go to 23D Decimal numbers PAGE 271

4 Quarterly (four times per year, for example electricity bill, water rates)

▶ To convert from a quarterly amount to a weekly amount, multiply by 4 (to get the yearly amount), then divide by 52 (the number of weeks in a year).

For example:

To convert a quarterly electricity bill of $412 into an amount per week:

$$\$412 \times 4 \div 52 = \$31.692\,3\ldots\ldots = \$31.69 \text{ a week}$$

5 Yearly (for example council or shire rates, car registration, insurance)

▶ To convert from a yearly amount to a weekly amount, divide by 52 (the number of weeks in a year).

For example:

To convert an annual council rates bill of $1743 to an amount per week:

$$\$1743 \div 52 = \$33.519\,2\ldots = \$33.52 \text{ a week}$$

NOTE

When dividing by anything other than two:

- Round the number up only if the first unwanted digit is 5, 6, 7, 8 or 9.
- Round the number down only if the first unwanted digit is 0, 1, 2, 3 or 4.

WHAT TO DO 1.1

1 Convert these fortnightly amounts into weekly amounts.

a $234	**b** $128	**c** $146.50	**d** $211.40
e $204.75	**f** $28.43	**g** $126.57	**h** $228.91

2 Convert these monthly amounts into weekly amounts.

a $400	**b** $500	**c** $146.50	**d** $37
e $250.40	**f** $112.30	**g** $126.57	**h** $228.91

3 Convert these bimonthly amounts into weekly amounts.

a $390	**b** $234	**c** $158.60	**d** $102.40

4 Convert these quarterly amounts into weekly amounts.

a $130	**b** $520	**c** $230.50	**d** $145.90

5 Convert these yearly amounts into weekly amounts.

a $442	**b** $152	**c** $1508.56	**d** $2553.20

1B Your personal budget

Imagine you are 18 years old. You have just completed Year 12 and are preparing to either go on to further study or find a job. In order to be ready for either possibility, you need to prepare a budget based on the assumption that you will be living away from home and paying more than $100 per week in rent.

INCOME

Income is money received from other people. This can be in the form of a wage or salary, payments like Youth Allowance from the government, or an allowance from your parents. Your total income is the sum of all the money you receive from all sources. The first step in preparing a budget is to determine what your income will be.

Youth Allowance

The Youth Allowance offers financial help for people aged 16 to 24 years. Payments are made through Centrelink. You may be eligible if you are:

- 16 to 21 years old and looking for full-time work or undertaking approved activities
- 18 to 24 years old and studying full-time
- 16 and 17 years old and have completed Year 12 or equivalent, need to live away from home in order to study, or are considered independent for Youth Allowance
- 16 to 24 years old and undertaking a full-time Australian Apprenticeship.

NOTE

To keep things simple, assume that there are no other dependent children in your family.

Youth Allowance (per fortnight) as of January 2015

Family situation	Full allowance
Single, with no children, under 18 years and living at parental home	$233.60
Single, with no children, 18 years or more and living at parental home	$281.00
Single, with no children, 18 years or more and required to live away from parental home	$426.80

Two other factors that determine how much Youth Allowance you are able to claim are your parents' combined income and your own income.

In order for you to get the full Youth Allowance as of January 2015, your parents needed to earn less than $50 151 annually. For every $1 over this amount, your Youth Allowance is reduced by 20 cents per fortnight.

You are also allowed to earn some money while receiving Youth Allowance.

- You can earn up to $427 before tax per fortnight before your payment is affected.
- If you earn over $427 and up to $512 per fortnight, your payment reduces by 50 cents for each dollar you earn over $427.
- If you earn more than $512 per fortnight, your payment reduces by $42.50 plus 60 cents for each dollar you earn over $512.
- If you are single, under 18 years and living at home, your payment reduces to $0 once your income reaches the maximum of $837.00 per fortnight.

If you need to check your status, visit this website: http://www.humanservices.gov.au/customer/enablers/centrelink/youth-allowance/eligibility-for-youth-allowance

Net income

Net income is the amount of money you have to spend after tax has been taken out. Your annual income while on Youth Allowance is likely to be under the tax threshold so you will probably not need to pay tax. But, in order to get an idea of the effect of income tax on how much you have to spend, follow these steps.

Step 1: Multiply your calculated weekly income by 52 to find the annual gross income.

Step 2: Go to the Australian Tax Office Tax Calculator: http://atotaxcalculator.com.au/. Enter the gross annual income into the calculator and select the current tax year.

Step 3: The calculator will give you the weekly and annual values of your net income after tax and other deductions.

NOTE

Another chapter in this book deals with taxation.

WHAT TO DO 1.2

1 Use the table below to find the Youth Allowance entitlement per fortnight for a single 17-year-old student who is living at home and studying full-time. The maximum Youth Allowance per fortnight is $233.60. Assume that her parents' combined annual income is $50 550 and that she has a part-time job where her average earnings are $440 per fortnight.

Youth Allowance (after parental income test)		
A	Parental income (per annum)	$
B	Subtract $50 151	$
C	Divide the answer in B by 5 (or multiply by 20 cents)	$
Youth Allowance (after personal earnings)		
D	Your income (per fortnight)	$
E	If a student or apprentice earning between $427 and $512 a fortnight, subtract $427 from D	$
F	Divide the answer in E by 2 (or multiply by 50 cents)	$
G	Divide the answer in C by 26	$
Youth Allowance entitlement per fortnight		
H	Subtract amounts F and G from $233.60. This is your youth allowance per fortnight.	$

2 In groups of three, draw up a table with the following headings. Now consider the situation of a typical 17-year-old person, living at home and studying full-time. List all the income sources, the pre-tax income from each source and how often it is received. Your group must decide the typical income sources and amounts for a person of your age. Next, use the previous procedures to work out the weekly equivalent from each income source. Finally, add up the amounts in the fourth column to give you a total weekly income.

Income source	Amount	Frequency	Weekly equivalent
Youth Allowance			
Part-time job			

EXPENSES

> **NOTE**
> Water rates, car registration and insurance are also examples of committed expenses. This means a set amount must be paid.

An expense is money spent to buy goods and services. Food and household items, transport, entertainment, clothing, education, rent and bills are all expenses. Expenses can be divided into two types.

- *Committed expenses* are those a person has agreed to pay for (or is stuck with), often in advance. Committed expenses include rent and utility bills.
- *Discretionary expenses* are those that a person can decide to do without or reduce. Discretionary expenses include entertainment, clothing, types of food and household items. This means that the amount spent is a matter of choice.

Committed expenses

> **obook**
> An Excel spreadsheet template to help you calculate an electricity bill is available on your obook.

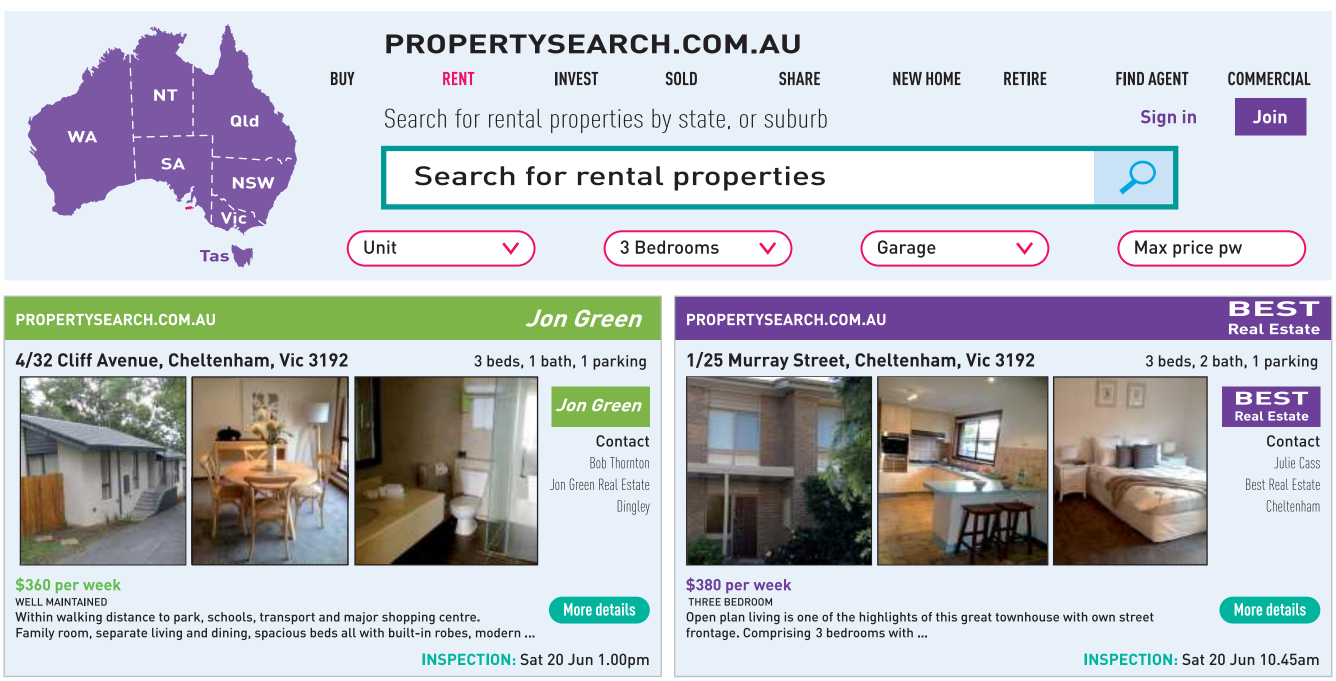
The main committed expense may be rent. You will need to find somewhere to live and you will probably need to share this expense with house mates.

Services you use in your home, such as electricity, gas, water and telecommunications, are often referred to as *utilities*. The cost of these services depends on several factors, including equipment rental fees and usage. Telecommunications covers the fixed line telephone and internet connections, although many households now only rely on their mobile phones for communication and data access. Failing to pay utility bills by the due date may result in the services being disconnected. Having these services reconnected after you have paid your bills often costs an additional reconnection fee.

Examples of comitted expense bills are provided in What to do 1.3. Questions relating to each bill will help you work out how much you can expect to pay for each service.

WHAT TO DO 1.3

1 Use a real-estate website to find a three-bedroom flat or house that you would all like to live in. Record the rent in the committed expenses table in question **5**.



2 Consider the electricity bill shown below.

- **a** What is the amount payable for this electricity bill?
- **b** What is the billing period (number of days)? Is the bill bimonthly or quarterly?
- **c** What is the rate for peak usage? Is the amount charged for peak usage correct?
- **d** Add up the energy usage to check if the total amount charged is correct.
- **e** For this bill, what is the average daily amount of electricity used?
- **f** Briefly explain what the graph indicates about electricity usage. Give reasons for your answer.
- **g** Based on this bill, what would be the average weekly electricity cost per person for a household of three? Enter this value into the committed expenses table in question **5**.

> **NOTE**
> To check the cost, multiply the energy used (number of kWh) by the rate (cost in cents per kWh).
> kWh means kilowatt hours.

FIRST LIGHT energy

Quarterly Electricity Account

LOCATION: 71 Winter Grove, SUMMERTON

Customer Number	0126485
Due Date	17 June 2015
Amount Payable	$679.15

Previous Amount Payable		**621.41**
Payment Received — Thank You		–621.41 Cr
Electricity (23/02/2015 to 23/05/2015)	**$617.41**	
Total GST Payable 10%	61.74	
Total Charges including GST		679.15
Total Amount Payable		**$679.15**

Energy Used & Costs

METER ID	THIS READING	–	LAST READING	=	ENERGY USED	x	RATE	=	COST
Peak Energy Rate — Contract (23/02/2015 to 23/05/2015)									
EDX009745/001	595.6		0.0		595.6 kWh		48.50c		$288.87
Shoulder Energy Charge — Contract (23/02/2015 to 23/05/2015)									
EDX009745/002	950.2		0.0		950.2 kWh		20.40c		$193.84
Off-Peak (Night Rate) Energy Rate — Contract (23/02/2015 to 23/05/2015)									
EDX009745/003	290.7		0.0		290.7 kWh		12.90c		$37.50
Electricity Service Availability Charge			90 days				108.00c/day		$97.20
Total Electricity Before GST					**1836.5 kWh**				**$617.41**

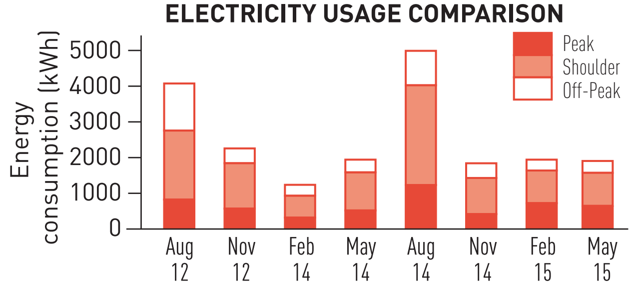
PowerSmart Home Electricity Usage Summary

Supply Period:
23 February 2015 to 23 May 2015 — 89 days

USAGE BREAKDOWN

Peak	596 kWh	32.44%
Shoulder	950 kWh	51.71%
Off-Peak	291 kWh	15.84%
TOTAL ENERGY	**1837 kWh**	

ELECTRICITY USAGE COMPARISON



3 a Consider the gas bill shown below. What is the amount of the gas bill?
b What is the billing period of the bill? Is the bill bimonthly or quarterly?
c What is the total amount of gas used in both cubic metres and megajoules?
d Is the amount charged correct? (Remember how you checked the amount for the electricity bill.)
e What is the average amount of gas used daily for this bill?
f Briefly explain what the graph indicates about gas usage for this household. Give reasons for your answer.
g Based on this bill, what would be the average weekly gas cost per person for a household of three? Enter this value into your committed expenses table in question **5**.

> **obook**
> An Excel spreadsheet template to help you calculate a natural gas bill is available on your obook.

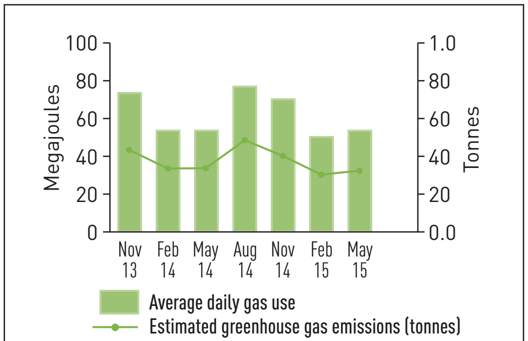
GREEN EDGE — Gas account

Your account number	0123456
Due date	**27 May 2015**
Total amount due	**$187.87**

Your account summary

Supply period	2 Feb 2015 to 3 May 2015
Previous balance	$183.47
Payment received	$183.47 cr
Balance brought forward	$0.00
New charges and credits (see details below)	
Usage and supply charges	$170.79
Total GST for new charges	$17.08
Total amount due	**$187.87**

Your overall picture



Usage

Average usage per day in this billing period	**53.30 MJ**
Same time last year	**53.33 MJ**
Average cost per day	**$2.06**
ESTIMATED GREENHOUSE GAS EMISSIONS FOR THIS BILL	**0.3 tonnes**

Your account in detail

DPI	51408525661
Reading type	Actual read on 3 May 2015 for 91 days
Tariff description	Residential Standard
Meter number	EA186971

Previous reading	Current reading	Units (m^3)	Multiplier value	Heating factor	Conversion	Usage MJ
7486	7611.01	125.01	1	38.27165	1.013730	4850

Usage and supply charges		
Gas consumption 3740 MJ @ $0.02967	$110.97	
Next 1110 MJ @ $0.01758	$19.51	
Supply charge	$40.31	
Total usage and supply charges		$170.79
Total GST for new charges		$17.08
Total amount due		**$187.87**

4 **a** Consider the water bill shown below. What is the amount owed?
b What is the billing period? Is the bill bimonthly or quarterly?
c What is the total amount of water used?
d What is the rate charged for water usage? Is this correct?
e What is the rate charged for sewage disposal? Is this correct?
f Add up the items in the bill. Check if the total is correct.
g What is the average amount of water used daily?
h Briefly explain what the graph indicates about water usage. Give reasons for your answers.
i Based on this bill, what would be the average weekly water cost per person for a household of three? Enter this value into your committed expenses table in question **5**.

PORT PHILLIP WATER

METER READING DETAILS

Meter number	Current reading	Previous reading	Consumption (kL)
ABCD12345	250	201	49

Customer Number	**22988505**
Due Date	**23 July 2015**
Amount	**$231.05**

ACCOUNT DETAILS

Usage charges (GST does not apply)	
For period 01/04/2015 to 30/06/2015 (91 days)	
Water usage	
27.93 kL @ $2.5044 per kL =	$69.95
Sewage disposal	
20.95 kL @ $1.8686 per kL =	$39.15
Total Usage Charges	**$109.10**
Service Charge Details (GST does not apply)	
Water Service Charge	$27.19
Sewerage Service Charge	$94.76
Total Service Charges	**$121.95**
Total Current Charges	**$231.05**

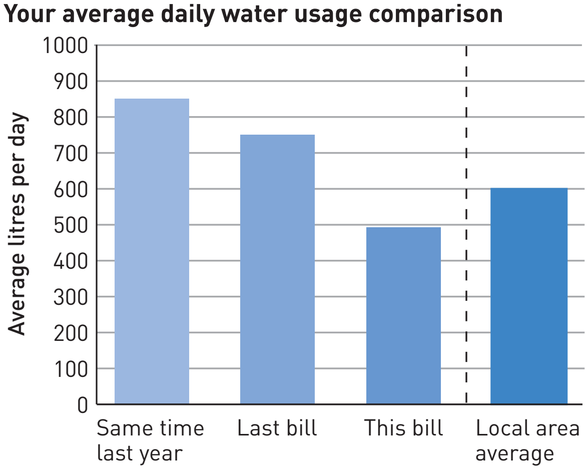
YOUR CHARGES EXPLAINED

Water Usage
Recorded by your water meter, this charge covers the amount of water used at your property. This charge is billed per kilolitre (kL) of water you use.

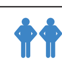
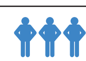
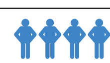
Service Charges
Your water and sewerage service charges are fixed charges for access to our water supply and sewerage systems. They also help us maintain, renew and expand these systems so we can continue to provide you with high-quality drinking water and safe sewage removal now and into the future.

NOTICEBOARD

Your average daily water usage comparison



Targets for water-efficient households

People per household	Average daily water use per person (in L)
	791
	396
	264
	198
	158
	132

5 In the same group of three from the previous exercise, draw up a table such as the one below. Add in all your committed expenses from the answers to the previous three questions. Use the rent value previously determined. Complete the table.

Committed expenses	Amount	Frequency	Weekly equivalent
Rent			
Electricity			
Gas			
Water			
	Total weekly committed expenses		

Discretionary expenses

Discretionary expenses are expenses that are not essential. This means they can be reduced or avoided completely. One way to reduce a discretionary expense might be to eat baked beans on toast for dinner instead of ordering a home delivery meal. Another way might be to wait until a movie is available for rent rather than going to see it at the cinema. Or you may choose a new mobile phone plan that costs less.

What to do 1.4 will help you establish what level of discretionary spending you are likely to have living away from home.

WHAT TO DO 1.4

1 As a class, make a list of all the items that an average 18-year-old needs to spend money on. Begin with these five main categories. Add more categories if you can think of any that you consider to be important.

- food and household
- clothing
- transport
- education/training/job seeking
- entertainment

2 In your group of three, work out the total costs per week of each of the categories.

- You can bring shopping dockets from home covering all household purchases for a week as a way of determining the household expenses.
- Think about the clothes you wear and how often you replace them. It is best to work out how much is spent on clothes in an entire year first, and then work out the weekly equivalent.

3 Fill in a table of discretionary expenses based on the one below.

Category	Amount	Frequency	Weekly equivalent
Food and household			
Clothing			
Transport			
Education/training/job seeking			
Entertainment			
Other			
	Total weekly discretionary expenses		

DEFICIT OR SURPLUS?

Which is greater, your net income or your total expenses? If your expenses are more than your income, your answer will be negative. This is called a deficit. You must reduce or eliminate some of the expenses until the answer is zero or positive. If your income is more than your expenses, your answer will be positive. This is called a surplus.

WHAT TO DO 1.5

1 When you have achieved a surplus budget, write up your findings and submit them to your teacher for assessment. Your submission must include:
   - an income table
   - a net income calculation
   - a committed expenses table
   - a statement of rules for phone usage/payment
   - a discretionary expenses table
   - a zero or surplus budget calculation
   - lists of items you consider are necessary expenses. Do not supply individual prices for the items.

PROJECT 1

obook
assess
TEACHER

BUDGETING TO MOVE OUT OF HOME

Can you afford to move out of home and get your own place?

A project worksheet for this chapter is available on the Teacher obook/assess. Ask your teacher to print it out for you. Completing this activity will help you make the right choices when budgeting for all the costs you will need to cover when renting your own place.

LIFE SKILLS

CHAPTER 2

Managing your saving and spending

ARE YOU READY?

Complete the questions below to see if you are ready to start this chapter or if you need to brush up on your skills first.

- Part 1 should be completed *with* a calculator.
- Part 2 should be completed *without* a calculator.

If there are any questions you cannot answer, the link below each question will direct you to a related Mathematical Skills chapter where you can revise basic concepts and get up to speed.

PART 1 WITH A CALCULATOR

1 Find the value of the following.

a $12 + 5 =$ **b** $9 \times 8 =$
c $2.5 \times 6 =$ **d** $20 \div 5 =$

23A Fundamental concepts

2 Find the value of the following.

a $3 + (4 - 1) - 5 =$
b $1000 + 10 \times 10 \times 73 =$
c $500 + 45 \times 1200 \div 73 =$

23A Fundamental concepts

3 Find the value of the '2' in each of the following numbers.

a 125 **b** 12.5
c 12.05 **d** 512

23D Decimal numbers

4 Find 5% of each of the following numbers.

a 10 **b** 90
c 555 **d** 1009

24B Percentage of a quantity

5 Round the following numbers correct to 2 decimal places.

a 37.1485 **b** 37.1513
c 37.1499 **d** 37.1555

23E Rounding with a calculator

6 Set your calculator to round to 3 decimal places and find the answers to the following.

a 1.0004×12.4
b $777.676\ 767 \times 4.1$
c 77.676×4.186

23E Rounding with a calculator

PART 2 WITHOUT A CALCULATOR

7 Convert the following to percentages.

a 25 out of 100 **b** 0.81 out of 1
c $\frac{1}{2}$ **d** $\frac{1}{4}$

24A Converting to percentages

8 Find the value of the following.

a $2 + 3 \times 9 =$
b $2 \times 50 \div 10 - 5 =$
c $20 \div 5 \times 4 =$
d $5 + 15 - 7 =$

28A Back to basics

9 Simplify the following fractions.

a $\frac{18}{12}$ **b** $\frac{25}{10}$
c $\frac{48}{12}$ **d** $\frac{144}{12}$

28B Fractions

10 Find 5% of the following amounts.

a \$300 **b** 11 g
c 3500 m **d** 620 L

24B Percentage of a quantity

11 Write the following fractions as decimals.

a $\frac{1}{2}$ **b** $\frac{1}{3}$
c $\frac{1}{8}$ **d** $\frac{1}{5}$

28B Fractions

12 Convert the following to percentages of 1.

a 0.05 **b** 0.2
c 0.5 **d** 0.67

28D Percentages

2A Bank accounts

Bank accounts, credit cards and mobile phones are very useful tools when used wisely. However, two of these, mobile phones and credit cards, are the source of major financial problems and bankruptcies for many young people. Often these problems occur because the fees and charges add up in unexpected ways. The purpose of this chapter is to:

- show you how to choose and manage a bank account so that you can minimise the fees you will be charged for its use
- help you understand simple interest and compound interest
- help you understand your credit card statement and calculate the true cost of an item bought on credit
- estimate the full cost of running a mobile phone before the bill arrives.

THE 100-POINT CHECK

To open a bank account in Australia, government legislation states that you must provide a sufficient number of items as proof of your identity. This is often referred to as the 100-point check. Under this check you are awarded certain points for producing particular items of identification. You must provide a combination of items that produce a total of 100 points or more.

The different items and their points are listed in the table on the facing page.

- You are only allowed to use one of the primary identification documents.
- You do not score additional points for more than one document of the same type.
- Documents in Category 3 must contain either a photograph or signature that can be matched to the signatory (that is you).
- Additional items can be awarded 25 points in Category 5.

WHAT TO DO 2.1

1 Read through the 100-point checklist shown on the facing page. This is similar to those used throughout Australia to open a bank account. List the documents you have or have access to.

2 For those documents you have or have access to, add up the allocated points per item. Do you have enough points to open a bank account?

3 If you don't have at least 100 points, list the additional documents you think it would be easiest for you to get, and find out how you would get them.

4 As a class, develop a list of places and ways to obtain additional documents.

Category	Identification method	Points value
Primary identification documents		
1	• Birth certificate/birth extract • Australian passport (current or expired within the previous 2 years, but not cancelled) • Australian Citizenship Certificate • International passport (current or expired within the previous 2 years, but not cancelled) • Other document of identity having same characteristics as a passport, such as a diplomatic or refugee document.	70
Secondary identification documents		
2	You are a known customer or have a written reference from the bank, building society or credit union you are applying to for an account, and you have held an account there for at least 12 months.	40
3	You have verification of your name from *one* of the following: • driver's licence • student card • firearm licence • identification card issued to a public employee • identification card issued to a recipient of a Commonwealth, state or territory benefit (such as a pension card).	40
4	You have verification of your name and address from *one* of the following: • a mortgage document • a current or previous employer within the last 2 years • a rating authority (for example, land rates) • a land title.	35
5	You have verification of your name by other secondary identification sources relating to the signatory: • marriage certificate (for confirming maiden name only) • credit card • Medicare card • telephone account • council rates notice • land tax.	25
6	You have verification of your name and address from *one* of the following: • electoral roll • landlord or estate agent if signatory lives in rented premises • records of a public utility.	25
7	You have verification of your name and date of birth from *one* of the following: • records of a primary, secondary or tertiary institution attended by the signatory in the last 10 years • the records of a professional or trade association of which the signatory is a member.	25

NOTE

You are only allowed to use one of the primary identification documents.

SOME KEY BANKING TERMS

Once you've opened your bank account, it's important that you understand a number of key terms that are commonly used when dealing with your bank.

WHAT TO DO 2.2

1 In groups of three, find the meaning of the commonly used banking terms listed below. Write a glossary describing these terms in your own words.

Common banking terms:

Deposit	EFTPOS
Withdrawal	Internet banking
Debit	Overdrawn account
Credit	Direct debit
ATM	BSB number
Paypass/Paywave	Overdraft
Transaction authentication number (TAN)	

2 As a class, share these descriptions and create a final list of definitions.

BANK FEES AND CHARGES

It is important to understand bank fees and make sure that you avoid unnecessary fees and are not overcharged.

WHAT TO DO 2.3

1 Form groups of three or four. Within your group, decide how often a typical person would:
  a make withdrawals from a bank account (also where and how)
  b make deposits to a bank account (also where and how)
  c pay bills by cash, EFTPOS, bankcard, internet or phone.

2 Obtain a bank account information pack from your teacher or from the internet. As a group, make a summary list of the operating fees and charges for that account. Call it an Account Summary Report.

3 Using the number of transactions your group decided on in question **1**, calculate the total cost per month of operating the bank account. Add this calculation to your group's Account Summary Report.

4 Add a note to your Account Summary Report of the costs involved if this account becomes overdrawn.

5 Suggest ways in which this cost could be reduced. Add this to the bottom of your Account Summary Report.

6 Select one member of your group as a spokesperson to give a brief summary of the account that you have analysed to the rest of the class.

2B Borrowing money

When you borrow money, you commonly pay a fee, called *interest*, that is based on:

- the amount that you have borrowed
- the length of time between borrowing the money and fully repaying it.

SIMPLE INTEREST

Interest is generally calculated as a percentage of the money you borrow (the principal amount) multiplied by the length of time you borrow the money. Percentage rates are normally expressed as a per annum (p.a.) rate. Thus a 10% p.a. rate means that you pay an additional 10% of the principal for each year that you have the borrowed money.

$$\text{Interest} = \text{principal} \times \frac{\text{interest (\%)}}{100} \times \text{time}$$

$$\text{Total owed} = \text{principal} + \text{interest}$$

If you borrow money at a simple interest rate (sometimes called flat rate), you pay a fixed amount of interest as part of your total repayments.

NOTE

The term 'per annum' comes from Latin, meaning per year.

EXAMPLE 2B-1 Simple interest

a If you borrow $1000 at a flat 10% p.a. for exactly 1 year, what interest will you pay? What will be your total repayment?

b If you borrow $1000 at a flat 10% p.a. for only 182 days, what interest will you pay? What will be your total repayment?

a $\text{Interest} = \$1000 \times \frac{10}{100} \times 1$

$= \$100$

$\text{Total repayment} = \$1000 + \$100$

$= \$1100$

b $\text{Interest} = 1000 \times \frac{10}{100} \times \frac{182}{365}$

$= \$49.86$

$\text{Total repayment} = \$1000 + \$49.86$

$= \$1049.86$

NEED SOME PRACTICE?

Go to 34A Simple interest (flat rate) PAGE 401

WHAT TO DO 2.4

1 Calculate the simple interest on the following amounts. What would be the total repayment?

a $2000 at 5% p.a. interest for 2 years

b $500 at 7.5% p.a. interest for 5 years

c $1500 at 3% p.a. interest for 10 years

d $1500 at 10% p.a. interest for 5 years

2 Taking each of the amounts above, work out the amount to be repaid (principal plus interest):

i half-yearly **ii** quarterly **iii** fortnightly.

COMPOUND INTEREST

obook
An Excel spreadsheet template to help you calculate compound interest is available on your obook.

Would it make any difference to Example 2B-1 part **a** if you were to calculate the interest owed on a weekly basis and add it to the principal at the end of each week rather than wait until the money is repaid at the end of the year?

First week: $1000 + \left(1000 \times \frac{10}{100} \times \frac{7}{365}\right) = \$1001.92$

Second week: $1001.92 + \left(1001.92 \times \frac{10}{100} \times \frac{7}{365}\right) = \$1003.84$

Third week: $1003.84 + \left(1003.84 \times \frac{10}{100} \times \frac{7}{365}\right) = \$1005.77$

This is often easier to calculate in table form.

NEED SOME PRACTICE?
Go to 34C Compound interest (by tables) PAGE 405

Balance at start	Interest owed	Balance at the end (starting balance plus interest)
$1000	$1.92	$1001.92
$1001.92	$1.92	$1003.84
$1003.84	$1.93	$1005.77

Notice that, in the third row, the interest owed has increased just a little. The more times you calculate it in this way, the more the little bits add up. If you were to continue this to the end of the 52-week year, the total owed would be $1105.06.

As you can see, calculating the interest in this way gives a slightly larger amount to be repaid at the end of the year. In practice, when paying off compound interest loans, interest for the period is calculated first, then the repayment is subtracted as shown in Example 2B-2.

NOTE
Remember to set up a table of repayments.

EXAMPLE 2B-2 Compound interest

Consider a loan of $2000 dollars, with monthly repayments of $400. The bank charges 12% interest per year. How long will it take to pay off the loan?

This means that the interest rate per month is 12% per 12 months = 1% per month.

End of month	Old balance	+ Interest	−Repayment	New balance
0	$0	$0	$0	$2000
1	$2000	1% × 2000 = $20	$400	$1620.00
2	$1620	1% × 1620 = $16.20	$400	$1236.20
3	$1236.20	1% × 1236.20 = $12.36	$400	$848.56
4	$848.56	1% × 848.56 = $8.49	$400	$457.05
5	$457.05	1% × 457.20 = $4.57	$400	$61.62
6	$61.62	1% × 61.62 = $0.62	$62.24	$0.00

The loan is paid off at the end of the sixth month.

NOTE
The last payment is less than $400. Also, the total amount paid is more than if the bank charges simple interest.

This form of interest calculation is called compound interest, and is the basis for most bank and credit card loans.

WHAT TO DO 2.5

1 You want to take out a personal loan of $1500 at 12% compound interest for 6 months to buy a new refrigerator. You will make monthly repayments.
  a What is the monthly repayment amount? (Divide the loan into 5 equal parts.)
  b What is the total amount you will pay back over the 6 months? (Set up a table.)
  c If you only borrowed $1000, and pay the same monthly amount as in part **a** above:
    i how quickly would you now pay off the loan?
    ii what would be the new total amount you have to pay back?

NEED SOME PRACTICE?
Go to 34C Compound interest (by tables) PAGE 405

2 You want to buy a car, borrowing $5000 over 2 years at a compound interest rate of 10% p.a. with monthly repayments.
  a What is the monthly repayment amount?
  b How much in total will you pay back over the 2 years?
  c If, instead, you paid half the repayment amount every fortnight:
    i would this change the total time it takes to repay the loan?
    ii would this change the total amount you pay back?

NOTE
If paying at the end of each month for 2 years there will be 23 payments.

2C Credit cards

Credit cards are issued by banks and financial institutions as a convenient way for consumers to purchase goods and services from vendors. The vendor is paid by the bank and the bank recovers the money from the cardholder. Various fees and interest charges apply. The cardholder receives a monthly statement and must make a minimum payment of 2–5% of the balance owing. If you do not pay the account in full by the due date (for example, you only make the minimum payment), any amount outstanding will be carried over to the next statement and interest charges will apply. Credit cards usually have a higher rate of interest than other consumer loans. Different rates of interest apply to purchases and cash advances for most cards.

There is often an annual fee for the use of the card, and fees may be charged for exceeding the credit limit or making late payments. Many cards offer interest-free days under certain conditions.

NOTE
A vendor is anyone who provides goods or services.

CHOOSING A CREDIT CARD

Credit cards can be grouped into three categories:

- those that offer interest-free periods with low or no fees, but do not have any 'reward points'
- those that offer interest-free periods and 'reward points', but charge a monthly, annual or per transaction fee
- those that charge interest from the day the purchase is made, but charge no extra (or few) account-keeping fees.

WHAT TO DO 2.6

1 a Work in groups of three. Choose one of the three different categories of credit cards listed on the previous page. Find a typical example of this type of credit card by investigating bank, credit union or credit card company websites.
  b Make a list of all fees and charges, including the interest rate charged, for this card.

2 Decide on the average monthly number of uses and the amount spent using the card (in dollars). Calculate the total cost of using the credit card for 1 year in the following situations.
  a You pay all the outstanding amount when it is due.
  b There is an outstanding balance of $1000 every month for the full 12 months, and you pay only your calculated average monthly amount each time the statement arrives.

3 As a group, write a short report summarising your findings.

UNDERSTANDING YOUR STATEMENT

Once a month the card owner is sent a statement. The statement details the purchases made and the balance owing. Understanding this statement is vital if you are to ensure that the charges you pay are kept to a minimum.

The statement on the facing page details the transactions for a credit card with up to 55 days interest free (between purchase and due date). Cash advances on this card will accrue interest from the day of the transaction.

WHAT TO DO 2.7

1 Use the credit card statement on the facing page to find the:
  a daily percentage rate
  b credit limit
  c due date for payment
  d minimum payment
  e balance on 22/4/15
  f meaning of CR.

2 How many days after the statement date is the minimum payment due?

3 How many interest-free days were given on:
  a Raymor, 17/04?
  b Kidcab clothing, 10/05?

4 a Multiply 0.05191 by 365 to obtain the annual percentage rate.
  b If the annual percentage rate printed on the statement was 18.95%, how is it different from your answer?

5 Find the:
  a closing balance
  b payments and credits
  c credit charge
  d available credit.

6 What percentage (to 2 decimal places) of the closing balance is the minimum payment?

CREDIT CARD STATEMENT

Statement begins	10 April 2015
Statement ends	10 May 2015
Account number	XXXX 1234 5678 9000
Overdue amount due now	$0
Amount due	$1782.35
Payment due date	4 June 2015
Minimum amount due	$53.00

Credit limlt	$5000.00
Available credit	$3217.65

Opening balance at 10 April	New transactions and charges	Payment received	Closing balance
$711.78	+$1782.35	−$711.78	$1782.35

DATE OF TRANSACTION	TRANSACTION DETAILS	AMOUNT	BALANCE
11/04/15	OPENING BALANCE	711.78	711.78
17/04/15	RAYMOR BROS, RICHMOND	200.00	911.78
20/04/15	DAVID JONES, SOUTHLAND	59.90	971.68
21/04/15	MYER, MELBOURNE	89.75	1061.43
21/04/15	MYER, MELBOURNE	24.95 CR	1036.48
21/04/15	KIDCAB CLOTHING, ST KILDA	152.80	1189.28
21/04/15	PAYMENT	711.78 CR	477.50
24/04/15	TELSTRA, MELBOURNE	89.30	566.80
26/04/15	IKEA RICHMOND EFT RICHMOND	130.00	696.80
01/05/15	BBC HARDWARE, ST KILDA	14.10	710.90
01/05/15	RAYMOR BROS PTY LTD, ESSENDON	788.00	1498.90
03/05/15	PLAYCRAFT, RICHMOND	168.00	1666.90
04/05/15	HENRYS MANCH, MELBOURNE	72.85	1739.75
10/05/15	KIDCAB CLOTHING, RICHMOND	42.60	1782.35

Interest charged on purchases	Purchase rate 18.95%	Daily rate 0.05191
Interest charged on cash advances	Cash advance rate 21.40%	Daily rate 0.05860

Minimum payment warning	If you only make minimum payments each month, you will pay more interest and it will take you longer to pay off your balance.

Checking your credit card statement

If you keep copies of your credit card transactions, it is easy to check them off against your credit card statement. It is very wise to do this. If you do not check the transactions, you may not notice any fraudulent use of your credit card or accidental double-charging for the same transaction.

CREDIT CARD TRAPS

The trap to avoid when using credit cards is having large unpaid bills over any period of time. You can make big savings if you buy at sales, but only if purchases on credit are repaid as soon as possible. Paying off credit cards as soon as possible saves more than paying off a house mortgage because of the higher interest rates involved.

If you don't have to borrow in order to buy things, credit cards can be used for convenience, so that you do not have to carry cash around with you, you have a monthly record of major purchases, and you use the suppliers' free-credit period.

Most credit cards and store cards do not charge interest if your account is paid by the due date (usually 25 days from the statement date). This means that you can obtain up to 55 days free credit if you buy at the beginning of your statement period. But, if you do not pay the bill within that period, interest is charged, usually from the date of purchase.

The trap that catches many credit card users is to have an unpaid balance over a continuing period. This removes, or greatly reduces, the chance of making the best use of the free-credit period on most cards. With the high interest rates for this form of borrowing, it is difficult to get off the credit 'merry-go-round' if you get caught.

WHAT TO DO 2.8

1 Read the section above on credit card traps and answer these questions.
   a What is the main trap to avoid when using credit cards?
   b How can you take maximum advantage of using a credit card?
   c What is the maximum number of days of interest-free credit you can get by using a credit card?

2D Mobile phones

Mobile phones are a fantastic way to keep in touch with friends and family, anywhere, any time. However, the charges for using these services (phone plans) can unexpectedly add up to very large amounts if you choose the wrong plan. This section is designed to help you identify the ways in which you might use the phone and choose the right plan.

WHAT DO YOU USE YOUR PHONE FOR?

Answer the following questions before you begin the process of selecting a phone plan.

- How many calls do you make per day or week, and during what times do you make the majority of calls?
- How long are your calls?
- How many SMS messages do you send, and at what times do you send most of them?
- Do you send and receive photographs?
- Do you use other 'premium' services (such as sports scores)?
- How long will you keep the phone?
- How much data do you need to access your emails and downloads?
- Can you change the mix of services or the plan if your usage pattern changes?

TYPES OF PHONE PLANS

There are basically four types of phone plan.

- Prepaid: The user pays an amount in advance and can make calls, send SMS, etc. until the prepaid amount runs out. This is much the same as using a pay phone with coins or a phone card.
- Low use: This plan is designed for users who generally limit themselves to receiving calls and to making short calls rarely or only in emergencies.
- Medium use: This plan is intended for users who m,.use the phone regularly, but not heavily, generally only for short phone conversations, a few SMS messages and the occasional 'premium' service.
- High use: This is the plan for users who heavily and frequently use most, if not all, of the services offered by the phone company.

WHAT TO DO 2.9

1 Work in groups of four. As a group, decide on one set of answers to the questions at the start of this section.

2 Research the set of mobile phone plans offered by a mobile phone provider. Make sure you choose a different mobile phone provider to those chosen by the other groups in your class.

3 Categorise the plans from the provider into the four types described previously. Choose one example of each type of plan from your group's chosen provider.

4 Calculate the cost of running a phone for 1 year on each of the four plans chosen, based on the usage pattern you decided on in question **1**.

5 As a group, write a report on your findings.

PROJECT 2

obook
assess
TEACHER

MAKING A SAVING

Don't let the bills stop you!

A project worksheet for this chapter is available on the Teacher obook/assess. Ask your teacher to print it out for you. Completing this activity will help you make the right choices when deciding your own financial future.

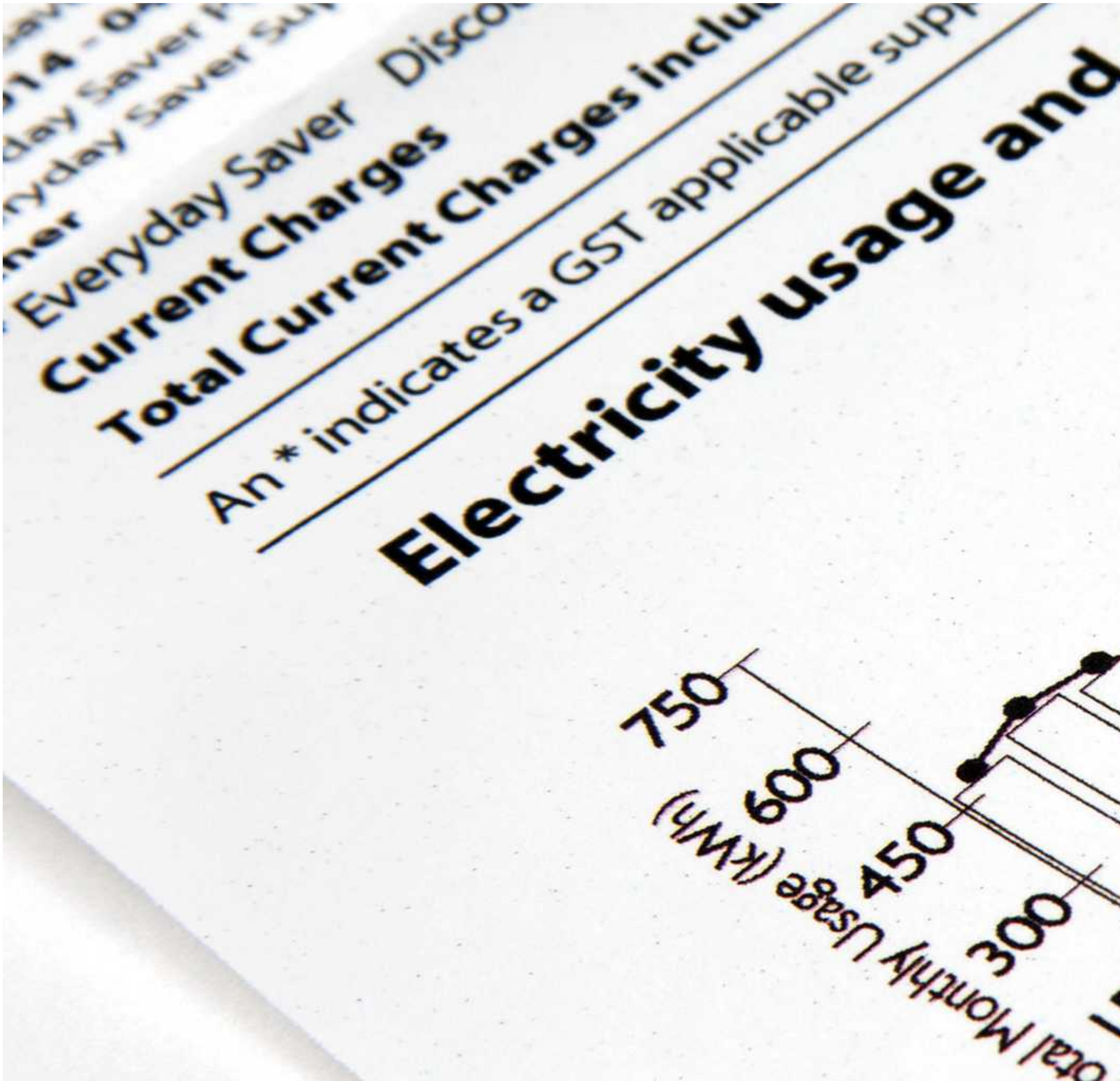
CHAPTER 3

Understanding your bills

3A Electricity bills

3C Water bills

3B Natural gas bills



ARE YOU READY?

Complete the questions below to see if you are ready to start this chapter or if you need to brush up on your skills first.

- Part 1 should be completed *with* a calculator.
- Part 2 should be completed *without* a calculator.

If there are any questions you cannot answer, the link below each question will direct you to a related Mathematical Skills chapter where you can revise basic concepts and get up to speed.

PART 1 WITH A CALCULATOR

1 Find the value of the following.

a $138 + 91 \times 2.2227 =$
b $0.75 \times 1200 \times 90 =$
c $240 \times 0.9 =$
d $200 \div 1000 =$

23A Fundamental concepts

2 If a bill has an average billing period of 91 days, approximately how many of these bills will you get in 1 year?

A 2 bills **B** 3 bills
C 4 bills **D** 6 bills

23A Fundamental concepts

3 If a bill has an average billing period of 30 days, approximately how many of these bills will you get in 1 year?

A 2 bills **B** 3 bills
C 6 bills **D** 12 bills

23A Fundamental concepts

4 Complete the following table (the first row is done for you).

Prefix	Symbol	Times base unit
kilo-	k	1000 ×
milli-	m	
	M	
centi-		

25A Units of measurement

5 The unit of power is:

A watt **B** gram
C Newton **D** ampere

25A Units of measurement

PART 2 WITHOUT A CALCULATOR

6 Find the value of the following.

a $2400 \times 24 =$
b $240 \times 7 =$
c $2400 \div 240 =$
d $260 \times 0.75 =$

28A Back to basics

7 Convert the following fractions of an hour to decimals.

a $\frac{1}{2}$ h **b** $\frac{1}{4}$ h
c $\frac{3}{4}$ h **d** $\frac{1}{6}$ h

28C Decimals

8 Round the following numbers correct to 2 decimal places.

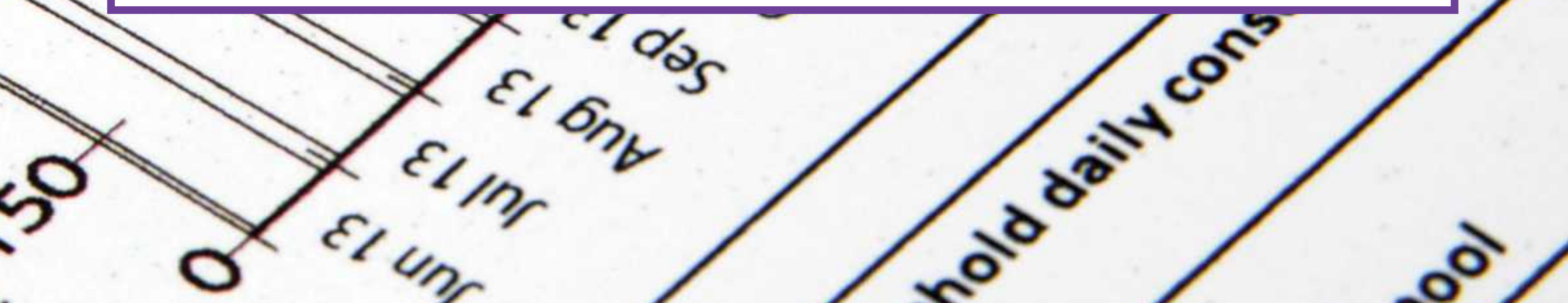
a 7.709 **b** 25.863
c 37.1499 **d** 0.017

28C Decimals

9 Add the following prices. Estimate the total cost.

\$49.99, \$30.50, \$11.50, \$132.99, \$59.99

23F Estimation



3A Electricity bills

obook
An Excel spreadsheet template to help you calculate an electricity bill is available on your obook.

Have you ever wondered how electricity charges work, or how much it costs to run your stereo? Perhaps you have just moved out of home, and the electricity, gas and water bills are too much for your budget. This chapter is about understanding the measures and charges on your amenity bills, and finding ways to reduce them.

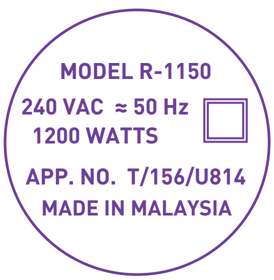
In brief, electricity suppliers generate energy that is carried in wires to your home using electrons. When you switch on your hairdryer you are taking the energy from the electrons and heating air. The same applies when you turn on the TV, except that you are now using the energy to make the picture. In fact, the supply companies do not care what you do with the energy, so long as you pay for it.

The rate at which you use this energy is measured in watts (W). The faster you use the energy, the more watts you use. For instance, a 100 W globe uses energy four times faster than a 25 W globe, and a 2500 W heater uses energy at one hundred times the rate of a 25 W globe.

APPLIANCE WATTAGE

NEED SOME PRACTICE?
Go to 25A Units of measurement PAGE 291

The wattage of an item will generally be on the manufacturer's label. Somewhere on the back, side or bottom of an electrical appliance you will find the manufacturer's label. This will give all sorts of details, but what you are looking for is the wattage. This is the number of watts the appliance uses when it is running.



Hairdryer, rated at 1200 W, 240 V

As the number of watts needed by an appliance can be quite large or small, metric prefixes are used to make it easier. The power rating label on the right shows mW. The 200 mW means 200 milliwatts, where milli means 'one-thousandth'.

Thus, 200 mW $= \frac{200}{1000}$ W $= 0.2$ W

Pastime Clock Company
Model number Q1145
POWER SOURCE 240 V
200 mW

WHAT TO DO 3.1

1 Convert the following measurements to watts.

a a 1 kW toaster
b a hot-water service labelled as 3.6 kW
c a 500 MW generator
d a 4 mW light sensor

2 Look around the classroom and make a list of all the electrical appliances in it, with their wattages. If the wattage is not listed, note the values for voltage and current; this will be dealt with in the next section.

In What to do 3.1 you may have noticed some appliances do not give the wattage, but instead only give values for voltage and current. To calculate the wattage (power) given the voltage and current, use this formula:

$$\text{Power} = \text{voltage} \times \text{current}$$

where power is in watts (W), voltage is in volts (V) and current is in amps (A).

EXAMPLE 3A-1 Finding wattage

A toaster has a label listing 240 V and 7 A. What is the toaster's wattage?

$$\begin{aligned}\text{Power} &= \text{voltage} \times \text{current}\\ &= 240 \times 7\\ &= 1680 \text{ W}\end{aligned}$$

The toaster is 1680 W.

NOTE

This course does not cover:
- 415 V appliances
- transformer packs if their output is not given in watts.

WHAT TO DO 3.2

1 Below is a set of labels from appliances that give voltage and current values instead of wattages. Find the wattage of each appliance.

Appliance	Voltage	Current	Calculated wattage
Stereo amplifier	240 V	0.9 A	
Electric kettle	240 V	9 A	
Clock radio	240 V	25 mA	

2 Find the wattage of the appliances in What to do 3.1 that only listed voltage and current values.

THE WATT-HOUR

You are now able to find out the energy usage of any household appliance. But what if you run a 2400 W fan heater for half an hour or for an hour? Have you used more energy in running it for an hour than for half an hour? Electricity companies would certainly be keen to charge you twice as much if you use it for twice as long.

The supply companies take the wattage you are using at any one time and multiply it by the number of hours you use it for. That is:

$$\text{Watt-hours} = \text{watts} \times \text{hours}$$

So the 2400 W heater that ran for 1 hour used:

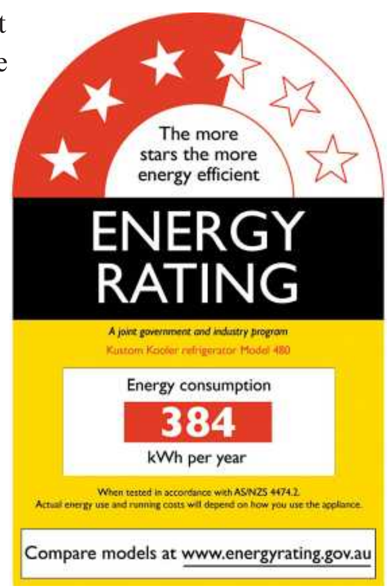
$$2400 \times 1 = 2400 \text{ watt-hours}$$

When it was run for half an hour it used:

$$2400 \times 0.5 = 1200 \text{ watt-hours}$$

As you can see, the numbers quickly get rather large, which is why the measure on your electricity bill is in kilowatt-hours.

$$\text{Kilowatt-hour} = \text{watt} \times \frac{\text{hour}}{1000}$$



WHAT TO DO 3.3

1 This is a table of appliance wattages. Work out the kilowatt-hours of energy that have been used.

Appliance	Wattage	Time run	Watt-hours	Kilowatt-hours
Heater	2400W	3 h		
Computer	250 W	30 min		
Stereo	260W	45 min		
Clock radio	6 W	24 h		
Toaster	1000 W	3 min		
Light bulb (incandescent)	100 W	6 h		
Light bulb (CFL)	15 W	6 h		
Light bulb (LED)	6 W	6 h		
Hot-water service	3.6 kW	4 h		

NOTE

Incandescent light bulbs are traditional bulbs that generate light by passing electricity through a wire filament.

Compact fluorescent lamp (CFL) bulbs are energy-efficient globes that work the same way as a standard fluorescent tube, only on a much smaller scale.

Light-emitting diode (LED) bulbs are the most energy efficient and longest-lasting bulbs.

NEED SOME PRACTICE?

Go to 23D Decimal numbers PAGE 271

TARIFFS: CHARGE FOR USE

The charge per kilowatt-hour is called the tariff. Electricity supply companies charge different tariffs, depending on how much you use and at what time you use it. Different electricity retailers in Victoria have different tariff structures. Most are based on paying different rates per kWh depending on the time of day you use the electricity. The tariff structures from one retailer are detailed below.

Tariff structure 1: for homes with off-peak water heaters

General domestic rate	All lights and power points in the home, any time	First 1020 kWh: 27.709c Remainder: 25.883c
Controlled load or off-peak (11 pm to 7 am)	Hot-water services, heat banks, underfloor heating, etc. (on only between 11 pm to 7 am)	All: 17.908c

Tariff structure 2: for homes with multi-rate metering

Peak rate (7 am to 11 pm weekdays)	All electrical items in the home	First 1020 kWh: 27.709c Remainder: 25.883c
Off-peak (11 pm to 7 am and all weekend)	All electrical items in the home	All: 18.634c

Tariff structure 3: for homes with smart meters

Peak rate (24/7)	All electrical items in the home	First 1020 kWh: 27.709c Remainder: 25.883c

Below is part of a sample electricity bill showing consumption and tariffs.

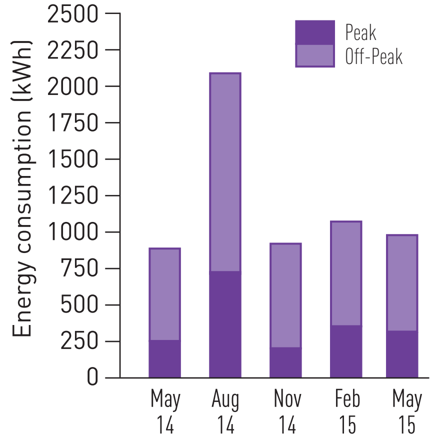
PowerSmart Home Electricity Usage Summary

Supply Period:
23 February 2015 to 23 May 2015 — 90 days

USAGE BREAKDOWN

Peak at 27.709c per kWh	290 kWh	$80.36
Off-peak at 18.634c per kWh	700 kWh	$130.44
Other charges	see over	$126.32
TOTAL	**990 kWh**	**$337.12**

ELECTRICITY USAGE COMPARISON



WHAT TO DO 3.4

1 Add a column to the table in What to do 3.3 called 'Cost per hour'. Work out the cost of running each of the appliances listed for 1 hour. Use the following rates based on tariff structure 1.
- 27.709 cents per kWh for general domestic rate
- 17.908 cents per kWh for off-peak rate for the hot-water service

COST PER HOUR, DAY, WEEK AND QUARTER

- Does it matter if you forget to turn off a light overnight?
- How much does it cost to run an electric heater each night?
- What will it roughly cost for the electricity bill in the house you have just moved into?

The answers to all these questions involve an estimation or measurement of your usage over a period of time.

The cost of running an appliance for an hour is:

$$\text{Cost per hour} = \frac{\text{appliance wattage}}{1000} \times 1\text{ (hour)} \times \text{tariff price (cents)}$$

NEED SOME PRACTICE?
Go to 23C Fractions of quantities
PAGE 269

NEED SOME PRACTICE?
Go to 23E Rounding with a calculator
PAGE 273

EXAMPLE 3A-2 Cost per hour

Using the general domestic rate of 27.709 cents per kWh for a smart meter, work out the cost per hour to run a 2400 W fan heater.

$$\begin{aligned}\text{Cost per hour} &= \frac{\text{appliance wattage}}{1000} \times 1\text{ (hour)} \times \text{tariff price (cents)}\\ &= \frac{2400}{1000} \times 1 \times 27.709\\ &= 66.5\text{c per hour}\end{aligned}$$

The cost of running an appliance for a day involves an estimation of how long the appliance is used per day. What to do 3.5 below gives a list of common household appliances, their wattages and an estimate of how long they are run each day.

Having worked out the running cost per day, determining the cost of running an appliance for a week or a quarter is easy.

- To get the weekly cost, multiply the daily cost by 7.
- To get the quarterly cost (the usual time period for electricity bills), multiply the weekly cost by 13.

Remember, appliances such as refrigerators, stoves, hot-water services and steam irons do not run all the time they are switched on. A rough rule of thumb is to say they run for about half the time they are turned on. For instance, a refrigerator is turned on all day every day, but it would only run about 12 hours a day.

WHAT TO DO 3.5

1 Work out how much these appliances cost per day to run. (The first is done for you.) Add two other appliances of your choice, and calculate the cost per day to run each. You could do this using a spreadsheet.

Appliance	Wattage	Hours run per day	kWh per day	Cost per day at 27c/kWh
Clock radio	6 W	24	$\frac{6}{1000} \times 24 = 0.144$	$0.144 \times 27.709 = 3.99$c
Hairdryer	1000 W	$\frac{1}{2}$		
Stereo	260 W	3		
Television	300 W	3		
Washing machine	360 W	1		
Kitchen light	100 W	5		
Your choice				
Your choice				

2 For the appliances listed in the previous table, calculate the weekly and quarterly running costs. If you used a spreadsheet to answer question **1**, add the columns needed.

ESTIMATING THE POSSIBLE BILL

You and two friends have just moved into a house together, having never lived away from home before. How much will the electricity bill be? There are two ways to estimate this.

- Calculate the power usage per quarter for every appliance you intend to use. This is possible, but tedious, even using a spreadsheet.
- Read the electricity meter at the beginning and end of a 1-week period, and multiply this by the 13 weeks to get a quarterly bill estimate. Add to this the quarterly service charge (about $100) and you will have a fairly good estimate of the forthcoming bill.

A typical example of a smart meter is shown on the right. To read the meter:

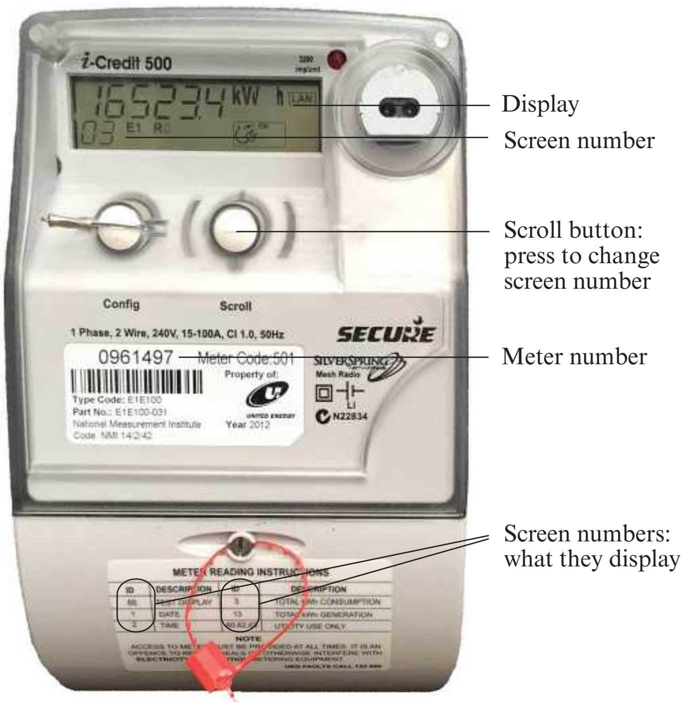
Step 1: Locate the Scroll button.

Step 2: Locate the label showing the screen numbers and what they show.

Step 3: Press the Scroll button until the screen number displays the total kWh consumed. Record this number.

Step 4: You may need to repeat this process to record the off-peak usage if you have an off-peak rate.

If you have a three-rate tariff (peak/shoulder/off-peak) and are unsure as to how to collect the usage data, you may need to refer to your electricity retailer's website or consult your teacher.



WHAT TO DO 3.6

NEED SOME PRACTICE?
Go to 25A Units of measurement PAGE 291

1 Below are readings from the electricity meters for three households at the beginning and end of a 1-week period. Estimate the quarterly bill for each household using:

- 27.709 cents per kWh peak rate
- 18.634 cents per kWh off-peak rate
- $138.82 for the quarterly service charge.

a *House 1:* Peak rate meter readings (no off-peak appliances)
Start of day 1: 00003 End of day 7: 00067

b *House 2:* Peak rate meter readings (no off-peak appliances)
Start of day 1: 52781 End of day 7: 52803

c *House 3:* (has both peak and off-peak tariffs)
Peak rate meter readings:
Start of day 1: 08267 End of day 7: 08330
Off-peak rate meter readings:
Start of day 1: 3661 End of day 7: 3689

3B Natural gas bills

obook
An Excel spreadsheet template to help you calculate a natural gas bill is available on your obook.

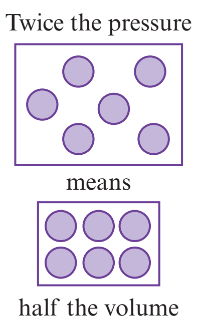
NOTE
Mega (M) means 'one million'.

Natural gas is formed from the decomposed remains of ancient buried swamps. In many cities in Australia, natural gas is piped directly from the gas fields to your home.

Gas tariffs are based on the amount of energy you get from burning the gas. The joule is the unit used for gas bills. As the joule is a very small unit, the megajoule (MJ) is used as the measure of energy on bills. The amount of gas you receive, and its quality, may vary over time. Gas companies use two factors to ensure that you pay for exactly what you get: the heating value and the pressure factor. Both are quoted on all gas bills.

Heating value

Natural gas is a natural product. This means the quality of the gas can vary slightly from time to time, resulting in the gas burning slightly hotter or cooler. A correction, called the heating value, is made for this. Poorer quality gas has a heating value of around 36, whereas higher quality gas can have a heating value of around 40.



Pressure factor

The pressure within the gas distribution system is not uniform. It can vary depending on what part of the city you live in, or as a result of changes made at the source. Increasing the pressure of a gas squeezes it into a smaller space: you still have the same number of gas molecules. It is the number of molecules that you pay for, not the volume. This is what the pressure factor is designed to correct for.

Reading a gas meter

Modern gas meters are simple to read. They use a display similar to the odometer in a car to tally up all the gas used (in cubic metres) since the meter was installed. The meter reader takes the reading at the start of the billing period and again at the end, and by subtraction finds the amount used over that time. Billing periods are usually 8 weeks.

NEED SOME PRACTICE?
Go to 30B Capacity PAGE 356

Calculating the gas usage charge

Gas usage is calculated using this formula:

Megajoules used = cubic metres × heating value × pressure factor

The charge is then calculated by multiplying the usage by the tariff.

Cost = megajoules used × appropriate tariff rate per megajoule

NOTE
On gas bills, the reading at the start of the period is called the previous reading, and the reading at the end is called the current reading.

Reading type	Actual reading on 3 May 2015 for 63 days
Tariff description	Residential Standard
Meter number	EA186971

Previous reading	Current reading	Units m^3	Heating value	Pressure factor	Usage MJ	Cost $
7486	7611.01	125.01	38.27165	1.013730	4850	

Usage and supply charges	
Gas Consumption 3740 MJ @ $0.02967	$110.97
Next 1110 MJ @ $0.01758	$19.51
Supply Charge	$40.31
Total usage and supply charges	**$170.79**

WHAT TO DO 3.7

1 The following are meter readings at the start and end of an 8-week period. Find the amount of gas used.

	Previous meter reading	Current meter reading	Cubic metres used
a	11925	12126	
b	2640	2693	
c	5483	5631	
d	6172	6210	

2 Fill in the table below to show the gas usage charges on the meters from question **1**. Assume a gas tariff of 29.67 cents per megajoule, a heating value of 38.27165 and a pressure factor of 1.01373.

	Cubic metres	Heating value	Pressure factor	Megajoules used	Usage charge (cost of gas)
a					
b					
c					
d					

3 This spreadsheet calculates the amount to be paid for gas usage. Use it to determine the cost of gas for the four meter readings.

	A	B	C	D
1	**Previous reading**	11925	Cubic metres used	=B2-B1
2	**Current reading**	12126	Megajoules used	=D1*B3*B4
3	**Heating value**	38.27165	Cost ($)	
4	**Pressure factor**	1.01373		
5	**Gas tariff (c)**	29.67		

NOTE

Compare the heating value and pressure factor used here with those for the area in which you live.

Staggered tariffs and energy conservation

Some tariffs are designed to encourage energy conservation. One way this is done is by charging more per megajoule for people who use a lot of gas. For example, one Melbourne supplier charges approximately 27.53 cents per megajoule for the first 4000 MJ and 29.67 cents per megajoule for any amount over that. The spreadsheet above can be modified for a staggered tariff situation.

	A	B	C	D
1	**Previous reading**	11925	Cubic metres used	=B2-B1
2	**Current reading**	12126	Megajoules used	=D1*B3*B4
3	**Heating value**	38.27165		
4	**Pressure factor**	1.01373		
5	**Lower gas tariff (c)**	27.53		
6	**Higher gas tariff (c)**	29.67		
7	**Cost ($)**	=IF(D2>4000,4000*B5/100+(D2-4000)*B6/100,D2*B5/100)		

3C Water bills

NOTE
Kilo (k) means 'one thousand'.

Like electricity bills, water bills are based on the amount you use. Also, like electricity, the litre is too small a unit, so your bill is based on kilolitres (kL). But, unlike electricity, the billing system is quite complex, so this section focuses on one part only: the amount of water you use, and ways to reduce it.

WATER USAGE

Fill in the table in What to do 3.8 for your household. For the shower and garden sprinkler, you need to calculate the amount of water used by multiplying the amount per minute by the number of minutes used. To do this you need to think about how long you spend in the shower and for how long you spend watering the garden.

Once you have worked out how much water your family uses in a year, think of practical ways to reduce it. For example, what is the result of shortening showers by one minute each? Exploring these possibilities is easily done by creating a simple spreadsheet.

WHAT TO DO 3.8

1 Complete the following table.

Water use	Average water used (L)	Number of times used per week	Amount used per week	per year
Shower (non-adjustable spray)	15 L/min			
Shower (adjustable spray)	7 L/min			
Bath	150 L			
Toilet full flush only style	11 L			
Toilet (full flush)	6 L			
Toilet (half flush)	3 L			
Washing machine (full)	120 L			
Washing machine (half)	60 L			
Dishwasher	35 L			
Garden sprinkler	16 L/min			
	Total water used			

WASTING WATER: A DRIPPING TAP

How many times have you been told to 'Turn the tap off'? Does it really matter if a little bit of water goes down the drain? Try the experiment in What to do 3.9 to find out.

WHAT TO DO 3.9

1 Use a beaker or measuring jug with volumes printed on the side of it. In the science laboratory or your sink at home, turn the tap on so that it drips in each of the four ways described in the table.

Drops per minute (approx.)	Time (min)	Volume (L)	Volume per day (L)	Volume per year (L)
30		0.1		
60		0.1		
120		0.1		
Continuous dribble		0.1		

- a Time how long it takes to get 100 mL (0.1 L) of water into the beaker in each case.
- b Calculate how much water is wasted per day in each case.
- c Calculate how much water is wasted per year in each case.
- d How many taps do you have in your home?
- e How much water might you be losing in a year if you let all the taps drip?
- f If the average home uses 240 000 L of water per year, what would the percentage increase be for a household that left its shower dripping at 30 drops per minute?

PROJECT 3

obook
assess
TEACHER

BILL MANAGEMENT

Don't let your bills control you!

A project worksheet for this chapter is available on the Teacher obook/assess. Ask your teacher to print it out for you. Completing this activity will help you make the right choices when managing your bills for your home or your business.

CHAPTER 4
Household shopping

ARE YOU READY?

Complete the questions below to see if you are ready to start this chapter or if you need to brush up on your skills first.

- Part 1 should be completed *with* a calculator.
- Part 2 should be completed *without* a calculator.

If there are any questions you cannot answer, the link below each question will direct you to a related Mathematical Skills chapter where you can revise basic concepts and get up to speed.

PART 1 WITH A CALCULATOR

1 Find the value of the following.

a $3.85 + 4 \times 2.50 =$

b $0.1 \times 50 \times 9 =$

c $240 \times 0.9 =$

d $7.50 \div 700 =$

23A Fundamental concepts

2 Find the value of the following.

a 10% of \$1.20

b 33% of \$55

c $\frac{1}{3}$ of \$120

d 25% more than 1 kg

24B Percentage of a quantity

3 The base unit of mass is:

A watt **B** gram

C Newton **D** kilogram

25A Units of measurement

4 How many grams are there in the following?

a 0.5 kg **b** 0.25 kg

c 1.5 kg **d** 0.025 kg

25A Units of measurement

5 Complete the following table (the first row is done for you).

Prefix	Symbol	Times base unit
milli-	mm	$\frac{1}{1000} \times$
kilo-		
	c	

25A Units of measurement

PART 2 WITHOUT A CALCULATOR

6 Find the value of the following.

a $2400 \div 24 =$

b $250 \times 4 =$

c $2400 \div 240 + 60 =$

d $200 \times 0.5 =$

28A Back to basics

7 Convert the following fractions to decimals.

a $\frac{1}{3}$ **b** $\frac{1}{4}$

c $\frac{1}{5}$ **d** $\frac{1}{6}$

28B Fractions

8 Round the following numbers correct to 2 decimal places.

a 15.203 **b** 1.234 567

c 2.4567 **d** 0.222 555

28C Decimals

9 Round the following amounts to the nearest five cents.

a \$1.99 **b** 56 cents

c 4×99 cents **d** 5 times \$2.10

23D Decimal numbers

10 Find each of the following.

a What would be the new weight if 85 kg is increased by 10%?

b How much would you have if \$50 is decreased by 10%?

c A \$150 dress is reduced to \$120. By what percentage has the price decreased?

24C Percentage change

4A Making a shopping list

From time to time everyone needs to shop for food, household products for the bathroom and laundry, electrical goods, soft furnishings, plus many more items.

Have you ever thought about the shopping that you do at the supermarket?

- What would an average weekly grocery list contain and what would it cost to buy?
- What is the best quantity of food to buy, without running back to the shop each day, or having food go stale?
- What are the ways to reduce the weekly food bill yet still maintain a healthy diet?

This chapter deals with issues such as these, as well as a range of other important considerations.

A shopping list is essential when buying groceries at the supermarket. It is very easy to forget some of the items that you wish to buy, and you do not want to double up on what is already in the cupboard. So, it is best to write a list of items that you can add to as you think of them, usually when you notice that quantities are low.

Cheese
Full-cream milk
Wholemeal bread
T-bone steak
Chicken sausages
Toothpaste
Canned peaches
Frozen peas
Fruit juice
Tomatoes
Potatoes
Bananas
Grapes
Detergent
Coffee
Raw sugar

ITEMS TO INCLUDE

Each household has a different set of preferences for grocery items. Modern householders sometimes use a spreadsheet for their shopping lists, with items listed in various categories.

WHAT TO DO 4.1

1 For a 1-week period, make a list of all items bought by your household. Quantities and brand names are not required. Use the following categories:

Food	Laundry	Bathroom	Other

2 Ask whoever does the shopping for your household what other items are kept in your house that fit these categories, but were not bought in that week. Make a separate list of these using the same headings as in question **1**.

3 Many people now purchase grocery items via the internet. A shopping list is provided that includes brand names, sizes (mL or g) and prices. A fee is charged for the service of purchasing your items and delivering them to you. Locate such sites on the internet and explore their possibilities.

SETTING UP HOUSE

Suppose you have just moved out from home and you are to house share with two or three other people. The group decides to equally share the costs of all food, cleaning and laundry needs. For all of you it is the first time away from home, so you will be setting up your basic food items and things such as cleaning supplies from scratch.

For current food and grocery pricing, you could start with: http://www2.woolworthsonline.com.au/ or http://shop.coles.com.au/online/national/

WHAT TO DO 4.2

1 In groups of three or four, set up a shopping list for your household. Use the categories previously described.

2 Either visit a local supermarket or use an internet shopping service to find prices for each item on your list, for both large- and small-sized quantities. Also find comparable generic items and record their prices as well.

3 Create a four-column table like the one below (possibly done on a spreadsheet) and fill in the groups. Complete the purchasing list with sizes and prices.

Item	Packet size (g, kg, L, etc.)	Price for brand name item	Price for generic item
Food			
	small		
	large		
	small		
	large		
	…		
Laundry			
	small		
	large		
	…		

4B Comparing prices

When buying loose fruit and vegetables, comparing prices is easy. The prices are usually given as price per kilogram or price per item.

NEED SOME PRACTICE?
Go to 25A Units of measurement
PAGE 291

For certain supermarkets, the total price of a product is displayed as a unit price on shelf labels. But this was not always the case, so shopping for groceries when the food comes in different-sized packets or containers could be hard. For products sold by weight, you need to find the price per unit of weight; that is, c/g or $/kg.

NEED SOME PRACTICE?
Go to 23B Fractions
PAGE 267

NEED SOME PRACTICE?
Go to 23D Decimal numbers
PAGE 271

EXAMPLE 4B-1 Deciding value by weight

If you have a 100 g tin of tuna at $1.89 and a 200 g tin of the same brand on special at $2.89, which is the better value?

100 g tin is $\frac{\$1.89}{100\text{ g}} = 1.89\text{c/g}$

200 g tin is $\frac{\$2.89}{200}\text{ g} = 1.45\text{c/g}$

If would be better to buy the 200 g tin on special, rather than two 100 g tins.

WHAT TO DO 4.3

1 In the following cases, work out a price per unit of weight and then decide which is the better buy.
  - a White-O laundry powder costs $6.98 for a 1.25 kg packet and $9.98 for a 2 kg packet.
  - b A particular brand of canned peaches has an 825 g can that costs $4.70 and a 415 g can that costs $3.34.
  - c A 500 g block of cheese costs $8.69 and a 200 g block of the same brand cheese costs $5.99.
  - d Baked beans cost 70 cents for a 130 g can, $2 for a 220 g can and $3.78 for a 680 g can.
  - e One brand of cream-assorted biscuits costs $7.30 for a 500 g packet and another brand costs $6.98 for a 450 g packet.

Some items, such as drinks and ice-cream, are sold in containers that display a clear statement of their capacities in litres (L) or millilitres (mL). For products sold in containers of given capacity, you need to find the price per capacity (c/mL or $/L).

EXAMPLE 4B-2 Deciding value by capacity

A certain brand of soy sauce comes in 250 mL bottles at $7.50 and in 400 mL bottles at $11.20. Which is the better buy?

The 250 mL bottle is $\frac{750\text{c}}{250\text{ mL}} = 3.0\text{c/mL}$

The 400 mL bottle is $\frac{1120\text{c}}{400\text{ mL}} = 2.8\text{c/mL}$

The larger bottle is the better buy on this occasion.

WHAT TO DO 4.4

1 In the following cases, work out a price per unit of capacity and then decide which is the better buy.
  a A 200 mL bottle of taco sauce costs $4.58 and a 600 mL bottle costs $9.24.
  b A 300 mL bottle of tomato sauce costs $3.80 and a 980 mL bottle costs $11.44.
  c A 500 mL bottle of soft drink costs $3.10 and a 375 mL bottle costs $2.60.
  d A 1.25 L bottle of a cola drink costs $2.56 and a 3 L bottle costs $6.78.

How do you compare the prices of two items that come in containers marked by one manufacturer in mL and by the other manufacturer in litre (L)?

To do this, you first need to convert the items to the same units.

- To work in mL, convert litres to mL.
- To work in grams, convert kilograms to grams.

EXAMPLE 4B-3 Comparing mixed units

Compare values for coffee that is sold at $6.18 for a 200 g jar and $23.12 for a 2 kg tin. Which is the better buy?

Step 1: First decide on which units to use.
In this case, cents per gram.

Step 2: A 200 g jar is $\frac{618\text{c}}{200\text{ g}} = 3.09\text{c/g}$

A 2 kg tin $= 2 \times 1000\text{ g} = 2000\text{ g}$

A 2000 g tin is $\frac{2312\text{c}}{2000\text{ g}} = 1.156\text{c/g}$

The large tin is a far cheaper buy.

NOTE

$1\text{ kg} = 1000\text{ g}$

$1\text{ g} = \frac{1}{1000}\text{ kg}$

$1\text{ L} = 1000\text{ mL}$

$1\text{ mL} = \frac{1}{1000}\text{ L}$

WHAT TO DO 4.5

1 A common brand of tomato sauce comes in 1 L bottles costing $2.88, and the generic equivalent comes in 980 mL bottles at $2.65. Compare the two buys in:
  a cents per mL b cents per L c $ per L.

2 A 1 kg block of tasty cheese costs $13.19 and a 250 g block costs $4.59. Compare the cost in:
  a cents per g b cents per kg c $ per kg.

3 A soy drink comes in containers of 250 mL at $1.26, 500 mL at $1.26 or 1 L at $2.13. Compare the prices in:
  a cents per mL b cents per L c $ per L.

4C Shopping sensibly

You notice that a 1.5 kg packet of breakfast cereal has a price comparison in cents per gram that makes it the best buy. It is also on special for 20% off its normal price. But, its use-by date is only 18 days away.

Can you consume it before the use-by date so that none of it is wasted?

Buying in large amounts (called bulk buying) is a great way to save money, but only if waste is minimised.

CONSUMPTION RATES AND TIMES

If, on average, you eat 50 g of breakfast cereal each day, your consumption rate is 50 g/day. Your consumption rate for milk could be 650 mL/day.

Consumption time, in days, can be determined using the formula:

$$\text{Consumption time} = \frac{\text{quantity bought}}{\text{daily consumption rate}}$$

EXAMPLE 4C-1 Comparing consumption times

For the 1.5-kg packet of breakfast cereal with a use-by date that is 18 days away, find the consumption times if:

a you alone eat it at a rate of 50 g/day
b you and your housemates eat it at a rate of 125 g/day.

a Consumption time is $\frac{1500 \text{ g}}{50 \text{ g/day}} = 30$ days

b Consumption time is $\frac{1500 \text{ g}}{125 \text{ g/day}} = 12$ days

WHAT TO DO 4.6

1 Over a period of 2 weeks, keep accurate records of your consumption of:
a breakfast cereal b bread (in slices) c milk

NOTE
A bread roll will count as 3 slices of bread.

2 Find your consumption rate per day for each item in question **1**.

3 How long would it take to consume:
a a 1 kg packet of breakfast cereal if the household consumes it at an average rate of 125 g per day?
b a 2 kg bag of sugar if the household consumes it at an average rate of 25 g per day?
c an 11 L bottle of spring water if it is consumed at a rate of 330 mL per day?

4 How long would it take to use a 10 kg bag of flour if the flour is used at a rate of 80 g per week?

CONSUMPTION GRAPHS

Graphs are an easy way to compare the rate of consumption.

NEED SOME PRACTICE?
Go to 35D
Linear modelling
PAGE 419

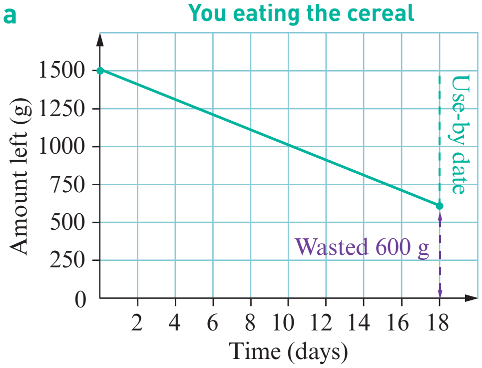
EXAMPLE 4C-2 Consumption graphs

For the 1.5 kg packet of breakfast cereal with a use-by date that is 18 days away, draw consumption graphs if:a

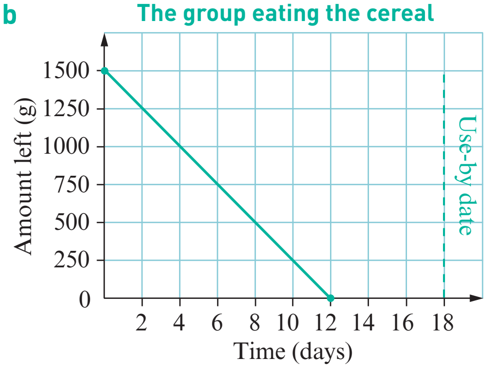
a you alone eat it at a rate of 50 g/day

b you and your housemates eat it at a rate of 125 g/day.

a



b



If you are rigid about the use-by date, you would throw out 600 g if you were eating the cereal alone, thus your savings are not what you first thought they would be.

$$\frac{600}{1500} \times 100\% = 40\% \text{ wasted}$$

WHAT TO DO 4.7

1 One dozen eggs have a use-by date that is 2 weeks from now. If you use three eggs a week, is this a good buy?

2 A 770 g jar of mayonnaise has a use-by date that is 8 months from now. Assuming that, in your household of two, each person uses four servings a week with an average serving size of 3 g, would you use all the mayonnaise in time?

3 A 4 L bottle of tomato sauce has a use-by date that is 6 months from now.

- **a** If a family uses approximately six 20 mL serves per week, would they use it all in time?
- **b** Use a graph to illustrate part **a**, with time in weeks on the horizontal axis.

4 A 1 kg tin of coffee has a use-by date of exactly 12 months ahead and costs \$33.45. The average consumption of coffee per cup is 1 g. Draw a line graph, with time on the horizontal axis (weeks), for these situations.

- **a** One light drinker of coffee consumes 4 cups a week.
- **b** Two medium drinkers of coffee each have 2 cups per day.
- **c** Three heavy drinkers of coffee each have 4 cups per day.

ESTIMATING HOW MUCH YOU EAT OR DRINK

It is often interesting to estimate how much you eat and drink over an extended period, such as a week, month, year or decade.

WHAT TO DO 4.8

1 How much liquid does a person drink in a year?
   a Put a mark near the top of a glass and fill it with water to this height. Measure the volume of water using a kitchen measuring jug or a measuring cylinder. If you drink hot drinks such as tea, coffee or chocolate, do the same for the cup you wish to use.
   b Over a 5-day period, record the number of glasses or cups you drink.
   c Total the quantity consumed and calculate the daily consumption rate.
   d Estimate the amount you would drink for 1 year.
2 Choose a food item, such as breakfast cereal. Over 1 week, weigh how much you put into your bowl each day using kitchen scales.
   a Calculate your average daily consumption rate.
   b Estimate your consumption for a year.
   c Find out the cost of the cereal you eat and estimate the cost of eating it for 1 year.
3 Many products give the weight or volume of the suggested serving size on the side or the back of the packet. Examine several such packets and record the relevant details. Choose one item you would consume regularly and estimate what quantity you would consume, and how much it would cost over 1 year.

FINAL SELECTION AND COSTING

Now that you have a better idea of quantities, brands and product pricing, you can put together and submit a report on the items you want for your imaginary household.

WHAT TO DO 4.9

1 In your original groups from What to do 4.1, use your shopping list and price data to decide what quantities and brands of products you wish to have. Set up a table with columns listing:
   - items you have chosen to buy
   - sizes or amounts for each item that you will buy
   - prices of each item
   - use-by dates for each item (where applicable).
2 Calculate the total cost of all items.
3 Prepare a report on why you have chosen particular sizes and brands. A spreadsheet would be useful here.

4D Shopping trends survey

The purpose of this survey is to design, carry out and analyse what people do when shopping. The survey is to be given only to adults, chosen at random, who shop for groceries, electrical goods, clothing, cars, and so on. Consider these questions.

1 When you shop for food, do you keep a balanced diet in mind? Yes No
2 When you buy foods, do you consider which are perishable and non-perishable? Yes No
3 Do you buy extra non-perishable items when they are on special? Yes No
4 Do you find that frozen vegetables are cheaper than fresh ones? Yes No
5 How often do you use frozen vegetables? Never 20% 40% 60% 80% Always
6 Savings can be made by buying fruit and vegetables in season when they are in plentiful supply and are cheapest. Do you take advantage of this? Yes No
7 The price of meat depends largely on supply. Do you buy different types of meat (such as lamb, beef, pork) more often when the prices are low? Yes No
8 Do you freeze meat? Yes No
9 Do you travel considerable distances to buy ‘specials’? Yes No
10 If the answer to question **9** is Yes, have you considered fuel and time costs? Yes No
11 When buying items such as electrical goods and cars, do you ‘haggle’ (barter) over price? Yes No
12 If the answer to question **11** is Yes, by how much do you haggle below the asking price? %

WHAT TO DO 4.10

1 In groups discuss these ideas for a survey.
- What other features should be included?
- What other questions should be included and how should they be written?
- How would you randomly choose the people you need to include in the survey?
- Who will gather the data?
- Who will analyse the data?
- Who will write up the summary of results?

2 Compile a survey document and carry out the actual survey. Analyse the data and write the report.

PROJECT 4

obook
assess
TEACHER

SHOPPING SAVVY

Getting a bargain!

A project worksheet for this chapter is available on the Teacher obook/assess. Ask your teacher to print it out for you. Completing this activity will help you make the right choices when shopping either for your business or for your own home.

CHAPTER 5
Measuring drug dosages

ARE YOU READY?

Complete the questions below to see if you are ready to start this chapter or if you need to brush up on your skills first.

- Part 1 should be completed *with* a calculator.
- Part 2 should be completed *without* a calculator.

If there are any questions you cannot answer, the link below each question will direct you to a related Mathematical Skills chapter where you can revise basic concepts and get up to speed.

PART 1 WITH A CALCULATOR

1 Find the value of the following.

a $3 \times \frac{1}{2} =$

b $0.001 \times 1000 =$

c $240 \times 0.9 =$

d $1000 \div \frac{1}{2} =$

23A Fundamental concepts

2 Find half of each of the following numbers.

a 750 **b** 2004

c 1088 **d** 0.78

23C Fractions of quantities

3 The basic metric unit of capacity is:

A fluid ounce **B** milligram

C gram **D** litre

25A Units of measurement

4 Complete the following table (the first row is done for you).

Prefix	Symbol	Times base unit
kilo-	k	1000 ×
milli-	m	
micro-		

25A Units of measurement

5 Convert the following.

a 1 g = ___ mg

b 100 mg = ___ g

c 1 L = ___ mL

25C Conversion of units

6 Find 25% of each of the following numbers.

a 750 **b** 2200

c 1088 **d** 0.78

24B Percentage of a quantity

PART 2 WITHOUT A CALCULATOR

7 Find the value of the following.

a $20 \times 10 =$ **b** $250 \times 5 - 7 =$

c $(8 - 5) \times 3 =$ **d** $2300 \div 23 =$

28A Back to basics

8 Convert the following fractions to decimals.

a $\frac{1}{2}$ **b** $\frac{2}{3}$

c $\frac{1}{5}$ **d** $\frac{1}{25}$

28B Fractions

9 Convert the following decimals to fractions.

a 0.3333 **b** 0.75

c 0.25 **d** 0.125

28C Decimals

10 Round the following numbers correct to 3 decimal places.

a 2.709 49 **b** 0.8636

c 370.1499 **d** 10.6777

23D Decimal numbers

11 500 mg is ____ g?

A 500 **B** 50

C 5 **D** 0.5

25A Units of measurement

5A Tablets and capsules

Have you ever taken aspirin, paracetamol or antibiotics? Chances are that you have. You may often think that taking medications is natural and safe. However, you need to take the correct amount: too little of the medication and it will not have its desired effect; too much and there could be serious side effects.

This chapter is intended for people who may have to care for children, whether as a parent, older sibling or childcare worker. It is important, as measuring and administering drugs to children safely means getting doses right every time. To ensure the amount of medication is correct, it is necessary to be able to accurately measure and calculate using weight and volumes.

Many medications are sold in tablet or capsule form with dosages counted in either half or whole tablets. For teenagers and adults, this is the usual method of taking medicine. In this section you look at several types of 'over the counter' solid medicines.

To perform the investigations in this chapter, you need to have access to lists of ingredients of common medicines (either from the side of the packet or from the internet). You will also need several different forms of medicine cups and scales that measure in gram or half gram divisions. An active ingredient is the part of a tablet that is the actual medicine. Other ingredients hold the tablet together or give it a certain taste. The unit of measurement is usually mg for milligrams and µg (often written as mcg) for micrograms.

WHAT TO DO 5.1

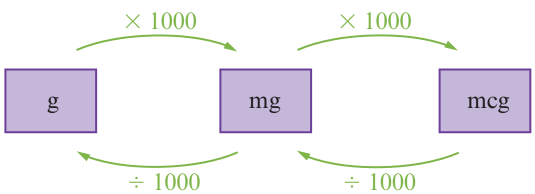
Complete the table below for each of the medicines listed.

Generic name	Brand names (3 or 4 each)	Active ingredient(s)	Amount of active ingredient per tablet	Unit of measurement
Paracetamol		Paracetamol		
Aspirin				
Ibuprofen				
Antacid				
Antihistamine (allergy)				
Vitamin (not multivitamin)				

5B Converting units of measurement

NEED SOME PRACTICE?
Go to 25C Conversion of units
PAGE 297

To have a better understanding of how much medicine is needed to have the desired effect, you can convert each of the quoted amounts to grams. The diagram below will help you understand the concept of unit conversion. The arrows show you which way to move the decimal point as well as whether to multiply or divide. This is for the conversion of units of weight from gram to milligram to microgram.



Use the conversion information to change the amount of active ingredient from the quoted unit to grams. Then work out the total dosage in grams.

EXAMPLE 5B-1 Changing units

Paracetamol tablets usually contain 500 mg of paracetamol. If the recommended dosage is two tablets, what is the dosage in grams?

$\frac{500}{1000} = \frac{1}{2}$ g — Divide by 1000 to convert to grams.

$2 \times \frac{1}{2} = 1$ g — Multiply by 2 as the dosage is 2 tablets.

The dosage is 1 g.

WHAT TO DO 5.2

1 Change each to micrograms.
- a 4 mg
- b 0.5 mg
- c 0.045 mg
- d 40 mg

2 Change each to grams.
- a 20 mg
- b 550 mg
- c 1.5 mg
- d 1500 mg

3 Complete the table below for each of the medicines listed. You will need to read the labels to determine the recommended dosages.

Generic name	Active ingredient per tablet	Active ingredient per tablet (g)	Total dosage (g)
Paracetamol	500 mg	0.5 g	1 g
Aspirin			
Ibuprofen			
Antacid			
Antihistamine			
Vitamin			

5C Determining the dosage

For children, many medications are provided in liquid form. This is for two reasons.

- It is easier for young children to swallow liquids than solid tablets.
- The effective dosage depends on the child's body weight.

To determine dosages for children, you need to base the volume given on the weight of the child (according to age). Most medicines give detailed instructions of dosages based on children's age and weight.

WHAT TO DO 5.3

1 Consider each of the following instruction sheets from the back of children's (also known as paediatric) medications similar to those examined earlier in this chapter. Discuss why these medicines give both an age range and a weight range for each dosage.

Liquid medicine dosage

Medication	Age range (years)	Weight range (kg)	Dosage (mL)	Amount of active ingredient per mL
Paracetamol	1–2	10–12	3–4	48 mg
	2–3	12–14	4	
	3–4	14–16	4–5	
	4–5	16–18	5–6	
	5–6	18–20	6	
	6–7	20–22	6–7	
	7–8	22–25	7–8	
	8–9	25–28	8–9	
	9–10	28–32	9–10	
	10–11	32–36	10–11	
	11–12	36–41	11–13	
Ibuprofen	0.5–1		2.5	20 mg
	1–3	10–15	5	
	4–6	16–21	7.5	
	7–9	22–29	10	
	10–12	30–41	15	
Antihistamine	1–2		2.5	5 mg
	2–12	Up to 30 kg	5	
		Over 30 kg	10	
Iron supplement	Under 2		2.5	1 mg of iron
	Over 2		5	

2 For the table below, find the percentage of active ingredient in each medicine and fill in the second column. Convert these volume dosages to amounts of active ingredient for each weight range using the formula:

Active ingredient per dosage = amount of active ingredient per mL × dosage volume

You will need to check the bottle to find the recommended dosage.

Generic name	Active ingredient in the bottle (%)	Volume of active ingredient in the bottle (mL)	Age range (years)	Active ingredient per dose (mg)
Paracetamol			1–2	
			2–3	
			3–4	
			4–5	
			5–6	
			6–7	
			7–8	
			8–9	
			9–10	
			10–11	
			11–12	
Ibuprofen			0.5–1	
			1–3	
			4–6	
			7–9	
			10–12	
Antihistamine			2.5	
			5	
			10	
			Up to 30 kg	
			Over 30 kg	
Iron supplement			Under 2	
			Over 2	

3 Of the medications in the table above, which one is most sensitive to ages or weights? Remember that children's weight increases with age.

5D Measuring the dosage

MEASURING DOSAGES WITH SCALES

As you will have noticed, most of the dosages for the common medicines are quite small. Have you ever tried to measure out weights this small before?

What to do 5.4 will give you an idea of just how small the weights used in medicines may be when you relate them to common objects.

WHAT TO DO 5.4

1 Working in groups of three or four, collect the equipment listed. Fill in the table.

You will need:

- a set of scales that measures grams or half grams
- a box of Tic Tacs or similar
- a box of Smarties or M&Ms
- a teaspoon
- some sugar or salt.

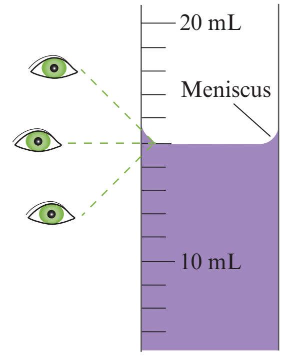
Item	Weight of one	Number that make up 1 g
Tic Tac		
Smartie or M&M		
1 level teaspoon of sugar or salt		

MEASURING DOSAGES BY CUP AND SYRINGE

Medicine cups and syringes are often used to work out medicine dosages for children and adults. It is important to read the scales cleary before administering any dosage.

There is always a small error due to the limit of reading of the measuring instrument. Two probems that may occur are due to:

- Parallax error: this can occur if your eye is not directly level with the scale on the measuring instrument when you read off the value. Otherwise you may read off the wrong value due to the angle you are looking at.
- The meniscus: this is the curve in the upper surface of a liquid close to the edges of the container, caused by surface tension. It can be either convex or concave, depending on the liquid and the surface. Always measure the level at the bottom of the curve.



WHAT TO DO 5.5

1 In groups of three or four, collect as many different examples of medicine cups and oral syringes as you can. To do this you may need to check your home medicine cabinet, ask the school nurse or enquire at your local pharmacy.

2 For each cup or syringe you have chosen, draw a labelled diagram showing the relative sizes and the volume markings. Also write the dimensions of each in millimetres.

Sample cup	Base diameter	Top diameter	Height	Diagram
1				
2				
3				
4				

Sample syringe	Base diameter	Height	Diagram
1			
2			
3			
4			

PROJECT 5

obook
assess
TEACHER

PAIN RELIEF

Making medication easier!

A project worksheet for this chapter is available on the Teacher obook/assess. Ask your teacher to print it out for you. Completing this activity will help you make the right choices when looking after your own and your family's health.

CHAPTER 6

Cooking by measure

ARE YOU READY?

Complete the questions below to see if you are ready to start this chapter or if you need to brush up on your skills first.

- Part 1 should be completed *with* a calculator.
- Part 2 should be completed *without* a calculator.

If there are any questions you cannot answer, the link below each question will direct you to a related Mathematical Skills chapter where you can revise basic concepts and get up to speed.

PART 1 WITH A CALCULATOR

1 Find the value of the following.

a $\frac{1}{2}$ of 250 = **b** $\frac{1}{8}$ of 500 =

c $240 \times 0.75 =$ **d** $1000 \div \frac{1}{4} =$

23A Fundamental concepts

2 Find one third of each of these numbers.

a 500 **b** $\frac{3}{4}$

c $\frac{2}{3}$ **d** 0.66

23C Fractions of quantities

3 The basic metric unit of mass is:

A milligram **B** ounce

C gram **D** litre

25A Units of measurement

4 How many grams are there in:

a 1.5 kg? **b** 0.75 kg?

c 1000 mg? **d** 50 mg?

25C Conversion of units

5 Find 66% of each of the following numbers.

a 750 **b** 2000

c 467 **d** 555

24B Percentage of a quantity

6 In a particular recipe, butter, sugar and flour are mixed together in the ratio of 1 : 1 : 2. If there is 500 g of butter, what weight of sugar and flour do you need?

24D Ratios

PART 2 WITHOUT A CALCULATOR

7 Find the value of the following.

a $4 \times 1.25 =$ **b** $2 \times \frac{1}{3} =$

c $\frac{1}{2} \times 250 =$ **d** $4 \times 1\frac{1}{2} =$

28A Back to basics

8 Convert the following fractions to decimals.

a $\frac{1}{3}$ **b** $\frac{1}{4}$

c $\frac{4}{5}$ **d** $\frac{2}{3}$

28B Fractions

9 Convert the following decimals to fractions.

a 0.5 **b** 0.125

c 0.667 **d** 0.44

28C Decimals

10 $\frac{3}{4}$ of a litre is ____ mL?

A 1200 **B** 750

C 500 **D** 34

23C Fractions of quantities

11 Round the following to the nearest 10 cents.

a $1.99 **b** 56 cents

c 4×99 cents **d** $5 \times \$2.10$

28C Decimals

12 Find the following amounts.

a 85 kg increased by 30%

b $100 decreased by 15%

24C Percentage change

6A Cooking measurements

The secret to being a good cook is simple. You need to be able to:

- accurately measure the ingredients, and be familiar with the basic units so that any incorrectly measured ingredient will seem 'wrong' before it is added
- use whatever measuring implements are available in the kitchen you are in
- adapt recipe sizes as required, be it an intimate dinner for two or an 18th birthday party for 50 people.

This chapter is designed to teach you skills, while helping you enjoy a few tasty snacks along the way.

NEED SOME PRACTICE?
Go to 25A Units of measurement
PAGE 291

Cooking, being a long-practised art, uses many different ways of measuring out the ingredients. In Australia, the metric system is the standard, and modern cookbooks are written using metric measures. The commonly used metric units in cooking are:

- weight in grams
- volume in millilitres
- temperature in degrees Celsius (°C).

MEASURING THE INGREDIENTS

There are four basic types of cooking ingredients:

- solid items, such as meat, that are measured by weight
- dry ingredients, such as flour, that can be measured by either weight or volume
- liquid ingredients, such as milk or water, that are measured by volume
- discrete items, such as eggs, that you count out.

If the required quantity does not need to be totally accurate, some recipes may make it easier by specifying 'two large potatoes' rather than '250 g of potato'. In this way you don't need to chop off and waste the perfectly good potato that is in excess of 250 g.

Quick measures

NOTE
Margarine and butter are generally bought in 500 g and 250 g tubs or blocks.

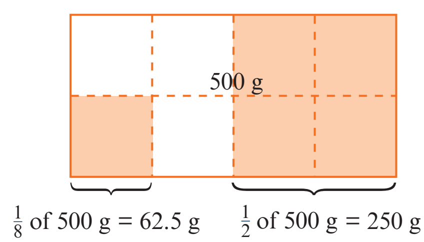
Most cooks will use the following quick method for 'weighing' margarine or butter.

For a 500 g block:

250 g is $\frac{1}{2}$ of 500 g

125 g is $\frac{1}{4}$ of 500 g

62.5 g ($\approx$ 60 g) is $\frac{1}{8}$ of 500 g



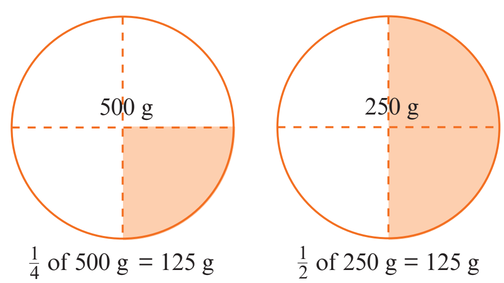
NEED SOME PRACTICE?
Go to 23C Fractions of quantities
PAGE 269

For a 500 g or 250 g tub:

125 g is $\frac{1}{4}$ of 500 g

125 g is $\frac{1}{2}$ of 250 g

These amounts can be quickly measured by using a new tub or block of margarine and marking equally spaced divisions on it. Then use the appropriate portion(s) for the weight that you need.



BASIC VOLUME CONVERSIONS

Because cups and spoons are more commonly available in kitchens than scales and measuring cylinders, there are some basic volume conversions for metric standard cups and spoons. These are shown in the table below.

Metric measure	Volume in millilitres	Abbreviation
1 cup	250 mL	1 c
$\frac{1}{2}$ cup	125 mL	$\frac{1}{2}$ c
$\frac{1}{3}$ cup	83.3 mL	$\frac{1}{3}$ c
$\frac{1}{4}$ cup	62.5 mL	$\frac{1}{4}$ c
metric tablespoon	20 mL	Tb
metric dessertspoon	10 mL	ds
metric teaspoon	5 mL	ts
$\frac{1}{2}$ teaspoon	2.5 mL	$\frac{1}{2}$ ts
$\frac{1}{4}$ teaspoon	1.25 mL	$\frac{1}{4}$ ts

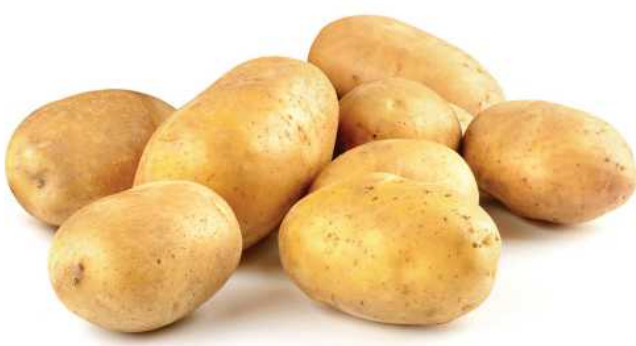
WHAT TO DO 6.1

1 Each of the following ingredients is to be measured accurately for the recipe.

potatoes, milk, eggs, sugar, butter, flour,
rice, tomatoes, sultanas, bread, onions

Classify the ingredients into one (or more) of the four measurement categories listed on the previous page.

2 If less accuracy was required, which items listed in question **1** could be moved into the discrete items category?



6B Cooking by weight or volume

NOTE

'Weight' and 'mass' are generally used interchangeably. The term 'weight' is most often used for cooking and shopping.

While many cooks will defend to the death whether it is better to measure ingredients by weight or volume, in practice it depends on the accuracy of the measuring implements available to you, and your familiarity with them. In the following exercise you will try both forms of measurement in two recipes.

COOKING BY WEIGHT

In What to do 6.2 you will be making muffins where the recipe is based on weight measurements.

WHAT TO DO 6.2

Muffins

Equipment

- kitchen scales capable of measuring to 10 g
- metric spoon measures
- mixing bowl
- flour sifter
- fork
- wooden spoon
- muffin trays
- spatula

Ingredients

- 1 × 55 g egg
- 250 g milk
- 60 g of vegetable oil
- 250 g self-raising flour
- 60 g sugar

Cooking instructions

1. Heat the oven to 180°C.
2. Lightly coat (grease) the muffin trays with butter.
3. Lightly beat the egg in the mixing bowl with a fork.
4. Add the vegetable oil and milk to the egg mix.
5. Sift the flour and add it to the mixture with the sugar, stirring until the flour is moist; do not overmix. The batter should be slightly lumpy.
6. Put the mixture into muffin trays, filling each to two-thirds depth.
7. Bake for 20 to 25 min or until golden brown, and loosen the muffins immediately from the trays when removed from the oven. Place the muffins on a plate.
8. They are best eaten warm with butter.

COOKING BY VOLUME

In What to do 6.3 you will be making honey oat fingers where the recipe is based on volume measurements.

WHAT TO DO 6.3

Honey oat fingers

Equipment

28 × 18 × 4 cm metal cooking tray (not 'non-stick')
mixing bowl
flour sifter
wooden spoon
saucepan
set of metric cup measures
set of metric spoon measures

Ingredients

$\frac{3}{4}$ c plain flour
$\frac{1}{4}$ ts bicarbonate of soda
1 c rolled oats
1 c coconut
$\frac{1}{2}$ c sugar
1 Tb honey
$\frac{1}{2}$ c butter or margarine
$\frac{1}{2}$ c finely chopped dates

Cooking instructions

1. Heat the oven to 180°C.
2. Sift the flour and bicarbonate of soda together into a mixing bowl.
3. Add the oats, coconut, dates and sugar to the mixture, and mix well.
4. Place the honey and butter or margarine into a large saucepan and heat gently until the butter melts.
5. Remove the saucepan from the stove, and immediately add this to the dry ingredients. Mix thoroughly.
6. Press the mixture evenly into the cooking tray, and place in the preheated oven.
7. Cook for 12 to 15 min, until golden brown.
8. Cut into slices and allow to cool in the tray.

6C Measuring by weight or volume

You have tried weight measurement and volume measurement in the last two recipes. What to do 6.4 will show you some of the relationships between the two.

Equipment

accurate analytic scales, measuring correct to 2 decimal places
standard spring balance kitchen scales with an analogue dial, measuring to 5 kg
metric measuring cup and spoon set

WHAT TO DO 6.4

1 a Consider the ingredients listed in the table below. Accurately measure out the specified volume.
b Weigh the measured volume on:
i the analytic scales
ii the kitchen scales.
c Fill in the table as you go.

Ingredient	Volume	Weight using analytic scales	Class average	Weight using kitchen scales	Class average
Flour	2 cups				
	1 cup				
Rice	$\frac{1}{2}$ cup				
Cinnamon	1 teaspoon				
Sugar	1 cup				
	$\frac{1}{4}$ cup				
Sultanas	1 cup				
	$\frac{1}{4}$ cup				
Margarine	$\frac{1}{2}$ cup				
Water	1 cup				
	1 L				

2 Collate each set of results on the board and fill in the 'class average' columns. Would you include widely variant data in calculating your averages? Using your own results and the class averages, discuss as a class these questions.
   a At what point did the kitchen scales become unreliable?
   b For ingredients being measured in small quantities, is it better to measure them by weight or by volume?
   c Are there ingredients that are better measured by weight?
   d What general conclusions can you list about weight and volume measuring in the kitchen?

USING A MIXTURE OF WEIGHT AND VOLUME

Choosing whether to cook by weight or volume may be an individual's choice and may depend on what measuring devices you have available. In many cases it may be best to choose a mixture of weight and volume such as in the following recipe.

WHAT TO DO 6.5

Mini cakes

Equipment

full range of metric volume and weight measures
deep patty pan to make approximately 12 cakes
mixing bowl
wooden spoon
spatula
mixer (optional)

Ingredients

$\frac{1}{4}$ c butter (62 g)
$\frac{1}{4}$ c sugar (62 g)
1 egg
125 g self-raising flour
3 Tb milk
$\frac{1}{4}$ ts vanilla essence

Cooking instructions

1 Heat the oven to 160°C.
2 Grease the patty pan and put aside.
3 Mix the butter and sugar together.
4 Add the egg and beat well.
5 Sift the flour and salt and add alternately with the milk, a third at a time, mixing in gently with the wooden spoon. Add the vanilla essence last.
6 Spoon into the patty pan.
7 Bake until golden brown (approximately 15 min).

6D Accuracy and rounding

> **NEED SOME PRACTICE?**
> Go to 23C Fractions of quantities PAGE 269

As you saw in What to do 6.4, the limits to how accurately you can measure are determined by what measuring device you use. If you could afford analytic scales for the kitchen, you could weigh everything to the nearest 1000th of a gram, which is quite unnecessary for cooking. For spring balance kitchen scales, the nearest 25 g is typical.

ANALOGUE READOUTS

> **NOTE**
> A rule of thumb for any analogue scale is that it is safe to read only to the nearest written scale division.

Analogue measuring devices typically have a circular scale and a needle. Examples include car speedometers and tachometers, clock faces with hands, and older-style kitchen scales. There are many other forms of analogue measuring devices. Even a ruler is an analogue measuring device. What defines an analogue measuring device is that no matter how small the divisions in the scale are, you can always read between the divisions.

ROUNDING

If you had only an ordinary set of kitchen scales, it would be silly to attempt to read such values as 62 g, 743 g or 234 g. You would need to round the measurements to the nearest readable value on the scales.

To do this, follow these steps.

Step 1: Find the level of accuracy of the measuring device.

Step 2: Determine which readable number is closest to the required amount.

Step 3: Round the required amount to the nearest readable measurement.

> **NEED SOME PRACTICE?**
> Go to 23D Decimal numbers PAGE 271

EXAMPLE 6D-1 Required accuracy

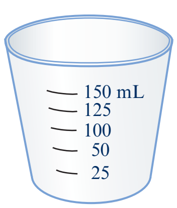
A kitchen scale has an accuracy of 20 g and the weight you want is given as 264 g. What would be the required amount?

The accuracy of 20 g means that only the values of 20, 40, 60, 80, 100, 120, and so on can be reasonably read.
264 g is 4 g from 260 g
264 g is 16 g from 280 g.
Since 264 g is closest to 260 g, the required amount is 260 g.

WHAT TO DO 6.6

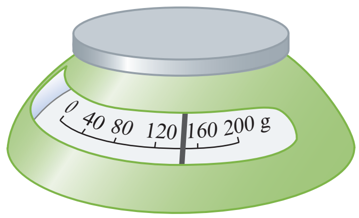
1 For the following analogue scales, state the smallest amount that can be reasonably read with certainty.

a

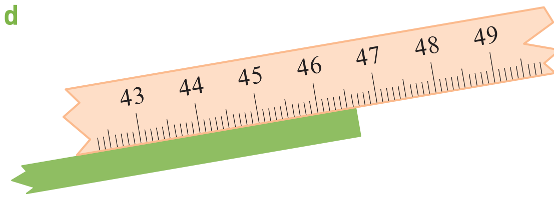


b

c



d



2 You want to measure 123 mL in a glass cup measure that is calibrated in 10 mL divisions. Round this volume, keeping in mind the accuracy of the measure.

3 You have a kitchen scale with 25 g divisions. What is the closest weight to 387 g that you can measure?

CHOOSING THE RIGHT MEASURING DEVICE

You need to choose the right measuring device to suit your requirements. In What to do 6.4, you would have found it impossible to accurately measure the weight of a teaspoon of cinnamon with ordinary kitchen scales.

How much difference do you think 'nearly a third less of the ingredient' will make? The percentage change in the required amount versus the measurable amount is critical.

$$\text{Percentage change} = \frac{\text{measurement amount}}{\text{required amount}} \times 100\%$$

EXAMPLE 6D-2 Percentage change

Can you measure 72 g of sultanas on scales with 50 g divisions?

$$\begin{aligned}\text{Percentage change} &= \frac{\text{measurement amount}}{\text{required amount}} \times 100\% \\ &= \frac{50}{72} \times 100\% \\ &= 69\%\end{aligned}$$

If you assume that any recipe allows a variation of 10% to 15%, then this gives a variation of 69%, which means the flavour may be seriously affected.
This means you will have to find a different set of scales with smaller divisions or convert to a spoon measure using the weight–volume equivalents table on page 67.

NEED SOME PRACTICE?
Go to 24C Percentage change PAGE 282

WHAT TO DO 6.7

1 For the following quantities, calculate the percentage change in the required amount caused by using the specified measuring instrument. State whether it is reasonable to use that measuring instrument.

Ingredient	Amount required	Measuring instrument used
Sugar	165 g	Scales with 100 g divisions
Salt	55 g	Scales with 5 g divisions
Milk	275 mL	Jug with 25 mL divisions

6E Weight and volume conversions

The scales have just broken down or the glass measuring jug was accidentally smashed on the floor. There is no time to replace it before the guests arrive. What do you do? The table on the opposite page is a valuable guide when you need to make quick changes in measurement style.

VOLUME-TO-WEIGHT CONVERSION

Follow these steps using the table on the opposite page.

Step 1: Pick the ingredient from the list of items.

Step 2: From the top row, choose the volume measurement you were to use.

Step 3: Using a ruler or similar item to guide you, read off the weight equivalent of the ingredient's volume.

For example, to determine the weight equivalent of 1 cup of uncooked rice, look at the table and see that it is 200 g.

WEIGHT-TO-VOLUME CONVERSION

There are two ways this can be done using the table on the opposite page.

If the weight you want is written in the body of the table and you want the volume, such as the volume of 250 g of brown sugar, follow these steps.

Step 1: Read across the brown sugar column until you reach the weight of 250 g.

Step 2: Read up to the measures column to find the volume, in this case 250 g of brown sugar is 1 metric cup.

If the exact weight you want does not appear in the body of the table and you want the volume, such as the volume of 500 g of sultanas, follow these steps.

Step 1: Find the weight of 1000 mL of sultanas, in this case 640 g.

Step 2: Divide the required weight of sultanas by the weight of 1000 mL of sultanas and multiply that by 1000.

$$\text{Volume of ingredient (mL)} = \frac{\text{weight required (g)}}{\text{weight 1000 mL volume (g)}} \times 1000$$

Step 3: Round to the accuracy of the measuring device.

NEED SOME PRACTICE?
Go to 23D Decimal numbers PAGE 271

EXAMPLE 6E-1 Finding volume

What is the volume of 400 g of chopped nuts?

Step 1: 1000 mL of chopped nuts = 500 g

Step 2:
$$\text{Volume of ingredient (mL)} = \frac{\text{weight required (g)}}{\text{weight 1000 mL volume (g)}} \times 1000$$
$$= \frac{400}{500} \times 1000$$
$$= 800 \text{ mL}$$

Step 3: If using cup measures it is $\frac{800}{250} = 3.2 \text{ cups} = 3\frac{1}{5} \text{ cups}$

If using mL it is 800 mL.

Weight–volume equivalents

Items	1 ts	$\frac{1}{2}$ Tb or 2 ts	1 Tb	$\frac{1}{4}$ c	$\frac{1}{3}$ c	$\frac{1}{2}$ c	$\frac{2}{3}$ c	$\frac{3}{4}$ c	1 c	$1\frac{1}{2}$ c	2c	$2\frac{1}{2}$ c	3 c	$3\frac{1}{2}$ c	4 c
Volume measures	5 mL	10 mL	20 mL	62 mL	83 mL	125 mL	167 mL	187 mL	250 mL	375 mL	500 mL	625 mL	750 mL	875 mL	1000 mL
Flour, all kinds: wheaten (plain and self-raising), wholemeal, corn, rice, arrowroot				31 g	42 g	62 g	83 g	93 g	125 g	187 g	250 g	312 g	375 g	437 g	500 g
Macaroni				31 g	42 g	62 g	83 g	93 g	125 g	187 g	250 g	312 g	375 g	437 g	500 g
Oats, rolled				23 g	30 g	45 g	60 g	67 g	90 g	135 g	180 g	225 g	270 g	315 g	360 g
Coconut, grated				16 g	22 g	33 g	43 g	48 g	65 g	98 g	130 g	162 g	195 g	227 g	260 g
Breadcrumbs (fresh)				15 g	20 g	30 g	40 g	45 g	60 g	90 g	120 g	150 g	180 g	210 g	240 g
Fats, solids (such as butter, dripping)	5 g	10 g	20 g	62 g	83 g	125 g	167 g	187 g	250 g	375 g	500 g	625 g	750 g	875 g	1000 g
Sugar, brown, moist	5 g	10 g	20 g	62 g	83 g	125 g	167 g	187 g	250 g	375 g	500 g	625 g	750 g	875 g	1000 g
Sugar, icing, sifted	4 g	8 g	16 g	50 g	67 g	100 g	133 g	150 g	200 g	300 g	400 g	500 g	600 g	700 g	800 g
Syrups (such as thick, honey, treacle)				31 g	42 g	63 g	83 g	94 g	125 g	187 g	250 g	312g	375 g	437 g	500 g
Rice, uncooked				50 g	67 g	100 g	133 g	150 g	200 g	300 g	400 g	500 g	600 g	700 g	800 g
Peas, dried, split				50 g	67 g	100 g	133 g	150 g	200 g	300 g	400 g	500 g	600 g	700 g	800 g
Lentils, uncooked				50 g	67 g	100 g	133 g	150 g	200 g	300 g	400 g	500 g	600 g	700 g	800 g
Coffee beans				50 g	67 g	100 g	133 g	150 g	200 g	300 g	400 g	500 g	600 g	700 g	800 g
Nuts, chopped				31 g	42 g	63 g	83 g	94 g	125 g	187 g	250 g	312 g	375 g	437 g	500 g
Dried fruits, small (such as currants, sultanas)				40 g	53 g	80 g	107 g	120 g	160 g	240 g	320 g	400g	480 g	580 g	640 g
Dried fruits, large, chopped (such as apricots, dates)				31 g	42 g	62 g	83 g	93 g	125 g	187 g	250 g	312 g	375 g	437 g	500 g
Cheese, grated		5 g	10 g	31 g	42 g	62 g	83 g	93 g	125 g	187 g	250 g	312 g	375 g	437 g	500 g

WHAT TO DO 6.8

Coconut drops

Convert the following recipe from weight to volume measures and make the coconut drops.

NEED SOME PRACTICE?
Go to 25A Units of measurement
PAGE 291

Equipment

- oven tray
- flour sifter
- mixing bowl
- wooden spoon
- metric cups and spoons
- mL measures

Ingredient

- 125 g self-raising flour
- $\frac{1}{4}$ ts salt
- 83 g butter
- 65 g desiccated coconut
- 62 g caster sugar
- 1 egg
- 40 g milk

NOTE

Milk has a similar volume per weight as water: 1 mL weighs 1 g.

Cooking instructions

1. Heat the oven to 180°C.
2. Grease the oven tray.
3. Sift the flour and salt.
4. Rub butter, flour and salt together with your fingertips until the mixture feels like fine breadcrumbs.
5. Mix in the coconut and caster sugar.
6. Stir in the egg and milk to make a stiff mixture.
7. Drop teaspoonfuls of the mixture onto the greased oven tray.
8. Bake for 15 to 20 min.
9. Cool on a cake rack.

6F Scaling up or scaling down

There are many occasions when a recipe is too small, or too large, for the number of people being catered for. On these occasions you need to alter the quantities to suit your needs. Remember that all ingredients must be increased (scaled up) or decreased (scaled down) by the same ratio.

WHAT TO DO 6.9

Scaling down a banana cake

The following is a recipe that makes two 175 mm sandwich tin cakes. Reduce the ingredients by half to make one cake only.

Equipment

one 175 mm sandwich tin
mixing bowl
wooden spoon
metric spoon
cup measures
metric scales
flour sifter
spatula

Ingredients

125 g butter
$\frac{3}{4}$ c caster sugar
$\frac{1}{2}$ ts vanilla essence
2 eggs
3 medium-sized bananas
$1\frac{1}{2}$ c self-raising flour
pinch of salt
1 ts bicarbonate of soda (dissolved in 2 Tb of milk)

Cooking instructions

1. Heat the oven to 190°C.
2. Grease the cake tin with butter.
3. Mix the butter and caster sugar together until they form a smooth mixture.
4. Add the vanilla essence.
5. Add the eggs, and mix well.
6. Thoroughly mash the bananas, and beat them into the mixture.
7. Sift the flour and salt, and mix it into the mixture using the wooden spoon.
8. Add the milk and soda mixture, and gently fold it in with the wooden spoon.
9. Tip the mixture into the greased sandwich tin and bake for 25 to 30 min.

PROJECT 6

obook
assess
TEACHER

MEASURING YUM AND NOT SO YUM

Getting it right the first time!

A project worksheet for this chapter is available on the Teacher obook/assess. Ask your teacher to print it out for you. Completing this activity will help you make the right choices when using all types of 'recipes', not just for food.

CHAPTER 7

Standard drinks and reaction time

ARE YOU READY?

Complete the questions below to see if you are ready to start this chapter or if you need to brush up on your skills first.

- Part 1 should be completed *with* a calculator.
- Part 2 should be completed *without* a calculator.

If there are any questions you cannot answer, the link below each question will direct you to a related Mathematical Skills chapter where you can revise basic concepts and get up to speed.

PART 1 WITH A CALCULATOR

1 Find the value of each of the following.

a $\frac{1}{2}$ of 25 = **b** $4 \times 375 =$

c $750 \div 7.7 =$ **d** $4000 \div 21 =$

23A Fundamental concepts

2 Find one fifth of each of the following numbers.

a 500 **b** $\frac{3}{4}$

c $\frac{2}{3}$ **d** 0.66

23C Fractions of quantities

3 The metric unit most commonly used for measuring drink volumes is:

A fluid ounce **B** milligram

C millilitre **D** litre

25A Units of measurement

4 Find 14% of each of the following numbers.

a 750 **b** 660

c 2800 **d** 542

24B Percentage of a quantity

5 Write the formulas used to calculate:

a the area of a square

b the volume of a square box

c the area of a circle

d the volume of a ball.

27A Area

30A Volume

PART 2 WITHOUT A CALCULATOR

6 Find the value of the following.

a $4 \times 12.5 =$ **b** $750 \div 100 =$

c $6 + 3 - 8 \times 2 =$ **d** $6 + 4 \div 3 \times 2 =$

28A Back to basics

7 Convert the following fractions to decimals.

a $\frac{1}{8}$ **b** $\frac{4}{8}$

c $\frac{1}{5}$ **d** $\frac{2}{5}$

28C Decimals

8 Convert the following decimals to fractions.

a 0.25 **b** 0.111

c 0.167 **d** 0.0025

28C Decimals

9 Convert these decimals to percentages of 1.

a 0.15 **b** 0.23

c 0.75 **d** 0.0025

28D Percentages

10 Convert these fractions to percentages of 1.

a $\frac{1}{2}$ **b** $\frac{2}{3}$

c $\frac{3}{4}$ **d** $\frac{1}{5}$

28D Percentages

11 375 mL is ____ L.

A 3.75 **B** 0.375

C 375 **D** 37.5

25A Units of measurement

7A Standard drinks and containers

Beer, wine and spirits are consumed around the world to celebrate many occasions. Champagne is often used for toasts at a 21st birthday or a wedding, wine is served with dinner, and for many people a beer is a sure sign the working week has ended.

The ingredient in alcoholic drinks that makes them alcoholic is called ethanol. When most people say ‘alcohol’ they mean ‘ethanol’. This chapter also uses the word alcohol.

In recent years there has been an effort to make people think about how much they drink and what they do afterwards. Alcohol has been recognised as being a major contributor to road and other accidents and to many public health problems. This chapter explores issues relating to the amount of alcohol in commercially manufactured drinks sold around Australia.

WHAT IS A ‘STANDARD’ DRINK?

> **NOTE**
> The definition of a standard drink varies from country to country. For example, in Austria it is 6 g of alcohol, but in Japan it is 19.75 g.

The standard drink is used in many countries to quantify alcohol intake. In Australia, a standard drink contains approximately 10 g or 12.5 mL of ethanol. One standard drink always contains the same amount of alcohol regardless of serving size or the type of alcoholic beverage.

Alcoholic beverages are typically divided into three classes – beers, wines and spirits – and typically contain between 3% and 40% alcohol by volume. There are many different types of alcoholic beverage sold commercially, including spirits, fortified wines (port and sherry), wines, regular beer, cider, low-alcohol beer and a variety of mixed drinks.

The amount of ethanol in each of these types of drink differs, so the volume of a standard drink varies depending on the size and type of drink. The following table gives some common drinks and serving sizes.

Common drinks and serving sizes

Drink	Light beer	Regular beer	Wine	Fortified wine	Spirits
% alcohol	2.7%–3.5%	4.5%–5.9%	9%–15%	18%–22%	38%–50%
Common serving method	 can	 pot	 wine glass	 port glass	 shot glass
Standard serving size	375 mL	285 mL	100 mL	60 mL	30 mL

VOLUMES OF STANDARD DRINKS AND CONTAINERS

The following investigation looks at the volume of standard drinks and containers. It is best carried out in a laboratory or kitchen. In pairs, work through the following set of activities, recording the information as needed. You will need:

8 to 12 plastic cups
8 to 12 medicine cups (40 mL size)
2 measuring cylinders (200 mL)
2 shot glasses
2 round whisky tumblers
2 octagonal whisky tumblers
2 port glasses
2 plastic or glass champagne flutes
2 red wine glasses
2 white wine glasses
2 pot glasses (285 mL)
2 soft-drink cans or stubbies (375 mL)

NOTE

Spirits, wines and beers have different percentages of alcohol. Some values are quoted in the table below.

WHAT TO DO 7.1

1 Complete the last column of the following standard drinks table using the formula:

$$\text{Volume of alcohol per drink (mL)} = \frac{\text{standard drink volume (mL)} \times \text{alcohol (\%)}}{100}$$

Type of drink	Name for standard drink size	Volume of a standard drink (mL)	Alcohol (%)	Volume of alcohol per drink (mL)
Spirits (100 proof)	Nip	30	50	
Spirits (80 proof)	Nip	30	40	
Fortified wine	Port glass	60	18	
Champagne	Flute	110	11.5	
White wine	Glass	110	11.5	
Red wine	Glass	100	12.5	
Beer (regular)	Pot/can/stubby	285	4.8	
Beer (light)	Pot/can/stubby	465	2.7	

2 **Spirits measures**

Step 1: Spirits such as whisky, vodka and gin are measured in nips. One nip is 30 mL. Pour what you think is 30 mL of water into a plastic cup and mark the level on the outside of the cup with a felt-tip pen. Now pour the water into the measuring cup and write the actual volume.

Step 2: Calculate how many standard (30 mL) drinks this represents.

Shot glasses

Step 1: Use the measuring cup to pour 30 mL of water into the shot glass. Does 30 mL fit into a shot glass? If not, measure the volume of the shot glass.

Step 2: If there is room to spare, fill the shot glass to the rim and measure the total volume.

Step 3: Calculate how many standard (30 m L) drinks this represents.

Round tumblers (straight sides)

Step 1: Fill the tumbler to a level you would consider normal for whisky or a similar spirit. Measure this volume.

Step 2: Calculate how many standard (30 mL) drinks this represents.

Step 3: Measure the diameter and the height of the tumbler.

Step 4: Now calculate the radius of the tumbler using: radius $= \frac{\text{diameter}}{2}$

Step 5: Calculate the volume of the tumbler using: volume $= \pi \times (\text{radius})^2 \times$ height.

Step 6: Fill the tumbler with water and then pour the water into the beaker to find its volume. Is it more or less than you calculated? Why?

Step 7: If this glass was filled to the top, how many standard drinks would this represent?

NEED SOME PRACTICE?
Go to 30A Volume
PAGE 351

NOTE

$\pi \approx 3.142$

Octagonal tumblers (straight sides)

Step 1: Fill the tumbler to a level you would consider normal for whisky or a similar spirit. Measure this volume.

Step 2: Calculate how many standard (30 mL) drinks this represents.

Step 3: Find the centre of the base of the tumbler by drawing four diagonals, as shown. This produces eight identical triangles that make up the octagon.

Step 4: Measure the base length and the perpendicular height of one of the triangles.

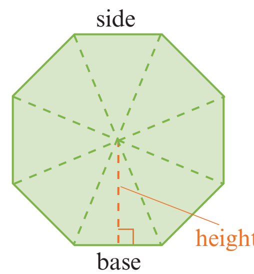
Step 5: Measure the height of the tumbler.

Step 6: Calculate the volume of the tumbler using:

Volume $= 8 \times \frac{1}{2} \times$ base $\times$ perpendicular height $\times$ tumbler height

Step 7: Fill the tumbler with water and then pour the water into the beaker to find its volume. Is it more or less than you calculated? Why?

Step 8: If this glass was filled to the top, how many standard drinks would this represent?



3 Fortified wine measures

Step 1: Measure the diameter and depth of the port glass.

Step 2: Now fill the port glass with water and measure the volume.

Step 3: Calculate the volume of a cone with the same diameter and depth. First divide the diameter by 2 to find the radius. Use the formula for the volume of a cone:

Volume $= \frac{1}{3} \times \pi \times (\text{radius})^2 \times$ height

How close is the port glass to the shape of a cone?

Step 4: Refill the port glass with 70 mL of water. Measure the distance from the top of the glass to the top of the water.

Step 5: The number of standard drinks a port glass contains when completely full can be found by dividing the volume when full by 70. How many standard drinks does a full port glass hold?

NEED SOME PRACTICE?
Go to 30A Volume
PAGE 351

4 Wine measures

Wine is served in wine glasses to a level of about 100 mL. Red wine glasses are generally larger than white wine glasses, but are still only filled to about 100 mL. Champagne is generally served in a flute glass.

Step 1: Pour what you think is 100 mL of water into a flute glass. Mark the level with a felt-tip pen. Pour the water into a beaker and write down the actual volume.

Step 2: Measure the diameter and depth of each wine glass.

Step 3: Fill each glass with water and measure the total volume of each glass using a measuring cylinder.

Step 4: Record the diameter, depth and volume of the flute, red wine and white wine glasses.

Step 5: Calculate the volume of half a sphere with the same diameter as the red wine glass. First divide the diameter by 2 to find the radius. Use the formula for the volume of a sphere:

$$\text{Volume} = \frac{2}{3} \times 3\pi \times (\text{radius})^3$$

Step 6: The number of standard drinks a wine glass contains when completely full can be found by dividing the volume when full by 100. How many standard drinks does each wine glass hold?

NEED SOME PRACTICE?
Go to 30A Volume PAGE 351

5 Beer measures

Regular beers

Step 1: Measure the diameter and the height of a beer glass (a pot).

Step 2: Measure the volume that the glass can hold using the measuring cylinder.

Step 3: Now measure one standard beer (265 mL) into each glass.

Step 4: Calculate the difference in volume from a standard drink (265 mL) to a full-capacity glass (a pot holds 285 mL).

Volume difference = volume of glass − volume of standard drink

Step 5: Calculate how much of a standard drink of beer this difference represents.

$$\text{Percentage of a standard drink} = \frac{\text{volume difference}}{265} \times 100$$

Step 6: Using the increase in volume of a full-capacity glass (pot) found above and the amount of alcohol in a standard glass (265 mL) of regular beer, calculate the number of standard drinks (12.5 mL of alcohol) that are in a 285 mL pot of regular beer.

Low-alcohol beers

Step 1: Fill the can or stubbie provided with water. Pour this into the 285 mL pot glass. Now use the measuring cylinder to find the volume left in the can.

Step 2: What fraction of beer was left over from the can?

Volume difference = volume of a can (375 mL) − volume of glass (285 mL)

$$\text{Fraction left} = \frac{\text{volume difference}}{\text{volume of can}} \times 100$$

Step 3: What amount of alcohol (in both mL of alcohol and standard drinks) would be in a 375 mL stubby/can of low-alcohol beer?

Step 4: What amount of alcohol (in both mL of alcohol and standard drinks) would be in a 285 mL pot of low-alcohol beer?

NEED SOME PRACTICE?
Go to 23C Fractions of quantities PAGE 269

NOTE
Find the fraction that represents 375 mL of a 465 mL standard drink of low-alcohol beer, then work out the alcohol volume and number of standard drinks.

6 Collecting the information

Using the information you have gathered from these activities, work out the number of standard drinks in each of the containers in the table (some may seem quite unrealistic). Write your method of calculation for each one. Complete the following table.

	80 proof spirits	Fortified wine	Wine	Regular beer	Light beer
750 mL bottle					
2 L cask					
4 L cask					
375 mL bottle					
1.5 L magnum					

7B Blood alcohol level and body weight

One of the main ways in which the amount of alcohol in a person's body can vary is the combination of how many drinks they have had recently and their body weight. It is, however, important to note that there are many other factors that can affect a person's blood alcohol level or concentration, such as age, gender, how much they have eaten, how tired or stressed they may be, and so on.

Under no circumstances should the information and calculations provided in this chapter be considered a reliable or legal means of assessing blood alcohol levels. All information is offered as a guide only. Actual results can vary widely between individuals, depending on body weight, gender, metabolism, the amount of food or water consumed, and many other factors.

NOTE

Blood alcohol concentration (BAC) is the amount of alcohol present in the bloodstream. A BAC of 0.05% means that there is 0.05 g of alcohol in every 100 mL of blood.

In all Australian states and territories, a blood alcohol concentration (BAC) of 0.05% is the maximum legal limit for people driving cars. Drivers with Learner Permits or Provisional Licences must have a blood alcohol level of 0.00%.

The following table displays the number of drinks consumed within 2 hours by people of different weights to reach three blood alcohol levels.

- The unshaded numbers indicate possible blood alcohol levels up to 0.05%.
- The grey shaded numbers indicate blood alcohol levels between 0.05% and 0.09%.
- Orange shaded numbers indicate blood alcohol levels over 0.10%.

Body mass (kg)	Drinks in 2 hours					
45	1	2	3	4	5	6
55	1	2	3	4	5	6
64	1	2	3	4	5	6
73	1	2	3	4	5	6
82	1	2	3	4	5	6
91	1	2	3	4	5	6
100	1	2	3	4	5	6
109	1	2	3	4	5	6

WHAT TO DO 7.2

1 Use the table on the previous page to find how many drinks in an hour would place you in each of these three blood alcohol levels.

a under 0.05%

b between 0.05 and 0.09%

c over 0.10%

2 Another way of determining blood alcohol levels is by using this formula.

Blood alcohol concentration % = 0.0012 × number of drinks × alcohol (mL)/drink

Use this formula and information from the standard drinks table developed in question **1** What to do 7.1 to complete the following table. To calculate the volume of alcohol in each standard drink, you can use the results from the table or reuse this formula:

$$\text{Volume of alcohol per drink (mL)} = \frac{\text{standard drink volume (mL)} \times \text{alcohol (\%)}}{100}$$

Type of drink	Standard volume (mL)	Alcohol (%)	Volume of alcohol per drink (mL)	Number of drinks in 1 hour 1	2	3	4	5	6
Spirits (100 proof)		50							
Spirits (80 proof)		40							
Fortified wine		18							
Champagne		11.5							
White wine		11.5							
Red wine		12.5							
Beer (regular)		4.8							
Beer (light)		2.7							

Shade the cells of your table into the three ranges (less than 0.05%, between 0.05% and 0.09% and greater than 0.10%) referring to the table on the previous page.

3 In groups of three or four discuss how closely your table matches the table on the previous page. What might cause the differences between the tables?

4 What is the maximum number of each drink that a driver with the same weight as yourself could have in an hour to keep them under the 0.05 limit?

NOTE

To ensure you are safe to drive, drink far less than the maximum number of drinks. It is best to have a designated driver who stays well under the legal limit for the whole night.

STAYING UNDER THE LIMIT

The maximum number of drinks a person can consume before being 'over the limit' is commonly stated as two in the first hour for men and one for women. It will take only one drink in each hour after this to keep the person over the limit.

Assigning designated drivers who do not drink for the night is by far the best method that can be used to avoid exceeding the 0.05 limit. Another method used by some people is to alternate between regular beer and low-alcohol beer. What to do 7.3 looks at this method and makes some conclusions about its effectiveness.

Beginning from zero blood alcohol, the total amount of alcohol that can be drunk by an 'average' male in 1 hour to stay under 0.05 is 25 mL of alcohol. After that, their body will burn off the equivalent of only one standard drink per hour.

When the total amount of alcohol consumed in 1 hour is more than 25 mL, even an 'average' man will be over the 0.05 limit. For a woman, this is reduced to as little as 12.5 mL. Will the next drink put you over the limit?

WHAT TO DO 7.3

1 Look back at What to do 7.1 and What to do 7.2 and note the volume of alcohol per drink (mL) for a 285 mL pot of regular beer (1.1 standard drinks) and for a low-alcohol beer (0.6 standard drinks).

2 a Complete the following table and fill in the blank spaces to show the total amount of alcohol consumed in each of these four different situations.
Situation 1: Drinks regular beer only.
Situation 2: Drinks light beer only.
Situation 3: Alternates between regular beer and light beer.
Situation 4: Alternates between light beer and regular beer.

	Number of drinks in 1 hour					
	1	2	3	4	5	6
Situation 1: regular beer						
Situation 2: light beer						
Situation 3: regular then light beer						
Situation 4: light then regular beer						

Shade the cell that puts the drinker over the 0.05 limit in each situation.

b Does the strategy of alternating between light and regular beer have any effect?

c Is there any difference between starting with a light or a regular beer, as shown in situations 3 and 4?

7C Reaction time and stopping distance

Alcohol is recognised as being a major contributor to road and other accidents. It has the effect of slowing down the overall functioning and reflex responses of a person, impairing their judgement and increasing their reaction time.

Under normal conditions a car does not come to a complete stop the instant you want it to. Two factors control how far a car travels between you seeing a problem up ahead and the car coming to a standstill. These are:

- reaction time: the time it takes for the driver to react and hit the brake pedal
- braking distance: the distance the car travels while the brakes are being applied.

Stopping distance = reaction-time distance + braking distance

REACTION TIME

Reaction time is the amount of time between seeing and acting. In What to do 7.4 you will measure reaction times in groups of four: a driver, a tester, a recorder and an observer.

WHAT TO DO 7.4

Set up the equipment so that the light can be switched on manually, starting the timer and signalling the driver to brake. The timer stop switch is then activated by the brake pedal.

Equipment

an electronic timer (measuring to hundredths of a second) with a remote switching capacity pedal simulator and a light that suits the timer mechanism

obook
Instructions on how to build this simulation are available on your obook.

1 *Test A:* Foot covering the brake
The driver is to have their foot resting on the brake pedal, and when the tester switches on the light, they respond by braking as quickly as possible. The recorder is to note the timer value. Repeat this test four more times, noting down the time for each trial. Average the five values for reaction time, and write this in column 2 of the table below.

NOTE
Drivers may need a few practice attempts first.

2 *Test B:* Foot moving from accelerator to brake
The procedure is the same as for Test A, except that the driver is to have their foot resting on the accelerator pedal. When the light flashes, they have to move to the brake pedal and press it. Do this test five times, and average the values obtained. Record this value in column 4 of the table.

Record sheet for reaction-time exercise

Student name	Reaction time (Test A)	Reaction distance (Test A 60 km/h)	Reaction time (Test B)	Reaction distance (Test B 60 km/h)

NOTE

The reaction time for drivers unaffected by alcohol, drugs or fatigue has been found to be about 0.5 s.

Calculating reaction-time distance

As you have a good estimate of your personal reaction time, you can now find how far you would have travelled in that time (before you hit the brake pedal). To do this, multiply your reaction time in seconds by the speed in metres per second. To convert from km/h to m/s, multiply by 1000 then divide by 3600.

NEED SOME PRACTICE?

Go to 25A Units of measurement PAGE 291

EXAMPLE 7C-1 Reaction-time distance

If you are travelling at 30 km/h and your reaction time is 0.5 s, what is your reaction-time distance?

Step 1: $30 \text{ km/h} = \frac{30 \times 1000}{3600} \text{ m/s}$ Multiply by 1000 and divide by 3600.

$= 8.333 \text{ m/s}$

Step 2: Reaction-time distance $=$ reaction time (s) $\times$ speed (m/s)

$= 0.5 \times 8.33$

$= 4.165 \text{ m}$

Your reaction-time distance is 4.165 m before you hit the brake pedal.

WHAT TO DO 7.5

1 Convert 60 km/h to m/s. Using your own reaction times, finish the table in What to do 7.4 by calculating your reaction-time distance for 60 km/h for Test A and Test B.

2 Use your personal reaction-time distance from Test B to calculate your reaction-time distances at each of the following speeds. Remember, this is the distance covered before the car even begins to slow down.

Speed in km/h	30	40	50	60	70	80	90	100	110	120
Speed in m/s	8.33									
Reaction distance (m)	4.165									

Simple reaction-time distance is the measure of a fully prepared person waiting for a signal. This is what you measured in the previous exercises. In the real world, many things can distract or impair a driver, resulting in much longer reaction times. As a class, discuss possible ways that your reaction time may be increased, and list these on the board.

BRAKING DISTANCE

Braking distance is the distance you travel after you have applied the brakes as the car slows to a stop. On a good bitumen surface, with a car in perfect condition, it takes about 21 m to stop while travelling at 60 km/h. This is longer on wet roads or gravel surfaces.

You can calculate the effects of different road surfaces on braking distances using the following formula:

$$D = \frac{V}{1000} \times (210 + 97R)$$

where D is the distance travelled in m
V is the speed in km/h
R is the surface factor

Surface factor	
Dry asphalt	1.4
Wet asphalt	1.7
Gravel	2.1
Hard snow	6.7
Ice	14.4

WHAT TO DO 7.6

1 Calculate the braking distances (m) for each of the speed and surface conditions listed in the table below.

Speed (km/h)	Dry asphalt	Wet asphalt	Gravel	Snow	Ice
30					
40					
50					
60					
70					
80					
90					
100					
110					
120					

2 Find your total stopping distances for the conditions above. Complete a table like the one below.

Stopping distance = reaction-time distance + braking distance

Condition	Personal reaction distance	Braking distance	Stopping distance
30 km/h, dry asphalt			
50 km/h, dry asphalt			
60 km/h, dry asphalt			
80 km/h, dry asphalt			

PROJECT 7

obook
assess
TEACHER

TESTING YOUR ALCOHOL

Bar ethics?

A project worksheet for this chapter is available on the Teacher obook/assess. Ask your teacher to print it out for you. Completing this activity will help you make the right choices for yourself and others when planning events where alcohol will be served.

CHAPTER 8

Planning an event

ARE YOU READY?

Complete the questions below to see if you are ready to start this chapter or if you need to brush up on your skills first.

- Part 1 should be completed *with* a calculator.
- Part 2 should be completed *without* a calculator.

If there are any questions you cannot answer, the link below each question will direct you to a related Mathematical Skills chapter where you can revise basic concepts and get up to speed.

PART 1 WITH A CALCULATOR

1 Find the value of each of the following.

a $18 \div 2 + 6 \times (3 + 2) =$

b $20 \div 6 - 6 \times (3 - 2) =$

c $32 - 2 \times 6 \times (3 \div 2) =$

d $81 \div 2 + 6 \div (3 + 2) =$

23A Fundamental concepts

2 Find one quarter of each of these numbers.

a 600 **b** $\frac{3}{4}$

c $\frac{2}{3}$ **d** 0.90

23C Fractions of quantities

3 A television costs $2185 and is sold for $2500. The percentage of profit is:

A 21% **B** 26%

C 14% **D** 41%

24B Percentage of a quantity

4 Jim buys a table for $1500 and sells it for $1000. The percentage loss is:

A 13% **B** 33%

C 66% **D** 77%

24B Percentage of a quantity

5 Two friends want to invest in a unit in a ratio of 7 : 8. If the unit costs $90 000, how much should each contribute?

A $45 000 and $45 000

B $70 000 and $20 000

C $42 000 and $48 000

D $50 000 and $40 000

24D Ratios

PART 2 WITHOUT A CALCULATOR

6 Find the value of each of the following.

a $20 \div 2 + 6 \times (3 + 2) =$

b $(32 - 5) \div 3 \times 12 =$

c $(45 - 5) \div 5 \times 12 =$

d $(60 - 4) \div 7 \times 2 =$

28A Back to basics

7 Convert the following fractions to decimals.

a $\frac{1}{2}$ **b** $\frac{2}{3}$

c $\frac{3}{4}$ **d** $\frac{1}{5}$

28C Decimals

8 Convert these decimals to fractions.

a 0.18 **b** 0.023

c 0.175 **d** 0.61

28C Decimals

9 Convert these fractions to percentages.

a $\frac{1}{8}$ **b** $\frac{4}{8}$

c $\frac{1}{5}$ **d** $\frac{2}{5}$

28D Percentages

10 Convert these decimals to percentages.

a 0.19 **b** 0.029

c 0.195 **d** 0.91

28D Percentages

11 In simplest form, the ratio of 750 mL : 2 L is:

A 3 : 8 **B** 75 : 20

C 3 : 4 **D** 75 : 2000

24D Ratios

8A Planning and costing an event

Planning an event involves many types of calculations, such as the cost of food, drinks, venue hire, decorations and staff. Whether you have a party at home or in a function venue, decisions need to be made about how many guests, the seating, and food and drink.

If you decide to have a function at home, you need to consider many factors. Do you have enough space? Should you hire a marquee? How much does the catering company charge? Who will serve the food? Will seating, crockery and cutlery be provided or will they cost extra?

Catering companies and event managers have developed formulas to estimate the amount of food and drink required by a set number of people based on the type and length of function. These formulas are used to calculate the cost of the food and drink.

COMPARING PACKAGES

Many function centres and catering companies offer a set price per head for food and drink, and you will need to check any additional fees such as room hire, security and penalty charges if your event goes over time. Many centres offer all-inclusive packages. It is important to compare prices before settling on a venue or function centre.

EXAMPLE 8A-1 Comparing packages

The Bolaca Hotel is offering a birthday package for $1100, which includes room hire and food for 60 people. The alternative is to pay $15 per person and $250 for room hire. Which is the cheaper option?

The cost of food for 60 people at $15 per head is:
$\$15 \times 60 = \$900$
The cost for room hire plus the cost per head for food is:
$\$900 + \$250 = \$1150$
The cheaper option is the $1100 package.

NEED SOME PRACTICE?
Go to 23A Fundamental concepts PAGE 263

WHAT TO DO 8.1

1 The Silver Arm function centre is offering a birthday package for $2000, which includes room hire and food for 100 people. The alternative is to pay $20 per person and $200 for room hire. Which is the cheaper option?

2 The Sea Shell function centre is offering a birthday package for $1500, which includes room hire and food for 40 people. The alternative is to pay $35 per person and $100 for room hire. Which is the cheaper option?

CATERING NEEDS

If you are having an event at home or you are self-catering at a particular site, it is important that you order the right amount of food. Too much food means that it is wasted and an unnecessary expense is added to your event. Not enough food may leave your guests hungry and they leave the event disappointed. A popular choice of food at events is finger food or canapés. They are simple to prepare and can be eaten standing up. Some basic rules for serving canapés are given in the following tables.

Number of canapés required

Length of party	Number of canapés per person
Up to an hour	6
One hour	8
One and a half hours	10
Two hours	12

Type of canapés required

Number of guests	Number of different canapés
Less than 35 guests	5
35 to 60 guests	7
Over 60 guests	9

EXAMPLE 8A-2 Catering needs

Natasha and Thomas are having canapés at the start of their wedding for 50 guests. They want enough canapés and enough variety for one and a half hours.

a How many canapés will they need?

b How many different types should they be offering?

Use the tables above to answer the questions.

a They need 10 canapés per person: $10 \times 50 = 500$ pieces.

b They should offer at least 7 different options for canapés.

WHAT TO DO 8.2

1 Lucy is having canapés at her birthday for one and a half hours and she has invited 100 guests.

a Using the table above, how many canapés will she need?

b How many different types should she be offering?

2 Ryan is having canapés at his 21st birthday for one and a half hours and has invited 120 guests.

a Using the table above, how many canapés will he need?

b How many different types should he be offering?

8B Calculating food and drink packages

Larger events such as weddings, birthday parties and Christmas break-ups are often held in function centres and restaurants. They are usually catered for using a set menu instead of having everyone order individual meals that all need to be served at once.

Function centres, catering companies and restaurants usually offer food and drink packages where they charge a set amount for a meal and a set amount for a limited choice of drinks. There may be two meal choices that are given to alternate guests, or guests may have a choice of one, two or more meal options. Packages are often provided for events of less than 5 hours, with an extra cost per hour being charged for events going over the 5 hours.

Catering companies calculate food and beverage packages using these formulas:

Total cost of food = number of guests × cost of package per person

Total cost of drinks = number of guests × cost of package per person

Below are a set of food and drink packages offered by the Blue Shell reception centre. These costs are to be used in the following calculations.

Food packages

Package options	Cost per person
Bronze package (two-course sit-down dinner)	$45
Silver package (three-course sit-down dinner)	$52
Gold package (five-course sit-down dinner)	$68

Extra options: platters

Platter type	Cost per platter
Antipasto platter	$50
Seasonal fruit platter	$50
Cheese platter	$55
Seasonal seafood platter	$85
Chargrilled Turkish loaf with a selection of dips	$30

Drink packages (4-hour packages)

Package options	Cost per person
Standard beverage package (beer, wine, soft drink, orange juice, tea, coffee and hot chocolate)	$34
Premium beverage package (includes standard plus champagne)	$40
Deluxe beverage package (includes standard plus spirits)	$48

EXAMPLE 8B-1 Finding costs

Tony has 120 guests coming to his birthday party at the Blue Shell reception centre. Use the tables on the previous page to answer these questions.

a Find the cost of the silver package for food.
b Find the cost of the premium beverage package.
c The function centre has round tables that seat eight people. How many tables will be required?
d Tony wants a seasonal fruit platter on each table. Calculate the extra cost.
e Calculate the total cost of food and drink for the birthday.

a Cost = number of guests × cost of silver package per person
= 120 guests × \$52
= \$6240

b Cost = number of guests × cost of premium beverage package per person
= 120 guests × \$40
= \$4800

c Tables required = number of guests ÷ number of people per table
= 120 ÷ 8
= 15 tables

d Cost of platters = number of tables × cost of platter
= 15 tables × \$50
= \$750

e Total cost = cost of food + cost of drink + cost of platters
= \$6240 + \$4800 + \$750
= \$11 790

WHAT TO DO 8.3

1 Jim has 110 guests coming to his birthday party at the Blue Shell reception centre.
a Find the cost of the gold package for food.
b Find the cost of the deluxe beverage package.
c The function centre has round tables that seat 10 people. How many tables will he need?
d Jim wants an antipasto platter on each table. Calculate the extra cost.
e Calculate the total cost of food and drink at the birthday party.

2 Kelly is having her 18th birthday party at the Blue Shell reception centre with 85 guests.
a Find the cost of the bronze package for food.
b Find the cost of the standard beverage package.
c The function centre has round tables that seat 12 people. How many tables will she need?
d Kelly wants a cheese platter on each table. Calculate the extra cost.
e Calculate the total cost of food and drink for the birthday party.

EXAMPLE 8B-2 Comparing costs

Andros and Mei are getting married and have been comparing prices at two function centres that offer comparable packages. They are having 100 guests.
Golden Bird Function Centre: offers a premium food package for \$47 per person and a drinks package for \$23 per person.
Brassy Bear Function Centre: offers a premium food package for \$51 per person and a drink package for \$18 per person.
Calculate the cost of each package and decide which option is cheaper.

NEED SOME PRACTICE?
Go to 23A Fundamental concepts PAGE 263

Golden Bird Function Centre
Total cost = (food package cost + drink package cost) × number of guests
= (\$47 + \$23) × 100
= \$70 × 100 = \$7000
Brassy Bear Function Centre
Total cost = (food package cost + drink package cost) × number of guests
= (\$51 + \$18) × 100
= \$69 × 100= \$6900
The Brassy Bear Function Centre is cheaper by \$100.

FOOD AND DRINK COSTS PER HOUR

When holding a shorter event, it may sometimes be more cost-effective to choose a package that charges for a set time rather than paying for a fixed-time event. Additional charges will apply for any time over the set amount.

EXAMPLE 8B-3 Costs per hour

The local law firm has booked a beer and wine with finger food package for their end of year function. They are expecting 100 people to attend. They have chosen a package for \$19.50 per person for one and a half hours. For every extra half hour they are charged \$5 for each person. If the event goes for two and a half hours, how much will the function cost?

Cost for 100 guests:
For one and a half hours = \$19.50 × 100
= \$1950
An additional half hour = \$5 × 100
= \$500
Two additional half hours = 2 × \$500
= \$1000
Total cost = \$1950 + \$1000
= \$2950
Total cost of the function is \$2950.

WHAT TO DO 8.4

1 a If a food package is \$62 per person and the drink package is \$39 per person, calculate the costs for:
i 100 guests ii 120 guests iii 150 guests.
b If a food package is \$85 per person and the drink package is \$54 per person, calculate the costs for:
i 80 guests ii 110 guests iii 240 guests.

2 The Mojo company is having a casual Christmas party for their 800 staff. They decide on a food and drink package for \$21.59 for the first hour and \$8 for any extra half hours. What is the cost if the party goes for:
a 1 hour b $1\frac{1}{2}$ hours c 2 hours

STAFFING RATIOS

Function centres, hotels and cafés have standard ratios of how many staff are required to work at particular times or events. Too many staff would add to costs and affect profit, whereas not enough staff would cause customer dissatisfaction and may lead to lack of future custom. Staff ratios differ depending on the type and time of an event, such as a sit-down or cocktail event. Example 8B-3 shows how event ratios are calculated.

EXAMPLE 8B-4 Staffing ratios

If the number of waiting staff required for a cocktail party is 1 server for every 30 guests, how many waiting staff are needed at a cocktail event with:
a 60 guests? b 150 guests? c 250 guests? d 500 guests?

a The server staff to guest ratio is 1 : 30. Thus 60 guests would need 2 server staff.
b 150 guests would require $\frac{150}{30} = 5$ staff
c 250 guests would require $\frac{250}{5} = 8.333$ staff (this would round up to 9 staff)
d 500 guests would require $\frac{500}{30} = 16.67$ staff (this would round up to 17 staff)

NEED SOME PRACTICE?
Go to 24D Ratios
PAGE 285

NOTE
Always remember that when it comes to people, you round up.

WHAT TO DO 8.5

1 The number of bar staff required for a sit-down dinner is 1 bartender to 100 guests. How many bar staff are required for an event with:
a 99 guests? b 101 guests? c 250 guests?

2 The number of food servers for a sit-down buffet lunch is 1 server to every 25 people. How many food servers are required for an event with:
a 40 guests? b 75 guests? c 120 guests?

3 The number of waiting staff required for a buffet breakfast is 1 server to 35 guests. How many waiting staff are required for an event with:
a 25 guests? b 65 guests? c 105 guests?

STAFF WAGES

Some people may decide to have special events in their own home and have a catering company come to them. The table below shows some rates catering companies may charge to have people come to your home to cook, serve food or tend a bar.

Staff rates: minimum of 4 hours

	Monday–Friday	Saturday	Sunday	Public holiday
Chef	\$50	\$56	\$56	\$85
Server (waiter/waitress)	\$44	\$50	\$56	\$81
Bartender	\$46	\$48	\$54	\$79

EXAMPLE 8B-4 Staff wages

Calculate how much it would cost to pay a chef, a server and a bartender for 4 hours on a Saturday.

Cost for a chef: $\$56 \times 4 = \$224$
Cost for a waiter: $\$50 \times 4 = \$200$
Cost for a bartender: $\$48 \times 4 = \$192$
Total cost for staff $= \$224 + \$200 + \$192$
$= \$616$

WHAT TO DO 8.6

1 Calculate how much it would cost to pay the following staff.
  a a chef for 5 hours on a Monday
  b a waiter for 4 hours on a Saturday
  c a bartender for 3 hours on a Sunday
  d a chef for 5 hours on a public holiday

2 Jane and Sandeep are having their wedding in a marquee. The catering company has quoted them \$135 per head for a food and drink package for 100 guests, but they have to pay for staff separately. The catering company requests the following staffing ratios: 1 chef per 40 guests, 1 waiter per 20 guests and 1 bartender per 100 guests.
  a How many staff will they require?
  b Using the table of staff rates above, how much will the staff cost?
  c How much will the catering company charge for the food and drink package plus the staff?

8C Calculating profit and loss

Break-even analysis is a tool that can be used to evaluate whether a business venture or, in this case, a social event will be profitable for the company or provider. It can also be used to help generate a selling price for a product or a ticket to an event. Before looking at break-even analysis, it is important to understand some vital terms.

NEED SOME PRACTICE?
Go to 33A Profit and loss PAGE 385

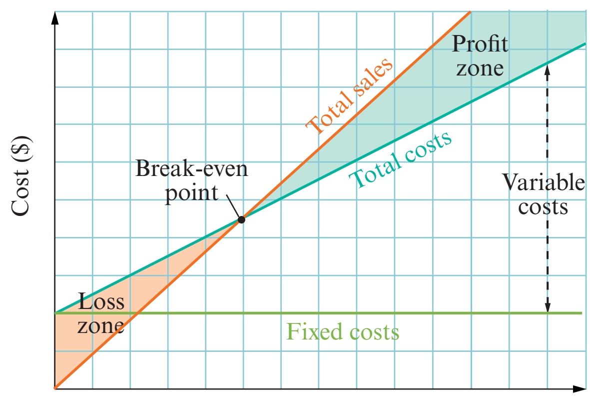
- Fixed costs are costs that are constant and are incurred regardless of the number of sales or products produced. For example, to rent a cupcake shop costs $400 per week regardless of how many cupcakes are sold; the rent still needs to be paid.
- Variable costs are costs that change depending on the number of sales. For example, the more cupcakes that are sold, the more ingredients that need to be purchased.
- The break-even point is where the revenue (income) equals the expenses (costs); that is, the point where no profit is made but nothing is lost either.

  To calculate the break-even point, use this formula:

$$\text{Break-even point} = \frac{\text{fixed cost}}{\text{selling price} - \text{variable costs}}$$

The break-even point can also be represented graphically as shown, where the relationship between fixed costs, variable costs and profit can be easily seen.

Profit and loss



PROFIT AND LOSS

Sometimes you need to calculate the amount of profit and percentage profit made if a certain number of people come to an event or you are making a product for sale. You may also need to determine the costs and profit to work out how much you should charge for an item or event.

Profit = selling price − cost price
Loss = cost price − selling cost

Cost price is how much it costs you to produce the item and selling price is how much you are selling the item for.

Formulas for the percentage profit or loss are:

$$\% \text{ profit} = \frac{\text{profit}}{\text{cost price}} \times 100\%$$

$$\% \text{ loss} = \frac{\text{loss}}{\text{cost price}} \times 100\%$$

EXAMPLE 8C-1 Calculating profit or loss

The Year 11 ball is coming up and your teacher has booked a function centre for the event with all profits of the event going towards a local charity. The venue is charging \$250 venue hire and \$45 per person for food and non-alcoholic drinks. There will be \$200 spent on promotion, including posters, invitations for staff and students, and tickets. There will also be \$550 spent on a DJ and \$100 on a photo booth with props. The tickets to the event will be sold for \$70.

a What is the fixed cost for the event?
b What are the variable costs if 100 people buy tickets?
c Calculate the ticket sales if 100 people buy tickets.
d Calculate the break-even point.
e Draw a break-even graph.
f Calculate the amount of profit made if 100 people come to the event.
g Calculate the percentage profit made if 100 people come to the event.

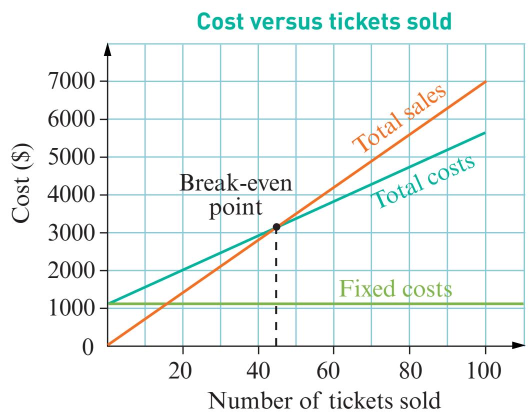
a The total fixed cost covers all the items except the food and drink costs.
Fixed costs = \$250 (hire) + \$200 (promotion) + \$550 (DJ) + \$100 (photo booth)
= \$1100

b The variable costs are food and drink for 100 people.
Variable costs = $\$45 \times 100 = \$4500$

c Ticket sales = $\$70 \times 100 = \$700$

d $$\text{Break-even point} = \frac{\text{fixed cost}}{\text{selling price} - \text{variable costs}}$$
$$= \frac{\$1100}{\$70 - \$45} = 44 \text{ tickets}$$
Any sales above 44 tickets will make a profit.

e



f Cost price = variable cost + fixed costs = \$4500 + \$1100 = \$5600
Selling price = ticket price × ticket sales = $\$70 \times 100 = \$7000$
Profit = selling price − cost price
= \$7000 − \$5600= \$1400

g $$\%\text{ profit} = \frac{\text{profit}}{\text{cost price}} \times 100\%$$
$$= \frac{\$1400}{\$5600} \times 100\% = 25\%$$

NEED SOME PRACTICE?
Go to 35D Linear modelling
PAGE 419

WHAT TO DO 8.7

1 The Year 12 valedictory dinner is on Friday night at a local function centre with all profits of the event going towards a local charity. The venue is charging $400 venue hire and $30 per person for food and non-alcoholic drinks. Tickets and promotion (posters, invitations for staff and students) will cost $150. The DJ for the event will cost $70 per hour. The photographer and photo booth will cost $45 per hour. The event is over 4 hours and tickets are to be sold for $70 each.
   - a What is the fixed cost for the event?
   - b What are the variable costs if 100 people buy tickets?
   - c Calculate the ticket sales for 100 people.
   - d Calculate the break-even point.
   - e Draw a break-even graph.
   - f Calculate the amount of profit made if 100 people come to the event.
   - g Calculate the percentage profit made.

2 Belalor Secondary College is holding their end of year Christmas play and they have booked a local theatre. They have the following expenses: theatre hire $110 per hour, technical staff $52 per hour, lighting and sound director $60 per hour, truck hire (delivery of set/props to and from theatre) $200, set construction $1500, costume hire and purchase $2500, make-up for performers $200 and promotion (posters, billboards and ticket construction) $250. Tickets to the event are $18.
   - a What are the fixed costs?
   - b If the event lasts for 4 hours, what are the fixed costs?
   - c What are the variable costs?
   - d Calculate the ticket sales if 400 people buy tickets.
   - e Calculate the break-even point.
   - f Draw a break-even graph.
   - g Calculate the amount of profit and percentage profit made if 350 people come to the event.

PROJECT 8

obook
assess
TEACHER

MAKING A NIGHT OF IT

Fun time for all?

A project worksheet for this chapter is available on the Teacher obook/assess. Ask your teacher to print it out for you. Completing this activity will help you make the right choices when organising a family or business event.

FINANCE

CHAPTER 9

Calculating your tax

ARE YOU READY?

Complete the questions below to see if you are ready to start this chapter or if you need to brush up on your skills first.

- Part 1 should be completed *with* a calculator.
- Part 2 should be completed *without* a calculator.

If there are any questions you cannot answer, the link below each question will direct you to a related Mathematical Skills chapter where you can revise basic concepts and get up to speed.

PART 1 WITH A CALCULATOR

1 Find the value of each of the following.

a $4 \times 376.25 =$

b $2387 \div 7.7 =$

c $4000 \times 2\frac{1}{8} =$

d $(87 \times 4.3) \div 125 =$

23A Fundamental concepts

2 Find one quarter of each of these numbers.

a 500 **b** $\frac{1}{2}$

c 2387 **d** 0.36

23C Fractions of quantities

3 Find 1.25% of each of these numbers.

a 157 234 **b** 97 630

c 2423 **d** 180 354

24B Percentage of a quantity

4 Find the value of the following.

a 10% of $12 000

b 33% of $55 000

24B Percentage of a quantity

5 Set your calculator to automatically round to 3 decimal places. Check that it does so by finding the answer to $6 \times 1.765\,321 =$

23E Rounding with a calculator

6 **a** What is gross pay?

b What is net pay?

c What is PAYG tax?

33F Deductions from earnings

PART 2 WITHOUT A CALCULATOR

7 Find the value of each of the following.

a $10 \times 12.53 =$

b $45 \div 15 =$

c $16 + 4 - 2 \times 6 =$

d $16 + \{(4 - 2) \times 6\} =$

28A Back to basics

8 Find 10% of each of the following numbers.

a 16 000 **b** 44 237

c 2423 **d** 180 354

28D Percentages

9 Convert the following fractions to decimals.

a $\frac{1}{7}$ **b** $\frac{1}{6}$

c $\frac{4}{9}$ **d** $\frac{2}{9}$

28B Fractions

10 Convert the following decimals to fractions.

a 0.75 **b** 0.333

c 0.44 **d** 0.88

28C Decimals

11 Convert these decimals to percentages of 1.

a 0.36 **b** 0.5

c 0.97 **d** 0.67

28D Percentages

12 Convert these fractions to percentages of 1.

a $\frac{1}{8}$ **b** $\frac{4}{5}$

c $\frac{1}{3}$ **d** $\frac{2}{5}$

28D Percentages

9A Taxable income

This chapter examines the meaning of taxable income and assesses how much income tax must be paid by an individual. Most people need to lodge a tax return each year, but there are some exceptions. If you had tax taken from any payment you received, you almost certainly need to lodge a tax return. If your taxable income was less than $18 200 during the 2014–15 income year, you may not need to lodge a tax return.

Taxation rates, levies and taxation laws change every year. This chapter uses rates current at the time of publication. Visit the Australian Taxation Office (ATO) website for current taxation information (www.ato.gov.au).

NEED SOME PRACTICE?
Go to 33F Deductions from earnings PAGE 398

Taxable income is calculated as:

Taxable income = total income − allowable deductions

TOTAL INCOME (GROSS INCOME)

The total income figure must include income obtained from all sources during the current financial year (1 July to 30 June). Total income could include:

- wages, salaries, commissions, bonuses and allowances
- pensions and any government payments
- any investment income, such as dividends, interest earned from financial institutions (or private loans) such as bank accounts, rental from investment properties
- capital gains from selling real estate, shares and managed fund investments
- business, partnership and trust income.

People who earn wages or salaries have tax deducted from their wages each time they are paid by their employers. This tax is called PAYG (pay-as-you-go) tax. The employer pays the tax to the Australian Taxation Office (ATO) on behalf of the employees.

ALLOWABLE TAX DEDUCTIONS

Tax deductions are allowable for certain expenses, mostly those used earning an income. Allowable deductions vary from one profession to another. Allowable deductions include:

- some or all of the cost of tools, equipment or other assets that help earn your income
- vehicle and other travel expenses directly connected with your work; not claims for normal trips between home and work
- buying and cleaning occupation-specific clothing, protective clothing and unique or distinctive uniforms
- self-education expenses if your study is work-related or if you receive a taxable bonded scholarship
- safety equipment you have purchased to be used in your employment
- expenses incurred in earning interest, dividends or other investment income (such as rental from investment properties)
- donations of $2 or more to approved charities
- union membership fees
- tax agent fees
- other expenses incurred in earning an income.

NOTE
Taxation forms are available online or may be completed by the individual or by their tax agent for a fee.

CALCULATING TAXABLE INCOME

To determine taxable income, you need to list all taxable income and subtract all allowable deductions.

EXAMPLE 9A-1 Calculating taxable income

During the financial year, Ashna received \$59 365.35 in wages and received a bonus of \$2500. She obtained \$1238.13 interest from a joint bank account with her husband. She had allowable deductions of \$458 for car use and \$300 for her uniform and its cleaning. She paid her tax agent a fee of \$135.

a What was Ashna's total income?

b What were her allowable deductions?

c What is her taxable income?

a Total income $= \$59\,365 + \$2500 + \frac{\$1238}{2}$

$= \$62\,484$

b Allowable deductions $= \$458 + \$300 + \$135$

$= \$893$

c Taxable income $=$ total income $-$ allowable deductions

$= \$62\,484 - \$893$

$= \$61\,591$

NEED SOME PRACTICE?
Go to 33F Deductions from earnings
PAGE 398

NOTE
Only whole dollar amounts are used in tax calculations; that is, no cents.

MEDICARE LEVY

The Medicare levy is an additional tax of 2% paid by most taxpayers to cover some of the cost of the public health system. If your income is below a certain threshold, the levy is reduced, and in some cases you may not have to pay the levy at all.

The table below shows how to calculate the Medicare levy for the 2014–15 tax year for a single person who has no dependants.

Taxable income for an individual	Rate of Medicare levy
Up to \$20 896	0
\$20 896–\$26 121	10% of the difference between your taxable income and \$20 896
Over \$26 121	2% of your taxable income

NEED SOME PRACTICE?
Go to 24B Percentage of a quantity
PAGE 281

For people who are paid wages or salaries (PAYG employees), the Medicare levy is taken out of their pay as part of their tax. Thus they are paying the Medicare levy throughout the year, out of their weekly, fortnightly or monthly pay.

There are many rules that apply to the Medicare levy with regards to reduction or exemption and other factors. For example, the thresholds in the table above will vary for families with dependants, pensioners and for people who do not qualify for Medicare benefits. For detail on the thresholds for these groups, go to the website https://www.ato.gov.au/Individuals/Medicare-levy/.

NOTE
PAYG stands for pay as you go. Employers deduct tax on a weekly, fortnightly or monthly basis.

Medicare surcharge

A Medicare surcharge, which is in addition to the Medicare levy, applies to individuals who have incomes above $90 000 (in 2014–15) where the individual does not have private health insurance. For details go to the website https://www.ato.gov.au/Individuals/Medicare-levy/Medicare-levy-surcharge/.

WHAT TO DO 9.1

1 Imagine you are a tax agent. Determine the taxable income for the following six people, given their income and deduction figures below. This must be accurate as you will be using these figures later in the chapter.

	A Munn	P Smith	L Jaura	S Pulgies	M Devine	P Ng
Wage/salary	$46 146	$51 649	$33 428	$95 236	$84 376	$135 096
Allowances	$1 029	$804		$1 448		
Interest	$172	$856	$478	$1 454	$4 212	$2 768
Bonuses			$2 000		$1 000	$7 000
Uniforms	$300	$275		$190		$770
Union fees	$250	$297	$144		$636	
Donations	$100	$146		$950	$1 260	$3 400
Use of car		$760		$3 274		$1 884
Tax agent fee	$140	$180	$140	$270	$240	$360
Total income						
Total deductions						
Taxable income						

2 Examine the current Tax Pack for individuals published by the Australian Taxation Office (either ask the ATO to send you a copy or examine the details online). In particular, look for allowable deductions.

3 Select two different jobs or professions and examine which allowable deductions apply to them.

4 Visit the internet site for the Australian Taxation Office at www.ato.gov.au and check out the details of the Medicare levy and the Medicare surcharge.

9B Calculating income tax

The calculation for how much income tax must be paid to the Australian Taxation Office is done using a table that is generally updated annually. The following table gives income tax rates for the 2014–15 financial year. To check the tax rates for the current year go to https://www.ato.gov.au/Rates/Individual-income-tax-rates/.

Income	Marginal tax rate	Tax payable
\$0 – \$18 200	0%	Nil
\$18 201 – \$37 000	19%	19 c for each \$1 over \$18 200
\$37 001 – \$80 000	32.5%	\$3572 plus 32.5c for each \$1 over \$37 000
\$80 001 – \$180 000	37%	\$17 547 plus 37c for each \$1 over \$80 000
\$180 001 and above	47%	\$54 547 plus 47c for each \$1 over \$180 000

These income tax rates do not include the Medicare levy, but they do include the 2% Temporary Budget Repair Levy for incomes over \$180 000, taking the top rate to 47%.

Example 9B-1 shows how this tax table is used to calculate the income tax payable by an individual taxpayer for given earnings.

EXAMPLE 9B-1 Calculating tax payable

What tax is payable for the following individuals (not including the Medicare levy)?

a Pardeep who has a taxable income of \$17 245
b Sabina who has a taxable income of \$45 046
c Mark who has a taxable income of \$188 462

Use the income tax table above to answer these questions.

a \$17 245 is below the minimum taxable threshold of \$18 200, so Pardeep will not pay any tax.

b \$45 046 lies between \$37 000 and \$80 000. *3rd row of income tax table*
Sabina pays \$3572 plus 32.5c for each \$1 over \$37 000.
\$45 046 − \$37 000 = \$8046
\$8046 × 0.325 = \$2615
\$3572 + \$2615 = \$6187
Sabina's tax payable is \$6187.

c \$188 462 lies above \$180 000. *5th row of income tax table*
Mark pays \$54 547 plus 47c for each \$1 over \$180 000.
\$188 462 − \$180 000 = \$8462
\$8462 × 0.47 = \$3977
\$54 547 + \$3131 = \$20 678
Mark's tax payable is \$58 524.

NEED SOME PRACTICE?
Go to 28A
Back to basics
PAGE 325

WHAT TO DO 9.2

1 Calculate the tax payable (not including the Medicare levy or surcharge) for the six people for whom you have found the taxable incomes in the table from What to do 9.1.

2 Assuming no deductions, determine the income tax payable (not including the Medicare levy or surcharge) by:
  a the prime minister, with an annual salary of $507 338
  b a doctor, with an annual salary of $178 000
  c an applied mathematician, receiving $75 000 annually
  d an electrician, receiving $82 500 annually.

CALCULATING TAX PAYABLE USING A SPREADSHEET

obook
An Excel spreadsheet template to help you calculate tax payable is available on your obook.

You can set up a spreadsheet to calculate the tax payable on various incomes.

Step 1: Into cell A1 type 'Taxable income'.

Step 2: Into cell B1 type '$35 641.00'.

Step 3: Into cell A2 type 'Income tax payable'.

Step 4: Into cell B2 type the following formula. Check it very carefully.

```
=IF(B1>180000,(B1-180000)*0.47+54547,
IF(B1>80000,(B1-80000)*0.37 +17547,
IF(B1>37000,(B1-37000)*0.32.5+3572,
IF(B1>18200,(B1-18200)*0.19,IF(B1>0,0)))))
```

Make sure you widen column B in your spreadsheet to accommodate the size of the formula. You should get the following spreadsheet.

	A	B	C
1	**Taxable income**	$35 641.00	
2	**Income tax payable**	$3313.79	
3			

WHAT TO DO 9.3

1 Use your spreadsheet to check the income tax payable for Pardeep, Sabina and Mark from Example 9B-1.

2 Use your spreadsheet to check the income tax payable for the six people from the table in What to do 9.1. Does this match your answers in What to do 9.1?

3 Someone says to you that they pay tax at '47 cents in the dollar'. Discuss the misleading feature(s) of this statement.

4 Find the starting salaries for various professions, jobs and apprenticeships. Determine the income tax payable (assuming no deductions) and the approximate take-home pay.

9C Extra tax or refund?

You are now at the stage where you can calculate whether you have to pay extra tax to the Australian Taxation Office or whether you will receive a refund. At each pay, your employer takes out PAYG tax instalments and you need to check if these match your estimates.

The Australian Taxation Office provides taxation instalment schedules that give employers the amount of tax to deduct from an employee's wages. These schedules are available for weekly, fortnightly and monthly pay intervals.

The total of the PAYG instalments appears on the employee's group certificate supplied to the employee at the end of the financial year. Here is a typical group certificate. Notice that Janine had $20 678 tax taken from her pay during the year. Also notice that her gross payment for the year is $88 462.

PAYG payment summary – individual non-business

Payment summary for the year ending 30 June 2015

Payee details:

Janine Wills
3 Plum Street
Jamville
Vic. 3999

NOTICE TO PAYEE

If this payment summary shows an amount in the total tax withheld box you must lodge a tax return. If no tax was withheld you may still have to lodge a tax return.

For more information on whether you have to lodge, or about this payment and how it is taxed, you can:

- Visit www.ato.gov.au
- Refer to TaxPack
- Phone 13 28 61 between 8.00 am and 6.00 pm (EST) Monday to Friday

Period of payment: Day/Month/Year 01/07/2014 To Day/Month/Year 30/06/2015

Payee's tax file number: 333 444 555

TOTAL TAX WITHHELD $ 20678

		Lump sum payments		Type
Gross payments	$ 88462	A $		
CDEP payments	$	B $		
Reportable fringe benefits amount	$	D $		
Reportable employer superannuation contributions	$	E $		
Total allowances	$	Total allowances are not included in Gross payments above.		

FINAL CALCULATIONS

To calculate if there is extra tax to pay or a refund, follow these steps.

Step 1: Find the taxable income.

Step 2: Find the tax payable on the taxable income using the table in Section 9B.

Step 3: If the tax payable is greater than the tax instalments, extra tax must be paid.
If the tax payable is less than the tax instalments, a refund is due.

Step 4: Calculate the extra tax payable or find the refund.

EXAMPLE 9C-1 Extra tax or a refund

Jasmine receives an annual salary of \$28 694 and earns an extra \$1475 for playing drums in a rock band over the tax year. Her allowable deductions total \$976. During the year, she has \$3354.40 taken out of her salary in PAYG tax instalments. Will Jasmine get a tax refund or will she have to pay extra tax to the ATO?

Step 1: Taxable income = total income − allowable deductions
= \$28 694 + \$1475 − \$976
= \$29 193

Step 2: Tax payable = (\$29 193 − \$18 200) × 0.19 (2nd row of income tax table)
= \$10 993 × 0.19
= \$2088.67

Step 3: Tax instalments = \$3354.40
The tax paid is greater than the tax payable so a refund is due.

Step 4: Refund = \$3354.40 − \$2088.67
= \$1265.73

WHAT TO DO 9.4

1 Phillip is an arborist. His annual salary is \$68 457 and he receives incomes of \$4124 and \$1256 from other sources. His allowable tax deductions total \$1973, and during the year he paid \$2480.26 in PAYG tax instalments. Follow the four steps above.
  a What is Phillip's taxable income?
  b Find the tax payable on Phillip's taxable income.
  c Does he have to pay extra tax or will he get a refund?
  d How much is the refund or extra tax payable?

2 Examine the group certificate shown on the next page.
  a What is the gross income?
  b What is the taxable income if deductions are \$3164?
  c What PAYG tax instalments have been paid?
  d Find the tax payable on the taxable income.
  e Is a tax refund due or extra tax payable?
  f How much is the refund or extra tax payable?

PAYG payment summary – individual non-business

Payment summary for the year ending 30 June 2015

Payee details:

Mary-Anne Glen
3 Jones Street
Smithville
Vic. 3000

NOTICE TO PAYEE

If this payment summary shows an amount in the total tax withheld box you must lodge a tax return. If no tax was withheld you may still have to lodge a tax return.

For more information on whether you have to lodge, or about this payment and how it is taxed, you can:

- Visit www.ato.gov.au
- Refer to TaxPack
- Phone 13 28 61 between 8.00 am and 6.00 pm (EST) Monday to Friday

	Day/Month/Year		Day/Month/Year
Period of payment	01/07/2014	To	30/06/2015

Payee's tax file number 444 555 6666

TOTAL TAX WITHHELD $ 20347

		Lump sum payments		Type
Gross payments	$ 87567	A $		
CDEP payments	$	B $		
Reportable fringe benefits amount	$	D $		
Reportable employer superannuation contributions	$	E $		
Total allowances	$			

Total allowances are not included in Gross payments above.

3 Find the extra tax payable or refund due for the original six people in the table in What to do 9.1, given that their annual PAYG tax instalments were:

A Munn	$7042	P Smith	$8013
L Jaura	$2500	S Pulgies	$19 371
M Devine	$22 104	P Ng	$42 050

PROJECT 9

obook
assess
TEACHER

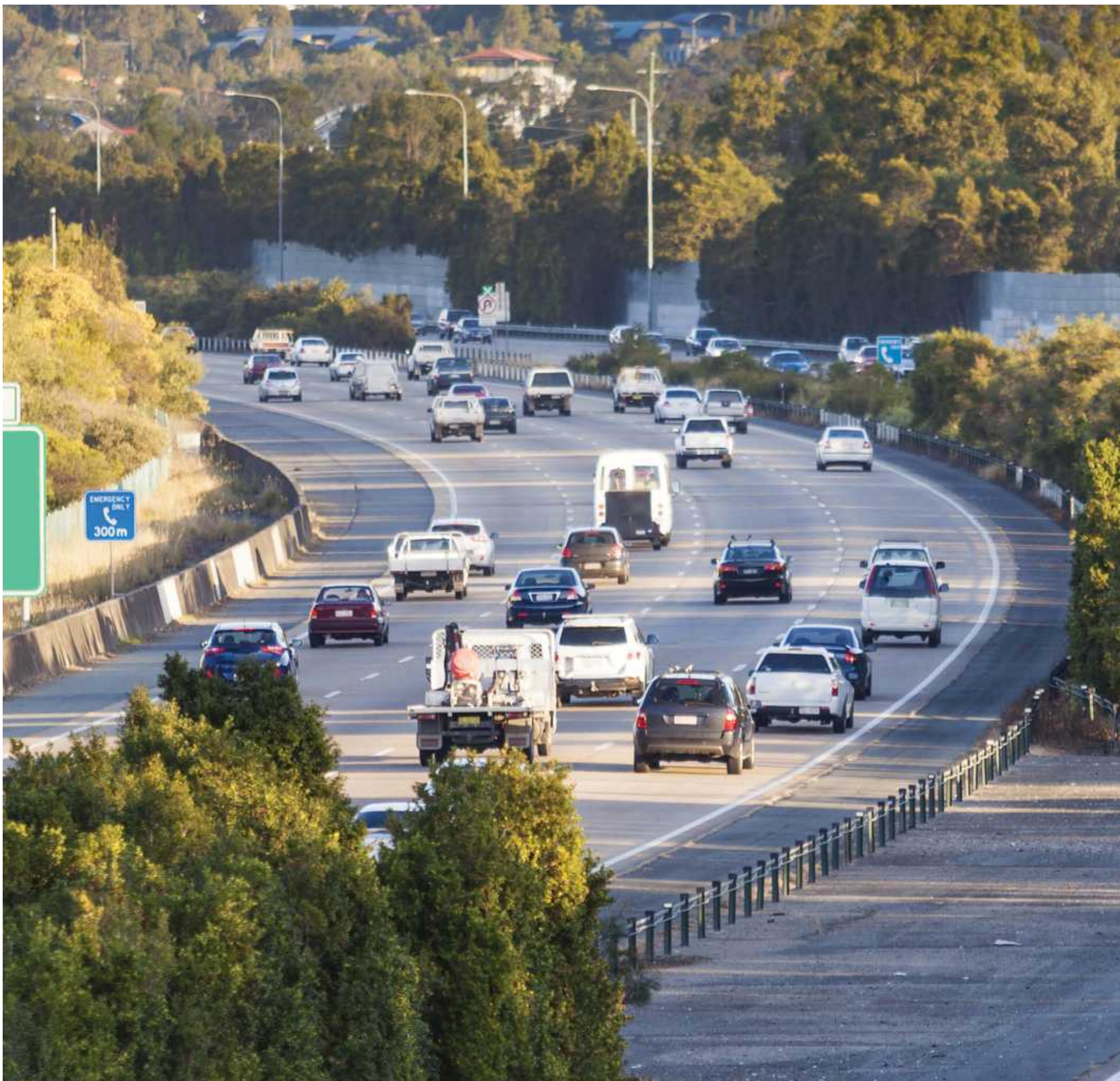
END OF FINANCIAL YEAR DELIGHTS

Taking the 'yuck' out of tax!

A project worksheet for this chapter is available on the Teacher obook/assess. Ask your teacher to print it out for you. Completing this activity will help you make the right choices when organising, keeping and maintaining your tax records.

CHAPTER 10

Buying and running a car



ARE YOU READY?

Complete the questions below to see if you are ready to start this chapter or if you need to brush up on your skills first.

- Part 1 should be completed *with* a calculator.
- Part 2 should be completed *without* a calculator.

If there are any questions you cannot answer, the link below each question will direct you to a related Mathematical Skills chapter where you can revise basic concepts and get up to speed.

PART 1 WITH A CALCULATOR

1 Find the value of each of the following.

a $\frac{1}{4}$ of 3458 =

b $7 + 12 \times 3.2 =$

c $25 \div 7.7 =$

d $754.8 \times 50 =$

23A Fundamental concepts

2 The metric unit most commonly used in Australia for measuring the volume of petrol used in a car is:

A kilometre **B** litre

C pint **D** barrel

25A Units of measurement

3 Find 17% of each of the following numbers.

a 750 **b** 660

c 860 **d** 1220

24B Percentage of a quantity

4 If the fuel gauge reading shown on the right is from a 45 L tank, approximately how much fuel is left in the tank?

25B Reading instruments and meters

5 Using a table, calculate the final cost of a \$5000 car loan for 3 years at 15% compound interest calculated annually.

34C Compound interest (by tables)

PART 2 WITHOUT A CALCULATOR

6 Find the value of each of the following.

a $6 \times 33.333 =$

b $77 \div 7 =$

c $12 - 8 + 4 \times 5 =$

d $(12 - 8 + 4) \times 5 =$

28A Back to basics

7 Write the value of the 3 in these numbers.

a 4321 **b** 3056

c 0.893 **d** 2003.06

28A Back to basics

8 Convert these fractions to decimal numbers.

a $3\frac{1}{4}$ **b** $\frac{6}{10}$

c $\frac{3}{5}$ **d** $1\frac{1}{2}$

28A Back to basics

9 Round 23 455 to the nearest:

a 10 **b** 100

23D Decimal numbers

10 Find the value of each of the following.

a 10% of 50 **b** 25% of 60

c 5% of 200 **d** 40% of 160

28D Percentages

11 Round the following numbers correct to 2 decimal places.

a 345.678 48 **b** 2.123 456

c 2.345 608 **d** 0.000 93

23D Decimal numbers

10A How much you can spend

Your own car. Freedom! No more buses. No more trams. No more, 'Mum, Dad, can I borrow your car or be driven somewhere?' Owning a car is the goal of many teenagers and young adults. But what does it cost to own a car? Can you afford it? Even if you have a car, do you use it all the time and forget about public transport, or are there times when taking the bus or the train is better?

There are many factors and costs involved in car ownership. This chapter looks at some of the costs of buying and running a car to help you begin to answer these questions. Section 10C lists several of the major costs of running a car for a year. If you live in an area where there is no public transport, or only limited services, then buying a car may not be a choice but a necessity. This section examines how much you can afford to spend on a car.

EXTRA COSTS

When buying a car, remember that the final price you pay the seller won't be your total out-of-pocket expense. Some of the extra costs are listed below.

Registration transfer

In Victoria, it will cost around $36.40 to transfer a car registration via a private sale, but it will be cheaper (around $18.50) if the car is purchased from a dealer. Note that some private sellers choose to sell their cars as the vehicle registration is about to expire, or has already expired, to save on the costs of renewal. So you may need to add the registration renwal as well as transfer to your purchase price.

Certificate of Roadworthiness

Any car sold in Victoria needs to have a Certificate of Roadworthiness. The cost will depend on the condition of the car. It is usually the seller's responsibility to provide and pay for a Certificate of Roadworthiness.

Stamp duty

Stamp duty is a state tax that is based on the market value of the vehicle or the price paid for the car. As of 1 July 2015, the stamp duty on new cars is:

- $6.40 for every $200 or part thereof (for a market value up to $63 184 or less)
- $10.40 for every $200 or part thereof (for a value over $63 184).

For used cars, stamp duty is $8.40 for every $200 or part thereof.

Car history report

If you are buying a used car, it may be worth running a vehicle check on the car's history. For about $37, you can run the Vehicle Identification Number (VIN) through a number of state and national databases to find out if a vehicle has any loans against it, has been written off or stolen, as well as getting a list of all known odometer readings to ensure that the car is what the seller says it is.

Don't buy a lemon!

You have found the car you want to buy, in the colour you want, with the accessories you want and even at a price you think you can afford. Do you accept the seller's word that it is a good car or should you have it checked by a licensed mechanic?

There are two checks that can be made on a car. The costs of these are given below for the RACV. In your calculations for this chapter, you should include the cost of a pre-purchase inspection.

- pre-purchase test for $218
- comprehensive (full workshop) inspections for $260

The other road assistance organisations have similar charges and policies at comparable costs.

BORROWING COSTS

Many people who buy a car do so by placing a deposit of their own money and taking out a loan from a bank or some other lending institution to pay the balance. Over a period of time, usually between 1 and 5 years, they pay back the money they have borrowed with interest. The amount of money a person can repay each month is what determines the length of time they take to pay off the loan and what sort of car they buy.

Deposit

Most lending institutions prefer people to pay a deposit on purchases that they need a loan for. Typically, the deposit is 10% of the total cost of the car.

$$\text{Deposit} = \text{total cost} \times 0.1$$

Balance

The balance is the amount owing after the deposit has been paid. This is the size of the loan you will need.

$$\text{Balance} = \text{total cost} - \text{deposit}$$

Loan repayments by table

Loans for items like cars are called personal loans. The interest payable on a personal loan is usually calculated on the outstanding (unpaid) balance each month. There are several ways of finding out how much the monthly payments should be. Here you will examine two methods: loan repayments tables and using a spreadsheet.

Some lenders have tables that show the interest rate and the term (the length of time the loan is for). In the body of the table is the amount of each monthly repayment for every $1000 of the original loan amount. The main disadvantages of interest rate tables are that they can be difficult to read and do not include all possible interest rate values.

Another way of calculating the monthly repayment on a loan is to use a spreadsheet. A spreadsheet can be tailored exactly to your situation and can be varied until you find the right mix of interest rate and term to suit you.

Loan term (months)	Annual interest rate 10.00%	10.50%	11.00%	11.50%	12.00%	12.50%	13.00%	13.50%	14.00%
12	$87.9159	$88.1486	$88.3817	$88.6151	$88.8488	$89.0829	$89.3173	$89.5520	$89.7871
18	$60.0571	$60.2876	$60.5185	$60.7500	$60.9820	$61.2146	$61.4476	$61.6811	$61.9152
24	$46.1449	$46.3760	$46.6078	$46.8403	$47.0735	$47.3073	$47.5418	$47.7770	$48.0129
30	$37.8114	$38.0443	$38.2781	$38.5127	$38.7481	$38.9844	$39.2215	$39.4595	$39.6984
36	$32.2672	$32.5024	$32.7387	$32.9760	$33.2143	$33.4536	$33.6940	$33.9353	$34.1776
42	$28.3168	$28.5547	$28.7939	$29.0342	$29.2756	$29.5183	$29.7621	$30.0071	$30.2532
48	$25.3626	$25.6034	$25.8455	$26.0890	$26.3338	$26.5800	$26.8275	$27.0763	$27.3265
54	$23.0724	$23.3162	$23.5615	$23.8083	$24.0566	$24.3064	$24.5577	$24.8104	$25.0647
60	$21.2470	$21.4939	$21.7424	$21.9926	$22.2444	$22.4979	$22.7531	$23.0098	$23.2683

NEED SOME PRACTICE?
Go to 34E Repaying a compounding loan
PAGE 408

EXAMPLE 10A-1 Calculating monthly repayments

Use the previous loan repayment table to find what would be the exact amount of a monthly repayment for a loan amount of $5500, with an interest rate of 11% compounding over a 24-month period.

Step 1: Find the 11.00% interest column.

Step 2: Go down until you reach the 24-month loan term row.

Step 3: This cell is your repayment per $1000, in this case $46.6078.

Step 4: Multiply this repayment per $1000 by the loan amount of $5500.
$\$46.6078 \times \$5500 = \$256\,342.9$

Step 5: Divide the answer by $1000 = $256.34

The monthly repayments would be $256.34.

It is important to know how to use interest rate tables so that you can understand what other people are charging you. If they make a mistake, you are the one who will pay, and no one else will be there to check if the car dealer's calculations are correct.

WHAT TO DO 10.1

1 From a newspaper or on the internet, find an advertisement for the sort of car you would like to buy. It should be one that you also think you can afford soon after leaving school.
  a For the car you have chosen, calculate the dollar amount of a 10% deposit.
  b Calculate the balance payable on your chosen car.

2 For your loan amount, work out the monthly repayments you would need to make in each of these situations.
  a 10% and for 3 years b 12.5% and for 4 years c 14% and for 5 years.

3 Using the previous loan repayment table, how much would you have to pay per month if you bought your car with no deposit at 14% interest compounding over 4 years?

Loan repayments by spreadsheet

The spreadsheet below can be used to calculate the month-by-month balance on a personal loan. The example is set up to calculate payments starting from an original balance of $1000. This will allow you to make sure your spreadsheet is correct by comparing it against the values in the previous loan repayment table.

When setting up the spreadsheet, you will need to enter the actual amounts in each of these cells.

In Cell B1, type in the total cost of the car ($1500).

In Cell B4, type in the amount you can pay up front ($500).

In Cell B8, type in the interest rate that is being charged on the loan (10%).

In Cell B10, type in the number of years that the loan is to last (1 year).

In Cell G3, type in the amount you think you can repay monthly.

Every other cell in the spreadsheet is to be calculated by the formulas given below. Click and fill down a column to fill all the cells below the first formula cell. When the new balance for any given cost reaches zero, you have found a combination of term and payment that will pay off your loan. Use the following table to complete your own spreadsheet.

	A	B	C	D	E	F	G	H
1	**Cost ($)**	**1500**		**Month**	**Balance**	**Interest**	**Payment per month**	**New balance**
2				0				=B6
3				1	=H2	=E3*($B$8/1200)	Write value	=E3+F3-G3
4	**Deposit ($)**	500		2	=H3	=E4*($B$8/1200)	=G3	=E4+F4-G4
5				3	=H4	=E5*($B$8/1200)	=G4	=E5+F5-G5
6	**Balance ($)**	=B1-B4		4	=H5	=E6*($B$8/1200)	=G5	=E6+F6-G6
7				5	=H6	=E7*($B$8/1200)	=G6	=E7+F7-G7
8	**Rate (%)**	10		6	=H7	=E8*($B$8/1200)	=G7	=E8+F8-G8
9				7	=H8	=E9*($B$8/1200)	=G8	=E9+F9-G9
10	**Term (year)**	1		8	=H9	=E10*($B$8/1200)	=G9	=E10+F10-G10

obook
An Excel spreadsheet to help you calculate loan repayments is available on your obook.

Your result should look like this.

	A	B	C	D	E	F	G	H
1	**Cost ($)**	**1500**		**Month**	**Balance**	**Interest**	**Payment per month**	**New balance**
2				0				1000
3				1	1000	8.333333	87.9159	920.42
4	**Deposit ($)**	500		2	920.42	7.670145	87.9159	840.17
5				3	840.17	7.001431	87.9159	840.17
6	**Balance ($)**	1000		4	759.26	6.327143	87.9159	677.67
7				5	677.67	5.647237	87.9159	595.40
8	**Rate (%)**	10		6	595.40	4.961665	87.9159	512.45
9				7	512.45	4.27038	87.9159	428.80
10	**Term (year)**	1		8	428.80	3.573334	87.9159	344.46

CAN YOU AFFORD THIS CAR?

Now that you have a monthly repayment, can you afford to pay it? You may already have worked through Chapter 1: Balancing your budget. If so, convert your monthly repayments to a weekly amount.

Find the weekly amount you need to put aside from your income using the formula:

$$\text{Weekly amount} = \frac{\text{monthly amount} \times 12}{52}$$

Can you spare this amount of cash every week for the entire term of the loan? If not, try a longer term or find a cheaper car. For example, Ali buys a very cheap car for $1500 with a deposit of $500. He decides to pay it off over 1 year at an interest rate of 10%. This means that his monthly repayments are $87.92.

From the interest table, this means that he needs to put aside:

$$\text{Weekly amount} = \frac{\$87.92 \times 12}{52} = \$20.29$$

Does your spreadsheet come up with the same amount?

WHAT TO DO 10.2

NOTE

Personal interest rates are always higher than home loan interest rates. Do not confuse the two.

1 Once you have checked that your spreadsheet works properly, put in the values for the car you selected at the start of the chapter.
   - **a** Try this first for an interest rate of 10%.
   - **b** Now find a current interest rate for personal loans from a newspaper or TV advertisement or from the internet.

10B Calculating fuel costs

The second major factor in being able to afford a car is the cost of fuel. To reliably estimate how much fuel will cost, you need to consider three variables:

- the average distance you travel each week
- the fuel consumption of the car you buy
- the price of fuel.

WEEKLY TRAVEL ESTIMATE

Where do you go in a week? How often do you go to each of these places?

Ali lives in Buckland Road and works in Potter Street. She has marked the positions of her house and workplace as stars on the map. She then traces the roads and streets that she uses to get from home to work.

Using the scale at the bottom of the map, Ali measures the distance between home and work to be about 5 km. Check her measurements and see if you can get a more accurate estimate of the distance.

You may need to work with several maps to find your own distances.

WHAT TO DO 10.3

1 Make a table and list all the places that you go to most weeks. Use this table to help you get started. In column 2 write how many times you go to each place in a week.

Place	Number of visits/week	Distance from home (km)
School		
Friends' places		
Work (part-time)		
Relatives' homes		
Shopping mall		
Sporting venues		

2 Using a street directory and a ruler, measure the distance from your home to each place. Fill in column 3. Check your answers using a service such as Google maps or a GPS.

NEED SOME PRACTICE?
Go to 31B Scale drawings PAGE 362

NEED SOME PRACTICE?
Go to 25B Reading instruments and meters PAGE 294

EXAMPLE 10B-1 Weekly travel estimate

Use the table below to write an expression for finding out how far Ali travels in a week. Include the number of trips made to each place and back, and the distance to each place and back. Discuss this as a group and determine the total distance.

Place	Number of visits/week	Distance from home (km)
School	5	3
A friend's place	1	2
Another friend's place	1	10
Work (part-time)	3	5
A relative's home	1	7
Shopping mall	1	5
A sporting venue	3	3

$$\begin{aligned}\text{Total distance} &= (5 \times 3 + 2 + 10 + 3 \times 5 + 7 + 5 + 3 \times 3) \times 2 \\ &= 126 \text{ km}\end{aligned}$$

FUEL USED

The fuel consumption of a car is the amount of fuel (L) a car uses to travel 100 km.

$$\text{Fuel consumption (L/100 km)} = \frac{\text{litres used}}{\text{100 km travelled}}$$

The table below lists the average fuel consumption for some cars with different-sized engines. For the car you have chosen, identify the engine size and use the table to find the fuel used for both city and highway cycles.

Average fuel consumption for different-sized cars

Engine size	City cycle (L/100 km)	Highway cycle (L/100 km)
Light (1.3 to 1.5 L)	8.4	6.9
Small (1.5 to 1.8 L)	9.5	7.7
Medium (2 to 2.4 L)	11.3	8.9
Upper medium (3 to 3.8 L)	14.5	11.2
4-wheel drive (3.3 to 4.6 L)	18.7	14.7

The city cycle is the distance travelled in towns or cities, where stopping and starting is frequent. The speed limits for city-cycle driving are 80 km/h and under. To work out how many litres of fuel you will use in city-cycle driving per week, use the formula:

$$\text{Fuel used (L)} = \text{distance travelled (km)} \times \frac{\text{city-cycle consumption (L)}}{100}$$

The highway cycle is the distance travelled on highways, where the speed limits are over 80 km/h and stopping is rare. This includes freeways, tollways and good country roads. To work out how many litres of fuel you will use in highway-cycle driving per week, use the formula:

$$\text{Fuel used (L)} = \text{distance travelled (km)} \times \frac{\text{highway-cycle consumption (L)}}{100}$$

EXAMPLE 10B-2 Weekly fuel usage

Of Ali's 126 km travelled per week, 92 km are in the city cycle and 34 km are in the highway cycle. Find her weekly fuel usage (L) if she has a medium-sized car.

Fuel used (city) $= 92 \times \frac{11.3}{100} = 10.4$ L

Fuel used (highway) $= 34 \times \frac{8.9}{100} = 3.0$ L

Total fuel used in a week $= 10.4 + 3.0 = 13.4$ L

PRICE OF FUEL

The price of fuel varies on a daily basis. It may also vary between service stations along the same road on the same day. Many cars run on 91-octane unleaded fuel, although there are a lot of cars that use higher grades (such as 95- or 98-octane or premium) for which you pay extra. Many cars also use diesel fuel.

It is now possible to find the fuel costs per week.

Weekly fuel cost = total fuel used (L) × cost/L

EXAMPLE 10B-3 Weekly fuel costs

Ali sees the following prices at her nearest (and cheapest) service station. How much does she need to spend on 13.4 L fuel if she chooses 91-octane unleaded petrol?

Fuel	Cost
ULP (91-octane)	127.9c
ULP (95-octane)	138.9c
ULP (98-octane)	144.9c
Diesel	146.5c
LPG	81.8c

13.4 L × \$1.279/L = \$17.14 per week

WHAT TO DO 10.4

1 On your way to or from school, watch for and record the prices of 91-octane unleaded petrol at five different service stations. Find the average cost per litre for unleaded petrol.

2 Add up your weekly repayment cost, weekly fuel cost and the cost of a pre-purchase inspection. You will need to convert the inspection cost to a weekly amount.

10C Other major costs

In addition to the cost of buying and running a car, there are a number of other costs associated with owning a car. Some are discussed below.

Registration

In order to legally drive a car on public roads in all states and territories across Australia, the car must be registered. Registration fees can vary depending on the make, model and size of the car, but an average registration fee is around $760 per year. A large portion of the registration fee in Victoria goes towards funding the Traffic Accident Commission (TAC), which manages a fund to pay for the rehabilitation of people who have suffered injuries from accidents on Victorian roads.

Vehicle damage insurance

There are two main types of insurance to cover vehicle damage.

- Third-party property insurance covers you for damage that your car does to other people's property. It does not cover damages to your car in the event of an accident. Insurance costs depend on the market value of the car, the age and gender of the owner, the experience of the driver and the area where the car resides.
  For a medium-risk suburb and a car worth $3000, third-party property insurance may cost around $500 per year for an 18-year-old male and around $420 per year for an 18-year-old female.
- Comprehensive insurance covers damage to both the car and other property you damage, and your own vehicle. For drivers under 20 years old, this type of insurance can be prohibitively expensive. It is often better to drive a cheaper car than needing comprehensive insurance to replace your car in the event of an accident that is your fault.

Servicing and repairs

Every 6 months or 8000–10 000 km, your car will need to be serviced to keep it running well. The cost of an average service at a garage is around $200. A car usually needs to be serviced about twice a year.

Cars wear out and parts need to be replaced. For example, tyres need to be replaced every 3 or 4 years (between 40 000 km and 60 000 km) as a general rule. Tyres cost an average of $200 each, although prices can vary enormously depending on the vehicle and choice of tyre.

Other parts also need to be replaced during the life of the car and can be expensive. It is not possible to predict when a part will need replacing, so a car owner needs to have money on hand just in case. Repair costs depend on several factors, such as the make and model of the car, the brand of the replacement part, the company you choose to deal with and the cost of the labour (set by the garage).

Some common mechanical repairs may include the brakes (front disc, back drum), the muffler, the tyres, the injector pump or injector overhaul, the front ball joint (steering) or a pair of front suspension struts. You may wish to get a quote from a dealer for one of these items for your chosen make and model car.

Speeding fines

A speeding fine is another cost that you may incur. If you are a conscientious driver, you will never have to pay a speeding fine. However, for the purpose of this section, assume that you receive a speeding fine once in the first 12 months of owning your car. Use the formula below in a spreadsheet to randomly select the speed range you are booked at.

=INT(RAND()*6+1)

How would you do this on your calculator?

Penalties for speeding (as at 1 June 2015)

	Excess over the speed limit	Penalty	Demerit points	Automatic licence suspension
1	By less than 10 km/h	$185	1	
2	10 km/h – 24 km/h	$295	3	
3	25 km/h – 29 km/h	$406	4	1 month
4	30 km/h – 34 km/h	$480	4	1 month
5	35 km/h – 39 km/h	$554	6	6 months
6	40 km/h – 44 km/h	$627	6	6 months
7	45 km/h or more	$738	8	12 months

NOTE

A maximum of 12 points can be lost in a 12-month period before a full licence is revoked. P-plate drivers have more severe rules.

WHAT TO DO 10.5

1 Estimate the weekly cost of running the car you want to buy.

2 For a particular make/model of car, compare the vehicle damage insurance costs for a 19-year-old male and a 19-year-old female for:
 a third-party property insurance
 b comprehensive insurance.

3 Compare this with the cost of a weekly myki ticket to travel the same distance on trains, trams and buses. Is it worth it?

Weekly myki ticket as of Jan 2015

Fare	Zone 1 and 2	Zone 2
Full fare	$37.60	$26
Concession	$8.80	$13

PROJECT 10

KEEPING THE CAR

Are my running costs affordable?

A project worksheet for this chapter is available on the Teacher obook/assess. Ask your teacher to print it out for you. Completing this activity will help you make the right choices when buying and maintaining your own car.

obook
assess
TEACHER

CHAPTER 11

Your carbon footprint

ARE YOU READY?

Complete the questions below to see if you are ready to start this chapter or if you need to brush up on your skills first.

- Part 1 should be completed *with* a calculator.
- Part 2 should be completed *without* a calculator.

If there are any questions you cannot answer, the link below each question will direct you to a related Mathematical Skills chapter where you can revise basic concepts and get up to speed.

PART 1 WITH A CALCULATOR

1 Find the value of each of the following.

a 1 + 25 + 298 + 70 + 11 700 + 23 900 =

b (10.7 × 11 700 + 12 × 23 900) ÷ 1000 =

c (10 + 25) × 298 − 1000 =

d (12 + 3.5) × 25 − 1000 =

23A Fundamental concepts

2 Set your calculator to automatically round to 2 decimal places and check that it does so by finding the answer to 6 × 1.135 79 =

23E Rounding with a calculator

3 There are ___ kilograms in 1 metric tonne.

A 1000 000 **B** 100 000

C 10 000 **D** 1000

25A Units of measurement

4 Complete the following table (the first row is done for you).

Prefix	Symbol	Times base unit
kilo-	k	1000 ×
giga-	G	
mega-		

25A Units of measurement

5 a Car A is 40% more fuel efficient than car B. If car B uses 1245 L of fuel to travel 10 000 km, how many litres does car A use to travel the same distance?

b Petrol prices are expected to rise by 33% over 12 months. If the current price is $1.20 per L, what is the new price?

24C Percentage change

6 A hairdryer is rated at 2000 W and 240 V.

a How many kilowatts (kW) does this hairdryer draw?

b If this hairdryer is run for half an hour, how many kilowatt hours (kWh × number of hours) have been used?

25A Units of measurement

PART 2 WITHOUT A CALCULATOR

7 Find the value of each of the following.

a (10 − 5) ÷ (2 + 3) =

b 10 − 5 ÷ 2 + 3 =

c 1200 + 23 000 − 1500 =

d 100 + 40 000 − 1000 =

28A Back to basics

8 Find 10% of each of the following numbers.

a 16 000 **b** 44 237

c 2423 **d** 180 354

28D Percentages

9 Convert the following fractions of an hour to decimals.

a $\frac{1}{3}$ h **b** $\frac{1}{4}$ h

23D Decimal numbers

10 Write the value of the 5 in these numbers.

a 4321.005 **b** 4.501

c 245 **d** 33 350

28E Powers of 10

11 You need to add up the following weights, in tonnes. Estimate the total weight.

95 t, 12 t, 2 t, 8.3 t, 2.8 t

23F Estimation

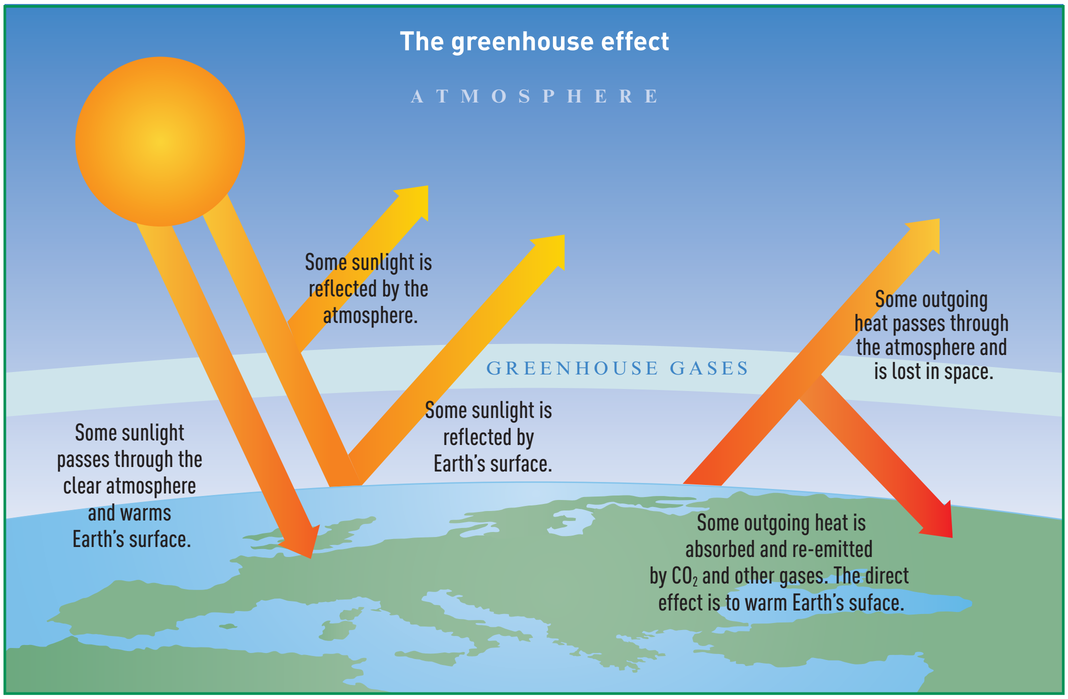
11A CO_2 equivalents and climate change

The effects of human-induced climate change are now being acknowledged worldwide, and most countries are putting strategies in place to try to combat its effects. From turning on the lights at home or work to driving a car or buying a newly manufactured product, almost everything involves the use of energy to do, produce or deliver. Since most energy in Australia is still produced by burning fossil fuels (mainly coal, oil and gas), carbon dioxide (CO_2) is released as a by-product of this energy production. As a result, the rising levels of CO_2 and other greenhouse gases in the atmosphere act like the glass roof of a greenhouse: more of the Sun's light that would have reflected back into space becomes trapped inside the atmosphere and then heats up Earth's surfaces (see the simplified greenhouse effect diagram on the next page).

The primary greenhouse gases in Earth's atmosphere are water vapour, carbon dioxide, methane, nitrous oxide and ozone. Having some CO_2 (and other greenhouse gases) in the atmosphere is very important: without any the planet would be completely frozen over! But it is the increasing amounts, due to human activity, that are added to the natural levels that is the issue.

The consequences of this rise in CO_2 and other greenhouse gas levels are important. These include the following:

- More energy is available to drive weather systems, resulting in larger and more ferocious storms, plus disturbance of the established weather patterns, making some areas of the globe hotter, some colder, some wetter and some drier.
- More CO_2 is dissolved into the ocean, acidifying the sea and reducing the capacity of corals and shellfish to build their bodies.
- The ocean's waters are expanding as they warm due to increased atmospheric heat, leading to rising sea levels that threaten entire nations on low-lying islands in the Pacific and Indian oceans.



- The increased atmospheric heat is disturbing the equilibrium of melting/refreezing of glaciers and the polar ice caps, resulting in a net loss of ice coverage on Earth. Not only does the loss of ice caps and glaciers raise the levels of the oceans, it also means the white areas of the Earth are becoming smaller; darker areas absorb heat more that white areas, thereby causing even more heating of the planet.

As you can imagine, many areas of mathematics are involved in the understanding of the greenhouse effect, global warming and climate change. This chapter looks at the mathematics needed to work out answers to two parts of this problem:

- Just how much greenhouse gas do you release from your homes and by driving your cars?
- Are there ways you can reduce the amount of greenhouse gas you are responsible for?

GREENHOUSE GAS EMISSIONS

In 2005, an international agreement (the Kyoto Protocol) to limit human-created greenhouse gas emissions came into effect. This agreement covered six of the major greenhouse gases. CO_2 is by far the largest volume of greenhouse gas emitted by humans, but the other five listed on the next page can have, for the same mass, many times the effect of CO_2. The greenhouse effect of each greenhouse gas can be expressed in terms of the climate change effects of an equivalent mass of CO_2. This is sometimes referred to as CO_2 equivalents or CO_2-e.

The six gases and their greenhouse potential compared to the same mass of CO_2 are shown in the following table.

Kyoto Protocol greenhouse gases	Chemical formula	Greenhouse effect (compared to 1 kg of CO_2)	Uses/sources
Carbon dioxide	CO_2	1 kg	Fossil fuel combustion, land-use change, cement
Methane	CH_4	25 kg	Natural gas, enteric fermentation, anaerobic decomposition
Nitrous oxide	N_2O	298 kg	Fertilisers, fossil fuel combustion
Perfluorocarbons	C_3F_8	8830 kg	Electronics, cathodes for aluminium manufacture
Hydrofluorocarbons	CHF_3 (HFC_{23})	14 800 kg	Refrigerant
Sulfur hexafluoride	SF_6	22 800 kg	High-voltage switchgear, manufacture of magnesium

As can be seen from the above table, CO_2 is not the only greenhouse gas emitted when burning fossil fuels. For example, nitrous oxide (N_2O) is also a by-product from burning fossil fuels. Although N_2O is released in far smaller quantities than CO_2, its effect is quite large because every molecule of N_2O has 298 times the greenhouse effect of a molecule of CO_2.

NOTE

In this chapter, the greenhouse gas effects from the burning of coal, gas and petrol will always be expressed as CO_2 equivalents or CO_2-e.

To simplify the calculations, the greenhouse effect of each of these gases can be expressed in terms of the climate change effects of an equivalent mass of CO_2. This is sometimes referred to as the global warming potential (GWP) of a greenhouse gas. The unit for GWP is CO_2-e.

EXAMPLE 11A-1 Global warming potential

Express the global warming potential of 10 kg of methane in CO_2-e.

$10 \text{ kg} \times 25 = 250$ kg of CO_2-e

The global warming potential of 10 kg of methane is 250 kg CO_2-e.

WHAT TO DO 11.1

1 Calculate the global warming potential, in CO_2-e, for each of the following.

a 5 kg of methane

b 0.5 kg of nitrous oxide

c 0.1 kg of HFC_{23}

d 1 g of sulfur hexafluoride

2 Discuss how society may be able to reduce emissions of some of the greenhouse gases listed in the Kyoto Protocol table above.

11B Measuring CO_2-e emissions

CO_2-e FROM ELECTRICITY

In Australia, power stations are most commonly powered with superheated steam created by burning the fossil fuels of brown coal, black coal or gas. Each of these fossil fuels produces different amounts of CO_2 and other greenhouse gases for the same amount of electrical energy generated. Each year, the Australian Government produces tables of the latest values. In Victoria, the majority of electricity is generated by burning brown coal. Because brown coal has the lowest energy value of all the types of coal, it emits the highest amount of CO_2-e for every unit of electrical energy produced.

Each state uses a mix of fossil fuels and renewable sources (wind, solar panels, hydroelectricity). As a result, each state emits very different amounts of CO_2-e per kilowatt hour (kWh) of electricity produced. Australian Department of the Environment figures for values of CO_2-e/kWh for electricity supplied in each Australian state in 2014 are given in the table below.

State/territory	kg CO_2-e/kWh
Victoria	1.34
New South Wales and Australian Capital Territory	0.99
Queensland	0.93
Western Australia (South West grid only)	0.83
South Australia	0.72
Northern Territory	0.78
Tasmania	0.23

Greenhouse gas emissions are usually expressed as tonnes of CO_2-e per annum.To calculate your household's annual CO_2-e from electricity use, follow these steps.

Step 1: Find *all* your electricity bills for the past 12 months. For the majority of households there are four quarterly electricity bills per year (one every 3 months).

Step 2: Find the total kWh used per bill.
For a single-rate tariff: this will be the total kWh on the bill.
For two-rate tariffs: add up the peak and off-peak subtotals.
For three-rate tariffs: add up the peak, shoulder and off-peak subtotals.

Step 3: Add the four quarterly bill totals to get your annual kWh usage.
For households with a solar grid-connect system that exports excess electricity generation to the grid, there will be a total kWh on each bill for the amount of electricity exported to the grid. Add up the exported kWh listed for your system for the four quarterly bills, then subtract this amount from your annual kWh usage before calculating your annual CO_2-e.

Step 4: Multiply your annual kWh total by the appropriate kg CO_2/kWh for your state, shown in the table above. This will give you the annual CO_2-e, in kg, for your household electricity usage.

Step 5: Divide your total by 1000 to express your annual CO_2-e total in tonnes.

NEED SOME PRACTICE?
Go to 25A Units of measurement
PAGE 291

NEED SOME PRACTICE?
Go to 25C Conversion of units
PAGE 297

WHAT TO DO 11.2

1 The table on the right shows the peak and off-peak kWh usage from 1 year's worth of electricity bills from a typical household. Use this data to calculate the CO_2-e for electricity use in tonnes per annum.

Billing period	kWh used per billing period	
1 Jan – 31 Mar	Peak: 728	Off-peak: 564
1 Apr – 30 June	Peak: 273	Off-peak: 682.6
1 July – 30 Sept	Peak: 546	Off-peak: 1001
1 Oct – 31 Dec	Peak: 364	Off-peak: 637

CO_2-e FROM NATURAL GAS

NOTE

In New South Wales, where a lot of energy is expended to extract the gas, the amount of CO_2-e emitted per GJ is larger.

The table below shows the kg CO_2-e per gigajoules (GJ) of gas energy for natural gas that is piped to the home for different states in Australia. There are large variations in the values per state, depending on the way the gas is extracted and the distance it is piped.

State/territory	kg CO_2-e/GJ
Victoria	55.23
New South Wales and Australian Capital Territory	64.13
Queensland	60.03
Western Australia	55.33
South Australia	61.73

NEED SOME PRACTICE?

Go to 25A Units of measurement PAGE 291

NEED SOME PRACTICE?

Go to 25C Conversion of units PAGE 297

To calculate your household's annual CO_2-e from gas, follow these steps.

Step 1: Find *all* your gas bills for the past 12 months. For the majority of households, there are six bimonthly gas bills per year (one every 2 months).

Step 2: Find the total MJ of gas used per bill (gas is charged by megajoule (MJ) used).

Step 3: Add up the bill MJ totals. This will give you the annual MJ used.

Step 4: Divide your MJ value by 1000 to convert this to GJ (gigajoules).

Step 5: Multiply your annual GJ of gas used for your state's kg CO_2-e/GJ as shown in the table above. This will give you the annual CO_2-e for your household gas usage.

WHAT TO DO 11.3

1 The table on the right shows a household's gas bill usage totals for 1 year. Use these values to answer these questions.

a What is this household's total natural gas MJ usage for the year?

b Convert this household's annual natural gas usage to GJ.

c Calculate this household's annual CO_2-e, in kg, for this gas usage if they are in Victoria.

d Calculate this household's annual CO_2-e, in kg, for this gas usage if they are in New South Wales.

e Convert your answers in part **c** and **d** to tonnes CO_2-e.

Billing period	MJ used for billing period
16 Dec – 15 Feb	1949.37 MJ
16 Feb – 15 Apr	1762.36 MJ
16 Apr – 15 Jun	5800.8 MJ
16 Jun – 15 Aug	9101.33 MJ
16 Aug – 15 Oct	7597.36 MJ
16 Oct – 15 Dec	5205.15 MJ

CO_2-e FROM VEHICLE FUELS

The table gives the approximate CO_2-e per 1000 L of petrol, diesel and LPG.

Fossil fuel	kg CO_2-e per 1000 L
Petrol	2469.92
Diesel	2899.25
LPG	1716.10

NOTE

These values apply to all vehicles built after 2004.

To calculate the annual CO_2-e from vehicle fuel, follow these steps.

Step 1: Find the amount of fuel used in the year. This can be done in several ways:

- Record how many litres of petrol, diesel or LPG you fill the car with each time you visit the service station for 1 year. This is the most accurate method, but it is time-consuming. (You have to wait a year to get a result!)
- Record how many litres of petrol, diesel or LPG you fill the car with each time you visit the service station for a typical week or month. Multiply this value by 52 or 12 as appropriate. This gives a good estimate for the vehicle, provided the week or month chosen is typical for the usage of the car.
- Estimate the approximate number of kilometres you have travelled in 1 year and find the average L/100 km value for the car over the previous 500 km to 1000 km. (This value is often stored in the vehicle trip computer.) This will provide a reasonable estimate, but it is not as accurate as the previous two methods unless you can get a true value for the kilometres travelled.
- Estimate the approximate number of kilometres you travel in 1 year and use the L/100 km fuel economy value from the manufacturer. This is the least accurate method, but it is often the only possible one if the car is new, or if you don't have easy access to the vehicle.

Step 2: Divide the fuel total in litres by 1000 to convert the litre value to kilolitres (kL).

Step 3: Multiply this kL value by the relevant kg CO_2-e per 1000 L. This will give the annual CO_2-e in kg for your car.

WHAT TO DO 11.4

1. A car uses 1342 L of petrol in 1 year. Calculate its annual greenhouse gas emissions in kg CO_2-e.
2. A car uses 87 L of petrol in a 1-month period.
   - a Approximately how many litres of fuel would this car use in 1 year?
   - b What would be this car's approximate annual greenhouse gas emissions in kg CO_2-e?
3. A car travels 9850 km in a year and for the last 875 km averaged 9.7 L/100 km. If this L/100 km value is typical for the year, what would be the approximate annual greenhouse gas emissions in kg CO_2-e for this car?
4. A car uses 943 L of diesel fuel in 1 year. Calculate its annual greenhouse gas emissions in kg CO_2-e.
5. A car uses 1475 L of LPG in 1 year. Calculate its annual greenhouse gas emissions in kg CO_2-e.

11C Estimating your carbon footprint

The calculation of the greenhouse gas emissions from your lifestyle, often referred to as carbon accounting, should cover the life cycle of every product you buy or use (from digging the raw materials out of the ground to disposing of the products when they are worn out), as well as every aspect of your lifestyle. This includes travel, garbage, water provision, the energy you use in the workplace, and so on. Deciding on where to stop including energy inputs, as well as the difficulty of collecting such data, can make exact carbon accounting for anything seem impossible.

As a result, any carbon account is a compromise between a complete estimate of the greenhouse emission and what is possible to estimate with the data available. If you set reasonable boundaries on what is and is not going to be accounted for, you can come up with an approximation of your part impact, sometimes called your carbon footprint, on the environment in terms of greenhouse gases.

To estimate your carbon footprint for the areas of household energy and private travel that you have been researching, it is suggested you set up a multi-sheet spreadsheet.

WHAT TO DO 11.5

1 Open an Excel spreadsheet, save it as 'Carbon footprint'. With the cursor over the title 'Sheet 1', right-click and select 'Rename' from the menu. Name the sheet 'Summary'.
Click on the ⊕ symbol next to Summary tab to open a second sheet.
With the cursor over the title 'Sheet 2', right-click to select 'Rename' and name the sheet 'Electricity'.
Repeat this process to create a third sheet named 'Nat Gas' and a fourth sheet named 'Fuel'. You are now ready to set up each sheet.

NOTE
Remember to save your spreadsheet often as you follow the instructions. The instructions apply to Microsoft Excel 2013.

2 Electricity sheet

Open the Electricity sheet and type in the headings as shown below.

	A	B	C	D	E	F
1	**Electricity**	**kWh**	**kg CO2-e/kWh**	**kg CO2-e**	**t CO2-e**	
2	Electricity bill Qtr 1			0		
3	Electricity bill Qtr 2			0		
4	Electricity bill Qtr 3			0		
5	Electricity bill Qtr 4			0		
6			Electricity: t CO2-e/annum =		0	t CO2-e

In Cell D2, type in the formula =SUM(B2*C2).
In Cell D3, type in the formula =SUM(B3*C2).
Repeat this for cells D4 and D5 to multiply the kg CO_2-e figure for your electricity supply by the kWh total for each bill.
Click on Cell E6 and enter the formula =SUM(D2:D5)/1000.
Use your carbon from electricity values from What to do 11.2. Enter the kWh totals for each bill in Cells B2 to B5, and the kg CO_2-e per kWh value for your state in Cell C2.

obook
An Excel spreadsheet template for What to do 11.5 is available on your obook.

3 Natural gas sheet

Open the Nat Gas sheet and type in the headings as shown below.

	A	B	C	D	E	F	G	H
1	Gas	Usage	kg CO2-e/kGJ	Divide to convert MJ to GJ	GJ of gas	kg CO2-e	t CO2-e	
2	Gas bill 1 in MJ			1000	0	0		
3	Gas bill 2 in MJ			↓Fill down	0	0		
4	Gas bill 3 in MJ				0	0		
5	Gas bill 4 in MJ				0	0		
6	Gas bill 5 in MJ				0	0		
7	Gas bill 6 in MJ				0	0		
8			Natural gas: t CO2-e/annum =				0	t CO2-e

In Cell E, type in the formula =SUM(B2/D2).
Repeat this for cells E3 to E7 to convert the bill MJ values to GJ.
In Cell F2, type in the formula =C2*E2). Repeat this for cells F3 to F7.
This will multiply the kg CO_2-e per GJ figure for your natural gas supply by the GJ energy on each gas bill.
Click on Cell G8 and enter the formula =SUM(F2:F7)/1000.
Use your carbon from natural gas values from What to do 11.3. Enter the MJ totals for each gas bill in Cells B2 to B7 and the kg CO_2-e per GJ value for Victoria in Cell C2.

4 Fuel sheet

Open the Fuel sheet and type in the headings as shown below.

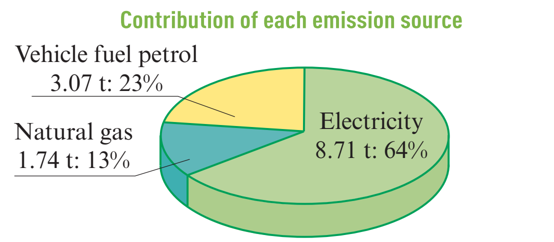
	A	B	C	D	E	F	G	H
1	Vehicle fuels	Usage	kg CO2-e/1000 L	Divide to convert to kL	kL	kg CO2-e	t CO2-e	
2	Petrol used in 1 year (L)			1000	0.00	0.00	0.00	
3	Diesel used in 1 year (L)			↓Fill down	0.00	0.00	0.00	
4	LPG used in 1 year (L)				0.00	0.00	0.00	
5			Vehicle fuels t CO2-e/annum =				0.00	t CO2-e

In Cell E2, type in the formula =B2/D2. Repeat for cells E3 and E4 as previously described. This converts the fuel volumes to kL.
In Cell F2, type in the formula =C2*E2. This will multiply the kg CO_2-e per kL figure for petrol to give the kg CO_2-e for 12 months' petrol usage.
In Cell F3, type in the formula =C3*E3. This will multiply the kg CO_2-e per kL figure for diesel to give the kg CO_2-e for 12 months' diesel usage.
In Cell F4, type in the formula =C4*E4. This will multiply the kg CO_2-e per kL figure for LPG to give the kg CO_2-e for 12 months' LPG usage.
Click on Cell G2 and enter the formula =F2/1000. This gives the total CO_2-e for petrol in tonnes per annum. Repeat for cells G3 and G4.
Click on Cell G5 and enter the formula =SUM(G2:G4). This gives the total CO_2-e for all vehicle fuels in tonnes per annum.
Use the CO_2-e per 1000 L values from the table on page 123. Enter the litre totals from questions **1**, **4** and **5** in What to do 11.4 in Cells B2 to B4 and the kg CO_2-e per 1000 L for each fuel type in Cell C2 to C4.

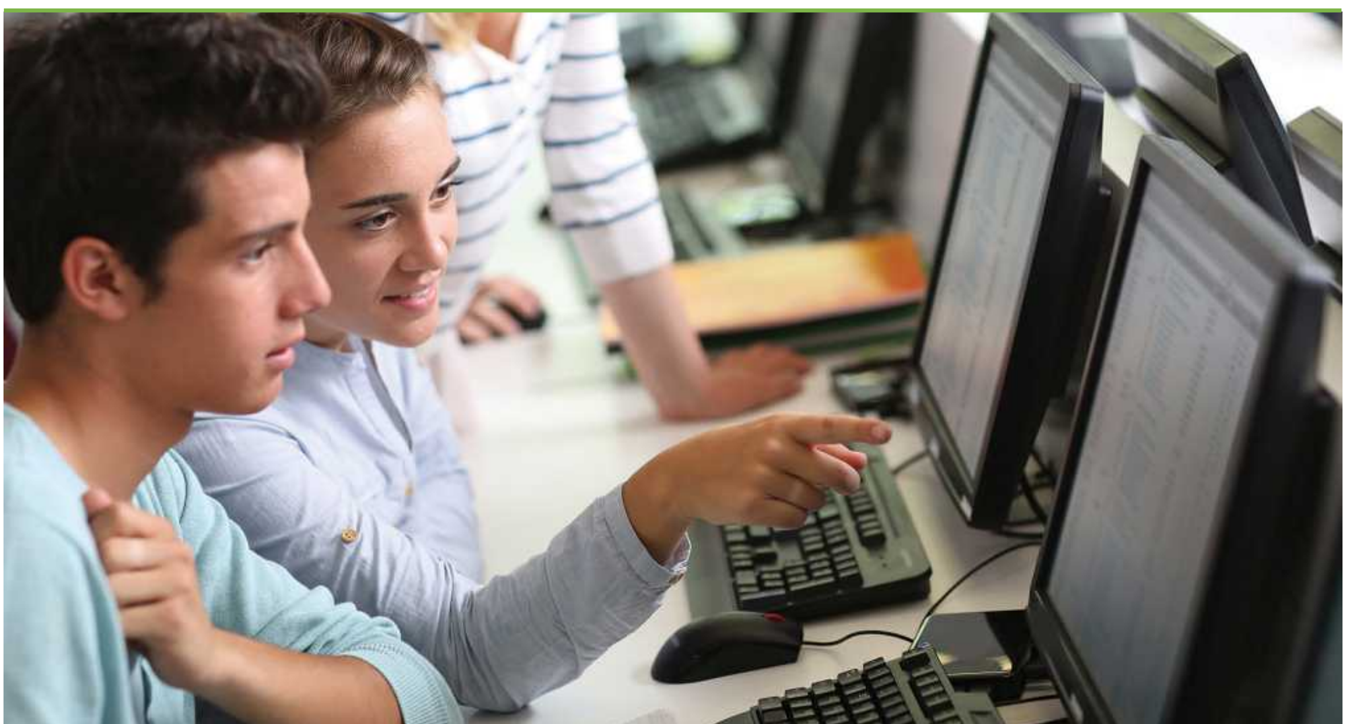
5 Summary page

Open the Summary sheet. In cell A1, type 'Summary page for household and private vehicle travel greenhouse gas emissions' and type in the headings as shown below.

	A	B	C	D	E	F	G
1	**Summary page for household and private vehicle travel greenhouse gas emissions**						
2	Annual CO2-e for electricity (tonne)						t CO2-e
3	Annual CO2-e for gas (tonne)						t CO2-e
4	Annual CO2-e for vehicle fuels (tonne)						t CO2-e
5							
6	**Total annual carbon footprint for electricity, gas and personal transport in tonnes CO2-e**						t CO2-e
7							
8							
9							
10							
11							
12							
13							



Click on Cell F2, type in = then open the Electricity sheet and click on cell E6 on that sheet. That will link the total value of tonne CO_2-e in that cell to the summary page.
Click on Cell F3, type in = then open the Nat Gas sheet and click on cell G8 on that sheet. That will link the total value of tonne CO_2-e in that cell to the summary page.
Click on Cell F4, type in = then open the Fuel sheet and click on cell G5 on that sheet. That will link the total value of tonne CO_2-e in that cell to the summary page.
In Cell F6, type =SUM(F2:F4).
If you wish, you can experiment with inserting a chart to graphically present the data. In this example, a 3-D pie chart has been inserted to show the relative contribution of each fossil fuel to this household's carbon footprint.



11D Reducing your carbon footprint

Now that you have identified and calculated the major greenhouse gas emissions and their sources for you and your household, you are ready to begin identifying and modelling some possible measures for reducing these emissions. The easiest way to do this is to estimate the reductions that may be made in electricity, gas or vehicle use and enter these new values into the spreadsheet created in Section 11C.

WHAT TO DO 11.6

1 Work in groups of three to five. Open your spreadsheet 'Carbon footprint' and save it as 'Carbon footprint-reduction measures'.

2 Discuss and make a list of possible ways to reduce the greenhouse emissions from household electricity, gas and vehicle fuel. Suggestions could include buying a more fuel-efficient car (or even an electric car), walking to replace some car trips, swapping to more efficient globes in the house, turning off a little-used second refrigerator, installing solar hot water or solar photovoltaic (electric) panels, and so on.

3 Select three to five of these suggestions (one per member of the group) and calculate the 'before' and 'after' values to change in the spreadsheet from Section 11C.

4 a Enter the new values individually into your 'Carbon footprint-reduction measures' spreadsheet, and let the spreadsheet recalculate the contribution of the reduction measure to the total greenhouse gas emissions for the household. Make a note of this new figure.

b Using all the reduction measures together, let the spreadsheet recalculate the contribution of this collection of reduction measures to the total greenhouse gas emissions for the household. Make a note of this new figure.

5 As a group, prepare and present a short report on your findings. Your report should explore what are potentially the most effective and least effective measures for household greenhouse gas reduction.

PROJECT 11

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TREADING LIGHTLY

Reducing your impact

A project worksheet for this chapter is available on the Teacher obook/assess. Ask your teacher to print it out for you. Completing this activity will help you make the right choices regarding your impact on the environment.

CHAPTER 12
Travelling in Australia

12A Travelling by road

12B Accommodation and meals

ARE YOU READY?

Complete the questions below to see if you are ready to start this chapter or if you need to brush up on your skills first.

- Part 1 should be completed *with* a calculator.
- Part 2 should be completed *without* a calculator.

If there are any questions you cannot answer, the link below each question will direct you to a related Mathematical Skills chapter where you can revise basic concepts and get up to speed.

PART 1 WITH A CALCULATOR

1 Find the value of each of the following.

a $\frac{1}{3}$ of 192 =

b $14.2 \times 36 + 9 =$

c $75 \times 4.850 =$

d $10.9100 \times 450 =$

23A Fundamental concepts

2 Find one third of the following numbers.

a 500 **b** 65

c $2\frac{1}{3}$ **d** $\frac{5}{8}$

23C Fractions of quantities

3 The metric unit most commonly used in Australia for measuring distances between cities is:

A mile **B** metre

C kilometre **D** nanometre

25A Units of measurement

4 Find 21% of each of these numbers.

a 750 **b** 660

24B Percentage of a quantity

5 Round these numbers correct to 2 decimal places.

a 34.345 **b** 1.67

c 391.12345 **d** 66.666

23E Rounding with a calculator

6 Calculate the value of 38.26×1.37 correct to 2 decimal places.

23E Rounding with a calculator

PART 2 WITHOUT A CALCULATOR

7 Find the value of each of the following.

a $3 \times 33.333 =$

b $81 \div 9 =$

c $12 + 8 - 4 \times 4 =$

d $(12 + 8 - 4) \times 4 =$

28A Back to basics

8 Convert the following fractions to decimals.

a $\frac{1}{7}$ **b** $\frac{2}{7}$

c $\frac{3}{7}$ **d** $\frac{4}{7}$

28C Decimals

9 Find the following percentages.

a 5% of 100 **b** 25% of 80

c 10% of 365 **d** 70% of 70

28D Percentages

10 Round these numbers correct to 3 decimal places.

a 345.678 48 **b** 2.123 456

c 2.345 608 **d** 0.000 93

23D Decimal numbers

11 A distance of 1155 m is best rounded as:

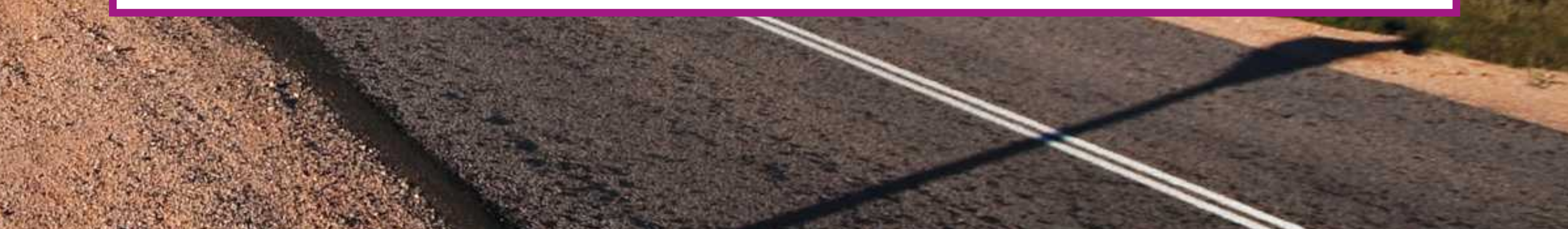
A 1.0 km **B** 1.1 km

C 1.2 km **D** 1.3 km

25C Conversion of units

12 Add up the following distances, in km. Estimate the total distance.
952 km, 12 km, 29 km, 830 km, 380 km

23F Estimation



12A Travelling by road

During a lifetime, most people will travel away from home for a holiday. The trip may be within the state in which they live or it may be to another state. The cost of such a trip can vary greatly depending on the chosen method of travel and on the accommodation.

It is a good idea to research and plan your trip beforehand. You should research:

- the cost of travel
- the cost of accommodation alternatives
- how much money you need in total for the entire trip.

The alternatives for travelling in Australia are by road, by rail or by air. This chapter examines the cost of a holiday by road. In the suggested projects, other means of transport may need to be examined.

The most common modes of travelling by road are:

- using your own car
- travelling by bus
- hiring a car.

USING YOUR OWN CAR

If your car has been regularly maintained and is totally roadworthy with acceptable tyres, the only holiday costs for the car should be the cost of the fuel you need and possibly oil. Your fuel cost will depend on:

- the price of petrol, LPG or diesel
- your car's fuel consumption.

NOTE

The fuel consumption of your car will vary depending on whether you travel in the city or the country. As a rule of thumb, driving in the country uses roughly 20%–30% less fuel than driving in the city.

The fuel consumption of a car is the amount of fuel (L) a car uses to travel 100 km. To measure the fuel consumption of your car, follow these steps.

Step 1: Fill your tank to capacity and record the odometer reading.

Step 2: When next you buy fuel, fill the tank again and record the litres added as well as the new odometer reading.

Step 3: The difference between the two odometer readings is the kilometres travelled.

Step 4: Use this formula:

$$\text{Fuel consumption (L/100 km)} = \frac{\text{litres used}}{\text{100 km travelled}}$$

NEED SOME PRACTICE?

Go to 24E Rates PAGE 288

EXAMPLE 12A-1 Fuel consumption

If the first odometer reading is 134598 and the second odometer reading is 134947, when 45.6 L of fuel are used, find the car's fuel consumption in L/100 km.

Distance travelled = 134 947 − 134 598 = 349 km

$$\text{100 km travelled} = \frac{349}{100} = 3.49$$

$$\begin{aligned}\text{Fuel consumption (L/100 km)} &= \frac{\text{litres used}}{\text{100 km travelled}}\\ &= \frac{45.6\ \text{L}}{3.49}\\ &= 13.1\ \text{L/100 km}\end{aligned}$$

The table below shows the fuel consumption for an average car by engine size.

Average car	Fuel consumption (L/100 km)
4-cylinder manual	5.1
4-cylinder automatic	6.2
6-cylinder manual	9.4
6-cylinder automatic	10.9
8-cylinder manual	13.3
8-cylinder automatic	15.3

The formula for calculating the number of litres of fuel used is:

$$\text{Fuel used (L)} = \text{distance travelled (km)} \times \frac{\text{fuel consumption (L)}}{100}$$

The price of fuel varies on a daily basis and depends on where you are travelling. The formula for calculating the fuel cost is:

$$\text{Fuel cost} = \text{fuel used (L)} \times \text{cost/L}$$

EXAMPLE 12A-2 Fuel cost

Using the fuel consumption table above, what are the fuel costs of travelling 450 km in a 6-cylinder automatic car if petrol costs \$1.34 per litre?

$$\begin{aligned}\text{Fuel used (L)} &= 450 \times \frac{10.9}{100} \\ &= 49.05\text{ L}\end{aligned}$$

$$\begin{aligned}\text{Fuel cost} &= 49.05 \times \$1.34 \\ &= \$65.73\end{aligned}$$

WHAT TO DO 12.1

1 Using the fuel consumption table above, find the number of litres of fuel used for a return trip from Melbourne to Sydney, travelling an estimated 2100 km, using a car that is:
  a a 4-cylinder manual
  b an 8-cylinder automatic.

2 Find the total fuel cost (at \$1.34/L) for a 4000 km return trip from Melbourne to Brisbane using a car that is:
  a a 6-cylinder automatic
  b a 6-cylinder manual.

3 Using an average \$1.59/L to allow for higher country fuel prices, how much should you budget for fuel if you wish to do a round trip from Melbourne to Adelaide to Alice Springs to Melbourne over a total of 6000 km using a car that is:
  a a 4-cylinder automatic?
  b an 8-cylinder automatic?

USING A BUS

Travelling by bus can be a cheap way to explore places within Australia. Most population centres can be reached by bus. The disadvantages of bus travel are that you are confined to the one seat for the entire journey and, if it is overnight travel, it may be difficult to sleep while sitting up. Also, with the decrease in airfares in recent years, buses are travelling less often as the cost is not necessarily that much cheaper.

Finding a bus fare

The best way to find bus fares is to use the websites of those companies offering interstate travel. One site that may be of use is http://www.greyhound.com.au/express/bookings.

Your teacher may also give you some other current websites to use for What to do 12.2.

When you want to find correct bus fares:

- Make sure you specify the exact date you want to travel: fares can vary by time of day, date or season.
- Check the site ticketing conditions, such as children under 4 years old may be able to travel free if they do not occupy a seat.
- Student/pensioner/children (age 4 to 14 years) concessions may also be available on some journeys. Check the site ticketing conditions to see what applies.

WHAT TO DO 12.2

1 Use a bus company website listing fares to find the full adult bus fare for a one-way trip from:
   a Melbourne to Sydney
   b Melbourne to Darwin
   c Adelaide to Perth
   d Brisbane to Melbourne.

2 Use a different bus company website listing fares from that in question **1** to find the cost of the return bus trip between:
   a Melbourne and Cairns
   b Melbourne and Alice Springs
   c Darwin and Perth
   d Canberra and Darwin.

3 The O'Farrell family travel from Melbourne to Adelaide and back by bus. They have two children, Vicki (1 month old) and Kerri (12 years). Vicki will sit on her mother's lap for the journey. A student discount may apply for Kerri, and Vicki may be able to travel for free. Using the two different bus companies from questions **1** and **2**, calculate the cost of the O'Farrell family's trip to Adelaide and back.

NOTE

Include in your answer to question **3** your recommendation as to which company the family should travel with based on the lowest cost and/or travel times available.

USING A HIRE CAR

Another method of travel is to use a hire car. With hire cars you have to pay for the fuel you use plus the daily hire charge. Hire cars are particularly useful when you wish to save time by flying to and from your destination, and you need a car to drive when you get there. For example, if you lived in Geelong and you wished to spend 12 days' holiday on the Gold Coast and have only a 14-day holiday, you would certainly fly and use a hire car.

The daily charge is less if you hire a car for longer periods. Below is a typical daily hire schedule for hire cars (GST included).

Vehicle	Fuel consumption (L/100 km)	Hire charge ($/day) 1–6 days	7–20 days	21+days
4-cylinder manual	5.1	72	67	57
4-cylinder automatic	6.2	77	72	61
4-cylinder wagon	6.8	84	79	67
6-cylinder manual	9.4	103	96	82
6-cylinder automatic	10.9	114	104	89
6-cylinder wagon	11.4	118	109	93
6-cylinder minivan	12.3	132	123	105

EXAMPLE 12A-3 Cost of car hire

If you hire a 6-cylinder automatic car for $104 per day, use it for 9 days and travel 2578 km, with petrol costing $1.35 per litre, what are the total costs?

Car hire: 9 days $\times$ $104 = $936.00

Fuel used (L) $= 2578 \times \frac{10.9}{100} = 281.0$ L

Fuel cost = 281.0 $\times$ $1.35 = $379.35

Total costs = $936.00 + $379.35

= $1315.35

WHAT TO DO 12.3

1 Find the total cost of hiring a car and the fuel used in the following cases.

	Vehicle	Number of days	Number of km	Fuel cost
a	4-cylinder automatic	5	600	
b	6-cylinder wagon	17	4500	
c	4-cylinder manual	12	1700	
d	6-cylinder automatic	22	5300	
e	6-cylinder minivan	28	6200	

NOTE

Fuel costs can change on a daily or weekly basis. Find the actual cost today and add the values to the last column.

12B Accommodation and meals

Another major expense when travelling is the cost of your accommodation while away. This cost can vary enormously depending on what type and standard of accommodation you require. Food costs must also be considered, particularly if you intend to eat differently than you do at home. Remember that if you stayed at home, you would still be paying some food costs.

ACCOMMODATION COSTS

NOTE

Star ratings show the type of guest experience you can expect and the relevant price – whether you want to be luxuriously indulged (5-star) or have a no-frills but safe and secure stay sleeping in a clean bed (1-star).

The cost of accommodation will depend on the type of accommodation chosen, such as motels, hotels, holiday flats, cabins, guest houses, caravans or camping. It will also depend on what standard of accommodation you choose (for example, 1-star versus 5-star) and when you choose to travel, as accommodation rates are generally higher in the busy or holiday season.

With so much variation in accommodation costs, it is difficult to accurately portray costs in a table. However, for the sake of the following exercises, the table below sets out fairly typical rates for various types of accommodation around Australia.

Accommodation costs

Type	Single/night	Double/night	Extra person/night	Weekly
Motel (country)	$120	$135	$20	–
Motel (city)	$220	$250	$25	–
Cabins/caravans	$70	$80	$5	$360
Holidays flats	$100	$120	–	$495
Camping	$14 per person	–		

EXAMPLE 12B-1 Accommodation costs

Using the previous table of accommodation costs, find the accommodation costs for a family of four people (two adults and two children) on a 14-day trip when they will be spending 9 nights in country motels and 5 nights in city motels.

Country motel

Cost of double per night = \$135

Cost per extra people (2 × \$20) = \$40

Cost for 9 nights (\$135 + \$40) × 9 = \$175 × 9

= \$1575

City motel

Cost of double per night = \$250

Cost per extra people (2 × \$25) = \$50

Cost for 5 nights (\$250 + \$50) × 5 = \$300 × 5

= \$1500

Total cost of accomodation = \$1575 + \$1500 = \$3075

WHAT TO DO 12.4

1 Using the previous table of accommodation costs, find the accommodation costs for:

a a single person on holiday in the country for 9 nights staying in a cabin

b a single person on holiday in the country for 6 nights and in the city for 7 nights staying in motels

c a couple hiring a holiday flat for 3 weeks

d a couple who stay 8 days in country motels and 5 days in city motels

e a couple with one child who stay 6 nights in a city motel, 3 days in cabins and camp for 2 nights.

FOOD COSTS

Your food costs when travelling will vary greatly depending on your preferences and where you are travelling. There are two main options.

- You buy your meals, for example at restaurants, hotels/motels or takeaways.
- You prepare your own meals.

The cost of buying your meals will depend on where you choose to eat. Takeaway food is generally cheaper than restaurant or motel/hotel meals. Restaurant prices can also vary greatly depending on the type of restaurant. Prices can also vary considerably depending on where you are staying. Holiday resorts often charge quite high prices for meals.

Preparing your own meals is the cheapest option of all because your only cost is the food purchased. However, many people prefer not to cook when on holidays and it is sometimes not possible to prepare your own meals (for example, most motel rooms do not have cooking facilities). It may also be difficult to find a supermarket or butcher when you are staying in an unfamiliar town.

With so much variation in food costs, it is difficult to accurately portray these in a table. However, the following table sets out a fairly typical or average cost for the various food options on a daily basis.

Daily food costs

Options	Adult	Child
Buy meals		
Restaurant	$60	$30
Motel/hotel	$50	$25
Fast food	$35	$18
Takeaway	$30	$18
Prepare meals		
Food costs	$20	$12

NOTE

Remember, these food costs are daily, and you need to take into account breakfast, lunch and dinner.

EXAMPLE 12B-2 Food costs

Using the previous table of daily food costs, find the food costs for the family of four people (two adults and two children) on their 14-day trip, if they split their meals between 50% eaten in motel restaurants, 30% fast food and 20% takeaway.

Motel meals

Adult: $2 \times \$50 \times 14 \times 50\% = \$700.00$ Child: $2 \times \$25 \times 14 \times 50\% = \$350.00$

Total motel meals: $\$700 + \$350 = \$1050$

Fast food

Adult: $2 \times \$35 \times 14 \times 30\% = \$294.00$ Child: $2 \times \$18 \times 14 \times 30\% = \$151.20$

Total fast food: $\$294 + \$151.20 = \$445.20$

Takeaway

Adult: $2 \times \$30 \times 14 \times 20\% = \$168.00$ Child: $2 \times \$18 \times 14 \times 20\% = \$100.80$

Total takeaways: $\$168 + \$100.80 = \$268.80$

Total cost of food $= \$1050 + \$445.20 + \$268.80 = \$1764$

WHAT TO DO 12.5

1 Find the food cost for a family of two adults and three children on a 3-week holiday, who split their meals between 40% eating in motels, 30% fast food and 30% takeaway.

2 A couple drive from Melbourne to Adelaide for their holidays. On their way, they stay 3 nights in motels and spend 2 weeks in Adelaide in a holiday flat on a weekly rate. They also spend 3 nights in motels on their return journey. They prepare all their own meals while in the holiday flat, but they eat at motels while travelling to and from Adelaide. Calculate their total food and accommodation costs.

3 Two couples go for a 3-week holiday around the countryside of New South Wales. Couple A go camping and prepare their own meals; couple B stay in motels and eat 50% of their meals in restaurants and the other 50% in motels. How much more will couple B pay for their food and accommodation?

4 A family of five (two adults and three children) travel from Melbourne to southern Queensland for a 3-week vacation. The journey there and back takes 7 days, during which they eat and sleep in motels, using one room per night. While at their holiday location, they stay in a caravan park in an on-site van. They pay for this on a weekly basis. While in the caravan park, they eat fast food 30% of the time, takeaway food 30% of the time and prepare their own food the rest of the time. Calculate the family's total accommodation and food expenses while on holidays.

PROJECT 12

TOURING LIKE A NOMAD?

Seeing the sights!

A project worksheet for this chapter is available on the Teacher obook/assess. Ask your teacher to print it out for you. Completing this activity will help you make the right choices when planning your travels around Australia.

obook
assess
TEACHER

CHAPTER 13

Travelling overseas

ARE YOU READY?

Complete the questions below to see if you are ready to start this chapter or if you need to brush up on your skills first.

- Part 1 should be completed *with* a calculator.
- Part 2 should be completed *without* a calculator.

If there are any questions you cannot answer, the link below each question will direct you to a related Mathematical Skills chapter where you can revise basic concepts and get up to speed.

PART 1 WITH A CALCULATOR

1 Find the value of each of the following.

a $400 \times 25.59 =$

b $2000 \div \$1.049 =$

c $\$30 + 2000 \times \$1.089 =$

d $400 \times \$32.65 + \$20 =$

e $7 \times \$56 + 7 \times \$65 + 7 \times \$50 + \$657 =$

23A Fundamental concepts

2 Find one fifth of the following numbers.

a 500 **b** 65

c $2\frac{1}{3}$ **d** $\frac{5}{8}$

23C Fractions of quantities

3 The metric unit that is most commonly used for measuring distances between countries is:

A mile **B** metre

C kilometre **D** nanometre

25A Units of measurement

4 Find 31% of each of the following numbers.

a 750 **b** 660

c 5900 **d** 6

24B Percentage of a quantity

5 Round the following numbers correct to 3 decimal places.

a 34.3455 **b** 1.6789

c 391.123 455 **d** 766.6008

23D Decimal numbers

PART 2 WITHOUT A CALCULATOR

6 Find the value of each of the following.

a $0.8 \times \$20 + \$10 =$

b $100 \times \$2.20 + \$5 =$

c $12 \times 0.6 - 6 =$

d $1.5 \times 30 - 25 =$

28C Decimals

7 Estimate the value of the following.

a $5000 \times \$0.78 =$

b $200 \times \$0.78 =$

c $80 \times \$1.2545 =$

d $1 \times \$2.05 =$

23F Estimation

8 Round these numbers correct to 3 decimal places.

a 345.67848 **b** 2.123456

c 2.345608 **d** 0.00093

23D Decimal numbers

9 If a car travels at 80 km/h, how far will it travel in 3 h?

A 90 km **B** 6000 m

C 240 km **D** 2400 km

28A Back to basics

10 A distance of 2155 m is best rounded as:

A 2.0 km **B** 2.1 km

C 2.2 km **D** 2.3 km

25C Conversion of units

13A Foreign currency exchange

Travelling to countries overseas can be a fantastic experience and a lot of fun, but it can also be a little more complicated than travelling within Australia. Travelling overseas means you will encounter different languages, cultures, food and currencies (money). Leaving Australia also means you will need to comply with the local laws and entry requirements of the different countries you visit.

The cost of overseas travel will depend on which country you visit, how you travel, and what type of holiday you choose. However, your main costs are transportation (getting there and getting around), accommodation, meals and shopping.

In addition to the obvious costs, there are also a number of other costs you need to be aware of before you travel. These include things such as:

- passport fees (including visa costs for some countries)
- departure taxes (when leaving Australia and some other countries)
- travel insurance (optional but recommended)
- travel guidebooks or ebooks.

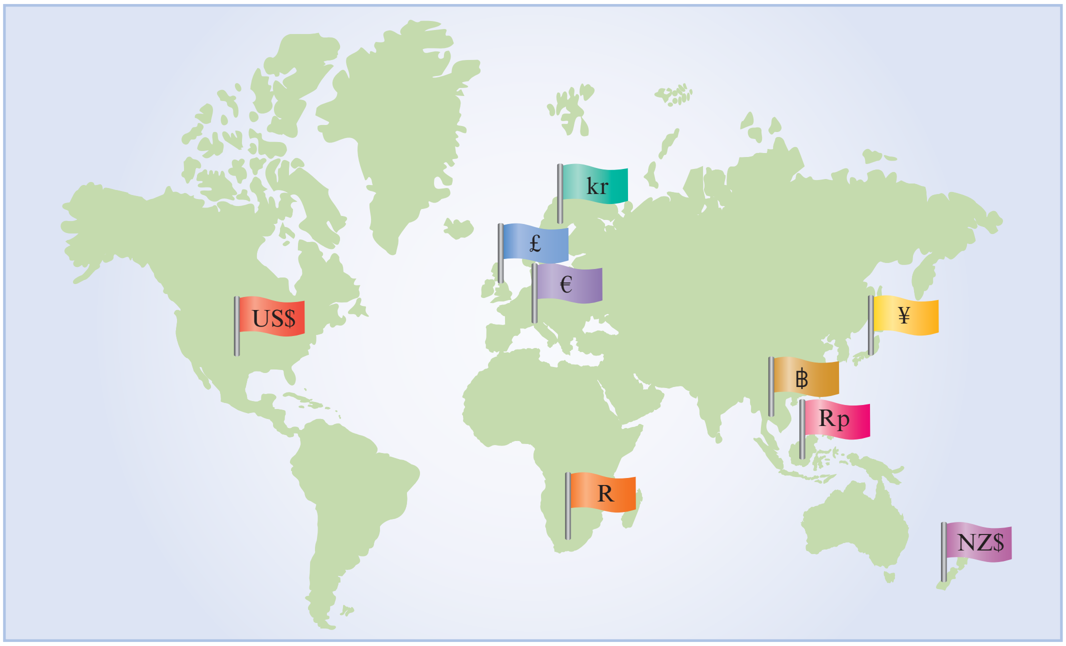
When you are travelling in another country, you need to use the local currency of that country to pay for the things you buy. To do this, you will need to exchange your Australian dollars for the equivalent amount of local currency. The value of your Australian dollars in local currency is calculated using a system known as international exchange rates. Exchange rates show the relationship between the values of currencies in different countries all around the world.

For example, if you are travelling to the USA, one Australian dollar (A$1.00) might only be worth 78 US cents. This means it will cost you about A$1.28 to buy one US dollar (US$1.00). So if something costs US$10, you will have to pay A$12.80.

The exchange rates in a particular country are usually given in terms of the value of the foreign currency in relation to one unit of the local currency. Exchange rates are published daily in newspapers and displayed in bank windows and at travel agencies. They are also widely available on currency exchange and travel websites. Exchange rates can vary daily, so if you are travelling overseas, it pays to check them regularly.

UNDERSTANDING EXCHANGE RATES

Use the world map below and the flags to identify the location of each of the following countries and regions. The currencies for these countries are shown in the exchange rate table below.



Value of one Australian dollar (A$)

Country/region	Currency name	Symbol	Exchange rate	Bank selling rate
European Union	euro	€	0.7103	0.6925
Indonesia	rupiah	Rp	10288.7	10 031.48
Japan	yen	¥	95.0755	92.6986
New Zealand	dollar	NZ$	1.069 94	1.043 19
South Africa	rand	R	9.2972	9.0648
Sweden	krona	kr	6.5644	6.4003
Thailand	baht	฿	26.1957	25.5408
United Kingdom	pound sterling	£	0.505	0.492
USA	dollar	US$	0.7823	0.7627

The exchange rate for the Australian dollar and various countries will vary daily. This table was correct for mid 2015. Look on the internet for current exchange rates.

BUYING FOREIGN CURRENCY

When heading off to a foreign country for a holiday, you will need to obtain foreign currency. You will be selling A$, so look at the exchange rate in the above table. To find out how much foreign currency you will get for your A$, multiply the number of dollars you have by the exchange rate of a particular foreign currency.

Foreign currency being bought = A$ being sold × exchange rate

EXAMPLE 13A-1 Buying foreign currency

Convert 400 Australian dollars to Thai baht.

$$\begin{aligned}\text{Thai baht being bought} &= \text{A\$ being sold} \times \text{exchange rate}\\ &= \$400 \times 26.1957\\ &= 10\,478 \text{ baht}\end{aligned}$$

Check by estimating: $\$400 \times 25 \approx 10\,000$ baht

WHAT TO DO 13.1

1 You have $400 to spend in each of these four countries listed below. How much local money can be purchased for:

a the UK? b Indonesia? c the USA? d Italy?

2 Estimate each answer listed in question **1** and show how you would do the estimation.

3 You are going to Japan for an exchange for 1 month, and have A$1000 to spend. How much is this in yen?

4 You are going to spend 1 month in the United Kingdom and 2 months in Europe and have A$2400. If you divide the money according to the time spent in each locality, how many pounds (£) and euros (€) will you have to spend?

NOTE

You are normally supplied with two cards, so if your handbag or backpack is stolen, you have another card in your luggage. Cards can also be replaced if necessary.

TRAVEL MONEY CARDS

When travelling overseas, many people prefer to use travel money cards rather than carrying cash. Travel money cards can be used like a debit card to withdraw money from ATMs and banks. The main advantages are that:

- they are more convenient than carrying large amounts of money
- they provide protection in case of accidental loss or theft
- they cost less than using your normal credit or debit card as you are not charged extra for every purchase.

Travel cards are purchased from a bank before departing Australia. A wide range of currencies can be held on the card, but it is advisable to take the currency of the country you are visiting or a widely acceptable currency like US dollars or euros (€).

Usually the bank providing the travel card charges a small set fee of approximately $20, plus 2–3% of the total value by giving you a lower exchange rate (a hidden cost). You should organise the travel card at least a few working days before you leave.

$$\text{Cost of travel card} = \frac{\text{amount of foreign currency}}{\text{bank's selling rate for travel cards}} + \$20$$

If you are going for a short trip and you don't have time to get a travel card, you can use most Australian savings and credit cards overseas, but the banks charge you every time you withdraw money or use a credit card. You can also take Australian dollars and change them in other countries, but it is not wise to carry large amounts of money.

EXAMPLE 13A-2 Using a travel money card

How much will it cost in Australian dollars to get a travel card loaded with UK£2000, if the conversion rate charge for travel cards is 0.4924?

$$\text{Total cost of travel card} = \frac{\text{amount of foreign currency}}{\text{applicable bank currency selling rate}} + \$20$$
$$= \frac{\pounds 2000}{0.4924} + \$20$$
$$= \$4081.74$$

WHAT TO DO 13.2

1 Refer to the bank selling rate column in the table on page 141 to calculate the cost, in A$, of purchasing a travel card for:

a ฿20 000
b NZ$1500
c US$3000
d £1500
e €800
f ¥250 000.

CONVERTING THE COST

When you are in a foreign country buying goods with the local currency, you will want to be able to work out how much something is costing you in Australian dollars, otherwise you will have no way of knowing if it is a bargain or a rip-off. To do this, apply the same principle that you used in Example 13A-2.

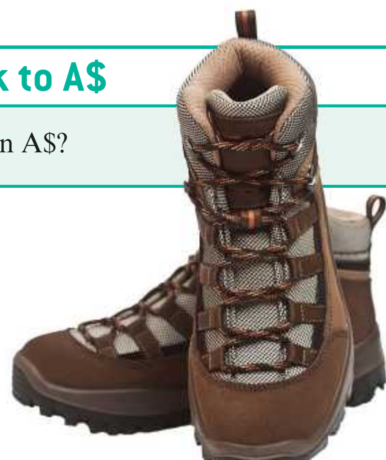
EXAMPLE 13A-3 Converting back to A$

If a pair of boots cost €80, how much will that be in A$?

Exchange rate for € to A$ is 0.7103.

$$\text{Cost in A\$} = \frac{€80}{0.7103}$$
$$= \$112.63$$

The boots would be A$112.63.



The method in Example 13A-2 is fine if you have a calculator handy, but it is not easy to divide by 0.7103 or even 0.7 in your head. Another way of estimating cost is to divide A$1 by the selling price for the currency. For € this would be:

$$\frac{\text{A\$1}}{0.7103} = 1.4079 \approx 1.5 \text{ (for simplicity)}$$

So when you are in Europe, you can just multiply the euro value by 1.5 to get a quick estimate of cost. It is worth doing this simple calculation when you arrive in a new country so that you can quickly estimate the cost of things without always needing your calculator. You can also look up the conversion rates on the internet.

EXAMPLE 13A-4 Estimating the cost in A$

Using A$1.5 as the approximate value of 1€, estimate, without a calculator, the appoximate cost in A$ of a dinner costing €10. Show your working.

Actual cost in A$ would be $\frac{€10}{0.7103} = A\$14.08$

The approximate cost would be €10 × 1.5 = A$15.

WHAT TO DO 13.3

1 Calculate how much it would cost you in A$, rounded to the nearest cent, to purchase:
- a a bowl of noodles in Thailand for ฿50
- b a camera in Germany for €190
- c running shoes in New York for US$75
- d a bottle of cola in Sweden for 17.05 kr.

2 How much in A$ would you pay for goods that cost:
- a 100 kr?
- b NZ$200?
- c ฿80?
- d US$352?
- e €1?
- f £1?

3 Using A$1.5 as the approximate value of 1 €, estimate, without a calculator, the approximate cost of the following items in A$. Show your working.
- a shoes €25
- b coffee €3
- c youth hostel bed €15
- d dress €40

BUYING AUSTRALIAN CURRENCY

When coming back home to Australia after your holiday, you will need to sell any foreign currency you have left and buy A$ with it. To find out how many A$ you will get for your foreign currency, divide the number of dollars you have by the exchange rate of the particular foreign currency.

$$\text{Australian dollars being bought} = \frac{\text{foreign currency being sold}}{\text{exchange rate}}$$

EXAMPLE 13A-5 Buying Australian currency

If you have NZ$365 and want to buy A$, how much will you receive?

The exchange rate for A$ to NZ$ is 1.069 94.

Australian dollars bought = $\frac{NZ\$365}{1.069\ 94} = A\$341.14$

WHAT TO DO 13.4

1 How much will you receive in Australian dollars if you exchange:
- a US$200?
- b ฿1500?
- c Rp1 200 000?
- d £350?

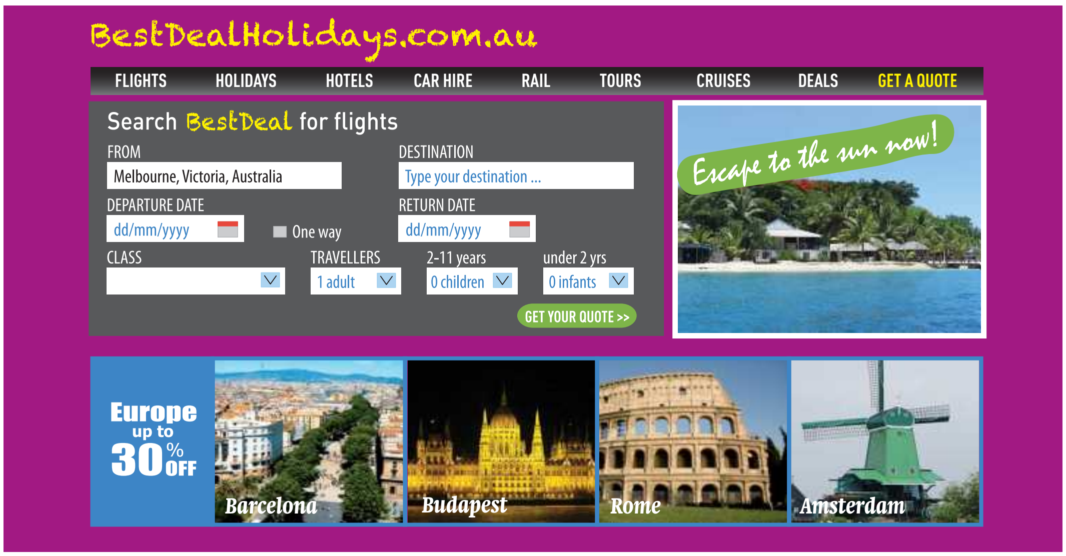
13B Airfares

The amount of money that you will need to pay for an airline ticket depends largely on two factors:

- the place you want to go to
- the time of year you want to travel.

To find out the cost, you can visit an airline or travel company website and enter your destination and dates of travel. Certain times of the year will cost more depending on the season or special events like Christmas and school holidays. You can buy a ‘one-way’ ticket or a ‘return’ ticket’ (which is the fare to your destination and back).

Many people go to a travel agent to help organise their travel, but you still need to check out prices to ensure you are not being overcharged.



WHAT TO DO 13.5

1 Google an airline or travel company and find out how much it will cost you to go from Melbourne to New York on 18 September and return on 10 October.

2 What is the cost of a one-way fare for a person who flies to London departing on:
  a 18 January? b 8 July? c 20 September?

3 Find the cheapest return air fare for two adults and two children to Bali at the following times. If luggage is extra, assume that each adult will take 20 kg of checked luggage.
  a January b 26 December for 2 weeks c March

4 If business class is 75% more than economy class, find (to the nearest dollar) the return business-class airfare for:
  a low shoulder b high season c low season.

5 Find the total cost for a family of two adults, a 14-year-old child, an 8-year-old child and a 6-month-old child flying to Paris from Sydney, departing 12 October. The youngest child will not occupy a seat.

13C Accommodation and meals

The other major expenses while travelling overseas are the cost of accommodation and meals. The extent of this cost depends on:

- where you travel (for example, Asia is cheaper than Europe or America)
- what standard of accommodation and food you want.

The range of costs for accommodation and meals is enormous. For current rates and recommendations, look at travel guides, either a book or an online subscription. Books are good as you won't always be able to access the internet. However, the following table shows some approximate costs, in A$, for one adult per day.

NOTE

Low-cost meals include buying food at the supermarket and fast-food restaurants.

Mid-range meals include average restaurants.

Overseas accommodation and meal costs (A$)

Region	Accommodation		Meals	
	Low-cost (hostel)	Mid-range (average hotel)	Low-cost meals	Mid-range meals
India/Pakistan	\$9	\$30	\$6	\$15
Southeast Asia	\$15	\$30	\$10	\$25
Europe	\$50	\$140	\$30	\$45
USA/Canada	\$60	\$120	\$30	\$65
New Zealand	\$40	\$80	\$30	\$65

EXAMPLE 13C-1 Cost of accommodation and food

How much would it cost a person holidaying in Europe for 10 days, staying in low-cost hostels and eating in low-cost cafés?

Accommodation: $10 \times \$50 = \$500$

Meals: $10 \times \$30 = \$300$

Total cost $= \$500 + \$300 = \$800$

WHAT TO DO 13.6

1 Calculate the accommodation and food costs for the following trips.

   a 3 weeks in Europe with low-cost accommodation and meals

   b 4 weeks in India with mid-range accommodation and meals

   c 6 weeks in the USA with low-cost accommodation and mid-range meals

   d 4 weeks in Southeast Asia with mid-range accommodation and low-cost meals

   e 10 weeks in New Zealand with mid-range accommodation and meals

2 A family of two adults and two children travel to Europe for 10 weeks. They use mid-range accommodation and meals. How much would it cost? Assume that children's costs are half the adult costs.

3 A couple want to travel through Southeast Asia for 3 months (13 weeks). They only have \$4800 for their accommodation and food costs. What are the best types of accommodation and food they can afford?

13D Other costs

Passport fees

All overseas travellers must have a passport. Australian passports are issued by the government and are valid for 10 years. The fee for an Australian passport in 2015 was $250 for adults and $125 for children.

Departure taxes

Anyone leaving Australia has to pay a departure tax. This is usually included in the airfare, or paid at the international airport. In 2015 the tax was $55 per person. Other countries also charge departure tax; for example, when you leave Japan, the departure tax is ¥2040 at Narita Airport, ¥2650 at Kansai Airport and ¥945 at Fukuoka Airport.

Immunisation costs

Travelling to some countries may lead to exposure to diseases that are not common in Australia, such as cholera, malaria and typhoid. Before leaving Australia, it is recommended that travellers immunise against certain diseases depending on where they are travelling. This will involve a trip to a doctor and you may have to pay a consultation fee plus the cost of the vaccination. Details of the costs will be available from your doctor.

Visa costs

Some countries require travellers to have an entry permit before they are allowed entry into the country. These permits are available from the embassies or consuls of the country you wish to visit and are called visas. A visa should be obtained either by your travel agent or by yourself before you leave Australia.

For example, a tourist to India will need a visa, costing $81.60. A tourist to Thailand can stay for up to 30 days without needing a visa, but a person visiting Thailand on business will need a visa. The cost and the requirements for visas change frequently, so before you travel you will need to check with your travel agent or on the internet.

Transport within an overseas country

You could travel by road (hire car, bus, campervan), by rail or by air, depending on your requirements. Time considerations are very important. Costs may be researched through travel agents or on the internet.

Travel insurance

Most overseas travellers take out some form of travel insurance to cover them for unforeseen incidents while overseas. This insurance usually covers:

- personal liability
- loss of luggage
- accidental death
- medical and related expenses
- loss of deposits or payment due to cancellation
- excess on hire car rentals.

Many different policies are available and you should choose carefully. The following table gives a typical scale of travel insurance premiums.

Travel insurance

Time away	Super		Standard		Budget	
	Single	Family	Single	Family	Single	Family
8 days	$97	$180	$87	$162	$44	$80
15 days	$127	$238	$114	$214	$71	$131
22 days	$155	$292	$144	$271	$86	$159
31 days	$195	$365	$174	$326	$102	$188
6 weeks	$227	$427	$205	$385	$127	$236
10 weeks	$298	$560	$265	$498	$161	$298
3 months	$341	$642	$310	$584	–	–
6 months	$500	$941	$456	$856	–	–
12 months	$839	$1578	$804	$1512	–	–

The three categories of insurance (super, standard and budget) give different levels of cover. Super insurance would provide insurance for a higher amount than standard or budget. The cost of insurance also varies according to where you are travelling; for example, insurance for Southeast Asia would be different to Europe.

WHAT TO DO 13.7

1 Calculate the travel insurance costs for the following people.
  a Lynn, who takes standard single cover and is away for 3 months.
  b Ian, who takes budget single cover and is away for 3 weeks.
  c Tracy, who takes super single cover and is away for 6 months.
  d The Menzel family, who take standard family cover and are away for 6 weeks.
  e The Tan family, who take budget family cover and are away for 2 weeks.

13E Case study: a holiday in Bali

This section briefly examines the itinerary and costs of a 7-day holiday to Bali for three friends. Refer to the previous sections so that you do not overlook any features and costs.

PLANNING THE HOLIDAY

Ari (19), Sally (18) and Tran (18) are planning a 7-day holiday to the Indonesian island of Bali to celebrate completing Year 12. Travelling as a group, they can save money on accommodation and transport.

First, they buy a guidebook and visit a travel agent to collect brochures on holidaying in Bali. Next, they research some great deals on the internet to plan their itinerary. Sally's friend Ari went on a similar holiday to Bali a year ago, so they asked him for some advice and recommendations. Ari was a great help when it came to allocating the time it would take to travel between destinations and also to see what they want.

After much discussion, the friends decide on staying at the following places. A friend's suggestions on 'not to be missed' ventures in Bali are also shown below under the activity column. A cycling trip around Ubud is essential. Research the cost of undertaking these experiences.

Day	Accommodation	Cost	Activity
1	Kuta	$26	
2	Kuta	$26	Surfing, shopping
3	Kuta	$26	
4	Ubud	$57 for 3	Bike ride
5	Ubud	$57 for 3	Mountains
6	Seminyak	$114 for 3 or Rp500 000	Relax/shopping
7	Seminyak	$114 for 3 or Rp500 000	Relax/shopping

PURCHASING AIRFARES

The table shows typical air fares to Bali. Prices are variable so it is good to look at all the optional dates you can fly. To see a spreadsheet of prices over several weeks, go to an airline website (rather than a travel agent website) to find the cheapest days to fly. Then you can book through an agent or a website. The prices in the table are in $A.

Departure from Melbourne	Fare	Departure from Bali	Fare
15 December	$379	22 December	$239
23 December	$1132	30 December	$279
3 January	$529	10 January	$539

WHAT TO DO 13.8

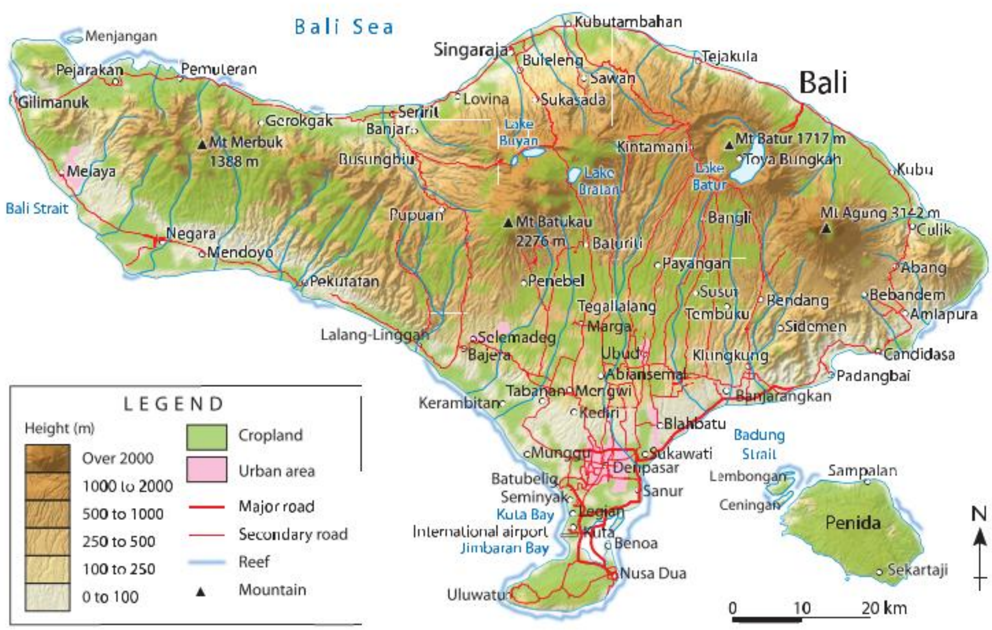
1 Which departure date listed in the table above will give the cheapest combined flights for 1 week's holiday?

2 Round the flight costs to estimate how much each of the return fares will be. Show your working.

3 Now look at flights for February and March. What are the cheapest return fares you can find? Why would fares be cheaper in February and March?

MAPS AND CALCULATING DISTANCES

It is good to understand the general layout of your destination and distances between major attractions on the island, both in kilometres and the amount of time taken to travel. In some countries, the major destinations may be hours apart, requiring a day's travel, but in Bali the distances are usually quite short, so it is possible to change location and fit in an activity in one day and still have time to relax. Below is a map of Bali. Using the scale at the bottom of the map, you can estimate the distances.



EXAMPLE 13E-1 Cost of accommodation and food

a Use the scale on the map of Bali to estimate the distance from Kuta to Ubud.
b If the average speed on Bali roads is 40 km/h, how long will the trip take?
c If the taxis cost Rp50 000 per hour, how much is this in Rp and in A$?

a Mark the edge of a piece of paper with each straight section of road. Use the map scale to estimate the distance from Kuta to Ubud. It is about 40 km.

b The average speed is 40 km/h over 40 km $= \frac{40 \text{ km}}{40 \text{ km/h}} = 1 \text{ h}$

c The taxi cost for 1 h is Rp50 000.

To convert to \$A: $\frac{\text{Rp}50\ 000}{10\ 288.7} = \text{A\$}4.86$

NEED SOME PRACTICE?
Go to 31B Scale drawings PAGE 362

NOTE
The quick way to do this division without a calculator is to cancel zeros.

WHAT TO DO 13.9

1 Trace the map of Bali into your notebook. Mark the following: Denpasar, Kuta, Seminyak, the airport, Ubud, Jimbaran Bay, Nusa Dua, Singaraja, Candidasa, Mt Batukau, Bangli.

2 Using the scale on the map and the details in Example 13E-1, estimate the:
a distance **b** time taken **c** cost in Rp **d** cost in A$
i from Ubud to Candidasa
ii from the airport to Kuta, allowing half an hour for heavy traffic
iii from Denpasar to Bangli.

4 List all the features of the holiday and the costs in A$ for the group on a spreadsheet. Be careful to convert to A$ whenever the prices are in rupiah. Use the exchange rate in Section 13A. Include columns to help you organise the trip, such as who is responsible for organising each aspect of the trip and notes on the progress of the planning. Also include:
- the airfares
- the accommodation costs
- an estimate of total food costs using the table in Section 13C
- the total travel insurance costs (in A$) from the table in Section 13D
- other costs from Section 13D, such as passports, departure tax, immunisation, visa costs, transport costs
- an allowance of, say, $300 each for unseen costs and spending money.

PROJECT 13

obook
assess
TEACHER

THE WORLD TOUR

Going around the world?

A project worksheet for this chapter is available on the Teacher obook/assess. Ask your teacher to print it out for you. Completing this activity will help you make the right choices when planning overseas travel.

CHAPTER 14

Maps, bearings and surveys

ARE YOU READY?

Complete the questions below to see if you are ready to start this chapter or if you need to brush up on your skills first.

- ▶ Part 1 should be completed *with* a calculator.
- ▶ Part 2 should be completed *without* a calculator.

If there are any questions you cannot answer, the link below each question will direct you to a related Mathematical Skills chapter where you can revise basic concepts and get up to speed.

PART 1 WITH A CALCULATOR

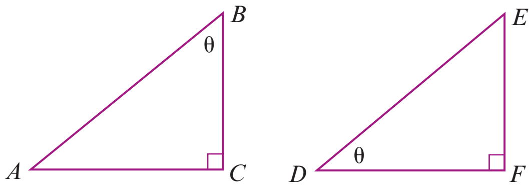
1 Find the value of each of the following.

a 0.75 of 3458 = **b** 6 + 12 × 3.2 =
c 30 ÷ 4.3 = **d** 32 + 42 =

23A Fundamental concepts

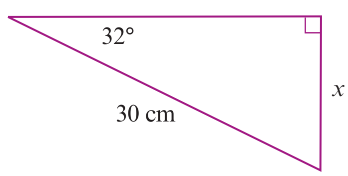
2 On the triangles shown below, label the:

a hypotenuse
b adjacent side to θ
c opposite side to θ



32B Right-angled triangle trigonometry

3 Use sine to find the value of x correct to 2 decimal places.



32C Finding unknown sides

4 Which metric unit is commonly used to measure the distances between the world's capital cities?

A mile **B** kilometre
C nautical mile **D** all of the above

25A Units of measurement

PART 2 WITHOUT A CALCULATOR

5 Find the value of each of the following.

a 45 + 90 + 22.5 = **b** −1.63 + 0 =
c 150 − 10 × 5 = **d** 32 + 4.2 =

28A Back to basics

6 Round these numbers correct to 3 decimal places.

a 110.2345 **b** 212.314 56
c 2.345 608 **d** 0.936 67

23D Decimal numbers

7 Convert the following to percentages.

a 25 **b** 0.3
c $\frac{5}{8}$ **d** $\frac{2}{3}$

28D Percentages

8 Find the value of each of the following.

a 10% of 150 **b** 25% of 160
c 5% of 200 **d** 20% of 550

24B Percentage of a quantity

9 The estimate answer to 21 × 49 is:

A 490 **B** 2100
C 1000 **D** 9800

23F Estimation

10 Add up the following lengths (in metres). Estimate, to the nearest metre, the total approximate length.
4.2 m, 6 m, 1.8 m, 12.7 m.

23F Estimation

14A Parts of a map

To travel from one place to another, people use maps. Maps can help us lay out a housing development or park correctly. They also help us navigate across town or across the world. The preparation of accurate maps is very important. People who prepare and develop maps are called surveyors and cartographers.

Surveying is the measurement of land for the production of maps. Surveyors are needed wherever there is new land development or buildings are to be constructed. If the boundaries of a property are not laid out clearly when land is being developed, it can be a very expensive process to fix.

This chapter looks at the basics of reading and preparing small-scale maps using some simple techniques.

WHAT IS A MAP?

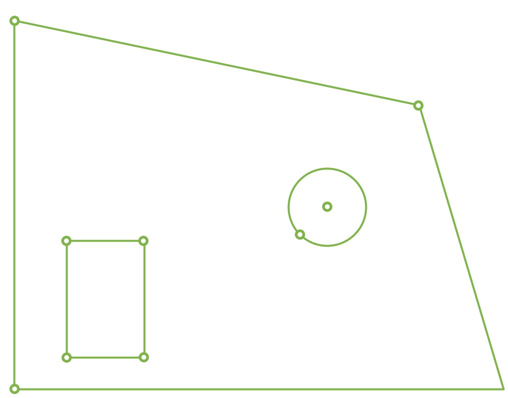
A map is a simplified plan of an area that uses symbols and colours to represent the features in the real world. Maps represent areas viewed from directly above (the plan view). Map-makers, known as cartographers, simplify the information from the real world and add additional information such as place names and borders. They use a legend to help unlock the information on the map. Symbols and colours are used in legends to help map-readers quickly recognise the features, for example green for parks, blue for water and lines to represent roads.

There are six key features all maps need to contain, known as BOLTSS:

- Border: to show the extent of the map
- Orientation: a direction arrow
- Legend: an explanation of the map symbols
- Title: a description of what the map is showing
- Scale: a way of showing the area represented by the map
- Source: the information used to produce the map.

WHAT TO DO 14.1

This diagram is an example of a poor map. It is supposed to be a map of a backyard with a shed and a tree in it. As a class, discuss the following questions.



1 What information is provided by the map in its current state?

2 What information is not provided by the map in its current state?

3 What is missing from the map?

4 What is wrong with the map?

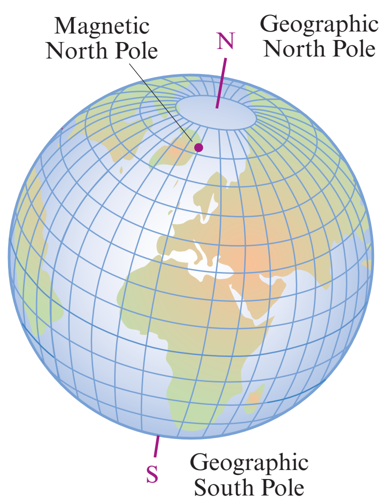
14B Getting your bearings

One of the most important features of any map is direction. The four main directions on a compass are north, south, east and west.

No matter where you are on Earth, when you hold a magnetic compass in your hand it will point towards the Magnetic North Pole. Magnetic north is the direction in which a free-swinging magnet, like a compass needle, will always point. Once the location of north is established, you can find the other points of the compass. Using compass points is an accurate method of finding a location because whichever direction you are facing, the compass directions always remain the same.

True north is the actual direction of Earth's geographic North Pole. Direction indicators on maps point to the geographic North and South poles. These are the furthest points north and south of the Equator.

For most places on Earth, the direction of true north is not the same as magnetic north. The changing position of the magnetic poles is thought to be due to Earth's rotation and the movement of minerals within Earth's mantle.



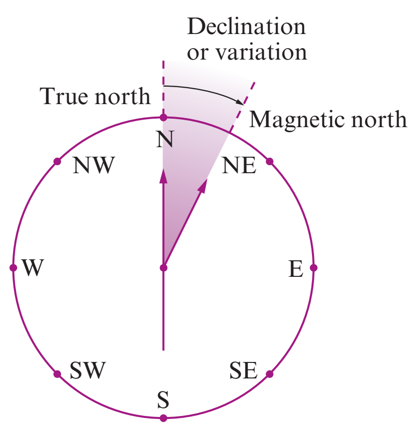
One accurate way to find true north is to measure the direction of magnetic north and then add an angle called a declination.

True north = magnetic north + declination angle

Tables of declinations are produced each year in all major places in the world. Declinations for various times and places can also be calculated. The National Geophysical Data Center provides an online calculator at http://www.ngdc.noaa.gov/geomag-web/#declination.

The table below shows the declinations for each of the capital cities in Australia calculated for 11 December 2014. For example, Melbourne has a declination of 11.6°, which means that magnetic north is 11.6° east of true north in Melbourne.

City	Magnetic declination
Adelaide	7.95°E
Brisbane	10.93°E
Canberra	12.28°E
Darwin	3.12°E
Melbourne	11.6°E
Perth	1.63°W
Sydney	12.45°E



WHAT TO DO 14.2

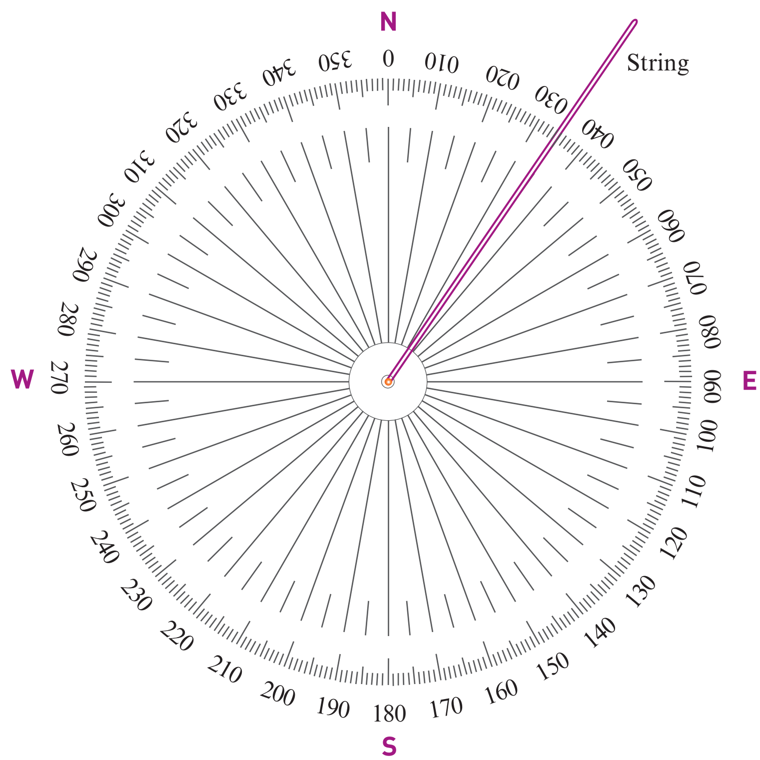
1 Use a compass to determine the direction of the Magnetic North Pole.

2 Use the table of declinations on the previous page to find the declination angle you need to add. You may wish to use the National Geophysical Data Center website given on the previous page.

3 What is significant about the declination for Perth? How does it affect calculations for true north?

MEASURING BEARINGS

Bearings are angles measured clockwise from true north. Bearings have values between 0° and 360°. The protractor below will allow you to measure and mark bearings on blocks of land for surveying. Surveyors use an instrument called a theodolite that works on the same principles but is more accurate.



WHAT TO DO 14.3

1 In groups of three or four, photocopy the protractor shown above onto A4 or A3 paper. Enlarge it as much as possible. Notice that north, south, east and west are marked and that the angles increase clockwise from zero.

2 Paste the protractor onto a sheet of stiff card or thin board. Press a drawing pin into the centre of the protractor so that there is just enough room to slip some string between the head of the drawing pin and the card. Tie one end of a ball of string around the drawing pin.

3 You will use your protractor in What to do 14.4.

14C Traverse surveying

Traverse surveying is a simple method for measuring the important distances on a block of land for preparing a map. With traverse surveying, the surveyor walks along a straight line, often a diagonal of the area to be surveyed, measuring distances along this line and distances to major features when they are at right angles to the line. The measurements are recorded in the surveyor's notebook or fieldbook.

LAYING OUT A BLOCK

In What to do 14.4, measurements are given for a block that you can set out on your schoolyard or oval. Work in groups of three or four.

WHAT TO DO 14.4

Step 1: Lay out a string line across the longest diagonal of the block to be surveyed. This is called the traverse. From points *A* to *F* is 15 m long.

Step 2: Establish the position of true north from end *A* of this diagonal.

Step 3: Use your protractor from What to do 14.3 to measure the bearing of the traverse from this end. Record the value.

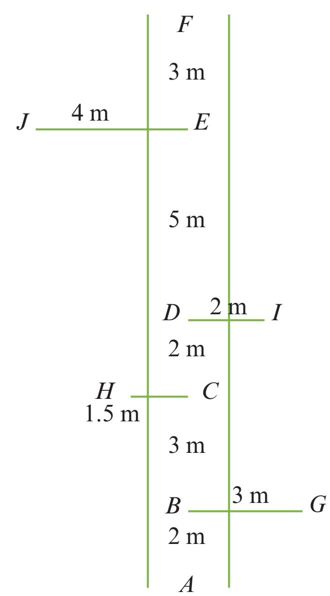
Step 4: Mark out the lengths along the traverse, as given in the diagram, using a trundle wheel or tape measure.

Step 5: Run perpendicular string lines from the traverse to all the important points on the block of land; that is, points *G*, *H*, *I* and *J*. Make sure that the lines are longer than the measurements needed.

Step 6: Now carefully measure the distances from the traverse to each point.

Step 7: Place a circular bin at point *I* and measure its circumference with a tape measure. Calculate the radius of the object using the following formula.

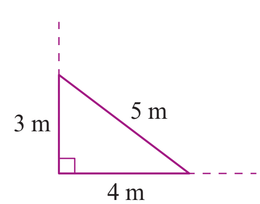
$$\text{Radius} = \frac{\text{circumference}}{2 \times \pi}$$



SURVEYING A BASKETBALL COURT

In What to do 14.4, the measurements were given to you. In What to do 14.5 you will need to measure the lengths yourself. Work in groups of five or six.

All the corners of a basketball court are right angles. This makes checking accuracy of a traverse survey easy. Check that the corners of the basketball court are, in fact, 90° using a tape measure to measure distances of 3 m, 4 m and 5 m and applying Pythagoras' theorem.



NOTE

The lengths can be shorter or longer than 3 m, 4 m and 5 m, as long as they are all multiplied by the same number.

Rather than take every possible measurement of the basketball court, it is only necessary to take measurements for one half of the court. The other half is a mirror image. An accurate map of the half-court can be drawn by measuring the positions of only nine points. At most you should need only eleven points.

WHAT TO DO 14.5

Drawing a sketch

As a group, decide which points you need to measure on the basketball court. To make recording the distances easier, draw a rough sketch on a sheet of A4 paper. Represent the traverse by drawing two parallel lines running the page length about 2 cm or 3 cm apart.

> **NOTE**
> For this exercise the three-point line on the basketball court will be ignored.

Step 1: Set up a traverse line between two opposite corners using a string line. So as not to damage the court, two students will need to hold the line in place at either end while the other lines are placed and all the measurements are taken.

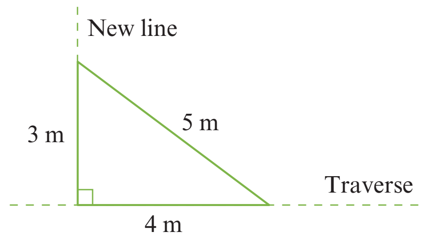
Step 2: Measure the bearings of the points using a magnetic compass and your protractor.

Step 3: Measure off distances along the traverse, writing them down as you go. At the end of the traverse you should have a measure of the length of the traverse line.

Step 4: Measure the perpendicular distances from the traverse to each point of interest on the court.

Setting perpendicular string lines

Mark the point where the line is to meet the traverse. Use a tape measure or string to set up a 3 m, 4 m and 5 m triangle with the right angle between the traverse and the new string line. The result is a right angle.



Drawing a plan of the basketball half-court

Drawing a map or plan can also be done in steps. The following steps can be applied to any plan or map drawing.

> **NEED SOME PRACTICE?**
> Go to 31B Scale drawings PAGE 362

Step 1: Decide on the paper size you will need. For the basketball court, a 1 cm : 1 m scale is easy to use. Although an A4 page may be sufficient, it is better to work with an A3 page.

Step 2: Draw in 1 cm margins around your page.

Step 3: In the bottom corner write the scale of the diagram to real life. In this example, it is 1 cm : 1 m or 1 : 100.

Step 4: Arrange your page so that the longest side of your paper will also be parallel to the traverse line of the diagram.

Step 5: Find the centres of your top and bottom margin lines. From these centre points, draw a faint centre line.

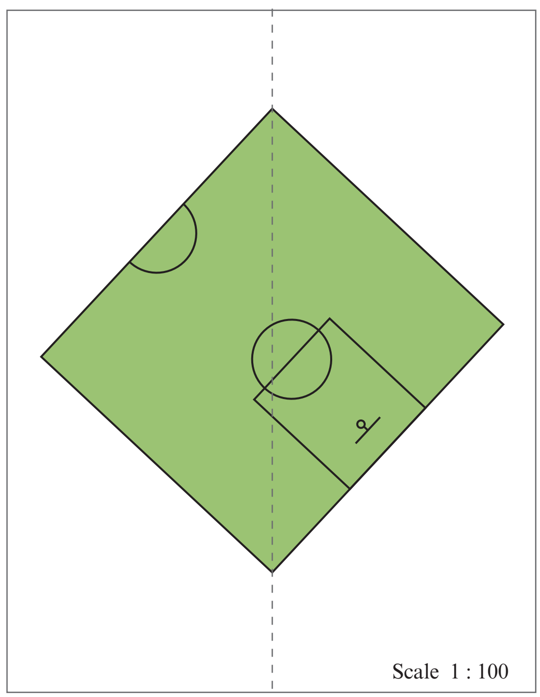
Step 6: Find the centre of the centre line.

Step 7: Halve the length of the traverse you measured and draw lines of this length to scale on either side of the centre point.

Step 8: Label one end of the traverse line as the start and the other as the end.

Step 9: Using the measurements from the field survey and a ruler and set square, fill in the rest of the map.

Your basketball half-court will look like this diagram, only larger. Check that the angles of your diagram are all right angles. If they are, then you have both measured and mapped the court accurately.

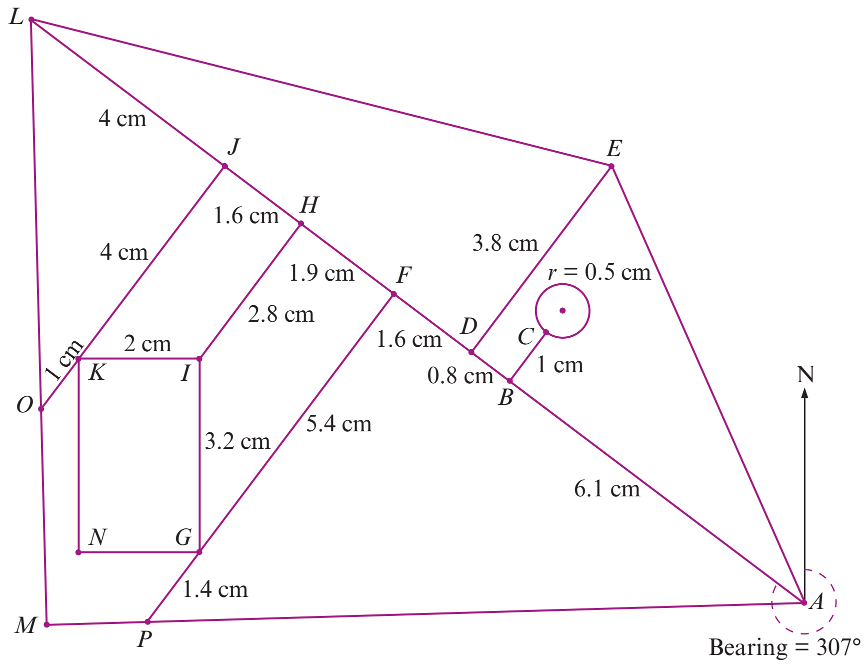


14D Calculating sides and corner angles

REDRAWING THE FIRST MAP

The map in What to do 14.1 has now been redrawn to include the types of features we have been learning about. This time it includes length measurements and a traverse bearing. However, it is not drawn to scale.

Traverse surveying is often used when parts of a block of land cannot be measured directly. This is the case with the corner marked M, as the shed is in the way. In this example, you could just measure the lengths of the boundary sides instead, but this is not always possible. It is possible, however, to find these lengths by calculations using trigonometry.

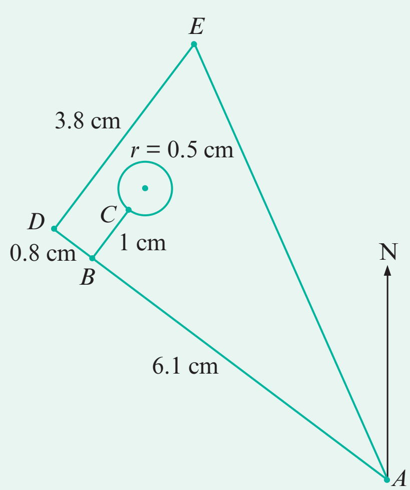


EXAMPLE 14D-1 Finding the length of *EA*

Triangle DAE has a right angle at D.

a Find the size of the angle at A, correct to 2 decimal places, using the inverse tangent function.

b Now find the length of the side EA using the sine ratio.



a $\angle EAD = \tan^{-1}\left(\dfrac{\text{opposite side}}{\text{adjacent side}}\right)$

$= \tan^{-1}\left(\dfrac{3.8}{6.9}\right) = 28.84°$ Adjacent side is $6.1 + 0.8 = 6.9$ cm

b $\sin 28.84° = \dfrac{\text{opposite side}}{\text{hypotenuse}} = \dfrac{3.8}{AE}$

$AE = \dfrac{3.8}{0.4824} = 7.88$ cm

NEED SOME PRACTICE?
Go to 32B Right-angled triangle trigonometry
PAGE 375

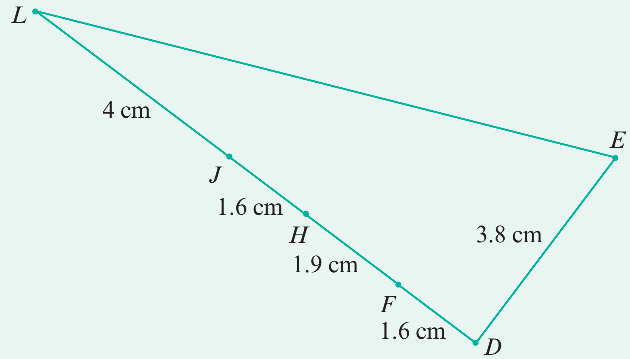
NEED SOME PRACTICE?
Go to 32D Finding unknown angles
PAGE 380

EXAMPLE 14D-2 Finding the length of *LE*

Triangle *DLE* has a right angle at *D*. The total length of *LD* is 9.1 cm.

a Find the size of the angle at *L*, correct to 2 decimal places, using the inverse tangent function.

b Find the length of the side *LE* using the cosine ratio.



a $\angle DLE = \tan^{-1}\left(\frac{\text{opposite side}}{\text{adjacent side}}\right)$

$= \tan^{-1}\left(\frac{3.8}{9.1}\right) = 22.66°$

b $\cos 22.66° = \frac{\text{adjacent side}}{\text{hypotenuse}} = \frac{9.1}{LE}$

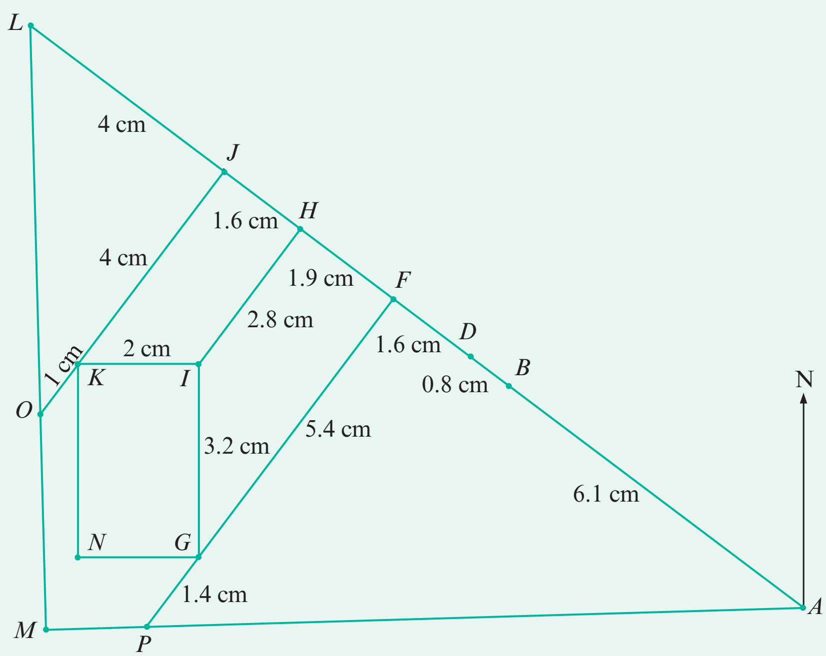
$LE = \frac{9.1}{0.9228} = 9.86 \text{ cm}$

EXAMPLE 14D-3 Finding the size of angle *M*

a Find the angle at *A* using the triangle *PFA*. Angle *PFA* is a right angle.

b Find the angle at *L* using the triangle *LJO*. Angle *LJO* is a right angle.

c Hence find the size of the angle at *M*. Is it a right angle?



a $\angle FAP = \tan^{-1}\left(\frac{\text{opposite side}}{\text{adjacent side}}\right) = \tan^{-1}\left(\frac{6.8}{8.5}\right) = 38.66°$

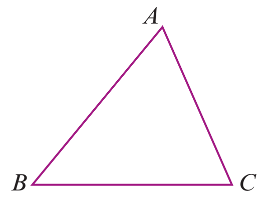
b $\angle OLJ = \tan^{-1}\left(\frac{\text{opposite side}}{\text{adjacent side}}\right) = \tan^{-1}\left(\frac{5}{4}\right) = 51.34°$

c The angle at *M* is $180° - (38.66° + 51.34°) = 90°$

The sine rule is one method used to find sides and angles in non-right-angled triangles. It states that the ratio of the length of any side of a triangle to the sine of the angle opposite it, is the same as that for the other two sides and opposite angles.

In triangle ABC:

$$\frac{BC}{\sin A} = \frac{AC}{\sin B} = \frac{AB}{\sin C}$$



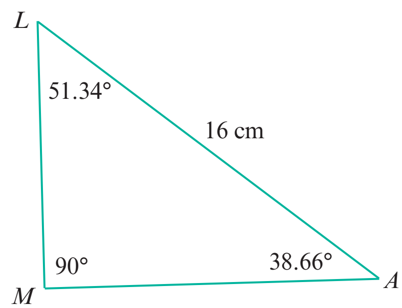
NEED SOME PRACTICE?
Go to 32C Finding unknown sides PAGE 377

EXAMPLE 14D-4 Finding the length of *LM* and *MA*

Use the sine rule to find the length of LM and MA.

Mark the values of the angles at L, M, A and the length of traverse LA.

$LA = 4 + 1.6 + 1.9 + 1.6 + 0.8 + 6.1 = 16$ cm



a Using the sine rule: $\frac{LA}{\sin M} = \frac{LM}{\sin A}$

Thus $LM = \frac{LA \times \sin 38.66°}{\sin 90°}$

$= \frac{16 \text{ cm} \times 0.6247}{1}$

$= 10$ cm

b Using the sine rule: $\frac{MA}{\sin L} = \frac{LM}{\sin A}$

Thus $MA = \frac{LM \times \sin 51.34°}{\sin 38.66°}$

$= \frac{10 \text{ cm} \times 0.7809}{0.6247}$

$= 12.5$ cm

NOTE

Check all calculations with careful measurement of your scale diagram.

WHAT TO DO 14.6

1 Using the techniques learnt in this chapter and using a scale of 1 cm : 1 m, draw an accurate map for the example block of land in this chapter using the measurements given.

2 Use what you have learnt to find the perpendicular from M to the traverse LA.

PROJECT 14

obook assess TEACHER

GET YOUR BEARINGS

Trail, hike or bike?

A project worksheet for this chapter is available on the Teacher obook/assess. Ask your teacher to print it out for you. Completing this activity will help you make the right choices when travelling off-road.

CHAPTER 15

Diet and nutrition

ARE YOU READY?

Complete the questions below to see if you're ready to start this chapter, or if you need to brush up on your skills first.

- Part 1 of the test should be completed *with* a calculator.
- Part 2 of the test should be completed *without* a calculator.

If there are any questions you cannot answer, the link below each question will direct you to a related Mathematical Skills chapter where you can revise basic concepts and get up to speed.

PART 1 WITH A CALCULATOR

1 Find the value of each of the following.

a 14 + 12 =
b 23 + 14 × 325 =
c 2150 + 400 + 65 + 4 × 445 =
d 4(3 × 370) + 960 =

23A Fundamental concepts

2 Find a quarter of these quantities.

a 1875 **b** 34.6

23C Fractions of quantities

3 Set your calculator to automatically round to 1 decimal place. Check that it does so by finding the answer to 5 × 1.765 432 =

23E Rounding with a calculator

4 How many ____ µg (mcg) are in 1 g?

A 1 000 000 **B** 100 000
C 10 000 **D** 1000

25A Units of measurement

5 Calculate the following.

a What is 25% of 0.0002?
b If 50% of something weighs 300 mg, what will the full amount weigh?
c Reduce 2200 by 10%.

24C Percentage change

6 What is the volume of 3 metric cups (250 mL) of liquid?

A 3 L **B** 1.5 L
C 1 L **D** 0.75 L

30A Volume

7 There are about 15 shelled pistachio nuts in 25 g. If Rita eats 45 shelled pistachio nuts, how many grams has she eaten?

24D Ratios

PART 2 WITHOUT A CALCULATOR

8 Find the value of each of the following.

a 750 + 8 × 450 + 1200 =
b (500 + 250) × 5 =
c 12 + 11.4 =
d 12 − 11.4 =

28A Back to basics

9 Convert the following fractions to decimals.

a $\frac{2}{4}$ **b** $\frac{1}{3}$
c $\frac{3}{8}$ **d** $\frac{3}{4}$

28C Decimals

10 Write the value of the 5 in the number 4321.05

28E Powers of 10

11 Convert the following.

a 5.4 kg = ___ g
b 667 g = ___ kg
c 5200 µm = ___ mg
d 8.4 g = ___ µm

25C Conversion of units

12 Add up the following weights. Estimate the total weight.
50 g, 350 g, 225 g, 75 g, 500 g

23F Estimation

15A Food groups and serving sizes

Eating healthily and maintaining an appropriate body weight can be viewed as a balance between food intake (energy in) and activity (energy out), while maintaining a balanced nutrient intake. Gaining or losing weight results from taking in too much or too little energy from the food you eat. Nutritional balance is a matter of eating the right proportions of the different types of food. Maintaining a healthy weight with the temptations of cake shops and fast-food outlets can be hard.

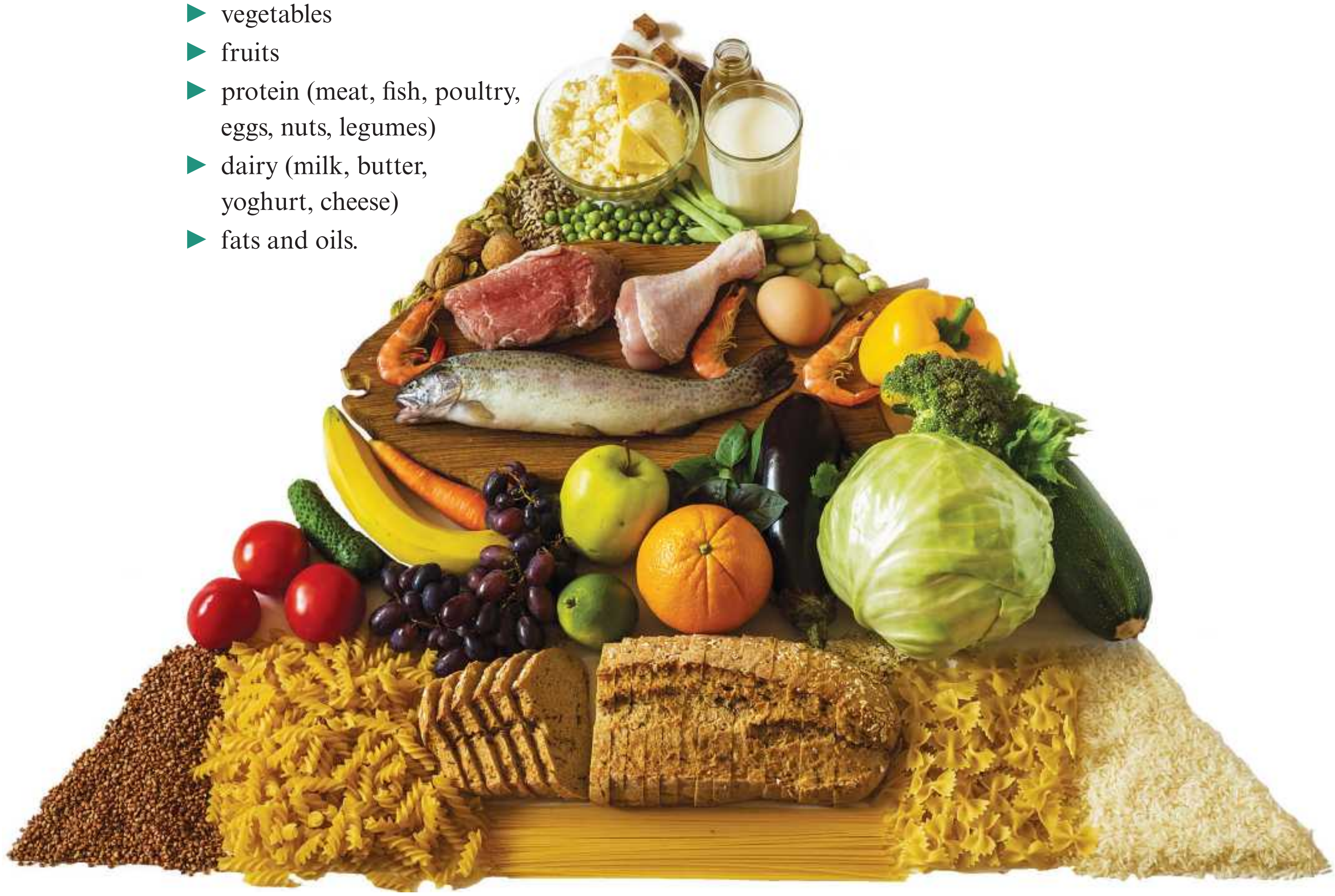
NOTE

A vegetarian diet that eliminates all animal-based proteins and products and includes only nuts and legumes as protein should not be followed without expert guidance.

This chapter is about determining the proportions of food types to eat in a healthy diet; understanding, measuring and calculating food energy values from commonly available sources; and providing help in making estimates of personal energy outputs. It does not contain enough information to support anyone wishing to start on a major weight-loss or a weight-gain program, or for those intending to begin specialised diets. Students interested in such programs *must* first seek specialist advice, and should only undertake such programs with ongoing supervision.

Your body needs a mixture of nutrients to grow and maintain itself properly. These nutrients include proteins, fats, sugars, minerals, vitamins and fibre. They are available in different proportions in the various types of food. As a result, a number of food groups have been defined based on the types of nutrients they contain. These food groups are:

- grains (bread, cereals, rice, pasta, noodles)
- vegetables
- fruits
- protein (meat, fish, poultry, eggs, nuts, legumes)
- dairy (milk, butter, yoghurt, cheese)
- fats and oils.



NEED SOME PRACTICE?
Go to 23B Fractions PAGE 267

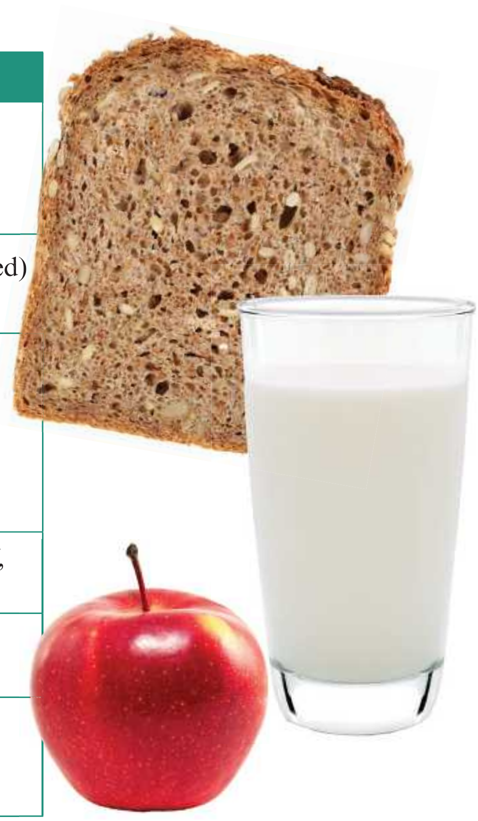
To maintain a healthy weight and enough energy to do your day-to-day activities, you need to:

- have a balanced food intake that is made up of a mixture of all the food groups
- consume enough energy according to your age and activity level so that you can maintain a healthy weight
- limit your intake of foods containing saturated fat, added salt and sugars, and alcohol.

SERVINGS

A normal food intake involves consuming a number of 'servings' of each food group per day. A typical set of recommendations for a 19 to 50-year-old with sedentary to moderate activity level is as follows.

Food group	Servings per day	Typical size of one serving
Grains	6–8	1 slice of bread 30 g dry cereal $\frac{1}{2}$ cup cooked rice or pasta
Vegetables	4–5	$\frac{1}{2}$ cup cut vegetables (raw or cooked) 1 cup raw leafy vegetables
Fruit	4–5	1 medium fruit $\frac{1}{4}$ cup dried fruit $\frac{1}{2}$ cup canned or frozen fruit $\frac{1}{2}$ cup whole-fruit juice
Protein	1–2	100 g cooked lean lamb, lean beef, lean chicken or fish
Dairy	2–3	1 cup of low-fat milk 25 g full-fat cheese
Fats and oils	2–3	1 teaspoon margarine 1 tablespoon mayonnaise 1 teaspoon vegetable oil



There are many publications that outline dietary guidelines, including the Australian Government's publication *Eat for Health: Australian Dietary Guidelines* or visit the website: https://www.eatforhealth.gov.au.

WHAT TO DO 15.1

1 For one day, write down all the food and drink types and quantities you consume from when you first get out of bed in the morning until you go to sleep that night.

2 Draw up a table similar to the one above, and sort each food and drink item you consumed into the six food groups named.

3 Calculate how many serves you consumed for each food group on that day.

15B Nutritional information

NEED SOME PRACTICE?
Go to 24B Percentage of a quantity
PAGE 281

NEED SOME PRACTICE?
Go to 25A Units of measurement
PAGE 291

NEED SOME PRACTICE?
Go to 30A Volume
PAGE 351

NEED SOME PRACTICE?
Go to 30C Mass
PAGE 358

In Australia, government legislation requires the manufacturers of all packaged foods to provide two types of information on the product packet. These are:

- a list of the ingredients in the product
- nutritional information about the product as a whole.

INGREDIENTS LIST

All packaged foods must show the ingredients in them as a list. The major requirements of this ingredients list are:

- The list is to be written in descending order (by weight). This means that the first listed ingredient has the greatest weight in the product, and the last listed ingredient has the least. For example, if sugar is listed at the start of the list, the product contains a greater proportion (by weight) of sugar than any other ingredient.
- Where an ingredient is regarded as a key ingredient, it must have the percentage by weight listed next to the ingredient name. For example, if a product is called 'apricot yoghurt', the percentage of apricot in the yoghurt must be listed beside the ingredient.
- Compound ingredients (ingredients that are made up of two or more ingredients), must have the ingredients listed in brackets beside the compound ingredient name. For example, spaghetti must have (flour, egg, water) listed.

WHAT TO DO 15.2

1 Examine the ingredients list from a breakfast cereal called 'Wholegrain sultana and apple delight' shown on the right and answer the following questions.

a Which ingredient is present in the largest amount, by weight, in this breakfast cereal?

b Which ingredient is present in the smallest amount, by weight, in this breakfast cereal?

c Why has the manufacturer been required to list the percentage of 'apple pieces' in this cereal, but not the percentage of sugar, even though sugar is present in a larger quantity than the apple pieces?

INGREDIENTS
Wholegrain cereals (55%)
Sultanas (10%)
Sugar
Apple pieces (4%)
Concentrated apricot purée
Pectin
Malt extract
Honey
Iron
Vitamins (riboflavin, niacin, folate, thiamin)

NUTRITION INFORMATION LABEL

In addition to the ingredients list, food manufacturers must provide a 'nutrition information' panel. It is a legal requirement that the nutrition information on every product looks the same and provides the same information, regardless of the type of product.

The nutrition information must be listed as both per serving size and per 100 g (see the sample label on the right). The serving size is determined by the manufacturer and varies depending on the type of product. The items of nutrition information that must be listed include:

- energy (displayed as kJ)
- protein
- total fats (including saturated fats and/or trans fats as a component of the total)
- total carbohydrates (including dietary fibre and/or sugars as a component of the total)
- sodium (salt)
- vitamins and minerals (individually).

Nutrition information
Servings per package – 16
Average serving size – 45 g (¾ metric cup[1])

	Average quantity per serving	% daily intake[2] per serving	Per serve with ½ cup skim milk	Quantity per 100 g
Energy	710 kJ	8%	900 kJ	1570 kJ
Protein	4.0 g	8%	8.7 g	9.0 g
Fat, total	0.6 g	0.9%	0.8 g	1.4 g
– saturated	0.1 g	0.6%	0.3 g	0.3 g
Carbohydrate	34.6 g	11%	41.1 g	77.0 g
– sugars	8.1 g	9%	14.5 g	17.9 g
Dietary fibre	3.0 g	10%	3.0 g	6.7 g
Sodium[3]	36 mg	2%	92 mg	80 mg
		% RDI[4]		
Thiamin (Vit B_1)	0.28 mg	25%	0.33 mg	0.61 mg
Riboflavin (Vit B_2)	0.42 mg	25%	0.68 mg	0.94 mg
Niacin	2.5 mg	25%	2.6 mg	5.6 mg
Folate	50 µg	25%	56 µg	111 µg
Iron	3.0 mg	25%	3.1 mg	6.7 mg

1 Cup measurement is approximate and is only to be used as a guide. If you have any specific dietary requirements please weigh your serving.
2 % daily intakes are based on an average adult diet of 8700 kJ. Your daily intakes may be higher or lower depending on your energy needs.
3 36 mg of sodium per serve is equivalent to 0.1 mg salt..
4 % recommended dietary intake (Aust/NZ) per serving.

WHAT TO DO 15.3

1 Use the above nutrition information panel to answer the following questions.
  a What is the size (in g) of the manufacturer's recommended serving size?
  b What is the total mass of fat in this serving?
  c What is the percentage of total fat in this serving?
  d Why is there 0.2 g more total fat in the serve with skim milk than for the 'average quantity per serving'?

VITAMINS AND MINERALS IN FOODS

Most people eat a balanced diet that includes fruit, vegetables, dairy, cereals and meat, and get the required vitamins and minerals in their food intake. Vitamins and minerals are essential parts of your diet and without them you can become sick. For example:

- a lack of vitamin A can affect eyesight, skin and hair
- a lack of thiamine (vitamin B_1) can cause beriberi (a disease affecting heart function and loss of muscle strength)
- a lack of vitamin D can cause bone disease: rickets in children and osteomalacia (soft bones) in adults (vitamin C aids absorption of vitamin D)
- a lack of vitamin C causes scurvy
- a lack of calcium leads to osteoporosis (vitamin C aids absorption of calcium)
- a low iron count produces anaemia (a condition causing tiredness and pale skin)
- a low iodine level during childhood can cause stunted growth, deaf-mutism and reduced IQ, and lead to thyroid problems in adulthood.

Some vitamins and minerals are harder to come by than others. As a result, some people with certain medical conditions or illnesses, or people with very unbalanced diets, can become deficient in one or more vitamin or mineral. These people can benefit from taking vitamin and mineral supplements.

NOTE

RDI stands for recommended daily intake. It is an indicator of how much the body needs to function properly. Either too much or too little can cause illness in many cases.

In the past, some populations were chronically deficient in certain vitamins and minerals. For example, in 19th-century England, rickets was a common condition characterised by bent legs due to soft bones. The most common cause of rickets is a lack of vitamin D or calcium in the diet. Both are essential for strong and healthy bones. The main sources of vitamin D are sunlight (your skin produces vitamin D when it is exposed to the sun) and foods such as oily fish, eggs and fortified breakfast cereals.

Today, government legislation requires the addition of some vitamins and minerals as supplements to prepared foods. Examples include adding folate to bread flour and iodine to salt. Also, some food manufacturers choose to add certain vitamins and/or minerals to their products. These include vitamin A, vitamin B_1 (thiamin), vitamin B_2 (riboflavin), vitamin C, calcium and iron. Governments also list recommended dietary intakes of essential nutrients.

WHAT TO DO 15.4

1 Examine the nutrition information panel on a number of prepared foods to look for the vitamin and mineral content of the product. For each, complete a table such as below.

Supplement	Food added to	Amount added	% RDI/serve	Serving size
Vitamin A				
Vitamin C				
Thiamin (B_1)				
Calcium				
Iron				
Folate/folic acid				

NOTE

Breakfast cereal is a common food product that has added vitamins and minerals. Fruit juices often have added vitamin C and certain milks have added calcium.

2 For each supplement, calculate the amount (g) that corresponds to the RDI using:

$$\text{Amount of vitamin or mineral (g)} = \frac{100}{\%\text{RDI}} \times \text{amount per serve}$$

Find the amount of each food you need to eat as your only source of the supplement.

Supplement	% RDI/serve	Serving size	Total RDI	Amount of food
Vitamin A				
Vitamin C				
Thiamin (B_1)				
Calcium				
Iron				
Folate/folic acid				

NOTE

Remember to convert mg or μg to g before using the formula.

NEED SOME PRACTICE?

Go to 25C Conversion of units
PAGE 297

15C Energy content of foods

The two commonly used units for measuring the energy content of food are the calorie (cal) and the joule (J).

- The joule is the standard metric unit used in Australia, but because the joule is a small unit, the kilojoule (kJ) is normally used to express food energy values:
  1000 joules = 1 kilojoule
- The calorie (cal) is the older, non-SI unit that is still sometimes used for food energy values. It is the standard unit used in food and diet materials coming from the USA.

  The conversion between food energy units is:
  100 calories = 418 kilojoules
  The kilojoule (kJ) is used as the unit in this chapter.

ENERGY CONTENT OF PACKAGED FOODS

For prepared and packaged food, the best way to find the number of kJ per serve (or 100 g) is to refer to the nutrition information panel on the packet. There are two ways you can calculate the approximate number of kJ you consume from this information:

- Find how many of the manufacturer's 'serves' you have consumed, and multiply this by the number of kJ per serve.
- Divide the weight of the amount you have consumed (in g) by 100 and multiply this by the number of kJ in 100 g.

EXAMPLE 15C-1 Energy content of packaged food

a If the nutrition information panel shows that a 45 g serve of cereal is 710 kJ and you eat 70 g, how many kilojoules of cereal have you eaten?

b If the nutrition information panel shows that a 100 g serve of the cereal is 1570 kJ and you again eat 70 g, how many kilojoules of cereal have you eaten?

a Number of serves is $\frac{70}{45} = 1.5556$

kJ eaten is $1.5556 \times 710 = 1104$ kJ

b kJ eaten is $\frac{70}{100} \times 1570 = 1099$ kJ

The nutritional information figures in Example 15C-1 were taken from the label in Section 15B. The 5 kJ difference between the two calculations is most likely due to the manufacturer rounding the values in either the per serve or 100 g values. The difference between the answers is only about 0.06% of the average person's kilojoule intake.

WHAT TO DO 15.5

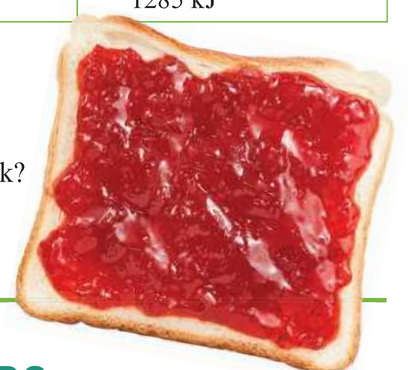
1 Convert the following to kJ.

a 1000 J
b 1 MJ
c 250 J
d 2200 calories
e 1000 calories
f 100 calories

2 Use the nutritional information shown below to answer the following questions.

Product	Energy per serve (kJ)	Serving size	Energy per 100 g
Margarine	240	10 g	2400 kJ
White bread	280	1 slice	400 kJ
Raspberry jam	144	15 g	958 kJ
Tomato soup	400	1 cup (250 g)	350 kJ
Fruit and nut muesli	950	50 g	1900 kJ
Muesli bar	550	30 g	1800 kJ
Apple pie	840	75 g	1120 kJ
Ice-cream	450	50 g	900 kJ
Skim milk	438	250 mL (1 cup)	175 kJ/100 mL
Full-cream milk	556	250 mL (1 cup)	222 kJ/100 mL
Orange juice (unsweetened)	450	250 mL (1 cup)	180 kJ/100 mL
Cola soft drink	600	375 mL (1.5 cups)	160 kJ/100 mL
Corn chips	1000	50 g	2000 kJ
Cheese slices	270	21 g	1285 kJ

a How many kJ are there in 1 cup (250 g) of tomato soup?
b How many kJ are there in 1 cup of skim milk?
c How many kJ are there in one serve of muesli with 1 cup of skim milk?
d How many kJ are there in one serve of muesli with 1 cup of full-cream milk?
e How many kJ are there in a slice of white bread spread with 5 g of margarine and 25 g of raspberry jam?



ENERGY CONTENT OF FRESH FOODS

Fresh food from the vegetable market or a butcher does not come with nutrition information labels. To find the kilojoule values of fresh food such as fruit, vegetables and meats, as well as basic ingredients (like flour and sugar), you will need to refer to tables or the internet. The next table gives the average kJ values for a variety of common foodstuffs. For items not in the table, your best sources are reference books or reputable websites. Not all information sourced on the internet is equally reliable. For nutrition information, you must always check that the data is provided by a reputable organisation.

Government bodies generally provide good information, as well as many health-focused not-for-profit organisations. However, some not-for-profit organisations, in particular some industry lobby sources, may provide old and/or biased information based on the food industry segment they represent.

Food	Serving	kJ per serving	kJ/100 g
Wheat flour: white	1 cup (125 g)	1849.65	1480
Cream: full cream (35% milk fat)	1 Tb (15 g)	213	1442
Cream: light (18% milk fat)	1 Tb (15 g)	110	730
Cheese	25 g	422	1690
Fruit			
Apple	1 medium: 140 g	308	220
Apricot	1 fruit: 35 g	70	200
Avocado	200 g	1350	675
Orange: navel	1 fruit: 140 g	288	206
Mandarin	1 fruit: 76 g	170	224
Pear	1 small: 148 g	360	243
Peach	1 medium: 150 g	245	163
Grapes	10 (50 g)	143	286
Blackberries	1 cup (144 g)	260	180
Blueberries	1 cup (148 g)	356	240
Strawberries	1 cup (144 g)	194	135
Vegetables			
Carrots	100 g	135	135
Potatoes	100 g	360	360
Pumpkin	100 g	50	50
Spinach	100 g	100	100
Celery	100 g	34	34
Mushrooms	100 g	15	15
Tomatoes	100 g	75	75
Lettuce	100 g	55	55
Meat, fish and eggs			
Egg	63 g	380	600
Lamb: lean leg, raw	100 g	587	587
Lamb: forequarter chop, lean, raw	100 g	1125	1125
Kangaroo: fillet	100 g	414	414
Beef: rump steak, lean, raw	100 g	511	511
Chicken: skinless breast	100g	444	444
White fish: steamed	120 g	430	360
Atlantic salmon: raw	198 g	1673	845
Nuts and seeds			
Peanuts	25 g	607	2428
Cashews	15 g	360	2400
Almonds: dried	10 g	250	2500
Walnuts: roasted	10 g	282	2820
Pistachio: shelled, roasted and salted	25 g	354	1416
Sunflower: hulled	15 g	358	2390
Sesame: dried	6 g	145	2400

HOW MANY KILOJOULES DID YOU EAT TODAY?

It is now time to estimate how many kilojoules you take in over one day, counting all the food and drink that you consume. Nutritional balance is eating the right proportions of the different types of food. Gaining or losing weight is the result from taking in too much or too little energy from the food you eat. You will be able to see how your daily food intake fits these requirements.

WHAT TO DO 15.6

1 For one day, keep a record of all the food and drink that you consumed: both food types and drink volumes. A sample table is provided below to assist you. Make a copy of this onto an A4 page to fill it out over the day.

2 For all packaged foods and drinks, use the nutrition information panel to fill in the kJ per 100 g value.

3 For fresh foods, write the weight of the food you ate in the table, then find the kJ per 100 g value from either the previous table or a reliable source.

4 For each food you ate, calculate the kJ value for the amount you ate. Put this value in the right-hand column.

5 Add up all the values in the right-hand column to give the total kJ you ate on that day.

Food	Weight/volume consumed	kJ/100 g or kJ/mL	kJ consumed for day
Breakfast			
Lunch			
Dinner			
Snacks			
Soft drinks/juices, etc.			
Total kJ for day			

15D Energy expenditure

You can now calculate the food energy you consume, but how many kJ do you actually need? If you eat more than you need, the excess kJ will be stored in your body as fat. If you eat a constant slight excess of food, this will lead to a continued accumulation of fat and result in obesity and the health-related issues that can go with being overweight.

HOW MUCH ENERGY DO YOU USE A DAY?

Research shows that children, adolescents and adults have quite different food energy intake needs. This also varies for males and females, as well as depending on the person's metabolism and their activity level.

The table below shows one set of suggested kJ intakes for average body weight, based on gender, age and activity level.

Age and gender	kJ requirement
Children (1–3 years): average activity	5 000
Children (4–6 years): average activity	6 300
Children (7–9 years): average activity	7 500
Children (10–12 years): average activity	8 800
13–15-year-old girls: average activity	9 200
13–15-year-old boys: average activity	10 500
16–18-year-old girls: average activity	9 200
16–18-year-old boys: average activity	12 600
Women: light work	8 000
Women: medium work	9 200
Women: heavy work	11 750
Men: light work	9 200
Men: medium work	11 750
Men: heavy work	14 250

WHAT TO DO 15.7

1 A 25-year-old woman works in an office during the day and works out in a gym for 1 hour, 4 days a week. Which category in the previous table would she best fit into?

A women: light work B women: medium work C women: heavy work D none of these

2 A 27-year-old man works as a builder's labourer carrying materials around a building site. According to the previous table, approximately how many kJ a day would he need to consume?

3 Use the suggested kJ intake table to estimate your personal daily kJ requirement.

PUTTING IT ALL TOGETHER

Despite everything you hear, read about in books or in the media, or see on television, weight loss and weight gain has a simple mathematical formula that underlies all programs designed to help people lose or gain weight:

$$\text{Weight change} = \text{energy input} - \text{energy output}$$

If the answer is positive, you will gain weight. If the answer is negative, you will lose weight; it is as simple as that. Your energy input and energy output do not have to balance every day. It is having a balance over time that will help you stay at a healthy weight for the long term.

Mind you, if you are a growing adolescent, the answer is supposed to be positive, while for an adult you would expect it to be around zero. It is also important that the 'energy input' part is made up of the right mix of the various food components you need for your body to grow and maintain itself properly, as is maintaining an adequate intake of the essential vitamins and minerals.

You burn a certain number of kJ just by breathing air and digesting food. You also burn a certain number of kJ through your daily routine. An important part of maintaining energy balance is the amount of physical activity that you do. People who are more physically active burn more kJ than those who are less active.

This section is designed to put all the previous parts of this theme together to find out whether your personal food intake is appropriate to your needs.

WHAT TO DO 15.8

1 This table shows the amount and types of food eaten by a 17-year-old boy who attends school 5 days a week and does not do any sports out of school time. Find the kJ content of each food item listed.

Food	Weight/volume consumed	kJ/100 g	kJ consumed for day
Breakfast			
Fruit and nut muesli with skim milk	50 g and 1.2 cup skim milk		
Unsweetened orange juice	1 cup		
Lunch			
Apple	1 medium (140 g)		
Sandwich: bread	2 slices		
1 slice of cheese	21 g		
1 can cola	375 mL		
2 muesli bars	60 g		
Dinner			
Lean rump steak	350 g		
Potato: steamed	100 g		
Pumpkin: steamed	75 g		
Spinach	50 g		
Apple pie	100 g		
Ice-cream	50 g		
Snacks			
Corn chips	200 g pack		
Total kJ for day			

a Complete this table to find the total kJ intake for the day.
b Check the food intake for the number of serves of the major food groups.
c Write a short summary of this person's diet and make suggestions as to:
   i what may happen if this diet is typical of their lifestyle and they follow it for the next few years
   ii what changes they could make to improve the diet.

PROJECT 15

obook
assess
TEACHER

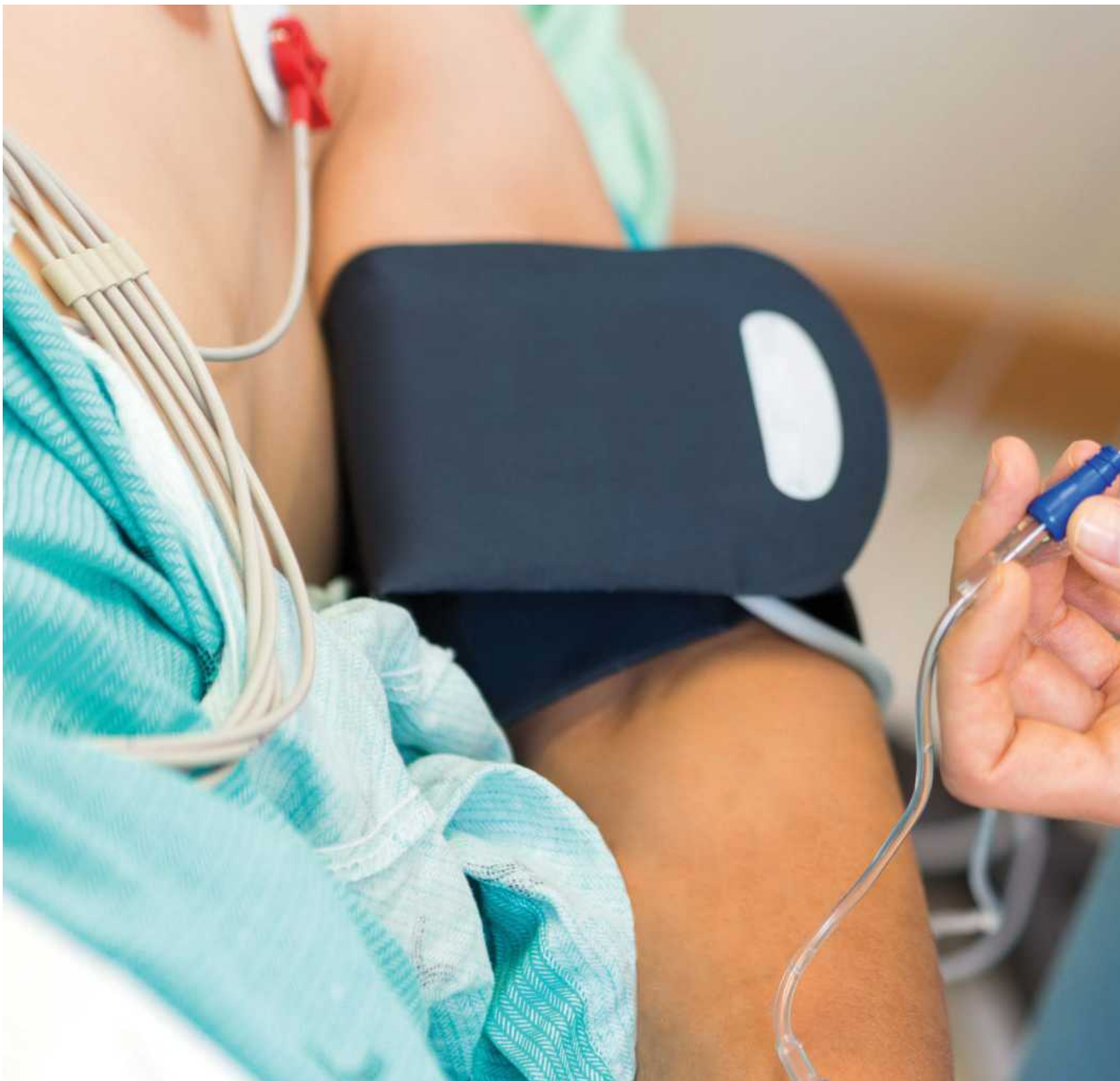
FUELLING YOURSELF

Moderation is the key!

A project worksheet for this chapter is available on the Teacher obook/assess. Ask your teacher to print it out for you. Completing this activity will help you make the right choices when selecting foods for your diet and the diet of your family.

CHAPTER 16

Calculating clinical dosages



ARE YOU READY?

Complete the questions below to see if you are ready to start this chapter, or if you need to brush up on your skills first.

- Part 1 should be completed *with* a calculator.
- Part 2 should be completed *without* a calculator.

If there are any questions you cannot answer, the link below each question will direct you to a related Mathematical Skills chapter where you can revise basic concepts and get up to speed.

PART 1 WITH A CALCULATOR

1 Find the values of the following.

a $88 \times 10 =$ **b** $0.85 \times 1000 =$
c $750 \div 100 =$ **d** $0.50 \div 1000 =$

23D Decimal numbers

2 Convert the following.

a 3.5 kg = ___ g
b 250 g = ___ kg
c 2000 mcg = ___ mg
d 250 g = ___ mcg

25C Conversion of units

3 Find the values of the following.

a $20 \times 8 =$ **b** $0.2 \times 80 =$
c $0.65 \times 0.08 =$ **d** $65 \times 12 =$

23D Decimal numbers

4 Convert to decimals correct to 1 decimal place.

a $\frac{125}{6} =$ **b** $\frac{1}{6} =$
c $\frac{25}{4} =$ **d** $\frac{25}{3} =$

23E Rounding with a calculator

PART 2 WITHOUT A CALCULATOR

5 Find the values of the following.

a $56 \times 10 =$ **b** $56 \times 1000 =$
c $5.6 \times 100 =$ **d** $5.6 \times 1000 =$

28C Decimals

6 Find the values of the following.

a $568 \div 10 =$ **b** $56.8 \div 1000 =$
c $0.568 \div 100 =$ **d** $\frac{5.68}{1000} =$

28C Decimals

7 Complete the following.

a 1 L = ___ mL
b 1 g = ___ mg
c 1 mg = ___ mcg

25C Conversion of units

8 Convert the following.

a 3.4 kg = ___ g
b 555 g = ___ kg
c 250 mcg = ___ mg
d 6.4 g = ___ mcg

25C Conversion of units

9 Find the values of the following.

a $4 \times 8 =$ **b** $0.4 \times 8 =$
c $0.4 \times 0.08 =$ **d** $40 \times 12 =$

28C Decimals

10 Round each number correct to the nearest whole number, 1 and 2 decimal places.

a 1.75, ___, ___, ___
b 0.8755, ___, ___, ___

23D Decimal numbers

11 Convert to decimals correct to 2 decimal places.

a $\frac{1}{4} =$ **b** $\frac{2}{3} =$
c $\frac{65 \text{ mL}}{6 \text{ h}} =$ **d** $\frac{250 \text{ g}}{125 \text{ g}} =$

28C Decimals

12 Simplify and multiply where required.

a $\frac{8}{12} =$ **b** $\frac{60}{120} =$
c $\frac{750 \text{ g}}{250 \text{ g}} \times \frac{5 \text{ mL}}{1} =$ **d** $\frac{500 \text{ mL}}{4 \text{ doses}} =$

28B Fractions

16A Drug calculations

Being able to flawlessly and efficiently perform drug calculations is an essential component of becoming a practising nurse, paramedic or midwife. Before graduating, all health science professionals are required to pass a final competency test (100%). This is not difficult to achieve, but it may require some effort to relearn basic number skills without a calculator. To reach this point of proficiency you will need two things:

- the relevant basic numeracy
- to be able to understand and use the key concepts of 'proportionality' and the following formula:

$$\text{Volume required} = \frac{\text{strength required}}{\text{stock strength}} \times \text{volume of stock}$$

Competency with drug calculations is perhaps best acquired by doing many drug calculations. Even when you are sure you understand them, you should continue to do and check many questions to achieve the accuracy required.

The main aim of practising these drug calculations is that they will become second nature when you come to apply them in the clinical situation, such as at an accident or in the hospital. To make sure the amount of medication is correct, it is necessary to be able to calculate ratios, decimals and fractions and to be able to read and interpret graphs, tables and charts.

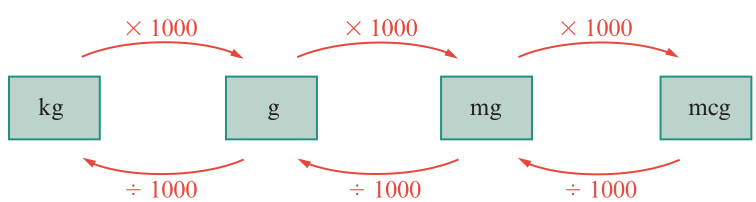
UNIT CONVERSION

Paramedics, nurses and midwives need to be very familiar with undertaking unit conversions without using a calculator before they can calculate correct doses so that they do not harm their patients. A milligram is 1000 times larger than a microgram, so imagine the consequences of mixing up the units.

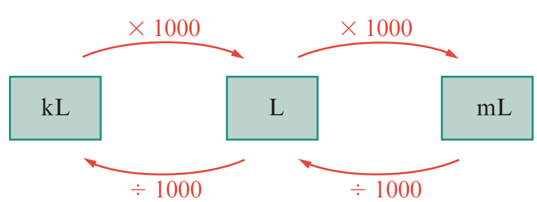
NEED SOME PRACTICE?
Go to 25C Conversion of units
PAGE 297

The diagrams below will help you understand the concept of unit conversion and allow you to have a permanent picture in your mind to refer to when you want to convert units. The arrows show you which way to move the decimal point as well as whether to multiply or divide.

This is the conversion for units of mass.



This is the conversion for units of capacity.



EXAMPLE 16A-1 Unit conversions

Convert the following.

a 0.2 kg to g
b 225 g to kg
c 2.5 g to mg
d 200 mL to L

a Larger to smaller
$0.2 \text{ kg} \times 1000 = 0200 \text{ g}$
$= 200 \text{ g}$
Multiply by 1000.

b Smaller to larger
$225 \text{ g} \div 1000 = 0.225 \text{ kg}$
Divide by 1000.

c Larger to smaller
$2.5 \text{ g} \times 1000 = 2500 \text{ mg}$
Multiply by 1000.

d Smaller to larger
$200 \text{ mL} \div 1000 = 0.200 \text{ L}$
$= 0.2 \text{ L}$
Divide by 1000.

NEED SOME PRACTICE?
Go to 25C Conversion of units PAGE 297

WHAT TO DO 16.1

1 Convert the following.
a 1 g = ___ mg b 1 g = ___ mcg c 1.2 L = ___ mL d 1 L = ___ mL

2 Change each to grams.
a 2.5 kg b 0.75 kg c 1.375 kg d 0.025 kg

3 Change each to kilograms.
a 2000 g b 55 g c 175 g d 5 g

4 Change each to litres.
a 285 mL b 4500 mL c 5 mL d 60 mL

5 Change each to millilitres.
a 4.5 L b 0.5 L c 0.25 L d 0.005 L

6 Change each to micrograms.
a 2 mg b 0.25 mg c 0.040 mg d 20 mg

7 Change each to grams.
a 25 mcg b 250 mcg c 1.5 mcg d 1200 mcg

8 Convert the following.
a 27 mcg = ___ mg b 250 mL = ___ L c 1200 mcg = ___ g d 1.2 kg = ___ g

9 Convert the following.
a 3.6 mg = ___ mcg b 3.6 g = ___ mcg c 3600 mL = ___ L d 1 g = ___ mg

16B Oral dosages

Many medications are in the form of a tablet, capsule or liquid. The liquid may be a solution, suspension, syrup or elixir. Suspensions should always be shaken thoroughly before measuring the required volume.

Check that the stock strength and the strength required are given in the same units for a particular dose; that is, that both strengths are in g, mg or mcg (µg). Include the units in the calculation as a check.

Other terms used for strength required are 'prescribed', 'ordered', 'to be taken' and 'dosage required'. Other terms for stock strength are 'stock available', 'on hand', 'strength available' and the mL contained.

NEED SOME PRACTICE?

Go to 28B Fractions PAGE 326

The general formula for working out dosages involves working out the fraction of the dose required and therefore the amount:

$$\text{Volume required} = \frac{\text{strength required}}{\text{stock strength}} \times \text{volume of stock}$$

EXAMPLE 16B-1 Volume required

Clancy has been prescribed 250 mg of amoxycillin, which is available as a stock strength of 1 g in 25 mL. What volume of amoxycillin should he be given?

$$\text{Volume required} = \frac{\text{strength required}}{\text{stock strength}} \times \text{volume of stock}$$

$$= \frac{250\text{ mg}}{1\text{ g}} \times \frac{25\text{ mL}}{1}$$

$$= \frac{250\text{ mg}}{1000\text{ mg}} \times \frac{25\text{ mL}}{1}$$ Change 1 g to 1000 mg.

$$= \frac{1}{4} \times \frac{25\text{ mL}}{1}$$ Cancel the units.

$$= \frac{25}{4}\text{ mL}$$

$$= 6.25\text{ mL}$$ Convert to a decimal.

NOTE

This volume works out to be $\frac{1}{4}$ of 25 mL or 6.25 mL.

WHAT TO DO 16.2

1 A patient is prescribed 750 mg of paracetamol orally. The stock is available in 500 mg tablets. Calculate the number of tablets required.

2 How many 20 mg tablets are required for a dose of 40 mg?

3 If 1 g of paracetamol is prescribed and the stock on hand is 500 mg tablets, how many tablets are required?

4 A patient is prescribed 450 mg of aspirin and the available stock is 300 mg tablets. How many tablets should be prescribed?

5 A patient is prescribed 500 mg of penicillin available in a syrup of 125 mg/5 mL. What volume is to be given?

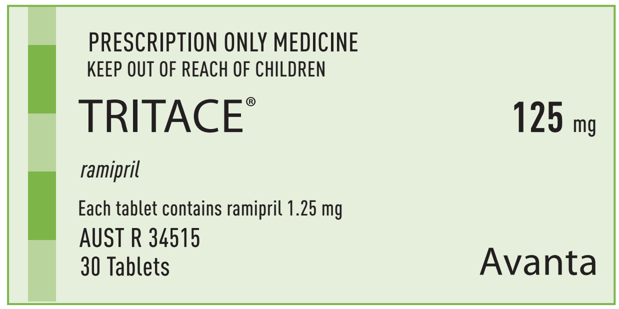
6 A patient is prescribed 180 mg of paracetamol available in a suspension of 125 mg/mL. Calculate how much should be given to the patient.

7 35 mg of chlorpromazine has been ordered for a patient and the stock available is a syrup of 25 mg/5 mL. How much should be given to the patient?

8 Calculate the volume of penicillin to be given if the prescribed penicillin amount is 1250 mg and it is available as a mixture of 250 mg/5 mL.

9 Flynn has been prescribed 400 mg of penicillin orally and the stock strength for the syrup is 125 mg/5 mL. What volume should be given?

10 Abi has been prescribed 2.5 mg of ramipril. Use the medicine label shown to determine how many Titrace tablets should be given.



16C Strength according to volume

To be able to give a patient the correct dose, it is important to understand how much of the active ingredient in the medicine is contained in a specific volume of liquid or in a tablet.

EXAMPLE 16C-1 Strength of solution

A mixture of penicillin contains 150 mg in 5 mL. How many mg of penicillin are contained in:

a 10 mL? **b** 30 mL? **c** 1 mL?

The mixture is 150 mg in 5 mL.

a

× 2	150 mg	in	5 mL	× 2
	300 mg	in	10 mL	

There are 300 mg in 10 mL.

b

× 6	150 mg	in	5 mL	× 6
	900 mg	in	30 mL	

There are 900 mg in 30 mL.

c

÷ 5	150 mg	in	5 mL	÷ 5
	30 mg	in	1 mL	

There are 30 mg in 1 mL.

WHAT TO DO 16.3

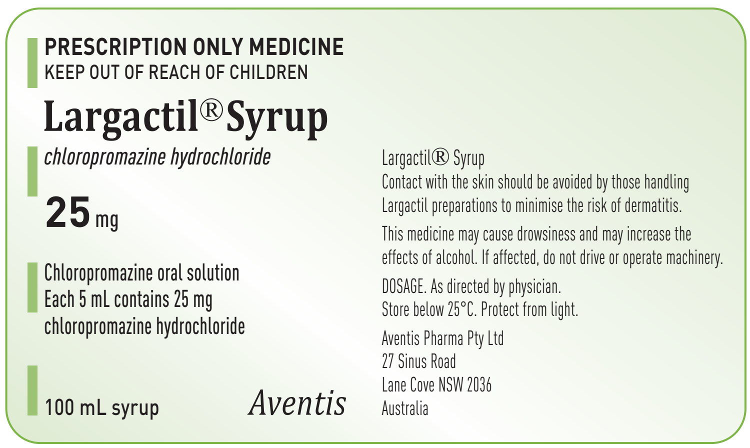
1. A solution contains morphine hydrochloride at 5 mg/mL. How many milligrams of morphine hydrochloride are in:
   - a 5 mL?
   - b 20 mL?
   - c 1 mL?
   - d 2.5 mL?

2. A tablet contains 500 mg of paracetamol. How much paracetamol is contained in:
   - a 2 tablets?
   - b $\frac{1}{2}$ tablet?

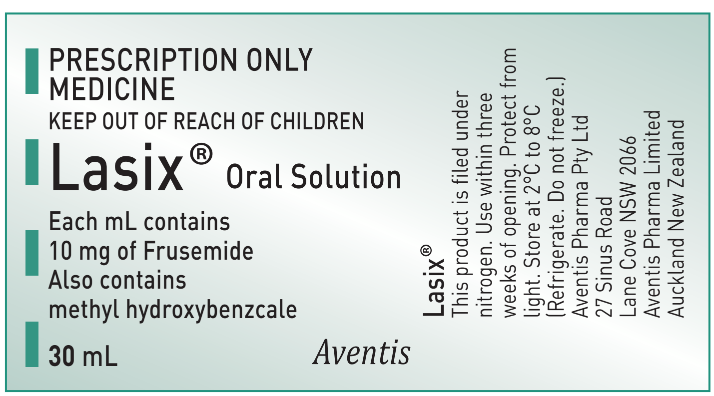
3. If a suspension contains penicillin at 125 mg/5 mL, how much is contained in:
   - a 10 mL?
   - b 15 mL?
   - c 40 mL?
   - d 35 mL?

4. A solution contains 10 mg/mL of furosemide (frusemide). How much furosemide is contained in:
   - a 1 mL?
   - b 20 mL?
   - c 5 mL?
   - d 4 mL?

5. Use the medicine packet label shown to answer the following questions.
   - a How much chloropromazine hydrochloride is in 5 mL of Largactil?
   - b How much chloropromazine hydrochloride is in 1 mL of Largactil?
   - c Calculate the volume required for a dose of 40 mg. Show your working using the formula for volume required.



6. Use the medicine packet label shown to answer the following questions.
   - a How much furosemide (frusemide) is in 5 mL of Lasix?
   - b How much furosemide is in 1 mL of Lasix?
   - c Calculate the volume of Lasix required for a dose of 40 mg of furosemide. Show your working using the formula for volume required.



16D Paediatric dosages

Paediatrics is the branch of medicine that deals with the medical care of infants, children and adolescents. Children are often more sensitive to medications than adults, so factors such as the age, weight and height of a child become much more important when determining the correct dosage of medication to be administered.

There are two main ways of calculating paediatric dosages:

- dosage based on body weight
- dosage based on surface area.

CALCULATING DOSAGE BASED ON BODY WEIGHT

There are wide variations in the heights and weights of children at any given age. Thus medical dosages are often calculated according to a child's body weight rather than age.

EXAMPLE 16D-1 Dosage based on body weight

A child is prescribed penicillin. The recommended dosage is 50 mg/kg/day for four doses per day. Calculate the size of a single dose if the child's weight is 20 kg.

$$\begin{aligned}\text{Required dose} &= \text{child's weight (kg)} \times \text{unit dose (per kg)}\\ &= 20\text{ kg} \times 50\text{ mg}\\ &= 1000\text{ mg/day}\\ \frac{1000\text{ mg}}{4\text{ doses}} &= 250\text{ mg per single dose}\end{aligned}$$

WHAT TO DO 16.4

1 A 32 kg child is prescribed flucloxacillan at 100 mg/kg/day, four doses per day. How much should she receive each day?

2 A 30 kg child is prescribed erythromycin at 40 mg/kg/day to be given in four doses. How much erythromycin should each dose contain?

3 40 mg/kg/day, in four doses per day, of chloramphenicol has been ordered for a child called Milly who weighs 40 kg. How much should she receive in each dose?

4 50 mg/kg/day of cephalothin has been prescribed for Clancy who weighs 30 kg. He requires three doses per day. How much should he receive in each dose?

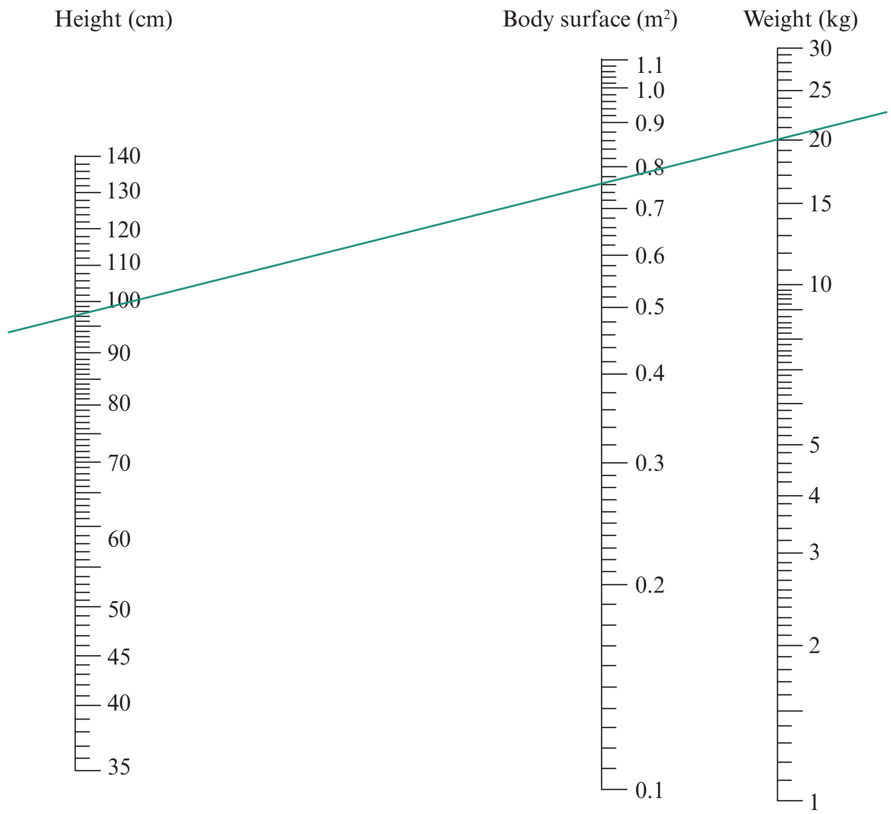
5 Flynn, who weighs 12 kg, is to be given ampicillin, 80 mg/kg/day. How much should he have in each dose if he is given four doses a day?

CALCULATING DOSAGE USING SURFACE AREA

In some cases, such as for chemotherapy, dosages are calculated according to body surface area using a nomogram. A nomogram is a means of determining the surface area of a child by measuring his or her weight and height.

NEED SOME PRACTICE?
Go to 28C Decimals PAGE 331

The following nomogram can be used to estimate the body surface area of infants and young children. To find the body surface area of a child with a weight of 20 kg and a height of 97 cm, use a ruler to join the height (length) of 97 cm and weight of 20 kg. The value where the ruler crosses body surface area (BSA) scale is 0.76 m².



NOTE

Someone in your class may have a younger brother or sister who you can test this on.

EXAMPLE 16D-2 Checking your surface area

The area of your hand (palm and fingers) is approximately 1% of your total body surface area. Check this by testing it on a young child and comparing the value to the value you get on the nomogram above.

Step 1: Trace around the outside of the child's hand (fingers together) on a piece of 2 mm graph paper.
Step 2: Count squares, or split the handprint into a series of rectangles, and find the total area of their hand.
Step 3: Multiply this by 100 to get the surface area of their body.
Step 4: Measure their weight and mark it on the weight scale on the nomogram.
Step 5: Measure their height and mark it on the height scale on the nomogram.
Step 6: Join the marked points with a ruler and read the value for body surface area (BSA) from the middle scale.
Step 8: Compare the results and comment on the how close or far apart they are.

EXAMPLE 16D-3 Dosage based on surface area

A child has been prescribed cytarabine. The recommended dosage is 120 mg/m². Stock strength is 100 mg/10 mL and her body surface area is 0.60 m². What volume of the medication should she be given?

Use the volume required formula to find the dose per m², then multiply by the body surface area of the child.

$$\text{Volume required} = \frac{\text{strength required}}{\text{stock strength}} \times \text{volume of stock} = \frac{SR}{SS} \times V$$

$$= \frac{120\text{ mg}}{100\text{ mg}} \times \frac{10\text{ mL}}{1}$$

$$= 12\text{ mL}$$

$$\text{Dose} = 12\text{ mL} \times 0.6\text{ m}^2$$

$$= 7.2\text{ mL}$$

The volume of the medication that should be given is 7.2 mL.

WHAT TO DO 16.5

1 The height and weight of different young children are given. Determine the body surface area using the previous nomogram. Answers should be estimated correct to 2 decimal places.

- a height 70 cm, weight 6.0 kg
- b height 72cm, weight 8.5 kg
- c height 85 cm, weight 11.5 kg
- d height 87cm, weight 14.5 kg
- e height 90 cm, weight 14.3 kg
- f height 77 cm, weight 11.4 kg
- g height 70 cm, weight 9.3 kg
- h height 68 cm, weight 10.2 kg
- i height 130 cm, weight 16.3 kg
- j height 120 cm, weight 20.2 kg

2 A child has been prescribed doxorubicin for leukaemia and the recommended dosage is 30 mg/m². The child's body surface area has been assessed at 0.40 m². Stock solution is 10 mg/10 mL. Calculate the volume of doxorubicin to be given.

3 Cytarabine has been prescribed. The recommended dosage is 10 mg/m² and the body surface area of the child is 0.45 m². The stock strength is 100 mg/5 mL. Determine the volume to be given.

PROJECT 16

obook
assess
TEACHER

THE RIGHT AMOUNT

Dosage, dosage, dosage!

A project worksheet for this chapter is available on the Teacher obook/assess. Ask your teacher to print it out for you. Completing this activity will help you make the right choices when measuring dosages.

CHAPTER 17

Types of design

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PART 1 WITH A CALCULATOR

1 Find the value of each of the following.

a $((8 \times 2) - (4 \times 2) \times 2) \div 2 =$

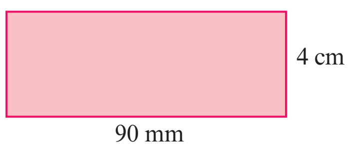
b $12 + 7 - 4 \times 2 + 2 =$

c $6 \times 3.8 \times 2 + 5 \times 54 \div 15 =$

d $(10 \times 6) - (4 + 2) \times 2 =$

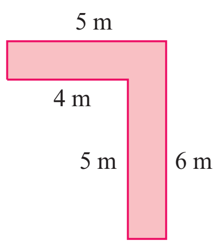
23A Fundamental concepts

2 Calculate the area of the following rectangle.



27A Area

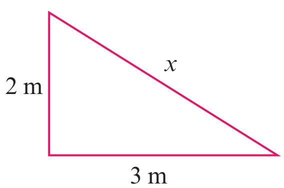
3 Calculate the area of the following composite shape.



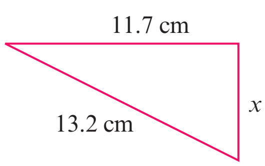
27A Area

4 Use Pythagoras' theorem to find the unknown side in each triangle.

a



b



32A Pythagoras' theorem

PART 2 WITHOUT A CALCULATOR

5 Find the value of each of the following.

a $12 - 8 \div 4 =$

b $5 + 5 \times 5 =$

c $12 \div 6 \times 4 =$

d $5 + 5 - 7 =$

28A Back to basics

6 Convert the following decimals to fracions.

a 0.667 **b** 0.55

c 0.9 **d** 0.15

23D Decimal numbers

7 Convert the following units.

a 4560 cm to m

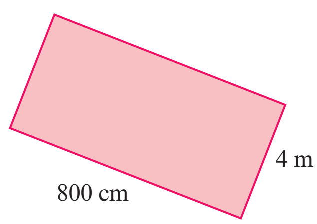
b 2.4 km to cm

c 0.66 km to mm

d 6000 mm to m

25A Units of measurement

8 What is the area of the following rectangle?



A 3200 cm^2 **B** 320 m^2

C 32 m^2 **D** 32 cm^2

27A Area

9 Add up the following prices for materials. Estimate the total cost.

\$59.99, \$35.50, \$11.50, \$32, \$59.99, \$3.99

23F Estimation

17A Interior design

Interior design is the practice, art and process of decorating the inside of a room or a building. The main principles of interior design include balance, rhythm, harmony, emphasis, colour, proportion and scale.

Balance is the even distribution or even arrangement of objects, either physically or visually, to reach a state of calm and order. Balance in interior design is achieved in one of three ways:

- through symmetrical (formal) balance
- through asymmetrical (informal) balance
- through radial balance.

SYMMETRICAL BALANCE

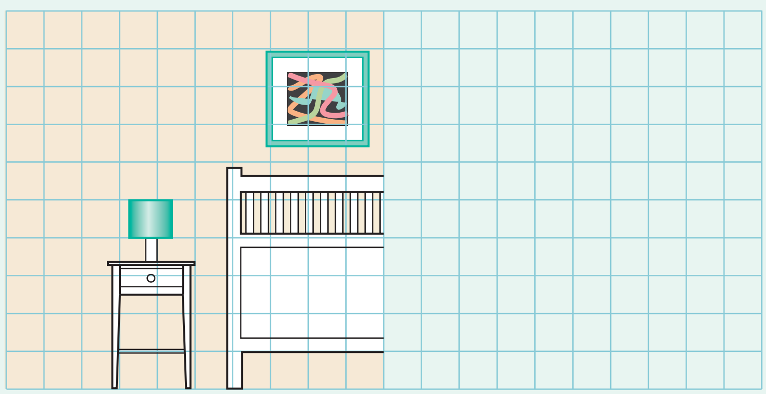
Symmetrical balance is achieved when items are actually repeated or mirrored along a central axis. Symmetry can be achieved through the use of pattern, the arrangement of furniture, by fixtures and by the use of colour.

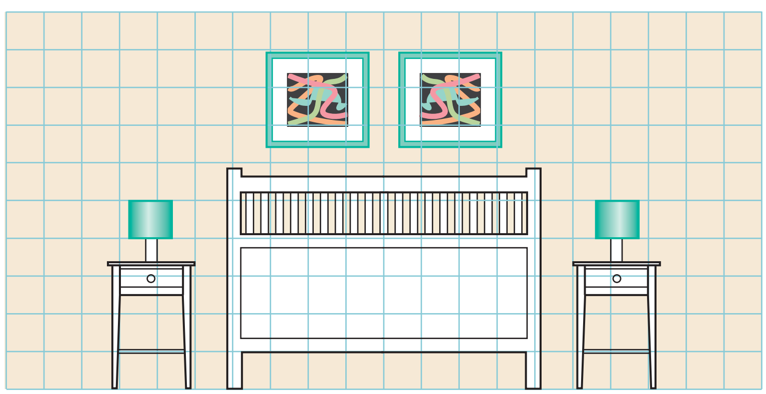
Symmetry not only happens in interior design, it also happens in the construction of units, apartments and town houses. The external appearance of a house, as well as the interior floor plan, may be a mirror image.

NEED SOME PRACTICE?
Go to 31B
Scale drawings
PAGE 362

EXAMPLE 17A-1 Symmetrical design

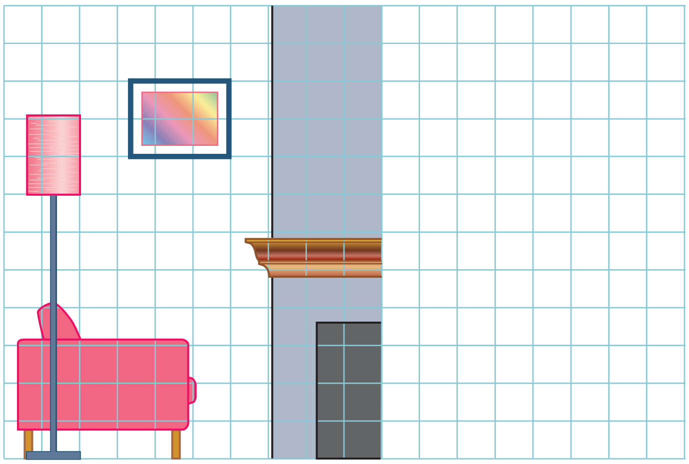
Complete the symmetrical design of the room shown below.



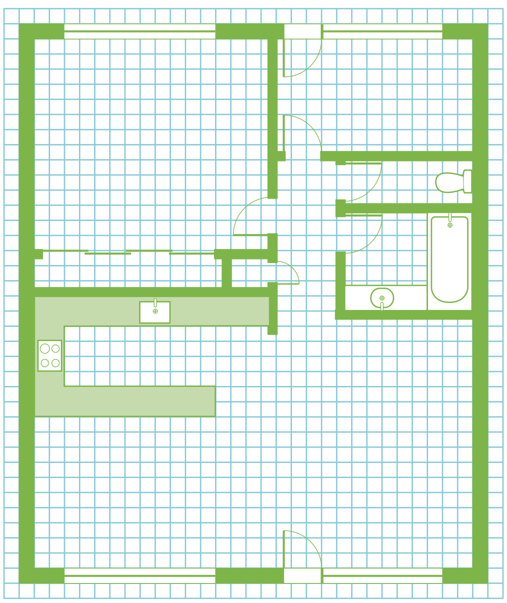


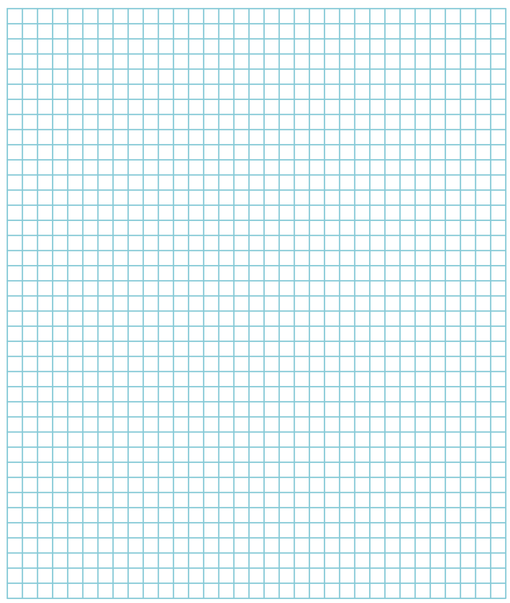
WHAT TO DO 17.1

1 Complete the symmetrical view for the room below.



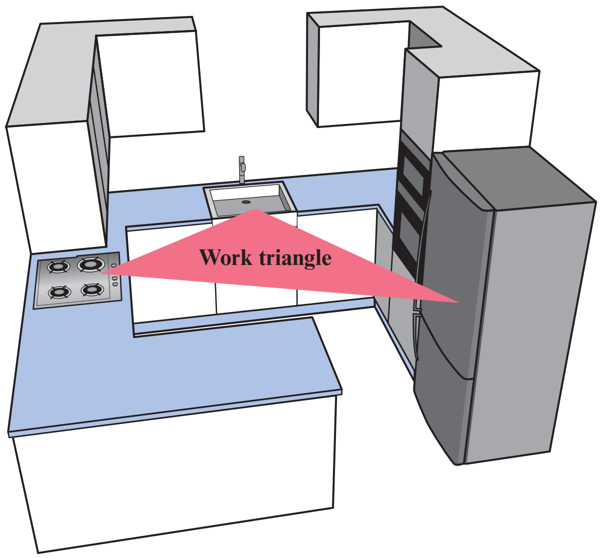
2 Create a second unit on the right that is a symmetrical mirror image of the unit shown on the left.





KITCHEN DESIGN

When designing a kitchen, one of the most important considerations is function; that is, how efficient a kitchen is to work in. The kitchen work triangle is a concept used to assess the efficiency of a kitchen layout. The key tasks in a home kitchen are carried out between the cooktop, the sink and the refrigerator. The idea is that when these three elements are in close (but not too close) proximity to one other, the kitchen will run efficiently, making your life easier and reducing unnecessary movement.

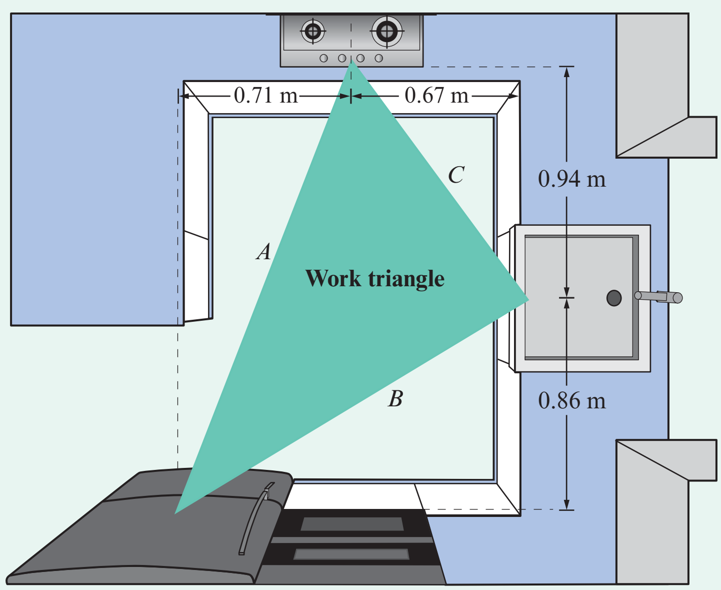


Two important rules kitchen designers use when designing kitchens are:

- No side of the triangle should be less than 1.2 m or more than 3 m.
- The sum of all three sides of the triangle should be between 4 m and 8 m.

EXAMPLE 17A-2 Checking kitchen design

Calculate the length of sides *A*, *B* and *C* of the work triangle and check them against the rules above. If any side does not meet the requirements, suggest a solution. Find the total of all the sides and determine if it meets the requirements.



EXAMPLE 17A-2 continued **Checking kitchen design**

Use Pythagoras' theorem $h = \sqrt{a^2 + b^2}$ to calculate the length of the sides of the work triangle.

Side A: $A = \sqrt{0.71^2 + (0.94 + 0.86)^2}$
$= \sqrt{0.71^2 + 1.8^2}$
$= \sqrt{0.504 + 3.24}$
$= \sqrt{3.744}$
$= 1.93$ m

Side B: $B = \sqrt{0.86^2 + (0.71 + 0.67)^2}$
$= \sqrt{0.86^2 + 1.38^2}$
$= \sqrt{0.74 + 1.9}$
$= \sqrt{2.64}$
$= 1.63$ m

Side C: $C = \sqrt{0.67^2 + 0.94^2}$
$= \sqrt{0.449 + 0.884}$
$= \sqrt{1.33}$
$= 1.15$ m

Sides A and B are between 1.2 m and 3 m.

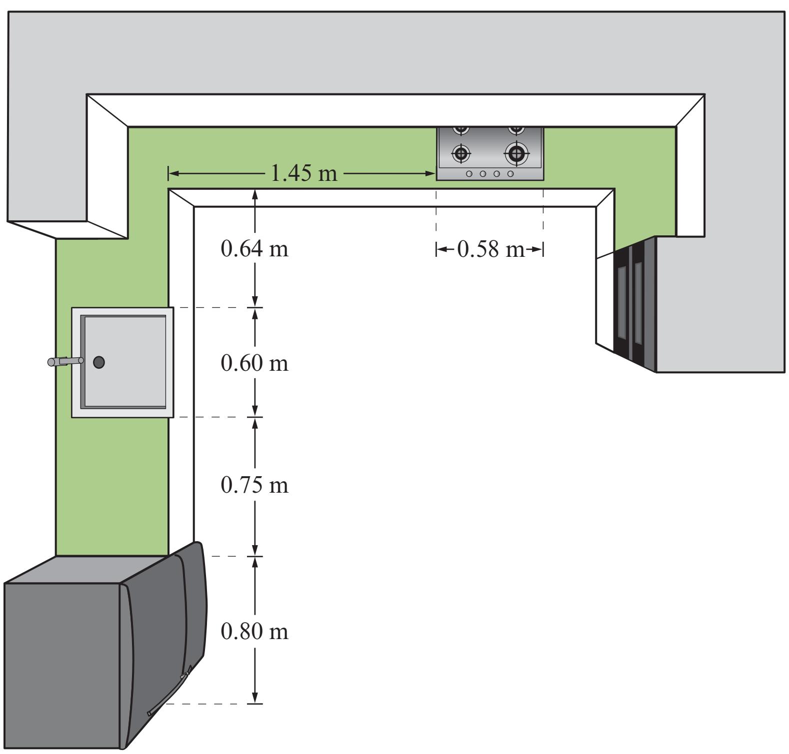
Side C is less than 1.2 m. To fix this you would move the sink down or move the cooktop to the left.

The total of all sides is 1.93 + 1.63 + 1.15 = 4.71 m.
This is bigger than the minimum of 4 m and less than the maximum of 8 m.

NEED SOME PRACTICE?
Go to 32A Pythagoras' theorem PAGE 371

WHAT TO DO 17.2

1 Using the diagram below, identify the kitchen triangle and calculate the length of each of the sides. Calculate the sum of the kitchen triangle and check it against the rules.



17B Product design

Planning before you start building a project helps you get the end product you want with the least amount of work and for the least cost. Cutting lists can be used to help you plan for and order the materials required. They also help in working out the best use of the materials available.

An example of a cutting list is shown below. Costs can be calculated by using the required amount of material and multiplying by its unit cost.

Part	Number	Length (mm)	Width (mm)	Thickness (mm)	Material
Shelves	4	400	200	20	Pine
Legs	6	400	70	35	Pine
Supports	4	200	70	35	Pine

NEED SOME PRACTICE?
Go to 25C Conversion of units PAGE 297

EXAMPLE 17B-1 Calculating cost

Calculate the cost of the shelves in the cutting list above, using a cost price of \$20 per square metre for the pine.

Step 1: Convert the dimensions from millimetres to metres.
$400 \text{ mm} \div 1000 = 0.4 \text{ m}$ and $200 \text{ mm} \div 1000 = 0.2 \text{ m}$

Step 2: Find the area of the shelves.
$0.2 \text{ m} \times 0.4 \text{ m} = 0.08 \text{ m}^2$

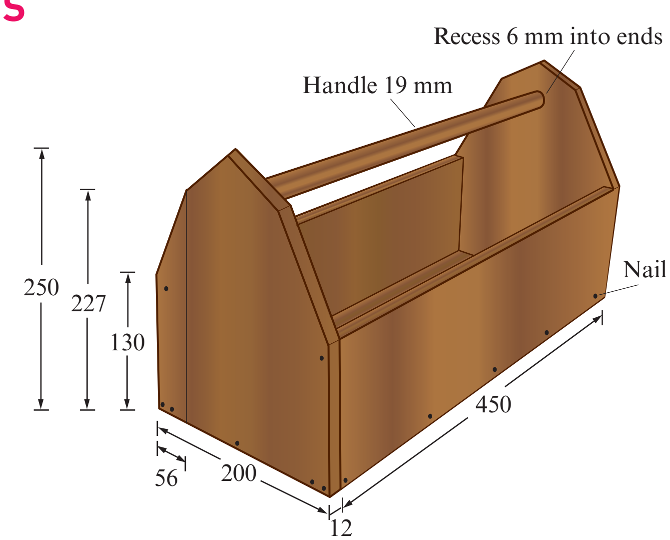
Step 3: Multiple the area required for one shelf by the cost per square metre.
$0.08 \text{ m}^2 \times \$20 = \$1.60$

Step 4: Multiple the area by the number of shelves.
$4 \times \$1.60 = \$6.40$

The cost of the shelves is \$6.40.

CALCULATING COSTS

To find the total cost of producing an item, you need to calculate the cost of all materials needed, keeping the wastage as low as possible. For example, to calculate the cost of producing the wooden toolbox shown, use the following steps.



Step 1: First check the diagram. No units are given on the diagram, so what does the 450 represent? You assume that all measurements are in mm.

Step 2: What materials besides wood are required? You need nails and probably wood glue. It is hard to tell how many nails from the diagram, so you have to estimate. A pack of 100 nails costs \$2.30 and 250 mL of PVA wood glue costs \$4.99.

Step 3: Using the toolbox diagram, complete the table below.

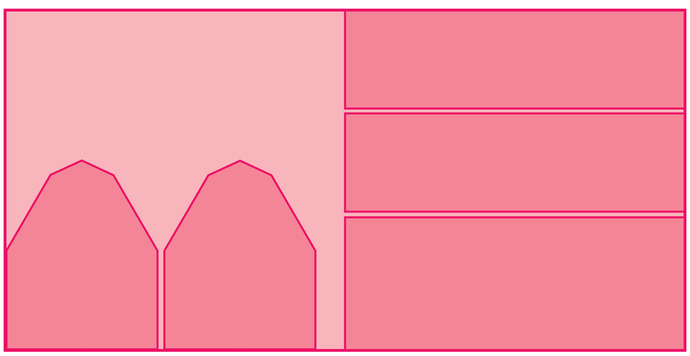
Part	Number	Length (mm)	Width (mm)	Thickness (mm)	Material
Base	1	450	176	12 mm	12 mm plywood
Sides	2	450	130	12 mm	12 mm plywood
Ends	2	250	200	12 mm	12 mm plywood
Handle	1	462		19 mm	19 mm dowel

NOTE

Base width
= 200 − 12 − 12
= 176 mm

Handle length
= 450 + 6 + 6
= 462 mm

Step 4: Create a layout diagram of the pieces required to make the toolbox from a piece of 900 mm by 450 mm plywood board. Notice that there are gaps between each of the pieces. This is to allow for the width of the saw. Saw widths are typically between 3 mm and 5 mm.



Step 5: To calculate the cost of constructing the toolbox, use the following formulas. First you will need to convert millimetre dimensions to metres.

$$\begin{aligned}\text{Cost of plywood} &= \text{area} \times \text{unit cost}\\ &= (0.9 \times 0.45) \times \$8.42\\ &= 0.405 \times \$8.42\\ &= \$3.41\end{aligned}$$

$$\begin{aligned}\text{Cost of dowel} &= \text{length} \times \text{unit cost}\\ &= 0.462 \times \$3.88\\ &= \$1.79\end{aligned}$$

NEED SOME PRACTICE?

Go to 27A Area
PAGE 311

Step 6: Fill in a table such as the one shown below.

Material	Dimension	Typical unit cost	Total cost
12 mm plywood	area	\$8.42/m^2	\$3.41
19 mm dowel	length	\$3.88/m	\$1.79
Pack of 100 nails		\$2.30	\$2.30
PVA glue		\$4.99	\$4.99
		Total cost of toolbox	\$12.49

NOTE

You need to buy the whole piece of 900 mm by 450 mm plywood no matter how much you need to make the toolbox.

NOTE

It is easier and often more accurate to work in millimetres and convert to metres at the end.

Step 7: Calculate how much of a 900 mm × 450 mm sheet is wasted using this formula:

$$\text{Wastage (\%)} = \frac{\text{area of whole sheet} - \text{area required to make toolbox}}{\text{area of whole sheet}} \times 100$$

Area of whole sheet = 900 × 450 = 405 000 mm^2

Area of tool box = (area of ends × 2) + (area of sides × 2) + area of base

To calculate the area of the toolbox break it up into shapes.

Area of base = 450 ×176 = 79 200 mm^2

Area of sides = 450 ×130 = 58 500 mm^2

To calculate the area of the ends, break each into three composite shapes: a rectangle, a triangle and a trapezium.

Area of end = area of rectangle + area of trapezium + area of triangle

$= (\text{length} \times \text{width}) + \frac{1}{2}(\text{side} + \text{side}) \times \text{height} + \frac{1}{2}(\text{base} \times \text{height})$

$= (200 \times 130) + \frac{1}{2}(200 + 88) \times 97 + \frac{1}{2}(88 \times 23)$

$= 26\,000 + 13\,968 + 1012$

$= 40\,980 \text{ mm}^2$

NOTE

Trapezium height
= 227 – 130
= 97 mm
Trapezium sides are 200 mm and 200 – (2 × 56)
= 200 – 112
= 88 mm
Triangle height
= 250 – 227
= 23 mm

Area of toolbox = (area of ends × 2) + (area of sides × 2) + area of base

$= (40\,980 \times 2) + (58\,500 \times 2) + 79\,200$

$= 81\,960 + 117\,000 + 79\,200$

$= 278\,160 \text{ mm}^2$

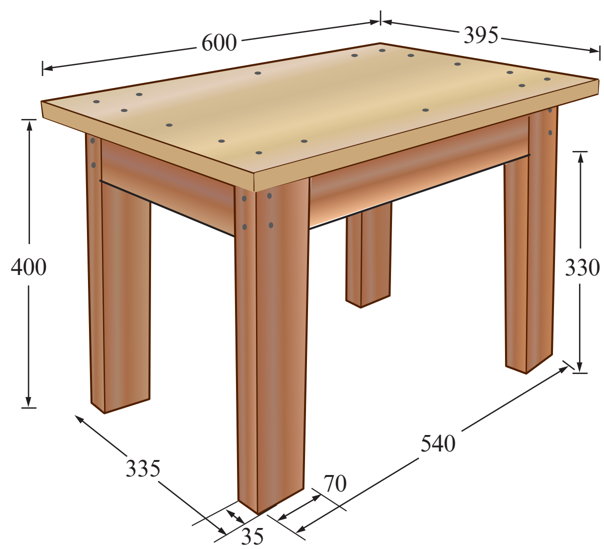
$= 0.278\,160 \text{ m}^2$

$$\text{Percentage wasted} = \frac{0.405 - 0.278\,16}{0.405} \times 100\% = 31\%$$

31% of the 900 mm × 450 mm plywood sheet is wasted.

WHAT TO DO 17.3

1 A carpenter needs to make a coffee table for his workshop. He draws a diagram so that he doesn't make any mistakes and can make the table as cheaply as possible. Calculate the materials and cost of making this coffee table.



a Complete the cutting list table below using the dimensions shown on the diagram.

Part	Number required	Length (mm)	Width (mm)	Thickness (mm)	Material
Legs	4		35	70	Pine planks
Rails: short	2		35		Pine planks
Rails: long	2				Pine planks
Top	1			17	Pine board

b Calculate the cost of constructing the coffee table given the costs and formulas below.

Cost of pine board = area × unit cost
Cost of pine planks = length × unit cost

Material	Dimension	Typical unit cost	Total cost
17 mm pine board	area	$8.42/m^2	
70 × 35 mm pine planks	length	$1.50/m	
Pack of 100 screws		$2.30	
		Total cost of coffee table	

c If a sheet of pine board is 2400 mm by 1200 mm, calculate how much is wasted using this formula:

$$\text{Wastage (\%)} = \frac{\text{area of whole sheet} - \text{area required to make coffee table}}{\text{area of whole sheet}} \times 100$$

REDUCING WASTE

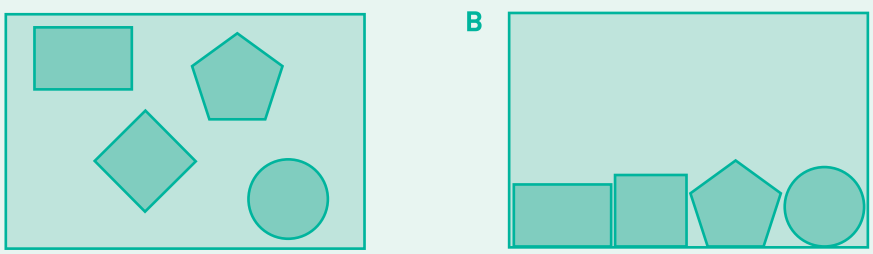
Reducing the waste material in woodwork projects has many benefits. It reduces the cost of the project, thus increasing the profit, reducing the environmental impact and reducing the cost of waste disposal. Wastage can be reduced by:

- measuring pieces accurately to reduce having to recut
- creating a template and placing the template on the material as close together as possible (this is not always possible as you often have to work with the wood grain)
- selecting faultless timber where possible (avoiding knots and splitting).

EXAMPLE 17B-2 Reducing wastage

Which of these layouts has the least amount of wastage?

A B



Layout B has less wastage, as the larger piece of scrap material is more useful. It is easier to cut more shapes from one large piece than from a series of smaller pieces.

There are three important features to choosing timber.

- The type of timber chosen is based on what it is to be used for, such as softwood or hardwood, and timber grade: visual grading (F) or machine grading (MGP).
- The size of the timber cross-section is given with the larger dimension followed by the smaller dimension, such as 90 mm × 45 mm rather than 45 mm × 90 mm. Measurements of the cross-section are given in millimetres.
- The timber length is often measured in metres and timber is usually sold as standard lengths increasing by 300 mm increments, such as 1800 mm, 2100 mm, 2400 mm up to 6000 mm. This may differ from location to location.

NOTE

Working with standard lengths may be a technique to reduce overall wastage and to bring costs down.

EXAMPLE 17B-3 Choosing timber lengths

Jimeoin needs two pieces of wood each of 1 m length.

a Is this a standard length?

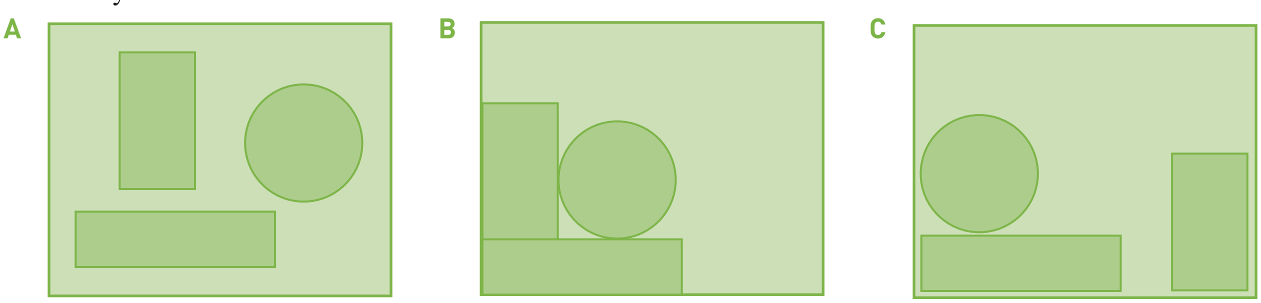
b What is the best way of buying these pieces in order to reduce wastage?

a As wood is often bought in standard lengths of multiples of 300 mm, 1000 mm or 1 m is not a multiple of 300 mm, thus it is not a size regularly available.

b Available lengths that are multiples of 300 mm would be 900 mm, 1200 mm, 1500 mm, 1800 mm and 2100 mm. The 900 mm length is not long enough. One option would be to buy two 1200 mm lengths and cut each by 200 mm, giving a total wastage of 400 mm. Another option is to buy one length of 2100 mm and cut it into two 1000 mm length, giving only 100 mm of wastage.

WHAT TO DO 17.4

1 Which layout has the least amount of waste?



2 Which of the following lengths are standard lengths that would be readily available?

a 1400 mm **b** 1700 mm **c** 1800 mm **d** 1750 mm

e 2100 mm **f** 2500 mm **g** 2400 mm **h** 3000 mm

3 Pedro is making two shelves, each 1300 mm long. Would he have less wastage if he bought one piece 2700 mm long and divided it into two, or if he bought two pieces each 1500 mm long?

4 Jessica is making a wooden table top and she needs four pieces of wood each 1 m in length. Would she have less wastage if she bought four 1200 mm pieces or if she bought one 4200 mm piece and cut it into four?

17C Fashion design

Whether you are making clothing or furnishings, it is important to get your measurements right. Buying too much may not be a problem, but it adds to the cost of making the product. Buying too little may mean that you cannot make what you have planned. This may result in another trip to the distributor, wasting time and money.

Fabric is sold per metre length with the width varying depending on the brand and style. Some common fabric widths are 90 cm, 112 cm, 122 cm, 148 cm and 150 cm.

Calculating the cost of fabric will give you an insight into how much a garment will cost to make, so you can decide whether it is worth making the garment yourself or otherwise purchasing a similar ready-made garment.

EXAMPLE 17C-1 Cost of fabric

Calculate the cost of making a dress if the fabric is \$15.95 per metre and the pattern requires 2.4 m of fabric.

$$\begin{aligned}\text{Cost of fabric} &= \text{price of fabric} \times \text{length required}\\ &= \$15.95 \times 2.4\\ &= \$38.28\end{aligned}$$

The fabric to make the dress would cost \$38.28.

NEED SOME PRACTICE?
Go to 23A Fundamental concepts
PAGE 263

EXAMPLE 17C-2 Comparing costs

Which of these fabrics is the most expensive per square metre?
Fabric A costs \$13.50 and is 90 cm wide.
Fabric B costs \$19.95 per m and is 150 cm wide.

Fabric A

Step 1: Convert the fabric width to metres.

$90 \text{ cm} \div 100 = 0.9 \text{ m}$

Step 2: Calculate the area of fabric based on the purchase of a 1 m length.

$0.9 \text{ m} \times 1 \text{ m} = 0.9 \text{ m}^2$

Step 3: Cost per square metre $= \dfrac{\$13.50}{0.9}$

$= \$15.00$

Fabric B

Step 1: Convert the fabric widths to metres.

$150 \text{ cm} \div 100 = 1.5 \text{ m}$

Step 2: Calculate the area of fabric based on the purchase of a 1 m length.

$1.5 \times 1 \text{ m} = 1.5 \text{ m}^2$

Step 3: Cost per square metre $= \dfrac{\$19.95}{1.5}$

$= \$13.30$

The most expensive fabric per square metre is fabric A, at \$15 per square metre.

NOTE

Remember that there are 100 cm in 1 m.

WHAT TO DO 17.5

Use the information in the tables below to answer the following questions.

Fabric needed to make a basic dress

Size	Fabric width 90 cm	Fabric width 112 cm	Fabric width 150 cm
6	265 cm	210 cm	155 cm
8	265 cm	210 cm	155 cm
10	270 cm	215 cm	160 cm
12	270 cm	215 cm	170 cm
14	275 cm	215 cm	180 cm
16	280 cm	220 cm	190 cm
18	285 cm	230 cm	200 cm
20	290 cm	235 cm	205 cm

Cost of fabric

Fabric width	Cost per metre
90 cm	$9.95
112 cm	$12.98
150 cm	$14.50

1 How much fabric would you need to make:
   a a size 12 dress with fabric 112 cm wide?
   b a size 6 dress with fabric 150 cm wide?
   c a size 20 dress with fabric 90 cm wide?

2 a Calculate the cost per square metre of the fabrics above to determine which of the following is the most expensive per square metre.
      i 90 cm at $9.95 per metre
      ii 112 cm at $12.98 per metre
      iii 150 cm at $14.50 per metre
   b Which fabric is the least expensive?

3 Use the cost of fabric from the table above to calculate the cost of the fabric required for the following.
   a a size 14 dress with a fabric width of 112 cm
   b a size 14 dress with a fabric width of 90 cm
   c a size 14 dress with a fabric width of 150 cm

As with many other industries, it is important to reduce wastage. Having a layout plan will allow you to position the pattern pieces in the most economical way on your fabric, as close together as possible. Some fabrics may have a grain, which refers to the way the fabric is woven or knitted together. Cutting the fabric on different grains will affect the way the fabric will hang and drape. Patterns often have a grain line to allow you to orientate the pattern correctly on the fabric. This will affect the way you need to cut your fabric to reduce wastage.

COST PRICE

If you are making a product to sell, there are many factors to consider when determining the cost price. These include the cost of materials to make the product, the labour costs and the cost of equipment needed to make the product. The price must be high enough to ensure that all costs are covered so that you can make a profit, while at the same time be low enough to encourage potential buyers to purchase the items. If the price is higher than that of similar products on the market, buyers may choose the cheaper alternatives and not your items.

EXAMPLE 17C-3 Cost price

Natasha enjoys making handbags so has decided to earn a little extra pocket money by making handbags to sell online. To make each handbag she needs:

- 1.5 m of fabric at $16.95/m^2$
- 1.5 m of lining fabric at $9.95/m^2$
- cotton reel at $6.95.

It takes her 3 hours to make a bag and Natasha values her labour at $15.00 per hour. How much will it cost to make each handbag?

Total cost of materials = fabric + lining + cotton
= (1.5 × $16.95) + (1.5 × $9.95) + $6.95
= $25.43 + $14.93 + $6.95
Total cost of materials = $47.31
Labour cost = hours taken × cost per hour
= 3 × $15
= $45
Total cost = cost of materials + labour costs
= $47.31 + $45
= $92.31
It will cost Natasha $92.31 to make a handbag.

WHAT TO DO 17.6

1 Nellie is making some necklaces to sell at a local market. To make the necklace she needs:
Beads: 40 cents each (six required for each necklace)
Beading thread: $8.00 per reel 9.0 m (50 cm required for each necklace)
Clasp pack: $6.00 for 10 (one required per necklace)
Pendant: $3.50 each (one required per necklace)
Nellie thinks her labour is worth $16 per hour and each necklace takes half an hour to make.
Calculate the cost of making one necklace.

2 Michelle wants to see if it will cost more to buy a dress than to make it. She purchased 2.20 m of fabric for $15.95 per metre, a pattern for $15.95, a reel of cotton for $9.95 and a bag of beading for $7.50. She thinks the dress will take her 6 hours to make and she considers her time is worth $12 per hour. The same dress retails for $250. Would it be cheaper for her to make the dress or buy the ready-made dress?

MARK-UP

Mark-up is the difference between the cost of producing an item and its selling price; it is added by someone selling the product in order to make a profit. Mark-up also needs to cover any additional costs of making and marketing the item. Mark-up can either be determined as a percentage or by using a set number. Clothing is often marked up by 50% to 75%, and some fashion labels may be marked up by as much as 350%. Shoes can be marked up by 50% to 500%, whereas cosmetics generally only have a small mark-up of 60% to 80%.

NEED SOME PRACTICE?

Go to 24A Converting to percentages PAGE 279

The formulas used are:

$$\text{Retail price (\$)} = \text{cost price (\$)} \times \frac{100\,\% + \text{mark-up}\,\%}{100}$$

$$\text{Mark-up (\$)} = \text{retail price (\$)} - \text{cost price (\$)}$$

$$\text{Mark-up (\%)} = \frac{\text{mark-up (\$)}}{\text{cost price (\$)}} \times 100$$

EXAMPLE 17C-4 Mark-up

a Find the retail price of clothing with a cost price of $300 that has a mark-up percentage of 25%.

b Determine the mark-up amount in dollars for part **a**.

a $\text{Retail price (\$)} = \text{cost price} \times \frac{100\% + \text{mark-up}\%}{100}$

$= \$300 \times \frac{100\% + 25\%}{100}$

$= \$300 \times 1.25 = \$375$

b $\text{Mark-up (\$)} = \text{retail price} - \text{cost price}$

$= \$375 - \$300 = \$75$

WHAT TO DO 17.7

1 Find the retail price of the following.
  - a Cost price = $250, mark-up of 125%
  - b Cost price = $175, mark-up of 74%
  - c Cost price = $365, mark-up of 35%
  - d Cost price = $140, mark-up of 66%
  - e Cost price = $105, mark-up of 150%
  - f Cost price = $132, mark-up of 95%

2 Calculate the mark-up ($) for all retail prices in question **1**.

3 Calculate the mark-up percentage (%) of these.
  - a Mark-up ($) = $50, cost price ($) = $50
  - b Mark-up ($) = $75, cost price ($) = $25
  - c Mark-up ($) = $105, cost price ($) = $30
  - d Mark-up ($) = $23, cost price ($) = $45
  - e Mark-up ($) = $70, cost price ($) = $75
  - f Mark-up ($) = $24, cost price ($) = $29

PROJECT 17

obook
assess
TEACHER

DESIGNING AWESOME

Waste not, want not!

A project worksheet for this chapter is available on the Teacher obook/assess. Ask your teacher to print it out for you. Completing this activity will help you make the right choices to reduce waste and make a profit.

CHAPTER 18

Fitness and speed

ARE YOU READY?

Complete the questions below to see if you are ready to start this chapter, or if you need to brush up on your skills first.

- Part 1 should be completed *with* a calculator.
- Part 2 should be completed *without* a calculator.

If there are any questions you cannot answer, the link below each question will direct you to a related Mathematical Skills chapter where you can revise basic concepts and get up to speed.

PART 1 WITH A CALCULATOR

1 Find the value of each of the following.

a $(-15 \times 2) \times (5 + 15) =$

b $(10 \times 3.8 \div 2) + (5 \times 54 \div 15) =$

c $12 \times 2 - (6 + 4)^2 + 9 \div 3 =$

d $(7^2 + 4) - 4 \times 6 =$

23A Fundamental concepts

2 Find 17% of each of the following numbers.

a 750 **b** 660

c 860 **d** 1220

24B Percentage of a quantity

3 Convert the following units.

a 1.3 km to m

b 23 600 mm to m

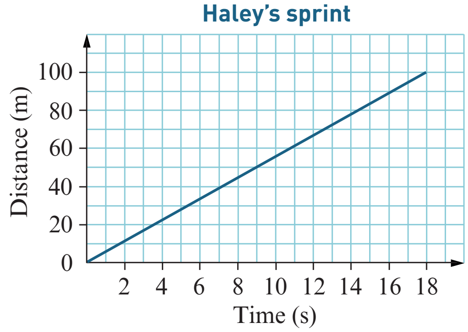
c 45 000 cm to km

d 600 m to km

25A Units of measurement

4 Anwer these questions from the graph.

Haley's sprint



a How far does Hayley run in 9 s?

b How long does it take her to run 100 m?

c What is her average speed?

35D Linear modelling

PART 2 WITHOUT A CALCULATOR

5 Find the value of each of the following.

a $12 - 8 \div (4 + 4) =$

b $(-7 + 3) \times (3 + 4) =$

c $12 \div 6 \times 4 =$

d $8 \times (20 - 12) =$

28A Back to basics

6 Convert the following decimals to percentages.

a 0.60 **b** 0.54

c 0.90 **d** 0.18

23D Decimal numbers

7 Convert the following decimals to fractions.

a 0.65 **b** 0.45

c 0.8 **d** 0.25

23D Decimal numbers

8 Convert the following units.

a 4550 cm to m

b 2.4 km to m

c 1000 m to km

d 6000 mm to m

25A Units of measurement

9 Write the value of the 1 in these numbers.

a 4231.0 **b** 31 445

c 463.001 **d** 4361.02

28E Powers of 10

10 Add up the following times. Estimate the total time.

1 min, 3 min 30 s, 4 min 20 s, 2 min 10 s

23F Estimation

18A Body mass index (BMI)

NOTE

The terms 'weight' and 'mass' are generally used interchangeably. Mass is a measure of the amount of material in an object; weight is a measure of how hard gravity is pulling on that object.

The body mass index (BMI) is a mathematical formula that is used to indicate if a person's mass is healthy, too light or too heavy. The interest in an index that measures body fat came with increasing obesity in prosperous Western societies. This index is used by health-care professionals to more objectively discuss body fat issues with their clients.

$$\text{BMI} = \frac{\text{mass}}{\text{height}^2}$$

where mass is in kilograms and height is in metres.

Depending on height and mass, a person can belong to one of these categories.

- A BMI lower than 18.5, which suggests the person is underweight.
- A BMI from 18.5 up to 25, which may indicate optimal weight.
- A BMI from 25 up to 30, which may indicate that the person is overweight.
- A BMI from 30 upwards, which suggests that the person is obese.

BMI values can be controversial and may be misleading if you are muscular, as muscle weighs more than fat. Athletes, who tend to have atypical muscle-to-fat ratios, may have a BMI that places them in a higher category despite having a healthy level of body fat.

EXAMPLE 18A-1 Understanding BMI

Nick Riewoldt, one of St Kilda's greatest players, is physically at the top of his game. His height is 1.93 m and his weight is 96 kg. What is his BMI? What does it indicate?

$$\text{BMI} = \frac{\text{mass}}{\text{height}^2}$$

$$= \frac{96}{1.93^2}$$ Substitute Nick's weight and height into the BMI equation.

$$= 25.77$$

Based on the BMI categories, Nick Riewoldt is considered overweight.
As muscle weighs more than fat, this BMI has placed him in the overweight category despite having a healthy level of body fat. Do you think this is correct?

WHAT TO DO 18.1

1 Calculate the BMI of the following celebrities.

Celebrity	Height (m)	Weight (kg)
Angelina Jolie	1.73	58
Brad Pitt	1.80	78
Beyoncé	1.69	59
Megan Fox	1.63	52
Jack Black	1.68	88

Celebrity	Height (m)	Weight (kg)
Ashton Kutcher	1.89	80
Nicole Kidman	1.79	58
Lady Gaga	1.55	49
Cameron Diaz	1.75	58
Jennifer Lawrence	1.75	63

2 Calculate the BMI of the following athletes.

Athlete	Height (m)	Weight (kg)
Lionel Messi	1.69	67
Gareth Bale	1.83	74
Danica Patrick	1.57	45
Rafael Nadal	1.85	85
Roger Federer	1.85	88

Athlete	Height (m)	Weight (kg)
Cristiano Ronaldo	1.86	83
Michael Phelps	1.97	88
Serena Williams	1.75	68
Maria Sharapova	1.88	59
Daniel Ricciardo	1.80	64

3 The table below shows the mean BMI of people above 20 years.

Country		1984	1989	1994	1999	2004	2009
Australia	Male	25.2	25.7	26.2	26.6	27.0	27.7
	Female	24.1	24.7	25.4	25.9	26.4	27.0
Canada	Male	25.6	26.0	26.4	26.9	27.2	27.5
	Female	24.5	25.0	25.5	26.0	26.5	26.8
Ethiopia	Male	19.5	19.5	19.6	19.8	20.0	20.4
	Female	19.0	19.2	19.5	19.9	20.4	20.8
France	Male	24.8	24.0	25.1	25.3	25.6	25.9
	Female	24.2	24.3	24.5	24.6	24.8	24.8
Tonga	Male	27.0	27.9	28.8	29.6	30.4	31.1
	Female	28.3	29.7	31.0	32.2	33.3	34.5
USA	Male	25.9	26.5	27.1	27.6	28.1	28.5
	Female	25.5	26.2	26.8	27.4	28.0	28.4

a Find the BMI range for each of the countries listed above. What does it show?
  i For males, which country shows the greatest change of BMI over the time period?
  ii For females, which country shows the least change in BMI over the time period?

b Draw line graphs for both males and females. What would a relative straight line indicate?

c Which country shows the most variation over the years?

18B Heart rate and respiration rate

You heart rate is the number of times that your heart beats within 1 minute (measured as beats per minute or bpm). To measure your heart rate, simply check your pulse. To check your pulse in your neck, place your index and third fingers on your neck to the side of your windpipe. To check your pulse at your wrist, place two fingers between the bone and the tendon over your radial artery, which is located on the thumb side of your wrist. Do not use your thumb to take your pulse as it has its own pulse that you may feel also.

Your heart rate, when taken after resting, can give an indication of your level of fitness. Heart rates generally range from 60 to 100 beats per minute for adults. Lower rates can imply a more efficient heart function and better cardiovascular fitness, and well-trained athletes might have a normal resting heart rate closer to 40 beats a minute. The lowest recorded heart rate was 28 beats per minute for Spanish cyclist Miguel Indurain.

Resting heart rate for men

Age	18–25	26–35	36–45	46–55	56–65	65+
Athlete	49–55	49–54	50–56	50–57	51–56	50–55
Excellent	56–61	55–61	57–62	58–63	57–61	56–61
Good	62–65	62–65	63–66	64–67	62–67	62–65
Above average	66–69	66–70	67–70	68–71	68–71	66–69
Average	70–73	71–74	71–75	72–76	72–75	70–73
Below average	74–81	75–81	76–82	77–83	76–81	74–79
Poor	82+	82+	83+	84+	82+	80+

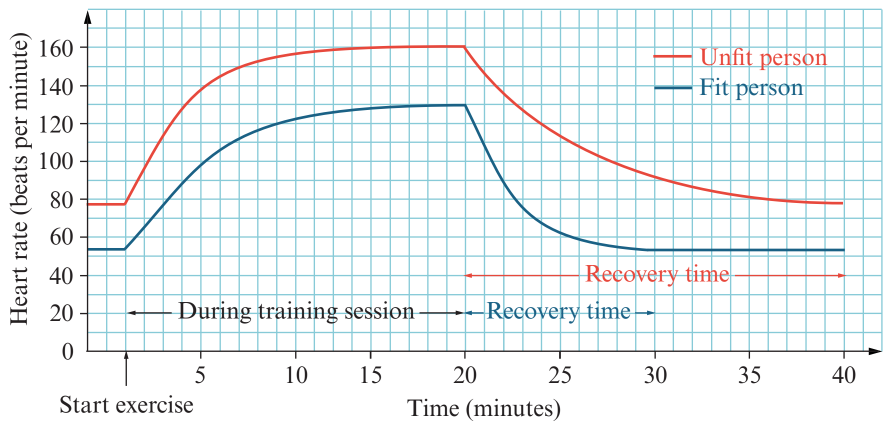
Resting heart rate for women

Age	18–25	26–35	36–45	46–55	56–65	65+
Athlete	54–60	54–59	54–59	54–60	54–59	54–59
Excellent	61–65	60–64	60–64	61–65	60–64	60–64
Good	66–69	65–68	65–69	66–69	65–68	65–68
Above average	70–73	69–72	70–73	70–73	69–73	69–72
Average	74–78	73–76	74–78	74–77	74–77	73–76
Below average	79–84	77–82	79–84	78–83	78–83	77–84
Poor	85+	83+	85+	84+	84+	84+

Factors that can affect heart rate are:

- age (the average heart rate decreases with age)
- body size (heart rate increases with size)
- gender (an average woman would have a higher heart rate than a man)
- body position (heart rate is lower when lying down than standing)
- excitement or stress (heart rate can increase)
- activity (heart rate increases during exercise)
- fitness level (very fit people have lower heart rates).

A typical heart rate response to exercise for both a fit and an unfit person are shown.



- Pre-exercise: The person is at rest. The heart rate is constant.
- Exercise session: The heart beats faster to pump blood around the body, sending energy and oxygen to the muscles being used. The heart rate rapidly increases.
- Recovery: The heart rate decreases over time until it returns to the resting heart rate.

A fit person's heart is more efficient than an unfit person's heart. They have a lower resting heart rate before they begin to exercise, take longer to reach maximum heart rate, and their maximum heart rate will be lower than for an unfit person. When they stop exercising, a fit person's heart will return to their resting heart rate more quickly.

An unfit person's heart is less efficient and will beat faster even while at rest, take less time to reach maximum heart rate and return to resting heart rate more slowly.

NOTE

Training sessions should be at least 20 to 30 minutes, with the aim of raising and maintaining training heart rate at higher level during this period.

NEED SOME PRACTICE?

Go to 35B Slope PAGE 414

MAXIMUM HEART RATE

Everyone's maximum heart rate differs and varies with age. As a rough estimation of a person's maximum heart rate, use the formula:

Men = 220 − age   Women = 226 − age

Thus the older you are, the lower your maximum heart. It is not uncommon to see maximum heart rates well over 200 during maximal exercise tests.

EXAMPLE 18B-1 Maximum heart rate

John is 17 years old. Calculate his maximum heart rate.

Maximum heart rate = (220 − age)
= (220 − 17) = 203

John's maximum heart rate is around 203.

TRAINING HEART RATE ZONES

When starting an exercise program it is important to know what you want to achieve. Do you want to increase your stamina or endurance? Or even lose weight? One way of improving this is to monitor your heart rate and keeping it within a certain 'target zone' for at least 20 to 30 minutes a few times a week.

Basic mathematical formulas are used to find the upper and lower heart rate you need to be within to reach your goal. The values are expressed as a percentage of maximum heart rate (for example, 70% of HRmax).

The target heart rate zone ranges are based on the metabolic systems in your body that fuel your muscles during exercise, and how hard you want to train. If you're not fit or you're just beginning an exercise program, aim for the lower end of your target zone (50%), then gradually build up the intensity. If you're healthy and want a vigorous intensity program, aim for the higher end of the zone.

Standard target heart rate zones are given in the table below. The intensity level is the percentage of your maximum heart rate.

Exercise level	Intensity level	Benefit
Very light exercise (recovery)	50%–60%	Maintain a healthy heart and fitness
Light exercise	60%–70%	Burn fat and calories
Moderate exercise	70%–80%	Increase stamina and endurance
Hard exercise (conditioning)	80%–90%	Fitness conditioning, muscle building and athletic training
Maximum exercise (athletic)	90%–100%	Athletic training and endurance

EXAMPLE 18B-2 Heart rate target zones

Rhys is a 30-year-old male who wants to lose weight. He has a fitness trainer who wants to find out the following.

a What is his maximum heart rate?
b What training intensity should he be aiming for?
c What should his lower target heart rate be?
d What should his upper target heart rate be?

a Maximum heart rate = (220 − age)
= (220 − 30)
= 190

b 60%–70% intensity

c Lower target rate should be 60% of his maximum heart rate of 190.
Lower target rate = 0.6 × 190
= 114

d Upper target rate should be 70% of his maximum heart rate of 190.
Upper target rate = 0.7 × 190
= 133

WHAT TO DO 18.2

1 Calculate the maximum heart rate for the following people.

a 15-year-old female b 20-year-old male c 27-year-old female d 34-year-old male
e 49-year-old female f 53-year-old male g 67-year-old female h 78-year-old male

2 Rianna is a 25-year-old female who wants to increase her stamina and endurance.

a What is her maximum heart rate?
b What training intensity should she be aiming for?
c What should her lower target heart rate be?
d What should her upper target heart rate be?

3 Complete the table below.

Age (years)	Maximum heart rate		Target heart rate at 60% intensity		Target heart rate at 75% intensity		Target heart rate at 90% intensity	
	Male	Female	Male	Female	Male	Female	Male	Female
15								
20								
25								
30								
35								
40								

RESPIRATION

Physical activity works your muscles harder as your lungs need more oxygen. To get more oxygen to your cells, your heart rate and breathing rate increase.

The respiration rate is the number of breaths per minute. For adults, the average respiration rate at rest is about 14 breaths per minute. During exercise this increases to about 35 breaths per minute.

The breath volume refers to the average volume of air breathed in (or out) for each breath. For adults, the breath volume at rest is about 0.6 L and during exercise it may increase to as much as 5 L.

A person's ventilation is the volume of air breathed in per minute.

Ventilation = breath volume × respiration rate

EXAMPLE 18B-3 Measuring ventilation

Aysha at rest has a breath volume of 0.6 L with 14 breaths per minute. Calculate Aysha's ventilation.

$$\begin{aligned}\text{Ventilation} &= \text{breath volume} \times \text{respiration rate}\\ &= 0.6 \times 14\\ &= 8.4\ \text{L/min}\end{aligned}$$

Aysha's ventilation is around 8.4 L/min.

VO_2 MAX

VO_2 max is the greatest volume of oxygen that can be breathed in per minute per kilogram of body mass during exercise. VO_2 max is the most important factor in determining your ability to perform sustained exercise; that is, it is the best indicator of your fitness and endurance. VO_2 is measured in mL/kg/min.

Theoretically, the more oxygen you can use during high-level exercise, the more energy you can produce. This is often the case with elite endurance athletes who typically have very high VO_2 max values. Other than genetic factors, the three components that have the greatest influence on VO_2 max are:

- age (it is highest for a 20-year-old, then decreases with age)
- gender (it is higher for women than men)
- altitude (reduced oxygen at higher altitudes will decrease VO_2 max).

One way of calculating VO_2 max is by using this formula reflecting your resting heart rate and age:

$$VO_2 \text{ max} = 15.3 \times \frac{\text{maximum heart rate}}{\text{resting heart rate}}$$

where heart rate is in beats/minute and VO_2 max is in mL/kg/min.

A more accurate way of calculating VO_2 max is in a sports performance laboratory, usually exercising on a treadmill or bicycle, where all the oxygen going in and out is collected and recorded.

EXAMPLE 18B-4 Calculating VO_2 max

Calculate the VO_2 max of a 20-year-old who has a resting heart rate of 72 beats per minute.

NEED SOME PRACTICE?
Go to 23D Decimal numbers PAGE 271

$$\begin{aligned}\text{Maximum heart rate} &= (220 - \text{age})\\ &= (220 - 20) = 200\end{aligned}$$

$$\begin{aligned}VO_2 \text{ max} &= 15.3 \times \frac{\text{maximum heart rate}}{\text{resting heart rate}}\\ &= 15.3 \times \frac{200}{72}\\ &= 15.3 \times 2.78\\ &= 42.5\end{aligned}$$

VO_2 max is 42.5 mL/kg/min.

The table below shows the VO_2 max measures for various sports participants at the highest level. The higher the VO_2 max values, the fitter the person. Thus they can exercise more intensely than people who are not as well conditioned. The VO_2 max values are given in mL/kg/min.

Activity	Males VO_2 max	Females VO_2 max
Cross-country skiers	83	63
Long-distance runners	80	60
Middle-distance runners	78	59
Speed skaters	73	55
Cyclists	72	56
Rowers	63	52
Hockey players	61	50
Soccer players	58	–
AFL players	55	–
Active young people	51	42

WHAT TO DO 18.3

1 Determine the ventilation for the following people.
   - a Joshua after light activity, with a breath volume of 2.0 L and a respiration rate of 30 breaths per minute
   - b Monica at rest, with a breath volume of 0.7 L and a respiration rate of 12 breaths per minute
   - c Trang at rest, with a breath volume of 0.65 L and a respiration rate of 18 breaths per minute
   - d Jasmine after vigorous activity, with a breath volume of 4.8 L and a respiration rate of 35 breaths per minute

2 Calculate the VO_2 max of the following people.
   - a Nguyen is a 29-year-old male with a resting heart rate of 64 beats per minute.
   - b Adrian is 18-year-old male with a resting heart rate of 72 beats per minute.
   - c Ping is 49-years-old female with a resting heart rate of 81 beats per minute.

3 Which of the following are true statements?
   - a VO_2 max for females seems to be approximately 75% to 80% of that for males for the same activity.
   - b People who wish to be performers at the highest level in endurance sports need higher VO_2 max ratings.
   - c Fitter people do not necessarily have high VO_2 max ratings.

18C Speed

Speed is the rate at which someone or something moves or operates or is able to move or operate. The most common units for calculating speed are km/h or m/s.

The formula used to calculate speed is:

$$\text{Speed} = \frac{\text{distance}}{\text{time}}$$

where speed is in metres/second
distance is in metres
time is in seconds.

This formula can be rearranged to find the time taken and distance covered.

$$\text{Time} = \frac{\text{distance}}{\text{speed}}$$

$$\text{Distance} = \text{speed} \times \text{time}$$

NEED SOME PRACTICE?
Go to 35C Straight-line graphs
PAGE 417

EXAMPLE 18C-1 Average speed

A male Olympic 100 m sprinter runs the distance in 9.92 seconds.

a What is his average speed?

b Draw a line graph of his run.

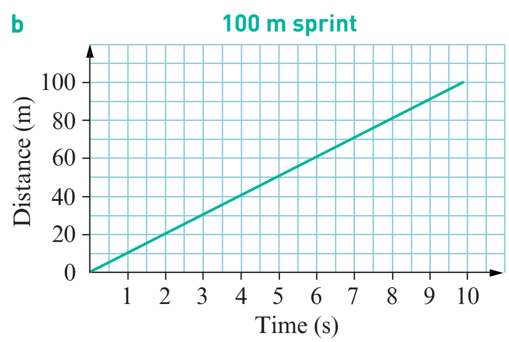
a $\text{Speed} = \frac{\text{distance}}{\text{time}}$

$= \frac{100}{9.92}$

≈ 10.08 m/s

Average speed is about 10.08 m/s.

b



NEED SOME PRACTICE?
Go to 25C Conversion of units
PAGE 297

The formula for speed is given with the units metres and seconds. There may be times when you want to convert to km/h or back again.

- To convert from m/s to km/h, divide by 1000 then multiply by 3600.
- To convert from km/h to m/s, multiply by 1000 then divide by 3600.

EXAMPLE 18C-2 Converting speed

Convert 60 km/h to m/s.

$60 \times 1000 = 60\,000$ — To convert from km to m multiply by 1000.

$= \frac{60\,000}{3600}$ — To convert from h to s divide by 3600.

$= 16.67$ m/s

WHAT TO DO 18.4

1 Convert the following to m/s.

a 30 km/h	b 37 km/h	c 42 km/h	d 49 km/h
e 56 km/h	f 61 km/h	g 68 km/h	h 73 km/h
i 84 km/h	j 89 km/h	k 99 km/h	l 108 km/h

2 Convert the following to km/h.

a 5 m/s	b 18 m/s	c 23 m/s	d 29 m/s
e 37 m/s	f 45 m/s	g 52 m/s	h 58 m/s
i 64 m/s	j 70 m/s	k 75 m/s	l 80 m/s

3 Use the previous formulas for speed, time and distance to calculate the following.

a If a field is 100 m long and it takes a person 20 s to run its length, at what speed is the person running in km/h?

b A cricketer bowls a ball 85 m to the batter and it takes 4 s for the ball to reach the batter. How fast is the ball travelling in km/h?

c If you drive a bike at 100 km/h for 6 h, how far will you have travelled?

d If you ride at 12 m/s for 15 min, how far will you go?

e You drive 3900 km from Melbourne to Perth. If your average speed is 100 km/h, how much time will you spend driving?

f A bullet travels at 850 m/s. How long will it take a bullet to go 1 km assuming constant speed?

g The fastest train in the world moves at 500 km/h. How far will it go in 3 h?

4 Calculate the average speed, in metres per seconds, of each of the track athletes listed in the events below.

Athlete	Country	Time (min:s)	Event	Year
Usain Bolt	Jamaica	9.58	100 m	2009
Usain Bolt	Jamaica	19.19	200 m	2009
Michael Johnston	USA	43.18	400 m	1999
Florence Griffith Joyner	USA	10.49	100 m	1988
Florence Griffith Joyner	USA	21.34	200 m	1988
Marita Koch	Germany	47.60	400 m	1985
David Lekuta Rudisha	Kenya	1:40.91	800 m	2012
Hicham El Guerrouj	Morocco	3:26.00	1500 m	1998
Jarmila Kratochvilova	Czech Republic	1:53.28	800 m	1983
Yunxia Qu	China	3:50.46	1500 m	1993

TIMING EVENTS

In athletics, cycling and swimming events, the performance of the participant is measured to the nearest hundredth of a second (and sometimes to the nearest thousandth).

A timing watch or clock is used and a typical display would be:

0° 21′ 31.16″

which reads 21 min 31.16 s.

hours : minutes : seconds

As speed is usually in km/h or m/s, sometimes it is necessary to convert the units to hours or seconds.

Using your calculator:

0 [° ′ ″] 21 [° ′ ″] ► 31.16 [° ′ ″] [=] 0.3586556 hours

then [×] 3600 [=] 1291.16 seconds

WHAT TO DO 18.5

1 What are the times for the following watch displays?

a 0° 7′ 16.42″ b 0° 0′ 26.06″ c 1° 27′ 45.06″

2 Convert each of the times given in question **1** to both and hours and seconds.

3 In a 100 m sprint, the competitors recorded the following times.

Marita 0°0′13.56″ Inger 0°0′15.82″ Tash 0°0′11.82″ Marion 0°0′11.63″

a Arrange the competitors in order from first to fourth place.

b If the current women's world record for the 100 m sprint is 10.49 s (set by Florence Griffith Joyner in 1988), calculate by how much each competitor needs to reduce her time to match this record.

c Marita improved her personal best time by 0°0′00.42″. Calculate her new personal best.

d Tash was 0°0′01.05″ slower than her personal best. Calculate her personal best time.

4 Below are the results of the men's 10 000 m for the Olympic Games from 1972 to 2012.

Year	Competitor (Gold medal)	Country	Time
1972	Lasse Viren	Finland	27:38.40
1976	Lasse Viren	Finland	27:40.38
1980	Miruts Yifter	Ethiopia	27:42.70
1984	Alberto Cova	Italy	27:47.54
1988	Brahim Boutayeb	Morocco	27:21.46
1992	Khalid Skah	Morocco	27:46.70
1996	Haile Gebrselassie	Ethiopia	27:07.34
2000	Haile Gebrselassie	Ethiopia	27:18.20
2004	Kenenisa Bekele	Ethiopia	27:05.10
2008	Kenenisa Bekele	Ethiopia	27:01.17
2012	Mo Farah	Great Britain	27:30.42

NOTE
Times are in minutes and seconds.

a Draw a line graph of these results. On the time axis, record times over 27 min.
b How much quicker was Bekele's best time than Viren's best time?
c Find Farah's time in both hours and seconds.

PROJECT 18

obook assess TEACHER

MY FITNESS

Am I a machine?

A project worksheet for this chapter is available on the Teacher obook/assess. Ask your teacher to print it out for you. Completing this activity will help you make the right choices when considering your own health and your family's health.

CHAPTER 19
Renovating and redecorating

ARE YOU READY?

Complete the questions below to see if you are ready to start this chapter, or if you need to brush up on your skills first.

- Part 1 should be completed *with* a calculator.
- Part 2 should be completed *without* a calculator.

If there are any questions you cannot answer, the link below each question will direct you to a related Mathematical Skills chapter where you can revise basic concepts and get up to speed.

PART 1 WITH A CALCULATOR

1 Find the value of each of the following.

a $2.0 + 1.0 + 2.5 + 3.0 + 4.5 =$

b $7.35 \times 3.15 + 4 \times 3.2 =$

c $6 \times 3.4 \times 2 + 4 \times 3.4 \times 1.5 \times 2 =$

23D Decimal numbers

2 A plank of wood is 150 mm wide and 4 m long. In square metres, what is the surface area of one side of this plank?

27A Area

3 A house plan is drawn to a scale of 1 : 100. This means that 10 mm on the drawing is:

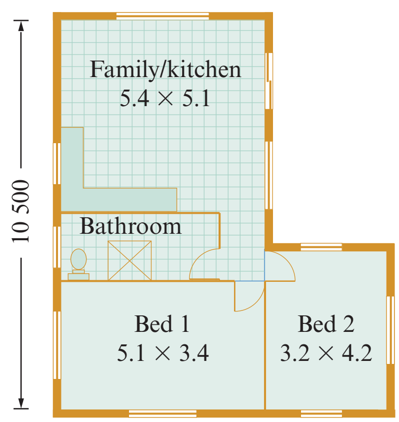
A 0.1 m **B** 10 m
C 10 cm **D** 1000 mm

31B Scale drawings

4 Use the house plan below to answer the following questions (in m^2).

a What is the floor area of bedroom 1?

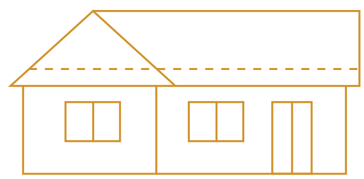
b What is the total floor area of the house?



27A Area

5 What part of a set of house plans is shown in the diagram below?

A a floor plan **B** a block plan
C an elevation **D** a detail drawing



31C Building plans

PART 2 WITHOUT A CALCULATOR

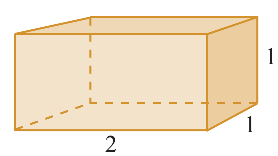
6 Find the value of each of the following.

a $12 + 8 + 14.5 =$ **b** $34 - 3 \times 5 =$

c $14 \div 7 \times 14 =$ **d** $19 + 5 - 7 =$

28A Back to basics

7 Draw a possible net for this solid.



27B Surface area

8 **a** What is the value of the 1 in 4321?

b Convert $1\frac{1}{3}$ to a decimal number.

c Round 6.17 m to the nearest tenth of a metre.

23D Decimal numbers

9 Add up the following paint and materials prices. Estimate the total cost.

\$49.99, \$35, \$17.50, \$32, \$5.99, \$3.99

23F Estimation

19A House plans

Are you sick of your shabby old bedroom with its boring 1980s colour scheme? Perhaps you're interested in buying your first home, described by the real estate agent as a 'renovator's delight'?

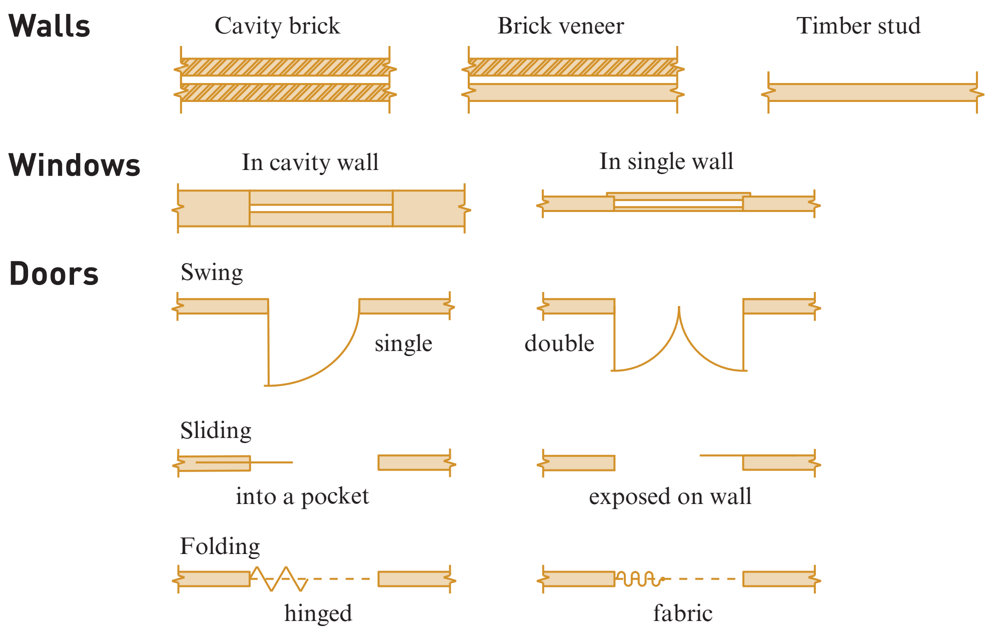
One way or the other, you have a room or a whole house in need of renovation or redecorating! If you decide to paint, you may think that getting a quote is all you need to do. But how do you know whether you are paying more than you need to? Maybe you cannot afford the quote and decide to do it yourself to avoid paying someone else.

Whether you hire a painter or decide to do it yourself, you need to know how to accurately estimate the areas and quantities of paint needed. The basic skills needed are described in this chapter.

To start any work on a house, you need an accurate listing of the sizes and shapes involved. The best place to find this information is on the plans of the house. Remember, house plans have to be used by everyone, from the original designer of the house, the builders and other tradespeople constructing the house, to the interior decorators and the owners themselves.

To enable this diverse group to communicate with each other, a set of conventions is used when drawing house plans so everyone can understand the same set of drawings. A part of this set of conventions is shown below.

NEED SOME PRACTICE?
Go to 31C
Building plans
PAGE 366



NEED SOME PRACTICE?
Go to 31B
Scale drawings
PAGE 362

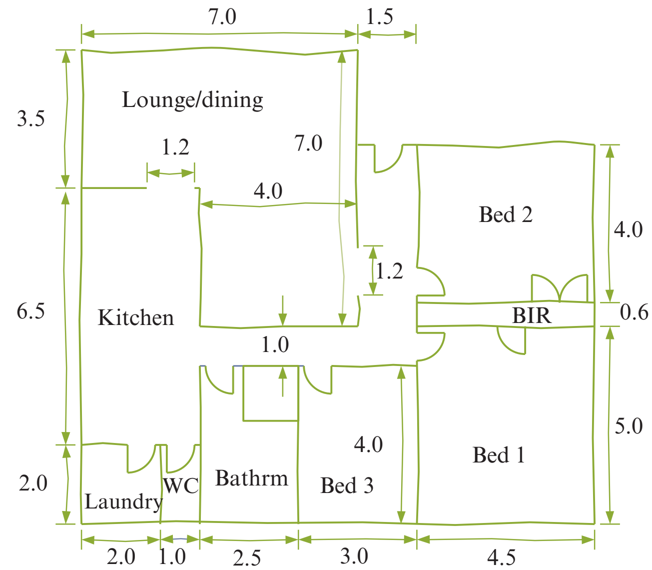
If you had the house built, you will have a set of plans available. However, if you buy an existing house, it may not be easy to obtain a set of accurate plans, and you may need to draw a set of plans of your own.

Drawing a house floor plan to scale

Rule of thumb: The commonly used scale on house plans is 1 : 100. This means that every 1 mm represents 100 mm (or 10 cm) in real life. So 1 cm (plan) = 1 m (real life).

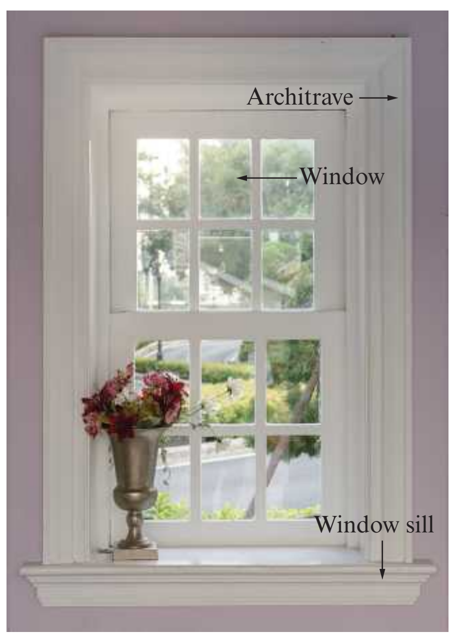
WHAT TO DO 19.1

1 This rough sketch shows the floor plan of a house. Your task is to properly draw it to a 1 : 100 scale. Notice that:
- the outer walls of the house are brick veneer
- the inner walls are timber stud
- the measurements are given in metres.



19B Internal painting

The simplest of redecoration jobs is repainting a bedroom. In the building trade, the architraves, skirting, window sills and picture rails are often collectively called the trim.

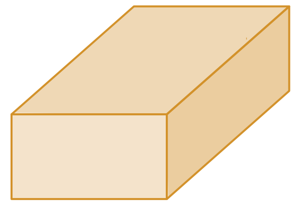


Colour choice in repainting a room is very much an individual thing. Some people paint a room in one colour and let their furniture and accessories (such as curtains, rugs, lamps) be the features. In older houses with fancy skirting, architraves, cornices and ceiling roses, any or all of these items may be used as features, and each painted in several different colours. Feature walls are also popular.

PARTS OF A ROOM

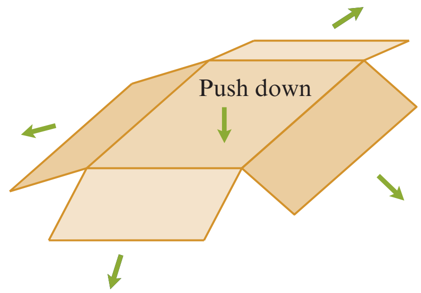
NEED SOME PRACTICE?
Go to 31B Scale drawings PAGE 362

Most rooms are the shape of rectangular boxes. Boxes have six faces. For a room, how many internal faces are usually painted? One? Three? Five? All of them? Discuss this as a class and identify the faces by name.



Blowing up a room

Now imagine that a room has its floor removed and its walls slide outwards from the bottom until the ceiling is flat on the ground as shown in the diagram. You can then label each rectangle as a wall or ceiling and add the measurements for each of the sides.

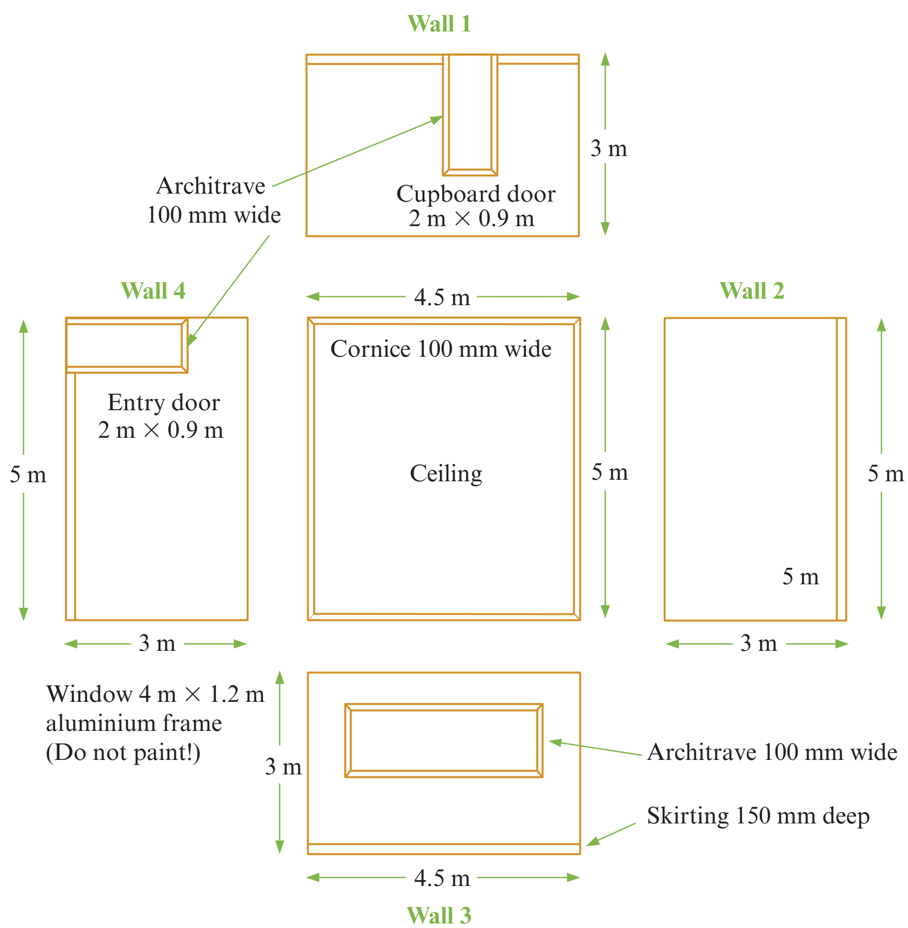


CALCULATING THE PAINTED AREAS OF A ROOM

NEED SOME PRACTICE?
Go to 27A Area PAGE 311

NEED SOME PRACTICE?
Go to 29C Making solids from nets PAGE 344

Here is a series of sketches of the walls and ceiling of the bedroom you plan to paint with the size of each part to be painted shown.

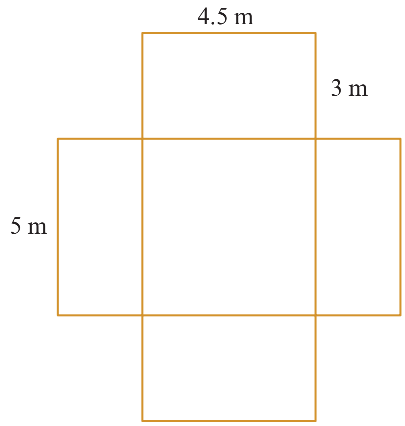


Draw a flattened-out diagram of the room plan shown on A4 paper in a 1.5 cm to 1 m scale. Complete the table below to help you with the scale.

Actual distance	Diagram distance
3 m	___ cm
4.5 m	___ cm
5 m	___ cm

Use your flattened-out model and draw in the trim (cornice, architraves and skirting).

If you cut out the diagram and fold the sides, you will form a model of your room.



NOTE

If the height of the room is 3 m, the height on the diagram is $3 \times 1.5 = 4.5$ cm.

NOTE

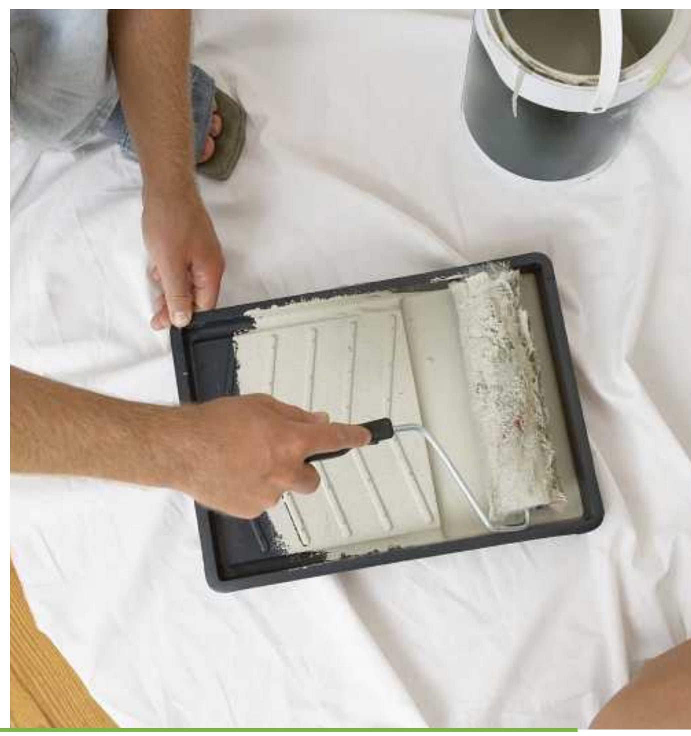
Remember that the area of a rectangle is found by multiplying its length and width.

WHAT TO DO 19.2

1 From the diagram on the previous page, work out the exact area of each of the items listed in this table.

Feature	Part	Length (m)	Width (m)	Area (m^2)
Ceiling	actual			
	cornice			
Wall 1	wall			
	architrave			
	skirting			
	door			
Wall 2	wall			
	skirting			
Wall 3	wall			
	architrave			
	skirting			
	window			
Wall 4	wall			
	architrave			
	skirting			
	door			

2 What would you do about the doors and windows that you counted in your wall calculations? You may use them to help you calculate the total areas of wall, ceiling and trim paint.



The previous calculations could also be done on a spreadsheet. To make this spreadsheet usable for any rectangular room, list all five items (wall, architrave, skirting, door and window) for each wall. If an item is not present on that wall, leave the length and width blank so that no area is calculated or counted.

	A	B	C	D	E
1	**Feature**	**Part**	**Length (m)**	**Width (m)**	**Area (m^2)**
2	Ceiling	actual			
3		cornice			
4	Wall 1	wall			
5		architrave			
6		skirting			
7		window			
8		door			
9	Wall 2	wall			
10		architrave			
11		skirting			
12		window			
13		door			
14	Wall 3	wall			
15		architrave			
16		skirting			
17		window			
18		door			
19	Wall 4	wall			
20		architrave			
21		skirting			
22		window			
23		door			
24		**Total areas**			
25			Ceiling	=E2+E3	
26			Walls	=E4+E9+E14+E19−E5−E6−E7−E8−E10 −E11−E12−E13−E15−E16−E17−E18−E20 −E21−E22−E23	
27			Doors and trims	=E5+E6+E8+E10+E11+E13+E15+E16+E18 +E20+E21+E23	

HOW MUCH PAINT IS NEEDED?

obook
An Excel spreadsheet template to help you calculate the amount of paint needed is available on your obook.

Having worked out the area for each type of paint, you now need to calculate how much paint to buy. A general rule of thumb is that 1 L of paint covers 15 m^2 with one coat. Most cans of paint display a 'coverage' value.

So, in general, number of litres of paint $= \frac{\text{area}}{15}$

The calculations could be done by extending the spreadsheet above to include volume of paint or by using the following table.

Section	Area (m^2)	Volume for one coat (L)	Volume for two coats (L)
Ceiling			
Wall			
Doors and trim			

WHAT TO DO 19.3

1 For the bedroom example on page 220, assume a coverage of 15 m^2 per coat. The ceiling and cornice will be one colour, the wall will be a second colour, and the skirting, architraves and doors will be a third colour. Calculate the volumes of each type of paint required for:

a one coat

b two coats.

2 To update your spreadsheet to include paint volumes for one and two coats, add cells F24, F25, F26, G24, G25 and G26.

WHAT TO BUY

Just like milk and soft drinks, paint comes in containers of certain sizes. The standard sizes for paint are 250 mL (samples), 500 mL, 1 L, 4 L, 10 L, 15 L and 20 L. Some small paint manufacturers offer non-standard sizes such as 2 L and 5 L. Wall and ceiling paints rarely come in 250 mL or 500 mL tins.

There are also several different types and finishes of paint available. For the sake of this exercise, you will assume that only water-based paints will be used. Paint finishes can range from high gloss to completely flat. You will also assume that the ceiling will be done in a flat finish, the walls will be done in a satin (sometimes called semi-gloss), and the trim will be done in a full gloss. The table below shows some typical prices for paint.

Ceiling paint (flat)	Wall paint (semi-gloss)	Trim paint (full gloss)
1 L \$21	1 L \$30	250 mL \$12
4 L \$35	4 L \$50	500 mL \$28
10 L \$110	10 L \$155	1 L \$42
15 L \$160	15 L \$199	4 L \$81

WHAT TO DO 19.4

1 Determine which tin sizes you will need to buy to paint the bedroom shown in Section 19B. To achieve a quality finish, you will need to paint two coats on all surfaces. Choose the tins on the basis of the most economical combination of sizes.

19C Floor coverings

Having painted the bedroom, you now realise how worn and out of place the old carpet looks. So what can you do about it? How much would it be to change the floor coverings for the whole house? You need to go back to your floor plan to decide what is to be done.

Kitchen benches are often fixed to the floor so a floor covering is usually not required under them. What other areas of the house do not require floor coverings?

WHAT TO DO 19.5

1 Using the floor plan you drew in What to do 19.1, calculate the floor areas and complete this table.

NOTE

Add the built-in wardrobe (BIR) measurements to Bedroom 2 measurements.

Room	Floor area (m^2)
Bathroom	
Bedroom 1	
Bedroom 2	
Bedroom 3	
Lounge/dining	
Hallways	
Laundry	
Kitchen	19.5
Toilet (WC)	

2 Use this table to help decide what floor coverings you will use.

NOTE

Carpet is usually priced by the broadloom metre, which is generally 3.66 m or 4 m wide. Thus you need to convert to square metres when comparing carpet prices to other floor coverings.

Covering	Type	Typical cost per m^2
Carpet	Synthetic	$45 laid
	Synthetic and wool blend	$65 laid
	Wool	$95 laid
Vinyl	Patterned	$55 laid
	Plain	$50 laid
Tiles	Slate	$85 materials only
	Ceramic	$54 materials only
	Cork	$48 materials only
Timber	Polish and seal existing floor	$45 completed
	Timber laminate	$110 completed

3 Having decided on what floor coverings you would like to use in each room, make a list of your choices and add a brief description of the reasons for each choice. Use the categories of cost, practicality and looks.

4 Using the areas calculated for each room, calculate the cost per room of your choice, and the total cost for the whole house. When choosing your floor coverings, price is only one consideration. What else do you need to take into account when deciding between alternatives?

19D Working to a budget

Having gone through this project, what if the total cost is over your budget? It would be good to change to a cheaper floor covering in a room or two and recalculate the total, but this may have to be done several times before the budget figure is reached.

A quick way to work out your costings is by using a two-sheet spreadsheet. The following exercise will produce a spreadsheet that can be used for almost any three-bedroom house with rectangular rooms, and could be adapted to a house of any size.

WHAT TO DO 19.6

obook

An Excel spreadsheet template for What to do 19.6 is available on your obook.

1 Set up an Excel floor cost calculator by following these steps.

Step 1: Open an Excel spreadsheet, and on Sheet 1 enter the information as below.

	A	B	C
1	**Floor covering**	**Cost/m^2 ($)**	**Labour/m^2 ($)**
2	Synthetic carpet	45	0
3	Synthetic and wool blend carpet	65	0
4	Wool carpet	95	0
5	Patterned vinyl	55	0
6	Plain vinyl	50	0
7	Slate tiles	85	35
8	Ceramic tiles	54	35
9	Cork tiles	48	35
10	Polished existing floorboards	45	0
11	Timber laminate floorboards	110	0

Step 2: Click on the new sheet symbol ⊕ on the lower left next to the label Sheet 1. On Sheet 2, type in the headings in Row 1 and Column A of Sheet 2 as below.

	A	B	C	D	E	F
1	**Room**	**Floor covering**	**Cost/m^2 ($)**	**Room area (m^2)**	**Labour/m^2 ($)**	**Cost/room ($)**
2	Bathroom					
3	Bedroom 1					
4	Bedroom 2					
5	Bedroom 3					
6	Lounge/dining					
7	Hallway					
8	Laundry					
9	Kitchen					
10	Toilet (WC)					

Step 3: Click on cell B2 of Sheet 2.
Click on the Developer menu, select Insert.
Select, under Form Controls, the Combobox symbol. 
Click back on cell B2. A resizeable box will appear.
Resize this box to suit the cell size.
Right click on the Combobox and select Format Control from the menu.
Select Input range.
Click on the Sheet1 tab (Sheet 1! Should now be in the input range line.)
Click on cell A2 in Sheet 1, the cursor now becomes a +.
Hold the Ctrl key down and drag the cursor down to highlight cells A2 to A11.

Step 4: Click on Format Control box.
Go to Cell link and type in B2.
Click the OK button.
Repeat Steps 3 and 4 for each cell in column B.

NOTE

You must remember that for B2, you will need to refer to the current cell in column B.

Step 5: On Sheet 2, move across to column C. Click on cell C2.
Type = INDEX (into the cell.
Click on Sheet 1.
Highlight (or outline) cells B2 to B11.
Press F4 (to make reference absolute).
Type , (a comma).
Click on Sheet 2.
Delete the text Sheet 2! and replace it with B2.
Type in) (close the brackets).
Press Enter.
Highlight (or outline) cells C2 to C10.
Hold down Control and press D.

Step 6: Type in the room areas in column D.

Step 7: Move across to column E. Click on cell E2.
Type = INDEX(.
Click on Sheet 1.
Highlight (or outline) cells C2 to C11.
Press F4 (make reference absolute).
Type , (a comma).
Click on Sheet 2.
Delete the phrase Sheet 2! and replace it with B2.
Type in) (close the brackets).
Press Enter.
Highlight (or outline) cells E2 to E10.
Hold down Control and press D (Fill down).

Step 8: In Column F, select cell F2 and type =C2*D2+E2*D2.
Highlight (or outline) cells F2 to F10
Hold down Control and press D (Fill down).

Step 9: In cell D11, type in Total floor covering cost:
In cell F11, type = SUM(F2:F10) and press Enter.

	A	B	C	D	E	F
1	**Room**	**Floor covering**	**Cost/m² ($)**	**Room area (m²)**	**Labour/m² ($)**	**Cost/room ($)**
2	Bathroom	Ceramic tiles	54	10	35	890.00
3	Bedroom 1	Wool carpet	95	22.5	0	2137.50
4	Bedroom 2	Wool carpet	95	20.7	0	1966.50
5	Bedroom 3	Wool carpet	95	12	0	1140.00
6	Lounge/dining	Slate tiles	85	38.5	35	4620.00
7	Hallway	Polished boards	45	12.4	0	558.00
8	Laundry	Ceramic tiles	54	4	35	356.00
9	Kitchen	Ceramic tiles	54	19.5	35	1735.50
10	Toilet (WC)	Ceramic tiles	54	2	35	178.00
11				**Total floor covering cost**		

2 Now review your choices for What to do 19.5 questions **2**, **3** and **4** using your spreadsheet.

PROJECT 19

obook
assess
TEACHER

BUILDING IDEAS

Reality TV show?

A project worksheet for this chapter is available on the Teacher obook/assess. Ask your teacher to print it out for you. Completing this activity will help you make the right choices when renovating your room, house or business.

CHAPTER 20

Modelling and building

20A Planning your job

20B Building your model

ARE YOU READY?

Complete the questions below to see if you are ready to start this chapter, or if you need to brush up on your skills first.

- Part 1 should be completed *with* a calculator.
- Part 2 should be completed *without* a calculator.

If there are any questions you cannot answer, the link below each question will direct you to a related Mathematical Skills chapter where you can revise basic concepts and get up to speed.

PART 1 WITH A CALCULATOR

1 Find the value of each of the following.

a $2.100 + 1.05 + 0.65 + 3.0 - 0.75 =$

b $7.35 + 3.15 \times 4 + 3.2 =$

c $6 + 3.5 \times 4.15 - 5 \times 2 \times 3 =$

23D Decimal numbers

2 Set your calculator to automatically round to 3 decimal places. Check by finding the answer to $5 \times 3.234\,567$.

23E Rounding with a calculator

3 Under the SI (metric) system, which units should be used to show the dimensions on a house plan?

A mm, cm **B** m, dm

C m, mm **D** feet, inches

25A Units of measurement

4 A plank of wood is 150 mm wide and 1.4 m long. In square metres, what is the surface area of one side of this plank?

27A Area

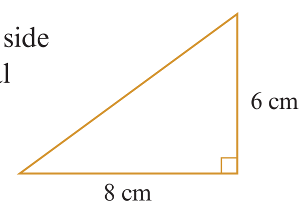
5 For a scale of 1 : 100, 100 mm on the drawing equals _____ mm in real life.

A 0.1 m **B** 1 m

C 10 m **C** 100 m

31B Scale drawings

6 Find the unknown side correct to 1 decimal place.



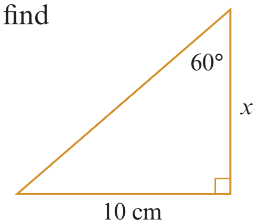
32C Finding unknown sides

7 The formula used to find the length of the unknown side in this triangle is:

A $\sin 60° = 10 \div x$

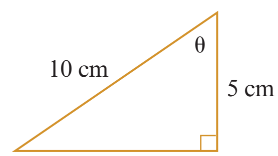
B $\cos 60° = 10 \div x$

C $\tan 60° = 10 \div x$



32A Pythagoras' theorem

8 Find the missing angle (θ) in the right-angled triangle shown.



32D Finding unknown angles

PART 2 WITHOUT A CALCULATOR

9 Find the value of each of the following.

a $12 + 8 \times 4.5 =$ **b** $(33 \div 3) \times 5 =$

c $14 \div 7 \times 4 =$ **d** $9 + 5 - 7 =$

28A Back to basics

10 Convert the following decimals to fractions.

a 0.666 67 **b** 0.25

c 0.8 **d** 0.125

23D Decimal numbers

11 Write the value of the 2 in these numbers.

a 4231.0 **b** 32 445

c 461.002 **d** 4361.02

28E Powers of 10

12 Estimate the approximate total length of the following lengths:
4.9 m, 2.5 m, 0.75 m, 500 mm.

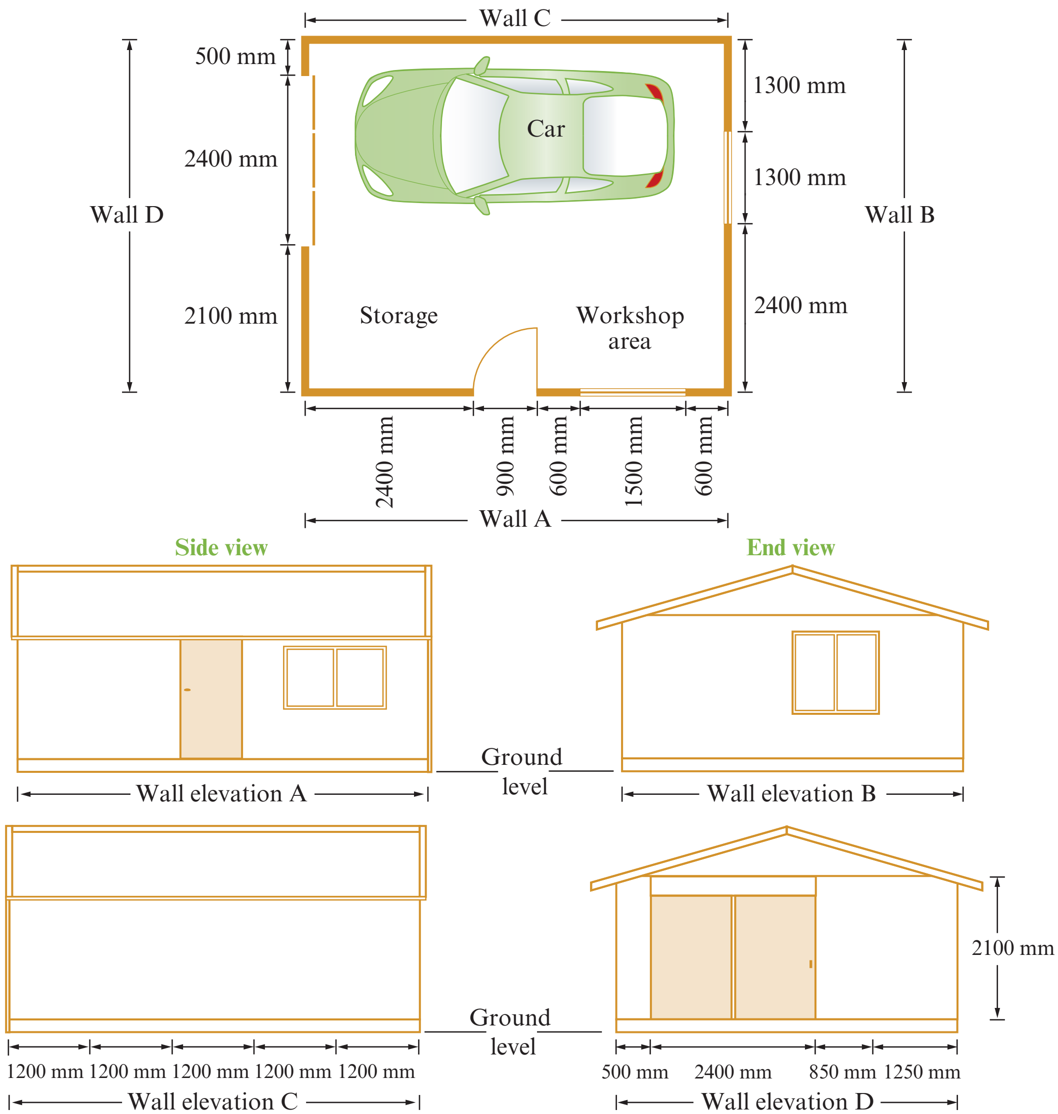
23F Estimation

20A Planning your job

Have you ever been interested in building a model aircraft or car? Maybe you have always wanted to build a model sailing ship, but the plans are gathering dust in a drawer? Perhaps you're even intending to become a carpenter and build houses for a living? The common thread of these three is the ability to read scale plans and accurately translate them into a three-dimensional object. To do this you must become familiar with a variety of mathematical skills.

NEED SOME PRACTICE?
Go to 31C
Building plans
PAGE 366

This chapter will help you develop these skills by learning to read building plans, then constructing a scale model of a garage frame. Shown below is a set of plans and elevations for a garage–workshop.

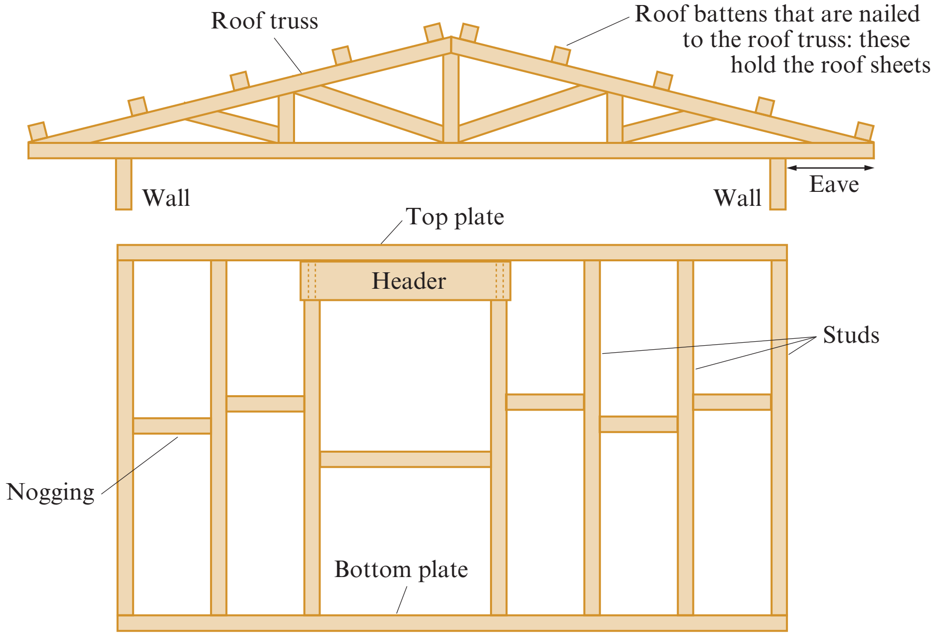


WHAT TO DO 20.1

1 Study the plans above carefully, and as a class discuss what information you can and cannot get from them.

BUILDING TERMINOLOGY

Before you start constructing your model garage–workshop, you need to first understand a few building structure terms.



DETERMINING THE BUILDING SEQUENCE

The steps in the building process happen in a set order. You cannot put on the roof tiles, for instance, before the roof structure is up. This may seem obvious, but how many times have you assembled something and had to go back a step or more to put in a forgotten item? Thinking ahead is the key, and to do this requires an appreciation of all the requirements of the job.

WHAT TO DO 20.2

1 The steps in the building process for this garage are listed below in a jumbled order. In groups of two or three, discuss the steps listed, and put them in what you think is an appropriate job order.
- Put on the roofing sheets (such as corregated iron).
- Build the walls.
- Lay out the outline of the building on the ground.
- Draw the plans.
- Build the roof.
- Put on the wall cladding (such as weatherboards).
- Lay the concrete floor slab.
- Wire the power points and lights.
- Survey the site.
- Estimate the types and amounts of materials needed.

DRAWING THE PLANS

The model garage is to be built as walls and roof frames using a 1 : 25 scale, and assembled on a base board that represents the concrete slab. To do this you need an accurate set of scale drawings of these frames. The next exercise is to be done in groups of four people. Each member is to draw (and later build) one wall frame and one copy of the roof truss.

WHAT TO DO 20.3

1 The walls

The following sets of plans are for the frames for the four walls of the garage. Your job is to redraw them using a 1 : 25 scale. In this way you will be able to easily translate the plans directly to your model (building on a 1 : 1 scale from the plans).

Step 1: Assign one of the four walls to each member of the group.

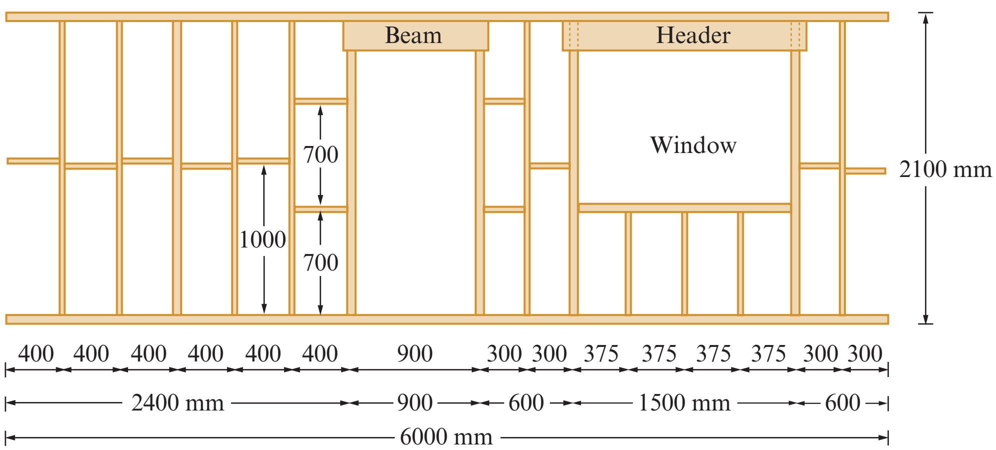
Step 2: Each group member is to draw their wall on an A4 page accurately using a 1 : 25 scale.

Step 3: One member of the group is to also draw a floor plan of the garage to a 1 : 25 scale.

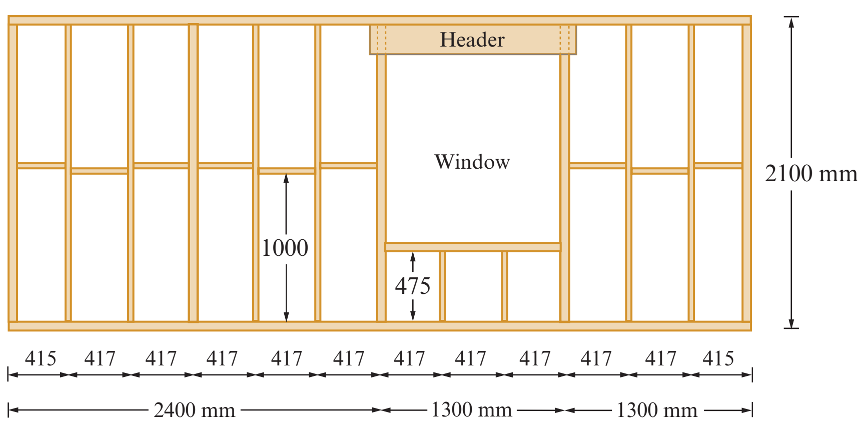
NOTE

Accuracy at this point is critical. Any error here will cause the finished products to not fit together!

Wall A



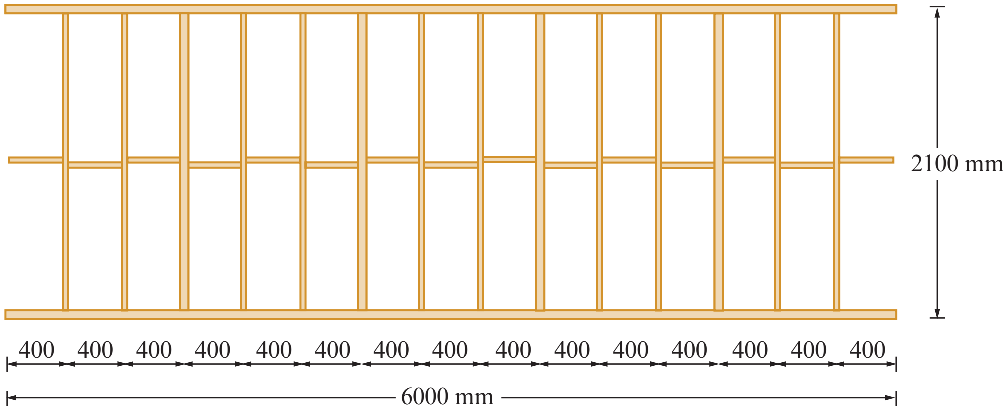
Wall B



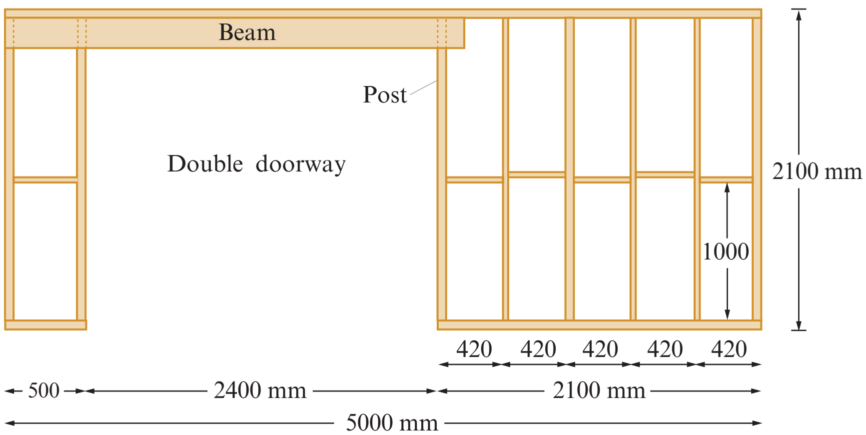
NEED SOME PRACTICE?
Go to 31B Scale drawings PAGE 362

NEED SOME PRACTICE?
Go to 31C Building plans PAGE 366

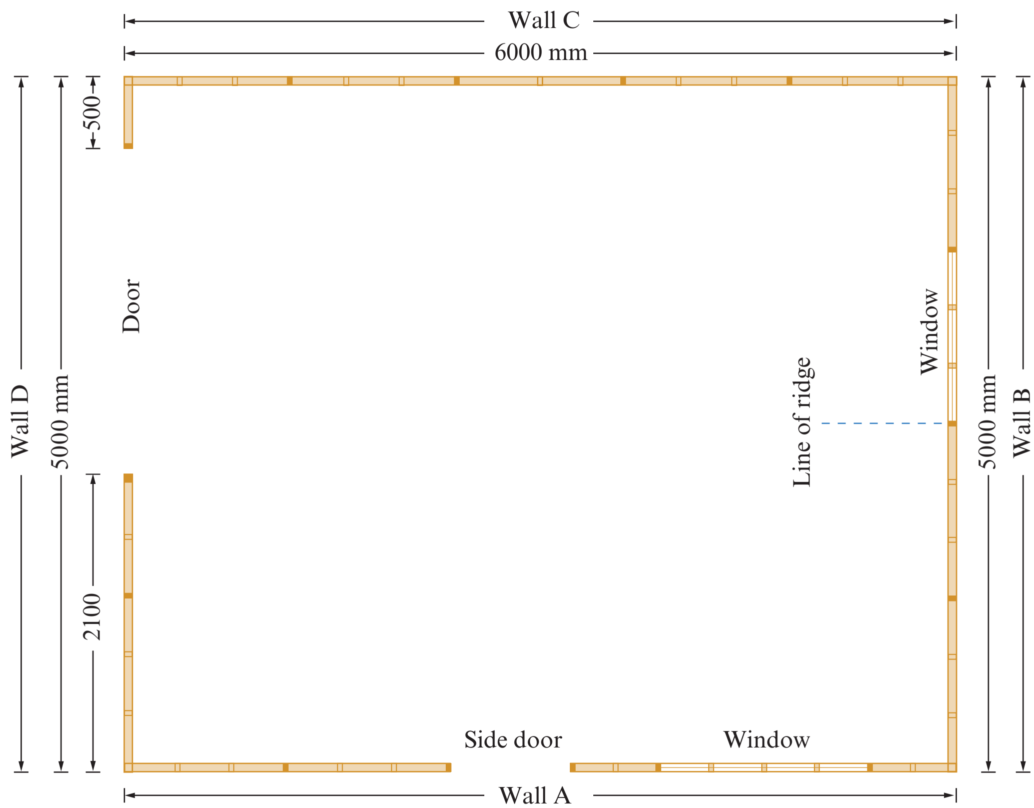
Wall C



Wall D



Floor plan



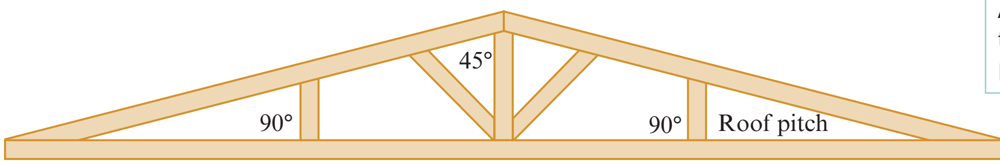
2 The roof

When building a roof, the width from eave to eave and the roof pitch (the angle at which the roof slopes) are both determined before the job starts. However, the actual length of the parts of the truss need to be calculated when it is being built. In the case of your garage, the roof trusses will be 6 m wide and have a pitch chosen from the table.

Roof type	Roof pitch
Tile roof	22.5°
Corrugated iron roof	15°
Loft-style roof	45°

Step 1 Choose a roof pitch for your group to use and calculate the height of your model roof. Below is a diagram of the layout for a typical roof truss.

NEED SOME PRACTICE?
Go to 32E Applying trigonometry PAGE 381



Step 2 Draw the roof truss for the garage using a 1 : 25 scale, showing the lengths of all components. You will use four roof trusses in the construction of the garage, so four copies of this plan will need to be made. Each member of the group is to make one truss.

QUANTITY ESTIMATION

Before you start any job, you need to know the quantity of material required. In this case, you will be cutting 5 mm strips of 2.5 mm thick balsa wood to use as the studs, plates and noggings in the frame. Headers are to be made from 10 mm wide strips of the 2.5 mm balsa wood. What to do 20.4 takes you through the quantity estimation process.

WHAT TO DO 20.4

1 Each member of the building team is to calculate what length of studs, noggings, top and bottom plates is needed for their wall frame and roof truss in the model. It will help to copy and fill in the following table. You could also use a spreadsheet. Because the model is to be the same size as the plans you have drawn, you are building on a 1 : 1 scale. This makes measuring and cutting easy as there are no scale conversions.

Item in frame	Number required	Length of in model	Total length required	Total length required in full-size garage
Studs				
Noggings				
Top plate				
Bottom plate				
Truss members				
		Total length to cut = sum of column 4, rows 1 to 5		
Headers				

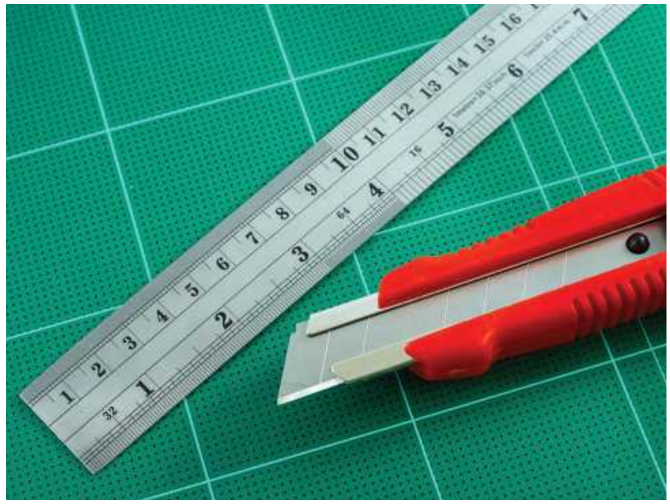
2 Having calculated the total length of 5 mm strips of timber to be cut, work out how many strips need to be cut from the 2.5 mm × 75 mm × 900 mm piece of balsa wood supplied.

20B Building your model

It is important to carefully work out what materials and tools you need to build the model garage. The following list should cover some of these. Is there anything missing?

Tools

- Utility knife
- Steel ruler
- A cork flooring tile glued to 20 mm chipboard or similar to use as the base board
- Dressmaking pins
- A board to do the cutting on



Materials

- Balsa wood, 2.5 mm × 75 mm × 900 mm
- Quick-setting glue
- 12 mm thick craftwood (or similar) to use as the garage–workshop floor
- A roll of clear sticky-back plastic (such as Contact)

BUILDING THE WALLS AND TRUSSES

Each member of the building team is to construct the stud wall and truss they drew up.

WHAT TO DO 20.5

1 The floor plan is to be stuck to a piece of 12 mm craftwood or similar. This is your 'concrete slab floor'.

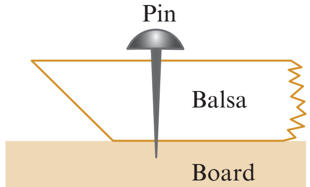
2 Place your wall frame plan on the cork tile, and cover the plan and tile with the clear Contact. Cut your top and bottom plates. Mitre-cut them at 45° as shown in diagram **A**. Place them, on edge, over their positions on the plan using dressmaking pins. Make sure your mitres point down as shown in diagram **B**.

NOTE

Remember the old carpenter's motto: 'Measure twice, cut once'.

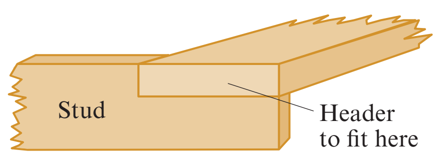
A

B



3 Cut the studs and glue them to the top and bottom plates, again positioning them using dressmaking pins until they dry.

4 Install the headers. Notch the studs so that the headers fit flush with the face of the stud, as shown in the diagram. Continue the process until all the studs, noggings and headers are installed.



Side view of stud

5 Now carefully remove the completed wall frame from the board and place it safely aside.

6 Repeat the process using the roof truss plan. (This time, place the 'timber' flat on the plan.)

NEED SOME PRACTICE?

Go to 32A Pythagoras' theorem PAGE 371

NEED SOME PRACTICE?

Go to 32C Finding unknown sides PAGE 377

NEED SOME PRACTICE?

Go to 32D Finding unknown angles PAGE 380

Pre-assembly checks

You are almost ready to assemble your garage. However, just as a builder would, you must check that the frames are correct before erecting them. First check the wall frames.

Test 1: Are the sides, top plates and bottom plates the correct lengths? Measure them.

Test 2: Are the walls square?

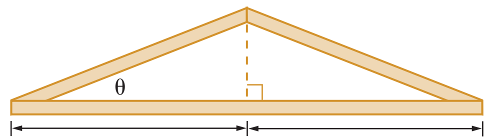
If Test 1 is correct, then you can do Test 2 by comparing the length of the diagonals. Why would this not work if the frame failed Test 1?

Next check the roof trusses.

Test 1: Are the top and bottom lengths correct?

Test 2: Is the centre of the roof pitch really in the centre?

This can be checked by drawing a perpendicular line from the base to the top, and measuring from the line to each end of the truss. Are the two dimensions the same (see the diagram)?



Test 3: Is the roof at the correct angle? Measure the length of the perpendicular used in Test 2 and the length from this to the outside end. Since this is a right-angled triangle, you can use the tan ratio to check the angle.

NOTE

$\tan \theta = \dfrac{\text{opposite}}{\text{adjacent}}$

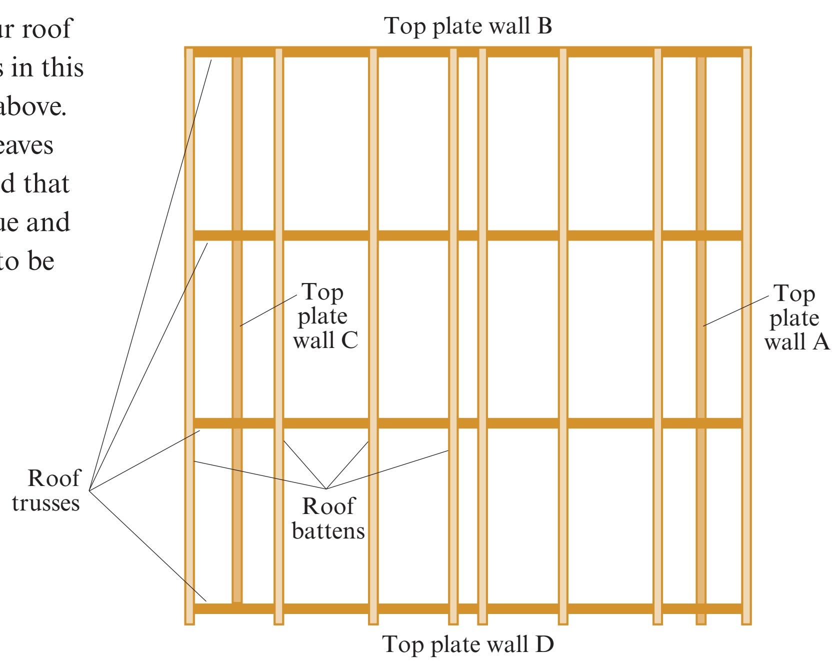
ASSEMBLING THE GARAGE

If the frames pass the tests above, you can now assemble the model garage onto the floor plan that is glued to the board provided (the 'concrete slab').

WHAT TO DO 20.6

1 *The walls:* Place glue at the attachment points and sit the assembled wall frames together on their corresponding places on the floor plan. Pin the frames together until the glue dries.

2 *The roof:* Carefully glue and pin the four roof trusses to the top plate of the garage, as in this plan of the roof structure shown from above. The trusses are 6000 mm wide and the eaves are 50 mm. Cut the pieces of balsa wood that will be used for the roof battens and glue and pin them to the roof trusses. There are to be eight battens.

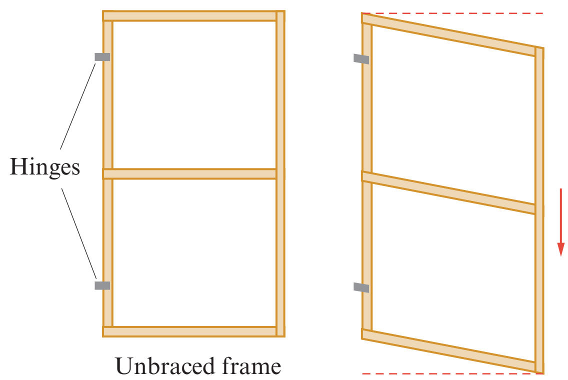


NEED SOME PRACTICE?

Go to 31C Building plans PAGE 366

THE GARAGE DOORS

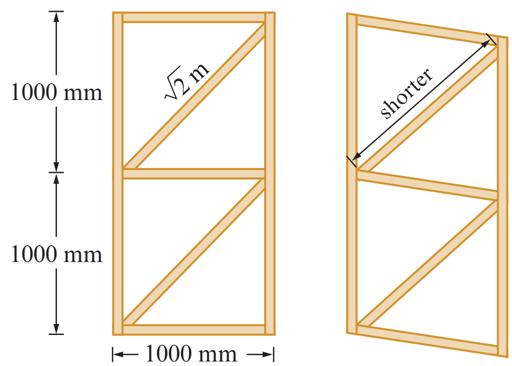
Doors and gates that are hinged from the side will eventually sag under their own weight unless precautions are taken (see the diagrams on the right).



NEED SOME PRACTICE?
Go to 32B Right-angled triangle trigonometry
PAGE 375

NEED SOME PRACTICE?
Go to 32C Finding unknown sides
PAGE 377

The usual method to prevent this is to use diagonal bracing. For the door to sag now, the diagonal brace would have to become shorter. Provided it is strong enough, this would not happen.

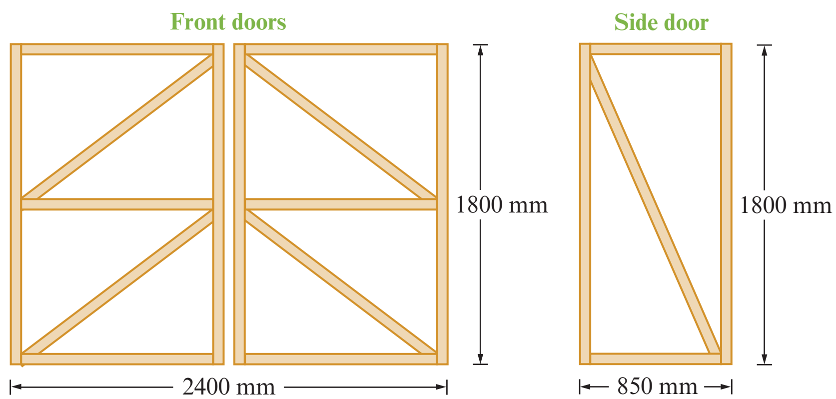


NEED SOME PRACTICE?
Go to 32E Applying trigonometry
PAGE 381

WHAT TO DO 20.7

1 The front doors and the side door for the garage are to be different widths. The sizes are given on the diagram on the right. Calculate the lengths of the diagonals.

2 Draw a detailed end plan of the door frame to a 1 : 10 scale. The frame is to be made of 25 mm square-section steel tube.



PROJECT 20

obook
assess
TEACHER

MODEL MAP

Contour it?

A project worksheet for this chapter is available on the Teacher obook/assess. Ask your teacher to print it out for you. Completing this activity will help you read and use scale drawings.

CHAPTER 21

Landscaping your yard

ARE YOU READY?

Complete the questions below to see if you are ready to start this chapter, or if you need to brush up on your skills first.

- Part 1 should be completed *with* a calculator.
- Part 2 should be completed *without* a calculator.

If there are any questions you cannot answer, the link below each question will direct you to a related Mathematical Skills chapter where you can revise basic concepts and get up to speed.

PART 1 WITH A CALCULATOR

1 Find the value of each of the following.

a 2.105 + 1.05 + 0.65 + 3.0 − 0.7 =
b 7.35 + 3.15 ÷ 4 + 3.2 =
c 6 + 3.5 × 215 − 5 × 2 × 3 =
d 1.005 + 1.23 + 54.1 + 22.309 + 7.780 =

23D Decimal numbers

2 Calculate the following. Your answer should be expressed as a fraction, not a decimal.

a $\frac{1}{3} + \frac{1}{3} =$ **b** $\frac{1}{2} + \frac{3}{4} =$
c $1\frac{1}{4} + 2\frac{3}{8} =$ **d** $5\frac{1}{4} - 2\frac{1}{5} =$

23B Fractions

3 There are ____ mm in 1 km?

A 1 000 000 **B** 100 000
C 10 000 **D** 1000

25A Units of measurement

4 Find the area of a circle with diameter:

a 5 mm **b** 0.6 cm
c 5.06 m **d** 10 km.

27A Area

5 How many litres would a container 200 mm × 350 mm × 100 mm hold?

A 0.7 L **B** 7.0 L
C 70 L **D** 700 L

30A Volume

6 For a scale of 1 : 50, 10 mm on the drawing equals _____ mm in real life.

A 50 mm **B** 500 mm
C 5000 mm **D** 5 m

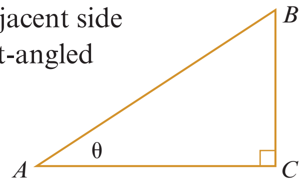
31B Scale drawings

7 Which of the following sets of three-side lengths will be a right-angled triangle?

A 1, 2, 3 **B** 3, 4, 5
C 5, 6, 7 **D** 3, 6, 12

32A Pythagoras' theorem

8 Which is the adjacent side to θ in this right-angled triangle?



A *AB*
B *BC*
C *AC*

32B Right-angled triangle trigonometry

PART 2 WITHOUT A CALCULATOR

9 Find the value of each of the following.

a 42 − 8 × 4 = **b** 5 + 5 ÷ 5 =
c 12 × 6 ÷ 4 = **d** 5 + 7 − 7 =

28A Back to basics

10 Convert the following decimals to fractions.

a 0.333 **b** 0.35
c 0.6 **d** 0.125

28C Decimals

11 Write the value of the 3 in these numbers.

a 4231.0 **b** 32 445
c 421.003 **d** 4321.03

28E Powers of 10

12 Estimate the approximate total length of the following lengths:
4.25 m, 6.5 m, 750 mm, 500 mm.

23F Estimation

21A Measuring the backyard

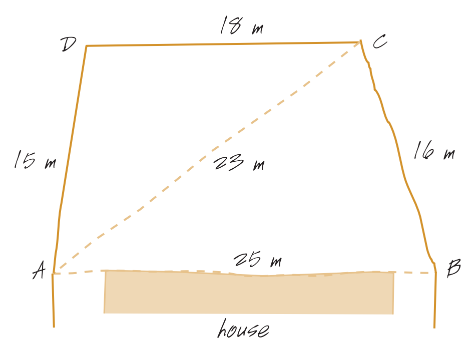
Anyone who buys a house will eventually need to put some effort into maintaining the yard. While this may only mean mowing the lawn and weeding the garden, there are many other tasks and improvements that can be done. It is often less expensive and more satisfying to do most of the work yourself. This chapter will explore some of the options and develop the mathematical skills that are needed to do the job. To begin with, a guided example will be used. After that there will be an opportunity to put the techniques and knowledge you have learnt into practice for yourself.

The sensible way to start the project is to measure the yard so that an accurate scale drawing can be made and important features or areas can be marked.

DRAWING A PLAN

NOTE
A quadrilateral-shaped yard can be split into two triangles by measuring a diagonal.

Measure the backyard with a tape measure and record the measurements on paper as a rough sketch. Without proper surveying equipment it is difficult to measure angles but easy to measure lengths. If you are particularly unlucky, the corners will not be at right angles. Your rough sketch might look like this.



NOTE
A3 is double the size of A4.

An A4 sheet of paper is approximately 30 cm by 21 cm. If you want the plan to fit onto an A4 sheet, what would be an appropriate scale? What would be appropriate for an A3 sheet? Draw the plan in pencil to start with in case you need to make corrections.

NEED SOME PRACTICE?
Go to 31B Scale drawings PAGE 362

NEED SOME PRACTICE?
Go to 27A Area PAGE 311

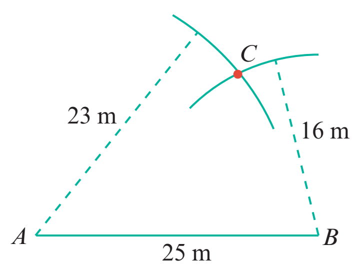
EXAMPLE 21A-1 Constructing triangles

Construct a triangle for the above plan with side lengths of 16 m, 23 m and 25 m.

Step 1: Draw one side (to scale) using a pencil and ruler, for example the 25 m side AB.

Step 2: At end A, draw an arc of radius 23 m. At end B, draw an arc of radius 16 m.

Step 3: Where the arcs meet, locate C, and ABC is the triangle you want.



WHAT TO DO 21.1

1 From the rough sketch, using pencil, ruler and compass, construct an accurate scale drawing of the backyard.

2 Calculate the area of the backyard using Heron's formula to find the area of the triangle with sides a, b and the triangle with sides c and a, c and d.

$A = \sqrt{s \times (s-a) \times (s-b) \times (s-c)}$ where $s = \dfrac{a+b+c}{2}$.

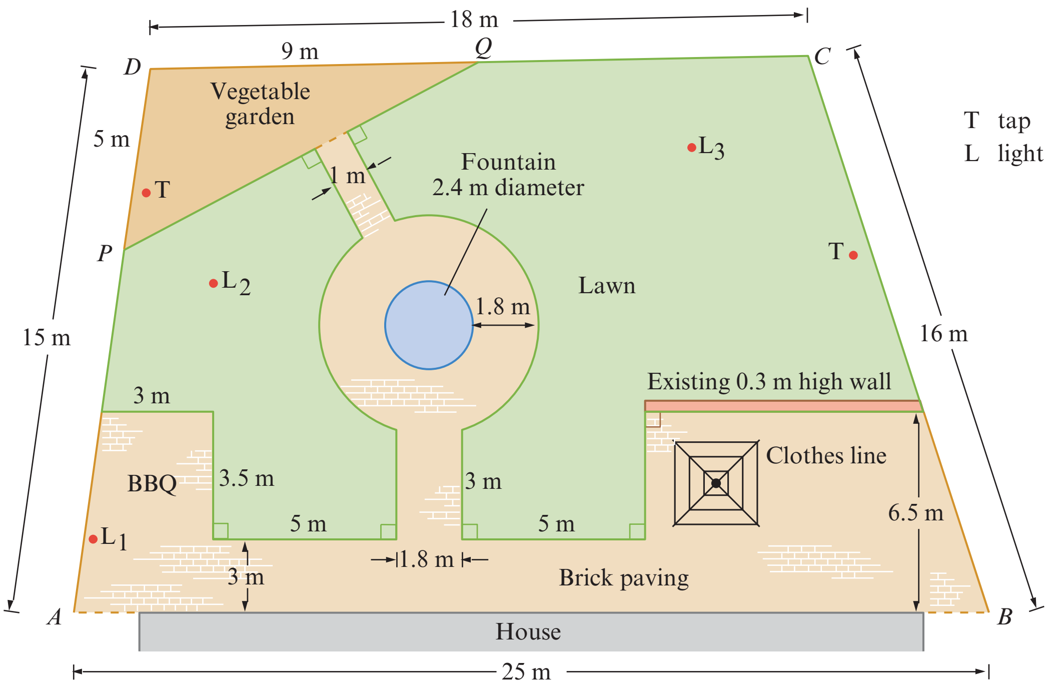
obook
An Excel spreadsheet template to help you apply Heron's formula is available on your obook.

21B Planning the backyard features

NEED SOME PRACTICE?
Go to 27A Area
PAGE 311

NEED SOME PRACTICE?
Go to 28A
Back to basics
PAGE 325

Now that the dimensions and area of the property are known, some decisions about what can be done to it may be made. For this example, a vegetable garden, an outdoor barbecue area, an ornamental fountain and a clothes line are to be added. The paths and the area around the barbecue, the clothes line and the fountain are to be paved. The remainder, except for the vegetable garden, is to be seeded or turfed with lawn. Three lights are to be installed and two taps. The plan is shown below.



WHAT TO DO 21.2

1 Add the suggested backyard features shown above (drawn to scale) to the scale diagram of the backyard from question **1** in What to do 21.1.

21C Order of tasks

Care must be taken to do all tasks in a logical order. For example, you would not build the fountain before installing the plumbing to it or electricity to run the pump. A numbered list showing the order is important, or a flow chart could be drawn.

Schedule

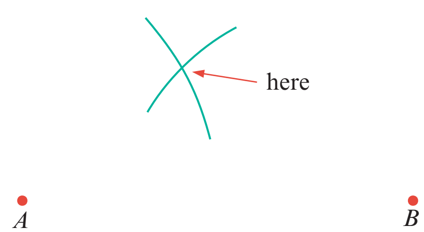
- A plumber needs to install pipes and taps as shown on the plan, and install the pipe to the fountain and the plumbing for it.
- An electrician needs to run a power line from the house to the fountain (the pump can be connected later) and a power line from the house to the three backyard lights.
- The concrete foundation needs to be made for the fountain.
- The clothes line and fountain can then be installed.
- All the brick paving areas need to be marked out.
- The lawn and vegetable garden areas need to be rotary hoed.
- The watering system for the lawn needs to be installed.
- A plumber can install the fountain pump and an electrician can connect power to the fountain pump. The electrician also needs to install the lights.
- The vegetable garden can be constructed and the lawn seeded (or use instant turf).

The ordering of tasks has some flexibility. Is the order of tasks above acceptable? Are the tasks easy to follow? Are there any essential tasks missing from the list?

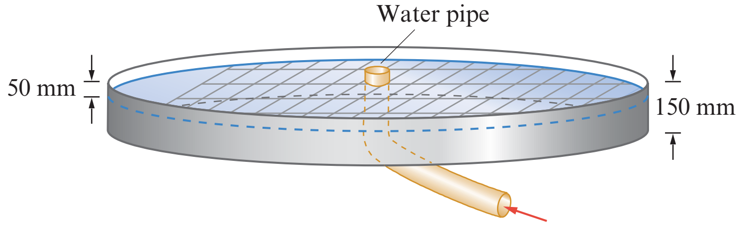
WATER FOUNTAIN

Choose a fountain that is appropriate to the size of the garden and your budget. Before the plumber can start, you must mark, with a peg, the centre of the circular foundation. To do this, use your accurate scale drawing.

You could use points A and B and a tape measure.



Next you need to build a foundation for the fountain. Specifications show that the fountain's foundation (2.4 m in diameter) is to be reinforced with steel mesh and is to be 150 mm thick as shown in the diagram. The steel mesh is to be 50 mm down from the top of the slab.



WHAT TO DO 21.3

1 a Find the area of the top surface of the foundation using:
   Area of circle = $\pi \times (\text{radius})^2$

  b Find the volume of the concrete required using (all units are in metres):
   Volume of concrete needed = area of foundation × height of foundation

NEED SOME PRACTICE?
Go to 27A Area PAGE 311

NEED SOME PRACTICE?
Go to 30A Volume PAGE 351

2 Find the total cost of the foundation if you lay it yourself, given the following costs:
- the circular framework (to hold the concrete in place while it dries) costs $55 to hire
- ready-mixed concrete costs $135 per m^3
- the reinforcing mesh costs $49 for a 3 m × 2.4 m sheet (you will need this much to cut the circle from).

CLOTHES LINE

The clothes line is square, but needs to be placed in the yard with thought to the circle it marks out. It must be a sufficient distance away from the house, fences and trees so that on windy days the clothes do not touch these objects. It also should be placed where it receives maximum sunshine.

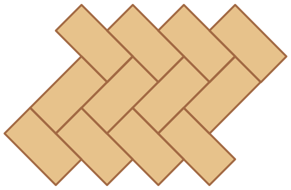
WHAT TO DO 21.4

1 Clothes lines come in different sizes to suit different requirements. A selection based on size and price has to be made. Obtain a clothes-line brochure that gives the dimensions of the line and how deep the vertical pipe must be placed.

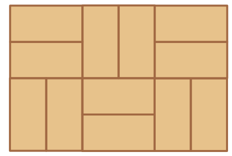
2 On your plan, locate the point where the vertical pipe of the line is to be placed and give instructions on how to find this point in the yard. For stability, the hole should be deep and the base concreted in.

BRICK PAVING

Brick paving is an attractive alternative to concrete. Over time, well-maintained brick paving often looks better as concrete frequently cracks and discolours. There are three common brick layout patterns used. These are:

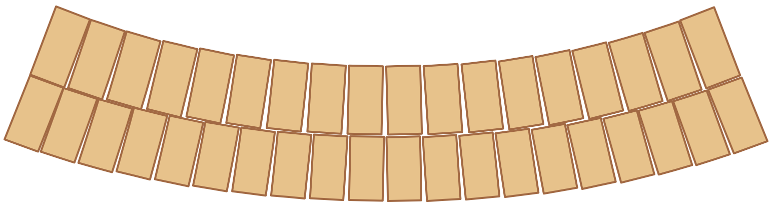


Herringbone



Basketweave

Many other patterns with different types of bricks exist and should be investigated. The path surrounding the fountain needs special attention. Bricks in radial alignment are common around circular things.



Cement pavers cost about \$27/m^2 and clay pavers cost about \$45/m^2.

WHAT TO DO 21.5

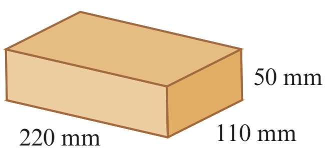
1 Use your scale diagram to calculate as accurately as possible the area to be paved. Remember:

Area of rectangle = length × width

Area of triangle = $\frac{1}{2}$ × base × height

Area of circle = π × (radius)^2

2 a A standard brick paver has the dimensions shown. What is the area of the upper face?



b How many pavers are needed for every square metre of paving?

$$\text{Number of pavers} = \frac{1}{\text{area of one paver in m}^2}$$

c How many pavers are needed for the backyard project?

3 The bricks are laid on a sand base about 75 mm thick.

a What volume, in m^3, of paving sand is needed for the project?

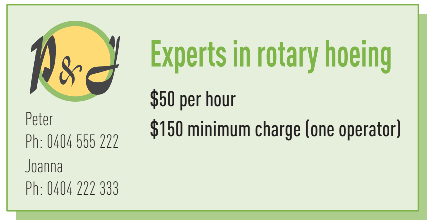
Volume = area × depth

b Find the cost if the paving sand is \$10.50/m^3 delivered.

ROTARY HOEING

The area of the yard that remains is to be the vegetable garden and the lawn area. This should be rotary hoed to loosen the existing soil and make it ready for planting.

Peter and Joanna's business card shows the following charges. Their service guarantees hoeing a minimum of 100 m^2 per hour.



WHAT TO DO 21.6

1 Find the area to be rotary hoed.

2 What will Peter and Joanna charge for the hoeing?

3 Before hoeing, an average of 50 mm depth of topsoil is spread over the site. It costs \$30.00/m^3 delivered. You will wheelbarrow it in and spread it manually.

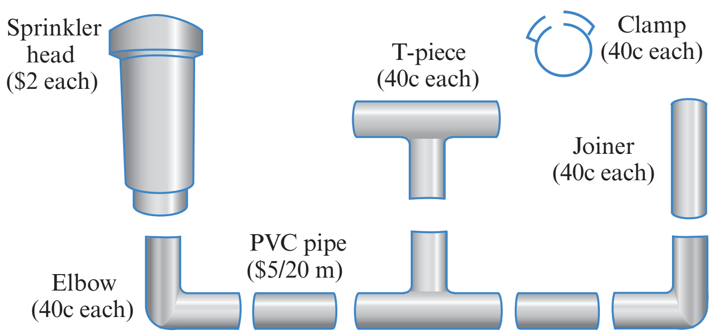
   a How many cubic metres of topsoil are needed?

   b How much will the topsoil cost?

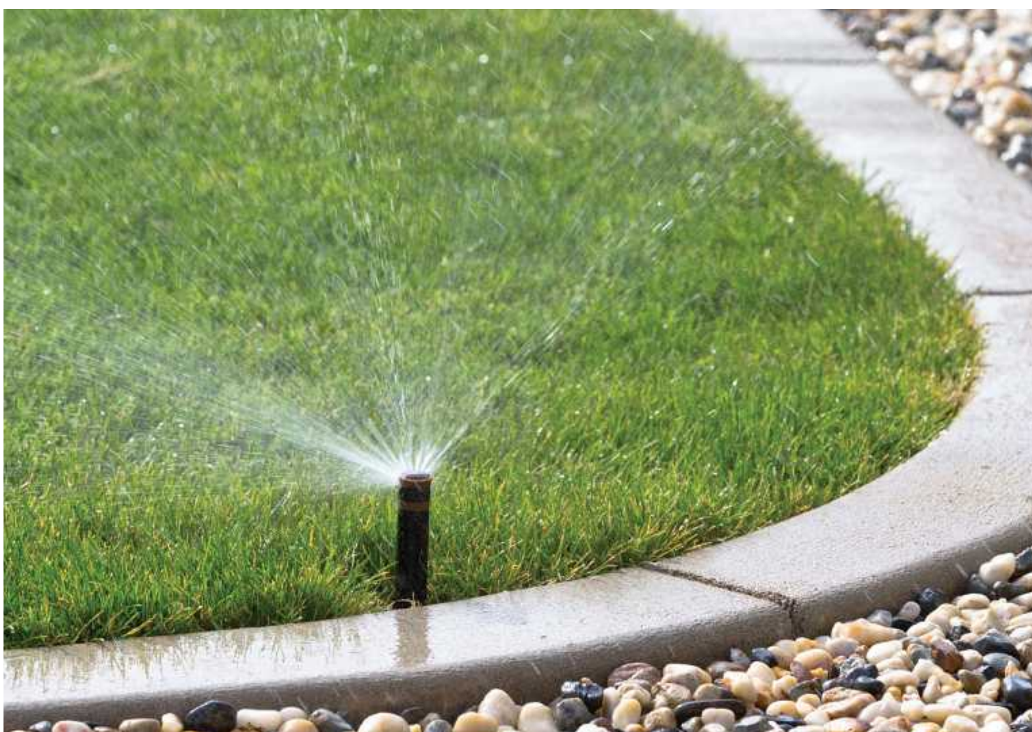
NOTE
You have already calculated the area of the backyard and the area to be paved.

WATERING SYSTEM

An automatic watering system can be easily installed before the lawn is planted. You decide to use the sprinkler heads that spread water in a circular pattern to a radius of 3 m in windless conditions, and can be opened to any angle required. Other parts and costs are given in the diagram.



A four- or six-station control box that is powered by a 9V battery is to be used. This sets the watering time for the sprinklers and the time when they come on. It is water pressure controlled. While the cost varies between brands, the average cost for a control box is about \$300.

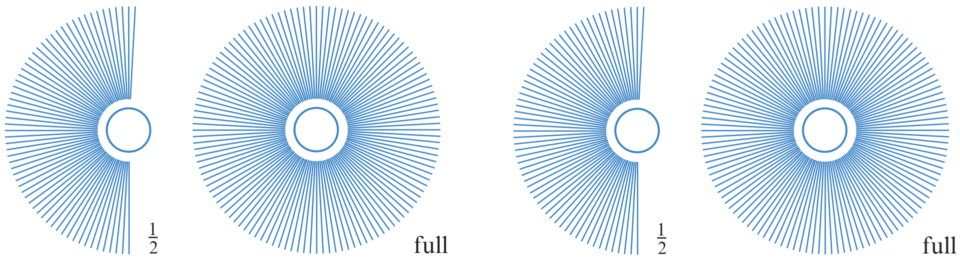


WHAT TO DO 21.7

1 Get your teacher to photocopy your scale diagram. On your copy, use a compass to draw circles (or parts of circles) to cover the lawn area with considerable overlap.

2 There are six possible lines that should have, at most, three full sprinklers on each. For example, the pattern shown adds to three and so is fine. All lines should connect at the right-hand side tap where the control box will also be located.

NEED SOME PRACTICE?
Go to 23B Fractions
PAGE 267



a Connect your system with piping to the control box and show this clearly on the plan.

b Use a code, such as T for T-piece, SP for sprinkler head and EL for elbow, on your diagram to show all pieces needed except for clips.

3 Cost the whole system, displaying the data in a spreadsheet.

VEGETABLE GARDEN, LAWN AREA AND BARBECUE

The lawn needs to be established, either by seeding or using instant lawn. You also need to look at buying a barbecue and maybe some outdoor furniture.

WHAT TO DO 21.8

1 Find the area of the vegetable garden using Heron's formula for finding the area of triangle with sides a, b and c. Use your scale drawing from What to do 21.2 to find the length of the third side.

$$A = \sqrt{s \times (s-a) \times (s-b) \times (s-c)} \text{ where } s = \frac{a+b+c}{2}$$

2 What area of the backyard is to be lawn? (You have already calculated the area to be rotary hoed and the area of the vegetable patch).

3 Lawn seed costs \$26 per kg packet and each packet should cover 25 m^2. Use this to determine how many packets of lawn seed are needed for your lawn.

4 Instant lawn costs \$8/m^2. Find the cost of using instant lawn and compare this with the cost of using seed.

5 Obtain brochures and costings for a gas barbecue and an outdoor table plus six chairs.

21D Costing the project

A spreadsheet is a very useful way to cost the project. Using the skills you have gained using spreadsheets in previous chapters, create a spreadsheet that includes every part of the backyard project.

WHAT TO DO 21.9

1 Here is a start to the spreadsheet. Some costs are to be calculated or researched and added into column D. Use the spreadsheet to find the total cost of the whole project.

	A	B	C	D	E
1	**What**	**Detail**	**Number**	**Cost/item**	**Cost**
2	Plumber	first visit	1	$175	=C2*D2
3	Electrician	first visit	1	$285	
4	Fountain	form hire	1	$55	↓
5		mesh	1		fill down
6		concrete	1	$735	
7		fountain kit	1		
8	Clothes line	line kit	1		
9	Paving	bricks (m^2)			
10		sand (m^3)			
11	Rotary hoeing				
12	Water system	pipe pieces			
13		battery			
14	Vegetable garden	lattice			
15	Lawn area	seeding			
16		turf			
17	Barbecue	gas unit			
18	Furniture	table and chairs			

obook
An Excel spreadsheet template for What to do 21.9 is available on your obook.

PROJECT 21

obook
assess
TEACHER

LANDSCAPING JOY

Getting your hands dirty in the garden

A project worksheet for this chapter is available on the Teacher obook/assess. Ask your teacher to print it out for you. Completing this activity will help you make the right choices when planning your own garden projects.

CHAPTER 22

Automotive calculations

ARE YOU READY?

Complete the questions below to see if you're ready to start this chapter, or if you need to brush up on your skills first.

- Part 1 of the test should be completed *with* a calculator.
- Part 2 of the test should be completed *without* a calculator.

If there are any questions you cannot answer, the link below each question will direct you to a related Mathematical Skills chapter where you can revise basic concepts and get up to speed.

PART 1 WITH A CALCULATOR

1 Find the value of each of the following.

a 2.105 − 1.050 + 2.503 + 3.02 − 0.7 =

b 7.35 × 3.15 − 4 + 3.2 =

c 10 + 3.5 × 10.4 + 5 × 2.5 + 23 =

23D Decimal numbers

2 Set your calculator to automatically round to 4 decimal places. Check that it does so by finding the answer to 5 × 1.765 432.

23E Rounding with a calculator

3 **a** There are ___ μm (micrometres) in 1 m.

b There are ___ μm (micrometres) in 1 km.

25A Units of measurement

4 Calculate the following percentage changes.

a A $40 item is reduced by 15%. What is its new cost?

b You hear that petrol prices are expected to rise by 45% over the next 12 months. If the current price is $1.29 per litre, what is petrol predicted to cost in 12 months?

24C Percentage change

5 A steel sheet is to be cut to 150 mm wide and 750 mm long. What is the top surface area of this sheet? Answer in cm^2.

27B Surface area

6 What is the area of a circle that is 1 m in diameter? Answer in m^2.

27A Area

7 A paint mixture is to be made by combining white and cyan in the ratio of 10 : 1. If there are 3 L of white paint, how much cyan paint is to be added?

24D Ratios

8 What is the volume of a cylinder 100 mm in diameter and 200 mm high?

A 1 570 796 L **B** 6.28 L

C 1.57 L **D** 0.157 L

30A Volume

PART 2 WITHOUT A CALCULATOR

9 Find the value of each of the following.

a 7 − 8 ÷ 4 + 5=

b (5 + 5) × 5 =

c 10 − 42 − 7 + 56 =

28A Back to basics

10 Convert the following fractions to decimals.

a $\frac{2}{3}$ **b** $\frac{1}{2}$ **c** $\frac{3}{5}$ **d** $\frac{1}{4}$

28C Decimals

11 Write the value of the 4 in these numbers.

a 4231.0 **b** 32 455

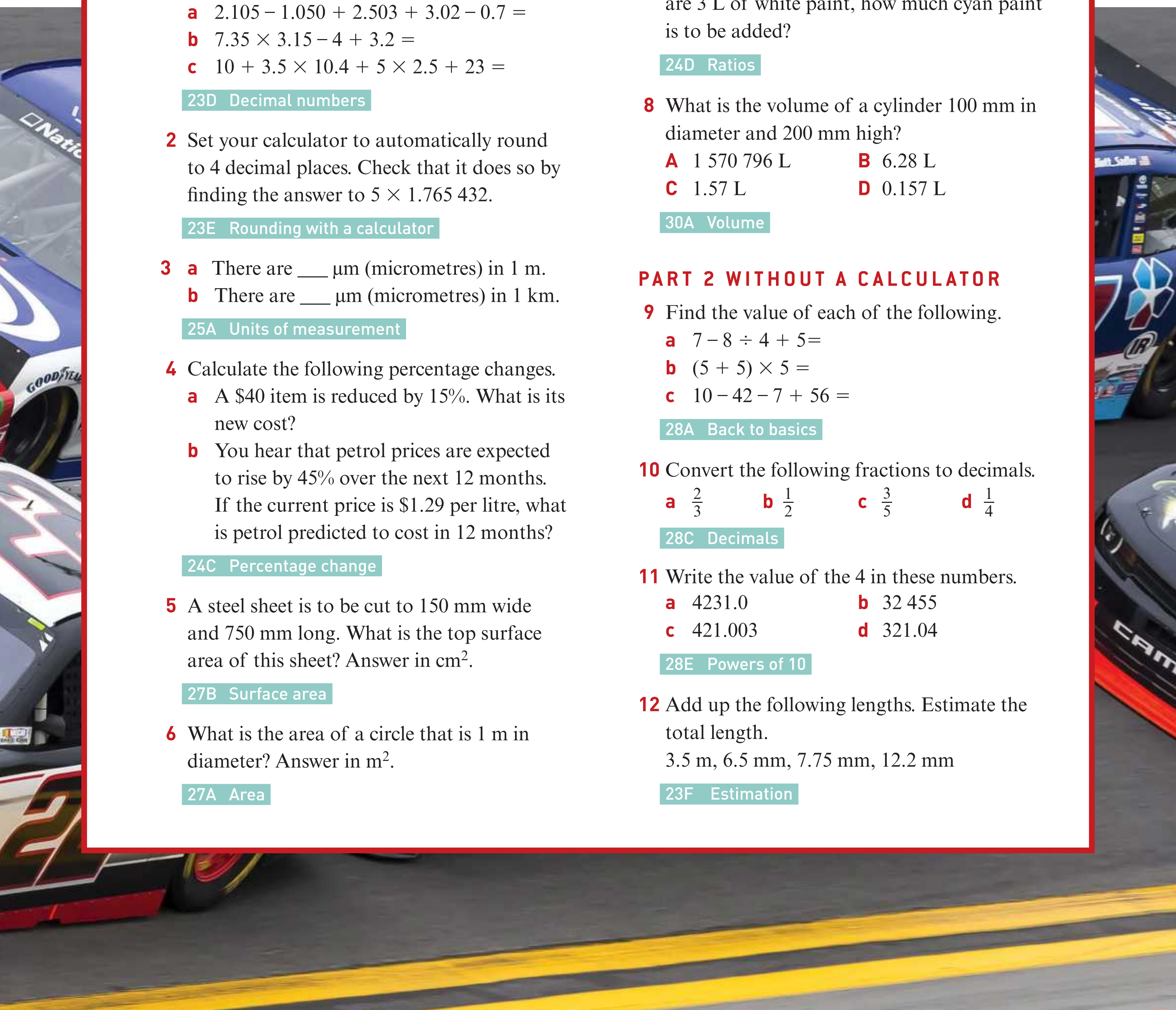
c 421.003 **d** 321.04

28E Powers of 10

12 Add up the following lengths. Estimate the total length.

3.5 m, 6.5 mm, 7.75 mm, 12.2 mm

23F Estimation



22A The engine

Cars are a part of everyday life in most countries in the modern world. The car industry and its associated businesses make up a sizeable proportion of the economy for many countries. For many other people, though, cars are just fun! The design, modification and maintenance of cars is a hobby enjoyed by young and old alike and can also make an interesting career.

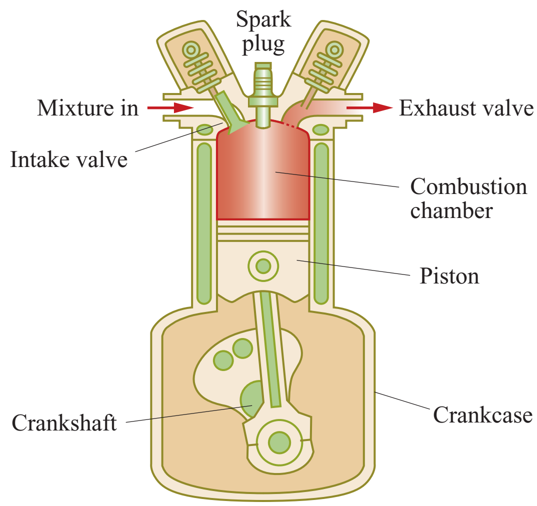
This topic explores several of the major systems that are needed to make cars work and the mathematical principles behind them. At the end of this topic you should have an understanding of how each of these systems functions and be able to calculate typical values for different types of cars.

NOTE

Internal combustion refers to the burning of the fuel inside the motor itself.

Almost all vehicles on the road today are powered by internal combustion engines. For most cars, the engine type is the four-stroke or Otto cycle engine. For cars like the Mazda RX-7, the engine is a rotary (or Wankel) engine. Lawnmowers, go-karts and some motorcycles use a two-stroke engine, whereas vans and trucks often rely on a diesel-powered version of the four-stroke motor. In this chapter, the Otto cycle engine will be used in diagrams and discussions. Most of the terms and calculations are similar for the other engine types.

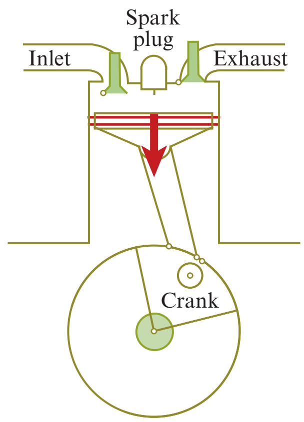
Internal combustion engines have one or more hollow cylinders in which pistons move up and down. Valves let gases into and out of the cylinder at specified times and, for petrol engines, a spark plug is used to ignite the fuel–air mixture. The piston is connected through a set of linkage pins to the crankshaft, which transfers the mechanical energy to the wheels through a series of gears and the shafts.



The four strokes of the cycle in a four-stroke engine are intake, compression, power and exhaust. The complete cycle requires two revolutions of the crankshaft.

Intake stroke

During intake, the piston starts at the top of the cylinder and, as it moves down, the inlet valve opens and the fuel–air mixture is sucked into the cylinder. At the bottom of this stroke, the inlet valve closes.

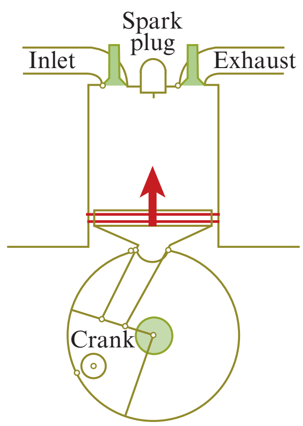


NOTE

Remember, each part of the Otto cycle covers one half of a full turn of the crankshaft, so a full four-stroke cycle comprises two 360° turns of the crankshaft to which the pistons are attached.

Compression stroke

During compression, both the inlet and exhaust valves are closed and the piston moves up from the bottom of the cylinder. This compresses the fuel–air mixture as the volume of the cylinder above the piston decreases.

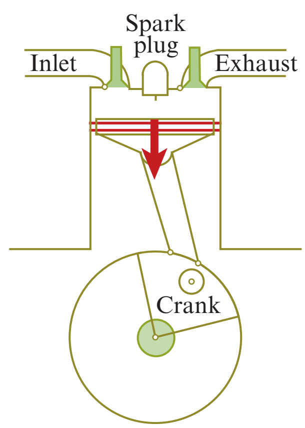


NEED SOME PRACTICE?

Go to 30A Volume PAGE 351

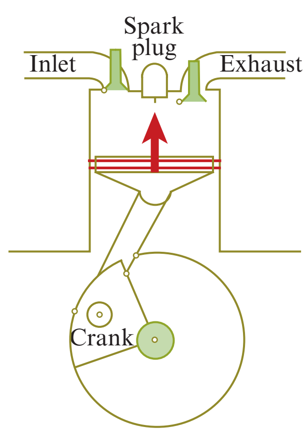
Power stroke

In the power stroke, both the inlet and exhaust valves remain closed and the spark plug is fired as the piston passes the top of its first full turn of the crankshaft. The explosion of the fuel–air mixture forces the piston down again and powers the engine for a half turn.



Exhaust stroke

In the exhaust stroke, the exhaust valve opens and the piston moves upwards again, forcing the waste products (CO_2 and other pollutants) out of the cylinder and into the exhaust system. As the piston reaches the top of the exhaust cycle, the exhaust valve closes and the intake stroke begins again.



NEED SOME PRACTICE?
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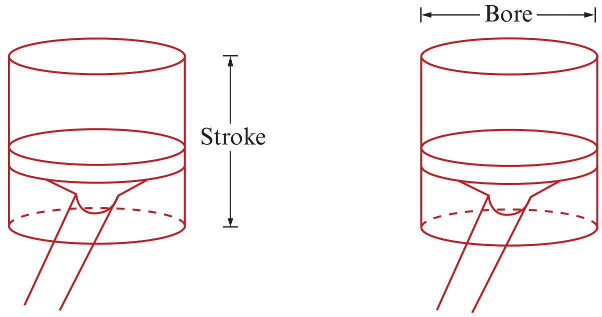
MEASUREMENTS

The diameter of the cylinder is called its bore and the distance through which the piston travels is called the stroke.



The volume of an engine cylinder is calculated using the following formula:

$$\text{Cylinder volume (or swept area)} = \pi \times (\tfrac{1}{2} \times \text{bore})^2 \times \text{stroke}$$

This is identical to the mathematical formula:

$$\text{Volume} = \pi \times \text{radius}^2 \times \text{height}$$

However, the terms used are specific to engines.

Something specific to engines is the term 'swept area' to refer to cylinder volume. Engine size is measured by the amount of volume that is 'swept' by the piston in its cylinder between its topmost and lowest points, multiplied by the number of pistons in the engine.

$$\text{Total volume of an engine} = \text{number of cylinders} \times \text{cylinder volume}$$

Older Australian vehicle motors are commonly measured in imperial units, while newer motor dimensions (and all European motors) are generally given in metric units.

Because the shape of the cylinder is just that, a cylinder, it is easy to calculate the exact engine size using the formula given above for calculating the volume of a cylinder.

All you need to know are:

- the bore (diameter) of the cylinder
- the stroke of the piston (total length of the cylinder between the top and bottom points of the piston's movement)
- the number of cylinders in the engine.

The manuals for almost all cars produced before the 1980s quote engine dimensions and capacity (volume) in inches and cubic inches. Anyone who is interested in working with cars, either professionally or as a hobby, needs to be able to convert between imperial and metric measurements. At various stages in this topic, conversion ratios will be quoted.

The automotive industry now generally uses litre for this purpose.

Some imperial–metric conversions

Imperial	Metric
1 inch (″)	2.54 cm or 25.4 mm
1 cubic inch (cu in)	16.39 cm^3 (cc)
61 cu in	1 L

WHAT TO DO 22.1

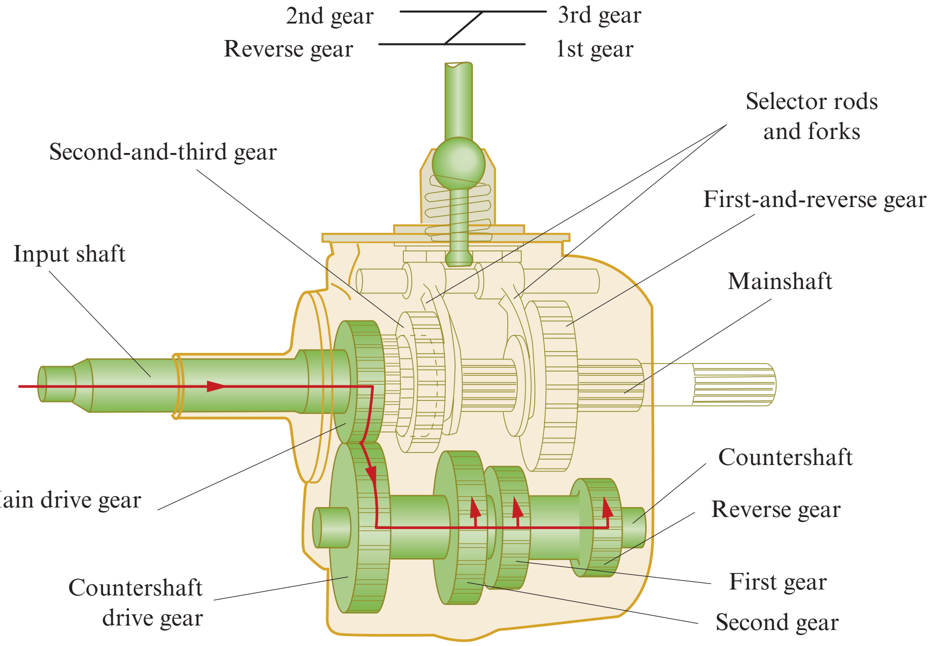
1 Calculate the cylinder and engine capacity for the classic vehicle motors listed in the table below. The bore, stroke and number of cylinders are given.

Motor	Year	Bore	Stroke	Number of cylinders	Cylinder volume (imperial)	Cylinder volume (metric)	Exact engine volume Imperial	Exact engine volume Metric
Holden								
130	1949–56	3.125″	3.000″	6				
138	1960–63	3.062″	3.125″	6				
173	1971–76	3.500″	3.000″	6				
186	1966–84	3.625″	3.000″	6				
202	1971–86	3.625″	3.250″	6				
Volkswagen 1949–70								
1100		2.953″	2.52″	4				
1200		3.031″	2.52″	4				
1500		3.27″	2.72″	4				
Ford (Falcon)								
144		3.5″	2.5″	6				
170		3.5″	2.94″	6				
200		3.86″	3.126″	6				

2 Learner and provisional drivers are limited in the vehicles they are allowed to drive, according to the vehicle size and type. You can find the latest Victorian requirements on the VicRoads website, and the vehicle list on the Redbook website.

22B Torque

The energy produced by the engine needs to be delivered to the wheels so that the car can move. If the engine was directly connected to the wheel, the engine would have to spin very fast in order for the car to travel at a good speed. Instead, gears are used to change the amount of torque (turning force) that is delivered to the wheels. Below is a diagram of a simple three-speed manual gearbox. Modern manual gearboxes are commonly five- or six-speed.



For the gearbox shown:

- In first gear, the purpose is to deliver more power than speed, to get the car moving.
- In top gear, engine revolutions and the drive shaft turn the same number of times per minute. Top gear is designed for speed on flat roads.

When going uphill, more torque is needed, so it is necessary to drop down a gear or two to keep the car from losing speed.

NEED SOME PRACTICE?

Go to 24D Ratios PAGE 285

Below are three typical sets of gear ratios for different types of engines.

Gear	Small engine	Large engine 1	Large engine 2
1st	3.5 : 1	2.8 : 1	1.8 : 1
2nd	2 : 1	1.8 : 1	1.5 : 1
3rd	1.4 : 1	1.3 : 1	1.3 : 1
4th	1 : 1	1 : 1	1 : 1

NOTE

For simplicity, consider that the crankshaft inputs directly to the gearbox, rather than through the countershaft/mainshaft.

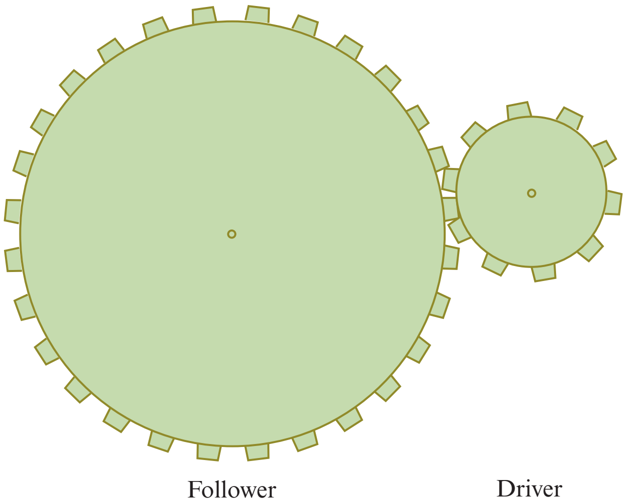
For example, in the table above, first gear with a small engine has a ratio of 3.5 : 1. This means two things:

- The crankshaft turns three-and-a-half times for every turn of the driveshaft.
- There are three-and-a-half times more teeth on the gear connected to the driveshaft (*follower*) than on the gear from the crankshaft (*driver*).

WHAT TO DO 22.2

1 Use the table on the previous page to determine which gear is shown in each of the following diagrams.

a



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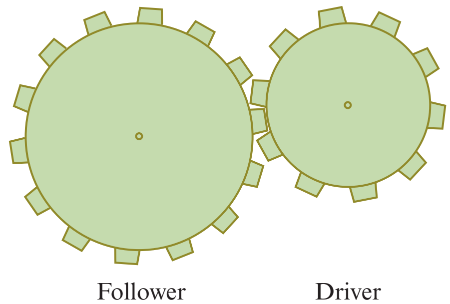

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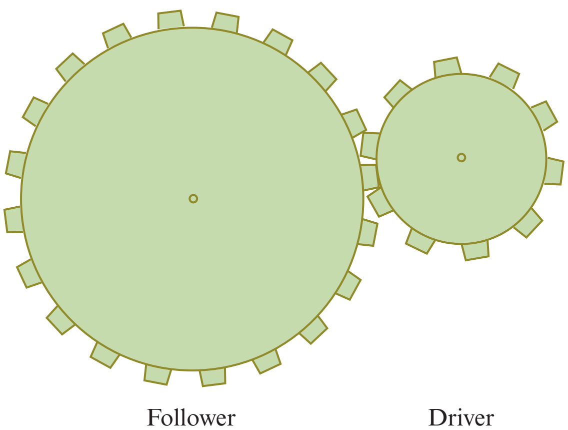
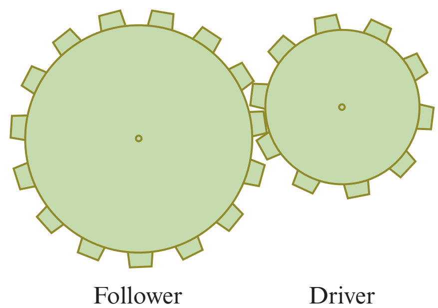
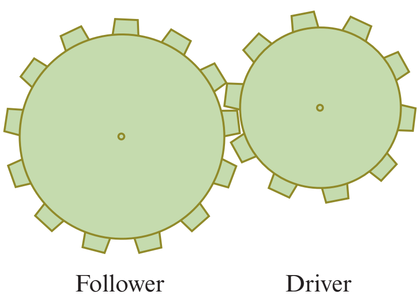


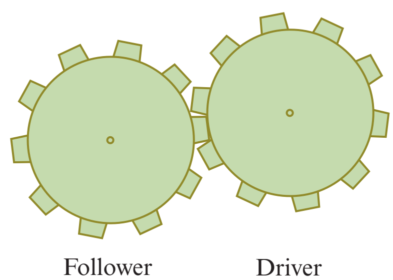
d



e



f



2 Fill in the table on the right for the number of teeth on each cog for these ratios.

	Driver	Follower
1st small	20	
3rd large		52
2nd small	20	
4th		30

The power from the engine is next transferred along the driveshaft to the differential and through the differential to the axle and finally to the wheels. The number of teeth on the differential is different to that on the driveshaft, and increases the number of times the wheels turn for each engine revolution. For example, for a car with an axle (or 'diff') ratio of 4 : 1 in top gear, each time the wheels turn once the engine has turned four times.

WHAT TO DO 22.3

1 Calculate the combined effect of the gearbox and the axle ratio for an axle ratio of 4 : 1 for the three typical sets of gear ratios for the different types of engines given in the table on the previous page.

Gear	Small engine	Large engine 1	Large engine 2
1st	14 : 1		
2nd	8 : 1		
3rd		4 : 1	
4th	4 : 1		

CONVERTING RPM TO DISTANCE TRAVELLED

Most cars operate at between 2000 and 4000 revolutions per minute (rpm). This means that the crankshaft turns 2000 to 4000 times in every minute. In the previous calculations, in 1st gear for a small engine, the car wheels spin 14 times less than this. If the circumference of the wheels is also known, then it is possible to work out how far the car travels for a given rpm.

Take the example of a car with a wheel diameter of 630 mm. Calculate the circumference of the tyres using:

$$\text{Circumference} = \pi \times \text{diameter}$$

$$\text{Wheel circumference} = \pi \times 630 = 1979 \text{ mm or } 1.98 \text{ m.}$$

If the engine is operating at 3000 rpm in 4th gear, the wheels are turning at $\frac{1}{4}$ of this:

$$\text{Wheel revolutions} = \frac{3000 \text{ rpm}}{4} = 750 \text{ revolutions}$$

The distance travelled in 1 min is the number of wheel revolutions by the circumference.

$$\text{Distance} = 750 \times 1.98 \text{ m} = 1485 \text{ m or } 1.485 \text{ km}$$

WHAT TO DO 22.4

1 Practise the calculations for the conditions given below.

a Engine is operating at 2000 rpm, in third gear for a small engine with a wheel diameter of 630 mm.

b Engine is operating at 2000 rpm, in second gear for a small engine with a wheel diameter of 650 mm.

c Engine is operating at 3000 rpm, in third gear for a large engine with a wheel diameter of 720 mm.

d Engine is operating at 2500 rpm, in fourth gear for a large engine with a wheel diameter of 2500 mm.

22C Tolerances

How straight is straight enough? How flat is flat enough? Exactly how close to the given dimension do you really have to drill that hole? Tolerance is a specification for how much variation from ideal measurements can be tolerated in your work.

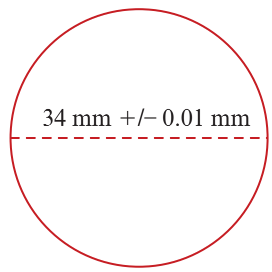
Tolerances can vary from 1 µm or 2 µm (micrometres) or even less for high-precision scientific equipment, to anything up to 1 cm or more for some building work.

NOTE

A micrometre (µ) is one millionth of a metre.

For many pieces of work, a detail drawing will be given for to you to follow. In the drawing's specifications, or in the actual measurements, a tolerance from the exact measurement will commonly be given. If you produce work that is outside this specified tolerance value, your work will be rejected by the customer and you will not be paid!

Often, mechanical drawings show lines with arrows and other symbols that indicate distances and other measurements that must be held to a certain tolerance. For example, the diameter of a hole to be drilled is 34 mm +/– 0.01 mm. This means that an acceptable 34 mm hole can range from 33.99 mm to 34.01 mm as shown in an engineering drawing such as this.



Percentage tolerances

Tolerances are not always specified as an absolute value. They can sometimes be specified as a percentage. For example, you may be given 50 m +/– 5%. As 5% of 50 is 2.5, this would indicate that any length within a range of 47.5 m to 52.5 m is acceptable.

NEED SOME PRACTICE?

Go to 24C Percentage change PAGE 282

Different upper and lower tolerances

The upper and lower tolerances specified may not always be the same. For example, the electrical standard for Australia specifies the nominal 230V AC voltage as 230 +10%/– 6%.

This means:

- the highest voltage allowed is 230 + 10% = 253 volts
- the lowest voltage allowed is 230 – 6% = 216.2 volts.

WHAT TO DO 22.5

1 Calculate the minimum and maximum for the following dimensions with tolerance specifications.

a 185 mm +/– 10 mm b 200 m +/– 5 m c 196 mm +/– 0.005 mm

2 Calculate the minimum and maximum for the following dimensions with percentage tolerance specifications.

a 185 mm +/– 1% b 200 m +/– 10% c 0.4 mm +/– 0.005%

3 Calculate the minimum and maximum for the following values with percentage tolerance specifications.

a 185 mm +5%/–1% b 200 Ω +10%/–5% c 400 m +1%/–0.2%

22D Changing wheel and tyre sizes

If you are ever given a ticket for speeding in a 60 km/h zone, the excuse that your speedometer read that you were only travellling at 60 km/h will not get you off paying the fine. Speedometers are set by the manufacturer to record the number of turns of the wheels on the road. If the diameter, and therefore the circumference, of the wheels change, the speedometer will not be accurate.

NEED SOME PRACTICE?
Go to 23A Fundamental concepts
PAGE 263

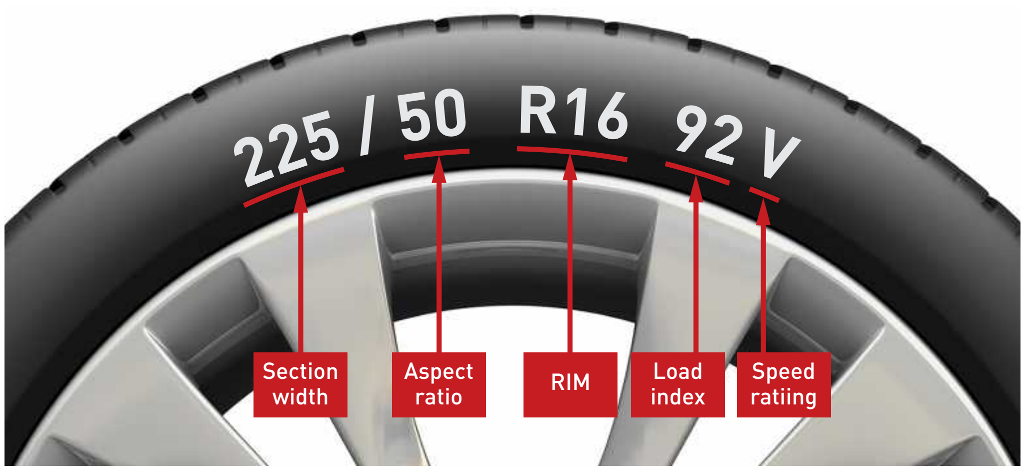
NEED SOME PRACTICE?
Go to 25A Units of measurement
PAGE 291

It is possible to work out if the speedo needs adjusting using the formula:

$$\text{Actual speed} = \frac{\text{new tyre diameter}}{\text{set (old) tyre diameter}} \times \text{speedo reading}$$

To work out the tyre diameter set by the manufacturer (the old diameter), it is necessary to find the tyre description either from the manual or from the side of one of the original tyres. Take, for example, a tyre displaying this information: 255/50R16.

- The first number, 255, is the tyre (section) width in mm (255 mm).
- The second number, 50, is the aspect ratio, which is a comparison of the tyre's section height with its section width (50 indicates the height is 50% of its width).
- R indicates radial ply construction.
- The third number, 16, is the diameter of the tyre's inner rim in inches (16 inches).



EXAMPLE 22D-1 Tyre diameter

What is the overall diameter of the tyre displaying this information: 255/50 R 16?

To work out the overall tyre diameter in mm use:

$$\begin{aligned}\text{Overall diameter} &= 2 \times \text{width} \times \text{aspect ratio} + \text{rim diameter} \times 25.4\\ &= 2 \times 255 \times 0.50 + 16 \times 25.4\\ &= 661.4 \text{ mm}\end{aligned}$$

NOTE

Convert the aspect ratio to a decimal. Convert inches to mm by multiplying by 25.4.

The car tyre diameter can change due to wear. When a tyre becomes worn, the tyre diameter decreases. To work out the diameter of a tyre, place the car on a jack and raise one of the wheels off the ground. Wrap a piece of string once around the tyre and measure the length. This is the tyre's circumference. To find the diameter, divide the measured circumference by π (3.1418).

$$\text{Diameter} = \frac{\text{circumference}}{\pi}$$

Another way a tyre diameter can change is to use a different make and model of tyre from the one the manufacturer recommends. If this is the case, the diameter of the new tyre can be worked out by reading the tyre wall and making a new set of calculations.

WHAT TO DO 22.6

1 Calculate the actual speed of a car that has the following tyre changes if the speedo reading is 100 km/h.

a 185/70 R 13 to 175/65 R 14

b 175/65 R 14 to 185/70 R 13

c 195/65 R 14 to 215/60 R 14

d 215/60 R 14 to 195/65 R 14

e 205/55 R 15 to 195/50 R 16

f 195/50 R 16 to 205/55 R 15

2 What is the overall diameter of a tyre displaying the following information?

a 185/70 R 13
b 175/65 R 14
c 195/65 R 14
d 215/60 R 15
e 205/55 R 15
f 195/50 R 16

3 This 2005 Renault Clio has a four-cylinder engine with the following quoted specifications:

Displacement	1.6 L
Bore and stroke	79.5 × 80.5 mm
Compression ratio	10 : 1
Standard tyres	175/65 R 14

a What is the exact volume of the engine correct to 1 decimal place? How would you explain any difference between the quoted capacity and calculated capacity?

b What is the combustion chamber volume for this engine?

c To increase acceleration, the owner is considering changing the tyres to 15 inch rims (195/50 R 15 tyres). If the top comfortable cruising speed of the car with the original wheels is 110 km/h, what will it be with the 15 inch rims?

PROJECT 22

obook
assess
TEACHER

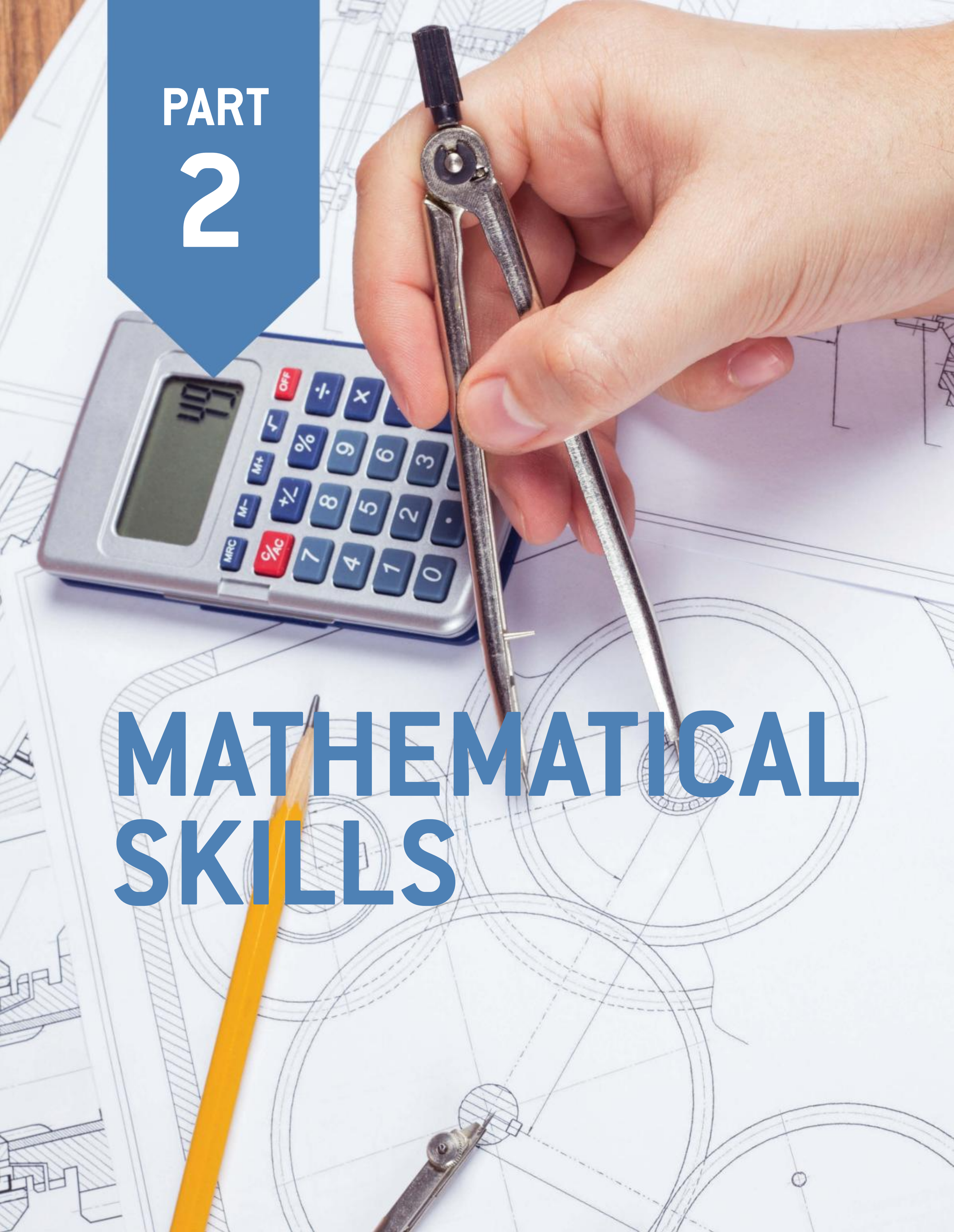
DRIVING YOU ROUND THE BEND

I don't have time for this … or do I?

A project worksheet for this chapter is available on the Teacher obook/assess. Ask your teacher to print it out for you. Completing this activity will help you see how information and communication technology can make things easier in the workplace.

PART 2

MATHEMATICAL SKILLS

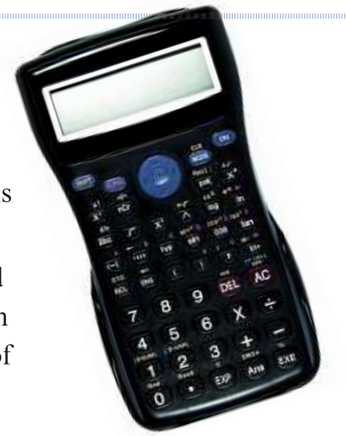


MATHEMATICAL SKILLS

CHAPTER 23

Fractions, decimals and rounding

23A Fundamental concepts



Being competent at working with numbers, including percentages, fractions and decimals, is vital to avoiding costly errors and potential scams in both your work and your personal life.

In this chapter we will review various forms of numbers and perform operations with them. A scientific calculator is essential in this work.

Mathematics has a number of basic concepts and underlying rules. These could be called the 'grammar' of mathematics and must not be violated. This first section is to review some fundamental concepts. Errors may result from overlooking any of these basic rules.

POSITIVE AND NEGATIVE NUMBERS

Adding positive numbers

Mostly we deal with positive numbers and therefore we do not normally put a *plus sign* (+) in front of them.

For example, we would say 'I have 10 dollars' and not 'I have +10 dollars'. This means that the normal sum of 10 + 10 = 20 can also be written as (+10) + (+10) = +20.

NOTE

A positive number is any number that is greater than 0.
A negative number is any number that is less than 0.

Adding negatives/subtracting positives

Is it possible to have less than nothing?

Imagine you are bankrupt. You have nothing. You borrow \$100 from a friend and lose it. You are now \$100 worse off than having nothing. We could say you have –\$100.

A debt of \$100 plus another \$100 debt would be a \$200 debt. A debt of \$100 could be written as –\$100, so a debt of \$100 and another debt of \$100 could be written as –\$100 + (–\$100) = –\$200.

Notice that $100 + (-40) = 100 - 40 = 60$

and that $100 - (+40) = 100 - 40 = 60$

Subtracting negatives

Suppose that you are \$200 in debt; that is, you have –\$200 in your bank account. The bank sends you a reminder about this debt, but someone repays it for you. Taking away the –\$200 means that you are no longer in debt; that is, –\$200 – (–\$200) = 0.

When compared with –\$200 + \$200 = 0, this gives us a rule for two negatives. For example, $100 - (-10) = 100 + 10 = 110$.

Examples like $-4 + 5 = 1$ and $+5 - 4 = 1$ show us that the **order** of addition and subtraction *does not matter*. So, for $-3 + 2 - 1 + 7 - 2 + 11$ we could rearrange it to $11 + 7 + 2 - 3 - 1 - 2$, which is much easier to solve.

NOTE

Rule for two positive signs:
Whenever two positive signs are together we add.

Rule for opposite signs:
Whenever a positive and a negative sign are together we subtract.

Rule for two negative signs:
Whenever two negative signs are together we add.

EXERCISE 23.1

1 Find the simplified value of:

a $200 + (-50)$

b $50 + (-100)$

c $-50 + (-100)$

d $-50 - (+100)$

e $-100 - (+50)$

f $25 + (-30)$

g $15 - (+30) + (-2)$

h $10 + 7 + (-3)$

i $-10 + (+20) + (-10)$

j $-1 + (-1) + (-1) + (+1) + (+1)$

2 Find the simplified value of:

a $16 + (-4)$

b $-4 + (-2)$

c $20 - (-15) - (+5)$

d $-17 + (-3) - (-20)$

e $-1 + (-4) - (-3) + (-1)$

f $+11 + (+11) + (-11) + (+11) - (+11) - 11 - (-11)$

g $+8 - (-7) + (-9) - (+6)$

h $17 + (-13) - (-8) - (+6) + (-6)$

i $150 - 300 - (-125) + (+50)$

j $-15 - (+1) + (-20) - (-17) + 10 + (-3) + (+18)$

Multiplying and dividing positives and negatives

- When two numbers of the same sign are multiplied, the answer is positive.
  For example: $+6 \times (+2) = 12$ and $-6 \times (-2) = 12$.
- When two numbers of opposite sign are multiplied, the answer is negative.
  For example: $+6 \times (-2) = -12$ and $-6 \times (+2) = -12$.
- When two numbers of the same sign are divided, the answer is positive.
  For example: $-6 \div (-2) = 3$ and $+6 \div (+2) = 3$.
- When two numbers of opposite sign are divided, the answer is negative.
  For example: $+6 \div (-2) = -3$ and $-6 \div (+2) = -3$.

EXERCISE 23.2

1 Find the simplified value of:

a $12 \times (-3)$

b $-12 \times (-3)$

c $-1 \div (-1)$

d $-15 \div 3$

e $-2 \times (-3) \times (-6) \times (-2)$

f $-312 \times (-15)$

g $-81 \div (+3)$

h $16 \times (-2) \times (-10)$

i $2 \times (-1)$

j $-1 \times (-1) \times (-1) \times 1 \times (-2)$

2 Find the simplified value of:

a $172 \times (-100)$

b $15 - (-3) + (-9)$

c $-1 \div (-1) \div (-1)$

d $-8 + (-9) - (-1)$

e $-2 \times (+3) \times (-1) \times (-100)$

f $-12 \div 2$

g $-5 + (-5) + (+20) - (-1)$

h $3 \times (-4) \div 12$

i $-12 + 100 - (-13)$

j $-7 \times (-1) \times 1$

ORDER OF OPERATIONS

When adding and subtracting, the order in which we do the operations makes no difference to the final result.

For example, $3 + 4 - 2 - 1 = 6$ and $4 - 1 - 2 + 3 = 6$ also.

However, when we have an expression that contains a mix of multiplying and dividing with addition and subtraction, the order *does* make a difference.

For example, to find the value of $6 + 2 \times 4$:

- if we do $6 + 2$ first and then multiply by 4, we will get $8 \times 4 = 32$
- whereas if we do 2×4 first and then add 6, we will get $8 + 6 = 14$.

Clearly there is a problem. We can overcome it by adopting the BODMAS rule. This rule is generally given as:

B **B**rackets first.

For example: $(6 + 2) \times 4 = 8 \times 4$
$= 32$

O **O**rders next (that is, powers and square roots, etc.). For simplicity in this chapter, we use the **O** in BODMAS to represent 'overs' or fractions and workings.

For example: $\frac{12-3}{3} \times 4 = \frac{9}{3} \times 4$
$= 3 \times 4$
$= 12$

D **D**ivision next.

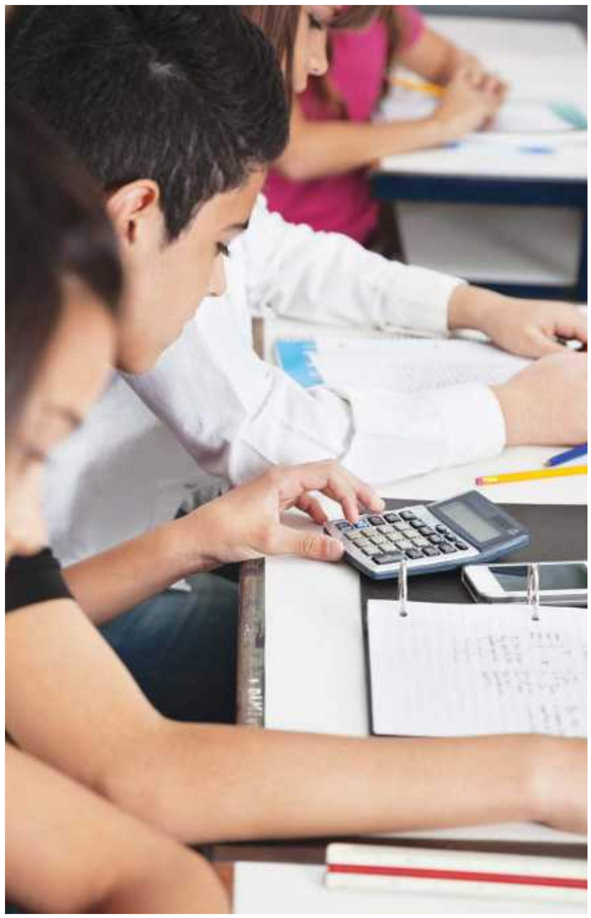
For example: $2 + 8 \div 4 = 2 + 2$
$= 4$

M **M**ultiplication next.

For example: $2 + 8 \times 4 = 2 + 32$
$= 34$

A **A**ddition next.

S **S**ubtraction next.



NOTE

Always work out from the innermost bracket first.

NOTE

Division and multiplication rank equally and go from left to right.

Addition and subtraction rank equally and go from left to right.

EXAMPLE 23A-1 Order of operations

Simplify the following.

a $(3 + 2) \times 4$ **b** $\{4 + (3 \times 2)\} \div 5$

a $(3 + 2) \times 4 = 5 \times 4$
$= 20$

b $\{4 + (3 \times 2)\} \div 5 = \{4 + 6\} \div 5$
$= 10 \div 5$
$= 2$

NOTE

Remember that $3(a + b) = 3a + 3b$.

EXAMPLE 23A-2 Further order of operations

Simplify the following.

a $6 \times 4 + 5 \times 2$

b $3 + \frac{12 - 3}{3} \times 4 \div (8 - 6)$

a $6 \times 4 + 5 \times 2 = 24 + 10$ Multiplication is before addition.
$= 34$

b $3 + \frac{12 - 3}{3} \times 4 \div (8 - 6) = 3 + \frac{12 - 3}{3} \times 4 \div 2$ Brackets are done first.
$= 3 + \frac{9}{3} \times 4 \div 2$ Overs are done next.
$= 3 + 3 \times 4 \div 2$ Division is done next.
$= 3 + 3 \times 2$ Multiplication is done next.
$= 3 + 6$
$= 9$

EXERCISE 23.3

1 Find the simplified value of the following.

a $4 + 7 \times 2 - 17$
b $-1 - 16 \div 4 + 5$
c $4 - (-3 \times 37) + 9$
d $15 \times \{(-3) + (-4) - (-15)\}$
e $8 \times \{11 + 16 - (-12)\}$
f $3(-4 + 7)$
g $4(9 - 4 + 5)$
h $4 + (27 \times -3) + 100$
i $15 - 3 \times 8 + 10$
j $(16 \times 4) - (15 \times 3)$
k $25 - 3 \times 4 + 10$
l $(36 \times 6) - (55 \times 2)$

2 Find the simplified value of the following.

a $\{(6 + 2) \div 4\} + (-2)$
b $(5 + 4) \times (10 - 1)$
c $3(-2 \times -2) + (-4 \times -3)$
d $\{(8 - 4)7\} + 3$
e $\{9(20 - 13)\} - 10\{5 - (-1)\}$
f $-2 + (-2) + 4(1 - 1)$
g $15 + (3 \times -5)$
h $\{20(3 + 16)\} + (-10 \times 30)$
i $(16 \div 4 + 5) \times 9$
j $\{10(5 - 5)\} + \{(4 \times 3) - 12\}$
k $15 - 3 \times 8 + 10$
l $\{5(2 - 5)\} + \{(18 \times 3) - 8\}$

3 Find the simplified value of the following.

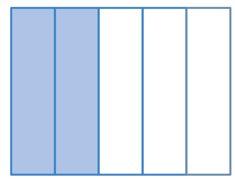
a $3 \times 24 \div 6 + 10$
b $(16 + 4) + 2 \times 3 - 20$
c $9 - 6 \times 4 + 17$
d $-15 \times (-3) - 6 \times 4 + 10$
e $3 \times \frac{4 + 2}{6} - 5 \times 3(4 + 5)$
f $6 + 12 \div 3 \times 6 + 10$
g $9 + 6 \times 24 \div 8 - 3$
h $8 - (-4 \times 2) \div 8 - 1$
i $9 + 6 \times 8 \div (-8) - 3$
j $-4 - (-16) \times 2 - 40 \div 40 \div (-4)$
k $-9 - (-9) \times 2 - 30 \div 30 \div (-1)$
l $8 - (-7) \times 10 - 40 \div 4 \div (-5)$

23B Fractions

A fraction consists of two whole numbers that are separated by a horizontal bar (called a vinculum). Fractions represent parts of a whole.

For example, in the diagram shown on the right there are five rectangles of equal size and two of them are shaded.

So, the shaded region represents the fraction $\frac{2}{5}$, as there are two parts shaded out of the whole (five parts).



The fraction key on a calculator

A key such as $a\frac{b}{c}$ is on most scientific calculators. This key is used to simplify number fractions.

Loading a fraction into a calculator

To load $\frac{2}{5}$, press 2 $a\frac{b}{c}$ 5. The display shows this as 2⌋5.

To load $\frac{13}{8}$, press 13 $a\frac{b}{c}$ 8. The display shows this as 13⌋8.

SIMPLIFYING FRACTIONS

After a fraction has been loaded, the = key can be pressed to simplify the fraction.

EXAMPLE 23B-1 Simplifying fractions

Simplify the following fractions.

a $\frac{2}{4}$ **b** $\frac{36}{45}$ **c** $\frac{13}{5}$

	Calculator	*Display*	*Answer*
a $\frac{2}{4}$	2 $a\frac{b}{c}$ 4 =	1⌋2	$\frac{1}{2}$
b $\frac{36}{45}$	36 $a\frac{b}{c}$ 45 =	4⌋5	$\frac{4}{5}$
c $\frac{13}{5}$	13 $a\frac{b}{c}$ 5 =	2⌋3⌋5	$2\frac{3}{5}$

EXERCISE 23.4

1 Use your calculator to simplify the following fractions.

a $\frac{3}{6}$ **b** $\frac{6}{3}$ **c** $\frac{8}{22}$ **d** $\frac{28}{7}$

e $\frac{15}{35}$ **f** $\frac{12}{16}$ **g** $\frac{32}{40}$ **h** $\frac{9}{15}$

i $\frac{56}{70}$ **j** $\frac{15}{7}$ **k** $\frac{21}{4}$ **l** $\frac{236}{19}$

CONVERTING A FRACTION TO A PERCENTAGE

There are many times when we want to be able to convert fractions to percentages, particularly when we want to compare values.

EXAMPLE 23B-2 Converting a fraction to a percentage

Convert the following fractions to a percentage.

a $\frac{3}{5}$ **b** $\frac{36}{45}$

	Calculator	*Display*	*Answer*
a $\frac{3}{5}$	3 [$a\frac{b}{c}$] 5 [×] 100 [=]	60	60%
b $\frac{36}{45}$	36 [$a\frac{b}{c}$] 45 [×] 100 [=]	80	80%

EXERCISE 23.5

1 Use your calculator to convert these fractions to percentages.

a $\frac{3}{4}$ **b** $\frac{2}{5}$ **c** $\frac{13}{20}$

d $\frac{17}{25}$ **e** $\frac{12}{8}$ **f** $\frac{4}{7}$

ADDING AND SUBTRACTING FRACTIONS

Adding and subtracting fractions is easily performed using a scientific calculator.

For example, to find $\frac{2}{3} + \frac{4}{5}$ press:

2 [$a\frac{b}{c}$] 3 [+] 4 [$a\frac{b}{c}$] 5 [=]

Likewise, to find $\frac{7}{8} - \frac{3}{5}$ press:

7 [$a\frac{b}{c}$] 8 [−] 3 [$a\frac{b}{c}$] 5 [=].

EXAMPLE 23B-3 Adding and subtracting fractions

Use your calculator to find:

a $\frac{3}{5} + \frac{2}{3}$ **b** $\frac{4}{7} - \frac{1}{3}$

	Calculator	*Display*	*Answer*
a $\frac{3}{5} + \frac{2}{3}$	3 [$a\frac{b}{c}$] 5 [+] 2 [$a\frac{b}{c}$] 3 [=]	1 ⌋ 4 ⌋ 15	$1\frac{4}{15}$
b $\frac{4}{7} - \frac{1}{3}$	4 [$a\frac{b}{c}$] 7 [−] 1 [$a\frac{b}{c}$] 3 [=]	5 ⌋ 21	$\frac{5}{21}$

NOTE

The calcuator key sequence is exactly the same order as the written expression.

EXAMPLE 23B-4 Subtracting mixed numbers

Use your calculator to find $2\frac{1}{3} - 1\frac{3}{4}$.

Press 2 [$a\frac{b}{c}$] 1 [$a\frac{b}{c}$] 3 [−] 1 [$a\frac{b}{c}$] 3 [$a\frac{b}{c}$] 4 [=]

The display is [7 ⌋ 12] which is $\frac{7}{12}$.

EXERCISE 23.6

1 Calculate the following.

a $\frac{2}{3} + \frac{3}{4}$ b $\frac{3}{7} + \frac{5}{8}$ c $\frac{6}{7} + \frac{3}{4}$ d $\frac{3}{4} + \frac{2}{7}$

e $\frac{1}{2} - \frac{1}{3}$ f $\frac{3}{4} - \frac{2}{3}$ g $\frac{4}{5} - \frac{2}{7}$ h $\frac{11}{12} - \frac{2}{5}$

2 Calculate the following.

a $1\frac{1}{2} + 2\frac{1}{3}$ b $\frac{3}{4} + 1\frac{2}{3}$ c $2\frac{1}{3} + \frac{4}{5}$ d $1\frac{1}{2} + \frac{2}{3} + 1\frac{2}{5}$

e $1\frac{1}{2} - \frac{2}{3}$ f $1\frac{1}{3} - \frac{3}{4}$ g $2\frac{1}{2} - 1\frac{2}{3}$ h $1\frac{1}{3} + 1\frac{2}{5} - \frac{3}{4}$

23C Fractions of quantities

Often in business-type calculations we have to find a fraction of a particular quantity, such as $\frac{2}{5}$ of a profit of \$13 384 or \$5353.60

NOTE

'Of' is replaced by the multiplication sign ×.

EXAMPLE 23C-1 A fraction of a quantity

Find $\frac{3}{5}$ of \$2350.

$\frac{3}{5}$ of \$2350 $= \frac{3}{5} \times$ \$2350 = \$1410

Calculator: 3 [$a\frac{b}{c}$] 5 [×] 2350 [=]

EXAMPLE 23C-2 Fractions of two quantities

How much do you get for a share of $\frac{1}{3}$ of \$45 690 and $\frac{2}{5}$ of \$63 450?

Total profit $= \frac{1}{3}$ of \$45 690 $+ \frac{2}{5}$ of \$63 450

$= \frac{1}{3} \times$ \$45 690 $+ \frac{2}{5} \times$ \$63 450 = \$40 610

Calculator: 1 [$a\frac{b}{c}$] 3 [×] 45690 [+] 2 [$a\frac{b}{c}$] 5 [×] 63450 [=]

EXERCISE 23.7

1 Jagadec and Mahesh needed to calculate $\frac{3}{7}$ of \$800.

Jagadec pressed 3 ÷ 7 × 800 = which gave the answer \$342.857 142 9.

Mahesh pressed 3 $a\frac{b}{c}$ 7 × 800 = which gave \$342$\frac{6}{7}$.

Why is Jagadec's method more convenient?

2 Find the following.

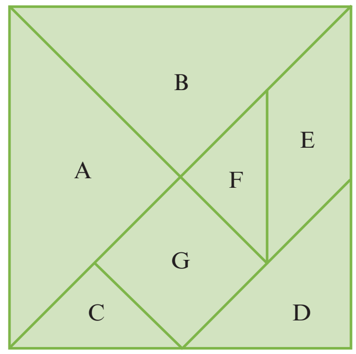
a $\frac{2}{5}$ of \$360 b $\frac{4}{7}$ of 840 t c $\frac{4}{11}$ of 3300 km

d $\frac{5}{12}$ of \$1620 e $\frac{7}{13}$ of 351 kg f $\frac{19}{23}$ of \$1288

3 The seven-piece tangram shown on the right is a Chinese puzzle that dates back to ancient times.

a What fraction is each piece of the whole?

b If the puzzle is made of a 15 cm × 15 cm piece of plastic, what is the area of each piece?



4 Wei has interests in two businesses: $\frac{1}{4}$ share in Xenon calculators and $\frac{1}{3}$ share in Frizby's Flowers. Profits to be distributed are \$56 420 from Xenon and \$38 103 from Frizby's Flowers. How much will Wei be paid in total from these two businesses?

5 Ivan and Petra invest in shares together. Ivan gets $\frac{1}{3}$ of the dividends from Sunraysia shares and $\frac{3}{4}$ of the dividend from AMP shares.

a What fraction of the dividend does Petra get for each share type?

b If the dividends paid for their investments are \$2716 for Sunraysia and \$5614 for AMP, find the amounts that Ivan and Petra receive as individuals.

6 John and Amy bought a business. John bought 4 of the 9 shares. Amy bought the remainder. John paid \$28 200 for his shares.

a How many shares did Amy buy?

b What fraction of the business did each person buy?

c How much did Amy pay for her shares?

23D Decimal numbers

The decimal point of a number is used to separate the part of the number that is greater than 1 from the part that is less than 1.

For example: 83.67

greater than 1 | less than 1

decimal point

83.67 is a shortened way of writing $80 + 3 + \frac{6}{10} + \frac{7}{100}$ or $83 + \frac{67}{100}$.

EXAMPLE 23D-1 Number values

Write the value of the '6' in the following numbers.

a 32 614.8 **b** 29.0651

a The value of 6 in 32 614.8 is 600.

32 614.8

100s 10s 1s

b The value of 6 in 29.0651 is $\frac{6}{100}$.

29.0651

$\frac{1}{10}$s $\frac{1}{100}$s

CONVERTING FRACTIONS TO DECIMALS

EXAMPLE 23D-2 Converting fractions to decimals

Use your calculator to convert these fractions to decimals.

a $\frac{7}{16}$ **b** $\frac{167}{11}$

a $\frac{7}{16} = 0.4375$

Calculator

7 ÷ 16 =

b $\frac{167}{11} = 15.181\,818\ldots$

167 ÷ 11 =

ROUNDING NUMBERS

We round numbers whenever we need fewer digits than are given by the calculator.

For example, we would say that the distance between two towns is 59 km rather than the actual distance of 58.367 km. The degree to which we round a number depends on the accuracy that we believe is appropriate.

NOTE

To round to a certain number of decimals or figures, look at the next digit:

- if it is 0, 1, 2, 3 or 4, do not change the digit being considered
- if it is 5, 6, 7, 8 or 9, increase the considered digit by one.

EXAMPLE 23D-3 Rounding numbers

Round 76 382 to the nearest:

a 10 **b** 100 **c** 1000

a 76 382 ≈ 76 380 to the nearest 10, as 82 is closer to 80 than to 90.

b 76 382 ≈ 76 400 to the nearest 100, as 382 is closer to 400 than to 300.

c 76 382 ≈ 76 000 to the nearest 1000, as 6382 is closer to 6000 than to 7000.

NOTE

The symbol ≈ means 'is approximately equal to'.

Rounding decimal numbers

Consider rounding 37.1485 to 1 decimal place. The following steps are recommended if you find rounding difficult.

Step 1: Circle the digit in the place to be rounded.
37.(1)485

Step 2: Look at the next digit to the right of the circled digit and underline it.
37.(1)<u>4</u>85

Step 3: Remove all digits to the right of the underlined digit.
37.(1)<u>4</u>

Step 4: If the underlined digit is 0, 1, 2, 3 or 4, remove it.
37.1
If the underlined digit is 5, 6, 7, 8 or 9, remove it and add 1 to the circled digit.
37.2

EXAMPLE 23D-4 Rounding decimal numbers

Write 365.417 correct to:

a 1 decimal place **b** 2 decimal places.

a To 1 decimal place:
365.(4)<u>1</u>7 ≈ 365.4 — The underlined digit is 1, so remove the right digits.

b To 2 decimal places:
365.4(1)<u>7</u> ≈ 365.42 — The underlined digit is 7, so remove it and add 1 to the circled digit (1 + 1 = 2).

EXERCISE 23.8

1 Write the value of the 7 in each of the following numbers.

a 37 142 b 2 700 631 c 10.704 d 8.0072 e 0.038 711

2 Convert the following to decimal numbers.

a $\frac{3}{5}$ b $\frac{7}{8}$ c $1\frac{2}{5}$ d $3\frac{1}{3}$

e $2\frac{2}{3}$ f $2\frac{5}{9}$ g $\frac{7}{12}$ h $\frac{11}{6}$

i $\frac{17}{17}$ j $\frac{113}{9}$ k $\frac{257}{11}$ l $\frac{437}{99}$

3 Round the following numbers to the nearest 10.

a 356 b 4629 c 53 924 d 267 189

4 Round the following numbers to the nearest 100.

a 356 b 4629 c 53 924 d 267 189

5 Round the following numbers to the nearest 1000.

a 1382 b 54 987 c 64 299 d 398 989

6 Round the numbers below to:

i 1 decimal place ii 2 decimal places iii 3 decimal places.

a 36.3824 b 0.192 645 c 119.4367

d 11.9909 e 51.797 92 f 65 656.6566

23E Rounding with a calculator

FIXING THE NUMBER OF DECIMAL PLACES

If you have to do several calculations and need to give your answers to a certain (fixed) number of decimal places each time, your can set up your calculator to automatically do this for you by using the [FIX] function.

On the TI-30X II calculator, press 2nd then use the ► and ◄ keys to select the required number of decimal places. Press = to fix the number of decimal places.

Other calculators may use the following procedure: press 2nd or MODE and repeat the key until FIX appears in the display. Choose FIX and the number of decimal places.

Note: If you choose 3 decimal places, 0.000 appears in the display.

NOTE
If your calculator does not give you this result, see your teacher.

Unfixing

To unfix the number of decimal places on the TI-30X II, press 2nd and use the ► or ◄ keys to select F. Press = to unfix the decimal places.

On other calculators, repeat the 2nd or MODE until the normal mode is obtained.

EXAMPLE 23E-1 Rounding with a calculator

a Calculate the value of 38.26×53.07 to 2 decimal places.
b Calculate the value of $8.179 \div 0.0138$ to 1 decimal place.

a First get [FIX] on your calculator, then select 2.
Press 38.26 [×] 53.07 [=].
The answer 2030.46 will appear in the display.

b First get [FIX] on your calculator, then select 1.
Press 8.179 [÷] 0.0138 [=].
The answer 592.7 will appear in the display.

EXERCISE 23.9

1 When denominators are other than a single number we must take great care.

To demonstrate this, consider the following attempts at finding $\frac{20}{2 \times 5}$ and $\frac{36}{12 - 3}$.

For $\frac{20}{2 \times 5}$, Jon pressed 20 [÷] 2 [×] 5 [=].

For $\frac{20}{2 \times 5}$, Frank pressed 20 [÷] [(] 2 [×] 5 [)] [=].

For $\frac{36}{12 - 3}$, Sarah pressed 36 [÷] 12 [−] 3 [=].

For $\frac{36}{12 - 3}$, Melanie pressed 36 [÷] [(] 12 [−] 3 [)] [=].

a Without using a calculator, find the values of $\frac{20}{2 \times 5}$ and $\frac{36}{12 - 3}$.
b Is Jon or Frank correct?
c Is Sarah or Melanie correct?
d Explain in your own words what you have learnt from parts **a**, **b** and **c**.

2 Give the order the calculator keys need to be pressed to calculate the following.

a 3.67×11.4 **b** $8.704 + \frac{6.93}{0.74}$ **c** $\frac{8.704 + 6.93}{0.74}$ **d** $\frac{0.74}{8.704 + 6.93}$

3 Set your calculator to give answers to 1 decimal place and then find:

a 3.85×4.62 **b** $63.82 \div 7.45$ **c** $3.675 + \frac{11.291}{5.67}$
d $\frac{3.675 + 11.291}{5.67}$ **e** $\frac{17.65}{3 - 0.271}$ **f** $\pi \times (5.67)^2$

4 Set your calculator to give answers to 2 decimal places and then find:

a 2.17×18.29 **b** $\frac{21.7}{18.29}$ **c** $(13.29)^2 \times 15.67$
d $\frac{(16.2)^2}{5.71 \times 3.68}$ **e** $\frac{1}{3} \times \pi \times (6.92)^2$ **f** $\frac{\sqrt{11}}{2.77}$

5 Set your calculator to give answers to 3 decimal places and then find:

a $12.8 \times \sqrt[3]{8.7}$ **b** $\dfrac{\sqrt{7.621}}{4}$ **c** $5107 \div (3.92)^4$

d $\pi \times 3.82 \times 9.76$ **e** $\dfrac{(6.24)^4}{5.7 - 3.821}$ **f** $\frac{4}{3} \times \pi \times (11.73)^3$

6 Solve the following problems.

a A petrol tank holds exactly 64 L. Find the cost of filling an empty tank if petrol costs \$1.32/L.

b A racewalker completes 20 km in 1 h 42 min.

i Write 1 h 42 min as a fraction (in hours).

ii Find the average speed of the runner.

c A courier receives 37c/km. What does the courier receive for a trip of 1079 km?

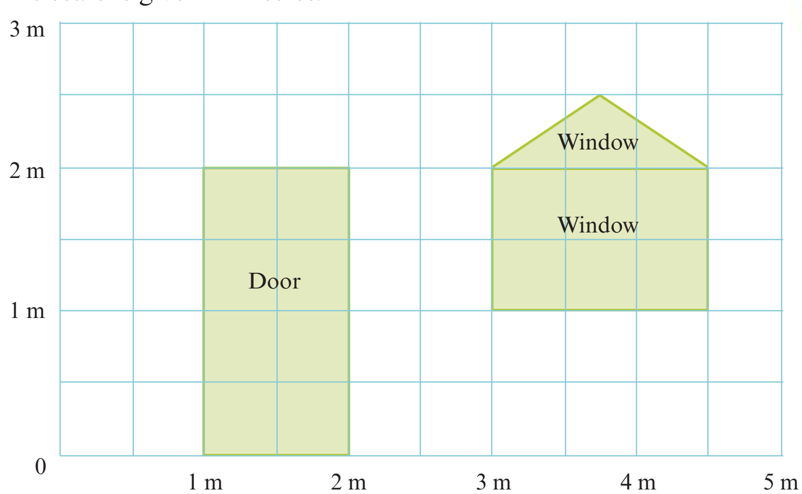
d Gold is valued at \$37.62/g. Find the value of a 3.462 kg ingot of gold.

e A swimmer averages 29.43 s for each lap of a 50 m pool. Find the total time taken, in minutes and seconds, by the swimmer if she swam 1500 m.

> **NOTE**
> $\text{Average speed} = \dfrac{\text{total distance travelled}}{\text{total time taken}}$

f A gold nugget sells for \$43 607 and weighs 1.389 kg. How much is 1 g of gold worth on the day of sale?

g Below is a diagram showing the brick wall of a house. The scale is given in metres.



i What fraction of the total wall area is glass (window)?

ii What fraction of the total wall area is wood (door)?

iii What fraction of the total wall area is brick (wall)?

23F Estimation

NOTE

Remember that estimation is quickly finding a number that is close enough to the right answer for the purpose you need it for. It is an application of the rounding skills that you practised in section 23E.

In everyday life, estimation is an important skill. From letting people know roughly when you are going to arrive at a restuarant for dinner to knowing if the restuarant has accidentally overcharged you in the final bill – estimating can quickly save you both frustration and money!

EXAMPLE 23F-1 Estimation in real life

You are buying three items. The first is \$13.35, the second is \$3.50 and the third is \$9.17. The shop assistant asks for \$27.82. Is this right?

We round each price up or down to the nearest dollar:

Step 1: 13.35 rounds downwards to 13
3.50 rounds upwards to 4
9.17 rounds downwards to 9

Step 2: You can then add in your head:
$13 + 4 + 9 = 26$

Therefore, you should be paying close to \$26 and not \$27.82. The sales assistant probably typed \$5.30 instead of \$3.50 in this case.

Estimation is also a key to getting good marks on a test. If you know roughly what the answer should be by making a quick estimate, you can check if you have mistyped a number or sum into the calculator.

EXAMPLE 23F-2 Estimation for improving exam results

You use your calculator to multiply 102 × 34 and get 408. Is this right?

To estimate the answer, follow these steps:
Step 1: Round 102 to 100 and 34 to 30.
Step 2: You can then work it out in your head: 100 × 30 = 3000.
As both rounded number are slightly more than 100 and 30, the actual answer is going to be slightly more than 3000: you know instantly that 408 is WRONG.
The actual answer is 3468.

EXERCISE 23.10

1 You buy six items at $3.95 and are asked for $27.65. Is this correct? If not, what is the most likely mistake the sales assistant made?

2 a You get 127.05 when you calculate 10.5 × 12.1. Without using a calculator, estimate if this answer is likely.
b Do the calculation with a calculator and see if this answer is correct.

3 Complete the following table from a selection of answers from an exam.

Sum	Your estimate	Candidate's answer	Is the answer likely to be correct? (Yes/No)
80 ÷ 7.9		10.13	
9.9 × 28		2772	
$\frac{53 \times 5}{25}$		1	
6.5 + 5 × 4		46	

4 You want to plant a row of trees. The row is 58 m long. The trees should be approximately 6 m apart. How many trees do you need to buy?

5 You are travelling in heavy traffic and are running late. You have 4.8 km to drive but have taken 4 min to travel around 1 km. You ask your passenger to call your friends at the restaurant and let them know you are running late. What remaining travel time estimate will you give them?

CHAPTER 24

Percentage, ratios and rates

24A Converting to percentages

Percentages are used every day in business, statistics, sport and many other contexts. Percentages express amounts as parts of 100. Per cent actually means 'per 100'. Thus:

- 1 per cent (1%) means 1 per 100
- 100 per cent (100%) means 100 out of 100.

For example, 23% means 23 parts out of every 100.

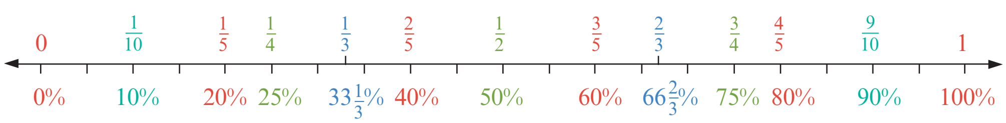
This can be expressed as $23\% = \frac{23}{100}$

Percentages are used in newspapers, magazines, advertising and in shop sales all the time, so it's important that you understand exactly what they mean and how to use them. In particular, you need to know how fractions and decimals convert to percentages.

The following table shows conversions of some simple fractions to decimals and percentages. These are very common and are worth committing to memory.

Half	Thirds	Quarters	Fifths
$\frac{1}{2} = 0.5 = 50\%$	$\frac{1}{3} = 0.33 = 33\frac{1}{3}\%$	$\frac{1}{4} = 0.25 = 25\%$	$\frac{1}{5} = 0.2 = 20\%$
	$\frac{2}{3} = 0.66 = 66\frac{2}{3}\%$	$\frac{3}{4} = 0.75 = 75\%$	$\frac{2}{5} = 0.4 = 40\%$
			$\frac{3}{5} = 0.6 = 60\%$
			$\frac{4}{5} = 0.8 = 80\%$

A number line can also be used to represent fractions and percentages.



To convert a fraction or decimal to a percentage we multiply by 100%. For example, to convert 2.5 to a percentage is $2.5 \times 100\% = 250\%$.

EXAMPLE 24A-1 Converting to percentages

Convert the following amounts to percentages.

a 0.358 **b** $\frac{11}{16}$

a $0.358 = 0.358 \times 100\%$

$= 35.8\%$

b $\frac{11}{16} = \frac{11}{16} \times 100\%$

$= 68.75\%$

Calculator

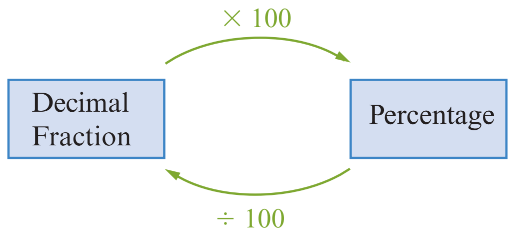
0.358 × 100 =

11 ÷ 16 × 100 =

NOTE

Rule of thumb: Percentages greater than 100 will give a final amount greater than the original. Percentages less than 100 will give a final amount less than the original.

This chart may help when converting fractions and decimals to percentages.



EXAMPLE 24A-2 Converting from percentages

a Convert 57.82% to a decimal number.

b Convert 6% to a fraction.

a $57.82\% = \frac{57.82}{100}$

$= 0.5782$

b $6\% = \frac{6}{100}$

$= \frac{3}{50}$

EXAMPLE 24A-3 Converting using a calculator

Convert $\frac{7}{11}$ to a percentage correct to 2 decimal places.

First get [FIX] on your calculator, then select 2.

Press 7 [÷] 11 [×] 100 [=].

The answer 63.64 will appear in the display.

EXERCISE 24.1

1 Convert the following to percentages of 100.

a 0.78 **b** 0.367 **c** 0.907 **d** 2.0135 **e** 1.4283

2 Convert the following to percentages.

a $\frac{5}{8}$ **b** $\frac{7}{8}$ **c** $\frac{3}{16}$ **d** $\frac{19}{25}$ **e** $\frac{3}{32}$

3 Convert the following percentages of 100 to decimals.

a 61.8% **b** 29.67% **c** 2.84% **d** 115% **e** 108.9%

4 Convert the following fractions to percentages of 1 correct to 2 decimal places.

a $\frac{7}{11}$ **b** $\frac{8}{15}$ **c** $\frac{3}{17}$ **d** $\frac{17}{31}$ **e** $\frac{27}{37}$

5 Convert the following to percentages correct to 3 decimal places.

a $\frac{3}{11}$ **b** $\frac{8}{13}$ **c** $\frac{4}{29}$ **d** $\frac{18}{23}$ **e** $\frac{28}{37}$

24B Percentage of a quantity

To find the percentage of a quantity, such as 8% of 245 kg:

- Replace the word 'of' by ×.
- Write the percentage as a decimal (for example 8% is 0.08).

EXAMPLE 24B-1 Percentage of a quantity

a Determine 8% of 245 kg. **b** Determine 14% of $325.

a $8\% \text{ of } 245 \text{ kg} = 0.08 \times 245$
$= 19.6 \text{ kg}$

Calculator

8 [÷] 100 [×] 245 [=]

b $14\% \text{ of } \$325 = 0.14 \times \$325$
$= 45.5$

14 [÷] 100 [×] 325 [=]

EXERCISE 24.2

1 Determine the following percentages.

a 12% of $300 **b** 27% of 11 t **c** 76% of 3400 m
d 45% of 630 L **e** 63% of 450 kg **f** 11.8% of $15 360

2 Abel must pay 35% of $45 000 to buy into a business venture. How much does he need to pay?

3 Jason's water ration is 24% of 2500 kL. How much water is he able to use?

4 Amanda is investing in two wineries. She wishes to buy 1.5% of Shiro Winery and 2.3% of Rieschard. The managements of Shiro and Rieschard are selling the wineries for $785 000 and $1.3 million respectively. What money must Amy raise to buy into these wineries?

24C Percentage change

Sometimes we need to find the percentage change when something has been increased or decreased by a percentage, such as the change in value of an object.

- To increase a quantity by 15%, we multiply by (100 + 15)%.
- To decrease a quantity by 15%, we multiply by (100 − 15)%.

EXAMPLE 24C-1 Percentage increase

Increase 85 kg by 20%.

New value = 120% of 85 kg	100 + 20 = 120
= 1.20 × 85 kg	Divide 120 by 100 to convert to a decimal.
= 102 kg	

Calculator: 120 [÷] 100 [×] 85 [=]

EXAMPLE 24C-2 Percentage decrease

Decrease $2500 by 2.3%.

New value = 97.7% of $2500	100 − 2.3 = 97.7
= 0.977 × $2500	Divide 97.7 by 100 to convert to a decimal.
= $2442.50	

Calculator: 97.7 [÷] 100 [×] 2500 [=]

EXERCISE 24.3

1 What percentage would you multiply by in order to increase a quantity by:
a 10% b 25% c 8% d 2.3% e 11.8%?

2 What percentage would you multiply by in order to decrease a quantity by:
a 10% b 25% c 8% d 2.3% e 11.8%?

3 Find the percentage increase.
a $3400 by 15% b 720 m by 38% c 3500 t by 2%

4 Find the percentage decrease.
a $3400 by 15% b 3.29 km by 12% c 625 km by 20%

Recall that:

- $\% \text{ increase} = \dfrac{\text{increase}}{\text{original value}} \times 100\%$
- $\% \text{ decrease} = \dfrac{\text{decrease}}{\text{original value}} \times 100\%$

EXAMPLE 24C-3 Finding the percentage increase

A menswear store increases the price of suits from \$160 to \$185. What is their percentage increase in price correct to 1 decimal place?

$$\begin{aligned}\text{Increase} &= \text{final value} - \text{original value}\\ &= \$185 - \$160 = \$25\\ \%\text{ increase} &= \frac{\text{increase}}{\text{original value}} \times 100\%\\ &= \frac{25}{160} \times 100\%\\ &= 15.6\%\end{aligned}$$

EXAMPLE 24C-4 Finding the percentage decrease

Due to several factors, the profits of a café in the Melbourne CBD fell from \$73 842 in 2015 to \$56 597 in 2016. Find the percentage decrease in profits over this period correct to 1 decimal place.

$$\begin{aligned}\text{Decrease} &= \text{original value} - \text{final value}\\ &= \$73\,842 - \$56\,597 = \$17\,245\\ \%\text{ decrease} &= \frac{\text{decrease}}{\text{original value}} \times 100\%\\ &= \frac{17\,245}{73\,842} \times 100\%\\ &= 23.4\%\end{aligned}$$

EXERCISE 24.4

1 The manager of a timber supply business increases all timber prices by 5% due to rises in business costs. Adjust the following prices per metre to the nearest 5 cents.

a Tasmanian oak 150 × 50 at \$12.60

b Pine 100 × 100 at \$16.75

c Mahogany 200 × 75 at \$25.10

d Teak 300 × 100 at \$50.25

2 You have been asked by your boss to adjust the prices of electrical goods for the '15% off everything' sale starting tomorrow. Adjust the following prices accordingly.

a Smart TV set at $999 **b** Blu-ray player at $150 **c** Toaster at $50

d Hi-fi system at $2460 **e** Refrigerator at $1140 **f** Freezer at $745

3 Determine the percentage increase when:

a $63 becomes $90 **b** 350 mL becomes 1.4 L

c 3.6 km becomes 4.5 km **d** 140 kg becomes 196 kg.

4 Determine the percentage decrease when:

a $3600 falls to $2700 **b** 7.5 t falls to 4500 kg

c 3.2 km falls to 2500 m **d** $37 500 falls to $24 000.

5 Determine the percentage change (increase or decrease) when:

a your profit of $31 766 in 2014 became $46 707 in 2015

b you sell dresses for $89 each instead of $99 each

c you sell software for $34.95 instead of $29.95

d your supplier of shoes sells them to you at $132 a pair instead of $149 a pair.

6 At an 'At least 25% off everything sale', the following price tags were observed.

i

ii

iii

a Check that the store manager is keeping to the advertising commitment for the given items.

b If a customer bought one of each of these items, what would be their overall percentage saving on the original marked prices?

c Discussion: Is the store manager guilty of false advertising?

7 Solve the following problems.

a In a certain school, 35% of the 840 students own computers. How many students own computers?

b In its first month, a piglet increased in weight from 3.4 kg to 7.8 kg. What was the percentage increase?

c A rainwater tank is 62% full and has a capacity of 75 kL. Determine the volume of water in the tank.

d When full, the MCG has a seating capacity of 105 000 people. If the stadium is 67% full, how many people are present?

24D Ratios

A ratio is a way of comparing like quantities. For example, if Allen has three tennis balls and Jake has one tennis ball, we can write this as a ratio:

Allen : Jake = 3 : 1

If we double the number of tennis balls for each person so that Allen now has six and Jake two, we could still write this as a ratio of 3 : 1 because for every one ball Jake has, Allen has three. Although it is correct to say 6 : 2, we can use the simpler ratio of 3 : 1.

EXAMPLE 24D-1 Writing ratios

Convert the statement 'Sandy has $9 and Leslie has $15' to a ratio in simplest form.

Sandy : Leslie = 9 : 15 — Divide each number by 3.
= 3 : 5

USING RATIOS

Ratios are used constantly in industry. Metals such as brass are a combination of other metals (copper and zinc) in a particular ratio. Paint colours are made by mixing tinters in a given ratio to the base colour.

NOTE
Notice that the numbers must be written in the same order as the words.

EXAMPLE 24D-2 Using ratios

A certain paint colour is made by combining three tints in the ratio:

Tint 1 1 part
Tint 2 2 parts
Tint 3 4 parts

or Tint 1 : Tint 2 : Tint 3 = 1 : 2 : 4.

If 70 mL of paint is needed in total, how much of each tint is used?

Total tint parts = 1 + 2 + 4 = 7

7 parts is 70 mL, so 1 part is 10 mL.

The amount of tint needed is 10 mL of Tint 1, 20 mL of Tint 2 and 40 mL of Tint 3.

SIMPLIFYING RATIOS

If Amid and Poh weigh 90 kg and 60 kg respectively, the ratio of their weights is 90 : 60. We can omit the units as they are the same in both cases.

Ratios can be converted to simplest form by multiplying or dividing each side in the ratio by the same number. The ratio 90 : 60 can be written in its simplest form as 3 : 2.

$$\begin{aligned}90 : 60 &= 90 \div 30 : 60 \div 30\\ &= 3 : 2\end{aligned}$$

NOTE
You can use your calculator to simplify ratios.

The ratio $1\frac{1}{3} : 2$ can be simplified by multiplying both numbers by 3 to obtain:

$$\begin{aligned}1\tfrac{1}{3} : 2 &= 1\tfrac{1}{3} \times 3 : 2 \times 3\\ &= 4 : 6\\ &= 2 : 3 &&\text{Dividing each number by 2.}\end{aligned}$$

EXAMPLE 24D-3 Simplifying ratios

Write 750 mL : 2 L in simplest form.

NOTE
Each number must be in the same unit before you can simplify the ratio.

$$\begin{aligned}750\text{ mL} : 2\text{L} &= 750\text{ mL} : 2000\text{ mL}\\ &= 750 : 2000\\ &= 3 : 8\end{aligned}$$

Calculator

750 [$a\frac{b}{c}$] 2000 [=] gives [3 ⌋ 8]

RATIOS AS PARTS OF A WHOLE

In the ratio $A : B = 3 : 7$, A has 3 parts and B has 7 parts, making 10 parts in total. If we wish to divide an amount of money between persons A and B:

$$A \text{ gets } \frac{3}{10} \text{ and } B \text{ gets } \frac{7}{10}$$

EXAMPLE 24D-4 Parts of a whole

Two business partners invest in their company in the ratio of 7 : 8. The business makes \$90 000 profit in one year. Divide the profit according to the ratio 7 : 8.

For the ratio 7 : 8 there are 7 + 8 = 15 parts.

$$\begin{aligned}\text{First share} &= \frac{7}{15} \text{ of } \$90\,000\\ &= \frac{7}{15} \times \$90\,000\\ &= \$42\,000\end{aligned}$$

$$\begin{aligned}\text{Second share} &= \frac{8}{15} \text{ of } \$90\,000\\ &= \frac{8}{15} \times \$90\,000\\ &= \$48\,000\end{aligned}$$

Check: \$42 000 + \$48 000 = \$90 000

EXERCISE 24.5

NOTE

Remember to convert to the same units if necessary.

1 Write the following as ratio statements in simplest form.
- a In a class, nine people play basketball for every three who play hockey.
- b Janice is three times as old as her daughter Briony.
- c Sasha is 150 cm tall whereas Ky is 1.8 m tall.
- d The weight of a fully grown cow is 520 kg whereas a horse's weight is 680 kg.
- e A netball costs \$28 while a football costs \$48.

2 Find, in simplest form, the ratio of:
- a \$1.80 to 45 cents
- b 3200 m to 4 km
- c 900 g to 1.2 kg
- d 3 weeks to 14 days
- e 75 cm to 2.5 m
- f 85 min to 2 h

3 Divide the following into the ratio amounts.
- a \$34 000 in the ratio 6 : 11
- b 720 kg in the ratio 3 : 5
- c 840 km in the ratio 5 : 7
- d 48 000 L in the ratio 5 : 11
- e 162.5 t in the ratio 2 : 3
- f \$360 000 in the ratio 2 : 7

4 Jason wishes to make 600 mL of detergent from detergent concentrate. Water is added to the concentrate in the ratio 5 : 1. How much concentrate is required?

5 Parents leave \$324 000 to their children in the ratio of 2 : 3 : 4. How much does each child receive?

6 In a club of 120 players, the ratio of basketballers to netballers is 5 : 7. Assuming no one plays both sports, how many play each sport?

7 Jarvis and Danuta buy a music business for \$480 000. If they invest in the business in the ratio of 11 : 5 and share the work and responsibilities in the same ratio, how much does each pay?

8 To tile the floor of the main foyer at Hewson High in a particular pattern, white, grey and black tiles are needed in the ratio of 4 : 3 : 2. If 480 grey tiles are needed, how many white and black tiles must be purchased?

9 A chef mixes sultanas, flour, sugar and egg white in the ratio of 4 : 6 : 1 : 2 (by weight) to make his special dessert. If the egg white weighs 100 g, what is the weight of each of the other ingredients?

24E Rates

A rate is a comparison between quantities of different kinds. The most commonly used rate is speed, where:

$$\text{Average speed} = \frac{\text{distance travelled}}{\text{time taken}}$$

That is, speed is a rate that compares two quantities – distance travelled and time taken. For example, if a car travels 380 km in 5 h:

$$\text{Average speed} = \frac{380\text{ km}}{5\text{ h}} = 76\text{ km/h}$$

which means that in 1 hour, the car travels 76 km.

Water usage is another rate commonly used. A rate of 250 kL/h means that 250 kL of water is used in 1 hour.

NOTE
1 kL = 1 kilolitre
= 1000 L

EXAMPLE 24E-1 Rate of flow

Water flows out of a tank at a rate of 750 L every 15 min. What is the rate in L/min?

$$\begin{aligned}\text{Rate} &= \frac{750\text{ L}}{15\text{ min}}\\ &= 50\text{ L/min}\end{aligned}$$

NOTE
L/min is litres per minute or litres/minutes.

EXAMPLE 24E-2 Rate of growth

The weight of a baby saltwater crocodile increases at 0.4 kg per month. What will be the weight increase after 14 months?

$$\begin{aligned}\text{Weight increase} &= \text{rate} \times \text{number of months}\\ &= \frac{0.4\text{ kg}}{\text{month}} \times 14\text{ months}\\ &= 5.6\text{ kg}\end{aligned}$$

EXAMPLE 24E-3 Finding rates

Given that 1 m/s = 3.6 km/h, find:

a 80 m/s in km/h **b** 100 km/h in m/s

a
$$\begin{aligned}1\text{ m/s} &= 3.6\text{ km/h}\\ 80\text{ m/s} &= 3.6 \times 80\text{ km/h}\\ &= 288\text{ km/h}\end{aligned}$$

b
$$\begin{aligned}1\text{ m/s} &= 3.6\text{ km/h}\\ 1\text{ km/h} &= \frac{1}{3.6}\text{ m/s}\\ &\approx 0.2778\text{ m/s}\\ 100\text{ km/h} &\approx 0.2778\text{ m/s} \times 100\\ &\approx 27.78\text{ m/s}\end{aligned}$$

NOTE
$\approx$ is a symbol that means approximately equal to.

EXERCISE 24.6

1 Express each of the following as a rate.

a 780 km is travelled in 13 h.

b 920 L of water flow every 4 min.

c 320 fish are caught in 4 h.

d 4.8 t of grain is reaped in 6 h.

2 a If water flows at a rate of 250 kL/week, how much water flows in:

i 1 week? ii 13 weeks? iii 2 days?

b The weight of pigs increases at a rate of 4.2 kg/month. What is the weight increase in:

i 1 month?

ii 4 months?

iii 10 days?

NOTE
Assume 1 month is 30 days.

3 Solve the following problems.

a A car travels at 80 km/h for 4.5 h. How far does it travel?

b Water runs out of a bath at a rate of 120 L/min. How long would it take to drain a bath containing 400 L of water?

4 a Show that 1 kg/day ≈ 41.7 g/h.

b Use the answer to part **a** to convert:

i 20 kg/day to g/h

ii 60 g/h to kg/day

NOTE
1 day = 24 h
1 kg = 1000 g

5 Flow rates can be critical for operating irrigation systems on large farms. If a farmer sets her irrigation sprinklers to deliver 10 000 L/h evenly over 1 ha, what rainfall per hour would this represent? That is, how deep is the water across the 1 ha?

NOTE
1 ha = 10 000 m^2

CHAPTER 25
Measurement and units

25A Units of measurement

From early history, humans have found the need to measure things such as the distance between two towns, the amount of grain produced, the weight of rare metals, the time of the day and so on.

UNITS OF MEASURE IN THE ANCIENT WORLD

In ancient times, the body ruled when it came to measuring. The length of a foot, the width of a finger and the distance of a step were all accepted units of measurement.

- From around 2700 BCE, the ancient Egyptians used the cubit (the length of the forearm from the elbow to the tip of the middle finger) to measure distance.
- From around 27 BCE, the Romans used the pace (the length of a footstep) to measure distance. A Roman mile was equal to 100 paces.

When it came to measuring units of weight, ancient civilisations invented the first scales.

- From around 2000 BCE, civilisations in the Indus River Valley (modern-day Pakistan) used weighing scales to measure amounts of precious metals such as gold. Those first weighing scales were actually balances, using two plates attached to an overhead beam fixed to a pole. Things were measured by placing the object being weighed on one plate and a weight-setting stone on the other, and adjusted until equilibrium was reached.

UNITS OF MEASURE IN AUSTRALIA

From the arrival of European settlers in Australia in 1788 to as recently as the 1970s, the official units of measurement in Australia were based on the British imperial measurement system. The most commonly used units of were:

- tons, pounds and ounces for weight
- miles, yards, feet and inches for length
- fluid ounces, pints and gallons for volume.

These units of measure were used in many parts of the world for centuries, especially in those countries that were part of the British Empire. With the increase in international trade during the 20th century, the need for a common measurement system became more important. Australia began the change in 1970 by adopting the worldwide standard of the metric system for most measurement units.

NOTE

The Système International (SI) is an internationally adopted metric system based on the metre for length, the kilogram for mass and the second for time.

Metric system

The metric system was developed in France in 1789 and uses the decimal system (base 10) to move from one unit to another. It has three base units that are in frequent use.

Basic metric units

Base unit	Abbreviation	Used for measuring
metre	m	length
gram	g	mass
second	s	time

There are a number of other units that are commonly used.

NOTE

Be careful to use capital letters correctly in abbreviations. For example, the abbreviation for joules is J not j.

Other metric units

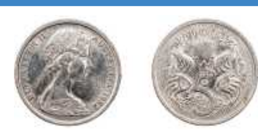
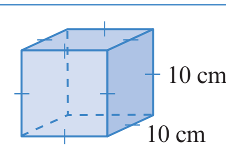
Unit	Abbreviation	Used for measuring
litre	L	capacity
tonne	t	heavy masses
square metre	m^2	area
cubic metre	m^3	volume
metres per second	m/s	speed (velocity)
newton	N	force
joule	J	energy
watt	W	power

This table gives examples to help you visualise some common units of measurement.

NOTE

Be careful with your spelling of metre and meter! Remember that:

- a metre is a unit of length
- a meter is an instrument for measuring quantities.

Unit	Description	Image
1 mm	The thickness of a 5-cent coin	
1 cm	The width of a fingernail	
1 km	Twice around an AFL oval boundary	
1 g	The mass of a paper clip or $\frac{1}{5}$ of an A4 sheet of paper	
1 L	The amount of water that would fill a container with inside measurements of 10 cm by 10 cm by 10 cm	

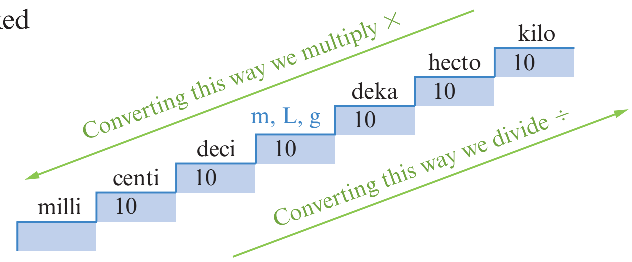
Large and small quantities

The following table gives the prefixes used to represent quantities. Some are used more commonly that others.

Prefix	Symbol	Definition
giga-	G	one billion (1 000 000 000) times the base unit
mega-	M	one million (1 000 000) times the base unit
kilo-	k	one thousand (1000) times the base unit
centi-	c	one hundredth $\left(\frac{1}{100}\right)$ of the base unit
milli-	m	one thousandth $\left(\frac{1}{1000}\right)$ of the base unit
micro-	μ	one millionth $\left(\frac{1}{1\,000\,000}\right)$ of the base unit
nano-	n	one billionth $\left(\frac{1}{1\,000\,000\,000}\right)$ of the base unit
pico-	p	one trillionth $\left(\frac{1}{1\,000\,000\,000\,000}\right)$ of the base unit

The original metric system looked like this. For example, to convert 250 kg to g we multiply by 1000.

250 kg = 250 × 1000 g

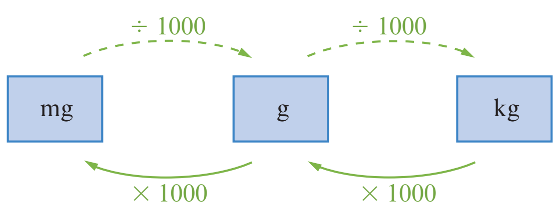


NOTE

In Australia we do not generally use deci, deka and hecto in our metric system.

The following conversion chart is for mg to kg. Remember, when converting from:

- smaller units to larger units, divide by the conversion factor
- larger units to smaller units, multiply by the conversion factor.



EXERCISE 25.1

1 Convert the following.

a 460 000 mg to g
b 5 kg to g
c 6 L to mL
d 7500 μm to mm

2 What unit would you use to measure the following?

a the weight of a large ship
b the memory of a computer
c the capacity of a dam
d the size of a pinhead

3 Fifty kilograms can be written as 50 kg in the metric system. Write abbreviated forms for:

a eighty-seven seconds
b seventy-five centimetres
c thirty-seven kilometres
d twenty-three thousand litres
e nine milligrams
f three thousand megalitres.

25B Reading instruments and meters

Over the course of an average day, most people come into contact with a range of different instruments and meters. Whether you're checking the speed of your car, reading the fuel gauge or looking at the temperature on a thermometer, it's important that you can read and accurately interpret the information given. In this section we will practise reading a number of common instruments and meters.

EXERCISE 25.2

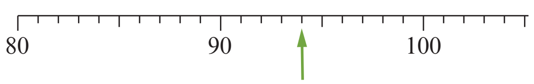
1 Find, by measurement, the length of each line interval AB in:

i millimetres **ii** centimetres.

a A ——— B **b** A ——— B **c** A ——— B

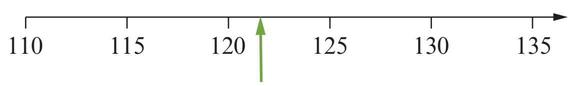
2 Estimate the readings on the following meters.

a

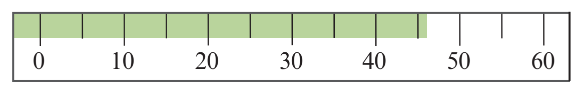


b

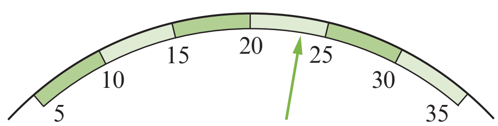
c



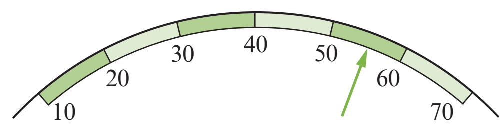
d



e

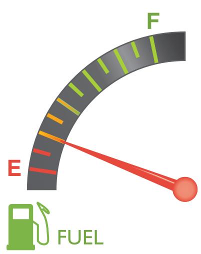


f

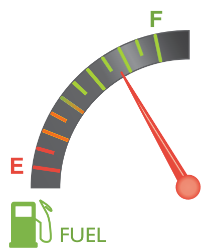


3 Estimate the fraction of fuel remaining.

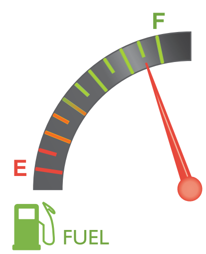
a



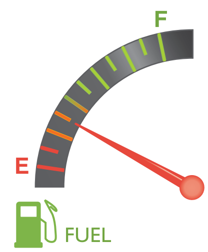
b



c



d



e

f

g

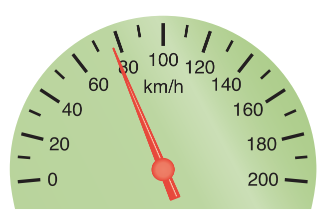
h

4 Estimate the speed from the following speedometers.

a

b

c



A tachometer is an instrument that measures the speed at which a shaft or disc in a motor rotates. This is measured in revolutions per minute (rpm).

EXAMPLE 25B-1 Measuring engine speed

This tachometer shows the speed of an engine in thousands of revolutions per minute (× 1000 rpm). Find the speed of the engine for the tachometer shown.

The meter reading is approximately $3\frac{1}{4}$.

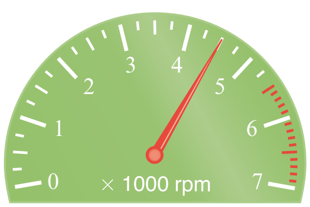
Speed of engine $= 3\frac{1}{4} \times 1000$ $\quad$ $\frac{1}{4}$ is 0.250.

$= 3250$ rpm

EXERCISE 25.3

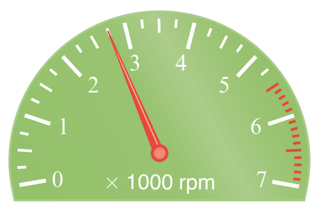
1 Find the speed of the following engines.

a



b

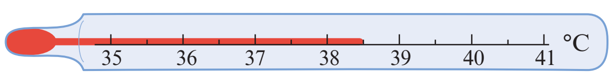
c



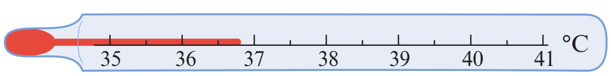
d

2 Read the following thermometers.

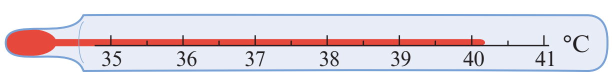
a



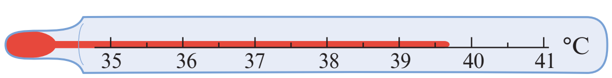
b



c



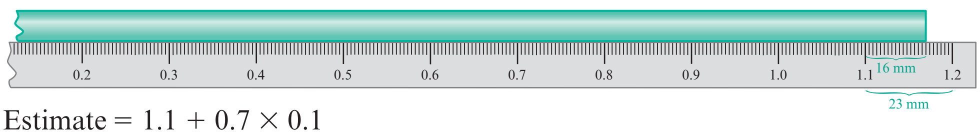
d



EXAMPLE 25B-2 Measuring without a scale

By measuring the ruler distance, estimate the length of the rod shown correct to 2 decimal places.

The rod's length is $1.1 + \frac{16}{23}$ of 0.1.

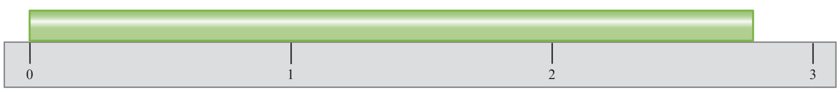


$$\begin{aligned}\text{Estimate} &= 1.1 + 0.7 \times 0.1 \\ &\approx 1.1 + 0.07 \\ &\approx 1.17\end{aligned}$$

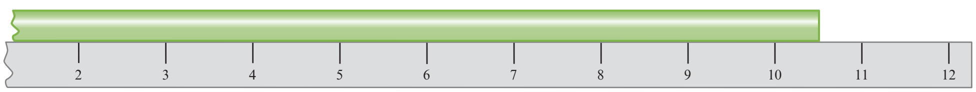
EXERCISE 25.4

1 Using the method shown in Example 25B-2, and your calculator, estimate the length of the following rods.

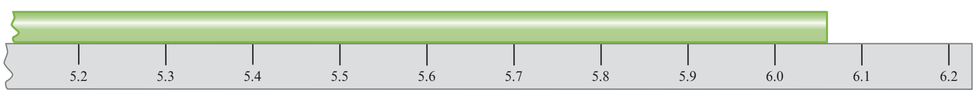
a



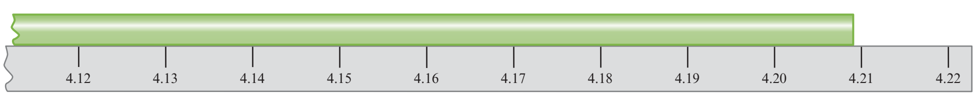
b



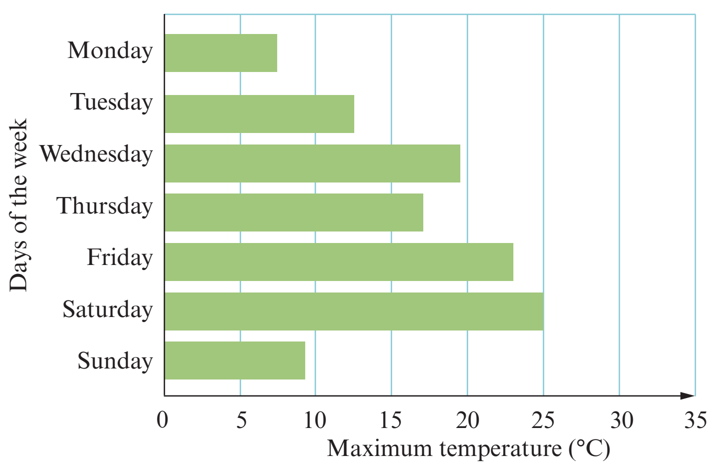
c



d



2 Using the method shown in Example 25B-2, estimate the maximum temperature for each day of the week shown in this graph.



25C Conversion of units

Many times we need to compare items so it is important to be able to convert between units. Metric units are easy to work with as they are all multiples of ten. We can convert between the various units by multiplying or dividing by the conversion factor. Remember:

- When converting from smaller units to larger units, divide by the conversion factor.
- When converting from larger units to smaller units, multiply by the conversion factor.

CONVERTING UNITS OF LENGTH

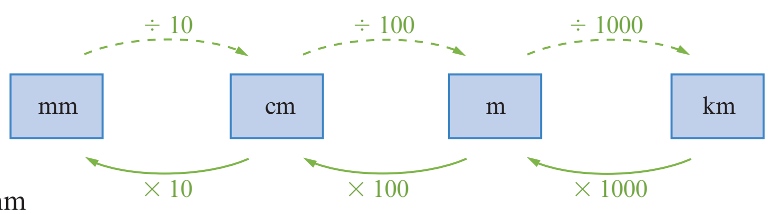
The following are the basic units of length and the relationship between the units. You may also find the conversion diagram useful when converting lengths.

$1\text{ mm} = \frac{1}{10}\text{ cm} = \frac{1}{1000}\text{ m}$

$1\text{ cm} = 10\text{ mm} = \frac{1}{100}\text{ m}$

$1\text{ m} = 100\text{ cm} = 1000\text{ mm} = \frac{1}{1000}\text{ km}$

$1\text{ km} = 1000\text{ m} = 100\ 000\text{ cm} = 1\ 000\ 000\text{ mm}$



EXAMPLE 25C-1 Converting units of length

Convert the following.

a 32.702 m to cm

b 67 140 cm to km

a Larger to smaller — *Multiply by 100.*

$$\begin{aligned} 32.702\text{ m} &= 32.702 \times 100\text{ cm} \\ &= 3270.2\text{ cm} \end{aligned}$$

b Smaller to larger — *Divide by 100 and then by 1000.*

$$\begin{aligned} 67\ 140\text{ cm} &= 67140 \div 100 \div 1000\text{ km} \\ &= 0.6714\text{ km} \end{aligned}$$

EXERCISE 25.5

1 Convert the following.

a 3250 cm to m **b** 274 mm to cm **c** 7125 m to km

d 89.3 m to cm **e** 93.21 cm to m **f** 6.315 m to km

2 Convert the following.

a 512 cm to mm **b** 2745 km to m **c** 2.839 km to cm

d 39.7 m to mm **e** 0.32 km to cm **f** 2.31 km to mm

3 **a** If there are 45 coils of fencing wire, each 275 m long, how many kilometres of wire are there in total?

b Jordan has 3.85 km of copper wire and needs to cut it into 1.5 cm lengths to be used in electric toasters. How many lengths can he make?

CONVERTING UNITS OF MASS

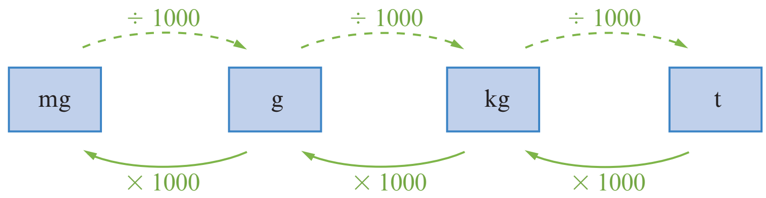
The following are the basic units of mass. You may find the conversion diagram useful when converting masses.

$1\text{ mg} = \frac{1}{1000}\text{ g}$

$1\text{ g} = 1000\text{ mg}$

$1\text{ kg} = 1000\text{ g}$

$1\text{ t} = 1000\text{ kg}$



NOTE

The abbreviation used for tonne is t.

EXAMPLE 25C-2 Converting units of mass

Convert the following.

a 2.25 g to mg

b 83 700 000 g to t

a Larger to smaller — Multiply by 1000.

$2.25\text{ g} = 2.25 \times 1000\text{ mg}$

$= 2250\text{ mg}$

b Smaller to larger — Divide by 1000 and then by 1000.

$83\,700\,000\text{ g} = 83\,700\,000 \div 1000 \div 1000\text{ t}$

$= 83.7\text{ t}$

EXERCISE 25.6

1 Convert the following.

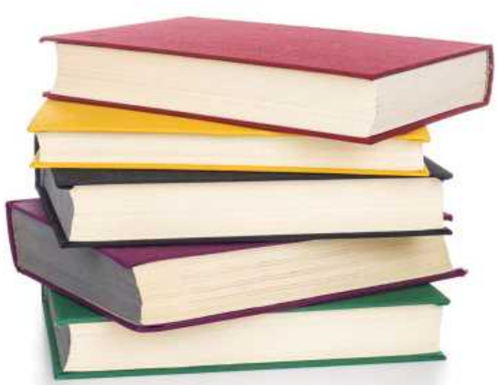
a 3200 g to kg
b 1.87 t to kg
c 47 835 mg to kg
d 4653 mg to g
e 2.83 t to g
f 0.0632 t to g
g 74 682 g to t
h 1.7 t to mg
i 91 275 g to kg

2 **a** In peppermint-flavoured sweets, 1 g of peppermint extract is used per sweet. How many sweets can be made from a drum containing 0.15 t of peppermint extract?

b A publisher produces a book weighing 856 g. There are 6000 books to be printed and transported interstate.

i How many tonnes of books are to be sent?

ii If the transport costs are $450 per tonne, what will be the total cost of sending the books?



CONVERTING UNITS OF AREA

The following are the basic units of area. You may also find the conversion diagram useful when converting areas.

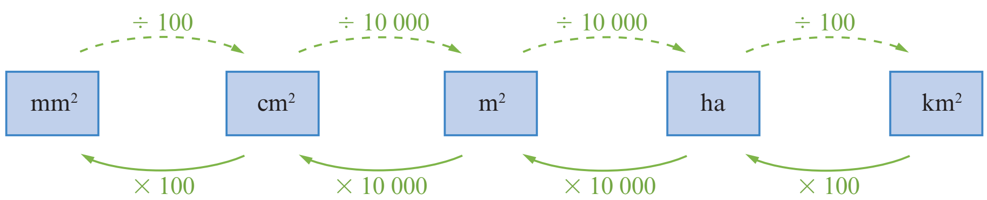
$1 \text{ mm}^2 = \frac{1}{100} \text{ cm}^2$

$1 \text{ cm}^2 = 100 \text{ mm}^2$

$1 \text{ m}^2 = 10\,000 \text{ cm}^2$

$1 \text{ ha} = 10\,000 \text{ m}^2$

$1 \text{ km}^2 = 1\,000\,000 \text{ m}^2 = 100 \text{ ha}$



EXAMPLE 25C-3 Converting units of area

Convert the following.

a $650\,000 \text{ m}^2$ to ha **b** 2.5 m^2 to mm²

a Smaller to larger

$650\,000 \text{ m}^2 = 650\,000 \div 10\,000 \text{ ha}$

$= 65 \text{ ha}$

Divide by 10 000.

b Larger to smaller

$2.5 \text{ m}^2 = 2.5 \times 10\,000 \times 100 \text{ mm}^2$

$= 2\,500\,000 \text{ mm}^2$

Multiply by 10 000 and then by 100.

EXERCISE 25.7

1 Convert the following.

a 23 mm^2 to cm² **b** 3.6 ha to m² **c** 726 cm^2 to m²

d 7.6 m^2 to mm² **e** 8530 m^2 to ha **f** 0.354 ha to cm²

g 13.54 cm^2 to mm² **h** 432 m^2 to cm² **i** $0.004\,82 \text{ m}^2$ to mm²

2 **a** I have purchased a 4.2 ha property. Council regulations allow me to have five free-range chickens for every 100 m^2. How many free-range chickens am I allowed to have?

b Sam purchased 100 m^2 of dress material, and needs to cut it into rectangles of area 2000 cm^2. This can be done without wastage. How many rectangles can Sam cut out?

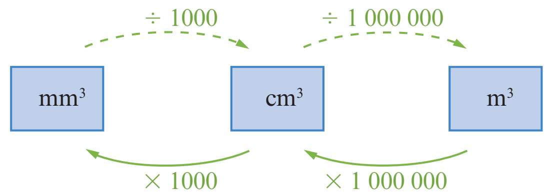
CONVERTING UNITS OF VOLUME

The following are the basic units of volume. You may also find the conversion diagram useful when converting volumes.

$1 \text{ mm}^3 = \frac{1}{1000} \text{ cm}^3$

$1 \text{ cm}^3 = 1000 \text{ mm}^3$

$1 \text{ m}^3 = 1\,000\,000 \text{ cm}^3$



EXAMPLE 25C-4 Converting units of volume

Convert the following.

a 0.137 m^3 to cm^3 **b** $8\,000\,000 \text{ cm}^3$ to m^3

a Larger to smaller — Multiply by 1 000 000.

$0.137 \text{ m}^3 = 0.137 \times 1\,000\,000 \text{ cm}^3$
$= 137\,000 \text{ cm}^3$

b Smaller to larger — Divide by 1 000 000.

$8\,000\,000 \text{ cm}^3 = 8\,000\,000 \div 1\,000\,000 \text{ m}^3$
$= 8 \text{ m}^3$

EXERCISE 25.8

1 Pablo was convinced that $1 \text{ m}^3 = 1000 \text{ cm}^3$. Sanshi told him to go back to basics. She said that 1 m^3 is the volume of a 1 m by 1 m by 1 m box and 1 m = 100 cm. Follow Sanshi's instructions and convince yourself that $1 \text{ m}^3 = 1\,000\,000 \text{ cm}^3$.

2 Convert the following.

a $39\,100\,000 \text{ cm}^3$ to m^3 **b** $469\,000 \text{ cm}^3$ to m^3 **c** 3.82 m^3 to cm^3 **d** 0.0179 m^3 to cm^3

3 **a** Thirty thousand ingots of lead, each of volume 250 cm^3, are required by a battery manufacturer. How many cubic metres of lead does the manufacturer need to purchase?

b A manufacturer of lead sinkers for fishing has 0.237 m^3 of lead. If each sinker is 5 cm^3 in volume, how many sinkers can be made?

UNITS OF CAPACITY

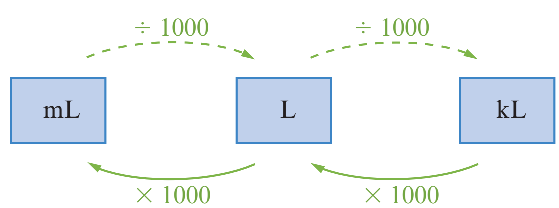
Volume refers to the amount of space occupied by an object. Capacity refers to the quantity, usually of liquid, that something can hold. Following are the basic units of capacity. You may also find the conversion diagram useful.

$1 \text{ mL} = 1 \text{ cm}^3$

$1 \text{ L} = 1000 \text{ mL} = 1000 \text{ cm}^3$

$1000 \text{ L} = 1 \text{ kL}$

$1 \text{ kL} = 1 \text{ m}^3$



EXAMPLE 25C-5 Converting units of capacity

Convert the following.

a 4.2 L to mL **b** 36 800 L to kL

a Larger to smaller — Multiply by 1000.

$4.2\text{ L} = 4.2 \times 1000\text{ mL}$
$= 4200\text{ mL}$

b Smaller to larger — Divide by 1000.

$36\,800\text{ L} = 36\,800 \div 1000\text{ kL}$
$= 36.8\text{kL}$

EXAMPLE 25C-6 Converting volume and capacity

Convert the following.

a 9.6 L to cm^3 **b** 3240 L to m^3

a Larger to smaller — Multiply by 1000.

$9.6\text{ L} = 9.6 \times 1000\text{ cm}^3$
$= 9600\text{ cm}^3$

b Smaller to larger — Divide by 1000.

$3240\text{ L} = 3240 \div 1000\text{ m}^3$
$= 3.24\text{ m}^3$

NOTE
Remember:
$1\text{ L} = 1000\text{ cm}^3$
$1\text{ m}^3 = 1000\text{ L}$

EXERCISE 25.9

1 Convert the following.

a 3.76 L to mL **b** 47 320 L to kL
c 3.5 kL to L **d** 0.423 L to mL
e 0.054 kL to mL **f** 58 340 mL to kL

2 **a** A chemist makes up 20 mL bottles of eye drops from a drum of eye-drop solution. If the full drum has an internal volume of 0.0275 kL, how many bottles of eye-drop solution can the chemist fill?

b 1000 dozen bottles of wine, each of capacity 750 mL, need to be filled from wine tanks of capacity 1000 L for export. How many tanks are needed?

3 Convert the following.

a 83 kL to m^3 **b** 3200 mL to cm^3 **c** 2300 cm^3 to L
d 7154 m^3 to L **e** 0.46 kL to m^3 **f** 4.6 kL to cm^3

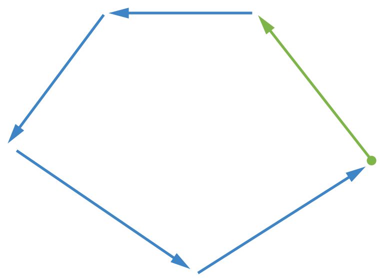
4 **a** What is the capacity of a bottle of volume 25 cm^3? Answer in mL.
b Find the volume of a tank if its capacity is 32 kL. Answer in m^3.
c How many litres are there in a tank of volume 7.32 m^3?

CHAPTER 26

Calculating length

26A Perimeter

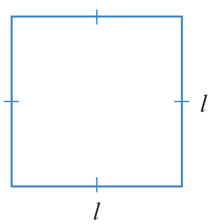
The perimeter of a figure is the measurement of the distance around its boundary. One way of thinking about perimeter is to imagine walking around a property. Start at one corner and walk around the boundary. When you arrive back at your starting point the perimeter is the distance you have walked.



A polygon is a figure that has straight lines as sides. For a polygon, the perimeter is obtained by adding the lengths of all of its sides.

Following is a summary of previously used perimeter formulas.

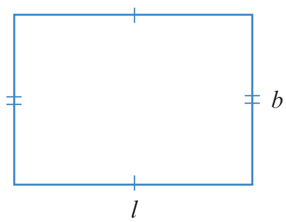
Square



Perimeter = $4l$

l = length

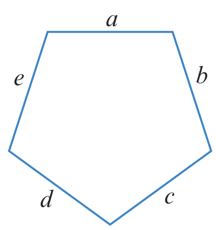
Rectangle



Perimeter = $2l + 2b$

l = length, b = breadth

Polygon



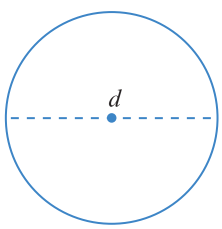
Perimeter = $a + b + c + d + e$

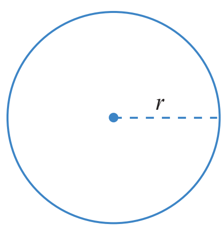
Circle

For a circle, the perimeter has a special name, the *circumference*, and we use either:

Circumference $= \pi \times \text{diameter} = \pi d$

or Circumference $= 2 \times \pi \times \text{radius} = 2\pi r$





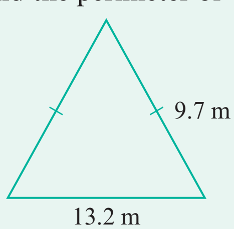
NOTE

Remember that π is a Greek letter that represents the number 3.141 592 6...

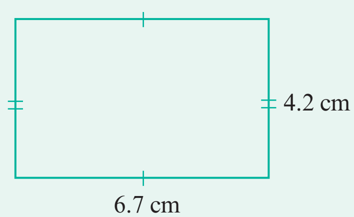
EXAMPLE 26A-1 Finding perimeters

Find the perimeter of the following shapes.

a



b

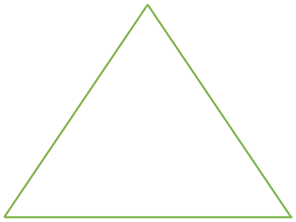


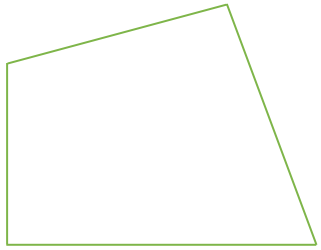
a $P = 13.2 + 2 \times 9.7$ m
$= 32.6$ m

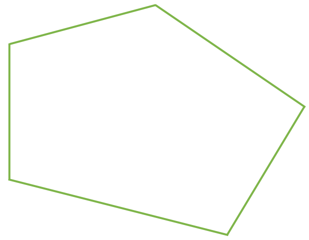
b $P = 2 \times 4.2 + 2 \times 6.7$
$= 21.8$ cm

EXERCISE 26.1

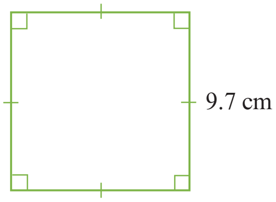
1 Measure, with your ruler, the lengths of the sides of these figures and then find each perimeter.

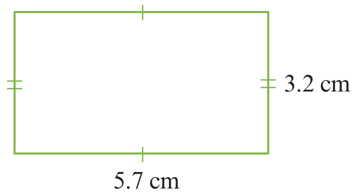
a


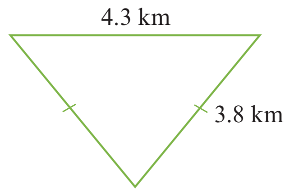
b


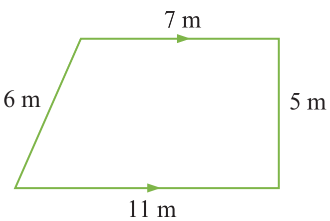
c


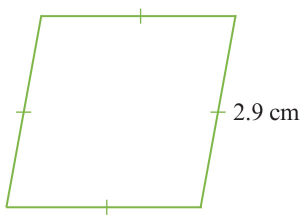
2 Find the perimeter of each of the following shapes.

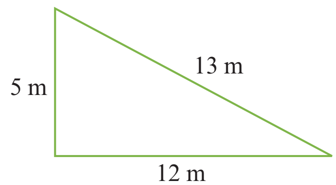
a


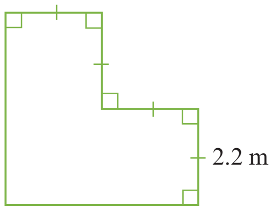
b


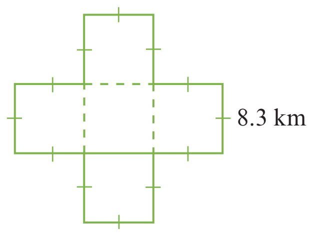
c


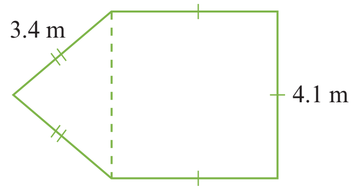
d


e


f


g


h


i


3 An equilateral triangular paddock has sides of 450 m and is fenced with three strands of wire, where the wire costs \$0.08 per metre.
 a Find the perimeter of the paddock.
 b What is the the total length of wire needed?
 c How much is the total cost of the wire?

NOTE
An equilateral triangle has three equal sides.

NOTE
Draw neat diagrams to help you understand these questions.

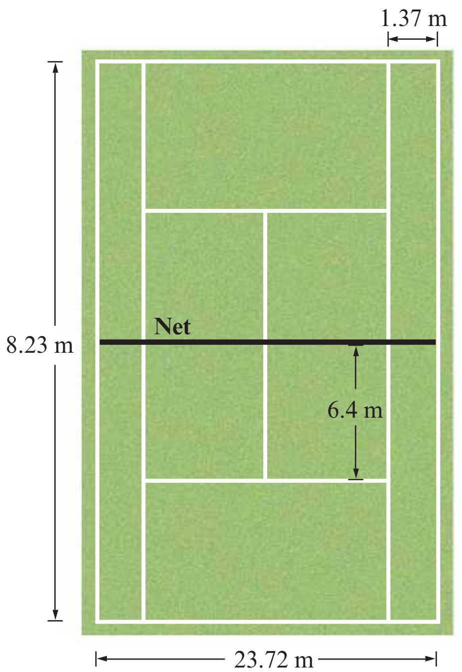
4 A rectangular swimming pool is 50 m long and 20 m wide and has brick paving 3 m wide around it. Find the outer perimeter of the brick paving.

5 A runner jogs 10 times around a rectangular housing estate. The estate is 1.08 km by 420 m. How far has the runner travelled?

6 A tennis court has the dimensions shown below.

a What is the perimeter of the court?

b Find the total length of all the marked lines (not the net).



EXAMPLE 26A-2 Finding the circumference

Find the circumference of the following circles correct to 2 decimal places.

a diameter 13.8 m

b radius 3.7 km.

a $C = \pi d$
$= \pi \times 13.8$
≈ 43.35 m

b $C = 2\pi r$
$= 2 \times \pi \times 3.7$
$= 23.25$ km

NOTE

Remember to use the [FIX] mode on your calculator to help you round correct to 2 decimal places.

EXERCISE 26.2

1 Find the circumference of the following circles correct to 2 decimal places.

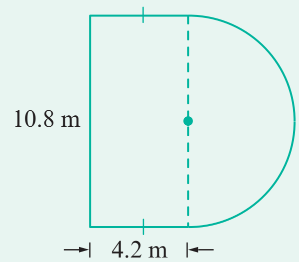
a diameter 13.2 cm
b radius 8.6 m
c diameter 115 m

2 Find the circumference of the following circles correct to 3 decimal places.

a radius 0.85 km
b diameter 7.2 m
c radius 235 cm

EXAMPLE 26A-3 Finding a more complex perimeter

Calculate the perimeter of this shape correct to 2 decimal places.



NOTE

≈ means approximately equals.

Diameter of semicircle: $d = 10$

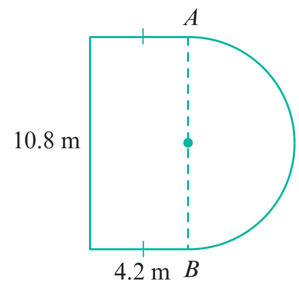
Distance A to B around semicircle $= \frac{1}{2} \times \pi \times d$

$= \frac{1}{2} \times \pi \times 10.8$

≈ 16.96 m

Perimeter $\approx 10.8 + 2 \times 4.2 + 16.96$ m

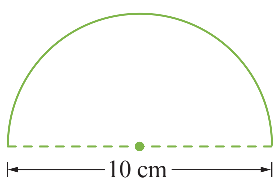
≈ 36.16 m



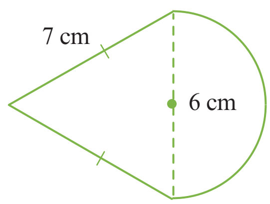
EXERCISE 26.3

1 Find the perimeter of these shapes correct to 2 decimal places.

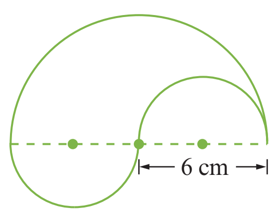
a



b

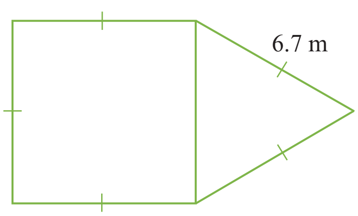


c

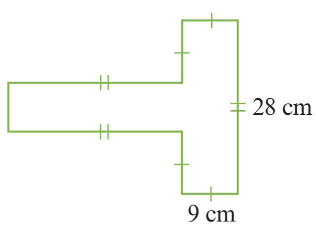


2 Copy each diagram and mark all unknown measurements on it before finding the perimeter.

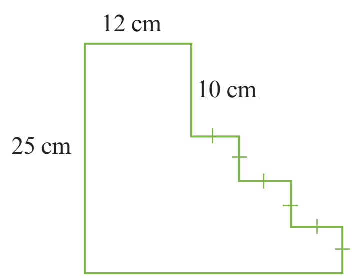
a



b



c



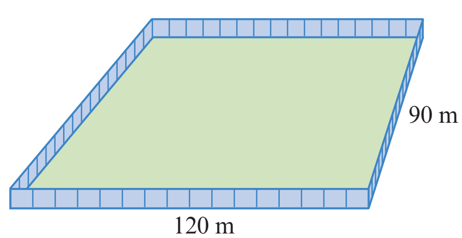
3 A small housing block is 40 m by 16 m and can be fenced for \$25.75 a metre. Find the cost of the fence.

4 You have purchased an industrial block of land that is 120 m long and 90 m wide. You decide to use fence panels that are 1.2 m wide to enclose the property. (The gate is also made of these panels.)

a Find the total perimeter of the block.

b Find the number of panels required.

c If each panel costs \$16.25, find the cost of the panelling to enclose the property.



5 The local council needs to build a retaining wall along a 1.5 km long embankment. The wall is to be made of old railway sleepers, which are 2.5 m long. The wall needs to be three sleepers high.

a What is the total length of sleepers required?

b How many sleepers are needed?

c If each sleeper weighs (on average) 63 kg, what mass (in tonnes) of sleepers is needed?

d If a truck can carry 15 t of sleepers, how many truckloads are needed?

6 A triathlon course has three legs:

Leg 1: 1.2 km swim

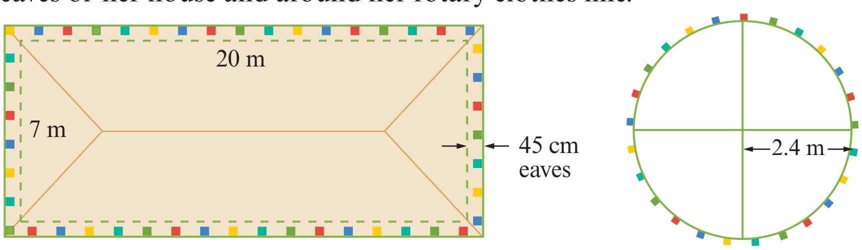
Leg 2: 8.3 km bicycle ride

Leg 3: 6.3 km run.

a Find the total distance covered.

b What is the average speed of a contestant who took 1 h 10 min to complete the course?

7 Jan is having a party and wishes to string coloured lights all around the eaves of her house and around her rotary clothes line.



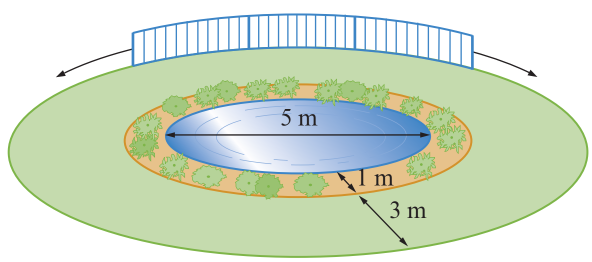
a Calculate the length of the lighting needed to go under the eaves.

b Calculate the circumference of the clothes line.

c Calculate the total length of lighting needed.

d If the lighting costs $6.75 per 9 m length to hire, determine the total cost of hiring the necessary length. (Full 9 m lengths must be hired as there are no part lengths.)

8 At Bushby Park there is a 5 m diameter circular pond that is surrounded by a 1 m wide garden bed and then a 3 m wide lawn. A safety fence is placed around the lawn with posts every 3 m and a gateway that is 1.84 m wide. The gate is wrought iron.



a How much safety fence is needed?

b How many posts are needed?

c If the posts cost $15.75 each and the safety fence costs $18.35 per metre, calculate the total cost of the fence (excluding the gate).

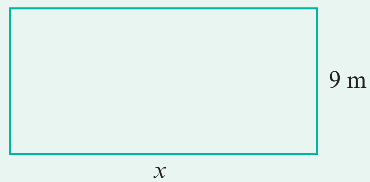
26B Finding unknown lengths

Sometimes we are required to find a dimension (side, radius or diameter) of a figure given other dimensions and its perimeter. To do this we give the unknown side a name or label. This label is usually a letter. The size of the unknown length can be found using algebra.

Algebra is a part of mathematics in which letters and other general symbols are used to represent numbers and quantities in formulas and equations.

EXAMPLE 26B-1 Finding the side length

This rectangle has a perimeter of 48 m. Find x.



$$P = 2l + 2b \text{ where } P = 48,\ l = x \text{ and } b = 9.$$

$$48 = 2x + 2 \times 9$$

$$48 = 2x + 18$$

$$48 - 18 = 2x + 18 - 18$$ Subtract 18 from both sides.

$$30 = 2x$$

$$\frac{30}{2} = \frac{2x}{2}$$ Divide both sides by 2.

$$x = 15$$

The length is 15 m.

EXAMPLE 26B-2 Finding the radius

Find the radius of a circle with circumference 11.7 m.

NOTE 6.283 is a good approximation of 2π.

$$C = 2\pi r$$

$$11.7 = 2 \times \pi \times r$$

$$\approx 6.283 \times r$$

$$\frac{11.7}{6.283} \approx \frac{r}{6.283}$$ Divide both sides by 6.283.

$$r \approx 1.86$$

The radius is approximately 1.86 m.

You can use a spreadsheet to find the answer for Example 26B-2, as shown on the opposite page.

Step 1: Type Radius into cell A1 and Circumference into cell B1.

Step 2: Type 1.00 into cell A2 and the formula = A2+0.01 into cell A3.

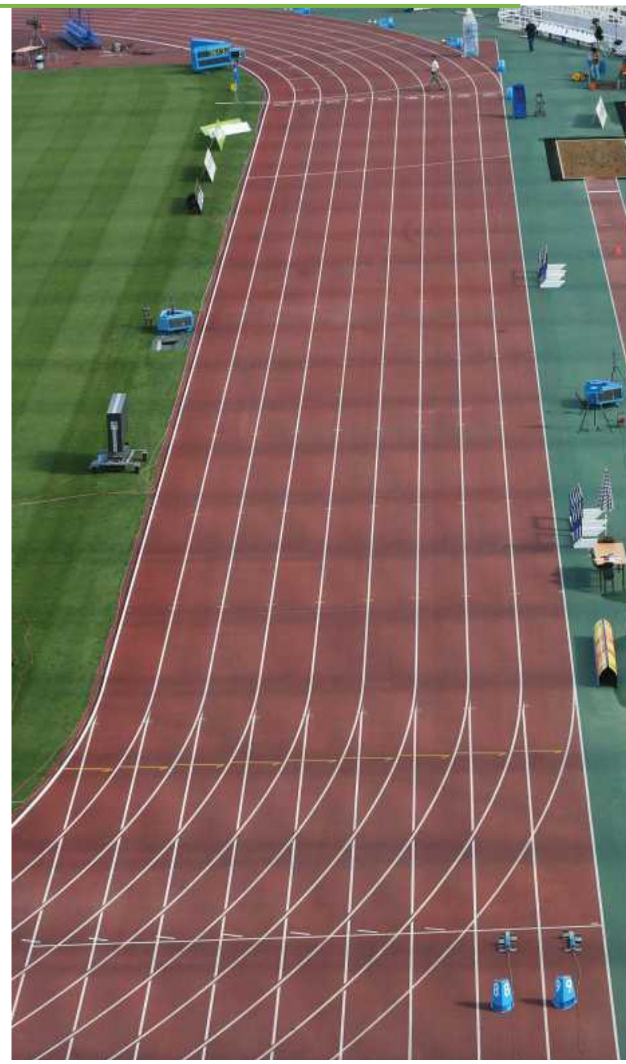
Step 3: Fill down Column A starting with a radius of 1, going up in increments of 0.01.

Step 4: Type = 6.283*A2 into cell B2 and fill down.

Step 5: Column B now calculates the circumference for each value of the radius.

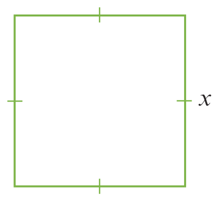
	A	B	C	D
1	Radius	Circumference		
2	1.00	=6.283*A2		
3	=A2+0.01			
4				
5	↓Fill down	↓Fill down		
6				
7				

EXERCISE 26.4

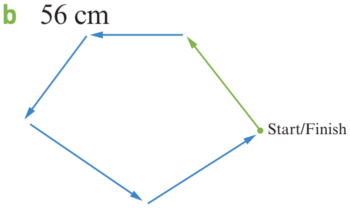


1 Find x for each shape based on the perimeter given.

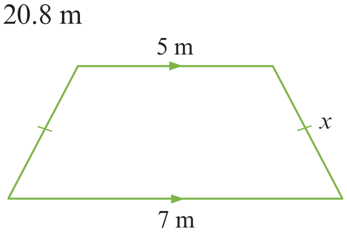
a 34 cm



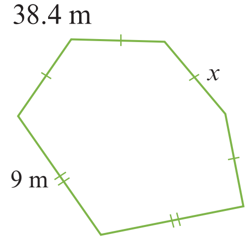
b 56 cm



c 20.8 m

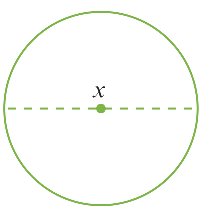


d 38.4 m

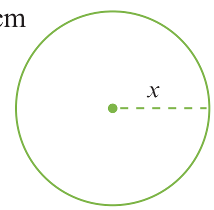


2 Find x given that each circle has the following circumference.

a 37.38 m



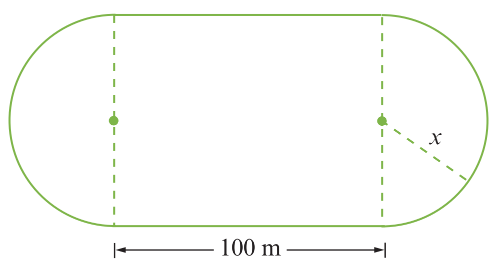
b 32.67 cm



3 a Use a spreadsheet to find the diameter of a circle with a circumference of 20 m correct to 1 decimal place.

b Use a spreadsheet to find the radius of a circle with a circumference of 213 cm correct to 2 decimal places.

4 An athletics track has a perimeter of 400 m and straight sides of 100 m. Find the value of x, the radius of the 'bends'. You could solve an equation or use a spreadsheet.

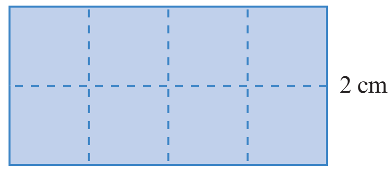


CHAPTER 27

Calculating area and surface area

27A Area

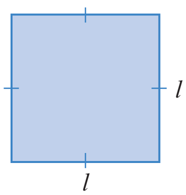
The area within a shape is the number of square units enclosed by the shape. For example, this rectangle encloses 8 cm^2.



AREA FORMULAS

The basic formulas for area are as follows.

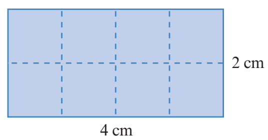
Square



l = length

Area = length × length = l^2

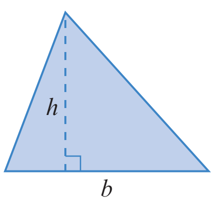
Rectangle



l = length, b = breadth

Area = length × breadth = $l \times b$

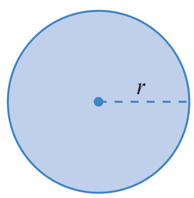
Triangle



h = height, b = base

Area = $\frac{1}{2}$(base × height) = $\frac{1}{2}(b \times h)$

Circle



r = radius

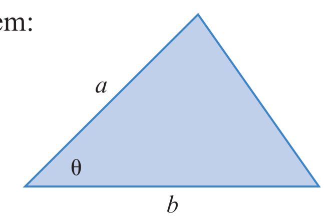
Area = π × radius^2 = $\pi \times r^2$

Other useful triangle formulas

Following are two other useful formulas for finding the area of triangles.

- Given two sides and the angle between them:

  Area = $\frac{1}{2} \times a \times b \times \sin\theta$

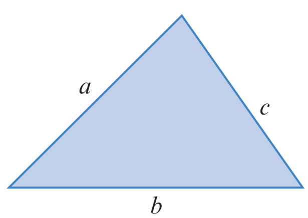


- Given all three sides:

  Area = $\sqrt{s(s-a)(s-b)(s-c)}$

  where $s = \dfrac{a+b+c}{2}$

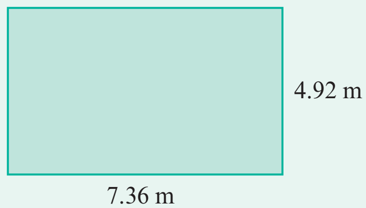
  This is called Heron's formula.



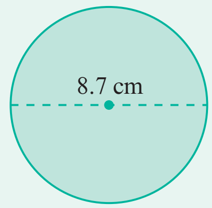
EXAMPLE 27A-1 Finding areas of simple shapes

Find the area of these simple shapes correct to 2 decimal places.

a



b



a $A = l \times b$

$= 7.36 \times 4.92$

$\approx 36.21 \text{ m}^2$

b $A = \pi \times \text{radius}^2$

$r = 8.7 \div 2 = 4.35$

$A = \pi \times (4.35)^2$

$\approx 59.45 \text{ cm}^2$

Calculator

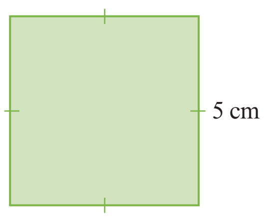
7.36 [×] 4.92 [=]

[π] [×] 4.35 [x^2] [=]

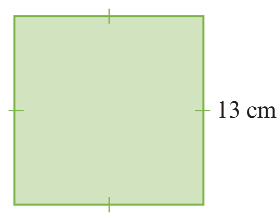
EXERCISE 27.1

1 Find the area of each shape correct to 2 decimal places if necessary.

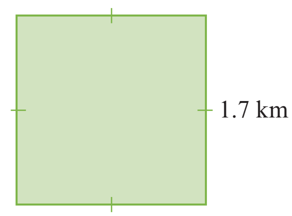
a



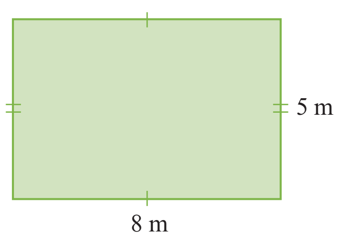
b



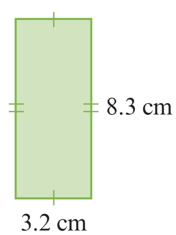
c



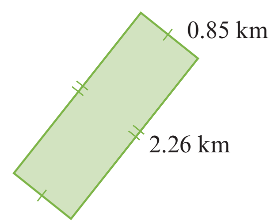
d



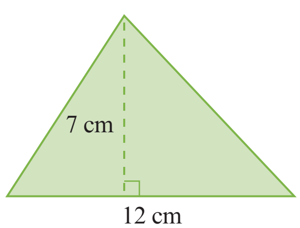
e



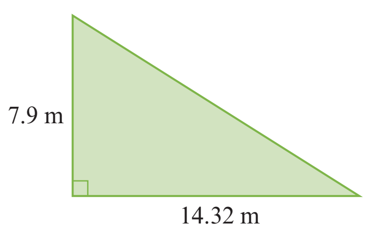
f



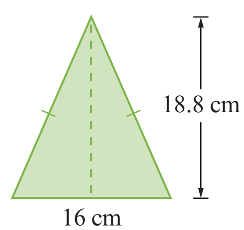
g



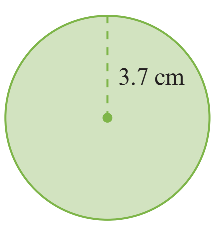
h



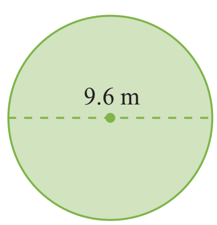
i



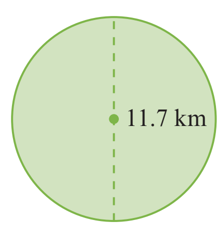
j



k

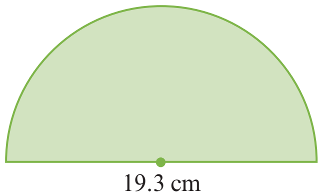


l

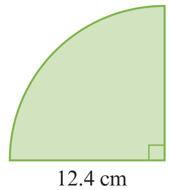


2 Calculate the area of these figures correct to the nearest square centimetre. A complete turn of a circle is 360°.

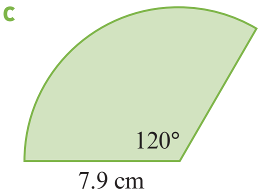
a



b

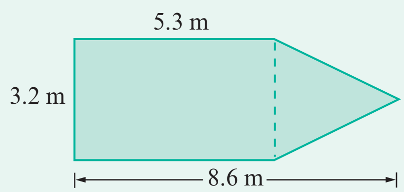


c

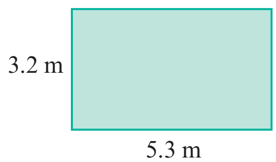


EXAMPLE 27A-2 Finding the area of a complex figure

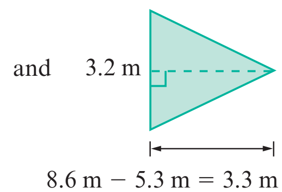
Calculate the area of this shape.



The required area is the sum of:



and



A = area of rectangle + area of triangle

$= l \times b + \frac{1}{2}(b \times h)$

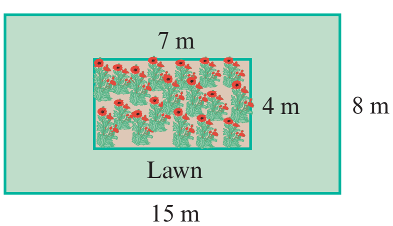
$= 5.3 \times 3.2 + \frac{1}{2} \times 3.2 \times 3.3$

$= 22.24\ \text{m}^2$

EXAMPLE 27A-3 Finding areas by subtraction

A 15 m by 8 m rectangular garden consists of a lawn with an inner area (7 m × 4 m) of flowers. Find the area of lawn.

NOTE It helps to draw a diagram first.



Area of large rectangle $= 15 \times 8$

$= 120\ \text{m}^2$

Area of flower rectangle $= 7 \times 4$

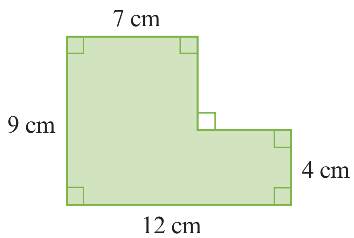
$= 28\ \text{m}^2$

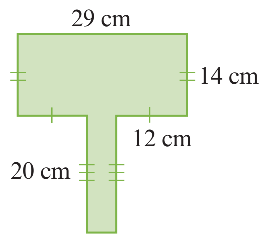
Area of lawn $= 120\ \text{m}^2 - 28\ \text{m}^2$

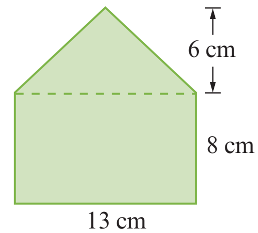
$= 92\ \text{m}^2$

EXERCISE 27.2

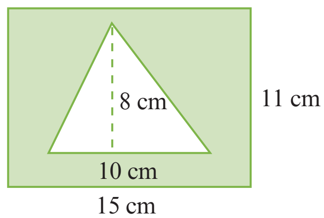
1 Calculate the area of the following composite shapes.

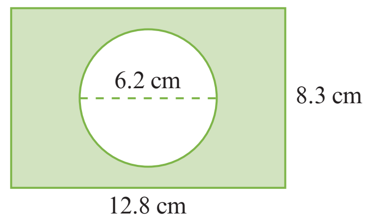
a


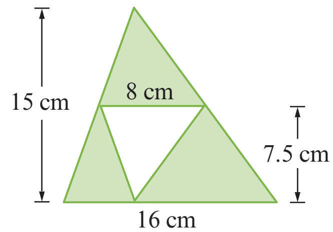
b


c


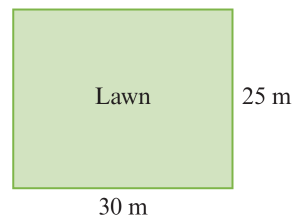
2 Calculate the area of the following shaded regions in square centimetres.

a


b


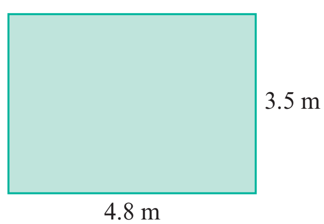
c


3 A rectangle of lawn is 30 m × 25 m and has a 1 m wide path around it. Draw a diagram and find the area of the concrete path.



EXAMPLE 27A-4 Costing an area

A room in a house is 3.5 m by 4.8 m. Find the cost of tiling the floor if tiles cost \$65 per laid square metre.



$$A = l \times b$$
$$= 4.8 \times 3.5$$
$$= 16.80\ \text{m}^2$$

Cost $= 16.80 \times \$65$

$= \$1092$

Cost of tiling the floor is \$1092.

EXERCISE 27.3

1 A wheat paddock is 1.2 km by 0.65 km.

  a Find the area of the paddock in hectares.

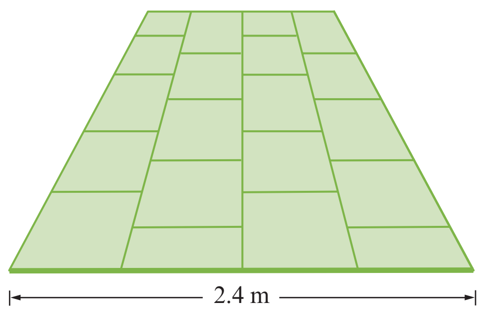
  b Find the total cost of planting the wheat crop at $79 per hectare.

2 Concrete pavers suitable for a driveway are:

  Type A: 0.6 m × 0.6 m

  Type B: 0.6 m × 0.3 m

  A driveway is to be 2.4 m wide and 18 m long. The pavers are laid on sand that costs $18 a tonne. A tonne of sand covers 17.5 m^2 to the required depth. Sand must be purchased as a multiple of 0.2 of a tonne.



  a Calculate the area of the driveway.

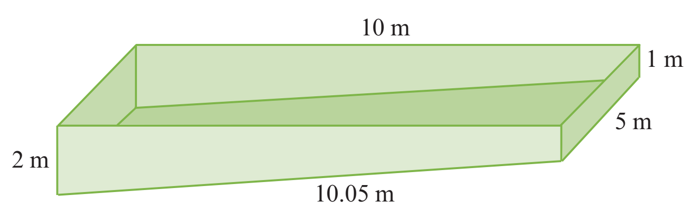
  b Explain why you need four Type B pavers for the given pattern in the diagram.

  c How many Type A pavers are needed?

  d How much sand is needed?

  e If the pavers cost $18.25 per square metre, find the total cost of the pavers and the sand.

3 A swimming pool with the dimensions shown is to be lined with tiles that are 25 cm × 20 cm. The tiles cost $21.50 per square metre.



  a Find the total surface area of the four walls and bottom of the pool.

  b How many tiles are needed for 1 m^2 of coverage?

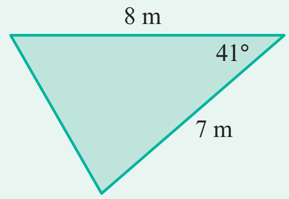
  c How many tiles are needed to complete the task?

  d What is the total cost of tiling the pool given that adhesive costs $2.35 per m^2 and labour costs $35 per m^2?

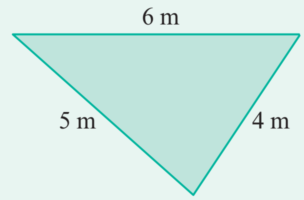
EXAMPLE 27A-5 Finding areas of triangles

Find the area of each triangle correct to 2 decimal places.

a



b



a Given two sides and the angle between them:

$A = \frac{1}{2} \times a \times b \times \sin\theta$

$= \frac{1}{2} \times 8 \times 7 \times \sin 41°$

$\approx 18.37\ \text{m}^2$

Caculator: 1 [÷] 2 [×] 8 [×] 7 [×] [sin] 41 [=]

b Given all three sides we use Heron's formula:

$A = \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \frac{a+b+c}{2}$

$s = \frac{4+5+6}{2} = 7.5$

$A = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{7.5(7.5-4)(7.5-5)(7.5-6)}$

$= \sqrt{7.5 \times 3.5 \times 2.5 \times 1.5}$

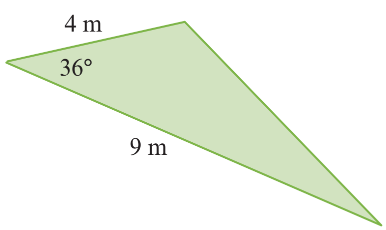
$\approx 9.92\ \text{m}^2$

Caculator: [√] [(] 7.5 [×] 3.5 [×] 2.5 [×] 1.5 [)] [=]

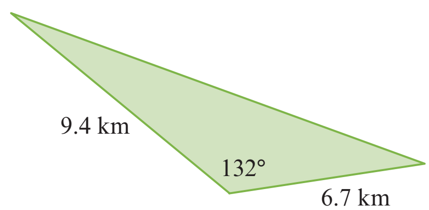
EXERCISE 27.4

1 Find the area of each triangle correct to 2 decimal places.

a

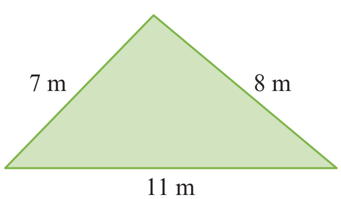


b

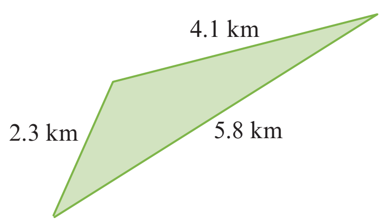


2 Use Heron's formula to find the area of each triangle correct to 1 decimal place.

a



b



A spreadsheet for Heron's formula

We can use a spreadsheet to find the area of a triangle using Heron's formula.

Key the titles, data and formula shown into the appropriate cells. Here we use the triangle from Example 27A-5 part **b**.

	A	B
1	a	4
2	b	5
3	c	6
4	s	=(B1+B2+B3)/2
5		
6	A	=SQRT(B4*(B4-B1)*(B4-B2)*(B4-B3))

NOTE

Remember that every multiplication must be entered using *.

The result shown below agrees with the answer given in Example 27A-5 part **b**.

	A	B
1	a	4
2	b	5
3	c	6
4	s	7.5
5		
6	A	9.9216

obook

An Excel spreadsheet template to help you apply Heron's formula is available on your obook.

EXERCISE 27.5

1 Find, using the above spreadsheet, the area of triangles with sides of length:

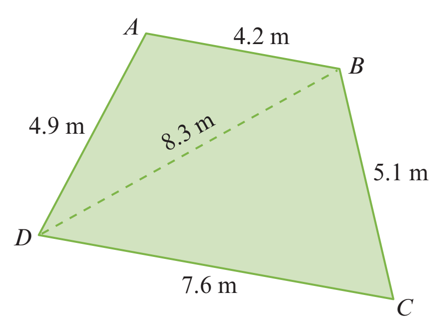
a 6 cm, 7 cm, 8 cm   **b** 1.2 km, 1.7 km, 1.8 km

NOTE

For part **a**, replace B1 with 6, B2 with 7 and B3 with 8.

2 A triangle has one side of 7.1 m, another of 10.3 m and an area of 25 m^2. Use your spreadsheet with trial and error replacements in cell B3 to find the length of the third side of the triangle. Give your answer correct to 2 decimal places.

3 Use your spreadsheet to help you find the area of the lawn with dimensions as shown below.



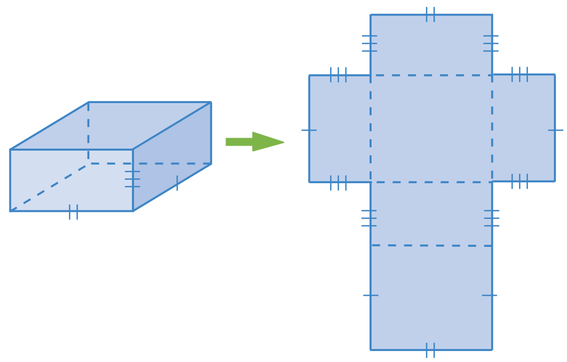
27B Surface area

SOLIDS WITH PLANE FACES

A plane face is a two-dimensional shape: it has width and breadth but no thickness. The surface area of a three-dimensional figure with plane faces is the sum of the areas of these faces.

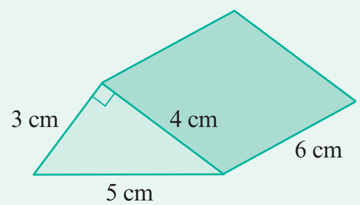
This means that the surface area is the same as the area of the net required to make the three-dimensional figure or how the shape would look if it were opened out flat. For example:

Surface area = area of the net shown.

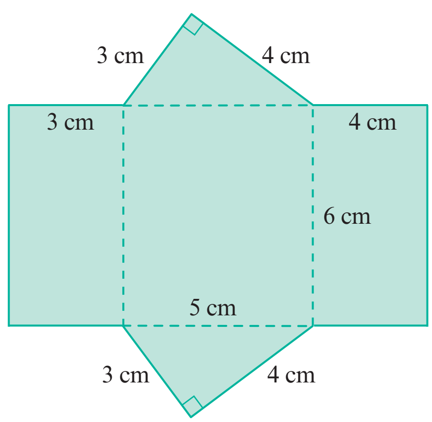


EXAMPLE 27B-1 Surface area of a triangular prism

Find the surface area of this triangular prism.



First draw the net.
The prism has:
two identical right-angle triangular faces
one rectangular 4 cm × 6 cm face (right top)
one rectangular 3 cm × 6 cm face (left top)
one rectangular 5 cm × 6 cm face (bottom).



$$
\begin{aligned}
\text{Surface area} &= 2 \times (\tfrac{1}{2} \times 3 \times 4) + (4 \times 6) + (3 \times 6) + (5 \times 6) \\
&= 12 + 24 + 18 + 30 \text{ cm}^2 \\
&= 84 \text{ cm}^2
\end{aligned}
$$

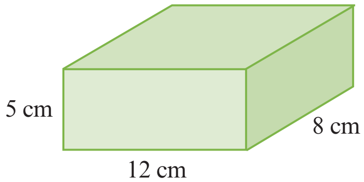
EXERCISE 27.6

1 Find the surface area of the following cubes with side lengths of:

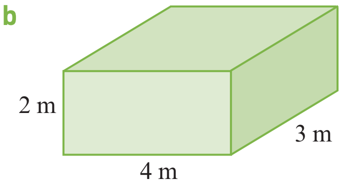
a 5 cm **b** 4.2 cm **c** 8.5 mm

2 Find the surface area of the following rectangular prisms.

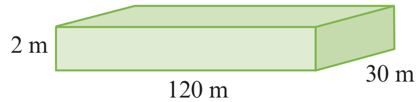
a



b

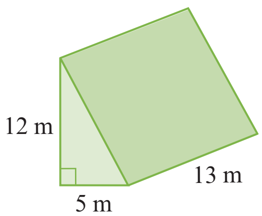


c

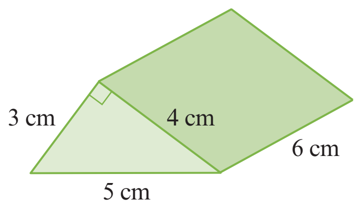


3 Find the surface area of the following triangular prisms.

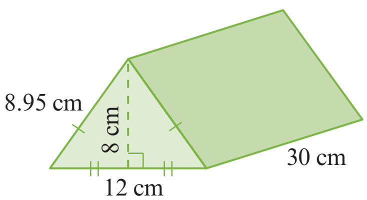
a



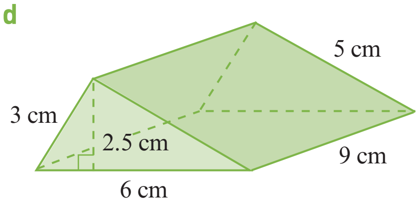
b

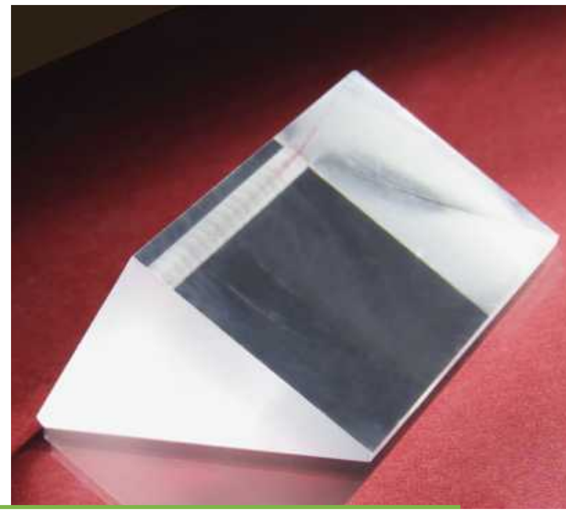


c



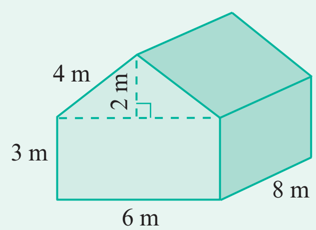
d





EXAMPLE 27B-2 Surface area of a composite solid

Find the surface area of this prism.



Area of the front face and back face is:

two rectangular 3 m × 6 m faces = 2 × 3 × 6 = 36 m^2

two identical right-angle triangular faces = $2 \times \frac{1}{2} \times 6 \times 2 = 12$ m^2

Area of the base is:

one rectangular 6 m × 8 m face = 48 m^2

Area of both sides is:

two rectangular 3 m × 8 m faces = 2 × 3 × 8 = 48 m^2

two rectangular 4 m × 8 m faces = 2 × 4 × 8 = 64 m^2

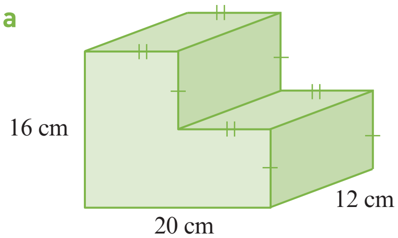
Total surface area = 36 + 12 + 48 + 48 + 64 m^2

= 208 m^2

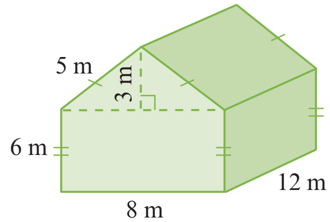
EXERCISE 27.7

1 Find the surface area of the following prisms.

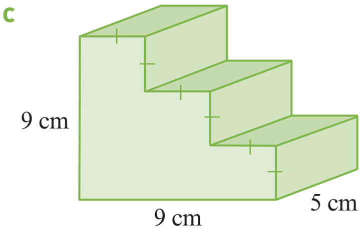
a



b



c



2 You wish to find the cost of painting the outside of a rectangular garden shed 8 m × 5 m × 3 m with a flat roof where 1 L of paint costs $12.50 and each litre covers 5 m^2.

a Draw a sketch of the shed and show its dimensions.

b How many faces are to be painted?

c Find the total surface area to be painted.

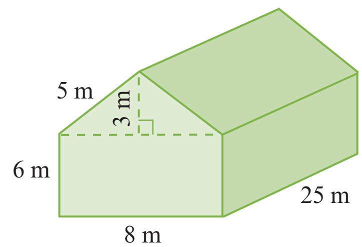
d Find the total cost of the paint given that only 1 L tins are available.

3 A shadehouse with dimensions as shown on the right is to be covered with shadecloth. The cloth costs $4.75 per square metre.

a Find the area of each end of the shadehouse.

b Find the total area to be covered with cloth.

c Find the total cost of the cloth needed given that 5% more than the calculated amount is necessary for wrapping around the piping substructure.

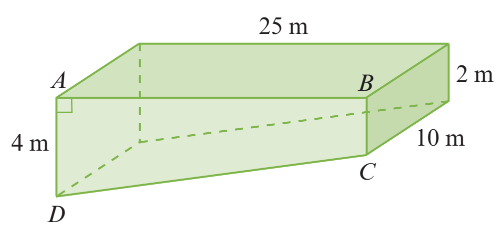


4 The diagram shows a 25 m swimming pool that is to be completely tiled inside.

a Find the area of *ABCD* by splitting it into a 2 m high rectangle and a triangle.

b Find the total inner surface area.

c Tiles and the tiling job cost $56/m^2. Find the total cost.

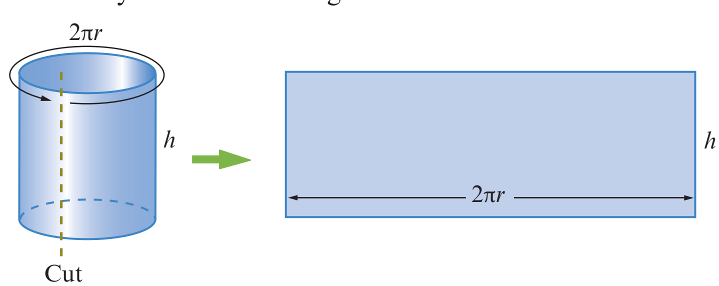


CYLINDERS, SPHERES AND CONES

NOTE

Peel the label off a cylindrical can and notice the shape when the label is flattened. The length of the rectangle is the same as the circumference of the cylinder.

The net of a hollow cylinder is a rectangle formed from the curved surface.



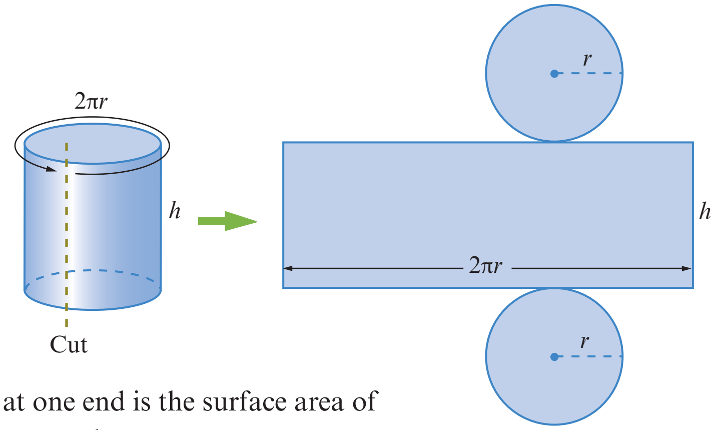
The outer surface area of a hollow cylinder is the area of a rectangle with sides $2\pi r$ and h.

$$A = 2\pi r \times h$$

The net of a closed cylinder is a rectangle formed from the curved surface plus two circular ends.

The outer surface area of a closed cylinder is the area of a rectangle with sides $2\pi r$ and h plus the area of two circles with radius r.

$$A = 2\pi rh + 2\pi r^2$$



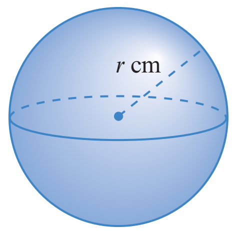
The outer surface area of a cylinder hollow at one end is the surface area of the curved surface plus the area of the circle at one end.

$$A = 2\pi rh + \pi r^2$$

Sphere

For a sphere of radius r units, the surface area is given by:

$$A = 4\pi r^2$$



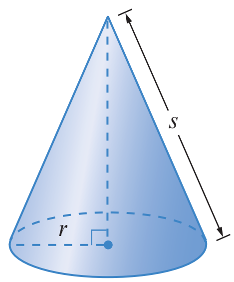
Cone

For a hollow cone of base radius r units and slant height s units, the surface area of curved surface is given by:

$$A = \pi rs$$

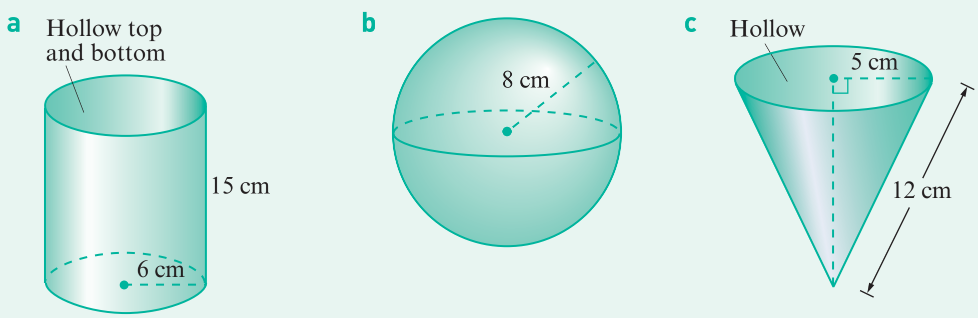
For a closed cone, the surface area of curved surface and base is given by:

$$A = \pi rs + \pi r^2$$



EXAMPLE 27B-3 Finding outer surface areas

Find the outer surface area of these objects correct to 1 decimal place.



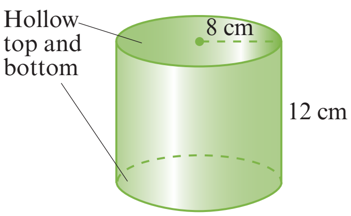
a $A = 2\pi rh$
$= 2 \times \pi \times 6 \times 15$
$\approx 565.5 \text{ cm}^2$

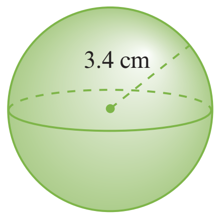
b $A = 4\pi r^2$
$= 4 \times \pi \times 8^2$
$\approx 804.2 \text{ cm}^2$

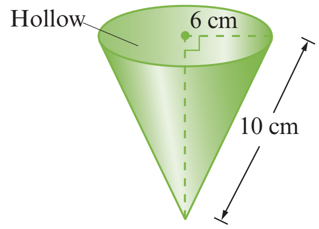
c $A = \pi rs$
$= \pi \times 5 \times 12$
$= 188.5 \text{ cm}^2$

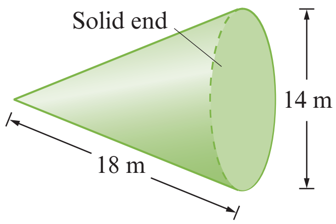
EXERCISE 27.8

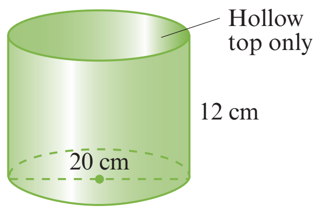
1 Find the outer surface area of the following objects correct to 1 decimal place.

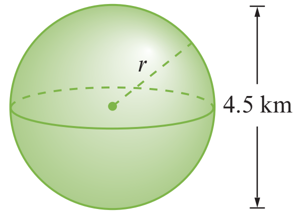
a 

b 

c 

d 

e 

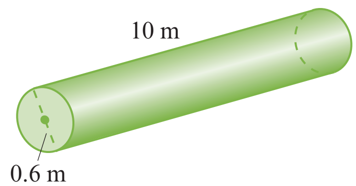
f 

2 A wharf is to have 24 cylindrical concrete pylons, each with a diameter of 0.6 m and 10 m long. Each is to be coated with a salt-resistant material.

a Find the total surface area of one pylon.

b Coating the pylons with the material costs \$45.50 per m^2. Find the cost of coating one pylon.

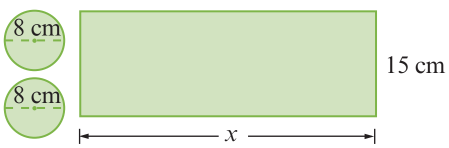
c Find the total cost for coating the 24 pylons.



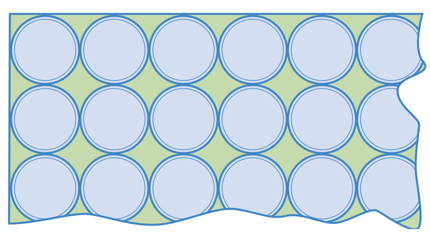
3 Soup is to be packaged in cans of 15 cm height and a base diameter of 8 cm.

a Find the total surface area of one can.

b Each can is made from three pieces of metal; two discs and a rectangle. Find the length of the rectangle; that is, find x.



c The discs are cut from an 80 cm by 80 cm sheet of metal. How many cans can be made using discs from one of these sheets?



d The rectangles for making the curved surface come from sheets that are 75 cm wide. How long must the 75 cm wide sheet be so that enough rectangles are made to match the discs from part **c**?

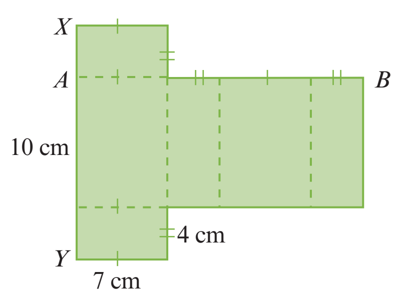
4 a The net shown is for a rectangular prism. Find the surface area.

b Find the distances AB and XY.

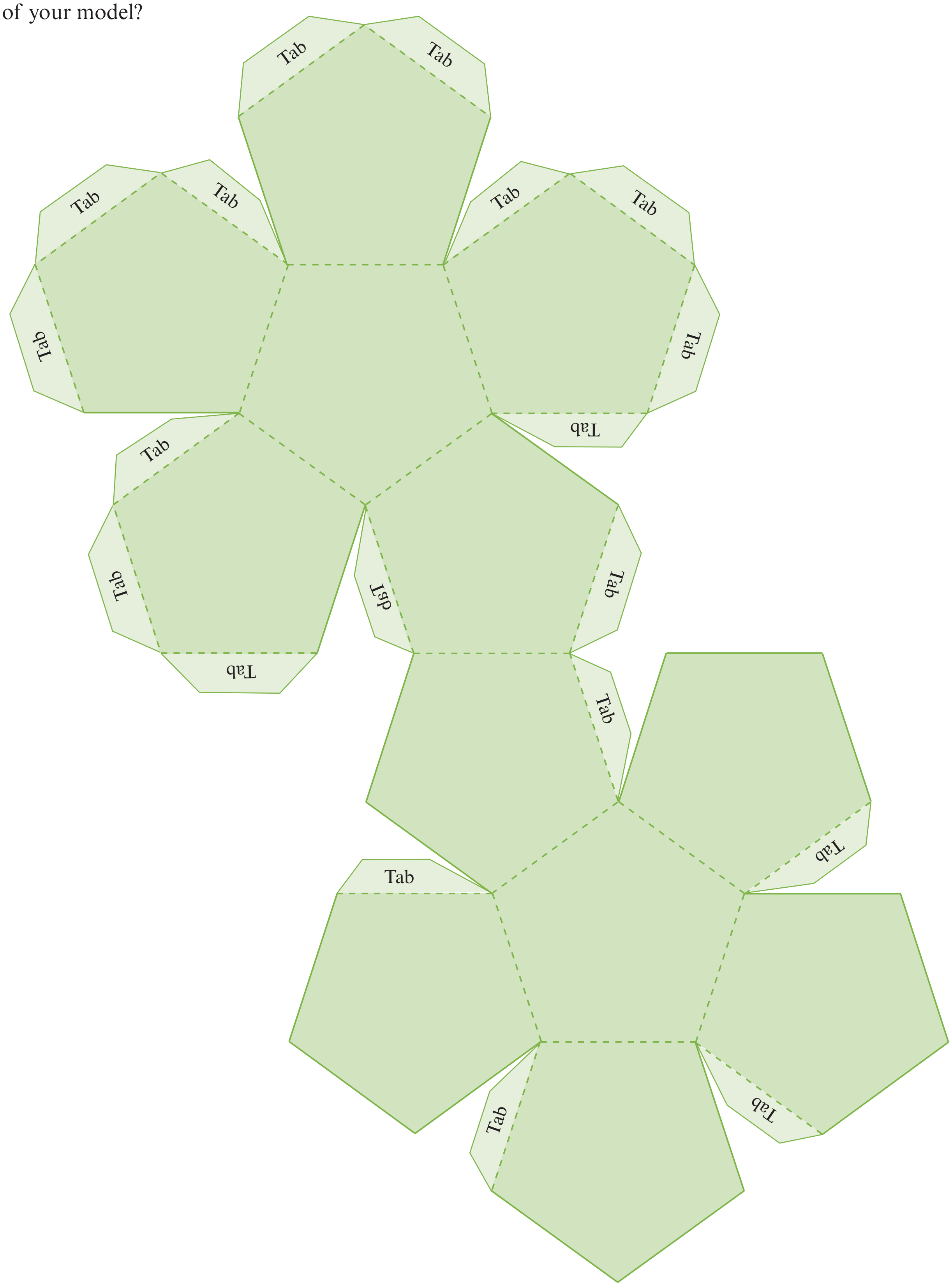
c How many complete prisms can be cut out from a 110 cm by 106 cm sheet of cardboard?

d Does the following formula give the correct answer to part **b**? Why or why not?

$$\text{Number of boxes} = \frac{\text{total area of cardboard}}{\text{area of one box}}$$



5 This is the net of one of the platonic solids: a dodecahedron. The tabs are to allow a paper copy to be glued together. The dotted lines are edges. Enlarge this diagram using a photocopier and make the three-dimensional object. How would you calculate the total surface area? What is the total surface area of your model?



MATHEMATICAL SKILLS

CHAPTER 28

Without a calculator

28A Back to basics

Many jobs require you to make quick, accurate calculations, and in some of these you must be able to calculate whether or not you have a calculator available. For jobs such as nurses or ambulance officers, getting the mathematics right can be a matter of life or death. As a result, a number of course selection tests require you to be able to do mathematics without a calculator.

Doing mathematical calculations without a calculator is a skill in itself, and is often very different from the way you would do the same mathematical calculations with a calculator. This skills practice is designed to help you both improve your performance in these career selection tests and to perform day-to-day mathematics when the calculator is missing or the batteries have gone flat.

In Section 23A we looked at the order of operations, and the acronym BODMAS was used. In mental arithmetic, we need to remember this whenever we are doing a calculation. It may be the ability to apply BODMAS that can give a pass or fail in an entrance exam.

BODMAS revision

B **B**rackets first: work out the inside of all brackets first.

O **O**rders next (that is, powers and square roots, etc.). For simplicity in this book, we use the **O** in BODMAS to represent 'overs' or fractions and workings.

D **D**ivision next.

M **M**ultiplication next.

A **A**ddition next.

S **S**ubtraction next.

EXAMPLE 28A-1 Order of operations

Calculate the following without using a calculator.

a $(3 + 4) \times (5 + 6)$ **b** $\frac{3+4}{7} \times \frac{4+6}{5}$ **c** $3 \times \frac{10-4}{6} + 5 \times 2(1 + 2)$

a $(3 + 4) \times (5 + 6) = 7 \times 11$
$= 77$

Solve inside the brackets first.

b $\frac{3+4}{7} \times \frac{4+6}{5} = \frac{7}{7} \times \frac{10}{5}$
$= 1 \times 2$
$= 2$

Do the overs first.

c $3 \times \frac{10-4}{6} + 5 \times 2(1 + 2) = 3 \times \frac{6}{6} + 5 \times 2(3)$
$= 3 \times 1 + 5 \times 6$
$= 3 + 30$
$= 33$

Do brackets and overs first.

EXERCISE 28.1

1 Calculate the following without a calculator.

a $2 + (4 \times 6)$

b $3 + (4 - 2) - 5$

c $(6 + 2) \div 4 - 2$

d $16 \div (6 + 2) + 8$

e $3 \times 30 \div 10 - 5$

f $16 - 5 \times 2 + 6$

g $\frac{6+6}{2} \times 10 \div 2 - 5$

h $10 \div 2 \times \frac{8+4}{1}$

i $(6 + 4) - (5 \times 2)$

j $6 + 3 - 8 \div 2$

k $\frac{10-4}{3} + \frac{7-4}{6}$

l $\frac{10-4}{3} - \frac{7-4}{6}$

28B Fractions

In Mathematical Skills Chapter 23, we covered calculating with fractions using a calculator. Working with fractions without a calculator is quite a different skill. Many nursing calculations require the creation and use of fractions. Many of these calculations involve medication dosages – a mistake here can be fatal! Remember:

- The top number in a fraction is called the numerator.
- The bottom number in a fraction is called the denominator.

SIMPLIFYING FRACTIONS

Finding the smallest denominator is sometimes called simplifying a fraction.

Consider the following fractions: $\frac{25}{75} = \frac{5}{15} = \frac{1}{3}$

Each of these fractions represent the same amount, but do you remember how to change from one to the other mathematically?

To reduce the size of the denominator without affecting the meaning of the fraction, we divide both the numerator and the denominator by the same number.

In the above example: $\frac{25 \div 5}{75 \div 5} = \frac{5}{15}$ and $\frac{5 \div 5}{15 \div 5} = \frac{1}{3}$

EXAMPLE 28B-1 Simplifying a fraction

Simplify the fraction $\frac{50}{100}$.

$\frac{50 \div 10}{100 \div 10} = \frac{5}{10}$ Divide both the numerator and the denominator by 10.

$\frac{5 \div 5}{10 \div 5} = \frac{1}{2}$ Divide both the numerator and the denominator by 5.

Of course, you may have noticed a quicker way to find this answer: dividing both the numerator and the denominator by 50 to begin with. Simplifying fractions is always a matter of dividing the same number into both the numerator and the denominator. It is your choice as to the number you start with – so long as it will divide into both.

Increasing the size of the denominator

As you will see in this section, being able to increase the size of the denominator without changing the meaning of the fraction is very useful when adding or subtracting two or more fractions.

In the previous section, we simplified a fraction by dividing both the numerator and the denominator by the same number. If we want to increase the size of the denominator without affecting what the fraction represents, we simply multiply both the numerator and the denominator by the same number.

EXAMPLE 28B-2 Finding the missing number

Find the missing number $\frac{1}{3} = \frac{\square}{15}$.

$$\frac{1}{3} = \frac{1 \times 5}{3 \times 5}$$

As $3 \times 5 = 15$, multiply the numerator and the denominator by 5.

$$= \frac{5}{15}$$

EXERCISE 28.2

1 Simplify the following fractions.

a $\frac{25}{35}$ b $\frac{9}{12}$ c $\frac{6}{18}$ d $\frac{450}{1000}$ e $\frac{24}{36}$

f $\frac{27}{36}$ g $\frac{14}{42}$ h $\frac{56}{80}$ i $\frac{560}{800}$ j $\frac{18}{27}$

2 Complete the following.

a $\frac{2}{3} = \frac{\square}{30}$ b $\frac{1}{5} = \frac{\square}{100}$ c $\frac{5}{6} = \frac{\square}{30}$ d $\frac{4}{9} = \frac{\square}{63}$

e $\frac{4}{7} = \frac{\square}{42}$ f $\frac{1}{12} = \frac{\square}{144}$ g $\frac{4}{6} = \frac{\square}{54}$ h $\frac{4}{11} = \frac{\square}{110}$

ADDING AND SUBTRACTING FRACTIONS

The trick to adding and subtracting fractions is for all the fractions involved to have the same denominator.

EXAMPLE 28B-3 Adding with different denominators

Find $\frac{2}{3} + \frac{5}{6}$.

$$\frac{2}{3} = \frac{2 \times 2}{3 \times 2} = \frac{4}{6}$$

Multiply both the numerator and the denominator by 2.

$$\frac{4}{6} + \frac{5}{6} = \frac{9}{6}$$

Now we can rewrite the problem with the same denominators.

$$= 1\frac{3}{6}$$

Simplify.

$$= 1\frac{1}{2}$$

EXERCISE 28.3

1 Complete the following additions and subtractions.

a $\frac{1}{2} + \frac{3}{4}$ b $\frac{2}{3} + \frac{2}{9}$ c $\frac{3}{4} - \frac{3}{8}$ d $\frac{9}{20} - \frac{2}{5}$

e $\frac{1}{6} + \frac{10}{66}$ f $\frac{2}{7} + \frac{13}{21}$ g $\frac{3}{8} + \frac{9}{24}$ h $\frac{4}{9} - \frac{27}{72}$

IMPROPER FRACTIONS

Improper fractions have the numerator equal to or larger than the denominator: for example $\frac{3}{2}$ and $\frac{3}{3}$. Any fraction with the same number on the top and bottom equals 1.

For easier understanding it is often useful to convert improper fractions back to a whole number and a fraction (a mixed number). You will have noticed in the previous section that when adding fractions the answer is often an improper fraction and we have converted it back to a mixed number.

The following examples use pizzas to illustrate the process. The denominator tells you the number of slices that each pizza has been cut into. The numerator tells you the total number of pizza slices you have.

EXAMPLE 28B-4 Simplifying a mixed number

Express $\frac{3}{2}$ as a mixed number.

Each pizza has been cut into two slices, and there are three slices in total.

$$\frac{3}{2} = \frac{2}{2} + \frac{1}{2}$$
$$= 1 + \frac{1}{2}$$
$$= 1\frac{1}{2}$$

EXAMPLE 28B-5 Simplifying a mixed number

Express $\frac{9}{4}$ as a mixed number.

Each pizza has been cut into four slices, and there are nine slices in total.

$$\frac{9}{4} = \frac{4}{4} + \frac{4}{4} + \frac{1}{4}$$
$$= 1 + 1 + \frac{1}{4}$$
$$= 2\frac{1}{4}$$

A mixed number is when you have a whole number and a fraction together, such as $3\frac{1}{3}$. When performing calculations there are also times when you need to turn a mixed number into an improper fraction. We use our pizza slice example again.

EXAMPLE 28B-6 Finding a mixed number

Find the missing number $3\frac{1}{4} = \frac{\square}{4}$.

Each pizza has been cut into four slices.

$$3\frac{1}{4} = \frac{4}{4} + \frac{4}{4} + \frac{4}{4} + \frac{1}{4}$$

$$= \frac{4 + 4 + 4 + 4}{4}$$

We add just like any other addition of fractions.

$$= \frac{13}{4}$$

NOTE

Remember that a fraction with the same numerator and denominator always equals one.

EXERCISE 28.4

1 Express these improper fractions as mixed numbers.

a $\frac{5}{3}$ **b** $\frac{9}{6}$ **c** $\frac{14}{5}$ **d** $\frac{9}{3}$

e $\frac{48}{20}$ **f** $\frac{70}{33}$ **g** $\frac{335}{33}$ **h** $\frac{2055}{500}$

2 Express these mixed numbers as improper fractions.

a $2\frac{1}{3}$ **b** $3\frac{3}{8}$ **c** $4\frac{1}{5}$ **d** $10\frac{1}{2}$

e $2\frac{1}{6}$ **f** $10\frac{1}{8}$ **g** $5\frac{1}{18}$ **h** $43\frac{1}{2}$

3 Perform the following additions and subtractions.

a $\frac{5}{7} + \frac{3}{4}$ **b** $\frac{14}{7} + \frac{3}{21}$ **c** $\frac{4}{5} - \frac{1}{7}$ **d** $\frac{6}{7} - \frac{3}{4}$

e $\frac{3}{4} + 1\frac{1}{3}$ **f** $1\frac{1}{3} - \frac{3}{4}$ **g** $1\frac{1}{2} + \frac{2}{3} - 1\frac{1}{6}$ **h** $1\frac{1}{2} - \frac{2}{3} + 2\frac{1}{4}$

MULTIPLYING FRACTIONS

From time to time, a calculation will call for multiplying two fractions. To use a simple real-world example, if you have half a pizza and wish to eat half of it now and save the rest for later, how much pizza will you eat now?

Intuitively, you know this is a quarter of the pizza. Mathematically it is $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$.

Perhaps you have a quarter of a pizza, and wish to eat half of it.

Again, intuitively, you know this is an eighth. Mathematically it is $\frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$.

The simple rule to remember when multiplying fractions is to multiply the numerators together and the denominators together.

For example: $\frac{1}{2} \times \frac{1}{2} = \frac{1 \times 1}{2 \times 2} = \frac{1}{4}$ and $\frac{1}{2} \times \frac{1}{4} = \frac{1 \times 1}{2 \times 4} = \frac{1}{8}$.

EXAMPLE 28B-7 Multiplying fractions

You have two-thirds of a pizza, and wish to share this equally between two people. How much of the pizza does each person get?

$\frac{2}{3} \times \frac{1}{2} = \frac{1 \times 2}{2 \times 3} = \frac{2}{6}$

$\frac{2}{6} = \frac{2 \div 2}{6 \div 2} = \frac{1}{3}$ Divide both the numerator and the denominator by 2.

Each person gets a third of a pizza.

EXERCISE 28.5

1 Perform the following multiplications, simplifying the answers where you can.

a $\frac{1}{2} \times \frac{4}{5}$ b $\frac{1}{3} \times \frac{3}{4}$ c $\frac{5}{3} \times \frac{2}{3}$ d $\frac{5}{4} \times \frac{4}{5}$

e $\frac{4}{5} \times \frac{5}{6}$ f $\frac{1}{6} \times \frac{6}{7}$ g $2\frac{1}{2} \times \frac{3}{4}$ h $3\frac{1}{5} \times \frac{5}{12}$

28C Decimals

In Mathematical Skills Chapter 23, we used calculators to convert fractions to decimals and vice versa. A useful way of looking at decimals is to realise they are only a special sort of fraction, where the denominator is limited to multiples of 10.

To differentiate decimals from all other fractions, they are written in a different form.

0.1 is really $\frac{1}{10}$

0.12 is really $\frac{12}{100}$

0.123 is really $\frac{123}{1000}$

FRACTIONS TO DECIMALS

Converting fractions to decimals involves nothing more than converting the fraction's denominator to a multiple of 10.

EXAMPLE 28C-1 Converting fractions to decimals

Convert the following fractions to a decimal number.

a $\frac{1}{2}$ **b** $\frac{1}{4}$

a $\frac{1}{2} = \frac{1 \times 5}{2 \times 5}$ Multiply the numerator and denominator by 5.

$= \frac{5}{10}$ Convert the fraction over 10 to decimal form.

$= 0.5$

b $\frac{1}{4} = \frac{1 \times 25}{4 \times 25}$ Multiply the numerator and denominator by 25.

$= \frac{25}{100}$ Convert the fraction over 100 to decimal form.

$= 0.25$

NOTE In part **a**, 2 divides evenly into 10 to give 5. In part **b**, 4 does not divide evenly into 10, but divides evenly into 100 to give 25.

But what if we must convert $\frac{1}{3}$ to a decimal? As 3 does not divide evenly into 10, 100 or even 1000, what do we do?

In this case, you need to remember a set of basic fraction to decimal conversions. The smallest number of these you can get away with are listed in the table on the right.

For example, to get the decimal equivalent:

- of $\frac{2}{3}$, multiply $\frac{1}{3}$ by 2 ($2 \times 0.333 = 0.666$)
- of $\frac{3}{5}$, multiply $\frac{1}{5}$ by 3 ($3 \times 0.2 = 0.6$).

These basic conversions must be committed to memory. If you remember all of these, you may never need to do the calculations given in the previous examples.

Fraction	Decimal
$\frac{1}{2}$	0.5
$\frac{1}{3}$	≈0.333
$\frac{1}{4}$	0.25
$\frac{1}{5}$	0.2
$\frac{1}{6}$	≈0.167
$\frac{1}{7}$	≈0.143
$\frac{1}{8}$	0.125
$\frac{1}{9}$	≈0.111

NOTE The fractions $\frac{1}{3}$, $\frac{1}{6}$, $\frac{1}{7}$ and $\frac{1}{9}$ are all rounded correct to 3 decimal places.

DECIMALS TO FRACTIONS

Since a decimal is nothing more than a fraction using a denominator that is limited to a multiple of 10, it is easy to convert them to fractions.

EXAMPLE 28C-2 Converting decimals to fractions

a Express 0.1 as a fraction.
b Express 0.456 as a fraction.

a $0.1 = 1$ out of 10 parts or $\frac{1}{10}$.

b $0.456 = \frac{456}{1000}$ There are three digits so we need three zeros after the 1.

$\frac{456 \div 2}{1000 \div 2} = \frac{228}{500}$ Divide the numerator and the denominator by 2.

$\frac{228 \div 2}{500 \div 2} = \frac{114}{250}$ Divide the numerator and the denominator by 2.

$\frac{114 \div 2}{250 \div 2} = \frac{57}{125}$ Divide the numerator and the denominator by 2.

EXERCISE 28.6

1 Express the following fractions as decimals.

a $\frac{3}{4}$ **b** $\frac{4}{25}$ **c** $\frac{2}{3}$ **d** $\frac{4}{7}$
e $6\frac{1}{3}$ **f** $4\frac{1}{4}$ **g** $2\frac{1}{9}$ **h** $\frac{5}{2}$

2 Express the following decimals as fractions.

a 0.3 **b** 0.5 **c** 0.12 **d** 0.158
e 0.201 **f** 0.333 **g** 0.143 **h** 0.25

28D Percentages

Percentage is also a special sort of fraction where the denominator is always 100. The difference between a percentage and a decimal or a fraction is that a percentage is written as 'the number part out of 100' followed by the symbol %.

1 per cent (1%) is really the fraction $\frac{1}{100}$.

10 per cent (10%) is really the fraction $\frac{10}{100}$.

100 per cent (100%) is really the fraction $\frac{100}{100}$.

DECIMALS AND FRACTIONS TO PERCENTAGES

To convert a decimal to a percentage, multiply by 100.

EXAMPLE 28D-1 Converting decimals to percentages

Express 0.234 as a percentage.

$0.234 \times 100 = 23.4\%$

To convert a fraction to a percentage, follow these steps.

Step 1: Convert the fraction to a decimal value.

Step 2: Multiply the decimal value by 100 to get the percentage.

EXAMPLE 28D-2 Converting fractions to percentages

Express $\frac{2}{3}$ as a percentage.

$\frac{2}{3} = 0.666\ldots$ Convert the fraction to a decimal value.

$0.666 \times 100 \approx 66.67\%$ Multiply the decimal value by 100 to get a percentage.

EXERCISE 28.7

1 Express the following decimal numbers as percentages.

a 0.05 **b** 0.5 **c** 0.15 **d** 0.2
e 0.9 **f** 0.95 **g** 0.272 **h** 0.965

2 Express the following fractions as percentages.

a $\frac{2}{3}$ **b** $\frac{2}{7}$ **c** $\frac{4}{5}$ **d** $\frac{23}{50}$
e $\frac{14}{20}$ **f** $\frac{14}{25}$ **g** $\frac{14}{50}$ **h** $\frac{87}{100}$

PERCENTAGES TO DECIMALS AND FRACTIONS

To convert a percentage to a decimal, follow these steps.

Step 1: Divide the percentage by 100.

Step 2: Express this in decimal notation.

EXAMPLE 28D-3 Converting percentages to decimals

Express 15% as a decimal.

$15\% = \frac{15}{100}$ Divide the percentage by 100.

$= 0.15$ Express this in decimal notation.

EXAMPLE 28D-4 Converting percentages to decimals

Express 15.8% as a decimal.

$15.8\% = \frac{15.8}{100}$ — Divide the percentage by 100.

$\frac{15.8 \times 10}{100 \times 10} = \frac{158}{1000}$ — Multiply the numerator and the denominator by 10.

$= 0.158$ — Express this in decimal notation.

So $15.8\% = 0.158$

NOTE

We cannot have a mixed fraction and decimal. We need complete fractional notation. To do this here we need to convert 15.8 to a whole number.

To convert a percentage to a fraction is the reverse process of that described previously for fractions to percentages.

Step 1: Divide the percentage by 100 (move the decimal point two places to the *left*).

Step 2: Convert the decimal to a fraction.

Step 3: Simplify the fraction if possible.

EXAMPLE 28D-5 Converting percentages to fractions

Express 10% as a fraction.

$\frac{10}{100} = 0.1$ — Divide the percentage by 100.

$0.1 = \frac{1}{10}$ — Convert the decimal to a fraction over 10.

EXAMPLE 28D-6 Converting percentages to fractions

Express 10.5% as a fraction.

$\frac{10.5}{100} = 0.105$ — Divide the percentage by 100.

$0.105 = \frac{105}{1000}$ — Convert the decimal to a fraction over 1000.

$\frac{105 \div 5}{1000 \div 5} = \frac{21}{200}$ — Divide the numerator and the denominator by 5.

EXERCISE 28.8

1 Express the following percentages as decimals.

a 10% b 2% c 34.5% d 12% e 23.05%

f 16% g 9% h 0.5% i 0.05% j 9.65%

2 Express the following percentages as fractions.

a 25% b 11% c 6% d 42.9% e 16.7%

f 75% g 35.3% h 20% i 33% j 99%

28E Powers of 10

When expressing very large or very small numbers, it is more convenient to write them in 'powers of 10' notation. Being able to easily use this form of writing numbers is important when using metric units.

For example, 20 000 can be written as $2 \times 10\,000$ or 20×1000.

In powers of 10 notation these become:

$$20\,000 = 2 \times 10\,000 = 2 \times 10^4$$

or

$$20\,000 = 20 \times 1000 = 20 \times 10^3$$

When using the metric system, we generally use only the powers of 10 that are multiples of 3: these are the blue numbers in the table below.

Thus, when dealing with large or small numbers to be used with metric units, we convert the number to the nearest power of 10 that is a multiple of 3. In the example above, we would use 20 000 as 20×10^3.

The table below shows the powers of 10 and the numbers they represent.

Power of 10 form	Number	Metric multiple
10^9	1 000 000 000	giga- (G)
10^6	1 000 000	mega- (M)
10^5	100 000	
10^4	10 000	
10^3	1000	kilo- (k)
10^2	100	
10^1	10	
10^0	1	
10^{-1}	0.1	
10^{-2}	0.01	
10^{-3}	0.001	milli- (m)
10^{-4}	0.0001	
10^{-5}	0.00 001	
10^{-6}	0.000 001	micro- (μ)

EXERCISE 28.9

1 Convert the following numbers to the nearest power of 10 that is a multiple of 3.

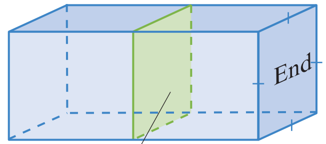
a 30 000 b 245 000 000 c 245 000 d 0.002

e 0.0004 f 4532 g 0.043 h 0.2×10^{-2}

CHAPTER 29
Technical drawing

29A Naming shapes

Solids that have identical polygonal ends joined by rectangular faces are known as *prisms.* Consider a box with square ends. If the solid is cut parallel to the end, the cross-section is identical at the ends. Prisms are named according to the shape of their flat ends (cross-section or base).

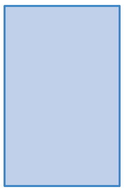


Square cross-section

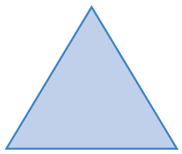
> **NOTE**
> The flat end is sometimes called the base.

This iPhone is close to a rectangular prism. This Toblerone is an example of a triangular prism. This pencil is an example of a hexagonal prism.

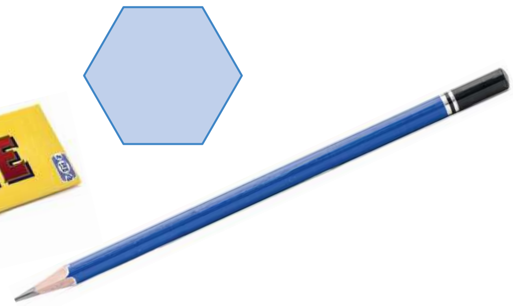
Rectanglar prism



Triangular prism

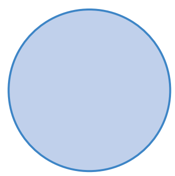


Hexagonal prism



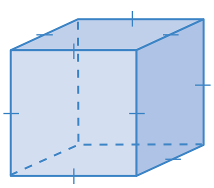
Cylinders

Although cylinders have uniform cross-sections, they are not prisms, as their other face is curved. Many objects are cylindrical, such as batteries.

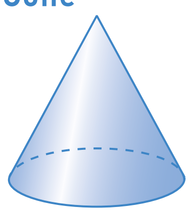


The names of other useful solids are given below.

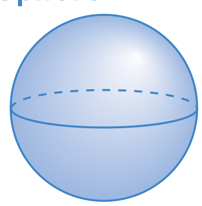
Cube



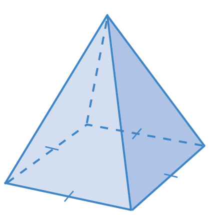
Cone



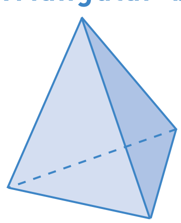
Sphere



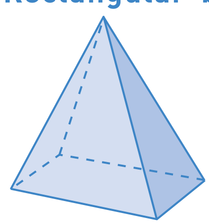
Square-based



Triangular-based



Rectangular-based



EXERCISE 29.1

1 Some of the solids above are called tapering solids. Why do think they are called tapering and which ones are they?

29B Drawing solids

Drawing a solid on a sheet of paper can cause some difficulties. Trying to represent something that has three dimensions (length, breadth and depth) on a plane that has only two dimensions (length and breadth) is not easy. The methods used to draw a solid on a two-dimensional plane are called projections or perspective drawings.

DRAWING RECTANGULAR PRISMS

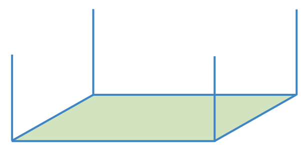
A rectangular prism has six rectangular faces. The simplest method of representing this solid is freehand. However, to represent more precise rectangular prisms, we can use oblique projection, isometric projection and perspective drawing. Building the solids first with cubes will make them easier to draw.

Freehand

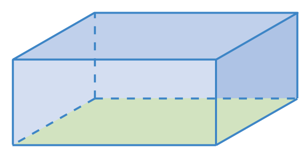
We can use the following steps to draw a rectangular prism freehand.

Step 1: Draw a parallelogram to represent the base. We use parallelograms to represent square and rectangular faces when drawing solids. Can you see why?

Step 2: Draw four lines of the same height vertically from each of the corners of the base.



Step 3: Complete the prism by drawing another parallelogram for the top. What do you notice about this parallelogram?



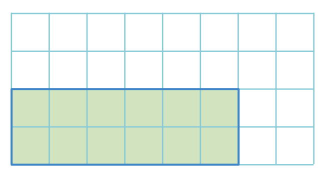
NOTE

The dotted lines shown here mean that the edge cannot actually be seen from this viewing angle.

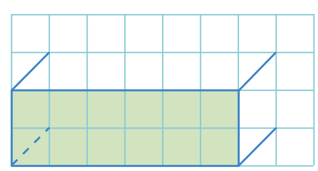
Oblique projection

This is the most common method used to illustrate more precise rectangular prisms. It uses lines that are slanting (oblique) projection. Squared grid paper helps to show angles of 45°. The steps below illustrate the method.

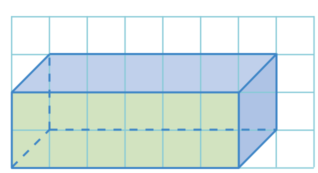
Step 1: Draw the front face of the solid.



Step 2: From each vertex (corner) draw lines back at an angle of 45° for each of the edges, making their lengths slightly shorter than they should actually be.

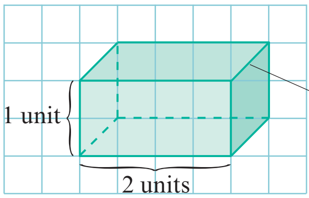


Step 3: Complete the drawing by joining the appropriate vertices (corners).



EXAMPLE 29B-1 Drawing an oblique projection

Draw an oblique projection for a rectangular box, 2 units long by 1 unit wide by 1 unit high (that is, a $2 \times 1 \times 1$ prism).



EXERCISE 29.2

1 Draw a freehand sketch of the following.
   a triangular-based pyramid
   b cylinder
   c cone
   d square-based pyramid
   e hexagonal prism
   f triangular prism

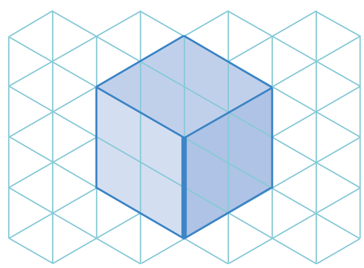
2 Draw an oblique projection for the following rectangular prisms.
   a $1 \times 1 \times 2$
   b $1 \times 2 \times 1$
   c $2 \times 2 \times 1$
   d $2 \times 1 \times 2$

3 Draw an oblique projection for the following.
   a triangular prism
   b pentagonal prism
   c hexagonal prism

Isometric projection

Isometric projections of rectangular objects are drawn starting with an edge. Lines are then drawn going back at 60° from that edge.

Special isometric graph paper, made up of the same-sized equilateral triangles joined together, is used for drawing solids in this form. Some isometric paper uses dots for the vertices of the triangles.

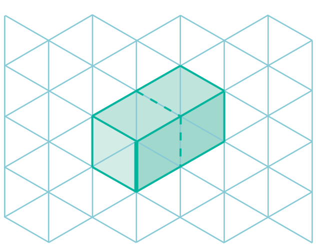


NOTE

For the cube shown, the front edge is shown darker, and this is our starting line.

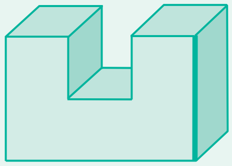
EXAMPLE 29B-2 Drawing an isometric projection

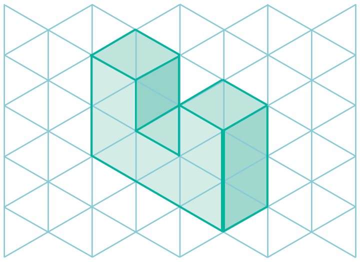
Draw an isometric projection for a rectangular box $1 \times 2 \times 1$.



EXAMPLE 29B-3 Drawing isometric projections

Draw an isometric projection for the following shape. Use the darker line as the starting edge.





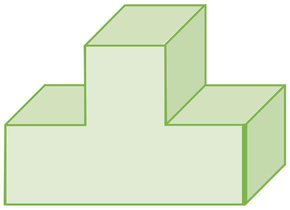
EXERCISE 29.3

1 Draw an isometric projection for the following rectangular prisms.

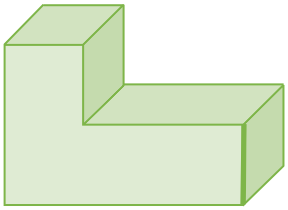
a $1 \times 1 \times 2$ prism b $1 \times 2 \times 1$ prism c $2 \times 2 \times 1$ prism d $2 \times 1 \times 2$ prism

2 Draw isometric projections for these shapes that are each 1 unit deep. (Use the darker line as the starting edge.) You may find it useful to build the shapes with cubes before you draw them.

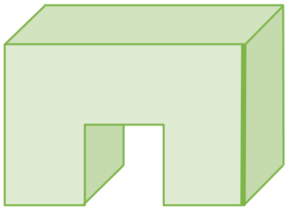
a

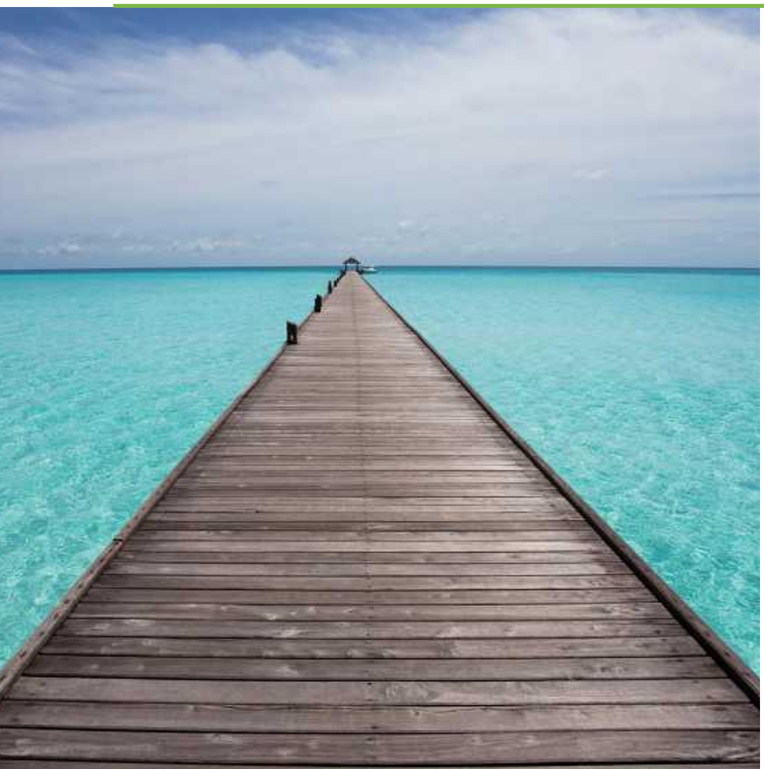


b



c





PERSPECTIVE DRAWINGS

Isometric drawings can be criticised as being non-realistic. In photographs, objects that are closer appear larger. Objects off in the distance appear to become smaller and smaller until they almost disappear.

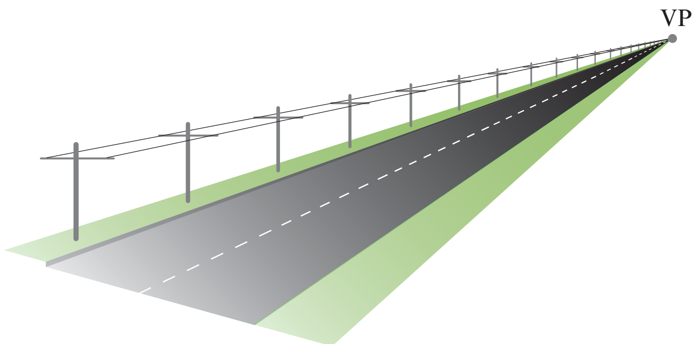
Perspective drawings are an attempt to make objects look as we see them and, in fact, as a photograph would capture them. A sense of depth is created by making objects smaller the further they are from the spectator, and parallel lines are made to converge (meet at a point) as they recede into the distance.

Perspective drawings have one vanishing point (VP) or two vanishing points (VP_1 and VP_2) that lie on the horizon. They are where the lines seem to vanish.

One-point perspective

In one-point perspective, as the distance from the spectator increases:

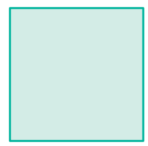
- the road edges get closer together and meet at VP
- the telegraph poles decrease in size
- the spaces between the poles also get smaller.



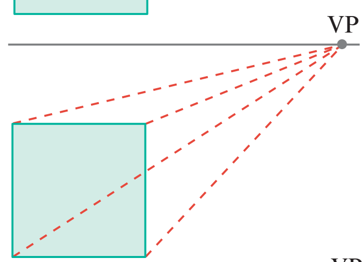
EXAMPLE 29B-4 A cube in one-point perspective

Draw a cube using one-point perspective.

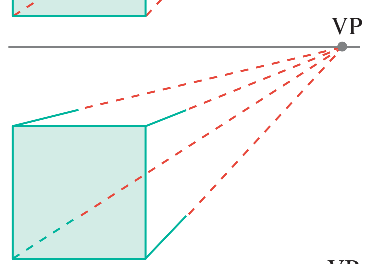
Step 1: Draw the front (nearest) face.



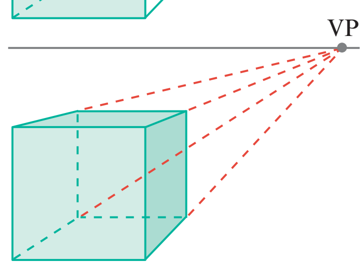
Step 2: Locate a vanishing point (VP) and draw dotted lines from each vertex.



Step 3: Draw the cubes sides from the vertices towards the vanishing point shorter than their actual length.

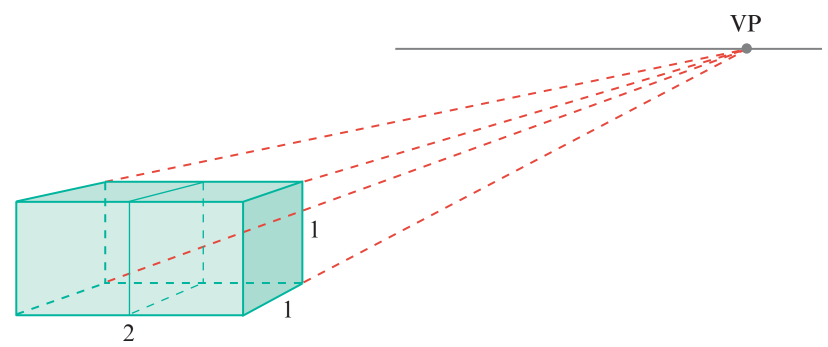


Step 4: Join the remaining vertices to complete the solid.



EXAMPLE 29B-5 A box in one-point perspective

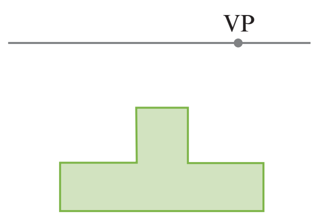
Make a perspective drawing of a rectangular box with the dimensions 2 × 1 × 1.



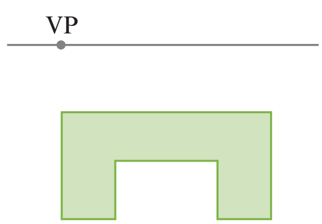
EXERCISE 29.4

1 Draw these uniform solids using the vanishing point given.

a



b

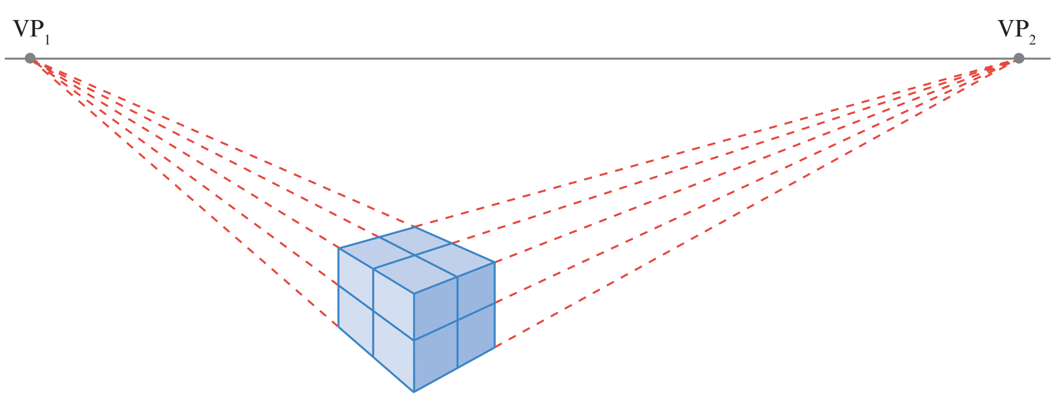


2 Make a perspective drawing of these rectangular prisms.

a 1 × 2 × 1 b 1 × 1 × 2 c 2 × 2 × 1 d 2 × 1 × 2

Two-point perspective

Consider a 2 × 2 × 2 cube. Notice the use of two vanishing points.

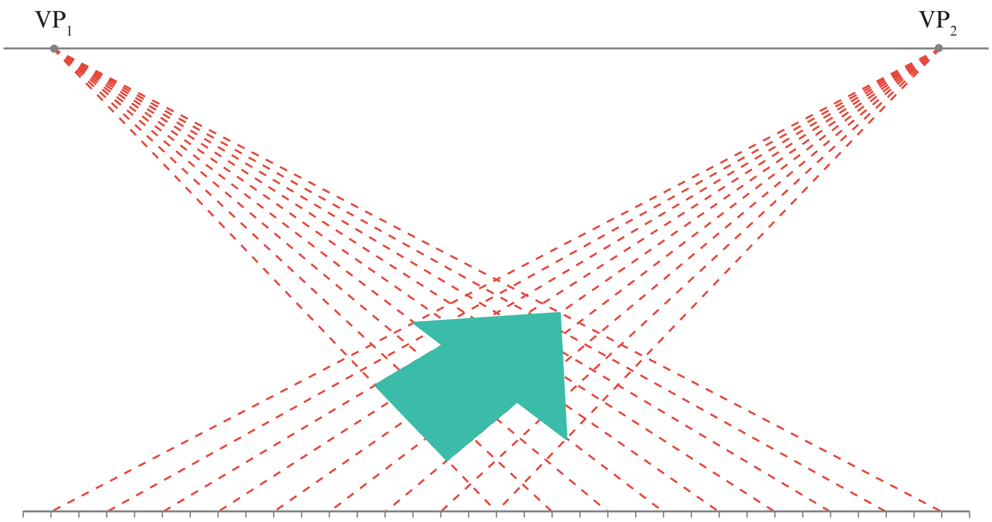


In two-point perspective drawings, the only true measurements are those on the common vertical line. All other measurements are affected by the perspective. Technical drawers use measuring lines or grids to draw the correct perspective lengths.

EXAMPLE 29B-6 An arrow in two-point perspective

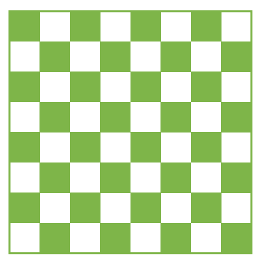
Draw an arrow using two-point perspective.

Step 1: Mark two vanishing points on the horizon, VP_1 and VP_2.
Step 2: Draw a base line. Mark off equal intervals on the base line.
Step 3: Draw lines from the base line to VP_1 and VP_2, creating a grid.
Step 4: The arrow is then drawn on the grid as shown below.

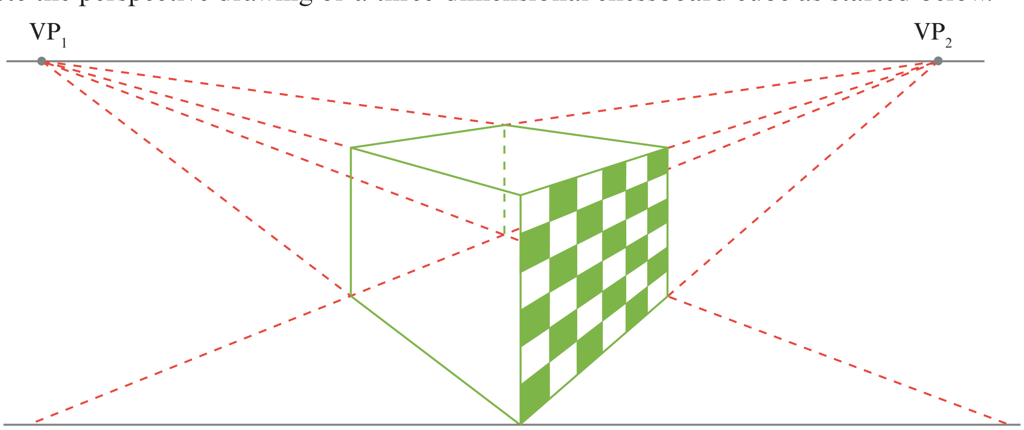


EXERCISE 29.5

1 Make a two-point perspective drawing of the chessboard shown. Show all grid lines as in Example 29B-6 above.

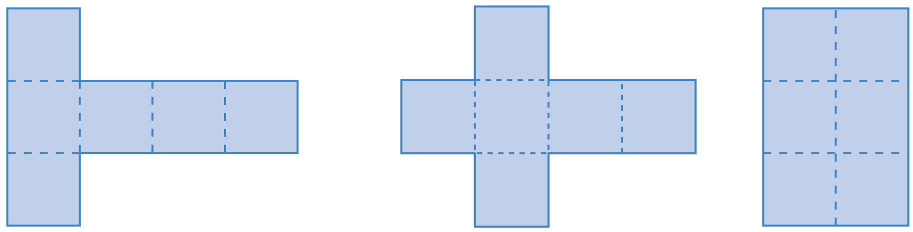


2 Complete the perspective drawing of a three-dimensional chessboard cube as started below.



29C Making solids from nets

A net is a shape or figure that is drawn in a plane and can be cut out and folded so that it forms a solid. For example, a cube has six faces, each of which is a square. Thus the net for a cube must have six squares. However, not all arrangements of the six squares in a plane can be folded to form a cube. Consider these possible arrangements:

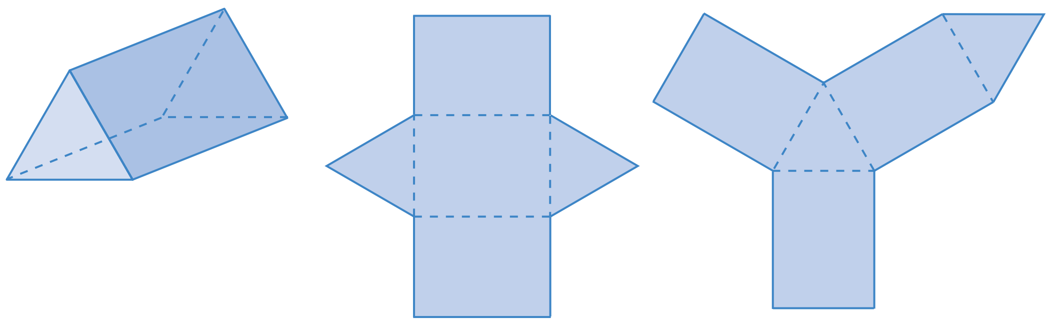


Net 1 Net 2 Net 3

Nets 1 and 2 can be folded into a cube, but net 3 cannot be folded into a cube.

Nets can be drawn for all rectangular and non-rectangular prisms, as well as pyramids. Just as there is more than one net for a cube, other shapes can also have more than one net each.

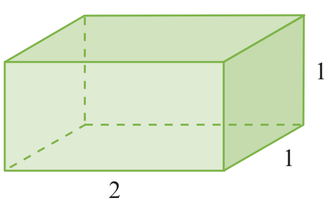
For example, this triangular prism can have either of the nets shown below.



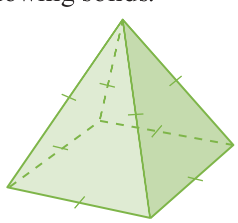
EXERCISE 29.6

1 Draw a possible net for each of the following solids.

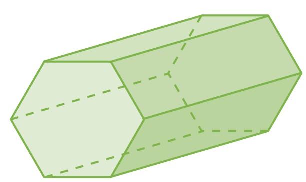
a



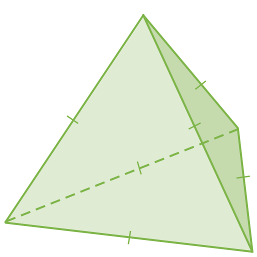
b



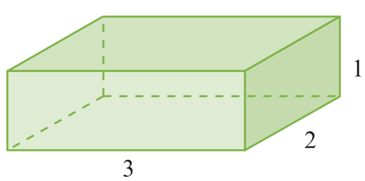
c



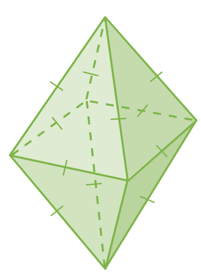
d



e

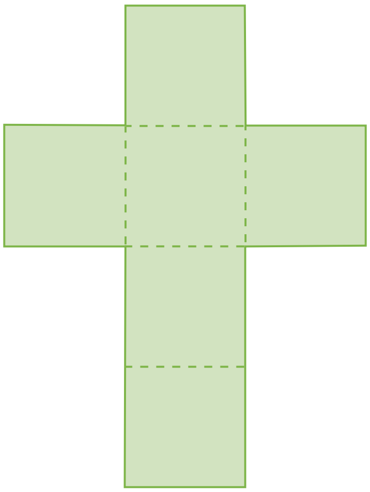


f

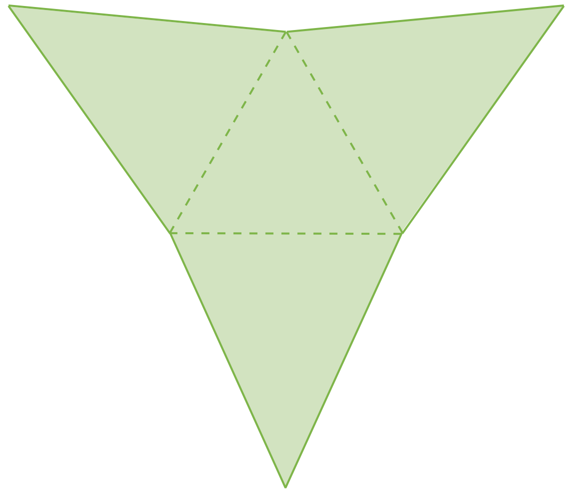


2 Get your teacher to photocopy and enlarge the following nets. Cut out each net and make it into a three-dimensional solid. Stick the edges together with tape.

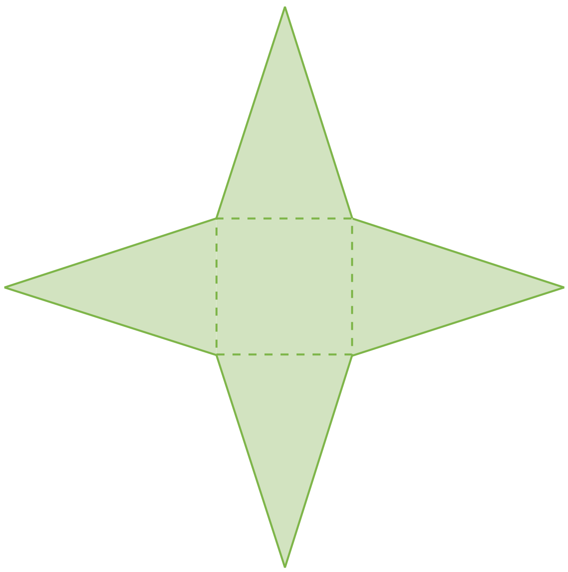
a



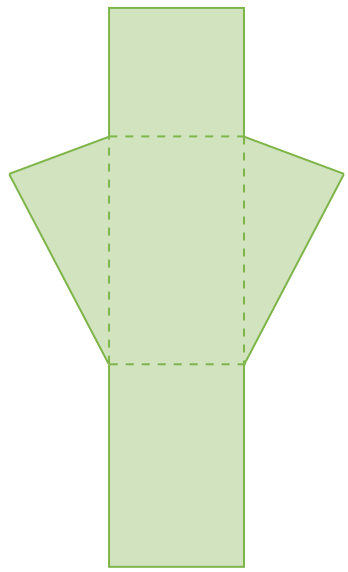
b



c



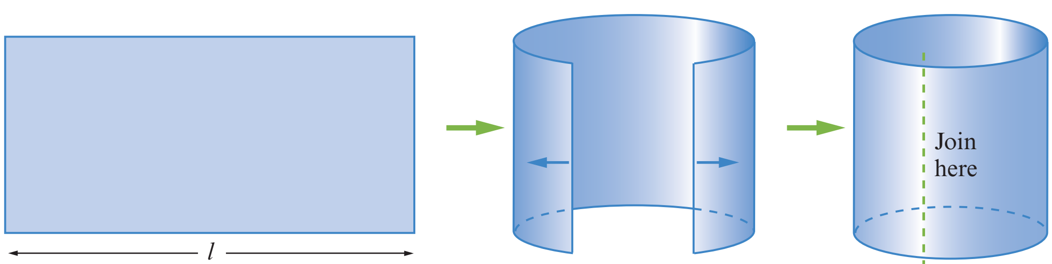
d



MAKING OTHER SHAPES FROM NETS

Cylinders

A cylinder can be made by bending a rectangular sheet of paper and joining the edges.



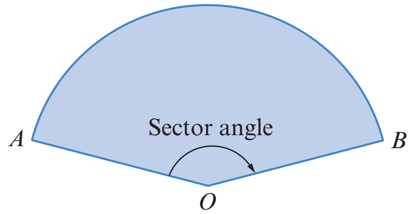
NOTE

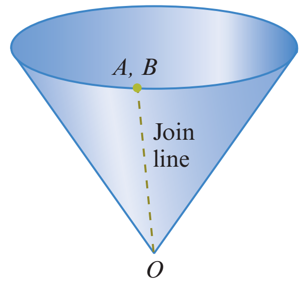
The length of the rectangular sheet of paper, l, is the circumference of the circular disks.

To create the circular disks to sit at each end of the cylinder, the radius is obtained by solving the equation $2\pi r = l$.

Cones

As shown below, a cone can be made by using a sector of a circle. The smaller the sector angle, the sharper the cone will be at O.



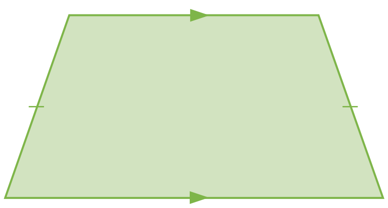


EXERCISE 29.7

1 Make a cylinder of height 8 cm and radius 4 cm from paper. You do not need to make circular ends for it.

2 Using circles of radius 5 cm, make cones with sector angles of 180°, 90° and 270°.

3 True or false? The net below can be used to make a lampshade.
Check your answer by making the net from paper.



4 Draw a net that could be used to make a lampshade similar to the one shown on the right.

29D Different views of objects

NOTE
You will find it easier to draw the solids if you build them with cubes first.

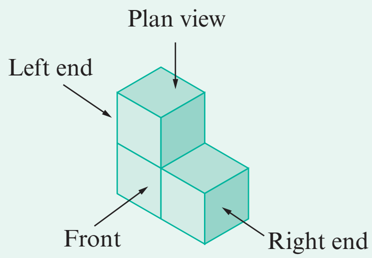
Plans and elevations are used to draw objects accurately.

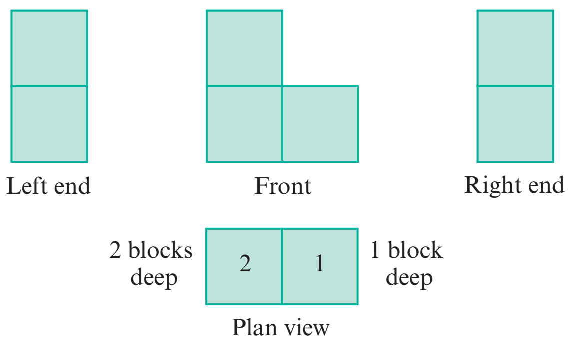
- A plan is the view looking vertically down on the object, as though you are looking at it from directly above.
- The elevation is the view looking horizontally at the object, as though you are looking at it from directly in front.
- The end elevations are the views from the right end and the left end, as though you are looking at it from each side.

Remember, when you are looking directly at a face of an object you will not be able to see how many blocks are behind it.

EXAMPLE 29D-1 Drawing different views

Draw this object from different views, showing the plan view, the left end, the front and the right end views.

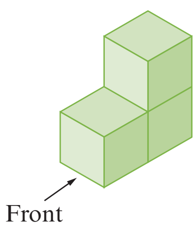




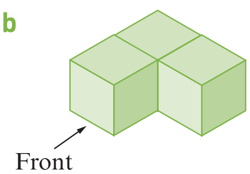
EXERCISE 29.8

1 Draw plan and elevation views for the following solids.

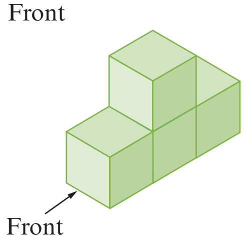
a



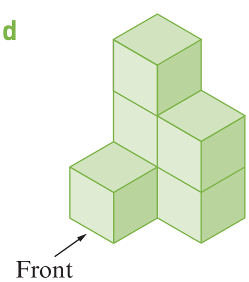
b



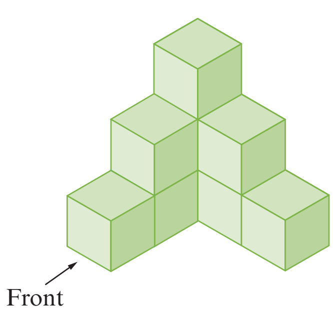
c



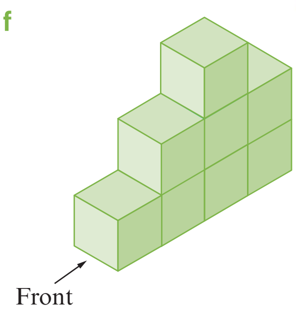
d

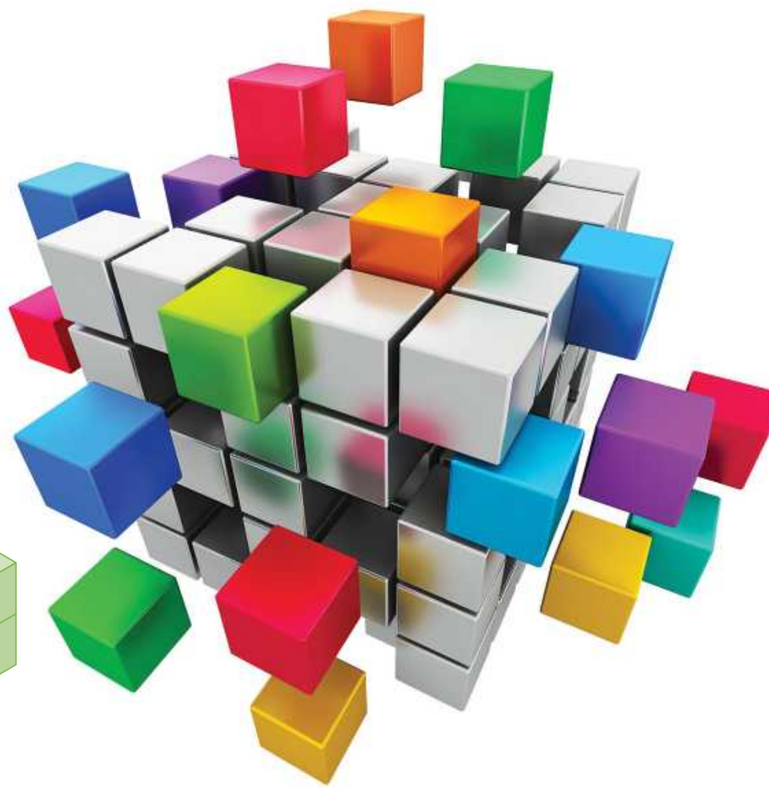


e



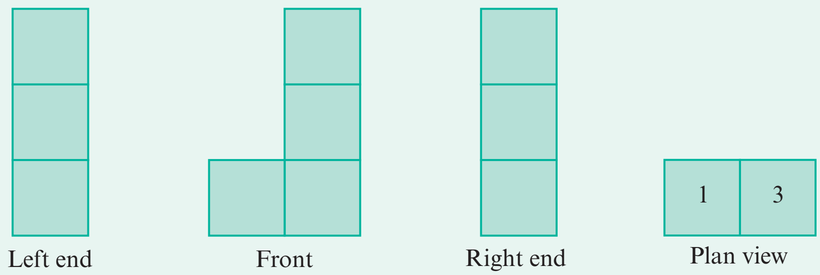
f

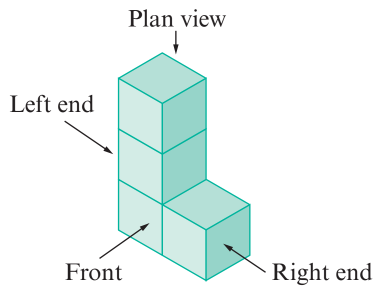




EXAMPLE 29D-2 Drawing an oblique projection

Use the following views to draw an oblique projection of the solid.

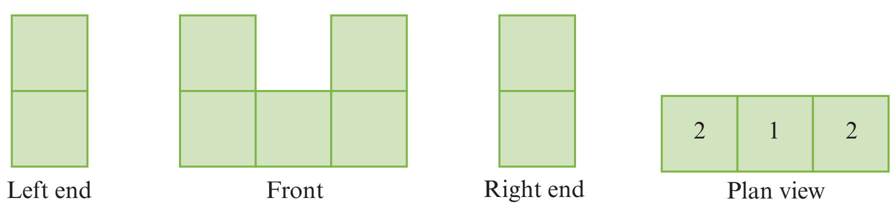




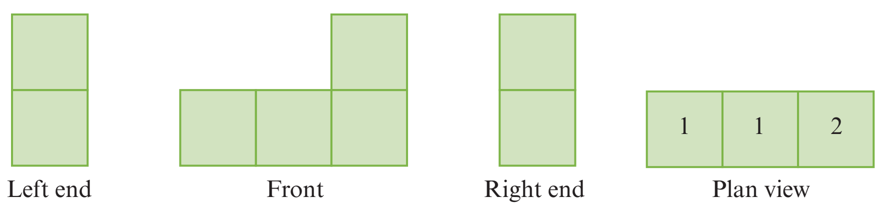
EXERCISE 29.9

1 Use the plan and elevation views to draw oblique projections of the following solids.

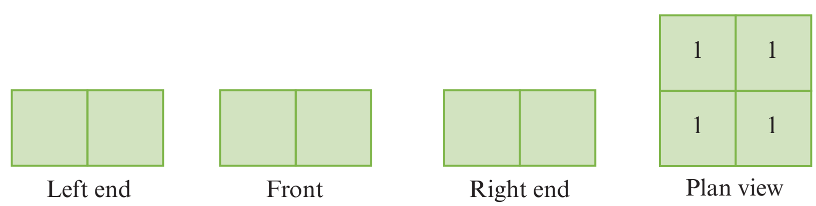
a



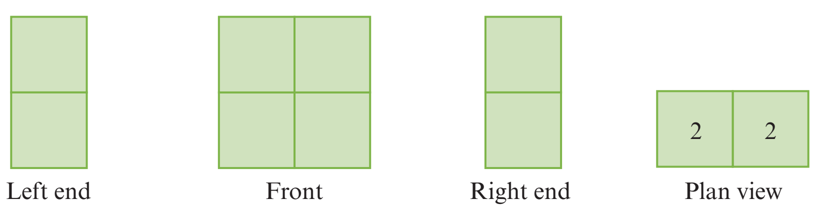
b



c

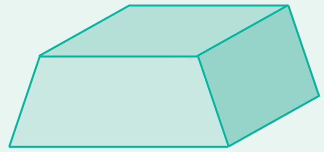


d



EXAMPLE 29D-3 Plan, front and side elevations

Draw the plan, front elevation and side elevation of the solid shown.



The plan of a solid is the view seen from directly above.

The front elevation is the view from directly in front.

The side elevation is the view from the side.

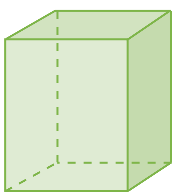
NOTE

If one elevation is the same as another, such as the front and rear elevations here, they do not need to be drawn.

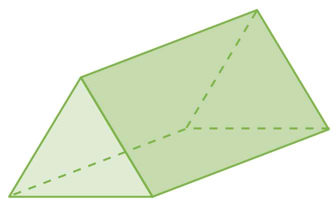
EXERCISE 29.10

1 Draw the plan, front elevation and end elevation of the following solids.

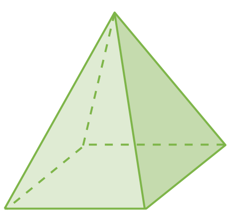
a



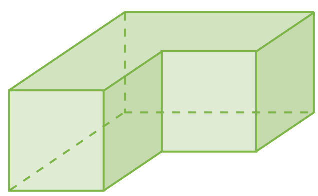
b



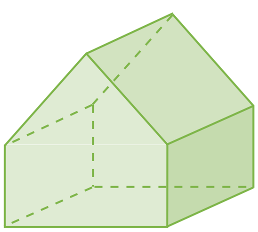
c



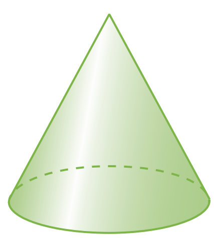
d



e



f



2 Draw a solid of your own choosing (not too complicated) and draw the plan, front elevation and end elevation of it.

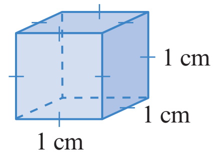
3 Objects that are closer appear larger and objects in the distance appear to become smaller until they almost disappear. How can the vanishing point help you estimate the size of distant objects?

CHAPTER 30
Volume, capacity and mass

30A Volume

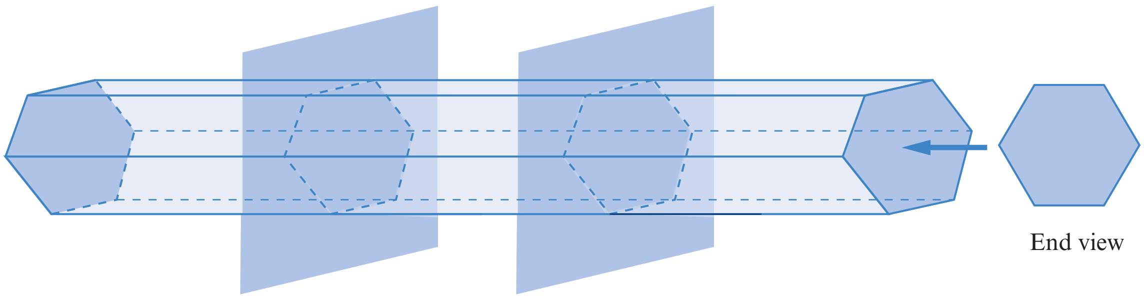
Volume is the measure of the amount of space inside a solid figure, such as a cube, ball, cylinder or pyramid.

Units of volume are always 'cubic'; that is, the volume of an object is the number of single unit cubes that would fit into the same space. For example, a cubic centimetre is equal to 1 cm × 1 cm × 1 cm.

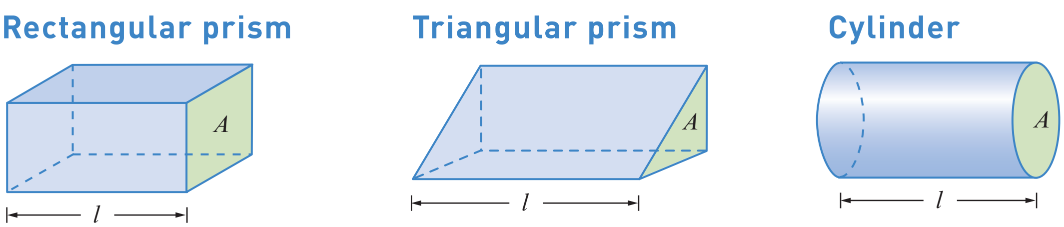


UNIFORM SOLIDS

For solids of uniform cross-section, cuts made perpendicular to the length are uniform; that is, they are always the same shape and size. For example, the hexagonal prism below has the same hexagonal cross-section when viewed from either end.



Further examples of solids with uniform cross-sections are:



obook
An Excel spreadsheet template to help you calculate the volume of a cylinder is available on your obook.

For all solids of uniform cross-section:

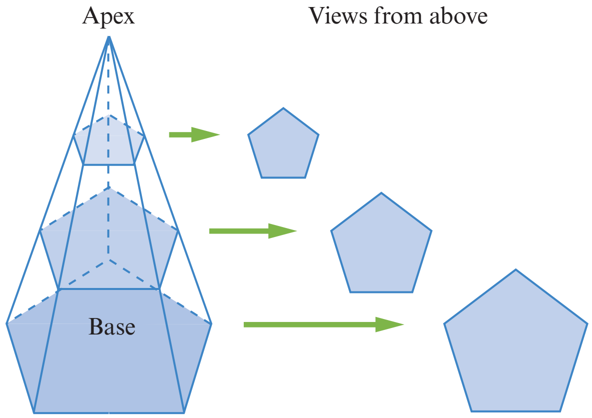
$$\text{Volume} = \text{area of cross-section} \times \text{length}$$

$$V = Al$$

where A represents the cross-sectional area and l represents the length of the solid.

TAPERING SOLIDS

For all solids that taper to a point, cuts made parallel to the base are not uniform, but they are similar (enlargements of each other). For example, this pyramid has the same shape cross-sections, but they are of different sizes.

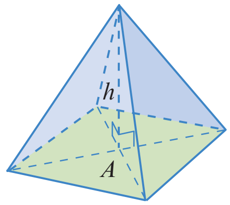


NOTE
The apex is the highest vertex in a geometric solid.

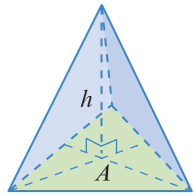
> **NOTE**
> A tetrahedron is a triangular-based pyramid.

Examples of tapering solids include:

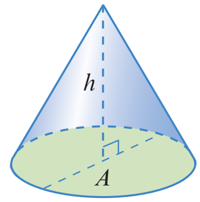
Pyramid



Tetrahedron



Cone



In all cases of tapering solids:

$$\text{Volume} = \tfrac{1}{3}(\text{area of base}) \times \text{height}$$

$$V = \tfrac{1}{3}Ah$$

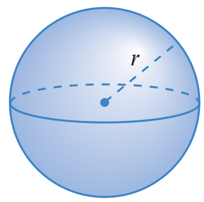
where A represents the cross-sectional area and h represents the height of the solid.

Sphere

For a sphere of radius r:

$$\text{Volume} = \tfrac{4}{3} \times \pi \times (\text{radius})^3$$

$$V = \tfrac{4}{3}\pi r^3$$

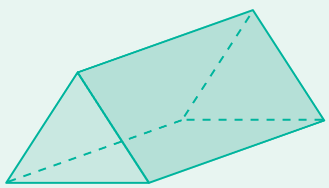


EXAMPLE 30A-1 Cross-section of solids

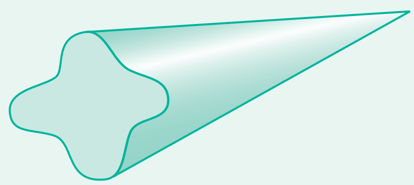
Consider each of the following shapes.

- **i** State whether it is uniform or tapering.
- **ii** If the shape is uniform, sketch the cross-section.

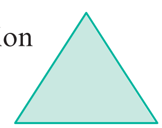
a



b



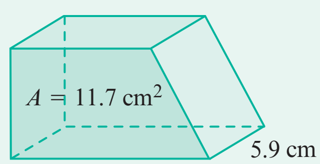
a **i** Uniform
ii Triangular cross-section



b **i** Tapering
ii Shape is not uniform.

EXAMPLE 30A-2 Volume of a uniform solid

Calculate the volume of this uniform solid correct to 1 decimal place.



$V = Al$ — This is a solid of uniform cross-section.

$= \text{area of cross-section} \times \text{length}$

$= 11.7\ \text{cm}^2 \times 5.9\ \text{cm}$

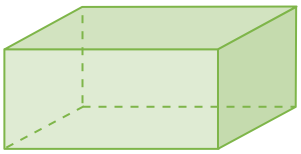
$= 69.03\ \text{cm}^3 \approx 69.0\ \text{cm}^3$

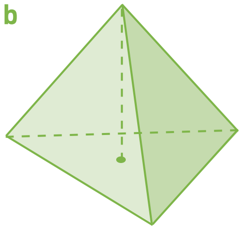
EXERCISE 30.1

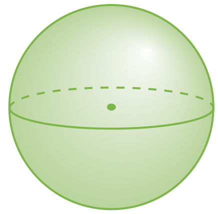
1 Consider each of the following solids.

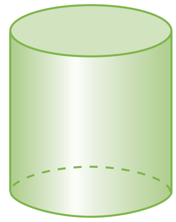
i State whether it is uniform or tapering.

ii If the shape is uniform, sketch the cross-section.

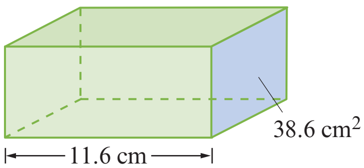
a


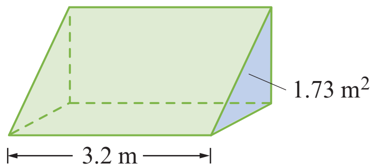
b


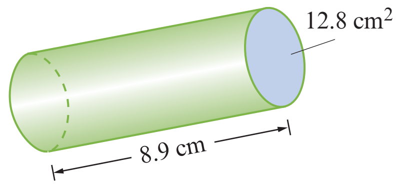
c


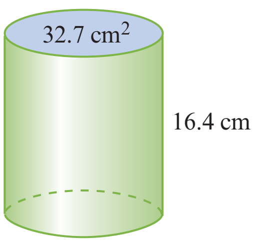
d


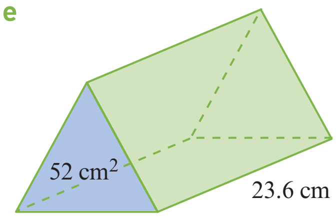
2 Calculate the volumes of these solids correct to 2 decimal places.

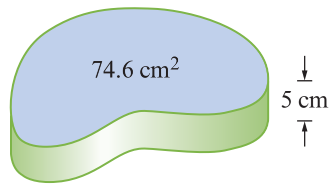
a


b


c


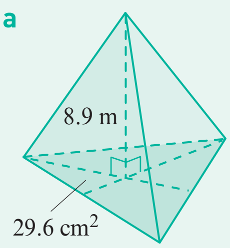
d


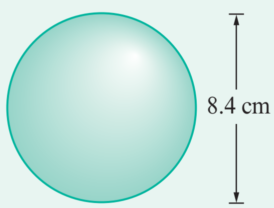
e


f


EXAMPLE 30A-3 Volume of other solids

Calculate the volume of these solids correct to 2 decimal places.

a


b


a The solid tapers to a point.

$V = \frac{1}{3}Ah$

$= \frac{1}{3} \times 29.6 \times 8.9 \text{ m}^3$

$\approx 87.81 \text{ m}^3$

b The sphere has $r = 4.2$ cm.

$V = \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \pi \times (4.2)^3$

$\approx 310.34 \text{ m}^3$

Calculator:

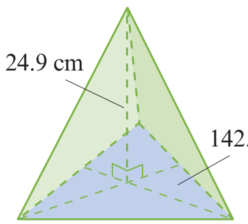
1 [$a\frac{b}{c}$] 3 [×] 29.6 [×] 8.9 [=]

4 [$a\frac{b}{c}$] 3 [×] [π] [×] 4.2 [^] 3 [=]

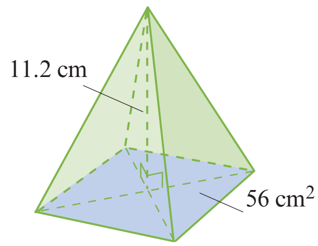
EXERCISE 30.2

1 Calculate the volume of the following solids correct to 2 decimal places.

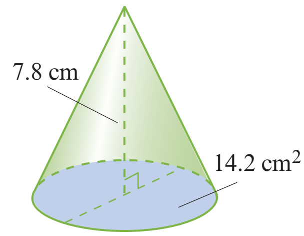
a



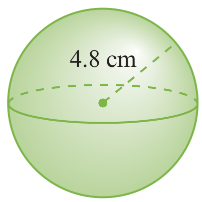
b



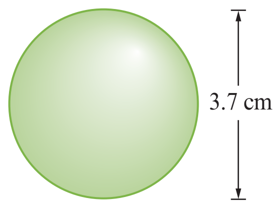
c



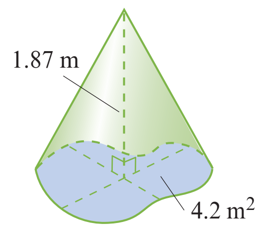
d



e

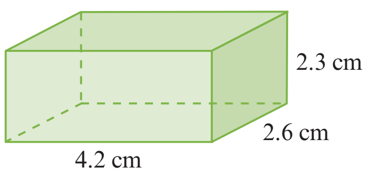


f

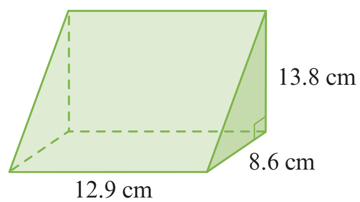


2 Calculate the volume of the following solids correct to 2 decimal places.

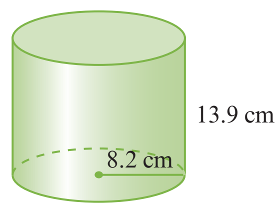
a



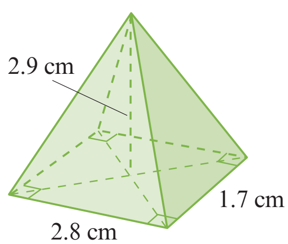
b



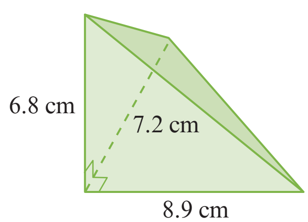
c



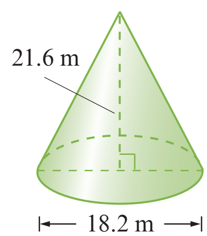
d



e

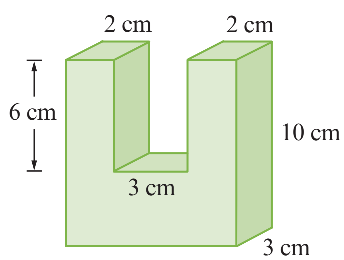


f

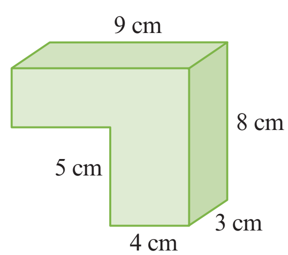


3 Calculate the volume of the following solids.

a



b



4 A circular cake tin has a radius of 20 cm and a height of 7 cm. When cake mix was added to the tin, the mixture's height was 2 cm. After the cake was cooked, it rose to 1.5 cm below the top of the tin.

a Sketch these two situations.

b Find the volume of the cake mix.

c Find the volume of the cooked cake.

d What was the percentage increase in the volume of the cake after it was cooked?

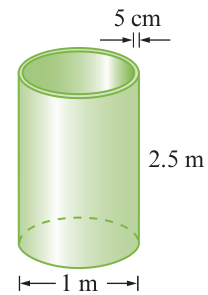
Remember: Percentage increase $= \frac{\text{increase}}{\text{original}} \times 100\%$

5 The Water Supply department uses huge concrete pipes with the dimensions shown in the diagram on the right to drain stormwater away.

a What is the outside radius of a pipe?

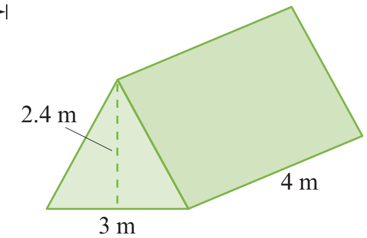
b What is the inside radius of a pipe?

c Find the volume of concrete necessary to make one pipe.



NOTE Think of the volume of the large cylinder minus the volume of the small cylinder.

6 I decided to go camping with friends. My tent has the dimensions as shown. One of my friends is very fussy about health and insists that each person in the tent should have 3 m³ of space. If I agree to my friend's condition, how many people could occupy the tent?



7 The management of a chocolate factory wishes to reduce the size of their 15 cm × 10 cm × 2 cm blocks by 10% for each measurement.

a Draw a diagram of the original block (a rectangular prism).

b What would be the new dimensions of the block?

c Find the volume of the old block.

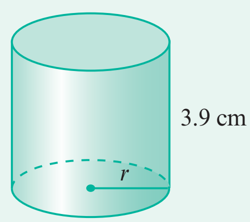
d Find the volume of the new block.

e What has been the percentage reduction in volume?

f If wrapping costs 20 cents per block regardless of size and the old block was sold at $2.70, what should the new block sell for?

EXAMPLE 30A-4 Finding the radius

Find the value of r, correct to 2 decimal places, if this solid has a volume of 54.03 cm³.



$$V = \text{area of cross-section} \times \text{length}$$

$$54.03 = \pi \times r^2 \times 3.9$$

$$\frac{54.03}{\pi \times 3.9} = \frac{\pi \times r^2 \times 3.9}{\pi \times 3.9}$$ Divide both sides by $\pi \times 3.9$.

$$\frac{54.03}{\pi \times 3.9} = r^2$$

$$r = \sqrt{\frac{54.03}{\pi \times 3.9}}$$

$$\approx 2.10$$

Calculator: 2nd √ 54.03 ÷ (π × 3.9) =

NOTE The π × 3.9 must be in brackets as it is all the denominator. Also both brackets need to be closed, as the TI-30X II automatically starts a bracket after the √.

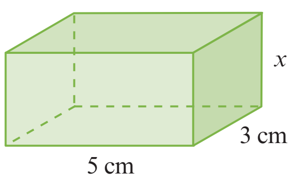
In Example 30A-4, a spreadsheet or graphics calculator could be used to solve the equation $54.03 = \pi \times r^2 \times 3.9$. The layout for the spreadsheet could be as follows.

	A	B	C
1	Radius	$\pi \times r^2 \times 3.9$	
2	1	=3.142*A2^2*3.9	
3	=A2+1		
4	↓Fill down	Fill down until the cell contents are as close as possible to 54.03.	
5			
6			

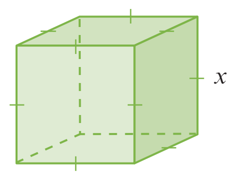
EXERCISE 30.3

1 Find x for the following solids correct to 1 decimal place.

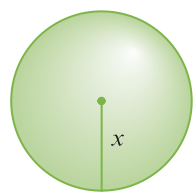
a Volume = 40 cm^3



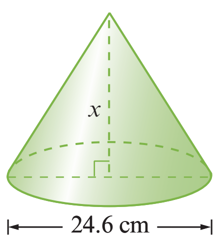
b Volume = 34.01 cm^3



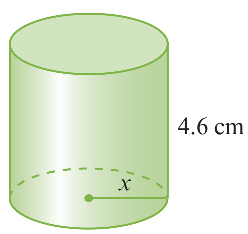
c Volume = 706 cm^3



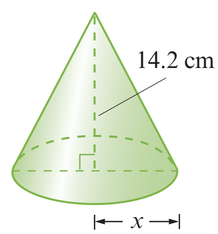
d Volume = 73.62 cm^3



e Volume = 43.75 cm^3



f Volume = 128.0 cm^3



30B Capacity

The capacity of a container is the quantity of fluid it can hold.

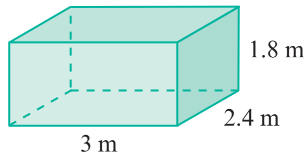
- 1 millilitre (mL) of fluid fills a 1 cm × 1 cm × 1 cm container = 1 cm^3 (1 mL = 1 cm^3).
- 1 litre (L) of fluid fills a 10 cm × 10 cm × 10 cm container (1 L = 1000 cm^3).
- 1000 litres (1 kL) of fluid fills a 1 m × 1 m × 1 m container (1000 L = 1 kL = 1 m^3).

EXAMPLE 30B-1 Capacity

How many kL of water would a 3 m × 2.4 m × 1.8 m tank hold when full?

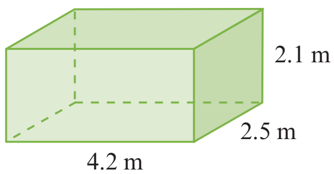
V = area of cross-section × height
$= (3 \times 2.4) \times 1.8\ \text{m}^3$
$= 12.96\ \text{m}^3$

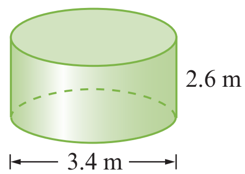
The capacity of the tank is 12.96 kL.

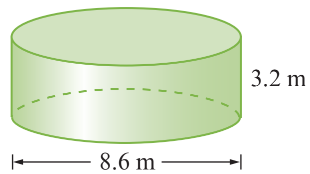


EXERCISE 30.4

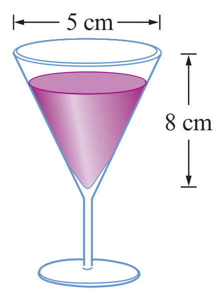
1 Find the capacity, in kL, of the following tanks correct to 2 decimal places.

a


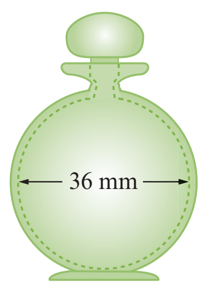
b


c


2 A conical wine glass has dimensions as shown on the right.
 a How many mL does the glass hold if it is 75% full?
 b If the wine is poured into a cylinder of the same diameter, how high will it rise?



3 A new perfume in a 36 mm (internal) diameter spherical bottle comes onto the market.
 a If the bottle is full, calculate its capacity in mL.
 b If the bottle costs $25 and the bottle and its contents cost $105, what is the cost of the perfume per bottle?
 c How much does the perfume cost per mL?



4 Which container of orange juice would give the better value:
 A a cylindrical carton of diameter 16 cm and height 19.9 cm costing $5.75?
 B a rectangular cask measuring 20 cm × 15 cm × 10 cm costing $4.50?

5 A roof has a surface area of 110 m^2. All the water falling on the roof goes into a cylindrical tank of base diameter 4 m. One night 12 mm of rain falls on the roof.
 a Find the volume of water that fell on the roof.
 b How many kL of water entered the tank?
 c By how much did the water level rise in the tank?

30C Mass

The mass of an object is the amount of matter in it.

- 1 gram (g) is the mass of 1 mL of pure water.
- 1 kilogram (kg) is the mass of 1 L of pure water.
- 1 tonne (t) is the mass of 1000 L of pure water.

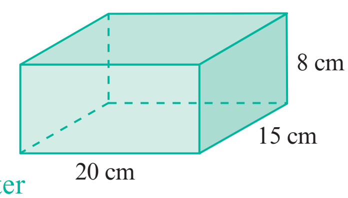
Water has a density of 1 kg/L. Some materials have higher densities as they weigh more for the same volume, and some have lower densities. Objects with densities lower than water float in water. For example, ice in a drink floats to the top.

Table of some densities

Substance	Density (g/mL)	Substance	Density (g/mL)
Cork	0.24	Water	1.00
Pine	0.65	Sugar	1.59
Olive oil	0.91	Steel	7.82
Ice	0.92	Mercury	13.55
Paper	0.93	Uranium	18.97

EXAMPLE 30C-1 Mass of water

Find the mass of water that will fill a rectangular container with internal measurements 20 cm × 15 cm × 8 cm.

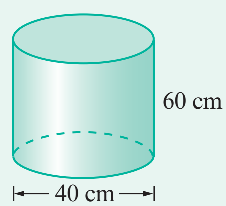


$V = 20 \times 15 \times 8$
$= 2400\ \text{cm}^3$
Capacity $= 2400$ mL $1\ \text{cm}^3 = 1$ mL
$= 2.4$ L
Mass $= 2.4$ kg   1 kg is the mass of 1 L of water

EXAMPLE 30C-2 Mass of a chemical

NOTE Fluids other than water may be heavier or lighter than water.

A chemical that is 1.6 times denser than water fills this drum. What mass of chemical is in the drum?



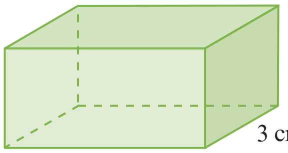
$V =$ area of cross-section × length
$V = \pi \times 20^2 \times 60$ cm   Area of circle $= \pi r^2$
$= 75\,398.22\ \text{cm}^3$
Capacity $= 75\,398.22$ mL ≈ 75.40 L
Mass $= 75.40 \times 1.6$ kg   1 L water = 1 kg
≈ 120.64 kg

EXERCISE 30.5

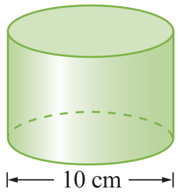
Internal measurements are given in all of the following problems.

1 Find the mass of water that fills these containers.

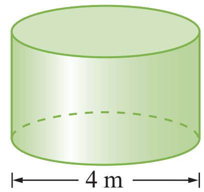
a



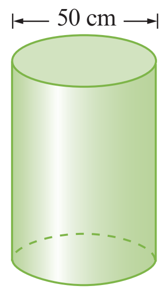
b



c

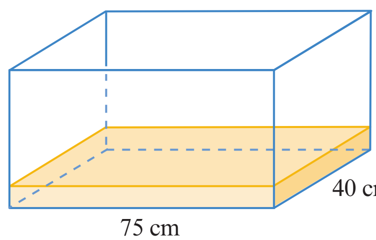


2 Olive oil is transported in drums of the size shown. The density of the oil is 91% that of pure water. Find the mass of oil within a full drum.



3 A brick has a mass of 4.5 kg. How many bricks could be carried in a truck which had a load limit of 12.33 t?

4 A glass fish tank has the dimensions shown. Sand is placed in the tank to a depth of 3 cm. Water is then added and filled to a depth of 3 cm from the top of the tank. The sand is 3.7 times denser than water.



- **a** Find the volume of sand.
- **b** Find the volume of water.
- **c** What is the mass of water?
- **d** What is the mass of sand?
- **e** Find the total mass of the tank and its contents if the mass of the glass is 15.6 kg.

5 In a 500 mL container, water comes up to the 350 mL marking and the oil floating on top comes up to the 400 mL marking. Estimate the combined mass of the water and oil if the oil is 86% the density of water.

CHAPTER 31
Scales and plans

31A Enlargements and reductions

Computer programs or a photocopier can be used to enlarge and reduce the size of a drawing without changing its shape. The size of an enlargement or reduction is described by its scale factor. We call the scale factor k.

- A 170% enlargement increases the size by 170% or 1.7 $k = 1.7$.

increased by 70% becomes

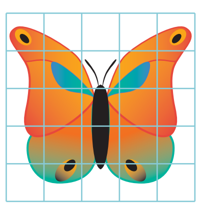
- A 70% reduction decreases the size by 70% or 0.7$k = 0.7$.

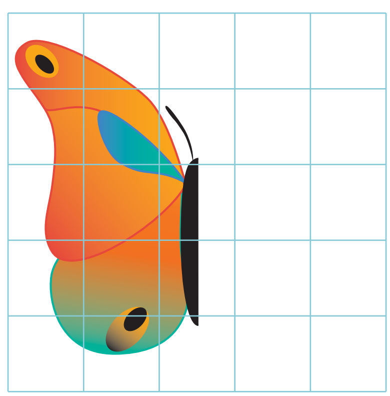
reduced by 70% becomes

EXAMPLE 31A-1 Enlarging an object

Enlarge this drawing to twice its original size.

First place a 5 mm × 5 mm grid over the original drawing.

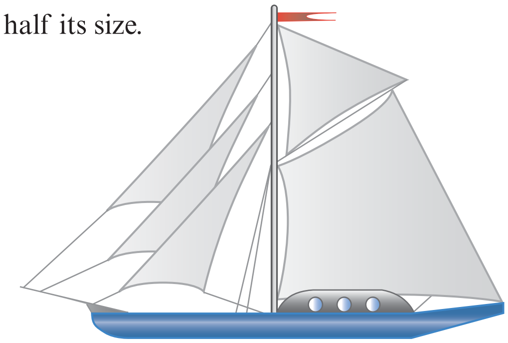




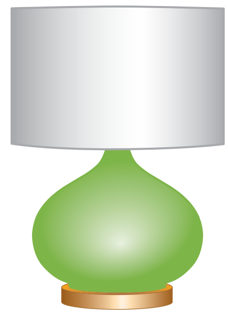
To enlarge the drawing to twice the original size (by a scale factor of 2), use a 10 mm × 10 mm grid. Copy each part of the drawing from the 5 × 5 grid squares onto the 10 × 10 grid squares. The enlarged picture is started here for you.

EXERCISE 31.1

1 Redraw this sailboat to:
  a twice its size
  b half its size.



2 Redraw this lamp to:
  a 150% of its size
  b 70% of its size.



31B Scale drawings

Almost everything that is made by manufacturers, builders or any other tradespeople starts as a drawing. Obviously it is too hard to draw, let alone use, a full-size drawing of a ship that is 100 m long! You can also imagine the difficulty if a watchmaker had to work from a drawing the same size as the watch. In the previous section we saw that it is easy to enlarge or reduce a drawing, but how do we show by how much it has been enlarged or reduced?

Because people from many different jobs and backgrounds use plans and drawings, a standard way of showing how much bigger or smaller the drawing is than the real object has been developed.

EXPRESSING SCALE RATIOS

A scale ratio is a way of showing by how much a drawing or plan has been enlarged or reduced. It is written in the following form: two numbers separated by a colon.

Scale □ : □

Number of units on drawing   Number of units in real-life

- The scale 1 : 100 means 1 mm on the drawing represents 100 mm actual length.
- The scale 1 : 50 means 1 mm on the drawing represents 50 mm actual length.
- The scale 1 : 1 means 1 mm on the drawing represents 1 mm actual length; that is, the plan is drawn full size.

Some examples are shown in this table.

Length measured from drawing	Scale	Actual length
60 mm	1 : 100	6000 mm (equals 6 m)
35 mm	1 : 50	1750 mm
15 mm	1 : 1	15 mm

EXERCISE 31.2

1 Complete the following table.

Length measured from drawing	Scale	Actual length
17 mm	1 : 2	
25 mm	1 : 5	
100 mm	1 : 10	
38 mm	1 : 50	
95 mm	1 : 200	

2 The lines in the table below are the lines on a plan. For each line, measure its length in mm and work out the actual length it represents using the following scales. The first one has been done for you.

	Dimensioned line	Line length	Scale		
			1 : 10	1 : 25	1 : 40
		50 mm	50 cm	1.25 m	2 m
a					
b					
c					
d					
e					
f					
g					
h					
i					

CONVERTING MEASUREMENTS TO PLAN SIZES

When measuring to draw a plan, say of an existing house, we need to measure the house and convert these measurements to the plan size. To do this, we simply take the real-life sizes and divide them by the second number in the scale ratio. Remember:

- If you want your drawing's final length to be in mm, measure the full-size object in mm.
- If you want your drawing's final length to be in cm, measure the full-size object in cm.

EXAMPLE 31B-1 Converting measurements

The outside wall of a house is 15 m long. What line length will be used to represent the wall length (in millimetres) given that a 1 : 100 scale is to be used?

15 m = 15 × 1000 mm = 15 000 mm
Line length = 15 000 mm ÷ 100 = 150 mm

EXERCISE 31.3

1 Complete the following table by finding the length to be drawn on the plan given the scale and the length measured on the job. The first one has been done for you.

Length to be drawn on the plan	Scale	Length measured on the job
150 ÷ 2 = 75 mm	1 : 2	150 mm
	1 : 5	500 mm
	1 : 10	560 mm
	1 : 50	5000 mm
	1 : 200	150 000 mm

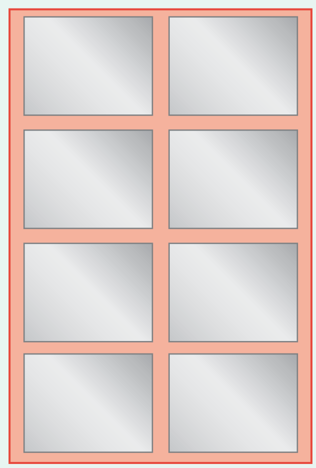
2 The following are lengths from real-life situations. Your job is to draw a line using the scales given.

a 10 500 mm 1 : 100 scale
b 9750 mm 1 : 100 scale
c 5000 mm 1 : 50 scale
d 4450 mm 1 : 200 scale
e 890 mm 1 : 10 scale
f 75 mm 1 : 1 scale
g 3750 mm 1 : 10 scale
h 150 mm 1 : 2 scale

EXAMPLE 31B-2 Finding actual dimensions

This is a scale drawing of a building. Use your ruler and the scale given to find:

a the real height of the building
b the real width of the building
c the real height of the first-floor windows (to the bottom sill).



Scale is 1 : 200

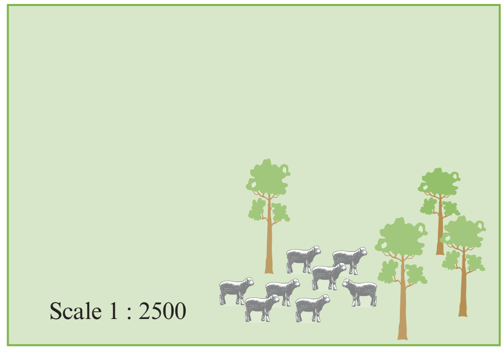
a Scaled height = 60 mm
Real height = 200 × 60 mm
= 12 000 mm = 12 m

b Scaled width = 40 mm
Real width = 200 × 40 mm
= 8000 mm = 8 m

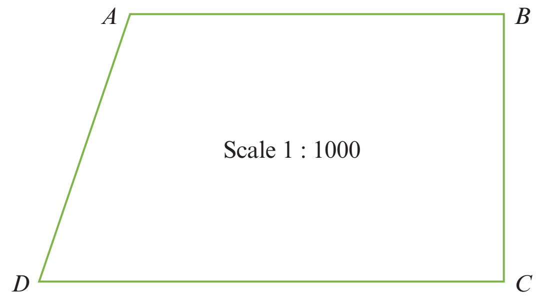
c Scaled height first-floor windows = 13 mm
Real height = 200 × 13 mm
= 2600 mm = 2.6 m

EXERCISE 31.4

1 The diagram on the right is a scale drawing of a rectangular field. Using your ruler and the scale given, find the real length of the breadth of the field.

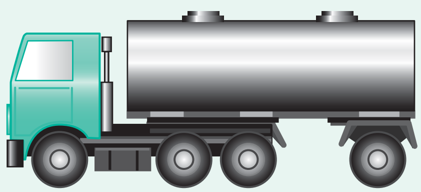


2 This is a scale drawing of a parking area. Use the scale given to find the real dimensions of the parking area.



EXAMPLE 31B-3 Converting measurements

Find the scale used in this drawing of a truck. The actual length of the truck is 5.4 m.



The length of the drawing is 54 mm. Real length is 5.4 m = 5400 mm.
We divide the scaled length by the real length.

$$\text{Scale} = \frac{\text{scaled length}}{\text{real length}}$$

$$= \frac{54 \text{ mm}}{5400 \text{ mm}}$$

$$= \frac{1}{100} \text{ or } 1 : 100$$

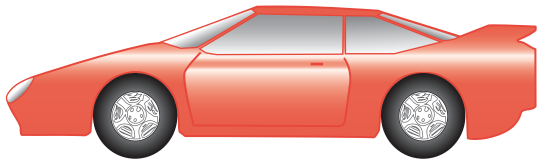
EXERCISE 31.5

1 The diagram is a scale drawing of a car.

a If the real length of the car is 5.6 m, find the scale used.

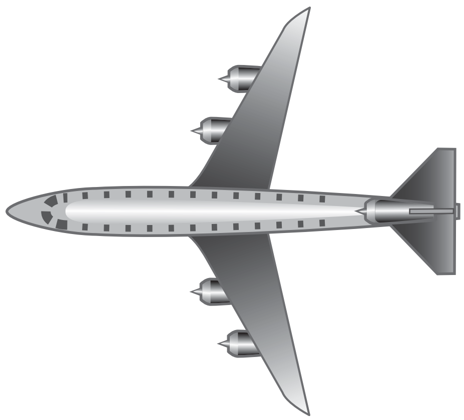
b Calculate the height of the car above the ground.

c What is the diameter of the wheel?



2 a The actual length of the plane in the scale diagram is 60 m. What is the scale?

b What is the actual wingspan of the plane?



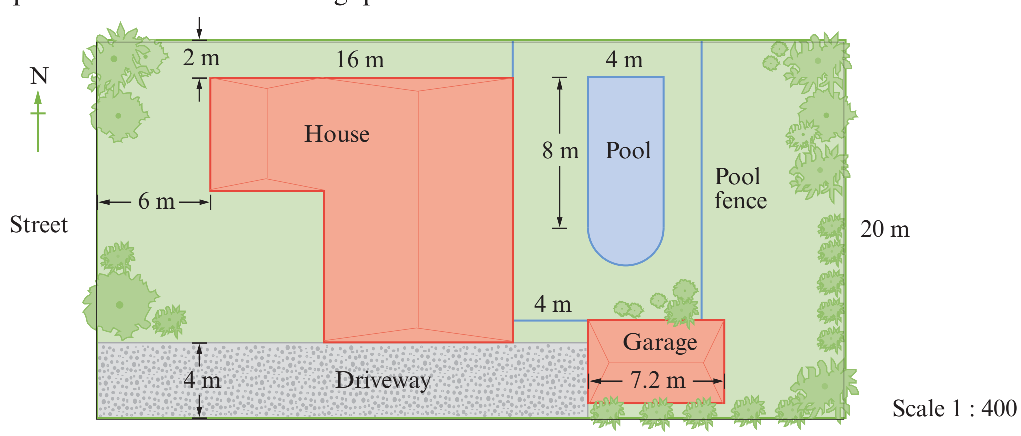
31C Building plans

Building plans are scale drawings of houses or other buildings. They include a site plan, a floor plan and side elevations so that all construction details can be determined.

A site plan is a drawing of the block of land showing the position of the residence and any other buildings or main features on the block.

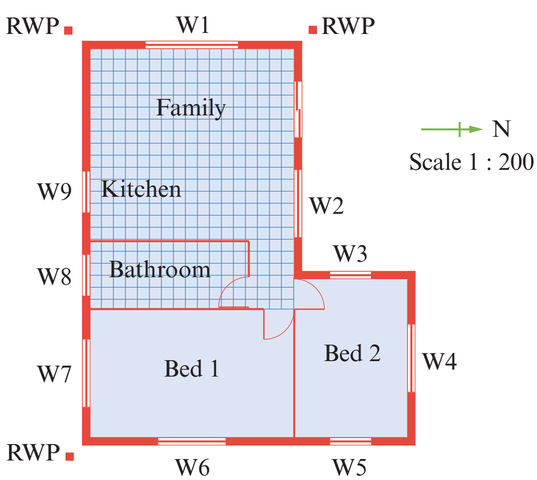
EXERCISE 31.6

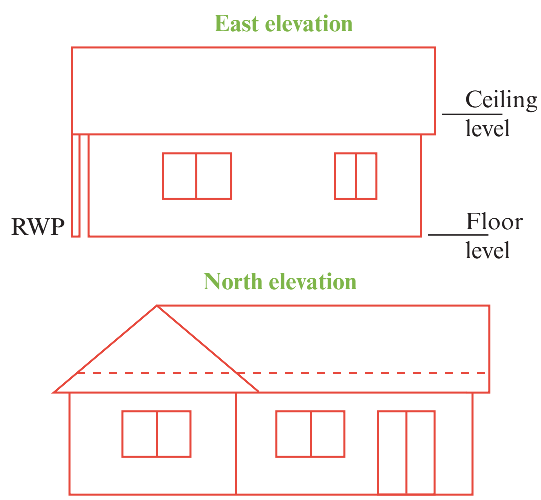
1 Use this site plan to answer the following questions.



a What are the dimensions of the block?
b Find the length of the driveway to the garage.
c What the distance between the house and the fence on the:
  i northern boundary? ii southern boundary?
d How far is the house from the street?
e How far is the garage wall from the eastern boundary?
f Calculate the area of the garage, to the nearest square metre.
g Find the total length of the pool fence.
h Calculate the area of land taken up by the pool, to the nearest square metre.
i If the pool has an average depth of 1.2 m, calculate the volume of water the pool could hold.
j Find the amount of water in the pool, in kilolitres.
k How many blocks of land of this size would fit into an area of 1 hectare? Remember: 1 ha = 1000 m^2.

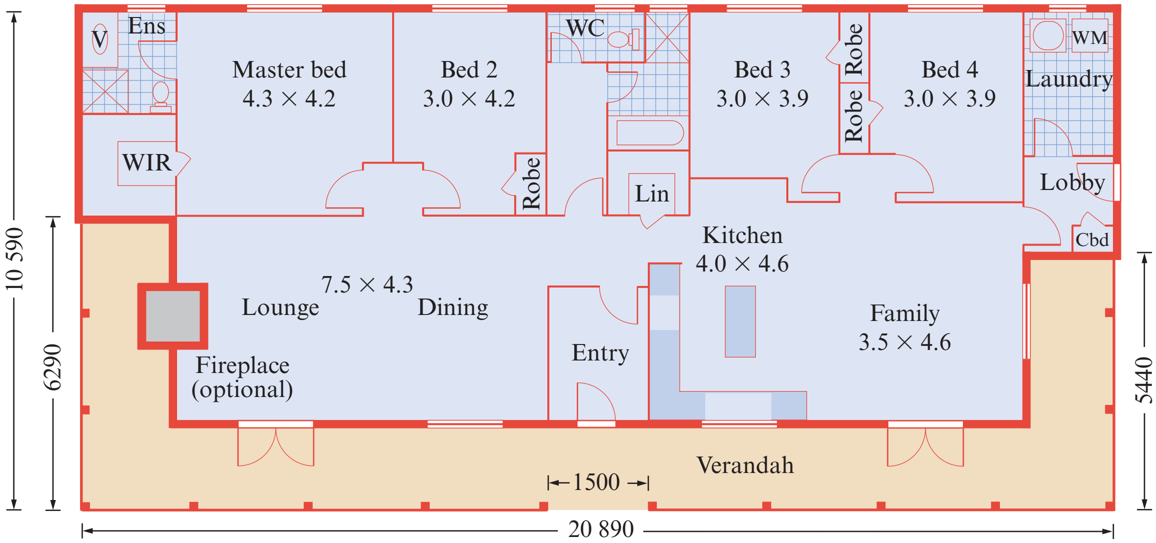
2 The diagrams below show the floor plan and elevations of a small holiday house. Use the plan to answer the following questions.





a How many of each of the following are there in the holiday house?
  i windows ii hinged doors iii sliding doors
b How many windows can be seen in the:
  i north elevation? ii east elevation?
  iii south elevation? iv west elevation?
c What is the scale used?
d Find the dimensions of:
  i the bathroom ii bedroom 1 iii bedroom 2.
e What is the size of windows W2, W5, W6 and the sliding door?
f Find the height of the ceiling above the floor.
g What is the maximum height of the roof above floor level?
h How far do the eaves extend beyond the walls of the house in the east elevation?
i Draw the south elevation.
j What area of land does the house occupy?
k If the house is built on a reinforced concrete slab of depth 250 mm, calculate the amount of concrete needed for the slab, to the nearest cubic metre.

3 Below is the floor plan of a kit home.



a Find the following symbols on the floor plan and discuss the meaning of each symbol.
   i WIR   ii LIN   iii ENS   iv WC   v V   vi WM

b i How many hinged doors does the house have (excluding doors to cupboards and wardrobes)?
   ii How many windows does the house have?

c How many vertical posts support the verandah roof?

d Find the number of wardrobes needed.

e What is the length and width of the house (including the verandah)?

f Guttering is needed around the four sides of the house. Calculate the total length of guttering needed.

g Find the area of the house, including the verandah, to the nearest square metre.

h If built with weatherboard cladding, the house costs \$180 300 to complete. What is the cost per square metre of the house, to the nearest dollar?

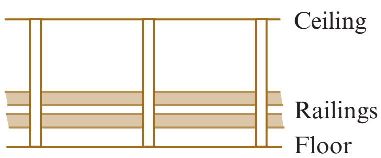
i If constructed on a concrete slab, 250 mm thick, what volume of concrete, in cubic metres, is needed?

j Calculate the scale used for this floor plan.

k Use the scale to find the dimensions of the laundry.

l Calculate the area of the ensuite to the master bedroom.

m A double railing made of timber is built around the verandah, as shown below. Calculate the number of metres of timber needed (leave a gap in front of the house entry and ignore the width of the posts).



n Bedrooms 2, 3 and 4 are to be carpeted so that the least amount of carpet is used. If carpet comes in rolls 3.6 m wide, how many metres of carpet will be needed?

o The master bedroom is also to be carpeted.

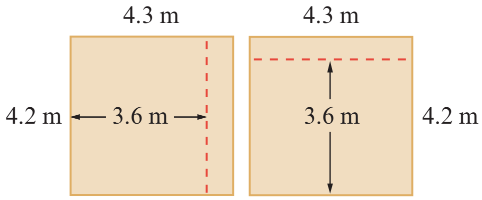


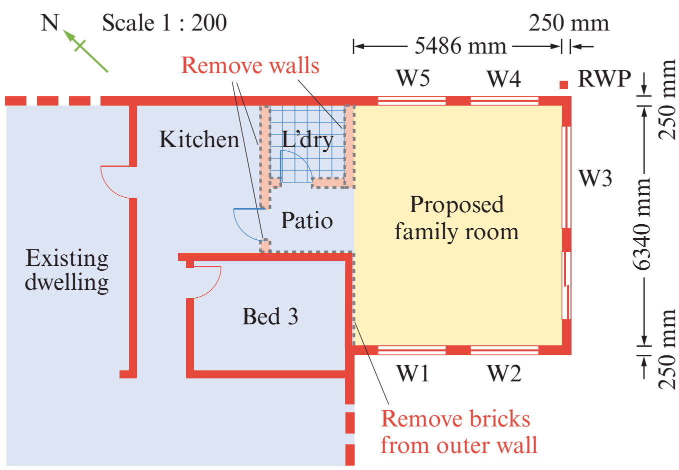
Diagram A Diagram B

 i If the carpet is laid as in diagram A, how many metres of carpet would be needed?

 ii If the carpet is laid as in diagram B, how many metres of carpet would be needed?

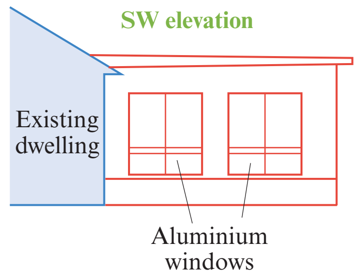
p Find the total cost of carpeting the four bedrooms if carpet costs $95 per linear metre.

4 These diagrams are the plan and elevations for a proposed house extension.

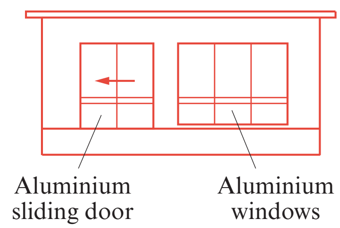


a What is the scale on the plan and elevations?
b What are the internal dimensions of the proposed family room?
c Find the thickness of the walls of the extension.
d What length of existing wall has to be removed?
e Determine the dimensions of the new kitchen.
f What length of guttering is needed across the rear of the family room?
g What length of downpipe is needed at the rear of the extension?
h What is the height of the ceiling above the floor?
i Use the plan and elevations to determine the dimensions of windows W1, W3 and W4.
j What is the size of the sliding door?
k Calculate the area of the new room in square metres.
l Calculate the area of the new kitchen in square metres.

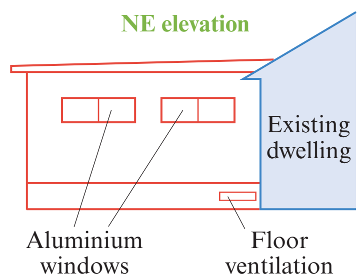
SW elevation



SE elevation



NE elevation



CHAPTER 32
Triangle calculations

32A Pythagoras' theorem

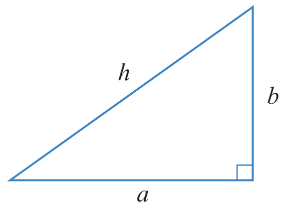
A mathematical formula is a special type of equation that shows the relationship between different variables. In mathematical formulas, variables are letters that stand for numbers we want to find out but don't know yet.

One of the most important mathematical formulas is known as Pythagoras' theorem.

> **NOTE**
> In mathematics, a *theorem* is a statement that has been proved to be true.

PYTHAGORAS' THEOREM

Pythagoras' theorem states that, for a right-angled triangle, the length of the hypotenuse squared is equal to the sum of the squares of the lengths of the other two sides. The hypotenuse is opposite the right angle and is always the longest side. Pythagoras' theorem is:



$$h^2 = a^2 + b^2$$

> **NOTE**
> The hypotenuse is the longest side of a right-angled triangle. It is always the side opposite the right angle.

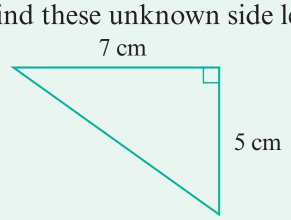
- The theorem is used to find the length of the third side of a right-angled triangle, given the other two sides.
  For example, knowing a and b $\quad h = \sqrt{a^2 + b^2}$
  For example, knowing a and h $\quad b = \sqrt{h^2 - a^2}$
- The theorem can also be used to determine whether a triangle is right-angled, given only the length of its sides.

In the building and construction industry, Pythagoras' theorem is constantly used to determine unknown lengths of sides of triangles. It is also used to check for right angles or near right angles.

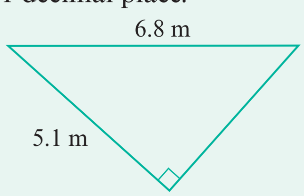
EXAMPLE 32A-1 Finding the unknown side length

Find these unknown side lengths correct to 1 decimal place.

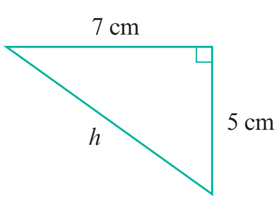
a



b

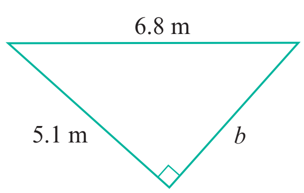


a



$$h = \sqrt{5^2 + 7^2}$$
$$\approx 8.6 \text{ cm}$$

b



$$h = \sqrt{6.8^2 - 5.1^2}$$
$$\approx 4.5 \text{ cm}$$

Calculator

a 2nd √ (5 x^2 + 7 x^2) =

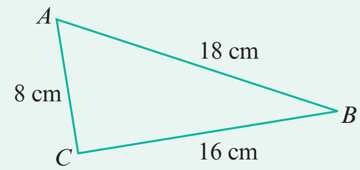
b 2nd √ (6.8 x^2 − 5.1 x^2) =

> **NOTE**
> ≈ means 'approximately equal to'.

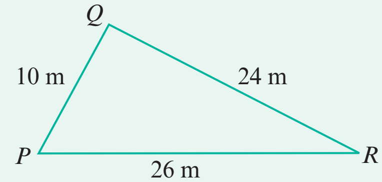
EXAMPLE 32A-2 Is it a right-angled triangle?

Which of the following triangles are actually right-angled triangles?

a



b



The hypotenuse, h, is the longest side.
For the triangle to be right-angled, the hypotenuse squared must equal the sum of squares of the two shorter sides.

a $h^2 = 18^2 = 324$

$a^2 + b^2 = 8^2 + 16^2$

$= 64 + 256$

$= 320 \neq 324$

Triangle ABC is not right-angled.

b $h^2 = 26^2 = 676$

$a^2 + b^2 = 10^2 + 24^2$

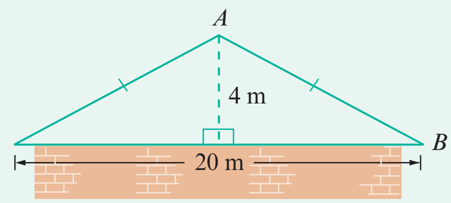
$= 100 + 576$

$= 676$

Triangle PQR is right-angled at Q.

EXAMPLE 32A-3 Practical application

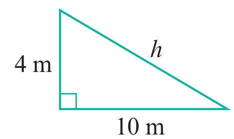
Find the length of the truss AB for this roof structure to 2 decimal places.



$h = \sqrt{4^2 + 10^2}$ Use Pythagoras' rule.

$\approx 10.7703\ldots$

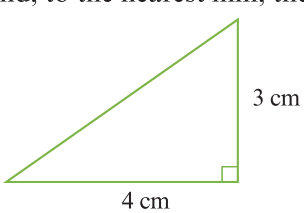
$AB \approx 10.77$ m



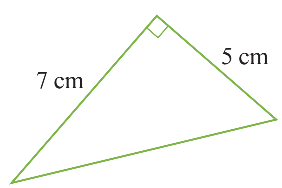
EXERCISE 32.1

1 Find, to the nearest mm, the length of each unknown side.

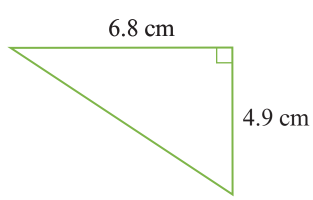
a



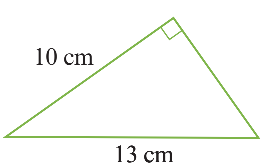
b



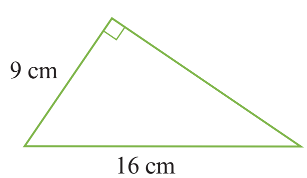
c



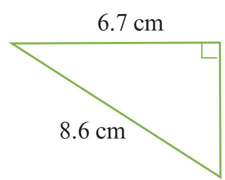
d



e

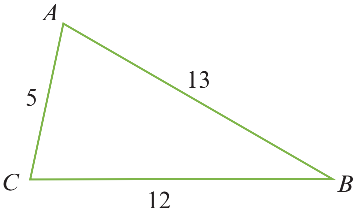


f

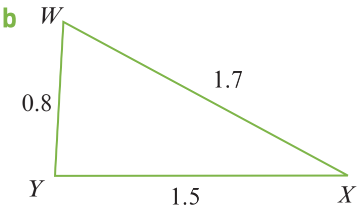


2 Which of the following triangles are right-angled? This question can be done using the method of Example 32A-2 or you could measure out the lengths in the schoolyard and then use the 3 : 4 : 5 rule (any triangle whose sides are in the ratio 3 : 4 : 5 is a right-angled triangle) to check for right angles. (The diagrams are not drawn to scale.)

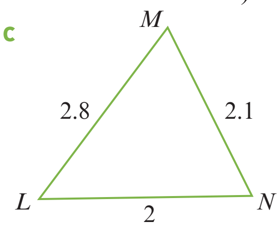
a



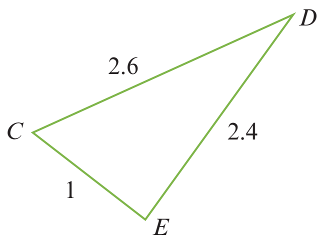
b



c



d



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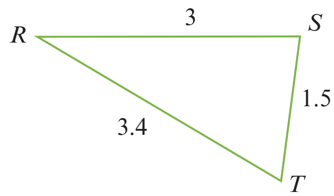

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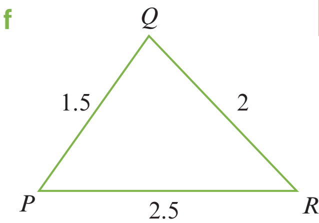
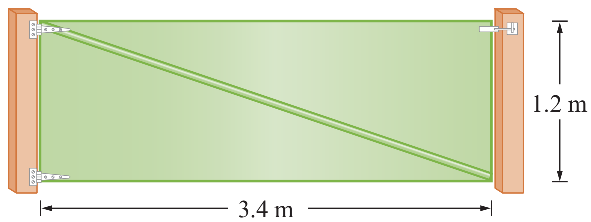


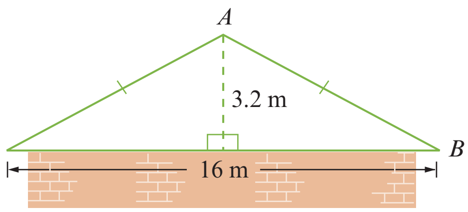
> **NOTE**
> The 3 : 4 : 5 rule: measure 3 m from the corner in one direction, then 4 m from the corner in the other direction. If the distance between is 5 m it is a right-angled triangle.

3 A metal gate is 3.4 m wide and 1.2 m high. To maintain its square corners (right-angled) and for strength purposes, a diagonal support is added. How long is the diagonal support?

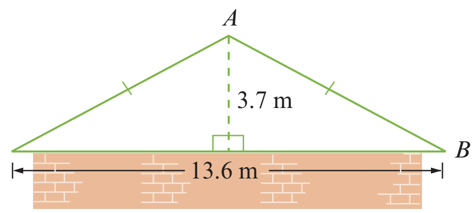


4 Find the length of the truss *AB* for the following roof structures.

a

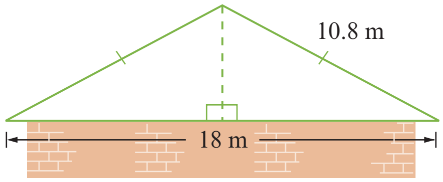


b

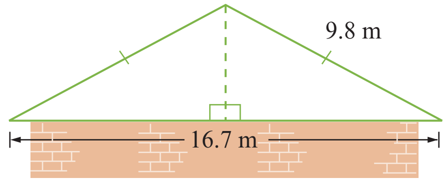


5 How high is the peak of the roof above the walls in the following roof structures?

a



b



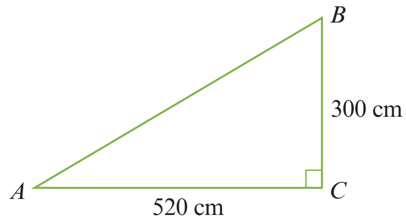
6 A farmer wanted to build a 4-strand wire fence around a triangular property. The fence is to be along all three sides of the property.

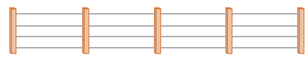
a Find the length of side AB.

b Find the perimeter of the property.

c Find the total length of wire required.

d If the wire costs \$29.50 for 100 m, find the total cost of the wire for the fence.





7 A yacht sails 23 km south and then 18 km east.

a Draw a set of axes and mark them north, east, south and west.

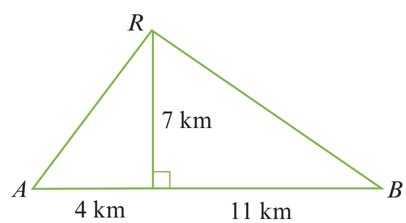
b Draw a fully labelled diagram of the yacht's course.

c How far is the yacht from its starting point?

8 A reservoir, R, supplies two towns, A and B, with water. New piping is required from R to each town. The towns are connected by a straight road, AB, as shown, and the reservoir is 7 km from this road.

a Find the distances AR and BR to the nearest metre.

b Find the total cost of the new pipelines, given that each 100 m will cost \$2550.

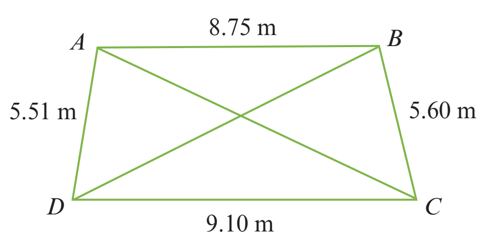


9 A carpenter is contracted to refurbish a shop. All the fittings are prefabricated and have only right-angles. In order to minimise his workload, the carpenter needs to start in the corner closest to 90°.

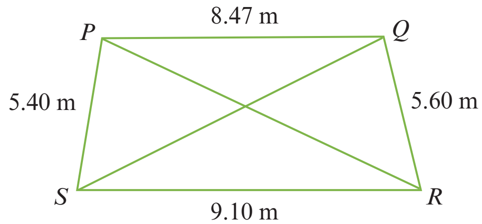
a Which corner should he start from, given the floor dimensions shown on the diagram?

Diagonal $AC = 10.21$ m

Diagonal $BD = 10.81$ m



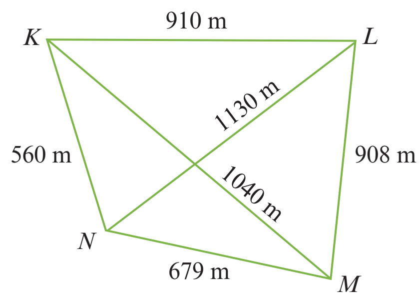
b If the measurements of another room's floor are as shown below, in which corner should the carpenter start?
Diagonal $PR = 9.75$ m
Diagonal $QS = 10.94$ m



10 A block of land has the dimensions shown.

a Draw a scale diagram of the block. (A pair of compasses is essential.)

b Determine which angle of figure *KLMN* is closest to 90°.

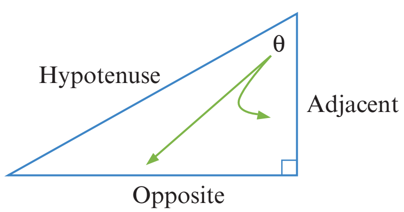


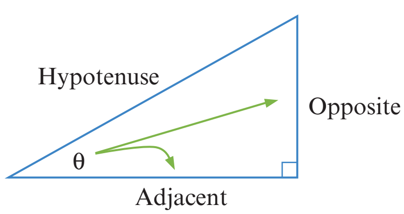
32B Right-angled triangle trigonometry

Trigonometry is a branch of mathematics that studies relationships involving lengths and angles of triangles. Trigonometry is used to find unknown sides and angles of right-angled triangles.

For a right-angled triangle, with given angle θ, first locate the hypotenuse (the longest side). Then locate the side opposite θ, called the opposite side, and the side next to θ (not the hypotenuse), called the adjacent side.

NOTE
Adjacent means 'next to'.





TRIGONOMETRIC FORMULAS

The following formulas can be used to calculate the unknown sides and angles in right-angled triangles, provided that sufficient information is given.

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} \qquad \tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

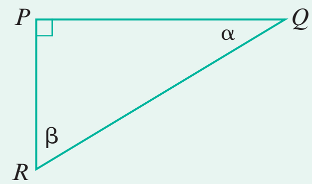
To help remember the above definitions, try using the mnemonic: SOH CAH TOA.

NOTE
You can use these abbreviations to save time:
- Hyp for the hypotenuse
- Opp for the opposite side
- Adj for the adjacent side.

EXAMPLE 32B-1 Naming sides

For this triangle, name the:

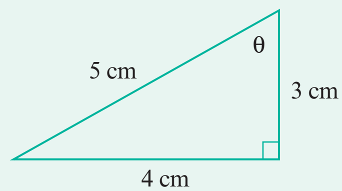
a hypotenuse **b** side opposite α
c side adjacent α **d** side opposite β
e side adjacent β



a QR **b** PR **c** PQ
d PQ **e** PR

EXAMPLE 32B-2 Finding trigonometric ratios

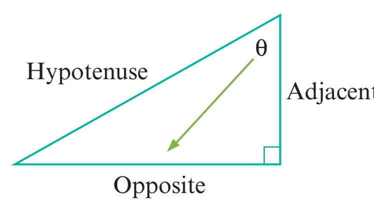
For this triangle, find sin θ, cos θ and tan θ.



$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{4}{5} = 0.8$$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{3}{5} = 0.6$$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{4}{3} = 1.3333\ldots$$

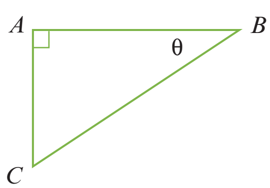


EXERCISE 32.2

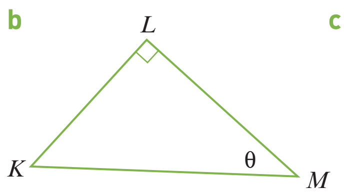
1 For each of these triangles, name the:

i hypotenuse **ii** side opposite θ **iii** side adjacent to θ

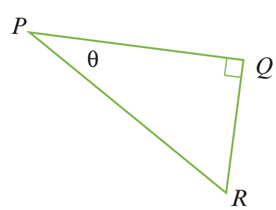
a



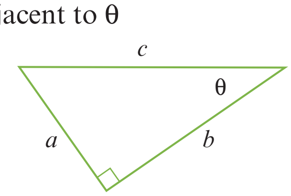
b



c



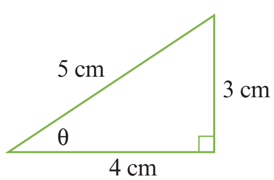
d



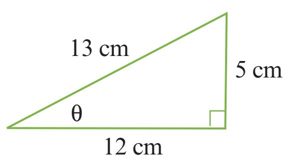
2 For each of these triangles, find:

i sin θ **ii** cos θ **iii** tan θ

a



b



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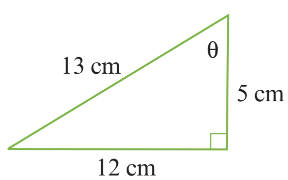
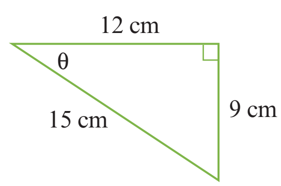


d



32C Finding unknown sides

Unknown sides and angles can be determined using a scientific calculator. For this it is essential that your calculator is set in the degree mode.

The DRG key enables you to do this. If DEG is not in the display, press DRG and select DEG from the choices on the screen.

When using trigonometry to find the lengths of sides of triangles, equations will arise where we need to find x, such as:

$$\sin 37° = \frac{x}{11} \qquad \text{or} \qquad \cos 42° = \frac{13}{x}$$

In general:

- if $a = \frac{x}{b}$ then $x = a \times b$
- if $a = \frac{b}{x}$ then $x = \frac{b}{a}$

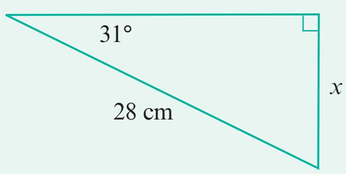
For example:

- if $\sin 37° = \frac{x}{11}$ then $x = 11 \times \sin 37°$
- if $\cos 42° = \frac{13}{x}$ then $x = \frac{13}{\cos 42°}$

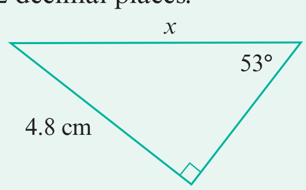
EXAMPLE 32C-1 Sine rule

Use sine to find each value of x correct to 2 decimal places.

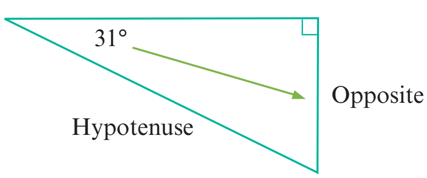
a



b



a



$$\sin 31° = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$= \frac{x}{28}$$

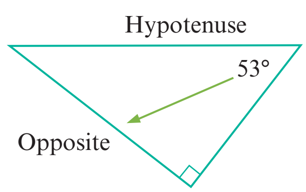
$$x = 28 \times \sin 31°$$

$$\approx 14.42$$

Calculator

28 × sin 31 =

b



$$\sin 53° = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$= \frac{4.8}{x}$$

$$x = \frac{4.8}{\sin 53°}$$

$$\approx 6.01$$

4.8 ÷ sin 53 =

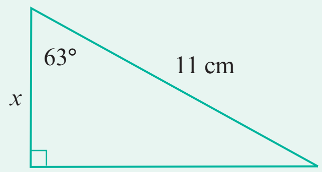
NOTE

Remember to set your calculator to display DEG before starting any calculations.

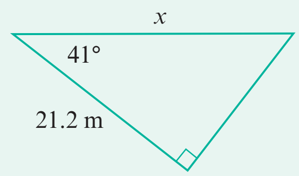
EXAMPLE 32C-2 Cosine rule

Use cosine to find each value of x correct to 2 decimal places.

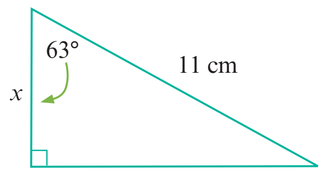
a



b



a

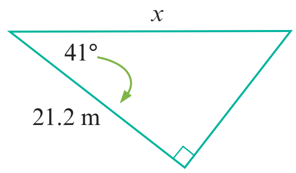


$$\cos 63° = \frac{\text{adjacent}}{\text{hypotenuse}}$$
$$= \frac{x}{11}$$
$$x = 11 \times \cos 63°$$
$$\approx 4.99$$

Calculator

11 × cos 63 =

b



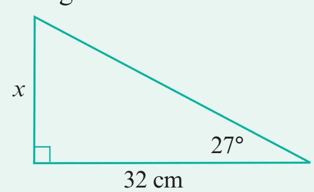
$$\cos 41° = \frac{\text{adjacent}}{\text{hypotenuse}}$$
$$= \frac{21.2}{x}$$
$$x = \frac{21.2}{\cos 41°}$$
$$\approx 28.09$$

21.2 ÷ cos 41 =

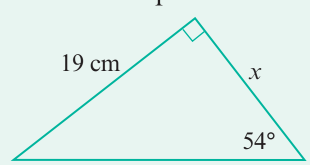
EXAMPLE 32C-3 Tangent rule

Use tangent to find each value of x correct to 2 decimal places.

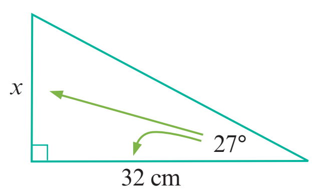
a



b



a

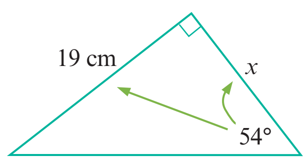


$$\tan 27° = \frac{\text{opposite}}{\text{adjacent}}$$
$$= \frac{x}{32}$$
$$\therefore x = 32 \times \tan 27°$$
$$\approx 16.30$$

Calculator

32 × tan 27 =

b



$$\tan 41° = \frac{\text{opposite}}{\text{adjacent}}$$
$$= \frac{19}{x}$$
$$\therefore x = \frac{19}{\tan 54°}$$
$$\approx 13.80$$

19 ÷ tan 54 =

EXERCISE 32.3

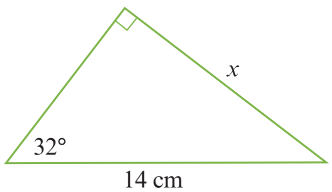
1 Using your calculator, find x correct to 1 decimal place.

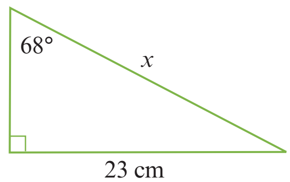
a $\sin 29° = \frac{x}{6}$ **b** $\cos 38° = \frac{x}{12}$ **c** $\tan 49° = \frac{x}{3}$ **d** $\cos 39° = \frac{3}{x}$

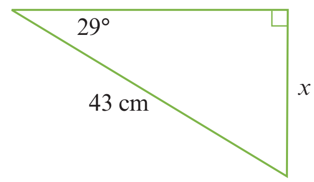
e $\sin 46° = \frac{11}{x}$ **f** $\tan 83° = \frac{14}{x}$ **g** $\tan 6° = \frac{x}{7}$ **h** $\cos 11.2° = \frac{15}{x}$

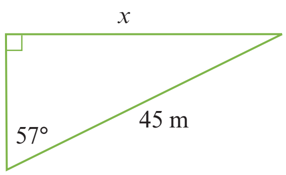
i $\sin 58.1° = \frac{1.61}{x}$ **j** $\cos 21.1° = \frac{x}{3.2}$ **k** $\sin 82.7° = \frac{6}{x}$ **l** $\tan 70.8° = \frac{24}{x}$

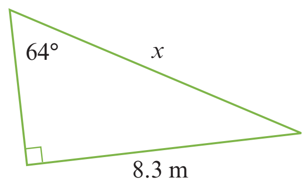
2 Use sine to find the value of x correct to 2 decimal places.

a


b


c


d


e


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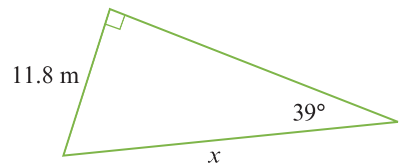
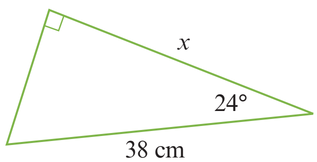


3 Use cosine to find the value of x correct to 2 decimal places.

a


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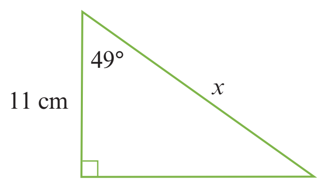

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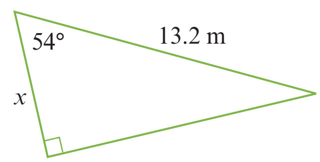

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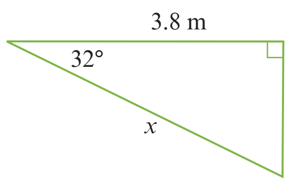

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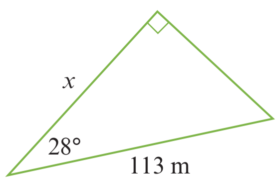

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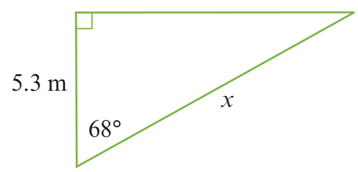
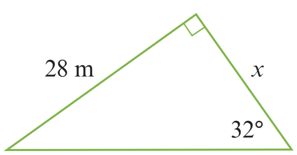


4 Use tangent to find the value of x correct to 2 decimal places.

a


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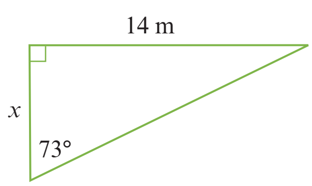
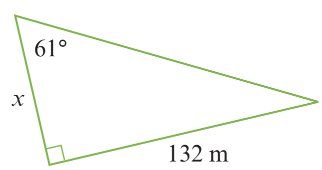
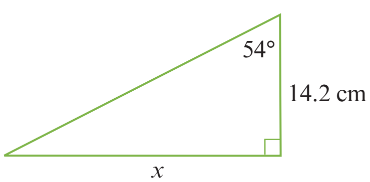
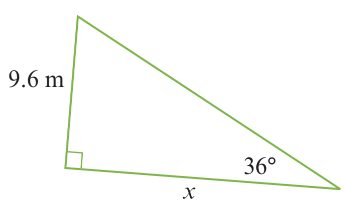


c


d


e


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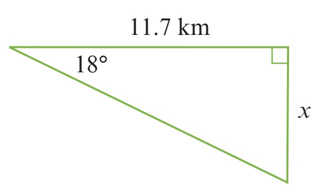


32D Finding unknown angles

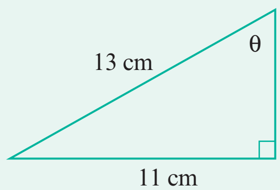
You can work backwards on a calculator to find an angle from one of the trigonometric ratios by using these key combinations: SHIFT sin or SHIFT cos or SHIFT tan. These may appear on your calculator display as $\tan^{-1}$ or $\sin^{-1}$ or $\cos^{-1}$.

- If $\sin\theta = \frac{a}{b}$ then $\theta = \sin^{-1}\left(\frac{a}{b}\right)$
- If $\cos\theta = \frac{a}{b}$ then $\theta = \cos^{-1}\left(\frac{a}{b}\right)$
- If $\tan\theta = \frac{a}{b}$ then $\theta = \tan^{-1}\left(\frac{a}{b}\right)$

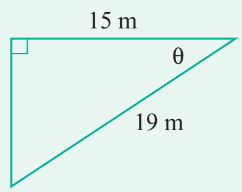
EXAMPLE 33D-1 Finding unknown angles

Find θ correct to 2 decimal places.

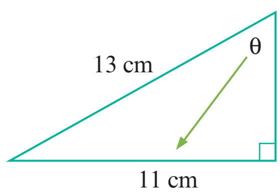
a



b



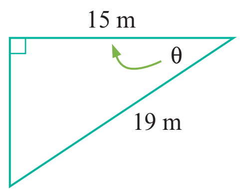
a



$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{11}{13}$$

$$\theta = \sin^{-1}\left(\frac{11}{13}\right) \approx 57.80°$$

b



$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{15}{19}$$

$$\theta = \cos^{-1}\left(\frac{15}{19}\right) \approx 37.86°$$

Calculator

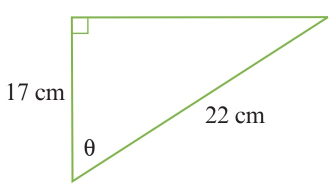
a 2nd sin (11 ÷ 13) =

b 2nd cos (15 ÷ 19) =

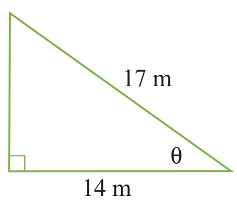
EXERCISE 32.4

1 Find the value of θ to the nearest degree.

a



b



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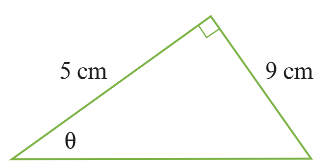

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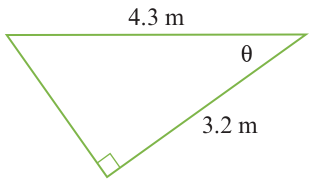
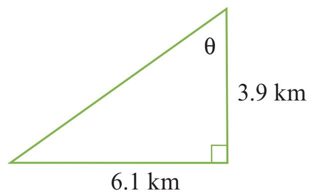


e



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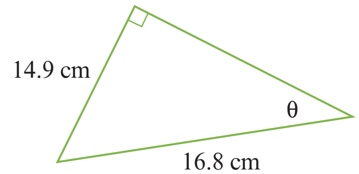
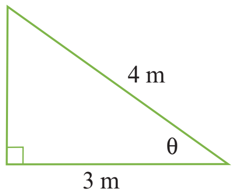
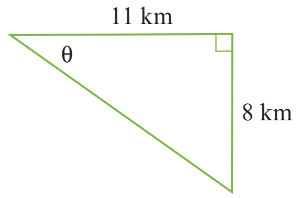


2 Find the value of θ correct to 1 decimal place.

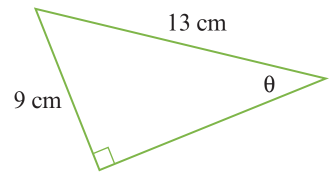
a



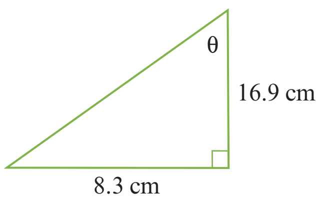
b



c



d



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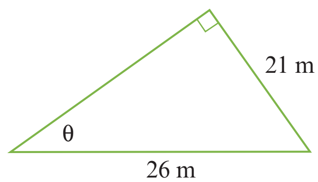

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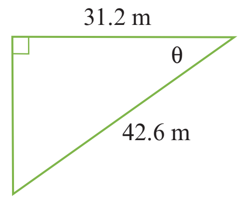
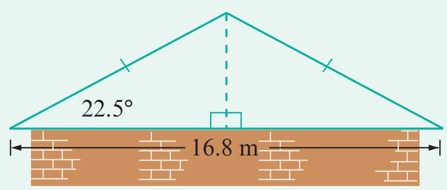


32E Applying trigonometry

Trigonometric formulas can be used in practical situations to calculate the unknown sides and angles in right-angled triangles. Previously we found the length of a roof truss given the height of the roof above the walls, and the height of the roof above the walls given the length of the truss. In the following example we find the height of the roof above the walls given the pitch of the roof.

EXAMPLE 32E-1 Solving practical problems

The roof structure of a house has a pitch of 22.5° as shown.
How high is the top of the roof above the walls to the nearest cm?



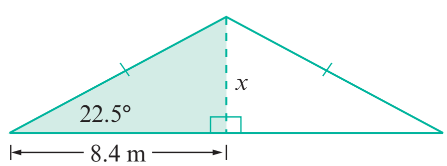
In the shaded right-angled triangle:

$$\tan 22.5° = \frac{\text{opposite}}{\text{adjacent}}$$

$$= \frac{x}{8.4}$$

$$x = 8.4 \times \tan 22.5°$$

$$\approx 3.479 \ldots$$

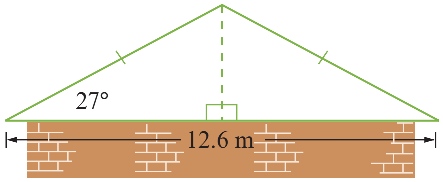


The top of the roof is 3.48 m above the walls.

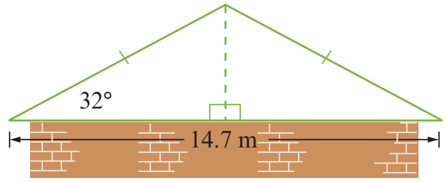
EXERCISE 32.5

1 For each roof structure, find the height of the roof above the walls.

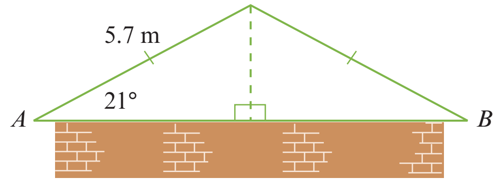
a



b



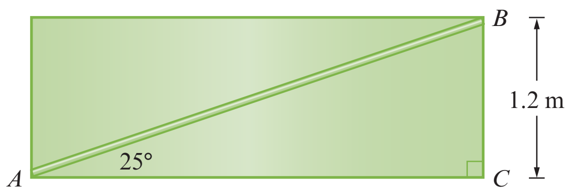
2 Find the length of the roof beam *AB* to the nearest centimetre.



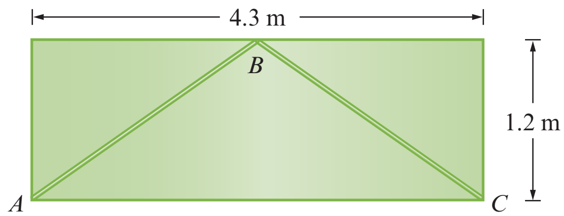
3 A rectangular gate has the shape shown. The diagonal strut, *AB*, makes an angle of 25° to the longer side *AC*.

a How wide is the gate?

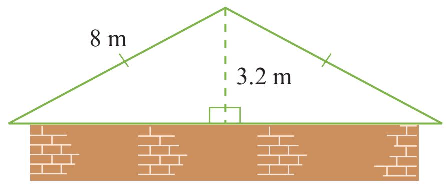
b Find the total length of metal needed to make the gate.



4 A gate is 4.3 m long and 1.2 m high. Two supports, *AB* and *BC*, are used as strengtheners. Find the angle that *AB* makes with *AC*.



5 Local council regulations state that, for safety reasons, the pitch angle of a roof must not exceed 22°. Will the roof structure shown be acceptable to the local council?

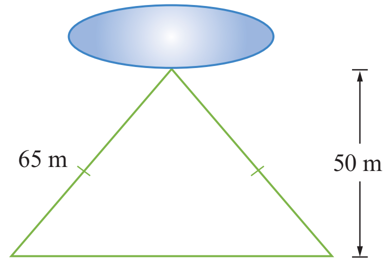


6 A window cleaner has a 6 m long ladder and, for safety reasons, the greatest (maximum) angle the ladder can make with the ground is 70°.

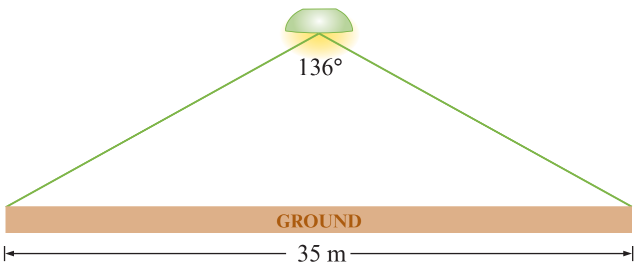
a Draw a side-on diagram of the ladder leaning against the building.

b What distance up the wall can the ladder reach?

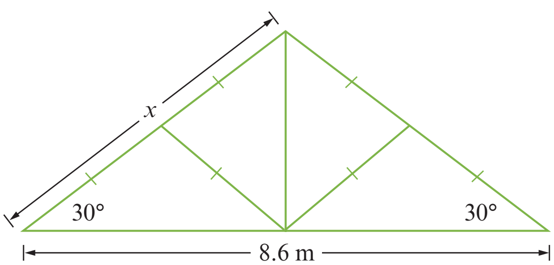
7 An advertising balloon is anchored 50 m above the ground by two ropes, each of length 65 m. What angle do the ropes make with the ground?



8 'Perfect' illumination occurs for a special lamp within a 136° angle. If a width of 35 m must be illuminated, how high above the ground must the lamp be placed?



9 A diagram of a timber roof truss is shown below. Eighteen of these trusses are needed to make the roof structure of a holiday house.



a Find x, to the nearest cm.

b What is the total length of timber needed to make one truss?

c What is the total length of timber needed to make all the trusses?

CHAPTER 33

Business calculations

33A Profit and loss

Whether a business makes a profit or a loss on an item they are selling is the difference between what it costs them to make or buy that item (the cost price) and the amount they are able to sell it for (the selling price).

- The business will make a *profit* if the selling price is higher than the cost price.
- The business will make a *loss* if the selling price is lower than the cost price.

For example, if an item costing \$100 is sold for \$120, a \$20 profit is made, whereas if it is sold for \$85, a \$15 loss is made.

If an item is marked up by, say, 20%, then its selling price will be 20% above its cost price. Profit and loss as a percentage of the cost price can be determined using the following formula:

$$\%\text{ profit} = \frac{\text{profit}}{\text{cost price}} \times 100\%$$

$$\%\text{ loss} = \frac{\text{loss}}{\text{cost price}} \times 100\%$$

EXAMPLE 33A-1 Calculating profit

A Blu-ray player costs \$185 and is sold for \$250.

a Calculate the profit.

b Calculate the profit as a percentage of the cost price.

a $\text{Profit} = \text{selling price} - \text{cost price}$

$= \$250 - \$185 = \$65$

b $\%\text{ profit} = \frac{\text{profit}}{\text{cost price}} \times 100\%$

$= \frac{65}{185} \times 100\%$

$\approx 35.14\%$

EXAMPLE 33A-2 Calculating loss

Suni pays \$1000 for a mountain bike, but after a few rides decides she does not need it and sells it for \$725.

a Calculate the loss.

b Express this loss as a percentage of the cost price.

a $\text{Loss} = \text{buying price} - \text{selling price}$

$= \$1000 - \$725 = \$275$

b $\%\text{ loss} = \frac{\text{loss}}{\text{cost price}} \times 100\%$

$= \frac{275}{1000} \times 100\%$

$\approx 27.5\%$

EXERCISE 33.1

1 For the following, determine whether a profit or loss has occurred and by how much.
   a Selling at $40 after buying for $25.
   b Selling at $65 after buying for $80.
   c Buying at $1175 and selling for $1550.
   d Buying at $2369 and selling for $2100.

2 A bicycle is bought for $340 and is sold for $450.
   a Calculate the profit or loss.
   b Find the percentage profit or loss on the sale.

3 A used-car firm pays $30 000 for a car and, due to financial difficulties, has to sell it immediately for $27 000. Find the loss incurred and express this loss as a percentage of the cost price.

4 A young couple purchased an old established home for $480 000. Over the next 6 months they spent another $60 000 on renovations and new furnishings. However, due to a change in employment, they were forced to sell the home within the year and received only $515 000 for the sale.
   a What was the loss incurred?
   b What was the loss as a percentage of the total cost?

5 A 30-m roll of wire mesh was bought wholesale for $216. If it was sold for $8.50 per metre, find the profit and express it as a percentage of the wholesale (cost) price.

EXAMPLE 33A-3 Profit on cost price

A smart TV set costs $845 and is marked up by 40%. What profit will be made on the cost price?

$$\text{Profit} = 40\% \text{ of cost price}$$
$$= \frac{40}{100} \times \$845 = \$338$$

EXERCISE 33.2

1 Find the profit made on the cost price of each of the following items.
   a a $425 barbecue marked up by 30%
   b a $830 computer marked up by 25%
   c a $1250 refrigerator marked up by 15%
   d a $15 690 ute marked up by 12%

33B Percentage change using a multiplier

One method for increasing \$400 by 30% requires two steps:

- First find 30% of \$400.
- Then add the result to \$400.

  However, we can solve this problem in one step by using a multiplier.

- To increase an amount by 30% is to multiply by (100 + 30)% = 130%. This is equivalent to a multiplier of 1.3.

  Thus \$400 increased by 30% is \$400 × 1.3 = \$520.

EXAMPLE 33B-1 Finding the multiplier

What multiplier corresponds to:

a a 25% increase

b a 15% decrease?

a 100% becomes 125% (100 + 25)%.
The multiplier is 1.25.

b 100% becomes 85% (100 − 15)%.
The multiplier is 0.85.

NOTE

Remember: 125% is $\frac{125}{100}$ or 1.25.

EXAMPLE 33B-2 Using the multiplier

a Increase \$400 by 27%.

b Decrease \$156 by 8%.

a The multiplier is 127% or 1.27. — Use 100% + 27%.
New amount = \$400 × 1.27
= \$508

b The multiplier is 92% or 0.92. — Use 100% − 8%.
New amount = \$156 × 0.92
= \$143.52

NOTE

Notice that the multiplier is:

- more than 1 for an increase
- less than 1 for a decrease.

EXERCISE 33.3

1 Find the multiplier that corresponds to a:

a 14% increase
b 14% decrease
c 37% increase
d 23% decrease
e 8.2% increase
f 7.9% decrease

2 Perform these calculations.

a Increase \$180 by 15%
b Increase \$365 by 4%
c Decrease \$450 by 24%
d Decrease \$5460 by 7%
e Increase \$62 310 by 5%
f Decrease \$14 735 by 2.8%

NOTE

To make two percentage changes in succession, we use two multipliers.

EXAMPLE 33B-3 Using two multipliers

Increase \$2500 by 10% and then decrease by 15%.

To increase by 10%, the multiplier is 110% = 1.1.

To decrease by 15%, the multiplier is 85% = 0.85.

The overall multiplier is 1.1×0.85.

$$\begin{aligned}\text{New amount} &= \$2500 \times 1.1 \times 0.85 \\ &= \$2337.50\end{aligned}$$

When using a percentage to increase a cost price to get a selling price, again a multiplier can be used:

Selling price = cost price × multiplier

EXAMPLE 33B-4 Finding the selling price

A warehouse owner buys a refrigerator for \$750 and marks it up by 35%. At what price does the owner sell the refrigerator?

$$\begin{aligned}\text{Selling price} &= \text{cost price} \times \text{multiplier} \\ &= \$750 \times 135\% && 100\% + 35\% = 135\% \\ &= \$750 \times 1.35 && \text{Multiplier} = 1.35 \\ &= \$1012.50\end{aligned}$$

The refrigerator was sold for \$1012.50.

EXERCISE 33.4

1 a Increase \$536 by 10% and then by 15%.
  b Decrease \$655 by 8% and then by 6%.
  c Decrease \$5400 by 9% and then increase by 9%.
  d Increase \$8795 by 7.5% and then decrease by 9.75%.

2 A leather coat costs a fashion store \$250 and they sell it at a profit of 40% of the cost. What is the selling price of the coat?

3 Used car dealer Cheap Cars paid \$8600 for a car but had to sell it for a 16% loss. At what price did they sell the car?

4 A real-estate company buys a block of units for \$1 250 000 and spends \$150 000 on renovations and repairs. Three months later the company was able to sell the units at a profit of 12% on their total investment. Find the sale price for the block of units.

33C Goods and services tax (GST)

From mid-2000, a federal tax, known as the *goods and services tax* (GST), is payable on most goods (such as all new goods) and services (such as hospitality, electrical work, plumbing, building, etc.). It is calculated at the rate of 10% of the purchase price of the goods or services. The price including the GST is written as 'price including GST'. The price excluding the GST is written as 'price excluding GST'. Most unprepared food items such as meat and vegetables are GST free.

In Australia all marked retail prices subject to GST must include the GST on the price tag. Wholesalers generally quote the price before adding the GST.

NOTE

'Price including GST' means the price after the GST has been added.
'Price excluding GST' means the price before the GST has been added.

EXAMPLE 33C-1 Calculating GST on excluded items

Calculate the GST that would be payable on the following (GST excluded).

a a microwave oven for \$350 **b** a \$2500 plumbing bill

a GST = 10% of \$350
= 0.1 × \$350
= \$35

b GST = 10% of \$2500
= 0.1 × \$2500
= \$250

EXAMPLE 33C-2 Including GST on payable items

A shopkeeper buys jumpers for \$40 that she wishes to sell at cost price including GST. How much would she sell the jumpers for to include GST?

Price including GST is the cost price plus 10%.

GST = 10% of \$40
= 0.1 × \$40 = \$4
\$40 + 4 = \$44

or

Price including GST = 110% of \$40
= 1.1 × \$40
= \$44

She must sell the jumper for \$44.

EXERCISE 33.5

1 Calculate the GST payable on the following (GST excluded).

a a smartphone for \$850
b a pair of trousers for \$85
c a briefcase for \$98.50
d a computer for \$1295
e a \$160 lawnmowing service
f an electrician's invoice for \$450

2 What is the GST-inclusive price on goods if a shopkeeper wishes to receive the following money on the sale?

a \$35
b \$80
c \$105
d \$668
e \$2350
f \$128.95

RULE OF 11

Generally the price on an item is quoted with GST already added (GST included). The amount of GST included can be calculated using the Rule of 11:

$$\text{GST included} = \text{selling price} \div 11$$

In Example 33C-2, the price of the shirt, including GST, was $44.

$$\text{GST} = \$44 \div 11 = \$4$$

EXAMPLE 33C-3 Rule of 11

How much GST is paid if the price is $99 (GST included)?

$$\text{GST} = \$99 \div 11 \quad \text{Rule of 11}$$
$$= \$9$$

The amount of GST is $9.

EXERCISE 33.6

1 Find the GST on each of the following goods.

a


b


c


2 At a hardware store, Jenni bought items marked at the following prices (all GST included).
$7.15, $3.50, $17.35, $8.75, $155, $21.85

a Find the total cost of her purchases.

b How much GST did Jenni pay on these purchases?

33D Discount

In order to attract customers or to get rid of old stock, many businesses reduce the price of an article from that shown on the price tag (called the marked price). The amount of money by which the cost of an item is reduced is called a discount. The discount is often stated as a percentage of the marked price (original selling price).

For a discount of x%, we multiply by $(100 - x)$%. Therefore, the discount is a percentage decrease. For example, 34% discount means that the article sells for $(100 - 34)$% of the marked price or 66% of the original price.

Selling price = marked price − discount

EXAMPLE 33D-1 Finding the price after discount

If the marked price of an Xbox is $460 and a 25% discount is offered, find the actual selling price.

To decrease $460 by 25% we multiply by (100 − 25)% = 75%.

Selling price = 75% of marked price
= 75% of $460
= 0.75 of $460 — 75% is 0.75 in decimal form.
= 0.75 × $460 — 'Of' indicates ×.
= $345

EXERCISE 33.7

1 **a** If the marked price of an LCD widescreen television is $1400 and a 15% discount is offered, find the actual selling price.

b A furniture distributor advertises kitchen tables at a marked price of $680 with a 25% discount for the first 20 customers. What is the actual selling price if you are one of the first 20 customers?

2 Complete the following table.

	Marked price	Discount	Selling price	Discount as a % of the marked price
a	$125	$25		
b	$240			25%
c	$2.75			20%
d		65 cents	$2.35	
e	$150		$120	

33E Payment for work done

The main source of income for most Australians is the money they receive from working. In return for work done, or services provided, they receive a payment from their employer. This payment represents their earnings from their job.

> **NOTE**
> Sometimes a low wage, called a retainer, is paid in addition to a commission.

Earnings are given different names according to the basis on which the amount to be paid is calculated. Earnings can be based on:

- the number of hours worked (wages)
- the number of items produced (piecework)
- a fixed amount regardless of the number of hours worked (salary)
- a percentage of the value of goods or services sold (commission).

A business may pay several different types of earnings to its employees. For example, a retail business would pay a salary to its executives, wages to the shop assistants and may pay commission to some of its salespeople.

WAGES

When earnings are based on the amount of time worked, they are referred to as wages. The wage payment is calculated according to the number of hours worked multiplied by the hourly rate. It is therefore important to accurately record the hours that each employee has worked so that the wage payment is correct.

The recording of employee hours may be done in several ways. In smaller companies, employees may manually write down their starting and finishing times (including the lunch break) or a timekeeper may be assigned to record these times. The details are collected each pay period and the hours worked added up for each employee.

Larger corporations may use the traditional time clocks. Each employee has a clock card that records the time of day when pushed into a time clock. This is done each time the employee arrives and leaves, and thus a record is made of the hours spent at work. The cards are collected each pay period and are used to calculate the number of hours that each employee has worked. Some systems use ID cards or badges, and the times are recorded electronically and entered into an accounting database.

Overtime

Employees who earn wages are expected to work a certain number of hours each day or each week, as negotiated in their workplace agreement. Overtime is paid to people who work additional hours and is usually paid at a higher rate. The most common rates of overtime payment are time-and-a-half and double-time.

- Time-and-a-half means that the employee is paid one-and-a-half times more than the normal rate. For example, if the normal rate of pay is \$25/h, then the employee would be paid $\$25 \times 1\frac{1}{2} = \$37.50$/h.
- Double-time means that the employee is paid twice the normal hourly rate. For example, if the normal hourly rate is \$25, then the employee would be paid $\$25 \times 2 = \$50$/h.

Overtime may be calculated on either a daily basis or a weekly basis. For overtime on a daily basis, the normal working week is divided into a number of hours per day. Any time worked over these daily hours is considered overtime. Any time worked on the weekend is also considered overtime, either at time-and-a-half or double-time rates.

For overtime on a weekly basis, overtime is considered to be the hours worked in excess of the normal weekly hours regardless of when they were worked. Any time worked on the weekend may also be considered as overtime either at time-and-a-half or double-time rates. In this section we will consider overtime on a weekly basis only.

EXAMPLE 33E-1 Determing weekly wages

Employees at John's Shed Factory work a $37\frac{1}{2}$-hour week from Monday to Friday. Time-and-a-half is paid for overtime during the week and for the first 4 hours on Satudrday. Double time is paid for the rest of Saturday and all hours on Sunday. Calculate the weekly wage of an employee, whose normal pay rate is $18.90 per hour, if he worked these hours:
Monday 8, Tuesday 7, Wednesday 9, Thursday $6\frac{1}{2}$, Friday 8, Saturday 6, Sunday 4

	Normal	Time-and-a-half	Double-time
Mon–Fri	$37\frac{1}{2}$	1	0
Saturday	0	4	2
Sunday	0	0	4
Total (h)	**$37\frac{1}{2}$**	**5**	**6**
Rate	$18.90	$28.35	$38.70
Wages	$708.75	$141.75	$226.80

Total wage for the week is $708.75 + $141.75 + $226.80
= $1077.30

Awards and the national minimum wage

In Australia, wage rates for different industries are often collectively agreed and set. These agreements are called awards. For people not covered by an award, the Fair Work Commission sets the minimum hourly rates for adults and juniors. In July 2015, the minimum adult wage rate was $17.29 per hour or $656.90 for a 38-hour week before tax. Casual employees covered by the national minimum wage also get at least a 25 per cent casual loading. If you ever need to check your status, visit this website:
http://www.fairwork.gov.au/Pay/minimum-wages

Junior rates

For both awards and the minimum wage, junior wages (wages for those under 21) are set as a percentage of the relevant adult hourly rate. For the minimum wage, a rough rule of thumb is 10% less for each year of age below 21 (the minimum working age is 15).To find your relevant junior minimum wage based on your industry, visit this website:
http://www.fairwork.gov.au/Pay/Minimum-wages/junior-pay-rates

EXERCISE 33.8

1 Convert these normal hourly rates to the rates at which overtime would be paid at time-and-a-half.

a $24.50 b $30.70 c $22.28
d $29.40 e $35.16 f $18.90

2 Convert these normal hourly rates to the rates at which overtime would be paid at double-time.

a $19.24 b $18.86 c $30.50
d $16.90 e $21.25 f $27.95

3 Tomash and Sabina also work for John's Shed Factory in Example 33E-1. Calculate their week's wages.

a Tomash, whose normal rate is $24 per hour, worked these hours:
Monday 9, Tuesday $7\frac{1}{2}$, Wednesday 8, Thursday 8, Friday $8\frac{1}{2}$, Saturday 7, Sunday 2

b Sabina, whose normal rate is $19.50 per hour, worked these hours:
Monday 7, Tuesday $6\frac{1}{2}$, Wednesday 9, Thursday 9, Friday $8\frac{2}{3}$, Saturday 8, Sunday 0

4 The Speed Cycle Factory works a 38-hour week from Monday to Friday. Any hours worked over this from Monday to Friday are paid at time-and-a-half, as are the first 3 hours on Saturday. For the rest of Saturday and Sunday, double-time rates apply. Calculate the following employees' week's wages.

a P Hussan, whose normal rate is $21.50 per hour, worked:
Monday 7, Tuesday 8, Wednesday 8, Thursday 8, Friday 10, Saturday 4, Sunday 4

b K Jacobs, whose normal rate is $23.85 per hour, worked:
Monday 8, Tuesday 7, Wednesday 7, Thursday 10, Friday 9, Saturday 2, Sunday 6

PIECEWORK

NOTE

Those working at home are referred to as outworkers.

Sometimes employees get paid for what they produce (the number of items) rather than strictly for the hours worked. Piecework payment is mainly found in the clothing industry, where production is divided into a number of separate operations (such as sewing, cutting, etc.) and each operation is given a standard time to complete. Workers operating under this system either work in a factory or from home.

Piecework payment is based on either:

- the number of operations completed each day, multiplied by the standard time to complete each operation, multiplied by the hourly rate, or
- the normal daily rate of pay, whichever is higher.

EXAMPLE 33E-2 Piecework pay

Cynthia is a machinist at a shirt factory where she sews one pocket to each shirt. Her hourly rate is $18.75 and the standard time for this operation is 0.018 hours. How much did she earn on Tuesday if she completed 528 pockets in 8 hours?

Hours worked based on operation $= 0.018 \times 528 = 9.504$ h

Actual hours worked $= 8$ h

Tuesday's pay $= 9.504 \times \$18.75$ Using the higher figure

$= \$178.20$

EXERCISE 33.9

1 Below are standard times that apply to the production of jeans.

Task	Standard time allowed (h)
Attach hip pocket	0.022
Hem leg	0.008
Attach rivets	0.006
Attach waistband	0.02
Attach belt loops	0.011
Hem back pocket	0.0085

a Calculate the standard hours worked if the number of hip pockets attached each day is:
i 402 ii 369 iii 457 iv 296

b Calculate the daily earnings of the following employees who worked 8-hour days at $21.90 an hour:
i J Clough who attached 417 waistbands
ii K Donaldson who hemmed 1057 back pockets
iii S Dean who attached 1556 rivets
iv M Black who attached 748 belt loops.

2 Roy works part-time as a machinist attaching rivets to jeans (0.006 hours per rivet) at $20.33 per hour. If his production and hours worked last week were as follows, calculate his earnings for the week.

	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
Items produced	865	714	792	879	914	523
Hours worked	5	4	$4\frac{1}{2}$	$5\frac{1}{2}$	5	4

SALARIES

Employees on a salary are paid a set amount per period regardless of the number of hours that they work. They do not get paid for any overtime they may work. Salaries are normally stated in terms of an annual amount, but can be easily converted to monthly, fortnightly, weekly or hourly amounts.

EXAMPLE 33E-3 Converting salaries

Convert an annual salary of $84 000 to amounts that are:

a monthly
b fortnightly
c weekly
d hourly (assume a 36-hour week)

a Monthly salary $= \frac{\$84\ 000}{12} = \$7000$

b Fortnightly salary $= \frac{\$84\ 000}{26} = \$3230.77$

c Weekly salary $= \frac{\$84\ 000}{52} = \$1615.38$

d Hourly salary $=$ weekly salary $\div 36 = \frac{\$1615.38}{36} = \$44.87$

NOTE
Monthly is per month, not every 4 weeks!

EXERCISE 33.10

1 Convert the following annual salaries to monthly, weekly and hourly rates given a $37\frac{1}{2}$-hour week.

a $36 800 b $48 450 c $75 295

2 Convert the following weekly salaries to fortnightly, annual and hourly rates given a 38-hour week.

a $868.00 b $575.50 c $1058.00

3 Which pays more (assuming a 40-hour week):

a $873.35 per week or $45 000 annually?

b a junior on $14.70 per hour or $29 600 annually?

c an apprentice on $32 448 annually or $15.60 per hour?

COMMISSION

Commission is a form of earnings where the employee is paid a certain percentage or amount based on what they have sold. It is generally paid to salespeople such as real-estate agents, car salespeople or commercial travellers. Sometimes a salesperson may be wholly paid on a commission basis, but it is more common for them to be on a basic or flat salary (called a retainer) plus commission.

EXAMPLE 33E-4 Working out commission

Stephen is paid a commission of 3.5% on sales of television and sound equipment plus a retainer of $420 a week. In March, Stephen sold $82 750 worth of stock. Calculate Stephen's total pay for March.

NOTE

1 month does not equal 4 weeks.

Retainer = $420 × 4 = $1680

Commission = $82 750 × 0.035 — Convert 3.5% to a decimal.

= $2896.25

Total = $1680 + $2896.25 = $4576.25

EXERCISE 33.11

1 Calculate the monthly earnings for the following cases. The retainer is paid weekly.

	Retainer	Commission	Sales
a	$225	3%	$126 525
b	$350	$21\frac{1}{2}$%	$194 000
c	$268	$4\frac{1}{4}$%	$93 248

2 Alfonso is a commercial traveller and is paid a salary of $400 per week plus 5% commission on any sales over $50 000 each month. If Alfonso sold $63 400 worth of goods in 1 month, how much did he earn that month?

3 Stefanie is a car salesperson who is paid $450 per week plus 8% commission on the value of all the second-hand cars she sells. The value is calculated after deducting the trade-in price. During the month of May she sells the following cars. How much will Stephanie earn in May?

Car type	Car price	Trade-in
Falcon	$13 450	$6 800
Mazda	$8 800	–
Commodore	$12 750	$6 500
Hyundai	$16 800	$10 000

4 Glen is a commercial traveller selling soft drinks and is paid $420 per week plus commission on his monthly sales. Glen's commission is calculated on the scale as shown.

Sales value	Commission rate
Up to $10 000	No commission
$10 001 to $20 000	Nil plus 3% of sales over $10 000
$20 001 to $30 000	$300 plus 2.5% of sales over $20 000
$30 001 to $40 000	$550 plus 2% of sales over $30 000
$40 001 and over	$750 plus 1% of sales over $40 000

Calculate Glen's earnings if he achieved the following sales:

a January sales $59 247
b February sales $49 381
c March sales $32 467
d April sales $21 328

33F Deductions from earnings

The wage, salary or commission that is paid each period is referred to as the gross pay. It is the total that has been earned during the period. However, this is not the amount you actually receive in your pay as deductions are made from the gross pay. The amount you receive after deductions is called net pay (or take-home pay).

$$\text{Gross pay} - \text{deductions} = \text{net pay}$$

Various types of deductions can be made from your gross pay. Most are voluntary. The most common types of deductions are PAYG tax, voluntary superannuation contributions, private medical insurance, union fees and social club payments. Tax is the only compulsory deduction.

NOTE

Employees may also elect to have some of their pay paid straight into a selected bank account as a form of saving.

PAYG tax

Employers are required by law to make deductions each pay period for income tax. The tax deducted is collected by the employer and paid to the government each month. The amount of tax deducted each pay period depends on the employee's gross pay and the current tax rates. This rate is set out in tax tables that the ATO sends to employers so that they deduct the correct amount each pay period. Rates are available on the ATO website: https://www.ato.gov.au/Rates/Tax-tables/

The amount of PAYG (pay as you go) tax deducted will also be affected by:

- the Medicare Levy paid by all income earners over a certain base level of income
- HELP fees that have accrued from tertiary study
- holiday loading, a bonus paid to some workers, usually a $17\frac{1}{2}\%$ loading on 4 weeks' earnings, to compensate for overtime and allowances lost while they are taking annual leave.

Voluntary superannuation contributions

Superannuation is a scheme in which regular payments are placed into a superannuation (investment) fund by the employer (and often the employee) to provide retirement benefits such as pensions and lump sum payouts. Your employer must contribute an amount equivalent to 9.5% (in 2014–15) of your gross pay into a superannuation fund of your choice (up to the maximum annual superannuation contributions base) on a regular basis.

For example, if an employee earns \$978 a week, the employer has to pay a 9.5% contribution to superannuation weekly, so 9.5% of \$978 is paid into the fund each week (\$92.91). If the employee makes a voluntary contribution of \$40 a week into the fund, then \$92.91 + \$40 = \$132.91 is paid into the fund each week and a deduction of \$40 is made from the employee's pay.

Private medical insurance

Many employees elect to take out private medical insurance (as distinct from Medicare) and have the premiums deducted from their pay. The amount deducted will depend on the type of cover. The employee must arrange with their employer for this money to be paid directly to the private health insurance fund of the employee's choice.

Union fees

Many employees belong to a union and the union membership fee is deducted from their pay each period and passed on to the union. Fees vary from one union to another – some are flat fees whereas others are calculated as a percentage of the members' yearly salary.

Social club contribution

Many businesses have a social club that is funded through deductions from employees' pay and the money collected is used to pay for various social functions.

EXAMPLE 33F-1 Calculating bring-home pay

Calculate the net weekly pay for an employee whose gross weekly pay is $875.23. Deductions are $3.85 for union fees, a 3% voluntary superannuation contribution, $18.75 for private medical cover, $1.25 for the social club and $128.55 for PAYG tax.

Gross weekly pay		$875.23
Deductions		
PAYG tax	$128.55	
Union fees	$3.85	
Voluntary superannuation (3% of $875.23)	$26.26	
Private medical insurance	$18.75	
Social club	$1.25	
Total	$178.66	−$178.66
Net pay		$696.57

EXERCISE 33.12

1 Calculate the net pay for the employees of Juni's Motor Repairs. (Their award pays holiday loading.)

	Employee	Gross pay ($)	Deductions ($)					Total deductions	Net pay ($)
			Tax[1]	PMI[2]	Union	3% V Sup[3]	SC[4]		
a	H Jones	886.45		–	4.23		1.75		
b	R Stagg (junior)	429.32		15.23	4.23		1.75		
c	C Cheng	750.40		22.13	4.23		1.75		
d	N Hare (part-time)	288.17		–	4.23		–		

Notes: 1 PAYG tax rates are available on the ATO website.
2 PMI is private medical insurance.
3 3% V Sup is a voluntary contribution of 3% of gross wages to superannuation.
4 SC is social club contribution.

2 Vicki is paid $19 an hour for a normal working week of 38 hours. Last week she worked 4 hours overtime at time-and-a-half and 4 hours at double-time. Her union fees are $430 per year and social club fees are $104 per year. She has basic private (single) health insurance of $28.20 a week and pays an additional 4% of her earnings in voluntary superannuation contributions. Calculate her net pay for last week.

MATHEMATICAL SKILLS

CHAPTER 34

Calculating interest

34A Simple interest (flat rate)

Whenever money is lent, the person lending the money (the lender) usually charges a fee to the person receiving the money (the borrower). This fee is called the interest and represents the cost of using another person's money. The borrower has to repay the amount borrowed (the principal) plus the interest charged for using that money.

The amount of interest charged on a loan depends on the amount borrowed, how long it is borrowed for and the interest rate. There are two ways of calculating interest: simple interest and compound interest.

Simple interest is calculated on the full amount borrowed for the entire period of the loan. For example, if \$2000 is borrowed at a flat rate of 8% p.a. for 3 years, the interest payable for 1 year is:

$$\begin{aligned} 8\% \text{ of } \$2000 &= \frac{8}{100} \times \$2000 \\ &= \$160 \end{aligned}$$

So, for 3 years the interest would be:

$$\left(\frac{8}{100} \times \$2000\right) \times 3 = \frac{\$2000 \times 8 \times 3}{100} = \$480$$

From examples such as this we can construct the simple interest formula:

$$\text{Interest} = \frac{\text{principal} \times \text{rate} \times \text{time}}{100}$$

where interest is paid in \$
principal is the amount borrowed in \$
rate is the flat rate of interest per year
time is the length of the loan in years

To find the principal, if the interest paid, rate and time are known. we use:

$$\text{Principal} = \frac{100 \times \text{interest}}{\text{rate} \times \text{time}}$$

This is useful if you want to calculate the amount borrowed.

To find the rate of interest, if the principal, interest paid and time are known, we use:

$$\text{Rate} = \frac{100 \times \text{interest}}{\text{principal} \times \text{time}}$$

This is useful if you want to calculate the rate of interest charged.

To find the time of the loan, if the principal, interest and rate are known, we use:

$$\text{Time} = \frac{100 \times \text{interest}}{\text{principal} \times \text{rate}}$$

This is useful if you want to calculate the time needed to discharge (pay off) the loan.

NOTE

Per annum (p.a.) means per year.

EXAMPLE 34A-1 Finding the simple interest

What is the simple interest on a loan of \$5000 at a flat rate of 7% p.a. over 18 months?

$$\text{Interest} = \frac{\text{principal} \times \text{rate} \times \text{time}}{100} = \frac{5000 \times 7 \times 1.5}{100} = \$525.00$$

where principal = \$5000
rate = 7%
time = 18 months = 1.5 years

The simple interest is \$525.

EXERCISE 34.1

1 Calculate the interest on a loan of:
- a \$1000 at a flat rate of 6% p.a. over a 3-year period
- b \$9500 at a flat rate of 8.3% p.a. over a 15-month period
- c \$20 000 at a flat rate of 6.5% p.a. over 5 years and 7 months
- d \$6000 at a flat rate of 7.3% p.a. over an 8-month period.

NOTE

Time must be in years. So, 5 years and 7 months is $5\frac{7}{12}$ years.

2 Which of the following loans charge less interest?
- A \$25 000 at a flat rate of 7% p.a. for 4 years
- B \$25 000 at a flat rate of 6.75% p.a. for $4\frac{1}{2}$ years

EXAMPLE 34A-2 Finding the amount borrowed

How much is borrowed if a flat rate of 7.5% p.a. results in an interest charge of \$4500 after 5 years?

$$\text{Principal} = \frac{100 \times \text{interest}}{\text{rate} \times \text{time}} = \frac{100 \times 4500}{7.5 \times 5} = \$12\ 000$$

where rate = 7.5%
interest = \$4500
time = 5 years

The amount borrowed is \$12 000.

Calculator: 100 [×] 4500 [÷] [(] 7.5 [×] 5 [)] [=]

EXERCISE 34.2

1 How much is borrowed if:
- a a flat rate of 7.5% p.a. results in an interest charge of \$825 after 5 years?
- b a flat rate of 7% p.a. results in an interest charge of \$7560 after 4 years?

2 An investor wants to earn \$2500 in 7 months. How much would he need to invest given that the interest rate is $8\frac{3}{4}$% (flat rate)?

EXAMPLE 34A-3 Finding the interest rate

If you wanted to earn \$5000 in interest on a 4-year loan of \$16 000, what flat rate of interest would you need to charge, correct to 2 decimal places?

$$\text{Rate} = \frac{100 \times \text{interest}}{\text{principal} \times \text{time}}$$

where interest = \$5000
principal = \$16 000
time = 4 years

$$= \frac{100 \times 5000}{16\,000 \times 4}$$

$$\approx 7.81$$

The rate of interest needed is 7.81% p.a.

EXERCISE 34.3

1 What flat rate of interest needs to be charged if you want to earn:

a \$900 after $4\frac{1}{2}$ years on \$5000?

b \$918 after 18 months on \$6800?

2 What rate of interest would need to be charged on a loan of \$18 000, if you wanted to earn \$2970 interest over 2 years?

3 A student wants to spend \$4500 on a car in 18 months' time. She has already saved \$4000, which she deposits into a flat-rate interest account. What interest rate must the account pay to enable her to reach her target?

EXAMPLE 34A-4 Finding the time of the loan

How long would it take to earn interest of \$4590 on a loan of \$12 000 if a flat rate of 8.5% p.a. is charged?

$$\text{Time} = \frac{100 \times \text{interest}}{\text{principal} \times \text{rate}}$$

where interest = \$4590
principal = \$12 000
rate = 8.5%

$$= \frac{100 \times 4590}{12\,000 \times 8.5}$$

$$= 4.5 \text{ years}$$

The loan would have a $4\frac{1}{2}$ year period.

NOTE

Time is the time period or 'length' of the loan. It is always expressed in years for this formula.

EXERCISE 34.4

1 How long would it take to earn interest of:

a \$4125 on a \$10 000 loan at a flat rate of 8.5% p.a.?

b \$8244 on a \$22 900 loan at a flat rate of 6% p.a.?

2 If you deposited \$6000 in an investment account that paid a flat rate of 6.75% p.a., how long would it take to earn \$1500 in interest?

34B Repayments on flat-rate loans

Whenever money is borrowed, it has to eventually be repaid along with the interest owing. The repayment of the amount owed (principal plus interest) is normally done by making regular (usually equal) payments over the length of the loan (weekly, monthly). The regular payment made includes both the principal and interest and is calculated by dividing the total amount repaid by the number of repayment periods.

$$\text{Amount of regular payment} = \frac{\text{total repayments}}{\text{number of repayments}} = \frac{\text{principal} + \text{interest}}{\text{number of repayments}}$$

When buying an item, such as a car, where finance through a bank or finance company is needed, it is important to check that you are making the correct repayments.

EXAMPLE 34B-1 Finding the repayment amount

Find the monthly repayments on a $12 000 flat-rate loan at 6% p.a. over 4 years.

Step 1: Calculate the interest on the loan.

$$\text{Interest} = \frac{\text{principal} \times \text{rate} \times \text{time}}{100}$$

$$= \frac{12\,000 \times 6 \times 4}{100} = \$2880$$

where principal = $12 000
rate = 6%
time = 4 years

Step 2: Number of repayments $= 4 \times 12 = 48$

Step 3: $$\text{Monthly repayment} = \frac{\text{principal} + \text{interest}}{\text{number of repayments}}$$

$$= \frac{12\,000 + 2880}{48} = \$310$$

The monthly repayments are $310.

EXERCISE 34.5

1 Sam buys a car from Henry's car yard. Henry is arranging $2500 finance. He tells Sam that the flat-rate loan at 8.25% interest will be repaid with equal monthly repayments of $115 over 3 years.
  a What interest is being charged on the loan over the 3 years?
  b For how many months does the loan operate?
  c Calculate the amount of the regular repayment Sam should pay on the loan. Should Sam accept the loan?

2 Calculate the monthly repayments on a flat-rate loan of $8500 at 8.5% p.a. over 4 years.

3 Calculate the amount of each repayment on a $20 000 flat-rate loan at 5.5% p.a. over 8 years repaid quarterly (every 3 months).

4 Kelly obtains a flat-rate loan of $8000 for 3 years at 7.5% p.a. If the loan is repaid 6 monthly, what size is each repayment?

34C Compound interest (by tables)

With compound interest, the interest is added to the principal (compounded) each period so that the principal on which interest is calculated continues to grow throughout the length or life of the loan. This differs from simple interest where the principal remains the same over the entire loan period.

Consider a compound interest investment of \$10 000 for 3 years where the interest is paid annually (once a year) and the interest rate is 8% p.a. The money stays with the investment company for the length of the loan and no payments are made until the 3 years are completed.

At the end of the first year the \$10 000 is increased by 8%.

Interest for year 1 = 8% of \$10 000
= $0.08 \times$ \$1000
= \$800 (which is the interest accrued for year 1)

The new value of the account is \$10 000 + \$800 = \$10 800.

At the end of year 2 the \$10 800 is also increased by 8%.

Interest for year 2 = 8% of \$10 800
= $0.08 \times$ \$10 800
= \$864 (which is more than for year 1)

The new value of the account is \$10 800 + \$864 = \$11 664.

Finally, at the end of year 3 the \$11 664 is also increased by 8%.

Interest for year 3 = 8% of \$11 664
= $0.08 \times$ \$11 664
= \$933.12 (which is more than for year 2)

The final value of the account is \$11 664 + \$933.12 = \$12 597.12.

Since the \$10 000 has increased to \$12 597.12:

Total interest earned = \$12 597.12 − \$10 000
= \$2597.12

If this investment was one where simple interest applied, the interest earned would have been $3 \times$ \$800 = \$2400, which is less than for a compound interest investment. The above calculations can be laid out in table form as in Example 34C-1.

EXAMPLE 34C-1 Finding the repayment amount

Calculate the final value of a \$10 000 compound interest investment after 3 years at 8% p.a. with interest calculated annually.

Time (years)	Amount	Interest at 8%
0	\$10 000	\$10 000 × 0.08 = \$800.00
1	\$10 000 + \$800.00 = \$10 800	\$10 800 × 0.08 = \$864.00
2	\$10 800 + \$864.00 = \$11 664	\$11 664 × 0.08 = \$933.12
3	\$11 664 + \$933.12 = \$12 597.12	

The final value of the investment is \$12 597.12.

EXERCISE 34.6

1 Find the final value of a compound interest investment of:
  a $2500 after 3 years at 6% p.a. with interest calculated annually
  b $4000 after 4 years at 7% p.a. with interest calculated annually
  c $8250 after 4 years at 8.5% p.a. with interest calculated annually.

2 Find the total interest earned for the following compound interest investments.
  a $750 after 2 years at 6.8% p.a. with interest calculated annually
  b $3350 after 3 years at 7.25% p.a. with interest calculated annually
  c $12 500 after 4 years at 8.1% p.a. with interest calculated annually

3 Xiao Ming invests $12 000 into an account that pays 7% p.a. compounded annually.
  a Find the value of her account after 2 years.
  b Find the total interest earned after 2 years.

4 Tikki places $5000 in a fixed-term investment account that pays 5.6% p.a. compounded annually.
  a How much will she have in her account after 3 years?
  b What interest has she earned over this period?

34D Compound interest (by formula)

Consider the previous example of $10 000 being invested over a 3-year period where the interest is paid annually and the rate is 8% p.a. The $10 000 is to be increased by 8% at the end of year 1.

End of year 1 value = $10 000 × 1.08 — 100% becomes 108%.

This new value must be increased by a further 8% at the end of year 2.

End of year 2 value = ($10 000 × 1.08) × 1.08

At the end of year 3, the final amount, A, is:

A = $10 000 × 1.08 × 1.08 × 1.08 — Multiply three times by 1.08.

= $10 000 × $(1.08)^3$

Using your calculator:

10000 [×] 1.08 [^] 3 [=] is $12 597.12 as before.

The total interest earned is $12 597.12 − $10 000 = $2597.12.

Compound interest formula

From examples like this one, we construct the compound interest formula.

$$\text{Amount} = \text{principal} \times (1 + \text{rate})^{\text{number}}$$

where amount is the final amount reached
principal is the amount originally borrowed or invested
rate is the interest rate per compound period
number is the number of periods (number of times the interest is compounded)

EXAMPLE 34D-1 Compound interest

$17 000 is deposited into a fixed-term investment account that pays 9% p.a. compounded annually for a 5-year period. Find, to the nearest cent:

a the final amount in the account **b** the total interest earned.

a Final amount = principal × (1 + rate)$^{\text{number}}$ where principal = $17 000
= $17 000 × $(1.09)^5$ rate = 9% = 0.09
= $26 156.61 number = 5 years

Calculator: 17000 × 1.09 ^ 5 =

b Interest earned = final amount − principal
= $26 156.61 − $17 000
= $9156.61

EXERCISE 34.7

1 Use the formula or table method to find the final value of each investment with interest componding annually.

a $3000 after 4 years at 7% p.a. **b** $7500 after 6 years at 8.3% p.a.

2 Find the interest earned on each investment with interest componding annually.

a $12 000 after 5 years at 6.7% **b** $17 300 after 7 years at 8.2%

3 Nindi invests $30 000 into an account that pays 5.2% interest compounded annually.

a How much will he have in his account after 7 years, assuming no withdrawals?

b How much interest will he earn?

4 Which investment would earn you more interest on an $8000 investment for 5 years?

A one that pays 8% p.a. simple interest **B** one that pays 7.5% p.a. compound interest

Spreadsheet: compounding amounts

Suppose you have $10 000 and wish to invest it for several years at a fixed compounding rate of 10% p.a. For an increase in value of 10%, the multiplier is 110%.

Interest = 100% + 10% = 1 + 0.1 = 1.1

At the end of year 1, the new amount = $10 000 × 1.1 = $11 000

At the end of year 2, the new amount = $11 000 × 1.1 = $12 100, and so on.

obook An Excel spreadsheet template to help you calculate compounding interest is available on your obook.

EXERCISE 34.8

1 Complete the table below for the $10 000 investment at a fixed compounding rate of 10% p.a. for up to 10 years. Then set this up as a spreadsheet as shown on the next page.

Number of years	0	1	2	3	4	5	6	7	8	9	10
Amount	$10 000	$11 000	$12 100								

2 For the spreadsheet on the left, type in the first three columns as shown. Fill down columns A and B from row 5 to row 20. You should see the spreadsheet on the right.

	A	B	C
1	**Year**	**Amount**	**Rate**
2	0	10000.00	10%
3	=A2+1	=B2*(1+$C$2)	
4	=A3+1	=B3*(1+$C$2)	
5	=A4+1	=B4*(1+$C$2)	
6	↓Fill down	↓Fill down	

	A	B	C
1	**Year**	**Amount**	**Rate**
2	0	10000.00	10%
3	1	11000.00	
4	2	12100.00	
5	3	13310.00	
6	↓Fill down	↓Fill down	

a Investigate what happens with other investment rates, such as 6%, 8%, 12%, 7.13%, by replacing cell C2 with any of these.

b Suppose you want your investment to double in 6 years. By trial and error in cell C2, find the interest rate that would achieve this, correct to 2 decimal places.

Extension

c You have $35 000 to invest and wish to know the fixed interest rate that will enable your investment to earn $10 000 in 8 years. (You do not need to set up a new spreadsheet, just replace 10000 in B2 with 35000.)

34E Repaying a compounding loan

> **obook**
> An Excel spreadsheet template to help you calculate reducing-balance loan repayments is available on your obook.

The method of repaying compound interest loans is shown in the following table and spreadsheet.

Suppose $2000 is to be repaid in monthly repayments of $400 at an interest rate of 12% p.a. compounding monthly. (This means that the interest charged is 1% per month.) The outstanding balance at the end of each month must have 1% interest added to it before the $400 repayment is taken.

The interest rate per month is the interest rate per year divided by 12. This creates a reduced balance still owing, as we can see in the following table.

End of month	Old balance	+ Interest	Repayment	New balance
0	$0.00			$2000.00
1	$2000.00	1% of $2000.00 = $20.00	$400.00	$1620.00
2	$1620.00	1% of $1620.00 = $16.20	$400.00	$1236.20
3	$1236.20	1% of $1236.20 = $12.36	$400.00	$848.56
4	$848.56	1% of $848.56 = $8.49	$400.00	$457.05
5	$457.05	1% of $457.05 = $4.57	$400.00	$61.62
6	$61.62	1% of $61.62 = $0.62	$62.24	$0.00

We see that the loan is repaid at the end of the sixth month. There are five monthly repayments of $400 each and a final one of $61.62 + $0.62 = $62.24.

Spreadsheet: reducing-balance loan repayments

The previous table of results shows that the balance owed at the end of each month can be easily obtained using the spreadsheet.

	A	B	C	D
1	End of period	Balance owed		
2	0	2000	rate %	12
3	=A2+1	=B2+$D$4/100*B2–$D$6	repayments per year	12
4	=A3+1	=B3+$D$4/100*B3–$D$6	rate % period	=D2/D3
5	↓Fill down	↓Fill down		
6			repayment amount	400
7				
8				

This will appear as follows.

	A	B	C	D
1	End of period	Balance owed		
2	0	2000	rate %	12
3	1	1620.00	repayments per year	12
4	2	1236.20	rate % period	1
5	3	848.56		
6	4	457.05	repayment amount	400
7	5	61.62	← residual	
8	6	–337.76		

However, most reducing-balance loans are such that:

- The periodic repayments are equal.
- There is no residual: the residual is the final payment amount, usually not the fixed repayment amount per period.

EXERCISE 34.9

1 Load the spreadsheet above, filling down columns A and B to create the above data.

2 Use trial and error replacements of cell D6 to create a zero (or very close to it) residual at B5.

3 Fill down columns A and B further to extend the loan period to 18 months. Replace D2 with 8.3 (for an 8.3% interest rate). Replace B2 with 3500. Use trial and error to create a zero residual at the end of period 18. For a $3500 reducing-balance loan at 8.3% p.a. over 18 months, what monthly repayment is needed to pay off the loan?

4 Use your spreadsheet to find the weekly repayments on a $20 000 reducing-balance loan over an 8-month period at 7.2% p.a.

CHAPTER 35

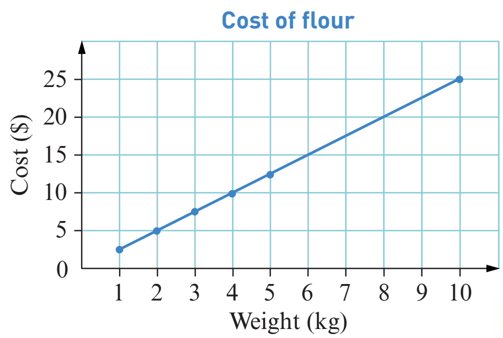
Graphical representation

35A Graphing lines from tables

We frequently find situations that can be modelled using straight line (linear) graphs. Sometimes when we plot points on a set of axes, they turn out to be linear or very close to linear. For example, a brand of flour can be purchased in bulk at $2.50 per kilogram and the following table shows the cost for several given weights of flour.

Weight (kg)	1	2	3	4	5	10
Cost ($)	2.50	5.00	7.50	10.00	12.50	25.00

Since the cost depends on the number of kilograms bought, we decide to graph the cost (dependent variable) on the vertical y-axis against the number of kilograms (independent variable) on the horizontal x-axis.



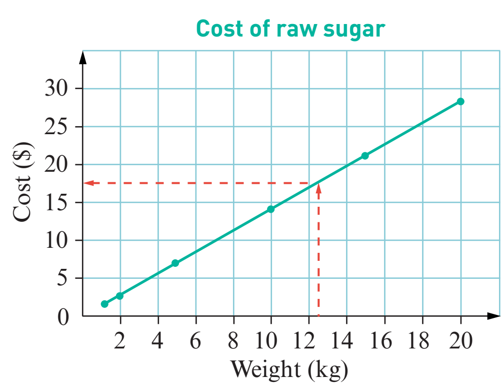
EXAMPLE 35A-1 Weight-versus-cost graph

Bulk raw sugar is sold for $1.40 per kilogram. The table below shows weight versus cost for the sugar.

Weight (kg)	1	2	5	10	15	20
Cost ($)	1.40	2.80	7.00	14.00	21.00	28.00

a Draw the graph of weight versus cost.
b Use the graph to find the cost of 12.5 kg of sugar.

a As the cost depends on the number of kilograms, it is the dependent variable and is put on the vertical (y) axis. The number of kilograms is the independent variable and is put on the horizontal (x) axis.



b Draw a line from 12.5 kg on the horizontal axis to the graph, then across to the vertical axis. The value is halfway between $15 and $20.
Thus 12.5 kg of sugar would cost about $17.50.

EXERCISE 35.1

1 Wine is sold in bulk for $5/L. This table shows litres bought versus cost of the wine.

Quantity (L)	1	2	3	4	5	6
Cost ($)	5	10	15	20	25	30

a Which is the dependent variable? Draw the graph of quantity versus cost.

b Use the graph to find the cost of 3.5 L of wine.

2 Chocolate frogs are sold for $15/kg. The table shows kilograms versus cost for the chocolate frogs.

Weight (kg)	1	2	3	4	5	6
Cost ($)	15	30	45	60	75	90

a Which is the dependent variable? Draw the graph of weight versus cost.

b Use the graph to find the cost of 4.5 kg of chocolate frogs.

3 Coffee beans cost $26/kg. The table shows weight in kilograms versus cost for the coffee beans.

Weight (kg)	1	2	3	4	5	6
Cost ($)	26	52	78	104	130	156

a Which is the dependent variable? Draw the graph of weight versus cost.

b Use the graph to find the cost of 5.5 kg of coffee beans.

EXAMPLE 35A-2 Distance-versus-cost graph

The cost of hiring a 40-passenger bus is a $150 booking fee plus $9.60/km.

a Complete this table of values for hiring the bus.

Distance (km)	0	10	20	30	40
Cost ($)					

b Sketch the graph of cost per kilometre.

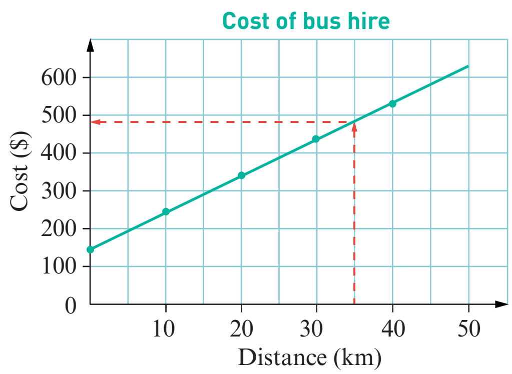
c Find the cost of a journey of 35 km.

EXAMPLE 35A-2 continued **Distance-versus-cost graph**

a For a journey of 0 km, cost = $150
For a journey of 10 km cost = $150 + $9.60 × 10 = $246
For a journey of 20 km, cost = $150 + $9.60 × 20 = $342
For a journey of 30 km, cost = $150 + $9.60 × 30 = $438
For a journey of 40 km, cost = $150 + $9.60 × 40 = $534
The table of values for hiring the bus is:

Distance (km)	0	10	20	30	40
Cost ($)	150	246	342	438	534

b



c The cost for a 35-km journey is about $480. (It is actually $486.)

NOTE
Remember to label the axes and use even divisions on the scale.

EXERCISE 35.2

1 The cost of hiring a taxi is a $3 flag fall and $2.40 per kilometre travelled.

a Complete the table of costs for taxi hire.

Distance (km)	0	10	20	30	40
Cost ($)	3	27			

b Draw the graph showing the cost of hiring a taxi.

c How much does it cost to travel 25 km?

2 Jon drove from Kangaroo Creek to Melbourne and every hour he travelled 90 km. His distance to Melbourne at any stage was given by distance = 540 km − 90 × time, where the time is hours since starting the trip.

a Complete this table.

Time (h)	0	1	2	3	4	5
Distance to Melbourne (km)	540	450				

b Draw the graph showing what distance is covered per hour (with time on the horizontal axis).

c How far is Kangaroo Creek from Melbourne?

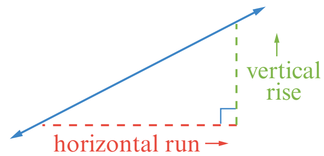
d How long did Jon take to travel to Melbourne?

e How far did Jon still have to travel after $4\frac{1}{2}$ hours?

35B Slope

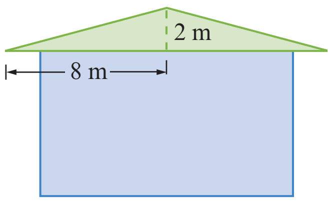
The word 'slope' or 'gradient' is often used to describe the degree of steepness of roads, railway tracks, house roofs, hills, etc. We measure slope using the following formula.

$$\text{Slope} = \frac{\text{vertical rise}}{\text{horizontal run}}$$



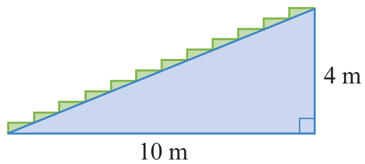
The following diagrams indicate slopes of varying amounts. The steeper the slope, the larger the gradient.

House roof



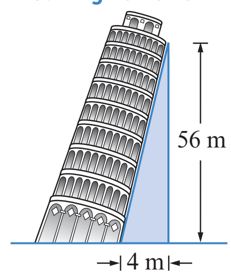
$$\text{Slope} = \frac{2}{8} = \frac{1}{4}$$

Escalator



$$\text{Slope} = \frac{4}{10} = \frac{2}{5}$$

Leaning Tower of Pisa



$$\text{Slope} = \frac{56}{4} = 14$$

FINDING SLOPES

Slope or gradient is also used when talking about the steepness of a line graph. When line segments are drawn on graph paper, we can easily determine the slope of the segments by drawing horizontal and vertical lines to complete a right-angled triangle.

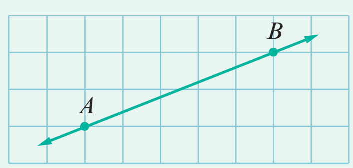
We can determine slopes by adopting the following procedure.

- Choose two convenient points on the graph (such as where it crosses the grid).
- Starting from the left-hand point, move horizontally to determine the number of units across (run) to the second point and then move vertically up or down to determine the number of units (rise) to the graph.
- Determine the slope using $\frac{\text{rise}}{\text{run}}$.

Notice that for a horizontal line graph, the vertical rise is 0, therefore the slope is 0.

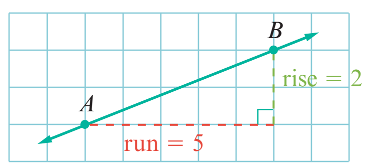
EXAMPLE 35B-1 Finding slopes

Find the slope of AB.



$$\text{Slope of } AB = \frac{\text{rise}}{\text{run}}$$

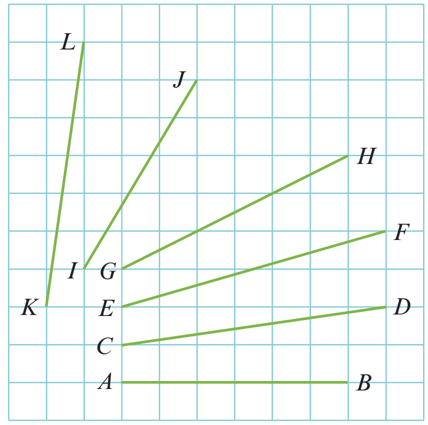
$$= \frac{2}{5}$$



EXERCISE 35.3

1 **a** Complete this table to find the slope of each of these line segments (or part lines).

Line segment	Run	Rise	Slope
AB			
CD			
EF			
GH			
IJ			
KL			



b Complete the following.

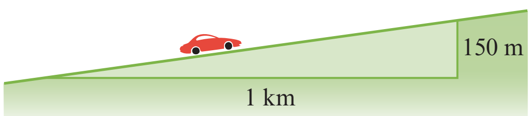
i The slope of a horizontal line is ______.

ii The slope of a vertical line is ______ or ______.

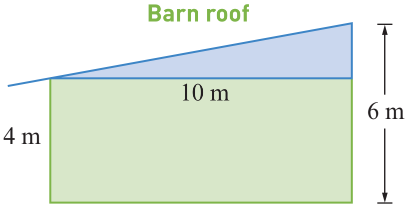
iii As the line segments become steeper, their slopes ______.

2 Find the slope of the following.

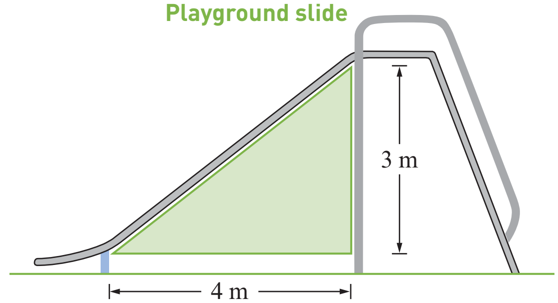
a Road up a hill



b Barn roof

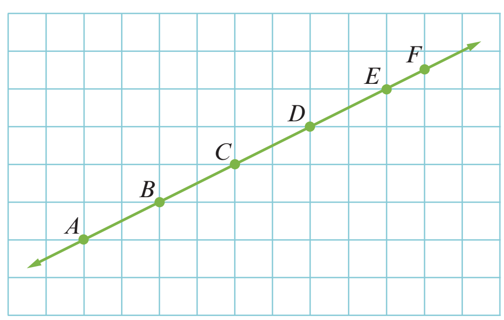


c Playground slide



3 **a** Complete the following table.

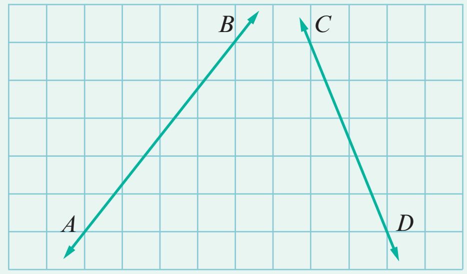
Line segment	Run	Rise	$\frac{\text{Rise}}{\text{Run}}$
BC	2	1	$\frac{1}{2}$
DE			
AC			
BE			
AE			
AF			

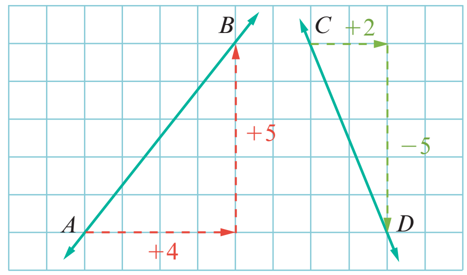


b State, in sentence form, any conclusions drawn from the graph and table.

EXAMPLE 35B-2 Positive and negative slopes

Determine the slope of AB and CD.





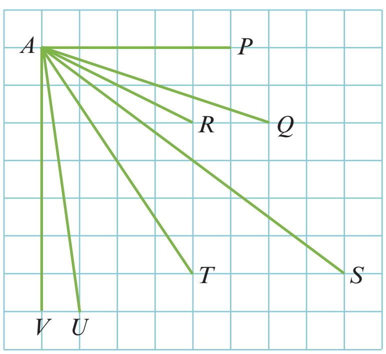
$$\text{Slope of } AB = \frac{\text{rise}}{\text{run}}$$
$$= \frac{+5}{+4} = \frac{5}{4}$$
$$\text{Slope of } CD = \frac{\text{rise}}{\text{run}}$$
$$= \frac{-5}{+2} = -\frac{5}{2}$$

EXERCISE 35.4

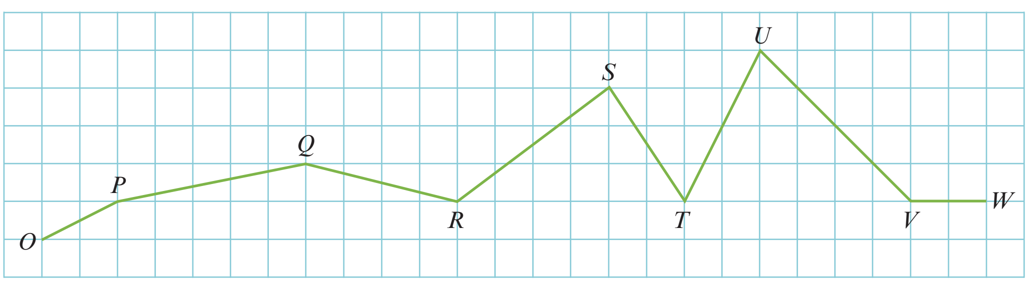
1 Determine the slope of these line segments.

a AP b AQ c AR

d AS e AT f AU



2 Imagine that you are walking across the countryside from O to W (from left to right).



a Indicate when you are going uphill.

b Indicate when you are going downhill.

c Where is the steepest positive slope?

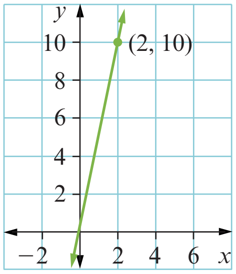
d Where is the steepest negative slope?

e Where is the slope zero?

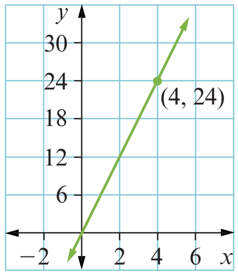
f When is the slope not zero but least?

3 Find the slope of these line graphs.

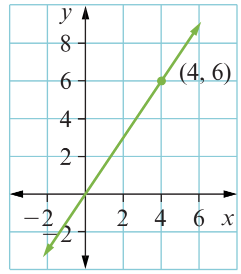
a



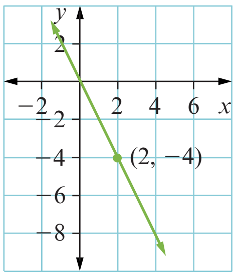
b



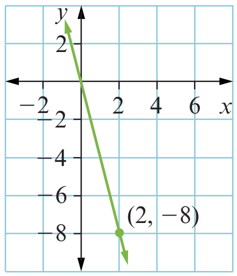
c



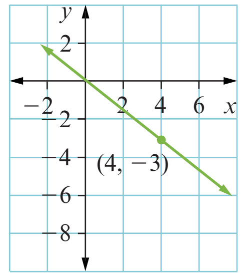
d



e



f



35C Straight-line graphs

The point where a straight-line graph meets the y-axis is called the y-intercept.

If a straight-line graph has slope m and y-intercept b, then the rule (equation) that connects the x- and y-coordinates for any point on the line is:

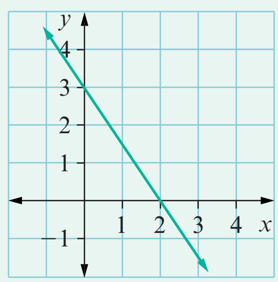
$$y = mx + b$$

EXAMPLE 35C-1 Straight-line graph equations

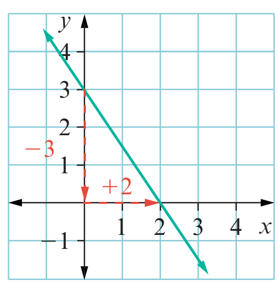
a Find the gradient of this line.

b Find where the graph cuts the vertical axis.

c Write the equation of the graph.



This graph does not pass through the origin. Draw a right-angled triangle in the same way as previously.



a Gradient $= \dfrac{\text{rise}}{\text{run}}$

$= \dfrac{-3}{+2}$

$= -\dfrac{3}{2}$ This is m.

b The graph cuts the vertical axis at 3. This is b.

c Substituting into $y = mx + b$, we get the equation $y = -\frac{3}{2}x + 3$.

NOTE

Note that the gradient is negative.

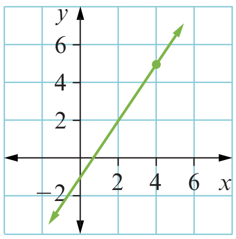
EXERCISE 35.5

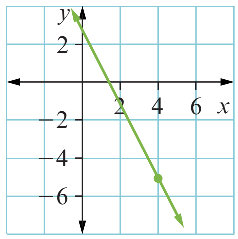
1 For the following graphs, find:

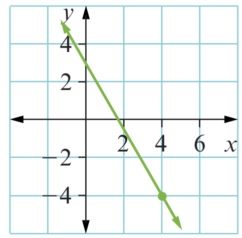
i the gradient of each line

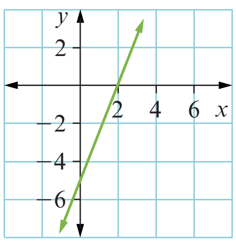
ii where it cuts the vertical axis

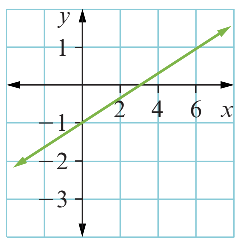
iii the equation of the graph.

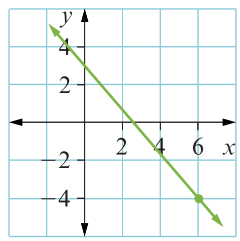
a


b


c


d


e


f


EXAMPLE 35C-2 Drawing linear graphs

Draw the linear graph modelled by the equation $y = 2x + 3$.

Substitute x values into the equation $y = 2x + 3$.

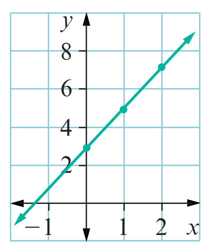
When $x = 0$, $y = 2 \times 0 + 3 = 3$

When $x = 1$, $y = 2 \times 1 + 3 = 5$

When $x = 2$, $y = 2 \times 2 + 3 = 7$

We write these as a table of values.

x	0	1	2
y	3	5	7



Use the table to plot the points and draw a straight line through them.

EXERCISE 35.6

1 Complete this table of values for each graph and sketch the graph.

a $y = 3x + 1$

b $y = 2x + 2$

c $y = -2x - 1$

d $y = 5x + 2$

x	0	1	2
y			

2 Sketch these straight-line graphs.

a $y = 2x + 7$

b $y = -3x - 1$

c $y = -x + 3$

d $y = \frac{1}{2}x - 1$

e $y = -\frac{3}{4}x + 1$

f $y = 0.4x + 1.2$

35D Linear modelling

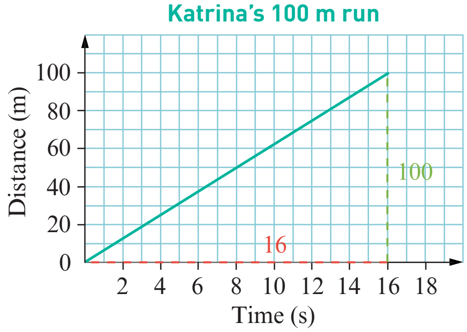
The gradient and intercept on the vertical axis of linear graphs have meaning in practical situations. A straight-line graph shows a direct relationship between the two variables.

EXAMPLE 35D-1 Understanding the gradient

At the athletics carnival, Katrina runs the 100 m sprint in 16 s. It is assumed that she runs at a constant rate.

a Sketch a straight-line graph to model Katrina's run.
b Find the gradient.
c Explain the meaning of the gradient.

a Use two points to draw the graph. They are (0, 0) and (16, 100).



b Draw in a triangle.

$$\text{Gradient} = \frac{\text{rise}}{\text{run}} = \frac{100}{16} = 6.25 \text{ m/s}$$

c The gradient is speed in m/s. Katrina runs at an average speed of 6.25 m/s.

EXERCISE 35.7

1 Craig drives 200 km in 4 h at a constant speed.

a Draw a graph of Craig's drive.
b How far does Craig drive in $1\frac{1}{2}$ hours?
c Find the gradient of the graph.
d What are the units of the gradient in this case?

2 On a particular day, 100 Australian dollars buy 78 US dollars.

a Draw a conversion graph with Australian dollars on the x-axis.
b How many Australian dollars are needed to purchase US$50?
c Find the gradient of the graph.
d What is the meaning of the gradient in this context?
e Extend your graph up to A$500. Is it still accurate as an exchange rate model?

NOTE
Extending a graph and using this extension is called extrapolation.

EXAMPLE 35D-2 Meaning of gradient and intercept

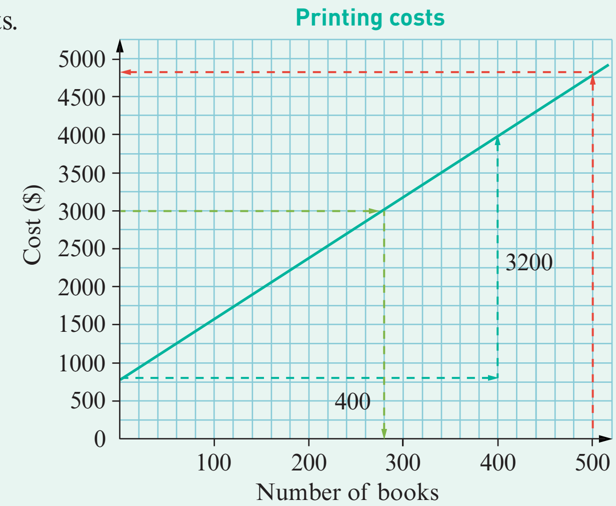
This graph models printing costs.

a Find the cost of printing 500 books.

b How many books can be printed for $3000?

c Find the gradient. What is its meaning?

d Find the y-intercept. What is its meaning?



a Printing 500 books would cost about $4800 (see red dashed line).

b $3000 would buy about 280 books (see green dashed line).

c Draw in a triangle to find the gradient.

$$\text{Gradient} = \frac{3200}{400} = 8$$

The gradient is 8. The units are $ per book, so the gradient is the cost per book.

d The y-intercept is approximately 800. The units are dollars. This means that the cost to print 0 books is about $800. This is the set-up cost.

EXERCISE 35.8

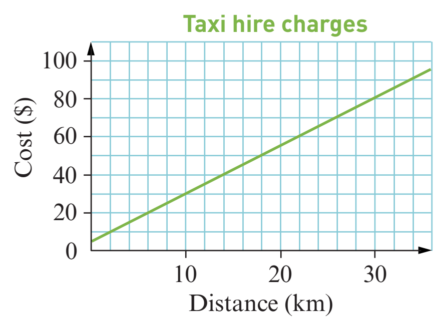
1 Here is a graph modelling taxi charges.

a Find the cost of travelling 25 km.

b How far can you travel for $40?

c Find the gradient. What is the meaning of the gradient?

d Find the intercept on the vertical axis. What is its meaning?



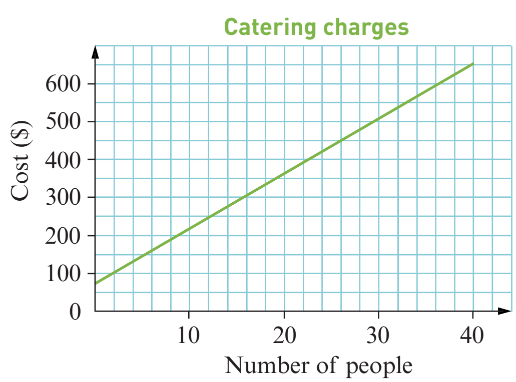
2 This graph models catering charges.

a How much would it cost for 35 people?

b How many people could eat for $300?

c Find the gradient. What is the meaning of the gradient?

d Find the intercept on the vertical axis. What is its meaning?



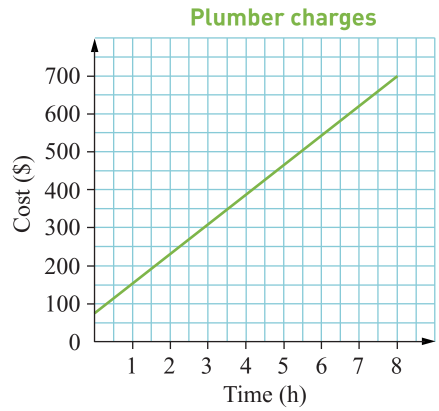
3 Here is a graph modelling a plumber's charges.

a How much will the plumber charge for $4\frac{1}{2}$ hours?

b Find the number of hours worked if the charge is \$300.

c Find the gradient. What is the meaning of the gradient?

d Find the intercept on the vertical axis. What is its meaning?



35E Scatter plots and line of best fit

Sometimes information given in a table does not lie exactly in a straight line, particularly when inaccurate measurements have been made. When the points are plotted they may be scattered, and so we call the graph of the points a scatter plot. We often try to draw a line that best fits the points and call it the line of best fit.

ESTIMATING THE LINE OF BEST FIT BY EYE

The line of best fit can be estimated by eye. We can use a ruler to judge how a straight line might best represent the scatter plot points.

EXAMPLE 35E-1 Line of best fit by eye

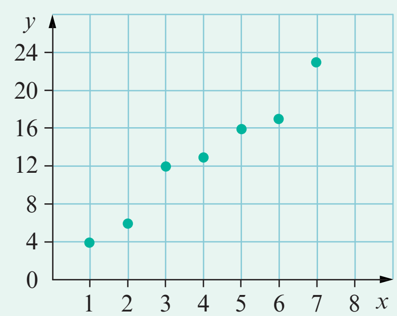
Consider the scatter plot shown.

a Draw a line of best fit by eye.

b Estimate the slope of the line using the coordinates of two points.

c Find the y-intercept by extending the line to the y-axis.

d Find the equation of the line.



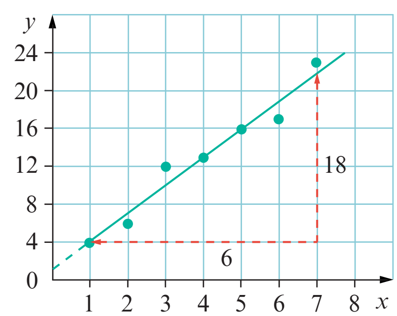
a The line of best fit is shown.

b Using points (x, y), the slope of the line is:

$$\frac{\text{rise}}{\text{run}} = \frac{18}{6}$$
$$= 3$$

c The extended line cuts the y-axis at about 1.

d The equation of the line is $y = 3x + 1$.



NOTE

Often we can try to draw a line that has the same number of points on each side.

EXERCISE 35.9

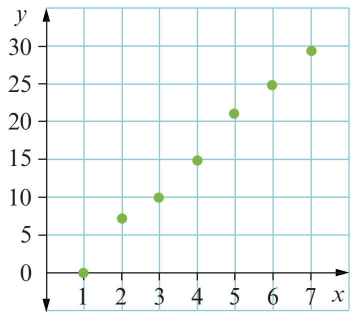
1 Accurately copy each graph and draw the line of best fit by eye.

  i Estimate the slope of the line using the coordinates of two points.

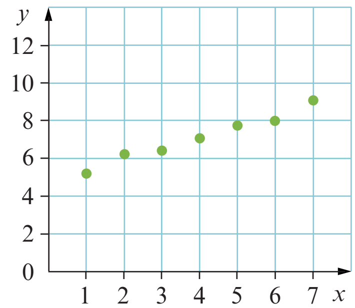
  ii Find the y-intercept by extending the line to the y-axis.

  iii Find the equation of the line.

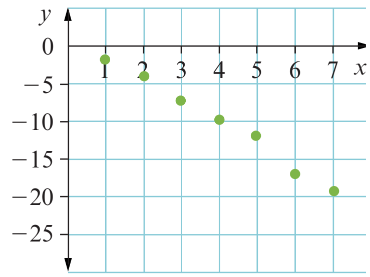
a



b



c

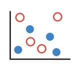


SPREADSHEET APPLICATION

obook An Excel spreadsheet template to help you calculate line of best fit is available on your obook.

Spreadsheets and graphics calculators can be used to find the line of best fit more quickly and accurately than was done in the previous exercise. These instructions to draw a line of best fit and find the linear equation are for Excel 2013. The information used is from the Example 35E-1. The data for this example is put into the spreadsheet below.

Step 1: Enter the data into the spreadsheet.

Step 2: Click on INSERT tab.
In the Charts section, click on the Scatter chart icon. 
Select the first Chart subtype  (the top left option).

Step 3: Right-click on any point in the data series.

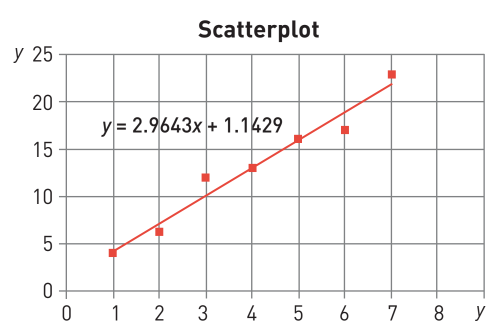
Step 4: Select Add Trendline. (The Trendline Options panel should appear; if not, select it and select Linear if this did not happen automatically.)

Step 5: Also in Trendline Options, select Display Equation on Chart.

Step 6: Save your spreadsheet with an appropriate file name.

The spreadsheet has drawn the line of best fit and shows the equation of this line. The gradient is 2.96 and the intercept is 1.14, so the equation is $y = 2.96x + 1.14$.

	A	B	C	D	E	F
1	x	y				
2	1	4				
3	2	6				
4	3	12				
5	4	13				
6	5	16				
7	6	17				
8	7	23				



EXERCISE 35.10

1 Using a spreadsheet, draws lines of best fit and find the equations for these tables of values.

a

x	1	2	3	4	5	6
y	12	16.5	22.6	27	33	39

b

x	1	2	3	4	5	6
y	4.8	4.1	2.9	1.8	1.1	0.2

c

x	1	2	3	4	5	6
y	20	25	29	37	42	46

d

x	1	2	3	4	5	6
y	30	26	22	19	14	10

35F Correlation

When dealing with a linear type of relationship, we use a concept known as correlation to decide how close the relationship is to a straight line.

Statisticians have devised some techniques to measure the correlation (strength and direction) of the linear relationship between two variables.

POSITIVE CORRELATION

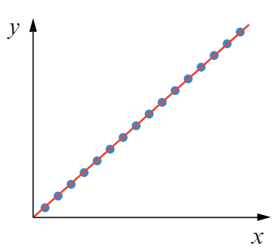
An association between two variables is described as a positive correlation if an *increase* in one variable results in an *increase* in the other, or as a *decrease* if one variable results in a *decrease* in the other in an approximately linear manner.

The strength of the association is best measured with the correlation coefficient (r) that ranges between 0 and 1.

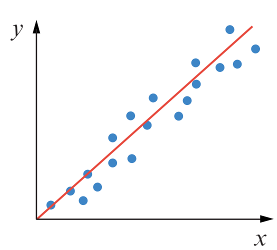
- An r value of 0 suggests that there is no linear association present (or no correlation).
- An r value of 1 suggests that there is a perfect positive linear association present. This means that the graph will be a straight line with a positive slope.

The correlation between the height and weight of people is positive and lies between 0 and +1, as taller people are generally heavier than shorter people. However, it is not an example of perfect positive correlation as not all short people are of light weight.

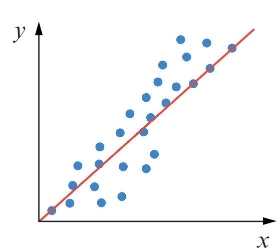
The r values between 0 and 1 show how close the association is to linear. Examples of scatter diagrams for positive correlation are shown below.



$r = +1$



$r \approx +0.8$



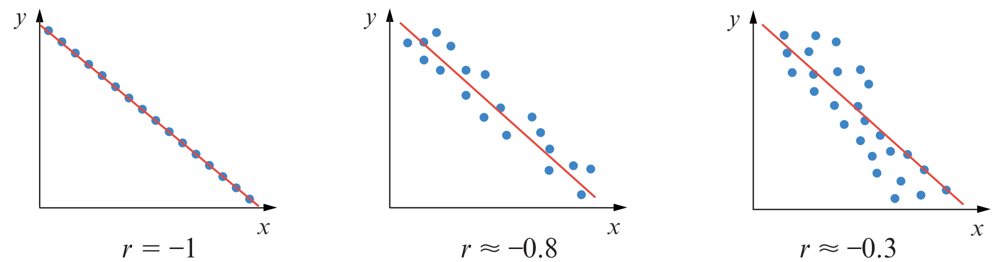
$r \approx +0.3$

NEGATIVE CORRELATION

An association between two variables is described as a negative correlation if an *increase* in one variable results in a *decrease* in the other, or as a *decrease* if one variable results in an *increase* in the other in an approximately linear manner.

The strength of the association is best measured with the correlation coefficient (r) that ranges between 0 and −1.

- An r value of −1 suggests that there is a perfect negative linear association present. This means that the graph will be a straight line with a negative slope. Examples of scatter diagrams for negative correlation are shown below.



Interpreting the correlation coefficient (r) using r^2

The main difficulty is in describing the strength of the association (that is, how close it is to a straight line) given the r value.

The following rules of thumb may help describe the strength of correlation using the coefficient of determination value, r^2, or the square of the correlation coefficient, r.

$r^2 = 0$	No correlation	$0 < r^2 < 0.25$	Very weak correlation
$0.25 \leqslant r^2 < 0.50$	Weak correlation	$0.50 \leqslant r^2 < 0.75$	Moderate correlation
$0.75 \leqslant r^2 < 0.90$	Strong correlation	$0.90 \leqslant r^2 < 1$	Very strong correlation
$r^2 = 1$	Perfect correlation		

obook
An Excel spreadsheet template to help you calculate r and r^2 is available on your obook.

Calculating r and r^2 on a spreadsheet

Step 1: Open an Excel spreadsheet and enter this data. The data relates to the extension of a spring (in cm) against the mass hung on it (in g).

	A	B	C	D
1	**Extension of spring (cm)**	**Mass (g)**		
2	0	18.3		
3	50	20.0		
4	100	22.4		
5	150	23.9		
6	200	26.2		
7	250	28.6		

Step 2: Save your spreadsheet as Correlation coeff.xls.

Step 3: In cell C5 type =CORREL(A2:A7,B2:B7) (this is r).
In cell C6 type =C5^2 (this is r^2).

The answer 0.9979… in cell C5 shows that r is positive. The answer 0.9959… in cell C6 indicates that there is a very strong correlation. Thus we say that there is a very strong positive correlation between the force applied to the spring and its length.

NOTE
Two variables may be highly correlated, but this does not mean that one causes the other. For example, damage caused by fire and the number of firefighters fighting the fire are highly correlated, but one does not cause the other. Both are related to a third variable, size of the fire.

EXERCISE 35.11

1 Getrid Inc. has been trying out a new chemical to control the number of lawn beetles in soil. Use a spreadsheet to determine the strength and direction of the correlation between the amount of chemical used and the number of beetles surviving per m^2 of lawn given the following data.

	Chemical (g)	Number surviving per m^2
Lawn A	2	11.2
Lawn B	5	6.4
Lawn C	6	3.9
Lawn D	3	5.7
Lawn E	9	2.8

obook
An Excel spreadsheet template to help you complete Exercise 35.11 is available on your obook.

2 Tomatoes are sprayed with a pesticide–fertiliser mix. The table gives the annual yield of tomatoes per plant (in kg) for various spray concentrations (in mL of pesticide–fertiliser/L of water).

Concentration (mL/L)	3	5	6	8	9	11
Yield per plant (kg)	6.7	9.0	10.3	12.0	12.4	15.0

a Draw a scatter plot for this data.
b Determine the r and r^2 values.
c Describe the association between yield and spray concentration.

3 The table shows the annual cherry yield and incidence of frosts data for a cherry-growing farm over a 7-year period.

Number of frosts	10	16	17	23	27	32	36
Cherry yield (t)	7.2	4.2	5.5	4.5	3.6	3.8	4.4

a Draw a scatter plot for this data.
b Determine the r and r^2 value.
c Describe the association between cherry yield and the number of frosts.

4 A building company is building houses in a new subdivision and completes them at the rate of 32 per month in fine weather. The data gives the number of houses completed when there are x days of rain in the month, for the first 6 months of the year.

Days of rain (x)	0	2	3	5	8	10	13
Number of houses completed (y)	32	31	30	27	22	20	15

a Draw a scatter plot for this data.
b Determine the r and r^2 value.
c Describe the association between the number of houses completed and the number of days of rain.

CHAPTER 36
Representing data

36A Data collection

In today's modern world, the amount of data (information) that is recorded is staggering. Every day data is collected and analysed relating to all kinds of things, such as the number of people living in different countries, how much they earn, how many of them are unemployed, the number of children they have, the number who have access to mobile phones and the internet, and the websites they access.

Think of anything happening in the world right now, and chances are that data is being collected on it. Government departments, businesses and scientific research bodies are all examples of groups that use statistics in one way or another.

When studying statistics, the entire group of people, objects or things being analysed is called the population. Gathering statistical information properly is vitally important. If gathered incorrectly, any resulting analysis of the data would almost certainly lead to incorrect conclusions. The gathering of statistical data may take the form of:

- a *census*, where information is collected from the whole population
- a *survey*, where information is collected from a much smaller group of the population, called a sample.

When using a sample, it is important that the information gathered is representative of the entire population. If the sample is too small, the data obtained is likely to be less reliable than that obtained from larger samples. For accurate information when sampling, it is essential that the individuals involved in the survey are randomly chosen from the population, and the number of individuals in the sample is large enough.

EXAMPLE 36A-1 Effect of sample size

Consider tossing a coin. Comment on the situation of making deductions about the results, in general, after tossing the coin:

a once only **b** 10 times **c** 1000 times.

a If we toss the coin once only and the result is a head, say, we could falsely conclude that 'when tossing a coin, the result is always a head'.

b If we toss the coin 10 times this could also lead to an incorrect deduction as it could result in nine heads (although unlikely) and you might be tempted to conclude that 'when tossing a coin, the result is a head 90% of the time'.

c Tossing the coin 1000 times gives a far more reliable picture. A result of 517 heads and 483 tails would still result in the reliable conclusion that 'when tossing a coin, heads will occur about 50% of the time'.

As a class, discuss the following.

- How would you randomly select 12 members of the public to stand for jury duty?
- How would you randomly select four numbers between 0 and 36 on a roulette wheel?
- When conducting a survey to find out the percentage of people who believe the AFL grand final should always be played at the MCG, would it be a good idea to ask a section of the crowd at the grand final this year?

EXAMPLE 36A-2 Estimation from a sample

157 out of 4500 wombats were captured and 11 were found to have eye problems.

a What is the population size?
b What size is the sample?
c What fraction of the population is estimated to have eye problems?
d Estimate the number of wombats in the population that have eye problems.

a The population size is 4500 wombats.
b The sample size is 157 wombats.
c Estimated fraction $= \dfrac{\text{number with property in sample}}{\text{sample size}} = \dfrac{11}{157}$

$\frac{11}{157}$ of the population are estimated to have eye problems.

d Estimated number $= \dfrac{\text{number with property in sample}}{\text{sample size}} \times$ population size.

$$= \frac{11}{157} \times 4500 \approx 315$$

About 315 wombats in the population have eye problems.

EXERCISE 36.1

1 A factory produces 5000 microprocessors per week. A random sample of 400 revealed that two were faulty.
a What size is the population?
b What size is the sample?
c How many microprocessors in the sample were not faulty?
d Estimate the total number of microprocessors produced in a week that are not faulty.

2 1150 householders were selected at random from the electoral roll and asked whether they would vote for the Australian Labor Party. The survey revealed that 620 answered ‘Yes’.
a If there are 16 million people in Australia over the age of 18, estimate how many would answer ‘No’.
b What percentage of Australians over 18 would answer ‘Yes’ in your estimation?

3 An alpine lake contains trout. On one particular day, a fisheries officer caught 600 trout. They were then tagged and released back into the lake. A fortnight later, 350 trout were caught and of these 28 had tags.
a Estimate the number of trout in the alpine lake.
b In calculating your estimate, what assumptions have you made?

36B Frequency tables

The organisation and display of data in a table or graph can help us make sensible comparisons. It may tell us what has happened in the past and help us predict the future.

Statistical information is often organised and presented in the form of a frequency table that shows individual values and the number of times each value occurs. A frequency table may also include columns for relative frequency and percentage relative frequency. The relative frequency of an event is given by:

$$\text{Relative frequency} = \frac{\text{frequency}}{\text{sample size}}$$

NOTE

The frequency of an event is the number of times it occurs in the sample.

SINGLE-VARIABLE DATA

EXAMPLE 36B-1 Frequency tables

The scores out of 10 for 30 students in a mental arithmetic test are listed below.

9 10 8 4 7 9 4 7 6 9 6 5 9 7 5 7 8 5 7 8 5 8 6 6 10 7 5 9 9 8

Draw a frequency table. Include a relative frequency column (to 3 decimal places).

Score	Tally	Frequency	Relative frequency
4	||	2	0.067
5	~~||||~~	5	0.167
6	||||	4	0.133
7	~~||||~~ |	6	0.200
8	~~||||~~	5	0.167
9	~~||||~~ |	6	0.200
10	||	2	0.067

← Relative frequency of 4

$$= \frac{\text{frequency}}{\text{sample size}}$$

$$= \frac{2}{30}$$

$$\approx 0.067$$

EXERCISE 36.2

1 The following data represents the number of mint drops in a packet.

28 32 29 32 33 29 31 32 27 28 27 30 26 31 27 28 32 33 28 29

31 32 28 31 30 29 30 27 32 29 32 31 29 32 31 27 28 29 27 31

a Tabulate the data, including columns for number, tally, frequency and relative frequency.

b What percentage of packets contain 30 or more mint drops?

2 A survey was conducted to find the number of matches in matchboxes. The results obtained from the first 60 boxes were:

47 52 49 51 50 47 50 49 48 51 49 50 50 48 50 48 49 50 52 47

52 48 50 51 49 50 51 50 51 52 52 52 50 50 48 50 51 50 49 51

49 48 50 50 51 49 50 48 50 51 47 50 48 50 51 48 49 50 49 50

a Tabulate the data in a frequency table, including a column for relative frequency.

b What percentage of matchboxes contained less than 50 matches?

3 A manufacturer produced a new heating element for a toaster that is expected to last 100 hours. To test the life of the new element, 50 toasters were selected at random and tested. The results were:

106 97 94 107 105 93 104 99 110 102 108 104 96 104 100 98 109
106 101 99 101 104 107 106 95 94 100 102 103 94 93 101 105 107
96 100 99 97 95 98 102 105 107 105 94 93 105 102 101 101

a Tabulate the data in columns, including columns for frequency and relative frequency.
b How many times did a result of 100 or more hours occur?
c What percentage of the results were 104 or 105 hours?
d What percentage of the results were less than 100 hours?

GROUPED DATA

Sometimes the observed data consists of many observations over a large range of values. If this is so, group the data into classes and determine the frequency of each class.

NOTE

For data with a large range of values recorded (such as from 0% to 100%), we group the data into classes such as 0–9, 10–19, 20–29, etc.

EXAMPLE 36B-2 Frequency table for grouped data

The following scores were obtained for a test that measured intelligence.

Test score	70–79	80–89	90–99	100–109	110–119	120–129	130–139
Frequency	9	28	43	62	37	14	7

Prepare a relative frequency table. Find the percentage of students who scored:

a less than 90 for the test b 110 or more for the test.

a For scores less than 90:
4.5% are 70–79
14.0% are 80–89
18.5% of students scored less than 90.

b For scores 110 or more:
18.5% are 110–119
7.0% are 120–129
3.5% are 130–139
29.0% of students scored 110 or more.

Test score	Frequency	Relative frequency
70–79	9	0.045
80–89	28	0.140
90–99	43	0.215
100–109	62	0.310
110–119	37	0.185
120–129	14	0.070
130–139	7	0.035
Total	200	1.000

EXERCISE 36.3

1 A group of young footballers were invited to participate in a distance kicking competition. These results were obtained.

Distance (m)	20–29	30–39	40–49	50–59	60–69
Number	3	26	41	14	6

a Tabulate the data to include a column for relative frequency.
b How many footballers kicked less than 40 m?
c What percentage of the footballers were able to kick at least 50 m?

2 A plant inspector takes a random sample of 2-week-old seedlings from a nursery and measures their height to the nearest mm. The results are shown in the table on the right.

Height (mm)	Frequency
300–324	12
325–349	18
350–374	42
375–399	28
400–424	14
425–449	6

a Add a relative frequency column to the table.

b How many of the seedlings are 400 mm or taller?

c What percentage of the seedlings are between 350 mm and 399 mm?

d The total number of seedlings in the nursery is 1462. Estimate the number of seedlings that measure:

i less than 400 mm

ii between 375 mm and 424 mm.

36C Graphical representation

In order to see differences in data more easily, we can display the data on a graph of some kind. We will examine the following ways of displaying data:

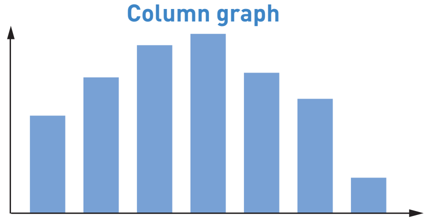
- frequency column graphs/histograms
- stem-and-leaf displays.

COLUMN GRAPHS AND HISTOGRAMS

Column graphs and histograms both have the following features:

- on the vertical axis we have the frequency of occurrence
- on the horizontal axis we have the range of scores
- column widths are equal
- the height varies according to the frequency.

Column graphs are used whenever the data is categorical. For example, for data on the number of students in a class with a particular eye colour, we would use a column graph. There are gaps between the columns, and the gaps are usually about half the width of the columns.



NOTE

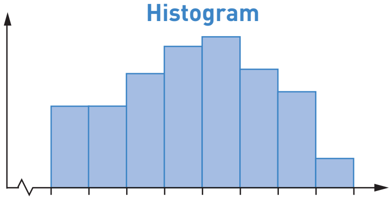
A categorical variable is one that describes a characteristic or names the categories into which the variable can be sorted.

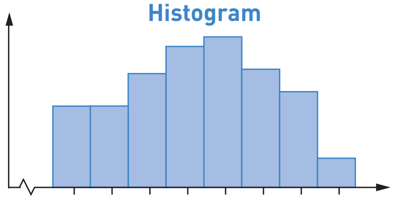
Histograms are used whenever the data is numerical. Numerical variables have a numerical value and can be discrete or continuous. For example, for data on the weights of Year 11 students we would use a histogram. There are no gaps between the columns.

NOTE

Continuous data is a value within a possible range of values.

Discrete data is an exact value.





EXAMPLE 36C-1 Drawing a histogram

The table shows the maximum daily temperature recorded (to the nearest °C) for a country town in February.

Maximum temperature (C°)	31	35	36	37	38	39	40	41	42
Frequency	1	2	5	7	8	3	0	1	1

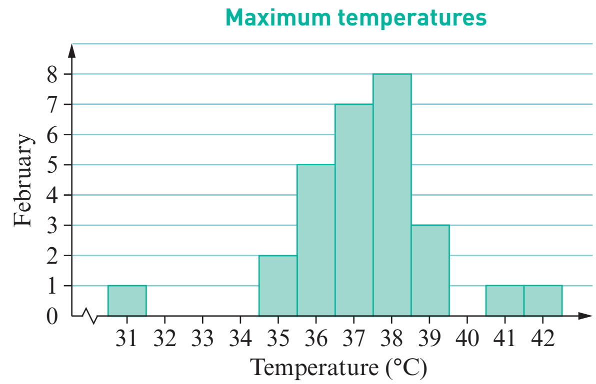
a Is the data discrete or continuous?

b Construct an appropriate histogram or column graph.

NOTE Any graph such as this must have a title, labels on the axes and scales.

a Temperature is continuous as it is possible to get temperatures like 36.2°C and 37.8°C. But, in the table, the values given are discrete values (rounded to the nearest degree).

b Since the data is numerical, we draw a histogram.



When the data covers a large range of values, it is desirable to group the data into classes, and graph the frequency of each class.

EXAMPLE 36C-2 Grouped data classes

The following weights (in kg) are for 26 students in a mathematics class.

101	91	83	84	72	93	67	85	79	87	78	89	68
80	107	70	77	85	64	95	76	87	74	68	59	82

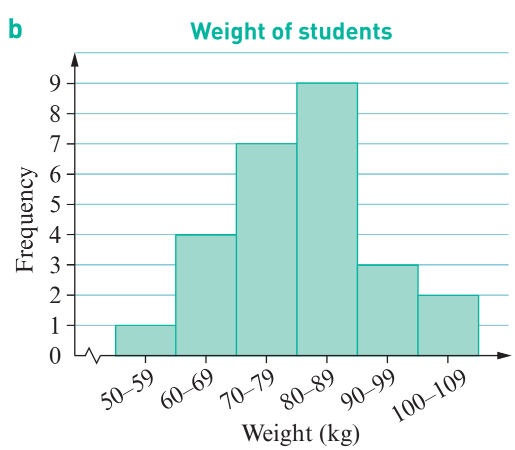
a Using classes 50–59, 60–69, 70–79, etc., tabulate the data in a frequency table.

b Construct a histogram to display the data.

a

Weight (kg)	Tally	Frequency
50–59	|	1
60–69	||||	4
70–79	~~||||~~ ||	7
80–89	~~||||~~ ||||	9
90–99	|||	3
100–109	||	2

b



EXERCISE 36.4

1 For each data set, state whether a histogram or a column graph should be used. Draw the appropriate graph.

a Most appealing car colour

Colour	White	Red	Blue	Green	Other
Frequency	38	27	19	18	11

b The heights of 25 hockey players (to the nearest cm)

Height (cm)	120–129	130–139	140–149	150–159	160–169
Frequency	1	2	7	14	1

c The time taken to make a pizza (to the nearest minute)

Time (min)	6	7	8	9	10	11	12	13
Frequency	1	0	1	3	11	22	7	1

d 'Personal best' times for 50 marathon runners

Time (min)	120–129	130–139	140–149	150–159	160–169
Frequency	1	10	26	11	2

STEM-AND-LEAF DISPLAYS

When data is grouped and displayed as a column graph or histogram, the original individual scores may be lost. To avoid this loss of data, a stem-and-leaf plot is often drawn instead. A stem-and-leaf plot uses all the the original data. The leaves are placed in ascending order.

A stem-and-leaf plot is shown for the data from Example 36C-2 (weights in kg).

	Stem	Leaf	
a stem →	5	9	← a leaf
	6	4 7 8 8	← This row represents 64, 67, 68 and 68.
	7	0 2 4 6 7 8 9	
	8	0 2 3 4 5 5 7 7 9	
	9	1 3 5	
	10	1 7	The scale (unit = 1) tells us the place value of each leaf.

NOTE

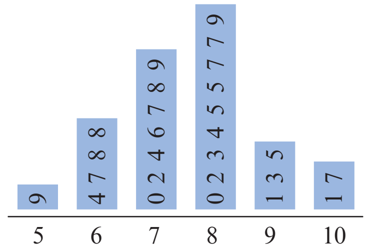
If the scale was 'unit = 0.1' then 6 | 4 7 8 8 would represent 6.4, 6.7, 6.8 and 6.8.

NOTE

Notice that the lengths of the stems are in the same ratio as the heights of the columns.

Compare the histogram from Example 36C-2 with the stem-and-leaf plot. Rotating the diagram 90° we see the shape of a column graph.

Stem	Leaf
5	9
6	4 7 8 8
7	0 2 4 6 7 8 9
8	0 2 3 4 5 5 7 7 9
9	1 3 5
10	1 7



EXAMPLE 36C-3 Grouped data classes

A greengrocer recorded the weights of all canteloupes sold on a particular day. Construct a stem-and-leaf plot for the data shown below (in kg).

1.1 1.6 0.7 2.5 3.9 2.6 1.4 1.7 1.8 3.1
3.1 2.5 4.3 3.2 2.5 1.9 1.6 0.8 3.4 2.1

The stem-and-leaf display for cantaloupes sold (in kg) is shown on the right. Unit = 0.1.

Stem	Leaf
0	7 8
1	1 4 6 6 7 8 9
2	1 5 5 5 6
3	1 1 2 4 9
4	3

EXERCISE 36.5

1 The weights of 24 soccer players were recorded (to the nearest kg) and this data was obtained.

72 63 90 70 67 71 89 64 93 86 86 84 66 78 75 89 80 91 81 72 87 72 84 87

Construct a stem-and-leaf display of the data.

2 The time taken (in hours) by farmers to travel to their nearest town centre is given below.

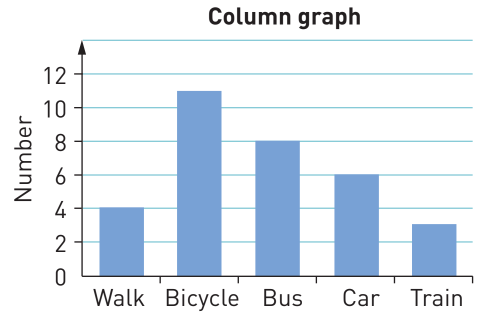
0.7 2.4 0.9 1.2 4.1 3.0 3.6 2.8 1.8 2.7 3.2 2.4 1.3 2.5

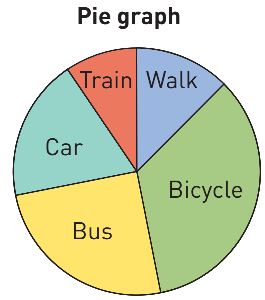
Construct a stem-and-leaf display of the data, stating the scale used.

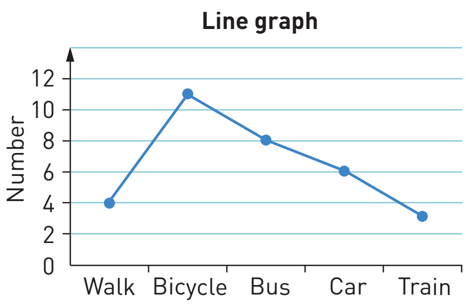
USING A SPREADSHEET FOR GRAPHING

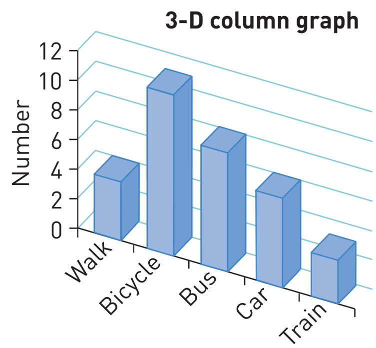
A spreadsheet can be used to draw various types of statistical graphs. Here are several graphs drawn from a spreadsheet of data collected about students' method of transport to school.

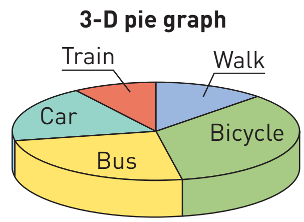
	A	B
1	**Method of travel**	**Number**
2	Walk	4
3	Bicycle	11
4	Bus	8
5	Car	6
6	Train	3











Graphing one distribution

If you want to draw a frequency column graph of the most appealing car colour data from question **1a** in Exercise 36.4, follow these steps using MS Excel 2013.

Step 1: Start a new spreadsheet and type in the data as shown on the right.

	A	B	C
1	**Colour**	**Frequency**	
2	White	38	
3	Red	27	
4	Blue	19	
5	Green	18	
6	Other	11	

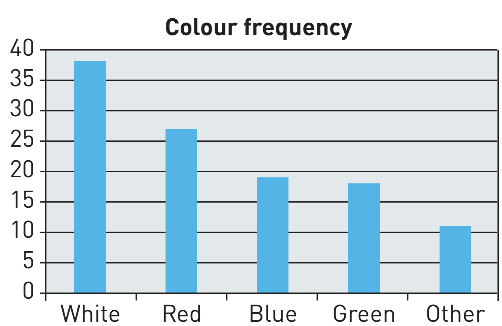
Step 2: Highlight the area shown.

Step 3: Click on the INSERT tab from the menu bar.

Step 4: Click on the graphs symbol to open the column graphs option.

Step 5: Choose the left-hand 2-D column option as shown. This will insert a column graph like this into the spreadsheet.

Step 6: Click on the 'Chart Title' label in the graph and type in a title.



Comparing two distributions

We add a second frequency to the previous data for car colours.

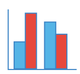
Colour	White	Red	Blue	Green	Other
Frequency 1	38	27	19	18	11
Frequency 2	15	13	8	11	4

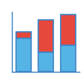
	A	B	C
1	**Colour**	**Frequency 1**	**Frequency 1**
2	White	38	15
3	Red	27	13
4	Blue	19	8
5	Green	18	11
6	Other	11	4

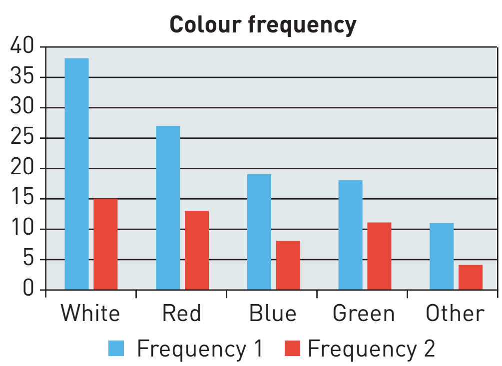
Step 1: Type the Frequency 2 data into column C and highlight the three columns as shown.

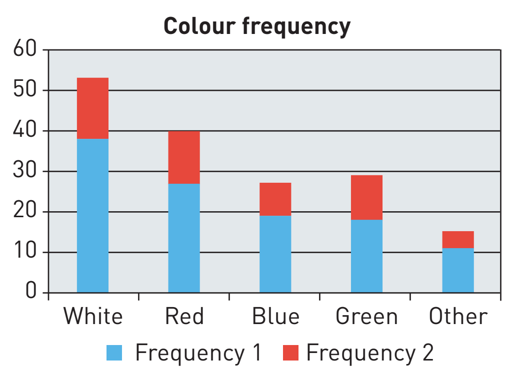
Step 2: As previously, click on the INSERT tab from the menu bar.

Step 3: Click on the graphs symbol to open the column graphs option.

Step 4: Choose this column option for the graph shown below left. 

Step 5: Choose this column option for the graph shown below right. 





36D Measuring the middle of the data

We are all familiar with sporting averages that are used to measure the performances of those playing a sport. The mean (or average) is the most commonly used measure of the middle. Two other measures, the mode and median, are also useful on occasions.

THE MEAN

The mean of a set of numbers is found by dividing the sum of the numbers by how many numbers there are in the set. We often use the term average, but the term mean is more precise. The symbol $\overline{x}$ (read 'x bar') is normally used to represent the mean.

$$\text{Mean} = \frac{\text{sum of all the numbers}}{\text{numbers in the set}}$$

The following examples show how the mean is calculated from raw data and from tabulated data.

EXAMPLE 36D-1 Finding the mean from raw data

Jose and Anthea both play goal shooter for their respective netball teams. Given their performances for the season were as follows, who has the higher mean?
Jose has played 11 games and scored: 14, 22, 17, 31, 15, 19, 24, 28, 26, 35, 29.
Anthea has played 8 games and scored: 17, 21, 36, 19, 16, 28, 26, 32.

$$\text{Jose's mean} = \frac{14 + 22 + 17 + 31 + 15 + 19 + 24 + 28 + 26 + 35 + 29}{11} = \frac{260}{11}$$
$$\approx 23.64$$
$$\text{Anthea's mean} = \frac{17 + 21 + 36 + 19 + 16 + 28 + 26 + 32}{8} = \frac{195}{8}$$
$$\approx 24.38$$

Anthea scored the higher mean number of goals.

EXAMPLE 36D-2 Finding the mean from tabulated data

This table shows the number of aces served by tennis players in their first set of a tournament. Determine the mean number of aces.

Number of aces	1	2	3	4	5	6
Frequency	4	11	18	13	7	2

Number of aces (x)	Frequency (f)	$f \times x$
1	4	4
2	11	22
3	18	54
4	13	52
5	7	35
6	2	12
Total	55	179

2 aces occurred 11 times. Instead of adding $2 + 2 + 2 + \ldots$ 11 times, we simply calculate 11×2.

$$\text{Mean} = \frac{\text{number of aces}}{\text{number of sets}} = \frac{179}{55} \approx 3.25$$

EXERCISE 36.6

1 Below are the points scored by two basketball teams over a 12-match series.
Team A: 91, 76, 104, 88, 73, 55, 121, 98, 102, 91, 114, 82
Team B: 87, 104, 112, 82, 64, 48, 99, 119, 112, 77, 89, 108
Which team had the higher mean score?

2 A survey of 50 students revealed the following number of siblings per student.

1 1 3 2 2 2 0 0 3 2 0 0 1 3 3 4 0 0 5 3 3 0 1 4 5
1 3 2 2 0 0 1 1 5 1 0 0 1 2 2 1 3 2 1 4 2 0 0 1 2

What is the mean number of siblings per student?

3 This table shows the average monthly rainfall for a city.

Month	J	F	M	A	M	J	J	A	S	O	N	D
Average rainfall	38	27	19	18	11	38	27	19	18	11	18	11

Calculate the mean average monthly rainfall for this city.

4 A manufacturer of toothpicks maintains that each packet contains 60 toothpicks. To test this, a quality control inspector tested 100 boxes and found the distribution shown in the table on the right.

a Find the mean number of toothpicks.

b Comment on these results in relation to the manufacturer's claim.

Number of toothpicks	Frequency
56	8
57	11
58	14
59	18
60	21
61	8
62	12
63	8

5 A total of 51 packets of chocolate almonds were opened and their contents counted. The table below gives the distribution of the number of almonds per packet sampled. Find the mean of the distribution.

Number in packet	Frequency
32	6
33	8
34	9
35	13
36	10
37	3
38	2

Using a calculator to find the mean

The easiest method of using your calculator to find the mean of a set of data is to simply add the data and divide by their number.

What happens if you forget to enter the (and)? Why?

For example, to find the mean of 1, 2, 3, 4, 5, 6, 7, press:

(1 + 2 + 3 + 4 + 5 + 6 + 7) ÷ 7 =

This will give the mean of 4.

However, this can be tedious, especially when there are many data points or when you are working from a frequency table. Most scientific calculators have a statistics mode.

To find the mean of the data using the statistics mode of the TI-30X II calculator, we use the following method.

Step 1: Set the calculator to statistics mode by pressing 2nd [STAT] and select 1-VAR using the ▲ and ▼ keys. Press ENTER.

Step 2: Press DATA to begin data entry.

Step 3: Enter the first data point (X1) and press ▼. Enter the data point's frequency and press ▼.

Step 4: Repeat Step 3 until all the data points and their frequencies have been entered. Press ENTER to save the last data entry. For example:

x	f
11	2
12	5
13	18
…	

Key in 11 ▼ 2 ▼
12 ▼ 5 ▼
13 ▼ 18 ▼
…, etc.

Step 5: Press STATVAR to display the results. Show each using the ▲ and ▼ keys.

n number of data points
x mean of all values
S_x sample standard deviation
σ_x population standard deviation
Σx sum of all values
Σx^2 sum of all x^2 values

Removing old data

Usually calculators will erase previous data when you switch out of, and then back into, statistics mode.

- To clear all data and exit statistics mode, press 2nd [EXIT STAT] ENTER.
- To clear the data and remain in statistics mode, press 2nd [STAT] and then select CLRDATA using the ▲ and ▼ keys.

NOTE

If you have a set of data in the calculator and you wish to enter a new set, it is essential that the old set is removed before you enter the new set.

EXERCISE 36.7

1 Find, using your calculator, the mean of the following sets of scores.

a 23, 24, 25, 26, 27, 28, 29, 30

b 7, 19, 5, 14, 13, 18, 21, 14, 11, 13, 15, 8

c 161, 156, 172, 183, 166, 184, 177, 162

2 Use your calculator to find the mean of the following sets of data. Each time, put in the score first, then the frequency.

a

Score	Frequency
22	1
23	6
24	12
25	17
26	8
27	2

b

Score	Frequency
18.0	3
18.1	7
18.2	13
18.3	21
18.4	5
18.5	1
18.7	2

NOTE

How useful is the mean? On a hot day (42°C), if a person has their head in the sun and their feet in a refrigerator (4°C), the mean temperature between the two is a comfortable 23°C. But how comfortable would that be?

Using a spreadsheet to find the mean

Spreadsheets have built-in functions that help you when handling data. Below on the left is a spreadsheet containing the marks obtained in a test out of 50. We can find the mean of the marks in cells B2 down to B6 by using the mean function; that is, the formula =AVERAGE(B2:B6) or =AVG(B2:B6).

The spreadsheet on the right gives the result.

	A	B
1	Test	Mark=
2	1	33
3	=A2+1	23
4	=A3+1	45
5	=A4+1	18
6	=A5+1	44
7		
8	MEAN:	=AVERAGE(B2:B6)

	A	B
1	Test	Mark
2	1	33
3	2	23
4	3	45
5	4	18
6	5	44
7		
8	MEAN:	32.6

EXERCISE 36.8

1 Use a spreadsheet to find the mean of the data in Exercise 36.7, question **1**.

THE MEDIAN

Another measure of the middle of a distribution is the median. The median is the middle score of the distribution, determined when the scores are placed in order of size from smallest to largest. For example, the median of the scores 11, 3, 2, 7, 6, 5, 8, 7, 4 is obtained by first writing them in order as 2, 3, 4, 5, 6, 7, 7, 8, 11 and noticing that the middle score is a 6. Thus the median is 6.

If there is an even number of scores in a sample, there are two middle scores. In such a case we average the two middle scores to find the median. For example, the set of scores 11, 3, 2, 7, 6, 5, 8, 7, 4, 3 is first written in order 2, 3, 3, 4, 5, 6, 7, 7, 8, 11. The two middle scores are 5 and 6.

$$\text{Median} = \frac{5 + 6}{2} = 5.5$$

EXAMPLE 36D-3 Finding the median

Find the median of:

a 5, 7, 6, 7, 6, 3, 4, 6, 5, 9, 7, 6, 11, 2, 4

b 2, 0, 1, 6, 5, 4, 3, 8, 2, 1, 7, 6, 3, 6, 4, 9, 2, 10

a In order of size the scores are 2, 3, 4, 4, 5, 5, 6, 6, 6, 6, 7, 7, 7, 9, 11

Median = 8th score = 6

b In order of size the scores are 0, 1, 1, 2, 2, 2, 3, 3, 4, 4, 5, 6, 6, 6, 7, 8, 9, 10

$$\text{Median} = \frac{4 + 4}{2} = 4$$

EXERCISE 36.9

1 Find the median of the following sets of scores.

a 3, 4, 6, 7, 8, 8, 10

b 3, 4, 6, 7, 8, 8, 10, 12

c 3, 4, 4, 5, 5, 5, 5, 6, 6, 6, 7, 8, 8, 9, 9

d 7, 11, 14, 12, 8, 8, 10, 9, 6, 9, 9, 9, 13, 5, 6, 7, 13, 8, 8, 9

2 This season's cricket scores for two openers are:

Gina: 3, 11, 48, 13, 0, 6, 41, 18, 16, 86, 17, 12

Sari: 21, 2, 31, 28, 16, 31, 29, 20, 17, 22, 14, 19

a Find the median score for each batswoman.

b Find the mean score for each batswoman.

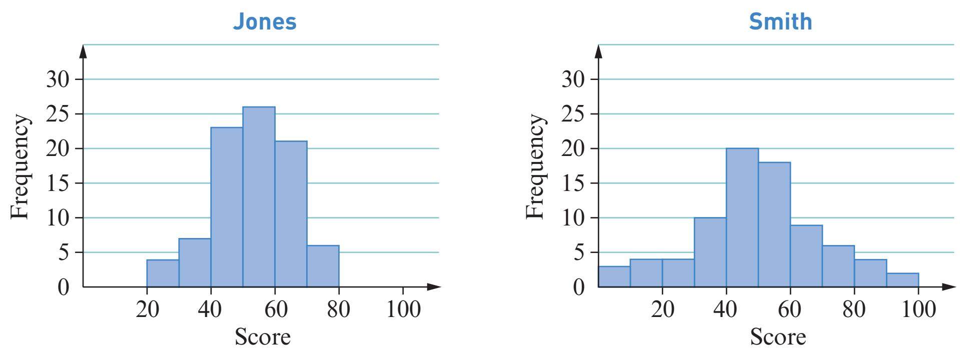
c Which measure of the middle seems more appropriate for cricket?

3 Discuss the following statement: 'In real-estate sales, the median is used as a measure of the middle of a distribution for sales within an area.' Why is this done?

36E Measuring spread

Another way in which we could measure data in order to gain a better understanding of it would be to measure the dispersion or spread of the data.

Consider the first-class innings of two cricketers, Smith and Jones, illustrated in the following histograms. Although Smith and Jones may have similar averages, Smith's scores are more widely spread.



The three common measures of spread are range, interquartile range and standard deviation. We discuss range and interquartile range here.

RANGE

The range of a set of numbers is the difference between the highest and the lowest scores. Consider the following two sets of data:

Set 1: 2, 6, 8, 9, 9, 10, 10, 10 where the range is $10 - 2 = 8$

Set 2: 4, 5, 7, 8, 10, 10, 10, 10 where the range is $10 - 4 = 6$

Each set consists of 8 scores and has the same mean (both means are 8). However, the data is not identical as the spread or range of scores is different.

So, by quoting the range, we can indicate the difference between the spread of the scores in the two sets of data.

INTERQUARTILE RANGE (IQR)

Another way of measuring spread is to use the interquartile range (IQR). To find the interquartile range:

- First write the scores in order of size from smallest to largest.
- Find Q_1 (the first quartile or lower quartile) below which are $\frac{1}{4}$ of the scores.
- Find Q_3 (the third quartile or upper quartile) below which are $\frac{3}{4}$ of the scores.
- Calculate the interquartile range: $\text{IQR} = Q_3 - Q_1$.
- Q_2, the middle or second quartile, is also the median.

You will find that calculations for Q_1 and Q_3 vary depending on whether you use a spreadsheet add-on, a graphics calculator, or an approximate method as outlined on the next page for small samples.

EXAMPLE 36E-1 Approximate interquartile range (IQR)

Find the interquartile range of the following data sets.

a 2, 3, 5, 6, 6, 6, 7, 7, 8, 9, 9

b 2, 3, 5, 6, 6, 6, 7, 7, 8, 9, 9, 10

c 2, 3, 5, 6, 6, 6, 7, 7, 8, 9, 9, 10, 11

The scores are all in order of size.

a 2 3 **5** 6 6 6 7 7 **8** 9 9 is an odd-sized sample of 11.

We include the median in both the top and bottom half of the scores.

$Q_1 = 5$ $Q_3 = 8$

IQR $= 8 - 5 = 3$

b 2 3 **5** **6** 6 6 7 7 **8** **9** 9 10 is an even-sized sample of 12.

$Q_1 = \frac{5 + 6}{2} = 5\frac{1}{2}$ $Q_3 = \frac{8 + 9}{2} = 8\frac{1}{2}$

IQR $= 8\frac{1}{2} - 5\frac{1}{2} = 3$

c 2 3 **5** **6** 6 6 7 7 8 **9** **9** 10 11 is an odd-sized sample of 13.

$Q_1 = \frac{5 + 6}{2} = 5\frac{1}{2}$ $Q_3 = 9$

IQR $= 9 - 5\frac{1}{2} = 3\frac{1}{2}$

EXERCISE 36.10

1 Find the range and interquartile range of:

a 3, 4, 4, 5, 5, 5, 5, 6, 6, 6, 7, 8, 8, 9, 9

b 9, 8, 8, 13, 7, 6, 15, 13, 9, 9, 9, 6, 9, 10, 8, 8, 7, 11, 12, 14, 8

c 21, 25, 22, 25, 28, 26, 25, 29, 24, 27, 28, 20, 21, 24, 25, 22

2 Two salespeople sell the following number of plasma TVs each day over a period of a month.

Dougal: 3, 5, 4, 6, 7, 6, 8, 4, 8, 6, 10, 8, 7, 4, 4, 4, 5, 6, 7

Sylvia: 4, 7, 7, 7, 8, 8, 5, 4, 3, 8, 7, 6, 7, 6, 4, 4, 6, 3

a Find the range and interquartile range of each set of sales.

b Comment on any differences in the data.

3 Gather data on your own that compares two people, objects or scores. Compare the distributions using the mean, median, range and interquartile range.

4 Consider the scores: 3, 6, 7, 5, 7, 6, 9, 6, 5, 8, 4, 4, 3, 6, 7, 8, 29.

a Is there an outlier for this data? What is it?

b Find the mean and median of the given data.

c Find the mean and median of the given data without the outlier.

d Which measure of the centre is more affected by the outlier?

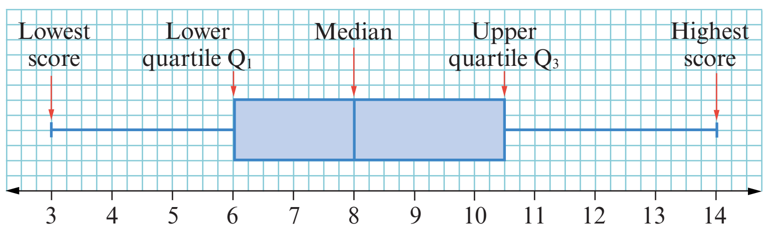
36F Box-and-whisker plots

A box-and-whisker plot consists of a rectangle (the box) and two lines, one on either side of the box (called the whiskers). It is drawn using the five special numbers:

- the lowest score
- the highest score
- the median
- the lower quartile, Q_1
- the upper quartile, Q_3

The box shows the spread of the middle half of the ranked data. For example, if the lowest and highest scores are 3 and 14 respectively and the median is 8, Q_1 is 6 and Q_3 is $10\frac{1}{2}$, the box would run from 6 to $10\frac{1}{2}$.

The box-and-whisker plot for this data is shown below.



Box-and-whisker plots are also very useful when comparing two sets of data.

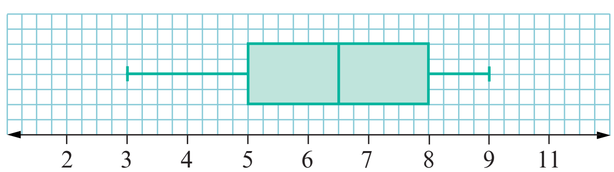
EXAMPLE 36F-1 Drawing a box-and-whisker plot

Draw a box-and-whisker plot for these scores: 5, 7, 4, 6, 6, 7, 8, 3, 9, 8.

In ranked order, the scores are:

3 4 5 6 6 7 7 8 8 9

lowest — Q_1 — median $= 6\frac{1}{2}$ — Q_3 — highest



Notice that the range and IQR are easily obtained from the box-and-whisker plot.

Range $= 9 - 3 = 6$

IQR $= 8 - 5 = 3$

EXERCISE 36.11

1 Draw box-and-whisker plots for ranked data with the following values.

   a Highest score 40, lowest score 15, median 28, upper quartile 32, lower quartile 23

   b Highest score 3.4, lowest score 1.1, median 2.1, upper quartile 2.8, lower quartile 1.9

   c Highest score 28, lowest score 6, median 10, upper quartile 18, lower quartile 7

2 Draw a box-and-whisker plot for the following data sets.

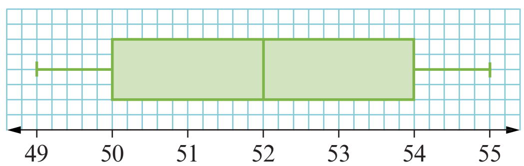
   a 2, 3, 4, 4, 4, 5, 5, 6, 6, 7, 8, 11

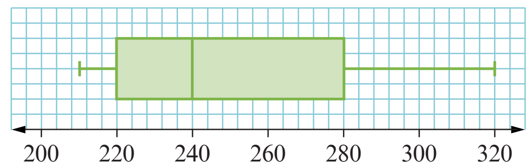
   b 5, 6, 7, 8, 8, 8, 8, 9, 9, 10, 11, 25

3 From the following box-and-whisker plots, find the:

   i range   ii upper quartile   iii lower quartile

   iv interquartile range   v median   vi outlier

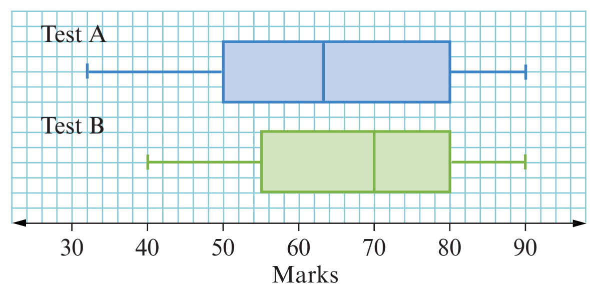
a


b


NOTE

An outlier is an individual observation that falls well outside the data pattern. It is tempting to leave it out, but not before the reason for the outlier has been investigated.

4 The box-and-whisker plots below compare the results of a science test (Test A) and a retest (Test B) on the same topic.



   a Write a brief account comparing the medians, ranges and IQR values.

   b Do you think that the students have improved their understanding of the topic, given the retest?

5 Two groups of students are given pairs of shoes to wear to school. The first group (X) have original rubber-soled shoes and the second group (Y) have new synthetic rubber-soled shoes. The data below shows the thickness of the soles of the shoes (in mm) after 6 months.

   Group X: 3, 5, 6, 4, 5, 6, 2, 7, 3, 4, 4, 6, 5, 5, 5, 7, 6, 4, 4, 3, 6, 5, 4, 2

   Group Y: 4, 6, 5, 4, 3, 5, 6, 6, 7, 6, 6, 4, 5, 7, 8, 6, 7, 5, 3, 6, 6, 7, 5

   a Find the median, and lower and upper quartiles for each distribution.

   b Draw a comparative box-and-whisker plot of each set of data.

   c Find the range and IQR of each distribution.

   d Is the new synthetic rubber on the soles of shoes an improvement? Give evidence.

CHAPTER 37

Interpreting data

37A Data and society

At one time, collecting and analysing data in order to find solutions to problems was the responsibility of statisticians, scientists and mathematicians. Today this is no longer the case. Retailers, marketing departments and small business owners all want to effectively improve their sales and customer experience by making use of the vast amount of data collected by their point-of-sale systems and websites. They are keen to analyse this data so that they can make decisions that are supported by evidence, not just gut feeling.

Understanding and interpreting data correctly is also becoming a more important life skill. In a world full of competing information and advertising, it is more important than ever to be good at interpreting data. Think of the last time you bought a mobile phone or a mobile phone plan. Interpreting the different plans and data usage allowances is really important in order to separate fact from fiction. Interpreting competing political messages and promises is another reason why we need to be good at interpreting data.

In Chapters 35 and 36, we looked at ways of collecting, representing and analysing data using statistics and graphs. In this chapter, we will look at combining these skills to learn how to make reliable, evidence-based conclusions by:

- asking answerable questions
- collecting appropriate data
- representing the data usefully
- making meaningful interpretations of the data
- presenting supportable conclusions.

EXERCISE 37.1

1 In small groups, choose one of the areas below and make a list of topics or issues that might require data to be collected, analysed and compared. There may be more than one group doing each topic, but your teacher should make sure that no two groups cover exactly the same issues.
   - Medicine (such as infection rates, cost of treatment)
   - Political debates in the news (such as immigration rates, cost of living, unemployment rates)
   - Personal life (such as income, spending habits)
   - Data usage (such as mobile phones and the internet)
   - Another topic of your choice

   a For the area chosen, briefly describe the topic.
   b List the types of data that may need to be collected and compared.
   c Suggest some problems that could be encountered in the collection or analysis of some of the data needed.

2 As a class, write up on the board each group's chosen topic. Discuss and refine the description, data types and possible problems found.

37B Comparison of data sets

Meaningful data can be organised into a table and then graphed. The way in which data varies, or is distributed, is often of great importance. The shape of the graph for a set of data is called its distribution.

The easiest way to see what the distribution is like is to graph either its stem-and-leaf plot or its histogram. To compare two sets of data (two distributions) we could examine:

- back-to-back stem-and-leaf plots
- side-by-side histograms
- back-to-back bar graphs.

Back-to-back stem-and-leaf plot

NOTE

A stem-and-leaf plot uses all the original data.

In Chapter 36 we plotted stem-and-leaf displays for a single data set. A back-to-back stem-and-leaf plot displays two sets of data on the same plot using the same stems with the leaves placed in ascending order to the left and right of the stem as shown (leaf unit = 1 unit).

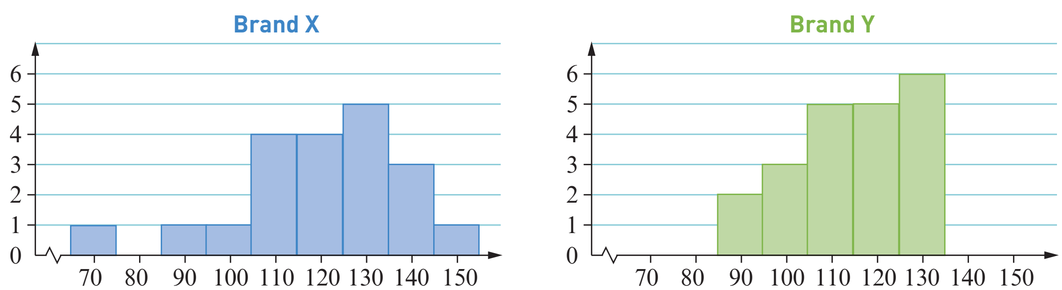
Brand X	Stem	Brand Y
1	7	
	8	
0	9	2
6	10	0 1 6
7 6 6 6	11	0 1 2 4 5
8 8 2 2	12	0 3 3 6 8
8 8 3 3 0	13	1 2 2 6 6 7
7 6 3	14	
6	15	

Side-by-side histograms

NOTE

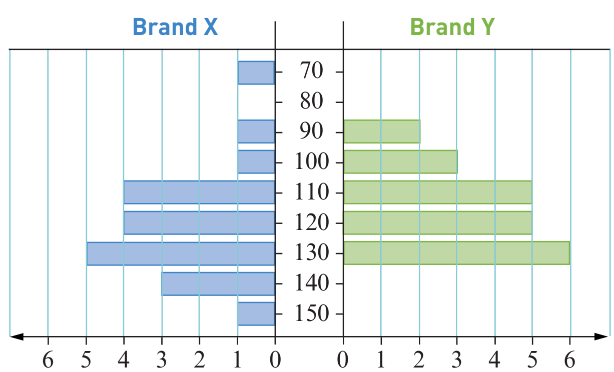
Close inspection of the graphs helps you to understand the distribution better. However, more than this is required to make meaningful conclusions.

In Chapter 36 we plotted column graphs and histograms. In the spreadsheet application we compared two sets of data on the one column graph. We can also compare two sets of data using two histograms that have the same scales.



Back-to-back bar graphs

A bar graph is a column graph with the columns running horizontally instead of vertically. A back-to-back bar graph can be used to display two sets of data on the same graph left and right.



EXERCISE 37.2

1 Read and discuss the following activity as a class.

A study was conducted to see which of two brands of smoke alarm (Spark and Sure Fire) was more effective. To measure how effective they were, it was decided to ask the measurable question:

'What is the length of time each alarm takes to react to the presence of smoke?'

Spark

Sure Fire

The following process was used to test the smoke alarms:

- 30 units of each smoke alarm were purchased. These were randomly allocated into pairs (one of each brand) and each pair was installed the same distance from a smoke source.
- The smoke source was a toaster that had a piece of bread inserted. The bread was allowed to burn until both alarms responded.
- The time (to the nearest second) from when the toaster was switched on to when each alarm responded was recorded.

The following data (in seconds) was obtained.

Response times Spark:

61	64	53	78	57	82	48	69	64	81	90	83	51	74	95
70	82	70	75	63	60	55	58	71	97	82	61	66	104	71

Response times Sure Fire

112	96	107	86	71	106	94	83	79	94	98	60	86	71	66
84	85	74	103	83	80	80	90	89	65	91	96	84	71	94

A back-to-back stem-and-leaf plot was then drawn (leaf unit = 1 second).

Spark	Stem	Sure Fire
8	4	
8 7 5 3 1	5	
9 6 4 4 3 1 1 0	6	0 5 6
8 5 4 1 1 0 0	7	1 1 1 4 9
3 2 2 2 1	8	0 0 3 3 4 4 5 6 6 9
7 5 0	9	0 1 4 4 4 6 6 8
4	10	3 6 7
	11	2

2 Based on these results, which brand of smoke alarm is better? Why? Discuss as a group and record your answers on the board.

Treatment of outliers

An outlier is an individual observation that falls well outside the overall pattern. An outlier may distort the overall picture and, as such, it is tempting to leave it out. The process of removing outliers is common, but not before the reason for the outlier being there has been investigated. If no reason can be found, outliers should be left in the data.

Outliers normally result from faulty equipment or human error. However, in some cases, they open up a new area to explore that could be critical to the study.

EXERCISE 37.3

1 Two taxi drivers, Hassam and Jordon, are friendly rivals. Each claims that he is the more successful driver. They agreed to randomly select 25 days on which they worked and record the 'daily fare totals'. The data collected to the nearest dollar was:

Hassam	194	99	188	208	95	168	205	196	116	132	153	205	191
	182	118	140	183	155	93	154	190	223	147	233	270	
Jordon	260	152	127	163	180	161	110	153	139	142	161	97	116
	129	215	241	110	159	147	174	162	158	223	160	139	

a Produce back-to-back stem-and-leaf plots of this data.

b Explain why the 'amount of money collected per hour' would have been a better variable to use than the 'daily fare totals'.

c Produce back-to-back stem-and-leaf plots for the 'amount of money collected per hour' given this data.

Hassam ($ per hour)	17.27	11.31	15.72	18.92	9.55	12.98	19.12	16.69	11.68
	15.84	12.81	24.03	15.03	12.95	20.09	18.64	18.94	13.92
	11.69	15.52	15.21	18.26	12.25	18.59	22.79		
Jordon ($ per hour)	23.70	13.30	12.18	14.20	15.74	14.01	10.05	10.05	12.20
	13.50	18.64	13.29	12.65	13.54	8.83	11.09	12.29	18.94
	20.08	13.84	14.57	13.34	13.44	13.63	14.18		

2 A new drug that is claimed to lower the cholesterol level in humans has been developed. A heart specialist was interested to know if the claims made by the company selling the drug were accurate. He enlisted the help of 50 of his patients. They agreed to take part in an experiment in which 25 of them would be randomly allocated the new drug and the other 25 would take an identical looking pill that was actually a placebo (a sugar pill that will have no effect at all).

All participants had their cholesterol level measured before starting the course of pills. Then at the end of the 2 months of taking the drug or placebo, they had their cholesterol level measured again. The data collected by the doctor is given below.

Cholesterol levels of all participants before the experiment:

7.1	8.2	8.4	6.5	6.5	7.1	7.2	7.1	6.1	6.0	8.5	5.0	6.7	7.3	8.9	6.2	6.3
7.1	8.4	7.4	7.6	7.5	6.6	8.1	6.2	7.0	8.1	8.4	6.4	7.6	8.6	7.5	7.9	6.2
6.8	7.5	6.0	5.0	8.3	7.9	6.7	7.3	6.0	7.4	7.4	8.6	6.5	7.6	6.3	6.2	

Cholesterol levels at the end of the trial by the 25 participants who took the drug:

4.8	5.6	4.7	4.2	4.8	4.6	4.8	5.2	4.8	5.0	4.7	5.1	4.4
4.7	4.9	6.2	4.7	4.7	4.4	5.6	3.2	4.4	4.6	5.2	4.7	

Cholesterol levels at the end of the trial by the 25 participants who took the placebo:

7.0	8.4	8.8	6.1	6.6	7.6	6.5	7.9	6.2	6.8	7.5	6.0	5.7
8.3	7.9	6.7	7.3	6.1	7.4	8.4	6.6	6.5	7.6	6.1	8.2	

a Produce a single stem-and-leaf plot for the cholesterol levels of all participants before the experiment.

b Produce a back-to-back stem-and-leaf plot for the data returned from the group who took the drug and those who took the placebo. Position it on your page so that this data can be easily compared to all the measurements before the experiment began.

3 Plant fertilisers come in many different brands. There are, however, essentially two types: organic and inorganic. A student was interested in finding out whether radish plants responded better to organic or inorganic fertiliser. He prepared three identical plots of ground, named plots A, B and C, in his father's garden and planted 40 radish seeds in each plot.

After planting, each plot was treated in an identical manner, except for the way they were fertilised. Cost prevented him using a variety of fertilisers, so he chose only one organic and one inorganic fertiliser.

Plot A received no fertiliser, plot B received the organic fertiliser as prescribed on the packet, and plot C received the inorganic fertiliser as prescribed on the packet. The student collected data on the length of the foliage (measured to the nearest centimetre) of the individual plants that survived up until the end of the experiment.

Data from plot A	2	7	29	9	10	8	36	36	42	32	32	32	30	38	39	38
	50	34	41	39	40	12	14	35	35	42	25	32	30	34	22	
Data from plot B	51	54	56	41	50	47	47	46	48	52	47	58	56	63	66	
	54	48	48	53	47	45	58	34	33	46	29	28	20	34		
Data from plot C	55	76	65	61	67	69	68	64	76	59	69	70	76	43	70	
	62	60	58	79	65	68	68	63	54	61	72	58	77	66	65	
	56	79	70	75	60	39	47	50								

a Produce a single stem-and-leaf plot for the lengths returned from the plants from plot A.

b Produce a back-to-back stem-and-leaf plot for the data returned from plots B and C. Position it on your page so that this data can be easily compared to that of plot A.

37C Analysing data

In any worthwhile comparison, all the facts must be known and careful, honest analysis undertaken. Wrong deductions can be made if all the facts are not known. For example, if Hayden scored 6359 runs at cricket and Harvey scored 3394, you would be tempted to conclude that Hayden was the better batsman. If you were then told that Hayden played 151 innings and Harvey played 43, you would reverse your decision after calculating Hayden's average as 42.1 and Harvey's as 78.9. If you were then told that Hayden's runs were scored in test matches and Harvey's cricketing career was in a D-grade district competition, you would again reverse your decision.

When you are convinced that it is fair to compare the data, follow these steps.

Step 1: Produce appropriate graphics.
Step 2: Look for outliers and treat them properly.
Step 3: Describe and compare the shape of each distribution.
Step 4: Describe and compare the centre of each distribution (the mean, the median).
Step 5: Describe and compare the spread of each distribution (the range, the interquartile range).
Step 6: Write out your conclusions.

Be careful when drawing conclusions. When analysing data from a sample you cannot make definite statements. It is better to make statements such as:

'The data suggests that ...' or 'on average ...' or 'the data supports the conjecture ...'.

Drawing box-and-whisker plots

In Chapter 36 you drew box-and-whisker plots, which are also very useful when comparing two sets of data.

- First rank the data in order.
- Then determine the median, the lower quartile (Q_1) and the upper quartile (Q_3). The box shows the spread of the middle half of the ranked data.
- The whiskers show the lowest score and the highest score (the range).

EXERCISE 37.4

1 Special chocolate hearts are enclosed in red shiny wrappers and cost a lot more than bulk chocolate hearts. Bulk chocolate hearts are not wrapped and come in a large container. Jamna suspects that:

- the weight of the special chocolate hearts varies little from one to the next, whereas the weight of the bulk chocolate hearts varies considerably
- special chocolate hearts are lighter than the bulk chocolate hearts.

Jamna bought one of each type from 20 supermarkets and weighed them to the nearest tenth of a gram.

These were her results (to the nearest tenth of a gram).

Weights of 20 special chocolate hearts:

11.8	12.1	12.0	12.0	12.2	12.3	12.1	12.3	12.2	12.3
12.0	12.0	12.0	12.1	11.9	12.1	12.3	12.0	12.0	12.0

Weights of 20 bulk chocolate hearts:

14.7	14.7	14.7	14.6	14.6	14.6	14.6	14.6	14.6	14.6
14.5	14.5	14.5	14.5	14.5	15.2	15.2	15.1	14.5	14.5

2 For the taxi drivers Hassam and Jordon, rank the driver's 'amount of money collected per hour' data in order. Determine the the median, the lower quartile (Q_1) and the upper quartile (Q_3). Plot and compare the box plots.

3 Enter the cholesterol data from Exercise 37.3 question **2**. Plot and compare the box plots.

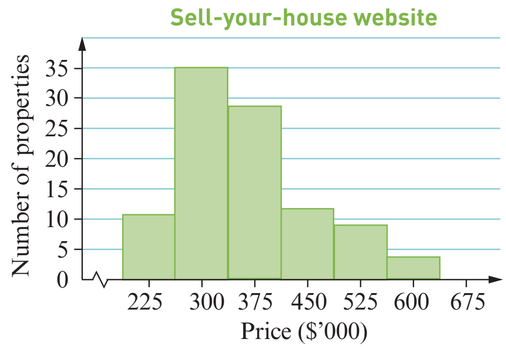
4 Enter the fertiliser data from Exercise 37.3 question **3**. Plot and compare the box plots.

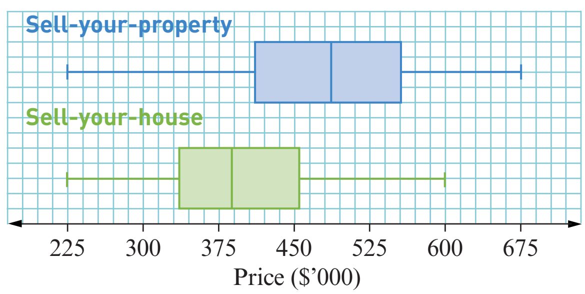
37D Reporting conclusions

CASE STUDY: REAL-ESTATE ADVERTISING

A homeowner, Franco, was interested in whether two different real-estate websites that advertised houses for sale had generally similar prices, or whether one website advertised houses in their local region at generally higher prices. He asked his statistician daughter, Julia, what he should do to investigate this situation. She suggested that she could obtain, at random, the asking prices for 100 advertised houses on each website that were in the same region, and would analyse the data and give him a written report.

The results from Julia's research were:





Statistics

Sell-your-property website	
Mean	$481 500
Median	$484 200
Mode	$450 000
Range	$450 000
Interquartile range	$144 780
Lower quartile, Q_1	$441 600
Upper quartile, Q_3	$556 200
$P_{90} - P_{10}$	$309 000
Standard deviation	
Sample	$111 630
Population	$111 090

Sell-your-house website	
Mean	$388 400
Median	$386 700
Mode	$300 000
Range	$375 000
Interquartile range	$119 040
Lower quartile, Q_1	$331 500
Upper quartile, Q_3	$450 600
$P_{90} - P_{10}$	$251 520
Standard deviation	
Sample	$95 580
Population	$95 100

NOTE

The mode and standard deviation given in this analysis are not discussed in this course.

NOTE

The $P_{90} - P_{10}$ is a measure of spread over the middle 80% of scores.

Sample report

To: Franco (Dad)
From: Julia

Re: Comparison of asking prices of homes advertised on the *Sell-your-property* and the *Sell-your-house* websites. ← Statement of topic

In response to your request to investigate whether the *Sell-your-property* and the *Sell-your-house* websites contain houses for sale at generally similar prices or otherwise, I offer the following information.

I randomly selected 100 homes advertised on *Sell-your-property* and 100 homes advertised on *Sell-your-house* over a period of 3 weeks. ← Sample sizes

No abnormally high or low house prices were found in the asking prices of homes collected from either website. The *Sell-your-property* price distribution was approximately symmetrical, but the *Sell-your-house* price distribution indicated skewness to more lower priced houses. ← Graphs, no outliers

The median asking price in the *Sell-your-property* website sample was \$484 200 compared with a much lower \$386 700 for the *Sell-your-house* website sample. ← See the results table for median.

The asking prices in the *Sell-your-property* sample showed far more variation than the *Sell-your-house* website. Interquartile ranges were \$144 780 and \$119 040 respectively. ← See the results table for interquartile range.

Hence, we can conclude that the analysis of our samples supports the conjecture that the asking prices of houses on the *Sell-your-property* website are considerably higher, in general, than houses advertised on the *Sell-your-house* website. ← Conclusion

The fact that I have collected data over only a 3-week period could place some doubt over whether my findings are reflective of all time periods.

Please contact me if you require further help in this matter.

EXERCISE 37.5

1 Write a report similar to the one provided above for one of the following.

a The taxi driver data on the ‘amount of money collected per hour’ from question **1** in Exercise 37.3

b The smoke alarm data from question **1** in Exercise 37.2

c The cholesterol data from question **2** in Exercise 37.3

ANSWERS

CHAPTER 1 Balancing your budget

Are you ready?

1 a 1000 b 110 c 14 d −10
2 a 195 b 68 c 0.174 d 13
3 A
4 a $27\frac{1}{2}$ b $24\frac{1}{2}$
5 a 60 b 1 ÷ 6 × 360
6 a 5 units b 5 tenths, 0.5
c 5 hundredths, 0.05 d 5 hundred, 500
7 a 37.1 b 37.2 c 37.1 d 37.2
8 a 75% b 30.5% c 62.5% d 66.7%
9 a 1 b 9 c 55.5 d 100.9
10 a $18 b 2.2 g c 1925 m d 217 L
11 a 102 kg b $95
12 B

CHAPTER 2 Managing your saving and spending

Are you ready?

1 a 17 b 72 c 15 d 4
2 a 1 b 8300 c 1239.73
3 a 20 b 2 c 2 d 2
4 a 0.5 b 4.5 c 27.75 d 50.45
5 a 37.15 b 37.15 c 37.15 d 37.16
6 a 12.405 b 3188.475 c 325.152
7 a 25% b 81% c 50% d 25%
8 a 29 b 5 c 16 d 13
9 a 1.5 or $\frac{3}{2}$ b 2.5 or $\frac{5}{2}$ c 4 d 12
10 a $15 b 0.55 g c 175 m d 31 L
11 a 0.5 b 0.333… c 0.125 d 0.2
12 a 5% b 20% c 50% d 67%

CHAPTER 3 Understanding your bills

Are you ready?

1 a 340.2657 b 81 000 c 216 d 0.2
2 C 3 D
4

Prefix	Symbol	Times base unit
kilo-	k	1000 ×
milli-	m	$\frac{1}{1000}$ ×
mega-	M	1 000 000 ×
centi-	c	$\frac{1}{100}$ ×

5 A
6 a 57 600 b 1680
c 10 d 195
7 a 0.5 h b 0.25 h
c 0.75 h d 0.167 h
8 a 7.71 b 25.86
c 37.15 d 0.02
9 Around $280 to $285

CHAPTER 4 Household shopping

Are you ready?

1 a 13.85 b 45
c 216 d ≈0.0107
2 a 12c b $18.15
c $40 d 1.25 kg
3 B
4 a 500 g b 250 g c 1500 g d 25 g
5

Prefix	Symbol	Times base unit
milli-	m	$\frac{1}{1000}$ ×
kilo-	k	1000 ×
centi-	c	$\frac{1}{100}$ ×

6 a 100 b 1000 c 70 d 100
7 a 0.333 b 0.25 c 0.2 d 0.167
8 a 15.20 b 1.23 c 2.46 d 0.22
9 a $2 b 55c c $3.95 d $10.50
10 a 93.5 kg b $40 c 20%

CHAPTER 5 Measuring drug dosages

Are you ready?

1 a 1.5 or $1\frac{1}{2}$ b 1 c 216 d 2000
2 a 375 b 1002 c 544 d 0.39
3 D
4

Prefix	Symbol	Times base unit
kilo-	k	1000 ×
milli-	m	$\frac{1}{1000}$ ×
micro-	mcg or μ	$\frac{1}{1\,000\,000}$ ×

5 a 1000 mg b 0.1g c 1000 mL
6 a 187.5 b 550 c 272 d 0.195
7 a 200 b 1243 c 9 d 100

8 a 0.5 b ≈0.667 c 0.2 d 0.04
9 a $\frac{1}{3}$ b $\frac{3}{4}$ c $\frac{1}{4}$ d $\frac{1}{8}$
10 a 2.709 b 0.864
c 370.150 d 10.678
11 D

CHAPTER 6 Cooking by measure

Are you ready?

1 a 125 b 62.5 c 180 d 4000
2 a 166.67 b $\frac{1}{4}$ c $\frac{2}{9}$ d 0.22
3 C
4 a 1500 g b 750 g c 1 g d 0.05 g
5 a 495 b 1320 c 308.22 d 366.3
6 500 g of sugar and 1000 g (or 1 kg) of flour
7 a 5 b $\frac{2}{3}$ c 125 d 6
8 a 0.333 b 0.25 c 0.8 d 0.667
9 a $\frac{1}{2}$ b $\frac{1}{8}$ c $\frac{2}{3}$ d $\frac{11}{25}$
10 B
11 a $2 b 60c c $4 d $10.50
12 a 110.5 kg b $85

CHAPTER 7 Standard drinks and reaction time

Are you ready?

1 a 12.5 b 1500 c ≈97.4 d ≈190.48
2 a 100 b $\frac{3}{20}$ c $\frac{2}{15}$ d 0.132
3 C
4 a 105 b 92.4 c 392 d 75.88
5 a length × width b length × width × height
c $\pi \times r^2$ d $\frac{4}{3} \times \pi \times r^3$
6 a 50 b 7.5 c −7 d 8.7
7 a 0.125 b 0.5 c 0.2 d 0.4
8 a $\frac{1}{4}$ b $\frac{1}{9}$ c $\frac{1}{6}$ d $\frac{1}{400}$
9 a 15% b 23% c 75% d 0.25%
10 a 50% b 66.7% c 75% d 20%
11 B

CHAPTER 8 Planning an event

Are you ready?

1 a 39 b −2.667 c 14 d 41.7
2 a 150 b $\frac{3}{16}$ c $\frac{1}{6}$ d 0.225
3 C 4 B 5 C
6 a 40 b 108 c 96 d 16
7 a 0.5 b 0.667 c 0.75 d 0.2
8 a $\frac{9}{50}$ b $\frac{23}{1000}$ c $\frac{7}{40}$ d $\frac{61}{100}$
9 a 12.5% b 50% c 20% d 40%
10 a 19% b 2.9 c 19.5% d 91%
11 A

CHAPTER 9 Calculating your tax

Are you ready?

1 a 1505 b 310
c 8500 d 2.9928
2 a 125 b 0.125
c 596.75 d 0.09
3 a 1965.425 b 1220.375
c 30.2875 d 2254.425
4 a $1200 b $18 150
5 10.592
6 a Total pay before deductions
b Total pay after deductions
c Pay-as-you-go tax deducted each time you are paid by an employer.
7 a 125.3 b 3 c 8 d 28
8 a 1600 b 4423.7
c 242.3 d 18 035.4
9 a 0.143 b 0.167
c 0.444 d 0.222
10 a $\frac{3}{4}$ b $\frac{1}{3}$
c $\frac{11}{25}$ d $\frac{22}{25}$
11 a 36% b 50% c 97% d 67%
12 a 12.5% b 80% c 33.3% d 40%

CHAPTER 10 Buying and running a car

Are you ready?

1 a 864.5 b 45.4 c 3.247 d 37 740
2 B
3 a 127.5 b 112.2 c 146.2 d 207.4
4 ≈34 L
5

Time (years)	Amount	Interest at 15%
0	$5000	$5000 × 0.15 = $750
1	$5000 + $750 = $5750	$5750 × 0.15 = $862.50
2	$5750 + $862.50 = $6612.50	$6612.50 × 0.15 = $991.88
3	$6612.50 + $991.88 = $7604.38	

6 a 199.998 b 11
c 24 d 40
7 a 3 hundred, 300 b 3 thousand, 3000
c 3 thousandths, 0.003 d 3 units, 3
8 a 3.25 b 0.6
c 0.6 d 1.5
9 a 23 460 b 23 500
10 a 5 b 15
c 10 d 64
11 a 345.68 b 2.12
c 2.35 d 0.00

CHAPTER 11 Your carbon foot footprint

Are you ready?

1 a 35 994 b 411.99 c 9430 d −612.5
2 6.81
3 D
4

Prefix	Symbol	Times base unit
kilo-	k	1000 ×
giga-	G	1 000 000 000 ×
mega-	M	1 000 000 ×

5 a 747 L b ≈$1.60
6 a 2 kW b 1 kW
7 a 1 b 10.5 c 22 700 d 39 100
8 a 1600 b 4423.7 c 242.3 d 18 035.4
9 a ≈0.33 h b 0.25 h
10 a 5 thousandths, 0.005 b 5 tenths, 0.5 c 5 units, 5 d fifty, 50
11 ≈120 t

CHAPTER 12 Travelling in Australia

Are you ready?

1 a 64 b 520.2 c 363.75 d 4909.5
2 a $166\frac{2}{3}$ b $21\frac{2}{3}$ c $\frac{7}{9}$ d $\frac{5}{24}$
3 C
4 a 157.5 b 138.6
5 a 34.35 b 1.67 c 391.12 d 66.67
6 52.42
7 a 99.999 b 9 c 4 d 64
8 a 0.143 b 0.286 c 0.429 d 0.571
9 a 5 b 20 c 36.5 d 49
10 a 345.678 b 2.123 c 2.346 d 0.001
11 C
12 2200

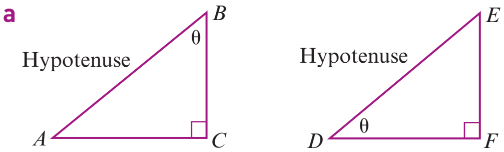
CHAPTER 13 Travelling overseas

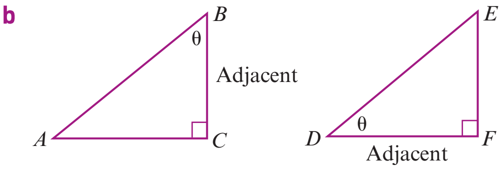
Are you ready?

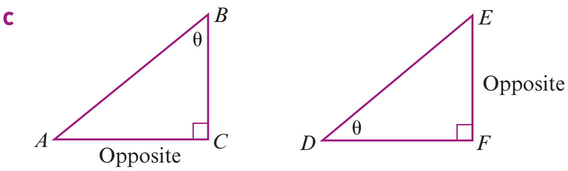
1 a 10 236 b 1906.5777 c $2208 d $13080 e $1854
2 a 100 b 13 c 0.4667 d 0.125
3 C
4 a 232.5 b 204.6 c 1829 d 1.86
5 a 34.346 b 1.679 c 391.123 d 766.601
6 a 26 b $225 c 1.2 d 20
7 a ≈$3500 ($3900) b ≈$160 ($156) c ≈$80 ($100.36) d ≈$2 ($2.05)
8 a 345.678 b 2.123 c 2.346 d 0.001
9 C
10 C

CHAPTER 14 Maps, bearings and surveys

Are you ready?

1 a 2593.5 b 44.4 c ≈6.98 d 74
2 a


b


c


3 15.9 cm
4 B
5 a 157.5 b −1.63 c 100 d 36.2
6 a 110.235 b 212.315 c 2.346 d 0.937
7 a 25% b 30% c 62.5% d 66.67%
8 a 15 b 20 c 5 d 110
9 C
10 ≈25 m

CHAPTER 15 Diet and nutrition

Are you ready?

1 a 26 b 4573 c 4395 d 5400
2 a 468.75 b 8.65
3 8.8
4 A
5 a 0.000 05 b 600 mg c 1980
6 D
7 75 g
8 a 5550 b 3750 c 23.4 d 0.6
9 a 0.5 b 0.333 c 0.375 d 0.75
10 5 hundredths, 0.05
11 a 5400 g b 0.667 kg c 5.2 mg d 8 400 000 μg
12 1200 g

CHAPTER 16 Calculating clinical dosages

Are you ready?

1 a 880 b 850 c 7.5 d 0.0005
2 a 3500 g b 0.25 kg c 2 mg d 250 000 000 mcg
3 a 160 b 16 c 0.052 d 780
4 a 20.8 b 0.2 c 6.3 d 8.3
5 a 560 b 56 000 c 560 d 5600
6 a 56.8 b 0.0568 c 0.005 68 d 0.005 68

7 a 1000 mL b 1000 mg
c 1000 mcg
8 a 3400 g b 0.555 kg
c 0.25 mg d 6 400 000 mcg
9 a 32 b 3.2 c 0.032 d 480
10 a 2, 1.8, 1.75 b 1, 0.9, 0.88
11 a 0.25 b 0.67
c 10.83 mL/h d 2.00
12 a $\frac{2}{3}$ b $\frac{1}{2}$
c 15 mL d 125 mL/dose

CHAPTER 17 Types of design

Are you ready?

1 a 0 b 13 c 63.6 d 48
2 3600 mm^2 or 36 cm^2
3 10 m^2
4 a 3.61 m to 2 decimal places
b 6.11 cm to 2 decimal places
5 a 10 b 30 c 8 d 3
6 a $\frac{2}{3}$ b $\frac{11}{20}$ c $\frac{9}{10}$ d $\frac{3}{20}$
7 a 45.60 m b 240 000 cm
c 660 000 mm d 6 m
8 C 9 ≈$204

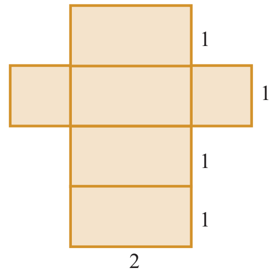
CHAPTER 18 Fitness and speed

Are you ready?

1 a −600 b 37
c −73 d 29
2 a 127.5 b 112.2
c 146.2 d 207.4
3 a 1300 m b 23.6 m
c 0.45 km d 0.6 km
4 a 50 m b 18 s c 5.56 m/s
5 a 11 b −28 c 8 d 64
6 a 60% b 54% c 90% d 18%
7 a $\frac{13}{20}$ b $\frac{9}{20}$ c $\frac{4}{5}$ d $\frac{1}{4}$
8 a 4.55 m b 2400 m
c 1 km d 6 m
9 a 1, 1 unit b 1000, 1 thousand
c 0.001, 1 thousandth d 1, 1 unit
10 11 min

CHAPTER 19 Renovating and redecorating

Are you ready?

1 a 13 b 35.9525 c 81.6
2 0.6 m^2 3 D
4 a 17.34 m^2 b 66.99 m^2
5 C
6 a 34.5 b 19 c 28 d 17
7


8 a 1 unit b 1.333 c 6.2 m
9 ≈$145

CHAPTER 20 Modelling and building

Are you ready?

1 a 6.05 b 23.15 c −9.475
2 16.173 3 C 4 0.21 m^2
5 C 6 10 cm 7 C 8 60°
9 a 48 b 55 c 8 d 7
10 a $\frac{2}{3}$ b $\frac{1}{4}$ c $\frac{4}{5}$ d $\frac{1}{8}$
11 a 2 hundred, 200 b 2 thousand, 2000
c 2 thousandths, 0.002 d 2 hundredths, 0.02
12 ≈9 m

CHAPTER 21 Landscaping your yard

Are you ready?

1 a 6.105 b 11.3375 c 728.5 d 86.424
2 a $\frac{2}{3}$ b $1\frac{1}{4}$ c $3\frac{5}{8}$ d $3\frac{1}{20}$
3 A
4 a ≈19.63 mm^2 b ≈0.283 cm^2
c ≈20.11 m^2 d ≈78.54 km^2
5 B 6 B 7 B 8 C
9 a 10 b 6 c 18 d 5
10 a $\frac{1}{3}$ b $\frac{7}{20}$ c $\frac{3}{5}$ d $\frac{1}{8}$
11 a thirty, 30 b thirty thousand, 30 000
c 3 thousandths, 0.003 d 3 hundredths, 0.03
12 12 m

CHAPTER 22 Automotive calculations

Are you ready?

1 a 5.878 b 22.3525 c 81.9
2 8.8272
3 a 1 000 000 μm b 1 000 000 000 μm
4 a $34 b $1.87
5 1125 cm^2 6 0.785 m^2
7 300 mL or 0.3 L 8 C
9 a 10 b 50 c 17
10 a 0.667 b 0.5 c 0.6 d 0.25
11 a 4 thousand, 4000 b four hundred, 400
c four hundred, 400 d 4 hundredths, 0.04
12 ≈3.5 m

CHAPTER 23 Fractions, decimals and rounding

EXERCISE 23.1

1 a 150 b −50 c −150 d −150
e −150 f −5 g −17 h 14
i 0 j −1

2 a 12 b −6 c 30 d 0
e −3 f 11 g 0 h 0
i 25 j 6

EXERCISE 23.2

1 a −36 b 36 c 1 d −5
e 72 f 4680 g −27 h 320
i −2 j 2

2 a −17 200 b 9 c −1 d −16
e −600 f −6 g 11 h −1
i 101 j 7

EXERCISE 23.3

1 a 1 b 0 c 124 d 120
e 312 f 9 g 40 h 23
i 1 j 19 k 23 l 106

2 a 0 b 81 c 24 d 31
e 3 f −4 g 0 h 80
i 81 j 0 k 1 l 31

3 a 22 b 6 c 2 d 31
e −132 f 40 g 24 h 8
i 0 j $28\frac{1}{4}$ k 10 l 80

EXERCISE 23.4

1 a $\frac{1}{2}$ b 2 c $\frac{4}{11}$ d 4
e $\frac{3}{7}$ f $\frac{3}{4}$ g $\frac{4}{5}$ h $\frac{3}{5}$
i $\frac{4}{5}$ j $2\frac{1}{7}$ k $5\frac{1}{4}$ l $12\frac{8}{19}$

EXERCISE 23.5

1 a 75% b 40% c 65%
d 68% e 150% f $57\frac{1}{7}$%

EXERCISE 23.6

1 a $1\frac{5}{12}$ b $1\frac{3}{56}$ c $1\frac{17}{28}$ d $1\frac{1}{28}$
e $\frac{1}{6}$ f $\frac{1}{12}$ g $\frac{18}{35}$ h $\frac{31}{60}$

2 a $3\frac{5}{6}$ b $2\frac{5}{12}$ c $3\frac{2}{15}$ d $3\frac{17}{30}$
e $\frac{5}{6}$ f $\frac{7}{12}$ g $\frac{5}{6}$ h $1\frac{59}{60}$

EXERCISE 23.7

1 Because he gives the answer as a decimal that can be easily converted to cents.

2 a $144 b 480 t c 1200 km
d $675 e 189 kg f $1064

3 a Parts A and B: 25%
Parts D, E and G: 12.5%
Parts C and F: 6.25%

b Parts A and $B = \frac{1}{4} = 56.25\text{ cm}^2$
Parts D, E and $G = \frac{1}{8} = 28.13\text{ cm}^2$
Parts C and $F = \frac{1}{16} = 14.06\text{ cm}^2$

4 $26 806

5 a Petra gets $\frac{2}{3}$ of Sunrasyia and $\frac{1}{4}$ of AMP.
b Ivan $5115.83, Petra $3214.17

6 a 5 b John $\frac{4}{9}$, Amy $\frac{5}{9}$ c $35 250

EXERCISE 23.8

1 a 7000 b 700 000 c $\frac{7}{10}$
d $\frac{7}{1000}$ e $\frac{7}{10\,000}$

2 a 0.6 b 0.875
c 1.4 d 3.333 3…
e 2.666 6… f 2.555 5…
g 0.583 3… h 1.833 3…
i 1 j 12.555 5…
k 23.363 6… l 4.414 1…

3 a 360 b 4630
c 53 920 d 267 190

4 a 400 b 4600
c 53 900 d 267 200

5 a 1000 b 55 000
c 64 000 d 399 000

6 a i 36.4 ii 36.38 iii 36.382
b i 0.2 ii 0.19 iii 0.193
c i 119.4 ii 119.44 iii 119.437
d i 12.0 ii 11.99 iii 11.991
e i 51.8 ii 51.80 iii 51.798
f i 65 656.7 ii 65 656.66 iii 65 656.657

EXERCISE 23.9

1 a 2 and 4 b Frank c Melanie
d A denominator with at least one operation should be placed in brackets.
For example $\frac{20}{2 \times 5}$ should be written as $\frac{20}{(2 \times 5)}$

2 a 3.67 [×] 11.4 [=]
b 8.704 [+] 6.93 [÷] 0.74 [=]
c 8.704 [+] 6.93 [=] [÷] 0.74 [=]
or [(] 8.704 [+] 6.93 [)] [÷] 0.74 [=]
d 0.74 [÷] [(] 8.704 [+] 6.93 [)] [=]

3 a 17.8 b 8.6 c 5.7
d 2.6 e 6.5 f 101.0

4 a 39.69 b 1.19 c 2767.70
d 12.49 e 50.15 f 1.20

5 a 26.326 b 0.690 c 21.628
d 117.129 e 806.885 f 6760.560

6 **a** \$84.48
b **i** $1\frac{7}{10}$ h **ii** 11.76 km/h
c \$399.23 **d** \$130240.44
e 14 min 42.9 s **f** \$31.39/g
g **i** $\frac{1}{8}$ **ii** $\frac{2}{15}$ **iii** $\frac{89}{120}$

EXERCISE 23.10

1 It is incorrect. The sales assistant probably did 7 times instead of 6 times.
2 **a** The estimate is $10 \times 12 + 12 \times 0.5 = 126$, which is pretty close.
b The answer is correct.
3

Sum	Your estimate	Candidate's answer	Is the answer likely to be correct? (Yes/No)
$80 \div 7.9$		10.13	Yes
9.9×28		2772	No
$\frac{53 \times 5}{25}$		1	No
$6.5 + 5 \times 4$		46	No

4 10 trees
5 About 20 min

CHAPTER 24 Percentage, ratios and rates

EXERCISE 24.1

1 **a** 78% **b** 36.7% **c** 90.7%
d 201.35% **e** 142.83%
2 **a** 62.5% **b** 87.5% **c** 18.75%
d 76% **e** 9.375%
3 **a** 0.618 **b** 0.2967 **c** 0.0284
d 1.15 **e** 1.089
4 **a** 63.64% **b** 53.33% **c** 17.65%
d 54.84% **e** 72.97%
5 **a** 27.273% **b** 61.538% **c** 13.793%
d 78.261% **e** 75.676%

EXERCISE 24.2

1 **a** \$36 **b** 2.97 t **c** 2584 m
d 283.5 L **e** 283.5 kg **f** \$1812.48
2 \$15 750
3 600 kL
4 \$41 675

EXERCISE 24.3

1 **a** 110% **b** 125% **c** 108%
d 102.3% **e** 111.8%
2 **a** 90% **b** 75% **c** 92%
d 97.7% **e** 88.2%
3 **a** \$3910 **b** 993.60 m **c** 3570 t
4 **a** \$2890 **b** 2.8952 km **c** 500 km

EXERCISE 24.4

1 **a** \$13.25 **b** \$17.60
c \$26.35 **d** 52.75
2 **a** \$849.15 **b** \$127.50 **c** \$42.50
d \$2091 **e** \$969 **f** \$633.25
3 **a** ≈42.9% **b** 300% **c** 25% **d** 40%
4 **a** 25% **b** 40% **c** ≈21.9% **d** 36%
5 **a** ≈47.0% increase **b** ≈10.1% decrease
c ≈16.7% increase **d** ≈11.4% decrease
6 **a** **i** Yes **ii** No **iii** Yes
b ≈25.85%
c Part **ii** is slightly less than a 25% discount.
7 **a** 294 students **b** ≈129.4%
c 46.5kL **d** 70 350 people

EXERCISE 24.5

1 **a** basketball : hockey = 3 : 1
b Janice : Briony = 3 : 1
c Sasha : Ky = 5 : 6
d cow : horse = 13 : 17
e netball : football = 7 : 12
2 **a** 4 : 1 **b** 4 :5 **c** 3 : 4
d 3 : 2 **e** 3 : 10 **f** 17 : 24
3 **a** \$12 000, \$22 000 **b** 270 kg, 450 kg
c 350 km, 490 km **d** 15 kL, 33 kL
e 65 t, 97.5 t **f** \$80 000, \$280 000
4 100 mL
5 \$72 000, \$108 000, \$144 000
6 50 basketballers, 70 netballers
7 Jarvis \$330 000, Danuta \$150 000
8 640 white tiles, 320 black
9 200 g, 300 g, 50 g

EXERCISE 24.6

1 **a** 60 km/h **b** 230 L/min
c 80 fish/h **d** 0.8 t/h
2 **a** **i** 250 kL **ii** 3250 kL **iii** 71.43 kL
b **i** 4.2 kg **ii** 16.8 kg **iii** 1.4 kg
3 **a** 360 km **b** 3 min 20 s
4 **a** 1 kg/day = 1000 g/24 h
= 41.666 667 g/h ≈ 41.7 g/h
b **i** 833.3 g/h **ii** 1.44 kg/day
5 1 mm/h

CHAPTER 25 Measurement and units

EXERCISE 25.1

1 **a** 460 kg **b** 5000 g
c 6000 mL **d** 7.5 mm
2 **a** tonnes **b** gigabytes
c megalitres **d** millimetres
3 **a** 87 s **b** 75 cm **c** 37 km
d 23 000 L **e** 9 mg **f** 3000 ML

EXERCISE 25.2

1 a i 20 mm; ii 2 cm
b i 30 mm; ii 3 cm
c i 45 mm; iii 4.5 cm

2 a 94 b 37.5 c 122
d 46 e 23 f 56

3 a $\frac{2}{10}$ b $\frac{3}{8}$ c $\frac{9}{10}$
d $\frac{3}{10}$ e $\frac{3}{4}$ f $\frac{1}{8}$
g $\frac{11}{16}$ h $\frac{9}{16}$

4 a 70 km/h b 25 km/h c 78 km/h

EXERCISE 25.3

1 a 4500 rpm b 2500 rpm
c 2750 rpm d 4200 rpm

2 a 38.5°C b 36.8°C
c 40.2°C d 39.7°C

EXERCISE 25.4

1 a ≈2.7 b 10.5
c 6.06 d ≈4.209

2 Monday 7.5°C, Tuesday 12.5°C, Wednesday 19°C, Thursday 17°C, Friday 23°C, Saturday 25°C, Sunday 9°C

EXERCISE 25.5

1 a 32.5 m b 27.4 cm
c 7.125 km d 8930 cm
e 0.9321 m f 0.006 315 km

2 a 5120 mm b 2 745 000 m
c 283 900 cm d 39 700 mm
e 32 000 cm f 2 310 000 mm

3 a 12.375 km b 256 666 lengths

EXERCISE 25.6

1 a 3.2 kg b 1870 kg
c 0.047 835 kg d 4.653 g
e 2 830 000 g f 63 200 g
g 0.074 682 t h 1 700 000 000 mg
i 91.275 kg

2 a 150 000 sweets
b i 5.136 t ii \$2311.20

EXERCISE 25.7

1 a 0.23 cm^2 b 36 000 m^2
c 0.0726 m^2 d 7 600 000 mm^2
e 0.853 ha f 35 400 000 cm^2
g 1354 mm^2 h 4 320 000 cm^2
i 4820 mm^2

2 a 2100 chickens b 500 rectangles

EXERCISE 25.8

2 a 39.1 m^3 b 0.469 m^3
c 3 820 000 cm^3 d 17 900 cm^3

3 a 7.5 m^3 b 47 400 sinkers

EXERCISE 25.9

1 a 3760 mL b 47.32 kL
c 3500 L d 423 mL
e 54 000 mL f 0.058 34 kL

2 a 1375 bottles b 9 tanks

3 a 83 m^3 b 3200 cm^3
c 2.3 L d 7 154 000 L
e 0.46 m^3 f 4 600 000 cm^3

4 a 25 mL b 32 m^3
c 7320 L

CHAPTER 26 Calculating length

EXERCISE 26.1

1 a 112 mm b 125 mm c 103 mm

2 a 38.8 cm b 17.8 cm c 11.9 km
d 29 m e 11.6 cm f 30 m
g 17.6 m h 99.6 km i 19.1 m

3 a 1350 m b 4050 m c \$324

4 164 m

5 30 km

6 a 63.9 m b 135.12 m

EXERCISE 26.2

1 a 41.47 cm b 54.04 m c 361.28 m

2 a 5.341 km b 22.619 m c 1476.549 cm

EXERCISE 26.3

1 a 15.71 cm b 23.42 cm c 37.70 cm

2 a 33.5 m b 130 cm c 104 cm

3 \$2884

4 a 420 m b 350 panels
c \$5687.50

5 a 4.5 km b 1800 sleepers
c 113.4 t d 7.56, thus 8 truckloads

6 a 15.8 km b 13.54 km/h

7 a 57.6 m b 15.08 m
c 72.68 m d \$60.75

8 a 39 m b 14 posts
c \$936.15

EXERCISE 26.4

1 a $x = 8.5$ cm b $x = 11$ cm
c $x = 4.4$ m d $x = 5.1$ m

2 a 11.898 m b 5.2 cm

3 a 6.4 m b 33.90 cm

4 $x = 31.83$ m

CHAPTER 27 Calculating area and surface area

EXERCISE 27.1

1 a 25 cm² b 169 cm²
c 2.89 km² d 40 m²
e 26.56 cm² f ≈1.92 km²
g 42 cm² h ≈56.56 m²
i 150.4 cm² j ≈43.01 cm²
k ≈72.38 m² l ≈107.51 km²
2 a ≈146 cm² b ≈121 cm²
c ≈65 cm²

EXERCISE 27.2

1 a 83 cm² b 506 cm² c 143 cm²
2 a 125 cm² b ≈76.05 cm² c 90 cm²
3 114 m²

EXERCISE 27.3

1 a 78 ha b $6162
2 a 43.2 m²
b 18 m = 30 × 0.6 m = 29 × 0.6 + 2 × 0.3
Need 4 × 0.6 m (Type B), two at one end and two at other end of 2 rows (one end is shown)
c 118 d 2.6 t
e $835.20
3 a 95.25 m² b 20 tiles
c 1905 tiles d ≈$5605.46

EXERCISE 27.4

1 a ≈10.58 m² b ≈23.40 km²
2 a ≈27.9 m² b ≈3.7 km²

EXERCISE 27.5

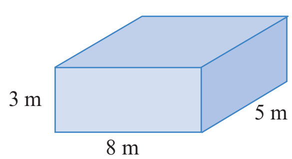
1 a 20.33 cm² b 0.98 km²
2 7.05 m
3 26.73 m²

EXERCISE 27.6

1 a 150 cm² b 105.84 cm² c 433.5 mm²
2 a 392 cm² b 52 m² c 7800 m²
3 a 450 m² b 84 cm²
c 993 cm² b 141 cm²

EXERCISE 27.7

1 a 1344 cm² b 480 m² c 288 cm²
2 a



b 5 faces c 118 m² d $300
3 a 60 m² b 670 m² c ≈$3341.63
4 a 75 m² b 461 m² c $25 816

EXERCISE 27.8

1 a ≈603.2 cm² b ≈145.3 cm²
c ≈188.5 cm² d ≈549.8 m²
e ≈1068.1 cm² f ≈63.6 km²
2 a ≈19.42 m² b $883.61
c $21 206.64
3 a ≈477.52 cm² b ≈25.13 cm
c 50 cans d ≈251.3 cm
4 a 276 cm²
b $AB = 22$ cm, $XY = 18$ cm
c 37 prisms
d No, as there is some wasted cardboard.

CHAPTER 28 Without a calculator

EXERCISE 28.1

1 a 26 b 0 c 0 d 10
e 4 f 12 g 25 h 60
i 0 j 5 k 2.5 l 1.5

EXERCISE 28.2

1 a $\frac{5}{7}$ b $\frac{3}{4}$ c $\frac{1}{3}$ d $\frac{45}{100} = \frac{9}{20}$
e $\frac{2}{3}$ f $\frac{3}{4}$ g $\frac{2}{6} = \frac{1}{3}$ h $\frac{7}{10}$
i $\frac{7}{10}$ j $\frac{2}{3}$
2 a $\frac{20}{30}$ b $\frac{20}{100}$ c $\frac{25}{30}$ d $\frac{28}{63}$
e $\frac{24}{42}$ f $\frac{12}{144}$ g $\frac{36}{54}$ h $\frac{40}{110}$

EXERCISE 28.3

1 a $\frac{5}{4} = 1\frac{1}{4}$ b $\frac{8}{9}$ c $\frac{3}{8}$ d $\frac{1}{20}$
e $\frac{21}{66} = \frac{7}{22}$ f $\frac{19}{21}$ g $\frac{18}{24} = \frac{3}{4}$ h $\frac{5}{72}$

EXERCISE 28.4

1 a $1\frac{2}{3}$ b $1\frac{3}{6} = 1\frac{1}{2}$ c $2\frac{4}{5}$
d 3 e $2\frac{8}{20} = 2\frac{2}{5}$ f $2\frac{4}{33}$
g $10\frac{5}{33}$ h $4\frac{55}{500} = 4\frac{11}{100}$
2 a $\frac{7}{3}$ b $\frac{27}{8}$ c $\frac{21}{5}$ d $\frac{21}{2}$
e $\frac{13}{6}$ f $\frac{81}{8}$ g $\frac{91}{18}$ h $\frac{87}{2}$
3 a $\frac{41}{28} = 1\frac{13}{28}$ b $\frac{45}{21} = 2\frac{1}{7}$ c $\frac{23}{35}$
d $\frac{3}{28}$ e $\frac{25}{12} = 2\frac{1}{12}$ f $\frac{7}{12}$
g 1 h $\frac{37}{12} = 3\frac{1}{12}$

EXERCISE 28.5

1 a $\frac{2}{5}$ b $\frac{1}{4}$ c $1\frac{1}{9}$
d 1 e $\frac{2}{3}$ f $\frac{1}{7}$
g $\frac{15}{8} = 1\frac{7}{8}$ h $\frac{8}{6} = 1\frac{1}{3}$

EXERCISE 28.6

1 a 0.75 b 0.16 c 0.667
d 0.572 e 6.333 f 4.25
g 2.111 h 2.5

2 a $\frac{3}{10}$ b $\frac{5}{10} = \frac{1}{2}$ c $\frac{12}{100} = \frac{3}{25}$
d $\frac{158}{1000} = \frac{79}{500}$ e $\frac{201}{1000}$ f $\frac{1}{3}$
g $\frac{1}{7}$ h $\frac{1}{4}$

EXERCISE 28.7

1 a 5% b 50% c 15% d 20%
e 90% f 95% g 27.2% h 96.5%

2 a 66.67% b 28.6% c 80% d 46%
e 70% f 56% g 28% h 87%

EXERCISE 28.8

3 a 0.1 b 0.02 c 0.345
d 0.12 e 0.2305 f 0.16
g 0.09 h 0.005 i 0.0005
j 0.0965

4 a $\frac{1}{4}$ b $\frac{11}{100}$ c $\frac{3}{50}$ d $\frac{429}{1000}$
e $\frac{167}{1000}$ f $\frac{3}{4}$ g $\frac{353}{1000}$ h $\frac{1}{5}$
i $\frac{33}{100}$ j $\frac{99}{100}$

EXERCISE 28.9

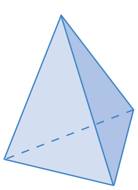
1 a 30×10^3 b 245×10^6 c 245×10^3
d 2×10^{-3} e 400×10^{-6} f 4.532×10^3
g 43×10^{-3} h 2×10^{-3}

CHAPTER 29 Technical drawing

EXERCISE 29.1

1 The solids are called tapering solids because the shape tapers (gets narrower) to a point. The tapering solids are the square-based pyramid, the triangular-based pyramid, the cone and the rectangular-based pyramid.

EXERCISE 29.2

1 a

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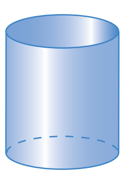
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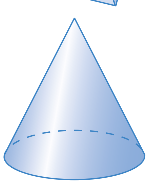
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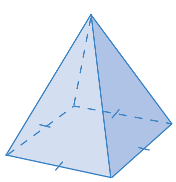
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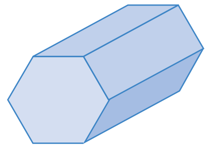
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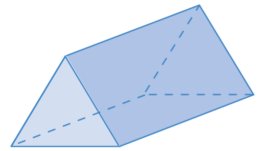
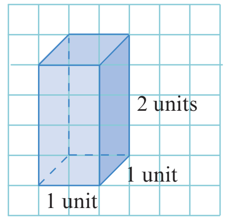


2 a


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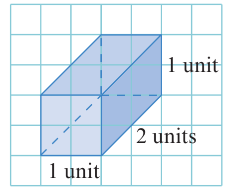

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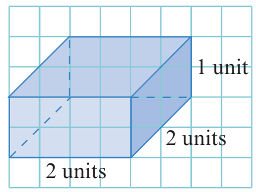
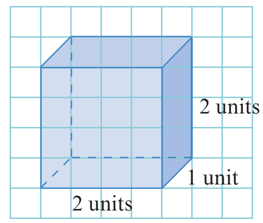


d


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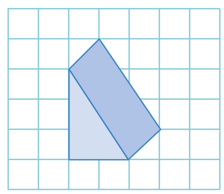
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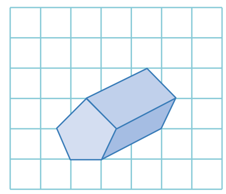
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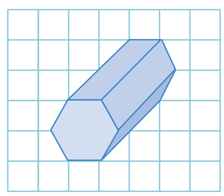
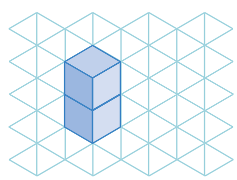


EXERCISE 29.3

1 a

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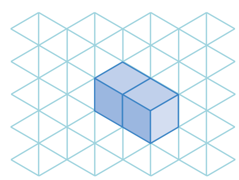
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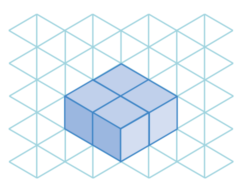
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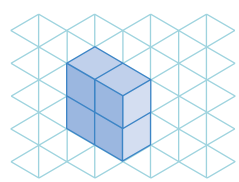

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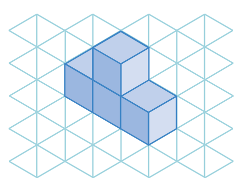
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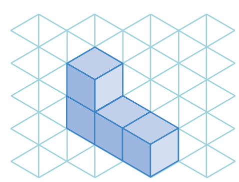
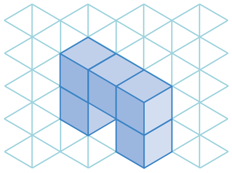
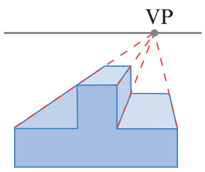


c

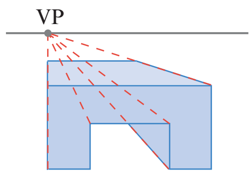


EXERCISE 29.4

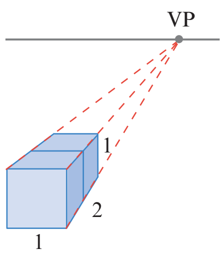
1 a



b



2 a



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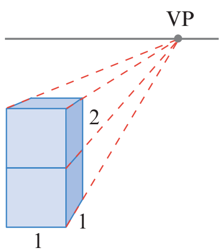
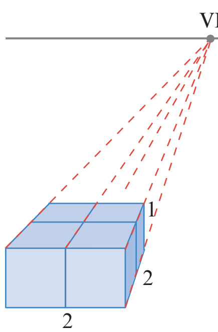


c



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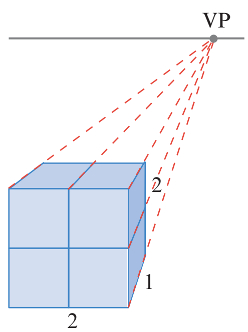
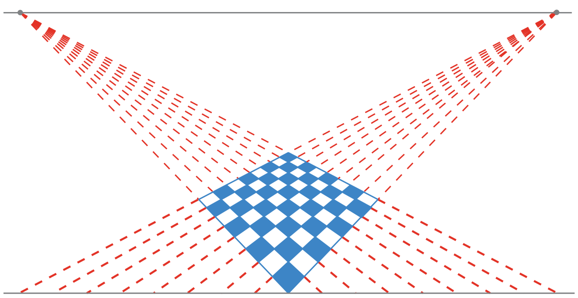
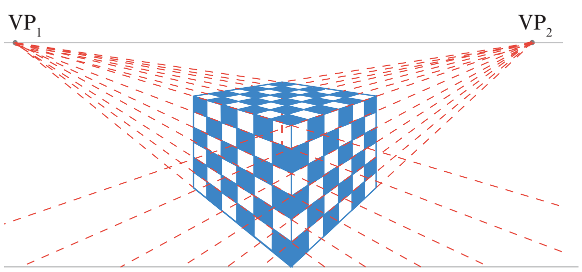


EXERCISE 29.5

1

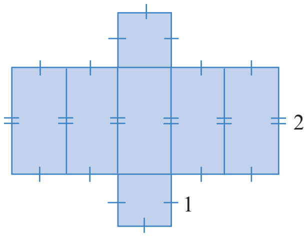


2

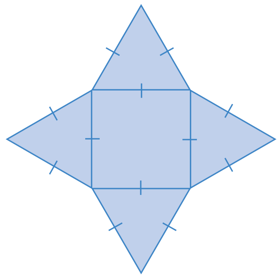


EXERCISE 29.6

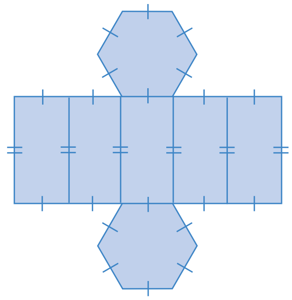
1 a



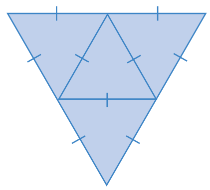
b



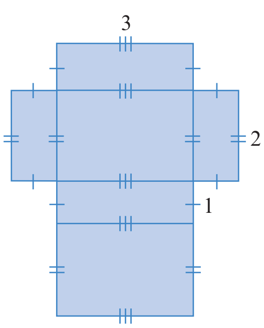
c



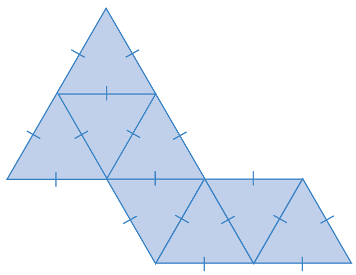
d



e

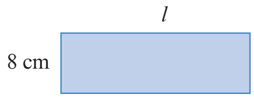


f



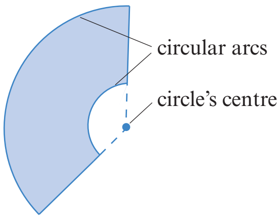
EXERCISE 29.7

1 You need to find the length of the cylinder.
$l = 2 \times \pi \times 4 \approx 25.1$ cm



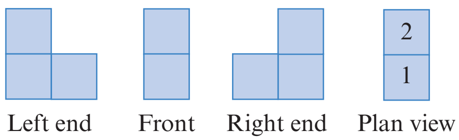
3 False, make it and see.

4

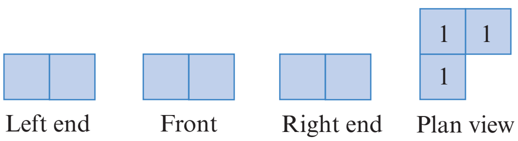


EXERCISE 29.8

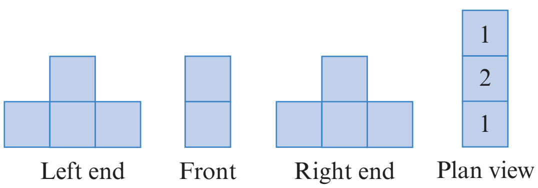
1 a



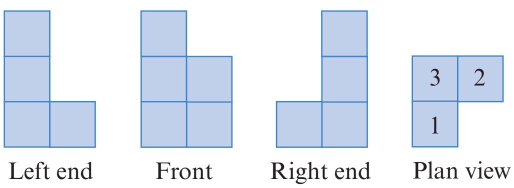
b



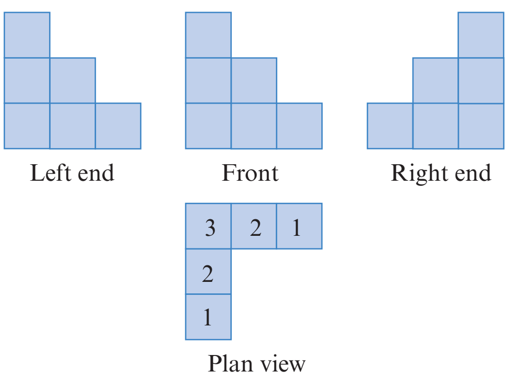
c



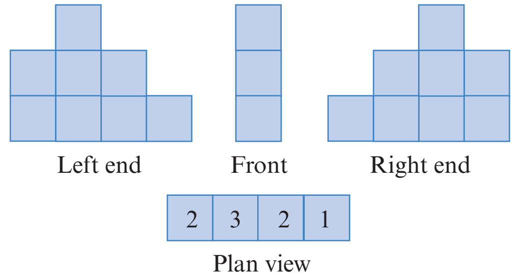
d



e

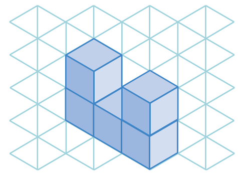


f

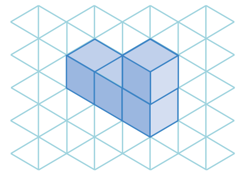


EXERCISE 29.9

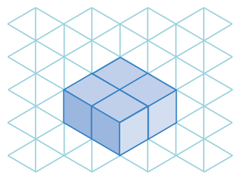
1 a



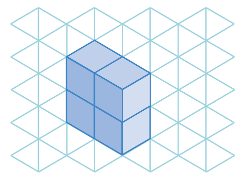
b



c

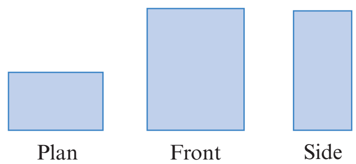


d

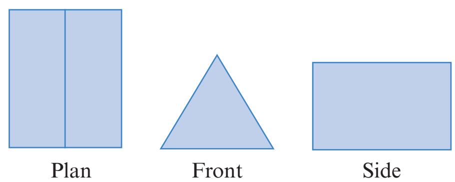


EXERCISE 29.10

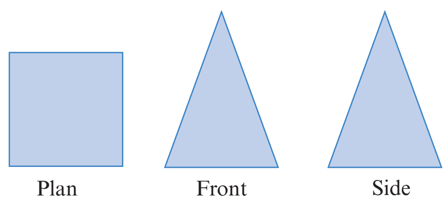
1 a



b



c



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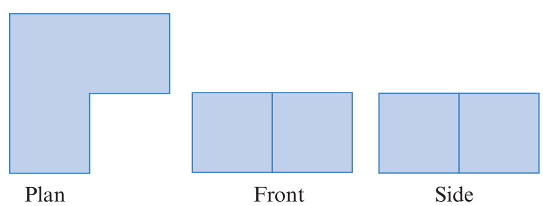
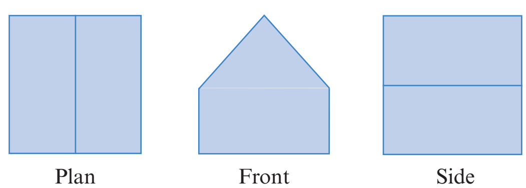
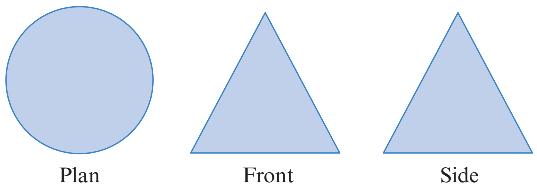


e

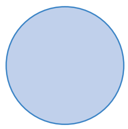


f


CHAPTER 30 Volume, capacity and mass

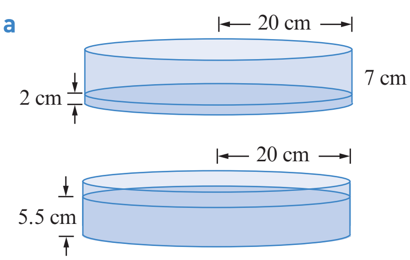
EXERCISE 30.1

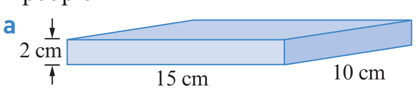
1 a i uniform
ii


b i tapering
c i neither
d i uniform
ii


2 a $447.76\ cm^3$ b $5.54\ m^3$
c $113.92\ cm^3$ d $536.28\ cm^3$
e $1227.2\ cm^3$ f $373\ cm^3$

EXERCISE 30.2

1 a $1181.09\ cm^3$ b $209.07\ cm^3$
c $36.92\ cm^3$ d $463.25\ cm^3$
e $26.52\ cm^3$ f $2.62\ m^3$
2 a $25.12\ cm^3$ b $765.49\ cm^3$
c $2936.25\ cm^3$ d $4.60\ cm^3$
e $72.62\ cm^3$ f $1873.12\ m^3$
3 a $156\ cm^3$ b $141\ cm^3$
4 a


b $2513.27\ cm^3$ c $6911.50\ cm^3$
d 275%
5 a 0.5 m b 0.45 m
c $0.3731\ m^3$
6 4 people
7 a


b 13.5 cm × 9 cm × 1.8 cm
c $300\ cm^3$ d $218.7\ cm^3$
e 27.1% reduction f $2.02 or $2.00

EXERCISE 30.3

1 a $x = 2.7$ cm b $x = 3.2$ cm
c $x = 5.5$ cm d $x = 0.4647$ cm
e $x = 1.7$ cm f $x = 2.9$ cm

EXERCISE 30.4

1 a 22.05 kL b 23.61 kL c 185.88 kL
2 a 39.3 mL b 2 cm
3 a 24.4 mL b $80 c $3.28/mL
4 Cylindrical container: $1.44/L
Rectangular cask: $1.50/L
The cylinder is better value.
5 a $1.32\ m^3$ b 1.32 kL c 10.5 cm

EXERCISE 30.5

1 a 30 g b 628.3 g c 41.47 t
2 142.9 kg
3 2740 bricks
4 a $9000\ cm^3$ b $72\,000\ cm^3$
c 72 kg d 33.3 kg
e 120.9 kg
5 350 g water + 43 g oil = 393 g

CHAPTER 31 Scales and plans

EXERCISE 31.1

1 a The ship needs to be drawn with double its length and height.
b The ship needs to be drawn with half its length and height.
2 a The lamp needs to be drawn with one-and-a-half times its length and height.
b The lamp needs to be drawn with 0.7 times its length and height.

EXERCISE 31.2

1

Diagram length	Scale	Actual length
17 mm	1 : 2	34 mm
25 mm	1 : 5	125 mm or 12.5 cm
100 mm	1 : 10	1000 mm or 1 m
38 mm	1 : 50	1900 mm or 1.9 m
95 mm	1 : 200	19 000 mm or 19 m

2

	Line length	1 : 10	1 : 25	1 : 40
a	25 mm	25 cm	62.5 cm	1 m
b	40 mm	40 cm	1 m	1.6 m
c	75 mm	75 cm	1.875 m	3 m
d	60 mm	60 cm	1.5 m	2.4 m
e	20 mm	20 cm	50 cm	80 cm
f	68 mm	68 cm	1.7 m	2.72 m
g	37 mm	37 cm	92.5 cm	1.48 m
h	73 mm	73 cm	1.825 m	2.92 m
i	46 mm	46 cm	1.15 m	1.84 m

EXERCISE 31.3

1

Length drawn on plan	Scale	Length measured
75 mm	1 : 2	150 mm or 15 cm
100 mm or 10 cm	1 : 5	500 mm or 50 cm
56 mm	1 : 10	560 mm or 56 cm
100 mm or 10 cm	1 : 50	5000 mm or 5 m
750 mm or 75 cm	1 : 200	150 000 mm or 150 m

2 **a** 105 mm **b** 97.5 mm **c** 100 mm
d 22.25 mm **e** 89 mm **f** 75 mm
g 375 mm **h** 75 mm

EXERCISE 31.4

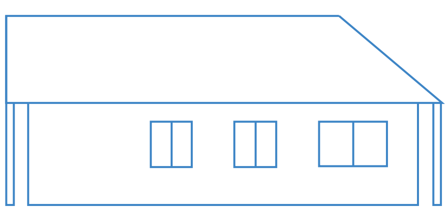
1 65 mm × 45 mm, so the field is 162.5 m × 112.5 m.
2 AB = 50 m, BC = 35 m, CD = 61 m, AD = 38 m

EXERCISE 31.5

1 **a** Scale is 1 : 80 **b** 1.6 m **c** 88 cm
2 **a** Scale is 1 : 1000 **b** 50 m

EXERCISE 31.6

1 **a** 40 m × 20 m **b** 26 m
c **i** 2 m **ii** 4 m
d 6 m **e** 5.8 m **f** 29 m^2 **g** 15 m
h 38 m^2 **i** 46 m^3 **j** 46 kL **k** 12
2 **a** **i** 9 **ii** 3 **iii** 1
b **i** 2 **ii** 2 **iii** 3 **iv** 1
c 1 : 200
d **i** 4.2 m × 1.8 m **ii** 5.4 m × 3.4 m
iii 4.4 m × 3.0 m
e 1.8 m × 1.6 m, 1.0 m × 1.0 m, 1.8 m × 1.6 m, 1.4 m × 2.2 m
f 3 m **g** 4.8 m **h** 0.4 m
i
South elevation



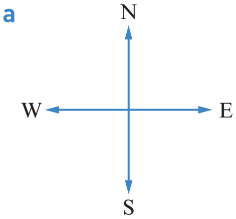
j 72.6 m^2 **k** 18 m^3
3 **a** WIR: walk-in robe, LIN: linen press, ENS: ensuite, WC: toilet, V: vanity unit, WM: washing machine
b **i** 17 **ii** 11
c 14 **d** 3 plus the WIR
e 20.89 m × 10.59 m **f** ≈63 m
g 221 m^2 **h** \$816/m^2 **i** 55.3 m^3
j 1 : 164 **k** 2.9 m × 1.8 m
l 8.7 m^2 **m** 62 m **n** 12 m
o **i** 8.4 m **ii** 8.6 m
p Using A, \$1938; using B, \$1957

4 **a** 1 : 200 **b** 6.34 m × 5.486 m
c 250 mm **d** 10 m
e 5.4 m × 3.8 m **f** 8 m
g 3.0 m **h** 2.8 m
i W1 is 2.0 m × 1.7 m
W3 is 2.0 m × 2.6 m
W5 is 0.6 m × 1.7 m
j 2 m × 1.7 m **k** 34.8 m^2
l 20.5 m^2

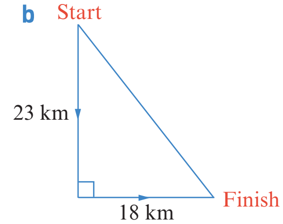
CHAPTER 32 Triangle calculations

EXERCISE 32.1

1 **a** 5 cm **b** 8.6 cm
c 8.4 cm **d** 8.3 cm
e 13.2 cm **f** 5.4 cm
2 **a** Right-angled at C **b** Right-angled at Y
c Not right-angled **d** Right-angled at E
e Not right-angled **f** Right angle at Q
3 3.606 m
4 **a** 8.616 m **b** 7.741 m
5 **a** 5.970 m **b** 5.130 m
6 **a** 600.3 m **b** ≈1420 m
c ≈5681 m **d** \$1675.95
7 **a**



b



c 29.21 km
8 **a** AR = 8.062 km, BR = 13.038 km
b \$538 050
9 **a** In $\triangle ABD$, $AB^2 + AD^2 - BD^2 \approx -9.93$
$BC^2 + DC^2 - BD^2 \approx -2.69$
$AB^2 + BC^2 - AC^2 \approx 3.68$
$AD^2 + CD^2 - AC^2 \approx 8.926$
$\therefore BC^2 + DC^2 - BD^2$ is closest to 0.
$\angle BCD$ is closest to 90°.
b $\angle QRS$
10 **b** $\angle LMN$

EXERCISE 32.2

1 **a** **i** BC **ii** AC **iii** AB
b **i** KM **ii** KL **iii** LM
c **i** PR **ii** QR **iii** PQ
d **i** c **ii** a **iii** b
2 **a** **i** $\frac{3}{5}$ **ii** $\frac{4}{5}$ **iii** $\frac{3}{4}$
b **i** $\frac{5}{13}$ **ii** $\frac{12}{13}$ **iii** $\frac{5}{12}$
c **i** $\frac{12}{13}$ **ii** $\frac{5}{13}$ **iii** $\frac{12}{5}$
d **i** $\frac{9}{15} = \frac{3}{5}$ **ii** $\frac{12}{15}$ **iii** $\frac{9}{12} = \frac{3}{4}$

EXERCISE 32.3

1 a $x = 2.9$ b $x = 9.5$ c $x = 3.5$
d $x = 3.9$ e $x = 15.3$ f $x = 1.7$
g $x = 0.7$ h $x = 15.3$ i $x = 1.9$
j $x = 3.0$ k $x = 6.0$ l $x = 8.4$

2 a $x = 7.42$ cm b $x = 24.81$ cm
c $x = 20.85$ cm d $x = 37.74$ m
e $x = 9.23$ m f $x = 18.75$ m

3 a $x = 34.71$ cm b $x = 16.77$ cm
c $x = 7.76$ m d $x = 4.48$ m
e $x = 99.77$ m f $x = 14.15$ m

4 a $x = 44.81$ m b $x = 4.28$ m
c $x = 73.17$ m d $x = 19.54$ cm
e $x = 13.21$ m f $x = 3.80$ km

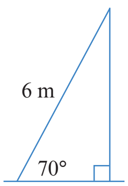
EXERCISE 32.4

1 a $\theta \approx 39°$ b $\theta \approx 35°$
c $\theta \approx 61°$ d $\theta \approx 42°$
e $\theta \approx 57°$ f $\theta \approx 62°$

2 a $\theta \approx 41.4°$ b $\theta \approx 36.0°$
c $\theta \approx 43.8°$ d $\theta \approx 26.2°$
e $\theta \approx 53.9°$ f $\theta \approx 42.9°$

EXERCISE 32.5

1 a 3.21 m b 4.59 m
2 10.64 m
3 a 2.573 m b 10.385 m
4 29.17°
5 No, as angle is ≈23.58°
6 a



b 5.638 m
7 50.28°
8 7.07 m
9 a $x \approx 4.965$ m b 25.978 m
c 467.6 m

CHAPTER 33 Business calculations

EXERCISE 33.1

1 a Profit of $15 b Loss of $15
c Profit of $375 d Loss of $269
2 a Profit of $110 b ≈32.4%
3 Loss is $3000, 10%
4 a $25 000 b ≈4.6%
5 Profit is $39, ≈18.1%

EXERCISE 33.2

1 a $127.50 b $207.50
c $187.50 d $1882.80

EXERCISE 33.3

1 a 1.14 b 0.86 c 1.37
d 0.77 e 1.082 f 0.921
2 a $207 b $379.60
c $342 d $5077.80
e $65 425.50 f $14 322.42

EXERCISE 33.4

1 a $678.04 b ≈$566.44
c $5356.26 d $8532.80
2 $350
3 $7224
4 $1 568 000

EXERCISE 33.5

1 a $85 b $8.50 c $9.85
d $129.50 e $16 f $45
2 a $38.50 b $88 c $115.50
d $734.80 e $2585 f $141.85

EXERCISE 33.6

1 a $12.64 b $2.72 c $12.91
2 a $213.60 b $19.42

EXERCISE 33.7

1 a $1190 b $510

2

	Marked price	Discount	Selling price	Discount as a % of the marked price
a	$125	$25	$100	20%
b	$240	$60	$180	25%
c	$2.75	55 cents	$2.20	20%
d	$3	65 cents	$2.35	≈21.7%
e	$150	$30	$120	20%

EXERCISE 33.8

1 a $36.75 b $46.05
c $33.42 d $44.10
e $52.74 f $28.35
2 a $38.48 b $37.72
c $61.00 d $33.80
e $42.50 f $55.90
3 a $1410 b ≈$1082.25
4 a $1225.50 b $1371.38

EXERCISE 33.9

1 a i 8.844 standard hours
ii 8.118 standard hours
iii 10.054 standard hours
iv 6.512 standard hours
b i $182.65 ii $196.76
iii $204.46 iv $180.19
2 $593.84

EXERCISE 33.10

1 a $3066.67, $707.69, $18.87
  b $4037.50, $931.73, $24.85
  c $6274.58, $1447.98, $38.61
2 a $1736, $45 136, $22.84
  b $1151, $29 926, $15.14
  c $2116, $55 016, $27.84
3 a $873.35 per week
  b $14.70 per hour
  c the same

EXERCISE 33.11

1 a $4695.75 b $43 110.00
  c $5035.04
2 $2270
3 $4080
4 a $2622.47 b $2523.81
  c $2279.34 d $2013.20

EXERCISE 33.12

1 Students' answers will vary due to current tax rates.
2 Net wage = $910.01 – current tax

CHAPTER 34 Calculating interest

EXERCISE 34.1

1 a $180 b ≈$985.63
  c ≈$7258.33 d $292
2 $25 000 at 7% for 4 years

EXERCISE 34.2

1 a $2200 b $27 000
2 ≈$48 979.59

EXERCISE 34.3

1 a 4% p.a. b 9% p.a.
2 8.25% p.a.
3 $8\frac{1}{3}$% p.a.

EXERCISE 34.4

1 a 4.853 years = 4 years and 312 days
  b 6 years
2 3.704 years = 3 years and 257 days

EXERCISE 34.5

1 a $618.75
  b 36 months
  c $86.63. No, as Sam should pay $86.63 per month, not $115.
2 $237.29 per month
3 $900 per quarter
4 $1633.33 every 6 months

EXERCISE 34.6

1 a $2977.54 b $5243.18 c $11 433.33
2 a $105.47 b $782.73 c $4569.19
3 a $13 738.80 b $1738.80
4 a $5887.92 b $887.92

EXERCISE 34.7

1 a $3932.39 b $12 101.30
2 a $4596.00 b $12 735.64
3 a $42 779.08 b $12 779.08
4 $3200 (simple), $3485.03 (compound), thus use compound interest investment.

EXERCISE 34.8

1 $13 310.00, $14 641.00, $16 105.10, $17 715.61, $19 487.17, $21 435.89, $23 579.48, $25 937.42
2 a Total at 6 years would be:
  10% = $17 715.61, 6% = $14 185.19
  8% = $15 868.74, 12% = $19 738.23
  7.13% = $15 117.04
  b 12.25% c 3.19%

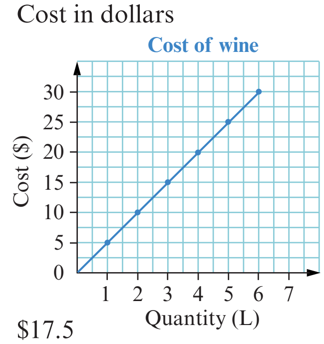
EXERCISE 34.9

2 ≈$412.08 3 ≈$207.47
4 ≈$639.38

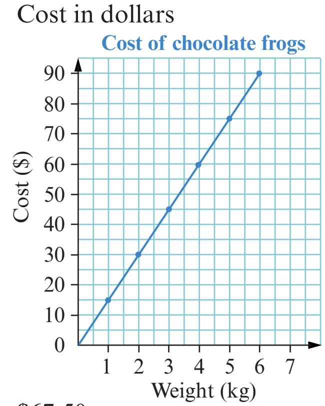
CHAPTER 35 Graphical representation

EXERCISE 35.1

1 a Cost in dollars

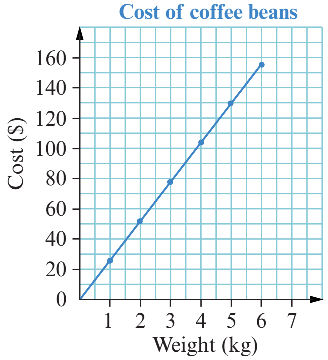


  b $17.5
2 a Cost in dollars



  b $67.50

3 a Cost in dollars



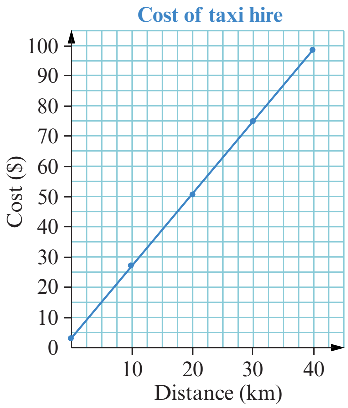
b ≈$142

EXERCISE 35.2

1 a

Distance (km)	0	10	20	30	40
Cost ($)	3	27	51	75	99

b

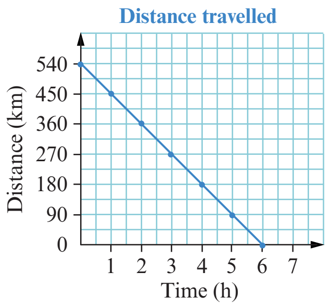


c $63

2 a

Time (h)	0	1	2	3	4	5	6
Distance (km)	540	450	360	270	180	90	0

b



c 540 km **d** 6 h **e** 135 km

EXERCISE 35.3

1 a

Line segment	Run	Rise	Slope
AB	6	0	0
CD	7	1	$\frac{1}{7}$
EF	7	2	$\frac{2}{7}$
GH	6	3	$\frac{1}{2}$
IJ	3	5	$\frac{5}{3}$
KL	1	7	7

b i zero
ii infinite or undefined
iii increase

2 a 0.15 **b** 0.2 **c** $\frac{3}{4}$ or 0.75

4 a

Line segment	Run	Rise	$\frac{\text{Rise}}{\text{Run}}$
BC	2	1	$\frac{1}{2}$
DE	2	1	$\frac{1}{2}$
AC	4	2	$\frac{1}{2}$
BE	6	3	$\frac{1}{2}$
AE	8	4	$\frac{1}{2}$
AF	9	$4\frac{1}{2}$	$\frac{1}{2}$

b A straight line has constant slope.

EXERCISE 35.4

1 a 0 **b** $-\frac{1}{3}$ **c** $-\frac{1}{2}$
d $-\frac{3}{4}$ **e** $-\frac{3}{2}$ **f** −7

2 a OP, PQ, RS, TU **b** QR, ST, UV
c TU **d** ST
e VW **f** PQ

3 a 5 **b** 6 **c** $\frac{3}{2}$
d −2 **e** −4 **f** $-\frac{3}{4}$

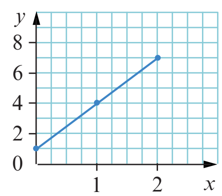
EXERCISE 35.5

1 a i Gradient $= \frac{6}{4} = \frac{3}{2}$ **ii** −1
iii $y = \frac{3}{2}x - 1$
b i Gradient = −2 **ii** 3
iii $y = -2x + 3$
c i Gradient $= -\frac{7}{4}$ **ii** 3
iii $y = -\frac{7}{4}x + 3$
d i Gradient $= \frac{5}{2}$ **ii** −5
iii $y = \frac{5}{2}x - 5$
e i Gradient $= \frac{1}{3}$ **ii** −1
iii $y = \frac{1}{3}x - 1$
f i Gradient $= -\frac{7}{6}$ **ii** 3
iii $y = -\frac{7}{6} + 3$

EXERCISE 35.6

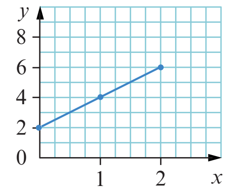
1 a

x	0	1	2
y	1	4	7



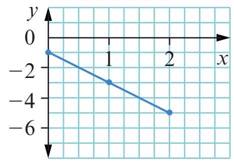
b

x	0	1	2
y	2	4	6



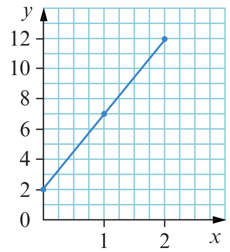
c

x	0	1	2
y	−1	−3	−5

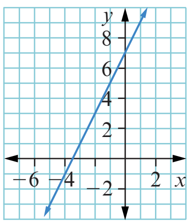


d

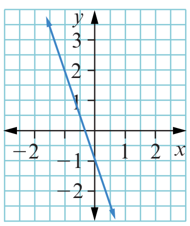
x	0	1	2
y	2	7	12



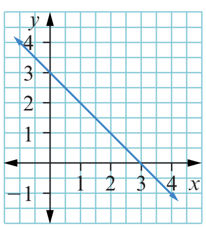
2 a $y = 2x + 7$



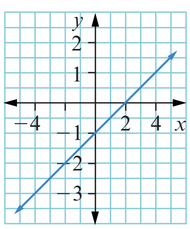
b $y = -3x - 1$



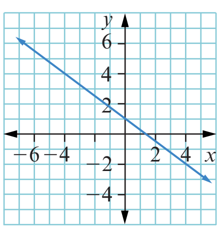
c $y = -x + 3$



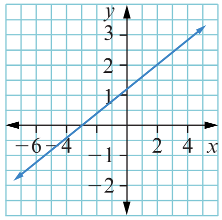
d $y = \frac{1}{2}x - 1$



e $y = -\frac{3}{4}x + 1$

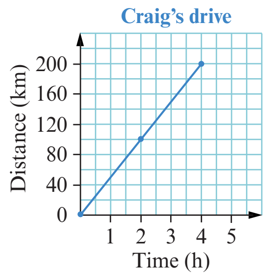


f $y = 0.4x + 1.2$



EXERCISE 35.7

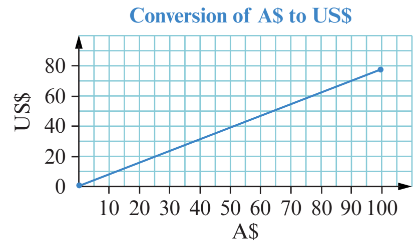
1 a



b 75 km **c** 50

d km/h

2 a



b \$65 **c** 0.78

d It is the conversion rate.

e Yes

EXERCISE 35.8

1 a ≈\$68 **b** 14 km

c 2.5, which is the cost/km

d \$5, the fixed charge

2 a ≈\$575 **b** ≈16 people

c ≈14, which is approximately \$14/person

d ≈\$75, the fixed cost

3 a ≈\$425 **b** ≈3 h

c ≈770, \$77/h charged

d \$80, call-out fee

EXERCISE 35.9

1 a **i** ≈5 **ii** −5 **iii** $y = 5x - 5$

b **i** 0.67 **ii** 4 **iii** $y = 0.67x + 4$

c **i** −3 **ii** 2 **iii** $y = -3x + 2$

EXERCISE 35.10

1 a $y = 5.40x + 6.13$ **b** $y = -0.95x + 5.80$

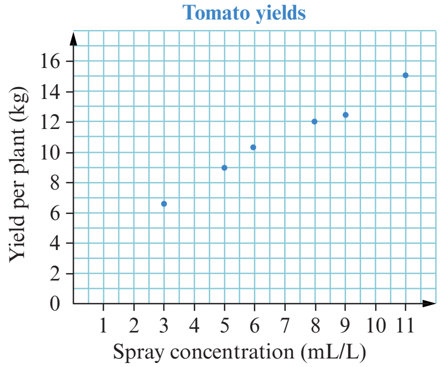
c $y = 5.4x + 14.27$ **d** $y = -3.97x + 34.07$

EXERCISE 35.11

1 $r = -0.8429$, $r^2 = 0.7105$

Negative, moderate correlation

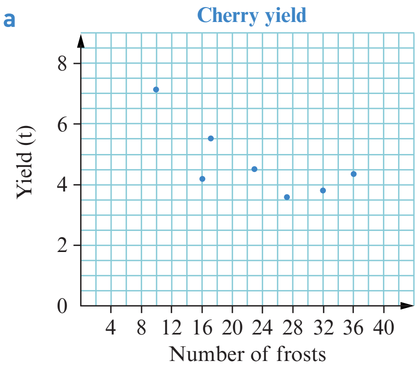
2 a



b $r = 0.9943, r^2 = 0.9886$

c Positive, very strong association

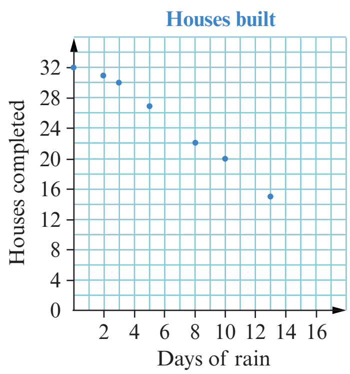
3 a



b $r = -0.7228, r^2 = 0.5223$

c Negative, moderate association

4 a



b $r = -0.9932, r^2 = 0.9864$

c Negative, very strong association

CHAPTER 36 Representing data

EXERCISE 36.1

1 a 5000 per week **b** 400
c 398 **d** 4975

2 a 7.374 million **b** 53.9%

3 a 7500

b Assumed it was random sampling. The caught fish were released elsewhere in the lake, and moved at random in the lake.

EXERCISE 36.2

1 a

Score	Tally	Frequency	Relative frequency
26	|	1	0.025
27	𝍸 |	6	0.150
28	𝍸 |	6	0.150
29	𝍸 ||	7	0.175
30	|||	3	0.075
31	𝍸 ||	7	0.175
32	𝍸 |||	8	0.200
33	||	2	0.050
	Total	40	1.000

b 50%

2 a

Score	Tally	Frequency	Relative frequency
47	||||	4	0.067
48	𝍸 ||||	9	0.150
49	𝍸 𝍸	10	0.167
50	𝍸 𝍸 𝍸 𝍸 |	21	0.350
51	𝍸 𝍸	10	0.167
52	𝍸 |	6	0.100
	Total	60	1.000

b 38.3%

3 a

Score	Tally	Frequency	Relative frequency
93	|||	3	0.06
94	||||	4	0.08
95	||	2	0.04
96	||	2	0.04
97	||	2	0.04
98	||	2	0.04
99	|||	3	0.06
100	|||	3	0.06
101	𝍸	5	0.10
102	||||	4	0.08
103	|	1	0.02
104	||||	4	0.08
105	𝍸	5	0.10
106	|||	3	0.06
107	||||	4	0.08
108	|	1	0.02
109	|	1	0.02
110	|	1	0.02
	Total	50	1.000

b 32 times **c** 18%

d 36%

EXERCISE 36.3

1 a

Distance	Frequency	Relative frequency
20–29	3	0.033
30–39	26	0.289
40–49	41	0.456
50–59	14	0.156
60–69	6	0.067
Total	90	1.001

b 29 **c** 22.2%

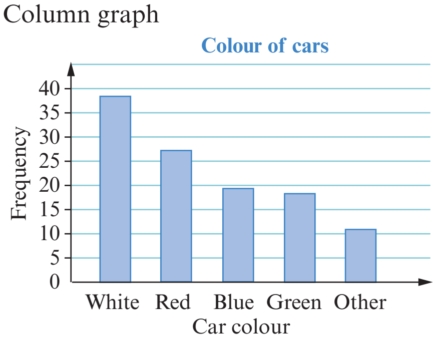
2 a

Height (mm)	Frequency	Relative frequency
300–324	12	0.100
325–349	18	0.150
350–374	42	0.350
375–399	28	0.233
400–424	14	0.117
425–449	6	0.050
Total	120	1.000

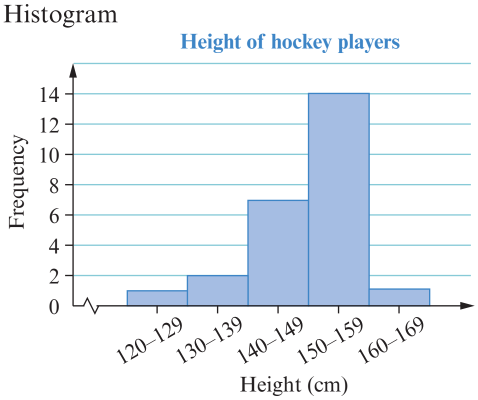
b 20 **c** 58.3%
d i 1218 **ii** 512

EXERCISE 36.4

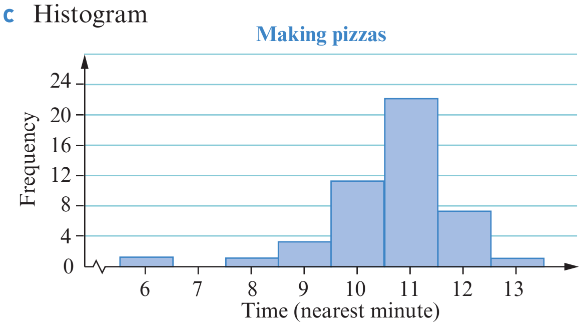
1 a Column graph



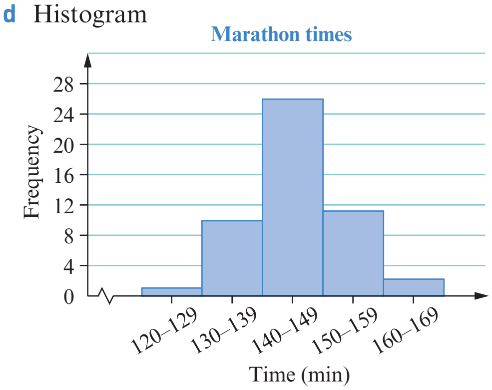
b Histogram



c Histogram



d Histogram



EXERCISE 36.5

1 Unit = 1

2 Unit = 0.1

Stem	Leaf
0	7 9
1	2 3 8
2	4 4 5 7 8
3	0 2 6
4	1

EXERCISE 36.6

1 Team A mean = 91.25, Team B mean = 91.75
Team B had the higher mean score.
2 1.7 siblings per student
3 21.25 mm
4 a 59.45 toothpicks
b Slightly lower than manufacturer's claim
5 34.59 almonds per packet

EXERCISE 36.7

1 a 26.5 **b** 13.17 **c** 170.13
2 a 24.67 **b** 18.26

EXERCISE 36.8

1 a 26.5 **b** 13.17 **c** 170.13

EXERCISE 36.9

1 a 7 **b** 7.5 **c** 6 **d** 9

2 a Median score: Gina 14.5, Sari 20.5
 b Mean score: Gina 22.58, Sari 20.83
 c The mean seems more appropriate.

3 Student responses will vary, but points covered in the discussion should include:
- The mean would be affected by very large or very small house prices (outliers).
- The median ignores outliers.

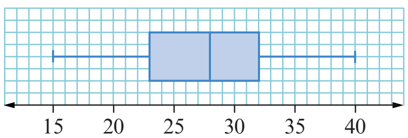
EXERCISE 36.10

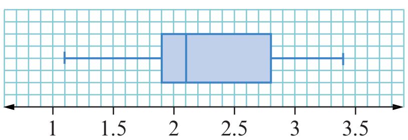
1 a Range = 6, IQR = 3
 b Range = 9, IQR = 3.5
 c Range = 9, IQR = 4.5

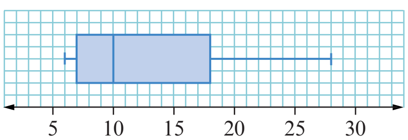
2 a Dougal: range = 7, IQR = 3
 Sylvia: range = 5, IQR = 3
 b The IQRs are the same, but the ranges are different; Sylvia's is smaller.

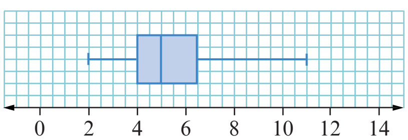
4 a Yes, 29
 b Mean = 7.24, median = 6
 c Mean = 5.88, median = 6
 d The mean is affected more by the outlier.

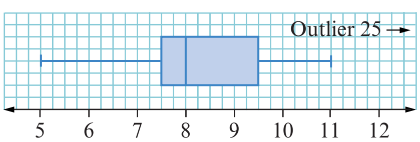
EXERCISE 36.11

1 a


 b


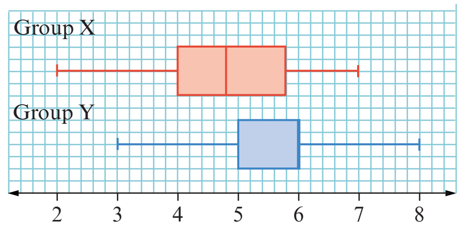
 c


2 a


 b


3 a i Range = 6 ii Upper quartile = 54
 iii Lower quartile = 50 iv IQR = 4
 v Median = 52 vi No outlier
 b i Range = 110 ii Q_3 = 280
 iii Q_1 = 220 iv IQR = 60
 v Median = 240 vi No outlier

4 a Test B has the higher median: 70 compared to 63.
 Test A has the larger range: 58 compared to 50.
 Test A has the larger IQR: 30 compared to 25.
 b Yes, but not by much.

5 a Group X: median = 5, Q_1 = 4, Q_3 = 6
 Group Y: median = 6, Q_1 = 5, Q_3 = 6
 b


 c Group X: range = 5, IQR = 2
 Group Y: range = 5, IQR = 1
 d Yes, as the majority of data has thickness of 1 mm more with the new synthetic rubber.

CHAPTER 37 Interpreting data

EXERCISE 37.1

2 Students' responses will vary but some points covered in the discussion should include:
- time to collect all the data would take too long
- cost to collect data will be large
- the need to select a sample to make it easier to do
- issues with outliers.

EXERCISE 37.2

1 Student responses will vary but may include the following information:
 Spark: Range = 56, IQR = 21, median = 70, mean = 71.17
 Sure Fire: Range = 52, IQR = 15, median = 85.5, mean = 85.93

2 Spark, since it reacts faster on average.

EXERCISE 37.3

1 a Unit = dollars

Jordon	Stem	Hassam
7	09	3 5 9
	10	
6 0 0	11	6 8
9 7	12	
9 9	13	2
7 2	14	0 7
9 8 3 2	15	3 4 5
3 2 1 1 0	16	8
4	17	
0	18	2 3 8
	19	0 1 4 6
	20	5 5 8
5	21	
3	22	3
	23	3
1	24	
	25	
0	26	
	27	0

b We cannot tell if the driver did many small trips each day or just one or two large trips if we have the total daily fare. The amount of money collected per hour will tell us how busy each driver was throughout the day.

c Leaf unit = cents

Jordon	Stem	Hassam
83	08	
	09	55
05 05	10	
09	11	31 68 69
65 29 20 18	12	25 81 95 98
84 63 54 50 44 34 30 29	13	92
57 20 18 01	14	
74	15	03 21 52 72 84
	16	69
	17	27
94 64	18	26 59 64 92 94
	19	12
08	20	09
	21	
	22	79
70	23	
	24	03

2 a Unit = 0.1

Stem	Cholesterol levels before
5	0 0
6	0 0 0 1 2 2 2 2 2 3 3 4 5 5 5 6 7 7 8
7	0 1 1 1 1 2 3 3 4 4 4 5 5 5 6 6 6 9 9
8	1 1 2 3 4 4 4 5 6 6 9

b Unit = 0.1

Placebo	Stem	Drug
	3	2
	4	2 4 4 4 6 6 7 7 7 7 7 7 8 8 8 8 9
7	5	0 1 2 2 6 6
8 7 6 6 5 5 2 1 1 1 0	6	2
9 9 6 6 5 4 3 0	7	
8 4 4 3 2	8	

3 a Unit = 1

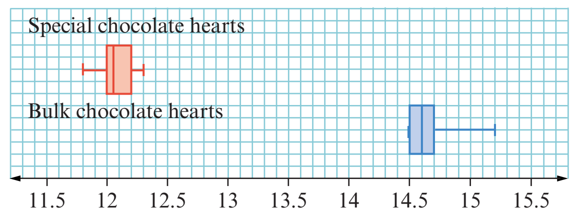
Stem	Plot A
0	2 7 8 9
1	0 2 4
2	2 5 9
3	0 0 2 2 2 2 4 4 5 5 6 6 8 8 9 9
4	0 1 2 2
5	0
6	
7	

b Unit = 1

Plot B	Stem	Plot C
	0	2
	1	
9 8 0	2	
4 4 3	3	9
8 8 8 7 7 7 7 6 6 5 1	4	3 7
8 8 6 6 4 4 3 2 1 0	5	0 4 5 6 8 8 9
6 3	6	0 0 1 1 2 3 4 5 5 5 6 7 8 8 8 9 9
	7	0 0 0 2 5 6 6 6 7 9 9

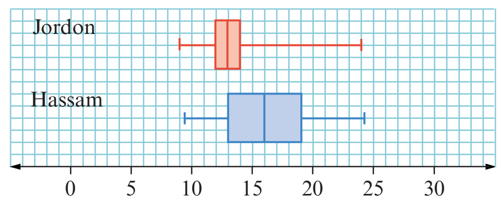
EXERCISE 37.4

1

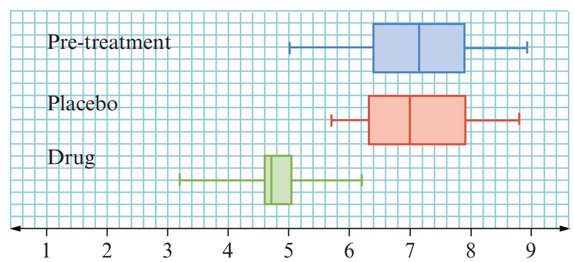


The special chocolate hearts are much lighter than the bulk chocolates. Both have the same IQR but the bulk chocolate hearts have a larger weight range.

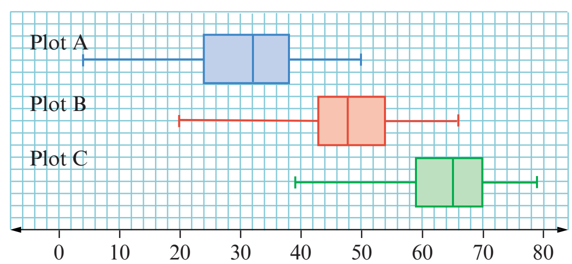
2



3



4



EXERCISE 37.5

1 Students' answers will vary, but should include:

- a statement of topic
- sample sizes
- graphs and whether there are outliers or not
- discussion of medians (or means)
- discussion of range and/or IQR
- conclusion.

Index

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