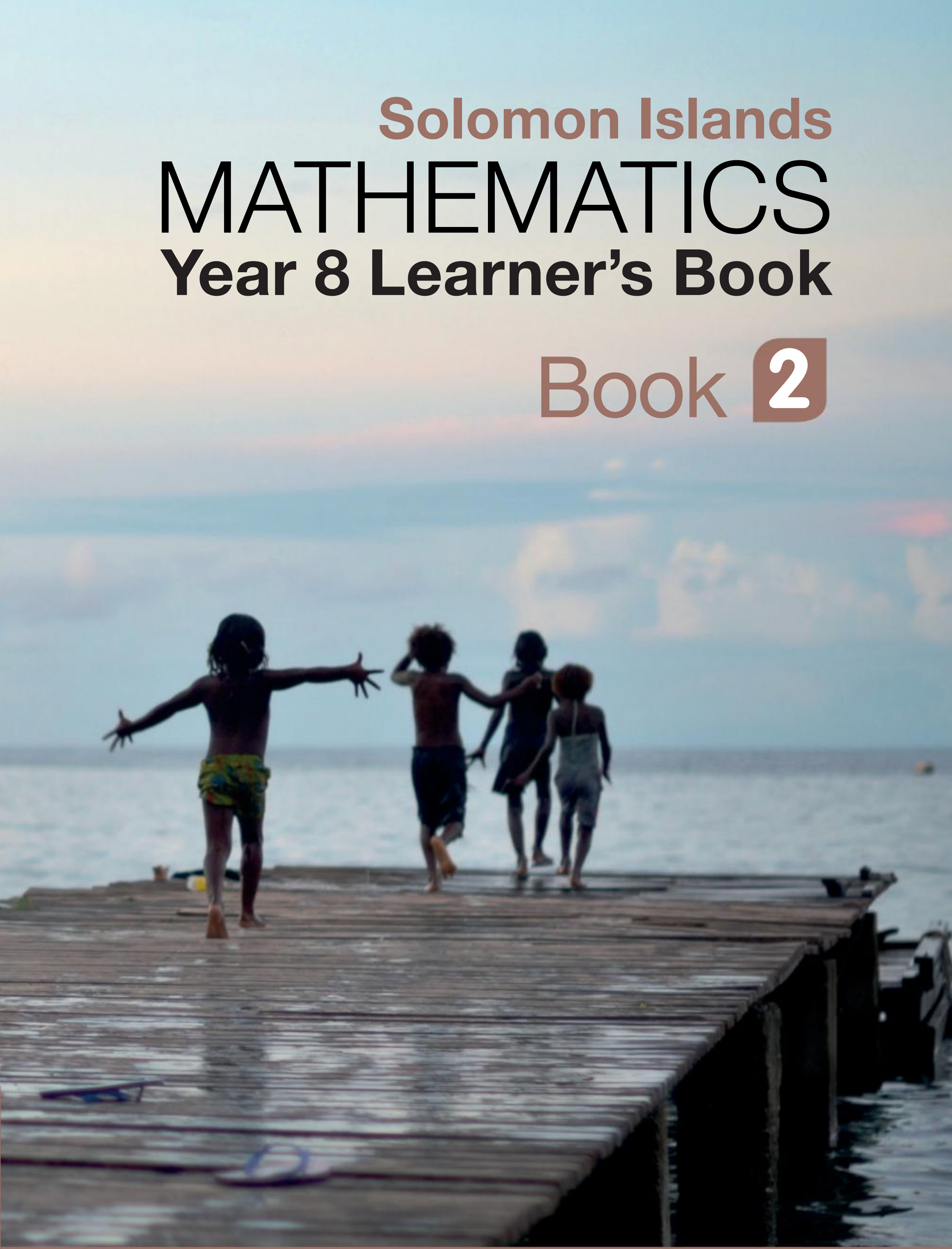
Solomon Islands

MATHEMATICS

Year 8 Learner's Book

Book 2



ALWAYS LEARNING

PEARSON

Solomon Islands

MATHEMATICS

Year 8 Learner's Book

Book 2

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Area and Volume 134

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Polyhedra and Networks 172

Answers 200

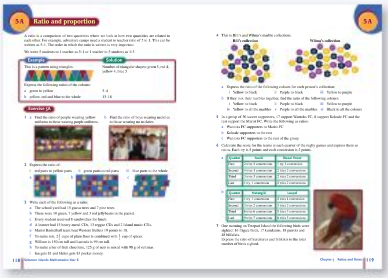
HOW TO USE THIS BOOK

The **Solomon Islands Mathematics** series has been written to cover the General Learning Outcomes of the Solomon Islands Secondary Mathematics Syllabus Years 7 to 9.

CHAPTER 4 Algebra Symbols

Chapter opening pages

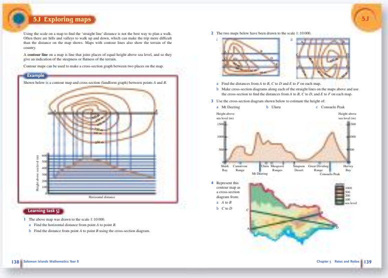
Chapter opening pages include a contemporary or historical context for the content and provide learners with a list of the skills that are covered in the chapter.



5A Ratio and proportion

Theory and exercise sections

Theory and exercise sections contain explanations, examples and exercises designed to develop understanding of concepts and provide opportunities for students to practise new skills.

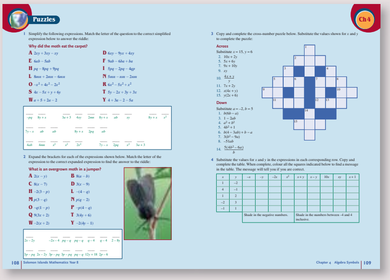


5J Exploring maps

Explorations

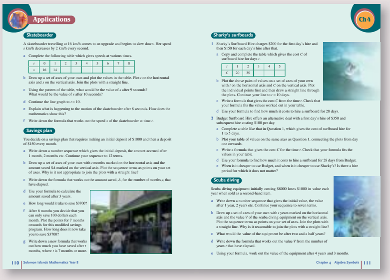
Explorations are scattered throughout the chapters, allowing students to work independently on non-standard problems and construct their own understandings.

These features are found at the end of each chapter



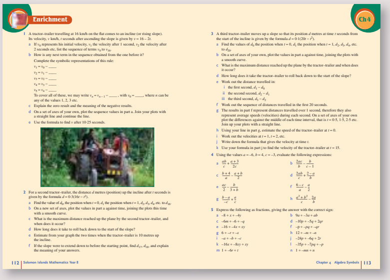
Puzzles

Puzzles are included for extra skills practice.



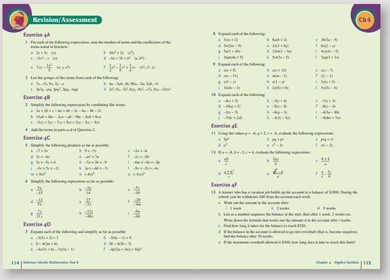
Application

Application sections investigate and apply mathematical ideas in a creative way and provide activities for a range of learner abilities.



Enrichment

Enrichment sections contain challenging tasks for learners to apply and extend their understanding of concepts.



Revision questions

Revision questions provide opportunities for learners to consolidate understanding of concepts.

Solomon Islands Mathematics Year 8 Learner's Book

Introduction

This book is written to help you learn Mathematics by actively participating in a variety of activities. The book has a total of 14 chapters. Each chapter focuses on a particular topic from one of the strands in the Solomon Islands Junior Secondary Mathematics Syllabus. The strands are *Number*, *Measurement*, *Algebra*, *Geometry*, *Statistics and Probability*. We hope that the activities in this book will encourage you to learn Mathematics effectively, and to gain enjoyment and enrichment from the topics and contexts involved.

Chapter organisation

The chapter order provides opportunities to revise topics studied in earlier years, to learn new knowledge and skills, and to review and develop your understanding throughout the school year.

The Number strand

The chapters that will further develop your number skills include *Percentages*, *Ratios and Rates* and *Indices*. You will continue your understanding of numbers, the ways they are represented and the quantities for which they stand. You will develop accuracy, efficiency and confidence in calculating, both mentally and on paper. You will refine your ability to estimate and to make approximations, and to be alert to the reasonableness of results and measurements. It is important to maintain your competency in the four basic operations, and to apply them confidently with whole numbers, directed numbers, decimals and fractions. By extending these skills to percentages, ratios and rates, and indices, you will be able to apply Mathematics to solving problems in real-life contexts.

The Measurement strand

The chapters that will further develop your measuring skills include *Length and Perimeter* and *Area and Volume*. You will need to apply the standard metric units to a range of practical situations, and be able to use a range of tools to measure accurately and with precision. You will need to understand and use appropriate formulae to calculate areas and volumes and apply them to real-life problems. You will also learn to estimate measurements so that you will know whether you receive approximately the right amount when buying goods in a shop.

The Algebra strand

The chapters that will develop your algebra skills in Year 8 are *Algebra Symbols*, *Equations and Inequations* and *The Coordinate Plane*. You will learn to recognise patterns and relationships in Mathematics and the real world, and be able to generalise from them. You will develop the ability to think abstractly and to use symbols, notation, and graphs and diagrams, to represent and communicate mathematical relationships, concepts and generalisations. You will also have opportunities to use algebraic expressions confidently to solve practical problems.

The Geometry strand

The chapters that will develop your geometric skills include *Polygons and Parallel Lines*, *Transformations*, *Pythagoras* and *Polyhedra and Networks*. Geometry is concerned with size, shape, position and the properties of space. This year you will gain knowledge of geometrical relations in both two and three dimensions, and recognise and appreciate their occurrence in the environment. You will develop spatial awareness, and the ability to recognise and make use of the geometrical properties and the symmetries of everyday objects. You will also appreciate how to use geometric models as aids to solve practical problems in time and space.

The Statistics and Probability Strand

There is one chapter for your study of *Statistics* and another for *Probability*. Statistics deals with the collecting, organising, presenting and interpreting of numerical information (data). Our newspapers are full of statistical information, and it is important that you can understand statements and graphs to check the accuracy of the conclusions. Your study of Probability will help you describe the chance of various events occurring. Games of chance are a fun way of learning about probability but the concepts can be applied to many real-life situations when outcomes are uncertain, and so the study of Probability develops our decision-making skills.

How to learn Mathematics

As you work through the chapters you will be asked to work on your own, work with a partner or in a group, and sometimes with the whole class. Therefore, you must be willing to participate actively in all the tasks and not rely on the teacher or friends for answers. When you actively participate you will learn a great deal as well.

Making mistakes

Learning Mathematics is a skill, like riding a bicycle. You cannot learn to ride a bicycle by just listening to the teacher telling you how to ride, you can only learn by doing it. Nobody has learnt to ride a bicycle without falling off many times. Making mistakes is part of the learning process and this is also true for Mathematics. The more familiar you are with the topic, the fewer mistakes you are likely to make. Like bicycle riding, Mathematics learning needs lots of practice and the exercises in this book are designed to help you practice until you become confident with each new skill. Homework is a chance to further practice the skills learnt in class, and what you can’t do on your own, you can ask your teacher or a friend the next day.

Developing skills

Mathematics is more than a series of facts and rules. It involves understandings and skills that can be applied to new situations. After each lesson it is useful to reflect on your learning and in particular about the problem-solving strategies that you used that day. Those same strategies may be useful for other problems in the future. And if you discover a new skill, show it to a friend. Not only will your friend benefit, but it will help you remember it too!

Suggested teaching plan for the Year 8 Learner's Book

Semester 1

Weeks	Sub-strands	Allocated Times
	Number	
1	Chapter 1: Percentages	3 Weeks
2		
3		
	Measurement	
4	Chapter 2: Length and Perimeter	3 Weeks
5		
6		
	Geometry	
7	Chapter 3: Polygons and Parallel Lines	2 Weeks
8		
	Algebra	
9	Chapter 4: Algebra Symbols	2 Weeks
10		
	Number	
11	Chapter 5: Ratios and Rates	3 Weeks
12		
13		
	Probabilty and Statistics	
14	Chapter 6: Statistics	3 Weeks
15		
16		
	Geometry	
17	Chapter 7: Transformations	3 Weeks
18		
19		
20	*Mid-Year Examinations*	1 Week
Mid-year Holidays		

Semester 2

Weeks	Sub-strands	Allocated Times
	Algebra	
21	Chapter 8: Equations and Inequations	3 Weeks
22		
23		
	Number	
24	Chapter 9: Indices	2 Weeks
25		
	Algebra	
26	Chapter 10: The Coordinate Plane	3 Weeks
27		
28		
	Geometry	
29	Chapter 11: Pythagoras	3 Weeks
30		
31		
	Probability and Statistics	
32	Chapter 12: Probability	3 Weeks
33		
34		
	Measurement	
35	Chapter 13: Area and Volume	3 Weeks
36		
37		
	Geometry	
38	Chapter 14: Polyhedra and Networks	2 Weeks
39		
40	*Final Examinations*	1 Week
End-of-year Holidays		

CHAPTER

8

Equations and Inequations

Equations and Inequations

Equations or mathematical models are used to solve real-life problems. For example, during the colonial days before the Solomon Islands was independent, people who applied to the police force had to meet a height requirement before they could be accepted into the force. An inequation can be formed to illustrate this requirement, given that only those people who were 170 cm (1.7 m) tall or more could be accepted. If the pronumeral h is used to represent a person's height in centimetres, then the inequation is $h \geq 170$. We can set up and solve equations using algebra. More complex equations or models are used in design and engineering.

This chapter covers the following skills:

- Solving equations by inspection
- Solving equations by flow charts
- Solving equations by inverse operations
- Using equations to model real-world problems
- Solving inequations
- Exploring the coordinate plane

Specific Learning Outcome (SLO)

Learners should be able to:

8.8.1.1 Define the term 'equation'. Equation: two expressions that are equal for some values of the pronumeral(s).

8.8.2.1 Solve simple equations by inspection.

8.8.3.1 Solve equations with one unknown using the 'flow chart' method.

8.8.4.1 Solve equations using the 'inverse operations' method.

8.8.5.1 Solve equations requiring two or three steps.

8.8.6.1 Define the term 'inequation'. Inequation: an expression where a pronumeral represents a range of values.

8.8.7.1 Use inequality symbols: $<$, $>$, $\geq$, and $\leq$ to write relationships between expressions.

8.8.8.1 Use a number line to represent inequations.

8.8.9.1 Solve inequation problems by applying inverse operation procedures.

8.8.9.2 Use equations and inequations to solve word problems.

8A Solving equations by inspection

Equations can be written using pronumerals to represent unknown numbers. For example, $5x + 4 = 14$ is an equation and true only for $x = 2$. Simple equations can be solved by inspection. Putting the equation into words is a useful strategy here.

Example	Solution
1 Solve $x + 7 = 11$. What number increased by 7 gives 11?	$x + 7 = 11$ $4 + 7 = 11$ so $x = 4$
2 Solve $y - 2 = 3$. What number when decreased by 2 gives 3?	$y - 2 = 3$ $5 - 2 = 3$ so $y = 5$
3 Solve $a + a + a = 12$. What number when added to itself twice, gives 12?	$a + a + a = 12$ $3a = 12$ $3 \times 4 = 12$ so $a = 4$
4 Solve $2y = 14$. What number when doubled gives 14?	$2y = 14$ $2 \times 7 = 14$ so $y = 7$
5 Solve $\frac{p}{4} = 8$. What number when divided by 4 gives 8?	$\frac{p}{4} = 8$ $\frac{32}{4} = 8$ so $p = 32$

Exercise 8A

1 Determine the value of the symbol in each of the following:

a $a + 15 = 20$ $a = ?$

b $b - 9 = 8$ $b = ?$

c $c - 10 = 26$ $c = ?$

d $d + 21 = 30$ $d = ?$

e $e - 2 = 19$ $e = ?$

f $f + 37 = 50$ $f = ?$

g $g + 154 = 181$ $g = ?$

h $h - 57 = 26$ $h = ?$

i $i + 219 = 302$ $i = ?$

2 Determine the value of the symbol in each of the following:

a $k + k + k = 21$

b $m + m + m + m = 36$

c $p + p + p + p + p = 10$

d $r + r + r + r + r = 55$

e $t + t + t + t + t = 60$

f $v + v + v + v + v + v = 42$

g $x + x + x = 21$

h $y + y + y + y + y = 30$

i $m + m + m + m + m = 25$

3 Solve the following equations by inspection:

a $x + 5 = 11$

b $y + 12 = 21$

c $7 + m = 13$

d $9 + a = 17$

e $r + 12 = 17$

f $t + 5 = 9$

g $10 + q = 18$

h $22 + p = 30$

i $x + 7 = 19$

j $s + 6 = 11$

k $b + 2 = 8$

l $y + 13 = 17$

m $m + 3 = 16$

n $a + 80 = 220$

o $e + 45 = 156$

p $f + 9 = 110$

q $x + 19 = 35$

r $y + 16 = 31$

s $p + 21 = 45$

t $25 + x = 46$

4 Solve the following equations by inspection:

- **a** $x - 7 = 9$
- **b** $s - 8 = 11$
- **c** $b - 5 = 8$
- **d** $y - 13 = 7$
- **e** $m - 13 = 14$
- **f** $a - 18 = 20$
- **g** $e - 24 = 6$
- **h** $f - 9 = 11$
- **i** $10 - b = 8$
- **j** $13 - c = 6$
- **k** $9 - d = 7$
- **l** $20 - a = 14$
- **m** $39 - m = 19$
- **n** $45 - n = 22$
- **o** $33 - p = 8$
- **p** $50 - q = 32$

5 Solve the following equations by inspection:

- **a** $6x = 12$
- **b** $9y = 27$
- **c** $8a = 32$
- **d** $h \times 5 = 30$
- **e** $12 \times m = 60$
- **f** $8 \times n = 56$
- **g** $p \times 4 = 12$
- **h** $q \times 15 = 45$
- **i** $17x = 34$
- **j** $65s = 195$
- **k** $2b = 88$
- **l** $13y = 52$
- **m** $30 \times m = 900$
- **n** $7 \times p = 280$
- **o** $e \times 45 = 135$
- **p** $f \times 19 = 114$

6 Solve the following equations by inspection:

- **a** $x \div 3 = 12$
- **b** $y \div 2 = 26$
- **c** $a \div 9 = 12$
- **d** $h \div 12 = 30$
- **e** $12 \div m = 6$
- **f** $z \div 12 = 7$
- **g** $135 \div p = 27$
- **h** $77 \div q = 7$
- **i** $\frac{y}{6} = 3$
- **j** $\frac{x}{7} = 2$
- **k** $\frac{m}{10} = 2$
- **l** $\frac{n}{8} = 7$
- **m** $\frac{p}{5} = 4$
- **n** $\frac{q}{9} = 3$
- **o** $\frac{r}{11} = 4$
- **p** $\frac{t}{12} = 6$

7 Write an equation for each of the following statements, then solve it:

- **a** I think of a number x then add twelve and the result is twenty.
- **b** I think of a number y then add sixteen and the result is thirty.
- **c** I think of a number a then subtract eight and the result is six.
- **d** I think of a number b then subtract eighteen and the result is fifteen.

8 Write an equation to represent the information, then determine the answer to each of the following:

- **a** Six more than a number is eight. What is the number?
- **b** Eleven less than a number is twenty. What is the number?
- **c** One-half of a number is fifty. What is the number?
- **d** Eight more than four times a number is twenty. What is the number?
- **e** Five less than three times a number is sixteen. What is the number?
- **f** The square root of a number is five. What is the number?

9 For each equation, determine by substitution whether the given solution in brackets is correct:

- **a** $x + 15 = 38 \quad [x = 22]$
- **b** $y - 12 = 7 \quad [y = 9]$
- **c** $5m - 15 = 10 \quad [m = 4]$
- **d** $2n + 3 = 15 \quad [n = 6]$
- **e** $\frac{b}{6} = 2 \quad [b = 12]$
- **f** $\frac{50}{p} = 10 \quad [p = 10]$
- **g** $\frac{q}{5} + 8 = 10 \quad [q = 10]$
- **h** $\frac{100}{b} - 1 = 4 \quad [b = 25]$
- **i** $\frac{3x + 1}{4} = 7 \quad [x = 9]$
- **j** $\frac{4x - 2}{3} = 7 \quad [x = 4]$
- **k** $\frac{m}{3} + \frac{1}{3} = 4 \quad [m = 11]$
- **l** $\frac{n}{8} - \frac{1}{8} = 1 \quad [n = 9]$

8B Solving equations with flow charts

This method displays the steps used to compile an equation. It is used where the unknown pronumeral appears *only once* in the equation. The solution is obtained by reversing the steps.

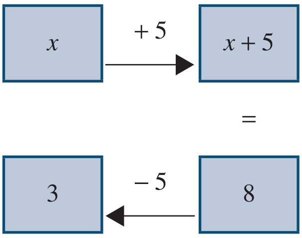
Example

Solve the following:

a $x + 5 = 8$

Solution

Display the flow chart:

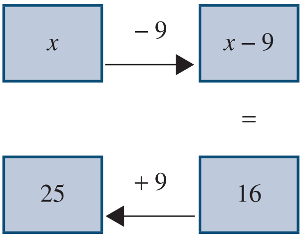


Reverse the flow and use 'inverse' or opposite operations

The solution is then $x = 3$, which is easily checked by inspection.

b $x - 9 = 16$

Display the flow chart:



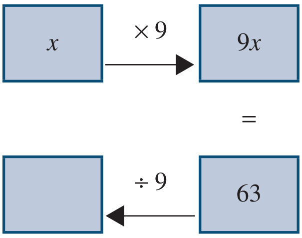
Reverse the flow and use 'inverse' or opposite operations

The solution is then $x = 25$, which is easily checked by inspection.

Exercise 8B

For questions 1–3, complete the details to solve the equations.

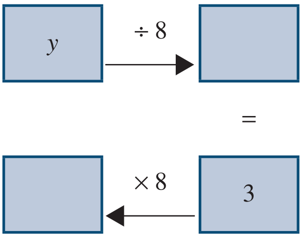
1 Solve $9x = 63$. Complete the flow chart:



Reverse the flow and use 'inverse' or opposite operations.

The solution is then $x =$ _____

2 Solve $\frac{y}{8} = 3$. Complete the flow chart:



The solution is then $y =$ _____

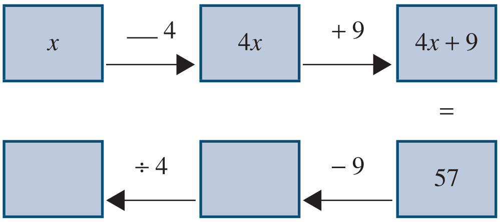
3 Solve each of the following equations by first drawing a flow chart and then reversing the flow:

a $p + 8 = 19$
b $10 + a = 26$ (Hint: Rewrite as $a + 10 = 26$.)
c $x + 10 = 17$
d $y + 15 = 31$
e $r - 9 = 13$
f $a + 8 = 20$
g $b - 14 = 10$
h $m - 6 = 9$

4 Solve each of the following equations by drawing a flow chart and then reversing the flow:

a $x \times 6 = 18$
b $y \times 8 = 40$
c $m \times 12 = 108$
d $9a = 54$
e $7b = 35$
f $8c = 32$
g $\frac{p}{7} = 4$
h $\frac{q}{9} = 3$
i $\frac{r}{12} = 5$
j $\frac{m}{9} = 4$
k $\frac{n}{3} = 13$
l $\frac{p}{8} = 12$
m $\frac{c}{4} = 3$
n $\frac{a}{5} = 11$
o $\frac{p}{7} = 13$

5 Solve $4x + 9 = 57$. Complete the flow chart:



The solution is then $x =$ _____

6 Solve each of the following equations, which have several steps, by drawing a flow chart and reversing the flow:

a $2x + 9 = 15$
b $4y + 6 = 30$
c $7a + 4 = 67$
d $3m - 5 = 13$
e $8n - 10 = 78$
f $9p - 8 = 28$
g $2x + 4 = 10$
h $2y + 6 = 0$
i $6a + 5 = 17$
j $5m + 1 = 11$
k $2n - 10 = 12$
l $9p - 2 = 34$
m $2x + 5 = 7$
n $4y + 6 = 46$
o $7a + 4 = 39$

7 Write equations for the following statements and then solve them by using a flow chart:

a I think of a number x, multiply it by 5 and the result is 20.
b I think of a number y, divide it by 8 and the result is 6.
c I think of a number n, multiply it by 8, then add 5 to get a result of 37.
d I think of a number q, multiply it by 9 then subtract 8 and get a result of 55.
e I think of a number m, multiply it by 6, then subtract 2 to get a result of 40.
f I think of a number s, multiply it by 9 then subtract 1 and get a result of 80.
g I think of a number x, divide it by 5, then subtract 3 to get a result of 2.
h I think of a number x, multiply it by 3, then divide by 4 and finally add 5 to get a result of 11.

8C Solving equations using inverse operation

The inverse operations used in solving an equation when using a flow chart can also be applied directly to solve it using algebra. Remember to keep the 'balance' of the equation to treat both sides of the equals sign the same way.

Example	Solution
1 I think of a number x and add 5 to get a result of 11. The equation is $x + 5 = 11$.	We subtract 5 from *both sides* to keep the 'balance': $x + 5 = 11$ $x + 5 - 5 = 11 - 5$ $x = 6$ Solution is $x = 6$.
2 I think of a number n and subtract 7 to get a result of 13. The equation is $n - 7 = 13$.	We add 7 to both sides: $n - 7 = 13$ $n - 7 + 7 = 13 + 7$ $n = 20$ Solution is $n = 20$.
3 A certain number y is multiplied by 8 to get a result of 32. The equation is $8y = 32$.	We divide both sides by 8: $8y = 32$ $\frac{8y}{8} = \frac{32}{8}$ $y = 4$ Solution is $y = 4$.
4 A certain number q divided by 12 gives a result of 8. The equation is $\frac{q}{12} = 8$.	We multiply both sides by 12: $\frac{q}{12} \times 12 = 8 \times 12$ Solution is $q = 96$.

Exercise 8C

1 Solve the following equations algebraically, showing the inverse operation steps presented in the worked examples:

a $x + 4 = 16$
b $13 + y = 24$ (Hint: Rewrite as $y + 13 = 24$.)
c $m - 7 = 15$
d $n - 19 = 6$
e $18 = p - 5$
f $p - \frac{1}{2} = 3\frac{1}{4}$
g $q + 2{\cdot}5 = 4{\cdot}75$
h $t - 3{\cdot}25 = 1{\cdot}75$
i $m + \frac{3}{4} = 1\frac{1}{2}$
j $q - 2\frac{1}{2} = 1\frac{3}{4}$
k $x - 1\frac{2}{3} = 3\frac{1}{3}$
l $p + 3{\cdot}5 = 8{\cdot}75$
m $s + 2{\cdot}25 = 3{\cdot}75$
n $t + 3{\cdot}5 = 6{\cdot}75$

2 Solve the following equations algebraically, showing the inverse operation steps presented in the worked examples. Note that part g onwards will involve answers which are simple fractions or decimals:

a $4x = 12$
b $6y = 72$
c $9n = 45$
d $3g = 33$
e $11p = 44$
f $5m = 60$
g $2r = 5$
h $3s = 7$
i $7t = 11$
j $4z = 10$
k $8q = 44$
l $6h = 22$
m $10x = 5$
n $12y = 8$
o $24z = 18$
p $6m = 15$
q $2n = 1{\cdot}8$
r $3t = 2{\cdot}7$
s $5a = 2{\cdot}5$
t $0{\cdot}5x = 1{\cdot}5$

3 Solve the following equations algebraically, showing the inverse operation steps presented in the worked examples:

a $\frac{x}{9} = 8$ **b** $\frac{a}{13} = 3$ **c** $\frac{b}{7} = 12$

d $\frac{p}{6} = 16$ **e** $\frac{q}{15} = 7$ **f** $\frac{r}{18} = 3$

g $\frac{m}{4} = 2\frac{1}{2}$ **h** $\frac{n}{3} = 1{\cdot}5$ **i** $\frac{p}{4} = 2\frac{3}{4}$

4 Six soft-serve ice creams at Sarah's Ice Cream cost me $36.

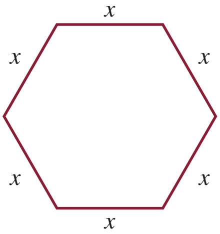
a Let p be the cost of one ice cream, and then write an equation and solve it for p.

b Let c be the change obtained from $50, and then write an equation and solve it for c.

5 When the price of a movie ticket P is divided by 11, the amount of Goods and Services Tax (GST) is calculated as $1·05. Write an equation for P (in cents), and then solve it to find the full price of a movie ticket.

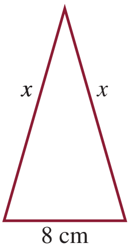
6 Using the perimeters of the following figures, write equations and solve to find x.

a



Perimeter is 216 cm

b



Perimeter is 38 cm

8D Solving two- and three-step equations

Equations with more than one operation can be solved by using the same method of inverse (opposite) operations. Undo each step in the order in which you would build up the equation following the method of Exercise 8B Question 5.

Example	Solution
1 Solve the equation $3x + 2 = 14$.	$3x + 2 = 14$ Subtract 2 from both sides: $3x + 2 - 2 = 14 - 2$ $3x = 12$ Divide both sides by 3: $\frac{3x}{3} = \frac{12}{3}$ $x = 4$
2 I think of a number y, multiply it by 3, then divide the result by 4 to get an answer of 6. The equation is $\frac{3y}{4} = 6$ or alternatively $\frac{3}{4}y = 6$.	$\frac{3y}{4} = 6$ Multiply both sides by 4: $\frac{3y}{4} \times 4 = 6 \times 4$ $3y = 24$ Divide both sides by 3: $\frac{3y}{3} = \frac{24}{3}$ $y = 8$
3 I think of a number x, multiply it by 5, divide the result by 4 and finally add 6 to get an answer of 16. The equation is $\frac{5x}{4} + 6 = 16$ or alternatively $\frac{5}{4}x + 6 = 16$.	$\frac{5x}{4} + 6 = 16$ Subtract 6 from both sides: $\frac{5x}{4} + 6 - 6 = 16 - 6$ $\frac{5x}{4} = 10$ Multiply both sides by 4: $\frac{5x}{4} \times 4 = 10 \times 4$ $5x = 40$ Divide both sides by 5: $\frac{5x}{5} = \frac{40}{5}$ $x = 8$

Exercise 8D

1 Solve the equations below:

a $3x + 4 = 31$
b $8y - 9 = 31$
c $9m - 3 = 60$
d $7 + 3n = 19$
e $10 + 5q = 35$
f $15 + 12a = 39$
g $18 + 6b = 36$
h $45 + 5c = 80$
i $5x - 7 = 33$
j $2r + 3 = 4$
k $6s + 5 = 14$
l $7a - 6 = 14$

2 Solve these two-step equations:

a $\frac{5x}{4} = 10$ **b** $\frac{3y}{4} = 9$ **c** $\frac{2z}{3} = 4$

d $\frac{3}{5}a = 6$ **e** $\frac{2}{9}b = 8$ **f** $\frac{5}{2}c = 10$

g $\frac{2}{3}m = 5$ **h** $\frac{4}{5}n = 2$ **i** $\frac{3p}{2} = 4$

3 Solve the following equations:

a $\frac{x}{6} + 5 = 7$ **b** $\frac{y}{7} + 9 = 12$ **c** $\frac{z}{2} - 6 = 8$

d $\frac{1}{8}m + 7 = 9$ **e** $\frac{1}{4}n - 2 = 4$ **f** $\frac{1}{12}p - 1 = 5$

g $\frac{q}{2} + \frac{3}{4} = 2$ **h** $\frac{1}{4}r - \frac{1}{2} = 1$ **i** $\frac{1}{3}s + 2 = 3\frac{2}{3}$

4 Solve these equations:

a $\frac{2x}{3} + 1 = 5$ **b** $\frac{3a}{4} + 2 = 8$ **c** $\frac{4n}{3} + 3 = 15$

d $\frac{5n}{3} - 1 = 9$ **e** $\frac{4p}{5} + 1 = 9$ **f** $\frac{2q}{7} - 3 = 1$

g $\frac{5}{6}m - 7 = 3$ **h** $\frac{2}{5}z + 4 = 10$ **i** $\frac{3}{8}t - 4 = 2$

5 Write an equation for each of the following statements and then solve it:

a I think of a number x, multiply it by 5, then subtract 15 to get a result of 50.

b Four times a certain number x divided by 5 gives a result of 8.

c A certain number y is divided by 5, then 7 is added to give a result of 12.

d Three times a certain number y is divided by 9, then 3 is subtracted to give a result of 2.

6 LC Dragons, that runs fortnightly trips to Kirakia, sells tickets for d dollars each. On a particular trip, the company decided to give a group of 10 learners a discount of \$70 each. If the total amount paid was \$1900.00, write an equation involving d and solve it to find the normal price d for a ticket sold by LC Dragons.

7 At Nicky Fast Food near Town Ground, I bought a drink, which cost \$8 dollars, and four doughnuts for a total of \$12. If p is the price of a doughnut, write an equation involving p and solve it to find the cost of a single doughnut.

8 For a group of learners to stay at the Pacific Casino, a special deal is negotiated so that the group pays only three-quarters of the usual price \$$P$. If the group pays \$126, write an equation involving P and solve it to find the usual price.

8E Inequations

An **inequation** occurs where two algebraic expressions are related by one of the following four **inequality signs**:

$<$ meaning 'less than' $\leq$ meaning 'less than or equal to'
$>$ meaning 'greater than' $\geq$ meaning 'greater than or equal to'

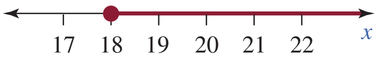
Some simple inequation statements are $3 < 7$ and $8 > 6$. Note that the inequality signs always point to the smaller of the two quantities, or equivalently the wider end opens up beside the larger of the two quantities.

Example

Solution

1 If x is the age at which a person can obtain a driving licence, write an inequation for x and illustrate the solution on a number line.

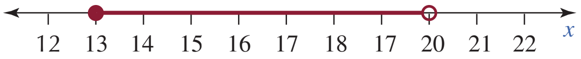
$x \geq 18$



Note that a 'filled in' or 'closed' end circle indicates that the end number is included.

2 If x is the age at which you are called a teenager, write an inequation for x and show the solution on a number line.

x has to be 13 or more but less than 20. Reading from the centre to the left and then to the right, we have $13 \leq x < 20$



Note that an 'empty' or 'open' end circle indicates that the end number is not included.

3 A number y is multiplied by 3, then 2 is subtracted to give a result which is 10 or more.

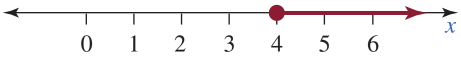
Write the inequation and solve it, showing the solution on a number line.

We apply the previously covered algebraic steps:

Start with $3y - 2 \geq 10$

$$3y - 2 + 2 \geq 10 + 2$$
$$3y \geq 12$$
$$\frac{3y}{3} \geq \frac{12}{3}$$
$$y \geq 4$$

That is, any number which is 4 or more satisfies the inequation.



Exercise 8E

1 Use the symbols $<$, $>$ or $=$ to correctly complete the following statements:

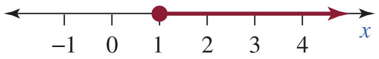
a $15 + 12$ ____ 2×5
b 3×6 ____ $40 \div 2$
c $4 + 8$ ____ 6×2
d $16 - 5$ ____ $24 \div 3$
e $4 \times 0{\cdot}5$ ____ $20 \div 10$
f $36 + 27$ ____ 6×7
g 9×12 ____ $216 \div 2$
h $4^2 + 1$ ____ $20 - 2$
i $5^2 + 12^2$ ____ 13^2
j $(2 + 3)^2$ ____ $2^2 + 3^2$

2 Draw a number line to represent the set of possible values for x:

a $x > 3$ **b** $x < -1$ **c** $3 < x \leq 6$ **d** $-1 \leq x < 5$

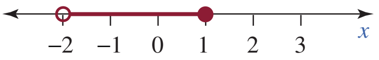
3 Write down the correct inequality for x which is represented by the following number lines:

a



b

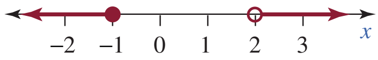
c



d

e

f



4 Write each of the following statements as an inequation, and show the solution on a number line:

- **a** x is less than 10
- **b** y is more than 8
- **c** z is 5 or less
- **d** m is 7 or more
- **e** x is greater than 3 but less than 6
- **f** x is between 4 and 8
- **g** x is 7 or more but less than 10
- **h** y is greater than 5 but less than or equal to 11
- **i** The price of petrol p varies from 128 cents per litre to 148 cents per litre inclusive.
- **j** In the Solomon Islands, the age a at which you can get a driving license is 18 years or more.
- **k** The age a of the people attending the Water Front Night Club on a particular Saturday night was greater than 13 but less than or equal to 20.
- **l** The time taken to get to school t varies from 20 to 30 minutes inclusive. (Inclusive means including the end values.)
- **m** The speed s at which I can ride my bike varies from zero to 35 kilometres per hour inclusive.

n The speed s at which I can drive my car in Point Cruz, Honiara, is any speed greater than zero but not more than 50 kilometres per hour.

o The discount d I receive when I pay cash can be anything from 5 to 10 per cent inclusive.

5 Solve the following inequations:

a $x + 5 < 9$ **b** $y - 8 > 10$ **c** $z - 15 \geq 16$ **d** $m + 7 \leq 12$

e $n + 19 < 21$ **f** $p - 17 \leq 3$ **g** $4x \geq 20$ **h** $12y > 72$

i $6z < 33$ **j** $9m \leq 36$ **k** $5p < 45$ **l** $10q > 55$

m $\frac{x}{7} \geq 5$ **n** $\frac{y}{13} \leq 4$ **o** $\frac{z}{15} > 4$ **p** $\frac{1}{4}m < 6$

q $\frac{1}{2}n \geq 9$ **r** $\frac{1}{3}p < 8$ **s** $\frac{2x}{3} \leq 12$ **t** $\frac{3y}{5} > 10$

u $\frac{4z}{3} < 8$ **v** $\frac{2}{5}a \leq 4$ **w** $\frac{7}{6}b < 14$ **x** $\frac{3}{4}c \geq 6$

6 Solve the following inequations by using the setting out of earlier sections:

a $4x + 2 \geq 8$ **b** $5 + 2y < 13$ **c** $6z - 2 \leq 22$

d $2a + 5 < 12$ **e** $10b + 9 \geq 24$ **f** $5c - 8 < 15$

g $\frac{m}{7} + 5 \geq 8$ **h** $\frac{n}{6} - 9 < 2$ **i** $\frac{p}{15} - 3 \geq 2$

j $\frac{1}{4}x + 2 < 3$ **k** $\frac{1}{8}y - 9 \geq 1$ **l** $\frac{1}{10}z - 5 \leq 4$

m $\frac{2x}{3} + 4 \leq 14$ **n** $\frac{3y}{5} - 2 > 7$ **o** $\frac{4z}{3} - 7 < 5$

p $\frac{2}{5}a + 3 \leq 5$ **q** $\frac{7}{6}b - 2 < 12$ **r** $\frac{3}{4}c - 9 \geq 3$

7 Write an inequation for each of the following statements and then solve it. Illustrate your solution on a number line:

- **a** I think of a number x, multiply it by 5 and then subtract 12 to get a result greater than 18.
- **b** Four times a certain number x divided by 5 gives a result less than 16.
- **c** A certain number y is divided by 4, then 7 is added to give a result of 9 or more.
- **d** Three times a certain number y is divided by 4, and 1 is subtracted to give a result greater than 8.

8 When I withdraw \$100 from my bank account, the balance is still in excess of \$175. Letting b be the original balance, write an inequation for b and solve it.

9 I have d dollars in my pocket and my friend has 6 dollars more than me. Together we have less than 16 dollars. Write an inequation for d and solve it.

10 The time t minutes taken for me to get to school is usually greater than 15 minutes, but is no more than 25 minutes. Write an inequation for t and show its solution on a number line.

11 During the Pacific Arts Festival that was held in Honiara in 2012, volunteers had to be greater than 15 years of age but not more than 60 years of age to work in the festival. If a was a person's age in years, write an inequality for a. Display the solution on a number line.

Puzzles

1 Determine the value of each symbol, then place the corresponding letters in the answers below to solve the riddle:

Why do seagulls fly over the sea?

$A - 20 = 52$
$B \times 3 = 24$
$C \div 3 = 1$
$D \times 4 = 44$
$E \div 6 = 3$
$F + 4 = 19$
$G - 7 = 43$
$H \div 4 = 11$
$I \div 11 = 2$
$J \div 3 = 10$
$K \times 5 = 45$
$L + 8 = 31$
$M + 6 = 22$
$N + 4 = 25$
$O + 1 = 14$
$P \times 2 = 70$
$Q \times 3 = 51$
$R + 9 = 11$
$S - 25 = 3$
$T \times 2 = 64$
$U - 7 = 20$
$V \div 10 = 2$
$W \times 1 = 5$
$Y - 48 = 4$

22	15		32	44	18	52		15	23	18	5	
13	20	18	2		32	44	18		8	72	, 52	
32	44	18	52		5	13	27	23	11		8	18
8	72	50	18	23	28							

2 Solve the equations, then match the letter to the answer below to find the answer to the riddle:

What did the alien say to the garden?

A $\frac{a}{3}+1=5$

D $3d+1=4$

E $3e-10=11$

K $\frac{k}{2}-1=4$

M $4m-10=6$

O $4o-5=15$

R $\frac{1}{2}r+1=5$

T $3t-24=3$

U $3u+1=7$

W $\frac{1}{2}w+1=4$

Y $\frac{1}{3}y+1=2$

Z $3z-3=13$

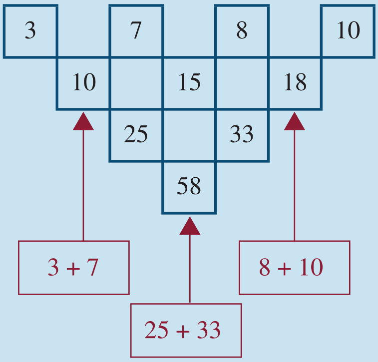
___ ___ ___ ___   ___ ___   ___ ___

9 12 10 7   4 7   9 5

___ ___ ___ ___   ___ ___ ___ ___ ___ ___

3 5 2 8   6 7 7 1 7 8

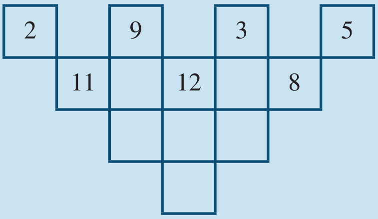
3 Number pyramids

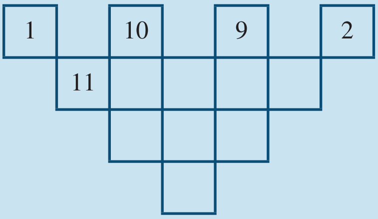


Number pyramids are problems which look like upside-down pyramids and involve finding the sum (adding) of two numbers.

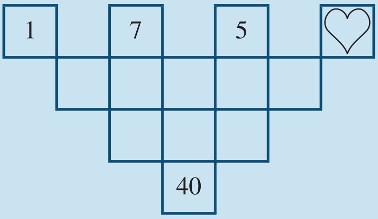
They are completed by adding two numbers, which are next to each other, then placing this total into the box below. This continues down until the last box is completed.

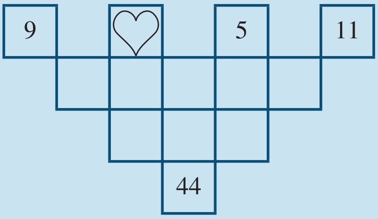
a Copy and complete the following number pyramids:

i


ii


b Determine the value of the missing symbol in each of the following number pyramids:

i


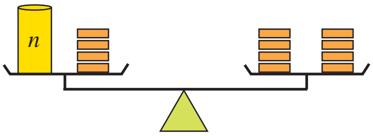
ii


Applications

In this section we will solve equations using ‘balance diagrams’. Balances of this type have been used in banks to manually count large numbers of coins.

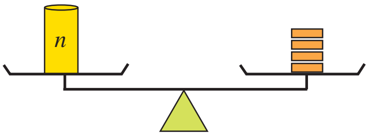
Example

1 The equation $n + 4 = 8$ can be represented by a lightweight container of n dollar coins plus four separate coins on the left, balanced by eight coins on the right. Solve for n.

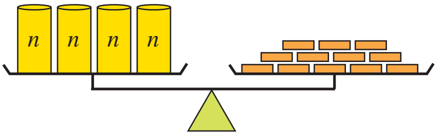


Solution

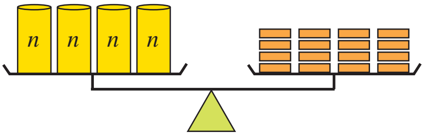
Subtracting 4 (coins) from both sides, just as you would with the equation, gives the solution $n = 4$.



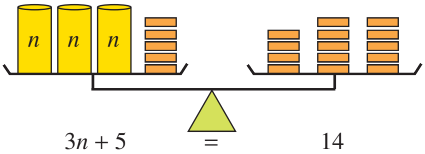
2 The equation $4n = 12$ can be represented with four lightweight containers of n dollar coins on the left, balanced by 12 coins on the right. Solve for n.



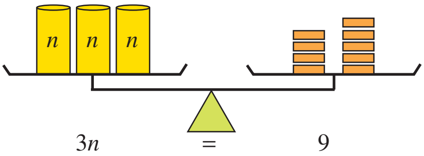
Dividing both sides into groups of 4, just as you would divide both sides of an equation by 4, gives the solution $n = 3$.



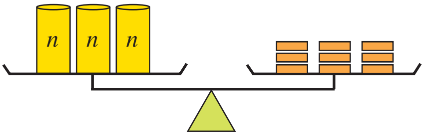
3 The equation $3n + 5 = 14$ can be repesented by the following balance diagram. Solve for n.



Subtracting 5 from both sides:



and then dividing each side into groups of 3:



gives the solution $n = 3$.

1 Solve the following equations by drawing balance diagrams:

a $n + 7 = 13$

b $n + 8 = 12$

c $6 + x = 15$

d $4y = 16$

e $5x = 35$

f $8n = 24$

2 Solve the following equations by drawing balance diagrams:

a $3x + 4 = 22$

b $2n + 3 = 17$

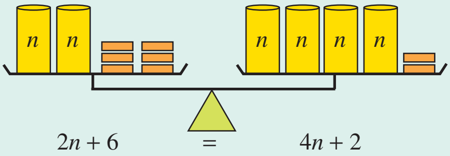
c $6n + 4 = 16$

d $5 + 2n = 11$

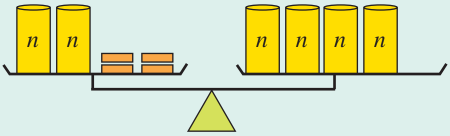
e $9 + 4x = 13$

f $10 + 5n = 20$

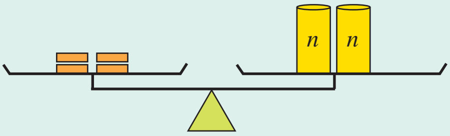
3 Write equations to represent the following sequence of balance diagrams, then solve the equation $2n + 6 = 4n + 2$.



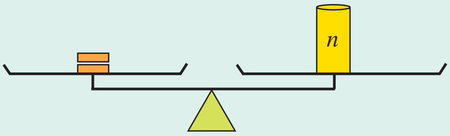
Take 2 from both sides:



Take $2n$ from both sides:



Divide both sides into groups of 2:



4 Solve the following equations using balance diagrams:

a $n + 6 = 3n + 2$

b $2n + 1 = n + 5$

c $2n + 6 = 3n + 2$

d $4n + 6 = 2n + 12$

e $4n + 8 = 6n$

f $6n = 5n + 6$

5 Write an equation for each of the following statements and solve it:

a Ten more than a certain number is equal to 3 times that number.

b If I add 8 to a certain number the value is equal to 5 times that number.

c I think of a number n, multiply it by 7 and then add 5. The answer is equivalent to 8 times the number I thought of plus 2.

d Six times a certain number x plus 5 gives the same answer as 3 times the same number plus 17.

e Three times a certain number x plus 6 equals the same number plus 10.

f Five times a certain number x plus 9 is equal to 3 times the same number plus 17.

Enrichment

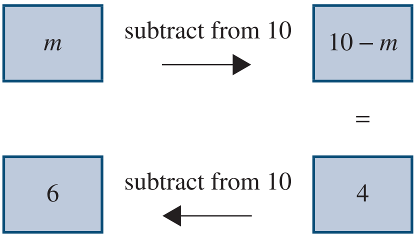
In this section we will cover extended applications of flow charts to solve more complicated linear equations. The only condition is that the unknown pronumeral (symbol) must appear only once.

Example

Solution

1 Solve $10 - m = 4$.

Display the flow chart:

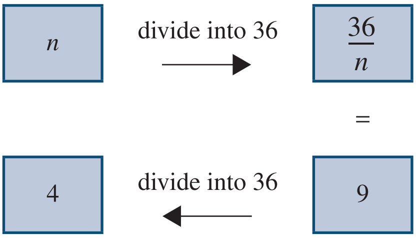


This is a *special case* in which the pronumeral is subtracted from a number; the *inverse operation* when flowing back is the *same operation*.

The solution is $x = 6$.

2 Solve $\frac{36}{n} = 9$.

Display the flow chart:



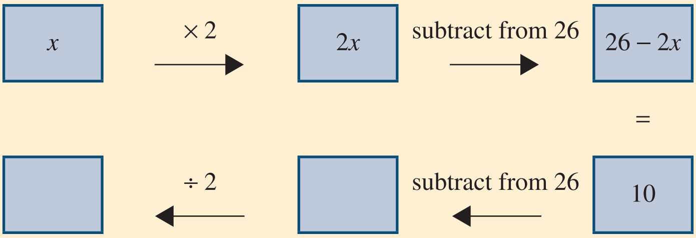
This is the *other* special case in which the pronumeral is divided into a number; the *inverse operation* when flowing back is the *same operation*.

The solution is $x = 4$.

Extending the basic ideas

1 I think of a number x, multiply it by two and subtract the result from twenty-six to get an answer of ten. Write an equation that models this process and solve it to find x, by completing the following flow chart.

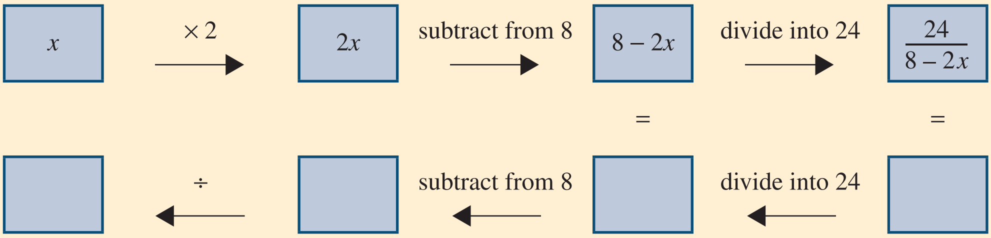
The equation is $26 - 2x =$ ______________.



The solution is $x =$ ______________.

2 A certain number x is multiplied by two and the answer is subtracted from eight to give a result that when divided into twenty-four gives a final answer of twelve. Write an equation that models this process and solve it to find x, by completing the following flow chart.

The equation is $\frac{24}{8-2x}$ = ______________.



Solution is x = _____. Verify by substituting: $8 - 2x$ _____ gives 2, divided into 24 gives 12.

3 Use a flow diagram to solve the following equations and then verify your solutions by substitution:

a $34 - a = 20$ **b** $23 - b = 10$ **c** $16 - m = 9$

d $10 - 3x = 4$ **e** $45 - 2y = 9$ **f** $52 - 8m = 12$

g $45 - 4n = 35$ **h** $29 - 6p = 15$ **i** $125 - 12q = 74$

4 Use a flow diagram to solve the following equations and then verify your solutions by substitution:

a $\frac{36}{x} = 9$ **b** $\frac{81}{y} = 27$ **c** $\frac{64}{z} = 4$

d $\frac{16}{20 - 3x} = 2$ **e** $\frac{18}{12 - y} = 3$ **f** $\frac{44}{33 - 2z} = 2$

g $\frac{18}{12 - 3p} = 6$ **h** $\frac{27}{18 - 5q} = 9$ **i** $17 - \frac{50}{15 - 2m} = 12$

5 Write equations for the following statements and solve them to find the unknown pronumerals:

a A certain number x is multiplied by seven and the result is subtracted from fifty to give a result of eight.

b A certain number y is multiplied by four and the result is subtracted from eighteen to give a result of ten.

c A certain number p is multiplied by six and the result is subtracted from twenty-five to give a result of seven.

d A certain number y is multiplied by seven and the result subtracted from twenty. When this answer is divided into twenty-four, a final result of four is obtained.

e A certain number q is multiplied by three and the result is subtracted from nineteen. This answer is divided into fourteen to get a final answer of two.

f A certain number m is multiplied by four and the result is subtracted from eighteen. This answer is divided into twelve to give a final result of 1·5.

Revision/Assessment

Exercise 8A

1 Solve the following equations by inspection:

a $x + 6 = 13$ **b** $9 + y = 14$ **c** $z - 7 = 11$

d $m - 9 = 21$ **e** $16 - q = 10$ **f** $20 - p = 12$

g $7m = 56$ **h** $n \times 5 = 45$ **i** $\frac{p}{4} = 3$

j $\frac{q}{7} = 8$ **k** $\frac{36}{r} = 3$ **l** $\frac{28}{s} = 4$

2 Check by substitution whether the given solutions to the following equations are correct:

a $3x - 4 = 11 \quad [x = 5]$ **b** $\frac{y}{4} + 5 = 10 \quad [y = 20]$

Exercise 8B

3 Solve the following equations by using a flow chart:

a $m + 6 = 11$ **b** $p - 13 = 7$ **c** $8r = 48$

d $\frac{t}{7} = 3$ **e** $4q + 3 = 19$ **f** $6p - 7 = 41$

Exercise 8C

4 Solve the following equations, this time setting out the appropriate algebraic steps:

a $x + 12 = 21$ **b** $y - 17 = 18$ **c** $9z = 72$

d $8m = 44$ **e** $\frac{m}{5} = 4$ **f** $\frac{n}{8} = \frac{1}{2}$

Exercise 8D

5 Solve the following equations setting out the appropriate algebraic steps:

a $6x + 2 = 50$ **b** $20 + 3y = 32$ **c** $8z - 4 = 68$

d $\frac{5x}{2} = 10$ **e** $\frac{2y}{3} = 6$ **f** $\frac{3}{4}q = 6$

g $\frac{z}{3} + 5 = 7$ **h** $\frac{1}{5}m + 7 = 9$ **i** $\frac{1}{4}n - 1 = 3$

j $\frac{3p}{5} + 8 = 14$ **k** $\frac{4}{5}q + 5 = 13$ **l** $\frac{3}{7}r - 2 = 4$

6 Write an equation for each of the following statements, then solve it:

a I think of a number x, multiply it by 7, and add 4 to get a result of 60.

b Three times a certain number y divided by 4 gives a result of 6.

c A certain number y divided by 2 then added to 5 gives a result of 14.

d From three times a certain number n divided by 2, 3 is subtracted to give result of 12.

e Three times a certain number q divided by 4, minus 5 gives a result of 1.

7 At the cinema, a group of 10 learners was given a total discount of \$20 when they each bought an ice cream. If they paid a total of \$25, find the price of an individual ice cream before any discount was applied.

Exercise 8E

8 Use the symbols <, >, or = to correctly complete the following statements:

a $16 + 13$ _____ 4×6

b 4×5 _____ $80 \div 2$

c 12×6 _____ $144 \div 2$

d $5^2 - 4$ _____ 6×4

9 Write each of the following statements as an inequation and show the solution on a number line:

a x is less than 13

b z is greater than 8

c y is 10 or more but less than 15

d m is between 20 and 30

e x is greater than 5 but less than 8

f x is either less than 4 or greater than 10

10 Write each of the following statements as an inequation:

a The number of goals g by which I expect my football team to win next weekend is at least 5 but no more than 8.

b The mark m that I expect to get on my next Mathematics test is more than 60% but no more than 100%.

c Learners in Year 8 are at least 11 but no more than 13 years old.

d To get a discount ski ticket, a person must be less than 15 or greater than 70 years of age.

11 Solve the following inequations by using the appropriate setting out:

a $x + 9 < 12$

b $z - 13 \leq 8$

c $5z > 15$

d $7m > 42$

e $\frac{p}{5} \geq 4$

f $\frac{1}{4}q \leq 11$

g $\frac{3n}{4} \geq 6$

h $\frac{5}{3}t \leq 10$

i $\frac{2}{5}s \geq 4$

12 Solve the following inequations, showing appropriate setting out:

a $3x + 2 > 17$

b $5y - 7 \leq 8$

c $\frac{z}{5} - 4 \geq 0$

d $\frac{1}{4}m + 3 > 9$

e $\frac{2p}{5} + 9 < 13$

f $\frac{3}{4}q - 8 \leq 7$

13 Write an inequation for each of the following statements and then solve it. Illustrate your solution on a number line:

a I think of a number n, multiply it by 6, then add 2 to get a result greater than 50.

b Four times a certain number y divided by 12 gives a result greater than 3.

c A certain number q divided by 9, plus 8 gives a result of 10 or more.

d Twice a certain number p divided by 8, minus 1 gives a result of 1 or less.

14 When I add d dollars to my bank account, which had a balance of \$100, the new balance is between \$125 and \$148. Write an inequation for d and solve it. Show your solution on a number line.

CHAPTER 9

Indices

Indices

These dancers at the Pacific Arts Festival are from the islands of Rennell and Bellona. Unlike most other people in the Solomon Islands, the people of Rennell and Bellona have language and customs that originated in Polynesia rather than Melanesia. The Polynesian triangle stretches from the southern Solomon Islands to Hawaii in the northeast Pacific and to New Zealand in the south Pacific. Polynesians are great navigators. While it is only 233 km from Rennell to Honiara, it is 5857 km to Hawaii. Polynesians used the sun and stars to navigate. The sun is approximately 149 600 000 km from the earth and the nearest star that could be used for navigation is 4·38 light years away. Mathematicians prefer to use indices to write large numbers, so the distance of the earth to the sun is usually written as 1.496×10^8 km.

This chapter covers the following skills:

- Representing numbers and expressions in index form
- Working with the index laws as applied to numbers and algebraic expressions
- Working with, and graphing, simple exponential relations in modelling situations
- Exploring scientific notation
- Explorating negative powers

Specific Learning Outcome (SLO)

Learners should be able to:

8.9.1.1 Identify the terms 'base', 'power', 'index' and 'exponent':

Power/Index Number

$$\text{Base} \longrightarrow a^3$$

8.9.2.1 Write numbers in index notation.

8.9.2.2 Write index numbers in expanded form.

8.9.2.3 Expand and evaluate numbers given in an index form.

8.9.3.1 Identify some of the laws that are used to manage Indices:

1. Same base when multiplied, ADD powers
2. Same base when divided, SUBTRACT powers
3. Square root of a number is the same as a base with the power of $\frac{1}{2}$.

8.9.4.1 Simplify indices using the index laws:

- $a^n \times a^p = a^{n+p}$
- $a^n \div a^p = a^{n+p}$
- $b^1 = b$
- $b^0 = 1$

8.9.5.1 Expand brackets with indices and simplify using the index laws with brackets:

- $(b^n)^p = b^{n \times p}$
- $(a \times b)^n = a^n \times b^n$
- $\left(\frac{a}{b}\right)^p = \frac{a^p}{b^p}$

8.9.6.1 Define scientific notation.

8.9.6.2 Identify numbers written using scientific notation:

Scientific notation: Numbers that are written as a product of a decimal number between 1 and 10, and an integral power of 10.

8.9.7.1 Convert large ordinary numbers to scientific notation.

8.9.7.2 Convert small decimal numbers (less than 1) to scientific notation.

8.9.8.1 Use tables and graphs to explore the behaviour of exponential numbers.

9A Index numbers

3^4 is an example of an index number. 3 is called the **base** and 4 is called the **index**, the power, the exponent or the logarithm.

$3^4 = 3 \times 3 \times 3 \times 3$
$3^4 = 81$

3^4 is a number in index form.
$3 \times 3 \times 3 \times 3$ is the same number in expanded form.
81 is the value of the index.
Note that 3^1 means 3.

Example	Solution
1 Write $(-3)^4$ in expanded form.	$(-3)^4 = -3 \times -3 \times -3 \times -3$
2 Evaluate $(-3)^4$.	$(-3)^4 = -3 \times -3 \times -3 \times -3$ $= {}^+9 \times {}^+9$ $= 81$
3 Write -3^4 in expanded form.	$-3^4 = -(3 \times 3 \times 3 \times 3)$
4 Evaluate -3^4.	$-3^4 = -(3 \times 3 \times 3 \times 3)$ $= -81$
5 Write as an index number and evaluate: base 2, power 3.	$2^3 = 2 \times 2 \times 2 = 8$
6 Solve $7^x = 343$.	By trial and error: $7 \times 7 = 49$ $7 \times 7 \times 7 = 343$ so $7^3 = 343$ $x = 3$
7 Solve $x^5 = 32$.	By trial and error: $2 \times 2 \times 2 \times 2 \times 2 = 32$ so $2^5 = 32$ $x = 2$

Exercise 9A

1 Write the following in index form:

a 3×3
b $5 \times 5 \times 5 \times 5$
c 10
d $1 \times 1 \times 1 \times 1 \times 1 \times 1$
e 2×2
f $9 \times 9 \times 9 \times 9 \times 9$
g $7 \times 7 \times 7$
h $0 \times 0 \times 0 \times 0 \times 0 \times 0 \times 0 \times 0$
i 12×12
j $6 \times 6 \times 6 \times 6$
k $4 \times 4 \times 4 \times 4 \times 4$
l $8 \times 8 \times 8 \times 8 \times 8 \times 8 \times 8$
m $1 \times 1 \times 1 \times 1$
n 10×10
o $11 \times 11 \times 11$
p $3 \times 3 \times 3 \times 3 \times 3 \times 3$
q $-3 \times -3 \times -3$
r $-5 \times -5 \times -5 \times -5$
s $-2 \times -2 \times -2$
t $-4 \times -4 \times -4 \times -4$

2 Write the following in expanded form and then evaluate:

a 3^2 **b** 4^2 **c** 10^3 **d** 2^5
e 5^2 **f** 7^1 **g** 1^7 **h** 0^5
i 10^4 **j** 8^5 **k** 4^3 **l** 2^1
m 8^2 **n** 1^5 **o** 2^6 **p** 9^3

3 Write the following as index numbers and evaluate:

a base 3, power 4 **b** base 6, index 3 **c** base 7, exponent 2
d base 5, index 3 **e** base zero, power 4 **f** base 1, index 5
g base 2, index 3 **h** base 5, logarithm 4 **i** base 10, index 6
j base 3, exponent 6 **k** base 11, power 3 **l** base 4, index 2

4 Evaluate the following:

a $(-1)^1$ **b** $(-1)^2$ **c** $(-1)^3$ **d** $(-1)^4$
e $(-2)^1$ **f** $(-2)^2$ **g** $(-2)^3$ **h** $(-2)^4$
i $(-3)^3$ **j** $(-3)^4$ **k** $(-3)^5$ **l** $(-3)^6$

5 Evaluate the following:

a -4^2 **b** -5^3 **c** $-(-3)^2$ **d** $-(-5)^3$
e -2^5 **f** -3^6 **g** $-(-2)^7$ **h** $-(-10)^4$
i -3^3 **j** -5^2 **k** $-(-3)^5$ **l** $-(-10)^6$

6 **a** Evaluate $(-1)^2$, $(-1)^4$, $(-1)^6$, $(-1)^8$.
b What do you notice about the signs of the answers when the powers are even?

7 **a** Evaluate $(-1)^1$, $(-1)^3$, $(-1)^5$, $(-1)^7$.
b What do you notice about the signs of the answers when the powers are odd?

8 Complete the following to obtain each number in index form:

a $8 = 2^{\square}$ **b** $25 = 5^{\square}$ **c** $49 = 7^{\square}$ **d** $27 = 3^{\square}$
e $10 = 10^{\square}$ **f** $81 = 9^{\square}$ **g** $81 = 3^{\square}$ **h** $1000 = 10^{\square}$
i $10\,000 = 10^{\square}$ **j** $64 = 8^{\square}$ **k** $64 = 4^{\square}$ **l** $64 = 2^{\square}$
m $100 = 10^{\square}$ **n** $512 = 8^{\square}$ **o** $256 = 2^{\square}$ **p** $2401 = 7^{\square}$

9 The following are called indicial equations. Solve them for x by inspection or trial and error:

a $2^x = 16$ **b** $3^x = 27$ **c** $6^x = 216$ **d** $5^x = 125$
e $2^x = 1024$ **f** $6^x = 1296$ **g** $3^x = 177\,147$ **h** $5^x = 15\,625$
i $9^x = 81$ **j** $3^x = 243$ **k** $10^x = 100\,000$ **l** $5^x = 625$
m $4^x = 1024$ **n** $6^x = 1296$ **o** $7^x = 343$ **p** $11^x = 14\,641$

10 Solve the following equations by inspection or trial and error, for the base number x:

a $x^2 = 16$ **b** $x^2 = 25$ **c** $x^2 = 81$ **d** $x^2 = 144$
e $x^3 = 27$ **f** $x^3 = 64$ **g** $x^3 = 216$ **h** $x^3 = 125$
i $x^4 = 16$ **j** $x^4 = 81$ **k** $x^3 = 1000$ **l** $x^7 = 128$
m $x^2 = 169$ **n** $x^5 = 3125$ **o** $x^5 = 7776$ **p** $x^6 = 729$

9B Exploring index laws

When dealing with index numbers of the same base, simple rules exist for multiplication and division, and for raising index numbers to further powers.

Learning task 9B

1 Copy and complete this table of selected base two index numbers:

1	2—	256	2—
2	2^1	512	2—
4	2^2	1 024	2—
8	2^3	2 048	2—
16	2—	4 096	2—
32	2—	8 192	2—
64	2—	16 384	2—
128	2—	32 768	2—

First index law for multiplication

Find 32 × 64.

Convert to an index number: $32 \times 64 = 2^5 \times 2^6$
Write in expanded form: $= (2 \times 2 \times 2 \times 2 \times 2) \times (2 \times 2 \times 2 \times 2 \times 2 \times 2)$
$= 2^{5+6}$
Add the powers: $= 2^{11}$
Use the table above: $= 2048$

2 Copy and complete these calculations:

a 16 × 128 = 2— × 2— = 2— = _____
b 8 × 4096 = 2— × 2— = 2— = _____
c 128 × 128 = 2— × 2— = 2— = _____
d 16 × 64 × 32 = 2— × 2— × 2— = 2— = _____
e 32 × 32 × 32 = 2— × 2— × 2— = 2— = _____

Second index law for division

Find 2048 ÷ 32.

Convert to an index number: $2048 \div 32 = 2^{11} \div 2^5$

Write in expanded form:

$$= \frac{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times \cancel{2}^1 \times \cancel{2}^1 \times \cancel{2}^1 \times \cancel{2}^1 \times \cancel{2}^1}{\cancel{2}_1 \times \cancel{2}_1 \times \cancel{2}_1 \times \cancel{2}_1 \times \cancel{2}_1}$$

Five of the 2s cancel, leaving six 2s: $= 2^{11-5}$
Subtract the powers: $= 2^6$
Use the table above: $= 64$

3 Copy and complete these calculations:

a 128 ÷ 16 = 2— ÷ 2— = 2— = _____
b 4096 ÷ 512 = 2— ÷ 2— = 2— = _____
c 16 384 ÷ 256 = 2— ÷ 2— = 2— = _____
d 32 768 ÷ 128 = 2— ÷ 2— = 2— = _____
e 1024 ÷ 128 = 2— ÷ 2— = 2— = _____
f 8192 ÷ 2048 = 2— ÷ 2— = 2— = _____

The first two index laws may be used in combination problems.

4 Copy and complete the following:

a $\frac{4096\times 1024}{8192}=\frac{2\text{—}\times 2\text{—}}{2\text{—}}=\frac{2\text{—}}{2\text{—}}=2\text{—}\div 2\text{—}=2\text{—}=\_\_\_\_\_$

b $\frac{512\times 8192}{32\,768}=\frac{2\text{—}\times 2\text{—}}{2\text{—}}=\frac{2\text{—}}{2\text{—}}=2\text{—}\div 2\text{—}=2\text{—}=\_\_\_\_\_$

c $\frac{64\times 2048\times 256}{16\,384}=\frac{2\text{—}\times 2\text{—}\times 2\text{—}}{2\text{—}}=\frac{2\text{—}}{2\text{—}}=2\text{—}\div 2\text{—}=2\text{—}=\_\_\_\_\_$

d $\frac{1024\times 16\times 4096}{32\,768}=\frac{2\text{—}\times 2\text{—}\times 2\text{—}}{2\text{—}}=\frac{2\text{—}}{2\text{—}}=2\text{—}\div 2\text{—}=2\text{—}=\_\_\_\_\_$

e $\frac{2048\times 8192}{16\,384\times 512}=\frac{2\text{—}\times 2\text{—}}{2\text{—}}=\frac{2\text{—}}{2\text{—}}=2\text{—}\div 2\text{—}=2\text{—}=\_\_\_\_\_$

f $\frac{128\times 4096}{2048\times 256}=\frac{2\text{—}\times 2\text{—}}{2\text{—}}=\frac{2\text{—}}{2\text{—}}=2\text{—}\div 2\text{—}=2\text{—}=\_\_\_\_\_$

5 Powers of numbers can be managed using another index law. Find 32^3.

As index numbers this is $(2^5)^3 = 2^5\times 2^5\times 2^5 = 2^{5\times 3} = 2^{15} = 32\,768$.

Note then that when the index number 2^5 is raised to a further power we multiply the powers.

Complete the following:

a $4^5=(2\text{—})^5=2\text{—}^{\times 5}=2\text{—}=\_\_\_\_\_$

b $16^2=(2\text{—})^2=2\text{—}^{\times 2}=2\text{—}=\_\_\_\_\_$

c $64^2=(2\text{—})\text{—}=2\text{—}^{\times}\text{—}=2\text{—}=\_\_\_\_\_$

d $32^3=(2\text{—})\text{—}=2\text{—}^{\times}\text{—}=2\text{—}=\_\_\_\_\_$

e $128^2=(2\text{—})\text{—}=2\text{—}^{\times}\text{—}=2\text{—}=\_\_\_\_\_$

f $4^7=(2\text{—})\text{—}=2\text{—}^{\times}\text{—}=2\text{—}=\_\_\_\_\_$

6 Square roots of numbers can be found by raising to the power of one-half. Find $\sqrt{1024}$.

As index numbers this is $\sqrt{1024}=1024^{\frac{1}{2}}=(2^{10})^{\frac{1}{2}}=2^{10\times\frac{1}{2}}=2^5=32$.

Complete the following:

a $\sqrt{4}=4^{\frac{1}{2}}=(2^2)^{\frac{1}{2}}=2^{2\times\frac{1}{2}}=2^{\text{—}}=\_\_\_\_\_\_\_$

b $\sqrt{16}=16^{\frac{1}{2}}=(2^{\text{—}})^{\frac{1}{2}}=2^{\text{—}\times\frac{1}{2}}=2^{\text{—}}=\_\_\_\_\_\_\_$

c $\sqrt{64}=64^{\frac{1}{2}}=(2^{\text{—}})^{\frac{1}{2}}=2^{\text{—}\times\frac{1}{2}}=2^{\text{—}}=\_\_\_\_\_\_\_$

7 Complete the following summary of index laws used (to a base of 2):

a $2^6\times 2^8=2^{\text{—}}$ We retain the base and _______ the powers.

b $2^{12}\div 2^7=2^{\text{—}}$ We retain the base and _______ the powers.

c $(2^5)^3=2^{\text{—}}$ We retain the base and _______ the powers.

d $2^0=\_\_\_\_$ Two or in fact any number to the power of zero is equal to _______.

9C Index laws with algebraic terms

The first four index laws are shown below, where *b* represents the base of the index.

- $b^n \times b^m = b^{n+m}$ For like base numbers, retain the base and add the powers.
- $b^n \div b^m = b^{n-m}$ For like base numbers, retain the base and subtract the powers.
- $b^1 = b$ Any base to the power of one is just the base number.
- $b^0 = 1$ Any base to the power of zero equals 1.
- Ordinary numbers multiply and divide as usual.

Example	Solution
1 Simplify:	
a $3^3 \times 3^2$	$3^3 \times 3^2 = 3^{3+2}$ $= 3^5$
b $x^7 \times x^3$	$x^7 \times x^3 = x^{10}$
c $4x^2 \times 6x^5$	$4x^2 \times 6x^5 = 4 \times 6 \times x^2 \times x^5$ $= 24 \times x^7$ $= 24x^7$
d $5xy^4 \times 7x^3y^6$	$5xy^4 \times 7x^3y^6 = 5 \times 7 \times x^1 \times x^3 \times y^4 \times y^6$ $= 35 \times x^4 \times y^{10}$ $= 35x^4y^{10}$
2 Simplify $7^5 \div 7^2$ using index laws and then evaluate.	$7^5 \div 7^2 = 7^{5-2}$ retain base, add powers $= 7^3$ $= 343$ $7 \times 7 \times 7$
3 Simplify $x^{12} \div x^7$ using index laws.	$x^{12} \div x^7 = x^5$ retain base, subtract powers
4 Simplify $\dfrac{12x^4y^7}{3x^3y^5}$ using index laws.	Treating each group separately: $\dfrac{12x^4y^7}{3x^3y^5} = \dfrac{12}{3} \times \dfrac{x^4}{x^3} \times \dfrac{y^7}{y^5}$ $= 4xy^2$

Exercise 9C

1 Simplify the following by using index laws, and write the answer as a basic numeral:

a $2^3 \times 2^2$ **b** $4^2 \times 4^1$ **c** $3^2 \times 3^4$ **d** $5^2 \times 5$
e $10^2 \times 10^4$ **f** $7^2 \times 7^0$ **g** $7^2 \times 7^1$ **h** $7^2 \times 7$
i $2^2 \times 2^3 \times 2^4$ **j** $3^2 \times 3^1 \times 3$ **k** $3^2 \times 3^1 \times 3^0$ **l** $5^2 \times 5 \times 5^0$

2 Simplify the following to an answer in index form:

a $x^2 \times x^5$ **b** $y^3 \times y^0$ **c** $m^6 \times m^2$ **d** $x^1 \times x^5$
e $a \times a^5$ **f** $a^0 \times a^5$ **g** $a^1 \times a^5$ **h** $b^0 \times b$
i $x^2 \times x^3 \times x^8$ **j** $n \times n^3 \times n^4$ **k** $n^0 \times n^1 \times n$ **l** $p^5 \times p^2 \times p^6$
m $a^3 \times a^5 \times a$ **n** $x^2 \times x^5 \times x^3$ **o** $y \times y^1 \times y^3$ **p** $m \times m^2 \times m^3$
q $r \times r^2 \times r^5$ **r** $s^0 \times s^1 \times s^3$ **s** $t^1 \times t^4 \times t^3$ **t** $n \times n^2 \times n^5$

3 Simplify the following:

a $3x^4 \times 6x^2$ **b** $4x^2 \times 6x^5$ **c** $9x^4 \times 5x^3$

d $5x^2 \times 2x^5$ **e** $3y^2 \times 6y^1$ **f** $8z^0 \times 5z^5$

g $7a \times a^5$ **h** $b^3 \times 8b^4$ **i** $2x^3 \times 5x^2 \times x^8$

j $3b^0 \times b^2 \times 5b^1$ **k** $7y^2 \times 4y^2 \times 2y^2$ **l** $2x^0 \times x^1 \times x^2$

4 Simplify the following:

a $2x^2y^3 \times 4x^3y^2$ **b** $3x^1y^3 \times 5x^2y^4$

c $2x^2y^2 \times 6x^2y^2$ **d** $2a^3b^3 \times 6a^2b^2$

e $4m^1n^2 \times m^3n^5$ **f** $p^1q^3 \times 5p^2q^4$

g $4x^0y^3 \times 2x^2y^4 \times 5x^4y$ **h** $6x^2y^3 \times 4x^3y^2 \times 2x^2y^3$

i $x^2y^3 \times 4xy^0 \times 2x^1y^3$ **j** $x^0y^1 \times 3x^3y^1 \times 8xy$

5 Simplify the following by using the index laws, then evaluate:

a $3^4 \div 3^3$ **b** $5^7 \div 5^2$ **c** $7^4 \div 7^2$

d $9^2 \div 9$ **e** $10^6 \div 10^3$ **f** $4^2 \div 4^0$

g $9^2 \div 9^2$ **h** $10^3 \div 10^3$ **i** $8^7 \div 8^5$

6 Simplify the following:

a $x^9 \div x^7$ **b** $y^8 \div y^6$ **c** $z^8 \div z^2$

d $b^5 \div b^2$ **e** $x^1 \div x^0$ **f** $p \div p^0$

g $q^6 \div q^6$ **h** $b^6 \div b^3$ **i** $y^{10} \div y^2$

7 Simplify the following as far as possible:

a $\frac{12x^5y^8}{4x^3y^5}$ **b** $\frac{24x^7y^4}{8x^5y^3}$ **c** $\frac{8x^5y^9}{2x^3y^7}$ **d** $\frac{15x^8y^6}{3x^4y^2}$

e $\frac{14r^3s^3}{7rs^3}$ **f** $\frac{6mn}{3m^0n^1}$ **g** $\frac{9p^2q^1}{3^0pq^0}$ **h** $\frac{8a^3b^4}{8^0a^2b^4}$

8 Complete the following index problems:

a $\frac{12x^6y^7}{20x^3y^5} = \frac{12}{20} \times \frac{x^6}{x^3} \times \frac{y^7}{y^5} = \frac{3}{5} \times x^{\_\_} \times y^{\_\_} = \frac{3}{5}x^{\_\_}y^{\_\_} =$ or $\frac{3x^{\_\_}y^{\_\_}}{5}$

b $\frac{10x^4y^9}{15x^2y^7} = \frac{10}{15} \times \frac{x^4}{x^2} \times \frac{y^9}{y^7} = \frac{\_\_}{\_\_} \times x^{\_\_} \times y^{\_\_} = \frac{\_\_}{\_\_}x^{\_\_}y^{\_\_} =$ or $\frac{\_\_x^{\_\_}y^{\_\_}}{\_\_}$

c $\frac{8x^5y^7}{16x^3y^7} = \frac{8}{16} \times \frac{x^5}{x^3} \times \frac{y^7}{y^7} = \frac{\_\_}{\_\_} \times x^{\_\_} \times y^{\_\_} =$ _____ or _____

d $\frac{12x^6y^8}{20x^3y^7} = \frac{3}{5} \times \frac{x^6}{x^3} \times \frac{y^8}{y^7} = \frac{\_\_}{\_\_} \times x^{\_\_} \times y^{\_\_} =$ _____ or _____

9 Simplify the following expressions:

a $\frac{8a^6b^4}{20b^3}$ **b** $\frac{15m^5n^8}{18nm^2}$ **c** $\frac{12st^6r^2}{14rs}$ **d** $\frac{21a^2b^4c^6}{24c^3ab^2}$

9D Managing brackets: Other index laws

Three extra laws that involve brackets are as follows:

$(b^n)^m = b^{n\times m}$	When raising to a further power, multiply the powers.
$(a\times b)^m = a^m\times b^m$	Each term inside the brackets is raised to the power.
$\left(\frac{a}{b}\right)^p = \frac{a^p}{b^p}$	Each term inside the brackets is raised to the power.

Example	Solution
1 Expand the brackets and simplify:	
a $(x^3)^4$	$\begin{aligned}(x^3)^4 &= x^3\times x^3\times x^3\times x^3\\ &= x^{12}\end{aligned}$ or using the index law: $\begin{aligned}(x^3)^4 &= x^{3\times 4}\\ &= x^{12}\end{aligned}$
b $(xy)^6$	$\begin{aligned}(xy)^6 &= (x^1y^1)^6\\ &= x^6y^6\end{aligned}$
c $\left(\frac{x}{y}\right)^5$	$\begin{aligned}\left(\frac{x}{y}\right)^5 &= \left(\frac{x^1}{y^1}\right)^5\\ &= \frac{x^5}{y^5}\end{aligned}$
d $(x^2y)^4$	$\begin{aligned}(x^2y)^4 &= (x^2y^1)^4\\ &= x^{2\times 4}y^{1\times 4}\\ &= x^8y^4\end{aligned}$
2 Remove brackets from $\left(\frac{x^3}{y^2}\right)^4$.	$\begin{aligned}\left(\frac{x^3}{y^2}\right)^4 &= \frac{x^{3\times 4}}{y^{2\times 4}}\\ &= \frac{x^{12}}{y^8}\end{aligned}$
3 Remove brackets from $(2x^2y)^3\times 5xy^2$ and simplify.	$\begin{aligned}(2x^2y)^3\times 5xy^2 &= (2^1x^2y^1)^3\times 5x^1y^2\\ &= 2^{1\times 3}x^{2\times 3}y^{1\times 3}\times 5x^1y^2\\ &= 2^3x^6y^3\times 5x^1y^2\\ &= 8\times 5\times x^6\times x^1\times y^3\times y^2\\ &= 40x^7y^5\end{aligned}$
4 Simplify $\frac{(3x^2y)^3}{9y^2x^6}$.	$\begin{aligned}\frac{(3x^2y)^3}{9y^2x^6} &= \frac{3^3x^6y^3}{9y^2x^6}\\ &= \frac{27x^6y^3}{9x^6y^2}\\ &= 3x^0y\\ &= 3y\end{aligned}$

Exercise 9D

1 Expand the brackets and evaluate the following:

a $(2^3)^2$ **b** $(4^2)^1$ **c** $(5^2)^0$

d $(5^0)^4$ **e** $(10^3)^2$ **f** $(3^2)^2$

g $(64)^{\frac{1}{2}}$ **h** $(49)^{\frac{1}{2}}$ **i** $(81)^{\frac{1}{2}}$

2 Expand the brackets and simplify where possible:

a $(x^2)^3$ **b** $(y^3)^5$ **c** $(m^6)^2$ **d** $(x^4)^2$

e $(a^0)^3$ **f** $(a^8)^0$ **g** $(a^0)^0$ **h** $(x^y)^0$

3 Expand the brackets and simplify where possible:

a $(a \times b)^3$ **b** $(xz)^5$ **c** $(mn)^1$ **d** $(pq)^0$

e $(5b)^2$ **f** $(9x)^2$ **g** $(10m)^3$ **h** $(3y)^4$

i $(x^3y^2)^4$ **j** $(m^5n^3)^2$ **k** $(ab^5)^4$ **l** $(a^3b)^2$

m $(2y^2z^2)^4$ **n** $(3q^3r)^3$ **o** $(5s^2t^2)^0$ **p** $(8mn^2)^2$

4 Expand the brackets and evaluate:

a $\left(\frac{4}{5}\right)^2 = \frac{4^2}{5^2} = \frac{\square}{\square}$ **b** $\left(\frac{2}{3}\right)^3$ **c** $\left(\frac{1}{2}\right)^4$ **d** $\left(\frac{5}{6}\right)^2$

e $\left(\frac{1}{4}\right)^3$ **f** $\left(\frac{4}{7}\right)^2$ **g** $\left(\frac{1}{10}\right)^6$ **h** $\left(\frac{5}{8}\right)^0$

5 Remove brackets from the following expressions and simplify where possible:

a $\left(\frac{a}{b}\right)^4$ **b** $\left(\frac{x}{y}\right)^2$ **c** $\left(\frac{m}{n}\right)^6$ **d** $\left(\frac{p}{q}\right)^0$

e $\left(\frac{x^2}{y^3}\right)^2$ **f** $\left(\frac{m^3}{n^4}\right)^5$ **g** $\left(\frac{a^2}{b}\right)^6$ **h** $\left(\frac{a}{b^3}\right)^4$

i $\left(\frac{1}{z^2}\right)^3$ **j** $\left(\frac{2}{z^2}\right)^4$ **k** $\left(\frac{3x}{2y^2}\right)^2$ **l** $\left(\frac{4m^5}{3n^3}\right)^2$

6 Simplify the following as far as possible using index laws:

a $(3x^2y^3)^3 \times 2xy^2$ **b** $(2a^2b)^2 \times 4a^3b^2$ **c** $3m^4n^2 \times (2m^2n^4)^3$

d $(2p^2q)^2 \times (5p^3q^2)^2$ **e** $(3rs)^2 \times (2r^5s^3)^3$ **f** $(5st^5)^2 \times (s^5t)^3$

g $(5p^3q)^3 \times ({}^{-}2pq^2)^4$ **h** $-4(rs)^3 \times 3(rs^2)^2$ **i** $(-st)2 \times (2s^2t)^3$

7 Simplify the following as far as possible by using the index laws:

a $\frac{(3x^2y)^3}{3x^2y^2}$ **b** $\frac{(4m^2n)^2}{8m^3n}$ **c** $\frac{50p^4q^6}{(5p^2q^2)^2}$

d $\frac{72r^3s^4}{(3rs^2)^2}$ **e** $\frac{(4x^3y^2)^3}{(2x^2y^3)^2}$ **f** $\frac{(9y^3x^4)^2}{(3x^2y)^3}$

9E Exploring scientific notation

Learning task 9E

1 **a** Write as whole numbers:

i 4×10^{1} **ii** 5×10^{2} **iii** 7×10^{0}

iv $6{\cdot}3 \times 10^{3}$ **v** 124×10^{2} **vi** $0{\cdot}4 \times 10^{1}$

b Convert these numbers to whole numbers:

i There are 10^{2} cents in a dollar.

ii It is expected that there will be 577×10^{3} people living in the Solomon Islands in 2014.

iii Lake Tegano, on Rennell, is the largest lake (155×10^{2} hectares) in the South Pacific.

iv The average annual rainfall on Guadalcanal's southern coast is $1{\cdot}25 \times 10^{4}$ mm.

v Mt Veve on Kolombangara Island is $1{\cdot}77 \times 10^{3}$ m high.

vi USS Kanawa, sunk by Japanese bombs near Kokomtambu Island, was a $1{\cdot}45 \times 10^{4}$ armed tanker.

vii Santa Cruz Island has an area of 660×10^{0} sq km.

Some numbers are difficult to read and write if they represent very large or very small values. So they are often written using standard form or scientific notation. For example, instead of writing the distance of the earth from the sun as 149 600 000 km, in scientific notation this written as $1{\cdot}496 \times 10^{8}$ km.

Note that scientific notation is written in two parts:

- A number between 1 and 10
- Multiplied by a power of 10

2 **a** Copy and complete the table and convert the numbers to powers of 10:

Number	Power of 10	Number	Power of 10
1		0·1	
10	10^{1}	0·01	10^{-2}
100	10^{2}	0·001	10^{-3}
1000		0·0001	
10 000		0·000 01	
100 000		0·000 001	
1 000 000		0·000 000 1	

b Describe any patterns that you see in the table.

10 can be written as	$1{\cdot}\overset{\curvearrowright}{0} \times 10^{1}$	move decimal point 1 place to the right: 10
100 can be written as	$1{\cdot}\overset{\curvearrowright}{0}\overset{\curvearrowright}{0} \times 10^{2}$	move decimal point 2 places to the right: 100
1000 can be written as	$1{\cdot}\overset{\curvearrowright}{0}\overset{\curvearrowright}{0}\overset{\curvearrowright}{0} \times 10^{3}$	move decimal point 3 places to the right: 1000
0·1 can be written as	${\cdot}\overset{\curvearrowleft}{1}{\cdot}0 \times 10^{-1}$	move decimal point 1 place to the left: 0·1
0·01 can be written as	${\cdot}\overset{\curvearrowleft}{0}\overset{\curvearrowleft}{1}{\cdot}0 \times 10^{-2}$	move decimal point 2 places to the left: 0·01
0·001 can be written as	${\cdot}\overset{\curvearrowleft}{0}\overset{\curvearrowleft}{0}\overset{\curvearrowleft}{1}{\cdot}0 \times 10^{-3}$	move decimal point 3 places to the left: 0·001

c Complete these:

- **i** 10 000 can be written as $1\cdot0 \times$ _______
- **ii** 1 000 000 can be written as $1\cdot0 \times$ _______
- **iii** 0·000 001 can be written as $1\cdot0 \times$ _______

Example	Solution
1 Convert the following numbers to scientific notation:	
a 23 000	$2\cdot3\,000\cdot = 2\cdot3 \times 10^4$
b 4 500 000	$4\cdot500\,000\cdot = 4\cdot5 \times 10^6$
2 Write the following as numbers:	
a $5\cdot1 \times 10^4$	$5\cdot1000 \times 10^4 = 51\,000$
b $6\cdot8 \times 10^6$	$6\cdot800\,000 \times 10^6 = 6\,800\,000$

3 Convert the following numbers to scientific notation:

a 240 **b** 34 000 **c** 567 000 **d** 12

e 900 000 **f** 4 500 000 **g** 387 000 **h** 4500

4 Write the following as numbers:

a $2\cdot4 \times 10^3$ **b** $6\cdot4 \times 10^4$ **c** $2\cdot3 \times 10^6$ **d** $3\cdot5 \times 10^2$

e $5\cdot96 \times 10^1$ **f** $6\cdot12 \times 10^7$ **g** $1\cdot23 \times 10^5$ **h** $6\cdot78 \times 10^8$

Example	Solution
3 Convert the following numbers to scientific notation:	
a 0·04	$0\cdot04 = 4\cdot0 \times 10^{-2}$
b 0·0056	$0\cdot0056 = 5\cdot6 \times 10^{-3}$
4 Write the following as numbers:	
a $5\cdot4 \times 10^{-2}$	$05\cdot4 \times 10^{-2} = 0\cdot054$
b $2\cdot8 \times 10^{-4}$	$\cdot0002\cdot8 \times 10^{-4} = 0\cdot000\,28$

5 Convert the following numbers to scientific notation:

a 0·035 **b** 0·06 **c** 0·000 16 **d** 0·000 078

e 0·016 **f** 0·000 278 **g** 0·2 **h** 0·45

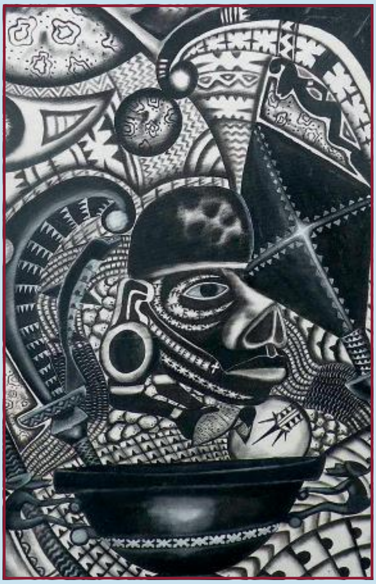
6 Write the following as numbers:

a $2\cdot5 \times 10^{-2}$ **b** $1\cdot4 \times 10^{-1}$ **c** $8\cdot3 \times 10^{-3}$ **d** $6\cdot5 \times 10^{-4}$

e $1\cdot5 \times 10^{-5}$ **f** $4\cdot1 \times 10^{-7}$ **g** $1\cdot2 \times 10^{-6}$ **h** $2\cdot8 \times 10^{-9}$

Puzzles

1 Simplify the following expressions. Match the letter to the correct index number below to solve the riddle:



What did one magnet say to the other magnet?

A $2 \times 2 \times 2 \times 2$

C 6×6

E $2 \times 2 \times 2 \times 2 \times 2$

I $3 \times 3 \times 3$

O 5×5

R $7 \times 7 \times 7 \times 7 \times 7 \times 7$

T 4

U $6 \times 6 \times 6 \times 6$

V $10 \times 10 \times 10 \times 10$

Y $5 \times 5 \times 5 \times 5$

Z $3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$

___ 5^4	___ 5^2	___ 6^4		___ 2^4	___ 7^6	___ 2^5		___ 10^4	___ 2^5	___ 7^6	___ 5^4
___ 2^4	___ 4^1	___ 4^1	___ 7^6	___ 2^4	___ 6^2	___ 4^1	___ 3^3	___ 10^4	___ 2^5		

2 Simplify the following expressions using the first index rule. Match the letter to the correct expression below to solve the riddle:

Which is lighter: the Sun, the Earth or the Moon?

A $x^4 \times x^3$

C $x^2 \times x^3$

D $2x \times 4x^2 \times x^3$

E $x \times x^2 \times x^3$

H $3x \times 4x$

I $2x^2 \times 3x$

N $2x^2 \times 4x^6$

R $x \times 5x \times x^4$

S $3x^4 \times 2x^4$

T $3x^4 \times 3x^2$

U $4x \times 2x^2 \times 3x^4$

Y $5x^4 \times 2x^2$

___ $9x^6$	___ $12x^2$	___ x^6		___ $6x^8$	___ $24x^7$	___ $8x^8$		___ $6x^3$	___ $9x^6$
___ $5x^6$	___ $6x^3$	___ $6x^8$	___ x^6	___ $6x^8$					
___ x^6	___ x^7	___ x^5	___ $12x^2$		___ $8x^6$	___ x^7	___ $10x^6$		

3 Simplify the expressions below. Match the expression to the correct letter to solve the riddle:

How do you see flying saucers?

A $\frac{8x^3y^3}{2xy^2}$ **B** $\frac{50a^4b^2}{25ab}$ **E** $\frac{6m^4n}{2m^4}$

I $\frac{15p^8q^5}{3p^8q^3}$ **M** $\frac{64x^2y}{16xy}$ **P** $\frac{14p^3q^3}{7pq^2}$

Q $\frac{75x^3y^5}{25xy}$ **R** $\frac{25a^7b}{5a^5b}$ **T** $\frac{18c^5d}{9c^4d}$

U $\frac{36a^4b^3}{6a^3b^3}$ **W** $\frac{45x^4y}{15x^2y}$ **Z** $\frac{32c^3d^2}{8c^2}$

$2c$ $5a^2$ $5q^2$ $2p^2q$ $6a$ $2p^2q$ $4x^2y$

$3x^2$ $4x^2y$ $5q^2$ $2c$ $3n$ $5a^2$

4 Use guess and check to complete this cross-number puzzle:

1		2		3		■	4
	■		■		■	5	
6		7			■	8	
	■		■	9	10		
11	12		■	13		■	
14		■	15			16	■
■	17			■	18		19
20		■	21				

Across

1. 845^2
5. $\sqrt{625}$
6. 209^2
8. $\sqrt{7396}$
9. $60^2 + 33^2$
11. 26^2
13. $\sqrt[3]{110\,592}$
14. $\sqrt{8464}$
15. $36^2 - 10^2$
17. $\sqrt{40\,401}$
18. 6^3
20. $\sqrt[3]{3375}$
21. $37^3 - 22^2$

Down

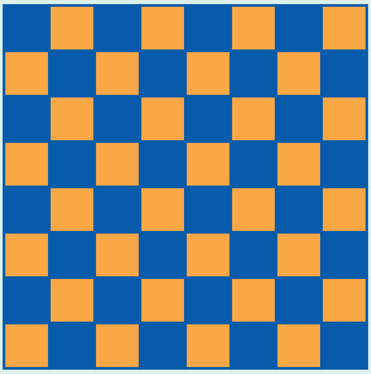
1. 89^3
2. 36^3
3. 521^2
4. 236^2
5. $2 \times 3^2 \times 4^2$
10. 41^3
12. 85^2
15. $\sqrt{13\,225}$
16. $16^2 + 12^2 + 6^3$
19. $13^2 - 10^2$

Applications

Chessboard reward

In a famous tale, a king rewarded a servant for loyal service.

The servant requested 1 grain of wheat for the first square of a chessboard, and then double that amount for the next square:
2 grains of wheat for the second square.
4 grains of wheat for the third square.
8 grains of wheat for the fourth square.
16 grains of wheat for the fifth square and so on.



a Copy and complete the table below for the first 10 squares:

Square	Grains of wheat on square	Power of 2	Total number of grains of wheat
1	1	2^0	1
2	2	2^1	3
3	4	2^2	7
4	8	2^3	15
5	16	2^4	
6			
7			
8			
9			
10			

b Write a general rule to represent the number of grains of wheat for the *n*th square.

c Use your rule to calculate the number of grains of wheat for the sixty-fourth square.

d Write a general rule that gives that total number of grains of wheat for up to and including the *n*th square. (Hint: Study the values in the last two columns.)

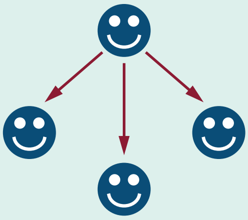
e How many grains of wheat in total would the servant receive?

f Do some research in the library or on the Internet to estimate of the weight of the wheat and the current price of a kilogram of wheat.

g Estimate the total value that the wheat would be worth today. Was it a clever request?

Triads

A new microscopic organism has been discovered and named a ‘triad’. Triads reproduce by dividing into three exact replicas of themselves.



Successive generations are shown by the number pattern:
1, 3, 9, 27…….

a Find a rule to represent the number of triads, t, in n generations.

b Use the rule to find the number of triads in the twelfth generation.

c After how many generations would there be 6561 triads?

Balls

Collect a range of balls that have a spherical shape when inflated.

a Estimate the radius of each of the balls.

Surface area of a sphere: $A = 4\pi r^2$

Volume of a sphere: $V = \frac{4}{3}\pi r^3$

b Copy and complete the table below and calculate the surface area and volume of each ball by using the formulas above and a calculator. Round to the nearest whole number.

c Divide the surface area by the volume and look for a pattern. Find an expression for the ratio in terms of the radius of the balls.

Ball	Radius (cm)	Surface area (cm²)	Volume (cm³)	$\frac{\text{Volume}}{\text{Surface area}}$
Marble				
Ping pong				
Squash				
Tennis				
Cricket				
Volley				
Basketball				
Beach ball				

Enrichment

Negative indices

$3^4 \div 3^6$ can be simplified using index laws:

$$\begin{aligned} 3^4 \div 3^6 &= 3^{4-6} \\ &= 3^{-2} \end{aligned}$$

$3^4 \div 3^6$ can be simplified by cancelling:

$$\begin{aligned} 3^4 \div 3^6 &= \frac{3^4}{3^6} \\ &= \frac{\cancel{3} \times \cancel{3} \times \cancel{3} \times \cancel{3}}{3 \times 3 \times (\cancel{3} \times \cancel{3} \times \cancel{3} \times \cancel{3})} \\ &= \frac{1}{3 \times 3} \\ &= \frac{1}{3^2} \end{aligned}$$

The two answers must agree so $3^{-2} = \frac{1}{3^2}$.

Any base to a negative power = 1 ÷ the base to the positive power.

> In general, $b^{-p} = \frac{1}{b^p}$.

Example	**Solution**
1 Express with positive powers and evaluate:	
a 5^{-3}	$5^{-3} = \frac{1}{5^3} = \frac{1}{5 \times 5 \times 5} = \frac{1}{125}$
b 8^{-1}	$8^{-1} = \frac{1}{8^1} = \frac{1}{8}$
c $\frac{1}{4^{-3}}$	$\frac{1}{4^{-3}} = 4^3 = 4 \times 4 \times 4 = 64$
d $\frac{1}{2^{-1}}$	$\frac{1}{2^{-1}} = 2^1 = 2$
2 Simplify these index expression calculations:	
a $16x^{-2}y^3 \times (2xy^{-1})^{-2}$	$16x^{-2}y^3 \times (2xy^{-1})^{-2}$ $= 16x^{-2}y^3 \times 2^{-2}x^{-2}y^2$ removing brackets $= 16 \times 2^{-2} \times x^{-2} \times x^{-2} \times y^3 \times y^2$ $= 16 \times \frac{1}{2^2} \times x^{-4} \times y^5$ $= 16 \times \frac{1}{4} \times \frac{1}{x^4} \times y^5$ $= \frac{4y^5}{x^4}$

Example

b $25x^4y^2 \div 50(x^2y)^{-3}$

Solution

$$25x^4y^2 \div 50(x^2y)^{-3}$$
$$= \frac{25x^4y^2}{50x^{-6}y^{-3}}$$
$$= \frac{25}{50} \times \frac{x^4}{x^{-6}} \times \frac{y^2}{y^{-3}}$$
$$= \frac{x^{10}y^5}{2}$$

1 Write the following with positive indices and evaluate:

a 3^{-2} b 5^{-1} c 2^{-5} d 7^{-1}

e 10^{-1} f 10^{-3} g 6^{-2} h 1^{-3}

2 Write the following with positive indices and evaluate:

a $\frac{1}{4^{-2}}$ b $\frac{1}{3^{-1}}$ c $\frac{1}{10^{-5}}$ d $\frac{1}{5^{-3}}$

e $\frac{1}{12^{-2}}$ f $\frac{1}{1^{-6}}$ g $\frac{1}{2^{-6}}$ h $\frac{1}{10^{-1}}$

3 Simplify the following, expressing your answers with positive indices:

a $32x^{-1}y^2 \times (4xy^{-1})^{-2}$ b $75x^4y^{-2} \times (5xy^{-1})^{-2}$

c $(3m^{-2}n)^{-4} \times 81mn^4$ d $(6mn^3)^{-2} \times 216mn^{-7}$

e $(7^{-1}a^{-2}b^3)^{-2} \times (ab^{-1})^{-5}$ f $(2a^3b^{-2})^{-1} \times (6a^{-1}b)^2$

4 Simplify the following, expressing your answers with positive indices:

a $100x^{-5}y^6 \div 50(x^3y^{-2})^{-3}$ b $27(m^{-2}n)^{-2} \div 3mn^{-4}$

c $\frac{50m^3n^4}{25m^{-1}n^2}$ d $\frac{8a^4b^4}{48a^5b^3}$ e $\frac{4a^2b^{-1}}{(2a^{-1}b^2)^{-2}}$ f $\frac{(10x^2y^{-1})^{-2}}{(20x^{-3}y^2)^{-1}}$

5 Find two solutions for the following index equations:

a $x^2 = 9$ b $x^2 = 100$ c $x^2 = 1$ d $x^2 = 81$

e $x^4 = 16$ f $x^4 = 81$ g $x^4 = 625$ h $x^4 = 10\,000$

i $x^6 = 64$ j $x^6 = 4069$ k $x^6 = 15\,625$ l $x^6 = 1\,000\,000$

6 Find all solutions for the following index equations. Clearly state whether the equations have one or two solutions:

a $x^2 = 121$ b $x^2 = 225$ c $x^2 = 400$ d $x^2 = 169$

e $x^3 = 8$ f $x^3 = 27$ g $x^3 = 125$ h $x^3 = 1000$

i $x^5 = 32$ j $x^5 = 243$ k $x^5 = 7776$ l $x^5 = 100\,000$

7 Can you solve the following index equations? Give reasons for your answers:

a $x^2 = {}^{-}1$ b $x^2 = {}^{-}100$ c $x^2 = {}^{-}25$ d $x^2 = {}^{-}81$

e $x^3 = {}^{-}1$ f $x^3 = {}^{-}8$ g $x^3 = {}^{-}27$ h $x^3 = {}^{-}1000$

Revision/Assessment

Exercise 9A

1 Evaluate the following indices:

a 5^2 **b** $(-4)^2$ **c** -6^2 **d** $(-6)^2$

e 10^4 **f** $(-10)^3$ **g** $(-2)^4$ **h** $-(-2)^5$

2 Find *x* in the following:

a $100 = 10^x$ **b** $36 = 6^x$ **c** $x^3 = 27$ **d** $x^4 = 16$

e $x^3 = -8$ **f** $x^3 = -1$ **g** $7^x = 49$ **h** $10^x = 100\,000$

3 Write down the following index numbers and work them out to basic numerals:

a base 5, power 2 **b** base 7, index 2 **c** base 2, exponent 3

d base 3, logarithm 4 **e** base 12, index 1 **f** base 100, power 0

Learning task 9B

4 Carry out these base 2 calculations, expressing your answers as basic numerals:

a $2^2 \times 2^3 = 2^{\text{—}} =$ _______

b $2^{12} \div 2^8 = 2^{\text{—}} =$ _______

c $\dfrac{2^5 \times 2^4}{2^7} = \dfrac{2^{\text{—}}}{2^{\text{—}}} = 2^{\text{—}} \div 2^{\text{—}} = 2^{\text{—}} =$ _______

d $(2^2)^3 = 2^{\text{—}} =$ _______

e $\sqrt{64} = (2^{\text{—}})^{\frac{1}{2}} = 2^{\text{—} \times \frac{1}{2}} = 2^{\text{—}} =$ _______

Exercise 9C

5 Simplify the following by using the index laws:

a $x^5 \times x^2$ **b** $y \times y^7$ **c** $z \times z^1 \times z^0$

d $4x^4 \times 3x^2$ **e** $3x^3y^4 \times 5x^3y^2$ **f** $6x^5y^2 \times 9y^3x^2$

g $x^9 \div x^7$ **h** $y^{12} \div y^6$ **i** $x^{10} \div x^{10}$

j $\dfrac{12x^6y^6}{3x^3y^5}$ **k** $\dfrac{8mn}{2m^0n^1}$ **l** $\dfrac{10x^4y^7}{20x^3y^5}$

Exercise 9D

6 Remove brackets and write the answer as a basic numeral:

a $(2^3)^2$ **b** $(1000^0)^6$ **c** $(10^2)^3$ **d** $(9^2)^{\frac{1}{2}}$

e $(5^3)^0$ **f** $(5^0)^3$ **g** $(2^3)^3$ **h** $(4^2)^3$

7 Remove brackets from the following expressions and simplify where possible:

a $(x^3)^2$ **b** $(y^5)^3$ **c** $(m^2)^7$ **d** $(x^6)^2$

e $(a^0)^7$ **f** $(a^4)^0$ **g** $(m^0)^0$ **h** $(m^n)^0$

i $(x^3)^2$ **j** $(x^2)^3$ **k** $((m^2)^4)^3$ **l** $((a^1)^2)^3$

8 Remove brackets from the following and simplify where possible:

a $(ab)^4$ b $(xz)^9$ c $(mn)^0$ d $(pq)^1$
e $(6b)^2$ f $(4x)^2$ g $(12m)^3$ h $(3y)^4$
i $(x^5y)^4$ j $(m^6n^2)^2$ k $(6y^3z^2)^3$ l $(2q^2r)^3$

9 Remove brackets and write the answer as a basic numeral:

a $\left(\frac{3}{5}\right)^2$ b $\left(\frac{1}{6}\right)^3$ c $\left(\frac{1}{3}\right)^4$ d $\left(\frac{5}{7}\right)^2$

10 Remove brackets from the following expressions and simplify where possible:

a $\left(\frac{p}{q}\right)^7$ b $\left(\frac{m}{n}\right)^3$ c $\left(\frac{x^3}{y^2}\right)^2$ d $\left(\frac{r^4}{t^3}\right)^5$
e $\left(\frac{1}{z^3}\right)^2$ f $\left(\frac{3}{x^2}\right)^3$ g $\left(\frac{2x}{3y^2}\right)^2$ h $\left(\frac{3m}{4n^5}\right)^2$

11 Simplify the following as far as possible using index laws:

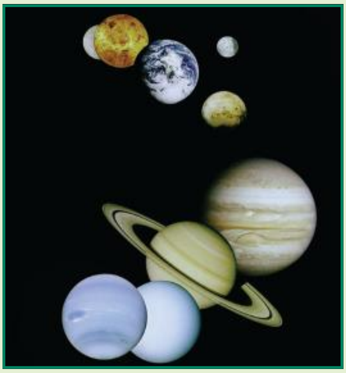
a $(2xy^2)^2 \times 2xy^3$ b $5m^2n^4 \times (2m^4n)^3$ c $(3pq)^2 \times (5pq^3)^2$
d $3(pq)^2 \times (2p^2q)^4$ e $5(x^3y^2)^3 \times 10xy^2$ f $3(m^2n)^3 \times (4m^2n)^2$

12 Simplify the following as far as possible using index laws:

a $\frac{(4xy)^3}{4x^2y^2}$ b $\frac{75p^6q^6}{(5p^3q^2)^2}$ c $\frac{(3x^2y^2)^3}{(3x^2y^3)^2}$

13 Copy and complete the table below and rewrite all the numbers in scientific notation:

Facts about Uranus	
Distance from the Sun (km)	2 871 000 000
Diameter at the equator (km)	51 118
Light from the Sun (min)	150
Orbit (Earth years)	84
Number of moons	15
Mass (Earth = 1)	14·5
Gravity (Earth = 1)	0·79



CHAPTER

10

The Coordinate Plane

The Coordinate Plane

There are many islands in the Solomon Islands. Long before the Europeans came, people of different islands were able to travel from one island to another using their observations of winds, currents, birds and stars to guide them. While some voyages were for trading purposes, others involved tribal fighting and even headhunting. In the western part of the Solomons, the Tomoko was used as a war canoe that carried warriors to conquer other tribes in the Solomons. These canoes were very fast and could be paddled all the way from Marovo to Rendova, Roviana and finally to Gizo in less than a day. While early Pacific voyagers used star charts constructed with coconut ribs as their maps, today navigators use the coordinate plane as the basis for maps and charts with the axes based on lines of longitude and latitude.

This chapter covers the following skills:

- Plotting points on a set of Cartesian axes
- Plotting sets of points that follow simple mathematical rules
- Developing and graphing simple linear and non-linear relations that model real-life situations
- Performing reflections about the line $y = x$
- Graphing straight lines using the intercept method
- Graphing straight lines using the gradient and *y*-interecept method

Specific Learning Outcome (SLO)

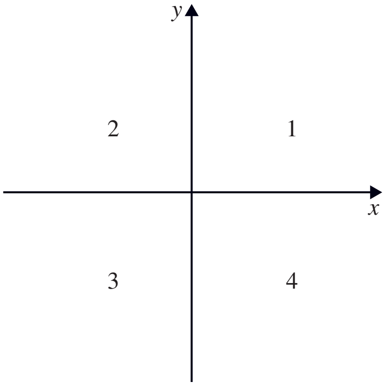
Learners should be able to:

8.10.1.1	Identify and name key components of the Cartesian plane: *x*- and *y*-axes, and origin.
8.10.2.1	Plot coordinate pairs of points and shapes on a coordinate plane.
8.10.3.1	List the coordinates of points that are in linear alignment.
8.10.3.2	Deduce a rule or equation for a linear set of coordinates.
8.10.3.3	Deduce a rule or equation from a set of coordinates given in tables.
8.10.4.1	Identify patterns of coordinates to determine values for *m* and *c* for linear equations in the form $y = mx$.
8.10.5.1	Deduce the linear equations from tables of coordinates.
8.10.5.2	Find the rule or equation from a given graph.
8.10.6.1	Define a linear relation.
8.10.6.2	Plot graphs by substituting *x*-values into an equation to find the corresponding *y*-values.
8.10.7.1	Complete tables according to given rules and plot the points on a set of Cartesian axes.
8.10.8.1	Identify the properties of horizontal and vertical linear graphs.
8.10.9.1	Deduce the equations of vertical and horizontal linear graphs.
8.10.10.1	Sketch or plot linear graphs by calculating the *y*- and *x*-intercepts.
8.10.11.1	Define the term ‘gradient’.
8.10.11.2	Calculate a gradient by dividing the rise by the run between two convenient points on a line. Formula: $m = \frac{\text{rise}}{\text{run}}$
8.10.12.1	Calculate gradient of given graphs, either positive or negative
8.10.13.1	Identify the equation for straight line: $y = mx + c$, then find the gradient and *y*-intercept.
8.10.14.1	Sketch linear graphs using the gradient and the *y*-intercept.
8.10.15.1	Find linear equations by substituting values ‘*m*’ and ‘*c*’ in the equation: $y = mx$.
8.10.15.2	Write equation for give linear graphs by finding the gradient and *y*-intercepts.

10A Plotting points on the Cartesian plane

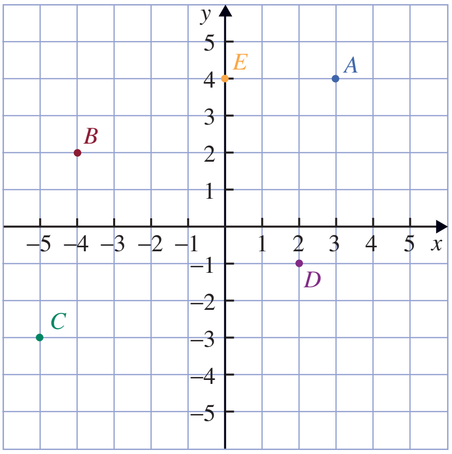
The Cartesian plane is made up of two number lines that intersect at right angles. The horizontal number line is generally called the *x*-axis and the vertical one is usually called the *y*-axis. We use *x*- and *y*-coordinates in the form (*x*, *y*) to locate the positions of points in the plane.

The Cartesian plane is also called the coordinate plane. The two axes divide the plane into four quadrants, numbered anticlockwise from the top right-hand side as shown.



Example

1



Solution

Point *A* has the coordinates (3, 4), as its position is 3 along the *x*-axis and 4 along the *y*-axis.

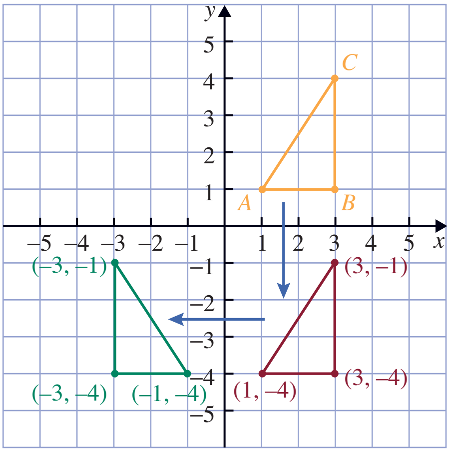
Point *B* has coordinates (–4, 2).
Point *C* has coordinates (–5, –3).
Point *D* has coordinates (2, –1).
Point *E* has coordinates (0, 4).

2 **a** Plot the three points *A*(1, 1), *B*(3, 1), *C*(3, 4) and connect them to form a right-angled triangle.

b Next translate (shift) the triangle down five units and draw its new position—often called the image.

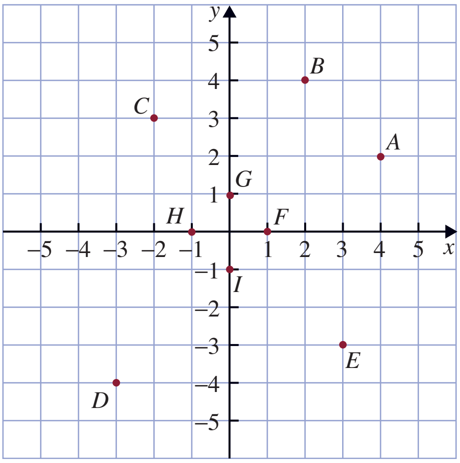
c Write down the coordinates of the vertices in this new position.

d Finally reflect (flip) the image triangle about the *y*-axis, draw its new position and label the coordinates of its vertices.



Exercise 10A

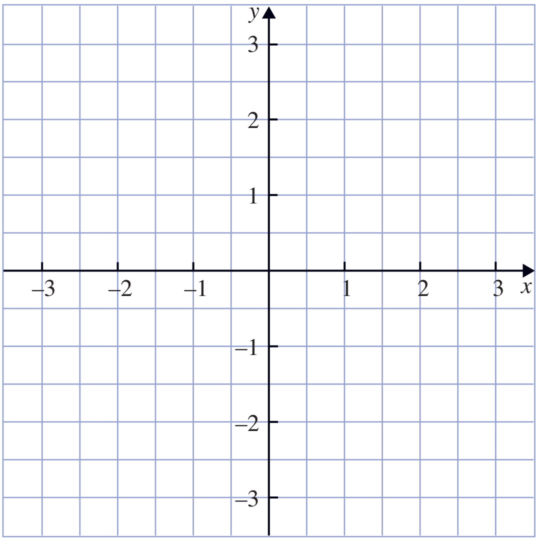
1 Write down the coordinates of the points identified by the letters *A*–*I*:



2 On the axes provided plot the following points, clearly labelling each point with the correct capital letter:

a $A(1, 0)$ $B(0, 2)$ $C(-1, 0)$ $D(0, -2)$. Connect the points to form a rhombus.

b $E(2, 0)$ $F(0, 1)$ $G(-2, 0)$ $H(0, -1)$. Connect these points to form another rhombus.



3 On a similar set of axes plot the following points, clearly labelling each point with the correct capital letter:

a $A(-2, -2)$ $B(0, 2)$ $C(1, 0)$. Connect the points to form a triangle.

b $D(-3, -3)$ $E(-3, 3)$ $F(3, 3)$ $G(3, -3)$. Connect these points to form a square.

c $H(-2, -2)$ $I(-2, 2)$ $J(0, -1)$. Connect the points to form a triangle.

d $K(-2, 1)$ $L(2, 1)$ $M(2, -1)$ $N(-2, -1)$. Connect these points to form a rectangle.

4 Draw up a set of Cartesian axes of your own on graph paper. Let 1 centimetre represent 1 unit, and allow the values on both the x-axis and the y-axis to run from –6 to +6.

- **a** Plot the points (2, 0), (6, 5), (6, 0).
- **b** Connect the points to form a right-angled triangle.
- **c** Translate the triangle 1 unit up, draw its new position, and write down the new coordinates of its vertices
- **d** Now reflect the triangle in part c about the y-axis and draw in its new position.
- **e** Label clearly the coordinates of the vertices in this new position.
- **f** Now reflect the triangle in part d about the x-axis.
- **g** Label each vertex with its coordinates.

5 Draw up a set of Cartesian axes of your own on graph paper, letting 1 centimetre represent 1 unit. Allow the values on both the x-axis and the y-axis to run from –7 to +7.

- **a** Plot the points (–7, –7), (–3, –3), (0, 0), (3, 3), (7, 7).
- **b** Join the points with a straight line. The equation to this line is $y = x$.
- **c** Plot the points (1, 3), (0, 7), (4, 6) and join them to form an isosceles triangle.
- **d** Plot the points (3, 1), (7, 0), (6, 4) and join them to form another isosceles triangle. Notice that the two triangles are symmetrically placed about the straight line. We say that one is the reflection of the other about the straight line.
- **e** Next plot the points (–1, –2), (–1, –5), (–3, –5) and join them to form a triangle.
- **f** Reflect this triangle about the line $y = x$, and carefully draw the reflected or image triangle.
- **g** Write down the coordinates of the vertices of the image triangle in part f.

6 Draw up a set of Cartesian axes of your own on graph paper, letting 1 centimetre represent 1 unit. Allow the values on both the x-axis and the y-axis to run from –8 to +8.

- **a** Plot the points (0, 0), (2, 8), (2, 4), (6, 6) and join them with straight lines.
- **b** Plot the points (–8, –8), (–2, –2), (0, 0), (2, 2), (8, 8).
- **c** Join these points using a ruler to form a straight line. This is the line $y = x$.
- **d** Reflect the shape in the line $y = x$.
- **e** Now reflect the shape in the y-axis and the x-axis, then in the y-axis again, to include all four quadrants of the Cartesian plane.
- **f** Colour in the shape to show an interesting pattern.

7
- **a** On a set of Cartesian axes of your own, draw a simple shape of your own choice. The shape could be a simple polygon, a complicated star or original design.
- **b** Write down the coordinates of its vertices.
- **c** Draw in the line $y = x$.
- **d** Reflect your shape across the line $y = x$.
- **e** Write down the coordinates of the vertices of the image of your shape.
- **f** What is the relationship between the coordinates of any vertex and those of its reflected image?

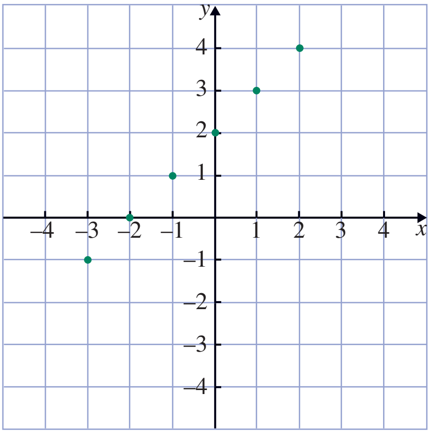
Linear patterns and simple rules

Sets or groups of points often have rules that relate the x and y values in each coordinate pair (x, y).

Example

Find the rule that relates the x and y values for each set of plotted points:

a

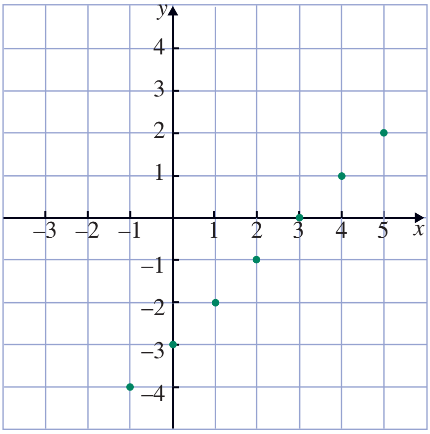


Solution

The coordinates of the plotted points are (–3, –1), (–2, 0), (–1, 1), (0, 2), (1, 3), (2, 4).

In each pair (x, y), the y-coordinate is 2 more than the x-coordinate, so we write $y = x + 2$. This is the rule for working out the y value in each pair using the x value.

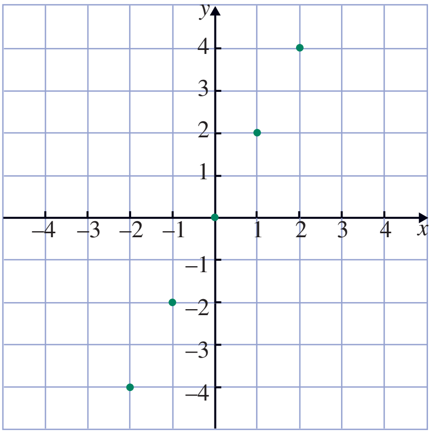
b



The coordinates of the plotted points are (–1, –4), (0, –3), (1, –2), (2, –1), (3, 0), (4, 1), (5, 2).

In each pair (x, y), the y-coordinate is 3 less than the x-coordinate, so we write $y = x - 3$, the rule for working out the y value in each pair using the x value.

c



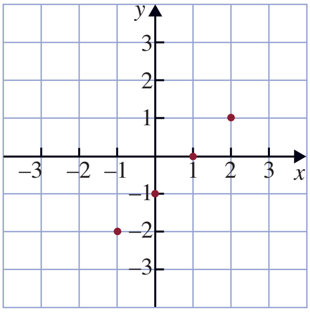
The coordinates of the plotted points are (–2, –4), (–1, –2), (0, 0), (1, 2) (2, 4).

In each pair (x, y), the y-coordinate is twice the x-coordinate, so we write $y = 2x$, the rule for working out the y value in each pair using the x value.

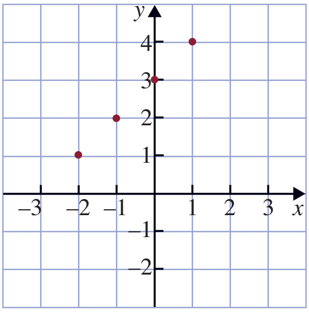
Exercise 10B

1 List the coordinates of each of the plotted points, then write down the rules that relate y to x:

a

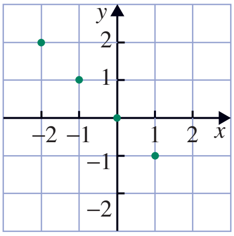


b

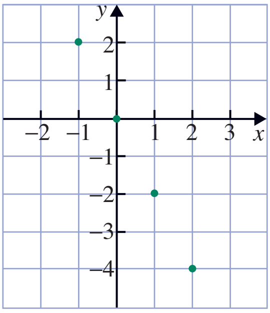


2 List the coordinates of each of the plotted points, then write down the rules that relate y to x:

a



b



3 Write down the rule that relates y to x:

a

x	–2	–1	0	1	2
y	–10	–5	0	5	10

b

x	–2	–1	0	1	2
y	–12	–6	0	6	12

c

x	–8	–4	0	4	8
y	–2	–1	0	1	2

d

x	–4	–2	0	2	4
y	–6	–3	0	3	6

4 Write down the rule that relates y to x:

a

x	–2	–1	0	1	2
y	8	4	0	–4	–8

b

x	–2	–1	0	1	2
y	10	5	0	–5	–10

c

x	–8	–4	0	4	8
y	4	2	0	–2	–4

d

x	–6	–3	0	3	6
y	2	1	0	–1	–2

e

x	–2	–1	0	1	2
y	–9	–4	1	6	11

(Hint: Refer back to Question 3a.)

f

x	–2	–1	0	1	2
y	–13	–8	–3	2	7

Combination rules 10C

In some sets of ordered pairs, the y value is often worked out by multiplying the x value by a certain number 'm' and then a fixed number 'c' is added or subtracted.

Example

1 Find the rule relating x to y:

x	–2	–1	0	1	2
y	–7	–5	–3	–1	1

2 2 2 2

Solution

Work out the difference between any two y values whose x values are separated by an increase of one.

$1 - -1 = 2$, or take $-1 - -3 = 2$. This means $y = 2x + c$.

Work out c by substituting any pair of values. $(0, -3)$ is easiest:

$-3 = 2 \times 0 + c$, so $c = -3$

The rule is $y = 2x - 3$. This is also referred to as the **equation** that relates y to x.

2 Find the rule relating x to y:

x	–2	–1	0	1	2
y	10	7	4	1	–2

–3 –3 –3 –3

Work out the difference between any two y values whose x values are separated by an increase of one.

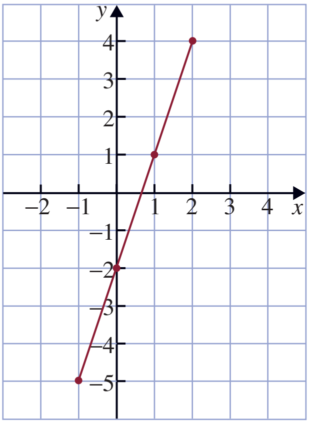
$1 - 4 = -3$, or take $7 - 10 = -3$. This means $y = -3x + c$.

Work out c by substituting any pair of values.

$(0, 4)$ is easiest, then $4 = -3 \times 0 + c$, that is $c = 4$.

The rule is $y = -3x + 4$.

3 Find the rule relating y to x for the points on the line shown:



x	–1	0	1	2
y	–5	–2	1	4

3 3 3

Working out $4 - 1 = 3$, or $1 - -2 = 3$, we have $y = 3x + c$.

Substitute any (x, y). $(0, -2)$ is easiest:

$-2 = 3 \times 0 + c$, so $c = -2$

The rule is $y = 3x - 2$.

Exercise 10C

1 Write down the rules that relate y to x in each of the following:

a

x	–1	0	1	2	3
y	–1	1	3	5	7

b

x	–1	0	1	2	3
y	–6	–2	2	6	10

c

x	–1	0	1	2	3
y	–3	2	7	12	17

d

x	–1	0	1	2	3
y	–1	2	5	8	11

e

x	–1	0	1	2	3
y	0·5	1	1·5	2	2·5

f

x	–1	0	1	2	3
y	–1	0·5	2	3·5	5

2 Write down the rules that relate y to x in each of the following:

a

x	–1	0	1	2	3
y	6	4	2	0	–2

b

x	–1	0	1	2	3
y	6	3	0	–3	–6

c

x	–1	0	1	2	3
y	–3	–7	–11	–15	–19

d

x	–1	0	1	2	3
y	–5	–6	–7	–8	–9

e

x	–1	0	1	2	3
y	2·5	2	1·5	1	0·5

f

x	–1	0	1	2	3
y	0	–0·25	–0·5	–0·75	–1

3 Write down the rules that relate y to x in each of the following tables and then complete each table:

a

x	–1	0	1	5	9
y		–1	2		

b

x	–2	0	1	6	15
y		5	7		

c

x	–5	0	4	5	8
y			5	4	

d

x	–6	0	7	10	11
y				–41	–45

4 Write down the rules that relate y to x in each of the following tables and then complete each table:

a

x	–2	0	2	4	6
y	0	1	2	3	

b

x	–2	0	2	4	6
y	–2	–1	0		2

c

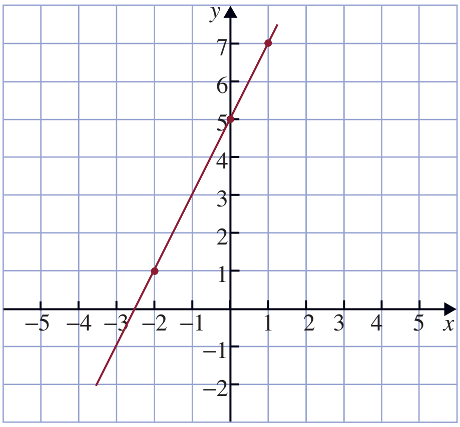
x	–3	0	3	6	9
y		2	3	4	

d

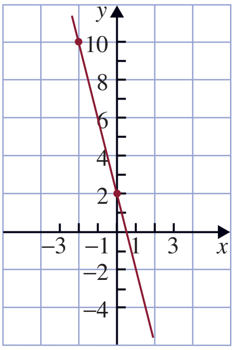
x	–6	0	6	12	18
y	3		–1		–5

5 Write down the equations for each of the following lines, using the points given on the line:

a



b



6 **a** On a set of axes of your own, plot the points (–2, 2), (–1, 4), (0, 6), (1, 8), (2, 10).

b Join the plotted points with a line and extend the line in both directions.

c What are the coordinates of the next three points in the pattern?

d Write down the rule that relates the y values to the x values.

7 The following table gives the cost of hiring a surfboard for a number of days:

Days t	3	4	5	6
Cost C	50	60	70	80

a Write down the rule that relates the cost C to the number of days hire t.

b Use your formula to find the cost of 2 weeks hire.

8 The following table gives the temperature T (°C) outside a ski lodge t hours after midnight:

Hours t	1	2	3	5	6
Temp. T	–2	–2·5	–3	–4	–4·5

a Write down the rule that relates the temperature T to the number of hours t after midnight.

b Use your formula to predict the temperature at 7:30 am. Do you think that this result is realistic?

9 The amount A in my bank account at various days of the month t is tabulated as

Days t	10	11	12	13	14
Amount A	200	175	150	125	100

a Write down the rule that relates the amount A to the number of days t.

b Use your formula to predict the status of the account after 20 days. Is the account overdrawn? If so by how much?

c When will the balance of my account be zero?

d When will the account reach its overdraft limit of –\$275?

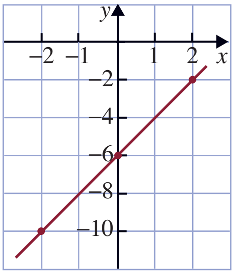
10D Plotting lines using a rule

A **linear relation** is a set of points which when plotted fall on a straight line.

Often the graph of a linear relation is drawn starting from the rule that relates the x and y values in each coordinate pair.

Example

1 Plot the graph of $y = 2x - 6$.



Solution

We need to work out the coordinates of a few points—at least two in order to position the line.

Choosing $x = –2, 0, 2$, for example, we find the matching y values:

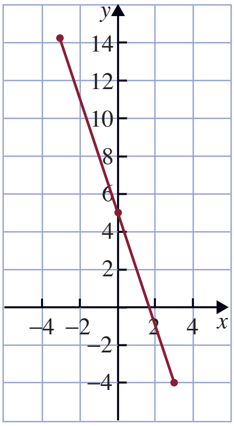
$x = –2, y = 2 \times –2 – 6 = –4 – 6 = –10$
Plot (–2, 10).

$x = 0, y = 2 \times 0 – 6 = 0 – 6 = –6$
Plot (0, –6).

$x = 2, y = 2 \times 2 – 6 = 4 – 6 = –2$
Plot (2, –2).

We draw a line through the points and extend it in both directions.

2 Plot the graph of $y = –3x + 5$.



Obtain some points to plot the line.

Taking $x = –3, 0, 3$ we find the matching y values:

$x = –3, y = –3 \times –3 + 5 = 9 + 5 = 14$

$x = 0, y = –3 \times 0 + 5 = 0 + 5 = 5$

$x = 3, y = –3 \times 3 + 5 = –9 + 5 = –4$

Instead of writing the values as coordinates, we can tabulate:

x	–3	0	3
y	14	5	–4

Exercise 10D

1 Complete the following tables using the given rules, then plot the points on a set of Cartesian axes of your own. Join the points with a straight line, extending it in each direction.

a $y = 2x$

x	–1	0	1	2
y				

b $y = 3x$

x	–1	0	1	2
y				

c $y = -x$

x	-2	-1	0	1
y				

d $y = -2x$

x	-1	0	1	2
y				

2 Complete the following tables using the given rules, then plot the points on a set of Cartesian axes of your own. Join the plots with a straight line, and extend the line in each direction.

a $y = 2x + 4$

x	-1	0	1	2
y				

b $y = 3x - 2$

x	-1	0	1	2
y				

c $y = -x + 3$

x	-2	-1	0	1
y				

d $y = -2x - 5$

x	-1	0	1	2
y				

3 Complete the ordered pairs, then graph the straight lines that represent the following rules:

a $y = x + 4$ (–2, 2), (0, ___), (2, ___)

b $y = x - 2$ (–2, ___), (0, ___), (2, ___)

c $y = -x + 3$ (–2, ___), (0, ___), (2, ___)

d $y = -x - 1$ (–2, ___), (0, ___), (2, ___)

e $y = 4x$ (–2, ___), (0, ___), (2, ___)

f $y = -3x$ (–2, ___), (0, ___), (2, ___)

g $y = 2x - 4$ (–2, ___), (0, ___), (2, ___)

h $y = -2x + 3$ (–2, ___), (0, ___), (2, ___)

4 The weekly cost C, in dollars, of running my motorbike is given by the rule $C = 0{\cdot}15d + 30$, where d is the distance I travel in kilometres, and \$30 covers the weekly ‘standing’ costs of insurance, registration, servicing etc.

a Complete the following table, which gives the cost for various distances travelled:

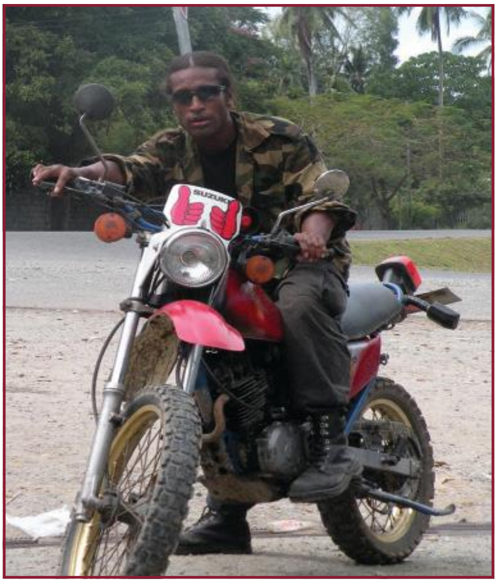
d	40	60	80
C			

b On a set of axes of your own, plot the points whose coordinates are given by the values in the table.

c Join the plots with a straight line, extending it back to the vertical axis (where $d = 0$), and continuing it on beyond $d = 80$.

d Use the rule to find the exact cost of travelling 100 km in a week.

e Find the distance travelled if the cost was \$60 for the week.



5 Penguins survive in freezing climates. The temperature T°C at a penguin colony, t hours after midnight is given by the rule $T = -0{\cdot}5t - 1$.

t	0	1	2	3	4	5	6
T							

- **a** Complete the table, which gives the temperature up to 6 am.
- **b** Plot the points whose coordinates are given by the values in the table on a set of axes of your own.
- **c** Join the plotted points with a straight line. Do not extend the line.
- **d** From your graph, read off the temperature at 5:30 am.
- **e** Use the rule (formula) that relates T to t, to find the exact temperature at 5:30 am.

6 The cost of hiring an Ice Bula machine for a class fundraising effort is \$70. Ice Bula is sold at \$2 each, so that the profit \$$P$ made when n serves of Ice Bula are sold is given by $P = 2n - 70$.

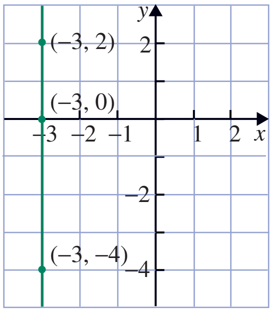
n	0	10	20	30	40	50	60
P							

- **a** Complete the table above, which gives the profit when various numbers of Ice Bula are sold.
- **b** Explain the meaning of the negative profit entries in the table.
- **c** On a set of axes of your own, plot the points whose coordinates are given by the values in the table.
- **d** Join the plotted points with a straight line. (Do not continue the line to the left for negative values of n.)
- **e** What minimum number of Ice Bula needs to be sold for the effort to 'break even', that is to avoid making a loss?
- **f** How many Ice Bulas must be sold to achieve a \$100 profit?

Horizontal and vertical lines 10E

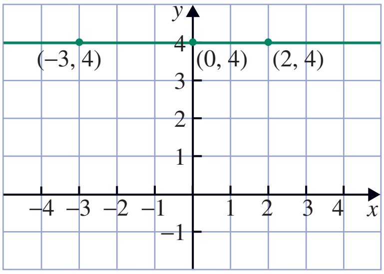
Example **Solution**

1 A vertical line passing through –3 on the x-axis has the equation $x = –3$. All points on this line will have coordinates of the form $(–3, y)$, where y can be any number that locates the position of the point against the y-axis.



2 A horizontal line passing through the point (0, 4) on the y-axis has the equation $y = 4$.

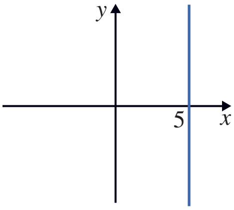
All points on this line will have coordinates $(x, 4)$, where x can be any value that locates the position of the point against the x-axis.



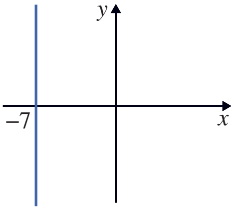
Exercise 10E

1 Write down the equations to each of the following lines:

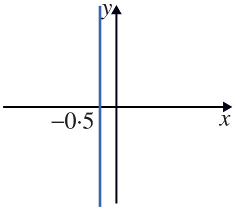
a



b

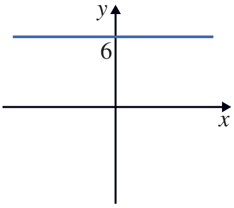


c

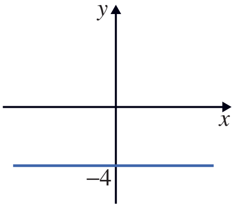


2 Write down the equations to each of the following lines:

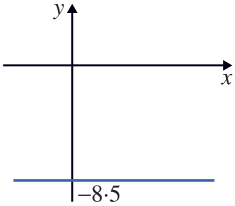
a



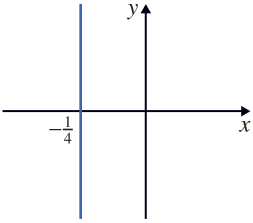
b



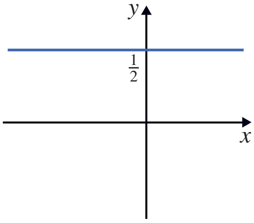
c



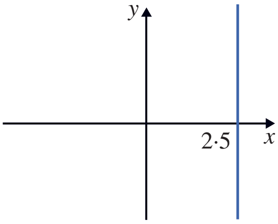
d



e



f



3 On a set of axes of your own, sketch the graphs of the straight lines with equations:

a $x = 10$ **b** $x = -2$ **c** $x = 0{\cdot}25$ **d** $x = 0$

4 On a set of axes of your own, sketch the graphs of the straight lines with equations:

a $y = 8$ **b** $y = -4$ **c** $y = -0{\cdot}25$ **d** $y = 0$

5 **a** Write down the equation of the line that is parallel to the x-axis and passes through the point:

i (2, 1) **ii** (–7, 3) **iii** (3, –5)

b Write down the equation of a line that is parallel to the y-axis and passes through the point:

i (2, –1) **ii** (–3, 4) **iii** (–0·75, 0)

6 **a** Join the points (0, 3), (1, 3), (2,2), (2, 1), (3, 0) on a set of Cartesian axes.

b Reflect the points in the lines $x = 0$, $y = 0$ and $x = 0$ again to include all four quadrants.

c Join the points (–1, –1) to (0, –2).

d Reflect in the line $x = 0$.

e Draw a dot at (–1, 1).

f Reflect in the line $x = 0$.

g Colour in your design.

7 **a** Draw up a pair of Cartesian axes of your own on graph paper. Using a scale of 1 centimetre to represent 1 unit, graph the following lines (on the one set of axes):

i $x = 2$ and $x = 6$ **ii** $y = -3$ and $y = 1$

b Shade in the square bordered by these four lines.

c Label the coordinates of the vertices.

d Reflect the square about the line $y = x$ to obtain the image.

e Write down the coordinates of the vertices of the image.

f What is the connection between the coordinates of the vertices of the square and those of its image?

8 **a** Draw up a pair of Cartesian axes of your own on graph paper. Using a scale of 1 centimetre to represent 1 unit, graph the following lines (on the one set of axes):

i $x = -2$ **ii** $y = -2$ **iii** $y = -x$

b Shade in the triangle bordered by these three lines.

c Label the coordinates of the vertices.

d Reflect the triangle about the line $y = x$ to obtain the image.

e Write down the coordinates of the vertices of the image.

f What is the connection between the coordinates of the vertices of the triangle and those of its image?

9 **a** On a set of Cartesian axes of your own, shade the region enclosed by the lines $x = 1$, $x = 3$, $y = 3$ and $y = 5$.

b Draw the reflection of this region about the line $y = x$.

c Label the coordinates of the corners of this reflected region.

Using the x- and y-intercepts

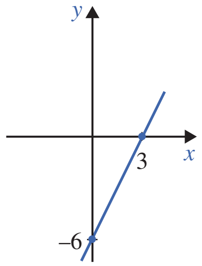
10F

The two most useful points to plot when graphing a straight line of the form $y = mx + c$ are the x-intercept and the y-intercept. This method does not work for equations of the form $y = mx$, $y = c$ and $x = c$, which are best dealt with by using other methods.

Example

Solution

1 Sketch the line $y = 2x - 6$ by finding the intercepts.



Substitute $x = 0$ into the equation and solve to find the y-intercept:

$y = 2 \times 0 - 6 = -6$

Substitute $y = 0$ and solve to find the x-intercept:

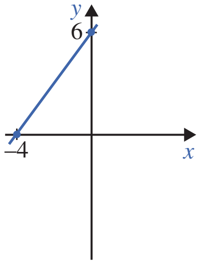
$0 = 2x - 6$

$6 = 2x$

$x = 3$

Plot the two intercepts, and draw a straight line through them.

2 Sketch the line $3x - 2y + 12 = 0$ by finding the intercepts.



First subtract 12 from both sides of the equation:

$3x - 2y + 12 - 12 = 0 - 12 = -12$

The equation is now $3x - 2y = -12$

Substitute $x = 0$ into the equation and solve to find the y-intercept:

$3 \times 0 - 2y = -12$

$-2y = -12$

$y = 6$

Substitute $y = 0$ into the equation and solve to find the x-intercept:

$3x - 2 \times 0 = -12$

$3x = -12$

$x = -4$

Plot the two intercepts (0, 6) and (–4, 0), and draw a straight line through them.

Exercise 10F

1 On a set of axes of your own, sketch the following straight lines using the intercept method:

a $y = x + 2$

b $y = x - 3$

c $y = 2x - 6$

d $y = \frac{1}{2}x + 3$

e $y = -x + 7$

f $y = -3x + 6$

g $y = -6x - 6$

h $y = -\frac{1}{4}x - 4$

i $2y - 3x = 6$

j $2y - x = 9$

k $2y - x + 6 = 0$

l $x - 2y - 5 = 0$

2 The temperature T°C on the beach t hours after midday is given by $T = 18 + 2t$.

a Find the T- and t-intercepts and use them to sketch the graph of T against t.

b What is the temperature at midday and at 2 pm?

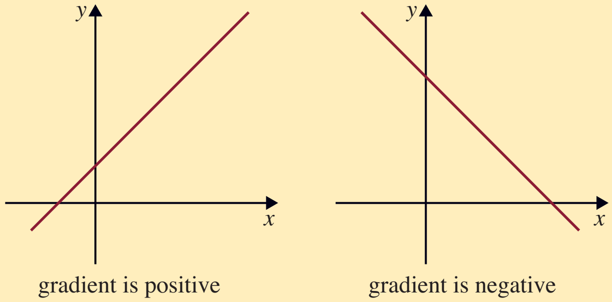
c According to this model, when will the temperature rise to 30°C?

d Comment on how realistic the model is.

10G Gradients of straight lines

The **gradient** or steepness of a straight line is the rate at which the line rises or falls. It can be calculated by dividing the **rise** by the **run** between two convenient points on the line. *m* is the symbol commonly used to represent the gradient of a straight line.

$$m = \frac{\text{rise}}{\text{run}}$$

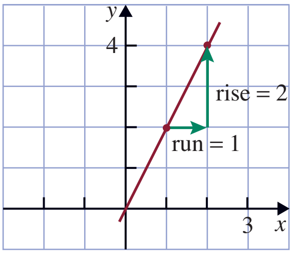


Example

1 The gradient of the line passing through the points (1, 2) and (2, 4) is

$$m = \frac{\text{rise}}{\text{run}} = \frac{2}{1} = 2$$

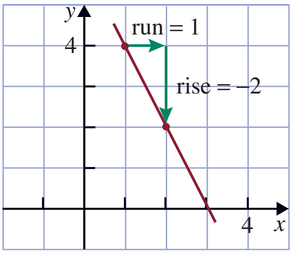
Note that the gradient is positive, as the line is showing an 'up-hill' slope to the right.



2 The gradient of the line passing through the points (1, 4) and (2, 2) is

$$m = \frac{\text{rise}}{\text{run}} = \frac{-2}{1} = -2$$

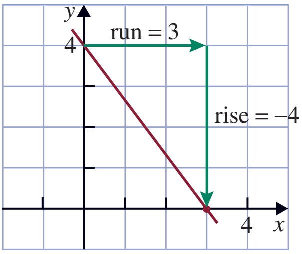
Note there is a 'fall' or negative rise.



3 The gradient of the line passing through the points (0, 4) and (3, 0) is

$$m = \frac{\text{rise}}{\text{run}} = \frac{-4}{3} \text{ or } -1\frac{1}{3}$$

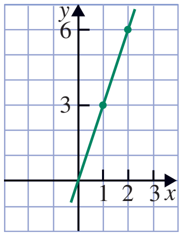
Note again that the gradient is negative, as the line is showing a downhill slope to the right.



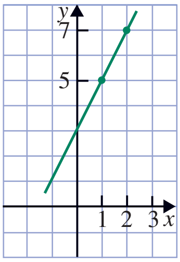
Exercise 10G

1 Find the gradients of the following lines:

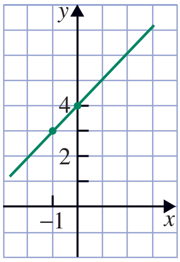
a



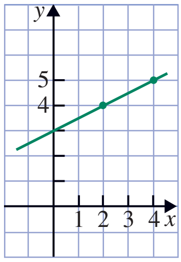
b



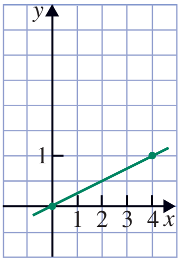
c



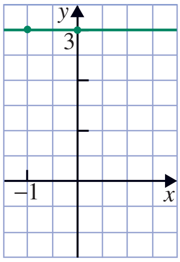
d



e

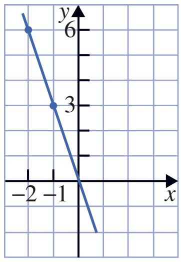


f

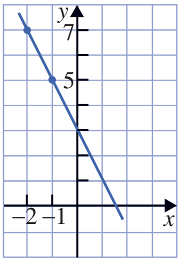


2 Find the gradients of the following lines:

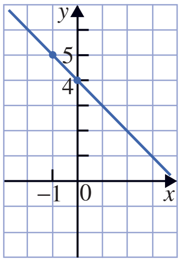
a



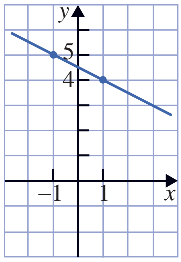
b



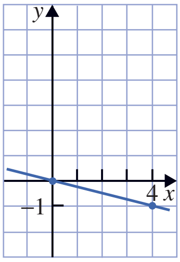
c



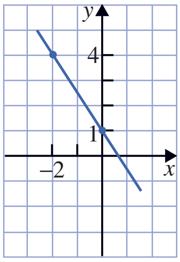
d



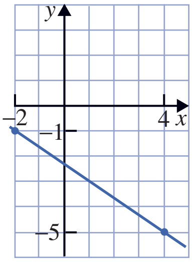
e



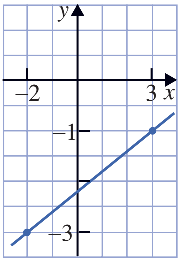
f



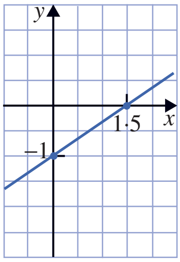
g



h



i

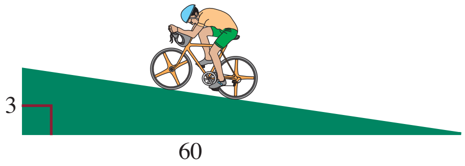


3 On a set of axes of your own, plot each of the following coordinate pairs and then find the gradient of the line joining each pair:

- **a** (1, 3), (2, 5)
- **b** (2, 2), (5, 8)
- **c** (–2, –1), (2, 7)
- **d** (–1, 3), (2, 0)
- **e** (1, –3), (3, –7)
- **f** (1, 1), (3, –5)
- **g** (–2, 0), (0, 4)
- **h** (0, 6), (2, 0)
- **i** (0, –5), (5, 0)

4 A cyclist rides up an incline that rises 3 metres for every 60 metres it runs.

- **a** What is the gradient of the incline?
- **b** How much will the incline rise for a run of 120 metres?



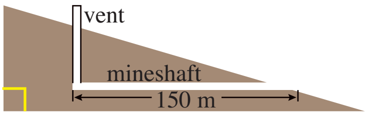
5 A hill has a vertical rise of 100 m for every kilometre along the horizontal.

- **a** Draw a triangle to represent the information.
- **b** Calculate the gradient.

6 A valley has a vertical drop of 75 m for every 2 kilometres along the horizontal.

- **a** Draw a diagram to represent the information.
- **b** Calculate the gradient.

7 A horizontal mine shaft is bored into the side of a mountain whose gradient is 0·4. If an air vent is to be put in 150 metres along the shaft directly up to the surface, how long will the air vent be?



8 Another horizontal mineshaft is bored into a mountain which rises at a gradient of 0·25. The air vent is 200 m directly below the surface of the mountain. How far along the mineshaft is the vent?

9 A ramp rises 4 metres for every 32 metres that it runs forwards. What is the gradient of the ramp?

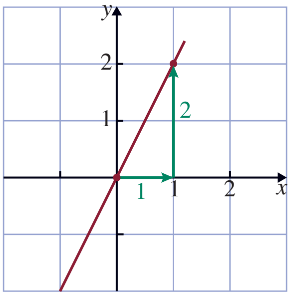
The gradient and y-intercept 10H

The general equation for a straight line is $y = mx + c$.
m is the gradient of the line
c is the y-intercept.

Lines in the form of $y = mx + c$ can be sketched using the gradient and y-intercept method.

Example **Solution**

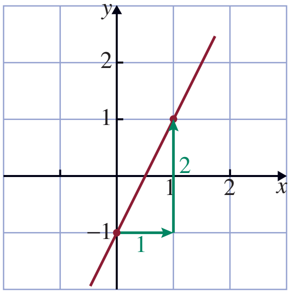
1 Sketch $y = 2x$.



$m = 2$ and $c = 0$

To graph the line, start at zero on the y-axis, run forward one step, rise two steps, then draw a line through the two positions.

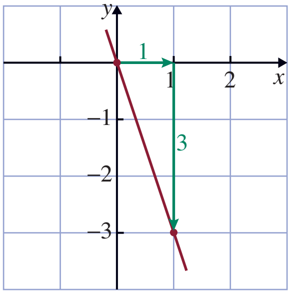
2 Sketch $y = 2x - 1$.



$m = 2$ and $c = -1$

To graph the line, start at –1 on the y-axis, run forward one step, rise two steps, then draw a line through the two positions.

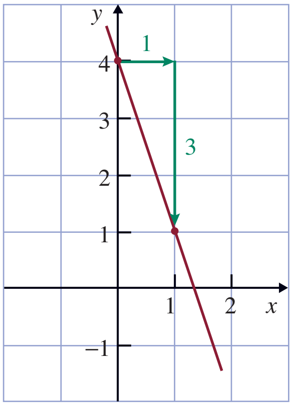
3 Sketch $y = -3x$.



$m = -3$ and $c = 0$

To graph the line, start at zero on the y-axis, run forward one step, drop three steps, then draw a line through the two positions.

4 Sketch $y = -3x + 4$.



$m = -3$ and $c = 4$

To graph the line, start at four on the y-axis, run forward one step, drop three steps, then draw a line through the two positions.

Exercise 10H

1 **a** On a set of axes of your own, graph the following straight lines by using the gradient and the y-intercept:

i $y = x$ **ii** $y = 2x$ **iii** $y = 3x$ **iv** $y = 4x$

v $y = 5x$ **vi** $y = 6x$ **vii** $y = \frac{1}{2}x$ **viii** $y = \frac{1}{4}x$

b What do you notice about the slope of each line as:

i m increases in value? **ii** m decreases in value?

2 **a** On a separate set of axes, graph the following straight lines by using the gradient and the y-intercept:

i $y = -x$ **ii** $y = -2x$ **iii** $y = -3x$ **iv** $y = -4x$

v $y = -5x$ **vi** $y = -6x$ **vii** $y = -\frac{1}{2}x$ **viii** $y = -\frac{1}{4}x$

b What do you notice about the slope of each line when m is negative?

3 **a** On a new set of axes, graph the following straight lines by using the gradient and the y-intercept:

i $y = 2x$ **ii** $y = 2x + 1$ **iii** $y = 2x + 2$ **iv** $y = 2x + 5$

b What happens to the basic graph of $y = 2x$ when c increases?

4 **a** On a new set of axes, graph the following straight lines by using the gradient and the y-intercept:

i $y = -2x$ **ii** $y = -2x - 1$ **iii** $y = -2x - 2$ **iv** $y = -2x - 5$

b What happens to the basic graph of $y = -2x$ when c is negative?

5 **a** On a separate set of axes of your own, graph the following straight lines by using the gradient and the y-intercept:

i $y = -3x$ **ii** $y = -3x + 2$ **iii** $y = -3x - 1$ **iv** $y = -3x - 4$

b Describe what happens to $y = -3x$ as the value of c changes.

6 Sketch the line that has:

a a gradient of 4 and a y-intercept of –1 **b** a gradient of –1 and a y-intercept of 2

c a gradient of 7 and a y-intercept of zero **d** a gradient of –3 and a y-intercept of –1

7 State the gradient m and y-intercept c, then sketch each line.

a $y = 2x - 9$ **b** $y = 5x + 9$ **c** $y = x - 3$

d $y = -3x + 8$ **e** $y = -7x$ **f** $y = 5 - 2x$

g $y = 4 - 8x$ **h** $y = 6 + 3x$ **i** $y = 4 - x$

j $y = 1 - x$ **k** $y = \frac{3}{4}x + 2$ **l** $y = -\frac{1}{2}x + 2$

m $y = 6 - x$ **n** $y = \frac{3x}{2} + 2$ **o** $y = \frac{x}{4} + 1$

p $y = \frac{x}{2} - 2$ **q** $y = \frac{x}{3} - 5$ **r** $y = 6 - \frac{3x}{2}$

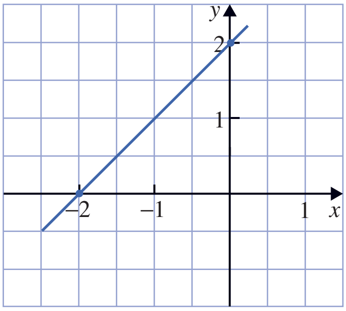
s $y = 2 - \frac{4}{5}x$ **t** $y = 1 + \frac{2}{3}x$ **u** $y = \frac{1}{2} - \frac{1}{2}x$

Finding equations of lines 10I

The equation of a straight line can be found by using the gradient and the *y*-intercept of the line.

Example

Find the equation of the line shown.



Solution

$y = mx + c$

$m = \frac{\text{rise}}{\text{run}} = \frac{2}{2} = 1$

$c = 2$

Equation is $y = x + 2$.

Exercise 10I

1 Write down the equations of the lines with:

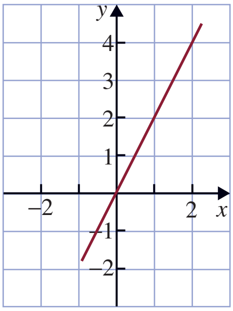
a $m = 3, c = 4$ **b** $m = 5, c = –1$ **c** $m = –3, c = 2$

d $m = \frac{1}{2}, c = 0$ **e** $m = 0, c = 4$ **f** $m = \frac{3}{4}, c = –\frac{1}{2}$

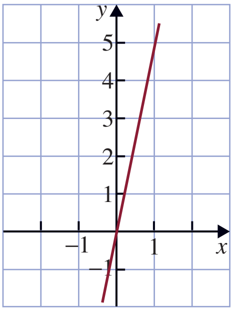
g $m = \frac{3}{2}, c = 4$ **h** $m = –\frac{4}{3}, c = –\frac{1}{3}$ **i** $m = –\frac{5}{2}, c = \frac{3}{2}$

2 Find the gradient and the *y*-intercept, and write down the equations to the following lines:

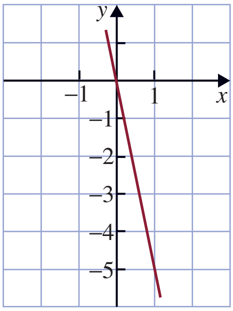
a



b

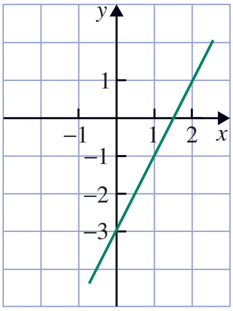


c

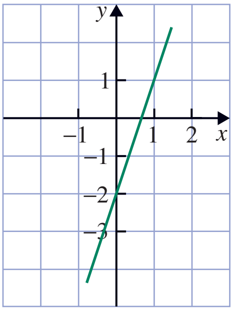


3 Find the gradient and the *y*-intercept, and write down the equations to the following lines:

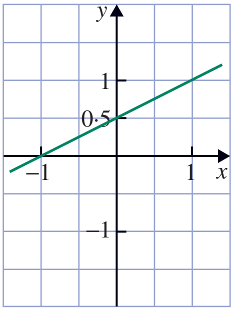
a



b



c

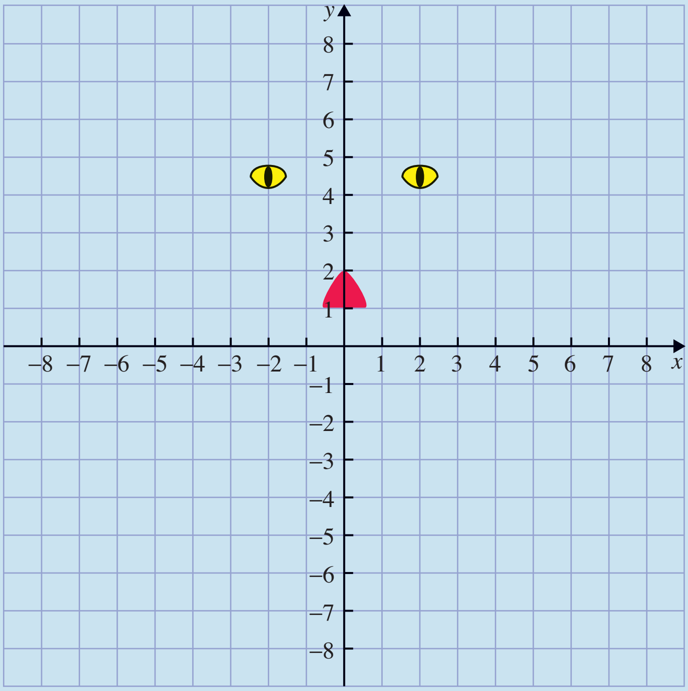


Puzzles

1 Mark and join the points on the set of axis shown below to reveal a picture:

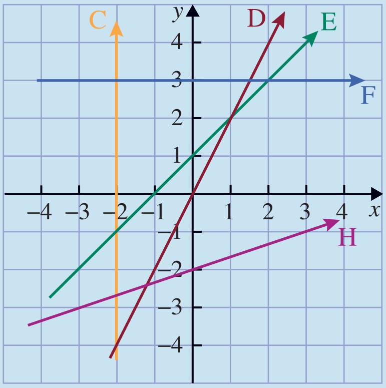
START (0, 7·5) (–2, 7) (–4, 6) (–5, 5) (–5, 3) (–4, 1) (–2·5, –1) (–2, –2·5) (0, –3) (2, –2·5) (2·5, –1) (4, 1) (5, 3) (5, 5) (4, 6) (2, 7) (0, 7·5)

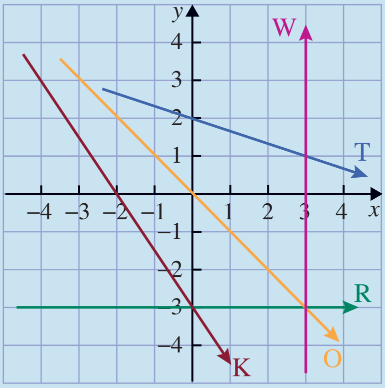
START (0, –4·5) (–4, –2·5) (–5, –3) (–5, –4) (–4, –4·5) (–3, –4) (0, –5·5) (3, –4) (4, –4·5) (5, –4) (5, –3) (4, –2·5) (0, –4·5), (–3, –6) (–4, –5·5) (–5, –6) (–5, –7) (–4, –7·5) (0, –5·5) (4, –7·5) (5, –7) (5, –6) (4, –5·5) (3, –6) (0, –4·5)



2 Write down the equations of the lines drawn on the following set of axes. Match the letter of the line to the correct equation shown below to solve the riddle:

Why were the playing cards brought on board the pirate ship?





___ $y = 3$ ___ $y = -x$ ___ $y = -3$ ___ $y = -\frac{x}{3} + 2$ ___ $y = \frac{x}{3} - 2$ ___ $y = x + 1$

___ $y = 2x$ ___ $y = x + 1$ ___ $x = -2$ ___ $y = \frac{-3x}{2} - 3$ ___ $x = -2$ ___ $y = -3$ ___ $y = x + 1$ ___ $x = 3$

3 Shown below is a series of lines, each with a coordinate point that falls on the line. Some of the numbers are missing and have been replaced by letters. Find the value of each letter, and match it to the correct number below to find the names of some famous pirates.

$y = 7$	(2, **A**)	$x = -3$	(**D**, 4)
$y = x + 4$	(**E**, 6)	$y = x - 2$	(–2, **G**)
$y = -x + 3$	(–3, **H**)	$y = -x - 1$	(–1, **I**)
$y = -x$	(**J**, 8)	$y = -3x$	(–1, **L**)
$y = 2x - 4$	(1, **N**)	$y = -2x + 3$	(**O**, 1)
$y = 5x$	(**P**, 20)	$y = x - 5$	(**R**, 3)
$y = -x + 1$	(**S**, –4)	$y = 4x + 22$	(**T**, –2)
$y = 3x$	(**V**, –15)		

___ ___ ___ ___   ___ ___ ___ ___   ___ ___ ___ ___ ___ ___
3 1 –2 –4   –8 1 6 –2   5 0 3 –5 2 8

___ ___ ___   ___ ___ ___ ___ ___ ___   ___ ___ ___ ___
7 –2 –3   4 0 8 7 –6 2   4 2 –6 2

Applications

Web page production costs

A learner who starts designing web pages pays a one-off cost of $2500 for set-up software. He estimates that each web page will cost him an extra $300 for various on-line costs. The cost C for producing n web pages is tabulated as follows:

n	0	2	4	6	8	10	12	14	16
C	2500	3100							

a Complete the table showing the costs of producing up to 16 web pages.

b Draw up a set of axes of your own with n represented on the horizontal axis and C on the vertical axis, and plot the points in the table above. Join these plots with a straight line.

The learner plans to charge customers $500 for each completed web page.

c Complete the following table, which lists the revenue R obtained from selling n web pages.

n	0	2	4	6	8	10	12	14	16
R	0	1000							

d On the same set of axes, plot the points given in the revenue table. Connect the points with a straight line.

e How many web pages need to be designed in order to show a profit?

f Write down the rule that relates C to n.

g Write down the rule that relates R to n.

h What are the coordinates of the point of intersection of the two lines?

i The profit P is equal to the revenue R minus the cost C. Work out the rule that gives the profit P from the number of web pages sold n.

Reflections

a On a Cartesian diagram where both the x- and y-axes run from –5 to 5, plot the points (–3, 0) and (0, 3) and draw the line L through these two points.

b Find the gradient and y-intercept of this line, and so write down its equation.

c Rule in the line $y = x$ by connecting the points (–2, –2), (0, 0), (2, 2) and extending the line in both directions.

d Reflect the line L about $y = x$.

e Write down the gradient and y-intercept of the image line, and so write down its equation in the form $y = mx + c$.

f The reflected line in part d is now rotated 90° anticlockwise about its y-intercept. Find the equation of this new line.

Bicycle hire

John's Bicycle was hired for $100 by the Tenaru National Secondary School for its annual school bazaar. It is proposed to charge students $0·50 for a 5-minute session of bicycle riding. Let $$P$ represent the profit when n sessions are used by the students.

a Copy and complete the table below, which gives the profit when various numbers of sessions are used.

n	0	50	100	150	200	250	300	350	400	450
P	–100	–75								

b Draw up a set of axes of your own with n represented on the horizontal axis and P on the vertical axis, and plot the points given in the table.

c Connect the plotted points with a straight line, and continue it on from $n = 450$.

d What is the 'break even' number of sessions needed to cover the hire of the bicycle?

e Rule in the horizontal line $P = 40$, and use it to read from the graph the number of sessions needed to obtain a profit of $40.

f Rule in the vertical line $n = 275$, and use it to read from the graph the profit when 275 sessions are used.

g Write down the rule relating P to n. It will take the form $P = \_\_\_\_\_ n - \_\_\_\_\_$

h Use the formula in part g to find the profit when 600 sessions are used.

Enrichment

1 Relations which take the form $y = mx + c$ are called linear relations because when values are tabulated and points plotted, the plots lie on a straight line whose gradient is m, and y-intercept is c. Other rules relating x and y can provide plots which lie on curves.

The cost of hiring a mini bus to take up to 24 students to a beach camp is \$48. The cost is to be shared between those going.

a Complete the following table, which gives the cost C for each student when there are $n = 1, 2, 3, \ldots 24$ students to share the cost.

n	1	2	3	4	6	8	10	12	16	20	24
C	48										2

b On a set of Cartesian axes of your own, with n the horizontal axis and C the vertical axis, plot the points given in the table.

c Sketch a smooth curve that passes through the points. The curve is actually part of a hyperbola.

d Rule in the line $n = 18$ (a vertical line through 18 on the n-axis). At what value of C does the line intersect the curve? What then is the approximate cost per person if 18 students share the cost?

e Rule in the line $C = \$9{\cdot}60$ (a horizontal line through 9·6 on the C-axis). At what value of n does the line intersect the curve? What then is the minimum number of students needed to share the cost if the charge is to be no more than \$9·60 per head?

f When any n value and its corresponding C value are multiplied together, what answer is always obtained? The rule relating the two variables is then $n \times C =$ ____ or equivalently $C = \frac{\_\_}{\_\_}$. Write this rule beside the graph.

g Use the rule found in part f to determine the cost per head if 14 students share the trip to the beach camp.

2 The solution to the simultaneous pair of linear equations $y = 3x + 1$ and $y = 2x - 1$ can be found by graphing each equation on a set of axes and reading off the coordinates of the point of intersection. The lines can be graphed by using either a brief table, or by using the gradient and y-intercept method.

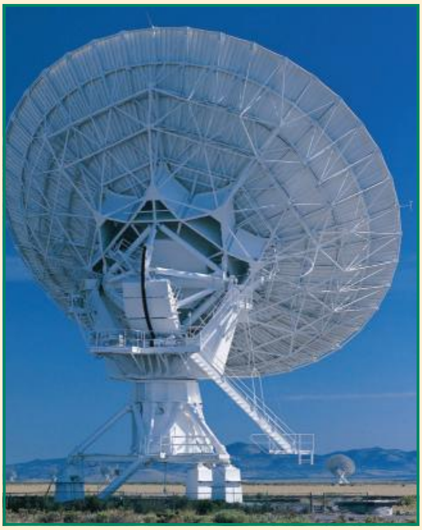
a Solve graphically the pair of equations $y = 3x + 1$ and $y = 2x - 1$. $x =$ _____, $y =$ _____

b Solve graphically the pair of equations $y = -2x$ and $y = x - 6$.

c Solve graphically the pair of equations $y = x + 6$ and $y = -2x + 3$

d Solve graphically the pair of equations $y = 3$ and $y = 4x - 5$.

e Solve graphically the pair of equations $y = 2x + 4$ and $x = -1$.



3 A cross-section of a satellite dish is modelled by the rule $y = 0{\cdot}1 \times x^2$, where x and y are measured in metres. The first entry is calculated by substitution: $y = 0{\cdot}1 \times (-4)^2 = 0{\cdot}1 \times 16 = 1{\cdot}6$.

a Copy and complete the table below. Notice the symmetry of the y values.

b On a set of axes of your own, plot the points represented by the values in the table.

c Join up the plots with a smooth curve. This curve is a parabola.

d How deep is the dish in the centre?

e Rule in the line $x = 2{\cdot}5$. How deep is the dish 2·5 metres out from the centre?

f Rule in the line $y = 1$. How far out from the centre must you be if you are at a depth of 0·6 metres down from the top edge?

x	–4	–3	–2	–1	0	1	2	3	4
y	1·6								

4 Ian and Liz decide to walk to the local swimming pool, which is half a kilometre away.

Liz begins to walk at a speed of 40 metres per minute. Ian starts 2 minutes later and walks at a speed of 50 metres per minute.

a Copy and complete the following table which shows how many metres Liz has walked after various minutes:

t (min)	0	1	2	3	4	5	6	7	8	9	10
d (metres)	0	40	80								

b On a set of axes of your own, plot the points given in this table. Set out t values along the horizontal axis and d values up the vertical axis. Join the plots with a straight line. Label and scale your axes clearly.

c Copy and complete this table which shows how many metres Ian has walked after various minutes.

t (min)	0	1	2	3	4	5	6	7	8	9	10
d (metres)	0	0	0	50							

d Plot these values on the same set of axes. Using a different coloured pencil, connect the plots.

e From the two graphs, find out when Ian meets up with Liz. How far has each walked when they meet?

Revision/Assessment

Exercise 10A

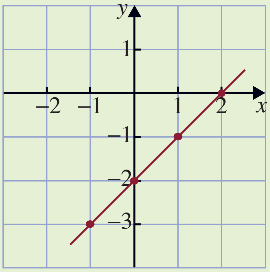
1 Draw up a set of Cartesian axes of your own on graph paper. Let 1 centimetre represent 1 unit, and the values on both the x- and y-axes to run from –10 to +10.

- a Plot the points (2, 1), (4, 1), (4, 3) and connect them to form a right-angled triangle.
- b Translate the triangle one unit to the right. Draw its new position, and write down the new coordinates of its vertices.
- c Now reflect the triangle in part b about the y-axis and draw in its new position. Label clearly the coordinates of the vertices in this new position.
- d Finally reflect the triangle in part c about the line $y = x$. Label each vertex with its coordinates.

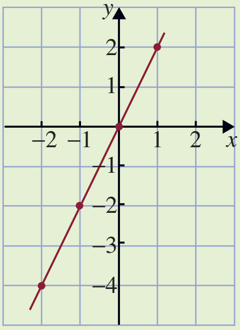
Exercise 10B

2 List the coordinates of some of the plotted points on the graphs below, then write down the rule that relates each y value to its x value.

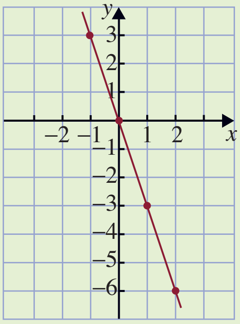
a



b



c



3 The coordinates of sets of points are listed in table form. Write down the rule that relates the y values to the x values in each case:

a

x	–2	–1	0	1	2
y	6	5	4	3	2

b

x	–2	–1	0	1	2
y	1	0·5	0	–0·5	–1

Exercise 10C

4 Write down the rules that relate y to x in each of the following:

a

x	–1	0	1	2	3
y	5	7	9	11	13

b

x	–1	0	1	2	3
y	5	2	–1	–4	–7

Exercise 10D

5 Complete the following tables by using the given rules, then plot each set of points on a pair of Cartesian axes of your own. Join the plots with a straight line, extending the line in each direction. Label each line with its equation.

a $y = 2x - 1$

x	–1	0	1	2
y				

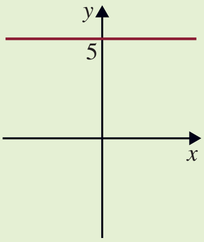
b $y = –3x - 2$

x	–1	0	1	2
y				

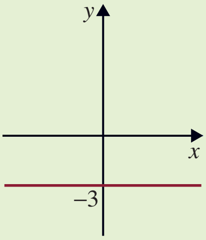
Exercise 10E

6 Write down the equations to each of the following lines.

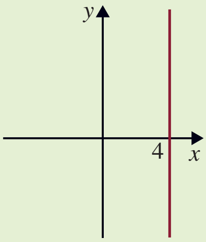
a



b



c



Exercise 10F

7 On a set of axes of your own, sketch the following straight lines using the incercept method:

a $y = x - 3$ **b** $y = -x + 2$ **c** $2y - x = 5$ **d** $x - 2y - 3 = 0$

Exercise 10G

8 On a set of Cartesian axes, plot the points (1, 4) and (2, 8), and then work out the gradient of the line that passes through these points.

9 On the same set of axes, repeat the steps of Question 8 using the points (–1, 5) and (2, –4).

10 Find the gradient of the line which joins (0, 6) and (3, 0).

Exercise 10H

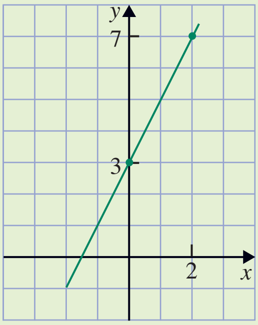
11 On a set of axes of your own, graph the following straight lines by using the gradient and the *y*-intercept.

a $y = x - 1$ **b** $y = 2x + 1$ **c** $y = -3x + 2$ **d** $y = -\frac{1}{2}x + 2$

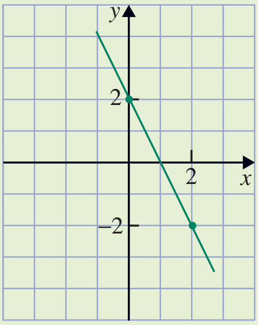
Exercise 10I

12 By reading off the gradient and the *y*-intercept, write down the equations to the following lines.

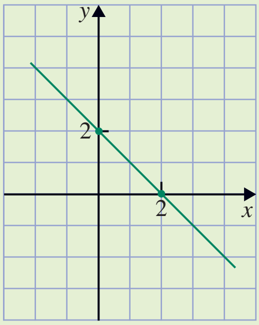
a



b



c



CHAPTER 11 Pythagoras

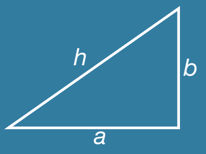
Pythagoras

Carpenters know the importance of using right-angled triangles to construct strong buildings. A rectangular frame is weak and needs a diagonal strut to strengthen it and maintain its shape. The carpenters used these principles when building this house for the South Pacific Arts Festival at the National Museum and Cultural Centre in Honiara in 2012.

Right-angled triangles have been important to many civilisations since the earliest times. The Egyptian farmers used a knotted rope with a ratio of 3 : 4 : 5 to resurvey their rectangular fields following the annual floods of the Nile River. The most famous Greek mathematician, Pythagoras, studied the properties of right-angled triangles with his students and an important theorem was named after him. This theorem continues to be studied by learners all over the world as it has many practical applications.

This chapter covers the following skills:

- Identifying the hypotenuse and adjacent sides in a right-angled triangle
- Understanding and writing the mathematical connection between the sides of a right-angled triangle in algebraic form
- Using Pythagoras' theorem to find side lengths in right-angled trianglesPythagoras' theorem



$h^2 = a^2 + b^2$

- Recognising some Pythagorean triads
- Applying Pythagoras' theorem to practical situations
- Using Pythagoras' theorem to find sides in shapes containing right-angled triangles
- Finding the lengths between points plotted on a Cartesian coordinate system
- Using Pythagoras' theorem to find missing lengths in three-dimensional problems

Specific Learning Outcome (SLO)

Learners should be able to:

8.11.1.1 State Pythagoras' theorem and its equation: $h^2 = a^2 + b^2$

8.11.1.2 Measure the lengths of the sides of various right-angle triangles and apply them to Pythagoras' theorem.

8.11.2.1 Explain Pythagoras theorem in terms of three sides of a right-angle triangle: $h^2 = a^2 + b^2$

8.11.2.2 Write equations for given triangles that relate all the three sides of the right-angle triangle.

8.11.3.1 Identify and define the term 'hypotenuse'.

8.11.3.2 Calculate the length of the hypotenuse using Pythagoras' theorem.

8.11.4.1 Define a Pythagorean triad.

8.11.4.2 Identify sets of three numbers that form Pythagorean triads, e.g. 3, 4, 5; 9, 12, 15 etc.

8.11.5.1 Find the length of the missing sides of given triangles using Pythagoras' theorem.

8.11.5.2 Solve practical questions using Pythagoras' theorem.

8.11.6.1 Define and identify the term 'irrational number'.

8.11.6.2 Define numbers in exact form as a surd or square root.

8.11.7.1 Calculate the length of a missing side of a right-angled triangle expressing the answer in exact form.

8.11.8.1 Define a composite shape and identify examples of composite shapes.

8.11.9.1 Find missing lengths indicated by pronumerals in given composite shapes.

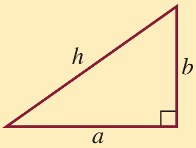
8.11.10.1 Calculate the lengths of line segments between two points on a graph using Pythagoras' theorem.

8.11.11.1 Calculate the dimensions of the cross-sections of solids using Pythagoras' theorem.

8.11.11.2 Investigate composite areas of squares and triangles using formulas.

11A Exploring Pythagoras' theorem

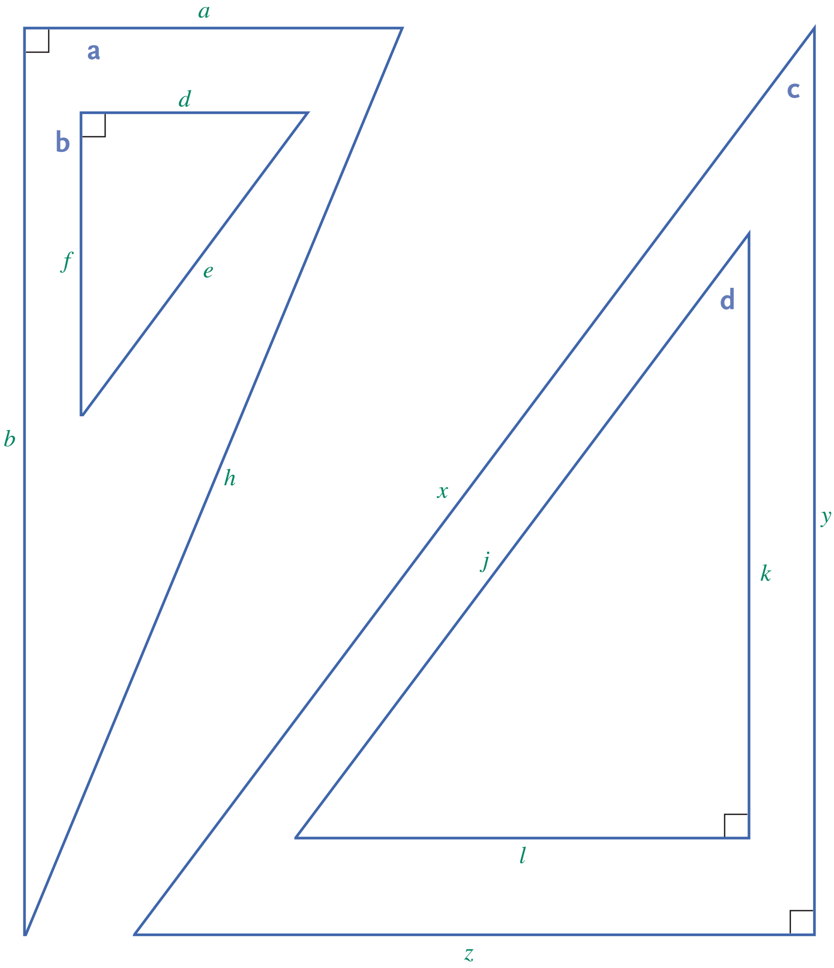
The lengths of the sides of right-angled triangles are connected by a rule which is attributed to Pythagoras, hence it is called **Pythagoras' theorem**. The longest side of a right-angled triangle is called the hypotenuse and is denoted by the letter h.



$$h^2 = a^2 + b^2$$

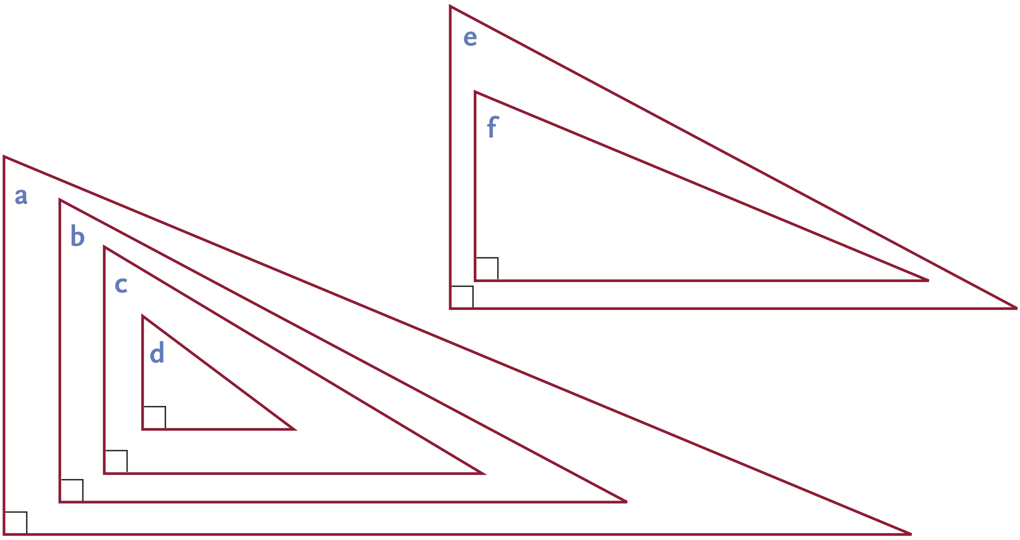
Learning task 11A

1 Measure the lengths of these right-angled triangles to the nearest centimetre and record the lengths in the table that follows. Show that Pythagoras' theorem holds for each triangle:

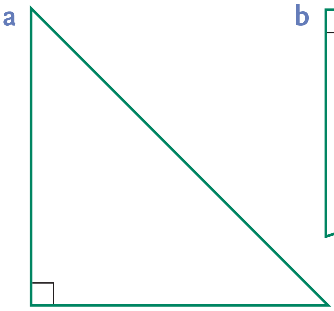


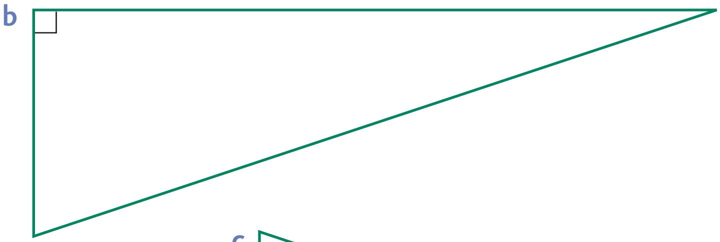
Triangle	Length	Length	Length	Pythagoras' theorem
a	a: 5 cm	b: 12 cm	h: 13 cm	$a^2 + b^2$ $\quad$ h^2 $= 5^2 + 12^2$ $\quad$ $= 13^2$ $= 25 + 144$ $\quad$ $= 13 \times 13$ $= 169$ $\quad$ $= 169$ $\therefore 5^2 + 12^2 = 13^2$ $\therefore a^2 + b^2 = h^2$
b	d:	f:	e:	
c	y:	z:	x:	
d	k:	l:	j:	

2 Measure these triangles to the nearest half centimetre where necessary and show that Pythagoras' theorem holds for each:



3 Measure the lengths of these triangles and show that Pythagoras' theorem holds for each:



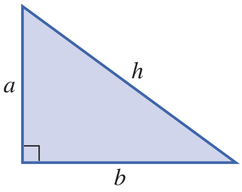


11B Stating Pythagoras' theorem

When using Pythagoras' theorem to find the length of sides in right-angled triangles, set up the equation in the form $h^2 = a^2 + b^2$, where h is the length of the hypotenuse and a and b are the lengths of the sides adjacent to the right angle.

Example

Construct an equation that connects the sides of the following triangle:



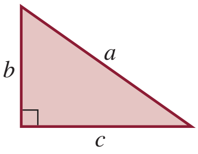
Solution

Equation:
$h^2 = a^2 + b^2$

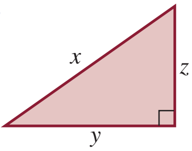
Exercise 11B

1 Write the equations that connect the sides of these triangles:

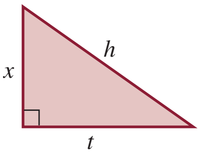
a



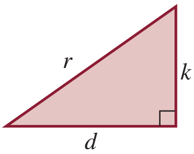
b



c

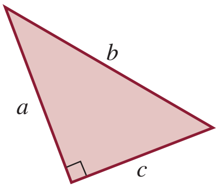


d

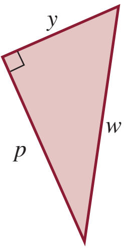


2 Write the equations that connect the sides of these triangles:

a



b



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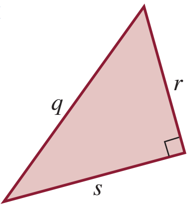

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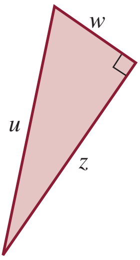
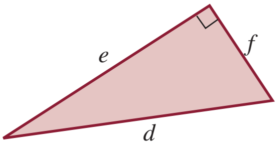
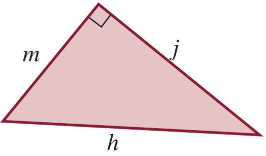


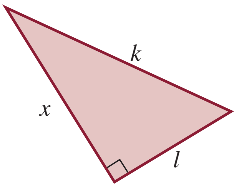
e



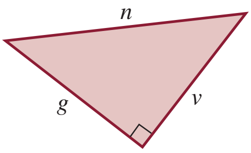
f



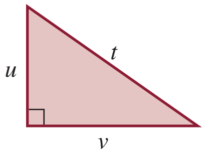
g



h



i



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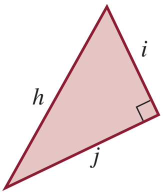
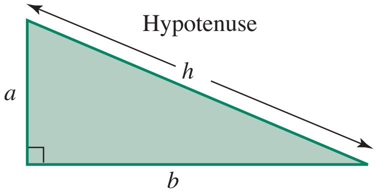


Finding the length of the hypotenuse

11C

The rule that connects the length of the three sides of a right-angled triangle is called **Pythagoras' theorem**. The rule states that the square of the hypotenuse is equal to the sum of the squares of the two shorter sides.

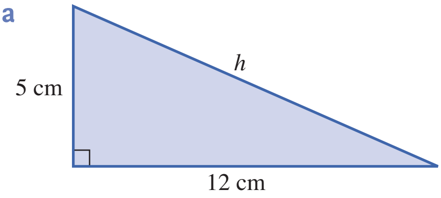


$$h^2 = a^2 + b^2$$

Example

Find the length of the hypotenuse in each of these triangles:

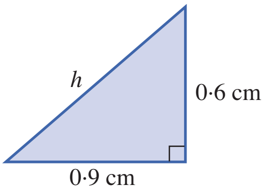
a



Solution

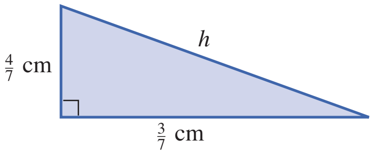
$$\begin{aligned} h^2 &= a^2 + b^2 \\ h^2 &= 5^2 + 12^2 \\ &= 25 + 144 = 169 \\ \therefore\ h &= \sqrt{169} = 13 \text{ cm} \end{aligned}$$

b



$$\begin{aligned} h^2 &= a^2 + b^2 \\ h^2 &= 0{\cdot}6^2 + 0{\cdot}9^2 \\ &= 0{\cdot}36 + 0{\cdot}8 \\ &= 1{\cdot}17 \\ \therefore\ h &= \sqrt{1{\cdot}17} = 1{\cdot}08 \text{ cm} \end{aligned}$$

c

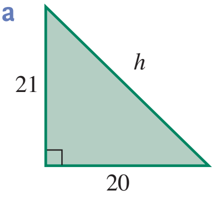


$$\begin{aligned} h^2 &= a^2 + b^2 \\ h^2 &= \left(\frac{4}{7}\right)^2 + \left(\frac{3}{7}\right)^2 \\ &= \frac{16}{49} + \frac{9}{49} = \frac{25}{49} \\ \therefore\ h &= \sqrt{\frac{25}{49}} = \frac{5}{7} \text{ cm} \end{aligned}$$

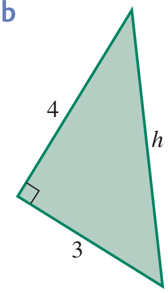
Exercise 11C

1 Find the length of the hypotenuse of each of these triangles, for which all lengths are in centimetres:

a



b



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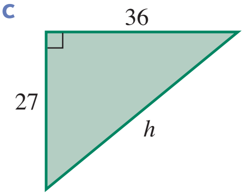
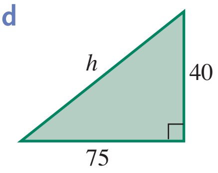
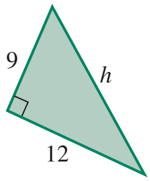


d



e



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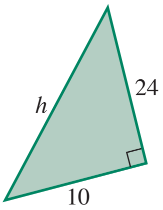
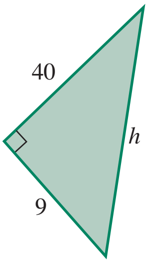
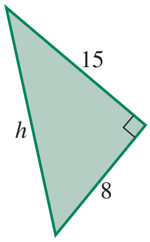


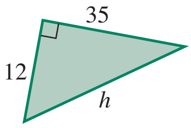
g



h



i



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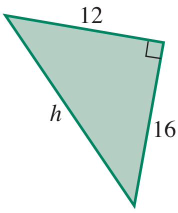
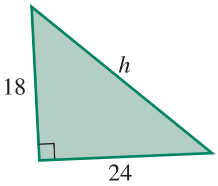
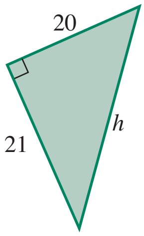


k

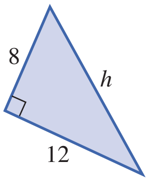


l

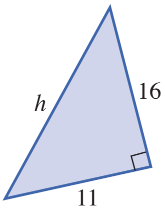


2 Find the length of the hypotenuse of these triangles, giving your answers correct to 1 decimal place. All lengths are in metres:

a



b



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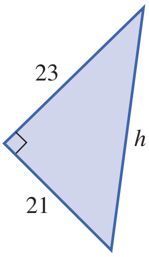

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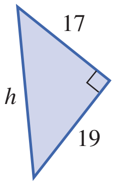

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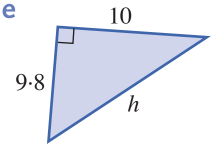
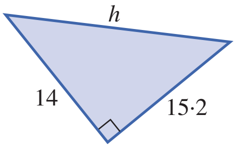


f



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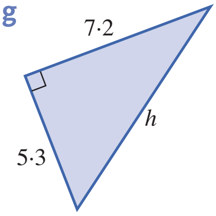

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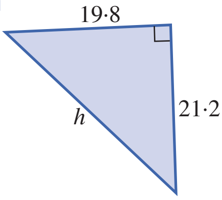
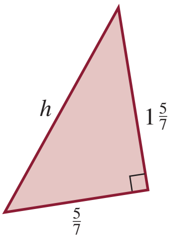
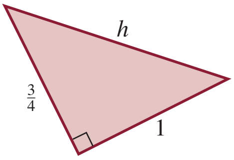


3 Find the length of the hypotenuse, giving your answers in fraction form. All lengths are in metres:

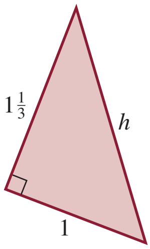
a



b



c



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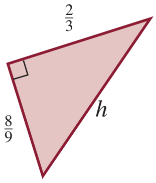

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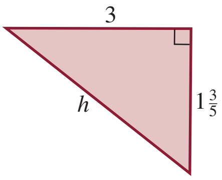

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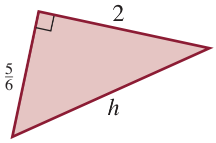

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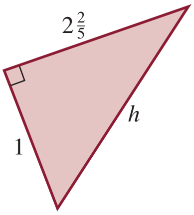

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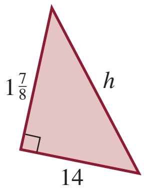
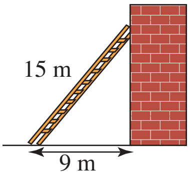
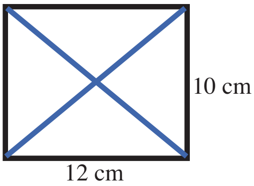


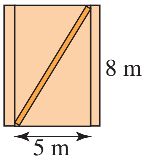
4 A ladder that is 15 metres long leans up against a tall wall. If the foot of the ladder is 9 metres from the base of the wall, how far up the wall will the ladder reach, to the nearest centimetre?



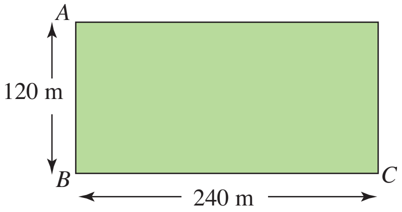
5 The flag shown has black and blue lines drawn on it. If the flag is 12 cm long and 10 cm wide, find the total length of the lines, to the nearest millimetre.



6 A length of wood is to be used to brace a rectangular wall. If the wall measures 5 metres by 8 metres, find the length of the brace, to the nearest millimetre.



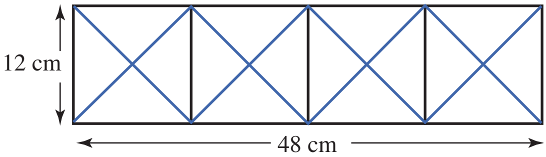
7 Daniel walks from A to B to C around the outside of this paddock. Kikolo walks through the paddock from A to C in a straight line. Find the extra distance, to the nearest metre, that Daniel walks compared to Kikolo.



8 Sussie runs diagonally across a rugby field that measures 70 metres by 100 metres. How far will she run, to the nearest centimetre?

9 Find the length of the longest side of a flag that is in the shape of a right-angled triangle, with perpendicular sides that are 1·2 metres and 0·9 metres long.

10 Find the length of the blue lines inside the four squares, expressing your answer correct to the nearest millimetre:

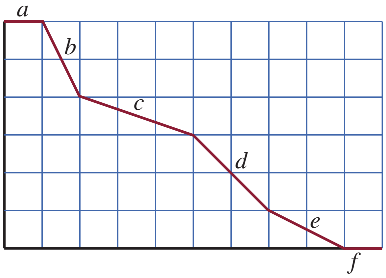


11 This is a plan of a walk down a hill. It has been drawn on a grid with 10 metres between each line. Sections *a* and *f* are the starting and finishing levels, while sections *b*, *c*, *d* and *e* are the slopes.

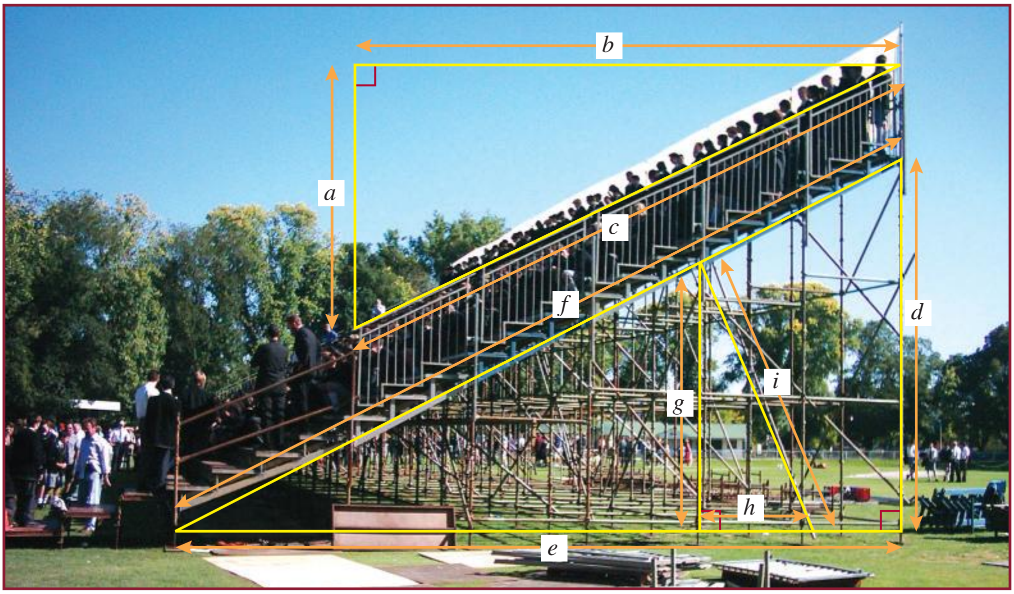
a Find the length of the following sections, correct to the nearest centimetre:

i *b* **ii** *c* **iii** *d* **iv** *e*

b Find the total distance of the walk, correct to the nearest centimetre.



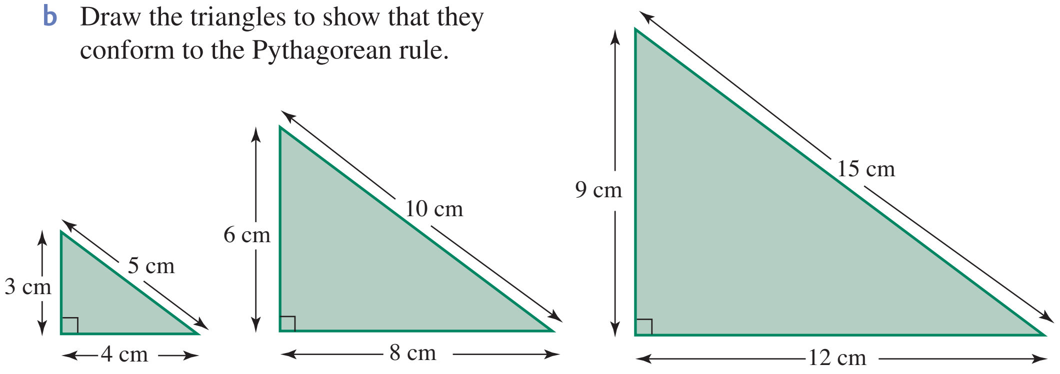
12 The photograph shows the structure used for a photograph of the 'whole school'. Estimate lengths *a*, *b*, *d*, *e*, *g*, *h*, using the height of appropriate adults in the photo, then use Pythagoras's theorem to calculate lengths *c*, *f* and *i*. Give all answers correct to the nearest centimetre.



Pythagorean triads 11D

A **Pythagorean triad** or triple is a set of three numbers, a, b and h, which conform to the rule $a^2 + b^2 = h^2$.

Example	Solution
1 Show that the numbers 3, 4, 5 form a Pythagorean triad.	To be a Pythagorean triad $3^2 + 4^2 = 5^2$ $9 + 16 = 25 = 5^2$ $\therefore$ 3, 4, 5 is a Pythagorean triad.
2 **a** Using the Pythagorean triad 3, 4, 5, generate further triads which are:	
i twice as large	$2 \times 3 \quad 2 \times 4 \quad 2 \times 5$ $6 \quad 8 \quad 10$ $6^2 + 8^2 = 100 = 10^2$ So 6, 8, 10 is a Pythagorean triad.
ii three times as large	$3 \times 3 \quad 3 \times 4 \quad 3 \times 5$ $9 \quad 12 \quad 15$ $9^2 + 12^2 = 225 = 15^2$ So 9, 12, 15 is a Pythagorean triad.
b Draw the triangles to show that they conform to the Pythagorean rule.	



Exercise 11D

1 Show that the numbers written in this order form Pythagorean triads:

a 5, 12, 13 **b** 6, 8, 10 **c** 7, 24, 25 **d** 8, 15, 17

e 10, 24, 26 **f** 9, 12, 15 **g** 9, 40, 41 **h** 14, 48, 50

i 16, 30, 34 **j** 18, 80, 82 **k** 27, 36, 45 **l** 21, 28, 35

2 Find x, the missing number in the following Pythagorean triads:

a 12, 35, x **b** 8, 15, x **c** 11, 60, x **d** 13, 84, x

e 15, 20, x **f** 12, 16, x **g** 16, 30, x **h** 18, 80, x

i 18, x, 82 **j** 21, x, 35 **k** 28, x, 53 **l** 28, x, 100

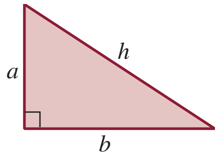
3 Using the following Pythagorean triads, produce new ones which are:

i twice as large **ii** three times as large **iii** four times as large

a 21, 28, 35 **b** 37, 684, 685 **c** 8, 15, 17

11E Finding the length of a perpendicular side

Pythagoras' theorem can be used to find the length of a perpendicular side when the hypotenuse and the other perpendicular side are known. Always start by writing the rule, placing the length of the hypotenuse first.



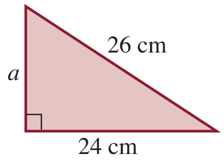
$h^2 = a^2 + b^2$
$\therefore a^2 = h^2 - b^2$
$\therefore a = \sqrt{h^2 - b^2}$

Learners without calculators are advised to make use of the table of square roots (Appendix) to help them with their calculations.

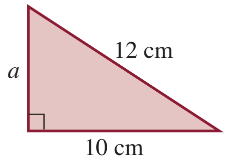
Example

Find the length of the missing side in these triangles:

a



b



Solution

a

$h^2 = a^2 + b^2$
$26^2 = a^2 + 24^2$
$676 = a^2 + 576$
$a^2 = 676 - 576 = 100$
$a = \sqrt{100} = 10$ cm

b

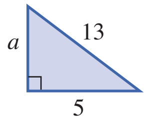
$h^2 = a^2 + b^2$
$12^2 = a^2 + 10^2$
$144 = a^2 + 100$
$a^2 = 144 - 100 = 44$
$a = \sqrt{44} = 6{\cdot}6$ cm $= 66$ mm

6·6 cm is the length correct to the nearest millimetre.

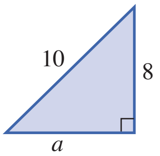
Exercise 11E

1 Find the length of the missing side in these triangles, for which all lengths are given in metres:

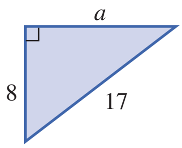
a



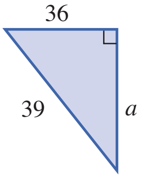
b



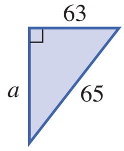
c



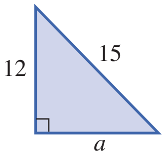
d



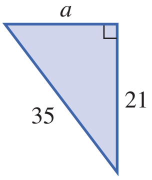
e



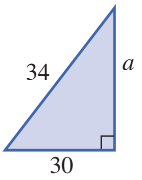
f



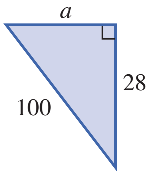
g



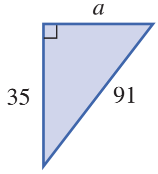
h



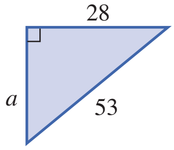
i



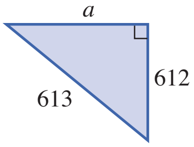
j



k

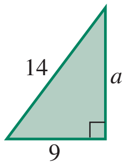


l

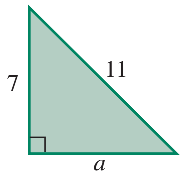


2 Find the value of the pronumeral expressed to 2 decimal places; all lengths are given in centimetres:

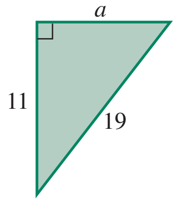
a



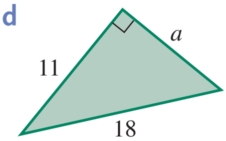
b



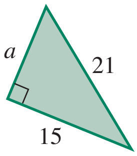
c



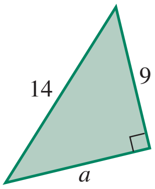
d



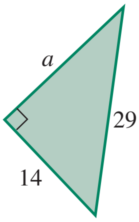
e



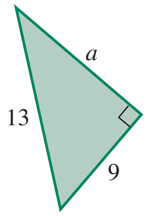
f



g

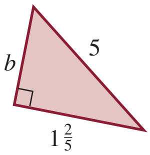


h



3 Find the value of b expressed in fraction form:

a



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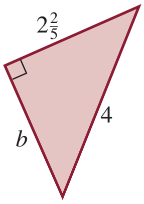

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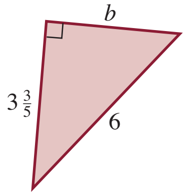

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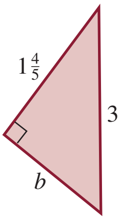

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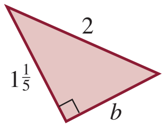

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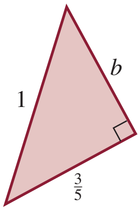

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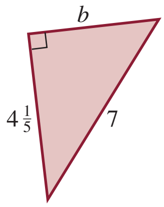

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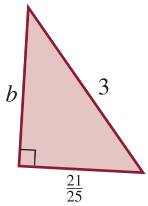
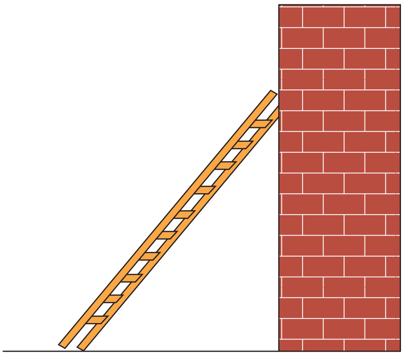


4 A 5-metre-long ladder rests up against a wall.



a How far up the wall will it reach when the foot of the ladder is the following distances from the base of the wall?

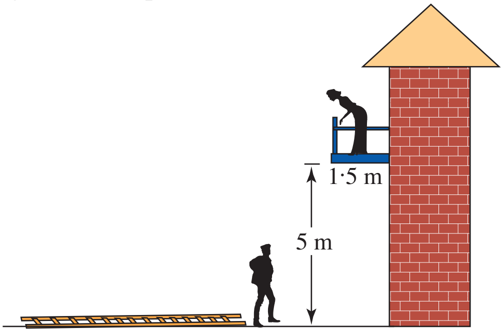
Give answers correct to the nearest centimetre.

- **i** 1 metre
- **ii** 2 metres
- **iii** 3 metres

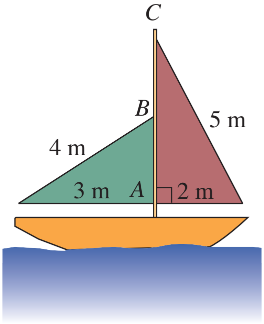
b How far from the foot of the wall will the ladder rest when it reaches the following distances from the bottom of the wall?

- **i** $1\frac{1}{2}$ metre
- **ii** $2\frac{1}{2}$ metres
- **iii** $3\frac{1}{2}$ metres

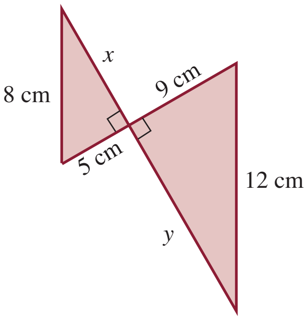
5 Romeo wants to give a rose to Juliet, who is standing on a balcony. He has a ladder which is 8 metres long and he plans to rest the ladder on the edge of the balcony. What distance from the base of the building should he place the foot of the ladder?



6 A red and a green sail are attached to a small boat. Find the height of each sail to the nearest centimetre.



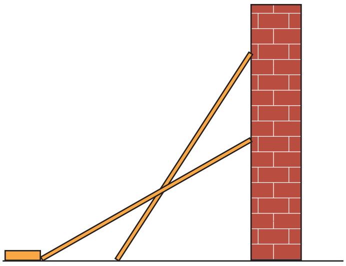
7 Find the total length of gold wire needed to make this brooch which is in the shape of two right-angled triangles. Give the answer to the nearest millimetre.



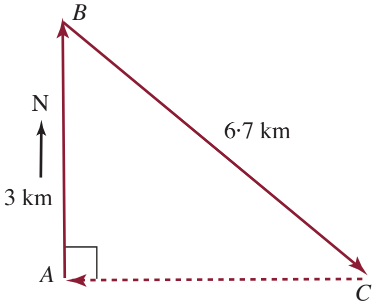
8 Mark all dimensions on the diagram then complete the question.

a A 12-metre-long ladder stands against a brick wall. If the foot of the ladder is 5 m from the base of the wall, how high up the wall will the ladder reach?

b If the foot of the ladder slips back 2 metres along the ground until it contacts a heavy box, find the height up the wall that the ladder now reaches, to the nearest centimetre.



9 Dovey walks to point B due north of point A, a distance of 3 km. He then proceeds to point C, which is due east of point A. Find the distance he needs to walk from point C directly to point A, to the nearest metre.



Pythagoras' theorem in exact form 11F

Irrational numbers such as $\sqrt{2}$ and $\sqrt{3}$ have decimal equivalents with an infinite number of decimal places and no repeating pattern; e.g. $\sqrt{2} = 1{\cdot}414\,213\,6\ldots$

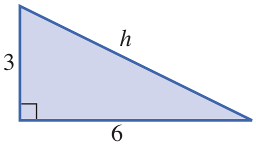
Rather than working with such inexact decimal numbers, it is better to write irrational numbers in their square root surd form. This is called giving an answer in an exact way or in **exact form** and the rules of surds can be used to fully simplify the answers.

Example

Solution

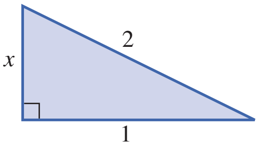
Find the length of the missing side expressed in exact form:

a



$$h^2 = a^2 + b^2$$
$$h^2 = 3^2 + 6^2$$
$$h^2 = 9 + 36$$
$$h^2 = 45$$
$$h = \sqrt{45} = \sqrt{9 \times 5}$$
$$h = 3\sqrt{5}$$

b

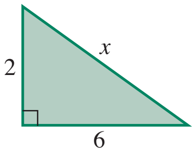


$$h^2 = a^2 + b^2$$
$$2^2 = x^2 + 1^2$$
$$4 = x^2 + 1$$
$$3 = x^2 \text{ or } x^2 = 3$$
$$\therefore\ x = \sqrt{3}$$

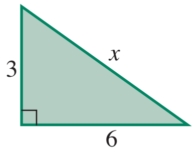
Exercise 11F

1 Find the lengths of the missing sides, expressing the answers in exact form:

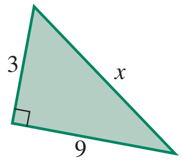
a



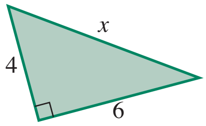
b



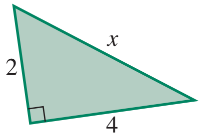
c



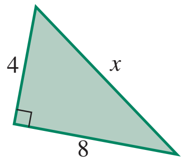
d



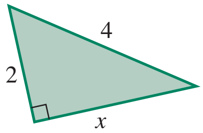
e



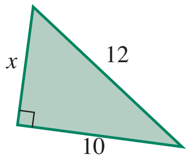
f



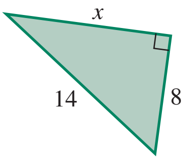
g



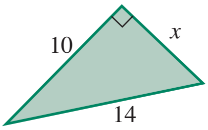
h



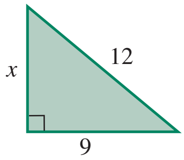
i



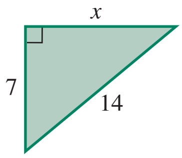
j



k

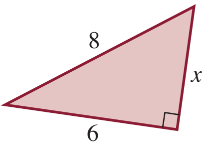


l

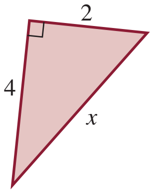


2 Find the lengths of the missing sides, expressing the answers in exact form:

a



b



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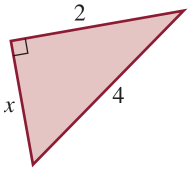
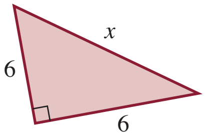
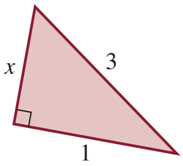


d



e



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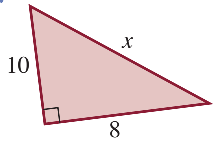
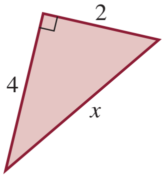
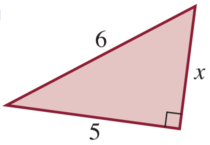


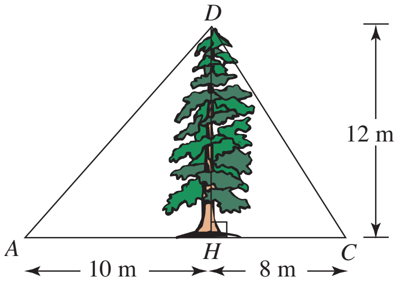
g



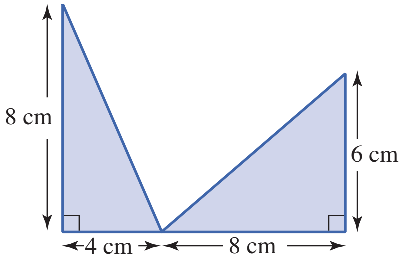
h



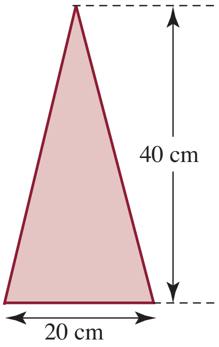
3 A 12-metre-tall tree is supported by two wires anchored to the ground 10 m and 8 m respectively from the trunk of the tree. Express the total length of the wires in exact form.



4 Find the perimeter of this shape expressed in exact form.

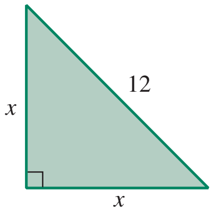


5 This isosceles triangle is 40 cm tall and has a base of 20 cm. Find the length of the sloping side and express the perimeter of the triangle in exact form.

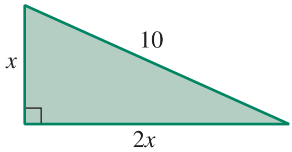


6 Find x:

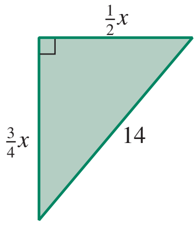
a



b



c

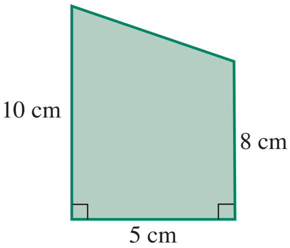


Composite shapes 11G

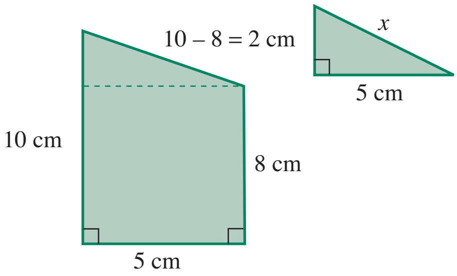
The things around us are often made up of a number of different shapes. When there are right-angled triangles, Pythagoras' theorem can be used to find unknown lengths. Divide the shapes into simple parts and work on each individually.

Example

1 Find the perimeter of this shape in exact form:



Solution



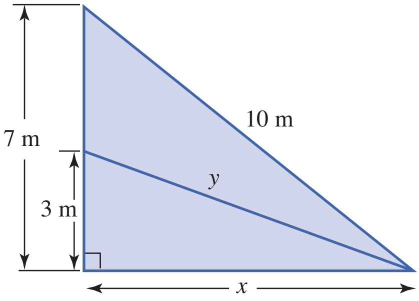
$$x^2 = 2^2 + 5^2$$
$$= 4 + 25 = 29$$
$$x = \sqrt{29}$$

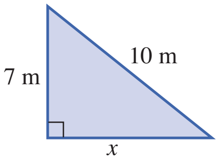
Perimeter of shape $= 10 + 5 + 8 + \sqrt{29}$
$= 23 + \sqrt{29}$

2 Find the lengths marked x and y:

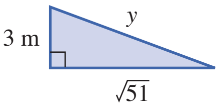
a in exact form

b correct to 1 decimal place





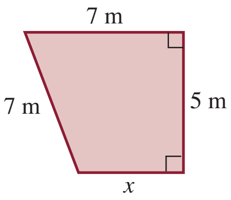
$$x^2 + 7^2 = 10^2$$
$$x^2 + 49 = 100$$
$$\therefore x^2 = 51$$
$$\therefore x = \sqrt{51}$$

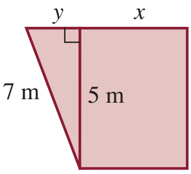


$$y^2 = 3^2 + (\sqrt{51})^2$$
$$= 9 + 51 = 60$$
$$\therefore y = \sqrt{60}$$
$$= 2\sqrt{15} \text{ cm}$$

$x = \sqrt{51} = 7{\cdot}1$ cm
$y = \sqrt{60} = 7{\cdot}7$ cm
correct to
1 decimal place

3 Find the length x in exact form and correct to 1 decimal place:





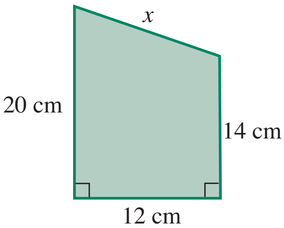
$$y^2 + 5^2 = 7^2$$
$$y^2 + 25 = 49$$
$$y^2 = 24$$
$$y = \sqrt{24}$$
$$= 2\sqrt{6}$$
$$x + 2\sqrt{6} = 7$$
$$\therefore x = 7 - 2\sqrt{6} = 2{\cdot}1 \text{ m}$$

11G

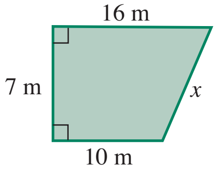
Exercise 11G

1 Find the length of the side marked x in the following shapes, expressing the answer to 2 decimal places:

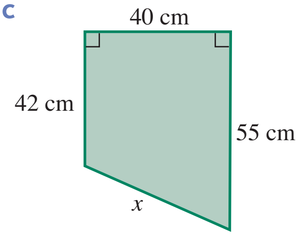
a



b

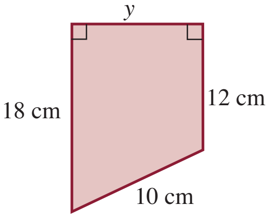


c

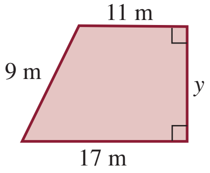


2 Find the length of the side marked y, expressed in exact form:

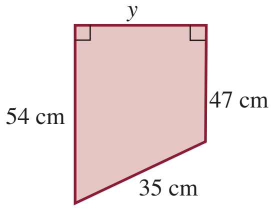
a



b

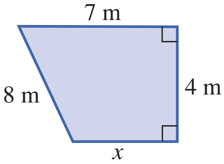


c

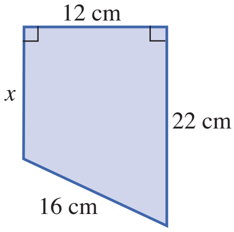


3 Find the length of the side marked x, expressed in exact form:

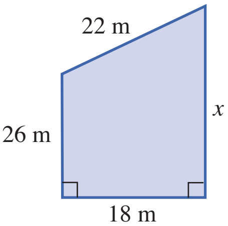
a



b



c

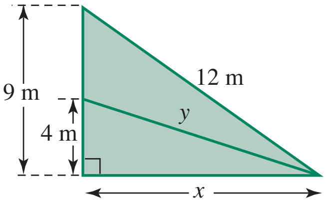


4 Find the length of the sides marked x and y:

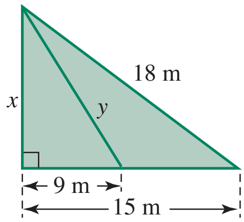
i in exact form

ii expressed correct to 2 decimal places

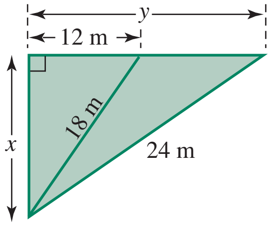
a



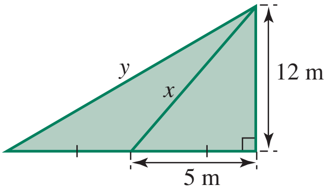
b



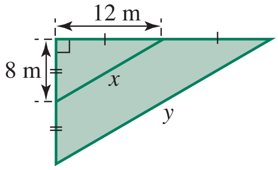
c



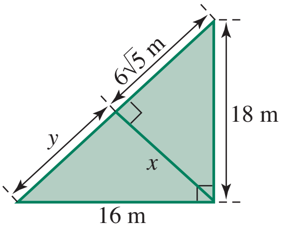
d



e



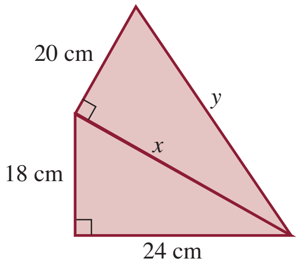
f



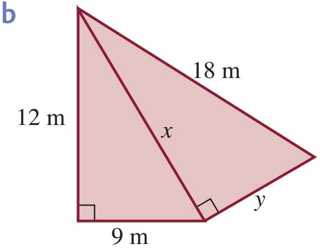
5 Find the length of the sides marked x and y:

i in exact form **ii** expressed correct to 2 decimal places

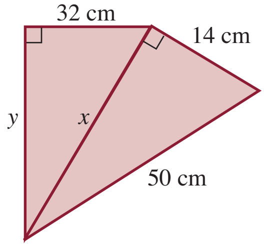
a



b

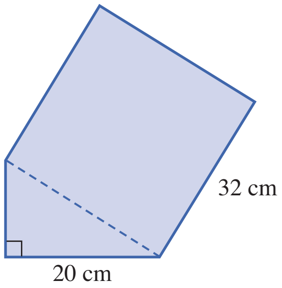


c

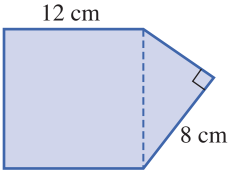


6 Find the perimeter of these shapes, which are made up of a right triangle and a square, expressing the answers in exact form:

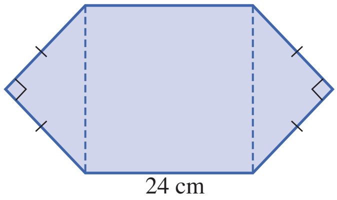
a



b

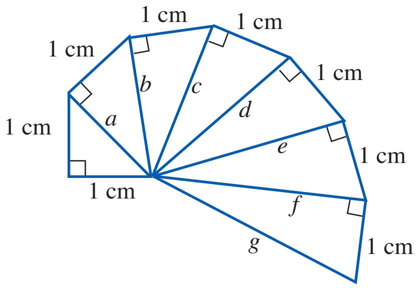


c

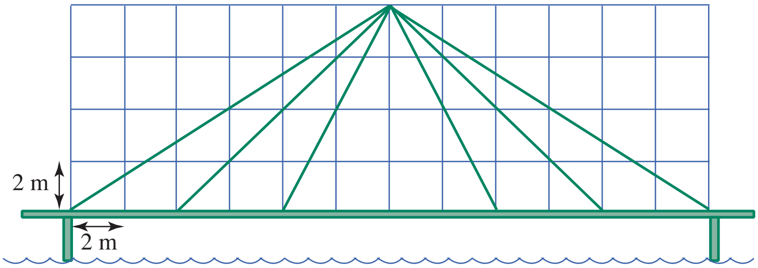


7 Find the values of the lines a–g in exact form and hence find the total area of the shape expressed:

i in exact form **ii** correct to 2 decimal places



8 This is a plan of a new steel section to be used in the construction of a bridge. If the grid lines are set at 2-metre intervals, find the total length of steel cable required to the nearest centimetre:

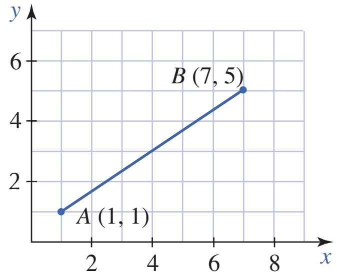


11H Coordinate geometry

Pythagoras' theorem can be used to find the distance between two points by constructing a right-angled triangle and finding the length of the hypotenuse.

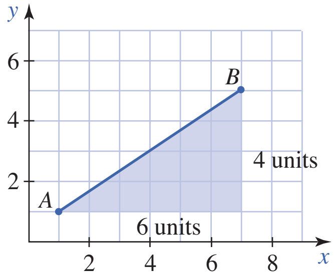
Example

Find the length of the line segment $\overline{AB}$ which joins points $A(1, 1)$ and $B(7, 5)$:



Solution

Make a right-angled triangle by using the lengths between the points, then use Pythagoras' theorem to find the length of $\overline{AB}$.

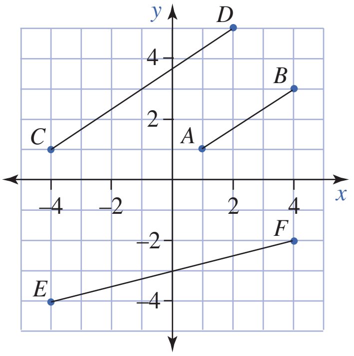


$$\begin{aligned} d\overline{AB} &= \sqrt{6^2 + 4^2} = \sqrt{36 + 16} = \sqrt{52} \\ &= 2\sqrt{13} \text{ units} \\ &= 2\sqrt{13} \approx 7{\cdot}21 \text{ units} \end{aligned}$$

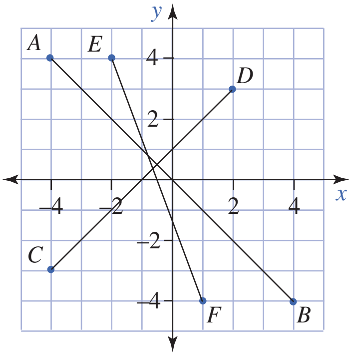
Exercise 11H

1 Find the length of the line segments joining the points marked on the axes, expressing the answers in exact form and correct to 2 decimal places:

a



b



2 Plot the following points onto a set of axes and find the distance between them, expressing the answers in decimal form correct to 2 decimal places:

a $A(1, 3)$ and $B(2, 6)$

b $C(2, 8)$ and $D(3, 6)$

c $E(0, 0)$ and $F(5, 8)$

d $F(1, 8)$ and $G(5, 4)$

e $H(–1, 2)$ and $J(3, 5)$

f $K(–2, 3)$ and $L(–4, 7)$

g $M(–2, 4)$ and $N(–6, –1)$

h $P(–3, –2)$ and $Q(–7, –6)$

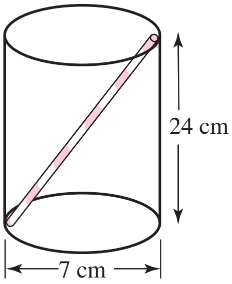
Pythagoras in three dimensions

11I

When working with solids in three dimensions, cross-sections need to be taken to find required lengths. These cross-sections often reveal right-angled triangles. It is most important to draw accurate and neat diagrams which show all known lengths relevant to the situation.

Example

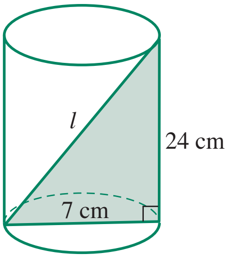
1 Find the length of the longest straw that can be placed into this cylinder with height 24 cm and diameter of 7 cm.



Solution

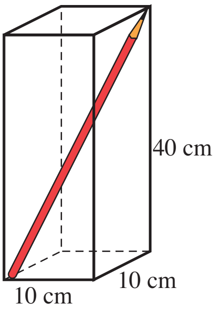
Use the length of the straw as the hypotenuse of the triangle and draw the right-angled triangle.

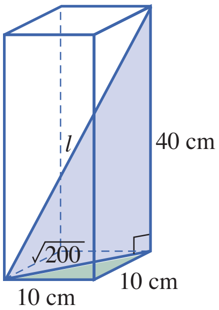
Apply Pythagoras' theorem to find the length of the straw:

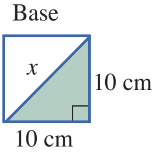


$$
\begin{aligned}
l^2 &= 24^2 + 7^2 \\
&= 625 \\
l &= \sqrt{625} \\
&= 25 \text{ cm}
\end{aligned}
$$

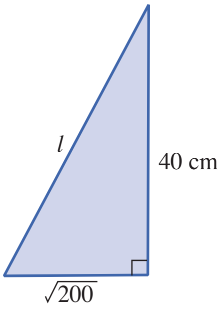
2 Find the length l of the longest pencil that would fit inside this pencil case.







$$
\begin{aligned}
x^2 &= 10^2 + 10^2 \\
&= 200 \\
x &= \sqrt{200} \\
&= 10\sqrt{2}
\end{aligned}
$$



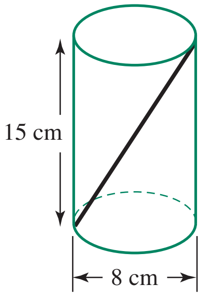
$$
\begin{aligned}
l^2 &= 40^2 + (\sqrt{200})^2 \\
&= 1600 + 200 \\
&= 1800 \\
l &= \sqrt{1800} \\
&= 30\sqrt{2} \\
&= 42\cdot4 \text{ cm}
\end{aligned}
$$

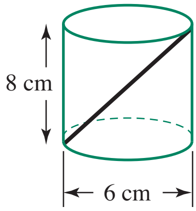
Longest pencil is approximately 42·4 cm.

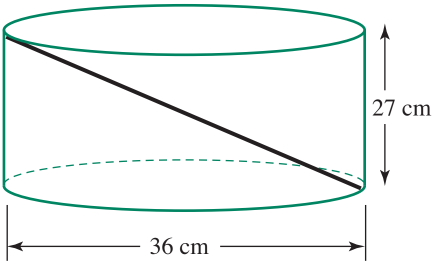
11I

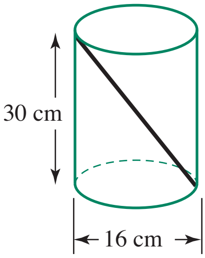
Exercise 11I

1 Calculate the length of the longest rods that will fit in these cylinders:

a


b


c


d


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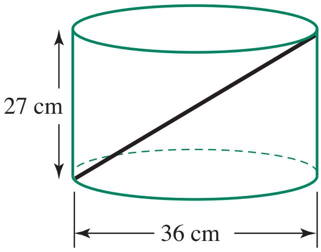
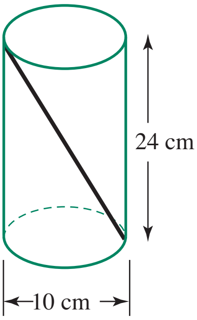
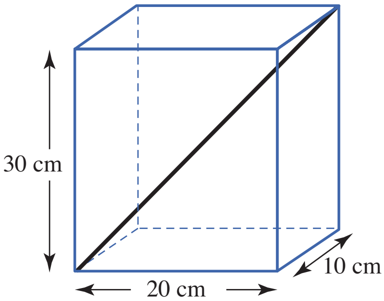
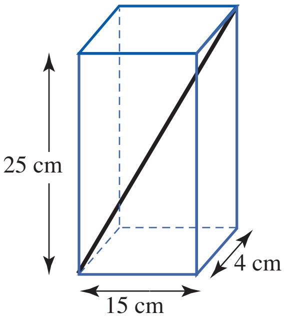
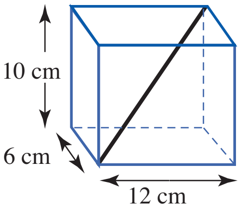


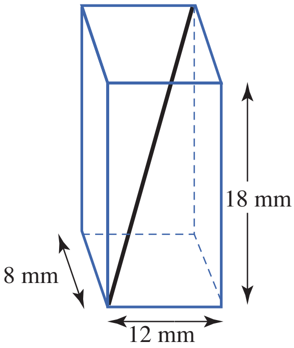
f


2 Calculate the length of the longest rod that will fit inside these boxes, expressing the answers in exact form and correct to 2 decimal places:

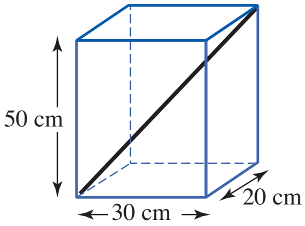
a


b


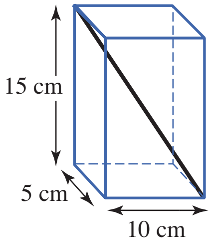
c


d


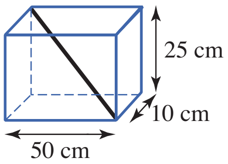
e



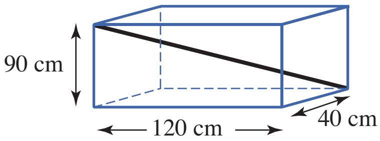
f



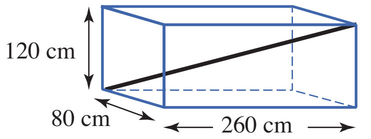
g



h



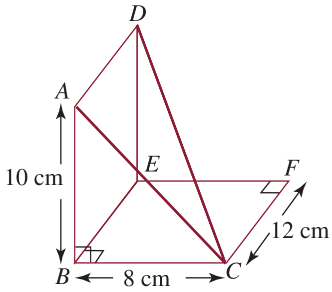
i



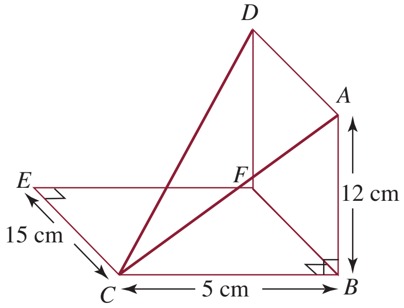
3 Calculate the length of the lines $\overline{AC}$ and $\overline{CD}$, expressed:

i in exact form **ii** correct to 2 decimal places

a



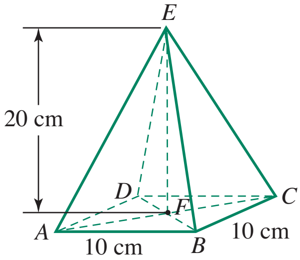
b



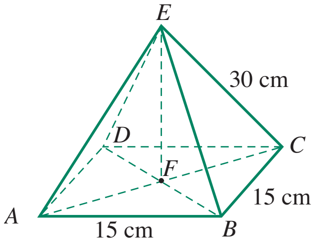
4 Find the length of the sides $\overline{AE}$, $\overline{AC}$, $\overline{AF}$ and $\overline{EF}$ in each of these solids, expressed:

i in exact form **ii** correct to 2 decimal places

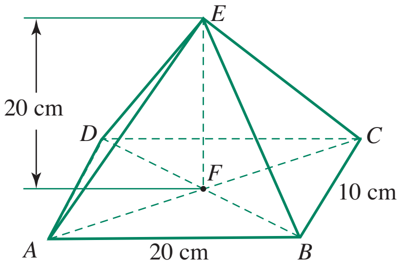
a



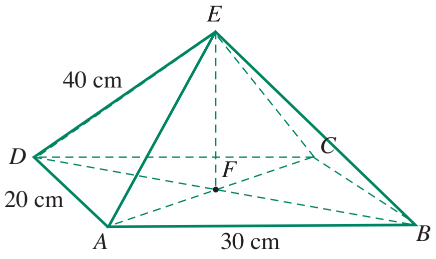
b



c



d

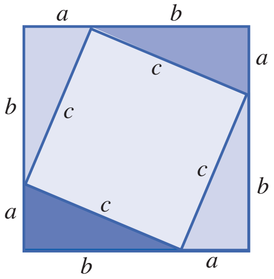


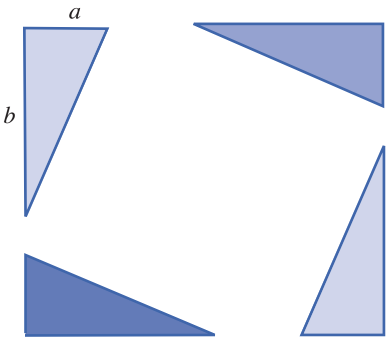
11J Exploring areas of triangles

Learning task 11J

1 The following two squares are the same size.

- This square has four identical triangles and a smaller square in the middle.





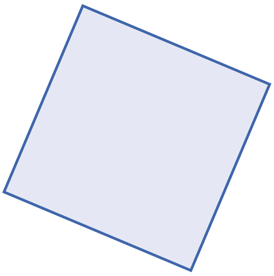
Area of triangle

$= \frac{1}{2} \times \text{base} \times \text{height}$

$= \frac{1}{2} \times a \times b = \frac{1}{2}ab$

There are four triangles, so area of triangles

$= 4 \times \frac{1}{2}ab =$ ______

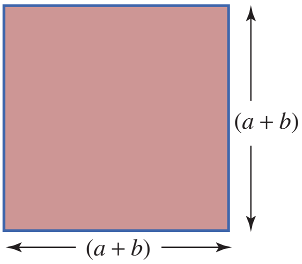


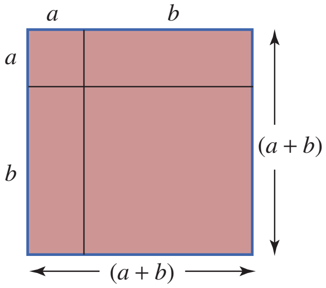
Area of smaller square

$= c \times c =$ _____

- Total area of square $= c^2 + 2ab$

- This square has side lengths of size $(a + b)$.





- Area of red square

$= (a + b)(a + b)$

$= a^2 +$ ______ $+$ ______ $+ b^2$

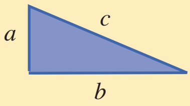
$= a^2 +$ ______ $+ b^2$

- The area of the original squares must be the same because the squares are identical:

$\therefore c^2 + 2ab = a^2 +$ ______ $+ b^2$

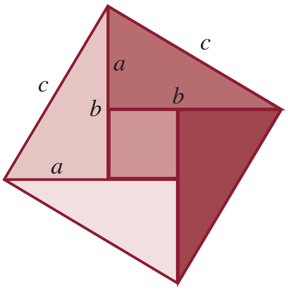
$c^2 = a^2 +$ _____ $+ b^2 -$ _____

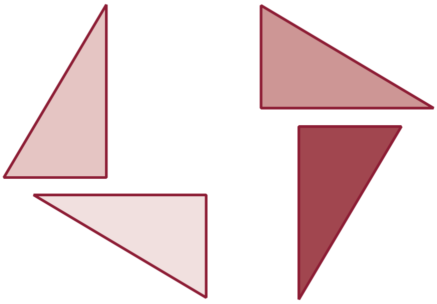
$c^2 =$ _____ $+$ ______



2 The following two squares are the same size.

- This square has four identical triangles and a small square in the middle.





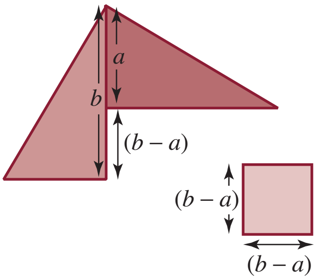
Area of triangle

$= \frac{1}{2} \times \text{base} \times \text{height}$

$= \frac{1}{2} \times a \times b = \frac{1}{2}ab$

There are four triangles, so area of triangles

$= 4 \times \frac{1}{2}ab =$ ______

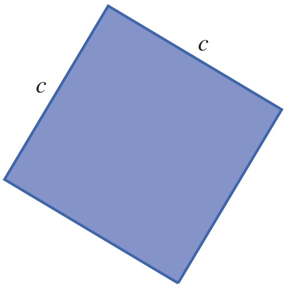


Area of small square

= ______ × ______ = ______

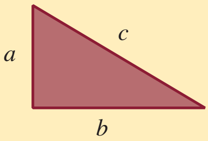
- Total area of the large square
  = area of triangles + area of small square
  = ______ + ______

- This square has sides lengths of size *c*.



- Area of the blue square
  = ______ × ______ = ______

- The area of the squares must be the same as the squares are identical.
  $\therefore c^2 =$ ______ + ______



Puzzles

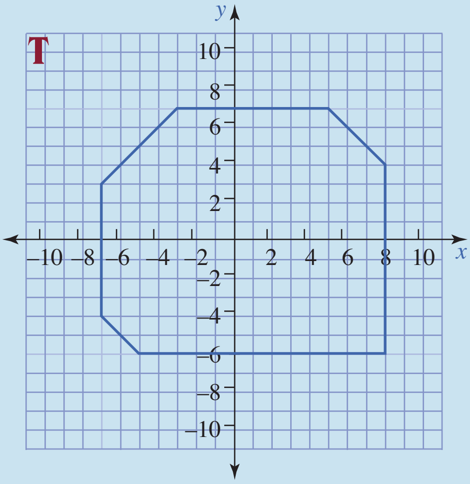
1 Find the value of the missing Pythagorean triad, then match the corresponding letter to the correct number below to find the answer to the riddle:

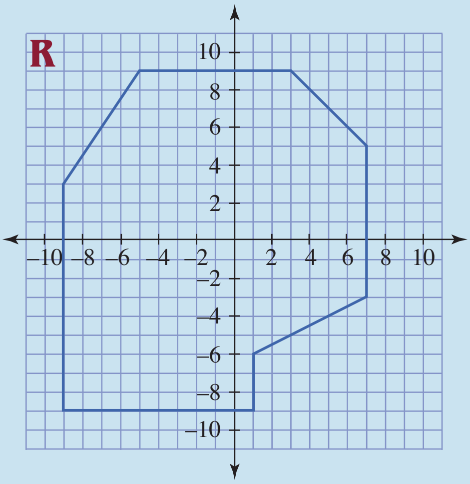
What is worse than finding a worm in your apple?

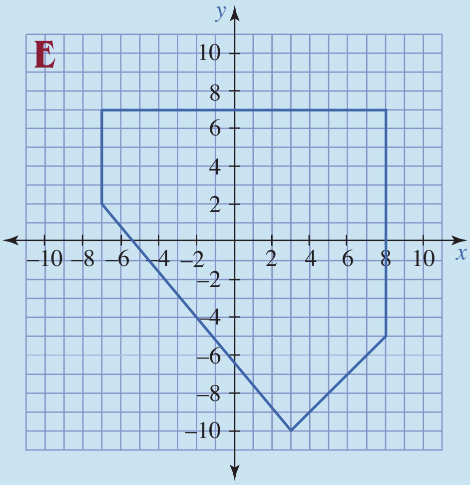
20, 99, **A**
23, 264, **D**
25, 60, **F**
27, 364, **G**
40, **H**, 401
30, **I**, 50
36, **L**, 325
16, **N**, 34
M, 36, 39
O, 360, 362
R, 15, 17
W, 4704, 4705

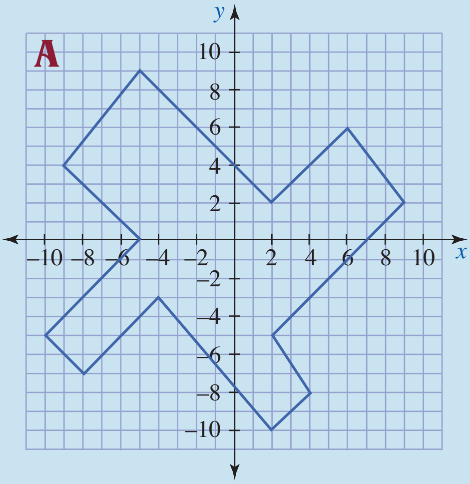
___	___	___	___	___	___	___				
65	40	30	265	40	30	365				
___	___	___	___		___		___	___	___	___
399	101	323	65		101		97	38	8	15

2 Calculate the perimeter of each shape below. Write the perimeters and the corresponding letters in ascending order. The letters spell out the name of the prefix meaning one million-million times: ___ ___ ___ ___

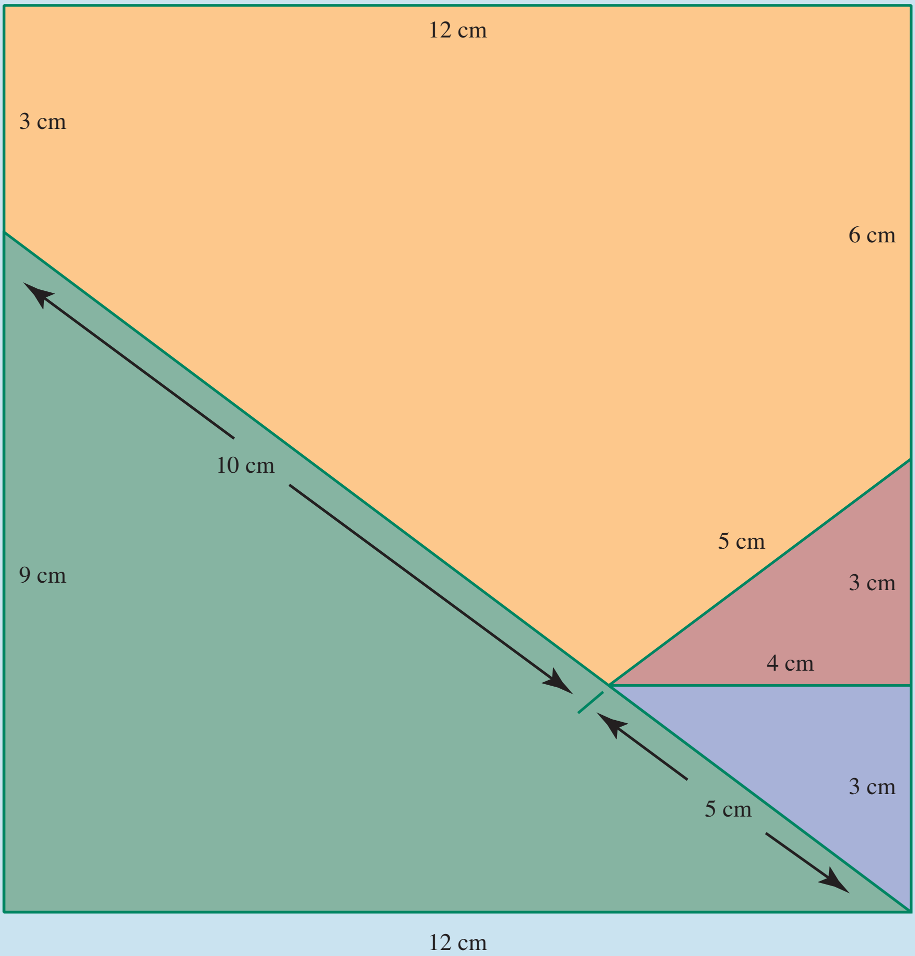








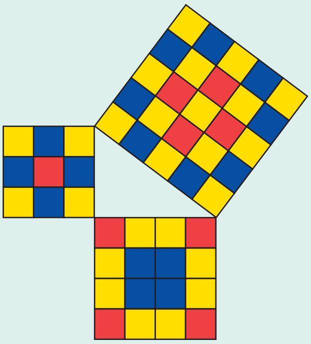
3 Copy the shapes below onto cardboard and cut out each triangle. Rearrange them to form a rectangle 9 cm by 16 cm. Show how Pythagoras' theorem works for the new shape. What other shapes can you make?



Applications

Pythagoras' theorem and areas on each side

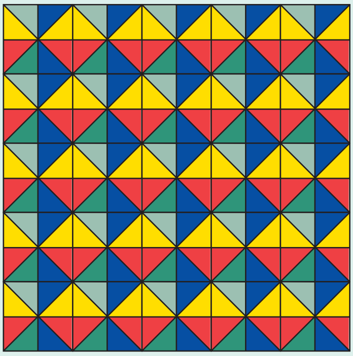
The theorem of Pythagoras describes the relationship between the sides of a right-angled triangle, using the areas of the square on each side.

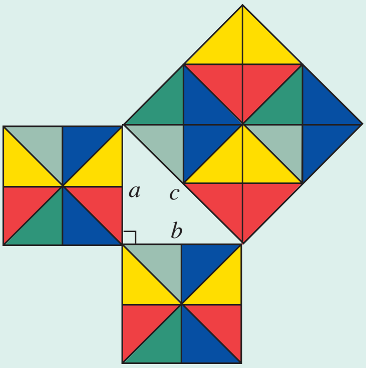


a Make a copy and cut out the squares shown which have been drawn on the perpendicular sides of the triangle. Glue them onto the large square which has been drawn on the hypotenuse of this special triangle.

Describe how this demonstrates Pythagoras' theorem for this triangle.

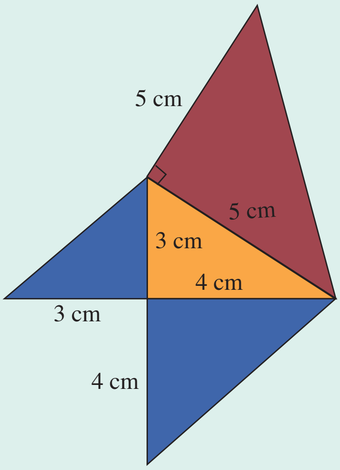
b Show how this ancient tiling pattern can be used to illustrate Pythagoras' theorem.



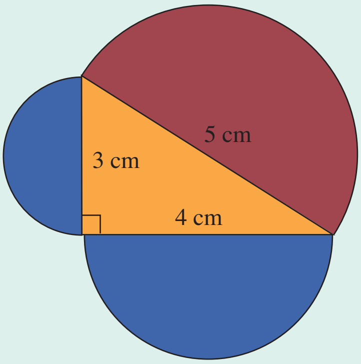


c Find the area of each of the blue shapes and show that their combined area is equal to the area of the red shape:

i



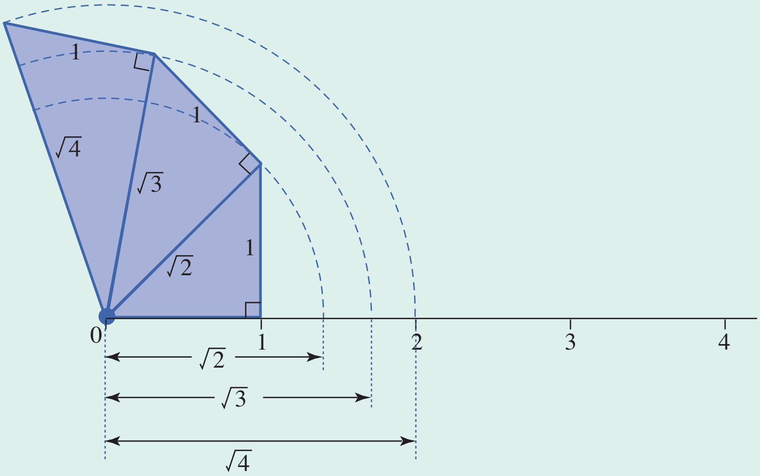
ii



Plotting irrational numbers on a number line

Irrational numbers such as $\sqrt{2}$ and $\sqrt{3}$ *cannot* be expressed as an exact decimal. By using right-angled triangles, lengths such as these can be placed onto a number line. This means that irrational numbers are part of the *real number system*, as they can be represented as lengths on a number line.

The lengths are made by using the 0 on the number line as the centre of a series of circles. The radii are determined by using a sequence of right-angled triangles stacked in the way shown below. Continue the sequence of triangles to place the numbers from $\sqrt{2}$ to $\sqrt{16}$ on a number line.

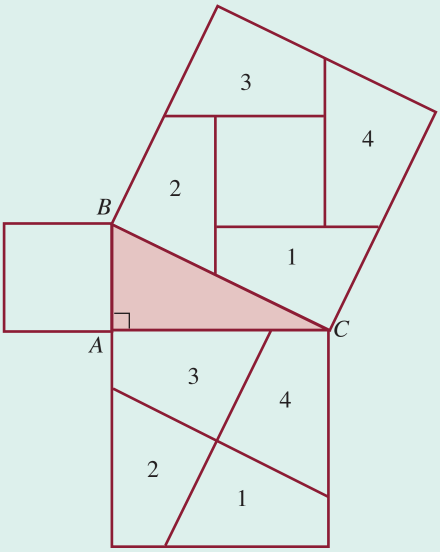


Perigal's dissection

On a piece of cardboard draw a right-angled triangle and construct squares on each side.

Follow these instructions to divide the square on *AC* into the four pieces shown.

- Find the centre of the square drawn on the line *AC*.
- Draw two lines through this centre point, one parallel to *BC*, the other perpendicular to *BC*.
- Cut out the squares on the sides *AB* and *AC*. Cut the square on *AC* into the four pieces numbered 1–4.
- Arrange these shapes as indicated on the square drawn on side *BC*.
- How does this illustrate Pythagoras' theorem?



Enrichment

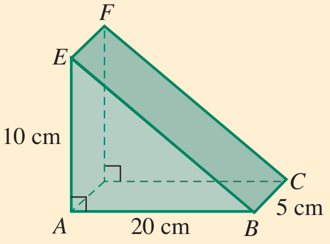
1 Find the edge length of each side of the cubes whose main diagonals are:

a 48 cm **b** 36 cm **c** 108 cm **d** 144 cm **e** 180 cm

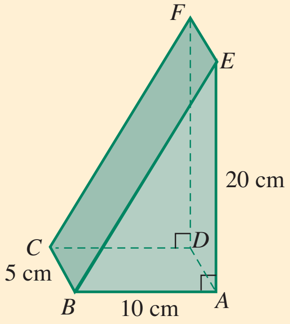
2 Find the lengths of the lines $\overline{BE}$, $\overline{BF}$, $\overline{AC}$, $\overline{AF}$, $\overline{BD}$, $\overline{ED}$ expressed:

i in exact form **ii** correct to 2 decimal places

a



b



3 The diagonals of a rhombus are 36 cm and 24 cm long.

a Draw a diagram of the rhombus, marking the known side lengths.

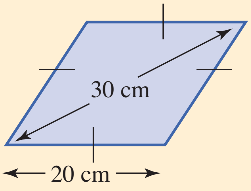
b Find the perimeter of the rhombus expressed in exact form and correct to 2 decimal places.

4 **a** Show that the triangle whose sides are 13, 13, 10 has the same area as the triangle whose sides are 13, 13, 24. What Pythagorean triad does this question use?

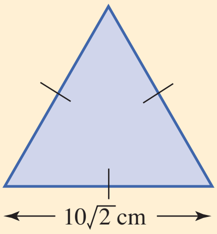
b Find other pairs of isosceles triangles with integral sides whose areas are equal.

5 Find the area of these regular shapes expressed in exact form:

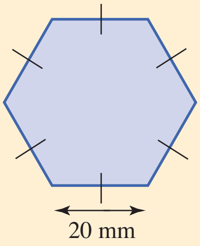
a



b

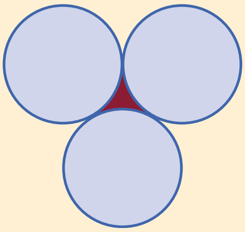


c

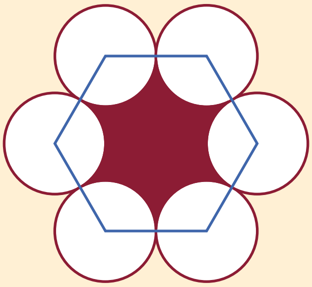


6 Find, in exact form, the red areas below, formed when circles with radius 12 cm are placed as shown:

a

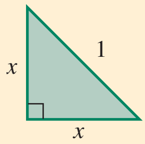


b



7 Find the lengths of the sides expressed in exact form with rational denominators where required:

a



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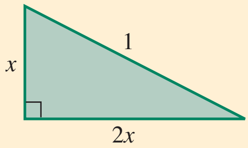

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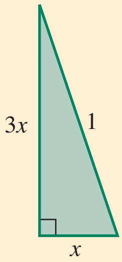
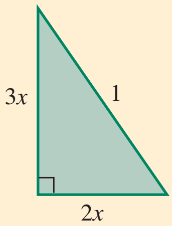


d



8 **a** Substitute values of n from 2 to 12 in the expressions $2n$, $n^2 - 1$, and $n^2 + 1$ in the table below.

b Show that the numbers are Pythagorean triads.

n	2	3	4	5	6	7	8	9	10	11	12
$2n$											
$n^2 - 1$											
$n^2 + 1$											

c Use algebra to show that the side lengths are those of a right-angled triangle $2n$, $n^2 - 1$, and $n^2 + 1$.

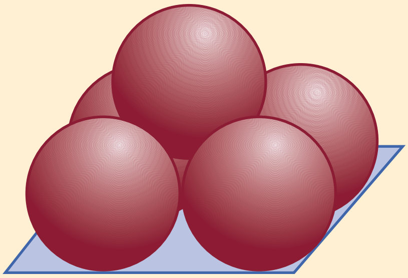
9 **a** Substitute values of n from 3 to 12 in the expressions $4n$, $n^2 - 4$, and $n^2 + 4$ in the table below.

b Show that the numbers are Pythagorean triads.

n	3	4	5	6	7	8	9	10	11	12
$4n$										
$n^2 - 4$										
$n^2 + 4$										

c Use algebra to show that the side lengths are those of a right-angled triangle $4n$, $n^2 - 4$, and $n^2 + 4$.

10 Four balls each with a radius of 10 cm are placed on a table so that each one contacts two others. A fifth ball of the same size is placed on top of them so that it contacts all four balls. Find the distance from the top of the fifth ball to the table, expressing the answer in exact form.

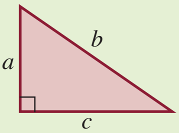


Revision/Assessment

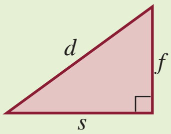
Exercise 11B

1 Write the equations which connect the sides of these triangles:

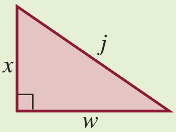
a



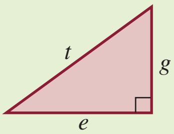
b



c



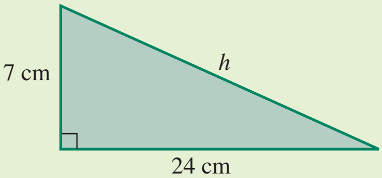
d



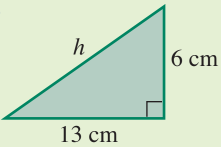
Exercise 11C

2 Find the length of the hypotenuse of the following triangles, expressed to 2 decimal places where necessary:

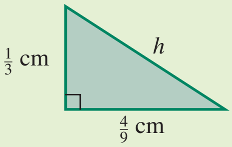
a



b

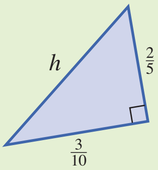


c

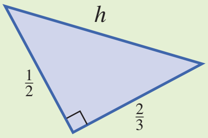


3 Find the length of the hypotenuse of each triangle, giving your answers in fraction form:

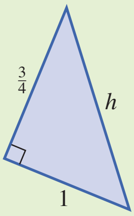
a



b



c



Exercise 11D

4 Show that the numbers 21, 28, 35 form a Pythagorean triad.

5 Find the missing number that makes 9, x, 41 a Pythagorean triad.

6 Using the Pythagorean triad 12, 35, 37, produce new triads which are:

i twice as large **ii** three times as large

Show that these new triads conform to the rule.

7 Using the following Pythagorean triads, produce new triads which are:

i twice as large **ii** five times as large

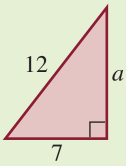
Show that these new triads conform to the rule.

a 16, 63, 65 **b** 24, 32, 40 **c** 13, 84, 85 **d** 11, 60, 61

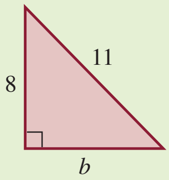
Exercise 11E

8 Find the value of the pronumeral expressed to 2 decimal places:

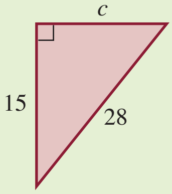
a



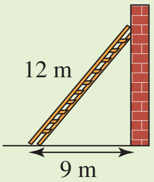
b



c

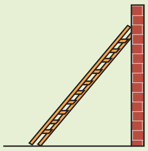


9 A ladder is 12 metres long and leans up against a tall wall. If the foot of the ladder is 9 metres from the base of the wall, how far up the wall will the ladder reach? Give the answer to the nearest centimetre.



10 A 7-metre-long ladder rests up against a wall. How far up the wall will it reach when its foot is the following distance from the base of the wall? Give the answers to the nearest centimetre:

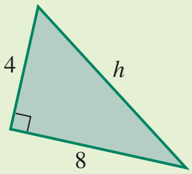
a 1 metre **b** 2 metres **c** 3 metres



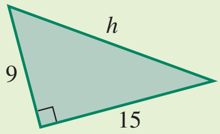
Exercise 11F

11 Find the lengths of the missing sides, expressed in exact or surd form:

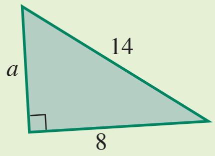
a



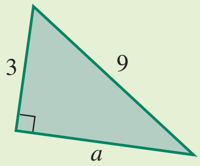
b



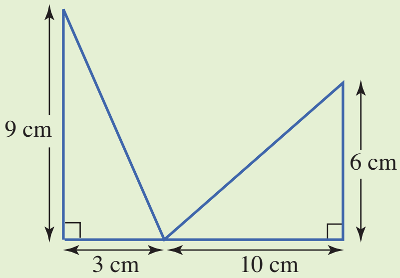
c



d



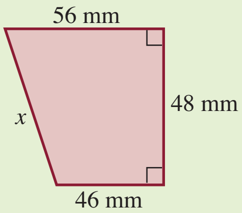
12 Find the perimeter of this shape, expressed in exact or surd form:



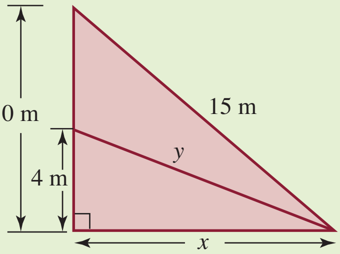
Exercise 11G

13 Find the length of the unknown sides, expressing the answers correct to 2 decimal places and in exact form:

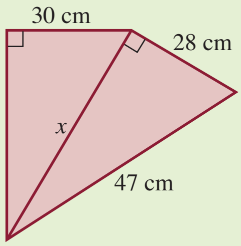
a



b



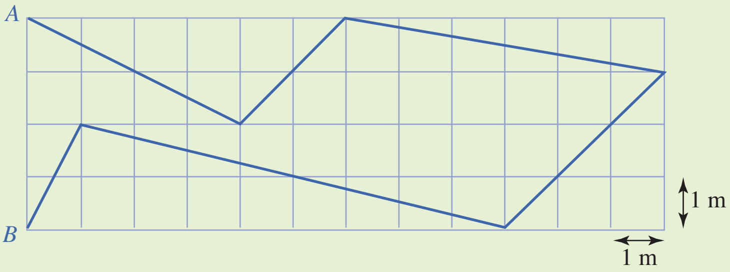
c



14 When blindfolded, Sally walks in a series of straight lines starting from A and finishing at B, shown on the 1-metre square grid below. Find the total distance that she walked expressed:

a in exact form

b correct to the nearest centimetre



Exercise 11H

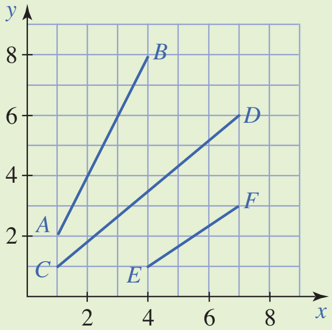
15 Find the length of the following line segments. Give the answers correct to 1 decimal place and in exact form:

i $\overline{AB}$

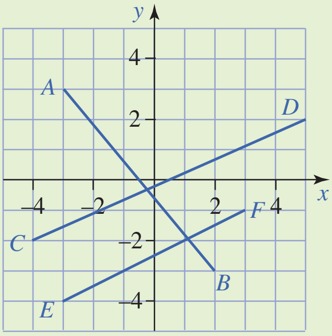
ii $\overline{CD}$

iii $\overline{EF}$

a



b



16 Find the length of the line segments between these points, expressing the answers in exact form:

a $A(1, -1)$ and $B(3, 5)$

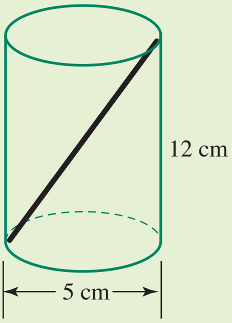
b $C(-1, 3)$ and $D(2, 8)$

c $E(1, -3)$ and $F(-2, 0)$

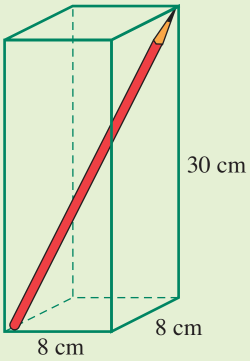
Exercise 11I

17 Find the length of the longest object that can be placed into these containers, expressing the answers to the nearest centimetre:

a

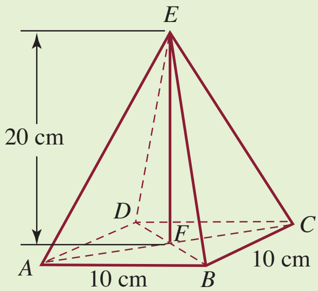


b



18 Find the length of the lines, expressing the answers in exact form:

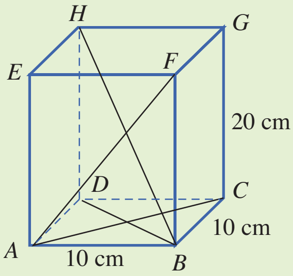
a $\overline{AC}$ **b** $\overline{AF}$ **c** $\overline{BD}$ **d** $\overline{BE}$



19 Find the length of the sides, $\overline{AC}$, $\overline{AF}$, $\overline{BH}$ and $\overline{BD}$ in these solids, expressed:

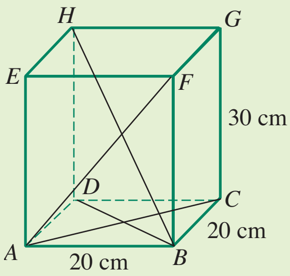
a in exact form

b correct to 2 decimal places

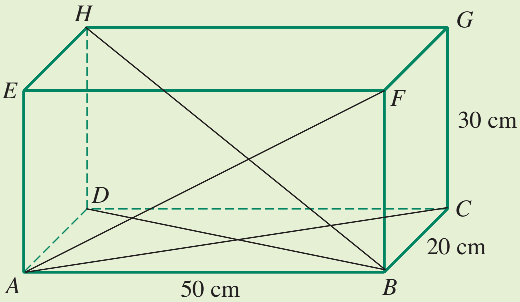


20 Find the length of the lines $\overline{AC}$, $\overline{AF}$, $\overline{BD}$ and $\overline{BH}$ in the following, expressing the answers in exact form:

a



b



CHAPTER 12

Probability

Probability

Travelling between Savo and Honiara using an OBM is a daily activity for many Savo islanders. The photograph shows some people about to push the boat out for a morning trip to Honiara. The sea is calm but before departure, the people would be wise to check the weather forecast to check whether it will be safe to make the crossing. They rely on an accurate prediction of the weather to help them decide whether it will be safe to travel today, or whether they need to postpone their trip until there is a better chance of fine weather. They use their knowledge and experience to calculate the probability of making it safely to Honiara and back home to Savo in a day.

This chapter covers the following skills:

- Using set notation
  - – $\in$ means is 'an element of'
  - – $\notin$ means is 'not an element of'
  - – ξ represents the universal set
  - – $\cap$ denotes the intersection of two sets
  - – $\cup$ denotes the union of two sets
  - – $\varnothing$ or $\{\ \}$ represents an empty or null set
- Using probability notation
  - – $\Pr(A) = \dfrac{n(A)}{n(\xi)}$ or

    $\Pr(A) = \dfrac{\text{number of elements } A}{\text{number of elements in the universal set}}$
  - – The probability of the complement of A is $\Pr(A')$ and $\Pr(A') = 1 - \Pr(A)$
- Drawing and interpreting Venn diagrams
- Displaying sample spaces using tree diagrams
- Using information statistics to predict the likelihood of an event
- Simulating experiments to estimate probabilities

Specific Learning Outcome (SLO)

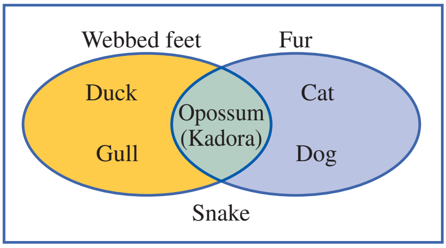
Learners should be able to:

8.12.1.1	Group objects or elements according to given criteria.
8.12.2.1	Group objects or elements into different classes or groups using circles.
8.12.3.1	Define the term 'sets'.
8.12.3.2	Group together elements to create sets.
8.12.4.1	Write sets of elements by using brackets and name them using capital letters.
8.12.5.1	Define and identify elements of sets.
8.12.5.2	Identify the symbol $\in$ as meaning 'an element of'.
8.12.6.1	Define the term 'universal set' and identify its symbol: ξ
8.12.7.1	Define the term 'complement of a set' and identify its symbol.
8.12.7.2	Identify elements that are complement to the given set.
8.12.8.1	Define the empty or null set and identify its symbol: $N = \{\ \}$ or $N = \varnothing$
8.12.9.1	Define 'intersection' and identify its symbol: $\cap$
8.12.9.2	Find elements that are intersected in given sets.
8.12.10.1	Define 'union' and identify its symbol: $\cup$
8.12.10.2	Find sets that are in union.
8.12.11.1	Find the probability of an outcome that is given in sets.
8.12.11.2	Calculate the probability of an outcome from sets using the formula $\Pr(A) = \dfrac{n(A)}{n(\xi)}$
8.12.12.1	Define and identify a sample space for compound events.
8.12.12.2	Use a simple grid to represent the total Sample Space for simple events.
8.12.13.1	Calculate the probabilities of compound events using grid.
8.12.14.1	Use a tree diagram to find probabilities of given events.
8.12.14.2	Construct tree diagram to display sample spaces of compound events and calculate their probabilities.
8.12.15.1	Find the probability of events occurring with playing cards.
8.12.16.1	Estimate probabilities using simulations.
8.12.16.2	Find the probabilities of events occurring in a sample proportionate to the total quantities, groups, or population.

12A Exploring sets

Items or elements collected together in groups are called sets. Elements can be grouped together according to certain criteria, so that they all have a common feature. One way of displaying sets of elements is a Venn diagram, in which a circle encloses all the elements in a set.

For example, this Venn diagram represents two sets: animals with webbed feet and animals with fur.



Learning task 12A

1 Working in pairs, select either the 'Creepies and crawlies' or the 'Sports' list.
  - a Write each of the items in the list on a separate piece of paper.
  - b Remove a group that has something in common.

    For example:
    Wasp, mosquito, fly and butterfly—all have wings
    Tennis, badminton and volleyball—all are played over a net
  - c Let your partner name the group and explain why they were chosen.
  - d Replace the removed cards.
  - e Now let your partner pull out a different group that has something in common.
  - f You name the group and explain why they were chosen.
  - g Are there any things in both groups?
  - h Are there any things in neither group?

Creepies and crawlies	Sports
■ Spider	■ Tennis
■ Fly	■ Football
■ Butterfly	■ Rugby
■ Worm	■ Cricket
■ Wasp	■ Netball
■ Bee	■ Basketball
■ Caterpillar	■ Soccer
■ Beetle	■ Basketball
■ Ant	■ Badminton
■ Mosquito	■ Volleyball

2 Display the two groups on an A3 sheet of paper using a circle for each group. Show where they overlap. Include any items that were not in either group, outside the circles.

3 Choose a third group from the list.
  - a Are there any items in all three groups?
  - b Are there any items in two groups?
  - c Are there any items in just one group?
  - d Are any items in none of the groups?

  Display the three groups on an A3 sheet of paper using circles for each group. Show where they overlap. Include any items that do not fit into a group, outside the circles.

Sets and Venn diagrams 12B

A **set** is a collection of things called elements. We can list all the elements of the set or use a description. We usually write a set in curly brackets and use a capital to denote the set.

Example	Solution
1 Write the letters of the alphabet as a set that is listed.	$A = \{a, b, c, d, e, f, g, h, i, j, k, l, m, n, o, p, q, r, s, t, u, v, w, x, y, z\}$
2 Write the letters of the alphabet as a set that is described.	$A =$ {letters of the alphabet}
3 Write the vowels as a set that is listed.	$V = \{a, e, i, o, u\}$
4 Write the vowels as a set that is described.	$V =$ {vowels}

The symbol $\in$ means is 'an element of' and the symbol $\notin$ means is 'not an element of'.

Example	Solution
5 Use set notation to show that the letter a is a vowel.	$a \in V$ We say that the letter a is an element of the set of vowels.
6 Use set notation to show that the letter b is not a vowel.	$b \notin V$ We say that the letter b is not an element in the set of vowels.

The set that includes all the possible outcomes is called the **universal set**. We use the symbol ξ to represent the universal set. In the example above A is the universal set.

The **complement** of a set is all the elements that are in the universal set but not in the given set. We use the symbol $'$ after the set name.

Example	Solution
7 List the set of letters that are not vowels.	$\xi =$ {letters in the alphabet} $V = \{a, e, i, o, u\}$ $V' = \{b, c, d, f, g, h, j, k, l, m, n, p, q, r, s, t, v, w, x, y, z\}$

A set with no elements is called an **empty set** or **null set** and we use the symbols $\varnothing$ or $\{\ \}$.

Example	Solution
8 List the set of numbers in the alphabet N.	$N = \{\ \}$ or $N = \varnothing$.

The symbol *n* is used to stand for 'the number of elements in'.

Example

9 State the number of elements in the set of vowels.

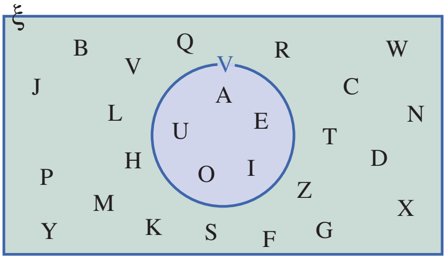
Solution

$V = \{a, e, i, o, u\}$

$n(V) = 5$

Venn diagrams can be used to display sets. The universal set, ξ, is a box and all other sets are circles. Elements that are members of the set go in the circle, and things that are not members of the set go outside the circle. The Venn diagram shown represents:

ξ = {letters in the alphabet}
V = {vowels}



When two sets have elements in common or overlap, we say that they intersect. The symbol used to represent an **intersection** is ∩.

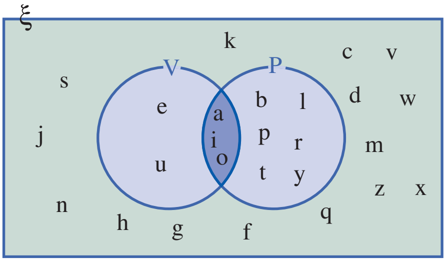
Example

10 Represent the set of letters in *V* and *P* in a Venn diagram and list the set $P \cap V$:

$V = \{\text{vowels}\} = \{a, e, i, o, u\}$

$P = \{\text{the letters in the word probability}\}$
$= \{p, r, o, b, a, i, l, t, y\}$

Solution



$P \cap V = \{a, i, o\}$

The combination of two sets is called the **union** and is denoted by the symbol ∪.

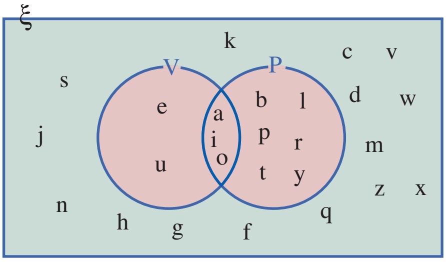
Example

11 Represent the set of letters in *V* and *P* in a Venn diagram and list the set $P \cup V$:

$V = \{\text{vowels}\} = \{a, e, i, o, u\}$

$P = \{\text{the letters in the word probability}\}$
$= \{p, r, o, b, a, i, l, t, y\}$.

Solution

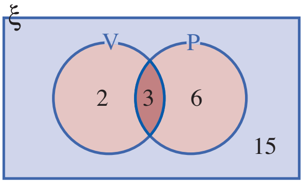


$P \cup V = \{a, b, e, i, l, o, p, r, t, u, y\}$

You do not always need to list the elements. In some cases, the number of elements in each set is required.

Example

12 State the number of elements in the sets: $V, P, P \cup V, P \cap V$.



Solution

$n(V) = 5$
$n(P) = 9$
$n(P \cup V) = 11$
$n(P \cap V) = 3$

Exercise 12B

1 State True or False to the following:

- **a** $\text{Banana} \in \{\text{fruits}\}$
- **b** $\text{Spider} \in \{\text{insects}\}$
- **c** $4 \in \{\text{even numbers}\}$
- **d** $\text{Soccer} \in \{\text{Olympic sports}\}$
- **e** $5 \notin \{\text{prime numbers}\}$
- **f** $\text{Peru} \notin \{\text{European countries}\}$

2 Let $M = \{\text{months}\}$
$T = \{\text{months with exactly 30 days}\}$
$R = \{\text{months with the letter r in their name}\}$

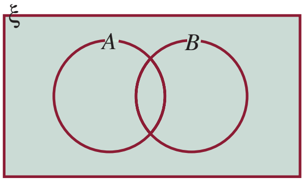
- **a** List M, the universal set. Find $n(\xi)$.
- **b** List the elements of T. Find $n(T)$.
- **c** List all the elements of R. Find $n(R)$.
- **d** List the elements of T'. Find $n(T')$.
- **e** List all the elements of R'. Find $n(R')$.

3 Let $N = \{\text{natural or counting numbers from 1 to 20}\}$ $O = \{\text{odd numbers from 1 to 20}\}$
$E = \{\text{even numbers from 1 to 20}\}$ $P = \{\text{prime numbers from 1 to 20}\}$

- **a** List the elements in N, O, E and P.
- **b** List the elements in:
  - **i** $P \cap E$
  - **ii** $P \cap O$
  - **iii** P'
  - **iv** O'
- **c** Find:
  - **i** $n(P \cap E)$
  - **ii** $n(P \cap O)$
  - **iii** $n(P')$
  - **iv** $n(O')$
- **d** What word could be used to describe the set $O \cap E$?
- **e** What is the set $O \cup E$?

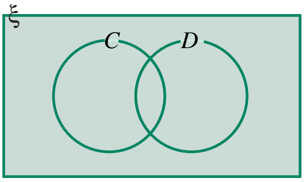
4 Copy the Venn diagram shown and shade the following:

- **a** A
- **b** B
- **c** $A \cap B$
- **d** $A \cup B$
- **e** A'
- **f** B'



5 **a** Copy the Venn diagram and place the numbers in the sets below into the correct sets:

$\xi = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$
$C = \{1, 2, 3, 4\}$
$D = \{2, 4, 6, 8\}$



b List the sets:

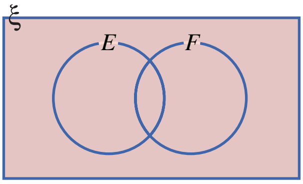
i $C \cap D$ **ii** $C \cup D$ **iii** C' **iv** D'

c Find:

i $n(C \cap D)$ **ii** $n(C \cup D)$ **iii** $n(C')$ **iv** $n(D')$

6 **a** Copy the Venn diagram and place the elements below into the correct sets:

$\xi = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$
$E = \{1, 3, 5, 7, 9\}$
$F = \{5, 6, 7, 8\}$



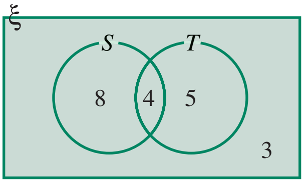
b List the sets:

i $E \cap F$ **ii** $E \cup F$ **iii** E' **iv** F'

c Find:

i $n(E \cap F)$ **ii** $n(E \cup F)$ **iii** $n(E')$ **iv** $n(F')$

7 The Venn diagram below shows the sports preferences for a class of Year 8 learners. Use it to find the following:



S = soccer players
T = tennis players

a How many learners are in the class?

b How many learners play:

i soccer?
ii only tennis?
iii soccer and tennis?
iv neither soccer nor tennis?

8 Sue is a triathlete and trains every day of the week. She swims on Monday, Wednesday and Friday. She cycles on Tuesday, Friday, Saturday and Sunday.

a Represent this information as a Venn diagram.

b Use your Venn diagram to find:

i the days on which Sue swims and cycles
ii the days on which Sue just swims
iii the days on which Sue just cycles
iv the day on which Sue neither swims nor cycles.

9 Sam is a runner and trains every day of the week. He jogs each day during the week and works out in the gym on Tuesdays, Thursdays and on the weekend.

a Represent this information as a Venn diagram.

b Use your Venn diagram to find:

i the days on which Sam jogs and works out in the gym
ii the days on which Sam jogs only
iii the days on which Sam works out in the gym only
iv the days on which Sam has no exercise.

Sets and probability 12C

Set notation can be used to find the probability of an outcome. We find probabilities by considering all the equally likely possibilities. To work out the probability of an event we then need to consider what fraction of possibilities gives a favourable result.

The probability of obtaining an element in set A at random is written as $\Pr(A)$:

$$\Pr(A) = \frac{n(A)}{n(\xi)} = \frac{\text{number of elements in set } A}{\text{number of elements in the universal set}}$$

Example

1 If $V = \{\text{vowels}\}$, and $\xi = \{\text{the letters in alphabet}\}$, find the probability of randomly selecting a vowel from the set of letters in the alphabet.

Solution

$V = \{\text{a, e, i, o, u}\}$

$n(V) = 5$

$n(\xi) = 26$

The probability of a vowel is written as $\Pr(V)$.

$$\Pr(V) = \frac{n(V)}{n(\xi)} = \frac{\text{number of elements in set } V}{\text{number of elements in the universal set}} = \frac{5}{26}$$

2 A letter is chosen at random from the letters of the alphabet. What is the probability that a letter chosen is in the word probability?

$P = \{\text{the letters in the word probability}\}$
$= \{\text{p, r, o, b, a, i, l, t, y}\}$

$$\Pr(P) = \frac{n(P)}{n(A)} = \frac{9}{26}$$

3 A letter is chosen at random from the letters of the alphabet. What is the probability that a letter chosen is not a vowel?

$V' = \{\text{b, c, d, f, g, h, j, k, l, m, n, p, q, r, s, t, v, w, x, y, z}\}$

$$\Pr(V') = \frac{n(V')}{n(\xi)} = \frac{21}{26}$$

We are certain that a letter chosen at random is either a vowel or not a vowel. There are no other options, so $\Pr(V') + \Pr(V) = 1$ or $\Pr(V') = 1 - \Pr(V)$.

$$\Pr(V') = 1 - \Pr(V) = 1 - \frac{5}{26} = \frac{21}{26}$$

12C

We can use Venn diagrams to assist us with some more difficult probability problems.

Example **Solution**

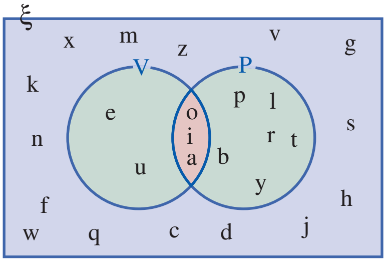
4 If a letter is chosen at random from the alphabet, find:

a $\Pr(V \cap P)$, which is the probability that a letter is a vowel and in the word probability.

$$\Pr(V \cap P) = \frac{\text{number in the overlap}}{\text{total in the Venn diagram}} = \frac{3}{26}$$

b $\Pr(V \cup P)$, which is the probability that a letter is a vowel or in the word probability.

$$\Pr(V \cap P) = \frac{\text{total number in the circles}}{\text{total in the Venn diagram}} = \frac{11}{26}$$



Exercise 12C

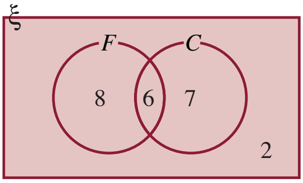
1 If a letter is chosen at random from the alphabet, find:

- **a** Pr(letter is in the word Guadalcanal)
- **b** Pr(letter is a consonant)
- **c** Pr(letter is not a consonant)

2 If a die is thrown, find:

- **a** Pr(score is even)
- **b** Pr(score is a prime number)
- **c** Pr(score is even and prime)
- **d** Pr(score is not prime)

3 The Venn diagram shows the languages studied by a class of Year 8 learners. Use it to find the following:



F = French
C = Chinese

- **a** How many learners are in the class?
- **b** How many studied French?
- **c** How many studied only Chinese?
- **d** What is the probability that a learner studies Chinese?
- **e** What is the probability that a learner doesn't study French?
- **f** What is the probability that a learner studies neither French nor Chinese?

4 Let ξ = {months}
R = {months with the letter r in their name}
Y = {months with the letter y in their name}

a Display this in a Venn diagram.

b Use your Venn diagram to find:

i $\Pr(R \cap Y)$ **ii** $\Pr(R \cup Y)$

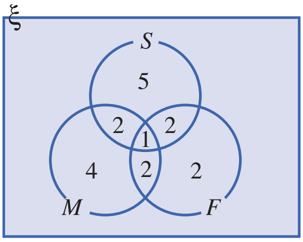
5 Let ξ = {natural or counting numbers from 1 to 50}
T = {multiples of 3}
F = {multiples of 4}

a Represent this information as a Venn diagram.

b If a number is chosen at random, find:

i $\Pr(T)$ **ii** $\Pr(F)$ **iii** $\Pr(T')$
iv $\Pr(F')$ **v** $\Pr(T \cap F)$ **vi** $\Pr(T \cup F)$

6 The Venn diagram shows the results of a survey of how Year 8 learners spent their weekend.



S = played sport
M = went to movies
F = visited family

a How many learners were surveyed?

b How many did all three activities?

c How many only went to the movies?

d What is the probability that a learner played sport?

e How many learners visited family or played sport but didn't go to the movies?

f What is the probability that a learner didn't play sport?

7 Let ξ = {numbers from 1 to 30}
Let T = {factors of 40}
Let F = {factors of 24}
Let S = {factors of 60}

If a number is chosen at random, find:

a $\Pr(T)$ **b** $\Pr(F)$ **c** $\Pr(S)$ **d** $\Pr(T')$ **e** $\Pr(F')$
f $\Pr(S')$ **g** $\Pr(T \cap F)$ **h** $\Pr(T \cap S)$ **i** $\Pr(T \cap F \cap S)$

8 A survey of 40 households found that 25 read the *Solomon Star* newspaper, 18 read the *Island Sun* newspaper and 5 read both newspapers.

a Represent this in Venn diagram.

b How many households read neither paper?

c What is the probability that a household reads both newspapers?

d What is the probability that a household reads either the *Solomon Star* or the *Island Sun* newspapers?

e What is the probability that a household didn't read the *Island Sun* newspaper?

f What is the probability that a household reads the *Island Sun* but not the *Solomon Star* newspaper?

g What is the probability that a household reads only one newspaper?

12D Sample space grids

Gambling, using games of chance with playing cards or dice, encouraged mathematicians to develop the laws of probability. Gambling using dice was very popular in the Solomon Islands during colonial days. They used one six-sided die to gamble by separating the six numbers into two sets: {1, 2, 3} and {4, 5, 6}. All players chose a number to bet on. If, for example, the roller of the die bet on the number 5 and it came up, then he won all the bets from players who bet on 1, 2 or 3. However, if a player bet a 4 or 6, he had his bet returned.

Another game required players to roll two dice. Bets could be won by either rolling a double or a total of 7. The probabilities of winning this game can be calculated by drawing a grid. The grid below represents the sample space when two 6-sided dice are rolled. Rolls that have a total of 7 are marked with a circle. The doubles are marked with a cross.

1st die	1	2	3	4	5	6
2nd die 1	×	•	•	•	•	○
2	•	×	•	•	○	•
3	•	•	×	○	•	•
4	•	•	○	×	•	•
5	•	○	•	•	×	•
6	○	•	•	•	•	×

The probability of a throw with a total of 7:

$$\Pr(\text{total of } 7) = \frac{n(\text{total of } 7)}{n(\xi)} = \frac{6}{36} = \frac{1}{6}$$

The probability of a double:

$$\Pr(\text{double}) = \frac{n(\text{double})}{n(\xi)} = \frac{6}{36} = \frac{1}{6}$$

The probability that a sum of the score on two dice is 10 can be found in the same way.

1st die	1	2	3	4	5	6
2nd die 1	•	•	•	•	•	•
2	•	•	•	•	•	•
3	•	•	•	•	•	•
4	•	•	•	•	•	×
5	•	•	•	•	×	•
6	•	•	•	×	•	•

The probability of a total of 10

$$= \Pr(\text{total of } 10) = \frac{n(\text{total of } 10)}{n(\xi)} = \frac{3}{36} = \frac{1}{12}$$

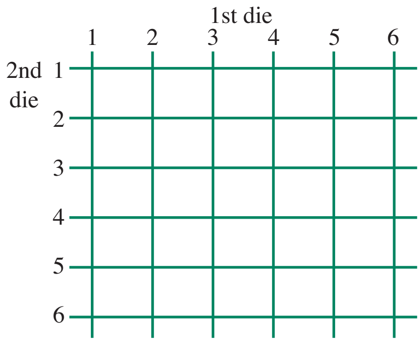
Grids can be used to represent the sample space of compound events where two events occur, such as rolling two dice, tossing two coins, choosing two random numbers or dealing two cards. Compound events with more than two events are usually represented by a tree diagram, which is discussed in the next section.

Grids can be simplified to show the possible outcomes on the top and side of the grids. Elements in the sample space are represented by the intersections of the lines.

Example

1 Use a simplified grid to represent the sample space when two dice are rolled.

Solution

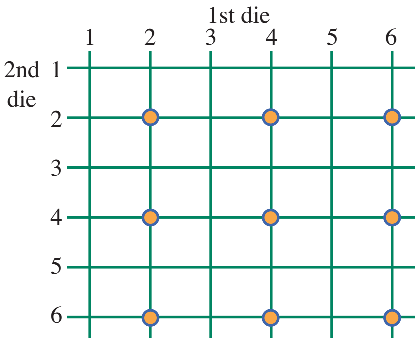


2 Find the probability of obtaining two even numbers.

Pr(evens)

$$= \frac{9}{36}$$

$$= \frac{1}{4}$$



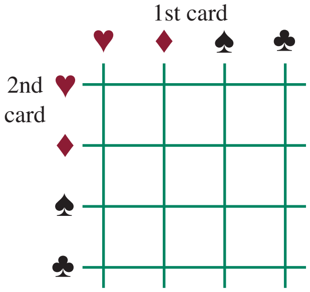
Exercise 12D

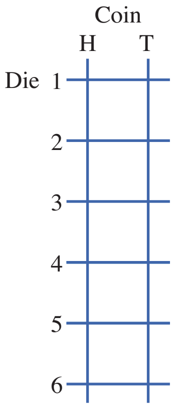
1 Use the grid in Example 1 to find the probability of rolling with two dice:

i a total of 7 ii a double iii a total of 7 and a double

2 This grid represents the sample space when two cards are drawn from a pack of 52 playing cards with replacement and the suit is noted. Use the grid to find the probability of:

- a two hearts
- b two red cards
- c two cards of the same suit
- d two cards of different suits
- e a spade and a club in that order
- f a spade and a club in any order
- g a heart and a black card in any order





3 This grid shows all the possibilities when a coin is tossed and a die is thrown. Use the grid to find the probability of:

- a a tail and a 5
- b a head and an odd number
- c a tail and an even number
- d a tail on the coin
- e a 3 on the die
- f a number greater than 4 and a tail
- g a number less than 2 and a head

4 There are two stacks of cards. One of the stacks has cards numbered from 1 to 4, the other has cards numbered from 1 to 10. A student is asked to choose a card from each pile.

- a Draw a grid to display the 40 possible outcomes.
- b Use your grid to find these probabilities:
  - i Each card shows the same number.
  - ii The sum of the numbers on the cards is 9.
  - iii The sum of the numbers on the cards is greater than 10.
  - iv The number 3 is on a card.
  - v The numbers have a difference of 5.
  - vi The numbers have a product of 12.

5 During Health Week held at the Central Hospital Grounds, people are encouraged to eat a lot of fruits and vegetables. The campaign suggests that if people eat one item from each group of fruits and vegetable each day, this will make a big difference in their lives as far as their health is concerned.
Group A—Vegetables: beans, cucumber, tomato, melon and eggplant
Group B—Fruits: banana, pawpaw, pineapple, guava and orange

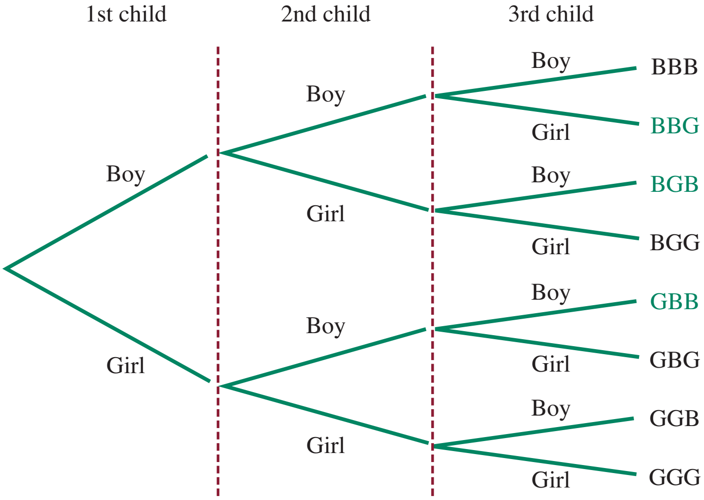
- a Display in a grid all the possible choices for a person.
- b Use your grid to find the probability that a random person will choose the following combinations of vegetables and fruits, assuming all choices are equally likely.
  - i beans and pawpaw
  - ii cucumber and banana
  - iii tomato and guava
  - iv melon and pineapple
  - v eggplant and orange

6 In the game of craps, people bet on the sum of the score on two dice. The player wins if the total score on the dice is 7 or 11.

- a Draw a grid to represent two dice being thrown.
- b Use the grid to predict the probability of winning at craps.

Tree diagrams

Tree diagrams are often used to display the sample space of compound events of three or more events. The branches of the tree represent all possible outcomes for each event. The tree diagram below represents all possible outcomes in a three-child family.



Example

Use the tree diagram above to find the probability of having only one girl in three-child family.

Assume there is an equal chance of having a boy or a girl.

$\Pr(G) = \Pr(B) = \frac{1}{2}$

Solution

When we travel along the branches we find there are 8 different combinations for three-child families:

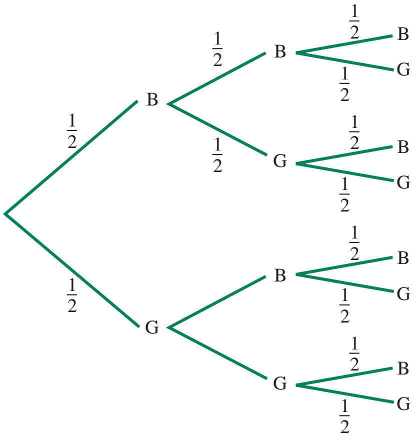
BBB
BBG
BGB
BGG
GBB
GBG
GGB
GGG

Only 3 of the 8 were combinations with only one girl.

Pr(one girl in a three-child family) = $\frac{3}{8}$

Solution

Alternatively, the probability can be found using a weighted tree diagram:



$$\Pr(\text{BBG}) = \tfrac{1}{2} \times \tfrac{1}{2} \times \tfrac{1}{2} = \tfrac{1}{8}$$

$$\Pr(\text{BGB}) = \tfrac{1}{2} \times \tfrac{1}{2} \times \tfrac{1}{2} = \tfrac{1}{8}$$

$$\Pr(\text{GBB}) = \tfrac{1}{2} \times \tfrac{1}{2} \times \tfrac{1}{2} = \tfrac{1}{8}$$

Pr(one girl in a three child family)

$$= \tfrac{1}{8} + \tfrac{1}{8} + \tfrac{1}{8} = \tfrac{3}{8}$$

Exercise 12E

1 Copy the tree diagram shown in the example above and extend it to show all the possible outcomes for 4-child families. What is the probability of a family with:

a 4 boys? **b** only one girl? **c** at least one girl?

2 At a wedding the following foods were available and served randomly to the guests.

Starter: pumpkin soup or corn soup
Main: fish or chicken or beef
Dessert: fruit salad or ice cream or banana cake

a Show all the possible combinations by using a tree diagram.

b How many different possible arrangements are available?

c How many arrangements have banana cake as a dessert?

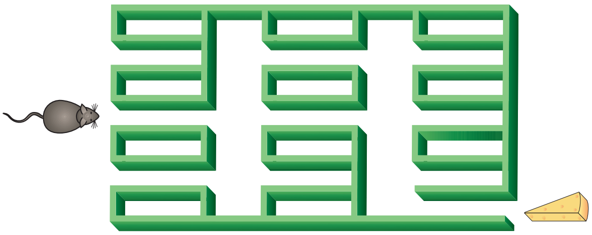
d If one person is chosen at random, what is the probability that they have:

- **i** a banana cake as a dessert?
- **ii** beef as a main course?
- **iii** pumpkin soup as a starter?
- **iv** ice cream as a dessert and fish as a main?
- **v** pumpkin soup or corn soup?

3 At a job interview, an applicant was asked three multiple-choice questions each with three possible answers, A, B and C. Assuming that the applicant had to guess the correct answers (B, C, A), what is the probability she got:

a all correct answers? **b** all wrong answers? **c** exactly one correct answer?
d at least one correct answer? **e** two correct answers?

4 In a laboratory experiment mice are put into the maze to see if they are able to find the cheese. They are unable to turn round in a closed passage. What is the probability that a mouse would reach the cheese by randomly selecting a passage?



Exploring a pack of cards 12F

In games of cards we often use a standard pack of cards. In a standard pack of cards:

- the 52 cards are divided into four suits: hearts, diamonds, spades and clubs
- the hearts and diamonds are red
- the clubs and spades are black
- each suit is made up of 13 cards: 3 face cards and 10 numbered cards
- the numbered cards are from 1 to 10
- the cards numbered 1 are called aces
- the face cards are jacks, queens and kings.

Learning task 12F

1 If one card is drawn at random from a pack of cards, find the following probabilities:

a Pr(a heart)
b Pr(a face card)
c Pr(a black card)
d Pr(not a spade)
e Pr(a red king)
f Pr(the queen of hearts)
g Pr(not a queen)
h Pr(a number card)
i Pr(card is a number above 5)
j Pr(an ace)
k Pr(a black ace)
l Pr(not a red ace)
m Pr(a black number card)
n Pr(a red, even numbered card)

2 If two cards are drawn at random from a pack of cards, with replacement, find the probability of the following events. Use a grid to display the sample space if necessary:

a Pr(two face cards)
b Pr(a face card and a number card)
c Pr(two diamonds)
d Pr(two cards not diamonds)
e Pr(two red cards)
f Pr(a spade and a diamond)
g Pr(two kings)
h Pr(king and queen in that order)
i Pr(two number cards)
j Pr(two number cards that add to 5)

3 If three cards are drawn at random from a pack of cards, with replacement, find the probability of the following events. Use a tree diagram to display the sample space if necessary:

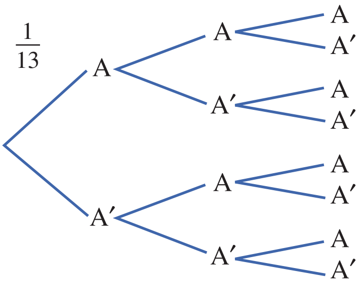
a Pr(three face cards)
b Pr(three clubs)
c Pr(three aces)
d Pr(three cards not aces)
e Pr(three red cards)
f Pr(king, queen and jack in that order)
g Pr(three number cards)
h Pr(a face card and two number cards)

4 The tree diagram shown represents the possible outcomes when three cards are dealt from a pack of 52 cards with replacement. There is a $\frac{1}{13}$ chance of selecting an ace, A.

a What is the probability of not selecting an ace?

b Copy the tree diagram and write the probabilities of the outcomes on each branch of the tree.

c Multiply the probabilities along each branch to find the probability of obtaining three aces.



12G Estimating proportions using sampling

Sampling is a technique used when we want to obtain some information about a large group and we are only able to investigate part of the group. We can also use sampling to find proportions in probability. This application of probability is often used in quality control in industry.

Example

A bag contains 500 lollies; some are red and some are black. A student takes a sample of 20 lollies from the bag in order to estimate the number of red lollies in the bag.

In the sample there are 12 red lollies and 8 black lollies.

a What is the probability of choosing a red lolly from the sample?

Solution

Pr(red lolly in the sample)

$$= \frac{\text{number of red lollies in the sample}}{\text{total number of lollies in the sample}}$$

$$= \frac{12}{20}$$

$= 0{\cdot}6$

b What would be a good estimation for the proportion of red lollies in the bag?

We would expect that the probability of a red lolly in the sample would be about the same as the probability of a red lolly in the bag.

Proportion of red lollies in the bag
= Pr(red lollies in the sample)
$= 0{\cdot}6$
= 60%

c How many red lollies would you expect there to be in the bag?

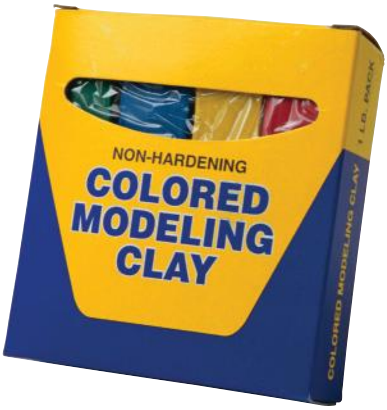
Expected number of red lollies in the bag
= 60% of 500
$= 0{\cdot}6 \times 500$
= 300

This method gives us an estimation of the number of red lollies in the bag. The only way we would know for sure is to count them all. Our estimate would be more accurate if we took a larger sample.

Quality control in manufacturing plants relies on sampling to test the quality of the manufacturing process.

Samples of the products are taken at regular intervals to check on standards. For example, the number of matches in a box or the amount of soft drink in a bottle is tested to ensure that defects are kept to a minimum.

Exercise 12G



1 A machine that makes modelling clay produces red, yellow, blue and green clay. The new owner is unsure as to what proportion of each colour is produced and takes a sample of 60 pieces of clay. In the sample there were 24 red, 12 yellow, 18 blue and 6 green.

   a How many of each colour would you expect to find in a pack of 12 pieces of clay?

   b How many of each colour would you expect to find in a box of 500 pieces of clay?

   c What is the probability that the machine produces a piece of red clay?

2 A jar contains 750 lollies; some of them are red and some are yellow. A sample of 15 lollies had 6 red and 9 yellow. (These were then returned to the jar.)

   a What proportion of the lollies in the sample were red?

   b What proportion of the lollies in the jar would you expect to be red?

   c How many of the lollies in the jar would you expect to be red?

   d How many of the lollies in the jar would you expect to be yellow?

3 A child takes a bag of 40 lollies from the jar in Question 2. How many yellow lollies would she expect to have?

4 A school principal needed to know how many girls were at the school of 1800 learners. He was unable to count them so instead took a sample of 30. In the sample there were 18 boys and 12 girls. Based on this sample, what would be a good estimate for the number of girls in the school?

5 A toy manufacturer needed to check the quality of the toys produced. There are two possible faults in the toys: faulty wire connections or missing parts. The quality controller takes a sample of 40 toys and finds one with faulty wire connections, two with missing parts and one that has both faults.

   a What is the probability that a randomly selected toy has:

      i faulty wire connections?

      ii missing parts?

      iii both faulty wire connections and missing parts?

      iv has a fault?

   b A customer orders 500 toys; how many toys would be expected to have:

      i faulty wire connections?

      ii missing parts?

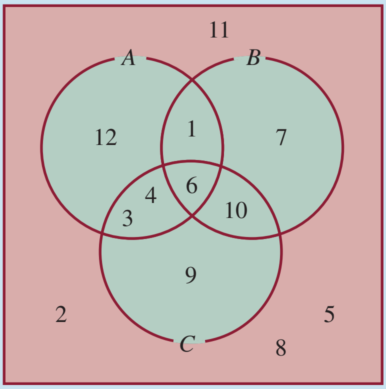
      iii both faulty wire connections and missing parts?

      iv a fault?

   c Another customer orders 800 toys but will return them if there are more than 100 with faults or more than 10 with both faults. Do you think the order will be returned? Explain your answer.

Puzzles

1 The red letters represent sets of numbers shown in the Venn diagram. Match the correct letter to the answer below to find the answer to the question:



What is the name of the fifth geometric body with 12 regular pentagons, discovered by Hippasos, who was consequently drowned by the Pythagoreans?

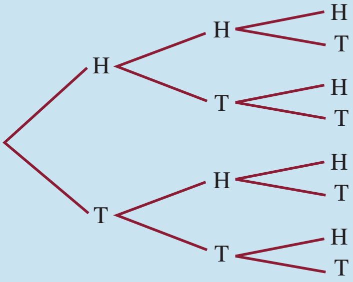
A {1, 3, 4, 6, 12}
C {1, 6, 7, 10}
D {3, 4, 6, 9, 10}
E {3, 4, 6}
L {1}
O {1, 2, 5, 7, 8, 11, 12}
R {2, 5, 7, 8, 9, 10, 11}
G {6, 10}
N {6}
P {1, 3, 4, 6, 7, 10, 12}
T {1, 3, 4, 6, 9, 10, 12}
H {1, 6}

$A \cup B$	$A \cap C$	$A \cap B \cap C$	$A \cup C$		A	$B \cap C$	C'	$A \cap B \cap C$	A	$A \cap B \cap C'$	
C	C'	C	$A \cap C$	B	A	$A \cap B$	$A \cap C$	C	A'	C'	$A \cap B \cap C$

2 A coin is tossed three times. The tree diagram represents the eight possible outcomes. Find the probability of the three throws being as shown below.
Match the correct letter to the answer below to find the answer to the question:

What term is used to represent something that is based on the number 60?

A Pr(3 same)
G Pr(HHT in that order)
L Pr(0, 1, 2 or 3 H)
S Pr(0, 1 or 3 H)
E Pr(2 H)
I Pr(at least 1 H)
M Pr(first is H)
X Pr(4 H)

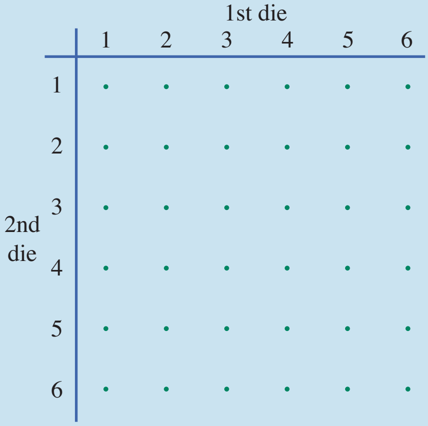


$\frac{5}{8}$	$\frac{3}{8}$	0	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{5}{8}$	$\frac{7}{8}$	$\frac{1}{2}$	$\frac{1}{4}$	1

3 Two dice are rolled. The grid below represents the 36 possible outcomes. Find the probability that the two numbers are as shown below. Match the letter to the correct probability, to answer the question:

What is the value of the number 2×10^{27} in words?

C Pr(both even)
I Pr(add to 6)
L Pr(add to 11 or more)
N Pr(difference of 1)
O Pr(product of 10)
S Pr(first > second)
T Pr(different)
W Pr(add to 12)



$\frac{5}{6}$	$\frac{1}{36}$	$\frac{1}{18}$		$\frac{1}{18}$	$\frac{1}{4}$	$\frac{5}{6}$	$\frac{5}{36}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{5}{36}$	$\frac{1}{18}$	$\frac{5}{18}$	$\frac{5}{12}$

4 Two cards are drawn at random from a pack of 52 playing cards, with replacement. Find the probability of the outcomes shown below. Match the letter to the correct probability, to answer the question:

Who commanded the building of the ancient Egyptian city of Alexandria in about 350 BC?

A Pr(2 ♥)
D Pr(2 aces)
E Pr(2 red aces)
G Pr(2 black cards)
H Pr(2 picture cards)
L Pr(ace then a red king)
N Pr(queen ♥ then black card)
R Pr(2 cards same)
T Pr(red or black)
X Pr(3 queen ♥)

$\frac{1}{16}$	$\frac{1}{338}$	$\frac{1}{676}$	0	$\frac{1}{16}$	$\frac{1}{104}$	$\frac{1}{169}$	$\frac{1}{676}$	$\frac{1}{52}$
1	$\frac{9}{169}$	$\frac{1}{676}$		$\frac{1}{4}$	$\frac{1}{52}$	$\frac{1}{676}$	$\frac{1}{16}$	1

Applications

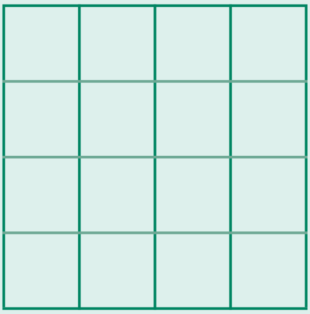
Simulations and sampling

Use playing cards to estimate the probability that two randomly selected learners are born on the same day of the week. Take a sample of pairs of friends in your class and record how many were born on the same day. Compare the results.

Four-child families

Use four coins to simulate an experiment to estimate the probability that children in a four-child family are all the same sex. Draw a tree diagram to find the theoretical probability that children in a four-child family are all the same sex and compare your results.

Multiplication Bingo



a Draw up a 4 by 4 Bingo card.

b Fill the table with numbers from 1 to 16.

c The bingo caller throws two dice and calls the numbers on the dice.

d Cross out the product of the numbers if it is on your card. Record the numbers.

e Keep playing until the first person crosses out all their numbers.

f Check your card. What numbers are left? Was it impossible to get any of the numbers on your card? Which numbers were more likely to come up?

No cream buns at the Hot Bread Shop

Imagine you are planning to visit the Hot Bread Shop to buy some cream buns. Sometimes, the buns are not ready and other times they are sold out. What is the probability that you will buy buns the next time you visit the Hot Bread Shop?

- Use one die.
- Let the number 1 and 2 represent the buns are not ready, 3 & 4 represent they have sold out, and 5 & 6 represent you successfully buy buns on that visit.
- Throw the die 50 times and record the number of 5s and 6s you roll.
- Calculate the probability you will buy a cream bun on your next visit to the Hot Bread Shop.

Who reads what?

Survey a sample of 50 families in your school to find out which newspapers they read in their homes. Estimate the proportions of the newspapers sold locally. Check the results of your survey with your local newspaper seller.

Colour combinations

Sam owns three shirts and three pairs of jeans as shown. The shirts are identical except for the colours, which are red, blue and green. The jeans are also identical except for the colours, which are red, blue and green. Sam grabs a shirt and a pair of jeans in the dark.

a Draw a tree diagram to show how many colour combinations are possible.

b What is the probability that he will wear a shirt and jeans of the same colour?

c Sam believes the old saying 'green and blue should never be seen unless there is a colour in between'. What is the probability that he is wearing green and blue?

Flags of the world

Choose a continent or region of the world and find the flags of the countries in that continent or region.

a Classify the flags according to similar features. For example, use colours, patterns such as stars, stripes, bands or logos. You should use two or three different classifications and place the flags into sets.

b Use a Venn diagram to display the different flags. Prepare a poster of your results and display it on the wall of your classroom.

c Design a flag for new country in the region. Use different colours and designs so that the flag is clearly distinguishable.

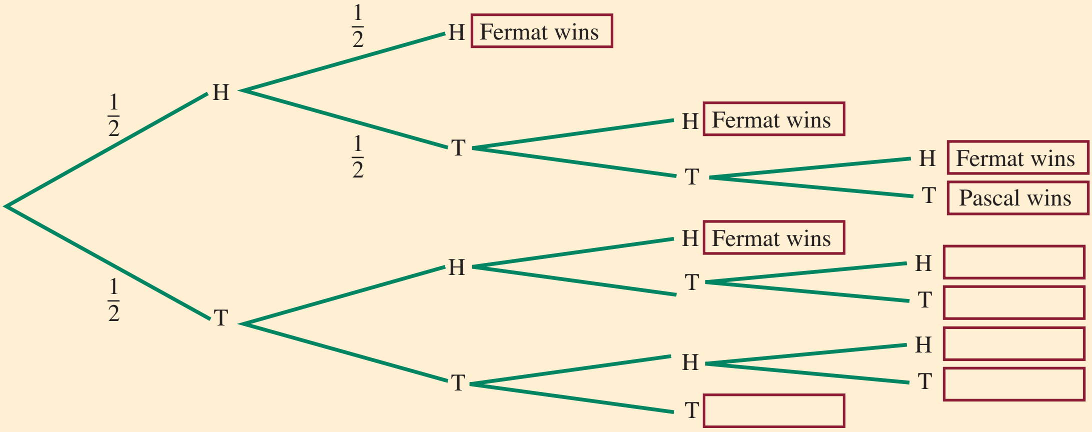
Enrichment

1 To investigate 'The Problem of Points', Pascal and Fermat planned to toss a coin 20 times. If the coin landed on heads, Fermat got a point, tails and Pascal got a point. The first to 10 points would win 100 francs. The game was interrupted when Fermet was winning 8 points to 7. How should the prize money be split, given that the outcomes heads and tails are equally likely? Fermat calculated that his chance of winning was $\frac{11}{16}$ and Pascal's was $\frac{5}{16}$ and arrived at a split of 68·75 francs to 31·25 francs in his favour.

a Copy and complete the tree diagram to represent the information.

b Multiply the probabilities along the branches to find the probability of each outcome.

c Explain how Fermat arrived at his answer.



2 A bag contains 5 red marbles and 3 green marbles. Two marbles are selected.

a Find the probability that both are red if the first one is replaced.

b Find the probability that both are red if the first one is not replaced.

3 Ben enjoys watching his local soccer teams play and based on previous experiences he estimates that the probability that the seniors, intermediates and juniors win are:

$$\Pr(\text{seniors win}) = 0{\cdot}2$$
$$\Pr(\text{intermediates win}) = 0{\cdot}7$$
$$\Pr(\text{juniors win}) = 0{\cdot}4$$

a What is the probability that all three teams win?

b What is the probability that one team wins?

c What is the probability that at least one team wins?

d What is the probability that two teams win?

4 In horse racing, a quinella is picking the winning horse in each of four races. What are the chances of winning a quinella if you randomly select horses and there are 7, 14, 11 and 8 horses in the races? Do you need to prepare a tree diagram for this question?

5 In a group of 20 learners 12 like tennis, 18 like basketball and some like both. They all like at least one of the sports. If one person is chosen at random, what is the probability that they:

a like tennis? b don't like tennis? c like basketball? d like both sports?

6 The probability of Sue solving a maths problem is $\frac{2}{3}$. She sits a test of three similar questions. Use a tree diagram to represent the information, and find the probability of Sue solving:

a none of the questions

b only one of the questions

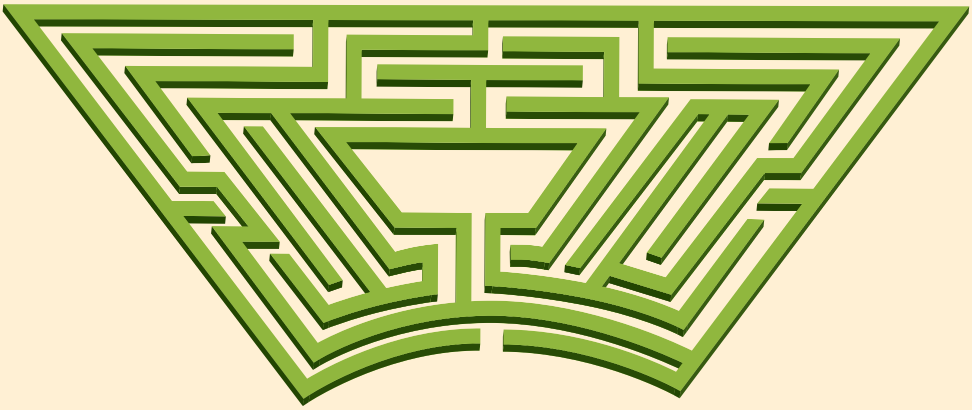
c two or more questions

d all of the questions

7 **a** The map of the Hampton Court Maze is shown. Copy the map and find a path from the outside to the centre, without retracing your steps.

b Are different paths possible?

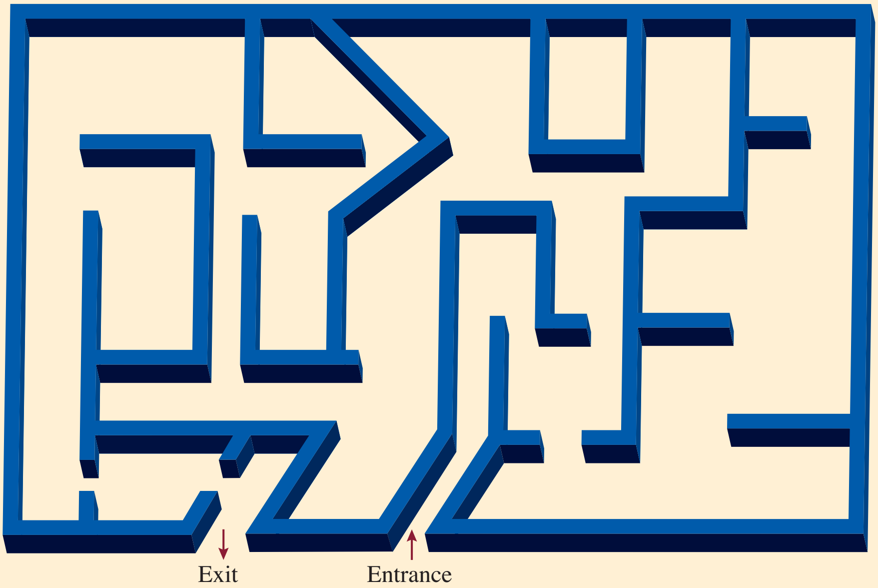
c Draw a tree diagram to represent the maze. Each branch in the tree represents a point in the maze where two or more paths are possible.



8 **a** The map of a maze from an amusement park is shown. Copy the map and find a path from the entrance to the exit without retracing your steps.

b Are different paths possible?

c Draw a tree diagram to represent the maze, so that each branch in the tree represents a point in the maze where two or more paths are possible.



Revision/Assessment

Exercise 12B

1 Are the following true or false?

a $q \in$ {vowels}

b $17 \in$ {prime numbers}

c Utupua $\in$ {Solomon Islands}

d Amy $\notin$ {boys' names}

e Blue $\notin$ {colours of the rainbow}

2 Let ξ = {numbers from 1 to 10}
Let F = {factors of 48}
Let T = {factors of 36}

a List the elements in F.

b List the elements in T.

c List the elements in $F \cap T$.

d List the elements in F'.

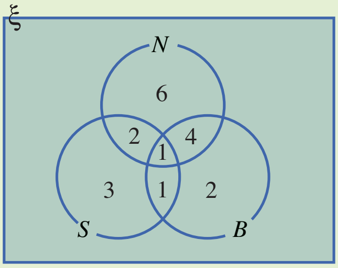
e List the elements in T'.

f List the elements in $F \cup T$.

3 Display the information in Question 2 in a Venn diagram.

Exercise 12C

4 The diagram below shows the sports played by a group of Year 8 learners:



N = netball
V = basketball
S = soccer

a How many learners are in the group?

b How many play all three sports?

c How many learners play soccer?

d If one learner is chosen at random, what is the probability that the learner:

  i plays three sports?

  ii plays netball?

  iii doesn't play basketball?

5 A bag contains 50 marbles: 20 are red, 12 are green and 18 are black. What is the probability that a marble selected at random is:

a red?

b green?

c not black?

6 The spinning wheel shown has the numbers 1–64 equally spaced around the wheel. The wheel is spun and lands on a number. Calculate the probability the wheel lands on:

a 6 **b** 60

c an even number **d** a multiple of 10

e a number greater than 19 **f** a factor of 60

g not a factor of 60 **h** a 2-digit number

Exercise 12D

7 **a** Display in a grid all the possible outcomes when two dice are thrown.

b What is the probability that the difference between the numbers thrown is 2?

c What is the probability that the product of the numbers thrown is 12?

8 In a group of friends there are 5 boys called Arnold, Basil, Comelios, David and Edwin. They often partner the girls in their group to local parties. The girls are Rosaria, Georgina, Aubrey and Eunice and they never miss a party. Assume that the girls randomly select their partners.

a Display all the possible partnerships in a grid.

b What is the probability that Comelios partners Eunice?

c What is the probability that Arnold gets a partner for the next ball?

d What is the probability that David doesn't partner Georgina?

Exercise 12E

9 At a wedding the following courses were available and served randomly to the guests:
Starter: pumpkin soup or banana soup
Main: beef, chicken or cassava pudding
Dessert: cheesecake or chocolate cake

a How many different possible arrangements are available?

b How many arrangements have chocolate cake as a dessert?

c If one person is chosen at random, what is the probability that the person:

- **i** Has chocolate cake as a dessert?
- **ii** Has beef as a main?
- **iii** Has pumpkin soup as a starter?
- **iv** Has cheesecake as a dessert and fish as a main?
- **v** Has pumpkin soup or pudding?

Learning task 12F

10 What is the probability that a card selected at random from a pack of cards is:

a a jack? **b** a diamond? **c** the jack of diamonds?

Exercise 12H

11 In a sample of 40 toys, three were found to be faulty. In an order of 2000 toys, how many toys would you expect to be faulty?

CHAPTER

13

Area and Volume

Area and Volume

Going to the beach at the weekend is a popular family activity throughout the Solomon Islands. Old and young enjoy picnics, the chance to relax, and to have fun on and in the sea. Swimming with blow-up tubes, balls and dugout canoes make for a most enjoyable day. Both balls and tubes have interesting mathematical properties. They are made from rubber or plastic sheets that determine their surface area. However, when inflated with air, they also have a volume, and the capacity and sizes of the balls or tubes determine whether they will support a person's weight when floating in the sea. In this chapter you will explore further the properties of area and volume that apply to many objects we use every day.

This chapter covers the following skills:

- Recognising common metric units of volume
- Obtaining areas by counting squares in order to develop new rules for the area of regular shapes
- Calculating the area of a triangle or one-half of the area of a suitable rectangle
- Calculating areas of shapes based on rectangles and triangles
- Finding areas using formulas
- Area of rectangle = length × width
- Area parallelogram = base × height
- Area triangle = $\frac{1}{2} \times \text{base} \times \text{height}$
- Area trapezium = $\frac{1}{2}(a + b) \times \text{height}$
- Area circle = πr^2
- Total surface area of a solid is the sum of the area of each of its faces
- Finding volume using a formula:Volume of prism = area base × height
- Finding volumes of shapes based on rectangular prisms

Specific Learning Outcome (SLO)

Learners should be able to:

8.13.1.1 Define 'area'.

8.13.1.2 Estimate areas of given shapes by counting the number of unit squares that would fit inside the shape.

8.13.2.1 Identify a rectangle and its properties.

8.13.2.2 Use the formula to calculate the area of rectangles: Area = base × height

8.13.3.1 Identify a parallelogram and its properties.

8.13.4.1 Use the formula to calculate the area of parallelograms: Area = base × height

8.13.5.1 Identify a triangle and its properties.

8.13.5.2 Use the formula to calculate the area of triangles:
Triangle: Area = $\frac{1}{2}$ base × height

8.13.6.1 Identify a trapezium and its properties.

8.13.6.2 Formulate the formula to calculate the area of a trapezium.

8.13.7.1 Use the formula to calculate the area of trapeziums: Area = $\frac{1}{2}(a + b) \times \text{height}$

8.13.8.1 Estimate the areas of irregular shapes by combining parts of the shape to make unit squares.

8.13.9.1 Calculate the area of circles using the formula: Area = πr^2

8.13.10.1 Define and identify compound shapes: Compound shapes: shapes that made up of a number of different shapes.

8.13.10.2 Identify a compound shape and the different shapes that make it compound.

8.13.11.1 Use formulae of different shapes to calculate the area of compound shapes.

8.13.12.1 Identify a prism and its properties.

8.13.13.1 Identify the shapes that make up the surface areas of prisms.

8.13.14.1 Calculate the surface area of each face of a prism.

8.13.14.2 Calculate the total surface area of prisms.

8.13.15.1 Define the term 'volume'.

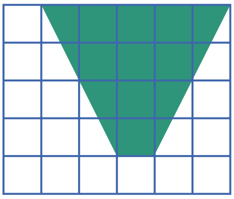
8.13.16.1 Calculate the volume of prisms and cuboids by using the formulae:
Volume = area of base × height;
Volume = $L \times W \times H$.

13A Using grids to find area

The **area** of a shape is the amount of two-dimensional space within it. We find the area of simple shapes by estimating the number of squares that can fit inside them. Area is measured in square units.

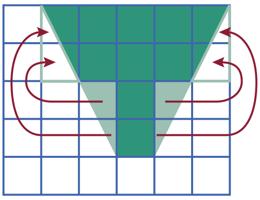
Example

The sides of the squares used in the grid below are each 1 centimetre. Use the squares to find the area of the shaded shape.

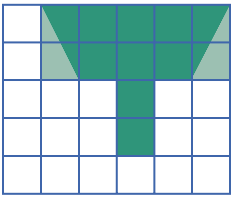


Solution

Step 1: Count the number of whole squares—each square is 1 square centimetre.



Step 2: Combine the parts of squares to make whole squares.



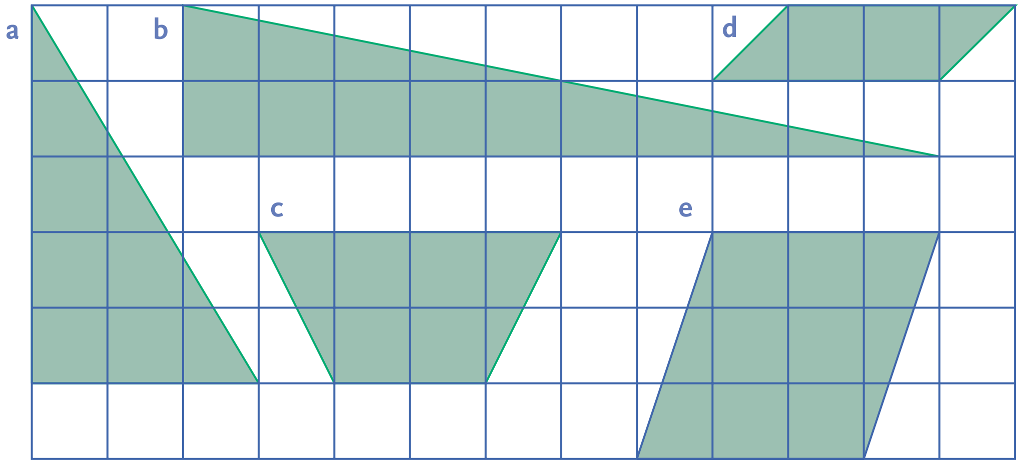
Step 3: Add these new whole squares to those in Step 1.

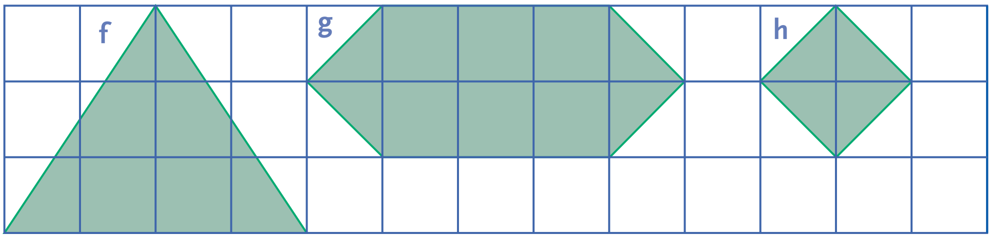
Step 4: State the number of square centimetres—this is the area of the shape.

Area = 12 square centimetres = 12 cm^2

Exercise 13A

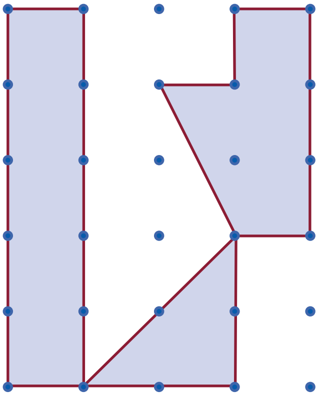
1 Find the area of the following shapes, which are drawn on centimetre grid paper:





2 The following shapes have been drawn on centimetre dot paper. Use the dots to draw a grid over the shapes and find the shaded area of each shape:

a



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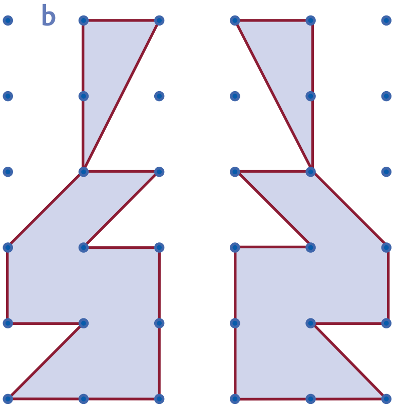

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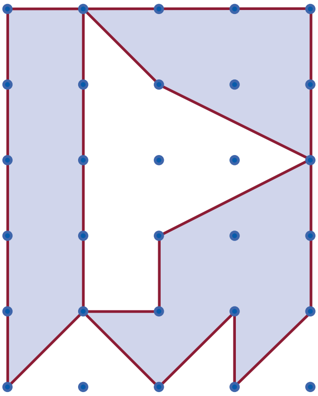

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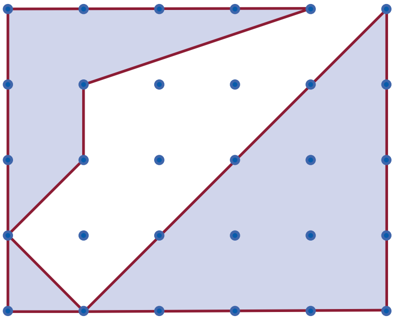

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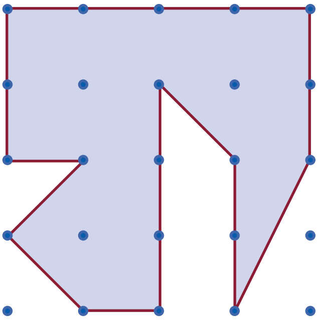

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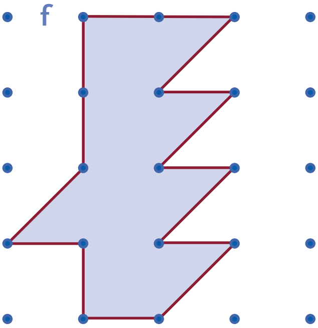

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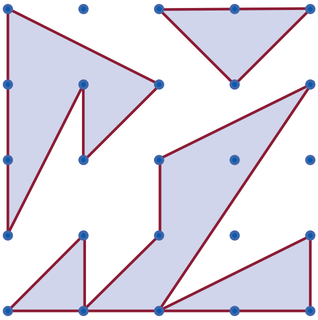

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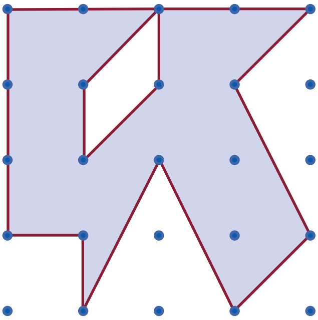
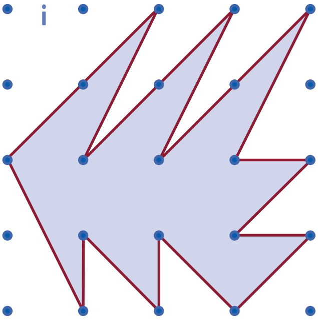
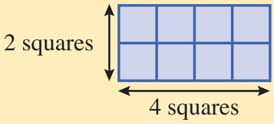


i



13B Areas of rectangles

We can use a short cut to find the area of a rectangle.



For this rectangle, it is easy to count the eight squares inside. There are clearly 2 lots of 4 squares or 4 lots of 2 squares: $2 \times 4 = 4 \times 2 = 8$

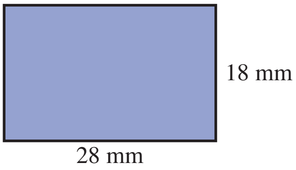
To find the area of a rectangle multiply the length by the width, but make sure that the units are the same.

Area = length × width

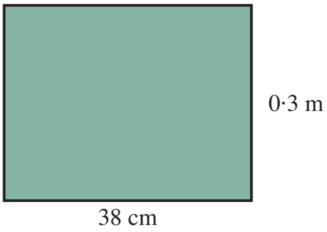
Example

Find the areas of these shapes:

a



b



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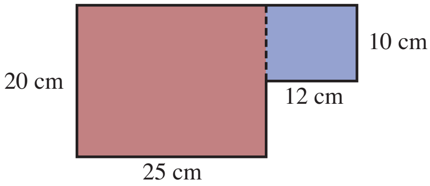

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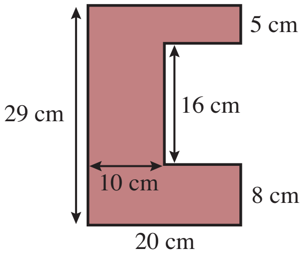


Solution

a

Area = length × width

$= 28 \times 18$

$= 504$ mm^2 (504 square millimetres)

b

First convert the width to centimetres:

$0{\cdot}3 \times 100 = 30$ cm

Area = length × width

$= 38 \times 30$

$= 1140$ cm^2

c

The shape is divided into two areas:

Area red $= 20 \times 25$

$= 500$ cm^2

Area blue $= 12 \times 10$

$= 120$ cm^2

Total area $= 500 + 120$

$= 620$ cm^2

d

Area of enclosing rectangle

$= 29 \times 20$

$= 580$ cm^2

Area of cutout rectangle

$= 16 \times 10$

$= 160$ cm^2

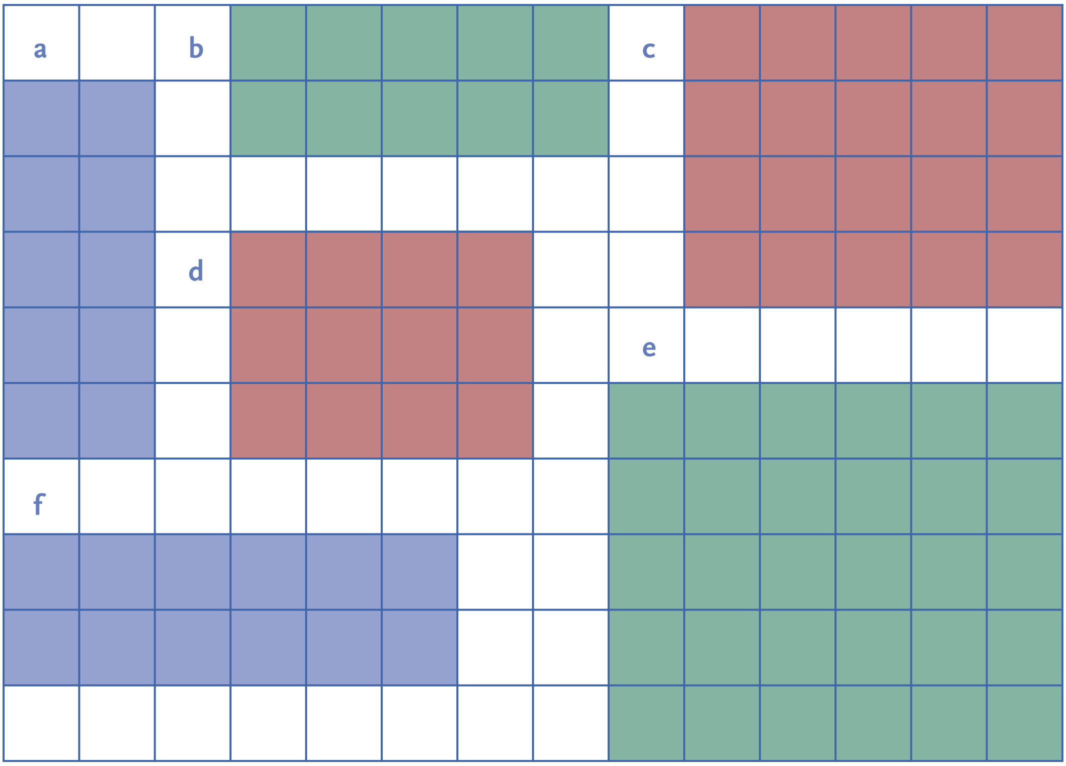
Shaded area $= 580 - 160$

$= 420$ cm^2

Exercise 13B

1 Find the area of each of the following rectangles:

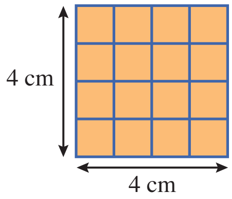
i Count the squares. **ii** Use the rule Area = length × width.



2 Find the area of each of the following rectangles.

i Count the squares. **ii** Use the rule.

a



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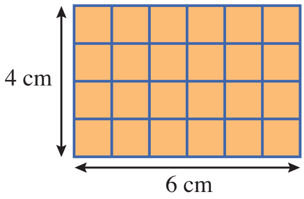

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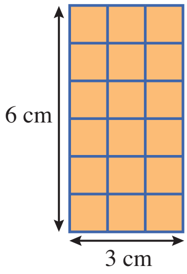
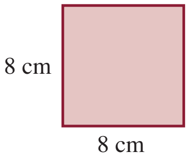


3 Find the areas of the following rectangles:

a



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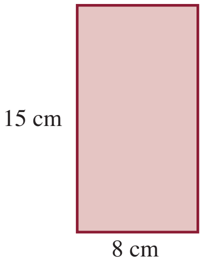

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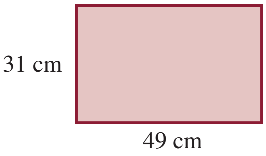

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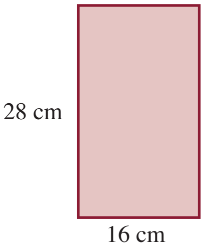

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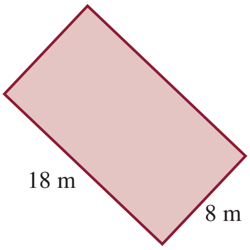

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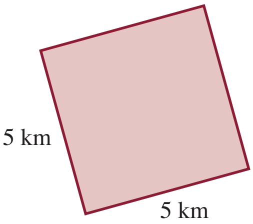
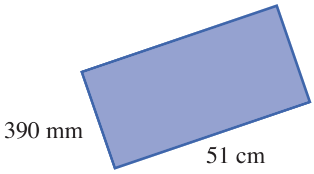


4 Find the area of the following shapes, expressed in square centimetres:

a



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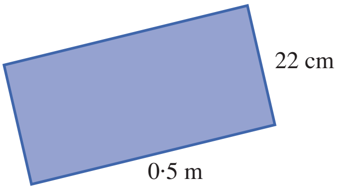

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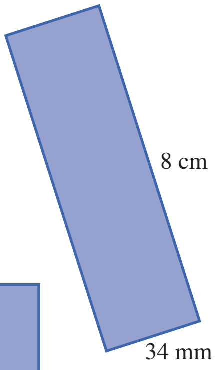
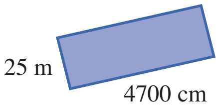
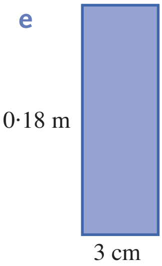


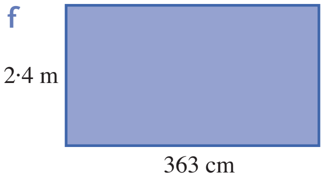
d



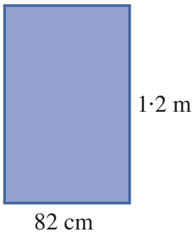
e



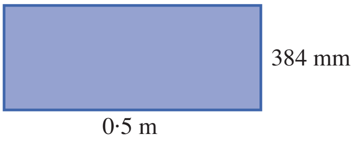
f



g

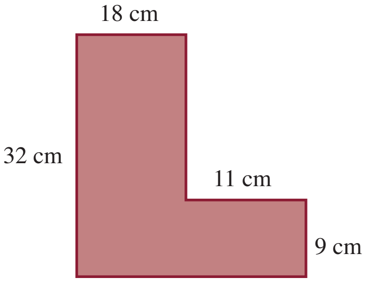


h

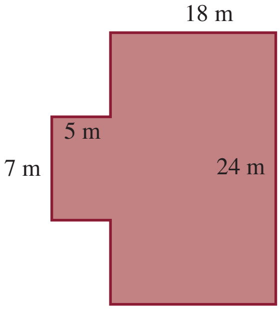


5 Divide the following shapes into rectangles and use the rule to find the total area:

a



b



6 Find the area of:

a a rectangle with length 78 mm and width 126 mm

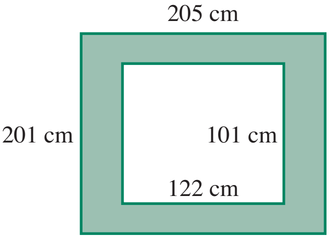
b a rectangle with length 1·89 km and width 850 m, in square kilometres

c a rectangle 3·04 cm by 26 mm, in square millimetres

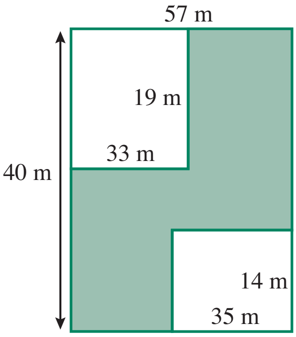
d a square with side length 8·9 m

7 Find the shaded area in the following shapes:

a



b

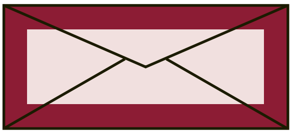


8 A rectangular backyard measures 14 metres by 20 metres. Three garden beds measure 1·2 m by 5 m, 7 m by 4·5 m and 3·8 m by 14 m. The rest of the backyard is grass.

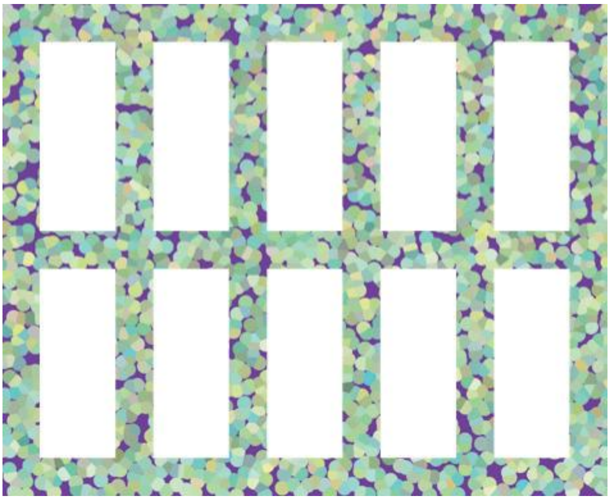
a Find the area of the garden beds.

b Find the area of grass in the backyard.

9 A rectangular envelope that is 15 cm by 25 cm has a rectangle drawn inside it 3 cm from the outside edge. The margin is coloured red, find its area.



10 This is an abstract painting in which white rectangles are painted over a coloured background. Each white rectangle measures 15 cm by 24 cm and the painting is 102 cm by 60 cm.



a Find the area of:

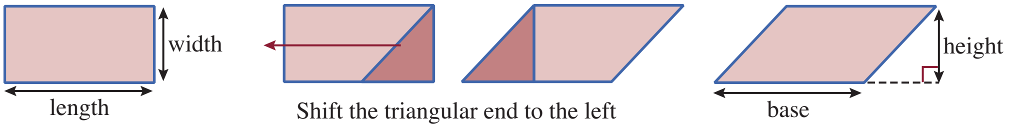
i white paint showing

ii coloured paint that is used.

b If the rectangles are evenly spaced both across the painting as well as from top to bottom, find the widths of the coloured parts of the painting.

13C Areas of parallelograms

A parallelogram can be made from a rectangle by shifting a triangular piece as shown below. The rule for finding the area of a parallelogram is similar to that for a rectangle.

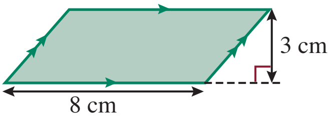


Rectangle	**Parallelogram**
Area = length × width	Area = base × height

Example

1 Find the area of each parallelogram:

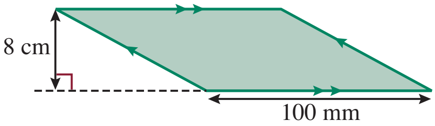
a



As the units are the same, the rule can be used straight away.

Area = base × height = 8 × 3 = 24 cm^2

b



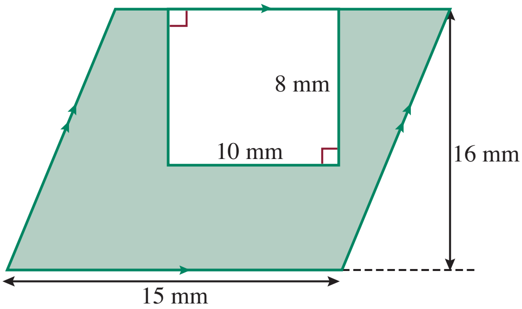
Change the units of the base to centimetres to make the units the same.

100 mm ÷ 10 = 10 cm

Use the rule:

Area = base × height = 10 × 8 = 80 cm^2

2 Find the shaded area:



Solution

Shaded area = area large parallelogram − area small rectangle

Area of parallelogram = 15 × 16
= 240 mm^2

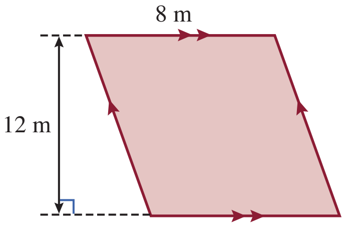
Area of rectangle = 10 × 8
= 80 mm^2

Shaded area = 240 − 80
= 160 mm^2

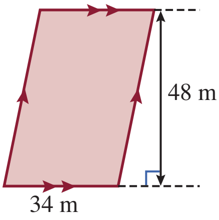
Exercise 13C

1 Find the area of each of the following parallelograms:

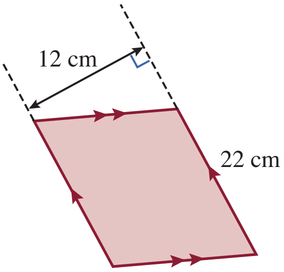
a



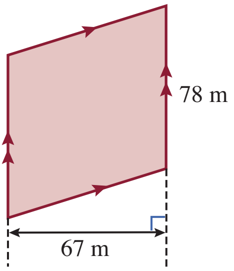
b



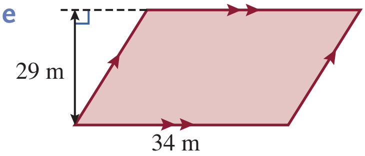
c



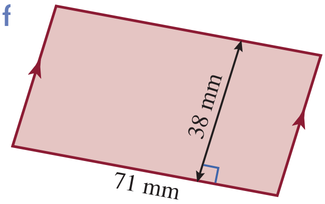
d



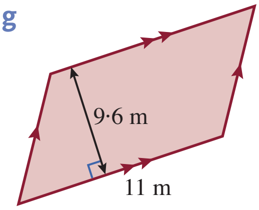
e



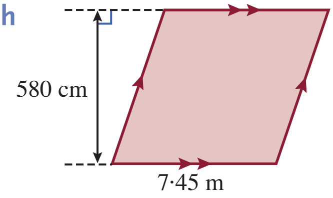
f



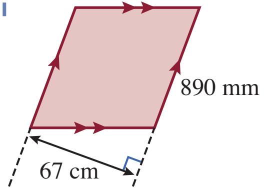
g



h

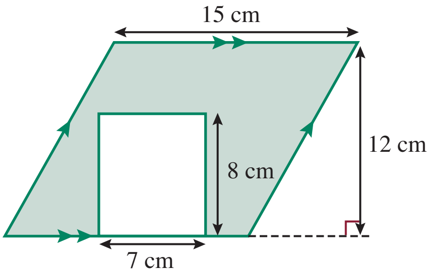


i

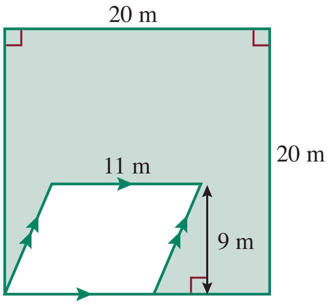


2 Find the shaded area of the following shapes:

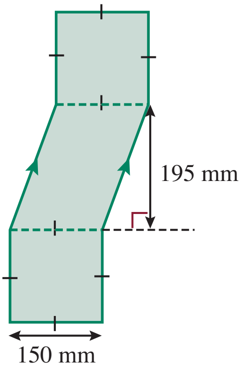
a



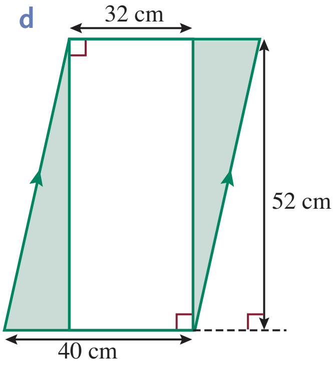
b



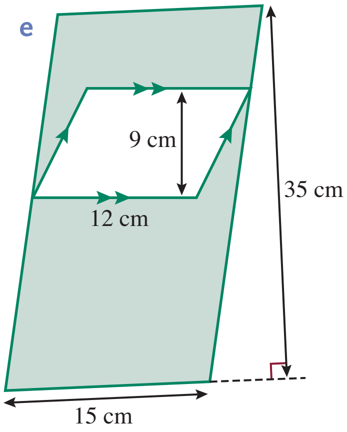
c



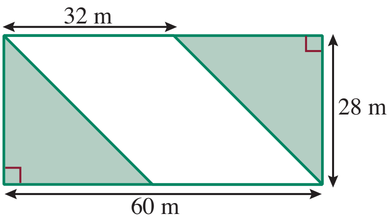
d



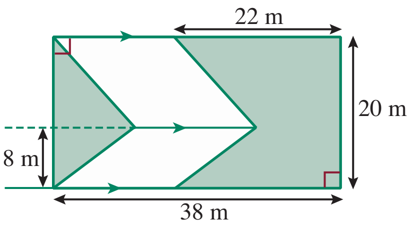
e



f

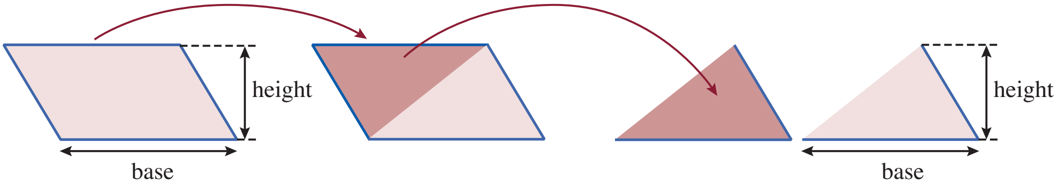


g



13D Areas of triangles

Two identical triangles can be joined together to make a parallelogram as shown below. The area of a triangle is half that of the resulting parallelogram.



Cut the parallelogram exactly in half

Triangle is flipped which is exactly the same as the other half

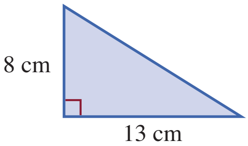
Area of each triangle is half that of the parallelogram

Parallelogram	**Triangle**
Area = base × height	Area = $\frac{1}{2}$ × base × height

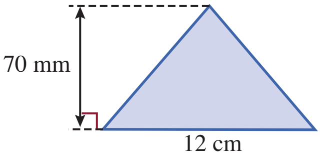
Example

Find the area of:

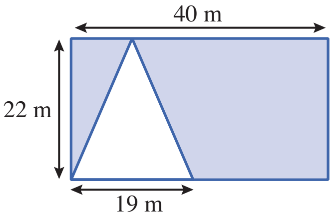
a the triangle



b the triangle



c the shaded region



Solution

a Height = 8 cm, base = 13 cm
The units are the same so use the rule.

$$\begin{aligned}\text{Rule: Area} &= \tfrac{1}{2} \times \text{base} \times \text{height}\\ &= \tfrac{1}{2} \times 13 \times 8\\ &= 52\ \text{cm}^2\end{aligned}$$

b Height = 70 mm, base = 12 cm
Change the height to centimetres and then use the rule.

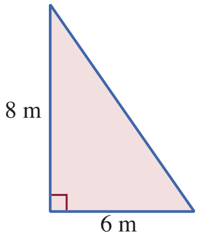
$$\begin{aligned}\text{Rule: Area} &= \tfrac{1}{2} \times \text{base} \times \text{height}\\ &= \tfrac{1}{2} \times 12 \times 7\\ &= 42\ \text{cm}^2\end{aligned}$$

c Shaded area
= area of rectangle − area of triangle
$= 40 \times 22 - \frac{1}{2} \times 19 \times 22$
$= 880 - 209$
$= 671\ \text{m}^2$

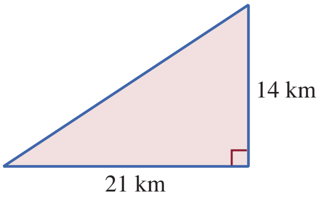
Exercise 13D

1 Find the area of these triangles:

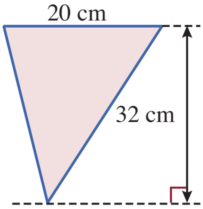
a



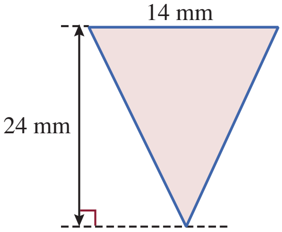
b



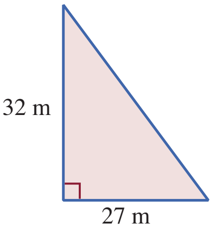
c



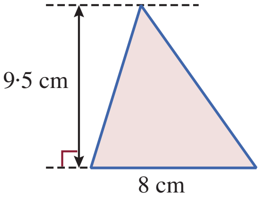
d



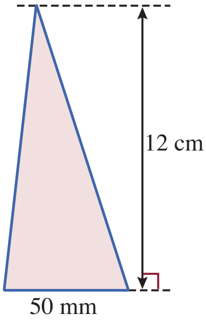
e



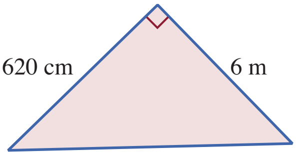
f



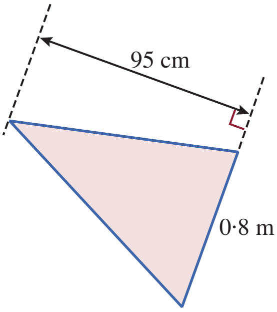
g



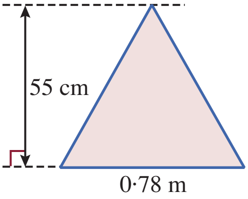
h



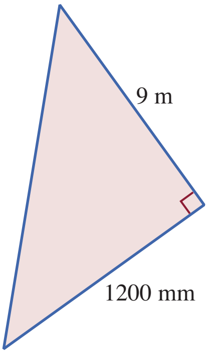
i



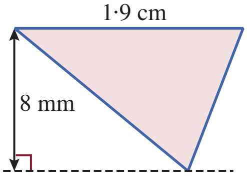
j



k

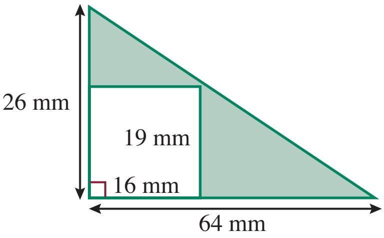


l

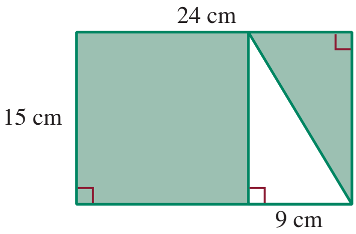


2 Find the coloured area in each of the following:

a

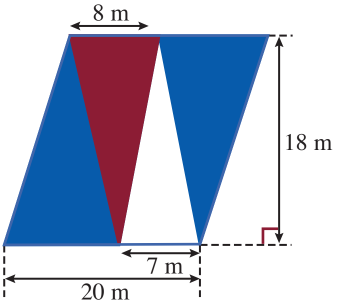


b

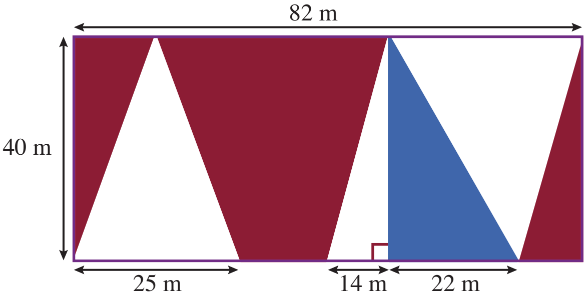


3 Find the area of red, white and blue paint for each of these paintings.

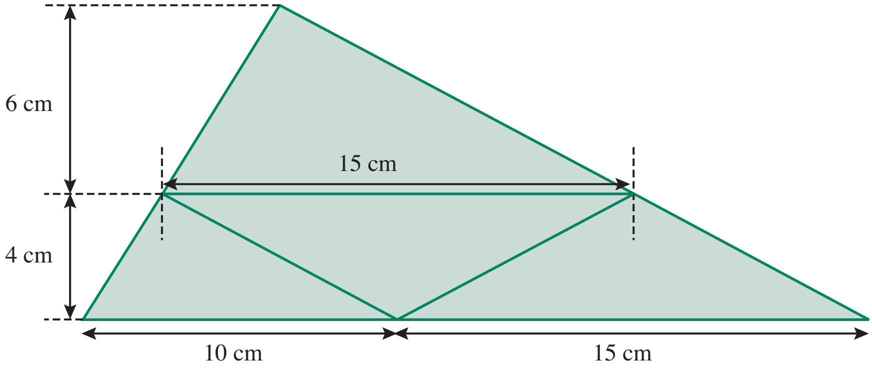
a



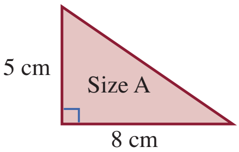
b

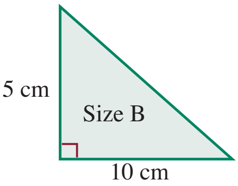


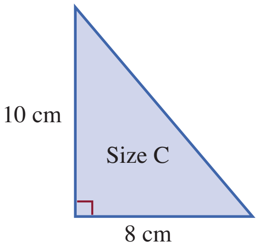
4 Find the area of each triangle inside the large triangle, and show that the sum of the areas of the small triangles is the same as the area of the large triangle.



5 Ceramic tiles are stocked in the following standard triangular sizes at the local tile shop.







Size A: \$800 per square metre   Size B: \$1000 per square metre   Size C: \$1200 per square metre

a Find the area of each tile.

b Find the number of tiles of each type needed to cover 1 m^2.

c Find the cost of one tile of each type.

d Find the cost of using size A tiles to cover these areas:

- **i** 2 m by 5 m
- **ii** 2·6 m by 3·5 m
- **iii** 620 cm by 240 cm

e Find the cost of using size B tiles to cover these areas:

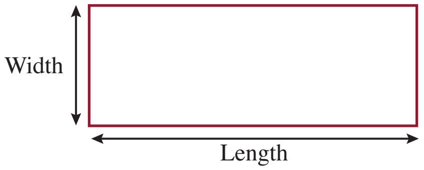
- **i** 4 m by 5 m
- **ii** 5·2 m by 2·8 m
- **iii** 1200 cm by 650 cm

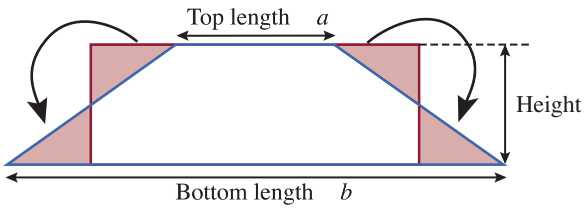
f Find the cost of using size C tiles to cover these areas:

- **i** 10 m by 20 m
- **ii** 12 m by 18 m
- **iii** 240 cm by 380 cm

Areas of trapeziums 13E

A **trapezium** is a four-sided figure with one pair of parallel sides. It can be thought of as being a rectangle that has been pushed in from both sides. The height of the trapezium is the same as the width of the rectangle, and the length of the rectangle is the same as the average of the top and base of the trapezium. The formulas are shown below.



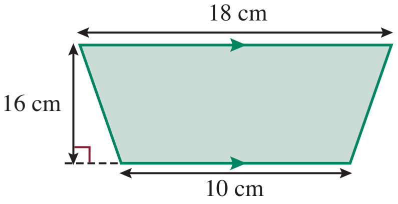


Rectangle	**Trapezium**
Area = length × width	Area = $\frac{1}{2}(a+b) \times$ height

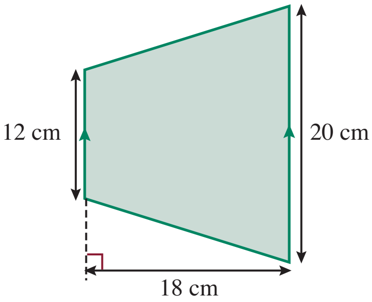
Example

Find the area of these trapeziums:

a



b



Solution

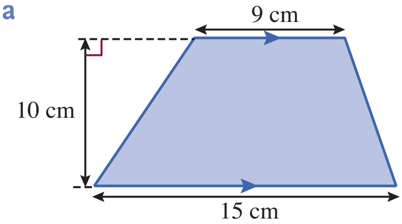
$$\begin{aligned} \text{Area} &= \tfrac{1}{2}(a+b) \times \text{height} \\ &= \tfrac{1}{2}(18+10) \times 16 \\ &= 224 \text{ cm}^2 \end{aligned}$$

$$\begin{aligned} \text{Area} &= \tfrac{1}{2}(a+b) \times \text{height} \\ &= \tfrac{1}{2}(12+20) \times 18 \\ &= 288 \text{ cm}^2 \end{aligned}$$

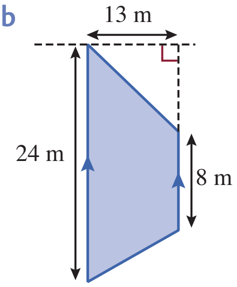
Exercise 13E

1 Find the area of the following trapeziums:

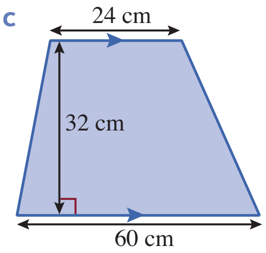
a



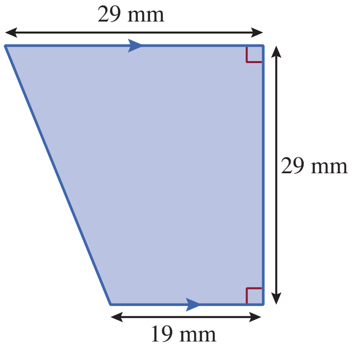
b



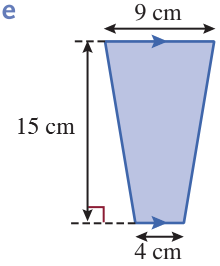
c



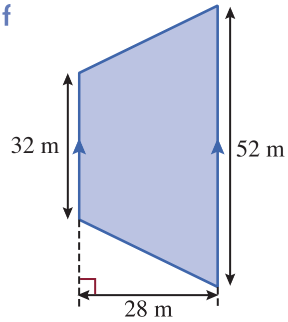
d



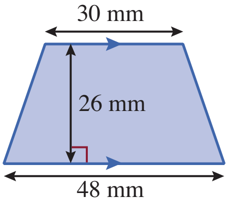
e



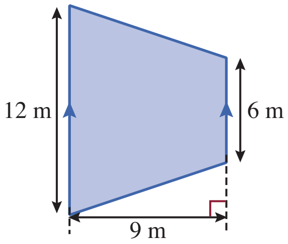
f



g

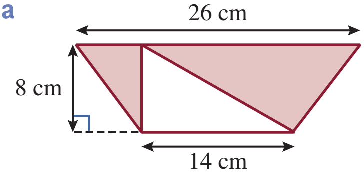


h

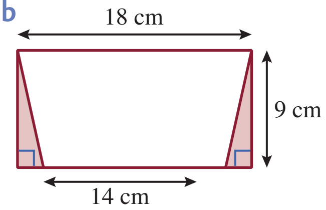


2 Find the shaded area in the following:

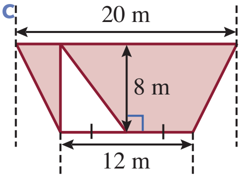
a



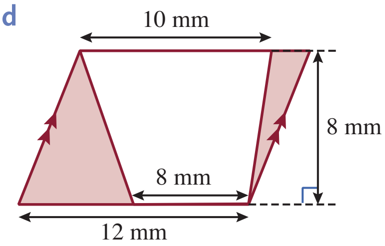
b



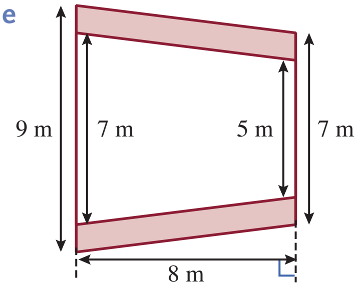
c



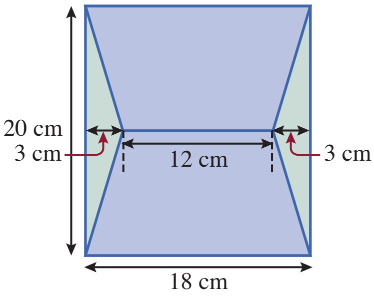
d



e



3 This rectangle contains two triangles and two trapeziums. Find the area of the rectangle and of each triangle and trapezium. Show that the sum of the areas of the triangles and the trapeziums is equal to the area of the rectangle.



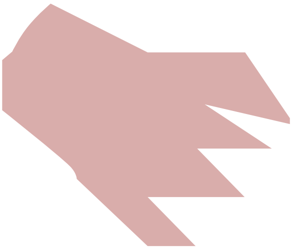
Areas of irregular shapes

13F

Usually the shapes we find in nature are irregular and don't fit exactly over a grid. We can find the approximate areas of these shapes by combining parts to make whole squares.

Example

Lucinda left this footprint in the garden when on her way to a fancy dress party. Estimate the area of her footprint using a five-centimetre grid.



Solution

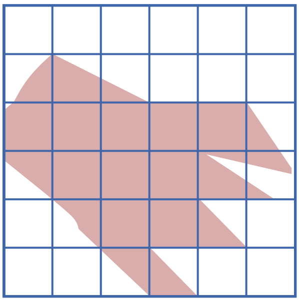
Place a five-centimetre grid over the shape. Each square is 5 cm by 5 cm = 25 cm^2.

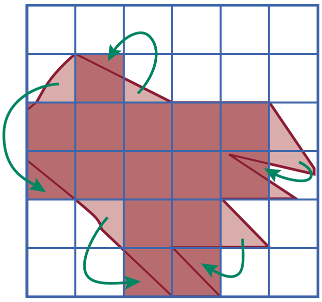
Count the whole squares: 10

Combine the part squares to make whole squares: approximately 5

$$\begin{aligned}\text{Area} &\approx 10 + 5\\ &\approx 15 \text{ squares}\end{aligned}$$

15 squares × 25 cm^2 = 625 cm^2

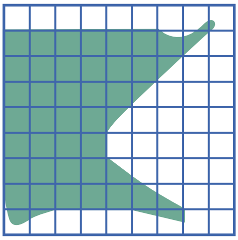




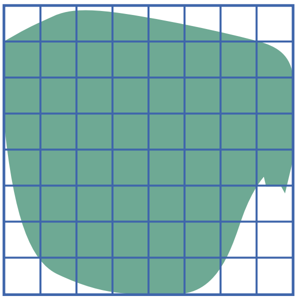
Exercise 13F

1 Find the area, in square units, of the ink blots shown in the following grids:

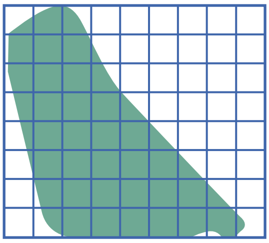
a



b

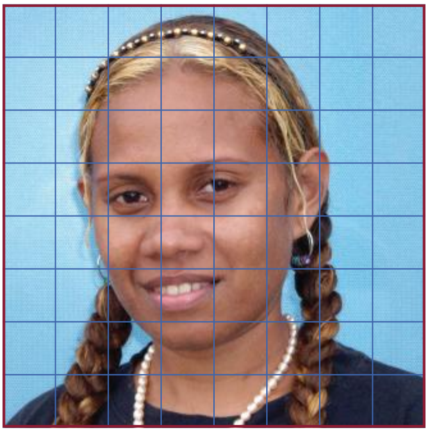


c

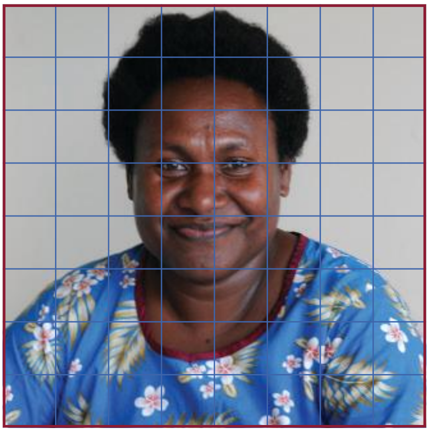


2 Use the grids to estimate the area, in square centimetres, taken up by the people in the photographs below.

a

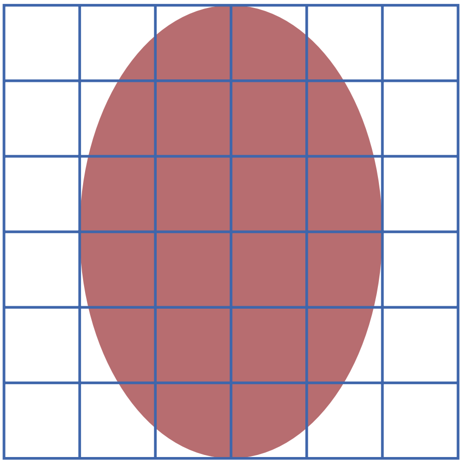


b

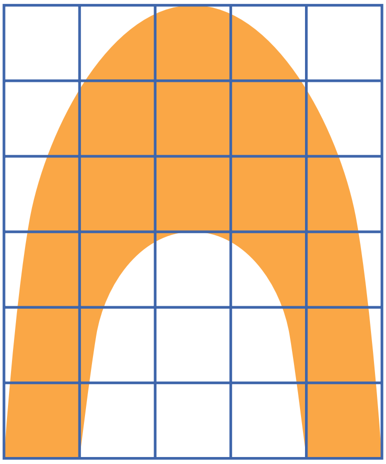


3 Use the centimetre grids below to estimate the area in square centimetres of the following shapes:

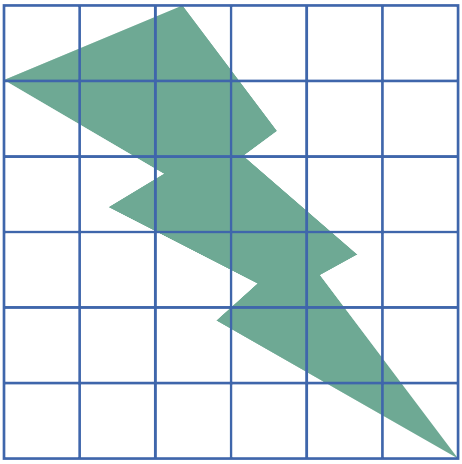
a



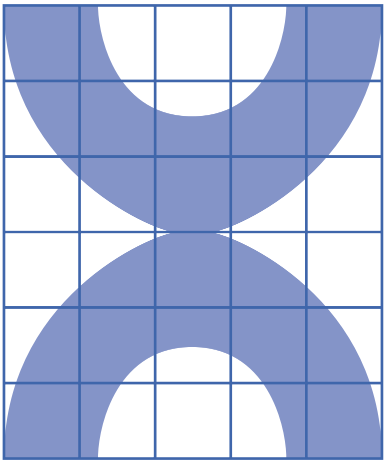
b



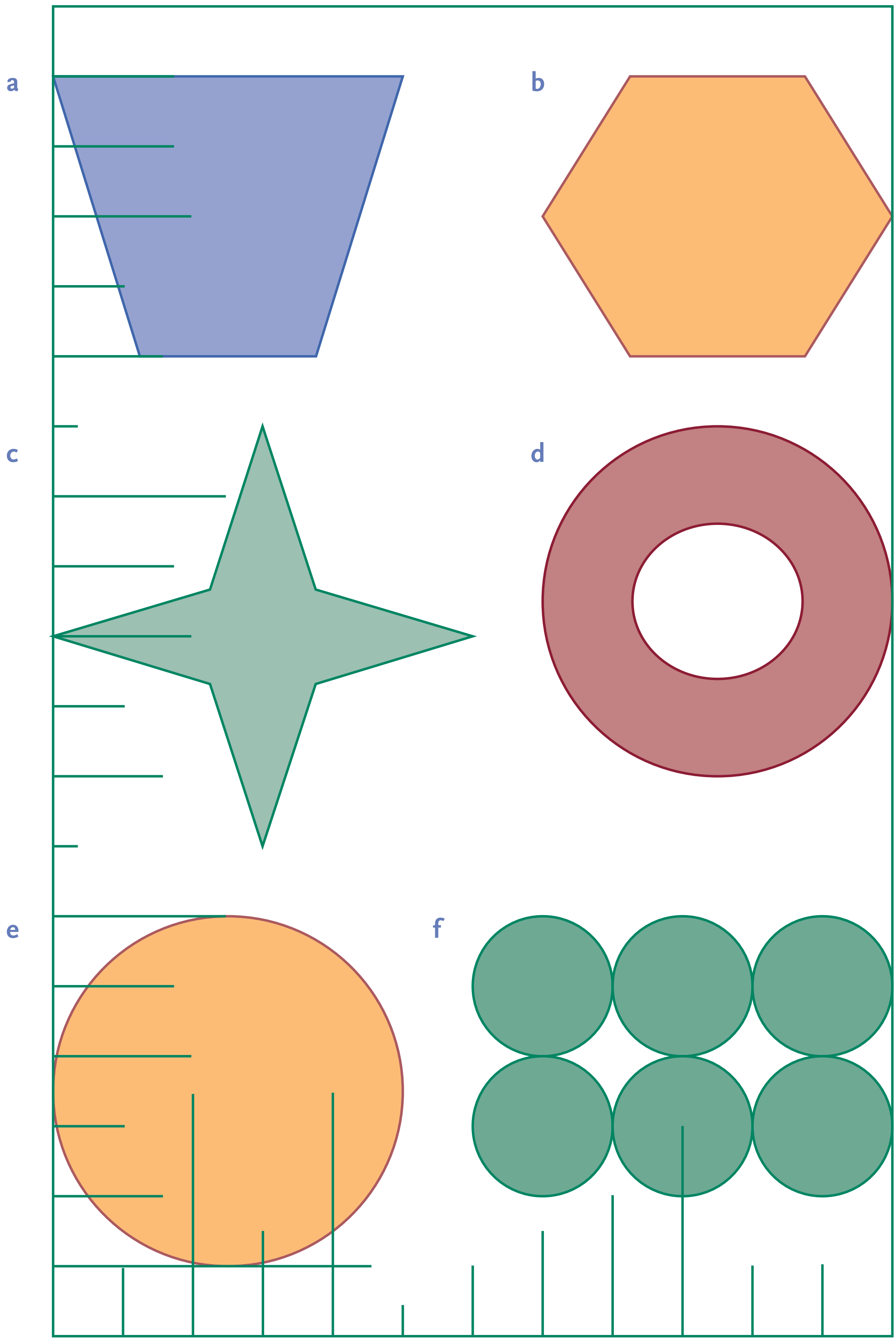
c



d

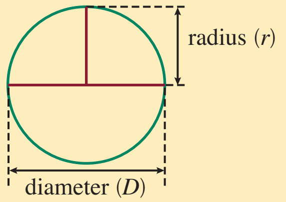


4 Trace these shapes into your exercise book and by completing the centimetre grids, use them to estimate the area of each shape.



13G Area of circles

Finding the areas of a shape with curved sides by counting squares only provides an approximate answer. The most accurate way to find the area of a circle is to use a formula. To use the formula we need to know either the radius or the diameter of the circle.



Rule for area of circle

$A = \pi r^2$

where π is approximately $3{\cdot}14$.

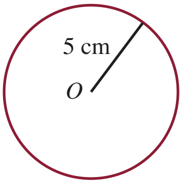
For accuracy use a calculator, which gives a better approximation for π as $3{\cdot}141\,592\,7$.

Answers in exact form are given in terms of π.

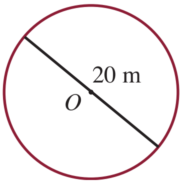
Example

Find the area of these circles, expressing your answers to 2 decimal places and in exact form:

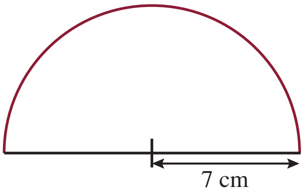
a



b



c



Solution

a

$A = \pi r^2$

$= \pi \times 5 \times 5$

$= 25\pi$ cm^2 in exact form

$= 78{\cdot}54$ cm^2 correct to 2 dp

b

Diameter is 20 m, radius = 10 m

$A = \pi r^2$

$= \pi \times 10 \times 10$

$= 100\pi$ m^2 in exact form

$= 314{\cdot}16$ m^2 correct to 2 dp

c

Radius = 7 cm

Area of half circle

$= \frac{1}{2} \times \pi r^2$

$= \frac{1}{2} \times \pi \times 7 \times 7$

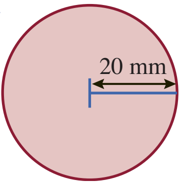
$= 24\frac{1}{2}\pi$ cm^2 in exact form

$= 76{\cdot}97$ cm^2 correct to 2 dp

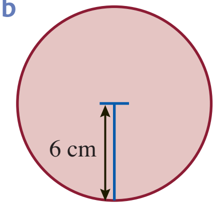
Exercise 13G

1 Find the area of these circles expressed **i** in exact form and **ii** correct to 2 decimal places:

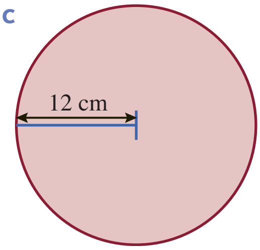
a



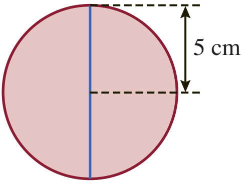
b



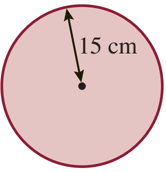
c



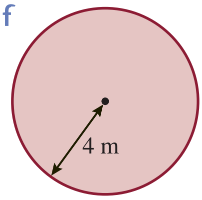
d



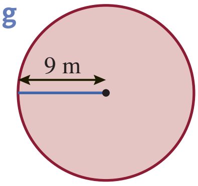
e



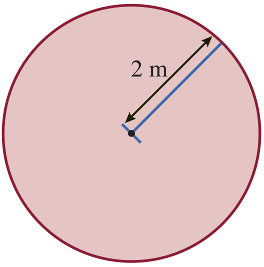
f



g

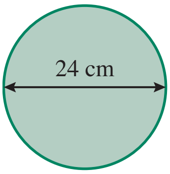


h

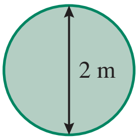


2 Find the area of these circles expressed **i** in exact form and **ii** correct to 1 decimal place:

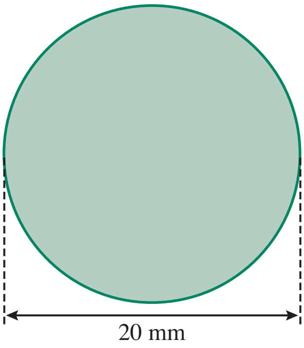
a



b



c



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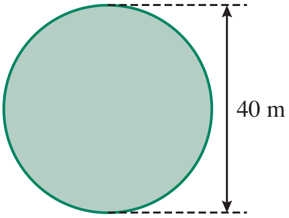
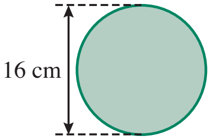


e



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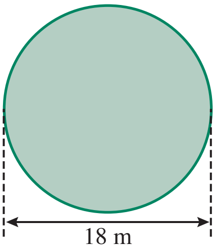
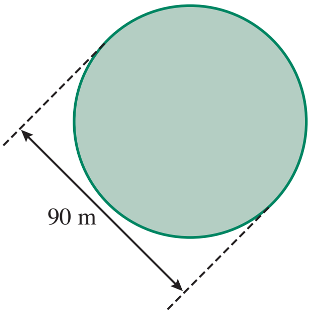
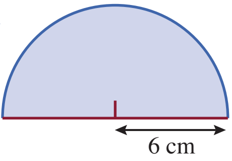


g

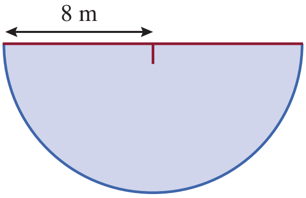


3 Find the area of the following shapes expressed **i** in exact form and **ii** correct to 2 decimal places:

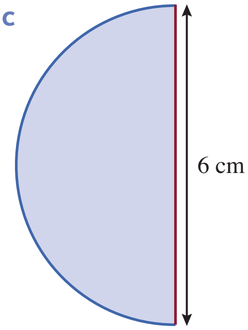
a



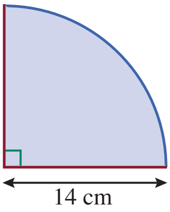
b



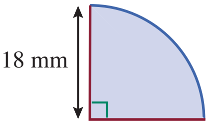
c



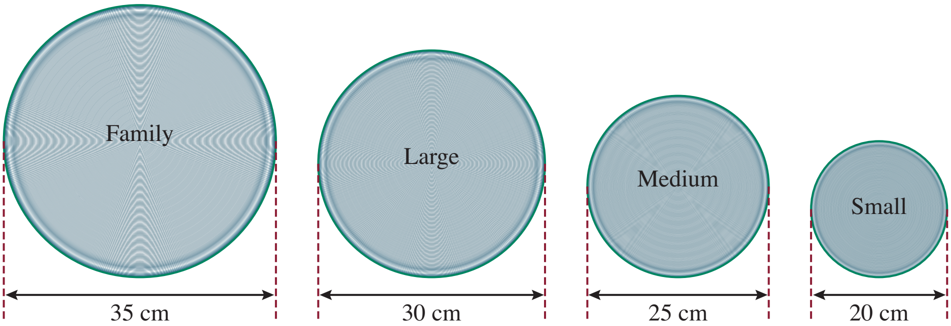
d



e



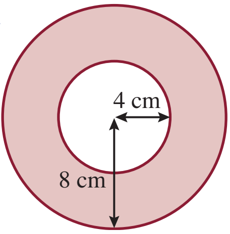
4 The local pizza shop sells the following sizes of pizza:



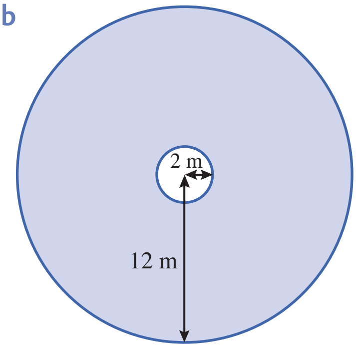
Use the above measurements to find the area of each size of pizza to the closest square centimetre.

5 Find the area of the coloured regions by subtracting the area of the smaller circle from that of the larger circle:

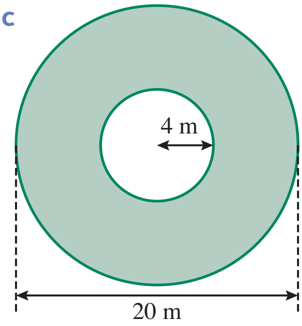
a



b



c



6 Find the areas of the circles shown in the photographs below:

a Dartboard of radius 20 cm

b Drum of radius 32 cm

c Fob watch of radius 1·8 cm

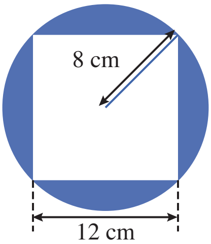
d Soccer medal of radius 2·4 cm

Areas of compound shapes 13H

Compound shapes are made up of a number of different shapes. Their area is found by adding or subtracting the areas of the various shapes.

Example

1 Find the coloured area of the following shape expressed in exact form and correct to 1 decimal place:



Solution

Area of circle: $A = \pi r^2$

$= \pi \times 8 \times 8$

$= 64\pi$

$= 201.1 \text{ cm}^2$

Area of square: $A = l^2$

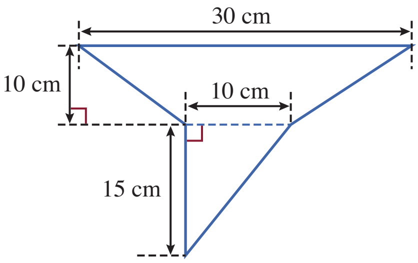
$= 12 \times 12$

$= 144 \text{ cm}^2$

Shaded area $= 64\pi - 144$

$= 57.1 \text{ cm}^2$

2 Find the total area of this shape:



Area of trapezium:

$$A = \frac{(30 + 10)}{2} \times 10$$

$= 200 \text{ cm}^2$

Area of triangle: $A = \dfrac{(10 \times 15)}{2}$

$= 75 \text{ cm}^2$

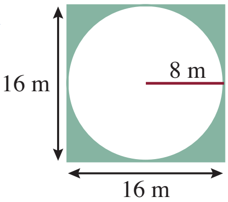
Total area $= 200 + 75$

$= 275 \text{ cm}^2$

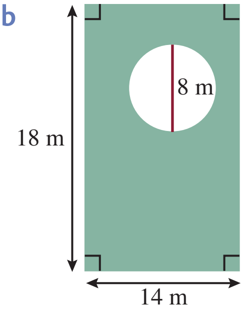
Exercise 13H

1 Find the shaded area of the following, expressed correct to 1 decimal place:

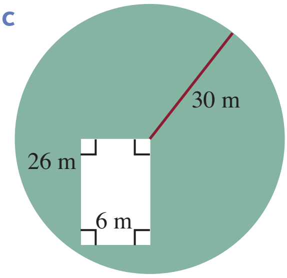
a



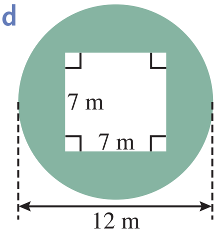
b



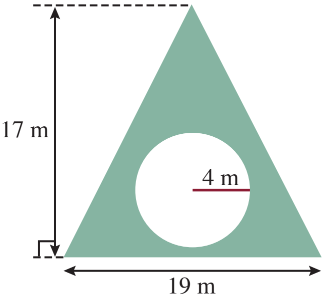
c



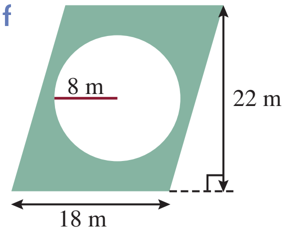
d



e

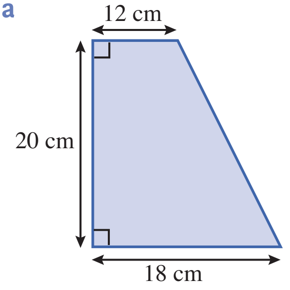


f

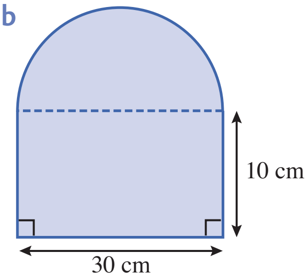


2 Find the area of each of the following shapes, correct to 2 decimal places:

a



b



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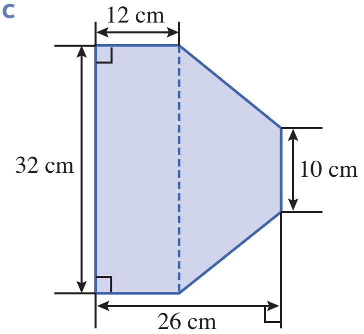

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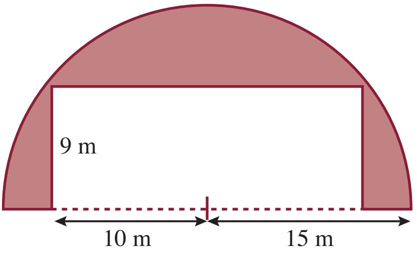

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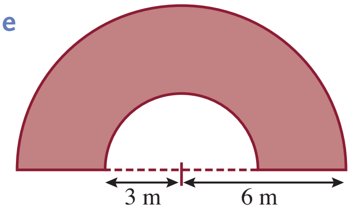

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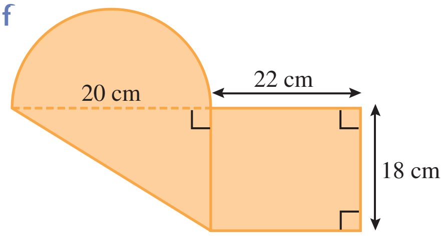

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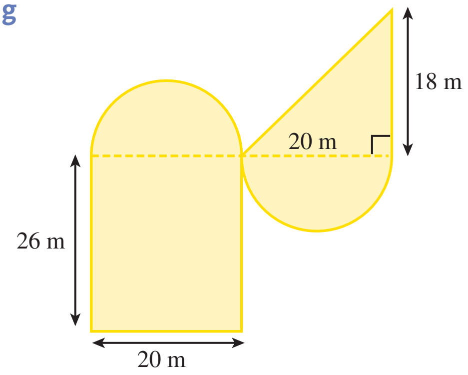
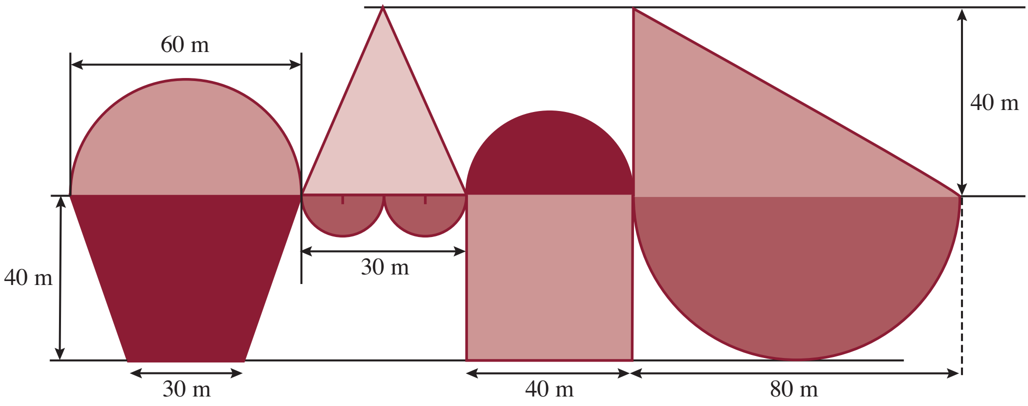
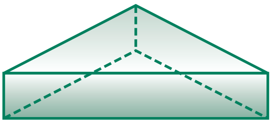


3 Find the area of the following shape:



Total surface area of prisms 13I

A **prism** is a solid that has straight sides. Its top surface is the same as its bottom surface.

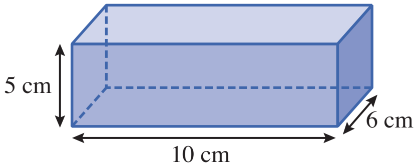


The total **surface area** of a prism is the sum of the areas of its surfaces (faces). When the sides of the prism are folded flat, the faces can be seen and the areas are easier to find.

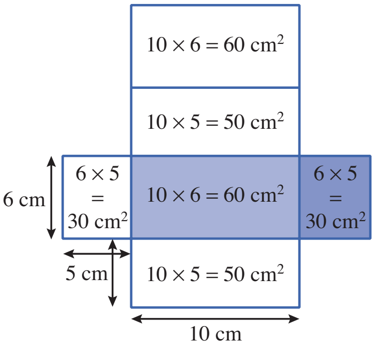
Example

Find the total surface area of these solids:

a

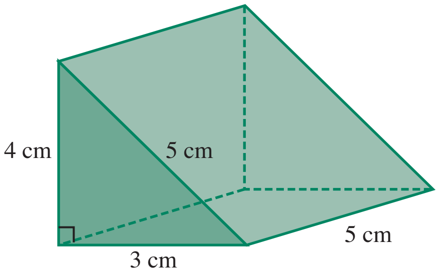


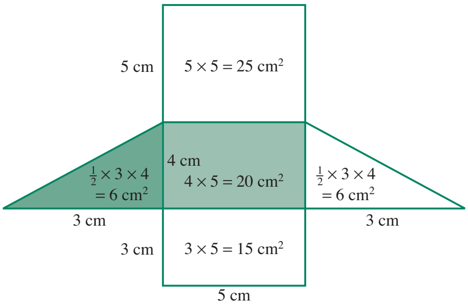
Solution



Total surface area
$= 60 + 60 + 50 + 50 + 30 + 30 = 280 \text{ cm}^2$

b



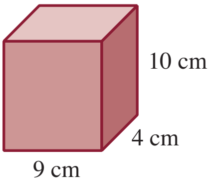


Total surface area
$= 25 + 20 + 15 + 6 + 6 = 72 \text{ cm}^2$

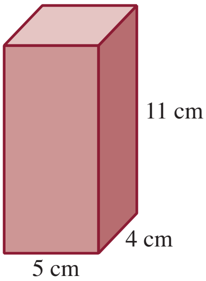
Exercise 13I

1 Find the total surface area of these solids:

a



b



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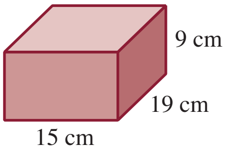
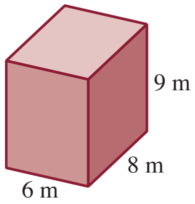
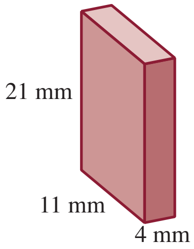


d



e



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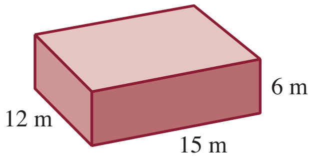
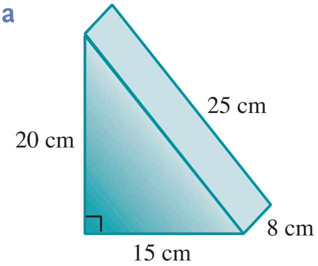


2 Find the total surface area of these prisms:

a



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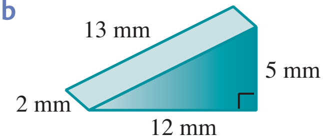

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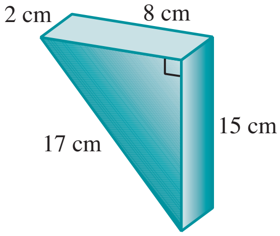

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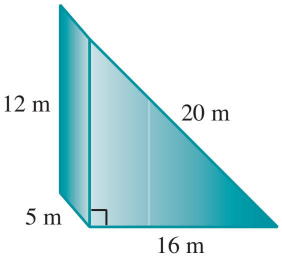

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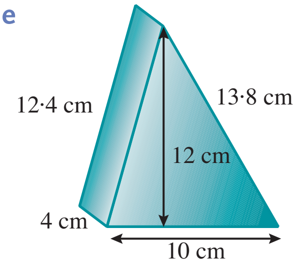

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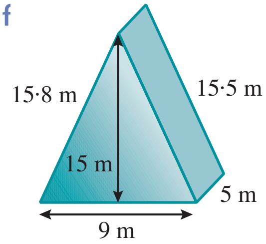
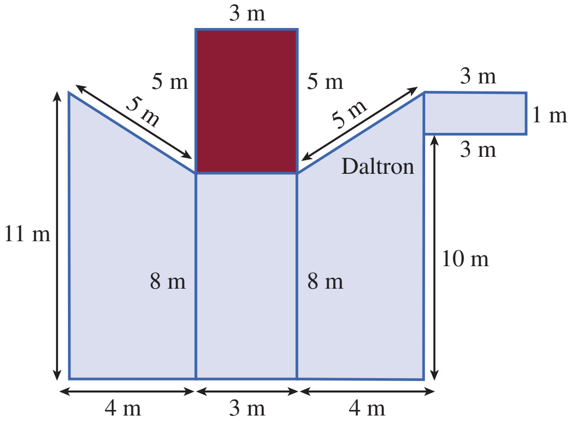


3 Shoe boxes are perfectly wrapped so that there is no overlap. Find the area of paper that is needed to wrap a shoe box with the following height, length and width:

a 12 cm, 30 cm and 16 cm **b** 20 cm, 20 cm and 18 cm **c** 16 cm, 28 cm and 10 cm

4 Daltron plans to repaint the outside of a tower that is part of their store. The tower can be modelled as a triangular prism on top of a cuboid. The vertical walls will be painted grey and the sloping roof will be painted red. The painter drew the diagram to show the faces that will be painted. Calculate the area of each colour that needs to be painted.



Volumes of prisms

13J

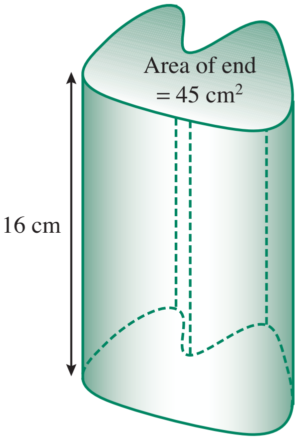
The volume of a solid is the amount of three-dimensional space inside it. To find the volume of a prism, multiply the base area by the height.

Volume = area of base × height

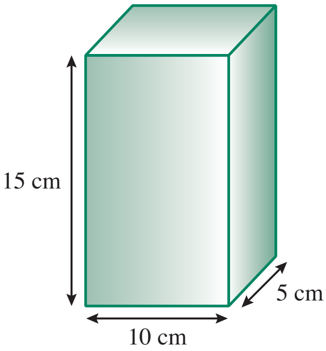
Example

Find the volume of these solids:

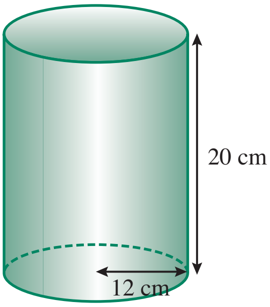
a



b



c



Solution

a

$$\begin{aligned}\textbf{Volume} &= \textbf{area of base} \times \textbf{height}\\ &= 45 \times 16\\ &= 720 \text{ cm}^3\end{aligned}$$

b

$$\begin{aligned}\text{Area of base} &= 10 \times 5\\ &= 50 \text{ cm}^2\end{aligned}$$

Volume = area of base × height

$$\begin{aligned}\therefore \text{ Volume} &= 50 \times 15\\ &= 750 \text{ cm}^3\end{aligned}$$

c

$$\begin{aligned}\text{Area of base} &= \pi r^2\\ &= \pi \times 12 \times 12\\ &= 144\pi \text{ cm}^2\end{aligned}$$

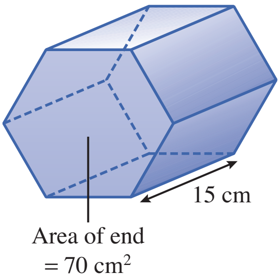
Volume = area of base × height

$$\begin{aligned}\therefore \text{ Volume} &= 144\pi \times 20\\ &= 2880\pi \approx 904{\cdot}79 \text{ cm}^3\end{aligned}$$

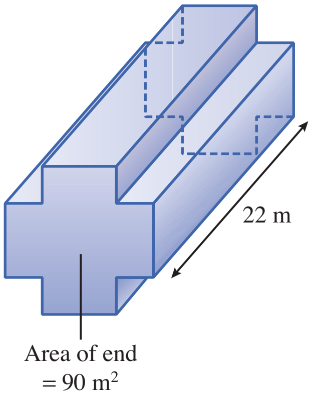
Exercise 13J

1 Find the volume of each prism:

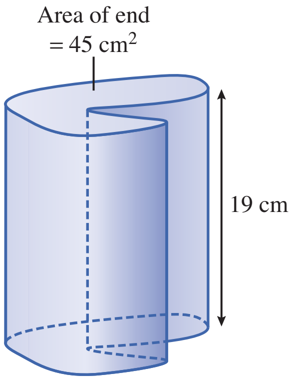
a



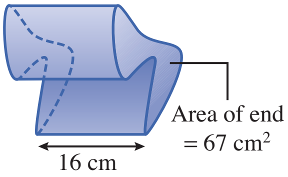
b



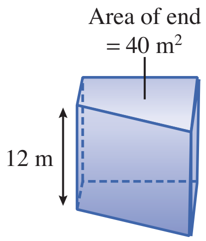
c



d



e



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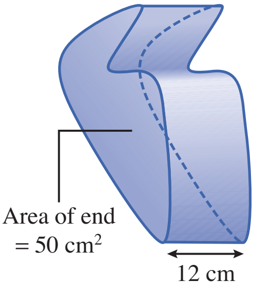
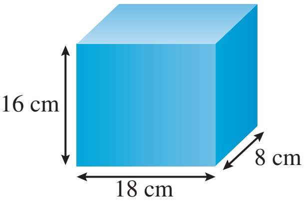


2 Find the volume of each cuboid:

a



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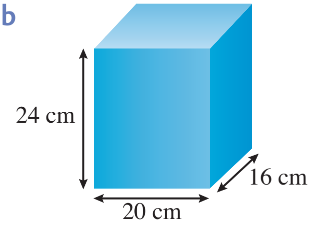

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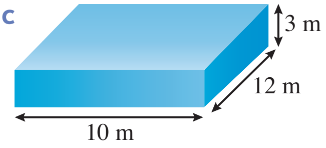

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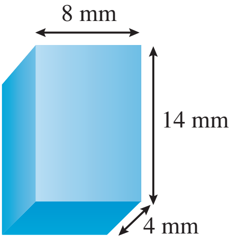

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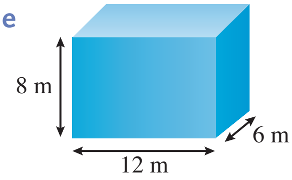

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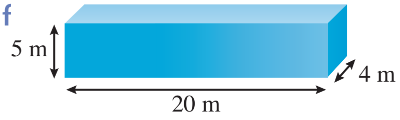
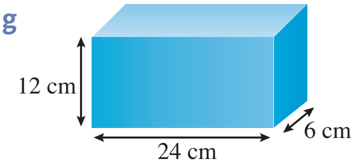
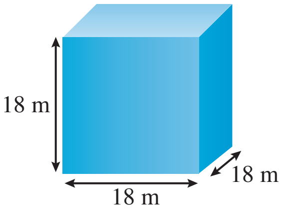


g

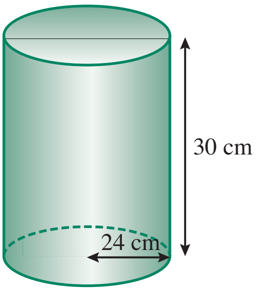


h



3 Find the volume of each cylinder expressed correct to 2 decimal places:

a



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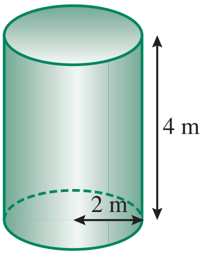

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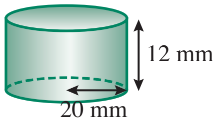

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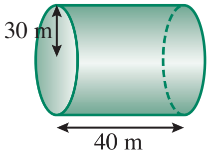

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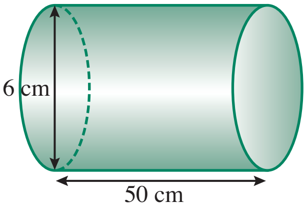

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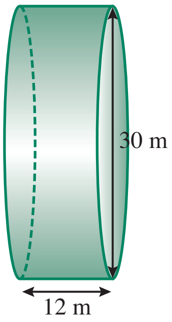

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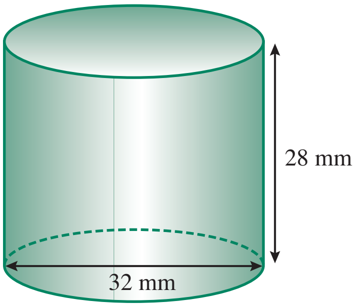

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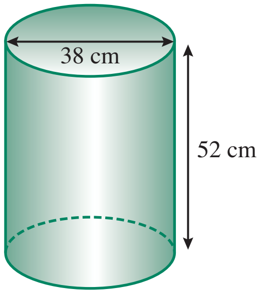
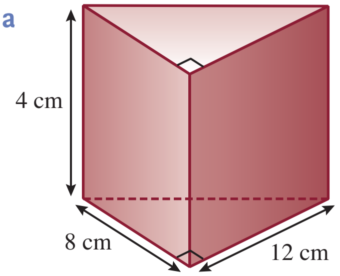


4 Find the volume of these solids:

a



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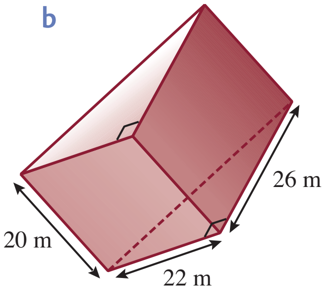

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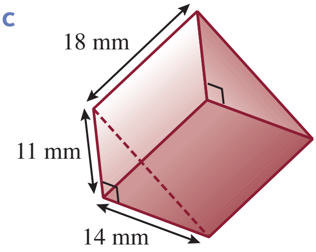
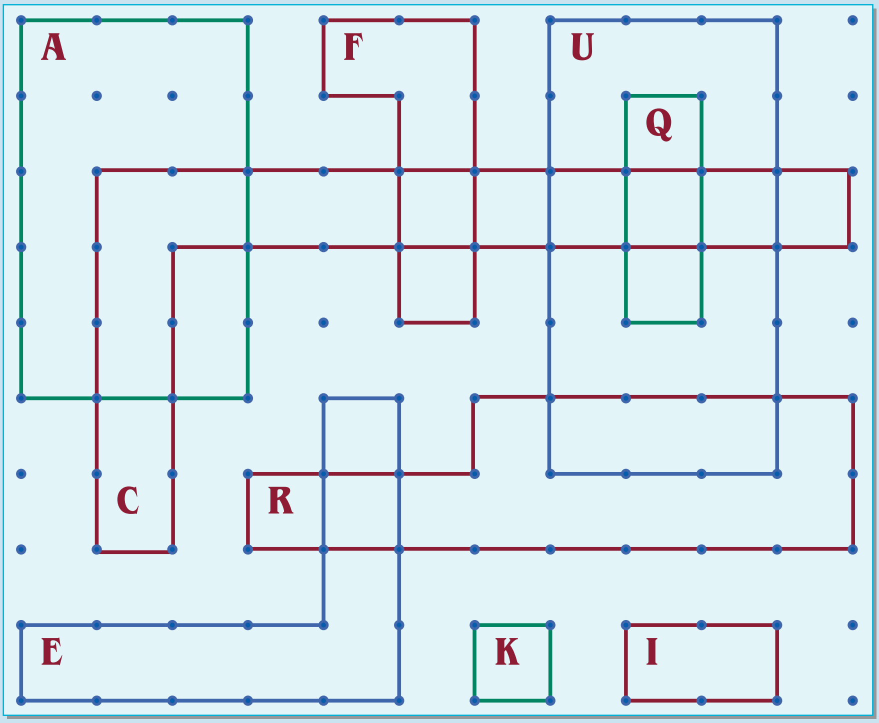


Puzzles

1 Use the centimetre grid to find the area of the shapes below. Match the letters to the correct area in cm^2 to solve the riddle:

What do you get if you cross a duck with a box of matches?

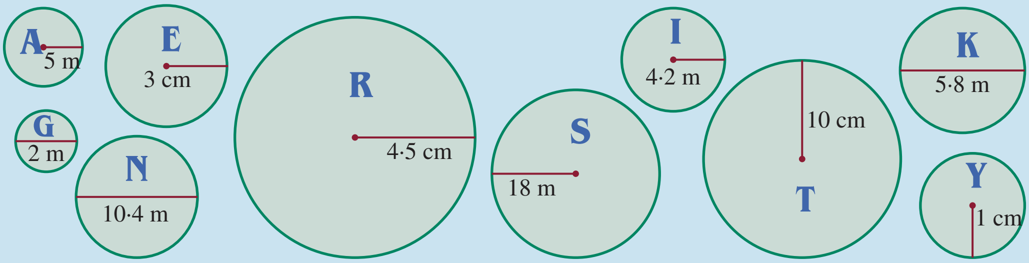


___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___

15 5 2 13 8 3 18 15 14 1 8 13

2 Find the area of the circles below correct to the nearest whole number, then match the letters to the answers to solve the riddle:

What do you get when you cross a bee with a skunk?



___ ___ ___ ___ ___ ___ ___

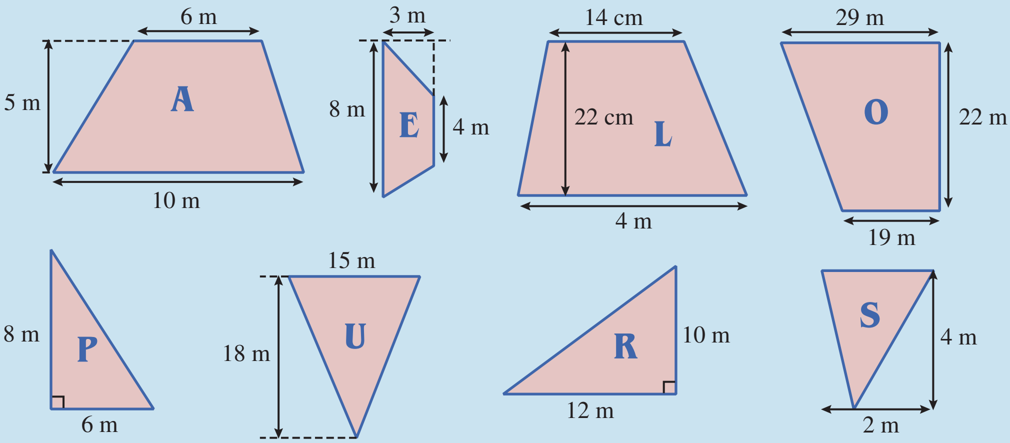
79 m^2 1018 m^2 314 cm^2 55 m^2 85 m^2 26 m^2 3 cm^2

___ ___ ___ ___ ___ ___ ___

1018 m^2 314 cm^2 55 m^2 85 m^2 3 m^2 28 m^2 64 m^2

3 Find the area of the following trapeziums and triangles. Match the letter to the correct area below to solve the riddle:

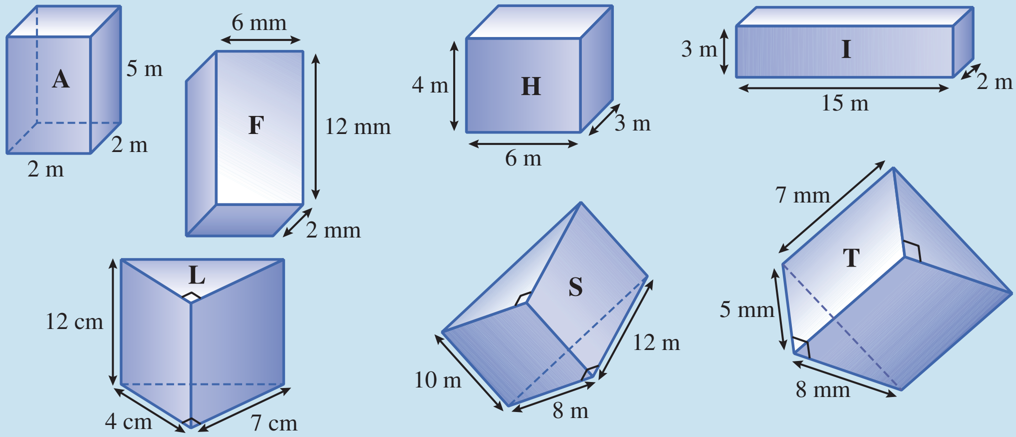
What do you get when you cross a cat with a lemon?



____		____	____	____	____		
40 m^2		60 m^2	18 m^2	40 m^2	4554 cm^2		
____	____	____	____	____	____	____	____
4 m^2	528 cm^2	135 m^2	60 m^2	24 m^2	135 m^2	4 m^2	4 m^2

4 Find the volume of the following prisms, then match the letter to the correct volume to solve the riddle:

What do you get when you cross sardines with a steamroller?



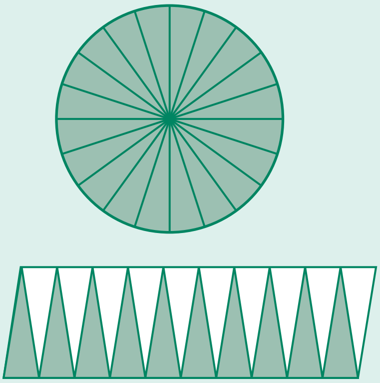
____		____	____	____	____		____	____	____	____
20 m^3		144 mm^3	168 cm^3	20 m^3	140 mm^3		144 mm^3	90 m^3	480 m^3	72 m^3

Applications

Pizza deals

Go down to your local pizza shop and measure the diameter of the different sizes of pizzas: small, medium, large, family etc. and work out the area of each pizza. Use the price of the one type of pizza to find the value of each pizza size in terms of dollars per square centimetre. Choose a pizza with a different topping. Compare the value in dollars per square centimetre for the different types of pizza. What effect do the different toppings have on the price per square centimetre for each size of pizza?

Estimating the area of a circle



Draw a large circle on cardboard. Divide it into 20 sectors as shown here.

Shade in the sectors and then cut them out.

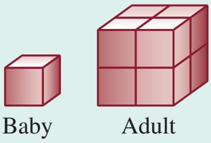
Each sector is nearly a triangle. Arrange these to form a parallelogram. Measure the length and height of the parallelogram and work out its area. Repeat this process with circles of different sizes.

Explain your results in terms of the radius of the circle. Paste your results onto a poster and describe your findings.

What would happen if you divided the circle into only 4 sectors? What about 40 sectors?

Draw diagrams to explain how the number of sectors affects the accuracy of your findings.

Baby in the car



When cars are left in the sun during summer the air inside can reach temperatures of 60 and 70 degrees Celsius. This is very dangerous, especially for children and animals left in these cars. These conditions are not as bad for adults because it relates to the size and volume of people of different ages.

Work in groups and collect nine cubes each measuring 2 cm by 2 cm by 2 cm.

Use a single cube to represent a baby and a $2 \times 2 \times 2$ group of cubes to represent an adult. Calculate the volume and surface area for the 'baby' and the 'adult'. Calculate the ratio of surface area to volume for each.

Discuss how this ratio might influence the rate of dehydration. Use your blocks to make more realistic models representing babies and adults, and calculate the surface area and volume of these.

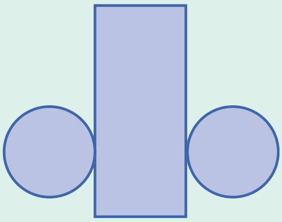
Packaging products

Collect a number of cardboard containers such as tissue boxes, biscuit packages or chocolate wrappers.

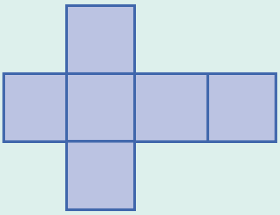
- **a** Measure the length and width of each side and find the area of each face.
- **b** Find the total surface area for each package. Fold the sides down and glue the package onto a poster.
- **c** Find the volume of each container.
- **d** Use a table to display your results and compare the volume with total surface area of the container.
- **e** Comment on your findings.

Nets for prisms

The net of a cylinder is shown here. The two circles are used for the top and bottom and the rectangle is wrapped around to make the sides.

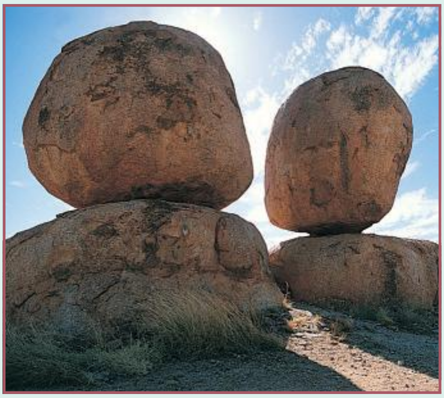


- **a** Use A4-sized dot paper to make a cylinder with the largest possible volume.
- **b** Calculate the surface area of the cylinder.
- **c** Use A4-sized dot paper to make a cuboid with the largest possible volume.
- **d** Calculate the surface area of the cuboid.



Volume of irregular solids

Solids such as rocks are usually found in odd shapes. Their volume can best be measured by using a measuring cylinder.

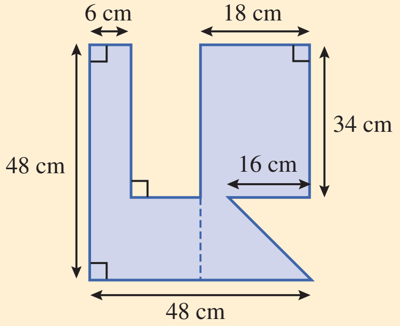


- **a** Collect small objects that will each fit inside the measuring cylinder—make sure that they won't float on water.
- **b** Half fill the measuring cylinder with water and read the level on the scale.
- **c** Carefully place the object into the measuring cylinder and read the new level. The difference between the levels is a measure of the volume of the objects in millilitres. One millilitre is the same as 1 cubic centimetre.
- **d** Use this fact to find the volume of your objects in cubic centimetres.

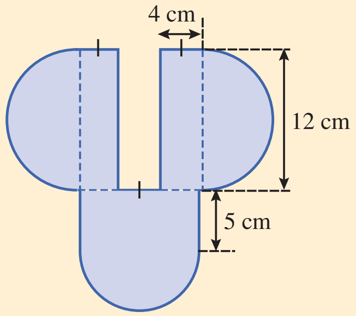
Enrichment

1 Find the area of these shapes to the nearest square centimetre:

a



b

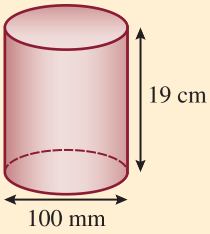


2 Find the total surface area and volume of:

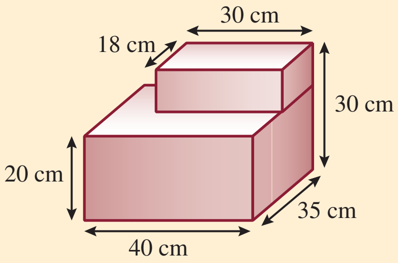
- **a** a cube with side length of 21 cm
- **b** a cube with side length 38 mm
- **c** a cuboid with dimensions 10 cm by 15 cm by 20 cm
- **d** a prism with an end area of 89 square metres, height of 24 metres and sides that have a total area of 192 square metres.

3 Find the volume and total surface areas of these solids:

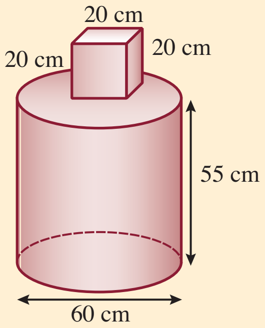
a



b



c



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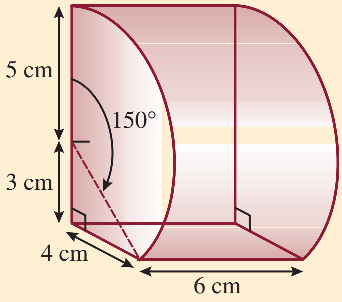
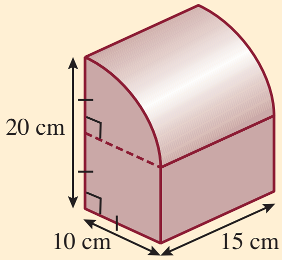
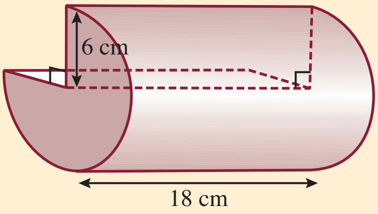


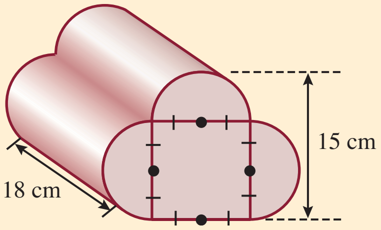
e



f



g

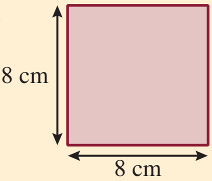


4 A white circle is to be placed inside each of the shapes shown below:

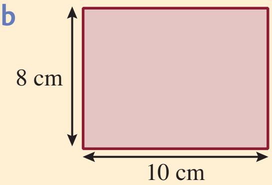
i Find the area of the largest circle that will fit inside each shape.

ii Find the coloured area that will still show.

a



b



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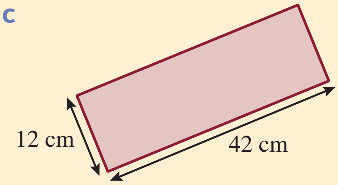

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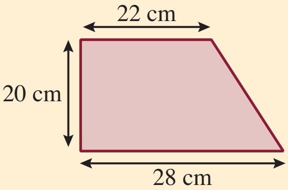

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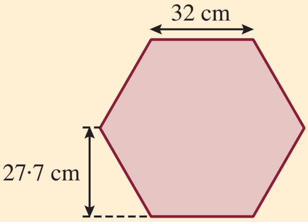
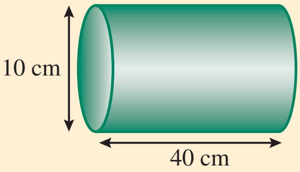


5 Draw a net for each of the following solids and mark on it the lengths of the sides. Use these net diagrams to find the total surface area of these shapes:

a



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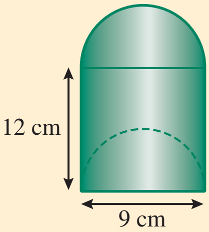
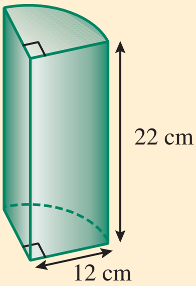
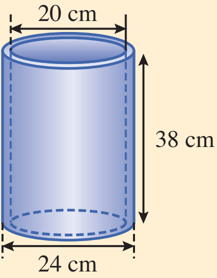


c



6 Find the volume and total surface area of these figures:

a



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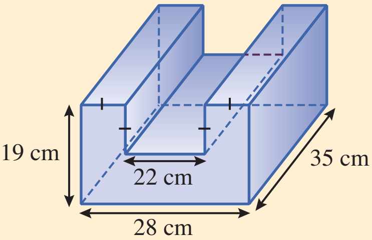

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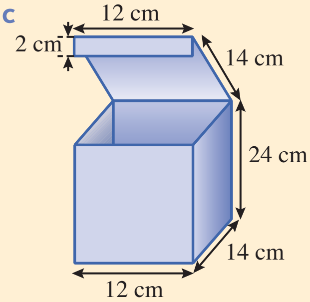
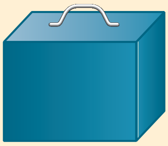


7 When flying in America, the hand luggage that may be carried in the plane is limited in size so that the width, height and length measurements when added up may not be larger than 150 cm. Two standard bags are allowed:

Bag A: the length is 30 cm more than the width

Bag B: length is twice the width



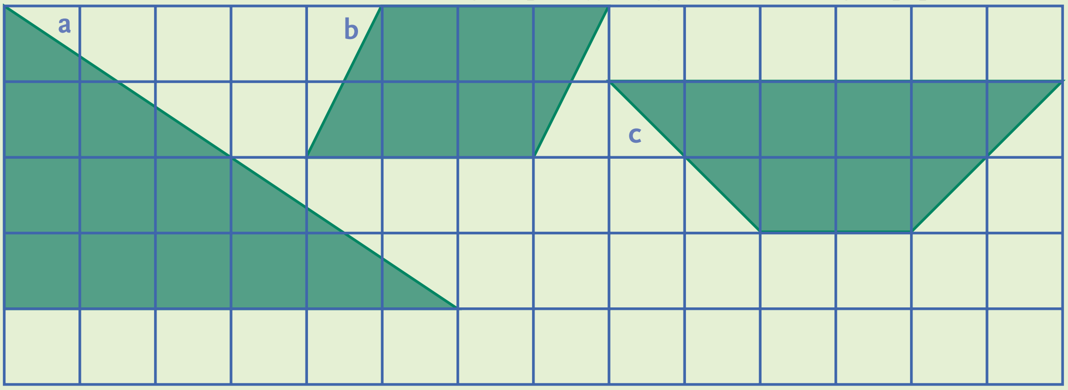
a Find the dimensions of each type of bag with the maximum volume.

b Find the surface area of each bag.

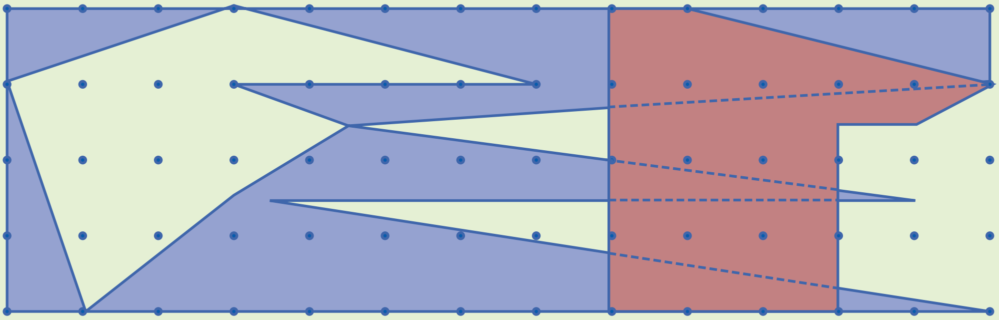
Revision/Assessment

Exercise 13A

1 Find the area of each of the following shapes by using the centimetre grid paper:

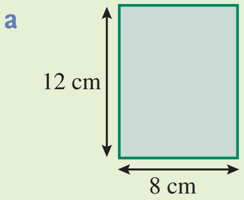


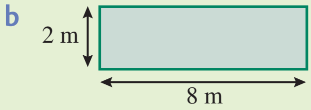
2 Use the dots to draw a grid over the shapes and estimate the area of the blue and the pink shapes.

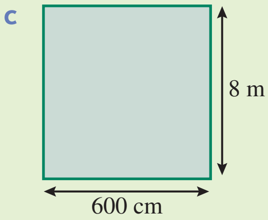


Exercise 13B

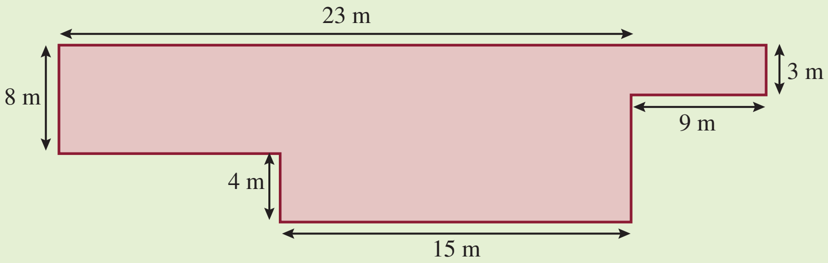
3 Find the area of these rectangles:







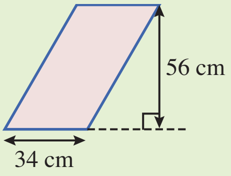
4 Divide this shape into rectangles and find its area:



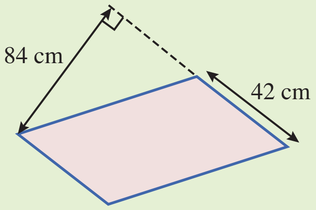
Exercise 13C

5 Find the area of these parallelograms:

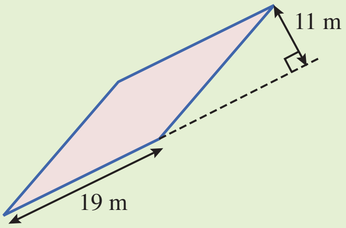
a



b

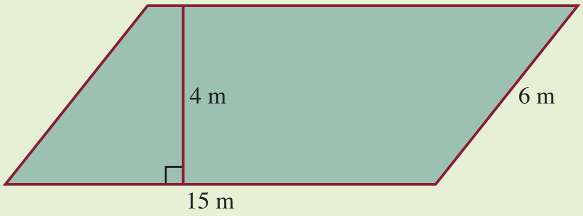


c



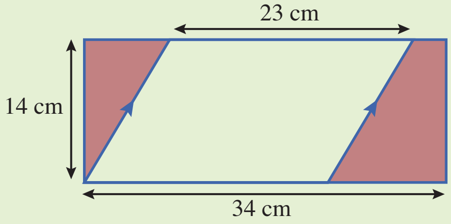
6 A garden plot in the shape of a parallelogram has a short side of length 6 metres and a long side of 15 metres. A 4-metre rope stretches across the garden as shown.

Find the area of the plot.

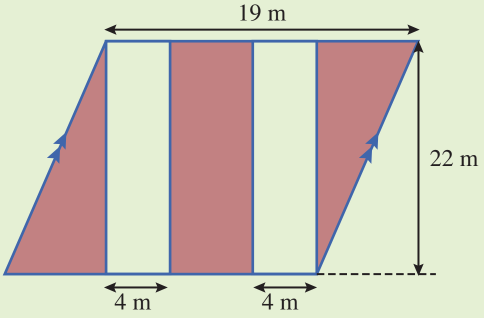


7 Find the coloured area in these shapes:

a



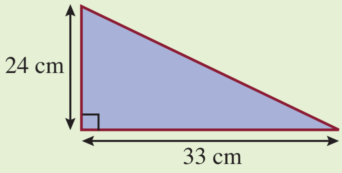
b



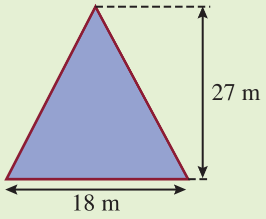
Exercise 13D

8 Find the area of these triangles:

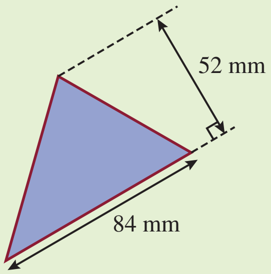
a



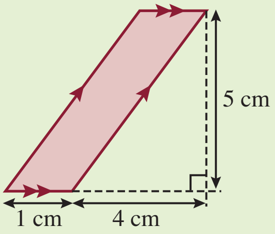
b



c

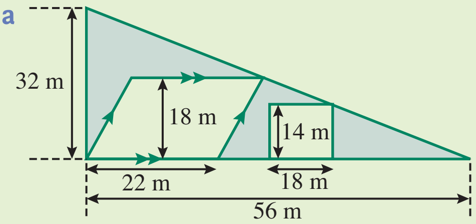


9 A puzzle contains parallelogram tiles with a height of 5 cm and a base of 1 cm as shown here. The tiles are placed into a square frame of side length 20 cm. Find the area covered by the tiles and the area of the board that is not covered.



10 Find the coloured areas in the shapes below:

a



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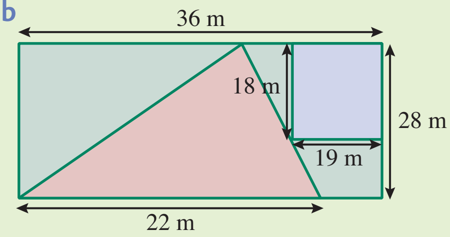
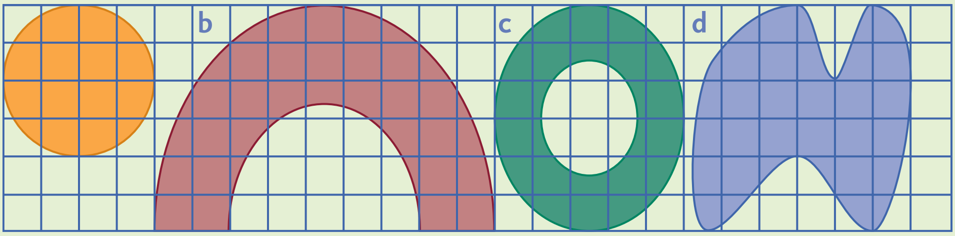


Exercise 13F

11 Use the grid to estimate the area of these shapes:

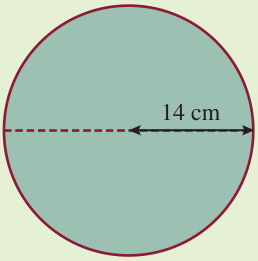
a



Exercise 13G

12 Find the area of these circles:

a



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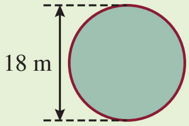

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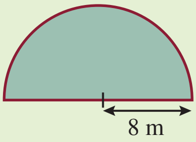
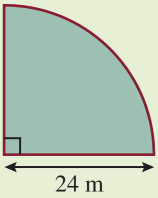


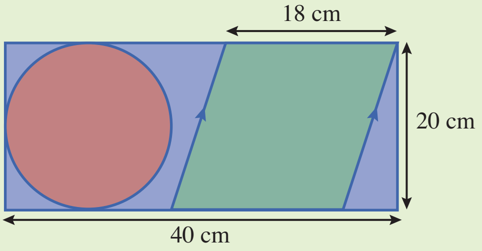
d



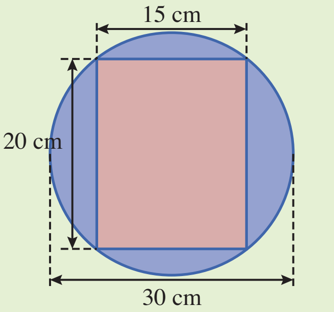
Exercise 13H

13 Find the coloured areas in the following shapes:

a



b



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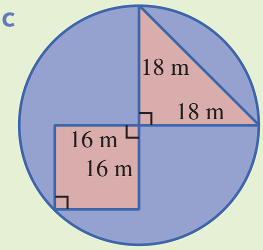

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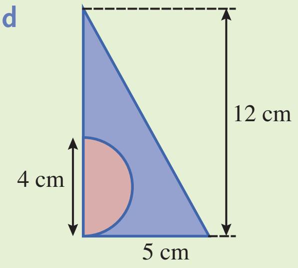
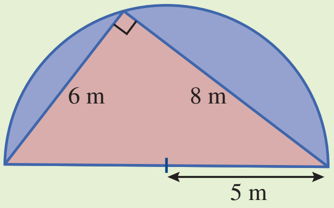


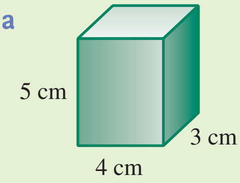
e



Exercise 13I

14 Find the total surface area of the following cuboids:

a



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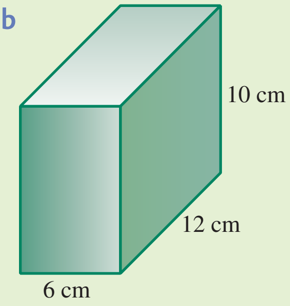
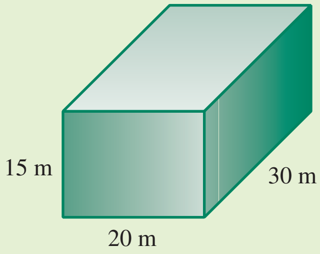
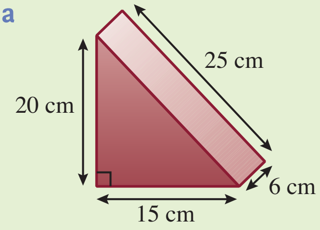


c

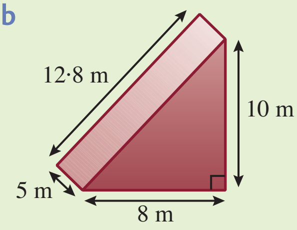


15 Find the total surface area of these prisms:

a



b



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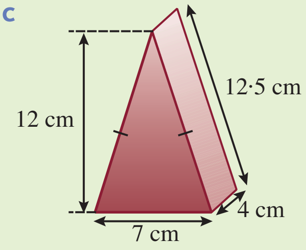
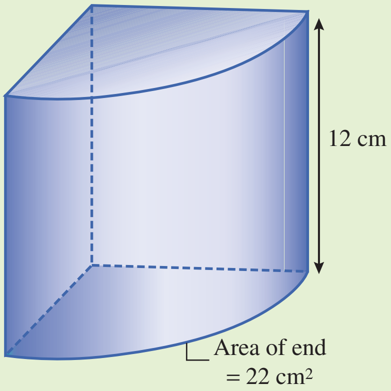


Exercise 13J

16 Find the volume of these solids:

a



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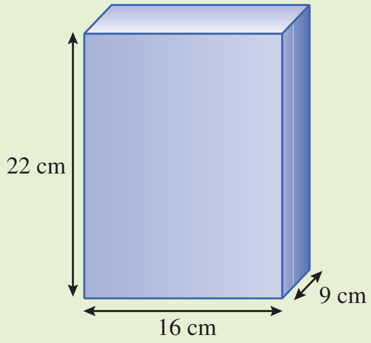
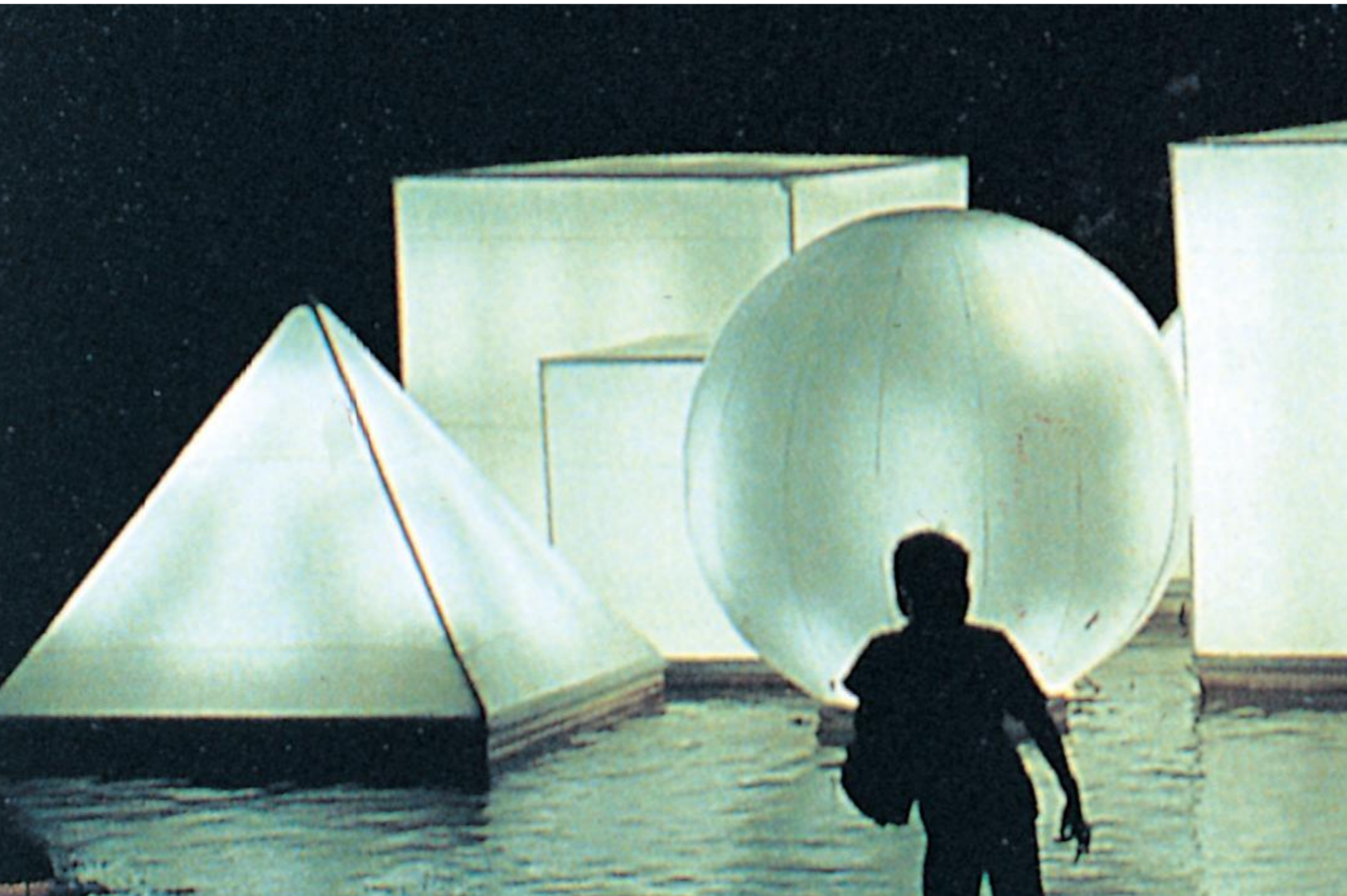


CHAPTER

14

Polyhedra and Networks



Polyhedra and Networks

Our world is three dimensional. Many of the solid shapes around us are combinations of three-dimensional figures such as prisms, pyramids, cones, cylinders and spheres. Graphic artists can create animations of 3D objects and scenes by using computer software. The artist draws the top, front and side views of the object and places them within a scene. The computer automatically draw all the objects from chosen viewpoints.

This chapter covers the following skills:

- Using polyhedra definitions
- Drawing 3D shapes using isometric drawing
- Visualising and drawing three-dimensional shapes
- Drawing one-point and two-point perspectives
- Defining polyhedra
- Recognising and drawing nets of selected polyhedra
- Using networks to show vertices, paths, traversability and the shortest path
- Constructing a solid from its net and verifying Euler's rule

Specific Learning Outcome (SLO)

Learners should be able to:

8.14.1.1 Define and identify solid shapes: cube, pyramid, triangular-based prism, tetrahedron, cylinder, cone, cuboid and sphere.

8.14.2.1 Identify three categories of solid shapes and their corresponding shapes: prisms, pyramids and solids with circular shapes.

8.14.2.2 Define the terms 'prism' and 'pyramid'.

8.14.2.3 Sketch the various solid shapes.

8.14.3.1 Cut sheets of cardboard to produce solid shapes.

8.14.3.2 Define the terms 'polyhedron' and 'polyhedra'.

8.14.3.3 Identify the five regular polyhedra and their corresponding nets.

8.14.4.1 Draw nets for a variety of prisms and pyramids.

8.14.4.2 Cut and fold nets to produce solid shapes.

8.14.5.1 Identify the lengths, widths and heights of various solids.

8.14.6.1 Identify three views that can be produced by drawing cubes on dot paper: isometric drawings showing corners; oblique drawings showing a front view; orthogonal drawings showing three separate views.

8.14.6.2 Draw cubes and composite solids on dot paper.

8.14.7.1 Draw perspective drawings of solids using one- and two-point perspectives.

8.14.8.1 Identify the properties of polyhedra.

8.14.8.2 Identify and define different parts of polyhedra: edge, vertex, face.

8.14.9.1 Name the five regular polyhedra called Platonic solids.

8.14.10.1 Define the term 'planar network'.

8.14.10.2 Draw a planar network for a cube.

8.14.11.1 Explain the term 'order of vertex'.

8.14.11.2 State the order for each vertex and the total order for a network.

8.14.12.1 Explain the following terms: network, vertices (nodes), edges (paths), regions, traversability.

8.14.13.1 Identify network vertices that have odd or even degrees.

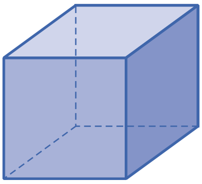
8.14.13.2 Find the shortest path for networks given as diagrams or maps.

8.14.13.3 Draw and construct networks with given number of vertices and edges.

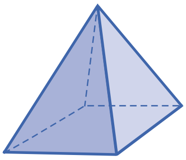
14A Solid shapes

To represent three-dimensional shapes on a two-dimensional surface, we can draw the visible edges using a solid line and the hidden edges with a broken, or dotted, line. Some important solid shapes are shown below.

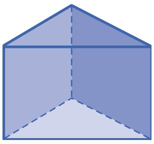
Cube



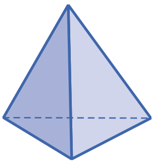
Pyramid



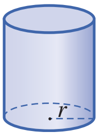
Triangular-based prism



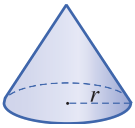
Tetrahedron



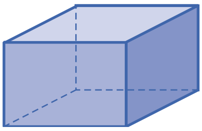
Cylinder



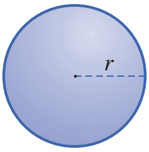
Cone



Cuboid



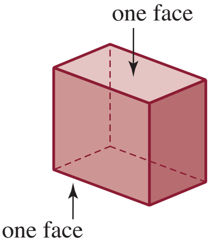
Sphere



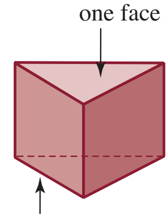
Solid shapes fall into the categories of prisms, pyramids and other solids with circular shapes such as spheres, cones and cylinders.

Prisms

Prisms are named after the shape of the base. A prism has two faces which are parallel and the same shape and size. All other faces are rectangles.



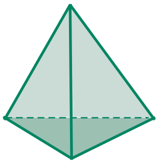
Rectangular prism



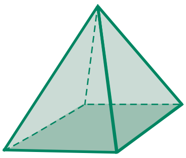
Triangular prism

Pyramids

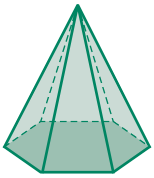
Pyramids are named after the shape of the base. A pyramid has a polygon base; all other faces are triangles. Right pyramids have the apex or vertex directly above the centre of the base. Skewed or slanted pyramids have an apex not directly above the centre of the base.



Triangular-based pyramid
(3-sided)



Square-based pyramid
(4-sided)

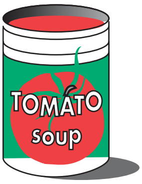


Hexagonal-based pyramid
(6-sided)

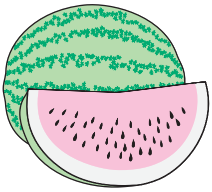
Exercise 14A

1 Look closely at the figures shown below and identify the solid shapes you can see in each:

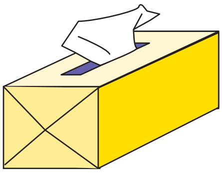
a



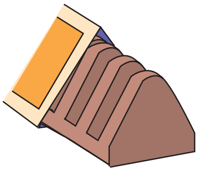
b



c

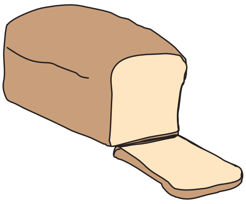


d

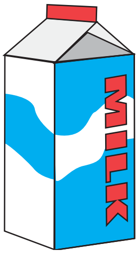


e

f



g



h

2 In your workbook draw an accurate diagram of the following:

a A cube

b A cone

c A cylinder

d A tetrahedron

e A triangular-based prism

f A pyramid

g A sphere

h A cuboid

i Two cubes stacked one on top of the other

j A pyramid stacked on top of a cube

k Two cuboids joined with a common base

l Two cylinders joined with a common base

3 Make a neat and accurate drawing of four prisms with a base that is a:

- **a** square
- **b** right-angle triangle
- **c** pentagon
- **d** trapezium

4 Make a neat and accurate drawing of four pyramids with a base that is a:

- **a** rectangle
- **b** equilateral triangle
- **c** hexagon
- **d** rhombus

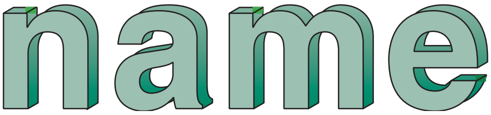
5 **a** What is the difference between a right pyramid and a skewed pyramid?

b Draw a neat and accurate sketch of two different skewed or slanted pyramids.

6 What is the difference between a prism and a pyramid?

7 Look around your classroom. Can you list ten items, or parts of items, that are in the shape of a prism?

8 Write your name in letters that are three-dimensional prisms.



9 **a** How many surfaces does this cylinder have?

b Can you name the shape of each surface?

c Name five items stored or packed in cylinders.

10 In your workbook carefully draw an accurate sketch of the following solids, using solid and dotted lines and labelling all side lengths:

- **a** Cube of length 4 cm
- **b** Sphere of radius 3·3 cm
- **c** Cylinder of radius 2·5 cm, height 5·8 cm
- **d** Cone of radius 3·5 cm, height 4·6 cm
- **e** Cuboid of length 5 cm, width 4 cm, height 3 cm
- **f** Tetrahedron with edge length 2 cm
- **g** Cube of length 5·8 cm
- **h** Sphere of radius 3·6 cm
- **i** Cylinder of radius 1·5 cm, height 6·3 cm
- **j** Cone of radius 2·9 cm, height 4·3 cm
- **k** Cuboid of length 7 cm, width 5 cm, height 2 cm
- **l** Tetrahedron with edge length 3·6 cm

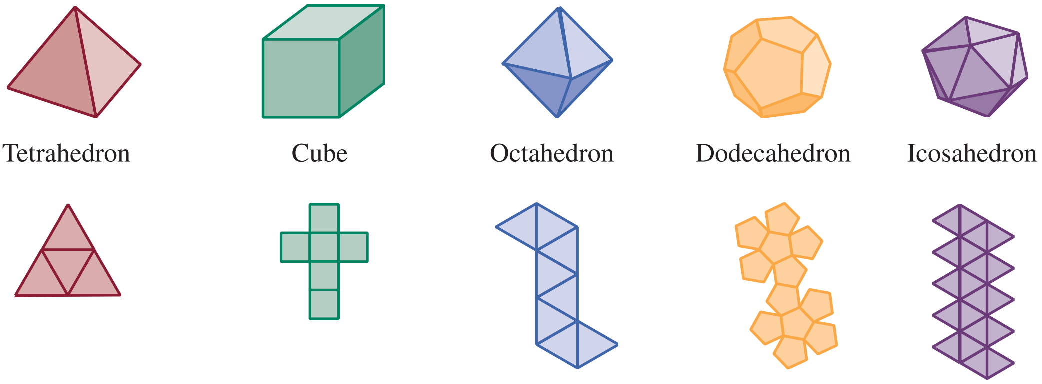
14B

Nets for solid shapes

The net of a solid shape is the two-dimensional representation that can be folded to produce a three-dimensional solid.

Most packaging boxes are cut from a single sheet of cardboard which is folded and taped into shape. The shape cut out from the cardboard to form the box before it is folded is called the net of the prism.

There are five regular polyhedra. Each face of the regular polyhedron is a congruent regular polygon with the same number of faces meeting at each vertex, or point. The polyhedra are shown with their nets below.



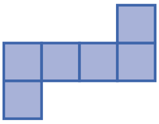
Exercise 14B

1 Draw an accurate net of:

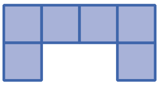
- **a** a cube
- **b** a cuboid
- **c** a cylinder
- **d** a triangular-based prism
- **e** a pyramid
- **f** a tetrahedron

2 **a** Which of the following nets will fold to make a cube?

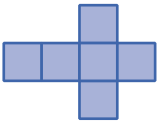
A



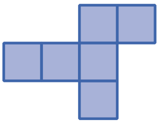
B



C



D

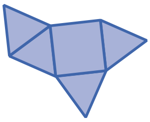


b Make a net from part a that does not fold into a cube.

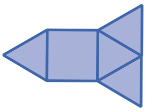
c Explain in a sentence why one of the nets from part a does not make a cube.

3 **a** Which of the following nets will fold to make a pyramid?

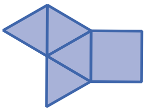
A



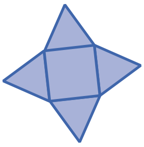
B



C



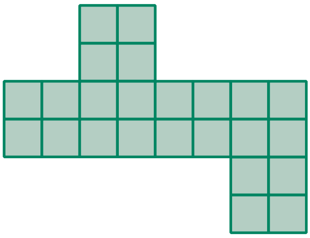
D



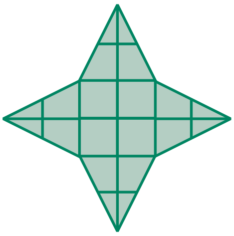
b Modify one of the nets above so that it does not fold to make a pyramid.

c Make a net of your modified pyramid to illustrate your answer to part b.

4 Cut out a larger version of the net shown and fold it to make a cube.

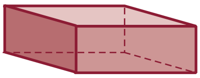


5 Cut out a larger version of the net shown and fold it to make a pyramid.

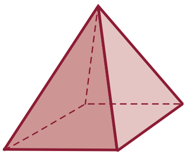


6 Match each of the nets with a polyhedron:

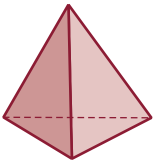
a



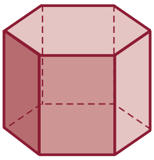
b



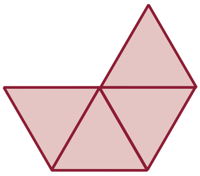
c



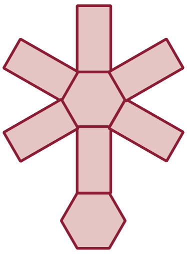
d



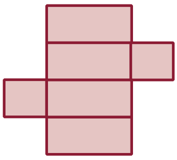
1



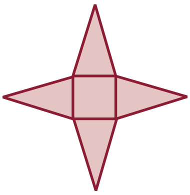
2



3



4



7 **a** Make an accurate three-dimensional drawing of a prism with a triangular base.

b Draw an accurate net of your prism on cardboard. Cut it out and fold to form a prism.

c Now experiment with your model and see how many different nets are possible for a triangular-based prism.

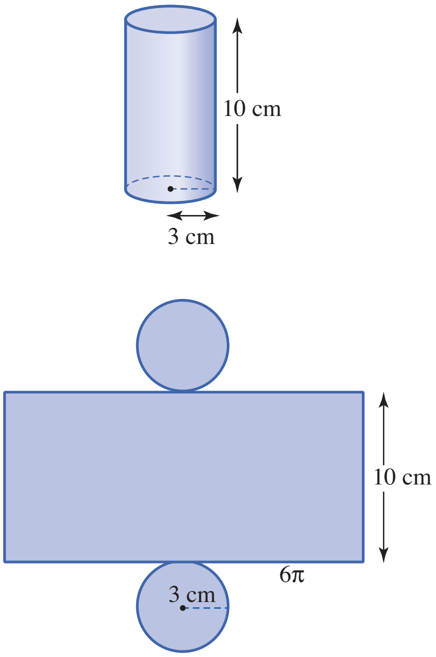
8 a Make a three-dimensional drawing of a pyramid with a square base.

  b Draw an accurate net of your pyramid on cardboard. Cut it out and fold it to form a pyramid.

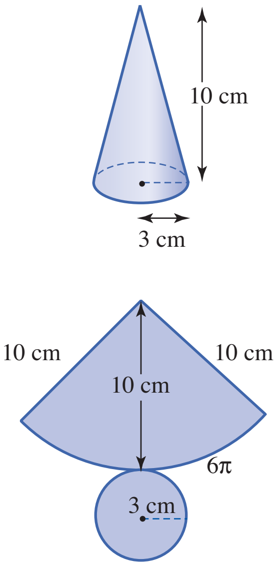
  c Now experiment with your model and see how many different nets are possible for a square-based pyramid.

9 Use the nets of the cylinder and cone shown below to make accurate models with these measurements:

a



b



10 a Design a net to form a cylinder and cone of height 15 cm and radius 5 cm.

  b Cut out the nets from cardboard and fold to form a model.

  c Use the models to estimate the relative volumes of the two solids.

  d What are the shapes of their cross-sections when made parallel or perpendicular to the base?

11 The roof structure of a church tower is shown here. Can you name its shape?

14C Drawing 3D shapes

Three-dimensional (3D) shapes have the dimensions length, width and height. They are evident in houses, city buildings, drink containers and some confectionary wrappers. Many three-dimensional models are made from two-dimensional nets or frames. Some computer animations are based on wire-frame models made up of polygons.

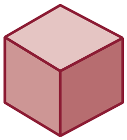
In this section you will use dot paper to visualise and construct 3D shapes based on cubes.

How you draw 3D shapes depends on which view you wish to represent.

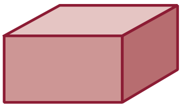
Isometric drawings show a corner view.

Oblique drawings show a front view.

Orthogonal drawings shown three separate views: front, side and top.



Corner view
or isometric drawing

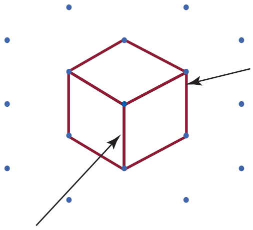


Front view or
oblique drawing

Example

1 Draw a cube using isometric paper.

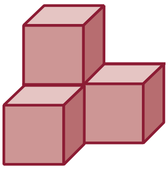
Solution

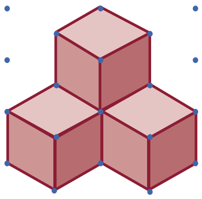


Step 2:
Then draw in side edges and top face

Step 1:
Start with vertical edge of cube

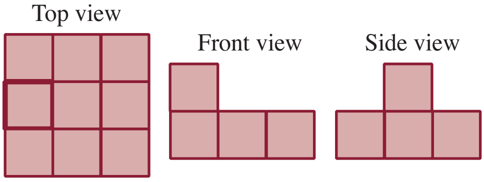
2 Draw this stack of four cubes using isometric paper. Note that one cube is hidden.





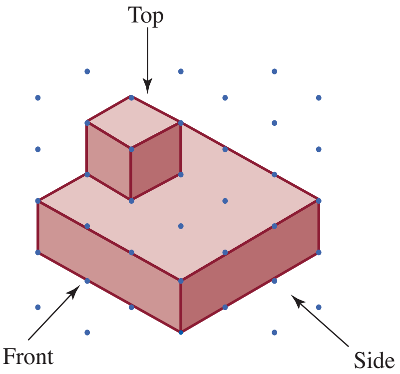
Example

3 The orthogonal drawing below shows the side, top and front view of a shape. Redraw it on isometric paper.



Note the bold lines on the top view, which indicate extra height or depth. In this case, the cubes are stacked two high.

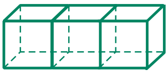
Solution



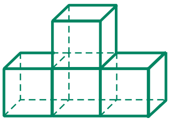
Exercise 14C

1 How many cubes are there in each of the following solids? Use isometric paper to draw each of the solids:

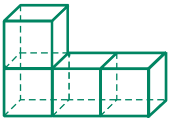
a



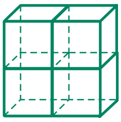
b



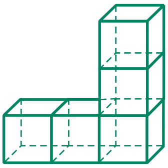
c



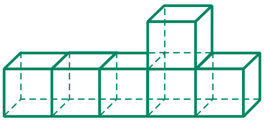
d



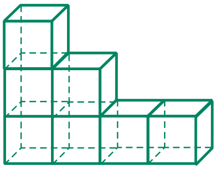
e



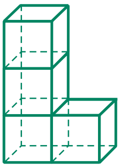
f



g

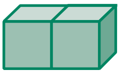


h

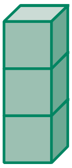


2 Make orthogonal drawings of these shapes. Draw their front, side and top views:

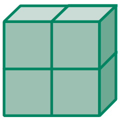
a



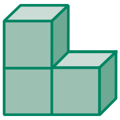
b



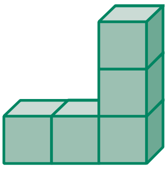
c



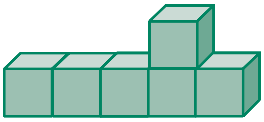
d



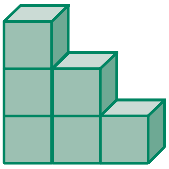
e



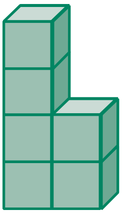
f



g



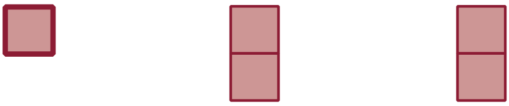
h



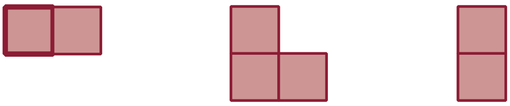
3 The orthogonal drawings show the top, front and side views. Draw the corner view of these shapes by using isometric paper:

Top view Front view Side view

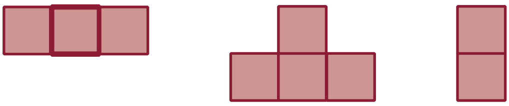
a



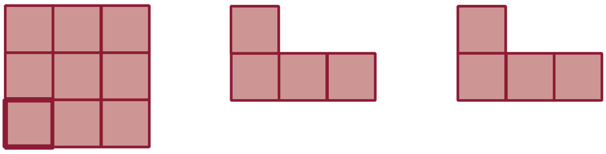
b



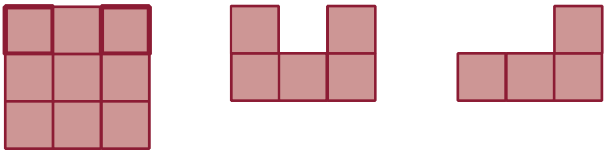
c



d

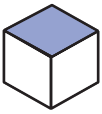


e

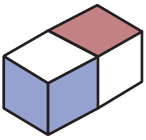


4 Redraw on isometric paper each of the shapes shown below, adding one cube to the faces shaded in blue, taking away any cube that has a face shaded in red:

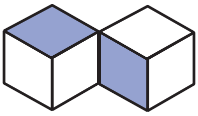
a



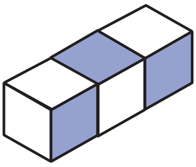
b



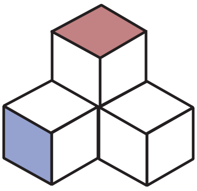
c



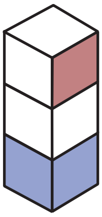
d



e



f



5 Solid shapes are made by stacking cubes on top of each other. The height of the cubes in each stack is shown below. Use isometric paper to draw the following shapes:

a

3	2	3
1	2	1

b

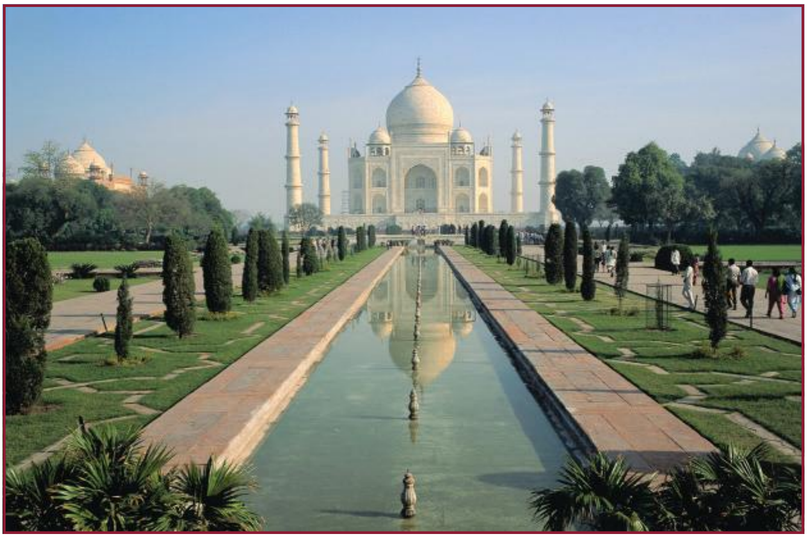
3	2	1
2	1	

c

1	1
2	2

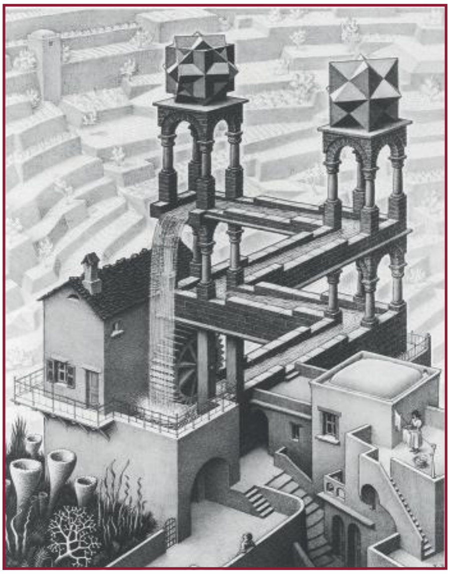
Perspective drawings

Perspective drawings are used in two-dimensional drawings to represent depth of three-dimensional objects.



Your field of vision depends on how close you are to an object. The closer you are to an object, the more of your field of view is taken up so it appears larger. As you move away from an object, the angle of your field of vision keeps getting smaller.

This is why posts appear to converge in the distance. The point at which distant lines appear to converge is called the **vanishing point**. Vanishing points make an image look like it is three-dimensional. We say it has perspective.



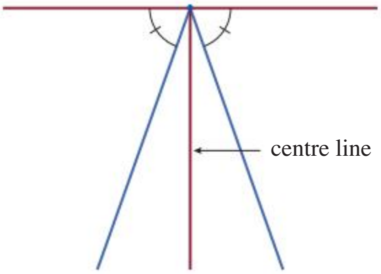
Most paintings have some kind of perspective in order to look more realistic. The Dutch graphic artist, Escher, created some famous images without correct perspective. Does the water really flow up-hill in this picture called *Waterfall*?

Two methods for creating perspective drawings are the one-point and the two-point perspective.

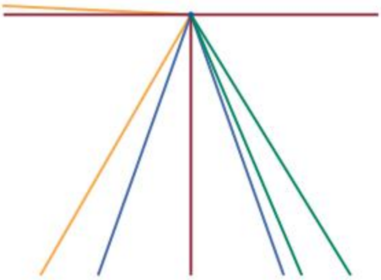
One-point perspective: A roadside scene

- Draw the horizon with the vanishing point in the middle.

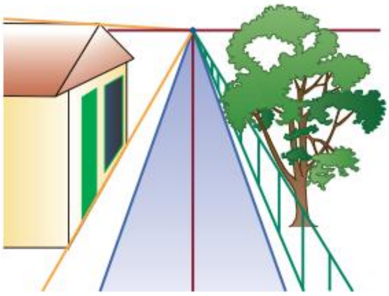
- Draw your centre line in pencil and two lines coming out of the vanishing point to represent the road. Make sure that the lines are at equal angles to the centre line.



- Draw in two lines on the right side to represent the fence.
- Draw two lines on the left side to represent the row of houses.



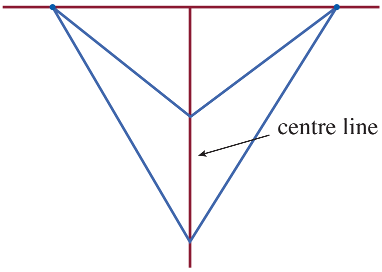
Fill in the details to complete the picture.



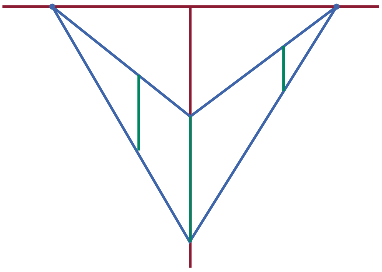
Two-point perspective: A box of tissues

- Draw the horizon with two vanishing points.

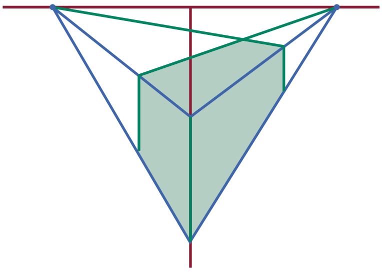
- Draw in the front edge of the box and join each end to the vanishing points.



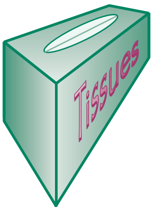
- Add in the two other visible edges of the box.



- Join the corners of the box to the opposite vanishing point.



- Add details to your box and rub out the other lines.

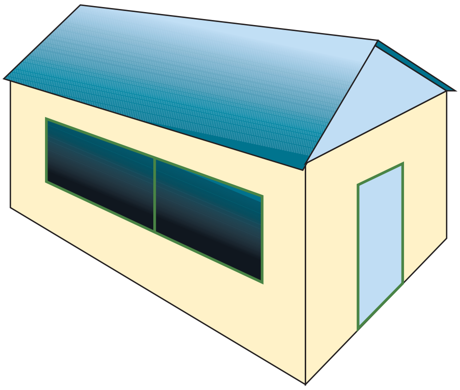


Exercise 14D

1 Copy these shapes into your books, then draw in the horizon line and the two vanishing points:

a

b



2 Draw one-point perspective drawings of the following scenes:

a

b

3 Draw two-point perspective drawings of the following objects:

a

b

c

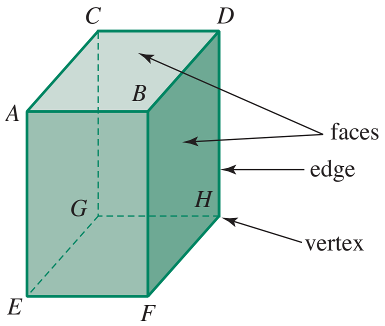
4 Using one-point or two-point perspective draw the following:

- **a** A rectangular prism such as your textbook or a box of tissues
- **b** The front of your home or your school
- **c** Your dream home
- **d** A building of the future
- **e** A highway scene with a road, trees and buildings
- **f** A boat wharf
- **g** A fish crate

Polyhedra definitions 14E

A **polyhedron** is a three-dimensional shape with faces that are polygons. You will recall that a polygon is an enclosed shape with straight sides. The **edge** of a polyhedron is a line segment where two faces meet. A **vertex** is the point where two or more edges meet. Capital letters are sometimes used to represent the vertices of the solid.

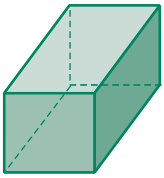
In three-dimensional shapes, the **face** is the flat part that is enclosed by edges.



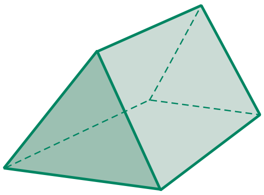
Example

1 A cuboid has six faces. Draw a cuboid.

Solution

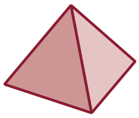


2 A triangular-based prism has five faces. Show these on a diagram.

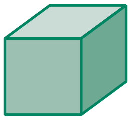


There are five regular polyhedra. All of their faces are congruent regular polygons with the same number of faces meeting at each vertex.

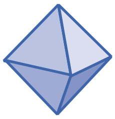
Tetrahedron



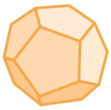
Cube (Hexahedron)



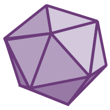
Octahedron



Dodecahedron



Icosahedron

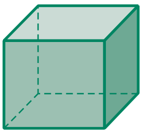


They are also called **platonic solids** after the Ancient Greek philosopher Plato (427–347 BC).

A planar network is one in which the edges do not cross. So it follows that platonic solids are three-dimensional solids that can be represented in two dimensions without any edges crossing.

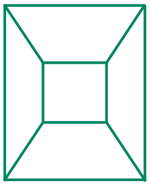
Example

3 Draw the planar nework of a three-dimensional cube.



Solution

Its two-dimensional planar network diagram:

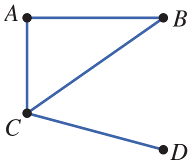


Vertices = 8
Edges = 12
Faces or regions = 6

The **order** of a vertex is equivalent to the total number of edges connected to the vertex. The total order of a network is equal to twice the total number of edges.

Example

4 State the order of each vertex and the total order of the network:



Solution

The order of $A = 2$, $B = 2$, $C = 3$, $D = 1$

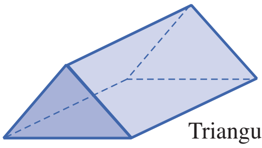
The total order of the network

$= 2 + 2 + 3 + 1$

$= 8$

or 2×4 edges $= 8$

Exercise 14E



Triangular prism

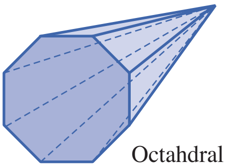
1 **a** How many faces does this prism have?

b Name the shape of each face.

c How many edges and vertices does it have?

d How many edges meet at each vertex?

e State the order of each vertex in the prism.



Octahdral pyramid

2 **a** How many faces does this pyramid have?

b Name the shape of each face.

c How many edges and vertices does it have?

d How many edges meet at each vertex?

e State the order of each vertex in the prism.

3 **a** Using the diagram of regular polyhedra on the previous page, copy and complete the following table in your workbook:

Name of the platonic solid	Number of faces (F) or regions	Number of vertices (V)	Number of edges (E)
Tetrahedron			
Cube			
Octahedron			
Dodecahedron			
Icosahedron			

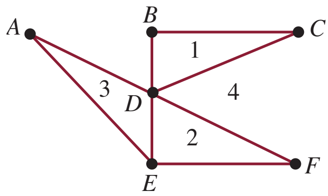
A mathematician named Leonhard Euler found that in any planar network the vertices, faces and edges of a platonic solid are linked by the rule $F + V = E + 2$.

b Verify this rule for each of the platonic solids in the table above.

Networks and their applications 14F

Often when we travel from one place to another we have the choice of more than one route. Your village has many pathways or roads leading through it. When we have a system of edges and vertices drawn to represent a number of possible pathways from one place to another, we say we have a **network**. A network can be represented in a diagram using vertices and edges or nodes and paths as shown below. Vertices can be labelled as odd or even, depending on the number of edges that intersect at them.

A network can be represented by using nodes (vertices), paths (edges) and regions.

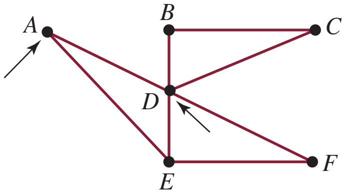


This network has:

- 6 vertices or nodes A, B, C, D, E, F
- 8 edges or paths
- 4 regions (3 inside and 1 outside)

Example

1 State the degree of the vertices indicated. Give a reason for your answer.



Solution

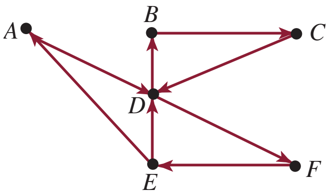
Vertex A is even with degree 2 because it has two edges.

Vertex D is odd with degree 5 because it has five edges.

A network is known as **traversable** if each edge can be traced without lifting the pencil or retracing the same edge. A network is traversable if it has zero or two odd vertices.

Example

2 Is this network traversable?



Solution

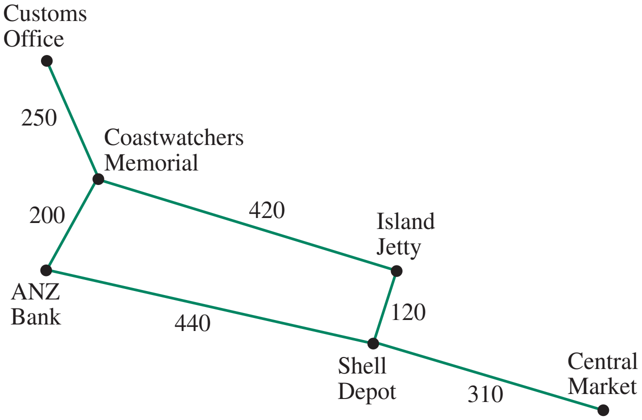
Yes, you can traverse the network without taking your pencil off the diagram or going over any edge more than once. The path would be:

$E \to D \to B \to C \to D \to F \to E \to A \to D$.

Networks can be used to find the **shortest path** through them, for example, the shortest route to deliver a message to all members of the church committee or the quickest method of cooking taro.

Example

3 The diagram below shows the harbour and Mendana Avenue in Honiara. Find the shortest path from the Customs Office to the Central Market. Distances are in metres.



Solution

There are two possible routes from the Customs Office to the Central Market.

Route 1: Customs Office–Coastwatchers Memorial–Island Jetty–Shell Depot–Central Market
$250 + 420 + 120 + 310 = 1100$ m

Route 2: Customs Office–Coastwatchers Memorial–ANZ Bank–Shell Depot–Central Market
$250 + 200 + 440 + 310 = 1200$ m

The shortest route is route 1.

This exercise can be applied to scheduling work on a building site, or preparing the ingredients and cooking the dinner so that everything is ready on time.

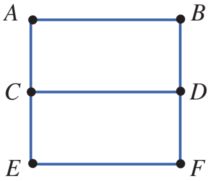
Exercise 14F

1 What is meant when a network is described as traversable?

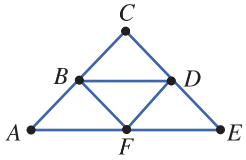
2 What is the difference between an odd and an even vertex?

3 For each network determine the number of odd and even vertices and state whether the network is traversable:

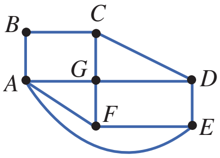
a



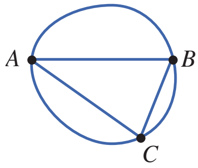
b



c



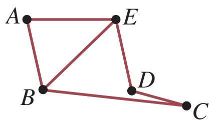
d



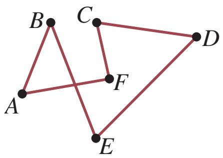
4 Summarise the relationship between odd and even vertices and traversable networks.

5 Use your summary from Question 4 to determine a route through the following networks:

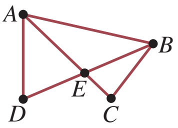
a



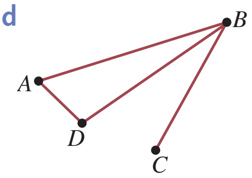
b



c



d

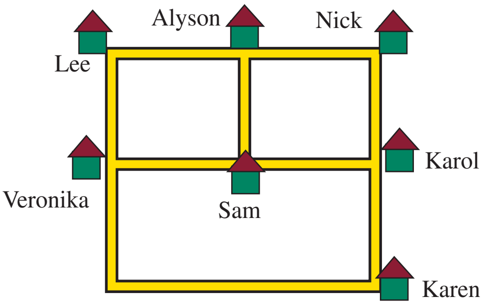


6 Draw a network with:

- **a** five vertices and eight edges
- **b** three vertices and nine edges
- **c** two vertices and three edges
- **d** ten vertices and twelve edges
- **e** three vertices with order = 2 and one with order = 3
- **f** two vertices with order = 3 and three vertices with order = 2

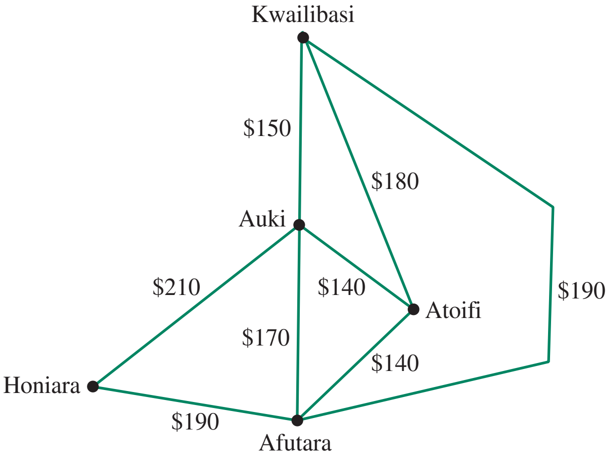
7 Seven people live in the same neighbourhood.

- **a** Represent them using a network diagram and determine the number of odd and even vertices.
- **b** Is the network traversable?



8 A Member of Parliament wishes to charter a plane to take him to meetings on Malaita at Auki, Atoifi and Afutara, and also in Honiara from his home at Kwailibasi. The charter company has provided a network diagram showing the airfares available between these places.

- **a** Copy this network into your book and determine the number of odd and even vertices.
- **b** Is the network traversable?
- **c** Calculate a path beginning and ending in Kwailibasi that visits all the meeting places.
- **d** Calculate the cheapest cost for his journey.



Puzzles

1 Find each of the words below in the word-search grid. You should be left with nine letters that spell out the name of important three-dimensional solids.

ISOMETRIC
CYLINDER
PYRAMID
PRISM
BOX
NET
VERTEX
CUBOID
BASE
CONE
CUBES
GRID
EDGE
VIEW
LINE

C	Y	L	I	N	D	E	R
I	P	G	O	L	I	N	E
R	D	R	L	Y	B	O	X
T	D	I	O	B	U	C	E
E	C	D	M	W	N	E	T
M	U	H	E	A	E	D	R
O	B	A	S	E	R	I	E
S	E	D	G	E	R	Y	V
I	S	A	M	S	I	R	P

The three-dimensional solids are _____________________________.

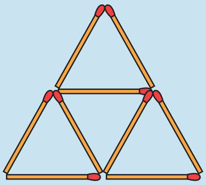
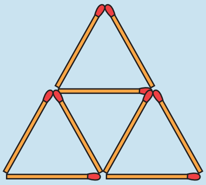
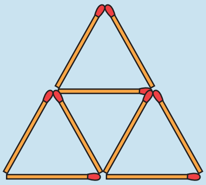
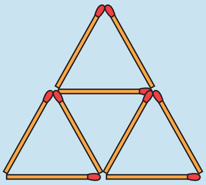
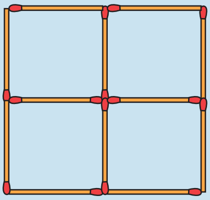
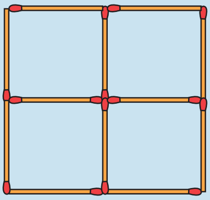
2 Make as many words as you can from the letters of ‘rectangular prism’.

rectangular prism

- A word should have at least three letters and should make sense.
- 20 words is excellent.
- Give yourself an extra point for each word with six or more letters.

Some words to get you started: care sang spit great tangle

3

■ Use nine matches to make this shape. ■ Remove three matches to leave one triangle.	
■ Use nine matches to make this shape. ■ Remove four matches to leave two triangles.	
■ Use nine matches to make this shape. ■ Remove three matches to leave two triangles.	
■ Use nine matches to make this shape. ■ Remove two matches to leave two triangles.	
■ Use 12 matches to make this shape. ■ Remove five matches to leave two squares.	
■ Use 12 matches to make this shape. ■ Remove two matches to leave two squares.	

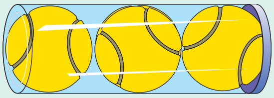
Applications

Poster

Design a poster for your favourite sport team or rock band using three-dimensional letters for the titles.

Economical packaging

Tennis balls are packaged in a plastic cylinder as shown.



a Find the diameter of a standard tennis ball and calculate the height and radius of the container.

b Draw a net and construct a container from stiff cardboard.

c Which do you think is greater, the space taken up by the three balls or the space remaining in the cylinder?

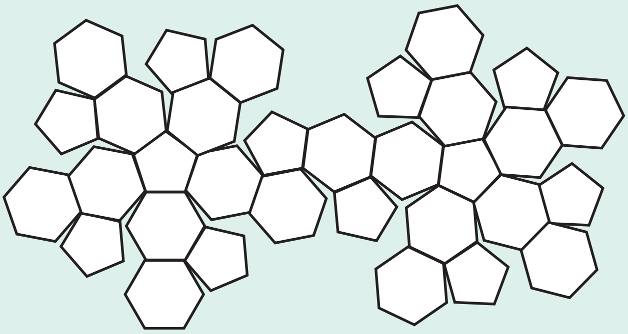
d If the cylinder was increased so that it was large enough to contain four tennis balls, find:
  i the minimum amount of cardboard required to make a net and construct the container
  ii the wasted volume, i.e. the volume of the container not taken up by the four tennis balls.

Soccer balls

A soccer ball is made from the net of a polyhedron which is made up of hexagons and pentagons.

a What is the name of the polyhedron?

b Enlarge the net on cardboard and include tabs to glue together. Colour the faces black and white.



Packaging

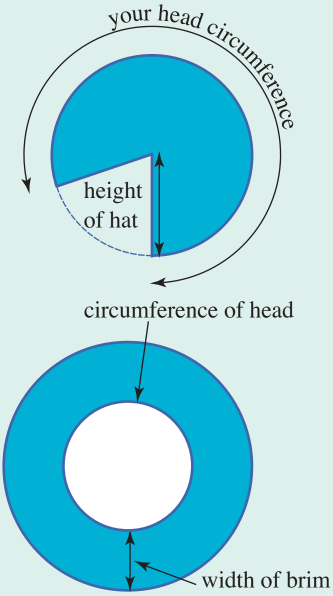
Most products we buy today come in packages of different shape and sizes.

- a What are the most important factors when designing packaging?
- b Which sort of products have rectangular-based packages, and why are some products cylindrical?
- c Why do you rarely see products in cubic packages?
- d Choose one product that comes in a package. Open out the package and draw the net for the shape.
- e Do you think the package is designed well? How could the product be changed to improve strength and durability, ease of handling and storing, cost effectiveness of materials, space for logos and visual appeal?
- f Design your own package and draw a 3D perspective drawing and a 2D net. List the positive features of your design.

Party hats

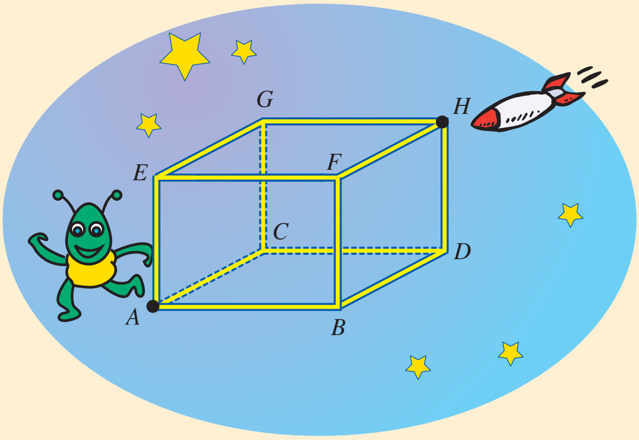
To make a hat for yourself follow these steps:

- a Measure the circumference of you head.
- b Decide how high you wish the hat to be.
- c Cut out the net from a circle of cardboard using the dimensions shown. Don't forget to include tabs to glue together.
- d Cut out the brim using the dimensions shown. Don't forget to include tabs.
- e Glue your hat together and decorate in your favourite colours.



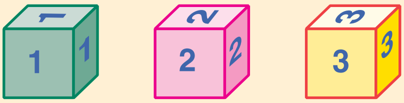
Enrichment

1 A computer game involves a Martian called Martin who is trapped in a system of tunnels in outer space. The tunnels form the 12 edges of a cube. Martin starts at point *A* and wants to travel to point *H* where his spaceship awaits. He can only travel along each tunnel once and never returns to point *A*.



- **a** Find the minimum number of tunnels required to reach his spaceship.
- **b** Find the maximum number of tunnels required to reach his spaceship.
- **c** Is it possible to travel along exactly four tunnels?
- **d** Investigate all possible paths. How many different paths are possible?

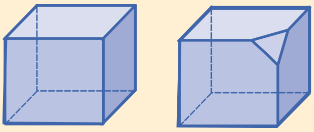
2 A learner entering secondary school is given three cubes. The first has the number 1 marked on every face. The second has a number 2 marked on every face. The third has a number 3 marked on every face.



The learner stacks the cubes side by side:

- **a** How many different numbers can be formed using all three cubes?
- **b** How many different numbers can be made if the learner is given a fourth cube marked with the number 4 on each of the faces?

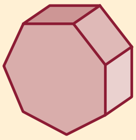
3 A truncated solid has a slice removed from a vertex. This slice results in an additional face in the shape of a polygon as shown.



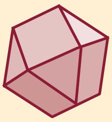
- **a** If pieces were sliced off all the vertices of a tetrahedron, what shape would be formed?
- **b** If pieces were sliced off all the vertices of a octahedron, what shape would be formed?

4 For each of the solids shown below, draw the original solid before it was truncated.

a

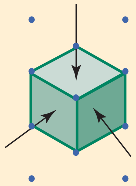
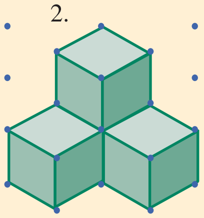


b



5 A series of shapes is made by adding one cube to the top and two front faces of a cube. Use isometric paper to draw the next shape in the pattern, then copy and complete the table to show the number of cubes in the first three shapes.

a

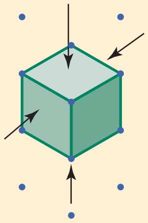
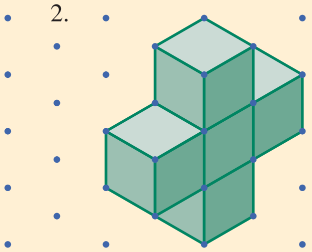
Number in series (n)	1. Add cubes 	2. 	3.
Number of cubes (c)	1	4	

b From your table predict the number of cubes needed to make the next three shapes in the series.

c Can you find a rule that represents the pattern where c is the number of cubes and n is the number in the series?

6 A second pattern is made by adding one cube to the top, bottom and two side faces of a cube. Use isometric paper to draw the next shape in the pattern, then copy and complete the table to show the number of cubes in the first three shapes.

a

Number in series (n)	1. Add cubes 	2. 	3.
Number of cubes (c)	1	5	

b From your table predict the number of cubes needed to make the next three shapes in the series.

c Can you find a rule that represents the pattern where c is the number of cubes and n is the number in the series?

Revision/Assessment

Exercise 14A

1 In your workbook carefully draw the following solids using solid and dotted lines, and labelling all side lengths:

- a Cube of length 2 cm
- b Sphere of radius 4·3 cm
- c Cylinder of radius 3·5 cm, height 5·8 cm
- d Cone of radius 3 cm, height 4 cm
- e Cuboid of length 6 cm, width 5 cm, height 4 cm
- f Tetrahedron with edge length 3 cm

Exercise 14B

2 a Draw a net that will fold to make a cube.

b Draw a net with the same number and shape of faces but which will not fold to make a cube. Describe in a sentence why it will not work.

3 a Draw a net that will fold to make a square-based pyramid.

b Draw a net with the same number of faces but which will not fold to make a square-based pyramid. Describe in a sentence why it will not work.

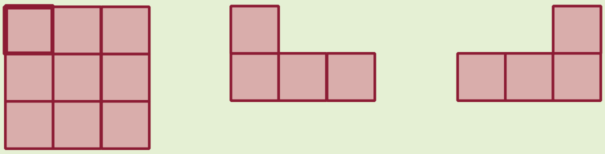
4 a Draw a net that will fold to make a tetrahedron.

b Draw a net with the same number and shape of faces but which will not fold to make a tetrahedron. Describe in a sentence why it will not work.

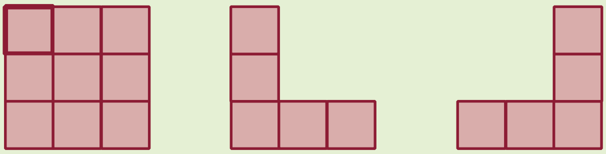
Exercise 14C

5 The orthogonal drawings below show the top, front and side view of shapes. Draw the corner view by using isometric paper:

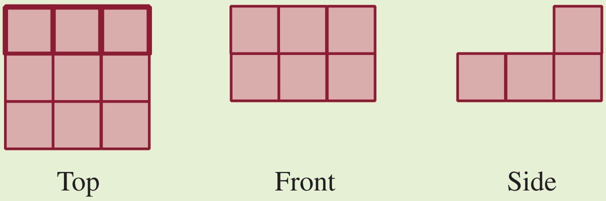
a



b



c

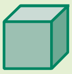


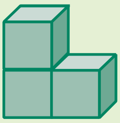
Exercise 14D

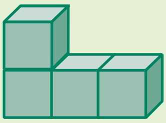
6 Draw a one-point perspective picture of a railway line.

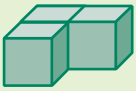
7 Draw a two-point perspective picture of a box of matches.

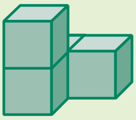
8 Draw the top, front and side views of each shape:

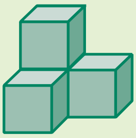
a 

b 

c 

d 

e 

f 

Exercise 14E

9 Make a neat and accurate drawing of four prisms with the following bases:

a pentagon

b isosceles triangle

10 Make a neat and accurate drawing of four pyramids with the following bases:

a square

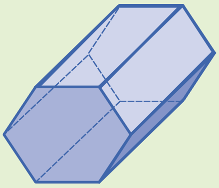
b trapezium

11 a How many faces does this prism have?

b Name the shape of each face.

c How many edges and vertices does it have?

d How many edges meet at each each vertex?

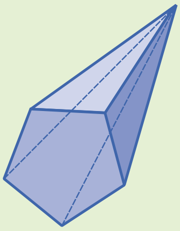


12 a How many faces does this pyramid have?

b Name the shape of each face.

c How many edges and vertices does it have?

d How many edges meet at each each vertex?



Exercise 14F

13 Explain the difference between an odd and an even vertex.

14 Draw networks that have the following number of vertices and edges:

	Vertices	Edges
a	5	8
b	8	12
c	7	9
d	5	6
e	4	5

Answers

These are selected answers only. A set of Fully Worked Solutions can be found in the Teacher's Resource.

Chapter 8

Exercise 8A

1 a $a = 5$ b $b = 17$ c $c = 36$ d $d = 9$
e $e = 21$ f $f = 13$ g $g = 27$ h $h = 83$
i $i = 83$

2 a $k = 7$ b $m = 9$ c $p = 2$ d $r = 11$
e $t = 12$ f $v = 7$ g $x = 7$ h $y = 6$
i $m = 5$

3 a $x = 6$ b $y = 9$ c $m = 6$ d $a = 8$
e $r = 5$ f $t = 4$ g $q = 8$ h $p = 8$
i $x = 12$ j $s = 5$ k $b = 6$ l $y = 4$
m $m = 13$ n $a = 140$ o $e = 111$ p $f = 101$
q $x = 16$ r $y = 15$ s $p = 24$ t $x = 21$

4 a $x = 16$ b $s = 19$ c $b = 13$ d $y = 20$
e $m = 27$ f $a = 38$ g $e = 30$ h $f = 20$
i $b = 2$ j $c = 7$ k $d = 2$ l $a = 6$
m $m = 20$ n $n = 23$ o $p = 25$ p $q = 18$

5 a $x = 2$ b $y = 3$ c $a = 4$ d $h = 6$
e $m = 5$ f $n = 7$ g $p = 3$ h $q = 3$
i $x = 2$ j $s = 3$ k $b = 44$ l $y = 4$
m $m = 30$ n $p = 40$ o $e = 3$ p $f = 6$

6 a $x = 36$ b $y = 52$ c $a = 108$ d $h = 360$
e $m = 2$ f $z = 84$ g $p = 5$ h $q = 11$
i $y = 18$ j $x = 14$ k $m = 20$ l $n = 56$
m $p = 20$ n $q = 27$ o $r = 44$ p $t = 72$

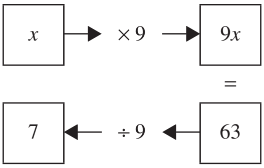
7 a $x = 8$ b $y = 14$ c $a = 14$ d $b = 33$

8 a $x = 2$ b $x = 31$ c $x = 100$ d $x = 3$
e $x = 7$ f $x = 25$

9 a Incorrect b Incorrect c Incorrect
d Correct e Correct f Incorrect
g Correct h Incorrect i Correct
j Incorrect k Correct l Correct

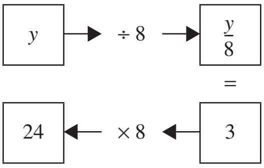
Exercise 8B

1



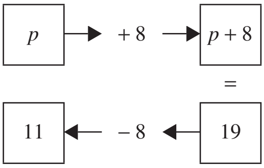
$x = 7$

2



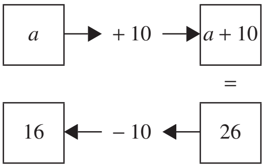
$y = 24$

3 a



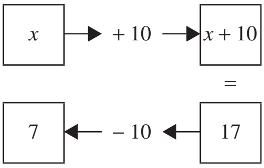
$p = 11$

b



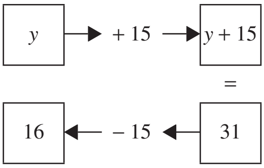
$a = 16$

c



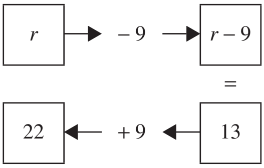
$x = 7$

d



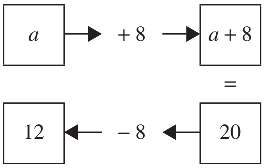
$y = 16$

e



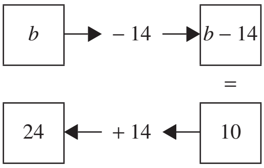
$r = 22$

f



$a = 12$

g



$b = 24$

h

$$\begin{tikzcd} \boxed{m} \arrow[r] & -6 \arrow[r] & \boxed{m-6} \arrow[d, phantom, "="] \\ \boxed{15} & +6 \arrow[l] & \boxed{9} \arrow[l] \end{tikzcd}$$

$m = 15$

4 a

$$\begin{tikzcd} \boxed{x} \arrow[r] & \times 6 \arrow[r] & \boxed{6x} \arrow[d, phantom, "="] \\ \boxed{3} & \div 6 \arrow[l] & \boxed{18} \arrow[l] \end{tikzcd}$$

$x = 3$

b

$$\begin{tikzcd} \boxed{y} \arrow[r] & \times 8 \arrow[r] & \boxed{8y} \arrow[d, phantom, "="] \\ \boxed{5} & \div 8 \arrow[l] & \boxed{40} \arrow[l] \end{tikzcd}$$

$y = 5$

c

$$\begin{tikzcd} \boxed{m} \arrow[r] & \times 12 \arrow[r] & \boxed{12m} \arrow[d, phantom, "="] \\ \boxed{9} & \div 12 \arrow[l] & \boxed{108} \arrow[l] \end{tikzcd}$$

$m = 9$

d

$$\begin{tikzcd} \boxed{a} \arrow[r] & \times 9 \arrow[r] & \boxed{9a} \arrow[d, phantom, "="] \\ \boxed{6} & \div 9 \arrow[l] & \boxed{54} \arrow[l] \end{tikzcd}$$

$a = 6$

e

$$\begin{tikzcd} \boxed{b} \arrow[r] & \times 7 \arrow[r] & \boxed{7b} \arrow[d, phantom, "="] \\ \boxed{5} & \div 7 \arrow[l] & \boxed{35} \arrow[l] \end{tikzcd}$$

$b = 5$

f

$$\begin{tikzcd} \boxed{c} \arrow[r] & \times 8 \arrow[r] & \boxed{8c} \arrow[d, phantom, "="] \\ \boxed{4} & \div 8 \arrow[l] & \boxed{32} \arrow[l] \end{tikzcd}$$

$c = 4$

g

$$\begin{tikzcd} \boxed{p} \arrow[r] & \div 7 \arrow[r] & \boxed{\frac{p}{7}} \arrow[d, phantom, "="] \\ \boxed{28} & \times 7 \arrow[l] & \boxed{4} \arrow[l] \end{tikzcd}$$

$p = 28$

h

$$\begin{tikzcd} \boxed{q} \arrow[r] & \div 9 \arrow[r] & \boxed{\frac{q}{9}} \arrow[d, phantom, "="] \\ \boxed{27} & \times 9 \arrow[l] & \boxed{3} \arrow[l] \end{tikzcd}$$

$q = 27$

i

$$\begin{tikzcd} \boxed{r} \arrow[r] & \div 12 \arrow[r] & \boxed{\frac{r}{12}} \arrow[d, phantom, "="] \\ \boxed{60} & \times 12 \arrow[l] & \boxed{5} \arrow[l] \end{tikzcd}$$

$r = 60$

j

$$\begin{tikzcd} \boxed{m} \arrow[r] & \div 9 \arrow[r] & \boxed{\frac{m}{9}} \arrow[d, phantom, "="] \\ \boxed{36} & \times 9 \arrow[l] & \boxed{4} \arrow[l] \end{tikzcd}$$

$m = 36$

k

$$\begin{tikzcd} \boxed{n} \arrow[r] & \div 3 \arrow[r] & \boxed{\frac{n}{3}} \arrow[d, phantom, "="] \\ \boxed{39} & \times 3 \arrow[l] & \boxed{13} \arrow[l] \end{tikzcd}$$

$n = 39$

l

$$\begin{tikzcd} \boxed{p} \arrow[r] & \div 8 \arrow[r] & \boxed{\frac{p}{8}} \arrow[d, phantom, "="] \\ \boxed{96} & \times 8 \arrow[l] & \boxed{12} \arrow[l] \end{tikzcd}$$

$p = 96$

m

$$\begin{tikzcd} \boxed{c} \arrow[r] & \div 4 \arrow[r] & \boxed{\frac{c}{4}} \arrow[d, phantom, "="] \\ \boxed{12} & \times 4 \arrow[l] & \boxed{3} \arrow[l] \end{tikzcd}$$

$c = 12$

n

$$\begin{tikzcd} \boxed{a} \arrow[r] & \div 5 \arrow[r] & \boxed{\frac{a}{5}} \arrow[d, phantom, "="] \\ \boxed{55} & \times 5 \arrow[l] & \boxed{11} \arrow[l] \end{tikzcd}$$

$a = 55$

o

$$\begin{tikzcd} \boxed{p} \arrow[r] & \div 7 \arrow[r] & \boxed{\frac{p}{7}} \arrow[d, phantom, "="] \\ \boxed{91} & \times 7 \arrow[l] & \boxed{13} \arrow[l] \end{tikzcd}$$

$p = 91$

5

$$\begin{tikzcd} \boxed{x} \arrow[r, "\times 4"] & \boxed{4x} \arrow[r, "+9"] \arrow[d, phantom, "="] & \boxed{4x+9} \arrow[d, phantom, "="] \\ \boxed{12} & \boxed{48} \arrow[l, "\div 4"'] & \boxed{57} \arrow[l, "-9"'] \end{tikzcd}$$

$x = 12$

6 **a**

$$\begin{tikzcd} \boxed{x} \arrow[r, "\times 2"] & \boxed{2x} \arrow[r, "+9"] \arrow[d, phantom, "="] & \boxed{2x+9} \arrow[d, phantom, "="] \\ \boxed{3} & \boxed{6} \arrow[l, "\div 2"'] & \boxed{15} \arrow[l, "-9"'] \end{tikzcd}$$

$x = 3$

b

$$\begin{tikzcd} \boxed{y} \arrow[r, "\times 4"] & \boxed{4y} \arrow[r, "+6"] \arrow[d, phantom, "="] & \boxed{4y+6} \arrow[d, phantom, "="] \\ \boxed{6} & \boxed{24} \arrow[l, "\div 4"'] & \boxed{30} \arrow[l, "-6"'] \end{tikzcd}$$

$y = 6$

c

$$\begin{tikzcd} \boxed{a} \arrow[r, "\times 7"] & \boxed{7a} \arrow[r, "+4"] \arrow[d, phantom, "="] & \boxed{7a+4} \arrow[d, phantom, "="] \\ \boxed{9} & \boxed{63} \arrow[l, "\div 7"'] & \boxed{67} \arrow[l, "-4"'] \end{tikzcd}$$

$a = 9$

d

$$\begin{tikzcd} \boxed{m} \arrow[r, "\times 3"] & \boxed{3m} \arrow[r, "-5"] \arrow[d, phantom, "="] & \boxed{3m-5} \arrow[d, phantom, "="] \\ \boxed{6} & \boxed{18} \arrow[l, "\div 3"'] & \boxed{13} \arrow[l, "+5"'] \end{tikzcd}$$

$m = 6$

e

$$\begin{tikzcd} \boxed{n} \arrow[r, "\times 8"] & \boxed{8n} \arrow[r, "-10"] \arrow[d, phantom, "="] & \boxed{8n-10} \arrow[d, phantom, "="] \\ \boxed{11} & \boxed{88} \arrow[l, "\div 8"'] & \boxed{78} \arrow[l, "+10"'] \end{tikzcd}$$

$n = 11$

f

$$\begin{tikzcd} \boxed{p} \arrow[r, "\times 9"] & \boxed{9p} \arrow[r, "-8"] \arrow[d, phantom, "="] & \boxed{9p-8} \arrow[d, phantom, "="] \\ \boxed{4} & \boxed{36} \arrow[l, "\div 9"'] & \boxed{28} \arrow[l, "+8"'] \end{tikzcd}$$

$p = 4$

g

$$\begin{tikzcd} \boxed{x} \arrow[r, "\times 2"] & \boxed{2x} \arrow[r, "+4"] \arrow[d, phantom, "="] & \boxed{2x+4} \arrow[d, phantom, "="] \\ \boxed{3} & \boxed{6} \arrow[l, "\div 2"'] & \boxed{10} \arrow[l, "-4"'] \end{tikzcd}$$

$x = 3$

h

$$\begin{tikzcd} \boxed{y} \arrow[r, "\times 2"] & \boxed{2y} \arrow[r, "+6"] \arrow[d, phantom, "="] & \boxed{2y+6} \arrow[d, phantom, "="] \\ \boxed{-3} & \boxed{-6} \arrow[l, "\div 2"'] & \boxed{0} \arrow[l, "-6"'] \end{tikzcd}$$

$y = -3$

i

$$\begin{tikzcd} \boxed{a} \arrow[r, "\times 6"] & \boxed{6a} \arrow[r, "+5"] \arrow[d, phantom, "="] & \boxed{6a+5} \arrow[d, phantom, "="] \\ \boxed{2} & \boxed{12} \arrow[l, "\div 6"'] & \boxed{17} \arrow[l, "-5"'] \end{tikzcd}$$

$a = 2$

j

$$\begin{tikzcd} \boxed{m} \arrow[r, "\times 5"] & \boxed{5m} \arrow[r, "+1"] \arrow[d, phantom, "="] & \boxed{5m+1} \arrow[d, phantom, "="] \\ \boxed{2} & \boxed{10} \arrow[l, "\div 5"'] & \boxed{11} \arrow[l, "-1"'] \end{tikzcd}$$

$m = 2$

k

$$\begin{tikzcd} \boxed{n} \arrow[r, "\times 2"] & \boxed{2n} \arrow[r, "-10"] \arrow[d, phantom, "="] & \boxed{2n-10} \arrow[d, phantom, "="] \\ \boxed{11} & \boxed{22} \arrow[l, "\div 2"'] & \boxed{12} \arrow[l, "+10"'] \end{tikzcd}$$

$n = 11$

l

$$\begin{tikzcd} \boxed{p} \arrow[r, "\times 9"] & \boxed{9p} \arrow[r, "-2"] \arrow[d, phantom, "="] & \boxed{9p-2} \arrow[d, phantom, "="] \\ \boxed{4} & \boxed{36} \arrow[l, "\div 9"'] & \boxed{34} \arrow[l, "+2"'] \end{tikzcd}$$

$p = 4$

m

$$\begin{tikzcd} \boxed{x} \arrow[r, "\times 2"] & \boxed{2x} \arrow[r, "+5"] \arrow[d, phantom, "="] & \boxed{2x+5} \arrow[d, phantom, "="] \\ \boxed{1} & \boxed{2} \arrow[l, "\div 2"'] & \boxed{7} \arrow[l, "-5"'] \end{tikzcd}$$

$x = 1$

n

$$\begin{tikzcd} \boxed{y} \arrow[r, "\times 4"] & \boxed{4y} \arrow[r, "+6"] \arrow[d, phantom, "="] & \boxed{4y+6} \arrow[d, phantom, "="] \\ \boxed{10} & \boxed{40} \arrow[l, "\div 4"'] & \boxed{46} \arrow[l, "-6"'] \end{tikzcd}$$

$y = 10$

o

$$\begin{tikzcd} \boxed{a} \arrow[r, "\times 7"] & \boxed{7a} \arrow[r, "+4"] \arrow[d, phantom, "="] & \boxed{7a+4} \arrow[d, phantom, "="] \\ \boxed{5} & \boxed{35} \arrow[l, "\div 7"'] & \boxed{39} \arrow[l, "-4"'] \end{tikzcd}$$

$a = 5$

7 **a** $x = 4$ **b** $y = 48$ **c** $n = 4$ **d** $q = 7$
e $m = 7$ **f** $s = 9$ **g** $x = 25$ **h** $x = 8$

Exercise 8C

1 **a** $x = 12$ **b** $y = 11$ **c** $m = 22$ **d** $n = 25$
e $p = 23$ **f** $p = 3\frac{3}{4}$ **g** $q = 2\cdot25$ **h** $t = 5$
i $m = \frac{3}{4}$ **j** $q = 4\frac{1}{4}$ **k** $x = 5$ **l** $p = 5\cdot25$
m $s = 1\cdot5$ **n** $t = 3\cdot25$

2 **a** $x = 3$ **b** $y = 12$ **c** $n = 5$ **d** $g = 11$
e $p = 4$ **f** $m = 12$ **g** $r = 2\frac{1}{2}$ **h** $s = 2\frac{1}{3}$
i $t = 1\frac{4}{7}$ **j** $z = 2\frac{1}{2}$ **k** $q = 5\frac{1}{2}$ **l** $h = 3\frac{2}{3}$
m $x = \frac{1}{2}$ **n** $y = \frac{2}{3}$ **o** $z = \frac{3}{4}$ **p** $m = 2\frac{1}{2}$
q $n = 0\cdot9$ **r** $t = 0\cdot9$ **s** $a = 0\cdot5$ **t** $x = 3$

3 **a** $x = 72$ **b** $a = 39$ **c** $b = 84$
d $p = 96$ **e** $q = 105$ **f** $r = 54$
g $m = 10$ **h** $n = 4\cdot5$ **i** $p = 11$

4 **a** $6p = 36, p = 6$ **b** $c = 50 - 6p, c = \$14$

5 $\frac{p}{11} = G, p = 1155$ cents

6 **a** $x = 36$ cm **b** $x = 15$ cm

Exercise 8D

1 **a** $x = 9$ **b** $y = 5$ **c** $m = 7$ **d** $n = 4$
e $q = 5$ **f** $a = 2$ **g** $b = 3$ **h** $c = 7$
i $x = 8$ **j** $r = \frac{1}{2}$ **k** $s = 1\frac{1}{2}$ **l** $a = 2\frac{6}{7}$

2 **a** $x = 8$ **b** $y = 12$ **c** $z = 6$
d $a = 10$ **e** $b = 36$ **f** $c = 4$
g $m = 7\frac{1}{2}$ **h** $n = 2\frac{1}{2}$ **i** $p = 2\frac{2}{3}$

3 **a** $x = 12$ **b** $y = 21$ **c** $z = 28$
d $m = 16$ **e** $n = 24$ **f** $p = 72$
g $q = 2\frac{1}{2}$ **h** $r = 6$ **i** $s = 5$

4 **a** $x = 6$ **b** $a = 8$ **c** $n = 9$
d $n = 6$ **e** $p = 10$ **f** $q = 14$
g $m = 12$ **h** $z = 15$ **i** $t = 16$

5 **a** $x = 13$ **b** $x = 10$ **c** $y = 25$ **d** $y = 15$

6 $1900 = 10(d - 70), 190 = d - 70$
Normal price, $d = \$260$

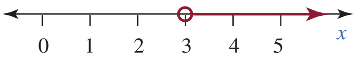
7 $12 = 8 + 4p, 4 = 4p$. Each doughnut cost \$1.

8 $126 = p \times \left(\frac{3}{4}\right)$
$126 \times \left(\frac{4}{3}\right) = p$
Usual price = \$168

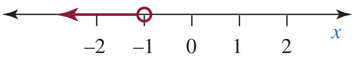
Exercise 8E

1 **a** $15 + 12 > 2 \times 5$ **b** $3 \times 6 < 40 \div 2$
c $4 + 8 = 6 \times 2$ **d** $16 - 5 > 24 \div 3$
e $4 \times 0\cdot5 = 20 \div 10$ **f** $36 + 27 > 6 \times 7$
g $9 \times 12 = 216 \div 2$ **h** $4^2 + 1 < 20 - 2$
i $5^2 + 12^2 = 13^2$ **j** $(2 + 3)^2 > 2^2 + 3^2$

2 **a**



b

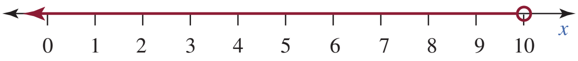


c

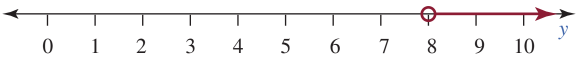
d

3 **a** $x \geq 1$ **b** $x < 2$ **c** $-2 < x \leq 1$
d $0 \leq x \leq 3$ **e** $0 < x < 3$ **f** $-1 \geq x > 2$

4 **a** $x < 10$

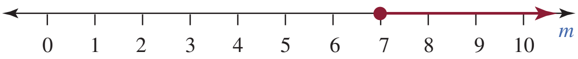


b $y > 8$



c $z \leq 5$

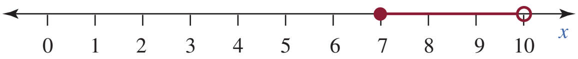
d $m \geq 7$



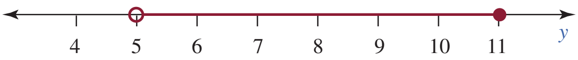
e $3 < x < 6$

f $4 < x < 8$

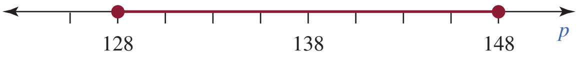
g $7 \leq x < 10$



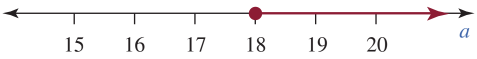
h $5 < y \leq 11$



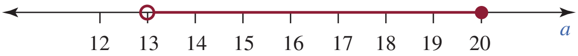
i $128 \leq p \leq 148$



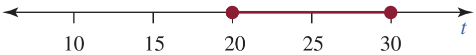
j $a \geq 18$



k $13 < a \leq 20$



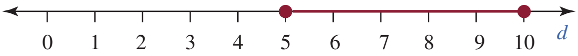
l $20 \leq t \leq 30$



m $0 \leq s \leq 35$

n $0 < s \leq 50$

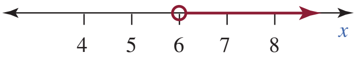
o $5 \leq d \leq 10$



5 a $x < 4$ b $y > 18$ c $z \geq 31$ d $m \leq 5$
e $n < 2$ f $p \leq 20$ g $x \geq 5$ h $y > 6$
i $z < 5\frac{1}{2}$ j $m \leq 4$ k $p < 9$ l $q > 5\frac{1}{2}$
m $x \geq 35$ n $y \leq 52$ o $z > 60$ p $m < 24$
q $n \geq 18$ r $p < 24$ s $x \leq 18$ t $y > 16\frac{2}{3}$
u $z < 6$ v $a \leq 10$ w $b < 12$ x $c \geq 8$

6 a $x \geq 1\frac{1}{2}$ b $y < 4$ c $z \leq 4$ d $a < 3\frac{1}{2}$
e $b \geq 1\frac{1}{2}$ f $c < 4\frac{3}{5}$ g $m \geq 21$ h $n < 66$
i $p \geq 75$ j $x < 4$ k $y \geq 80$ l $z \leq 90$
m $x \geq 80$ n $y > 15$ o $z < 9$ p $a \leq 5$
q $b < 12$ r $c \geq 16$

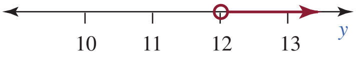
7 a $x > 6$



b $x < 20$

c $y \geq 8$

d $y > 12$

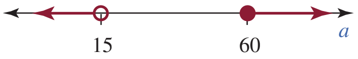


8 $b > 275$
The original balance was greater than $275.

9 $d < 5$
The amount d is less than $5.

10 $15 < t \leq 25$

11 $15 < a < 60$



Applications

1 a $n = 6$ b $n = 4$ c $x = 9$
d $y = 4$ e $x = 7$ f $n = 3$
2 a $x = 6$ b $n = 7$ c $n = 2$
d $n = 3$ e $x = 1$ f $n = 2$
3 $n = 2$
4 a $n = 2$ b $n = 4$ c $n = 4$
d $n = 3$ e $n = 4$ f $n = 6$
5 a $x = 5$ b $x = 2$ c $n = 3$
d $x = 4$ e $x = 2$ f $x = 4$

Enrichment

1 $x = 8$ **2** $x = 3$
3 a $a = 14$ b $b = 13$ c $m = 7$
d $x = 2$ e $y = 18$ f $m = 5$
g $n = 2\frac{1}{2}$ h $p = 2\frac{1}{3}$ i $q = 4\frac{1}{4}$
4 a $x = 4$ b $y = 3$ c $z = 16$
d $x = 4$ e $y = 6$ f $z = 5\frac{1}{2}$
g $p = 3$ h $q = 3$ i $m = 2\frac{1}{2}$
5 a $x = 6$ b $y = 2$ c $p = 3$
d $y = 2$ e $q = 4$ f $m = 2·5$

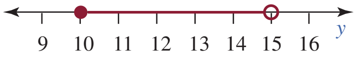
Revision/Assessment

1 a $x = 7$ b $y = 5$ c $z = 18$
d $m = 30$ e $q = 6$ f $p = 8$
g $m = 8$ h $n = 9$ i $p = 12$
j $q = 56$ k $r = 12$ l $s = 7$
2 a Correct b Correct
3 a $m = 5$ b $p = 20$ c $r = 6$
d $t = 21$ e $q = 4$ f $p = 8$
4 a $x = 9$ b $y = 35$ c $z = 8$
d $m = 5\frac{1}{2}$ e $m = 20$ f $n = 4$
5 a $x = 8$ b $y = 4$ c $z = 9$ d $x = 4$
e $y = 9$ f $q = 8$ g $z = 6$ h $m = 10$
i $n = 16$ j $p = 10$ k $q = 10$ l $r = 14$
6 a $x = 8$ b $y = 8$ c $y = 18$ d $n = 10$
e $q = 8$
7 Each ice cream usually costs $4·50.
8 a $16 + 13 > 4 \times 6$ b $4 \times 5 < 80 \div 2$
c $12 \times 6 = 144 \div 2$ d $5^2 - 4 < 6 \times 4$

9 **a** $x < 13$

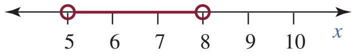
b $z > 8$

c $10 \le y < 15$

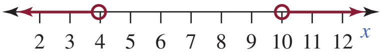


d $20 < m < 30$

e $5 < x < 8$



f $x < 4, x > 10$



10 **a** $5 \le g \le 8$ **b** $60 < m \le 100$

c $11 \le x \le 13$ **d** $x < 15, x > 70$

11 **a** $x < 3$ **b** $z \le 21$ **c** $z > 3$

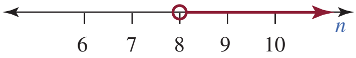
d $m > 6$ **e** $p \ge 20$ **f** $q \le 44$

g $n \ge 8$ **h** $t \le 6$ **i** $s \ge 10$

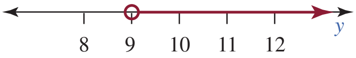
12 **a** $x > 5$ **b** $y \le 3$ **c** $z \ge 20$

d $m > 24$ **e** $p < 10$ **f** $q \le 20$

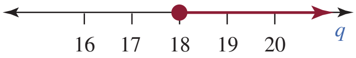
13 **a** $n > 8$



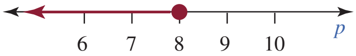
b $y > 9$



c $q \ge 18$



d $p \le 8$



14 $125 < 100 + d < 148$

$25 < d < 48$

Between \$25 and \$48 was added to bank account.

Chapter 9

Exercise 9A

1 **a** 3^2 **b** 5^4 **c** 10^1 **d** 1^6 **e** 2^2

f 9^5 **g** 7^3 **h** 0^8 **i** 12^2 **j** 6^4

k 4^5 **l** 8^7 **m** 1^4 **n** 10^2 **o** 11^3

p 3^6 **q** $(-3)^3$ **r** $(-5)^4$ **s** $(-2)^3$ **t** $(-4)^4$

2 **a** 9 **b** 16 **c** 1000 **d** 32

e 25 **f** 7 **g** 1 **h** 0

i 10 000 **j** 32 768 **k** 64 **l** 2

m 64 **n** 1 **o** 64 **p** 729

3 **a** 81 **b** 216 **c** 49

d 125 **e** 0 **f** 1

g 8 **h** 625 **i** 1 000 000

j 729 **k** 1331 **l** 16

4 **a** –1 **b** 1 **c** –1 **d** 1

e –2 **f** 4 **g** –8 **h** 16

i –27 **j** 81 **k** –243 **l** 729

5 **a** –16 **b** –125 **c** –9

d 125 **e** –32 **f** –729

g 128 **h** –10 000 **i** –27

j –25 **k** 243 **l** –1 000 000

6 **a** 1

b When the powers are even the answers are positive.

7 **a** –1

b When the powers are odd the answers are negative.

8 **a** $8 = 2^3$ **b** $25 = 5^2$

c $49 = 7^2$ **d** $27 = 3^3$

e $10 = 10^1$ **f** $81 = 9^2$

g $81 = 3^4$ **h** $1000 = 10^3$

i $10\,000 = 10^4$ **j** $64 = 8^2$

k $64 = 4^3$ **l** $64 = 2^6$

m $100 = 10^2$ **n** $512 = 8^3$

o $256 = 2^8$ **p** $2401 = 7^4$

9 **a** $x = 4$ **b** $x = 3$ **c** $x = 3$ **d** $x = 3$

e $x = 10$ **f** $x = 4$ **g** $x = 11$ **h** $x = 6$

i $x = 2$ **j** $x = 5$ **k** $x = 5$ **l** $x = 4$

m $x = 5$ **n** $x = 4$ **o** $x = 3$ **p** $x = 4$

10 **a** $x = 4$ **b** $x = 5$ **c** $x = 9$ **d** $x = 12$

e $x = 3$ **f** $x = 4$ **g** $x = 6$ **h** $x = 5$

i $x = 2$ **j** $x = 3$ **k** $x = 10$ **l** $x = 2$

m $x = 13$ **n** $x = 5$ **o** $x = 6$ **p** $x = 3$

Learning task 9B

1

1	2^0	256	2^8
2	2^1	512	2^9
4	2^2	1 024	2^{10}
8	2^3	2 048	2^{11}
16	2^4	4 096	2^{12}
32	2^5	8 192	2^{13}
64	2^6	16 384	2^{14}
128	2^7	32 768	2^{15}

2 **a** $16 \times 128 = 2^4 \times 2^7 = 2^{11} = 2048$
b $8 \times 4096 = 2^3 \times 2^{12} = 2^{15} = 32\,768$
c $128 \times 128 = 2^7 \times 2^7 = 2^{14} = 16\,384$
d $16 \times 64 \times 32 = 2^4 \times 2^6 \times 2^5 = 2^{15} = 32\,768$
e $32 \times 32 \times 32 = 2^5 \times 2^5 \times 2^5 = 2^{15} = 32\,768$

3 **a** $128 \div 16 = 2^7 \div 2^4 = 2^3 = 8$
b $4096 \div 512 = 2^{12} \div 2^9 = 2^3 = 8$
c $16\,384 \div 256 = 2^{14} \div 2^8 = 2^6 = 64$
d $32\,768 \div 128 = 2^{15} \div 2^7 = 2^8 = 256$
e $1024 \div 128 = 2^{10} \div 2^7 = 2^3 = 8$
f $8192 \div 2048 = 2^{13} \div 2^{11} = 2^2 = 4$

4 **a** $\dfrac{4096 \times 1024}{8192} = \dfrac{2^{12} \times 2^{10}}{2^{13}} = \dfrac{2^{22}}{2^{13}} = 2^{22-13}$
$= 2^9 = 512$

b $\dfrac{512 \times 8192}{32\,768} = \dfrac{2^9 \times 2^{13}}{2^{15}} = \dfrac{2^{22}}{2^{15}} = 2^{22-15} = 2^7 = 128$

c $\dfrac{64 \times 2048 \times 256}{16\,384} = \dfrac{2^6 \times 2^{11} \times 2^8}{2^{14}} = \dfrac{2^{25}}{2^{14}}$
$= 2^{25-14} = 2^{11} = 2048$

d $\dfrac{1024 \times 16 \times 4096}{32\,768} = \dfrac{2^{10} \times 2^4 \times 2^{12}}{2^{15}} = \dfrac{2^{26}}{2^{15}}$
$= 2^{26-15} = 2^{11} = 2048$

e $\dfrac{2048 \times 8192}{16\,384 \times 512} = \dfrac{2^{11} \times 2^{13}}{2^{14} \times 2^9} = \dfrac{2^{24}}{2^{23}} = 2^{24-23}$
$= 2^1 = 2$

f $\dfrac{128 \times 4096}{2048 \times 256} = \dfrac{2^7 \times 2^{12}}{2^{11} \times 2^8} = \dfrac{2^{19}}{2^{19}} = 2^{19-19} = 2^0 = 1$

5 **a** $4^5 = (2^2)^5 = 2^{2 \times 5} = 2^{10} = 1024$
b $16^2 = (2^4)^2 = 2^{4 \times 2} = 2^8 = 256$
c $64^2 = (2^6)^2 = 2^{6 \times 2} = 2^{12} = 4096$
d $32^3 = (2^5)^3 = 2^{5 \times 3} = 2^{15} = 32\,768$
e $128^2 = (2^7)^2 = 2^{7 \times 2} = 2^{14} = 16\,384$
f $4^7 = (2^2)^7 = 2^{2 \times 7} = 2^{14} = 16\,384$

6 **a** $\sqrt{4} = 4^{\frac{1}{2}} = (2^2)^{\frac{1}{2}} = 2^{2 \times \frac{1}{2}} = 2^1 = 2$

b $\sqrt{16} = 16^{\frac{1}{2}} = (2^4)^{\frac{1}{2}} = 2^{4 \times \frac{1}{2}} = 2^2 = 4$

c $\sqrt{64} = 64^{\frac{1}{2}} = (2^6)^{\frac{1}{2}} = 2^{6 \times \frac{1}{2}} = 2^3 = 8$

7 **a** $2^6 \times 2^8 = 2^{14}$ We retain the base and add the powers.
b $2^{12} \div 2^7 = 2^5$ We retain the base and subtract the powers.
c $(2^5)^3 = 2^{15}$ We retain the base and multiply the powers.
d $2^0 = 1$ Two or in fact *any number* to the power of zero is equal to 1.

Exercise 9C

1 **a** 32 **b** 64 **c** 729
d 125 **e** 1 000 000 **f** 49
g 343 **h** 343 **i** 512
j 81 **k** 27 **l** 125

2 **a** x^7 **b** y^3 **c** m^8 **d** x^6 **e** a^6 **f** a^5
g a^6 **h** b^1 **i** x^{13} **j** n^8 **k** n^2 **l** p^{13}
m a^9 **n** x^{10} **o** y^5 **p** m^6 **q** r^8 **r** s^4
s t^8 **t** n^8

3 **a** $18x^6$ **b** $24x^7$ **c** $45x^7$ **d** $10x^7$
e $18y^3$ **f** $40z^5$ **g** $7a^6$ **h** $8b^7$
i $10x^{13}$ **j** $15b^3$ **k** $56y^6$ **l** $2x^3$

4 **a** $8x^5y^5$ **b** $15x^3y^7$ **c** $12x^4y^4$
d $12a^5b^5$ **e** $4m^4n^7$ **f** $5p^3q^7$
g $40x^6y^8$ **h** $48x^7y^8$ **i** $8x^4y^6$
j $24x^4y^3$

5 **a** 3 **b** 3125 **c** 49 **d** 9 **e** 1000
f 16 **g** 1 **h** 1 **i** 64

6 **a** x^2 **b** y^2 **c** z^6 **d** b^3 **e** x^1
f p **g** $q^0 = 1$ **h** b^3 **i** y^8

7 **a** $3x^2y^3$ **b** $3x^2y^1$ **c** $4x^2y^2$ **d** $5x^4y^4$
e $2r^2$ **f** $2m^1$ **g** $9p^1q^1$ **h** $8a$

8 **a** $\frac{3}{5}x^3y^2$ or $\frac{3x^3y^2}{5}$ **b** $\frac{3}{5}x^2y^2$ or $\frac{2x^2y^2}{3}$
c $\frac{1}{2}x^2$ or $\frac{x^2}{2}$ **d** $\frac{3}{5}x^3y$ or $\frac{3x^3y}{5}$

9 **a** $\frac{2a^6b}{5}$ **b** $\frac{5m^3n^7}{6}$ **c** $\frac{6rt^6}{7}$ **d** $\frac{7ab^2c^3}{8}$

Exercise 9D

1 **a** $2^6 = 64$ **b** $4^2 = 16$
c $5^0 = 1$ **d** $5^0 = 1$
e $10^6 = 1\,000\,000$ **f** $3^4 = 81$

g $2^3 = 8$ **h** $7^1 = 7$

i $3^2 = 9$

2 **a** x^6 **b** y^{15} **c** m^{12} **d** x^8

e 1 **f** 1 **g** 1 **h** 1

3 **a** a^3b^3 **b** x^5z^5 **c** mn

d 1 **e** $25b^2$ **f** $81x^2$

g $1000m^3$ **h** $81y^4$ **i** $x^{12}y^8$

j $m^{10}n^6$ **k** a^4b^{20} **l** a^6b^2

m $16y^8z^8$ **n** $27q^9r^3$ **o** 1

p $64m^2n^4$

4 **a** $\frac{16}{25}$ **b** $\frac{8}{27}$ **c** $\frac{1}{16}$

d $\frac{25}{36}$ **e** $\frac{1}{64}$ **f** $\frac{16}{49}$

g $\frac{1}{1\,000\,000}$ **h** 1

5 **a** $\frac{a^4}{b^4}$ **b** $\frac{x^2}{y^2}$ **c** $\frac{m^6}{n^6}$ **d** 1

e $\frac{x^4}{y^6}$ **f** $\frac{m^{15}}{n^{20}}$ **g** $\frac{a^{12}}{b^6}$ **h** $\frac{a^4}{b^{12}}$

i $\frac{1}{z^6}$ **j** $\frac{16}{z^8}$ **k** $\frac{9x^2}{4y^4}$ **l** $\frac{16m^{10}}{9n^6}$

6 **a** $54x^7y^{11}$ **b** $16a^7b^4$ **c** $24m^{10}n^{14}$

d $100p^{10}q^6$ **e** $72r^{17}s^{11}$ **f** $25s^{17}t^{13}$

g $2000p^{13}q^{11}$ **h** $-12r^5s^7$ **i** $8s^8t^5$

7 **a** $9x^4y$ **b** $2mn$ **c** $2q^2$

d $8r$ **e** $16x^5$ **f** $3x^2y^3$

Learning task 9E

1 **a** **i** 40 **ii** 500 **iii** 7

iv 6300 **v** 124 000 **vi** 4

b **i** 100 **ii** 577 000 **iii** 15 500

iv 12 500 **v** 1770 **vi** 14 500

vii 660

2 **a**

Number	Power of 10	Number	Power of 10
1	10^0	0·1	10^{-1}
10	10^1	0·01	10^{-2}
100	10^2	0·001	10^{-3}
1000	10^3	0·0001	10^{-4}
10 000	10^4	0·000 01	10^{-5}
100 000	10^5	0·000 001	10^{-6}
1 000 000	10^6	0·000 000 1	10^{-7}

b When the index is positive it is equal to the number of zeros after the digit 1. When the index is negative it is equal to one less than the number of zeros between the digit 1 and the decimal point.

c **i** $1·0 \times 10^4$ **ii** $1·0 \times 10^6$ **iii** $1·0 \times 10^{-6}$

3 **a** $2·4 \times 10^2$ **b** $3·4 \times 10^4$

c $5·67 \times 10^5$ **d** $1·2 \times 10^1$

e $9·0 \times 10^5$ **f** $4·5 \times 10^6$

g $3·87 \times 10^5$ **h** $4·5 \times 10^3$

4 **a** 2400 **b** 64 000

c 2 300 000 **d** 350

e 59·6 **f** 61 200 000

g 123 000 **h** 678 000 000

5 **a** $3·5 \times 10^{-2}$ **b** $6·0 \times 10^{-2}$

c $1·6 \times 10^{-4}$ **d** $7·8 \times 10^{-5}$

e $1·6 \times 10^{-2}$ **f** $2·78 \times 10^{-4}$

g $2·0 \times 10^{-1}$ **h** $4·5 \times 10^{-1}$

6 **a** 0·025 **b** 0·14

c 0·0083 **d** 0·000 65

e 0·000 015 **f** 0·000 000 41

g 0·000 001 2 **h** 0·000 000 002 8

Applications

Chessboard reward

a

Square	Grains of wheat on square	Power of 2	Total number of grains of wheat
1	1	2^0	1
2	2	2^1	3
3	4	2^2	7
4	8	2^3	15
5	16	2^4	31
6	32	2^5	63
7	64	2^6	127
8	128	2^7	255
9	256	2^8	511
10	512	2^9	1023

b Grains of wheat in nth square $N = 2^{n-1}$

c $n = 64$. So the number of grains of wheat $= 2^{63} = 9·223\,372\,037 \times 10^{18}$

d Total $= 2^n - 1$

e The total number of grains of wheat $= 2^{64} - 1 = 1·844\,674\,407 \times 10^{19} - 1$

Triads

a $t = 3^{n-1}$

b $n = 12$. So $t = 3^{12-1} = 177\,147$ triads in the twelfth generation

c There would be 6561 triads after 9 generations.

Balls

c $\frac{A}{V} = \frac{4\pi r^2}{\frac{4}{3}\pi r^3} = \frac{3}{r}$

The ratio of surface area to volume is $\frac{3}{r}$.

Enrichment

1 **a** $\frac{1}{9}$ **b** $\frac{1}{5}$ **c** $\frac{1}{32}$ **d** $\frac{1}{7}$
e $\frac{1}{10}$ **f** $\frac{1}{1000}$ **g** $\frac{1}{36}$ **h** 1

2 **a** 16 **b** 3 **c** 100 000 **d** 125
e 144 **f** 1 **g** 64 **h** 10

3 **a** $\frac{2y^4}{x^3}$ **b** $3x^2$ **c** m^9
d $\frac{6}{mn^{13}}$ **e** $\frac{49}{ba}$ **f** $\frac{18b^4}{a^5}$

4 **a** $2x^4$ **b** $9m^3n^2$ **c** $2m^4n^2$
d $\frac{b}{6a}$ **e** $16b^3$ **f** $\frac{y^4}{5x^7}$

5 **a** $x^2 = 9$
$\therefore x = \pm 3$
b $x^2 = 100$
$x = \pm 10$
c $x^2 = 1$
$\therefore x = \pm 1$
d $x^2 = 81$
$\therefore x = \pm 9$
e $x^4 = 16$
$\therefore x = \pm 2$
f $x^4 = 81$
$\therefore x = \pm 3$
g $x^4 = 625$
$\therefore x = \pm 5$
h $x^4 = 10\,000$
$\therefore x = \pm 10$
i $x^6 = 64$
$\therefore x = \pm 2$
j $x^6 = 4096$
$\therefore x = \pm 4$
k $x^6 = 15\,625$
$\therefore x = \pm 5$
l $x^6 = 1\,000\,000$
$\therefore x = \pm 10$

6 **a** $x^2 = 121$ There are 2 solutions: $x = \pm 11$
b $x^2 = 225$ There are 2 solutions: $x = \pm 15$
c $x^2 = 400$ There are 2 solutions: $x = \pm 20$
d $x^2 = 169$ There are 2 solutions: $x = \pm 13$
e $x^3 = 8$ There is 1 solution: $x = +2$
f $x^3 = 27$ There is 1 solution: $x = +3$
g $x^3 = 125$ There is 1 solution: $x = +5$
h $x^3 = 1000$ There is 1 solution: $x = +10$
i $x^5 = 32$ There is 1 solution: $x = +2$
j $x^5 = 243$ There is 1 solution: $x = +3$
k $x^5 = 7776$ There is 1 solution: $x = +6$
l $x^5 = 100\,000$ There is 1 solution: $x = +10$

7 **a** $x^2 = -1$ This cannot be solved; square numbers are always positive.
b $x^2 = -100$ This cannot be solved; square numbers are always positive.
c $x^2 = -25$ This cannot be solved; square numbers are always positive.
d $x^2 = -81$ This cannot be solved; square numbers are always positive.
e $x^3 = -1$ There is 1 solution: $x = -1$
f $x^3 = -8$ There is 1 solution: $x = -2$
g $x^3 = -27$ There is 1 solution: $x = -3$
h $x^3 = -1000$ There is 1 solution: $x = -10$

Revision/Assessment

1 **a** 25 **b** 16 **c** –36 **d** 36
e 10 000 **f** –1000 **g** 16 **h** 32

2 **a** $x = 2$ **b** $x = 2$ **c** $x = 3$
d $x = 2$ **e** $x = -2$ **f** $x = {}^{-}1$
g $x = 2$ **h** $x = 5$

3 **a** 25 **b** 49 **c** 8
d 81 **e** 12 **f** 1

4 **a** 32 **b** 16 **c** 4 **d** 64 **e** 8

5 **a** x^7 **b** y^8 **c** z^2
d $12x^6$ **e** $15x^6y^6$ **f** $54x^7y^5$
g x^2 **h** y^6 **i** 1
j $4x^3y$ **k** $4m$ **l** $\frac{xy^2}{2}$

6 **a** 64 **b** 1 **c** 1 000 000
d 9 **e** 1 **f** 1
g 512 **h** 4096

7 **a** x^6 **b** y^{15} **c** m^{14} **d** x^{12}
e 1 **f** 1 **g** 1 **h** 1
i x^6 **j** x^6 **k** m^{24} **l** a^6

8 **a** a^4b^4 **b** x^9z^9 **c** 1
d pq **e** $36b^2$ **f** $16x^2$
g $1728m^3$ **h** $81y^4$ **i** $x^{20}y^4$
j $m^{12}n^4$ **k** $216y^9z^6$ **l** $8q^6r^3$

9 **a** $\frac{9}{25}$ **b** $\frac{1}{216}$ **c** $\frac{1}{81}$ **d** $\frac{25}{49}$

10 **a** $\frac{p^7}{q^7}$ **b** $\frac{m^3}{n^3}$ **c** $\frac{x^6}{y^4}$ **d** $\frac{r^{20}}{t^{15}}$
e $\frac{1}{z^6}$ **f** $\frac{27}{x^6}$ **g** $\frac{4x^2}{9y^4}$ **h** $\frac{9m^2}{16n^{10}}$

11 **a** $8x^3y^7$ **b** $40m^{14}n^7$ **c** $225p^4q^8$
d $48p^{10}q^6$ **e** $50x^{10}y^8$ **f** $48m^{10}n^5$

12 **a** $16xy$ **b** $3q^2$ **c** $3x^2$

13

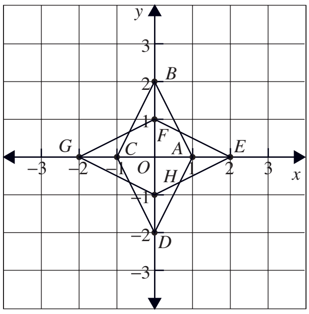
Facts about Uranus		Scientific notation
Distance from the Sun (km)	2 871 000 000	$2{\cdot}871 \times 10^9$
Diameter at equator (km)	51 118	$5{\cdot}1118 \times 10^4$
Light from the Sun (min)	150	$1{\cdot}5 \times 10^2$
Orbit (Earth years)	84	$8{\cdot}4 \times 10^1$
Number of moons	15	$1{\cdot}5 \times 10^1$
Mass (Earth = 1)	14·5	$1{\cdot}45 \times 10^1$
Gravity (Earth = 1)	0·79	$7{\cdot}9 \times 10^{-1}$

Chapter 10

Exercise 10A

1 $A(4, 2)$	$B(2, 4)$	$C(-2, 3)$	$D(-3, -4)$
$E(3, -3)$	$F(1, 0)$	$G(0, 1)$	$H(-1, 0)$
$I(0, -1)$			

2



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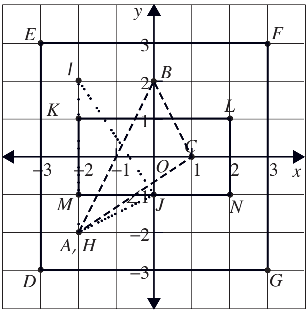

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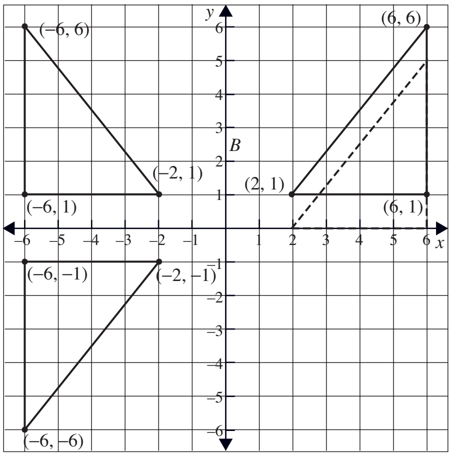

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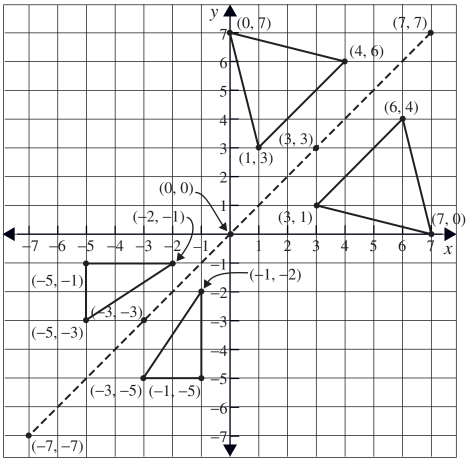
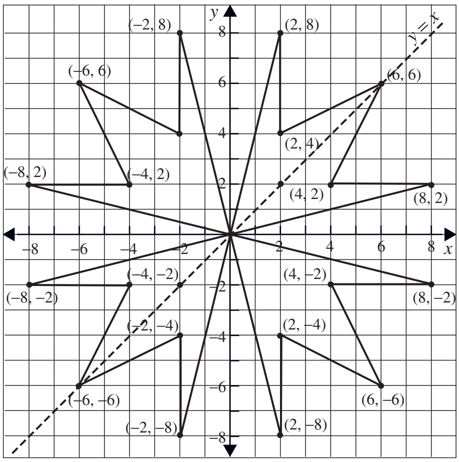


g (−2, −1), (−5, −1), (−5, −3)

6



7 f The coordinates are swapped around.
The x value becomes the y value.
The y value becomes the x value.

Exercise 10B

1 **a** Coordinates are (–1, –2), (0, –1), (1, 0), (2, 1)
$y = x - 1$

b Coordinates are (–2, 1), (–1, 2), (0, 3), (1, 4)
$y = x + 3$

2 **a** Coordinates are (–2, 2), (–1, 1), (0, 0), (1, –1)
$y = -x$

b Coordinates are (–1, 2), (0, 0), (1, –2), (2, –4)
$y = -2x$

3 **a** $y = 5x$ **b** $y = 6x$

c $y = \frac{1}{4}x$ **d** $y = \frac{3}{2}x$

4 **a** $y = -4x$ **b** $y = -5x$ **c** $y = -\frac{1}{2}x$

d $y = -\frac{1}{3}x$ **e** $y = 5x + 1$ **f** $y = 5x - 3$

Exercise 10C

1 **a** $y = 2x + 1$ **b** $y = 4x - 2$

c $y = 5x + 2$ **d** $y = 3x + 2$

e $y = 0{\cdot}5x + 1$ **f** $y = 1{\cdot}5x + 0{\cdot}5$

2 **a** $y = -2x + 4$ **b** $y = -3x + 3$

c $y = -4x - 7$ **d** $y = -x - 6$

e $y = -0{\cdot}5x + 2$ **f** $y = -0{\cdot}25x - 0{\cdot}25$

3 **a** $y = 3x - 1$

x	–1	0	1	5	9
y	–4	–1	2	14	26

b $y = 2x + 5$

x	–2	0	1	6	15
y	1	5	7	17	35

c $y = -x + 9$

x	–5	0	4	5	8
y	14	9	5	4	1

d $y = -4x - 1$

x	–6	0	7	10	11
y	23	–1	–29	–41	–45

4 **a** $y = 0{\cdot}5x + 1$

x	–2	0	2	4	6
y	0	1	2	3	4

b $y = 0{\cdot}5x - 1$

x	–2	0	2	4	6
y	–2	–1	0	1	2

c $y = \frac{1}{3}x + 2$

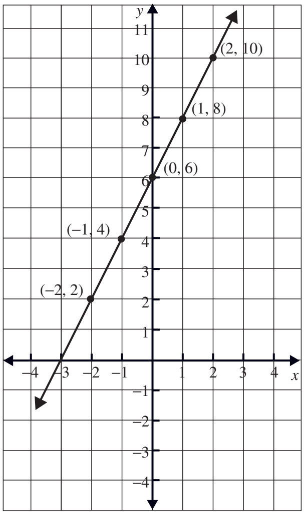
x	–3	0	3	6	9
y	1	2	3	4	5

d $y = \frac{-1}{3}x + 1$

x	–6	0	6	12	18
y	3	1	–1	–3	–5

5 **a** $y = 2x + 5$ **b** $y = -4x + 2$

6 **a & b**



c (3, 12), (4, 14), (5, 16) **d** $y = 2x + 6$

7 **a** $C = 10t + 20$ **b** The hire would cost $160.

8 **a** $T = -0{\cdot}5t - 1{\cdot}5$

b The temperature would be –5·25°C
This wouldn't be realistic as the Sun would have risen and the temperature would no longer be dropping.

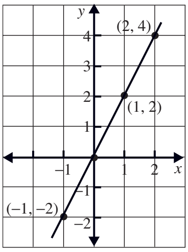
9 **a** $A = -25t + 450$

b The amount would be –$50.
The account is overdrawn by $50.

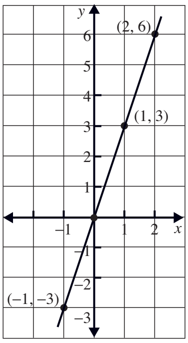
c $t = 18$ **d** $t = 29$

Exercise 10D

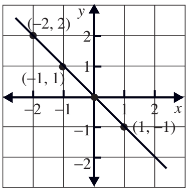
1 **a**



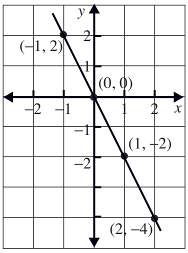
b



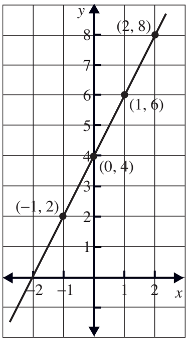
c



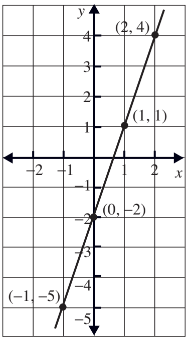
d



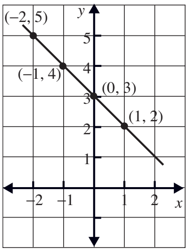
2 a



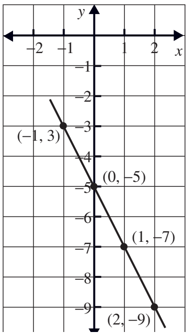
b



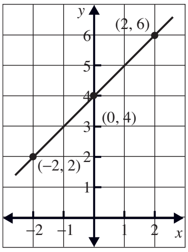
c



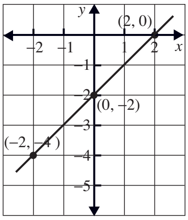
d



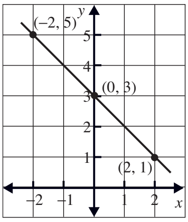
3 a (–2, 2), (0, 4), (2, 6)



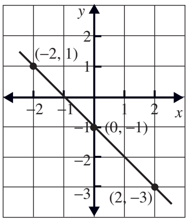
b (–2, –4), (0, –2), (2, 0)



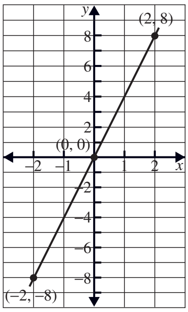
c (–2, 5), (0, 3), (2, 1)



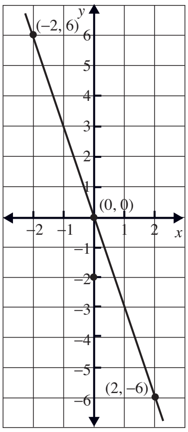
d (–2, 1), (0, –1), (2, –3)



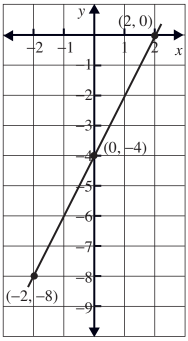
e (–2, –8), (0, 0), (2, 8)



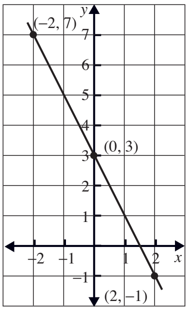
f (–2, 6), (0, 0), (2, –6)



g (–2, –8), (0, –4), (2, 0)



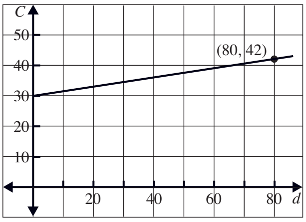
h (–2, 7), (0, 3), (2, –1)



4 a

d	40	60	80
C	36	39	42

b & c



d $d = 100$

$C = 0{\cdot}15 \times 100 + 30 = 45$

The cost would be \$45.

e $C = 60$

$60 = 0{\cdot}15 \times d + 30$

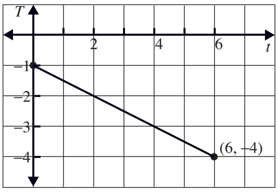
$\therefore\ d = 200$

The distance travelled would be 200 km.

5 a

t	0	1	2	3	4	5	6
T	–1	–1·5	–2	–2·5	–3	–3·5	–4

b & c



d The temperature at 5:30 am is –3·75°C.

e $t = 5\cdot5$

$T = -0\cdot5 \times 5\cdot5 - 1 = -3\cdot75$

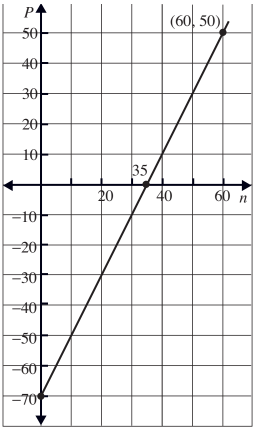
The temperature at 5:30 am is –3·75°C.

6 a

n	0	10	20	30	40	50	60
P	–70	–50	–30	–10	10	30	50

b If 35 or fewer Ice Bulas are sold, the class will make a loss as this is the cost of making the Ice Bulas.

c & d



e 35 serves need to be sold to break even

f $P = 100$

$100 = 2 \times n - 70$

$\therefore n = 85$

85 serves need to be sold.

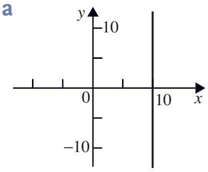
Exercise 10E

1 a $x = 5$ **b** $x = -7$ **c** $x = -0\cdot5$

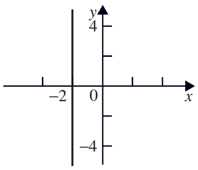
2 a $y = 6$ **b** $y = -4$ **c** $y = -8\cdot5$

d $x = -\frac{1}{4}$ **e** $y = \frac{1}{2}$ **f** $x = 2\cdot5$

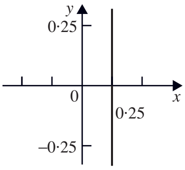
3 a



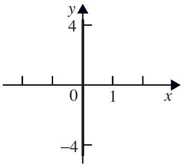
b



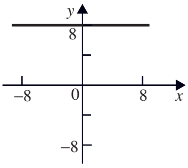
c



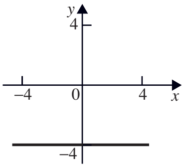
d



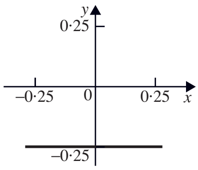
4 a



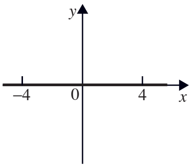
b



c



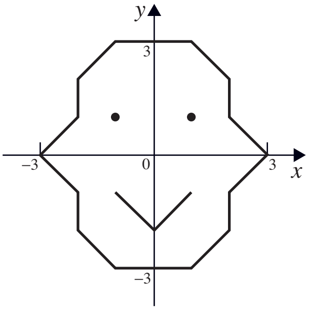
d



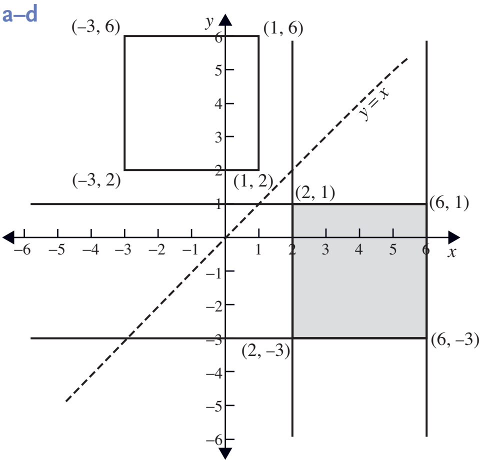
5 a i $y = 1$ **ii** $y = 3$ **iii** $y = -5$

b i $x = 2$ **ii** $x = -3$ **iii** $x = -0\cdot75$

6



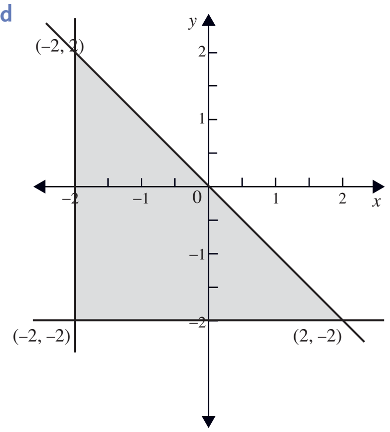
7 a–d



e (1, 2), (1, 6), (–3, 6) (–3, 2)

f In the image the coordinates are reversed. The x value becomes the y value and y value becomes the x value.

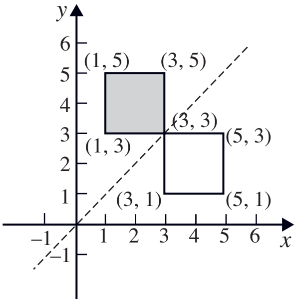
8 **a–d**



e (–2, 2), (–2, –2), (2, –2)

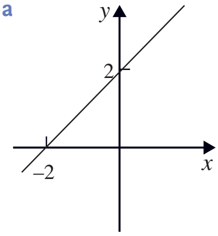
f In the image the coordinates are reversed.

9

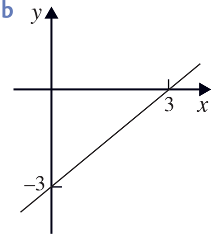


Exercise 10F

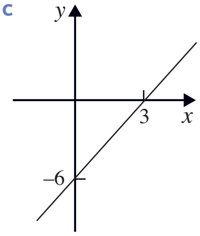
1 **a**



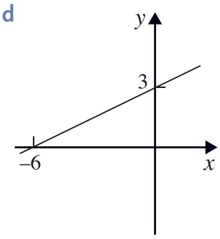
b



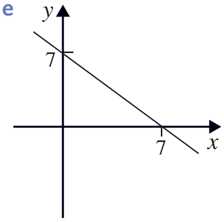
c



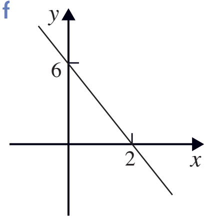
d



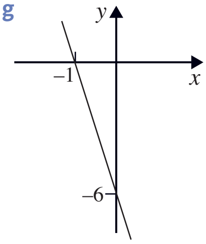
e



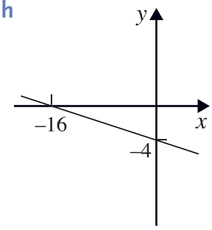
f



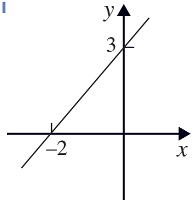
g



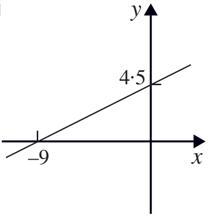
h



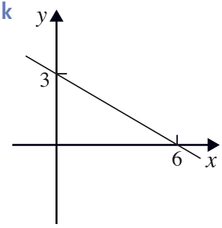
i



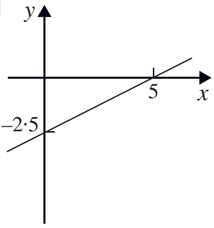
j



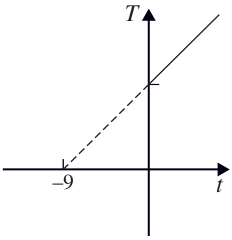
k



l



2 **a**



b Midday 18°C, 2 pm 22°C

c 6 pm

d Not realistic, temperature should be falling later in afternoon.

Exercise 10G

1 **a** $m = \frac{3}{1} = 3$

b $m = \frac{2}{1} = 2$

c $m = \frac{1}{1} = 1$

d $m = \frac{1}{2} = 0{\cdot}5$

e $m = \frac{1}{4} = 0{\cdot}25$

f $m = \frac{0}{1} = 0$

2 **a** $m = \frac{-3}{1} = -3$

b $m = \frac{-2}{1} = -2$

c $m = \frac{-1}{1} = -1$

d $m = \frac{-1}{2} = -0{\cdot}5$

e $m = \frac{-1}{4} = -0{\cdot}25$

f $m = \frac{-3}{2} = -1{\cdot}5$

g $m = -\frac{2}{3}$

h $m = \frac{2}{5}$

i $m = \frac{2}{3}$

3 **a** $m = \frac{2}{1} = 2$

b $m = \frac{6}{3} = 2$

c $m = \frac{8}{4} = 2$

d $m = \frac{-3}{3} = -1$

e $m = \frac{-4}{2} = -2$

f $m = \frac{-6}{2} = -3$

g $m = \frac{4}{2} = 2$

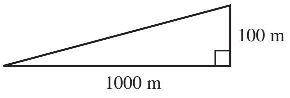
h $m = \frac{-6}{2} = -3$

i $m = \frac{5}{5} = 1$

4 **a** $m = \frac{3}{60} = 0{\cdot}05$

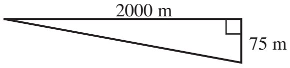
b If the incline runs 120 m it will rise
$0{\cdot}05 \times 120 = 6$ m

5 **a**



b $m = \frac{100}{1000} = 0{\cdot}1$

6 **a**



b $m = \frac{-75}{2000} = -0{\cdot}0375$

7 $m = 0{\cdot}4$
rise $= m \times$ run $= 0{\cdot}4 \times 150 = 60$ m
The air vent will be 60 m long.

8 $m = 0{\cdot}25$

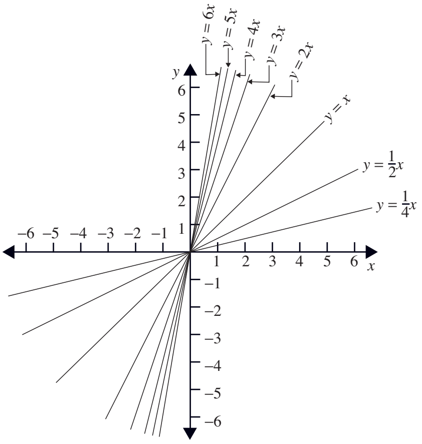
run $= \frac{\text{rise}}{m} = \frac{200}{0{\cdot}25} = 800$ m

The vent is 800 m along the mine shaft.

9 $m = \frac{1}{8}$

Exercise 10H

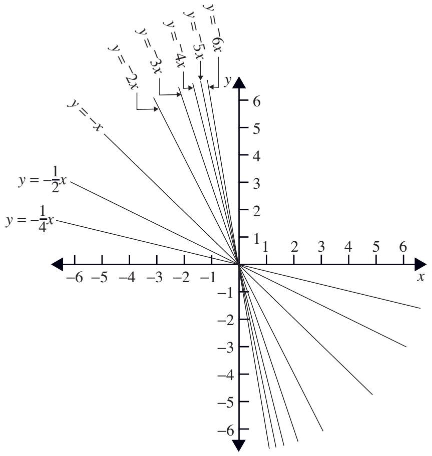
1 **a**



b **i** As m increases in value the lines become steeper.

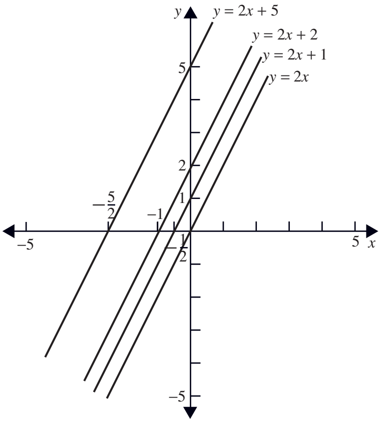
ii As m decreases in value and approaches 0 the slope is less steep.

2 **a**



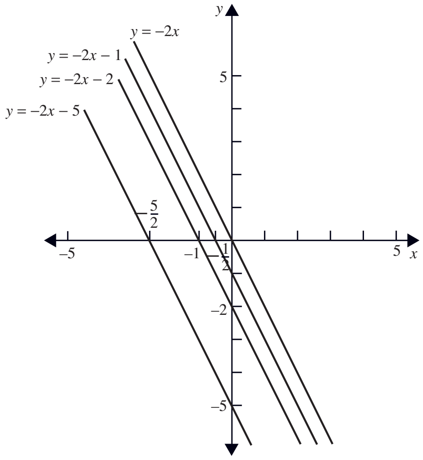
b When m is negative the slope is downhill.

3 **a**



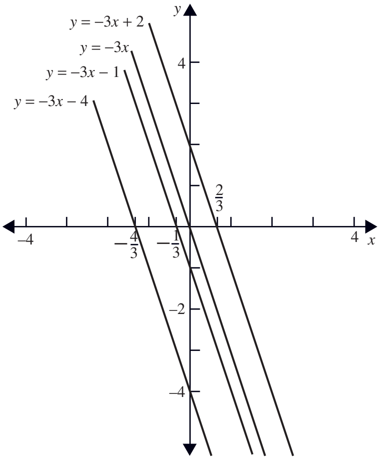
b When c increases the graph moves directly up, remaining parallel to $y = 2x$.

4 a



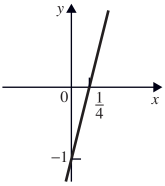
b When *c* decreases the graph moves directly down, remaining parallel to $y = –2x$.

5 a

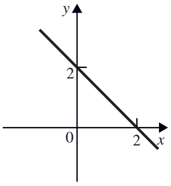


b When *c* is positive the graph moves up; when it is negative it moves down, and the more negative it becomes the further down it moves.

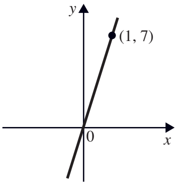
6 a



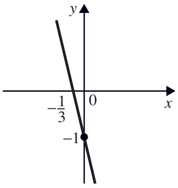
b



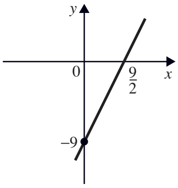
c



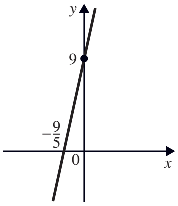
d



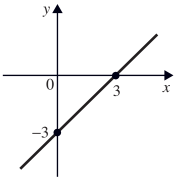
7 a $m = 2, c = –9$



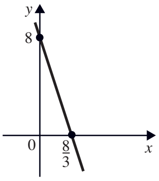
b $m = 5, c = 9$



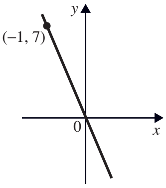
c $m = 1, c = –3$



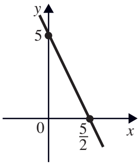
d $m = –3, c = 8$



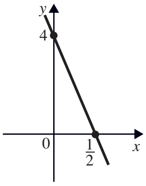
e $m = –7, c = 0$



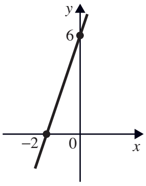
f $m = –2, c = 5$



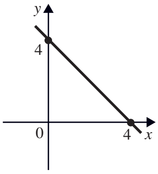
g $m = –8, c = 4$



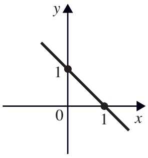
h $m = 3, c = 6$



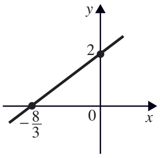
i $m = –1, c = 4$



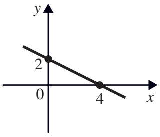
j $m = –1, c = 1$



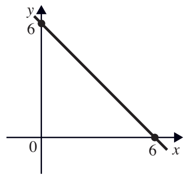
k $m = \frac{3}{4}, c = 2$



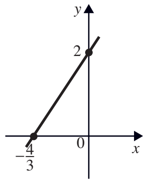
l $m = –\frac{1}{2}, c = 2$



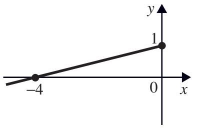
m $m = -1$, $c = 6$



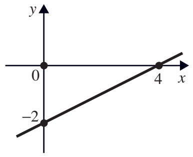
n $m = \frac{3}{2}$, $c = 2$



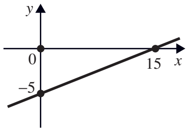
o $m = \frac{1}{4}$, $c = 1$



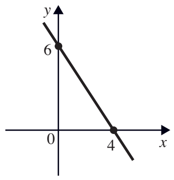
p $m = \frac{1}{2}$, $c = -2$



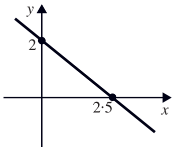
q $m = \frac{1}{3}$, $c = -5$



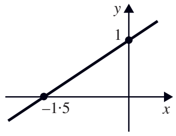
r $m = -\frac{3}{2}$, $c = 6$



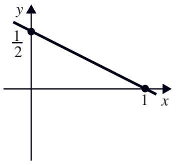
s $m = -\frac{4}{5}$, $c = 2$



t $m = \frac{2}{3}$, $c = 1$



u $m = -\frac{1}{2}$, $c = \frac{1}{2}$



Exercise 10I

1 **a** $y = 3x + 4$

b $y = 5x - 1$

c $y = -3x + 2$

d $y = \frac{1}{2}x$

e $y = 4$

f $y = \frac{3}{4}x - \frac{1}{2}$

g $y = \frac{3}{2}x + 4$

h $y = -\frac{4}{3}x - \frac{1}{3}$

i $y = -\frac{5}{2}x + \frac{3}{2}$

2 **a** $m = 2$
$c = 0$
$y = 2x$

b $m = 5$
$c = 0$
$y = 5x$

c $m = -5$
$c = 0$
$y = -5x$

3 **a** $m = 2$
$c = -3$
$y = 2x - 3$

b $m = 3$
$c = -2$
$y = 3x - 2$

c $m = \frac{1}{2}$

$c = \frac{1}{2}$

$y = \frac{1}{2}x + \frac{1}{2}$

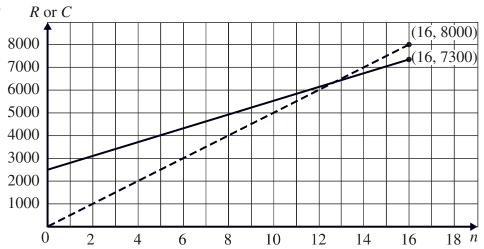
Applications

Web page production costs

a

n	0	2	4	6	8	10	12	14	16
C	2500	3100	3700	4300	4900	5500	6100	6700	7300

b



c

n	0	2	4	6	8	10	12	14	16
R	0	1000	2000	3000	4000	5000	6000	7000	8000

e 13 web pages need to be designed to show a profit.

f $m = 300$
$c = 2500$
$C = 300n + 2500$

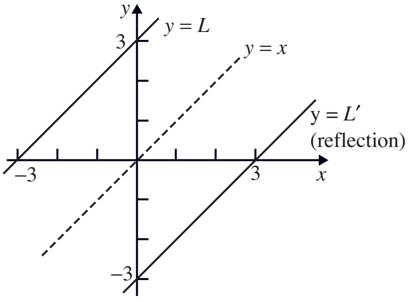
g $m = 500$
$c = 0$
$R = 500n$

h When $R = C$, $500n = 300n + 2500$
$\therefore\ 200n = 2500$
$\therefore\ n = 12{\cdot}5$
$\therefore\ R = 500 \times 12{\cdot}5 = 6250$
Therefore the point of intersection is (12·5, 6250)

i $P = 500n - 300n - 2500$
$P = 200n - 2500$

Reflections

a



b $m = 1$
$c = 3$
$y = x + 3$

e $m = 1$
$c = -3$
$y = x - 3$

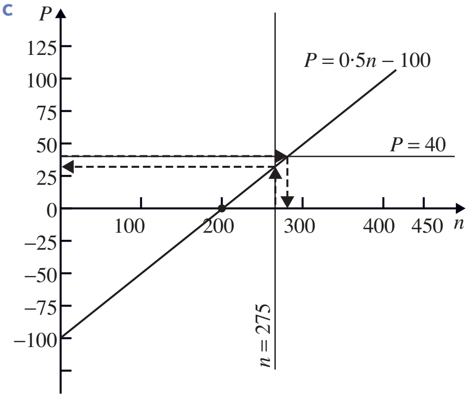
f $m = 1$
$c = -3$
$y = -x - 3$

Bicycle hire

a

n	P
0	–100
50	–75
100	–50
150	–25
200	0
250	25
300	50
350	75
400	100
450	125

b & c



d The break even number of sessions is 200.

e 280 sessions would be needed to obtain a profit of $40.

f The profit would be $37·50

g $P = 0{\cdot}5 \times n - 100$

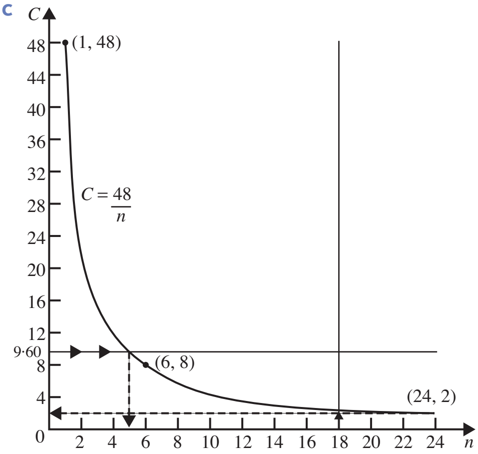
h $n = 600$
$P = 0{\cdot}5 \times 600 - 100$
$\therefore P = 200$
The profit would be $200.

Enrichment

1 a

n	C
1	48
2	24
3	16
4	12
6	8
8	6
10	4·8
12	4
16	3
20	2·4
24	2

b & c



d The approximate cost per person with 18 people would be $2·65.

e The minimum number of learners needed to share the cost would be 5 learners.

f When n is multiplied by C the answer is always 48.

$\therefore n \times C = 48$

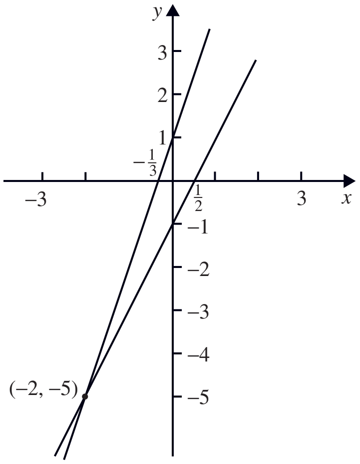
$\therefore C = \frac{48}{n}$

g $n = 14$

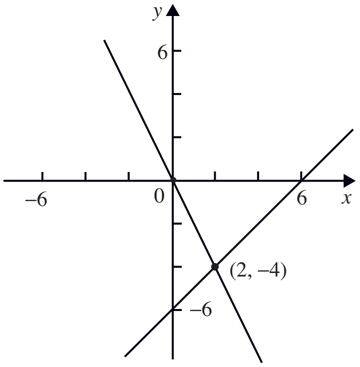
$\therefore C = \frac{48}{14} = 3{\cdot}43$

The cost would be $3·45 per person.

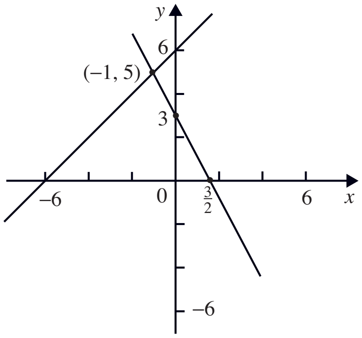
2 **a** $x = -2, y = -5$



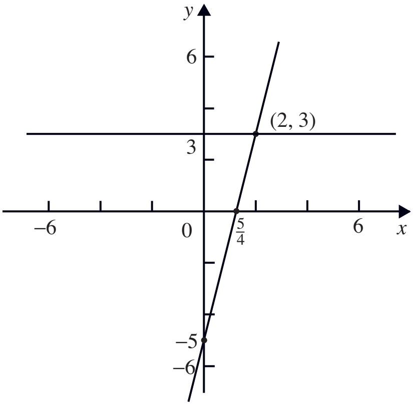
b $x = 2, y = -4$



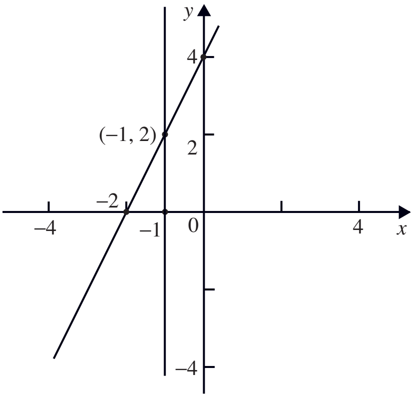
c $x = -1, y = 5$



d $x = 2, y = 3$



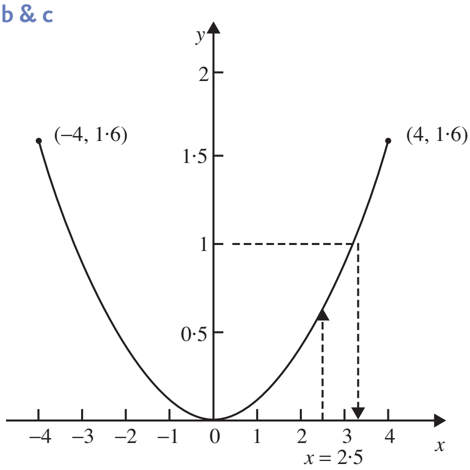
e $x = -1, y = 2$



3 **a**

x	–4	–3	–2	–1	0	1	2	3	4
y	1·6	0·9	0·4	0·1	0	0·1	0·4	0·9	1·6

b & c



d The dish is 1·6 m deep in the centre.

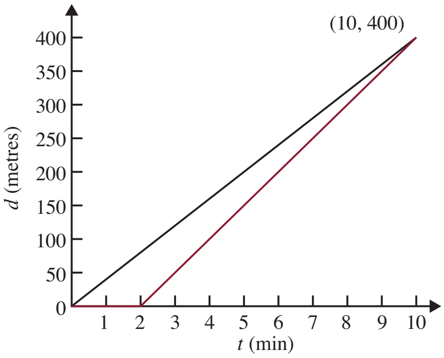
e The dish is 0·625 m deep.

f You must be 3·16 m from the centre.

4 **a**

t (min)	d (metres)
0	0
1	40
2	80
3	120
4	160
5	200
6	240
7	280
8	320
9	360
10	400

b



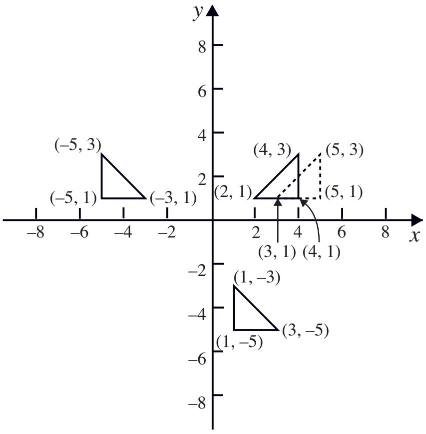
c

t (min)	d (metres)
0	0
1	0
2	0
3	50
4	100
5	150
6	200
7	250
8	300
9	350
10	400

e Ian meets up with Liz after 10 minutes, when they have each walked 400 m.

Revision/Assessment

1



2 **a** The coordinates are
(–1, –3), (0, –2), (1, –1), (2, 0)
$y = x - 2$

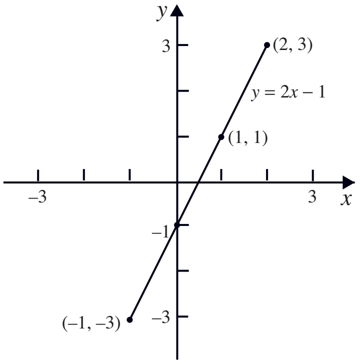
b The coordinates are
(–2, –4), (–1, –2), (0, 0), (1, 2)
$y = 2x$

c The coordinates are
(–1, 3), (0, 0), (1, –3), (2, –6)
$y = -3x$

3 **a** $y = -x + 4$ **b** $y = -0{\cdot}5x$

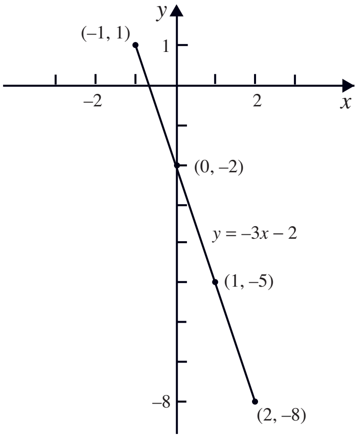
4 **a** $y = 2x + 7$ **b** $y = -3x + 2$

5 **a**



x	–1	0	1	2
y	–3	–1	1	3

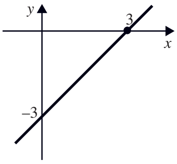
b



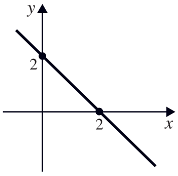
x	−1	0	1	2
y	1	−2	−5	−8

6 **a** $y = 5$ **b** $y = -3$ **c** $x = 4$

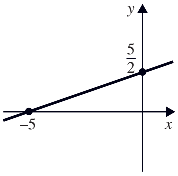
7 **a**



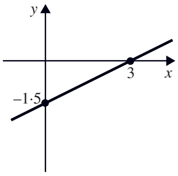
b



c

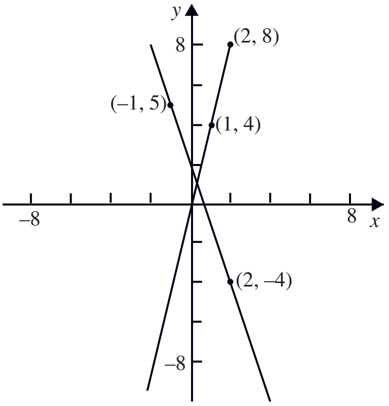


d



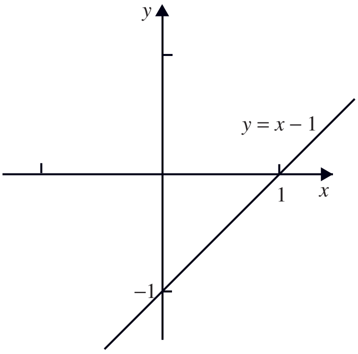
8 $m = \frac{4}{1} = 4$

9 $m = \frac{-9}{3} = -3$

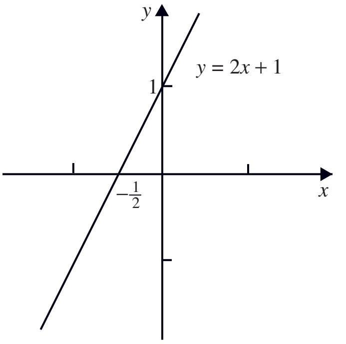


10 $m = \frac{-6}{3} = -2$

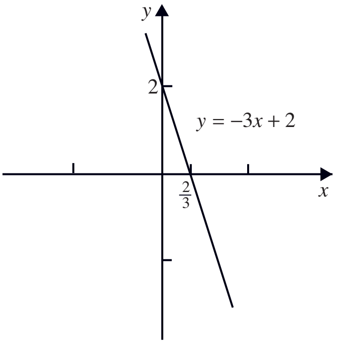
11 **a**



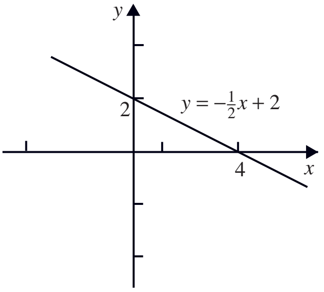
b



c



d



12 **a** $m = 2$
$c = 3$
$y = 2x + 3$

b $m = -2$
$c = 2$
$y = -2x + 2$

c $m = -1$
$c = 2$
$y = -x + 2$

Chapter 11

Learning task 11A

1 **a** $5^2 + 12^2 = 13^2$ **b** $3^2 + 4^2 = 5^2$
c $12^2 + 9^2 = 15^2$ **d** $8^2 + 6^2 = 10^2$
2 **a** $5^2 + 12^2 = 13^2$ **b** $4^2 + 7{\cdot}5^2 = 8{\cdot}5^2$
c $3^2 + 5^2 \approx 5{\cdot}9^2$ **d** $1{\cdot}5^2 + 2^2 = 2{\cdot}5^2$
e $4^2 + 7{\cdot}5^2 = 8{\cdot}5^2$ **f** $6^2 + 2{\cdot}5^2 = 6{\cdot}5^2$
3 **a** $4^2 + 4^2 = 5{\cdot}66^2$ **b** $3^2 + 9^2 \approx 9{\cdot}5^2$
c $6^2 + 2^2 \approx 6{\cdot}3^2$

Exercise 11B

1 **a** $a^2 = b^2 + c^2$ **b** $x^2 = y^2 + z^2$
c $h^2 = x^2 + t^2$ **d** $r^2 = d^2 + k^2$
2 **a** $b^2 = a^2 + c^2$ **b** $w^2 = y^2 + p^2$
c $q^2 = r^2 + s^2$ **d** $u^2 = w^2 + z^2$
e $d^2 = e^2 + f^2$ **f** $h^2 = m^2 + j^2$
g $k^2 = x^2 + l^2$ **h** $n^2 = g^2 + v^2$
i $t^2 = u^2 + v^2$ **j** $h^2 = i^2 + j^2$

Exercise 11C

1 **a** 29 cm **b** 5 cm **c** 45 cm **d** 85 cm
e 15 cm **f** 26 cm **g** 41 cm **h** 17 cm
i 37 cm **j** 20 cm **k** 30 cm **l** 29 cm
2 **a** 14·4 m **b** 19·4 m **c** 31·1 m **d** 25·5 m
e 14·0 m **f** 20·7 m **g** 8·9 m **h** 29·0 m
3 **a** $1\frac{6}{7}$ m **b** $1\frac{1}{4}$ m **c** $1\frac{2}{3}$ m **d** $1\frac{1}{9}$ m
e $3\frac{2}{5}$ m **f** $2\frac{1}{6}$ m **g** $2\frac{3}{5}$ m **h** $14\frac{1}{8}$ m
4 12·00 m (1200 cm)
5 Blue lines ≈ 2 × 15·6 ≈ 31·2 cm or 312 mm
Black lines = 44 cm or 440 mm
6 9·434 m **7** 91·67 m **8** 12·07 m
9 1·5 m **10** 135·8 cm
11 **a** **i** 22·36 m **ii** 31·62 m
iii 28·28 m **iv** 22·36 m
b 114·62 m
12 $a = 3{\cdot}6$ m, $b = 7{\cdot}6$ m, $c = 8{\cdot}4$ m, $d = 5{\cdot}4$ m,
$e = 11{\cdot}5$ m, $f = 12{\cdot}7$ m, $g = 4$ m, $h = 1{\cdot}8$ m, $i = 4{\cdot}4$ m

Exercise 11D

1 **a** $5^2 + 12^2 = 13^2$ **b** $6^2 + 8^2 = 10^2$
c $7^2 + 24^2 = 25^2$ **d** $8^2 + 15^2 = 17^2$
e $10^2 + 24^2 = 26^2$ **f** $9^2 + 12^2 = 15^2$
g $9^2 + 40^2 = 41^2$ **h** $14^2 + 48^2 = 50^2$
i $16^2 + 30^2 = 34^2$ **j** $18^2 + 80^2 = 82^2$
k $27^2 + 36^2 = 45^2$ **l** $21^2 + 28^2 = 35^2$
2 **a** 37 **b** 17 **c** 61 **d** 85
e 25 **f** 20 **g** 34 **h** 82
i 80 **j** 28 **k** 45 **l** 96
3 **a** **i** 42, 56, 70 **ii** 63, 84, 105 **iii** 84, 112, 140
b **i** 74, 1368, 1370
ii 111, 2052, 2055
iii 148, 2736, 2740
c **i** 16, 30, 34 **ii** 24, 45, 51 **iii** 32, 60, 68

Exercise 11E

1 **a** 12 m **b** 6 m **c** 15 m **d** 15 m
e 16 m **f** 9 m **g** 28 m **h** 16 m
i 96 m **j** 84 m **k** 45 m **l** 35 m
2 **a** 10·72 cm **b** 8·49 cm **c** 15·49 cm
d 14·25 cm **e** 14·70 cm **f** 10·72 cm
g 25·40 cm **h** 9·38 cm
3 **a** $4\frac{4}{5}$ **b** $3\frac{1}{5}$ **c** $4\frac{4}{5}$ **d** $2\frac{2}{5}$
e $1\frac{3}{5}$ **f** $\frac{4}{5}$ **g** $5\frac{3}{5}$ **h** $2\frac{22}{25}$
4 **a** **i** 4·90 m **ii** 4·58 m **iii** 4·00 m
b **i** 3·57 m **ii** 4·33 m **iii** 4·77 m
5 7·74 m **6** $\overline{AB} \approx 2{\cdot}65$ m, $\overline{AC} \approx 4{\cdot}58$ m
7 48·1 cm
8 **a** 10·91 m **b** 9·75 m
9 5·990 km

Exercise 11F

1 **a** $2\sqrt{10}$ **b** $3\sqrt{5}$ **c** $3\sqrt{10}$
d $2\sqrt{13}$ **e** $2\sqrt{5}$ **f** $4\sqrt{5}$
g $2\sqrt{3}$ **h** $2\sqrt{11}$ **i** $2\sqrt{33}$
j $4\sqrt{6}$ **k** $3\sqrt{7}$ **l** $7\sqrt{3}$
2 **a** $2\sqrt{7}$ **b** $2\sqrt{5}$ **c** $2\sqrt{3}$
d $6\sqrt{2}$ **e** $2\sqrt{2}$ **f** $2\sqrt{41}$
g $3\sqrt{2}$ **h** $\sqrt{11}$
3 $2\sqrt{61} + 4\sqrt{13}$ m **4** $36 + 4\sqrt{5}$ cm
5 $20 + 20\sqrt{17}$ cm
6 **a** $6\sqrt{2}$ **b** $2\sqrt{5}$ **c** $4\sqrt{13}$

Exercise 11G

1 **a** 13·42 cm **b** 9·22 m **c** 42·06 cm
2 **a** 8 cm **b** $3\sqrt{5}$ m **c** $14\sqrt{6}$ cm
3 **a** $7 - 4\sqrt{3}$ m **b** $22 - 4\sqrt{7}$ cm
c $26 + 4\sqrt{10}$ m

4 **a** $x = 3\sqrt{7} \approx 7{\cdot}94$ m
$y = \sqrt{79} \approx 8{\cdot}89$ m

b $x = 3\sqrt{11} \approx 9{\cdot}95$ m
$y = 6\sqrt{5} \approx 13{\cdot}42$ m

c $x = 6\sqrt{5} \approx 13{\cdot}42$ m
$y = 6\sqrt{11} \approx 19{\cdot}90$ m

d $x = 13 \approx 13{\cdot}00$ m
$y = 2\sqrt{61} \approx 15{\cdot}62$ m

e $x = 4\sqrt{13} \approx 14{\cdot}42$ m
$y = 8\sqrt{13} \approx 28{\cdot}84$ m

f $x = 12 \approx 12{\cdot}00$ m
$y = 4\sqrt{7} \approx 10{\cdot}58$ m

5 **a** $x = 30 = 30{\cdot}00$ cm $y = 10\sqrt{13} \approx 36{\cdot}06$ cm

b $x = 15 = 15{\cdot}00$ m $y = 3\sqrt{11} \approx 9{\cdot}95$ m

c $x = 48 = 48{\cdot}00$ cm $y = 16\sqrt{5} \approx 35{\cdot}78$ cm

6 **a** $116 + 4\sqrt{39}$ cm **b** $44 + 4\sqrt{5}$ cm

c $48 + 48\sqrt{2}$ cm

7 $a = \sqrt{2}$ cm $\approx 1{\cdot}41$ cm $b = \sqrt{3}$ cm $\approx 1{\cdot}73$ cm

$c = 2$ cm $d = \sqrt{5}$ cm $\approx 2{\cdot}24$ cm

$e = \sqrt{6}$ cm $\approx 2{\cdot}45$ cm $f = \sqrt{7}$ cm $\approx 2{\cdot}65$ cm

$g = 2\sqrt{2}$ cm $\approx 2{\cdot}83$ cm

i Area $= \frac{1}{2}(3 + \sqrt{2} + \sqrt{3} + \sqrt{5} + \sqrt{6} + \sqrt{7})$

ii $6{\cdot}74$ cm^2

8 $68{\cdot}94$ m

Exercise 11H

1 **a** $d(\overline{AB}) = \sqrt{13} \approx 3{\cdot}61$ units
$d(\overline{CD}) = 2\sqrt{13} \approx 7{\cdot}21$ units
$d(\overline{EF}) = \sqrt{68} = 2\sqrt{17} \approx 8{\cdot}25$ units

b $d(\overline{AB}) = \sqrt{128} = 8\sqrt{2} \approx 11{\cdot}31$ units
$d(\overline{CD}) = \sqrt{72} = 6\sqrt{2} \approx 8{\cdot}49$ units
$d(\overline{EF}) = \sqrt{73} \approx 8{\cdot}54$ units

2 **a** 3·16 **b** 2·24 **c** 9·43 **d** 5·66
e 5 **f** 4·47 **g** 6·40 **h** 5·66

Exercise 11I

1 **a** 17 cm **b** 10 cm **c** 45 cm
d 34 cm **e** 45 cm **f** 26 cm

2 **a** $l = 10\sqrt{14} \approx 37{\cdot}42$ cm

b $l = \sqrt{866} \approx 29{\cdot}43$ cm

c $l = 2\sqrt{70} \approx 16{\cdot}73$ cm

d $l = 2\sqrt{133} \approx 23{\cdot}07$ mm

e $l = 10\sqrt{38} \approx 61{\cdot}64$ cm

f $l = 5\sqrt{14} \approx 18{\cdot}71$ cm

g $l = 5\sqrt{129} \approx 56{\cdot}79$ cm

h $l = 10\sqrt{241} \approx 155{\cdot}24$ cm

i $l = 20\sqrt{221} \approx 297{\cdot}32$ cm

3 **a** $d\overline{AC} = 2\sqrt{41} \approx 12{\cdot}81$ cm
$d\overline{CD} = 2\sqrt{77} \approx 17{\cdot}55$ cm

b $d\overline{AC} = 13$ cm
$d\overline{CD} = \sqrt{394} \approx 19{\cdot}85$ cm

4 **a** $d\overline{AE}$ **i** $15\sqrt{2}$ cm **ii** 21·21 cm
$d\overline{AC}$ **i** $10\sqrt{2}$ cm **ii** 14·14 cm
$d\overline{AF}$ **i** $5\sqrt{2}$ cm **ii** 7·07 cm
$d\overline{EF} = 20$ cm

b $d\overline{AC}$ **i** $15\sqrt{2}$ cm **ii** 21·21 cm
$d\overline{AE} = 30$ cm
$d\overline{AF}$ **i** $\frac{15}{2}\sqrt{2}$ cm **ii** 10·61 cm
$d\overline{EF}$ **i** $15\sqrt{\frac{7}{2}}$ cm **ii** 28·06 cm

c $d\overline{AC}$ **i** $10\sqrt{5}$ cm **ii** 22·36 cm
$d\overline{AE}$ **i** $5\sqrt{5}$ cm **ii** 11·18 cm
$d\overline{AF}$ **i** $55\sqrt{21}$ cm **ii** 22·91 cm
$d\overline{EF} = 20$ cm

d $d\overline{AE} = 40$ cm
$d\overline{AC}$ **i** $10\sqrt{13}$ cm **ii** 36·06 cm
$d\overline{AF}$ **i** $5\sqrt{13}$ cm **ii** 18·03 cm
$d\overline{EF}$ **i** $5\sqrt{51}$ cm **ii** 35·71 cm

Enrichment

1 **a** 4 cm **b** $2\sqrt{3}$ cm **c** 6 cm
d $4\sqrt{3}$ cm **e** $2\sqrt{15}$ cm

2 **a** $d\overline{BE} = 10\sqrt{5} \approx 22{\cdot}36$ cm
$d\overline{BF} = 5\sqrt{21} \approx 22{\cdot}91$ cm
$d\overline{AC} = 5\sqrt{17} \approx 20{\cdot}62$ cm
$d\overline{AF} = 5\sqrt{5} \approx 11{\cdot}18$ cm
$d\overline{BD} = 5\sqrt{17} \approx 20{\cdot}62$ cm
$d\overline{ED} = 5\sqrt{5} \approx 11{\cdot}18$ cm

b $d\overline{BE} = 10\sqrt{5} \approx 22{\cdot}36$ cm
$d\overline{BF} = 5\sqrt{21} \approx 22{\cdot}91$ cm
$d\overline{AC} = 5\sqrt{5} \approx 11{\cdot}18$ cm
$d\overline{AF} = 5\sqrt{17} \approx 20{\cdot}62$ cm
$d\overline{ED} = 5\sqrt{17} \approx 20{\cdot}62$ cm
$d\overline{BD} = 5\sqrt{5} \approx 11{\cdot}18$ cm

3 **b** $P = 24\sqrt{13} \approx 86{\cdot}53$ cm

4 **a** 5, 12, 13

b

Triad	Triangles
3, 4, 5	5, 5, 8 and 5, 5, 6
7, 24, 25	25, 25, 48 and 25, 25, 14

5 **a** $150\sqrt{7}$ cm^2 **b** $50\sqrt{3}$ cm^2 **c** $600\sqrt{3}$ mm^2

6 a $(144\sqrt{3}+72\pi)\text{cm}^2$ b $(216\sqrt{3}-288\pi)\text{cm}^2$

7 a $\frac{\sqrt{2}}{2}$ b $\frac{\sqrt{5}}{5}$ c $\frac{\sqrt{10}}{10}$ d $\frac{\sqrt{13}}{13}$

8 a

n	$2n$	n^2-1	n^2+1
2	$2\times2=4$	$2^2-1=3$	$2^2+1=5$
3	$2\times3=6$	$3^2-1=8$	$3^2+1=10$
4	$2\times4=8$	$4^2-1=15$	$4^2+1=17$
5	$2\times5=10$	$5^2-1=24$	$5^2+1=26$
6	$2\times6=12$	$6^2-1=35$	$6^2+1=37$
7	$2\times7=14$	$7^2-1=48$	$7^2+1=50$
8	$2\times8=16$	$8^2-1=63$	$8^2+1=65$
9	$2\times9=18$	$9^2-1=80$	$9^2+1=82$
10	$2\times10=20$	$10^2-1=99$	$10^2+1=101$
11	$2\times11=22$	$11^2-1=120$	$11^2+1=122$
12	$2\times12=24$	$12^2-1=143$	$12^2+1=145$

c $(2n)^2+(n^2-1)^2=4n^2+n^4-2n^2+1$
$=n^4+2n^2+1$
$(n^2+1)^2=n^4+2n^2+1$
$\therefore (2n)^2+(n^2-1)^2=(n^2+1)^2$

9 a

n	$4n$	n^2-4	n^2+4
3	$4\times3=12$	$3^2-4=5$	$3^2+4=13$
4	$4\times4=16$	$4^2-4=12$	$4^2+4=20$
5	$4\times5=20$	$5^2-4=21$	$5^2+4=29$
6	$4\times6=24$	$6^2-4=32$	$6^2+4=40$
7	$4\times7=28$	$7^2-4=45$	$7^2+4=53$
8	$4\times8=32$	$8^2-4=60$	$8^2+4=68$
9	$4\times9=36$	$9^2-4=77$	$9^2+4=85$
10	$4\times10=40$	$10^2-4=96$	$10^2+4=104$
11	$4\times11=44$	$11^2-4=117$	$11^2+4=125$
12	$4\times12=48$	$12^2-4=140$	$12^2+4=148$

c $(4n)^2+(n-4)^2=16n^2+n^4-8n^2+16$
$=n^4+8n+16$
$(n^2+4)^2=n^4+8n+16$
$\therefore (4n)^2+(n-4)^2=(n^2+4)^2$

10 $20+10\sqrt{2}$ cm

Revision/Assessment

1 a $b^2=a^2+c^2$ b $d^2=f^2+s^2$
c $j^2=x^2+w^2$ d $t^2=e^2+g^2$

2 a 25 cm b 14·32 cm c $\frac{5}{9}$ cm

3 a $\frac{1}{2}$ b $\frac{5}{6}$ c $1\frac{1}{4}$

4 $21^2+28^2=35^2$ **5** 40

6 i 24, 70, 74 ii 36, 105, 111

7 a i 32, 126, 130 ii 80, 315, 325
b i 48, 68, 80 ii 120, 160, 200
c i 26, 168, 170 ii 65, 420, 425
d i 22, 120, 122 ii 55, 300, 305

8 a 9·75 b 7·55 c 23·64

9 7·94 m

10 a 6·93 m b 6·71 m c 6·32 m

11 a $4\sqrt{5}$ b $3\sqrt{34}$ c $2\sqrt{33}$ d $6\sqrt{2}$

12 $28+3\sqrt{10}+2\sqrt{34}$ cm

13 a $x=2\sqrt{601}\approx 49{\cdot}03$ mm
b $x=5\sqrt{3}\approx 11{\cdot}18$ m c $x=5\sqrt{57}\approx 37{\cdot}75$ cm
$y=\sqrt{141}\approx 11{\cdot}87$ m $y=5\sqrt{21}\approx 22{\cdot}91$ cm

14 a $3\sqrt{5}+5\sqrt{2}+\sqrt{37}+2\sqrt{17}$ m
b 28·11 m

15 a i $d\overline{AB}=\sqrt{45}=3\sqrt{5}\approx 6{\cdot}7$
ii $d\overline{CD}=\sqrt{61}\approx 7{\cdot}8$
iii $d\overline{EF}=\sqrt{13}\approx 3{\cdot}6$
b i $d\overline{AB}=\sqrt{61}\approx 7{\cdot}8$
ii $d\overline{CD}=\sqrt{97}\approx 9{\cdot}8$
iii $d\overline{EF}=\sqrt{45}=3\sqrt{5}\approx 6{\cdot}7$

16 a $2\sqrt{10}$ b $\sqrt{34}$ c $3\sqrt{2}$

17 a 13 cm b 32 cm

18 a $10\sqrt{2}$ cm b $5\sqrt{2}$ cm
c $10\sqrt{2}$ cm d $9\sqrt{5}$ cm

19 $d\overline{AC}$ i $10\sqrt{2}$ cm ii 14·14 cm
$d\overline{AF}$ i $10\sqrt{5}$ cm ii 22·36 cm
$d\overline{BH}$ i $10\sqrt{6}$ cm ii 24·49 cm
$d\overline{BD}$ i $10\sqrt{2}$ cm ii 14·14 cm

20 a $d\overline{AC}=20\sqrt{2}$ cm b $d\overline{AC}=10\sqrt{29}$ cm
$d\overline{AF}=10\sqrt{13}$ cm $d\overline{AF}=10\sqrt{34}$ cm
$d\overline{BD}=20\sqrt{2}$ cm $d\overline{BD}=10\sqrt{29}$ cm
$d\overline{BH}=10\sqrt{17}$ cm $d\overline{BH}=10\sqrt{38}$ cm

Chapter 12

Exercise 12B

1 a True b False c True
d True e False f True

2 **a** M = {January, February, March, April, May, June, July, August, September, October, November, December}
$n(\xi) = 12$

b T = {April, June, September, November}
$n(T) = 4$

c R = {January, February, March, April, September, October, November, December}
$n(R) = 8$

d T' = {January, February, March, May, July, August, October, December}
$n(T') = 8$

e R' = {May, June, July, August}
$n(R') = 4$

3 **a** $N = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20\}$

$O = \{1, 3, 5, 7, 9, 11, 13, 15, 17, 19\}$

$E = \{2, 4, 6, 8, 10, 12, 14, 16, 18, 20\}$

$P = \{2, 3, 5, 7, 11, 13, 17, 19\}$

b **i** $P \cap E = \{2\}$

ii $P \cap O = \{3, 5, 7, 11, 13, 17, 19\}$

iii $P' = \{1, 4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20\}$

iv $O' = \{2, 4, 6, 8, 10, 12, 14, 16, 18, 20\}$

c **i** $n(P \cap E) = 1$

ii $n(P \cap O) = 7$

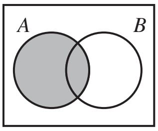
iii $n(P') = 12$

v $n(O') = 10$

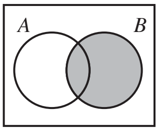
d $O \cap E$ = Null set = Ø

e $O \cup E = N$ = Universal set = ξ

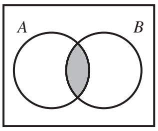
4 **a**



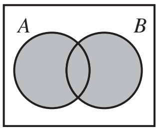
b



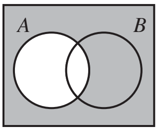
c



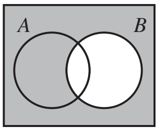
d



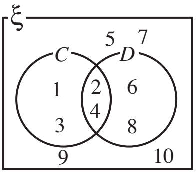
e



f



5 **a**



b **i** $C \cap D = \{2, 4\}$

ii $C \cup D = \{1, 2, 3, 4, 6, 8\}$

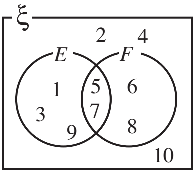
iii $C' = \{5, 6, 7, 8, 9, 10\}$

iv $D' = \{1, 3, 5, 7, 9, 10\}$

c **i** $n(C \cap D) = 2$ **ii** $n(C \cup D) = 6$

iii $n(C') = 6$ **iv** $n(D') = 6$

6 **a**



b **i** $E \cap F = \{5, 7\}$

ii $E \cup F = \{1, 3, 5, 6, 7, 8, 9\}$

iii $E' = \{2, 4, 6, 8, 10\}$

v $F' = \{1, 2, 3, 4, 9, 10\}$

c **i** $n(E \cap F) = 2$ **ii** $n(E \cup F) = 7$

iii $n(E') = 5$ **iv** $n(F') = 6$

7 **a** 20 learners are in the class

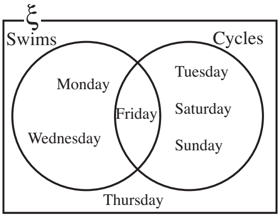
b **i** 12 play soccer

ii 5 play only tennis

iii 4 play soccer and tennis

iv 3 don't play soccer or tennis

8 **a**



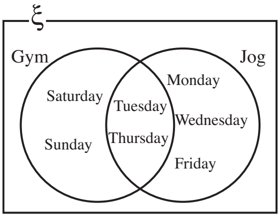
b **i** On Friday Sue swims and cycles.

ii On Monday and Wednesday Sue just swims.

iii On Tuesday, Saturday and Sunday Sue just cycles.

iv On Thursday Sue doesn't swim or cycle.

9 **a**



b **i** On Tuesday and Thursday Sam jogs and works out in the gym.

ii On Monday, Wednesday and Friday Sam jogs only.

iii On Saturday and Sunday Sam works out in the gym only.

iv There is no day Sam doesn't exercise.

Exercise 12C

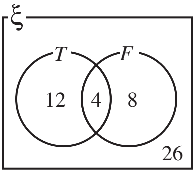
1 **a** $\frac{7}{26}$ **b** $\frac{21}{26}$ **c** $\frac{5}{26}$

2 **a** $\frac{1}{2}$ **b** $\frac{1}{2}$ **c** $\frac{1}{6}$ **d** $\frac{1}{2}$

3 **a** 23 **b** 14 **c** 7 **d** $\frac{13}{23}$ **e** $\frac{9}{23}$ **f** $\frac{2}{23}$

4 **b** **i** $\frac{1}{6}$ **ii** $\frac{5}{6}$

5 **a**



5 **b** **i** $\frac{8}{25}$ **ii** $\frac{6}{25}$ **iii** $\frac{17}{25}$

iv $\frac{19}{25}$ **v** $\frac{2}{25}$ **vi** $\frac{12}{25}$

6 **a** 18 **b** 1 **c** 4 **d** $\frac{5}{9}$ **e** 9 **f** $\frac{4}{9}$

7 **a** $\Pr(T) = \frac{7}{30}$ **b** $\Pr(F) = \frac{7}{30}$

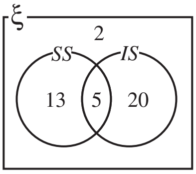
c $\Pr(S) = \frac{10}{30} = \frac{1}{3}$ **d** $\Pr(T') = \frac{23}{30}$

e $\Pr(F') = \frac{23}{30}$ **f** $\Pr(S') = \frac{2}{3}$

g $\Pr(T \cap F) = \frac{2}{15}$ **h** $\Pr(T \cap S) = \frac{1}{5}$

i $\Pr(T \cap S \cap F) = \frac{1}{10}$

8 **a**



SS = {people who read the *Solomon Star*}
IS = {people who read the *Island Sun*}

b 2 **c** $\frac{1}{8}$ **d** $\frac{19}{20}$ **e** $\frac{3}{8}$ **f** $\frac{1}{2}$ **g** $\frac{33}{40}$

Exercise 12D

1 $\frac{1}{3}$

2 **a** $\frac{1}{16}$ **b** $\frac{1}{4}$ **c** $\frac{1}{4}$ **d** $\frac{3}{4}$

e $\frac{1}{16}$ **f** $\frac{1}{8}$ **g** $\frac{1}{4}$

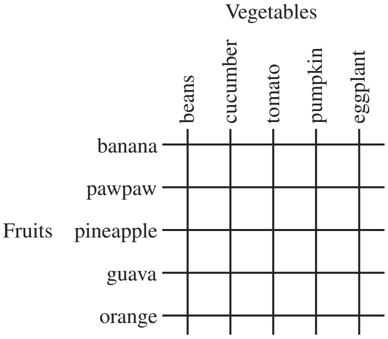
3 **a** $\frac{1}{12}$ **b** $\frac{1}{4}$ **c** $\frac{1}{4}$ **d** $\frac{1}{2}$

e $\frac{1}{6}$ **f** $\frac{1}{6}$ **g** $\frac{1}{12}$

4 **b** **i** $\frac{1}{10}$ **ii** $\frac{1}{10}$ **iii** $\frac{1}{4}$

iv $\frac{13}{40}$ **v** $\frac{1}{10}$ **vi** $\frac{3}{40}$

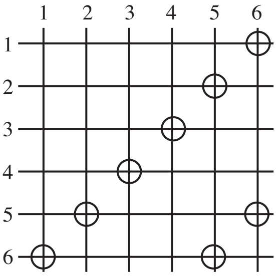
5 **a**



b **i** $\frac{1}{25}$ **ii** $\frac{1}{25}$ **iii** $\frac{1}{25}$

iv $\frac{1}{25}$ **v** $\frac{1}{25}$

6 **a**



b $\frac{2}{9}$

Exercise 12E

1 **a** $\frac{1}{16}$ **b** $\frac{1}{4}$ **c** $\frac{15}{16}$

2 **b** 18 **c** 6

d **i** $\frac{1}{3}$ **ii** $\frac{1}{3}$ **iii** $\frac{1}{2}$ **iv** $\frac{1}{9}$ **v** 1

3 **a** There are 27 different arrangements possible: $\frac{1}{27}$

b $\frac{8}{27}$ **c** $\frac{12}{27}$ **d** $\frac{19}{27}$ **e** $\frac{2}{9}$

4 Pr(getting through the first set of passages) $= \frac{2}{3}$

Pr(getting through the second set of passages) $= \frac{2}{3}$

Pr(getting through the third set of passages) $= \frac{1}{4}$

Pr(getting to the cheese) $= \frac{2}{3} \times \frac{2}{3} \times \frac{1}{4} = \frac{4}{36} = \frac{1}{9}$

Learning task 12F

1 **a** $\frac{1}{4}$ **b** $\frac{3}{13}$ **c** $\frac{1}{2}$ **d** $\frac{3}{4}$ **e** $\frac{1}{26}$

f $\frac{1}{52}$ **g** $\frac{12}{13}$ **h** $\frac{10}{13}$ **i** $\frac{5}{13}$ **j** $\frac{1}{13}$

k $\frac{1}{26}$ **l** $\frac{25}{26}$ **m** $\frac{5}{13}$ **n** $\frac{5}{26}$

2 **a** $\frac{9}{169}$ **b** $\frac{60}{169}$ **c** $\frac{1}{16}$ **d** $\frac{9}{16}$ **e** $\frac{1}{4}$

f $\frac{1}{8}$ **g** $\frac{1}{169}$ **h** $\frac{1}{169}$ **i** $\frac{100}{169}$ **j** $\frac{4}{169}$

3 **a** $\frac{27}{2197}$ **b** $\frac{1}{64}$ **c** $\frac{1}{2197}$ **d** $\frac{1728}{2197}$

e $\frac{1}{64}$ **f** $\frac{1}{2197}$ **g** $\frac{100}{2197}$ **h** $\frac{900}{2197}$

4 **a** $\frac{12}{13}$ **c** $\frac{1}{2197}$

Exercise 12G

1 **a** Expected number of red pieces in box of twelve

$= \frac{2}{5} \times 12 = 4{\cdot}8$

Expected number of yellow pieces in twelve

$= \frac{1}{5} \times 12 = 2{\cdot}4$

Expected number of blue pieces in twelve

$= \frac{3}{10} \times 12 = 3{\cdot}6$

Expected number of blue pieces in twelve

$= \frac{1}{10} \times 12 = 1{\cdot}2$

b Expected number of red pieces in 500

$= \frac{2}{5} \times 500 = 200$

Expected number of yellow pieces in 500

$= \frac{1}{5} \times 500 = 100$

Expected number of blue pieces in 500

$= \frac{3}{10} \times 500 = 150$

Expected number of blue pieces in 500

$= \frac{1}{10} \times 500 = 50$

c Pr(machine produces a red piece)

= Pr(red piece in sample) $= \frac{2}{5}$

2 **a** $\frac{6}{15}$ **b** $\frac{6}{15}$ **c** 300 **d** 450

3 24 **4** 720

5 **a** **i** $\frac{1}{20}$ **ii** $\frac{3}{40}$ **iii** $\frac{1}{40}$ **iv** $\frac{1}{10}$

b **i** 25 **ii** 37·5 **iii** 12·5 **iv** 50

c Expected number with faults in 800 $= \frac{1}{10} \times 800 = 80$

Expected number with both faults $= \frac{1}{40} \times 800 = 20$

Expect more than 10 with both faults, so expect that order will be returned.

Applications

Four-child families

Pr(all the same sex) $= 2 \times \left(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}\right) = \frac{1}{8}$

Multiplication Bingo

f It is impossible to get 7, 11, 13, 14, 17, 19, 21, 22, 23, 26, 27, 28, 29, 31, 32, 33, 34, 35

The numbers 6 and 12 are the most likely to come up.

Colour combinations

a RR, RB, RG, BR, BB, BG, GR, GB, GG

b $\frac{1}{63}$ **c** $\frac{2}{9}$

Enrichment

1 **c** Pr(Fermat wins)

$= \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$

$+ \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$

$= \frac{1}{4} + 2 \times \left(\frac{1}{8}\right) + 3 \times \left(\frac{1}{16}\right) = \frac{11}{16}$

Pr(Pascal wins) = 1 – Pr(Fermat wins)

$= 1 - \frac{11}{16} = \frac{5}{16}$

To split the money:

Fermat receives $\frac{11}{16} \times 100 = 68{\cdot}75$ francs

Pascal receives $\frac{5}{16} \times 100 = 31{\cdot}25$ francs

2 a $\frac{25}{64}$ b $\frac{20}{56}$

3 a 0·056

b Pr(only seniors win) = 0·036
Pr(only intermediates win) = 0·336
Pr(only juniors win) = 0·096
Pr(only one team wins) = 0·468

c Pr(at least one team wins) = 1 – Pr(no teams win)
= 0·856

d Pr(two teams win) = 0·332

4 $\frac{1}{8624}$

5 a $\frac{3}{5}$ b $\frac{2}{5}$ c $\frac{9}{10}$ d $\frac{1}{2}$

6 a $\frac{1}{27}$ b $\frac{2}{9}$ c $\frac{20}{27}$ d $\frac{8}{27}$

7 b Yes, there are two different paths.

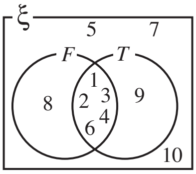
8 b No, there is only one path.

Revision/Assessment

1 a False b True c True
d True e False

2 a $F = \{1, 2, 3, 4, 6, 8\}$
b $T = \{1, 2, 3, 4, 6, 9\}$
c $F \cap T = \{1, 2, 3, 4, 6\}$
d $F' = \{5, 7, 9, 10\}$
e $T' = \{5, 7, 8, 10\}$
f $F \cup T = \{1, 2, 3, 4, 6, 8, 9\}$

3

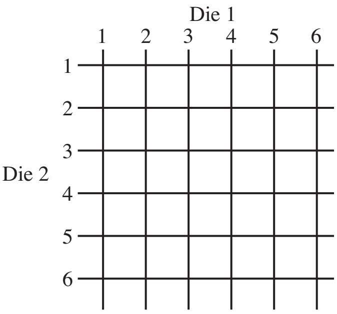


4 a There are 19 people in the group.
b 1 person plays all three sports.
c 7 learners play soccer.
d i $\frac{1}{19}$ ii $\frac{13}{19}$ iii $\frac{11}{19}$

5 a $\frac{2}{5}$ b $\frac{6}{25}$ c $\frac{16}{25}$

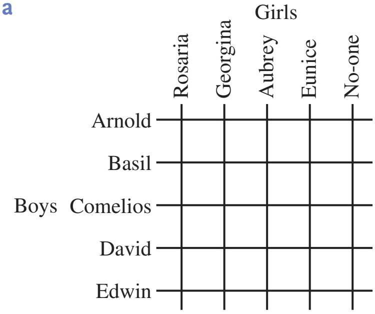
6 a $\frac{1}{64}$ b $\frac{1}{64}$ c $\frac{1}{2}$ d $\frac{3}{32}$
e $\frac{45}{64}$ f $\frac{3}{16}$ g $\frac{13}{16}$ h $\frac{55}{64}$

7 a



b $\frac{2}{9}$ c $\frac{1}{9}$

8 a



b $\frac{1}{25}$ c $\frac{4}{5}$ d $\frac{24}{25}$

9 a 12 arrangements are possible.
b 6 arrangements have mousse as a dessert.
c i $\frac{1}{2}$ ii $\frac{1}{3}$ iii $\frac{1}{2}$ iv 0 v $\frac{2}{3}$

10 a $\frac{1}{13}$ b $\frac{1}{4}$ c $\frac{1}{52}$

11 150

Chapter 13

Exercise 13A

1 a 7·5 cm^2 b 10 cm^2 c 6 cm^2
d 3 cm^2 e 9 cm^2 f 6 cm^2
g 8 cm^2 h 2 cm^2

2 a 11 cm^2 b 11 cm^2 c 12·5 cm^2
d 12·5 cm^2 e 11·5 cm^2 f 6·5 cm^2
g 7·5 cm^2 h 10 cm^2 i 8 cm^2

Exercise 13B

1 a 10 cm^2 b 10 cm^2 c 20 cm^2
d 12 cm^2 e 30 cm^2 f 12 cm^2

2 a 16 cm^2 b 24 cm^2 c 18 cm^2

3 a 64 cm^2 b 120 cm^2 c 1519 cm^2
d 448 cm^2 e 144 m^2 f 25 km ^{2}

4 a 1989 cm^2 b 1100 cm^2
c 27·2 cm^2 d 11 750 000 cm^2
e 54 cm^2 f 87 120 cm^2
g 9840 cm^2 h 1920 cm^2
5 a 675 cm^2 b 467 m^2
6 a 9828 mm^2 b 1·6065 km^2
c 790·4 mm^2 d 79·21 m^2
7 a 28 883 cm^2 b 1163 m^2
8 a 90·7 m^2 b 189·3 m^2
9 204 cm^2
10 a i 3600 cm^2 ii 2520 cm^2
b Width of coloured parts as vertical rows 4·5 cm for horizontal rows 4 cm

Exercise 13C

1 a 96 m^2 b 1632 m^2 c 264 cm^2
d 5226 m^2 e 986 m^2 f 2698 mm^2
g 105·6 m^2 h 43·21 m^2 i 5963 cm^2
2 a 124 cm^2 b 301 m^2 c 74 250 mm^2
d 416 cm^2 e 417 cm^2 f 784 m^2
g 440 m^2

Exercise 13D

1 a 24 m^2 b 147 km^2 c 320 cm^2
d 168 mm^2 e 432 m^2 f 38 cm^2
g 30 cm^2 h 18·6 m^2 i 3800 cm^2
j 2145 cm^2 k 5·4 m^2 l 76 mm^2
2 a 528 mm^2 b 292·5 cm^2
3 a Red 72 m^2 b Red 1340 m^2
White 63 m^2 White 1500 m^2
Blue 225 m^2 Blue 440 m^2
4 45 cm^2, 20 cm^2, 30 cm^2, 30 cm^2; Sum = 125 cm^2
Area of large triangle = 0·5 × 25 × 10 = 125 cm^2
5 a Area: Size A = 0·5(5 × 8) = 20 cm^2
Size B = 0·5(5 × 10) = 25 cm^2
Size C = 0·5(8 × 10) = 40 cm^2
b Tiles per 1 m^2: Size A = 500 tiles
Size B = 400 tiles
Size C = 250 tiles
c Cost per tile: Size A = $16·00
Size B = $2·50
Size C = $4·00
d i 10 m^2, ∴ cost = $8000
ii 9·1 m^2, ∴ cost = $7280
iii 14·88 m^2, ∴ cost = $11 904·00
e i 20 m^2, ∴ cost = $20 000
ii 14·56 m^2, ∴ cost = $14 560
iii 78 m^2, ∴ cost = $78 000
f i 200 m^2, ∴ cost = $240 000
ii 216 m^2, ∴ cost = $259 200
iii 9·12 m^2, ∴ cost = $10 944

Exercise 13E

1 a 120 cm^2 b 208 m^2 c 1344 cm^2
d 696 mm^2 e 97·5 cm^2 f 1176 m^2
g 1014 mm^2 h 81 m^2
2 a 104 cm^2 b 18 cm^2 c 104 m^2
d 24 mm^2 e 16 m^2
3 Area of rectangle = 360 cm^2
Area of triangle = 30 cm^2
Area of trapezoid = 150 cm^2
Sum of areas = 360 cm^2

Exercise 13F

1 a 39 units^2 b 50 units^2 c 33 units^2
2 a 70 square cm b 76 square cm
3 a 19 cm^2 b $15\frac{1}{2}$ cm^2 c $9\frac{1}{2}$ cm^2
d 15 cm^2
4 a 16 cm^2 b 15 cm^2 c $8\frac{3}{4}$ cm^2
d 16 cm^2 e $19\frac{1}{2}$ cm^2 f 18 cm^2

Exercise 13G

1 a i 400π ii 1256·64 mm ^{2}
b i 36π ii 113·10 cm^2
c i 144π ii 452·39 cm^2
d i 25π ii 78·54 cm^2
e i 225π ii 706·86 cm^2
f i 16π ii 50·27 m^2
g i 81π ii 254·47 m^2
h i 4π ii 12·57 m^2
2 a i 144π ii 452·4 cm^2
b i π ii 3·1 m^2
c i 100π ii 314·2 mm ^{2}
d i 400π ii 1256·6 m^2
e i 64π ii 201·1 cm^2
f i 81π ii 254·5 m^2
g i 2025π ii 6361·7 m^2
3 a i 18π ii 56·55 cm^2
b i 32π ii 100·53 m^2
c i $\frac{9}{2}\pi$ ii 14·14 cm^2
d i 49π ii 153·94 cm^2
e i 81π ii 254·47 mm^2
4 Family: 962 cm^2 Large: 707 cm^2
Medium: 491 cm^2 Small: 314 cm^2
5 a 150·80 cm^2 b 439·82 m^2 c 263·89 m^2
6 a 1256·64 cm^2 b 3216·99 cm^2
c 10·18 cm^2 d 18·10 cm^2

Exercise 13H

1 a 54·9 m^2 b 201·7 m^2 c 2671·4 m^2
d 64·1 m^2 e 111·2 m^2 f 194·9 m^2

2 a 300 cm^2 b 653·43 cm^2 c 678 cm^2
d 173·43 m^2 e 42·41 m^2 f 733·08 cm^2
g 1014·16 m^2

3 Area of shape 1 = 3213·7 m^2
Area of shape 2 = 776·7 m^2
Area of shape 3 = 2228·3 m^2
Area of shape 4 = 4113·3 m^2
Total area = 10 332 m^2

Exercise 13I

1 a 332 cm^2 b 238 cm^2 c 1182 cm^2
d 348 m^2 e 718 mm^2 f 684 m^2

2 a 780 cm^2 b 120 mm^2 c 200 cm^2
d 432 m^2 e 264·8 cm^2 f 336·5 m^2

3 a 2064 cm^2 b 2240 cm^2 c 1776 cm^2

4 Red (sides) = 78 m^2 Blue (top) = 35·1 m^2

Exercise 13J

1 a 1050 cm^3 b 1980 m^3 c 855 cm^3
d 1072 cm^3 e 480 m^3 f 600 m^3

2 a 2304 cm^3 b 7680 cm^3 c 360 m^3
d 448 mm^3 e 576 m^3 f 400 m^3
g 1728 cm^3 h 5832 m^3

3 a 54 286·72 cm^3 b 50·27 m^3
c 15 079·64 mm^3 d 113 097·34 m^3
e 1413·72 cm^3 f 8482·30 m^3
g 22 518·94 mm^3 h 58 973·98 cm^3

4 a 192 cm^3 b 5720 m^3 c 1386 mm^3
d 146 250 m^3 e 4200 cm^3 f 2376 mm^3

Enrichment

1 a 1376 cm^2 b 325·65 cm^2

2 a TSA = 2646 cm^2 b TSA = 8664 mm^2
V = 9261 cm^3 V = 54 872 mm^3
c TSA = 1300 cm^2 d TSA = 370 m^2
V = 3000 cm^3 V = 2136 m^3

3 a TSA = 753·98 cm^2 b TSA = 6760 cm^2
V = 1492·26 cm^3 V = 33 400 cm^3
c TSA = 17 622·1 cm^2 d TSA = 227·99 cm^2
V = 163 508·84 cm^3 V = 232·35 cm^3
e i TSA = 1192·70 cm^2 f i TSA = 894·59 cm^2
ii V = 2678·10 cm^3 ii V = 1526·81 cm^3
g i TSA = 1463·85 cm^2
ii V = 3920·57 cm^3

4 a i Circle radius 4 cm, 50·3 cm^2
ii 13·7 cm^2
b i Circle radius 4 cm, 50·3 cm^2
ii 29·73 cm^2
c i Circle radius 6 cm, 113·1 cm^2
ii 390·9 cm^2
d i Circle radius 10 cm, 314·2 cm^2
ii 185·8 cm^2
e i Circle radius 27·7 cm, 2410·5 cm^2
ii 248·7 cm^2

5 a 1413·7 cm^2 b 341·3 cm^2
c 1168·9 cm^2

6 a V = 5252·7 cm^3 b V = 16 310 cm^3
TSA = 5529·2 cm^2 TSA = 4432 cm^2
c V = 4032 cm^3
TSA = 1608 cm^2

7 Bag A: Volume of the bag = 114 081·04 cm^3
a 36·5 cm, 47·0 cm, 66·5 cm
b Total surface area = 14 536·5 cm^2
Bag B: Volume of the bag = 111 111$\frac{1}{9}$ cm^3
a 33$\frac{1}{3}$ cm, 50 cm, 66$\frac{2}{3}$ cm
b Total surface area = 14 444$\frac{4}{9}$ cm^2

Revision/Assessment

1 a 12 cm^2 b 6 cm^2 c 8 cm^2

2 Pink: 13 cm^2 Blue: 21 cm^2

3 a 96 cm^2 b 16 m^2 c 48 m^2

4 271 cm^2

5 a 1904 cm^2 b 3528 cm^2 c 209 m^2

6 60 m^2

7 a 154 cm^2 b 242 m^2

8 a 396 cm^2 b 243 m^2 c 2184 mm^2

9 Area of one tile = 5 cm^2

Frame will hold 16 tiles along the width and 4 on length: A = 320 cm^2

Area uncovered = 80 cm^2

10 a Green 248 m^2 b Green 358 m^2
Pink 308 m^2
Blue 342 m^2

11 a 12 squares b 27 squares
c 18 squares d 23 squares

12 a 615·75 cm^2 b 254·47 m^2
c 100·53 m^2 d 452·39 m^2

13 a Pink 314·2 cm^2
Green 360 cm^2
Purple 125·8 cm^2
b Pink 300 cm^2
Purple 406·9 cm^2
c Purple 599·9 m^2
Pink 418 cm^2
d Purple 23·7 cm^2
Pink 6·3 cm^2

e Pink 24 m^2
Purple 15·3 m^2

14 a 94 cm^2 b 504 cm^2 c 2700 m^2

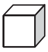
15 a 660 cm^2 b 234 m^2 c 212 cm^2

16 a 264 cm^3 b 3168 cm^3

Chapter 14

Exercise 14A

1 a Cylinder b Sphere
c Rectangular prism d Triangular prism
e Cone and hemisphere
f Rectangular prism or cuboid
g Rectangular prism or cuboid and triangular prism
h Cylinders

2 a

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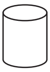
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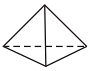
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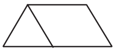
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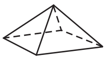
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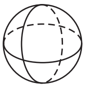
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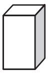
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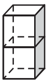
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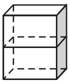
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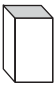
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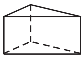
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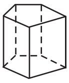
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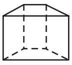

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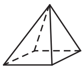
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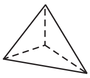
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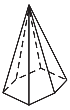
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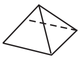
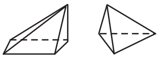


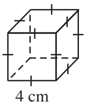
5 a A right pyramid has its apex above the centre of the base.
b


6 A pyramid has a polygon as its base. All its other faces are triangles which meet at the apex. A prism has two congruent end faces which are polygons and all the other faces are rectangles.

7 Check your list with your teacher.

8 Show your partner.

9 a 3
b 2 circles as end faces and 1 rectangle curved around
c tennis balls, tomato soup, cat food, lipstick, hairspray, jam

10 a


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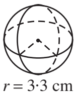

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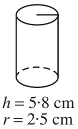

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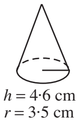

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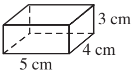

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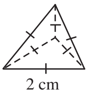

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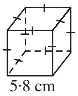

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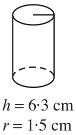

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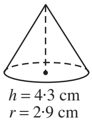

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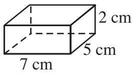

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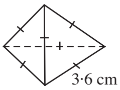

Exercise 14B

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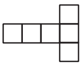
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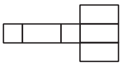
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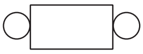
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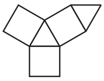
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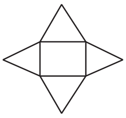
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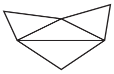
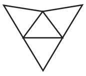


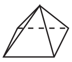
2 a A, C and D
b Show your teacher.
c After you fold the four squares, both ends fold to the same place.

3 a A, B, C and D.

6 a 3 b 4 c 1 d 2

7 a

b

c Show your answers to your partner.

8 a

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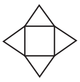
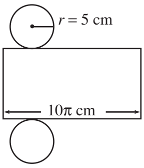

c Show your answers to your partner.

10 a


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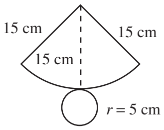


c The volume of the cylinder is three times the volume of the cone.

d Cross-section of cylinder is a circle or a rectangle. Cross-section of a cone is a circle and a triangle.

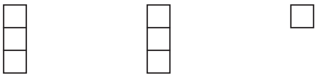
11 The tower roof is a pyramid.

Exercise 14C

1 a 3 b 4 c 4 d 4
e 5 f 6 g 7 h 4

2 Front view Side view Top view

a


b


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d


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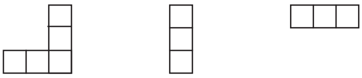

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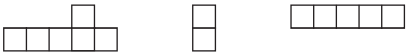

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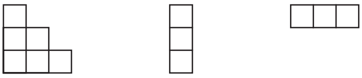

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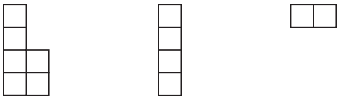
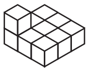


3 a

b

c

d

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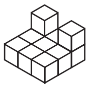
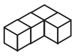


4 a

b

c

d

e

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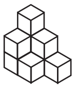
5 a

b

c


Exercise 14E

1 a 5 faces

b Rectangles and triangles

c 9 edges and 6 vertices

d 3 edges at each vertex

e All 6 vertices are of order 3.

2 a 9 faces

b Triangles and an octagon

c 9 vertices and 16 edges

d 3 and 8

e 8 vertices of order 3 and 1 of order 8.

3 a

Name of platonic solid	**Number of faces (F)**	**Number of vertices (V)**	**Number of edges (E)**
Tetrahedron	4	4	6
Cube	6	8	12
Octahedron	8	6	12
Dodecahedron	12	20	30
Icosahedron	20	12	30

b Tetrahedron $F + V = E + 2$
$4 + 4 = 6 + 2$

Cube $F + V = E + 2$
$6 + 8 = 12 + 2$

Octahedron $F + V = E + 2$
$8 + 6 = 12 + 2$

Dodecahedron $F + V = E + 2$
$12 + 20 = 30 + 2$

Isosahedron $F + V = E + 2$
$20 + 12 = 30 + 2$

Exercise 14F

1 A network is traversable if each edge can be traced without lifting the pencil or retracing the same edge.

2 An odd vertex has an odd number of edges that intersect there.

An even vertex has an even number of edges that intersect there.

3 a traversable; 2 odd vertices and 4 even vertices

b traversable; 6 even vertices

c not traversable; 4 odd vertices and 3 even vertices

d traversable; 3 even vertices

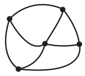
4 A network is traversable only if it has zero or two odd vertices.

5 a traversable, 2 odd vertices

b traversable, no odd vertices

c traversable, 2 odd vertices

d traversable, 2 odd vertices

6 a

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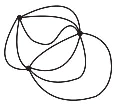
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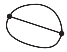
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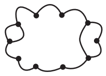
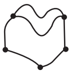
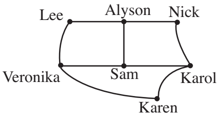


e 1 odd vertex is impossible

f 

7 a 

b No, it has 3 odd vertices.

8 a 2 odd vertices and 3 even vertices

b Yes

c Route Kwailibasi, Atoifi, Afutara, Honiara, Auki, Kwailibasi

d $870

Applications

Packaging

a Cost of production, usable volume, type of product, looks, durability, environment friendly

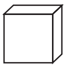
b Powder-based products, e.g. washing detergents. Liquid products are usually found in cylindrical packaging.

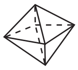
c Difficult to hold

Enrichment

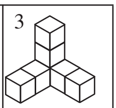
1 a 3 b 7 c not possible d 18

2 a 6 b 24

4 a 

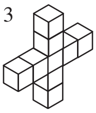
b 

5 a

Number in series (n)	1 	2 	3 
Number of cubes (c)	1	4	7

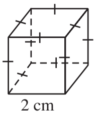
b 10, 13, 16 c $c = 3n - 2$

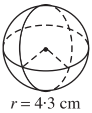
6 a

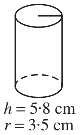
Number in series (n)	1 	2 	3 
Number of cubes (c)	1	5	9

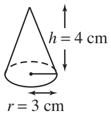
b 13, 17 and 21 cubes c $c = 4n - 3$

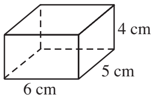
Revision/Assessment

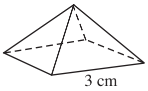
1 a 

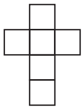
b 

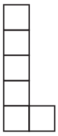
c 

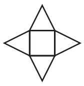
d 

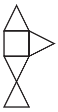
e 

f 

2 a 

b 

3 a 

b 

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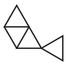
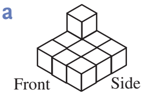
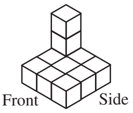
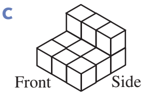
b 

5 a 

b 

c 

8

	Front view	Side View	Top view
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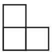
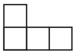
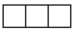
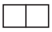
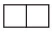
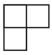
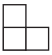
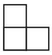
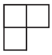
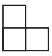
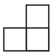
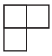
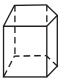
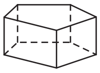
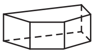
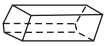
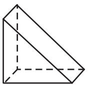
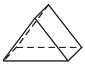
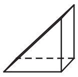
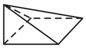
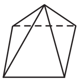
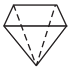
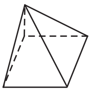
b			
c			
d			
e			
f			

9 a    

b    

10 a    

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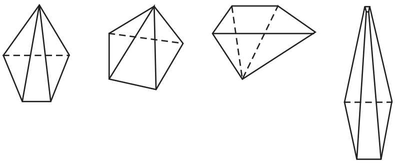


11 **a** 8

b 2 hexagons, 6 rectangles

c 18 edges,12 vertices

d 3

12 **a** 6

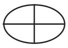
b 5 triangles, 1 pentagon

c 6 vertices, 10 edges

d 3 edges at all vertices except at one which has 5

13 An even number of edges meet at an even vertex and an odd number of edges meet at odd vertex.

14 **a**



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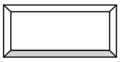
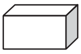
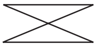


c



d



e

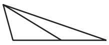


Table of Square Roots

N	√N	N	√N	N	√N	N	√N	N	√N	N	√N
1	1.000	**51**	7.141	**101**	10.050	**151**	12.288	**201**	14.177	**251**	15.843
2	1.414	**52**	7.211	**102**	10.100	**152**	12.329	**202**	14.213	**252**	15.875
3	1.732	**53**	7.280	**103**	10.149	**153**	12.369	**203**	14.248	**253**	15.906
4	2.000	**54**	7.348	**104**	10.198	**154**	12.410	**204**	14.283	**254**	15.937
5	2.236	**55**	7.416	**105**	10.247	**155**	12.450	**205**	14.318	**255**	15.969
6	2.449	**56**	7.483	**106**	10.296	**156**	12.490	**206**	14.353	**256**	16.000
7	4.646	**57**	7.550	**107**	10.344	**157**	12.530	**207**	14.388	**257**	16.031
8	2.828	**58**	7.616	**108**	10.392	**158**	12.570	**208**	14.422	**258**	16.062
9	3.000	**59**	7.681	**109**	10.440	**159**	12.610	**209**	14.457	**259**	16.093
10	3.162	**60**	7.746	**110**	10.488	**160**	12.649	**210**	14.491	**260**	16.124
11	3.317	**61**	7.810	**111**	10.536	**161**	12.689	**211**	14.526	**261**	16.155
12	3.464	**62**	7.874	**112**	10.583	**162**	12.728	**212**	14.560	**262**	16.186
13	3.606	**63**	7.937	**113**	10.630	**163**	12.767	**213**	14.595	**263**	16.217
14	3.742	**64**	8.000	**114**	10.677	**164**	12.806	**214**	14.629	**264**	16.248
15	3.873	**65**	8.062	**115**	10.724	**165**	12.845	**215**	14.663	**265**	16.279
16	4.000	**66**	8.124	**116**	10.770	**166**	12.884	**216**	14.697	**266**	16.310
17	4.123	**67**	8.185	**117**	10.817	**167**	12.923	**217**	14.731	**267**	16.340
18	4.243	**68**	8.246	**118**	10.863	**168**	12.962	**218**	14.765	**268**	16.371
19	4.359	**69**	8.307	**119**	10.909	**169**	13.000	**219**	14.799	**269**	16.401
20	4.472	**70**	8.367	**120**	10.955	**170**	13.038	**220**	14.832	**270**	16.432
21	4.583	**71**	8.426	**121**	11.000	**171**	13.077	**221**	14.866	**271**	16.462
22	4.690	**72**	8.485	**122**	11.045	**172**	13.115	**222**	14.900	**272**	16.492
23	4.796	**73**	8.544	**123**	11.091	**173**	13.153	**223**	14.933	**273**	16.523
24	4.899	**74**	8.602	**124**	11.136	**174**	13.191	**224**	14.967	**274**	16.553
25	5.000	**75**	8.660	**125**	11.180	**175**	13.229	**225**	15.000	**275**	16.583
26	5.099	**76**	8.718	**126**	11.225	**176**	13.267	**226**	15.033	**276**	16.613
27	5.196	**77**	8.775	**127**	11.269	**177**	13.304	**227**	15.067	**277**	16.643
28	5.292	**78**	8.832	**128**	11.314	**178**	13.342	**228**	15.100	**278**	16.673
29	5.385	**79**	8.888	**129**	11.358	**179**	13.379	**229**	15.133	**279**	16.703
30	5.477	**80**	8.944	**130**	11.402	**180**	13.416	**230**	15.166	**280**	16.733
31	5.568	**81**	9.000	**131**	11.446	**181**	13.454	**231**	15.199	**281**	16.763
32	5.657	**82**	9.055	**132**	11.489	**182**	13.491	**232**	15.232	**282**	16.793
33	5.745	**83**	9.110	**133**	11.533	**183**	13.528	**233**	15.264	**283**	16.823
34	5.831	**84**	9.165	**134**	11.576	**184**	13.565	**234**	15.297	**284**	16.852
35	5.916	**85**	9.220	**135**	11.619	**185**	13.602	**235**	15.330	**285**	16.882
36	6.000	**86**	9.274	**136**	11.662	**186**	13.638	**236**	15.362	**286**	16.912
37	6.083	**87**	9.327	**137**	11.705	**187**	13.675	**237**	15.395	**287**	16.941
38	6.164	**88**	9.381	**138**	11.747	**188**	13.711	**238**	15.427	**288**	16.971
39	6.245	**89**	9.434	**139**	11.790	**189**	13.748	**239**	15.460	**289**	17.000
40	6.325	**90**	9.487	**140**	11.832	**190**	13.784	**240**	15.492	**290**	17.029
41	6.403	**91**	9.539	**141**	11.874	**191**	13.820	**241**	15.524	**291**	17.059
42	4.481	**92**	9.592	**142**	11.916	**192**	13.856	**242**	15.556	**292**	17.088
43	6.557	**93**	9.644	**143**	11.958	**193**	13.892	**243**	15.588	**293**	17.117
44	6.633	**94**	9.695	**144**	12.000	**194**	13.928	**244**	15.620	**294**	17.146
45	6.708	**95**	9.741	**145**	12.042	**195**	13.964	**245**	15.652	**295**	17.176
46	6.782	**96**	9.798	**146**	12.083	**196**	14.000	**246**	15.684	**296**	17.205
47	6.856	**97**	9.849	**147**	12.124	**197**	14.036	**247**	15.716	**297**	17.234
48	6.928	**98**	9.899	**148**	12.166	**198**	14.071	**248**	15.748	**298**	17.263
49	7.000	**99**	9.950	**149**	12.207	**199**	14.107	**249**	15.780	**299**	17.292
50	7.071	**100**	10.000	**150**	12.247	**200**	14.142	**250**	15.811	**300**	17.321

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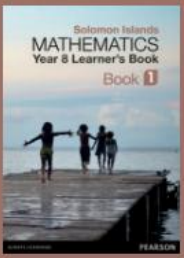

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