VICTORIAN CURRICULUM VERSION 2.0

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SECOND EDITION

OXFORD MATHS 7

Series Consultant: Thomas Christiansen
Helen Sorenson Alexander Blanksby
Morgan Levick Echo Gu Jennifer Nolan
Melanie Koetsveld Sonja Stambulic Robert Bell

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CONTENTS

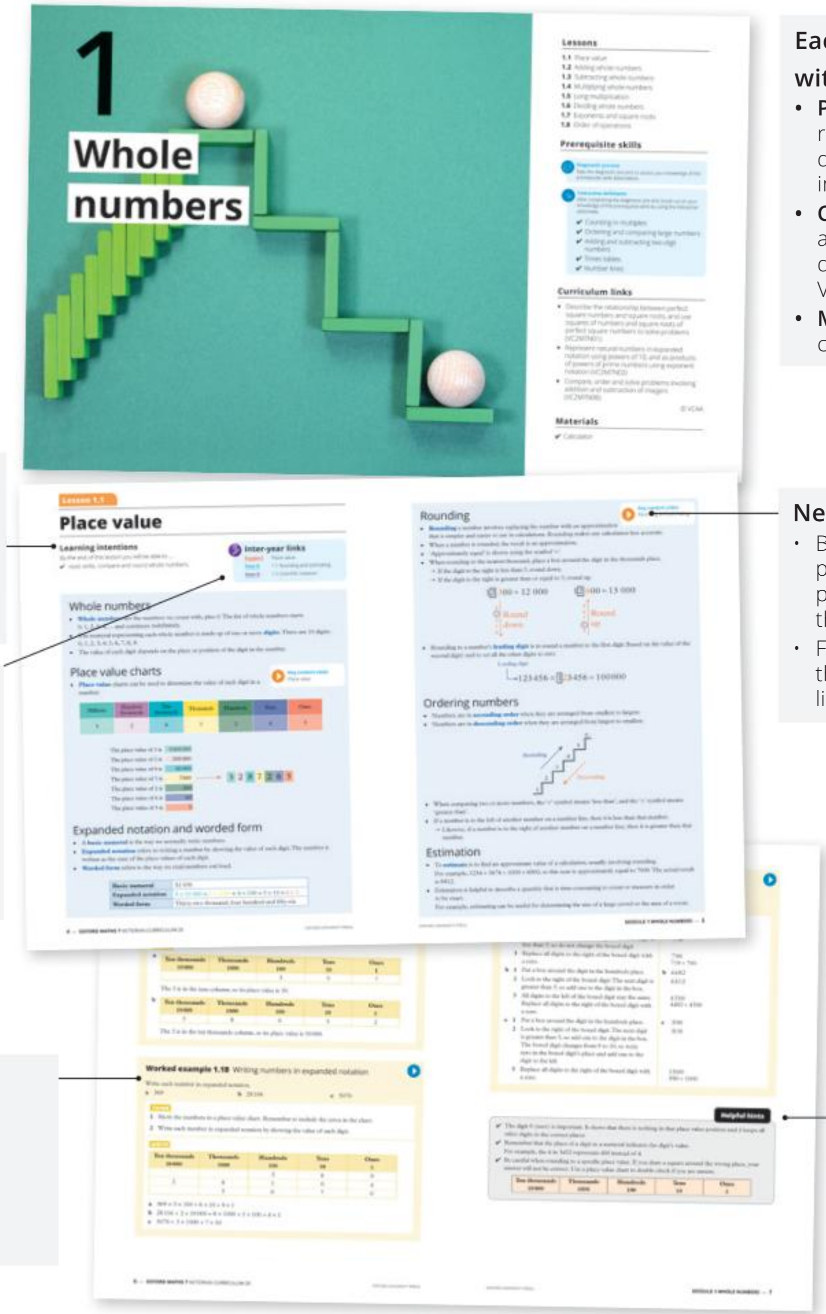
CONTENTS

Introducing Oxford Maths 7–10 (Second edition)

Oxford Maths 7–10 Victorian Curriculum utilises an innovative suite of print and digital resources to guide students on a focused mathematics journey. The series makes maths accessible to students with differing levels of understanding, increasing engagement by giving learners the opportunity to achieve success at their own skill level while also providing comprehensive syllabus coverage.

Key features of Student Books

> Complete access to all digital resources.
> Australian Maths Trust (AMT) spreads offer unique questions designed to challenge students and build engagement.
> STEAM projects encourage inter-disciplinary thinking.
> Semester reviews provide an opportunity to revise key concepts from each semester.
> NAPLAN practice allows students to revise numeracy skills for the National Assessment Program.



Each module opens with:

- **Prerequisite skills** with reference to an online diagnostic pre-test and interactive skillsheets.
- **Curriculum links** to all relevant content descriptions in the Victorian Curriculum.
- **Materials** used to complete the exercises.

Learning intentions

- Signpost the foundational skills being developed in each lesson.

Inter-year links

- Provide easy access to support and extension material from each of the 7–10 Student Books as students build knowledge year on year.

New theory

- Backed by the latest pedagogical research to promote engagement with the material.
- Filled with precise diagrams that bring key concepts to life and aid understanding.

Worked examples

- Outline a step-by-step thought process for solving essential questions with direct reference to the exercises.

Helpful hints

- Provide additional strategies for tackling problems.
- Highlight important elements of the theory.
- Point out common misconceptions.

Differentiated learning pathways

- Each lesson is separated into three pathways, tailoring for students of all skill levels.
- Each pathway can be assigned based on results of the diagnostic pre-tests that are recommended at the beginning of every module.

Understanding and fluency

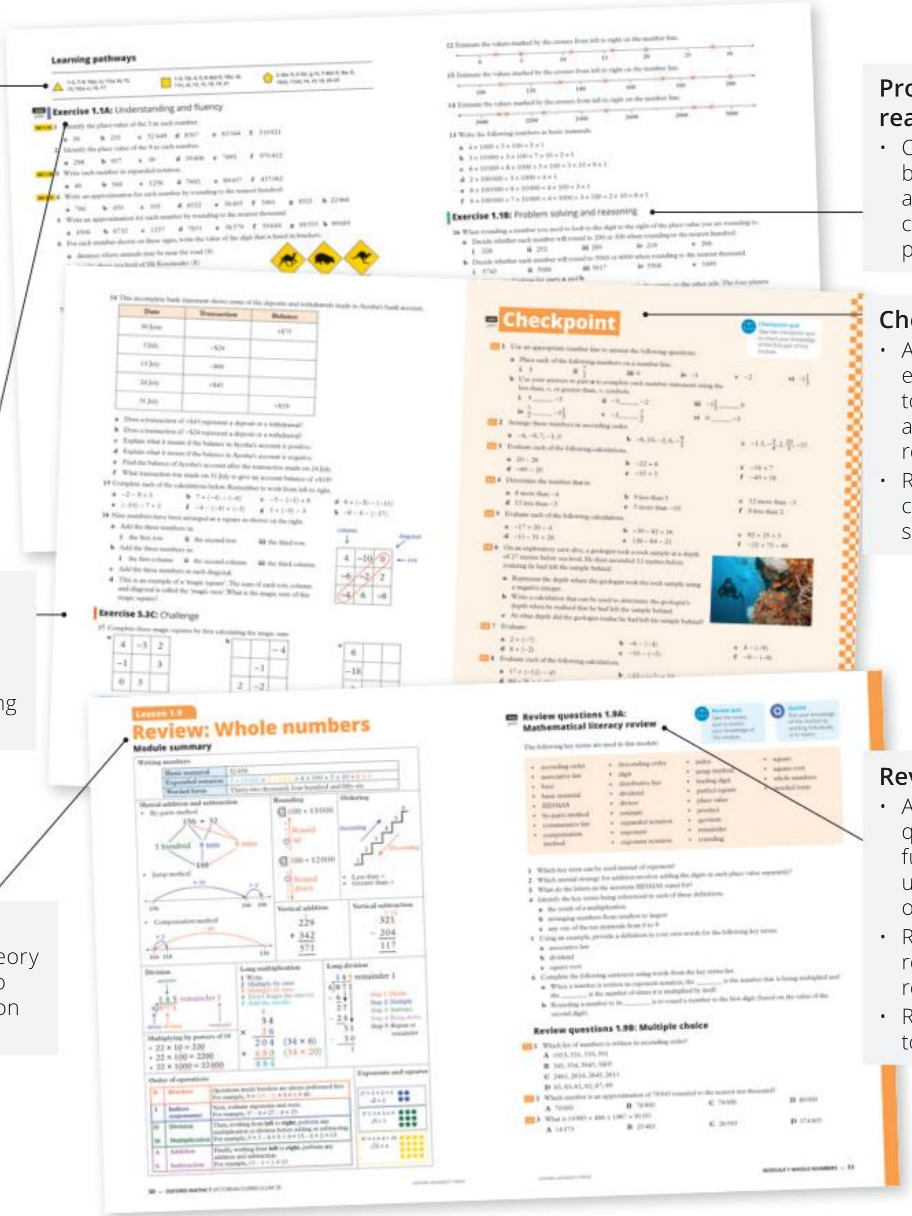
- Basic exercises dedicated to practising key concepts.

Challenge

- Advanced exercises designed to build engagement and anticipate future learning outcomes.

Module summary

- Condenses all the theory from each lesson into one accessible revision page.



Learning pathways

Exercise 1.1A: Understanding and fluency

Exercise 1.1B: Problem solving and reasoning

Checkpoint

Exercise 5.3C: Challenge

Lesson 1.9

Review: Whole numbers

Module summary

Review questions 1.9A: Mathematical literacy review

Review questions 1.9B: Multiple choice

Problem solving and reasoning

- Comprehensive exercises bring together new ideas and provide engaging contexts from real-world problems.

Checkpoint

- A section in the middle of each module dedicated to summarising key skills and encouraging memory retention.
- Reference to an online checkpoint quiz to gauge student progress.

Review questions

- Additional practice questions to further consolidate understanding at the end of each module.
- Reference to an online review quiz to track results.
- Reference to Quizlet test to revise new terminology.

Integrated STEAM projects

- Take the hard work out of cross-curricular learning with engaging STEAM projects. Two fully integrated projects are included at the end of each book in the series, and are scaffolded and mapped to the Science, Maths and Humanities curricula. The same projects also feature in the corresponding Oxford Humanities and Oxford Science series to assist cross-curricular learning.

[STEAM project 1]

How can we reduce contaminants in local waterways so that biodiversity in the area is protected?

HUMANITIES

MATHS

SCIENCE

Problem-solving through design thinking

- Each STEAM project investigates a real-world problem that students are encouraged to problem-solve using design thinking.

Full digital support

- Each STEAM project is supported by a wealth of digital resources, including student booklets (to scaffold students through the design-thinking process of each project), videos to support key concepts and skills, and implementation and assessment advice for teachers.

oxforddigital

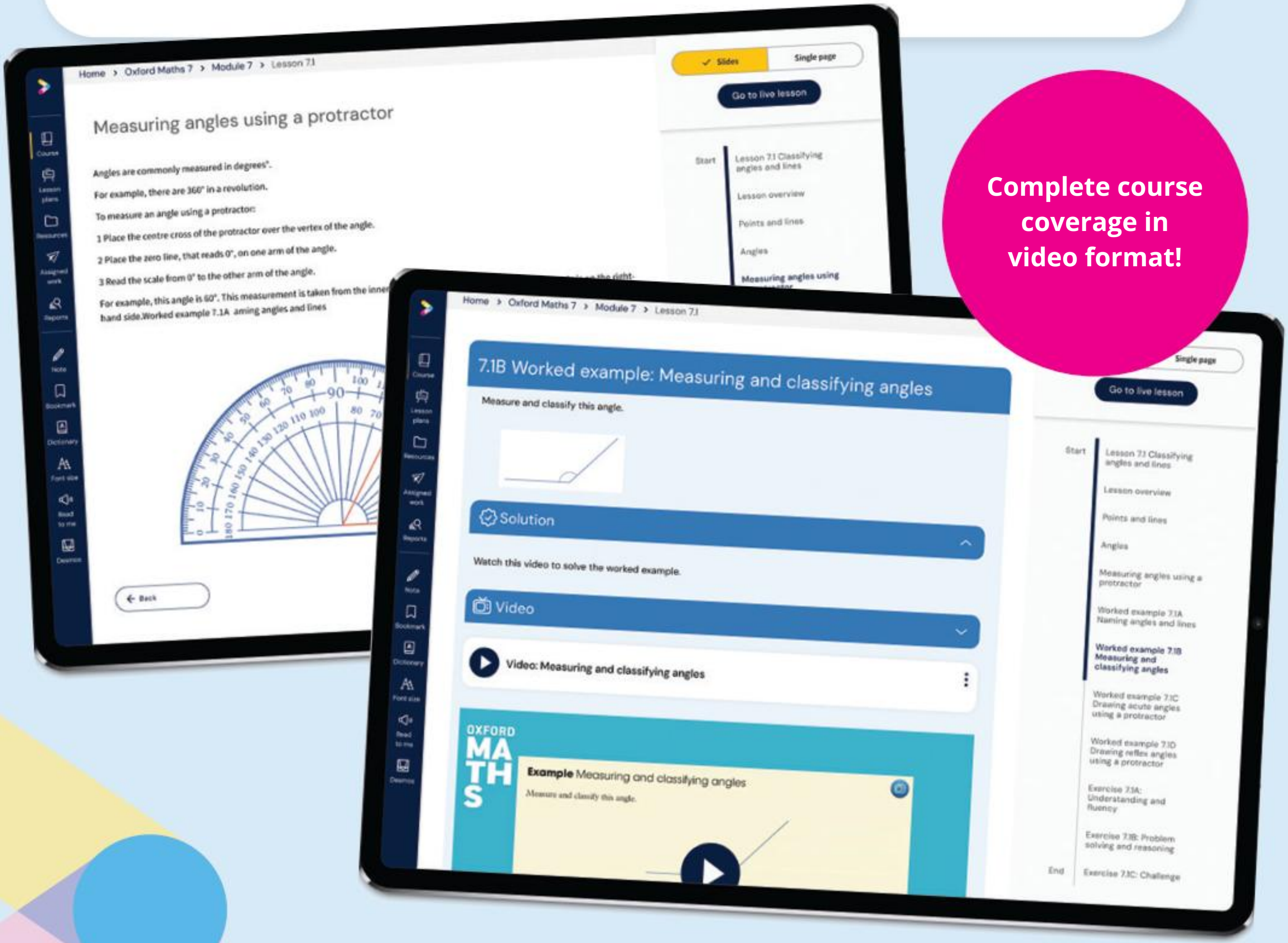
This Student Book is supported by a range of digital resources and features for students and teachers on Oxford Digital.

Key features for students

Student digital access allows you to:

- **view** all the Student Book content in a fully accessible, reflowable format that's delivered in bite-sized chunks so you can work at you own pace
- **highlight**, **take notes**, **bookmark** pages, or define words with the built-in **Australian Oxford Dictionary**
- **watch** hundreds of short **content videos** and **worked example videos** to help you revise anything you don't understand, catch up on things you've missed or complete your homework
- **complete** thousands of interactive questions and quizzes as you work through the content and get the answers and results sent to you instantly
- **use** a suite of **Desmos calculators and graphing tools** to help you complete your work.

Complete course coverage in video format!



Measuring angles using a protractor

Angles are commonly measured in degrees°.

For example, there are 360° in a revolution.

To measure an angle using a protractor:

1 Place the centre cross of the protractor over the vertex of the angle.

2 Place the zero line, that reads 0°, on one arm of the angle.

3 Read the scale from 0° to the other arm of the angle.

For example, this angle is 60°. This measurement is taken from the inne

hand side.Worked example 7.1A aming angles and lines

7.1B Worked example: Measuring and classifying angles

Measure and classify this angle.

Solution

Watch this video to solve the worked example.

Video

Video: Measuring and classifying angles

Example Measuring and classifying angles

Measure and classify this angle.

- Lesson 7.1 Classifying angles and lines
- Lesson overview
- Points and lines
- Angles
- Measuring angles using a protractor
- Worked example 7.1A Naming angles and lines
- **Worked example 7.1B Measuring and classifying angles**
- Worked example 7.1C Drawing acute angles using a protractor
- Worked example 7.1D Drawing reflex angles using a protractor
- Exercise 7.1A: Understanding and fluency
- Exercise 7.1B: Problem solving and reasoning
- Exercise 7.1C: Challenge

Key features for teachers

Teacher digital access allows you to:

- **elevate** your teaching and reduce planning and preparation time with **Live Lesson mode**. This Premium feature is an Australian first that lets you upgrade from traditional print-based lesson plans to **fully integrated, perfectly sequenced and timed interactive lessons complete with classroom activities** that drive engagement and participation
- **personalise** learning for every student and **differentiate** content based on student strengths and weaknesses. Assign support or extension resources to any student in any year level using **inter-year links**
- **identify gaps in understanding** with diagnostic quizzes that generate personalised learning pathway reports for every student based on ability. Learning pathway reports provide clear learning goals and exercises for every lesson including interactive resources designed to target misconceptions and give each student what they need to address them!
- **begin every lesson** with ready-made learning intentions and success criteria
- **revolutionise** your planning, marking and reporting with powerful analytics on student performance and progress:
  - **Assessment report** shows how students are performing in each online interactive assessment, providing instant feedback for teachers about areas of understanding
  - **Curriculum report** summarises student performance against specific curriculum content descriptors and curriculum codes
  - **Skills report** displays students' understanding of prerequisite skills in mathematics.

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Quizlet

NAPLAN-style practice assessments

NAPLAN (National Assessment Program – Literacy and Numeracy) is an annual assessment for students in Years 3, 5, 7 and 9. NAPLAN is made up of tests in the areas of:

- reading
- writing
- conventions of language (spelling, grammar and punctuation)
- numeracy.

From 2022, all NAPLAN tests are being conducted online. OUP has developed a number of NAPLAN-style practice assessments* for this title, which can be accessed on Oxford Digital.

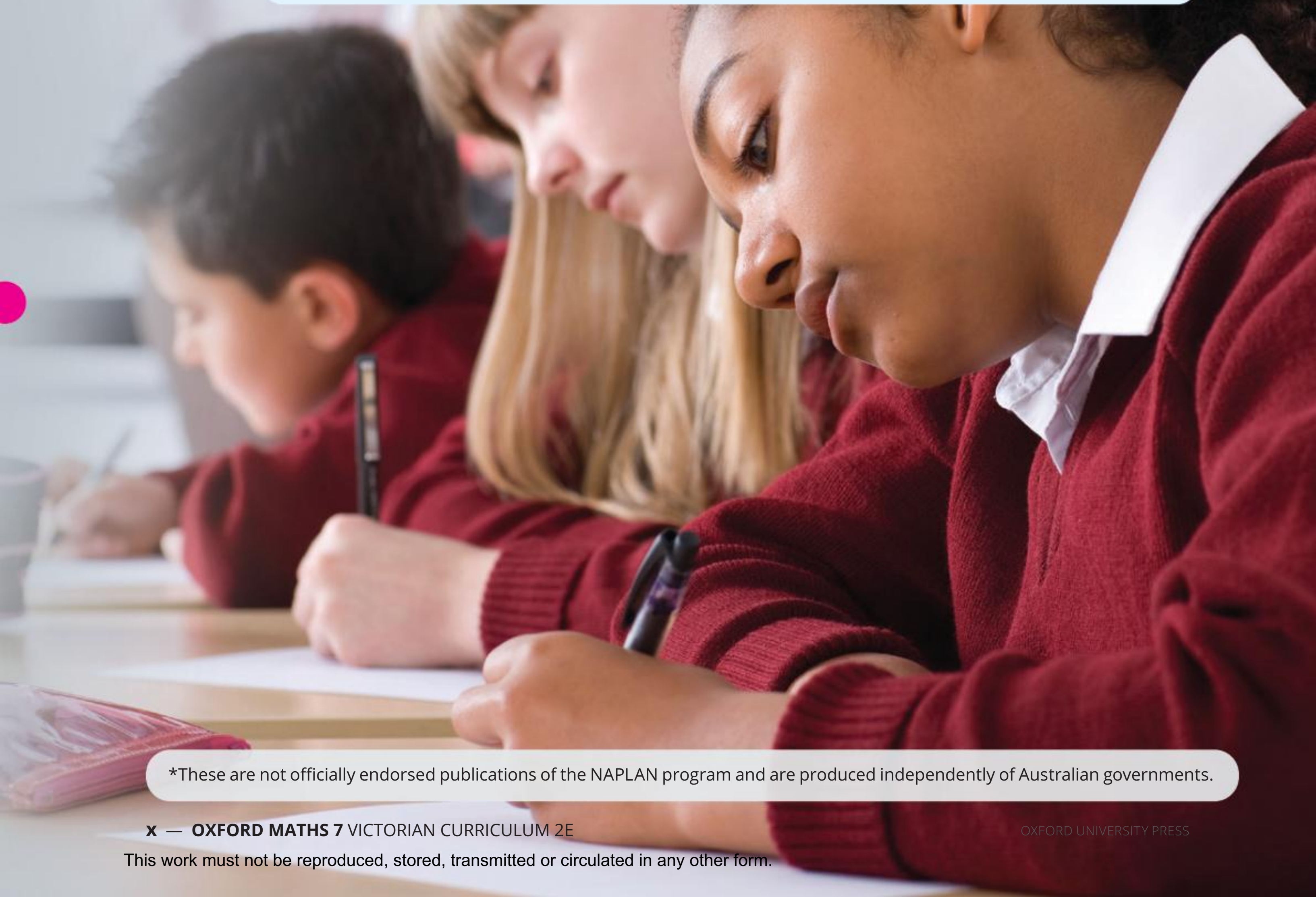
Access the practice tests on oxforddigital

Practice test 1
Non-calculator

Practice test 2
Non-calculator

Practice test 1
Calculator

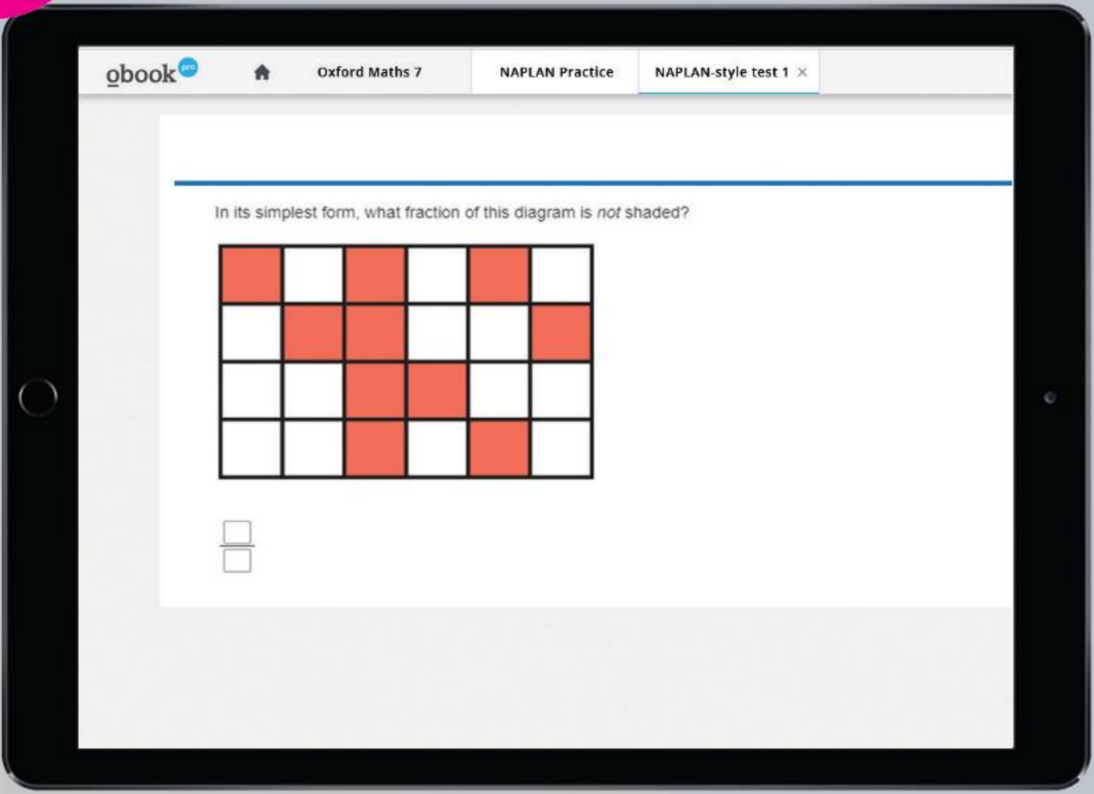
Practice test 2
Calculator



*These are not officially endorsed publications of the NAPLAN program and are produced independently of Australian governments.

Online practice

There are four online practice assessments available on Oxford Digital.

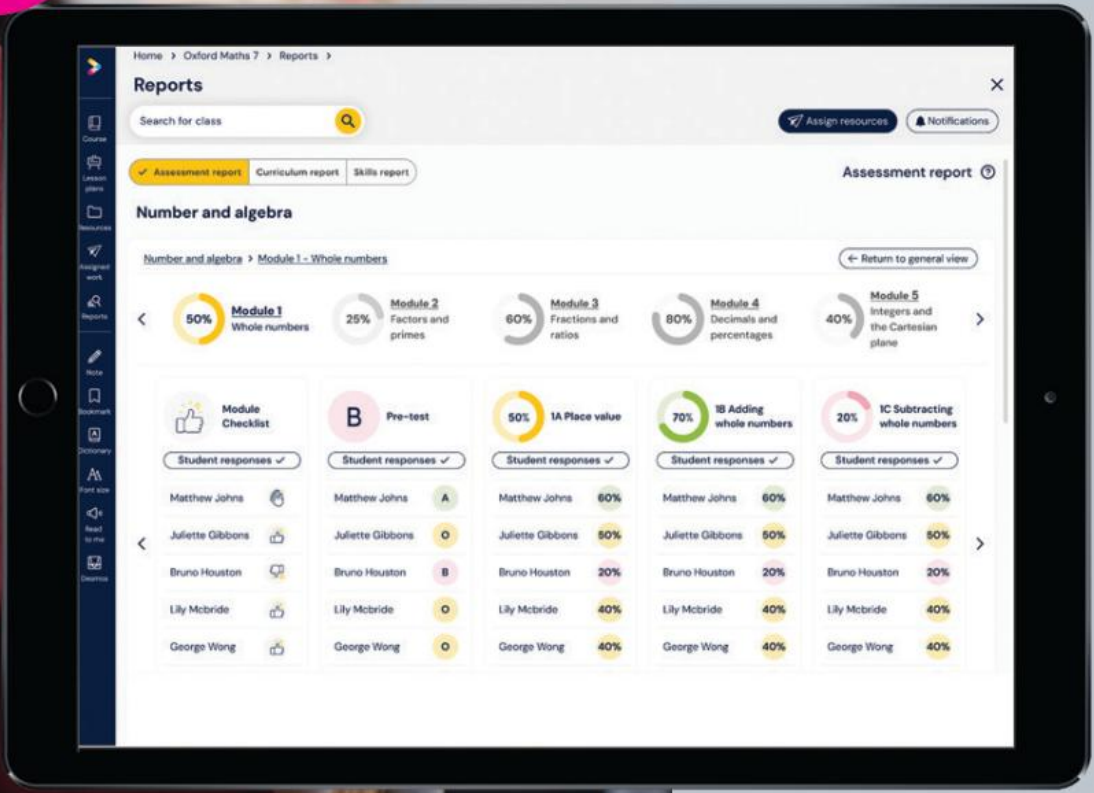


In its simplest form, what fraction of this diagram is *not* shaded?

☐
☐

Reporting

The results from your practice assessments can be found under the Reports tab on Oxford Digital.



Reports

Assessment report

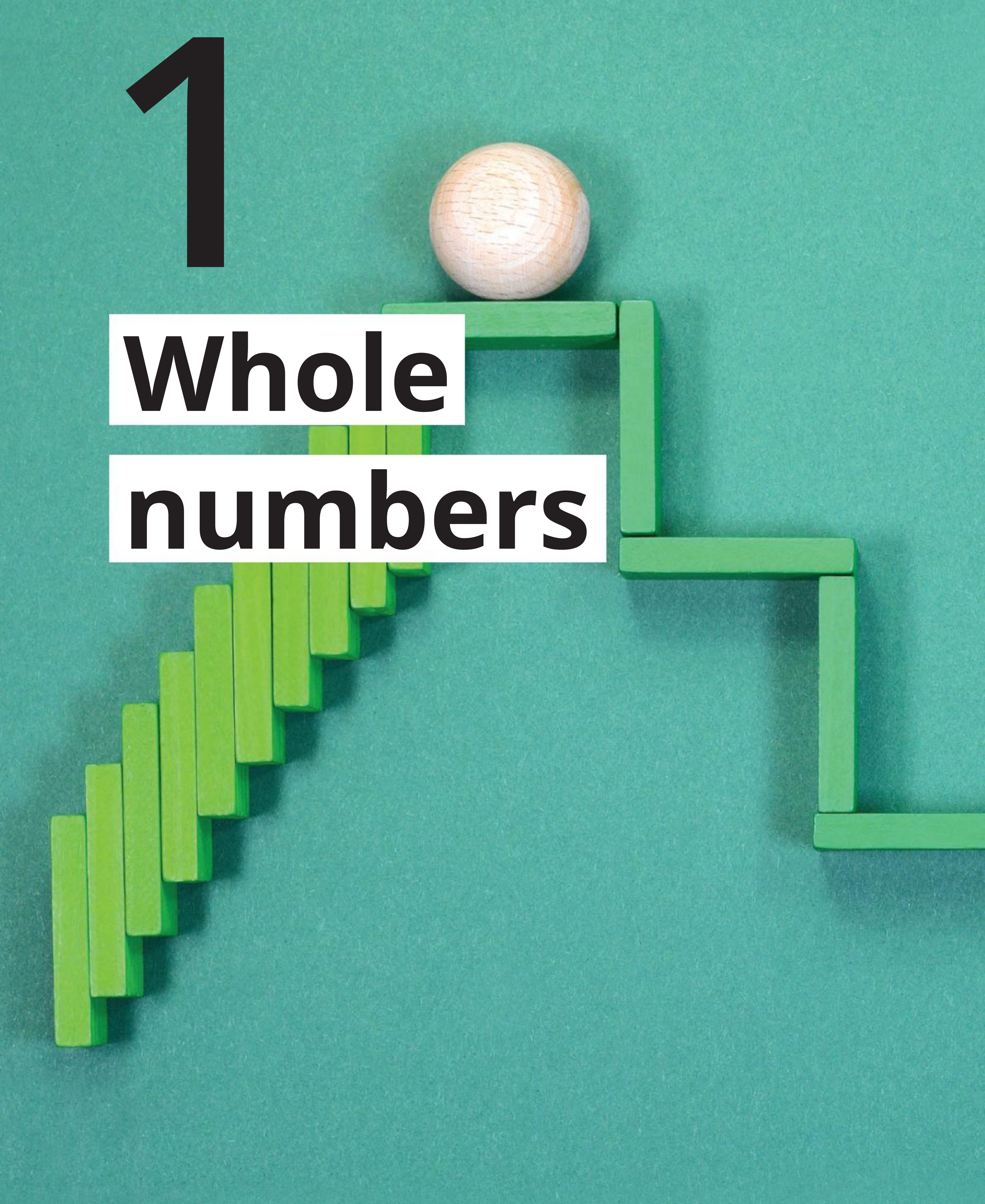
Number and algebra

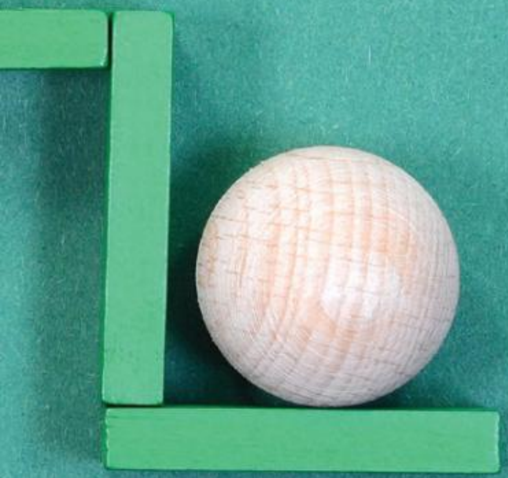
50% Module 1 Whole numbers	25% Module 2 Factors and primes	60% Module 3 Fractions and ratios	80% Module 4 Decimals and percentages	40% Module 5 Integers and the Cartesian plane

	Module Checklist	B Pre-test	50% 1A Place value	70% 1B Adding whole numbers	20% 1C Subtracting whole numbers
	Student responses ✓	Student responses ✓	Student responses ✓	Student responses ✓	Student responses ✓
Matthew Johns	✋	A	60%	60%	60%
Juliette Gibbons	👍	O	50%	50%	50%
Bruno Houston	👎	B	20%	20%	20%
Lily Mcbride	👍	O	40%	40%	40%
George Wong	👍	O	40%	40%	40%

1

Whole numbers





Lessons

Prerequisite skills

Diagnostic pre-test
Take the diagnostic pre-test to assess your knowledge of the prerequisite skills listed below.

Interactive skillsheets
After completing the diagnostic pre-test, brush up on your knowledge of the prerequisite skills by using the interactive skillsheets.

- ✔ Counting in multiples
- ✔ Ordering and comparing large numbers
- ✔ Adding and subtracting two-digit numbers
- ✔ Times tables
- ✔ Number lines

Curriculum links

- Describe the relationship between perfect square numbers and square roots, and use squares of numbers and square roots of perfect square numbers to solve problems (VC2M7N01)
- Represent natural numbers in expanded notation using powers of 10, and as products of powers of prime numbers using exponent notation (VC2M7N02)
- Compare, order and solve problems involving addition and subtraction of integers (VC2M7N08)

© VCAA

Materials

✔ Calculator

Lesson 1.1

Place value

Learning intentions

By the end of this lesson you will be able to ...

✔ read, write, compare and round whole numbers.

Inter-year links

Support	Place value
Year 8	1.1 Rounding and estimating
Year 9	1.5 Scientific notation

Whole numbers

- **Whole numbers** are the numbers we count with, plus 0. The list of whole numbers starts 0, 1, 2, 3, 4, ... and continues indefinitely.
- The numeral representing each whole number is made up of one or more **digits**. There are 10 digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.
- The value of each digit depends on the place or position of the digit in the number.

Place value charts

Key content video
Place value

- **Place value** charts can be used to determine the value of each digit in a number.

Millions	Hundred thousands	Ten thousands	Thousands	Hundreds	Tens	Ones
3	2	8	7	2	6	3

The place value of 3 is 3 000 000
The place value of 2 is 200 000
The place value of 8 is 80 000
The place value of 7 is 7000
The place value of 2 is 200
The place value of 6 is 60
The place value of 3 is 3

Expanded notation and worded form

- A **basic numeral** is the way we normally write numbers.
- **Expanded notation** refers to writing a number by showing the value of each digit. The number is written as the sum of the place values of each digit.
- **Worded form** refers to the way we read numbers out loud.

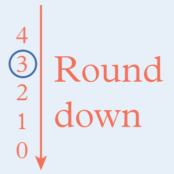
Basic numeral	32 456
Expanded notation	3 × 10 000 + 2 × 1000 + 4 × 100 + 5 × 10 + 6 × 1
Worded form	Thirty-two thousand, four hundred and fifty-six

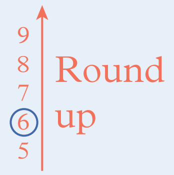
Rounding

Key content video
Rounding and estimating

- **Rounding** a number involves replacing the number with an approximation that is simpler and easier to use in calculations. Rounding makes any calculation less accurate.
- When a number is rounded, the result is an approximation.
- ‘Approximately equal’ is shown using the symbol ‘≈’.
- When rounding to the nearest thousand, place a box around the digit in the thousands place.
  - → If the digit to the right is less than 5, round down.
  - → If the digit to the right is greater than or equal to 5, round up.

$$1\boxed{2}\,300 \approx 12\,000 \qquad 1\boxed{2}\,600 \approx 13\,000$$





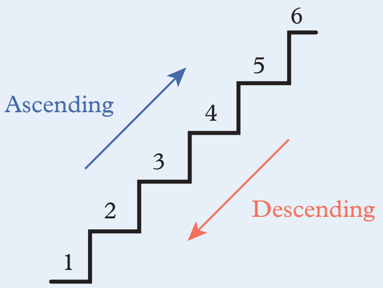
- Rounding to a number’s **leading digit** is to round a number to the first digit (based on the value of the second digit) and to set all the other digits to zero.

Leading digit

$$123\,456 = \boxed{1}23\,456 \approx 100\,000$$

Ordering numbers

- Numbers are in **ascending order** when they are arranged from smallest to largest.
- Numbers are in **descending order** when they are arranged from largest to smallest.



- When comparing two or more numbers, the ‘<’ symbol means ‘less than’, and the ‘>’ symbol means ‘greater than’.
- If a number is to the left of another number on a number line, then it is less than that number.
  - → Likewise, if a number is to the right of another number on a number line, then it is greater than that number.

Estimation

- To **estimate** is to find an approximate value of a calculation, usually involving rounding.
  For example, 1234 + 5678 ≈ 1000 + 6000, so this sum is approximately equal to 7000. The actual result is 6912.
- Estimation is helpful to describe a quantity that is time consuming to count or measure in order to be exact.
  For example, estimating can be useful for determining the size of a large crowd or the area of a room.

Worked example 1.1A Identifying the place value of a digit in a number

Identify the place value of the 5 in each number.

a 357 **b** 58 032

THINK

1 Show the numbers in a place value chart. Remember to include the zeros in the chart.

2 Consider the place value of the 5.

WRITE

a

Ten thousands **10 000**	**Thousands** **1000**	**Hundreds** **100**	**Tens** **10**	**Ones** **1**
		3	5	7

The 5 is in the tens column, so its place value is 50.

b

Ten thousands **10 000**	**Thousands** **1000**	**Hundreds** **100**	**Tens** **10**	**Ones** **1**
5	8	0	3	2

The 5 is in the ten thousands column, so its place value is 50 000.

Worked example 1.1B Writing numbers in expanded notation

Write each number in expanded notation.

a 369 **b** 28 104 **c** 5070

THINK

1 Show the numbers in a place value chart. Remember to include the zeros in the chart.

2 Write each number in expanded notation by showing the value of each digit.

WRITE

Ten thousands **10 000**	**Thousands** **1000**	**Hundreds** **100**	**Tens** **10**	**Ones** **1**
		3	6	9
2	8	1	0	4
	5	0	7	0

a $369 = 3 \times 100 + 6 \times 10 + 9 \times 1$

b $28\,104 = 2 \times 10\,000 + 8 \times 1000 + 1 \times 100 + 4 \times 1$

c $5070 = 5 \times 1000 + 7 \times 10$

Worked example 1.1C Rounding a number

Write an approximation for each number by rounding to the nearest hundred.

a 719 **b** 4482 **c** 990

THINK	**WRITE**
a **1** Put a box around the digit in the hundreds place.	**a** $\boxed{7}19$
2 Look to the right of the boxed digit. The next digit is less than 5, so do not change the boxed digit.	$\boxed{7}19$
3 Replace all digits to the right of the boxed digit with a zero.	$\boxed{7}00$ $719 \approx 700$
b **1** Put a box around the digit in the hundreds place.	**b** $4\boxed{4}82$
2 Look to the right of the boxed digit. The next digit is greater than 5, so add one to the digit in the box.	$4\boxed{4}82$
3 All digits to the left of the boxed digit stay the same. Replace all digits to the right of the boxed digit with a zero.	$4\boxed{5}00$ $4482 \approx 4500$
c **1** Put a box around the digit in the hundreds place.	**c** $\boxed{9}90$
2 Look to the right of the boxed digit. The next digit is greater than 5, so add one to the digit in the box. The boxed digit changes from 9 to 10, so write zero in the boxed digit's place and add one to the digit to the left.	$\boxed{9}90$
3 Replace all digits to the right of the boxed digit with a zero.	$1\boxed{0}00$ $990 \approx 1000$

Helpful hints

- ✔ The digit 0 (zero) is important. It shows that there is nothing in that place value position and it keeps all other digits in the correct places.
- ✔ Remember that the place of a digit in a numeral indicates the digit's value.
  For example, the 4 in 3452 represents 400 instead of 4.
- ✔ Be careful when rounding to a specific place value. If you draw a square around the wrong place, your answer will not be correct. Use a place value chart to double check if you are unsure.

Ten thousands	**Thousands**	**Hundreds**	**Tens**	**Ones**
10 000	**1000**	**100**	**10**	**1**

Learning pathways

△ 1–5, 7–9, 10(a, c), 11(a, b), 12, 13, 15(a–c), 16, 17

■ 1–6, 7(b, d, f), 8–9(d–f), 10(c, d), 11(c, d), 13, 15, 18, 19, 21

⬟ 2–3(e, f), 4–5(c, g, h), 7–8(d–f), 9(e, f), 10(d), 11(d), 14, 15, 18, 20–23

ANS p496

Exercise 1.1A: Understanding and fluency

WE 1.1A **1** Identify the place value of the 5 in each number.

a 56 **b** 235 **c** 52 649 **d** 8507 **e** 65 344 **f** 516 921

2 Identify the place value of the 9 in each number.

a 298 **b** 957 **c** 39 **d** 59 406 **e** 7891 **f** 970 412

WE 1.1B **3** Write each number in expanded notation.

a 46 **b** 568 **c** 1256 **d** 7692 **e** 89 457 **f** 457 082

WE 1.1C **4** Write an approximation for each number by rounding to the nearest hundred.

a 780 **b** 450 **c** 103 **d** 6522 **e** 38 405 **f** 5991 **g** 8521 **h** 22 968

5 Write an approximation for each number by rounding to the nearest thousand.

a 4506 **b** 8732 **c** 1257 **d** 7851 **e** 36 579 **f** 50 644 **g** 89 555 **h** 99 689

6 For each number shown on these signs, write the value of the digit that is listed in brackets.

a distance where animals may be near the road (9)

b height above sea level of Mt Kosciuszko (8)

c distance to:

- **i** Alice Springs (1)
- **ii** Tennant Creek (5)
- **iii** Darwin (1)

7 Write these numbers as basic numerals.

a sixty-two thousand

b nine hundred and seventy-eight

c three hundred and four

d two hundred and fifty thousand, one hundred and twelve

e twelve thousand, five hundred and forty-three

f nine thousand and twenty-six

8 Write these numbers in worded form and expanded notation.

a 362 **b** 7215 **c** 45 733 **d** 234 601 **e** 6 420 058 **f** 55 555

9 Complete the following number sentences by writing < or > in the space provided.

a 8530 ________ 7503 **b** 46 249 ________ 64 249 **c** 317 294 ________ 37 294

d 709 ________ 4503 **e** 5678 ________ 5876 **f** 10 462 ________ 10 248

10 Write each list of numbers in ascending order (from smallest to largest).

a 58, 72, 9, 40, 88, 15, 28 **b** 856, 805, 890, 806, 846

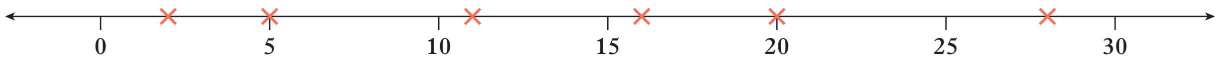
c 625, 9472, 6105, 10 417, 9902 **d** 2374, 23 074, 23 704, 234, 2347

11 Write each list of numbers in descending order (from largest to smallest).

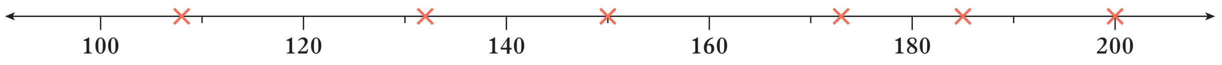
a 870, 8000, 87, 1800, 807 **b** 3999, 3909, 399, 309, 3099

c 72 156, 75 126, 75 561, 75 516 **d** 2 567 291, 256 291, 1 967 219

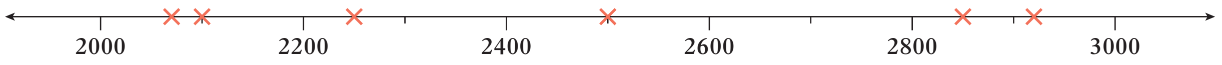
12 Estimate the values marked by the crosses from left to right on the number line.



13 Estimate the values marked by the crosses from left to right on the number line.



14 Estimate the values marked by the crosses from left to right on the number line.



15 Write the following numbers as basic numerals.

a 4 × 1000 + 5 × 100 + 3 × 1

b 3 × 10 000 + 3 × 100 + 7 × 10 + 2 × 1

c 6 × 10 000 + 8 × 1000 + 5 × 100 + 3 × 10 + 9 × 1

d 2 × 100 000 + 3 × 1000 + 4 × 1

e 8 × 100 000 + 9 × 10 000 + 4 × 100 + 3 × 1

f 6 × 100 000 + 7 × 10 000 + 4 × 1000 + 3 × 100 + 2 × 10 + 6 × 1

Exercise 1.1B: Problem solving and reasoning

16 When rounding a number you need to look to the digit to the right of the place value you are rounding to.

a Decide whether each number will round to 200 or 300 when rounding to the nearest hundred.

i 228 **ii** 252 **iii** 280 **iv** 219 **v** 266

b Decide whether each number will round to 5000 or 6000 when rounding to the nearest thousand.

i 5743 **ii** 5086 **iii** 5617 **iv** 5508 **v** 5499

c Explain your thinking for parts **a** and **b**.

17 The diameter of a planet is the distance from one side, through the centre, to the other side. The four planets closest to the Sun are shown below with their diameters.


Mercury
4878 km


Venus
12 104 km


Earth
12 756 km


Mars
6794 km

a Write their diameters as approximate values by rounding to the nearest thousand.

b Draw a number line and mark these approximate values on it. Label each value with the name of the planet.

c Use your approximate values to write a sentence comparing the sizes of the planets.

18 Consider the image on the right which labels the diameter of the Moon.

a Write these measurements as approximate values.

i the diameter of the Moon, by rounding to the nearest thousand

ii the distance from Earth to the Moon, if the distance between them is 334 402 km, by rounding to the nearest hundred thousand

b Is the distance from Earth to the Moon approximately 10, 100 or 1000 times greater than the diameter of the Moon? Explain your thinking.

19 Every whole number consists of one or more digits.

- **a** How many different two-digit numbers can you make from 3 and 5 if you cannot repeat digits? List them in ascending order.
- **b** How many different two-digit numbers can you make from 3 and 5 if you can repeat digits? List them in descending order.

20 Every whole number consists of one or more digits.

- **a** How many different two-digit numbers can you make from 2, 4 and 7 if you cannot repeat digits? List them in descending order.
- **b** How many different two-digit numbers can you make from 2, 4 and 7 if you can repeat digits? List them in ascending order.

Exercise 1.1C: Challenge

21 Consider the digits 1, 3, 6 and 9. Use these digits to write:

- **a** the largest four-digit number without repeating any digits
- **b** the smallest four-digit number if digits can be repeated
- **c** the largest even number without repeating any digits
- **d** all the four-digit numbers between 3620 and 6350 if no digits can be repeated.

22 Consider the place value chart. The digit in the thousands column is currently missing.

Millions	Hundred thousands	Ten thousands	Thousands	Hundreds	Tens	Ones
1	2	3	*	0	2	9

- **a** When rounded to the nearest hundred thousand, what will the number be?
- **b** What digit, or digits, placed in the thousands column will give the number 1 230 000 when rounded to the nearest ten thousand?
- **c** Suppose the digit in the thousands column is 9, and 760 000 is added to the number in the table. Determine the result rounded to the nearest ten thousand.
- **d** Suppose the digit in the thousands column is 9, and 760 000 is subtracted from the number in the table. Determine the result rounded to the nearest ten thousand.

23 A multi-billionaire designed a special new lock for his safe. The lock is reset to a different five number code every two minutes. To crack the code, you must round each number so that the sum of the digits is closest to 20.

For example, 48 922 would be rounded to 48 900 as $4 + 8 + 9 = 21$, and rounding to the nearest hundred in this case gives the closest digit sum to 20.

Can you crack the code in the given time limit of two minutes?

Online resources:

- **Interactive skillsheet** Place value
- **Interactive skillsheet** Rounding whole numbers
- **Worksheet** Place value
- **Investigation** Roman numerals
- **Quick quiz** 1.1

Lesson 1.2

Adding whole numbers

Learning intentions

By the end of this lesson you will be able to ...

✔ add whole numbers using mental strategies and the vertical algorithm.

Inter-year links

Support	Adding whole numbers
Year 8	1.2 Adding and subtracting whole numbers

Key content video
Adding whole numbers

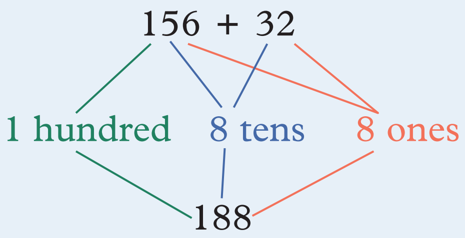
Addition properties

- The **associative law** of addition states that regardless of how numbers are grouped, the answer does not change.
  For example, $(2 + 4) + 6 = 2 + (4 + 6)$.
- The **commutative law** of addition states that the order in which numbers are added does not change the answer.
  For example, $2 + 4 = 4 + 2$.

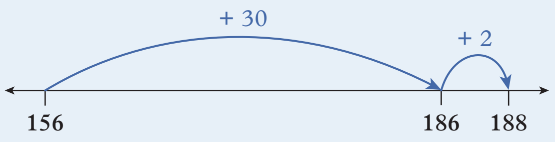
Addition (+)
Sum
Plus
More than
Increase
Total
Together

Mental addition strategies

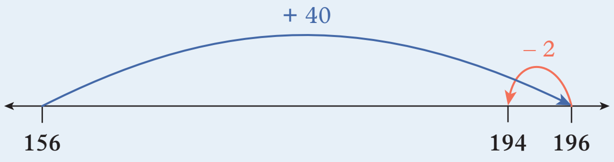
- The **by-parts method** involves adding the digits in each place value separately.
  For example, $156 + 32 = 100 + (50 + 30) + (6 + 2)$
  $= 188$.



- The **jump method** involves breaking down one of the given numbers, then adding each part of the number in stages.
  For example, $156 + 32 = 156 + 30 + 2$
  $= 188$.



- The **compensation method** involves rounding one number to make the calculations easier. The amount required to round the number is then added or subtracted, depending on if the number was rounded up or down.
  For example, $156 + 38 = 156 + 40 - 2$
  $= 194$.



Vertical addition algorithm

- In the vertical addition algorithm, numbers are written one under the other with the digits lined up according to their place value.
  - → Always start by adding the digits in the ones column first, followed by the tens column, then the hundreds column and so on.

$$\begin{array}{r} {}^{1} \\ 229 \\ +\ 342 \\ \hline 571 \\ \hline \end{array} \longleftarrow \text{answer}$$

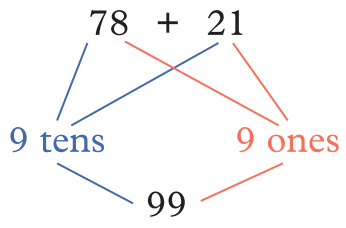
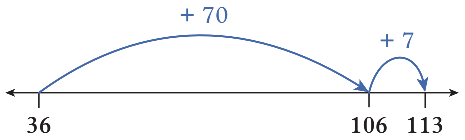
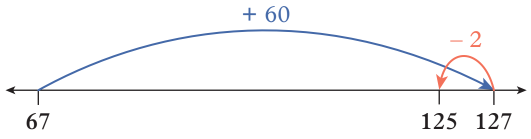
Ones column: 9 + 2 = 11, carry the 1
Tens column: 2 + 4 + (1) = 7
Hundreds column: 2 + 3 = 5

- Regrouping occurs when there is a result of 10 or more in one column.
  - → For example, 11 is the same as 1 ten and 1 one, so a result of 11 is the same as placing 1 in the tens column and 1 in the ones column. The 1 ten is represented by the orange 1 as it has been regrouped from the ones column.

Worked example 1.2A Using mental strategies to add two numbers

Work out each sum using the mental strategy given in brackets.

a 78 + 21 (by-parts method)
b 36 + 77 (jump method)
c 67 + 58 (compensation method)

THINK	**WRITE**
a **1** Add the tens together. **2** Add the ones together. **3** Combine the two answers to get the final answer.	**a**  78 + 21 = 99
b **1** Choose a starting number, then break down the second number into tens and ones. **2** Add the tens to the starting number followed by the ones.	**b**  36 + 77 = 113
c **1** Choose a starting number, then round the second number to the nearest 10. **2** Add the rounded number to the starting number, then add or subtract the amount needed to round the second number.	**c**  67 + 58 = 125

Worked example 1.2B Using vertical addition to add two numbers

Calculate 634 + 891 using the vertical addition algorithm.

$$\begin{array}{r} 634 \\ +\ \underline{891} \\ \hline \end{array}$$

THINK

1 Add the digits in the ones column: 4 + 1 = 5.

2 Add the digits in the tens column: 3 + 9 = 12.

Write 2 in the tens column of the answer and write 1 above the hundreds column.

3 Add the digits in the hundreds column: 1 + 6 + 8 = 15.

WRITE

$$\begin{array}{r} \overset{1}{6}34 \\ +\ 891 \\ \hline 1525 \\ \hline \end{array}$$

Worked example 1.2C Using vertical addition to add three numbers

Use the vertical addition algorithm to calculate 3108 + 547 + 1619.

THINK

1 Set out the addition problem by lining up digits according to place value.

2 Add the digits in the ones column: 8 + 7 + 9 = 24.

Write 4 in the ones column of the answer and write 2 above the tens column.

3 Add the digits in the tens column: 2 + 0 + 4 + 1 = 7.

4 Add the digits in the hundreds column: 1 + 5 + 6 = 12.

Write 2 in the hundreds column of the answer and write 1 above the thousands column.

5 Add the digits in the thousands column: 1 + 3 + 0 + 1 = 5.

WRITE

$$\begin{array}{r} \overset{1}{3}1\overset{2}{0}8 \\ 547 \\ +\ 1619 \\ \hline 5274 \\ \hline \end{array}$$

Helpful hints

- ✔ Don't forget that addition is commutative – reordering sums can make them a lot easier! For example, 12 + 123 455 = 123 455 + 12.
- ✔ Make sure that your place value columns are aligned so that you add together the correct digits.
- ✔ When you are adding multiple large numbers together, it can be helpful to estimate your answer using rounding to ensure you have the right answer. You can compare your two results and easily see if you have made a mistake.

Learning pathways

△ 1–12, 14, 16, 20(a)

■ 1–8, 13, 15, 17, 18

⬠ 1(d), 2(d), 3(d), 4(g, h), 7(e–h), 8(d–f), 9–12, 17–20

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Exercise 1.2A: Understanding and fluency

WE 1.2A **1** Use the by-parts method to work out each sum. Draw the corresponding diagram to support your answer.

a 458 + 102 **b** 75 + 23 **c** 34 + 57 **d** 49 + 810

2 Use the jump method to work out each sum. Draw the corresponding diagram to support your answer.

a 103 + 46 **b** 112 + 13 **c** 21 + 94 **d** 66 + 22

3 Use the compensation method to work out each sum. Draw the corresponding diagram to support your answer.

a 48 + 56 **b** 274 + 27 **c** 26 + 58 **d** 79 + 89

4 Use any mental strategy to work out each sum.

a 50 + 30 **b** 120 + 60 **c** 360 + 20 **d** 25 + 13

e 16 + 61 **f** 56 + 34 **g** 65 + 35 **h** 123 + 345

5 Use any mental strategy to work out each sum.

a 20 + 40 + 10 **b** 500 + 100 + 300 **c** 18 + 12 + 50 **d** 73 + 7 + 20

e 52 + 11 + 27 **f** 35 + 29 + 35 **g** 115 + 205 + 80 **h** 203 + 203 + 203

WE 1.2B **6** Calculate each sum using the vertical addition algorithm.

a $\begin{array}{r} 243 \\ +\ 715 \\ \hline \\ \hline \end{array}$ **b** $\begin{array}{r} 538 \\ +\ 154 \\ \hline \\ \hline \end{array}$ **c** $\begin{array}{r} 758 \\ +\ 461 \\ \hline \\ \hline \end{array}$ **d** $\begin{array}{r} 697 \\ +\ 805 \\ \hline \\ \hline \end{array}$

7 Use the vertical addition algorithm to calculate each of the following.

a 641 + 478 **b** 157 + 296 **c** 2438 + 5160 **d** 3762 + 1489

e 2175 + 485 **f** 96 + 5743 **g** 16 407 + 782 **h** 8009 + 35 714

WE 1.2C **8** Use the vertical addition algorithm to calculate each of the following.

a 385 + 461 + 723 **b** 749 + 218 + 837 **c** 927 + 56 + 614

d 869 + 21 + 70 **e** 2764 + 8293 + 451 **f** 7605 + 246 + 38

9 Evaluate the following without using a calculator.

a 23 748 + 61 392 + 53 709 **b** 82 407 + 9385 + 411

c 34 715 + 306 937 + 8256 **d** 947 + 600 411 + 85 103

10 Evaluate the following without using a calculator.

a 28 + 7640 + 459 + 7 + 834 + 2406 **b** 92 762 + 547 + 85 + 2942 + 6073

11 Check your answers to questions **9** and **10** with a calculator.

Exercise 1.2B: Problem solving and reasoning

12 You decide to train for a local cycling race. On the first weekend you cycle 32 km, on the second you cycle 45 km and on the third you cycle 59 km. To find the total distance covered over the three weekends, you can write the calculation in a number of ways.

a Decide if the following statements are true or false.

i 32 + 45 + 59 = (3 + 4 + 5) + (2 + 5 + 9)

ii 32 + 45 + 59 = 30 + 40 + 60 + 2 + 5 − 1

iii 32 + 45 + 59 = 59 + (30 + 40) + (2 + 5)

b List two other ways to write the calculation. **c** Calculate the total distance covered.

13 There are many different ways of adding whole numbers to reach a given target.

a Identify two two-digit numbers that add to 53.

b Identify two three-digit numbers that add to 386 and meet these conditions.

i both numbers are odd

ii both numbers are even

c Identify three four-digit whole numbers that add to 5207. Suggest another set of three four-digit numbers that add to the same total.

14 Riley is going to make enchiladas for his family for dinner. The recipe says that it will take 20 minutes preparation time and 55 minutes baking time.

a How long will it take for Riley to have dinner ready?

b Riley starts preparing the meal at 5:15 pm. What time will it be ready for the family to eat?

15 The Tour de France is an annual international cycling race that finishes in Paris, France. In 2011, Cadel Evans became the first Australian to win the race, following the itinerary shown below.

Date in 2011	Start and finish	Distance
2 July	Passage du Gois → Mont des Alouettes	192 km
3 July	Les Essarts → Les Essarts (team time trial)	23 km
4 July	Olonne-sur-Mer → Redon	198 km
5 July	Lorient → Mûr-de-Bretagne	172 km
6 July	Carhaix → Cap Fréhel	165 km
7 July	Dinan → Lisieux	226 km
8 July	Le Mans → Châteauroux	218 km
9 July	Aigurande → Super-Besse Sancy	189 km
10 July	Issoire → Saint-Flour	208 km
11 July	Le Lioran Cantal	rest
12 July	Aurillac → Carmaux	158 km
13 July	Blaye-les-Mines → Lavaur	168 km
14 July	Cugnaux → Luz-Ardiden	211 km
15 July	Pau → Lourdes	152 km
16 July	Saint-Gaudens → Plateau de Beille	169 km
17 July	Limoux → Montpellier	192 km
18 July	Département de la Drôme	rest
19 July	Saint-Paul-Trois-Châteaux → Gap	163 km
20 July	Gap → Pinerolo	179 km
21 July	Pinerolo → Galibier Serre-Chevalier	200 km
22 July	Modane Valfréjus → Alpe-d'Huez	110 km
23 July	Grenoble → Grenoble (time trial)	42 km
24 July	Créteil → Paris Champs-Élysées	95 km

a Answer these questions without using a calculator.

i How far did the competitors ride in the first five days of the race?

ii How far did the competitors ride before their first rest day?

iii How far did the cyclists ride between leaving Gap and finishing in Paris?

iv What was the total distance covered in the Tour de France?

b Check your answers using a calculator.

c Research the latest Tour de France. What was the total distance covered?

16 To enter the stadium for a soccer match, patrons used one of four gates.

a Estimate the number of people who attended the match by first rounding the numbers at each gate to the nearest thousand.

Gate	Number of people
A	8759
B	9042
C	10 365
D	11 008

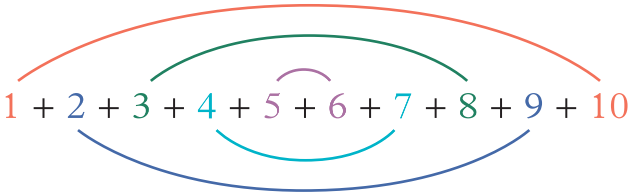
b Calculate the exact number of people who attended the match.

c Which answer would a sports commentator be more likely to use when reporting on the match?

17 Consider adding the numbers from 1 to 10.

a Add the numbers in order: 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 = ?

b This calculation can also be completed by first pairing the numbers. Write the smallest number and the largest number together, then the second smallest and second largest together and so on.



Complete this calculation for adding the numbers from 1 to 10.

(1 + 10) + (2 + 9) + (3 + ___) + (4 + ___) + (5 + ___)
= ___ + ___ + ___ + ___ + ___
= ___

c Which strategy did you find easier? Justify your answer.

18 Use the strategy of grouping numbers in suitable pairs to add the numbers from 1 to 20.

19 Grouping can also be used to find the sum of an odd amount of numbers.

a Use the strategy of grouping numbers in suitable pairs to add these numbers.

i from 1 to 9 **ii** from 1 to 19

b Explain what is different about using this strategy with an odd number of numbers.

c Use a suitable strategy to add these numbers.

i from 2 to 8 **ii** from 3 to 17 **iii** from 5 to 25

Exercise 1.2C: Challenge

20 Consecutive numbers are numbers that follow each other, for example, 3 and 4.

a Identify two consecutive numbers that add to 53.

b Identify three consecutive numbers that add to 114.

c Identify four consecutive numbers that add to 190.

d What strategy did you use to find the sets of consecutive numbers?

Online resources:

Worksheet Adding numbers

Investigation Abracadabra with magic squares

Desmos activity Adding whole numbers

Quick quiz 1.2

Lesson 1.3

Subtracting whole numbers

Learning intentions

By the end of this lesson you will be able to ...

✔ subtract whole numbers using mental strategies and the vertical algorithm.

Inter-year links

Support	Subtracting whole numbers
Year 8	1.2 Adding and subtracting whole numbers

Key content video
Subtracting whole numbers

Subtraction properties

- Subtraction of two or more numbers is not associative. This means that two numbers cannot be grouped and subtracted before subtracting from the other number.

  For example, (8 – 5) – 2 = 1 and 8 – (5 – 2) = 5.
- Subtraction of two or more numbers is not commutative. This means that two or more numbers cannot be subtracted in any order to achieve the same result.

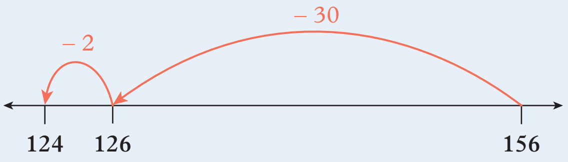
  For example, 7 – 4 is not the same as 4 – 7.

Subtraction (–)
Difference
Take away
Less than
Fewer
Reduce
Minus

Mental subtraction methods

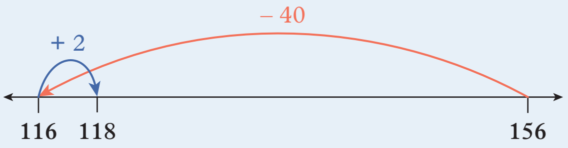
- The jump method and the compensation method can also be applied to subtraction.
  - → Jump method for subtraction

    For example, 156 – 32 = 156 – 30 – 2
    = 124.



  - → Compensation method for subtraction

    For example, 156 – 38 = 156 – 40 + 2
    = 118.



Vertical subtraction algorithm

- In the vertical subtraction algorithm, the number to be subtracted is written under the other number with the digits lined up according to their place value.
  - → If a column is subtracting a larger digit from a smaller digit, regroup 1 from the column on the left as 10 for the column where the subtraction is occurring.
  - → Always start by subtracting the digits in the ones column first, followed by the tens column, then the hundreds column and so on.

$$\begin{array}{r} 3\overset{1}{\cancel{2}}\overset{11}{\cancel{1}} \\ -\ 204 \\ \hline 117 \\ \hline \end{array} \longleftarrow \text{answer}$$

Ones column: Take 10 from the tens column
11 – 4 = 7
Tens column: 1 – 0 = 1
Hundreds column: 3 – 2 = 1

Worked example 1.3A Using mental strategies to subtract two numbers

Work out each difference using the mental strategy given in brackets.

a 81 – 56 (jump method) **b** 78 – 49 (compensation method)

THINK

a **1** Choose a starting number, and then break down the second number into tens and ones.

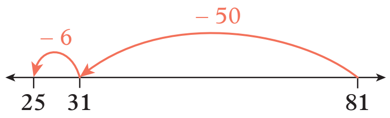
2 Subtract the tens from that number followed by the ones.

b **1** Choose a starting number, then round the second number to the nearest 10.

2 Subtract the rounded number from the starting number, then add or subtract the amount needed to round the second number.

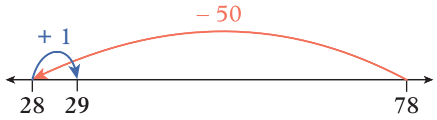
WRITE

a



81 – 56 is 25.

b



78 – 49 is 29.

Worked example 1.3B Using vertical subtraction to subtract two numbers

Calculate 426 – 281 using the vertical subtraction algorithm.

$$\begin{array}{r} 426 \\ -\ 281 \\ \hline \\ \hline \end{array}$$

THINK

1 Subtract the digits in the ones column: 6 – 1 = 5.

2 Subtract the digits in the tens column: 2 – 8.

Take 1 from the hundreds column by reducing the hundreds to 3 and increasing the tens to 12. Subtract the digits in the tens column: 12 – 8 = 4.

3 Subtract the digits in the hundreds column: 3 – 2 = 1.

WRITE

$$\begin{array}{r} \overset{3}{\cancel{4}}\overset{12}{\cancel{2}}6 \\ -\ 281 \\ \hline 145 \\ \hline \end{array}$$

Worked example 1.3C Using vertical subtraction to subtract numbers with different numbers of digits

Use the vertical subtraction algorithm to calculate 788 – 27.

THINK

1 Set out the subtraction problem by lining up digits according to place value.

2 Subtract the digits in the ones column: 8 – 7 = 1.

3 Subtract the digits in the tens column: 8 – 2 = 6.

4 Subtract the digits in the hundreds column: 7 – 0 = 7. Recall that if there is no digit in a place then it has a value of zero.

WRITE

$$\begin{array}{r} 788 \\ -\ \ 27 \\ \hline 761 \\ \hline \end{array}$$

Helpful hints

- ✔ When performing vertical subtraction, make sure that your place value columns are aligned so that you subtract the correct numbers. See the example on the right.
- ✔ Subtraction is not commutative. Unlike with addition problems, you cannot change the order of the numbers around the sign and achieve the same result.
  For example, 12 – 4 is not the same as 4 – 12.
- ✔ Read each question carefully. You may find some subtraction questions easy to calculate in your head. Don't find yourself performing lots of regrouping and end up with an answer as easy to find as 1!
  For example, 101 – 99 requires lots of work if you use the vertical subtraction algorithm, but you might be able to see that the answer is 2 straight away!

$$\begin{array}{r} 654 \\ -\ 321 \\ \hline 333 \\ \hline \end{array}$$

Learning pathways

△ 1–6, 7(d, e, f), 9–12, 14, 17

■ 1–4, 6(a–d), 7–10, 11(a–c), 13–15, 18

⬟ 1(d), 2(d), 4(g, h), 6(g, h), 8, 11, 13, 15, 16, 18, 19

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Exercise 1.3A: Understanding and fluency

WE 1.3A **1** Use the jump method to work out each difference. Draw a corresponding diagram to support your answer.

a 60 – 20 **b** 78 – 26 **c** 65 – 52 **d** 184 – 96

2 Use the compensation method to work out each difference. Draw a corresponding diagram to support your answer.

a 85 – 19 **b** 69 – 28 **c** 129 – 67 **d** 281 – 48

3 Use any mental strategy to work out each difference.

a 60 – 40 **b** 300 – 200 **c** 170 – 30 **d** 250 – 40

e 710 – 20 **f** 178 – 58 **g** 270 – 35 **h** 393 – 281

4 Use any mental strategy to work out each problem.

a 50 + 20 – 10 **b** 700 + 100 – 300 **c** 14 + 16 – 20 **d** 98 + 12 – 1

e 570 + 25 – 80 **f** 36 – 12 + 8 **g** 63 – 41 + 20 **h** 400 – 50 + 85

WE 1.3B **5** Calculate each difference using the vertical subtraction algorithm.

a $\begin{array}{r} 58 \\ -\ 25 \\ \hline \\ \hline \end{array}$ **b** $\begin{array}{r} 87 \\ -\ 46 \\ \hline \\ \hline \end{array}$ **c** $\begin{array}{r} 624 \\ -\ 359 \\ \hline \\ \hline \end{array}$ **d** $\begin{array}{r} 406 \\ -\ 134 \\ \hline \\ \hline \end{array}$

6 Use the vertical subtraction algorithm to calculate each of the following.

a 56 – 14 **b** 81 – 36 **c** 456 – 234 **d** 938 – 651

e 624 – 185 **f** 3846 – 1724 **g** 7508 – 5631 **h** 8135 – 6479

WE 1.3C **7** Use the vertical subtraction algorithm to calculate each of the following.

a 6327 – 215 **b** 5962 – 647 **c** 2475 – 728

d 7836 – 908 **e** 4025 – 462 **f** 5002 – 431

8 Determine the following differences without using a calculator.

a 36 274 – 28 093 **b** 508 246 – 137 651

c 211 537 – 36 409 **d** 81 752 – 8362

9 Evaluate the following without using a calculator.

a 58 945 – 45 120 – 1205 **b** 101 101 – 45 751 – 458 – 2378

10 Check your answers to question **9** with a calculator.

11 For each calculation:

i estimate the answer by first rounding each number to its leading digit

ii use pen and paper to work out the exact answer

iii use a calculator to check the result you obtained for part **ii**.

a 61 + 48 – 77 **b** 254 – 123 + 448

c 4708 – 369 + 532 **d** 29 071 + 8275 – 17 466

e 77 – 25 + 89 – 60 + 41 **f** 809 + 1252 – 754 – 36

Exercise 1.3B: Problem solving and reasoning

12 The longest river in the world is the Nile in Africa, with a length of 6650 km. The longest river in Australia is the Barka (the Darling River), with a length of 2740 km. To find the difference in length between these two rivers, you can write the calculation in a number of ways.

a Decide if the following statements are true or false.

i 6650 – 2740 = 6000 – 2000 – 600 – 700 – 50 – 40

ii 6650 – 2740 = 6650 – 2000 – 700 – 40

iii 6650 – 2740 = 6650 – 3000 – 260

iv 6650 – 2740 = 6700 – 50 – 2800 + 60

b List two other ways to write the calculation.

c Calculate the difference between the two river lengths.

13 The difference between two numbers can be found by subtracting one number from the other number.

a Identify two two-digit numbers that have a difference of 43.

b Identify two numbers that have a difference of 168 and meet these conditions:

i both numbers are odd

ii both numbers are even.

c Identify two three-digit numbers that have a difference of 286 and meet these conditions:

i one number must be more than twice the other

ii one number must be triple the other.

14 Josh is comparing the distance to travel by plane from Sydney to Rome using two different routes. One journey stops at Hong Kong to refuel, while another stops at Singapore.

Flight sector	Distance by air
Sydney–Hong Kong	7372 km
Sydney–Singapore	6288 km
Hong Kong–Rome	9307 km
Singapore–Rome	10 048 km

a Determine the total flight distance from Sydney to Rome if the plane stops at Hong Kong on the way.

b Determine the total flight distance from Sydney to Rome if the plane stops at Singapore on the way.

c Which flight distance is the shortest and by how much?

15 This 162-storey building named the Burj Khalifa, located in Dubai, United Arab Emirates, was completed in 2010.

a How does the height of this building compare with other structures? Find the difference in height between Burj Khalifa and each of the structures shown in this table.

Structure	Date completed	Height
Washington Monument (Washington DC, USA)	1884	169 m
Eiffel Tower (Paris, France)	1889	300 m
Empire State Building (New York, USA)	1931	381 m
Sydney Tower (Sydney, Australia)	1981	309 m
Petronas Towers (Kuala Lumpur, Malaysia)	1998	452 m
Taipei 101 (Taipei, Taiwan)	2003	509 m
Q1 (Gold Coast, Australia)	2005	323 m
Eureka Tower (Melbourne, Australia)	2006	297 m

b Which two structures could have their heights added to give a result closest to the height of Burj Khalifa?

c Which three structures could have their heights added to give a result closest to the height of Burj Khalifa?

16 The highest mountain in the world is Mount Everest, which has a height above sea level of 8848 m.

a How does this compare to Mount Kosciuszko, the highest mountain in Australia, at 2228 m above sea level?

b Compare the height of Mount Everest with the three mountains in our solar system listed below.

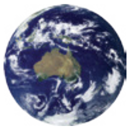
Mountain	Location	Height above surface
Mons Huygens	The Moon	4700 m
Maxwell Montes	Venus	11 000 m
Olympus Mons	Mars	21 171 m

c Determine the difference in height between the mountains listed for Mars and the Moon.

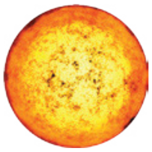
d Determine the difference in height between Mount Kosciuszko and Maxwell Montes.

17 There are 191 people travelling on a train. At the first station, 52 people leave the train and another 48 board the train. At the following station, 69 people get off the train and 75 get on. How many people are now on the train?

18 In May 2000, five planets in our solar system reached a point in their orbit around the Sun where they were roughly positioned in a straight line on the same side of the Sun (as viewed from the Sun). Earth was positioned on the opposite side of the Sun. The average distance from the Sun for each planet is shown on the right.



Earth
149 600 000 km



Sun

Mercury
57 909 000 km

Venus
108 200 000 km

Mars
227 940 000 km

Jupiter
778 400 000 km

Saturn
1 423 600 000 km

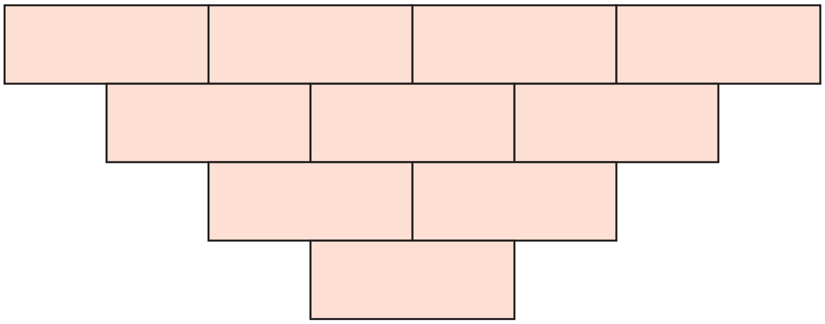
- **a** Use this information to find the distance between these planets.
  - **i** Jupiter and Saturn
  - **ii** Mercury and Saturn
  - **iii** Venus and Mercury
- **b** If the Sun has a diameter of about 1 392 000 km, what was the distance between Jupiter and Earth?
- **c** In June 2010, Uranus, Jupiter and Mercury were roughly lined up on one side of the Sun while Venus, Mars and Saturn were lined up on the other side. At this time, what was the approximate distance between:
  - **i** Jupiter and Mercury
  - **ii** Saturn and Venus
  - **iii** Jupiter and Saturn?
- **d** The film *War of the Worlds* is based on the idea that Martians travelled to Earth when the distance between Mars and Earth was the shortest. About what distance would the Martian spacecraft have travelled?

Exercise 1.3C: Challenge

19 Consider the diagram below.

You are to place the numbers 1 to 10 in each of the boxes in the diagram. Each box may contain one number, which should be equal to the difference of the two boxes above. There is more than one solution to this problem.

- **a** Explain why the number 10 must be placed in the top row.
- **b** How many different solutions can you find? Symmetrical solutions are considered to be the same.



Online resources:

Interactive skillsheet
Adding and subtracting whole numbers

Worksheet
Subtracting numbers

Investigation
Discoveries across the globe

Quick quiz
1.3

Lesson 1.4

Multiplying whole numbers

Learning intentions

By the end of this lesson you will be able to ...

✔ multiply whole numbers using mental strategies and short multiplication.

Inter-year links

Support	Multiplying whole numbers
Year 8	1.3 Multiplying and dividing whole numbers

Key content video
Multiplying whole numbers

Multiplication properties

- Multiplication is repeated addition, which can be reduced to multiplication of factors resulting in a **product**.

$$\underbrace{3 + 3}_{\text{repeated addition}} = \underbrace{2 \times 3}_{\text{factors}} = \underbrace{6}_{\text{product}}$$

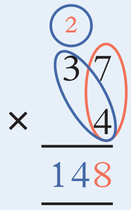
Multiplication (×)
Product
Times
Multiply
Groups of

- The associative law of multiplication states that regardless of how the numbers are grouped, the answer does not change.
  For example, (3 × 5) × 2 = 3 × (5 × 2).
- The commutative law of multiplication states that the order in which numbers are multiplied does not change the answer.
  For example, 14 × 2 = 2 × 14.
- The **distributive law** states that multiplication can be 'distributed' across a bracket. This means that large numbers can be broken down into a group of smaller numbers, which can then be multiplied separately.

$$\begin{aligned} 5 \times 32 &= 5 \times (30 + 2) \\ &= 5 \times 30 + 5 \times 2 \\ &= 150 + 10 \\ &= 160 \end{aligned}$$

Short multiplication

- Short multiplication involves multiplying a large number by a one-digit number.



1 Multiply 4 by 7 to get 28
2 Write the 8 in the ones column below
3 Add the 2 to the tens column above
4 Multiply 4 by 3 to get 12
5 Add the 2 tens to the 12 and get 14
6 Write 14 below

Multiplying by 10, 100 or 1000

- Multiplying a number by 10 increases each digit's place value by 1 column. Move all digits one place value space to the left and if there are no digits in any place then insert a zero.
- Multiplying a number by 100 increases each digit's place value by 2 columns. Move all digits two place value spaces to the left and if there are no digits in any place then insert a zero.
- Multiplying a number by 1000 increases each digit's place value by 3 columns. Move all digits three place value spaces to the left and if there are no digits in any place then insert a zero.

$$22 \times 1\underline{0} = 22\underline{0}$$
$$22 \times 1\underline{00} = 22\underline{00}$$
$$22 \times 1\underline{000} = 22\,\underline{000}$$

Worked example 1.4A Using mental strategies to multiply numbers

Work out each product using the multiplication strategy given in brackets.

a 8 × 16 (distributive law)

b 4 × 8 × 5 (associative law)

THINK

a **1** Break the second number into tens and ones: 16 = 10 + 6.

2 Multiply each part of the second number by the first number: 8 × 10 + 8 × 6.

3 Add the results together: 80 + 48 = 128.

b **1** Use brackets to group pairs of numbers that are easy to multiply together.

2 Multiply the grouped numbers together: 8 × 5 = 40.

3 Multiply the result by the third number: 4 × 40 = 160.

WRITE

a $8 \times 16 = 8 \times (10 + 6)$

$= 8 \times 10 + 8 \times 6$

$= 80 + 48$

$= 128$

b $4 \times 8 \times 5 = 4 \times (8 \times 5)$

$= 4 \times 40$

$= 160$

Worked example 1.4B Multiplying by a one-digit number

Use short multiplication to calculate 157 × 3.

THINK

1 Set out the multiplication problem by lining up digits according to their place value.

2 Multiply 3 by the 7 in the ones column. 3 × 7 = 21, put the 1 in the ones column below and put the 2 in the tens column above the 5.

3 Multiply 3 by the 5 in the tens column. 3 × 5 = 15, add the extra 2 to get 17. Put the 7 in the tens column below and put the 1 in the hundreds column above the 1.

4 Multiply 3 by the 1 in the hundreds column. 3 × 1 = 3, add the extra 1 to get 4. Put the 4 in the hundreds column below.

WRITE

$$\begin{array}{r} \overset{1}{1}\overset{2}{5}7 \\ \times \quad 3 \\ \hline 471 \\ \hline \end{array}$$

Worked example 1.4C Multiplying by 10, 100 or 1000

Calculate each product.

a 62×10 **b** 153×100 **c** 27×1000

THINK	**WRITE**
a To multiply by 10, move 62 one place value space to the left and insert a zero into the empty space.	**a** 62_ $62 \times 10 = 620$
b To multiply by 100, move 153 two place value spaces to the left and insert two zeros into the empty spaces.	**b** 153_ _ $153 \times 100 = 15\,300$
c To multiply by 1000, move 27 three place value spaces to the left and insert three zeros into the empty spaces.	**c** 27_ _ _ $27 \times 1000 = 27\,000$

Helpful hints

- ✔ Don't forget to keep the place value of the digits when using the distributive law.
  For example,
  $32 \times 3 = 30 \times 3 + 2 \times 3$
  $32 \times 3 \neq 3 \times 2 + 2 \times 3$.
- ✔ When you multiply by 10, you're not 'adding a zero'. Instead you're moving the digit one place to the left and then inserting a zero in the ones column. Think, 'insert a zero'.
- ✔ You may discover other multiplication techniques that work for you. For example, multiplying by 4 is the same as doubling a number twice, and multiplying by 8 is the same as doubling a number three times.

Learning pathways

△ 1–5, 6–7(a–d), 8–10, 12, 13(a, b)

🟨 1–7, 9–11, 13, 14

⬠ 2–4, 6, 7, 11–16

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p498

Exercise 1.4A: Understanding and fluency

1 Use your times tables knowledge to calculate each product.

a 7×4 **b** 9×2 **c** 12×11 **d** 6×7

WE 1.4A 2 Use the distributive law to work out each product. Break up the larger number into tens and ones and multiply them separately with the smaller number.

a 15×3 **b** 14×8 **c** 5×19 **d** 7×16

3 Use the associative law to work out each product. Use brackets to group pairs of numbers that are easy to multiply together.

a $9 \times 3 \times 2$ **b** $2 \times 6 \times 7$ **c** $8 \times 4 \times 9$ **d** $3 \times 5 \times 11$

WE 1.4B **4** Use short multiplication to calculate each product.

a 74 × 8 **b** 93 × 2 **c** 45 × 3 **d** 26 × 7

e 192 × 5 **f** 804 × 9 **g** 532 × 6 **h** 281 × 4

WE 1.4C **5** Calculate each product.

a 5 × 10 **b** 32 × 10 **c** 639 × 10 **d** 48 × 100

e 7 × 100 **f** 103 × 100 **g** 51 623 × 10 **h** 2784 × 1000

6 Calculate each product using any multiplication strategy.

a 4 × 3 × 10 **b** 6 × 7 × 10 **c** 2 × 10 × 9 **d** 15 × 4 × 100

e 8 × 5 × 1000 **f** 41 × 3 × 1000 **g** 356 × 1000 × 2 **h** 125 × 1000 × 4

7 Calculate each product. You can use the strategy of multiplying by 10 or 100 or 1000. Remember that 60 is the same as 10 × 6 or 6 × 10 (see the commutative law).

a 8 × 60 **b** 4 × 30 **c** 12 × 70 **d** 93 × 40

e 217 × 50 **f** 6345 × 20 **g** 52 × 800 **h** 428 × 300

8 Copy this table into your workbook.

First number × second number	Product	Number of zeros in the first number	Number of zeros in the second number	Number of zeros in the product
10 × 10				
100 × 10				
1000 × 10				
10 000 × 10				

a Write your answer to each multiplication in the product column.

b Complete each row by writing the number of zeros in the first number, the second number and the product.

c Can you see a pattern? Explain how this pattern provides a quick method of doing multiplications like this.

d Use this method to calculate each of these products.

i 10 × 100 **ii** 100 × 100 **iii** 100 × 1000 **iv** 1000 × 1000

9 Multiplications can be split up into easier calculations to perform.

a To find 4000 × 100, the calculation can be written as 4 × 1000 × 100 or 1000 × 100 × 4.

i Calculate 1000 × 100.

ii Multiply this result by 4 to obtain your final answer.

b Use this strategy to calculate each of these products.

i 300 × 10 **ii** 700 × 100 **iii** 6000 × 100 **iv** 2000 × 1000

v 100 × 50 **vi** 1000 × 400 **vii** 10 000 × 8000 **viii** 100 × 9000

10 Multiplications can be split up into easier calculations to perform.

a One way to calculate 300 × 20 is to calculate 3 × 100 × 2 × 10 or 3 × 2 × 100 × 10.

i Calculate 3 × 2.

ii Calculate 100 × 10.

iii Multiply the results you found in parts **i** and **ii** to find the answer to 300 × 20.

b Use this strategy to calculate each product.

i 400 × 20 **ii** 3000 × 30 **iii** 200 × 600 **iv** 9000 × 500

v 70 × 800 **vi** 600 × 4000 **vii** 30 000 × 7000 **viii** 800 × 20 000

11 Calculate each product.

a 40 × 10 × 200 **b** 60 × 900 × 3000 **c** 700 × 20 × 400 **d** 5000 × 300 × 80

Exercise 1.4B: Problem solving and reasoning

12 From a part-time job, James has saved $37 each week. To find the amount he has saved after 28 weeks, you can write the calculation in a number of ways.

- **a** Decide if the following statements are true or false.
  - **i** 37 × 28 = 37 × (20 + 8)
  - **ii** 37 × 28 = (30 + 7) × (20 + 8)
  - **iii** 37 × 28 = 28 × 37
  - **iv** 37 × 28 = 37 × 20 × 37 × 8
  - **v** 37 × 28 = 28 × 30 + 28 × 7
- **b** List two other ways to write the calculation.
- **c** Perform the calculation to find the total amount that James has saved.

13 Rory enjoys rowing with her team on the lake. She knows that for each full stroke, they glide 5 metres and for each half stroke they glide 3 metres.

- **a** If the team takes 230 full strokes, how far will they glide?
- **b** If the team takes 358 half strokes, how far will they glide?
- **c** If the lake is only 1000 metres wide, how many full strokes does the team need to take to cross the lake three times? Justify your answer.

14 The train to Brisbane from Rosewood has six carriages. During peak hour, all 88 seats per carriage are full.

- **a** How many seats are there on the train?
- **b** If there are 166 passengers standing in each carriage during peak hour, how many passengers are standing in the entire train?
- **c** How many people are there on the train during peak hour?
- **d** If another carriage containing 166 standing passengers and 88 sitting passengers was added to the train, how many people would there be on the seven-carriage train?

15 A local electrician is ordering light bulbs for a new apartment complex. Each apartment needs 43 light bulbs.

- **a** If there are 30 apartments in the complex, how many light bulbs does the electrician need to order?
- **b** It takes approximately 4 minutes to install a light bulb. How long will it take the electrician to install all of the light bulbs in the complex? Give your answer in minutes.
- **c** Show that it takes the electrician one hour to install 15 light bulbs.

Exercise 1.4C: Challenge

16 Multiplication can be represented visually in many different ways. One visual representation of 15 × 3 is:

$$15 \times 3 = 3 \begin{array}{|c|c|} \multicolumn{1}{c}{10} & \multicolumn{1}{c}{5} \\ \hline 30 & 15 \\ \hline \end{array} = 45$$

Draw five different visual representations of 24 × 6 by splitting up 24 into columns and 6 into rows.

Online resources:

Worksheet Multiplying by 10, 100 or 1000

Investigation Multiplying magic

Quick quiz 1.4

ANS
p499

Checkpoint

Checkpoint quiz
Take the checkpoint quiz to check your knowledge of the first part of this module.

1.1 **1** Write each number in worded form and expanded notation.
a 836 **b** 9654 **c** 629 000

1.1 **2** Write each number as a basic numeral.
a four hundred and nineteen
b ninety-six thousand, two hundred
c three hundred and twenty-five thousand, six hundred and fifty-six

1.1 **3** Write an approximation for each number by rounding to the nearest thousand.
a 9234 **b** 25 872 **c** 135 896

1.2 **4** Use any mental strategy to calculate each sum.
a 70 + 40 + 4 **b** 87 + 39 + 23 **c** 118 + 232 + 250

1.2 **5** Use the vertical addition algorithm to calculate each of the following.
a 654 + 387 **b** 6826 + 8347 **c** 34 763 + 365

1.2 **6** The first AFLW game was held on Friday 3 February 2017 at Princes Park in Melbourne. There was a crowd of 24 568 in attendance. The national television audience for the game was 896 000. What was the total number of people who saw the game?

1.3 **7** Use any mental strategy to calculate each difference.
a 350 – 60 **b** 586 – 46 **c** 287 – 143

1.3 **8** Use the vertical subtraction algorithm to calculate each of the following.
a 964 – 732 **b** 4863 – 568 **c** 8256 – 4737

1.3 **9** The original books in the Harry Potter series have the following number of pages:

The Philosopher's Stone – 223
The Chamber of Secrets – 251
The Prisoner of Azkaban – 317
The Goblet of Fire – 636
The Order of the Phoenix – 766
The Half-Blood Prince – 607
The Deathly Hallows – 607

a What is the total number of pages in the series?
b What is the difference in pages between the longest and the shortest books in the series?

1.4 **10** Use the distributive law to calculate each product.
a 13 × 7 **b** 6 × 18 **c** 23 × 8

1.4 **11** Calculate the product using any multiplication strategy.
a 536 × 9 **b** 437 × 40 **c** 675 × 700

1.4 **12** The list of ingredients for making 24 pancakes is below.
Gayi is planning to make pancakes for all the Year 7 students at her school. To have enough ingredients for one pancake each, Gayi needs six times the ingredients.

3 cups milk
2 eggs
4 teaspoons vanilla extract
4 cups self-raising flour
1 cup sugar
50 g butter

a How many Year 7 students are there at Gayi's school?
b How much of each ingredient does Gayi need?

Lesson 1.5

Long multiplication

Learning intentions

By the end of this lesson you will be able to …

✔ multiply whole numbers using long multiplication.

Inter-year links

Support	Multiplying whole numbers
Year 8	1.3 Multiplying and dividing whole numbers

Long multiplication

Key content video
Long multiplication

- In long multiplication, the digits are lined up according to place value.
- The process of long multiplication uses the distributive law to multiply the first number by the digits of the second number separately and then find their sum.

Write →	$\overset{2}{3}4$	
	$\times\ 26$	
Multiply by ones →	204	(34×6)
Multiply by tens → Don't forget the zero(s)	$+\ 680$	(34×20)
Add the results →	884	

Worked example 1.5A Using long multiplication to multiply a two-digit number by a two-digit number

Calculate the product using long multiplication.

$$\begin{array}{r} 6\,8 \\ \times\ 3\,7 \\ \hline \end{array}$$

THINK

1. Set up the multiplication problem by lining up the digits according to their place value.
2. Multiply 7 × 8 = 56. Write the 6 in the ones column below and write the 5 in the tens column above the 6.

   Multiply 7 × 6 = 42. Add the extra 5 to get 47. Write the 7 in the tens column and the 4 in the hundreds column below.
3. Start a new line and write 0 in the ones column, as the next step is to multiply by tens and there will be zero ones.
4. Multiply 3 × 8 = 24. Write the 4 in the tens column below and the 2 in the hundreds column next to the 6.
5. Multiply 3 × 6 = 18. Add the extra 2 to get 20 hundreds. Write the 0 in the hundreds column and the 2 in the thousands column below.
6. Add the results of the products and write the answer.

WRITE

$$\begin{array}{r} {\scriptstyle 2\ 5} \\ 6\,8 \\ \times\quad 3\,7 \\ \hline 4\,7\,6 \\ +\ 2\,0\,4\,0 \\ \hline 2\,5\,1\,6 \end{array}$$

68 × 37 = 2516

Worked example 1.5B Using long multiplication to multiply a three-digit number by a two-digit number

Calculate the product using long multiplication. $\begin{array}{r} 5\,4\,3 \\ \times \quad 8\,6 \\ \hline \end{array}$

THINK

1 Set out the multiplication problem by lining up the digits according to their place value.

2 Multiply by ones: 543 × 6 = 3258.

3 Multiply by tens:
543 × 80 = 43 440.

4 Add the results of the products and write the answer underneath.

WRITE

$$\begin{array}{cccccc}
 & & & 3 & 2 & \\
 & & & 2 & 1 & \\
 & & & 5 & 4 & 3 \\
\times & & & & 8 & 6 \\ \hline
 & & 3 & 2 & 5 & 8 \\
 & 4 & 3 & 4 & 4 & 0 \\ \hline
 & 4 & 6 & 6 & 9 & 8
\end{array}$$

543 × 86 = 46 698

Helpful hints

- ✔ Take care to line up digits by place value when setting up long multiplication problems to avoid making errors when adding the products together.
- ✔ Don't forget to write down the zeros! When multiplying by a multiple of 10, don't forget to insert a zero in the ones column.
- ✔ When multiplying a three-digit number by a three-digit number, insert two zeros in the third line of your calculation. In the first line you are multiplying by ones, in the second line you are multiplying by tens, and in the third line you are multiplying by hundreds.

$$\begin{array}{cccc}
 & & 1 & 2 \\
\times & & 3 & 4 \\ \hline
 & & 4 & 8 \\
 & 3 & 6 & 0 \\ \hline
 & 4 & 0 & 8
\end{array}$$

Learning pathways

△ 1–8, 9a(i, ii), 10–13

■ 1, 2(e–h), 3, 4(c, d), 5(e–h), 6, 7(e–h), 8–10, 12, 14, 16

⬟ 2(g, h), 3, 5(e–h), 6, 7(e–h), 8, 9, 12, 13, 15–18

ANS p499

Exercise 1.5A: Understanding and fluency

WE 1.5A 1 Calculate each product using long multiplication.

a $\begin{array}{r} 94 \\ \times\ \ 27 \\ \hline \end{array}$ **b** $\begin{array}{r} 13 \\ \times\ \ 11 \\ \hline \end{array}$ **c** $\begin{array}{r} 25 \\ \times\ \ 18 \\ \hline \end{array}$ **d** $\begin{array}{r} 37 \\ \times\ \ 22 \\ \hline \end{array}$

2 Multiply the following pairs of numbers using long multiplication.

a 38 × 15 **b** 62 × 24 **c** 46 × 32 **d** 85 × 73

e 65 × 48 **f** 56 × 81 **g** 99 × 18 **h** 24 × 92

3 Check your answers to question **2** using a calculator.

WE 1.5B 4 Calculate each product using long multiplication.

a $\begin{array}{r} 133 \\ \times\ \ 19 \\ \hline \end{array}$ **b** $\begin{array}{r} 205 \\ \times\ \ 34 \\ \hline \end{array}$ **c** $\begin{array}{r} 147 \\ \times\ \ 56 \\ \hline \end{array}$ **d** $\begin{array}{r} 281 \\ \times\ \ 64 \\ \hline \end{array}$

5 Multiply the following pairs of numbers using long multiplication.

a 415 × 23	**b** 283 × 54	**c** 123 × 37	**d** 231 × 56
e 782 × 49	**f** 506 × 81	**g** 1059 × 18	**h** 2654 × 42

6 Check your answers to question **5** using a calculator.

7 Multiply the following pairs of numbers using long multiplication. Hint: Your third multiplication line should end with two zeros. This is because you are multiplying by hundreds.

a 346 × 125	**b** 865 × 347	**c** 624 × 253	**d** 937 × 625
e 538 × 124	**f** 361 × 253	**g** 497 × 516	**h** 702 × 281

8 Check your answers to question **7** using a calculator.

9 An estimate is an approximate value of a calculation.

a Estimate these products by first rounding each number to the leading digit of each number, then multiplying.

i 591 × 82 **ii** 2175 × 93 **iii** 7856 × 304 **iv** 63 019 × 5647

b Check how close your estimations are to the exact result by performing the calculations.

Exercise 1.5B: Problem solving and reasoning

10 An average adult human heart beats around 72 times in a minute.

a How many times does it beat in one hour?

b How many times does it beat in one day?

11 A school has 25 students in each of its 32 classes.

a How many students are enrolled at this school?

b On a particular day, three students are away from 15 of the classes and two students are away from 11 of the classes. How many students are at school on this day?

12 Over the school holidays, a team of eight teenagers deliver take-away menus to homes near a pizza restaurant. How many menus are delivered in a week if each teenager visits 46 homes each day?

13 Galápagos tortoises move extremely slowly, covering a distance of about 260 m in 1 hour.

a What distance could a Galápagos tortoise travel in 12 hours?

b Compare this result with the distance that a human could walk in 12 hours, assuming humans walk about 4500 m per hour.

c The longest lifespan on record belongs to a male Galápagos tortoise kept in a British military fort for 154 years.

Assume there are 365 days in a year. Calculate how long he lived in:

- **i** months
- **ii** days
- **iii** hours.

14 Assume there are 365 days in a year.

a Calculate how old you will be at your next birthday in:

- **i** months
- **ii** days
- **iii** hours.

b Estimate how many hours there are between now and your next birthday.

15 Earth travels a distance of about 2 575 200 km each day. Assuming there are 365 days in a year, estimate, by rounding to the leading digits in your calculations, the distance Earth travels in one complete orbit around the Sun.

16 In 2017, around 392 000 hectares of vegetation was cleared in Queensland. When 392 000 hectares of vegetation is cleared, it is estimated that 45 million animals, including koalas, birds and reptiles, die from loss of habitat.

a If 392 000 hectares of vegetation is cleared every year for four years, what will be the total area of land cleared over this time period?

b Estimate how many animals will die if this occurs.

c If 392 000 hectares of vegetation is cleared one year and the area of vegetation cleared doubles each year, what is the total area of land that will be cleared after three years?

Exercise 1.5C: Challenge

17 Fill in the stars using only the digits 1, 2, 3, 4, 5 and 6 to solve the following problem. There may be more than one correct answer. Note: Each digit can be used more than once.

$$\begin{array}{rcccc} & & \star & \star & \star \\ \times & & & \star & \star \\ \hline & \star & \star & \star & \star \\ + & \star & \star & \star & 0 \\ \hline = & \star & \star & \star & \star \end{array}$$

18 Siân and Ajarné are building a model of a horse-riding arena. The arena is a rectangle that is 24 m long and 16 m wide with a fence around the perimeter.

a Determine the area of the arena. Recall that the area of a rectangle is the length multiplied by the width.

b Siân and Ajarné decide to build their model arena on a piece of cardboard. They have a square piece of cardboard with a side length of 30 cm.

  i What are the dimensions of the largest arena that can be built on the cardboard if they want the ratio of the length and width of the rectangle to be the same as the actual arena?

  ii Determine the area of the model arena.

  iii Determine the area of the cardboard that is not being used in the model.

c The girls decide to build the fence out of matchsticks. Each matchstick is 2 mm wide.

d How many matchsticks will they need? Recall that there are 10 millimetres in 1 centimetre.

Online resources:

Interactive skillsheet
Multiplying whole numbers

Investigation
Long multiplication

Quick quiz
1.5

Lesson 1.6

Dividing whole numbers

Learning intentions

By the end of this lesson you will be able to ...

✔ divide whole numbers using short and long division.

Inter-year links

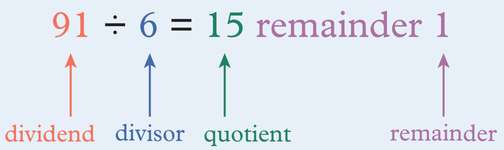
Support	Dividing whole numbers
Year 8	1.3 Multiplying and dividing whole numbers

Division

Key content video
Dividing whole numbers

- When performing a division, the **dividend** is the number that is divided by another number.
- The **divisor** is the number that divides the dividend.
- The **quotient** is the whole number result after the division has occurred.
- The **remainder** is the part of the dividend that is left over when the number is not a multiple of the divisor, and is always less than the divisor.

Division (÷)
Quotient
Divide
Shared
Distributed

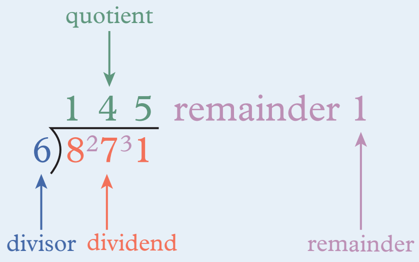


- One number is said to be exactly divisible by another number when there is no remainder (the remainder is zero).

  For example, 91 is not exactly divisible by 6, but 36 is exactly divisible by 6.

Short division

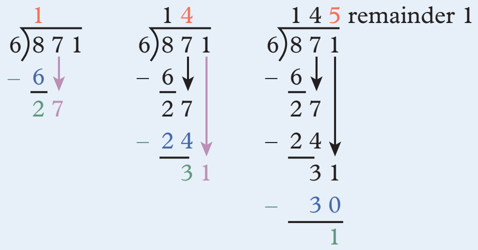
- Always start by dividing the leading digit of the dividend by the divisor. This is different from addition, subtraction and multiplication, where the calculation starts with the ones digits.



Long division

- Long division is the same process as short division; however, all the steps are recorded. The same five steps listed are repeated until the answer is found.

Step 1: Divide
Step 2: Multiply
Step 3: Subtract
Step 4: Bring down
Step 5: Repeat or remainder



Worked example 1.6A Using short division

Use short division to calculate each quotient and remainder.

a $4\overline{)98}$ **b** $7\overline{)1459}$

THINK

a **1** How many 4s in 9? Write 2 above the 9 on the quotient line. Work out the remainder. 4 × 2 = 8, so the remainder is 9 – 8 = 1. Write the 1 next to the 8 to get 18.

2 How many 4s in 18? Write 4 above the 8 on the quotient line. Work out the remainder. 4 × 4 = 16, so the remainder is 18 – 16 = 2.

b **1** How many 7s in 1? Write a zero above the 1. Underline the 1 and 4.

2 How many 7s in 14? Write 2 above the 4 on the quotient line.

3 How many 7s in 5? Write a zero above the 5. Underline the 5 and 9.

4 How many 7s in 59? Write 8 above the 9 on the quotient line. Work out the remainder. 7 × 8 = 56, so the remainder is 59 – 56 = 3.

WRITE

a $\begin{array}{r} 2\;\,4 \\ 4\overline{)9\,^{1}8} \end{array}$ remainder 2

98 ÷ 4 = 24 remainder 2

b $\begin{array}{r} 0208 \\ 7\overline{)\underline{14}\underline{59}} \end{array}$ remainder 3

1459 ÷ 7 = 208 remainder 3

Worked example 1.6B Using long division

Use long division to calculate 6492 ÷ 19.

THINK

1 List multiples of 19.

2 Divide: How many 19s in 6? Write a zero above the 6. Underline the 6 and 4.
How many 19s in 64? Write 3 above 4 on the quotient line.
Multiply: 19 × 3 = 57
Subtract: The remainder is 64 – 57 = 7.
Bring down the 9 and write it beside the remainder of 7.
Then, the next number to divide is 79.

3 Divide: How many 19s in 79? Write 4 above 9 on the quotient line.
Multiply: 19 × 4 = 76
Subtract: The remainder is 79 – 76 = 3.
Bring down the 2 and write it beside the remainder of 3.
Then, the next number to divide is 32.

4 Divide: How many 19s in 32? Write 1 above 2 on the quotient line.
Multiply: 19 × 1 = 19
Subtract: The remainder is 32 – 19 = 13.

5 Check that the remainder is less than the divisor and write the answer.

WRITE

$$\begin{array}{r} 0\;3 \\ 19\overline{)\underline{6\;4}\;9\;2} \\ -\underline{5\;7}\downarrow \\ 7\;9 \end{array}$$

$$\begin{array}{r} 0\;3\;4 \\ 19\overline{)6\;4\;9\;2} \\ -\underline{5\;7}\downarrow \\ 7\;9 \\ -\underline{7\;6}\downarrow \\ 3\;2 \end{array}$$

$$\begin{array}{r} 0\;3\;4\;1 \text{ remainder } 13 \\ 19\overline{)6\;4\;9\;2}\phantom{\text{ remainder } 13} \\ -\underline{5\;7}\downarrow\phantom{\text{ remainder } 13} \\ 7\;9\phantom{\text{ remainder } 13} \\ -\underline{7\;6}\downarrow\phantom{\text{ remainder } 13} \\ 3\;2\phantom{\text{ remainder } 13} \\ -\underline{1\;9}\phantom{\text{ remainder } 13} \\ 1\;3\phantom{\text{ remainder } 13} \end{array}$$

6492 ÷ 19 = 341 remainder 13

Helpful hints

- ✔ Remember that division starts from the left, dividing the largest place value digit first.
- ✔ Remainders and decimals are not the same thing! For example, remainder 2 is not the same as 0.2.
- ✔ Remember that 0341 = 341, as a zero in the first place value doesn't change the value of the number. However, 308 is not the same as 38. In this case, the zero shows that there are zero tens and keeps the 3 in the hundreds column and the 8 in the ones column.
- ✔ When dividing, the remainder must always be less than the divisor.
- ✔ When using long division, it can be helpful to have the list of steps handy until you are confident in the skill.

Step 1: Divide
Step 2: Multiply
Step 3: Subtract
Step 4: Bring down
Step 5: Repeat or remainder

Learning pathways

△ 1–4, 5(a–d), 6(a, b), 7–10, 12–14, 16(a, b)

□ 1–4, 5(a, c, e, g), 6, 7, 9–11, 15, 16(a, b), 18

⬠ 3(g, h), 4(c, d), 5(e–h), 6, 7, 10, 11, 13, 16–20

ANS p499

Exercise 1.6A: Understanding and fluency

1 For each division, identify:

i the dividend **ii** the divisor **iii** the quotient **iv** the remainder.

a 9 ÷ 2 = 4 remainder 1 **b** 17 ÷ 7 = 2 remainder 3 **c** 30 ÷ 5 = 6

WE 1.6A **2** Use short division to calculate each quotient and remainder.

a $2\overline{)13}$ **b** $5\overline{)23}$ **c** $3\overline{)17}$ **d** $10\overline{)25}$
e $8\overline{)38}$ **f** $4\overline{)26}$ **g** $9\overline{)66}$ **h** $6\overline{)48}$

3 Use short division to calculate each quotient and remainder.

a 538 ÷ 4 **b** 756 ÷ 6 **c** 172 ÷ 3 **d** 1229 ÷ 5
e 877 ÷ 5 **f** 489 ÷ 7 **g** 668 ÷ 8 **h** 1234 ÷ 4

4 Another way to write a division calculation is as a fraction.

For example, 57 ÷ 3 is the same as $\frac{57}{3}$. The horizontal line between the two numbers replaces the division sign. Perform each division using short division.

a $\frac{63}{9}$ **b** $\frac{1470}{6}$ **c** $\frac{658}{7}$ **d** $\frac{1251}{3}$

WE 1.6B **5** Use long division to calculate each quotient and remainder.

a 542 ÷ 21 **b** 739 ÷ 18 **c** 884 ÷ 26 **d** 798 ÷ 12
e 465 ÷ 22 **f** 881 ÷ 41 **g** 7462 ÷ 35 **h** 1234 ÷ 56

6 Use long division to calculate each quotient and remainder.

a 1658 ÷ 43 **b** 4509 ÷ 45 **c** 15 789 ÷ 13 **d** 12 023 ÷ 80

7 Evaluate each of the following.

a $\frac{272}{17}$ **b** $\frac{3350}{25}$ **c** $\frac{47\,136}{32}$ **d** $\frac{36\,088}{52}$

8 Copy this table.

First number ÷ second number	$\frac{\textbf{First number}}{\textbf{second number}}$	**Quotient**	**Number of zeros in the first number**	**Number of zeros in the second number**	**Number of zeros in the quotient**
10 ÷ 10	$\frac{10}{10}$				
100 ÷ 10	$\frac{100}{10}$				
1000 ÷ 10	$\frac{1000}{10}$				
10 000 ÷ 10	$\frac{10\,000}{10}$				
100 ÷ 100	$\frac{100}{100}$				
1000 ÷ 100	$\frac{1000}{100}$				
10 000 ÷ 100	$\frac{10\,000}{100}$				

a Write your answer to each division in the 'Quotient' column.

b Complete each row of the table by writing the number of zeros in the first number, the second number and the quotient.

c Can you see a pattern? Explain how this pattern provides a shortcut for dividing by multiples of 10.

d Use this method to work out the following quotients.

i 10 000 ÷ 10 **ii** 100 000 ÷ 100 **iii** 1000 ÷ 1000 **iv** 10 000 ÷ 1000

9 To work out 5000 ÷ 100, the calculation can be written as $\frac{5000}{100}$.

a Complete the following steps to calculate the value of this fraction.

i Calculate 1000 ÷ 100.

ii Multiply this result by 5 to obtain your final answer.

b Use this strategy to evaluate the quotient of each of the following.

i 200 ÷ 10 **ii** 600 ÷ 100 **iii** 9000 ÷ 100 **iv** 4000 ÷ 1000

v 800 ÷ 10 **vi** 3000 ÷ 100 **vii** 50 000 ÷ 1000 **viii** 6000 ÷ 10

10 One way to evaluate 600 ÷ 20 is to write the calculation as $\frac{600}{20}$.

a Complete the following steps to calculate the value of this fraction.

i Calculate 6 ÷ 2.

ii Calculate 100 ÷ 10.

iii Multiply the results from parts **i** and **ii** to obtain your final answer.

b Use this strategy to perform each division.

i $\frac{900}{30}$ **ii** $\frac{8000}{20}$ **iii** $\frac{1200}{60}$

iv $\frac{25\,000}{500}$ **v** 6000 ÷ 300 **vi** 80 000 ÷ 4000

vii 70 000 ÷ 700 **viii** 1600 ÷ 80 **ix** 2500 ÷ 50

11 An estimate is an approximate value of a calculation.

a Estimate the quotient of each of the following by first rounding each number to the first digit of each number before dividing. Hint: Use the strategy from question **10**.

i 627 ÷ 33 **ii** 5940 ÷ 18 **iii** 3852 ÷ 214 **iv** 83 490 ÷ 3795

b Use a calculator to check how close your estimations are to the exact results.

Exercise 1.6B: Problem solving and reasoning

12 Georgia is helping to arrange a birthday party for her younger brother. She has 165 sweets to share among 15 party bags. To work out the number of sweets in each party bag, you can describe the calculation in a number of ways.

a Which of the following number sentences correctly describes the information above?

A Divide 165 by 15.

B How many times does 165 go into 15?

C Find the quotient when 15 is divided by 165.

D Find the product of 165 and 15.

b List one other way to write the calculation.

c Determine the number of sweets in each party bag.

13 Chris can type (on his computer) at a rate of 68 words each minute.

a How long would it take him to type a message of 272 words?

b Explain how you could check the answer to this division problem using multiplication.

14 Charlotte swam 1200 m in a 25 m swimming pool. How many laps of the pool did she complete?

15 The ancestor of the common wombat and the southern hairy-nosed wombat is the giant wombat, a marsupial that disappeared from Australia around 40 000 years ago. On average, giant wombats were 3 m in length and 180 cm in height, with a mass of 2000 kg. The mass of an average common wombat is 37 kg. The photo shows a hairy-nosed wombat and its mass.

a How many common wombats, when their weights are added together, would be approximatively equivalent to one giant wombat?

b How many hairy-nosed wombats, when their weights are added together, would be approximatively equivalent to one giant wombat?

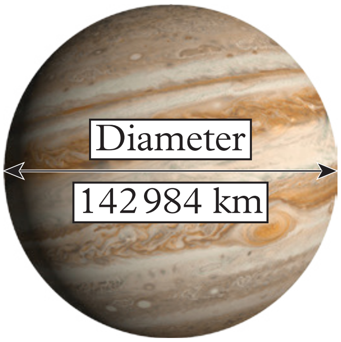
16 Isobel and Connor are helping to make food parcels for a charity organisation. There are 1350 bags of rice to be shared among 84 food parcels.

a How many bags of rice will be in each food parcel?

b How many bags of rice will be left over?

c Determine the number of food parcels that could be made up with an equal share of the bags of rice so that no bags are left over. There are many solutions to this question.

17 Earth orbits the Sun in about 365 days, while the planet Jupiter completes an orbit around the Sun in about 4333 Earth days. Earth has a diameter of approximately 12 756 km.



a Jupiter is shown on the right with its diameter. Estimate the number of times that Earth would complete a full orbit around the Sun for each one of Jupiter's orbits. First round each number to its leading digit.

b If Earth was drawn to the same scale as Jupiter is shown in the photo, approximately how many Earths would fit across the equator of Jupiter?

Exercise 1.6C: Challenge

18 An apartment building has twelve apartments that are numbered from 110 to 121. The manager has twelve key cards for the apartments numbered from 1 to 12.

The manager has forgotten which key card is for which apartment but knows that each apartment number is divisible by the number of its key card.

Determine which key cards pair with each apartment.

19 Below are the numbers 1 to 9. Using only the operations +, –, × and ÷, find at least one way to make 100 using all the digits.

1 2 3 4 5 6 7 8 9

+ – × ÷

= 100

20 The following is a long division problem showing all the working out steps and the result. There is no remainder. When this book was printed, the numbers were replaced with letters! Fortunately, each letter represents the same number throughout the problem. Construct the original problem.

```
    I A E D H E
A B)C D F G G C
   −A B
      B F
    − G C
      A A G
    − A I B
          J G
        − J I
            G C
          − G C
              I
```

Online resources:

Interactive skillsheet
Dividing whole numbers

Interactive skillsheet
Multiplying and dividing by powers of 10

Worksheet
Long division

Investigation
Lolly bags

Quick quiz
1.6

Lesson 1.7

Exponents and square roots

Learning intentions

By the end of this lesson you will be able to ...

✔ convert and evaluate numbers in exponent form and expanded form.

Inter-year links

Year 8	4.1 Exponents
Year 9	1.1 Exponents

Exponent form

Key content video
Exponents

- An **exponent** (or **index**) of a number indicates how many times a number is being multiplied by itself. The number that is being multiplied is called the **base**. A number is in **exponent form** (or exponent notation) when it is written as a base with an exponent. A number in exponent form is also known as a **power**.

$$\text{base} \longrightarrow \underbrace{2^{\overset{\text{exponent/index}}{\downarrow}3}}_{\text{exponent form}} = \underbrace{2 \times 2 \times 2}_{\text{expanded form}} = \underbrace{8}_{\text{basic numeral}}$$

→ In the example above, the number in exponent form is read as ‘2 to the power of 3’. Similarly, 3^4 is read as ‘3 to the power of 4’.

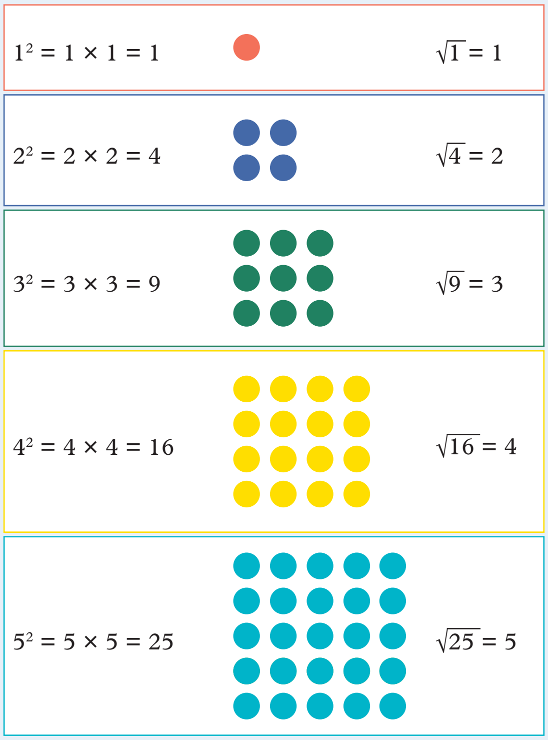
Perfect squares

Key content video
Squares and square roots

- To **square** a number is to multiply it by itself or ‘raise it to the power of 2’. For example, squaring 3, or raising 3 to the power of 2, can be written numerically as: $3^2 = 3 \times 3 = 9$.
- A **perfect square** (or square number) is a number that can be expressed as the product of a whole number multiplied by itself. Perfect squares can be represented in a square pattern. For example, $3 \times 3 = 9$, so 9 is a perfect square number.

Square roots

- The **square root** of a number is the number that when squared (raised to the power of 2) results in the original number.
- The symbol $\sqrt{}$ is used to denote the square root of a number. For example, $\sqrt{9} = 3$, because when 3 is multiplied by itself the result is 9.



Powers of 10

- A power of 10 has a base of 10 and a whole number exponent. Each power of 10 is equal to a basic numeral starting with 1 followed by the same number of zeros as in the exponent of the power. For example:
  $10^2 = 10 \times 10 = 100$
  $10^3 = 10 \times 10 \times 10 = 1000$
  → Intuitively, it follows that the value of 10^0 is 1.
- The values in the first row of a place value chart can be expressed as powers of 10. Therefore, whole numbers can be written in expanded notation using powers of 10.

Ten thousands **10 000 = 10^4**	**Thousands** **1000 = 10^3**	**Hundreds** **100 = 10^2**	**Tens** **10 = 10^1**	**Ones** **1 = 10^0**
4	7	1	0	5

$47\,105 = 4 \times 10^4 + 7 \times 10^3 + 1 \times 10^2 + 5 \times 10^0$

Worked example 1.7A Converting numbers to exponent form

Write each repeated multiplication in exponent form.

a $8 \times 8 \times 8 \times 8 \times 8 \times 8$

b $3 \times 3 \times 4 \times 4 \times 4 \times 4 \times 4$

THINK	**WRITE**
a Identify the base and the exponent. The base is the number that is repeatedly multiplied, 8. The exponent represents the number of times the base appears in expanded form, 6.	**a** $8 \times 8 \times 8 \times 8 \times 8 \times 8$ $= 8^6$
b Identify the base and the exponent. Notice that there are two different bases. Write the first base, 3, with its exponent, 2, multiplied by the second base, 4, with its exponent, 5.	**b** $3 \times 3 \times 4 \times 4 \times 4 \times 4 \times 4$ $= 3^2 \times 4^5$

Worked example 1.7B Calculating the value of a number in exponent form

Write each of the following in expanded form and calculate their values.

a 4^3

b $2^4 \times 3^2$

THINK	**WRITE**
a 1 Identify the base and the exponent. The base is 4 and the exponent is 3, so 4 is multiplied by itself 3 times.	**a** $4^3 = 4 \times 4 \times 4$
2 Perform the multiplication.	$= 16 \times 4$ $= 64$
b 1 Identify the base and the exponent. In this case, there are two bases, so each base needs to be expanded separately.	**b** $2^4 \times 3^2 = 2 \times 2 \times 2 \times 2 \times 3 \times 3$
2 Perform the multiplication.	$= 4 \times 4 \times 9$ $= 16 \times 9$ $= 144$

Worked example 1.7C Calculating the square root

Calculate the value of each of the following square roots.

a $\sqrt{16}$ **b** $\sqrt{144}$

THINK	**WRITE**
Use the times tables or square numbers to determine which number multiplied by itself is equal to the number under the square root symbol.	**a** $4 \times 4 = 16$ So, $\sqrt{16} = 4$ **b** $12 \times 12 = 144$ So, $\sqrt{144} = 12$

Helpful hints

- ✔ Don't confuse powers with multiplication!

  For example, $2^3 = 2 \times 2 \times 2$

  $2^3 \neq 2 \times 3$.
- ✔ Write your exponents carefully – they should be smaller than the base and sit on the right shoulder of the base. Your 4^2 should look different to your 42!

$4^2 \neq 42$

Learning pathways

▲	1–3, 4–7(a, c, e, g), 8, 9, 10(a, b), 11(1st, 2nd columns), 12, 13(a, b), 14, 15, 18	■	1–3, 4–7(b, d, f, h), 8, 9, 10(b, c), 11(2nd, 3rd columns), 12–17, 19, 20, 23	⬟	1(e, f), 3(e, f), 4–7(f, g, h), 8(c, d), 10(c, d), 11(d, f, h, l), 12, 13(c, d), 16, 18, 21, 22, 24, 25

ANS p500

Exercise 1.7A: Understanding and fluency

WE 1.7A **1** Write each repeated multiplication in exponent form.

a $5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5$
b $4 \times 4 \times 4 \times 4 \times 4 \times 4$
c $11 \times 11 \times 11$
d $20 \times 20 \times 20 \times 20 \times 20$
e $9 \times 9 \times 9 \times 9 \times 9 \times 9 \times 9 \times 9$
f $300 \times 300 \times 300 \times 300$

2 How would you read each answer obtained in question **1**? Write each answer using words.

3 Write each repeated multiplication in exponent form.

a $7 \times 7 \times 7 \times 9 \times 9 \times 9 \times 9$
b $4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 2 \times 2 \times 2$
c $3 \times 3 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5$
d $8 \times 8 \times 8 \times 8 \times 13 \times 13$
e $2 \times 2 \times 2 \times 2 \times 6 \times 6 \times 7 \times 7$
f $19 \times 19 \times 23 \times 23 \times 23 \times 23 \times 31$

WE 1.7B **4** Write each of the following in expanded form and calculate their values.

a 3^2 **b** 2^5 **c** 7^3 **d** 5^2 **e** 9^4 **f** 4^3 **g** 1^5 **h** 6^3

5 Calculate the value of each power.

a 2^4 **b** 3^4 **c** 7^2 **d** 5^3 **e** 11^3 **f** 12^2 **g** 10^3 **h** 16^2

6 Write each product in expanded form and calculate their values.

a $2^3 \times 4^2$ **b** $3^4 \times 5^2$ **c** $8^2 \times 10^3$ **d** $3^5 \times 1^4$
e $2^6 \times 7^2$ **f** $10^2 \times 3^3$ **g** $9^4 \times 11^2$ **h** $6^2 \times 3^2$

WE 1.7C **7** Calculate the value of each of the following square roots.

a $\sqrt{9}$
b $\sqrt{36}$
c $\sqrt{64}$
d $\sqrt{1}$
e $\sqrt{49}$
f $\sqrt{121}$
g $\sqrt{144}$
h $\sqrt{400}$

8 Without using a calculator, complete each statement.

a $11^2 = 121$, so $\sqrt{121} =$ ______
b $35^2 = 1225$, so $\sqrt{1225} =$ ______
c $61^2 = 3721$, so $\sqrt{\underline{\quad}} = 61$
d $298^2 = 88\,804$, so $\sqrt{\underline{\quad}} = 298$

9 Expanded form can be used when converting between powers of 10 and basic numerals.

a Write the following powers of 10 as basic numerals.

i 10^4
ii 10^6
iii 10^{12}
iv 10^9

b Write the following numbers as powers of 10.

i 100
ii 100 000
iii 1
iv 10 000 000

10 Write the following numbers in expanded form using powers of 10.

a 4129
b 19 088
c 51 535
d 403 086

Exercise 1.7B: Problem solving and reasoning

11 Evaluate each of the following by first working out the value of any numbers written in exponent form.

a $3^4 + 2^3$
b $10^2 + 5^2$
c $8^2 - 2^6$
d $4^3 - 1^9$
e $10^2 \div 5^2$
f $6^4 \div 3^3$
g $4^2 + 2^4 + 3^3$
h $9^2 + 2^2 - 1^5$
i $10^2 \times 3^2 \times 2^2$
j $4 \times 2^3 \times 5$
k $7^2 \times 8 \times 9^2$
l $9^3 \times 6^2 \times 8^1$

12 Which power is greater: 3^5 or 5^3?

13 Arrange the powers in each list from smallest to largest. You may like to use a calculator to help you.

a $3^2, 4^5, 2^3, 5^4$
b $7^6, 6^7, 1^{50}, 50^3$
c $3^{10}, 9^4, 6^5, 10^3$
d $8^3, 3^8, 4^6, 12^2$

14 State if each of the following is true or false.

a $12 < \sqrt{121} < 13$
b $8 < \sqrt{16} < 10$
c $7 < \sqrt{64} < 9$
d $11 < \sqrt{144} < 13$

15 The square root of a non-perfect square number is not a whole number. We can, however, find two consecutive whole numbers that the square root is between. For example, what is $\sqrt{10}$? Recall that $3^2 = 9$ and $4^2 = 16$. Since 10 is between the perfect squares 9 and 16, $\sqrt{10}$ is between 3 and 4. This is useful for estimating square roots.

State if each of the following is true or false.

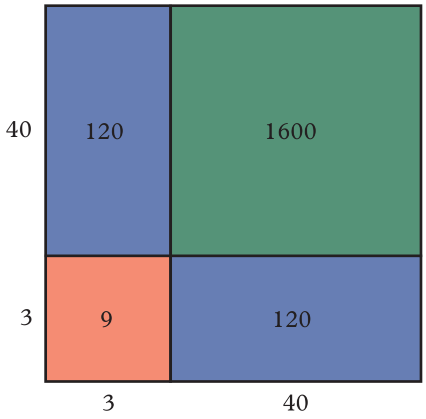
a $2 < \sqrt{7} < 3$
b $5 < \sqrt{26} < 6$
c $7 < \sqrt{30} < 8$
d $9 < \sqrt{80} < 10$

16 Write the closest whole numbers that each square root lies between.

a __ $< \sqrt{15} <$ __
b __ $< \sqrt{33} <$ __
c __ $< \sqrt{40} <$ __
d __ $< \sqrt{99} <$ __
e __ $< \sqrt{120} <$ __
f __ $< \sqrt{214} <$ __

17 Create a list of the first 20 square numbers.

18 The square of a two-digit number can be found using an area diagram. For example, to find 43^2 write $43 = 40 + 3$. Draw a square with side length 43 and break it into the four sections as shown. The area of the square is equal to $43^2 = (40 + 3)^2$. It is also equal to the sum of the areas of the orange and green squares and two copies of the blue rectangle. This means that $43^2 = 3^2 + 40^2 + 2 \times 3 \times 40 = 9 + 1600 + 240 = 1849$.



a Using this method, find:

i 27^2
ii 52^2
iii 99^2.

b Can you see a pattern? Explain how this pattern provides a method for finding the squares of two-digit numbers.

19 A colony of bacteria grows very quickly and triples its size each day.

	Size of colony (in millions)		
Number of days	**Expanded form**	**Exponent form**	**Number of bacteria (in millions)**
1	3	3^1	3
2	3 × 3	3^2	9
3	3 × 3 × 3		
4			
5			

- **a** Complete the table.
- **b** What is the size of the colony after:
  - **i** 6 days
  - **ii** 1 week
  - **iii** 2 weeks?
- **c** On what day is the colony:
  - **i** 27 times larger than on day 1
  - **ii** 243 times larger than on day 1
  - **iii** 6561 times larger than on day 1.

20 Dianna sends a text message to four of her friends. Each friend forwards it to another four people, who each then send it to another four people.

- **a** How many text messages in total have been sent?
- **b** Explain how exponents can be used to solve this problem.

21 Over the summer school holidays, Taylor is offered a part-time job for which he will earn $100 per week. The job is available for 4 weeks. Taylor decides to discuss a different payment plan with his prospective boss. 'How about paying me only $5 in the first week and then in each of the other weeks paying me five times as much as the week before.' The boss thinks this new plan might save her money. Which payment plan do you think the boss should go with? Use calculations to justify your answer.

22 Interesting patterns emerge when we investigate perfect squares. Consider the difference between two consecutive perfect squares. Recall that consecutive means one following directly after another.

- **a** The perfect squares are 1, 4, 9, 16, 25, … Find the difference between the first 10 pairs of consecutive perfect squares, starting with 1 and 4. Record your results in a table.
- **b** What do all these differences have in common?
- **c** In a number sequence, the second difference is determined by taking the difference between consecutive 'first differences'. Calculate the first five second differences for the sequence of perfect squares. What do you find?

23 Arrange the numbers 1 to 16 in pairs so that each pair adds up to a perfect square.

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16

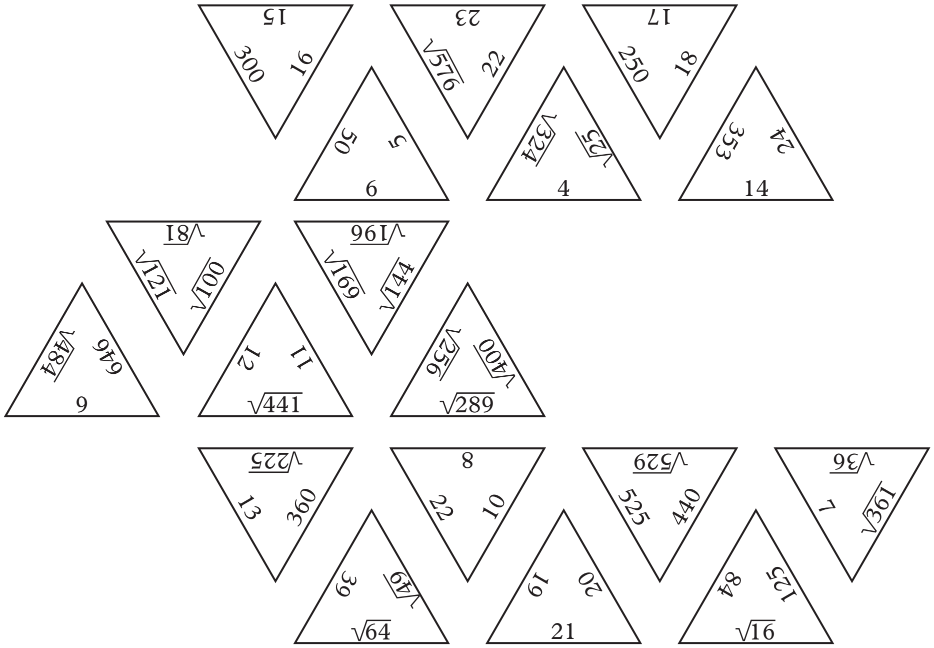
Exercise 1.7C: Challenge

24 An evil genius has discovered a new toxic serum that, when consumed, decreases a person's weight by taking the square root of their current weight, every hour on the hour.

The evil genius tests the serum on four brave volunteers.

- **a** At the time they consume the serum, Person A weighs 99 kg, Person B weighs 142 kg, Person C weighs 56 kg, and Person D weighs 82 kg. Without using a calculator, approximate the weight of each person after the first hour has passed.
- **b** Without using a calculator, approximate the weight of each person after the second hour has passed. Remember, the serum will decrease each person's weight by the square root of their new weight, not their original weight.
- **c** Without using a calculator, approximate the weight of each person after the third hour has passed.
- **d** Estimate how much each person will weigh after 10 hours.
- **e** Will the weight of each person ever reach 0 kg?
- **f** Determine a mathematical serum formula that could hypothetically decrease a person's weight to 0 kg after 13 hours.

25 Consider the following triangles. On the edges are square roots and basic numerals. Match the square roots to their result. Your finished puzzle should look like a diamond.
You may like to draw the triangles and cut them out so that you can move them around easily. For an extra hard challenge, do not use cut-out versions of the triangles to help you.



Online resources:

Interactive skillsheet Exponents

Interactive skillsheet Squares and square roots

Worksheet Exponent form

Worksheet Finding powers and square roots

Investigation Patterns and squared numbers

Quick quiz 1.7

Lesson 1.8

Order of operations

Learning intentions

By the end of this lesson you will be able to ...

✔ use BIDMAS to apply the correct order of operations in calculations.

Inter-year links

Support	Order of operations
Year 8	1.8 Order of operations

Order of operations

Key content video
Order of operations

- **BIDMAS** can be used to help remember the order of operations.

B	**Brackets**	Operations inside brackets are always performed first. For example, $5 \times (10 - 2) = 5 \times 8 = 40$.
I	**Indices (exponents)**	Next, evaluate exponents and roots. For example, $3^3 - 4 = 27 - 4 = 23$.
D **M**	**Division** **Multiplication**	Then working from **left** to **right,** perform any multiplication or division before adding or subtracting. For example, $5 \times 3 - 4 + 8 \div 4 = 15 - 4 + 2 = 13$.
A **S**	**Addition** **Subtraction**	Finally, working from **left** to **right,** perform any addition and subtraction. For example, $15 - 4 + 2 = 13$.

→ Operations at the same ranking are performed in order from left to right.

→ Where there is more than one set of brackets in the problem, perform the operations inside the innermost brackets first.

Worked example 1.8A Order of operations

Without using a calculator, evaluate each of the following.

a $3 + 8 \div 2 - \sqrt{4}$ **b** $7 \times 2^3 - 9 \times 5$

THINK	WRITE
a **1** Exponents and square roots are worked out before ×, ÷, + and −. Calculate $\sqrt{4} = 2$.	**a** $3 + 8 \div 2 - \sqrt{4}$ $= 3 + 8 \div 2 - 2$
2 Division is performed before addition. Calculate $8 \div 2 = 4$.	$= 3 + 4 - 2$
3 Working from left to right, perform the addition, then the subtraction.	$= 5$
b **1** Exponents and square roots are worked out before ×, ÷, + and −. Calculate $2^3 = 8$.	**b** $7 \times 2^3 - 9 \times 5$ $= 7 \times 8 - 9 \times 5$
2 Multiplication is performed before subtraction so, working from left to right, first calculate $7 \times 8 = 56$.	$= 56 - 9 \times 5$
3 Perform the next multiplication. Calculate $9 \times 5 = 45$.	$= 56 - 45$
4 Perform the subtraction.	$= 11$

Worked example 1.8B Order of operations with brackets

Without using a calculator, evaluate each of the following.

a $16 \div (10 - 2) + 3 \times 7$ **b** $8 + [12 \div (9 - 5)]^2 - 1$

THINK	**WRITE**
a **1** Perform any operations inside brackets first. Calculate $10 - 2 = 8$.	**a** $16 \div (10 - 2) + 3 \times 7$
2 Perform division and multiplication before addition. Working from left to right, first calculate $16 \div 8 = 2$.	$= 16 \div 8 + 3 \times 7$ $= 2 + 3 \times 7$
3 Perform multiplication before addition. Calculate $3 \times 7 = 21$.	$= 2 + 21$
4 Perform the addition.	$= 23$
b **1** There are two sets of brackets. Perform the operation in the innermost set of brackets first. Calculate $9 - 5 = 4$.	**b** $8 + [12 \div (9 - 5)]^2 - 1$ $= 8 + [12 \div 4]^2 - 1$
2 Perform the operation in the remaining set of brackets. Calculate $12 \div 4 = 3$.	$= 8 + 3^2 - 1$
3 Apply exponents before addition and subtraction. Calculate $3^2 = 9$.	$= 8 + 9 - 1$
4 Working from left to right, perform the addition, then the subtraction.	$= 17 - 1$ $= 16$

Helpful hints

BIDMAS

- ✔ Remember to use BIDMAS and not to just work from left to right.
- ✔ Remember to always complete any operations in brackets first!
- ✔ Calculators don't always get the order of operations correct. If you are using a basic calculator, you will need to type in the calculation in a way that follows the BIDMAS rules.
- ✔ When a division is immediately followed by a multiplication, as in $12 \div 4 \times 3$, use brackets to clarify whether you mean $(12 \div 4) \times 3 = 9$ or $12 \div (4 \times 3) = 1$.

Addition (+)	**Subtraction (−)**	**Multiplication (×)**	**Division (÷)**
Sum	Difference	Product	Quotient
Plus	Take away	Times	Divide
More than	Less than	Multiply	Shared
Increase	Fewer	Groups of	Distributed
Total	Reduce		
Together	Minus		

Learning pathways

△ 1–3(1st, 2nd columns), 4, 5(1st column), 6(a–c), 7–10, 14

■ 1, 2–3(3rd column), 4, 5(2nd column), 6(a, c, e), 7, 9, 11, 12, 14–16

⬟ 1(i–l), 2–3(3rd column), 4, 5–6(d, e, f), 7, 9, 12, 13, 16–18

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Exercise 1.8A: Understanding and fluency

WE 1.8A **1** Without using a calculator, evaluate each of the following.

a $6 + 15 \div 3$ **b** $9 - 4 \times 2$ **c** $7 + 3 \times 6 - 4$ **d** $12 \div 4 + 7 \times 8$

e $4 \times 4 + 6 \div 2$ **f** $2 + 9 \times 4 \div 6$ **g** $10 \times 5 \div 5 + 2$ **h** $12 \div 3 + 4 - 8$

i $7 + (25 \div 5) \times 3 + 2$ **j** $16 - (8 \div 4) \times 2 + 5$ **k** $6 \times 4 - 10 \times 2$ **l** $9 \times 3 - 6 \times 3 \div 2$

2 Following the correct order of operations, complete the calculations below.

- **a** $10 \times 6 - 5 \times 9$
- **b** $8^2 - 5 \times 3 + 11$
- **c** $21 - 3^2 \times 2 + 1$
- **d** $4 \times 3^3 - 6^2 \times 2 + 1$
- **e** $\sqrt{9} + 9 \times 8 - 2^3$
- **f** $34 - 3^3 + 2 \times 4^2$
- **g** $\sqrt{144} \div 12 + 2^2 - 5$
- **h** $22 - 2 \times 11 + 4^2 \div \sqrt{16}$
- **i** $7 - 3^2 + \sqrt{4} \times 3^3$
- **j** $(84 \div 7) \times 2^2 - 5 + \sqrt{36}$
- **k** $4 \times \sqrt{49} + 6^2 - \sqrt{81}$
- **l** $5 \times \sqrt{100} - (\sqrt{9} \div 3) \times 7^2$

WE 1.8B **3** Without using a calculator, evaluate each of the following. Remember to start with the innermost bracket.

- **a** $28 \div (12 - 5) + 2 \times 6$
- **b** $(9 + 3) \div (15 - 13)$
- **c** $4^3 - 10 \times (3 + 1)$
- **d** $\sqrt{64} \div 4 + (2 \times 3) + 7$
- **e** $\sqrt{100} \times 5 - 2 \times (2 + 3)$
- **f** $3 \times [8 + (2 \times 9 - 4)] + 20$
- **g** $[2 \times (3 + 4 - 5) + 4] \div 4$
- **h** $3 - 22 \div (4 + 7) + 2^3$
- **i** $8 \div [2 \times (4 - 3) + 2] + \sqrt{16}$
- **j** $2^3 + (5 - 3) \times 9 - \sqrt{4}$
- **k** $4^3 - [7 - 5 \times (3 + 8)] + 3$
- **l** $(5^2 - 5) \times (3^2 + 3) \times (4^2 - 4)$

4 Use the order of operations to complete this question.

- **a** Evaluate each of the following pairs of expressions.
  - **i** $(25 \div 5) \times 5$ and $25 \div (5 \times 5)$
  - **ii** $(30 \div 2) \times (2 + 1)$ and $30 \div [2 \times (2 + 1)]$
  - **iii** $[(22 + 10) \div 4] \times (7 - 5)$ and $(22 + 10) \div [4 \times (7 - 5)]$
- **b** Sam provided the following working to evaluate the expression $81 \div 9 \times 3$.

  $81 \div 9 \times 3 = 81 \div 3 \times 9 = 27 \times 9 = 243$.
  - **i** Explain what Sam did incorrectly.
  - **ii** Rewrite the original expression twice with brackets in different locations, as in part **a**.
  - **iii** Evaluate both of your expressions.

5 For each calculation:

- **i** find an estimate of the answer by first rounding each number to its leading digit
- **ii** use a calculator to work out the exact answer.
  - **a** $69 \times (28 + 11) - 51 \times 2$
  - **b** $(67 + 31) \div (12 - 5)$
  - **c** $815 \times (61 - 49) + 17 \times 3$
  - **d** $19 \times (9 - 4) + 3 \times (52 + 27)$
  - **e** $\sqrt{100} \times 7 + 2 \times (3^2 + 42)$
  - **f** $[13 \times (7 - 5) + 28] \times 2$

6 For each of the following:

- **i** write the problem using numbers and symbols, using brackets where necessary
- **ii** work out the result.
  - **a** 10 divided by 2, then subtracted from the product of 11 and 4
  - **b** the quotient of 12 and 4, all multiplied by 2 to the power of 3
  - **c** 9 to the power of 2 take away 3 to the power of 3, all doubled
  - **d** 9 subtracted from 17 then multiplied by the square root of 4, all shared between 2
  - **e** the product of the square root of 16 and 6, then divided by 2 to the power of 3
  - **f** 3 less than the difference of 14 and 7

Exercise 1.8B: Problem solving and reasoning

7 The average of a set of scores is calculated as follows:

average = (total sum of the scores) ÷ (number of scores)

For example, the average of 9, 10 and 14 is found by working out $(9 + 10 + 14) \div 3$.

- **a** Use the correct order of operations to find the average of 9, 10 and 14.
- **b** Find the average of each set of numbers.
  - **i** 15, 18 and 24
  - **ii** 3, 5, 6, 8, 9 and 11
  - **iii** 278 and 356
  - **iv** 7, 7, 11 and 15
  - **v** 50, 51, 52, 52 and 60
  - **vi** 50, 20, 30, 40, 60, 50 and 30

8 Shaz makes scooters and skateboards. She has made eight scooters and eight skateboards but needs to order wheels. If each scooter has two wheels and each skateboard has four wheels, how many wheels does Shaz need to order?

9 Ines is in charge of transporting four Year 7 classes to a local farm on a school excursion. She decides that they will hire coaches. Each class consists of 21 students and two teachers. One coach can carry 48 people including the driver.

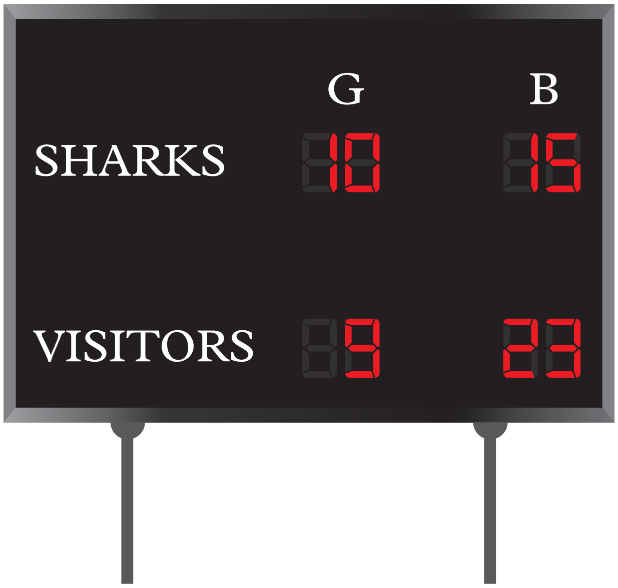
- **a** If each coach is driven by one of the teachers, how many coaches does Ines need to hire?
- **b** How many spare seats will there be?
- **c** The cost of hiring the coaches is shared evenly across all the students and teachers. If it costs $598 to hire one coach for a day, how much would each person need to pay?

10 Kyle goes to the store to buy ingredients for his cake. He buys three cartons of 12 eggs, but four eggs from each carton break on his way home. He returns to the store and buys another carton of 12 eggs. If all the eggs in the fourth carton are intact, how many unbroken eggs does Kyle have in total?

11 Jun is purchasing all the tennis balls for her club for the season. Each tennis student at the club needs eight balls for them to use during practice. At the busiest time of the day there are four classes running. There is a class with two students, a class with three students, a class with one student and a class with five students. How many tennis balls should Jun purchase?

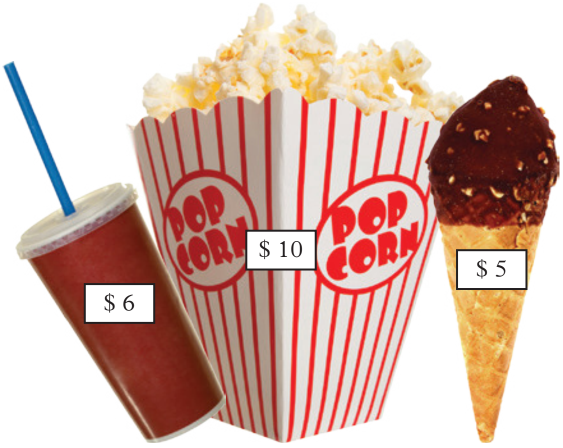
12 Sam is organising food for a movie night. He buys enough party pies for each attendee to have four pies. He buys 96 pies, but then finds out that two people cannot come to the movie night. How many people will attend the movie night?

13 In Australian Rules football, each team scores goals and behinds. Each goal has a value of six points and each behind has a value of one point. During a match, the scoreboard usually shows the number of goals, behinds and points both teams have scored. The image does not show the total number of points.



- **a** Write statements to show the total points scored by each team. Use the numbers and operations needed to calculate the total:
  - **i** points scored by the Sharks
  - **ii** points scored by the visitors.
- **b** Calculate the number of points each team scored.
- **c** At this stage of the match, which team is winning and by how much?

14 You and a group of friends go to the cinema. At the cinema kiosk, you order eight medium soft drinks, four jumbo containers of popcorn and seven ice creams. This photo on the right shows the prices for each item.



a Write a mathematical calculation to show how to work out the total cost of the food and drink.

b Calculate the total cost.

15 Jordan completed a worksheet on the order of operations. Identify the errors in Jordan's working.

a
$$\begin{aligned} 5 \times 7 - 2 \times (3 - 1)^2 - 1 &= 35 - 2 \times 2^2 - 1 \\ &= 35 - 4^2 - 1 \\ &= 35 - 16 - 1 \\ &= 18 \end{aligned}$$

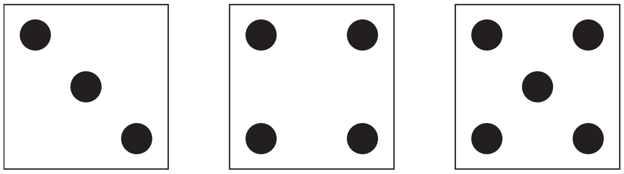
b
$$\begin{aligned} (3 + 4)^2 - 2^2 \times (5 - 2) &= 3 + 4^2 - 2^2 \times 3 \\ &= 3 + 16 - 4 \times 3 \\ &= 15 \times 3 \\ &= 45 \end{aligned}$$

Exercise 1.8C: Challenge

16 In the late 1780s, students in a class were asked to add the whole numbers from 1 to 100. A student named Johann Carl Friedrich Gauss (who later became a famous mathematician) correctly answered the question in less than one minute by using the strategy of adding pairs of numbers. Try this calculation yourself. Can you find the answer in less than one minute?

17 Shubhra tells Roneet to roll three dice and, regardless of which three numbers are rolled, he will be able to make all the numbers from 1 to 20 using only BIDMAS, including squares and square roots.

a Roneet rolls the three dice and the numbers that come up are: 3, 4 and 5. Use each of 3, 4 and 5 once only and BIDMAS to make the numbers from 1 to 20.



b Roll three dice and repeat part **a** with the numbers you roll.

c Is Shubhra telling the truth? Are there any sets of three numbers from 1 to 6 that cannot be used to make all the numbers from 1 to 20?

18 Simplify the following problems.

a $\dfrac{45}{8(9-8)-3} + \dfrac{2(4)^2 - 5\sqrt{4}}{100 \div 50}$

b $\dfrac{(6-5)^2 + (7-4)^2}{\sqrt{25} - \sqrt{9}}$

Online resources:

- **Interactive skillsheet** Order of operations
- **Worksheet** Using the correct order of operations
- **Investigation** Four 4s
- **Quick quiz** 1.8

Lesson 1.9

Review: Whole numbers

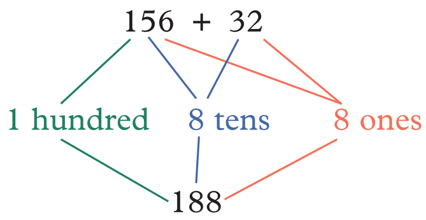
Module summary

Writing numbers

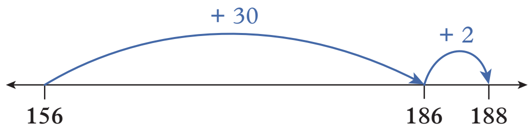
Basic numeral	32 456
Expanded notation	3 × 10 000 + 2 × 1000 + 4 × 100 + 5 × 10 + 6 × 1
Worded form	Thirty-two thousand, four hundred and fifty-six

Mental addition and subtraction

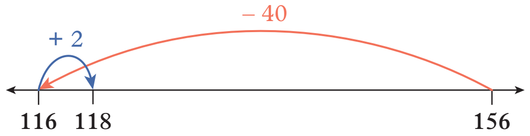
- By-parts method



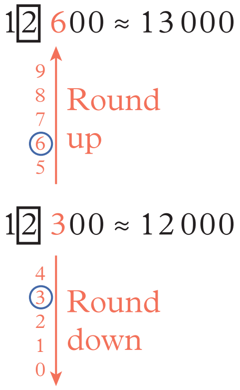
- Jump method



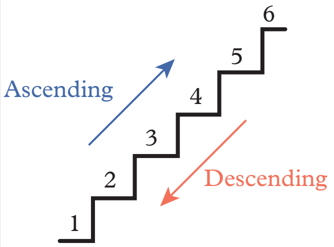
- Compensation method



Rounding



Ordering



- Less than <
- Greater than >

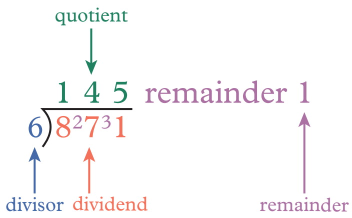
Vertical addition

$$\begin{array}{r} \overset{1}{2}29 \\ +\ 342 \\ \hline 571 \\ \hline \end{array}$$

Vertical subtraction

$$\begin{array}{r} 3\overset{1}{\cancel{2}}\overset{11}{\cancel{1}} \\ -\ 204 \\ \hline 117 \\ \hline \end{array}$$

Division



Multiplying by powers of 10

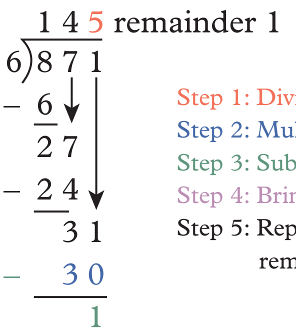
- $22 \times 1\underline{0} = 22\underline{0}$
- $22 \times 1\underline{00} = 22\underline{00}$
- $22 \times 1\underline{000} = 22\,\underline{000}$

Long multiplication

1. Write
2. Multiply by ones
3. Multiply by tens
4. Don’t forget the zero(s)
5. Add the results

$$\begin{array}{rrl} & \overset{2}{3}4 & \\ \times & 26 & \\ \hline & 204 & (34 \times 6) \\ + & 680 & (34 \times 20) \\ \hline & 884 & \\ \hline \end{array}$$

Long division



Step 1: Divide
Step 2: Multiply
Step 3: Subtract
Step 4: Bring down
Step 5: Repeat or remainder

Order of operations

B	**Brackets**	Operations inside brackets are always performed first. For example, 5 × (10 – 2) = 5 × 8 = 40
I	**Indices (exponents)**	Next, evaluate exponents and roots. For example, $3^3 - 4 = 27 - 4 = 23$
D **M**	**Division** **Multiplication**	Then, working from **left** to **right**, perform any multiplication or division before adding or subtracting. For example, 5 × 3 – 4 + 8 ÷ 4 = 15 – 4 + 2 = 13
A **S**	**Addition** **Subtraction**	Finally, working from **left** to **right**, perform any addition and subtraction. For example, 15 – 4 + 2 = 13

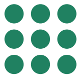
Exponents and squares

$2^2 = 2 \times 2 = 4$

$\sqrt{4} = 2$

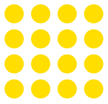
$3^2 = 3 \times 3 = 9$

$\sqrt{9} = 3$



$4^2 = 4 \times 4 = 16$

$\sqrt{16} = 4$



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Review questions 1.9A: Mathematical literacy review

Review quiz
Take the review quiz to assess your knowledge of this module.

Quizlet
Test your knowledge of this module by working individually or in teams.

The following key terms are used in this module:

- ascending order
- associative law
- base
- basic numeral
- BIDMAS
- by-parts method
- commutative law
- compensation method
- descending order
- digit
- distributive law
- dividend
- divisor
- estimate
- expanded notation
- exponent
- exponent form
- index
- jump method
- leading digit
- perfect square
- place value
- product
- quotient
- remainder
- rounding
- square
- square root
- whole numbers
- worded form

1 Which key term can be used instead of exponent?

2 Which mental strategy for addition involves adding the digits in each place value separately?

3 What do the letters in the acronym BIDMAS stand for?

4 Identify the key terms being referenced in each of these definitions.

- **a** the result of a multiplication
- **b** arranging numbers from smallest to largest
- **c** any one of the ten numerals from 0 to 9

5 Using an example, provide a definition in your own words for the following key terms.

- **a** associative law
- **b** dividend
- **c** square root

6 Complete the following sentences using words from the key terms list.

- **a** When a number is written in exponent form, the ________ is the number that is being multiplied and the ________ is the number of times it is multiplied by itself.
- **b** Rounding a number to its ________ is to round a number to the first digit (based on the value of the second digit).

Review questions 1.9B: Multiple choice

1.1 **1** Which list of numbers is written in ascending order?

A 1053, 531, 510, 501
B 345, 354, 3045, 3405
C 2461, 2614, 2641, 2611
D 85, 83, 81, 82, 87, 89

1.1 **2** Which number is an approximation of 78 945 rounded to the nearest ten thousand?

A 70 000 **B** 78 900 **C** 79 000 **D** 80 000

1.2 **3** What is 14 985 + 486 + 1987 + 9135?

A 14 373 **B** 25 483 **C** 26 593 **D** 174 805

1.3 **4** Which calculation gives an answer of 42?

A 96 – 42 **B** 1569 – 584 **C** 7859 – 7813 **D** 9026 – 8984

1.4 **5** Using the distributive law, 6 × 45 becomes:

A 6 × 40 + 6 × 50 **B** 6 × 40 + 6 × 5 **C** 45 × 6 + 4 × 5 **D** 45 × 6 + 40 × 6

1.5 **6** Which product has the greatest value?

A 48 × 49 **B** 13 × 78 **C** 26 × 54 **D** 89 × 21

1.6 **7** Which division does *not* have a remainder of 4?

A 79 ÷ 5 **B** 36 ÷ 8 **C** 43 ÷ 10 **D** 22 ÷ 6

1.7 **8** What is 5 × 5 × 5 × 8 × 8 in exponent form?

A $3^5 \times 2^8$ **B** $5^3 + 8^2$ **C** 40^5 **D** $5^3 \times 8^2$

1.8 **9** What is 5 + 4 × 9 – 3?

A 15 **B** 38 **C** 54 **D** 78

1.8 **10** $70 - (2 \times 3)^2 + 4$ is equal to:

A 38 **B** 62 **C** 49 **D** 64

Review questions 1.9C: Short answer

1.1 **1** Write each number in expanded notation.

a 467 **b** 7856 **c** 9030 **d** 15 452

1.1 **2** Write each list of numbers in ascending order.

a 5347, 547, 53 047, 57 **b** 87 605, 87 506, 87 056

1.1 **3** Write an approximation of each number by rounding to the nearest hundred.

a 392 **b** 9488 **c** 554 **d** 12 945

1.2 **4** Evaluate each sum.

a 75 + 49 **b** 83 + 32 **c** 561 + 224 **d** 85 + 33

1.2 **5** Evaluate each of the following.

a 467 + 56 + 7801 + 943 **b** 383 604 + 2557 + 16 092

1.2 **6** Find two three-digit whole numbers that add to 478 if:

a both numbers are odd **b** both numbers are even.

1.3 **7** Evaluate each of the following differences.

a 9564 – 5381 **b** 371 625 – 38 047

c 17 659 – 9816 **d** 101 011 – 59 678

1.3 **8** Evaluate each of the following.

a 4895 – 1625 – 325 **b** 978 – 486 – 239

1.4 **9** Use a mental strategy to evaluate each of the following.

a 38 × 11 **b** 41 × 21

c 5 × 8 × 7 **d** 72 × 14

e 2 × 48 × 4 **f** 91 × 5 × 2

1.4 **10** Calculate each product.

a 68×7

b 91×6

c 546×3

d 741×9

e 2458×8

f $17\,853 \times 4$

1.5 **11** Use a strategy to evaluate each of the following.

a 389×100

b 412×6000

c 3400×200

d $500 \times 300 \times 40$

e $25 \times 48 \times 4$

f $931 \times 50 \times 3$

1.5 **12** Determine the value of each of the following.

a 542×37

b 456×78

c 6135×29

d 9856×11

e $25 \times 96 \times 2$

f $122 \times 43 \times 5$

1.6 **13** Evaluate each of the following.

a $850 \div 6$

b $\frac{68\,445}{4}$

c $\frac{7521}{3}$

d $567\,204 \div 7$

1.6 **14** Evaluate each of the following.

a $45\,987 \div 11$

b $18\,423 \div 16$

c $\frac{5648}{23}$

d $\frac{78\,525}{45}$

1.6 **15** Use a strategy to evaluate each of the following.

a $7000 \div 10$

b $80\,000 \div 2000$

c $15\,000 \div 30$

d $210\,000 \div 700$

1.7 **16** Write each of the following in expanded form and calculate the value as a basic numeral.

a 2^6

b $5^3 \times 3^2$

c $2^3 \times 7^2$

d $6^2 \times 4^3$

1.7 **17** Write 10 000 in exponent form with a base of 10.

1.7 **18** Write each of these in exponent form and evaluate.

a seven squared

b the square of six

c the square of 19

d 8 squared

1.7 **19** Calculate the value of each of these square roots.

a $\sqrt{25}$

b $\sqrt{81} + 4^2$

c $\sqrt{64} - \sqrt{16}$

d $\sqrt{100} + \sqrt{9} - \sqrt{49} - \sqrt{36}$

1.8 **20** Use the correct order of operations to perform each of the following calculations.

a $7^2 - 6 \times 4 + 3$

b $8 \times (19 - 13) + 2 \times \sqrt{36}$

c $4 + 3 \times (5 - 2)^2$

d $16 - 2 \times (3 + 4) + 9$

1.8 **21** Evaluate each of the following.

a $6 \times [12 - (8 - \sqrt{4})]$

b $100 - [5 \times (3 - 2)]^2$

c $[6^3 \div (3^2 - \sqrt{9})] \div 9$

d $(3 \times 3)^2 - (2 \times 4)^2 + \sqrt{49}$

Review questions 1.9D: Mathematical modelling

1 Saina gives her dad the following question:

$20 + 10 \div 5 \times 6$

'Got it!', says Dad. 'The answer is 36.'

'Uh-uh, Dad, you're wrong', says Saina.

- **a** What did Saina's dad do incorrectly?
- **b** Add a pair of brackets to make the order of operations clear.
- **c** What is the correct answer?

2 Jordan wrote a list of the type and quantity of food he needed for his birthday party. He carefully noted the price per item during his shopping trip.

Item	Quantity	Price per item
Party Pie 24 pack	4	$4
Allen's mixed lollies	3	$3
Chips 100 g	4	$2
M&Ms 250 g	2	$4
Freddo frogs 20 pack	2	$5
Cocktail sausages 2 kg pack	1	$6
Frozen pizza	1	$10
Mudcake	3	$4

- **a** Calculate how much Jordan spent on each item.
- **b** How much did he spend in total?
- **c** If he paid with two $50 notes, how much change did he get?
- **d** What is the cheapest item on the list?
- **e** Which item did he spend the least money on? Is this different from the answer to part **d**? Explain why or why not.
- **f** Each bag of Allen's mixed lollies has 40 lollies. How many lollies in total does Jordan have?
- **g** Jordan wants lolly bags for himself and his friends with at least 10 lollies in each bag. What is the maximum number of friends he can invite?
- **h** Jordan adds Freddo frogs to the lolly bags. How many Freddo frogs does each friend get? How many Freddo frogs are left over?
- **i** Jordan and his friends need to be divided into equal groups for games. Using your answer from part **g**, what size could the groups be if there must be at least three people in each?

3 Tom and Kate are elite supercar drivers. They are competing in the next Melbourne 400 championships. Four races are held over four days and each race is 100 km long.

- **a** If there are 19 laps in each race, approximate the distance of each lap in kilometres.
- **b** If Tom has completed 246 km, which day of the event is it?
- **c** There are 16 turns in the track. Over the four days, how many turns will each driver make?
- **d** Lap times are usually around 2 minutes each. On Day 1 of the race, Kate's incredibly consistent lap time was 1 minute and 56 seconds, and Tom's consistent lap time was 2 minutes and 6 seconds. Determine each driver's total time for one day's race. Give your answer in minutes and seconds.

e On the second day of the race, Kate's lap time was a little faster at 1 minute 49 seconds but she took an extra long pit stop time of 3 minutes. Was Kate's total time shorter on Day 1 or Day 2? State the difference between the two times.

f For each car, pit stop teams can change between hard tyres and soft tyres. Hard tyres are not as fast as soft tyres, but they last longer on the track. Soft tyres provide the driver with more grip on the road so that they can go faster, but they don't last long before the driver must change the tyres. During a pit stop, the tyre must be changed from soft to hard, or from hard to soft.

On Day 3, Tom is completing each lap on a set of hard tyres at a consistent pace of 2 minutes and 1 second. With 4 laps to go, Tom can change his tyres again. Another pit stop will take 15 seconds, and he will complete each lap on the new soft tyres in 1 minute and 55 seconds. Should Tom pit stop again and, if so, how much more quickly will he complete the race?

Checklist

Now that you have completed this module, reflect on your ability to do the following.

I can do this	I need to review this
☐ Identify the place value of a digit in a number ☐ Write numbers in expanded notation ☐ Round whole numbers	☐ Go back to Lesson 1.1 Place value
☐ Use mental strategies to add whole numbers ☐ Use the vertical addition algorithm for large numbers	☐ Go back to Lesson 1.2 Adding whole numbers
☐ Use mental strategies to subtract whole numbers ☐ Use the vertical subtraction algorithm for large numbers	☐ Go back to Lesson 1.3 Subtracting whole numbers
☐ Use mental strategies to multiply whole numbers ☐ Find the product of two numbers using short multiplication ☐ Multiply whole numbers by powers of 10	☐ Go back to Lesson 1.4 Multiplying whole numbers
☐ Use long multiplication to multiply a two-digit number by a two-digit number ☐ Use long multiplication to multiply a three-digit number by a two-digit number	☐ Go back to Lesson 1.5 Long multiplication
☐ Calculate quotients using short division ☐ Calculate quotients using long division	☐ Go back to Lesson 1.6 Dividing whole numbers
☐ Convert numbers between exponent form and expanded form ☐ Evaluate numbers in exponent form ☐ Evaluate the square root of a number	☐ Go back to Lesson 1.7 Exponents and square roots
☐ Use BIDMAS to apply the correct order of operations in calculations ☐ Use BIDMAS to solve worded problems	☐ Go back to Lesson 1.8 Order of operations

2

Factors and primes

Lessons

Prerequisite skills

Diagnostic pre-test
Take the diagnostic pre-test to assess your knowledge of the prerequisite skills listed below.

Interactive skillsheets
After completing the diagnostic pre-test, brush up on your knowledge of the prerequisite skills by using the interactive skillsheets.

- ✔ Counting in multiples
- ✔ Times tables
- ✔ Multiplying and dividing whole numbers
- ✔ Exponents

Curriculum links

- Represent natural numbers in expanded notation using powers of 10, and as products of powers of prime numbers using exponent notation (VC2M7N02)

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Materials

✔ Calculator

Lesson 2.1

Number sequences

Learning intentions

By the end of this lesson you will be able to ...

✔ identify the rule of a sequence and find more terms.

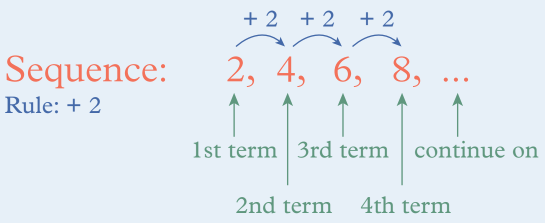
Inter-year links

Support	Number sequences
Year 8	1.4 Multiples, factors, exponents and roots

Number sequences

Key content video
Number sequences

- A **number sequence** is an ordered list of numbers with a set rule.
- A **term** is an individual number in a sequence.
- A **rule** is a description of the relationship between each term and the next term in a sequence.



Finding the rule for a sequence

- To find the rule for a sequence, consider all possible operations including:
  - → addition
  - → subtraction
  - → multiplication
  - → division
  - → exponents
  - → square roots.

Worked example 2.1A Identifying the rule in a sequence

Identify the rule in words for each sequence.

a 3, 6, 9, 12, ... **b** 136, 94, 52, 10, ... **c** 2, 4, 8, 16, 32, ...

THINK

1 Look for a pattern between the numbers in the sequence.

2 Check the pattern or rule works with all values.

3 Describe the pattern in a sentence.

WRITE

a Add 3

+ 3 + 3 + 3
3, 6, 9, 12, ...

Each term is 3 more than the term before.

b Subtract 42

− 42 − 42 − 42
136, 94, 52, 10, ...

Each term is 42 less than the term before.

c Multiply by 2

× 2 × 2 × 2 × 2
2, 4, 8, 16, 32, ...

Each term is double the value of the term before.

Worked example 2.1B Generating terms in a sequence

Generate the next two terms in each sequence.

a 11, 22, 33, 44, __, __.

b 800, 400, 200, 100, __, __.

THINK	**WRITE**
a **1** Identify the rule for the sequence. Each term is 11 more than the term before.	**a** + 11 + 11 + 11 + 11 + 11 11, 22, 33, 44, ___, ___, …
2 Add 11 to 44 for the first new term, then add 11 again for the second new term.	44 + 11 = 55 and 55 + 11 = 66 The next two terms in the sequence are 55 and 66.
b **1** Identify the rule for the sequence. Each term is divided by 2 to find the next term.	**b** ÷ 2 ÷ 2 ÷ 2 ÷ 2 ÷ 2 800, 400, 200, 100, ____, ____, …
2 Divide 100 by 2 for the first new term, then divide by 2 again for the second new term.	100 ÷ 2 = 50 and 50 ÷ 2 = 25 The next two terms in the sequence are 50 and 25.

Helpful hints

✔ You may think that sequences *always* increase as you move to the right, but sequences can *decrease* as well. For example, 24, 12, 6, 3, …

Learning pathways

▲ 1–7, 9, 10(a, b), 11–14

■ 1(d–f), 2(d–f), 3–6, 8–12, 14, 15

⬟ 1(d–f), 2(d–f), 4, 6, 9–16

ANS p503

Exercise 2.1A: Understanding and fluency

WE 2.1A **1** Identify the rule in words for each sequence.

a 4, 6, 8, 10, 12, …

b 9, 18, 27, 36, 45, …

c 5, 10, 15, 20, 25, …

d 24, 20, 16, 12, 8, …

e 810, 270, 90, 30, 10, …

f 4, 16, 64, 256, 1024, …

2 Generate the first three terms of the following sequences.

a Starting at 8, multiply by 3 to get the next term.

b Starting at 500, divide by 5 to get the next term.

c Starting at 23, subtract 7 to get the next term.

d Starting at 1000, add 200 to get the next term.

e Starting at 25, multiply by 2 then subtract 20 to get the next term.

f Starting at 18, add 6 then divide by 2 to get the next term.

WE 2.1B **3** Generate the next two terms in each sequence.

a 6, 12, 18, 24, ______, ______
b 7, 14, 21, 28, ______, ______
c 78, 72, 66, 60, ______, ______
d 1408, 704, 352, 176, ______, ______
e 91, 78, 65, 52, ______, ______
f 4, 16, 64, 256, _____, ______

4 Identify the sixth term in each sequence.

a 4, 8, 12, …
b 6, 11, 16, …
c 99, 93, 87, …
d 76, 61, 46, …
e 5, 10, 20, …
f 4096, 1024, 256, …

5 Find the missing terms in the following sequences.

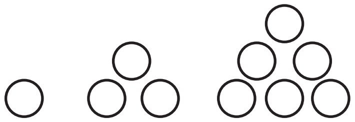
a 5, 12, ____, 26, ____, 40, 47, 54, …
b 53, 45, 37, ____, 21, 13, ____, …
c ____, 15, 24, 33, 42, 51, ____, …
d 300, ____, 250, 225, 200, ____, 150

6 Find the missing terms in the following sequences.

a 4, 8, ____, 32, 64, 128, ____, 512, …
b 4000, ____, 1000, 500, ____, 125, …
c ____, 15, 45, 135, 405, ____, …
d 729, 243, ____, ____, 9, 3, …

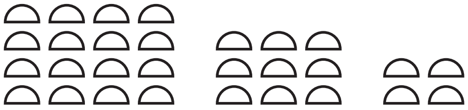
7 Consider the diagrams on the right.

a Draw the next term in this sequence.
b What is the rule for this sequence?



8 Consider the diagrams on the right.

a Draw the next term in this sequence.
b What is the rule for this sequence?



9 For each of the following:

i Generate the next few terms in the sequence.
ii Write the rule for the sequence.
iii Rewrite the sequence in exponent form. What is the twelfth term in the sequence in exponent form?

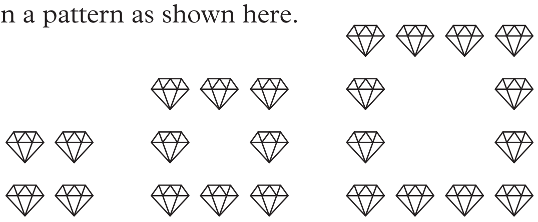
a 2, 4, 8, 16, 32, 64, ____, ____, ____, ____
b 3, 9, 27, ____, ____, ____
c 5, 25, 125, ____, ____, ____
d 10, 100, 1000, ____, ____, ____

Exercise 2.1B: Problem solving and reasoning

10 When given a rule and any term in a sequence, you can find any other term in the sequence.

a The fifth term in a sequence is 35. If the rule for this sequence is add 5, what is the first term?
b The eighth term in a sequence is 90. If the rule for this sequence is subtract 12, what is the third term?
c The sixth term in a sequence is 192. If the rule for this sequence is multiply by 2, what is the first term?
d The sixth term in a sequence is 81. If the rule for this sequence is divide by 3, what is the second term?

11 Trinh has a collection of diamonds. She arranges the diamonds in a pattern as shown here.



a Draw the next term in this sequence.
b Write down the sequence that represents the number of diamonds in each term.
c Identify the rule in words for the sequence.
d How many diamonds would be in the tenth term of this pattern?
e Trinh only has enough diamonds for exactly six terms in the sequence. How many diamonds does she have?

12 A perfect cube is the result of a positive whole number that is multiplied by itself twice. The first perfect cube is 1, as 1 × 1 × 1 = 1.

Write the sequence of the first 10 perfect cubes.

13 Chao planted a small tree in his backyard on New Year's Day. The tree was 60 cm tall when he planted it. He measured how tall the tree was at the start of each month. Over the year, Chao found that the tree grew by 8 cm each month. How tall was the tree on New Year's Day of the following year?

14 Naisha made a video of herself performing a new dance. She liked it so much that the following day she shared the video with four of her friends. Naisha's friends were impressed with her video so the next day each of her friends shared it with four new people.

Determine the following if this sharing pattern continues in this way.

a How many people will see Naisha's video for the first time on the seventh day after she created it?

b How many people in total will see the video in the seven days, including Naisha?

15 Saina's dad asks her, 'I am thinking of a sequence that begins 4, 16, … What is the rule for the sequence, Saina?' She replies, 'Don't be silly Dad. You need at least three terms of a sequence to find the rule!'

Who is correct, Saina or her dad? Give a reason for your answer.

Exercise 2.1C: Challenge

16 Arvin works in a fruit and vegetable shop. He is given the task of arranging 200 apples on a display shelf. Arvin decides that he will arrange the apples in rows. Each row of apples will have two fewer apples than the previous row.

a Find a pattern for 200 apples that follows the rule: subtract 2.

b How many rows will there be in the display?

c How many apples will there be in the bottom row?

d How many apples will there be in the top row?

e Is there more than one pattern that has a total of 200 apples and follows the rule subtract 2? Why or why not?

Online resources:

Interactive skillsheet
Number sequences

Investigation
The cost of stationery

Quick quiz
2.1

Lesson 2.2

Divisibility rules

Learning intentions

By the end of this lesson you will be able to ...

✔ determine if a number is divisible by 2, 3, 4, 5, 6, 8, 9 or 10.

Inter-year links

Support	Dividing whole numbers
Year 8	1.3 Multiplying and dividing whole numbers

Divisibility

- When performing a division, if the remainder is zero then we can say that the dividend is divisible by the divisor. For example, $108 \div 9 = 12$, so 108 is divisible by 9, but 91 is not divisible by 6 as there is a remainder of 1.

$$\frac{\overset{\text{dividend}}{\overset{\downarrow}{108}}}{\underset{\text{divisor}}{\underset{\uparrow}{9}}} = \underset{\text{quotient}}{\underset{\uparrow}{12}}$$

$$\underset{\text{dividend}}{\underset{\uparrow}{91}} \div \underset{\text{divisor}}{\underset{\uparrow}{6}} = \underset{\text{quotient}}{\underset{\uparrow}{15}} \text{ remainder } \underset{\text{remainder}}{\underset{\uparrow}{1}}$$

Divisibility rules

Key content video
Divisibility rules

- We can use divisibility rules to test if one number is divisible by another.

÷ 2 The last digit of the number is even.	÷ 3 The sum of all the digits in the number is divisible by 3.	÷ 4 The number made by the last two digits is divisible by 4.
÷ 5 The last digit is 5 or 0.	÷ 6 The number is divisible by 2 and 3.	÷ 7 There is no easy trick for 7.
÷ 8 The number made by the last three digits is divisible by 8.	÷ 9 The sum of all the digits in the number is divisible by 9.	÷ 10 The last digit is 0.

Worked example 2.2A Using the divisibility rules

Check if 845 026 is divisible by each of the following numbers and provide a reason using the divisibility rules.

a 2 **b** 3 **c** 4

THINK	**WRITE**
a Check if the last digit of the number is even.	**a** 845 02$\boxed{6}$ Since the last digit is even, 845 026 is divisible by 2.
b Check if the sum of all the digits in the number is divisible by 3.	**b** $8 + 4 + 5 + 0 + 2 + 6 = 25$ Since 25 is not exactly divisible by 3, 845 026 is not divisible by 3.
c Check if the number made by the last two digits is divisible by 4.	**c** 845 0$\boxed{26}$ Since 26 is not exactly divisible by 4, 845 026 is not divisible by 4.

Worked example 2.2B Determining divisibility of numbers

Determine if the first number is divisible by the second number.

a 1218 and 8 **b** 43 938 and 6

THINK	**WRITE**
a To determine if the number is divisible by 8, check if the number made by the last three digits is divisible by 8.	**a** 1$\boxed{218}$ $\begin{array}{r} 2\ \ 7 \text{ remainder } 2 \\ 8\overline{)21^{5}8} \end{array}$ Since 218 is not exactly divisible by 8, 1218 is not exactly divisible by 8.
b To determine if the number is divisible by 6, check if the number is divisible by 2 and 3.	**b** 43 93$\boxed{8}$ Since the last digit is even, it is divisible by 2. $4 + 3 + 9 + 3 + 8 = 27$ Since 27 is divisible by 3, 43 938 is divisible by 3. Since 43 938 is divisible by 2 and 3, it is also divisible by 6.

Helpful hints

✔ Be careful when checking for divisibility. If your calculations are incorrect, you might come to the wrong conclusion.

Learning pathways

▲ 1–7, 9, 11, 12 ■ 2, 4–9, 12–16 ⬟ 2, 4–6, 9, 10, 12–18

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Exercise 2.2A: Understanding and fluency

WE 2.2A **1** Check if 546 is divisible by each of the following numbers and provide a reason using the divisibility rules.

a 2 **b** 3 **c** 4 **d** 5
e 6 **f** 8 **g** 9 **h** 10

2 Check if 8640 is divisible by each of the following numbers and provide a reason using the divisibility rules.

a 2 **b** 3 **c** 4 **d** 5
e 6 **f** 8 **g** 9 **h** 10

3 Check if 65 190 is divisible by each of the following.

a 2 **b** 3 **c** 4 **d** 5
e 6 **f** 8 **g** 9 **h** 10

4 Check if 321 272 is divisible by each of the following.

a 2 **b** 3 **c** 4 **d** 5
e 6 **f** 8 **g** 9 **h** 10

WE 2.2B **5** Determine if the first number is divisible by the second number.

a 315 814 and 4 **b** 516 696 and 8 **c** 274 752 and 6
d 26 535 and 9 **e** 367 168 and 8 **f** 74 074 and 6

6 State whether the following statements are true or false.

a 432 is divisible by 6 and 8 **b** 816 is divisible by 3 and 4
c 7265 is divisible by 2 and 5 **d** 6561 is divisible by 8 and 9

Exercise 2.2B: Problem solving and reasoning

7 The Year 7 captains have organised a sausage sizzle. They have ordered 1050 sausages and want to make sure each student can have an equal number of sausages with no leftovers.

a Use the divisibility tests to determine whether the captains can provide 2, 3 or 4 sausages per person without wastage.

b Determine which of these would be the best option for a class of 350 students.

8 Anais has 208 pipe cleaners. She plans to use all of them to make legs for either grasshoppers or octopuses to sell at the school fair. Given that grasshoppers have 6 legs and octopuses have 8 legs, which should she choose to have no pipe cleaners left over?

9 Yanni orders chocolate chips for his cookie business and receives 7398. He wants to guarantee that his customers receive either 8 or 9 chocolate chips in each cookie, while also having no chocolate chips left over. How many chocolate chips should go in each cookie?

10 An Australian soft toy company is exporting 1662 toy quokkas all over the world. They can send the toy quokkas in boxes of 6 or 9. To avoid having any toy quokkas left over, which should they choose?

11 Hayley knows how to test for divisibility by 2, 3, 4 and 5. Explain how she could use two of these divisibility tests to check whether 14 805 is divisible by 15.

12 Scott tells you that if a number is divisible by 4 and 6 then it will also be divisible by 24. Is Scott correct? Give a reason for your answer.

13 Invent a divisibility test for 12. Explain why your test works.

14 There are 60 minutes in an hour.

a Do 118 440 minutes make up an exact number of hours?

b Which divisibility rules could you use to check this?

15 The following is a divisibility test for 11.

→ Starting at 0, add the units digit and subtract the tens digit.

→ Continue from right to left, alternately adding and subtracting the digits of the number you started with. Remember to include any 0 digits in the sum.

→ If the result is divisible by 11, then the original number is also divisible by 11.

For example, consider 10 648.

The alternating sum is 8 – 4 + 6 – 0 + 1 = 11.

11 is divisible by 11.

Therefore, 10 648 is divisible by 11.

Check to see if these numbers are divisible by 11.

a 53 411

b 962 016

Exercise 2.2C: Challenge

16 Find digits that go in the boxes to make a five-digit number divisible by 450.

373☐☐

17 A number is called a Harshad number if it is divisible by the sum of its digits. For example, 18 is a Harshad number because 18 is divisible by 1 + 8 = 9.

a Explain why all whole numbers between 1 and 9 are Harshad numbers.

b Find all Harshad numbers less than 60.

c Are all numbers divisible by 6 Harshad numbers? Give a reason for your answer.

d Are all numbers divisible by 10 Harshad numbers? Give a reason for your answer.

18 A teacher writes a mystery number on a white board. They then ask their class to list the divisors of the number, one divisor per student.

The first student says, 'The number is divisible by 2.'

The second student says, 'The number is divisible by 3.'

The third student says, 'The number is divisible by 4.'

…

The 30th student says, 'The number is divisible by 31.'

The teacher then tells the class, 'Two students were wrong when they made their claim. These students spoke one after the other.'

Which two students spoke incorrectly?

Online resources:

Interactive skillsheet
Divisibility rules

Investigation
No remainders left over

Quick quiz
2.2

Lesson 2.3

Multiples and the lowest common multiple

Learning intentions

By the end of this lesson you will be able to …

✔ find multiples of a number and determine the lowest common multiple of two numbers.

Inter-year links

Support	Multiples and factors
Year 8	1.4 Multiples, factors, exponents and roots

Key content video
Multiples and the lowest common multiple

Multiples

- The **multiples** of a given whole number result from multiplying that number by another whole number.

  For example, the multiples of 3 are 3, 6, 9, 12, 15, 18, …
- There are infinitely many multiples of any given whole number.
- Multiples can be written in a multiplication table. The multiples of the orange numbers are shown in blue below. For example, the multiples of 2 can be read in both the row and column labelled by 2.

×	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9
2	2	4	6	8	10	12	14	16	18
3	3	6	9	12	15	18	21	24	27
4	4	8	12	16	20	24	28	32	36
5	5	10	15	20	25	30	35	40	45
6	6	12	18	24	30	36	42	48	54
7	7	14	21	28	35	42	49	56	63
8	8	16	24	32	40	48	56	64	72
9	9	18	27	36	45	54	63	72	81

Lowest common multiple

- **Common multiples** are multiples that appear in the multiples list of two or more different whole numbers.

  For example, the multiples of 3 are 3, 6, 9, 12, 15, 18, 21, 24, … and the multiples of 4 are 4, 8, 12, 16, 20, 24, … so the first three common multiples of 3 and 4 are 12, 24 and 36.
- The **lowest common multiple (LCM)** is the lowest (first) multiple that is common in the multiples list of two or more numbers.

  For example, the multiples of 3 are 3, 6, 9, 12, 15, 18, 21, 24, … and the multiples of 4 are 4, 8, 12, 16, 20, 24, … so the LCM of 3 and 4 is 12.
- To find the LCM of 2 and 3:
  1 List the multiples of 2.
  2 List the multiples of 3.
  3 Circle the common multiples of 2 and 3.
  4 Find the lowest common multiple.

Multiples of 2: 2, 4, 6, 8, 10, 12, …
Multiples of 3: 3, 6, 9, 12, 15, 18, …
LCM: 6

Worked example 2.3A Finding multiples

List the first four multiples of each number.

a 4

b 7

THINK

a For the first four multiples, multiply 4 by 1, 2, 3 and 4.

b For the first four multiples, multiply 7 by 1, 2, 3 and 4.

WRITE

a $4 \times 1 = 4$
$4 \times 2 = 8$
$4 \times 3 = 12$
$4 \times 4 = 16$
The first four multiples of 4 are 4, 8, 12 and 16.

b $7 \times 1 = 7$
$7 \times 2 = 14$
$7 \times 3 = 21$
$7 \times 4 = 28$
The first four multiples of 7 are 7, 14, 21 and 28.

Worked example 2.3B Finding the lowest common multiple (LCM)

Find the lowest common multiple (LCM) of 12 and 30.

THINK

1 List the multiples of 12.

2 List the multiples of 30.

3 Circle the multiples that are the same in the two lists.

4 Find the lowest number in the list of common multiples.

WRITE

Multiples of 12: 12, 24, 36, 48, 60, 72, 84, 96, 108, 120, …
Multiples of 30: 30, 60, 90, 120, 150, 180, …
The LCM is 60.

Helpful hints

✔ Lowest common multiples are not necessarily ‘low’ in value, they can be quite large numbers.
✔ There is no such thing as a ‘highest common multiple’ as any given number has infinitely many multiples.

Learning pathways

△ 1–8, 9(b, d, f, h), 10–15

□ 1(b, c, f, h), 2, 4, 6, 7(a, c, e, f), 8, 9, 12, 14, 16–18

⬠ 3, 5, 7(e, g), 8, 9(c, e, h), 12, 16–20

ANS p505

Exercise 2.3A: Understanding and fluency

WE 2.3A **1** List the first four multiples of each number.

a 10   **b** 6   **c** 9   **d** 5   **e** 11   **f** 30

g 25   **h** 13   **i** 40   **j** 22   **k** 14   **l** 31

2 List all the multiples of 8 between 23 and 65.

3 List all the multiples of 7 between 50 and 100.

4 Find the first four common multiples of 4 and 5.

5 Find the first six common multiples of 3 and 4.

6 The common multiple of two numbers can be found by listing the multiples of each number.

a List the multiples of 5 and 6 up to 60.

b Identify the common multiples of 5 and 6 in the lists from part **a**.

c What is the lowest common multiple of 5 and 6?

WE 2.3B **7** Find the lowest common multiple (LCM) of each pair of numbers.

a 2 and 5   **b** 4 and 7   **c** 6 and 9   **d** 8 and 12

e 5 and 8   **f** 7 and 10   **g** 15 and 20   **h** 8 and 6

8 Find the LCM of each group of numbers.

a 2, 3 and 4   **b** 5, 6 and 20   **c** 3, 4 and 9   **d** 4, 6 and 8

e 5, 3 and 2   **f** 30, 9 and 2   **g** 6, 7 and 8   **h** 2, 3, 4 and 5

9 Find the LCM of each pair of numbers.

a 12 and 8   **b** 14 and 4   **c** 16 and 5   **d** 12 and 11

e 18 and 16   **f** 24 and 8   **g** 20 and 80   **h** 50 and 7

10 Find the multiple of 8 that is closest to each number.

a 85   **b** 110   **c** 220   **d** 300

11 Find the multiple of 14 that is closest to each number.

a 50   **b** 130   **c** 210   **d** 330

12 Find the multiple of 45 that is closest to each number.

a 120   **b** 230   **c** 310   **d** 430

Exercise 2.3B: Problem solving and reasoning

13 Sunny and Jai have made the same number of cupcakes for a fundraising bake sale. Sunny cooked her cupcakes in batches of 9 and Jai cooked his in batches of 11. What is the smallest possible number of cupcakes that each of them could have baked?

14 Daquan has piano lessons on every fifth day and cricket training on every third day. If he has both piano lessons and cricket training on August 5, on what date will he next have both a piano lesson and cricket training?

15 Every fourth paling of this fence is painted blue.

- **a** The fence is made up of 84 palings in total. How many palings are painted blue?
- **b** The fence is to be extended to 128 palings in total. How many more palings will need to be painted blue?

16 According to the timetable, a particular train is expected to depart from the station every 12 minutes. The first train of the day departs at 5:12 am.

- **a** List all the times this train should depart from the station in the first hour of service.
- **b** What is the departure time of:
  - **i** the third train of the day
  - **ii** the sixth train of the day?
- **c** What number train will depart from the station at 8:24 am? Explain your strategy for determining the solution.

17 Kiara is planning to donate a number of hats and water bottles to Year 2 students at a school in Papua New Guinea. The hats are sold in packs of 8 and the water bottles are sold in packs of 13. What is the smallest number of packs of hats and water bottles that she can purchase so that there will be exactly one hat per water bottle?

18 Batman, Superman and Wonder Woman decide to race through a city to determine who is the fastest. Batman uses the speed booster in the Batmobile to pass 7 blocks per boost. Superman leaps through the air at 3 blocks per jump. Wonder Woman runs 5 blocks at a time, then flies 5 blocks at a time, repeating this process.

- **a** If they all land at the finish line at the same time, what is the shortest possible distance of the race in blocks?
- **b** How many speed boosters did Batman use in the race?
- **c** How many leaps did Superman make in the race?
- **d** How many blocks did Wonder Woman run and how many blocks did she fly?

Exercise 2.3C: Challenge

19 Do the numbers 3 and 4 have a highest common multiple? Give a reason for your answer.

20 Consecutive numbers are numbers that follow each other, for example, 7 and 8.

- **a** Find three consecutive numbers (apart from 3, 4 and 5) such that the first number is a multiple of 3, the second is a multiple of 4, and the third is a multiple of 5. Describe the general rule used to determine these sets of numbers.
- **b** Find four consecutive numbers (apart from 6, 7, 8 and 9) such that the first number is a multiple of 6, the second is a multiple of 7, the third is a multiple of 8, and the fourth is a multiple of 9. Describe the general rule used to determine these sets of numbers.

Online resources:

Interactive skillsheet
Lowest common multiple

Investigation
Neon signs

Quick quiz
2.3

ANS
p505

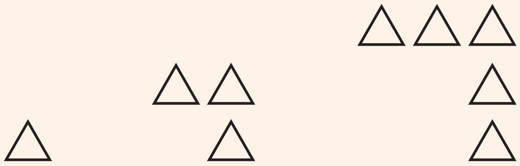
Checkpoint

Checkpoint quiz
Take the checkpoint quiz to check your knowledge of the first part of this module.

2.1 **1** Identify the rule in words for each sequence.
- **a** 5, 7, 9, 11, 13, …
- **b** 12, 24, 36, 48, …
- **c** 3, 6, 12, 24, …

2.1 **2** Generate the first three terms of the following sequences.
- **a** Starting at 9, multiply by 4 to get the next term.
- **b** Starting at 280, subtract 20 to get the next term.
- **c** Starting at 36, add 12 to get the next term.
- **d** Starting at 2000, divide by 5 to get the next term.

2.1 **3** Consider the diagrams on the right.
- **a** Draw the next term in this sequence.
- **b** What is the rule for this sequence?
- **c** How many triangles are in the fifth term?



2.1 **4** Theo is reading a book and has already finished the first five pages. Each day, he reads four more pages.
- **a** How many pages will Theo have read by the end of the fifth day?
- **b** If there are 89 pages in the book, how much longer will it take Theo to finish the book?

2.2 **5** Check if the number 157 464 is divisible by each of the following.
- **a** 2 **b** 3 **c** 4 **d** 5
- **e** 6 **f** 8 **g** 9 **h** 10

2.2 **6** Determine if the first number is divisible by the second number.
- **a** 319 999 and 9 **b** 816 966 and 8 **c** 653 184 and 6

2.2 **7** State whether the following statements are true or false.
- **a** 30 240 is divisible by 9 and 8.
- **b** 34 565 is divisible by 4 and 5.
- **c** 50 625 is divisible by 15.

2.2 **8** A book company has printed 39 366 copies of its latest book called *Swings and Roundabouts*. The company has boxes that hold nine books and boxes that hold eight books. Use divisibility tests to find which boxes it should use to ship the books if it wants every box to be full of books.

2.3 **9** List the first five multiples of each number.
- **a** 7 **b** 12 **c** 15 **d** 24

2.3 **10** A multiple of a number is the result of multiplying that number by another whole number.
- **a** List all the multiples of 6 between 1 and 100.
- **b** List all the multiples of 9 between 1 and 100.
- **c** List all the common multiples of 6 and 9 between 1 and 100.
- **d** What is the LCM of 6 and 9?

2.3 **11** Find the LCM of each pair of numbers.
- **a** 3 and 5 **b** 6 and 8 **c** 7 and 12

2.3 **12** Amiya has netball training on every fourth day and work on every fifth day. If she has both netball training and work on Monday:
- **a** how many days until she next has both on the same day
- **b** on what day of the week will she next have them both?

Lesson 2.4

Factors and the highest common factor

Learning intentions

By the end of this lesson you will be able to ...

✔ find all factors of a number and determine the highest common factor of two numbers.

Inter-year links

Support	Multiples and factors
Year 8	1.4 Multiples, factors, exponents and roots
Year 9	2.3 Factorising using the HCF

Key content video
Factors and the highest common factor

Factors

- The **factors** of a whole number are the whole numbers which divide exactly into that number.
  For example, the factors of 12 are 1, 2, 3, 4, 6 and 12.
- The divisibility rules are helpful when looking for factors of large numbers.

÷ 2	÷ 3	÷ 4	÷ 5	÷ 6
The last digit of the number is even.	The sum of all the digits in the number is divisible by 3.	The number made by the last two digits is divisible by 4.	The last digit is a 5 or 0.	The number is divisible by 2 and 3.

- **Factor pairs** are two numbers that when multiplied give a certain product. It is often easier to find factors in pairs.
  For example, the factor pairs that give a product of 12 are 1 and 12, 2 and 6, and 3 and 4.

Highest common factor

- A **common factor** is a number that divides exactly into two or more whole numbers.
  For example, 12 and 9 have a common factor of 3.
- The **highest common factor (HCF)** is the greatest factor that is common to two or more given numbers.
  For example, the factors of 8 are 1, 2, 4, 8 and the factors of 12 are 1, 2, 3, 4, 6, 12, so the HCF of 8 and 12 is 4.
- To find the HCF of 16 and 24:
  1 List all factors of 16.
  2 List all factors of 24.
  3 Circle the common factors of 16 and 24.
  4 Find the highest common factor.

Factors of 16: 1, 2, 4, 8, 16
Factors of 24: 1, 2, 3, 4, 6, 8, 12, 24
HCF: 8

Worked example 2.4A Finding factor pairs

Complete the following factor pairs.

a 15 = 1 × 15 or 3 × ____

b 18 = 1 × ____ or 2 × ____ or 3 × ____

THINK

a Divide the number by the given factor to find the other half of the factor pair.

b Divide the number by each of the given factors to find the other halves of the factor pairs.

WRITE

a 15 ÷ 3 = 5
15 = 1 × 15 or 3 × 5

b 18 ÷ 1 = 18
18 ÷ 2 = 9
18 ÷ 3 = 6
18 = 1 × 18 or 2 × 9 or 3 × 6

Worked example 2.4B Finding factors

Find all the factors of each number.

a 48

b 16

THINK

a **1** 1 × 48 = 48, so place the 1 on the left and the 48 on the right.

2 2 × 24 = 48, so place the 2 after 1 on the left and the 24 on the right, just before 48.

3 3 × 16 = 48, so place the 3 on the left and the 16 on the right.

4 4 × 12 = 48, so place the 4 on the left and the 12 on the right.

5 5 is not a factor of 48 as 48 is not divisible by 5.

6 6 × 8 = 48, so place the 6 on the left and the 8 on the right.

7 7 is not a factor of 48 as 48 is not divisible by 7.

8 The next possible factor is 8, which has already appeared in the list, so all the factors have been found.

b **1** 1 × 16 = 16 and 2 × 8 = 16, so place them in the table.

2 Using the divisibility rule for 3, 1 + 6 = 7, which is not divisible by 3, so 3 is not a factor of 16.

3 4 × 4 = 16, which is a perfect square. Only write 4 once.

4 16 is not divisible by 5, 6 or 7, so all the factors have been found.

WRITE

a

1		48

1	2		24	48

1	2	3		16	24	48

1	2	3	4		12	16	24	48

1	2	3	4	6		8	12	16	24	48

Factors of 48 are 1, 2, 3, 4, 6, 8, 12, 16, 24, 48.

b

1	2			8	16

1	2	4		8	16

Factors of 16 are 1, 2, 4, 8, 16.

Worked example 2.4C Finding the highest common factor

Find the highest common factor (HCF) of 24 and 36.

THINK

1 List all the factors of 24 and all the factors of 36.

2 Circle the common factors: the factors that appear in both lists.

3 Identify the highest common factor: the common factor with the greatest value.

WRITE

Factors of 24: ①, ②, ③, ④, ⑥, 8, ⑫, 24
Factors of 36: ①, ②, ③, ④, ⑥, 9, ⑫, 18, 36
The HCF is 12.

Helpful hints

- ✔ Be careful not to mix up HCF and LCM!
- ✔ Make sure you use a systematic method for finding all the factors; it is easy to miss one or two.
- ✔ Each factor should only appear once in your list, so the factors of perfect squares should only be recorded once.
  For example, 4 × 4 = 16 gives a single factor of 4.

Learning pathways

△ 1–3, 4(d–i), 5(a, d, f, g), 6–8, 10(c–f), 11(c–f), 12–14, 17

□ 3, 4(d–i), 5(b, e, h), 6(d, e, f), 9, 10(e–h), 11(e–h), 12, 15–19

⬠ 3(d, e, f, i, l), 4(d, h, k), 5(c, i), 6(d, f), 10(g, h), 11(g, h), 12(b, d), 15, 19–22

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Exercise 2.4A: Understanding and fluency

WE 2.4A **1** Complete the following factor pairs.

a 20 = 1 × 20 or 4 × ____
b 24 = 1 × ____ or 2 × ____ or 3 × ____
c 32 = 1 × ____ or ____ × 16 or ____ × ____
d 28 = 1 × ____ or ____ × 14 or 4 × ____
e 49 = 1 × ____ or ____ × 7
f 50 = ____ × 50 or 2 × ____ or 5 × ____

WE 2.4B **2** Find all the factors of each number.

a 15 **b** 18 **c** 9 **d** 12 **e** 10 **f** 14

3 Find all the factors of each number.

a 20 **b** 24 **c** 32 **d** 28 **e** 49 **f** 50
g 6 **h** 66 **i** 56 **j** 81 **k** 27 **l** 100

4 Find all the factors of each number. Use the divisibility tests to check if a number is a factor.

a 44 **b** 54 **c** 60 **d** 64 **e** 75 **f** 105
g 120 **h** 132 **i** 200 **j** 98 **k** 168 **l** 231

5 Use your answers to question **3** to find the common factors of each set of numbers.

a 6 and 20
b 81 and 27
c 50 and 100
d 56 and 100
e 6 and 81
f 32 and 56
g 6, 24 and 27
h 20, 28 and 32
i 28, 49 and 56

6 Find the factors between 10 and 30 of each of the following numbers.

a 26
b 55
c 42
d 63
e 70
f 36

7 The factors of a whole number are the whole numbers which divide exactly into that number.

a List the factors of 18.
b List the factors of 36.
c List the common factors of 18 and 36.
d What is the highest common factor (HCF) of 18 and 36?

8 The factors of a whole number are the whole numbers which divide exactly into that number.

a List the factors of 21.
b List the factors of 45.
c List the common factors of 21 and 45.
d What is the HCF of 21 and 45?

9 The factors of a whole number are the whole numbers which divide exactly into that number.

a List the factors of 12.
b List the factors of 30.
c List the common factors of 12 and 30.
d What is the HCF of 12 and 30?

WE 2.4C **10** Find the HCF of each pair of numbers.

a 8 and 24
b 15 and 27
c 24 and 42
d 5 and 20
e 36 and 32
f 45 and 30
g 63 and 35
h 50 and 100

11 Find the HCF of each group of numbers.

a 4, 8 and 12
b 10, 25 and 30
c 16, 24 and 32
d 15, 6 and 27
e 42, 36 and 18
f 20, 30 and 50
g 38, 26 and 14
h 9, 6, 12 and 18

12 Find the LCM and the HCF of each group of numbers.

a 12 and 30
b 10 and 35
c 4, 12 and 16
d 2, 9 and 18

Exercise 2.4B: Problem solving and reasoning

13 Sophie is thinking of an odd number that is a factor of 24 and is not 1. What number is she thinking of?

14 Max is arranging 18 pieces of sushi on a plate for his guests. He would like to arrange them so that there are the same number of pieces in each row.

a Suggest one way that Max could arrange the sushi on a plate. You may like to present your answer using a diagram.
b How many possible arrangements are there? Describe the arrangements that are possible.

15 Archie has 24 tickets to the cinema and Veronica has 42 tickets to the school dance. They decide to sell them in identical packs that each contain a number of both types of tickets. What is the largest number of identical packs they can make if all tickets are used and there are no tickets left over?

16 For a school fete, Hanna and Josh are deciding how to divide up a batch of 120 cookies so that there is an equal number of cookies in each bag. Suggest at least three ways this could be done.

17 Describe the differences between multiples and factors.

18 The highest common factor is the greatest factor that is common to two or more given numbers.

a What is the highest common factor of 18 and 45?

b What is the 'lowest common factor' of 18 and 45?

c Explain why the 'lowest common factor' of two or more numbers is not particularly useful. Consider the lowest common factors of a few different pairs of numbers.

19 Tobias is making bunches of balloons for his little brother's birthday party. He has 56 purple balloons, 64 green balloons and 104 blue balloons. He wants all the bunches to be identical.

a What is the greatest number of bunches Tobias can make if he uses every balloon?

b How many of each colour will be in each bunch?

c How many balloons will be in each bunch?

Exercise 2.4C: Challenge

20 Find the smallest whole number that has only:

a one factor **b** two factors **c** three factors **d** four factors **e** five factors **f** six factors.

21 The HCF of two numbers is 16. The smaller of the two numbers is 64 less than the larger number. Both numbers are less than 100. Find the two numbers that fit this description.

22 An abundant number is a number that is smaller than the sum of its factors (excluding 1 and itself). The number 12 is the first abundant number. Find all the abundant numbers between 1 and 60.

Online resources:

Interactive skillsheet
Highest common factor

Worksheet
Finding the highest common factor

Quick quiz
2.4

Lesson 2.5

Prime and composite numbers

Learning intentions

By the end of this lesson you will be able to …

✔ determine if a number is prime or composite and find the prime factors of positive whole numbers.

Inter-year links

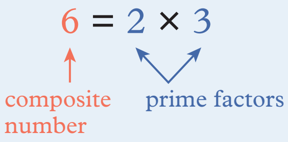
Support	Prime and composite numbers
Year 8	5.7 Factorising
Year 9	2.3 Factorising using the HCF

Key content video
Prime and composite numbers

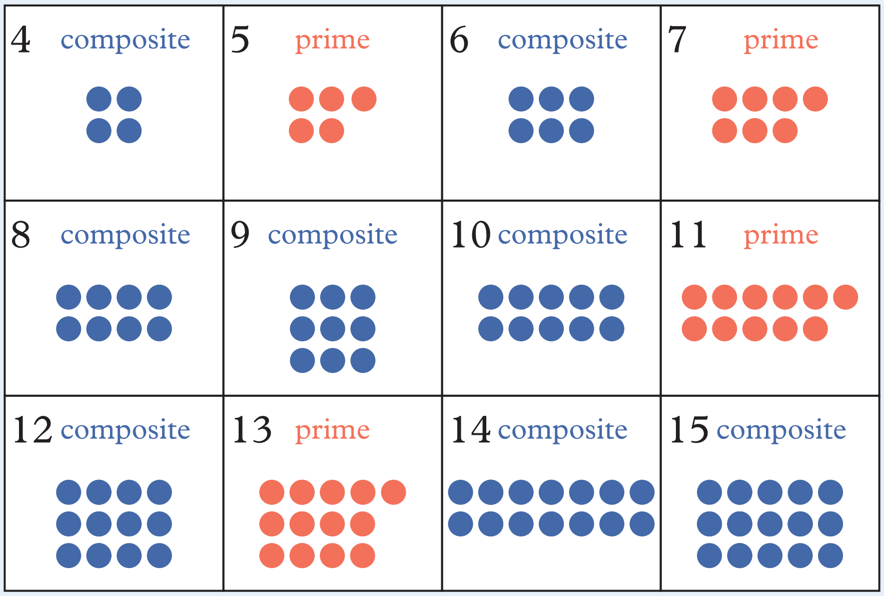
Prime numbers

- A **prime number** is a positive whole number greater than 1 that has exactly two factors: itself and 1.
  For example, the only factors of 7 are 1 and 7, so 7 is a prime number.
- The divisibility rules can be used to quickly check if a number is a prime.
  For example, if a number (other than 2) is even, then it is divisible by 2 and not a prime number.

Composite numbers



- A **composite number** is a positive whole number that is exactly divisible by at least one whole number other than itself and 1.
- All composite numbers can be written as the product of **prime factors**, which are factors that are prime numbers.
- 1 is neither prime nor composite because it only has one factor.
- 0 is neither prime nor composite because it is not a positive number.
- Composite numbers can be arranged in a rectangular array of dots.



Worked example 2.5A Identifying prime and composite numbers

State whether each number is prime or composite. Give a reason for your answer.

a 19 **b** 48 **c** 57

THINK	WRITE
a Determine if 19 has any factors other than 1 and itself.	**a** 19 is a prime number because it only has two factors: 1 and itself.
b Determine if 48 has any factors other than 1 and itself. 48 is an even number, so it is divisible by 2.	**b** 48 is a composite number since it has more than two factors.
c Determine if 57 has any factors other than 1 and itself. 57 is divisible by 3.	**c** 57 is a composite number since it has more than two factors.

Worked example 2.5B Finding prime factors

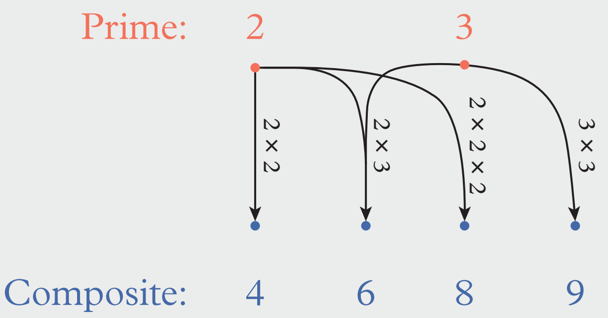
List the prime factors of:

a 40 **b** 16.

THINK	WRITE
a List all the factors of 40. Identify the prime numbers.	**a** Factors: 1, 2, 4, 5, 8, 10, 20, 40 The prime factors of 40 are 2 and 5.
b List all the factors of 16. Identify the prime numbers.	**b** Factors: 1, 2, 4, 8, 16 The only prime factor of 16 is 2.

Helpful hints

- ✔ 2 is the only even number that is also a prime number.
- ✔ Although most even numbers are composite numbers, not all odd numbers are primes. For example, 21 is an odd number but it has four factors, 1, 3, 7 and 21, so it is not prime.
- ✔ Remember that 0 and 1 are neither prime nor composite.
- ✔ All composite numbers can be written as the product of prime factors. On the right you can see how the first two prime numbers, 2 and 3, make up the first four composite numbers.



Learning pathways

▲ 1–7, 8(a, b), 9(a, b), 10–13

■ 3–4, 5(g–l), 6(g–l), 7–9, 12–16

⬟ 5(j, k, l), 6(j, k, l), 7, 8(c, d), 9(c, d), 12, 13, 15, 17, 18

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Exercise 2.5A: Understanding and fluency

1 The prime numbers from 1 to 20 are 2, 3, 5, 7, 11, 13, 17 and 19.

- **a** List all the factors of each of the prime numbers from 1 to 20.
- **b** How many factors does each prime number have?

2 The composite numbers from 1 to 20 are 4, 6, 8, 9, 10, 12, 14, 15, 16, 18 and 20.

- **a** List all the factors of each of the composite numbers from 1 to 20.
- **b** How many factors does each composite number have?
- **c** Do composite numbers have more, fewer or the same number of factors as prime numbers?

3 List all the factors of each of the following numbers.

a 24 **b** 49 **c** 73 **d** 125 **e** 37 **f** 82

4 Use your answers to question **3** to decide whether each number is prime or composite.

WE 2.5A **5** State whether each of the following numbers is prime or composite. Give a reason for your answer.

a 14 **b** 28 **c** 27 **d** 47 **e** 38 **f** 79
g 18 **h** 43 **i** 77 **j** 59 **k** 131 **l** 87

WE 2.5B **6** List the prime factors of each of the following numbers.

a 4 **b** 10 **c** 18 **d** 35 **e** 16 **f** 20
g 27 **h** 40 **i** 42 **j** 56 **k** 33 **l** 64

Exercise 2.5B: Problem solving and reasoning

7 Describe the difference between prime numbers and composite numbers.

8 Find two prime numbers that add to give each of the following numbers.

a 24 **b** 36 **c** 82 **d** 144

9 Find two prime numbers that multiply to give each of the following numbers.

a 21 **b** 55 **c** 26 **d** 115

10 Bobby is writing out the factors of 162 in factor pairs, but he only wants to record pairs that contain composite numbers. Which of the following factor pairs contains only composite numbers?

A 2 × 81 **B** 3 × 54 **C** 6 × 27 **D** 1 × 162

11 A prime number has exactly two factors: itself and 1.

- **a** Write down all the prime numbers between 10 and 30.
- **b** What do these numbers have in common?
- **c** Are all prime numbers odd? Use an example to justify your answer.

12 Explain why the number 1 is a special number that is neither prime nor composite.

13 Answer true or false to each statement. Give a reason for your answers.

- **a** All even numbers are prime numbers.
- **b** All odd numbers are prime numbers.

- **c** The two smallest prime numbers are 2 and 3.
- **d** All composite numbers are even numbers.
- **e** The product of two prime numbers is always even.
- **f** The product of two prime numbers is always a composite number.
- **g** The number 0 is neither a prime number nor a composite number since it is even.
- **h** Between 1 and 30, there are more composite numbers than prime numbers.

14 Zeke is the head of a manufacturing business that packages mugs into boxes of 30. He wants to pack the mugs in a single rectangular layer.

The dimensions of the arrangement shown on the right are 2 by 6.

- **a** What are the dimensions of all the rectangles Zeke can make with his 30 mugs?
- **b** Why would he choose to pack the mugs in groups of 30 instead of 29 or 31?
- **c** If Zeke wanted to pack boxes of 36, list all the possible arrangements he can choose from if he decided that he could also stack them in multiple layers. Use a systematic approach to determine all possible options.

15 A pair of prime numbers that differ by 2 are called twin primes. For example, 3 and 5 are twin primes, as $5 - 3 = 2$.

- **a** List the three other pairs of twin primes that occur between 1 and 20.
- **b** How many pairs of twin primes occur between 21 and 100? List all the twin primes between 21 and 100.

Exercise 2.5C: Challenge

16 What is the largest three-digit prime number?

17 How many three-digit prime numbers can you make using all three digits 1, 3 and 5?

18 After finishing a week-long investigation into endangered animals, the 220 Year 7 students at OUP Secondary School each created a card with information about one of the animals they had studied.
On Friday afternoon, Ms. Press decided to play a game and placed all 220 cards face down on a table.
Ms. Press then lined up all the students and gave the following instructions:

- The first student in line must flip over every card.
- The second student in line must flip over every second card (cards 2, 4, 6, 8, etc.).
- The third student in line must flip over every third card (cards 3, 6, 9, 12, etc.).
- The fourth student in line must flip over every fourth card (cards 4, 8, 12, 16, etc.).
- and so on until the very last student flips over the last card (student 220).

After each of the 220 students in Year 7 took their turns flipping over the cards, how many cards were face up?
Hint: Try to spot a pattern in the number of cards which will be face up.

Online resources:

Interactive skillsheet Prime and composite numbers	**Worksheet** Identifying prime and composite numbers	**Investigation** Encryption codes	**Quick quiz** 2.5

Lesson 2.6

Prime factorisation

Learning intentions

By the end of this lesson you will be able to ...

✔ find the prime factorisation of whole numbers, and use it to find the lowest common multiple or highest common factor of two numbers.

Inter-year links

Support	Multiples and factors
Support	Prime and composite numbers
Year 8	5.7 Factorising
Year 9	1.1 Exponents

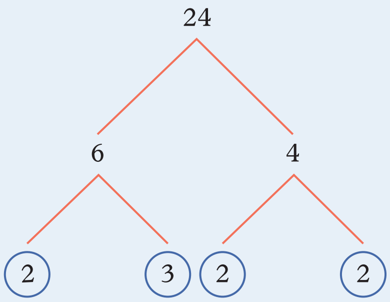
Prime factorisation

Key content video
Prime factorisation

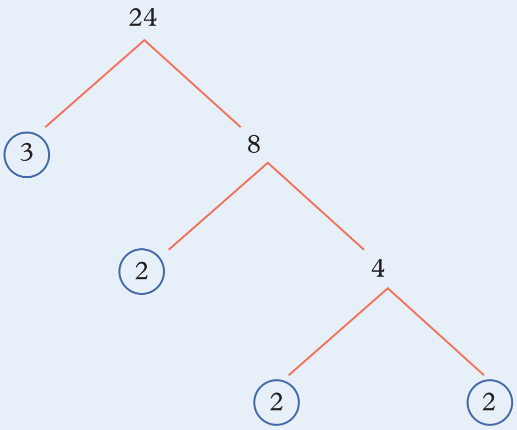
- The **prime factorisation** of a whole number is the product of the primes that make up that number. For example, 2 × 3 is the product of the primes that make up 6, so 2 × 3 is the prime factorisation of 6.
- The fundamental theorem of arithmetic states:
  1 Every number greater than 1 is either a prime number or can be written as a product of prime numbers.
  For example, $24 = 2 \times 2 \times 2 \times 3 = 2^3 \times 3$
  2 There is only one prime factorisation for every composite number.
  For example, $24 = 2^3 \times 3$ is the unique prime factorisation for 24.

Factor trees

- Prime factorisation can be performed using **factor trees**. In factor trees, composite numbers are broken down into factor pairs until prime numbers are found.
- To find the prime factorisation of a number using factor trees:
  1 Write the initial number at the top of the tree.
  2 On the branches splitting the initial number in two, write a factor pair that when multiplied give the initial number.
  3 Repeat step **2** for any composite factors until the numbers at the ends of the branches are all prime.
  4 Write the initial number as a product of its prime factors. Simplify the products using exponents and write the bases in ascending order.



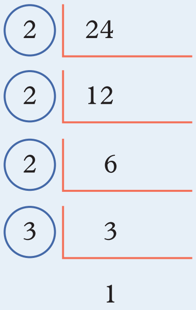
$$\begin{aligned}24 &= 2 \times 3 \times 2 \times 2\\ &= 2^3 \times 3\end{aligned}$$



$$\begin{aligned}24 &= 3 \times 2 \times 2 \times 2\\ &= 2^3 \times 3\end{aligned}$$

Repeated division

- Prime factorisation can also be performed using repeated division. In repeated division, the initial composite number is divided by primes until the result is 1.
- To find the prime factorisation of a number using repeated division:
  1 Divide the initial number by its lowest prime factor.
  2 Divide the quotient by its lowest prime factor.
  3 Repeat the division process until the quotient is 1.
  4 Write the initial number as a product of its prime factors. Simplify the products using exponents and write the bases in ascending order.



$24 = 2 \times 2 \times 2 \times 3$
$= 2^3 \times 3$

Worked example 2.6A Finding prime factorisation using factor trees

Use a factor tree to find the prime factorisation of each number.

a 36 **b** 72

THINK

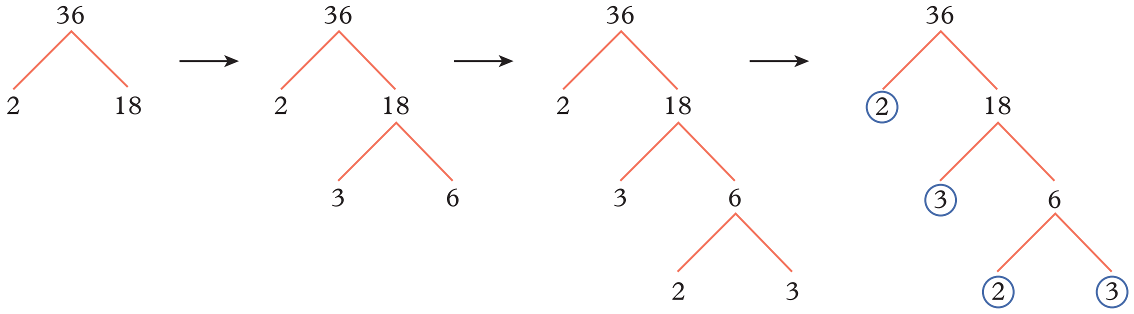
1 Split the composite number into two smaller factors. If no factor pairs are obvious, divide by its lowest prime factor to identify a factor pair. The lowest prime factor of even numbers is always 2.

2 Find a factor pair of any resulting composite factors. Repeat the process until all the factors are prime.

3 Write the initial composite number as a product of its prime factors. Remember to simplify the products using exponents and place the bases in ascending order: 2, 3, 5, ...

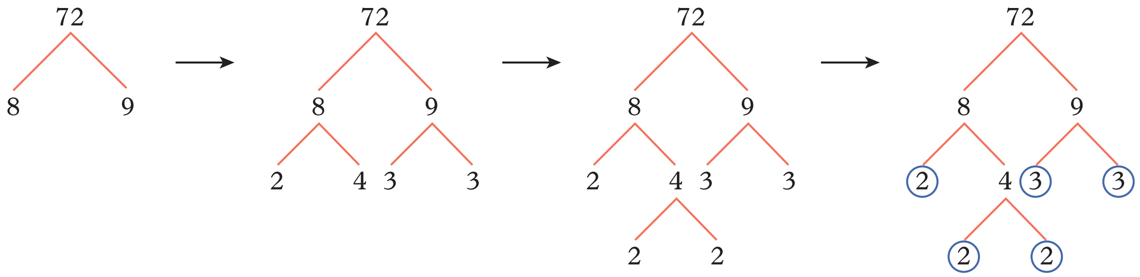
WRITE

a



$36 = 2 \times 3 \times 2 \times 3$
$= 2^2 \times 3^2$

b



$72 = 2 \times 2 \times 2 \times 3 \times 3$
$= 2^3 \times 3^2$

Worked example 2.6B Finding prime factorisation using repeated division

Use repeated division to find the prime factorisation of each number.

a 12 **b** 210

THINK

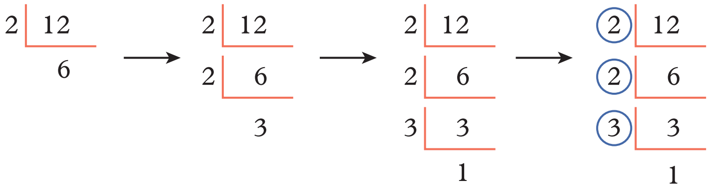
1 Divide the composite number by its lowest prime factor. The divisibility rules can be used to quickly identify a prime factor.

2 Divide the resulting composite number by a prime factor. Repeat the process until the quotient is 1. Only divide the quotient by prime numbers.

3 Write the initial composite number as a product of its prime factors. Simplify the products using exponents and place the bases in ascending order: 2, 3, 5, …

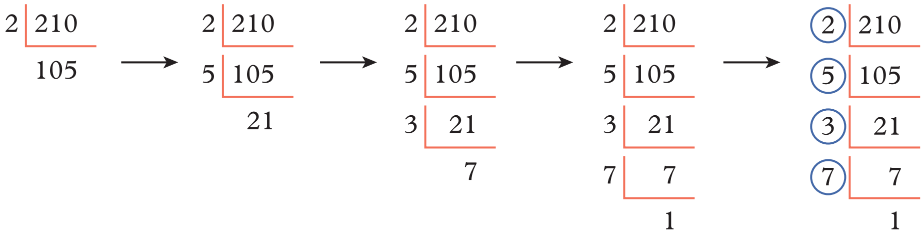
WRITE

a



$$12 = 2 \times 2 \times 3$$
$$= 2^2 \times 3$$

b



$$210 = 2 \times 3 \times 5 \times 7$$

Helpful hints

- ✔ When finding factor pairs for your factor trees, remember that the numbers in a factor pair must multiply together to produce the composite number.
- ✔ When factorising using repeated division, ONLY divide composite numbers by prime numbers.
- ✔ When using the repeated division method, you can determine some factor pairs by dividing the initial composite number by each of its factors.

Learning pathways

△ 1, 2, 3(b, d, h), 4, 5, 6–7(1st, 2nd column), 8–11, 15(a, b), 16

■ 1(d–f), 2(d–h), 3(d–h), 4, 5(b, c), 6–7(3rd column), 9, 10, 12, 14–16, 18, 22

⬟ 1(d, f), 2(c, h, j), 3(c, h, i), 4, 5(c), 6–7(4th column), 9, 13, 14, 16–22

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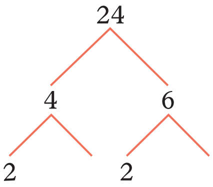
Exercise 2.6A: Understanding and fluency

1 Complete each factor tree and show each composite number as a product of prime factors.

a

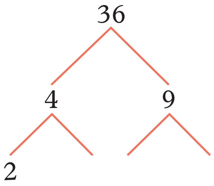


The prime factors of 24 are 2 and ____.

24 = 2 × 2 × ____ × ____

= 2^3 × ____

b

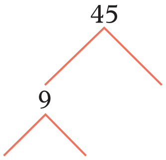


The prime factors of 36 are ____ and ____.

36 = 2 × ____ × ____ × ____

= 2^2 × ____

c

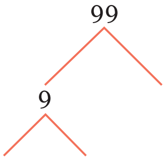


The prime factors of 45 are ____ and 5.

45 = ____ × 3 × ____

= ____ × ____

d

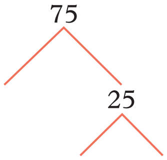


The prime factors of 99 are 3 and ____.

99 = 3 × ____ × ____

= ____ × ____

e

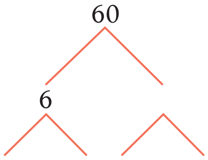


The prime factors of 75 are ____ and ____.

75 = ____ × ____ × ____

= 3 × ____

f



The prime factors of 60 are ____, ____ and ____.

60 = ____ × ____ × ____ × ____

= ____ × ____ × ____

WE 2.6A **2** Use a factor tree to find the prime factorisation of each number.

a 26
b 32
c 50
d 63
e 78
f 58
g 84
h 92
i 144

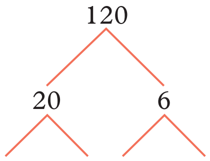
3 Use a factor tree to find the prime factorisation of each number.

a 48
b 96
c 135
d 182
e 168
f 285
g 244
h 416
i 450

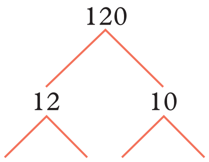
4 Factor trees can be used to find the prime factorisation of any positive whole number.

a Complete each factor tree for the composite number 120. Remember to continue until all of the prime factors are found.

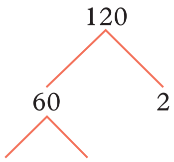
i



ii



iii



b Write the prime factorisation for each tree. Do the initial factors change the result of each prime factorisation?

5 Complete each repeated division and show each composite number as a product of prime factors.

a

2	80
2	40
2	20
☐	10
☐	☐
	1

The prime factors of 80 are 2 and ____.

80 = 2 × 2 × ____ × ____ × ____

= 2^4 × ____

b

5	225
☐	45
3	☐
☐	☐
	1

The prime factors of 225 are ____ and ____.

225 = 3 × 3 × ____ × ____

= 3^2 × ____

c

2	570
☐	☐
☐	☐
19	19
	1

The prime factors of 570 are 2, ____, ____ and ____.

570 = 2 × ____ × ____ × ____

= ____ × ____ × ____ × ____

WE 2.6B **6** Use repeated division to find the prime factorisation of each number.

a 28 **b** 56 **c** 44 **d** 85
e 132 **f** 52 **g** 250 **h** 90
i 72 **j** 220 **k** 100 **l** 400

7 Use repeated division to find the prime factorisation of each number.

a 228 **b** 560 **c** 324 **d** 845
e 343 **f** 512 **g** 154 **h** 960
i 740 **j** 1140 **k** 798 **l** 1716

8 Decide if the following products are the prime factorisation of each composite number. Use the fundamental theorem of arithmetic to justify your answer.

a 24 = 2 × 2 × 2 × 3

b 36 = $2^2 \times 3^2$ and 36 = $2^3 \times 3$

c 19 = 2 × 17

d 100 = 4 × 5^2

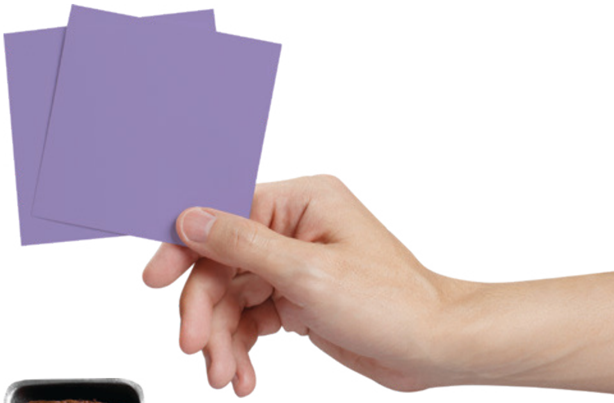
9 What is the smallest number that has:

a four different prime factors

b five prime factors

c six different prime factors?

Exercise 2.6B: Problem solving and reasoning



10 Erin is making tickets for her International Women's Day trivia night. She wants to cut square tickets that are the same size from a sheet of paper which measures 240 cm by 330 cm. What is the largest size the square tickets can be if Erin uses all the paper? Use each number's prime factorisation to help you.

11 A chocolate cake mixture has been baked in a large rectangular cake tin. The chef wishes to cut the cake into 36 individual serves.

a Describe how prime factors can be used to divide the cake into 36 equal pieces.

b Draw a diagram to show a way of cutting the cake using prime factors.

c Draw a diagram to show one more way of cutting the cake into 36 equal pieces.

12 Using only the product of the prime factors 2, 3 and 5, make the following numbers.

a 100 **b** 180 **c** 1000

13 Using only the product of the prime factors 3, 5 and 7, make the following numbers.

a 315 **b** 441 **c** 1575

14 Find the smallest whole number that is divisible by 2, 5 and 9.

15 We can use prime factorisation to find the HCF of a pair of numbers.

1. First, find the prime factorisation of each number.
2. Circle the prime factors that are common to both numbers.
3. Multiply them together to find the HCF.

For example,

$$\begin{aligned} 24 &= \enclose{circle}{2} \times 2 \times 2 \times \enclose{circle}{3} \\ 30 &= \enclose{circle}{2} \times \enclose{circle}{3} \times 5 \\ \text{HCF} &= \enclose{circle}{2} \times \enclose{circle}{3} \\ &= 6 \end{aligned}$$

Use prime factorisation to find the HCF of each pair of numbers.

a 20 and 42 **b** 35 and 45 **c** 72 and 64 **d** 84 and 96

16 Ming has made 45 purple (blueberry) and 60 green (pistachio) macarons. She sorts them into bags so that each bag has the same number of purple macarons and the same number of green macarons with none of either colour left over.

a Use prime factors to find the HCF of 45 and 60.

b What is the smallest number of macarons that could be in each bag?

17 The school library has been given 55 science fiction and 75 fantasy books. They plan to sort the books into groups that will be given as prizes in a reading competition. Each prize is to have the same number of science fiction books and the same number of fantasy books, with none left over. How many books will be in each prize? Use prime factorisation to help you determine the answer.

18 We can use prime factorisation to find the LCM of a pair of numbers.

1 First, find the prime factorisation of each number. Write them using exponents.
2 Circle the highest exponent of each prime factor listed.
3 Multiply them together to find the LCM.

For example,

$$\begin{aligned} 72 &= \enclose{circle}{2^3} \times \enclose{circle}{3^2} \\ 60 &= 2^2 \times 3 \times \enclose{circle}{5} \\ \text{LCM} &= \enclose{circle}{2^3} \times \enclose{circle}{3^2} \times \enclose{circle}{5} \\ &= 360 \end{aligned}$$

Use prime factorisation to find the LCM of each pair of numbers.

a 20 and 42 **b** 35 and 45 **c** 75 and 64 **d** 84 and 96

19 Find the smallest whole number that 80 can be multiplied by to result in a perfect square, and state that perfect square.

20 Two dairy farmers, Yonas and Samara, are transporting their entire herds between paddocks. Yonas is moving his cows in groups of 21 and Samara is moving her cows in groups of 34. If each farmer has the same number of cows, what is the smallest number of cows in each herd?

21 Two different buses leave the city at midday for the Blue Mountains. The first bus makes a stop every 25 minutes and the second bus makes a stop every 30 minutes. After they leave the city, when will the buses next stop at the same time?

Exercise 2.6C: Challenge

22 Some four-digit numbers can be written as the product of a three-digit number and a two-digit number using each of the digits 1 to 9 exactly once. Use prime factorisation to help find the two factors in these products. There is more than one correct answer to two of the products. Try to find all the answers!

$$\begin{array}{cccc} & \square & \square & \square \\ \times & & \square & \square \\ \hline \boxed{4} & \boxed{3} & \boxed{9} & \boxed{6} \end{array} \qquad \begin{array}{cccc} & \square & \square & \square \\ \times & & \square & \square \\ \hline \boxed{5} & \boxed{3} & \boxed{4} & \boxed{6} \end{array}$$

$$\begin{array}{cccc} & \square & \square & \square \\ \times & & \square & \square \\ \hline \boxed{5} & \boxed{7} & \boxed{9} & \boxed{6} \end{array}$$

Online resources:

Interactive skillsheet
Prime factorisation

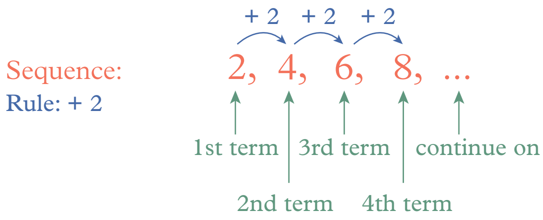
Quick quiz
2.6

Lesson 2.7

Review: Factors and primes

Module summary

Number sequences



- To find the rule for a sequence, consider all possible operations including:
  - addition
  - subtraction
  - multiplication
  - division
  - exponents
  - square roots.

Divisibility rules

÷ 2 The last digit of the number is even.	÷ 3 The sum of all the digits in the number is divisible by 3.	÷ 4 The number made by the last two digits is divisible by 4.
÷ 5 The last digit is 5 or 0.	÷ 6 The number is divisible by 2 and 3.	÷ 7 There is no easy trick for 7.
÷ 8 The number made by the last three digits is divisible by 8.	÷ 9 The sum of all the digits in the number is divisible by 9.	÷ 10 The last digit is 0.

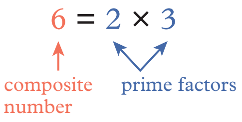
Lowest common multiple

Multiples of 2: 2, 4, ⑥, 8, 10, ⑫, …
Multiples of 3: 3, ⑥, 9, ⑫, 15, 18, …
LCM: 6

Highest common factor

Factors of 16: ①, ②, ④, ⑧, 16
Factors of 24: ①, ②, 3, ④, 6, ⑧, 12, 24
HCF: 8

Prime numbers



- 1 is neither prime nor composite because it only has one factor.
- 0 is neither prime nor composite because it is not a positive number.

The fundamental theorem of arithmetic

1 Every number greater than 1 is either a prime number or can be written as a product of prime numbers.

For example,

$$24 = 2^3 \times 3$$

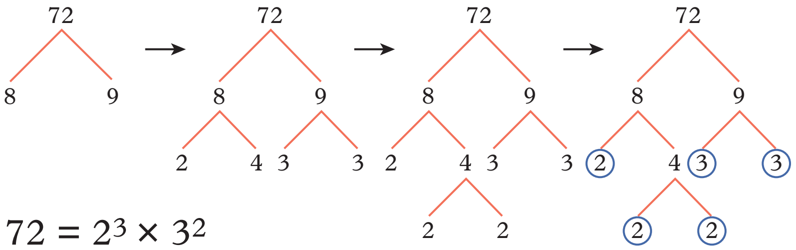
2 There is only one prime factorisation for every composite number.

For example,

$$24 = 2^3 \times 3$$

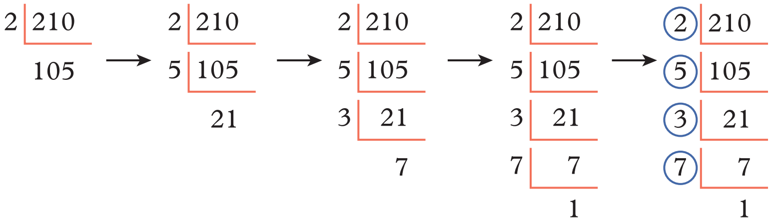
is the *unique* factorisation of 24.

Factor trees



$$72 = 2^3 \times 3^2$$

Repeated division



$$210 = 2 \times 3 \times 5 \times 7$$

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Review questions 2.7A: Mathematical literacy review

Review quiz
Take the review quiz to assess your knowledge of this module.

Quizlet
Test your knowledge of this module by working individually or in teams.

The following key terms are used in this module:

- common factor
- common multiple
- composite number
- dividend
- divisor
- factor
- factor pair
- factor tree
- highest common factor
- lowest common multiple
- multiple
- number sequence
- prime factor
- prime factorisation
- prime number
- quotient
- remainder
- rule
- term
- whole number

1 Explain the difference between a factor and a multiple.

2 When performing a division, what is the quotient?

3 Answer true or false to the following statements.
  a There is a common factor between every pair of positive whole numbers.
  b Every positive whole number is a prime factor of a composite number.
  c There is a unique prime factorisation of every composite number.

4 Identify the key terms being referenced in each of these definitions.
  a a positive whole number that is divisible by at least one whole number other than itself and 1
  b a description of the relationship between each term and the next term in a sequence

5 Using an example, provide a definition in your own words for the following key terms.
  a prime number
  b lowest common multiple

6 Complete the following sentences using words from the key terms list.
  a When performing a division, if the _______ is zero then we can say the _______ is divisible by the _______.
  b A _______ is a number that divides exactly into two or more _______.

Review questions 2.7B: Multiple choice

2.1 **1** The missing term in the sequence 6, 12, 18, ___, 30, 36, … is:
A 24 **B** 6 **C** 42 **D** 50

2.1 **2** The fourth term in a sequence is 16. If the rule for this sequence is subtract 6, then the first term is:
A 6 **B** 22 **C** 34 **D** 40

2.2 **3** A number that is divisible by 3 and 5 is:
A 65 **B** 125 **C** 99 **D** 135

2.2 **4** A number that is divisible by 3 and 8 is:
A 9072 **B** 243 **C** 846 **D** 4096

2.3 **5** A number that is *not* a common multiple of 6 and 16 is:
A 24 **B** 48 **C** 96 **D** 144

2.3 **6** The lowest common multiple of 9 and 5 is:
A 90 **B** 45 **C** 30 **D** 14

2.4 **7** What is the highest common factor of 10 and 25?
A 5 **B** 10 **C** 25 **D** 50

2.4 **8** Which of the following is *not* a factor of 60?

A 4 **B** 8 **C** 12 **D** 15

2.5 **9** Which number is *not* a prime number?

A 11 **B** 25 **C** 37 **D** 53

2.5 **10** Which one of the following is a prime number?

A 51 **B** 29 **C** 39 **D** 63

2.5 **11** What is the prime factorisation of 60?

A $3 \times 4 \times 5$ **B** $2^2 \times 15$ **C** $2^2 \times 3 \times 5$ **D** $2 \times 3 \times 10$

2.6 **12** Which one of the following composite numbers has the most prime factors?

A 20 **B** 24 **C** 30 **D** 36

Review questions 2.7C: Short answer

2.1 **1** Find the missing terms in these sequences.

a 14, 18, ___, 26, ___, 34, … **b** 75, ___, 65, 60, 55, 50, …

c ___, 10, 20, 40, 80, ___, … **d** 176, 88, 44, ___, ___, …

2.1 **2** On Monday Sasha notices 5 ants in her kitchen, on Tuesday she sees 10 ants and on Wednesday there are 20 ants. If this pattern continues, how many ants will be in Sasha's kitchen the following Monday?

2.2 **3** Check if the number 13 824 is divisible by:

a 2 **b** 4 **c** 6 **d** 8

2.2 **4** The Year 7 captains have organised a cupcake fundraiser. They have ordered 2625 cupcakes. They want to make sure each student can have an equal number of cupcakes with no leftovers.

a Use the divisibility tests to determine whether the captains can provide 2, 3, 4, 5, 6 or 7 cupcakes per person without wastage.

b Determine which of these would be the best option if there are 375 students.

2.3 **5** Write all the multiples of 4 between 22 and 45.

2.3 **6** There are 100 houses in a street. The houses are numbered from 1 to 100. All the houses that are a multiple of 6 have a dog. All the houses that are multiples of 8 have a cat. Which houses have both a dog and a cat?

2.3 **7** Find the lowest common multiple of each group of numbers.

a 12 and 18 **b** 6, 9 and 15

2.4 **8** List all the factors of 80.

2.4 **9** Susie has 24 soft toys. She arranges them on her bed in rows so that there are the same number of toys in each row. Suggest three different ways Susie could arrange her toys.

2.4 **10** Find the highest common factor of each group of numbers.

a 12 and 18 **b** 6, 9 and 15

2.4 **11** Find the first three numbers greater than 1 that have:

a a factor of 7

b four different factors

c an odd number of factors.

2.5 **12** Find two different prime numbers that add to give each of the following numbers.

a 14 **b** 21 **c** 26 **d** 30

2.5 **13** Find two prime numbers that multiply to give each of the following numbers.

a 15 **b** 33 **c** 34 **d** 58

2.6 **14** All positive whole numbers greater than 1 are either prime numbers or composite numbers.

- **a** Is 40 a prime or composite number? Give a reason to support your answer.
- **b** Draw a factor tree and list the prime factors of 40.
- **c** Write 40 as a product of its prime factors in exponent form.

2.6 **15** An op shop has 48 shirts and 36 jumpers to give away. The staff sort clothes into bags so that each bag has the same number of shirts and jumpers without any left over.

- **a** Use prime factors to find the HCF of 36 and 48.
- **b** How many bags will there be if the least number of items are placed in each bag?
- **c** How many pieces of clothing in total will be in each bag?

Review questions 2.7D: Mathematical modelling

1 Tahlia is an animal photographer. She has 280 photos of mammals and 168 photos of birds. Tahlia plans to make a book from her photos.

- **a** How many photos does Tahlia have in total?
- **b** Tahlia wants to have either exactly six or eight photos on every page. Which number does she choose so that she has no photos left over?
- **c** How many pages are in her book?
- **d** How many photos of mammals should Tahlia put on each page if she wants to make sure all pages in the book contain the same number of mammal photos?
- **e** How many photos of birds should Tahlia put on each page if she wants to make sure all pages in the book contain the same number of bird photos?
- **f** Every fifth page of the book has photos of sea mammals and every sixth page has birds of prey. Which pages would contain photos of both sea mammals and birds of prey?
- **g** Every fourth page has photos of big cats. Which pages would contain photos of big cats, sea mammals and birds of prey?

2 Simon is having a group of friends over for pizza. To help Simon calculate how many slices he needs, he decides to use prime factors.

He thinks about splitting the first pizza into 12 equal pieces. The product of prime factors for 12 is $2 \times 2 \times 3$. Therefore, he needs to cut the pizza into two equal halves, each of the halves in half again, and then each of the quarters into three equal pieces of pizza.

- **a** On the next pizza, he decides to have eight slices. Create a factor tree for the number 8.
- **b** List the product of prime factors of 8.
- **c** Describe the order of cuts Simon will need to make in the pizza to get eight equal slices.
- **d** The next pizza is very big, and Simon thinks he can get 15 slices out of the pizza. Draw a factor tree for the number 15.
- **e** List the product of prime factors of 15.
- **f** Describe the order of cuts Simon will need to make in the pizza to get 15 equal slices.

3 Eratosthenes of Cyrene was a Greek mathematician, geographer, poet and astronomer. Among his many achievements, he created a simple process for finding prime numbers which is known as the sieve of Eratosthenes.

Consider the number grid on the right, which details all the whole numbers from 2 to 100.

	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

a Circle the number 2 and cross out all the other multiples of 2 in the grid (4, 6, 8, …). You may spot a pattern in the numbers to help do this quickly.

b Circle the next uncovered number (3) and cross out all the other multiples of this number in the grid (6, 9, 12, …) that have not already been crossed out.

c Repeat part **b** until all the numbers in the grid are either circled or crossed out.

d Explain why all the circled numbers in the grid are prime numbers, and why all the crossed out numbers in the grid are composite numbers.

e Use your sieve of Eratosthenes to list all the prime numbers between 2 and 100.

Checklist

Now that you have completed this module, reflect on your ability to do the following.

I can do this	**I need to review this**
☐ Find the rule between terms in a sequence ☐ Find more terms in a sequence	☐ Go back to Lesson 2.1 Number sequences
☐ Determine if a number is divisible by 2, 3, 4, 5, 6, 8, 9 or 10	☐ Go back to Lesson 2.2 Divisibility rules
☐ Find multiples of a number ☐ Find the lowest common multiple of two numbers	☐ Go back to Lesson 2.3 Multiples and the lowest common multiple
☐ Find all the factors of a number ☐ Determine the highest common factor of two or more numbers	☐ Go back to Lesson 2.4 Factors and the highest common factor
☐ Determine if a number is prime or composite ☐ Find the prime factors of positive whole numbers	☐ Go back to Lesson 2.5 Prime and composite numbers
☐ Express whole numbers as a product of prime factors ☐ Find the lowest common multiple of two numbers using prime factorisation ☐ Find the highest common factor of two numbers using prime factorisation	☐ Go back to Lesson 2.6 Prime factorisation

3

Fractions and ratios

Lessons

Prerequisite skills

Diagnostic pre-test
Take the diagnostic pre-test to assess your knowledge of the prerequisite skills listed below.

Interactive skillsheets
After completing the diagnostic pre-test, brush up on your knowledge of the prerequisite skills by using the interactive skillsheets.

✔ Benchmark fractions
✔ Adding and subtracting whole numbers
✔ Multiplying and dividing whole numbers
✔ Highest common factor
✔ Number lines

Curriculum links

- Find equivalent representations of rational numbers and represent positive and negative rational numbers and mixed numbers on a number line (VC2M7N03)
- Multiply and divide fractions and decimals using efficient mental and written strategies, and digital tools (VC2M7N05)
- Use the 4 operations with positive rational numbers, including fractions and decimals, to solve problems using efficient mental and written calculation strategies (VC2M7N06)
- Recognise, represent and solve problems involving ratios (VC2M7N09)

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Materials

✔ Calculator

Lesson 3.1

Fractions

Learning intentions

By the end of this lesson you will be able to ...

✔ identify fractions from various representations and express one quantity as a fraction of another.

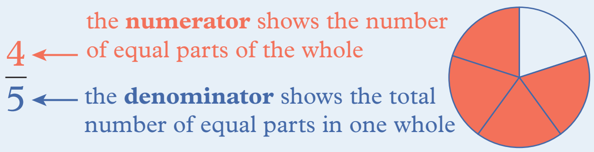
Inter-year links

Support	Fractions
Year 8	2.1 Fractions

Key content video
Understanding fractions

Fractions

- A **fraction** is a part or a portion of a whole, or multiple wholes.



- **Proper fractions** have a **numerator** that is less than the **denominator**.

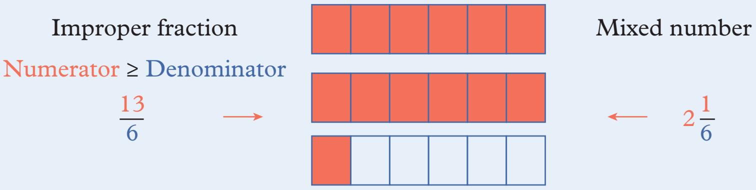
  Some examples are $\frac{3}{5}, \frac{2}{6}, \frac{11}{15}$.

- **Improper fractions** have a numerator that is greater than or equal to the denominator.

  Some examples are $\frac{13}{6}, \frac{5}{5}, \frac{31}{8}$.

- **Mixed numbers** consist of both a whole number and a proper fraction.

  For example, $2\frac{1}{6} = 2 + \frac{1}{6}$.



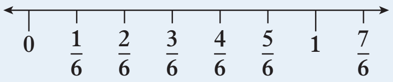
- Whole numbers can be written as fractions:
  - → where the denominator is 1,

    for example, $5 = \frac{5}{1}, 2 = \frac{2}{1}, 100 = \frac{100}{1}$
  - → or the denominator divides into the numerator without a remainder,

    for example, $\frac{2}{2} = 1, \frac{9}{3} = 3, \frac{28}{7} = 4$.

Fractions on a number line

- Fractions can be written on a number line.
  - → Number lines can be used to compare fractions.



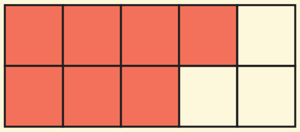
Fractions of a set

- Fractions can show the proportion or number of a group of objects within a larger set of objects.
  - → A **set** is a defined group of objects or numbers.

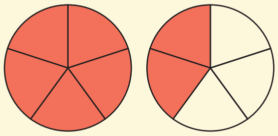
    For example, here is a set of 5 counters. There are 2 blue counters, so $\frac{2}{5}$ of the counters are blue.

Worked example 3.1A Identifying fractions from a diagram

a The rectangle represents a whole. What fraction is shaded of the whole shape?



b Each circle represents a whole. What fraction is shaded of the whole shape?



THINK

1 Count the number of equal parts in the whole shape.

2 Count the number of shaded parts in the shape.

3 Write the number of shaded parts as the numerator, and the number of parts in each whole as the denominator.

WRITE

a The shape is cut into 10 parts.
The shape has 7 shaded parts.
The fraction of the shape that is shaded is $\frac{7}{10}$.

b The shape is cut into 5 parts. There are two shapes.
There are 7 shaded parts.
The fraction of the shape that is shaded is $\frac{7}{5}$.

Worked example 3.1B Identifying fractions from a number line

Label the fractions $\frac{3}{4}, \frac{5}{4}, 2\frac{1}{4}$ on a number line.

THINK

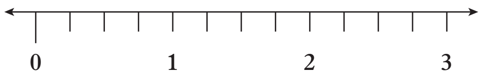
1 Draw a number line from 0 to 3, making sure the smallest and largest fractions are between the numbers on the scale.

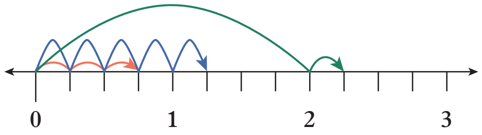
Divide the number line into segments of equal lengths; the number of segments between two whole numbers should be the same as the denominator. Divide each section between two whole numbers into 4 parts.

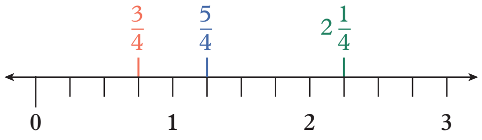
2 Find the location of the fractions on the number line by counting up to the fraction from 0.

3 Mark and label the fractions on the number line.

WRITE







Worked example 3.1C Identifying fractions from a set

Identify the fraction that is described by the following statement.

There are 17 correct answers on a quiz containing 25 questions.

THINK	**WRITE**
1 Determine the total number of objects in the set. There are 25 questions in the quiz.	There are 25 questions in the quiz.
2 Determine the number of objects in the group within the set. There are 17 correct answers in the quiz.	There are 17 correct answers in the quiz.
3 Write the number of objects in the group within the set as the numerator, and the number of objects in the set as the denominator.	$\frac{17}{25}$ of the answers to the quiz were correct.

Helpful hints

✔ Although fractions are made up of a numerator and a denominator, a fraction is just one value.

✔ Be careful when selecting the important information in a question. Try to determine the total number of objects in the set first. Make sure you write this as the denominator.

Learning pathways

△ 1(1st, 2nd column), 2–9, 10(b, d, f), 12–14, 15(a–c)

■ 1(3rd, 4th column), 2, 4–5, 7–10, 12, 15–17, 18(a–c)

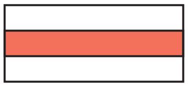
⬠ 1(4th column), 2, 5(e–h), 8, 10, 12, 15, 18–20

ANS p510

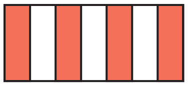
Exercise 3.1A: Understanding and fluency

WE 3.1A **1** What fraction of each shape is shaded?

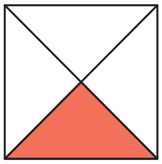
a



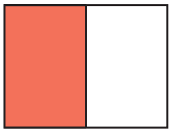
b



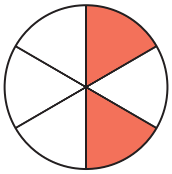
c



d



e



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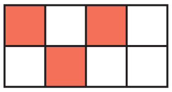

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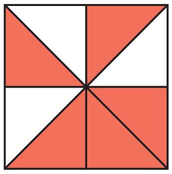

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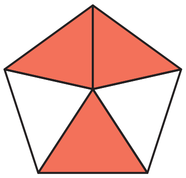
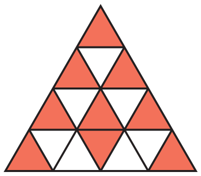


i



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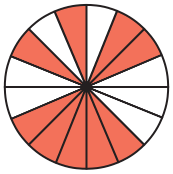

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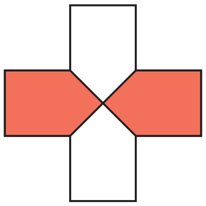

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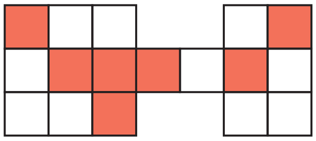


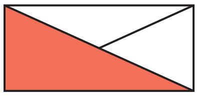
2 What fraction of each shape in question **1** is *not* shaded?

3 Which diagram correctly displays that $\frac{1}{3}$ of the shape has been shaded? Provide a reason for your selection.

A

B

C



4 Consider the following fractions: $\frac{5}{2}, 2\frac{3}{5}, \frac{7}{8}, 1\frac{5}{9}, \frac{17}{2}, \frac{16}{31}, \frac{29}{15}, 12\frac{1}{7}, \frac{13}{22}$.

a Which of the above fractions are proper fractions?

b Which of the above fractions are improper fractions?

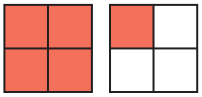
c Which of the above fractions are mixed numbers?

5 If one whole shape represents 1 whole, write the number that is represented by the shading in each diagram as:

i an improper fraction

ii a mixed number.

a



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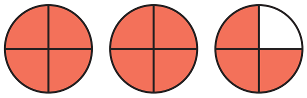

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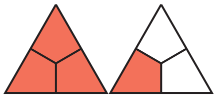

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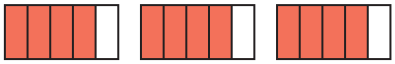

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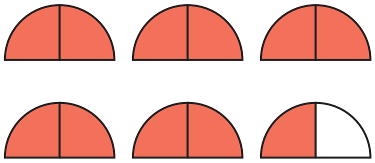

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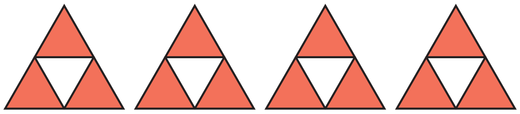

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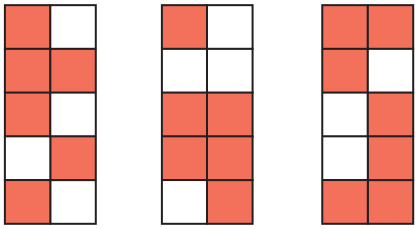

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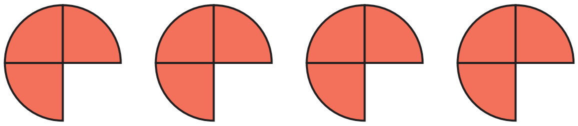
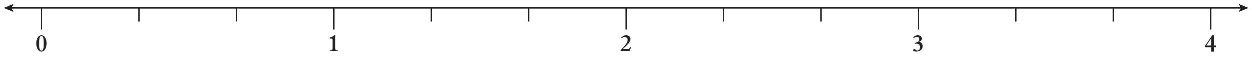


6 The number line below has been divided into four equal intervals from 0 to 4. Each interval has been further subdivided into three equal parts.



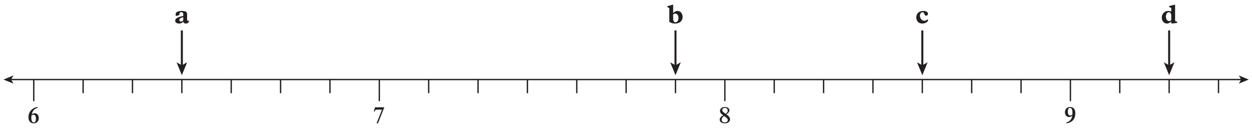
a What fraction is the smallest interval?

b Label each mark on the number line with the value it represents.

WE 3.1B **7** Label the fractions $\frac{2}{5}, \frac{5}{5}, \frac{8}{5}, \frac{13}{5}$ and $\frac{17}{5}$ on a number line.

8 Label the fractions $7\frac{3}{4}, \frac{36}{4}, 9\frac{2}{4}, \frac{33}{4}$ and $\frac{29}{4}$ on a number line.

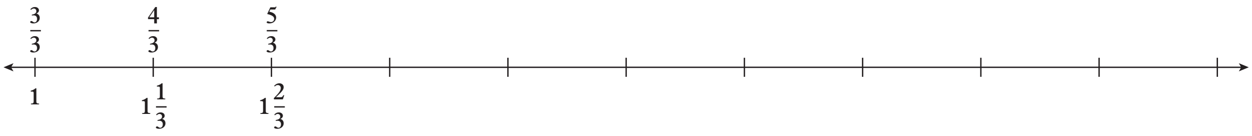
9 Write down the values represented by **a–d** on the number line.



WE 3.1C **10** Identify the fraction that is described by each statement.

- **a** 12 correct answers on a quiz containing 23 questions
- **b** the number of months consisting of 31 days in a year
- **c** $23 change from a $50 note
- **d** a basket containing 41 balls, where 8 are red
- **e** 17 episodes of a 22-episode series that have been watched
- **f** 21 slices of pizza where each slice is one-eighth of a pizza

11 The number line below has been divided into equal intervals. The first three points have been labelled using improper fractions and mixed numbers.



- **a** Complete the marked number line using both improper fractions and mixed numbers.
- **b** Write down the fraction that is:
  - **i** the number that is $\frac{1}{3}$ to the right of $\frac{5}{3}$
  - **ii** the improper fraction that is $\frac{2}{3}$ to the left of 4
  - **iii** the mixed number that is 1 whole to the right of $\frac{7}{3}$.

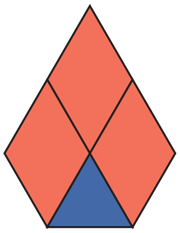
12 What is the difference between a proper and an improper fraction?

13 What is the difference between an improper fraction and a mixed number?

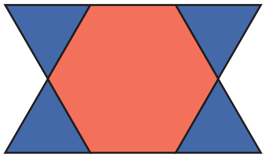
Exercise 3.1B: Problem solving and reasoning

14 If the whole shape is 1 whole, determine the fraction of each colour. Hint: How many blue triangles can fit inside the other shapes?

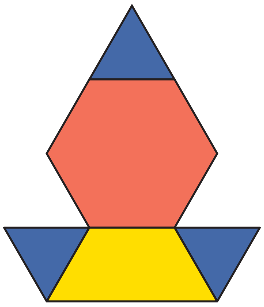
a



b



c



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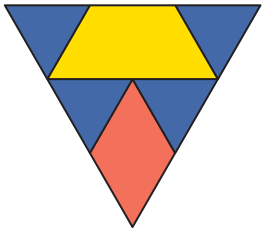
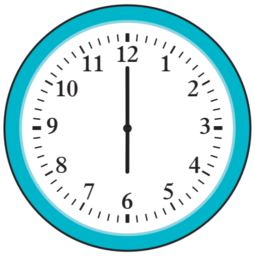
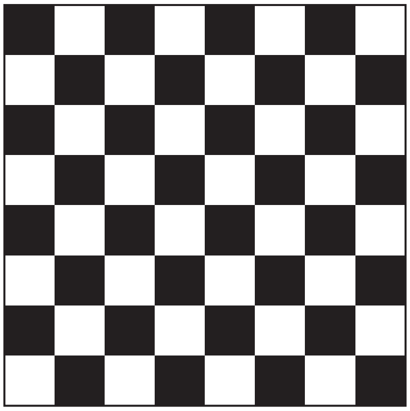


15 There are 60 minutes in 1 hour. What fraction of an hour is:

- **a** 30 minutes
- **b** 15 minutes
- **c** 17 minutes
- **d** 31 minutes
- **e** 119 minutes (as an improper fraction)
- **f** 311 minutes (as a mixed number)?

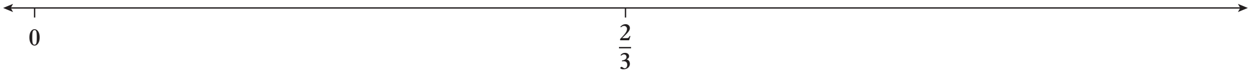


16 Determine what fraction of the shape is shaded. Explain your reasoning.



17 Copy the following number lines and mark where the number 1 should be positioned. Explain your reasoning.

a



b

18 Setting up a number line for fractions with large denominators is hard to do accurately without grid paper or a ruler. We can make it easier for denominators that are composite numbers by dividing the number line into smaller intervals. For example, to divide a number line into 12 equal intervals we can first divide it into halves, then quarters, and finally twelfths.

a Draw a number line from 0 to 1. Mark $\frac{6}{12}$ halfway between 0 and 1.

b Mark $\frac{3}{12}$ halfway between 0 and $\frac{6}{12}$, and mark $\frac{9}{12}$ halfway between $\frac{6}{12}$ and 1.

c Divide each small interval into three equal parts. These are the remaining twelfths.

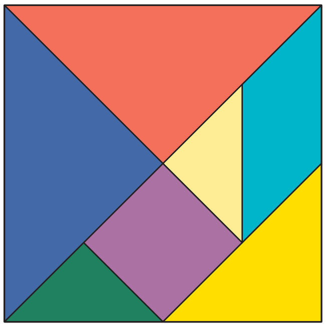
A number line can be marked for other denominators in similar ways.

d By repeatedly halving, divide a number line between the numbers 2 and 3 into 16 equal parts.

e Explain how you could divide a number line between the numbers 0 and 1 into 15 equal parts.

Exercise 3.1C: Challenge

19 I am a fraction. My denominator has exactly three factors, and my numerator is the cube of a prime number. The difference between my denominator and my numerator is 1. What fraction am I?



20 The puzzle on the right is called a tangram. It is made up of two large triangles, one medium triangle, two small triangles, one square and one parallelogram.

a What fraction of the whole tangram is the small triangle?

b What fraction of the whole tangram is the medium triangle?

c What fraction of the whole tangram is the large triangle?

Online resources:

Interactive skillsheet
Identifying fractions

Investigation
Apple pies

Desmos activity
Shading a fraction of a shape

Quick quiz
3.1

Lesson 3.2

Equivalent fractions

Learning intentions

By the end of this lesson you will be able to ...

✔ identify and simplify equivalent fractions using various methods including fraction walls and number lines.

Inter-year links

Support	Equivalent fractions
Year 8	2.1 Fractions

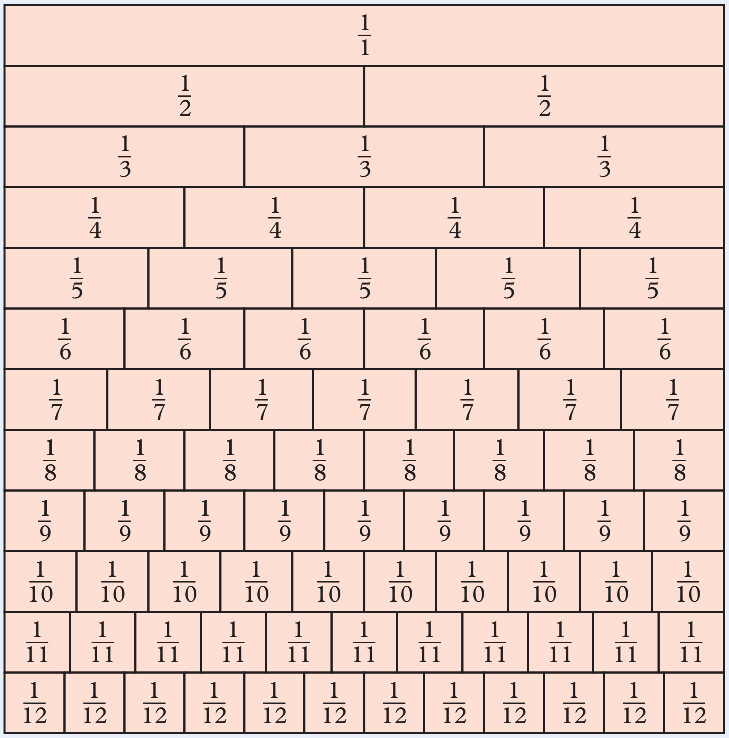
Equivalent fractions

Key content video
Equivalent fractions

- **Equivalent fractions** are fractions that have the same numerical value. They mark the same point on the number line.

  For example, $\frac{1}{2}, \frac{2}{4}, \frac{3}{6}$ and $\frac{6}{12}$ all mark the same point on the number line, so they are equivalent fractions.
- A fraction is equivalent to 1 if the numerator and the denominator are the same.

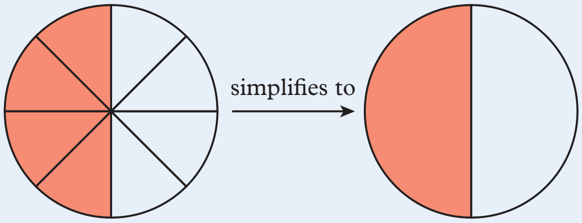
  For example, $\frac{1}{1}, \frac{2}{2}, \frac{3}{3}, \ldots, \frac{10}{10}, \ldots, \frac{43}{43}, \ldots, \frac{101}{101}$ are all equivalent to 1.
- A **fraction wall** is a visual representation to help compare and identify fractions, set out in the form of a wall.



- An equivalent fraction is produced by multiplying or dividing the numerator and the denominator of a fraction by the same value.

$$\frac{2}{3} \underset{\times 5}{\overset{\times 5}{=}} \frac{10}{15} \qquad \frac{12}{10} \underset{\div 2}{\overset{\div 2}{=}} \frac{6}{5}$$

Simplifying fractions



- Fractions in their **simplest form** have a highest common factor of 1 between the numerator and the denominator.
  For example, $\frac{3}{4}$ is in its simplest form, as the highest common factor (HCF) between 3 and 4 is 1.
- There are two ways to simplify fractions:
  - → use repeated division until the only common factor between the numerator and the denominator is 1,

    for example, $\frac{36}{72} = \frac{\cancel{36}^{18}}{\cancel{72}^{36}}$

    $= \frac{\cancel{18}^{3}}{\cancel{36}^{6}}$

    $= \frac{\cancel{3}^{1}}{\cancel{6}^{2}}$

    $= \frac{1}{2}$
  - → find the HCF of the numerator and the denominator and then divide the numerator and the denominator by the HCF,

    for example, the HCF of 36 and 72 is 36, so divide both the numerator and denominator by 36.

    $$\frac{36}{72} = \frac{\cancel{36}^{1}}{\cancel{72}^{2}}$$

    $$= \frac{1}{2}.$$
- For mixed numbers, only simplify the fraction part; the whole number stays the same.

  For example, $2\frac{12}{24} = 2\frac{\cancel{12}^{1}}{\cancel{24}^{2}}$

  $= 2\frac{1}{2}.$

Worked example 3.2A Writing equivalent fractions

Complete each set of equivalent fractions.

a $\frac{14}{8} = \frac{}{4}$

b $\frac{5}{13} = \frac{15}{}$

THINK

1 Find a relationship between the two known numerators and denominators. Consider if the equivalent fraction has been found by using multiplication or division.

2 Use the same relationship to obtain the missing numerator or denominator of the second fraction.

WRITE

a $\frac{14}{8} \underset{\div 2}{=} \frac{}{4}$

$\frac{14}{8} \overset{\div 2}{\underset{\div 2}{=}} \frac{7}{4}$

b $\frac{5}{13} \overset{\times 3}{=} \frac{15}{}$

$\frac{5}{13} \overset{\times 3}{\underset{\times 3}{=}} \frac{15}{39}$

Worked example 3.2B Simplifying fractions using repeated division

Write each fraction in its simplest form using repeated division.

a $\frac{80}{144}$

b $3\frac{54}{72}$

THINK	WRITE
a **1** Find a common factor of 80 and 144. They are both even numbers, so 2 is a common factor. **2** Divide both the numerator and the denominator by 2. **3** Find another common factor and repeat the process. Repeat until the only common factor is 1.	**a** $\frac{80}{144} = \frac{\cancel{80}^{40}}{\cancel{144}^{72}}$ $= \frac{\cancel{40}^{20}}{\cancel{72}^{36}}$ $= \frac{\cancel{20}^{10}}{\cancel{36}^{18}}$ $= \frac{\cancel{10}^{5}}{\cancel{18}^{9}}$ $= \frac{5}{9}$
b **1** Find a common factor of 54 and 72. They both have digit sums that are multiples of 3, so 3 is a common factor. **2** Leave the whole number as it is and divide both the numerator and the denominator by 3. **3** Find another common factor and repeat the process. Repeat until the only common factor is 1.	**b** $3\frac{54}{72} = 3\frac{\cancel{54}^{18}}{\cancel{72}^{24}}$ $= 3\frac{\cancel{18}^{3}}{\cancel{24}^{4}}$ $= 3\frac{3}{4}$

Worked example 3.2C Simplifying fractions using the HCF

Write each fraction in its simplest form using the HCF.

a $\frac{21}{49}$

b $2\frac{45}{65}$

THINK	WRITE
a **1** Find the highest common factor (HCF) of the numerator and the denominator. **2** Divide both the numerator and the denominator by the HCF.	**a** Factors of 21: 1, 3, ⑦, 21 Factors of 49: 1, ⑦, 49 HCF = 7 $\frac{21}{49} = \frac{\cancel{21}^{3}}{\cancel{49}^{7}}$ $= \frac{3}{7}$
b **1** Find the HCF of the numerator and the denominator of the fraction component. **2** Leave the whole number as it is and divide both the numerator and the denominator by the HCF.	**b** Factors of 45: 1, 3, ⑤, 9, 15, 45 Factors of 65: 1, ⑤, 13, 65 HCF = 5 $2\frac{45}{65} = 2\frac{\cancel{45}^{9}}{\cancel{65}^{13}}$ $= 2\frac{9}{13}$

Helpful hints

- ✔ Remember that ‘equivalent’ is another way of saying ‘equal to’.
- ✔ Don’t be fooled by fractions that have odd or prime numbers; sometimes they can be simplified. For example, $\frac{3}{15} = \frac{1}{5}$.
- ✔ Use the divisibility rules from Module 2 to help you find common factors. The larger the common factor, the faster you’ll find the equivalent fraction in its simplest form.

Learning pathways

▲ 1(a, c, e), 2–9, 11–13

■ 3–6, 7(b, d), 8, 10, 11, 14, 15(a, b)

⬟ 3(e, f), 4(b, d, f), 5(b, d, f), 7(d), 8, 11, 13–17

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Exercise 3.2A: Understanding and fluency

1 Using the fraction wall, determine all of the fractions on the wall that are equivalent to:

a $\frac{1}{2}$ **b** $\frac{3}{4}$ **c** $\frac{3}{12}$

d $\frac{2}{5}$ **e** $\frac{2}{3}$ **f** $\frac{7}{7}$

WE 3.2A 2 Complete each set of equivalent fractions.

a $\frac{4}{9} = \frac{\quad}{36}$ **b** $\frac{24}{8} = \frac{12}{\quad}$ **c** $2\frac{5}{11} = 2\frac{\quad}{33}$

d $\frac{42}{70} = \frac{\quad}{10}$ **e** $\frac{144}{48} = \frac{\quad}{12}$ **f** $3\frac{2}{8} = 3\frac{\quad}{72}$

g $\frac{2}{3} = \frac{\quad}{9} = \frac{\quad}{90}$ **h** $\frac{5}{6} = \frac{\quad}{18} = \frac{20}{\quad}$ **i** $\frac{2}{5} = \frac{10}{\quad} = \frac{\quad}{15}$

3 Complete each set of equivalent fractions.

a $5\frac{1}{13} = 5\frac{\quad}{39} = 5\frac{5}{\quad}$

b $\frac{\quad}{10} = \frac{2}{20} = \frac{\quad}{30} = \frac{8}{\quad}$

c $\frac{8}{\quad} = \frac{24}{21} = \frac{\quad}{35} = \frac{\quad}{77}$

d $3\frac{3}{5} = 3\frac{\quad}{30} = 3\frac{9}{\quad} = 3\frac{\quad}{70}$

e $\frac{1}{12} = \frac{6}{\quad} = \frac{8}{\quad} = \frac{\quad}{144} = \frac{15}{\quad} = \frac{\quad}{240}$

f $\frac{2}{3} = \frac{\quad}{6} = \frac{10}{\quad} = \frac{\quad}{30} = \frac{\quad}{75} = \frac{400}{\quad}$

WE 3.2B 4 Write each fraction in its simplest form using repeated division.

a $\frac{10}{16}$ **b** $\frac{28}{36}$ **c** $\frac{63}{56}$ **d** $2\frac{12}{30}$ **e** $5\frac{33}{36}$ **f** $6\frac{15}{75}$

WE 3.2C 5 Write each fraction in its simplest form using the HCF.

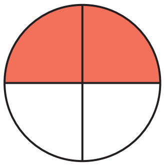
a $\frac{11}{33}$ **b** $\frac{30}{48}$ **c** $\frac{116}{94}$ **d** $7\frac{20}{45}$ **e** $4\frac{56}{64}$ **f** $3\frac{16}{54}$

6 List and find groups of equivalent fractions.

$\frac{1}{3}$	$\frac{5}{10}$	$\frac{12}{15}$	$\frac{6}{18}$	$\frac{17}{25}$
$\frac{30}{45}$	$\frac{16}{48}$	$\frac{18}{54}$	$\frac{20}{60}$	$\frac{52}{156}$

7 What fraction of each shape is shaded? Simplify your answers.

a



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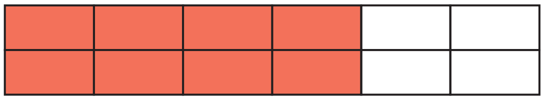
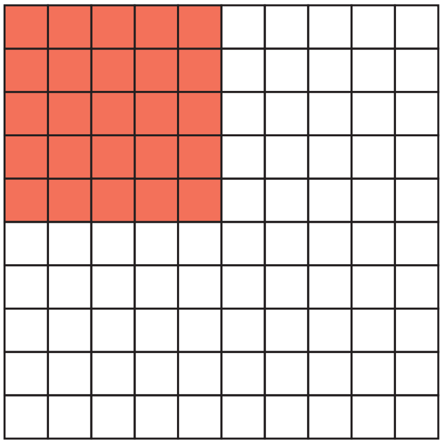
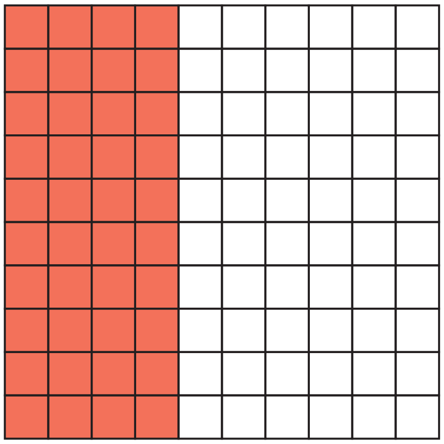


c



d

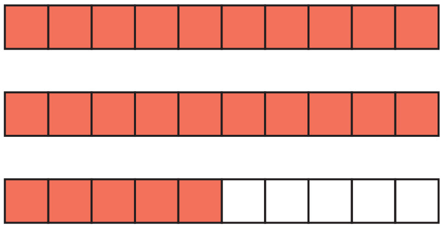


8 If each shape represents 1 whole, write the number that is represented by the shading in each diagram as:

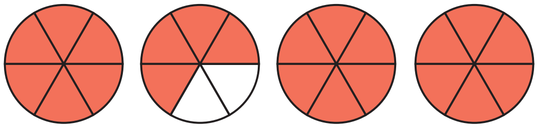
i a simplified improper fraction

ii a simplified mixed number.

a



b



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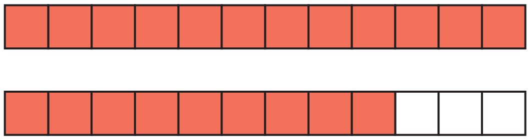


Exercise 3.2B: Problem solving and reasoning

9 Ingrid is making toy wallabies. She has 60 buttons to use as eyes. So far she has made 13 wallabies. What fraction of the buttons does she have left? Write your answer in its simplest form.

10 Larni scored 22 of her team's 34 goals. What fraction of the team's goals did Larni score? Write your answer in its simplest form.

11 Cristiano spent 75 minutes of a 90-minute football game on the field. What fraction of time, in its simplest form, did he spend off the field?

12 Find the values of the missing numbers.

a $\frac{27}{18} = \frac{\quad}{50}$

b $\frac{120}{\quad} = \frac{48}{54}$

c $\frac{33}{39} = \frac{143}{\quad}$

13 Keiynan is $\frac{5}{6}$ of the way through his 300-page book. If he reads 10 more pages, what fraction is he through the book? Write your answer in its simplest form.

14 The Thomson Reservoir is the largest water storage reservoir in Melbourne. When it is full, it can hold enough water to fill the Melbourne Cricket Ground (MCG) 628 times.

a If there was only enough water to fill the MCG 471 times, what fraction of the reservoir contains water?

b A state of emergency is declared when the reservoir's water level is below $\frac{1}{4}$.
How many MCGs could be filled at that water level?

15 A fraction can be simplified by dividing the numerator and the denominator by the highest common factor (HCF). Therefore, the HCF can be determined by finding the product of all common factors used in the simplification process.

For example, $\frac{80}{144} \overset{\div 2}{=} \frac{\cancel{80}^{40}}{\cancel{144}^{72}} \overset{\div 2}{=} \frac{\cancel{40}^{20}}{\cancel{72}^{36}} \overset{\div 2}{=} \frac{\cancel{20}^{10}}{\cancel{36}^{18}} \overset{\div 2}{=} \frac{\cancel{10}^{5}}{\cancel{18}^{9}} = \frac{5}{9}$.
The HCF of 80 and 144 is $2 \times 2 \times 2 \times 2 = 16$.

Find the HCF of the following numbers by simplifying fractions.

a 42 and 315 **b** 252 and 270 **c** 3024 and 3780

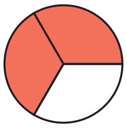
Exercise 3.2C: Challenge

16 When simplifying by using repeated division, especially when the HCF is not obvious or not easy to determine, a useful strategy is to divide by prime numbers until there is a remainder. That is, divide by 2 until one or both of the numbers are not even, then divide by 3 until the digit sum of one or both of the numbers is not a multiple of 3, then divide by 5 until one or both of the numbers do not end in a 0 or a 5, and so on.

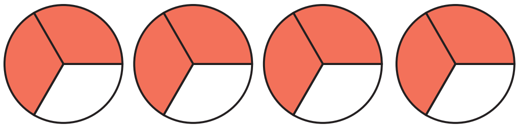
Simplify the following fractions using repeated division.

a $\frac{48\,600}{4800}$ **b** $\frac{121\,500}{202\,500\,000}$ **c** $\frac{196}{2058}$ **d** $\frac{161\,051}{2310}$

17 Here is a diagram showing $\frac{2}{3}$ of the whole circle shaded.



Here are four more of the same diagram. However, the fraction shaded depends on what is considered the whole.



Simplifying where possible, determine what fraction is shaded if:

a one whole is considered as one full circle

b one whole is considered as two full circles

c one whole is considered as four full circles

d one whole is considered as $\frac{1}{3}$ of a circle

e one whole is considered as $\frac{2}{3}$ of a circle

f one whole is considered as $\frac{4}{3}$ of a circle.

Online resources:

Interactive skillsheet
Equivalent fractions

Worksheet
Simplifying fractions

Investigation
Quilt design

Quick quiz
3.2

Lesson 3.3

Ordering fractions

Learning intentions

By the end of this lesson you will be able to ...

✔ compare and order fractions with different numerators and denominators using equivalent fractions.

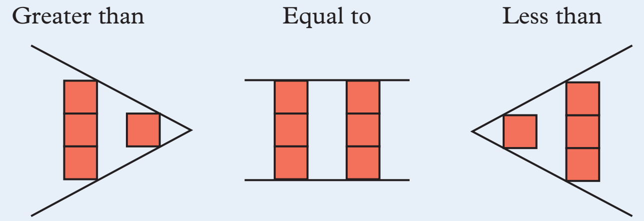
Inter-year links

Year 8 2.1 Fractions

Ordering fractions

Key content video
Ordering fractions

- ‘Less than’ is shown using the symbol ‘<’.
- ‘Greater than’ is shown using the symbol ‘>’.
- **Unit fractions** are fractions that have a numerator of 1. The larger the denominator of a unit fraction, the smaller the fraction.



3 > 1 3 is greater than 1

3 = 3 3 is equal to 3

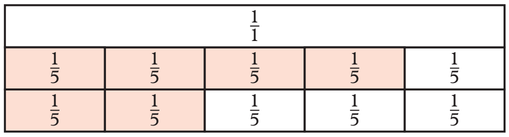
1 < 3 1 is less than 3

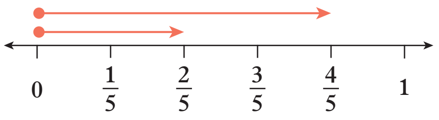
Ordering and comparing methods

Same denominator

To compare $\frac{2}{5}$ and $\frac{4}{5}$, think about having 2 parts, each one-fifth of a whole, and 4 of the same sized parts of the same sized whole.

$\frac{2}{5} < \frac{4}{5}$

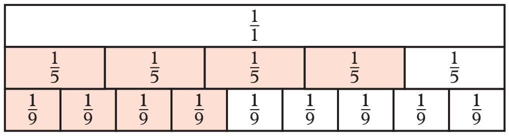


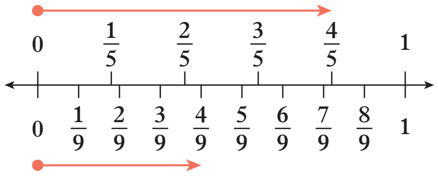


Same numerator

To compare $\frac{4}{5}$ and $\frac{4}{9}$, remember that $\frac{1}{5}$ is greater than $\frac{1}{9}$, so $\frac{4}{5}$ is 4 large parts compared to the 4 smaller parts of $\frac{4}{9}$.

$\frac{4}{9} < \frac{4}{5}$





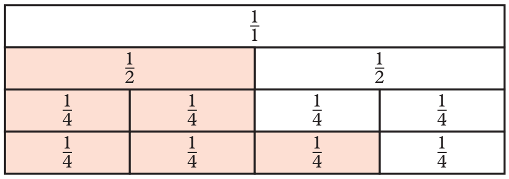
Ordering and comparing fractions using equivalent fractions

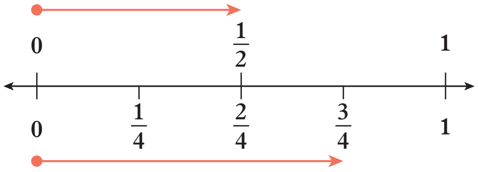
- A **common denominator** is a common multiple of the denominators of two or more fractions.

Common denominator

To compare $\frac{1}{2}$ and $\frac{3}{4}$, change $\frac{1}{2}$ to an equivalent fraction with a denominator of 4. Then $\frac{1}{2}$ can be easily compared with $\frac{3}{4}$.

$$\frac{1}{2} = \frac{2}{4}$$
$$\frac{2}{4} < \frac{3}{4}$$





- The **lowest common denominator (LCD)** is the lowest common multiple of the denominators in two or more fractions.

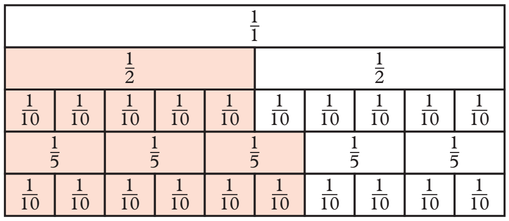
Lowest common denominator

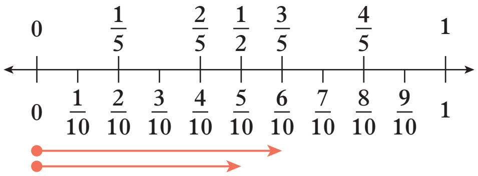
To compare $\frac{1}{2}$ and $\frac{3}{5}$, find the LCD and multiply the numerator and denominator of each fraction to get an equivalent fraction. Then compare the fractions with the same denominator.

The LCD of 2 and 5 is 10.

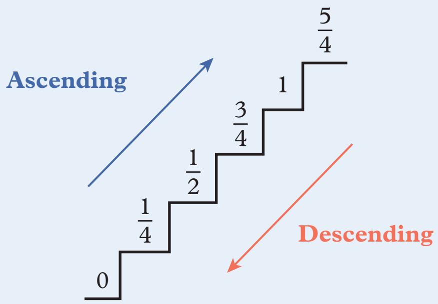
$$\frac{1}{2} \underset{\times 5}{\overset{\times 5}{=}} \frac{5}{10}$$

$$\frac{3}{5} \underset{\times 2}{\overset{\times 2}{=}} \frac{6}{10}$$





- Fractions in ascending order are ordered from smallest to largest.
- Fractions in descending order are ordered from largest to smallest.

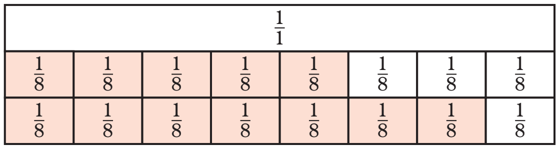


Worked example 3.3A Comparing fractions with the same denominator

Which fraction is greater: $\frac{5}{8}$ or $\frac{7}{8}$? Write your answer as an inequality.

THINK

Check if the fractions have the same denominator, same numerator, or neither. These fractions have the same denominator, so $\frac{5}{8}$ is less than $\frac{7}{8}$ as it has fewer of the same parts of the whole.



WRITE

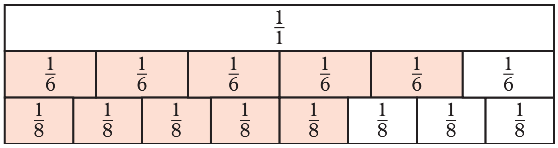
$\frac{5}{8} < \frac{7}{8}$

Worked example 3.3B Comparing fractions with the same numerator

Which fraction is greater: $\frac{5}{8}$ or $\frac{5}{6}$? Write your answer as an inequality.

THINK

Check if the fractions have the same denominator, same numerator, or neither. These fractions have the same numerator, so $\frac{5}{6}$ is greater than $\frac{5}{8}$ as it has 5 greater parts of $\frac{1}{6}$ compared to 5 smaller parts of $\frac{1}{8}$.



WRITE

$\frac{5}{8} < \frac{5}{6}$

Worked example 3.3C Comparing fractions with different denominators and numerators

Which fraction is greater: $\frac{5}{7}$ or $\frac{11}{14}$? Write your answer as an inequality.

THINK

1 Compare fractions by converting one fraction to an equivalent fraction that has either the same denominator or the same numerator as the other fraction. Since 14 is a multiple of 7, find an equivalent fraction to $\frac{5}{7}$ with a denominator of 14.

2 Compare the fractions with the same denominator and write the answer using the original fractions.

WRITE

$$\frac{5}{7} \underset{\times 2}{\overset{\times 2}{=}} \frac{10}{14}$$

$\frac{10}{14} < \frac{11}{14}$, therefore $\frac{5}{7} < \frac{11}{14}$

Worked example 3.3D Ordering fractions by finding the lowest common denominator

Which fraction is greater: $\frac{5}{6}$ or $\frac{7}{9}$? Write your answer as an inequality.

THINK	WRITE
1 Find the LCM of the denominators by listing the multiples of 6 and 9. Identify the lowest multiple that appears in both lists.	Multiples of 6: 6, 12, 18, 24, … Multiples of 9: 9, 18, 27, 36, … The LCM is 18. So, 18 is the LCD.
2 For each fraction, find the equivalent fraction with a denominator equal to the LCM. 6 multiplied by 3 is 18, so multiply the numerator and denominator of $\frac{5}{6}$ by 3. 9 multiplied by 2 is 18, so multiply the numerator and denominator of $\frac{7}{9}$ by 2.	$\frac{5}{6} \underset{\times 3}{\overset{\times 3}{=}} \frac{15}{18}$ $\qquad$ $\frac{7}{9} \underset{\times 2}{\overset{\times 2}{=}} \frac{14}{18}$
3 Compare the fractions with the same denominator and write the answer using the original fractions.	$\frac{15}{18} > \frac{14}{18}$, therefore $\frac{5}{6} > \frac{7}{9}$

Helpful hints

✔ Remember to consider both the numerator and the denominator of fractions before comparing fractions.

✔ Multiplying two denominators together will find a common denominator but not always the lowest common denominator.

Learning pathways

▲ 1–4, 5(a, c), 6(a, b), 7–11, 13, 14, 15(a, b)

■ 3(b, d, f), 5(b, d), 6(c, d), 7, 8, 9(b, c), 10(b, c), 11, 12, 15–17

⬟ 5(b, d), 6(c, d), 8, 9(c, d), 10(c, d), 11, 12, 15–20

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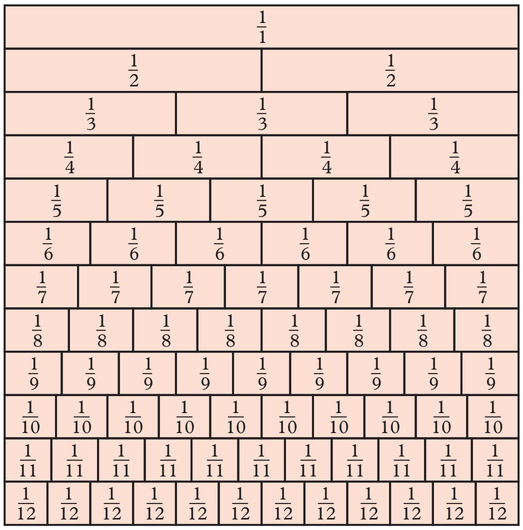
Exercise 3.3A: Understanding and fluency

1 Use the fraction wall to decide which fraction is greater.

- **a** $\frac{2}{3}$ or $\frac{5}{8}$
- **b** $\frac{7}{10}$ or $\frac{9}{12}$
- **c** $\frac{2}{12}$ or $\frac{1}{5}$
- **d** $\frac{5}{6}$ or $\frac{7}{8}$



WE 3.3A **2** Identify the greater fraction in each pair. Write your answer as an inequality, with the greater fraction first.

a $\frac{3}{6}$ or $\frac{5}{6}$

b $\frac{7}{5}$ or $\frac{9}{5}$

c $\frac{93}{15}$ or $\frac{32}{15}$

d $1\frac{3}{8}$ or $1\frac{7}{8}$

e $3\frac{9}{20}$ or $3\frac{11}{20}$

f $\frac{108}{4}$ or $\frac{180}{4}$

WE 3.3B **3** Identify the greater fraction in each pair. Write your answer as an inequality, with the greater fraction first.

a $\frac{2}{3}$ or $\frac{2}{4}$

b $\frac{16}{44}$ or $\frac{16}{45}$

c $\frac{77}{100}$ or $\frac{77}{12}$

d $2\frac{5}{6}$ or $2\frac{5}{7}$

e $4\frac{13}{20}$ or $4\frac{13}{15}$

f $\frac{55}{17}$ or $\frac{55}{30}$

4 Use a number line to answer the following questions.

a Draw a number line and mark the position of each of these fractions.

$$\frac{12}{7}, \frac{10}{7}, \frac{15}{7}, \frac{5}{7}, \frac{21}{7}, \frac{32}{7}, \frac{1}{7}, \frac{31}{7}$$

b List the fractions in ascending order (smallest to largest) using the less than sign '<'. What do you notice?

5 Write each list of fractions in ascending order (smallest to largest) using the less than sign.

a $\frac{8}{11}, \frac{2}{11}, \frac{15}{11}, \frac{5}{11}$

b $\frac{16}{7}, \frac{15}{7}, \frac{6}{7}, \frac{14}{7}$

c $\frac{10}{5}, \frac{10}{13}, \frac{10}{15}, \frac{10}{3}$

d $\frac{77}{59}, \frac{77}{101}, \frac{77}{70}, \frac{77}{89}$

6 Write each list of fractions in descending order (largest to smallest).

a $\frac{1}{5}, 1\frac{2}{5}, 3\frac{2}{5}, 1\frac{4}{5}, 2\frac{1}{5}, \frac{4}{5}$

b $\frac{6}{5}, \frac{6}{2}, \frac{6}{6}, \frac{6}{4}, \frac{6}{3}, \frac{6}{8}$

c $\frac{67}{67}, \frac{1}{67}, \frac{100}{67}, \frac{76}{67}, \frac{33}{67}$

d $\frac{38}{40}, \frac{38}{60}, \frac{38}{2}, \frac{38}{37}, \frac{38}{19}, \frac{38}{59}$

WE 3.3C **7** Which fraction in each pair is greater? Write your answer as an inequality.

a $\frac{3}{6}$ or $\frac{5}{12}$

b $\frac{7}{10}$ or $\frac{30}{40}$

c $\frac{93}{15}$ or $\frac{32}{5}$

d $\frac{43}{48}$ or $\frac{7}{8}$

e $1\frac{9}{20}$ or $1\frac{55}{100}$

f $\frac{23}{2}$ or $\frac{180}{14}$

WE 3.3D **8** Determine which fraction is greater by finding their lowest common denominator (LCD). Write your answer as an inequality.

a $\frac{5}{6}$ or $\frac{8}{9}$

b $\frac{10}{20}$ or $\frac{11}{14}$

c $2\frac{15}{24}$ or $2\frac{14}{16}$

d $\frac{12}{7}$ or $\frac{13}{5}$

e $\frac{6}{17}$ or $\frac{5}{9}$

f $\frac{17}{16}$ or $\frac{18}{17}$

9 Write each list of fractions in ascending order.

a $\frac{7}{5}, \frac{2}{5}, \frac{4}{3}, \frac{1}{3}, \frac{8}{3}, \frac{16}{5}$

b $\frac{5}{8}, \frac{11}{4}, \frac{2}{6}, \frac{7}{6}, \frac{5}{4}, \frac{9}{8}$

c $\frac{2}{9}, \frac{4}{3}, \frac{11}{9}, \frac{5}{6}, \frac{7}{3}, \frac{9}{6}$

d $\frac{8}{15}, \frac{5}{4}, \frac{6}{3}, \frac{11}{10}, \frac{9}{20}, \frac{10}{6}$

10 Write each list of fractions in descending order.

a $\frac{3}{2}, \frac{8}{5}, \frac{7}{6}, \frac{10}{6}, \frac{5}{2}, \frac{12}{5}$

b $\frac{1}{8}, \frac{11}{12}, \frac{5}{6}, \frac{3}{8}, \frac{7}{12}, \frac{2}{6}$

c $\frac{10}{7}, \frac{9}{8}, \frac{3}{4}, \frac{3}{2}, \frac{7}{8}, \frac{6}{7}$

d $\frac{2}{9}, \frac{1}{3}, \frac{5}{7}, \frac{4}{3}, \frac{11}{7}, \frac{13}{9}$

11 Complete each number statement using the less than, <, or greater than, >, symbols.

a $\frac{1}{3}$ ______ $\frac{1}{4}$

b $\frac{2}{5}$ ______ $\frac{5}{2}$

c $4\frac{5}{6}$ ______ $\frac{23}{3}$

d $\frac{42}{5}$ ______ $7\frac{6}{7}$

e $\frac{7}{9}$ ______ $\frac{6}{7}$

f $8\frac{5}{6}$ ______ $\frac{49}{5}$

12 Without changing denominators, complete each number statement using the less than, <, or greater than, >, symbols.

a $\frac{12}{13}$ ______ $\frac{13}{14}$

b $\frac{4}{5}$ ______ $\frac{3}{4}$

c $\frac{1}{6}$ ______ $\frac{2}{7}$

d $\frac{3}{9}$ ______ $\frac{3}{10}$

e $\frac{4}{19}$ ______ $\frac{4}{17}$

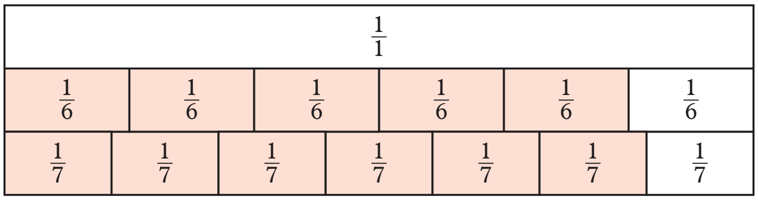
f $\frac{5}{16}$ ______ $\frac{5}{26}$

Exercise 3.3B: Problem solving and reasoning

13 Philip was comparing two of his test results. He scored $\frac{22}{25}$ on the first test and $\frac{45}{50}$ on the second. On which test did Philip perform better?

14 Zach and Melanie are comparing their results after an afternoon of archery. From 35 attempts, Zach scored 20 bullseyes. Melanie scored 24 bullseyes from 40 attempts. Who hit the bullseye on the archery target with more accuracy? Justify your answer.

15 We can compare fractions that both have a numerator that is one away from a whole. The fraction with a larger denominator has smaller parts, so will be missing a smaller part.



$\frac{5}{6} < \frac{6}{7}$

Write the inequality that compares the following pairs of fractions.

a $\frac{5}{6}, \frac{8}{9}$ **b** $\frac{17}{16}, \frac{18}{17}$ **c** $\frac{100}{101}, \frac{99}{100}$ **d** $\frac{48}{47}, \frac{47}{46}$

16 The following inequalities and explanations may or may not be correct. Write the correct inequalities and give explanations.

a $\frac{5}{8} > \frac{5}{7}$ since $8 > 7$ **b** $\frac{70}{34} > \frac{9}{10}$ since $70 > 9$ **c** $\frac{17}{5} < \frac{23}{5}$ since $17 < 23$

17 Sophie and Tilly are practising shooting goals in netball. Sophie shoots 29 goals from 40 attempts and Tilly shoots 25 goals from 30 attempts. Sophie says, 'I don't want to make you feel bad, Tilly, but I am obviously the more accurate shooter since I got 29 goals and you only got 25.' Tilly replies, 'Well I don't want to make you feel bad about your maths and shooting skills, but I am more accurate!' Which student is correct? Justify your answer.

Exercise 3.3C: Challenge

18 Place in ascending order.

$\frac{21}{5}$	$\frac{15}{4}$	$\frac{17}{8}$	$\frac{25}{6}$	$\frac{60}{9}$	$\frac{11}{2}$	$\frac{10}{3}$

19 Place in ascending order.

$\frac{98}{99}$	$\frac{997}{999}$	$\frac{9996}{9999}$	$\frac{99\,995}{99\,999}$	$\frac{999\,994}{999\,999}$

20 A certain fraction is greater than $\frac{1}{3}$ and less than $\frac{3}{5}$. How many possible whole number values are there for the numerators if the denominator of this fraction is:

a 15 **b** 30 **c** 20?

Online resources:

Interactive skillsheet
Ordering fractions

Quick quiz
3.3

Lesson 3.4

Mixed numbers and improper fractions

Learning intentions

By the end of this lesson you will be able to …

✔ convert between mixed numbers and improper fractions and identify them on a number line.

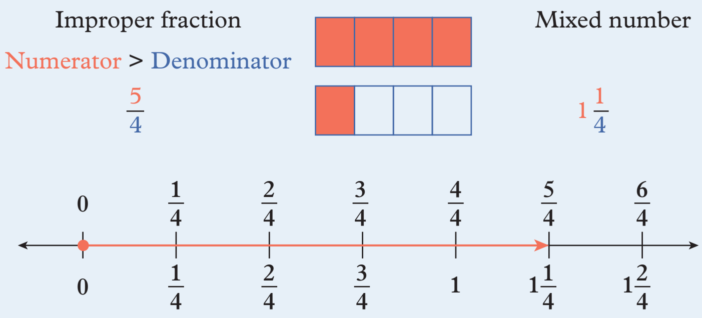
Inter-year links

Year 8 2.1 Fractions

Mixed numbers and improper fractions

Key content video
Mixed numbers and improper fractions

- Mixed numbers can be written as improper fractions. Improper fractions can also be written as mixed numbers.
- Mixed numbers consist of both a whole number and a proper fraction. For example, $1\frac{1}{4} = 1 + \frac{1}{4}$.



Improper fraction to mixed number

- Improper fractions show how many parts of equal size are present. Converting improper fractions to mixed numbers can show how many wholes can be created from those equal parts and how many parts are left over.
- To convert an improper fraction to a mixed number:
  1. Divide the numerator by the denominator.
  2. Write down the whole number answer and place the remainder over the denominator.
  3. Simplify the fraction if possible.

$$\frac{13}{4} = 13 \div 4 = 3 \text{ remainder } 1$$

$$= 3\frac{1}{4}$$

Mixed number to improper fraction

- Mixed numbers consist of a whole number and a fraction. Converting mixed numbers to improper fractions allows us to see how many equal parts there are in the whole number and the fraction.
  1. Multiply the denominator by the whole number and add the numerator.
  2. Write the result over the denominator.

$$\overset{+}{\underset{\times}{3}}\frac{1}{4} = \frac{(4 \times 3) + 1}{4} = \frac{13}{4}$$

Worked example 3.4A Converting improper fractions to mixed numbers

Convert the improper fraction $\frac{26}{6}$ to a mixed number.

THINK	WRITE
1 Divide the numerator by the denominator.	$\frac{26}{6} = 4$ remainder 2
2 Write down the whole number answer and place the remainder over the denominator.	$4\frac{2}{6} = 4\frac{\cancel{2}^{1}}{\cancel{6}_{3}}$
3 Simplify the fraction.	$= 4\frac{1}{3}$

Worked example 3.4B Converting mixed numbers to improper fractions

Convert the mixed number $3\frac{2}{7}$ to an improper fraction.

THINK	WRITE
1 Multiply the denominator by the whole number and add the numerator.	$\overset{+}{\underset{\times}{3}}\frac{2}{7} = \frac{(7 \times 3) + 2}{7}$
2 Write the result over the denominator.	$= \frac{23}{7}$

Helpful hints

- ✔ An improper fraction can be simplified before or after writing it as a mixed number.
- ✔ It can be easier to simplify after writing an improper fraction as a mixed number as the numerator will be much smaller.
  For example, $\frac{12}{8} = 1\frac{\cancel{4}^{1}}{\cancel{8}_{2}} = 1\frac{1}{2}$ or $\frac{\cancel{12}^{3}}{\cancel{8}_{2}} = \frac{3}{2} = 1\frac{1}{2}$.

Learning pathways

▲ 1–3, 4(a, b), 5–6, 7(a, c, e, g), 8–11 ■ 1, 2, 4(c, d), 6, 7, 9, 10, 12, 14 ⬟ 1(e, f, k, l), 2(e, f, k, l), 4(d), 7, 9, 12–15

ANS p512

Exercise 3.4A: Understanding and fluency

WE 3.4A **1** Convert each improper fraction to a mixed number.

a $\frac{7}{5}$ **b** $\frac{26}{3}$ **c** $\frac{25}{9}$

d $\frac{43}{10}$ **e** $\frac{75}{8}$ **f** $\frac{88}{13}$

g $\frac{22}{7}$ **h** $\frac{17}{4}$ **i** $\frac{49}{6}$

j $\frac{50}{11}$ **k** $\frac{67}{5}$ **l** $\frac{111}{2}$

WE 3.4B **2** Convert each mixed number to an improper fraction.

a $1\frac{2}{9}$ **b** $3\frac{1}{6}$ **c** $5\frac{8}{9}$

d $2\frac{6}{11}$ **e** $10\frac{4}{5}$ **f** $8\frac{3}{4}$

g $2\frac{3}{7}$ **h** $12\frac{1}{3}$ **i** $4\frac{5}{8}$

j $5\frac{3}{10}$ **k** $7\frac{1}{12}$ **l** $2\frac{4}{15}$

3 State whether the following statements are true or false.

a To convert an improper fraction to a mixed number, divide the numerator by the denominator, write down the whole number answer and place the remainder over the denominator.

b An improper fraction is smaller than 1.

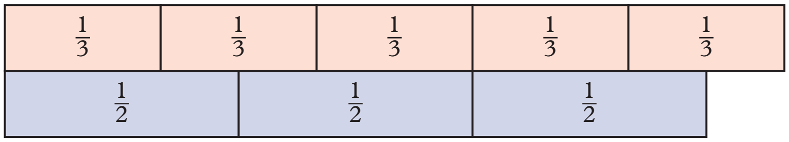
c A mixed number can have an improper fraction part.

d $\frac{45}{18} \neq 2\frac{8}{16}$

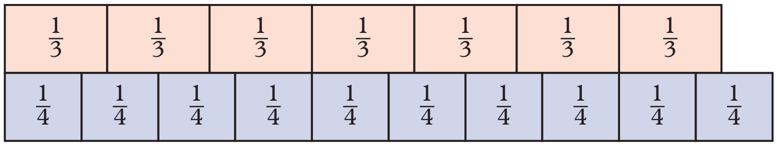
e An improper fraction can be simplified before or after converting it to a mixed number.

4 Write an inequality that compares the two fractions shown in each diagram. Provide your answers as mixed numbers or improper fractions.

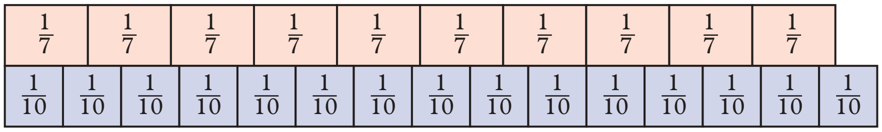
a



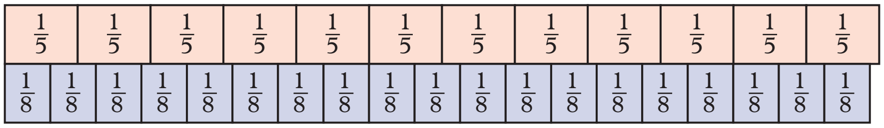
b



c



d



5 Convert each fraction to an improper fraction (if necessary) and use the symbols < or > to make a true statement.

a $2\frac{4}{7}$ ______ $\frac{17}{7}$

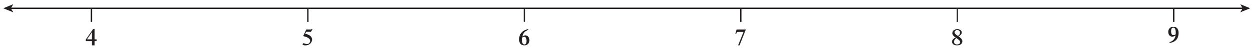
b $8\frac{2}{3}$ ______ $\frac{28}{3}$

c $\frac{38}{11}$ ______ $2\frac{6}{11}$

d $9\frac{4}{5}$ ______ $\frac{51}{5}$

6 Just like proper fractions, mixed numbers and improper fractions can be placed on a number line.

a Copy the number line and divide each interval into three equal parts. Label each of these parts using mixed numbers.



b Convert each fraction to a mixed number, simplifying where necessary, and then mark its position on the number line drawn in part **a**.

i $\frac{23}{3}$

ii $\frac{20}{3}$

iii $5\frac{2}{6}$

iv $7\frac{4}{6}$

v $\frac{24}{4}$

vi $\frac{12}{3}$

c Hence, arrange the fractions in part **b** in ascending order.

7 Convert both fractions to mixed numbers and use the < or > symbols to make a true statement.

a $\frac{28}{3}$ ______ $\frac{17}{5}$

b $\frac{49}{6}$ ______ $\frac{38}{4}$

c $\frac{13}{2}$ ______ $\frac{26}{9}$

d $\frac{28}{5}$ ______ $\frac{61}{10}$

e $\frac{42}{7}$ ______ $\frac{63}{9}$

f $\frac{27}{6}$ ______ $\frac{15}{4}$

g $\frac{32}{5}$ ______ $\frac{63}{8}$

h $\frac{21}{2}$ ______ $\frac{41}{4}$

Exercise 3.4B: Problem solving and reasoning

8 Leica buys eight packets of trading cards to share equally with four of her friends.

a Write a mixed number to represent how many packets Leica and each of her friends receive.

b If each packet has 20 trading cards, how many cards does each person receive?

9 Sava is on a week-long camping trip with five friends. He has bought nine packets of marshmallows to toast on the campfire each night.

a Write a mixed number to represent how many packets of marshmallows are allocated to each person.

b If each packet contains 10 marshmallows, how many does each person receive?

10 The following conversions have mistakes. Explain the mistake and correct the calculation.

a $3\frac{1}{5} = \frac{3+1}{5} = \frac{4}{5}$

b $\frac{14}{5} = 3\frac{1}{5}$

c $6\frac{2}{3} = \frac{2}{6 \times 2 + 3} = \frac{2}{15}$

d $3\frac{4}{7} = \frac{7}{25}$

11 Darius is a chocaholic. His favourite type of chocolate comes in a block of 24 pieces. One day Darius ate $2\frac{2}{3}$ blocks of chocolate. Determine how many pieces of chocolate Darius ate.

12 Asha wrote the following calculation to convert $\frac{18}{7}$ to a mixed number:

$$\begin{aligned}\frac{18}{7} &= \frac{14+4}{7}\\ &= \frac{14}{7}+\frac{4}{7}\\ &= \frac{14}{7}+\frac{4}{7}\\ &= 2\frac{4}{7}\end{aligned}$$

a Use this method to write the following improper fractions as mixed numbers.

i $\frac{8}{3}$ **ii** $\frac{17}{2}$ **iii** $\frac{11}{6}$ **iv** $\frac{47}{10}$

b Explain how this method relates to the method used in **Worked example 3.4A**.

The reverse also works for converting a mixed number to an improper fraction.

$$\begin{aligned}2\frac{4}{7} &= 2+\frac{4}{7}\\ &= \frac{14}{7}+\frac{4}{7}\\ &= \frac{14+4}{7}\\ &= \frac{18}{7}\end{aligned}$$

c Use this method to write the following improper fractions as mixed numbers.

i $3\frac{2}{5}$ **ii** $1\frac{6}{17}$ **iii** $9\frac{1}{2}$ **iv** $9\frac{7}{10}$

d Explain how this method relates to the method used in **Worked example 3.4B**.

13 Georgina, Roisin and Vanessa decide to order large pizzas for a class lunch. Each pizza will be cut into eight equal slices. The girls estimate that each person will eat two slices and there will be a maximum of 22 people.

a Write the number of pizzas required as:

i an improper fraction

ii a mixed number.

b Will each person be able to receive the estimated two slices of pizza if five large pizzas have been ordered?

c The girls' budget does not allow them to order more than five pizzas. What is the minimum number of equal slices each pizza can be cut into, if everyone is to receive two slices?

d What fraction of a pizza will be left over if the pizzas are cut in the way suggested in part **c**?

e On the day of the lunch, three people were away and $4\frac{5}{8}$ of the pizzas were eaten. If each pizza was cut into eight slices, how many slices were eaten?

Exercise 3.4C: Challenge

14 Write $\frac{69\,615}{31\,122}$ as a simplified mixed number.

15 There are mixed numbers in the form $a\frac{b}{c}$, where a represents a whole number, b represents the numerator and c represents the denominator. After converting this mixed number into an improper fraction, the fraction becomes $\frac{ab}{c}$. For example, $9\frac{3}{10} = \frac{93}{10}$. In this case $a = 9$, $b = 3$, $c = 10$. List more fractions like this.

Online resources:

Interactive skillsheet
Mixed numbers and improper fractions

Worksheet
Mixed numbers and improper fractions

Investigation
Sandwich fractions

Quick quiz
3.4

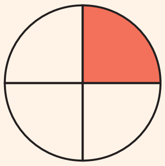
ANS p513

Checkpoint

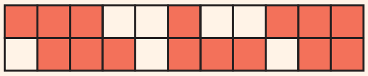
Checkpoint quiz
Take the checkpoint quiz to check your knowledge of the first part of this module.

3.1 **1** What fraction of each shape is shaded?

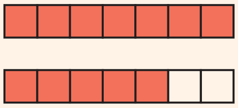
a



b

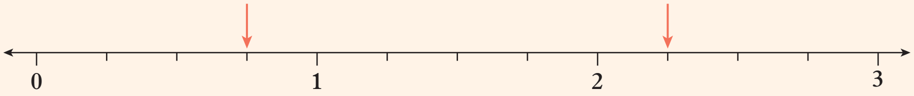


c

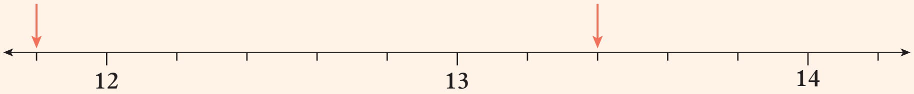


3.1 **2** Write the fractions indicated on each number line. Use mixed numbers where required.

a



b



3.1 **3** Write the fraction represented by each statement.

a Lachlan has collected 17 of the 20 trading cards in a set.
b Sal has surveyed 51 of the 100 people required.
c Katharine made 16 attempts at kicking a goal and got 15 in.

3.2 **4** Complete each set of equivalent fractions.

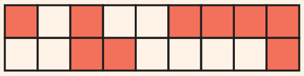
a $\frac{12}{5} = \frac{}{15}$ **b** $\frac{20}{} = \frac{4}{7}$ **c** $\frac{18}{27} = \frac{}{3} = \frac{}{6}$

3.2 **5** Simplify the following fractions.

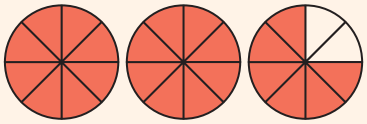
a $\frac{28}{56}$ **b** $\frac{88}{55}$ **c** $7\frac{40}{48}$

3.2 **6** Write the simplified fraction that the shaded section represents. Each shape represents one whole.

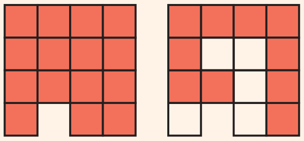
a



b



c



3.3 **7** Write the correct inequality to compare each pair of fractions.

a $\frac{87}{121}, \frac{78}{121}$ **b** $\frac{55}{8}, \frac{55}{9}$ **c** $\frac{3}{4}, \frac{5}{6}$

3.3 **8** Write the following lists of fractions in ascending order.

a $\frac{9}{17}, 1\frac{5}{17}, \frac{20}{17}, \frac{1}{17}$ **b** $\frac{2}{3}, \frac{1}{6}, \frac{3}{4}, \frac{7}{12}$ **c** $5\frac{1}{8}, 5\frac{1}{3}, 5\frac{1}{4}, 5\frac{1}{6}$ **d** $\frac{15}{9}, \frac{13}{4}, \frac{17}{6}, \frac{19}{12}$

3.4 **9** Write these mixed numbers as simplified improper fractions.

a $2\frac{3}{5}$ **b** $6\frac{5}{7}$ **c** $12\frac{35}{40}$

3.4 **10** Write these improper fractions as simplified mixed numbers.

a $\frac{19}{3}$ **b** $\frac{67}{8}$ **c** $\frac{180}{21}$

Lesson 3.5

Adding and subtracting fractions

Learning intentions

By the end of this lesson you will be able to ...

✔ add and subtract proper fractions, improper fractions and mixed numbers.

Inter-year links

Support	Adding and subtracting fractions
Year 8	2.2 Adding and subtracting fractions

Key content video
Adding and subtracting fractions

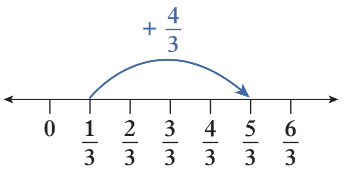
Adding and subtracting like and unlike fractions

- **Like fractions** have the same denominators and **unlike fractions** have different denominators. For example, $\frac{2}{3}$ and $\frac{4}{3}$ are like fractions, whereas $\frac{2}{3}$ and $\frac{5}{4}$ are unlike fractions.

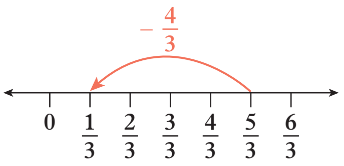
Adding and subtracting like fractions

1 Add or subtract the numerators and keep the same denominator.
2 Simplify the fraction if required.

$$\frac{1}{3}+\frac{4}{3}=\frac{5}{3}$$



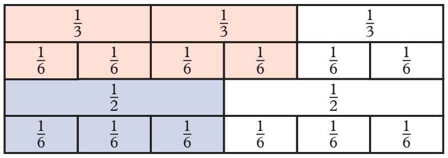
$$\frac{5}{3}-\frac{4}{3}=\frac{1}{3}$$



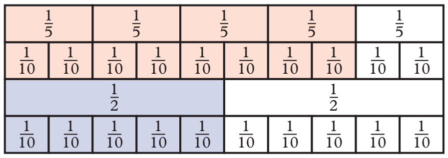
Adding and subtracting unlike fractions

1 Convert into equivalent fractions with a common denominator.
2 Add or subtract the numerators and keep the same denominator.
3 Simplify the fraction if required.

$$\frac{2}{3}+\frac{1}{2}=\frac{4}{6}+\frac{3}{6}$$
$$=\frac{7}{6}$$



$$\frac{4}{5}-\frac{1}{2}=\frac{8}{10}-\frac{5}{10}$$
$$=\frac{3}{10}$$



Adding and subtracting mixed numbers

Adding and subtracting mixed numbers	
1 Convert mixed numbers into improper fractions. **2** Convert the improper fractions into equivalent fractions with a common denominator. **3** Add or subtract the numerators and keep the same denominator. **4** Convert the answer into a mixed number and simplify if possible.	$2\frac{1}{4}+3\frac{1}{5}=\frac{9}{4}+\frac{16}{5}$ $=\frac{45}{20}+\frac{64}{20}$ $=\frac{109}{20}$ $=5\frac{9}{20}$
	$2\frac{3}{4}-1\frac{1}{2}=\frac{11}{4}-\frac{3}{2}$ $=\frac{11}{4}-\frac{6}{4}$ $=\frac{5}{4}$ $=1\frac{1}{4}$

- Before adding and subtracting fractions, it can be helpful to estimate the answer. Consider if the answer will be greater than or equal to $\frac{1}{2}$, 1, 2 etc. For mixed numbers, you can estimate by adding or subtracting just the whole number parts.

Worked example 3.5A Adding and subtracting like fractions

Evaluate the following.

a $\frac{3}{8}+\frac{7}{8}$ **b** $\frac{9}{14}-\frac{2}{14}$ **c** $\frac{1}{5}+\frac{3}{5}-\frac{2}{5}$

THINK	WRITE
a **1** Add the numerators and keep the same denominator. **2** Simplify the fraction by dividing the numerator and denominator by the HCF.	**a** $\frac{3}{8}+\frac{7}{8}=\frac{10}{8}$ $=\frac{\cancel{10}^{5}}{\cancel{8}^{4}}$ $=\frac{5}{4}$
b **1** Subtract the numerators and keep the same denominator. **2** Simplify the fraction by dividing the numerator and denominator by the HCF.	**b** $\frac{9}{14}-\frac{2}{14}=\frac{7}{14}$ $=\frac{\cancel{7}^{1}}{\cancel{14}^{2}}$ $=\frac{1}{2}$
c **1** Remember the order of operations. Addition and subtraction are both the last operation to be applied, so work from left to right. **2** Simplify the fraction. The HCF of 2 and 5 is 1, so the fraction is already in its simplest form.	**c** $\frac{1}{5}+\frac{3}{5}-\frac{2}{5}=\frac{4}{5}-\frac{2}{5}$ $=\frac{2}{5}$

Worked example 3.5B Adding and subtracting unlike fractions

Evaluate the following.

a $\frac{5}{6}+\frac{1}{2}$ **b** $\frac{11}{5}-\frac{6}{4}$

THINK	**WRITE**
a 1 Convert into equivalent fractions with a common denominator. The LCM of 6 and 2 is 6. Change $\frac{1}{2}$ to an equivalent fraction with a denominator of 6.	**a** $\underset{\times 3}{\overset{\times 3}{\frac{1}{2} = \frac{3}{6}}}$
2 Add the numerators and keep the same denominator.	$\frac{5}{6}+\frac{1}{2}=\frac{5}{6}+\frac{3}{6}$ $=\frac{\cancel{8}^{4}}{\cancel{6}^{3}}$
3 Simplify the fraction by dividing the numerator and denominator by the HCF.	$=\frac{4}{3}$
b 1 Convert into equivalent fractions with a common denominator. The LCM of 5 and 4 is 20. Change both fractions to equivalent fractions with a denominator of 20.	**b** $\underset{\times 4}{\overset{\times 4}{\frac{11}{5} = \frac{44}{20}}}$ $\underset{\times 5}{\overset{\times 5}{\frac{6}{4} = \frac{30}{20}}}$
2 Subtract the numerators and keep the same denominator.	$\frac{11}{5}-\frac{6}{4}=\frac{44}{20}-\frac{30}{20}$ $=\frac{\cancel{14}^{7}}{\cancel{20}^{10}}$
3 Simplify the fraction by dividing the numerator and denominator by the HCF.	$=\frac{7}{10}$

Worked example 3.5C Adding and subtracting mixed numbers

Evaluate the following.

a $1\frac{4}{9}+2\frac{2}{9}$ **b** $4\frac{1}{3}-2\frac{1}{4}$

THINK	**WRITE**
a 1 Convert the mixed numbers into improper fractions.	**a** $\overset{+}{\underset{\times}{1}}\frac{4}{9}+\overset{+}{\underset{\times}{2}}\frac{2}{9}=\frac{13}{9}+\frac{20}{9}$
2 Add the numerators and keep the same denominator.	$\frac{13}{9}+\frac{20}{9}=\frac{33}{9}$
3 Convert the answer into a mixed number and simplify if possible.	$=3\frac{\cancel{6}^{2}}{\cancel{9}^{3}}$ (33 ÷ 9 = 3 remainder 6) $=3\frac{2}{3}$
b 1 Convert the mixed numbers into improper fractions.	**b** $\overset{+}{\underset{\times}{4}}\frac{1}{3}-\overset{+}{\underset{\times}{2}}\frac{1}{4}=\frac{13}{3}-\frac{9}{4}$
2 Rewrite the improper fractions with a common denominator. The LCM of 3 and 4 is 12.	$\underset{\times 4}{\overset{\times 4}{\frac{13}{3} = \frac{52}{12}}}$ $\underset{\times 3}{\overset{\times 3}{\frac{9}{4} = \frac{27}{12}}}$
3 Subtract the numerators and keep the same denominator.	$\frac{13}{3}-\frac{9}{4}=\frac{52}{12}-\frac{27}{12}$ $=\frac{25}{12}$ (25 ÷ 12 = 2 remainder 1)
4 Convert the answer into a mixed number and simplify if possible.	$=2\frac{1}{12}$

Helpful hints

- ✔ The key to success in adding and subtracting fractions is a common denominator. When all fractions have the same denominator, you only need to add or subtract the numerators.
  For example, $\frac{5}{6}-\frac{2}{6}\neq\frac{5-2}{6-6}$.
- ✔ Sometimes you only need to find an equivalent fraction of one of your fractions for their denominators to be the same.
  For example, $\frac{1}{4}-\frac{1}{8}=\frac{2}{8}-\frac{1}{8}$.

Learning pathways

△ 1–5(1st, 2nd columns), 6–9, 10(a, b), 11–13, 18(a, c)

■ 4(b, d, f, g), 5–6, 8–10, 14, 15, 17, 18(b, c, d), 19(a)

⬟ 4–5(3rd column), 6, 8(g, h), 9–10, 14, 16, 17, 18(b, d), 19–21

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Exercise 3.5A: Understanding and fluency

1 Evaluate the following. Where possible, write your answer as a mixed number.

a $1+\frac{1}{4}$ **b** $2+\frac{1}{2}$ **c** $\frac{3}{5}+3$

d $12-\frac{1}{3}$ **e** $5-\frac{7}{12}$ **f** $13-\frac{1}{13}$

g $2+4\frac{2}{3}$ **h** $3+2\frac{1}{2}$ **i** $4\frac{3}{5}+3$

j $2-1\frac{11}{33}$ **k** $53-28\frac{3}{12}$ **l** $30-3\frac{2}{13}$

WE 3.5A **2** Find the result of each sum. Where possible, write your answer as a mixed number.

a $\frac{1}{5}+\frac{2}{5}$ **b** $\frac{4}{7}+\frac{2}{7}$ **c** $\frac{5}{12}+\frac{3}{12}$

d $\frac{5}{18}+\frac{6}{18}$ **e** $\frac{3}{37}+\frac{16}{37}$ **f** $\frac{9}{23}+\frac{16}{23}$

g $\frac{15}{11}+\frac{2}{11}+\frac{8}{11}$ **h** $\frac{2}{13}+\frac{4}{13}+\frac{5}{13}$ **i** $\frac{23}{100}+\frac{7}{100}+\frac{14}{100}$

3 Find the result of each difference. Where possible, write your answer as a mixed number.

a $\frac{11}{7}-\frac{5}{7}$ **b** $\frac{25}{21}-\frac{8}{21}$ **c** $\frac{19}{13}-\frac{5}{13}$

d $\frac{13}{15}-\frac{3}{15}$ **e** $\frac{26}{27}-\frac{25}{27}$ **f** $\frac{21}{33}-\frac{5}{33}$

g $\frac{48}{17}-\frac{13}{17}-\frac{10}{17}$ **h** $\frac{55}{3}-\frac{47}{3}-\frac{7}{3}$ **i** $\frac{21}{8}-\frac{2}{8}-\frac{15}{8}$

WE 3.5B **4** Evaluate the following. When required, simplify your answer.

a $\frac{2}{3}+\frac{1}{6}$ **b** $\frac{2}{12}+\frac{5}{6}$ **c** $\frac{1}{2}-\frac{3}{8}$

d $\frac{3}{4}-\frac{3}{16}$ **e** $\frac{2}{15}+\frac{4}{5}$ **f** $\frac{4}{30}+\frac{7}{10}$

g $\frac{5}{8}-\frac{5}{48}$ **h** $\frac{3}{26}+\frac{1}{13}$ **i** $\frac{17}{23}-\frac{1}{69}$

5 Calculate the following.

a $\frac{1}{2} + \frac{2}{3}$ **b** $\frac{3}{5} - \frac{1}{4}$ **c** $\frac{3}{8} + \frac{5}{6}$

d $\frac{5}{9} - \frac{1}{2}$ **e** $\frac{3}{4} + \frac{2}{3}$ **f** $\frac{1}{3} - \frac{1}{5}$

g $\frac{4}{7} + \frac{1}{2}$ **h** $\frac{5}{6} - \frac{3}{5}$ **i** $\frac{6}{7} - \frac{1}{3}$

6 Calculate the following.

a $\frac{2}{3} + \frac{11}{12} + \frac{3}{4}$ **b** $\frac{3}{5} + \frac{13}{20} + \frac{12}{15}$ **c** $\frac{7}{8} + \frac{5}{3} - \frac{5}{12}$

d $\frac{5}{6} - \frac{1}{2} + \frac{3}{8}$ **e** $\frac{21}{5} + \frac{8}{3} - \frac{13}{10}$ **f** $\frac{8}{5} - \frac{11}{8} + \frac{1}{4}$

7 Consider the four calculations (i, ii, iii, and iv) and four diagrams (I, II, III, and IV).

a Match each diagram with the correct calculation. Determine which diagram and which calculation does not have a match.

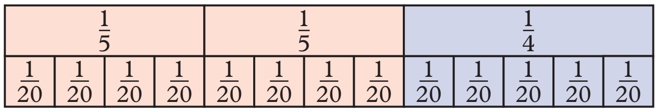
b Write the answer to each calculation. Use the diagrams to assist you.

c Write a calculation and its answer for the diagram without a match.

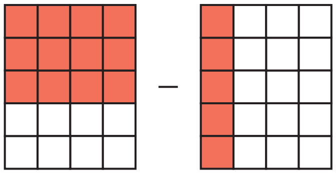
d Draw a diagram (fraction wall or number line) for the calculation without a match.

i $\frac{2}{5} - \frac{1}{4}$ **ii** $\frac{3}{5} + \frac{1}{4}$ **iii** $\frac{2}{5} + \frac{1}{4}$ **iv** $\frac{3}{5} - \frac{1}{4}$

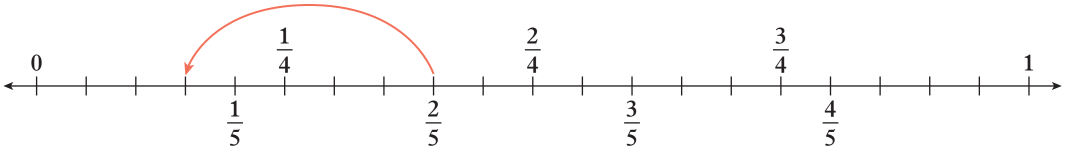
I



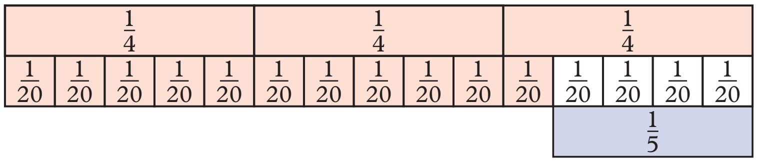
II



III



IV



WE 3.5C **8** Evaluate the following.

a $3\frac{2}{5} + 4\frac{1}{5}$ **b** $6\frac{4}{5} - 3\frac{2}{5}$ **c** $2\frac{8}{11} + 1\frac{5}{11}$ **d** $8\frac{1}{9} - 4\frac{4}{9}$

e $12\frac{2}{3} - 2\frac{1}{3} - 8\frac{2}{3}$ **f** $3\frac{2}{5} + 4\frac{1}{5} + 1\frac{4}{5}$ **g** $6\frac{5}{12} + 2\frac{1}{12} - 7\frac{11}{12}$ **h** $3\frac{1}{8} + 2\frac{5}{8} - 1\frac{7}{8}$

9 Evaluate the following.

a $2\frac{2}{3} + 1\frac{1}{2}$ **b** $4\frac{4}{7} - 1\frac{2}{3}$ **c** $3\frac{2}{5} + 2\frac{1}{4}$ **d** $5\frac{2}{3} - 3\frac{4}{5}$

e $6\frac{5}{6} - 4\frac{3}{10}$ **f** $3\frac{2}{14} + 2\frac{2}{21}$ **g** $1\frac{5}{6} + 5\frac{1}{2}$ **h** $4\frac{1}{6} - 2\frac{7}{24}$

10 Evaluate the following.

a $\frac{8}{3} + 5 + 1\frac{1}{8}$ **b** $1\frac{1}{7} + 2\frac{1}{2} - 3\frac{5}{14}$ **c** $\frac{28}{5} - 4\frac{1}{7} + 1\frac{1}{2}$ **d** $3\frac{5}{12} - \frac{10}{5} + \frac{17}{6}$

Exercise 3.5B: Problem solving and reasoning

11 Dean completed a 10 km run in $\frac{5}{7}$ of an hour, while Mark completed the run in $\frac{3}{4}$ of an hour.

a Which runner was faster?

b What was the difference between the two times?

12 A factory worker is employed to make T-shirts. On average, each T-shirt takes $3\frac{1}{2}$ minutes for cutting, $2\frac{4}{5}$ minutes for sewing and $4\frac{1}{6}$ minutes for finishing. What is the total time required to make one T-shirt?

13 Explain why the following calculations are wrong.

a $\frac{3}{4} + \frac{3}{4} = \frac{6}{8}$ **b** $\frac{3}{8} - \frac{1}{2} - \frac{1}{3} = \frac{1}{3}$

14 A group of friends organises to catch up for lunch and decides to order pizza. Each person estimates what fraction of a pizza he or she can eat. Luisa can eat $\frac{1}{3}$, Declan $\frac{1}{2}$, Nisa $\frac{3}{8}$, Tomas $\frac{2}{3}$ and Christopher $\frac{3}{2}$ of a pizza.

a How many full pizzas will need to be ordered?

b What fraction of a pizza will be left over?

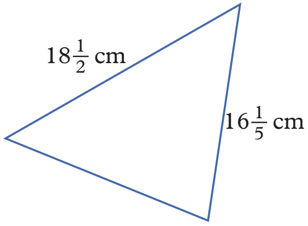
15 Laila is planning to travel overseas at the end of the year and needs to save money, so she devises a budget. She decides to allocate $\frac{2}{5}$ of her weekly pay to living expenses, $\frac{1}{3}$ to rent, $\frac{1}{8}$ to going out and the remainder to savings.

a Calculate what fraction of Laila's pay will go towards living expenses, rent and going out.

b What fraction of her pay will go towards savings? Explain your answer.

16 The total length around the outside of the triangle on the right is $49\frac{2}{5}$ cm.

The length measurements of two of the sides are known. What is the length of the unknown side?



17 While on camp, students were required to complete a 21 km bushwalk over three days. They covered $8\frac{2}{5}$ km on the first day and $7\frac{1}{3}$ km on the second day.

a What distance will they need to walk on the third day?

b How much further did they walk on the first day than on the second?

18 Complete each of these patterns by filling in the gaps. Explain the rule for each pattern.

a $\frac{2}{7}, \frac{5}{7}, \frac{8}{7}$, ______, ______, ______, ______ **b** $10, 11\frac{2}{5}, 12\frac{4}{5}$, ______, ______, ______, ______

c $\frac{19}{3}, \frac{16}{3}, \frac{13}{3}$, ______, ______, ______, ______ **d** $12\frac{14}{15}, 10\frac{10}{15}, 8\frac{6}{15}$, ______, ______, ______

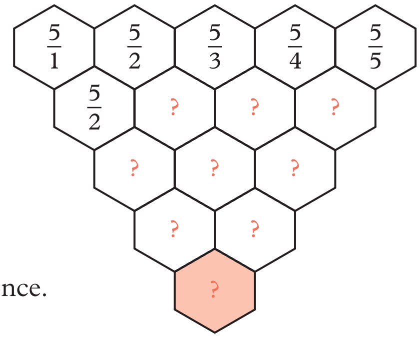
Exercise 3.5C: Challenge

19 Use a single pair of brackets to make each equation true.

a $\frac{1}{2} - \frac{5}{8} - \frac{2}{5} = \frac{11}{40}$ **b** $\frac{19}{28} - \frac{3}{7} + \frac{3}{14} = \frac{1}{28}$

20 Replace each question mark with the difference between the two fractions above it. What will the number at the bottom be?

Repeat this process with $\frac{9}{1}, \frac{9}{2}, \frac{9}{3}, \frac{9}{4}, \frac{9}{5}, \frac{9}{6}, \frac{9}{7}, \frac{9}{8}, \frac{9}{9}$ across the top. What will the number at the bottom of this triangle be?



21 Create a true statement with the form below using each digit from 1 to 9 once.

$$\square\frac{\square}{\square} + \square\frac{\square}{\square} = \square\frac{\square}{\square}$$

Online resources:

Interactive skillsheet
Adding and subtracting fractions

Worksheet
Adding and subtracting fractions

Investigation
Concoction cups

Desmos activity
The fraction challenge

Quick quiz
3.5

Lesson 3.6

Multiplying fractions

Learning intentions

By the end of this lesson you will be able to …

✔ multiply fractions by whole numbers, fractions and mixed numbers.

Inter-year links

Year 8	2.3 Multiplying and dividing fractions

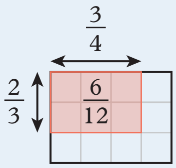
Multiplying fractions

Key content video Multiplying fractions

- When multiplying fractions, the denominators do not need to be the same.
- To multiply two fractions:

  1. Multiply the numerators.
  2. Multiply the denominators.
  3. Simplify the fraction.

$$\frac{2}{3}\times\frac{3}{4}=\frac{2\times 3}{3\times 4}$$
$$=\frac{6}{12}$$
$$=\frac{1}{2}$$



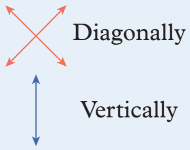
- When multiplying mixed numbers, convert the mixed numbers into improper fractions, multiply across, and give the result as a mixed number.

$$1\frac{3}{5}\times 2\frac{2}{7}=\frac{8}{5}\times\frac{16}{7}$$
$$=\frac{128}{35}$$
$$=3\frac{23}{35}$$

Cancelling

- When multiplying fractions, **cancelling** or dividing out common factors can simplify the process.
  → Cancelling can only be performed diagonally and vertically, not horizontally.

$$\frac{2}{3}\times\frac{3}{4}=\frac{2\times\cancel{3}^{1}}{\cancel{3}^{1}\times 4}$$
$$=\frac{\cancel{2}^{1}}{\cancel{4}^{2}}$$
$$=\frac{1}{2}$$



Multiplying by whole numbers

- Whole numbers can be written as fractions with a denominator of 1. For example, $8=\frac{8}{1}$.
- The number 1 can be written as a fraction where the numerator and denominator are equal.

  For example, $1=\frac{1}{1}=\frac{2}{2}=\frac{3}{3}=\frac{100}{100}\ldots$
- When multiplying any number or fraction by 1, the result remains the same.
- When asked to find a fraction 'of' a whole number or fraction, replace the 'of' with '×'.

  For example, $\frac{2}{3}$ of 20 is the same as $\frac{2}{3}\times 20$.

Worked example 3.6A Multiplying fractions with simplifying

Calculate the result of $\frac{3}{5} \times \frac{2}{6}$.

THINK	WRITE
1 Multiply the numerators, and then multiply the denominators.	$\frac{3}{5} \times \frac{2}{6} = \frac{3 \times 2}{5 \times 6}$
2 Simplify the fraction. 6 and 30 have a common factor of 6.	$= \frac{\cancel{6}^{1}}{\cancel{30}^{5}}$ $= \frac{1}{5}$

Worked example 3.6B Multiplying fractions with cancelling

Calculate the result of $\frac{8}{5} \times \frac{25}{12}$.

THINK	WRITE
1 Look diagonally and vertically for any common factors between the numerators and denominators. Cancel the 8 and 12 by dividing both numbers by 4. Then cancel the 25 and 5 by dividing both numbers by 5.	$\frac{8}{5} \times \frac{25}{12} = \frac{\cancel{8}^{2}}{\cancel{5}^{1}} \times \frac{\cancel{25}^{5}}{\cancel{12}^{3}}$
2 Multiply the numerators, and then multiply the denominators.	$= \frac{2 \times 5}{1 \times 3}$ $= \frac{10}{3}$

Worked example 3.6C Multiplying mixed numbers

Calculate the result of $1\frac{2}{3} \times 2\frac{7}{10}$.

THINK	WRITE
1 Convert each mixed number into an improper fraction.	$1\frac{2}{3} \times 2\frac{7}{10} = \frac{5}{3} \times \frac{27}{10}$
2 Look diagonally and vertically for any common factors between the numerators and denominators. Cancel the 5 and 10 by dividing both numbers by 5. Then cancel the 3 and 27 by dividing both numbers by 3.	$= \frac{\cancel{5}^{1}}{\cancel{3}^{1}} \times \frac{\cancel{27}^{9}}{\cancel{10}^{2}}$
3 Multiply the numerators, and then multiply the denominators.	$= \frac{1 \times 9}{1 \times 2}$ $= \frac{9}{2}$
4 Convert the improper fraction to a mixed number.	$= 4\frac{1}{2}$

Helpful hints

- ✔ When multiplying and dividing fractions, you do not need a common denominator.
- ✔ Always write whole numbers as fractions before multiplying.
- ✔ Always convert mixed numbers to improper fractions before multiplying.
- ✔ Cancelling out common factors of the numerators and denominators before multiplying means you are simplifying before multiplying (which is often easier since you are working with smaller numbers).

Learning pathways

△ 1–6, 7(a, b, g, h), 8(a, c, f, h), 10–12, 14, 15, 16(a), 17

□ 2(d, e, g, h), 3–4, 5(e–h), 6, 7(c, d, e, i, j, k), 8–10, 13, 16–18

⬠ 2–4(g–i), 5(g, h), 6(d, e, f), 7(d, e, f, j, k, l), 9–10, 13, 16–20

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Exercise 3.6A: Understanding and fluency

1 Use repeated addition to calculate the following products.

a $3 \times \frac{2}{5}$ **b** $4 \times \frac{2}{9}$ **c** $6 \times \frac{1}{6}$ **d** $2 \times \frac{11}{3}$

WE 3.6A **2** Calculate each of these products. Leave your answer as an improper fraction if applicable.

a $\frac{1}{5} \times \frac{3}{2}$ **b** $\frac{2}{3} \times \frac{1}{7}$ **c** $\frac{11}{3} \times \frac{2}{5}$

d $\frac{1}{3} \times \frac{4}{9}$ **e** $\frac{3}{8} \times \frac{7}{2}$ **f** $\frac{11}{13} \times \frac{6}{7}$

g $\frac{1}{3} \times \frac{7}{15}$ **h** $\frac{3}{10} \times \frac{9}{7}$ **i** $\frac{9}{2} \times \frac{9}{5}$

WE 3.6B **3** Calculate each of these products.

a $\frac{8}{9} \times \frac{1}{4}$ **b** $\frac{2}{3} \times \frac{6}{11}$ **c** $\frac{3}{5} \times \frac{10}{13}$

d $\frac{1}{8} \times \frac{4}{17}$ **e** $\frac{7}{9} \times \frac{12}{35}$ **f** $\frac{20}{27} \times \frac{9}{15}$

g $\frac{3}{4} \times \frac{18}{24}$ **h** $\frac{9}{35} \times \frac{20}{21}$ **i** $\frac{14}{28} \times \frac{8}{13}$

WE 3.6C **4** Calculate each of these products. Give your answer as a mixed number if applicable.

a $\frac{3}{5} \times 1\frac{4}{11}$ **b** $5\frac{2}{4} \times \frac{6}{7}$ **c** $5\frac{3}{5} \times 3\frac{1}{4}$

d $4\frac{2}{3} \times \frac{6}{2}$ **e** $\frac{7}{9} \times 3\frac{5}{7}$ **f** $5\frac{2}{3} \times 2\frac{2}{5}$

g $1\frac{3}{20} \times \frac{8}{5}$ **h** $3\frac{1}{3} \times 4\frac{1}{8}$ **i** $2\frac{4}{9} \times 3\frac{3}{16}$

5 Calculate each of these products.

a $\frac{3}{8} \times 24$ **b** $\frac{5}{11} \times 66$ **c** $\frac{12}{15} \times 4$ **d** $5 \times \frac{16}{25}$

e $9 \times \frac{1}{24}$ **f** $6 \times \frac{5}{36}$ **g** $\frac{8}{3} \times 15$ **h** $9 \times \frac{4}{81}$

6 Calculate:

a $\frac{2}{5}$ of $\frac{1}{8}$ **b** $\frac{3}{4}$ of $\frac{3}{7}$ **c** $\frac{2}{11}$ of $\frac{7}{9}$

d $\frac{9}{4}$ of $\frac{5}{11}$ **e** $2\frac{7}{13}$ of 26 **f** $5\frac{1}{2}$ of $3\frac{1}{3}$

7 To increase or decrease a number by a fraction, we can multiply the number by one plus or minus the fraction.
For example:

- to increase 12 by $\frac{1}{4}$: $12 \times \left(1 + \frac{1}{4}\right) = 12 \times \frac{5}{4} = 15$
- to decrease 12 by $\frac{1}{4}$: $12 \times \left(1 - \frac{1}{4}\right) = 12 \times \frac{3}{4} = 9$

Now try the following:

a increase 4 by $\frac{1}{2}$ **b** increase 9 by $\frac{2}{3}$ **c** increase 24 by $\frac{5}{6}$ **d** increase 18 by $\frac{5}{3}$
e increase $\frac{1}{2}$ by $\frac{1}{2}$ **f** increase $\frac{1}{3}$ by $\frac{4}{3}$ **g** decrease 4 by $\frac{1}{2}$ **h** decrease 6 by $\frac{1}{3}$
i decrease 12 by $\frac{5}{6}$ **j** decrease 100 by $\frac{13}{20}$ **k** decrease $\frac{1}{2}$ by $\frac{1}{3}$ **l** decrease $\frac{4}{3}$ by $\frac{1}{3}$

8 When multiplying three fractions together, start by multiplying the first two fractions together, then multiply the result by the third fraction. Calculate:

a $\frac{5}{9} \times \frac{9}{10} \times \frac{12}{17}$ **b** $\frac{3}{8} \times \frac{15}{26} \times \frac{8}{9}$ **c** $\frac{28}{5} \times \frac{11}{6} \times \frac{24}{7}$ **d** $1\frac{1}{12} \times \frac{8}{3} \times \frac{15}{4}$
e $\frac{22}{13} \times 10\frac{2}{5} \times 2\frac{1}{11}$ **f** $4\frac{1}{2} \times 5\frac{2}{5} \times \frac{21}{8}$ **g** $\frac{12}{5} \times 7\frac{1}{2} \times 3\frac{1}{4}$ **h** $6\frac{2}{3} \times 5\frac{1}{4} \times 1\frac{2}{7}$

9 Calculate the following. Hint: Remember the rules for the order of operations.

a $\frac{5}{8} + \frac{1}{2} \times \frac{3}{4}$ **b** $\frac{2}{3} \times \frac{4}{5} + \frac{8}{15}$ **c** $\frac{13}{6} - \frac{5}{3} \times \frac{5}{4}$ **d** $\frac{3}{8} + \frac{9}{4} \times \frac{11}{6} - \frac{8}{3}$

Exercise 3.6B: Problem solving and reasoning

10 Toby has a garage sale where he hopes to sell 450 used books he has collected. He manages to sell $\frac{7}{9}$ of the books. How many books is he left with?

11 Hilaire works at the local swimming pool as a lifeguard. Her shift begins at 3:45 pm and ends at 6:15 pm.

a Write the hours Hilaire works in one shift as:

i a mixed number **ii** an improper fraction.

b If Hilaire has the same shift five nights a week, calculate the total number of hours she works. Write your answer as a mixed number.

12 A store takes $\frac{2}{5}$ off the price of its televisions during the stocktake sales. If a television normally costs $6320, what will it cost during the stocktake sale?

13 Amie manages to complete $\frac{3}{8}$ of her $5\frac{1}{2}$ km run before stopping. How far did she run prior to stopping?

14 Multiplying a number by a unit fraction is the same as dividing the number by the denominator of the unit fraction.

For example, $\frac{1}{2} \times 10 = \frac{10}{2}$
$= 10 \div 2.$

a Write the following multiplications as divisions.

i $\frac{1}{3} \times 33$ **ii** $\frac{1}{8} \times 40$ **iii** $\frac{1}{101} \times 55$ **iv** $\frac{1}{7} \times \frac{4}{9}$

b Write the following divisions as multiplications.

i $56 \div 7$ **ii** $90 \div 10$ **iii** $92 \div 35$ **iv** $\frac{8}{11} \div 5$

For any fraction, we can use the associative property to do the multiplication in a different order.

For example,
$$\begin{aligned}\frac{3}{2} \times 10 &= 3 \times \left(\frac{1}{2} \times 10\right)\\ &= 3 \times \frac{10}{2}\\ &= 3 \times (10 \div 2)\\ &= 3 \times 5\\ &= 15.\end{aligned}$$

c Calculate the following using this method.

i $\frac{3}{8} \times 16$ **ii** $\frac{8}{5} \times 35$ **iii** $\frac{12}{7} \times 63$ **iv** $\frac{7}{6} \times \frac{18}{5}$

15 Identify whether the following statements are true or false.

- **a** When multiplying a whole number by a proper fraction, the answer is always smaller than the whole number.
- **b** When multiplying a whole number by an improper fraction or mixed number, the answer is always smaller than the whole number.

16 Investigate if multiplying a fraction by a fraction will make the answer smaller or greater than both of the fractions.

- **a** Calculate each of the following products. Determine if the product is less than both fractions, greater than both fractions or between both fractions.
  - **i** $\frac{2}{3} \times \frac{4}{5}$
  - **ii** $3 \times \frac{1}{7}$
  - **iii** $\frac{9}{4} \times \frac{11}{3}$
  - **iv** $\frac{5}{6} \times 2\frac{5}{6}$
  - **v** $\frac{8}{3} \times \frac{3}{4}$
  - **vi** $\frac{1}{9} \times \frac{1}{7}$
  - **vii** $3\frac{1}{3} \times \frac{10}{3}$
  - **viii** $\frac{5}{3} \times 4$
- **b** When will the product of two fractions be a value less than both fractions?
- **c** When will the product of two fractions be a value greater than both fractions?
- **d** When will the product of two fractions be a value between both fractions?
- **e** What value does one of the fractions need to be so that the product is the same value as one of the fractions?

Exercise 3.6C: Challenge

17 While correcting his students' homework, a teacher found three different answers to the question: 'Find $2\frac{4}{9} \times 3\frac{3}{8}$ in its simplest form'. The answers given were: $6\frac{12}{72}$, $8\frac{1}{4}$ and $6\frac{12}{17}$.

- **a** Which answer is correct?
- **b** Comment on the error that is likely to have been made in each incorrect case.

18 Saina's dad showed her the following multiplication:

$$\frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} \times \frac{4}{5} \times \frac{5}{6} \times \frac{6}{7} \times \frac{7}{8}$$

Dad said, 'Wow, that took me ages to work out!'

Saina said, 'No Dad, this is really easy!'

How did Saina turn this into an easy calculation?

19 Consider the following multiplication pattern:

$$\frac{1}{1} \times \frac{1}{2} \times \frac{1}{3} \times \frac{1}{4} \times \frac{1}{5} \times \ldots$$

- **a** Determine the value of the number by multiplying up to the number $\frac{1}{5}$.
- **b** Simplify the products in the table below.

Column	1	2	3	4	5
Value	$\frac{1}{1}$	$\frac{1}{1} \times \frac{1}{2}$	$\frac{1}{1} \times \frac{1}{2} \times \frac{1}{3}$	$\frac{1}{1} \times \frac{1}{2} \times \frac{1}{3} \times \frac{1}{4}$	$\frac{1}{1} \times \frac{1}{2} \times \frac{1}{3} \times \frac{1}{4} \times \frac{1}{5}$

- **c** As more fractions are multiplied in the pattern, does the result become greater or smaller?
- **d** What is the value that the pattern approaches but never actually gives as a result?

20 Create a true statement with the form below using seven of the digits from 1 to 9, without using a digit more than once.

$$\frac{\square}{\square} \times \frac{\square}{\square} = \square\frac{\square}{\square}$$

Online resources:

Worksheet
Multiplying fractions

Investigation
All things equal

Quick quiz
3.6

Lesson 3.7

Dividing fractions

Learning intentions

By the end of this lesson you will be able to …

✔ divide fractions by whole numbers, fractions and mixed numbers.

Inter-year links

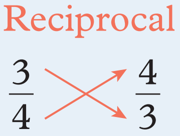
Year 8 2.3 Multiplying and dividing fractions

The reciprocal of a fraction

Key content video
Dividing fractions

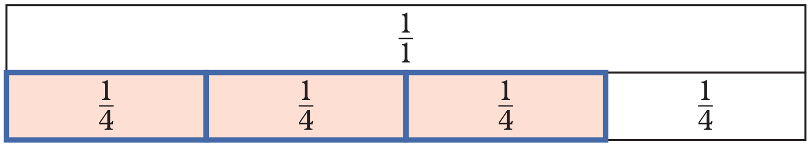
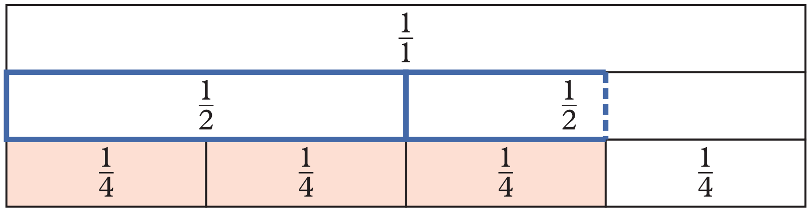
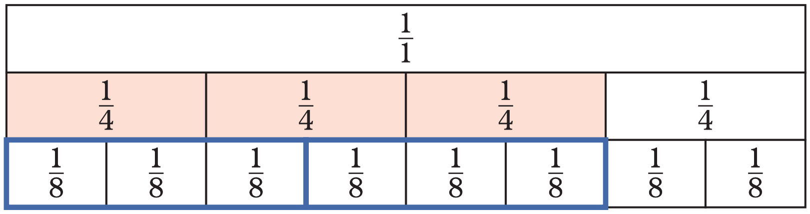
- When a fraction is multiplied by its **reciprocal**, the result is 1.

  For example, the reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$ as $\frac{2}{3} \times \frac{3}{2} = \frac{6}{6} = 1$.
- To find the reciprocal of a proper or improper fraction, invert the fraction by swapping the numerator and the denominator. For a mixed number, write it as an improper fraction first.



Dividing fractions by fractions

- Dividing fractions can be thought of as asking: ‘How many of one fraction fits into another fraction?’

$\frac{3}{4} \div \frac{1}{4} = 3$	How many $\frac{1}{4}$ are there in $\frac{3}{4}$?	
$\frac{3}{4} \div \frac{1}{2} = 1\frac{1}{2}$	How many $\frac{1}{2}$ are there in $\frac{3}{4}$?	
$\frac{3}{4} \div \frac{3}{8} = 2$	How many $\frac{3}{8}$ are there in $\frac{3}{4}$?	

- Dividing by a fraction is the same as multiplying by the reciprocal of the fraction. Keep the first fraction the same, and then multiply by the reciprocal of the second fraction.

  For example, $\frac{3}{4} \div \frac{3}{8} = \frac{3}{4} \times \frac{8}{3}$.

Dividing fractions by whole numbers

- Dividing by a whole number is the same as multiplying by the reciprocal of the whole number.
- A whole number can be written as a fraction with denominator 1, so the reciprocals of whole numbers are the unit fractions.

$$5 = \frac{5}{1} \xrightarrow{\text{Reciprocal}} \frac{1}{5}$$

Dividing fractions by mixed numbers

- When dividing fractions by mixed numbers, start by converting the mixed numbers into improper fractions, then divide the fractions by multiplying by the reciprocal. Provide the result as a mixed number or proper fraction.

For example, $1\frac{2}{3} \div 2\frac{1}{4} = \frac{5}{3} \div \frac{9}{4}$

$= \frac{5}{3} \times \frac{4}{9}$

$= \frac{20}{27}$.

Worked example 3.7A Finding the reciprocal of a fraction

Write the reciprocal of each value.

a $\frac{3}{7}$ **b** $\frac{1}{8}$ **c** 2 **d** $2\frac{3}{4}$

THINK	**WRITE**
a Invert the fraction. Swap the numerator and the denominator.	**a** $\frac{7}{3}$
b Invert the fraction. Simplify the result.	**b** $\frac{8}{1} = 8$
c Write the whole number as a fraction with a denominator of 1. Invert the fraction.	**c** $2 = \frac{2}{1}$. The reciprocal is $\frac{1}{2}$.
d Write the mixed number as an improper fraction. Invert the improper fraction.	**d** $2\frac{3}{4} = \frac{11}{4}$. The reciprocal is $\frac{4}{11}$.

Worked example 3.7B Dividing fractions by fractions

Calculate $\frac{1}{7} \div \frac{3}{5}$.

THINK	**WRITE**
1 Keep the first fraction the same and multiply by the reciprocal of the second fraction.	$\frac{1}{7} \div \frac{3}{5} = \frac{1}{7} \times \frac{5}{3}$
2 Multiply the numerators together and then multiply the denominators together.	$= \frac{1 \times 5}{7 \times 3}$ $= \frac{5}{21}$

Worked example 3.7C Quotients between fractions and whole numbers

Calculate the following quotients:

a $\frac{4}{5} \div 6$ **b** $6 \div \frac{3}{8}$.

THINK	WRITE
a **1** Keep the first fraction the same and multiply by the reciprocal of the second number. $6 = \frac{6}{1}$ so the reciprocal is $\frac{1}{6}$.	**a** $\frac{4}{5} \div 6 = \frac{4}{5} \times \frac{1}{6}$
2 Cancel the 4 and 6 by dividing both numbers by the common factor 2.	$= \frac{\cancel{4}^{2}}{5} \times \frac{1}{\cancel{6}^{3}}$
3 Multiply the numerators together and then multiply the denominators together.	$= \frac{2 \times 1}{5 \times 3}$ $= \frac{2}{15}$
b **1** Keep the first number the same and multiply by the reciprocal of the second fraction.	**b** $6 \div \frac{3}{8} = 6 \times \frac{8}{3}$
2 3 is a factor of 6, so cancel the 3 in the denominator.	$= \cancel{6}^{2} \times \frac{8}{\cancel{3}}$
3 Multiple the numerator by 2.	$= 2 \times 8$ $= 16$

Worked example 3.7D Dividing mixed numbers

Calculate $4\frac{3}{8} \div 2\frac{6}{7}$.

THINK	WRITE
1 Convert each mixed number to an improper fraction.	$4\frac{3}{8} \div 2\frac{6}{7} = \frac{35}{8} \div \frac{20}{7}$
2 Keep the first fraction the same and multiply by the reciprocal of the second fraction.	$= \frac{35}{8} \times \frac{7}{20}$
3 Cancel the 35 and 20 by dividing each by the common factor 5.	$= \frac{\cancel{35}^{7}}{8} \times \frac{7}{\cancel{20}^{4}}$
4 Multiply the numerators together and then multiply the denominators together.	$= \frac{7 \times 7}{8 \times 4}$ $= \frac{49}{32}$
5 Change the improper fraction to a mixed number.	$= 1\frac{17}{32}$

Helpful hints

- ✔ The phrase ‘Keep Change Flip’ can help you remember how to multiply fractions: KEEP the first fraction, CHANGE the division to multiplication, FLIP the second fraction.
- ✔ You can change any whole number into a fraction with a denominator of 1.
- ✔ Always simplify your fractions!

Learning pathways

△ 1–11 ■ 4–7, 9–14 ⬟ 5–7, 9, 11–16

ANS p515

Exercise 3.7A: Understanding and fluency

1 Rewrite each division problem in the form ‘How many ____ are there in ____?’

a $2 \div \frac{1}{8}$ **b** $4 \div \frac{1}{3}$ **c** $\frac{1}{2} \div \frac{1}{12}$ **d** $\frac{1}{3} \div \frac{1}{9}$

2 Write the following statements as a division. Then, use the fraction wall to find the quotient.

a How many $\frac{1}{9}$ are there in $\frac{2}{3}$?

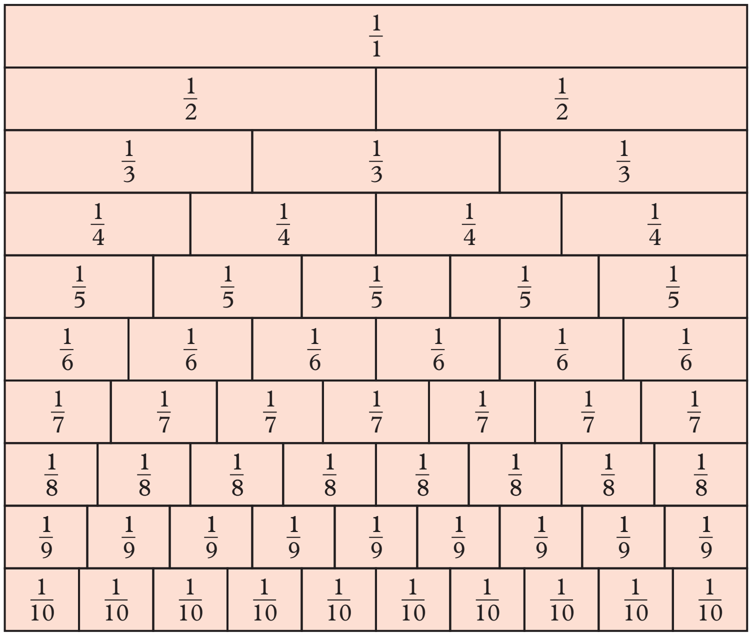
b How many $\frac{1}{5}$ are there in $\frac{3}{10}$?

c How many $\frac{2}{5}$ are there in $\frac{9}{10}$?

d How many $\frac{1}{3}$ are there in $\frac{5}{6}$?

e How many $\frac{3}{8}$ are there in 1?

f How many $\frac{2}{9}$ are there in $\frac{4}{6}$?



WE 3.7A 3 Write the reciprocal of each value.

a $\frac{65}{31}$ **b** $\frac{11}{3}$

c $\frac{2}{5}$ **d** $\frac{27}{4}$

e $\frac{1}{100}$ **f** 8

g $\frac{34}{1}$ **h** $\frac{65}{3}$

i $4\frac{2}{7}$ **j** $9\frac{1}{8}$

4 Using the fact that dividing by a number is equivalent to multiplying by its reciprocal, write each of these divisions as multiplications. Do not perform the multiplication.

a $\frac{4}{7} \div \frac{5}{9}$ **b** $\frac{8}{3} \div \frac{1}{9}$ **c** $\frac{3}{5} \div 4$ **d** $6 \div \frac{3}{10}$ **e** $9\frac{3}{8} \div \frac{3}{2}$

f $\frac{8}{3} \div 6\frac{7}{10}$ **g** $5\frac{1}{5} \div 7\frac{4}{9}$ **h** $3 \div 2\frac{3}{8}$ **i** $4\frac{5}{8} \div 10$ **j** $21 \div 32$

WE 3.7B 5 Calculate each of the quotients. Give your answer as a mixed number if applicable.

a $\frac{3}{8} \div \frac{2}{5}$ **b** $\frac{1}{12} \div \frac{1}{11}$ **c** $\frac{3}{4} \div \frac{3}{10}$ **d** $\frac{1}{9} \div \frac{4}{17}$ **e** $\frac{1}{10} \div \frac{4}{25}$

f $\frac{6}{7} \div \frac{8}{21}$ **g** $\frac{16}{15} \div \frac{12}{5}$ **h** $\frac{14}{3} \div \frac{7}{11}$ **i** $\frac{4}{13} \div \frac{20}{3}$ **j** $\frac{15}{27} \div \frac{11}{21}$

WE 3.7C 6 Calculate each of the quotients.

a $\frac{2}{9} \div 6$ **b** $\frac{3}{4} \div 9$ **c** $\frac{5}{2} \div 10$ **d** $\frac{8}{9} \div 2$ **e** $4 \div \frac{7}{5}$

f $6 \div \frac{1}{7}$ **g** $14 \div \frac{7}{6}$ **h** $12 \div \frac{8}{9}$ **i** $5 \div \frac{10}{13}$ **j** $18 \div \frac{21}{5}$

WE 3.7D 7 Calculate each of the quotients.

a $3\frac{1}{2} \div \frac{4}{5}$ **b** $1\frac{4}{7} \div \frac{5}{2}$ **c** $6\frac{1}{3} \div 1\frac{1}{4}$ **d** $7\frac{1}{5} \div \frac{22}{3}$

e $1\frac{2}{7} \div \frac{15}{21}$ **f** $2\frac{7}{9} \div \frac{4}{18}$ **g** $\frac{13}{4} \div 2\frac{6}{10}$ **h** $3\frac{3}{5} \div 1\frac{4}{5}$

i $4 \div 1\frac{1}{5}$ **j** $2\frac{1}{3} \div 5$ **k** $9\frac{1}{11} \div 10$ **l** $7\frac{3}{5} \div 19$

Exercise 3.7B: Problem solving and reasoning

8 Fill in the correct number to complete the following calculations.

a _____ $\times \frac{1}{3} = 6$

b _____ $\times \frac{2}{5} = 2$

c $\frac{5}{12} \times$ _____ $= \frac{1}{3}$

d $12 \times$ _____ $= 2\frac{2}{3}$

e _____ $\times \frac{1}{3} \times \frac{1}{2} = 6$

f $\frac{3}{2} \times$ _____ $\times \frac{1}{2} = \frac{15}{4}$

9 A batch of a chocolate pudding recipe requires $\frac{1}{6}$ kg of sugar. How many batches of chocolate pudding can be made from a 1 kg packet of sugar?

10 Over the school holidays, Brianna worked a total of 44 hours stacking shelves at the supermarket. If she only worked $5\frac{1}{2}$ hour shifts, how many shifts did she work?

11 Rhys shares $3\frac{3}{8}$ pizzas equally between eight friends and himself. What fraction of a pizza will each person receive?

12 A new housing estate is to be developed on $43\frac{1}{5}$ hectares of land. If each block measures $\frac{3}{5}$ hectares, how many blocks will there be?

13 Gianluca must pack 15 kg of apples into plastic bags that can hold a maximum of $1\frac{1}{3}$ kg.

a How many $1\frac{1}{3}$ kg bags can he fill?

b How many kilograms of apples will be left over?

14 Priya and Rosa run a cafe that serves high tea on Sunday afternoons. One whole sandwich can be cut into three finger sandwiches. The cafe's menu states that each customer receives four finger sandwiches.

a Write the fraction of a whole sandwich each customer receives as an improper fraction.

This Sunday they make 36 whole sandwiches.

b How many customers can they serve?

c They quickly realise they have more customers than expected, so they decide to cut each sandwich into five finger sandwiches instead of three. How many customers per day could they now serve?

d What fraction of a whole sandwich does each customer receive in this case?

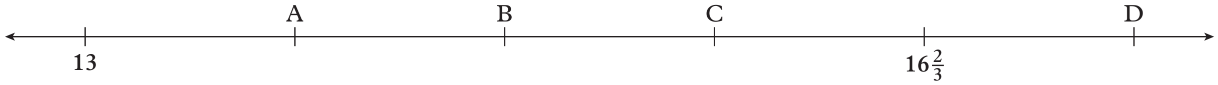
15 Sava has started an online business selling backpacks that he makes from recycled T-shirts. He needs $1\frac{2}{3}$ T-shirts to make 1 backpack.

a How many T-shirts does he need to make 30 backpacks?

b At one of his favourite op shops Sava finds 15 T-shirts that are perfect for his backpacks. How many backpacks will he be able to make from these T-shirts?

Exercise 3.7C: Challenge

16 Consider the number line below. If all intervals are the same size, state whether the following are true or false.



a $A = 14$

b $B > 15$

c $C = 15\frac{3}{4}$

d $D < 17\frac{1}{2}$

Online resources:

Interactive skillsheet Multiplying and dividing fractions

Worksheet Dividing fractions

Investigation Paper cuts

Quick quiz 3.7

Lesson 3.8

Ratios

Learning intentions

By the end of this lesson you will be able to ...

✔ compare and simplify ratios, and divide quantities into a ratio.

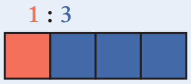
Inter-year links

Year 8	3.5 Ratios
Year 9	6.1 Ratios and rates

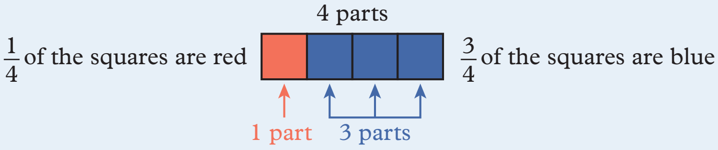
Ratios

Key content video
Understanding ratios

- A **ratio** is a comparison of two or more quantities of the same kind.
  - → Ratios are used to show the relationship between equal parts of the quantities. Each quantity is separated by a colon (:) read as ‘compared with’.
    For example, the ratio 1 : 3 reads ‘1 part compared with 3 parts’ or ‘1 to 3.’



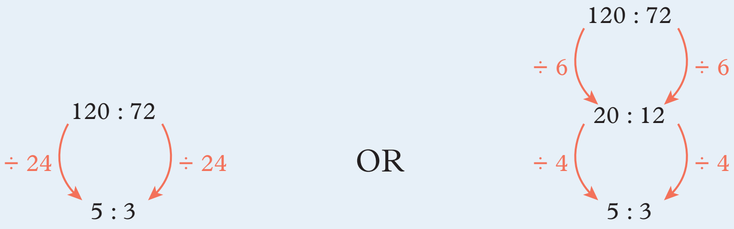
- A ratio must be written in the order of the given worded description.
  For example, one part cordial to five parts water is written as 1 : 5.
- Ratios can be viewed as fractions where the numerator is the number in the ratio and the denominator is the total number of parts.



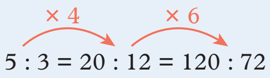
- Before writing a ratio, the numbers must be in the same unit of measurement.

Simplifying ratios

- A ratio is in its simplest form when the only common factor in the ratio is 1. Ratios can be simplified by:
  - → dividing the numbers in the ratio by the highest common factor,
  - → or by repeated division of common factors.



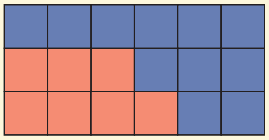
- Numbers in a ratio can be multiplied or divided by a value to create an equivalent ratio.



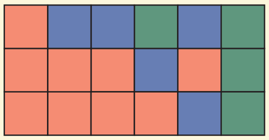
Worked example 3.8A Determining ratios from a diagram

Write a ratio for each of the following.

a red to blue tiles



b red to blue to green tiles



THINK	WRITE
1 Count the number of each coloured tile.	**a** There are 7 red tiles and 11 blue tiles. red : blue 7 : 11
2 Write the numbers as they appear in the question: red then blue then green.	**b** There are 9 red tiles, 5 blue tiles and 4 green tiles. red : blue : green 9 : 5 : 4
3 Write the ratio with a colon between each comparison.	

Worked example 3.8B Simplifying ratios

Write the ratio 35 : 56 in its simplest form.

THINK	WRITE
1 Find the highest common factor (HCF) of each number in the ratio.	The HCF of 35 and 56 is 7.
2 Simplify by dividing each number in the ratio by the HCF.	35 : 56 ÷ 7 ÷ 7 = 5 : 8
3 Write the ratio.	35 : 56 = 5 : 8

Worked example 3.8C Dividing a quantity into a ratio

Divide 20 into the ratio 2 : 3.

THINK	WRITE
1 Find the total number of parts in the ratio.	Number of parts = 2 + 3 = 5
2 Divide the quantity by the total number of parts.	20 ÷ 5 = 4
3 Multiply 4 by each part of the ratio.	4 × 2 = 8 4 × 3 = 12 The ratio 2 : 3 divides 20 into 8 and 12.

Helpful hints

- ✔ Keep the order of the numbers in the ratio the same as the order of the worded comparison. It tells you the order of the quantities you are comparing.
- ✔ Ratios are not written in size order, only in comparison order.

Learning pathways

△ 1–8, 9(a–d), 10–11, 14, 16, 18, 19 ■ 3–7, 8(d–f), 9, 11, 13, 15, 17–19, 21–23 ⬟ 6, 7, 8(d–f), 9, 11, 13, 15, 20–26

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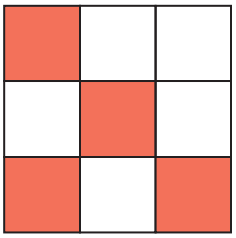
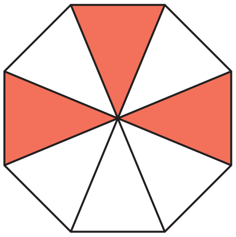
Exercise 3.8A: Understanding and fluency

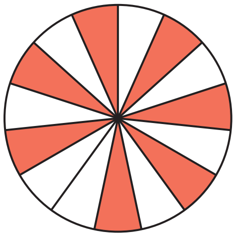
WE 3.8A **1** Write each comparison as a ratio of:

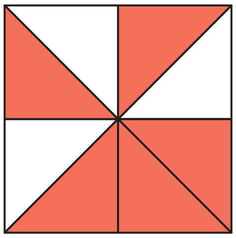
i the number of shaded parts to the number of non-shaded parts

ii the number of shaded parts to the total number of parts.

a


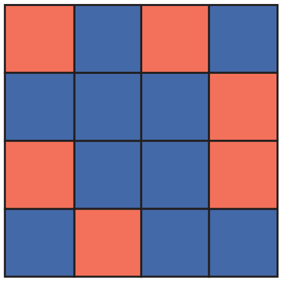
b


c


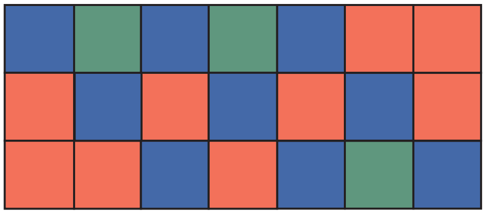
d


2 Write each ratio in the given order.

a What is the ratio of red to blue tiles?



b What is the ratio of blue to red to green tiles?



3 Write each comparison as a ratio:

a the number of red jelly beans compared with the number of yellow jelly beans

b the number of orange jelly beans compared with the total number of jelly beans

c the number of pink jelly beans compared with the number of yellow jelly beans compared with the number of blue jelly beans

d the number of red and blue jelly beans compared with the number of pink and purple jelly beans.

4 Write each comparison as a ratio in the given order.

a 132 cm to 207 cm **b** 16 kg to 35 kg

c 18 min to 11 min **d** $85 to $121

5 Write each comparison as a ratio in the given order.

- **a** Kayla spent 5 hours on Facebook and 1 hour doing homework.
- **b** Australia won three gold, four silver and nine bronze medals in the World Swimming Championships.
- **c** Twenty-eight students tried out for the volleyball team, while 23 students tried out for the netball team.
- **d** To make a drink of cordial, use one part cordial to four parts water.

WE 3.8B 6 Write each ratio in its simplest form.

a 21 : 6 **b** 12 : 18 **c** 52 : 64 **d** 36 : 96

e 88 : 48 **f** 40 : 45 **g** 65 : 13 : 39 **h** 22 : 132 : 44

7 A fruit bowl contains apples, mandarins and bananas and is shown in the image to the right. There are three bananas, three apples and one mandarin.

Write each comparison as a ratio in its simplest form:

- **a** the number of apples to the number of mandarins
- **b** the number of apples to the number of bananas
- **c** the number of mandarins to the number of bananas to the number of apples.

8 Complete the patterns of equivalent ratios.

a
3 : 2
6 : 4
15 : __

b
4 : 5
__ : 25
16 : __

c
__ : 210
10 : 70
2 : __

d
144 : __
24 : 9
8 : __

e
__ : 121
150 : __
15 : 11

f
147 : __
__ : 63
7 : 9

WE 3.8C 9 Divide the following amounts into the given ratio.

a 33 in 5 : 6 **b** 35 in 3 : 2 **c** 70 in 2 : 5 **d** 132 in 2 : 1

e 119 in 9 : 8 **f** 195 in 2 : 13 **g** 2304 in 8 : 1 : 9 **h** 1554 in 3 : 7 : 4

Exercise 3.8B: Problem solving and reasoning

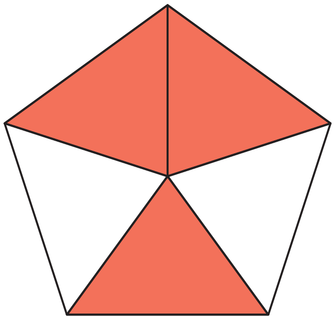
10 Max scored $\frac{87}{100}$ (or 87 out of 100) for his Geography test.

Write a ratio for:

- **a** the marks received compared to the total number of marks
- **b** the marks lost compared to the total number of marks
- **c** the marks received compared to the marks lost.

11 Look at the pentagon shown on the right.

- **a** What is the ratio of shaded sections to non-shaded sections?
- **b** What is the ratio of non-shaded sections to the total number of sections?
- **c** What is the ratio of shaded sections to the total number of sections?
- **d** Can any of the ratios in parts **a–c** be written as fractions? Explain your reasoning.
- **e** Can any of the ratios in parts **a–c** *not* be written as fractions? Explain your reasoning.
- **f** True or false: Ratios can be written as fractions when comparing the ratio of a part to its whole.



12 In a bag of 28 red and blue marbles, 16 marbles are red.

a How many marbles are blue? **b** Write the ratio of red to blue marbles in its simplest form.

13 Jesse and James bought a $13 lottery ticket together. James was a bit short on cash, so Jesse paid $8 and James paid the rest. Against all odds, they won $26 million dollars. They decide to share the money in the same ratio that they paid for the ticket. How much money do Jesse and James each get?

14 A breakfast cereal mix is made using 8 kg of oats, 4 kg of dried fruit and 12 kg of wheat flakes.

a Write the ratio of oats to dried fruit to wheat flakes in its simplest form.

b Write each ingredient as a fraction of the whole mix.

15 Geoff mixes 4 L of blue paint with 5 L of yellow paint to create a shade of green for his studio.

a Write the ratio of the amount of blue paint to the amount of green paint.

b How much green paint does Geoff have?

c What fraction of the new shade is the blue paint?

d A year later, Geoff notices marks on the wall that won't wipe off. He decides to repaint that area. If he bought a 2 L tin of blue paint and a 6 L tin of yellow paint, how much of the same shade of green paint can Geoff make?

16 Last week, Shannen completed extra chores around home and earned $32. This week, she earned $14 less. Write as a ratio, in its simplest form, the amount of last week's pocket money to this week's pocket money.

17 The table shows the attendance at each performance of the local school production.

Performance	Number of people attending
Thursday evening	486
Friday evening	549
Saturday matinee	390
Saturday evening	520

a Find the total attendance at the school production over the three days.

b Write as a ratio in its simplest form:

i the number attending Thursday's performance to the number attending Friday's performance

ii the number attending Saturday's matinee performance to the number attending Saturday's evening performance.

c If the venue can seat a maximum of 550 people, write a ratio for the total attendance to the total possible attendance over the three days.

18 When one number is missing from a pair of equivalent ratios, there are two different strategies you can use to find the missing number.

a Equivalent ratios can be found by multiplying or dividing the same number on both sides of a ratio. Fill in the missing numbers in the following conversions.

$$\begin{tikzcd} 12 \arrow[d, bend right, "\times\frac{3}{12}"'] & : & 36 \arrow[d, bend left, "\times\_\_"] \\ 3 & : & \_\_ \end{tikzcd} \qquad \begin{tikzcd} 12 & : & \_\_ \\ 3 \arrow[u, bend left, "\times\frac{12}{3}"] & : & 9 \arrow[u, bend right, "\times\_\_"'] \end{tikzcd}$$

b When one number is missing from a pair of equivalent ratios, the missing number could also be found by considering the relationship between the two numbers on both sides of the colon in the given ratio. Fill in the missing numbers in the following conversions.

$$\begin{tikzcd} 12 \arrow[rr, bend left, "\times\frac{36}{12}"] & : & 36 \\ 3 \arrow[rr, bend right, "\times\_\_"'] & : & \_\_ \end{tikzcd} \qquad \begin{tikzcd} 12 & : & \_\_ \arrow[ll, bend right, "\times\_\_"'] \\ 3 & : & 9 \arrow[ll, bend left, "\times\frac{12}{36}"] \end{tikzcd}$$

19 Is each pair of ratios equivalent? Give a reason for your answer.

a 7 : 8 and 21 : 24　　**b** 20 : 32 and 5 : 8　　**c** 63 : 18 and 9 : 2　　**d** 30 : 45 and 90 : 60

20 A two-ingredient dough recipe uses $1\frac{1}{4}$ cups of self-raising flour to 1 cup of Greek yoghurt and makes four servings. How many cups of self-raising flour and Greek yoghurt are needed to make 10 servings? Write your answers using mixed numbers.

21 The ratio of apple juice to soda water for a fruit punch recipe is 5 : 3. If the recipe requires 750 mL of apple juice, how much soda water is required?

22 Divide $300 using the ratio 4 : 6.

a Calculate the total number of parts in the ratio.

b Write each number in the ratio as a fraction of the total number of parts.

c Multiply each fraction from part **b** by the quantity ($300).

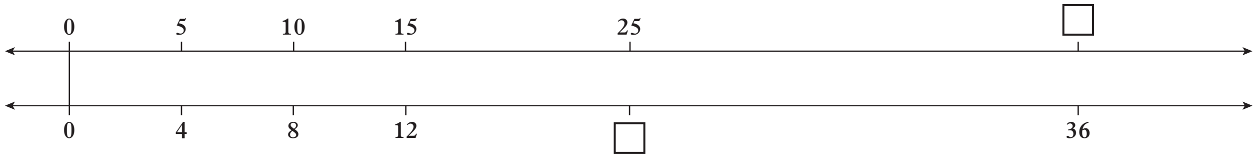
d Do the amounts found in part **c** add to $300?

e Decide if you prefer this method of dividing a quantity into a ratio.

23 Nadia divides her pocket money for April in the ratio 3 : 2 : 1 to cover buying a gift, entertainment expenses and savings. Calculate the amount she has allocated for entertainment expenses if her total pocket money for April is $84.

Exercise 3.8C: Challenge

24 Below is a double number line that is being used to show the relationship between the ratio 5 : 4 and ratios that are equivalent to it.



This can also be shown in a table.

0	5	10	15	25	
0	4	8	12		36

a Complete the double number line and ratio table.

b There are two calculations that produce the equivalent ratios 5 : 4, 10 : 8 and 15 : 12.

We can see that 5 × 3 = 15 and 4 × 3 = 12, but also 5 + 10 = 15 and 4 + 8 = 12 .

Explain how both calculations work in proving that the ratio 5 : 4 is equivalent to 15 : 12.

25 A bag has a number of green and orange marbles in it. The ratio of green to orange marbles is 5 : 8. When four green marbles are taken out of the bag, the ratio changes to 3 : 5. Find the number of marbles originally in the bag.

26 Itsuki has two tins of paint in different shades of purple. The first is a mix of purple and white paint in the ratio 1 : 1 in a 1 L tin. The second is a mix of purple and white paint in the ratio 3 : 1 in a 4 L tin.

a Which tin of paint has a darker shade?

b If Itsuki mixes both tins of paint together, how many litres of paint will she have?

c How much white paint is in the mixture?

d What is the ratio of purple and white paint in the mixture?

e Itsuki's friend Ito has two tins of paint as well. The first is a mix of purple and white paint in the ratio 4 : 1 in a 1 L tin. The second is a mix of purple and white paint in the ratio 1 : 1 in a 4 L tin.

i If Ito wants to add paint from the second tin to the first tin to match the colour of Itsuki's paint, what fraction of the second tin's paint should he add?

ii How much paint is left in the second tin?

Online resources:

Interactive skillsheet
Visual representations of ratios

Interactive skillsheet
Simplifying ratios

Interactive skillsheet
Dividing a quantity into a ratio

Investigation
Quilt patterns

Investigation
Writing ratios

Quick quiz
3.8

Lesson 3.9

Review: Fractions and ratios

Module summary

Types of fractions

- Proper fractions
  $\frac{3}{5}, \frac{2}{6}, \frac{11}{15}, \frac{3}{3}$
- Improper fractions
  $\frac{13}{6}, \frac{9}{5}, \frac{31}{8}$
- Mixed numbers
  $2\frac{1}{6}, 1\frac{2}{3}, 4\frac{5}{6}$

Equivalent fractions

$$\frac{2}{3} \overset{\times 5}{\underset{\times 5}{=}} \frac{10}{15} \qquad \frac{12}{10} \overset{\div 2}{\underset{\div 2}{=}} \frac{6}{5}$$

- The HCF of 36 and 72 is 36,

$$\frac{36}{72} = \frac{\cancel{36}^{1}}{\cancel{72}_{2}} = \frac{1}{2}$$

Improper fractions to mixed numbers

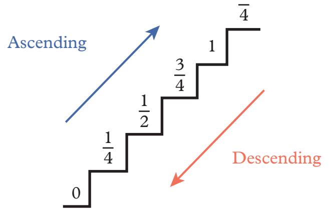
$$\frac{13}{4} = 13 \div 4 = 3 \text{ remainder } 1$$

$$= 3\frac{1}{4}$$

Mixed numbers to improper fractions

$$\overset{+}{\underset{\times}{3\frac{1}{4}}} = \frac{(4 \times 3) + 1}{4} = \frac{13}{4}$$

Ordering fractions



- To compare $\frac{1}{2}$ and $\frac{3}{5}$, find the LCD and multiply the numerator and denominator of each fraction to get an equivalent fraction. Then compare the fractions with the same denominator.

Multiplying fractions

1. Multiply the numerators together.
2. Multiply the denominators together.
3. Simplify the fraction.

$$\frac{2}{3} \times \frac{3}{4} = \frac{2 \times 3}{3 \times 4}$$
$$= \frac{6}{12}$$
$$= \frac{1}{2}$$

Reciprocal

$$\frac{3}{4} \rightarrow \frac{4}{3} \qquad 5 = \frac{5}{1} \rightarrow \frac{1}{5}$$

Dividing fractions

- Dividing by a fraction is the same as multiplying by the reciprocal of the fraction.
- Keep the first fraction the same, and multiply by the reciprocal of the second fraction.

$$\frac{3}{4} \div \frac{3}{8} = \frac{3}{4} \times \frac{8}{3}$$

Simplifying ratios

$$120 : 72 \xrightarrow[\div 24]{\div 24} 5 : 3$$

Add and subtract like fractions

1. Add or subtract the numerators and keep the same denominator.
2. Simplify the fraction.

$$\frac{1}{3} + \frac{4}{3} = \frac{5}{3}$$

Add and subtract unlike fractions

1. Convert into equivalent fractions with a common denominator.
2. Add or subtract the numerators and keep the same denominator.
3. Simplify the fraction.

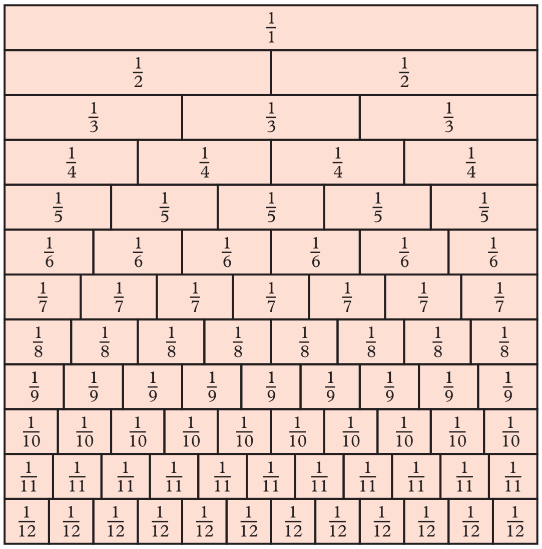
$$\frac{2}{3} + \frac{1}{2} = \frac{4}{6} + \frac{3}{6}$$
$$= \frac{7}{6}$$

Add and subtract mixed numbers

1. Convert mixed numbers into improper fractions.
2. Convert the improper fractions into equivalent fractions with a common denominator.
3. Add or subtract the numerators and keep the same denominator.
4. Convert the answer into a mixed number.
5. Simplify the fraction.

$$2\frac{1}{4} + 3\frac{1}{5} = \frac{9}{4} + \frac{16}{5}$$
$$= \frac{45}{20} + \frac{64}{20}$$
$$= \frac{109}{20}$$
$$= 5\frac{9}{20}$$

Fraction wall



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Review questions 3.9A: Mathematical literacy review

Review quiz
Take the review quiz to assess your knowledge of this module.

Quizlet
Test your knowledge of this module by working individually or in teams.

The following key terms are used in this module:

- cancelling
- common denominator
- denominator
- equivalent fraction
- fraction
- fraction wall
- highest common factor (HCF)
- improper fraction
- like fractions
- lowest common denominator (LCD)
- mixed numbers
- number line
- numerator
- proper fraction
- ratio
- reciprocal
- set
- simplest form
- simplify
- unit fraction
- unlike fractions

1 What are the differences between like fractions and unlike fractions?

2 Use fractions to represent the following quantities.

a a half
b two-thirds
c three quarters
d four and one-tenth

3 Write the following fractions in words.

a $\frac{1}{3}$
b $\frac{2}{9}$
c $\frac{2}{4}$
d $12\frac{1}{2}$

4 Identify the key terms being referenced in each of these definitions.

a fractions that have the same numerical value
b a common multiple of the denominators of two or more fractions

5 Provide a definition in your own words for the following key terms using an example.

a unit fraction
b reciprocal
c ratio

6 Complete the following sentences using words from the key term list.

a ________ have a numerator that is greater than or equal to the denominator. They can be written as ________, which consist of both a whole number and a ________.
b When a fraction is written in ________, the ________ between the numerator and the denominator is 1.
c When adding and subtracting like fractions, add and subtract the ________ and keep the same ________.

7 Identify whether the following statements are true or false. If a statement is false, briefly explain why.

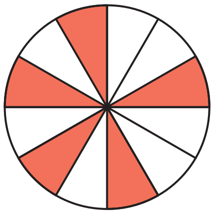
a A proper fraction has infinitely many equivalent fractions.
b All whole numbers can be written as fractions.
c Number lines and fraction walls can both be used to compare fractions.
d A unit fraction cannot be further simplified.
e The reciprocal of 0 is 0.
f Cancelling can only be performed when a fraction is multiplied by a whole number.
g A ratio can be used to compare up to three quantities.

Review questions 3.9B: Multiple choice

3.1 **1** A worksheet has 29 questions on it. If you have completed 17 of the questions, what fraction of the worksheet is complete?

A 17
B $\frac{29}{17}$
C 29
D $\frac{17}{29}$

3.1 **2** What fraction of the shape is coloured red?



A 5
B $\frac{5}{10}$
C $\frac{7}{12}$
D $\frac{5}{12}$

3.2 **3** Which of the following is an equivalent fraction to $\frac{2}{5}$?

A $\frac{3}{5}$
B $\frac{6}{15}$
C $\frac{2}{10}$
D $\frac{1}{10}$

3.2 **4** Simplify $\frac{120}{195}$.

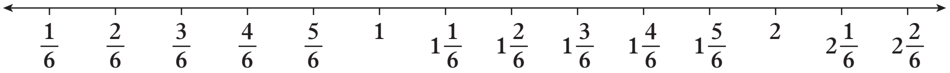
A 15
B $\frac{8}{13}$
C $\frac{40}{65}$
D $\frac{24}{39}$

3.3 **5** $1\frac{2}{6}, \frac{2}{3}, 1\frac{1}{2}, \frac{5}{12}$ in descending order is:

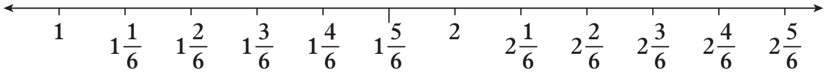
A $1\frac{1}{2}, 1\frac{2}{6}, \frac{2}{3}, \frac{5}{12}$
B $\frac{5}{12}, \frac{2}{3}, 1\frac{2}{6}, 1\frac{1}{2}$
C $1\frac{2}{6}, 1\frac{1}{2}, \frac{5}{12}, \frac{2}{3}$
D $\frac{2}{3}, \frac{5}{12}, 1\frac{1}{2}, 1\frac{2}{6}$

3.4 **6** Which of the following number lines is the most appropriate to compare the following fractions: $\frac{5}{6}, \frac{1}{3}, 1\frac{1}{3}, \frac{7}{6}, \frac{7}{3}$?

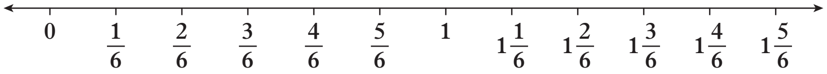
A



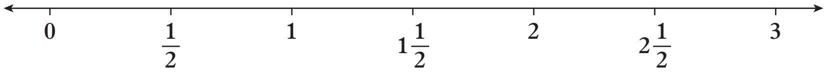
B



C



D



3.4 **7** Convert $3\frac{4}{7}$ to an improper fraction.

A $\frac{7}{7}$
B $\frac{4}{10}$
C $\frac{16}{7}$
D $\frac{25}{7}$

3.5 **8** What is $\frac{2}{5} + \frac{1}{3}$ equal to?

A $\frac{3}{8}$
B $\frac{2}{15}$
C $\frac{1}{2}$
D $\frac{11}{15}$

3.5 **9** What is $1\frac{1}{2} + 2\frac{5}{6} - 1\frac{2}{3}$?

A $2\frac{2}{3}$
B 4
C $2\frac{4}{5}$
D 6

3.6 **10** Which of the following pairs of fractions has a lowest common denominator that is not equal to the product of the denominators?

A $\frac{4}{12}, \frac{6}{7}$
B $\frac{3}{2}, \frac{2}{3}$
C $\frac{5}{8}, \frac{11}{12}$
D $\frac{14}{1}, \frac{2}{5}$

3.6 **11** What is $\frac{2}{7}$ of $\frac{8}{9}$?

A $\frac{16}{63}$ **B** $\frac{10}{16}$ **C** $\frac{6}{2}$ **D** $\frac{18}{56}$

3.6 **12** What is $\frac{14}{5} \times \frac{3}{21} \times \frac{25}{30}$ in its simplest form?

A 3 **B** $\frac{5}{3}$ **C** $\frac{3}{5}$ **D** $\frac{1}{3}$

3.7 **13** What is $\frac{7}{20} \div \frac{14}{5}$ equal to?

A $\frac{1}{2}$ **B** $\frac{1}{8}$ **C** $\frac{7}{15}$ **D** $\frac{21}{25}$

3.8 **14** What is a mixture of 100 mL cordial to 1.4 L of water written as a ratio in its simplest form?

A 100 : 1.4 **B** 100 : 1400 **C** 1 : 14 **D** 101.4

3.8 **15** Dario and Barbosa divide $150 in the ratio 3 : 2. Dario will get:

A $5 **B** $60 **C** $30 **D** $90

Review questions 3.9C: Short answer

3.1 **1** Mark the following fractions on a number line between the given values.

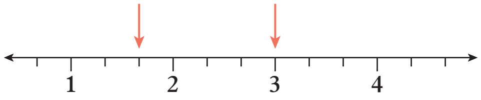
a $1\frac{3}{4}$ on a number line from 0 to 2

b $\frac{7}{12}$ on a number line from 0 to 1

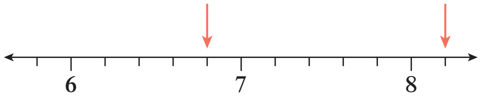
c $\frac{16}{3}$ on a number line from 2 to 6

3.1 **2** Write the improper fractions identified by the arrows.

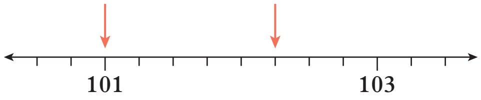
a



b



c



3.2 **3** For each of the following fractions, write an equivalent fraction using the given information.

a $\frac{2}{5}$ with a denominator of 25 **b** $\frac{9}{8}$ with a numerator of 27

c $\frac{3}{7}$ with a numerator of 21 **d** $\frac{50}{120}$ with a denominator of 24

3.2 **4** Simplify each fraction.

a $\frac{16}{20}$ **b** $\frac{5}{45}$ **c** $\frac{50}{100}$ **d** $\frac{21}{49}$

3.3 **5** Place these fractions in ascending order.

$\frac{1}{4}, \frac{2}{7}, \frac{3}{5}, 1\frac{2}{3}, \frac{9}{2}, \frac{11}{20}$

3.4 **6** Complete this table.

	Mixed number	Improper fraction
a	$2\frac{3}{8}$	
b		$\frac{11}{2}$
c		$\frac{17}{9}$
d	$5\frac{1}{3}$	

3.5 **7** Calculate:

a $\frac{1}{9}+\frac{4}{9}$ **b** $\frac{7}{11}-\frac{5}{11}$ **c** $\frac{2}{3}-\frac{1}{2}$ **d** $\frac{3}{10}+\frac{2}{5}$

3.5 **8** Calculate:

a $2\frac{5}{7}+3\frac{1}{7}$ **b** $2\frac{1}{5}-1\frac{2}{5}$ **c** $1\frac{3}{8}-\frac{1}{4}$ **d** $1\frac{4}{5}+2\frac{2}{3}$

3.6 **9** Calculate:

a $\frac{2}{5}\times\frac{4}{7}$ **b** $\frac{3}{10}\times\frac{5}{9}$ **c** $\frac{2}{3}\times\frac{10}{11}$ **d** $\frac{12}{5}\times\frac{4}{9}$

3.6 **10** Calculate:

a $\frac{2}{3}\times 1\frac{4}{5}$ **b** $2\frac{3}{7}\times\frac{1}{5}$ **c** $2\frac{1}{4}\times 3\frac{2}{3}$ **d** $1\frac{3}{8}\times 4\frac{1}{3}$

3.7 **11** Calculate:

a $\frac{3}{4}\div\frac{2}{3}$ **b** $\frac{25}{12}\div\frac{5}{3}$ **c** $\frac{49}{24}\div\frac{7}{16}$ **d** $\frac{1}{4}\div\frac{1}{3}$

3.7 **12** Calculate:

a $3\frac{4}{7}\div 2\frac{1}{7}$ **b** $\frac{8}{9}\div 1\frac{1}{3}$ **c** $3\frac{3}{5}\div\frac{6}{15}$ **d** $2\frac{3}{5}\div 1\frac{9}{15}$

3.8 **13** Use the photo of the Smarties to write each comparison as a ratio.

a the number of pink Smarties to the number of yellow Smarties

b the number of yellow Smarties to the number of green Smarties

c the number of orange Smarties to the total number of Smarties

d the number of orange and pink Smarties to the number of green Smarties

3.8 **14** Write as ratios in their simplest form.

a 24 : 36 **b** 15 : 20 **c** 75 : 120 **d** 56 : 4

3.8 **15** The ratio of snakes to lizards in a reptile zoo is 7 : 5. If there are 40 lizards, how many snakes are there in the zoo?

Review questions 3.9D: Mathematical modelling

1 A survey of 300 families found that 150 have one pet and 100 have two pets.

a What fraction of the families have one pet? Write this in its simplest form.

b What fraction of the families have two pets? Write this in its simplest form.

c Write a ratio for the number of families with one pet compared to those with two pets.

d If $\frac{1}{12}$ of the families have three or more pets, how many families is this?

e What fraction of the families do not have pets? How many families is this?

f A news reporter said that 5 in every 10 people have one pet, and 3 in every 10 people have two pets. Are they correct? Use your knowledge of equivalent fractions to support your answer.

2 Lili, Camila and Madelaine each bought half a cake in their favourite flavour.

a What fraction of a whole cake do they have altogether?

b Lili ate $\frac{1}{3}$ of her half-cake, Camila ate $\frac{3}{5}$ of her half-cake, and Madelaine ate $\frac{7}{15}$ of her half-cake. What fraction of a whole cake have they eaten in total?

c What fraction of a whole cake do they have left?

d What fraction of the amount of cake they started with do Lili, Camila and Madelaine have left?

e Their friend Casey says he can eat $\frac{3}{4}$ of the leftover cake. How much of a whole cake does Casey think he can eat?

f If Casey does eat $\frac{3}{4}$ of the leftover cake and Lili, Camila and Madelaine cannot ever eat more cake than they did in part **b**, who would be able to eat the rest of the cake tomorrow?

3 Keith found two bottles of 2 L juice in the fridge. One bottle is half empty, and the other is unopened.

a How much juice is left in the half empty bottle?

b Keith poured all the juice from the half empty bottle into a 3 L jug. What fraction of the jug is full?

c Keith drank $\frac{1}{4}$ L of juice and his friend Emma drank $\frac{1}{2}$ L of juice from the jug. How much juice is left in the jug?

d What fraction of the jug is empty?

e Emma opened the second bottle of juice to refill the jug to half full. How much juice did Emma add?

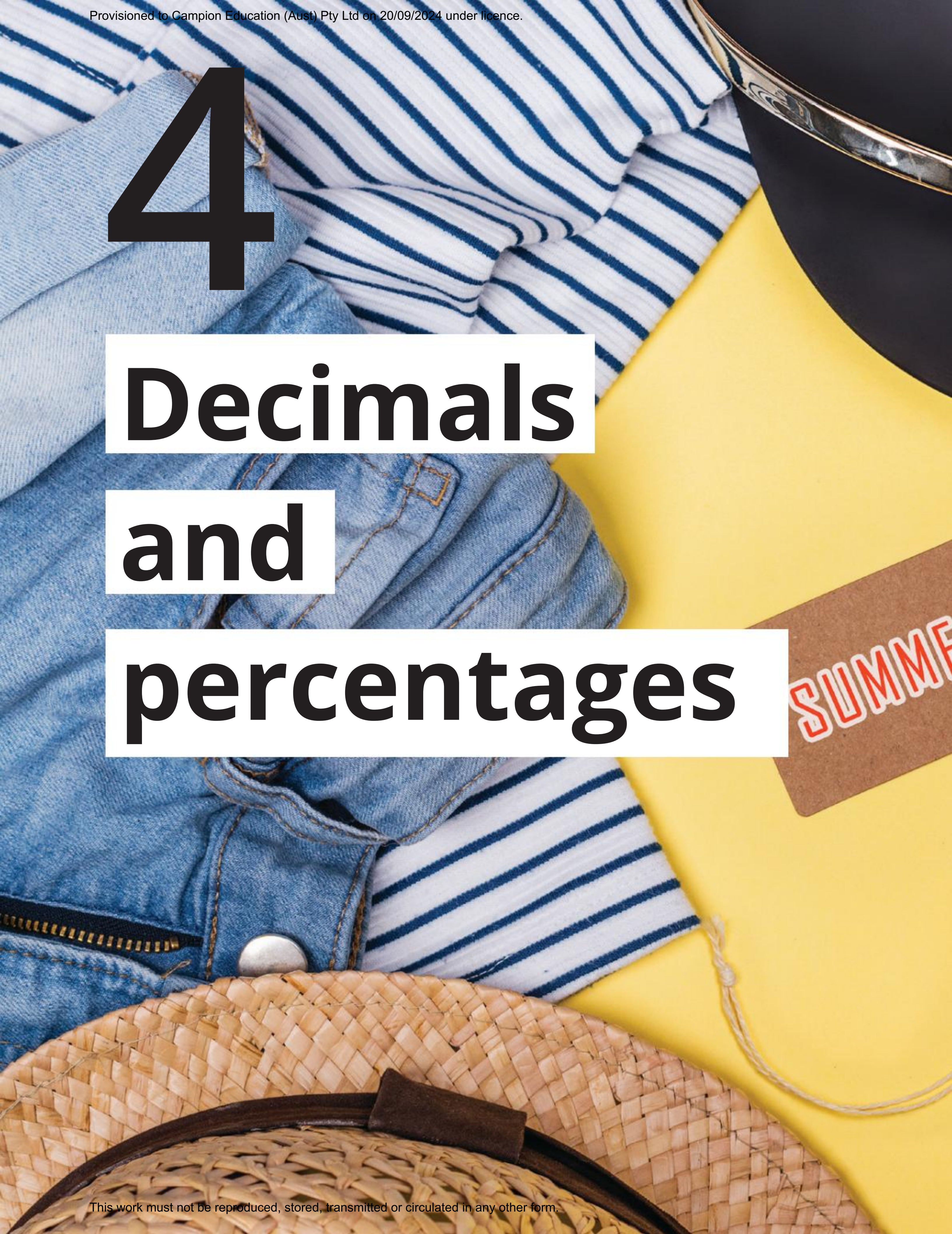
f What fraction of the juice bottle is full after the refill?

g Keith and Emma shared the juice in the jug in the ratio of 3 : 2. How much juice did Keith have in total?

Checklist

Now that you have completed this module, reflect on your ability to do the following.

I can do this	I need to review this
☐ Identify fractions from images, statements and number lines ☐ Express one quantity as a fraction of another	☐ Go back to Lesson 3.1 Fractions
☐ Recognise equivalent fractions on a fraction wall and number line ☐ Simplify fractions	☐ Go back to Lesson 3.2 Equivalent fractions
☐ Compare fractions with the same numerator or denominator ☐ Compare fractions with different numerators and denominators using equivalent fractions ☐ Order fractions in ascending and descending order	☐ Go back to Lesson 3.3 Ordering fractions
☐ Identify mixed numbers and improper fractions on a number line ☐ Convert improper fractions to mixed numbers ☐ Convert mixed numbers to improper fractions	☐ Go back to Lesson 3.4 Mixed numbers and improper fractions
☐ Add and subtract fractions with the same denominator ☐ Add and subtract fractions with different denominators ☐ Add and subtract mixed numbers	☐ Go back to Lesson 3.5 Adding and subtracting fractions
☐ Multiply fractions by whole numbers ☐ Multiply fractions by fractions ☐ Multiply mixed numbers	☐ Go back to Lesson 3.6 Multiplying fractions
☐ Identify the reciprocal of a fraction ☐ Divide proper and improper fractions ☐ Divide mixed numbers	☐ Go back to Lesson 3.7 Dividing fractions
☐ Compare quantities using ratios ☐ Simplify ratios ☐ Divide quantities into a ratio	☐ Go back to Lesson 3.8 Ratios



4

Decimals and percentages

Lessons

Prerequisite skills

Diagnostic pre-test

Take the diagnostic pre-test to assess your knowledge of the prerequisite skills listed below.

Interactive skillsheets

After completing the diagnostic pre-test, brush up on your knowledge of the prerequisite skills by using the interactive skillsheets.

- ✔ Place value
- ✔ Adding and subtracting whole numbers
- ✔ Multiplying whole numbers
- ✔ Dividing whole numbers
- ✔ Equivalent fractions

Curriculum links

- Find equivalent representations of rational numbers and represent positive and negative rational numbers and mixed numbers on a number line (VC2M7N03)
- Round decimals to a given accuracy appropriate to the context and use appropriate rounding and estimation to check the reasonableness of computations (VC2M7N04)
- Multiply and divide fractions and decimals using efficient mental and written strategies, and digital tools (VC2M7N05)
- Use the 4 operations with positive rational numbers, including fractions and decimals, to solve problems using efficient mental and written calculation strategies (VC2M7N06)
- Find percentages of quantities and express one quantity as a percentage of another, with and without digital tools (VC2M7N07)

Lesson 4.1

Decimals

Learning intentions

By the end of this lesson you will be able to ...

✔ identify the place value of a digit in a decimal and convert between fractions and decimals.

Inter-year links

Support	Fractions and decimals
Year 8	2.4 Decimals

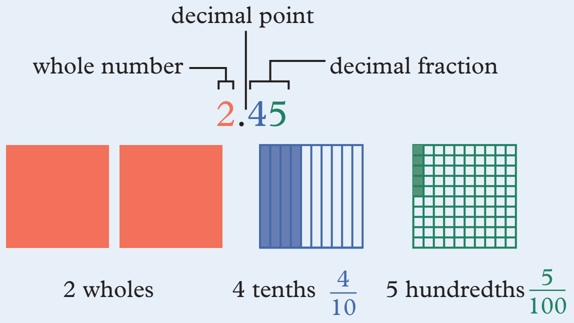
Key content video
Understanding decimals

Decimals

- A **decimal** is a number with a whole number part and a fraction part that are separated by a **decimal point**. The fraction part of a decimal is called a '**decimal fraction**', but we usually just refer to it as the decimal part of the number.
- A decimal is a useful way of writing fractions with denominators of 1, 10, 100, 1000, etc.

  For example,

  $\frac{2}{1} = 2$, $\frac{2}{10} = 0.2$, $\frac{2}{100} = 0.02$, $\frac{2}{1000} = 0.002$.



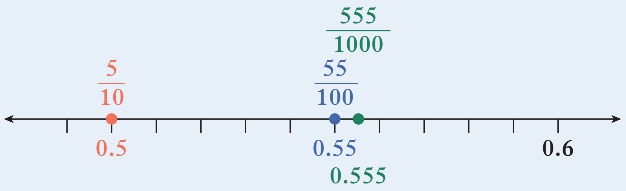
Value of decimals

- The value of each digit in a decimal depends on the place or position of the digit in the number. For example, the decimal 29.6183 is shown in the following place value chart.

Tens 10	Ones 1	.	Tenths $\frac{1}{10}$	Hundredths $\frac{1}{100}$	Thousandths $\frac{1}{1000}$	Ten-thousandths $\frac{1}{10\,000}$
2	9	.	6	1	8	3

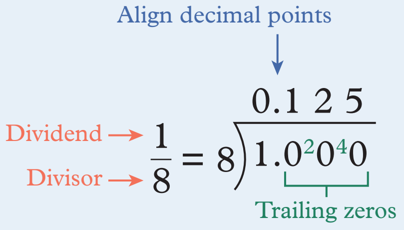
- Understanding the place value of the digit is important when converting between fractions and decimals.

0.5	5 tenths	$\frac{5}{10}$
0.55	55 hundredths	$\frac{55}{100}$
0.555	555 thousandths	$\frac{555}{1000}$



Converting fractions to decimals

- To convert from a fraction to a decimal, divide the numerator by the denominator. **Trailing zeros** can be added after the decimal point in the dividend until there are repeating digits in the answer or there is no remainder. Trailing zeros do not change the value of the number.



Worked example 4.1A Writing the place value of digits in a decimal

What is the place value of the digit 9 in each number?

a 5.7839 **b** 0.981 **c** 12.693

THINK

1 Draw a place value chart and place the digits into the right column according to their position relative to the decimal point.

2 Determine which column the number 9 is in to determine its place value.

WRITE

Tens 10	Ones 1	.	Tenths $\frac{1}{10}$	Hundredths $\frac{1}{100}$	Thousandths $\frac{1}{1000}$	Ten-thousandths $\frac{1}{10\,000}$
	5	.	7	8	3	9
	0	.	9	8	1	
1	2	.	6	9	3	

a Nine ten-thousandths or $\frac{9}{10\,000}$

b Nine tenths or $\frac{9}{10}$

c Nine hundredths or $\frac{9}{100}$

Worked example 4.1B Writing a fraction as a decimal when the denominator is a power of 10

Write the following fractions as decimals.

a $\frac{7}{10}$ **b** $\frac{13}{100}$ **c** $2\frac{81}{100}$

THINK

a Place the numerator in the place value column indicated by the denominator. There are 7 tenths, so place the 7 in the tenths column.

b There are 13 hundredths, so place the 1 in the tenths column and the 3 in the hundredths column.

c There are 2 ones and 81 hundredths, so place the 2 in the ones column, the 8 in the tenths column and the 1 in the hundredths column.

WRITE

Tens 10	Ones 1	.	Tenths $\frac{1}{10}$	Hundredths $\frac{1}{100}$	Thousandths $\frac{1}{1000}$	Ten-thousandths $\frac{1}{10\,000}$
	0	.	7			
	0	.	1	3		
	2	.	8	1		

a $\frac{7}{10} = 0.7$ **b** $\frac{13}{100} = 0.13$ **c** $2\frac{81}{100} = 2.81$

Worked example 4.1C Writing a decimal as a fraction

Write the following decimals as fractions.

a 1.6 **b** 0.42 **c** 12.098

THINK

1. Identify any whole numbers to the left of the decimal point.
2. Place the decimal part of the number as the numerator of the fraction. Use the place value of the last digit to determine the value of the denominator.
3. Simplify the fraction by finding common factors of the numerator and denominator.

WRITE

a $1.6 = 1\frac{6}{10}$

$= 1\frac{\cancel{6}^{3}}{\cancel{10}_{5}}$

$= 1\frac{3}{5}$

b $0.42 = \frac{42}{100}$

$= \frac{\cancel{42}^{21}}{\cancel{100}_{50}}$

$= \frac{21}{50}$

c $12.098 = 12\frac{98}{1000}$

$= 12\frac{\cancel{98}^{49}}{\cancel{1000}_{500}}$

$= 12\frac{49}{500}$

Worked example 4.1D Writing a fraction as a decimal using short division

Write $\frac{5}{8}$ as a decimal.

THINK

1. As the denominator of the fraction is not a power of 10, write the fraction as a division problem.
2. Work out the division using short division. Include trailing zeros after the decimal point until you can divide exactly without a remainder. Align the decimal points.

WRITE

$\frac{5}{8} = 5 \div 8$

$$\begin{array}{r} 0.6\,2\,\ 5 \\ 8\overline{)5.0^{2}0^{4}0} \end{array}$$

$\frac{5}{8} = 0.625$

Helpful hints

- ✔ Decimals and fractions are often thought of as two separate skills. They are actually two different ways of writing the same thing!
- ✔ If you have trouble determining the place value of a digit, draw a place value chart to help you.

Tens **10**	**Ones** **1**	**.**	**Tenths** $\frac{1}{10}$	**Hundredths** $\frac{1}{100}$	**Thousandths** $\frac{1}{1000}$	**Ten-thousandths** $\frac{1}{10\,000}$
1	2	.	3	4	5	6

Learning pathways

△ 1–6, 7(1st, 2nd columns), 8–10, 11(1st, 2nd columns), 12, 14, 16, 17

■ 1(g–l), 2, 4, 5, 7, 9–12, 13(b–d), 15–19, 20(a, b)

⬟ 1(i–k), 2(a, b, f), 7, 9–11, 13, 15, 18–21

ANS p517

Exercise 4.1A: Understanding and fluency

WE 4.1A **1** What is the place value of the digit 8 in each number?

a 28.61 **b** 304.128 **c** 927.835 **d** 16.0058

e 84.761 **f** 62.1852 **g** 821.793 **h** 71.085

i 0.0018 **j** 3.8024 **k** 8236.12 **l** 183.049

2 Write each of these as a decimal. If necessary, use a place value chart to help you.

a $5000 + 700 + 80 + 5 + \frac{1}{10} + \frac{9}{100} + \frac{5}{1000}$

b $400 + 90 + 6 + 0.2 + 0.08 + 0.003$

c 6 thousandths

d 5 ones, 7 tenths and 3 thousandths

e 3 tens, 6 ones and 7 hundredths

f 4 tens, 1 tenth and 7 ten-thousandths

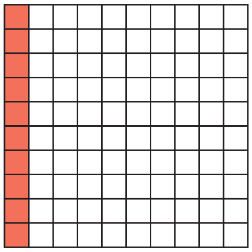
3 How many decimal places do each of these decimals have?

a 6.214 **b** 0.388 82 **c** 16.03 **d** 198.413 020

e 28 **f** 11.2 **g** 2.000 102 84 **h** 53.0

4 Describe the decimal shown in each hundredths grid.

a



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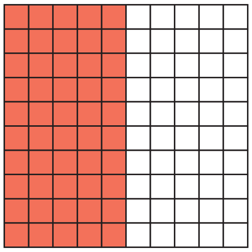

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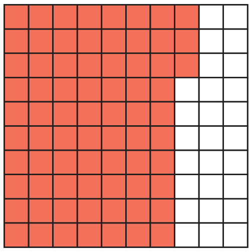

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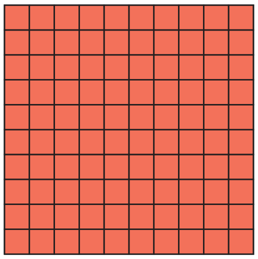
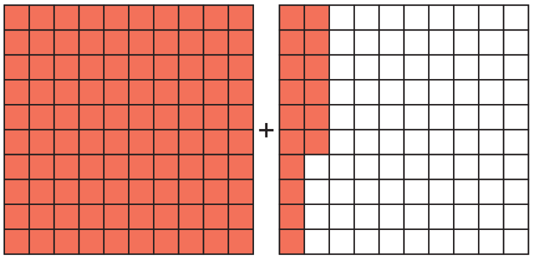
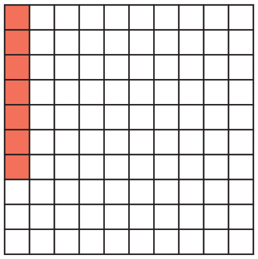


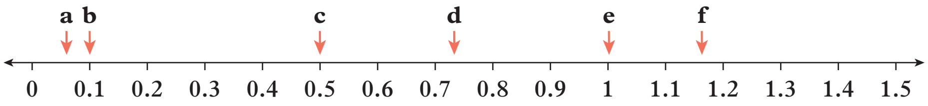
e



f



5 Match each decimal from question **4** to the equivalent arrow on the number line below.



WE 4.1B **6** Write each fraction as a decimal. If necessary, use a place value chart to help you.

a $\frac{7}{10}$ **b** $\frac{9}{10}$ **c** $\frac{33}{100}$ **d** $\frac{531}{1000}$

e $\frac{27}{100}$ **f** $\frac{11}{100}$ **g** $\frac{2978}{10\,000}$ **h** $\frac{804}{1000}$

7 Write each fraction as a decimal.

a $\frac{3}{100}$ **b** $\frac{17}{100}$ **c** $\frac{9}{10}$ **d** $\frac{2010}{10\,000}$

e $\frac{375}{10\,000}$ **f** $\frac{156}{10\,000}$ **g** $\frac{7}{100}$ **h** $\frac{921}{1000}$

i $\frac{42}{1000}$ **j** $\frac{3}{10\,000}$ **k** $\frac{1981}{100\,000}$ **l** $\frac{86}{10\,000}$

8 Write each decimal as a fraction with a denominator of 100, and then simplify.

a 0.25 **b** 0.50 **c** 0.35 **d** 0.88

e 0.23 **f** 0.08 **g** 0.12 **h** 0.95

WE 4.1C **9** Write each decimal as a fraction in its simplest form.

a 0.2 **b** 0.7 **c** 0.13 **d** 0.26

e 0.009 **f** 0.426 **g** 0.512 **h** 0.720

i 0. 0004 **j** 1.56 **k** 2.375 **l** 3.28

10 Write each decimal as a mixed number in its simplest form.

a 5.83 **b** 3.902 **c** 6.45 **d** 8.35

e 2.0008 **f** 3.9 **g** 12.88 **h** 1.706

WE 4.1D **11** Write each fraction as a decimal.

a $\frac{7}{8}$ **b** $\frac{3}{4}$ **c** $\frac{4}{5}$ **d** $\frac{9}{8}$

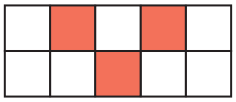
e $\frac{57}{4}$ **f** $\frac{15}{8}$ **g** $6\frac{3}{5}$ **h** $7\frac{1}{4}$

i $\frac{3}{20}$ **j** $\frac{1}{25}$ **k** $\frac{7}{50}$ **l** $\frac{9}{40}$

12 For each rectangle, write the area that is shaded as a:

i fraction **ii** decimal.

a



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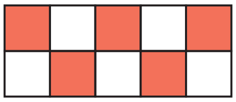
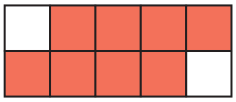


c



13 When the denominator of a fraction is a power of 10, the fraction can be written as a decimal without performing division; for example, $\frac{7}{10} = 0.7$. Convert the given fractions into tenths, hundredths or thousandths. Then write each equivalent fraction as a decimal.

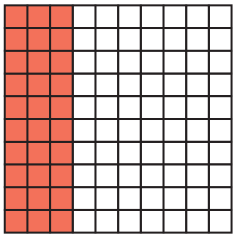
For example,
$$\begin{aligned}\frac{3}{5} &= \frac{3}{5} \times \frac{2}{2}\\ &= \frac{6}{10}\\ &= 0.6\end{aligned}$$

a $\frac{1}{5}$ **b** $\frac{3}{4}$ **c** $\frac{7}{20}$

d $\frac{4}{5}$ **e** $\frac{11}{50}$ **f** $\frac{2}{4}$

g $\frac{13}{20}$ **h** $\frac{1}{2}$ **i** $\frac{19}{50}$

Exercise 4.1B: Problem solving and reasoning



14 Look at the grid shown on the right. Three of the ten columns are shaded. The fraction that has been shaded is $\frac{3}{10}$.

- **a** Write the fraction as a decimal.
- **b** How many small squares are there in total? How many of these are shaded?
- **c** Write the number of squares that are shaded as a fraction.
- **d** Explain why your answers for parts **a** and **c** are the same.

15 Jacey bought some lollipops for a car trip: two raspberry, two apple, one cola, two watermelon and three strawberries & cream.

- **a** How many lollipops are there in total?
- **b** Write the number of raspberry lollipops as a fraction of the total number of lollipops.
- **c** Write your answer to part **b** as a decimal.
- **d** Write the number of each of the other lollipops as both a fraction and a decimal of the total number of lollipops.

16 You are offered 0.3 of this block of chocolate or a piece that is $\frac{1}{4}$ of the block of chocolate. Which portion is larger? Show your calculations.

17 Tim wrote that $\frac{42}{1000} = 0.42$. Can you help explain where he might have gone wrong?

18 Salwa is making a crocheted doily for a project at school. She has 4.25 m and $1\frac{4}{5}$ m of yarn.

- **a** Convert the decimal to a fraction so they are both in the same form.
- **b** How much yarn does Salwa have?
- **c** If she needs a total of 6 m of yarn, does she have enough?

Exercise 4.1C: Challenge

19 Look at the section between 7 cm and 8 cm on your ruler. It is divided into tenths, so each small mark represents $\frac{1}{10}$ of 1 cm or 0.1 cm.

- **a** If you magnified the section between 7.3 cm and 7.4 cm to see it divided into ten equal intervals, each interval would represent $\frac{1}{10}$ of $\frac{1}{10}$ cm. What is this as a decimal?
- **b** If you magnified the section between 7.30 cm and 7.31 cm to see it divided into ten equal intervals, each interval would represent $\frac{1}{10}$ of $\frac{1}{100}$ cm. What is this as a decimal?
- **c** If you repeated this process another two times, what would each of the smallest interval represent?

20 Write each of the following as a decimal.

- **a** $3400 + \frac{710}{10} + \frac{49}{100}$
- **b** $\frac{37}{10\,000} + \frac{230}{10} + \frac{54}{100}$
- **c** $\frac{283}{10} + \frac{1002}{100} + \frac{987}{1000}$
- **d** five million ten thousandths
- **e** ninety-nine thousand tenths, ninety-nine hundred hundredths, ninety-nine thousandths
- **f** twelve hundred forty-seven tenths, nine thousand five hundred and twenty-nine hundredths

21 Consider this list of fractions: $3\frac{7}{9}$, $3\frac{11}{12}$, $3\frac{8}{10}$, $3\frac{162}{200}$, $3\frac{7}{8}$. Which fraction is closest to 3.82?

Online resources:

- **Interactive skillsheet** Place value with decimals
- **Worksheet** Converting between fractions and decimals
- **Investigation** Target race
- **Quick quiz** 4.1
- **Interactive skillsheet** Fractions and decimals

Lesson 4.2

Ordering and rounding decimals

Learning intentions

By the end of this lesson you will be able to ...

✔ compare, order and round decimals.

Inter-year links

Support	Ordering decimals
Year 8	2.4 Decimals

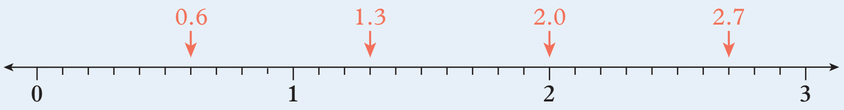
Key content video
Ordering and rounding decimals

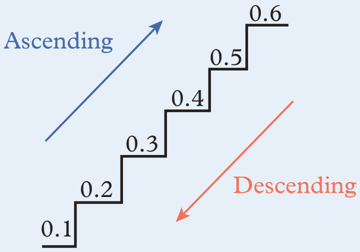
Ordering numbers

- To order decimals, consider the digits from left to right.
  For example, 5.673 and 5.689 have the same whole number and tenth values. 8 hundredths is larger than 7 hundredths, so 5.673 < 5.689.

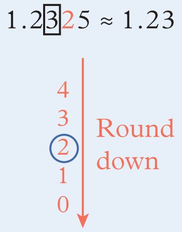
Ones	.	Tenths $\frac{1}{10}$	Hundredths $\frac{1}{100}$	Thousandths $\frac{1}{1000}$
5	.	6	7	3
5	.	6	8	9

- Decimals can be represented on a number line.
  - → To represent tenths, separate the line into ten equal parts between two whole numbers.





Rounding



- Rounding a number involves replacing the number with an approximation that is simpler and easier to use in calculations; however, this will also make any calculations less accurate.
  - → 'Approximately equal' is shown using the symbol '≈'.
- When rounding 1.2345,
  - → to the nearest tenth or one **decimal place**: $1.\boxed{2}345 = 1.2$
  - → to the nearest hundredth or two decimal places: $1.2\boxed{3}45 = 1.23$
  - → to the nearest thousandth or three decimal places: $1.23\boxed{4}5 = 1.235$
- **Accuracy** describes the closeness of a measurement to its exact value. The higher the degree of accuracy, the closer the measurement is to the exact value.

Worked example 4.2A Ordering decimals

Arrange the following decimals in ascending order.

3.14, 4.13, 1.34, 3.41

THINK	**WRITE**
1 Group the numbers by comparing the whole number parts. There are one 4, two 3s and one 1.	1.34, 3.14, 4.13 3.41
2 Compare the tenth values of numbers with the same whole number part. 4 tenths are larger than 1 tenth.	1.34, 3.14, 4.13 3.41
3 Write the decimals in ascending order, from smallest to largest.	1.34, 3.14, 3.41, 4.13

Worked example 4.2B Rounding decimals

Round 0.429 63 to:

a two decimal places

b three decimal places

c four decimal places.

THINK	**WRITE**
a 1 Draw a box around the second decimal place.	**a** $0.4\boxed{2}963$
2 Look at the digit to the right of the box. The digit to the right is 9. Add 1 to the digit in the box.	$0.4\boxed{2}963$
3 Discard all the digits to the right of the box. All digits to the left of the boxed digit stay the same.	$0.4\boxed{3}$ $0.42963 \approx 0.43$
b 1 Draw a box around the third decimal place.	**b** $0.42\boxed{9}63$
2 Look at the digit to the right of the box. The digit to the right is 6. Add 1 to the digit in the box.	$0.42\boxed{9}63$
3 As the boxed digit changes from 9 to 10, write zero in the boxed digit's place and add 1 to the place to the left.	$0.43\boxed{0}$
4 Discard all the digits to the right of the box. All digits to the left of the boxed digit stay the same. As three decimal places are requested, keep the trailing zero in the final answer.	$0.42963 \approx 0.430$
c 1 Draw a box around the fourth decimal place.	**c** $0.429\boxed{6}3$
2 Look at the digit to the right of the box. The digit to the right is 3. Do not change the digit in the box.	$0.429\boxed{6}3$ $0.429\boxed{6}$
3 Discard all the digits to the right of the box. All digits to the left of the boxed digit stay the same.	$0.42963 \approx 0.4296$

Helpful hints

- ✔ You may think that 0.000 19 is greater than 0.2 because there are more digits shown. Remember to compare from left to right. As 0 tenths is smaller than 2 tenths, 0.000 19 < 0.2.
- ✔ Make sure you consider the context before you round decimals.
- ✔ When asked for a specific number of decimal places, always include them, even if the last digits are trailing zeros.
  For example, 1.002 rounded to two decimal places is 1.00, not 1.

Learning pathways

△ 1–8, 10–12, 15, 17(a, e)

■ 2, 4–9, 11, 13, 16, 17, 18(a, b)

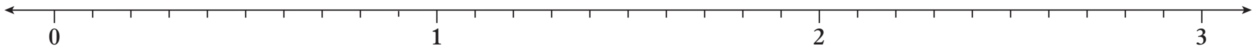
⬠ 2(d, f, h), 4, 5, 7–9, 13, 14, 16, 17(a, c, e), 18, 19

ANS p518

Exercise 4.2A: Understanding and fluency

1 Copy the number line below and mark the following numbers on it.

2.5, 1.8, 0.1, 0.8, 1.5, 1.0



2 Insert < (less than) or > (greater than) between each pair of decimals to make the statement true. Remember to compare each equivalent place value column of each number. Use a place value chart to help you.

a 4.73 ___ 3.473
b 3.21 ___ 3.24
c 9.6841 ___ 9.684 09
d 0.017 ___ 0.107
e 55.03 ___ 55.003
f 12.218 ___ 12.2108
g 6.9891 ___ 6.8999
h 1378.5 ___ 1378.499
i 800.78 ___ 800.778

WE 4.2A 3 Put the following decimals in ascending order.

a 9.10, 1.91, 1.90, 9.01
b 5.55, 5.5, 5.05, 0.55
c 4.567, 45.67, 4.0567, 4.5067
d 1, 0.001, 0.100 001, 0.101

4 Write each list of decimals in ascending order.

a 0.641, 0.461, 0.146, 0.328, 0.823, 0.238
b 2.341, 2.143, 2.431, 4.132, 1.234, 3.412
c 38.162, 36.218, 38.261, 36.812, 36.182, 32.861
d 18.4251, 18.2145, 18.4125, 18.1524, 18.4152, 18.2154

5 Write each list in the order written in brackets.

a $0.000\,04,\ 0.0003,\ 0.000\,030,\ 0.00\,040,\ \frac{2}{10\,000},\ \frac{21}{100\,000}$ (descending)
b $0.81,\ 0.801,\ \frac{81}{10},\ 0.8,\ 8.0,\ \frac{80\,001}{100\,000}$ (ascending)
c $\frac{1}{2},\ 0.51,\ \frac{501}{100},\ 0.0510,\ \frac{1}{5},\ 0.15$ (ascending)
d $3+\frac{2}{10}+\frac{6}{100},\ 3+\frac{25}{100},\ 3-\frac{25}{100},\ 3.75,\ 3.026,\ 3+\frac{6}{10}+\frac{2}{100}$ (descending)

WE 4.2B **6** Round 3.546 52 to:

a three decimal places **b** four decimal places **c** two decimal places.

7 Round each of these decimals to the number of decimal places indicated in the brackets.

a 82.631 (2) **b** 59.111 78 (3) **c** 0.263 418 (4)
d 0.266 66 (2) **e** 15.0003 (3) **f** 2.175 498 7 (6)
g 0.218 638 2 (3) **h** 165.545 47 (4) **i** 43.806 251 (5)

8 Round 168.246 588 to:

a the nearest hundred **b** the nearest ten **c** the nearest whole number
d the nearest tenth **e** the nearest hundredth **f** the nearest thousandth.

Exercise 4.2B: Problem solving and reasoning

9 Draw this number line. Each division on your number line should be 1 cm apart.

a Mark the decimals 23.15, 23.3, 23.45, 23.55, 23.7 and 23.95 on your number line.
b Which whole number is closest to each of the decimals less than 23.5?
c Which whole number is closest to each of the decimals greater than 23.5?
d Explain how you would round these decimals to the nearest whole number.

10 During an athletics carnival, Mark threw a javelin the following distances:

22.81 m, 21.98 m, 21.89 m and 22.76 m.

a What was Mark's shortest throw?
b What was Mark's longest throw?
c Explain how you compared the distances to decide which throw was longest and which was shortest.

11 Consider the decimals 12.021, 12.201 and 12.210.

a What is the place value of 0 in each number?
b What would happen if you wrote each number without its zero? For example, if you wrote 12.021 as 12.21.
c Zeros between non-zero digits, such as the zero in 12.021, are placeholder zeros. Why is it important that placeholder zeros are never left out?
d Zeros at the end of a decimal (such as the zero in 12.210) are trailing zeros. Does it matter if trailing zeros are left out? Explain your answer.
e Write the three numbers in ascending order.

12 Mia had to round 0.726 53 to three decimal places. Explain why her answer of 0.726 is incorrect.

13 Another way to reduce the number of decimal places is called truncating. To truncate a decimal, we drop all decimal places after a given place value. For example, truncating 0.123 45 after four decimal places is 0.1234.

a Truncate the following decimals to the given number of decimal places.
  i 5.418 52 to one decimal place and two decimal places
  ii 3.0014 to one decimal place and three decimal places
  iii 2.999 99 to two decimal places and one decimal place
  iv 0.003 124 1 to two decimal places and five decimal places
b This time, round (rather than truncate) each of the numbers in part **a** to the same number of decimal places.
c When do rounding and truncating give the same answer?
d When do rounding and truncating give different answers?

14 State the smallest number of decimal places you can round each pair of numbers to and be able to correctly state which is larger using the rounded numbers.

- **a** 0.1234 and 0.1324
- **b** 4.3567 and 4.4456
- **c** 0.000 234 158 and 0.000 023 415 8
- **d** 9.999 99 and 9.999 399

15 Explain why the following statements are incorrect. Give the correct statement.

- **a** $0.0002 > 0.1$
- **b** $0.001 < 0.001\,00$
- **c** $0.05 = 0.5$
- **d** $12.34 < 5.678$

16 Tyler has given the following reasons to justify the inequalities he has written. Decide whether each inequality is correct and whether his reason is correct.

- **a** $\frac{1}{8} > \frac{1}{5}$ because $\frac{1}{8}$ is equal to 0.8 and $\frac{1}{5}$ is equal to 0.5 and $0.8 > 0.5$
- **b** $0.34 < 0.6789$ because 34 is less than 6789
- **c** $0.04 > 0.0391$ because $\frac{4}{100} > \frac{3.91}{100}$

17 Consider the following situations. State how the bold values could be rounded and why.

- **a** Ronnie's dad got 32 L of petrol at 129.9 cents/L. It cost **$42.2175** and Ronnie's dad paid using a savings card.
- **b** Ronnie's mum got 30 L of petrol at 133.4 cents/L. It cost **$40.02** and Ronnie's mum paid using cash.
- **c** A hiker has to turn **32.2851°** clockwise and walk 20 m.
- **d** A rocket has to turn **32.2851°** clockwise and travel 20 000 km.
- **e** The height of a person is **1.592 412 m**.
- **f** The length of a bug is **1.592 412 cm**.

Exercise 4.2C: Challenge

18 In Australia most things are measured using the metric system. This means that we can think of smaller units, like m, in terms of larger units, like km, by rounding to the correct number of decimal places.

- **a** Round 58.348 cm to the nearest mm. Write your answer in cm.
- **b** Round 98.2124 L to the nearest mL. Write your answer in L.
- **c** Round 214.192 751 km to the nearest m. Write your answer in km.
- **d** Round 214.192 751 km to the nearest cm. Write your answer in km.

19 State the number of decimal places each of the following should be rounded to.

- **a** A number in centimetres rounded to the nearest millimetre
- **b** A number in litres rounded to the nearest millilitre
- **c** A number in kilometres rounded to the nearest metre
- **d** A number in kilometres rounded to the nearest centimetre

Online resources:

Interactive skillsheet Rounding decimals

Interactive skillsheet Ordering decimals

Worksheet Comparing decimals

Investigation Decimals in the library

Quick quiz 4.2

Lesson 4.3

Adding and subtracting decimals

Learning intentions

By the end of this lesson you will be able to …

✔ add and subtract decimals.

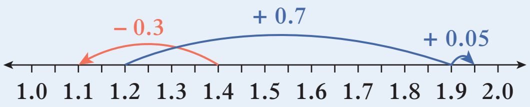
Inter-year links

Support	Adding and subtracting decimals
Year 8	2.5 Adding and subtracting decimals

Adding and subtracting decimals

Key content video
Adding and subtracting decimals

- Number lines are helpful to visualise the addition and subtraction of decimals. For example, the diagram shows 1.2 + 0.75 = 1.95 and 1.4 – 0.3 = 1.1.



Using the vertical algorithm	Using fractions
$\begin{array}{r} 3.02 \\ +\ 4.15 \\ \hline 7.17 \\ \hline \end{array}$	$3.02 + 4.15 = 3 + \frac{2}{100} + 4 + \frac{15}{100}$ $= 7 + \frac{17}{100}$ $= 7.17$

- Decimals can be added or subtracted in the same way as whole numbers using the vertical algorithm or by using fractions.
- When adding or subtracting decimals, the digits must be positioned in their correct place value columns and the decimal points must be lined up.
- Blank spaces in the columns may be replaced with trailing zeros to assist working out.

  For example, 3.2 + 3.002 can be written as $\begin{array}{r} 3.200 \\ +\ 3.002 \\ \hline 6.202 \end{array}$

Worked example 4.3A Using vertical addition to add decimals

Use vertical addition to calculate 13.21 + 3.089.

THINK	WRITE
1 Arrange the numbers vertically. Ensure the decimal points and the digits are lined up.	$\begin{array}{r} 13.21 \\ +\ \ 3.089 \\ \hline \\ \hline \end{array}$
2 Replace blank spaces with additional zeros so that each number contains the same number of decimal places.	$\begin{array}{r} 13.210 \\ +\ 03.089 \\ \hline \\ \hline \end{array}$
3 Add the digits in each column. Ensure the decimal point in the answer is lined up with the decimal points in the calculation.	$\begin{array}{r} 13.210 \\ +\ 03.089 \\ \hline 16.299 \\ \hline \end{array}$

Worked example 4.3B Using vertical subtraction to subtract decimals

Use vertical subtraction to calculate 29.43 – 16.785.

THINK	**WRITE**
1 Arrange the numbers vertically. Ensure the decimal points and the digits are lined up.	$\begin{array}{r} 29.43 \\ -16.785 \\ \hline \\ \hline \end{array}$
2 Replace any blank spaces with trailing zeros so that each number contains the same number of decimal places.	$\begin{array}{r} 29.430 \\ -16.785 \\ \hline \\ \hline \end{array}$
3 Subtract the digits in each column as you would for whole numbers. Ensure the decimal point in the answer is lined up with the decimal points in the calculation.	$\begin{array}{r} 2\overset{8}{\cancel{9}}.\overset{13}{\cancel{4}}\overset{12}{\cancel{3}}\overset{10}{\cancel{0}} \\ -\ 16.785 \\ \hline 12.645 \\ \hline \end{array}$

Helpful hints

✔ Make sure that the decimal points for each number are aligned when adding or subtracting decimals.

For example, $\begin{array}{r} 100\,000.3 \\ -\quad 12.000\,003 \\ \hline \\ \hline \end{array}$

✔ Remember to consider the place value of the digits in decimal fractions. Otherwise, you could make a mistake like 0.5 + 0.6 = 0.11 instead of 0.5 + 0.6 = 1.1.

Learning pathways

△ 1–12, 15–17, 20

■ 3–10, 13–14, 16, 18, 19, 22

⬟ 5, 8–10, 16, 19–23

ANS p519

Exercise 4.3A: Understanding and fluency

1 For each sum below:

i show the expression on a number line **ii** calculate the sum.

a 0.5 + 0.3 **b** 1.3 + 0.7 **c** 0.8 + 0.6 **d** 1.2 + 0.6

2 For each difference below:

i show the expression on a number line **ii** calculate the difference.

a 0.8 – 0.3 **b** 2 – 0.4 **c** 1.8 – 0.9 **d** 1.1 – 0.5

3 Estimate the result of each calculation by rounding each number to the nearest 10.

a $\begin{array}{r} 38.62 \\ +\ 49.59 \\ \hline \\ \hline \end{array}$ **b** $\begin{array}{r} 132.61 \\ +\ 183.86 \\ \hline \\ \hline \end{array}$ **c** $\begin{array}{r} 16.345 \\ +\ 175.928 \\ \hline \\ \hline \end{array}$ **d** $\begin{array}{r} 54.756 \\ +\ 237.309 \\ \hline \\ \hline \end{array}$

e $\begin{array}{r} 9.58 \\ 117.371 \\ +\ 22.65 \\ \hline \\ \hline \end{array}$ **f** $\begin{array}{r} 46.2863 \\ 1152.93 \\ +\quad 9 \\ \hline \\ \hline \end{array}$ **g** $\begin{array}{r} 120.146 \\ 11.06 \\ +\ 813.705 \\ \hline \\ \hline \end{array}$ **h** $\begin{array}{r} 8.05 \\ 261.841 \\ +\ 7832.5 \\ \hline \\ \hline \end{array}$

WE 4.3A **4** Evaluate each calculation in question **3** and check how the answers compare with your estimates.

5 Use vertical addition to calculate each of these.

a 12.582 + 9.81 + 13.62 **b** 1482 + 37.31 + 0.487

c 48.799 + 3.218 + 14 **d** 3826.1 + 0.4326 + 28.571

e 0.9453 + 128.62 + 40.1 + 8.3279 **f** 12.063 + 0.004 + 3.904

6 Estimate the result of each calculation by first rounding each number to the nearest 10.

a $\begin{array}{r} 79.84 \\ -\ 23.61 \\ \hline \\ \hline \end{array}$ **b** $\begin{array}{r} 163.88 \\ -\ 129.16 \\ \hline \\ \hline \end{array}$ **c** $\begin{array}{r} 487.6 \\ -\ 92.7 \\ \hline \\ \hline \end{array}$ **d** $\begin{array}{r} 33.7 \\ -\ 10.48 \\ \hline \\ \hline \end{array}$

e $\begin{array}{r} 1492.6 \\ -\ 15.92 \\ \hline \\ \hline \end{array}$ **f** $\begin{array}{r} 172 \\ -\ 58.73 \\ \hline \\ \hline \end{array}$ **g** $\begin{array}{r} 23.078 \\ -\ 7.49 \\ \hline \\ \hline \end{array}$ **h** $\begin{array}{r} 2031.703 \\ -\ 798.0912 \\ \hline \\ \hline \end{array}$

WE 4.3B **7** Evaluate each calculation in question **6** and check how the answers compare with your estimates.

8 Use vertical subtraction to find each difference.

a 172.91 – 48.984 **b** 16.872 – 9.385 **c** 2014.23 – 1998.76

d 414.9 – 389.76 **e** 17 – 10.325 **f** 14.008 – 3.67

9 Check your answers to questions **5** and **8** with a calculator.

10 Without using a calculator, evaluate each of the following. Remember to follow the order of operations (BIDMAS).

a 12 + 629.1 + 34.75 – 392.7 **b** 68.419 – 23.8 + 17.75

c 37.941 – 37.899 + 15.371 **d** 648 – (58.63 – 28.21)

e 192.62 – (154 + 38.39) **f** (84.7 + 93.2) – (128.8 – 109.6)

Exercise 4.3B: Problem solving and reasoning

11 Angelo needs the following lengths of fencing for repairs on his farm: 15.36 m, 28.42 m, 5.8 m and 17.1 m. State the total length of fencing Angelo requires.

12 Shanti bought five items priced at \$3.69, \$14.99, \$7.25, \$10.59 and \$5.99.

a Estimate the total cost of the five items.

b Use your answer from part **a** to estimate how much change she should receive from \$50.

c Find the sum of the five items and hence the exact amount of change she should receive.

d How accurate was your estimate from part **b**?

13 Natasha scored 8.3, 9.1, 8.5, 7.9 and 8.8 in her gymnastics competition.

a Calculate Natasha’s total score.

b Monique competed in the same gymnastics competition and received scores of 8.6, 7.5, 9.2, 8.3 and 8.5. Calculate Monique’s total score.

c Who received the higher overall score and what was the difference in their totals?

14 Jonathan, Karamo and Tan competed in the high jump championships, clearing heights of 1.43 m, 1.39 m and 1.38 m, respectively. If the school record for the high jump was 1.52 m, find the difference between each competitor's jump and the record.

15 Check the following calculations and describe any errors made.

a
$$\begin{array}{r} 1\ 2.7 \\ +\ 3.4\ 1 \\ \hline 4.6\ 8 \\ \hline \end{array}$$

b
$$\begin{array}{r} 9.8 \\ -\ 4.47 \\ \hline 5.47 \\ \hline \end{array}$$

c
$$\begin{array}{r} 6.00 \\ -\ 1.25 \\ \hline 5.75 \\ \hline \end{array}$$

d
$$\begin{array}{r} 5.3 \\ -\ 1.7 \\ \hline 4.4 \\ \hline \end{array}$$

16 Estimating the answer before using an algorithm can often help us avoid mistakes, prompting us to double-check our working. For each of the calculations complete the following:

i Write the calculation with the numbers each rounded to their nearest whole number.

ii Calculate an estimate using the rounded numbers and state whether your estimate should be higher or lower than the actual answer.

iii Use a calculator to determine the actual answer.

a 9.58 + 1.67 **b** 15.02 – 9.18 **c** 14 – 2.85 **d** 24.658 – 2.984 + 6.182

17 Matthew is planning to paint four bedrooms over a long weekend. He needs these amounts of paint: 4.2 L, 3.89 L, 2.5 L and 1.9 L.

a How many litres of paint will Matthew need?

b If the paint comes in 5 L tins, how many tins will he need to buy?

18 Melbourne's average total monthly rainfall for the month of September is 65.7 mm. In the past year, it rained on five days during the month of September, with daily totals of 28.7 mm, 12.8 mm, 1.3 mm, 0.9 mm and 20.98 mm. Comment on how this past year's monthly total compares with the average monthly total for September.

19 Adrian is a sales rep and spends much of his time on the road travelling from one country town to the next. He is required to record the car's odometer readings at the end of each day, as shown in this log sheet.

Day	Odometer reading	Distance travelled (km)
Start	28 071.5	0
Day 1	28 258.7	
Day 2	28 407.3	
Day 3	28 657.1	
Day 4	28 801.9	
Day 5	29 053.4	

a Complete the log sheet. You can use your calculator.

b What was the odometer reading at the beginning of Adrian's travels?

c On which day did Adrian travel the least distance? How far did he travel on this day?

d On which day did Adrian travel the greatest distance? How far did he travel on this day?

e Find the total distance Adrian travelled in the five days.

20 When travelling domestically you usually need to make sure that your checked luggage is less than 20 kg. Elio is checking in three bags, weighing 4.5 kg, 13.79 kg and 3.8 kg.

a What is the total weight of his checked luggage?

b How much weight will he need to remove to stay under 20 kg?

21 Elisa's monthly bank statement is shown below. An opening balance of $328.56 states how much money is in Elisa's account at the beginning of the month.

Date	Transaction details	Debit (−)	Credit (+)	Balance
28 Feb	Opening balance			328.56 CR
05 Mar	Personal cheque 531287		125.42	
08 Mar	FJ Jewellers	58.99		
09 Mar	Athletique Gym Membership	65.00		
16 Mar	ABWDL Smith St	250.00		
23 Mar	Officeworks	28.53		
30 Mar	Interest		1.78	
30 Mar	Monthly fee	5.00		
31 Mar	Closing balance			

- **a** Calculate the following.
  - **i** The balance after each transaction
  - **ii** The total debits
  - **iii** The total credits
  - **iv** The final balance at the end of the month
- **b** Elisa wishes to purchase a drone, retailing for $199. Comment on whether she will have enough money in her account at the end of the month.

Exercise 4.3C: Challenge

22 Calculate 100 – 0.009 245 – 0.524 13 – 0.140 907 – 12.380 718.

23 An alternative to rounding a decimal is to truncate the number by dropping all the decimal places after a given place value without changing the final digit. For example, 12.345 truncated to two decimal places is 12.34.

- **a** Truncate the following to the number of decimal places in the brackets.
  - **i** 462.2718 (1)
  - **ii** 1.2793 (3)
  - **iii** 0.009 149 (2)
  - **iv** 12.649 20 (2)
- **b** Estimate 7.218 24 + 2.742 + 17.682 76 by:
  - **i** rounding each number to zero decimal places
  - **ii** truncating each number to zero decimal places.
- **c** Evaluate 7.218 24 + 2.742 + 17.682 76. Give your answer:
  - **i** rounded to three decimal places
  - **ii** truncated to three decimal places.
- **d** Which estimate is the better estimate of the actual result: the rounded or the truncated estimate?
- **e** When will the rounded and truncated estimates be the same?
- **f** The sum of three numbers is estimated using two different methods; one by rounding each number to one decimal place and the other by truncating each number to one decimal place. Which estimate, if any, will always be greater than or equal to the other estimate? Explain your answer.
- **g** What is the maximum possible difference between the two estimates described in part **f**?
- **h** What is the maximum possible difference between the sum of three numbers rounded to one decimal place and the sum of the same three numbers truncated to two decimal places?

Online resources:

Interactive skillsheet
Adding and subtracting decimals

Worksheet
Adding and subtracting decimals

Investigation
Takeaway decimals

Desmos activity
Decimal diagrams

Quick quiz
4.3

Lesson 4.4

Multiplying decimals

Learning intentions

By the end of this lesson you will be able to …

✔ multiply decimals by powers of 10, whole numbers and decimals.

Inter-year links

Support	Multiplying decimals
Year 8	2.6 Multiplying and dividing decimals
Year 9	9.1 Wages and salaries

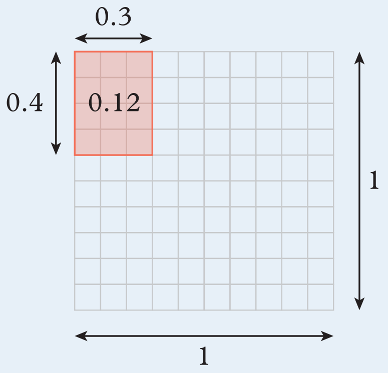
Key content video
Multiplying decimals

Multiplying decimals

- Decimals can be multiplied in the same way as whole numbers.
- The result of the product of two or more decimals has the same number of decimal places as the total number of decimal places in the original problem.
- To multiply decimals
  1. Count the decimal places in the problem.
  2. Multiply the decimals as whole numbers.
  3. Write the answer with this same number of decimal places. Add place-holding zeros where necessary.

$$\underset{\substack{\uparrow\\ \text{one}\\ \text{decimal}\\ \text{place}}}{2.\underline{5}} \times \underset{\substack{\uparrow\\ \text{two}\\ \text{decimal}\\ \text{place}}}{0.\underline{03}} = \underset{\substack{\uparrow\\ \text{the answer}\\ \text{will have a}\\ \text{total of three}}}{0.\underline{075}}$$

- Multiplication of decimals can also be represented using diagrams.



Multiplying by powers of 10

- Multiplying a number by 10 increases the place value of each digit by 1 column. Move the decimal point one place value space to the right and insert a place-holding zero where necessary.

$$5.23 \times 1\underline{0} = 52.3$$

- Multiplying a number by 100 increases the place value of each digit by 2 columns. Move the decimal point two place value spaces to the right and insert a place-holding zero where necessary.

$$5.23 \times 1\underline{00} = 523.0$$

- Multiplying a number by 1000 increases the place value of each digit by 3 columns. Move the decimal point three place value spaces to the right and insert a place-holding zero where necessary.

$$5.23 \times 1\underline{000} = 5230.0$$

Worked example 4.4A Multiplying decimals

Find each product.

a 0.7×0.03

b 0.12×6

THINK

1 Count the decimal places in the problem.

2 Multiply the decimals as whole numbers.

3 Write the answer with this same number of decimal places. Add zeros where necessary.

WRITE

a $0.\underline{7} \times 0.\underline{0}\,\underline{3}$

There are three decimal places in total.

$7 \times 3 = 21$

$0.\underline{021}$

$0.7 \times 0.03 = 0.021$

b $0.\underline{1}\,\underline{2} \times 6$

There are two decimal places in total.

$12 \times 6 = 72$

$0.\underline{72}$

$0.12 \times 6 = 0.72$

Worked example 4.4B Multiplying decimals using long multiplication

Use long multiplication to evaluate 18.6 × 4.9.

THINK

1 Estimate the result by rounding to the nearest whole number.

2 Count the decimal places in the problem. There are two decimal places in total.

3 Multiply the decimals as whole numbers using long multiplication.

4 Write the answer with this same number of decimal places.

5 Check that your estimate is a close approximation of the result and that the decimal point is in the correct position.

WRITE

$$\begin{aligned} 18.6 \times 4.9 &\approx 19 \times 5 \\ &= 95 \end{aligned}$$

$18.\underline{6} \times 4.\underline{9}$

$$\begin{array}{r} {\scriptstyle 3\;2\;\;\;} \\ {\scriptstyle 7\;5\;} \\ 186 \\ \times \quad 49 \\ \hline 1674 \\ +\,7440 \\ \hline 9114 \end{array}$$

$91.\underline{14}$

$18.6 \times 4.9 = 91.14$

Worked example 4.4C Multiplying by 10, 100 or 1000

Find each product.

a 2.27×100

b 15.3×1000

THINK

a When multiplying by 100, move the decimal point two place value spaces to the right. Don't write the decimal point if there are no digits other than zero after it.

b When multiplying by 1000, move the decimal point three place value spaces to the right. Add zeros where necessary.

WRITE

a $2.\underline{27}$

$2.27 \times 100 = 227$

b $15.\underline{300}$

$15.3 \times 1000 = 15\,300$

Helpful hints

- ✔ When multiplying by powers of ten, move the decimal point to the **right**.
- ✔ Don't try to line up the decimal point in the algorithm for multiplying. It won't give the correct result. Instead, remember to count the decimal places in the question and use the total in the answer.
  $0.\underline{1}\,\underline{2}\,\underline{3} \times 0.\underline{4}\,\underline{5}$ = Answer has five decimal places
- ✔ Multiplying a number by a decimal *less than 1 will decrease* the number. Multiplying a number by a decimal *greater than 1 will increase* the number.

Learning pathways

△ 1–3, 4–5(1st, 2nd columns), 6–11, 12(a, b, e), 13(a, b), 17

■ 2, 4–5(2nd, 3rd columns), 6, 7(g–l), 8, 9(a, b), 12, 13, 14(a, b), 15, 18

⬠ 2(g–i), 5, 7(3rd column), 8, 9, 12–14, 16, 18–20

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Exercise 4.4A: Understanding and fluency

1 How many decimal places will there be in the answer to each calculation?

a 1.56 × 2.489 **b** 0.4896 × 1.356 **c** 12.364 29 × 3.8
d 15.6308 × 46.75 **e** 0.0056 × 46.21 **f** 0.167 × 0.007

WE 4.4A 2 Use your knowledge of times tables and decimal places to evaluate each product.

a 0.2 × 0.6 **b** 0.3 × 0.8 **c** 0.6 × 0.7
d 0.08 × 0.9 **e** 0.004 × 0.9 **f** 0.7 × 0.7
g 1.2 × 0.8 **h** 0.006 × 0.04 **i** 0.0001 × 0.007

3 Evaluate each product.

a 3.8 × 7 **b** 9.8 × 4 **c** 0.63 × 5
d 1.14 × 8 **e** 4.0008 × 9 **f** 3.28 × 6
g 2.759 × 3 **h** 51.4 × 7 **i** 19.86 × 4

WE 4.4B 4 Use long multiplication to evaluate the following.

a 24.7 × 2.8 **b** 63.4 × 7.2 **c** 128.4 × 9.8
d 13.64 × 2.1 **e** 15.78 × 8.3 **f** 148.42 × 6.5
g 298.61 × 0.16 **h** 13.213 × 1.84 **i** 52.475 × 42.9

5 Evaluate without using a calculator.

a 1.37 × 24 **b** 2.41 × 36 **c** 13.21 × 15
d 8.64 × 72 **e** 92.84 × 0.002 **f** 186.71 × 0.04
g 0.05 × 248 **h** 0.006 × 617 **i** 20.03 × 52

6 Check your answers to questions **4** and **5** with a calculator.

WE 4.4C 7 Evaluate each product.

a 6 × 1000 **b** 42 × 100 **c** 0.1284 × 10
d 15.721 × 100 **e** 0.0004 × 1000 **f** 0.0702 × 100
g 6.1 × 10 **h** 15.0 × 10 **i** 0.31 × 1000
j 64.813 × 100 **k** 4.8 × 100 000 **l** 128.468 × 1 000 000

Exercise 4.4B: Problem solving and reasoning

8 Multiplying by 60 is the same as multiplying by 6 and then multiplying by 10.

For example, $4.86 \times 60 = 4.86 \times 6 \times 10$
$= 29.16 \times 10$
$= 291.6$

Use this strategy to determine each product.

a 12.98×700 **b** 37.2×8000 **c** 43.62×2400

d 106.2×320 **e** $6.5 \times 15\,000$ **f** $0.27 \times 45\,000$

9 Evaluate each of the following without using a calculator.

a $(2.387 + 4.21 - 2.7) \times 3.1$ **b** $6.85 \times 7.2 - (4.5 + 8.2)$

c $14.85 - (6.57 + 12.35) \times 0.1$ **d** $62.87 \times 0.002 + 17.8 \times 0.02$

10 Look at the calculations in the first column of this table.

Original calculation	Estimated answer	Answer
38 × 21	40 × 20 = 800	798
3.8 × 21		
38 × 2.1		
3.8 × 2.1		
380 × 210		

a What do the calculations have in common? **b** How are these calculations different?

c Complete the second column by estimating the answer. Round each number to its leading digit (first non-zero digit). The first one has been done for you.

d Complete the third column by using the digits 798, a decimal point and zeros if needed, to create a number close to the estimated answer obtained in part **c**.

e What is the relationship between the total number of decimal places in the original calculation and the number of decimal places in the answer?

11 Barry purchases 58.7 L of unleaded petrol at a cost of $1.32 per litre.

a Estimate the total cost of the petrol.

b Calculate the total cost of the petrol to the nearest cent.

c Find the difference between the estimated cost and the actual cost of the petrol purchased.

12 Calculate the cost of each of these items, rounding your answers to the nearest cent.

a 0.75 kg of cherries at $12.99 per kg

b 1.56 kg of Granny Smith apples at $3.90 per kg

c 2.78 kg of bananas at $3.50 per kg

d 1.2 kg of grapes at $6.90 per kg

e Three punnets of strawberries at $2.99 per punnet

f 2.89 kg oranges at $3.49 per kg

13 For each of the following calculations:

i estimate the product by rounding both numbers to the leading digit

ii calculate the actual product using a calculator

iii find the difference between the estimation and the actual product.

a 6.1295×9.1463 **b** 7561.2×256.94

c $0.004\,127 \times 1295.291$ **d** $0.850\,005 \times 0.006\,24$

14 We can see where to place the decimal point in a decimal multiplication problem by writing the calculation using fractions.

For example, $12.3 \times 0.02 = \frac{123}{10} \times \frac{2}{100}$

$= \frac{246}{1000}$

$= 0.246$

Use the above method for the following calculations.

a 0.07×0.003 **b** 200.3×0.005 **c** 1.74×6.21 **d** 9.99×2.21

15 The local gym offers its clients the following payment plans for a yearly membership.

- Option 1: a flat-fee payment of $645
- Option 2: monthly instalments of $59.45

a Determine how much Option 2 costs for a year.

b Which option is cheaper?

c How much money is saved if the cheaper option is chosen?

16 When a decimal is squared, the number of decimal places in the number is related to the number of decimal places in the answer.

a Calculate the value of each of the following.

i 0.2^2 **ii** 0.4^2 **iii** 0.25^2 **iv** 0.008^2 **v** 1.2^2

b Comment on the number of decimal places in the numbers to be squared and the number of decimal places in the answer. What do you observe?

17 The magnification on a microscope is 1000×. This means that objects appear 1000 times larger than they actually are. How wide would an insect's eye that is 0.008 mm in diameter appear when viewed through this microscope?

18 Colin is a plumber. He charges an initial fee of $120 and an additional $36.80 per hour.

a The Grant family needs a pipe to be unblocked. If this job takes Colin a total of 3.75 hours, how much will he charge?

b The Torcasios family requires a new hot water system to be installed. If this job takes Colin a total of 45 minutes, how much will he charge?

Exercise 4.4C: Challenge

19 The grid method can be used for multiplying decimals.

For example, $5.76 \times 21.823 = (5 + 0.7 + 0.06) \times (20 + 1 + 0.8 + 0.02 + 0.03)$.

×	**20**	**1**	**0.8**	**0.02**	**0.003**
5	100	5	4.0	0.10	0.015
0.7	14	0.7	0.56	0.014	0.0021
0.06	1.2	0.06	0.048	0.0012	0.000 18
Total	115.2	5.76	4.608	0.1152	0.017 28

Therefore, $5.76 \times 21.823 = 115.2 + 5.76 + 4.608 + 0.1152 + 0.017\,28 = 125.700\,48$.

Use the grid method to calculate the following products.

a 8.12×0.45 **b** 1.285×8.21 **c** 3.127×19.21

20 Use the digits 0–9 only once to write a product with a value as close to 7.9 as possible.

☐☐.☐☐☐ × ☐.☐☐☐☐

Online resources:

Interactive skillsheet Multiplying decimals

Worksheet Multiplying decimals

Investigation Burning joules

Desmos activity Multiplying with areas

Quick quiz 4.4

ANS
p520

Checkpoint

Checkpoint quiz
Take the checkpoint quiz to check your knowledge of the first part of this module.

4.1 **1** What is the place value of the digit 5 in each number?

a 81.475
b 9.5024
c 6.3915
d 12.0561

4.1 **2** Write the following decimals as fractions in their simplest form.

a 0.17
b 5.3
c 0.15
d 9.125

4.1 **3** Write the following fractions as decimals.

a $\frac{71}{100}$
b $\frac{12}{5}$
c $\frac{5}{8}$
d $11\frac{7}{20}$

4.2 **4** Complete the statements by filling in the missing symbol: >, = or <.

a 1.3 ____ 0.6
b 0.1 ____ 0.10
c 0.07 _____ 0.5
d 31.2729 _____ 31.8

4.2 **5** Put the following lists in the order written in brackets.

a 17.80, 1.708, 1.087, 7.108, 7.180 (ascending)
b 1.056 89, 15.689, 15.986, 1.5689, 1.5698 (descending)
c 0.7, 0.08, $\frac{9}{10}$, $\frac{18}{100}$, 0.20 (ascending)
d 1.4, $\frac{3}{2}$, $\frac{34}{20}$, $\frac{11}{5}$, 1.80 (descending)

4.2 **6** Round the following numbers to the decimal places written in brackets.

a 6.472 912 (two decimal places)
b 32.1825 (three decimal places)
c 0.003 047 2 (three decimal places)
d 99.987 192 (one decimal place)

4.3 **7** Calculate the following.

a 2.1 + 3.8
b 8.7 – 6.5
c 5.28 + 7.99
d 15.12 – 8.54

4.3 **8** Evaluate the following calculations.

a 47.91 + 38.27
b 203.4 + 87.49
c 83.47 – 21.36
d 421.8 – 39.72

4.4 **9** Calculate the following products.

a 5.91 × 10
b 0.004 12 × 100
c 0.124 × 1000
d 52.82 × 0.1

4.4 **10** Calculate the following products.

a 0.06 × 0.009
b 2.3 × 0.8
c 19.3 × 0.45
d 20.8 × 6.6

4.4 **11** Estimate each of the calculations by rounding each number to the nearest whole number.

a 3.48 – 1.28 + 9.19
b 9.14 + 10.4 × 0.6
c 37.1 × (15.2 + 4.8)
d (17.7 – 8.9) × (0.05 + 0.18)

4.4 **12** Evaluate each of the calculations in question **11**.

Lesson 4.5

Dividing decimals

Learning intentions

By the end of this lesson you will be able to …

- ✔ divide decimals by powers of 10, whole numbers and decimals.

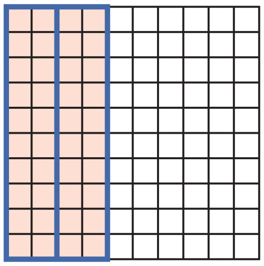
Inter-year links

Support	Dividing a decimal by a whole number
Year 8	2.6 Multiplying and dividing decimals

Dividing a decimal by a whole number

Key content video
Dividing a decimal by a whole number

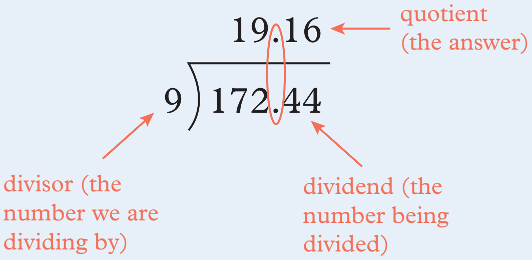
- 0.4 ÷ 2 is asking: 'How large are the groups when 4 tenths is divided into 2 equal groups?'



0.4 is shown on the hundredths grid.
0.4 is grouped into 2 groups of 20 hundredths.
So, there are 2 tenths or 0.2 in each group.
0.4 ÷ 2 = 0.2

- When dividing a decimal by a whole number, use the short division method used for dividing whole numbers.
- Align the decimal point in the answer with the dividend.

172.44 ÷ 9



- Trailing zeros can be added following the digits after the decimal point in the dividend if required.

Dividing by powers of 10

- Dividing a number by 10 decreases the place value of each digit by 1 column. Move the decimal point one place value space to the left and insert place-holding zeros where necessary.

$$5.23 \div 1\underline{0} = 0.523$$

- Dividing a number by 100 decreases the place value of each digit by 2 columns. Move the decimal point two place value spaces to the left and insert place-holding zeros where necessary.

$$5.23 \div 1\underline{00} = 0.0523$$

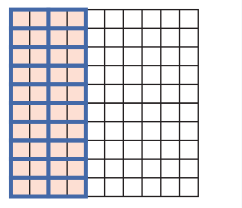
- Dividing a number by 1000 decreases the place value of each digit by 3 columns. Move the decimal point three place value spaces to the left and insert place-holding zeros where necessary.

$$5.23 \div 1\underline{000} = 0.00523$$

Dividing a decimal by a decimal

Key content video
Dividing a decimal by a decimal

- 0.4 ÷ 0.02 is asking: 'How many 2 hundredths are in 4 tenths?'



0.4 is shown on the hundredths grid.
0.4 is grouped into 20 groups of 2 hundredths.
So, there are 20 groups of 2 hundredths in 4 tenths.
0.4 ÷ 0.02 = 20

- To help divide by a decimal, you can create an equivalent problem by moving the decimal point in both the dividend and divisor the same number of spaces to the right or multiplying both the dividend and divisor by multiples of 10.
- To divide a decimal by a decimal:
  1. Convert the divisor to a whole number by moving the decimal point to the right.
  2. Move the decimal point the same number of spaces to the right in the dividend.
  3. Perform short division, adding trailing zeros where necessary.

$$\overbrace{1.2}^{\text{Dividend}} \div \overbrace{0.4}^{\text{Divisor}} = 12 \div 4 = 3$$

Worked example 4.5A Dividing a decimal by a whole number

Evaluate 16.5 ÷ 6.

THINK	WRITE
1 Evaluate the division in the same way as for whole numbers, using short division.	$\begin{array}{r} 2.\;7\;5 \\ 6\overline{)16.{}^{4}5{}^{3}0} \end{array}$
2 Align the decimal points and digits that have the same place value.	
3 Add trailing zeros after the last digit in the dividend to obtain an exact decimal answer without a remainder.	16.5 ÷ 6 = 2.75

Worked example 4.5B Dividing by 10, 100 or 1000

Evaluate each quotient.

a 12.5 ÷ 10 **b** 20.08 ÷ 100 **c** 3.14 ÷ 1000

THINK	WRITE
a When dividing by 10, move the decimal point one place value space to the left.	**a** 12.5 12.5 ÷ 10 = 1.25
b When dividing by 100, move the decimal point two place value spaces to the left.	**b** 20.08 20.08 ÷ 100 = 0.2008
c When dividing by 1000, move the decimal point three place value spaces to the left. Include place-holding zeros where necessary.	**c** 003.14 3.14 ÷ 1000 = 0.003 14

Worked example 4.5C Dividing by a decimal

Evaluate each quotient.

a 45 ÷ 1.2

b 16.482 ÷ 0.04

THINK

1 Convert the divisor to a whole number by moving the decimal point to the right.

2 Move the decimal point the same number of spaces to the right in the dividend. Add place-holding zeros where necessary.

3 Perform short division and add trailing zeros where necessary. Make sure the decimal points are lined up.

WRITE

a $45.\underset{\curvearrowright}{0} \div 1.\underset{\curvearrowright}{2} = 450 \div 12$

$$\begin{array}{r} 3\ \ 7.\ \ 5 \\ 12\overline{)\underline{45}\,^{9}0\,.^{6}0} \end{array}$$

$45 \div 1.2 = 37.5$

b $16.\underset{\curvearrowright\curvearrowright}{482} \div 0.\underset{\curvearrowright\curvearrowright}{04} = 1648.2 \div 4$

$$\begin{array}{r} 412.05 \\ 4\overline{)\underline{16}48.\underline{20}} \end{array}$$

$16.482 \div 0.04 = 412.05$

Helpful hints

- ✔ When dividing by a power of ten, move the decimal point to the **left**.
- ✔ When adding trailing zeros after the decimal, you are not changing the number. You are just writing a zero that was already implicitly there.
- ✔ Remember that trailing zeros should be added to the dividend until you can divide exactly, without a remainder.
- ✔ Dividing a number by a decimal *less than 1 will increase* the number. Dividing a number by a decimal greater *than 1 will decrease* the number.

Learning pathways

▲ 1–2(1st, 2nd columns), 3–15, 17, 22, 26

■ 1–2(c, f, i), 3(b, d, f), 4–6, 10–13, 16, 18, 20, 21, 23–27

⬟ 2(g–i), 3–5(e, f), 6, 10–12, 14–16, 18, 19, 23–28

ANS p521

Exercise 4.5A: Understanding and fluency

1 Evaluate each of the following using short division.

a $4\overline{)24.92}$ **b** $5\overline{)68.605}$ **c** $8\overline{)72.816}$

d $7\overline{)161.98}$ **e** $9\overline{)100.035}$ **f** $3\overline{)4.9194}$

g $9\overline{)26.2953}$ **h** $5\overline{)20.75}$ **i** $7\overline{)24.57}$

WE 4.5A 2 Perform each division using short division. Remember to add trailing zeros if needed.

a 15 ÷ 6 **b** 27 ÷ 4 **c** 63 ÷ 5

d 17.4 ÷ 4 **e** 12.3 ÷ 6 **f** 2.031 ÷ 5

g 4.83 ÷ 8 **h** 234.3 ÷ 6 **i** 56.23 ÷ 4

3 Calculate the cost of one item in each case.

a Seven exercise books cost $8.96
b Four tyres cost $870
c 12 pens cost $15.36
d 10 icy-poles cost $7.50
e 15 gym sessions cost $177
f Eight movie passes for $58

4 Calculate the cost of 1 kg in each case.

a 6 kg of oranges cost $23.28
b 5 kg of nectarines cost $24.90
c 3 kg of mangos cost $11.94
d 6 kg of scotch fillet costs $71.82
e 2 kg of capsicums cost $13.96
f 4 kg of apples cost $31.92
g 12 kg of firewood cost $20.40
h 7 kg chicken wings for $29.89

WE 4.5B **5** Evaluate each quotient.

a $678.543 \div 1000$
b $953.5 \div 100$
c $288.94 \div 10$
d $68\,942.1 \div 1000$
e $1509.5 \div 10\,000$
f $4.36 \div 100$

6 Dividing by 900 is the same as dividing by 9 and then dividing by 100.

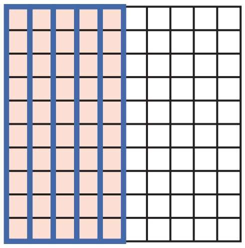
For example,
$$\begin{aligned} 5342.4 \div 900 &= (5342.4 \div 9) \div 100 \\ &= 593.6 \div 100 \\ &= 5.936 \end{aligned}$$

Use this strategy to calculate the following.

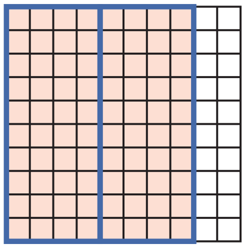
a $393.946 \div 700$
b $231\,120 \div 200$
c $16\,790.4 \div 30$
d $1248.9 \div 500$
e $307\,152 \div 5000$
f $8192.5 \div 200$

7 Each of these diagrams represents a decimal being divided by another decimal. Write the corresponding division problem for each diagram.

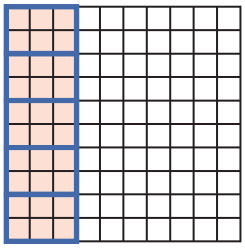
a



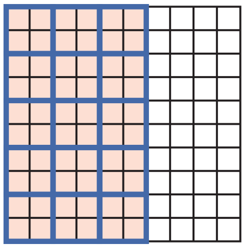
b



c



d



8 Determine answers to your equations in question **7**. Use the diagrams to help you.

9 Write an equivalent division problem for each of these so that you are dividing by a whole number.

a $62.356 \div 0.8$
b $9.84 \div 0.008$
c $19.5674 \div 0.06$
d $0.462 \div 0.3$
e $0.042 \div 0.0004$
f $0.005 \div 0.02$

WE 4.5C **10** Evaluate each quotient.

a $3.86 \div 0.2$
b $0.843 \div 0.003$
c $5.929 \div 0.07$
d $27.32 \div 0.0008$
e $42.39 \div 0.9$
f $7.296 \div 0.02$

11 Evaluate each quotient. Remember to use trailing zeros if needed.

a $9.524 \div 0.8$
b $1.735 \div 0.04$
c $32.475 \div 0.006$
d $0.007\,53 \div 0.000\,04$
e $1.369\,47 \div 0.0006$
f $0.0573 \div 0.004$

12 Following the correct order of operations, complete the calculations below.

a $(12.6 \div 0.003) \times 3.8$
b $(8.34 - 2.18) \div 0.28$
c $1.4^2 + 8.38 \div 0.004$
d $\sqrt{(0.0048 \div 0.03) \times 100}$
e $\sqrt{(3.2 \times 5.1) - (0.16 \times 2)} \div 0.008$
f $2.3 \times (1.08 \times 7.23) - (4.206 \div 1.2)^2$

Exercise 4.5B: Problem solving and reasoning

13 A group of Year 7 students decides to divide the $558.75 they have raised throughout the year equally between three charities. How much will each charity receive?

14 Pizza can be bought by the slice or as a whole pizza that is equivalent to six slices. Compare the cost per slice of the pizza shown on the right. Which option is better value and by how much?

15 Milk can be bought in different sizes. At a supermarket, a 1 L container of milk costs $2.97 and a 2 L container costs $4.98.

- **a** Calculate the cost for 1 L if you buy the 2 L container of milk.
- **b** Compare this cost to the cost of the 1 L container of milk. Which option is the best buy? Explain why it is better value.

16 Customers can buy single eggs or a carton of a dozen eggs at a health food store.

- **a** Compare the cost per egg in each case. Which option is better value?
- **b** How much would customers save per egg if they bought the better value option? Give your answer correct to the nearest cent.
- **c** Why do you think the store offers both options?

17 Calculate the cost of 100 g in each case. Where appropriate, round to the nearest cent.

- **a** 300 g of leg ham cost $11.19
- **b** 500 g of cheese cost $7.80
- **c** 900 g of green beans cost $4.76
- **d** 200 g of aniseed rings cost $2.99

18 A recipe requires 800 g of pasta sauce. Katie considers two options:

Firenze pasta sauce 400 g jar for $3.73
Milano pasta sauce 500 g jar for $4.09.

- **a** Compare the value of 100 g of each product. Round your answers to the nearest cent.
- **b** How many jars should Katie buy for her recipe? Which brand of sauce should Katie buy if she wants value for money? Explain.

19 Finn has the option of three different summer jobs: picking grapes, delivering pamphlets or packing boxes. He tries each of them out. He earns $51.20 picking grapes for 8 hours, $46.40 delivering pamphlets for 6 hours, and $32.50 packing boxes for 4 hours. Which job pays the best by the hour?

20 Lorena has the option of buying a weekly travel ticket for $13.75 or a daily ticket costing $2.95.

- **a** How much does her weekly ticket cost per day if she uses it five days a week?
- **b** How does the price of a daily ticket compare with that of a weekly ticket? Which is the better option?
- **c** Are there occasions when buying daily tickets would be better than buying a weekly ticket? Explain.

21 Simon bought a 500 g pack of tomatoes for \$3.98. Julia bought three tomatoes shown on the right, which weighed a total of 300 g.

a Calculate the amount paid by each person for 100 g of tomatoes. Where appropriate, round to the nearest cent.

b Who made the best buy?

22 How many 0.375 L bottles can be filled from a 32.25 L water dispenser?

23 Dividing by a decimal is often considered more difficult than other operations.

a Calculate:

i 200 ÷ 40 **ii** 200 ÷ 4 **iii** 200 ÷ 0.4 **iv** 200 ÷ 0.04

b What pattern do you notice as the divisor becomes smaller?

c What do you notice about the answers when you are dividing by a decimal less than 1?

d Is the following statement true or false?

When dividing by decimals less than 1, the smaller the divisor, the larger the answer.

24 Use your calculator to perform the first division, and then use the result to perform the second division by hand.

a **i** 7 654 394 ÷ 6158 **ii** 0.765 4394 ÷ 6.158

b **i** 4 309 524 297 ÷ 45 721 **ii** 43 095.242 97 ÷ 45.721

25 As part of an awareness-raising campaign for childhood cancer, a gold ribbon is to be cut into lengths and sold to supporters to wear.

a How many pieces of 0.15 m long ribbons can be cut from a 21.7 m spool of ribbon?

b Will any ribbon be left over? If so, how much?

26 Describe how you can decide which bottle of drink shown on the right is the better value for money. Is the 2 L bottle of orange soda or the 1.5 L bottle of cola the better buy?

27 The unitary method is a technique for solving problems by first finding the value of a single item, and then multiplying the value of a single item by the total number of items to find the value of a group of items.

Roland wants to buy 10 avocados. Brianna bought three avocados for \$6.30.

a How much did Brianna pay per avocado?

b Use your answer in part **a** to find how much Roland should pay for 10 avocados.

c Explain how this problem uses both division and multiplication.

Exercise 4.5C: Challenge

28 Ava walks 8 km in 1 hour and 19 minutes, Caleb walks 6 km in 1 hour and 7 minutes, and Isaac walks 10 km in 1 hour and 44 minutes.

a Who is the fastest walker?

b What is this person's pace in kilometres per hour? Give your answer correct to two decimal places.

Online resources:

- **Interactive skillsheet** Dividing decimals
- **Worksheet** Dividing decimals by a whole number
- **Investigation** Best buys – pasta sauce
- **Quick quiz** 4.5

Lesson 4.6

Percentages

Learning intentions

By the end of this lesson you will be able to ...

✔ identify percentages from various representations, and convert numbers from or to a percentage.

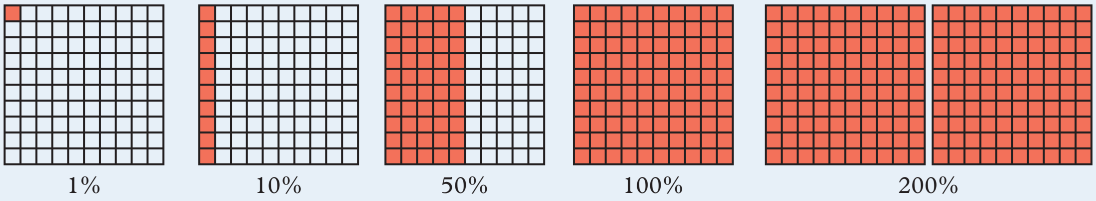
Inter-year links

Support	Fractions, decimals and percentages
Year 8	3.1 Percentages, decimals and fractions

Percentages

Key content video
Understanding percentages

- A **percentage** is a proportion of a whole that has been divided into 100 parts.



- The term '**per cent**' means 'per hundred' or 'out of 100'. The symbol for percentage is %.
  - → 75% also means $\frac{75}{100}$.
  - → 50% also means $\frac{50}{100}$ or exactly half.
  - → 200% also means $\frac{200}{100}$ or exactly 2.
- To write a fraction as a percentage, find an equivalent fraction with a denominator of 100.

  For example, $\frac{9}{25} = \frac{36}{100}$

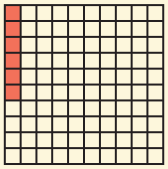
  $= 36\%$.
- To write a decimal as a percentage, multiply the decimal by 100 then add a % symbol.

  For example, 0.65 = 0.65 × 100% = 65%.

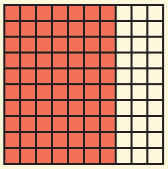
Worked example 4.6A Finding the percentage of a shaded figure

What percentage of these hundred squares has been shaded?

a



b



THINK

1 Count the number of sections that are shaded out of 100.

2 Write the shaded sections as a percentage.

WRITE

a There are 6 squares shaded out of 100.
6%

b There are 70 squares shaded out of 100.
70%

Worked example 4.6B Writing a fraction as a percentage

Write the following fractions as percentages.

a $\frac{19}{100}$ **b** $\frac{3}{2}$

THINK	WRITE
If the denominator of a fraction is not 100, first write an equivalent fraction with a denominator of 100. Once the denominator of the fraction is 100, write the digits of the numerator in front of the % symbol.	**a** $\frac{19}{100} = 19\%$ **b** $\frac{3}{2} = \frac{150}{100}$ $= 150\%$

Worked example 4.6C Writing a decimal as a percentage

Write the following decimals as percentages.

a 0.75 **b** 0.2 **c** 1.175

THINK	WRITE
To convert a decimal to a percentage, multiply by 100%. 'Move' the decimal point two places to the right and insert place-holding zeros in the empty spaces if necessary.	**a** $0.\underset{\smile\smile}{75} \times 100\% = 75\%$ **b** $0.\underset{\smile\smile}{20} \times 100\% = 20\%$ **c** $1.\underset{\smile\smile}{17}5 \times 100\% = 117.5\%$

Worked example 4.6D Writing a percentage as a fraction

Write each percentage as a fraction in its simplest form.

a 48% **b** 125%

THINK	WRITE
1 To change a percentage to a fraction, write the percentage as a fraction with a denominator of 100. **2** Then simplify the fraction.	**a** $48\% = \frac{48}{100}$ $= \frac{\cancel{48}^{12}}{\cancel{100}_{25}}$ $= \frac{12}{25}$ **b** $125\% = \frac{125}{100}$ $= \frac{\cancel{125}^{5}}{\cancel{100}_{4}}$ $= \frac{5}{4}$

Helpful hints

- ✔ Remember that one whole is equal to 100%. If you have a percentage that is greater than 100%, then you have more than one whole.
- ✔ Like decimals, percentages are also another way of writing fractions, although percentages represent a fraction with a denominator of 100.
- ✔ Remember that
  - → 6% refers to 6 hundredths
  - → 60% refers to 60 hundredths
  - → 600% refers to 600 hundredths
- ✔ Percentages are meaningless without context. You should ask yourself, ‘What is this a percentage of?’

Learning pathways

△ 1–3, 4–7(1st, 2nd columns), 8–12, 14, 17

🟨 3, 4–5(2nd, 3rd columns), 6–7(1st, 2nd columns), 9, 11, 13, 15–19

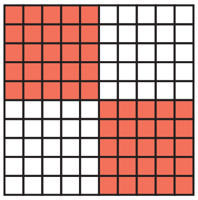
⬠ 3–5(3rd, 4th columns), 6–7(2nd, 3rd columns), 9, 11, 13, 15, 18–20

ANS p521

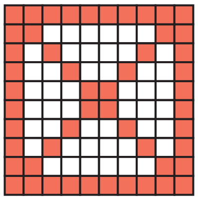
Exercise 4.6A: Understanding and fluency

WE 4.6A **1** What percentage of each square has been shaded?

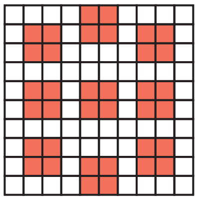
a



b



c



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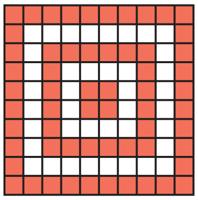
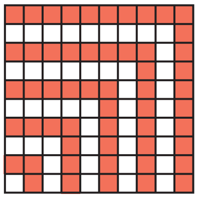
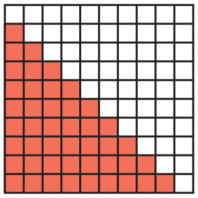


e



f



2 Write the percentage of each square in question **1** that is unshaded.

WE 4.6B **3** Write each fraction as a percentage.

a $\frac{23}{100}$ **b** $\frac{78}{100}$ **c** $\frac{35}{100}$ **d** $\frac{99}{100}$

e $\frac{13}{100}$ **f** $\frac{7}{100}$ **g** $\frac{82}{100}$ **h** $\frac{4}{100}$

4 Write each fraction as a percentage by first converting to an equivalent fraction with a denominator of 100.

a $\frac{9}{20}$ **b** $\frac{11}{50}$ **c** $\frac{7}{4}$ **d** $\frac{17}{25}$

e $1\frac{3}{5}$ **f** $\frac{5}{25}$ **g** $\frac{12}{20}$ **h** $\frac{46}{50}$

i $\frac{60}{50}$ **j** $2\frac{1}{4}$ **k** $\frac{160}{200}$ **l** $\frac{40}{200}$

WE 4.6C **5** Write each decimal as a percentage.

a 0.23
b 0.8
c 0.13
d 0.67
e 0.6
f 1.7
g 2.09
h 3.14
i 0.79
j 1.91
k 0.04
l 0.55

6 Write each percentage as a fraction with a denominator of 100.

a 7%
b 19%
c 43%
d 57%
e 3%
f 63%
g 11%
h 91%
i 79%
j 21%
k 99%
l 33%

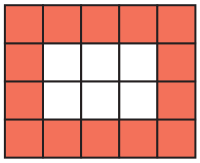
WE 4.6D **7** Write each percentage as a fraction in its simplest form.

a 64%
b 18%
c 27%
d 80%
e 5%
f 95%
g 10%
h 45%
i 60%
j 75%
k 35%
l 88%

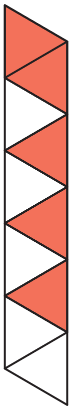
8 Complete these tasks for the shapes below.

i Count the number of sections that are shaded and the total number of sections.
ii Write the proportion that is shaded as a fraction.
iii Find an equivalent fraction that has a denominator of 100.
iv Write this as a percentage.
v Write this as a decimal.

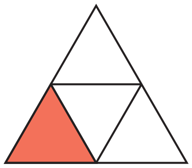
a



b



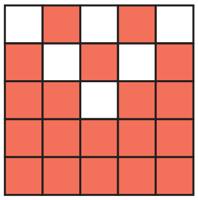
c



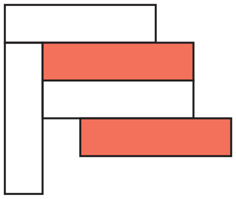
9 Complete these tasks for the shapes below.

i What percentage of each figure has been shaded?
ii Write your percentage as a decimal.

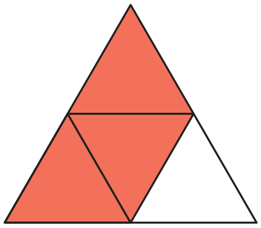
a



b



c



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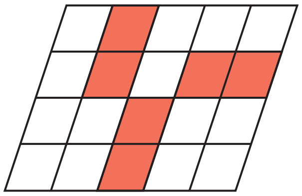
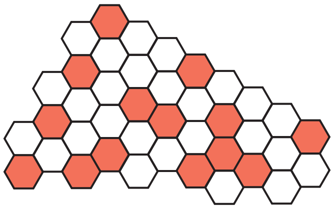
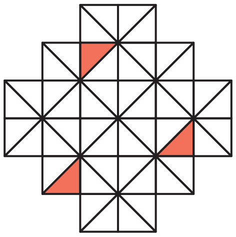


e



f



Exercise 4.6B: Problem solving and reasoning

10 Match each jug with the percentage to which it has been filled.

a

b

c

d

A 20% **B** 5% **C** 85% **D** 45%

11 Write a percentage to describe each situation by first finding an equivalent fraction that has a denominator of 100.

a Four students out of 10 have blonde hair.

b Eighteen people out of 20 come to a party.

c Two people out of 50 miss the train.

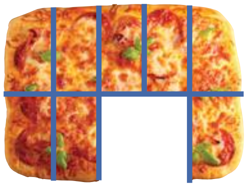
d One person out of five walks to school.

12 a Write the percentage eaten from each of the three pizzas (assume all pieces are equally-sized).

b Write the portion of pizza not eaten from each tray as a percentage.

c Write a brief explanation of how you calculated the percentages in part **a**.

i



ii

iii

13 Write, as a simplified ratio, the amount consumed to the amount remaining.

a Gerard drinks 60% of a box of 30 cans of lemonade.

b Jayne reads 75% of a series of 12 books.

c Jordan eats 45% of a muesli bar.

d Robert watches 18% of a movie.

14 While downloading songs to his phone, Geoff noticed a horizontal bar on the screen. The bar's shading changed according to what percentage of the song had downloaded.

a Draw a rectangle 10 cm long and 1.5 cm wide to represent the horizontal bar.

b Consider your answer to part **a**.

- **i** Why do you think you were instructed to draw the rectangle 10 cm long?
- **ii** What will each 1 cm represent along the rectangle?
- **iii** What will each 1 mm represent along the rectangle?

c Using the horizontal bar from part **a**, show how the bar would look when 15% of a song has been downloaded.

d Draw four rectangles with the same dimensions as part **a** to show these percentages.

i 85% **ii** 38% **iii** 23% **iv** 96%

15 Find the percentage that remains in each situation. Remember to add the portions of the ratio together to determine the whole.

a The ratio of homework problems Renee completes to the problems Renee does not complete is 6 : 4.

b The ratio of M&M's Elise eats to the M&M's Elise does not eat is 35 : 15.

c The ratio of levels on a game Xavier completes to the levels Xavier does not complete is 7 : 13.

d The ratio of spare batteries Peter uses to spare batteries Peter does not use is 1 : 3.

16 Reuben is having difficulty understanding how the two containers shown are both 50% full, yet the volume of liquid in each is different. Explain the relationship between the volume and the percentage of liquid in the containers.

17 Percentages can be more than 100%. This can only happen when it is possible to have more than a whole.

- **a** If 200% means double the whole amount, what does 300% mean?
- **b** To write 100% as a number, first write it as a fraction: $100\% = \frac{100}{100} = 1$.
  Use this method to write each percentage as a number.
  - **i** 200%
  - **ii** 350%
  - **iii** 491%
  - **iv** 375.4%

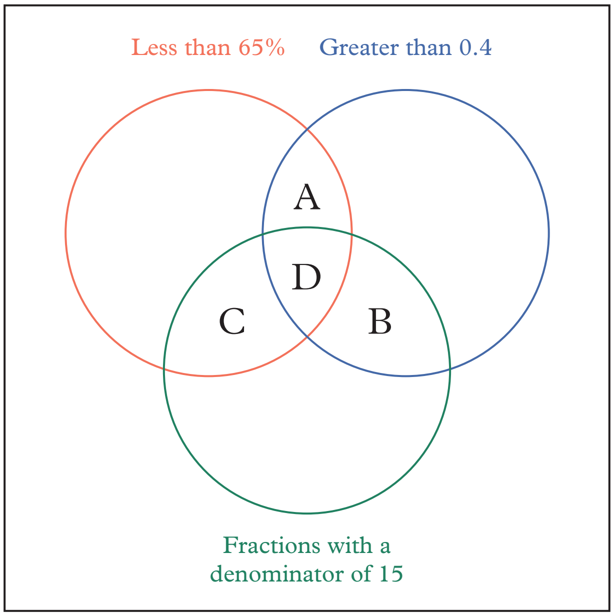
18 A hamburger from Greasy Joe's has 450% of the daily recommended fat intake.

- **a** Write this percentage as a number.
- **b** If you decide to get chips with the burger, the meal would have 5.25 times the amount of the daily recommended fat. Write this amount as a percentage.

Exercise 4.6C: Challenge

19 Lauren fills her glass of water to the brim. She empties out 20% of the water. She decides there's now too little water and adds 20% of the amount of water that is in the glass. Explain why the glass is not filled to the brim.

20 Below is a Venn diagram. Where two circles overlap, we consider the statement to be true for both groups.



- **a** Determine if there are any values that can appear in A. If so, state or describe the values in the region.
- **b** Determine if there are any values that can appear in B. If so, state or describe the values in the region.
- **c** Determine if there are any values that can appear in C. If so, state or describe the values in the region.
- **d** Determine if there are any values that can appear in D. If so, state or describe the values in the region.

Online resources:

Interactive skillsheet Percentages

Worksheet Writing percentages

Investigation Percentages with punch

Quick quiz 4.6

Lesson 4.7

Fractions, decimals and percentages

Learning intentions

By the end of this lesson you will be able to …

✔ convert between fractions, decimals and percentages and use the appropriate representation to solve problems.

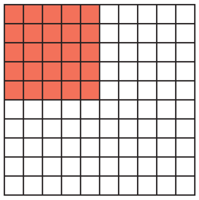
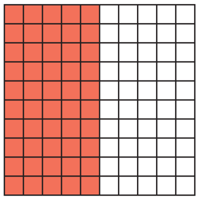
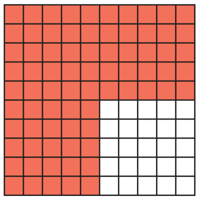
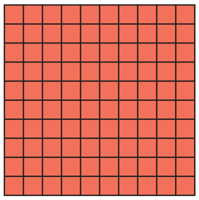
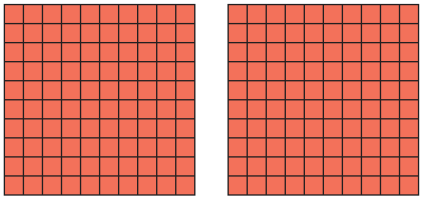
Inter-year links

Support	Fractions, decimals and percentages
Year 8	3.1 Percentages, decimals and fractions

Fractions, decimals and percentages

Key content video
Fractions, decimals and percentages

- Fractions, decimals and percentages are different ways of representing the same value.

Percentage	Fraction	Decimal	Diagram
25%	$\frac{25}{100} = \frac{1}{4}$	0.25	
50%	$\frac{50}{100} = \frac{1}{2}$	0.5	
75%	$\frac{75}{100} = \frac{3}{4}$	0.75	
100%	$\frac{100}{100} = 1$	1.0	
200%	$\frac{200}{100} = 2$	2.0	

Converting between fractions, decimals and percentages

	Fraction	**Decimal**
Percentage to …	Write the percentage as a fraction with a denominator of 100. $15\% = \frac{15}{100} = \frac{3}{20}$	Divide the in front of the percentage symbol by 100. $23.4\% = \frac{23.4}{100} = 0.234$

	Percentage	**Decimal**
Fraction to …	Write the fraction as an equivalent fraction with a denominator of 100. $\frac{7}{25} \overset{\times 4}{\underset{\times 4}{=}} \frac{28}{100} = 28\%$	Divide the numerator by the denominator. Add trailing zeros as required. Align decimal points $\begin{array}{r} 0.1\,2\,5 \\ \text{Dividend} \rightarrow \\ \text{Divisor} \rightarrow \end{array} \frac{1}{8} = 8\overline{)1.\underbrace{0^{2}0^{4}0}_{\text{Trailing zeros}}}$

	Percentage	**Fraction**
Decimal to …	Multiply the decimal by 100%. $0.\underset{\smile\smile}{75} \times 100\% = 75\%$	Place the decimal fraction as the numerator with a denominator of 10, 100, 1000, … with as many zeros as there are digits after the decimal point. $0.65 = \frac{65}{100} = \frac{13}{20}$

Worked example 4.7A Converting from a percentage

Fill in the blanks of the following table.

Percentage	Fraction	Decimal
40%		

THINK

1 To convert a percentage to a fraction, write the percentage as a fraction with a denominator of 100. Simplify the fraction.

2 To convert to a decimal, divide the number in front of % by 100.

WRITE

Percentage	Fraction	Decimal
40%	$\frac{\cancel{40}^{2}}{\cancel{100}^{5}} = \frac{2}{5}$	$\underset{\smile\smile}{40}\% = 0.4$

Worked example 4.7B Converting from a decimal

Fill in the blanks of the following table.

Percentage	Fraction	Decimal
		0.86

THINK

1 To convert a decimal to a percentage, multiply by 100%.

2 To convert a decimal to a fraction, place the decimal fraction part of the number as the numerator of the fraction. Make the denominator 10, 100, 1000, … with as many zeros as there are digits after the decimal point in the decimal fraction part of the number. Simplify the fraction.

WRITE

Percentage	Fraction	Decimal
$0.\underset{\smile}{86} = 86\%$	$\frac{\cancel{86}^{43}}{\cancel{100}^{50}} = \frac{43}{50}$	0.86

Worked example 4.7C Converting from a fraction

Fill in the blanks of the following table.

Percentage	Fraction	Decimal
	$\frac{1}{25}$	

THINK

1 Start by finding an equivalent fraction with a denominator of 100.

2 To convert a fraction to a percentage, use the equivalent fraction with a denominator of 100, and write the numerator as the percentage.

3 To convert to a decimal, use the equivalent fraction $\frac{4}{100}$, which is equal to 4 hundredths.

WRITE

Percentage	Fraction	Decimal
$\frac{1}{25} = \frac{1}{25} \times \frac{4}{4}$ $= \frac{4}{100}$ $= 4\%$	$\frac{1}{25}$	$\frac{4}{100} = 0.04$

Helpful hints

- ✔ Don't forget to simplify your fractions.
- ✔ Remember that it is easier to convert from a fraction to a decimal when the fraction has a denominator of 100.
- ✔ Remember that 6% is equal to '6 out of one hundred', which is '6 hundredths' and is equal to 0.06.

Learning pathways

△ 1–8, 10, 12

■ 4–7, 9–11, 13–14

⬟ 5–7, 9, 11, 13–15

ANS p523

Exercise 4.7A: Understanding and fluency

1 Complete the table to show the equivalent forms of each amount.

Percentage	Fraction	Decimal
	$\frac{1}{2}$	
		0.25
75%		
	$\frac{3}{10}$	
		0.125
62.5%		
	$\frac{1}{5}$	
		0.4
60%		

WE 4.7A **2** Write each percentage as a fraction and a decimal. Simplify your fractions where possible.

a 46% **b** 13% **c** 99%

d 25% **e** 20% **f** 50%

g 5% **h** 8% **i** 1%

WE 4.7B **3** Write each decimal as a percentage and a fraction. Simplify your fractions where possible.

a 0.28 **b** 0.88 **c** 0.15

d 0.62 **e** 0.45 **f** 0.72

g 0.09 **h** 0.04 **i** 0.3

WE 4.7C **4** Write each fraction as a percentage and a decimal.

a $\frac{51}{50}$ **b** $\frac{17}{20}$ **c** $\frac{14}{5}$

d $\frac{9}{4}$ **e** $\frac{13}{25}$ **f** $\frac{21}{2}$

g $\frac{405}{1000}$ **h** $\frac{83}{20}$ **i** $\frac{7121}{100}$

5 Write each percentage as a decimal and a fraction. Simplify your fractions where possible.

For example, $12.3\% = 0.123 = \frac{123}{1000}$.

a 23.84% **b** 19.65% **c** 46.7% **d** 3.09%

e 567.4% **f** 0.467% **g** 12.895% **h** 73.28%

i 200.5%

6 Write each decimal as a percentage and a fraction. Simplify your fractions where possible.

a 0.518 **b** 9.02 **c** 0.105 **d** 0.7

e 21.5 **f** 8.41 **g** 7.1 **h** 0.009

i 0.1765

7 Write each fraction as a percentage and a decimal. Remember you can use division to convert a fraction to a decimal.

For example, $\frac{2}{3} = 3\overline{)2.00}$

a $\frac{3}{8}$ **b** $\frac{3}{40}$ **c** $\frac{3}{80}$

d $\frac{3}{32}$ **e** $\frac{17}{8}$ **f** $\frac{91}{40}$

g $9\frac{37}{80}$ **h** $11\frac{29}{32}$ **i** $\frac{9}{80}$

Exercise 4.7B: Problem solving and reasoning

8 Eclectus parrots are found in north-eastern Australia. The male is green and the female is red and blue.

a Write the number of male parrots in the photo on the right as a fraction of the total number of parrots.

b What percentage of the group is:

  i male? **ii** female?

c Write each answer to part **b** as a decimal.

9 Converting a fraction to a percentage can also be done by finding the fraction of 100%.

For example, $\frac{3}{5} = \frac{3}{5}$ of $100\% = \frac{3}{5} \times 100\% = \frac{300}{5}\% = 60\%$.

Use this method to convert the following fractions to percentages.

a $\frac{17}{25}$ **b** $\frac{21}{10}$ **c** $\frac{43}{20}$ **d** $\frac{7}{8}$

10 Consider the following numbers:

109%	$\frac{38}{200}$	$\frac{38}{20}$	38%	3.8
$1\frac{9}{10}$	19%	1.9	1.09	190%
0.38	20%	$\frac{19}{95}$	$\frac{19}{5}$	0.2

a Determine which values are equivalent.

b For each value in the table, there should be an equivalent fraction, decimal and percentage. Identify the values that are missing a fraction, decimal or percentage representation, and write the value in that form.

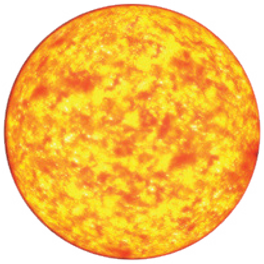
c Put the values in descending order using their decimal forms.

d Use appropriate equivalent values from parts **a** and **b** to perform the following calculations more efficiently.

  i $\frac{25}{19} \times 3.8$ **ii** $\frac{19}{95} \times 100$ **iii** $\frac{1}{19} \times 38\%$

11 The elements that make up the Sun are listed in the table below.

Element	Percentage of total mass
H (hydrogen)	71.0%
He (helium)	27.1%
O (oxygen)	0.97%
C (carbon)	0.40%
Si (silicon)	0.099%
N (nitrogen)	0.096%
Mg (magnesium)	0.076%
Ne (neon)	0.058%
S (sulphur)	0.040%
Fe (iron)	0.014%



a Write the percentage of hydrogen as a fraction and a decimal.

b If Fe and Mg are the only metals, find how much of the Sun is made of metal and write it as a:

i percentage **ii** decimal **iii** fraction.

c What do the percentages of the listed elements add to? Why do you think this might be?

12 Eliza has cycled 496 km of the gruelling 620 km Great Victorian Bike Ride. What percentage of the ride does she still have to complete?

13 Laura and Rosie were comparing their working of a particular problem. They were required to convert $17\frac{1}{8}\%$ to a decimal. Their setting out is slightly different; however, both methods produce the same result. Comment on whether the answer and setting out of each is correct.

Laura

$$\begin{aligned} 17\tfrac{1}{8}\% &= 17.125\% \\ &= \frac{17.125}{100} \\ &= \frac{17.125}{100} \times \frac{1000}{1000} \\ &= \frac{17\,125}{100\,000} \\ &= 0.171\,25 \end{aligned}$$

Rosie

$$\begin{aligned} 17\tfrac{1}{8}\% &= 17.125\% \\ &= \frac{17.125}{100} \\ &= 17.125 \div 100 \\ &= 0.171\,25 \end{aligned}$$

Exercise 4.7C: Challenge

14 What are the possible denominators, less than 100, of a simplified fraction such that the decimal and percentage forms have a fixed number of decimal places?

For example, the fraction $\frac{1}{2}$ is equivalent to 0.5 and 50%. They have a fixed number of decimal places. But $\frac{1}{3}$ is equivalent to $0.3\dot{3}3...$ and 33.33...% where both have a recurring decimal of 3.

15 Without using the same digit more than once, fill in the spaces to make a correct statement.

$$\frac{\square\square}{\square\square} = \square\square.\square\,\%$$

Online resources:

Interactive skillsheet Fractions, decimals and percentages

Worksheet Numbers as fractions, decimals and percentages

Investigation Dominations

Quick quiz 4.7

Lesson 4.8

Calculating percentages

Learning intentions

By the end of this lesson you will be able to …

✔ express one quantity as a percentage of another and find quantities using percentages.

Inter-year links

Support	Calculating percentages
Year 8	3.2 Calculating percentages
Year 9	9.2 Other forms of income

Expressing a quantity as a percentage of another

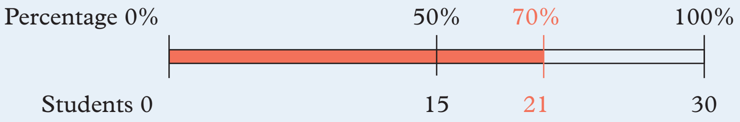
Key content video
Calculating percentages

- Percentages can be used in a social context to compare numbers and to represent fractions and ratios.
- To express one quantity as a percentage of another, write the equivalent fraction out of 100.
  For example, in a class of 30 students, 21 have a sister.

$$\frac{21}{30} \overset{\div 3}{\underset{\div 3}{=}} \frac{7}{10}$$

$$= \frac{70}{100}$$

$$= 70\%$$

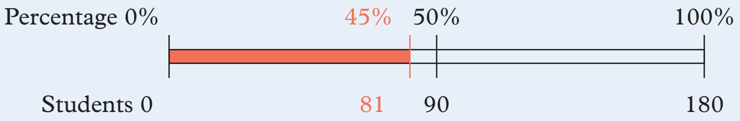


Percentage of a quantity

- To calculate a percentage of a quantity, multiply the percentage by the quantity.
  → Recall that when asking for a fraction or percentage 'of' an amount, replace the 'of' with the multiplication symbol, '×'.
- For example, 45% of 180 students like ice-cream more than frozen yoghurt.

$$45\% \text{ of } 180 = \frac{45}{100} \times \frac{180}{1}$$

$$= 81 \text{ students}$$



Worked example 4.8A Expressing a quantity as a percentage of another

Express 60 as a percentage of 150.

THINK	WRITE
1 Express the amount as a fraction of the total and simplify.	$\frac{60}{150} \overset{\div 30}{\underset{\div 30}{=}} \frac{2}{5}$
2 To convert a fraction to a percentage, write the fraction with a denominator of 100.	$= \frac{40}{100}$ $= 40\%$
3 Write your answer.	60 is 40% of 150.

Worked example 4.8B Calculating a percentage of a quantity

Calculate 12% of \$160.

THINK

1 Multiply the percentage by the quantity.

2 Convert the percentage to a decimal or fraction to make the calculation easier.

WRITE

$$\begin{aligned}12\% \text{ of } \$160 &= 0.12 \times \$160\\ &= \$19.20\end{aligned}$$

Helpful hints

✔ You can use fractions or decimals to find the percentage of a quantity. Both methods will give you the same answer.
For example, 21 out of 30 as a percentage is $\frac{21}{30}$ of 100%

$$\begin{aligned}&= \frac{21}{\cancel{30}^{3}} \times \frac{\cancel{100}^{10}}{1}\%\\ &= \frac{\cancel{21}^{7}}{\cancel{3}^{1}} \times \frac{10}{1}\%\\ &= 7 \times 10\%\\ &= 70\%\end{aligned}$$

✔ You can use the commutative law of multiplication to help you calculate percentages.
For example, to find 4% of 75, you could do the following:

$$\frac{4}{100} \times 75 = \frac{75}{100} \times 4 = \frac{3}{4} \times 4 = 3$$

Learning pathways

△ 1–7, 9, 10, 12, 13, 15, 18

□ 1–8, 11–18

⬠ 1–5(2nd column), 7, 11–19

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Exercise 4.8A: Understanding and fluency

WE 4.8A **1** Express the first number as a percentage of the second.

a 15 out of 20
b 25 out of 20
c 3 out of 5
d 3 out of 6
e 100 out of 50
f 90 out of 16
g 37 out of 40
h 7 out of 8

2 Calculate the percentage of each quantity.

a 5% of 3900
b 10% of 120
c 20% of 520
d 50% of 80
e 15% of 50
f 90% of 20
g 60% of 65
h 75% of 16

WE 4.8B **3** Calculate the percentage of each amount.

a 12% of \$40
b 17% of \$450
c 11% of \$500
d 15% of \$1326
e 8% of \$8750
f 9% of \$1870
g 22% of \$6750
h 25% of \$35.80

4 Calculate each amount. Write your answers as decimals.

a 2.5% of 180
b 5.5% of 70
c 75.8% of 120
d 20.4% of 150
e 42.8% of 65
f 17.5% of 1650
g 5.3% of 960
h 16.8% of 150
i 85.2% of 95
j 17.6% of 120
k 12.5% of 99
l 10.2% of 78

5 Calculate the percentage of each amount.

a 110% of 40
b 105% of 70
c 300% of 15.2
d 250% of 160
e 120% of 8.9
f 460% of 98
g 165% of 90
h 220% of 140

6 Look at the percentage problems and your answers to question **5**.

a What do you notice about each percentage compared to those in question **4**?

b Compare each answer to the corresponding original amount.

Exercise 4.8B: Problem solving and reasoning

7 Meredith is impressed at how quickly her mother can mentally calculate percentages. One of her mother's strategies is shown below.

> To obtain 10%, divide the amount by 10.
> This is because 10% can be written as $\frac{1}{10}$.
> So 10% of 72 is 7.2 (since 72 ÷ 10 = 7.2).

You can extend this idea to find other values. For example, to find 20% of an amount, find 10% and then double the value. Use this logic to copy and complete these sentences.

a To find 5% of an amount, first obtain 10% and then ________ the value.

b To find 30% of an amount, first obtain 10% and then ________ the value.

c To find 40% of an amount, first obtain 10% and then ________ the value.

8 A survey showed 92% of commuters were not satisfied with the public transport system.

a If 6500 commuters took part in the survey, how many people were not satisfied with the public transport system?

b How many of the 6500 commuters were satisfied with the system?

9 Lachlan scored 18 out of 25 on his first test and 23 out of 30 for his next test.

a Calculate what percentage he scored for his first test.

b Calculate what percentage he scored for his second test.

c Which test did Lachlan perform better in? Explain your answer.

10 Wally, Artemis and Megan plan to share a bag of lollies. Wally claims 55%, Artemis claims $\frac{1}{4}$, and Megan takes the rest.

a What proportion of the lollies does Megan get (·) as a decimal?

b How many lollies do each get if there are 460 lollies in the bag?

c Write the following ratios:

  i The lollies Wally receives to the lollies he does not receive.
  ii The lollies Wally receives to the lollies Artemis receives.
  iii The lollies Artemis receives to the lollies Megan receives.

11 The ratio 8 : 5 has some parts either added or subtracted so that the ratio becomes 1 : 1 after simplifying. (As a result of the addition and subtraction to the parts, some parts have been added or subtracted from the total as well.)

- **a** If the first part of the ratio is altered, what percentage of the original first part is the new first part?
- **b** If the second part of the ratio is altered, what percentage of the original second part is the new second part?

12 In business, the cost price is the cost paid to create or obtain a product, while the selling price is the price that the seller sells the product for. A profit is made when the selling price is greater than the cost price, while a loss is made when the cost price is greater than the selling price. To calculate profit and loss, we use the following formulas:

Profit = selling price − cost price   Loss = cost price − selling price

For each of the following situations:

- **i** state if a profit or loss has been made
- **ii** calculate the profit or loss.
  - **a** A chair that cost $50 is placed on eBay and sold for $87.
  - **b** A sandwich maker that was originally priced at $59 is sold for $45.
  - **c** A camera is bought for $935 and sold for $599.
  - **d** A car purchased for $28 899 is sold for $17 580.
  - **e** A supermarket buys milk at $0.80 per litre and sells it for $2.24 per litre.
  - **f** A toy bought at $8.75 is sold for $2.60.
  - **g** Scarves bought at $5.98 each are sold for $13.50 each.

13 Profit and loss can also be expressed as a percentage of the cost price, using the following formulas:

$$\text{Percentage profit} = \frac{\text{profit}}{\text{cost price}} \qquad \text{Percentage loss} = \frac{\text{loss}}{\text{cost price}}$$

Find the percentage profit or loss, correct to two decimal places, for each situation in question **12**. Remember to express each answer as a percentage.

14 Daphne purchased 40 picture frames for her store at $8.50 each. She plans to sell them for $12.50.

- **a** What is the expected profit on each frame?
- **b** 15% of the frames are damaged and so Daphne reduces the selling price of these to 50% of the planned selling price. How many frames are damaged and what is the new selling price for these frames?
- **c** Does Daphne make a profit or loss on the damaged frames? Calculate how much the profit or loss is.
- **d** If all 40 frames are sold, calculate the total profit she has made.
- **e** Find the percentage profit she has made.
- **f** Compare the value you got in part **e** with the percentage profit Daphne would have made if none of the frames were damaged.

15 Thomas and Ivo compare the increase in value of two properties to decide which made the greatest percentage profit. Thomas says the first property has the greatest percentage profit, but Ivo thinks the second has. Who is correct?

Property 1: valued at $695 000, sold for $963 000.

Property 2: valued at $1 200 000, sold for $1 600 000.

16 If we know the amount a percentage is equal to, we can work out the total amount.

For example, 12 is 30% of what number?

$$\begin{aligned}30\% &= \frac{3}{10}\\ &= \frac{3\times 4}{10\times 4}\\ &= \frac{12}{40}\end{aligned}$$

so 12 is 30% of 40.

a 36 is 50% of what number?

b 162 is 90% of what number?

c 126 is 210% of what number?

d 333 is 37% of what number?

Exercise 4.8C: Challenge

17 We can find a percentage of a percentage in the same way as we find a fraction of a fraction. For example, a half of a half is a quarter: 50% of 50% is 25%.

a Calculate 75% of 250% of 120% of 95% of 80.

b Calculate what percentage the answer from part **a** is out of 80.

18 A rectangle has its length reduced to 80% of its original length and its width reduced to 40% of its original width. Determine what percentage the area of the reduced rectangle is of the area of the original rectangle.

19 A demolition company is tasked with removing a skyscraper on a tight deadline. The company claims they can remove 17% of the building's original height each day.

a If the building is 247 m tall, how tall will the building be after the first day?

b How long will it take to remove the entire building?

c After an investigation, the company was found to be guilty of false claims. Instead of removing 17% of the building's original height per day, the company actually intended to remove 17% of the remaining height each day.

  i Under this new claim, will the company remove the building in more or less time than their original claim?

  ii How long will it take to remove the building according to the new claim?

d The company updates their claim to reflect the investigation, but adds a line claiming they will remove the remaining height of the building in one day if the height is less than 20 m. How long will it take to remove the building under the company's new claim?

Online resources:

Interactive skillsheet
Calculating percentages

Worksheet
Calculating the percentage of a quantity

Investigation
Calculating the football ladder

Quick quiz
4.8

Lesson 4.9

Review: Decimals and percentages

Module summary

Place value chart

Tens	Ones	.	Tenths	Hundredths	Thousandths	Ten-thousandths
10	1		$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$	$\frac{1}{10\,000}$

Adding and subtracting decimals

- When adding or subtracting decimals, the digits must be positioned in their correct place value columns and the decimal points must be lined up.

$$\begin{array}{r} 3.02 \\ +\ 4.15 \\ \hline 7.17 \\ \hline \end{array}$$

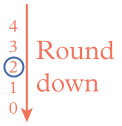
- Blank spaces in the columns may be replaced with additional trailing zeros to assist working out.

$$\begin{array}{r} 3.200 \\ +\ 3.002 \\ \hline 6.202 \\ \hline \end{array}$$

Rounding

$1.2\boxed{3}95 \approx 1.24$

$1.2\boxed{3}25 \approx 1.23$



Multiplying by 10s

$5.23 \times \underline{10} = 52.3$

$5.23 \times \underline{100} = 523.0$

$5.23 \times \underline{1000} = 5230.0$

Dividing by 10s

$5.23 \div \underline{10} = 0.523$

$5.23 \div \underline{100} = 0.0523$

$5.23 \div \underline{1000} = 0.00523$

Multiplying decimals

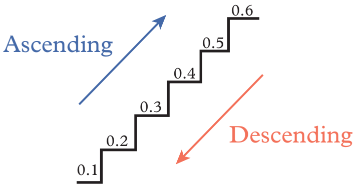
1. Count the decimal places in the problem.
2. Multiply the decimals as whole numbers.
3. Write the answer with this same number of decimal places. Add place-holding zeros where necessary.

$2.\underline{5}$ ← one decimal place
$\times 0.\underline{03}$ ← two decimal places
= three decimal places

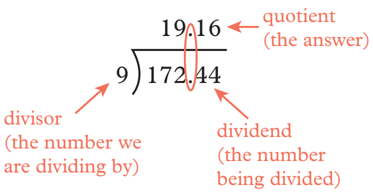
$$\begin{array}{r} \scriptstyle 1 \\ 25 \\ \times\ \ 3 \\ \hline 75 \end{array}$$

$2.5 \times 0.03 = 0.075$

Ordering



Dividing by a whole number



Dividing by a decimal

1. Convert the divisor to a whole number by moving the decimal point to the right.
2. Move the decimal point the same number of spaces to the right on the dividend.
3. Perform short division and add trailing zeros where necessary.

$$\overset{\text{Dividend}}{1.2} \div \overset{\text{Divisor}}{0.4} = 12 \div 4 = 3$$

Converting between fractions, decimals and percentages

	Fraction	Decimal
Percentage to …	Write the percentage as a fraction with a denominator of 100. $15\% = \frac{15}{100} = \frac{3}{20}$	Divide the number in front of the % symbol by 100 and remove the % symbol. $23.4\% = \frac{23.4}{100} = 0.234$

	Percentage	Decimal
Fraction to …	Write the fraction as an equivalent fraction with a denominator of 100. $\frac{7}{25} \overset{\times 4}{=} \frac{28}{100} = 28\%$	Divide the numerator by the denominator. Add trailing zeros as required. Dividend → 1, Divisor → 8 $\frac{1}{8} = \begin{array}{r} 0.1\,2\,5 \\ 8\overline{)1.0^{2}0^{4}0} \end{array}$ Trailing zeros

	Percentage	Fraction
Decimal to …	Multiply the decimal by 100%. $0.75 \times 100\% = 75\%$	Place the numbers after the decimal point in the numerator of the fraction and the denominator is 10, 100, 1000 … with as many zeros as there are digits after the decimal point. $0.65 = \frac{65}{100} = \frac{13}{20}$

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Review questions 4.9A: Mathematical literacy review

Review quiz
Take the review quiz to assess your knowledge of this module.

Quizlet
Test your knowledge of this module by working individually or in teams.

The following key terms are used in this module:

- accuracy
- ascending order
- decimal
- decimal fraction
- decimal place
- decimal point
- descending order
- digit
- dividend
- divisor
- fraction
- per cent
- percentage
- place-holding zeros
- place value
- product
- quotient
- remainder
- rounding
- trailing zeros
- whole number

1 Explain the difference between putting a list of decimals in ascending order and putting them in descending order.

2 When performing a division, what is the dividend and what is the divisor?

3 Identify the key terms being referenced in each of these definitions.
 a replacing a number with an approximation that is simpler and easier to use in calculations
 b a proportion of a whole divided into one hundred parts

4 Using an example, provide a definition in your own words for the following key terms.
 a decimal fraction
 b trailing zeros

5 Complete the following sentences using words from the key terms list.
 a The term ________ means ‘per hundred’ or ‘out of 100’.
 b ________ describes the closeness of a measurement to its exact value.

Review questions 4.9B: Multiple choice

4.1 **1** What is $\frac{1}{5}$ written as a decimal?
A 0.2 **B** 0.5 **C** 1.5 **D** 0.15

4.1 **2** What is 0.26 as a fraction in its simplest form?
A $\frac{13}{50}$ **B** $\frac{2}{6}$ **C** $\frac{26}{100}$ **D** 26%

4.1 **3** Which of the following does not equal 3.71?
A $3+\frac{7}{10}+\frac{1}{100}$ **B** $\frac{37}{10}+\frac{1}{100}$ **C** $\frac{371}{1000}$ **D** $\frac{301}{100}+\frac{700}{1000}$

4.2 **4** What is 13.054 8726 rounded to three decimal places?
A 13.0 **B** 13.054 **C** 13.055 **D** 13.0549

4.2 **5** Which statement is false?
A $12.345 > 12.309$
B $0.0006 > 0.000\,03$
C $48.2189 < 48.950$
D $183.506 < 183.099$

4.3 **6** Which has the smallest answer?
A $1.568 + 6.238 - 4.22$
B $1.456 + 0.853 + 1.0899$
C $10.9856 - 7.53 + 2.2$
D $1.451 + 1.254 + 0.684$

4.4 **7** What is 6.04×2.1?
A 1.2684 **B** 12.684 **C** 126.84 **D** 1268.4

4.5 **8** What is 8.21 ÷ 4?

A 2.0525 **B** 2.21 **C** 4.21 **D** 32.84

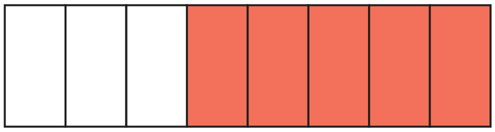
4.5 **9** If 10 apples cost $9.85, how much would six apples cost?

A $0.985 **B** $1.64 **C** $5.91 **D** $16.42

4.6 **10** Which has the largest answer?

A 23 ÷ 0.4 **B** 48 ÷ 1.2 **C** 60.5 ÷ 1.1 **D** 15 ÷ 0.3

4.6 **11** What percentage of this figure is shaded?



A $\frac{5}{8}\%$ **B** 5% **C** 50% **D** 62.5%

4.6 **12** What is 48% as a fraction in its simplest form?

A $\frac{48}{100}$ **B** $\frac{24}{50}$ **C** 0.48 **D** $\frac{12}{25}$

4.6 **13** What is $\frac{9}{20}$ written as a percentage?

A 9% **B** $\frac{9}{20}\%$ **C** 45% **D** 180%

4.7 **14** The fraction $\frac{7}{8}$ can also be written as:

A 78% **B** 7.8 **C** 0.75 **D** 87.5%

4.7 **15** What is 64.5% written as a decimal?

A 0.645 **B** 64.5 **C** 0.0645 **D** 64 500

4.7 **16** What is 0.1265 written as a percentage?

A 0.1265% **B** 12.65% **C** 1265% **D** 126 500%

4.8 **17** What is 25% of 220?

A 5500 **B** 245 **C** 195 **D** 55

Review questions 4.9C: Short answer

4.1 **1** What is the place value of the digit 7 in each number?

a 10.578 **b** 2.075 **c** 0.000 517 **d** 16.794

4.1 **2** Write each fraction as a decimal.

a $\frac{87}{100}$ **b** $\frac{117}{1000}$ **c** $\frac{3}{10}$ **d** $\frac{7}{100}$

4.1 **3** Write each fraction as a decimal.

a $\frac{16}{25}$ **b** $\frac{19}{50}$ **c** $\frac{15}{20}$ **d** $\frac{3}{8}$

4.2 **4** For each number:

i write the number of decimal places

ii round to two decimal places.

a 1589.231 57 **b** 12.709 66 **c** 0.9286 **d** 152.875 964 3

4.2 **5** Place this list in descending order.

12.5089, 12.8905, 18.0958, 12.0985, 10.9852

4.3 **6** Calculate:

a 23.25 + 84.15 + 13.06 **b** 1.025 + 3.508 + 4.999

c 14.562 – 12.869 **d** 5.239 – 2.506 + 1.5555

4.4 **7** Calculate:

a 12.63 × 100 **b** 15.3298 × 10 **c** 4.2 × 50 **d** 3.047 × 200

4.4 **8** Calculate:

a 0.7 × 0.3 **b** 0.9 × 0.04 **c** 1.1 × 0.08 **d** 0.08 × 0.04

4.4 **9** Calculate:

a 25.8 × 0.1 **b** 46.8 × 0.01 **c** 6.5 × 7.2 **d** 4.23 × 1.9

4.5 **10** Calculate:

a 78.94 ÷ 10 **b** 1256.359 ÷ 100 **c** 124.56 ÷ 10 **d** 189 765.6 ÷ 1000

4.5 **11** Calculate:

a 18.654 ÷ 2 **b** 94.56 ÷ 6 **c** 148.63 ÷ 5 **d** 327.45 ÷ 8

4.5 **12** Evie considers two packs of cheese: 500 g for $7.80 and 700 g for $12.60.

a Calculate the value of 100 g of each product.

b Which product is the best buy? Explain.

4.5 **13** Calculate:

a 6.2 ÷ 0.5 **b** 0.75 ÷ 0.2 **c** 8.62 ÷ 0.4 **d** 0.0081 ÷ 0.05

4.6 **14** Write each percentage as a fraction:

i with a denominator of 100

ii in its simplest form.

a 49% **b** 22% **c** 75% **d** 5%

4.6 **15** What percentage of a pie remains if Caleb eats 23% and Hogan eats 31%?

4.7 **16** Write each decimal as a fraction in its simplest form.

a 0.75 **b** 0.18 **c** 0.489 **d** 0.003

4.7 **17** Write each decimal as a percentage.

a 0.56 **b** 0.08 **c** 1.79 **d** 0.2748

4.7 **18** Write each percentage as a decimal.

a 78% **b** 46.5% **c** 9% **d** 10.679%

4.8 **19** Write each number as a percentage.

a $\frac{3}{5}$ **b** $\frac{7}{8}$ **c** 1 **d** $2\frac{3}{4}$

4.8 **20** Calculate each amount.

a 10% of 450 **b** 30% of 500 **c** 45% of 300

d 85% of 650 **e** 6% of 1499 **f** 41% of 347

4.8 **21** Calculate each amount.

a 272% of 550 km **b** 34.75% of 950 g **c** 5.8% of $120 **d** 16.2% of 450 L

Review questions 4.9D: Mathematical modelling

1 Alicia's birthday is coming soon.

a Rob, Carina, Belinda and Mario want to buy Alicia a bunch of white roses.

i Rob has $10.35, Carina has $8.25, Belinda has $9.90 and Mario has $10.80. How much money do they have altogether?

ii Their local florist, Fantastic Flowers, has white roses for $2.99 each. How many white roses can they buy?

b Fantastic Flowers also sells red roses. The cost of each white rose is $1.75 and the cost of each red rose is $2.10.

i If they continue to sell the white roses for $2.99 each, what is the percentage profit or loss that Fantastic Flowers will make on each white rose sold, rounded to one decimal place?

ii If they sell each red rose for $4.29, what is the percentage profit or loss that they make on each red rose sold, rounded to one decimal place?

c Rob, Carina, Belinda and Mario decide to buy a mixture of red and white roses for Alicia.
  - **i** Rob and Carina only buy white roses. How many white roses can they buy altogether?
  - **ii** How much money do Rob and Carina have left over?
  - **iii** Belinda and Mario only buy red roses. How many red roses can they buy altogether?
  - **iv** How much money do Belinda and Mario have left over?

d The four of them decide to pool their leftover money to buy as many extra roses as they can.
  - **i** How many and what colour roses do they buy?
  - **ii** How many roses will Alicia receive for her birthday from her four friends?

Checklist

Now that you have completed this module, reflect on your ability to do the following.

I can do this	I need to review this
☐ Identify the place value of a digit in a decimal ☐ Convert a decimal to a fraction ☐ Convert a fraction to a decimal	☐ Go back to Lesson 4.1 Decimals
☐ Compare and order decimals ☐ Round decimals to a given number of decimal places	☐ Go back to Lesson 4.2 Ordering and rounding decimals
☐ Add decimals ☐ Subtract decimals	☐ Go back to Lesson 4.3 Adding and subtracting decimals
☐ Multiply decimals by whole numbers ☐ Multiply a decimal by a power of 10 ☐ Multiply decimals by decimals	☐ Go back to Lesson 4.4 Multiplying decimals
☐ Divide decimals by whole numbers ☐ Divide a decimal by a power of 10 ☐ Divide decimals by decimals	☐ Go back to Lesson 4.5 Dividing decimals
☐ Identify percentages from various representations ☐ Convert fractions to percentages ☐ Convert decimals to percentages ☐ Convert percentages to fractions	☐ Go back to Lesson 4.6 Percentages
☐ Convert between fractions, decimals and percentages ☐ Choose between fractions, decimals and percentages to solve problems	☐ Go back to Lesson 4.7 Fractions, decimals and percentages
☐ Express one quantity as a percentage of another ☐ Find percentages of quantities	☐ Go back to Lesson 4.8 Calculating percentages

5

Integers and the Cartesian plane

Lessons

Prerequisite skills

Diagnostic pre-test
Take the diagnostic pre-test to assess your knowledge of the prerequisite skills listed below.

Interactive skillsheets
After completing the diagnostic pre-test, brush up on your knowledge of the prerequisite skills by using the interactive skillsheets.

- ✔ Number lines
- ✔ Mixed numbers and improper fractions
- ✔ Adding and subtracting whole numbers

Curriculum links

- Find equivalent representations of rational numbers and represent positive and negative rational numbers and mixed numbers on a number line (VC2M7N03)
- Compare, order and solve problems involving addition and subtraction of integers (VC2M7N08)
- Investigate, interpret and describe relationships between variables represented in graphs of functions developed from authentic data (VC2M7A04)

Materials

- ✔ Calculator
- ✔ Grid paper

Lesson 5.1

Negative numbers

Learning intentions

By the end of this lesson you will be able to ...

✔ use, compare and order positive and negative numbers.

Inter-year links

Support	Negative numbers
Year 8	1.5 Integers

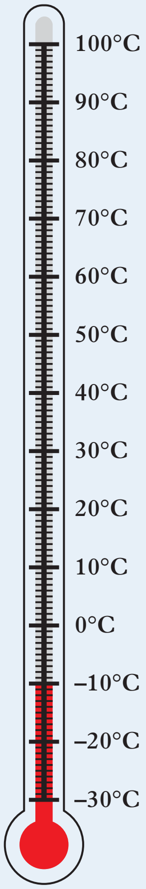
Key content video
Negative numbers

Negative numbers

- **Negative numbers** are less than zero in value.
  - → Negative numbers include a negative sign, −, placed directly before the first digit. For example, 'negative ten' is written as −10.
  - → Positive numbers, which are greater than zero in value, can be represented using a positive sign, +. The positive sign can also be left off. For example, 'positive ten' or 'ten' is written as +10 or 10.
- Although the negative sign and the minus sign look the same, the function of the symbol depends on its context.

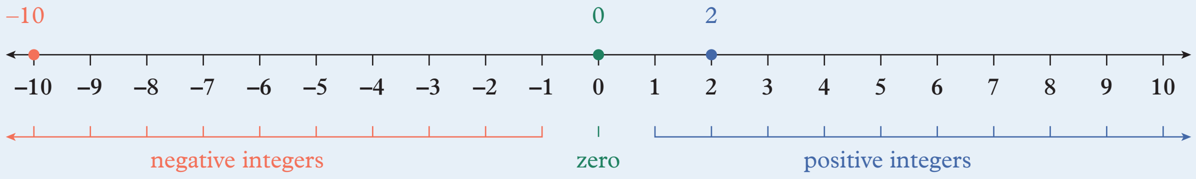
Example	Function of '−'	Read as
−8	Indicates the number is less than zero in value	'negative eight'
2 − 8	Indicates the operation of subtraction	'two minus eight'
−8 + 2	Indicates the number is less than zero in value	'negative eight plus two'

- In the real world, negative numbers are used to represent quantities that lie on a scale that goes below zero. For example, temperatures and bank balances can be represented by negative numbers.



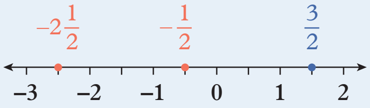
Integers

- In maths, different types of numbers are divided into sets. The set of **integers** includes:
  - → positive whole numbers
  - → negative whole numbers
  - → zero.
- Integers can be shown on a number line.
  - → The numbers on a number line increase in value going from left to right.
  - → The closer a number is to 0, the smaller the **magnitude** of the number.
  - → On a horizontal number line, negative numbers are located to the left of zero because they are less than zero in value.



Negative fractions and decimals

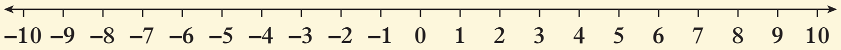
- As with negative whole numbers, a negative sign can be placed before a fraction or decimal to indicate that it is less than zero in value.
- Negative fractions and decimals are less in value than the whole number component of the fraction or decimal. For example, $-3\frac{1}{4}$ is less than -3, and -5.6 is less than -5.
- Negative fractions can be represented on a number line in a similar way to positive fractions.
- When representing negative mixed numbers on a number line, the value appears to the left of the negative whole number part.
  - → A helpful way to visualise where a negative mixed number lies on a number line is to think of it as counting to the left of zero. For example, $-2\frac{1}{2}$ can be located by counting two units to the left of 0, and then an additional half unit to the left of -2.



Worked example 5.1A Comparing positive and negative numbers

Locate the following numbers on the number line in order to complete each number statement using the less than, <, or greater than, >, symbols.

a 5 ______ −8 **b** −3 ______ $-\frac{1}{3}$



THINK

a **1** Locate each number on the number line. 5 is positive and located to the right of zero. −8 is negative and located to the left of zero.

2 Compare the value of the numbers. 5 is located farther to the right on the number line, so 5 is greater than −8.

3 Complete the statement using the correct symbol.

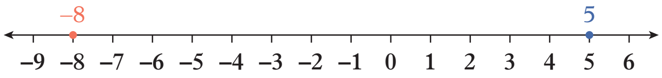
b **1** Locate each number on the number line. Both numbers are negative and located to the left of zero. $-\frac{1}{3}$ is located a third of the distance between 0 and −1 from 0.

2 Compare the value of the numbers. $-\frac{1}{3}$ is located farther to the right on the number line, so −3 is less than $-\frac{1}{3}$.

3 Complete the statement using the correct symbol.

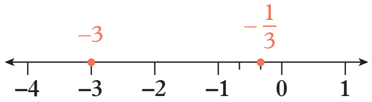
WRITE

a



$5 > -8$

b



$-3 < -\frac{1}{3}$

Worked example 5.1B Ordering positive and negative numbers

Arrange these numbers in ascending order.

$6, -\frac{7}{2}, -1$

THINK	**WRITE**
1 Locate the numbers on the number line. To locate the improper fraction on the number line, convert the improper fraction into a mixed number. $-3\frac{1}{2}$ is located halfway between −4 and −3.	$-7 \div 2 = -3$ remainder 1 $-\frac{7}{2} = -3\frac{1}{2}$ 
2 Arranging the numbers in 'ascending order' involves ordering the numbers from smallest to largest in value. The numbers increase in value reading left to right along the number line.	$-\frac{7}{2}, -1, 6$

Helpful hints

✔ Although the negative sign is identical to the minus sign, remember that the meaning of the symbol depends on its context!
For example, −8 − 2 is read as negative eight minus two.

Learning pathways

△ 1, 2, 3–4(1st, 2nd columns), 5–6(a–d), 7–9, 11–13, 17

■ 2, 3–4(2nd, 3rd columns), 5–6(d–f), 10, 12, 15–18, 20(a)

⬟ 3–4(g–i), 5–6(e, f), 10(d–f), 14, 15, 18–21

ANS p525

Exercise 5.1A: Understanding and fluency

1 Use an appropriate number line to answer the following questions.

a Place each of the following numbers on a number line.

i 2 **ii** 4 **iii** 0 **iv** −5 **v** −2 **vi** −1

b Which is the larger number in each of the following pairs of numbers?

i 2 and 4 **ii** 0 and −5 **iii** −2 and 4

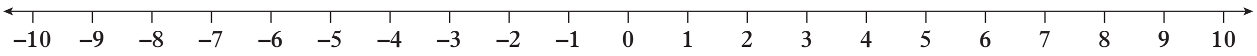
iv −2 and −5 **v** 4 and −1 **vi** −1 and −2

2 Represent the following positive and negative fractions and mixed numbers on a number line.

$-\frac{1}{2}, \frac{2}{3}, -2\frac{3}{4}, 2\frac{1}{3}, -\frac{7}{4}, -3\frac{1}{2}, \frac{10}{3}$

Hint: Divide the positive numbers into thirds and the negative numbers into quarters.

WE 5.1A **3** Locate the following numbers on the number line in order to complete each number statement using the less than, <, or greater than, >, symbols.



a −4 ______ 6

b 0 ______ −3

c −5 ______ −9

d −7 ______ $-\frac{1}{7}$

e −2 ______ $-2\frac{2}{3}$

f $-\frac{5}{4}$ ______ −2

g $-4\frac{2}{5}$ ______ $-\frac{20}{5}$

h $-\frac{7}{2}$ ______ $-3\frac{2}{3}$

i $-\frac{13}{3}$ ______ $-\frac{21}{5}$

4 Complete each number statement using the less than, <, or greater than, >, symbols.

a -40 ______ 30 **b** 0 ______ -22 **c** -65 ______ -55

d 78 ______ $-\frac{100}{2}$ **e** $-\frac{127}{10}$ ______ $-\frac{328}{100}$ **f** $-150\frac{79}{190}$ ______ 0

g $248\frac{23}{25}$ ______ $-300\frac{67}{70}$ **h** $-\frac{120}{7}$ ______ $15\frac{5}{7}$ **i** $-\frac{143}{9}$ ______ $-\frac{95}{6}$

WE 5.1B **5** Arrange these numbers in ascending order.

a $4, -6, 6, -2, 0$ **b** $-13, 10, -3, 4, -\frac{5}{2}$

c $-1, -\frac{2}{3}, 1, \frac{10}{3}, -12$ **d** $16, -5\frac{2}{3}, 10, -5, 5, -\frac{9}{4}$

e $-20, 25\frac{35}{40}, 25, -20\frac{4}{5}, -20.75, 25\frac{17}{20}$ **f** $-1, -\frac{12}{13}, 0.6, 1\frac{5}{6}, -\frac{12}{10}, -1.25$

6 Arrange these numbers in descending order.

a $2.1, -5, 4.8, -6, 8$ **b** $-35, -46.2, -12, 43, 80.8$

c $-4.7, 8.9, -12.5, 6.5, -2.3$ **d** $7.6, -4\frac{1}{3}, 15, -4.5, 12.5, -\frac{11}{2}$

e $-50.3, -36\frac{25}{45}, 48.9, -45\frac{4}{5}, -48.9, \frac{12}{5}$ **f** $-11, -\frac{120}{11}, -0.11, -11\frac{5}{6}, -\frac{110}{12}, -11.25$

7 List all the integers between −5 and 7 (not including −5 and 7).

8 List five fractions between −1 and 1 in ascending order.

9 List five mixed numbers between −5 and 5 in descending order.

Exercise 5.1B: Problem solving and reasoning

10 Write an integer to represent each of the following situations.

a The temperature at a ski resort is 3 degrees below zero.

b You have $360 in the bank.

c The lift stops at the second floor below the ground floor.

d Lake Eyre is 15 m below sea level.

e The top of Mt Kosciuszko is 2230 m above sea level.

11 Some thermometers have scales that show positive and negative temperatures.

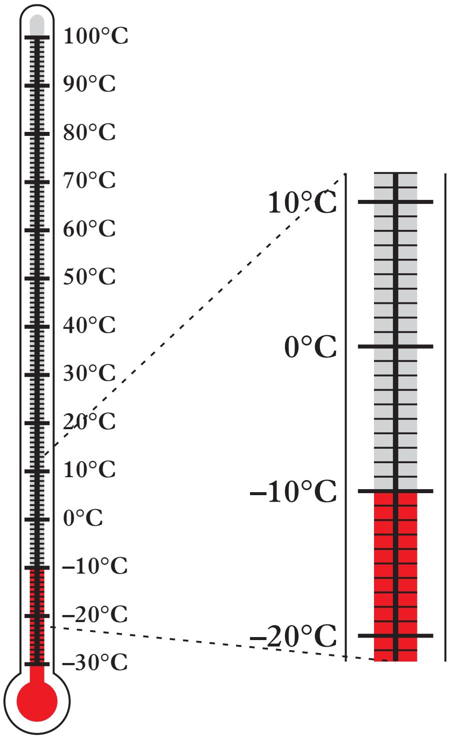
a What is the temperature shown on this thermometer?

b Determine whether each of the temperatures listed below is higher or lower than the temperature shown on the thermometer.

i 8°C **ii** 0°C

iii −15°C **iv** −8°C

c How can you tell whether a number is larger or smaller on this thermometer scale?



12 A group of King penguins and Gentoo penguins is living in a specially designed enclosure at the Sea Life Melbourne Aquarium. The temperature of their environment is kept between −10°C and 2°C.

a Show these two temperatures on a number line.

b Which temperature is lower?

c The pool contains salt water kept at a temperature of −1°C. List the three temperatures in ascending order.

d If the temperature of the air in the enclosure is −9°C, would it feel warmer or cooler to a penguin leaving the pool?

13 The panel of buttons in the lift of a multi-storey school is shown on the right.

- **a** What is the highest level you can travel to in the lift?
- **b** What is the lowest level you can travel to in the lift?
- **c** How many levels does the school have?
- **d** Which button most likely corresponds to ground level?
- **e** Describe the number of floors you move between, and in what direction, if you enter the lift at ground level and press the button labelled ‘3’.
- **f** Describe the number of floors you move between, and in what direction, if you enter the lift at ground level and press the button labelled ‘−2’.
- **g** Describe the number of floors you move between, and in what direction, if you enter the lift at level −1 and press the button labelled ‘3’.
- **h** What does the negative sign represent in this problem?

14 Automatic teller machines (ATMs) allow you to deposit money into and withdraw money from your bank account.

- **a** Katie checks her bank balance and sees that she has $100 in her account. Write this amount as a positive integer.
- **b** Katie’s bank allows her account to be overdrawn. This means that she can withdraw more money than she actually has in her account. If she withdraws $120 at an ATM, how much does she owe the bank?
- **c** Write her new bank balance as a negative integer.

15 In financial terminology, you are said to be ‘in the black’ if you have money and ‘in the red’ if you owe money. A group of friends either have money in their accounts or owe money to a bank in the following amounts.

Student	Amount
Paulo	$85.50
Kamilla	$98.05
Amad	$135.98
Jessica	$172.20

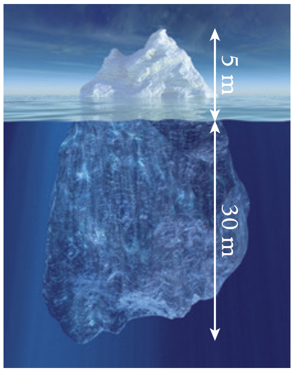
- **a** Write each friend’s bank balance as a positive or a negative number.
- **b** Who has the most money in the bank?
- **c** Who owes the most money to the bank?
- **d** List the bank balances in order of increasing value. Explain your reasoning.

16 Zero is sometimes described as a ‘reference point’, with the sign of a number indicating its direction from zero. Raj’s school is located 2 kilometres from his house. Raj’s mum is a doctor and works at a hospital located 9 kilometres from their house, in the opposite direction to Raj’s school.

- **a** If the hospital is in the positive direction, locate Raj’s school and the hospital where Raj’s mum works on the number line below.

- **b** What is the distance between Raj’s school and the hospital where Raj’s mum works?

17 In 1912, the luxury liner RMS *Titanic* sank on its maiden voyage after its hull was damaged by an iceberg, which lay mostly under water. If the surface of the water, referred to as ‘sea level’, is assigned a value of 0 metres, complete the following with reference to the figure on the right.



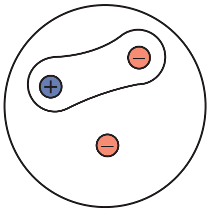
- **a** Represent the height of the part of the iceberg that is located above sea level using a positive integer.
- **b** Represent the length of the iceberg that lies under water using a negative integer.
- **c** What is the total height of this iceberg?
- **d** The wreck of the *Titanic* now lies on the sea floor, 3800 m below sea level. Write this value as a negative integer.

18 Gannets are large seabirds that fish by diving into the sea and pursuing their prey underwater. For this question take the surface of the water (sea level) to be at a height of 0 metres.

- **a** A gannet is flying $25\frac{1}{2}$ metres above sea level. Represent the gannet's height using a positive mixed number.
- **b** A squid is swimming $12\frac{1}{3}$ metres below sea level. Represent the squid's depth using a negative mixed number.
- **c** A shark is swimming $25\frac{9}{10}$ metres below the squid. How far below sea level is the shark? Explain your answer in words, or with a diagram.
- **d** Is the gannet or the shark closer to the squid? Use a number line to justify your answer.

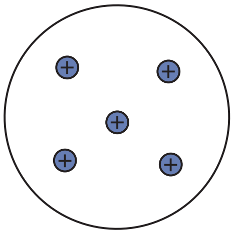
19 When something is electrically charged, it contains an imbalance of positive and negative electric charges. The total electric charge of an object can be determined by pairing as many positive and negative charges as possible and then counting the remaining positive or negative charges that are not paired.

For example, the object represented by the diagram on the right has a charge of −1, because there is one negative charge that is not paired with a positive charge.

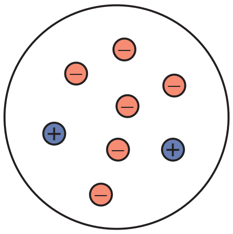


- **a** For each diagram below, pair as many positive and negative charges as possible and then determine the total electric charge of each object.

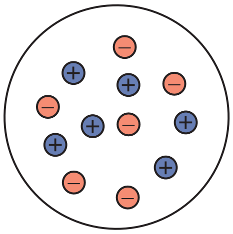
i



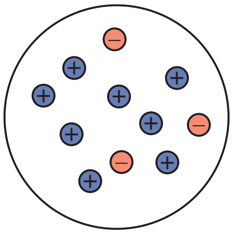
ii



iii



iv



- **b** Which diagrams have the same total electric charge? Explain how this is possible when they have different numbers of positive and negative charges.

Exercise 5.1C: Challenge

20 Find numbers that satisfy the following descriptions.

- **a** I have a single-digit negative integer and a single-digit positive integer that add together to give a positive integer. Justify your answer.
- **b** I have a single-digit negative integer and a two-digit positive integer. If I subtract one integer from the other, I get a negative integer. Justify your answer.

21 The coldest temperature that is physically possible is called 'absolute zero'. When measured in degrees Celsius, absolute zero is approximately −273°C.

The Celsius scale was invented as a way of measuring temperature so that 0°C is the freezing temperature of water. Scientists often use another scale for measuring temperature called the Kelvin scale. In the Kelvin scale, 0 kelvins (0 K) is set at absolute zero and each increase of 1 kelvin is equal to an increase of 1°C.

- **a** Is it physically possible for a temperature to be negative kelvins? Justify your answer.
- **b** Draw a number line to represent the Kelvin scale from 0 K to 300 K.
- **c** Mark your number line at the point that corresponds to 0°C. Write this temperature in kelvins.
- **d** Use your number line to convert the following temperatures from the Celsius scale (°C) to the Kelvin scale (K).
  - **i** −273°C
  - **ii** −10°C
  - **iii** 27°C

Online resources:

Interactive skillsheet
Ordering positive and negative numbers

Worksheet
Integers and number lines

Investigation
Weather at Thredbo

Quick quiz
5.1

Lesson 5.2

Adding and subtracting positive integers

Learning intentions

By the end of this lesson you will be able to …

✔ add and subtract positive integers.

Inter-year links

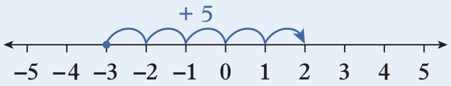
Support	Negative numbers
Year 8	1.6 Adding and subtracting integers

Adding positive integers

Key content video
Adding and subtracting positive numbers

- Adding a positive integer increases the value of the number, which corresponds to moving right on the number line.

  For example, to add 5, move five steps to the right:

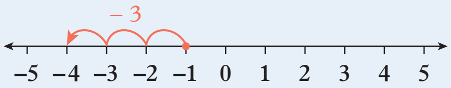


$-3 + 5 = 2$

Subtracting positive integers

- Subtracting a positive integer decreases the value of the number, which corresponds to moving left on the number line.

  For example, to subtract 3, move three steps to the left:

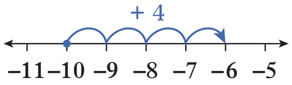


$-1 - 3 = -4$

Worked example 5.2A Adding and subtracting positive integers

Use the number line to calculate:

a $-10 + 4$ **b** $5 - 7$

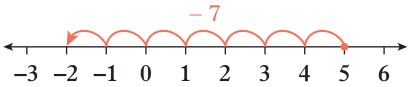
THINK	WRITE
a **1** Draw a number line. Make sure the first number in the calculation is included on the number line. **2** Start at the first number, then consider whether the second number will increase or decrease the value of the first number. Adding 4 will increase the value of the first number, so move four steps to the right. **3** Write down the answer.	**a**  $-10 + 4 = -6$

b **1** Draw a number line.

2 Start at the first number, then consider whether the second number will increase or decrease the value of the first number. Subtracting 7 will decrease the value, so move 7 steps to the left.

3 Write down the answer.

b



5 – 7 = –2

Worked example 5.2B Solving addition and subtraction problems

The temperature inside Helen's house is −4°C. She turns on the heater and, after half an hour, the temperature in the house has increased by 15°C.

a Write a calculation that can be used to determine the temperature of the house after half an hour.

b What will the temperature in the house be after half an hour?

THINK

a **1** Identify the starting temperature.

2 Identify the change in temperature and include the appropriate sign. An increase of 15°C means add 15, or + 15.

3 Write the calculation by combining the starting temperature and the change in temperature.

b Determine the answer by completing the calculation.

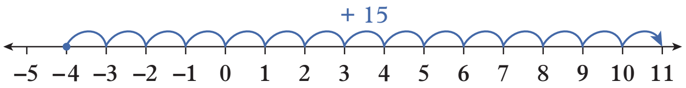
WRITE

a Starting temperature: −4°C

Change in temperature: + 15°C

−4 + 15

b



−4 + 15 = 11

After half an hour, the house will be 11°C.

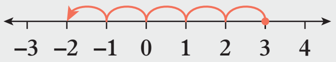
Helpful hints

✔ Avoid making assumptions about the sign of the answer; subtraction can produce positive numbers and addition can produce negative numbers.

For example, −5 + 3 = −2 and 5 − 3 = 2.

✔ Take care when crossing over 0!

For example, 3 − 5 = −2.



Learning pathways

△ 1–4, 5(a–c), 6, 7(a–c), 8, 11, 13, 14

□ 2–3(2nd, 3rd columns), 4, 5(d–f), 7(d–f), 8, 10, 12, 13, 15

⬠ 3(g–i), 5(d–f), 7(d–f), 9, 12–16

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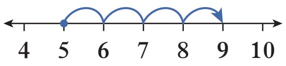
Exercise 5.2A: Understanding and fluency

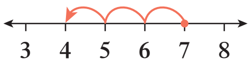
1 Complete each number sentence using the diagrams provided.

a



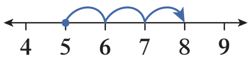
5 + 4 = ________

b



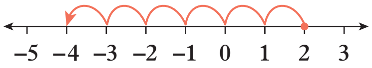
7 − 3 = ________

c



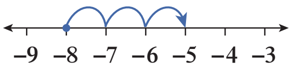
________ + 3 = ________

d



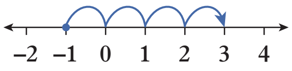
2 − ________ = ________

e



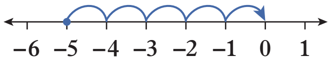
−8 + 3 = ________

f



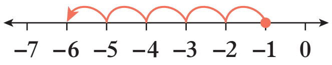
−1 + ________ = ________

g



________ + ________ = ________

h



________ − ________ = ________

WE 5.2A 2 Use a number line to calculate the following.

a 2 + 5 **b** −3 + 4 **c** −9 + 7

d 7 − 2 **e** −4 − 6 **f** −2 + 3

g 5 − 9 **h** −7 + 7 **i** −8 + 5

3 Complete each of the following calculations. Hint: Try visualising the calculation on a long number line.

a 10 − 15 **b** −18 + 5 **c** −11 + 6

d −50 − 30 **e** −68 + 9 **f** −55 + 28

g 62 − 72 **h** −77 + 79 **i** 123 – 255

4 Determine the number that is:

a 5 more than −3 **b** 7 less than 2 **c** 6 more than −6

d 8 less than −1 **e** 4 more than −5 **f** 3 less than 1.

5 Complete each of the following calculations.

a 4 + 9 − 7 **b** −3 – 6 + 13 **c** −24 + 11 + 9

d 12 – 11 + 2 **e** −110 – 33 − 11 **f** −18 + 112 – 6

6 What number, when added to each of the following numbers, gives 0?

a −4 **b** 7 **c** 1 **d** −10

7 Determine the missing numbers in each of the following calculations.

a 3 − ___ = −12 **b** −21 + ___ = −13 **c** ___ + 15 = 7

d −10 − ___ = −55 **e** ___ − 9 = −7 **f** 41 − ___ = −32

Exercise 5.2B: Problem solving and reasoning

8 Without performing each calculation, decide whether the result of the following is a positive integer, a negative integer or zero.

a −12 + 9 **b** −30 + 20 **c** 25 − 18

d −45 + 45 **e** −29 − 15 **f** −19 + 24

9 Describe how you tell by looking at the numbers in a sum whether the result is positive, negative or zero.

WE 5.2B **10** Party pies are stored in a freezer at −18°C. To serve them, the party pies are heated up by 98°C.

  a Write a calculation that can be used to determine the temperature of the pies after they have been heated up.

  b What will the temperature of the party pies be after they have been heated up?

11 A dolphin is swimming 2 m below the surface of the water and then leaps up a vertical distance of 3 m. How high above the water's surface does the dolphin reach?

12 The temperature at 6 am is −8°C. One hour later the temperature has increased by 5°C.

  a Write a calculation that can be used to determine the temperature at 7 am.

  b What will be the temperature at 7 am?

13 Geetika owes her sister $50.

  a Represent this amount as a negative integer.

  b Geetika pays back $35. Write a sum to represent this problem.

  c After paying back $35, how much does Geetika owe her sister?

14 Oscar has overdrawn his bank account by $26.

  a Represent this amount as a negative number.

  b Oscar deposits $40 into his account. Write a sum to determine Oscar's new account balance.

  c What is the balance of Oscar's account after he deposits the $40?

Exercise 5.2C: Challenge

15 Shawn is planning an international trip and has gathered some crucial flight information shown in the table below. Shawn departs from Sydney and transfers at Singapore, Dubai and Paris before arriving in Helsinki.

Departure local time	Flight duration	Time difference between arrival city and departure city	Arrival local time
Sydney Saturday 12:00	8 hours	2 hours behind	Singapore Saturday 18:00
Singapore Saturday 21:00	8 hours	**i** _____	Dubai Sunday 01:00
Dubai Sunday 03:30	8 hours	2 hours behind	Paris **ii** _____
Paris Sunday 14:00	4 hours	1 hour ahead	Helsinki **iii** _____

  a The key information of the first leg of his journey can be expressed with the equation: 12 + 8 + (−2) = 18. Use appropriate calculations to fill in the missing information in the table. Remember there are only 24 hours in a day.

  b What is the time difference between Helsinki and Sydney?

16 Two identical 4-sided dice are rolled, and the two numbers are added together. The list of all possible results is given below.

  −6, −4, −2, 0, 2, 4, 6

What are the numbers on the dice? Explain how you solved this problem.

Online resources:

Worksheet Adding integers using counters

Investigation Modelling addition and subtraction

Quick quiz 5.2

Lesson 5.3

Adding and subtracting negative integers

Learning intentions

By the end of this lesson you will be able to ...

✔ add and subtract negative integers.

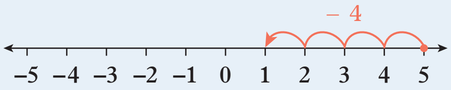
Inter-year links

Support	Negative numbers
Year 8	1.6 Adding and subtracting integers

Key content video
Adding and subtracting negative numbers

Adding negative integers

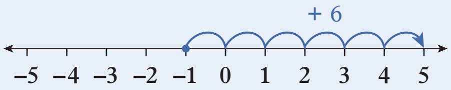
- Adding a negative integer decreases the value of the number and corresponds to moving left on the number line.
  For example, to add −4, move four steps to the left:



$$\begin{aligned} 5 + (-4) &= 5 - 4 \\ &= 1 \end{aligned}$$

Subtracting negative integers

- Subtracting a negative integer increases the value of the number and corresponds to moving right on the number line.
  For example, to subtract −6, move six steps to the right:



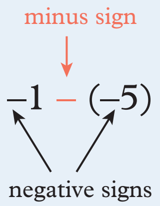
$$\begin{aligned} -1 - (-6) &= -1 + 6 \\ &= 5 \end{aligned}$$

Adding and subtracting negative integers

- Adding a negative integer is equivalent to subtracting a value.
- Subtracting a negative integer is equivalent to adding a value.

$+(-) = -$
$-(-) = +$

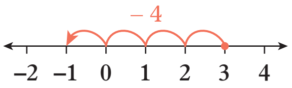
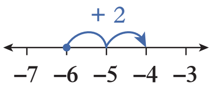
→ Note that brackets are being used to separate the operation and the negative sign of the second number.



Worked example 5.3A Adding and subtracting negative integers

Use the number line to calculate:

a $3 + (-4)$ **b** $-6 - (-2)$

THINK	WRITE
a **1** Simplify the calculation. Adding a negative integer is equivalent to subtracting a value, so replace $+(-)$ with $-$.	**a** $3 + (-4) = 3 - 4$ $= -1$
2 Draw a number line.	
3 Start at the first number, then subtract the value by moving left.	
b **1** Simplify the calculation. Subtracting a negative number is equivalent to adding a value, so replace $-(-)$ with $+$.	**b** $-6 - (-2) = -6 + 2$ $= -4$
2 Draw a number line.	
3 Start at the first number, then add the value by moving right.	

Worked example 5.3B Solving problems involving addition and subtraction of negative integers

A scuba diver is swimming at a depth of 8 metres below sea level when they spot a leatherback turtle swimming at a depth of 13 metres below sea level.

a Represent the depth of the diver and the depth of the turtle as negative integers.

b Write a difference calculation that can be used to determine the difference between the depth of the diver and the depth of the turtle.

c What is the difference between the depth of the turtle and the depth of the diver?

THINK	WRITE
a Identify the depth of the diver and the turtle.	**a** Diver: −8 metres Turtle: −13 metres
b To calculate the difference between the depth of the diver and the turtle, subtract the depth of the turtle from the depth of the diver.	**b** $-8 - (-13)$
c Determine the answer by simplifying, then completing the calculation.	**c** $-8 - (-13) = -8 + 13$ $= 5$ The difference between the depth of the turtle and the depth of the diver is 5 metres.

Helpful hints

- ✔ Write your calculations out in full and double-check any changes in the sign to avoid making errors in the arithmetic.
- ✔ When simplifying calculations involving negative numbers, only combine the signs located *between* the numbers.

The sign of the first number indicates the starting point of the calculation on the number line.

start from the left of 0

$$-1 - (-5) = -1 + 5$$

$$-(-) = +$$

Learning pathways

△ 1–3, 4–6(1st, 2nd columns), 8–9(1st, 2nd columns), 10, 12, 15(a–d)

■ 1–2(c, d), 3–5(b, d, f, g), 6(b, c, e), 7(a, b), 8(b, c, e), 9, 11, 14, 15(a–d), 16

⬟ 1(c, d), 3(g, h), 5–6(3rd column), 7(c, d), 8–9(3rd column), 13, 15–17

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Exercise 5.3A: Understanding and fluency

1 Simplify each of the following calculations by combining the operation with the negative sign to write an equivalent calculation.

a $-3 - (-5)$ **b** $6 + (-5)$ **c** $-2 + (-4)$ **d** $7 - (-1)$

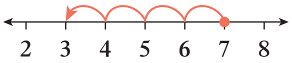
2 Complete each of the number sentences below using a plus or minus sign to write an equivalent calculation.

a $-3 + (-6) = -3$ __ 6 **b** $1 - (-3) = 1$ __ 3

c $8 - (-9) = 8$ __ 9 **d** $-5 + (-7) = -5$ __ 7

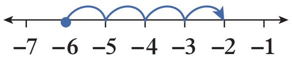
3 Complete each number sentence using the diagrams provided.

a



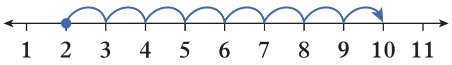
$7 + (-4) =$ ________

b



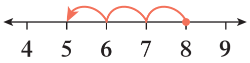
$-6 - (-4) =$ ________

c



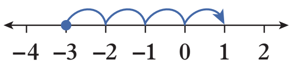
$2 - (-8) =$ ________

d



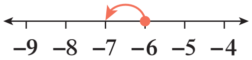
$8 +$ ________ $=$ ________

e



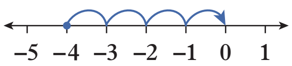
________ $- (-4) =$ ________

f



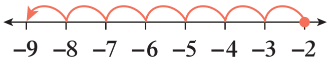
$-6 + (-1) =$ ________

g



________ − ________ = ________

h



________ + ________ = ________

WE 5.3A **4** Use the number line to calculate:

a $1 + (-8)$ **b** $-2 - (-3)$ **c** $1 - (-5)$

d $6 + (-5)$ **e** $-3 - (-1)$ **f** $-7 - (-7)$

g $4 - (-1)$ **h** $5 - (-5)$ **i** $-6 - (-9)$

5 Complete each of the following calculations.

a $20 + (-7)$
b $-24 - (-13)$
c $-14 + (-15)$
d $-13 + (-10)$
e $38 - (-27)$
f $-82 - (-14)$
g $21 + (-43)$
h $-115 - (-44)$
i $250 - (-50)$

6 Complete each of the following calculations.

a $11 + (-3) - 15$
b $-9 - (-15) + 22$
c $-33 + (-12) + (-6)$
d $41 - 31 + (-10)$
e $-15 - (-28) - (-12)$
f $-42 + (-124) - (-9)$

7 What integer should be subtracted from each of these to give a result of 0?

a -5
b $+2$
c -8
d 7

8 Determine the missing numbers in each of the following calculations.

a $12 - \_\_\_ = 25$
b $-7 + \_\_\_ = -22$
c $\_\_\_ + (-9) = -15$
d $-31 - \_\_\_ = -6$
e $39 - \_\_\_ = 106$
f $\_\_\_ - (-52) = -75$

Exercise 5.3B: Problem solving and reasoning

9 Without performing each calculation, decide whether the result of the following is a positive integer, a negative integer or zero.

a $13 + (-8)$
b $25 - (-35)$
c $-18 - (-15)$
d $-33 - (-19)$
e $53 + (-53)$
f $-26 + (-42)$

WE 5.3B **10** Food in the freezer is frozen to a temperature of −15°C, while food in the main section of the fridge is cooled to a temperature of 4°C.

a Write a difference calculation that can be used to determine the difference between the temperature in the main section of the fridge and the temperature in the freezer.

b What is the difference in temperature between the two sections of the fridge?

11 At Thredbo, the daily maximum and minimum temperatures were recorded over a week.

	Sun	**Mon**	**Tue**	**Wed**	**Thu**	**Fri**	**Sat**
Maximum temperature (°C)	7	3	8	2	−1	4	5
Minimum temperature (°C)	1	−2	−1	−3	−5	−2	0

a Calculate the difference between the minimum and maximum temperatures for each day of the week.

b Which day had the biggest range of temperature?

12 Ayesha has $74 in her bank account. Ayesha has an overdraft limit of $100, which means she is allowed to withdraw up to $100 more than a zero balance.

a Represent Ayesha's account balance as a positive integer.

b Ayesha withdraws some money from her account. Her account balance is now −$31. Write a difference calculation that can be used to determine the amount of money Ayesha withdrew from her account.

c How much money did Ayesha withdraw from her account?

13 The depth of a cave is determined by measuring the distance from its highest entrance to the lowest accessible point in the cave. Turquoise Cave has a depth of 57 metres and Amethyst Cave has a depth of 75 metres.

a Represent the depth of each cave using negative numbers.

b Subtract the depth of Turquoise Cave from Amethyst Cave.

c Subtract the depth of Amethyst Cave from Turquoise Cave.

d Explain why your answers to parts **b** and **c** are different.

e Which answer represents the difference in the depth of each cave? Explain.

14 This incomplete bank statement shows some of the deposits and withdrawals made to Ayesha's bank account.

Date	Transaction	Balance
30 June		+$75
5 July	−$24	
13 July	−$68	
24 July	+$45	
31 July		+$19

a Does a transaction of +$45 represent a deposit or a withdrawal?

b Does a transaction of −$24 represent a deposit or a withdrawal?

c Explain what it means if the balance in Ayesha's account is positive.

d Explain what it means if the balance in Ayesha's account is negative.

e Find the balance of Ayesha's account after the transaction made on 24 July.

f What transaction was made on 31 July to give an account balance of +$19?

15 Complete each of the calculations below. Remember to work from left to right.

a $-2 - 9 + 3$ **b** $7 + (-4) - (-6)$ **c** $-5 - (-1) + 8$ **d** $6 + (-3) - (-11)$

e $(-10) - 7 + 2$ **f** $-4 - (-4) + (-5)$ **g** $1 + (-9) - 3$ **h** $-8 - 6 - (-17)$

16 Nine numbers have been arranged as a square as shown on the right.

a Add the three numbers in:

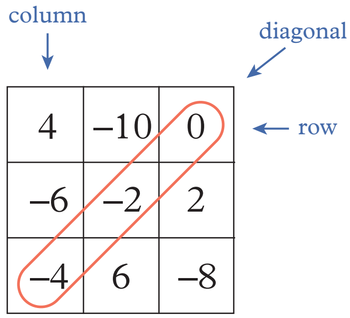
i the first row **ii** the second row **iii** the third row.

b Add the three numbers in:

i the first column **ii** the second column **iii** the third column.

c Add the three numbers in each diagonal.

d This is an example of a 'magic square'. The sum of each row, column and diagonal is called the 'magic sum'. What is the magic sum of this magic square?



4	−10	0
−6	−2	2
−4	6	−8

Exercise 5.3C: Challenge

17 Complete these magic squares by first calculating the magic sum.

a

4	−3	2
−1		3
0	5	

b

		– 4
	−1	
2	−2	

c

6		
−18		
3		−12

Online resources:

Interactive skillsheet
Adding and subtracting with negative integers

Investigation
Slide rule

Desmos activity
Adding integers

Quick quiz
5.3

Worksheet
Subtracting integers using counters

ANS
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Checkpoint

Checkpoint quiz
Take the checkpoint quiz to check your knowledge of the first part of this module.

5.1 **1** Use an appropriate number line to answer the following questions.

a Place each of the following numbers on a number line.
i 3 **ii** $\frac{7}{2}$ **iii** 0 **iv** −3 **v** −2 **vi** $-1\frac{1}{3}$

b Use your answers to part **a** to complete each number statement using the less than, <, or greater than, >, symbols.
i 3 ______ −3 **ii** −3______ −2 **iii** $-1\frac{1}{3}$ ______ 0
iv $\frac{7}{2}$ ______ $-1\frac{1}{3}$ **v** −2______ $\frac{7}{2}$ **vi** 0 ______ −3

5.1 **2** Arrange these numbers in ascending order.

a −4, −9, 7, −1, 0 **b** −6, 10, −2, 6, $-\frac{9}{2}$ **c** −1.5, $-\frac{3}{4}$, 2, $\frac{20}{3}$, −15

5.2 **3** Evaluate each of the following calculations.

a 20 − 26 **b** −22 + 6 **c** −16 + 7
d −60 − 20 **e** −35 + 5 **f** −40 + 18

5.2 **4** Determine the number that is:

a 8 more than −4 **b** 9 less than 5 **c** 12 more than −3
d 15 less than −5 **e** 5 more than −10 **f** 8 less than 2.

5.2 **5** Evaluate each of the following calculations.

a −17 + 20 − 4 **b** −30 − 42 + 16 **c** 92 + 25 + 5
d −11 − 31 + 28 **e** 136 − 84 − 21 **f** −22 + 75 − 49

5.2 **6** On an exploratory cave dive, a geologist took a rock sample at a depth of 27 metres below sea level. He then ascended 12 metres before realising he had left the sample behind.

a Represent the depth where the geologist took the rock sample using a negative integer.
b Write a calculation that can be used to determine the geologist's depth when he realised that he had left the sample behind.
c At what depth did the geologist realise he had left his sample behind?

5.3 **7** Evaluate each of the following calculations.

a 2 + (−7) **b** −6 − (−4) **c** 4 − (−9)
d 8 + (−2) **e** −10 − (−5) **f** −9 − (−9)

5.3 **8** Evaluate each of the following calculations.

a 17 + (−12) − 45 **b** −22 − (−7) + 19 **c** −45 + (−42) + (−9)
d 80 − 26 + (−51) **e** −11 − (−46) − (−57) **f** −79 + (−38) − (−21)

5.3 **9** Determine the missing numbers in each of the following calculations.

a 8 − ___ = 17 **b** ___ + (−22) = −12 **c** −11 + ___ = −47
d −20 − ___ = −8 **e** 25 − ___ = 99 **f** ___ − (−31) = −88

5.3 **10** A submarine is travelling at a depth of 12 metres below sea level. It descends a further 3 metres, and then ascends 5 metres.

a Represent this problem using a calculation.
b What is the final depth of the submarine?

Lesson 5.4

The Cartesian plane

Learning intentions

By the end of this lesson you will be able to ...

✔ identify and plot coordinate points on the Cartesian plane.

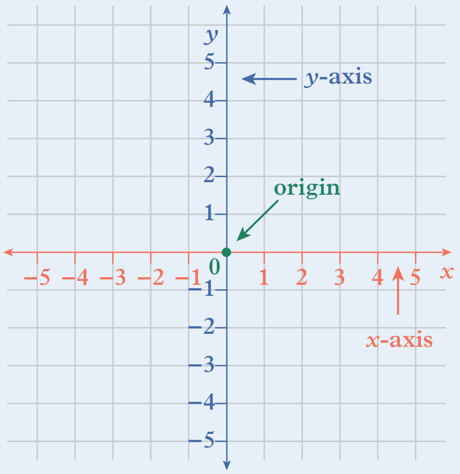
Inter-year links

Support	The Cartesian plane
Year 8	6.5 Plotting linear relationships
Year 9	3.3 Plotting linear relationships

Key content video
The Cartesian plane

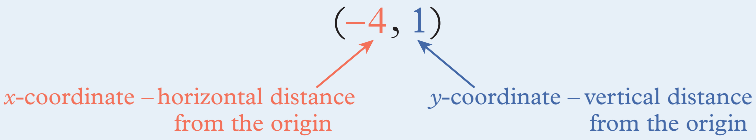
The Cartesian plane

- The **Cartesian plane** is made up of two number lines that cross at right angles and intersect at zero.
  - → The horizontal number line is called the ***x*-axis**.
  - → The vertical number line is called the ***y*-axis**.
  - → The point at which the two axes intersect at zero is called the **origin**.

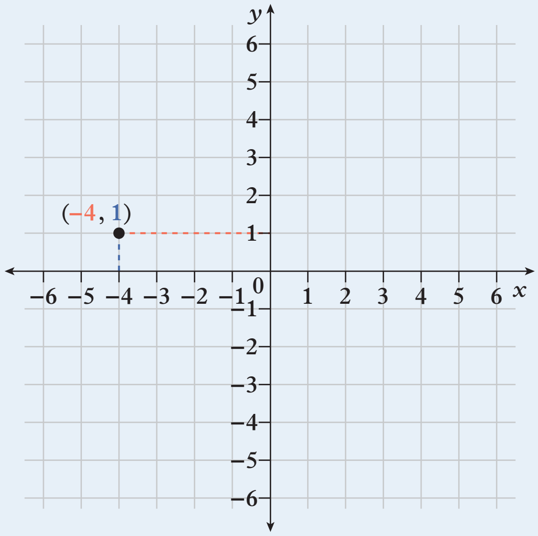


Cartesian coordinates

- **Cartesian coordinates** describe the position of a point on the Cartesian plane.
- The ***x*-coordinate** describes the point's horizontal distance from the origin.
- The ***y*-coordinate** describes the point's vertical distance from the origin.
- The x-coordinate is always listed first, followed by a comma, and then the y-coordinate. Coordinates are enclosed in brackets.

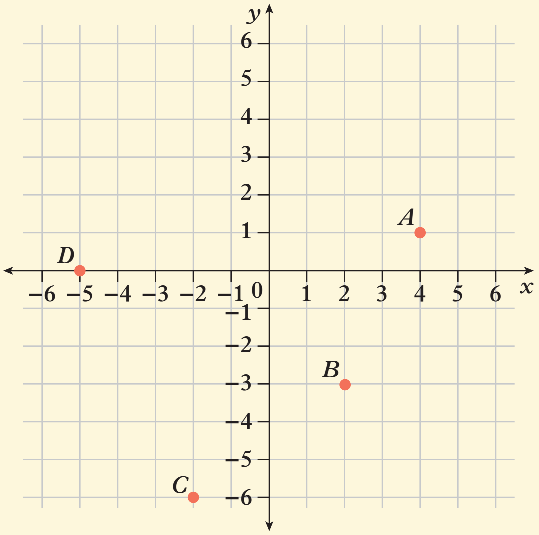


- The sign of each coordinate indicates its location relative to the origin.
  - → A positive x-coordinate indicates that the point is right of the origin.
  - → A negative x-coordinate indicates that the point is left of the origin.
  - → A positive y-coordinate indicates that the point is above the origin.
  - → A negative y-coordinate indicates that the point is below the origin.
- For example, the coordinates to the right describe the position of a point located:
  - → 4 units to the left of the origin
  - → 1 unit above the origin.



Worked example 5.4A Writing coordinate points

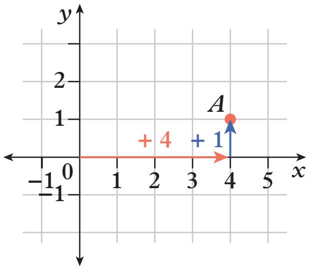
Write the coordinates of points *A*–*D* shown on the Cartesian plane below.



THINK

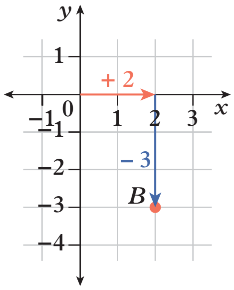
Point *A* is located 4 units to the right of the origin, so its *x*-coordinate is +4, or simply ‘4’, and 1 unit up, so its *y*-coordinate is +1, or simply ‘1’.

WRITE



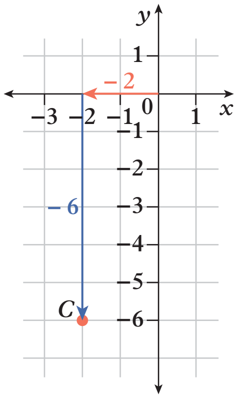
4 units right,
1 unit up
A(4, 1)

Point *B* is located 2 units to the right of the origin, so its *x*-coordinate is 2, and 3 units down, so its *y*-coordinate is −3.



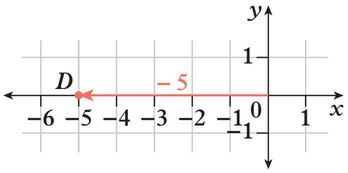
2 units right,
3 units down
B(2, −3)

Point *C* is located 2 units to the left of the origin, so its *x*-coordinate is −2, and 6 units down, so its *y*-coordinate is −6.



2 units left,
6 units down
C(−2, −6)

Point D is located 5 units to the left of the origin, so its x-coordinate is -5. As point D is located on the x-axis, its y-coordinate is 0.



5 units left
$D(-5, 0)$

Worked example 5.4B Plotting points on the Cartesian plane

Plot and label the following points on the Cartesian plane.

a $A(4, 3)$ **b** $B(-1, -5)$ **c** $C(0, 6)$ **d** $D(2, -2)$

THINK

1 Draw a Cartesian plane, with the x- and y-axes drawn large enough to include all the coordinates.

2 Mark each point with a dot and label the dot with its letter and coordinates.

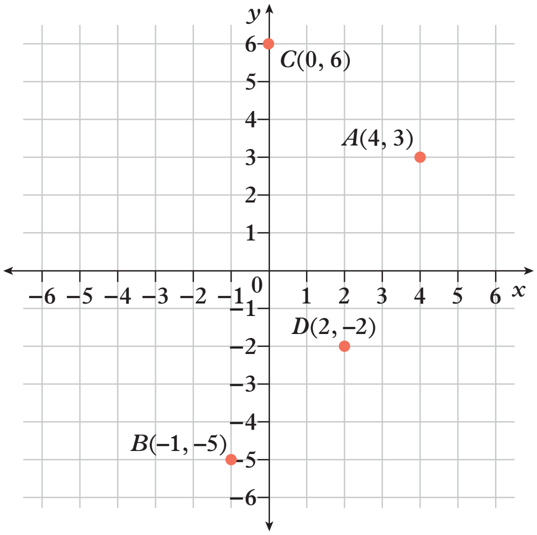
   a Point A has an x-coordinate of 4 and a y-coordinate of 3. Start at the origin, move 4 units to the right and 3 units up.

   b Point B has an x-coordinate of -1 and a y-coordinate of -5. Start at the origin, move 1 unit to the left and 5 units down.

   c Point C has an x-coordinate of 0 and a y-coordinate of 6. As the x-coordinate is 0, stay on the y-axis and move 6 units up.

   d Point D has an x-coordinate of 2 and a y-coordinate of -2. Start at the origin, move 2 units to the right and 2 units down.

WRITE



Helpful hints

✔ Take care when reading and writing coordinates – the x-coordinate is always listed first, followed by the y-coordinate.

✔ Remember that the sign of the x- and y-coordinates indicates the position of each coordinate relative to the origin.

	Position relative to the origin	
Sign	–	+
***x*-coordinate**	Left	Right
***y*-coordinate**	Down	Up

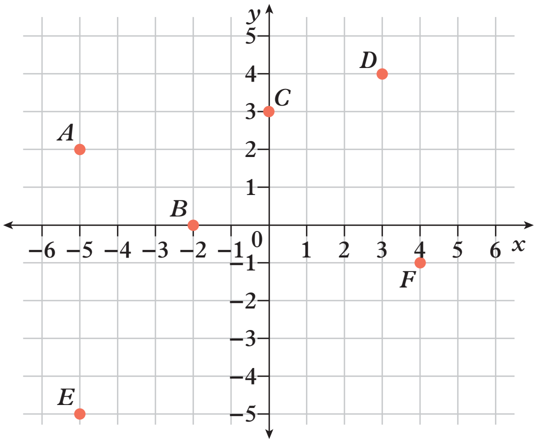
Learning pathways

▲ 1–5, 6(a), 7, 8, 10, 11(a, b) ■ 3–8, 10, 11 ⬟ 5, 6(b), 9, 10, 12

ANS p527

Exercise 5.4A: Understanding and fluency

1 Match the letter used to label each of the following coordinates on the Cartesian plane.

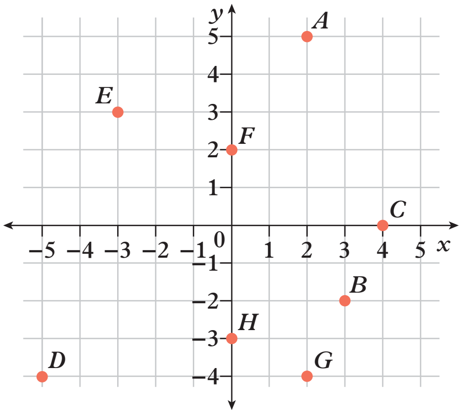


a (3, 4) b (−5, 2) c (4, −1) d (−5, −5) e (0, 3) f (−2, 0)

2 Which point in question **1** is:

a on the x-axis b on the y-axis?

WE 5.4A 3 Write the coordinates of the points A–H shown on the Cartesian plane.



4 List the points in question **3** that have:

a the same x-coordinate b the same y-coordinate.

WE 5.4B 5 Plot and label the following points on the Cartesian plane.

a $A(2, -3)$ b $B(-4, -4)$ c $C(-4, 3)$ d $D(-1, 0)$

e $E(0, -2)$ f $F(-3, 2)$ g $G(5, -4)$ h $H(0, 5)$

6 For each of the following sets of coordinates complete the following.

i Draw a Cartesian plane using an axes scale that is suitable for each set of points.

ii Label each point with its coordinates.

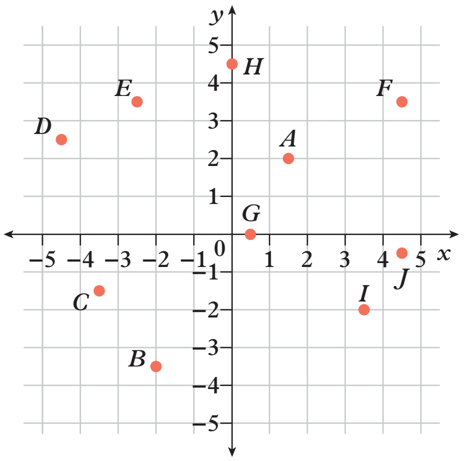
a $(-0.2, 0.4), (1.8, -0.6), (0.6, 0), (-1.2, -1.4), (0, 1.6), (2, 0.8)$

b $\left(1\frac{1}{2}, -2\frac{1}{2}\right), \left(0, -1\frac{1}{2}\right), \left(-\frac{1}{2}, 3\right), \left(-2\frac{1}{2}, -1\frac{1}{2}\right), \left(2\frac{1}{2}, \frac{1}{2}\right), \left(\frac{1}{2}, 0\right)$

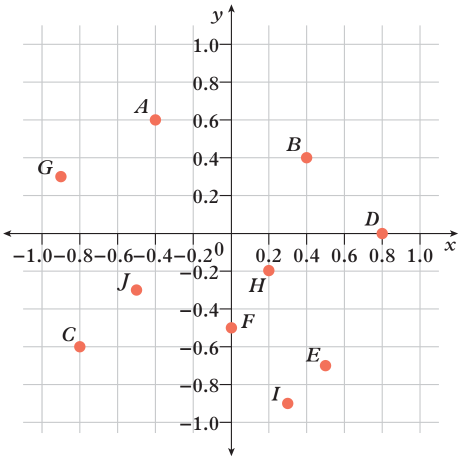
7 What are the coordinates of the origin?

8 Write the coordinates of the points *A*–*J* shown on each of the Cartesian planes below.

a



b

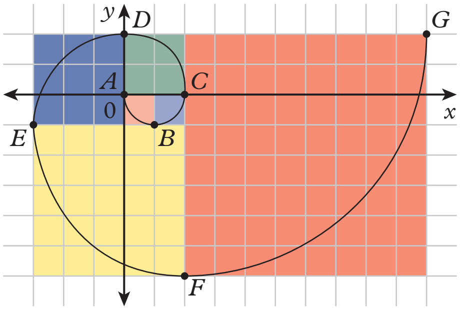


Exercise 5.4B: Problem solving and reasoning

9 Describe the location on the Cartesian plane of points with the following characteristics.

a an *x*-coordinate of 0 **b** a *y*-coordinate of 0 **c** both an *x*- and a *y*-coordinate of 0

10 The Fibonacci sequence is a mathematical pattern that begins with the numbers 0 and 1. Subsequent numbers in the sequence are derived by adding the two preceding ones together. The initial numbers in this sequence are: 0, 1, 1, 2, 3, 5, 8, 13, … . The Fibonacci sequence can also be used to create a Fibonacci spiral on the Cartesian plane as shown on the right.



a Write the coordinates of the points *A*–*G* labelled on the Cartesian plane.

b Each pair of points listed below is located on the opposite vertices (corners) of a square. Determine the side lengths of each square.

i *A* and *B* **ii** *B* and *C* **iii** *C* and *D*

iv *D* and *E* **v** *E* and *F* **vi** *F* and *G*

c Use the initial values of the Fibonacci sequence provided above to find the side length of the next square.

d Use your answer from part **c** to find the coordinate of the next point on the spiral.

Exercise 5.4C: Challenge

11 Draw a Cartesian plane with both axes extending from −5 to 5.

a Plot the points *A*(−3, 4) and *B*(−3, −5). **b** What is the distance between points *A* and *B*?

c Plot the point *C*(2, −5). **d** What is the distance between points *B* and *C*?

e Find the coordinates of point *D* so that the points *A*, *B*, *C* and *D* form the corners of a rectangle.

12 Plot two points with the coordinates (−2, 3) and (4, 3) on the Cartesian plane.

a What is the distance between the points?

b If a line joining these two points forms the side of a square, list the coordinates of the other two corners of the square.

c Part **b** has two possible answers. List the second possible answer.

d What is the vertical distance between the points listed in parts **b** and **c**?

Online resources:

Interactive skillsheet The Cartesian plane	**Investigation** Fishy business	**Desmos activity** Battle boats	**Quick quiz** 5.4
Worksheet The Cartesian plane		**Desmos activity** Hitting the bullseye	

Lesson 5.5

Line graphs

Learning intentions

By the end of this lesson you will be able to ...

✔ interpret line graphs and calculate speed from travel graphs.

Inter-year links

Support	Interpreting data
Year 8	6.8 Finding linear equations
Year 9	3.4 Gradient and intercepts

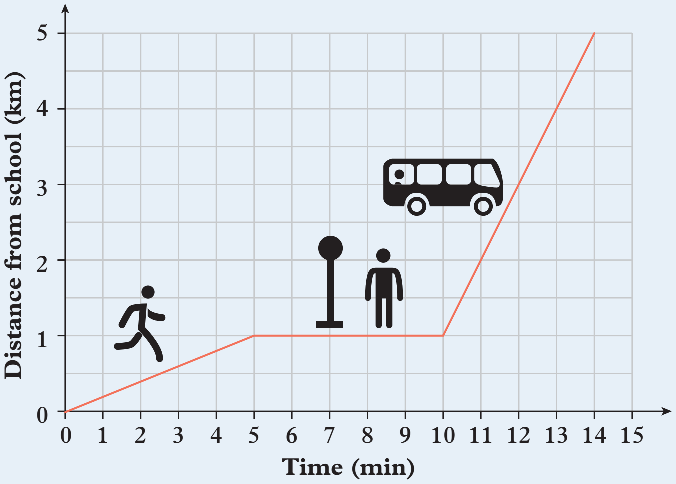
Key content video
Travel graphs

Line graphs

- **Line graphs** show the relationship between two quantities, with one quantity detailed on the horizontal axis and the other quantity detailed on the vertical axis.
  - → Each point on a line graph consists of two values: one value relating to the quantity on the horizontal axis, and one value relating to the quantity on the vertical axis.
  - → The quantity on the horizontal axis must be continuous, for example, time.
- When given a value of one of the quantities, we can trace up and across, or across and down, to read off the value of the quantity on the other axis.
  - → Depending on the data, some values may (or may not) relate to more than one value on the other axis. For example, there may be two times of day when the temperature is 20°C, but there is only one temperature for any given time of day.
- To interpret a line graph, consider how the value of one quantity changes as the value of the other quantity changes. The **slope** (or gradient) of a line graph at a given point indicates how quickly one quantity is changing compared to the other quantity.

Travel graphs

- **Travel graphs**, or **distance–time graphs**, detail the movement of a person or object over a period of time in relation to a fixed reference point. For example, the graph on the right displays the distance a person is from school over a period of 14 minutes.
  - → Time is given on the horizontal axis.
  - → Distance is given on the vertical axis.



Speed

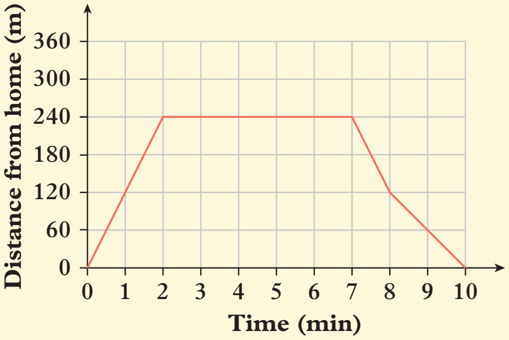
- The **speed** of an object is a measure of how far the object travels over a given period of time.
- Speed can be calculated by dividing the distance travelled by the time spent.

$$\text{speed} = \frac{\text{distance}}{\text{time}}$$

- The slope of any segment of a travel graph indicates the speed of travel for that segment.
  - → The steeper the slope, the greater the speed of travel.
- On a travel graph, a horizontal line segment indicates that there is no change in distance over time, therefore the person or object is **stationary** (not moving), with a speed of zero.

Worked example 5.5A Interpreting travel graphs

Laura walks to a cafe near her house for a coffee, and then walks home again. The travel graph on the right shows her distance from her house from the time she leaves for the cafe to the time she returns home.



a How far was Laura from home at:

i 0 minutes **ii** 2 minutes **iii** 8 minutes?

b What does the horizontal line indicate about the speed at which Laura was travelling over that time interval?

c Where was Laura located 4 minutes into her walk?

d How far is the cafe from Laura's house?

e How long did it take Laura to:

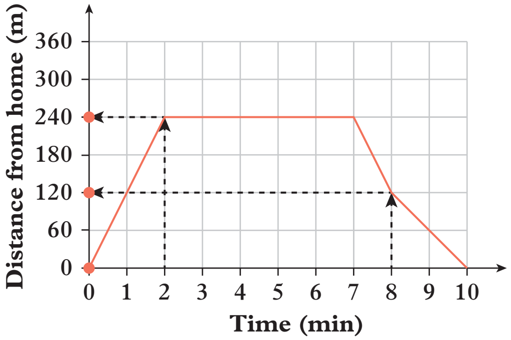
i walk to the cafe **ii** walk home from the cafe?

THINK

a Time is shown on the horizontal axis and distance is shown on the vertical axis. Locate each time on the horizontal axis and trace up to the corresponding point on the graph, then trace across to read the distance off the vertical axis.

WRITE

a



i 0 m **ii** 240 m **iii** 120 m

b A horizontal line indicates that there is no change in distance with respect to time. This means that Laura was stationary (not moving) with a speed of zero over that time interval.

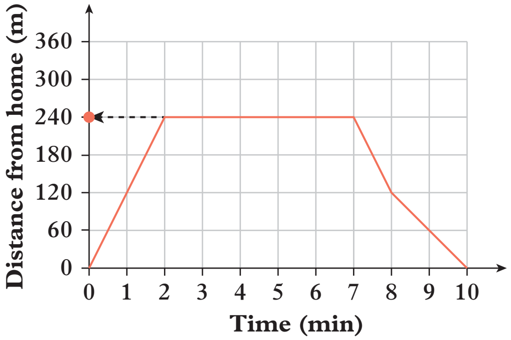
b The horizontal line indicates that Laura was stationary with a speed of zero over that time interval.

c Locate the time on the horizontal axis and trace up to the corresponding point on the graph. At 4 minutes, the graph is horizontal meaning that Laura is stationary, so we can assume that she was located at the cafe.

c 4 minutes into her walk, Laura was located at the cafe.

d As the horizontal line represents the time at which Laura is at the cafe, trace from the horizontal line across to the vertical axis to read the distance from Laura's house to the cafe.

d

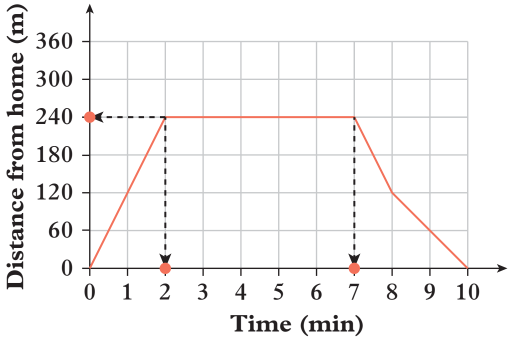


The cafe is 240 m from Laura's house.

e **i** As Laura started walking at 0 minutes, trace from the start of the horizontal line down to the horizontal axis to read the time it took Laura to walk to the cafe.

ii Trace from the end of the horizontal line down to the horizontal axis to read the time at which Laura started to walk home. The journey takes 10 minutes in total, so subtract the time from when she started to walk home from the total time for the duration of her walk home.

e

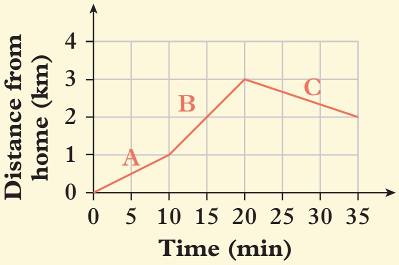


i 2 minutes

ii 10 − 7 = 3 minutes

Worked example 5.5B Calculating speed from a travel graph

The following travel graph shows Daniel's distance from his house over the duration of a jog.



Calculate Daniel's speed during interval B.

THINK	WRITE
1 Determine the duration of interval B by tracing from the graph along the horizontal axis. Interval B starts at 10 minutes and ends at 20 minutes.	duration $= 20 - 10$ $= 10$ minutes
2 Determine the distance travelled in interval B by tracing from the graph across to the vertical axis. At the start of interval B, Daniel is 1 km from his house and at the end of the interval, he is 3 km from his house.	distance $= 3 - 1$ $= 2$ km
3 Calculate the speed by dividing the distance travelled by the time taken to travel that distance.	speed $= \frac{\text{distance}}{\text{time}}$ $= \frac{2\text{ km}}{10\text{ minutes}}$ $= 0.20$ km per minute

Helpful hints

- ✔ Try reading travel graphs as you would a story – consider the significance of any distinctive features such as horizontal lines and changes in steepness.
- ✔ Read axes labels carefully – take note of the scale used and the units.

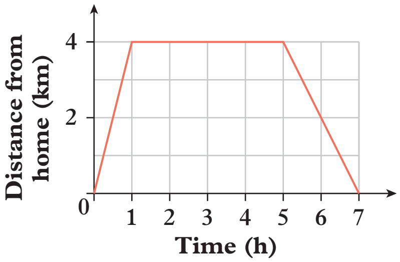
Learning pathways

△ 1–6, 9, 11 ■ 2, 4–6, 8, 9, 11, 12 ⬟ 4–6, 8, 9, 11–13

ANS p528

Exercise 5.5A: Understanding and fluency

WE 5.5A **1** The graph on the right represents Taisha's journey to and from school on a school day.



a What is the distance between Taisha's home and school?

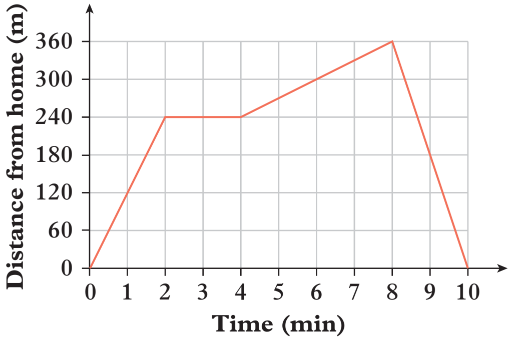
b How long was Taisha at school?

c How long did it take Taisha to:

i walk from home to school **ii** walk from school to home?

d According to the graph, what was the total distance she walked to and from school?

2 Levi leaves home, walks to a local shop to buy a bag of prawn crackers, then walks to his friend's house. Levi and his friend then jog back to Levi's house together. The travel graph below shows this journey.



a How far was Levi from home at:

i 0 minutes **ii** 1 minute **iii** 8 minutes?

b What does the horizontal line indicate about the speed at which Levi was travelling over that time interval?

c Where was Levi located:

i 3 minutes into his walk **ii** 8 minutes into his walk **iii** 10 minutes into his walk?

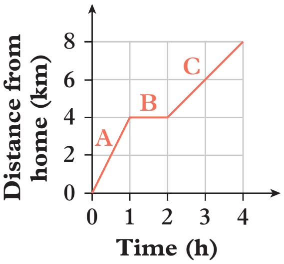
d How far are the following locations from Levi's house?

i the shop **ii** Levi's friend's house

e How long did it take Levi to:

i walk to the shop **ii** walk from the shop to his friend's house **iii** jog home from his friend's house?

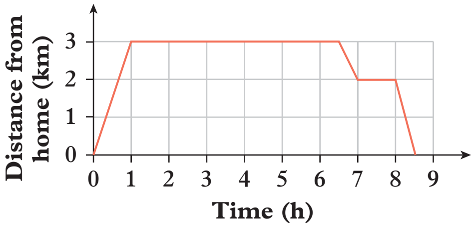
WE 5.5B **3** The following travel graph shows Sarah's journey from her home to her local thrift shop after stopping to visit her friend along the way.



Calculate Sarah's speed during:

a interval A **b** interval B **c** interval C.

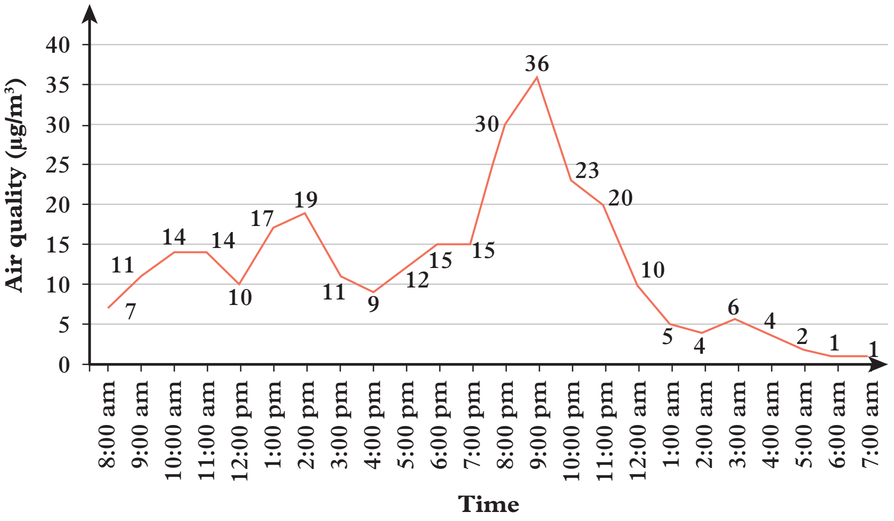
4 Nina walks from her home to school. On the way home, she stops for a dental appointment. The travel graph below represents her journey.



- **a** How far is Nina's school from her house?
- **b** If Nina left home at 7:30 am, at what time did she leave school to walk to the dentist?
- **c** How far did she travel from school to get to the dental clinic?
- **d** How long was Nina's dental appointment?
- **e** What time did Nina arrive home?
- **f** During which part of her journey did Nina walk the fastest? What was her speed?

5 The line graph below shows the air quality on a particular July day in Sydney. Air quality is measured in micrograms, µg, of particulate matter per cubic metre, m^3, of air – the higher the value of the air quality, the worse the quality of air.

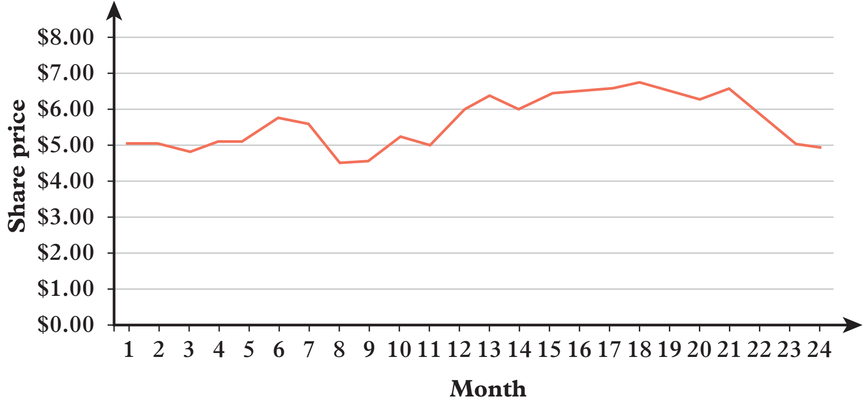
Air quality



- **a** When was the air quality at its worst on this particular day?
- **b** What was the air quality at 3:00 pm?
- **c** Find two different times when the air quality was the same at both times.
- **d** Over what time period was the air quality improving the fastest?

6 The following line graph details the price of Qantas shares over a two-year period.

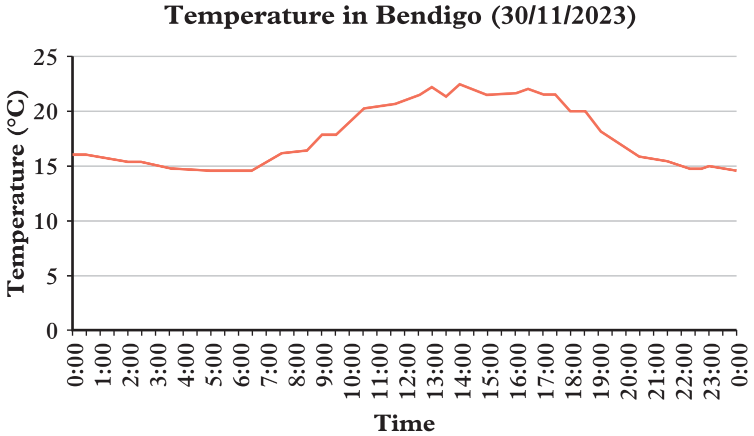
Qantas share price
(Dec. 2021–Nov. 2023)



- **a** Did the shares rise in value or fall in value between months 12 and 13?
- **b** In which month were the shares at their highest value?
- **c** Between which two months did the shares fall most in value?

Exercise 5.5B: Problem solving and reasoning

7 The temperature throughout a day in Bendigo has been recorded in the following graph.



a Between 8 am and 9 am did the temperature rise or fall? **b** Did the temperature on this day reach 25°C?

c At what time of day was the maximum temperature reached?

8 Alex's journey to and from school is shown on the right.

i Answer true or false to each statement.

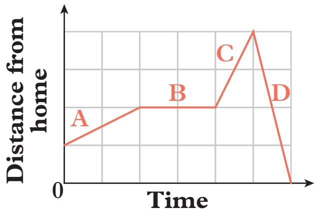
ii Use the graph to explain why each statement is true or false.

a Section A shows that Alex is walking home.

b Section B shows that Alex is stationary.

c Section C shows Alex walking slower than she did in section A.

d Section D shows Alex walking faster than she did in section C.

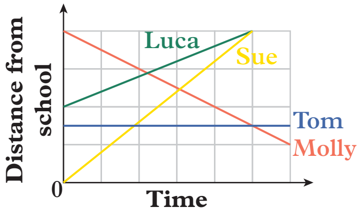


9 The travel graph on the right shows the journeys of four students.

a Which student is stationary or not moving?

b Which student is travelling the fastest?

c Which student is travelling in an opposite direction to two of the other students?

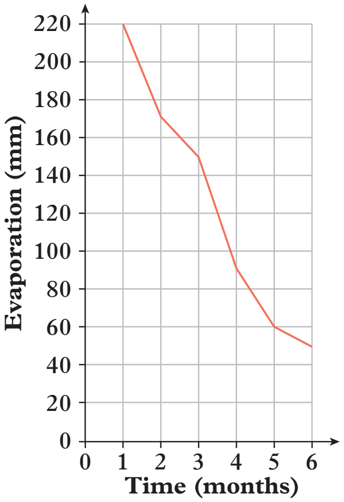


10 Farmers and water authorities use information about the evaporation rate of water to manage water storage effectively. The graph on the right shows data for evaporation recorded over the first six months of a year. The evaporation values, in millimetres, are obtained using a standard-sized container and equipment that measures the change in the water level due to evaporation.

a What was the total amount of evaporation in January, month 1?

b What was the total amount of evaporation in March?

c What does the slope of this graph represent?



This table shows the amount of evaporation for the second half of the year.

Month	Jul	Aug	Sep	Oct	Nov	Dec
Evaporation (mm)	59	67	84	106	150	199

d Copy the graph and extend the horizontal axis to include all the remaining months of the year.

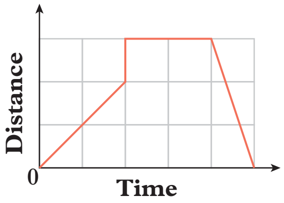
e Plot the evaporation values for July to December on your graph and join each point with a line.

f Which month of the year had the highest evaporation? What was the amount of evaporation in this month?

g Which month of the year had the lowest evaporation? What was the amount of evaporation in this month?

h Based on your answers to parts **f** and **g**, list two factors that affect the amount of evaporation.

11 Does the graph on the right represent a journey that could really take place? Explain why or why not.



12 Jeanne is a diving instructor. She uses GPS trackers to record the depths of scuba divers, and how long they are underwater.

Jeanne graphs the information from the GPS tracker on a particular day. To draw her graph, she takes note of the following information:

- the deck of the boat is 1.5 metres above the water
- the water is 20 metres deep
- scuba divers can be in the water for up to 45 minutes.

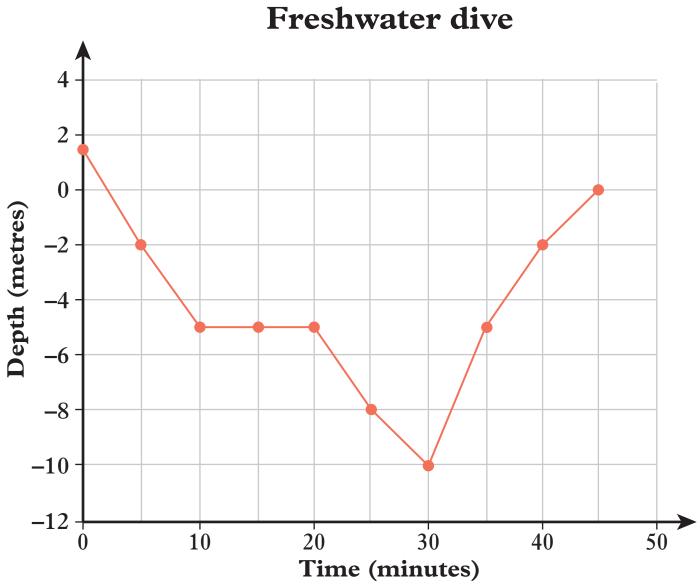
If time is shown on the horizontal axis and depth is shown on the vertical axis:

a what range of values should Jeanne use for the horizontal axis

b what range of values should Jeanne use for the vertical axis

c does either axis require negative numbers? Explain your answer.

On another day, Jeanne produces the graph below for a diver who travels to the bottom of a lake.



d How long did it take the diver to reach the bottom of the lake from the deck of the boat?

e Did the diver take longer to reach the bottom of the lake or to return to the surface of the water? Justify your answer.

f At one stage, the diver stops to take some measurements. At what depth did the diver make these measurements, and how long did the diver take to make the measurements?

Exercise 5.5C: Challenge

13 One morning at 9 am you begin to climb a high mountain, taking a walking track that leads to the peak. It is a hard walk and you have to stop and rest several times along the way.

You camp overnight on the peak of the mountain, then at 9 am the next morning, you begin to walk down the same walking track. Because you are walking downhill, it is easier and you reach the bottom of the mountain in much less time than it took you to reach the peak. Is there a single point along the track which you pass at exactly the same time on both days? Use a graph to help answer this question.

Online resources:

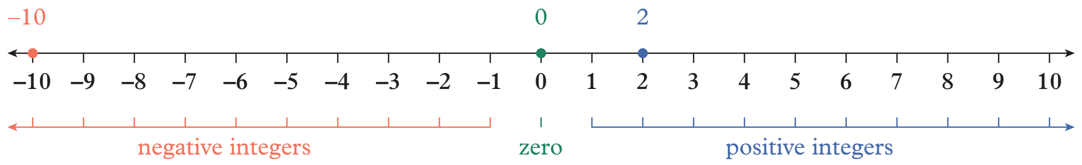
Interactive skillsheet Travel graphs	Worksheet Interpreting travel graphs	Investigation Utility bills	Quick quiz 5.5

Lesson 5.6

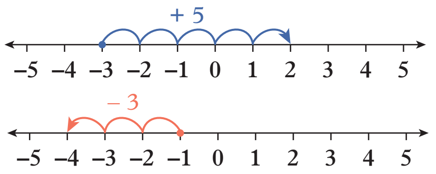
Review: Integers and the Cartesian plane

Module summary

Integers



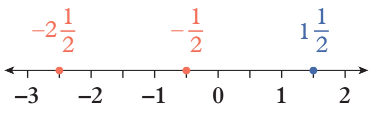
Adding and subtracting positive integers



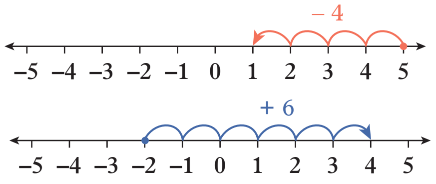
$-3 + 5 = 2$

$-1 - 3 = -4$

Negative fractions



Adding and subtracting negative integers



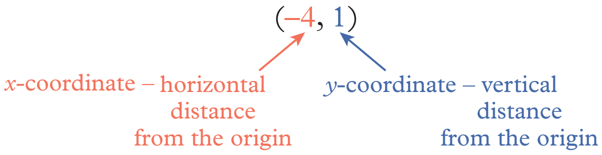
$$5 + (-4) = 5 - 4 = 1$$

$$-2 - (-6) = -2 + 6 = 4$$

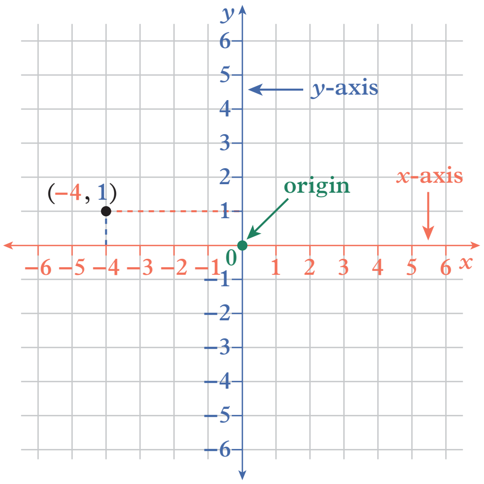
Negative sign and minus sign notation

Example	Read as
–8	'negative eight'
2 – 8	'two minus eight'
–8 + 2	'negative eight plus two'

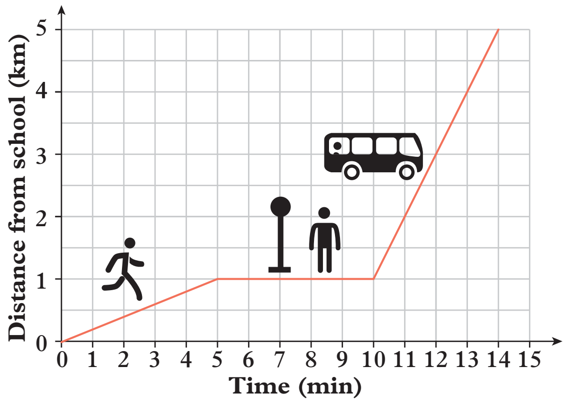
Cartesian coordinates



The Cartesian plane



Travel graphs



Speed

$$\text{speed} = \frac{\text{distance}}{\text{time}}$$

$$= \frac{60 \text{ kilometres}}{1 \text{ hour}}$$

$$= 60 \text{ kilometres per hour}$$

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Review questions 5.6A: Mathematical literacy review

Review quiz
Take the review quiz to assess your knowledge of this module.

Quizlet
Test your knowledge of this module by working individually or in teams.

The following key terms are used in this module:

- Cartesian coordinates
- Cartesian plane
- distance–time graph
- gradient
- integer
- line graph
- magnitude
- negative number
- origin
- point
- slope
- speed
- stationary
- travel graph
- *x*-axis
- *x*-coordinate
- *y*-axis
- *y*-coordinate

1 Identify the negative signs and minus signs in the following equation: $-3 - (-1) = 0 - 2$.

2 Complete the following sentences using words from the key term list.

a The closer a number is to 0, the smaller the ________ of the number.

b The ________ is made up of two number lines that cross at right angles and intersect at zero. The horizontal number line is called the ________, the vertical number line is called the ________, and the point at which the two axes intersect at zero is called the ________.

c ________ describe the position of a point on the Cartesian plane. The ________ describes the point's horizontal distance from the origin, and the ________ describes the point's vertical distance from the origin.

3 Which key terms can be used instead of the following terms?

a gradient

b travel graph

4 Identify the key term being referenced in each of these definitions.

a distance travelled per unit of time

b has a speed of 0

5 Identify whether the following statements are true or false. If a statement is false, briefly explain why.

a Integers include all positive numbers, negative numbers and zero.

b The sum of two negative numbers is always negative.

c The magnitude of a positive number is always greater than the magnitude of a negative number.

6 Write worded questions that could be represented by the following equations.

a $-3 - 7 = -10$

b $-2 + 20 = 18$

Review questions 5.6B: Multiple choice

5.1 **1** Which integer lies between −3 and 3?

A −5 **B** −1.5 **C** −1 **D** 1.5

5.1 **2** Which number is less than $-3\frac{3}{4}$ in value?

A $-4\frac{1}{3}$ **B** $-\frac{5}{2}$ **C** −1 **D** 2.5

5.1 **3** Which number is greater than $-\frac{26}{5}$ in value?

A −3.5 **B** −7.5 **C** −10.5 **D** −6

5.2 **4** Which of these calculations gives a positive result?

A $0 - 12$ **B** $-12 + 12$ **C** $12 - 24$ **D** $-12 + 24$

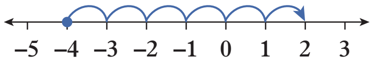
5.3 **5** What is the result of $-4 + (-6) - (-5)$?

A −10 **B** −5 **C** 5 **D** 7

5.3 **6** Which integer when added to 5 gives −3?

A −8 **B** −2 **C** 2 **D** 8

5.3 **7** Which calculation best represents this diagram?

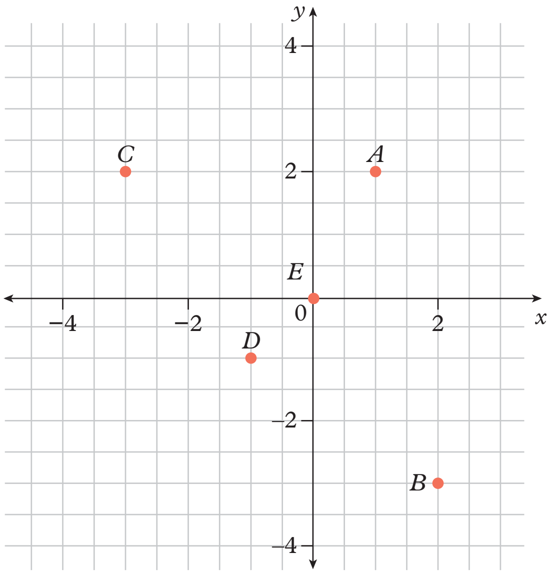


A −4 − 6 = 2 **B** −4 − (−6) = 2 **C** −4 − 2 = −6 **D** −4 + (−6) = 2

5.3 **8** Which two integers give a sum of −9 and a difference of 1?

A −4 and −5 **B** −4 and 5 **C** 4 and 5 **D** 4 and −5

5.4 **9** Which coordinates match the coordinates for point *C* on the Cartesian plane below?

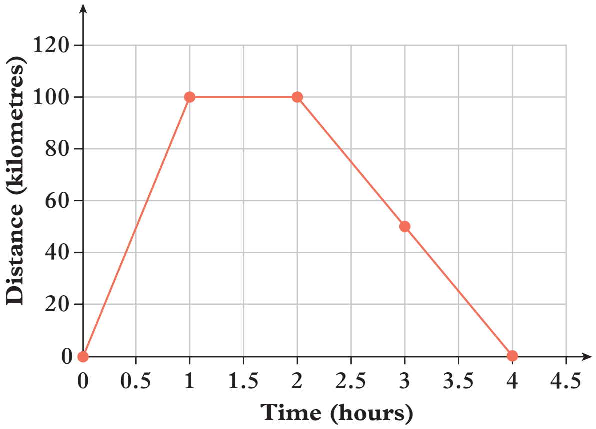


A (1, 2) **B** (2, −3) **C** (0, 0) **D** (−3, 2)

5.4 **10** Which of the following coordinates is located on the *x*-axis?

A (0, 0) **B** (0, −1) **C** (0, 1) **D** (1, 1)

5.5 **11** Anthony takes his car out for a drive. The travel graph for his drive is shown below.



The car is stationary from:

A 0 to 1 hours **B** 1 to 2 hours **C** 2 to 3 hours **D** 3 to 4 hours

Review questions 5.6C: Short answer

5.1 **1** Decide whether each statement is true or false.

a −3 > 3 **b** −12 < −5 **c** 0 > −7 **d** 1 < −1

5.1 **2** Arrange each list in ascending order.

a 25, −5, 0, 12, −10, 2 **b** −3, 9, 6, −6, 0, −12

5.1 **3** Arrange each list in descending order.

a $1, \frac{9}{5}, -2, -\frac{9}{10}, 0, 2\frac{2}{5}$ **b** $5\frac{1}{4}, -4.75, -\frac{15}{4}, 5.5, \frac{7}{2}, -2\frac{3}{4}$

5.1 **4** Write an integer to represent each of these situations.

a Death Valley in California, USA, is 86 metres below sea level.

b You have $615 in the bank.

c The top of Mt Everest is 8848 metres above sea level.

d Your bank account is overdrawn by $35.

5.2 **5** Use the number line to calculate:

a 1 − 9 **b** −7 + 8 **c** −8 − 6 **d** −10 + 9

5.2 **6** A scuba diver is 5.5 metres below sea level. If she dives a further 2 metres down, then rises by 4 metres, what is her final position?

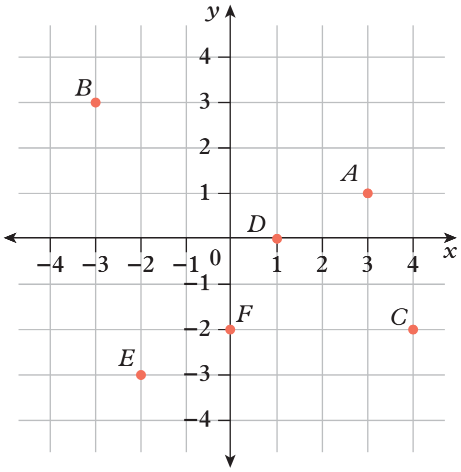
5.3 **7** Use the number line to calculate:

a −6 + (−3) **b** (−4) − (−5)

c 2 + (−4) **d** 11 + (−3)

5.3 **8** You have $25 in your bank account. The bank mistakenly deducts $40 from your account. They realise their mistake, but then make a second deduction of $35. What is the final balance of your bank account?

5.4 **9** Write the coordinates of the points *A*–*F* shown on the Cartesian plane on the right.

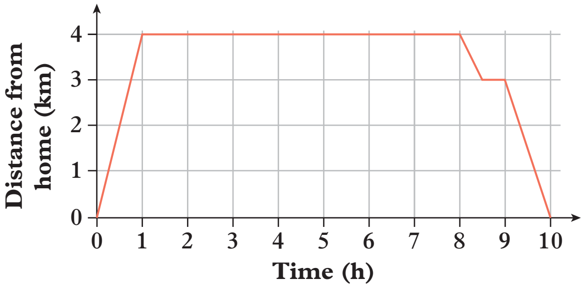


5.4 **10** Plot and label these points on the Cartesian plane.

a *A*(−4, −2) **b** *B*(−1, 4)

c *C*(0, 3.5) **d** $D\left(3\frac{1}{2}, -5\frac{1}{2}\right)$

5.5 **11** Jung walks from his home to school. On his way home from school, he stops at a shop with friends. The following graph represents his journey.



a How far does Jung walk from home to school?

b How long did Jung stop at the shop?

c How long did it take Jung to walk:

  i from home to school

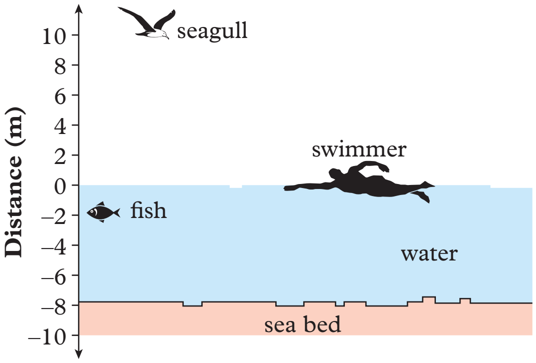
  ii from school to the shop

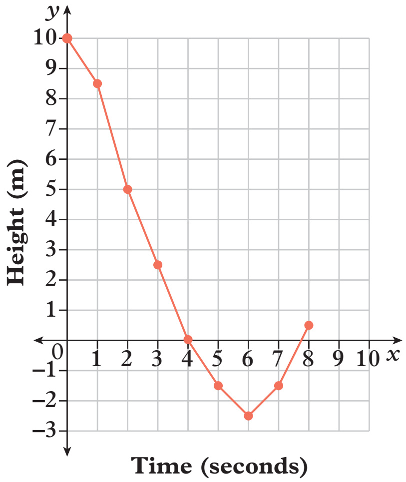
  iii from the shop to home?

d What was Jung's fastest walking speed?

Review questions 5.6D: Mathematical modelling

1 Refer to the scale shown in the following diagram to answer each of the questions.





a Represent the position of each of the following using an integer in metres.

- **i** the seagull
- **ii** the fish
- **iii** the swimmer
- **iv** the sea bed

b Calculate the vertical distance between the:

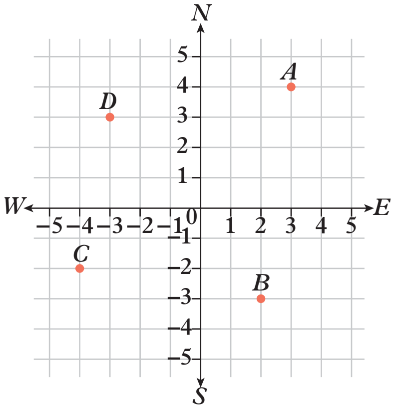
- **i** seagull and fish
- **ii** sea bed and fish.

c A stingray is swimming at three times the depth at which the fish is swimming. Represent its position using an integer.

d The seagull spots the fish under the water and dives towards it. The graph on the right above shows the height of the seagull after spotting the fish.

- **i** What does the slope of each segment of the graph represent?
- **ii** Describe the seagull's movements in the first 8 seconds after it spots the fish.
- **iii** If the fish was located 2.75 metres below the surface of the water 6 seconds after the seagull dived, did the seagull catch the fish?

2 Oliver creates a map of his house and the houses of his four best friends: Alex (A), Bian (B), Chi (C) and Diaz (D). The grid on the right represents the map.



a If Oliver's house is located at the origin:

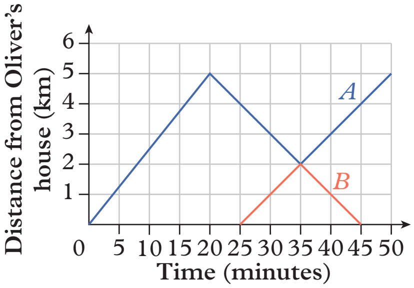
- **i** which of his friends lives furthest east of Oliver
- **ii** which of his friends lives furthest north of Oliver?

b If one square represents 1 kilometre:

- **i** how far west does Diaz live from Oliver
- **ii** how far south does Chi live from Oliver?

c If Bian moves 5 kilometres west and 2 kilometres south of his current home, what are the new coordinates of Bian's house?

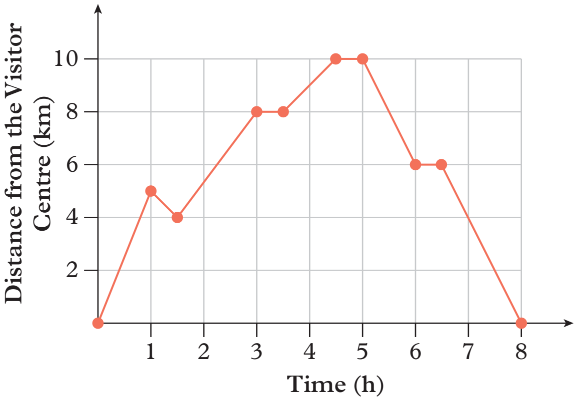
d Alex attended a party at Oliver's place. Upon returning home, he realised that he had forgotten his phone at Oliver's. He went back to retrieve it, but Oliver had already found the phone and decided to bring it to Alex himself. The graph on the right shows their distances from Oliver's home.



- **i** Which line represents Oliver's distance from home?
- **ii** How long did it take for Oliver to find Alex's phone after he left?
- **iii** What is the distance between Alex's home and Oliver's home?
- **iv** How far did Oliver travel before encountering Alex returning for his phone?
- **v** Who got home first in the end?

3 Mauna Kea in Hawaii is the ‘tallest’ mountain in the world with a total height of 10 210 metres. However, 6005 metres of Mauna Kea is located below sea level, so Mt Everest gets the title of ‘highest’ mountain in the world with a total height of 8848 metres, all of which is located above sea level.

- **a** What is the height of Mauna Kea above sea level?
- **b** How much higher above sea level is the peak of Mt Everest than the peak of Mauna Kea?
- **c** If the base of both mountains were located at sea level, how much higher would the peak of Mauna Kea be than the peak of Mt Everest?
- **d** A group of hikers climbed to the top of the Mauna Kea summit from the Visitor Centre. The graph on the right shows their journey.
  - **i** How many breaks did the group take?
  - **ii** How much time did the group spend at the summit?
  - **iii** The group departed at 7:30 am. What time did they return to the Visitor Centre?
  - **iv** After one member of the group lost their wallet, the group retraced their steps for 1 kilometre. When was the wallet found?
  - **v** During which part of the journey did the group hike at their fastest pace?



Checklist

Now that you have completed this module, reflect on your ability to do the following.

I can do this	I need to review this
☐ Represent positive and negative numbers on a number line ☐ Use negative numbers to represent real-life situations ☐ Compare and order positive and negative numbers	☐ Go back to Lesson 5.1 Negative numbers
☐ Add and subtract positive integers ☐ Solve problems involving addition and subtraction of positive integers	☐ Go back to Lesson 5.2 Adding and subtracting positive integers
☐ Add and subtract negative integers ☐ Solve problems involving addition and subtraction of negative integers	☐ Go back to Lesson 5.3 Adding and subtracting negative integers
☐ Identify and plot coordinate points on the Cartesian plane	☐ Go back to Lesson 5.4 The Cartesian plane
☐ Interpret travel graphs ☐ Calculate speed from travel graphs	☐ Go back to Lesson 5.5 Line graphs

Semester 1 review

ANS p530

Short answer

1 Write the following numbers in expanded notation.

a 4817 **b** 19 030 **c** 6^4 **d** 51^3

2 Evaluate the following.

a $\sqrt{36}$ **b** 99×240 **c** $345 + 698$

d 3^4 **e** $1000 - 248$ **f** $3234 \div 7$

3 Rounding involves replacing a number with an approximation.

a Round 135 678 to the nearest:

i ten

ii ten thousand

iii hundred thousand.

b Round 57.658 214:

i to the nearest ten

ii to the nearest tenth

iii correct to three decimal places

iv correct to two decimal places

v to the nearest whole number.

4 Calculate the following sums.

a $859 + 472$ **b** $\frac{5}{4} + \frac{3}{8}$ **c** $53.7 + 2.41$

d $(-15) + (-8)$ **e** $\frac{7}{9} + \frac{9}{2}$ **f** $0.67 + 0.43$

5 Calculate the following differences.

a $859 - 472$ **b** $\frac{5}{4} - \frac{3}{8}$ **c** $53.7 - 2.41$

d $(-15) - (-8)$ **e** $\frac{9}{2} - \frac{7}{9}$ **f** $0.67 - 0.43$

6 Calculate the following products.

a 375×73 **b** $\frac{8}{3} \times \frac{9}{16}$ **c** 2.7×0.41 **d** $5\frac{1}{2} \times 8\frac{2}{3}$

7 Calculate the following quotients.

a $498.4 \div 8$ **b** $\frac{15}{4} \div \frac{3}{2}$ **c** $10.2 \div 0.12$ **d** $4\frac{3}{4} \div 2\frac{3}{8}$

8 Write an integer, fraction, ratio or percentage to answer each of the following questions.

a Two-fifths of a class of 25 students said their family did not own a pet. How many of the students' families did not own a pet?

b Out of 20 power saws, 15 were faulty. What fraction of the power saws were faulty?

c 120 students were on a camp and 45% preferred to eat breakfast before 7:30 am. How many students preferred to eat breakfast before 7:30 am?

d This year, a company made $70 million in sales – up from $50 million last year. Write this year's sales as a percentage of last year's sales for the company.

e Sara was sharing lollies between herself and her younger brother Vishnu in the ratio 3 : 2. If she gave herself 15 lollies, how many lollies did she start with?

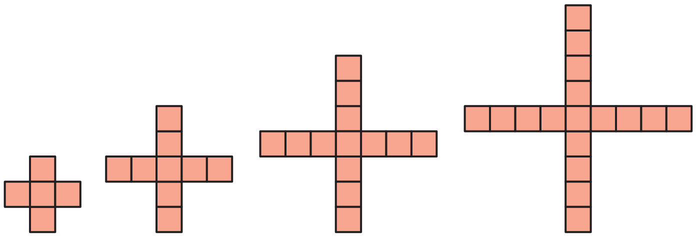
f On a Monopoly board, there are 22 properties, four railroads and two utilities available for purchase. Write the ratio of the number of properties to the total number of railroads and utilities.

9 Calculate the highest common factor and the lowest common multiple for each of the following pairs of numbers.

a 32 and 80 **b** 24 and 40 **c** 21 and 98 **d** 210 and 462

10 Describe the following sequences in words.

a



b $7, 1, -5, -11, \ldots$ **c** $3, 6, 12, 24, \ldots$ **d** $\frac{150}{7}, \frac{30}{7}, \frac{6}{7}, \frac{6}{35}, \ldots$

11 Decide whether the following numbers are prime or composite.

a 51 **b** 47 **c** 227 **d** 221

12 Use the divisibility rules to determine whether the following are true or false.

a 6 is a factor of 9252 **b** 4 is a factor of 3166

c 4427 is a multiple of 9 **d** 7265 is a multiple of 5

13 Evaluate each of the following.

a $\begin{array}{r} 479 \\ +\ 342 \\ \hline \end{array}$ **b** $\begin{array}{r} 416 \\ -\ 387 \\ \hline \end{array}$ **c** $\begin{array}{r} 257 \\ \times\ 63 \\ \hline \end{array}$ **d** $7\overline{)2394}$

14 Evaluate each of the following.

a $8 + 4 \times 7 - 3$ **b** $8 \div 4 \times 2 - 3 + 9$ **c** $\frac{2}{3} - \frac{5}{12} \div \frac{7}{4}$

d $5.2 \times 0.7 - 1.5$ **e** $(-5) - 2 - (-8) + (-4)$ **f** $\frac{2}{5} \times 4.8 + 3.81$

15 Simplify the following.

a $8\frac{4}{16}$ **b** $\frac{45}{18}$ **c** 55 : 75 **d** 91 : 35

16 Plot the following points on the Cartesian Plane: (–3, –7), (–2, –4), (–1, –1), (0, 2), (1, 5), (2, 8), (3, 11).

17 At 7 am the temperature was −1°C, at 1 pm the temperature was 18°C, and at 7 pm the temperature was −4°C.

a How much did the temperature increase from 7 am to 1 pm?

b How much did the temperature decrease from 1 pm to 7 pm?

18 Complete the table below. Where appropriate, express all fractions as improper fractions and round your answers to two decimal places.

Percentage	Fraction	Decimal
35%		
	$\frac{31}{4}$	
4.5%		
	$\frac{68}{7}$	
	$\frac{14}{25}$	
		6.42
	$\frac{13}{5}$	
		8.931

19 Put the following lists of numbers in ascending order.

a $\frac{9}{5}, \frac{8}{6}, \frac{5}{6}, \frac{9}{8}$ **b** 0.5, 5.5, 0.05, 5.0 **c** $5, -3, -4, 1$ **d** $-\frac{11}{25}, -0.41, -\frac{2}{5}, -\frac{3}{7}$

20 Write the following numbers as the product of their prime factors.

a 56 **b** 45 **c** 216 **d** 350

21 Calculate the following powers.

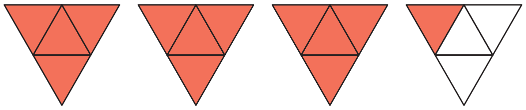
a 7^3 **b** 2^8 **c** 3^4 **d** 17^2

22 Plot the following sets of numbers on a single number line.

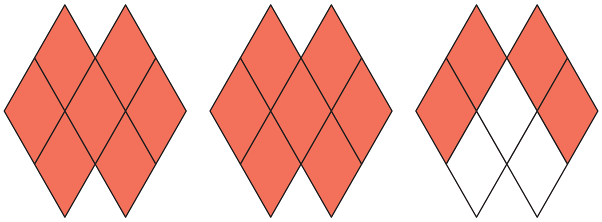
a 2, −3, 5, −1 **b** $\frac{3}{2}, \frac{3}{4}, -\frac{1}{2}, -\frac{5}{4}$ **c** −0.1, 0.05, 0.10, −0.2

23 If each composite shape below is considered to be a whole, express the shaded portion of each diagram in the form specified in brackets.

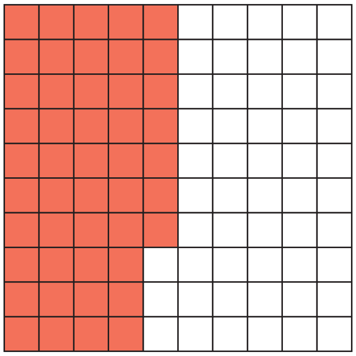
a (mixed number)



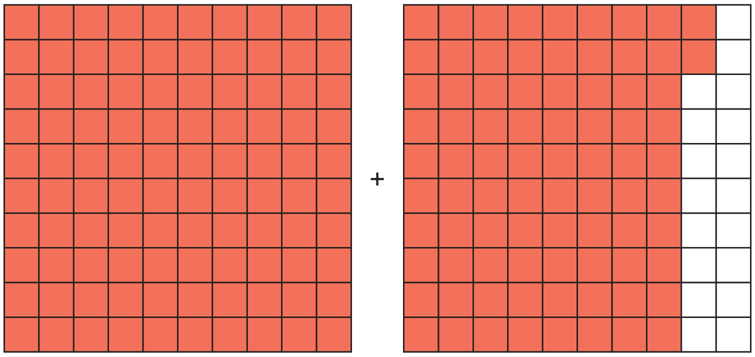
b (improper fraction)



c (percentage)



d (percentage)



24 Common multiples appear in the multiples list of two or more different whole numbers.

a How many common multiples of 8 and 12 are between 1 and 100?

b How many common factors do 120 and 140 share?

25 A car has an indicator that blinks every 2 seconds. The car in front of it has an indicator that blinks every 3 seconds. If both indicators begin to blink at the same time, how many times will they blink at the same time within a minute?

26 Use prime factorisation to answer the following questions.

a What is the smallest number divisible by 2, 3, 4, 5, 6, 7, 8, 9 and 10?

b What is the smallest number you can multiply by 350 so that the result is a square number?

Mathematical modelling

1 A gardener plants different coloured roses at one metre spacings to create a display. From the left-most edge of the flowerbed, a white rose (w) is planted every 2 m, a red rose (r) is planted every 3 m, and a pink rose (p) is planted in all the gaps where a white or red rose is not used. Where a white rose and a red rose would both be placed, a yellow rose (y) is used instead.

a Complete the sequence of the first 12 rose colours.

p, w, r, __, __, __, __, __, __, __, __, __

b Describe the rule for planting yellow roses.

c Describe the rule for planting pink roses.

d The gardener continues the pattern until the 50th metre.

i What is the colour of the last rose?

ii How many yellow roses are used?

2 A boutique bakery makes pastry packs of cannolis, eclairs and macarons. Each pack contains the same combination of pastries. The bakery makes 30 cannolis, 45 eclairs and 105 macarons a day to put into pastry packs.

a How many packs can be made? All pastries must be used.

b How many of each pastry is included in each pack?

c What fraction of the pack is macarons?

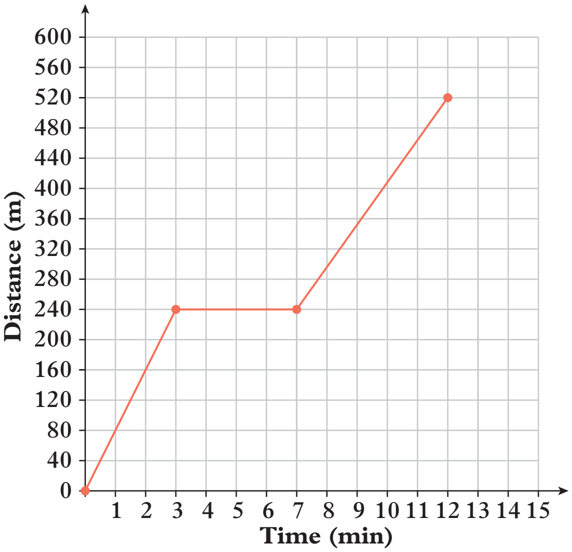
d It costs the bakery $1.20 to make a cannoli, $0.80 to make an eclair and $0.65 to make a macaron. How much does it cost to make all of the cannolis, eclairs and macarons for the pastry packs for a day?

e Each cannoli sells for $3.30, each eclair sells for $3.30 and each macaron sells for $2.60.

i How much do all of the packs made in a day sell for?

ii How much does each pack sell for?

3 Jason walks to and from the park. Jason's distance from his home on his way to the park is shown on the travel graph below. He stops partway to the park to meet up with his friend Kate.



a How far does Jason live from Kate?

b How far is the park from Jason's house?

c How far is the park from Kate's house?

d How long did it take Jason to get to the park?

e How long did Jason have to wait for Kate?

f After an hour at the park, Jason and Kate head home separately. Jason walks directly home. It takes him eight minutes to get home.

i Sketch a travel graph for Jason's trip home.

ii How fast, in m/min, did Jason walk home?

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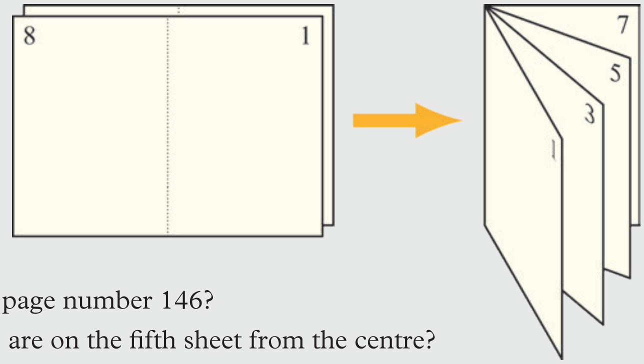
EXPLORATIONS 1

1 Newspapers

A newspaper is made of several A2 sheets folded in the middle so that each sheet makes four numbered pages. For example, two sheets make eight pages, numbered as shown.



a Friday's newspaper has 64 pages. One sheet has the page number 11 on it. What are the other three page numbers on this sheet?

b Saturday's newspaper is larger with 220 pages. What are the other three page numbers on the sheet that has page number 146?

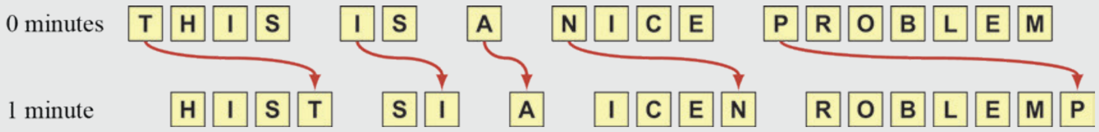
c Sunday's paper has 96 pages. What four page numbers are on the fifth sheet from the centre?

d The sum of the page numbers on one sheet in Monday's newspaper is 258. What is the last page number in this newspaper?

e Sometimes, to get the right number of pages, a single half-sheet of two pages needs to be used somewhere in the newspaper. Tuesday's newspaper has such an insert, and one of its pages is numbered 53. Anh opened the newspaper with its middle pages facing up. He noticed that there were twice as many sheets after the insert as there were before it. Find the last page number in this newspaper and explain why there is only one answer.

2 Word spins

You are making sentences from a collection of alphabet tiles. After one minute, you pick up the first letter of each word and place it at the end, as in the example shown.



As you continue to do this each minute, the whole collection of letters moves along from left to right but the length of each 'word' stays the same.

a How many minutes does it take for the sentence 'THIS IS A NICE PROBLEM' to return to normal?

b How many minutes does it take for the sentence 'MATHS HOTSHOTS CAN SOLVE HARDER STUFF' to return to normal?

c Make up your own sentence that will take exactly 2 hours and 12 minutes to return to normal.

d Using exactly 20 alphabet tiles, make up a sentence that takes the shortest possible time to return to normal.

e Using exactly 20 alphabet tiles, make up a sentence that takes the longest possible time to return to normal.

f In a variation of the game, not only do you move the first letter to the end of each word, but at the same time you also move the entire first word to the end of the sentence. Does this affect any of your answers for parts **a–e**?

3 Copy of a copy

The school photocopier has seven buttons that reduce, maintain or enlarge the area of a printed image:

50% 75% 80% 100% 120% 125% 150%

a Mr Tashibo wants to make a smaller copy of a Japanese alphabet poster. He runs the original through the copier at 80% and then runs the copy through at 75%. What is the effective percentage factor overall? Could he have achieved the same result using two different buttons?

b Ms Prash wants to enlarge a map by a factor of 200% but notices that this is not one of the options. Using the copier no more than three times, what is the closest she can get to achieving this?

c One day, Mr Richo notices that three out of the seven buttons are not working but, by using copies of copies as necessary, it is still possible to achieve all seven factors from the remaining four buttons. What could those four buttons be? Find all possibilities.

d Is it possible to achieve all seven factors if only three of the seven buttons are still working? Explain.

e A different copier has five buttons, including the standard 100%. Using copies of copies as necessary, each of the factors 90%, 80%, 70%, 60%, 50%, 40%, 30%, 20% and 10% can be achieved. What percentages could be on the other four buttons?

4 Ts and Rs

We define two numerical operations labelled T and R.

The effect of T is to add 1 to a number. For example, if we apply the operation T to the number 2 three times in a row, we obtain 3, then 4, then 5.

The effect of R is to find the negative reciprocal of a number – that is, divide 1 by that number and change it from positive to negative, or vice versa. For example, if we apply the operation R to 2 we obtain $-\frac{1}{2}$, and if we apply the operation R to $-\frac{3}{2}$ we obtain $\frac{2}{3}$. Note that R can never be applied to the number 0.

The operations T and R can be combined. For example, we can turn 0 into $\frac{2}{5}$ by successfully applying the operations T T T R T T R T:

$$0 \xrightarrow{\text{T}} 1 \xrightarrow{\text{T}} 2 \xrightarrow{\text{T}} 3 \xrightarrow{\text{R}} -\frac{1}{3} \xrightarrow{\text{T}} \frac{2}{3} \xrightarrow{\text{T}} \frac{5}{3} \xrightarrow{\text{R}} -\frac{3}{5} \xrightarrow{\text{T}} \frac{2}{5}.$$

a Starting with 2, list the numbers produced by successively applying the operations T R R T R T R T R.

b Find a sequence of operations that turns $\frac{3}{4}$ into $\frac{2}{3}$.

c Find a sequence of operations that turns 3 into 0.

d Explain how any positive integer can be turned into 0.

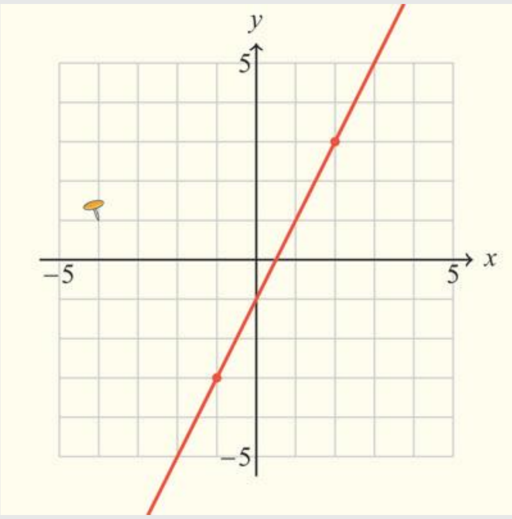
e Explain how 0 can be turned into any negative integer.

f Can any integer be turned into any other integer?

5 Fold and pierce

On a piece of paper, Sarafina draws a Cartesian plane and plots the points (–1, –3) and (2, 3). She then draws a line through the two points, all the way to the edge of the paper, as shown.

Sarafina carefully folds the paper along the line and places a pin through the point (–4, 1).



a Which other point does the pin go through?

b Sarafina unfolds the paper and draws a line through the two pin holes. She then folds the paper twice, once along the original line and once along the new line, and places the pin at (3, 0). Which other points does the pin go through?

c On a new piece of paper, Sarafina draws another Cartesian plane and draws a line through a point P on the x-axis and a point Q on the y-axis. She folds along the line and then places a pin, which goes through (4, –1) and (2, 5).
What are the coordinates of P and Q?

Explorations inspired by the Australian Maths Trust's competitions and programs: www.amt.edu.au

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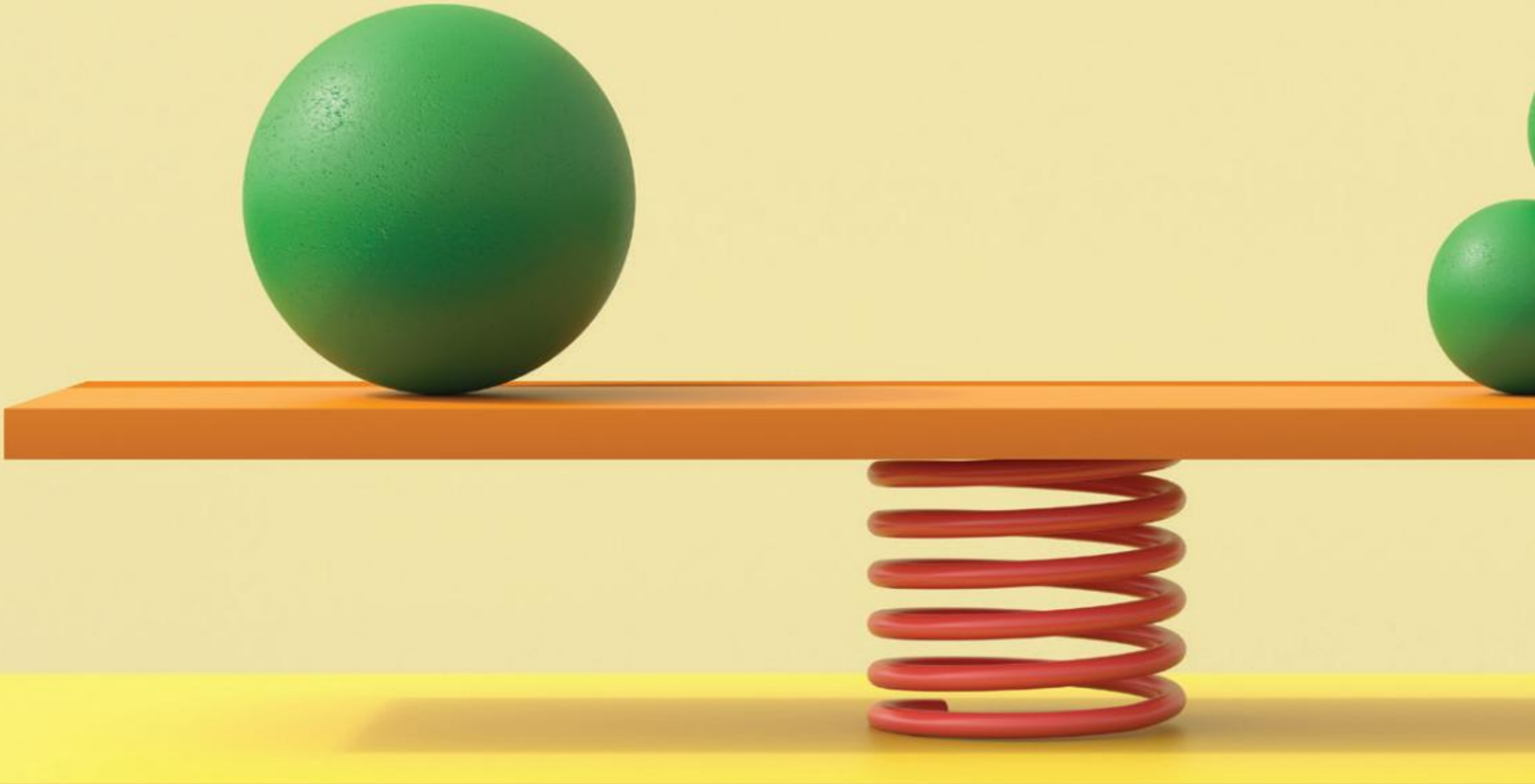
6

Algebra



Lessons

Prerequisite skills

Diagnostic pre-test
Take the diagnostic pre-test to assess your knowledge of the prerequisite skills listed below.

Interactive skillsheets
After completing the diagnostic pre-test, brush up on your knowledge of the prerequisite skills by using the interactive skillsheets.

- ✔ Times tables
- ✔ Order of operations
- ✔ Adding and subtracting with negative numbers
- ✔ The Cartesian plane

Curriculum links

- Recognise and use variables to represent everyday formulas algebraically and substitute values into formulas to determine an unknown (VC2M7A01)
- Apply the associative, commutative and distributive laws to aid mental and written computation, and formulate algebraic expressions using constants, variables, operations and brackets (VC2M7A02)
- Solve one-variable linear equations of increasing complexity with natural number solutions; verify equation solutions by substitution (VC2M7A03)
- Generate tables of values from visually changing patterns or the rule of a function; describe and plot these relationships on the Cartesian plane (VC2M7A05)
- Manipulate formulas involving several variables using digital tools, and describe the effect of systematic variation in the values of the variables (VC2M7A06)

Lesson 6.1

Variables

Learning intentions

By the end of this lesson you will be able to …

✔ use variables in formulas and rules to complete tables and plot points on the Cartesian plane.

Inter-year links

Support	Number sequences
Year 8	5.1 Pronumerals

Unknowns and pronumerals

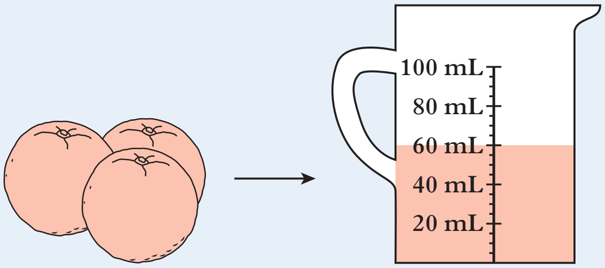
Key content video
Variables

- A **pronumeral** is a letter or symbol that is used in place of a number.
- In algebra, an **unknown** number is represented using a pronumeral.
  For example, an incomplete sum with a question mark can be rewritten using the pronumeral 'x'.

$$? + 3 = 5$$
$$\downarrow$$
$$x + 3 = 5$$

Variables, rules and formulas

- A **variable** is a quantity that can take on different values. Variables can be represented using pronumerals.
  For example, the number of oranges you put into a juicer affects the volume of orange juice that comes out. In this example, both the *number of oranges* and the *volume of juice* are variables.
- A **rule** describes the relationship between two or more variables.
  For example, the relationship between the above variables can be described by the rule *one orange makes 20 millilitres of juice*, or:
  volume of juice (in mL) = 20 × number of oranges



- A **formula** is a rule written using pronumerals.
  The above rule can be re-written as a formula by representing *volume of juice*, in mL, with the pronumeral v and the *number of oranges* with the pronumeral n.
  $v = 20 \times n$
- Formulas are often written with one side (usually the left) consisting only of a variable which does not appear on the other side. This variable is called the **subject**, and we say that it is written in terms of any other variables in the formula. In the formula above, v is the subject which is written in terms of n.

Function machines

- A **function machine** can be used to process mathematical operations.
- In a function machine, a value is inputted into the left-hand side, one or more operations are performed on that value, and the result is outputted at the right-hand side.
  For example, this function machine represents the formula $y = 2 \times x + 5$.

$$\boxed{x} \rightarrow \boxed{\times 2} \rightarrow \boxed{2 \times x} \rightarrow \boxed{+ 5} \rightarrow \boxed{y}$$

The value of x is the input into the function machine, and the value of y is the output.

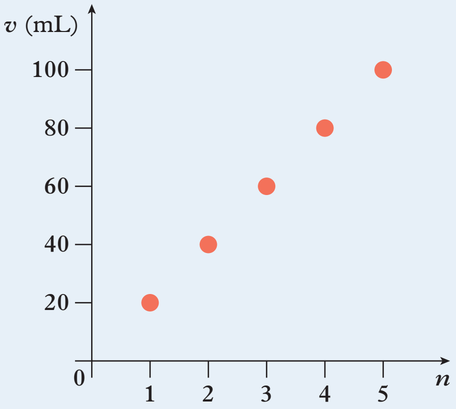
Tables of values and graphs

$v = 20 \times n$

n	1	2	3	4	5
v **(mL)**	20	40	60	80	100

- A **table of values** shows the relationship between two variables. The values of one variable are listed in the first row of the table, and the corresponding values of a second variable are listed in the second row of the table.
- The relationship between two variables can also be displayed in a graph, where one variable is placed on the horizontal axis, and the other variable is placed on the vertical axis.
- When the value of one variable depends on the value of another variable, the first variable is known as the **dependent variable** and the second variable is known as the **independent variable**.

  For example, the volume of juice depends on the number of oranges, so we can say that *volume of juice* is the dependent variable and *number of oranges* is the independent variable.
- In a table of values, the independent variable is usually listed in the top row and the dependent variable is listed in the second row.



- When plotting variables on a graph, the independent variable should be plotted on the horizontal axis and the dependent variable should be plotted on the vertical axis.
- If the points lie on a straight line on the Cartesian plane, the relationship between the variables is said to be linear.

Worked example 6.1A Using a rule to complete a table of values

The cost of one avocado is $4. Use this information to complete the table below.

Number of avocados	1	2	3	4	5
Total cost ($)					

THINK

1 One avocado costs $4, therefore two avocados cost 2 × $4 = $8, three avocados cost 3 × $4 = $12, and so on.

2 Write the total cost values in the table.

WRITE

$1 \times 4 = 4 \quad 2 \times 4 = 8 \quad 3 \times 4 = 12 \quad 4 \times 4 = 16 \quad 5 \times 4 = 20$

Number of avocados	1	2	3	4	5
Total cost ($)	4	8	12	16	20

Worked example 6.1B Using a function machine to complete a table of values

Use this function machine to complete the table below.

$$\boxed{x} \rightarrow \boxed{\times 4} \rightarrow \boxed{4 \times x} \rightarrow \boxed{+ 5} \rightarrow \boxed{y}$$

x	1	2	3	4	5
y					

THINK

1 Multiply each value of x by 4.

2 Add 5 to the results from step **1**.

3 Enter the answers into the table of values.

WRITE

x	1	2	3	4	5
× 4	$1 \times 4 = 4$	$2 \times 4 = 8$	$3 \times 4 = 12$	$4 \times 4 = 16$	$5 \times 4 = 20$
+ 5	$4 + 5 = 9$	$8 + 5 = 13$	$12 + 5 = 17$	$16 + 5 = 21$	$20 + 5 = 25$

x	1	2	3	4	5
y	9	13	17	21	25

Worked example 6.1C Plotting points from a table of values and recognising linear relationships

Consider the table of values on the right.

a List the coordinates of each point.

b Plot the points on the Cartesian plane.

c State whether or not the points lie on a straight line.

***x*-coordinate**	−3	−2	−1	0	1	2	3
***y*-coordinate**	−1	0	1	2	3	4	5

THINK

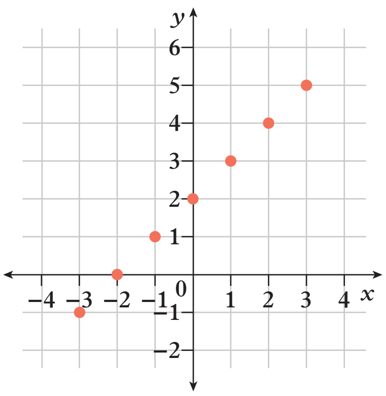
a Write out the coordinates for each point. List the x-coordinate first, followed by a comma, and then the y-coordinate.

b Draw a Cartesian plane, and then plot each point on the plane.

WRITE

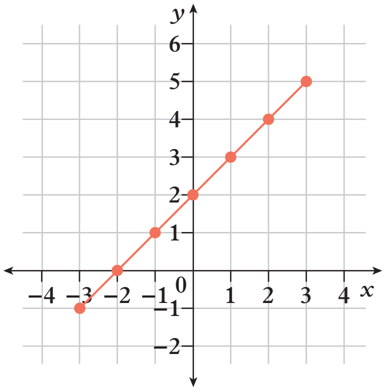
a (−3, −1), (−2, 0), (−1, 1), (0, 2), (1, 3), (2, 4), (3, 5)

b



c Examine the pattern formed by the points. Joining the points together shows that the points lie on a straight line.

c



The points lie on a straight line.

Helpful hints

✔ A variable is a number or quantity, not an object.
In the formula, $v = 20 \times n$, the pronumeral 'n' represents the *number* of oranges, not one single orange.

Learning pathways

△ 1–9, 11–13

□ 2, 3, 5, 7–8(c, d), 9–11, 13, 14, 16

⬠ 3, 5, 8, 9–11, 13–18

ANS p531

Exercise 6.1A: Understanding and fluency

WE 6.1A **1** The cost of one banana is \$2. Use this information to complete the table below.

Number of bananas	1	2	3	4	5
Total cost (\$)					

2 On a rainy day, the amount of water in a rainwater tank increases by 10 L every hour. Assuming the tank is initially empty, use this information to complete the table below.

Time (h)	1	2	3	4	5
Volume of water in the tank (L)					

3 Anna's car uses 9.2 L of petrol for every 100 km travelled. If her petrol tank initially contains 50 L of petrol, use this information to complete the table below.

Distance travelled (km)	0	100	200	300	400	500
Volume of petrol in the tank (L)	50					

4 The cost of a child's movie ticket is \$15. If n is the number of children and c is the total cost in dollars, draw a table that shows the total cost of buying 1, 2, 3, 4 and 5 child movie tickets.

5 Eli is watching a movie and trying to make his popcorn last as long as possible. He starts the movie with 150 pieces of popcorn and eats two pieces every minute.

If t is the time in minutes from the start of the movie and p is the number of pieces of popcorn left in Eli's bucket, draw a table that shows the number of pieces of popcorn left in Eli's bucket at 20-minute intervals over the first 100 minutes of the movie.

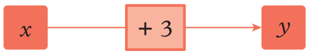
6 Match each rule **a–d** with its corresponding formula **I–IV**.

a Multiplying x by 7 is equal to y.
b Adding 7 to x is equal to y.
c Multiplying x by 7 then adding 7 is equal to y.
d Subtracting 7 from x is equal to y.

I $y = x + 7$
II $y = x - 7$
III $y = 7 \times x$
IV $y = 7 \times x + 7$

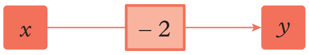
WE 6.1B 7 Use the function machines to find the values of y in each of the following tables.

a



x	1	2	3	4	5
y					

b

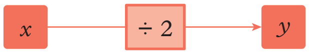


x	2	3	4	5	6
y					

c

x	0	1	2	3	4
y					

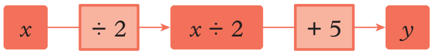
d



x	0	2	4	6	8
y					

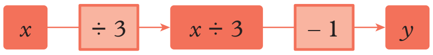
8 Use the function machines to complete the accompanying tables of values.

a



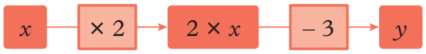
x	0	2	4	6	8
y					

b



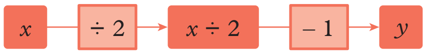
x	3	6	9	12	15
y					

c



x	1	2	3	4	5
y					

d



x	2	4	6	8	10
y					

WE 6.1C 9 For each of the tables of values below complete the following.

i List the coordinates for each point.
ii Plot the points on the Cartesian plane.
iii State whether or not the points lie on a straight line.

a

x	–3	–2	–1	0	1	2	3
y	–5	–4	–3	–2	–1	0	1

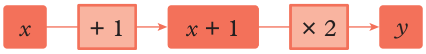
b

x	–3	–2	–1	0	1	2	3
y	4	–1	–4	–5	–4	–1	4

10 For each of the function machines below:

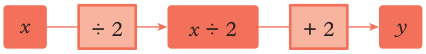
i complete the table of values
ii plot the points on the Cartesian plane
iii state whether or not the points lie on a straight line.

a



x	0	1	2	3	4
y					

b



x	0	1	2	3	4
y					

Exercise 6.1B: Problem solving and reasoning

11 Hira works at a cafe and receives a 50-cent tip from every customer she serves.

a Use this information to complete the table below.

Number of customers	1	2	3	4	5
Amount in tips earned ($)					

b What are the two variables in this relationship?

c Which variable is the independent variable and which variable is the dependent variable?

d How much money does Hira earn if she serves:

i five customers **ii** six customers

iii 12 customers?

e How many customers would Hira have to serve to earn:

i \$2 in tips **ii** \$10 in tips **iii** \$100 in tips?

12 The relationship between two variables can be represented by a rule, a table of values or a graph. Each representation has different advantages. Match each representation **a–c** to its corresponding property **I–III**.

a A rule **I** can be used to visually identify any patterns in the relationship between two variables.

b A table of values **II** can be used to calculate the value of an unknown variable.

c A graph **III** provides a sample of values for each variable.

13 A pasta machine flattens and stretches one piece of dough to produce a new piece that is longer. One piece of dough is 10 cm long when it goes into the pasta machine. It comes out of the machine 30 cm long. Another 20 cm piece goes into the machine and comes out 60 cm long. A third 30 cm piece goes into the machine and comes out 90 cm long.

a Write a sentence describing the relationship between the length of dough that goes into the machine and the length of dough that comes out of the machine.

b Complete the table below.

Length of dough in (cm)	10	20	30	40	50	60
Length of dough out (cm)						

c Calculate the length of dough:

i coming out of the machine if the length going in is 100 cm

ii going into the machine if the length coming out is 240 cm.

14 Consider the equation $x - 7 = 13$.

a Is x a variable? Explain your reasoning.

b Decide whether the pronumeral(s) in each of the following calculations are variables or represent a specific unknown number.

i $5 \times p = 10$ **ii** $2 \times a + 1 = 15$ **iii** $y = x - 5$

15 A newborn baby's weight increases by approximately 30 grams each day.

a What are the two variables in this situation?

b Which variable is the independent variable, and which is the dependent variable?

c After five days, by how much should a newborn baby's weight have increased?

d To write a rule for this situation, what piece of information do you need to find out?

16 The temperature at a ski resort was recorded every 2 hours starting at midnight. The results are shown below.

Time (number of hours after midnight)	0	2	4	6	8	10	12	14	16	18	20	22
Temperature (°C)	−4	−5	−6	−3	−1	3	4	7	5	4	0	−2

a Draw a Cartesian plane and label the horizontal axis as 'Time' and the vertical axis as 'Temperature'.

b Decide on the scale to use for your horizontal axis. Do you need negative numbers on the scale? Explain why or why not. Mark your scale on the time axis.

c Decide on the scale to use for the vertical axis. Mark your scale on the temperature axis.

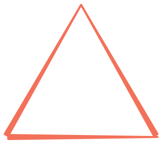
d List the coordinates for each point. Remember that the horizontal coordinate is always written first.

e Plot the points listed in the table of values.

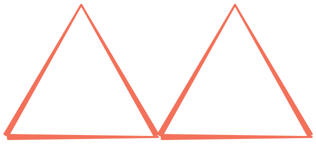
f What was the coldest recorded temperature? At what time of the day was

g What was the warmest recorded temperature? At what time of the day was this temperature recorded?

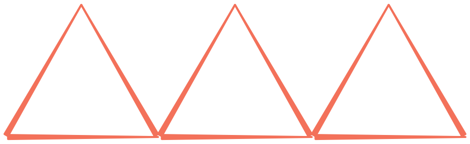
17 A pattern can be seen when shapes containing different numbers of triangles are made using toothpicks.



1 triangle



2 triangles



3 triangles

a Complete the table below to show the relationship between the number of triangles and the number of toothpicks needed to make the shape.

Number of triangles	1	2	3	4	5	6
Number of toothpicks						

b Write a rule in words which relates the number of triangles with the number of toothpicks.

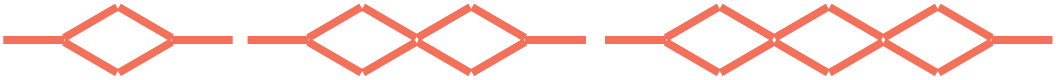
c Use the rule to find:

- **i** the number of toothpicks needed to make a shape containing 120 triangles
- **ii** the number of triangles that could be made with 90 toothpicks.

Exercise 6.1C: Challenge

18 The formula $y = 4 \times x + 2$ represents a relationship that relates to designs of a rollerblading track. Each track is made up of a different number of identical concrete slabs as shown below.

Aerial plan of some possible tracks



one diamond **two diamonds** **three diamonds**

a What does x represent?

b What does y represent?

c Describe the design of the largest rollerblading track that could be made using 22 concrete slabs.

d Design a new rollerblading track using a different shape. Write a formula to represent your design.

Online resources:

Interactive skillsheet
Variables

Investigation
Which internet provider is best?

Quick quiz
6.1

Lesson 6.2

Terms, expressions and equations

Learning intentions

By the end of this lesson you will be able to …

✔ identify and write terms, expressions and equations.

Inter-year links

Support	Writing formulas
Year 8	5.1 Pronumerals

Key content video
Terms, expressions and equations

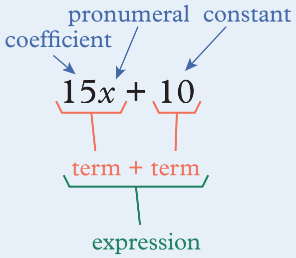
Algebraic notation

- Products are simplified by leaving out the multiplication sign and placing the number in front of the pronumeral.
- When a pronumeral is multiplied by 1, the 1 does not need to be included.
- Quotients are represented using fractions.

$$x \times 7 = 7 \times x = 7x$$
$$x \times 1 = 1 \times x = x$$
$$x \div 7 = \frac{x}{7}$$

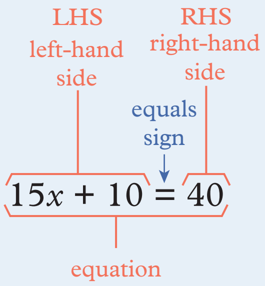
Terms

- In algebra, a **term** is a single pronumeral, number or the product or quotient of a number and one or more pronumerals. For example, p, $6a$, $5mn$, $-3x$, $\frac{1}{2}v$, 2.
- The **coefficient** of a term is the number by which that term is being multiplied. The coefficient includes the plus (+) or minus (−) sign that comes before the term. For example, the coefficient of $3x$ is 3, the coefficient of x is 1, and the coefficient of $-\frac{1}{2}x$ is $-\frac{1}{2}$.
- A **constant** is a term without any pronumerals. It is a fixed value. For example, 2, 75, −10.



Expressions and equations

- An **expression** is a quantity that is represented by a sequence of numbers and/or pronumerals that are connected by mathematical operations. This includes a single term or the sum or difference of two or more terms. For example, x, $a - 5$, $4f + 5gh + 6$.
- An **equation** is a mathematical statement that shows equality between the left-hand side (LHS) and the right-hand side (RHS) of the equation. For example, $y = 10x$, $x + 3 = 5$, $10p + 2 = 11q - 7$.
  → Equations have an equals sign, whereas expressions do not.
- The four operations in equations can be described using a range of vocabulary.



Addition (+)	Subtraction (−)	Multiplication (×)	Division (÷)
• Sum	• Difference	• Product	• Quotient
• Plus	• Take away	• Times	• Divide
• More than	• Less than	• Multiply	• Share
• Increase	• Fewer	• Groups of	• Distribute
• Total	• Reduce		
• Together	• Minus		
• And			

Worked example 6.2A Identifying terms, coefficients and constants

Consider the expression $3a - b + 7$.

a Determine the number of terms in the expression.

b Identify any variables in the expression.

c Identify the coefficient of each term that contains a variable.

d List any constant terms.

THINK	**WRITE**
a Count the number of terms. The terms are separated by a + or − sign.	**a** There are three terms: $3a$, $-b$ and 7.
b Identify the pronumerals in the expression.	**b** The variables are a and b.
c Identify the number by which each variable is being multiplied. As $3a = 3 \times a$, the coefficient of $3a$ is 3. As $-b = -1b$, the coefficient of b is -1.	**c** The coefficient of $3a$ is 3. The coefficient of b is -1.
d List the terms that do not contain a pronumeral.	**d** The constant term is 7.

Worked example 6.2B Writing expressions

Write an expression for each statement.

a The sum of a and 3.

b Twice the value of c.

THINK	**WRITE**
a Think about which operation is taking place. Finding 'the sum' means to add, so the sum of a and 3 is a plus 3.	**a** $a + 3$
b Finding twice the value of a number is to multiply by 2. Twice the value of c is 2 times c. Simplify the product by leaving out the multiplication sign and placing the number first.	**b** $2 \times c = 2c$

Worked example 6.2C Writing equations with one unknown

Write an equation for each statement. Use x for the unknown number.

a 10 less than a number is equal to 0.

b A quarter of a number is equal to 28.

THINK	**WRITE**
a '10 less than a number' means subtract 10 from the unknown number, x, to get 0.	**a** $x - 10 = 0$
b 'A quarter of a number' means divide the unknown number, x, by 4 to get 28. Remember division is written as a fraction.	**b** $x \div 4 = 28$ $\frac{x}{4} = 28$

Worked example 6.2D Writing equations with two unknowns

Write a formula for each statement.

a x divided by 3 is equal to y.

b y is 5 less than x.

THINK	**WRITE**
a Think carefully about the order of variables in the equation. ‘x is divided by 3’ means divide x by 3 to get y. Remember that quotients are written as fractions. In equations, the unknown with a coefficient of 1 is usually placed on the left-hand side of the equation.	**a** $x \div 3 = y$ $\frac{x}{3} = y$ $y = \frac{x}{3}$
b ‘y is 5 less than x’ means y is equal to x minus 5.	**b** $y = x - 5$

Helpful hints

- ✔ The coefficient of x is 1, not 0.
- ✔ As well as acting as an operator, minus signs can be read as negative signs and are therefore a property of the variables which they precede.
  For example, in the expression $5 - 3x$, the coefficient of x is -3.
- ✔ Remember subtraction and division are not commutative. For example, $x - 3$ is not the same as $3 - x$, and $\frac{x}{3}$ is not the same as $\frac{3}{x}$.
- ✔ You can use any letter or symbol you like to represent a variable; however, it is good practice to use a pronumeral that relates to the variable, for example, b for the number of bananas or c for the number of carrots.

Learning pathways

△ 1–9, 11–16	■ 1(b, e, h), 2, 3(e–h), 5, 7, 8, 9(e–h), 10, 11, 13, 15, 17, 19–24	⬟ 1(b, e, h), 2(h–l), 3(g, h), 8–10, 12, 15, 18, 19, 21–27

ANS p533

Exercise 6.2A: Understanding and fluency

WE 6.2A **1** For each expression:

- **i** Determine the number of terms in the expression.
- **ii** Identify any variables in the expression.
- **iii** Identify the coefficient of each term that contains a variable.
- **iv** List any constant terms.

a $5x + 6$

b $2a + 4b + 3$

c $m + 4$

d $2 + 11k$

e $4c + 7d$

f $13y$

g $f + 5$

h $6x + w + 1 + 3y$

i $8p + 10 + m$

2 Determine whether each statement is true or false. For the statements that are false, provide a reason for your answer.

a $y = 3x + 2$ is an expression
b $4d$ is a term
c $7k - 3$ is an expression
d the constant in $5a + 3$ is 5
e $m = n + 9$ is a formula
f the coefficient of x is 1
g $8n = 24$ is an expression
h the coefficient of $6y$ is 6
i the constant in $4c + 5$ is 5
j $8 + 3a$ is an equation
k $6x + 2$ has two terms
l $5 - 2y$ is an expression

3 State whether each of these is an expression or an equation.

a $3x + 2 = 5$
b $x - 6$
c $8 = 4 + y$
d $a + b + c = 11$
e $5 - 2abc$
f $d = 3f + 7$
g $k = 8m$
h $p + q + 2r + 5$

4 What do all the equations in question **3** have in common?

WE 6.2B **5** Write an expression for each statement.

a the sum of a and 7
b b divided by 2
c 5 less than c
d 3 times the value of d
e e subtracted from 8
f twice the value of f
g 4 divided by g
h 9 more than h
i the sum of k, m and n

6 How many terms are there in each expression in question **5**?

7 Write an expression for each statement. Use the pronumeral x to represent the unknown number.

a 8 more than a number
b 15 less than a number
c a number is multiplied by 3
d a number is divided by 7

8 Write an expression for each statement. Use the pronumeral n to represent the unknown number.

a a number is multiplied by 4 and then 5 is added to the result
b a number is divided by 3 and then 7 is added to the result
c a number is doubled and then 9 is subtracted from the result
d a number is multiplied by 6 and then 1 is subtracted from the result

WE 6.2C **9** Write an equation for each statement. Use x for the unknown number.

a 5 more than a number is equal to 11
b 3 less than a number is equal to 4
c a number multiplied by 6 is equal to 18
d a number divided by 2 is equal to 5
e twice a number is equal to 24
f a number is subtracted from 10 and the result is 8
g a number tripled plus 1 is equal to 7
h two-fifths of a number minus 8 is equal to 0

WE 6.2D **10** Write an equation for each statement.

a x divided by 6 is equal to y
b the sum of x and 11 is equal to 20
c y is 9 less than x
d b is 130 more than a
e q is 10 times the value of p
f a quarter of d is equal to c
g m is double the value of n
h f is three-tenths of the value of g

Exercise 6.2B: Problem solving and reasoning

11 Your friend takes 3 pens out of their pencil case. If there were originally p pens in your friend's pencil case, write a formula for the number of pens, n, left in your friend's pencil case.

12 Natasha has x dollars in her wallet. She receives \$20 as a gift. Write a formula for the amount of money, t, Natasha now has in her wallet.

13 Teresa has n Easter eggs. Write an expression to represent each situation below.

- **a** Her friend has 10 more eggs.
- **b** Her cousin has three times as many eggs.
- **c** Her younger brother has half as many eggs.
- **d** Her sister has four fewer eggs.

14 A recipe for scrambled eggs requires 2 eggs per person. Write a formula for the number of eggs, n, needed to make scrambled eggs for p people.

15 At the start of each year, Ying Ying is 5 years older than her little sister.

- **a** Identify the variables in this problem and select a pronumeral to represent each variable.
- **b** Use your pronumerals to write a formula that represents the relationship between the girls' ages.

16 In Australian rules football, each goal is worth 6 points and each behind is worth 1 point. Write a formula to represent the number of points a team scores, p, if they score g goals and b behinds.

17 Conor receives p dollars per week as pocket money.

- **a** How much pocket money does he receive:
  - **i** in a fortnight
  - **ii** in a year?
- **b** If he also earns w dollars per week for mowing his neighbour's lawn, how much does he receive each week?

18 The cost of hiring a canoe is \$15 per hour plus an extra booking fee of \$10. The total cost of hiring a canoe, c, can be represented using the equation $c = 15n + 10$.

- **a** Identify all the variables in this equation.
- **b** How many terms are there in the expression on the right-hand side of the equation?
- **c** What does the constant term represent?

19 The cost of hiring a car is made up of a daily rate plus a set fee for insur The total cost of hiring a car, c, can be calculated using the formula $c = 25d + 110$.

- **a** What variable does the pronumeral d represent in this formula?
- **b** How much does the insurance cost?
- **c** Excluding the set fee for insurance, how much does it cost to hire the car per day?
- **d** How much does it cost to hire the car for one week?

20 Chelsea and Olivia are planning a pizza party. A large pizza has 8 slices and costs \$25. There are n people coming to the party, and it is assumed that each person plans to eat three slices.

- **a** Write a formula in terms of n for the number of slices of pizza, s, to be eaten at the party.
- **b** Write a formula in terms of s for the number of pizzas, p, that Chelsea and Olivia should buy.
- **c** Assume the formula in part **b** results in p being a whole number. Write a formula for the total cost, c, of buying p pizzas for the party.
- **d** If p is not a whole number, how should Chelsea and Olivia determine the number of pizzas they need to buy?

21 Each week, your family buys 10 apples, seven bananas and six oranges from the local fruit market. The cost varies each week.

- **a** Identify all the variables in this problem.
- **b** How many terms would you expect there to be in an expression for the total weekly cost of fruit? Explain your reasoning.
- **c** Choose a pronumeral to represent the cost of each type of fruit.
- **d** Write a term for the cost of:
  - **i** the apples
  - **ii** the bananas
  - **iii** the oranges.
- **e** Write an expression for the total weekly cost of the fruit.
- **f** Let w = the total weekly cost of the fruit. Use your answer to part **e** to write a formula for this relationship.

22 Your family buys 500 grams of chicken and 300 grams of cheese from the deli every week. The cost varies each week. Write a formula for the relationship between the total weekly cost of chicken and cheese and the price, per gram, of chicken and cheese.

23 Entrance to the museum costs $10 per student. Students who also wish to attend an optional show at the planetarium need to pay an extra $5. Write a formula for the total cost of admission to the museum, including the additional planetarium costs, for a large group of Year 7 students.

Exercise 6.2C: Challenge

24 If y represents an unknown number, write an expression for the sum of the number multiplied by itself and 3. Hint: Use exponent form.

25 Which expression in each pair of expressions represents the larger number?

- **a** $x + 3$ and $x + 5$
- **b** $x + 1$ and $x - 1$
- **c** $x - 7$ and $x - 4$
- **d** $x - 2$ and x
- **e** $2x + 7$ and $2x + 4$
- **f** $3x - 6$ and $3x + 1$

26 ‘Like terms’ are terms which contain the same pronumerals in any order. For example, ab, $-ab$ and $10ba$ are like terms, but ab and $8ac$ are not like terms. Determine whether the following pairs of terms are like terms.

- **a** x and $8x$
- **b** $2b$ and $-a$
- **c** $-8a$ and $16a$
- **d** xy and $9xy$
- **e** yx and $-xy$
- **f** 8 and $8x$
- **g** 2 and -10
- **h** $-10abc$ and $-10ab$

27 Identify any like terms in the following lists of terms.

- **a** $x, y, -x, -y, 20x, -20y$
- **b** $a, ab, 10a, -ba, 9a, \frac{1}{2}ab$
- **c** $xyz, xzy, yxz, yzx, zxy, zyx$
- **d** $23mn, 14m, 18n, -nm, -3m$
- **e** $7a, 7b, 7c, 7d, 7e, 7f$
- **f** $9gh, 17hg, -45h, -32gh, 20g$
- **g** $abc, -cab, 10ab, -5bc, -6ba, 4cb$
- **h** $4x, 4, -12x, -12, 10x, 10$

Online resources:

Interactive skillsheet Expressions and equations

Interactive skillsheet Writing formulas

Worksheet Identifying terms, expressions and equations

Investigation School supplies

Quick quiz 6.2

Lesson 6.3

Order of operations in algebra

Learning intentions

By the end of this lesson you will be able to ...

✔ write algebraic expressions and equations using grouping symbols.

Inter-year links

Support	Writing formulas
Year 8	5.1 Pronumerals
Year 9	2.1 Simplifying

Key content video
Order of operations in algebra

Grouping symbols

- Operations inside **grouping symbols** are always performed first.

  Grouping symbols include brackets, the fraction line in a quotient, and the square root sign.

  For example:

  → brackets

  $$\begin{aligned}2(7-3) &= 2 \times (7-3)\\ &= 2 \times 4\\ &= 8\end{aligned}$$

  → fraction line

  $$\begin{aligned}\frac{7-3}{2} &= \frac{4}{2}\\ &= 2\end{aligned}$$

  → square root sign.

  $$\begin{aligned}2\sqrt{7-3} &= 2 \times \sqrt{7-3}\\ &= 2 \times \sqrt{4}\\ &= 2 \times 2\\ &= 4\end{aligned}$$

- The multiplication sign between numbers and brackets is usually left out.

  For example, $2 \times (x+1) = 2(x+1)$.

Order of operations and laws of arithmetic

- The order of operations also applies to algebraic expressions and equations.

B	**Brackets**	Operations inside grouping symbols are always performed first.
I	**Indices (or exponents)**	Next, simplify powers and square roots.
D **M**	**Division** **Multiplication**	Then working from left to right, perform any multiplication or division.
A **S**	**Addition** **Subtraction**	Finally, working from left to right, perform any addition or subtraction.

- All the laws of arithmetic apply to algebraic terms and expressions, this means:

	Arithmetic	**Algebra**
The associative law	$(3+5)+2 = 3+(5+2)$ $(3 \times 5) \times 2 = 3 \times (5 \times 2)$	$(a+b)+c = a+(b+c)$ $(a \times b) \times c = a \times (b \times c)$
The commutative law	$3+2 = 2+3$ $3 \times 2 = 2 \times 3$	$a+b = b+a$ $a \times b = b \times a$

Exponents

- Repeated multiplication of variables can be written in exponent form.
  For example, $a \times a \times a = a^3$.
- Exponents or powers only apply to the number or pronumeral immediately to the left of the power.
  Brackets can be used to apply the power to a whole term.
  Compare $2x^2 = 2 \times x^2 = 2 \times x \times x$ with $(2x)^2 = 2x \times 2x$.

Worked example 6.3A Writing expressions using grouping symbols

Write an expression for each statement, using brackets where appropriate.

a Add 2 to a, then multiply the result by 3
b The sum of a and b all divided by 5
c a is multiplied by 4 and the result is squared
d 7 less than half of a

THINK	**WRITE**
a 1 Think about which operation is taking place. ‘Add 2’ means +2.	**a** $a + 2$
2 Look for words that refer to the order of operations. ‘*Then* multiply *the result* by 3’ means the multiplication should be performed *after* the addition. Group $a + 2$ using brackets, then multiply the brackets by 3.	$(a + 2) \times 3$
3 Simplify the product by placing the number first and removing the multiplication sign.	$= 3(a + 2)$
b 1 Finding ‘the sum’ means to add, so the sum of a and b is a plus b.	**b** $a + b$
2 ‘*All* divided by 5’ means the whole sum should be divided by 5. Group the terms together using brackets, then divide the whole expression by 5.	$(a + b) \div 5$
3 Simplify the quotient by rewriting the expression as a fraction. The fraction line is a grouping symbol, so the brackets are no longer necessary.	$= \frac{a + b}{5}$
c 1 ‘Multiplied by 4’ means 4 times a. Simplify the product.	**c** $a \times 4 = 4a$
2 ‘*The result* is squared’ means the whole term should be squared. Brackets are required to square the whole term, including the coefficient.	$(4a)^2$
d ‘7 less’ means subtract 7 from ‘half of a’, which is a divided by 2. Simplify the quotient by writing it as a fraction.	**d** $a \div 2 - 7 = \frac{a}{2} - 7$

Worked example 6.3B Writing formulas using grouping symbols

Jasmine is doorknocking on her street to raise money for the Australian Red Cross. Her parents donate x dollars before she leaves the house. At the first house she visits, she collects \$10. At the second house she visits, her generous neighbour triples the amount of money in her collection tin.

a Identify the variables in the problem.

b Write an expression for the amount of money in Jasmine's tin after she leaves the first house.

c Write a formula for the total amount of money in Jasmine's collection tin, t, after she visits the second house.

THINK	**WRITE**
a Identify the variables in the problem.	**a** x = the initial amount of money in Jasmine's collection tin when she leaves the house t = the total amount of money in Jasmine's collection tin after she visits the second house
b Jasmine collects \$10 at the first house, so the amount of money in her tin is equal to her parents' donation, x, plus the \$10 she collects at the first house.	**b** $x + 10$
c **1** After Jasmine visits the second house, the total amount of money in the tin is 3 times the amount of money that was already in the tin. Brackets are necessary to group $x + 10$, the amount of money that was already in the tin. Simplify the product by placing the number first and removing the multiplication sign.	**c** $(x + 10) \times 3 = 3(x + 10)$
2 Write the formula by writing an equation for the relationship between the two variables. Equate the total amount of money in the tin, t, with the expression in terms of x.	The formula is $t = 3(x + 10)$.

Helpful hints

- ✔ Always consider the order of operations when dealing with algebraic expressions.

  BIDMAS

- ✔ The position of grouping symbols can change the order of operations – take care when placing brackets, fractions and square roots!

  For example,

  $2x + 1$ is not the same as $2(x + 1)$

  $\frac{x+1}{2}$ is not the same as $\frac{x}{2} + 1$

  $2\sqrt{x+1}$ is not the same as $2\sqrt{x} + 1$

Learning pathways

△ 1–10, 12

■ 1(d, h), 3(b, d, f, h), 5, 6(e–i), 8, 9, 11, 14–16

⬠ 3(b, d, f, h), 5(e–h), 6(h, i), 9, 11, 13–18

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Exercise 6.3A: Understanding and fluency

1 Operations can be performed on both numbers and pronumerals.

a Rewrite the following addition expressions as products. For example, $5 + 5 = 2 \times 5$.

i $7 + 7 + 7$ **ii** $10 + 10 + 10$ **iii** $x + x + x$

b Rewrite the following multiplication expressions as powers. For example, $5 \times 5 = 5^2$.

i $7 \times 7 \times 7$ **ii** $10 \times 10 \times 10$ **iii** $x \times x \times x$

c What did you notice about working with pronumerals in parts **a** and **b**?

2 Rewrite each expression using standard algebraic notation.

a $7 \times (x + 6)$ **b** $2 \times (x - 3)$ **c** $(x + 1) \times 5$ **d** $(x - 2) \times 8$

e $(x + 4) \div 7$ **f** $(x - 5) \div 11$ **g** $(9 + x) \div 2$ **h** $(1 - x) \div 6$

WE 6.3A 3 Write an expression for each statement, using brackets where appropriate.

a add 1 to *a*, then multiply the result by 10

b subtract 9 from *a*, then divide the result by 5

c *a* is first divided by 3, then 6 is added

d the sum of *a* and 21 is all divided by 7

e the product of 7 and *a* is all squared

f 12 less than *a* squared

g 4 less than the square root of *a*

h the square root of the product of 10 and *a*

4 The number sentence '7 less than half of *a*' is represented by the expression $\frac{a}{2} - 7$. Rewrite the number sentence to describe the expression $\frac{a - 7}{2}$.

5 Write number sentences to describe the order of operations in the following expressions and equations.

a $y = 7x + 10$ **b** $y = \frac{x + 12}{9}$ **c** $6 - \sqrt{x}$ **d** $q = \frac{p}{3} - 8$

e $\sqrt{2s - 1}$ **f** $s = t^2 - 1$ **g** $b = \frac{1}{2}(a^2 - 1)$ **h** $n = (m + 7)^2 - 16$

6 What is the first operation performed on *x* in each of the following expressions?

a $7(3x + 4)$ **b** $\frac{x - 5}{2} + 9$ **c** $5(x + 6) - 1$

d $\frac{3x - 2}{7}$ **e** $9\sqrt{\frac{x}{8} + 3}$ **f** $4(x^2 - 3) + 5$

g $\frac{x^2 + 7}{5} - 2$ **h** $3(8x - 1)^2$ **i** $\sqrt{x^2 + 12}$

WE 6.3B 7 An empty bus collects *p* passengers from the terminal and travels towards the first stop. At the first stop, 4 passengers get off. The number of passengers on the bus is doubled after picking people up at the second stop.

a Identify the variables in the problem.

b Write an expression for the number of passengers on the bus after it leaves the first stop.

c Write a formula for the total number of passengers on board the bus, *t*, after the second stop.

Exercise 6.3B: Problem solving and reasoning

8 Ada receives *m* dollars for her birthday. She spends $15 on a paint set and half of the remaining money on a canvas.

a Identify the variables in the problem.

b Write an expression for the amount of money in Ada's wallet after she buys a paint set.

c Write a formula for the amount of money Ada has left, *r*, after buying the paint set and canvas.

9 Explain the difference between the expressions $2x + 5$ and $2(x + 5)$.

10 A gardener divides a packet of seeds among four pots. He plants n seeds in each pot, then adds 3 more seeds to each pot. Write a formula for the total number of seeds planted, s.

11 Kai has x dollars in his wallet. His mum gives him another \$30 before he goes out. Kai gives half of his money to the cashier at the cinema and receives \$7 change. Write a formula for the amount of money Kai has left, y, after visiting the cinema.

12 Oscar has some friends over for a hotpot. They each put 3 dumplings in the hotpot. While Oscar and his friends are chatting, his little brother takes 5 of the dumplings for himself. Write a formula for the number of dumplings each person now gets to eat.

13 Suqi, Max and Nas sell hats at a craft market each week. A tenth of the money they make at the market goes to tax, \$75 goes to stall hire, and the remainder is divided equally between the three friends. Write a formula for the amount of money each person receives.

14 Celeste purchases equipment for her swimming club and bathers for each swimmer in the club. The equipment costs \$450 and each pair of bathers costs \$50. Write a formula for the total amount that each swimmer needs to pay if the cost of equipment and bathers is divided evenly among them.

15 A market garden has tomato plants arranged in rows of equal length. There are twice as many tomato plants in each row as there are rows. Write a formula for the total number of tomato plants in the garden.

16 The formula relating distance (d) in kilometres, speed (s) in kilometres per hour and time (t) in hours is $d = st$. Ananya visits her friend Sophie. She drives to and from Sophie's house at a speed of 50 km/h and spends 2 hours at Sophie's house. If Ananya is out for a total of t hours, write a formula to find how far Ananya lives from Sophie.

17 A square sports ground has light poles placed 3 metres apart along each side of the ground, starting at each corner of the ground.

- **a** Write a formula for the number of light poles placed along one side of the ground.
- **b** Write a formula for the number of light poles placed around the whole ground.
- **c** Explain why your answer to part **b** is not 4 times your answer to part **a**.

Exercise 6.3C: Challenge

18 Bob owns a coffee shop and sells coffee in three sizes: small, regular and large. He sells a small coffee for \$3.50, a regular coffee for \$4.00 and a large coffee for \$4.50. Bob sells twice as many small coffees as regular coffees, and four times as many large coffees as regular coffees. Write an expression for the total amount of money Bob makes from coffee sales using only one variable.

Online resources:

Interactive skillsheet Grouping symbols

Investigation Think of a number!

Quick quiz 6.3

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Checkpoint

Checkpoint quiz
Take the checkpoint quiz to check your knowledge of the first part of this module.

6.1 **1** The Blue Cup Cafe makes a profit of \$2 on every cup of coffee they sell.

a Use this information to complete the table below.

Number of cups of coffee	1	2	3	4	5
Profit (\$)					

b What are the two variables in this relationship?

c How much profit does The Blue Cup Cafe make if it sells:

i five cups of coffee **ii** 15 cups of coffee?

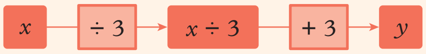
d How many cups of coffee would The Blue Cup Cafe have to sell to make a profit of:

i \$8 **ii** \$16?

6.1 **2** For the function machine below right, complete the following.

a Complete this table of values.

x	0	3	6	9	12
y					



b Plot the points on the Cartesian plane.

c State whether or not the points lie on a straight line.

6.2 **3** For each expression:

i determine the number of terms in the expression

ii identify any variables in the expression

iii identify the coefficient of each term that contains a variable

iv list any constant terms.

a $10x + 2$ **b** $\frac{1}{2}f$ **c** $a + 5b - 12$ **d** $x - y$

6.2 **4** Write an expression for each statement.

a the sum of t and 6 **b** 11 less than k

c 6 times the value of b **d** h divided by 9

6.2 **5** Write an equation for each statement.

a Half of x is equal to y. **b** 9 more than b is equal to c.

c 20 less than m is equal to n. **d** 6 times p is equal to q.

6.2 **6** Remy scores a total of 15 goals in n soccer games. If he scored the same number of goals in each game, write a formula for the number of goals Remy scores in each game, g.

6.3 **7** Write an expression for each statement using brackets where appropriate.

a 7 times the square root of the sum of a and 6

b b less than 8 all divided by 3

c c is divided by 9, then 6 is added to the result

d the sum of d and 4 all squared

6.3 **8** A teacher sets up four rows of n tables for a class test. She then realises she has not set up enough tables so adds two more tables to each row. Write an expression for the total number of tables set up for the test.

6.3 **9** Daniel is preparing fruit baskets for the apartments in his building. He is preparing one fruit basket for each apartment. Originally, there are p pieces of fruit in each basket, but Daniel decides to take one piece of fruit out of each basket to keep for himself. If there are three residents in each apartment, write an equation for the number of pieces of fruit each resident receives, n.

Lesson 6.4

Substitution

Learning intentions

By the end of this lesson you will be able to …

✔ substitute a value into an expression or equation and evaluate the result.

Inter-year links

Year 8 5.2 Substitution

Substitution

Key content video
Substitution

- **Substitution** involves replacing each pronumeral in an algebraic expression with a number.
- To **evaluate** an algebraic expression, substitute the known values of each pronumeral into the expression and work out the value of the expression.
  → It is important to follow the correct order of operations when evaluating an expression.
- Substitution can be used to find the value of an unknown variable when the values of all the other variables in an equation are known.

Let $x = 5$

$$x^2 + \frac{x}{3} = 5^2 + \frac{5}{3}$$

Worked example 6.4A Evaluating expressions containing one pronumeral

Evaluate each expression by substituting 4 for x.

a $x + 2$ **b** $3x - 1$

THINK	WRITE
a Replace x with 4 and complete the addition.	**a** $x + 2 = 4 + 2$ $= 6$
b Replace x with 4 and complete the operations in the correct order. Remember that $3x = 3 \times x$ and multiplication is performed before subtraction.	**b** $3x - 1 = 3 \times 4 - 1$ $= 12 - 1$ $= 11$

Worked example 6.4B Evaluating expressions containing more than one pronumeral

Evaluate each expression by letting $b = 3$ and $c = 5$.

a $7(c - b)$ **b** $\frac{b + c}{2}$

THINK	WRITE
a **1** Replace c with 5 and b with 3. **2** Perform the subtraction inside the brackets first, then the multiplication.	**a** $7(c - b) = 7(5 - 3)$ $= 7 \times 2$ $= 14$

b **1** Replace b with 3 and c with 5. **2** Remember the fraction line acts like a pair of brackets and groups b and c together. Perform the addition inside the fraction first, then the division.	**b** $\frac{b+c}{2} = \frac{3+5}{2}$ $= \frac{8}{2}$ $= 4$

Worked example 6.4C Using formulas

Find the value of y if $x = 3$.

a $y = \frac{x}{3}$ **b** $y = \frac{1}{3}(15 - x)$

THINK	WRITE
a To find the value of y, substitute $x = 3$ into the expression on the right-hand side of the equation and evaluate.	**a** $y = \frac{x}{3}$ $= \frac{3}{3}$ $y = 1$
b **1** Substitute $x = 3$ into the right-hand side of the equation and evaluate. Remember that $\frac{1}{3}(15 - x) = \frac{1}{3} \times (15 - x)$. **2** Perform the subtraction inside the brackets first, and then the multiplication.	**b** $y = \frac{1}{3}(15 - x)$ $= \frac{1}{3} \times (15 - 3)$ $= \frac{1}{3} \times 12$ $= \frac{12}{3}$ $y = 4$

Helpful hints

- ✔ Multiplication signs are left out of algebraic terms and expressions. The implied operation between a coefficient and a pronumeral, a coefficient and a set of brackets, or two pronumerals is *multiplication*.
  For example,
  $2x = 2 \times x$
  $2(x + 1) = 2 \times (x + 1)$
  $xy = x \times y$
- ✔ Always double-check for any changes in the sign when substituting negative numbers into an expression or equation. For example, when substituting $x = -2$ into the expression $10 - x$:

$$\begin{aligned} 10 - x &= 10 - (-2) \\ &= 10 + 2 \\ &= 12 \end{aligned}$$

Learning pathways

△ 1–6, 7(1st, 2nd columns), 8, 9–10(b, d, g, i), 11(a, c, f, g), 12, 14, 15

□ 4–6, 7(2nd, 3rd columns), 8, 9(f, h, j), 10, 11(c, e, h), 13, 16–18

⬠ 5(e, f), 6(d–f), 7(2nd, 3rd columns), 10(f–j), 11(f–h), 16–21

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Exercise 6.4A: Understanding and fluency

WE 6.4A **1** Evaluate each expression by substituting 6 for x.

a $x + 5$ **b** $x - 2$ **c** $3x$

d $\frac{x}{2}$ **e** $5x + 8$ **f** $21 - x$

2 Evaluate each expression by substituting 5 for y.

a $y + 12$ **b** $y - 1$ **c** $8y$

d $\frac{y}{5}$ **e** $3y - 2$ **f** $5 - y$

3 Evaluate $3k + 5$ by substituting each of these values for k.

a $k = 1$ **b** $k = 3$ **c** $k = 4$

d $k = 7$ **e** $k = 0$ **f** $k = \frac{1}{3}$

4 Evaluate $4m - 5$ by substituting each of these values for m.

a $m = 2$ **b** $m = 5$ **c** $m = 20$

d $m = 1$ **e** $m = 0$ **f** $m = \frac{1}{4}$

5 Evaluate each expression by substituting in the given value.

a $x + 10, \quad x = -1$ **b** $a + 3, \quad a = -2$

c $3 - g, \quad g = -1$ **d** $y + 7, \quad y = -7$

e $9 - k, \quad k = -5$ **f** $16 - m, \quad m = -4$

6 Evaluate each expression by substituting in the given value.

a $\frac{2}{3}x, \quad x = \frac{1}{3}$ **b** $10q, \quad q = \frac{1}{5}$

c $n + \frac{1}{2}, \quad n = \frac{1}{2}$ **d** $s - \frac{1}{4}, \quad s = \frac{2}{3}$

e $\frac{5}{2}b - 3, \quad b = \frac{1}{6}$ **f** $2y + \frac{3}{4}, \quad y = \frac{1}{4}$

WE 6.4B **7** Evaluate each expression by letting $a = 2$, $b = 3$ and $c = 6$.

a $a + 4$ **b** $b + c$ **c** $5c$

d $3a - c$ **e** $7a + b$ **f** $\frac{c}{b}$

g $6(a + c)$ **h** $\frac{a + b}{5}$ **i** $3(c - a)$

j $4b + 3c$ **k** $\frac{c + 4}{a}$ **l** $8a - 2b$

8 The term ab means $a \times b$. Rewrite the following terms using the multiplication symbol, ×.

a $2ab$ **b** a^2 **c** a^2b

9 Evaluate the following expressions by first rewriting the following terms using multiplication symbols, and then letting $m = 3$ and $n = 5$.

a m^2 **b** $2n^3$

c $m^2 + 5$ **d** $n^2 - 8$

e $4m^2$ **f** $2(n^2 + 3)$

g $9mn$ **h** $3n(m^2 + 1)$

i $\sqrt{3m}$ **j** $\sqrt{2n + 6}$

WE 6.4C **10** Find the value of y if $x = 4$.

a $y = 19 - 3x$ **b** $y = \frac{1}{4}x$ **c** $y = \frac{3}{2}x + 7$ **d** $y = \frac{1}{5}(x + 6)$ **e** $y = \frac{2}{3}(x - 7)$

f $y = \frac{4}{x}$ **g** $y = \frac{28}{x} + 3$ **h** $y = 10 - x^2$ **i** $y = 2x^2 - 40$ **j** $y = 3\sqrt{x}$

11 Find the value of p for each of these t values using the formula $p = 6t - 2$.

a $t = 1$ **b** $t = 9$ **c** $t = -2$ **d** $t = 6.4$

e $t = 5.3$ **f** $t = 0.8$ **g** $t = -\frac{1}{2}$ **h** $t = \frac{2}{3}$

Exercise 6.4B: Problem solving and reasoning

12 At the school canteen, a salad roll costs \$5 and a can of drink costs \$2.

- **a** Write an expression for the cost of r salad rolls and d cans of drink.
- **b** Evaluate the expression to find the cost of three salad rolls and five cans of drink.
- **c** Find the cost of eight salad rolls and seven cans of drink.

13 There are 12 months in one year, 10 years in a decade and 100 years in a century.

- **a** Write a formula that can be used to calculate the number of months, m, in a given number of years, y.
- **b** Use your formula to find the number of months in:
  - **i** 5 years
  - **ii** a decade
  - **iii** a century
  - **iv** 80 years.

14 The relationship between density, d, mass, m, and volume, v, can be represented by the formula $d = \frac{m}{v}$.

- **a** Calculate the values of d for these values of m and v.
  - **i** $m = 10, v = 2$
  - **ii** $m = 36, v = 9$
  - **iii** $m = 6.3, v = 3$
  - **iv** $m = 22, v = 4$
- **b** Find the average density (in grams per cm^3) of a 36 g chocolate bar that has a volume of 50 cm^3.

15 The relationship between distance travelled, d, average speed, s, and time, t, can be represented by the formula $d = st$.

Complete this table of values showing the distance travelled for different combinations of average speed and time of travel.

Average speed (km/h)	40	45	50	60	50	70
Time (h)	2	3	2.5	1.5	3	2.5
Distance (km)						

16 For each of the following tables of values:

- **i** fill out the table using the formula $d = st$.
- **ii** substitute the constant variable into the formula $d = st$.

a

Average speed (km/h)	40	40	40	40	40	40
Time (h)	1	2	3	4	5	6
Distance (km)						

b

Average speed (km/h)	40	45	50	55	60	65
Time (h)	5	5	5	5	5	5
Distance (km)						

17 In question **16**, you substituted $s = 40$ km/h and $t = 5$ hours into the formula $d = st$. Find two more pairs of values for s and t that result in the same total distance travelled.

18 The perimeter of a two-dimensional shape is equal to the total length of the shape's outside edge.

- **a** Consider a square with side lengths represented by *s*.
  - **i** Write a formula for the relationship between the total perimeter of a square, *p*, and its side length, *s*.
  - **ii** Use the formula to calculate the perimeter of a square with a side length of 2 cm.
- **b** A regular pentagon is a two-dimensional shape that has five sides of equal length.
  - **i** Write a formula for the relationship between the total perimeter of a pentagon, *p*, and the length of one side of the pentagon, *s*.
  - **ii** Use the formula to calculate the perimeter of a regular pentagon with a side length of 2 cm.
- **c** A regular polygon is a many-sided shape that has sides of equal length. Write a formula for the relationship between the perimeter of a polygon, *p*, that has *n* sides, and the length of one side of that polygon, *s*.

19 Alex has started working at the local pizza parlour. Alex's normal rate of pay is $15 per hour, but on Sundays he earns $18 per hour.

- **a** Write a formula for the amount of money Alex earns in a week.
- **b** In his first week, Alex works 40 hours from Monday to Friday and 3 hours on Sunday. Use your formula to determine how much Alex earned in this week.
- **c** At the beginning of the next week, Alex works for 5 hours on Sunday. How long would he have to work at the normal rate to earn the same amount of money as in the previous week? Give your answer to the nearest hour.
- **d** Alex does some research and discovers that it is illegal to pay him less than $21 per hour on Sundays. He brings this up with his manager, who agrees to the minimum rate of pay as well as paying him for the previous 2 weeks of stolen wages. Calculate the amount of money Alex is owed.

Exercise 6.4C: Challenge

20 Consecutive numbers are whole numbers that follow immediately after each other. For example, 1, 2 and 3 are consecutive numbers.

- **a** Write a formula that can be used to calculate the product of two consecutive numbers.
- **b** Use your formula to find the product of two consecutive numbers if one of the numbers is 23.
- **c** Explain why there is more than one answer to part **b**.

21 Consider an unknown two-digit number. Adding the sum of its digits to the product of its digits gives the original number.

- **a** Use algebra to find the two-digit number.
- **b** There is more than one solution to this problem. Find them all and explain how you know you have found all the solutions.

Online resources:

Interactive skillsheet Substitution	**Worksheet** Evaluating expressions	**Investigation** Heart rates	**Investigation** Formulas in spreadsheets	**Quick quiz** 6.4

Lesson 6.5

Solving equations by inspection

Learning intentions

By the end of this lesson you will be able to ...

✔ verify solutions to equations and solve simple linear equations.

Inter-year links

Year 8	6.2 Solving linear equations
Year 9	3.1 Solving linear equations

Key content video
Solving equations by inspection

Solving equations

- To **solve** an equation, you need to find the value (or values) that makes the equation true.

  For example, the **solution** to the equation $x + 3 = 5$ is $x = 2$ because $2 + 3 = 5$.

  → When solving equations, the equals sign should be viewed as an indication of equivalence. That is, an indication that the left-hand side of an equation is equal in value to the right-hand side of the equation.

  What value of x makes the left-hand side of the equation equal to the right-hand side of the equation?

$$\underbrace{x + 3}_{\substack{\text{left-hand} \\ \text{side} \\ \text{LHS}}} \overset{\text{equals sign}}{=} \underbrace{5}_{\substack{\text{right-hand} \\ \text{side} \\ \text{RHS}}}$$

Verifying solutions

- To verify a solution to an equation, substitute that value into the equation to see whether it makes a true statement.

  For example, $x = 2$ is the solution to the equation $x + 3 = 5$ because $2 + 3 = 5$ is a true statement.

Linear equations

- A **linear equation** is an equation where any variables are raised to the power of 1 and no two variables are multiplied together.

  For example, $y = x + 3$, $x + 1 = 3$ and $x + y + 1 = 0$ are linear equations.
- Linear equations with exactly one variable have a unique solution.
- Linear equations with more than one variable do not have a unique solution unless the values of all variables except one are specified.

Solution methods

- **Solving by inspection** involves mentally identifying the solution to an equation.
- To solve equations by inspection:
  1. Reword the equation as a question and identify a number which answers the question.
  2. Use substitution to check if the number is the solution to the equation.
- The **guess, check and improve** method involves using trial-and-error and should be used when the solution cannot be quickly mentally identified.
- To solve equations using the guess, check and improve method:
  1. Guess a possible solution.
  2. Use substitution to check if the number is the solution to the equation.
  3. Improve your guess and repeat steps 1 and 2 until the solution is found.

Worked example 6.5A Verifying a solution

Determine if $x = 11$ is the solution to $x - 4 = 7$.

THINK	**WRITE**
1 Write the equation.	$x - 4 = 7$
2 Substitute $x = 11$ into the expression on the left-hand side (LHS) of the equation and evaluate.	LHS $= x - 4$ $= 11 - 4$ $= 7$ $=$ RHS
3 Determine whether the statement is true or false by comparing the value of the left-hand side (LHS) to the value of the right-hand side (RHS) of the equation.	LHS = RHS
4 Write your answer. $11 - 4 = 7$ is a true statement, so $x = 11$ is the solution.	$x = 11$ is the solution to $x - 4 = 7$.

Worked example 6.5B Solving equations by inspection

Solve each equation by inspection.

a $a + 8 = 14$ **b** $4x = 20$ **c** $\frac{m}{5} = 9$

THINK	**WRITE**
a 1 Change the equation to a question. What number plus 8 is equal to 14?	**a** $a + 8 = 14$ Let $a = 6$
2 Check the solution using substitution.	LHS $= a + 8$ $= 6 + 8$ $= 14$ $=$ RHS LHS = RHS
3 Write the solution to the equation.	$a = 6$
b 1 Change the equation to a question. What number multiplied by 4 is equal to 20?	**b** $4x = 20$ Let $x = 5$
2 Check the solution using substitution.	LHS $= 4x$ $= 4 \times x$ $= 4 \times 5$ $= 20$ $=$ RHS LHS = RHS
3 Write the solution to the equation.	$x = 5$
c 1 Change the equation to a question. What number divided by 5 is equal to 9?	**c** $\frac{m}{5} = 9$ Let $m = 45$
2 Check the solution using substitution.	LHS $= \frac{m}{5}$ $= \frac{45}{5}$ $= 9$ $=$ RHS LHS = RHS
3 Write the solution to the equation.	$m = 45$

Worked example 6.5C Solving equations using the 'guess, check and improve' method

Use the 'guess, check and improve' method to solve $2x + 6 = 20$.

THINK

1 Set up a table to show each guess, the value of the left-hand side (LHS) of the equation and the value of the right-hand side (RHS) of the equation.
The first row of the table has been completed for a guess of $x = 3$.

2 Compare the value of the LHS and the value of the RHS and adjust your next guess accordingly. In this case, the LHS = 12 which is lower than the RHS = 20. The solution must therefore be higher than the first guess of $x = 3$.

3 Repeat step 2 until the value of the LHS = RHS.

4 Write the solution.

WRITE

Guess x	LHS $2x + 6$	RHS	LHS = RHS?
3	12	20	no

Guess x	LHS $2x + 6$	RHS	LHS = RHS?
3	12	20	no
8	22	20	no
7	20	20	yes

$x = 7$ is the solution to $2x + 6 = 20$.

Worked example 6.5D Using equations to solve worded problems

Jamie and Emma both collect basketball cards. Jamie has collected 8 more cards than Emma.

a Write an expression for the number of cards that Jamie has in his collection if Emma has x cards in her collection.

b If Jamie has 30 cards in his collection, write an equation that can be used to find how many cards Emma has in her collection.

c Solve the equation by inspection to find the value of x.

d How many cards does Emma have in her collection?

THINK

a Think about which operation is required. Jamie has 8 *more* cards than Emma, so Jamie has x plus 8 cards.

b Write an equation containing the variable x. If Jamie has 30 cards in his collection, the number of cards Emma has in her collection, x, plus 8 equals 30.

c Change the equation to a question. What number plus 8 is equal to 30? Check your solution using substitution.

d Emma has x cards in her collection.

WRITE

a $x + 8$

b $x + 8 = 30$

c $x + 8 = 30$

Let $x = 22$

$$\begin{aligned} \text{LHS} &= x + 8 \\ &= 22 + 8 \\ &= 30 \\ &= \text{RHS} \end{aligned}$$

LHS = RHS

$x = 22$

d Emma has 22 cards in her collection.

Helpful hints

- ✔ Try to think of the equals sign, =, as an indication that the left-hand side of an equation is equal in value to the right-hand side of the equation, rather than a command to ‘work out the answer’.
- ✔ Remember to define your pronumerals before writing equations to represent a worded problem.

Learning pathways

▲ 1, 2, 3(1st column), 4, 5(a, b), 6, 8, 10, 12(a–f), 14–16, 19

■ 2(d–i), 3(2nd column), 5–9, 11, 12(e–h), 13, 15, 16, 18, 21

⬟ 2(g–i), 3(n–r), 5(d), 8(c, d), 9, 11, 12(f–h), 17, 18, 20–23

ANS p536

Exercise 6.5A: Understanding and fluency

1 Substitute $x = 4$ into the expression on the left-hand side of each equation. Does this substitution result in a true statement?

a $x + 9 = 13$ **b** $x - 3 = 2$ **c** $9 - x = 1$

d $5x = 20$ **e** $\frac{x}{2} = 2$ **f** $3x = 7$

WE 6.5A **2** Determine if the value given in brackets for each pronumeral is the solution to the equation.

a $m - 8 = 5\ (m = 13)$ **b** $7p = 28\ (p = 3)$ **c** $k + 5 = 14\ (k = 8)$

d $11 - n = 7\ (n = 4)$ **e** $\frac{t}{4} = 6\ (t = 24)$ **f** $9y = 28\ (y = 3)$

g $6 + c = 7\ (c = 1)$ **h** $f - 4 = 4\ (f = 6)$ **i** $\frac{x}{5} = 1\ (x = 5)$

WE 6.5B **3** Solve each equation by inspection.

a $a + 7 = 12$ **b** $b - 1 = 5$ **c** $2c = 16$

d $\frac{d}{3} = 6$ **e** $5e = 50$ **f** $\frac{f}{8} = 2$

g $g - 6 = 3$ **h** $h + 4 = 11$ **i** $10 - i = 6$

j $7j = 56$ **k** $3 + k = 13$ **l** $l - 5 = 6$

m $\frac{20}{m} = 5$ **n** $\frac{x}{6} = 9$ **o** $x + 11 = 11$

p $9 = p - 2$ **q** $33 = 3q$ **r** $3 = \frac{t}{4}$

4 Complete this table to solve $4x + 11 = 35$ using the ‘guess, check and improve’ method.

Guess x	LHS $4x + 11$	RHS	LHS = RHS?
3	23	35	no
9	47	35	no
5		35	
7		35	

WE 6.5C **5** Use the ‘guess, check and improve’ method to solve each equation.

a $2x + 7 = 29$ **b** $3x - 5 = 19$ **c** $6x - 4 = 74$ **d** $7x + 9 = 121$

6 Which solution makes $4x + 3 = 11$ a true statement?

A $x = 1$ **B** $x = 2$ **C** $x = 3$ **D** $x = 4$

7 What value of x makes $2x + 1 = 7$ a true statement?

8 Solve each equation.

a $2x - 3 = 1$ **b** $5x + 2 = 22$ **c** $4x - 1 = 11$ **d** $9 - 2x = 7$

9 Write an equation that has a solution of $x = 6$ and includes:

- **a** addition
- **b** multiplication
- **c** subtraction and division
- **d** brackets, addition and subtraction.

10 For each of the following statements:

- **i** Write an equation. Use x to represent the unknown number.
- **ii** Solve the equation using any method.
- **iii** Check your solution by substituting the solution back into the equation.
  - **a** When 6 is added to a number, the result is 13.
  - **b** When a number is multiplied by 9, the result is 54.
  - **c** 7 less than a number is equal to 5.
  - **d** Twice a number is 16.
  - **e** A number is multiplied by 2, then 5 is added to give 23.
  - **f** 8 less than a number all times 3 is equal to 24.
  - **g** The sum of one quarter of a number and 2 is equal to 20.
  - **h** Half of the sum of a number and 3 is equal to 13.

Exercise 6.5B: Problem solving and reasoning

11 Consider the linear equation $y = 3x + 5$. Find the value of:

a y when $x = 1$ **b** y when $x = 12$ **c** x when $y = 11$ **d** x when $y = 20$

12 Consider the linear equation $10a - 2b = 20$. Determine the value of:

a b when $a = 4$ **b** a when $b = 5$ **c** b when $a = 2$ **d** a when $b = 25$

WE 6.5D **13** T-shirts are on sale for $15 each.

- **a** Write an expression for the cost of n T-shirts.
- **b** Write an equation to represent how many T-shirts you can buy with $60.
- **c** Solve the equation by inspection to find the value of n.
- **d** How many T-shirts can you buy with $60?

14 Nikita has $5 and buys a sausage in bread at a fundraising event.

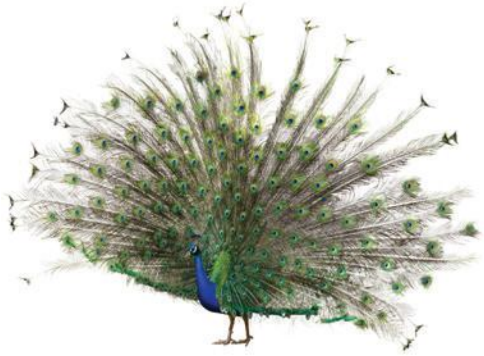
- **a** If a sausage in bread costs s dollars, write an expression for the amount of change Nikita receives.
- **b** Nikita receives $3 change. Write an equation that can be used to find the cost of a sausage in bread.
- **c** Solve the equation by inspection to find the value of s.
- **d** How much is a sausage in bread?

15 A bus has 17 unoccupied seats and a total seating capacity of 48.

- **a** Select and define a pronumeral to represent the unknown in this problem.
- **b** Write an equation to represent this problem.
- **c** Solve the equation by inspection.
- **d** Assuming there is one person in each of the occupied seats, how many people are on board the bus?

16 James earns \$12 an hour working at the local deli. He earns a total of \$336 in January.

- **a** Select and define a pronumeral to represent the unknown in this problem.
- **b** Use the pronumeral from part **a** to write an equation for the relationship between the total number of hours James worked, and the total amount of money he earned in January.
- **c** Use the equation from part **b** to determine how many hours he worked in January.



17 There are twice as many peacocks at the zoo as there are giraffes.

- **a** Write an equation to represent this problem.
- **b** If there are 8 peacocks at the zoo, use the equation from part **a** to determine how many giraffes there are at the zoo.

18 There are x cars and y motorbikes in a car park. Altogether, there are 15 vehicles and 42 wheels in the car park.

- **a** Write an equation to represent the total number of vehicles in the car park.
- **b** Write an equation to represent the total number of wheels in the car park.
- **c** Use the 'guess, check and improve' method to determine how many cars and motorbikes are in the car park.

19 In a volleyball tournament, Eleanor plays 4 more games than Jing. They play a total of 12 games. If Eleanor played x games and Jing played y games, complete the following.

- **a** Write an equation to represent the relationship between the number of games Eleanor played and the number of games Jing played.
- **b** Write an equation to represent the total number of games they played.
- **c** Use the 'guess, check and improve' method to determine how many games Eleanor and Jing each played.

20 Identify the most efficient method to solve each of the following equations. Justify your selection by referring to the operations in each equation.

a $10 - x = 2$ **b** $5(x + 3) = 4x$

Exercise 6.5C: Challenge

21 For each of the following equations:

- **i** reword the equation as a question as if you are solving by inspection
- **ii** determine whether the equation has a unique solution, no solutions or infinitely many solutions
- **iii** explain your answer to part **ii**.

a $x + 1 = x$ **b** $x + 2 = x + 3$ **c** $y = x$ **d** $y = x + 1$

22 Solve each equation by inspection.

a $\frac{x}{2} + 3 = 4$ **b** $\frac{x}{3} - 1 = 3$ **c** $\frac{x+1}{4} = 2$ **d** $\frac{x-2}{6} = 1$

23 Consider the equation $x^2 + 10 = 7x$.

- **a** Use the 'guess, check and improve' method to solve the equation.
- **b** There are two possible solutions to this equation. Find a second solution to this equation.
- **c** Compare this equation to the equation in question **3**. All the equations in question **3** have only one solution. What feature of the equation above generates a second solution?

Online resources:

Interactive skillsheet Solving equations by inspection

Worksheet Solving equations by inspection

Worksheet Using 'guess, check and improve' to solve equations

Quick quiz 6.5

Investigation Everyday groceries

Lesson 6.6

Solving equations using inverse operations

Learning intentions

By the end of this lesson you will be able to ...

✔ solve linear equations using inverse operations.

Inter-year links

Year 8	6.3 Solving linear equations with the unknown on both sides
Year 9	3.1 Solving linear equations

Key content video
Solving equations using inverse operations

Equivalent equations

- **Equivalent equations** are equations that have exactly the same solution (or solutions).

  For example, the following equations are equivalent because they all have the solution $x = 1$.

$$\begin{aligned} -3 \quad 2x + 3 &= 5 \quad -3 \\ \div 2 \quad 2x &= 2 \quad \div 2 \\ x &= 1 \end{aligned}$$

- The simplest equivalent equation to any equation is the solution, in which the pronumeral is the subject of the equation.

  For example, the solution to the equivalent equations listed above is represented by the equation $x = 1$.

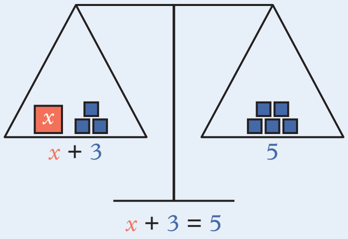
The balance model

- In the balance model, an equation is viewed as a set of balanced scales with each scale representing one side of the equation.

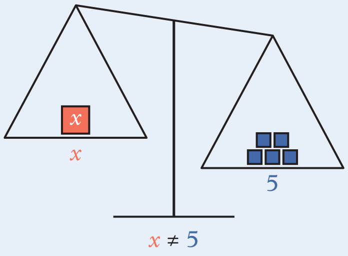
  To 'solve' an equation using the balance model:

  1. Remove items from the scale containing the pronumeral block until there is a single pronumeral block by itself on one side of the scales.
  2. Make the same changes to the other scale to re-balance the scales. With only one pronumeral on one scale, the scales represent the simplest equivalent equation, or the solution to the equation.

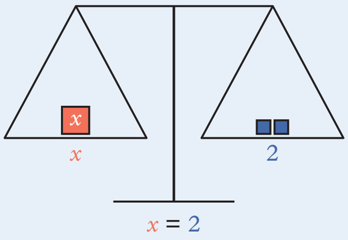
  For example, the diagram below represents the equation $x + 3 = 5$.



  To get x by itself, we remove three blue blocks from the left-hand scale.



  To re-balance, we also remove three from the right-hand scale.



  Once these changes have been made, it is clear that one x block has the same mass as 2 blue blocks, therefore $x = 2$.

- The balance model can be used to solve equations like $3x = 9$ by splitting the blocks on each scale into three groups and then removing blocks so that only one group remains on each scale.

Inverse operations

Operation	Inverse operation
+ 3	− 3
− 3	+ 3
× 3	÷ 3
÷ 3	× 3

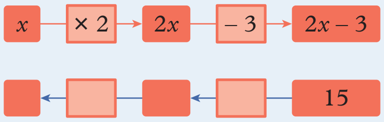
- **Inverse operations**, or opposite operations, reverse the effect of another operation.
- Adding 0 to an expression does not change the value of the expression. Addition and subtraction are inverse operations because adding and then subtracting (or subtracting and then adding) the same value results in 0.
- Multiplying an expression by 1 does not change the value of the expression. Multiplication and division are inverse operations because multiplying and then dividing by (or dividing and then multiplying by) the same value results in 1.

Backtracking

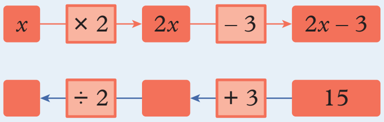
- **Backtracking** involves drawing a function machine to represent an equation, and then using inverse operations to work backwards to the solution.
- To solve an equation using backtracking, complete the following steps. This example demonstrates how to solve $2x - 3 = 15$.

  1 Draw a function machine to show how the expression is created from a single pronumeral.

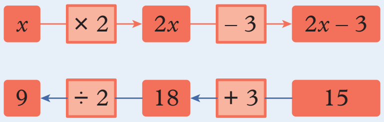
  2 Add a second function machine underneath and put the stand-alone value in the right-hand box of the second row.



  3 Identify the inverse operations to those on the top row.



  4 Perform the inverse operations to work backwards to the solution.



- When performing backtracking, each column of the function machines represents an equivalent equation.
- The first column represents the simplest equivalent equation, which contains the pronumeral by itself on one side of the equation and the solution on the other side.

Worked example 6.6A Solving a one-step equation using the balance method

Solve $x - 7 = 11$ using the balance method.

THINK	WRITE
1 Determine what operation needs to be undone so that the pronumeral (x) is by itself on one side of the equation. '− 7' needs to be undone.	$x - 7 = 11$
2 Undo '− 7' by performing the inverse operation of '+ 7' on both sides of the equation.	$x - 7 + 7 = 11 + 7$
3 Simplify the equation to get the solution.	$x = 18$
4 Check your solution.	When $x = 18$, $x - 7 = 18 - 7$ $= 11$

Worked example 6.6B Drawing a function machine to represent an expression

Draw a function machine to represent each expression.

a $6j - 1$ **b** $\frac{k + 5}{3}$

THINK

a **1** Write the variable in the first box.

2 Work out the first operation to be performed. The coefficient of $6j$ is 6, so the first operation is to multiply j by 6.

3 The next operation is performed on the entire contents of the second box. 1 needs to be subtracted from $6j$ to get $6j - 1$.

b **1** Write the variable in the first box.

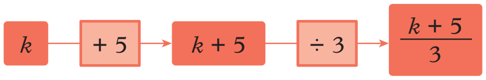
2 Work out the first operation to be performed. The fraction line groups $k + 5$ together, so the first operation is to add 5 to k.

3 The next operation is performed on the entire contents of the second box. $k + 5$ needs to be divided by 3 to get $\frac{k + 5}{3}$.

WRITE

a

b



Worked example 6.6C Solving a two-step equation using backtracking

Solve $2x + 5 = 11$ using backtracking.

THINK

1 Draw a function machine to show how the expression is created from a single pronumeral.

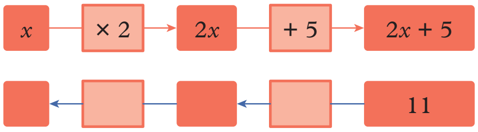
2 Add a second function machine underneath and put the stand-alone value in the right-hand box of the second row.

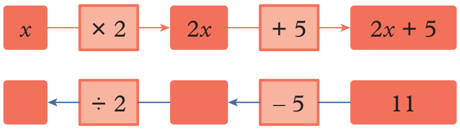
3 Identify the inverse operations to those on the top row. The inverse operation to '+ 5' is '– 5' and the inverse operation to '× 2' is '÷ 2'.

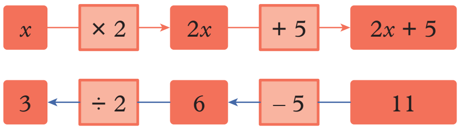
4 Perform the inverse operations to work backwards to the solution.

5 Write the solution.

WRITE







$x = 3$

Worked example 6.6D Solving a two-step equation with grouping symbols using backtracking

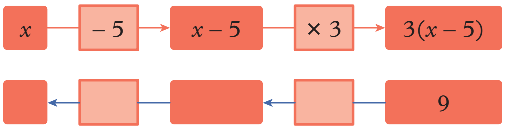
Solve $3(x - 5) = 9$ using backtracking.

THINK

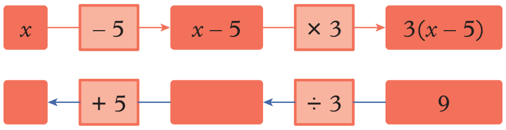
WRITE

1 Draw a function machine to show how the expression is created from a single pronumeral.

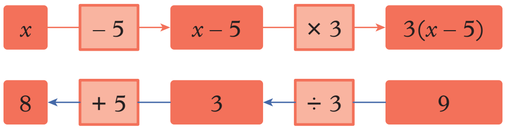
2 Add a second function machine underneath and put the stand-alone value in the right-hand box of the second row.



3 Identify the inverse operations to those on the top row. The inverse operation to '× 3' is '÷ 3' and the inverse operation to '– 5' is '+ 5'.



4 Perform the inverse operations to work backwards to the solution.



5 Write the solution.

$x = 8$

Helpful hints

✔ To find the solution, the pronumeral does not have to be on the left-hand side of the equation – as long as the pronumeral is by itself on one side of the equation, you've found the solution!

✔ Remember that in equations such as $1 - 2x = 3$, the coefficient of x is -2. The negative sign must also be taken into account when applying inverse operations.

✔ With practice, you may be able to determine the inverse operations you need to perform in order without having to draw a function machine.

Learning pathways

▲ 1–5, 6–9(1st, 2nd columns), 10, 12(a, c), 13(c–f), 14(a–f), 15, 16, 18(a, b), 21

■ 3–5, 6(g–l), 7(e–h), 8(e–h), 9(f–i), 11(g–l), 12(b, d), 13(e–h), 14(d–i), 16, 17, 19, 22, 24(a, b)

⬟ 7–9(f–h), 11(g–l), 13(e–h), 14(d–i), 18–20, 22–25

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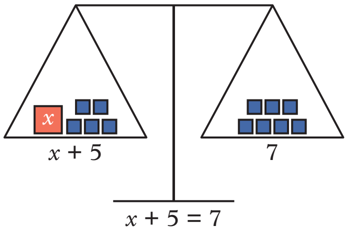
Exercise 6.6A: Understanding and fluency

1 Consider the diagram on the right. Each blue block has a mass of 1 g. The mass of the x block is unknown.



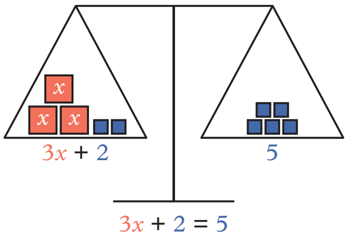
a What change do you need to make to the left-hand scale to get x by itself?

b What change should you make to the right-hand scale to keep the scales balanced?

c Draw a new diagram showing the scales after the changes are made in parts **a** and **b**.

d What is the mass of the x block?

2 Consider the diagram on the right. Each blue block has a mass of 1 g. The mass of the x block is unknown.



- **a** What two changes do you need to make to the left-hand scale to get x by itself? Hint: Remember you need to be able to make the same change to both the left and the right-hand scales. Begin by removing the blue blocks from the left-hand scale.
- **b** What two changes should you make to the right-hand scale to keep the scales balanced?
- **c** Draw a new diagram showing the scales after the changes are made in parts **a** and **b**.
- **d** What is the mass of the x block?

3 Identify the inverse operation to each of the operations listed below.

a $+5$ **b** -3 **c** $\times 8$ **d** $\div 2$

4 Solve each equation by completing the incomplete calculations below.

a
$$\begin{aligned} a + 5 &= 12 \\ a + 5 - \underline{\quad} &= 12 - \underline{\quad} \\ a &= \underline{\quad} \end{aligned}$$

b
$$\begin{aligned} b - 3 &= 8 \\ b - 3 + \underline{\quad} &= 8 + \underline{\quad} \\ b &= \underline{\quad} \end{aligned}$$

c
$$\begin{aligned} 8c &= 32 \\ \frac{8c}{\underline{\quad}} &= \frac{32}{\underline{\quad}} \\ c &= \underline{\quad} \end{aligned}$$

d
$$\begin{aligned} \frac{d}{3} &= 5 \\ \frac{d}{3} \times \underline{\quad} &= 5 \times \underline{\quad} \\ d &= \underline{\quad} \end{aligned}$$

5 Apply the inverse operation(s) given in the brackets to both sides of the following equations to solve each equation.

a $x + 7 = 12 \quad (-7)$

b $a - 9 = 1 \quad (+9)$

c $4p = 24 \quad (\div 4)$

d $\frac{s}{6} = 5 \quad (\times 6)$

e $10 + y = 33 \quad (-10)$

f $\frac{1}{2}f = 45 \quad \left(\div \frac{1}{2}\right)$

g $2d = 18 \quad (\div 2)$

h $3x + 1 = 10 \quad (-1, \text{then} \div 3)$

WE 6.6A **6** Solve the one-step equations below using the balance method. Verify your solutions using substitution.

a $x + 8 = 13$ **b** $x - 4 = 2$ **c** $3x = 24$ **d** $x - 5 = 25$

e $7x = 56$ **f** $x - 6 = 11$ **g** $3x = 36$ **h** $x + 1 = 8$

i $11x = 33$ **j** $x + 15 = 4$ **k** $4x = 120$ **l** $23 - x = 50$

WE 6.6B **7** Draw a function machine to build each expression.

a $5(x + 4)$ **b** $\frac{x}{3} - 2$ **c** $4x - 3$ **d** $\frac{x + 6}{8}$

e $9x + 1$ **f** $\frac{x}{4} + 6$ **g** $\frac{x - 5}{9}$ **h** $\frac{5x}{7}$

WE 6.6C **8** Solve the two-step equations below using backtracking.

a $4x + 3 = 27$ **b** $\frac{x}{5} + 1 = 3$ **c** $2x - 5 = 1$ **d** $\frac{x}{3} - 4 = 2$

e $\frac{x}{4} - 1 = 6$ **f** $7x + 1 = 29$ **g** $2 + 3x = 20$ **h** $\frac{x}{9} + 12 = 20$

WE 6.6D **9** Solve the two-step equations below using backtracking.

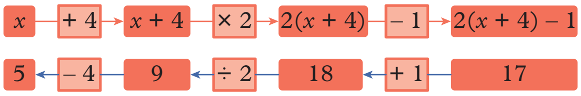
a $2(x + 3) = 16$ **b** $\frac{x - 4}{5} = 3$ **c** $3(x - 2) = 27$ **d** $\frac{x + 3}{7} = 2$

e $\frac{x - 1}{6} = 1$ **f** $\frac{x + 2}{3} = 6$ **g** $4(x - 7) = 12$ **h** $\frac{x - 5}{4} = 8$

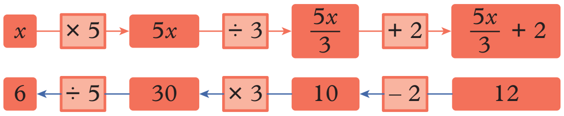
10 For each pair of function machines, write:

i the equation to be solved **ii** the solution to the equation.

a



b



11 Solve each of the following equations using inverse operations. Verify your solutions using substitution.

a $9x + 4 = 13$ **b** $\frac{a}{2} + 5 = 12$ **c** $7(x + 1) = 49$ **d** $\frac{x + 4}{3} = 5$

e $\frac{p}{6} - 3 = 4$ **f** $6(d - 4) = 30$ **g** $3f - 7 = 17$ **h** $\frac{z - 8}{7} = 3$

i $11x + 3 = 47$ **j** $\frac{t}{5} + 8 = 14$ **k** $\frac{q + 6}{4} = 1$ **l** $\frac{2y}{7} = 4$

Exercise 6.6B: Problem solving and reasoning

12 Consider the linear equation $y = \frac{x}{2} + 3$. Find the value of:

a y when $x = 2$ **b** y when $x = 10$ **c** x when $y = 5$ **d** x when $y = 0$

13 Solve the following linear equations by first substituting the bracketed values into each equation.

a $y = 3x - 10 \quad (y = 2)$ **b** $y = 2(x + 7) \quad (y = 20)$

c $v = \frac{n}{4} - 13 \quad (v = 7)$ **d** $x + 2y = 4 \quad (x = 4)$

e $3a - 5b = 3 \quad (b = 3)$ **f** $m = \frac{n + 5}{8} \quad (m = 0)$

g $y = \frac{x - 9}{3} \quad (y = 5)$ **h** $1 + \frac{q}{3} = 5 + p \quad (q = 3)$

14 What is the first operation performed on x in each expression?

a $7(3x + 4)$ **b** $\frac{x - 5}{2} + 9$ **c** $5(x + 6) - 1$

d $\frac{3x - 2}{7}$ **e** $9\left(\frac{x}{8} + 3\right)$ **f** $4(x - 3) + 5$

g $\frac{x + 7}{5} - 2$ **h** $3(8x - 1)$ **i** $\frac{6x + 3}{11}$

15 There are x mice in a cage. The pet-shop owner moves more mice into the cage so that there is now double the number of mice for sale. Monique buys 3 mice, leaving 15 mice in the cage.

a Write an equation to represent this problem.

b Solve this equation to determine how many mice were originally in the cage.

16 An animal shelter provides care to injured wildlife before they are released back into their natural habitat. There are n animals in the shelter on Monday. On Wednesday, 3 animals are brought in, half of the animals in the shelter are released on Friday, then 5 more animals are brought in on Saturday.

a Write an expression for the number of animals in the shelter at the end of Saturday.

b If there were 3 animals in the shelter on Monday, use the expression from part **a** to calculate the number of animals in the shelter at the end of Saturday.

17 You have some money in your wallet and your older sister gives you \$5 for cleaning her room. Since you are saving money to buy new runners, your mother agrees to match 'dollar for dollar' the amount you have. That is, she will give you extra money so that you now have twice the amount of money you had. You just have enough money to buy runners for \$136.

a Write an equation to represent this problem.

b Solve the equation to determine how much money you originally had in your wallet.

18 To solve each of the following problems, write an equation to represent the problem and solve the equation using inverse operations. Show all steps of your working.

- **a** A restaurant has an outdoor terrace it sets up with tables during summer. Each table seats four people. How many tables should the restaurant set up if 64 people book for Sunday lunch?
- **b** The cost of hiring a tennis court in the evening is \$6 plus \$15 per hour. For how many hours can you hire the tennis court if you have \$81?
- **c** You have \$20 and buy a drink for \$3. The cost of a standard pizza is \$9. Extra toppings are \$2 each. How many extra toppings could you get on a pizza?
- **d** Alisa is saving to buy a phone that costs \$369. She is able to save \$24 per week. If she currently has \$105, in how many weeks can she buy the phone?

19 Yuhan is planning to take a taxi to the airport. A taxi ride costs \$1.90 per minute in addition to a flag fall, or fixed starting price, of \$6.20.

- **a** Write an equation for the total cost of a taxi ride in terms of the length of the taxi ride.
- **b** According to Google maps, it should take Yuhan 32 minutes to get to the airport from his house. What is the total cost of a 32-minute taxi ride?
- **c** Yuhan's taxi gets stuck in traffic and the taxi ride ends up costing him \$80. Based on this cost, determine how long it took Yuhan to get to the airport to the nearest minute.

20 A chemistry student is boiling water using a Bunsen burner. The Bunsen burner increases the temperature of the water by 14.5°C per minute. The water initially measures 18°C.

- **a** Write an equation for the temperature of the water in terms of the length of time spent over the Bunsen burner.
- **b** What is the temperature of the water after 5 minutes?
- **c** How long does it take for the water to reach 100°C? Give your answer to the nearest minute.

21 Ting has a packet of 45 stickers. She gives some friends 5 stickers each and keeps 15 stickers for herself. Use algebra to determine how many of Ting's friends received stickers.

22 Ravi is a maths tutor and earns \$900 in June. This is \$100 less than twice the amount Ravi earned in May. Use algebra to determine how much money Ravi earned in May.

Exercise 6.6C: Challenge

23 Parker tells his friend Penelope to think of a whole number and then complete the following operations.

Step 1: Think of a number.

Step 2: Multiply by 4.

Step 3: Subtract 2.

Step 4: Subtract the number you first thought of.

Step 5: Subtract 4.

Step 6: Divide by 3.

Step 7: Add 2.

Explain why, no matter which number Penelope chooses, she will always get the number she first thinks of.

24 Solve the following equations for x.

a $\frac{8x}{3} + 1 = 9$ **b** $5(x + 3) - 2 = 33$ **c** $\frac{3(5x - 2)}{7} + 6 = 18$ **d** $4\left(\frac{x}{3} + 1\right) - 5 = 19$

25 Find the value of b in the following equation using any method.

$a + 3b = a + b + 5$

Online resources:

Interactive skillsheet Solving equations using inverse operations	**Worksheet** Solving equations using inverse operations	**Investigation** A balancing act	**Quick quiz** 6.6

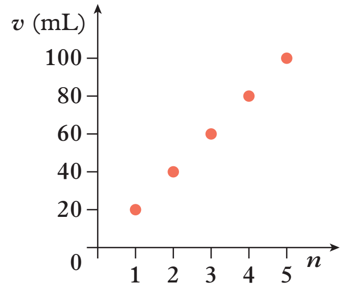
Lesson 6.7

Review: Algebra

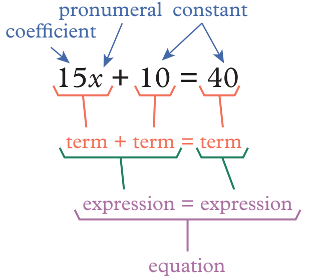
Module summary

Variables

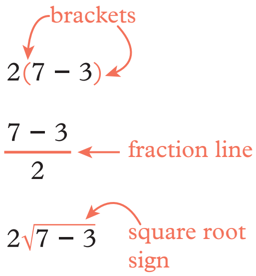
n	1	2	3	4	5
v (mL)	20	40	60	80	100



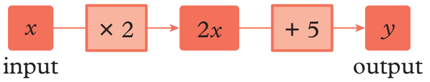
Terms, expressions and equations



Grouping symbols



Function machines



Order of operations

B	**Brackets**	Operations inside grouping symbols are always performed first.
I	**Indices (or exponents)**	Next, simplify powers and square roots.
D **M**	**Division** **Multiplication**	Then, working from left to right, perform any multiplication or division.
A **S**	**Addition** **Subtraction**	Finally, working from left to right, perform any addition or subtraction.

Algebraic notation

$x \times 7 = 7 \times x = 7x$

$x \times 1 = 1 \times x = x$

$x \div 7 = \frac{x}{7}$

Substitution

Let $x = 5$

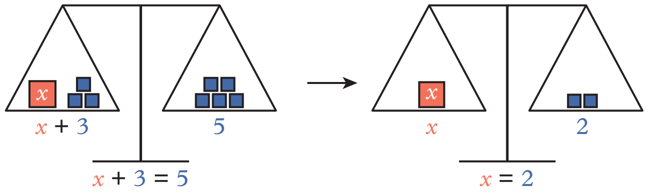
$x^2 + \frac{x}{3} = 5^2 + \frac{5}{3}$

Solving equations by inspection

$a + 8 = 14$
What number plus
8 is equal to 14?

$a + 8 = 14$
Let $a = 6$.
LHS $= a + 8$
$= 6 + 8$
$= 14$
$=$ RHS
LHS $=$ RHS
So, $a = 6$.

The balance model



The guess, check and improve method

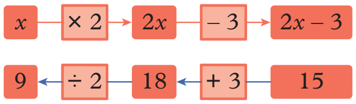
$2x + 6 = 20$

Guess x	LHS $2x + 6$	RHS	LHS = RHS?
3	12	20	no
8	22	20	no
7	20	20	yes

Inverse operations and backtracking

Operation	Inverse operation
+ 3	− 3
− 3	+ 3
× 3	÷ 3
÷ 3	× 3

$2x - 3 = 15$
$2x - 3 + 3 = 15 + 3$
$2x = 18$
$2x \div 2 = 18 \div 2$
$x = 9$



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Review questions 6.7A: Mathematical literacy review

Review quiz
Take the review quiz to assess your knowledge of this module.

Quizlet
Test your knowledge of this module by working individually or in teams.

The following key terms are used in this module:

- algebra
- associative law
- backtracking
- coefficient
- commutative law
- constant
- dependent variable
- difference
- equation
- equivalent equations
- evaluate
- expression
- formula
- function machine
- grouping symbols
- guess, check and improve
- independent variable
- inverse operations
- linear equation
- product
- pronumeral
- quotient
- rule
- solution
- solve
- solve by inspection
- subject
- substitution
- sum
- table of values
- term
- unknown
- variable
- verify

1 Using an example, provide a definition in your own words for the following key terms.

a variable
b expression
c equation
d coefficient
e term

2 Complete the following sentences using words from the key terms list.

a $x + y$ is the ________ of x and y, $x - y$ is the ________ of x and y, xy is the ________ of x and y, $\frac{x}{y}$ is the ________ where x is divided by y.

b ________ equations have exactly the same ________. This can be verified using ________.

c The value of an algebraic expression can be ________ by ________ the known values of each pronumeral into the expression and working out the value of the expression.

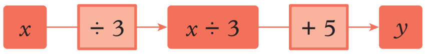
3 Write a mathematically correct sentence using the following words: inverse operation, linear equation, solution, solve, substitution, verify.

4 Identify the property being referenced in each of these equations from the key terms list.

a $a + b = b + a$
b $(a + b) + c = a + (b + c)$

Review questions 6.7B: Multiple choice

6.1 **1** Use this function machine to find the value of y with input $x = 3$.



A 14 **B** 1 **C** 8 **D** 6

6.2 **2** Which of the following statements is not true about the expression $4x - 3y + 1$?

A The coefficient of y is 3.
B The coefficient of x is 4.
C There are 3 terms.
D The constant term is 1.

6.2 **3** Which expression best represents the statement ‘the sum of a, b and c’?

A abc **B** $a + b + c$ **C** $ab + c$ **D** $a - b - c$

6.3 **4** I think of a number, multiply it by 4, then subtract 6 to get a result of 12. The equation that best represents this series of operations is:

A $4x - 6 = 12$ **B** $4(x - 6) = 12$ **C** $4x + 6 = 12$ **D** $4x = 12 - 6$

6.3 **5** The sum of a number and 6 is squared and the result is doubled. Which expression correctly represents this number sentence?

A $2n^2 + 6$ **B** $2(n^2 + 6)$ **C** $(2n + 6)^2$ **D** $2(n + 6)^2$

6.4 **6** If $a = 4$, $b = 3$ and $c = 1$, what does the expression $2a + b - c$ equal?

A 26 **B** 10 **C** 8 **D** 12

6.5 **7** Which one of the following equations is $x = 7$ a solution to?

A $x + 1 = 7$ **B** $2x = 14$ **C** $x + 1 = 6$ **D** $7x = 1$

6.6 **8** The value of x in the equation $5(x - 7) = 30$ is:

A -1 **B** 10 **C** 12 **D** 13

6.6 **9** What is the solution to $\frac{x - 6}{2} = 2$?

A $x = 2$ **B** $x = 4$ **C** $x = 8$ **D** $x = 10$

Review questions 6.7C: Short answer

6.1 **1** Gerald is collecting worms for his compost. He starts collecting worms at 1:00 pm in the afternoon. He collects 5 worms every 30 minutes.

a Identify the variables in this problem.

b Use this information to complete the table below.

Time	1:00 pm	1:30 pm	2:00 pm	2:30 pm	3:00 pm
Total number of worms collected					

6.1 **2** Plot the points from each table of values on the Cartesian plane. Identify whether the points lie on a straight line.

a

x	0	1	2	3	4
y	4	5	6	7	8

b

x	2	4	6	8	10
y	3	5	7	9	11

6.2 **3** For each expression, identify:

i the number of terms
ii any variables used
iii the coefficient of each term containing a variable
iv any constant terms.

a $7x + 3y$ **b** $a + 3$ **c** $5d + 2c + 9$ **d** $4 + e + 2t + 6s$

6.2 **4** Write an equation for each statement.

a A number multiplied by 7 equals 21.
b A number divided by 3 equals 6.
c 7 more than a number equals 10.
d 2 fewer than a number equals 17.

6.3 **5** Write an expression for each statement using grouping symbols where appropriate.

a Add 7 to x then multiply the result by 3
b The sum of p and 5 all divided by 13
c Half of c all squared
d 3 less than v squared

6.4 **6** Evaluate each expression by substituting $x = 1$, $y = 4$ and $z = 2$.

a $x + y + z$ **b** $2x + y$ **c** $3(y - z)$ **d** $\frac{xy}{z}$

6.4 **7** Find the value of b if $a = 3$ is substituted into each formula.

a $b = 3a + 5$ **b** $b = 4a - 7$ **c** $b = \frac{a}{3} + 1$ **d** $b = 2(a + 6)^2$

6.5 **8** Solve each equation by inspection.

a $4a = 28$ **b** $b - 10 = 21$ **c** $c + 8 = 11$ **d** $\frac{d}{2} = 5$

6.5 **9** Use the 'guess, check and improve' method to find the value of x in the equation below.

$$\frac{2(x - 10)}{5} = 4$$

6.6 **10** Solve each equation using inverse operations.

a $x + 9 = 21$

b $\frac{x}{4} = 12$

c $3x + 5 = 17$

d $2(x - 1) = 24$

e $\frac{x + 10}{3} = 2$

f $9 - \frac{x}{5} = 20$

6.6 **11** Check your answers to question **10** by substituting the solutions back into each equation.

Review questions 6.7D: Mathematical modelling

1 Santo is a session musician who earns money by playing back-up guitar in recording sessions. He charges \$250 per hour, plus a flat rate of \$300 for each recording session.

a Identify the variables in this problem.

b Select and define pronumerals to represent the variables in this problem.

c Write a formula for the relationship between these variables.

d Use the formula to determine how much Santo earns from a 5-hour recording session.

e Santo also plays at music festivals. For music festivals, he charges \$325 per hour with no flat-rate fee.

i Write a formula that can be used to calculate how much Santo earns from a given music festival.

ii Santo plays at a music festival and earns \$1950. How long did he play at the festival?

iii Santo is asked to play at a recording session and a festival on the same day. He decides to take whichever job pays more. The music festival will pay Santo for six hours and the recording session will pay him for seven hours. Which job does Santo choose?

iv If Santo only works one job, how many hours would he have to work in each job to earn the same amount of money?

2 Lena is calculating the total amount of money she received from selling tickets to her school play on Friday night. Child tickets are \$8, senior tickets are \$12, and adult tickets are \$16.

a Write a formula for the relationship between the money received (m), the number of child tickets (c), the number of senior tickets (s), and the number of adult tickets (a).

b Lena sells 9 more adult tickets than child tickets and 3 times as many senior tickets as she does child tickets. Represent the number of adult tickets and senior tickets using the variable c.

c Lena sells 10 child tickets. Determine the number of adult and senior tickets she sold.

d How much money did Lena make?

e The school put on a special matinee performance for the local primary schools where they charged \$3 per primary student. Accompanying adults received free tickets. Lena calculates a total of \$216 earnt from tickets to this performance. How many primary students attended the matinee show?

3 Liam has a big bag of lollies for his birthday party. Once his friends arrive, he hands out party bags with 3 lollies in each and keeps 12 lollies for himself.

- **a** Write an expression for the total number of lollies Liam gives away in terms of the number of friends at Liam's party.
- **b** Write an equation for the initial number of lollies in Liam's bag in terms of the number of friends at Liam's party.
- **c** Find the minimum number of lollies in Liam's bag initially. Justify your answer.
- **d** It is possible that Liam's bag contained more lollies than this minimum number. What other possible solutions can you find? Explain.
- **e** What is the pattern for the range of solutions you found?
- **f** If Liam had given out party bags containing five lollies each instead, what would be the minimum number of lollies in his bag to begin with?
- **g** What is the maximum number of friends Liam could have invited to his party if he had 112 lollies to begin with and wanted to give each friend 5 lollies and keep 12 lollies for himself?
- **h** What is the maximum number of friends Liam could have invited to his party if he had 100 lollies to begin with and wanted to give each friend 5 lollies and keep 12 lollies for himself?

Checklist

Now that you have completed this module, reflect on your ability to do the following.

I can do this	I need to review this
☐ Recognise and use variables in everyday formulas ☐ Use rules to complete tables of values ☐ Plot coordinate points from a table of values ☐ Recognise patterns on the Cartesian plane	☐ Go back to Lesson 6.1 Variables
☐ Identify and write terms, expressions and equations ☐ Write expressions and equations to represent worded problems	☐ Go back to Lesson 6.2 Terms, expressions and equations
☐ Write algebraic expressions and equations using grouping symbols	☐ Go back to Lesson 6.3 Order of operations in algebra
☐ Substitute values into expressions and equations ☐ Evaluate algebraic expressions	☐ Go back to Lesson 6.4 Substitution
☐ Use substitution to verify whether a value is a solution to an equation ☐ Solve simple equations by inspection ☐ Solve simple equations using the guess, check and improve method	☐ Go back to Lesson 6.5 Solving equations by inspection
☐ Identify inverse operations ☐ Solve equations using inverse operations	☐ Go back to Lesson 6.6 Solving equations using inverse operations

7

Angles and shapes

Lessons

Prerequisite skills

Diagnostic pre-test
Take the diagnostic pre-test to assess your knowledge of the prerequisite skills listed below.

Interactive skillsheets
After completing the diagnostic pre-test, brush up on your knowledge of the prerequisite skills by using the interactive skillsheets.

- ✓ Comparing angles less than 180°
- ✓ Classifying shapes
- ✓ Substitution
- ✓ Solving equations using inverse operations

Curriculum links

- Identify corresponding, alternate and co-interior relationships between angles formed when parallel lines are crossed by a transversal; use them to solve problems and explain reasons (VC2M7M04)
- Demonstrate that the interior angle sum of a triangle in the plane is 180° and apply this to determine the interior angle sum of other shapes and the size of unknown angles (VC2M7M05)
- Classify triangles, quadrilaterals and other polygons according to their side and angle properties; identify and reason about relationships (VC2M7SP02)

Materials

- ✓ Protractor
- ✓ Compass
- ✓ Ruler
- ✓ Calculator

Lesson 7.1

Classifying angles and lines

Learning intentions

By the end of this lesson you will be able to …

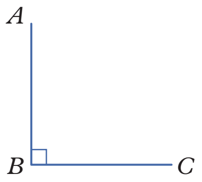
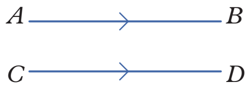
✔ identify, measure and classify angles and lines.

Inter-year links

Support	Understanding angles
Year 8	7.1 Angles

Points and lines

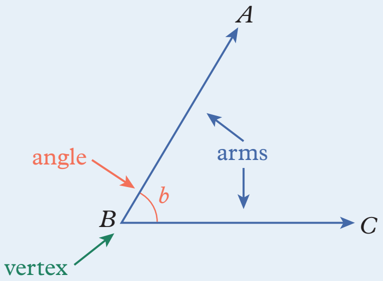
Key content video
Classifying angles and lines

Object	Definition	Example
Point	A location in space that has no length or width. Points are represented using a dot labelled with a capital letter.	
Line segment	A line joining two points. Line segments are named using a bar and the two end point labels.	 Name: $\overline{AB}$
Ray	A line that starts at a point and continues to infinity. Rays are named using an arrow, which indicates the direction of the ray.	 Name: $\overrightarrow{AB}$
Line	A line that continues to infinity in both directions. Lines are named using a double-headed arrow.	 Name: $\overleftrightarrow{AB}$
Perpendicular lines	Two lines that meet to form a right angle. Right angles are labelled with a small box where the lines meet. The symbol ⊥ means 'is perpendicular to'.	 $AB \perp BC$
Parallel lines	Two lines that are always the same distance apart so that they never meet. Parallel lines are labelled with matching arrowheads. The symbol || means 'is parallel to'.	 $AB \parallel CD$

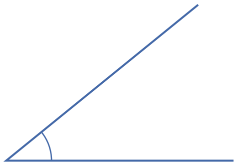
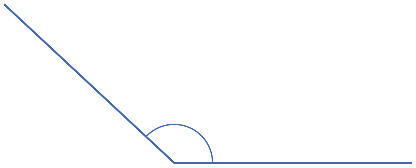
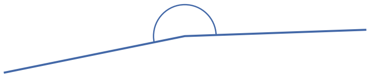
Angles

- When two lines or rays meet, they form an **angle**.
  - → Angles are marked using a portion of a circle called an **arc**.
  - → The two lines or rays are called the **arms** of the angle.
  - → The point at which the two lines meet is called the **vertex**.
- Angles can be named using:
  - → a pronumeral, letter or symbol that stands in for a number. In the following example the pronumeral b is standing in for the size of the angle.
  - → an angle symbol, ∠, followed by a capital letter representing a point on one of the arms of the angle, followed by a capital letter representing the vertex, followed by a capital letter representing a point on the other arm of the angle.

For example, in the diagram below, $b = \angle ABC = 60°$.



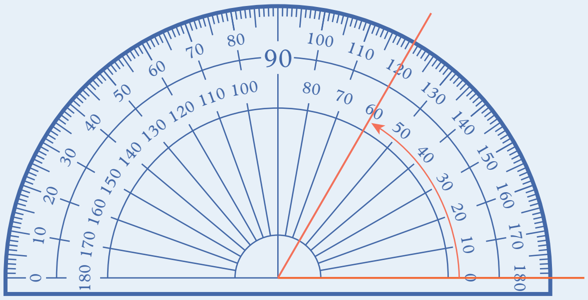
- Angles are classified according to their size.

Classification	Definition	Example
Acute angle	Between 0° and 90°	
Right angle	90°	
Obtuse angle	Between 90° and 180°	
Straight angle	180°	
Reflex angle	Between 180° and 360°	
Revolution	360°	

Measuring angles using a protractor

- Angles are commonly measured in **degrees**°. For example, there are 360° in a revolution.
- To measure an angle using a protractor:
  1. Place the centre cross of the protractor over the vertex of the angle.
  2. Place the zero line, that reads 0°, on one arm of the angle.
  3. Read the scale from 0° to the other arm of the angle.

  For example, this angle is 60°. This measurement is taken from the inner scale because the horizontal arm of the angle is on the right-hand side.

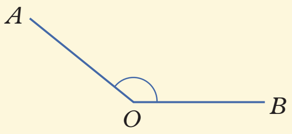


Worked example 7.1A Naming angles and lines

Name the objects below using standard mathematical notation.

a

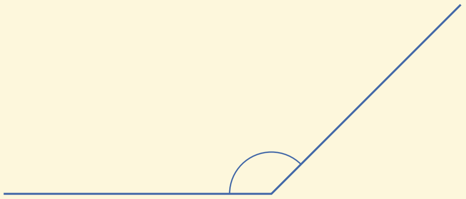
b

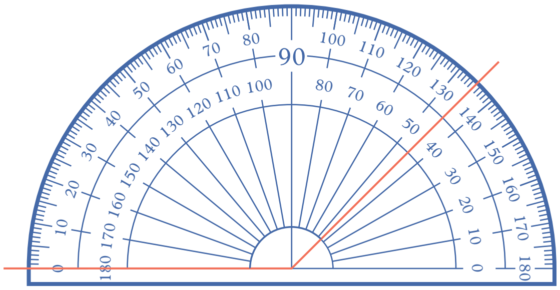


THINK	WRITE
a Identify the object. The line extends from point P to infinity, so this is a ray. Rays are denoted by an arrow symbol. The arrow symbol should indicate that the line extends from point P in the direction of Q.	**a** $\overrightarrow{PQ}$
b Identify the object. The circular marker at the vertex, O, indicates that this is an angle. Angles are denoted using an angle symbol, ∠. The vertex label, O, must be placed in the middle of the name.	**b** $\angle AOB$ (or $\angle BOA$)

Worked example 7.1B Measuring and classifying angles

Measure and classify this angle.



THINK	WRITE
1 Measure the angle using a protractor. Because the horizontal arm of the angle is on the left-hand side, use the scale that begins from 0° on the left-hand side of the protractor.	
2 Classify the angle. As 135° is greater than 90° but less than 180°, this is an obtuse angle.	The angle is 135°. This is an obtuse angle.

Worked example 7.1C Drawing acute angles using a protractor

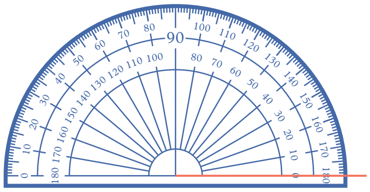
Use a protractor to draw an angle of 60°.

THINK

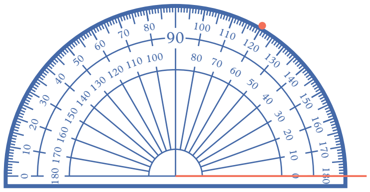
WRITE

1 Begin by drawing a horizontal line segment.

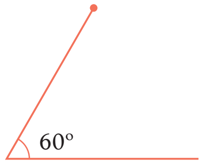
2 Place the protractor over the segment so that the centre cross aligns with one end of the segment, and the segment lies under either the protractor's left- or right-hand zero line.



3 Draw a point at the 60° mark on your protractor. Because the segment is aligned with the right-hand zero line, use the inner scale to measure 60°. If the segment is aligned with the left-hand zero line, the outer scale should be used.



4 Using a ruler, join the end of the horizontal segment with the point, and label the angle.



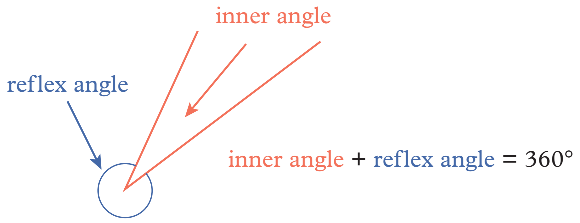
Worked example 7.1D Drawing reflex angles using a protractor

Use a protractor to draw an angle of 240°.

THINK

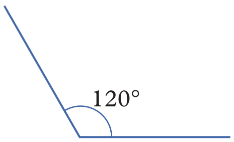
WRITE

1 A reflex angle cannot be drawn directly using a 180° protractor as shown in Worked example 7.1C. First determine the size of the acute or obtuse angle *inside* the reflex angle by subtracting the reflex angle from a full revolution, 360°.

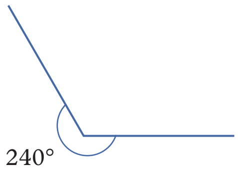


inner angle = 360° − 240°
= 120°

2 Use a protractor to draw the inner angle.



3 Mark the reflex angle on the underside of the obtuse angle.



Helpful hints

- ✔ When reading a protractor, remember to read from 0° and use the correct scale – the outer or inner scale. To ensure you use the correct scale, it can be helpful to visually determine whether an angle is acute, obtuse or reflex before measuring it with a protractor.
- ✔ When drawing angles, think carefully about which side of the line you should place the arc on – incorrect placement of the arc can turn an acute or obtuse angle into a reflex angle and vice versa!
- ✔ When labelling an angle, it can be helpful to indicate if the angle is a reflex angle. For example, write 'the reflex angle $\angle AOB$' to distinguish from the indicated obtuse angle $\angle AOB$ in **Worked example 7.1A** part **b**.

Learning pathways

△ 1–5, 6(a–c), 7–9, 10(a–c), 11(a, b), 12(a, c), 14(a–c)	■ 1–5, 6(b, d, e), 7–9, 10(b–e), 11–12(c, d), 15–17	⬟ 1–4, 6(d–f), 7(c, d), 9–12(c, d), 16–18

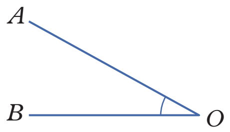
ANS p538

Exercise 7.1A: Understanding and fluency

WE 7.1A **1** Name the objects below using standard mathematical notation.

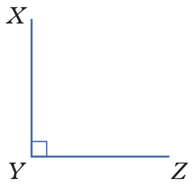
a

b

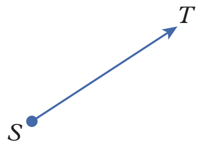


c

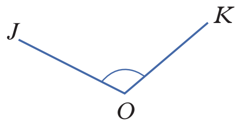
d



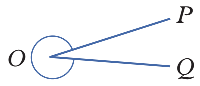
e



f



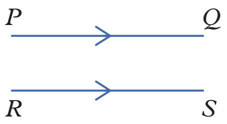
g



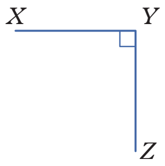
h

2 Use mathematical notation to describe these diagrams.

a

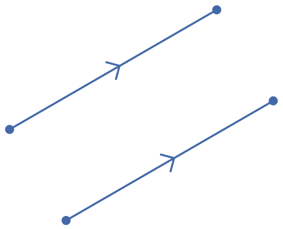


b

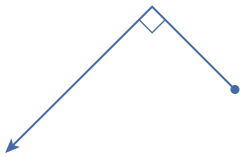


3 Use mathematical terms to describe these diagrams in words.

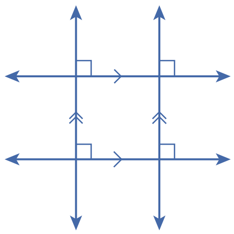
a



b



c



4 Draw a diagram of the following arrangements. Use standard mathematical symbols to illustrate the relationship between the lines in each diagram.

a two parallel rays

b a ray perpendicular to a segment

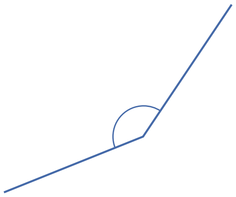
c five rays that radiate out from the same point

d two parallel lines with a line perpendicular to them

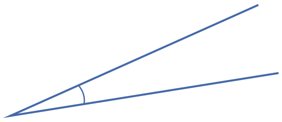
5 Consider the following angles.

- **a** By visual inspection, order each angle from largest to smallest.
- **b** Match each angle to one of the following descriptions: 90°, 180°, 360°, between 0° and 90°, between 90° and 180°, between 180° and 360°.
- **c** Classify each angle.

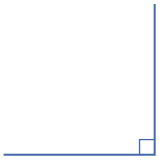
i



ii

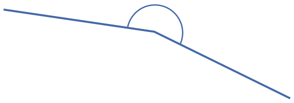


iii

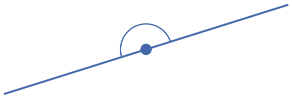


iv

v

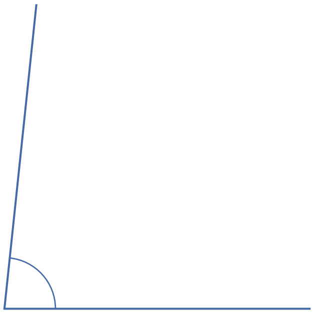


vi

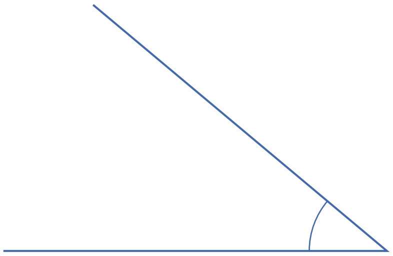


WE 7.1B **6** Measure and classify each of the angles below.

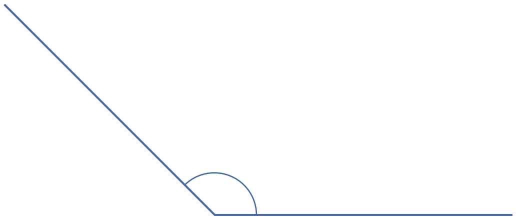
a



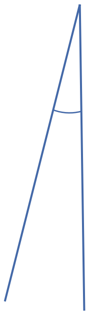
b



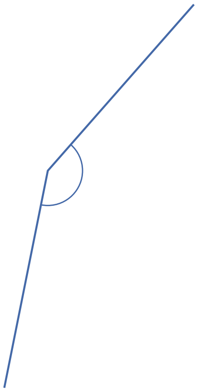
c



d



e



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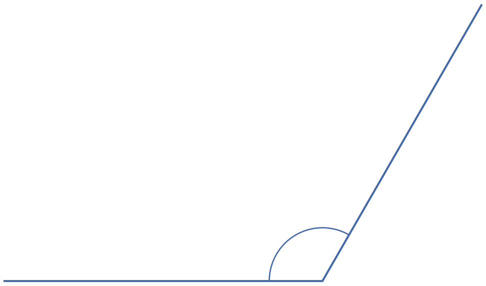
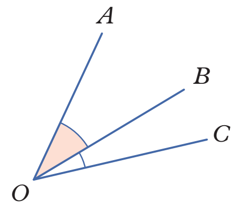
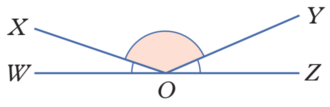


7 Name and classify the shaded angle in each of the following diagrams.

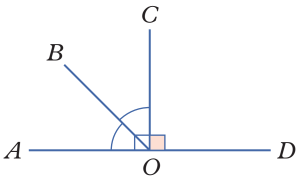
a



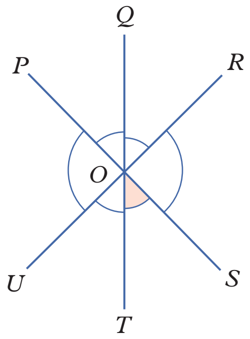
b



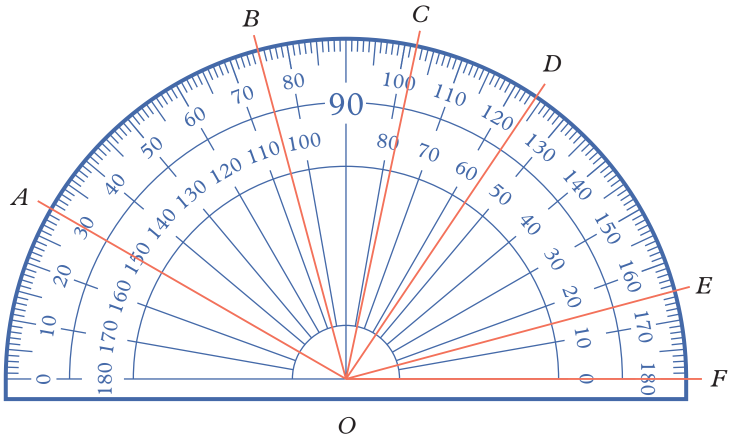
c



d



8 Consider the diagram below. You should read from the inner scale because the horizontal arm of the angle is aligned with the right-hand zero line, ∠*AOF* = 150°.



a What is the size of ∠*EOF*?

b What is the size of ∠*DOF*?

c Classify ∠*COF*.

d Name all the acute angles.

e Name all the obtuse angles.

9 Consider the following angles.

a 80° **b** 42° **c** 173° **d** 10°

i Without a protractor, use a ruler to draw an estimate of each of the angles listed above.

ii Now use a protractor to measure each of your drawings. Write the actual size of each angle next to each drawing. How many of your estimates were within 10° of the actual measurement?

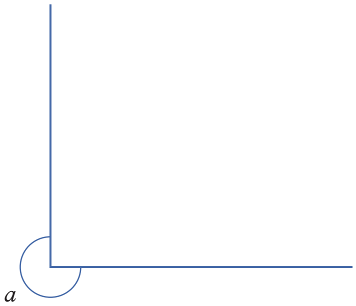
WE 7.1C **10** Use a protractor to draw the following angles.

a 45° **b** 120° **c** 165°

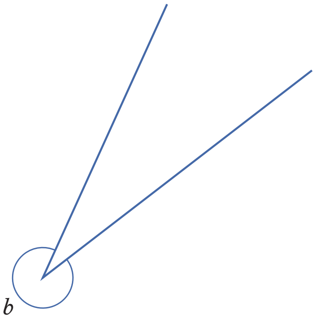
d 9° **e** 94° **f** 112°

11 Find the size of the reflex angle by first measuring the acute or obtuse angle.

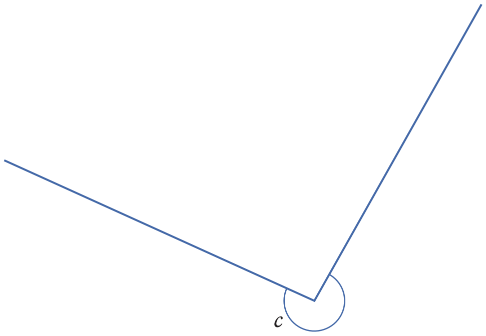
a



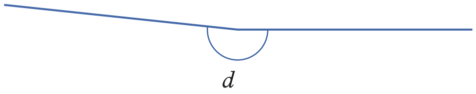
b



c



d

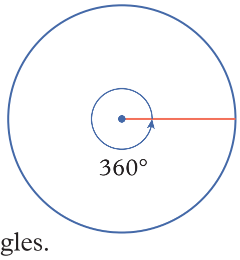


WE 7.1D **12** Use a protractor to draw the following angles.

a 270° **b** 200° **c** 335° **d** 185°

Exercise 7.1B: Problem solving and reasoning

13 Over 3000 years ago, Babylonian astronomers estimated that there were 360 days in a year and so they divided circles into 360 degrees (360°). We still use this system today.

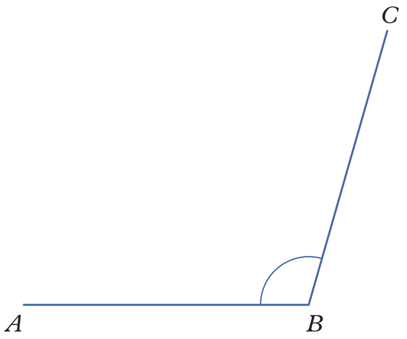


- **a** How many right angles make up a straight angle?
- **b** How many right angles make up a revolution?
- **c** How many straight angles make up a revolution?

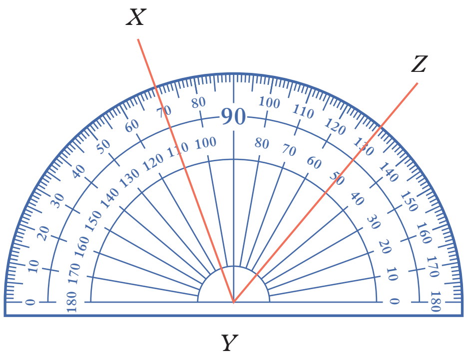
14 Classify the angles produced by adding together the following pairs of angles.

- **a** two right angles
- **b** two straight angles
- **c** a right angle and a straight angle
- **d** two obtuse angles

15 A distracted student measures the angle below as $\angle ABC = 74°$. What mistake did they make and what is the actual size of the angle?



16 Determine the size of $\angle XYZ$ without taking any further measurements.



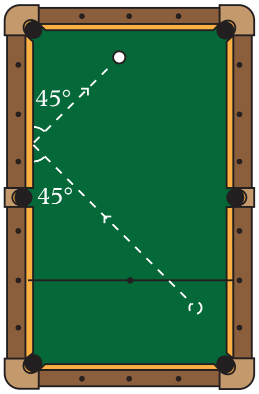
17 Angles are also used to measure the amount of turn when a person or object changes direction. A plane is flying from Adelaide to Broome. When it is halfway there, bad weather forces it to change direction and land in Perth. Use your protractor to measure the acute angle the plane must turn through to fly towards Perth.

Exercise 7.1C: Challenge

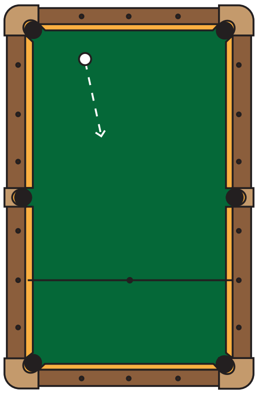
18 Pool players make use of angles when planning their shots. They know that when a ball bounces off the side of a table it rebounds at the same angle.

The ball on the pool table shown on the right hits the side of the table at an angle of 45° and bounces off at an angle of 45°.

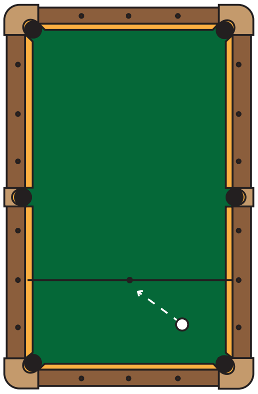
Use your knowledge of drawing angles to show which pocket the white ball will end up in for each of the scenarios below.



a



b



Online resources:

- **Interactive skillsheet** Classifying angles
- **Worksheet** Classifying lines
- **Investigation** Mini-golf challenge
- **Quick quiz** 7.1
- **Worksheet** Classifying angles
- **Worksheet** Measuring angles
- **Investigation** Constructing parallel and perpendicular lines

Lesson 7.2

Angles at a point

Learning intentions

By the end of this lesson you will be able to …

✔ identify and find angles at a point.

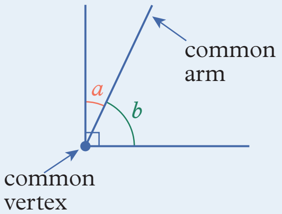
Inter-year links

Support	Understanding angles
Year 8	7.1 Angles

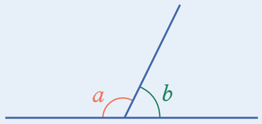
Key content video
Angles at a point

Pairs of angles

- **Adjacent angles** share a common arm and a common vertex, but do not overlap.
  For example, angles a and b are adjacent in each of the diagrams shown.



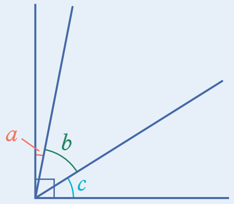
- **Complementary angles** are two angles that form a right angle or add to 90°.
  a is the *complement* of b because $a + b = 90^\circ$.
- **Supplementary angles** are two angles that form a straight angle or add to 180°.
  a is the *supplement* of b because $a + b = 180^\circ$.



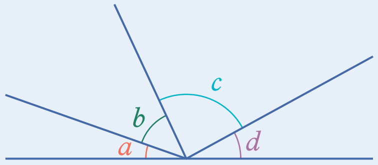
- Complementary and supplementary angles do not need to be adjacent to each other.

Angles at a point

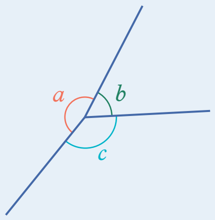
- Angles at a point that form a right angle add to 90°.
  For example, $a + b + c = 90^\circ$.



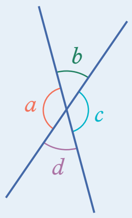
- Angles at a point that form a straight angle add to 180°.
  For example, $a + b + c + d = 180^\circ$.



- Angles at a point that form a revolution add to 360°.
  For example, $a + b + c = 360^\circ$.



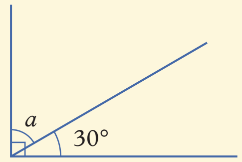
- Where two or more straight lines intersect, they form **vertically opposite angles**, or ‘X’ angles, that are equal in size.
  For example, $a = c$ and $b = d$.



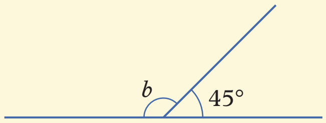
Worked example 7.2A Finding complementary and supplementary angles

Find the size of the unknown angle in each of the diagrams below.

a



b



THINK	WRITE
a 1 Identify the relationship between the unknown and known angles. *a* and 30° form a right angle.	**a** $a = 90^\circ - 30^\circ$
2 Calculate the size of the unknown angle. Angles that form a right angle add to 90°, so $a = 90^\circ - 30^\circ$.	$= 60^\circ$
b 1 Identify the relationship between the angles. *b* and 45° form a straight angle.	**b** $b = 180^\circ - 45^\circ$
2 Calculate the size of the unknown angle. Angles that form a straight angle add to 180°, so $b = 180^\circ - 45^\circ$.	$= 135^\circ$

Worked example 7.2B Finding the complement and the supplement

Complementary angles add to 90° and supplementary angles add to 180°.

a Find the complement of 42°.

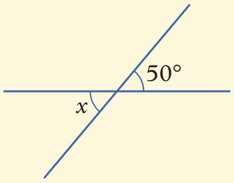
b Find the supplement of 42°.

THINK	WRITE
a Subtract 42° from 90° to find the complement of 42°.	**a** $90^\circ - 42^\circ = 48^\circ$ 48° is the complement of 42°.
b Subtract 42° from 180° to find the supplement of 42°.	**b** $180^\circ - 42^\circ = 138^\circ$ 138° is the supplement of 42°.

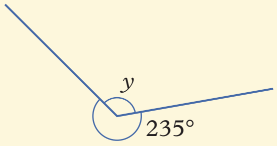
Worked example 7.2C Finding unknown angles at a point

Find the size of the unknown angle in each of the diagrams below.

a



b

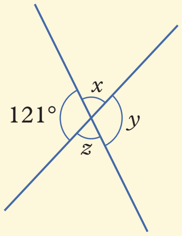


THINK	WRITE
a Identify the relationship between the unknown and known angles. *x* is vertically opposite to 50° and vertically opposite angles are equal in size.	**a** $x = 50^\circ$
b Identify the relationship between the angles. Angles around a point add to 360°. Subtract 235° from 360°.	**b** $y = 360^\circ - 235^\circ$ $= 125^\circ$

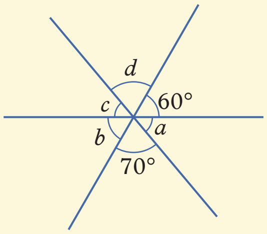
Worked example 7.2D Finding multiple unknown angles at a point

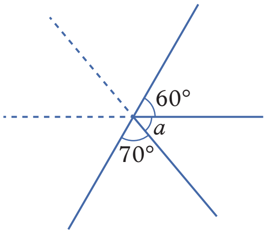
Find the size of each of the unknown angles in the diagrams below.

a



b



THINK	WRITE
a Identify any relationships between unknown and known angles.	**a**
1 y is vertically opposite to 121° and vertically opposite angles are equal in size.	$y = 121^\circ$
2 x and 121° are supplementary angles, so subtract 121° from 180° to calculate x.	$x = 180^\circ - 121^\circ$ $= 59^\circ$
3 z and x are vertically opposite and vertically opposite angles are equal in size.	$z = 59^\circ$
b Identify any relationships between the angles.	**b**
1 b is vertically opposite to 60°, d is vertically opposite to 70° and vertically opposite angles are equal in size.	$b = 60^\circ$ $d = 70^\circ$
2 70°, a and 60° form a straight angle, 180°. Subtract 70° and 60° from 180° to calculate a.  	$a = 180^\circ - 70^\circ - 60^\circ$ $= 50^\circ$
3 c is vertically opposite to a and vertically opposite angles are equal in size.	$c = 50^\circ$

Helpful hints

- ✔ Try not to confuse complementary and supplementary angles. Complementary angles add to 90° and supplementary angles add to 180°.
- ✔ Complementary and supplementary angles do not have to be adjacent – they are just pairs of angles that add up to 90° and 180°, respectively.
- ✔ Vertically opposite angles are angles that sit opposite each other where two lines meet; they do not have to be arranged vertically!

Learning pathways

△ 1–4, 6–8, 9(a–c), 10, 11(a, b), 15

■ 1(d–f), 2(b, d), 3, 5, 6(c–f), 8, 9(c–e), 10–13, 16

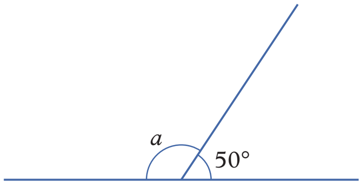
⬟ 1(e, f), 2(c, d), 5, 6(d, e, h), 8(e, f), 9(d–f), 11, 14, 16, 17

ANS p539

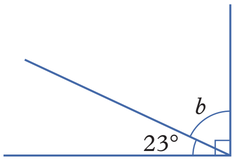
Exercise 7.2A: Understanding and fluency

WE 7.2A **1** Find the size of the unknown angle in each of the diagrams below.

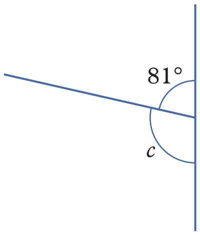
a



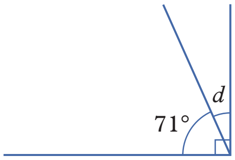
b



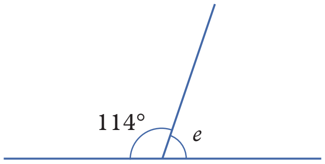
c



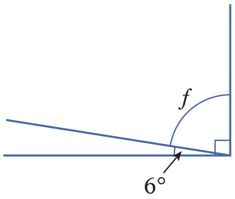
d



e

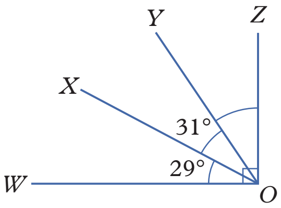


f

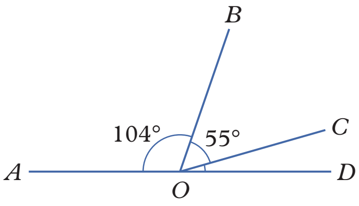


2 Calculate the size of the angle named in brackets in each of the diagrams below.

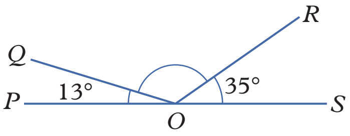
a (∠*YOZ*)



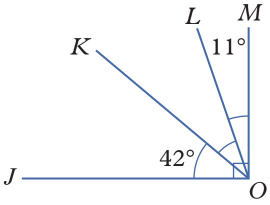
b (∠*COD*)



c (∠*QOR*)



d (∠*KOL*)



3 Name the two pairs of adjacent angles in each of the diagrams in question **2**.

WE 7.2B **4** Complementary angles add to 90° and supplementary angles add to 180°.

a Find the complement of these angles.

i 23° **ii** 77° **iii** 88° **iv** 9°

b Find the supplement of these angles.

i 158° **ii** 21° **iii** 100° **iv** 97°

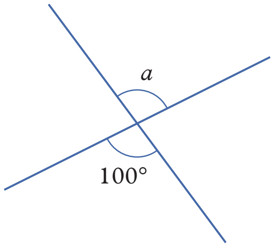
5 For each angle listed below, find the angle that is its:

i complement **ii** supplement.

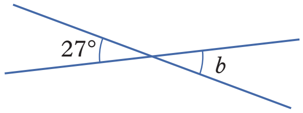
a 46° **b** 12° **c** 85° **d** 27°

WE 7.2C **6** Find the size of the unknown angle in each of the diagrams below.

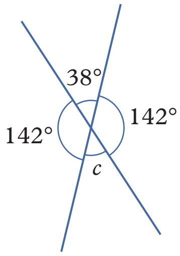
a



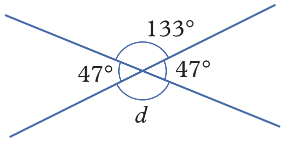
b



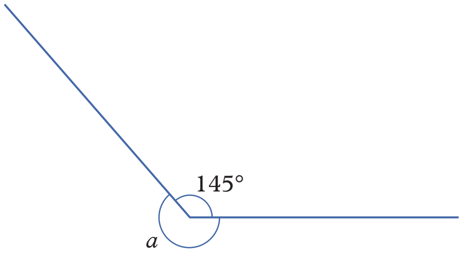
c



d



e



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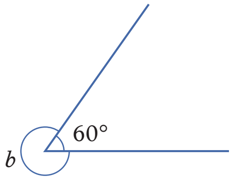
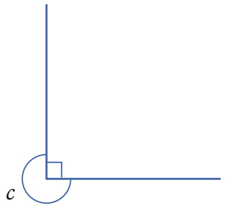
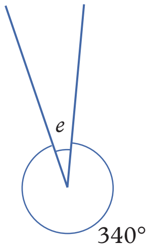


g



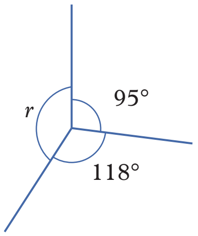
h



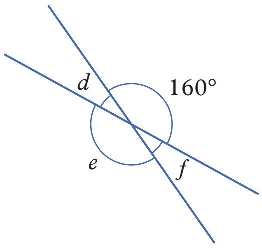
7 Are the pairs of angles in each diagram in question **6e–h** adjacent? Explain your answer.

WE 7.2D **8** Find the size of each of the unknown angles in the diagrams below.

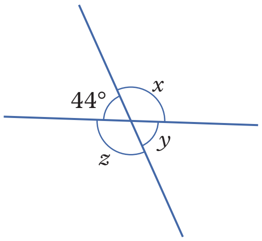
a



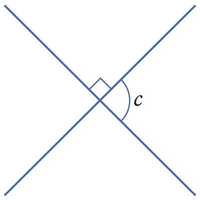
b



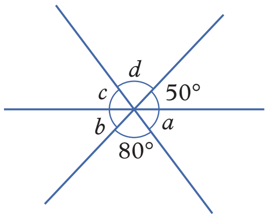
c



d



e



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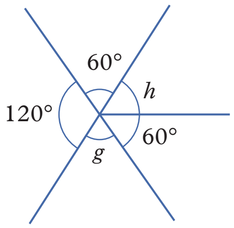
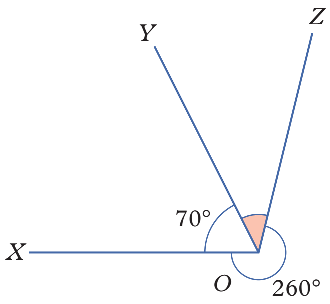
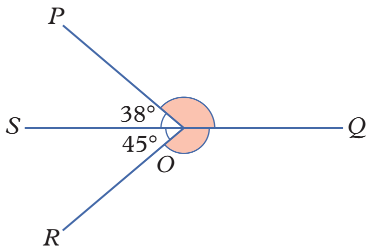


9 Name and determine the size of each of the shaded angles in the diagrams below.

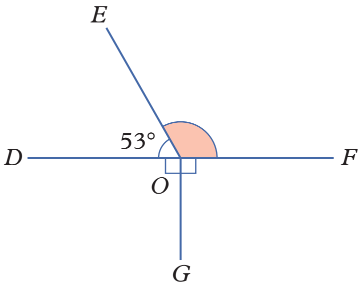
a



b



c



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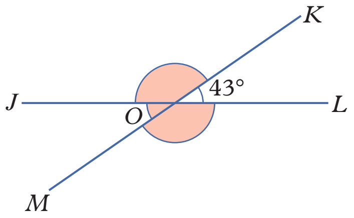

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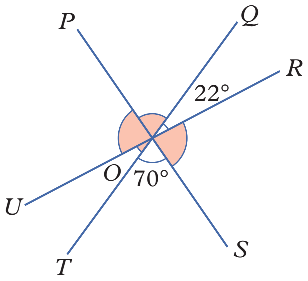

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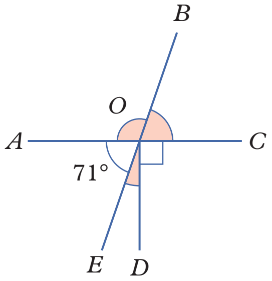


Exercise 7.2B: Problem solving and reasoning

10 For each of the turns described below:

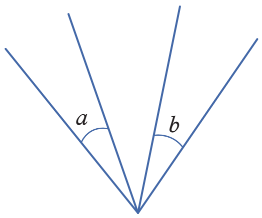
- **i** find the size of the remaining angle each person needs to turn through
- **ii** state whether the pairs of angles are complementary, supplementary or add to make a full revolution.
  - **a** Peter turns 56° in his car in an attempt to make a right-hand turn.
  - **b** Michelle completes 146° of a pirouette.
  - **c** Leigh completes $\frac{2}{3}$ of a left-hand turn on his bicycle.

11 Chelsea, Kane and Sara are sharing a large round pizza. Chelsea eats one piece, Kane eats four pieces and Sara eats three pieces.

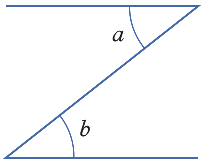
- **a** How many pieces do they need to cut the pizza into?
- **b** If every piece is equal in size, what is the smallest angle in each piece of pizza?
- **c** The next weekend, Chelsea's friend Ben joins them for dinner, so they decide to order two pizzas. This time Chelsea eats one piece, Kane eats four pieces, Sara eats two pieces and Ben eats three pieces. All the pizza is eaten.
  - **i** How many pieces is this in total?
  - **ii** What is the angle size of each piece of pizza? Remember that there are two pizzas to share and every piece is equal in size.

12 Adjacent angles share a common side and a common vertex, but do not overlap. Explain why *a* and *b* are not adjacent angles in each of the diagrams below.

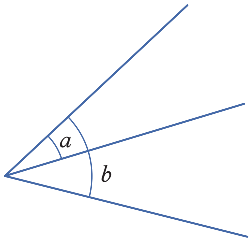
a



b

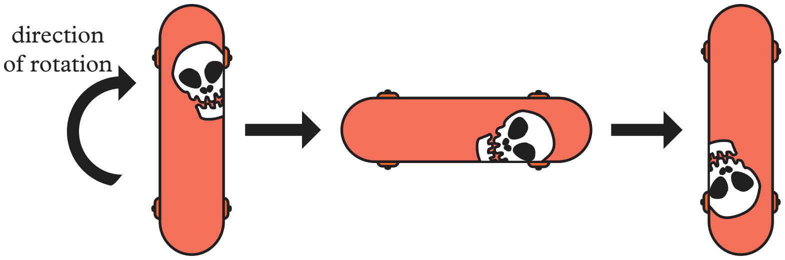


c



13 An aerial freestyle skier launches herself into the air but is knocked off balance by a gust of wind. She estimates that she has already turned through 415°. To land safely she needs to have turned through a multiple of 180°. How much further does she need to rotate if she is to land safely?

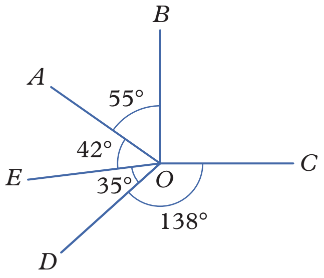
14 Skateboarding, snowboarding and wakeboarding tricks often involve rotation through the air. A '180 aerial' means the rider spins through 180° and then lands with the board facing the opposite direction.



Show the start and end position of a rider's board after the following aerial rotations.

a 360° **b** 540° **c** 720°

15 Consider the diagram on the right. Determine whether the following statements are true or false. Give a reason for your answer.



a $\angle BOC = 90°$

b $\angle AOB$ and $\angle DOE$ are complementary

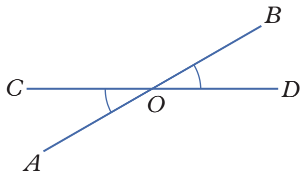
c $\angle EOA$ is the supplement of $\angle COD$

d $\angle AOB + \angle BOC + \angle COD + \angle DOE + \angle EOA = 360°$

16 Can two acute angles be supplementary? Explain.

Exercise 7.2C: Challenge

17 Use the diagram below and your knowledge of supplementary angles to explain why vertically opposite angles must be equal in size.



Online resources:

Interactive skillsheet Angles at a point

Worksheet Angles at a point

Investigation Sharing the poles

Quick quiz 7.2

Lesson 7.3

Angles and parallel lines

Learning intentions

By the end of this lesson you will be able to …

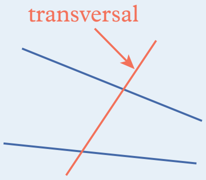
✔ identify and find angles on parallel lines and determine if two lines are parallel.

Inter-year links

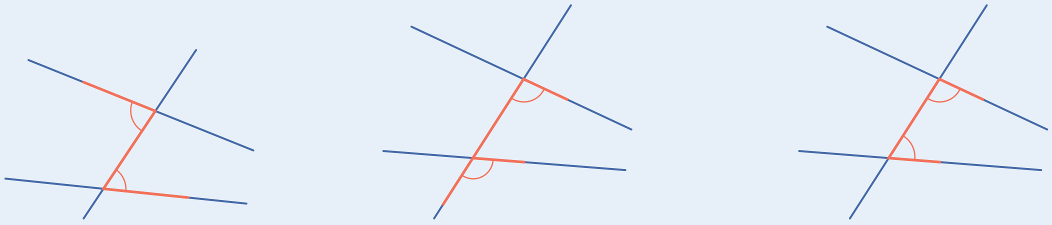
Support	Understanding angles
Year 8	7.1 Angles

Transversals

Key content video Angles and parallel lines



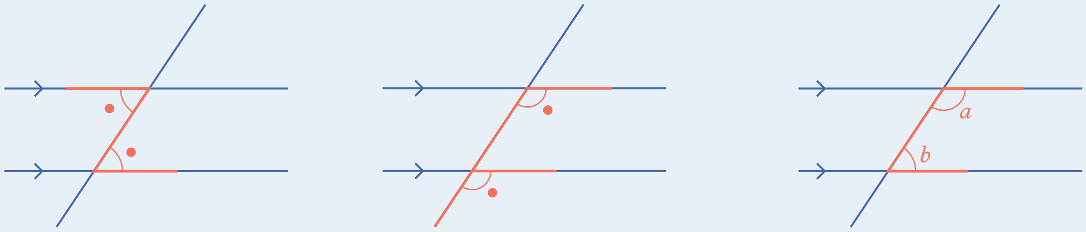
- A **transversal** is a straight line that crosses two or more lines. Transversals create a number of angles when they cross a set of lines.
  - → **Alternate angles**, or 'Z' angles, are on alternate, or opposite, sides of the transversal.
  - → **Corresponding angles**, or 'F' angles, are on the same side of the transversal and are both either above or below each of the crossed lines.
  - → **Co-interior angles**, or 'C' angles, are on the same side of the transversal between the crossed lines.



alternate angles corresponding angles co-interior angles

Transversals and parallel lines

- When a transversal crosses a pair of parallel lines:
  - → alternate angles, or 'Z' angles, are equal in size
  - → corresponding angles, or 'F' angles, are equal in size
  - → co-interior angles, or 'C' angles, are supplementary; that is, they add to 180°.

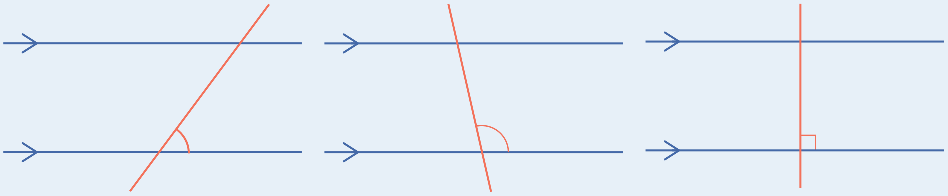


alternate angles corresponding angles co-interior angles

$a + b = 180°$

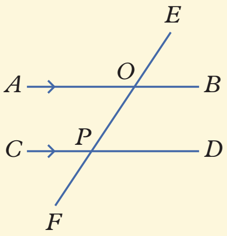
- These pairs of angles can be measured to determine if two lines are parallel – if any of the relationships listed above are true, then the lines crossed by the transversal are parallel.

- Transversals can form acute, obtuse or right angles with parallel lines.



Worked example 7.3A Identifying alternate, corresponding and co-interior angles

Consider the diagram below and identify the:



a transversal

b angle that is alternate to $\angle AOF$

c angle that corresponds to $\angle CPE$.

THINK

a A transversal is a straight line that crosses two or more lines.

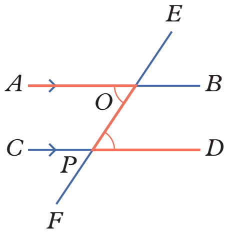
b Alternate angles are on opposite sides of the transversal and lie inside the two angles made by the letter 'Z'. Draw a 'Z' that incorporates $\angle AOF$. The other angle inside this 'Z' is $\angle EPD$, so $\angle AOF$ and $\angle EPD$ are alternate angles.

c Corresponding angles are on the same side of the transversal and lie inside the two angles made by the letter 'F'. Note that the F is upside down and reversed. Draw an 'F' that encloses $\angle CPE$. The other angle inside this 'F' is $\angle AOE$, so $\angle CPE$ and $\angle AOE$ are corresponding angles.

WRITE

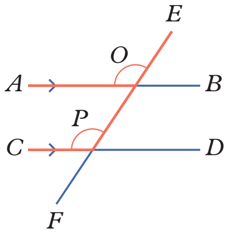
a EF is the transversal.

b



$\angle EPD$ is alternate to $\angle AOF$.

c



$\angle AOE$ is corresponding to $\angle CPE$.

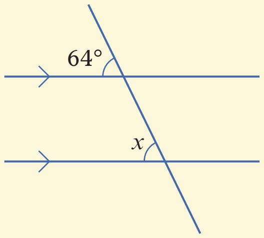
Worked example 7.3B Finding unknown alternate, corresponding and co-interior angles

For each of the following diagrams:

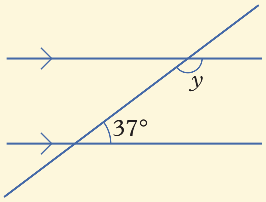
i name the relationship between the unknown and known angles

ii find the size of the unknown angle.

a



b



THINK

a **i** Identify the relationship between the unknown and known angles. *x* and 64° lie inside the two angles of the letter 'F', so they are corresponding angles.

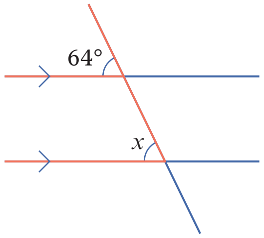
ii Corresponding angles formed by parallel lines are equal in size.

b **i** Identify the relationship between the unknown and known angles. *y* and 37° lie inside the two angles of the letter 'C', so they are co-interior angles.

ii Co-interior angles formed by parallel lines are supplementary, that is, they add to 180°, so $y = 180° - 37°$.

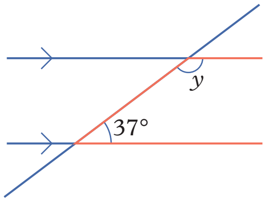
WRITE

a **i** *x* and 64° are corresponding angles.



ii $x = 64°$

b **i** *y* and 37° are co-interior angles.

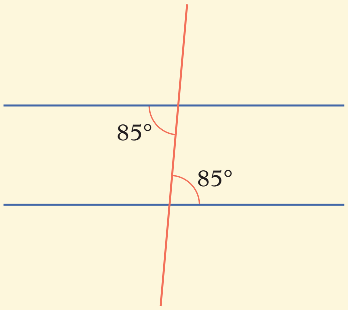


ii $y = 180° - 37°$

$= 143°$

Worked example 7.3C Determining if two lines are parallel using alternate, corresponding and co-interior angles

Decide whether the two blue lines in the diagram below are parallel. Explain your reasoning.



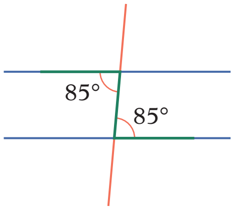
THINK

1 Identify the relationship between the labelled angles. The labelled angles lie inside the two angles of the letter 'Z', so they are alternate angles.

2 Determine whether the relationship between the angles holds. Alternate angles formed by parallel lines are equal in size. The alternate angles are equal in size, so the blue lines are parallel.

3 Write the answer and provide an explanation.

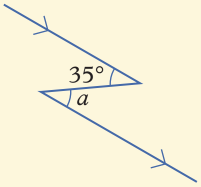
WRITE



The blue lines are parallel because the alternate angles are both 85°.

Worked example 7.3D Finding unknown alternate, corresponding and co-interior angles in complex diagrams

Find the size of the unknown angle below. Explain how you obtained your answer.



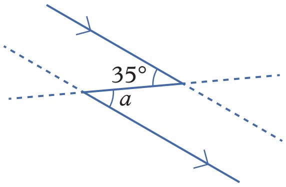
THINK

1 Identify and highlight any parallel and transversal lines in the diagram.

2 Identify the relationship between the unknown and known angles. a and 35° lie inside the angles of the letter 'Z', so they are alternate angles.

3 Write the answer and provide an explanation.

WRITE



a and 35° are alternate angles.

$a = 35°$ because alternate angles formed by parallel lines are equal in size.

Helpful hints

- ✔ Remember to look for the letter to determine which pair of angles you might be dealing with.
  - → Alternate angles form a 'Z'.
  - → Corresponding angles form an 'F'.
  - → Co-interior angles form a 'C'.
- ✔ Take care to identify the nature of the relationship between a pair of angles – when two lines are parallel, alternate and corresponding angles are equal in size, while co-interior angles are supplementary!

Learning pathways

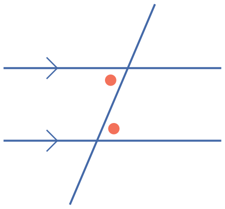
- △ 1–3, 5, 6, 7(a–d), 8(a–c), 9(b, e, g), 10
- ■ 1(e–i), 2, 3(a, e, f), 4(a, b), 5(c, d), 6(f–i), 7(d–g), 8(b, d, e), 9(d–g), 11, 13
- ⬟ 3, 4, 5(a), 6(f–i), 7(f–h), 8(d–f), 9(d–h), 12, 14

ANS p540

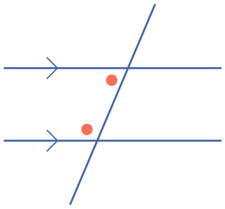
Exercise 7.3A: Understanding and fluency

1 Identify each pair of angles as alternate, corresponding or co-interior in each of the diagrams below.

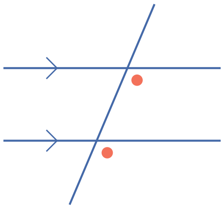
a



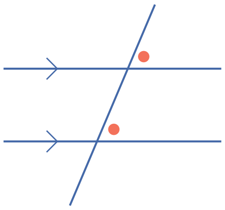
b



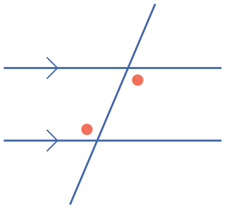
c



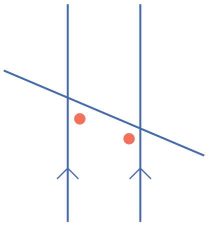
d



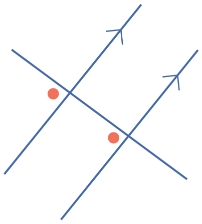
e



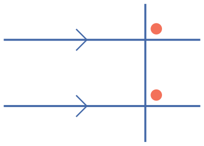
f



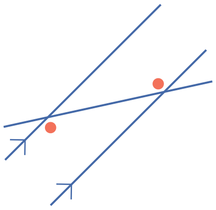
g



h



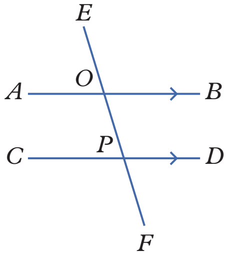
i



2 Use your knowledge of angles and parallel lines to state whether the pairs of angles in question **1** are equal in size or supplementary.

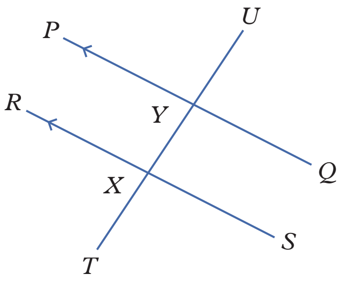
WE 7.3A **3** Consider the diagram below right and identify the:

- **a** transversal
- **b** angle that corresponds to $\angle EOB$
- **c** angle that is alternate to $\angle CPE$
- **d** angle that corresponds to $\angle AOE$
- **e** angle that is co-interior to $\angle AOF$
- **f** angle that is co-interior to $\angle EPD$
- **g** angle that is alternate to $\angle BOF$.



4 In the diagram on the right, identify and name:

- **a** the transversal
- **b** all the pairs of alternate angles
- **c** all the pairs of co-interior angles
- **d** all the pairs of corresponding angles.



5 Use the diagram given in question **3** to identify an angle that is:

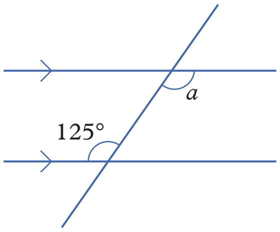
- **a** vertically opposite to $\angle AOF$
- **b** supplementary to $\angle FPD$
- **c** vertically opposite to $\angle EPD$
- **d** supplementary to $\angle CPE$.

WE 7.3B **6** For each of the following diagrams:

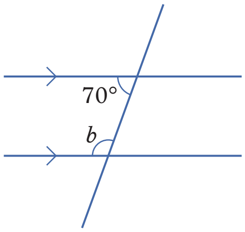
i name the relationship between the unknown and known angles

ii find the size of the unknown angle.

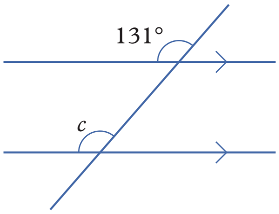
a



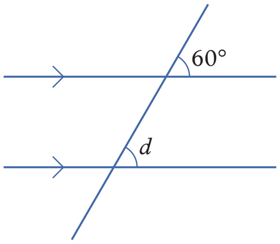
b



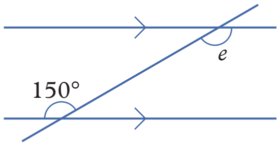
c



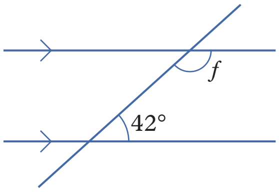
d



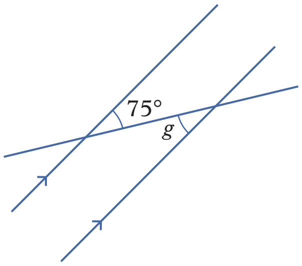
e



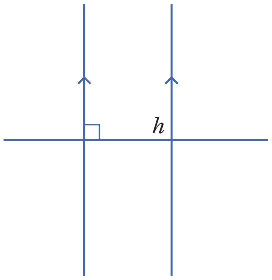
f



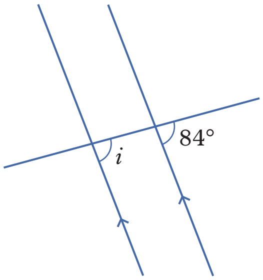
g



h

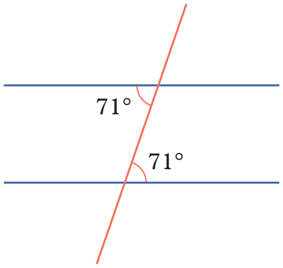


i

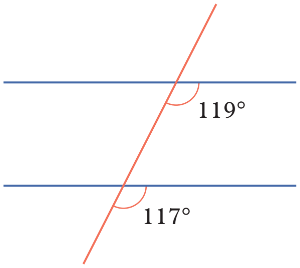


WE 7.3C **7** Decide whether the blue lines in each of the diagrams below are parallel. Explain your reasoning by referring to the relationship between the angles in the diagram.

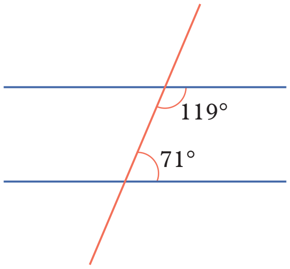
a



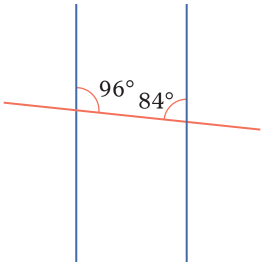
b



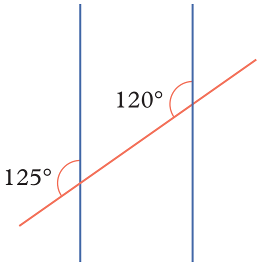
c



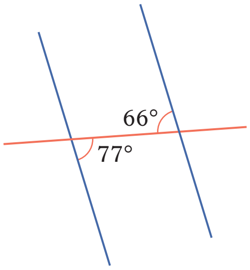
d



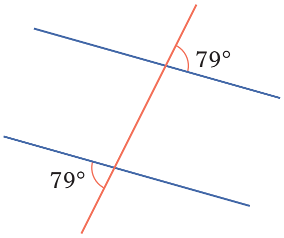
e



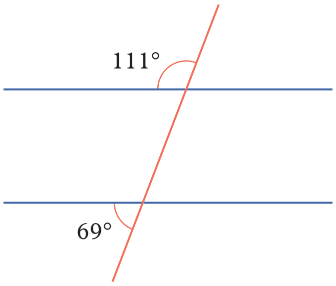
f



g



h



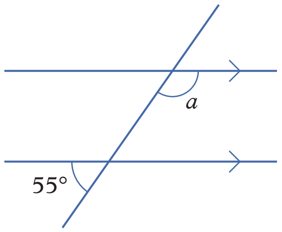
Exercise 7.3B: Problem solving and reasoning

8 For each of the following diagrams:

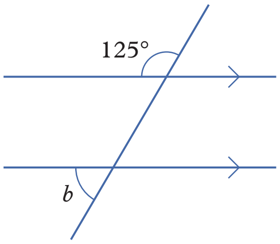
i name the relationship between the unknown and known angles

ii find the size of the unknown angle.

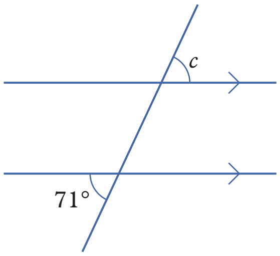
a



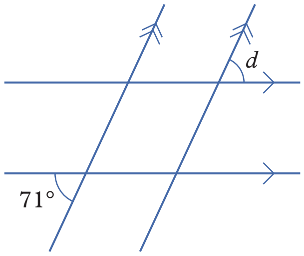
b



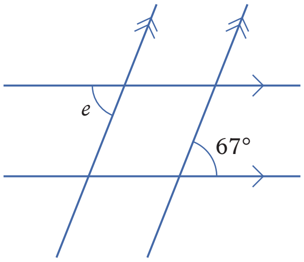
c



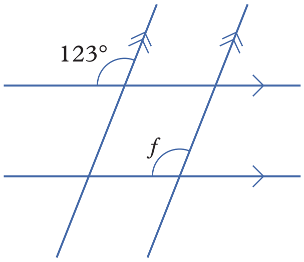
d



e

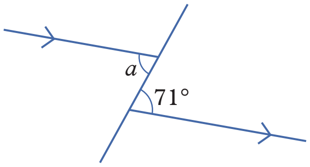


f

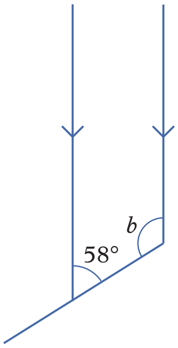


WE 7.3D **9** Find the size of each of the unknown angles. Explain how you obtained your answers.

a



b



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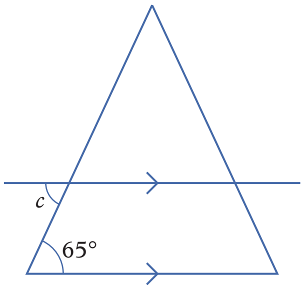

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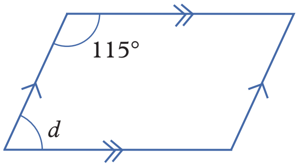

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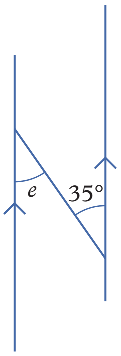

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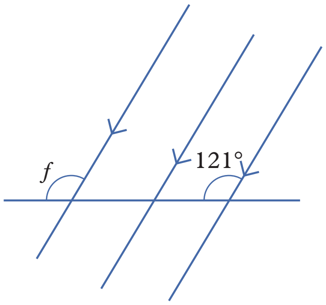

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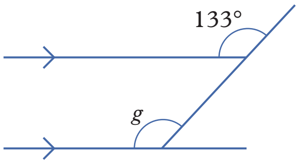

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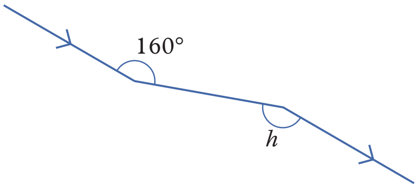
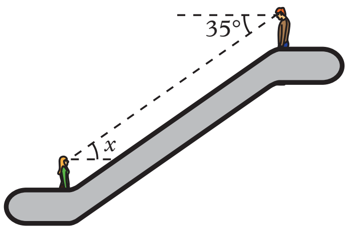
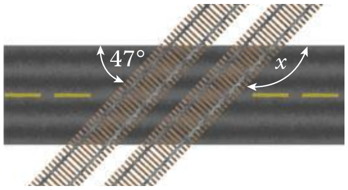


10 Cameron is standing on top of the escalators, looking down at Sarah. If Cameron is looking down at an angle of 35° below the horizontal, at what angle is Sarah looking up at him?



11 A road crosses a pair of railway lines as shown in the diagram. If a driver approaches the railway lines at an angle of 47° as shown, at what angle does the car leave the tracks?



12 Amal is putting up some paintings over a straight crack in the wall. She knows that the top painting is parallel with the edge of the ceiling.

a Find the size of the angles marked *a*, *b* and *c* for the paintings to be parallel with one another.

b Amal measures angle *c* and finds that it is 66°. Are the paintings parallel with one another? Explain your reasoning.

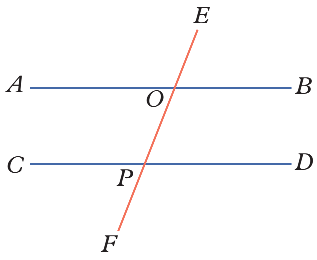
c By how much will she have to move the bottom painting in order to make it parallel to the top one?

13 A student draws the diagram below right and makes the following measurements with their protractor:

$\angle AOF = 81°$ $\quad\quad$ $\angle FPD = 99°$

a Assuming the student's measurements are accurate, are the two blue lines in the diagram parallel? Use the student's measurements to justify your answer.

b For the two blue lines to be parallel, $\angle AOF = \angle EPD$. This relationship is referred to as a condition that must be satisfied for the two blue lines to be parallel. Write two more conditions that could be used to show the two blue lines are parallel.



Exercise 7.3C: Challenge

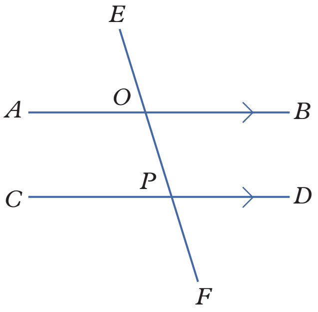
14 A mathematical proof is an argument that shows or proves that a statement is true. For example, the proof below shows that alternate angles formed by a transversal crossing parallel lines are equal in size.

$\angle FOB$ and $\angle CPE$ are alternate angles.
$\angle AOE = \angle FOB$ since vertically opposite angles are equal in size.
$\angle AOE = \angle CPE$ since corresponding angles on parallel lines are equal in size.
Therefore, $\angle FOB = \angle CPE$.

a Use the diagram to prove that co-interior angles are supplementary.

b Use the diagram to show that when a transversal runs through a pair of parallel lines:

- **i** all the acute angles formed by the transversal are equal in size
- **ii** all the obtuse angles formed by the transversal are equal in size
- **iii** if any one of the angles formed by the transversal is a right angle, then all other angles formed must be right angles.



Online resources:

Interactive skillsheet Angles and parallel lines

Worksheet Angles and parallel lines

Investigation The streets of New York

Desmos activity Transversals and parallel lines

Quick quiz 7.3

ANS
p541

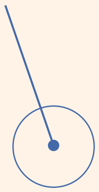
Checkpoint

Checkpoint quiz
Take the checkpoint quiz to check your knowledge of the first part of this module.

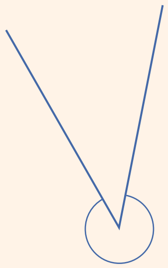
7.1 **1** Match the following labels to the corresponding diagram.

a acute **b** obtuse **c** reflex
d right **e** straight **f** revolution

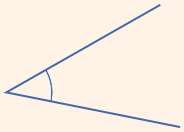
I



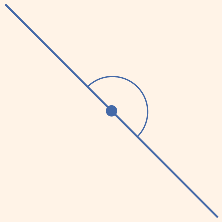
II



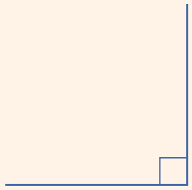
III



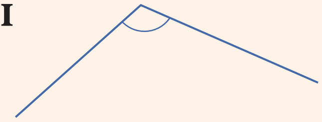
IV



V



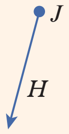
VI



7.1 **2** Identify and name the following objects using standard mathematical notation.

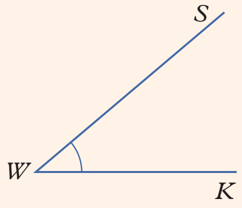
a

b



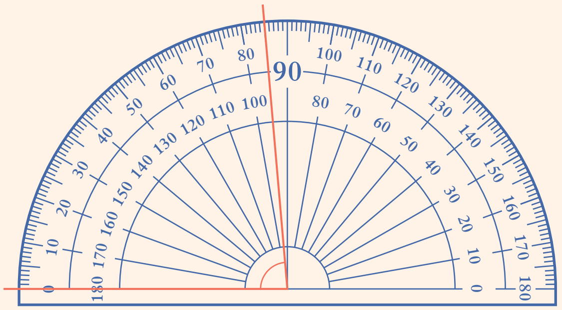
c

d

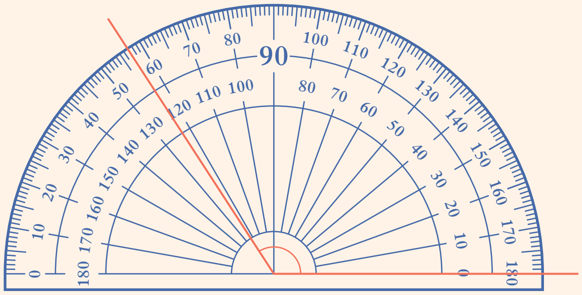


7.1 **3** State the size of the following angles.

a



b



7.2 **4** Decide if the following angles add to form:

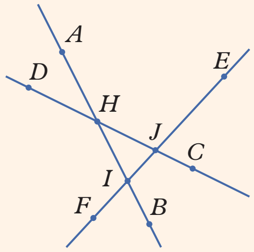
A a right angle **B** a straight angle **C** a revolution **D** a reflex angle

a 33° and 147° **b** 160° and 200° **c** 55°, 92° and 43° **d** 21°, 37° and 32°

7.2 **5** Calculate the complement and supplement of the following angles.

a 19° **b** 37° **c** 89°

7.2 **6** If ∠*AHD* = 37°, ∠*HIJ* = 70° and ∠*EJH* = 106° in the diagram on the right, determine the size of the following angles.

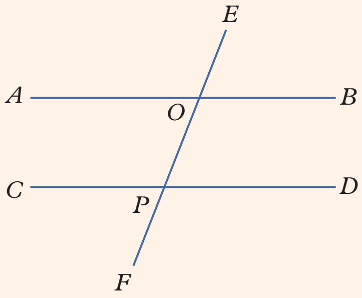


a ∠*AHJ* **b** ∠*IJC* **c** ∠*FIB*

7.3 **7** Determine whether the following statements are true or false.

a Two angles that form a straight angle are supplementary.
b Vertically opposite angles are equal in size.
c A transversal is a straight line that crosses two or more other lines.
d Corresponding angles are supplementary.
e Co-interior angles add to 90°.

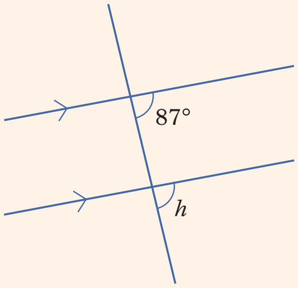
7.3 **8** State whether the angle pairs are alternate, corresponding, co-interior or vertically opposite in the diagram on the right.



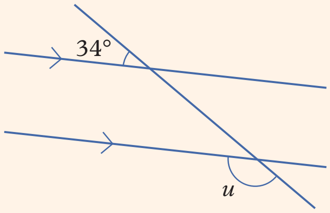
a ∠*FOB* and ∠*EPD* **b** ∠*AOF* and ∠*EPD*
c ∠*AOE* and ∠*FOB* **d** ∠*CPF* and ∠*AOF*

7.3 **9** Determine the values of the pronumerals in each of the diagrams below.

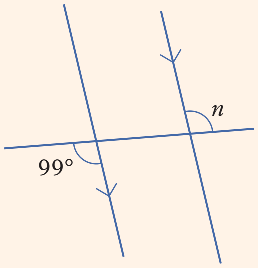
a



b

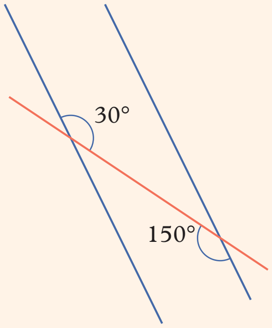


c

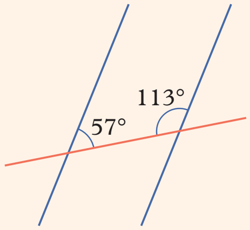


7.3 **10** Decide if the blue lines in each of the diagrams below are parallel. Give a reason for your answers.

a



b



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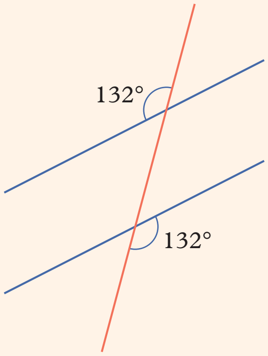
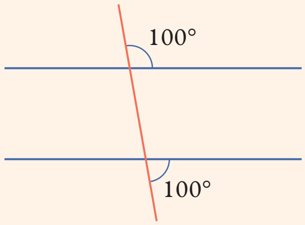


d



Lesson 7.4

Triangles

Learning intentions

By the end of this lesson you will be able to ...

✔ classify triangles by side lengths or angle sizes, and find unknown angles in a triangle.

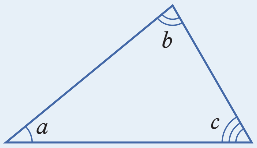
Inter-year links

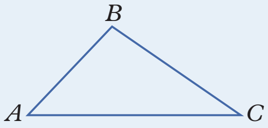
Year 8	7.2 Triangles
Year 9	5.1 Pythagoras' theorem

Triangles

Key content video
Classifying triangles

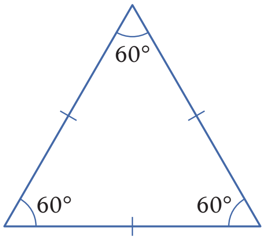
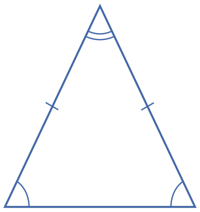
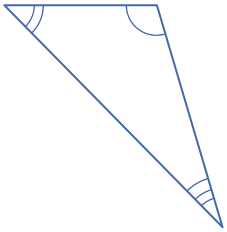
- A **triangle** is a two-dimensional (2D) shape with three straight sides.
- The **interior** or **internal angles** in a triangle add to 180°.
  $a + b + c = 180°$
- Triangles are named using a triangle symbol, followed by capital letters placed at each vertex or corner of the triangle. For example, $\triangle ABC$.



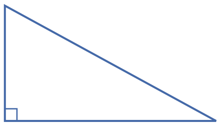
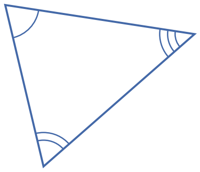
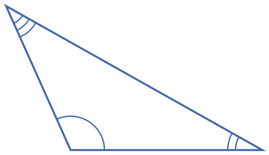


Classifying triangles

- Triangles can be classified by their side lengths and interior angles. In geometric diagrams:
  - → sides of equal length are marked using the same number of hatch marks or dashes
  - → angles of equal size are marked using the same number of arcs.
- All triangles can be classified into one of three categories:

Type of triangle	Properties	Example
Equilateral triangle	• three sides of equal length • three identical internal angles of 60°	
Isosceles triangle	• two sides of equal length • two internal angles that are equal in size	
Scalene triangle	• three sides of three different lengths • three different internal angles	

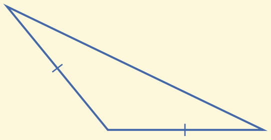
- Triangles can be classified further by considering their internal angle sizes:

Type of triangle	Properties	Example
Right-angled triangle	• one right angle, 90°	
Acute triangle	• three acute angles	
Obtuse triangle	• one obtuse and two acute angles	

- To fully classify a triangle, classify it by both its side lengths and angles. For example, a triangle may be an isosceles acute triangle.

Worked example 7.4A Classifying triangles

Fully classify this triangle.



THINK

1 Examine the sides of the triangle to determine how many are equal in length. Two sides of this triangle are equal in length, so it is an isosceles triangle.

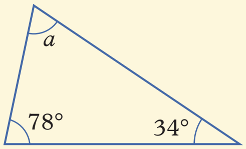
2 Examine the internal angles of the triangle. One internal angle is obtuse, so this is an isosceles obtuse triangle.

WRITE

Isosceles obtuse triangle

Worked example 7.4B Finding an unknown angle in a triangle

Find the size of the unknown angle in this triangle.



THINK

The internal angles in a triangle add to 180°. Subtract each of the known angles from 180° to calculate the size of the third angle, *a*.

WRITE

$$\begin{aligned} a &= 180° - 78° - 34° \\ &= 68° \end{aligned}$$

Helpful hints

✔ Try not to confuse the values of common angles – the sum of all internal angles in a triangle is 180°, *not* 360°!

Learning pathways

▲ 1–4, 5(a, b), 7, 10, 14(a)

■ 1(a, c, d, e, g), 2(b, d, f), 3(c–f), 4, 5, 6(a, b), 7, 8, 11, 12(a, b, e, f), 14(a–c)

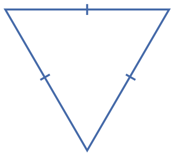
⬟ 1(e–i), 3–5, 6(c, d), 7, 9, 11, 12(c, e ,f), 13, 14(d–f), 15

ANS p541

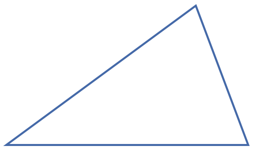
Exercise 7.4A: Understanding and fluency

WE 7.4A **1** Fully classify each of the triangles below.

a



b



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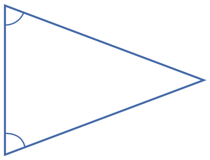

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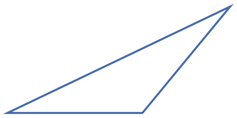

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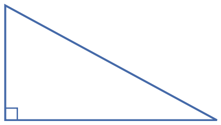

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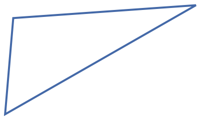

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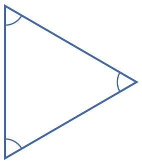

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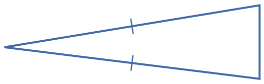
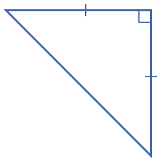
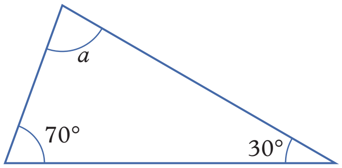


i

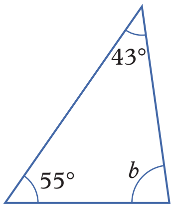


WE 7.4B **2** Find the size of the unknown angle in each of the triangles below.

a



b



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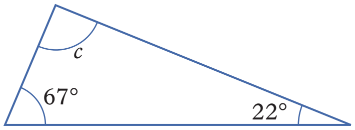

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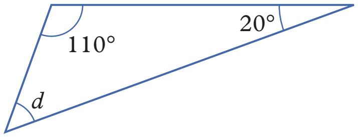

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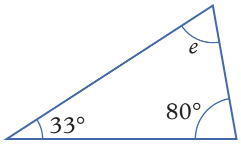

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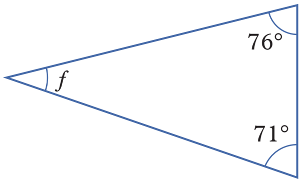
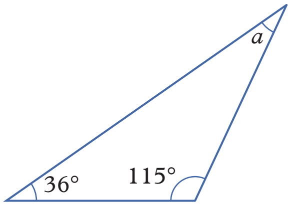
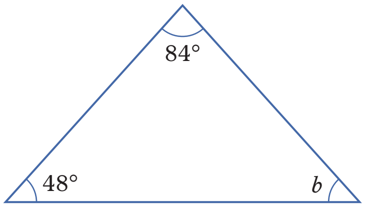


3 Fully classify each of the following triangles by first finding the sizes of the unknown angles.

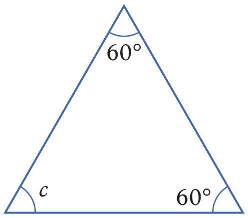
a



b



c



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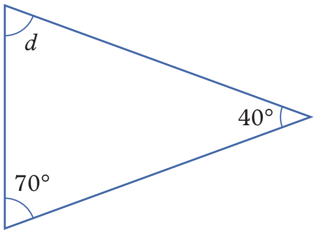
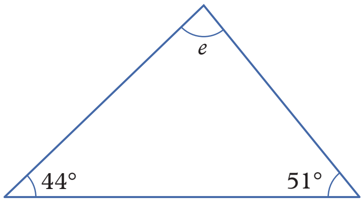


e



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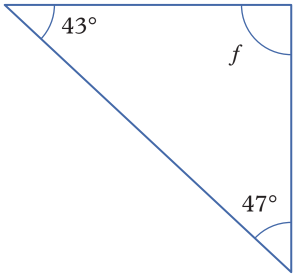
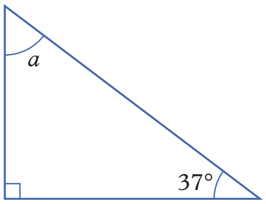


Exercise 7.4B: Problem solving and reasoning

4 Use your knowledge of the properties of a triangle to find the size of each labelled unknown angle.

a



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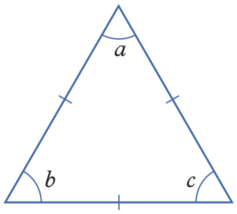
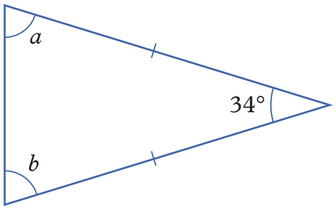


c



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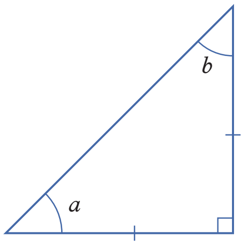

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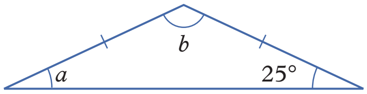

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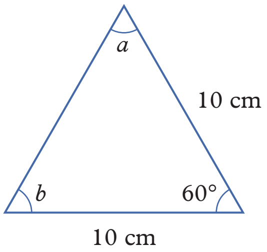
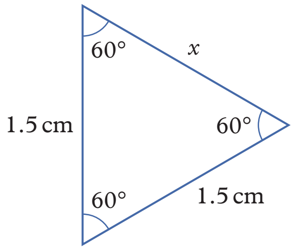


5 Use your knowledge of triangle properties to find the size of each labelled unknown side length.

a



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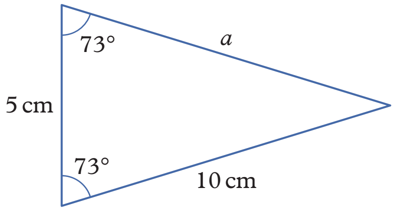

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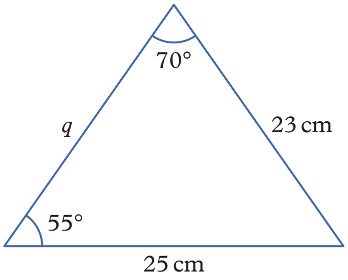

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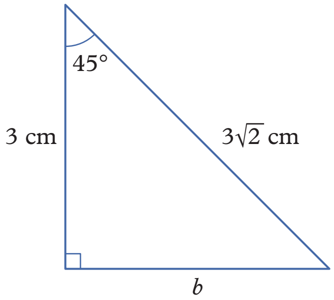


6 Draw a sketch of the following triangles, labelling all the internal angles. Include letter labels and hatch marks where appropriate. You are not required to draw the angles using a protractor.

a $\triangle ABC$, where $\angle ABC = 145^\circ$ and $\angle ACB = 25^\circ$

b $\triangle PQR$, where $\angle PQR = 60^\circ$ and $\angle PRQ = 60^\circ$

c $\triangle XYZ$, where $\angle XYZ = 23^\circ$ and $\angle YXZ = 67^\circ$

d $\triangle JKL$, where $\angle KJL = 50^\circ$ and $\angle JLK = 50^\circ$

7 The interior angles of a triangle add to 180°.

a Decide whether it is possible to draw a triangle with each of the following sets of interior angles.

i 90°, 60°, 30°
ii 100°, 35°, 35°
iii 60°, 60°, 60°
iv 5°, 10°, 165°
v 90°, 90°, 20°
vi 13°, 13°, 154°

b Classify the triangles that can be drawn.

8 Is it possible for a right-angled triangle to also be an isosceles triangle? Justify your answer.

9 Which of the three types of triangle (equilateral, isosceles or scalene) can have an obtuse angle? Explain your reasoning.

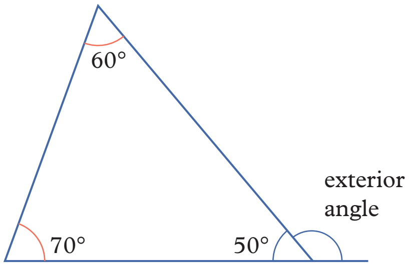
10 The number of sides of equal length and the number of internal angles that are equal in size in a triangle are related.

a Complete the table below.

Type of triangle	**Number of sides of equal length**	**Number of internal angles that are equal in size**
Equilateral		
Isosceles		
Scalene		

b Using the table from part **a**, describe the relationship between the number of sides of equal length and number of angles of equal size in a triangle.

11 The triangle on the right has had one of its sides extended and an exterior angle marked in.



a Why do you think this angle is called an 'exterior' angle?

b Use your knowledge about the sum of angles on a straight line to find the size of the exterior angle.

c Add the two opposite interior angles together (shown in orange). What do you find?

d Copy the triangle and extend the other two sides in the anti-clockwise direction in order to mark in the other two exterior angles.

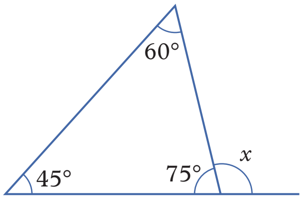
e Determine the size of the other two exterior angles.

f For each of these exterior angles, find the sum of the 'opposite' two interior angles. How do these relate to the size of each exterior angle?

g Write a sentence to describe the relationship between an exterior angle and the two interior angles opposite that exterior angle. Explain this relationship with reference to your knowledge of supplementary angles.

12 Use your findings from question **11** to calculate the size of angle *x* for these triangles.

a



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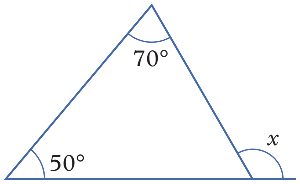

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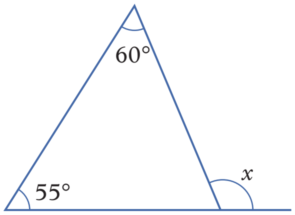

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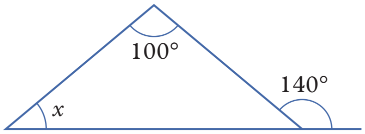

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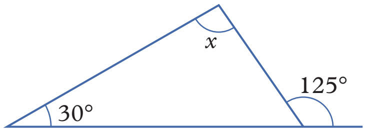

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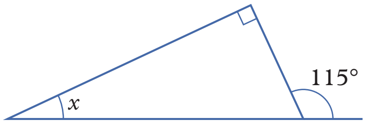


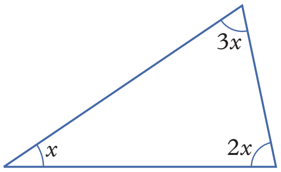
13 Look at the triangle in question **11**.

- **a** What is the sum of the exterior angles of this triangle?
- **b** Select two more triangles from question **12** and determine:
  - **i** the size of each of the exterior angles
  - **ii** the sum of the exterior angles.
- **c** Write a sentence describing your findings regarding the sum of the exterior angles of a triangle.

Exercise 7.4C: Challenge

14 Use your knowledge of triangle properties and solving algebraic equations to find the value of the pronumerals in these triangles.

a



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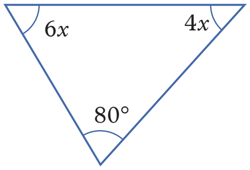

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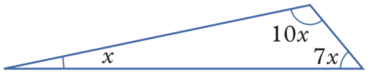

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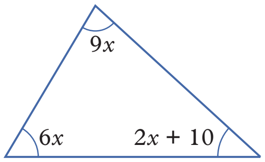
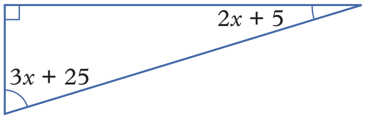


e



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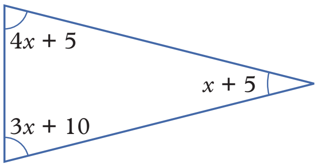
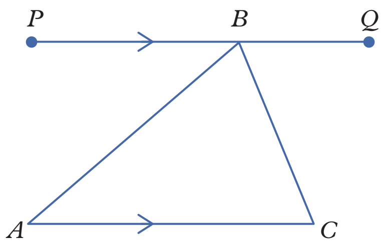


15 Consider the diagram below in which $\overline{PQ}$ is parallel to the base, $\overline{AC}$.



- **a** Recall the relationships between angles created by transversals in Lesson 7.3 and classify the following pairs of angles as alternate, corresponding or co-interior.
  - **i** $\angle PBA$ and $\angle BAC$
  - **ii** $\angle QBC$ and $\angle ACB$
- **b** Use your answers from part **a** to explain why

  $\angle BAC + \angle ABC + \angle ACB = 180°$.

Online resources:

Interactive skillsheet Triangle properties

Worksheet Classifying triangles

Investigation Mosaic triangles

Investigation Constructing triangles

Quick quiz 7.4

Quadrilaterals

Learning intentions

By the end of this lesson you will be able to ...

✔ classify triangles by side lengths or angle sides, and find unknown angles in a triangle.

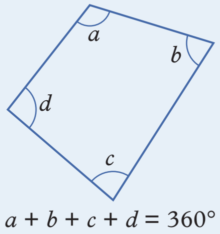
Inter-year links

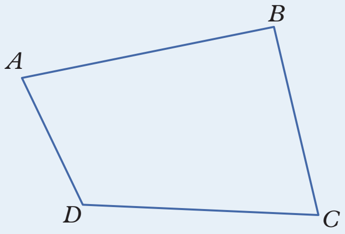
Year 8 9.6 Quadrilaterals

Quadrilaterals

Key content video
Classifying quadrilaterals

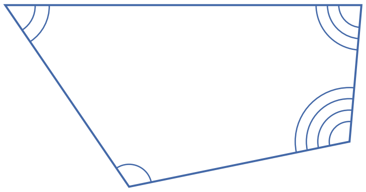
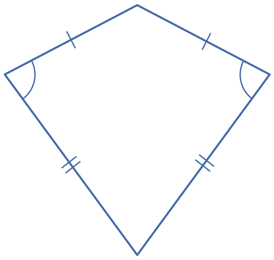
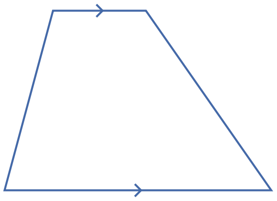
- A **quadrilateral** is a two-dimensional (2D) shape with four straight sides.
- The interior or internal angles in a quadrilateral add to 360°.
- Quadrilaterals can be named by listing their vertices in order. The vertices should be listed in either clockwise or anticlockwise order. For example, the quadrilateral below right can be named either *ABCD* or *ADCB*.



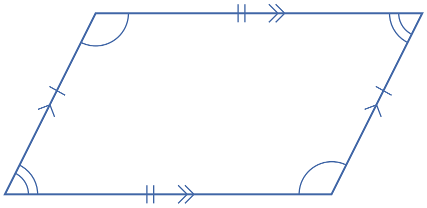


Classifying quadrilaterals

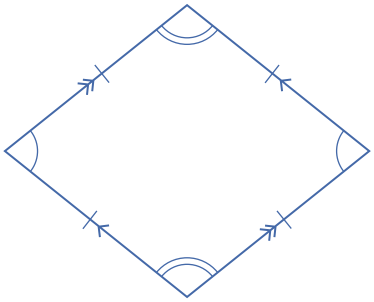
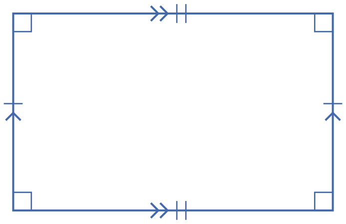
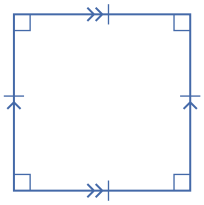
- Quadrilaterals can be classified by their side lengths and interior angles.

Type of quadrilateral	Properties	Example
Irregular quadrilateral	• four different interior angles • no parallel sides	
Kite	• two pairs of sides that are equal in length • one pair of opposite angles that are equal in size • no parallel sides	
Trapezium	• at least one pair of parallel sides	

- A parallelogram is a special type of trapezium.

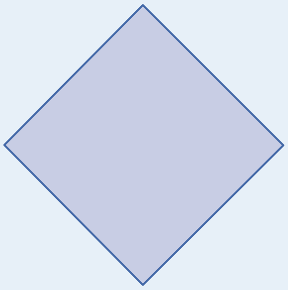
Type of quadrilateral	Properties	Example
Parallelogram	• two pairs of parallel sides • two pairs of opposite angles that are equal in size • opposite sides are of equal length	

- Three special types of parallelogram are:

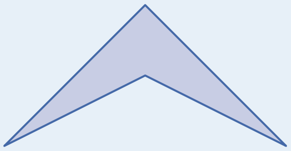
Rhombus	All parallelogram features plus: • four sides of equal length	
Rectangle	All parallelogram features plus: • four right angles	
Square	All parallelogram features plus: • four sides of equal length • four right angles	

Convex and non-convex quadrilaterals

- A **convex quadrilateral** has four interior angles less than 180°. A **non-convex quadrilateral** (or concave quadrilateral) has one interior reflex angle (an angle greater than 180°).



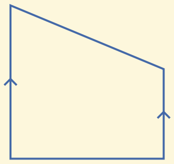
Convex quadrilateral



Non-convex quadrilateral

Worked example 7.5A Classifying quadrilaterals

Classify the quadrilateral on the right.



THINK

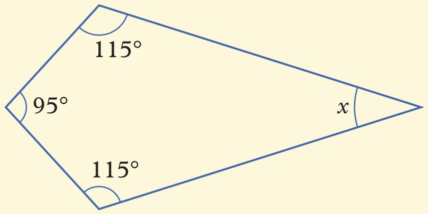
Examine the sides and angles of the quadrilateral to identify its special properties. The quadrilateral has one pair of parallel sides, so it is a trapezium.

WRITE

Trapezium

Worked example 7.5B Finding an unknown angle in a quadrilateral

Find the size of the unknown angle in the quadrilateral on the right.



THINK

The internal angles in a quadrilateral add to 360°. Subtract each of the known angles from 360° to calculate the fourth angle, x.

WRITE

$x = 360° - 115° - 115° - 95°$
$\quad = 35°$

Helpful hints

✔ Take care when reading diagram notation. Remember:
  - → Sides of equal length are marked using the same number of hatch marks.
  - → Parallel sides are marked using the same number of arrowheads.
  - → Angles of equal size are marked using the same number of arcs.

Learning pathways

▲ 1–5, 6(a–c), 8(a, b, f, h), 9, 10, 13–15

■ 1–3, 4–6(b–e), 8(b–g), 11, 12(a, b), 13, 17, 18(a–e), 19(a–c)

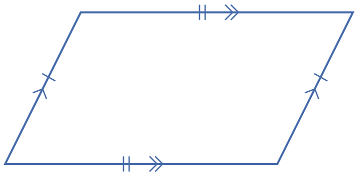
⬟ 1, 2, 4(b, e, f), 6–7(d–f), 8(g–i), 12(c), 13, 16, 18–20

ANS p542

Exercise 7.5A: Understanding and fluency

WE 7.5A **1** Classify each of these quadrilaterals.

a



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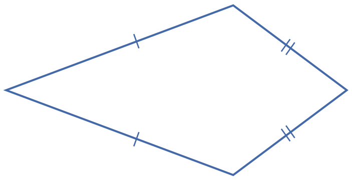

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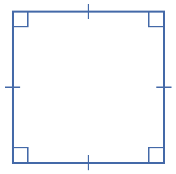

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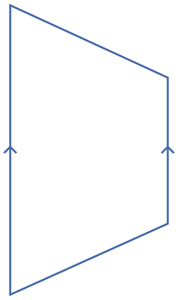

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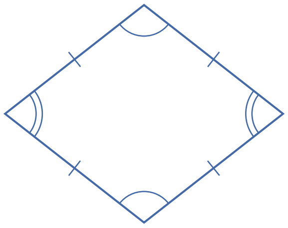

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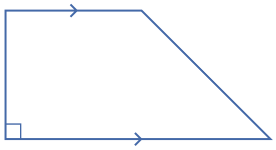
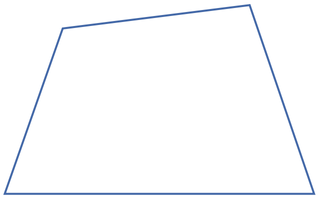


g



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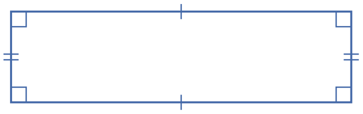
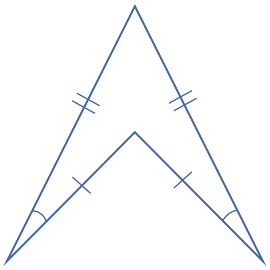


i



2 Match each of the following descriptions to one of the quadrilateral types.

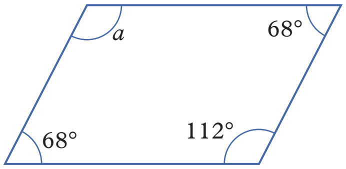
- **a** All sides are equal in length, two pairs of opposite angles are equal in size.
- **b** At least one pair of opposite sides is parallel.
- **c** Two pairs of opposite sides are parallel and equal in length; all internal angles are 90°.
- **d** Two pairs of opposite sides are parallel and equal in length; two pairs of opposite angles are equal in size.
- **e** All sides are equal in length; all angles are 90°.
- **f** Two pairs of adjacent sides are equal in length; one pair of opposite angles is equal.

3 Decide whether each statement is true or false.

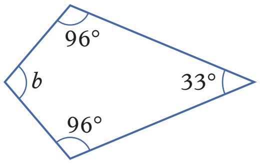
- **a** A rhombus has exactly four sides that are equal in length.
- **b** A kite has two pairs of opposite angles that are equal in size.
- **c** A trapezium has at least one pair of parallel sides.
- **d** Irregular quadrilaterals have exactly four sides that are of different lengths.

WE 7.5B **4** Find the size of the unknown angle in each of the quadrilaterals below.

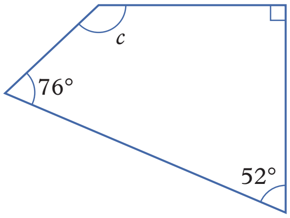
a



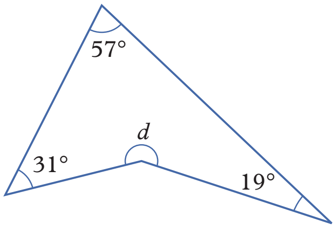
b



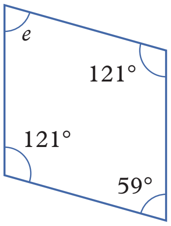
c



d



e



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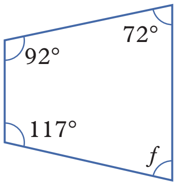
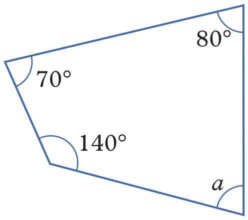


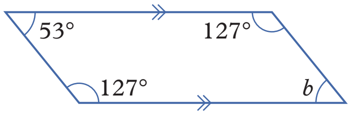
5 Classify the quadrilaterals in question **4** as either convex or non-convex.

6 Classify each of the following quadrilaterals, and then find the unknown angles.

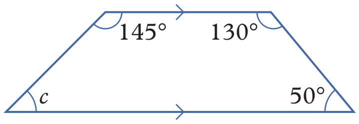
a



b



c



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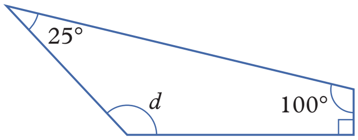

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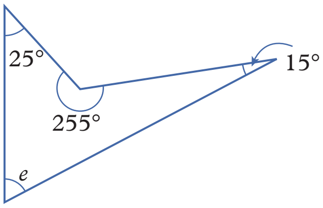

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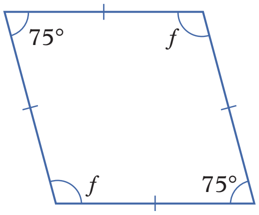
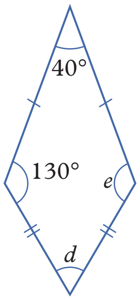


7 Classify the quadrilaterals in question **6** as either convex or non-convex.

Exercise 7.5B: Problem solving and reasoning

8 Use your knowledge of quadrilateral properties to find the size of each labelled unknown angle.

a



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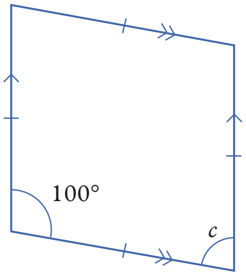

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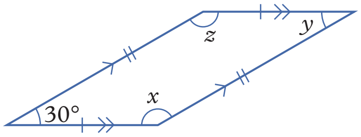

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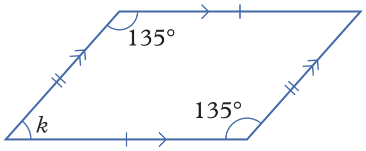

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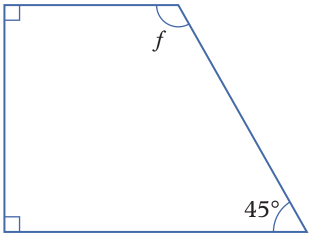

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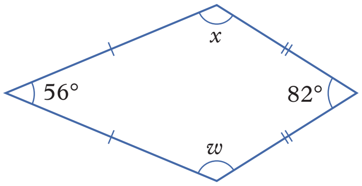

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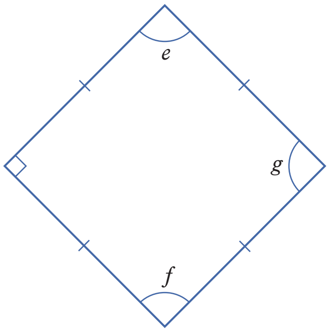
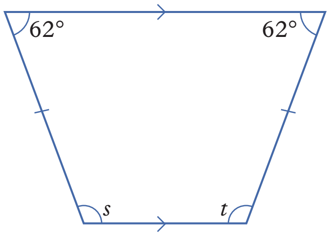
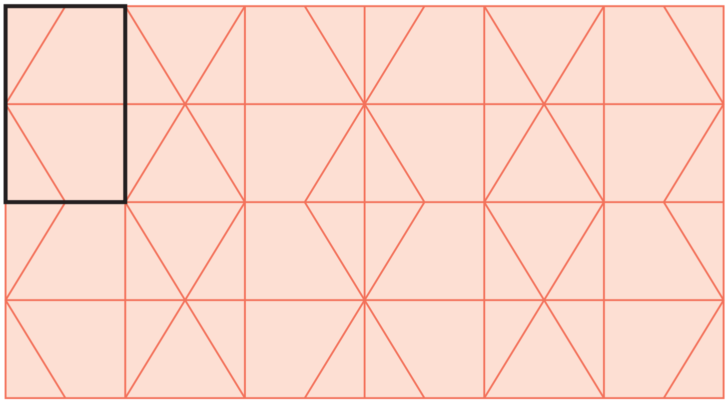


i

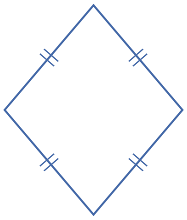


9 Identify and list the different types of quadrilaterals in this grid. Your list should include any larger, composite shapes, like the rectangle outlined below.



10 These shapes are commonly referred to as 'diamonds'. Classify each shape using their mathematical names.

a



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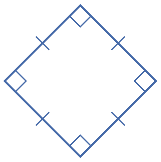
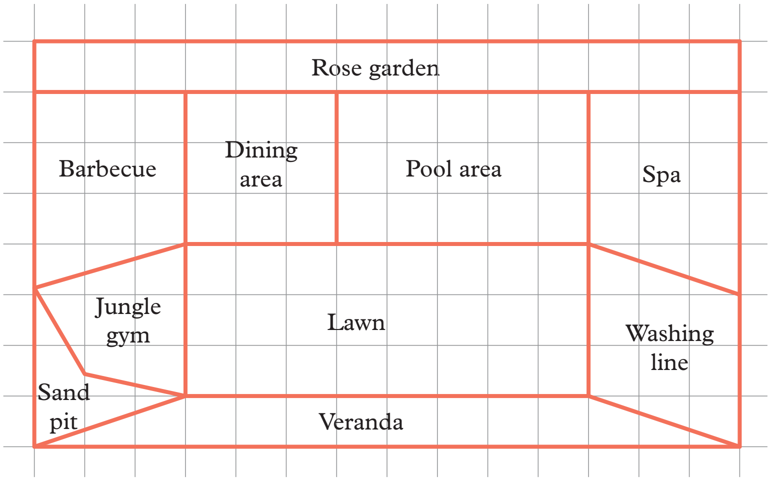
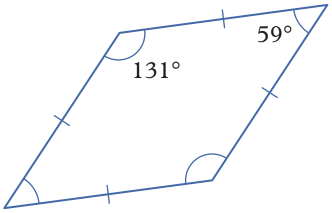


11 The Singh family is renovating their backyard. The backyard is rectangular, and their plans for the backyard involve a variety of quadrilaterals. Identify which type of quadrilateral each area of the plan makes, giving the most specific name possible.

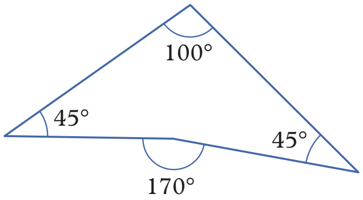


12 Each of the following diagrams contains at least one mistake. Identify the mistakes and explain why the diagrams are incorrect.

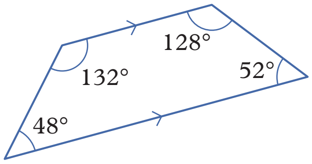
a Rhombus



b Kite



c Irregular quadrilateral



13 The following descriptions could be referring to more than one type of quadrilateral. List all the types of quadrilaterals that fit each description.

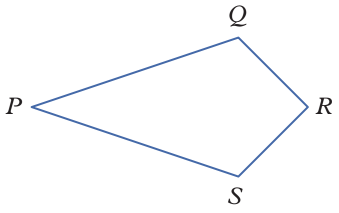
a All interior angles are right angles.
b All sides are equal in length.
c Two pairs of sides are equal in length.
d At least one pair of sides are parallel.
e Two pairs of opposite sides are equal in length.

14 Some of the special quadrilaterals are related. Answer the following questions and explain your answer by referring to the special features of each type of quadrilateral.

a Is a square a rectangle?
b Is a rectangle a square?
c Is a rectangle a parallelogram?
d Is a square a rhombus?

15 Which of the following could also be a name for quadrilateral *PQRS*?

A *PRQS*
B *PSRQ*
C *PSQR*
D *PRSQ*



16 Which types of quadrilaterals can be non-convex?

17 A convex quadrilateral has four angles less than 180° and a non-convex quadrilateral has an angle greater than 180° (a reflex angle). Explain why a quadrilateral cannot have an angle of 180°.

18 Every quadrilateral can be divided into two triangles by joining two of the vertices.

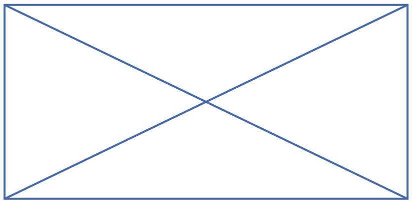
a Draw a rectangle in your workbook and draw a diagonal line joining two opposite corners of the rectangle. How many triangles have you formed?

b What is the sum of the angles in a triangle?

c Complete the following statement:

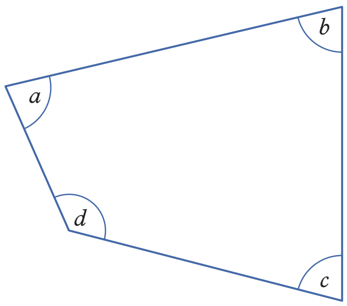
The sum of angles in a rectangle = 2 × ______° = ______°

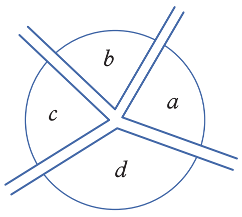
d A friend adds another diagonal line to the rectangle as shown below. This forms four triangles. Explain why it is not correct to say that the angle sum of a rectangle is 4 × 180° = 720°.



e How can the calculation in part **d** be corrected to show the true sum of angles in a rectangle?

f Another way to demonstrate the angle sum of a quadrilateral is to arrange the internal angles of a quadrilateral as shown in the diagrams below.





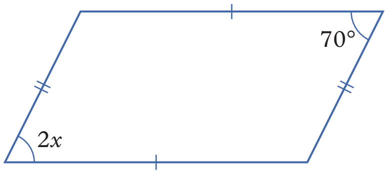
i What type of angle is formed by rearranging the internal angles of a quadrilateral as shown above?

ii How many degrees are in this type of angle? Does this match the angle sum you found in part **c**?

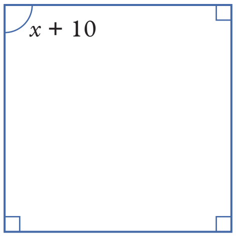
Exercise 7.5C: Challenge

19 Use your knowledge of quadrilateral properties and basic algebra to find the value of the pronumerals in these quadrilaterals.

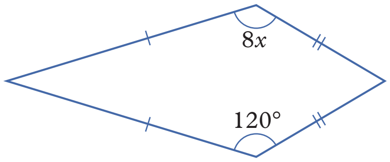
a



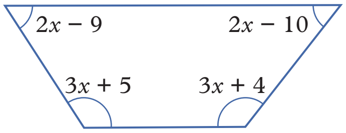
b



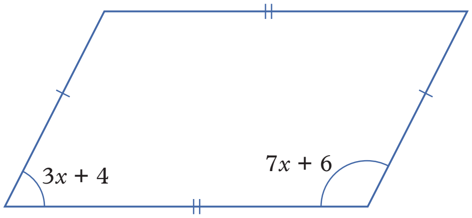
c



d



e



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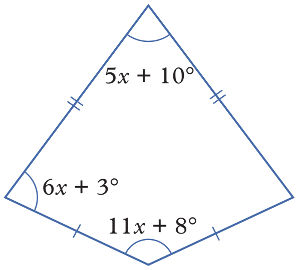
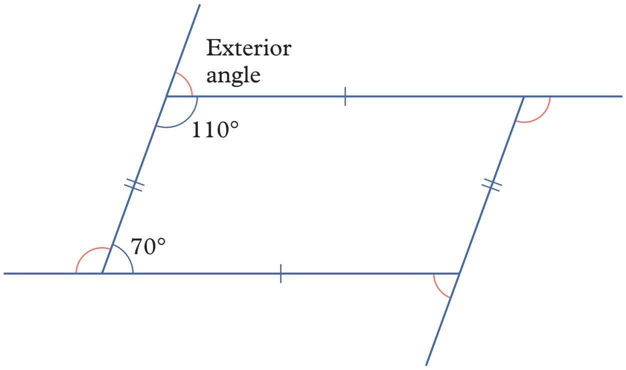


20 Exterior angles can be found for quadrilaterals as well as triangles.



a Calculate the four exterior angles of the above parallelogram.

b Add these angles together. What do you find? How does this compare to the sum of exterior angles for a triangle?

c Repeat the activity for at least two different types of quadrilaterals. Write a sentence about what you find.

Online resources:

Interactive skillsheet Quadrilateral properties	**Worksheet** Classifying quadrilaterals	**Investigation** Quadrilaterals everywhere	**Quick quiz** 7.5

Lesson 7.6

Polygons

Learning intentions

By the end of this lesson you will be able to ...

✔ classify, describe and name polygons.

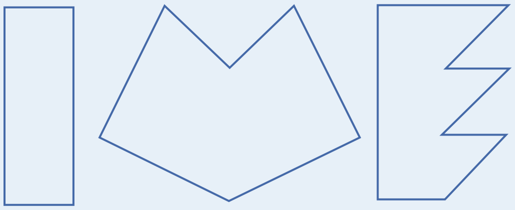
Inter-year links

Support Polygons

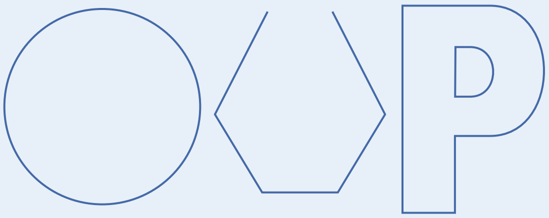
Polygons

Key content video
Polygons

- A **polygon** is a closed shape with straight sides.



Polygons



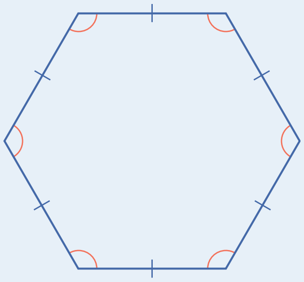
Non-polygons

- Polygons are named according to how many sides or interior angles they have.

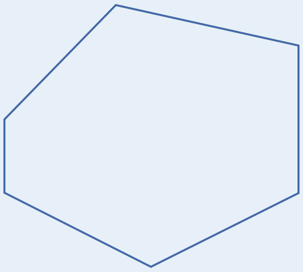
Greek/Latin prefix	Number of sides	Name of shape
penta-	5	pentagon
hexa-	6	hexagon
hepta-	7	heptagon
octa-	8	octagon
nona-	9	nonagon
deca-	10	decagon
dodeca-	12	dodecagon
poly-	many	polygon

→ Triangles (three sides) and quadrilaterals (four sides) are also polygons.

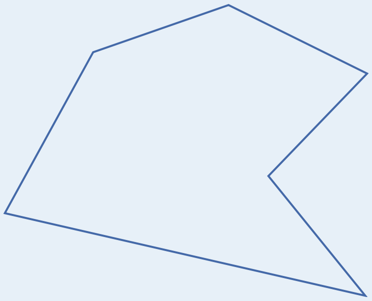
- **Regular polygons** have all sides equal in length and all angles equal in size. If a polygon is not regular, then it is an **irregular polygon**.
- **Concave polygons** have at least one interior reflex angle (greater than 180°). **Convex polygons** have all interior angles less than 180°. All regular polygons are convex polygons.



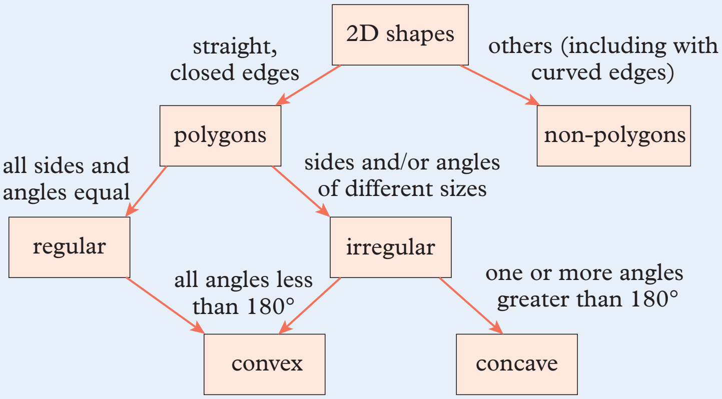
Regular, convex hexagon



Irregular, convex hexagon



Irregular, concave hexagon

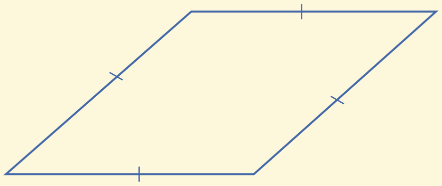


Worked example 7.6A Describing polygons

Decide if this polygon is:

a regular or irregular

b convex or concave.



THINK

a Examine the sides and angles. The angles are not equal in size, so the polygon is irregular.

b Examine the internal angles. There are no internal angles larger than 180°, so the polygon is convex.

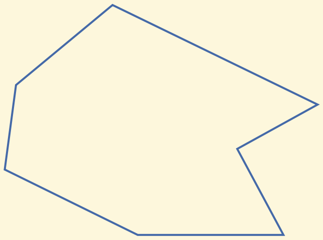
WRITE

a This polygon is irregular.

b This polygon is convex.

Worked example 7.6B Naming polygons

Describe and name this shape.



THINK

1 Examine the sides of the shape. All sides are straight, so it is a polygon. It has seven sides, so it is a heptagon.

2 All sides are not equal in length, so it is irregular.

3 Examine the angles of the shape. There is a reflex angle, so it is concave.

WRITE

The shape is an irregular, concave heptagon.

Helpful hints

- ✔ For a shape to be a polygon, all sides must be straight! Some common shapes, such as circles, are not polygons.
- ✔ An equilateral triangle is a regular polygon as all angles are equal in size and all side lengths are the same length.

Learning pathways

△ 1–4, 5(a, b, d), 6, 7

■ 1(b, e, f, i), 2(d–f), 3(d–f), 5–7, 8(a, b), 9

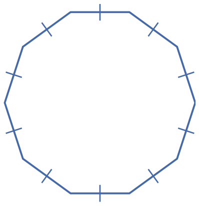
⬠ 1(b, e, f, i), 2(d–f), 3(e, f), 5(d–f), 7, 8, 10(c, d), 11

ANS p543

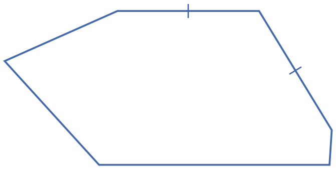
Exercise 7.6A: Understanding and fluency

1 Identify whether each shape is a polygon or a non-polygon.

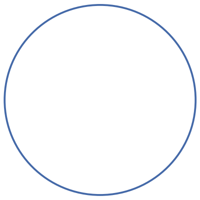
a



b



c



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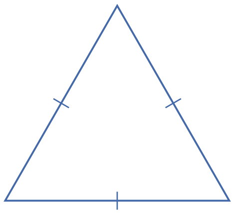

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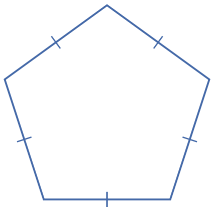

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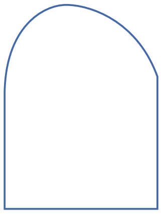

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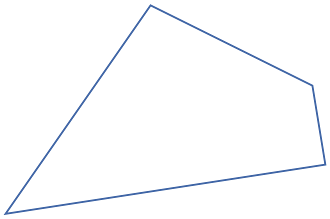

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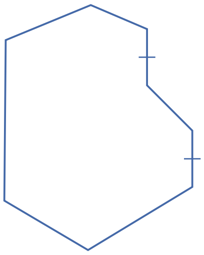
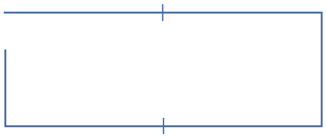


i

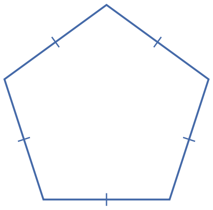


WE 7.6A **2** Decide if each polygon is:

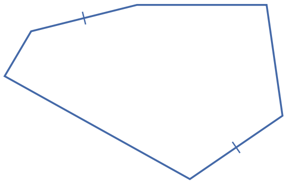
i regular or irregular

ii convex or concave.

a



b



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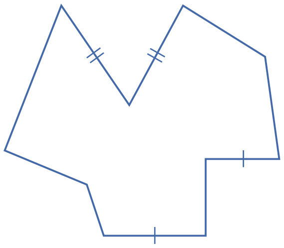

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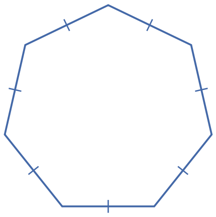
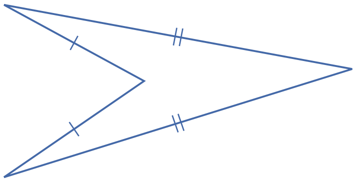
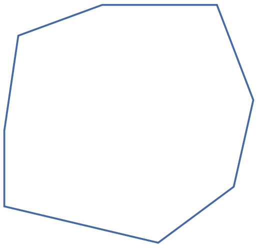


e

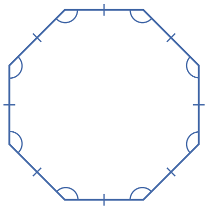


f



WE 7.6B **3** Describe and name each shape.

a



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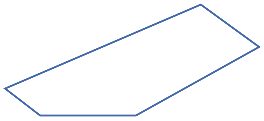

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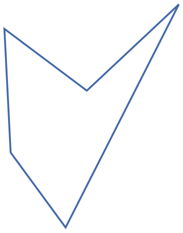

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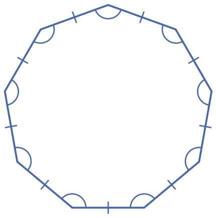

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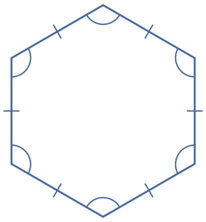

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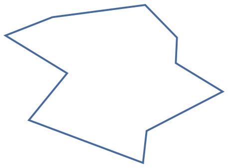


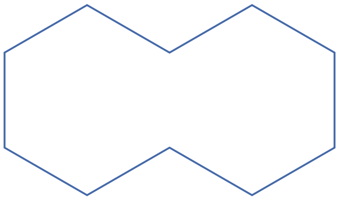
4 Consider each polygon only in question **1**.

- **i** State if the polygon is regular or irregular.
- **ii** Write the number of sides the polygon has.
- **iii** Does the polygon have an internal reflex angle?
- **iv** Give the polygon a name using your answers to parts **i**, **ii** and **iii**.

5 Use your knowledge of describing polygons to name each of the following shapes.

a

b



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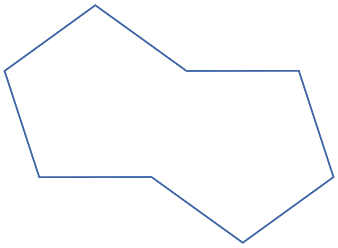

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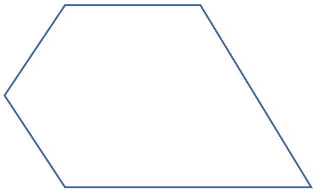

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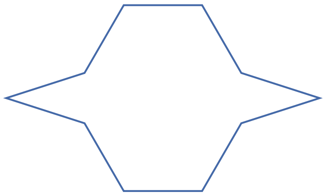

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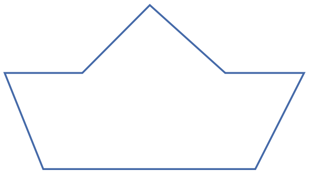
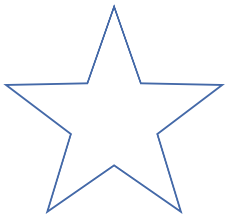


Exercise 7.6B: Problem solving and reasoning

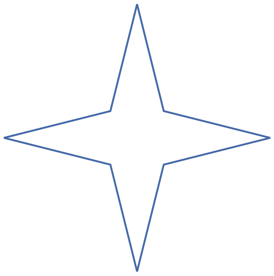


6 A familiar type of concave polygon is the star. The star on the right can be called either a five-pointed star or a concave decagon.

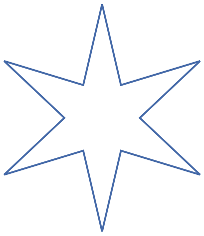
a Explain both methods for naming the star.

b Name the following star shapes according to each method.

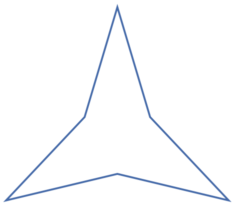
i



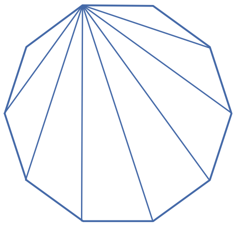
ii



iii



c What relationship can you see between the two methods for naming stars?



7 To calculate the interior angle sum of any polygon, divide the shape into triangles as shown in the decagon on the right.

a What property of triangles makes it useful to do this?

b How many sides does a decagon have?

c How many triangles are in the decagon?

d Use your answer from part **c** to calculate the interior angle sum in a decagon.

8 Consider each shape below and use the triangle method from question **7** to find:

i the number of triangles each shape can be divided into

ii the interior angle sum in each shape.

a a pentagon **b** a hexagon **c** a heptagon **d** a nonagon

9 Are triangles and quadrilaterals considered to be polygons? Explain your answer.

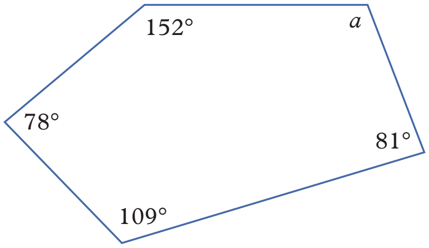
Exercise 7.6C: Challenge

10 Use your findings from question **7** to find the size of an individual internal angle in each of these shapes if they are regular polygons.

a pentagon **b** hexagon **c** heptagon **d** nonagon

11 Find the size of each unknown angle in these irregular polygons.

a



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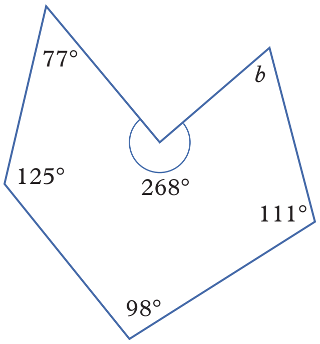

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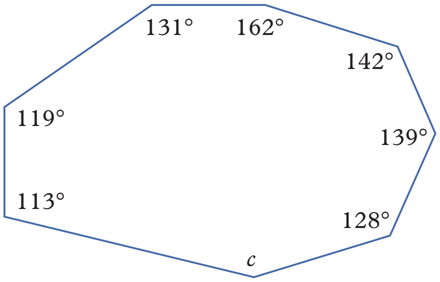


Online resources:

Interactive skillsheet
Polygons

Investigation
Constructing regular polygons

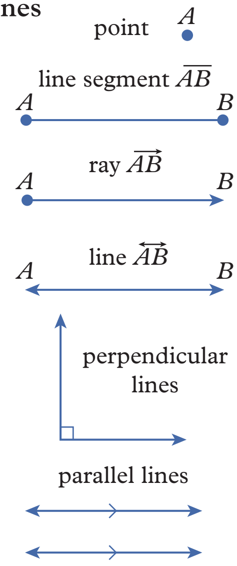
Quick quiz
7.6

Lesson 7.7

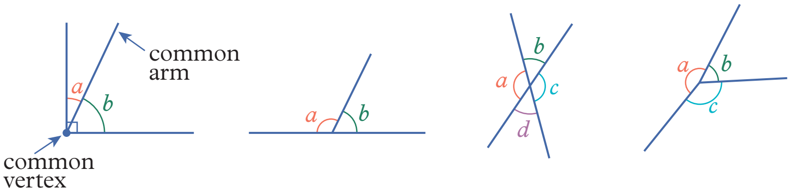
Review: Angles and shapes

Module summary

Lines



Angles around a point



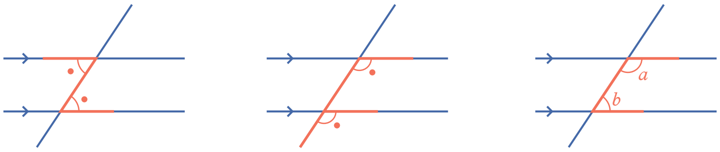
complementary $a + b = 90°$

supplementary $a + b = 180°$

vertically opposite $a = c$ and $b = d$

revolution $a + b + c = 360°$

Angles and parallel lines



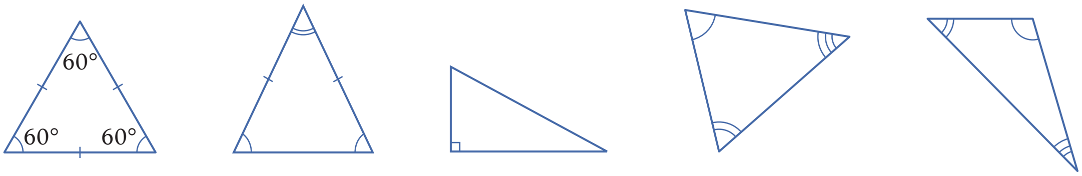
alternate ‘Z’

corresponding ‘F’

co-interior ‘C’ $a + b = 180°$

Triangles

$a + b + c = 180°$



equilateral

isosceles

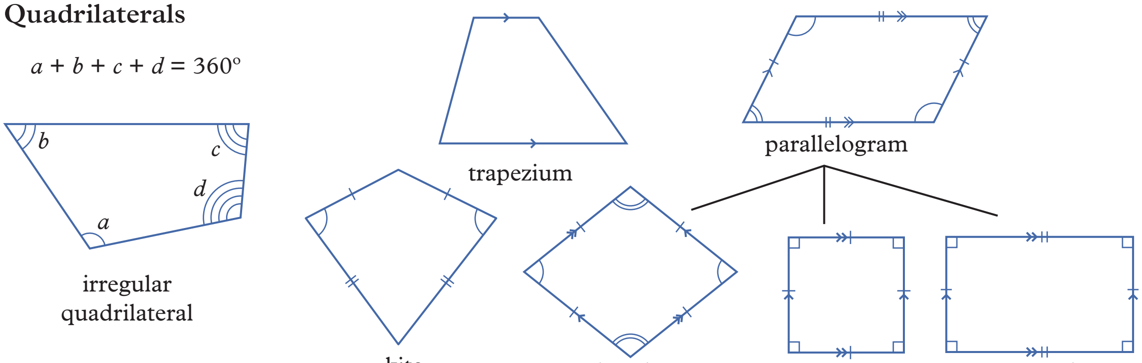
right-angled

scalene acute

scalene obtuse

Quadrilaterals

$a + b + c + d = 360°$



irregular quadrilateral

trapezium

parallelogram

kite

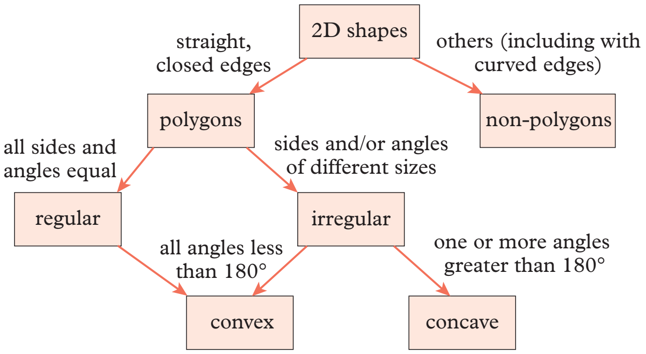
rhombus

square

rectangle

Polygons

Number of sides	Name
5	pentagon
6	hexagon
7	heptagon
8	octagon
9	nonagon
10	decagon
12	dodecagon



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Review questions 7.7A: Mathematical literacy review

Review quiz
Take the review quiz to assess your knowledge of this module.

Quizlet
Test your knowledge of this module by working individually or in teams.

The following key terms are used in this module:

- acute angle
- acute triangle
- adjacent angles
- alternate angles
- angle
- arc
- arms
- co-interior angles
- complementary angles
- concave polygon
- convex polygon
- convex quadrilateral
- corresponding angles
- degrees
- equilateral triangle
- interior angles
- internal angles
- irregular polygon
- irregular quadrilateral
- isosceles triangle
- kite
- line
- line segment
- non-convex quadrilateral
- obtuse angle
- obtuse triangle
- parallel lines
- parallelogram
- perpendicular lines
- point
- polygon
- quadrilateral
- ray
- rectangle
- reflex angle
- regular polygon
- revolution
- rhombus
- right angle
- right-angled triangle
- scalene triangle
- square
- straight angle
- supplementary angles
- transversal
- trapezium
- triangle
- vertex
- vertically opposite angles

1 Which unit are angles commonly measured in?

2 Which type of triangle has two sides of equal length?

3 Which type of quadrilateral has at least one pair of parallel sides?

4 Explain the difference between complementary angles and supplementary angles.

5 Identify the key terms being referenced in each of these definitions.

a a line joining two points

b a quadrilateral with two pairs of opposite sides that are parallel and equal in length, and four angles of 90°

c a closed shape with straight sides

6 Using an example, provide a definition in your own words for the following key terms.

a transversal

b right angle

c vertically opposite angles

7 Complete the following sentences using words from the key terms list.

a When two lines or _____ meet, they form an _____.

b _____ share a common arm and a common _____, but do not overlap.

Review questions 7.7B: Multiple choice

7.1 **1** In which of the following lists are the types of angles listed in order of decreasing size?

A revolution, obtuse, right

B right, straight, revolution

C reflex, obtuse, straight

D acute, obtuse, reflex

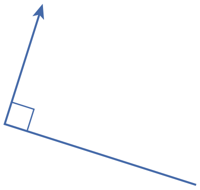
7.1 **2** The diagram on the right is best described as:

A a ray parallel to a line segment.

B two parallel rays.

C a line segment perpendicular to a ray.

D two perpendicular line segments.



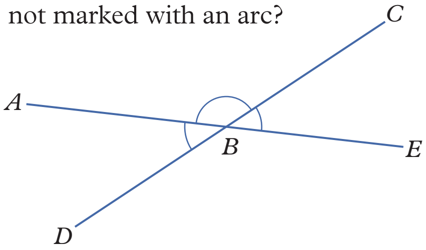
7.1 **3** In the diagram on the right, which of the following angles is not marked with an arc?

A ∠*ABC*

B ∠*DBA*

C ∠*CBA*

D ∠*DBE*



7.2 **4** Two angles that add to 90° are called:

A supplementary angles.

B right angles.

C perpendicular angles.

D complementary angles.

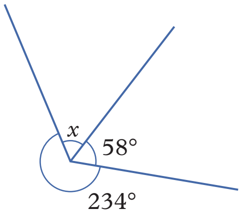
7.2 **5** What is the size of angle x in the diagram on the right?

A 42°

B 68°

C 122°

D 292°



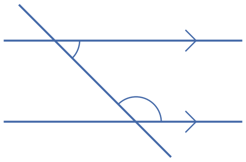
7.3 **6** The angles marked in the diagram on the right are:

A co-interior and supplementary.

B corresponding and complementary.

C co-interior and complementary.

D corresponding and supplementary.



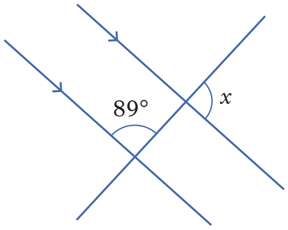
7.3 **7** The value of x in the diagram on the right is:

A 1°

B 89°

C 91°

D 101°



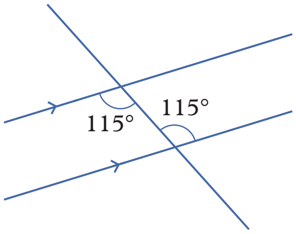
7.3 **8** Based only on the information in the diagram on the right, the lines are parallel because:

A the alternate angles are supplementary.

B the corresponding angles are equal.

C the co-interior angles are equal.

D the alternate angles are equal.



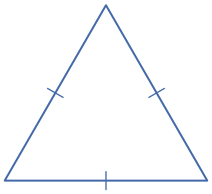
7.4 **9** The triangle on the right is:

A a scalene acute triangle.

B an isosceles acute triangle.

C an equilateral acute triangle.

D an equilateral right-angled triangle.



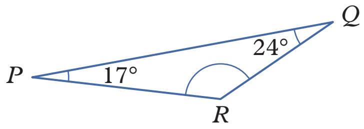
7.4 **10** In the diagram on the right, ∠*QRP* is equal to:

A 17°

B 139°

C 41°

D 24°



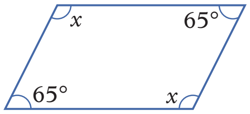
7.5 **11** The quadrilateral on the right is:

A a kite.

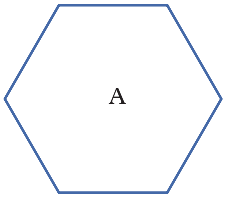
B a parallelogram.

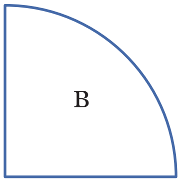
C a rhombus.

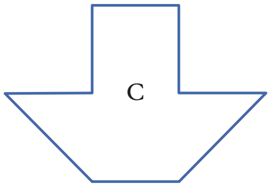
D a rectangle.

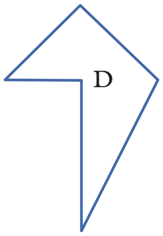


Use figures A–D to answer questions **12** and **13**.









7.6 **12** Which shape is not a polygon?

A A　　**B** B　　**C** C　　**D** D

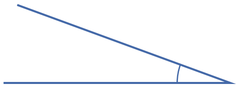
7.6 **13** Which shape is a pentagon?

A A　　**B** B　　**C** C　　**D** D

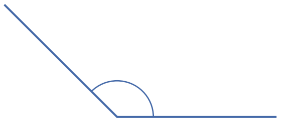
Review questions 7.7C: Short answer

7.1 **1** Measure and classify the angles below as acute, right, obtuse, straight, reflex or a revolution.

a

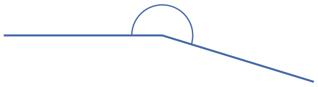


b



c

d

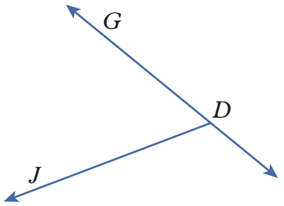


7.1 **2** Draw each of the following angles using a protractor.

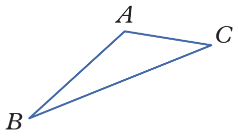
a 40°　　**b** 115°　　**c** 200°

7.1 **3** Identify and name all of the objects in the following diagrams.

a



b



7.2 **4** Complementary angles add to 90° and supplementary angles add to 180°.

a Find the complementary angle to:

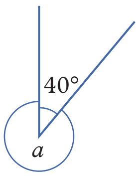
i 23°　　**ii** 79°.

b Find the supplementary angle to:

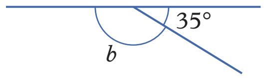
i 9°　　**ii** 127°.

7.2 **5** Find the size of the unknown angles in each of the diagrams below.

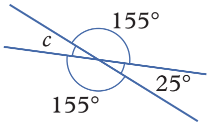
a



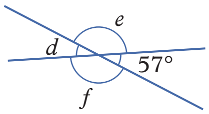
b



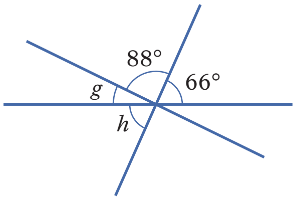
c



d



e



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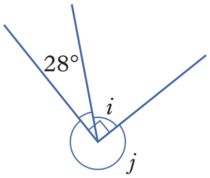


7.3 **6** Use this diagram to find:

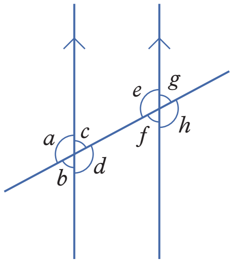
a the angle corresponding to *c*

b the alternate angle to *e*

c the co-interior angle to *f*.

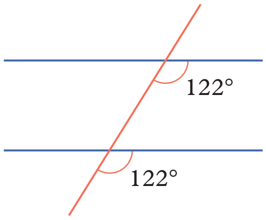
d If *f* = 77°, find:

i *b* **ii** *d* **iii** *a*.

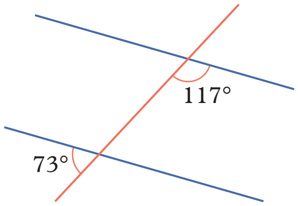


7.3 **7** Decide whether the blue lines in each diagram are parallel. Explain your reasoning.

a

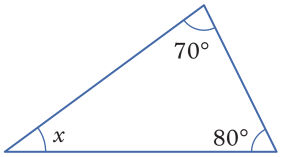


b

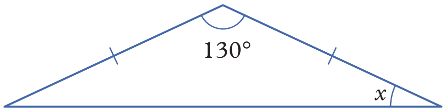


7.4 **8** Fully classify each of the triangles below and hence find the value of *x*.

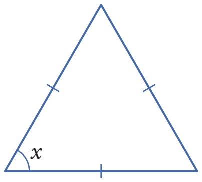
a



b



c



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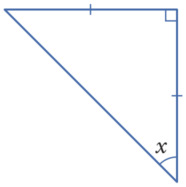
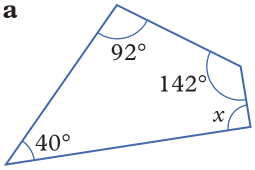


7.5 **9** Classify each of the quadrilaterals below and hence find the value of *x*.

a



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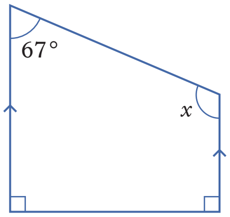
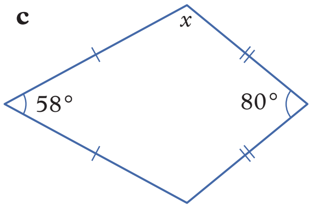


c



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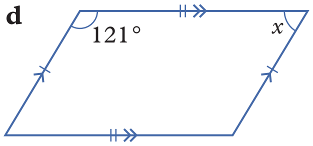
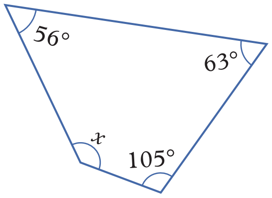
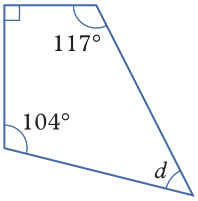


7.5 **10** Find the value of the pronumerals in each diagram below.

a



b



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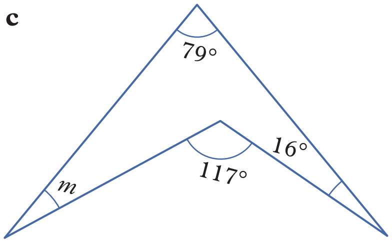


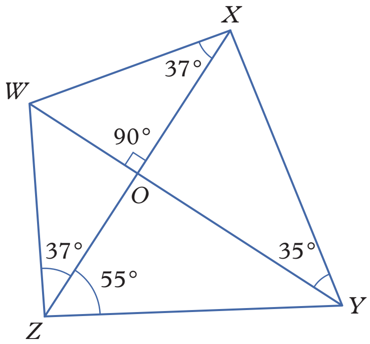
7.5 **11** Consider the diagram on the right where $\angle WXZ = 37°$, $\angle WZX = 37°$, $\angle OZY = 55°$ and $\angle OYX = 35°$.

a Find the size of the following angles.

i $\angle XWZ$ **ii** $\angle YOZ$

iii $\angle OYZ$ **iv** $\angle XOY$

b Classify the quadrilateral $WXYZ$.



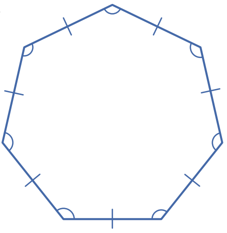
7.6 **12** Consider each of the following polygons.

i State whether it is regular or irregular.

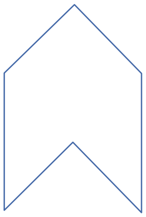
ii Write the number of sides it has.

iii Does it have an internal reflex angle?

a



b



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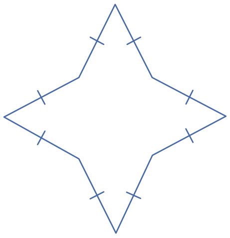

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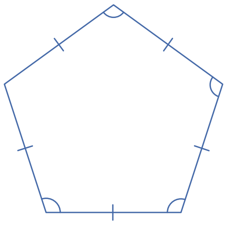
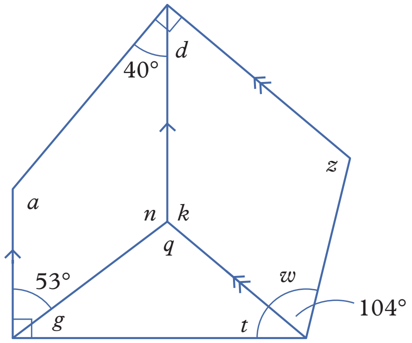


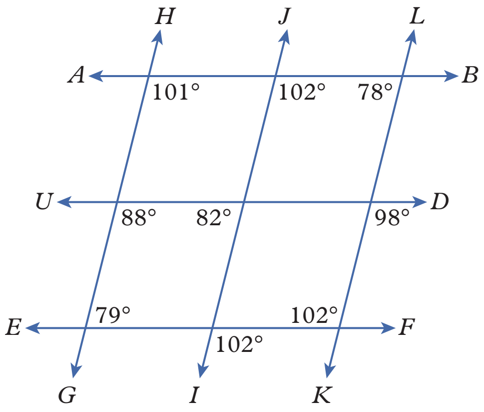
7.6 **13** Give each of the polygons in question **12** a name using your answers to parts **i**, **ii** and **iii**.

Review questions 7.7D: Mathematical modelling

1 The irregular pentagon below has been broken into two different quadrilaterals and a triangle. Find the value of all the pronumerals.



2 Identify and name the parallel lines in the diagram below.



Checklist

Now that you have completed this module, reflect on your ability to do the following.

I can do this	I need to review this
☐ Identify and name line segments, rays, lines, parallel lines and perpendicular lines ☐ Measure and draw angles using a protractor ☐ Identify and classify angles	☐ Go back to Lesson 7.1 Classifying angles and lines
☐ Identify complementary, supplementary and vertically opposite angles ☐ Find unknown angles using complementary and supplementary relationships ☐ Find unknown angles at a point	☐ Go back to Lesson 7.2 Angles at a point
☐ Identify alternate, corresponding and co-interior angles ☐ Find unknown angles by identifying alternate, corresponding and co-interior angles ☐ Determine if two lines are parallel using alternate, corresponding and co-interior angles	☐ Go back to Lesson 7.3 Angles and parallel lines
☐ Classify triangles using side lengths and angle sizes ☐ Find unknown angles using the sum of angles in a triangle	☐ Go back to Lesson 7.4 Triangles
☐ Identify and classify squares, rectangles, rhombuses, parallelograms, kites and trapeziums ☐ Find unknown angles using the sum of angles in a quadrilateral	☐ Go back to Lesson 7.5 Quadrilaterals
☐ Classify polygons as regular or irregular ☐ Classify polygons as convex or concave ☐ Describe and name polygons	☐ Go back to Lesson 7.6 Polygons

8

Objects and transformations

Lessons

Prerequisite skills

Diagnostic pre-test
Take the diagnostic pre-test to assess your knowledge of the prerequisite skills listed below.

Interactive skillsheets
After completing the diagnostic pre-test, brush up on your knowledge of the prerequisite skills by using the interactive skillsheets.

✔ Polygons
✔ The Cartesian plane

Curriculum links

- Represent three-dimensional objects in 2 dimensions; discuss and reason about the advantages and disadvantages of different representations (VC2M7SP01)
- Describe the effect of transformations of a set of points using coordinates in the Cartesian plane, including translations, reflections in an axis, and rotations about the origin (VC2M7SP03)

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Materials

✔ Ruler
✔ Isometric dot paper

Lesson 8.1

3D objects

Learning intentions

By the end of this lesson you will be able to ...

✔ identify and name 3D objects, and draw right prisms.

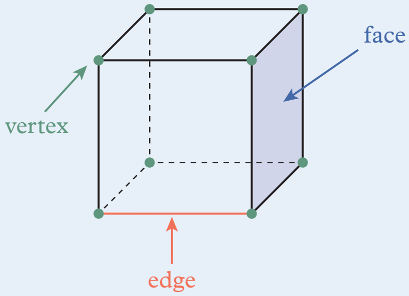
Inter-year links

Support	3D objects
Year 8	8.6 Three-dimensional space

Key content video
3D objects

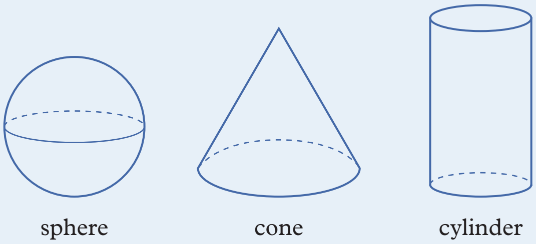
3D objects

- A **face** is a flat surface on a three-dimensional (3D) object.
  - → An **edge** is a line segment that joins two faces.
  - → A **vertex** (plural: vertices) is a corner point where three or more edges meet.



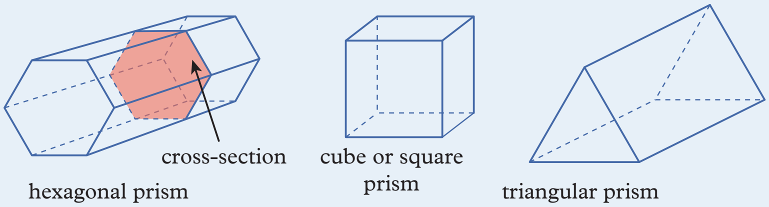
- A **cross-section** of a 3D object is the 2D shape obtained when cutting the object.
- A **polyhedron** (plural: polyhedra) is a 3D object with all faces being polygons.
  - → All polyhedra have straight edges.
  - → **Cylinders**, **cones** and **spheres** are examples of 3D objects that are not polyhedra.

Non-polyhedra



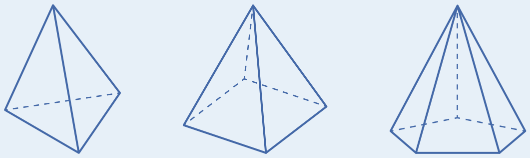
- A **prism** is a polyhedron with matching ends that are the same shape and size. The matching ends of a prism are joined by straight edges. In a **right prism**, the angles between the matching ends and the other faces are right angles.
  - → Prisms have a consistent cross-section throughout their length. That is, all cross-sections of the prism parallel to the matching ends are the same shape and size.
  - → Prisms are named after the shape at each matching end. For example, if the end shape is a triangle it is a triangular prism.
  - → The other faces (not end faces) of a right prism are rectangles.

Right prisms



- A **pyramid** is a polyhedron with a **base** and triangular faces joining the base which all meet at a single point.
  - → Pyramids are named after the shape of the base. For example, if the base is a pentagon it is a pentagonal-based pyramid.

Pyramids



triangular-based pyramid

square-based pyramid

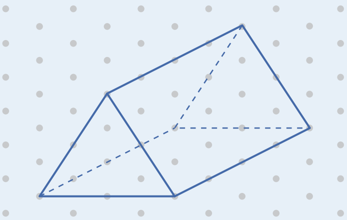
pentagonal-based pyramid

  - → A triangular-based pyramid may also be called a tetrahedron.

Drawing right prisms

- Isometric or square dot paper can be used to draw 3D shapes.
  - → Dashed lines should be used to show hidden edges.

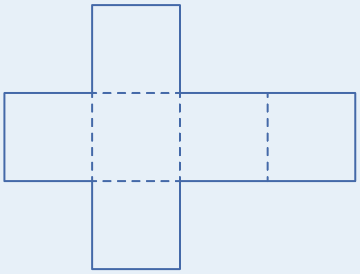
Triangular prism

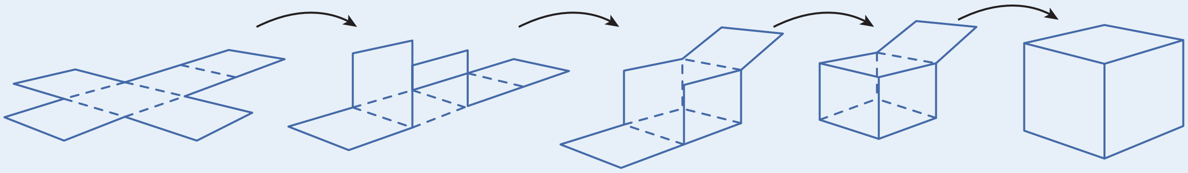


Nets

Key content video
Nets

- A **net** is a two-dimensional (2D) composite shape that can be folded to make a 3D object.
  - → In a net you can see all the different faces of a 3D object. These faces must be put together in such a way that they will fold to make this object.
  - → The folds in a net are indicated by dashed lines.

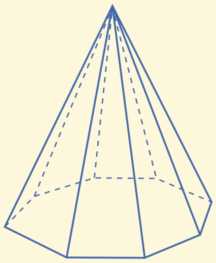




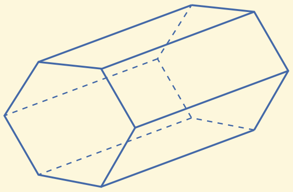
Worked example 8.1A Identifying and naming 3D objects

Name the following 3D objects.

a



b



THINK

a **1** Identify the object. This is a polyhedron with a base and triangular faces joining the base which all meet at a single point, so it is a pyramid.

2 Identify the shape of the base. The base has eight sides, so it is an octagon.

3 Name the object.

b **1** Identify the object. This is a polyhedron with matching ends that are the same shape and size, and straight edges joining these ends, so it is a prism.

2 Identify the shape of the base. The base has six sides, so it is a hexagon.

3 Name the object.

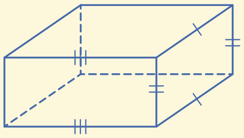
WRITE

a This is an octagonal-based pyramid.

b This is a hexagonal prism.

Worked example 8.1B Drawing nets

Draw a net for this rectangular prism.



THINK

1 Identify the number of faces on the object, as well as the shape of each face. This object has six rectangular faces. Visualise the object unfolding.

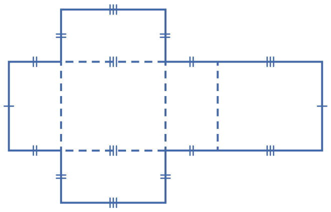
2 Draw the face at the bottom first; this will form the centre of the net.

3 Next, draw the faces which touch the bottom face. Each of these faces will be on a different side of the bottom face in the net.

4 Draw the final face, which will touch one of the other faces. Visualise the net folding back to the shape of the object.

5 Add equal side marks to the net, and ensure that all folds are indicated using dashed lines.

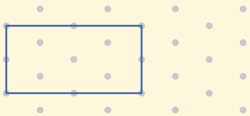
WRITE



Worked example 8.1C Using a 2D shape to draw a prism

Use isometric dot paper to draw a prism with the 2D shape as one of the matching ends.

Rectangular prism

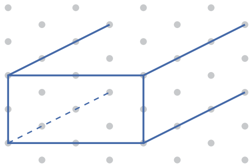


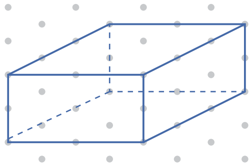
THINK

1 Draw a line from each vertex using the dots on the paper as a guide. Make sure each line is the same length and that all four lines are parallel. Show the hidden edge as a dashed line.

2 Complete the prism by drawing two horizontal lines and two vertical lines that join the vertices that have just been drawn. Show any hidden edges as dashed lines.

WRITE





Helpful hints

- ✔ The matching ends of a prism must be polygons. Many people mistakenly assume that cylinders are prisms as they have a consistent cross-section throughout their length. However, the matching ends of a cylinder are not polygons.
- ✔ When using isometric dot paper to draw 3D objects, the paper should be held so there are vertical columns of dots.
- ✔ If you are unsure whether a net will fold up into a 3D object, cut it out and try for yourself. Including tabs can also be helpful if you want to construct a 3D object.

Learning pathways

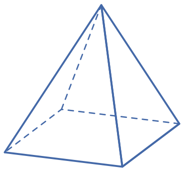
▲	1–6, 7(d–f), 8, 9(b, c, e), 10, 11(a, c, e), 12(b), 14, 16	■	2, 3, 7, 8, 9(a, b, f), 11(c, f), 12(a), 13, 15–18, 21	⬟	3(d–f), 7(d–f), 11(c, f), 12(b, c), 15–21

ANS p545

Exercise 8.1A: Understanding and fluency

Isometric dot paper to complete the isometric drawings can be found on your obook pro.

1 Look at this square-based pyramid.

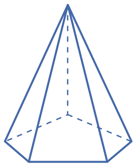


a How many faces does it have?

b How many edges does it have?

c How many vertices does it have?

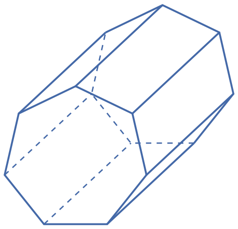
2 Look at this pentagonal pyramid.



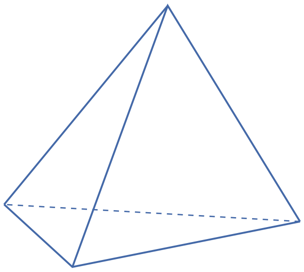
- **a** How many faces does it have?
- **b** How many edges does it have?
- **c** How many vertices does it have?

3 Identify whether the following 3D objects are polyhedra or non-polyhedra.

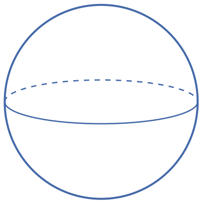
a



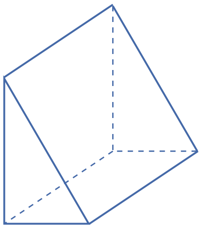
b



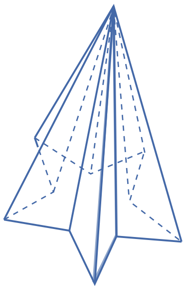
c



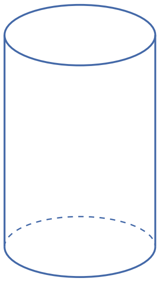
d



e



f



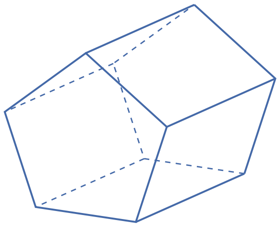
4 Which of the polyhedra in question **3** have a consistent cross-section throughout their length, parallel to the base?

5 What 2D shape forms the cross-section or base of each prism and pyramid in question **3**?

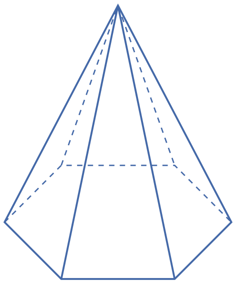
6 Name the prisms and pyramids in question **3**.

WE 8.1A 7 Name the following 3D objects.

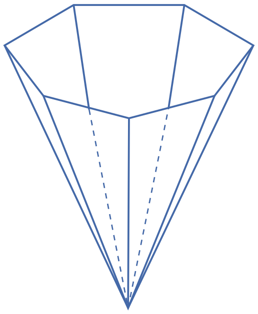
a



b



c



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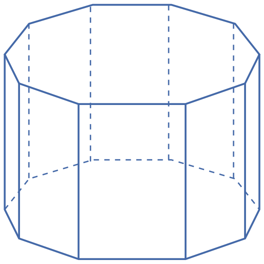
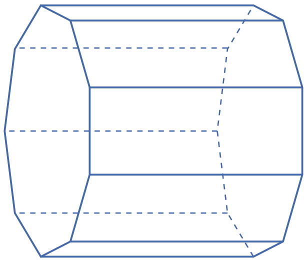


e



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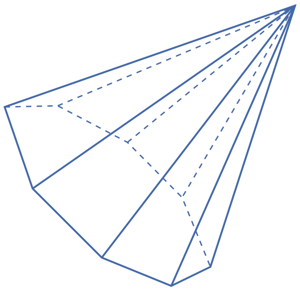


8 Provide the mathematical name for each of these real-life 3D objects.

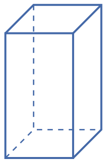
a

b

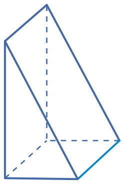
c

WE 8.1B **9** Draw a net for each of these 3D objects.

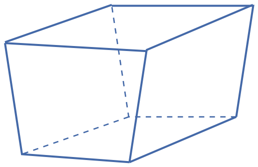
a



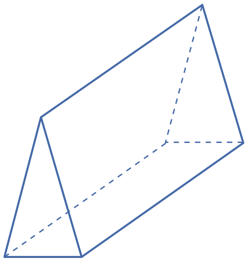
b



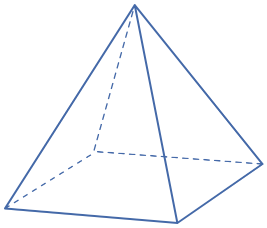
c



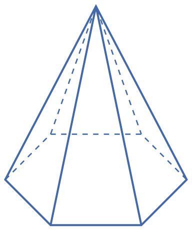
d



e



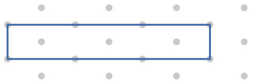
f



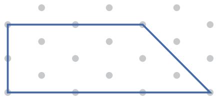
10 Nets can also be made for open objects. For example, this box has no lid. Redraw the nets for the objects in questions **9a** and **c** as if they were open objects.

WE 8.1C **11** Use isometric dot paper to draw a prism with the 2D shape as one of the matching ends.

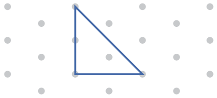
a Rectangular prism



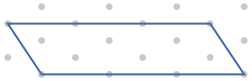
b Trapezium prism



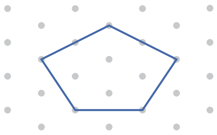
c Triangular prism



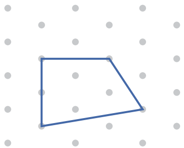
d Parallelogram prism



e Pentagonal prism



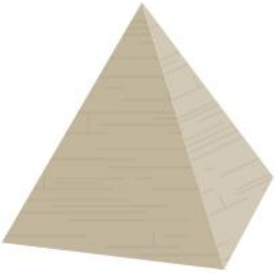
f Quadrilateral prism



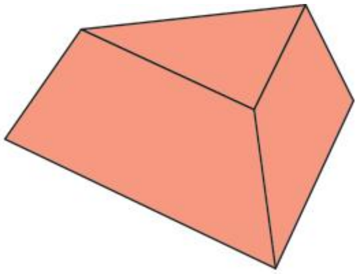
12 Use isometric dot paper to draw these 3D objects.

a

b



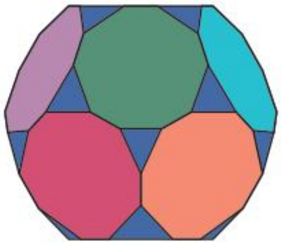
c



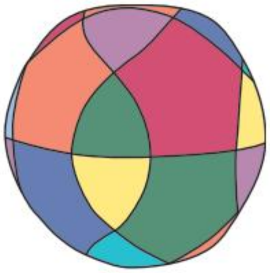
Exercise 8.1B: Problem solving and reasoning

13 Decide whether each 3D object is a polyhedron. Explain your answers.

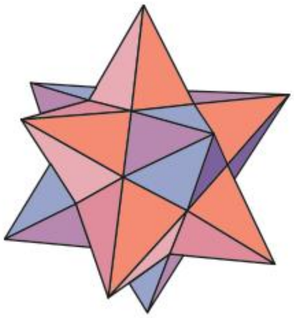
a



b



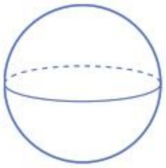
c



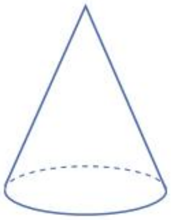
14 Consider these three 3D objects.

a Name each object.

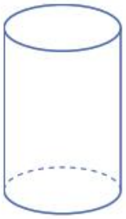
i



ii



iii

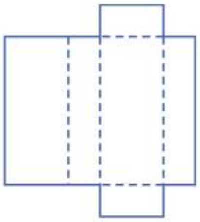


b Explain why they are not polyhedra.

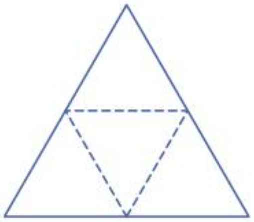
15 Is a soccer ball a polyhedron? Explain your answer.

16 What object can be constructed from each of these nets?

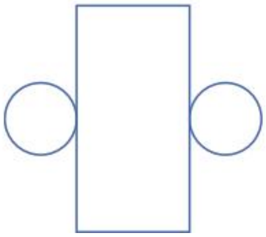
a



b

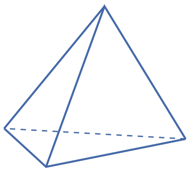
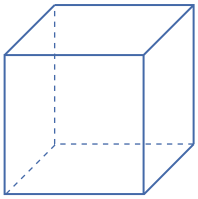
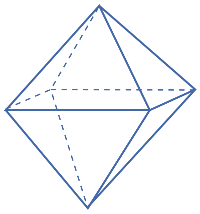
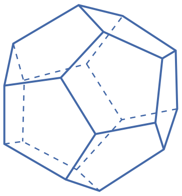
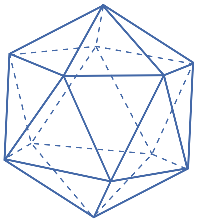


c



17 There is a group of special polyhedra named the Platonic solids. These polyhedra are all regular. This means that all their faces are the same shape, their edges are the same length and the vertices meet at the same angles.

a Copy and complete the table below.

	Tetrahedron	**Cube (Hexahedron)**	**Octahedron**	**Dodecahedron**	**Icosahedron**
Image					
Number of faces (*F*)				12	20
Name of polygon on face					

b Add two rows to the table and find the number of vertices (V) and the number of edges (E) for each Platonic solid.

c Add another row to the table and calculate $F + V - E$.

d What do you find?

18 Euler's rule states that for any polyhedron, $F + V - E = 2$.

a Is this true for all the Platonic solids above?

b Investigate Euler's rule further by finding $F + V - E$ for all the polyhedra in question **3**. What do you find?

19 Use Euler's rule to find the number of faces on each polyhedron, given the object has:

a 10 edges, 4 vertices

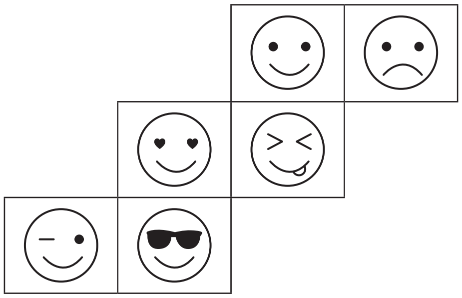
b 6 edges, 3 vertices

c 18 edges, 6 vertices

d 20 edges, 10 vertices.

Exercise 8.1C: Challenge

20 The net below is folded into a cube. It is placed on a table with the  face on top. Which face is at the bottom?



21 Two nets are distinct if they cannot be reflected or rotated to form each other. There are 11 different distinct nets for a cube. Draw them all.

Online resources:

- **Interactive skillsheet** Naming 3D objects
- **BLM** Isometric dot paper
- **Worksheet** Drawing 3D objects
- **Investigation** Isometric and perspective drawings
- **Quick quiz** 8.1

Lesson 8.2

2D representations of 3D objects

Learning intentions

By the end of this lesson you will be able to …

✔ draw plans and elevations of prisms and 3D objects.

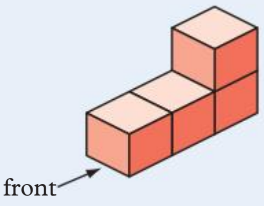
Inter-year links

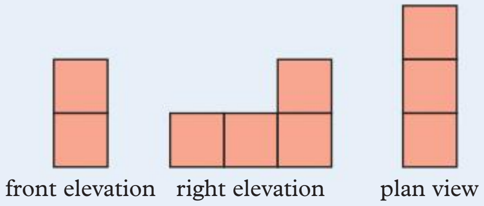
Support	3D objects
Year 8	8.6 Three-dimensional space

Plans and elevations

Key content video
Plans and elevations

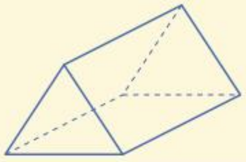
- **Plans** and **elevations** are 2D drawings of a 3D object from a single view.
- A plan view shows what a 3D object looks like when viewed from directly above the object. An elevation shows what a 3D object looks like when viewed from the front, side or back of the object.
- A set of plan and elevations consists of a front elevation, a side elevation (either left or right) and a plan view.
  → Back elevations and bottom views may also be included in a set of plan and elevations.
- A 3D object can be drawn from a set of plan and elevations.




front elevation right elevation plan view

Worked example 8.2A Drawing a plan and elevations of prisms

Draw a set of plan and elevations for this prism.



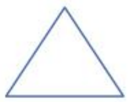
THINK

1 Picture viewing the object from the front. From this angle the view is a triangle. Draw the triangle as the front elevation.

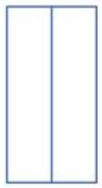
2 Picture viewing the object from the right-hand side. From this angle the view is a rectangle. Draw the rectangle as the right elevation.

3 Picture viewing the object from above. From this angle the view is a rectangle split in two. Draw this as the plan view.

WRITE

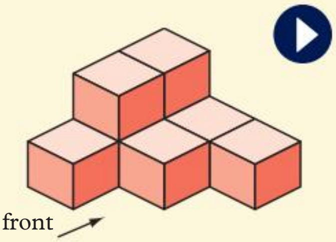

front elevation


right elevation


plan view

Worked example 8.2B Drawing a plan and elevations of 3D objects

Draw a set of plan and elevations for this object.



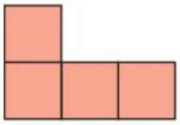
THINK

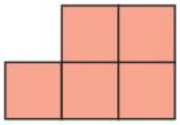
1 Picture viewing the object from the front, as indicated by the arrow. Three cubes are visible on the bottom and only one on the top. Draw this as the front elevation.

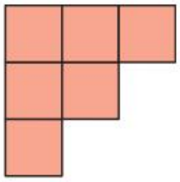
2 Picture viewing the object from the right-hand side. Three cubes are visible on the bottom and two on the top. Draw this as the right elevation.

3 Picture viewing the object from above. Three rows of cubes are visible: three in the top row, two in the next row and one in the bottom row. Draw this as the plan view.

WRITE


front elevation


right elevation


plan view

Helpful hints

Do not try to include perspective in your views! For example, when looking directly at a sloping edge (as in the right elevation of **Worked example 8.2A**), the slope will not be visible from that view.

Learning pathways

△ 1–4, 5(a, b), 6, 8, 9, 11, 12, 14(a)

■ 1, 2, 3(d–f), 4–8, 11, 12, 14(a–c)

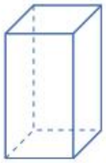
⬟ 3(d–f), 4, 5(c, d), 6, 7, 13, 14(d), 15, 16

ANS p546

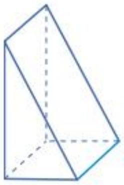
Exercise 8.2A: Understanding and fluency

WE 8.2A **1** Draw a set of plan and elevations for each of these prisms.

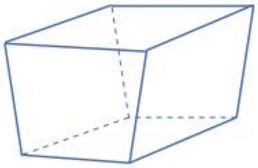
a



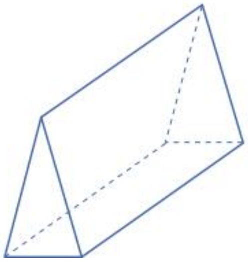
b



c



d



2 Draw a set of plan and elevations for each of these objects. Do not include the writing on the packaging or the dots on the die.

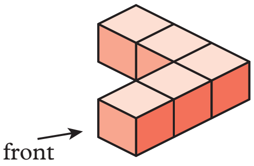
a

b

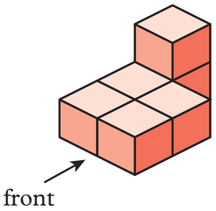
c

WE 8.2B **3** Draw a set of plan and elevations for each object.

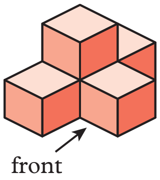
a



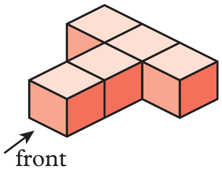
b



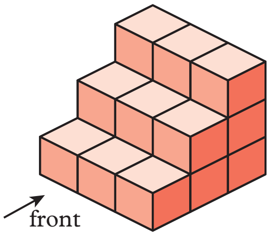
c



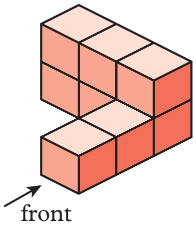
d



e

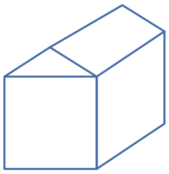


f

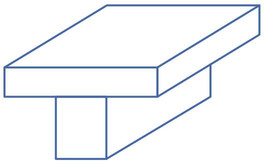


4 Draw a set of plan and elevations for each of these objects. Include the bottom views in your set of drawings.

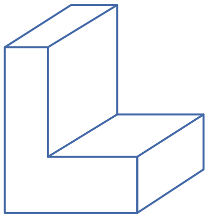
a



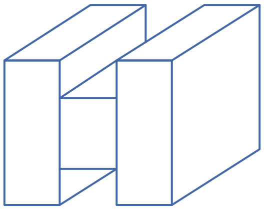
b



c

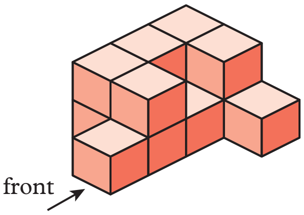


d

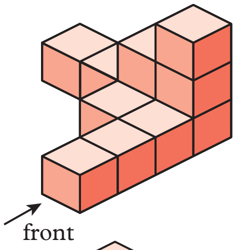


5 Draw a set of plan and elevations for each of these more complex objects. Include the back elevations in your set of drawings.

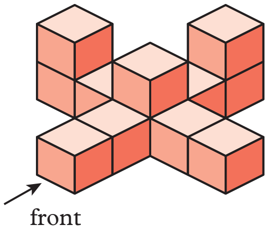
a



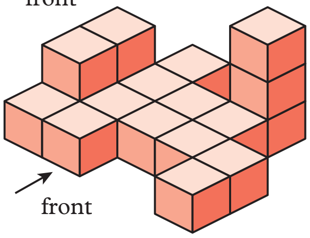
b



c

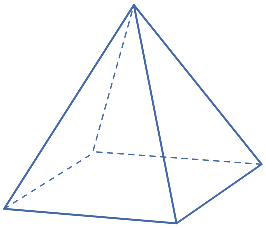


d

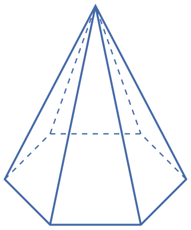


6 Draw a set of plan and elevations for each of these pyramids.

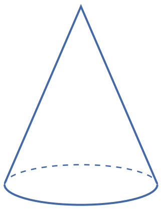
a



b

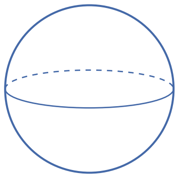


7 Draw a set of plan and elevations for this cone.

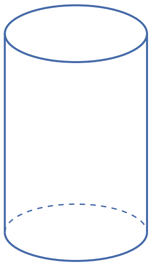


Exercise 8.2B: Problem solving and reasoning

8 What shapes are the front elevation, side elevation and plan view of a sphere?



9 Explain why the side elevation of an upright cylinder is a rectangle.



10 Draw a set of plan and elevations for this tent.

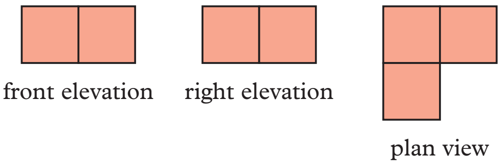
11 Consider the sets of drawings from your answers to question **4** that contain the bottom views as well as the plan views. Discuss the similarities and differences between the plan views and bottom views.

12 Consider the sets of drawings from your answers to question **5** that contain the back elevations as well as the front elevations. Discuss the similarities and differences between the front elevations and back elevations.

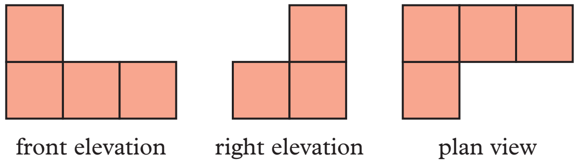
13 Explain why some people may decide to include the back elevations and/or bottom views in a set of plan and elevations, and some people may decide to leave them out.

14 Use each set of plan and elevations to construct the matching 3D object. Draw the objects on isometric or square dot paper.

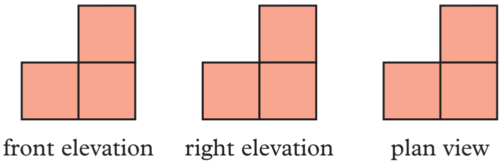
a



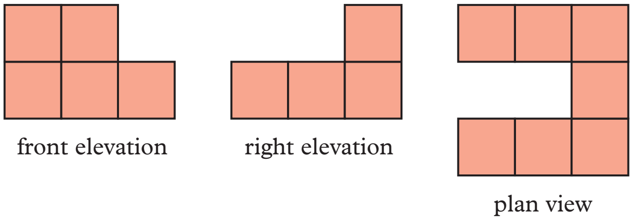
b



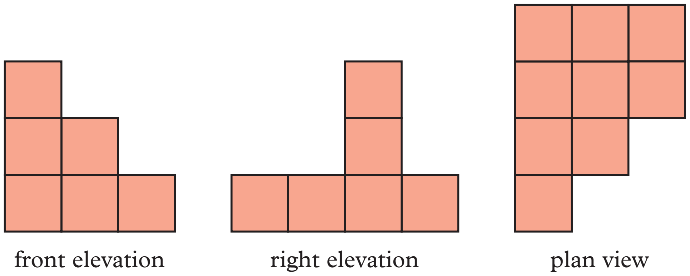
c



d



15 Use this set of plan and elevations to construct the matching 3D object. Draw the object on isometric or square dot paper.

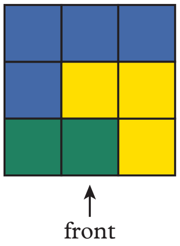


Exercise 8.2C: Challenge

16 An apartment block has the following plan view. The blue squares represent three storeys high. The green squares represent two storeys high. The yellow squares represent one storey high.

a Draw the front and right elevations.

b Draw a 3D version of the apartment block on isometric dot paper.



Online resources:

Interactive skillsheet Plans and elevations

Worksheet Plans and elevations

Investigation Buildings and other 3D structures

Quick quiz 8.2

Lesson 8.3

Translations

Learning intentions

By the end of this lesson you will be able to …

✔ describe and perform translations.

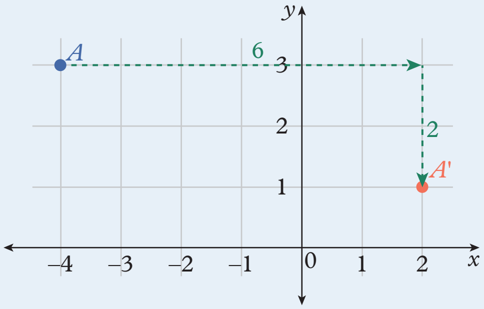
Inter-year links

Support	Transformations
Year 8	9.1 Transformations

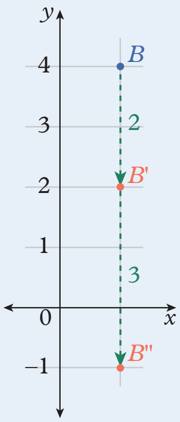
Key content video
Translations

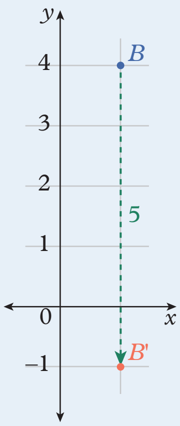
Translations

- A **translation** is a transformation which moves a point, line or shape without turning or changing the size. Translations on the Cartesian plane are recorded by counting how many units the object slides up or down and left or right.
  - → Translations along the x-axis slide the shape left or right.
  - → Translations along the y-axis slide the shape up or down.
  - → Point A has been translated 6 units to the right along the x-axis and 2 units down along the y-axis.



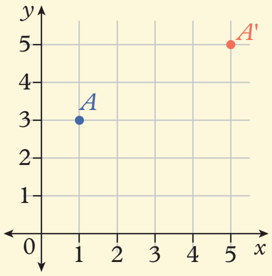
- A transformed object (also called an **image**) should always be given a name that indicates that it is a transformation – usually by adding a small dash to the top right of the original name. For example, the image of point A is called A' ('A prime').
  - → If an image is translated twice, label the second translation with two dashes; for example, A''.
- Multiple translations along the same axis can be combined into a single translation.
  - → For example, a translation of 2 units right followed by a translation of 1 unit left is the same as a translation of 1 unit right.
  - → For example, a translation of 2 units down followed by a translation of 3 units down is the same as a translation of 5 units down.





Worked example 8.3A Identifying a translation

Describe the translation shown in the following diagram.

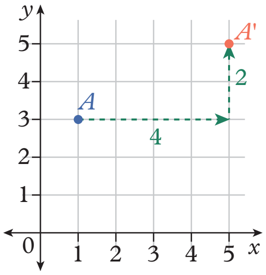


THINK

1 Count the number of units the point has been translated right or left.

2 Count the number of units the point has been translated up or down.

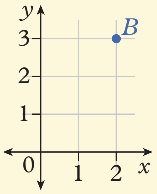
WRITE



The point has been translated 4 units right and 2 units up.

Worked example 8.3B Performing a translation

Translate this point 3 units to the right and 4 units down.



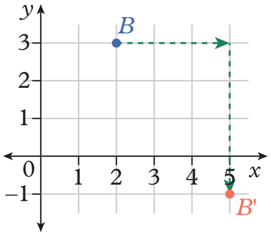
THINK

1 Count 3 units to the right.

2 Count 4 units down.

4 Label the translated image B' to show that it is a transformation of the original point.

WRITE



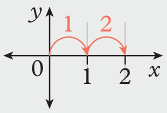
Worked example 8.3C Finding coordinates after a translation

A point P is at $(-3, 4)$ on the Cartesian plane. It is translated 6 units down and 10 units to the right. What are the coordinates of the translated point P'?

THINK	WRITE
1 If translating to the right, add the units to the x-coordinate. If translating to the left, subtract the units from the x-coordinate. **2** If translating up, add the units to the y-coordinate. If translating down, subtract the units from the y-coordinate.	$-3 + 10 = 7$ $4 - 6 = -2$ The coordinates of P' are $(7, -2)$.

Helpful hints

When identifying or performing a translation, do not start counting at the starting position. Start counting at the position of the first unit beyond the shape.



Learning pathways

△ 1–3, 4–5(a–d), 6–9, 10(a, b), 11(b)

■ 1–2(d–f), 3(b–d), 4(e–h), 5(d–f), 6, 7(c, d), 8, 10(c, d, e), 11(a–c), 12(a, b)

⬟ 2(e, f), 3(b–d), 4(e–h), 5(f), 6, 8(c), 10–11(e, f), 12(c), 13, 14

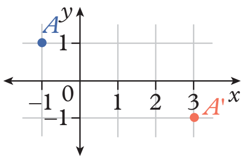
ANS p548

Exercise 8.3A: Understanding and fluency

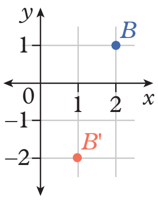
Grids to complete the Lesson 8.3 translations can be found in the Lesson 8.3 BLM.

WE 8.3A **1** Identify the translation shown in the following diagrams.

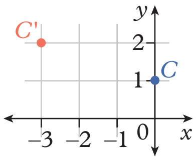
a



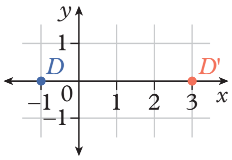
b



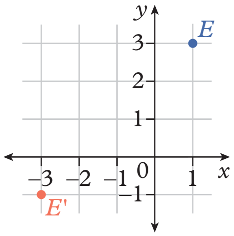
c



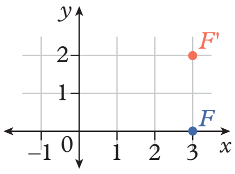
d



e

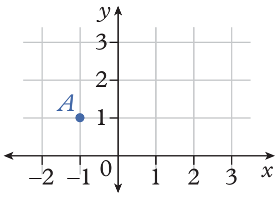


f

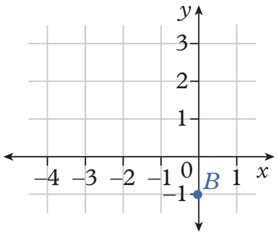


WE 8.3B **2** Translate the points by the given amount.

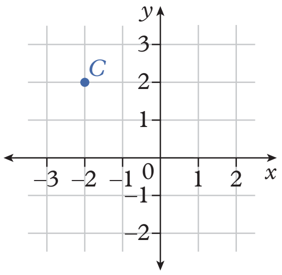
a 2 units right and 1 unit up



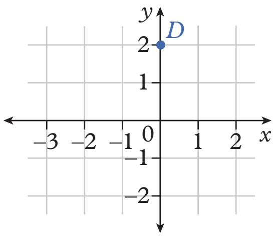
b 4 units left and 3 units up



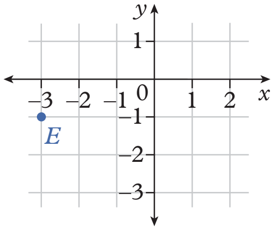
c 4 units down



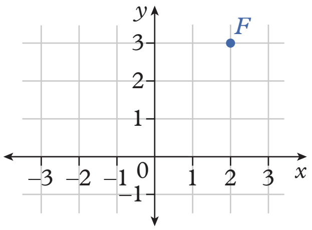
d 3 units left



e 4 units right and 2 units down



f 3 units left and 1 unit down



WE 8.3C **3** The coordinates of a translated point can be identified by adding or subtracting values to the coordinates of the original point.

a Point Q is at (2, 1) on the Cartesian plane. It is translated 7 units to the left and 4 units down. What are the coordinates of the translated point Q'?

b Point R is at (−5, 0) on the Cartesian plane. It is translated 2 units to the right and 5 units up. What are the coordinates of the translated point R'?

c Point S is at (2, −4) on the Cartesian plane. It is translated 12 units to the left and 4 units up. What are the coordinates of the translated point S'?

d Point T is at (1, 7) on the Cartesian plane. It is translated 1 unit to the right and 9 units down. What are the coordinates of the translated point T'?

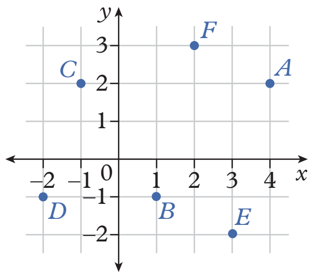
4 Describe the translation required to move the first point to the second point.

a $A(2, 1)$ to $A'(5, 5)$
b $B(-3, 3)$ to $B'(0, 4)$
c $C(5, 2)$ to $C'(1, -1)$
d $D(-2, 0)$ to $D'(2, -2)$
e $E(4, -1)$ to $E'(6, -3)$
f $F(-2, -3)$ to $F'(3, 3)$
g $G(2, -4)$ to $G'(-4, 2)$
h $H(0, 6)$ to $H'(-3, -4)$

5 Which single translation is equivalent to these combined translations?

a 2 units right and 5 units up, followed by 3 units right and 2 units down
b 5 units right and 1 unit up, followed by 4 units left and 6 units down
c 1 unit left and 2 units down, followed by 2 units left and 8 units down
d 4 units right and 3 units up, followed by 9 units right and 3 units up
e 2 units right and 4 units down, followed by 2 units right and 4 units up
f 7 units left and 1 unit down, followed by 1 unit left and 10 units up

6 Look at the following Cartesian plane.



a Which point has been translated 1 unit right and 4 units up from point *B*?

b Which point has been translated 1 unit left and 4 units down from point *A*?

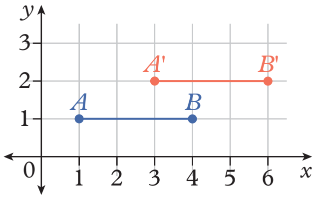
c Describe these translations.

i point *B* to point *C*	**ii** point *A* to point *B*
iii point *D* to point *A*	**iv** point *F* to point *E*
v point *C* to point *F*	**vi** point *E* to point *B*

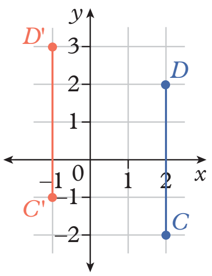
Exercise 8.3B: Problem solving and reasoning

7 Identify the translations shown in the following diagrams.

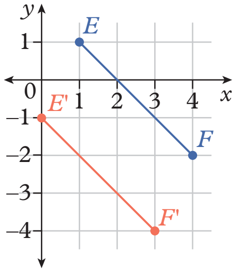
a



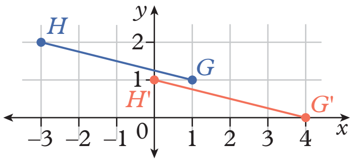
b



c

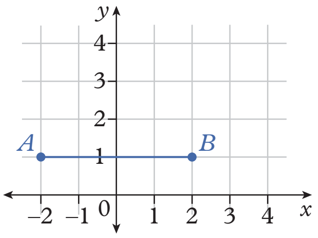


d

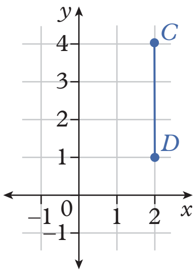


8 Translate these line segments by the given amounts.

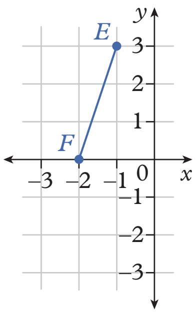
a 2 units right and 3 units up



b 3 units left and 2 units down



c 1 unit right and 3 units down



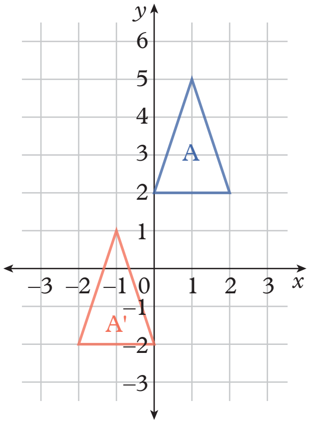
9 What translation is shown in the diagram on the right?

A 2 units right and 4 units up

B 4 units right and 2 units up

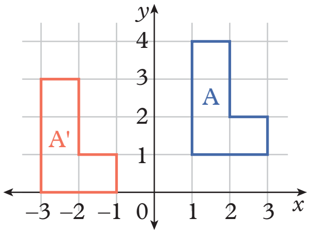
C 2 units right and 4 units down

D 2 units left and 4 units down

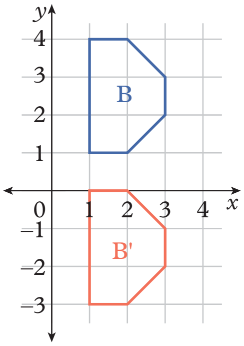


10 Identify the translations shown in the following diagrams.

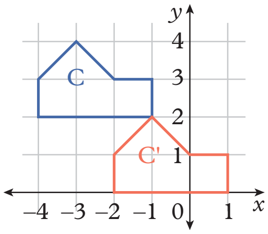
a



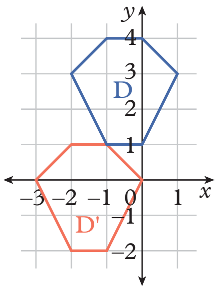
b



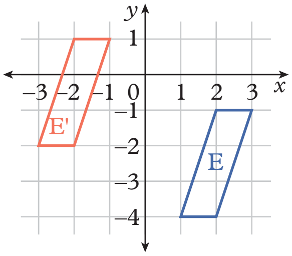
c



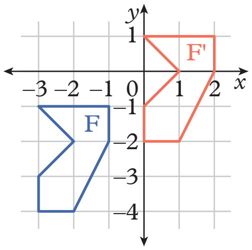
d



e

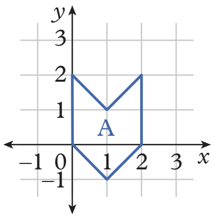


f

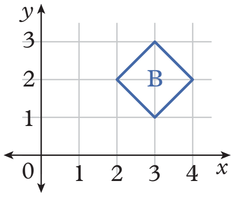


11 Translate the shapes by the given amount.

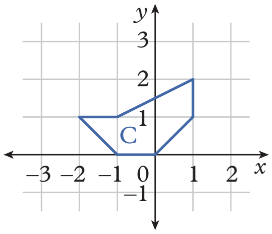
a 3 units left and 4 units up



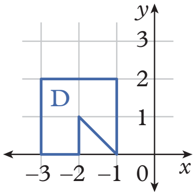
b 5 units left and 4 units down



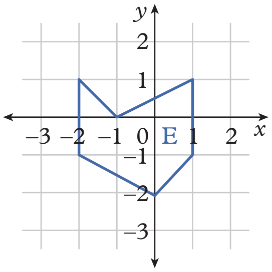
c 1 unit right and 1 unit down



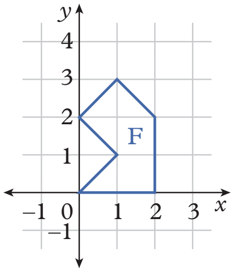
d 5 units up and 5 units right



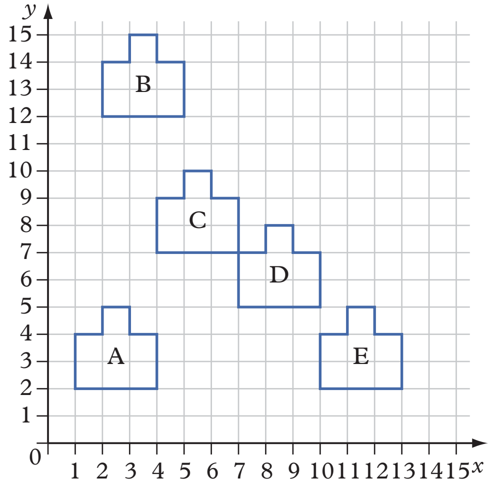
e 3 units down and 2 units right



f 2 units up and 2 units right



12 Look at the following shapes on the Cartesian plane.



- **a** Which shape has been translated 3 units right and 5 units up from shape A?
- **b** Jordan thinks that shape E has been translated 6 units right from shape A. Can you explain where Jordan may have gone wrong?
- **c** Describe these translations.
  - **i** Shape B to shape C
  - **ii** Shape B to shape E
  - **iii** Shape D to shape A
  - **iv** Shape E to shape B
  - **v** Shape D to shape C
  - **vi** Shape A to shape E

13 The figure in question **12** consists of an original shape and four mirror images. It was assumed that shape A was the original shape. Use the clues below to find the real original shape and the final mirror image.

Clue 1: The first image has been translated 3 units to the left and 2 units up from the original figure.

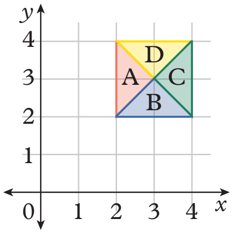
Clue 2: The second image has been translated 6 units to the right and 5 units down from the first image.

Clue 3: The third image has been translated 8 units to the left and 10 units up from the second image.

Clue 4: The final image has been translated 1 unit to the left and 10 units down from the third image.

Exercise 8.3C: Challenge

14 Using only translations of the four triangles, turn this square into a parallelogram that is not a square.



Online resources:

Interactive skillsheet Translations

BLM Lesson 8.3

Quick quiz 8.3

ANS p550

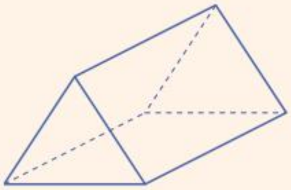
Checkpoint

Checkpoint quiz
Take the checkpoint quiz to check your knowledge of the first part of this module.

BLM
Checkpoint

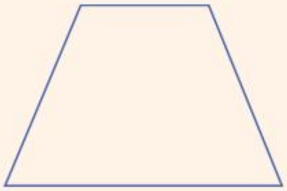
Grids to complete questions with transformations can be found in the Checkpoint BLM.

8.1 **1** Look at this triangular prism.



a How many faces does it have?
b How many edges does it have?
c How many vertices does it have?

8.1 **2** Use isometric dot paper to draw a right prism with this 2D shape as one of the matching ends.



8.1 **3** Look at each object and give its mathematical name.

a

b

c

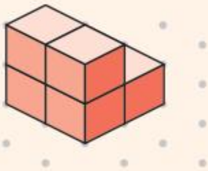
8.2 **4** Draw a set of plan and elevations for each of these objects.

a

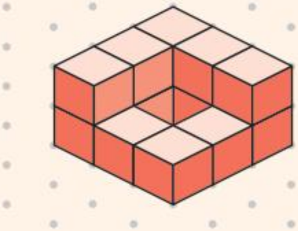
b

8.2 **5** Draw a set of plan and elevations for each object.

a



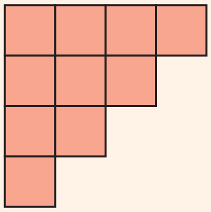
b

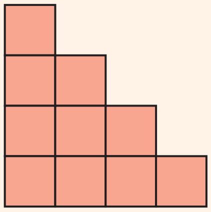


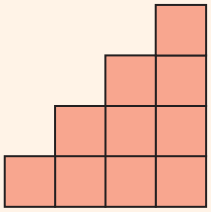
c

8.2 **6** Use each set of plan and elevations to construct at least one example of the matching 3D object. Draw the objects on isometric or square dot paper.

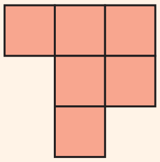
a

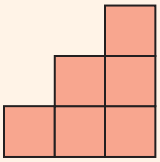

plan view

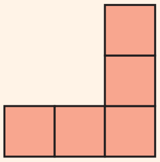

front elevation


right elevation

b


plan view


front elevation


right elevation

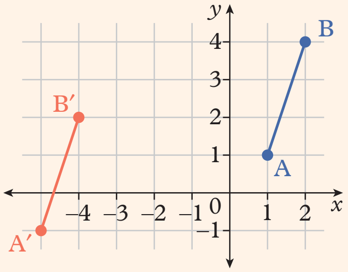
8.3 **7** The coordinates of a translated point can be identified by adding or subtracting values to the coordinates of the original point.

a Point U is at $(-3, -6)$ on the Cartesian plane. It is translated 8 units to the left and 4 units down. What are the coordinates of the translated point U'?

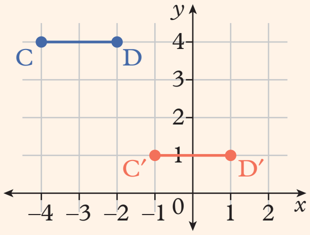
b Point V is at $(8, -4)$ on the Cartesian plane. It is translated 3 units to the left and 13 units up. What are the coordinates of the translated point V'?

8.3 **8** Identify the translations shown in the following diagrams.

a

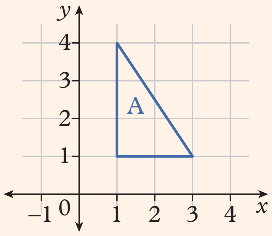


b

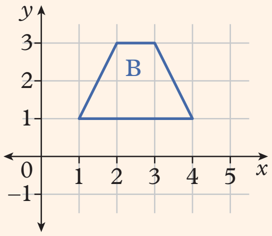


8.3 **9** Translate the shapes by the given amount.

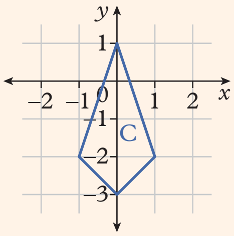
a 3 units left and 5 units down



b 6 units right and 1 unit up



c 3 units right and 2 units down



Lesson 8.4

Reflections and rotations

Learning intentions

By the end of this lesson you will be able to ...

✔ perform reflections and rotations.

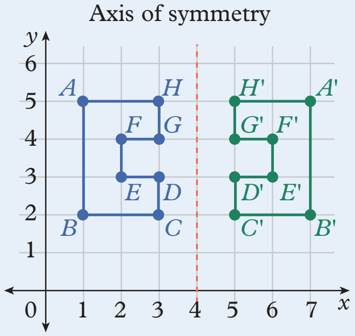
Inter-year links

Support	Transformations
Year 8	9.1 Transformations

Reflections

Key content video
Rotations and reflections

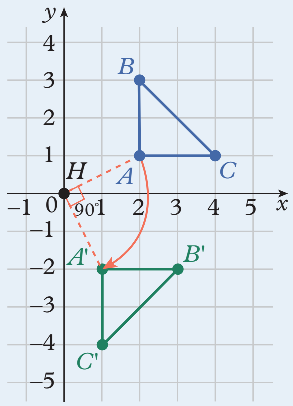
- **Reflections** are performed in an **axis of symmetry**. Each point in the image must be the same distance from the axis of symmetry as the corresponding original point.



- When performing a reflection, identify a single point to concentrate on. This is called the **point of focus**.
  → After reflecting one point you may be able to determine the position of the other reflected points in relation to this point. If not, proceed by reflecting a second point.

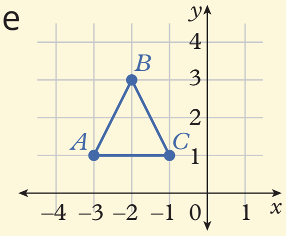
Rotations

- A **rotation** turns a shape about a fixed point. To perform a rotation, three details are required:
  1 The centre of rotation.
    - Usually the origin or a specified point.
  2 The angle of the rotation.
    - Between 0° and 360°.
  3 The direction of the rotation.
    - Clockwise or anticlockwise.



- As with reflections, start by rotating a single point (the point of focus), and continue to rotate single points until you can confidently draw the whole rotated shape.

Worked example 8.4A Performing a reflection on the Cartesian plane



Reflect this shape in the *y*-axis and give the coordinates of the vertices of the image.

THINK

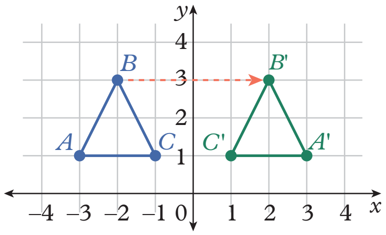
1 Choose a point of focus, *B*.

2 Reflect *B* in the *y*-axis. Label the reflected point *B*'.

3 Reflect the other points in the *y*-axis and draw the reflected shape.

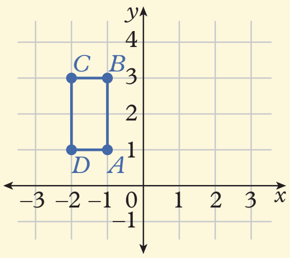
4 Give the coordinates of the image.

WRITE



The coordinates of the image are *A*'(3, 1), *B*'(2, 3) and *C*'(1, 1).

Worked example 8.4B Performing a rotation on the Cartesian plane



Rotate this shape 90° clockwise about the origin, *O*, and give the coordinates of the image.

THINK

1 Choose a point of focus, *A*. Draw a straight line connecting *A* to the origin.

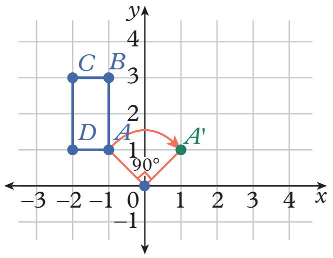
2 Using the line segment *OA*, draw a 90° angle clockwise about the origin. The distance from the origin to the rotated point, *A*', will be the same as the distance from the origin to *A*.

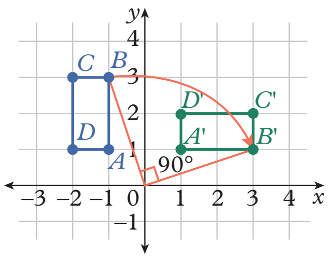
3 Choose a second point to rotate, *B*. Rotate *B* to 90° clockwise about the origin and label the rotated point *B*'.

4 If you can, draw the remaining points of the rotated shape. If needed, rotate a third point from the original shape. The rotated shape must be the same shape and size as the original shape.

5 Give the coordinates of the image.

WRITE





The coordinates of the image are *A*'(1, 1), *B*'(3, 1), *C*'(3, 2) and *D*'(1, 2).

Helpful hints

- ✔ Remember to give your image a name that identifies it as a transformation. For example, for a shape *B*, the image is named *B*'.
- ✔ After a reflection or a rotation, the interior angles and side lengths of the image will be the same as the interior angles and side lengths of the original shape.

Learning pathways

△ 1, 2, 4, 6, 7(a, b), 8(c), 10, 11(c), 12, 14

□ 1, 2, 4–6, 7(b, c), 8–10, 13–16

⬠ 1–2(b, d), 3(d), 5–6(b, d), 7(b, c), 13, 15–19

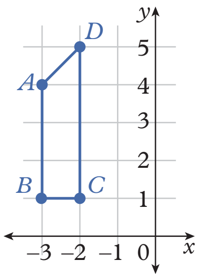
ANS p550

Exercise 8.4A: Understanding and fluency

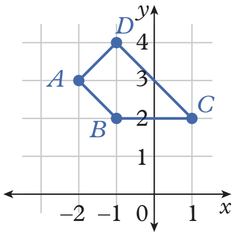
Grids to complete the Lesson 8.4 rotations and reflections can be found in the Lesson 8.4 BLM.

WE 8.4A **1** Reflect each of the shapes in the *y*-axis and give the coordinates of the images.

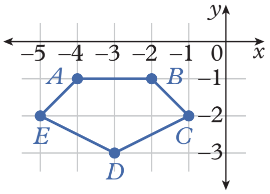
a



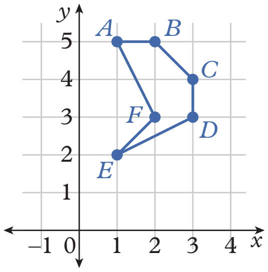
b



c



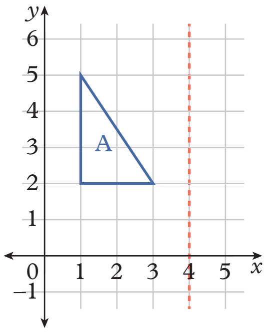
d



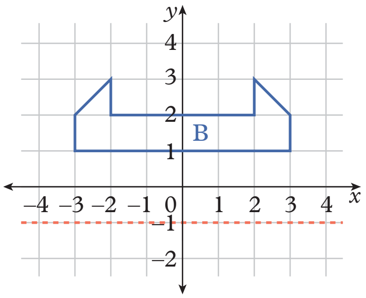
2 Reflect each of the shapes in question **1** in the *x*-axis and give the coordinates of the images.

3 Reflect each of these shapes in the axis of symmetry shown.

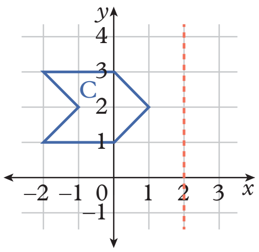
a



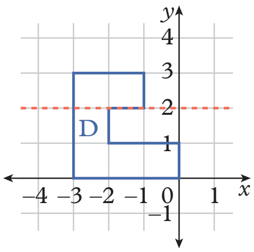
b



c

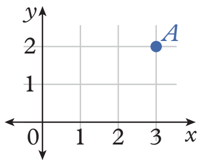


d

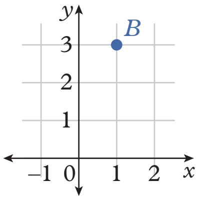


4 Rotate each of the points 90 degrees clockwise about the origin and give the coordinates of the images.

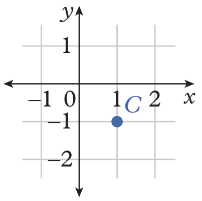
a



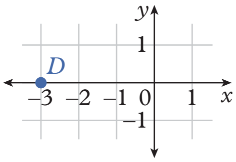
b



c

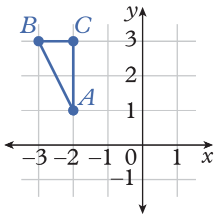


d

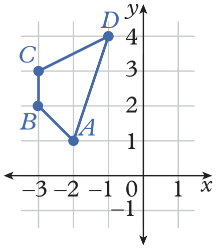


WE 8.4B **5** Rotate each of the shapes 90° clockwise about the origin and give the coordinates of the images.

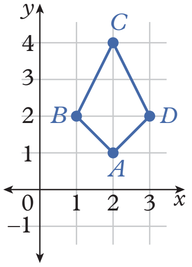
a



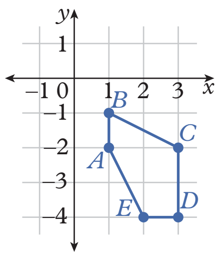
b



c



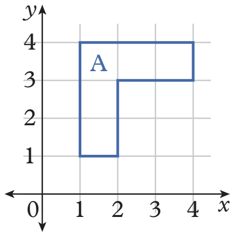
d



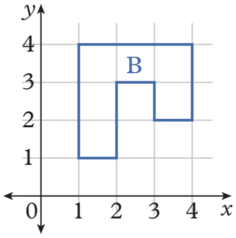
6 Rotate each of the shapes in question **5** 90° anticlockwise about the origin and give the coordinates of the images.

7 Rotate each shape 180° clockwise about the origin.

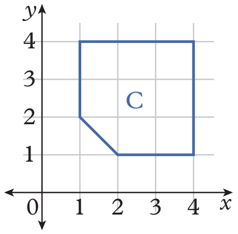
a



b



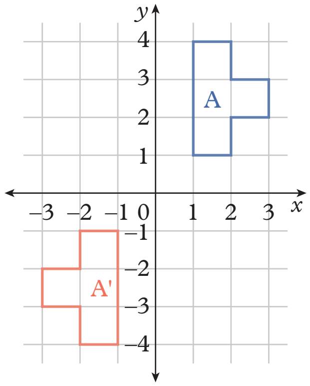
c



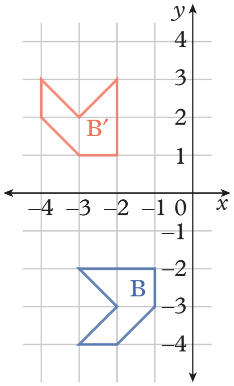
Exercise 8.4B: Problem solving and reasoning

8 Each of these shapes has been rotated clockwise about the origin. By how many degrees have they been rotated?

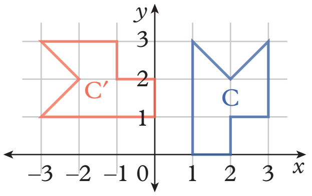
a



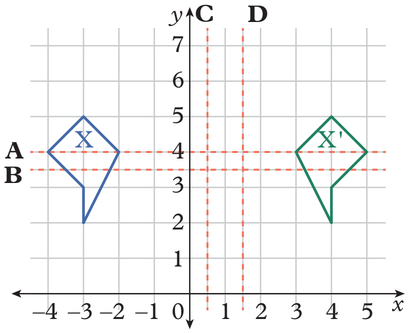
b



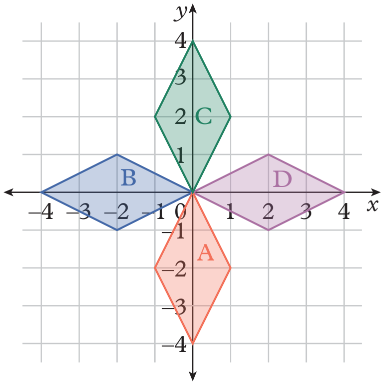
c



9 Which of the given lines has the shape been reflected in?

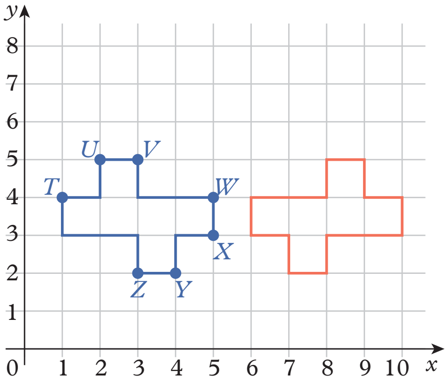


10 This figure is made up of a number of rotations of one shape. Describe the transformations for the shapes B, C and D given that they are rotations of shape A.

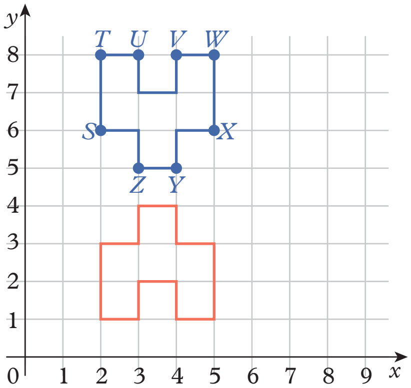


11 Write the coordinates of the point *V'* for each of these reflections.

a

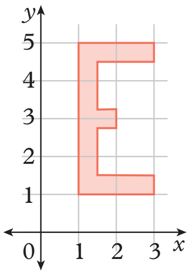


b

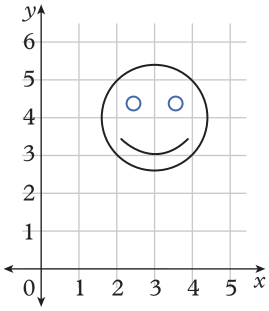


12 For each of these shapes, one half of the figure has been reflected in an axis of symmetry to create the full shape. Draw the axis of symmetry that has been used.

a

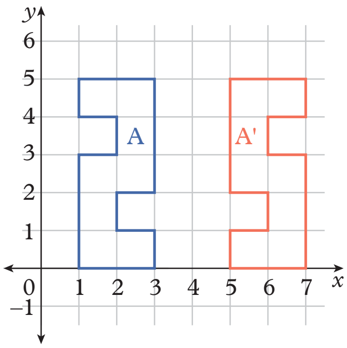


b

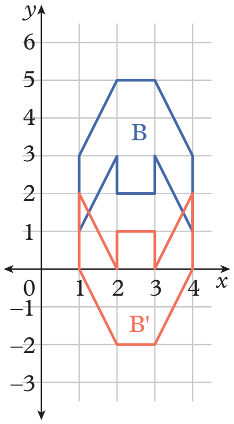


13 Draw the axis of symmetry for these reflections.

a



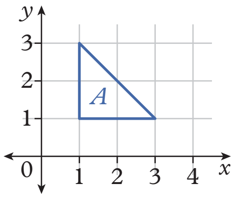
b



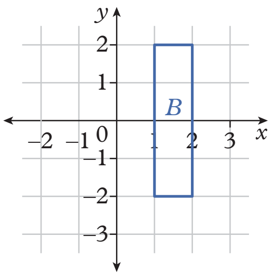
14 Explain why a rotation of 90° clockwise about the origin can also be described as a rotation of 270° anticlockwise about the origin.

15 Consider the following shapes on the Cartesian plane.

i



ii



a Rotate each shape 60° clockwise about the origin.

b Explain why it is more difficult to rotate a shape by 60° about the origin than it is to rotate a shape by 90° about the origin.

16 Consider the points $A(5, 1)$, $B(2, 6)$, $C(-3, 1)$ and $D(-2, -4)$.

a Plot the points on graph paper and rotate them by 90° clockwise about the origin.

b If the coordinates of a point are (x, y), what are the coordinates of the image after a rotation of 90° clockwise about the origin?

c Rotate the points by 90° anticlockwise about the origin.

d If the coordinates of a point are (x, y), what are the coordinates of the image after a rotation of 90° anticlockwise about the origin?

e Rotate the points by 180° clockwise about the origin.

f If the coordinates of a point are (x, y), what are the coordinates of the image after a rotation of 180° clockwise about the origin?

17 Consider the points (2, 2), (6, 2) and (4, 5).

a Plot the points on graph paper and join them to make a shape.

b Reflect the shape in the y-axis. What are the new coordinates?

c Reflect the original shape in the x-axis. What are the new coordinates?

d Rotate the original shape by 180° about the origin. What are the new coordinates?

Exercise 8.4C: Challenge

18 Draw a Cartesian plane from 0 to 10 on both the x and y axes and complete the following.

a Plot the following two quadrilaterals on the same Cartesian plane.

Vertices of first quadrilateral: (8, 2), (10, 4), (8, 6) and (6, 4)

Vertices of second quadrilateral: (4, 10), (2, 8), (4, 6) and (6, 8)

b Draw the line of symmetry between the two quadrilaterals.

c What do you notice about the pairs of coordinates that are reflections of each other in this line?

19 When given an image and a transformation, we determine the original location of the shape.

a A triangle is reflected in the x-axis, the new coordinates of the vertices are (1, 1), (2, 1) and (1, 2). What were the original coordinates?

b A square is reflected in the y-axis, the new coordinates of the vertices are (1, 1), (2, 1), (2, 2) and (1, 2). What were the original coordinates?

c A quadrilateral is rotated 90° clockwise about the origin, the new coordinates of the vertices are (8, 2), (10, 4), (6, 4) and (8, 6). What were the original coordinates?

Online resources:

Interactive skillsheet
Rotations

Interactive skillsheet
Reflections

BLM
Lesson 8.4

Investigation
Tetris

Quick quiz
8.4

Lesson 8.5

Combined transformations

Learning intentions

By the end of this lesson you will be able to ...

✔ perform combined transformations.

Inter-year links

Support	Transformations
Year 8	9.1 Transformations

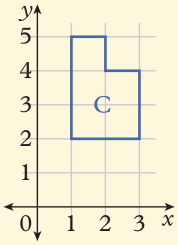
Combined transformations

Key content video
Combined transformations

- Translations, rotations and reflections (in axes of symmetries) are all examples of **isometric transformations**. Isometric transformations do not change the shape or size of a shape or object.
- When performing combined transformations, the order in which the transformations are performed is important. For example, a translation followed by a reflection will probably not result in the same image as the same reflection followed by the same translation.

Worked example 8.5A Performing a combined transformation

Translate this shape 3 units right and 1 unit down, and then reflect the translated shape in the x-axis.



THINK

1 Select a point of focus on shape C.

2 Translate the point of focus 3 units to the right and 1 unit down.

3 Redraw the shape starting from the point of focus.

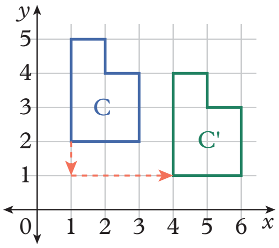
4 Label the new shape C' to show that it is a transformation of the original shape.

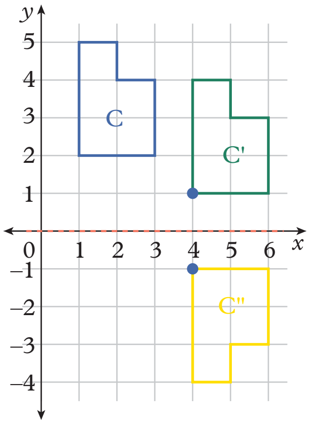
5 Reflect the point of focus of C' in the x-axis.

6 Reflect the other points of C' in the x-axis and draw the reflected shape.

7 Label the final shape C''.

WRITE





Learning pathways

△ 1–3, 5, 7, 8 ☐ 1–3, 5, 7, 9, 10 ⬠ 1, 4, 6, 7, 9–11

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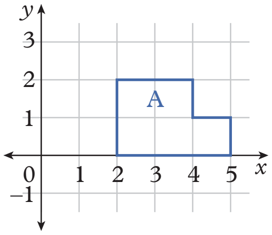
Exercise 8.5A: Understanding and fluency

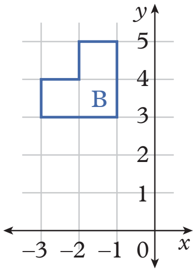
Grids to complete the Lesson 8.5 transformations can be found in the Lesson 8.5 BLM.

WE 8.5A **1** Perform the following combined transformations.

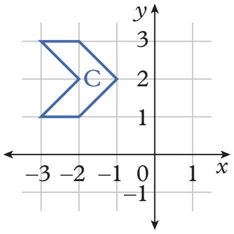
a Translate 1 unit left and 3 units up, and then reflect in the y-axis.



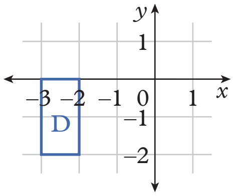
b Translate 3 units right and 3 units down, and then reflect in the x-axis.



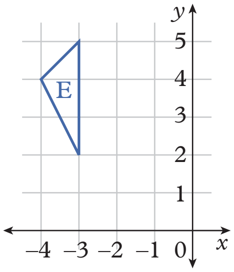
c Reflect in the y-axis, and then translate 3 units down.



d Reflect in the x-axis, and then translate 3 units right and 2 units up.

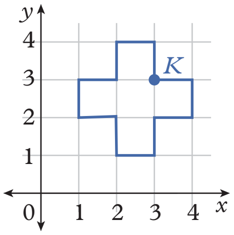


e Rotate 180° clockwise about the origin, and then translate 2 units left and 1 unit up.

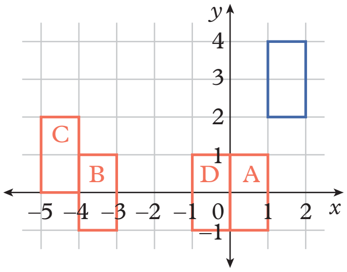


2 What will the coordinates of point K be after these two transformations?

→ Reflection in the y-axis
→ Translation of 3 units right and 2 units up

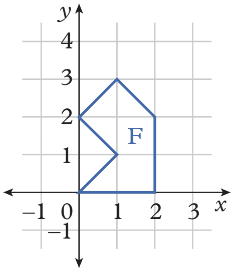


3 Which is the correct final position of the blue rectangle after it has been translated 2 units left and 3 units down, followed by a reflection in the y-axis?



4 The order in which combined transformations are performed is important.

a Translate this shape by 3 units right and 3 units up, before reflecting the translated image in the y-axis.



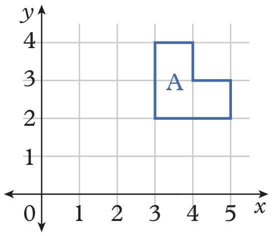
b Perform this combined transformation in the opposite order; that is, reflect the shape in the y-axis, and then translate the reflected image 3 units right and 3 units up.

c If the shape is first reflected in the y-axis, what translation would need to occur after to result in the same image as in part **a**?

Exercise 8.5B: Problem solving and reasoning

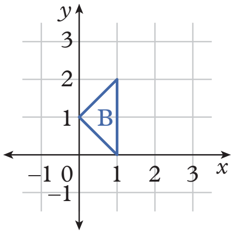
5 Transform the shape on the right according to the steps listed. Draw each shape.

Translate 3 units to the left and 4 units down, then rotate 90° clockwise about the origin, then reflect in the x-axis. Label the final image A‴.



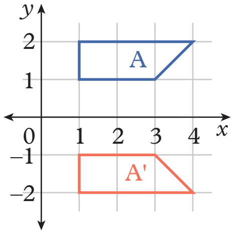
6 Transform this shape according to the steps listed. Draw each shape.

Translate 2 units to the right and 2 units up, then rotate 90° anticlockwise about the origin, then reflect in the x-axis. Label this image B‴.

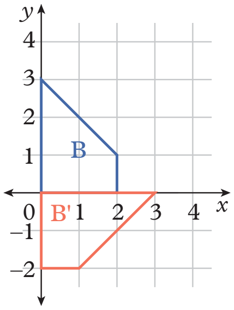


7 Identify each transformation shown as a single translation, rotation or reflection.

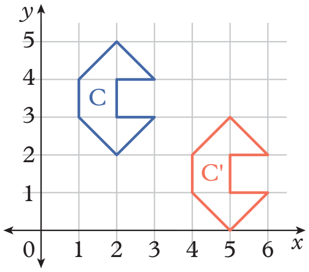
a



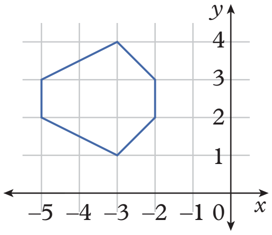
b



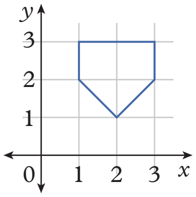
c



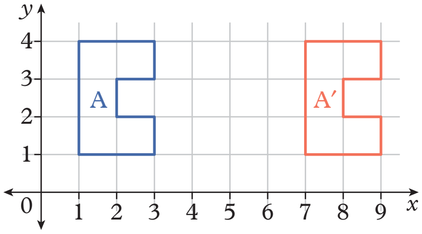
8 This shape is reflected in the *y*-axis and then reflected in the *x*-axis. What single transformation would produce the same result?



9 This shape is rotated by 180° about the origin and then reflected in the *x*-axis. What single transformation would produce the same result?



Exercise 8.5C: Challenge



10 Consider the transformation of shape A shown in this diagram.

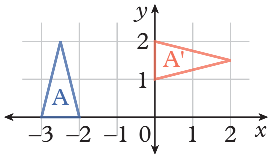
a This transformation can be the result of a single translation. State the translation required for this transformation.

b This transformation can also be the result of two reflections. If the shape is first reflected in the *y*-axis, in what line must the shape then be reflected in?

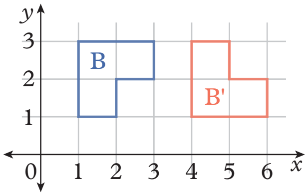
Hint: The required line is in the form *y* = __.

11 A rotation clockwise about the origin followed by a translation have been performed in the following diagrams. State the transformations.

a



b



Online resources:

BLM
Lesson 8.5

Investigation
Alien walk

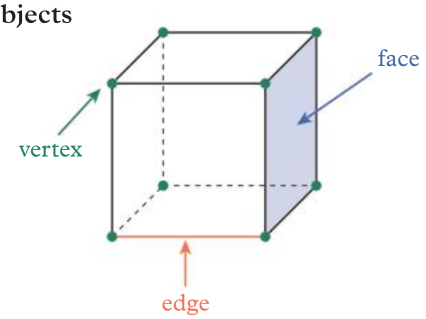
Quick quiz
8.5

Lesson 8.6

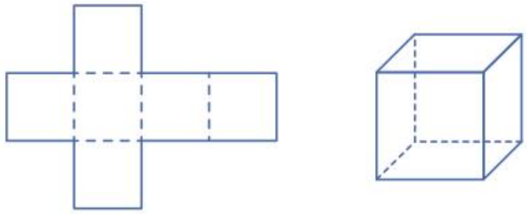
Review: Objects and transformations

Module summary

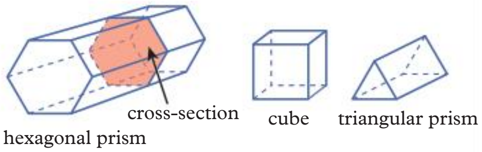
3D objects



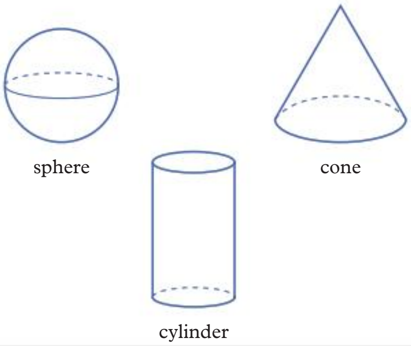
Nets



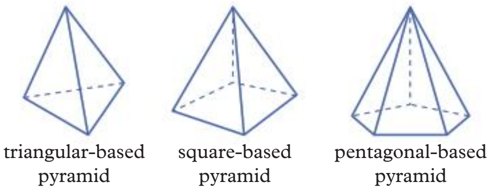
Prisms



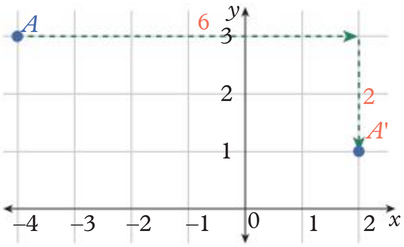
Non-polyhedra



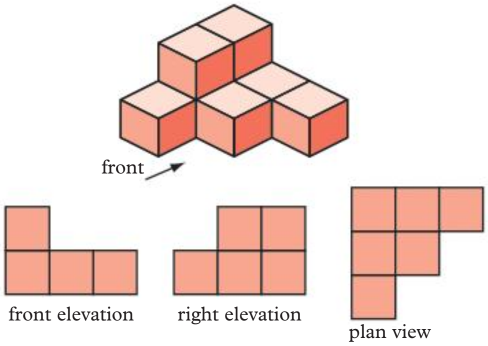
Pyramids



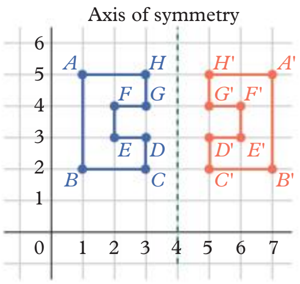
Translations



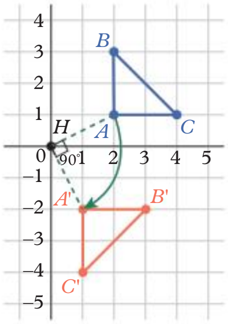
Plans and elevations



Reflections



Rotations



ANS
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Review questions 8.6A: Mathematical literacy review

Review quiz
Take the review quiz to assess your knowledge of this module.

Quizlet
Test your knowledge of this module by working individually or in teams.

The following key terms are used in this module:

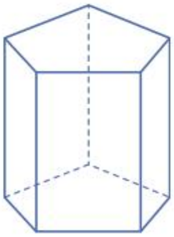
- anticlockwise
- axis of symmetry
- base
- Cartesian plane
- clockwise
- cone
- coordinate
- cross-section
- cylinder
- edge
- elevations
- face
- image
- isometric transformation
- net
- plans
- point of focus
- polyhedron
- prism
- pyramid
- reflection
- right prism
- rotation
- sphere
- tetrahedron
- translation
- vertex

1 Which three details are required to perform a rotation?
2 Explain the difference between a plan view and an elevation.
3 What characteristic distinguishes right prisms from other prisms?
4 Identify the key terms being referenced in each of these definitions.
   a a 3D object with all faces being polygons
   b a transformation which does not change the shape or size of a shape or object
5 Provide a definition in your own words for the following key terms.
   a net **b** point of focus
6 Complete the following sentences using words from the key term list.
   a A ________ is a transformation which moves a point, line or shape without turning or changing size.
   b Reflections are performed in an ________.

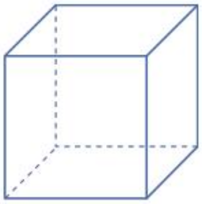
Review questions 8.6B: Multiple choice

Use Figures A–D to answer questions **1** and **2**.

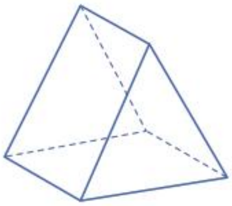
A



B



C

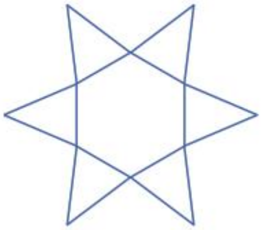


D

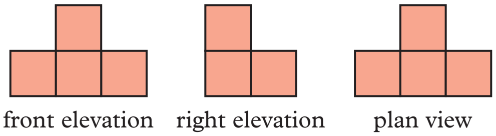
8.1 **1** Which of the 3D objects above is not a prism?
A A **B** B **C** C **D** D

8.1 **2** Which of the 3D objects above is not a polyhedron?
A A **B** B **C** C **D** D

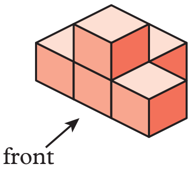
8.1 **3** Which 3D object does this net create?
A hexagonal pyramid
B hexagonal prism
C tetrahedron
D rectangular pyramid



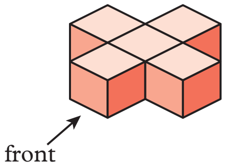
8.2 **4** Which 3D object could this set of plan and elevations represent?



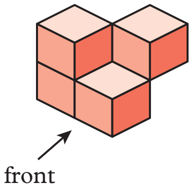
A



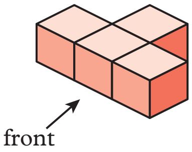
B



C

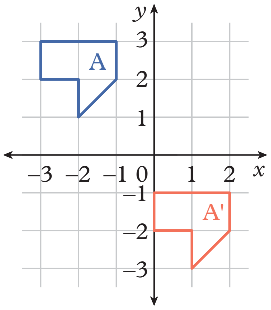


D



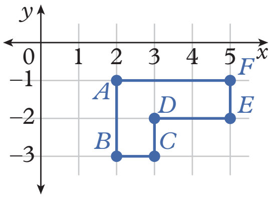
8.3 **5** What is the translation shown in this diagram?

- **A** 5 units right and 4 units down
- **B** 6 units right and 5 units down
- **C** 3 units right and 4 units down
- **D** 4 units right and 3 units down



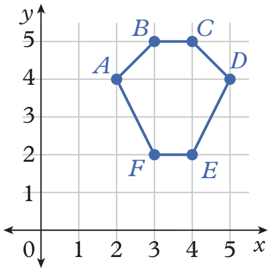
8.4 **6** What will be the coordinates of point *A*' if this shape is rotated 90° clockwise about the origin?

- **A** (1, 2)
- **B** (−1, 2)
- **C** (−1, −2)
- **D** (−2, −1)



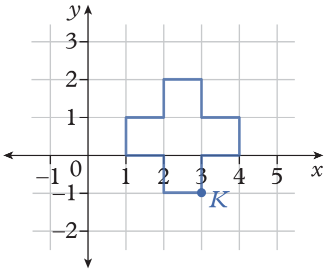
8.4 **7** What will be the coordinates of point *D*' if this shape is reflected in the *y*-axis?

- **A** (−5, 4)
- **B** (4, −5)
- **C** (5, −4)
- **D** (−5, −4)



8.5 **8** This shape is translated 3 units to the left and 4 units down, and then the translated image is reflected in the *x*-axis. What are the coordinates of vertex *K*" in the final image?

- **A** (5, 0)
- **B** (0, 5)
- **C** (0, 3)
- **D** (0, −5)



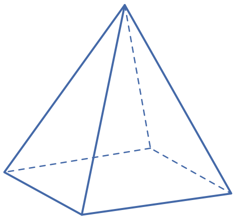
Review questions 8.6C: Short answer

BLM
Review

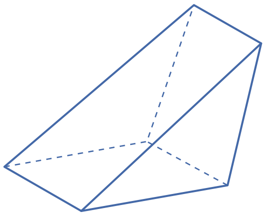
Grids to complete questions with transformations can be found in the Review BLM.

8.1 **1** Are these 3D objects prisms?

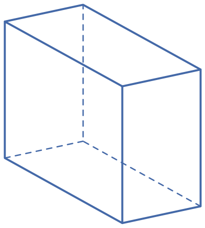
a



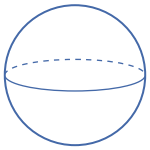
b



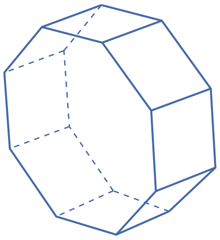
c



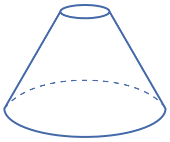
d



e

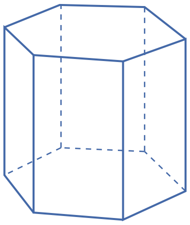


f

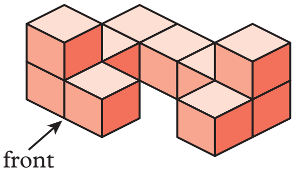


8.1 **2** Name each of the prisms from question **1**.

8.2 **3** Draw a set of plan and elevations for the following prism.

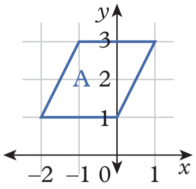


8.2 **4** Draw a set of plan and elevations for this 3D object.

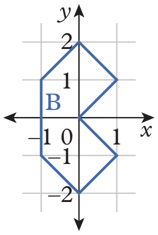


8.3 **5** Translate each of the shapes by the given amounts.

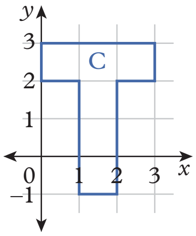
a 4 units down and 1 unit left



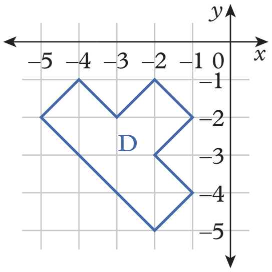
b 3 units up and 2 units left



c 1 unit up and 3 units right



d 2 units up and 5 units right



8.3 **6** Describe the translation required to move the first point to the second point.

a $A(7, 3)$ to $A'(6, 1)$

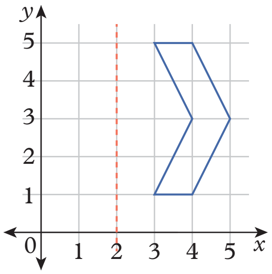
b $B(1, -2)$ to $B'(1, 5)$

c $C(-3, -2)$ to $C'(2, -1)$

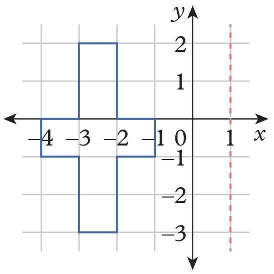
d $D(0, -1)$ to $D'(4, 5)$

8.4 **7** Reflect each of these shapes in the given axis of symmetry.

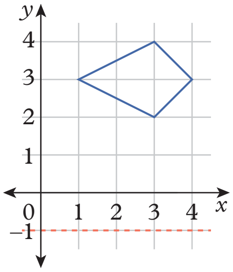
a



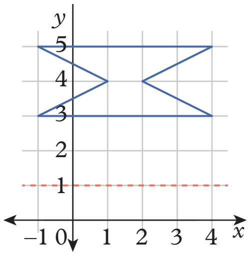
b



c

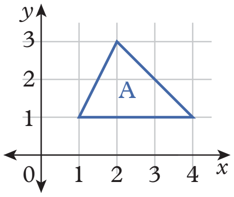


d

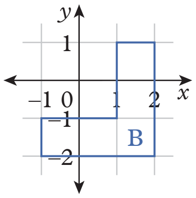


8.4 **8** Rotate each of these shapes 90° clockwise about the origin.

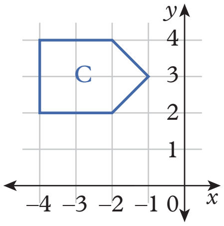
a



b



c



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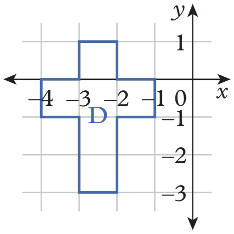
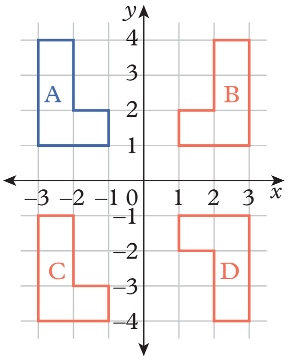


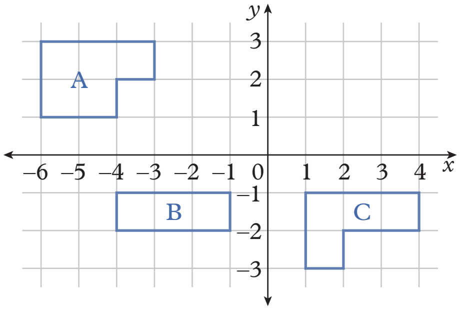
8.4 **9** Rotate each of the shapes in question **8** 180° anticlockwise about the origin.

8.5 **10** Consider the shapes shown on this grid. Describe the single transformation that produces each of the shapes B to D if the original shape is A.



Review questions 8.6D: Mathematical modelling

Consider the following shapes on the Cartesian plane.



a Transform each of the following shapes as instructed.

A: reflect in the *y*-axis

B: translate 7 units right and 5 units up

C: rotate 90° anticlockwise about the origin

b Use isometric dot paper to draw a prism, with the shape formed in part **a** being the base of the prism.

c How many **i** faces, **ii** edges, and **iii** vertices does your prism have?

d Draw a set of plan and elevations for your prism.

Checklist

Now that you have completed this module, reflect on your ability to do the following.

I can do this	I need to review this
☐ Identify and name 3D objects ☐ Draw right prisms	☐ Go back to Lesson 8.1 3D objects
☐ Draw plans and elevations of prisms ☐ Draw plans and elevations of 3D objects formed from combinations of prisms	☐ Go back to Lesson 8.2 2D representations of 3D objects
☐ Describe translations on the Cartesian plane ☐ Perform translations on the Cartesian plane	☐ Go back to Lesson 8.3 Translations
☐ Perform reflections in an axis on the Cartesian plane ☐ Perform rotations about the origin on the Cartesian plane	☐ Go back to Lesson 8.4 Reflections and rotations
☐ Perform combined transformations	☐ Go back to Lesson 8.5 Combined transformations

9

Length, area and volume

Lessons

Prerequisite skills

Diagnostic pre-test

Take the diagnostic pre-test to assess your knowledge of the prerequisite skills listed below.

Interactive skillsheets

After completing the diagnostic pre-text, brush up on your knowledge of the prerequisite skills by using the interactive skillsheets.

- ✔ Perimeter using a square grid
- ✔ Area using a square grid
- ✔ Volume using cubes
- ✔ Units of measurement
- ✔ Multiplying and dividing by powers of 10

Curriculum links

- Establish the formulas for areas of rectangles, triangles and parallelograms and use these in problem-solving (VC2M7M01)
- Solve problems involving the volume of right prisms including rectangular and triangular prisms, using established formulas and appropriate units (VC2M7M02)
- Describe the relationship between π and the circumference, radius and diameter of a circle (VC2M7M03)

Materials

- ✔ Calculator
- ✔ Grid paper

Lesson 9.1

Length and perimeter

Learning intentions

By the end of this lesson you will be able to ...

✔ convert between units of length and calculate perimeters.

Inter-year links

Support	Length and perimeter
Year 8	8.1 Length and perimeter

Key content video
Length and perimeter

Metric units

- The metric system is based on place value. Metric prefixes are added to units to describe different values.

Place value	thousands	hundreds	tens	units	tenths	hundredths	thousandths
Value	1000	100	10	1	0.1	0.01	0.001
Prefix	kilo-	hecto-	deca-	N/A	deci-	centi-	milli-
Abbreviation	k-	h-	da-	N/A	d-	c-	m-

Length

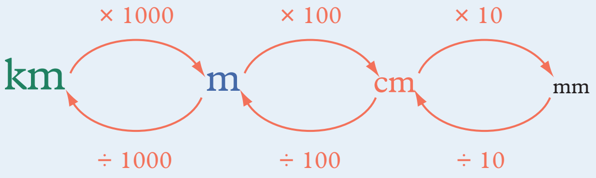
- **Length** is a measure of distance. To specify a length, you need a numerical value as well as a unit of length, for example, 3 kilometres.

Metric units of length	
Name	**Abbreviation**
millimetre	mm
centimetre	cm
metre	m
kilometre	km

- To convert between units of length, multiply or divide by the appropriate **conversion factor**.

Conversion factors:

1 cm = 10 mm
1 m = 100 cm
1 km = 1000 m



For example,

5 km = 5 × 1000
= 5000 m

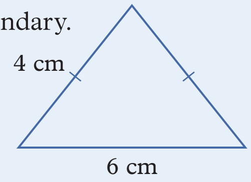
Perimeter

- The **perimeter** of a two-dimensional (2D) shape is the length of the shape's boundary.
- To calculate the perimeter of a shape, add the lengths of each side.

For example, the perimeter of this triangle is calculated as: 4 + 4 + 6 = 14 cm.

→ All length measurements must be in the same unit before being added.

→ Sides of equal length are marked using one or more hatch marks.



Worked example 9.1A Converting between units of length

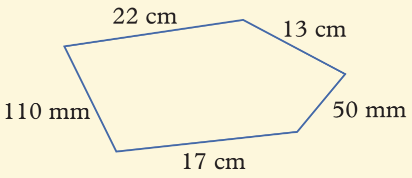
Complete these length conversions.

a 5.4 km = __________ m **b** 900 mm = __________ cm **c** 0.7 km = __________ mm

THINK	WRITE
a Identify the appropriate conversion factor. To convert from a larger to a smaller unit, multiply the length by the conversion factor.	**a** $1 \text{ km} = 1000 \text{ m}$ $5.4 \text{ km} = 5.4 \times 1000$ $= 5400 \text{ m}$
b Identify the appropriate conversion factor. To convert from a smaller to a larger unit, divide the length by the conversion factor.	**b** $1 \text{ cm} = 10 \text{ mm}$ $900 \text{ mm} = 900 \div 10$ $= 90 \text{ cm}$
c First, convert from km to m. Multiply the length by the conversion factor. Then convert from m to mm. Multiply the result from the last step by the conversion factor.	**c** $0.7 \text{ km} = 0.7 \times 1000$ $= 700 \text{ m}$ $= 700 \times 1000$ $= 700\,000 \text{ mm}$

Worked example 9.1B Calculating perimeter

Calculate the perimeter of the following shape in centimetres.



THINK	WRITE
1 Convert all measurements into the unit specified by the question. In this case it is centimetres.	$110 \text{ mm} = 110 \div 10$ $= 11 \text{ cm}$ $50 \text{ mm} = 50 \div 10$ $= 5 \text{ cm}$
2 Write out the sum of the side lengths. There are five sides to the shape so there should be five lengths in the sum.	$\text{perimeter} = 22 + 13 + 5 + 17 + 11$
3 Add all the lengths together. Include the appropriate unit in the answer.	$= 68 \text{ cm}$

Helpful hints

- ✔ To specify a length, you must include a unit of length as well as the numerical value. Make sure that your answers contain the correct units of length.
- ✔ When the units are the same you can leave them out of calculations. However, you must remember to include them in your final answer.

Learning pathways

△ 1(1st column), 2(a, b, e), 3(1st, 2nd columns), 4(a, c, f), 5, 6(a, c, e), 7(a, c, e), 8(a, b), 11, 12(a–c), 16(a–c), 17

■ 1(g–l), 2(b, e, f), 3(2nd column), 4(d–f), 5(c), 6(d–f), 7(e, f), 8(c, d), 9, 11, 12(c, g), 13, 18, 19, 22, 23

⬟ 2(d, f), 3(2nd column), 4(f), 6(d–f), 7(f), 8(d), 10, 11, 14, 15, 20–24

ANS p557

Exercise 9.1A: Understanding and fluency

WE 9.1A **1** Complete these length conversions.

a 8 km = ________ m

b 3 m = ________ cm

c 700 mm = ________ cm

d 500 cm = ________ m

e 16 cm = ________ mm

f 2000 m = ________ km

g 3.5 m = ________ cm

h 7.9 mm = ________ cm

i 0.06 km = ________ m

j 0.47 cm = ________ mm

k 4310 mm = ________ km

l 51.2 cm = ________ m

2 Complete these length conversions.

a 4 km = ________ m = ________ cm

b 12 000 mm = ________ cm = ________ m

c 73 m = ________ cm = ________ mm

d 8400 cm = ________ m = ________ km

e ________ mm = 250 cm = ________ m

f ________ cm = 362 m = ________ km

3 Convert these lengths to the units shown in brackets.

a 55 cm (mm)

b 27 000 m (km)

c 41 mm (cm)

d 6.2 m (cm)

e 3.6 km (m)

f 89 cm (m)

g 1.7 km (cm)

h 500 mm (m)

i 7000 cm (km)

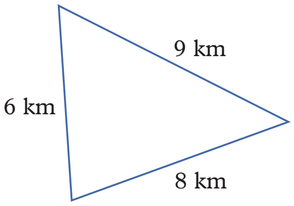
j 3.65 m (mm)

k 0.94 km (mm)

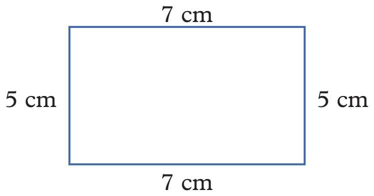
l 1800 mm (km)

4 Calculate the perimeter of each shape.

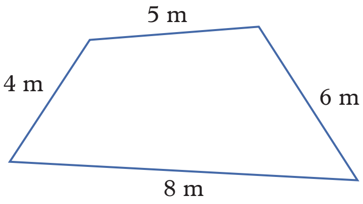
a



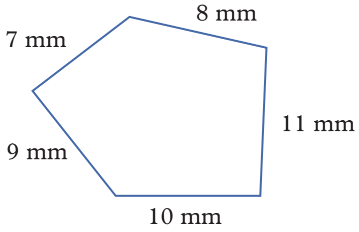
b



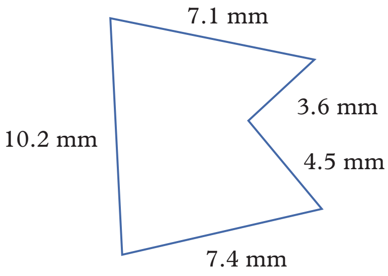
c



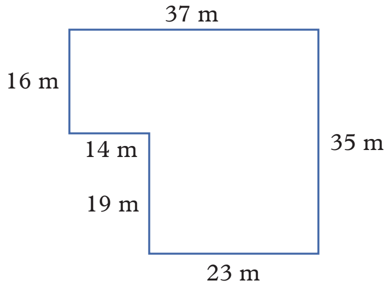
d



e

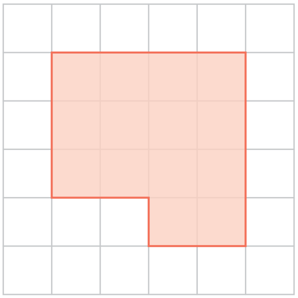


f



5 Calculate the perimeter of these shapes. Assume each square in the grid has a length of 1 cm.

a



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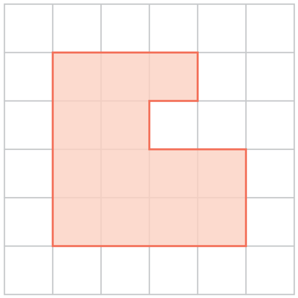

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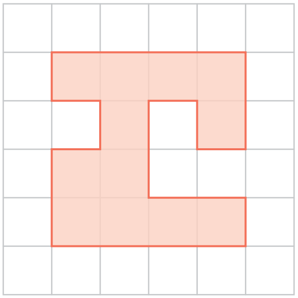
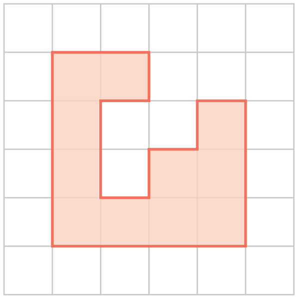
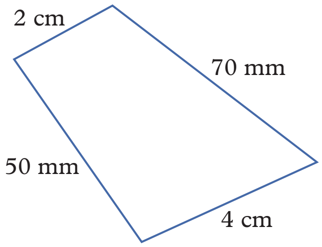


d

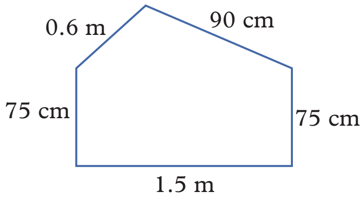


WE 9.1B **6** Calculate the perimeter of each shape in centimetres.

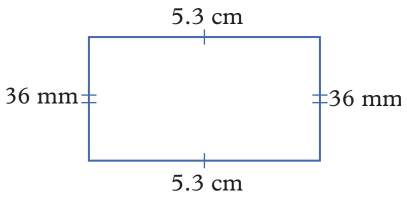
a



b



c



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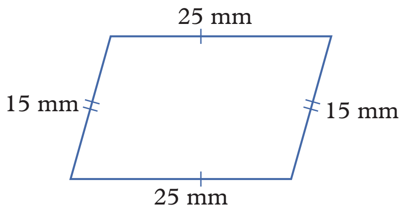

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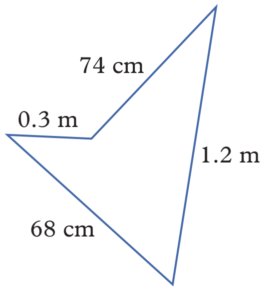
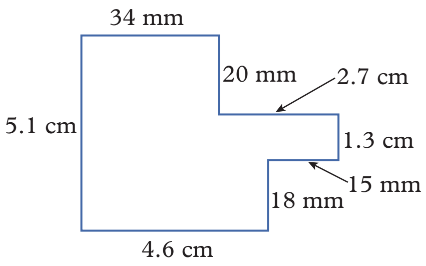


f

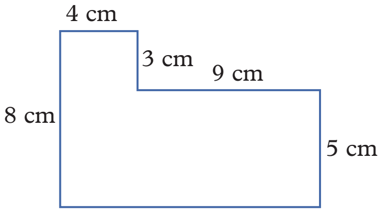


7 List the following length measurements in ascending order, from smallest to largest.

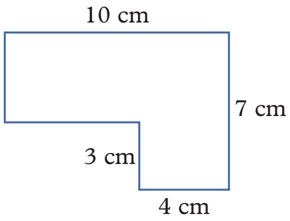
- **a** 5 cm, 15 mm, 60 mm
- **b** 2 m, 190 cm, 210 mm
- **c** 0.7 km, 701 m, 7001 cm
- **d** 82 mm, 8 cm, 0.8 m
- **e** 3 km, 3000 mm, 300 m, 30 cm
- **f** 5500 mm, 0.005 km, 5.7 m, 560 cm

8 Calculate the perimeter of each of the following shapes.

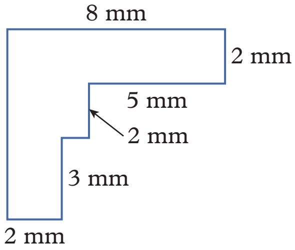
a



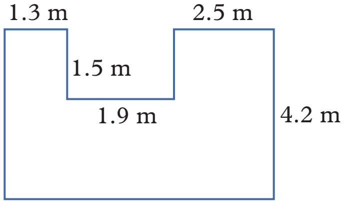
b



c



d



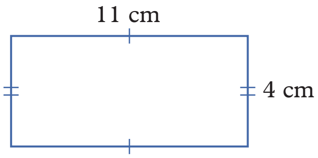
Exercise 9.1B: Problem solving and reasoning

9 A volleyball court is 18 m long and 9 m wide.

- **a** Draw a diagram of the court showing its measurements and the net.
- **b** Calculate its perimeter.
- **c** To warm up, each team jogs five laps around their half of the court. That is, the players jog along the outside lines and along the net on their side of the court. How far does a player jog during the warm-up?

10 A gardener plans to dig a rectangular herb garden with a border 36 m long. If the length of the garden is twice as long as the width, what are the dimensions?

11 A rectangle has length 11 cm and width 4 cm.

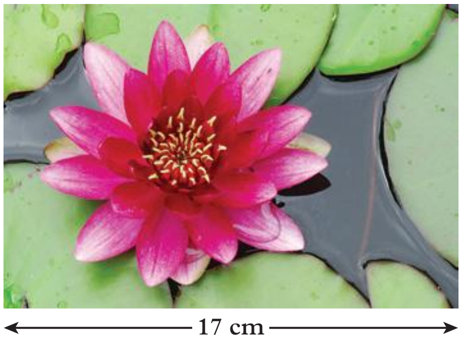


- **a** Calculate the perimeter of this rectangle.
- **b** If l is used to represent the length of a rectangle and w is used to represent the width of a rectangle, write a formula for finding the perimeter, P, of a rectangle.

12 Use the formula from question **11** to find the perimeter of these rectangles.

- **a** length 5 cm, width 3 cm
- **b** length 12 m, width 10 m
- **c** length 47 mm, width 35 mm
- **d** length 100 cm, width 24 cm
- **e** length 6 m, width 4.5 m
- **f** length 3.2 cm, width 1.9 cm
- **g** length 8.62 m, width 425 cm
- **h** length 15 mm, width 1.2 cm

13 The photo shown has a perimeter of 52 cm. What is the width of the photo?



14 If P is used to represent the perimeter of a rectangle and w is used to represent the width of a rectangle, write a formula for finding the length, l, of a rectangle.

15 A rectangular poster is 9.8 cm wide and has a perimeter of 50.8 cm. What is the length of the poster?

16 Some students made errors when they were converting length measurements. Explain the error in each case and give the correct answer.

a 55 cm = 5.5 mm

b 24 km = 2400 m

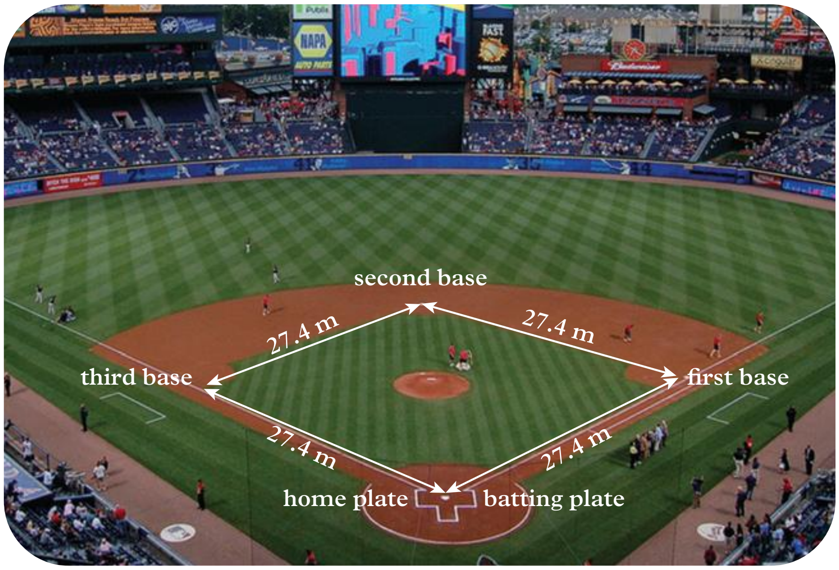
c 0.66 m = 60.6 cm

d 1.73 mm = 0.0173 cm

e 420 cm = 42 m

f 800 m = 0.08 km

17 In baseball, a batter must run from the batting plate to first base, then second base, third base and finally home. Use the measurements shown to answer the following.



a How far does a batter run to reach first base after hitting the ball?

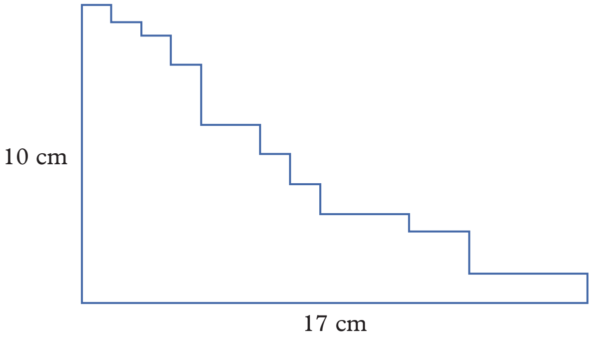
b How far does a batter run to reach third base after hitting the ball?

c How far does a batter run to reach the home plate when scoring a home run?

d Which of parts **a**, **b** and **c** is the same as the perimeter of the baseball diamond?

18 A square drink coaster has a perimeter of 342 mm. What is the length of the drink coaster?

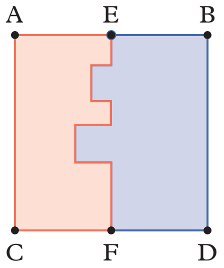
19 Consider the following shape.



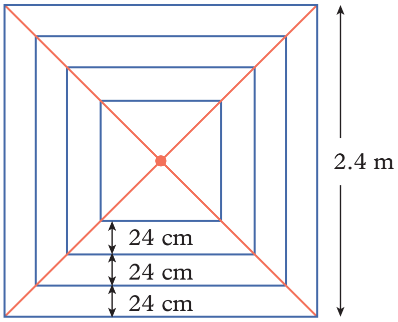
a Explain how you can calculate the perimeter of the shape when given only two side lengths. Note: All corners are right angles.

b What is the perimeter of the shape?

20 In the square shown below, E is at the midpoint of AB and F is at the midpoint of CD. Does the orange or blue shape have a larger perimeter or are their perimeters the same? Explain your answer.



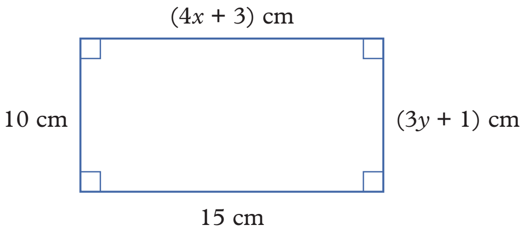
21 A Hills Hoist clothesline has four lines arranged in squares. Use the measurements shown on the diagram, which shows the clothesline from above, to calculate the total length of clothesline used.



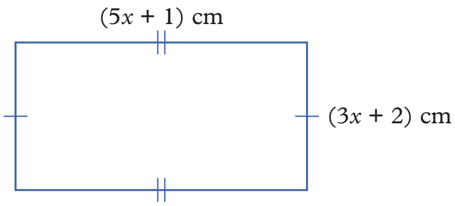
22 A rectangular swimming pool of length 7.5 m and width 4.6 m is to have a safety fence built around it. If the fence is to be built 2 m from the edge of the pool, calculate the total length of fencing needed, including the gate.

Exercise 9.1C: Challenge

23 Find the values of x and y.



24 If the perimeter of this shape is 38 cm, find the value of x.



Online resources:

Interactive skillsheet Converting between units of length

Worksheet Converting units of length

Investigation Metric and imperial units of length

Quick quiz 9.1

Interactive skillsheet Perimeter

Worksheet Calculating perimeter

Lesson 9.2

Circumference of a circle

Learning intentions

By the end of this lesson you will be able to ...

✔ calculate the radius, diameter and circumference of circles.

Inter-year links

Support	Length and perimeter
Year 8	8.4 Area of a circle

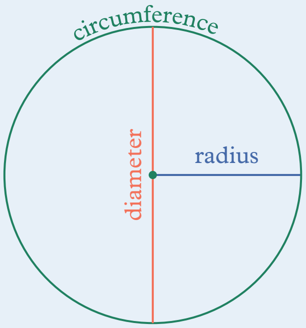
Key content video
Circumference

Features of circles

- The perimeter of a circle is called its **circumference** (C).
- The **diameter** of a circle (d) is the length from one point on the circle to another through the centre of the circle.
- The **radius** of a circle (r) is the length from the centre of the circle to any point on the circle.
  → The radius is half the length of the diameter.

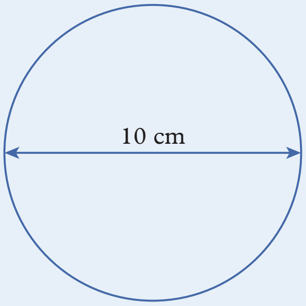
Calculating the circumference

- The ratio of the circumference of any circle to its diameter is equal to **π (pi)**.
  → π is approximately equal to 3.14.



- To find the circumference of a circle, multiply the diameter by π.
- For example, the circumference of the circle below is $\pi \times 10 \approx 31.4$ cm.

$$C = \pi d$$
$$C = 2\pi r$$

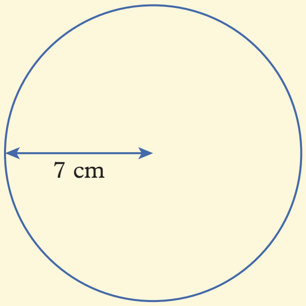


→ This formula can be rearranged to find the value of the radius or diameter, given the circumference.

$$d = \frac{C}{\pi} \qquad r = \frac{C}{2\pi}$$

Worked example 9.2A Finding the diameter

The following diagram shows the length of the radius of the circle. Find the length of the diameter of the circle.

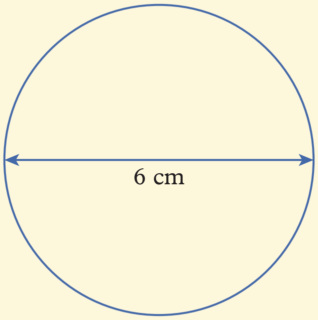


THINK	**WRITE**
1 The diameter is twice the length of the radius. Multiply the radius by two.	$d = 2r$ $= 2 \times 7$
2 Write your answer, including the appropriate unit.	$= 14$ cm

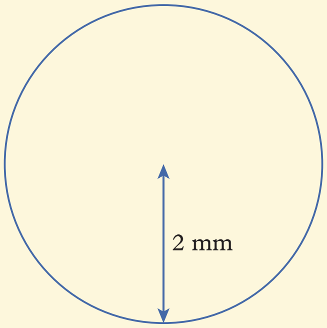
Worked example 9.2B Calculating the circumference

Given the following diameter and radius lengths, calculate the circumference of each circle, correct to two decimal places.

a



b



THINK	**WRITE**
a **1** Identify which formula to use.	**a** $C = \pi d$
2 Substitute the measurement for the diameter.	$= \pi \times 6$
3 Calculate using π on your calculator.	$= 18.849\,555\,9\ldots$
4 Round to two decimal places and include the appropriate unit.	≈ 18.85 cm
b **1** Identify which formula to use.	**b** $C = 2\pi r$
2 Substitute the measurement for the radius.	$= 2 \times \pi \times 2$
3 Calculate using π on your calculator.	$= 12.566\,370\,6\ldots$
4 Round to two decimal places and include the appropriate unit.	≈ 12.57 mm

Worked example 9.2C Using the circumference to calculate the radius

Calculate the radius of a circle, correct to two decimal places, given that its circumference is 60 mm.

THINK	**WRITE**
1 Identify both the given value and the required value.	$C = 60$ mm, $r = ?$
2 Identify which formula to use.	$C = 2\pi r$
3 Substitute the value for the circumference into the formula.	$60 = 2\pi r$
4 Rearrange the formula to make r the subject, and calculate the value of the radius on your calculator.	$r = \frac{60}{2\pi}$ $r = 9.549...$
5 Round to two decimal places and include the appropriate unit.	$r \approx 9.55$ mm

Helpful hints

- ✔ Unless instructed otherwise, use π on your calculator instead of an approximation such as 3.14. You should only round your answers at the final step of the calculations.
- ✔ Be careful to substitute the correct value into the correct formula. Remember that C stands for circumference, d stands for diameter and r stands for radius.

Learning pathways

△ 1–9, 10(a–d), 11, 13, 17

■ 1, 2, 3–4(b, e, f, i), 5, 7, 10(b, d, e, f), 12, 15–17

⬟ 3–4(f–i), 5, 7, 10(e, f), 14–18

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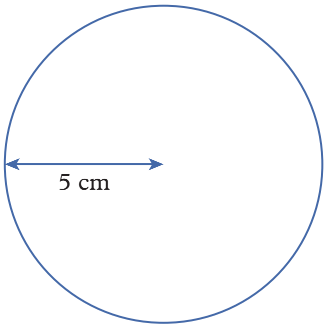
Exercise 9.2A: Understanding and fluency

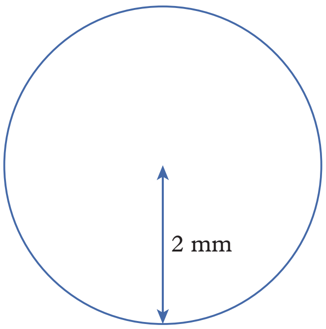
For all questions in this topic, use π on your calculator.

WE 9.2A **1** The following diagrams show the length of the radius of each circle. Find the length of the diameter of each circle.

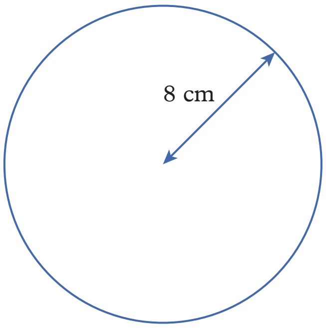
a



b

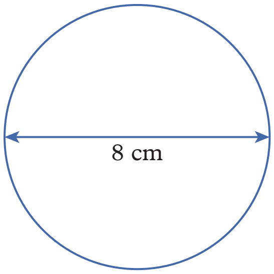


c



2 The following diagrams show the length of the diameter of each circle. Find the length of the radius of each circle.

a



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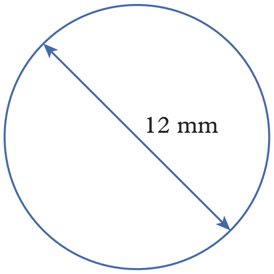

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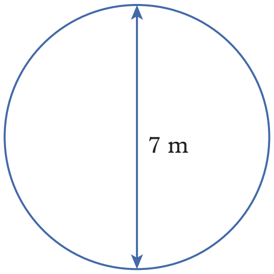
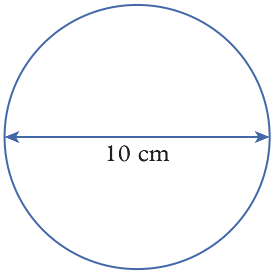


WE 9.2B **3** Given the following diameter and radius lengths, calculate the circumference of each circle, correct to two decimal places.

a



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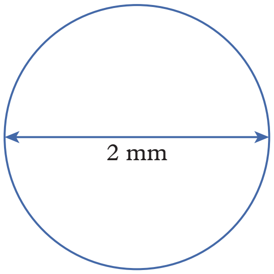

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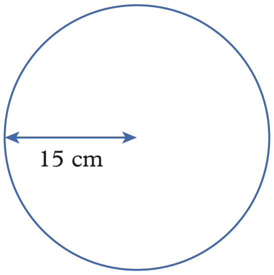

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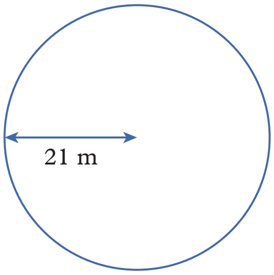

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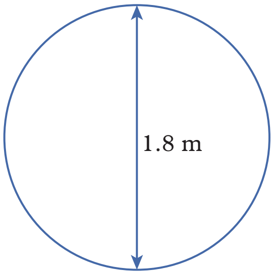

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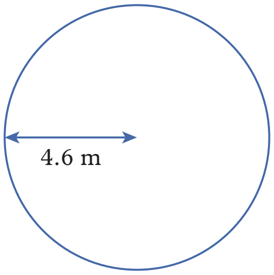

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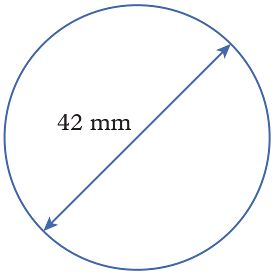

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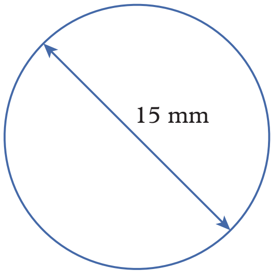
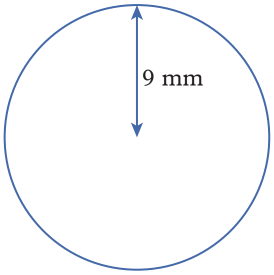


i



4 Find the circumference, correct to two decimal places, of a circle with:

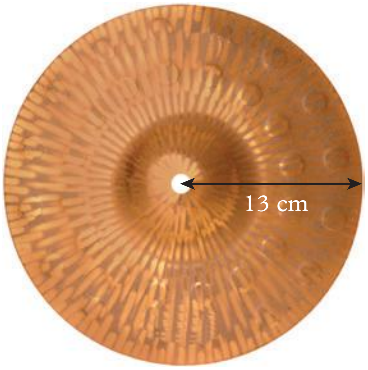
a radius 1.2 m
b diameter 2.5 cm
c diameter 6.4 mm
d radius 0.4 cm
e radius 19 cm
f radius 13 mm
g diameter 0.75 m
h diameter 27 cm
i radius 82 km.

WE 9.2C **5** Calculate the **i** diameter and **ii** radius of a circle, correct to two decimal places, given that its circumference is:

a 31.41 cm
b 62.82 cm
c 50 m
d 25 m
e 1 km
f 32 m
g $\frac{22}{7}$ km
h $\frac{28}{5}$ m
i $\frac{67}{11}$ cm.

6 Calculate the circumference of each object, correct to two decimal places.

a



b

c

d

7 Calculate the radius of the following objects, correct to two decimal places.

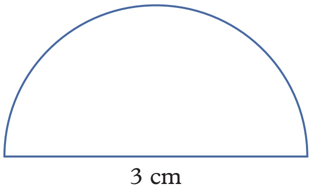
- **a** The circumference of a DVD is 377 mm.
- **b** The circumference of a frisbee is 44 cm.
- **c** The circumference of a small circular plate is 21 cm.
- **d** The circumference of a round cake is 63.8 cm.

Exercise 9.2B: Problem solving and reasoning

8 A trundle wheel is a device that is used to roughly measure distances. The wheel, with a circumference of 1 m, makes a clicking noise every time an indicator on the wheel makes a full revolution to indicate 1 m has been travelled.

- **a** The trundle wheel is used to measure the width of a road. It clicks 9 times and the indicator finishes halfway around the wheel. Approximately how wide is the road?
- **b** Determine the radius of the trundle wheel, correct to the nearest millimetre.

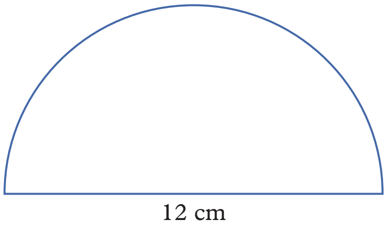
9 Consider this semicircle.



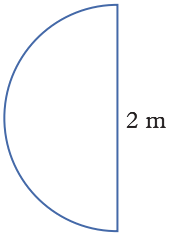
- **a** What percentage of a full circle is a semicircle?
- **b** What would be the circumference if it was a full circle? Give your answer correct to two decimal places.
- **c** Use your answers to parts **a** and **b** to find the length of the curved section of the semicircle.
- **d** What is the perimeter of the semicircle?

10 Use your understanding of circumference to find the perimeter of each of these shapes. Give your answers correct to two decimal places.

a



b



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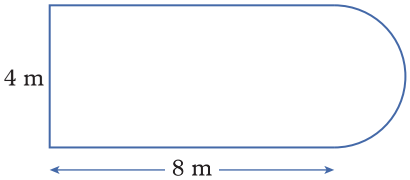

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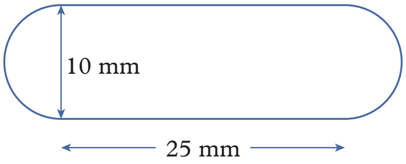

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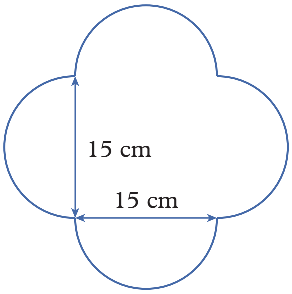

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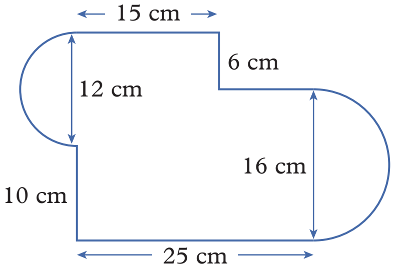


11 Jack goes for a bike ride with his younger sister Lily. If Jack's bike wheels have a diameter of 55 cm and Lily's have a diameter of 32 cm:

- **a** find the circumference of their wheels, correct to two decimal places
- **b** find how many times each wheel turns to travel 1 km to the nearest whole number.

12 Laura and Matt go for a ride on a merry-go-round. Laura chooses a horse 5.4 m from the centre and Matt chooses a horse 4.1 m from the centre.

- **a** Do these lengths relate to diameter or radius? Explain.
- **b** Find how far they will each travel in one revolution of the merry-go-round. Give your answer correct to two decimal places.
- **c** If the merry-go-round rotates six times in a ride, use your answer to part **b** to find the difference in the distance Matt and Laura travel.
- **d** Explain why, although Matt and Laura are on the same ride, Laura travels further than Matt.

13 Imagine you could tie a piece of string around the circumference of the Earth at the equator.

If Earth has a diameter at the equator of about 12 756 km, how much string would be needed? Give your answer correct to two decimal places.

14 Ellie bought Thomas a ring. She knows that his ring finger has a circumference of 60 mm. She buys him an S size ring, which has an inner diameter of 19.15 mm.

a Find the inner circumference of this ring, correct to two decimal places.

b Ellie gives the ring to Thomas and finds that it doesn't fit because it won't go over his knuckle. She tries both a T size ring (diameter of 19.56 mm), which is still slightly too small, and a U size ring (diameter of 19.96 mm), which is slightly too big.

What size range must the circumference of Thomas' knuckle be in? Hint: Find the circumferences of both the T and U size rings. Give your answers correct to two decimal places.

c Ellie discovers that these rings also come in half sizes.

i What would be the diameter of a ring size T$\frac{1}{2}$?

ii If the ring is a perfect fit, find the circumference of Thomas' knuckle, correct to two decimal places.

15 An A4 piece of paper has dimensions 210 × 297 mm. The paper can be rolled into the shape of a cylinder where one of the side lengths is the circumference of the circular ends. Determine the two possible radii for the circles that can be formed if the paper just touches itself after being rolled, correct to two decimal places.

16 π can be used to give exact value answers. For example, the circumference of a circle with a diameter of 2 cm is exactly 4π cm.

Calculate the circumferences of the following circles as exact values by keeping π in the answers.

a Circle with $d = 7.5$ m **b** Circle with $r = 6.4$ cm **c** Circle with $r = 1.7$ m

17 Calculate the **i** diameter and **ii** radius of a circle given that its circumference is exactly:

a 11π cm **b** 4.53π mm **c** $\frac{2\pi}{3}$ km.

Exercise 9.2C: Challenge

18 An amusement park has a new Ferris wheel. The Ferris wheel can be modelled as a circle with a radius of 54 m.

a What is the diameter of the Ferris wheel?

b If the lowest point of the Ferris wheel stands 5 m above ground level, how high above ground level is the highest point of the Ferris wheel?

c What is the circumference of the Ferris wheel, correct to two decimal places?

d How many carriages could fit on the Ferris wheel if each carriage takes up 1 m of space on the circumference, and there has to be at least 3 m between each carriage on the circumference?

e Use your answer from part **c** to calculate the distance between each carriage on the circumference of the Ferris wheel, correct to two decimal places.

f If the owner wanted to fit 100 carriages onto the Ferris wheel, assuming a minimum distance of 3 m between each carriage, by how much would the radius of the Ferris wheel have to increase, correct to two decimal places?

Online resources:

Interactive skillsheet Circumference

Worksheet Finding the circumference

Worksheet Identifying parts of a circle

Investigation Approximations of π

Quick quiz 9.2

Lesson 9.3

Area of a rectangle

Learning intentions

By the end of this lesson you will be able to …

✔ calculate the area of rectangles.

Inter-year links

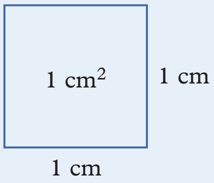
Support	Understanding area
Year 8	8.2 Area of rectangles and triangles
Year 9	5.2 Area of composite shapes

Key content video
Area of a rectangle

Area

- The **area** of a 2D shape is the amount of space enclosed by that shape.

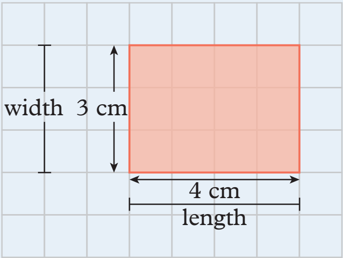
Metric units of area	
Name	**Abbreviation**
square millimetre	mm^2
square centimetre	cm^2
square metre	m^2
square kilometre	km^2



Area of a rectangle

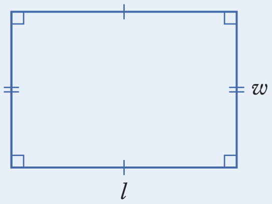
- Rectangles have two dimensions:
  - → The length of a rectangle is the length of its longest side.
  - → The **width** of a rectangle is the length of its shortest side.
- A rectangle can be divided into squares to calculate its area. The area of a rectangle is equal to the total area of the squares that fit inside it. This can be found by multiplying its length by its width.

Area of a rectangle = length × width.



→ Expressing the rule using pronumerals, where A = area of a rectangle, l = length and w = width, the formula for the area of a rectangle is:

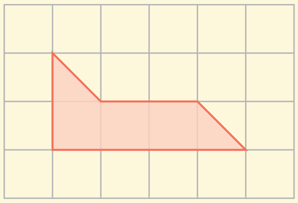
$$A = l \times w$$



→ The measurements for l and w must be in the same unit before they are substituted into the formula.

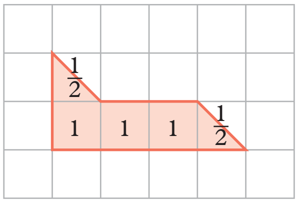
Worked example 9.3A Finding the area by counting squares

Find the exact area of this shape. Assume each square in the grid has an area of 1 cm^2.



THINK

Calculate the area of the shape by adding up the number of squares that make up the shape. As each square in the grid has an area of 1 cm^2, 4 squares = 4 cm^2.



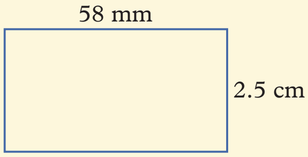
WRITE

3 squares + 2 half squares = 3 squares + 1 square
= 4 squares

Area = 4 cm^2

Worked example 9.3B Calculating the area of a rectangle using the formula

Calculate the area of this rectangle.



THINK

1 Identify the measurements for l and w. Convert all measurements into the same unit.

2 Substitute the values for l and w into the formula for the area of a rectangle.

3 Calculate the area of the rectangle. Remember to include the appropriate unit in the answer. As the length and width are in centimetres, the area will be in square centimetres, cm^2.

WRITE

l = 58 mm
= 5.8 cm
w = 2.5 cm

$A = l \times w$
$= 5.8 \times 2.5$

$= 14.5 \text{ cm}^2$

Alternatively, if w was converted into millimetres, A = 1450 mm^2.

Helpful hints

- ✔ The perimeter and area of 2D shapes are not the same thing! The perimeter is a measure of the length around the boundary of a shape, whereas the area is a measurement of the amount of space that the shape covers.
- ✔ When working with areas, the units will be square units. Remember to include a superscript 2 to indicate this, for example, cm^2.

Learning pathways

△ 1, 2, 3(a–c), 4(a, e, f), 5(a–c), 6(1st column), 7, 8, 9(a, e, f), 10, 11(a, b), 12, 15(a, b)

■ 1(d–f), 3(c, e, f), 4(e, f), 5(d–f), 6(2nd column), 7(c, d), 8, 9(e–h), 11, 13, 14, 15(c, d), 17, 18, 21

⬠ 1(e, f), 4(b, e, f), 5(d–f), 6(d, f), 7(c, d), 8, 9(f, h), 11(c, d), 13, 14, 16, 19–22

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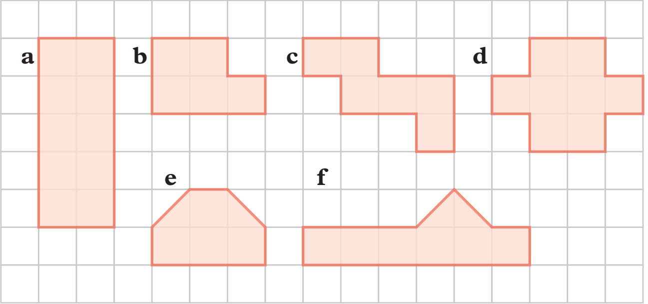
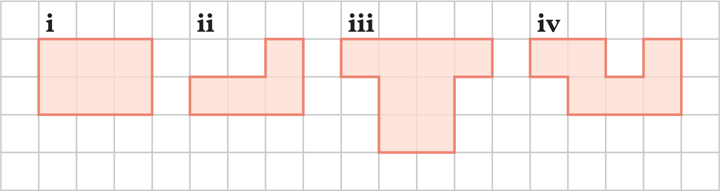
Exercise 9.3A: Understanding and fluency

WE 9.3A **1** Find the exact area of these shapes. Assume each square in the grid has an area of 1 cm^2.



2 These shapes are shown on 1-cm grid paper.



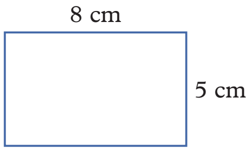
a Find the perimeter of each shape.

b Find the area of each shape.

c Explain the difference between finding the perimeter and finding the area of each shape.

WE 9.3B **3** Calculate the area of each rectangle.

a



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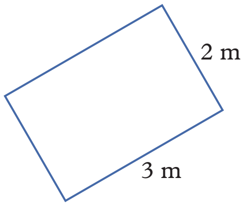

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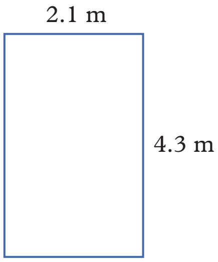

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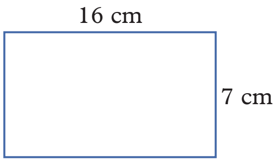

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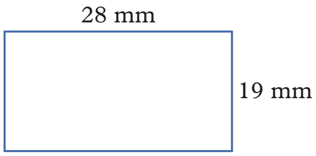

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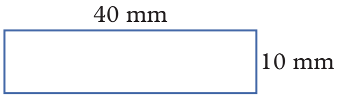


4 Find the area of the rectangles with these dimensions.

a length 9 cm, width 7 cm

b length 13 mm, width 4 mm

c length 6 m, width 1 m

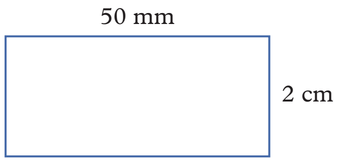
d length 10.4 cm, width 5 cm

e length 14.2 mm, width 9.7 mm

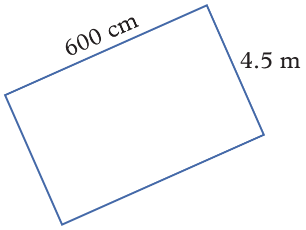
f length $17\frac{1}{2}$ m, width $5\frac{1}{4}$ m

5 Calculate the area of each rectangle.

a



b



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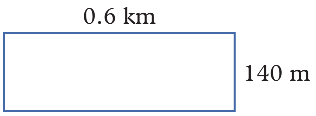

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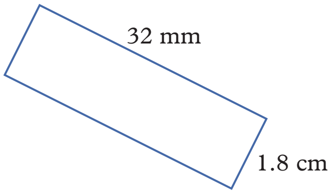

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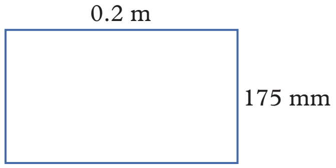
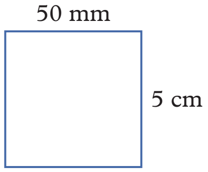


f



6 Find the area of each of the following rectangles.

a length 3 m, width 100 cm

b length 8 cm, width 50 mm

c length 4 km, width 280 m

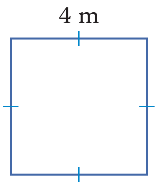
d length 600 mm, width 30 cm

e length 3.1 cm, width 20 mm

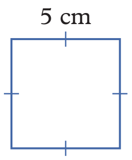
f length 8.4 cm, width 0.05 m

7 Calculate the area of each square.

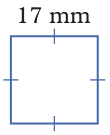
a



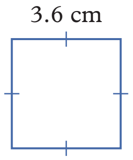
b



c



d



8 Use your working from question **7** to determine the formula for the area (A) of a square of length l.

9 Find the area of the squares with these side lengths.

a 7 m

b 15 cm

c 11 km

d 60 mm

e 6.4 cm

f 180 m

g 0.7 mm

h 0.53 km

Exercise 9.3B: Problem solving and reasoning

10 The shape shown in Figure A can be divided into two rectangles as shown in Figure B.

Figure A

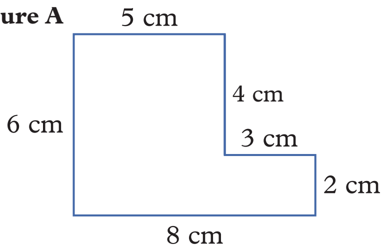
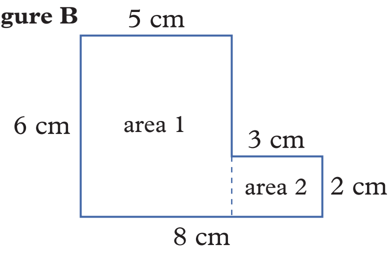


Figure B



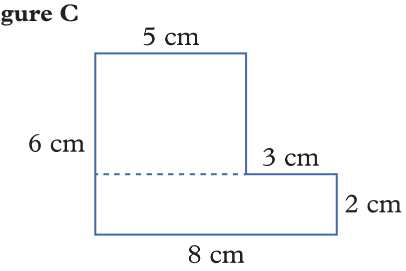
a Calculate the area of the rectangle labelled as area 1 in Figure B.

b Calculate the area of the rectangle labelled as area 2 in Figure B.

c What is the total area of the shape?

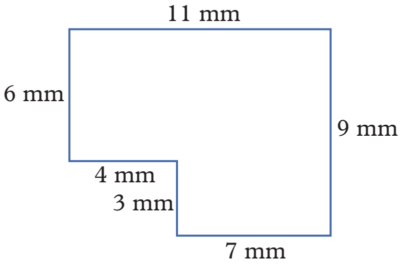
d Figure A can also be divided into two other rectangles, as shown in Figure C. Calculate the area of the shape using Figure C.

Figure C

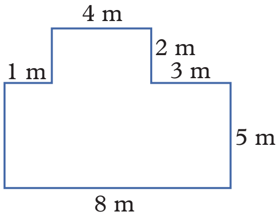


11 Calculate the area of these shapes by first dividing them into rectangles.

a



b



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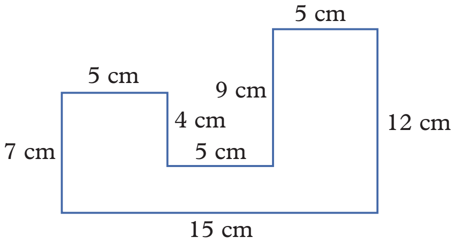

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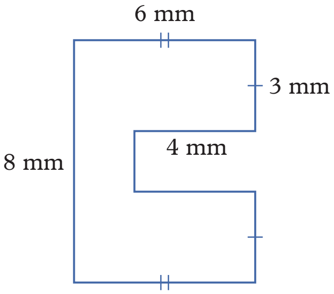
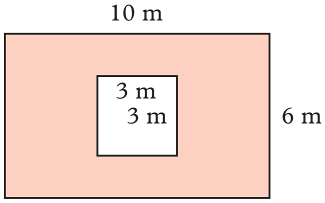


12 In questions **10** and **11**, the shapes are divided into separate rectangles and the areas of each rectangle are added together to find the total area of the shape.

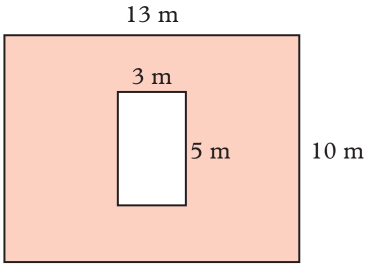


a Explain how you could calculate the area of the shaded region in this shape using subtraction instead of addition. Both shapes are rectangles.

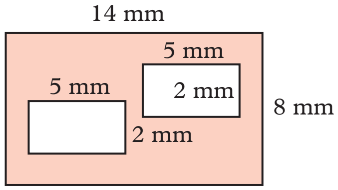
b What is its area?

13 Find the area of the shaded region in each of the following diagrams. All shapes are rectangles.

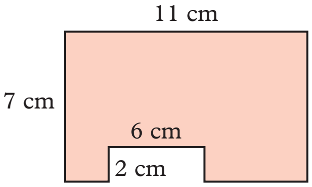
a



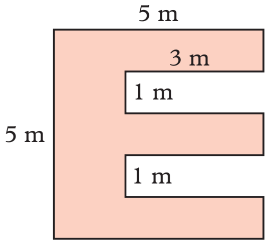
b



c



d



14 Use the images of these two phones to determine the following.

a What is the width of the screen on the phone in Figure A, if its area is 3750 mm^2?

b If the area of the screen on the phone in Figure B is 3000 mm^2, what is its length?

Figure A

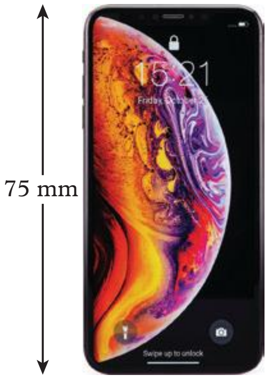
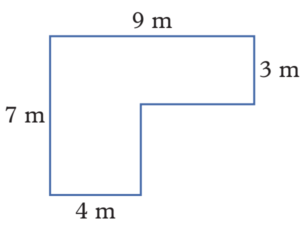


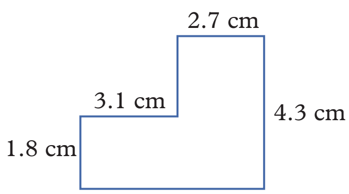
Figure B

15 Find **i** the area and **ii** the perimeter of each shape. Note: All corners are right angles.

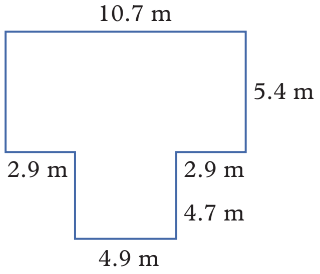
a



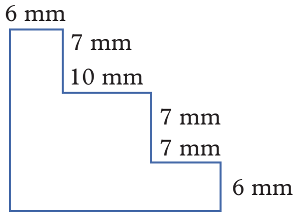
b



c



d



16 The perimeter of a rectangle with a length of 14 cm is 46 cm. What is its area?

17 The perimeter of a square is 80 m. What is its area?

18 The area of a square is 16 mm^2. What is its perimeter?

19 Explain why two rectangles with the same area do not always have the same perimeter. Give an example in your answer.

20 A tennis court has a playing area that is divided by lines drawn on the ground, as shown in the diagram on the right. Doubles matches use the full width of the court, whereas singles matches do not use the sidelines on either side of the court.

a Use the dimensions labelled in the diagram to calculate:

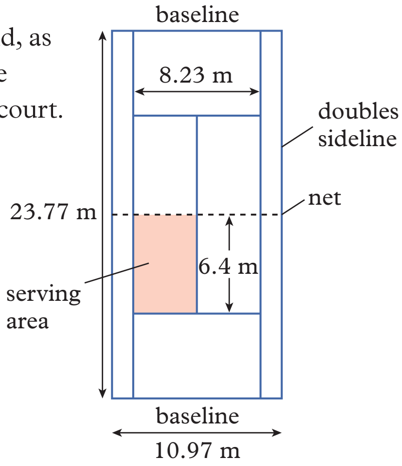
i the distance from the baseline to the net

ii the playing area used in a doubles match

iii the playing area used in a singles match

iv the area of the serving area.

b When it starts to rain, a rectangular waterproof cover is placed over the playing surface. If the cover extends beyond the court by 1.5 m in each direction, what area of waterproof material is required for the cover?



Exercise 9.3C: Challenge

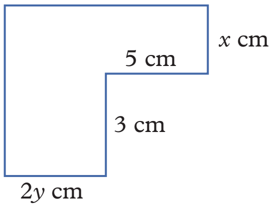
21 Is it possible for a rectangle's perimeter to have the same value in cm as its area in cm^2? Explain your answer.

22 Write four equivalent expressions for the area of the shape to the right.

a One that contains products added together.

b Two that contain a sum multiplied by a term added to another term.

c One that contains a subtraction.



Online resources:

Interactive skillsheet Area of a rectangle

Worksheet Area of a rectangle

Investigation Floor designs

Quick quiz 9.3

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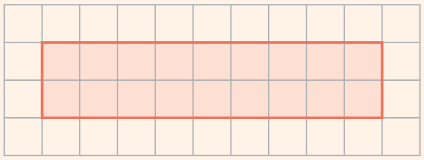
Checkpoint

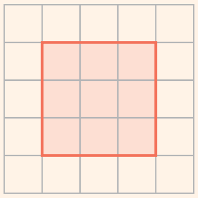
Checkpoint quiz
Take the checkpoint quiz to check your knowledge of the first part of this module.

9.1 **1** Calculate the perimeter of the following shapes shown on 1-cm grid paper.

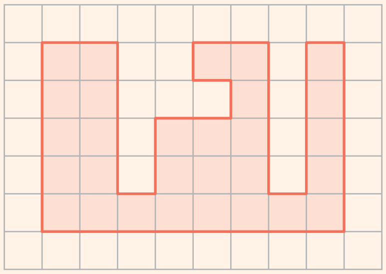
a



b



c

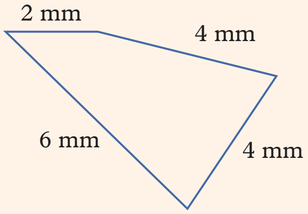


9.1 **2** Convert the following lengths to the unit in brackets.

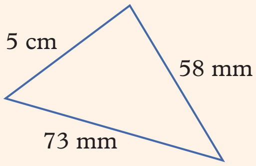
a 15 mm (cm) **b** 25 912 km (m) **c** 23 cm (m) **d** 0.3 m (mm)

9.1 **3** Calculate the perimeter of the following shapes.

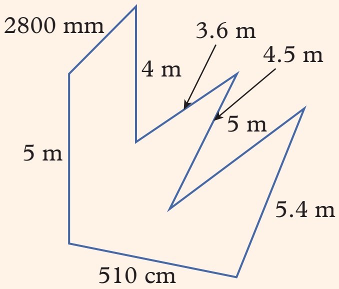
a



b



c



9.2 **4** Calculate the circumference of the following circles, and give your answers correct to two decimal places.

a Circle with diameter 31 m **b** Circle with radius 7 cm

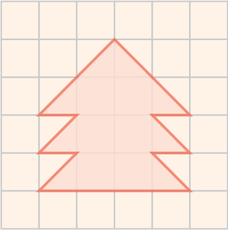
9.2 **5** Use the given information to find the missing lengths, correct to two decimal places.

a $C = 42.1$ m, find d **b** $C = 13$ cm, find r **c** $C = 20$ m, find d **d** $C = 100$ mm, find r

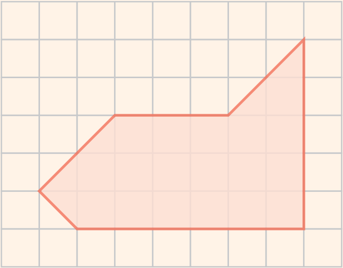
9.3 **6** Calculate the area of the shapes shown in question **1**.

9.3 **7** Calculate the area of the following shapes shown on 1-cm grid paper.

a

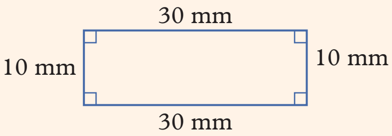


b

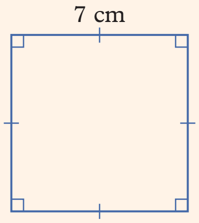


9.3 **8** Calculate the area of the following shapes.

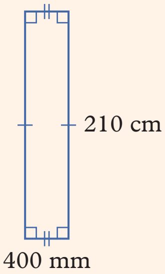
a



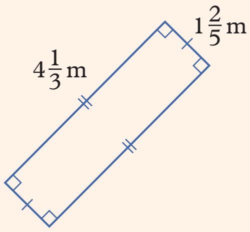
b



c



d



Lesson 9.4

Area of a parallelogram

Learning intentions

By the end of this lesson you will be able to …

✔ calculate the area of parallelograms.

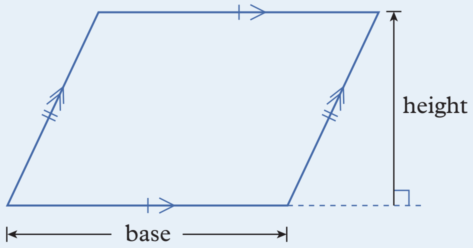
Inter-year links

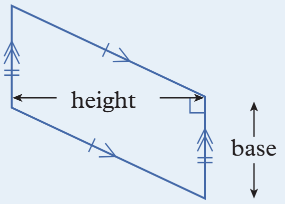
Support	Understanding area
Year 8	8.3 Area of quadrilaterals
Year 9	5.2 Area of composite shapes

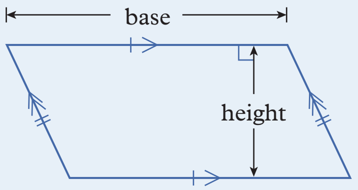
Area of a parallelogram

Key content video
Area of a parallelogram

- Parallelograms have two dimensions:
  - → The **base** is the length of one side of the parallelogram.
  - → The **height** is the perpendicular distance from the base to the opposite side of the parallelogram.

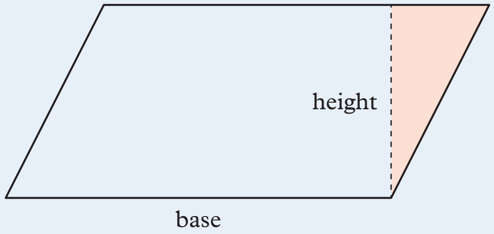


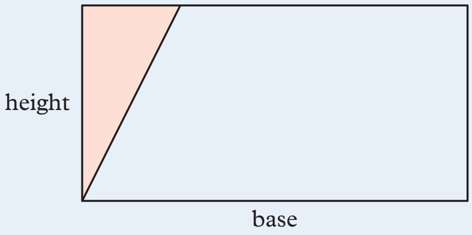




- Parallel sides are identified using matching arrowheads.
- Every parallelogram can be rearranged into a rectangle with the same base length and height. Therefore, the area of a parallelogram can be calculated using the rule:

Area of a parallelogram = base × height

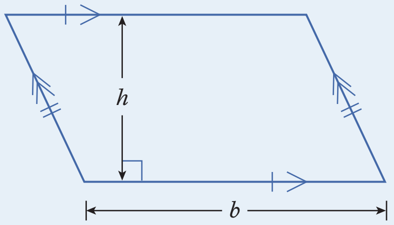




- Expressing the rule using pronumerals, where A = area of a parallelogram, b = base and h = height, the formula for the area of a parallelogram is:

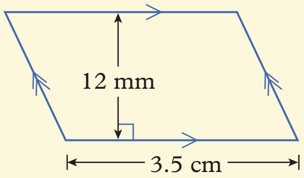
$$A = b \times h$$

  - → The base and height must be perpendicular; that is, they must have a 90-degree angle between them.
  - → The measurements for b and h must be in the same unit before they are substituted into the formula.



Worked example 9.4A Calculating the area of a parallelogram using the formula

Calculate the area of this parallelogram in square centimetres, cm^2.



THINK	**WRITE**
1 Identify the measurements for *b* and *h*. All measurements must be in the same unit before they are substituted into the formula. Convert the height into the units specified by the question.	$b = 3.5$ cm $h = 12$ mm $= 1.2$ cm
2 Substitute the values for *b* and *h* into the formula.	$A = b \times h$ $= 3.5 \times 1.2$
3 Calculate the area of the parallelogram. Remember to include the appropriate unit in the answer.	$= 4.2$ cm^2

Helpful hints

- ✔ The base can be any side of a parallelogram; it doesn't need to be at the bottom of a diagram.
- ✔ The height of a parallelogram is the perpendicular length between two parallel sides; it is not the length of the sloping edge! The height is used to calculate the area, and the sloping edges are used to calculate the perimeter.

Learning pathways

△ 1–6, 7(a–c), 8(a–c), 9, 11, 12

■ 2(c, d), 3–8, 10, 11, 13, 14, 15(a)

⬟ 3(d–f), 5–8, 11, 13, 15, 16

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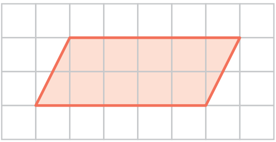
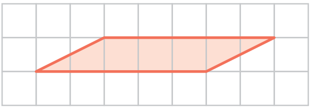
Exercise 9.4A: Understanding and fluency

1 Find the area of these parallelograms shown on 1-cm grid paper.

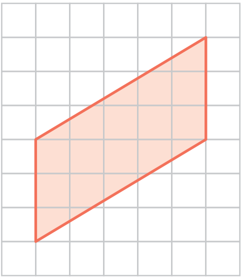
a



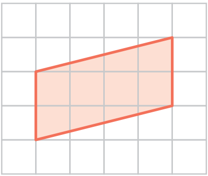
b



c

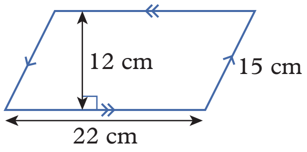


d



2 State the base length and the height of each of these parallelograms.

a



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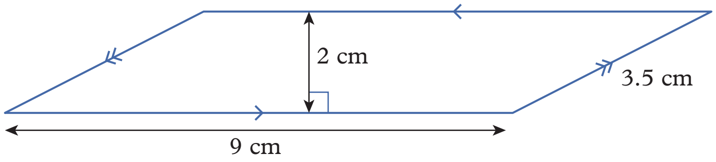

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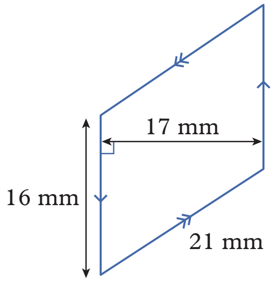

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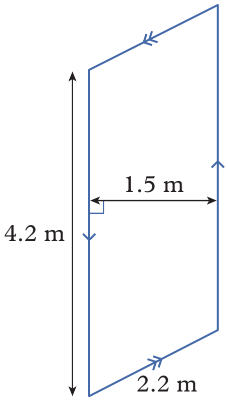
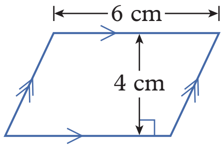
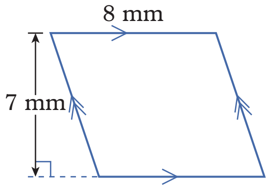


3 Calculate the area of these parallelograms.

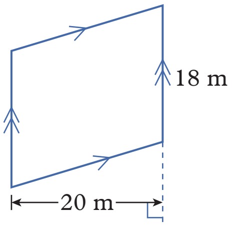
a



b



c



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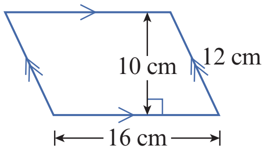
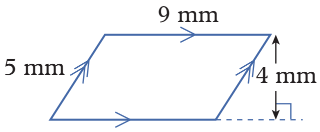
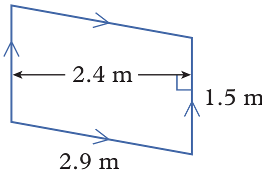


e



f



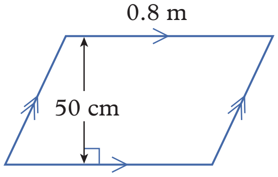
4 Find the area of the parallelograms with these dimensions.

- **a** base length 5 cm, height 3 cm
- **b** base length 10 mm, height 17 mm
- **c** base length 4 m, height 1.5 m
- **d** base length 2.5 cm, height 4.1 cm

5 Find the area of each parallelogram with these dimensions in the units specified in brackets.

- **a** base length 30 mm, height 5 cm (cm^2)
- **b** base length 58 m, height 200 cm (m^2)
- **c** base length 0.06 m, height 40 mm (mm^2)
- **d** base length 27.4 cm, height 27.4 mm (cm^2)

WE 9.4A 6 Calculate the area of each parallelogram in square centimetres, cm^2.

a


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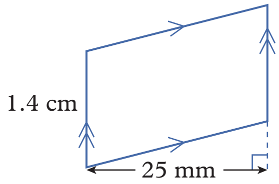

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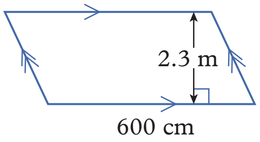

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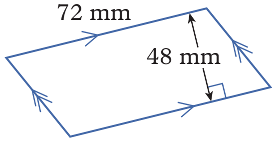

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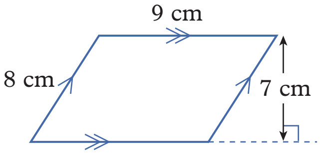

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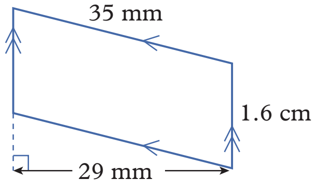
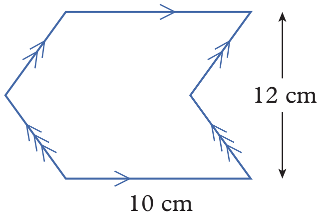


Exercise 9.4B: Problem solving and reasoning

7 Divide these shapes into two or more rectangles and/or parallelograms to calculate their area.

a


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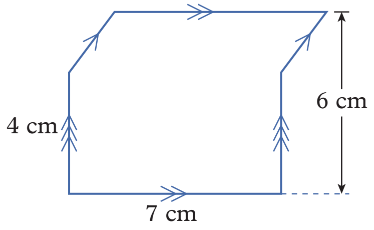
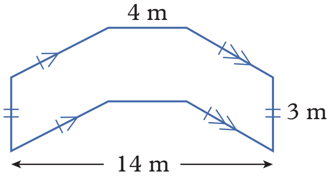


c


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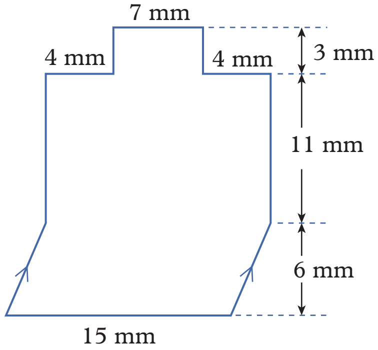
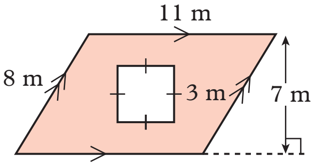
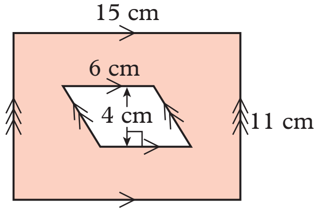
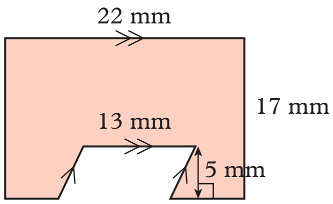


8 Find the area of the shaded region in each of these. Consider subtracting areas as well as adding areas to obtain your answer.

a


b


c


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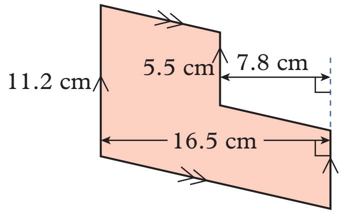
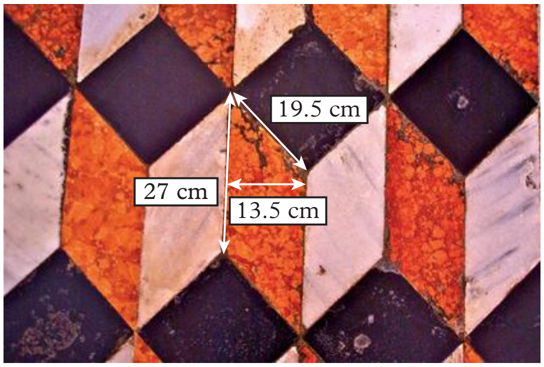


9 Use the measurements shown on this photo of floor tiles to calculate the area of a red tile.

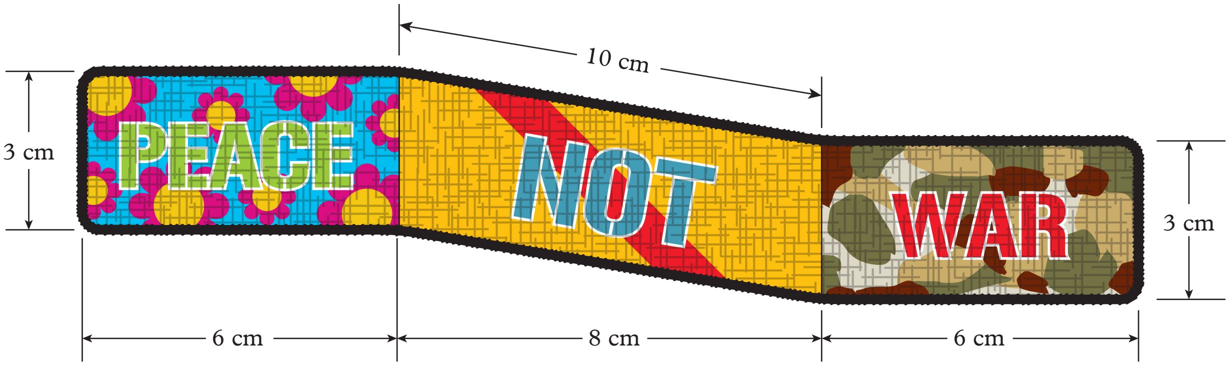


10 A stair rail is supported by five identical glass panels.

a Use the dimensions shown in the photo above to calculate the area of glass needed for all the glass panelling.

b If each glass panel were to be 10 cm taller, what area of glass would be needed?

11 A badge for a jacket is made by sewing three pieces of fabric as shown below. The centre piece is in the shape of a parallelogram and the other two pieces of fabric are rectangles.



a What area of fabric is needed for the parallelogram?

b What area of fabric is needed for each rectangle?

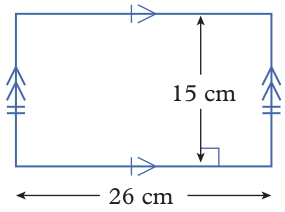
c Calculate the total area of fabric needed for this badge.

d Black cord is sewn around the outer edge of the badge to make it stand out on the jacket. Calculate how much black cord is needed.

12 The area of a parallelogram is 20 cm^2. If its height is 4 cm, how long is the base of the parallelogram?

13 The area of a parallelogram is 36 cm^2. If its base is 45 mm long, what is the height of the parallelogram?

14 Consider the rectangle shown.

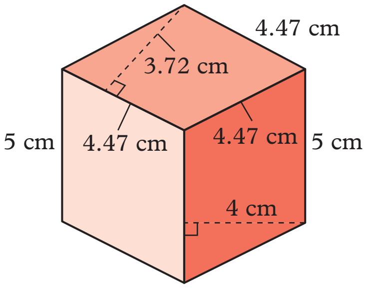


a Calculate its area using the formula for the area of a rectangle, $A = l \times w$.

b Calculate its area using the formula for the area of a parallelogram, $A = b \times h$.

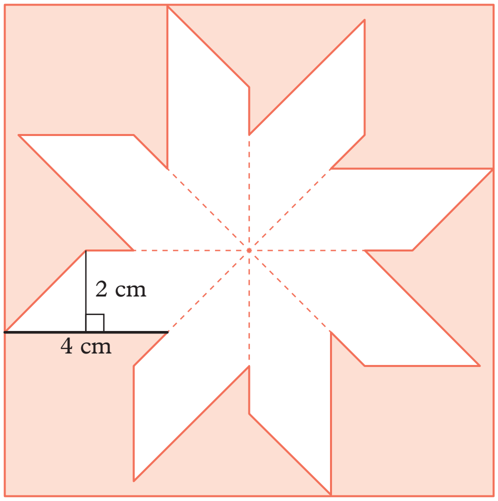
c Explain how you obtain the same answer in each case.

15 The following drawing of a rectangular prism is constructed using two identical parallelograms, and a rhombus representing one of the matching ends of the prism. A rhombus is a parallelogram with all sides of equal length. Determine the total area of the drawing.



Exercise 9.4C: Challenge

16 Eight identical parallelograms have been cut out of a square piece of paper as shown. Find the area of the remaining paper.



Online resources:

Interactive skillsheet Area of a parallelogram

Worksheet Area of a parallelogram

Investigation The secret of the parallelogram

Quick quiz 9.4

Lesson 9.5

Area of a triangle

Learning intentions

By the end of this lesson you will be able to ...

✔ calculate the area of triangles.

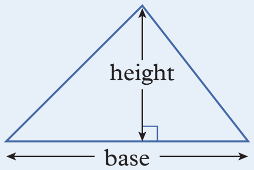
Inter-year links

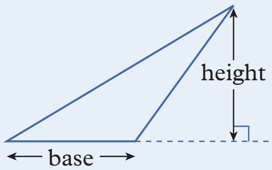
Support	Understanding area
Year 8	8.2 Area of rectangles and triangles
Year 9	5.3 Surface area of prisms

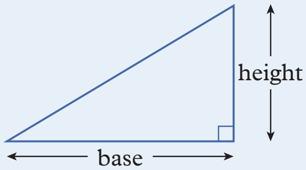
Area of a triangle

Key content video
Area of a triangle

- Triangles have two dimensions:
  - → The base is the length of the base of the triangle.
  - → The height is the perpendicular distance from the base to the opposite vertex of the triangle.

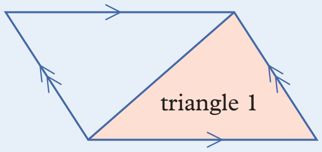


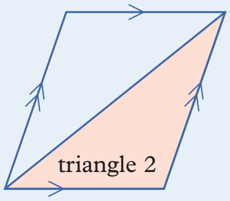


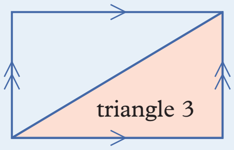


- Every triangle can be enclosed by a corresponding parallelogram or rectangle which has the same base length and height as the triangle.
- The corresponding parallelogram or rectangle is made up of exactly two of the triangles that it encloses, therefore the area of a triangle can be calculated using the rule:

$$\text{Area of a triangle} = \frac{1}{2} \times \text{base} \times \text{height}$$

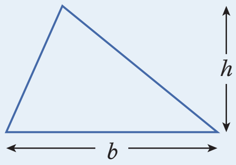






- Expressing the rule using pronumerals, where A = area of a triangle, b = base and h = height, the formula for the area of a triangle is:

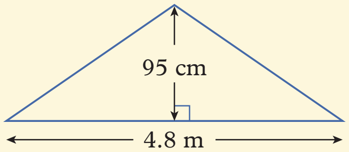
$$A = \frac{1}{2} \times b \times h$$



  - → The base and height must be perpendicular to each other.
  - → The measurements for b and h must be in the same unit before they are substituted into the formula.

Worked example 9.5A Calculating the area of a triangle using the formula

Calculate the area of this triangle in m^2.



THINK	**WRITE**
1 Identify the measurements for *b* and *h*. Convert the height into the units specified by the question.	$b = 4.8$ m $h = 95$ cm $= 0.95$ m
2 Substitute the values for *b* and *h* into the formula.	$A = \frac{1}{2} \times b \times h$ $= \frac{1}{2} \times 4.8 \times 0.95$
3 Calculate the area of the triangle. Remember to include the appropriate unit in the answer.	$= 2.28$ m^2

Helpful hints

The base can be any side of a triangle; it does not have to be at the bottom! Every triangle will have three potential bases, along with three corresponding perpendicular heights.

Learning pathways

△ 1, 2(a, c, e), 3–5, 6(a, c, e), 7, 9, 13(a–c)

□ 3(b, d, f), 4, 5, 6(b, d, f), 7, 8, 10, 12–14

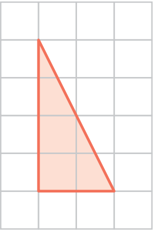
⬠ 4, 6(d–f), 7, 11–16

ANS p560

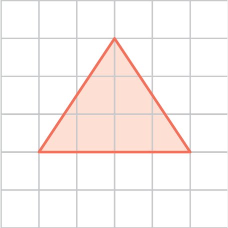
Exercise 9.5A: Understanding and fluency

1 Find the area of these triangles shown on 1-cm grid paper.

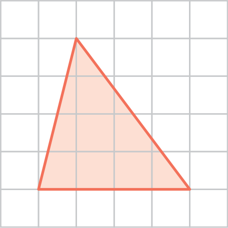
a



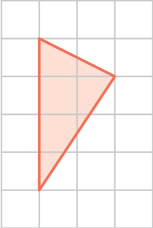
b



c



d



2 Find the area of the triangles with these dimensions.

a base 12 cm, height 9 cm

b base 24 mm, height 4 mm

c height 3 m, base 5 m

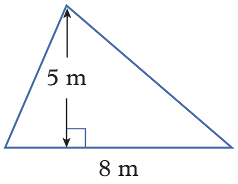
d base 8.6 cm, height 7 cm

e base 14.2 mm, height 9.7 mm

f height 15.4 m, base 17.03 m

3 State the base length and the height of each of these triangles.

a



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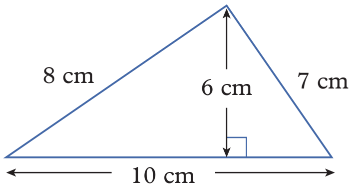
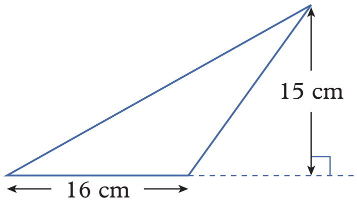
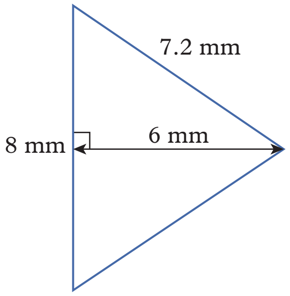


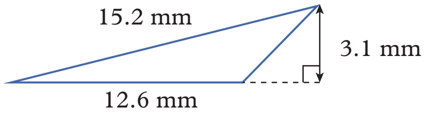
c



d



e



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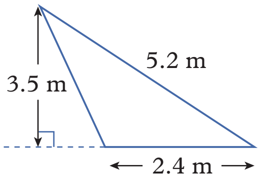
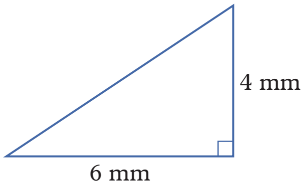


4 Calculate the area of each triangle.

a



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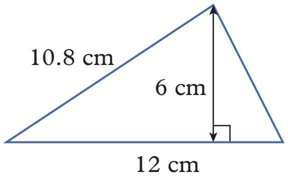
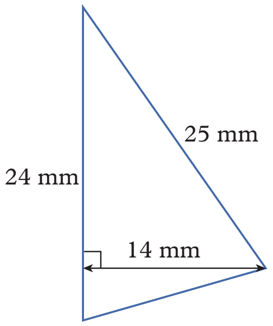


c



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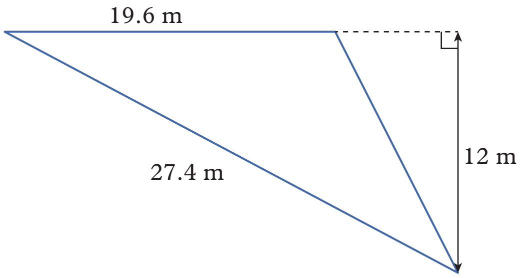
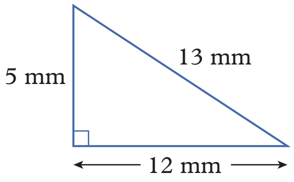


e



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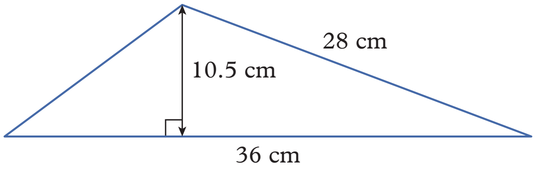
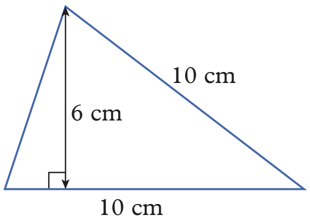
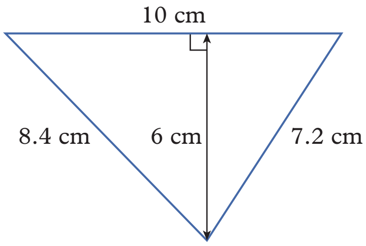


5 **a** Calculate the area of each of the following triangles.

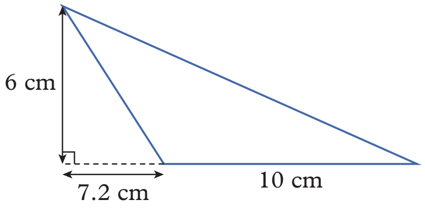
i



ii



iii



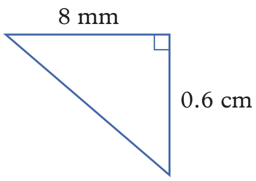
b Comment on your answers to part **a**.

6 Find the area of each triangle with these dimensions.

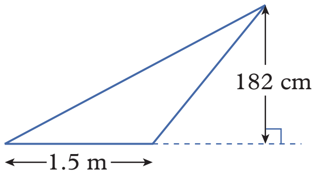
- **a** base 8 cm, height 30 mm
- **b** height 3 cm, base 16 mm
- **c** base 4 m, height 50 cm
- **d** base 310 cm, height 10 m
- **e** height 20 mm, base 0.7 cm
- **f** height 97 cm, base 2.8 m

WE 9.5A **7** Calculate the area of each triangle.

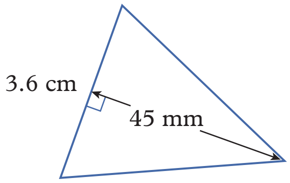
a



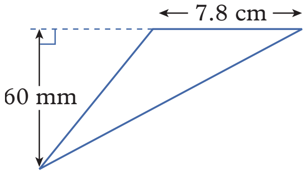
b



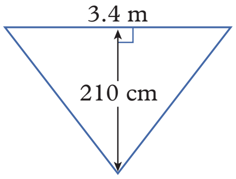
c



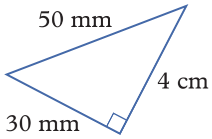
d



e

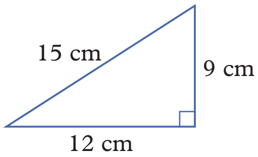


f



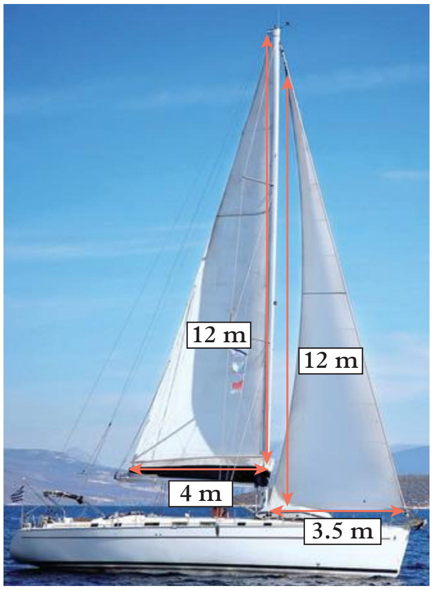
Exercise 9.5B: Problem solving and reasoning

8 Consider the triangle on the right.



- **a** A student has calculated the area of the triangle to be 90 cm^2. Explain the mistake that the student has made and find the correct area of the triangle.
- **b** Calculate the perimeter of the triangle.

9 Calculate the amount of sailcloth needed for the two triangular sails on the boat on the right.



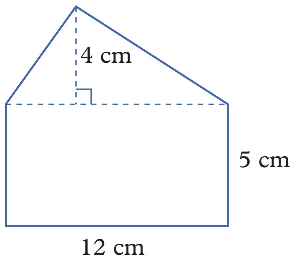
10 The area of a triangle is 20 cm^2. If its base is 10 cm long, what is the height of the triangle?

11 The area of a triangle is 400 mm^2. If its height is 2 cm, what is the base length in mm?

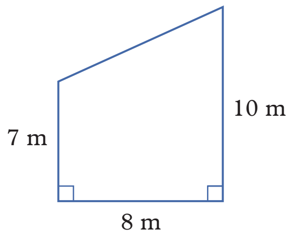
12 Use your understanding of how to find the area of a triangle to calculate the area of metal needed for the following stop sign.

13 A composite shape is made from two or more basic shapes. Calculate the area of each of the following composite shapes.

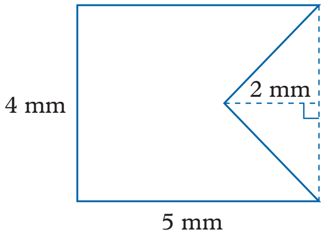
a



b



c



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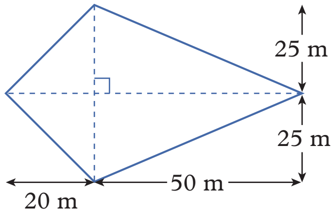

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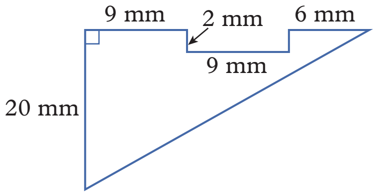

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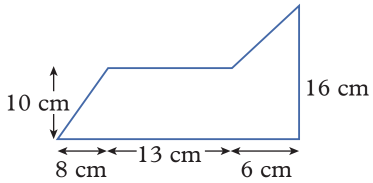
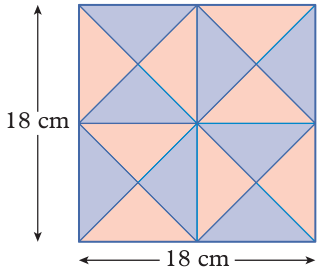
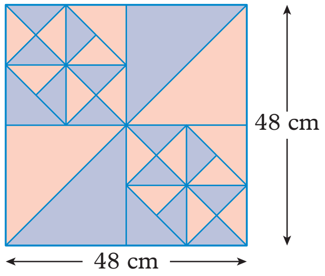


14 Calculate the total area that is shaded orange in each of the graphics below.

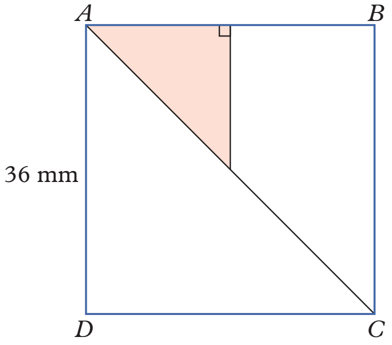
a



b

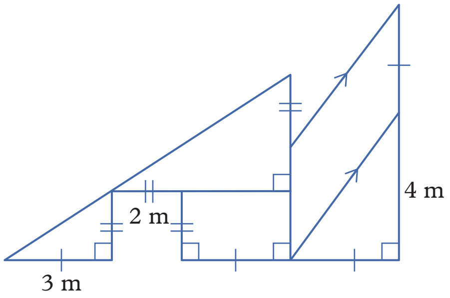


15 The square on the right has a side length of 36 mm. A vertical line has been drawn from the midpoint of side *AB* to meet the diagonal. What is the area of the shaded region?



Exercise 9.5C: Challenge

16 Find the area of this shape. Hint: Recreate the shape on square paper.



Online resources:

Interactive skillsheet
Area of a triangle

Worksheet
Area of a triangle

Investigation
Mighty mathematician, the superhero

Desmos activity
Exploring triangle area with Geoboards

Quick quiz
9.5

Lesson 9.6

Volume of right prisms

Learning intentions

By the end of this lesson you will be able to ...

✔ calculate the volume of right prisms.

Inter-year links

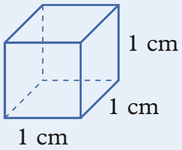
Support	Volume and capacity
Year 8	8.5 Volume and capacity
Year 9	5.5 Volume of prisms and cylinders

Volume

Key content video
Volume of prisms

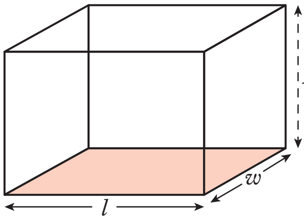
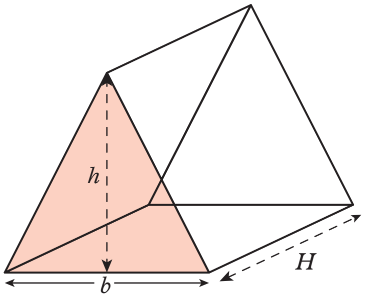
- The **volume** of a 3D object is the amount of three-dimensional space that it occupies.
- The metric units of volume include cubic millimetres (mm^3), cubic centimetres (cm^3) and cubic metres (m^3).

Metric units of volume	
Name	**Abbreviation**
cubic millimetre	mm^3
cubic centimetre	cm^3
cubic metre	m^3



- A prism is a 3D object with matching polygon ends that are the same shape and size. These matching ends are the base of the prism.
  - → Prisms have a consistent cross-section parallel to their base, throughout the height of the prism.
- A right prism is a prism where the base is perpendicular to the height.
  - → All of the prisms in this text are right prisms.
- The volume (V) of any prism can be found by multiplying the area of the base (A) by the height (H).

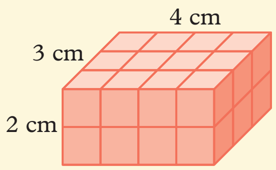
$$V = A \times H$$

Shape	Diagram	Formula
Rectangular prism		$V =$ (area of rectangle) × height of prism $=$ length × width × height of prism $= lwH$
Triangular prism		$V =$ (area of triangle) × height of prism $= \frac{1}{2} \times$ base of triangle × height of triangle × height of prism $= \frac{1}{2} bhH$

- All measurements must be in the same unit before they are substituted into the formula.

Worked example 9.6A Finding volume by counting cubes

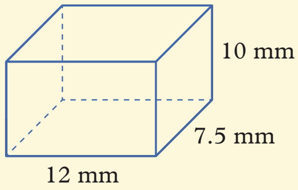
Find the volume of the rectangular prism.



THINK	**WRITE**
1 Work out the number of cubes in one layer of the prism.	$4 \times 3 = 12$ cubes in one layer of the prism
2 Calculate the volume of the rectangular prism by multiplying the volume of one layer by the number of layers in the prism. As each cube has a side length of 1 cm, the volume of one cube is 1 cm^3.	$12 \times 2 = 24$ cubes in the prism Volume of the rectangular prism = 24 cm^3

Worked example 9.6B Calculating the volume of a rectangular prism using the formula

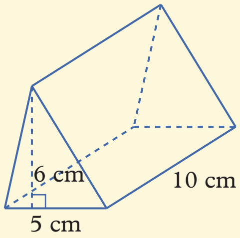
Calculate the volume of the rectangular prism.



THINK	**WRITE**
1 Identify the measurements for *l*, *w* and *H*. Check that each measurement is in the same unit.	$l = 12$ mm $w = 7.5$ mm $H = 10$ mm
2 Substitute the values for *l*, *w* and *H* into the formula.	$V = l \times w \times H$ $= 12 \times 7.5 \times 10$
3 Calculate the volume of the rectangular prism. Remember to include the appropriate unit in the answer.	$= 900$ mm^3

Worked example 9.6C Calculating volume of a triangular prism

Calculate the volume of the triangular prism.



THINK	**WRITE**
1 Identify the base of the prism. In this example the base is the triangular face. As we will be dealing with two heights, we will refer to them as *h* and *H*.	$b = 5$ cm, $h = 6$ cm
2 Calculate the area of the base, *A*. In this example use the formula for the area of a triangle, $A = \frac{1}{2}bh$.	$A = \frac{1}{2}bh$ $= \frac{1}{2} \times 5 \times 6$ $= 15\ \text{cm}^2$
3 Identify the value of the height of the prism and substitute the values for the area of the base and the height into the formula for the area of a prism.	$H = 10$ cm $V = AH$ $= 15 \times 10$
4 Calculate the volume of the triangular prism. Remember to include the appropriate unit in the answer.	$= 150\ \text{cm}^3$

Helpful hints

For rectangular prisms, any of the sides can be designated as the base, but remember that the height must be perpendicular (at right angles) to the base.

Learning pathways

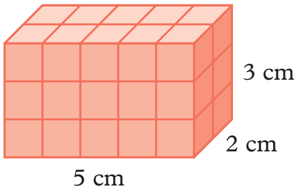
△ 1(a, b), 2–6, 8, 10, 11, 15, 18	■ 1(c, d), 2(b–e), 3(b, d), 4–11, 14–16, 18	⬠ 2(d–f), 3(d), 4, 5, 9, 12–19

ANS p560

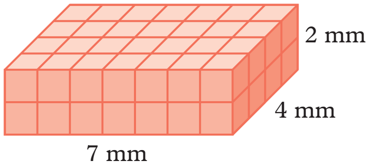
Exercise 9.6A: Understanding and fluency

WE 9.6A **1** Find the volume of each rectangular prism.

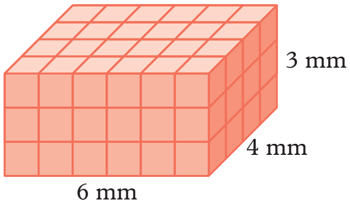
a



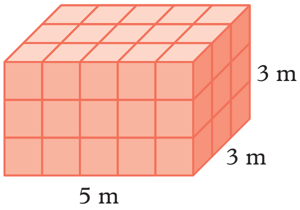
b



c

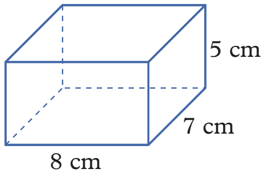


d

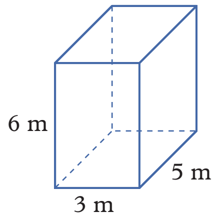


WE 9.6B **2** Calculate the volume of each rectangular prism.

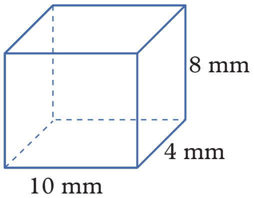
a



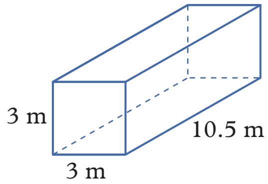
b



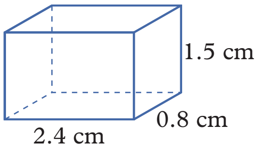
c



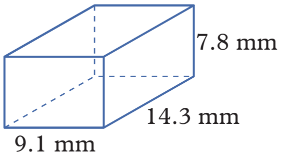
d



e

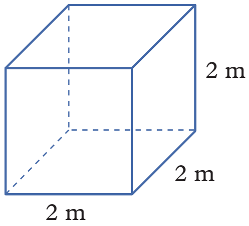


f



3 Calculate the volume of each cube.

a



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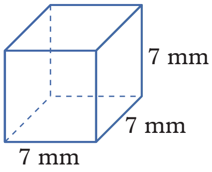

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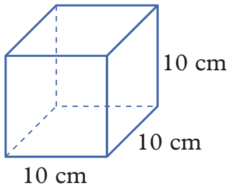
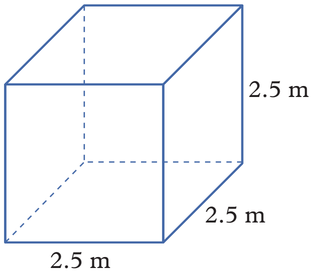
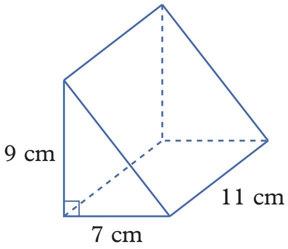


d

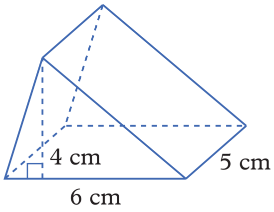


WE 9.6C **4** Calculate the volume of each triangular prism.

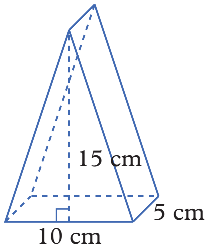
a



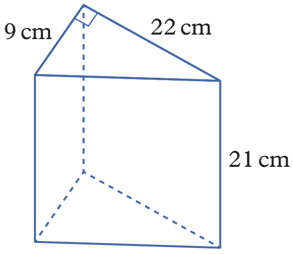
b



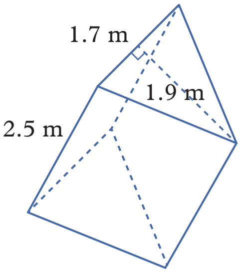
c



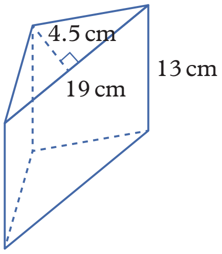
d



e



f



5 Calculate the volume of each prism.

- **a** rectangular prism 2.5 cm long, 4.5 cm wide and 6 cm high
- **b** triangular prism 9 mm high, 3.5 mm wide and with 12 mm base
- **c** rectangular prism 7 m long, 2 m wide and 11 m high
- **d** triangular prism 5.5 mm high, 8 mm wide and with 1.5 mm base
- **e** triangular prism 16 cm high, 5 cm wide and with 12 cm base
- **f** rectangular prism 11.5 mm long, 18 mm wide and 7 mm high

6 Calculate the volume of each object.

a

b

Exercise 9.6B: Problem solving and reasoning

7 A delivery of 3 m^3 of concrete is to be spread to form a driveway that is 7.5 m long and 3.2 m wide. How thick will the layer of concrete be?

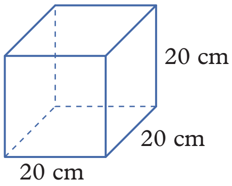
8 A prism-shaped office building has a floor area of 1200 m^2. If each floor of the building, including the ceiling, is 3.2 m in height, and there are 6 floors in the building, what is the volume of the building?

9 A sugar cube has the dimensions 1 cm by 1 cm by 1 cm.

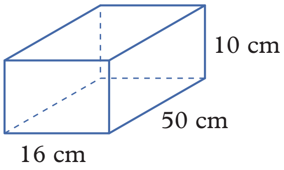
- **a** What is the volume of one sugar cube?
- **b** How many sugar cubes would fit into a box with dimensions 12 cm by 10 cm by 7 cm?

10 By calculating the volume of both of these boxes, demonstrate that differently shaped objects can have the same volume.

a

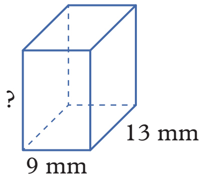


b



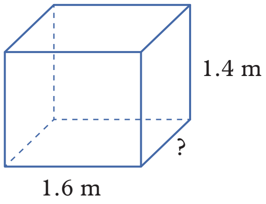
11 Find the missing length in these rectangular prisms.

a



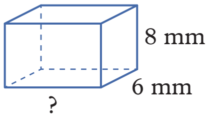
Volume = 1989 mm^3

b



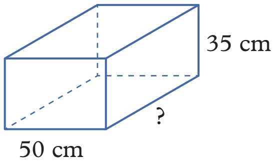
Volume = 2.912 m^3

c



Volume = 672 mm^3

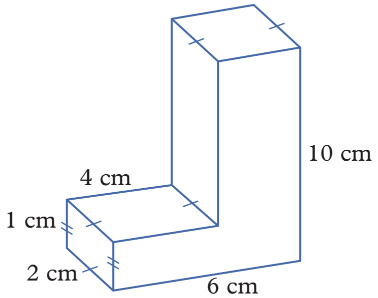
d



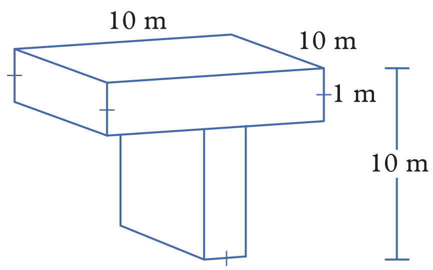
Volume = 0.14 m^3

12 Find the volume of these composite solids made up from rectangular prisms.

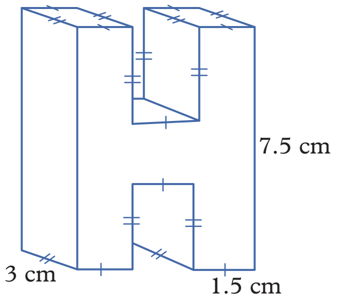
a



b



c



13 A new kind of insulation is to be fitted in the roof of a house. It is measured in cubic metres rather than square metres because it fills the entire roof (normally insulation is just a thick layer). The roof is in the form of a triangular prism. If the roof is 2 m high, 9 m wide and 14 m long, calculate:

a How much insulation is needed (in cubic metres)?

b How much it will cost if the insulation costs $30 per cubic metre?

14 A triangular prism has a height of 20 cm and a volume of 400 cm^3.

a What is the area of the triangular face?

b If the triangular face is 10 cm high, what is the base length?

15 The capacity of a 3D object is a measure of how much liquid that object can hold. A container with an internal volume of 1 cm^3 holds 1 mL of liquid, while a container with an internal volume of 1000 cm^3 holds 1 L of liquid.

a If a container has an internal volume of 235 cm^3, what is its capacity in mL?

b If a container has a capacity of 8.3 L, what is its internal volume?

16 A flower vase is in the shape of a rectangular prism. If the internal dimensions of the flower vase are 10 cm × 12 cm × 25 cm, what is the capacity of the flower vase in litres?

17 The mass of a solid object can be calculated by multiplying the volume by the density. If a solid steel cube has a side length of 6 cm and a density of 7.85 g/cm^3, what is the mass of the steel cube?

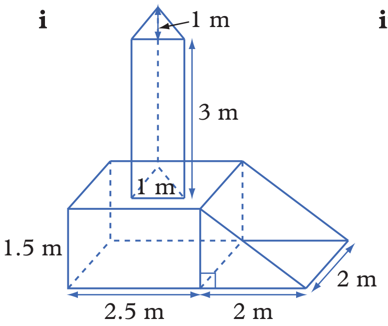
18 How many whole number sets of dimensions (in cm) are there for a rectangular prism with a volume of 20 cm^3? List them all.

Exercise 9.6C: Challenge

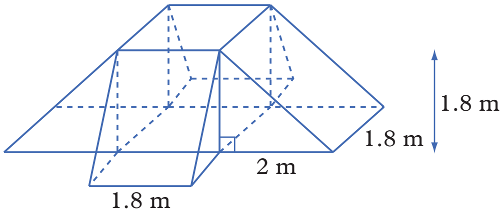
19 Andrew likes to make geometric sculptures.

a If his materials cost $2 per cubic metre, calculate the cost of the materials used in each sculpture.

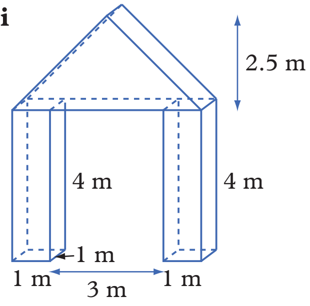
i



ii



iii



b Mel decides she wants to buy one of Andrew's pieces, and she wants to buy the lightest of the three sculptures. Use the following information to recommend which sculpture Mel should buy.

- Sculpture **i**: weighs 3 kg per cubic metre
- Sculpture **ii**: weighs 2 kg per cubic metre
- Sculpture **iii**: weighs 2.5 kg per cubic metre

Online resources:

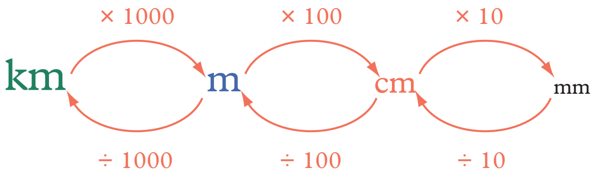
- **Interactive skillsheet** Volume of prisms
- **Worksheet** Volume of rectangular prisms
- **Worksheet** Volume of triangular prisms
- **Investigation** Volume of a building
- **Investigation** Fixed lengths and variable lengths
- **Quick quiz** 9.6

Lesson 9.7

Review: Length, area and volume

Module summary

Conversion factors

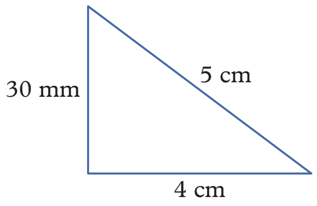


Metric units of length

1 cm = 10 mm
1 m = 100 cm
1 km = 1000 m

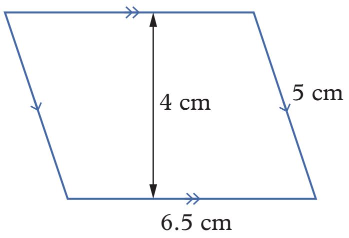
$5 \text{ km} = 5 \times 1000$
$= 5000 \text{ m}$

Perimeter



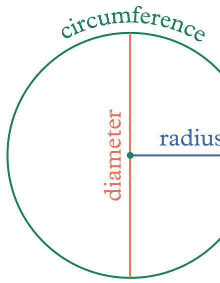
$P = 30 \text{ mm} + 4 \text{ cm} + 5 \text{ cm}$
$= 3 \text{ cm} + 4 \text{ cm} + 5 \text{ cm}$
$= 12 \text{ cm}$

Area of a parallelogram



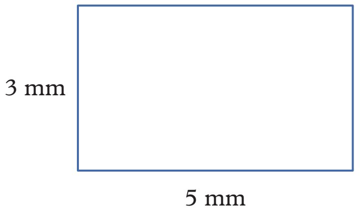
$A = bh$
$= 6.5 \times 4$
$= 26 \text{ cm}^2$

Circumference of a circle



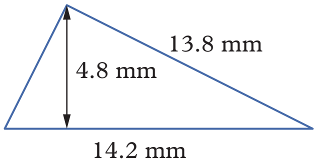
$C = \pi d$ or
$C = 2\pi r$

Area of a rectangle



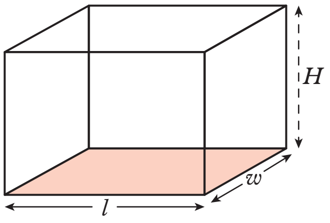
$A = lw$
$= 3 \times 5$
$= 15 \text{ mm}^2$

Area of a triangle



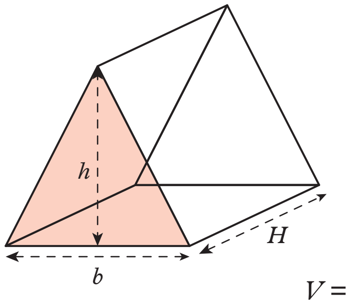
$A = \frac{1}{2}bh$
$= \frac{1}{2} \times 14.2 \times 4.8$
$= 34.08 \text{ mm}^2$

Volume of rectangular prism



$V = lwH$

Volume of triangular prism



$V = \frac{1}{2}bhH$

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Review questions 9.7A: Mathematical literacy review

Review quiz
Take the review quiz to assess your knowledge of this module.

Quizlet
Test your knowledge of this module by working individually or in teams.

The following key terms are used in this module:

- area
- base
- circle
- circumference
- conversion factor
- diameter
- height
- length
- metric units
- parallel
- parallelogram
- perimeter
- perpendicular
- pi (π)
- prism
- radius
- rectangle
- rectangular prism
- triangle
- triangular prism
- volume
- width

1 Explain how you can calculate the area of a parallelogram.

2 Explain how you can calculate the radius of a circle, if given the circumference.

3 Explain how you can convert a quantity in metres into kilometres.

4 Identify the key terms being referenced in each of these definitions.

a the length of a two-dimensional shape's boundary

b the amount of three-dimensional space a 3D object occupies

5 Using an example, provide a definition in your own words for the following key terms.

a circumference

b conversion factor

6 Complete the following sentences using words from the key terms list.

a The ________ of a circle is the length from the centre of the circle to any point on the circle.

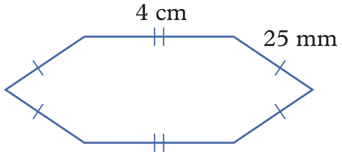
b The ________ of a triangle is the perpendicular distance from the ________ to the opposite vertex of the triangle.

Review questions 9.7B: Multiple choice

9.1 **1** Which length measurement is equivalent to 123 cm?

A 1230 mm **B** 12.3 m **C** 0.123 km **D** 12 300 m

9.1 **2** The perimeter of this shape is:



A 6.5 cm **B** 108 mm **C** 18 cm **D** 29 mm

9.2 **3** A circle has a diameter of 6 cm. What is its radius?

A 12 cm **B** 6π cm **C** 36π cm **D** 3 cm

9.3 **4** The area of a rectangle with length 9 cm and width 3 cm is:

A 6 cm^2 **B** 12 cm^2 **C** 24 cm^2 **D** 27 cm^2

9.3 **5** Which of these rectangles has the largest area?

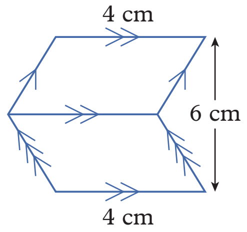
A length 75 mm width 45 mm

B length 11 mm width 6 cm

C length 19 mm width 4 cm

D length 17 mm width 75 mm

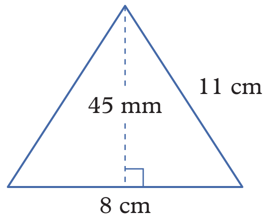
9.4 **6** The area of this shape is:



A 14 cm^2 **B** 24 cm^2 **C** 48 cm^2 **D** 144 cm^2

9.5 **7** The base length and height of this triangle respectively are:

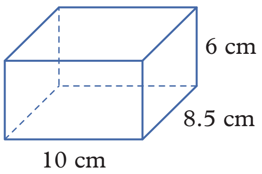
A 8 cm, 4.5 cm
B 8 cm, 11 cm
C 11 cm, 4.5 cm
D 4.5 cm, 8 cm



9.5 **8** What is the height of a triangle with a base length of 22 mm and an area of 6.16 cm^2?

A 5.6 mm **B** 5.6 cm **C** 27.104 cm **D** 56 cm

9.6 **9** What is the volume of this rectangular prism?



A 60 cm^3 **B** 85 cm^3 **C** 196 cm^3 **D** 510 cm^3

9.6 **10** What is the volume of a triangular prism with base area 2.5 cm^2 and length 10 cm?

A 2.5 cm^3 **B** 1.25 cm^3 **C** 25 cm^3 **D** 5 cm^3

Review questions 9.7C: Short answer

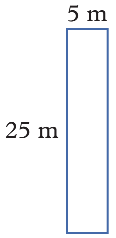
9.1 **1** Convert these units.

a 34 cm into mm **b** 170 cm into m

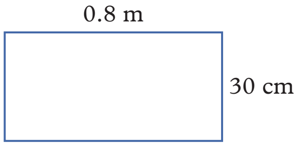
c 45 mm into cm **d** 8.7 km into m

9.1 **2** Calculate the perimeter of each rectangle.

a

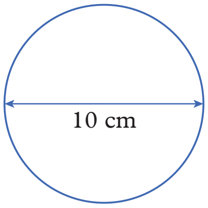


b

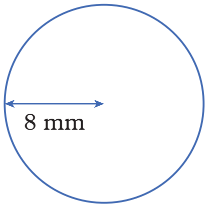


9.2 **3** Calculate the circumference of each circle, correct to two decimal places.

a

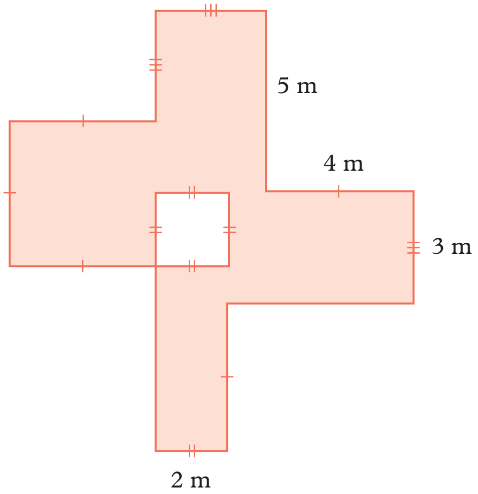


b



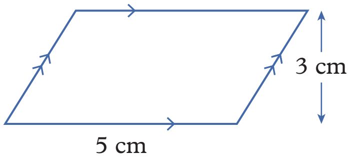
9.3 **4** Calculate the area of each rectangle shown in question **2**.

9.3 **5** Calculate the shaded area of this composite shape.

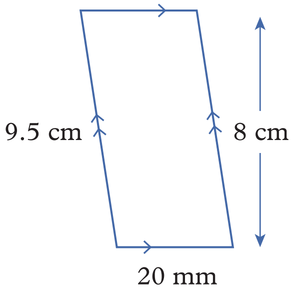


9.4 **6** Calculate the area of each parallelogram.

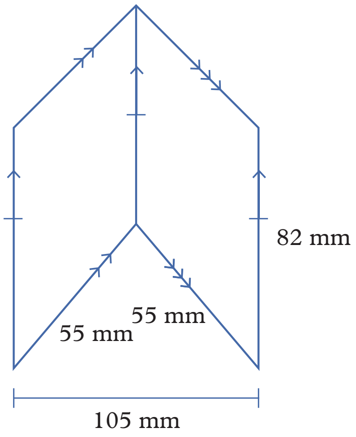
a



b

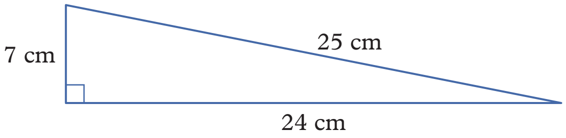


9.4 **7** Calculate the area of this composite shape.

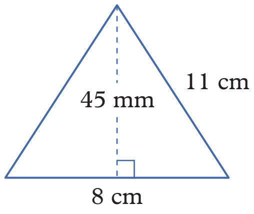


9.5 **8** Calculate the area of each triangle.

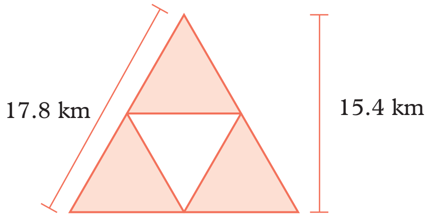
a



b

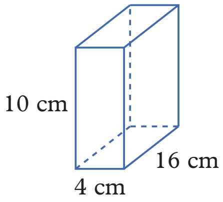


9.6 **9** Calculate the shaded area of this composite shape consisting of equilateral triangles.

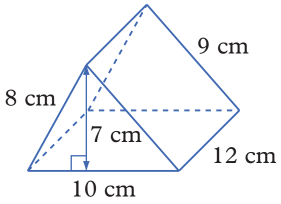


9.6 **10** Calculate the volume of each object.

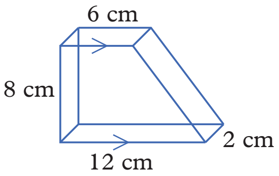
a



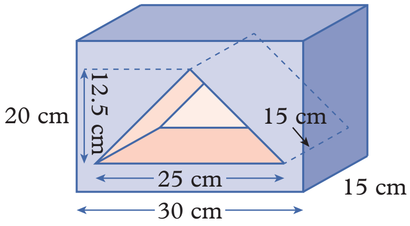
b



c

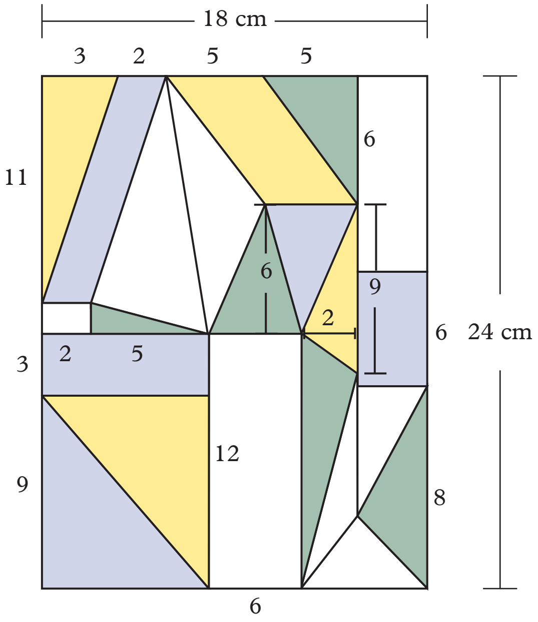


9.6 **11** Calculate the volume of clay in this brick.



Review questions 9.7D: Mathematical modelling

1 Gertrude is crafting a geometric painting using yellow and blue paints, which she mixes to make a green paint. She plans to use an 18 cm × 24 cm canvas. Her design is shown on the right. Note: All lengths are in cm.



- **a** Calculate the total yellow area.
- **b** Calculate the total blue area.
- **c** Calculate the total green area.
- **d** Calculate the total unpainted (white) area.
- **e** Gertrude bought 0.25 litres of yellow paint and half a litre of blue paint. How much paint does she have in cubic centimetres?
- **f** If the yellow and blue paint is mixed in the ratio 1 : 2 to make green paint, how much yellow and blue paint does she mix to make 0.45 L of green paint?
- **g** Gertrude gets her painting framed with a 2 cm wide frame. What is the perimeter of the framed painting?

2 This question considers a famous decimal number.

a Evaluate the following calculation.

$$\frac{4}{1} - \frac{4}{3} + \frac{4}{5} - \frac{4}{7} + \frac{4}{9}$$

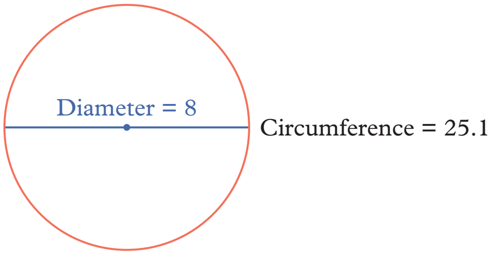
b Write the result from part **a** as a decimal, correct to two decimal places.

c The calculation in part **a** is part of a pattern. Explain how the pattern works.

d Determine the next two terms in the calculation if it continues to follow the pattern.

e Calculate the new result including the two extra terms from part **d**. Write your answer as a decimal and round your result to two decimal places.

f Consider the following circle.



i Divide the circumference by your result from part **e**. Round your result to two decimal places.

ii Multiply the diameter by your result from part **e**. Round your result to two decimal places.

iii Describe the relationship between the number from part **e** and the diameter and circumference of a circle.

iv Explain what would happen to the relationship between the circumference and the diameter if more terms were added to the calculation in part **a** and then the new value was used instead.

Checklist

Now that you have completed this module, reflect on your ability to do the following.

I can do this	I need to review this
☐ Convert between metric units of lengths ☐ Calculate the perimeter of 2D shapes	☐ Go back to Lesson 9.1 Length and perimeter
☐ Calculate the circumference of a circle ☐ Find the radius and diameter, given the circumference of a circle	☐ Go back to Lesson 9.2 Circumference of a circle
☐ Calculate the area of a rectangle using length and width	☐ Go back to Lesson 9.3 Area of a rectangle
☐ Calculate the area of a parallelogram using the formula	☐ Go back to Lesson 9.4 Area of a parallelogram
☐ Calculate the area of a triangle using the base and height	☐ Go back to Lesson 9.5 Area of a triangle
☐ Calculate the volume of rectangular and triangular prisms	☐ Go back to Lesson 9.6 Volume of right prisms

Computational Thinking

Curriculum link

Design algorithms involving a sequence of steps and decisions that will sort and classify sets of shapes according to their attributes, and describe how the algorithms work (VC2M7SP04)

Inter-year links

Year 8	Binary numbers
Year 8	Geometric proofs
Year 9	Triangles
Year 10	Solving non-linear equations with digital tools
Year 10	Graph algorithms

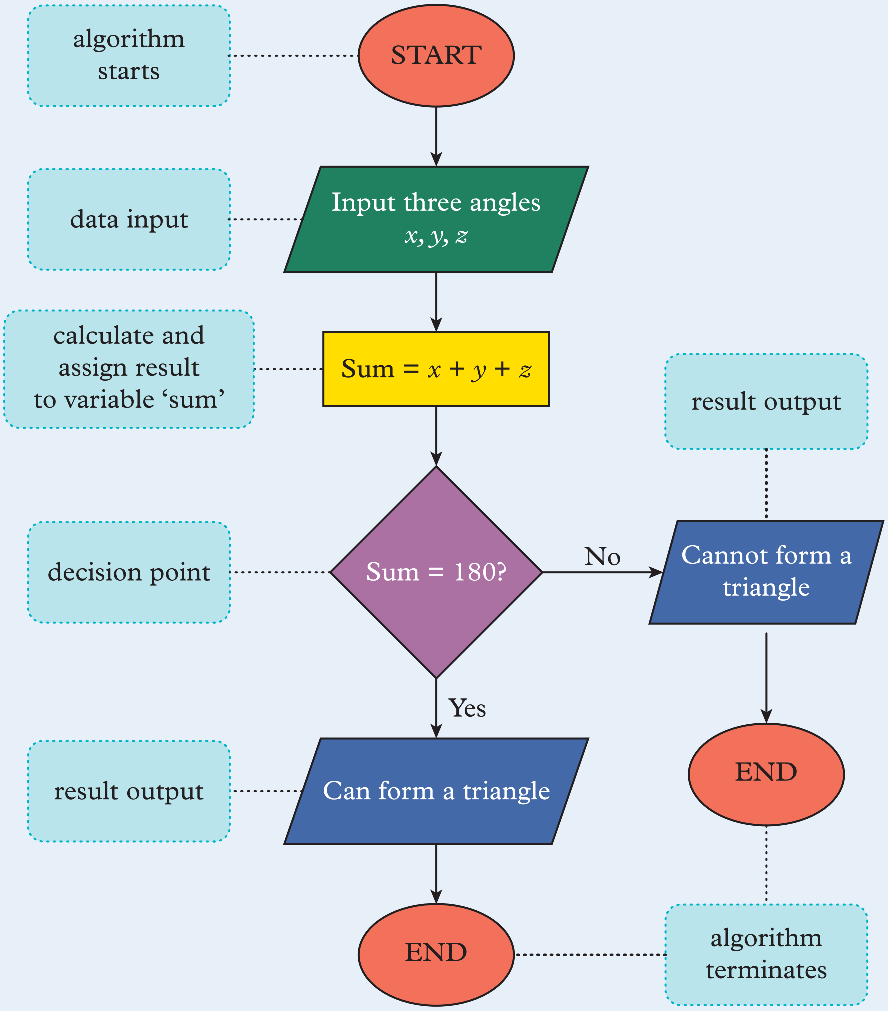
Flow charts

A flow chart is a graphical representation of a logical flow of procedures, which can be called an algorithm. Shapes and colours are used to represent different instructions and decisions.

The basic elements of a flow chart can include:

- the start and end of the flow chart
- arrows which show the order steps should be completed in
- data input
- results output
- an assignment, calculation or comparison operation
- a decision point that determines the next step to follow in the flow chart.

This flow chart is an algorithm that determines if three angles form the interior angles of a triangle and includes the basic elements of a flow chart.



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Check your understanding

1 Answer the following questions.

a How many of each of the following are there in the flow chart?

i required inputs **ii** possible outputs **iii** decision points

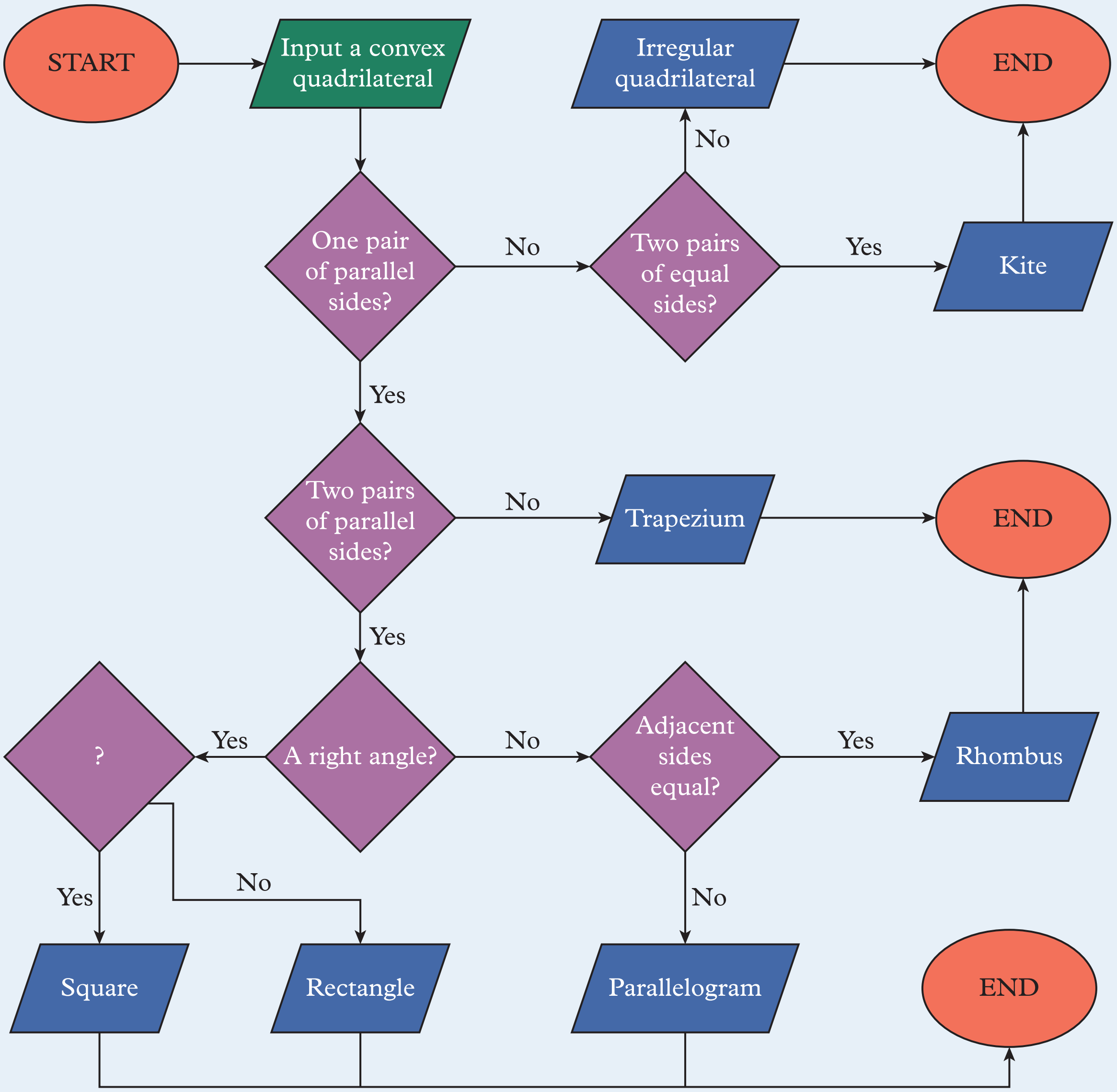
b What is the result of the flow chart for inputs of three angles 20°, 30°, 40°?

c What is the result of the flow chart for inputs of three angles 105°, 15°, 60°?

d Describe the test being applied to the input angles.

Classifying quadrilaterals

Recall the definitions and properties of quadrilaterals from Lesson 7.5. This flow chart can be used to classify quadrilaterals using their side and angle properties, but it is missing some information.

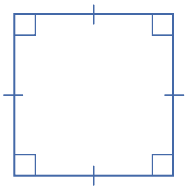


Check your understanding

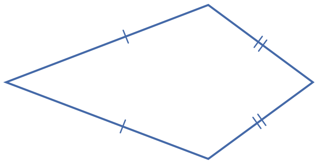
2 Answer the following questions.

- **a** What are the possible outputs of the flow chart?
- **b** Fill in a test (indicated by '?' above) that could be used to distinguish between a rectangle and a square.
- **c** Use the flow chart to classify the following quadrilaterals. Write the sequence of decisions as you go through the steps in the flow chart.

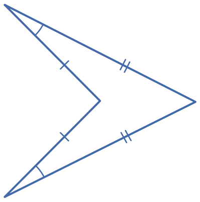
i



ii



iii



Task 1

The previous flow chart classifies convex quadrilaterals by having decision points first ask about side properties.

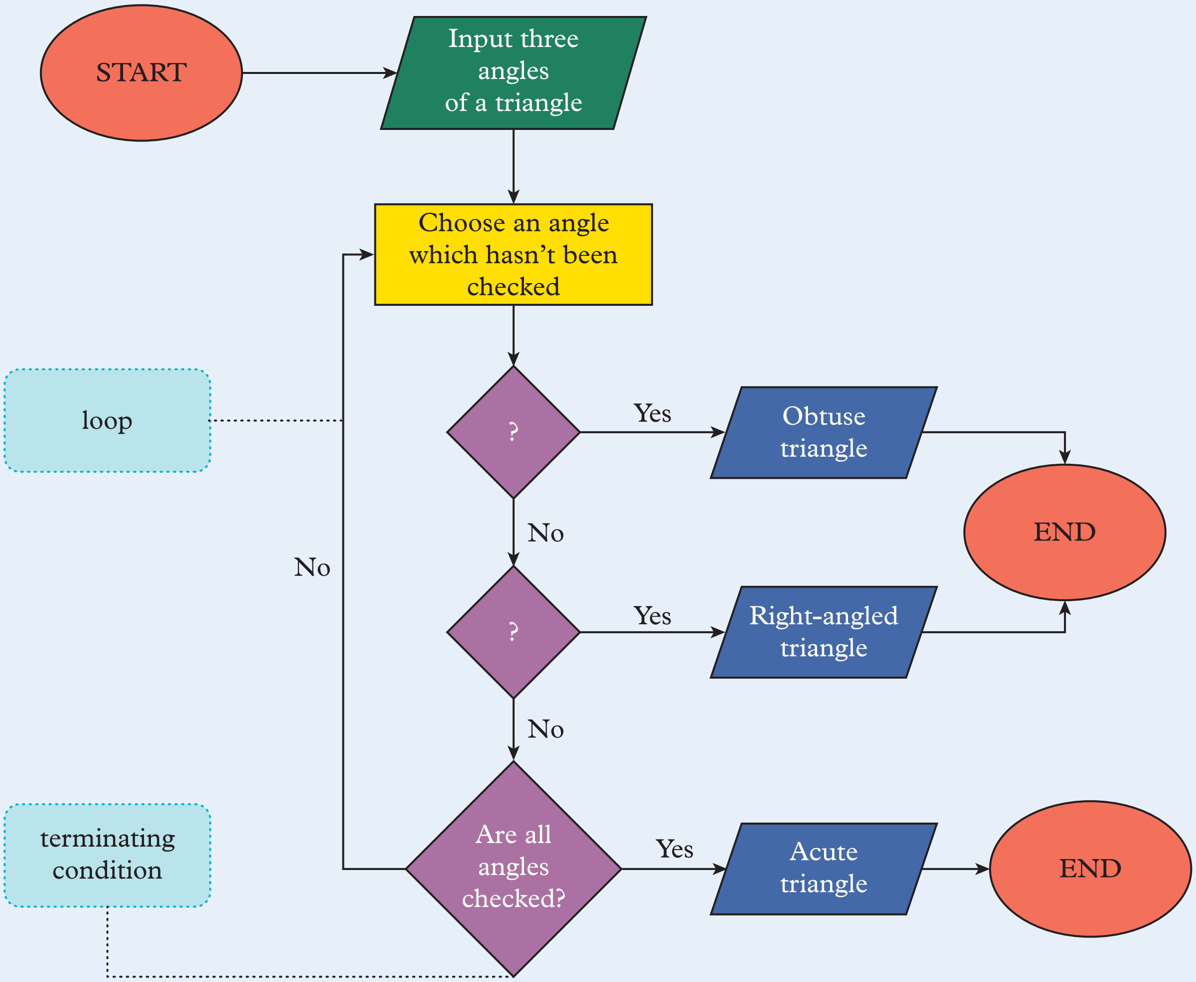
1. Create a similar flow chart which classifies convex quadrilaterals first by their angle properties.
2. How could you adapt your flow chart to input any quadrilateral, and then output that the quadrilateral is concave or, if it is convex, to classify it as before?

Loops

In an algorithm, some of the steps may need to be repeatedly executed until a terminating condition is met. This is called a loop. The terminating condition could be:

- when the number of iterations required is completed
- a specified value constrained on a variable is met
- the end of a flow chart where a conclusion is reached.

The flow chart below classifies a triangle as acute, right-angled or obtuse, based on the sizes of its interior angles. It includes a loop, indicated by the 'No' arrow which takes you back to an earlier decision point. Each iteration of the loop checks one of the input angles. The loop continues until all angles are checked. The decision point 'Are all angles checked?' is the terminating condition of the loop.



Check your understanding

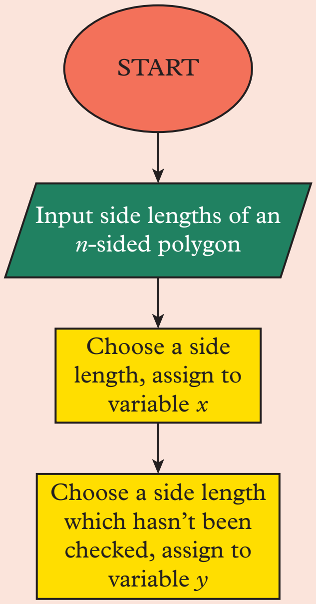
3 Answer the following questions.

- **a** Complete the two unfinished decision points (indicated by '?') with suitable tests.
- **b** Use this flow chart to classify triangles with the following internal angles. Write all steps as you follow the flow chart.
  - **i** 30°, 90°, 60°
  - **ii** 30°, 20°, 130°
  - **iii** 70°, 50°, 60°
- **c** What is the maximum number of times you go through the loop in the flow chart? Explain why this is the maximum.

Task 2

A regular polygon has all sides equal in length and all internal angles equal in size.

1. Create a flow chart to classify whether an *n*-sided polygon is a regular polygon based only on its side lengths. Ensure your flow chart includes a loop. The first part of the flow chart is provided here for you.
2. Use your flow chart to classify a polygon with side lengths 4, 4, 2 and 4.
3. Use your flow chart to classify a polygon with side lengths 5, 5, 5, 5 and 6.
4. What is the maximum number of times you go through the loop in your flow chart? Which polygons require the maximum number of loops?
5. Explain why it is not possible to use only the side lengths of a polygon to determine whether it is regular.
6. Add steps to your flow chart so that it accurately determines whether a polygon is regular.



Extension

Pseudocode is another way to represent an algorithm without using a particular coding language. Specific terms and formatting are used to represent the steps in the algorithm. The pseudocode on the right represents the same algorithm as one of the flow charts you have already considered.

```
Line number   Pseudocode
1             input angles $x$, $y$, $z$
2                 Sum ← $x + y + z$
3                 if Sum = 180 then
4                     print Can form a triangle
5                 else
6                     print Cannot form a triangle
7                 end if
```

1. Identify the flow chart this pseudocode corresponds to and redraw it in your workbook.
2. Label the line number of the pseudocode next to the corresponding component in the flow chart.
3. Describe the role of each of the terms in bold. Identify their corresponding elements in the flow chart.

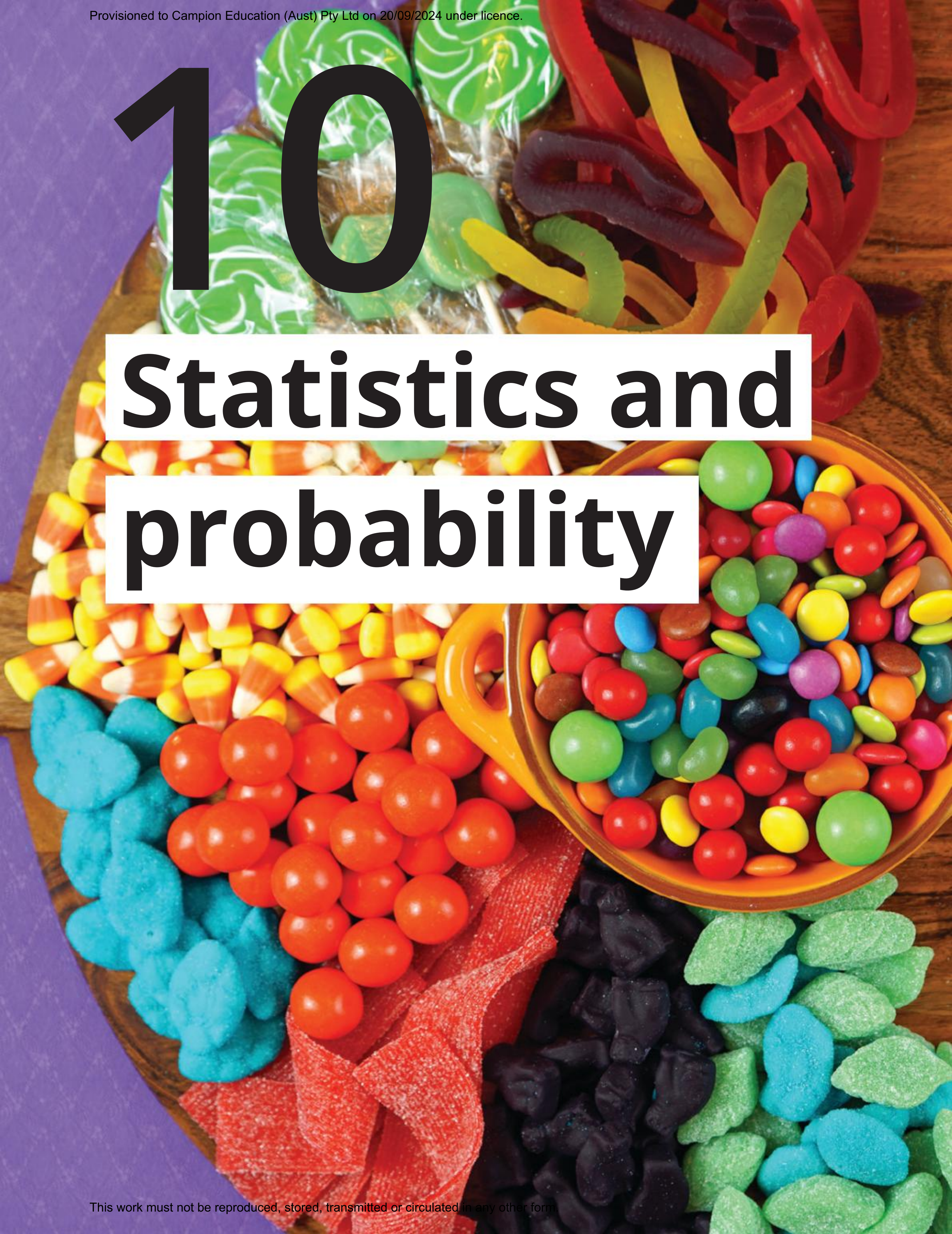
The pseudocode above can be modified to determine whether a triangle formed is obtuse, right angled or acute, by adding more lines of instructions. The following lines can be added to first check whether a triangle is obtuse.

```
if $x > 90$ or $y > 90$ or $z > 90$ then
    print Triangle is obtuse
```

4. What is the role of the bold term **or**? What feature does this replace in a flow chart you have already considered?
5. Copy the pseudocode into your workbook and add the extra lines of code. Don't forget to number the new lines.
6. Can you extend the algorithm further to classify whether the triangle is scalene, isosceles or equilateral?

10

Statistics and probability



Lessons

Prerequisite skills

Diagnostic pre-test
Take the diagnostic pre-test to assess your knowledge of the prerequisite skills listed below.

Interactive skillsheets
After completing the diagnostic pre-text, brush up on your knowledge of the prerequisite skills by using the interactive skillsheets.

- ✔ Ordering and comparing large numbers
- ✔ Substitution
- ✔ Fractions, decimals and percentages
- ✔ Equivalent fractions

Curriculum links

- Acquire data sets for discrete and continuous numerical variables and calculate the range, median, mean and mode … (VC2M7ST01)
- Create different types of displays of numerical data, including dot plots and stem-and-leaf plots, using software where appropriate; describe and compare the distribution of data … (VC2M7ST02)
- Plan and conduct statistical investigations for issues involving discrete and continuous numerical data, and data collected from primary and secondary sources … (VC2M7ST03)
- Identify the sample space for single-stage experiments; assign probabilities to the possible outcomes and predict relative frequencies for related experiments (VC2M7P01)
- Conduct repeated chance experiments and run simulations with a large number of trials using digital tools; compare predicted with observed results … (VC2M7P02)

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Materials

✔ Ruler ✔ Protractor ✔ Calculator

Lesson 10.1

Summary statistics

Learning intentions

By the end of this lesson you will be able to ...

✔ classify numerical data, calculate and interpret the summary statistics of a data set.

Inter-year links

Support	Understanding data
Year 8	10.2 Summary statistics
Year 9	7.4 Summary statistics from tables and displays

Key content video
Calculating summary statistics

Numerical data

- **Numerical data** can be counted or measured. There are two types of numerical data: discrete and continuous.
  - → **Discrete data** can be counted; for example, the number of fish in an aquarium.
  - → **Continuous data** can be measured; for example, the time it takes to listen to a song.

Summary statistics

- **Summary statistics** provide an overview of numerical data and can be used to help make sense of the data.
- The **mean** is a numerical average of the values in a data set. To find the mean, add all the values together, then divide by how many values are in the set.

  For example, for the data set 1, 2, 3, 4, 6:

$$\begin{aligned}\text{Mean} &= \frac{\text{sum of all data values}}{\text{total number of data values}}\\ &= \frac{1+2+3+4+6}{5}\\ &= 3.2\end{aligned}$$

- The **median** is the middle of a data set when the values are listed in order. The median divides a data set in half.
  - → If there is an even number of values in a data set, the median will be the average of the two middle values.

  For example,

$$2,\ 3,\ 4,\ 5,\ 6 \qquad\qquad 1,\ 2,\ 3,\ 4,\ 5,\ 6$$

$$\text{Median} = 4 \qquad \begin{aligned}\text{Median} &= \frac{3+4}{2}\\ &= 3.5\end{aligned}$$

- The **mode** is the most common value in a numerical data set. There can be more than one mode in a data set, and if there are no repeated values, there is no mode.

  For example,

1,2,3,4,4,5	1,2,2,3,4,4,5	1,2,3,4,5,6
Mode = 4	Mode = 2,4	No mode

- The mean, median and mode are measures of centre, as they represent a central point in a data set.
- The **range** is the difference between the largest (maximum) and smallest (minimum) values in a data set.

  Range = maximum value – minimum value
- An **outlier** is an extreme data point that is much higher or lower than the other values in a data set. For example, 100 is an outlier in the data set 1, 2, 3, 3, 100.

Worked example 10.1A Classifying numerical data

Classify the following numerical data as either discrete or continuous.

a The heights of a class of 26 Year 7 students in cm.

b The number of pens in your pencil case.

THINK

a Consider whether the data can be counted or measured. Heights can be measured, so this is continuous data.

b Consider whether the data can be counted or measured. The number of pens can be counted, so this is discrete data.

WRITE

a Heights of students are continuous data.

b The number of pens is discrete data.

Worked example 10.1B Calculating summary statistics

The lengths in cm of a school of fish are listed below.

6, 7, 13, 7, 12, 7, 8, 10, 11

a Find the mean. **b** Find the mode. **c** Find the median. **d** Find the range.

e A fish of length 15 cm is added to the data set. What is the new median?

THINK	**WRITE**
a Add the numbers together and divide by the total number of values.	**a** $\text{Mean} = \frac{\text{sum of all data points}}{\text{total number of data points}}$ $= \frac{6 + 7 + 13 + 7 + 12 + 7 + 8 + 10 + 11}{9}$ $= \frac{81}{9}$ $= 9$
b List the numbers in the data set in ascending order and identify the number which occurs most often.	**b** 6, 7, 7, 7, 8, 10, 11, 12, 13 Mode = 7
c List the numbers in the data set in ascending order and find the middle value.	**c** 6, 7, 7, 7, 8, 10, 11, 12, 13 Median = 8
d Identify the minimum and maximum values of the data set. Subtract the minimum value from the maximum value.	**d** Range = maximum value – minimum value = 13 – 6 = 7
e List the numbers in the data set in ascending order and find the middle value. As there are two middle values, add them together and divide the total by 2.	**e** 6, 7, 7, 7, 8, 10, 11, 12, 13, 15 $\text{Median} = \frac{8 + 10}{2}$ $= 9$

Helpful hints

- ✔ Order data sets first before calculating the median and mode. This will decrease the likelihood of errors!
- ✔ When using a calculator to find the mean, find the value of the total first before dividing. You can use brackets around the addition to ensure the sum is divided, not just the last number.
- ✔ A good method for making sure all data points are included when you re-order a data set is to put a dot above or below the values as you list them, and to count the number of data points at the end.

  For example,

  $\dot{4}$, 5, $\dot{2}$, 7, $\dot{1}$, 5

  1, 2, 4, _, _, _

Learning pathways

▲ 1–3, 4(a, c, e, g), 5, 6, 8, 9, 12, 14

■ 1, 3, 4(b, d, g, h), 5, 7, 9–11, 13, 16, 17, 21(a)

⬟ 1, 4(d, g, h), 5(c, d), 6, 9, 10, 15–18, 20, 21(b), 22

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Exercise 10.1A: Understanding and fluency

WE 10.1A **1** Classify the following numerical data as either discrete or continuous.

- **a** Weight
- **b** Time spent on the internet per week
- **c** Population
- **d** Hand span
- **e** Number of runs scored in a game of cricket
- **f** Number of spots on a leopard

WE 10.1B **2** A zoologist collected the following data on the number of young in the litters of several fat-tailed dunnarts (small mouse-like marsupials).

7 8 6 7 6 7 9 8 15 7 8

- **a** Find the mean.
- **b** Find the mode.
- **c** Find the median.
- **d** Find the range.
- **e** A new litter with 10 young is added to the data set. What is the new median?

3 Information was collected on the number of online games students play in a normal week. The results are below.

0 4 2 1 1 3 2 1 0 0 2 1 0
1 0 2 1 1 0 0 2 1 1 1 0 0

- **a** Find the mean, correct to two decimal places.
- **b** Find the median.
- **c** Find the mode.
- **d** Find the range.

4 Find the **i** mean, **ii** median, **iii** mode and **iv** range for each data set. Round your answers to two decimal places where appropriate.

- **a** 4, 8, 2, 5, 3, 7, 5, 7, 7, 7, 1, 2, 6
- **b** 1, 1, 1, 2, 5, 6, 8, 4, 5, 8, 6, 9, 6, 4
- **c** 1, 2, 3, 4, 5, 6, 7, 8, 9, 10
- **d** 4, 7, 5, 8, 3, 6, 9, 9, 1, 0, 3, 5, 4, 7, 2
- **e** 4, 8, 3, 7, 11, 15, 4, 9, 5
- **f** 17, 13, 11, 7, 5, 3, 2
- **g** 11, 17, 13, 9, 13, 5, 18, 12
- **h** 8, 5, 8, 6, 8, 1, 93, 5, 5, 5, 8

5 What will be the position of the median in an ordered set of data containing:

a 7 data points **b** 8 data points **c** 13 data points **d** 22 data points?

6 Match each summary statistic to the correct data set.

a Mean = 9	**I** 15, 8, 4, 7, 3, 10, 1
b Mode = 4	**II** 11, 4, 6, 8, 3
c Median = 7	**III** 3, 17, 8, 6, 14, 8, 7
d Range = 8	**IV** 1, 7, 4, 5, 9, 4, 2, 3, 3, 4, 10

Exercise 10.1B: Problem solving and reasoning

7 A bag of 8 tangerines weighs 720 g.

a What is the mean weight of the tangerines in the bag?

b A tangerine is added to the bag. State whether the mean increases, decreases, or stays the same if the tangerine weighs:

i 120 g **ii** 90 g **iii** 60 g **iv** 91 g.

8 Dorothy measured the heights of all her classmates and calculated the mean, median, mode and range of the heights. To write up her report, she needs to interpret the statistics by writing them in a sentence that describes what the statistics mean. State the statistic each statement is interpreting.

a The heights of my classmates were spread out over a 50 cm interval.

b The most common height was 150 cm.

c Half of the class had heights greater than 155 cm.

d The numerical average height of my classmates is 157 cm.

9 In a science experiment, different model carts are built to see how far they will move after the same force is applied to them. The following statistics were obtained.

Mean	Median	Mode	Range
6.4 m	6.2 m	4.0 m	10.1 m

Write a sentence to interpret each of the statistics.

10 Consider these two ordered data sets.

Set A: 8, 10, 12, 12, 12, 13, 14, 15, 18, 20

Set B: 3, 6, 8, 12, 12, 13, 17, 18, 21, 24

a Calculate the **i** mean, **ii** median and **iii** mode for each data set.

b Comment on your answer to part **a**.

11 A group of Year 7 students were asked how much money they spend on their mobile phones every month. The results, in dollars, are:

20, 50, 30, 10, 30, 20, 20, 15, 20, 30, 20, 20, 25.

a Find the mean, median, mode and range of the data set.

b Use your answers from part **a** to write a three-sentence description of the data.

12 After finishing the survey in question **11**, students were then asked how much pocket money they earned per week. The results, in dollars, were:

5, 5, 10, 20, 5, 5, 0, 5, 0, 5, 10, 5, 100.

a Find the mean, median and mode of the data set.

b What is the difference between the mean and median?

c Explain why the mean is not a good indicator of the centre of this data set.

d Would any of these students be able to afford to pay for their phone bill from question **11** using just their pocket money? Explain.

13 A company claims that the average salary of their employees is \$80 000. One of the employees, who earns much lower than this, seems suspicious of the average and decides to ask some of her colleagues what they earn. She gathers the following salaries, including her own.

\$42 000, \$51 000, \$39 000, \$40 000, \$45 000, \$60 000, \$25 000, \$33 000

a Calculate the mean and median salary of this sample of employees.

b Based on your answer from part **a**, do you think the claim that the average salary is \$80 000 is reasonable? Why or why not?

c It turns out that the chief executive officer and chief financial officer earn much more than the rest of the company.

- **i** Which average (mean or median) was probably used to calculate the \$80 000 figure? Why?
- **ii** Which summary statistic would be a better measure of the average salary? Why?

14 At the end of the year, two students received their exam scores for each of their subjects.

Yolanda's scores: 40%, 41%, 43%, 44%, 42%, 95%

Zoe's scores: 99%, 97%, 98%, 96%, 94%, 42%

Their parents asked how they went on average this year.

a To one decimal place, calculate the mean and median score:

- **i** Yolanda received
- **ii** Zoe received.

b To make themselves look better, which average (mean or median) should:

- **i** Yolanda use
- **ii** Zoe use?

c To give a more representative statistic, which average (mean or median) should:

- **i** Yolanda use
- **ii** Zoe use?

15 Two friends were comparing the delays on their train lines. Dylan argued that his train was worse; his train was once delayed by 20 minutes. Costa argued that his train was worse; his train is always delayed. They decided to record how many minutes late their trains were for two weeks. The results are below.

Dylan: 5 3 1 4 2 2 5 15 20 3 2 4 2 3

Costa: 4 6 8 5 6 7 7 9 8 8 7 9 9 9

a Whose train line had the largest range of delays? What was the range of this line's delays?

b Calculate the mean delay for each train, correct to two decimal places.

c Whose train do you think is more reliable? Justify your decision using the results from parts **a** and **b**.

16 Three zoologists examined 15 cheetahs to determine how many spots each cheetah had on its coat. The results of their survey are listed below. One zoologist claimed that the average number of spots is 466, another calculated the average of the data set to be 456, and the third said that it is 472. Use your knowledge of statistics to discuss why all three are correct.

450 495 413 456 517 472 436 485 476 499 486 444 456 424 481

17 Consider the following data set.

6 8 10 11 12 13

What number, when added to this data set, would make the mode, mean and median equal to each other?

18 Two local footballers were competing to be the leading goalkicker in the Central Suburbs Football League.

The number of goals each player scored in the 15 games of the season is listed below.

Player 1: 3, 5, 3, 2, 6, 4, 3, 1, 4, 4, 5, 3, 3, 2, 3

Player 2: 1, 3, 2, 4, 1, 4, 3, 1, 13, 4, 3, 5, 1, 2, 5

a Which player won the leading goalkicker award by kicking the most goals in the season?

b Find the mean, median and mode number of goals kicked by each player. Give your answers to two decimal places where necessary.

c Which player was the more consistent goalkicker? Explain your answer.

d There is an outlier in the number of goals Player 2 kicked. Identify the outlier and explain why this value is an outlier.

e Recalculate the mean and median number of goals kicked for Player 2 with the outlier removed from the data set. Give your answers to two decimal places where necessary.

f Considering your answers to parts **b** and **e**, discuss how the outlier affected the mean and median values.

19 A recent census found there is an average of 1.97 children in every Australian family.

a Which summary statistic do you think they used to get this number? Why?

b Braeden is confused as to how a family can have 1.97 children and how it is possible that every family has the same number of children. Explain the meaning of this average to Braeden.

Exercise 10.1C: Challenge

20 There are five values in a data set. They are all different positive integers. The median and the mean are both 4.

a Find all potential data sets that meet these criteria.

b Explain how you can be sure that you have found all potential data sets.

21 Find the **i** mean, **ii** median, **iii** mode, and **iv** range for each data set. Round your answers to two decimal places where appropriate.

a 7.4, 6.8, 9.2, 11.3, 8.1, 6.8, 4.6, 13.3, 10.5, 5.3

b 22.4, 28.0, 33.3, 19.7, 38.5, 42.9, 29.5, 21.3, 20.8, 16.4, 45.2, 51.1

22 Find the **i** mean, **ii** median, **iii** mode, and **iv** range for each data set.

a

Score	4	5	6	7	8
Frequency	2	3	3	4	6

b

Score	10	11	12	13	14	15
Frequency	1	3	2	5	3	5

Online resources:

Interactive skillsheet The mean

Interactive skillsheet The median

Interactive skillsheet The mode

Interactive skillsheet The range

Worksheet Mean, median, mode and range

Investigation Collecting numerical data

Quick quiz 10.1

Desmos activity Make it balance

Lesson 10.2

Describing data with dot plots

Learning intentions

By the end of this lesson you will be able to …

✔ interpret, describe and construct dot plots.

Inter-year links

Support	Data displays
Year 8	10.3 Analysing data
Year 9	7.3 Grouped data and histograms

Key content video
Describing data with dot plots

Frequency tables

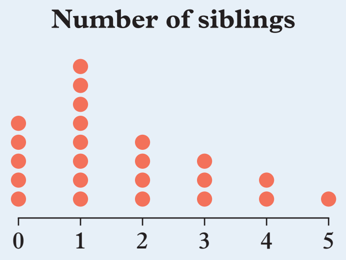
- The **frequency** of a data value is a measure of how often that data value occurs.
- A **frequency table** can be used to tally frequencies in a data set.
  - → For example, this frequency table shows a group of 18 people divided by age.

Age	Tally	Frequency
11	𝍸 |	6
12	|||	3
13	||	2
14	𝍸 ||	7
Total		**18**

- To construct a frequency table from a list of data values, create a table with three columns. The first column will be your numerical variable, the second will be a tally column and the third will record the frequency.
  - → Go through the list of data values, adding a tally mark in the appropriate row of the tally column for each piece of data. Every fifth tally mark should cross over the previous four marks to form groups of five.
  - → Use the tally marks to write the final values in the frequency column.

Dot plots

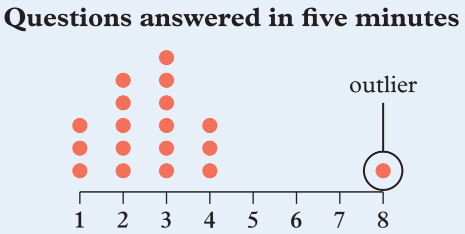
- **Dot plots** can be used to display numerical data. In a dot plot, the frequency of a value is indicated using dots.
- When constructing a dot plot:
  - → The dots should be equally sized.
  - → The dots should be vertically aligned in columns and equally spaced vertically.
  - → The spacing on the horizontal axis should be consistent.


Number of siblings

Summary statistics in dot plots

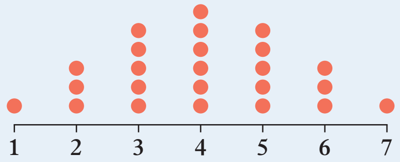
- The mean can be calculated by identifying the individual data values from a dot plot, and then using the formula: (sum of values) ÷ (number of values).
- The mode is the most common data value. It is the data value with the tallest column in a dot plot.
- The median can be found by identifying the individual data values on a dot plot, and then finding the middle value.
- The range is found by subtracting the maximum data value from the minimum data value.

- Outliers are easy to identify when numerical data is presented in a dot plot.

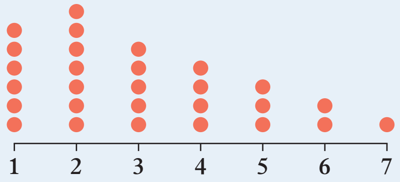


Describing data

- By displaying numerical data, you can identify the shape of the distribution of the data and use this to describe data sets.
- **Symmetric distributions** have a peak in the middle and are evenly spread on either side of the peak.

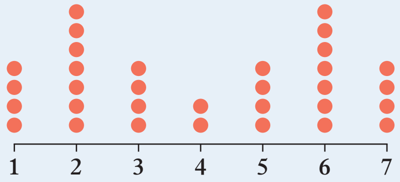


  → Distributions are rarely perfectly symmetric. When distributions are close to being symmetric, they can be described as 'approximately symmetric'.
- A **skewed distribution** has the data values clustered on one side of the distribution and a tail on the other side of the distribution.
  → A **positively skewed distribution** has the data values clustered on the left-hand side, with a tail on the right side of the distribution.



  → A **negatively skewed distribution** has the data values clustered on the right-hand side, with a tail on the left side of the distribution.

- A **bi-modal distribution** has two clear peaks.
  → The two peaks in a bi-modal distribution may have slightly different frequencies.



- In a symmetric (or approximately symmetric) distribution, the median and mean have similar values. In a skewed distribution, the mean and median are likely to be different from each other.
- If a distribution contains an outlier, this should be mentioned when describing the shape of the distribution.

Worked example 10.2A Drawing a dot plot

A group of students were asked how many times they went to the local park in the last week. Present the data below in a dot plot.

0, 1, 2, 0, 1, 4, 3, 0, 3, 1, 4, 3, 3, 3, 2, 3, 0, 4

THINK

1 Create a frequency table to record the data.

Number of visits	Tally	Frequency
0	||||	4
1	|||	3
2	|||	3
3	𝍸 |	6
4	|||	3

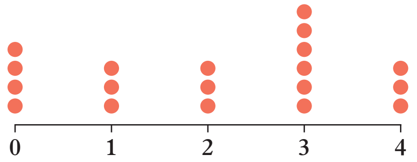
2 Draw the horizontal axis. The axis should be long enough to fit all the values, and there should be enough space above the axis to place the dots.

3 Place the dots in columns above the appropriate value. The dots should be aligned with the values and evenly spaced.

4 Include a title.

WRITE

Number of visits to the local park



Worked example 10.2B Identifying summary statistics from a dot plot

The dot plot on the right shows the number of attempts it took a group of learner drivers to pass their practical driving test.

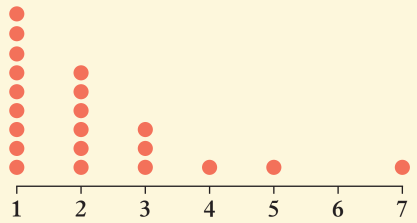
a How many learners were surveyed?

b What is the range of number of attempts?

c What is the mode number of attempts?

d What is the median number of attempts?

Number of attempts taken to pass the practical driving test



THINK

a Count the total number of dots on the plot.

b Subtract the lowest number of attempts from the highest number of attempts.

c Identify the number of attempts that has the highest frequency or number of dots.

d Count the dots to the median value of the data set. There are 21 values, so the median is the 11th value.

WRITE

a 21 learners were surveyed.

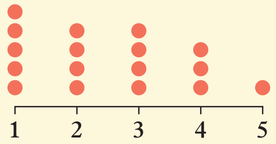
b Range = 7 – 1
= 6
The range of the number of attempts is 6.

c The mode number of attempts is 1.

d The median number of attempts is 2.

Worked example 10.2C Describing numerical data

Describe the shape of the distribution of the data shown in this dot plot.



THINK

1 Consider whether the distribution is symmetric, skewed or bi-modal. The data values in this distribution are clustered on the left-hand side with a tail on the right of the distribution, so this is a positively skewed distribution.

2 Identify whether there are any outliers. There are no outliers in this distribution.

WRITE

The distribution is positively skewed with no outliers.

Helpful hints

- ✔ Watch out for graphs that are misleading! Graphs with frequency scales that do not start from zero and graphs with inconsistent scales can be particularly misleading.
- ✔ In a dot plot, the individual data values are shown on the horizontal axis and should not be confused with the frequency, which is represented by the heights of the columns of dots.
- ✔ When drawing an axis, make sure your scale covers the minimum and maximum frequencies. All values on your scale should also be easy to read.

Learning pathways

△ 1–4, 6(a–e), 7(a–d), 8–10, 14

□ 2, 3, 5, 6, 7(c–f), 9, 10, 13–15

⬠ 3, 6, 7(c–f), 10–13, 15–17

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Exercise 10.2A: Understanding and fluency

1 Harry conducts a survey to find out how many siblings each member of his class has and collects the following data.

1, 2, 1, 0, 2, 2, 4, 1, 2, 1, 1, 3, 1, 2, 3, 0, 1, 2, 1, 1, 2, 2, 8, 1, 2, 1

- **a** Record the data in a frequency table.
- **b** How many people were surveyed?
- **c** What was the most common number of siblings?

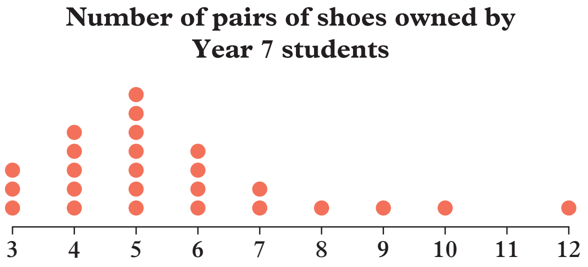
WE 10.2A 2 A group of students were asked how many days in a week they catch public transport. Present the data below in a dot plot.

5, 4, 1, 7, 0, 5, 2, 6, 5, 3, 5, 7, 6, 7, 5, 0, 1, 5, 7, 6, 0, 1, 2, 7, 5, 5, 5, 7

3 A group of students were asked how many days (on average) in a week they did more than 30 minutes of exercise. Present the data below in a dot plot.

2, 3, 0, 5, 1, 7, 0, 6, 3, 7, 4, 5, 1, 2, 1, 6, 4, 4, 1, 0, 2, 1, 5, 5, 3, 4

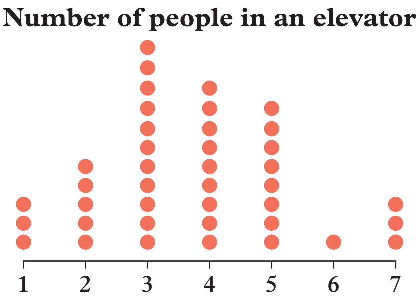
WE 10.2B 4 The following dot plot shows the number of pairs of shoes owned by members of a class of Year 7 students.



- **a** How many students are in the class?
- **b** What is the range of pairs of shoes owned?
- **c** What is the mode number of pairs of shoes owned?
- **d** What is the median number of pairs of shoes owned?

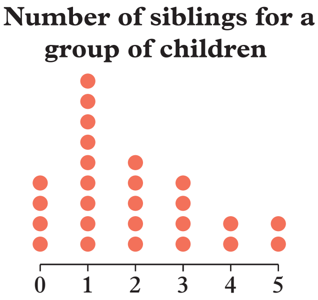
5 Consider the dot plot on the right.

- **a** What is the mode of the data?
- **b** What is the range of the number of people in an elevator at any one time?
- **c** Write the data from this graph as a list of numbers and use the list to find the mean and median of the data.



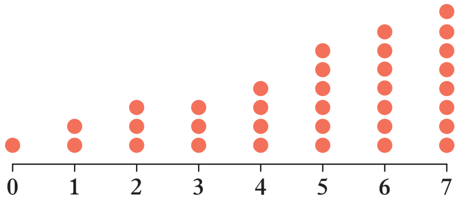
6 The dot plot on the right shows the number of siblings for a group of children.

- **a** How many children are in the group?
- **b** What is the range of the number of siblings?
- **c** What is the mode number of siblings?
- **d** What is the median number of siblings?
- **e** What is the mean number of siblings, correct to two decimal places?
- **f** Use the summary statistics to write two sentences about the number of siblings for this group of children.

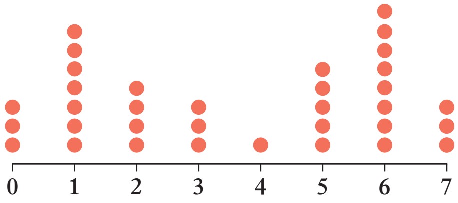


WE 10.2C 7 Describe the shape of the distribution of the data shown in these dot plots.

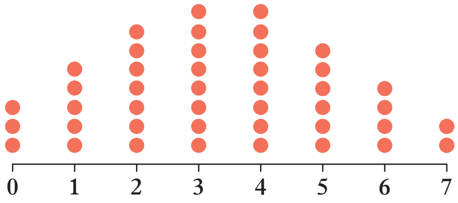
a



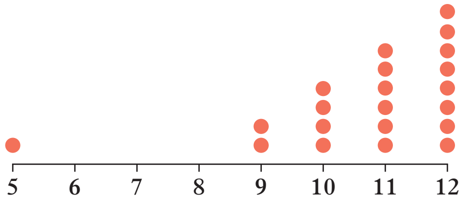
b



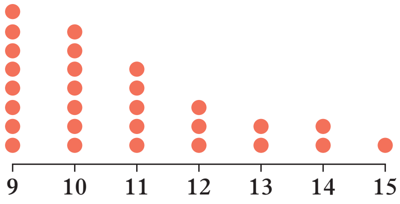
c



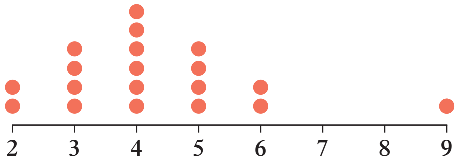
d



e



f



Exercise 10.2B: Problem solving and reasoning

8 Rachael has been keeping track of her scores out of 10 on her weekly maths quizzes. She decides to turn her list of scores into a dot plot which she can easily add new scores to. Her scores from her maths quizzes are:

8, 9, 8, 8, 6, 8, 7, 9, 8, 9.

a Create Rachael's dot plot.

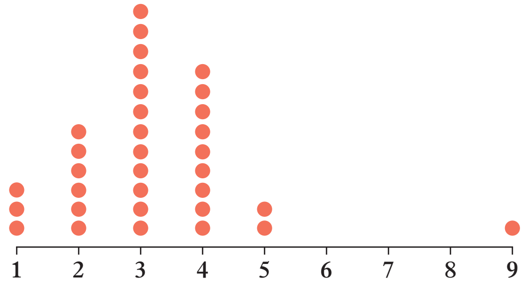
b Calculate Rachael's mean score.

c On the next maths quiz, Rachael scores 10 out of 10. Add this to her dot plot.

d Correct to two decimal places, how much has Rachael's mean maths score increased?

9 Salwa produced this dot plot showing the scores people recorded in an online game.

Scores recorded in an online game



a What is the value of the outlier?

b Salwa realises she misread the score of the outlier when plotting the graph. The score of the outlier is actually 3. If you were to redraw the dot plot based on this information, what change(s) would you need to make to the graph?

10 The ages of the people at a birthday party are listed below. Explain why a dot plot would not be a good choice to represent this data.

12, 14, 13, 13, 14, 12, 15, 13, 14, 15, 45, 14, 13, 13, 48, 14

11 The crowds for a local football team, rounding to the nearest hundred, are recorded below.

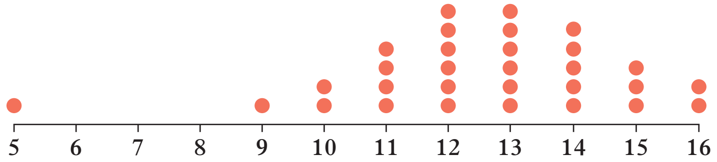
500, 400, 500, 700, 400, 600, 700, 700, 500, 600, 300, 500, 700, 600, 400, 600, 500, 500

a Present the data in a dot plot.

b Comment on the shape of the distribution of the data.

12 The dot plot below shows the number of car sales a dealership makes over a three-week period.

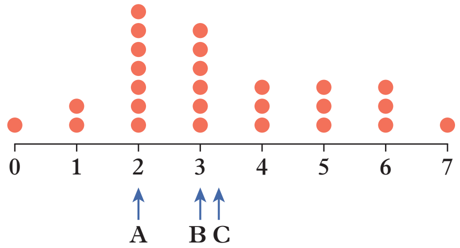
Number of sales



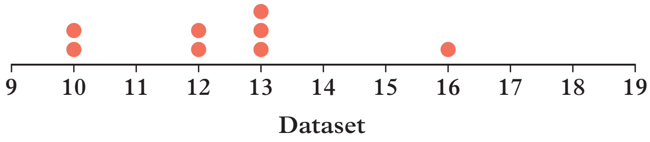
a Comment on the shape of the distribution of the data.

b Calculate the **i** mean, **ii** median, **iii** mode and **iv** range of the data.

13 The three arrows in the dot plot on the right represent the values of the mean, median and mode. Without performing any calculations, identify which arrow belongs to which summary statistic. Give reasons for your answer.

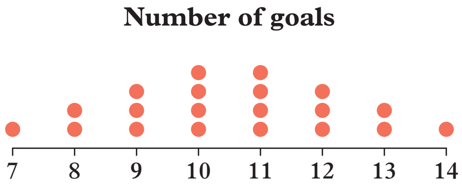


14 The dot plot below is missing one dot. Find a possible value of the missing dot for each of the following scenarios. In each case, state whether your answer is unique or if it is one answer out of several that are possible.



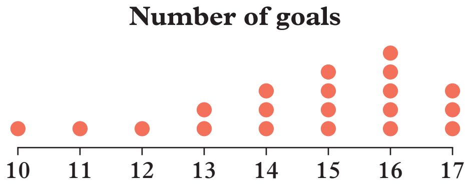
- **a** The maximum value is 17.
- **b** The mean is 12.
- **c** The median is 13.
- **d** The modes are 10 and 13.
- **e** The range is 7.
- **f** The mean is a whole number.

15 The number of goals Team A scores over a 20-game period is recorded in the dot plot below.



- **a** Comment on the shape of the distribution of goals Team A scores.
- **b** Calculate the mean and median number of goals Team A scores.

The number of goals Team B scores over a 20-game period is also recorded in a dot plot.



- **c** Comment on the shape of the distribution of goals Team B scores.
- **d** Calculate the mean and median number of goals Team B scores.
- **e** Complete the following sentences.
  - **i** In a symmetrical distribution, the mean and median are _______.
  - **ii** In a negatively skewed distribution, the mean is _______ the median.

Exercise 10.2C: Challenge

16 The bottom three rows of dots on a dot plot are removed and every column of dots still has dots remaining. Which statistics (mean, median, mode and range) will *not* change? Why?

17 Construct a dot plot with the following statistics:

- number of data points: 11
- mean: 3
- median: 3
- mode: 4
- range: 7

Online resources:

Interactive skillsheet
Dot plots

Interactive skillsheet
Describing data with dot plots

Worksheet
Dot plots

Investigation
TV audiences

Desmos activity
What's in a name?

Quick quiz
10.2

Lesson 10.3

Stem-and-leaf plots

Learning intentions

By the end of this lesson you will be able to …

✔ interpret, describe and construct stem-and-leaf plots.

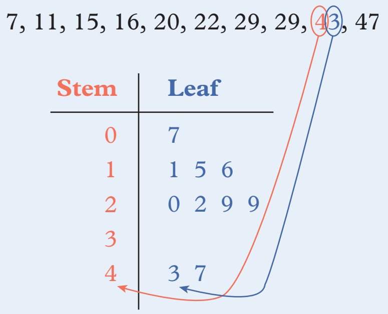
Inter-year links

Year 9 7.6 Comparing data

Stem-and-leaf plots

Key content video
Stem-and-leaf plots

- **Stem-and-leaf plots** or stem plots can be used to display numerical data. In stem-and-leaf plots:
    - → Each piece of data is split into two parts: a *stem* (the first digit or digits) and a *leaf* (the last digit).
    - → All stems have the same place value.
    - → A key is necessary to indicate the value of the data. For example, in the stem-and-leaf plot on the right, 1 | 5 = 15 indicates a stem of 1 and a leaf of 5, which represents the data point 15.
    - → The value of the leaves increases as they move away from the stem.
    - → Leaves should be evenly spaced and written in columns.



Key: 1 | 5 = 15

Split stem-and-leaf plots

- In a **split stem-and-leaf plot** each stem is split into two or more rows. For example, in the split stem-and-leaf plot below, the leaves with a value of 0 to 4 are put in the first row of each stem value, and the leaves with a value of 5 to 9 are put in the second row of each stem value.

Stem	Leaf
4	3 4
4	6 8 9
5	0 2 3 3 4
5	5 8 8
6	1 2 4
6	6

Key: 4|3 = 43

- Split stem-and-leaf plots should be used when the data does not have a wide spread or does not have many stem values.

Describing data with stem-and-leaf plots

- Stem-and-leaf plots can be used to describe the shape of the distribution of the data in the same way as a dot plot, although the data in a stem-and-leaf plot is displayed horizontally instead of vertically.
- A positively skewed distribution will have data values clustered at the top of the stem-and-leaf plot, with a tail at the bottom.
- A negatively skewed distribution will have data values clustered at the bottom of the stem-and-leaf plot, with a tail at the top.
- For example, the distribution of the data in the stem-and-leaf plot on the right is positively skewed.

Stem	Leaf
2	3 5 7 7 9
3	0 1 1 3 6 6 8 9
4	2 2 4 7 8 9
5	4 6
6	5
7	0

Key: 2|3 = 23

Worked example 10.3A Identifying summary statistics from a stem-and-leaf plot

Consider the stem-and-leaf plot on the right.

a Find the range of this data set.

b Find the mode of this data set.

c Find the median of this data set.

d Calculate the mean of this data set.

Stem	Leaf
0	2 5 8
1	0 7 7 9
2	2 6

Key: 1|9 = 19

THINK	WRITE
a Use the key to interpret the plot. Subtract the minimum value from the maximum value to find the range.	**a** Range = 26 − 2 = 24 The range of the data is 24.
b Identify the value which occurs most often.	**b** The mode of this data set is 17.
c Count to the median value of the ordered list of numbers. There are 9 values, so the median is the 5th value.	**c** 2, 5, 8, 10, 17, 17, 19, 22, 26 The median of this data set is 17.
d Add all the numbers together and divide by the total number of values.	**d** Mean $= \frac{2+5+8+10+17+17+19+22+26}{9}$ $= \frac{126}{9}$ $= 14$ The mean of this data set is 14.

Worked example 10.3B Constructing a stem-and-leaf plot

Arrange this set of data into a stem-and-leaf plot.

12, 17, 23, 46, 8, 34, 33, 15, 28, 22, 42, 10, 4, 35, 39, 50

THINK

1. Identify the minimum and maximum stem values. The minimum value is 4 and the maximum value is 50. 4 has a stem of ‘0’ and 50 has a stem of ‘5’.
2. Place all stems from 0 to 5 in a single vertical column.
3. Construct the leaves by entering the data points from the list into the appropriate row of the plot.
4. Rewrite the stem-and-leaf plot, putting the data points in ascending order as you move away from the stem. Make sure the numbers are evenly spaced.
5. Include a key.

WRITE

Unordered

Stem	Leaf
0	8 4
1	2 7 5 0
2	3 8 2
3	4 3 5 9
4	6 2
5	0

Ordered

Stem	Leaf
0	4 8
1	0 2 5 7
2	2 3 8
3	3 4 5 9
4	2 6
5	0

Key: 1|0 = 10

Worked example 10.3C Describing data in a stem-and-leaf plot

Describe the shape of the distribution of the data shown in this stem-and-leaf plot.

Stem	Leaf
0	7 9
1	6 6 8 9
2	0 2 3 7 7 8
3	1 1 4 6 8
4	3 5 9
5	0 3

Key: 1|4 = 14

THINK

1 Consider whether the distribution is symmetric, skewed or bi-modal. The data values in this distribution are approximately symmetrical.

2 Identify whether there are any outliers.

WRITE

The distribution is approximately symmetrical. There are no outliers in this distribution.

Helpful hints

- ✓ The minimum and maximum stems will not always have the same number of digits. For example, if a data set has a minimum value of 55 and a maximum value of 120, the minimum stem will be 5 and the maximum stem will be 12.
- ✓ When calculating summary statistics from a stem-and-leaf plot remember to use the full data values, not just the values of the leaves.
- ✓ Make sure you use the key to decode the data values. For example, 2|7 may represent 2.7, 27, 270 or 2700.
- ✓ Turning your page 90° anticlockwise allows you to view a stem-and-leaf plot in the same orientation as a dot plot. This can help identify the shape of the distribution.

Learning pathways

▲ 1–3, 4(a, b), 5, 6, 9, 10(a–c), 11

■ 2, 3, 4(c, d), 5, 7, 8, 10, 12, 13

⬟ 4(c, d), 6, 7, 8, 10–15

ANS p564

Exercise 10.3A: Understanding and fluency

1 The number of people at a local skate park over the course of a month is displayed in the stem-and-leaf plot on the right.

- **a** Add an appropriate key to the plot.
- **b** What was the smallest number of people at the park during the month?
- **c** What was the largest number of people?
- **d** What is the range of the number of people in the park for this month?
- **e** What is the mode for the number of people in the park for this month?
- **f** How many days were there in this month?

Stem	Leaf
1	4 8
2	2 5 5 6 9
3	1 1 2 4 5 6 6 7 7 7
4	4 4 8 9
5	1 2 3
6	0 0 1 6
7	4 8
8	
9	8

WE 10.3A 2 Consider the stem-and-leaf plot on the right.

a Find the range of the data.

b Find the mode of the data.

c Find the median of the data.

d Calculate the mean of the data, correct to two decimal places.

Stem	Leaf
0	1 5 6
1	2 8 9 9
2	1 4 8
3	0 1 4
4	3

Key: 2|1 = 21

3 Consider the stem-and-leaf plot on the right.

a Redraw the stem-and-leaf plot so that the data values are in ascending order.

b What is the smallest number in the data set?

c What is the largest number in the data set?

d What is the mode?

e Add the number 112 to the plot.

Stem	Leaf
5	1 2 9 8
6	4 6 7 8 2
7	1 1 5 8 6
8	3 3 4 9 2 2 1 3
9	
10	8

Key: 5|1 = 51

WE 10.3B 4 Arrange each set of data into a stem-and-leaf plot. Remember to include a key with each plot.

a 11, 14, 18, 19, 23, 23, 26, 31, 38, 42, 42, 43

b 13, 18, 22, 46, 54, 59, 31, 36, 42, 19, 4, 28

c 54, 34, 33, 65, 39, 27, 35, 51, 42, 49, 37, 53

d 41, 37, 53, 29, 31, 59, 67, 23, 41, 59, 29, 41

5 The ages of customers sitting in a sushi restaurant at lunch time were recorded and displayed below.

15	23	29	45	19	65	18	29	31	34	22
24	19	33	25	32	54	39	28	19	26	

a How old were the youngest and oldest customer in the restaurant?

b Draw a stem-and-leaf plot to represent this data. Remember to include a key.

c What was the most common customer age?

d What was the most common age bracket (10–19, 20–29, 30–39, etc.)?

6 The number of points scored by a junior basketball team in each match over a season are shown in the stem-and-leaf plot on the right.

a What is the range of the scores?

b What is the mode score?

c What is the median score?

d What is the mean score, correct to two decimal places?

Stem	Leaf
1	1 5 6
2	0 0 1 2 3 4 4 5
3	0 1 2 3 6 7 7
4	1 1 1 3
5	2 5
6	7

Key: 1|5 = 15

7 The ages of a sample of people that use Instagram on a daily basis are given below.

16	18	20	21	29	34	25	15	17	19	13	14	31
14	14	18	23	27	36	54	45	46	76	53	41	22
46	29	30	18	21	24	41	33	15	16	25	12	10

a Create a stem-and-leaf plot to display this data.

b What is the age of the youngest Instagram user in this sample group?

c What is the age of the oldest Instagram user in this sample group?

d What is the most common age bracket of people that use Instagram in this sample group?

WE 10.3C **8** Describe the shape of the distribution of the data shown in these stem-and-leaf plots.

a

Stem	Leaf
1	4 5 7 9 9
2	0 2 3 5 5 7 8
3	1 4 4 6 9 9
4	2 3 7
5	2 4
6	0

Key: 1|4 = 14

b

Stem	Leaf
6	3
6	6 7 9 9
7	2
7	6 9
8	0 0 3 4
8	8

Key: 6|3 = 6.3

c

Stem	Leaf
4	5
5	3 7
6	4 9
7	2 4 8 8
8	0 3 5 5 7
9	1 4 5 6 8 9 9

Key: 4|5 = 45

Exercise 10.3B: Problem solving and reasoning

9 Consider the split stem-and-leaf plot below.

Stem	Leaf
2	8
3	1 1 4
3	5 7 8 8 8
4	0 1 3 4
4	5 6 6
5	1

Key: 2|8 = 28

a Find the range of the data.

b Find the mode of the data.

c Find the median of the data.

d Calculate the mean of the data, correct to two decimal places.

10 A sample of feathertail gliders (tiny possums, native to Australia) were weighed to monitor their health. The results are shown in this stem-and-leaf plot.

Stem	Leaf
10	1 4 8
11	2 3 6 6
12	1 2 4 5 5 6 9
13	1 2 4 4 4 6 8 8 8 8
14	0 1
15	
16	1

Key: 10|1 = 10.1 g

a What is the mass of the lightest glider?

b What is the mass of the heaviest glider?

c Will said that the modal mass recorded was 138 g. Identify and explain his mistake, and then state the actual modal mass.

d Identify the outlier in this data set and state the mass of the heaviest glider, excluding the outlier.

e Another five feathertail gliders were weighed and their masses (in grams) are recorded below. Redraw the stem-and-leaf plot, including these masses in the plot.

14.3 12.9 13.1 13.6 11.0

11 The figures below indicate the approximate number of people lined up at a sample of cinemas an hour before the premiere of the latest Spider-Man movie.

120 250 360 80 150 430 570 220 160 130 170 240 220

60 110 140 200 130 120 100 180 220 260 130 150 100

A stem-and-leaf plot has been started on the right.

Stem	Leaf
0	6 8
1	
2	

Key: 1|2 = 120

a How is the key different from those you have seen before?

b Complete the stem-and-leaf plot.

c How many people were in the smallest line?

d How many people were in the largest line?

e Calculate the mean number of people lining up an hour before the premiere. Give your answer to the nearest whole number.

12 The schedule on the right lists the times a tram arrives at a certain stop in Brisbane on a Sunday.

a How is the schedule different from a normal stem-and-leaf plot?

b What is the earliest time the tram leaves the stop?

c At what time does the last tram leave the stop?

d How many trams run each hour from midday to 7 pm?

e Between which times do the shortest waiting periods occur?

f Between which times do the longest waiting periods occur?

Sunday	
AM	
6	
7	16 46
8	16 46
9	16 46
10	12 30 43 56
11	08 18 30 42 54
PM	
12	06 18 30 42 54
1	06 18 30 42 54
2	06 18 30 42 54
3	06 18 30 42 54
4	06 18 30 42 54
5	06 18 30 42 54
6	06 18 30 42 54
7	04 19 34
8	04 35
9	05 35
10	04 31
11	01 31 59

13 Consider the stem-and-leaf plot below. Identify and describe three things that need to be fixed.

Stem	Leaf
2	1 4 9 2
3	3 9 8 3 5
4	1 1 6 7 5 3 1
5	0 6 3
8	7 8
9	9 4 2 2

14 When displaying numerical data, describe the situations when it would be best to use a dot plot, a split stem-and-leaf plot and a standard stem-and-leaf plot to display the data?

Exercise 10.3C: Challenge

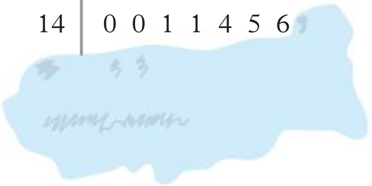
15 While working on an ordered stem-and-leaf plot worksheet, Sal spills his drink. The bottom section of the plot is smudged, obscuring three leaves, the final stem and the key. He already had the following information:

- The range was 0.026
- The mean was 0.14065

a Determine the missing maximum value.

b Determine all possible pairs of numbers for the two remaining illegible digits.

Stem	Leaf
12	9
13	3 4 4 8 8 8 9 9 9
14	0 0 1 1 4 5 6



Online resources:

Interactive skillsheet Stem-and-leaf plots

Worksheet Stem-and-leaf plots

Investigation Reaction times

Quick quiz 10.3

ANS
p565

Checkpoint

Checkpoint quiz
Take the checkpoint quiz to check your knowledge of the first part of this module.

10.1 **1** Classify the following numerical data as either discrete or continuous.

- **a** Number of pair of shoes a person owns
- **b** Weight of sugar required for a dessert recipe

10.1 **2** Match the summary statistics for numerical data to how it is calculated.

a	Mean	**I**	Find the number with the highest frequency.
b	Median	**II**	Split the data into two equal sized groups, find the middle value.
c	Mode	**III**	maximum value – minimum value
d	Range	**IV**	$\dfrac{\text{sum of all data points}}{\text{total number of data points}}$

10.1 **3** Calculate the following for the data sets (to two decimal places where needed).

i mean **ii** median **iii** mode **iv** range

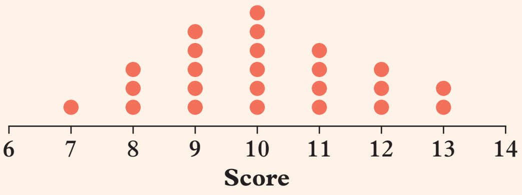
- **a** 6, 7, 7, 8, 9, 10, 12, 15
- **b** $2.5, 8.3, 9\frac{1}{7}, 5\frac{2}{3}, 2.5, 2.5, 1\frac{3}{14}$

10.1 **4** State the statistic each statement is interpreting.

- **a** Half of the students scored under 65 on the science test.
- **b** The most common weight was 1.2 kg.
- **c** The difference in height between the shortest and tallest basketballer is 46 cm.
- **d** If all the prizes were shared equally, everyone would receive $75.

10.2 **5** A group of 13 teenagers at a skatepark were comparing how many bones they had broken while skateboarding. The number of bones broken were 3, 1, 1, 2, 4, 2, 0, 1, 1, 7, 2, 1, 1. Display this data on a dot plot.

10.2 **6** Consider the dot plot on the right.



- **a** Calculate the mean, correct to two decimal places.
- **b** Calculate the median.
- **c** Calculate the mode.
- **d** Calculate the range.
- **e** Describe the shape of the distribution of the data.

10.3 **7** Construct a stem-and-leaf plot for the following data.

15, 42, 60, 27, 28, 12, 45, 37, 19, 28, 39, 31, 52, 35, 30, 40, 41, 47, 32, 23

10.3 **8** Consider the stem-and-leaf plot on the right.

- **a** What is the value of the data point represented by the 4 leaf on the 10 stem?
- **b** Calculate the mean.
- **c** Calculate the median.
- **d** Calculate the mode.
- **e** Calculate the range.
- **f** Describe the shape of the distribution of the data.

Stem	Leaf
9	5 7 8 8 9
10	0 1 2 4 7
11	1 3 5
12	4 6
13	0

Key: 9 | 5 = 95

Statistical Investigations

The statistical investigation process

Statistics can be used to answer a wide range of real-life questions. By collecting and analysing data, we can draw evidence-based conclusions, allowing us to make informed decisions.

A statistical investigation involves the following steps:

1. Identifying a question to investigate
2. Collecting data
3. Analysing the data
4. Reporting on the findings.

Data used in a statistical investigation can either be collected directly by the researcher (primary data) or taken from a reputable source (secondary data).

Curriculum link

Plan and conduct statistical investigations for issues involving discrete and continuous numerical data, and data collected from primary and secondary sources; analyse and interpret distributions of data and report findings in terms of shape and summary statistics. (VC2M7ST03)

Predicting the weather

Humans have been trying to accurately predict the weather for thousands of years. If we know what sort of weather to expect we can plan accordingly, making sure that we're prepared for heatwaves, droughts, heavy rain and other weather phenomena.

The Bureau of Meteorology was established in 1906 and provides weather services across Australia. Millions of Australians use the data collected by the Bureau of Meteorology to know what weather to expect today and in the future.

The Bureau of Meteorology's mission

The Bureau's mission is to provide Australians with the information they need to live safely and sustainably within their natural environment, which encompasses the atmosphere, ocean, water and land. To achieve this, the Bureau of Meteorology:

- monitors and reports on current environmental conditions;
- analyses and explains trends in environmental data;
- provides forecasts, warnings and long-term outlooks on environmental phenomena that affect the safety and prosperity of Australians; and
- fosters greater public understanding of environmental intelligence.

© Bureau of Meteorology

Australian Government

Bureau of Meteorology

Investigation

In this statistical investigation, we will look at rainfall in Alice Springs and how it changes over the course of a year. The following data comes from the Bureau of Meteorology and shows the number of days in each month with heavy rainfall (at least 10 millimetres) in Alice Springs in 2022.

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Days with at least 10 mm rain	5	2	0	0	0	2	0	0	1	1	2	3

Investigation questions

1 Explain why this is secondary data, not primary data.

2 How could you collect primary data about the number of days with heavy rain each month in your local area?

3 In Alice Springs, January had the most days with at least 10 millimetres of rain. Does this mean January had the most total rain of any month? Explain why or why not.

4 Describe how the rainfall in Alice Springs changes over the year. Which season appears to have the most rain?

Analysis

1 Analyse the data given in the table by calculating each of the following summary statistics.

- **a** Mean
- **b** Median
- **c** Mode
- **d** Range

2 Draw a dot plot of the data and use it to describe the shape of the distribution.

3 Write a report on your findings from this investigation. Discuss the summary statistics and the rainfall pattern.

Investigate further

Perform a similar investigation of the rainfall in your local area by collecting data from the Bureau of Meteorology (bom.gov.au/climate/data/). Compare the rainfall in your local area with the rainfall in Alice Springs.

Lesson 10.4

Describing probability

Learning intentions

By the end of this lesson you will be able to ...

✔ list the sample space of experiments and distinguish between equally likely outcomes and outcomes that are not equally likely.

Inter-year links

Support	Understanding probability
Year 8	10.4 Theoretical probability

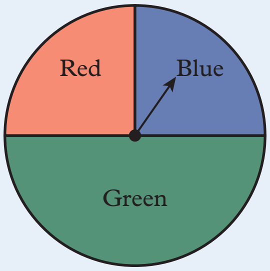
Probability terminology

Key content video
Describing probability

- A **trial** is a repeatable procedure with a result which is determined by chance.
- An **experiment** consists of one or more trials.
  For example, flipping a coin twice is an experiment which involves two trials.
- An **outcome** is a possible result of a trial or experiment.
  For example, if a coin is flipped once then the possible outcomes are H and T, whereas if a coin is flipped twice then the possible outcomes are HH, HT, TH and TT.
- The **sample space** of an experiment is a set of all the possible outcomes of that experiment.
  For example, the sample space for the experiment of rolling one die is: $\{1, 2, 3, 4, 5, 6\}$.
- An **event** refers to a single outcome or a group of outcomes in a sample space.
  For example, a single outcome is rolling a 2; a group of outcomes is rolling a 2 or rolling a 4.

Describing probability

- **Probability** describes the chance that a particular event will occur. It is expressed on a scale from impossible to certain.
  - → Events that are less likely to occur than to not occur can be described as *unlikely*.
  - → Events that are equally likely to occur or not occur can be described as *even chance*.
  - → Events that are more likely to occur than to not occur can be described as *likely*.
- **Equally likely outcomes** have the same probability of occurring.
  For example, it is equally likely that the spinner will land on blue or red, but not equally likely that the spinner will land on blue compared to green.



Fairness and bias

- When dealing with probabilities, we need to know whether the tools we are using are **fair** or **biased**.
  - → A fair tool will behave as you expect it to. For example, a fair coin will have an even chance of landing on heads or tails.
  - → A biased tool will favour one or more outcomes above others. For examples, a biased six-sided die may be weighted to land on 6 more often than 1.

Worked example 10.4A Listing the sample space

List the sample space for each of the following experiments.

a Flipping a coin

b Drawing a card from a standard deck of cards and recording the suit

THINK	WRITE
a There are two possible outcomes. Place these within curly brackets separated by commas.	**a** {head, tail}
b Although there are 52 cards in a deck, there are only four different suits. Place these within curly brackets separated by commas.	**b** {hearts, diamonds, clubs, spades}

Worked example 10.4B Describing probability

Describe the probability of the following events using one of the following five terms: impossible, unlikely, even chance, likely, certain.

a Rolling a six-sided die and obtaining a six

b Flipping a coin and obtaining a tail

c May following April

d Selecting a non-picture card from a deck of cards

THINK	WRITE
a Rolling a six is unlikely, but not impossible.	**a** Unlikely
b The chance of obtaining a tail is equal to the chance of obtaining a head. As there are two equally likely outcomes, this is an even chance.	**b** Even chance
c May always follows April.	**c** Certain
d Each suit is made up of 10 cards that do not have a picture, and 3 picture cards, so this is a likely event.	**d** Likely

Helpful hints

- ✔ If there are two outcomes in a sample space, this does not automatically mean they are equally likely to occur.

  For example, given the sample space {win the lottery, do not win the lottery} there is not an even chance you'll win!
- ✔ Events should not be described as impossible unless they will never happen, and events should not be described as certain unless they will definitely happen.

Learning pathways

△ 1(a–d), 2–6, 7(a–c), 10(a–d), 12(a, b), 13

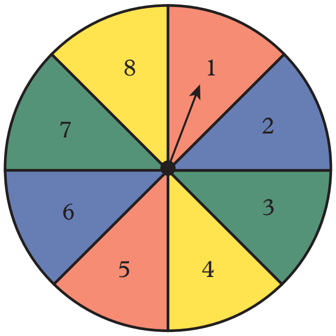
■ 1(a, b, e, f), 2(d–f), 3, 5–8, 10(d–f), 11, 12(a–c), 13

⬟ 1(e, f), 2(d, f), 5, 7, 9, 10(e, f), 11, 12(c, d), 14, 15

ANS p565

Exercise 10.4A: Understanding and fluency

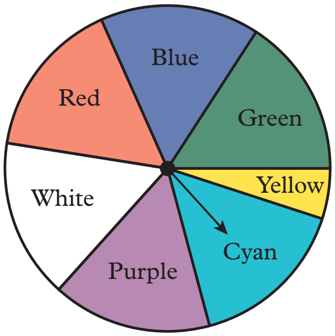
WE 10.4A **1** List the sample space for each of the following experiments and surveys.



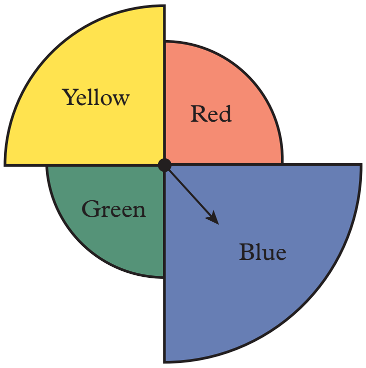
- **a** The spinner on the right is spun and the colour is recorded.
- **b** The spinner on the right is spun and the number is recorded.
- **c** Justine asks her friends what their favourite season is.
- **d** Lisa conducts a survey on the month in which her classmates were born.
- **e** Selecting one coin at random from a wallet containing at least one of every Australian coin.
- **f** Rishi conducts a survey about how many days a week people work.

2 State whether all of the outcomes of the following experiments are equally likely or not equally likely.

- **a** Spinner A is spun and the colour is recorded.
- **b** A card is drawn at random from a standard deck after someone has slipped a second ace of spades into the deck.
- **c** A fair 20-sided die labelled 1–20 is rolled.
- **d** Spinner B is spun and the colour is recorded.



Spinner A



Spinner B

- **e** A biased six-sided die labelled 1–6 is rolled such that a 6 is more likely to be rolled and a 1 is less likely to be rolled.
- **f** Without looking, a token is taken from a bag containing 15 small red tokens and 15 large blue tokens.

WE 10.4B **3** Describe the probability of each event using one of the following five terms: impossible, unlikely, even chance, likely, certain.

- **a** Rolling a 1, 2, 3, 4 or 5 on a standard six-sided die
- **b** A woman giving birth to triplets
- **c** Selecting a red card from a standard deck of cards containing 26 red and 26 black cards
- **d** Picking the winning option out of three equally likely choices
- **e** Having at least one hot day in summer in Australia

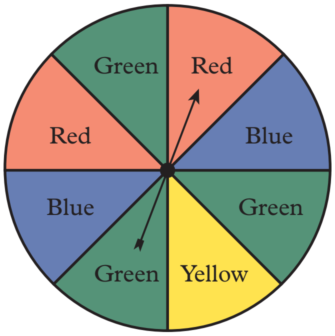
4 Match each event to the most appropriate probability.

a	Winning a coin toss	**I**	Likely
b	Rolling more than 2 on a standard six-sided die	**II**	Certain
c	Living to 100 years old	**III**	Even chance
d	Tuesday following Monday	**IV**	Unlikely

5 This spinner below is spun once. Provide an example of an outcome that is:

a unlikely **b** likely **c** impossible

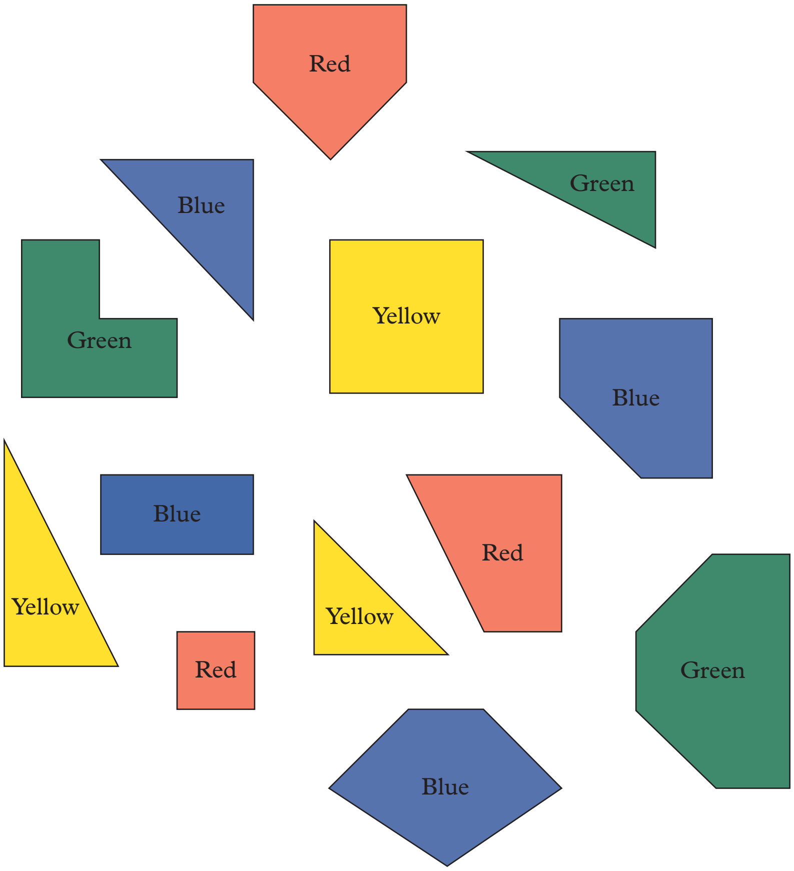
d even chance **e** certain.



6 Look at the picture of the polygons. List the sample space if:

a the polygons are grouped by colour

b the polygons are grouped by number of sides.



7 The number of elements in a sample space is equal to the number of possible outcomes. For example, the sample space for flipping a coin consists of two elements, {heads, tails}, because there are two possible outcomes: heads or tails. Find the number of elements in the sample space of the following experiments.

a Randomly choosing a letter from all the vowels in the alphabet

b Randomly choosing a day of the week

c Randomly drawing a letter from the word FIRST

d Randomly drawing a letter from the word TEXT

e Randomly selecting one counter out of a bag containing 8 red, 7 green and 10 blue coloured counters

Exercise 10.4B: Problem solving and reasoning

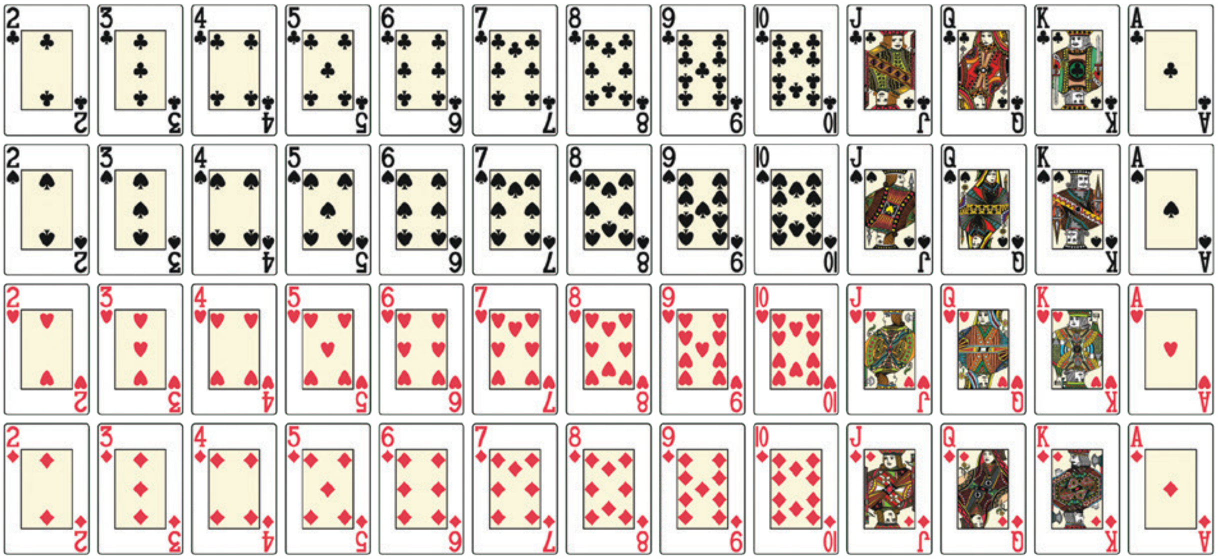
8 A fair 10-sided die labelled 1, 2, 2, 3, 4, 4, 5, 6, 6, 7 is rolled.

- **a** If each individual face is considered a different outcome, are the outcomes equally likely?
- **b** If each number is considered a different outcome, are the outcomes equally likely?

9 A board game lets you choose from a collection of differently labelled six-sided dice which are rolled to determine the number of spaces you move. For each of the following dice, determine:

- **i** if the die has equally likely outcomes based on the numbers
- **ii** the average (mean) die roll if each face is equally likely to appear.
  - **a** $\{0, 0, 0, 0, 10, 10\}$
  - **b** $\{0, 0, 0, 7, 7, 7\}$
  - **c** $\{1, 3, 3, 3, 4, 4\}$
  - **d** $\{-3, -2, -1, 1, 2, 3\}$
  - **e** $\{0, 1, 3, 5, 7, 9\}$

10 A standard deck of playing cards has four suits: clubs, spades, hearts and diamonds. There are 13 cards in each suit (Ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen, King) making a total of 52 cards.



Six cards are drawn at random from a standard deck of cards. State whether the following statements are true or false.

- **a** There will always be an equal number of red and black cards.
- **b** All four Aces can be drawn.
- **c** A set of four-of-a-kind can be drawn.
- **d** There will always be red and black cards drawn.
- **e** There will always be a card from each of the four suits drawn.

11 Not all experiments have outcomes that are equally likely. For example, you are equally likely to select a ‘B’ from the word ‘BALLOON’ as you are to select an ‘A’, but you are more likely to select a ‘B’ from the word ‘BABY’ than you are to select an ‘A’.

Determine whether or not the outcomes to the experiments in question **7** are equally likely.

12 The sample space of an experiment can continue indefinitely. For example, the number of goals scored in a soccer match is $\{0, 1, 2, 3, 4, \ldots\}$ or the temperature outside to the nearest integer is $\{\ldots, -2, -1, 0, 1, 2, \ldots\}$. The three dots '…' means to continue the pattern of the previous numbers in the set. Write the sample spaces of the following experiments which continue indefinitely.

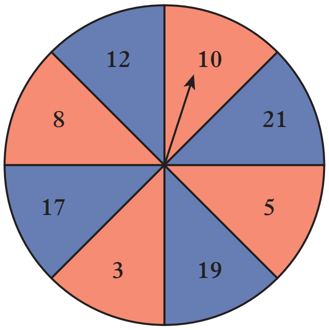
a A random number generator is programmed to print a positive multiple of 3.

b The score at the end of a game of Pac-Man where 10 points are scored for every pellet he eats.

c A person's elevation (distance above or below sea level) at a given point in time to the nearest tenth of a metre.

d The number of bacteria in a petri dish after some time has passed, where there is initially one bacterium and the number of bacteria doubles every hour.

13 This spinner with equally sized segments is spun once and the number it lands on is recorded. State whether each of these events are impossible, unlikely, even chance, likely or certain.



a Landing on an even number

b Landing on a number greater than 10

c Landing on a square number

d Landing on a multiple of 3

e Landing on a composite number

14 The events A, B, C, D and E all have different probabilities of occurring.

- E is less likely to occur than B but is more likely to occur than A.
- A is more likely to occur than C.
- D is certain to occur.

a Use the clues provided to place events A to E in order of likelihood to occur (from least likely to most likely).

b Which is the only event that might be impossible?

Exercise 10.4C: Challenge

15 The events A, B and C are either impossible, unlikely, even chance, likely, or certain, with no two events being in the same category. One, and only one, of the following statements about the events is false. Determine which statement is false and correct the statement.

1 Event C is likely to happen.
2 Event C is twice as likely as B.
3 Event A is twice as likely as C.
4 Event B is unlikely to happen.
5 Event A is certain to happen.

Online resources:

Interactive skillsheet Describing probability

Investigation Probability snap

Quick quiz 10.4

Lesson 10.5

Theoretical probability

Learning intentions

By the end of this lesson you will be able to …

✔ calculate theoretical probabilities.

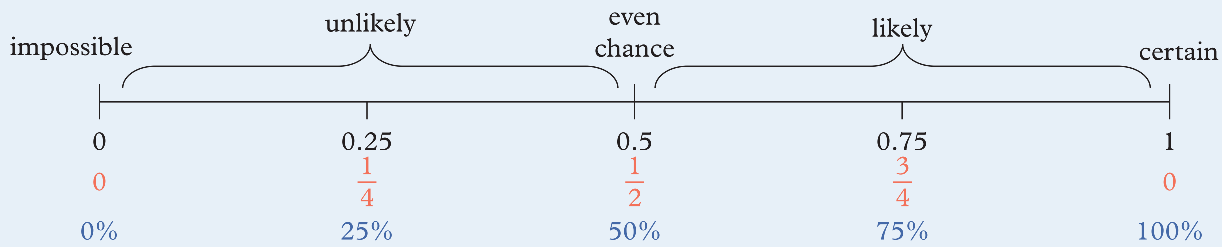
Inter-year links

Support	Understanding probability
Year 8	10.4 Theoretical probability
Year 9	8.1 Two-step chance experiments

Key content video
Calculating theoretical probability

Expressing probabilities

- All events have a probability between 0 (impossible) and 1 (certain) of occurring.
- Probabilities can be expressed as decimals, fractions and percentages.



Theoretical probability

- Events are made up of one or more outcomes. A **favourable outcome** is a result that you are interested in.
  For example, the event of ‘rolling an even number’ with a standard six-sided die consists of the favourable outcomes 2, 4 and 6.
- Pr(event) means the probability of the stated event occurring.
- The **theoretical probability** of an event occurring indicates how likely it is that you can expect that event to occur.
  → Theoretical probability does not tell us what *will* happen, but allows us to predict what *might* happen.
- Theoretical probability is based on mathematical reasoning, not experiments.
- When all outcomes are equally likely, the theoretical probability of an event is calculated by dividing the number of favourable outcomes by the total number of equally likely outcomes:

$$\text{Pr(event)} = \frac{\text{number of favourable outcomes}}{\text{number of equally likely outcomes}}$$

  → To calculate the theoretical probability of an event:
    1 Count all the possible equally likely outcomes.
    2 Count all the favourable outcomes that make up the event.
    3 Divide the number of favourable outcomes by the total number of equally likely outcomes.
- When comparing two events from the same experiment, the event with the higher probability is more likely to occur.
- The sum of the probabilities of all outcomes in the sample space will always be 1 or 100%.

Worked example 10.5A Calculating theoretical probabilities

A standard six-sided die is rolled.

a List the sample space.

b What is the probability of rolling a 4?

c What is the probability of rolling a number greater than 4?

THINK	**WRITE**
a List the possible outcomes in curly brackets.	**a** $\{1, 2, 3, 4, 5, 6\}$
b **1** Count the number of equally likely outcomes. There are six equally likely outcomes.	**b** $\Pr(\text{rolling a } 4) = \dfrac{\text{number of favourable outcomes}}{\text{number of equally likely outcomes}}$
2 Count the number of favourable outcomes. There is one favourable outcome (rolling a 4).	
3 Use the probability formula to calculate the required probability.	$= \dfrac{1}{6}$
c **1** Count the number of favourable outcomes. There are two favourable outcomes (rolling a 5 or a 6).	**c** $\Pr(\text{rolling number} > 4) = \dfrac{\text{number of favourable outcomes}}{\text{number of equally likely outcomes}}$
2 Use the probability formula to calculate the required probability.	$= \dfrac{2}{6}$
3 Simplify the fraction.	$= \dfrac{1}{3}$

Helpful hints

- ✔ Make sure all possible outcomes are equally likely before using the probability formula.
- ✔ List the sample space to avoid making errors when determining the number of favourable outcomes and the total number of equally likely outcomes.

Learning pathways

▲ 1, 3, 4, 5(a, b), 6, 7, 9, 12–14, 16, 18(b), 19

■ 2, 5, 8, 10, 11, 13(d–f), 15, 16, 18(a, b), 20, 21, 22(a, b)

⬟ 5, 8, 10(d, e), 11, 13(d–f), 17, 18, 20, 21(c, d), 22(c, d), 23

ANS p566

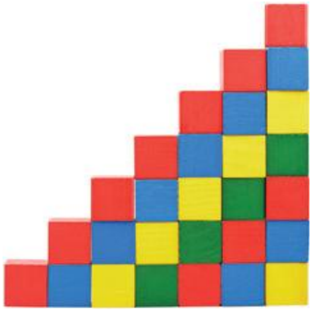
Exercise 10.5A: Understanding and fluency

1 Theo selects a block at random from the group of blocks on the right.

a What is the total number of equally likely outcomes?

b If Theo is looking to find the probability of selecting a red block, what is the number of favourable outcomes?

c What is the probability that Theo selects a red block?



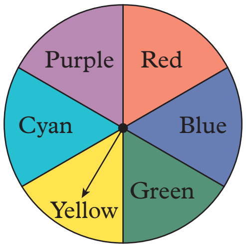
2 Consider selecting a block at random from the staircase of blocks on the right.

a What is the total number of equally likely outcomes?

b What is the probability of selecting a red block?

c What is the probability of selecting a green block?

WE 10.5A **3** The spinner with equally sized segments below is spun once and the result is recorded.



a List the sample space.

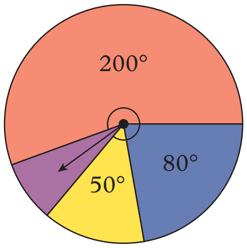
b What is the probability of landing on the yellow section?

c What is the probability of landing on a green or purple section?

4 A fair coin is flipped.

a What is the probability of getting a head? Give your answer as a percentage.

b What is the probability of getting a tail? Give your answer as a percentage.



5 To determine the theoretical probability of landing on a sector on a spinner, divide the angles at the centre by the number of degrees in a full turn.

a Determine the angle of the purple sector.

b Determine the theoretical probability of landing on the:

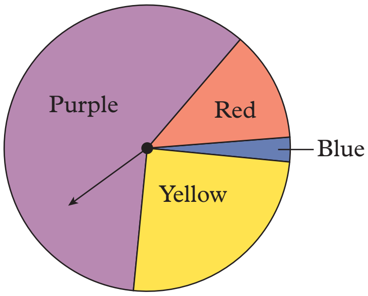
i purple sector **ii** yellow sector

iii blue sector **iv** red sector.

c When the angles at the centre of a spinner are not known, we can measure the angles using a protractor. For the spinner on the right, measure the angles at the centre of the:

i purple sector **ii** yellow sector

iii blue sector **iv** red sector.



d Determine the theoretical probability of this spinner landing on the:

i purple sector **ii** yellow sector

iii blue sector **iv** red sector.

6 A standard six-sided die is rolled.

a What is the probability of rolling a 2?

b What is the probability of rolling an odd number?

c What is the probability of rolling a number less than 5?

d What is the probability of rolling a multiple of 3 or a 4?

7 A card is selected at random from a standard deck and the suit is recorded.

a What is the probability of selecting a club?

b What is the probability of selecting a red card?

c What is the probability of selecting a green card?

8 A letter is selected at random from the word STATISTICS.

a What is the probability, given as a decimal, of selecting a T?

b What is the probability, given as a decimal, of selecting a C?

c What is the probability, given as a decimal, of selecting a vowel?

9 A box of Favourites chocolates contains six Crunchies, six Cherry Ripes, four Caramels and four Flakes. A Favourite is picked at random from the bag.

a How many Favourites are there in total?

b What is the probability, given as a decimal, of:

i picking a Crunchie? **ii** picking a Caramel?

10 A card is randomly drawn from a standard deck.

a What is the probability of selecting a picture card (Jack, Queen or King)?

b What is the probability of selecting an Ace?

c What is the probability of selecting the Queen of spades?

d What is the probability of selecting an even number from the deck?

e What is the probability of selecting a prime number from the deck?

11 The spinner from the game Twister is shown.

What is the probability, given as a percentage, of:

a spinning a yellow?

b spinning a left foot?

c spinning a blue right hand?

d spinning a green or red?

e spinning a right foot or blue (or both)?

12 A deck of UNO cards has four suits: red, yellow, green and blue. Each suit contains the cards 0–9, Reverse, Skip and Draw Two. There are two copies of all cards except the 0s. The deck also includes four standard Wild Cards and four Draw Four Wild Cards. This gives a total of 108 cards. A card is randomly selected from the deck.

a What is the probability of selecting a 0?

b What is the probability of selecting a 6?

c What is the probability of selecting a Reverse?

d What is the probability of selecting a Wild Card (standard) or Draw Four?

e What is the probability that a card selected will make another player draw cards?

13 Express the following probabilities using decimals.

a Muhammed has a 70% chance of kicking this goal.

b There is a $\frac{1}{4}$ chance it will rain tomorrow.

c Sylvia is 95% likely to pass her English exam.

d There is a $\frac{2}{5}$ chance that Hawthorn will win the Premiership.

e The likelihood that Priya will get the lead role is 65%.

f There is a 100% chance that Tuesday will follow Monday.

Exercise 10.5B: Problem solving and reasoning

14 Tess takes a handful of jellybeans from a packet and says that because there are four different colours of jellybeans, the probability of selecting a green jellybean is $\frac{1}{4}$. Describe Tess's mistake and calculate the correct probability of selecting a green jellybean from the handful shown on the right.

15 Can an outcome have a 200% chance of occurring? Explain your answer.

16 Explain why, when calculating theoretical probability using the formula, all the outcomes must be equally likely.

17 In 1889 Nintendo was founded to sell handmade hanafuda cards (hanafuda translates to 'flower cards'). The hanafuda deck contains 48 cards divided into 12 suits representing the months of the year. The cards are categorised into different types: 24 plains, 10 ribbons (3 with poetry, 3 blue, 4 plain red), 9 animals and 5 brights.

a Which month is most likely to be a plain card? What is the chance of drawing that month's plain card(s)?

b Which month is most likely to not be a plain card? What is the chance of drawing that month's plain card(s)?

c You've drawn a ribbon card, what is the chance that it has poetry on it?

d You've already drawn three plain cards, what is the chance you draw another one?

18 An event with a probability of 0.5 is five times more likely to occur than an event with a probability of 0.1. For each of these pairs of events, determine how many times more likely the second event is to occur than the first.

a Drawing a random card from a standard deck and checking if it is a spade or if it is red.

b Rolling a fair, standard, six-sided die and getting a 5 or getting an odd number.

c A bag contains 18 red marbles, 6 blue marbles and 3 pink marbles. A marble is picked at random and checked if:

- **i** it is red or if it is blue
- **ii** if it is pink or it is blue
- **iii** if it is red or it is pink.

19 A drawer contains 5 wood, 5 stone, 5 marble and 5 glass coasters. A coaster is pulled out of the drawer at random and its type {wood, stone, marble, glass} is noted.

- **a** Are the events equally likely?
- **b** The coaster is returned to the drawer after it is pulled out before the next one is pulled out. Are the events equally likely?
- **c** The coaster is not returned to the drawer after it is pulled out before the next one is pulled out. Are the events equally likely?
- **d** A different type of coaster is returned to the coasters after one is pulled out before the next one is pulled out. Are the events equally likely?

20 Determine if the following statements are true or false.

- **a** A fraction always gives the exact probability.
- **b** Percentages allow for comparisons more easily than fractions.
- **c** A decimal must always be rounded to give the probability.
- **d** Decimals are more useful than percentages when the probabilities are very close to 0 or 1 (e.g. 0.0001 and 0.99999).
- **e** Fractions always make the probability easy to interpret.

21 List all possible outcomes for each of the following experiments. Then determine the probability of each of these outcomes, giving your answers in the forms shown in the brackets. Finally, add together all the probabilities of the outcomes for each experiment.

- **a** A fair, standard, six-sided die is rolled and the number is recorded. (Fraction)
- **b** A fair coin is tossed and the result is recorded. (Decimal)
- **c** A counter is chosen from a bag containing 9 red and 16 blue counters and the colour is recorded. (Fraction)
- **d** A sticker is chosen from a shuffled pile containing 31 ‘Good Work’ stickers, 52 ‘Awesome’ stickers and 17 ‘Fabulous’ stickers, and the type is recorded. (Percentage)

22 Determine the probability of the following outcomes.

- **a** A biased die is rolled. The probability of rolling a 1 is 0.2, rolling a 2 is 0.1, rolling a 3 is 0.2, rolling a 4 is 0.1, rolling a 5 is 0.2. What is the probability of rolling a 6?
- **b** A rigged coin is flipped. The probability of the coin landing on heads is 100%. What is the probability of the coin landing on tails?
- **c** The probability of correctly choosing the 6 winning numbers from 1 to 49 in the lottery is 1 in 13 983 816. What is the probability of not choosing all 6 winning numbers?
- **d** A marble is picked out of a bag containing orange, pink and aqua marbles. The probability of picking an orange marble is $\frac{1}{3}$ and the probability of selecting a pink marble is $\frac{1}{4}$. What is the probability of selecting an aqua marble?

Exercise 10.5C: Challenge

23 A fair 10-sided die with the numbers 0–9 is rolled.

- **a** If the die is rolled once, what is the probability of:
  - **i** rolling an odd number
  - **ii** rolling a positive, even number
  - **iii** rolling a prime number?
- **b** Draw a table to show all possible outcomes when rolling the die twice. Hint: Put the numbers from one roll in the columns of the table and the numbers from the other roll in the rows of the table.
- **c** Use your answer to part **b** to help you find the probability of throwing a total of 15 when rolling the die twice.

Online resources:

Interactive skillsheet
Calculating theoretical probability

Worksheet
Calculating theoretical probability

Quick quiz
10.5

Lesson 10.6

Experimental probability

Learning intentions

By the end of this lesson you will be able to …

✔ calculate expected results and experimental probabilities.

Inter-year links

Support	Chance experiments
Year 8	10.5 Two-way tables
Year 9	8.4 Relative frequency

Key content video
Relative frequency

Experimental probability

- A **successful trial** is a trial where the desired event occurs.
  For example, an experiment performed to determine the probability of rolling a 6 on a standard die has a successful trial when a 6 is rolled.
- The **relative frequency** (or **experimental probability**) of an event occurring is determined by an experiment involving a number of trials.

$$\text{Relative frequency} = \frac{\text{number of successful trials}}{\text{total number of trials}}$$

  → The theoretical probability of an event can be used to predict the relative frequency. However, the relative frequency will not always be the same as the theoretical probability since experiments involve random chance.
- The theoretical probability of an event can be used to determine the expected number of successful trials:

$$\text{Expected number of successful trials} = \Pr(\text{event}) \times \text{total number of trials}$$

  For example, if the expected outcome of rolling a die 1000 times is required, this formula could be used instead of rolling a die 1000 times.
- **Simulation** involves using digital technology or simple devices like coins or dice to simulate real experiments.
  → Most calculators and spreadsheets have a built-in random number generator that can be used to perform simulations.

The law of large numbers

- For a small number of trials, the relative frequency may be significantly different from the theoretical probability.
- As the number of trials increases, the relative frequency is expected to approach (get close in value to) the theoretical probability. This is known as the **law of large numbers**.

Worked example 10.6A Calculating expected results and experimental probabilities

A fair coin is flipped 50 times.

a Calculate the theoretical probability of obtaining a head when a coin is flipped once.

b How many times would you expect to obtain a head based on the theoretical probability?

c When the experiment was carried out, 27 heads were obtained. Calculate the relative frequency of obtaining a head.

THINK

a Use the theoretical probability formula to calculate the probability of obtaining a head. There is one head and two possible outcomes: heads or tails.

b Calculate the expected number of successful trials by multiplying the theoretical probability by the number of trials in the experiment.

c Calculate the relative frequency by dividing the number of heads flipped by the total number of trials.

WRITE

a $\Pr(\text{head}) = \dfrac{\text{number of favourable outcomes}}{\text{total number of all possible outcomes}}$

$= \dfrac{1}{2}$

b Expected number of heads $= \Pr(\text{head}) \times \text{total number of trials}$

$= \dfrac{1}{\cancel{2}_{1}} \times \cancel{50}^{25}$

$= 25$

c Relative frequency $= \dfrac{\text{number of heads}}{\text{total number of trials}}$

$= \dfrac{27}{50}$

Helpful hints

- ✔ If you repeat an experiment more than once, you are likely to get different results! For example, if you flip a coin 10 times you won't get 5 heads every time you repeat the experiment.
- ✔ Remember to simplify fraction answers where possible.
- ✔ The expected number of successful trials for an experiment may not be a whole number. Instead it is the mean result if the experiment was repeated indefinitely. For example, in an experiment of five coin tosses the expected number of heads is 2.5, even though that is not a possible result of the experiment.

Learning pathways

△ 1–5, 9, 10, 13(a–d)

■ 3–7, 8–11, 13, 14(a–d)

⬟ 4, 6, 8–15

ANS p567

Exercise 10.6A: Understanding and fluency

WE 10.6A **1** A fair coin is flipped 20 times.

- **a** Calculate the theoretical probability of obtaining a tail.
- **b** How many times would you expect to obtain a tail based on the theoretical probability?
- **c** When the experiment was carried out, 12 tails were obtained. Calculate the relative frequency of obtaining a tail.

2 A standard six-sided die was rolled 60 times.

- **a** Calculate the theoretical probability of obtaining a 1.
- **b** How many times would you expect to obtain a 1 based on the theoretical probability?
- **c** When the experiment was carried out, eight 1s were obtained. Calculate the relative frequency of obtaining a 1.

3 A fair coin is flipped 20 times and 13 tails are obtained.

- **a** Calculate the relative frequency of obtaining a tail.
- **b** Compare this relative frequency to the theoretical probability of obtaining a tail.
- **c** If the coin was flipped 2000 times, would you expect the relative frequency of obtaining a tail to increase, decrease or stay the same? Explain.

4 A standard six-sided die is rolled 60 times and six 1s are obtained.

- **a** Calculate the relative frequency of obtaining a 1.
- **b** Compare this relative frequency to the theoretical probability of obtaining a 1.
- **c** If the die was rolled 6000 times, would you expect the relative frequency of obtaining a 1 to increase, decrease or stay the same? Explain.

5 To determine the shopping habits of people at a shopping centre, 100 people were surveyed as they left the shopping centre. They were asked: 'What was the main type of shopping you did from this list?' {groceries, clothing, electrical and entertainment, medical, window shopping (browsing)}. The data was tabulated as shown.

Type of shopping	**Frequency**
Groceries	61
Clothing	18
Electrical and entertainment	12
Medical	4
Window shopping	5

- **a** What is the probability that the next person surveyed will have mainly done grocery shopping based on the relative frequencies? Give your answer as a decimal.
- **b** The relative frequency of a shopper having mainly done clothes shopping is 18%. Why is the percentage equal to the frequency?
- **c** If 11 000 people were at the shopping centre on Tuesday, and the survey is representative of the population, then how many people would be expected to have been window shopping?

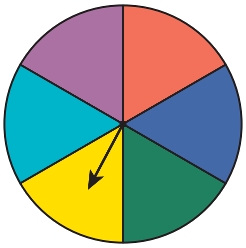
6 David was rolling a four-sided die and keeping track of the number of 1s, 2s and 3s he had rolled. He decided to determine the relative frequency of each number after 50 rolls but realised that he hadn't been tracking the number of 4s rolled. His table looks like this:

Number	1	2	3	4
Relative frequency	0.24	0.16	0.34	

- **a** Determine the relative frequency of the number 4 appearing.
- **b** Determine the amount of 4s he must have rolled.

Exercise 10.6B: Problem solving and reasoning

7 Jade flips a coin 50 times, recording 27 heads and 23 tails. April also records 27 heads and 23 tails when she flips a coin 50 times. Would you expect their friend Peter to record the same results if he flips a coin 50 times? Explain why or why not.



8 Maddy spins this spinner with equally sized segments 100 times and records the following results.

Colour	Purple	Red	Dark blue	Green	Yellow	Light blue
Frequency	13	16	20	19	14	18

a Calculate the relative frequency of landing on each colour.

b If Maddy spins the spinner once more, would it be more likely to land on green or yellow, or would each colour have the same probability of occurring? Explain your answer.

9 Eric rolled a 6-sided die 36 times and recorded his results in the following table.

Number	**Expected frequency**	**Actual frequency**	**Theoretical probability**	**Relative frequency**
1	6	3	$\frac{6}{36}$	$\frac{3}{36}$
2	6	5	$\frac{6}{36}$	$\frac{5}{36}$
3	6	7	$\frac{}{36}$	$\frac{}{36}$
4	6	5		
5	6	5		
6	6	11		

a Complete the table by calculating the theoretical probability and the relative frequency of throwing each number.

b Is the relative frequency of rolling a 6 larger or smaller than the theoretical probability?

c A small number of trials can lead to misrepresentative results. However, results can also be intentionally skewed, or distorted, by tampering with the equipment used in an experiment. To find out if this die is unfair, another 3600 trials were conducted.

The results of the additional trials are given in the table below. Would you say that this is a fair die? Why or why not?

Number	**Expected frequency**	**Actual frequency**
1	600	285
2	600	576
3	600	745
4	600	467
5	600	512
6	600	1015

10 In a monster-catching game, a rare monster is claimed to appear 5% of the time. You want to test the validity of that claim, so you ask your friend to join you in an experiment where you will encounter 50 monsters each and write down how many times the rare monster appears. You encountered the rare monster two times and your friend encountered it seven times.

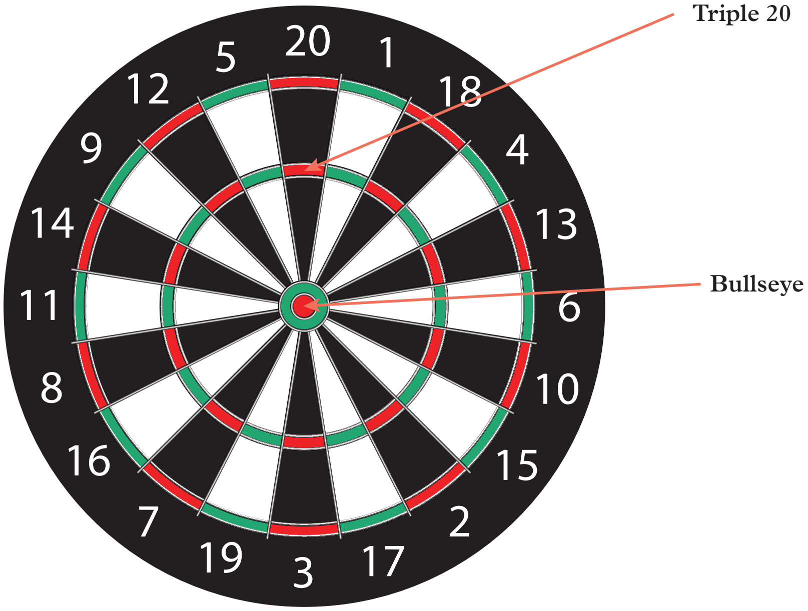
a Calculate the relative frequency of your sample and your friend's sample as a percentage.

b Do you believe the claim that the monster appears 5% of the time? Why or why not?

c A special form of the rare monster has a chance of appearing of 1 in 4096. Write this as a percentage to four decimal places.

d If you encounter the rare monster 4095 times, without seeing the special form, are you guaranteed to encounter the special form on your next encounter? Why or why not?

11 Pritesh plays darts. He wants to determine how likely it is to score a bullseye and a triple 20, provided the dart never misses the board. He takes a picture of his dart board and writes a computer program to define each pixel on the board as a point the dart can hit. He then commands the program to throw 1000 darts at the board randomly. Of the 1000 darts, 1 dart hits the bullseye and 5 darts hit the triple 20.



a Calculate, as a simplified fraction, the relative frequency of scoring:

i a bullseye **ii** a triple 20.

b Pritesh uses the computer program to find the number of pixels on the board for the bullseye and for the triple 20. The program prints the following number of pixels:

Total: 916 088, Bullseye: 1283, Triple 20: 4214.

Calculate, as a percentage to two decimal places, the probability of scoring:

i a bullseye **ii** a triple 20.

12 A game show has a large tub containing 10 000 coloured marbles. The aim of the game is to get the closest guess to the number of red marbles in the tub. You are allowed to take three samples of the marbles and let a machine count the number of red marbles and total marbles in the sample.

The first contestant Vernica takes her three samples. She gets 49 reds out of 87, 34 reds out of 57, and 43 reds out of 74.

a Calculate the relative frequency of each of Vernica's samples as a percentage, correct to the nearest percent.

b She decides to use her median relative frequency to estimate the number of red marbles. Calculate her estimate.

c The second contestant Jake takes his three samples. He gets 54 reds out of 95, 19 reds out of 32, and 31 reds out of 53.

i Calculate the relative frequency of each of Jake's samples as a percentage, correct to the nearest percent.

ii He decides to calculate the mean relative frequency to estimate the number of red marbles. Calculate his estimate.

iii Still not confident in his estimate, Jake decides to combine his three samples and calculate the relative frequency to the nearest hundredth of a percent. He uses this figure to re-estimate the number of red marbles. Calculate his second estimate.

iv Jake locks in his second estimate. If the number of red marbles in the tub was 5821, which contestant won? Did Jake choose his best estimate?

13 An experiment involves rolling two dice and recording the sum of the values shown on each die.

		Die 2					
		1	**2**	**3**	**4**	**5**	**6**
Die 1	**1**	2	3	4	5	6	7
	2	3	4				
	3						
	4						
	5						
	6						

a Complete the table.

b How many chances are there of rolling a total of 6?

c What is the theoretical probability of rolling a total of 6?

d How many times would you expect to roll a total of 6 if you rolled the dice 36 times?

e Perform the experiment and tally the results.

f Compare the results of your experiment to the theoretical probability of rolling a total of 6. Comment on the reasons for the similarities or differences between the two figures.

14 The chocolate company Mars often runs a promotion in which you can win a free Mars Bar. One out of every six wrappers is printed with a token for a free bar.

a What is the theoretical probability of winning a Mars Bar?

b What is the theoretical probability of *not* winning a Mars Bar?

c Using your theoretical probabilities, calculate how many Mars Bars you could expect to buy in order to win five free Mars Bars.

d Describe a probability experiment that has six possible outcomes and can be used to simulate the chance of winning a free Mars Bar.

e Conduct your experiment 30 times. How many free Mars Bars did you win?

f For better results, simulations should always be repeated. Repeat this experiment another four times and record the number of free Mars Bars you win each time.

g What are some of the advantages and disadvantages of running multiple simulations when conducting an experiment?

Exercise 10.6C: Challenge

15 A six-sided die was rolled a number of times. The relative frequency of each number appearing is shown below. Determine the smallest number of times the die could have been rolled to give these probabilities.

Number	1	2	3	4	5	6
Relative frequency	$\frac{1}{10}$	$\frac{1}{3}$	$\frac{1}{7}$	$\frac{1}{5}$	$\frac{1}{6}$	$\frac{2}{35}$

Online resources:

Interactive skillsheet Relative frequency

Worksheet Comparing experimental and theoretical probability

Investigation Rolling a 10-sided die

Quick quiz 10.6

Simulations

The Law of Large Numbers

The relative frequency of an event occurring can be determined by an experiment involving a number of trials.

$$\text{Relative frequency} = \frac{\text{number of sucessful trials}}{\text{total number of trials}}$$

There is a concept in probability called the Law of Large Numbers, which states that as the number of trials in a random chance experiment increases, the more likely it is that the relative frequency of an event will approach the theoretical probability of that event.

Take tossing coins, for example. If you toss a fair coin 10 times, then you might expect it to land on heads 5 times. However, it would not be surprising if it landed on heads 7 times or more. However, if you toss a coin 100 times, it is extremely unlikely it will land on heads 70 times or more.

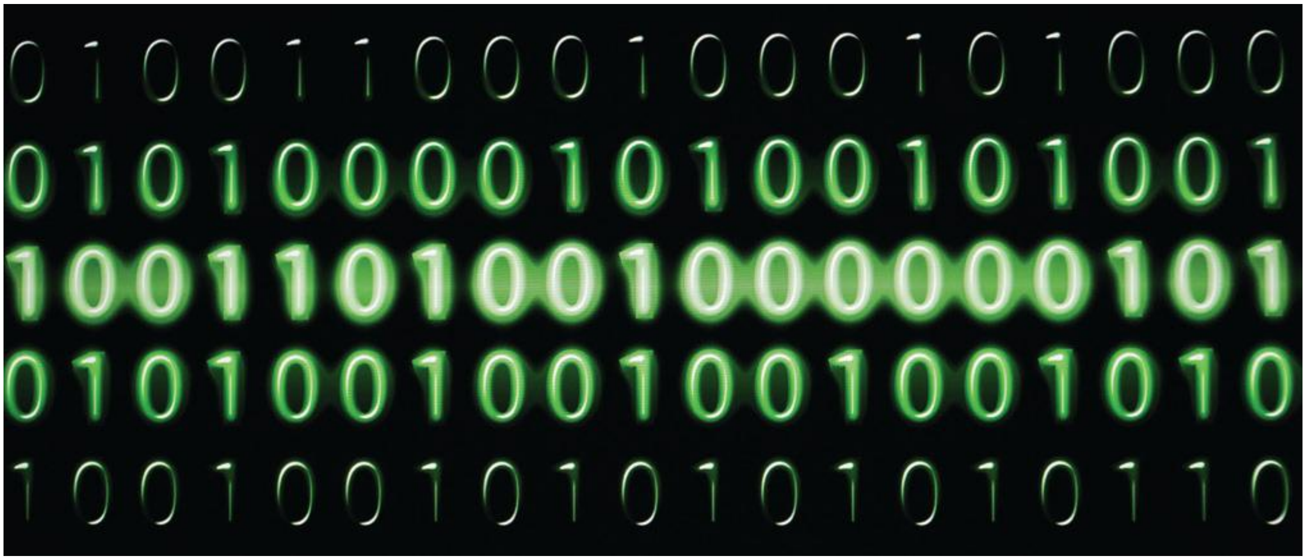
Curriculum link

Conduct repeated chance experiments and run simulations with a large number of trials using digital tools; compare predicted with observed results, explaining the differences and the effect of sample size on the outcomes. (VC2M7P02)

Simulation: Tossing coins

A simulation is a recreation of the random element of an experiment. For example, rather than physically tossing coins, we can simulate tossing coins using a computer. This saves us the time and effort of tossing coins and recording the results.

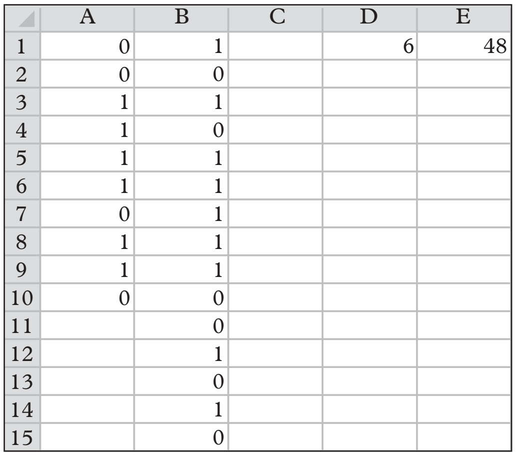
For this simulation, we will use a spreadsheet program to simulate tossing a coin many times. Using a spreadsheet program allows us to simulate tens or hundreds of coin tosses that would otherwise take a significant amount of time to perform and record. A simulation using a spreadsheet can also be performed multiple times with minimal effort. By repeating a simulation many times, we can see if our observations are one-offs or if they repeatedly occur.



Instructions

Open a new Microsoft Excel document or a similar spreadsheet program.

We will simplify the code we have to write by allowing 1s to represent heads and 0s to represent tails.

Code	**Explanation**
1 In cell A1, type in '=RANDBETWEEN(0,1)' and press Enter.	This randomly generates a 0 or 1 with equal chance, just like a coin toss lands on a tail or a head with equal chance.
2 Drag the bottom right corner of cell A1 down to cell A10.	This will add up all the non-empty entries in columns A and B. Your spreadsheet should look similar to the following image: 
3 Write the same formula in cell B1 and press Enter, this time dragging it down to B100.	
4 In cell D1, type in '=SUM(A:A)' and in cell E1, type in '=SUM(B:B)'.	
5 In cell D2, type in '=SUM(A:A)/10' and in cell E2, type in '=SUM(B:B)/100'.	This divides the sum by the total number of coin flips, which gives the relative frequency of heads.

	A	B	C	D	E
1	0	1		6	48
2	0	0			
3	1	1			
4	1	0			
5	1	1			
6	1	1			
7	0	1			
8	1	1			
9	1	1			
10	0	0			
11		0			
12		1			
13		0			
14		1			
15		0			

Simulation questions

1 Are the numbers in your spreadsheet different to the numbers in the image shown in the table? If so, is that a problem? Why or why not?

2 Referring to coin tosses, what does the number in cell D1 represent, and what does the number in cell E1 represent?

You may notice that every time you execute a formula, the whole spreadsheet refreshes. This can also be done manually by pressing F9. Refreshing the spreadsheet means all the formulas are executed again, and so all the randomly generated numbers are re-generated.

3 Repeatedly press F9 while focusing on the numbers in D2 and E2. What do you notice? How often is the number in D2 0.7 or higher? How often is the number in E2 0.7 or higher?

4 Write a short sentence explaining how the results of the simulation help to confirm the Law of Large Numbers.

Extension

Use a similar method to the one described in the instructions to simulate 1000 coin tosses. What do you notice about the relative frequency of heads from 1000 tosses?

Hint: The formula '=RANDARRAY(1000,1,0,1,TRUE)' can be used to simulate all 1000 tosses at once, without having to drag a cell all the way down to row 1000.

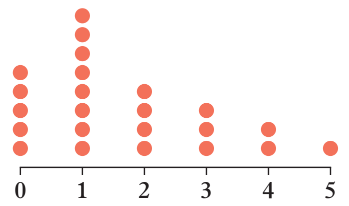
Lesson 10.7

Review: Statistics and probability

Module summary

Dot plots

Number of siblings



Summary statistics

$$\text{Mean (numerical average)} = \frac{2+3+4+4+6+7+10}{7}$$

$$= 5$$

Median (middle): 2, 3, 4, 4, 6, 7, 10

Mode (most common): = 2, 3, 4, 4, 6, 7, 10

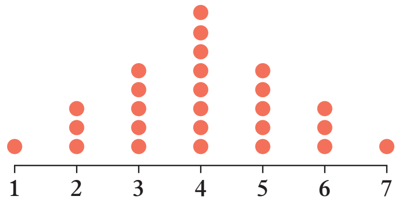
$$\text{Range} = 10 - 2$$

$$= 8$$

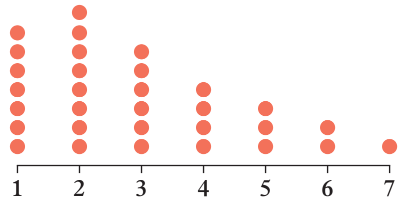
Outlier: 1, 2, 3, 3, 100

Describing data

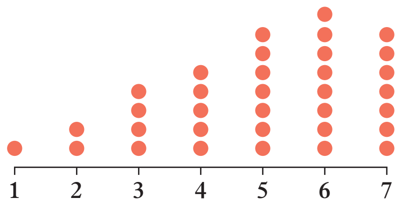
Symmetric



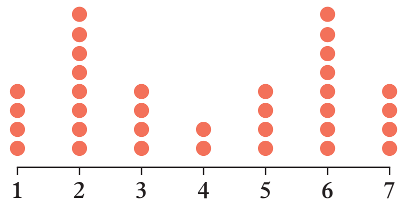
Positively skewed



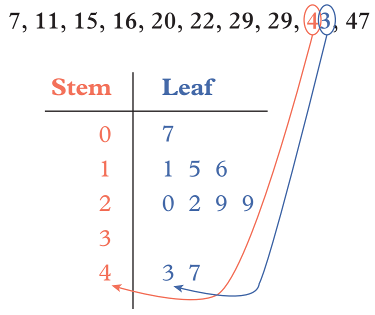
Negatively skewed



Bi-modal



Stem-and-leaf plot

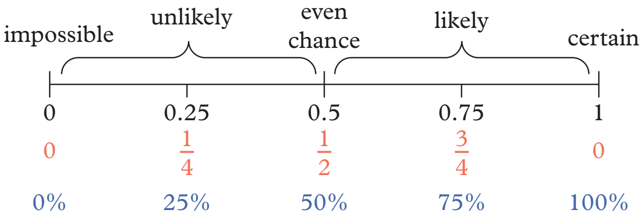


7, 11, 15, 16, 20, 22, 29, 29, 43, 47

Stem	Leaf
0	7
1	1 5 6
2	0 2 9 9
3	
4	3 7

Key: 1 | 5 = 15

Theoretical probability



For a standard 6-sided die:

Sample space = {1, 2, 3, 4, 5, 6}

Since each outcome is equally likely:

$$\text{Pr(rolling a 3)} = \frac{\text{number of favourable outcomes}}{\text{total number of possible outcomes}}$$

$$= \frac{1}{6}$$

Experimental probability

$$\text{Relative frequency} = \frac{\text{number of successful trials}}{\text{total number of trials}}$$

Expected number = Pr(event) × number of trials

ANS p568

Review questions 10.7A: Mathematical literacy review

Review quiz Take the review quiz to assess your knowledge of this module.

Quizlet Test your knowledge of this module by working individually or in teams.

The following key terms are used in this module:

- biased
- bi-modal distribution
- continuous data
- discrete data
- dot plot
- equally likely outcomes
- event
- experiment
- experimental probability
- fair
- favourable outcome
- frequency
- frequency table
- law of large numbers
- maximum
- mean
- median
- minimum
- mode
- negatively skewed distribution
- numerical data
- outcome
- outlier
- positively skewed distribution
- probability
- range
- relative frequency
- sample space
- simulation
- skewed distribution
- split stem-and-leaf-plot
- stem-and-leaf-plot
- successful trial
- summary statistics
- symmetric distribution
- theoretical probability
- trial

1 Explain the difference between a positively skewed distribution and a negatively skewed distribution.

2 Explain the difference between a split stem-and-leaf plot and a regular stem-and-leaf plot.

3 Explain the difference between discrete data and continuous data.

4 Identify the key terms being referenced in each of these definitions.

a the most common value in a numerical data set

b a repeatable procedure with a result which is determined by chance

5 Using an example, provide a definition in your own words for the following key terms.

a outlier

b sample space

6 Complete the following sentences using words from the key terms list.

a ________ are made up of one or more outcomes. A ________ is a result that you are interested in.

b The ________ is the difference between the (________) and (________) values in a data set.

Review questions 10.7B: Multiple choice

10.1 **1** Which of the following is an example of discrete numerical data?

A Your height

B Your finishing position in a race

C The number of pets that you have

D The colour of your hair

10.1 **2** What is the median of the data set: 5, 3, 4, 7, 9, 16, 5, 6, 8?

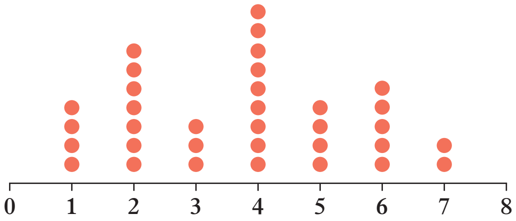
A 5 **B** 6 **C** 7 **D** 9

10.1 **3** What is the mean of the data set: 6, 9, 11, 7, 3, 15, 7, 14?

A 7 **B** 8 **C** 9 **D** 12

10.2 **4** What is the mode of the data presented in the dot plot below?

A 3 **B** 4 **C** 6 **D** 9



10.2 **5** What is the median of the data presented in the dot plot above?

A 2 **B** 3 **C** 3.5 **D** 4

10.3 **6** The key to a stem-and-leaf plot is 1|2 = 12. What is the leaf of the data point 48?

A 4 **B** 8 **C** 12 **D** 48

10.3 **7** How would you describe the shape of the distribution of the data shown in the split stem-and-leaf plot on the right?

A Symmetric

B Positively skewed

C Negatively skewed

D Bi-modal

Stem	**Leaf**
7	4
7	6 8
8	0 2 4
8	5 6 7
9	1 2 3 4
9	6 7 8 9 9

Key: 7|4 = 74

10.4 **8** How could the possibility of Friday following on from Thursday be described?

A Likely

B Even chance

C Unlikely

D Certain

10.5 **9** A letter is selected at random from the word MATHEMATICS. What is the probability, expressed as a fraction, of selecting an *M*?

A $\frac{1}{8}$ **B** $\frac{1}{11}$ **C** $\frac{1}{4}$ **D** $\frac{2}{11}$

10.6 **10** If a standard six-sided die is rolled 120 times, how many times would you expect to obtain a four?

A 4 **B** 20 **C** 30 **D** 120

Review questions 10.7C: Short answer

1 Consider the following data set: 7, 1, 8, 12, 71, 6, 14, 10, 2, 2, 3, 5, 9.

10.1 **a** Calculate the:

- **i** mean (to one decimal place)
- **ii** median
- **iii** mode
- **iv** range for this group of data.

b Explain why the mean is not the most appropriate summary statistic to represent this group of data.

10.1 **2** Ms Oxford set a maths test for her class and calculated the mean, median, mode and range of the results. Which statistic is represented by each statement about the results?

a More students got 42 than any other score.

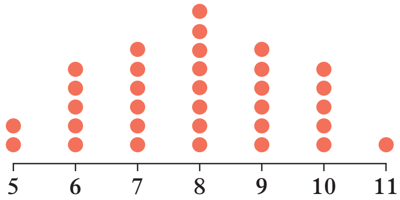
b Half of the students scored more than 39.

c The numerical average of the scores was 40.5.

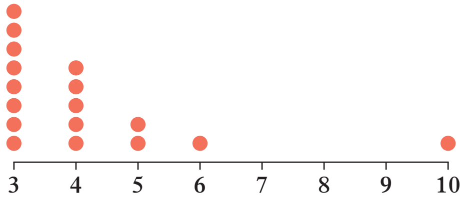
d The difference between the highest and lowest score was 50.

10.2 **3** Describe the shape of the distribution of the data shown in these dot plots.

a



b



10.2 **4** The number of times in a week that different families in a street went grocery shopping is listed below.

1, 2, 1, 3, 1, 7, 2, 1, 1, 1, 4, 1, 2, 1, 5, 1, 2, 1, 3, 2, 1, 1

a Present the data in a dot plot.

b Comment on the shape of the distribution of the data.

10.3 **5** Redraw the following stem-and-leaf plot as a split stem-and-leaf plot with each stem split into two rows.

Stem	Leaf
3	3 4 6 6 8 9
4	0 1 1 1 3 5 6 6 7 8 8 9 9 9
5	0 2 2 4 7 9

Key 3 | 4 = 34

10.3 **6** This stem-and-leaf plot on the right shows the lengths of 23 worms in a worm farm.

Stem	Leaf
8	1 5 9
9	2 2 3 5 6 8
10	1 1 1 2 2 4 6 7 8
11	0 1 2 2
12	7

Key 8 | 1 = 8.1 cm

a Write down the values represented by the leaves 4 and 9.

b Which stem has the highest frequency?

c What was the mean worm length, correct to the nearest tenth of a centimetre?

d What was the median worm length?

e What was the modal worm length?

f What was the range of worm lengths?

g Write two sentences to describe the lengths of the worms in the worm farm.

10.3 **7** Arrange the data below into a stem-and-leaf plot.

13 16 45 32 62 17 22 28 55 9 12 38 42
51 59 31 16 13 12 25 30 41 20 5 23 33

10.4 **8** For each of the following experiments:

i list the sample space

ii categorise the outcome in brackets as: impossible, unlikely, even chance, likely, certain.

- **a** flipping a fair coin (head)
- **b** rolling a standard, six-sided die (odd number)
- **c** selecting a card from a standard 52-card deck and recording the suit (diamond)
- **d** randomly selecting a counter from a bag containing 12 red, 7 blue and 2 white counters (red counter).

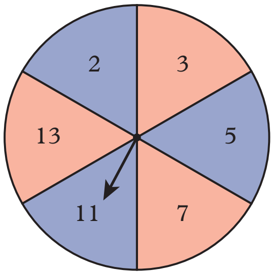
10.5 **9** Calculate the theoretical probability of:

a rolling a standard six-sided die and obtaining a 3

b flipping a fair coin and landing on tails

c selecting a consonant from the word REVISION.

10.5 **10** The spinner below with equally sized segments is spun once and the number the spinner lands on is recorded.



a List the sample space.

b What is the probability of landing on an odd number?

c What is the probability of landing on a number less than 10?

10.6 **11** A card is drawn from a standard deck of 52 cards. The suit is noted, then the card is returned to the deck and the deck is shuffled. The first 10 recorded results are:

hearts, clubs, hearts, clubs, diamonds, hearts, diamonds, spades, hearts, clubs

a Calculate the relative frequency, as a decimal, of obtaining:

- **i** a heart
- **ii** a spade
- **iii** a club
- **iv** a diamond
- **v** a red card (hearts or diamonds).

b Which relative frequency (or relative frequencies) from part **a** is (or are) closest to the theoretical probability of the same event?

Review questions 10.7D: Mathematical modelling

1 A survey was conducted on the number of movies a group of Year 7 students had watched in the past month. The results are below.

1 5 3 2 7 1 4 2 3 6 2 5 7 13 2 2 1 0 4 2 5 2 1 0 5 1

a What type of data is this?

b How many people were surveyed?

c Calculate the:

- **i** mean (to one decimal place)
- **ii** median
- **iii** mode
- **iv** range for this data set.

d Display this data and explain why you think this is the best choice to present this particular data set.

2 A bag contains 20 marbles. 10 are bronze, 5 are silver, 3 are gold, and the rest are iron. A marble will be pulled out of the bag at random and then replaced.

a State the sample space of the experiment.

b Calculate the theoretical probability of selecting an iron marble.

c A marble is picked from the bag and then replaced. This is repeated 12 times with the following outcomes: silver, iron, silver, bronze, bronze, bronze, bronze, bronze, silver, gold, silver, bronze.
Calculate the relative frequency of selecting an iron marble.

d Another 12 trials are repeated with the following outcomes:

bronze, iron, bronze, bronze, bronze, bronze, bronze, silver, iron, bronze, gold, bronze.

Calculate the relative frequency of selecting an iron marble.

e Give a reason why the relative frequencies are not equal to each other or to the theoretical probability in part **b**.

3 Ji opens his piggy bank and finds fifty coins inside:

20c, \$1, 50c, 50c, 5c, 20c, \$2, \$1, 10c, 20c,
50c, \$1, \$1, 5c, 20c, 50c, 20c, \$1, 5c, 10c,
\$2, 50c, 5c, 10c, 10c, 10c, \$1, \$2, 5c, 20c,
10c, \$1, \$1, \$2, 50c, 10c, 20c, 5c, 5c, 10c,
50c, 20c, \$1, 5c, \$2, 10c, 20c, 5c, \$1, 50c

a Record the data in a frequency table.

b What is the total value of the coins in Ji's piggy bank?

c What is the mean value of the coins in Ji's piggy bank?

d Ji picks out a coin at random. Assuming all coins have an equal chance of being picked, what is the probability that:

i a 50c coin is picked

ii a gold coin is picked

iii a coin worth 10c or less is picked.

e Ji picks out 20 coins at random, putting each coin he selects back into the pile before selecting another coin. He records the following results.

Coin	5c	10c	20c	50c	\$1	\$2
Frequency	3	3	5	5	3	1

i Which of the relative frequencies are higher than the corresponding theoretical probabilities of picking that coin?

ii Based on your answer to part **i**, is it reasonable to assume that all coins have an equal chance of being picked? Explain your answer.

Checklist

Now that you have completed this module, reflect on your ability to do the following.

I can do this	**I need to review this**
☐ Classify numerical data as discrete or continuous ☐ Calculate the mean, median, mode and range of a numerical data set ☐ Interpret summary statistics in the context of the data	☐ Go back to Lesson 10.1 Summary statistics
☐ Construct dot plots ☐ Calculate summary statistics from a dot plot ☐ Describe the shape of the distribution of data in a dot plot	☐ Go back to Lesson 10.2 Describing data with dot plots
☐ Construct stem-and-leaf plots ☐ Calculate summary statistics from a stem-and-leaf plot ☐ Describe the shape of the distribution of data in a stem-and-leaf plot	☐ Go back to Lesson 10.3 Stem-and-leaf plots
☐ Use probability terminology ☐ List sample spaces of experiments ☐ Distinguish between equally likely outcomes and outcomes that are not equally likely	☐ Go back to Lesson 10.4 Describing probability
☐ Calculate theoretical probabilities ☐ Express probabilities as fractions, percentages and decimals	☐ Go back to Lesson 10.5 Theoretical probability
☐ Calculate the expected results of an experiment ☐ Calculate experimental probabilities	☐ Go back to Lesson 10.6 Experimental probability

Semester 2 review

ANS p569

Short answer

1 Decide whether the following statements about $3x + \frac{1}{2}y - 4xy + 0.1$ are true or false. For the statements that are false, provide a reason for your answer.

- **a** It is an expression.
- **b** There are 3 terms.
- **c** The coefficient of the y term is $\frac{1}{2}$.
- **d** The coefficient of the xy term is 4.
- **e** The constant term is 0.1.

2 Consider the following function machine.

- **a** Use the function machine to complete the table of values below.

x	0	1	2	3	4
y					

- **b** List the values as a set of coordinates in the form (x, y).
- **c** Plot the coordinates on the Cartesian plane.

3 The cost of one mango is \$2.

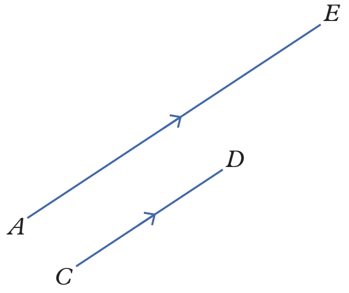
- **a** Draw a table which shows the total cost of buying 1, 2, 3, 4 and 5 mangoes.
- **b** Write a formula for the price, p, of buying m mangoes.
- **c** Use the formula to find the total cost when purchasing 12 mangoes.

4 Solve the following equations to find the value of x.

- **a** $2x = 10$
- **b** $x - 9 = 15$
- **c** $3x + 2 = 8$
- **d** $\frac{x - 6}{4} = 9$

5 Describe the following diagrams using standard naming conventions and correct mathematical terminology.

a



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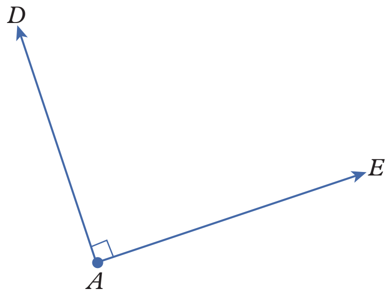
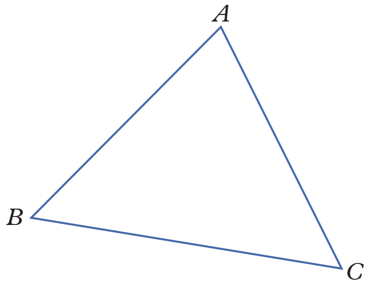


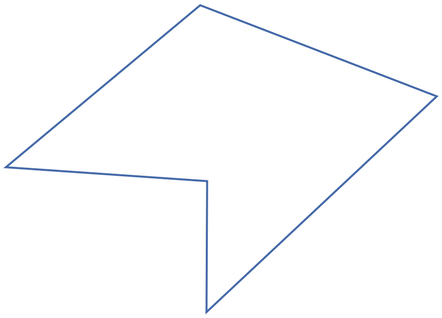
6 Consider the following triangle.



- **a** Measure $\angle ABC$ to the nearest degree.
- **b** If $\angle ACB = 54°$, what type of triangle is $\triangle ABC$?

7 State whether each of the following shapes are polygons or not.

a



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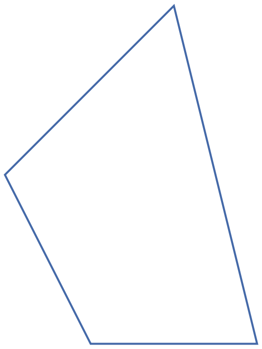

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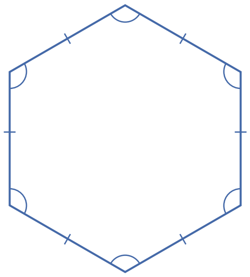

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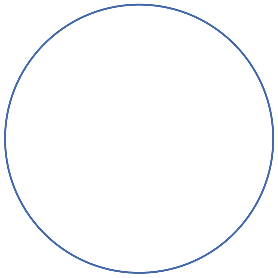

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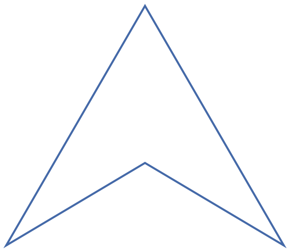

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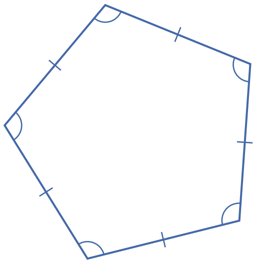
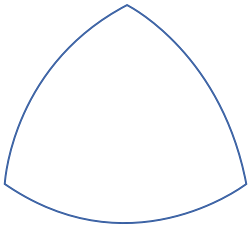


g



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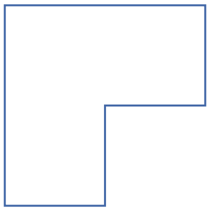
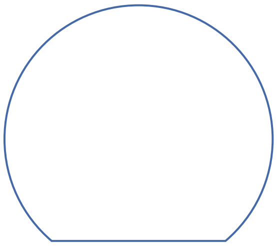


i



8 Complete the following for each of the polygons in question **7**.

- **i** State whether it is concave or convex.
- **ii** State whether it is regular or irregular.
- **iii** Give the mathematical name.

9 Draw two parallel lines cut by a transversal to form an acute angle of 35°. Show that your lines are parallel by measuring and labelling the alternate, corresponding and co-interior angles to your 35° angle.

10 Determine the size of the angles described below.

- **a** Complement of 19°
- **b** Supplement of 19°
- **c** Angle that is vertically opposite to an angle of 19°
- **d** Angle that is co-interior to an angle of 19° between parallel lines
- **e** Angle that is alternate to an angle of 19° between parallel lines

11 Write an expression, equation or formula for each of the following statements.

- **a** Five more than three lots of b
- **b** The product of three less than m and five is equal to n.
- **c** Seven more than one eighth of a number is equal to three. Let u be the number.
- **d** The sum of two numbers divided by 2 is equal to the product of the two numbers. Let p and q be the two numbers.
- **e** Nine lots of the difference between t and 8, where t is greater than 8, is equal to 41.

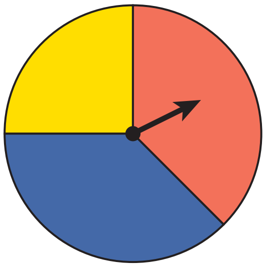
12 Evaluate the expression $a + bc + d$ by substituting each of the following sets of values.

- **a** $a = 2, b = 3, c = 9, d = 12$
- **b** $a = -2, b = 3, c = 9, d = -12$
- **c** $a = \frac{1}{2}, b = \frac{3}{4}, c = \frac{1}{9}, d = \frac{5}{12}$
- **d** $a = 2.5, b = 3.7, c = 0.9, d = 1.24$

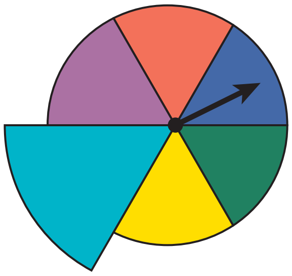
13 For each of the following, state if the chance of it occurring is impossible, unlikely, even chance, likely or certain.

- **a** You buy one ticket for the lottery and win first prize.
- **b** A number is chosen at random from 1 to 20 and the result is even.
- **c** A randomly generated polygon has straight edges.
- **d** A randomly selected chair at a furniture shop has four legs.

14 State whether the chance of each outcome is equally likely.



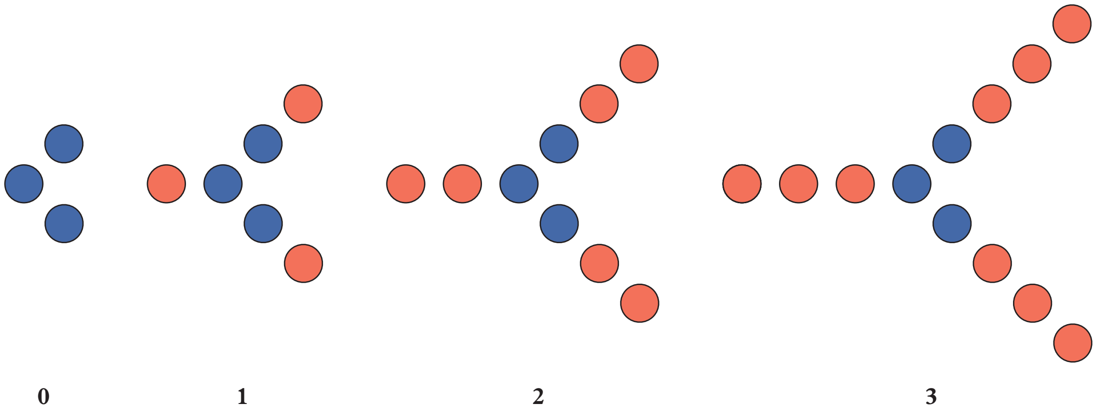
- **a** The pointer on the spinner with three segments (shown above right) is spun and the colour that the pointer lands on is recorded.
- **b** A fair coin is flipped, and it is noted if the coin landed on heads or tails.
- **c** A deck contains 20 brown cards, 10 blue cards and 10 purple cards. A card is drawn from the deck and the colour is noted.
- **d** The faces of a fair, 12-sided die are numbered 1, 2, 3, 4, 4, 6, 7, 8, 9, 10, 11, 12. The die is rolled and the number is noted.
- **e** The pointer on the spinner with 6 equally divided segments (shown below right) is spun and the colour that the pointer lands on is recorded.



15 For each of following numerical data sets:

- **i** state if the data collected is discrete or continuous
- **ii** calculate the median of the data
- **iii** calculate the range of the data.
  - **a** The number of siblings students in a Year 7 class have.
    1, 2, 1, 3, 0, 2, 1, 0, 1, 5, 2, 1, 2
  - **b** The time taken to run one lap of a 200-m sports track (in seconds).
    31, 25, 35, 38, 41, 28, 35, 30, 33, 44
  - **c** The level reached on the first attempt at a mobile game.
    3, 5, 1, 4, 3, 2, 1, 1, 4, 7, 2, 1, 3
  - **d** The weights of oranges in a bag (in grams).
    136, 128, 173, 169, 140, 152, 139, 141

16 Eve creates the following sequence of diagrams using blue and red counters.



- **a** Complete the table below using the diagrams above.

	0	1	2	3
Number of blue counters				
Number of red counters				

- **b** How does the number of blue counters change from one diagram to the next?
- **c** How does the number of red counters change from one diagram to the next?
- **d** Why does the numbering below the diagrams start at 0?
- **e** An expression relating to this sequence is $3r + 3$. Explain what each variable and term in the expression represents. What could the expression be used for?

17 Use the table below to solve the equation $3x + 2 = 5(x - 2)$.

Guess x	**LHS** $3x + 2$	**RHS** $5(x - 2)$	**LHS = RHS?**
2			
4			
6			
8			

18 A rectangle has a perimeter of 24 cm.

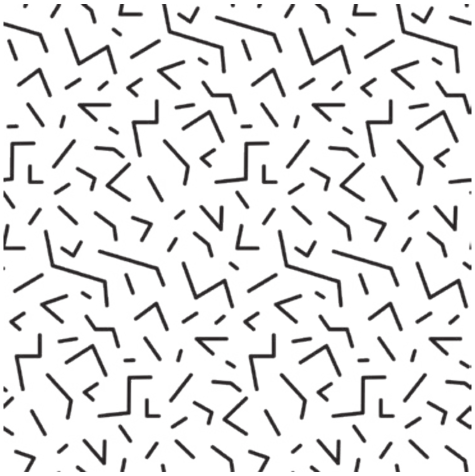
a If the length and width of the rectangle are whole numbers, state all possible pairs of length and width.

b Which pair of lengths creates the rectangle with the greatest area? What is this area?

c A different rectangle with side lengths also has a perimeter of 24 cm. If the length is twice as long as the width, calculate the dimensions of the rectangle.

19 A dressmaker measures a random sample of 12 angles from the print shown on the right. The angles are:

19°, 214°, 321°, 191°, 239°, 225°,
142°, 147°, 271°, 124°, 315°, 26°



a Calculate the following values for the sample, correct to two decimal places:

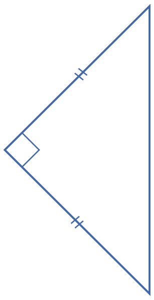
- **i** mean
- **ii** median
- **iii** mode.

b Determine the relative frequency of selecting:

- **i** an acute angle
- **ii** an obtuse angle
- **iii** a reflex angle.

20 State all the possible names of the triangles and quadrilaterals shown below.

a



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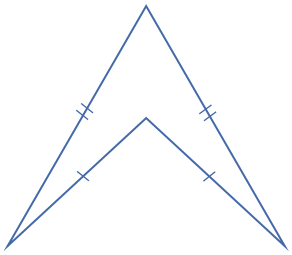
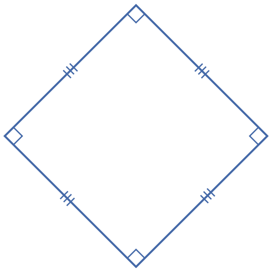


c

d



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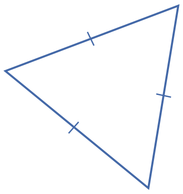

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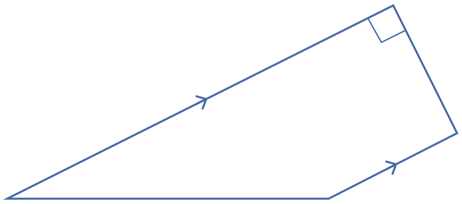


21 A rectangle has an area of 48 cm^2.

a If the length and width of the rectangle are whole numbers, state all possible pairs of length and width.

b Calculate the perimeter of each pair of length and width.

c Consider all the possible whole number side lengths. Calculate the mean and median side length.

d Calculate how many times larger the length is than the width of each possible pair.

e One of these rectangles is randomly selected. Calculate the probability that the selected rectangle:

i has a side length that is a multiple of 4

ii has a side length that is odd

iii has a perimeter greater than 30 cm.

f If a square has an area of 48 cm^2, the side length is $\sqrt{48}$ cm. Between what two whole numbers is this side length?

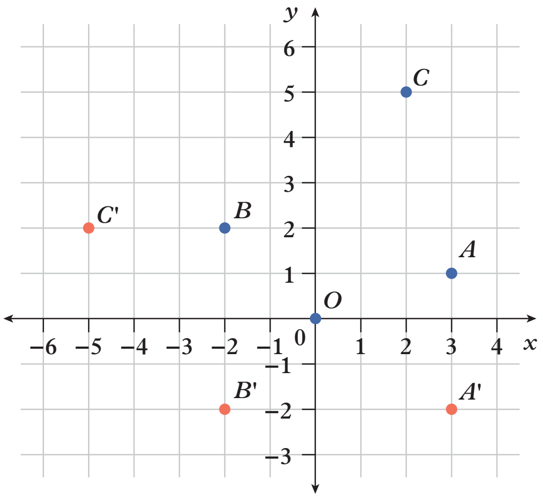
22 In the diagram on the right, each of the points A, B and C have been transformed using a different one of the following transformations to create their images A', B' and C':

→ a rotation of 90° about O either clockwise or anticlockwise

→ a reflection in the x-axis or in the y-axis

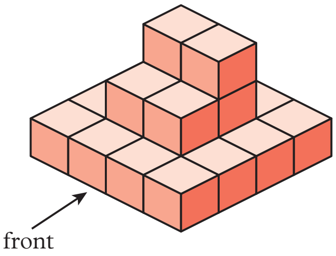
→ a horizontal translation or a vertical translation, but not both.

Determine which transformation has been applied to points A, B and C to create their images.

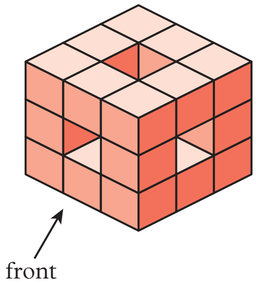


23 Draw the front elevation, side elevation, and plan view of the following 3D objects. For part **b**, assume that every face has the middle cube missing.

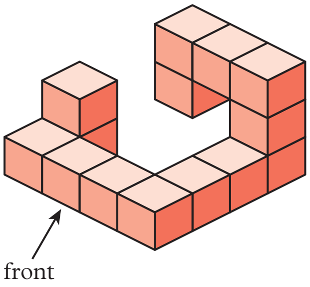
a



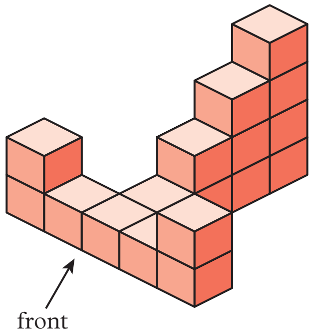
b



c



d



24 For each of the following:

i define the unknown variable

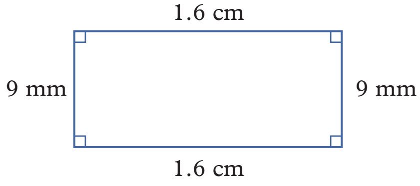
ii write an equation to represent the problem

iii solve the equation.

- **a** One of the co-interior angles between parallel lines is 50°. Determine the size of the other co-interior angle.
- **b** Three of the interior angles in a quadrilateral are 81.5°, 52.3° and 95.1°. Determine the size of the fourth interior angle.
- **c** Each side of a hexagon is 2.1 cm longer than the previous side. Determine the length of the shortest side if the perimeter is 51.9 cm.
- **d** The circumference of a circle is 6 cm, determine the length of the radius.
- **e** The area of a square is 81 cm^2. Determine the side length of the square.
- **f** The area of a triangle with a side length of 4 cm is 40 cm^2. Determine the length of the perpendicular height.
- **g** The volume of a cube is 64 cm^3. Determine the side length of the cube.
- **h** The mean of 3, 5, 8, 12, 4 and one other number is 7. Determine the value of the missing number.

25 Calculate the area of the following shapes.

a



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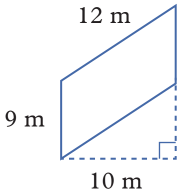

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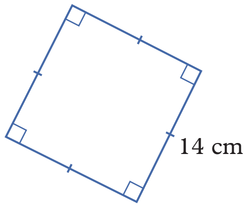

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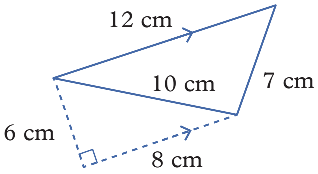

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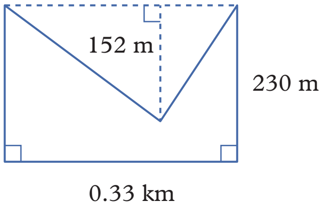

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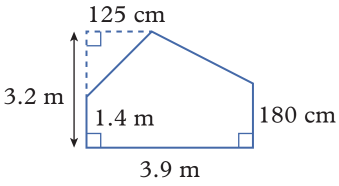
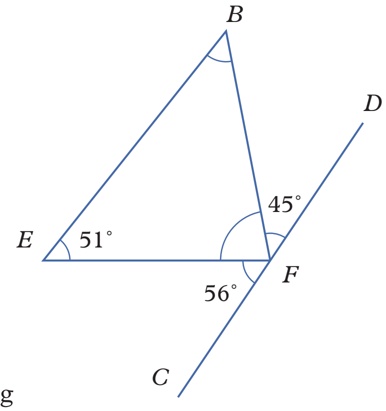


26 Consider the diagram shown on the right.

- **a** Use three letters to name the angle that is 56° in size.
- **b** Are $\overline{EB}$ and $\overline{CF}$ parallel? Use the angles in the diagram to justify your answer.
- **c** If $\angle CFD$ is a straight angle, determine the size of angle $\angle EFB$.
- **d** Determine the size of angle $\angle FBE$. Is ΔBFE an isosceles triangle? Why or why not?



27 Xavier and Yvette are trying to write equations that have a solution of 2. Xavier writes his equations using the pronumeral x and Yvette writes her equations using the pronumeral y. They currently have the following equations:

$4x = 8$	$x - 4 = -2$	$\frac{y}{9} = 18$	$y + 51 = 53$
$\frac{x+1}{3} = 1$	$9(y - 2) = 36$	$x + 5 = 3$	$7y - 30 = -16$

- **a** Determine which equations do not have a solution of 2.
- **b** Yvette argues that $y = 2$ is an equation with the solution $y = 2$. Explain why she is correct.
- **c** Xavier argues that $x = 2$ is the solution to the equation $x + x + x + x + x + x + x + x = x + x + x + x$. Use substitution to explain why he is incorrect.

28 A supermarket chain launched a 'collectathon' promotion, where customers were given limited edition stickers every time they shopped and encouraged to collect all 50. After three months, the supermarket chain surveyed customers at their Lilydale store about the number of unique stickers they had collected. The data they collected is:

15, 2, 32, 43, 39, 7, 37, 44, 12, 45,
49, 26, 20, 34, 21, 26, 33, 22, 48, 2

a Create a stem-and-leaf plot for this data.

b Calculate the:

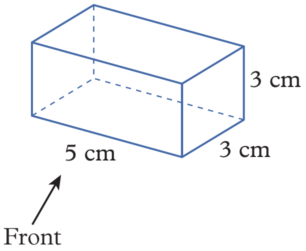
i mean **ii** median **iii** mode(s) **iv** range.

c Write two sentences that describe the number of stickers that surveyed customers have.

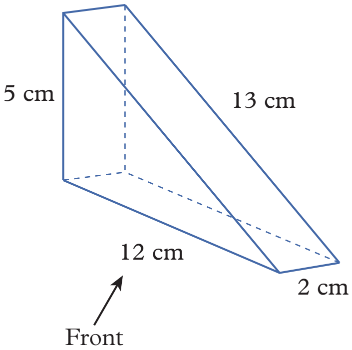
29 What must be added to the expression $4x + 3y + 9x + 8 + 2x - 8x - 5y - 15$ so that it is equal to 0?

30 Tyler is making two 3D models.

Object 1:



Object 2:



a Give the mathematical name of the two objects Tyler is making.

b Determine the number of **i** faces, **ii** edges and **iii** vertices for each object.

c Draw the front, side and top plans for both objects.

d Calculate the volume of both objects

Mathematical modelling

1 Matt uses the table below to record the height of his plant over the first few days of growth. He notices each day that the plant is twice as tall in centimetres as the number of days that have passed.

Days	1	2	3	4	5
Height (cm)	2	4	6		

a Write a formula for the height, h, of Matt's plant after d days.

b Use the formula to complete the table of values.

c Determine how many days it will take for the plant to reach a height of 22 cm.

d Matt gives a cutting of his plant to his friend, Niamh. When Niamh plants the cutting, it is 1 cm tall, and it grows 3 cm each day.

i Write a formula for the height, h, of Niamh's plant after d days.

ii Determine how many days it will take for Niamh's plant to reach a height of 22 cm.

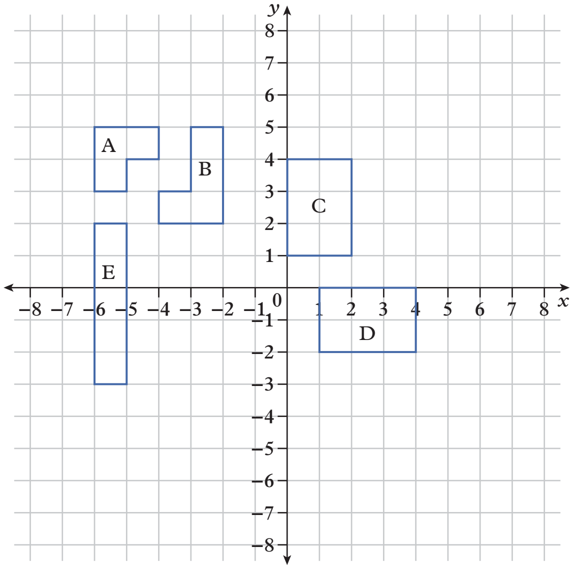
e To make her plant grow faster, Niamh feeds her plant Supergrowth plant food. The instructions on the packet state that the plant should be fed two-thirds of 4 less than the height of the plant each day in grams.

  i Write a formula for the number of grams, g, of Supergrowth that should be fed to the plant when it is h cm tall.

  ii How much Supergrowth should Niamh feed the plant when it is 22 cm tall?

2 Adriana and Beethal are designing an artwork. Here is their current plan.

BLM
Semester 2 review



a Complete their design by following their instructions:

- Reflect shape A in the y-axis.
- Reflect shape B in the x-axis.
- Rotate shape C 90° anticlockwise about the origin.
- Rotate shape D 90° clockwise about the origin.
- Translate shape E 11 units right and 1 unit up.
- Reflect the image of A (A') in the x-axis.

b What single transformation can be applied to shape A to get the image shape A"?

c State which of shapes A, B, C, D or E have:

  i the greatest area

  ii the greatest perimeter.

d State the mathematical name of shapes A and E.

3 A class of 30 students in a Year 7 game-design class have a competition to see who can clear the most levels of Pac-Man. The class recorded the highest level each player reached.

3, 7, 5, 2, 2, 7, 9, 8, 7, 10, 10, 1, 3, 5, 3,
9, 10, 5, 7, 2, 6, 2, 3, 4, 6, 2, 6, 9, 1, 5

a State whether the data collected is discrete or continuous?

b Create a frequency table and a dot plot for the data.

c Calculate the:

  i mean (to one decimal place)

  ii median

  iii mode

  iv range.

d Write two sentences that describe the highest levels the students reached.BLM

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EXPLORATIONS 2

1 Substitutes

What happens when you substitute the expression $2x + 1$ into itself? That is, think about the expression '$2 \times \square + 1$' and replace the box with another copy of '$2x + 1$'. Expanding brackets and simplifying, we get

$$\begin{aligned} 2(2x + 1) + 1 &= 4x + 2 + 1 \\ &= 4x + 3. \end{aligned}$$

a Starting with the answer $4x + 3$, show that substituting $2x + 1$ for x results in the expression $8x + 7$.

b What is the next answer if you substitute $2x + 1$ for x again?

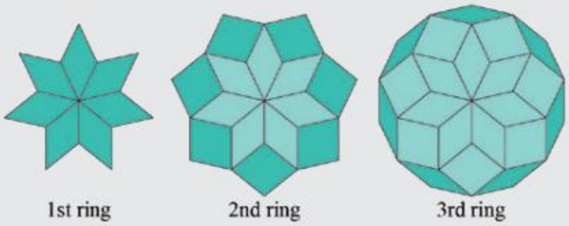
c Without doing the calculation, can you predict the next two answers when you repeat this substitution?

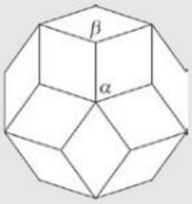
d Write down a general rule for the final expression when you repeat the substitution n times. Can you explain why this works?

e Investigate what expressions you get when you substitute $x + y$ into itself as follows: first substitute $x + y$ for x, then substitute $x + y$ for y, then substitute $x + y$ for x, and so on, alternating between x and y at each stage.

2 Rhombus rings

The first diagram shows a ring of seven identical rhombuses all sharing a common centre point. In the second diagram, another ring of seven rhombuses is added around the first. Likewise, a third ring is added in the third diagram. After this, no more rings can be added.





a The fourth diagram shows a pattern with five rhombuses in each ring. Find the size of the angles α and β.

b A pattern has eight rhombuses in each ring. What are the angles in each rhombus in the last ring?

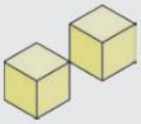
c In another pattern, each rhombus in the last ring has an angle of 20° and each rhombus in the second last ring has an angle of 60°. What are the angles in each rhombus in the third last ring? How many rings are there in total and how many rhombuses are in each ring?

d Let α be the size of the angles at the centre of a rhombus ring pattern. Show that rhombuses in the second ring have an angle of 2α and rhombuses in the third ring have an angle of 3α. Does this pattern continue?

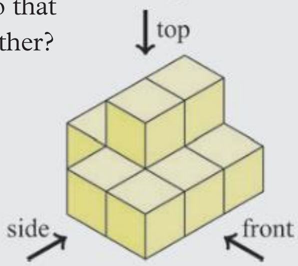
e The third ring of a rhombus ring pattern consists of squares. How many squares are there?

f A rhombus ring pattern has seven rings. How many rhombuses are there at the centre? Find all possible answers.

3 Ambiguous stacks



Isometric view is a convenient way to draw three-dimensional diagrams on a grid of equilateral triangles. But sometimes the diagrams are ambiguous; that is, you cannot tell exactly how the shapes fit together. Are these two cubes on the same level and one cube-width apart, is one floating above the other so that they touch at a corner, or is one floating further away so that their left faces are in line with each other?



Nine identical cubes are stacked on a table, as shown on the right. Cubes cannot float above the table and adjacent cubes must share an entire face.

a Draw the side view, as seen by looking in the direction of the arrow marked 'side'.

b Draw one possibility for the front view. Also draw the corresponding top view and the isometric view as seen from above but from the opposite corner.

c Draw a different possibility for the front view, the corresponding top view and the opposite isometric view. Hint: Some cubes may only *look* like they are touching!

d Using eight cubes, design two different stacks that have the same isometric view, one which is three cubes high and the other only two cubes high. Draw the common isometric view and the side, front and top views of each.

4 Laptagons

A *laptagon* is a right-angled shape with integer side lengths, in centimetres, which is formed from two overlapping rectangles. The overlap must have non-zero area but it cannot completely cover either of the two rectangles. So the first three diagrams show legitimate laptagons but the final two diagrams do not.

a Show how a 6 × 6 square and a 7 × 7 square can overlap to form one laptagon with perimeter 48 cm and another with perimeter 30 cm. Which other perimeters can be achieved using these squares?

b A laptagon with perimeter 32 cm is formed from two squares with 1 cm^2 of overlap. Find all possible sizes of the two squares.

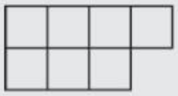
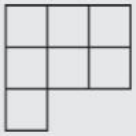
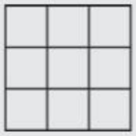
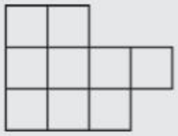
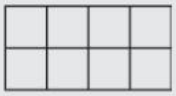
c A laptagon with perimeter 30 cm is formed from two squares with 12 cm^2 of overlap. Find all possible sizes of the two squares.

d A laptagon with perimeter 30 cm is formed from two rectangles (not necessarily squares) with 12 cm^2 of overlap. What is the largest possible area of the laptagon?

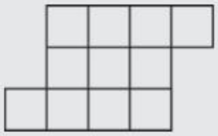
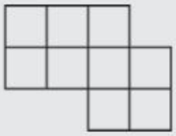
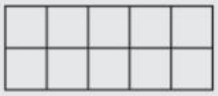
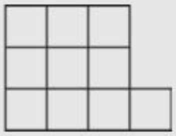
5 Magical averages

A grid consisting of *n* cells is called *magical* if it can be filled with the numbers 1 to *n* in such a way that the average of the numbers in every row and column is the same. Note that some rows or columns may only have one cell.

a Show that the following grids are all magical.

b Explain why the following grids are not magical.

c Investigate rectangular grids with two equal length rows. Can you come up with a conjecture (guess) about which are magical and which are not?

6 Plus printer

Pseudo-code is an informal way to describe a computer program without worrying about the precise symbols and commands that are needed to correctly write it up in a specific programming language. An example of pseudo-code is shown on the right.

```
enter a number n
loop
  if n is 0
    quit
  else if n is odd
    print "+"
    reduce n by 1
  else if n is even
    divide n by 2
```

a How many plus symbols are printed if *n* is 26?

b How many plus symbols are printed if *n* is 89?

c Find two consecutive values of *n*: one that prints six plus symbols, and the other that prints only one.

d Investigate whether it is possible for three consecutive values of *n* to print the same number of plus symbols.

Explorations inspired by the Australian Maths Trust's competitions and programs: www.amt.edu.au

[STEAM project 1]

How can we reduce contaminants in local waterways so that biodiversity in the area is protected?

Your task

Design a device that will reduce the contaminants that are entering your local waterway, in order to protect biodiversity in the area.

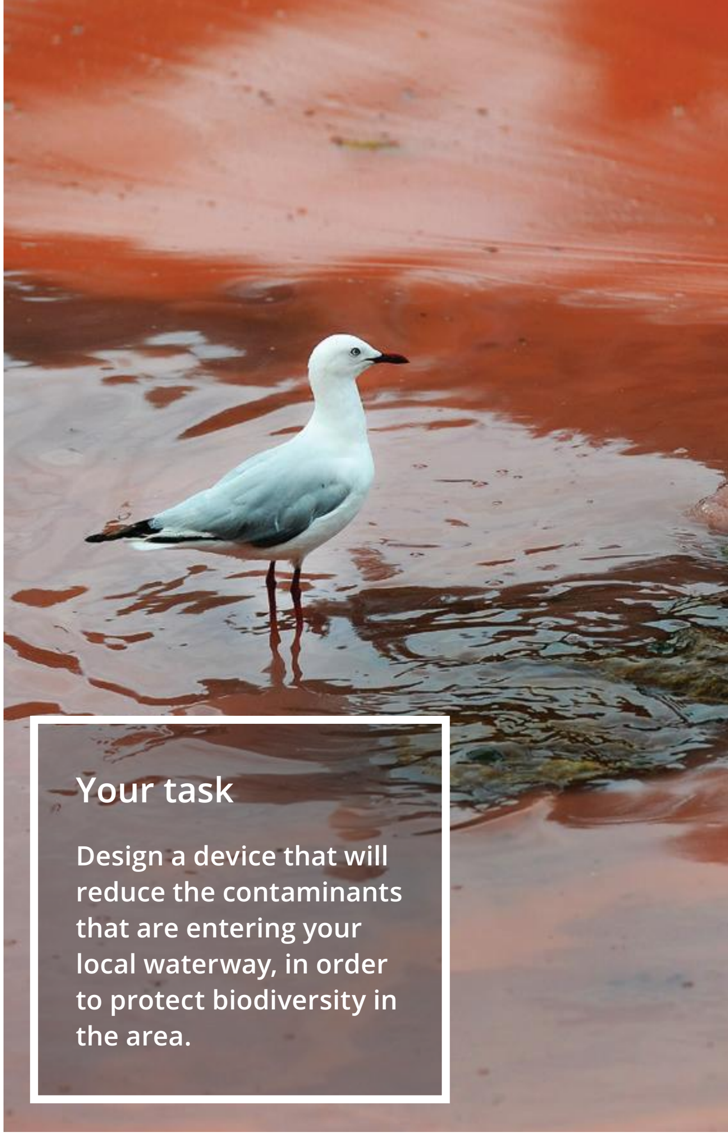


Figure 1 Algal blooms occur when an oversupply of nutrients in the water allows algae populations to quickly increase, covering the water's surface. Contaminants such as industrial fertiliser running into waterways can cause algal blooms. Algal blooms are often toxic to other aquatic life.

Australia is one of the driest continents in the world. But according to an Australian Government report, Australians consume more water per person than any other country, using an average of 100 000 L per person every year.

Water is an important resource in Australia, so it is critical to manage our waterways carefully.

Australia has many waterways, including rivers, groundwater systems, wetland environments and other human-made passages for water. Waterways are vital to our existence and are valuable economic assets.

Waterways play an important role in supporting biodiversity in our local areas, by providing habitats for wildlife (such as fish and turtles) and plants.

Humans rely on local waterways for drinking water, irrigation of crops, industrial processes and recreational activities. But sometimes these human activities can impact waterways, endangering the biodiversity of a local area.

Waterways also hold spiritual significance for many people. Due to the importance of local waterways to Aboriginal and Torres Strait Islander peoples, waterways are part of our cultural heritage.

We must, therefore, manage our waterways to maintain their complex ecosystems.

Water quality and contaminants

A contaminant is a substance that pollutes or poisons something. Contaminants can occur naturally, or be caused by humans – such as microplastics, pesticides and litter. Litter is an example of a physical contaminant, while pesticides are examples of chemical contaminants. There are many types of contaminants, which are often more heavily concentrated in industrial, urbanised or agricultural areas.

Human land-use and major weather events, such as floods and bushfires, can introduce contaminants into local waterways and affect the water quality.

When water becomes contaminated, it can affect the health of an entire ecosystem, leading to serious environmental issues, such as acidic soil or algal blooms. So it is important for all waterways to be managed to protect the organisms that rely on them to survive.

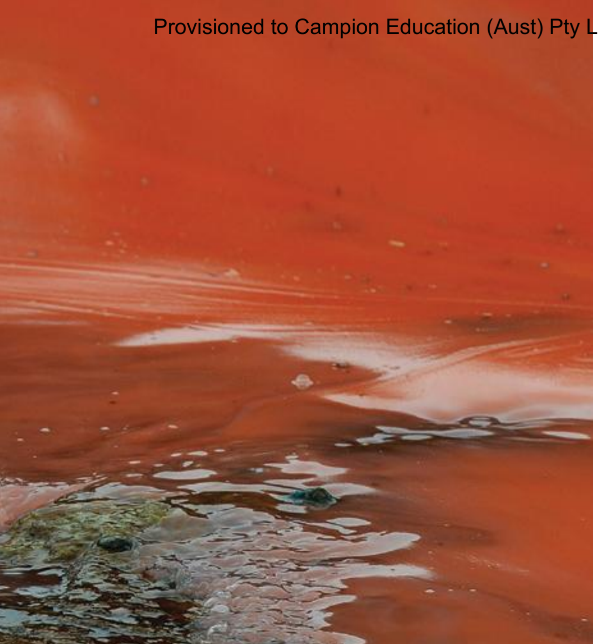


Figure 2 Litter is a contaminant in local waterways, such as the River Torrens in Adelaide.

HUMANITIES

In Geography this year, you will learn about water as a resource and how it connects places as it moves through the environment. You will also study the variability and scarcity of water in Australia.

In History, you will investigate the importance of water in sustaining ancient civilisations.

To complete this task successfully, you will need to investigate the health of a local waterway and the nearby land uses that may be introducing contaminants to the water. You will then need to research strategies that will help to reduce these contaminants.

MATHS

In Maths this year, you will consolidate your understanding of area and volume and different units for measuring them. You will learn to perform calculations involving fractions, decimals and percentages – both with and without digital technology.

To complete this task successfully, you will need to combine these mathematical skills with your understanding of chemical and physical changes. You can then determine the scale of the problem and design your prototype in detail. You may need to perform calculations that relate the concentrations of contaminants, the dimensions of your prototype, the volume of water that can and needs to be processed, and the quantity of contaminants that need to be disposed of.

You will find help for applying these maths skills in Module 3 'Fractions and ratios', Module 4 'Decimals and percentages', and Module 9 'Length, area and volume' of *Oxford Maths 7 Victorian Curriculum*.

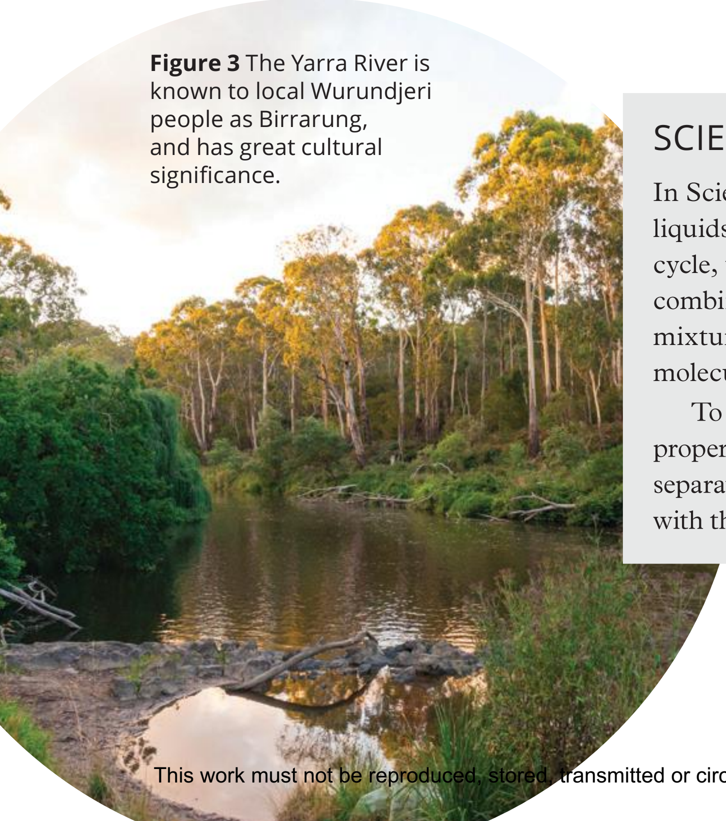


Figure 3 The Yarra River is known to local Wurundjeri people as Birrarung, and has great cultural significance.

SCIENCE

In Science this year, you will learn about how particles move in solids, liquids and gases. Each of these states of matter occurs during the water cycle, which is influenced by both nature and humans. When water is combined with other particles, it forms different types of solutions and mixtures. The unique properties of each particle (including the water molecule) can allow it to be isolated and purified once more.

To complete this task successfully, you will need to consider the properties of each contaminant, and how these properties can be used to separate the contaminants from the water. You will also need to be familiar with the scientific method, and understand how to conduct a fair test.

The design cycle

To successfully complete this task, you will need to complete each of the phases of the design cycle.

Discover

When designing solutions to a problem, you need to know who you are helping and what they need. The people you are helping, who will use your design, are called your end-users.

Consider the following questions to help you empathise with your end-users:

- Who am I designing for?
- What problems are they facing? Why are they facing them?
- What do they need? What do they not need?
- What does it feel like to face these problems?
- What words would you use to describe these feelings?

To answer these questions, you may need to investigate using different resources, or even conduct interviews or surveys.

Define

Before you start to design your device, you need to define the criteria that you will use to test that the problem is solved.

Define your version of the problem

Rewrite the problem so that you describe the group you are helping, the problem they are experiencing and the reason it is important to solve it. Use the following question as a guide:

'How can we help (the group) to solve (the problem) so that (the reason)?'

Determine the criteria

1 Define each contaminant that is present in the waterway. Describe the properties of each contaminant.

2 Describe how you could test whether the contaminant was present in the water.

3 Describe how the contaminant would affect the biodiversity of the area if it were not removed from the waterway.

Ideate

Once you know who you're designing for, and what the criteria are, it's time to get creative!

- Outline the criteria or requirements your device must fulfil (for example, the weight and height of your design).
- Brainstorm at least one idea per person that fulfils the criteria.
- Consider whether your idea will prevent contamination from occurring or solve the problem after it has already occurred.

Remember that there are no bad ideas at this stage. One silly thought could lead to a genius innovation!

Build

Draw each individual design for your device. Label each part of the design. Include the materials that will be used for its construction.

Include in the individual designs:

a the method you will use to isolate each contaminant

b the location of the device in the waterway.

If there is more than one separation method used in your design, identify the order in which you will carry out each method.

Present your design to your group. Use the criteria or requirements that you identified to decide which design your group will build.

Build the prototype

Build and test the prototype of your group's chosen device.

Use the following questions as a guideline for your prototype:

- What materials will you need to build your prototype?
- How will you test whether each step of the design is successful? What will the outcome of each step look like?
- How will you record the steps you use when testing your device?
- How will you record the details of each extracted contaminant?
- How will you dispose of the extracted contaminants after your project is completed?

Test

Use the scientific method to design and experiment with each separation method to ensure its success. You will need to control your variables between each test.

What criteria will you use to determine the success of your prototype?

Conduct your tests and record your results in an appropriate table.

Communicate

Present your design to the class as though you are trying to get your peers to invest in your device.

In your presentation, you will need to:

- explain why removing the contaminant is important for the local wildlife
- describe the key features of your design and how it will reduce the amount of contaminant in the waterways
- construct a labelled diagram of your prototype in the natural environment
- explain the principles that support your design – the importance of water in the local environment and in sustaining civilisations, how some waterways become contaminated and how these contaminants can be reduced
- estimate the number of devices needed to reduce contaminants in the waterways in your local area
- calculate the cost of implementing your design.

Online resources:

Student booklet
This helpful booklet will guide you step-by-step through the project.

What is the design cycle?
This video will help you to better understand each phase in the design cycle.

How to manage your project
This 'how-to' video will help you to manage your time throughout the design cycle.

How to pitch your idea
This 'how-to' video will help you with the 'Communicate' phase of your project.

[STEAM project 2]

How can we reduce waste so that we don't exploit resources?

In 2019, the United Nations estimated that every year 90 billion tonnes of resources (including fossil fuels, precious metals and non-metals) are extracted from the earth and turned into usable products. When these products are no longer used or wanted, only 9 per cent are recycled.

If we want to have enough resources left for future generations, humans cannot continue to extract materials from the earth in this way. It is unsustainable.

E-waste

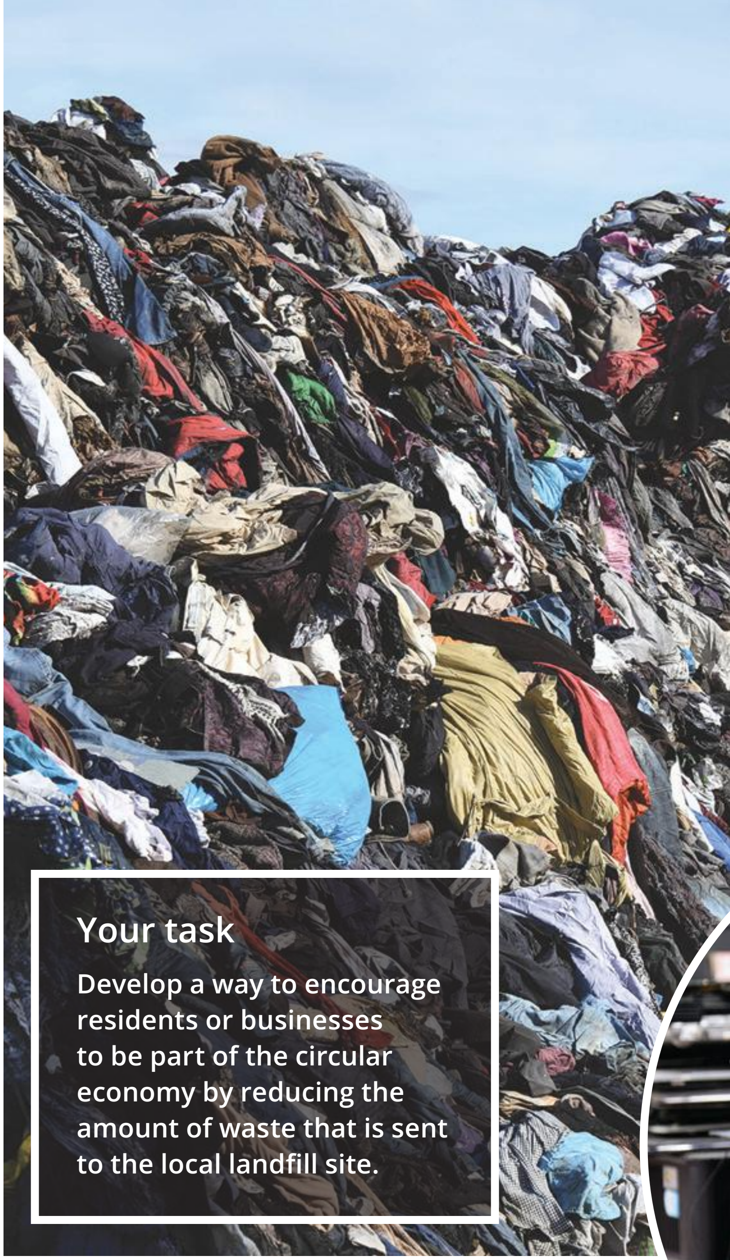
In Australia, millions of electronic devices are thrown away every year. This is known as e-waste. E-waste can be difficult to recycle, but it often contains valuable metals.

The average mobile phone contains 0.034 g of gold, 16 g of copper and 0.35 g of silver. When a phone is thrown into landfill (along with all the other e-waste that once used electricity or batteries) the materials can take many decades to break down (sometimes releasing toxins into the ground).

Recycling these materials reduces the need to mine new resources.

Fast fashion

Fast fashion is the term used for affordable clothing produced rapidly for the general public. It is designed and produced quickly to reflect current fashion trends. When fashion becomes outdated, the clothes or products that reflect that fashion are often thrown away. The Australian Bureau of Statistics (2020) identified that, on average, each person sends 23 kg of clothing to landfill each year. Over 60 per cent of this clothing is made of synthetic fibres (made from fossil fuels) that can take many years to biodegrade.



Your task

Develop a way to encourage residents or businesses to be part of the circular economy by reducing the amount of waste that is sent to the local landfill site.

Figure 1 In Australia, 6000 kg of textiles and clothing are dumped in landfill every 10 minutes.

What is a circular economy?

One potential solution to our unsustainable use of resources is a 'circular economy'. In a circular economy, resources are used and reused as much as possible. This benefits businesses because the longer a resource can be used, the more value it has. A circular economy is based on three key principles:

1 eliminate waste or pollution in the production of a product
2 keep products and materials in use
3 regenerate natural systems.

Some companies are already using the principles of a circular economy in the products they sell in Australia.

HUMANITIES

In Economics and Business this year, you will learn how consumers and producers respond to and influence each other in the market, particularly through price mechanisms. You will consider how resources are distributed, allocated or used in the production of food, clothing and electronics, and the growing consumer demand for sustainable products.

In Geography, you will investigate how the environment can affect the liveability of a place. You may survey the local area to understand the role of services and facilities provided to minimise, reduce and prevent waste, and strategies used to enhance liveability.

To complete this task successfully, you will need to consider the products you buy and the decisions businesses make when deciding what to produce, and the consequences when a product reaches its end-of-life. You will also need to gain an understanding of people's perceptions in your local area towards waste management, recycling and the importance of the environment in measuring liveability.

MATHS

In Maths this year, you will use fractions, percentages and decimals to represent numbers, and ratios between quantities. You will consolidate your knowledge of volume, learning about cubic units and how to convert between different units of volume. You will perform calculations with and without digital technology.

To complete this task successfully, you will need to quantify the problem, which will include using ratios or fractions to scale between individual, local, national and global situations. You will need to cost your solution, accounting for any costs saved by recycling valuable materials.

You will find help for applying these maths skills in Module 3 'Fractions and ratios', Module 4 'Decimals and percentages', and Module 9 'Length, area and volume' in *Oxford Maths 7 Victorian Curriculum*.

Figure 2 Electronic waste does not belong in landfill. Laptops that are no longer working or wanted are examples of e-waste.

SCIENCE

In Science this year, you will learn about separating mixtures, recycling different materials and disruptions to ecosystems. You will need to use your knowledge to consider the role of renewable and 'non-renewable' resources in different products, and how constantly sourcing new materials will affect the surrounding ecosystem.

To complete this task successfully, you may need to consider how the change in state of matter can aid a circular economy. You will also need to be familiar with the scientific method and understand how to conduct a fair test.

Figure 3 The average mobile phone contains gold, copper and silver.

The design cycle

To successfully complete this task, you will need to complete each of the phases of the design cycle.

Discover

When designing solutions to a problem, you need to know who you are helping and what they need. The people you are helping, who will use your design, are called your end-users. This stage involves thinking about the problem (not possible solutions).

Consider the following questions to help you empathise with your end-users:

- Who am I designing for?
- What problems are they facing? Why are they facing them?
- What do they need? What do they not need?
- Who is producing the waste? Why is the waste being produced?
- What does it feel like to face these problems?
- What words would you use to describe these feelings?

To answer these questions, you may need to investigate using different resources, or even conduct interviews or surveys.

Define

Before you start to design your solution, you need to define the criteria that you will use to test the success of your solution.

Define your version of the problem

Rewrite the problem so that you describe the group you are helping, the problem they are experiencing and the reason it is important to solve it. Use the following phrase as a guide.

‘How can we help (the group) to solve (the problem) so that (the reason)?’

Determine the criteria

1 Describe the product that is being used. How much product is needed for normal functions?
2 Describe the waste that is being produced. In what units could you measure the amount of waste? How could you estimate how much waste is currently being produced?
3 Describe the different things that currently happen to this waste. To what fraction of the waste does this happen?
4 Describe the criteria that you will use to measure the success of your design.

Ideate

Once you know who you’re designing for, and what the criteria are, it’s time to get creative!

As a group, brainstorm ways to solve the problem. Remember that there are no bad ideas at this stage. One silly thought could lead to a genius innovation!

Once you have many possible solutions, select three to five ideas and research whether these ideas have already been produced by someone else. If the prototype idea is already on the market, can you make a better version? If it’s not, what will be needed to make it?

Build

Draw your top two ideas. Label each part of the designs. Include the materials or skills required for their construction.

Include in the designs:

- a a description of how the users will interact with the prototype idea
- b a description of how the amount of waste will be decreased, and by roughly how much
- c a description of how the design will contribute to the circular economy
- d at least one advantage and disadvantage of each design.

Select one of the designs to take to the building and testing stage.

Build the prototype

You will need to build at least three versions of your prototype idea. The first version will be tested for usefulness; the second will be used to test or survey the group you are helping; the third will be used for the presentation.

Use the following questions as a guideline for your prototype idea.

- What skills will you need?
- How will you produce a physical version of your prototype idea?
- How will you collect data on the effectiveness of your idea?

Test

Prototype 1

Use the scientific method to design an experiment that will test the effectiveness and strength of your first prototype. You will test the prototype more than once so that you can compare, but you will need to control your variables between tests.

What criteria will you use to determine the success of your solution?

Conduct your tests and record your results in an appropriate table.

Prototype 2

If your prototype will be used to reduce waste, then you will need to generate a survey to test whether the prototype is appropriate for the user. (How would they use it? Would it make their work easier or harder? How likely do you think they are to buy it, and why? How will the prototype affect normal behaviours? How will the production of the prototype affect the environment?)

Prototype 3

Use the information you have obtained from testing the first two versions to adapt your last prototype to be more effective and usable for the group you are helping. You may want to use the first two prototypes to demonstrate how the design has been improved over time.

Communicate

Present your solution to the class as though your peers are going to invest their money. How will you convince them it is a good idea?

In the presentation, you will need to:

- explain why we need to reduce the amount of waste going to the local landfill
- describe the key features of your design and how they will reduce the amount of waste in the landfill, using calculations to justify a quantitative estimate of that reduction
- construct a labelled diagram of your prototype in the natural environment
- describe how the ecosystem will be impacted by your prototype idea
- explain the principles that support your design – such as the circular economy
- use calculations to estimate the cost of implementing your design.

Online resources:

Student booklet
This helpful booklet will guide you step-by-step through the project.

What is the design cycle?
This video will help you to better understand each phase in the design cycle.

How to manage your project
This 'how-to' video will help you to manage your time throughout the design cycle.

How to define a problem
This 'how-to' video will help you to narrow your ideas down and define a specific problem.

Answers

MODULE 1 Whole numbers

EX p8

1.1 Place value

1 a 50 b 5 c 50 000
d 500 e 5000 f 500 000

2 a 90 b 900 c 9
d 9000 e 90 f 900 000

3 a $4 \times 10 + 6 \times 1$
b $5 \times 100 + 6 \times 10 + 8 \times 1$
c $1 \times 1000 + 2 \times 100 + 5 \times 10 + 6 \times 1$
d $7 \times 1000 + 6 \times 100 + 9 \times 10 + 2 \times 1$
e $8 \times 10\,000 + 9 \times 1000 + 4 \times 100 + 5 \times 10 + 7 \times 1$
f $4 \times 100\,000 + 5 \times 10\,000 + 7 \times 1000 + 8 \times 10 + 2 \times 1$

4 a 800 b 500 c 100
d 6500 e 38 400 f 6000
g 8500 h 23 000

5 a 5000 b 9000 c 1000
d 8000 e 37 000 f 51 000
g 90 000 h 100 000

6 a 90 b 8
c i 10 ii 500 iii 1000

7 a 62 000 b 978 c 304
d 250 112 e 12 543 f 9026

8 a Three hundred and sixty-two
$3 \times 100 + 6 \times 10 + 2 \times 1$
b Seven thousand, two hundred and fifteen
$7 \times 1000 + 2 \times 100 + 1 \times 10 + 5 \times 1$
c Forty-five thousand, seven hundred and thirty-three
$4 \times 10\,000 + 5 \times 1000 + 7 \times 100 + 3 \times 10 + 3 \times 1$
d Two hundred and thirty-four thousand, six hundred and one
$2 \times 100\,000 + 3 \times 10\,000 + 4 \times 1000 + 6 \times 100 + 1 \times 1$
e Six million, four hundred and twenty thousand and fifty-eight
$6 \times 1\,000\,000 + 4 \times 100\,000 + 2 \times 10\,000 + 5 \times 10 + 8 \times 1$
f Fifty-five thousand, five hundred and fifty-five
$5 \times 10\,000 + 5 \times 1000 + 5 \times 100 + 5 \times 10 + 5 \times 1$

9 a $8530 > 7503$ b $46\,249 < 64\,249$
c $317\,294 > 37\,294$ d $709 < 4503$
e $5678 < 5876$ f $10\,462 > 10\,248$

10 a 9, 15, 28, 40, 58, 72, 88
b 805, 806, 846, 856, 890
c 625, 6105, 9472, 9902, 10 417
d 234, 2347, 2374, 23 074, 23 704

11 a 8000, 1800, 870, 807, 87
b 3999, 3909, 3099, 399, 309
c 75 561, 75 516, 75 126, 72 156
d 2 567 291, 1 967 219, 256 291

12 2, 5, 11, 16, 20, 28

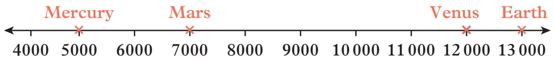
13 108, 132, 150, 173, 185, 200

14 2070, 2100, 2250, 2500, 2850, 2920

15 a 4503 b 30 372 c 68 539
d 203 004 e 890 403 f 674 326

16 a i 200 ii 300 iii 300
iv 200 v 300
b i 6000 ii 5000 iii 6000
iv 6000 v 5000
c If the number is 250 or greater, it is closer to 300.
If the number is 5500 or greater, it is closer to 6000.

17 a Mercury: 5000 km, Venus: 12 000 km,
Earth: 13 000 km, Mars: 7000 km
b



c Sample answer: Mars is slightly larger than Mercury, whereas Earth and Venus are nearly the same size and approximately twice as large as the smaller two planets.

18 a i 3000 km ii 300 000 km
b The distance from Earth to the Moon is approximately 100 times greater than the diameter of the Moon. The diameter of the Moon is approximately 3000 km. If you multiply this by 100 you get 300 000 km, which is approximately the distance from Earth to the Moon.

19 a Two different two-digit numbers; 35, 53
b Four different two-digit numbers; 55, 53, 35, 33

20 a Six different two-digit numbers; 74, 72, 47, 42, 27, 24
b Nine different two-digit numbers; 22, 24, 27, 42, 44, 47, 72, 74, 77

21 a 9631 b 1111 c 9316
d 3691, 3916, 3961, 6139, 6193, 6319

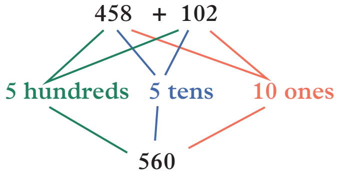
22 a 1 200 000 b 0, 1, 2, 3 or 4 c 2 000 000
d 480 000

23 23 570, 98 500, 79 500, 86 000

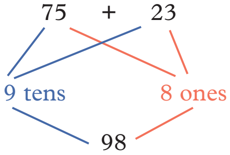
EX p14

1.2 Adding whole numbers

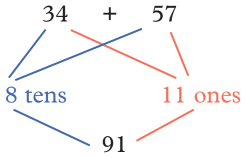
1 a 560



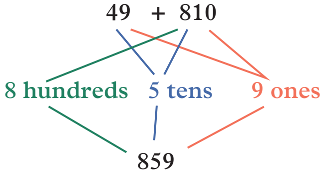
b 98



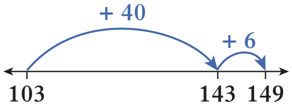
c 91



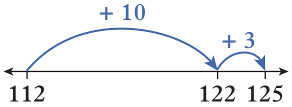
d 859



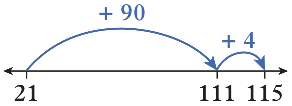
2 **a** 149



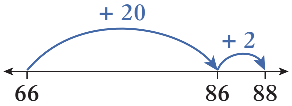
b 125



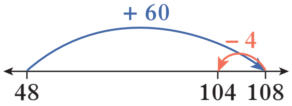
c 115



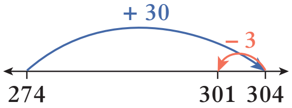
d 88



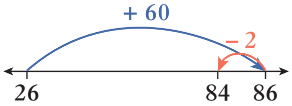
3 **a** 104



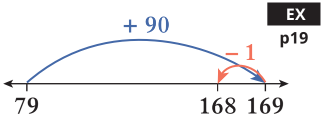
b 301



c 84



d 168



4 **a** 80 **b** 180 **c** 380
d 38 **e** 77 **f** 90
g 100 **h** 468

5 **a** 70 **b** 900 **c** 80
d 100 **e** 90 **f** 99
g 400 **h** 609

6 **a** 958 **b** 692 **c** 1219
d 1502

7 **a** 1119 **b** 453 **c** 7598
d 5251 **e** 2660 **f** 5839
g 17 189 **h** 43 723

8 **a** 1569 **b** 1804 **c** 1597
d 960 **e** 11 508 **f** 7889

9 **a** 138 849 **b** 92 203 **c** 349 908
d 686 461

10 **a** 11 374 **b** 102 409

11 See answers for questions **9** and **10**.

12 **a** **i** False **ii** True **iii** True
b Sample answer:
32 + 45 + 59 = (30 + 40 + 50) + (2 + 5 + 9)
c 136 km

13 **a** Sample answer: 42 + 11 = 53
b **i** Sample answer: 233 + 153 = 386
ii Sample answer: 214 + 172 = 386
c Sample answer: 1367 + 1289 + 2551 = 5207

14 **a** 75 mins **b** 6:30 pm

15 **a** **i** 750 km **ii** 1591 km **iii** 626 km
iv 3430 km
b See answers for part **a**.
c Answers will vary.

16 **a** 39 000 **b** 39 174
c Depends on the commentator. Some like to round the nearest ten thousand, some like to tell you the actual number.

17 **a** 55
b
(1 + 10) + (2 + 9) + (3 + 8) + (4 + 7) + (5 + 6)
= 11 + 11 + 11 + 11 + 11
= 55

18 210

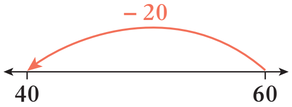
19 **a** **i** 45 **ii** 190
b There will be one number that is not part of a pair and will need to be added separately.
c **i** 35 **ii** 150 **iii** 315

20 **a** 26 and 27 **b** 37, 38 and 39
c 46, 47, 48 and 49
d Answers will vary.

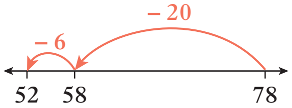
EX p19

1.3 Subtracting whole numbers

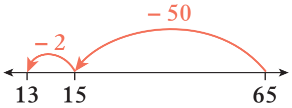
1 **a** 40



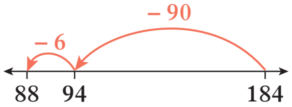
b 52



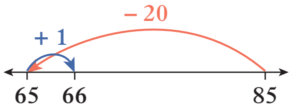
c 13



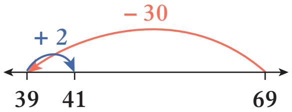
d 88



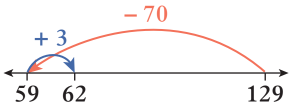
2 **a** 66



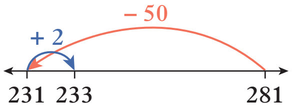
b 41



c 62



d 233



3 **a** 20 **b** 100 **c** 140
d 210 **e** 690 **f** 120
g 235 **h** 112

4 **a** 60 **b** 500 **c** 10
d 109 **e** 515 **f** 32
g 42 **h** 435

5 **a** 33 **b** 41 **c** 265
d 272

6 **a** 42 **b** 45 **c** 222
d 287 **e** 439 **f** 2122
g 1877 **h** 1656

7 **a** 6112 **b** 5315 **c** 1747
d 6928 **e** 3563 **f** 4571

8 **a** 8181 **b** 370 595 **c** 175 128
d 73 390

9 **a** 12 620 **b** 52 514

10 See answers for question **9**.

11 **a** **i** 30 **ii** 32
b **i** 600 **ii** 579
c **i** 5100 **ii** 4871
d **i** 18 000 **ii** 19 880
e **i** 120 **ii** 122
f **i** 1260 **ii** 1271

12 **a** **i** False **ii** True **iii** False
iv True
b Sample answer:
6650 – 2740 = 6700 – 2700 – 50 – 40
c 3910 km

13 **a** Sample answer: 95 – 52 = 43
b **i** Sample answer: 259 – 91 = 168
ii Sample answer: 578 – 410 = 168
c **i** Sample answer: 486 – 200 = 286
ii 429 – 143 = 286

14 **a** 16 679 km **b** 16 336 km
c The Singapore stopover is the shortest by 343 km.

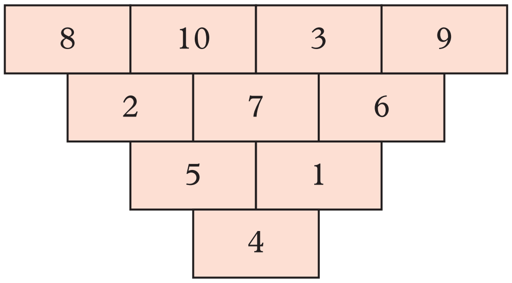
15 **a** Washington Monument: 659 m, Eiffel Tower: 528 m, Empire State Building: 447 m, Sydney Tower: 519 m, Petronas Towers: 376 m, Taipei 101: 319 m, Q1: 505 m, Eureka Tower: 531 m
b Taipei 101 and the Q1 give a total height of 832 m.
c Washington Monument, Empire State Building and Eureka Tower give a total height of 847 m.

16 **a** Mount Everest is 6620 m higher than Mount Kosciuszko.
b 4148 m higher, 2152 m lower, 12 323 m lower
c 16 471 m **d** 8772 m

17 193

18 **a** **i** 645 200 000 km
ii 1 365 691 000 km
iii 50 291 000 km
b 929 392 000 km
c **i** 720 491 000 km
ii 1 315 400 000 km
iii 2 203 392 000 km
d 78 340 000 km

19 **a** If the number 10 is on a lower row then the two numbers diagonally above it would need to have a difference of 10; however, this is not possible since 10 is the largest number in the set.
b



EX p25

1.4 Multiplying whole numbers

1 **a** 28 **b** 18 **c** 132 **d** 42
2 **a** 45 **b** 112 **c** 95 **d** 112
3 **a** 54 **b** 84 **c** 288 **d** 165
4 **a** 592 **b** 186 **c** 135 **d** 182
e 960 **f** 7236 **g** 3192 **h** 1124
5 **a** 50 **b** 320 **c** 6390 **d** 4800
e 700 **f** 10 300 **g** 516 230 **h** 2 784 000
6 **a** 120 **b** 420 **c** 180
d 6000 **e** 40 000 **f** 123 000
g 712 000 **h** 500 000
7 **a** 480 **b** 120 **c** 840
d 3720 **e** 10 850 **f** 126 900
g 41 600 **h** 128 400
8 **a, b**

First number × second number	Product	Number of zeros in the first number	Number of zeros in the second number	Number of zeros in the product
10 × 10	100	1	1	2
100 × 10	1000	2	1	3
1000 × 10	10 000	3	1	4
10 000 × 10	100 000	4	1	5

c The total number of zeros in the product is the sum of the number of zeros in the original calculation.
d **i** 1000 **ii** 10 000 **iii** 100 000
iv 1 000 000

9 **a** **i** 100 000 **ii** 400 000
b **i** 3000 **ii** 70 000 **iii** 600 000
iv 2 000 000 **v** 5000 **vi** 400 000
vii 80 000 000 **viii** 900 000

10 **a** **i** 6 **ii** 1000 **iii** 6000
b **i** 8000 **ii** 90 000 **iii** 120 000
iv 4 500 000 **v** 56 000 **vi** 2 400 000
vii 210 000 000 **viii** 16 000 000

11 **a** 80 000 **b** 162 000 000
c 5 600 000 **d** 120 000 000

12 **a** **i** True **ii** True **iii** True
iv False **v** True
b Sample answer: 37 × 28 = 37 × 20 + 37 × 8
c $1036

13 **a** 1150 m **b** 1074 m
c Divide 1000 m by the full stroke length of 5 m. Then multiply by 3. The result is 600.

14 **a** 528 **b** 996 **c** 1524
d 1778

15 **a** 1290 **b** 5160 minutes
c If it takes 4 minutes to install 1 light bulb, it takes 60 minutes to install 15 light bulbs as 4 × 15 = 60.

16 Sample answer:

$$24 \times 6 = \begin{array}{r|c|c|l} \multicolumn{1}{r}{} & \multicolumn{1}{c}{20} & \multicolumn{1}{c}{4} & \\ \cline{2-3} 6 & 120 & 24 & = 144 \\ \cline{2-3} \end{array} \qquad 24 \times 6 = \begin{array}{r|c|c|c|l} \multicolumn{1}{r}{} & \multicolumn{1}{c}{10} & \multicolumn{1}{c}{10} & \multicolumn{1}{c}{4} & \\ \cline{2-4} 6 & 60 & 60 & 24 & = 144 \\ \cline{2-4} \end{array}$$

$$24 \times 6 = \begin{array}{r|c|c|l} \multicolumn{1}{r}{} & \multicolumn{1}{c}{20} & \multicolumn{1}{c}{4} & \\ \cline{2-3} 3 & 60 & 12 & = 144 \\ \cline{2-3} 3 & 60 & 12 & \\ \cline{2-3} \end{array} \qquad 24 \times 6 = \begin{array}{r|c|c|c|l} \multicolumn{1}{r}{} & \multicolumn{1}{c}{10} & \multicolumn{1}{c}{10} & \multicolumn{1}{c}{4} & \\ \cline{2-4} 5 & 50 & 50 & 20 & = 144 \\ \cline{2-4} 1 & 10 & 10 & 4 & \\ \cline{2-4} \end{array}$$

$$24 \times 6 = \begin{array}{r|c|c|l} \multicolumn{1}{r}{} & \multicolumn{1}{c}{20} & \multicolumn{1}{c}{4} & \\ \cline{2-3} 2 & 40 & 8 & = 144 \\ \cline{2-3} 2 & 40 & 8 & \\ \cline{2-3} 2 & 40 & 8 & \\ \cline{2-3} \end{array}$$

EX p28

Checkpoint

1 **a** Eight hundred and thirty-six
$8 \times 100 + 3 \times 10 + 6 \times 1$
b Nine thousand, six hundred and fifty-four
$9 \times 1000 + 6 \times 100 + 5 \times 10 + 4 \times 1$
c Six hundred and twenty-nine thousand
$6 \times 100\,000 + 2 \times 10\,000 + 9 \times 1000$

2 **a** 419 **b** 96 200 **c** 325 656
3 **a** 9000 **b** 26 000 **c** 136 000
4 **a** 114 **b** 149 **c** 600
5 **a** 1041 **b** 15 173 **c** 35 128
6 920 568
7 **a** 290 **b** 540 **c** 144
8 **a** 232 **b** 4295 **c** 3519
9 **a** 3407 **b** 543
10 **a** 91 **b** 108 **c** 184
11 **a** 4824 **b** 17 480 **c** 472 500
12 **a** 144
b 18 cups milk, 12 eggs, 24 teaspoons vanilla extract, 24 cups self-raising flour, 6 cups sugar, 300 g butter

EX p30

1.5 Long multiplication

1 **a** 2538 **b** 143 **c** 450
d 814
2 **a** 570 **b** 1488 **c** 1472
d 6205 **e** 3120 **f** 4536
g 1782 **h** 2208
3 See answers for question **2**.
4 **a** 2527 **b** 6970 **c** 8232
d 17 984
5 **a** 9545 **b** 15 282 **c** 4551
d 12 936 **e** 38 318 **f** 40 986
g 19 062 **h** 111 468
6 See answers for question **5**.
7 **a** 43 250 **b** 300 155 **c** 157 872
d 585 625 **e** 66 712 **f** 91 333
g 256 452 **h** 197 262
8 See answers for question **7**.
9 **a** **i** 48 000 **ii** 180 000 **iii** 2 400 000
iv 360 000 000
b **i** 48 462 **ii** 202 275 **iii** 2 388 224
iv 355 868 293
10 **a** 4320 **b** 103 680
11 **a** 800 **b** 733
12 2576 menus
13 **a** 3120 m **b** 54 000 m
c **i** 1848 months
ii 56 210 days
iii 1 349 040 hours
14 Sample answer: If on your next birthday, you turn 12 years old:
a **i** 144 months
ii 4380 days
iii 105 120 hours
15 1 200 000 000 km
16 **a** 1 568 000 hectares
b 180 million animals
c 2 744 000 hectares
17 Sample answer:

$$\begin{array}{r} 243 \\ \times \quad 15 \\ \hline 1215 \\ +2430 \\ \hline =3645 \end{array}$$

18 **a** 384 m^2
b **i** 30 cm by 20 cm
ii 600 cm^2
iii 300 cm^2
c 500 matchsticks

EX p35

1.6 Dividing whole numbers

1 **a** **i** 9 **ii** 2 **iii** 4
iv 1
b **i** 17 **ii** 7 **iii** 2
iv 3
c **i** 30 **ii** 5 **iii** 6
iv 0
2 **a** 6 remainder 1 **b** 4 remainder 3
c 5 remainder 2 **d** 2 remainder 5
e 4 remainder 6 **f** 6 remainder 2
g 7 remainder 3 **h** 8 (or 8 remainder 0)
3 **a** 134 remainder 2 **b** 126
c 57 remainder 1 **d** 245 remainder 4
e 175 remainder 2 **f** 69 remainder 6
g 83 remainder 4 **h** 308 remainder 2
4 **a** 7 **b** 245
c 94 **d** 417
5 **a** 25 remainder 17 **b** 41 remainder 1
c 34 **d** 66 remainder 6
e 21 remainder 3 **f** 21 remainder 20
g 213 remainder 7 **h** 22 remainder 2
6 **a** 38 remainder 24 **b** 100 remainder 9
c 1214 remainder 7 **d** 150 remainder 23

7 **a** 16 **b** 134
c 1473 **d** 694

8 **a, b**

First number ÷ Second number	10 ÷ 10	100 ÷ 10	1000 ÷ 10	10 000 ÷ 10	100 ÷ 100	1000 ÷ 100	10 000 ÷ 100
$\frac{\textbf{First number}}{\textbf{Second number}}$	$\frac{10}{10}$	$\frac{100}{10}$	$\frac{1000}{10}$	$\frac{10\,000}{10}$	$\frac{100}{100}$	$\frac{1000}{100}$	$\frac{10\,000}{100}$
Quotient	1	10	100	1000	1	10	100
Number of zeros in the first number	1	2	3	4	2	3	4
Number of zeros in the second number	1	1	1	1	2	2	2
Number of zeros in the quotient	0	1	2	3	0	1	2

c The total number of zeros in the quotient is the difference in the number of zeros in the original calculation.

d **i** 1000 **ii** 1000 **iii** 1
iv 10

9 **a** **i** 10 **ii** 50
b **i** 20 **ii** 6 **iii** 90
iv 4 **v** 80 **vi** 30
vii 50 **viii** 600

10 **a** **i** 3 **ii** 10 **iii** 30
b **i** 30 **ii** 400 **iii** 20
iv 50 **v** 20 **vi** 20
vii 100 **viii** 20 **ix** 50

11 **a** **i** 20 **ii** 300 **iii** 20
iv 20
b **i** 19 **ii** 330 **iii** 18
iv 22

12 **a** A
b One possible answer is: How many times does 15 go into 165?
c 11

13 **a** 4 minutes **b** $4 \times 68 = 272$

14 48 laps

15 **a** 54 **b** 95

16 **a** 16 **b** 6
c 1350 food parcels if each contains one bag of rice. The next largest is 675 food parcels if each one contains 2 bags of rice.

17 **a** 10 **b** 11

18 110: 10, 111: 3, 112: 8, 113: 1, 114: 6, 115: 5, 116: 4, 117: 9, 118: 2, 119: 7, 120: 12, 121: 11

19 Sample answer:
$4 + 5 = 9$
$9 \times 6 = 54$
$7 \times 3 = 21$
$21 + 9 = 30$
$2 \times 8 = 16$
$30 + 16 = 46$
$46 \times 1 = 46$
$46 + 54 = 100$

20 Sample answer:

$$\begin{array}{r} 14\,654 \\ 18\overline{)263\,772} \\ -\underline{18} \\ 83 \\ -\underline{72} \\ 117 \\ -\underline{108} \\ 97 \\ -\underline{90} \\ 72 \\ -\underline{72} \\ 0 \end{array}$$

1.7 Exponents and square roots

EX p41

1 **a** 5^9 **b** 4^7 **c** 11^3
d 20^5 **e** 9^8 **f** 300^4

2 **a** 5 to the power of 9
b 4 to the power of 7
c 11 to the power of 3
d 20 to the power of 5
e 9 to the power of 8
f 300 to the power of 4

3 **a** $7^3 \times 9^4$ **b** $4^6 \times 2^3$
c $3^2 \times 5^6$ **d** $8^4 \times 13^2$
e $2^4 \times 6^2 \times 7^2$ **f** $19^2 \times 23^4 \times 31$

4 **a** $3 \times 3 = 9$
b $2 \times 2 \times 2 \times 2 \times 2 = 32$
c $7 \times 7 \times 7 = 343$
d $5 \times 5 = 25$
e $9 \times 9 \times 9 \times 9 = 6561$
f $4 \times 4 \times 4 = 64$
g $1 \times 1 \times 1 \times 1 \times 1 = 1$
h $6 \times 6 \times 6 = 216$

5 **a** 16 **b** 81 **c** 49
d 125 **e** 1331 **f** 144
g 1000 **h** 256

6 **a** $2 \times 2 \times 2 \times 4 \times 4 = 128$
b $3 \times 3 \times 3 \times 3 \times 5 \times 5 = 2025$
c $8 \times 8 \times 10 \times 10 \times 10 = 64\,000$
d $3 \times 3 \times 3 \times 3 \times 3 \times 1 \times 1 \times 1 \times 1 = 243$
e $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 7 \times 7 = 3136$
f $10 \times 10 \times 3 \times 3 \times 3 = 2700$
g $9 \times 9 \times 9 \times 9 \times 11 \times 11 = 793\,881$
h $6 \times 6 \times 3 \times 3 = 324$

7 **a** 3 **b** 6 **c** 8
d 1 **e** 7 **f** 11
g 12 **h** 20

8 **a** 11 **b** 35 **c** $\sqrt{3721}$
d $\sqrt{88\,804}$

9 **a** **i** 10 000 **ii** 1 000 000
iii 1 000 000 000 000 **iv** 1 000 000 000
b **i** 10^2 **ii** 10^5
iii 10^0 **iv** 10^7

10 **a** $4 \times 10^3 + 1 \times 10^2 + 2 \times 10^1 + 9 \times 10^0$
b $1 \times 10^4 + 9 \times 10^3 + 8 \times 10^1 + 8 \times 10^0$
c $5 \times 10^4 + 1 \times 10^3 + 5 \times 10^2 + 3 \times 10^1 + 5 \times 10^0$
d $4 \times 10^5 + 3 \times 10^3 + 8 \times 10^1 + 6 \times 10^0$

11 **a** 89 **b** 125 **c** 0
d 63 **e** 4 **f** 48
g 59 **h** 84 **i** 3600
j 160 **k** 31 752 **l** 209 952

12 3^5

13 **a** $2^3, 3^2, 5^4, 4^5$ **b** $1^{50}, 7^6, 50^3, 6^7$
c $10^3, 9^4, 6^5, 3^{10}$ **d** $12^2, 8^3, 4^6, 3^8$

14 **a** False **b** False **c** True
d True

15 **a** True **b** True **c** False
d False

16 **a** $3 < \sqrt{15} < 4$ **b** $5 < \sqrt{33} < 6$
c $6 < \sqrt{40} < 7$ **d** $9 < \sqrt{99} < 10$
e $10 < \sqrt{120} < 11$ **f** $14 < \sqrt{214} < 15$

17 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225, 256, 289, 324, 361, 400

18 **a** **i** 729 **ii** 2704 **iii** 9801
b If a two-digit number equals $10 \times a + b$, then $(10 \times a + b)^2 = 100 \times a^2 + b^2 + 2 \times a \times b$.

19 **a**

Number of days	1	2	3	4	5
Expanded form	3	3×3	$3 \times 3 \times 3$	$3 \times 3 \times 3 \times 3$	$3 \times 3 \times 3 \times 3 \times 3$
Exponent form	3^1	3^2	3^3	3^4	3^5
Number of bacteria (in millions)	3	9	27	81	243

b **i** 729 **ii** 2187 **iii** 4 782 969
c **i** 4 days **ii** 6 days **iii** 9 days

20 **a** 84 **b** $4 + 4^2 + 4^3$

21 The boss should pay Taylor \$100 per week, making a total of \$400. If the boss used the other method, the boss would have to pay Taylor \$5 + \$25 + \$125 + \$625 = \$780.

22 **a**

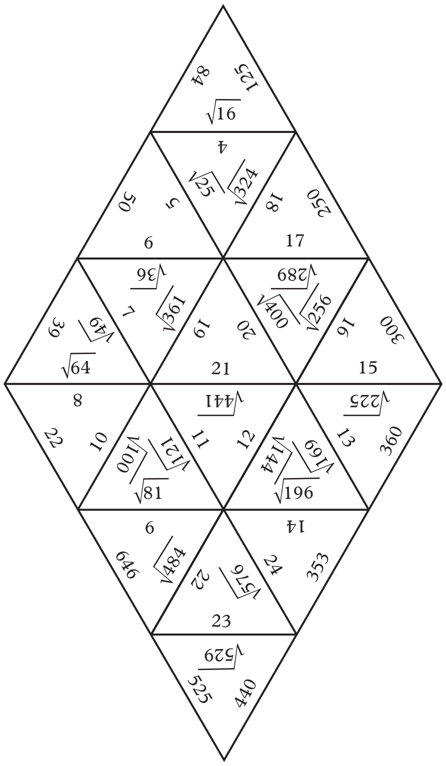
Consecutive perfect squares	**Difference**
1 and 4	$4 - 1 = 3$
4 and 9	$9 - 4 = 5$
9 and 16	$16 - 9 = 7$
16 and 25	$25 - 16 = 9$
25 and 36	$36 - 25 = 11$
36 and 49	$49 - 36 = 13$
49 and 64	$64 - 49 = 15$
64 and 81	$81 - 64 = 17$
81 and 100	$100 - 81 = 19$
100 and 121	$121 - 100 = 21$

b They are consecutive odd numbers.
c All of the second differences are equal to 2.

23 $1 + 8 = 9, 2 + 7 = 9, 3 + 6 = 9, 4 + 5 = 9,$
$9 + 16 = 25, 10 + 15 = 25, 11 + 14 = 25, 12 + 13 = 25$

24 **a** Person A weighs just below 10 kg, Person B weighs just below 12 kg, Person C weighs between 7 and 8 kg, Person D weighs just above 9 kg.
b Person A weighs just above 3 kg, Person B weighs between 3 and 4 kg, Person C weighs between 2 and 3 kg, Person D weighs just above 3 kg.
c Person A weighs just below 2 kg, Person B weighs just below 2 kg, Person C weighs between 1 and 2 kg, Person D weighs between 1 and 2 kg.
d Each person will weigh approximately 1 kg.
e No person will ever weigh 0 kg. Their weight will continue to get closer and closer to 1 kg for each hour for the rest of their life.
f Sample answer: Subtract one thirteenth of the initial weight every hour.

25 Sample answer:



EX p46

1.8 Order of operations

1 **a** 11 **b** 1 **c** 21
d 59 **e** 19 **f** 8
g 12 **h** 0 **i** 24
j 17 **k** 4 **l** 9

2 **a** 15 **b** 60 **c** 4
d 37 **e** 67 **f** 39
g 0 **h** 4 **i** 52
j 49 **k** 55 **l** 1

3 **a** 16 **b** 6 **c** 24
d 15 **e** 40 **f** 86
g 2 **h** 9 **i** 8
j 24 **k** 115 **l** 2880

4 **a** **i** 25 and 1
ii 45 and 5
iii 16 and 4
b **i** You can't switch the places of the 9 and 3 because the equation doesn't have brackets around the multiplication.
ii $(81 \div 9) \times 3$ and $81 \div (9 \times 3)$
iii 27 and 3

5 **a** **i** 2700 **ii** 2589
b **i** 20 **ii** 14
c **i** 8060 **ii** 9831
d **i** 340 **ii** 332
e **i** 168 **ii** 172
f **i** 100 **ii** 108

6 **a** **i** $11 \times 4 - (10 \div 2)$ **ii** 39
b **i** $(12 \div 4) \times 2^3$ **ii** 24
c **i** $(9^2 - 3^3) \times 2$ **ii** 108
d **i** $((17 - 9) \times \sqrt{4}) \div 2$ **ii** 8
e **i** $(\sqrt{16} \times 6) \div 2^3$ **ii** 3
f **i** $14 - 7 - 3$ **ii** 4

7 **a** 11
b **i** 19 **ii** 7 **iii** 317
iv 10 **v** 53 **vi** 40

8 48

9 **a** 2 **b** 4 **c** $13

10 36

11 88

12 22

13 **a** **i** Sharks: $10 \times 6 + 15 \times 1$
ii Visitors: $9 \times 6 + 23 \times 1$
b Sharks: 75, Visitors: 77
c Visitors by 2 points

14 **a** $8 \times 6 + 4 \times 10 + 7 \times 5$ **b** $123

15 **a** 2×2^2 cannot be simplified to 4^2. You'd need to apply the index before multiplication.
b $(3 + 4)^2$ simplifies to $(7)^2$ not $3 + 4^2$. Brackets should be applied first.
$3 + 16 - 4 \times 3$ simplifies to $3 + 16 - 12$. Multiplication is applied before addition and subtraction.

16 5050

17 **a** Sample answer:

1	$4^2 - 3 \times 5$
2	$3 + 4 - 5$
3	$(5 + 4) \div 3$
4	$5 + 3 - 4$
5	$(4^2 + 3^2) \div 5$
6	$5 + (4 - 3)$
7	$3 \times 4 - 5$
8	$4^2 \div (5 - 3)$
9	$3^2 \div (5 - 4)$
10	$3^2 + 5 - 4$
11	$3 \times 5 - 4$
12	$5 + 4 + 3$
13	$5^2 - 3 \times 4$
14	$4^2 - (5 - 3)$
15	$3 \times 5 \times 4^0$
16	$(5 + 3)^2 \div 4$
17	$(4 \times 5) - 3$
18	$5^2 - 4^4 + 3^3$
19	$3 \times 5 + 4$
20	$3^2 + 4^2 - 5$

c Yes, it doesn't work for 1, 1 and 1, for example.

18 **a** 20 **b** 5

EX p51

Review: Whole numbers

Review questions 1.9A: Mathematical literacy review

1 index

2 by-parts method

3 brackets, indices, division, multiplication, addition, subtraction

4 **a** product
b ascending order
c digit

5 **a** The associative laws of addition and multiplication state that in either addition or multiplication, regardless of how the numbers are grouped, the answer does not change. For example, $(2 + 4) + 6 = 2 + (4 + 6)$ and $(3 \times 5) \times 2 = 3 \times (5 \times 2)$.
b When performing a division, the dividend is the number that is being divided by another number. For example, when calculating $91 \div 6$, 91 is the dividend.
c The square root of a number is the number that when squared results in the original number. For example, $\sqrt{9} = 3$ because $3^2 = 9$.

6 **a** base, exponent.
b leading digit

Review questions 1.9B: Multiple choice

1 B **2** D **3** C **4** D
5 B **6** A **7** C **8** D
9 B **10** A

Review questions 1.9C: Short answer

1 **a** 4 × 100 + 6 × 10 + 7 × 1
b 7 × 1000 + 8 × 100 + 5 × 10 + 6 × 1
c 9 × 1000 + 3 × 10
d 1 × 10 000 + 5 × 1000 + 4 × 100 + 5 × 10 + 2 × 1
2 **a** 57, 547, 5347, 53 047
b 87 056, 87 506, 87 605
3 **a** 400 **b** 9500 **c** 600
d 12 900
4 **a** 124 **b** 115 **c** 785
d 118
5 **a** 9267 **b** 402 253
6 **a** Sample answer: 355 + 123 = 478
b Sample answer: 146 + 332 = 478
7 **a** 4183 **b** 333 578 **c** 7843
d 41 333
8 **a** 2945 **b** 253
9 **a** 418 **b** 861 **c** 280
d 1008 **e** 384 **f** 910
10 **a** 476 **b** 546 **c** 1638
d 6669 **e** 19 664 **f** 71 412
11 **a** 38 900 **b** 2 472 000 **c** 680 000
d 6 000 000 **e** 4800 **f** 139 650
12 **a** 20 054 **b** 35 568 **c** 177 915
d 108 416 **e** 4800 **f** 26 230
13 **a** 141 remainder 4 **b** 17 111 remainder 1
c 2507 **d** 81 029 remainder 1
14 **a** 4180 remainder 7 **b** 1151 remainder 7
c 245 remainder 13 **d** 1745
15 **a** 700 **b** 40 **c** 500
d 300
16 **a** 2 × 2 × 2 × 2 × 2 × 2 = 64
b 5 × 5 × 5 × 3 × 3 = 1125
c 2 × 2 × 2 × 7 × 7 = 392
d 6 × 6 × 4 × 4 × 4 = 2304
17 10^4
18 **a** $7^2 = 49$ **b** $6^2 = 36$ **c** $19^2 = 361$
d $8^2 = 64$
19 **a** 5 **b** 25 **c** 4
d 0
20 **a** 28 **b** 60 **c** 31
d 11
21 **a** 36 **b** 75 **c** 4
d 24

Review questions 1.9D: Mathematical modelling

1 **a** Saina's dad performed the calculations from left to right instead of using the order of operations.
b 20 + (10 ÷ 5) × 6
c 32
2 **a**

Item	Quantity	Price per item	Total cost per item
Party pie 24 pack	4	$4	$16
Allen's mixed lollies	3	$3	$9
Chips 100 g	4	$2	$8
M&Ms 250 g	2	$4	$8
Freddo frogs 20 pack	2	$5	$10
Cocktail sausages 2 kg pack	1	$6	$6
Frozen pizza	1	$10	$10
Mudcake	3	$4	$12

b $79 **c** $21 **d** chips
e Cocktail sausages. This is different from part **d** as he only bought one of these, whereas he bought four packets of chips.
f 120 lollies
g 11 friends (12 people in total)
h Three Freddo frogs each with four left over
i Four groups of 3, three groups of 4, two groups of 6
3 **a** 5 km
b The third day
c 1216 turns
d Kate's total time: 36 mins 44 seconds
Toms' total time: 39 mins 54 seconds
e Kate's total time on Day 2: 37 mins 31 seconds
Kate was faster on Day 1 by 47 seconds.
f Tom should pit stop again; he will be quicker by 9 seconds.

MODULE 2 Factors and primes

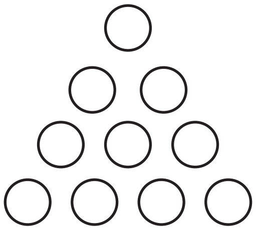
EX p59

2.1 Number sequences

1 **a** Add two **b** Add nine **c** Add five
d Subtract four **e** Divide by three
f Multiply by four
2 **a** 8, 24, 72 **b** 500, 100, 20 **c** 23, 16, 9
d 1000, 1200, 1400
e 25, 30, 40 **f** 18, 12, 9
3 **a** 30, 36 **b** 35, 42 **c** 54, 48
d 88, 44 **e** 39, 26 **f** 1024, 4096
4 **a** 24 **b** 31 **c** 69
d 1 **e** 160 **f** 4
5 **a** 19, 33 **b** 29, 5 **c** 6, 60
d 275, 175

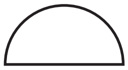
6 **a** 16, 256 **b** 2000, 250 **c** 5, 1215
d 81, 27

7 **a**



b Add a new row to the bottom containing one more shape than the previous bottom row.

8 **a**

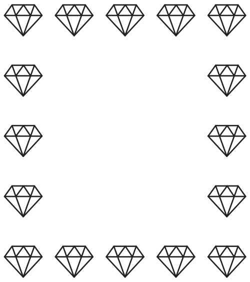


b Decrease the side length of the square by 1 shape.

9 **a** **i** 2, 4, 8, 16, 32, 64, 128, 256, 512, 1024
ii Multiply by 2
iii $2^1, 2^2, 2^3, 2^4, 2^5, 2^6, 2^7, 2^8, 2^9, 2^{10}$ and 2^{12}
b **i** 3, 9, 27, 81, 243, 729
ii Multiply by 3
iii $3^1, 3^2, 3^3, 3^4, 3^5, 3^6$ and 3^{12}
c **i** 5, 25, 125, 625, 3125
ii Multiply by 5
iii $5^1, 5^2, 5^3, 5^4, 5^5$ and 5^{12}
d **i** 10, 100, 1000, 10 000, 100 000, 1 000 000
ii Multiply by 10
iii $10^1, 10^2, 10^3, 10^4, 10^5, 10^6$ and 10^{12}

10 **a** 15 **b** 150 **c** 6
d 6561

11 **a**



b 4, 8, 12, …
c Starting at 4, add 4 to get the next term
d 40 **e** 84

12 1, 8, 27, 64, 125, 216, 343, 512, 729, 1000

13 156 cm

14 **a** 16 384 **b** 21 845

15 Sample answer: Saina is correct. There are many different sequences that begin 4, 16, … For example, 'starting at 4, add 12 to get the next term' and 'starting at 4, multiply by 4 to get the next term'. Note: Different sequences are still possible if you starting with three terms.

16 Sample answers:
a 53, 51, 49, 47 **b** 4
c 53 **d** 47
e Yes, patterns with the following starting numbers work: 101, 53, 44, 32, 29

EX p64

2.2 Divisibility rules

1 **a** Divisible by 2 as the last digit (6) is even
b Divisible by 3 as the sum of the digits (15) is divisible by 3
c Not divisible by 4 as the number formed from the last two digits (46) is not divisible by 4
d Not divisible by 5 as last digit is not 0 or 5
e Divisible by 6 as it is divisible by 2 and 3
f Not divisible by 8. Prove by division, or because it is not divisible by 4
g Not divisible by 9 as sum of the digits is not divisible by 9
h Not divisible by 10 as the last digit is not 0

2 **a** Divisible by 2 as the last digit (0) is even
b Divisible by 3 as the sum of the digits (18) is divisible by 3
c Divisible by 4 as 40 is divisible by 4
d Divisible by 5 as the last digit is 0
e Divisible by 6 as it is divisible by 2 and 3
f Divisible by 8 as 640 is divisible by 8
g Divisible by 9 as the sum of the digits is divisible by 9
h Divisible by 10 as the last digit is 0

3 **a** Divisible by 2
b Divisible by 3
c Not divisible by 4
d Divisible by 5
e Divisible by 6
f Not divisible by 8
g Not divisible by 9
h Divisible by 10

4 **a** Divisible by 2
b Not divisible by 3
c Divisible by 4
d Not divisible by 5
e Not divisible by 6
f Divisible by 8
g Not divisible by 9
h Not divisible by 10

5 **a** No **b** Yes **c** Yes
d No **e** Yes **f** No

6 **a** True **b** True **c** False
d False

7 **a** 2 and 3 **b** 3

8 Octopuses

9 9

10 6

11 Sample answer: If 14 805 is divisible by 3 and 5, then it is divisible by 15.

12 Sample answer: Scott is not correct. There are some numbers which are divisible by 4 and 6 but are not divisible by 24, for example 36.

13 Sample answer: If the sum of the all the digits in the number is divisible by 3 and the number made by the last two digits is divisible by 4, then the number is divisible by 12. This is because any number which is divisible by both 3 and 4 is also divisible by 12.

14 **a** Yes
  b Sample answer: If a number is divisible by 3, 4 and 5, then it is also divisible by 60 (the number of minutes in an hour).

15 **a** No **b** Yes

16 50

17 **a** The sum of the digits in a one-digit number is equal to the number, which is divisible by itself.
  b 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 12, 18, 20, 21, 24, 27, 30, 36, 40, 42, 45, 48, 50, 54
  c No, because 66 is divisible by 6 but not by 6 + 6 = 12.
  d No, because 130 is divisible by 10 but not by 1 + 3 + 0 = 4.

18 The students that say the number is divisible by 16 and by 17.

EX p68

2.3 Multiples and the lowest common multiple

1 **a** 10, 20, 30, 40 **b** 6, 12, 18, 24
  c 9, 18, 27, 36 **d** 5, 10, 15, 20
  e 11, 22, 33, 44 **f** 30, 60, 90, 120
  g 25, 50, 75, 100 **h** 13, 26, 39, 52
  i 40, 80, 120, 160 **j** 22, 44, 66, 88
  k 14, 28, 42, 56 **l** 31, 62, 93, 124

2 24, 32, 40, 48, 56, 64

3 56, 63, 70, 77, 84, 91, 98

4 20, 40, 60, 80

5 12, 24, 36, 48, 60, 72

6 **a** Multiples of 5: 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60
  Multiples of 6: 6, 12, 18, 24, 30, 36, 42, 48, 54, 60
  b 30, 60 **c** 30

7 **a** 10 **b** 28 **c** 18
  d 24 **e** 40 **f** 70
  g 60 **h** 24

8 **a** 12 **b** 60 **c** 36
  d 24 **e** 30 **f** 90
  g 168 **h** 60

9 **a** 24 **b** 28 **c** 80
  d 132 **e** 144 **f** 24
  g 80 **h** 350

10 **a** 88 **b** 112
  c 216 and 224 **d** 296 and 304

11 **a** 56 **b** 126 **c** 210
  d 336

12 **a** 135 **b** 225 **c** 315
  d 450

13 99

14 August 20

15 **a** 21 **b** 11

16 **a** 5:12 am, 5:24 am, 5:36 am, 5:48 am, 6:00 am, 6:12 am
  b **i** 5:36 am **ii** 6:12 am
  c 17th train. Sample answer: I noticed that 1 train arrives every 12 minutes, so 5 trains arrive every hour.

17 13 packs of hats and 8 packs of water bottles

18 **a** 105 blocks **b** 15 **c** 35
  d She runs 55 blocks and flies 50 blocks.

19 Sample answer: No, as the list of common multiples can extend indefinitely.

20 **a** Sample answer: 63, 64, 65. The first set is 3, 4 and 5. To find the next, add the LCM (60) giving 63, 64, 65, then 123, 124, 125 etc.
  b Sample answer: 3030, 3031, 3032, 3033. As before, the first set is 6, 7, 8 and 9. To find the next, add the LCM (504) giving 510, 511, 512, 513, then 1014, 1015, 1016, 1017, etc.

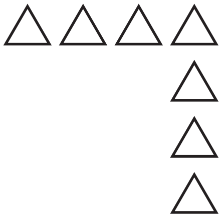
EX p70

Checkpoint

1 **a** Starting at 5, add 2 to get the next term
  b Starting at 12, add 12 to get the next term
  c Starting at 3, multiply by 2 to get the next term

2 **a** 9, 36, 144 **b** 280, 260, 240
  c 36, 48, 60 **d** 2000, 400, 80

3 **a**



  b Starting with one triangle, add two triangles (one on each arm) to get the next term.
  c 9

4 **a** 25 pages
  b 21 days

5 **a** Yes **b** Yes **c** Yes
  d No **e** Yes **f** Yes
  g Yes **h** No

6 **a** No **b** No **c** Yes

7 **a** True **b** False **c** True

8 The boxes that hold nine books

9 **a** 7, 14, 21, 28, 35 **b** 12, 24, 36, 48, 60
  c 15, 30, 45, 60, 75 **d** 24, 48, 72, 96, 120

10 **a** 6, 12, 18, 24, 30, 36, 42, 48, 54, 60, 66, 72, 78, 84, 90, 96
b 9, 18, 27, 36, 45, 54, 63, 72, 81, 90, 99
c 18, 36, 54, 72, 90
d 18

11 **a** 15 **b** 24 **c** 84

12 **a** 20 **b** Sunday

EX p73

2.4 Factors and the highest common factor

1 **a** 20 = 1 × 20 or 4 × 5
b 24 = 1 × 24 or 2 × 12 or 3 × 8
c 32 = 1 × 32 or 2 × 16 or 4 × 8
d 28 = 1 × 28 or 2 × 14 or 4 × 7
e 49 = 1 × 49 or 7 × 7
f 50 = 1 × 50 or 2 × 25 or 5 × 10

2 **a** 1, 3, 5, 15
b 1, 2, 3, 6, 9, 18
c 1, 3, 9
d 1, 2, 3, 4, 6, 12
e 1, 2, 5, 10
f 1, 2, 7, 14

3 **a** 1, 2, 4, 5, 10, 20
b 1, 2, 3, 4, 6, 8, 12, 24
c 1, 2, 4, 8, 16, 32
d 1, 2, 4, 7, 14, 28
e 1, 7, 49
f 1, 2, 5, 10, 25, 50
g 1, 2, 3, 6
h 1, 2, 3, 6, 11, 22, 33, 66
i 1, 2, 4, 7, 8, 14, 28, 56
j 1, 3, 9, 27, 81
k 1, 3, 9, 27
l 1, 2, 4, 5, 10, 20, 25, 50, 100

4 **a** 1, 2, 4, 11, 22, 44
b 1, 2, 3, 6, 9, 18, 27, 54
c 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60
d 1, 2, 4, 8, 16, 32, 64
e 1, 3, 5, 15, 25, 75
f 1, 3, 5, 7, 15, 21, 35, 105
g 1, 2, 3, 4, 5, 6, 8, 10, 12, 15, 20, 24, 30, 40, 60, 120
h 1, 2, 3, 4, 6, 11, 12, 22, 33, 44, 66, 132
i 1, 2, 4, 5, 8, 10, 20, 25, 40, 50, 100, 200
j 1, 2, 7, 14, 49, 98
k 1, 2, 3, 4, 6, 7, 8, 12, 14, 21, 24, 28, 42, 56, 84, 168
l 1, 3, 7, 11, 21, 33, 77, 231

5 **a** 1, 2 **b** 1, 3, 9, 27 **c** 1, 2, 5, 10, 25, 50
d 1, 2, 4 **e** 1, 3 **f** 1, 2, 4, 8
g 1, 3 **h** 1, 2, 4 **i** 1, 7

6 **a** 13, 26 **b** 11 **c** 14, 21
d 21 **e** 14 **f** 12, 18

7 **a** Factors of 18: 1, 2, 3, 6, 9, 18
b Factors of 36: 1, 2, 3, 4, 6, 9, 12, 18, 36
c 1, 2, 3, 6, 9, 18
d 18

8 **a** Factors of 21: 1, 3, 7, 21
b Factors of 45: 1, 3, 5, 9, 15, 45
c 1, 3
d 3

9 **a** Factors of 12: 1, 2, 3, 4, 6, 12
b Factors of 30: 1, 2, 3, 5, 6, 10, 15, 30
c 1, 2, 3, 6
d 6

10 **a** 8 **b** 3 **c** 6
d 5 **e** 4 **f** 15
g 7 **h** 50

11 **a** 4 **b** 5 **c** 8
d 3 **e** 6 **f** 10
g 2 **h** 3

12 **a** LCM = 60, HCF = 6
b LCM = 70, HCF = 5
c LCM = 48, HCF = 4
d LCM = 18, HCF = 1

13 3

14 **a** One possible answer is: three rows of six pieces.
b Six ways are possible: one row of 18 pieces, two rows of 9 pieces, three rows of 6 pieces, six rows of 3 pieces, nine rows of 2 pieces, 18 rows of 1 piece.

15 6

16 Sample answers include: four bags of 30 cookies, twelve bags of 10 cookies, six bags of 20 cookies.

17 Sample answer: A multiple of a number is the answer obtained when that number is multiplied by another whole number. A factor of a number is a number that divides exactly into the given number.

18 **a** 9
b 1
c Sample answer: Every lowest common factor is 1 since 1 is a factor of every number.

19 **a** 8
b 7 purple balloons, 8 green balloons, 13 blue balloons
c 28 balloons

20 **a** 1 **b** 2 **c** 4
d 6 **e** 16 **f** 12

21 80 and 16

22 12, 18, 20, 24, 30, 36, 40, 42, 48, 54, 56 and 60

EX p78

2.5 Prime and composite numbers

1 **a**

Number	Factors
2	1, 2
3	1, 3
5	1, 5
7	1, 7
11	1, 11
13	1, 13
17	1, 17
19	1, 19

b 2

2 **a, b**

Number	Factors	Number of factors
4	1, 2, 4	3
6	1, 2, 3, 6	4
8	1, 2, 4, 8	4
9	1, 3, 9	3
10	1, 2, 5, 10	4
12	1, 2, 3, 4, 6, 12	6
14	1, 2, 7, 14	4
15	1, 3, 5, 15	4
16	1, 2, 4, 8, 16	5
18	1, 2, 3, 6, 9, 18	6
20	1, 2, 4, 5, 10, 20	6

c More than prime numbers

3 **a** 1, 2, 3, 4, 6, 8, 12, 24 **b** 1, 7, 49
c 1, 73 **d** 1, 5, 25, 125
e 1, 37 **f** 1, 2, 41, 82

4 73 and 37 are prime numbers. The others are composite numbers.

5 **a** Composite; even number
b Composite; even number
c Composite; divisible by 3
d Prime; only two factors
e Composite; even number
f Prime; only two factors
g Composite; even number
h Prime; only two factors
i Composite; divisible by 7
j Prime; only two factors
k Prime; only two factors
l Composite; divisible by 3

6 **a** 2 **b** 2, 5 **c** 2, 3
d 5, 7 **e** 2 **f** 2, 5
g 3 **h** 2, 5 **i** 2, 3, 7
j 2, 7 **k** 3, 11 **l** 2

7 Sample answer: Prime numbers have only two factors: 1 and the number itself. Composite numbers have more than two factors.

8 Sample answers:
a 7 + 17 = 24 **b** 13 + 23 = 36
c 71 + 11 = 82 **d** 61 + 83 = 144

9 **a** 3 × 7 = 21 **b** 5 × 11 = 55
c 2 × 13 = 26 **d** 5 × 23 = 115

10 C

11 **a** 11, 13, 17, 19, 23, 29
b They are all odd numbers.
c All prime numbers are odd except for 2.

12 Sample answer: The number 1 has only one factor so it is not one of the prime numbers (which have two factors) and it is not one of the composite numbers (which have more than two factors).

13 **a** False, as 4 is an even number and it is not prime.
b False, as 9 is odd but is a composite number (not prime).
c True, as each number has only two factors.
d False, as 9 is a composite number but is odd.
e False, as 3 × 5 = 15 is composite and odd.
f True, as a composite number has more than two factors and therefore if two prime numbers are multiplied, the result is a composite number.
g False, being even is not the reason 0 is neither prime nor composite.
h True, as there are fewer prime numbers than composites.

14 **a** 1 by 30, 2 by 15, 3 by 10, and 5 by 6.
b Sample answer: They are prime numbers so can only be arranged into one long row to be a rectangle. The dimensions would be 1 by 29 and 1 by 31.
c 1 by 1 by 36, 1 by 2 by 18, 1 by 3 by 12, 1 by 4 by 9, 1 by 6 by 6, 2 by 2 by 9, 2 by 3 by 6, 3 by 3 by 4.

15 **a** 5 and 7, 11 and 13, 17 and 19
b Four: 29 and 31, 41 and 43, 59 and 61, 71 and 73

16 997

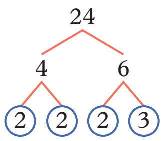
17 Sample answer: None, as per the divisibility rules 1 + 3 + 5 = 9, so any arrangement of the number will be divisible by 3 and 9.

18 14, these are the perfect squares.

EX p83

2.6 Prime factorisation

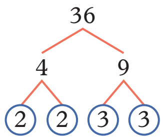
1 **a**



Prime factors: 2, 3

$24 = 2 \times 2 \times 2 \times 3 = 2^3 \times 3$

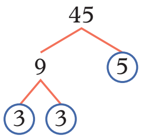
b



Prime factors: 2, 3

$36 = 2 \times 2 \times 3 \times 3 = 2^2 \times 3^2$

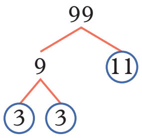
c



Prime factors: 3, 5

$45 = 3 \times 3 \times 5 = 3^2 \times 5$

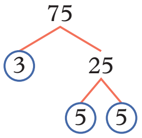
d



Prime factors: 3, 11

$99 = 3 \times 3 \times 11 = 3^2 \times 11$

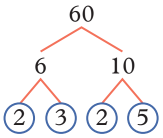
e



Prime factors: 3, 5

$75 = 3 \times 5 \times 5 = 3 \times 5^2$

f



Prime factors: 2, 3, 5

$60 = 2 \times 2 \times 3 \times 5 = 2^2 \times 3 \times 5$

2 **a** 2×13 **b** 2^5 **c** 2×5^2

d $3^2 \times 7$ **e** $2 \times 3 \times 13$ **f** 2×29

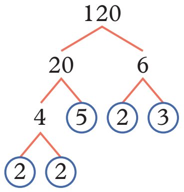
g $2^2 \times 3 \times 7$ **h** $2^2 \times 23$ **i** $2^4 \times 3^2$

3 **a** $2^4 \times 3$ **b** $2^5 \times 3$ **c** $3^3 \times 5$

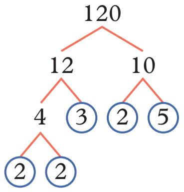
d $2 \times 7 \times 13$ **e** $2^3 \times 3 \times 7$ **f** $3 \times 5 \times 19$

g $2^2 \times 61$ **h** $2^5 \times 13$ **i** $2 \times 3^2 \times 5^2$

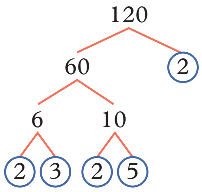
4 **a** **i** Sample answer:



ii

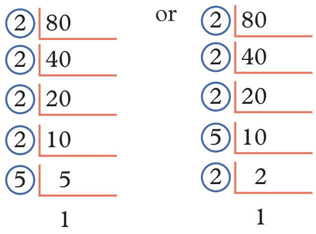


iii



b $120 = 2 \times 2 \times 2 \times 3 \times 5 = 2^3 \times 3 \times 5$. The initial dividing factors do not change the prime factorisation; it will always be the same.

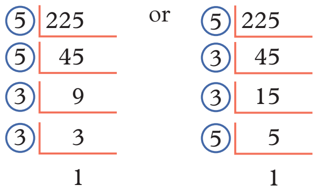
5 **a**



Prime factors: 2, 5

$80 = 2 \times 2 \times 2 \times 2 \times 5 = 2^4 \times 5$

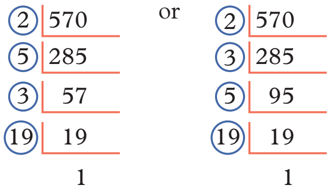
b



Prime factors: 3, 5

$225 = 3 \times 3 \times 5 \times 5 = 3^2 \times 5^2$

c



Prime factors: 2, 3, 5, 19

$570 = 2 \times 3 \times 5 \times 19$

6 **a** $2^2 \times 7$ **b** $2^3 \times 7$ **c** $2^2 \times 11$

d 5×17 **e** $2^2 \times 3 \times 11$ **f** $2^2 \times 13$

g 2×5^3 **h** $2 \times 3^2 \times 5$ **i** $2^3 \times 3^2$

j $2^2 \times 5 \times 11$ **k** $2^2 \times 5^2$ **l** $2^4 \times 5^2$

7 **a** $2^2 \times 3 \times 19$ **b** $2^4 \times 5 \times 7$ **c** $2^2 \times 3^4$

d 5×13^2 **e** 7^3 **f** 2^9

g $2 \times 7 \times 11$ **h** $2^6 \times 3 \times 5$ **i** $2^2 \times 5 \times 37$

j $2^2 \times 3 \times 5 \times 19$

k $2 \times 3 \times 7 \times 19$

l $2^2 \times 3 \times 11 \times 13$

8 **a** Yes, it is in the expanded notation.

b No, as there is only one prime factorisation for each number. The first option is the correct factorisation for 36.

c No, as 2×17 doesn't equal 19. 19 is a prime number, so it doesn't have a factorisation.

d No, as 4 is not a prime factor.

9 **a** $2 \times 3 \times 5 \times 7 = 210$

b $2^5 = 32$

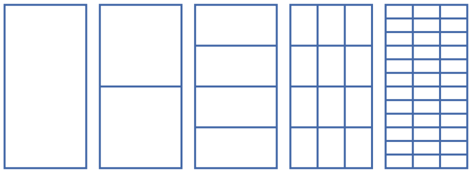
c $2 \times 3 \times 5 \times 7 \times 11 \times 13 = 30\,030$

10 The HCF = 30, so the tickets should be 30 cm by 30 cm.

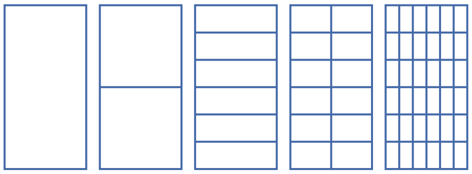
11 **a** $36 = 2 \times 2 \times 3 \times 3$.

Sample answer: This means the cake can be cut in half to produce two pieces, then each piece is cut in half again, then each of these pieces is cut into equal thirds and then each of these pieces into equal thirds again.

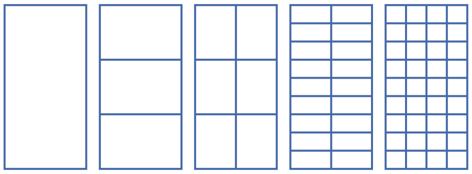
b Sample answer:



c Sample answer:



or



12 a $100 = 2^2 \times 5^2$

b $180 = 2^2 \times 3^2 \times 5$

c $1000 = 2^3 \times 5^3$

13 a $315 = 3^2 \times 5 \times 7$

b $441 = 3^2 \times 7^2$

c $1575 = 3^2 \times 5^2 \times 7$

14 90

15 a 2 **b** 5 **c** 8 **d** 12

16 a 15 **b** 7

17 26

18 a 420 **b** 315 **c** 4800 **d** 672

19 5; 400

20 714

21 2:30 pm

22

	1	5	7
×		2	8
4	3	9	6

	2	9	7
×		1	8
5	3	4	6

	1	3	8
×		4	2
5	7	9	6

	1	9	8
×		2	7
5	3	4	6

	4	8	3
×		1	2
5	7	9	6

EX p88

Review: Factors and primes

Review questions 2.7A: Mathematical literacy review

1 A factor of a given number is a whole number which divides exactly into the given number, whereas a multiple is the result of multiplying a given whole number by another whole number.

2 The quotient is the number obtained when dividing the dividend by the divisor.

3 a True

b False

c True

4 a composite number

b rule

5 a A prime number is a positive whole number greater than 1 that has only two factors: 1 and itself. For example, 7 is a prime number, as the only factors of 7 are 1 and 7.

b The lowest common multiple is the smallest number that is common in the multiples list of two or more numbers. For example, the lowest common multiple of 3 and 4 is 12, as the multiples of 3 are 3, 6, 9, 12, ... and the multiples of 4 are 4, 8, 12, ...

6 a remainder, dividend, divisor

b common factor, whole numbers

Review questions 2.7B: Multiple choice

1 A **2** C **3** D **4** A

5 A **6** B **7** A **8** B

9 B **10** B **11** C **12** C

Review questions 2.7C: Short answer

1 a 22, 30 **b** 70 **c** 5, 160 **d** 22, 11

2 640

3 a Yes **b** Yes **c** Yes **d** Yes

4 a 3, 5 and 7 **b** 7

5 24, 28, 32, 36, 40, 44

6 24, 48, 72, 96

7 a 36 **b** 90

8 1, 2, 4, 5, 8, 10, 16, 20, 40, 80

9 Sample answer: 2 rows of 12, 3 rows of 8, 4 rows of 6

10 a 6 **b** 3

11 a 7, 14, 21 **b** 6, 8, 10 **c** 4, 9, 16

12 a 3 and 11

b 2 and 19

c 3 and 23, 7 and 19

d 7 and 23, 11 and 19, 13 and 17

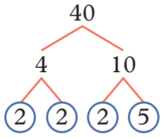
13 a 3 and 5 **b** 3 and 11

c 2 and 17 **d** 2 and 29

14 a Composite, as it has more than two factors

b Its prime factors are 2 and 5.

Sample answer:



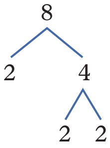
c $40 = 2^3 \times 5$

15 a 12 **b** 12 **c** 7

Review questions 2.7D: Mathematical modelling

1 **a** 448 **b** 8 **c** 56
d 5 **e** 3 **f** Page 30
g No pages

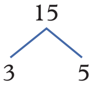
2 **a**



b 8 = 2 × 2 × 2

c Cut the pizza in half, cut each of the halves in half again, and then cut each of the four pieces in half again.

d



e 15 = 3 × 5

f Cut the pizza into three equal pieces, and then cut each of those pieces into five equal slices.

3 **a–c**

	②	③	~~4~~	⑤	~~6~~	⑦	~~8~~	~~9~~	~~10~~
⑪	~~12~~	⑬	~~14~~	~~15~~	~~16~~	⑰	~~18~~	⑲	~~20~~
~~21~~	~~22~~	㉓	~~24~~	~~25~~	~~26~~	~~27~~	~~28~~	㉙	~~30~~
㉛	~~32~~	~~33~~	~~34~~	~~35~~	~~36~~	㊲	~~38~~	~~39~~	~~40~~
㊶	~~42~~	㊸	~~44~~	~~45~~	~~46~~	㊼	~~48~~	~~49~~	~~50~~
~~51~~	~~52~~	(53)	~~54~~	~~55~~	~~56~~	~~57~~	~~58~~	(59)	~~60~~
(61)	~~62~~	~~63~~	~~64~~	~~65~~	~~66~~	(67)	~~68~~	~~69~~	~~70~~
(71)	~~72~~	(73)	~~74~~	~~75~~	~~76~~	~~77~~	~~78~~	(79)	~~80~~
~~81~~	~~82~~	(83)	~~84~~	~~85~~	~~86~~	~~87~~	~~88~~	(89)	~~90~~
~~91~~	~~92~~	~~93~~	~~94~~	~~95~~	~~96~~	(97)	~~98~~	~~99~~	~~100~~

d The circled numbers do not have any prime factors (other than 1) smaller than them, otherwise they would have been crossed out. Therefore, they are all prime numbers. The crossed-out numbers all have prime factors smaller than them (other than 1), and therefore they are all composite numbers.

e 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

MODULE 3 Fractions and ratios

EX p96

3.1 Fractions

1 **a** $\frac{1}{3}$ **b** $\frac{4}{7}$ **c** $\frac{1}{4}$
d $\frac{1}{2}$ **e** $\frac{1}{3}$ **f** $\frac{3}{8}$
g $\frac{5}{8}$ **h** $\frac{3}{5}$ **i** $\frac{9}{16}$
j $\frac{9}{16}$ **k** $\frac{2}{4} = \frac{1}{2}$ **l** $\frac{7}{17}$

2 **a** $\frac{2}{3}$ **b** $\frac{3}{7}$ **c** $\frac{3}{4}$
d $\frac{1}{2}$ **e** $\frac{2}{3}$ **f** $\frac{5}{8}$
g $\frac{3}{8}$ **h** $\frac{2}{5}$ **i** $\frac{7}{16}$
j $\frac{7}{16}$ **k** $\frac{2}{4} = \frac{1}{2}$ **l** $\frac{10}{17}$

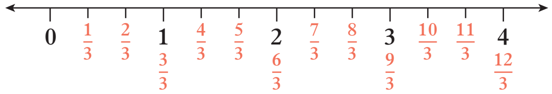
3 B. The rectangle is divided into three equal sections and one of the sections is shaded. The other two rectangles are not divided into equal sections.

4 **a** $\frac{7}{8}, \frac{16}{31}, \frac{13}{22}$ **b** $\frac{5}{2}, \frac{17}{2}, \frac{29}{15}$ **c** $2\frac{3}{5}, 1\frac{5}{9}, 12\frac{1}{7}$

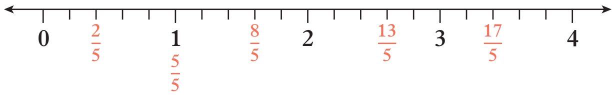
5 **a** **i** $\frac{5}{4}$ **ii** $1\frac{1}{4}$
b **i** $\frac{11}{4}$ **ii** $2\frac{3}{4}$
c **i** $\frac{4}{3}$ **ii** $1\frac{1}{3}$
d **i** $\frac{12}{5}$ **ii** $2\frac{2}{5}$
e **i** $\frac{11}{2}$ **ii** $5\frac{1}{2}$
f **i** $\frac{12}{4}$ **ii** 3
g **i** $\frac{19}{10}$ **ii** $1\frac{9}{10}$
h **i** $\frac{12}{3}$ **ii** 4

6 **a** $\frac{1}{3}$

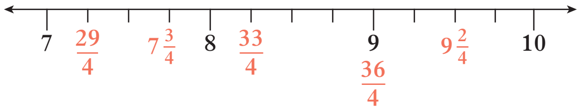
b



7



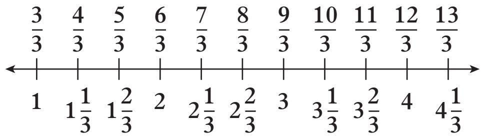
8



9 **a** $6\frac{3}{7}$ or $\frac{45}{7}$ **b** $7\frac{6}{7}$ or $\frac{55}{7}$ **c** $8\frac{4}{7}$ or $\frac{60}{7}$
d $9\frac{2}{7}$ or $\frac{65}{7}$

10 **a** $\frac{12}{23}$ **b** $\frac{7}{12}$ **c** $\frac{23}{50}$
d $\frac{8}{41}$ **e** $\frac{17}{22}$ **f** $\frac{21}{8}$

11 **a**



b **i** 2 **ii** $\frac{10}{3}$ **iii** $3\frac{1}{3}$

12 Sample answer: The numerator in a proper fraction is less than the denominator and the fraction is less than 1. The numerator in an improper fraction is greater than or equal to the denominator and the fraction is greater than or equal to 1.

13 Sample answer: Improper fractions are a fraction only. Mixed numbers are a combination of a whole number and a fraction.

14 a Blue: $\frac{1}{7}$, red: $\frac{6}{7}$

b Blue: $\frac{4}{10} = \frac{2}{5}$, red: $\frac{6}{10} = \frac{3}{5}$

c Blue: $\frac{3}{12} = \frac{1}{4}$, yellow: $\frac{3}{12} = \frac{1}{4}$, red: $\frac{6}{12} = \frac{1}{2}$

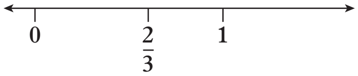
d Blue: $\frac{4}{9}$, red: $\frac{2}{9}$, yellow: $\frac{3}{9} = \frac{1}{3}$

15 a $\frac{1}{2}$ **b** $\frac{1}{4}$ **c** $\frac{17}{60}$

d $\frac{31}{60}$ **e** $\frac{119}{60}$ **f** $5\frac{11}{60}$

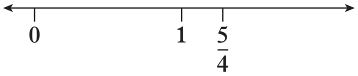
16 Sample answer: There are 8 rows and 8 columns, so there are 64 boxes. If every second box is shaded, we know that $\frac{1}{2}$ are shaded because there is an even number of boxes.

17 a



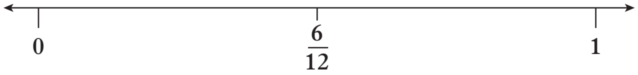
The space between $\frac{2}{3}$ and 1 should be half the size of the space between 0 and $\frac{2}{3}$.

b

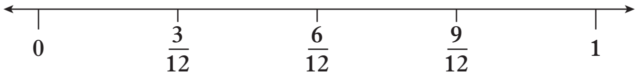


The space between 0 and 1 should be four times the size of the space between 1 and $\frac{5}{4}$.

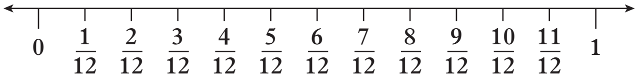
18 a



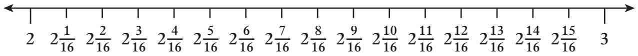
b



c



d



e Sample answer: Divide a number line between 0 and 1 into three equal parts, then divide each interval into five equal parts each representing one fifteenth.

19 $\frac{8}{9}$

20 a $\frac{1}{16}$ **b** $\frac{2}{16}$ or $\frac{1}{8}$ **c** $\frac{4}{16}$ or $\frac{1}{4}$

EX
p103

3.2 Equivalent fractions

1 a $\frac{2}{4}, \frac{3}{6}, \frac{4}{8}, \frac{5}{10}, \frac{6}{12}$ **b** $\frac{6}{8}, \frac{9}{12}$ **c** $\frac{1}{4}, \frac{2}{8}$

d $\frac{4}{10}$ **e** $\frac{4}{6}, \frac{6}{9}, \frac{8}{12}$

f $\frac{1}{1}, \frac{2}{2}, \frac{3}{3}, \frac{4}{4}, \frac{5}{5}, \frac{6}{6}, \frac{8}{8}, \frac{9}{9}, \frac{10}{10}, \frac{11}{11}, \frac{12}{12}$

2 a $\frac{4}{9} = \frac{16}{36}$ **b** $\frac{24}{8} = \frac{12}{4}$ **c** $2\frac{5}{11} = 2\frac{15}{33}$

d $\frac{42}{70} = \frac{6}{10}$ **e** $\frac{144}{48} = \frac{36}{12}$ **f** $3\frac{2}{8} = 3\frac{18}{72}$

g $\frac{2}{3} = \frac{6}{9} = \frac{60}{90}$ **h** $\frac{5}{6} = \frac{15}{18} = \frac{20}{24}$ **i** $\frac{2}{5} = \frac{10}{25} = \frac{6}{15}$

3 a $5\frac{1}{13} = 5\frac{3}{39} = 5\frac{5}{65}$

b $\frac{1}{10} = \frac{2}{20} = \frac{3}{30} = \frac{8}{80}$

c $\frac{8}{7} = \frac{24}{21} = \frac{40}{35} = \frac{88}{77}$

d $3\frac{3}{5} = 3\frac{18}{30} = 3\frac{9}{15} = 3\frac{42}{70}$

e $\frac{1}{12} = \frac{6}{72} = \frac{8}{96} = \frac{12}{144} = \frac{15}{180} = \frac{20}{240}$

f $\frac{2}{3} = \frac{4}{6} = \frac{10}{15} = \frac{20}{30} = \frac{50}{75} = \frac{400}{600}$

4 a $\frac{5}{8}$ **b** $\frac{7}{9}$ **c** $\frac{9}{8}$

d $2\frac{2}{5}$ **e** $5\frac{11}{12}$ **f** $6\frac{1}{5}$

5 a $\frac{1}{3}$ **b** $\frac{5}{8}$ **c** $\frac{58}{47}$

d $7\frac{4}{9}$ **e** $4\frac{7}{8}$ **f** $3\frac{8}{27}$

6 $\frac{1}{3}, \frac{6}{18}, \frac{16}{48}, \frac{18}{54}, \frac{20}{60}, \frac{52}{156}$

7 a $\frac{1}{2}$ **b** $\frac{2}{3}$ **c** $\frac{1}{4}$

d $\frac{2}{5}$

8 a i $\frac{5}{2}$ **ii** $2\frac{1}{2}$

b i $\frac{11}{3}$ **ii** $3\frac{2}{3}$

c i $\frac{7}{4}$ **ii** $1\frac{3}{4}$

9 $\frac{17}{30}$

10 $\frac{11}{17}$

11 $\frac{1}{6}$

12 a 75 **b** 135 **c** 169

13 $\frac{13}{15}$

14 a $\frac{3}{4}$ **b** 157

15 a 21 **b** 18 **c** 756

16 a $\frac{81}{8}$ or $10\frac{1}{8}$ **b** $\frac{3}{5000}$

c $\frac{2}{21}$ **d** $\frac{14\,641}{210}$ or $69\frac{151}{210}$

17 a $\frac{8}{3} = 2\frac{2}{3}$ **b** $\frac{4}{3} = 1\frac{1}{3}$ **c** $\frac{2}{3}$

d 8 **e** 4 **f** 2

EX
p109

3.3 Ordering fractions

1 a $\frac{2}{3}$ **b** $\frac{9}{12}$ **c** $\frac{1}{5}$

d $\frac{7}{8}$

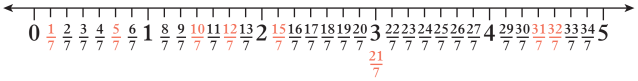
2 a $\frac{5}{6} > \frac{3}{6}$ **b** $\frac{9}{5} > \frac{7}{5}$ **c** $\frac{93}{15} > \frac{32}{15}$

d $1\frac{7}{8} > 1\frac{3}{8}$ **e** $3\frac{11}{20} > 3\frac{9}{20}$ **f** $\frac{180}{4} > \frac{108}{4}$

3 a $\frac{2}{3} > \frac{2}{4}$ **b** $\frac{16}{44} > \frac{16}{45}$ **c** $\frac{77}{12} > \frac{77}{100}$

d $2\frac{5}{6} > 2\frac{5}{7}$ **e** $4\frac{13}{15} > 4\frac{13}{20}$ **f** $\frac{55}{17} > \frac{55}{30}$

4 a



b $\frac{1}{7} < \frac{5}{7} < \frac{10}{7} < \frac{12}{7} < \frac{15}{7} < \frac{21}{7} < \frac{31}{7} < \frac{32}{7}$

The numerators are in ascending order.

5 **a** $\frac{2}{11} < \frac{5}{11} < \frac{8}{11} < \frac{15}{11}$
b $\frac{6}{7} < \frac{14}{7} < \frac{15}{7} < \frac{16}{7}$
c $\frac{10}{15} < \frac{10}{13} < \frac{10}{5} < \frac{10}{3}$
d $\frac{77}{101} < \frac{77}{89} < \frac{77}{70} < \frac{77}{59}$

6 **a** $3\frac{2}{5}, 2\frac{1}{5}, 1\frac{4}{5}, 1\frac{2}{5}, \frac{4}{5}, \frac{1}{5}$
b $\frac{6}{2}, \frac{6}{3}, \frac{6}{4}, \frac{6}{5}, \frac{6}{6}, \frac{6}{8}$
c $\frac{100}{67}, \frac{76}{67}, \frac{67}{67}, \frac{33}{67}, \frac{1}{67}$
d $\frac{38}{2}, \frac{38}{19}, \frac{38}{37}, \frac{38}{40}, \frac{38}{59}, \frac{38}{60}$

7 **a** $\frac{3}{6} > \frac{5}{12}$ **b** $\frac{7}{10} < \frac{30}{40}$ **c** $\frac{93}{15} < \frac{32}{5}$
d $\frac{43}{48} > \frac{7}{8}$ **e** $1\frac{9}{20} < 1\frac{55}{100}$ **f** $\frac{23}{2} < \frac{180}{14}$

8 **a** $\frac{5}{6} < \frac{8}{9}$ **b** $\frac{10}{20} < \frac{11}{14}$ **c** $2\frac{15}{24} < 2\frac{14}{16}$
d $\frac{12}{7} < \frac{13}{5}$ **e** $\frac{6}{17} < \frac{5}{9}$ **f** $\frac{17}{16} > \frac{18}{17}$

9 **a** $\frac{1}{3}, \frac{2}{5}, \frac{4}{3}, \frac{7}{5}, \frac{8}{3}, \frac{16}{5}$
b $\frac{2}{6}, \frac{5}{8}, \frac{9}{8}, \frac{7}{6}, \frac{5}{4}, \frac{11}{4}$
c $\frac{2}{9}, \frac{5}{6}, \frac{11}{9}, \frac{4}{3}, \frac{9}{6}, \frac{7}{3}$
d $\frac{9}{20}, \frac{8}{15}, \frac{11}{10}, \frac{5}{4}, \frac{10}{6}, \frac{6}{3}$

10 **a** $\frac{5}{2}, \frac{12}{5}, \frac{10}{6}, \frac{8}{5}, \frac{3}{2}, \frac{7}{6}$
b $\frac{11}{12}, \frac{5}{6}, \frac{7}{12}, \frac{3}{8}, \frac{2}{6}, \frac{1}{8}$
c $\frac{3}{2}, \frac{10}{7}, \frac{9}{8}, \frac{7}{8}, \frac{6}{7}, \frac{3}{4}$
d $\frac{11}{7}, \frac{13}{9}, \frac{4}{3}, \frac{5}{7}, \frac{1}{3}, \frac{2}{9}$

11 **a** $\frac{1}{3} > \frac{1}{4}$ **b** $\frac{2}{5} < \frac{5}{2}$ **c** $4\frac{5}{6} < \frac{23}{3}$
d $\frac{42}{5} > 7\frac{6}{7}$ **e** $\frac{7}{9} < \frac{6}{7}$ **f** $8\frac{5}{6} < \frac{49}{5}$

12 **a** $\frac{12}{13} < \frac{13}{14}$ **b** $\frac{4}{5} > \frac{3}{4}$ **c** $\frac{1}{6} < \frac{2}{7}$
d $\frac{3}{9} > \frac{3}{10}$ **e** $\frac{4}{19} < \frac{4}{17}$ **f** $\frac{5}{16} > \frac{5}{26}$

13 The second test

14 Melanie; $\frac{24}{40}$ compared to $\frac{20}{35}$

15 **a** $\frac{5}{6} < \frac{8}{9}$ **b** $\frac{17}{16} > \frac{18}{17}$ **c** $\frac{100}{101} > \frac{99}{100}$
d $\frac{48}{47} < \frac{47}{46}$

16 **a** A larger denominator with common numerators indicates a smaller fraction as the five pieces are smaller. $\frac{5}{8} < \frac{5}{7}$
b The numerators cannot be directly compared if the denominators are not the same. $\frac{70}{34} > \frac{9}{10}$
c This is correct as 17 parts of size $\frac{1}{5}$ is less than 23 parts of size $\frac{1}{5}$.

17 Tilly is correct as $\frac{25}{30}$ is greater than $\frac{29}{40}$.

18 $\frac{17}{8}, \frac{10}{3}, \frac{15}{4}, \frac{25}{6}, \frac{21}{5}, \frac{11}{2}, \frac{60}{9}$

19 $\frac{98}{99}, \frac{997}{999}, \frac{9996}{9999}, \frac{99\,995}{99\,999}, \frac{999\,994}{999\,999}$

20 **a** 3 **b** 7 **c** 5

EX p114 3.4 Mixed numbers and improper fractions

1 **a** $1\frac{2}{5}$ **b** $8\frac{2}{3}$ **c** $2\frac{7}{9}$
d $4\frac{3}{10}$ **e** $9\frac{3}{8}$ **f** $6\frac{10}{13}$
g $3\frac{1}{7}$ **h** $4\frac{1}{4}$ **i** $8\frac{1}{6}$
j $4\frac{6}{11}$ **k** $13\frac{2}{5}$ **l** $55\frac{1}{2}$

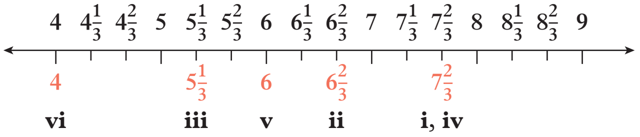
2 **a** $\frac{11}{9}$ **b** $\frac{19}{6}$ **c** $\frac{53}{9}$
d $\frac{28}{11}$ **e** $\frac{54}{5}$ **f** $\frac{35}{4}$
g $\frac{17}{7}$ **h** $\frac{37}{3}$ **i** $\frac{37}{8}$
j $\frac{53}{10}$ **k** $\frac{85}{12}$ **l** $\frac{34}{15}$

3 **a** True **b** False **c** False
d False **e** True

4 **a** $\frac{5}{3} > \frac{3}{2}$ or $1\frac{2}{3} > 1\frac{1}{2}$
b $\frac{7}{3} < \frac{10}{4}$ or $2\frac{1}{3} < 2\frac{1}{2}$
c $\frac{10}{7} < \frac{15}{10}$ or $1\frac{3}{7} < 1\frac{1}{2}$
d $\frac{12}{5} > \frac{19}{8}$ or $2\frac{2}{5} > 2\frac{3}{8}$

5 **a** $\frac{18}{7} > \frac{17}{7}$ **b** $\frac{26}{3} < \frac{28}{3}$ **c** $\frac{38}{11} > \frac{28}{11}$
d $\frac{49}{5} < \frac{51}{5}$

6 **a, b**



c $\frac{12}{3}, 5\frac{2}{6}, \frac{24}{4}, \frac{20}{3}, 7\frac{4}{6} = \frac{23}{3}$

7 **a** $9\frac{1}{3} > 3\frac{2}{5}$ **b** $8\frac{1}{6} < 9\frac{2}{4}$ **c** $6\frac{1}{2} > 2\frac{8}{9}$
d $5\frac{3}{5} < 6\frac{1}{10}$ **e** $6 < 7$ **f** $4\frac{3}{6} > 3\frac{3}{4}$
g $6\frac{2}{5} < 7\frac{7}{8}$ **h** $10\frac{1}{2} > 10\frac{1}{4}$

8 **a** $1\frac{3}{5}$ **b** 32

9 **a** $1\frac{1}{2}$ **b** 15

10 **a** Added the whole number to the numerator without first multiplying it by the denominator.
$3\frac{1}{5} = \frac{3 \times 5 + 1}{5} = \frac{16}{5}$
b Miscalculated the division 14 ÷ 5 as 3 remainder 1 (14 is one less than 3 × 5 = 15 not one more).
$\frac{14}{5} = 2\frac{4}{5}$
c Added the whole number multiplied by the numerator to the denominator. $6\frac{2}{3} = \frac{6 \times 3 + 2}{3} = \frac{20}{3}$
d Assumed that fractions always have a smaller numerator than denominator. $3\frac{4}{7} = \frac{25}{7}$

11 64 pieces

12 **a** **i** $2\frac{2}{3}$ **ii** $8\frac{1}{2}$ **iii** $1\frac{5}{6}$

iv $4\frac{7}{10}$

b The alternative method takes out the largest multiple of the denominator and considers the remainder as the numerator of the mixed fraction. This is the same as dividing by the denominator in the worked example method.

c **i** $\frac{17}{5}$ **ii** $\frac{23}{17}$ **iii** $\frac{19}{2}$

iv $\frac{97}{10}$

d The calculation for the numerator in the worked example is the same as in the alternative method, only it is completed with more steps.

13 **a** **i** $\frac{44}{8}$ **ii** $5\frac{4}{8}$ $\left(\text{or } 5\frac{1}{2}\right)$

b No, because there will only be 40 slices.

c 9 slices **d** $\frac{1}{9}$ **e** 37 slices

14 $2\frac{9}{38}$

15 Sample answer: $\frac{11}{10} = 1\frac{1}{10}$

EX
p117

Checkpoint

1 **a** $\frac{1}{4}$ **b** $\frac{15}{22}$ **c** $1\frac{5}{7}$ or $\frac{12}{7}$

2 **a** $\frac{3}{4}$ and $2\frac{1}{4}$ **b** $11\frac{4}{5}$ and $13\frac{2}{5}$

3 **a** $\frac{17}{20}$ **b** $\frac{51}{100}$ **c** $\frac{15}{16}$

4 **a** $\frac{12}{5} = \frac{36}{15}$ **b** $\frac{20}{35} = \frac{4}{7}$ **c** $\frac{18}{27} = \frac{2}{3} = \frac{4}{6}$

5 **a** $\frac{1}{2}$ **b** $\frac{8}{5}$ or $1\frac{3}{5}$ **c** $7\frac{5}{6}$

6 **a** $\frac{1}{2}$ **b** $2\frac{3}{4}$ or $\frac{11}{4}$ **c** $1\frac{2}{3}$ or $\frac{5}{3}$

7 **a** $\frac{87}{121} > \frac{78}{121}$ **b** $\frac{55}{8} > \frac{55}{9}$ **c** $\frac{3}{4} < \frac{5}{6}$

8 **a** $\frac{1}{17}, \frac{9}{17}, \frac{20}{17}, 1\frac{5}{17}$ **b** $\frac{1}{6}, \frac{7}{12}, \frac{2}{3}, \frac{3}{4}$

c $5\frac{1}{8}, 5\frac{1}{6}, 5\frac{1}{4}, 5\frac{1}{3}$ **d** $\frac{19}{12}, \frac{15}{9}, \frac{17}{6}, \frac{13}{4}$

9 **a** $\frac{13}{5}$ **b** $\frac{47}{7}$ **c** $\frac{103}{8}$

10 **a** $6\frac{1}{3}$ **b** $8\frac{3}{8}$ **c** $8\frac{4}{7}$

EX
p121

3.5 Adding and subtracting fractions

1 **a** $1\frac{1}{4}$ **b** $2\frac{1}{2}$ **c** $3\frac{3}{5}$

d $11\frac{2}{3}$ **e** $4\frac{5}{12}$ **f** $12\frac{12}{13}$

g $6\frac{2}{3}$ **h** $5\frac{1}{2}$ **i** $7\frac{3}{5}$

j $\frac{2}{3}$ **k** $24\frac{3}{4}$ **l** $26\frac{11}{13}$

2 **a** $\frac{3}{5}$ **b** $\frac{6}{7}$ **c** $\frac{2}{3}$

d $\frac{11}{18}$ **e** $\frac{19}{37}$ **f** $1\frac{2}{23}$

g $2\frac{3}{11}$ **h** $\frac{11}{13}$ **i** $\frac{11}{25}$

3 **a** $\frac{6}{7}$ **b** $\frac{17}{21}$ **c** $1\frac{1}{13}$

d $\frac{2}{3}$ **e** $\frac{1}{27}$ **f** $\frac{16}{33}$

g $1\frac{8}{17}$ **h** $\frac{1}{3}$ **i** $\frac{1}{2}$

4 **a** $\frac{5}{6}$ **b** 1 **c** $\frac{1}{8}$

d $\frac{9}{16}$ **e** $\frac{14}{15}$ **f** $\frac{5}{6}$

g $\frac{25}{48}$ **h** $\frac{5}{26}$ **i** $\frac{50}{69}$

5 **a** $1\frac{1}{6}$ **b** $\frac{7}{20}$ **c** $1\frac{5}{24}$

d $\frac{1}{18}$ **e** $1\frac{5}{12}$ **f** $\frac{2}{15}$

g $1\frac{1}{14}$ **h** $\frac{7}{30}$ **i** $\frac{11}{21}$

6 **a** $2\frac{1}{3}$ **b** $2\frac{1}{20}$ **c** $2\frac{1}{8}$

d $\frac{17}{24}$ **e** $5\frac{17}{30}$ **f** $\frac{19}{40}$

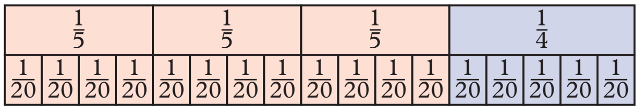
7 **a** i & III, iii & I, iv & II; ii and IV do not match

b **i** $\frac{3}{20}$ **ii** $\frac{17}{20}$ **iii** $\frac{13}{20}$

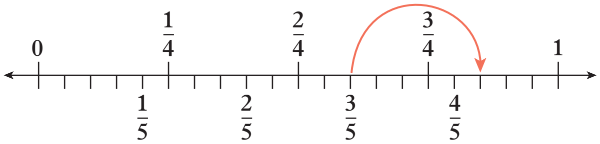
iv $\frac{7}{20}$

c $\frac{3}{4} - \frac{1}{5} = \frac{11}{20}$

d



or



8 **a** $7\frac{3}{5}$ **b** $3\frac{2}{5}$ **c** $4\frac{2}{11}$

d $3\frac{2}{3}$ **e** $1\frac{2}{3}$ **f** $9\frac{2}{5}$

g $\frac{7}{12}$ **h** $3\frac{7}{8}$

9 **a** $4\frac{1}{6}$ **b** $2\frac{19}{21}$ **c** $5\frac{13}{20}$

d $1\frac{13}{15}$ **e** $2\frac{8}{15}$ **f** $5\frac{5}{21}$

g $7\frac{1}{3}$ **h** $1\frac{7}{8}$

10 **a** $8\frac{19}{24}$ **b** $\frac{2}{7}$ **c** $2\frac{67}{70}$

d $4\frac{1}{4}$

11 **a** Dean **b** $\frac{1}{28}$ of an hour

12 $10\frac{7}{15}$ minutes

13 **a** The calculation added both the numerators and the denominators. When adding like fractions, only add the numerators and keep the denominator the same.

b The calculation subtracted both the numerators and the denominators. When subtracting unlike fractions, fractions need to be converted to equivalent fractions with a common denominator before being subtracted.

14 **a** 4 **b** $\frac{5}{8}$

15 **a** $\frac{103}{120}$

b $\frac{17}{120}$ will go towards savings.
This is the difference between Laila's pay (1) and her combined expenses.

16 $14\frac{7}{10}$ cm

17 **a** $5\frac{4}{15}$ km **b** $1\frac{1}{15}$ km

18 **a** $\frac{11}{7}, \frac{14}{7}$ (or 2), $\frac{17}{7}, \frac{20}{7}$; increases by $\frac{3}{7}$ each time.

b $14\frac{1}{5}, 15\frac{3}{5}, 17, 18\frac{2}{5}$; increases by $1\frac{2}{5}$ each time.

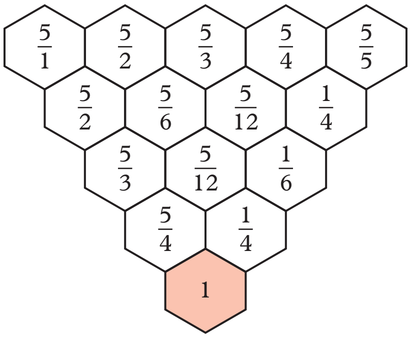
c $\frac{10}{3}, \frac{7}{3}, \frac{4}{3}, \frac{1}{3}$; decreases by $\frac{3}{3}$ (or 1) each time.

d $6\frac{2}{15}, 3\frac{13}{15}, 1\frac{9}{15}\left(\text{or } 1\frac{3}{5}\right)$; decreases by $2\frac{4}{15}$ each time.

19 **a** $\frac{1}{2} - \left(\frac{5}{8} - \frac{2}{5}\right) = \frac{11}{40}$

b $\frac{19}{28} - \left(\frac{3}{7} + \frac{3}{14}\right) = \frac{1}{28}$

20



The number at the bottom will also be 1.

21 Sample answer: $2\frac{4}{8} + 7\frac{1}{3} = 9\frac{5}{6}$

EX
p126

3.6 Multiplying fractions

1 **a** $\frac{6}{5}$ or $1\frac{1}{5}$ **b** $\frac{8}{9}$ **c** 1
d $\frac{22}{3}$ or $7\frac{1}{3}$

2 **a** $\frac{3}{10}$ **b** $\frac{2}{21}$ **c** $\frac{22}{15}$
d $\frac{4}{27}$ **e** $\frac{21}{16}$ **f** $\frac{66}{91}$
g $\frac{7}{45}$ **h** $\frac{27}{70}$ **i** $\frac{81}{10}$

3 **a** $\frac{2}{9}$ **b** $\frac{4}{11}$ **c** $\frac{6}{13}$
d $\frac{1}{34}$ **e** $\frac{4}{15}$ **f** $\frac{4}{9}$
g $\frac{9}{16}$ **h** $\frac{12}{49}$ **i** $\frac{4}{13}$

4 **a** $\frac{9}{11}$ **b** $4\frac{5}{7}$ **c** $18\frac{1}{5}$
d 14 **e** $2\frac{8}{9}$ **f** $13\frac{3}{5}$
g $1\frac{21}{25}$ **h** $13\frac{3}{4}$ **i** $7\frac{19}{24}$

5 **a** 9 **b** 30 **c** $3\frac{1}{5}$
d $3\frac{1}{5}$ **e** $\frac{3}{8}$ **f** $\frac{5}{6}$
g 40 **h** $\frac{4}{9}$

6 **a** $\frac{1}{20}$ **b** $\frac{9}{28}$ **c** $\frac{14}{99}$
d $1\frac{1}{44}$ **e** 66 **f** $18\frac{1}{3}$

7 **a** 6 **b** 15 **c** 44
d 48 **e** $\frac{3}{4}$ **f** $\frac{7}{9}$
g 2 **h** 4 **i** 2
j 35 **k** $\frac{1}{3}$ **l** $\frac{8}{9}$

8 **a** $\frac{6}{17}$ **b** $\frac{5}{26}$ **c** $35\frac{1}{5}$
d $10\frac{5}{6}$ **e** $36\frac{4}{5}$ **f** $63\frac{63}{80}$
g $58\frac{1}{2}$ **h** 45

9 **a** 1 **b** $\frac{16}{15}$ or $1\frac{1}{15}$ **c** $\frac{1}{12}$
d $\frac{11}{6}$ or $1\frac{5}{6}$

10 100

11 **a** **i** $2\frac{1}{2}$ hours **ii** $\frac{5}{2}$ hours
b $12\frac{1}{2}$ hours

12 $3792

13 $2\frac{1}{16}$ km

14 **a** **i** $33 \div 3$ **ii** $40 \div 8$ **iii** $55 \div 101$
iv $\frac{4}{9} \div 7$

b **i** $\frac{1}{7} \times 56$ **ii** $\frac{1}{10} \times 90$ **iii** $\frac{1}{35} \times 92$
iv $\frac{1}{5} \times \frac{8}{11}$

c **i** $3 \times (16 \div 8) = 6$
ii $8 \times (35 \div 5) = 56$
iii $12 \times (63 \div 7) = 108$
iv $7 \times \left(\frac{18}{5} \div 6\right) = \frac{21}{5}$

15 **a** True **b** False

16 **a** **i** $\frac{8}{15}$, less than both
ii $\frac{3}{7}$, between both
iii $\frac{33}{4}$, greater than both
iv $2\frac{13}{36}$, between both
v 2, between both
vi $\frac{1}{63}$, less than both
vii $\frac{100}{9}$ or $11\frac{1}{9}$, greater than both
viii $\frac{20}{3}$ or $6\frac{2}{3}$, greater than both

b When both are proper fractions (value between 0 and 1)

c When both are mixed numbers, whole numbers or improper fractions (value greater than 1)

d When one fraction is a proper fraction and the other is a mixed number or improper fraction (one value between 0 and 1, and the other value greater than 1)

e 1

17 **a** $8\frac{1}{4}$

b $6\frac{12}{72}$ is incorrect; the whole numbers and fractions were multiplied separately rather than converting the mixed numbers to improper fractions.

$6\frac{12}{17}$ is incorrect; as well as multiplying the whole numbers, the denominators of the fractions were added.

18 She used cancelling to remove most of the calculations.

See: $\frac{1}{\cancel{2}^1} \times \frac{\cancel{2}^1}{\cancel{3}^1} \times \frac{\cancel{3}^1}{\cancel{4}^1} \times \frac{\cancel{4}^1}{\cancel{5}^1} \times \frac{\cancel{5}^1}{\cancel{6}^1} \times \frac{\cancel{6}^1}{\cancel{7}^1} \times \frac{\cancel{7}^1}{8} = \frac{1}{8}$

19 **a** $\frac{1}{120}$

b

Column	1	2	3	4	5
Value	1	$\frac{1}{2}$	$\frac{1}{6}$	$\frac{1}{24}$	$\frac{1}{120}$

c Smaller

d Zero

20 Sample answer: $\frac{3}{2} \times \frac{5}{4} = 1\frac{7}{8}$

EX p132

3.7 Dividing fractions

1 **a** How many $\frac{1}{8}$s are there in 2?

b How many $\frac{1}{3}$s are there in 4?

c How many $\frac{1}{12}$s are there in $\frac{1}{2}$?

d How many $\frac{1}{9}$s are there in $\frac{1}{3}$?

2 **a** $\frac{2}{3} \div \frac{1}{9} = 6$ **b** $\frac{3}{10} \div \frac{1}{5} = 1\frac{1}{2}$

c $\frac{9}{10} \div \frac{2}{5} = 2\frac{1}{4}$ **d** $\frac{5}{6} \div \frac{1}{3} = 2\frac{1}{2}$

e $1 \div \frac{3}{8} = 2\frac{2}{3}$ **f** $\frac{4}{6} \div \frac{2}{9} = 3$

3 **a** $\frac{31}{65}$ **b** $\frac{3}{11}$ **c** $\frac{5}{2}$ or $2\frac{1}{2}$

d $\frac{4}{27}$ **e** 100 **f** $\frac{1}{8}$

g $\frac{1}{34}$ **h** $\frac{3}{65}$ **i** $\frac{7}{30}$

j $\frac{8}{73}$

4 **a** $\frac{4}{7} \times \frac{9}{5}$ **b** $\frac{8}{3} \times \frac{9}{1}$ **c** $\frac{3}{5} \times \frac{1}{4}$

d $\frac{6}{1} \times \frac{10}{3}$ **e** $9\frac{3}{8} \times \frac{2}{3}$ **f** $\frac{8}{3} \times \frac{10}{67}$

g $5\frac{1}{5} \times \frac{9}{67}$ or $\frac{26}{5} \times \frac{9}{67}$ **h** $\frac{3}{1} \times \frac{8}{19}$

i $4\frac{5}{8} \times \frac{1}{10}$ or $\frac{37}{8} \times \frac{1}{10}$ **j** $21 \times \frac{1}{32}$

5 **a** $\frac{15}{16}$ **b** $\frac{11}{12}$ **c** $2\frac{1}{2}$

d $\frac{17}{36}$ **e** $\frac{5}{8}$ **f** $2\frac{1}{4}$

g $\frac{4}{9}$ **h** $7\frac{1}{3}$ **i** $\frac{3}{65}$

j $1\frac{2}{33}$

6 **a** $\frac{1}{27}$ **b** $\frac{1}{12}$ **c** $\frac{1}{4}$

d $\frac{4}{9}$ **e** $2\frac{6}{7}$ **f** 42

g 12 **h** $13\frac{1}{2}$ **i** $6\frac{1}{2}$

j $\frac{30}{7}$

7 **a** $4\frac{3}{8}$ **b** $\frac{22}{35}$ **c** $5\frac{1}{15}$

d $\frac{54}{55}$ **e** $1\frac{4}{5}$ **f** $12\frac{1}{2}$

g $1\frac{1}{4}$ **h** 2 **i** $3\frac{1}{3}$

j $\frac{7}{15}$ **k** $\frac{10}{11}$ **l** $\frac{2}{5}$

8 **a** 18 **b** 5 **c** $\frac{4}{5}$

d $\frac{2}{9}$ **e** 36 **f** 5

9 6

10 8

11 $\frac{3}{8}$

12 72

13 **a** 11 bags **b** $\frac{1}{3}$ kg

14 **a** $\frac{4}{3}$ **b** 27 customers

c 45 customers **d** $\frac{4}{5}$

15 **a** 50 T-shirts **b** 9 backpacks

16 **a** False **b** False **c** True **d** False

EX p136

3.8 Ratios

1 **a** **i** 4 : 5 **ii** 4 : 9

b **i** 3 : 5 **ii** 3 : 8

c **i** 7 : 8 **ii** 7 : 15

d **i** 5 : 3 **ii** 5 : 8

2 **a** 6 : 10 **b** 9 : 9 : 3

3 **a** 9 : 7 **b** 2 : 57 **c** 2 : 14 : 1

d 19 : 6

4 **a** 44 : 69 **b** 16 : 35 **c** 18 : 11

d 85 : 121

5 **a** 5 : 1 **b** 3 : 4 : 9 **c** 28 : 23 **d** 1 : 4

6 **a** 7 : 2 **b** 2 : 3 **c** 13 : 16

d 3 : 8 **e** 11 : 6 **f** 8 : 9

g 5 : 1 : 3 **h** 1 : 6 : 2

7 **a** 3 : 1 **b** 1 : 1 **c** 1 : 3 : 3

8 **a** 3 : 2
6 : 4
15 : 10

b 4 : 5
20 : 25
16 : 20

c 30 : 210
10 : 70
2 : 14

d 144 : 54
24 : 9
8 : 3

e 165 : 121
150 : 110
15 : 11

f 147 : 189
49 : 63
7 : 9

9 **a** 15, 18 **b** 21, 14

c 20, 50 **d** 88, 44

e 63, 56 **f** 26, 169

g 1024, 128, 1152 **h** 333, 777, 444

10 **a** 87 : 100 **b** 13 : 100 **c** 87 : 13

11 **a** 3 : 2 **b** 2 : 5 **c** 3 : 5

d Parts **b** $\left(\frac{2}{5}\right)$ and **c** $\left(\frac{3}{5}\right)$ can be written as fractions as they are expressed as part of the whole.

e Part **a** cannot be written as a fraction as it is comparing shaded to non-shaded sections.

f True

12 **a** 12 **b** 4 : 3

13 Jesse: $16 million, James: $10 million

14 **a** 2 : 1 : 3 **b** $\frac{1}{3}, \frac{1}{6}, \frac{1}{2}$

15 **a** 4 : 9 **b** 9 L **c** $\frac{4}{9}$ **d** $4\frac{1}{2}$ L

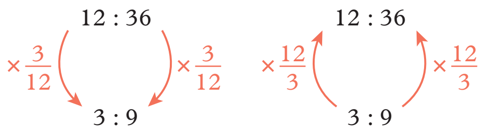
16 16 : 9

17 **a** 1945

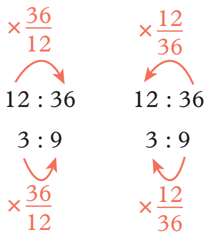
b **i** 54 : 61 **ii** 3 : 4

c 389 : 440

18 **a**



b



19 **a** Yes, since the ratios can be expressed in the same simplest form.

b Yes, since the ratios can be expressed in the same simplest form.

c No, since the ratios cannot be expressed in the same simplest form.

d No, since the ratios cannot be expressed in the same simplest form.

20 $3\frac{1}{8}$ cups of self-raising flour and $2\frac{1}{2}$ cups of Greek yoghurt

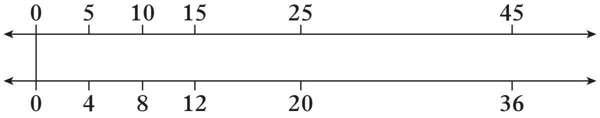
21 450 mL

22 **a** 10 **b** $\frac{4}{10}$ and $\frac{6}{10}$ or $\frac{2}{5}$ and $\frac{3}{5}$

c \$120 and \$180 **d** Yes

23 \$28

24 **a**



0	5	10	15	25	45
0	4	8	12	20	36

b Sample answer: We are adding multiples of the number to itself which is the same as multiplying by a whole number. They are the same ratio because we are adding the same number of multiples to each number in the ratio.

25 260 marbles

26 **a** The second tin **b** 5 L

c 1.5 L **d** 7 : 3

e **i** $\frac{1}{8}$ **ii** 3.5 L

EX
p141

Review: Fractions and ratios

Review questions 3.9A: Mathematical literacy review

1 Like fractions have the same denominators and unlike fractions have different denominators.

2 **a** $\frac{1}{2}$ **b** $\frac{2}{3}$ **c** $\frac{3}{4}$ **d** $4\frac{1}{10}$

3 **a** one-third **b** two-ninths

c two-quarters **d** twelve and a half

4 **a** equivalent fractions

b common denominator

5 **a** Unit fractions are fractions that have a numerator of 1. For example, $\frac{1}{3}$.

b The product of a number and its reciprocal is 1. The reciprocal can be found by first writing a number as a proper or improper fraction and then swapping the numerator and the denominator. For example, the reciprocal of $\frac{3}{2}$ is $\frac{2}{3}$.

c A ratio is a comparison of two or more quantities of the same kind; for example, when comparing the number of red jellybeans to blue jellybeans, the ratio might be 2 : 3.

6 **a** Improper fractions, mixed numbers, proper fraction

b the simplest form, highest common factor

c numerators, denominator

7 **a** True **b** True **c** True **d** True

e False. 0 does not have a reciprocal. The product of 0 and any number is always 0, not 1.

f False. Cancelling can be performed when a fraction is multiplied by a fraction or a whole number. For example, $\frac{2}{3} \times \frac{3}{5} = \frac{2}{5}$.

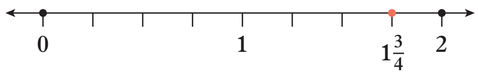
g False. A ratio can be used to compare two or more quantities. There is no upper limit to how many quantities can be compared. For example, when comparing the number of apples to pears to oranges to lemons, the ratio might be 1 : 2 : 3 : 4.

Review questions 3.9B: Multiple choice

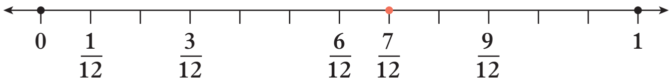
1 D **2** D **3** B **4** B
5 A **6** A **7** D **8** D
9 A **10** C **11** A **12** D
13 B **14** C **15** D

Review questions 3.9C: Short answer

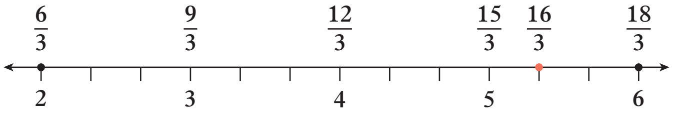
1 **a**



b



c



2 **a** $\frac{5}{3}, \frac{9}{3}$ **b** $\frac{34}{5}, \frac{41}{5}$ **c** $\frac{404}{4}, \frac{409}{4}$

3 **a** $\frac{10}{25}$ **b** $\frac{27}{24}$ **c** $\frac{21}{49}$ **d** $\frac{10}{24}$

4 **a** $\frac{4}{5}$ **b** $\frac{1}{9}$ **c** $\frac{1}{2}$ **d** $\frac{3}{7}$

5 $\frac{1}{4}, \frac{2}{7}, \frac{11}{20}, \frac{3}{5}, 1\frac{2}{3}, \frac{9}{2}$

6 **a** $\frac{19}{8}$ **b** $5\frac{1}{2}$ **c** $1\frac{8}{9}$ **d** $\frac{16}{3}$

7 **a** $\frac{5}{9}$ **b** $\frac{2}{11}$ **c** $\frac{1}{6}$ **d** $\frac{7}{10}$

8 **a** $5\frac{6}{7}$ **b** $\frac{4}{5}$ **c** $1\frac{1}{8}$ **d** $4\frac{7}{15}$

9 **a** $\frac{8}{35}$ **b** $\frac{1}{6}$ **c** $\frac{20}{33}$ **d** $1\frac{1}{15}$

10 **a** $1\frac{1}{5}$ **b** $\frac{17}{35}$ **c** $8\frac{1}{4}$ **d** $5\frac{23}{24}$

11 **a** $1\frac{1}{8}$ **b** $1\frac{1}{4}$ **c** $4\frac{2}{3}$ **d** $\frac{3}{4}$

12 **a** $1\frac{2}{3}$ **b** $\frac{2}{3}$ **c** 9 **d** $1\frac{5}{8}$

13 **a** 4 : 3 **b** 3 : 2 **c** 1 : 14 **d** 5 : 2

14 **a** 2 : 3 **b** 3 : 4 **c** 5 : 8 **d** 14 : 1

15 56

Review questions 3.9D: Mathematical modelling

1 **a** $\frac{1}{2}$ **b** $\frac{1}{3}$ **c** 3 : 2

d 25 **e** $\frac{1}{12}$, 25 families

f Correct about one pet, because 5 : 10 = 1 : 2; but not about two pets, because $\frac{3}{10} \neq \frac{1}{3}$.

2 **a** $\frac{3}{2}$ or $1\frac{1}{2}$ **b** $\frac{7}{10}$ **c** $\frac{4}{5}$

d $\frac{8}{15}$ **e** $\frac{3}{5}$

f Camila and Madelaine

3 **a** 1 L **b** $\frac{1}{3}$ **c** $\frac{1}{4}$ L

d $\frac{11}{12}$ **e** $1\frac{1}{4}$ L **f** $\frac{3}{8}$

g 900 mL

MODULE 4 Decimals and percentages

EX p151

4.1 Decimals

1 **a** Eight ones or 8

b Eight thousandths or $\frac{8}{1000}$

c Eight tenths or $\frac{8}{10}$

d Eight ten-thousandths or $\frac{8}{10\,000}$

e Eight tens or 80

f Eight hundredths or $\frac{8}{100}$

g Eight hundreds or 800

h Eight hundredths or $\frac{8}{100}$

i Eight ten-thousandths or $\frac{8}{10\,000}$

j Eight tenths or $\frac{8}{10}$

k Eight thousands or 8000

l Eight tens or 80

2 **a** 5785.195 **b** 496.283 **c** 0.006

d 5.703 **e** 36.07 **f** 40.1007

3 **a** 3 **b** 5 **c** 2

d 6 **e** 0 **f** 1

g 8 **h** 1

4 **a** 0.1 **b** 0.5 **c** 0.73

d 1 **e** 1.16 **f** 0.07

5 **a** 0.07 **b** 0.1 **c** 0.5

d 0.73 **e** 1 **f** 1.16

6 **a** 0.7 **b** 0.9 **c** 0.33

d 0.531 **e** 0.27 **f** 0.11

g 0.2978 **h** 0.804

7 **a** 0.03 **b** 0.17 **c** 0.9

d 0.2010 **e** 0.0375 **f** 0.0156

g 0.07 **h** 0.921 **i** 0.042

j 0.0003 **k** 0.019 81 **l** 0.0086

8 **a** $\frac{1}{4}$ **b** $\frac{1}{2}$ **c** $\frac{7}{20}$

d $\frac{22}{25}$ **e** $\frac{23}{100}$ **f** $\frac{2}{25}$

g $\frac{3}{25}$ **h** $\frac{19}{20}$

9 **a** $\frac{1}{5}$ **b** $\frac{7}{10}$ **c** $\frac{13}{100}$

d $\frac{13}{50}$ **e** $\frac{9}{1000}$ **f** $\frac{213}{500}$

g $\frac{64}{125}$ **h** $\frac{18}{25}$ **i** $\frac{1}{2500}$

j $1\frac{14}{25}$ **k** $2\frac{3}{8}$ **l** $3\frac{7}{25}$

10 **a** $5\frac{83}{100}$ **b** $3\frac{451}{500}$ **c** $6\frac{9}{20}$

d $8\frac{7}{20}$ **e** $2\frac{1}{1250}$ **f** $3\frac{9}{10}$

g $12\frac{22}{25}$ **h** $1\frac{353}{500}$

11 **a** 0.875 **b** 0.75 **c** 0.8

d 1.125 **e** 14.25 **f** 1.875

g 6.6 **h** 7.25 **i** 0.15

j 0.04 **k** 0.14 **l** 0.225

12 **a** **i** $\frac{3}{10}$ **ii** 0.3

b **i** $\frac{5}{10}$ or $\frac{1}{2}$ **ii** 0.5

c **i** $\frac{8}{10}$ or $\frac{4}{5}$ **ii** 0.8

13 **a** $\frac{1}{5} = \frac{2}{10} = 0.2$

b $\frac{3}{4} = \frac{75}{100} = 0.75$

c $\frac{7}{20} = \frac{35}{100} = 0.35$

d $\frac{4}{5} = \frac{8}{10} = 0.8$

e $\frac{11}{50} = \frac{22}{100} = 0.22$

f $\frac{2}{4} = \frac{50}{100} = 0.50$ or 0.5

g $\frac{13}{20} = \frac{65}{100} = 0.65$

h $\frac{1}{2} = \frac{5}{10} = 0.5$

i $\frac{19}{50} = \frac{38}{100} = 0.38$

14 a 0.3

b There are 100 small squares; 30 are shaded.

c $\frac{30}{100}$ or $\frac{3}{10}$

d 0.3 and 0.30 are the same as both are describing 3 tenths and 0 hundredths or 30 hundredths. Zeros on the end of the decimal to the right don't change the value of the number.

15 a 10 **b** $\frac{2}{10}$ or $\frac{1}{5}$

c 0.2 **d** Apple: $\frac{2}{10}$ or $\frac{1}{5} = 0.2$

Cola: $\frac{1}{10} = 0.1$

Watermelon: $\frac{2}{10}$ or $\frac{1}{5} = 0.2$

Strawberries & cream: $\frac{3}{10} = 0.3$

16 0.3 is greater than $\frac{1}{4}$ (which is equivalent to 0.25).

17 Sample answer: He has forgot to put a zero in the tenths place: 42 thousandths means 4 hundredths and 2 thousandths.

18 a 4.25 m and 1.8 m (or $4\frac{1}{4}$ m and $1\frac{4}{5}$ m)

b 6.05 m (or $6\frac{1}{20}$ m) **c** Yes

19 a 0.01 **b** 0.001 **c** 0.000 01

20 a 3471.49 **b** 23.5437 **c** 39.307

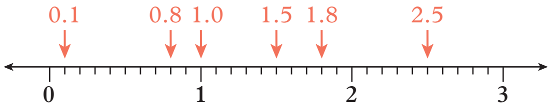
d 500 **e** 9999.099 **f** 219.99

21 $3\frac{162}{200} = 3\frac{81}{100}$

EX
p156

4.2 Ordering and rounding decimals

1



2 a > **b** < **c** >

d < **e** > **f** >

g > **h** > **i** >

3 a 1.90, 1.91, 9.01, 9.10

b 0.55, 5.05, 5.5, 5.55

c 4.0567, 4.5067, 4.567, 45.67

d 0.001, 0.100001, 0.101, 1

4 a 0.146, 0.238, 0.328, 0.461, 0.641, 0.823

b 1.234, 2.143, 2.341, 2.431, 3.412, 4.132

c 32.861, 36.182, 36.218, 36.812, 38.162, 38.261

d 18.1524, 18.2145, 18.2154, 18.4125, 18.4152, 18.4251

5 a 0.000 40, 0.0003, $\frac{21}{100\,000}$, $\frac{2}{10\,000}$, 0.000 04, 0.000 030

b 0.8, $\frac{80\,001}{100\,000}$, 0.801, 0.81, 8.0, $\frac{81}{10}$

c 0.0510, 0.15, $\frac{1}{5}$, $\frac{1}{2}$, 0.51, $\frac{501}{100}$

d 3.75, $3 + \frac{6}{10} + \frac{2}{100}$, $3 + \frac{2}{10} + \frac{6}{100}$, $3 + \frac{25}{100}$, 3.026, $3 - \frac{25}{100}$

6 a 3.547 **b** 3.5465 **c** 3.55

7 a 82.63 **b** 59.112 **c** 0.2634

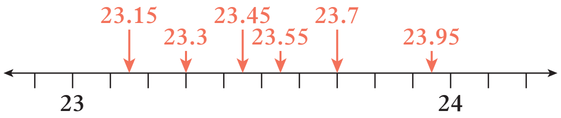
d 0.27 **e** 15.000 **f** 2.175 499

g 0.219 **h** 165.5455 **i** 43.806 25

8 a 200 **b** 170 **c** 168

d 168.2 **e** 168.25 **f** 168.247

9 a



b 23 **c** 24

d If the tenths digit is 0, 1, 2, 3 or 4, round to the whole number part of the number. If the tenths digit is 5, 6, 7, 8 or 9, add 1 to the whole number and discard the remaining digits (the decimal place digits).

10 a 21.89 m **b** 22.81 m

c Arrange in ascending order. The first number will be the shortest distance and the last number will be the largest.

11 a 0 tenths, 0 hundredths, 0 thousandths

b A different value would result. The zero holds the position of the digits in the correct place value. The result would be greater because zero is replaced by a bigger number in that place value.

c Zeros are needed so that the correct decimal can be represented with all digits in the right positions according to place value.

d Trailing zeros can be left out as they do not change the value of the decimal.

e 12.021, 12.201, 12.210

12 The digit in the next place value is a 5, meaning the digit in the third place value must have 1 added to it.

13 a i 5.4 and 5.41

ii 3.0 and 3.001

iii 2.99 and 2.9

iv 0.00 and 0.003 12

b i 5.4 and 5.42

ii 3.0 and 3.001

iii 3.00 and 3.0

iv 0.00 and 0.003 12

c When the number rounds down (the following digit is 0–4).

d When the number rounds up (the following digit is 5–9).

14 a 2 decimal places **b** 2 decimal places

c 4 decimal places **d** 3 decimal places

15 a Despite 2 > 1, the place value of the 2 (ten thousandths) is much smaller than the place value of the 1 (tenths). 0.0002 < 0.1

b Despite 1 < 100, the place value of the 1 is the same in both (thousandth), all other digits are 0. 0.001 = 0.001 00

c Despite 5 = 5, the place value of the 5 is not the same in both (hundredth and tenth). 0.05 < 0.5

d Despite 1234 < 5678, the first non-zero digits do not have the same place value (1 ten and 5 ones). 12.34 > 5.678

16 a Inequality is incorrect. Reasoning is incorrect – Tyler incorrectly converted the fractions to decimals $\frac{1}{8} = 0.125$ and $\frac{1}{5} = 0.2$.

b Inequality is correct. Reasoning is incorrect – Tyler did not compare using the correct place values 3400 < 6789.

c Inequality is correct. Reasoning is correct.

17 a To 2 decimal places to $42.22 as we do not pay amounts less than one cent on a card.

b To the nearest 5 cents to $40.00 as we do not pay amounts less than five cents using cash.

c To the nearest degree to 32° as the hiker only has to travel 20 m so will not be far off course.

d To 4 decimal places to 32.2851° as the rocket has to travel a very long distance and it is very expensive if it veers off course and crashes or does not have enough fuel to correct the error.

e To 2 decimal places to 1.592 m as we generally can't tell much difference between millimetre differences of height.

f To 3–4 decimal places to 1.592 cm or 1.5924 cm as it is likely the bug is being measured for a report and more accuracy for the comparison would be useful.

18 a 58.3 cm **b** 98.212 L

c 214.193 km **d** 214.19275 km

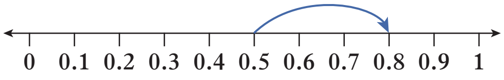
19 a 1 decimal place **b** 3 decimal places

c 3 decimal places **d** 5 decimal places

EX
p160

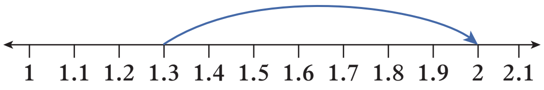
4.3 Adding and subtracting decimals

1 a i



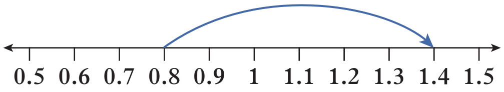
ii 0.8

b i



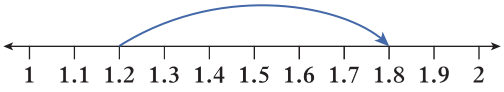
ii 2

c i



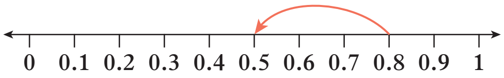
ii 1.4

d i



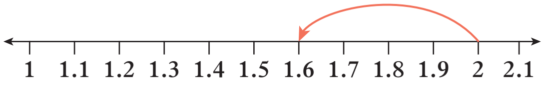
ii 1.8

2 a i



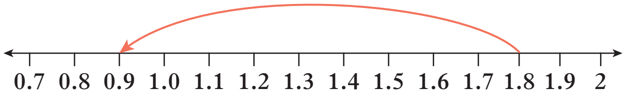
ii 0.5

b i



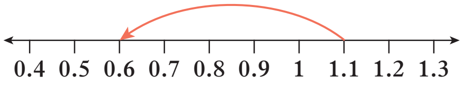
ii 1.6

c i



ii 0.9

d i



ii 0.6

3 a 90 **b** 310 **c** 200
d 290 **e** 150 **f** 1210
g 940 **h** 8100

4 a 88.21 **b** 316.47 **c** 192.273
d 292.065 **e** 149.601 **f** 1208.2163
g 944.911 **h** 8102.391

5 a 36.012 **b** 1519.797 **c** 66.017
d 3855.1036 **e** 177.9932 **f** 15.971

6 a 60 **b** 30 **c** 400
d 20 **e** 1470 **f** 110
g 10 **h** 1230

7 a 56.23 **b** 34.72 **c** 394.9
d 23.22 **e** 1476.68 **f** 113.27
g 15.588 **h** 1233.6118

8 a 123.926 **b** 7.487 **c** 15.47
d 25.14 **e** 6.675 **f** 10.338

9 See answers in questions **5** and **8**.

10 a 283.15 **b** 62.369 **c** 15.413
d 617.58 **e** 0.23 **f** 158.7

11 66.68 m

12 a $43

b $7

c $42.51, change is $7.49

d Only 49 cents away from the actual amount

13 a 42.6

b 42.1

c Natasha received the higher score by 0.5.

14 Jonathan 0.09 m, Kamaro 0.13 m, Tan 0.14 m

15 a The place values are not lined up, so the incorrect digits are added.

b The 7 in the second row has not been subtracted as the additional zero was not added above it.

c The 6 was not reduced to a 5 when borrowing from the ones column, so the calculation in this column is incorrect.

d The student could not perform 3 – 7 without negatives so instead performed 7 – 3 rather than taking 10 tenths from the 5 ones.

16 a i 10 + 2
ii 12, higher than the actual answer
iii 11.25
b i 15 – 9
ii 6, higher than the actual answer
iii 5.84
c i 14 – 3
ii 11, lower than the actual answer
iii 11.15
d i 25 – 3 + 6
ii 28, lower than the actual answer
iii 27.856

17 a 12.49 L
b Three tins

18 Total monthly rainfall of 64.68 mm is 1.02 mm below September average, so it is very close to the average.

19 a Day 1: 187.2 km; Day 2: 148.6 km; Day 3: 249.8 km; Day 4: 144.8 km; Day 5: 251.5 km
b 28 071.5 **c** Day 4, 144.8 km
d Day 5, 251.5 km **e** 981.9 km

20 a 22.09 kg
b More than 2.09 kg

21 a i $453.98, $394.99, $329.99, $79.99, $51.46, $53.24, $48.24
ii $407.52 **iii** $127.20 **iv** $48.24
b Elisa has $48.24 in her account at the end of the month; not enough to purchase a drone.

22 86.945

23 a i 462.2 **ii** 1.279 **iii** 0.00 **iv** 12.64
b i 28 **ii** 26
c i 27.643 **ii** 27.642
d the rounded estimate
e when all numbers round down
f The rounding estimate will always be greater than or equal to the truncating estimate, because when a value is rounded up it increases and when a value is rounded down it is equal to the truncated value.
g 0.3 **h** 0.15

EX p166

4.4 Multiplying decimals

1 a 5 **b** 7 **c** 6
d 6 **e** 6 **f** 6

2 a 0.12 **b** 0.24 **c** 0.42
d 0.072 **e** 0.0036 **f** 0.49
g 0.96 **h** 0.000 24 **i** 0.000 000 7

3 a 26.6 **b** 39.2 **c** 3.15
d 9.12 **e** 36.0072 **f** 19.68
g 8.277 **h** 359.8 **i** 79.44

4 a 69.16 **b** 456.48 **c** 1258.32
d 28.644 **e** 130.974 **f** 964.73
g 47.7776 **h** 24.311 92 **i** 2251.1775

5 a 32.88 **b** 86.76 **c** 198.15
d 622.08 **e** 0.185 68 **f** 7.4684
g 12.4 **h** 3.702 **i** 1041.56

6 See answers for questions **4** and **5**.

7 a 6000 **b** 4200 **c** 1.284
d 1572.1 **e** 0.4 **f** 7.02
g 61 **h** 150 **i** 310
j 6481.3 **k** 480 000 **l** 128 468 000

8 a 9086 **b** 297 600 **c** 104 688
d 33 984 **e** 97 500 **f** 12 150

9 a 12.0807 **b** 36.62
c 12.958 **d** 0.481 74

10 a All the calculations involve the same digits in the same order.
b The digits represent different place values in the different numbers.
c, d

Original calculation	Estimated answer	Answer
38 × 21	40 × 20 = 800	798
3.8 × 21	4 × 20 = 80	79.8
38 × 2.1	40 × 2 = 80	79.8
3.8 × 2.1	4 × 2 = 8	7.98
380 × 210	400 × 200 = 80 000	79 800

e The number of decimal places in the answer equals the total number of decimal places in the original calculation.

11 a $60 **b** $77.48
c The estimate is $17.48 lower.

12 a $9.74 **b** $6.08 **c** $9.73
d $8.28 **e** $8.97 **f** $10.09

13 a i 54 **ii** 56.062 245 85 **iii** 2.062 245 85
b i 2 400 000 **ii** 1 942 774.728 **iii** 457 225.272
c i 4 **ii** 5.345 665 957 **iii** 1.345 665 957
d i 0.0054 **ii** 0.005 304 031 2 **iii** 0.000 095 968 8

14 a 0.000 21 **b** 1.0015 **c** 10.8054 **d** 22.0779

15 a $713.40 **b** Option 1 **c** $68.40

16 a i 0.04 **ii** 0.16 **iii** 0.0625
iv 0.000 064 **v** 1.44
b The number of decimal places in the answer is twice the number of decimal places in the number to be squared.

17 8 mm

18 a $258 **b** $147.60

19 a 3.654 **b** 10.549 85 **c** 60.069 67

20 Sample answer: 14.679 × 0.5382 = 7.900 237 8

EX p169

Checkpoint

1 a Five thousandths or $\frac{5}{1000}$
b Five tenths or $\frac{5}{10}$
c Five ten-thousandths or $\frac{5}{10\,000}$
d Five hundredths or $\frac{5}{100}$

2 **a** $\frac{17}{100}$ **b** $5\frac{3}{10} = \frac{53}{10}$

c $\frac{3}{20}$ **d** $9\frac{1}{8} = \frac{73}{8}$

3 **a** 0.71 **b** 2.4 **c** 0.625

d 11.35

4 **a** $1.3 > 0.6$ **b** $0.1 = 0.10$

c $0.07 < 0.5$ **d** $31.2729 < 31.8$

5 **a** 1.087, 1.708, 7.108, 7.180, 17.80

b 15.986, 15.689, 1.5698, 1.5689, 1.056 89

c 0.08, $\frac{18}{100}$, 0.20, 0.7, $\frac{9}{10}$

d $\frac{11}{5}$, 1.80, $\frac{34}{20}$, $\frac{3}{2}$, 1.4

6 **a** 6.47 **b** 32.183 **c** 0.003 **d** 100.0

7 **a** 5.9 **b** 2.2 **c** 13.27 **d** 6.58

8 **a** 86.18 **b** 290.89 **c** 62.11 **d** 382.08

9 **a** 59.1 **b** 0.412 **c** 124 **d** 5.282

10 **a** 0.000 54 **b** 1.84 **c** 8.685 **d** 137.28

11 **a** $3 - 1 + 9 = 11$ **b** $9 + 10 \times 1 = 19$

c $37 \times (15 + 5) = 740$ **d** $(18 - 9) \times (0 + 0) = 0$

12 **a** 11.39 **b** 15.38 **c** 742 **d** 2.024

4.5 Dividing decimals

EX p172

1 **a** 6.23 **b** 13.721 **c** 9.102

d 23.14 **e** 11.115 **f** 1.6398

g 2.9217 **h** 4.15 **i** 3.51

2 **a** 2.5 **b** 6.75 **c** 12.6

d 4.35 **e** 2.05 **f** 0.4062

g 0.603 75 **h** 39.05 **i** 14.0575

3 **a** \$1.28 **b** \$217.50 **c** \$1.28

d \$0.75 **e** \$11.80 **f** \$7.25

4 **a** \$3.88 **b** \$4.98 **c** \$3.98 **d** \$11.97

e \$6.98 **f** \$7.98 **g** \$1.70 **h** \$4.27

5 **a** 0.678 543 **b** 9.535 **c** 28.894

d 68.9421 **e** 0.150 95 **f** 0.0436

6 **a** 0.562 78 **b** 1155.6 **c** 559.68

d 2.4978 **e** 61.4304 **f** 40.9625

7 **a** $0.5 \div 0.1$ **b** $0.8 \div 0.4$

c $0.3 \div 0.06$ **d** $0.6 \div 0.04$

8 **a** 5 **b** 2 **c** 5 **d** 15

9 **a** $623.56 \div 8$ **b** $9840 \div 8$ **c** $1956.74 \div 6$

d $4.62 \div 3$ **e** $420 \div 4$ **f** $0.5 \div 2$

10 **a** 19.3 **b** 281 **c** 84.7

d 34 150 **e** 47.1 **f** 364.8

11 **a** 11.905 **b** 43.375 **c** 5412.5

d 188.25 **e** 2282.45 **f** 14.325

12 **a** 15 960 **b** 22 **c** 2096.96

d 4 **e** 500 **f** 5.674 295

13 \$186.25

14 \$18.90 ÷ 6 = \$3.15. Whole pizza is better value by \$0.35 per slice.

15 **a** \$2.49

b 2 L container, as it is cheaper per litre.

16 **a** \$8.99 ÷ 12 ≈ \$0.75. The carton of eggs is better value.

b \$0.15

c Some people may want to buy less than 12 eggs.

17 **a** \$3.73 **b** \$1.56 **c** \$0.53 **d** \$1.50

18 **a** Firenze \$0.93, Milano \$0.82; Milano is better value

b Two, Firenze (\$7.46 compared to \$8.18)

19 Packing boxes at approximately \$8.13 an hour

20 **a** \$2.75

b If using it five days per week, weekly option is cheaper by 20 cents per day.

c If planning to travel less than five days per week, buying daily tickets will cost less than a weekly ticket.

21 **a** Simon \$0.80, Julia \$0.66 **b** Julia

22 86

23 **a** **i** 5 **ii** 50 **iii** 500

iv 5000

b The answer increases by a power of 10.

c The answer is larger than the dividend.

d True

24 **a** **i** 1243 **ii** 0.1243

b **i** 94 257 **ii** 942.57

25 **a** 144

b Yes, 0.1 m remaining

26 Divide cost by volume to find the cost of 1 L of each. Orange mineral water is the better buy (\$1.68 per L compared to \$1.80 per L for cola).

27 **a** \$2.10

b \$21.00

c The cost of one avocado is found by dividing \$6.30 by 3. Multiply by 10 for cost of 10 avocados.

28 **a** Ava

b 6.08 km/h

4.6 Percentages

EX p178

1 **a** 50% **b** 52% **c** 36%

d 60% **e** 55% **f** 45%

2 **a** 50% **b** 48% **c** 64%

d 40% **e** 45% **f** 55%

3 **a** 23% **b** 78% **c** 35%

d 99% **e** 13% **f** 7%

g 82% **h** 4%

4 **a** 45% **b** 22% **c** 175%

d 68% **e** 160% **f** 20%

g 60% **h** 92% **i** 120%

j 225% **k** 80% **l** 20%

5 **a** 23% **b** 80% **c** 13%

d 67% **e** 60% **f** 170%

g 209% **h** 314% **i** 79%

j 191% **k** 4% **l** 55%

6 **a** $\frac{7}{100}$ **b** $\frac{19}{100}$ **c** $\frac{43}{100}$

d $\frac{57}{100}$ **e** $\frac{3}{100}$ **f** $\frac{63}{100}$

g $\frac{11}{100}$ **h** $\frac{91}{100}$ **i** $\frac{79}{100}$

j $\frac{21}{100}$ **k** $\frac{99}{100}$ **l** $\frac{33}{100}$

7 **a** $\frac{16}{25}$ **b** $\frac{9}{50}$ **c** $\frac{27}{100}$

d $\frac{4}{5}$ **e** $\frac{1}{20}$ **f** $\frac{19}{20}$

g $\frac{1}{10}$ **h** $\frac{9}{20}$ **i** $\frac{3}{5}$

j $\frac{3}{4}$ **k** $\frac{7}{20}$ **l** $\frac{22}{25}$

8 **a** **i** 14 sections shaded out of a total of 20 sections

ii $\frac{14}{20}$ **iii** $\frac{70}{100}$ **iv** 70%

v 0.7

b **i** 5 sections shaded out of a total of 10 sections

ii $\frac{5}{10}$ **iii** $\frac{50}{100}$ **iv** 50%

v 0.5

c **i** 1 section shaded out of a total of 4 sections

ii $\frac{1}{4}$ **iii** $\frac{25}{100}$ **iv** 25%

v 0.25

9 **a** **i** 76% **ii** 0.76

b **i** 40% **ii** 0.4

c **i** 75% **ii** 0.75

d **i** 30% **ii** 0.3

e **i** 35% **ii** 0.35

f **i** 6% **ii** 0.06

10 **a** C **b** A **c** D **d** B

11 **a** 40% **b** 90%

c 4% **d** 20%

12 **a** **i** 20% **ii** 10% **iii** 30%

b **i** 80% **ii** 90% **iii** 70%

c Sample answer: Divide the number of pieces eaten by the total number of pieces and multiply by 100.

13 **a** 3 : 2 **b** 3 : 1 **c** 9 : 11

d 9 : 41

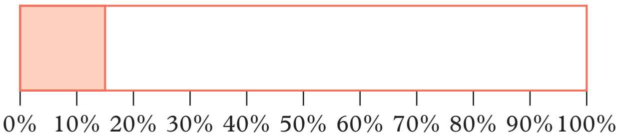
14 **a** See part **c**

b **i** 10 is factor of 100 so a denominator of 10 can easily be expressed as a denominator of 100.

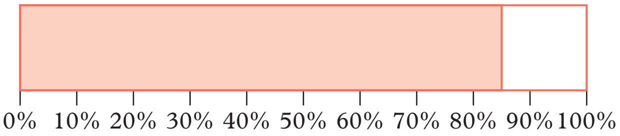
ii Each cm will represent 10%.

iii Each mm will represent 1%.

c 15% = $\frac{15}{100}$ so 15 mm out of 100 mm (or 1.5 cm out of 10 cm) needs to be shaded.



d **i**



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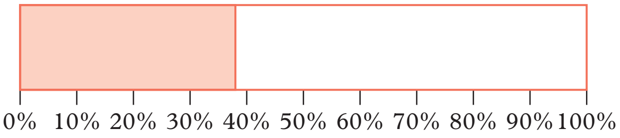
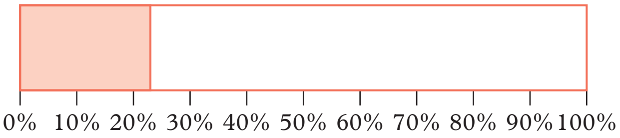
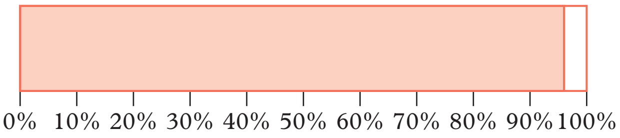


iii



iv



15 **a** 40% **b** 30% **c** 65% **d** 75%

16 Sample answer: Percentage is calculated by finding volume of liquid in each container as a fraction of the total volume that container holds. However, 50% of a large container is still going to be more that 50% of a small container.

17 **a** Three times the whole amount

b **i** 2 **ii** 3.5

iii 4.91 **iv** 3.754

18 **a** 4.5 **b** 525%

19 Sample answer: 20% of the full glass (100%) is more than 20% of the partially emptied glass (80%), so it will not fill back to the brim.

20 **a** The values that can appear here are between 0.4 and 0.65.

b The values that can appear here are fractions with a denominator of 15 and are greater than $\frac{6}{15}$.

c The values that can appear here are $\frac{9}{15}$, $\frac{8}{15}$, $\frac{7}{15}$, $\frac{6}{15}$, $\frac{5}{15}$, $\frac{4}{15}$, $\frac{3}{15}$, $\frac{2}{15}$, and $\frac{1}{15}$.

d The value in this region are $\frac{9}{15}$, $\frac{8}{15}$, and $\frac{7}{15}$.

EX p185

4.7 Fractions, decimals and percentages

1

Percentage	Fraction	Decimal
50%	$\frac{1}{2}$	0.5
25%	$\frac{1}{4}$	0.25
75%	$\frac{3}{4}$	0.75
30%	$\frac{3}{10}$	0.3
12.5%	$\frac{1}{8}$	0.125
62.5%	$\frac{5}{8}$	0.625
20%	$\frac{1}{5}$	0.2
40%	$\frac{2}{5}$	0.4
60%	$\frac{3}{5}$	0.6

2 **a** $\frac{23}{50}$, 0.46 **b** $\frac{13}{100}$, 0.13 **c** $\frac{99}{100}$, 0.99
d $\frac{1}{4}$, 0.25 **e** $\frac{1}{5}$, 0.2 **f** $\frac{1}{2}$, 0.5
g $\frac{1}{20}$, 0.05 **h** $\frac{2}{25}$, 0.08 **i** $\frac{1}{100}$, 0.01

3 **a** 28%, $\frac{7}{25}$ **b** 88%, $\frac{22}{25}$ **c** 15%, $\frac{3}{20}$
d 62%, $\frac{31}{50}$ **e** 45%, $\frac{9}{20}$ **f** 72%, $\frac{18}{25}$
g 9%, $\frac{9}{100}$ **h** 4%, $\frac{1}{25}$ **i** 30%, $\frac{3}{10}$

4 **a** 102%, 1.02 **b** 85%, 0.85 **c** 280%, 2.8
d 225%, 2.25 **e** 52%, 0.52 **f** 1050%, 10.5
g 40.5%, 0.405 **h** 415%, 4.15 **i** 7121%, 71.21

5 **a** $23.84 = 0.2384 = \frac{149}{625}$
b $19.65\% = 0.1965 = \frac{393}{2000}$
c $46.7\% = 0.467 = \frac{467}{1000}$
d $3.09\% = 0.0309 = \frac{309}{10\,000}$
e $567.4\% = 5.674 = 5\frac{337}{500}$
f $0.467\% = 0.00467 = \frac{467}{100\,000}$
g $12.895\% = 0.12895 = \frac{2579}{20\,000}$
h $73.28\% = 0.7328 = \frac{458}{625}$
i $200.5\% = 2.005 = 2\frac{1}{200}$

6 **a** 51.8%, $\frac{259}{500}$ **b** 902%, $9\frac{1}{50}$ **c** 10.5%, $\frac{21}{200}$
d 70%, $\frac{7}{10}$ **e** 2150%, $21\frac{1}{2}$ **f** 841%, $8\frac{41}{100}$
g 710%, $7\frac{1}{10}$ **h** 0.9%, $\frac{9}{1000}$ **i** 17.65%, $\frac{353}{2000}$

7 **a** 37.5%, 0.375
b 7.5%, 0.075
c 3.75%, 0.0375
d 9.375%, 0.093 75
e 212.5%, 2.125
f 227.5%, 2.275
g 946.25%, 9.4625
h 1190.625%, 11.906 25
i 11.25%, 0.1125

8 **a** $\frac{2}{5}$
b **i** 40% **ii** 60%
c **i** 0.4 **ii** 0.6

9 **a** 68% **b** 210% **c** 215% **d** 87.5%

10 **a** $1\frac{9}{10} = \frac{38}{20} = 1.9 = 190\%; \frac{38}{200} = 19\%;$
$\frac{19}{95} = 20\% = 0.2$
$\frac{19}{5} = 3.8; 1.09 = 109\%; 0.38 = 38\%$
b $\frac{38}{200} = 19\% = 0.19; \frac{19}{5} = 3.8 = 380\%$
$1.09 = 109\% = 1\frac{9}{100} = \frac{109}{100}; 0.38 = 38\% = \frac{19}{50}$
c 3.8, 1.9, 1.09, 0.38, 0.2, 0.19
d **i** $\frac{25}{19} \times 3.8 = \frac{25}{19} \times \frac{19}{5} = 5$
ii $\frac{19}{95} \times 100 = 20\% \times 100 = 20$
iii $\frac{1}{19} \times 38\% = \frac{1}{19} \times \frac{19}{50} = 0.02$

11 **a** $\frac{71}{100}$, 0.71
b **i** 0.09% **ii** 0.0009 **iii** $\frac{9}{10\,000}$
c 99.853%; values may be rounded and there may be other substances in the Sun.

12 20%

13 Both methods of working are correct.

14 2, 4, 5, 8, 10, 16, 20, 25, 32, 40, 50, 64, 80

15 One possible solution: $\frac{37}{40} = 92.5\%$

EX p189

4.8 Calculating percentages

1 **a** 75% **b** 125% **c** 60%
d 50% **e** 200% **f** 562.5%
g 92.5% **h** 87.5%

2 **a** 195 **b** 12 **c** 104
d 40 **e** 7.5 **f** 18
g 39 **h** 12

3 **a** \$4.80 **b** \$76.50 **c** \$55
d \$198.90 **e** \$700 **f** \$168.30
g \$1485 **h** \$8.95

4 **a** 4.5 **b** 3.85 **c** 90.96
d 30.6 **e** 27.82 **f** 288.75
g 50.88 **h** 25.2 **i** 80.94
j 21.12 **k** 12.375 **l** 7.956

5 **a** 44 **b** 73.5 **c** 45.6
d 400 **e** 10.68 **f** 450.8
g 148.5 **h** 308

6 **a** Each percentage is greater than 100%.
b The final amount is greater than the original amount.
7 **a** halve **b** triple **c** quadruple
8 **a** 5980 **b** 520
9 **a** 72% **b** 76.7%
c The second test, as his percentage is higher
10 **a** 0.2
b Wally: 253, Artemis: 115, Megan: 92
c **i** 11 : 9 **ii** 11 : 5 **iii** 5 : 4
11 **a** 62.5% **b** 160%
12 **a** **i** profit **ii** $37
b **i** loss **ii** $14
c **i** loss **ii** $336
d **i** loss **ii** $11 319
e **i** profit **ii** $1.44
f **i** loss **ii** $6.15
g **i** profit **ii** $7.52
13 **a** 74% **b** 23.73% **c** 35.94% **d** 39.1%
e 180% **f** 70.29% **g** 125.75%
14 **a** $4 **b** six frames; $6.25
c loss of $2.25 per frame **d** $122.50
e 36.03%
f undamaged 47.06%; damaged 36.03%
15 Thomas. Property 1 increased by 38.56%, Property 2 increased by 33.33%.
16 **a** 72 **b** 180 **c** 60 **d** 900
17 **a** 171 **b** 213.75%
18 32%
19 **a** 205.01 m **b** 6 days **c** More time
d Under the new claim, the building will never reach 0 m. After 20 days, the building will be less than 6 m tall.
e 15 days

EX
p194

Review: Decimals and percentages

Review questions 4.9A: Mathematical literacy review

1 Ascending order means from smallest to largest, whereas descending order means from largest to smallest.
2 When performing a division, the dividend is the value that is being divided, and the divisor is the value it is being divided by.
3 **a** Rounding **b** Percentage
4 **a** The part of a decimal that appears after the decimal point. For example, in 3.729, '729' is the decimal fraction.
b Zeros that are added at the end of a decimal, after the decimal point, that do not affect the value of the number. For example, 8.2500 has two trailing zeros.
5 **a** per cent
b Accuracy

Review questions 4.9B: Multiple choice

1 A **2** A **3** C
4 C **5** D **6** D
7 B **8** A **9** C
10 A **11** D **12** D
13 C **14** D **15** A
16 B **17** D

Review questions 4.9C: Short answer

1 **a** Hundredths **b** Hundredths
c Millionths **d** Tenths
2 **a** 0.87 **b** 0.117
c 0.3 **d** 0.07
3 **a** 0.64 **b** 0.38
c 0.75 **d** 0.375
4 **a** **i** Five **ii** 1589.23
b **i** Five **ii** 12.71
c **i** Four **ii** 0.93
d **i** Seven **ii** 152.88
5 18.0958 12.8905 12.5089 12.0985 10.9852
6 **a** 120.46 **b** 9.532 **c** 1.693
d 4.2885
7 **a** 1263 **b** 153.298 **c** 210
d 609.4
8 **a** 0.21 **b** 0.036 **c** 0.088
d 0.0032
9 **a** 2.58 **b** 0.468 **c** 46.8
d 8.037
10 **a** 7.894 **b** 12.563 59 **c** 12.456
d 189.7656
11 **a** 9.327 **b** 15.76 **c** 29.726
d 40.931 25
12 **a** $1.56, $1.80
b 500 g pack, cost per 100 g is cheaper
13 **a** 12.4 **b** 3.75 **c** 21.55
d 0.162
14 **a** **i** $\frac{49}{100}$ **ii** $\frac{49}{100}$
b **i** $\frac{22}{100}$ **ii** $\frac{11}{50}$
c **i** $\frac{75}{100}$ **ii** $\frac{3}{4}$
d **i** $\frac{5}{100}$ **ii** $\frac{1}{20}$
15 46%
16 **a** $\frac{3}{4}$ **b** $\frac{9}{50}$ **c** $\frac{489}{1000}$
d $\frac{3}{1000}$
17 **a** 56% **b** 8% **c** 179%
d 27.48%
18 **a** 0.78 **b** 0.465 **c** 0.09
d 0.106 79

19 **a** 60% **b** 87.5% **c** 100%
d 275%

20 **a** 45 **b** 150 **c** 135
d 552.5 **e** 89.94 **f** 142.27

21 **a** 1496 km **b** 330.125 g **c** $6.96
d 72.9 L

Review questions 4.9D: Mathematical modelling

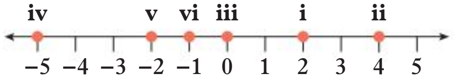
1 **a** **i** $39.30 **ii** 13
b **i** 70.9% profit **ii** 104.3% profit
c **i** 6 white roses **ii** $0.66
iii 4 red roses **iv** $3.54
d **i** 1 white rose **ii** 11 roses (7 white, 4 red)

MODULE 5 Integers and the Cartesian plane

EX p202

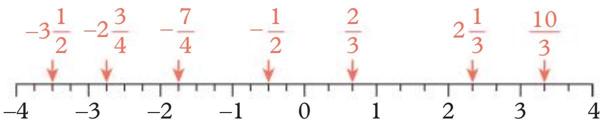
5.1 Negative numbers

1 **a**



b **i** 4 **ii** 0 **iii** 4
iv −2 **v** 4 **vi** −1

2



3 **a** $-4 < 6$ **b** $0 > -3$ **c** $-5 > -9$
d $-7 < -\frac{1}{7}$ **e** $-2 > -2\frac{2}{3}$ **f** $-\frac{5}{4} > -2$
g $-4\frac{2}{5} < -\frac{20}{5}$ **h** $-\frac{7}{2} > -3\frac{2}{3}$ **i** $-\frac{13}{3} < -\frac{21}{5}$

4 **a** $-40 < 30$ **b** $0 > -22$ **c** $-65 < -55$
d $78 > -\frac{100}{2}$ **e** $-\frac{127}{10} < -\frac{328}{100}$
f $-150\frac{79}{190} < 0$ **g** $248\frac{23}{25} > -300\frac{67}{70}$
h $-\frac{120}{7} < 15\frac{5}{7}$ **i** $-\frac{143}{9} < -\frac{95}{6}$

5 **a** −6, −2, 0, 4, 6
b $-13, -3, -\frac{5}{2}, 4, 10$
c $-12, -1, -\frac{2}{3}, 1, \frac{10}{3}$
d $-5\frac{2}{3}, -5, -\frac{9}{4}, 5, 10, 16$
e $-20\frac{4}{5}, -20.75, -20, 25, 25\frac{17}{20}, 25\frac{35}{40}$
f $-1.25, -\frac{12}{10}, -1, -\frac{12}{13}, 0.6, 1\frac{5}{6}$

6 **a** 8, 4.8, 2.1, −5, −6
b 80.8, 43, −12, −35, −46.2
c 8.9, 6.5, −2.3, −4.7, −12.5
d $15, 12.5, 7.6, -4\frac{1}{3}, -4.5, -\frac{11}{2}$
e $48.9, \frac{12}{5}, -36\frac{25}{45}, -45\frac{4}{5}, -48.9, -50.3$
f $-0.11, -\frac{110}{12}, -\frac{120}{11}, -11, -11.25, -11\frac{5}{6}$

7 −4, −3, −2, −1, 0, 1, 2, 3, 4, 5, 6

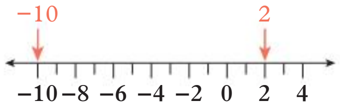
8 Sample answer: $-\frac{5}{6}, -\frac{3}{4}, -\frac{1}{2}, -\frac{1}{5}, \frac{1}{6}, \frac{3}{5}$

9 Sample answer: $2\frac{3}{5}, 2\frac{2}{7}, -1\frac{8}{9}, -2\frac{1}{6}, -2\frac{3}{4}, -4\frac{1}{4}$

10 **a** −3 **b** 360 **c** −2
d −15 **e** 2230

11 **a** −10°C
b **i** Higher **ii** Higher
iii Lower **iv** Higher
c A larger number is higher on the scale.

12 **a**



b −10°C
c −10°C, −1°C, 2°C
d Cooler

13 **a** Level 5 **b** Level −3
c Nine levels **d** 0
e You go up three levels.
f You go down two levels.
g You go up four levels.
h Levels that are below ground level.

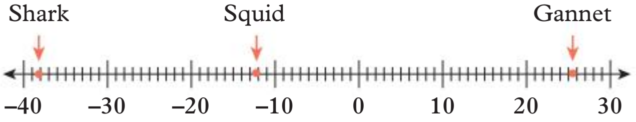
14 **a** 100 **b** $20 **c** −20

15 **a** Paulo: −85.50; Kamilla: 98.05; Amad: 135.98; Jessica: −172.20
b Amad **c** Jessica
d −172.20, −85.50, 98.05, 135.98. Numbers increase in value going left to right on the number line.

16 **a** Raj's school is at −2 and the hospital is at 9.
b 11 km

17 **a** 5 **b** −30 **c** 35 m **d** −3800

18 **a** $25\frac{1}{2}$ **b** $-12\frac{1}{3}$
c $-38\frac{7}{30}$ m. The shark is swimming $25\frac{9}{10}$ lower than the squid, so the shark is at a depth of $-12\frac{1}{3} - 25\frac{9}{10} = -38\frac{7}{30}$ m.
d The shark is closer to the squid.



19 **a** **i** +5 **ii** −4
iii 0 **iv** +5
b Diagrams **i** and **iv** have the same electric charge because the 3 negative charges in diagram **iv** cancel out 3 of the 8 positive charges, leaving it with a charge of +5.

20 **a** Sample answer: −1 and 2. The positive integer is larger in magnitude than the negative integer.
b Sample answer: −1 and 10. As long as you subtract the positive integer from the negative integer, the result

will be negative as subtracting value from a negative integer makes the negative integer more negative.

21 **a** No. The temperature cannot be negative kelvin because 0 K is the coldest temperature that is physically possible.

b, **c**

d **i** 0 K **ii** 263 K **iii** 300 K

EX p208

5.2 Adding and subtracting positive integers

1 **a** 5 + 4 = 9 **b** 7 – 3 = 4
c 5 + 3 = 8 **d** 2 – 6 = −4
e −8 + 3 = −5 **f** −1 + 4 = 3
g −5 + 5 = 0 **h** −1 – 5 = −6

2 **a** 7 **b** 1 **c** −2
d 5 **e** −10 **f** 1
g −4 **h** 0 **i** −3

3 **a** −5 **b** −13 **c** −5
d −80 **e** −59 **f** −27
g −10 **h** 2 **i** −132

4 **a** 2 **b** −5 **c** 0
d −9 **e** −1 **f** −2

5 **a** 6 **b** 4 **c** −4
d 3 **e** −154 **f** 88

6 **a** 4 **b** −7 **c** −1
d 10

7 **a** 15 **b** 8 **c** −8
d 45 **e** 2 **f** 73

8 **a** Negative integer **b** Negative integer
c Positive integer **d** Zero
e Negative integer **f** Positive integer

9 If the positive number is further from zero than the negative number, the sum will be positive. If the negative number is further from zero than the positive number, the sum will be negative. If the positive number and the negative number are the same distance from zero (opposite numbers), the sum will be zero.

10 **a** −18 + 98 **b** 80°C

11 1 m

12 **a** −8 + 5 **b** −3°C

13 **a** −50 **b** −50 + 35 **c** $15

14 **a** −26 **b** −26 + 40 **c** $14

15 **a** **i** 4 hours behind **ii** Sunday 09:30
iii Sunday 19:00
b Helsinki is 7 hours behind Sydney.

16 −3, −1, 1 and 3. Half of the highest result gives the highest number, 6 ÷ 2 = 3 and half the smallest result gives the smallest number, −6 ÷ 2 = −3. The second largest number is 4, which would be made up of the largest number, 3, as well as another dice number, in this case 1. Likewise, the next smallest number, −4, would be made up of the smallest number, −3, plus another dice number, in this case −1. Using a table to confirm all the results:

+	−3	−1	1	3
−3	−6	−4	−2	0
−1	−4	−2	0	2
1	−2	0	2	4
3	0	2	4	6

EX p212

5.3 Adding and subtracting negative integers

1 **a** −3 + 5 **b** 6 – 5
c −2 – 4 **d** 7 + 1

2 **a** −3 + (−6) = −3 – 6
b 1 – (−3) = 1 + 3
c 8 – (−9) = 8 + 9
d −5 + (−7) = −5 – 7

3 **a** 7 + (−4) = 3
b −6 − (−4) = −2
c 2 – (−8) = 10
d 8 + (−3) = 5
e −3 − (−4) = 1
f −6 + (−1) = −7
g −4 – (−4) = 0
h −2 + (−7) = −9

4 **a** −7 **b** 1 **c** 6
d 1 **e** −2 **f** 0
g 5 **h** 10 **i** 3

5 **a** 13 **b** −11 **c** −29
d −23 **e** 65 **f** −68
g −22 **h** −71 **i** 300

6 **a** −7 **b** 28 **c** −51
d 0 **e** 25 **f** −157

7 **a** −5 **b** 2
c −8 **d** 7

8 **a** −13 **b** −15 **c** −6
d −25 **e** −67 **f** −127

9 **a** Positive integer **b** Positive integer
c Negative integer **d** Negative integer
e Zero **f** Negative integer

10 **a** 4 – (−15) **b** 19°C

11 **a** Sun: 6°C, Mon: 5°C, Tue: 9°C, Wed: 5°C, Thu: 4°C, Fri: 6°C, Sat: 5°C
b Tuesday

12 **a** 74 **b** −31 – 74 **c** $105

13 **a** Turquoise Cave: −57 m, Amethyst Cave: −75 m
b −18 m
c 18 m

d Sample answer: If the negative number being subtracted is further from zero than the other negative number, the result will be positive. If the negative number being subtracted is closer to zero than the other negative number, the result will be negative.

e Sample answer: The positive number represents the difference in depth as negative differences have no meaning.

14 **a** Deposit **b** Withdrawal

c Ayesha has money in her account.

d Ayesha owes money to the bank.

e +$28 **f** Withdrawal of $9

15 **a** −8 **b** 9 **c** 4

d 14 **e** −15 **f** −5

g −11 **h** 3

16 **a** **i** −6 **ii** −6 **iii** −6

b **i** −6 **ii** −6 **iii** −6

c −6, −6 **d** −6

17 **a**

4	−3	2
−1	1	3
0	5	−2

b

1	0	−4
−6	−1	4
2	−2	−3

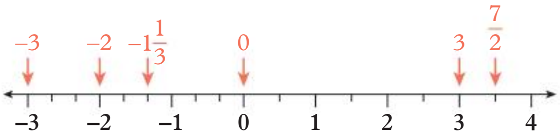
c

6	−6	−9
−18	−3	12
3	0	−12

EX p215

Checkpoint

1 **a**



b **i** 3 > −3 **ii** −3 < −2 **iii** $-1\frac{1}{3} < 0$

iv $\frac{7}{2} > -1\frac{1}{3}$ **v** $-2 < \frac{7}{2}$ **vi** 0 > −3

2 **a** −9, −4, −1, 0, 7

b $-6, -\frac{9}{2}, -2, 6, 10$

c $-15, -1.5, -\frac{3}{4}, 2, \frac{20}{3}$

3 **a** −6 **b** −16 **c** −9

d −80 **e** −30 **f** −22

4 **a** 4 **b** −4 **c** 9

d −20 **e** −5 **f** −6

5 **a** −1 **b** −56 **c** 122

d −14 **e** 31 **f** 4

6 **a** −27 m **b** −27 + 12

c 15 m below sea level

7 **a** −5 **b** −2 **c** 13

d 6 **e** −5 **f** 0

8 **a** −40 **b** 4 **c** −96

d 3 **e** 92 **f** −96

9 **a** −9 **b** 10 **c** −36

d −12 **e** −74 **f** −119

10 **a** −12 + (−3) + 5

b 10 m below sea level

EX p219

5.4 The Cartesian plane

1 **a** *D* **b** *A* **c** *F*

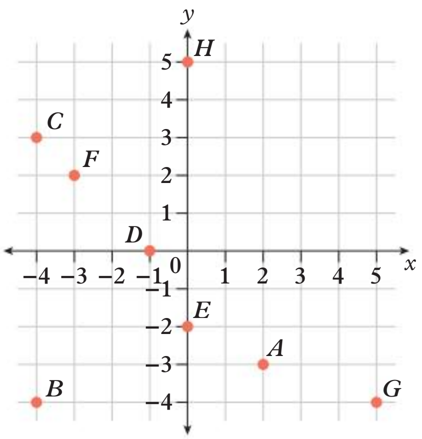
d *E* **e** *C* **f** *B*

2 **a** *B* **b** *C*

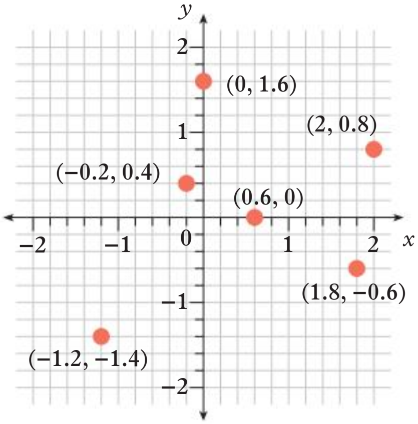
3 $A(2, 5), B(3, -2), C(4, 0), D(-5, -4), E(-3, 3), F(0, 2), G(2, -4), H(0, -3)$

4 **a** *F* and *H*; *A* and *G* **b** *D* and *G*

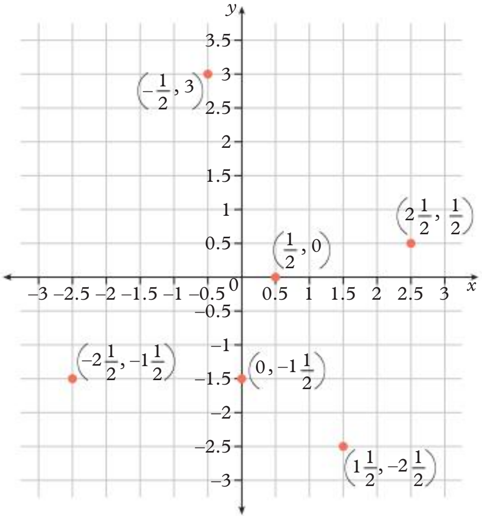
5



6 **a** **i, ii**



b **i, ii**



7 (0, 0)

8 **a** $A(1\frac{1}{2}, 2), B(-2, -3\frac{1}{2}), C(-3\frac{1}{2}, -1\frac{1}{2}), D(-4\frac{1}{2}, 2\frac{1}{2}),$
$E(-2\frac{1}{2}, 3\frac{1}{2}), F(4\frac{1}{2}, 3\frac{1}{2}), G(\frac{1}{2}, 0), H(0, 4\frac{1}{2}), I(3\frac{1}{2}, -2),$
$J(4\frac{1}{2}, -\frac{1}{2})$

b $A(-0.4, 0.6), B(0.4, 0.4), C(-0.8, -0.6), D(0.8, 0),$
$E(0.5, -0.7), F(0, -0.5), G(-0.9, 0.3), H(0.2, -0.2),$
$I(0.3, -0.9), J(-0.5, -0.3)$

9 **a** On the y-axis (vertical axis)

b On the x-axis (horizontal axis)

c At the intersection of the x-axis and the y-axis (the origin)

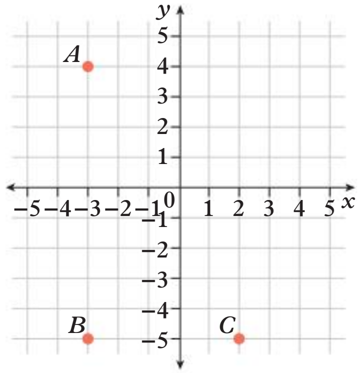
10 **a** $A(0, 0), B(1, -1), C(2, 0), D(0, 2), E(-3, -1),$
$F(2, -6), G(10, 2)$

b **i** 1 **ii** 1 **iii** 2
iv 3 **v** 5 **vi** 8

c 13 **d** (−3, −15)

11 **b** 9 units

a, c



d 5 units **e** $D(2, 4)$

12 **a** 6 units

b Sample answer: (−2, −3) and (4, −3)

c Other possible answer: (−2, 9) and (4, 9).

d 12 units

EX
p224

5.5 Line graphs

1 **a** 4 km **b** 4 hours

c **i** 1 hour **ii** 2 hours

d 8 km

2 **a** **i** 0 m **ii** 120 m **iii** 360 m

b Levi was stationary.

c **i** At a local shop
ii At his friend's house
iii At home

d **i** 240 m **ii** 360 m

e **i** 2 minutes
ii 4 minutes
iii 2 minutes

3 **a** 4 km/h **b** 0 km/h **c** 2 km/h

4 **a** 3 km **b** 2 pm **c** 1 km

d 1 hour **e** 4 pm

f From dental clinic to home; 4 km/h

5 **a** 9:00 pm **b** 11 μg/m^3

c Possible answers: 9:00 am and 3:00 pm, 10:00 am and 11:00 am, 12:00 pm and 12:00 am, 6:00 pm and 7:00 pm, 2:00 am and 4:00 am, 6:00 am and 7:00 am

d 9:00–10:00 pm

6 **a** rise **b** month 18

c months 7 and 8

7 **a** rise **b** no **c** 14:00 (2 pm)

8 **a** **i** False

ii The distance from home is increasing during this section.

b **i** True

ii The distance from home stays the same during this section.

c **i** False

ii The slope is steeper for section C than for section A.

d **i** True

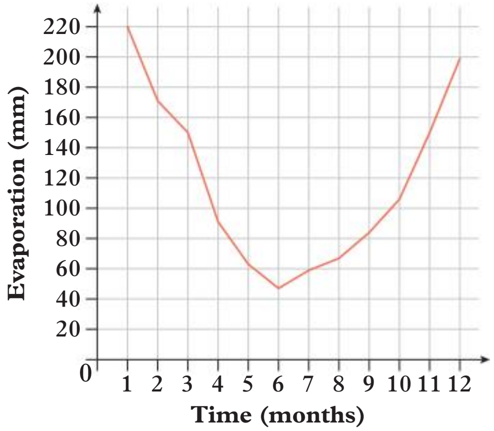
ii The slope is steeper for section D than for section C.

9 **a** Tom **b** Sue **c** Molly

10 **a** 220 mm **b** 150 mm

c The rate at which the evaporation level is changing

d, e



f January, 220 mm

g June, around 50 mm

h Sample answer: temperature, humidity and wind affect the amount of evaporation.

11 No; you cannot be in different places at the same time.

12 **a** 0 to 45

b –20 to 1.5

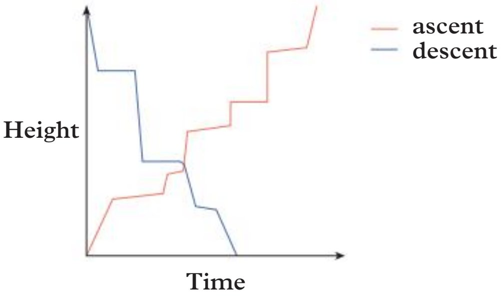
c Yes, the vertical axis requires negative values as it measures the depth of the dive above and below the surface of the water.

d 30 minutes

e To dive to the bottom of the lake. The descent took 30 minutes, whereas the ascent took 15 minutes.

f 5 m below the water, 10 minutes

13 Yes, there will be exactly one point as no matter the speed, the line graphs will intersect at one point.



EX p229

Review: Integers and the Cartesian plane

Review questions 5.6A: Mathematical literacy review

1 negative signs: –3 – (–1) = 0 – 2

minus signs: –3 – (–1) = 0 – 2

2 **a** magnitude

b Cartesian plane, *x*-axis, *y*-axis, origin

c Cartesian coordinates, *x*-coordinate, *y*-coordinate

3 **a** slope **b** distance–time graph

4 **a** speed **b** stationary

5 **a** False. Integers include zero, all positive whole numbers and their negative counterparts. Fractions and decimals are not integers.

b True.

c False. A negative number could have a greater magnitude than a positive number or have the same magnitude as a positive number. For example, the magnitude of –2 is the same as the magnitude of 2. The magnitude of –2 is greater than the magnitude of 1.

6 **a** Sample answer: If the temperature in a freezer is –3 and the temperature is turned down another 7 degrees, what is the new temperature?

b Sample answer: If Sam's opening bank balance is –2, what is his new bank balance after his parents transferred 20 dollars into his account?

Review questions 5.6B: Multiple choice

1 C	**2** A	**3** A	**4** D
5 B	**6** A	**7** B	**8** A
9 D	**10** A	**11** B	

Review questions 5.6C: Short answer

1 **a** False **b** True

c True **d** False

2 **a** –10, –5, 0, 2, 12, 25

b –12, –6, –3, 0, 6, 9

3 **a** $2\frac{2}{5}, \frac{9}{5}, 1, 0, -\frac{9}{10}, -2$

b $5.5, 5\frac{1}{4}, \frac{7}{2}, -2\frac{3}{4}, -\frac{15}{4}, -4.75$

4 **a** –86 **b** 615

c 8848 **d** –35

5 **a** –8 **b** 1

c –14 **d** –1

6 3.5 m below sea level

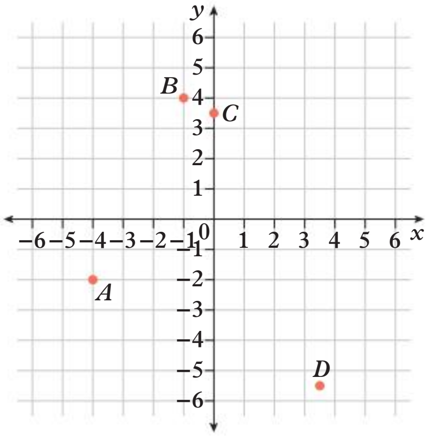
7 **a** –9 **b** 1

c –2 **d** 8

8 –$50

9 $A(3, 1), B(-3, 3), C(4, -2), D(1, 0), E(-2, -3), F(0, -2)$

10



11 **a** 4 km **b** 30 minutes

c **i** 1 hour **ii** 30 minutes **iii** 1 hour

d 4 km/h

Review questions 5.6D: Mathematical modelling

1 **a** **i** 10 m **ii** –2 m

iii 0 **iv** –8 m

b **i** 12 m **ii** 6 m

c –6 m

d **i** The rate of change of the height of the seagull

ii The seagull dived to the water, plunged under the water and then came back to the surface.

iii No

2 **a** **i** Alex **ii** Alex

b **i** 3 km **ii** 2 km

c (–3, –5)

d **i** *B* **ii** 25 minutes **iii** 5 km

iv 2 km **v** Oliver

3 **a** 4205 m **b** 4643 m **c** 1362 m

d **i** 3 **ii** half an hour **iii** 3:30 pm

iv 9:00 am **v** The first hour of their journey

EX
p234

SEMESTER 1 review

Short answer

1 **a** 4 × 1000 + 8 × 100 + 10 + 7
b 10 000 + 9 × 1000 + 3 × 10
c 6 × 6 × 6 × 6
d 51 × 51 × 51

2 **a** 6 **b** 23 760 **c** 1043
d 81 **e** 752 **f** 462

3 **a** **i** 135 680 **ii** 140 000 **iii** 100 000
b **i** 60 **ii** 57.7 **iii** 57.658
iv 57.66 **v** 58

4 **a** 1331 **b** $\frac{13}{8}$ **c** 56.11
d −23 **e** $\frac{95}{18}$ **f** 1.1

5 **a** 387 **b** $\frac{7}{8}$ **c** 51.29
d −7 **e** $\frac{67}{18}$ **f** 0.24

6 **a** 27 375 **b** $\frac{3}{2}$ **c** 1.107
d $47\frac{2}{3}$

7 **a** 62.3 **b** $\frac{5}{2}$ **c** 85
d 2

8 **a** 10 **b** $\frac{3}{4}$ **c** 54
d 140% **e** 25 **f** 11 : 3

9 **a** HCF = 16, LCM = 160
b HCF = 8, LCM = 120
c HCF = 7, LCM = 294
d HCF = 42, LCM = 2310

10 **a** Start with a cross made up of five squares and form each new shape by adding a square to each arm of the previous shape.
b Start with 7 and form each subsequent term by subtracting 6 from the previous term.
c Start with 3 and form each subsequent term by doubling the previous term.
d Start with $\frac{150}{7}$ and form each subsequent term, by dividing the previous term by 5.

11 **a** Composite **b** Prime
c Prime **d** Composite

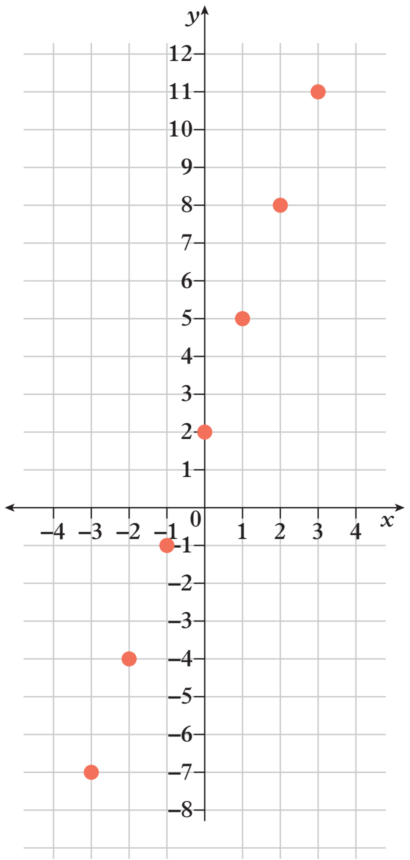
12 **a** True **b** False **c** False
d True

13 **a** 821 **b** 29 **c** 16 191
d 342

14 **a** 33 **b** 10 **c** $\frac{3}{7}$
d 2.14 **e** −3 **f** 5.73

15 **a** $8\frac{1}{4}$ **b** $\frac{5}{2}$ or $2\frac{1}{2}$ **c** 11 : 15
d 13 : 5

16



17 **a** 19°C **b** 22°C

18

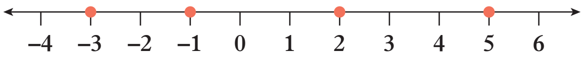
Percentage	Fraction	Decimal
35%	$\frac{7}{20}$	0.35
775%	$\frac{31}{4}$	7.75
4.5%	$\frac{9}{200}$	0.05
971.43%	$\frac{68}{7}$	9.71
56%	$\frac{14}{25}$	0.56
642%	$\frac{321}{50}$	6.42
260%	$\frac{13}{5}$	2.6
893.1%	$\frac{8931}{1000}$	8.93

19 **a** $\frac{5}{6}, \frac{9}{8}, \frac{8}{6}, \frac{9}{5}$
b 0.05, 0.5, 5.0, 5.5
c −4, −3, 1, 5
d $-\frac{11}{25}, -\frac{3}{7}, -0.41, -\frac{2}{5}$

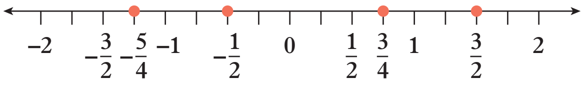
20 **a** $2^3 \times 7$ **b** $3^2 \times 5$ **c** $2^3 \times 3^3$
d $2 \times 5^2 \times 7$

21 **a** 343 **b** 256 **c** 81
d 289

22 **a**



b



c

23 **a** $3\frac{1}{4}$ **b** $\frac{18}{7}$ **c** 47% **d** 182%

24 **a** 4 **b** 6

25 11

26 **a** 2520 **b** 14

Mathematical modelling

1 **a** p, w, r, w, p, y, p, w, r, w, p, y

b Plant every 6 m, starting at 6 m.

c Plant the first 1 m, then 4 m, then 2 m, then 4 m, then 2 m, alternating 4 m and 2 m apart.

d **i** White **ii** 8

2 **a** 15

b 2 cannolis, 3 eclairs, 7 macarons

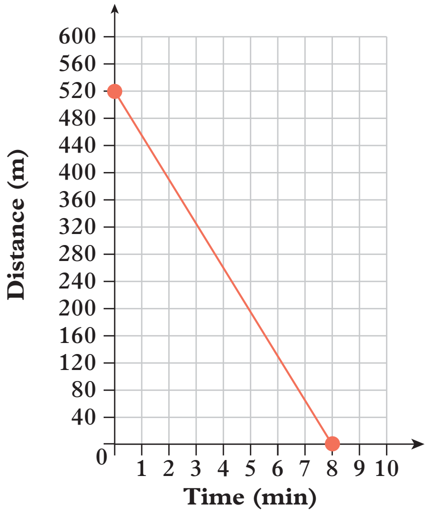
c $\frac{7}{12}$ **d** $140.25

e **i** $520.50 **ii** $34.70

3 **a** 240 m **b** 520 m **c** 280 m

d 12 min **e** 4 min

f **i**



ii 65 m/min

EX p238

EXPLORATIONS 1

1 **a** 12, 53 and 54 **b** 75, 76 and 145

c 39, 40, 57 and 58 **d** 128

e 158

2 **a** 28 minutes

b 60 minutes (HOTSHOTS only takes 4 minutes)

c All word-lengths must be factors of 132 and the factors 3, 4 and 11 must each occur in at least one word-length, e.g. THIS WAS COMPLICATED or I DO LIKE COMPLICATED THINGS

d 2 minutes using 1- and 2- letter words, e.g. IF IT IS TO BE IT IS UP TO ME (or shorter if non-sensical sentences are allowed)

e 420 minutes using 1-, 3-, 4-, 5- and 7-letter words, e.g. I CAN SOLVE THIS PROBLEM

f **a** 140 minutes

b unchanged

c answers will vary

d 4 minutes, e.g. HOTSHOTS LOVE THIS QUIZ

e 840 minutes, e.g. I TRY A TRICKIER PROBLEM

3 **a** 60%; yes, from 120% then 50%, or vice versa

b Approximately 195.3% from 125% three times

c 50%-80%-120%-125% or 50%-80%-125%-150%

d No: if there are only 3, they must be 50%, 80% and 125%, but then 150% cannot be achieved.

e One possibility is 50%-70%-80%-150%, but there are others.

4 **a** $2, 3, -\frac{1}{3}, 3, 4, -\frac{1}{4}, \frac{3}{4}, -\frac{4}{3}, -\frac{1}{3}, 3$

b R T T

c R T R T T R T T

d Apply R T, then repeatedly apply R T T as many times as necessary.

e Apply T R, then repeatedly apply R T R T R as many times as necessary.

f Yes: via part **d**, part **e** or repeatedly applying T, any number can be turned into 0 and vice versa.

5 **a** (4, −3)

b (−1, 2), (1, −4) and (−3, −2)

c $P(-3, 0)$ and $Q(0, 1)$

MODULE 6 Algebra

EX p245

6.1 Variables

1

Number of bananas	1	2	3	4	5
Total cost ($)	2	4	6	8	10

2

Time (h)	1	2	3	4	5
Volume of water in the tank (L)	10	20	30	40	50

3

Distance travelled (km)	0	100	200	300	400	500
Volume of petrol in the tank (L)	50	40.8	31.6	22.4	13.2	4

4

n	1	2	3	4	5
c ($)	15	30	45	60	75

5

t	0	20	40	60	80	100
p	150	110	70	30	0	0

6 **a** III **b** I **c** IV **d** II

7 a

x	1	2	3	4	5
y	4	5	6	7	8

b

x	2	3	4	5	6
y	0	1	2	3	4

c

x	0	1	2	3	4
y	0	4	8	12	16

d

x	0	2	4	6	8
y	0	1	2	3	4

8 a

x	0	2	4	6	8
y	5	6	7	8	9

b

x	3	6	9	12	15
y	0	1	2	3	4

c

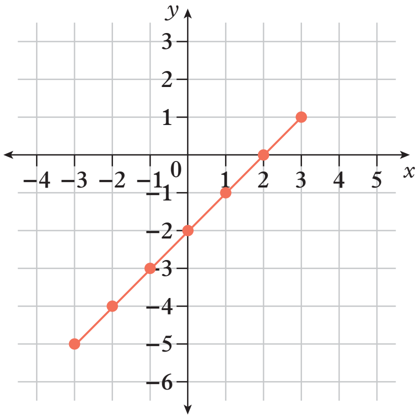
x	1	2	3	4	5
y	–1	1	3	5	7

d

x	2	4	6	8	10
y	0	1	2	3	4

9 a i (–3, –5), (–2, –4), (–1, –3), (0, –2), (1, –1), (2, 0), (3, 1)

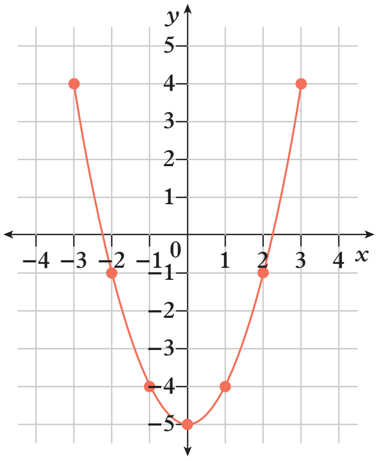
ii



iii The points lie on a straight line

b i (–3, 4), (–2, –1), (–1, –4), (0, –5), (1, –4), (2, –1), (3, 4)

ii

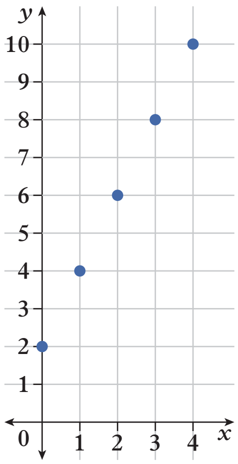


iii The points do not lie on a straight line

10 a i

x	0	1	2	3	4
y	2	4	6	8	10

ii

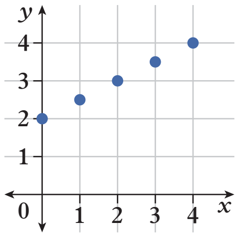


iii The points lie on a straight line.

b i

x	0	1	2	3	4
y	2	2.5	3	3.5	4

ii



iii The points lie on a straight line.

11 a

Number of customers	1	2	3	4	5
Amount of tips earned ($)	0.50	1.00	1.50	2.00	2.50

b Number of customers, amount of tips earned

c Independent variable: number of customers;
Dependent variable: amount in tips earned ($)

d **i** $2.50 **ii** $3.00 **iii** $6.00

e **i** 4 customers

ii 20 customers

iii 200 customers

12 a II **b** III **c** I

13 a The length of dough that comes out of the machine is three times the length of dough that goes into the machine.

b

Length of dough in (cm)	10	20	30	40	50	60
Length of dough out (cm)	30	60	90	120	150	180

c **i** 300 cm **ii** 80 cm

14 a No, *x* is not a variable because the value of *x* is fixed. For the calculation to be true, *x* = 20.

b **i** specific unknown number

ii specific unknown number

iii variable

15 **a** weight, number of days

b independent variable = number of days
dependent variable = weight

c 150 g

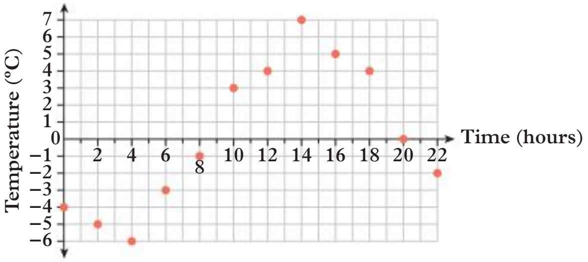
d initial weight

16 **b** Scale should be from 0 to 22 on the horizontal axis. Negative numbers not needed because negative time has no meaning.

c Scale should go from –6 to 7 on the vertical axis.

d (0, –4), (2, –5), (4, –6), (6, –3), (8, –1), (10, 3), (12, 4), (14, 7), (16, 5), (18, 4), (20, 0), (22, –2)

a, e



f –6°C at 4 am

g 7°C at 2 pm

17 **a**

Number of triangles	1	2	3	4	5	6
Number of toothpicks	3	6	9	12	15	18

b the number of toothpicks is three times the number of triangles

c **i** 360 toothpicks **ii** 30 triangles

18 **a** x represents the number of diamonds in a given track.

b y represents the number of concrete slabs required to make up a given track.

c The largest track that can be made using 22 concrete slabs would consist of 5 diamonds, joined end-to-end, with one horizontal track at the entrance of the first diamond and the exit of the last.

d Sample answer: The number in front of the x should be adjusted to match the number of sides of the chosen shape. For example, a hexagonal design can be represented by the formula $y = 6x + 2$.

EX
p251

6.2 Terms, expressions and equations

1 **a** **i** 2 **ii** x
iii 5 **iv** 6

b **i** 3 **ii** a, b
iii 2, 4 **iv** 3

c **i** 2 **ii** m
iii 1 **iv** 4

d **i** 2 **ii** k
iii 11 **iv** 2

e **i** 2 **ii** c, d
iii 4, 7 **iv** none

f **i** 1 **ii** y
iii 13 **iv** none

g **i** 2 **ii** f
iii 1 **iv** 5

h **i** 4 **ii** x, w, y
iii 6, 1, 3 **iv** 1

i **i** 3 **ii** p, m
iii 8, 1 **iv** 10

2 **a** False, it is an equation as it has an equals sign.

b True **c** True

d False, the constant is 3.

e True **f** True

g False, it is an equation as it has an equals sign.

h True **i** True

j False, it is an expression as there is no equals sign.

k True **l** True

3 **a** Equation **b** Expression
c Equation **d** Equation
e Expression **f** Equation
g Equation **h** Expression

4 They all have an equals sign (=).

5 **a** $a + 7$ **b** $\frac{b}{2}$ **c** $c - 5$
d $3d$ **e** $8 - e$ **f** $2f$
g $\frac{4}{g}$ **h** $h + 9$ **i** $k + m + n$

6 **a** 2 **b** 1 **c** 2
d 1 **e** 2 **f** 1
g 1 **h** 2 **i** 3

7 **a** $x + 8$ **b** $x - 15$
c $3x$ **d** $\frac{x}{7}$

8 **a** $4n + 5$ **b** $\frac{n}{3} + 7$
c $2n - 9$ **d** $6n - 1$

9 **a** $x + 5 = 11$ **b** $x - 3 = 4$ **c** $6x = 18$
d $\frac{x}{2} = 5$ **e** $2x = 24$ **f** $10 - x = 8$
g $3x + 1 = 7$ **h** $\frac{2}{5}x - 8 = 0$

10 **a** $y = \frac{x}{6}$ **b** $x + 11 = 20$ **c** $y = x - 9$
d $b = a + 130$ **e** $q = 10p$ **f** $c = \frac{1}{4}d$
g $m = 2n$ **h** $f = \frac{3}{10}g$

11 $n = p - 3$

12 $t = x + 20$

13 **a** $n + 10$ **b** $3n$
c $\frac{n}{2}$ **d** $n - 4$

14 $n = 2p$

15 **a** y = Ying Ying's age, s = her little sister's age

b $y = s + 5$

16 $p = 6g + b$

17 **a** **i** $2p$ **ii** $52p$

b $p + w$

18 **a** c = the cost of hiring a canoe, n = the number of hours

b 2

c The booking fee

19 **a** d = number of days **b** \$110
c \$25 **d** \$285

20 **a** $s = 3n$ **b** $p = \frac{s}{8}$
c $c = 25p$
d They should round p up to the next whole number.

21 **a** The cost of apples, the cost of bananas and the cost of oranges
b 3. The total cost is made up of the cost of 3 different fruits.
c One example is: let a represent cost of an apple, let b represent cost of a banana, let c represent cost of an orange.
d **i** $10a$ **ii** $7b$ **iii** $6c$
e $10a + 7b + 6c$
f $w = 10a + 7b + 6c$

22 If c = the cost of chicken per gram, d = the cost of cheese per gram, w = the total weekly cost of deli goods, $w = 500c + 300d$.

23 If n = the number of Year 7 students, p = the number of students who want to attend the planetarium, t = the total cost of admissions, $t = 10n + 5p$.

24 $y^2 + 3$

25 **a** $x + 5$ **b** $x + 1$ **c** $x - 4$
d x **e** $2x + 7$ **f** $3x + 1$

26 **a** Yes **b** No **c** Yes **d** Yes
e Yes **f** No **g** Yes **h** No

27 **a** $x, -x, 20x$ and $y, -y, -20y$
b $a, 10a, 9a$ and $ab, -ba, \frac{1}{2}ab$
c $xyz, xzy, yxz, yzx, zxy, zyx$
d $23mn, -nm$ and $14m, -3m$
e There are no like terms.
f $9gh, 17hg, -32gh$
g $abc, -cab$ and $10ab, -6ba$ and $-5bc, 4cb$
h $4x, -12x, 10x$ and $4, -12, 10$

EX p258

6.3 Order of operations in algebra

1 **a** **i** 3×7 **ii** 3×10 **iii** $3 \times x = 3x$
b **i** 7^3 **ii** 10^3 **iii** x^3
c Sample answer: Pronumerals act the same way as numbers. Multiplication, even when implied, is repeated addition, and exponents represent repeated multiplication.

2 **a** $7(x + 6)$ **b** $2(x - 3)$
c $5(x + 1)$ **d** $8(x - 2)$
e $\frac{x+4}{7}$ **f** $\frac{x-5}{11}$
g $\frac{9+x}{2}$ **h** $\frac{1-x}{6}$

3 **a** $10(a + 1)$ **b** $\frac{a-9}{5}$ **c** $\frac{a}{3} + 6$
d $\frac{a+21}{7}$ **e** $(7a)^2$ **f** $a^2 - 12$
g $\sqrt{a} - 4$ **h** $\sqrt{10a}$

4 7 less than a all divided by 2

5 **a** y is 10 more than 7 times x
b The sum of x and 12 all divided by 9 is y
c The square root of x less than 6
d q is 8 less than p divided by 3
e The square root of all of 2 times s minus 1
f s is 1 less than the square of t
g Half of 1 less than the square of a is b
h 16 less than the square of 7 more than m is n

6 **a** Multiply by 3 **b** Subtract 5 **c** Add 6
d Multiply by 3 **e** Divide by 8 **f** Square
g Square **h** Multiply by 8 **i** Square

7 **a** Number of passengers from the terminal, number of passengers on the bus
b $p - 4$ **c** $t = 2(p - 4)$

8 **a** Dollars received on birthday, remaining money
b $m - 15$ **c** $r = \frac{m-15}{2}$

9 $2(x + 5)$ is equal to $2 \times (x + 5)$, which means that all of the contents of the brackets are multiplied by 2. In $2x + 5$, only the x is multiplied by 2.

10 $s = 4(n + 3)$

11 $y = \frac{x+30}{2} + 7$

12 Sample answer: $n = \frac{3x-5}{x}$

13 Sample answer: $y = \frac{1}{3}\left(\frac{9}{10}x - 75\right)$

14 Sample answer: $y = \frac{450 + 50x}{x}$

15 Sample answer: $n = 2r^2$

16 Sample answer: $d = \frac{50(t-2)}{2}$

17 **a** Sample answer: $n = \frac{l}{3} + 1$
b Sample answer: $n = \frac{4l}{3}$
c Sample answer: If the number of light poles placed along one side of the ground was multiplied by 4, the light poles at each corner would be counted twice.

18 $29r$ (or $\frac{29}{2}s$ or $\frac{29}{4}l$)

EX p260

Checkpoint

1 **a**

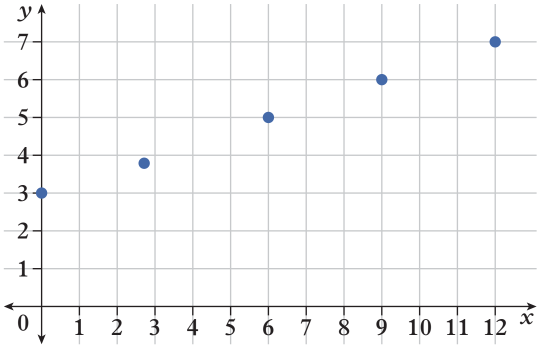
Number of cups of coffee	1	2	3	4	5
Profit (\$)	2	4	6	8	10

b Number of cups of coffee, profit
c **i** \$10 **ii** \$30
d **i** 4 **ii** 8

2 **a**

x	0	3	6	9	12
y	3	4	5	6	7

b



c The points lie on a straight line

3 **a** **i** 2 **ii** x
iii 10 **iv** 2
b **i** 1 **ii** f
iii $\frac{1}{2}$ **iv** none
c **i** 3 **ii** a, b
iii 1, 5 **iv** −12
d **i** 2 **ii** x, y
iii 1, −1 **iv** none

4 **a** $t + 6$ **b** $k - 11$
c $6b$ **d** $\frac{h}{9}$

5 **a** $y = \frac{x}{2}$ **b** $c = b + 9$
c $n = m - 20$ **d** $q = 6p$

6 $g = \frac{15}{n}$

7 **a** $7\sqrt{a} + 6$ **b** $\frac{8 - b}{3}$
c $\frac{c}{9} + 6$ **d** $(d + 4)^2$

8 $4(n + 2)$

9 $n = \frac{p - 1}{3}$

EX
p263

6.4 Substitution

1 **a** 11 **b** 4 **c** 18
d 3 **e** 38 **f** 15

2 **a** 17 **b** 4 **c** 40
d 1 **e** 13 **f** 0

3 **a** 8 **b** 14 **c** 17
d 26 **e** 5 **f** 6

4 **a** 3 **b** 15 **c** 75
d −1 **e** −5 **f** −4

5 **a** 9 **b** 1 **c** 4
d 0 **e** 14 **f** 20

6 **a** $\frac{2}{9}$ **b** 2 **c** 1
d $\frac{5}{12}$ **e** $-2\frac{7}{12}$ **f** $1\frac{1}{4}$

7 **a** 6 **b** 9 **c** 30
d 0 **e** 17 **f** 2
g 48 **h** 1 **i** 12
j 30 **k** 5 **l** 10

8 **a** $2 \times a \times b$ **b** $a \times a$ **c** $a \times a \times b$

9 **a** 9 **b** 250 **c** 14
d 17 **e** 36 **f** 56
g 135 **h** 150 **i** 3
j 4

10 **a** 7 **b** 1 **c** 13
d 2 **e** −2 **f** 1
g 10 **h** −6 **i** −8
j 6

11 **a** 4 **b** 52 **c** −14
d 36.4 **e** 29.8 **f** 2.8
g −5 **h** 2

12 **a** $5r + 2d$ **b** \$25 **c** \$54

13 **a** $m = 12y$
b **i** 60 **ii** 120 **iii** 1200
iv 960

14 **a** Calculate the value of d for these values of m and v.
i 5 **ii** 4 **iii** 2.1 **iv** 5.5
b 0.72 g/cm^3

15

Average speed (km/h)	40	45	50	60	50	70
Time (h)	2	3	2.5	1.5	3	2.5
Distance (km)	80	135	125	90	150	175

16 **a** **i**

Average speed (km/h)	40	40	40	40	40	40
Time (h)	1	2	3	4	5	6
Distance (km)	40	80	120	160	200	240

ii $d = 40t$

b **i**

Average speed (km/h)	40	45	50	55	60	65
Time (h)	5	5	5	5	5	5
Distance (km)	200	225	250	275	300	325

ii $d = 5s$

17 Sample answer: $s = 100$ and $t = 2$, $s = 50$ and $t = 4$

18 **a** **i** $p = 4s$ **ii** 8 cm
b **i** $p = 5s$ **ii** 10 cm
c $p = ns$

19 **a** $m = 15x + 18y$ where x is the number of hours Alex works during the week, y is the number of hours Alex works on Sunday and m is the amount of money Alex earns in a week
b \$654
c 38 hours
d \$24

20 **a** $y = x(x + 1)$ where x is the lower number and y is the product of the consecutive numbers
b 506 or 552
c The other number could either be 22 or 24.

21 a Possible answers are 19, 29, 39, 49, 59, 69, 79, 89 and 99.

b Sample answer: If the digits of the number are a and b respectively then this situation can be modelled by the equation $10a + b = a + b + ab$. The solution to this equation is $b = 9$, so any two-digit number ending in 9 satisfy the question.

EX p269

6.5 Solving equations by inspection

1 a True **b** False **c** False
d True **e** True **f** False

2 a Yes **b** No **c** No
d Yes **e** Yes **f** No
g Yes **h** No **i** Yes

3 a $a = 5$ **b** $b = 6$ **c** $c = 8$
d $d = 18$ **e** $e = 10$ **f** $f = 16$
g $g = 9$ **h** $h = 7$ **i** $i = 4$
j $j = 8$ **k** $k = 10$ **l** $l = 11$
m $m = 4$ **n** $x = 54$ **o** $x = 0$
p $p = 11$ **q** $q = 11$ **r** $t = 12$

4

Guess x	LHS $4x + 11$	RHS	LHS = RHS?
3	23	35	no
9	47	35	no
5	31	35	no
7	39	35	no
6	35	35	yes

So, $x = 6$

5 a $x = 11$ **b** $x = 8$
c $x = 13$ **d** $x = 16$

6 B

7 3

8 a $x = 2$ **b** $x = 4$
c $x = 3$ **d** $x = 1$

9 Sample answers:
a $x + 7 = 13$ **b** $4x = 24$
c $\frac{x-2}{4} = 1$ **d** $2(x+3) - 10 = 8$

10 a i $x + 6 = 13$ **ii** $x = 7$
b i $9x = 54$ **ii** $x = 6$
c i $x - 7 = 5$ **ii** $x = 12$
d i $2x = 16$ **ii** $x = 8$
e i $2x + 5 = 23$ **ii** $x = 9$
f i $3(x - 8) = 24$ **ii** $x = 16$
g i $\frac{1}{4}x + 2 = 20$ **ii** $x = 72$
h i $\frac{1}{2}(x + 3) = 13$ **ii** $x = 23$

11 a $y = 8$ **b** $y = 41$
c $x = 2$ **d** $x = 5$

12 a $b = 10$ **b** $a = 3$
c $b = 0$ **d** $a = 7$

13 a $15n$ **b** $15n = 60$ **c** $n = 4$
d Four T-shirts

14 a $5 - s$ **b** $5 - s = 3$ **c** $s = 2$
d \$2

15 a Let p = the number of occupied seats
b $p + 17 = 48$ **c** $p = 31$ **d** 31 people

16 a Let h = the number of hours James works in January
b $12h = 336$ **c** 28 hours

17 a $2g = p$ **b** 4 giraffes

18 a $x + y = 15$ **b** $4x + 2y = 42$
c 6 cars and 9 motorbikes

19 a $x = y + 4$ **b** $x + y = 12$
c Eleanor played 8 games and Jing played 4 games

20 a Sample answer: Solving by inspection as there is only one operation in the equation.

b Sample answer: Guess, check and improve as there is more than one operation in the equation.

21 a i What number plus 1 is equal to the same number?
ii No solutions
iii There is no number equal to itself plus 1.

b i What number plus 2 is equal to the same number plus 3?
ii No solutions
iii There is no number where you can add 2 or 3 and get the same total.

c i What number is equal to another number?
ii Infinitely many solutions
iii You can pick any number for y and then choose the same value for x.

d i What number is equal to another number plus 1?
ii Infinitely many solutions
iii You can pick any number for y and then choose that number minus 1 for x.

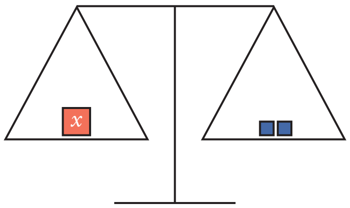
22 a $x = 2$ **b** $x = 12$ **c** $x = 7$ **d** $x = 8$

23 a, b $x = 2$ or $x = 5$
c The power of 2 on the x term.

EX p275

6.6 Solving equations using inverse operations

1 a Remove five blue blocks
b Remove five blue blocks
c

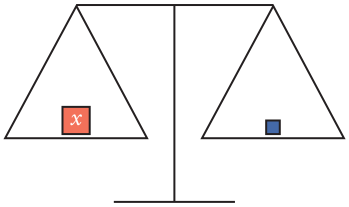


d 2 g

2 a Remove two blue blocks, then divide the number of blocks by three

b Remove two blue blocks, then divide the number of blocks by three

c



d 1 g

3 **a** − 5 **b** + 3
c ÷ 8 **d** × 2

4 **a** $a = 7$ **b** $b = 11$
c $c = 4$ **d** $d = 15$

5 **a** $x = 5$ **b** $a = 10$ **c** $p = 6$
d $s = 30$ **e** $y = 23$ **f** $f = 90$
g $d = 9$ **h** $x = 3$

6 **a** $x = 5$ **b** $x = 6$ **c** $x = 8$
d $x = 30$ **e** $x = 8$ **f** $x = 17$
g $x = 12$ **h** $x = 7$ **i** $x = 3$
j $x = -11$ **k** $x = 30$ **l** $x = -27$

7 **a** $x \rightarrow \boxed{+4} \rightarrow x + 4 \rightarrow \boxed{\times 5} \rightarrow 5(x + 4)$

b $x \rightarrow \boxed{\div 3} \rightarrow \dfrac{x}{3} \rightarrow \boxed{-2} \rightarrow \dfrac{x}{3} - 2$

c $x \rightarrow \boxed{\times 4} \rightarrow 4x \rightarrow \boxed{-3} \rightarrow (4x - 3)$

d $x \rightarrow \boxed{+6} \rightarrow x + 6 \rightarrow \boxed{\div 8} \rightarrow \dfrac{x + 6}{8}$

e $x \rightarrow \boxed{\times 9} \rightarrow 9x \rightarrow \boxed{\div 8} \rightarrow 9x + 1$

f $x \rightarrow \boxed{\div 4} \rightarrow \dfrac{x}{4} \rightarrow \boxed{+6} \rightarrow \dfrac{x}{4} + 6$

g $x \rightarrow \boxed{-5} \rightarrow x - 5 \rightarrow \boxed{\div 9} \rightarrow \dfrac{x - 5}{9}$

h $x \rightarrow \boxed{\times 5} \rightarrow 5x \rightarrow \boxed{\div 7} \rightarrow \dfrac{5x}{7}$

8 **a** $x = 6$ **b** $x = 10$ **c** $x = 3$
d $x = 18$ **e** $x = 28$ **f** $x = 4$
g $x = 6$ **h** $x = 72$

9 **a** $x = 5$ **b** $x = 19$ **c** $x = 11$
d $x = 11$ **e** $x = 7$ **f** $x = 16$
g $x = 10$ **h** $x = 37$

10 **a** **i** $2(x + 4) - 1 = 17$ **ii** $x = 5$
b **i** $\frac{5x}{3} + 2 = 12$ **ii** $x = 6$

11 **a** $x = 1$ **b** $a = 14$ **c** $x = 6$
d $x = 11$ **e** $p = 42$ **f** $d = 9$
g $f = 8$ **h** $z = 29$ **i** $x = 4$
j $t = 30$ **k** $q = -2$ **l** $y = 14$

12 **a** $y = 4$ **b** $y = 8$ **c** $x = 4$
d $x = -6$

13 **a** $x = 4$ **b** $x = 3$ **c** $n = 80$
d $y = 0$ **e** $a = 6$ **f** $n = -5$
g $x = 24$ **h** $p = -3$

14 **a** × 3 **b** − 5 **c** + 6
d × 3 **e** ÷ 8 **f** − 3
g + 7 **h** × 8 **i** × 6

15 **a** $2x - 3 = 15$ **b** 9

16 **a** $\frac{n + 3}{2} + 5$ **b** 8

17 **a** Let x = the original amount of money in your wallet, $2(x + 5) = 136$
b $x = 63$, \$63

18 Let x represent each variable.
a $4x = 64$, $x = 16$, 16 tables
b $15x + 6 = 81$, $x = 5$, 5 hours
c $2x + 12 = 20$, $x = 4$, 4 extra toppings
d $24x + 105 = 369$, $x = 11$, 11 weeks

19 **a** $c = 1.9m + 6.2$
b \$67 **c** 39 minutes

20 **a** $t = 14.5m + 18$
b 90.5°C **c** 6 minutes

21 6

22 \$500

23 Let x be the number Penelope first thinks of.
The calculation simplifies to x.

$$\begin{aligned}\frac{4x - 2 - x - 4}{3} + 2 &= \frac{3x - 6}{3} + 2\\ &= x - 2 + 2\\ &= x\end{aligned}$$

24 **a** $x = 3$ **b** $x = 4$ **c** $x = 6$
d $x = 15$

25 $b = \frac{5}{2}$

EX **p280**

Review: Algebra

Review questions 6.7A: Mathematical literacy review

1 Sample answer:

a A variable is a quantity that can have different values and can be represented with a word, symbol or pronumeral. For example, a can be used to represent the quantity of apples.

b An expression can be either a single term or the sum or difference of two or more terms. For example, $3a$ is an expression.

c An equation is a mathematical statement that asserts the equality of two expressions. For example, $3a = 3$ is an equation.

d The coefficient of a term is the number multiplied by the pronumeral. For example, 3 is the coefficient of a in $3a$.

e A term in algebra can be a single pronumeral, a single number, or the product of a number and one or more pronumerals. For example, in $3a + 3$, there are two terms, $3a$ and 3.

2 **a** sum, difference, product, quotient
b Equivalent, solutions, substitution
c evaluated, substituting

3 Sample answer: We perform inverse operations to solve a linear equation, then substitute the solution to verify if it satisfies the equation.

4 **a** The commutative law
b The associative law

Review questions 6.7B: Multiple choice

1 D **2** A **3** B **4** A **5** D
6 B **7** B **8** D **9** D

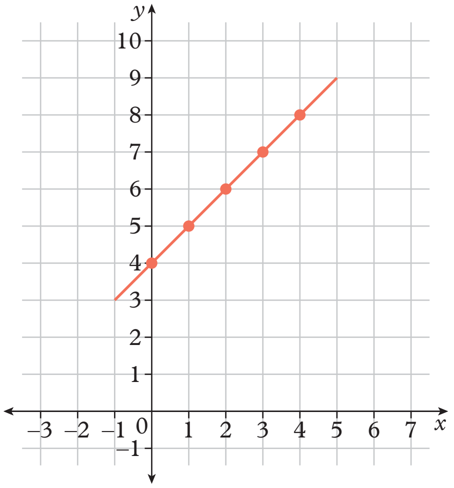
Review questions 6.7C: Short answer

1 **a** Time, total number of worms collected

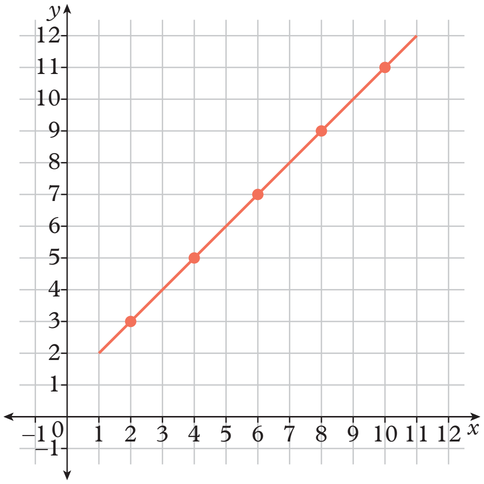
b

Time (pm)	1:00	1:30	2:00	2:30	3:00
Total number of worms collected	0	5	10	15	20

2 **a** The points lie on a straight line.



b The points lie on a straight line.



3 **a** **i** 2 **ii** x, y
iii 7, 3 **iv** none
b **i** 2 **ii** a
iii 1 **iv** 3

c **i** 3 **ii** d, c
iii 5, 2 **iv** 9
d **i** 4 **ii** e, t, s
iii 1, 2, 6 **iv** 4

4 **a** $7x = 21$ **b** $\frac{x}{3} = 6$
c $x + 7 = 10$ **d** $x - 2 = 17$

5 **a** $3(x + 7)$ **b** $\frac{p + 5}{13}$
c $\left(\frac{c}{2}\right)^2$ **d** $v^2 - 3$

6 **a** 7 **b** 6 **c** 6 **d** 2
7 **a** 14 **b** 5 **c** 2 **d** 162
8 **a** $a = 7$ **b** $b = 31$
c $c = 3$ **d** $d = 10$
9 $x = 20$
10 **a** $x = 12$ **b** $x = 48$ **c** $x = 4$
d $x = 13$ **e** $x = -4$ **f** $x = -55$

Review questions 6.7D: Mathematical modelling

1 **a** Number of hours, money earned
b Let c = the total amount that Santo charges for a given session, r = the number of hours in a specific recording session
c $c = 250r + 300$
d $1550
e **i** $f = 325h$, where f = the total amount that Santo charges and h = the number of hours
ii 6 hours
iii The recording session
iv 4 hours

2 **a** $m = 8c + 12s + 16a$
b $a = 9 + c, s = 3c$
c $a = 19, s = 30$
d $744
e 72 students

3 **a** $3f$
b $l = 3f + 12$
c Since the plural ‘friends’ is used, we assume that Liam has at least 2 friends. So the minimum is $l = 18$.
d If $f = 3$, then $l = 21$; if $f = 4$, then $l = 24$, etc.
e The possible solutions go up in increments of 3.
f 22 **g** 20 **h** 17

MODULE 7 Angles and shapes

7.1 Classifying angles and lines

EX p290

1 **a** AB **b** $\angle AOB$ (or $\angle BOA$)
c $\overleftrightarrow{CD}$ **d** $\angle XYZ$ (or $\angle ZYX$)
e $\overrightarrow{ST}$ **f** $\angle JOK$ (or $\angle KOJ$)
g The reflex angle $\angle POQ$ (or $\angle QOP$)
h $\angle LMN$ (or $\angle NML$)

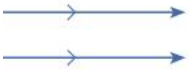
2 **a** $PQ \parallel RS$
b $XY \perp YZ$

3 **a** Two parallel intervals

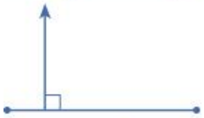
b A ray perpendicular to an interval

c Two parallel lines perpendicular to another two parallel lines

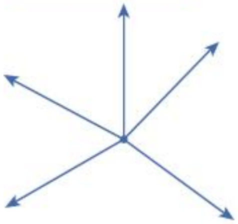
4 **a**



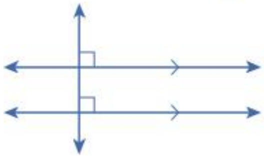
b



c



d



5 **a** **iv, v, vi, i, iii, ii**

b **i** Between 90° and 180°

ii Between 0° and 90°

iii 90° **iv** 360°

v Between 180° and 360° **vi** 180°

c **i** Obtuse **ii** Acute

iii Right angled **iv** Revolution

v Reflex **vi** Straight

6 Measurements within 2°–3° of the answers below are acceptable.

a 84°, acute **b** 40°, acute **c** 135°, obtuse

d 15°, acute **e** 150°, obtuse **f** 120°, obtuse

7 **a** $\angle AOB$ (or $\angle BOA$), acute

b $\angle XOY$ (or $\angle YOX$), obtuse

c $\angle COD$ (or $\angle DOC$), right angled

d $\angle SOT$ (or $\angle TOS$), acute

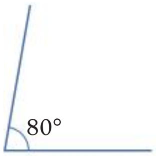
8 **a** 15° **b** 56°

c $\angle COF$ is an acute angle

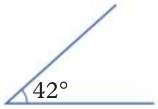
d $\angle AOB$, $\angle AOC$, $\angle BOC$, $\angle BOD$, $\angle COD$, $\angle COE$, $\angle COF$, $\angle DOE$, $\angle DOF$, $\angle EOF$

e $\angle AOD$, $\angle AOE$, $\angle AOF$, $\angle BOF$

9 **a**



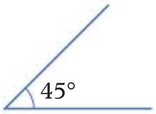
b



c

d

10 **a**



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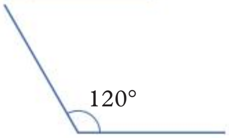
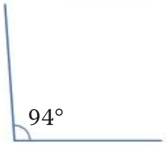


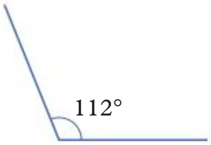
c

d

e



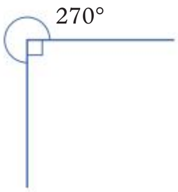
f



11 **a** $a = 270°$ **b** $b = 332°$

c $c = 265°$ **d** $d = 186°$

12 **a**



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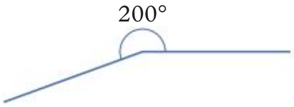

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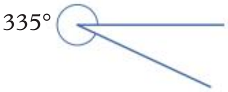
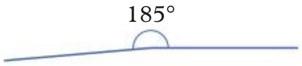


d



13 **a** 2 **b** 4 **c** 2

14 **a** Straight angle **b** Revolution

c Reflex angle **d** Reflex angle

15 The protractor was read from the right-hand zero line. $\angle ABC = 106°$.

16 $\angle XYZ = 60°$

17 80°

18 **a** Top right pocket

b Top left pocket

EX p298

7.2 Angles at a point

1 **a** $a = 130°$ **b** $b = 67°$ **c** $c = 99°$

d $d = 19°$ **e** $e = 66°$ **f** $f = 84°$

2 **a** $\angle YOZ = 30°$

b $\angle COD = 21°$

c $\angle QOR = 132°$

d $\angle KOL = 37°$

3 **a** $\angle WOX$ and $\angle XOY$; $\angle XOY$ and $\angle YOZ$

b $\angle AOB$ and $\angle BOC$; $\angle BOC$ and $\angle COD$

c $\angle POQ$ and $\angle QOR$; $\angle QOR$ and $\angle ROS$

d $\angle JOK$ and $\angle KOL$; $\angle KOL$ and $\angle LOM$

4 **a** **i** 67° **ii** 13° **iii** 2° **iv** 81°

b **i** 22° **ii** 159° **iii** 80° **iv** 83°

5 **a** **i** 44° **ii** 134°

b **i** 78° **ii** 168°

c **i** 5° **ii** 95°

d **i** 63° **ii** 153°

6 **a** $a = 100^\circ$ **b** $b = 27^\circ$ **c** $c = 38^\circ$

d $d = 133^\circ$ **e** $a = 215^\circ$ **f** $b = 300^\circ$

g $c = 270^\circ$ **h** $e = 20^\circ$

7 Yes, these pairs of angles are adjacent as they share a common side and a common vertex.

8 **a** $r = 147^\circ$ **b** $d = 20^\circ; e = 160^\circ; f = 20^\circ$

c $x = 136^\circ; y = 44^\circ; z = 136^\circ$ **d** $c = 90^\circ$

e $a = 50^\circ; b = 50^\circ; c = 50^\circ; d = 80^\circ$

f $g = 60^\circ; h = 60^\circ$

9 **a** $\angle YOZ = 30^\circ$

b $\angle POQ = 142^\circ, \angle QOR = 135^\circ$

c $\angle EOF = 127^\circ$

d $\angle JOK = 137^\circ, \angle JOM = 43^\circ, \angle LOM = 137^\circ$

e $\angle TOU = 22^\circ, \angle POU = 88^\circ, \angle POQ = 70^\circ, \angle ROS = 88^\circ$

f $\angle AOB = 109^\circ, \angle BOC = 71^\circ, \angle DOE = 19^\circ$

10 **a** **i** 34° **i** Complementary

b **i** 214° **ii** Full revolution

c **i** 30° **ii** Complementary

11 **a** 8 **b** 45°

c **i** 10 **ii** 72°

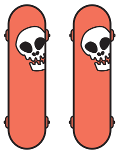
12 **a** The two angles do not share a common side.

b The two angles do not share a common vertex.

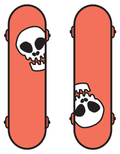
c The two angles overlap.

13 125°

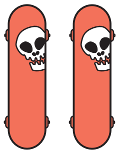
14 **a** start end



b start end



c start end



15 **a** True, the angles around a point must sum to 360°.
$360^\circ - 138^\circ - 35^\circ - 42^\circ - 55^\circ = 90^\circ$

b True, two complementary angles add to make 90°.
$55^\circ + 35^\circ = 90^\circ$

c True, two supplementary angles add to make 180°.
$42^\circ + 138^\circ = 180^\circ$

d True, $55^\circ + 90^\circ + 138^\circ + 35^\circ + 42^\circ = 360^\circ$.

16 No, the largest acute angle is less than 90°, therefore the sum of two acute angles must always be less than 180°.

17 Sample answer:
$\angle AOC$ and $\angle BOD$ are both supplementary to $\angle BOC$
hence: $180^\circ - \angle BOC = \angle AOC$
$180^\circ - \angle BOC = \angle BOD$
So, $\angle AOC = \angle BOD$

EX **p306**

7.3 Angles and parallel lines

1 **a** Alternate **b** Co-interior

c Corresponding **d** Corresponding

e Alternate **f** Co-interior

g Corresponding **h** Corresponding

i Alternate

2 The pairs of angles in **a**, **c**, **d**, **e**, **g**, **h** and **i** are equal in size. The pairs of angles in **b** and **f** are supplementary.

3 **a** EF **b** $\angle EPD$ **c** $\angle FOB$

d $\angle CPE$ **e** $\angle CPE$ **f** $\angle FOB$

g $\angle CPE$

4 **a** UT

b $\angle PYT$ and $\angle UXS$, $\angle QYT$ and $\angle RXU$

c $\angle RXU$ and $\angle PYT$, $\angle QYT$ and $\angle UXS$

d $\angle PYU$ and $\angle RXU$, $\angle PYT$ and $\angle RXT$, $\angle SXT$ and $\angle QYT$, $\angle SXU$ and $\angle QYU$

5 **a** $\angle EOB$ **b** $\angle DPE$ or $\angle CPF$ or $\angle AOF$ or $\angle BOE$

c $\angle CPF$ **d** $\angle DPE$, $\angle CPF$ or $\angle AOF$ or $\angle BOE$

6 **a** **i** Alternate **ii** 125°

b **i** Co-interior **ii** 110°

c **i** Corresponding **ii** 131°

d **i** Corresponding **ii** 60°

e **i** Alternate **ii** 150°

f **i** Co-interior **ii** 138°

g **i** Alternate **ii** 75°

h **i** Co-interior **ii** 90°

i **i** Corresponding **ii** 84°

7 **a** Yes, the alternate angles are equal.

b No, the corresponding angles are not equal.

c No, the co-interior angles are not supplementary.

d Yes, the co-interior angles are supplementary.

e No, the corresponding angles are not equal.

f No, the alternate angles are not equal.

g Yes, the angles vertically opposite the labelled angles are alternate angles and equal.

h Yes, the angles supplementary to the labelled angles are co-interior and supplementary.

8 **a** **i** Supplementary **ii** 125°

b **i** Supplementary **ii** 55°

c **i** Equal **ii** 71°

d **i** Equal **ii** 71°

e **i** Equal **ii** 67°

f **i** Equal **ii** 123°

9 **a** $a = 71^\circ$, alternate angles

b $b = 122^\circ$, co-interior angles

c $c = 65^\circ$, alternate angles

d $d = 65^\circ$, co-interior angles

e $e = 35^\circ$, alternate angles

f $f = 121^\circ$, corresponding angles

g $g = 133^\circ$, corresponding angles

h $h = 160^\circ$, alternate angles

10 35°

11 133°

12 a $a = b = c = 64^\circ$

b No, if the paintings were parallel then c would be 64°.

c Amal would have to move the left-hand side of the bottom painting down to make $c = 64^\circ$.

13 a $\angle AOF$ is corresponding to $\angle CPF$, and $\angle CPF$ and $\angle FPD$ are angles on a straight line (supplementary). Therefore, as $81^\circ+99^\circ= 180^\circ$, the two blue lines are parallel.

b Sample answer: $\angle BOF = \angle EPC$, $\angle CPE = \angle AOE$

14 a $\angle AOF$ and $\angle CPE$ are co-interior angles.

$\angle AOF = \angle CPF$ since corresponding angles on parallel lines are equal.

$\angle CPF + \angle CPE = 180^\circ$ since they are angles that form a straight angle.

$\angle AOF + \angle CPE = 180^\circ$ since $\angle AOP = \angle CPF$

Therefore, $\angle AOF$ and $\angle CPE$ are supplementary since they add to 180°.

b i Let $\angle AOE$ be an acute angle.

The corresponding angle $\angle CPE = \angle AOE$ since corresponding angles on parallel lines are equal, therefore it is also acute.

The alternate angle to $\angle CPE$ is $\angle FOB$, since alternate angles on parallel lines are equal. Therefore, $\angle FOB = \angle CPE = \angle AOE$ and they are all acute.

The corresponding angle to $\angle FOB$ is $\angle FPD$, since corresponding angles on parallel lines are equal. Therefore, $\angle FPD = \angle FOB = \angle CPE = \angle AOE$ and they are all acute.

The remaining four angles each form a straight angle with an acute angle, so must be the supplement of the acute angle. The supplement of an acute angle is always an obtuse angle.

ii Let $\angle AOE$ be an acute angle. We know that $\angle FPD = \angle FOB = \angle CPE = \angle AOE$ so they are all acute. The four remaining obtuse angles, $\angle BOE$, $\angle AOP$, $\angle DPO$ and $\angle CPF$ each form a straight angle with an acute angle, so must be the supplement of the acute angle. The supplement of an acute angle is an obtuse angle. Since all the acute angles are equal, each obtuse angle is the supplement of the same angle, and so they must also all be equal.

iii All angles on parallel lines are equal or supplementary. If any angle is 90°, then all equal angles must also be 90°, and all supplementary angles are also 90°. Therefore, all angles would equal 90°.

EX p310

Checkpoint

1 a III **b** VI **c** II

d V **e** IV **f** I

2 a Point, P

b Ray, $\overrightarrow{JH}$

c Line, $\overleftrightarrow{BY}$

d Angle, $\angle SWK$ (or $\angle KWS$)

3 a 85° **b** 123°

4 a B **b** C **c** D **d** A

5 a Complement = 71°, supplement = 161°

b Completement = 53°, supplement = 143°

c Complement = 1°, supplement = 91°

6 a $\angle AHJ = 143^\circ$

b $\angle IJC = 106^\circ$

c $\angle FIB = 70^\circ$

7 a True **b** True **c** True

d False **e** False

8 a Co-interior **b** Alternate

c Vertically opposite

d Corresponding

9 a $h = 87^\circ$ **b** $u = 146^\circ$ **c** $n = 99^\circ$

10 a Not parallel. The alternate angles are not equal.

b Not parallel. The co-interior angles are not supplementary.

c Parallel. The angles vertically opposite the labelled angles are alternate angles and equal.

d Not parallel. The angles vertically opposite to the labelled angles are co-interior and are not supplementary.

EX p314

7.4 Triangles

1 a Equilateral acute triangle

b Scalene acute triangle

c Isosceles obtuse triangle

d Scalene obtuse triangle

e Scalene right-angled triangle

f Scalene obtuse triangle

g Equilateral acute triangle

h Isosceles acute triangle

i Isosceles right-angled triangle

2 a $a = 80^\circ$ **b** $b = 82^\circ$ **c** $c = 91^\circ$

d $d = 50^\circ$ **e** $e = 67^\circ$ **f** $f = 33^\circ$

3 a $a = 29^\circ$, scalene acute triangle

b $b = 48^\circ$, isosceles acute triangle

c $c = 60^\circ$, equilateral acute triangle

d $d = 70^\circ$, isosceles acute triangle

e $e = 85^\circ$, scalene acute triangle

f $f = 90^\circ$, isosceles right-angled triangle

4 a $a = 53^\circ$ **b** $a = b = c = 60^\circ$

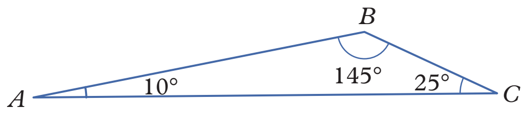
c $a = b = 73^\circ$ **d** $a = b = 45^\circ$

e $a = 25^\circ$; $b = 130^\circ$

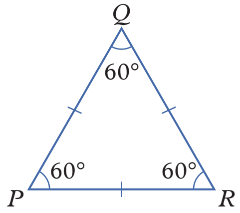
f $a = b = 60^\circ$

5 **a** $x = 1.5$ cm **b** $a = 10$ cm
c $q = 23$ cm **d** $b = 3$ cm

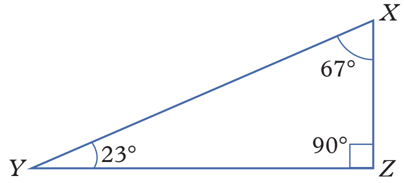
6 **a**



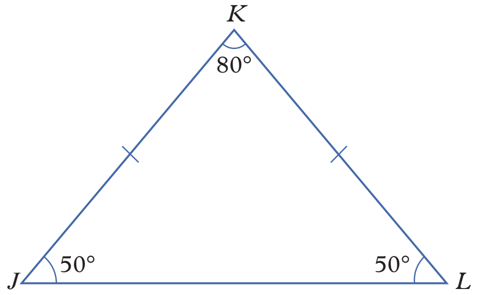
b



c



d



7 **a** **i** Possible
ii Not possible
iii Possible
iv Possible
v Not possible
vi Possible
b **i** Scalene right-angled triangle
iii Equilateral acute triangle
iv Scalene obtuse triangle
vi Isosceles obtuse triangle

8 Yes, a right-angled triangle with two 45° angles is also an isosceles triangle.

9 Either a scalene or an isosceles triangle can have an obtuse triangle.

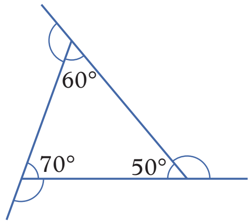
In an equilateral triangle all angles are 60°, so they are all acute angles.

10 **a**

Type of triangle	Number of sides of equal length	Number of internal angles that are equal in size
Equilateral	3	3
Isosceles	2	2
Scalene	0	0

b In any triangle, the number of sides of equal length is the same as the number of angles that are equal in size.

11 **a** It is outside the triangle. **b** 130°
c 130°. They add to the size of the exterior angle.
d **e** 110° and 120°

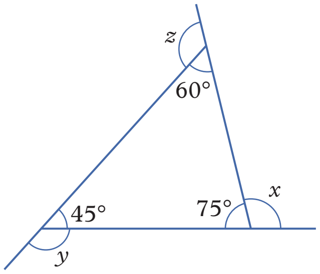


f 110° and 120°. They add to the size of the exterior angle.

g An exterior angle is equal to the sum of the two interior angles opposite that exterior angle. This is because an exterior angle is supplementary to the adjacent interior angle and the sum of interior angles in a triangle is 180°, so the two opposite interior angles are also supplementary to the interior angle adjacent to the exterior angle.

12 **a** $x = 105°$ **b** $x = 120°$ **c** $x = 115°$
d $x = 40°$ **e** $x = 95°$ **f** $x = 25°$

13 **a** 360°
b Sample answer:



i $x = 105°; y = 135°; z = 120°$
ii $x + y + z = 360°$
c The exterior angles of a triangle add to 360°.

14 **a** $x = 30°$ **b** $x = 10°$ **c** $x = 10°$
d $x = 10°$ **e** $x = 12°$ **f** $x = 20°$

15 **a** **i** Alternate **ii** Alternate
b $\angle PBA$, $\angle ABC$ and $\angle QBC$ are angles on a straight line, therefore $\angle PBA + \angle ABC + \angle QBC = 180°$.
As alternate angles on parallel lines are equal, $\angle BAC + \angle ABC + \angle ACB = 180°$.

EX p320

7.5 Quadrilaterals

1 **a** Parallelogram **b** Kite **c** Square
d Trapezium **e** Rhombus **f** Trapezium
g Irregular quadrilateral
h Rectangle **i** Kite

2 **a** Rhombus **b** Trapezium
c Rectangle **d** Parallelogram
e Square **f** Kite

3 **a** True **b** False **c** True
d False

4 **a** $a = 112°$ **b** $b = 135°$ **c** $c = 142°$
d $d = 253°$ **e** $e = 59°$ **f** $f = 79°$

5 **a** Convex **b** Convex **c** Convex
d Non-convex **e** Convex **f** Convex

6 **a** Irregular quadrilateral; $a = 70^\circ$
b Parallelogram; $b = 53^\circ$
c Trapezium; $c = 35^\circ$
d Irregular quadrilateral; $d = 145^\circ$
e Irregular quadrilateral; $e = 65^\circ$
f Rhombus; $f = 105^\circ$

7 **a** Convex **b** Convex **c** Convex
d Convex **e** Non-convex **f** Convex

8 **a** $d = 60^\circ, e = 130^\circ$ **b** $c = 80^\circ$
c $x = 150^\circ, y = 30^\circ, z = 150^\circ$ **d** $k = 45^\circ$
e $m = n = 90^\circ$
f $f = 135^\circ$
g $x = w = 111^\circ$
h $e = f = g = 90^\circ$
i $s = t = 118^\circ$

9 Trapezium, rectangle, rhombus, parallelogram, irregular quadrilateral

10 **a** Rhombus **b** Square

11 Barbecue: trapezium
Dining area: square
Jungle gym: irregular quadrilateral
Lawn: rectangle
Pool area: rectangle
Rose garden: rectangle
Sand pit: irregular quadrilateral
Spa: trapezium
Veranda: trapezium
Washing line: parallelogram

12 **a** Angles are incorrectly labelled. As opposite angles are equal in a rhombus, the labelled angles must sum to 180°, otherwise interior angle sum is not 360°.
b Angles are incorrectly labelled. The interior angle sum is 380°, not 360°.
c The diagram is a trapezium not an irregular quadrilateral.

13 **a** Rectangle or square
b Rhombus or square
c Rectangle, parallelogram, rhombus, or square
d Trapezium, parallelogram, rhombus, rectangle, or square
e Parallelogram, rectangle, rhombus or square

14 **a** Yes, all four angles are 90°
b No, all sides are not equal in length
c Yes, there are two pairs of parallel sides and the pairs of opposite angles are equal in size
d Yes, all four sides are of equal length

15 B

16 Irregular quadrilaterals and kites

17 The angles inside a shape are formed where two edges meet, whereas an angle of 180° is a straight angle.

18 **a** Two **b** 180° **c** $2 \times 180^\circ = 360^\circ$
d Not all of the triangles' internal angles align with the internal angles of quadrilateral. Each triangle has one internal angle located at the centre of the quadrilateral.
e Subtract 360°
f **i** Revolution **ii** 360°; yes

19 **a** $x = 35^\circ$ **b** $x = 80^\circ$ **c** $x = 15^\circ$
d $x = 37^\circ$ **e** $x = 17^\circ$ **f** $x = 12^\circ$

20 **a** 70°, 110°, 70°, 110°
b 360°. The exterior angles on a triangle and a quadrilateral both add to 360°.
c Sample answer: The exterior angles of all quadrilaterals add to 360°.

EX
p328

7.6 Polygons

1 **a** Polygon **b** Polygon
c Not a polygon **d** Polygon
e Polygon **f** Not a polygon
g Polygon **h** Polygon
i Not a polygon

2 **a** **i** Regular **ii** Convex
b **i** Irregular **ii** Convex
c **i** Irregular **ii** Concave
d **i** Regular **ii** Convex
e **i** Irregular **ii** Concave
f **i** Irregular **ii** Convex

3 **a** Regular, convex octagon
b Irregular, convex pentagon
c Irregular, concave pentagon
d Regular, convex nonagon
e Regular, convex hexagon
f Irregular, concave decagon

4 Shapes **a**, **b**, **d**, **e**, **g** and **h** from question **1** are polygons.
a **i** Regular **ii** Ten sides
iii No **iv** Regular, convex decagon
b **i** Irregular **ii** Six sides
iii No **iv** Irregular, convex hexagon
d **i** Regular **ii** Three sides
iii No
iv Regular, convex triangle or equilateral triangle
e **i** Regular **ii** Five sides
iii No **iv** Regular, convex pentagon
g **i** Irregular **ii** Four sides
iii No **iv** Irregular, convex quadrilateral
h **i** Irregular **ii** Eight sides
iii Yes **iv** Irregular, concave octagon

5 **a** Irregular, convex hexagon
b Irregular, concave decagon
c Irregular, concave octagon
d Irregular, convex pentagon
e Irregular, concave decagon
f Irregular, concave heptagon

6 **a** One name describes the number of convex points it has and the other name describes how many sides it has.

b **i** Four-pointed star or concave octagon

ii Six-pointed star or concave dodecagon

iii Three-pointed star or concave hexagon

c The number of points of the star is half the number of sides.

7 **a** Angles in a triangle add to 180°.

b 10 sides **c** Eight **d** 1440°

8 **a** **i** Three **ii** 540°

b **i** Four **ii** 720°

c **i** Five **ii** 900°

d **i** Seven **ii** 1260°

9 Yes; triangles and quadrilaterals have straight sides.

10 **a** 108° **b** 120°

c 128.57° **d** 140°

11 **a** $a = 120^\circ$ **b** $b = 41^\circ$ **c** $c = 146^\circ$

EX
p332

Review: Angles and shapes

Review questions 7.7A: Mathematical literacy review

1 degrees

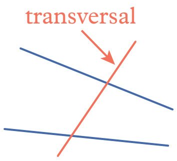
2 isosceles triangle

3 trapezium

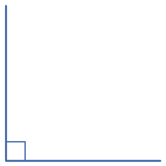
4 Complementary angles sum to 90°, whereas supplementary angles sum to 180°.

5 **a** line segment **b** rectangle **c** polygon

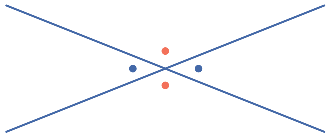
6 **a** A transversal is a line that crosses two or more other lines. For example,



b A right angle is an angle that is exactly 90°. For example,



c Vertically opposite angles are a pair of angles directly opposite each other formed when two or more straight lines intersect. For example,



7 **a** rays, angle

b Adjacent angles, vertex

Review questions 7.7B: Multiple choice

1 A 2 C 3 D 4 D

5 B 6 A 7 C 8 D

9 C 10 B 11 B 12 B

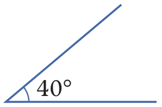
13 D

Review questions 7.7C: Short answer

1 **a** 20°, acute **b** 135°, obtuse

c 90°, right angle **d** 197°, reflex

2 **a**



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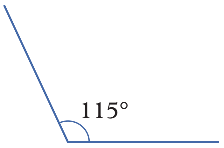
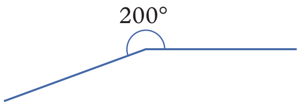


c



3 **a** Ray: $\overrightarrow{DJ}$ Line: $\overleftrightarrow{GD}$

b Triangle: $\triangle ABC$

4 **a** **i** 67° **ii** 11°

b **i** 171° **ii** 53°

5 **a** $a = 320^\circ$ **b** $b = 145^\circ$ **c** $c = 25^\circ$

d $d = 57^\circ, e = 123^\circ, f = 123^\circ$

e $g = 26^\circ, h = 66^\circ$

f $i = 62^\circ, j = 270^\circ$

6 **a** g **b** d **c** d

d **i** 77° **ii** 103° **iii** 103°

7 **a** Yes, the corresponding angles are equal.

b No, the corresponding angle to 117° is not supplementary to 73°.

8 **a** Scalene acute triangle, $x = 30^\circ$

b Isosceles obtuse triangle, $x = 25^\circ$

c Equilateral acute triangle, $x = 60^\circ$

d Isosceles right-angled triangle, $x = 45^\circ$

9 **a** Irregular quadrilateral, $x = 86^\circ$

b Trapezium, $x = 113^\circ$ **c** Kite, $x = 111^\circ$

d Parallelogram, $x = 59^\circ$

10 **a** $x = 136^\circ$ **b** $d = 49^\circ$ **c** $m = 22^\circ$

11 **a** **i** 106° **ii** 90° **iii** 35°

iv 90°

b Kite

12 **a** **i** Regular **ii** 7 **iii** No

b **i** Irregular **ii** 6 **iii** Yes

c **i** Irregular **ii** 8 **iii** Yes

d **i** Regular **ii** 5 **iii** No

13 **a** Regular, convex heptagon

b Irregular, concave hexagon

c Irregular, concave octagon

d Regular, convex pentagon

Review questions 7.7D: Mathematical modelling

1 $a = 140°$, $d = 50°$, $g = 37°$, $k = 130°$, $n = 127°$, $q = 103°$, $t = 40°$, $w = 64°$, $z = 116°$

2 $\overleftrightarrow{AB}$ is parallel to $\overleftrightarrow{EF}$ and $\overleftrightarrow{IJ}$ is parallel to $\overleftrightarrow{LK}$ as the alternate angles are equal (102°) and the co-interior angles are supplementary, $102° + 78° = 180°$

MODULE 8 Objects and transformations

EX p343

8.1 3D objects

1 **a** Five faces **b** Eight edges **c** Five vertices

2 **a** Six faces **b** Ten edges **c** Six vertices

3 **a** Polyhedra **b** Polyhedra **c** Not polyhedra
  d Polyhedra **e** Polyhedra **f** Not polyhedra

4 **a** and **d**

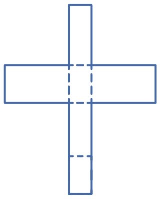
5 **a** Heptagon **b** Triangle **d** Triangle
  e Decagon or 5-pointed star

6 **a** Heptagonal prism
  b Triangular pyramid (or tetrahedron)
  d Triangular prism
  e Decagonal pyramid

7 **a** Pentagonal prism
  b Hexagonal pyramid
  c Heptagonal pyramid
  d Decagonal prism
  e Nonagonal prism
  f Octagonal pyramid

8 **a** Rectangular prism
  b Triangular pyramid (or tetrahedron)
  c Sphere

9 **a**



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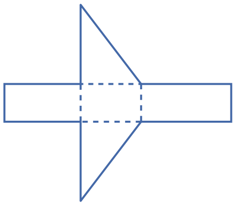
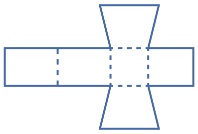
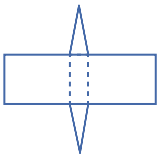


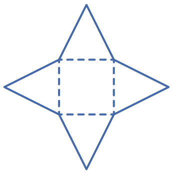
c



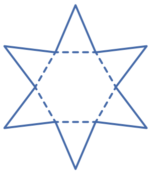
d



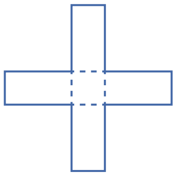
e



f



10 **a**



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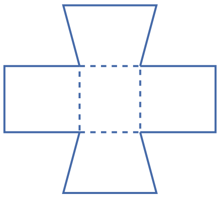
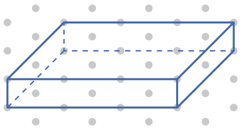
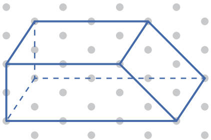


11 Sample answers:

a



b



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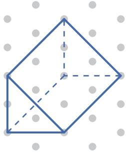

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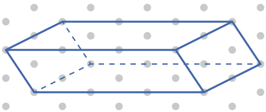

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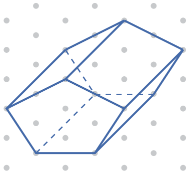

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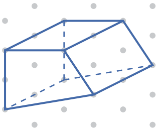
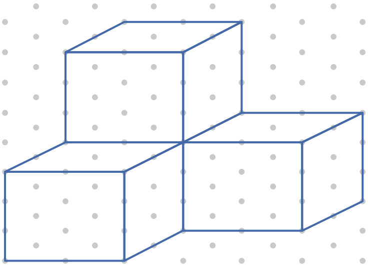
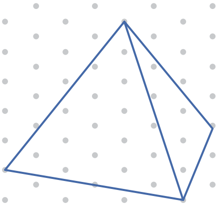


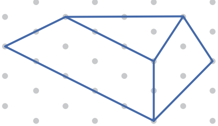
12 **a** Sample answer:



b Sample answer:



c Sample answer:



13 **a** Yes; it has flat faces, straight edges and vertices
  b No; it has some curved sides
  c Yes; it has flat faces, straight edges and vertices

14 **a** **i** Sphere
  ii Cone
  iii Cylinder
  b They have some curved sides.

15 Sample answer: no, the faces are slightly curved because of the pressure in the ball.

16 **a** Rectangular prism
  b Tetrahedron (triangular pyramid)
  c Cylinder

17 a–c

	Tetrahedron	**Cube (hexahedron)**
F	4	6
polygon	triangle	square
V	4	8
E	6	12
F + V − E	2	2

	Octahedron	**Dodecahedron**
F	8	12
polygon	triangle	pentagon
V	6	20
E	12	30
F + V − E	2	2

	Icosahedron
F	20
polygon	triangle
V	12
E	30
F + V − E	2

d $F + V - E = 2$

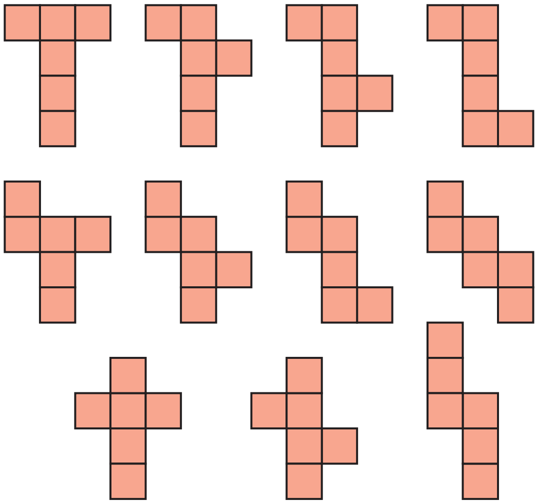
18 a Yes **b** The rule stays the same.

19 a 8 **b** 5 **c** 14

d 12

20 The smiley face with hearts for eyes

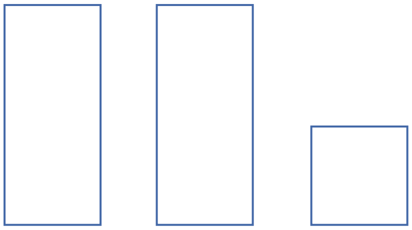
21



EX p349

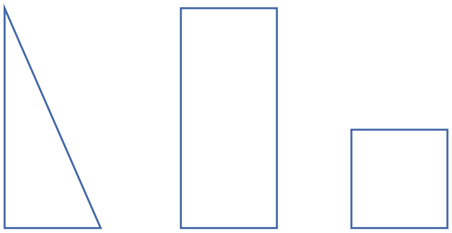
8.2 2D representations of 3D objects

1 a



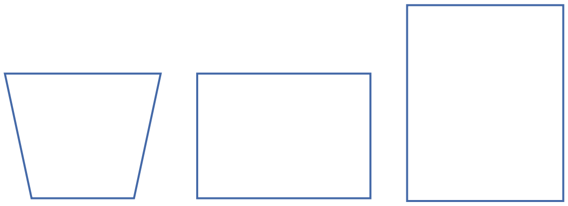
front elevation right elevation plan view

b



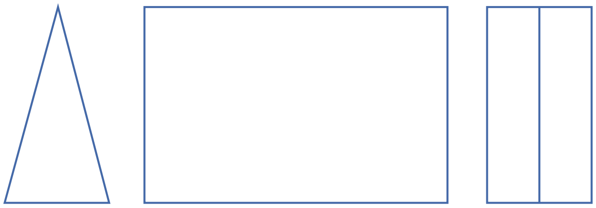
front elevation right elevation plan view

c



front elevation right elevation plan view

d

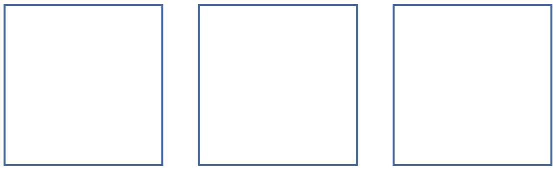


front elevation right elevation plan view

2 a

front elevation right elevation plan view

b

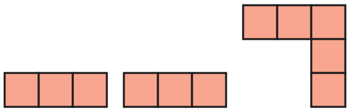


front elevation right elevation plan view

c

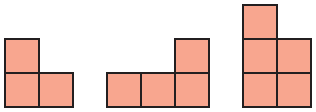
front elevation right elevation plan view

3 a



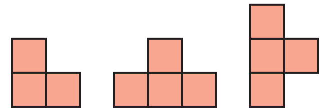
front elevation right elevation plan view

b



front elevation right elevation plan view

c



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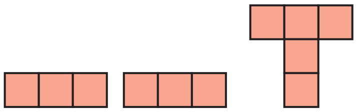
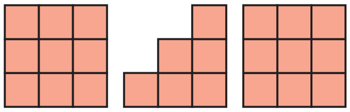
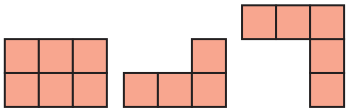


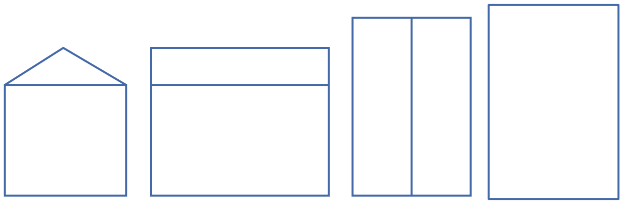
e



f



4 a



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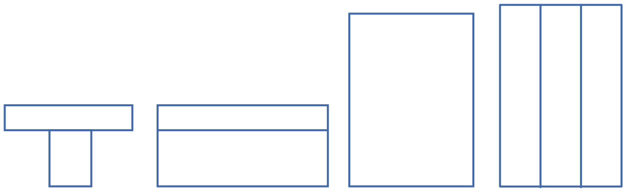
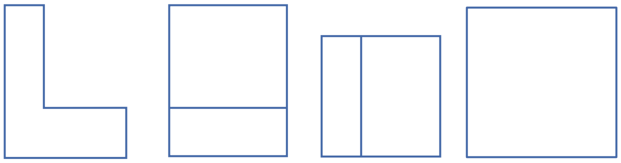
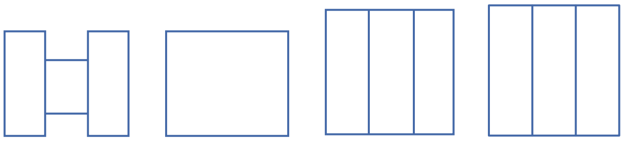


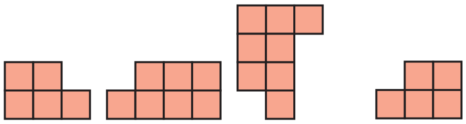
c



d



5 a



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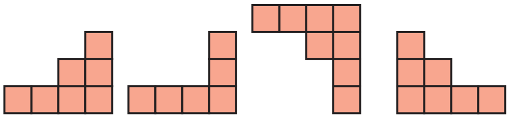

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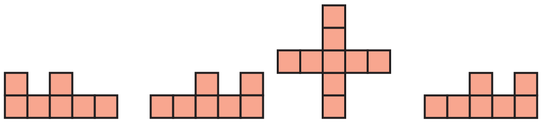
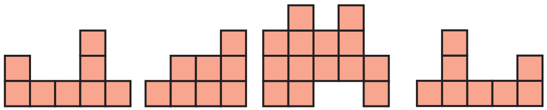
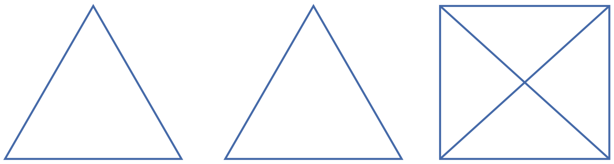


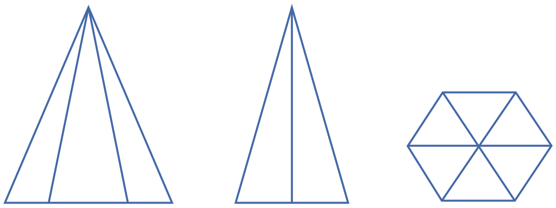
d



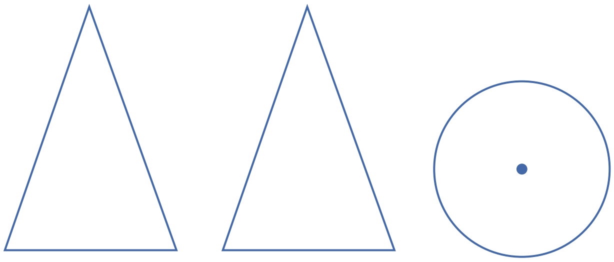
6 a



b



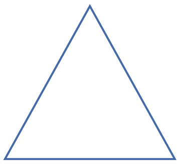
7

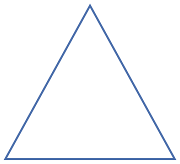


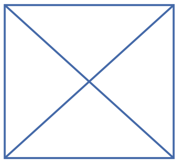
8 They are all circles.

9 When viewed from the side you don't see the curved edge and only see a rectangular shape.

10


front elevation


right elevation

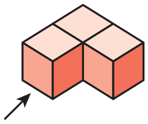

plan view

11 Sample answer: The plan and bottom views have the same shape, but different details are included in each view.

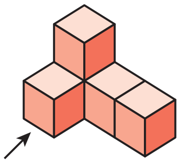
12 Sample answer: The shape of the back elevation is a mirror image of the front elevation, and the two views may also include different details.

13 Sample answer: Some people may decide to include as many views of a 3D object as possible, so as much detail as possible is shown. Other people may decide to only include as many views as necessary to be able to fully visualise the 3D object. Some views may not contain any new or different information.

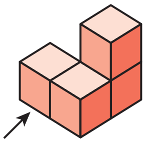
14 a



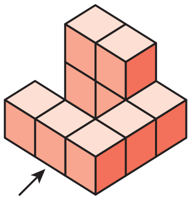
b



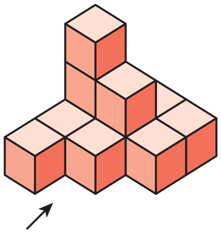
c



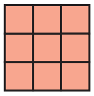
d

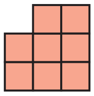


15

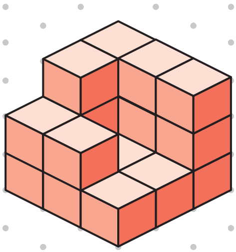


16 a


front elevation


right elevation

b



EX p355

8.3 Translations

1 **a** 4 units right and 2 units down

b 1 unit left and 3 units down

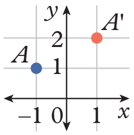
c 3 units left and 1 unit up

d 4 units right

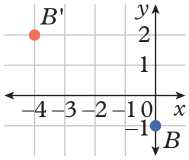
e 4 units left and 4 units down

f 2 units up

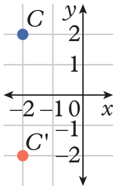
2 **a**



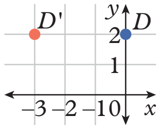
b



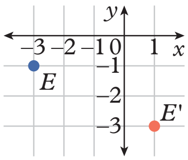
c



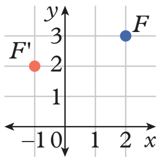
d



e



f



3 **a** (−5, −3) **b** (−3, 5) **c** (−10, 0)

d (2, −2)

4 **a** 3 units to the right and 4 units up

b 3 units to the right and 1 unit up

c 4 units to the left and 3 units down

d 4 units to the right and 2 units down

e 2 units to the right and 2 units down

f 5 units right and 6 units up

g 6 units left and 6 units up

h 3 units left and 10 units down

5 **a** 5 units right and 3 units up

b 1 unit right and 5 units down

c 3 units left and 10 units down

d 13 units right and 6 units up

e 4 units right

f 8 units left and 9 units up

6 **a** F

b E

c **i** 2 units left and 3 units up

ii 3 units left and 3 units down

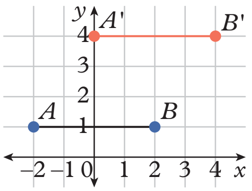
iii 6 units right and 3 units up

iv 1 unit right and 5 units down

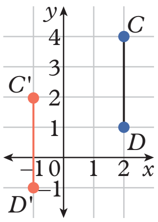
v 3 units right and 1 unit up
vi 2 units left and 1 unit up

7 **a** 2 units right and 1 unit up
b 3 units left and 1 unit up
c 1 unit left and 2 units down
d 3 units right and 1 unit down

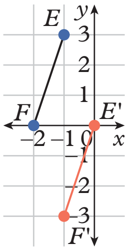
8 **a**



b



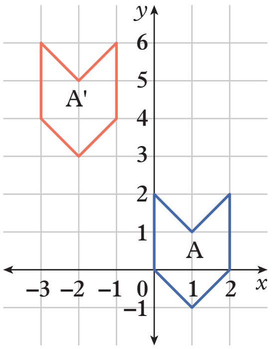
c



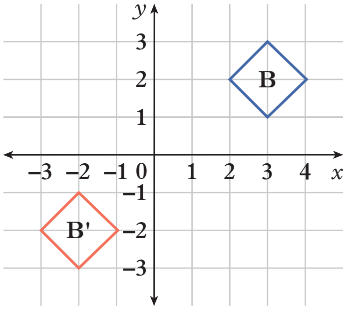
9 D

10 **a** 4 units left and 1 unit down
b 4 units down
c 2 units right and 2 units down
d 1 unit left and 3 units down
e 4 units left and 2 units up
f 3 units right and 2 units up

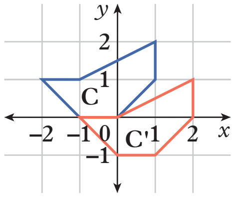
11 **a**



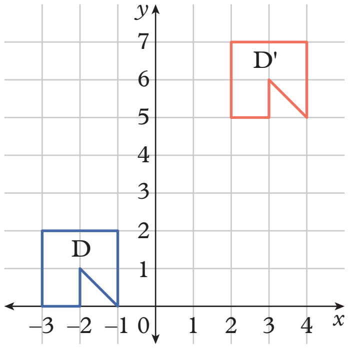
b



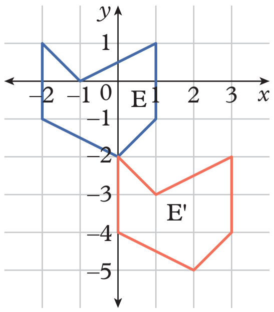
c



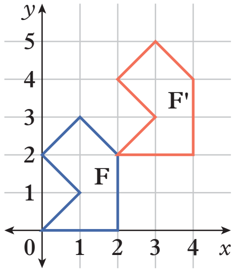
d



e



f



12 **a** Shape C
b Sample answer: Jordan has calculated the distance between the closest points on the two shapes; whereas, they should have counted the distance between matching vertices on the two shapes.
c **i** 2 units right and 5 units down
ii 8 units right and 10 units down
iii 6 units left and 3 units down
iv 8 units left and 10 units up
v 3 units left and 2 units up
vi 9 units right

13 Original figure = shape D, final image = shape A

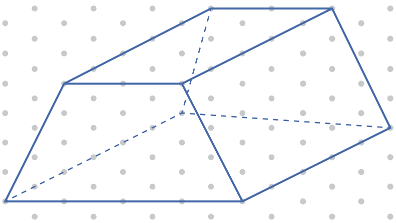
14 There are 8 possibilities: translate A and D or A and B 2 units to the right; or translate C and B or C and D 2 units to the left; or translate B and A or B and C 2 units up; or translate D and A or D and C 2 units down.

EX p360

Checkpoint

1 **a** 5 **b** 9 **c** 6

2

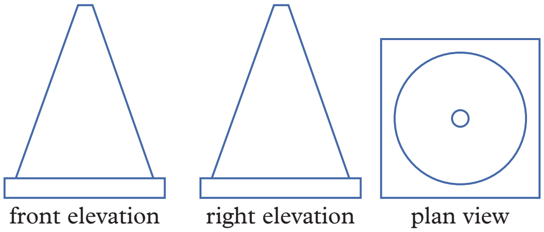


3 **a** Triangular pyramid (or tetrahedron)

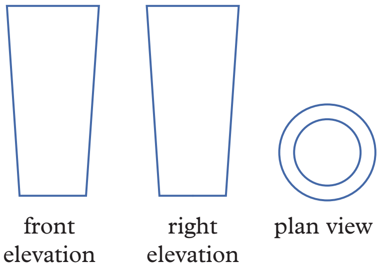
b Cylinder

c Sphere

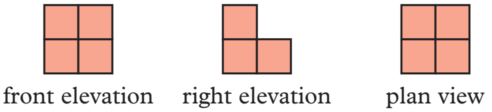
4 **a**



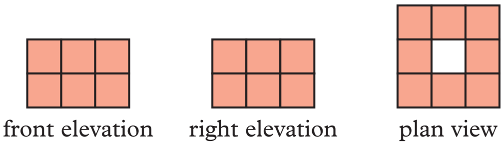
b



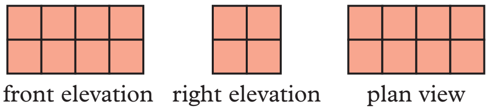
5 **a**



b

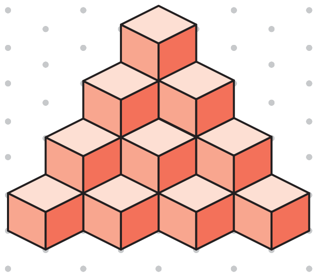


c

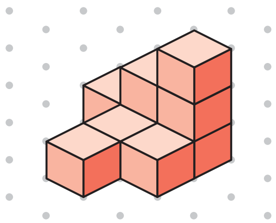


6 Sample answer:

a



b



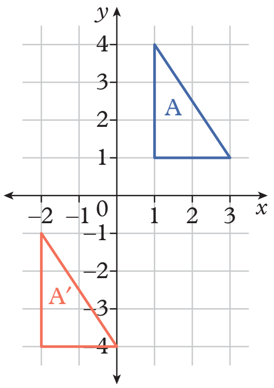
7 **a** (−11, −10)

b (5, 9)

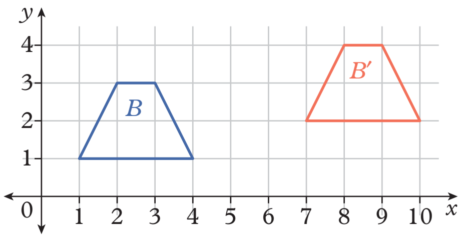
8 **a** 6 units left and 2 units down

b 3 units right and 3 units down

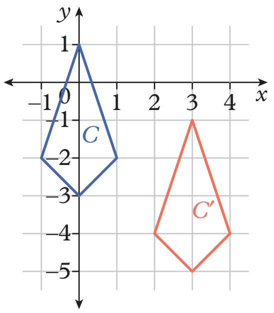
9 **a**



b



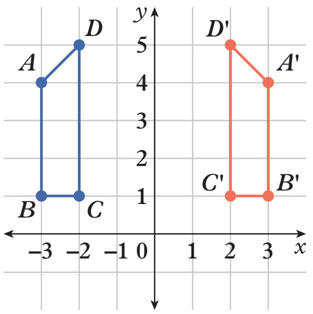
c



EX p364

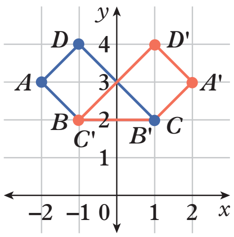
8.4 Reflections and rotations

1 **a**



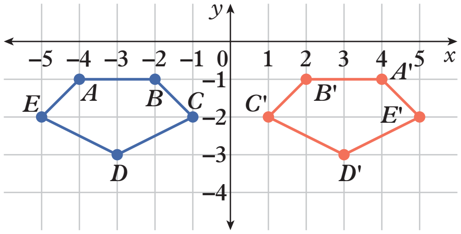
The coordinates of the image are $A'(3, 4)$, $B'(3, 1)$, $C'(2, 1)$, $D'(2, 5)$.

b



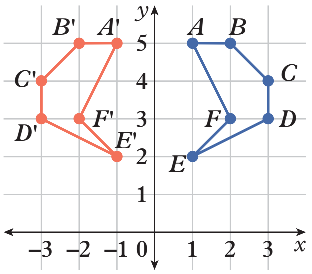
The coordinates of the image are $A'(2, 3)$, $B'(1, 2)$, $C'(-1, 2)$, $D'(1, 4)$.

c



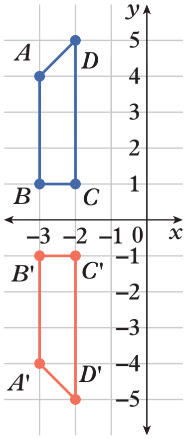
The coordinates of the image are $A'(4, -1)$, $B'(2, -1)$, $C'(1, -2)$, $D'(3, -3)$, $E'(5, -2)$.

d



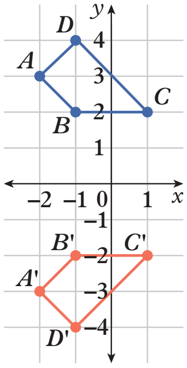
The coordinates of the image are $A'(-1, 5)$, $B'(-2, 5)$, $C'(-3, 4)$, $D'(-3, 3)$, $E'(-1, 2)$, $F'(-2, 3)$.

2 **a**



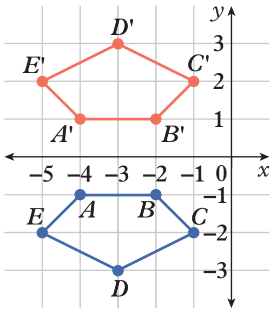
The coordinates of the image are $A'(-3, -4)$, $B'(-3, -1)$, $C'(-2, -1)$, $D'(-2, -5)$.

b



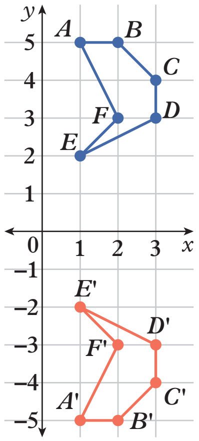
The coordinates of the image are $A'(-2, -3)$, $B'(-1, -2)$, $C'(1, -2)$, $D'(-1, -4)$.

c



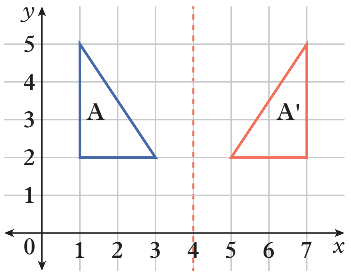
The coordinates of the image are $A'(-4, 1)$, $B'(-2, 1)$, $C'(-1, 2)$, $D'(-3, 3)$, $E'(-5, 2)$.

d

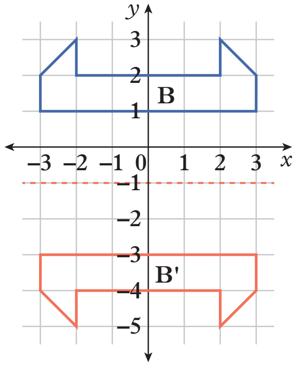


The coordinates of the image are $A'(1, -5)$, $B'(2, -5)$, $C'(3, -4)$, $D'(3, -3)$, $E'(1, -2)$, $F'(2, -3)$.

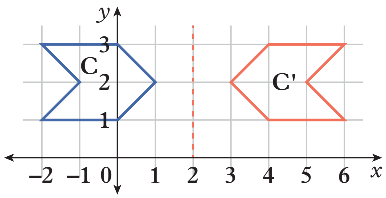
3 **a**



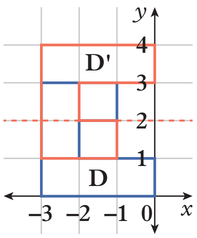
b



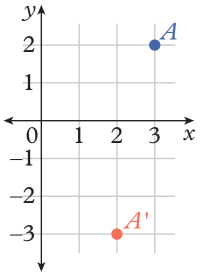
c



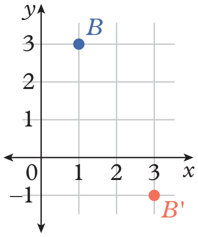
d



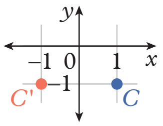
4 a



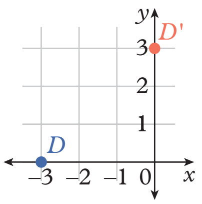
b



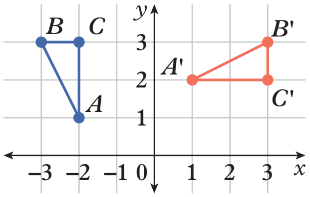
c



d

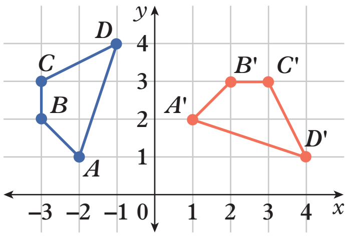


5 a



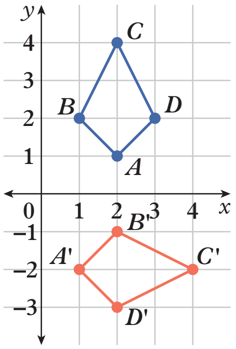
The coordinates of the image are $A'(1, 2)$, $B'(3, 3)$, $C'(3, 2)$.

b



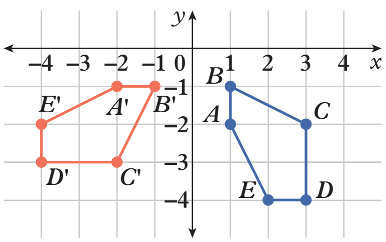
The coordinates of the image are $A'(1, 2)$, $B'(2, 3)$, $C'(3, 3)$, $D'(4, 1)$.

c



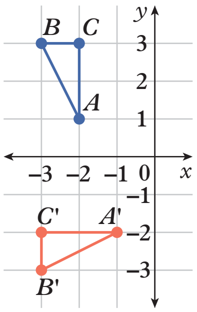
The coordinates of the image are $A'(1, -2)$, $B'(2, -1)$, $C'(4, -2)$, $D'(2, -3)$.

d



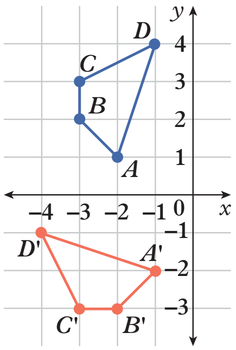
The coordinates of the image are $A'(-2, -1)$, $B'(-1, -1)$, $C'(-2, -3)$, D' $(-4, -3)$, $E'(-4, -2)$.

6 a



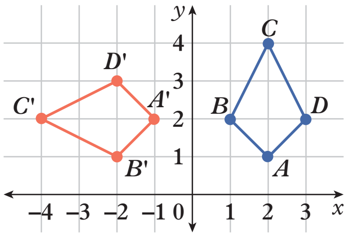
The coordinates of the image are $A'(-1, -2)$, $B'(-3, -3)$, $C'(-3, -2)$.

b



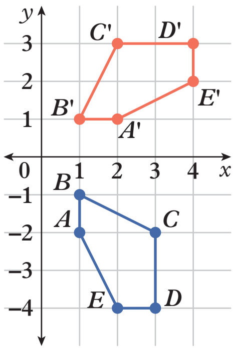
The coordinates of the image are $A'(-1, -2)$, $B'(-2, -3)$, $C'(-3, -3)$, $D'(-4, -1)$.

c



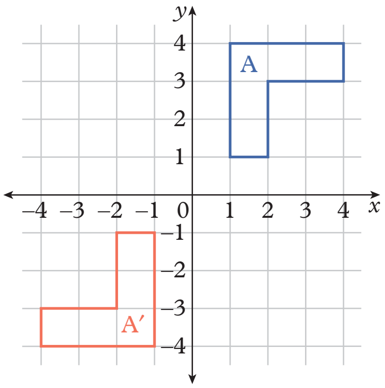
The coordinates of the image are $A'(-1, 2)$, $B'(-2, 1)$, $C'(-4, 2)$, $D'(-2, 3)$.

d

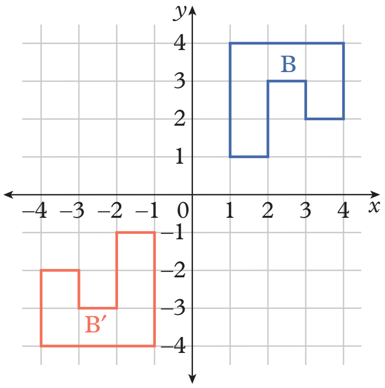


The coordinates of the image are $A'(2, 1)$, $B'(1, 1)$, $C'(2, 3)$, $D'(4, 3)$, $E'(4, 2)$.

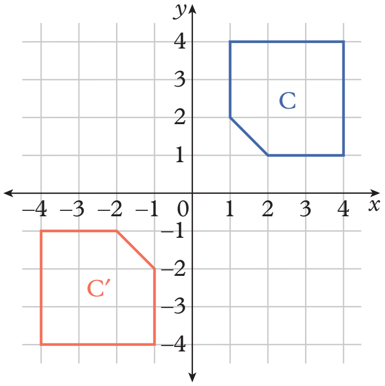
7 **a**



b



c



8 **a** 180° **b** 90° **c** 270°

9 C

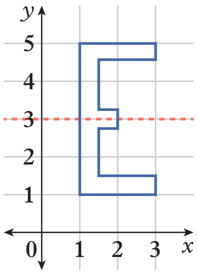
10 B: rotation of 90° clockwise (or 270° anticlockwise) about the origin

C: rotation of 180° clockwise (or 180° anticlockwise) about the origin

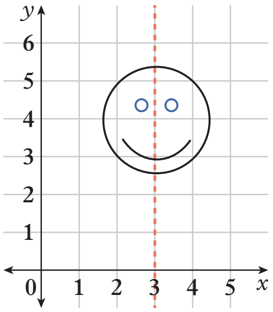
D: rotation of 90° anticlockwise (or 270° clockwise) about the origin

11 **a** (8, 5) **b** (4, 1)

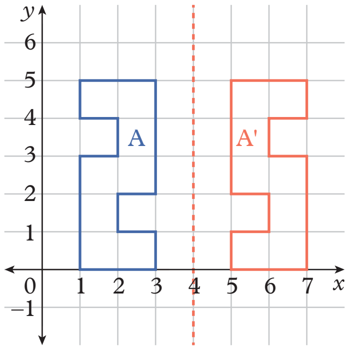
12 **a**



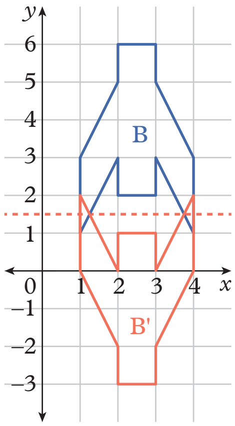
b



13 **a**

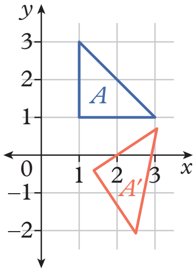


b

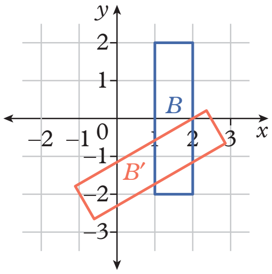


14 Sample answer: Turning 90°clockwise results in the same position as turning 270° anticlockwise because there are 360° in a full circle and 270° = 360° – 90°.

15 **a** **i**

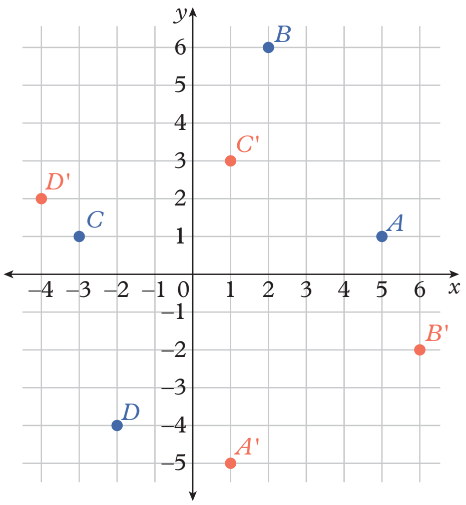


ii



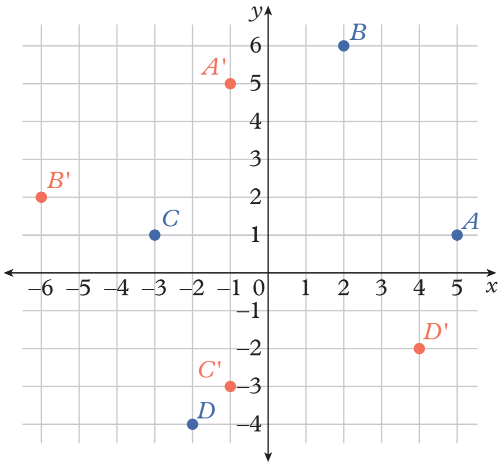
b Sample answer: When rotating by 60°, vertices which are at integer coordinates rotate to positions which are not integer coordinates, which makes it more difficult to determine the exact position of the image's vertices.

16 a



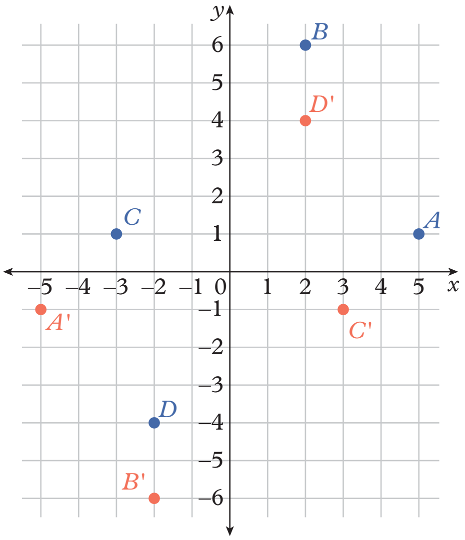
b (*y*, −*x*)

c



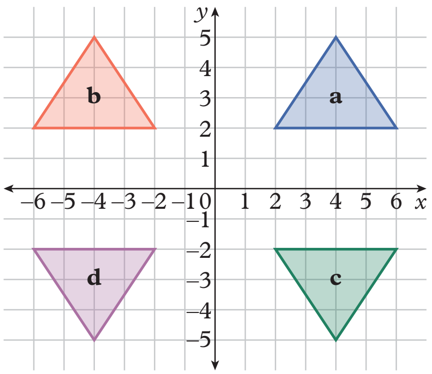
d (−*y*, *x*)

e



f (−*y*, −*x*)

17 a

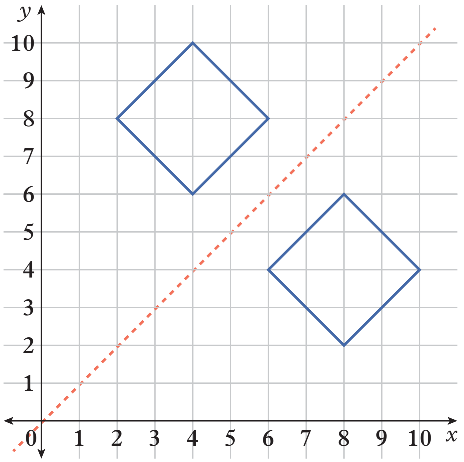


b (−2, 2), (−6, 2), (−4, 5)

c (2, −2), (6, −2), (4, −5)

d (−2, −2), (−6, −2), (−4, −5)

18 a, b



c The *x*- and *y*-coordinates have swapped places.

19 a (1, −1), (2, −1), (1, −2)

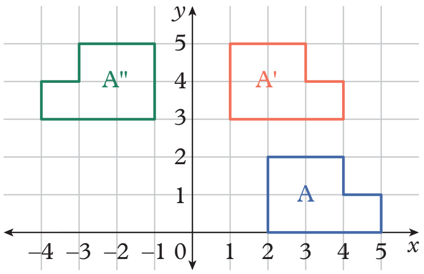
b (−1, 1), (−2, 1), (−2, 2), (−1, 2)

c (−2, 8), (−4, 10), (−4, 6), (−6, 8)

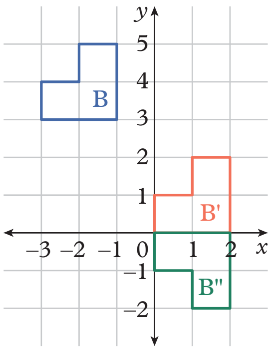
EX p369

8.5 Combined transformations

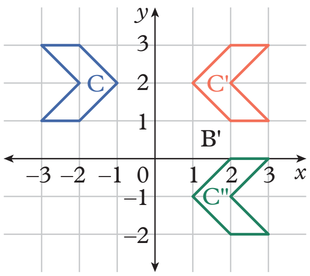
1 a



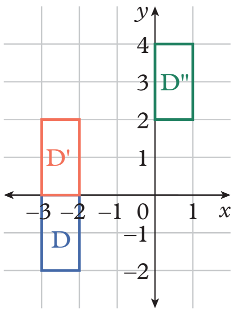
b



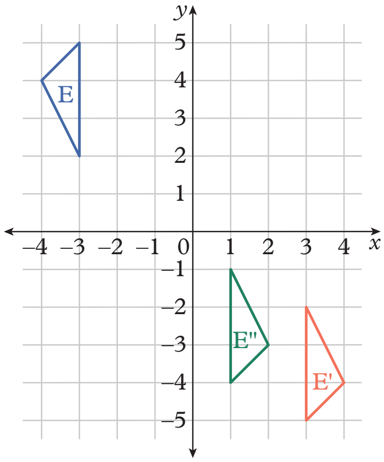
c



d



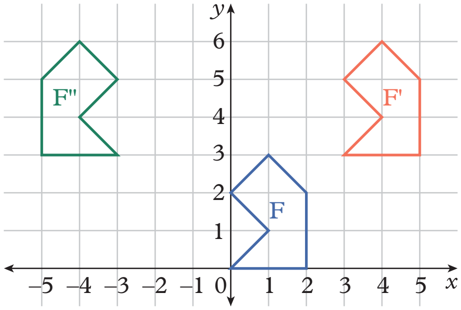
e



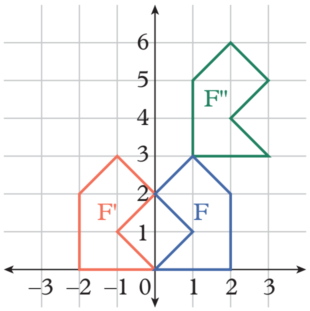
2 (0, 5)

3 A

4 **a**

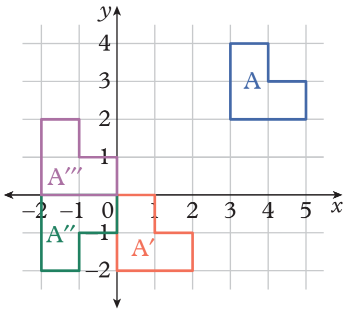


b

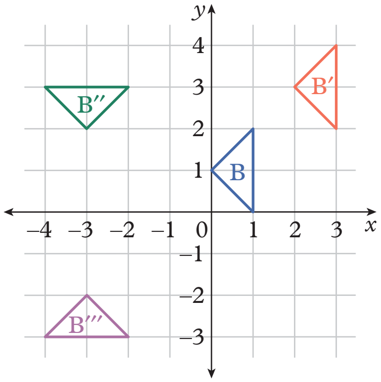


c 3 units left and 3 units up

5



6



7 **a** Reflection in the x-axis

b Rotation 90° clockwise (or 270° anticlockwise) about the origin

c Translation 2 units down and 3 units right

8 Rotation of 180° about the origin

9 Reflection in the y-axis or translation 4 units left

a Reflection in the x-axis

b Rotation 90° clockwise (or 270° anticlockwise) about the origin

10 **a** 6 units right **b** $y = 3$

11 **a** rotation 90° clockwise about the origin followed by a translation 1 unit down

b rotation 270° clockwise about the origin followed by a translation 7 units right

EX p373 Review: Objects and transformations

Review questions 8.6A: Mathematical literacy review

1 the centre of rotation, the angle of rotation and the direction of the rotation

2 A plan view shows what a 3D object looks like when viewed from directly above the object, while an elevation shows what a 3D object looks like when viewed from the front or side of the object.

3 In a right prism, the angles between the matching ends and the other faces are right angles.

4 **a** polyhedron

b isometric transformation

5 **a** a two-dimensional plan that can be folded to form a 3D object

b a point to concentrate on when reflecting or rotating a shape

6 **a** translation **b** axis of symmetry

Review questions 8.6B: Multiple choice

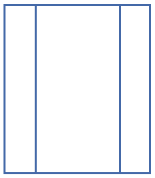
1 D **2** D **3** A **4** A
5 C **6** C **7** A **8** B

Review questions 8.6C: Short answer

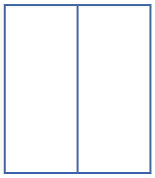
1 **a** No **b** Yes **c** Yes
  d No **e** Yes **f** No

2 **b** Triangular prism
  c Rectangular prism
  e Octagonal prism

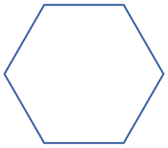
3



front elevation

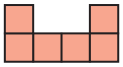


right elevation

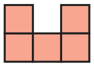


plan view

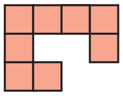
4



front elevation

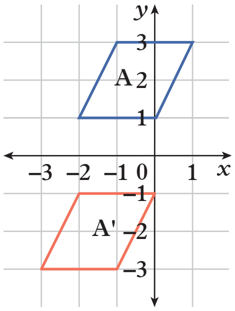


right elevation

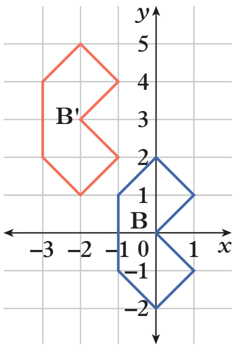


plan view

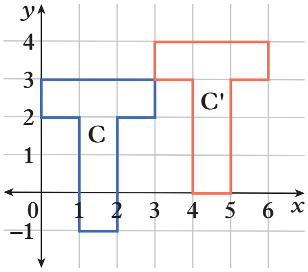
5 **a**



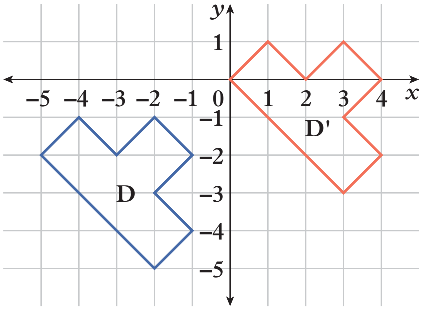
b



c

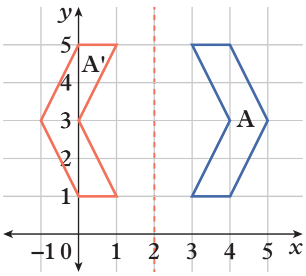


d

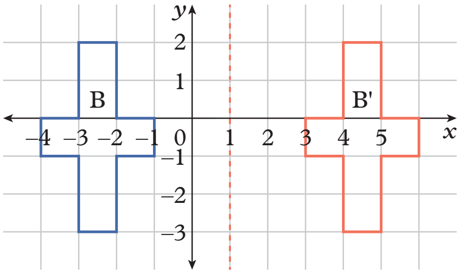


6 **a** 2 units down and 1 unit left
  b 7 units up
  c 1 unit up and 5 units right
  d 6 units up and 4 units right

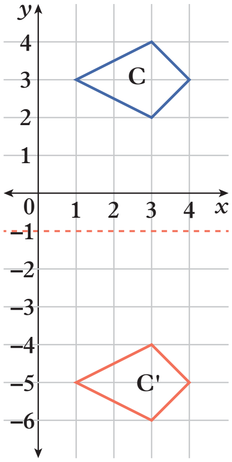
7 **a**



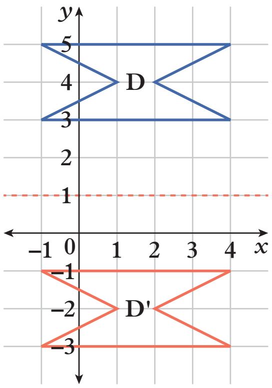
b



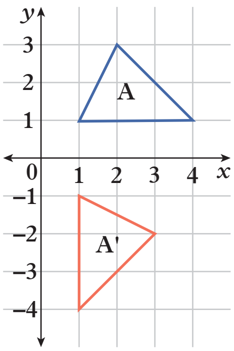
c



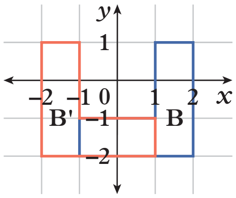
d



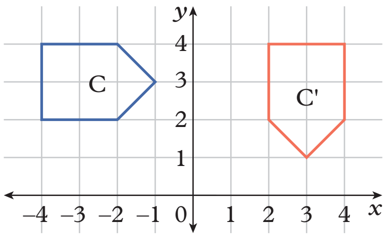
8 **a**



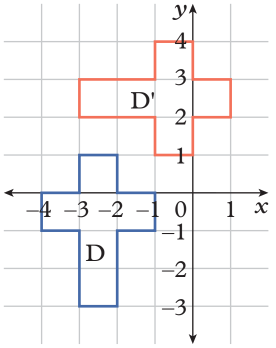
b



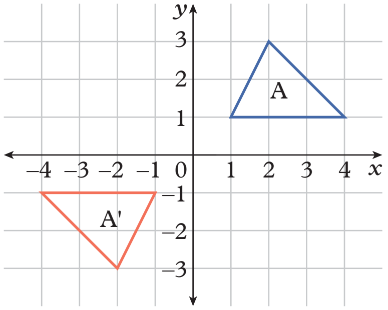
c



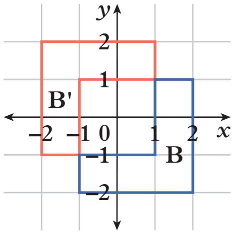
d



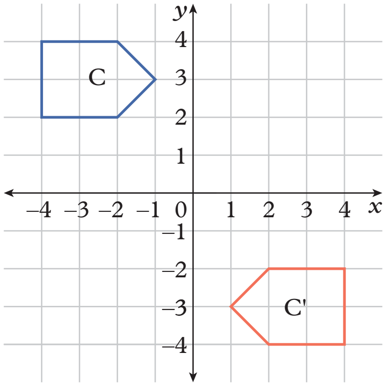
9 **a**



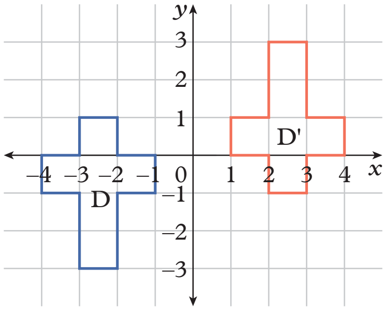
b



c



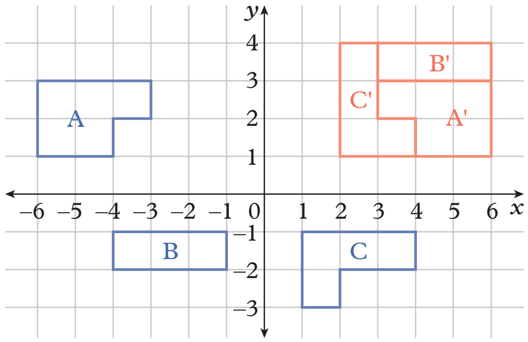
d



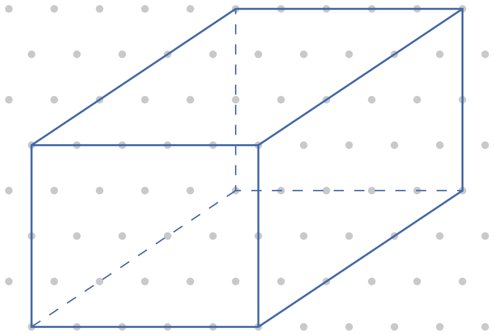
10 B: reflection in the *y*-axis
C: translation 5 units down
D: rotation 180° about the origin

Review questions 8.6D: Mathematical modelling

a

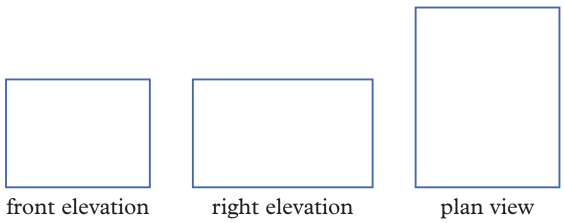


b Sample answer:



c **i** 6 **ii** 12 **iii** 8

d Sample answer:



MODULE 9 Length, area and volume

EX
p382

9.1 Length and perimeter

1 **a** 8 km = 8000 m
b 3 m = 300 cm
c 700 mm = 70 cm
d 500 cm = 5 m
e 16 cm = 160 mm
f 2000 m = 2 km
g 3.5 m = 350 cm
h 7.9 mm = 0.79 cm
i 0.06 km = 60 m
j 0.47 cm = 4.7 mm
k 4310 mm = 0.004 31 km
l 51.2 cm = 0.512 m

2 **a** 4 km = 4000 m = 400 000 cm
b 12 000 mm = 1200 cm = 12 m

c 73 m = 7300 cm = 73 000 mm
d 8400 cm = 84 m = 0.084 km
e 2500 mm = 250 cm = 2.5 m
f 36 200 cm = 362 m = 0.362 km

3 **a** 550 mm **b** 27 km **c** 4.1 cm
d 620 cm **e** 3600 m **f** 0.89 m
g 170 000 cm **h** 0.5 m **i** 0.07 km
j 3650 mm **k** 940 000 mm **l** 0.0018 km

4 **a** 23 km **b** 24 cm **c** 23 m
d 45 mm **e** 32.8 mm **f** 144 m

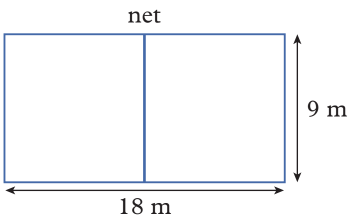
5 **a** 16 cm **b** 18 cm **c** 24 cm
d 22 cm

6 **a** 18 cm **b** 450 cm **c** 17.8 cm
d 8 cm **e** 292 cm **f** 22.4 cm

7 **a** 15 mm, 5 cm, 60 mm
b 210 mm, 190 cm, 2 m
c 7001 cm, 0.7 km, 701 m
d 8 cm, 82 mm, 0.8 m
e 30 cm, 3000 mm, 300 m, 3 km
f 0.005 km, 5500 mm, 560 cm, 5.7 m

8 **a** 42 cm **b** 34 cm **c** 30 mm
d 22.8 m

9 **a**



b 54 m **c** 180 m

10 12 m by 6 m

11 **a** 30 cm **b** $P = 2l + 2w$ or $P = 2(l + w)$

12 **a** 16 cm **b** 44 m **c** 164 mm
d 248 cm **e** 21 m **f** 10.2 cm
g 25.74 m or 2574 cm
h 54 mm or 5.4 cm

13 11 cm

14 $l = \frac{P - 2w}{2}$ or $l = \frac{P}{2} - w$

15 15.6 cm

16 **a** Divided by 10 instead of multiplying by 10 to convert from cm to mm. Correct answer is 55 cm = 550 mm.
b Multiplied by 100 instead of multiplying by 1000 to convert from km to m. Correct answer is 24 km = 24 000 m.
c Inserted a zero before multiplying by 100 to convert from m to cm. Correct answer is 0.66 m = 66 cm.
d Divided by 100 instead of dividing by 10 to convert from mm to cm. Correct answer is 1.73 mm = 0.173 cm.
e Divided by 10 instead of dividing by 100 to convert from cm to m. Correct answer is 420 cm = 4.2 m.
f Divided by 10 000 instead of dividing by 1000 to convert from m to km. Correct answer is 800 m = 0.8 km.

17 **a** 27.4 m **b** 82.2 m **c** 109.6 m **d** Part **c**

18 85.5 mm

19 **a** Sample answer: The sum of all the horizontal lengths of the staircase is equal to the length at the bottom and the sum of all of the vertical lengths of the staircase is equal to the height on the left.
b 54 cm

20 Sample answer: The shapes have the same perimeter. AE = BE, AC = BD and CF = DF. The length from E to F is the same for both shapes.

21 26.88 m

22 40.2 m

23 $x = 3, y = 3$

24 $x = 2$

EX p389

9.2 Circumference of a circle

1 **a** 10 cm **b** 4 mm **c** 16 cm

2 **a** 4 cm **b** 6 mm **c** 3.5 m

3 **a** 31.42 cm **b** 6.28 mm **c** 94.25 cm
d 131.95 m **e** 5.65 m **f** 28.90 m
g 131.95 mm **h** 47.12 mm **i** 56.55 mm

4 **a** 7.54 m **b** 7.85 cm **c** 20.11 mm
d 2.51 cm **e** 119.38 cm **f** 81.68 mm
g 2.36 m **h** 84.82 cm **i** 515.22 km

5 **a** **i** 10.00 cm **ii** 5.00 cm
b **i** 20.00 cm **ii** 10.00 cm
c **i** 15.92 m **ii** 7.96 m
d **i** 7.96 m **ii** 3.98 m
e **i** 0.32 km **ii** 0.16 km
f **i** 10.19 m **ii** 5.09 m
g **i** 1.00 km **ii** 0.50 km
h **i** 1.78 m **ii** 0.89 m
i **i** 1.94 cm **ii** 0.97 cm

6 **a** 81.68 cm **b** 113.10 cm **c** 163.36 mm
d 95.82 cm

7 **a** 60.00 mm **b** 7.00 cm **c** 3.34 cm
d 10.15 cm

8 **a** 9.5 m **b** 159 mm

9 **a** 50% **b** 9.42 cm **c** 4.71 cm
d 7.71 cm

10 **a** 30.85 cm **b** 5.14 m **c** 26.28 m
d 81.42 mm **e** 94.25 cm **f** 109.98 cm

11 **a** Jack: 172.79 cm, Lily: 100.53 cm
b Jack: 579 times, Lily: 995 times

12 **a** The given distances relate to radius, as the distance from centre to edge is radius.
b Matt: 25.76 m, Lauren: 33.93 m
c 49.01 m
d Remember that the longer the radius is, the greater the circumference.

13 40 074.16 km

14 **a** 60.16 mm

b Knuckle circumference is between 61.45 mm and 62.71 mm.

c **i** 19.76 mm **ii** 62.08 mm

15 33.42 mm and 47.27 mm

16 **a** 7.5π cm **b** 12.8π cm **c** 3.4π m

17 **a** **i** 11 cm **ii** 5.5 cm

b **i** 4.53 mm **ii** 2.265 mm

c **i** $\frac{2}{3}$ km **ii** $\frac{1}{3}$ km

18 **a** 108 m **b** 113 m **c** 339.29 m

d 84 carriages **e** 3.04 m **f** 9.66 m

EX p396

9.3 Area of a rectangle

1 **a** 10 cm^2 **b** 5 cm^2 **c** 6 cm^2

d 8 cm^2 **e** 5 cm^2 **f** 7 cm^2

2 **a** **i** 10 cm **ii** 10 cm **iii** 14 cm

iv 14 cm

b **i** 6 cm^2 **ii** 4 cm^2 **iii** 8 cm^2

iv 6 cm^2

c Sample answer: Area is the space inside a shape; perimeter is the total distance around the edge of a shape.

3 **a** 40 cm^2 **b** 6 m^2 **c** 9.03 m^2

d 112 cm^2 **e** 532 mm^2 **f** 400 mm^2

4 **a** 63 cm^2 **b** 52 mm^2 **c** 6 m^2

d 52 cm^2 **e** 137.74 mm^2 **f** $91\frac{7}{8}$ m^2

5 **a** 10 cm^2 or 1000 mm^2

b 27 m^2 or 270 000 cm^2

c 0.084 km^2 or 84 000 m^2

d 5.76 cm^2 or 576 mm^2

e 0.035 m^2 or 35 000 mm^2

f 25 cm^2 or 2500 mm^2

6 **a** 3 m^2 or 30 000 cm^2

b 40 cm^2 or 4000 mm^2

c 1.12 km^2 or 1 120 000 m^2

d 180 000 mm^2 or 1800 cm^2

e 6.2 cm^2 or 620 mm^2

f 42 cm^2 or 0.0042 m^2

7 **a** 16 m^2 **b** 25 cm^2

c 289 mm^2 **d** 12.96 cm^2

8 $A = l \times l$ or $A = l^2$

9 **a** 49 m^2 **b** 225 cm^2 **c** 121 km^2

d 3600 mm^2 **e** 40.96 cm^2 **f** 32 400 m^2

g 0.49 mm^2 **h** 0.2809 km^2

10 **a** 30 cm^2 **b** 6 cm^2 **c** 36 cm^2

d 20 cm^2 + 16 cm^2 = 36 cm^2

11 **a** 87 mm^2 **b** 48 m^2 **c** 110 cm^2

d 40 mm^2

12 **a** Find the area of the larger rectangle and subtract the area of the smaller rectangle.

b 51 m^2

13 **a** 115 m^2 **b** 92 mm^2 **c** 65 cm^2

d 19 m^2

14 **a** 50 mm **b** 60 mm or 6 cm

15 **a** **i** 43 m^2 **ii** 32 m

b **i** 17.19 cm^2 **ii** 20.2 cm

c **i** 80.81 m^2 **ii** 41.6 m

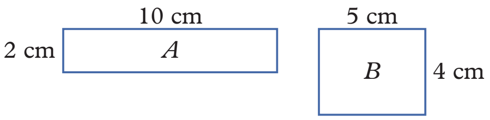
d **i** 292 mm^2 **ii** 86 mm

16 126 cm^2

17 400 m^2

18 16 mm

19 Sample answer: Area and perimeter are two different measures. Long, thin rectangles will have a larger perimeter than wider rectangles with the same area. For example, both of the rectangles below have an area of 20 cm^2, but rectangle A has a perimeter of 24 cm and rectangle B has a perimeter of 18 cm.



20 **a** **i** 11.885 m **ii** 260.7569 m^2

iii 195.6271 m^2 **iv** 26.336 m^2

b 373.9769 m^2

21 Sample answer: Yes; one possible rectangle has length 6 cm and width 3 cm. The perimeter is 18 cm and the area is 18 cm^2.

22 **a** $5x + 6y + 2xy$

b $x(2y + 5) + 6y$ and $2y(x + 3) + 5x$

c $(2y + 5)(x + 3) - 15$

EX p400

Checkpoint

1 **a** 22 cm **b** 12 cm **c** 44 cm

2 **a** 1.5 cm **b** 25 912 000 m

c 0.23 m **d** 300 mm

3 **a** 16 mm

b 18.1 cm = 181 mm

c 35.4 m

4 **a** 97.39 m **b** 43.98 cm

5 **a** 13.40 m **b** 2.07 cm

c 6.37 m **d** 15.92 mm

6 **a** 18 cm^2 **b** 9 cm^2 **c** 29 cm^2

7 **a** 10 cm^2 **b** 20.5 cm^2

8 **a** 300 mm^2 **b** 49 cm^2

c 8400 cm^2 **d** $6\frac{1}{15}$ m^2

EX p402

9.4 Area of a parallelogram

1 **a** 10 cm^2 **b** 5 cm^2 **c** 15 cm^2

d 8 cm^2

2 **a** Base length = 22 cm, height = 12 cm

b Base length = 9 cm, height = 2 cm

c Base length = 16 mm, height = 17 mm
d Base length = 4.2 m, height = 1.5 m
3 a 24 cm^2 **b** 56 mm^2 **c** 360 m^2
d 160 cm^2 **e** 36 mm^2 **f** 3.6 m^2
4 a 15 cm^2 **b** 170 mm^2 **c** 6 m^2
d 10.25 cm^2
5 a 15 cm^2 **b** 116 m^2
c 2400 mm^2 **d** 75.076 cm^2
6 a 4000 cm^2 **b** 3.5 cm^2 **c** 138 000 cm^2
d 34.56 cm^2 **e** 63 cm^2 **f** 4.64 cm^2
7 a 120 cm^2 **b** 42 cm^2 **c** 42 m^2
d 276 mm^2
8 a 68 m^2 **b** 141 cm^2 **c** 309 mm^2
d 141.9 cm^2
9 364.5 cm^2
10 a 3.3 m^2 **b** 3.85 m^2
11 a 24 cm^2 **b** 18 cm^2 **c** 60 cm^2
d 50 cm
12 5 cm
13 8 cm
14 a 390 cm^2 **b** 390 cm^2
c Sample answer: A rectangle is a special parallelogram where the height of the parallelogram equals the width of the rectangle and base of parallelogram is equal to length of rectangle. In both cases, the two dimensions (length and width in a rectangle, and base and height in a parallelogram) are at right angles to each other.
15 56.6284 cm^2
16 80 cm^2

EX p408

9.5 Area of a triangle

1 a 4 cm^2 **b** 6 cm^2 **c** 8 cm^2
d 4 cm^2
2 a 54 cm^2 **b** 48 mm^2 **c** 7.5 m^2
d 30.1 cm^2 **e** 68.87 mm^2 **f** 131.131 m^2
3 a Base length = 8 m, height = 5 m
b Base length = 10 cm, height = 6 cm
c Base length = 16 cm, height = 15 cm
d Base length = 8 mm, height = 6 mm
e Base length = 12.6 mm, height = 3.1 mm
f Base length = 2.4 m, height = 3.5 m
4 a 12 mm^2 **b** 36 cm^2 **c** 168 mm^2
d 117.6 m^2 **e** 30 mm^2 **f** 189 cm^2
5 a i 30 cm^2 **ii** 30 cm^2 **iii** 30 cm^2
b Sample answer: all these triangles have the same base length and height, so their areas are also the same.
6 a 12 cm^2 or 1200 mm^2
b 240 mm^2 or 2.4 cm^2
c 1 m^2 or 10 000 cm^2
d 155 000 cm^2 or 15.5 m^2
e 70 mm^2 or 0.7 cm^2
f 1.358 m^2 or 13 580 cm^2
7 a 24 mm^2 or 0.24 cm^2 **b** 13 650 cm^2 or 1.365 m^2
c 8.1 cm^2 or 810 mm^2 **d** 23.4 cm^2 or 2340 mm^2
e 3.57 m^2 or 35 700 cm^2 **f** 6 cm^2 or 600 mm^2
8 a The student used 15 cm as the height of the triangle instead of 9 cm. The correct area of the triangle is 54 cm^2.
b 36 cm
9 45 m^2
10 4 cm
11 40 mm
12 2880 cm^2
13 a 84 cm^2 **b** 68 m^2 **c** 16 mm^2
d 1750 m^2 **e** 222 mm^2 **f** 248 cm^2
14 a 162 cm^2 **b** 1152 cm^2
15 162 mm^2
16 31.5 m^2

EX p414

9.6 Volume of right prisms

1 a 30 cm^3 **b** 56 mm^3 **c** 72 mm^3
d 45 m^3
2 a 280 cm^3 **b** 90 m^3 **c** 320 mm^3
d 94.5 m^3 **e** 2.88 cm^3 **f** 1015.014 mm^3
3 a 8 m^3 **b** 343 mm^3 **c** 1000 cm^3
d 15.625 m^3
4 a 346.5 cm^3 **b** 60 cm^3 **c** 375 cm^3
d 2079 cm^3 **e** 4.0375 m^3 **f** 555.75 cm^3
5 a 67.5 cm^3 **b** 189 mm^3 **c** 154 m^3
d 33 mm^3 **e** 480 cm^3 **f** 1449 mm^3
6 a 60.5375 cm^3 **b** 2673 cm^3
7 0.125 m or 12.5 cm
8 23 040 m^3
9 a 1 cm^3 **b** 840
10 a V = 8000 cm^3
b V = 8000 cm^3
11 a 17 mm **b** 1.3 m
c 14 mm **d** 80 cm or 0.8 m^3
12 a 48 cm^3 **b** 190 m^3
c 74.25 cm^3
13 a 126 m^3 **b** $3780
14 a 20 cm^2 **b** 4 cm
15 a 235 mL **b** 8300 cm^3
16 3 L
17 47.1 g
18 Four sets: 20 cm by 1 cm by 1 cm, 10 cm by 2 cm by 1 cm, 5 cm by 4 cm by 1 cm, 5 cm by 2 cm by 2 cm
19 a i $24 **ii** $37.58 **iii** $28.50
b Sculpture **i** weighs 36 kg; sculpture **ii** weighs 37.584 kg and sculpture **iii** weighs 35.625 kg. She should buy sculpture **iii**.

EX p419

Review: Length, area and volume

Review questions 9.7A: Mathematical literacy review

1 multiply the length of the base by the height

2 divide the circumference by 2π

3 divide the quantity in metres by 1000

4 **a** perimeter **b** volume

5 **a** The circumference of a circle is the perimeter of a circle.

b A conversion factor is a multiplier for converting a quantity expressed in one set of units into another quantity expressed in a different set of units.

6 **a** radius **b** height, base

Review questions 9.7B: Multiple choice

1 A **2** C **3** D **4** D **5** A

6 B **7** A **8** B **9** D **10** C

Review questions 9.7C: Short answer

1 **a** 340 mm **b** 1.7 m **c** 4.5 cm

d 8700 m

2 **a** 60 m **b** 220 cm or 2.2 m

3 **a** 31.42 cm **b** 50.27 mm

4 **a** 125 m^2 **b** 2400 cm^2 or 0.24 m^2

5 56 m^2

6 **a** 15 cm^2 **b** 16 cm^2 or 1600 mm^2

7 8610 mm^2

8 **a** 84 cm^2 **b** 18 cm^2 or 1800 mm^2

9 102.795 km^2

10 **a** 640 cm^3 **b** 420 cm^3 **c** 144 cm^3

11 **a** 6656.25 cm^3

Review questions 9.7D: Mathematical modelling

1 **a** 87 cm^2 **b** 107.5 cm^2 **c** 59.5 cm^2

d 178 cm^2 **e** 750 cm^3

f 0.15 L of yellow and 0.3 L of blue

g 100 cm

2 **a** $\frac{1052}{315}$ or $3\frac{107}{315}$

b 3.34

c The calculation alternates between adding and subtracting each term. The denominators increase in odd numbers and the numerator is always 4.

d $-\frac{4}{11}+\frac{4}{13}$

e 3.28

f **i** 7.64

ii 26.24

iii The circumference divided by 3.28 is approximately equal to the diameter. The diameter multiplied by 3.28 is approximately equal to the circumference.

iv The relationships will produce more accurate approximations when more terms are added to the original calculation.

EX p424

Computational thinking

1 **a** **i** 3 (angles x, y, z)

ii 2 (Can/Cannot form a triangle)

iii 1 (sum = 180?)

b Cannot form a triangle

c Can form a triangle

d Whether the three angles sum to 180°, which is the interior angle sum of a triangle

2 **a** Irregular quadrilateral, Kite, Trapezium, Rhombus, Parallelogram, Rectangle, Square

b All sides equal?

c **i** Square. Yes, Yes, Yes, Yes

ii Kite. No, Yes

iii This quadrilateral is concave so it can't be input for this flowchart.

3 **a** To test for an obtuse triangle, use 'Angle > 90?'.
To test for a right-angled triangle, use 'Angle = 90?'.

b **i** **1** Start
2 Input angles: 30, 60, 90
3 Choose the first angle: 30
4 30 > 90: No
5 30 = 90: No
6 Not all angles are checked, choose the next angle: 90
7 30 > 90: No
8 30 = 90: Yes
9 Output: Right-angled triangle
10 Stop

ii **1** Start
2 Input angles: 30, 20, 130
3 Choose the first angle: 30
4 30 > 90: No
5 30 = 90: No
6 Not all angles are checked, choose the next angle: 20
7 20 > 90: No
8 20 = 90: No
9 Not all angles are checked, choose the next angle: 130
10 130 > 90: Yes
11 Output: Obtuse triangle
12 Stop

iii **1** Start
2 Input angles: 70, 50, 60
3 Choose the first angle: 70
4 70 > 90: No
5 70 = 90: No
6 Not all angles are checked, choose the next angle: 50
7 50 > 90: No
8 50 = 90: No

9 Not all angles are checked, choose the next angle: 60
10 60 > 90: No
11 60 = 90: No
12 All angled are checked.
13 Output: Acute triangle
14 Stop

c Three times. There are only three input angles, so you can check at most three angles.

MODULE 10 Statistics and probability

EX p432

10.1 Summary statistics

1 **a** Continuous **b** Continuous **c** Discrete
d Continuous **e** Discrete **f** Discrete

2 **a** 8 **b** 7 **c** 7
d 9 **e** 7.5

3 **a** 1.04 **b** 1 **c** 1 **d** 4

4 **a** **i** 4.92 **ii** 5
iii 7 **iv** 7
b **i** 4.71 **ii** 5
iii 1, 6 **iv** 8
c **i** 5.5 **ii** 5.5
iii No mode **iv** 9
d **i** 4.87 **ii** 5
iii 3, 4, 5, 7, 9 **iv** 9
e **i** 7.33 **ii** 7
iii 4 **iv** 12
f **i** 8.29 **ii** 7
iii No mode **iv** 15
g **i** 12.25 **ii** 12.5
iii 13 **iv** 13
h **i** 13.82 **ii** 6
iii 5, 8 **iv** 92

5 **a** The 4th value
b Between the 4th and 5th value
c The 7th value
d Between the 11th and 12th value

6 **a** III **b** IV **c** I **d** II

7 **a** 90 g
b **i** Increases **ii** Stays the same
iii Decreases **iv** Increases

8 **a** Range **b** Mode
c Median **d** Mean

9 Sample answer:
Mean: If all carts travelled the same distance, but the total distance travelled remained the same, then each cart would have travelled 6.4 m.
Median: Half the carts travelled more than 6.2 m.
Mode: The most common distance travelled was 4.0 m.
Range: The distances travelled by the carts were spread out over a 10.1 m interval.

10 **a** **i** Set 1: 13.4, Set 2: 13.4
ii Set 1: 12.5, Set 2: 12.5
iii Set 1: 12, Set 2: 12
b Two different numerical data sets can have the same mean, median and mode.

11 **a** Mean = $23.85, median = $20, mode = $20, range = $40
b Sample answer: More students spent $20 on their mobile phone than any other amount. The difference between the least and most amount spent on mobile phones each month was $40. The average spend was $23.85.

12 **a** Mean = $13.46, median = $5, mode = $5
b $8.46
c Sample answer: Only two students earn more than the mean. The student who gets $100 has had a big effect on the results. Most students earn much less than the mean.
d Sample answer: Most would be able to, but most would have no money left over for anything else; all except two students get $5 a week or more. $5 a week is about $20 a month, which is what most students spend on their mobile phone per month.

13 **a** Mean = $41 875, median = $41 000
b Sample answer: Both the mean and median are significantly less than $80 000, so this claim is hard to justify. However, this is only a sample of the employees, and this may be a sample that is not representative of the population.
c **i** The mean, as the two employees skewed the average much higher than it should have been.
ii The median, as 50% of employees will earn more than it and 50% of employees will earn less than it.

14 **a** **i** Mean = 50.8%, median = 42.5%
ii Mean = 87.7%, median = 96.5%
b **i** Mean **ii** Median
c **i** Median **ii** Median

15 **a** Dylan, 19 minutes
b Dylan: 5.07 minutes, Costa: 7.29 minutes
c One possible answer is that Costa's train is worse because on average it runs later than Dylan's, even though Dylan's train has had two very late departures.

16 Sample answer:
mean = 466, median = 472, mode = 456.
All three summary statistics can be described as the average.

17 10

18 **a** Player 2
b Player 1: mean = 3.4, median = 3, mode = 3
Player 2: mean = 3.47, median = 3, mode = 1
c Player 1, whose mean, median and mode are all similar values and who has a much smaller range (5 vs. 12).

d 13 is the outlier. This value is significantly different from all the other values in this data set.

e Mean = 2.79, median = 3

f The outlier made a significant difference to the mean, increasing the value from 2.79 to 3.47, but did not affect the median.

19 a The mean, because of the number of decimal places. The mode will be a whole number, and the median will be either a whole number or a whole number and a half.

b The mean is a numerical average that does not have to match any of the values in a data set. The mean is the value given if all the values in a data set are evenly distributed.

20 a 1, 2, 4, 5, 8 1, 2, 4, 6, 7 1, 3, 4, 5, 7
2, 3, 4, 5, 6

b The middle value must be 4, and the sum of the values must be 20; therefore, the remaining four values must sum to 16. There are only four potential data sets that meet these criteria.

21 a i 8.33 **ii** 7.75
iii 6.8 **iv** 8.7

b i 30.76 **ii** 28.75
iii No mode **iv** 34.7

22 a i 6.5 **ii** 7
iii 8 **iv** 4

b i 13.11 **ii** 13
iii 13 and 15 **iv** 5

EX p439 10.2 Describing data with dot plots

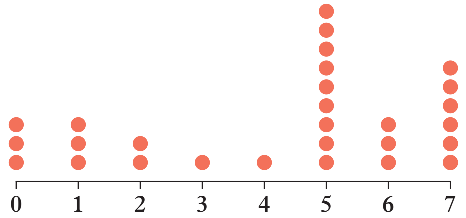
1 a

Number of siblings	Tally	Frequency
0	||	2
1	𝍸 𝍸 |	11
2	𝍸 ||||	9
3	||	2
4	|	1
5		0
6		0
7		0
8	|	1

b 26 **c** 1

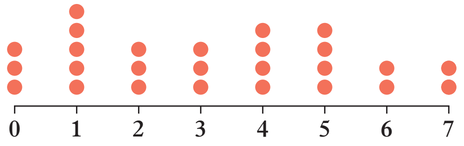
2

Number of days per week that students catch public transport



3

Number of days per week that students do more than 30 minutes of exercise



4 a 25 **b** 9 **c** 5
d 5

5 a 3 **b** 6
c Mean = 3.725, median = 4

6 a 26 **b** 5 **c** 1
d 1.5 **e** 1.88

f Sample answer: The number of siblings for this group of children is spread out over 5 siblings (from 0 to 5). The average number of siblings for this group of children is 1–2 (mode: 1, median: 1.5, mean 1.88).

7 a Negatively skewed

b Bi-modal

c Approximately symmetric

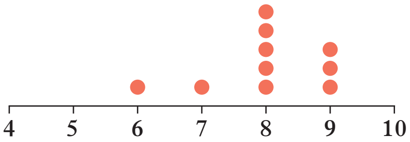
d Negatively skewed with an outlier

e Positively skewed

f Symmetric with an outlier

8 a

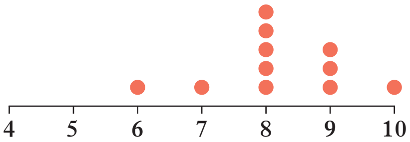
Weekly score



b 8

c

Weekly score



d 0.18

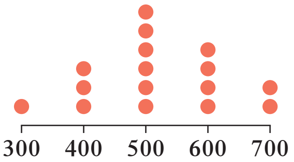
9 a 9

b Sample answer: Delete the dot for the score 9, remove the scores above 6 from the horizontal axis, and add 1 dot to the of the 3 column.

10 The range of the data is large (36) so there would need to be a long horizontal scale which would be impractical, with a lot of blank space between the 15 and 45 values.

11 a

Crowds for a local football team



b Approximately symmetric

12 **a** Approximately symmetric with an outlier

b **i** 12.5 **ii** 13 **iii** 13 **iv** 11

13 A = mode, B = median, C = mean

Sample answer: The mode is the highest column (A), the median must be either a whole number or whole number and a half (B) and the mean is the other value (C).

14 **a** 17 (unique) **b** 9 (unique)

c Any number greater than or equal to 13 (not unique)

d 10 (unique) **e** 9 or 17 (not unique)

f Any multiple of 9 (not unique)

15 **a** Symmetric

b Mean = 10.5, median = 10.5

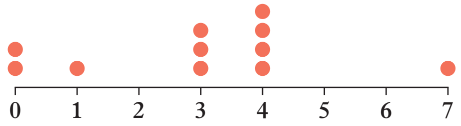
c Negatively skewed

d Mean = 14.6, median = 15

e **i** the same **ii** less than

16 Sample answer: The range and the mode. The range will not change as the highest and lowest values still have dots. The mode will not change as the tallest column will stay the tallest if the same amount is removed from each column. Since the total number of dots will change, the median and mean are likely to change (although they may not).

17 Sample answer:



Note: Other possible answers exist.

EX p445

10.3 Stem-and-leaf plots

1 **a** Sample answer: 1 | 4 = 14

b 14 **c** 98 **d** 84

e 37 **f** 31

2 **a** 42 **b** 19 **c** 20

d 20.79

3 **a**

Stem	Leaf
5	1 2 8 9
6	2 4 6 7 8
7	1 1 5 6 8
8	1 2 2 3 3 3 4 9
9	
10	8

Key: 5 | 1 = 51

b 51 **c** 108 **d** 83

e

Stem	Leaf
5	1 2 8 9
6	2 4 6 7 8
7	1 1 5 6 8
8	1 2 2 3 3 3 4 9
9	
10	8
11	2

Key: 5 | 1 = 51

4 **a**

Stem	Leaf
1	1 4 8 9
2	3 3 6
3	1 8
4	2 2 3

Key: 1 | 1 = 11

b

Stem	Leaf
0	4
1	3 8 9
2	2 8
3	1 6
4	2 6
5	4 9

Key: 1 | 3 = 13

c

Stem	Leaf
2	7
3	3 4 5 7 9
4	2 9
5	1 3 4
6	5

Key: 2 | 7 = 27

d

Stem	Leaf
2	3 9 9
3	1 7
4	1 1 1
5	3 9 9
6	7

Key: 2 | 3 = 23

5 **a** 15 and 65

b

Stem	Leaf
1	5 8 9 9 9
2	2 3 4 5 6 8 9 9
3	1 2 3 4 9
4	5
5	4
6	5

Key: 1 | 5 = 15

c 19 **d** 20–29

6 **a** 56 **b** 41 **c** 31

d 31.88

7 **a**

Stem	Leaf
1	0 2 3 4 4 4 5 5 6 6 7 8 8 8 9
2	0 1 1 2 3 4 5 5 7 9 9
3	0 1 3 4 6
4	1 1 5 6 6
5	3 4
6	
7	6

Key: 1 | 5 = 15

b 10 **c** 76 **d** 10–19

8 **a** Positively skewed

b Bi-modal

c Negatively skewed

9 **a** 23 **b** 38 **c** 38

d 39.18

10 **a** 10.1 g **b** 16.1 g

c Sample answer: Will did not read the key properly. It is 13.8 g.

d The outlier weighs 16.1 g. 14.1 g is the heaviest glider excluding the outlier.

e

Stem	Leaf
10	1 4 8
11	0 2 3 6 6
12	1 2 4 5 5 6 9 9
13	1 1 2 4 4 4 6 6 8 8 8 8
14	0 1 3
15	
16	1

Key: 10 | 1 = 10.1 g

11 **a** Each stem represents hundreds and each leaf represents tens instead of tens and ones (units).

b

Stem	Leaf
0	6 8
1	0 0 1 2 2 3 3 3 4 5 5 6 7 8
2	0 2 2 2 4 5 6
3	6
4	3
5	7

Key: 1 | 2 = 120

c 60 **d** 570 **e** 192

12 **a** The leaves have two digits instead of one.

b 7:16 am **c** 11:59 pm **d** 5

e 6:54 pm and 7:04 pm (10 minutes)

f 8:04 pm and 8:35 pm (31 minutes)

13 The plot is missing a key, the stem should not skip numbers and the leaves should be in ascending order.

14 It would be best to use a dot plot when the range of the values is small and there are repeated values.

It would be best to use a split stem-and-leaf plot when the data values only have a small number of stems.

It would be best to use a standard stem-and-leaf plot when the values are spread over a reasonable number of stems.

15 **a** 0.155

b 0.146 and 0.154; 0.147 and 0.153; 0.148 and 0.152; 0.149 and 0.151

EX p449

Checkpoint

1 **a** Discrete **b** Continuous

2 **a** IV **b** II **c** I **d** III

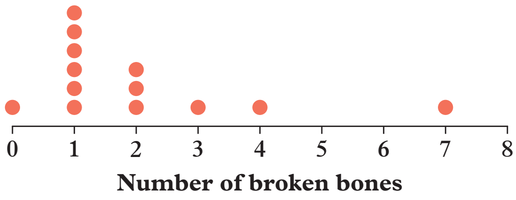
3 **a** **i** 9.25 **ii** 8.5 **iii** 7 **iv** 9

b **i** 4.55 **ii** 2.5 **iii** 2.5 **iv** $7\frac{13}{14}$

4 **a** median **b** mode **c** range

d mean

5



6 **a** 10.08 **b** 10 **c** 10

d 6 **e** Approximately symmetric

7

Stem	Leaf
1	2 5 9
2	3 7 8 8
3	0 1 2 5 7 9
4	0 1 2 5 7
5	2
6	0

Key: 1 | 2 = 12

8 **a** 104 **b** 107.5 **c** 103

d 98 **e** 35 **f** Positively skewed

EX p454

10.4 Describing probability

1 **a** {red, blue, green, yellow}

b {1, 2, 3, 4, 5, 6, 7, 8}

c {spring, summer, autumn, winter}

d {January, February, March, April, May, June, July, August, September, October, November, December}

e {5c, 10c, 20c, 50c, \$1, \$2}

f {0, 1, 2, 3, 4, 5, 6, 7}

2 **a** Not equally likely

b Not equally likely

c Equally likely

d Equally likely

e Not equally likely

f Not equally likely

3 **a** Likely **b** Unlikely

c Even chance

d Unlikely **e** Certain

4 **a** III **b** I **c** IV

d II

5 Sample answers:

a Landing on a yellow segment

b Landing on a red or green segment

c Landing on a purple segment

d Landing on a yellow or green segment

e Landing on a yellow, red, green or blue segment

6 **a** {red, blue, green, yellow}

b {3, 4, 5, 6}

7 **a** 5 **b** 7 **c** 5

d 3 (T, E and X are the different letters to choose from)

e 3

8 **a** Yes **b** No

9 **a** **i** Not equally likely **ii** $3\frac{1}{3}$

b **i** Equally likely **ii** 3.5

c **i** Not equally likely **ii** 3

d **i** Equally likely **ii** 0

e **i** Equally likely **ii** $4\frac{1}{6}$

10 **a** False **b** True **c** True

d False **e** False

11 **a** Equally likely **b** Equally likely

c Equally likely **d** Not equally likely

e Not equally likely

12 Sample answers:

a {3, 6, 9, ...}

b {0, 10, 20, 30, ...}

c {..., −0.2, −0.1, 0, 0.1, 0.2, ...}

d {1, 2, 4, 8, 16, ...}

13 **a** Unlikely

b Even chance

c Impossible

d Unlikely

e Even chance

14 **a** C, A, E, B, D

b C

15 Statement 1 is false. Event C has an even chance of occurring.

EX p459

10.5 Theoretical probability

1 **a** 25 **b** 5 **c** $\frac{5}{25} = \frac{1}{5}$

2 **a** 28 **b** $\frac{10}{28} = \frac{5}{14}$ **c** $\frac{4}{28} = \frac{1}{7}$

3 **a** {red, dark blue, green, yellow, light blue, purple}

b $\frac{1}{6}$ **c** $\frac{2}{6} = \frac{1}{3}$

4 **a** 50% **b** 50%

5 **a** 30°

b **i** $\frac{1}{12}$ **ii** $\frac{5}{36}$ **iii** $\frac{2}{9}$ **iv** $\frac{5}{9}$

c **i** 215° **ii** 90° **iii** 10° **iv** 45°

d **i** $\frac{43}{72}$ **ii** $\frac{1}{4}$ **iii** $\frac{1}{36}$ **iv** $\frac{1}{8}$

6 **a** $\frac{1}{6}$ **b** $\frac{1}{2}$ **c** $\frac{4}{6} = \frac{2}{3}$

d $\frac{3}{6} = \frac{1}{2}$

7 **a** $\frac{1}{4}$ **b** $\frac{1}{2}$ **c** 0

8 **a** 0.3 **b** 0.1 **c** 0.3

9 **a** 20

b **i** 0.3 **ii** 0.2

10 **a** $\frac{12}{52} = \frac{3}{13}$ **b** $\frac{4}{52} = \frac{1}{13}$ **c** $\frac{1}{52}$

d $\frac{20}{52} = \frac{5}{13}$ (2, 4, 6, 8, 10)

e $\frac{16}{52} = \frac{4}{13}$ (2, 3, 5, 7)

11 **a** 25% **b** 25% **c** 6.25%

d 50% **e** 43.75%

12 **a** $\frac{4}{108} = \frac{1}{27}$ **b** $\frac{8}{108} = \frac{2}{27}$ **c** $\frac{8}{108} = \frac{2}{27}$

d $\frac{8}{108} = \frac{2}{27}$ **e** $\frac{12}{108} = \frac{1}{9}$

13 **a** 0.7 **b** 0.25 **c** 0.95

d 0.4 **e** 0.65 **f** 1

14 Sample answer: although there are four different outcomes, each outcome is not equally likely. There are different numbers of each coloured jellybean. There are two green jellybeans out of a total of ten jellybeans, so the correct probability of selecting a green jellybean is $\frac{2}{10}$, which simplifies to $\frac{1}{5}$.

15 Sample answer: no, the maximum probability an outcome can have of occurring is 1, which is 100%.

16 Sample answer: if the outcomes are not all equally likely, then some would be more likely to occur than their theoretical probability and others would be less likely to occur than their theoretical probability. In the formula all outcomes are added equally to each other, showing that they have equal probability of occurring.

17 **a** December, $\frac{1}{16}$ **b** November, $\frac{1}{48}$

c $\frac{3}{10}$ **d** $\frac{7}{15}$

18 **a** 2 times **b** 3 times

c **i** $\frac{1}{3}$ times **ii** 2 times **iii** $\frac{1}{6}$ times

19 **a** Yes **b** Yes **c** No

d No

20 **a** True **b** True **c** False

d False **e** False

21 **a** $\frac{1}{6}+\frac{1}{6}+\frac{1}{6}+\frac{1}{6}+\frac{1}{6}+\frac{1}{6}=1$

b 0.5 + 0.5 = 1

c $\frac{9}{25}+\frac{16}{25}=1$

d 31% + 52% + 17% = 100%

22 **a** 0.2 **b** 0%

c 13 983 815 in 13 983 816 **d** $\frac{5}{12}$

23 **a** **i** $\frac{5}{10}=\frac{1}{2}$ **ii** $\frac{4}{10}=\frac{2}{5}$ **iii** $\frac{4}{10}=\frac{2}{5}$

b

		Second roll of die				
		0	**1**	**2**	**3**	**4**
First roll of die	**0**	0, 0	0, 1	0, 2	0, 3	0, 4
	1	1, 0	1, 1	1, 2	1, 3	1, 4
	2	2, 0	2, 1	2, 2	2, 3	2, 4
	3	3, 0	3, 1	3, 2	3, 3	3, 4
	4	4, 0	4, 1	4, 2	4, 3	4, 4
	5	5, 0	5, 1	5, 2	5, 3	5, 4
	6	6, 0	6, 1	6, 2	6, 3	6, 4
	7	7, 0	7, 1	7, 2	7, 3	7, 4
	8	8, 0	8, 1	8, 2	8, 3	8, 4
	9	9, 0	9, 1	9, 2	9, 3	9, 4

		Second roll of die				
		5	**6**	**7**	**8**	**9**
First roll of die	**0**	0, 5	0, 6	0, 7	0, 8	0, 9
	1	1, 5	1, 6	1, 7	1, 8	1, 9
	2	2, 5	2, 6	2, 7	2, 8	2, 9
	3	3, 5	3, 6	3, 7	3, 8	3, 9
	4	4, 5	4, 6	4, 7	4, 8	4, 9
	5	5, 5	5, 6	5, 7	5, 8	5, 9
	6	6, 5	6, 6	6, 7	6, 8	6, 9
	7	7, 5	7, 6	7, 7	7, 8	7, 9
	8	8, 5	8, 6	8, 7	8, 8	8, 9
	9	9, 5	9, 6	9, 7	9, 8	9, 9

c $\frac{4}{100}$ or $\frac{1}{25}$

EX p465

10.6 Experimental probability

1 **a** $\frac{1}{2}$ **b** 10 **c** $\frac{12}{20}=\frac{3}{5}$

2 **a** $\frac{1}{6}$ **b** 10 **c** $\frac{8}{60}=\frac{2}{15}$

3 **a** $\frac{13}{20}$

b The relative frequency is higher than the theoretical probability.

c Decrease; you would expect it to get closer to the theoretical probability.

4 **a** $\frac{6}{60}=\frac{1}{10}$

b The relative frequency is lower than the theoretical probability.

c Increase; you would expect it to get closer to the theoretical probability.

5 **a** 0.61

b The number of people surveyed was 100, so the frequency is also per 100.

c 550 people

6 **a** 0.26 **b** 13

7 Sample answer: It is possible that Peter could record the same results, although it is very unlikely. The results that Jade and April recorded will not have an effect on the results Peter records.

8 **a** purple: $\frac{13}{100}$, orange: $\frac{16}{100}=\frac{4}{25}$, dark blue: $\frac{20}{100}=\frac{1}{5}$, green: $\frac{19}{100}$, yellow: $\frac{14}{100}=\frac{7}{50}$, light blue: $\frac{18}{100}=\frac{9}{50}$

b Each colour would have the same probability of occurring. The past results have no impact on the result of the next spin.

9 **a**

Number	Expected frequency	Actual frequency	Theoretical probability	Relative frequency
1	6	3	$\frac{6}{36}$	$\frac{6}{36}$
2	6	5	$\frac{6}{36}$	$\frac{5}{36}$
3	6	7	$\frac{6}{36}$	$\frac{7}{36}$
4	6	5	$\frac{6}{36}$	$\frac{5}{36}$
5	6	5	$\frac{6}{36}$	$\frac{5}{36}$
6	6	11	$\frac{6}{36}$	$\frac{11}{36}$

b Larger

c Sample answer: The die is probably not fair; the 6 is rolled too many times.

10 **a** Your sample: 4%, your friend's sample: 14%

b Sample answer: Yes, 5% does not mean exactly five in every hundred. It means that each encounter has a 5% chance of the monster appearing. The sample percentages of 4% and 14% are both possible.

c 0.0244%

d Sample answer: No. 1 in 4096 does not mean exactly 1 in every 4096. It means that each encounter has a 1 in 4096 chance of encountering the special form. You may encounter the special form more or less frequently than once every 4096 encounters.

11 **a** **i** $\frac{1}{1000}$ **ii** $\frac{1}{200}$

b **i** 0.14% **ii** 0.46%

12 **a** 56%, 60%, 58% **b** 5800 red marbles

c **i** 57%, 59%, 58% **ii** 5800 red marbles

iii 5778 red marbles

iv Vernica won. Jake did not choose his best estimate, his first estimate was actually closer to the actual number.

13 **a**

		Die 2					
		1	**2**	**3**	**4**	**5**	**6**
Die 1	**1**	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

b 5 **c** $\frac{5}{36}$

d 5 **e, f** Answer will vary

14 **a** $\frac{1}{6}$ **b** $\frac{5}{6}$ **c** 30

d Rolling a die

e–f Answers will vary.

g Sample answer: The main advantage is that the more simulations you do, the closer the experimental probability is likely to be to the actual probability. The main disadvantage is that running a lot of simulations can take a lot of time, which in some circumstances can also mean a lot of money.

15 210

EX p473

Review: Statistics and probability

Review questions 10.7A: Mathematical literacy review

1 A positively skewed distribution has the data values clustered on the left-hand side with a tail on the right of the distribution, whereas a negatively skewed distribution has the data values clustered on the right-hand side with a tail on the left of the distribution.

2 In a split stem-and-leaf plot each stem is split into two or more rows, whereas in a regular stem-and-leaf plot each stem is unique.

3 Discrete data is numerical data that can be counted, whereas continuous data is numerical data that can be measured.

4 **a** mode

b trial

5 **a** An outlier is an extreme data point that is much higher or lower than the other values in a data set. For example, 100 is an outlier in the data set 1, 2, 3, 3, 100.

b The sample space of an experiment is a set of all the possible outcomes of that experiment. For example, the sample space for the experiment of rolling one six-sided die is: {1, 2, 3, 4, 5, 6}.

6 **a** Events, favourable outcome

b range, maximum, minimum

Review questions 10.7B: Multiple choice

1 C **2** B **3** C **4** B **5** D

6 B **7** C **8** D **9** D **10** B

Review questions 10.7C: Short answer

1 **a** **i** 11.5 **ii** 7 **iii** 2 **iv** 70

b The mean is skewed by a single large data point.

2 **a** Mode

b Median

c Mean

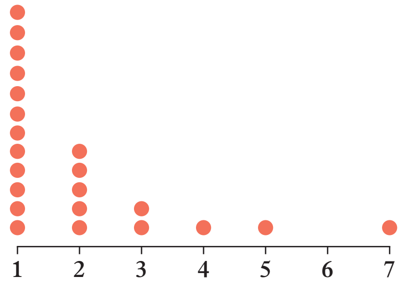
d Range

3 **a** Approximately symmetric

b Positively skewed with an outlier

4 **a**

Number of times families went grocery shopping



b Positively skewed

5

Stem	**Leaf**
3	3 4
3	6 6 8 9
4	0 1 1 1 3
4	5 6 6 7 8 8 9 9 9
5	0 2 2 4
5	7 9

Key: 3|4 = 34

6 **a** 10|4 = 10.4 cm, 8|9 = 8.9 cm

b The 10 stem **c** 10.1 cm **d** 10.1 cm

e 10.1 cm **f** 4.6 cm

g The lengths of the worms are spread out over 4.6 cm (from 8.1 cm to 12.7 cm). The average length of a worm is 10.1 cm (mode: 10.1 cm, median: 10.1 cm, mean: 10.1 cm).

7

Stem	**Leaf**
0	5 9
1	2 2 3 3 6 6 7
2	0 2 3 5 8
3	0 1 2 3 8
4	1 2 5
5	1 5 9
6	2

Key: 1|3 = 13

8 **a** **i** {head, tail} **ii** Even chance

b **i** {1, 2, 3, 4, 5, 6} **ii** Even chance

c **i** {club, diamond, heart, spade}

ii Unlikely

d **i** {red, blue, white} **ii** Likely

9 **a** $\frac{1}{6}$ **b** $\frac{1}{2}$ **c** $\frac{1}{2}$

10 **a** {2, 3, 5, 7, 11, 13}

b $\frac{5}{6}$ **c** $\frac{2}{3}$

11 **a** **i** 0.4 **ii** 0.1 **iii** 0.3

iv 0.2 **v** 0.6

b **iii** and **iv** are equally close

Review questions 10.7D: Mathematical modelling

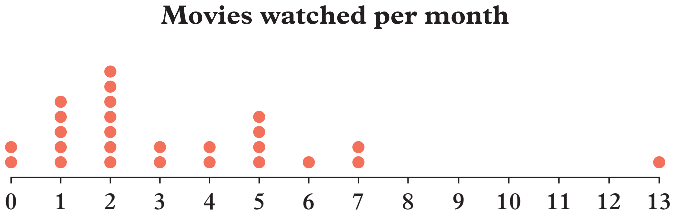
1 **a** Discrete numerical

b 26

c **i** 3.3 **ii** 2 **iii** 2

iv 13

d Sample answer: a dot plot allows you to see the range and spread of data clearly.



Movies watched per month

2 **a** {bronze, silver, gold, iron}

b $\frac{1}{10}$ **c** $\frac{1}{12}$ **d** $\frac{1}{6}$

e Sample answer: The theoretical probability does not guarantee that 1 in every 10 trials will be an iron marble; there will be variation in the number of iron marbles that get picked. The number of trials is not equal to a multiple of 10 and cannot be simplified to a tenth, so the relatively frequency cannot be equal to the theoretical probability.

3 **a**

Coin	Frequency
5c	9
10c	9
20c	9
50c	8
$1	10
$2	5

b $27.15

c 54.3c

d **i** $\frac{4}{25}$ **ii** $\frac{3}{10}$ **iii** $\frac{9}{25}$

e **i** 20c and 50c coins

ii No, the larger coins appear to be more likely to be selected than the smaller coins.

SEMESTER 2 review

EX p478

Short answer

1 **a** True

b False. There are 4 terms. Each term is something that is added or subtracted.

c True

d False. The coefficient is −4.

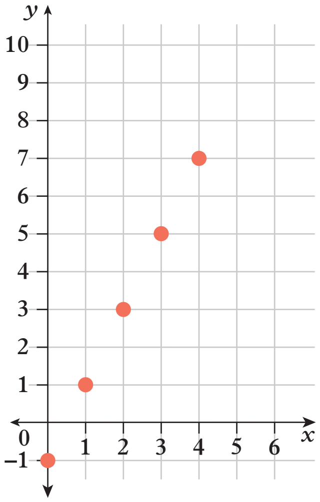
e True

2 **a**

x	0	1	2	3	4
y	−1	1	3	5	7

b (0, −1), (1, 1), (2, 3), (3, 5), (4, 7)

c



3 **a**

x	1	2	3	4	5
y	2	4	6	8	10

b $p = 2m$ **c** $24

4 **a** $x = 5$ **b** $x = 24$ **c** $x = 2$

d $x = 42$

5 **a** $\overline{AE}$ and $\overline{CD}$ are parallel line segments

b $\overrightarrow{AD}$ and $\overrightarrow{AE}$ are perpendicular rays

6 **a** 54° **b** Isosceles acute triangle

7 **a** Polygon **b** Polygon **c** Polygon

d Not a polygon **e** Polygon **f** Polygon

g Not a polygon **h** Polygon **i** Not a polygon

8 **a** **i** Concave **ii** Irregular

iii Irregular, concave pentagon

b **i** Convex **ii** Irregular

iii Irregular, convex quadrilateral

c **i** Convex **ii** Regular

iii Regular, convex hexagon

e **i** Concave **ii** Irregular

iii Irregular, concave quadrilateral

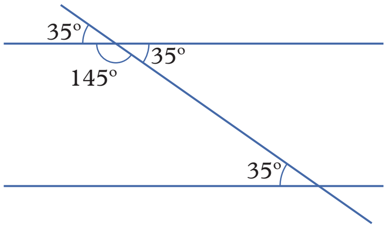
f **i** Convex **ii** Regular

iii Regular, convex pentagon

h **i** Concave **ii** Irregular

iii Irregular, concave hexagon

9 Sample answer: other orientations and labelled angles possible.



10 **a** 71° **b** 161° **c** 19°
d 161° **e** 19°

11 **a** $3b + 5$
b $5(m - 3) = n$
c $\frac{u}{8} + 7 = 3$
d $\frac{p+q}{2} = pq$
e $9(t - 8) = 41$

12 **a** 41 **b** 13 **c** 1 **d** 7.07

13 **a** Unlikely **b** Even chance
c Certain **d** Likely

14 **a** Not equally likely **b** Equally likely
c Not equally likely **d** Not equally likely
e Equally likely

15 **a** **i** Discrete
ii 1
iii 5
b **i** Continuous
ii 34
iii 19
c **i** Discrete
ii 3
iii 6
d **i** Continuous
ii 140.5
iii 45

16 **a**

	0	1	2	3
Number of blue counters	3	3	3	3
Number of red counters	0	3	6	9

b No change, constant value
c Add three counters
d Because there are 0 red counters in the first diagram. Since the number of blue counters is constant, it is easier to number the diagrams by looking at the red counters.
e The constant term 3 represents the number of blue counters in each diagram which stays constant. The variable r represents which number diagram it is after the first one which is labelled 0. The term $3r$ represents the total number of red counters in the r^{th} diagram. The expression represents the total number of counters in the r^{th} diagram.

17

Guess	LHS	RHS	LHS = RHS?
$x = 2$	8	0	No
$x = 4$	14	10	No
$x = 6$	20	20	Yes
$x = 8$	26	30	No

18 **a** 1 cm × 11 cm, 2 cm × 10 cm, 3 cm × 9 cm, 4 cm × 8 cm, 5 cm × 7 cm, 6 cm × 6 cm
b 6 cm × 6 cm = 36 cm^2
c 4 cm × 8 cm

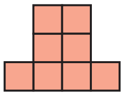
19 **a** **i** 186.17° **ii** 202.5° **iii** No mode
b **i** $\frac{1}{6}$ **ii** $\frac{1}{4}$ **iii** $\frac{7}{12}$

20 **a** Isosceles right-angled triangle
b Kite
c Scalene obtuse triangle
d Square, rhombus, rectangle, parallelogram, trapezium
e Equilateral acute triangle
f Trapezium

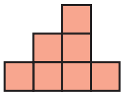
21 **a** 1 cm × 48 cm, 2 cm × 24 cm, 3 cm × 16 cm, 4 cm × 12 cm, 6 cm × 8 cm
b 98 cm, 52 cm, 38 cm, 32 cm, 28 cm
c Mean = 12.4 cm, median = 7 cm
d $48, 12, \frac{16}{3} = 5\frac{1}{3}, 3, \frac{4}{3} = 1\frac{1}{3}$
e **i** 1 **ii** $\frac{2}{5}$ **iii** $\frac{4}{5}$
f 6 and 7

22 *A*: a vertical translation
B: a reflection in the *x*-axis
C: a rotation of 90° about *O* anticlockwise

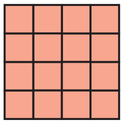
23 **a**



front elevation

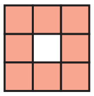


side elevation

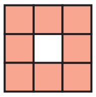


plan view

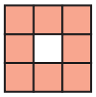
b



front elevation

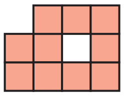


side elevation

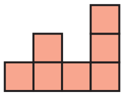


plan view

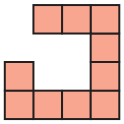
c



front elevation

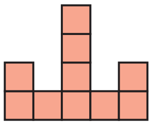


side elevation

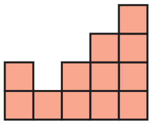


plan view

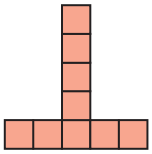
d



front elevation



side elevation



plan view

24 Pronumeral choices may vary.

- **a** **i** Let x be the size of the other co-interior angle.
  - **ii** $x + 50° = 180°$
  - **iii** $x = 130°$
- **b** **i** Let x be the size of the fourth interior angle.
  - **ii** $x + 81.5° + 52.3° + 95.1° = 360°$
  - **iii** $x = 131.1°$
- **c** **i** Let x be the length of the shortest side of the hexagon.
  - **ii** $x + (x + 2.1) + (x + 4.2) + (x + 6.3) + (x + 8.4) + (x + 10.5) = 51.9$
  - **iii** $x = 3.4$ cm
- **d** **i** Let r be the radius of the circle
  - **ii** $2\pi r = 6$ cm
  - **iii** $r = 0.95$ cm
- **e** **i** Let x be the side length of the square.
  - **ii** $x^2 = 81$ cm^2
  - **iii** $x = 9$ cm
- **f** **i** Let x be the length of the perpendicular height.
  - **ii** $\frac{1}{2} \times 4 \text{ cm} \times x = 40 \text{ cm}^2$
  - **iii** $x = 20$ cm
- **g** **i** Let x be the side length of the cube.
  - **ii** $x^3 = 64$ cm^3
  - **iii** $x = 4$ cm
- **h** **i** Let x be the value of the missing number.
  - **ii** $\frac{x + 3 + 5 + 8 + 12 + 4}{6} = 7$
  - **iii** $x = 10$

25
- **a** 1.44 cm^2 or 144 mm^2
- **b** 90 m^2
- **c** 196 cm^2
- **d** 36 cm^2
- **e** 50 820 m^2 or 0.050 82 km^2
- **f** 9.5 m^2 or 95 000 cm^2

26
- **a** ∠*CFE* or ∠*EFC*
- **b** No, as the alternate angles ∠*CFE* and ∠*FEB* are not equal.
- **c** ∠*EFB* = 79°
- **d** ∠*FBE* = 50°, not isosceles because there are not two equal angles.

27
- **a** $\frac{y}{9} = 18$, $9(y - 2) = 36$ and $x + 5 = 3$
- **b** Sample answer: $y = 2$ is an equation as it has two expressions written equal to each other. $y = 2$ is the solution of the equation as substituting $y = 2$ shows that the left-hand side is equal to the right-hand side.
- **c** Sample answer: Substituting $x = 2$ reveals that the left-hand side is equal to 16 and the right-hand side is equal to 8.

28 **a**

Stem	Leaf
0	2 2 7
1	2 5
2	0 1 2 6 6
3	2 3 4 7 9
4	3 4 5 8 9

Key: 1 | 2 = 12

- **b** **i** 27.85 **ii** 29 **iii** 2, 26 **iv** 47
- **c** Sample answer: The number of stickers is spread out over 47 stickers (from 2 stickers to 49 stickers). The average number of stickers is around 27 (modes: 2, 26, median: 29, mean 27.85).

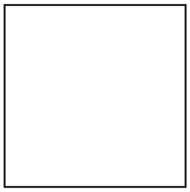
29 $-7x + 2y + 7$

30
- **a** Object 1: Rectangular prism
  Object 2: Triangular prism
- **b** **i** Object 1: 6, Object 2: 5
  - **ii** Object 1: 12, Object 2: 9
  - **iii** Object 1: 8, Object 2: 6

c Object 1:

Front elevation:

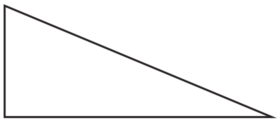
Side elevation:



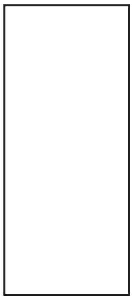
Plan view:

Object 2:

Front elevation:



Side elevation:



Plan view:

d Object 1: 45 cm^3

Object 2: 60 cm^3

Mathematical modelling

1 **a** $h = 2d$

b

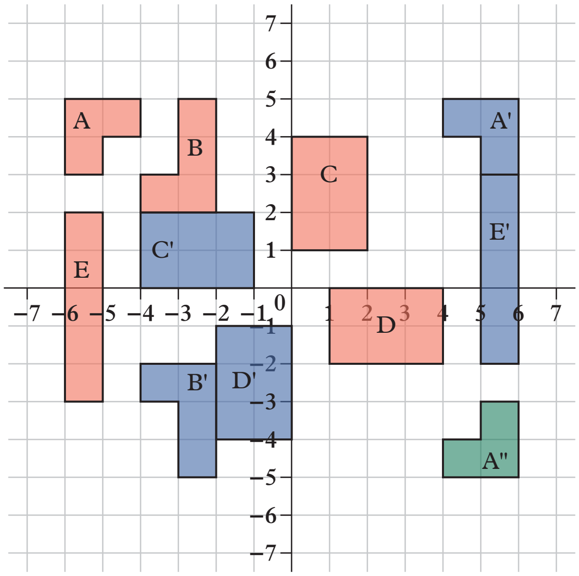
Days	1	2	3	4	5
Height (cm)	2	4	6	8	10

c 11 days

d **i** $h = 3d + 1$ **ii** 7 days

e **i** $g = \frac{2}{3}(h - 4)$ **ii** 12 g

2 **a**



b Rotation of 180° about the point (0, 0)

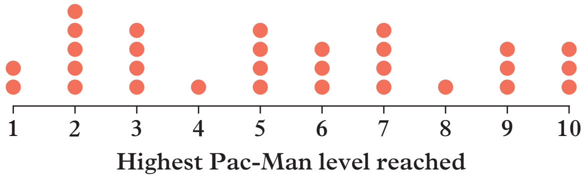
c **i** *C* and *D* **ii** *E*

d *A* is an irregular, concave hexagon. *E* is a rectangle.

3 **a** Discrete

b

Level	Frequency
1	2
2	5
3	4
4	1
5	4
6	3
7	4
8	1
9	3
10	3
Total	**30**



c **i** 5.3 **ii** 5 **iii** 2

iv 9

d Sample answer: The highest Pac-Man levels reached are spread out over 9 levels (from level 1 to level 10). The average highest Pac-Man level reached was about 5 (mode: 2, median 5, mean 5.3).

EX p486

EXPLORATIONS 2

1 **a** Simplify $4(2x + 1) + 3$

b $16x + 15$

c $32x + 31$ and $64x + 63$

d $(2^{n+1})x + (2n^{n+1} - 1)$

e $x + 2y$, $3x + 2y$, $3x + 5y$, $8x + 5y$, … ; the coefficients are the Fibonacci numbers

2 **a** $\alpha = 72°$ and $\beta = 144°$

b 45° and 135°

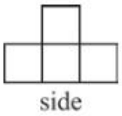
c 80° and 100°; four rings with nine rhombuses each

d Yes, the pattern continues (for as long as the resulting angle is less than 180°)

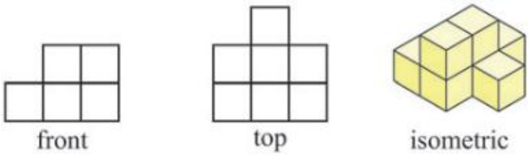
e 12

f 15 or 16

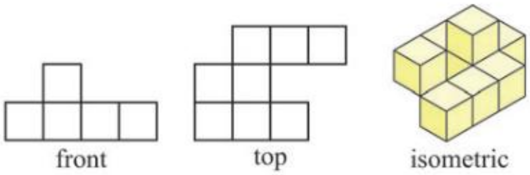
3 **a**



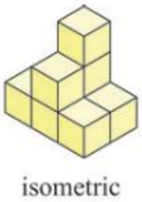
b



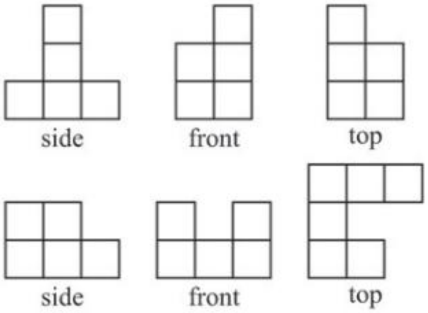
c



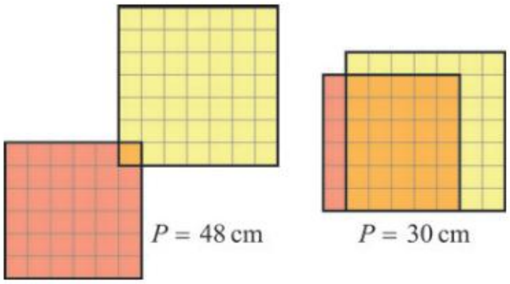
d



A reflection of this solution is also valid.



4 **a**



The only possible perimeters, in centimetres, are the even numbers between 30 and 48, inclusive.

b Side lengths 2 cm and 7 cm, 3 cm and 6 cm, or 4 cm and 5 cm

c Side lengths 4 cm and 7 cm, or 5 cm and 6 cm

d 55 cm^2

5 **a** Here is one possible solution for each grid. There are many others.

1	6	5	4
7	2	3	

1	6	5
7	2	3
4		

8	1	6
3	5	7
4	9	2

1	9		
6	2	7	5
8	4	3	

1	7	6	4
8	2	3	5

b The first and last column cannot contain the same number; the bottom-right cell cannot have the same number above it and to its left; the total of 55 cannot be shared equally among the two rows; the average of 5.5 cannot be placed in the bottom-right cell.

c The number of columns is even and greater than 2.

6 **a** 3

b 4

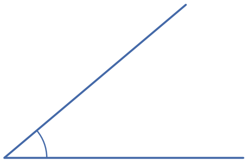
c 63 and 64

d No. The number of plus symbols is the same as the number of 1's in the binary representation of n.

Glossary

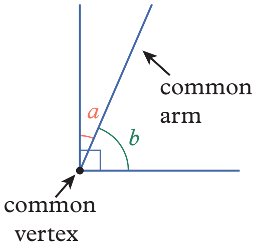
accuracy
the closeness of a measurement to its exact value

acute angle
angle greater than 0° but less than 90°

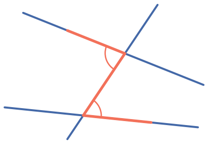


acute triangle
triangle with three acute angles

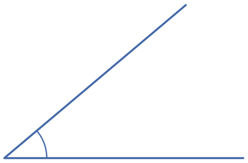
adjacent angles
angles that share a common arm and a common vertex



alternate angles
a pair of angles not adjacent to (next to) each other on opposite sides of a transversal between two lines, sometimes called 'Z angles'



angle
a measure of turn formed when two lines or rays meet at a point



arc
a portion of a circle; angles are usually marked using an arc

area
amount of space enclosed by a two-dimensional (2D) shape

arms
the lines or rays that form an angle

ascending order
arranging numbers from smallest to largest

associative law
a law which states that regardless of how three or more numbers are grouped, when added or multiplied, the result is the same; for example,
$(4 + 5) + 6 = 9 + 6 = 15$ and
$4 + (5 + 6) = 4 + 11 = 15$;
$(4 \times 5) \times 6 = 20 \times 6 = 120$ and
$4 \times (5 \times 6) = 4 \times 30 = 120$

axis of symmetry
a line in which a shape is reflected in; also called a line of symmetry

backtracking
representing an equation using a function machine, and using inverse operations to work backwards to the solution

base
1. for a value expressed in exponent form, the base is the number which is repeatedly multiplied; for example, 2^4 has a base of 2 and can be written as $2 \times 2 \times 2 \times 2$
2. in a 2D shape, it is a nominated side of the shape, where the base and height are perpendicular to each other. *See* height for diagram
3. in a 3D object, it is a nominated face of the object, where the base and height are perpendicular to each other

basic numeral
the way we normally write numbers using digits; for example, as a basic numeral, thirty-two is written as 32

biased
when one outcome is favoured over other outcomes in a probability experiment

BIDMAS
the conventional order in which operations are applied in a mathematical expression: Brackets, Indices (or exponents), Division & Multiplication, Addition & Subtraction

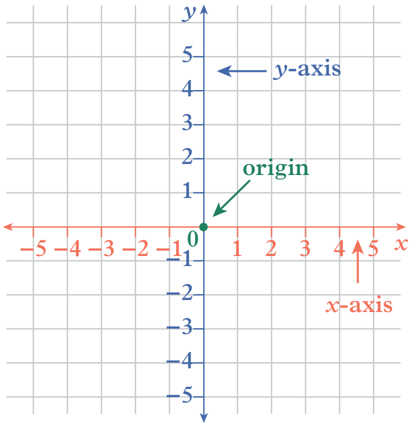
bi-modal distribution
a data distribution with two clear peaks

by-parts method
addition method involving adding the digits in each place-value separately

cancelling (simplifying)
dividing the numerator and the denominator of a fraction by the same factor (usually the highest common factor) to produce an equivalent fraction

Cartesian coordinates
pair of numbers that describe the position of a point on the Cartesian plane; for example, (3, 5) indicates an x-coordinate of 3 and a y-coordinate of 5

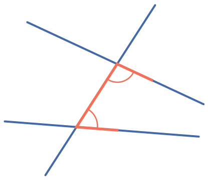
Cartesian plane
number plane or region formed by a pair of horizontal and vertical axes that allows any point to be described



circumference
perimeter of a circle

coefficient
the number by which a pronumeral (or pronumerals) is being multiplied in a term

co-interior angles
a pair of angles on the same side of a transversal between two lines, sometimes called 'C angles'



common denominator
a common multiple of the denominators of two or more fractions

common factors
factors that divide exactly into two or more whole numbers; for example, 12 and 9 have a common factor of 3

common multiples
multiples that appear in the multiples list for two or more different whole numbers; for example: multiples of 3 are 3, 6, 9, 12, 15, 18, 21, 24, … and multiples of 4 are 4, 8, 12, 16, 20, 24, …, so the first two common multiples of 3 and 4 are 12 and 24

commutative law
a law which states that the order in which two numbers are added or multiplied is not important; for example, 3 + 4 = 7 and 4 + 3 = 7; 3 × 4 = 12 and 4 × 3 = 12

compensation method
rounding one number to make calculations easier. The amount by which the number was originally rounded is then added or subtracted

complementary angles
angles that add to make a right angle (90°)

composite number
a positive whole number that is divisible by at least one whole number other than itself and 1; for example, 8 has factors of 1, 2, 4 and 8, and so is composite

concave polygon
polygon with at least one interior angle greater than 180°

cone
3D object that tapers from a circular base to a point

constant
a term without any pronumeral component; for example, the constant in the expression $3b - 4c + 10$ is 10

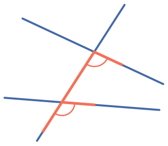
continuous data
numerical data that can be measured; for example, height of a plant

conversion factor
a multiplier for converting a quantity expressed in one set of units into another quantity expressed in a different set of units; for example, multiply by 10 to convert from centimetres to millimetres

convex polygon
polygon with interior angles that are all less than 180°

convex quadrilateral
quadrilateral with interior angles that are all less than 180°

corresponding angles
a pair of angles on the same side of a transversal and in corresponding positions on the two lines (both above a line or both below a line), sometimes called ‘F angles’



cross-section
a 2D shape obtained when cutting a 3D object

cylinder
3D object with a uniform circular cross-section

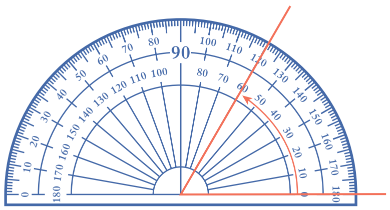
decimal
numbers that have a whole number component, which is to the left of the decimal point, and a decimal fraction component, which is to the right of the decimal point; for example, 3.456

decimal fraction
the part of a decimal number that appears after the decimal point

decimal place
position of a digit after the decimal point, where each decimal place represents a different place value

decimal point
dot that separates the whole number component and the decimal fraction of a decimal number; for example, 3.456

degrees
the unit of measure for angles; for example, 60 degrees is written as 60°, as shown on the protractor



denominator
bottom number of a fraction; for example, the denominator of $\frac{2}{3}$ is 3

dependent variable
a variable whose values depends on the value of another variable (the independent variable)

descending order
arranging numbers from largest to smallest

diameter
line segment from one side of a circle to the other through the centre of the circle; also defined as the length of this line segment

digit
a single symbol used to make numerals; the 10 digits are 0, 1, 2, 3, 4, 5, 6, 7, 8, 9

discrete data
numerical data that can be counted; for example, number of siblings

distance–time graph
See travel graph

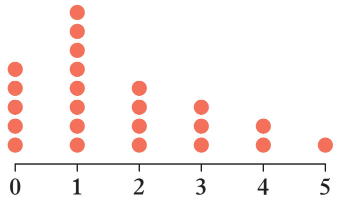
distributive law
states that the same result is obtained if you add the numbers inside a pair of brackets first before multiplying by the common factor, or multiply each number inside the brackets by the number outside the brackets before adding: $a(b + c) = ab + ac$

dividend
number that is being divided by another number (divisor); for example, 6.84 is the dividend, 2 is the divisor and 3.42 is the quotient in 6.84 ÷ 2 = 3.42

divisor
number that divides into another number (dividend); for example, in 15 ÷ 3 = 5, 3 is the divisor

dot plot
graph that presents every piece of data in a data set as a dot above a matching number on a horizontal scale

Number of siblings



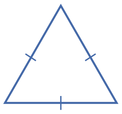
edge
line segment that joins two faces of a polyhedron. *See* polyhedron

elevation
a drawing that shows what a 3D object looks like when viewed from the front or side of the object

equally likely outcomes
outcomes that have the same probability of occurring

equation
two algebraic expressions separated by an equals sign; for example, $2d + 5 = 10$

equilateral triangle
triangle with all three sides equal in length and three identical internal angles of 60°



equivalent equations
equations that have the same solution(s)

equivalent fractions
different fractions that have the same numerical value; for example, $\frac{2}{3} = \frac{4}{6}$

estimate
an approximate value of a calculation, usually involving rounding

evaluate
calculate the numerical value of a term or expression after substitution; for example, evaluating $3x + 4$ when $x = 2$ gives 10

event
an outcome or group of outcomes in a sample space; for example, a single outcome of rolling a die is a 2; a group of outcomes is rolling a 2 or a 4

expanded notation
where a number is written as the sum of its place value components; for example, $3479 = 3000 + 400 + 70 + 9$

experiment
a repeatable procedure that has a clearly defined set of possible results

experimental probability
probability of an event based upon the results of a probability experiment. *See* relative frequency

exponent
for a value expressed in exponent form, the exponent indicates the number of times the base is written as a repeated multiplication; for example, 2^4 has an exponent of 4 and can be written as $2 \times 2 \times 2 \times 2$ in expanded form

exponent form
shorter form of writing a repeated multiplication, where a number is written with a base and an exponent; for example, 2^4 is written in exponent form

expression
a quantity that is represented by a sequence of numbers and/or pronumerals that are connected by mathematical operations

face
flat surface of a polyhedron, where each face is a polygon. *See* polyhedron

factor
whole number that divides exactly into a given whole number; for example, factors of 8 are 1, 2, 4 and 8

factor pair
two numbers that when multiplied give a certain product; for example, the factor pairs that give a product of 12 are 1 and 12, 2 and 6, 3 and 4

factor tree
tree diagrams where composite numbers are broken down into factor pairs until prime numbers are found

fair
when a tool behaves as you expect it to in a probability experiment

favourable outcome
an outcome within the sample space that we are interested in

formula
relationship or rule between two or more variables, usually represented using pronumerals; for example, $s = \frac{d}{t}$ is the formula for speed equals distance divided by time

fraction
a part or a portion of a whole, or multiple wholes

fraction wall
a visual representation to help compare and identify fractions, set out in the form of a wall

frequency
the number of times a particular value occurs in a data set

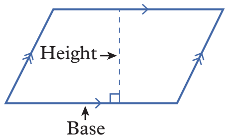
frequency table
a table organised to show data by recording the frequency of each value in the data set

function machine
a way of processing mathematical operations, where a value is inputted into the left-hand side and the result is outputted at the right-hand side

grouping symbols
brackets, roots and fraction lines that show where a group starts and ends, and help to show the order the mathematical operations are applied in

guess, check and improve
a trial-and-error strategy to solve simple equations; first guess a solution, then check if the number is the solution, and then improve your guess until the solution is found

height
perpendicular length measurement from the base to the top or end of a shape or object



highest common factor (HCF)
greatest factor that is common to two or more given numbers; for example, factors of 8 are 1, 2, 4, 8 and factors of 12 are 1, 2, 3, 4, 6, 12, so the HCF of 8 and 12 is 4

image
result of completing a transformation on a shape or object

improper fraction
fraction where the numerator is greater than (or equal to) the denominator; for example, $\frac{9}{7}$ or $\frac{6}{6}$

independent variable
a variable whose value isn't dependent on the value of any other variables

index (plural indices)

See exponent

integers

whole numbers that are positive, negative or zero

interior angle (internal angle)

angle inside a two-dimensional shape

inverse operation

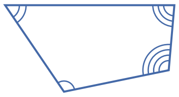
operation that reverses the effect of a previous operation; for example, multiplication and division are inverse operations and addition and subtraction are inverse operations

irregular polygon

polygon that does not have all sides equal in length or all angles equal in size

irregular quadrilateral

quadrilateral with four different internal angles and no parallel sides

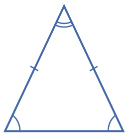


isometric transformation

type of transformation (such as translation, rotation or reflection) applied to a shape or object that does not change its shape or size

isosceles triangle

triangle with two sides equal in length, and equal angles opposite the equal sides

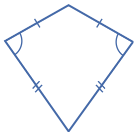


jump method

breaking down one of the given numbers, then adding or subtracting each part of the number in stages

kite

quadrilateral with two pairs of adjacent sides equal in length, one pair of opposite angles equal in size and no parallel sides



law of large numbers

a law which states that as the number of trials of an experiment increases, the relative frequency is likely to approach the theoretical probability

leading digit

first non-zero digit of a number; for example, the leading digit of 328 is 3

length

measurement of the longest side of a shape or object

like fractions

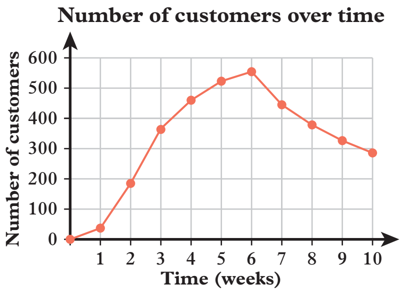
fractions with the same denominators; for example, $\frac{1}{6}$ and $\frac{5}{6}$. *See* unlike fractions

line

a line that continues to infinity in both directions, which is named using a double-headed arrow; for example, $\overleftrightarrow{AB}$

line graph

a graph that displays the relationship between two variables, where points are plotted and joined by a straight line. A continuous variable must be displayed on the horizontal axis; for example, time



line segment

a line joining two points, which is named using a bar and the two end points; for example, $\overline{AB}$

linear equation

an equation where any variables are raised to the power of 1 and no two variables are multiplied together; for example: $y = x + 3$

lowest common denominator (LCD)

lowest common multiple of the denominators of two or more fractions; for example, $\frac{1}{3}$ and $\frac{1}{2}$ have an LCD of 6

lowest common multiple (LCM)

lowest (first) multiple that is common in the multiples list of two or more numbers, excluding zero (0); for example, multiples of 3 are 3, 6, 9, 12, 15, 18, 21, 24, … and multiples of 4 are 4, 8, 12, 16, 20, 24, … so the LCM of 3 and 4 is 12

magnitude

size of a mathematical object; the magnitude of a number refers to its size independent of its sign; for example, −10 has a greater magnitude than 3

mean

a measure of the centre of a data set, also known as the average, calculated by adding all of the data values together and dividing by the number of values

median

a measure of the centre of a data set, determined by placing the data in ascending order (smallest to largest) to find the middle number of the data set

mixed number

made up of a whole number component and a proper fraction component; for example, $3\frac{4}{5}$

mode

the most common value in a numerical data set

multiple

result of multiplying a given number by a whole number; for example, multiples of 4 are 0, 4, 8, 12, 16, … (4 × 0, 4 × 1, 4 × 2, 4 × 3, 4 × 4, …)

negative number

a number that is less than zero; for example, −25

negatively skewed distribution

a data distribution with the data values clustered on the right (or top) side and a tail on the left (or bottom) side

net

two-dimensional plan that can be folded to form a 3D object

non-convex quadrilateral

quadrilateral with one interior reflex angle (greater than 180°); also called a concave quadrilateral

number sequence

an ordered list of numbers with a set rule; for example, 2, 4, 6, 8, 10, …

numerator

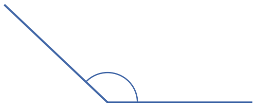
top number of a fraction; for example, the numerator of $\frac{2}{3}$ is 2

numerical data

data that can be counted or measured. *See* continuous data, discrete data

obtuse angle

angle greater than 90° but less than 180°



obtuse triangle

triangle with one obtuse and two acute angles

origin

point where the x- and y-axes intersect on a Cartesian plane, with coordinates (0, 0)

outcome

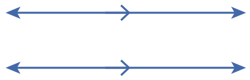
a possible result in an experiment or trial; for example, when rolling a die, there are six possible outcomes: 1, 2, 3, 4, 5 and 6

outlier

extreme piece of data that is much higher or lower than the rest of the data in a numerical data set

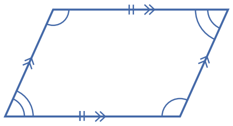
parallel lines

lines (or rays or segments) that are always the same distance apart so that they never meet. Parallel lines are labelled with matching arrowheads



parallelogram

quadrilateral with two pairs of opposite sides that are parallel and equal in length, and two pairs of opposite angles that are equal in size



per cent

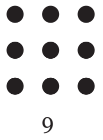
for every one hundred (or per hundred); for example, 20 per cent means 20 for every 100

percentage

a portion of a whole divided into one hundred parts, shown using the symbol %

perfect square

number that can be expressed as the product of two equal whole numbers; for example, $3 \times 3 = 9$, so 9 is a perfect square

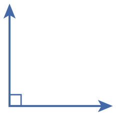


perimeter

length of the boundary of a two-dimensional (2D) shape

perpendicular lines

two lines (or rays or segments) that meet to form a right angle; perpendicular lines are labelled with a small box at the right angle



pi (π)

used in calculations involving circumference and area of a circle; it is the ratio of the circumference of a circle to its diameter; the symbol for pi is π

place value

the numerical value of a digit in a number depending on its position; for example, the digit 3 in 2316 represents 3 hundreds or 300, and in 15.34 it represents 3 tenths or $\frac{3}{10}$

plan

a drawing of a 3D object from a single view directly above the object

point

an exact position or location in space that has no length or width, which is represented by a dot labelled with a capital letter or coordinates

point of focus

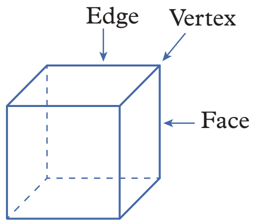
a point to concentrate around when reflecting or rotating a shape

polygon

a closed, 2D shape with straight sides

polyhedron (plural polyhedra)

3D object where each face is a polygon



positively skewed distribution

a data distribution with the data values clustered on the left (or bottom) side and a tail on the right (or top) side

power

a number written in exponent form

prime factorisation

the unique product of primes that make up a whole number

prime factors

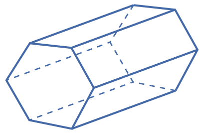
factors of a composite number that are prime numbers

prime number

a positive whole number greater than 1 that has exactly two factors, itself and 1; for example, the only factors of 7 are 1 and 7, so 7 is a prime number

prism

object with two ends that are identical polygons and joined by straight edges of equal lengths



probability

the chance that a particular event will occur; expressed on a scale from impossible to certain

product

result of a multiplication; for example, $10 \times 3 = 30$ where 30 is the product

pronumeral

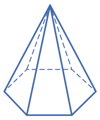
letter or symbol that takes the place of a number

proper fraction

fraction where the numerator is smaller than the denominator; for example, $\frac{4}{5}$

pyramid

a polyhedron with a base and triangular faces joining the base which all meet at a single point



quadrilateral

two-dimensional shape with four straight sides

quotient

result of a division; for example, 28 ÷ 4 = 7 where 7 is the quotient

radius (plural: radii)

line segment from the centre of a circle to the edge of the circle; also defined as the length of this line segment; it is half the diameter of the circle

range

a measure of spread determined by finding the difference between the highest and lowest values in a data set

ratio

comparison of two or more quantities of the same kind; for example, when comparing the number of red jellybeans to blue jellybeans, the ratio is 2 : 3

ray

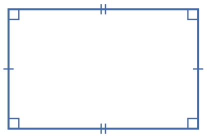
a line that starts at a point and continues to infinity; it is named using an arrow which indicates the direction of the ray; for example, $\overrightarrow{AB}$

reciprocal

found by first writing a number as a proper or improper fraction and then swapping the numerator and the denominator; for example, the reciprocal of $\frac{3}{4}$ is $\frac{4}{3}$

rectangle

quadrilateral with two pairs of opposite sides that are parallel and equal in length, and four angles of 90°

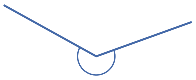


reflection

transformation where a shape or object is reflected (flipped) in an axis of symmetry to produce its exact mirror image

reflex angle

angle that is greater than 180° but less than 360°



regular polygon

polygon with all sides equal in length and all angles equal in size

relative frequency

the number of times a favourable outcome occurs divided by the total number of outcomes

$$\text{Relative frequency} = \frac{\text{number of occurrences}}{\text{total number of outcomes}}$$

remainder

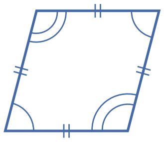
the part of the dividend that is left over when the number is not exactly divisible by the divisor; for example, 22 divided by 7 is 3 remainder 1

revolution

angle that is exactly 360°

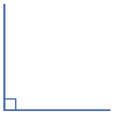
rhombus

quadrilateral with two pairs of opposite sides that are parallel, all four sides equal in length and two pairs of opposite angles that are equal in size



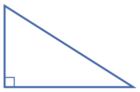
right angle

angle that is exactly 90°



right-angled triangle

triangle with two perpendicular sides, or one right angle, 90°



right prism

a prism with right angles between the matching ends and the other faces

rotation

transformation of a shape or object that is rotated (turned) around a fixed point, where the movement is described as an angle of rotation in either a clockwise or anticlockwise direction

rounding

replacing the number with an approximation that is simpler and easier to use in calculations, but which will also make any calculations less accurate; for example, 123 343 ≈ 120 000

rule

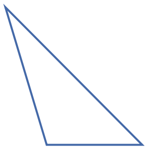
1. a description of the relationship between each term and the next term in a sequence
2. a description of the relationship between two or more variables

sample space

a list of all the different outcomes possible for a probability experiment; for example, the sample space when rolling a die is {1, 2, 3, 4, 5, 6}

scalene triangle

triangle with three sides of different lengths and three different internal angles



set

a defined group of objects or numbers

simplest form (fractions)

fractions with a highest common factor of 1 between the numerator and the denominator

simulation

using technology or simple devices like coins or dice to imitate real-life events

skewed distribution

a data distribution with data values clustered on one side and a tail on the other side

slope (gradient)

indicates how quickly one quantity is changing compared to another; shown by the steepness and direction of a line

solution
value of the pronumeral that makes an equation a true statement; that is, the left side and the right side of the equation are equal

solve
find the value of the pronumeral that makes an equation a true statement; that is, the left side and the right side of the equation are equal

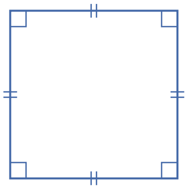
solving by inspection
mental strategy to solve simple equations by inspection or observation to identify a number that makes an equation true

speed
how fast something is travelling, measured as distance per unit of time

sphere
3D object shaped like a ball

split stem-and-leaf plot
a stem-and-leaf plot with each stem split into two or more rows

square
1 quadrilateral with all four sides equal in length, two pairs of opposite sides that are parallel and four angles of 90°



2 to raise to the power of 2; for example, $3^2 = 9$

square root
the number that when multiplied by itself is equal to the original number; for example, the square root of 9 (or $\sqrt{9}$) is 3

stationary
a speed of zero

stem-and-leaf plot (stem plot)
a display of data where each piece of data is split into two parts; the last digit becoming the leaf and the other digits becoming the stem

straight angle
angle that is exactly 180°

subject (of a formula)
a stand-alone variable which only appears on one side (usually the left) of a formula

substitution
replacement of a pronumeral with a given number; for example, substituting $x = 2$ in $3x + 4$ gives $3 \times 2 + 4$

successful trial
a probability trial where a favourable outcome is obtained

summary statistics
information such as the mean, median, mode and range that provides a summary of a numerical data set

supplementary angles
angles that add to make a straight angle (180°)

symmetric distribution
a data distribution with a peak in the middle and with the data spread evenly on either side of the peak

table of values
a table showing the relationship between two variables, with corresponding values listed in columns

term
1 a single pronumeral, number or the product or quotient of numbers and pronumerals for example, the expression $3b - 4c + 10$ has three terms

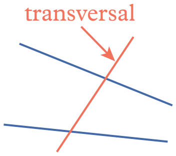
2 an individual number in a sequence

theoretical probability
probability based on mathematical reasoning, not experiments

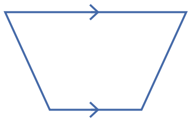
trailing zero
any zero to the right of the last non-zero digit after the decimal point in a decimal; for example, 3.900 has two trailing zeros

translation
transformation of a shape or object that is translated (moved) without turning or changing size, where the movement is often described as the number of units up or down and left or right

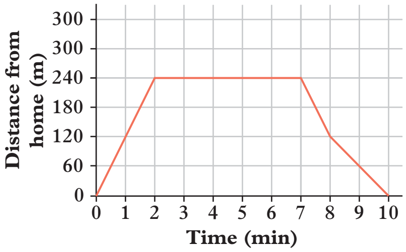
transversal
line that crosses (intersects) two or more lines



trapezium
quadrilateral with at least one pair of opposite sides that are parallel



travel graph
shows the movement of a person or object over a period of time in relation to a fixed reference point



trial
a single performance in a probability experiment; for example, rolling a die 50 times in a probability experiment is completing 50 trials

triangle
two-dimensional shape with three straight sides

unit fraction
a fraction of the form $\frac{1}{n}$, where n is a natural number; for example, $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots$

unknown
a number that is represented using a pronumeral; for example, the x in $2x = 4$

unlike fractions

fractions with different denominators; for example $\frac{2}{3}$ and $\frac{5}{4}$. *See* like fractions

variable

quantity that can take on different values and can be represented with a pronumeral

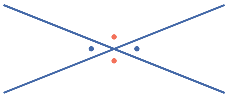
vertex (plural vertices)

1 corner point where three or more edges of a polyhedron meet. *See* polyhedron

2 the point at which two lines meet

vertically opposite angles

a pair of equal angles not adjacent to (next to) each other, formed where two lines intersect; sometimes called ‘X angles’



volume

amount of space that a 3D object occupies

whole numbers

the numbers which we use to count, plus zero: 0, 1, 2, 3, 4, ...

width

length measurement of the shortest side of a shape or object

worded form

the way we normally read numbers out loud; for example, four hundred and fifty-one is the worded form of 451

x-axis

horizontal number line that forms part of the Cartesian plane

x-coordinate

first of a pair of numbers (coordinates) that describes the point’s horizontal distance from the origin on the Cartesian plane

$(-4, 1)$

x-coordinate – horizontal distance from the origin

y-axis

vertical number line that forms part of the Cartesian plane

y-coordinate

second of a pair of numbers (coordinates) that describes the point’s vertical distance from the origin on the Cartesian plane

$(-4, 1)$

y-coordinate – vertical distance from the origin

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