Apex Exam Guide

Mathematical Methods

Year 12 QCE
Queensland Curriculum

2025 Edition

Edward Nyugen

Apex Exam Guide
Mathematical Methods
Year 12 QCE

Resource Hub Link: https://www.apexbooks.com.au/
- Subscribe for future updates.
- Contact details for any queries or feedback.
- Relevant QCE subject information.
- University scholarship information.
- Other useful information.

Key Features of the Apex Exam Guides
- Contains the past four years of QCE exam questions from 2023 to 2020, categorised based on the most up to date QCAA syllabus reviews.
- Significantly reduced overall cost compared to alternative book options.
- Future editions will be released with the most up to date QCE exams.
- Students can review chapters, specific questions or solutions at any given time throughout the course.
- Students can refer to relevant chapters to complete during class or for private study to prepare for topic tests or exams.
- Teachers can use these eBooks as a great reference point for lesson planning, developing internal assessments or when preparing class activities to ensure alignment with exam level questions.
- Mark allocation included to guide student time management.
- Fully worked answers from the QCE marking guide are included for feedback.
- Increase student confidence and enable high achievement.

How to Use Apex Exam Guides
Purchase of these eBook PDFs allows your students access to previous QCE exam questions, arranged by syllabus topics and answers from the marking guide. Revision for QCE assessments or exams becomes much more straightforward. Students can review chapters, specific questions or solutions at any given time throughout the course. Students can be referred to relevant chapters to complete during class or privately to prepare for topic tests or exams. These textbooks also act as a great reference for teachers regarding lesson planning, developing internal assessments or when preparing class activities to ensure alignment with exam level questions.

Books available in this series

Mathematics	Sciences	English
• Essential Mathematics • General Mathematics • Mathematical Methods • Specialist Mathematics	• Biology • Chemistry • Physics • Psychology	• English • Essential English • Literature
Humanities and Social Sciences	**Health and Physical Education**	**Technologies**
• Accounting • Ancient History • Business • Economics	• Health • Physical Education	• Design • Digital Solutions • Engineering

Acknowledgements
External Assessment Resources © State of Queensland (QCAA) 2024.

Contents

Unit 3 – Topic 1: The logarithmic function 2

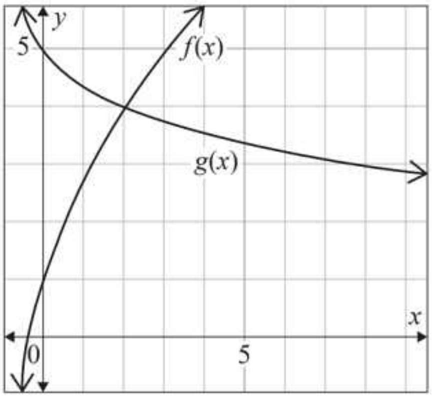
Paper 1 Section 1

2021 Paper 1 Section 1 Question 1 The logarithmic function 2	$2\log_{10}(x) - \log_{10}(3x)$ is equal to (A) $\log_{10}\left(\frac{x}{3}\right)$ (B) $\log_{10}\left(x^2 - 3x\right)$ (C) $\frac{2\log_{10}(x)}{\log_{10}(3x)}$ (D) $-\log_{10}(x)$

Paper 1 Section 2

2021 Paper 1 Section 2 Question 12 The logarithmic function 2	Solve for x in the following. a) $\log_2(5x + 7) = 5$ [2 marks] b) $\log_{10}(x + 3) + \log_{10}(x - 3) = \log_{10}(9x - 29)$ [3 marks]

2020 Paper 1 Section 2 Question 15 **The logarithmic function 2**	Solve the following equations. a) $4e^x = 100$ [1 mark] b) $2 \log_4 x - \log_4 (x - 1) = 1$ [3 marks]

2020 Paper 1 Section 2 Question 18 **The logarithmic function 2**	The function $f(x)$ has the form given by $f(x) = 3\log_2(x + a) + b$ The function $g(x)$ has the form given by $g(x) = -\log_3(x + c) + 5$ A section of the graphs of the two functions is shown.  Determine the values of a, b and c [6 marks]

2022 Paper 2 Section 1 Question 8

The logarithmic function 2

Determine the equation of the asymptote of the function $f(x) = \log_9(x - 3) - 4$.

(A) $x = -4$
(B) $x = -3$
(C) $x = 3$
(D) $x = 4$

2021 Paper 2 Section 1 Question 5

The logarithmic function 2

Solve for x given that $\log_3(x - 1) = 2$.

(A) 7
(B) 8
(C) 9
(D) 10

2020 Paper 2 Section 1 Question 2

The logarithmic function 2

The pH of a substance is a measure of its acidity and is given by the formula $\text{pH} = -\log_{10}[H^+]$ where $[H^+]$ is the concentration of hydrogen ions in moles per litre. If a solution has a pH equal to 0.2, the concentration of hydrogen ions in moles per litre is closest to

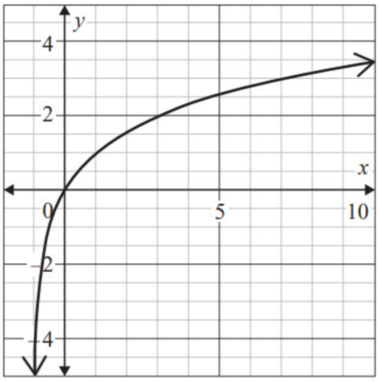
(A) 0.32
(B) 0.63
(C) 0.70
(D) 1.58

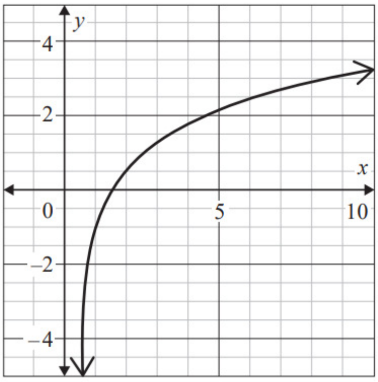
2020 Paper 2 Section 1 Question 4

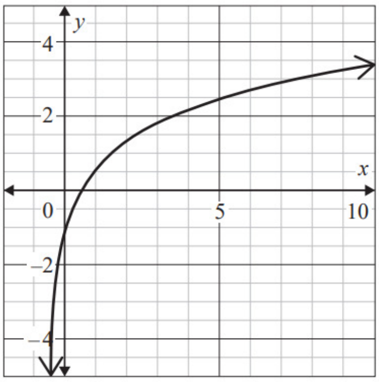
The logarithmic function 2

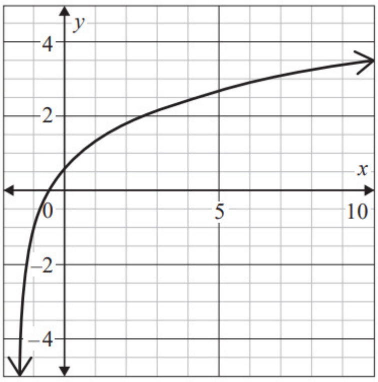
Consider the function $f(x) = \log_p(x + q)$ where $p > 1$ and $0 < q < 1$.

Which of the following could be the graph of $f(x)$?

(A)


(B)


(C)


(D)


Paper 2 Section 2

2021 Paper 2 Section 2 Question 15 **The logarithmic function 2**	A new internet search engine gives a ranking R to each website based on the function $R = \log_{10}(50h^2)$, where h is the number of hits (visits) the website has received. If a website currently has 100 hits, determine how many more hits they need to increase their ranking by 1. [4 marks]

Marking Guide – Paper 1 Section 1

2021 Paper 1 Section 1 Question 1 **The logarithmic function 2**	$2\log_{10}(x) - \log_{10}(3x)$ is equal to (A) $\log_{10}\left(\frac{x}{3}\right)$ (B) $\log_{10}\left(x^2 - 3x\right)$ (C) $\frac{2\log_{10}(x)}{\log_{10}(3x)}$ (D) $-\log_{10}(x)$ **Answer is A.**

Marking Guide – Paper 1 Section 2

2021 Paper 1 Section 2 Question 12

The logarithmic function 2

Solve for x in the following.

a) $\log_2(5x + 7) = 5$ [2 marks]

Sample Response	The response
Changing from log to index form $5x + 7 = 32$	• correctly establishes the linear equation [1 mark]
$5x = 25$ $x = 5$	• determines x [1 mark]

b) $\log_{10}(x + 3) + \log_{10}(x - 3) = \log_{10}(9x - 29)$ [3 marks]

Sample Response	The response
Using addition log law $\log_{10}(x^2 - 9) = \log_{10}(9x - 29)$	• correctly applies the log law [1 mark]
Equating and rearranging $x^2 - 9x + 20 = 0$	• establishes quadratic equation [1 mark]
Factorising $(x - 4)(x - 5) = 0$ $\rightarrow x = 4, 5$	• determines 2 solutions [1 mark]

2020 Paper 1 Section 2 Question 15

The logarithmic function 2

Solve the following equations.

a) $4e^x = 100$ [1 mark]

Sample Response	The response
$e^x = 25$ $x = \ln(25)$	• correctly determines x [1 mark]

b) $2\log_4 x - \log_4(x - 1) = 1$ [3 marks]

Sample Response	The response
Using log laws $\log_4 \frac{x^2}{x-1} = 1$	• correctly establishes equation using log laws [1 mark]
Change from log to index form $\frac{x^2}{x-1} = 4$ $x^2 - 4x + 4 = 0$	• correctly establishes the quadratic equation [1 mark]
Factorising $x = 2$	• determines x [1 mark]

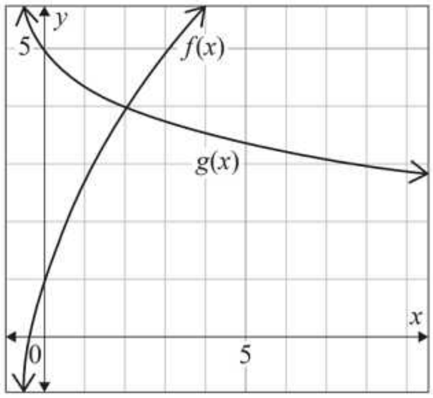
2020
Paper 1
Section 2
Question 18

The logarithmic function 2

The function $f(x)$ has the form given by $f(x) = 3\log_2(x + a) + b$

The function $g(x)$ has the form given by $g(x) = -\log_3(x + c) + 5$

A section of the graphs of the two functions is shown.



Determine the values of a, b and c [6 marks]

Sample Response	The response
Using y-intercepts $1 = 3\log_2(a) + b$ (i) $5 = -\log_3(c) + 5$ (ii) From (ii) $c = 1$	• correctly determines c [1 mark]
Using point of intersection (2, 4) $4 = 3\log_2(2 + a) + b$(iii) Solving simultaneously (i) and (iii) $b = 1 - 3\log_2(a)$(i) $b = 4 - 3\log_2(2 + a)$(ii)	• correctly establishes two equations in a and b [1 mark]
Equating $1 - 3\log_2(a) = 4 - 3\log_2(2 + a)$ $3\log_2(2 + a) - 3\log_2(a) = 3$	• correctly selects procedure to solve for unknowns [1 mark]
$3\ (\log_2\left(\frac{2 + a}{a}\right)) = 3$ $\frac{2 + a}{a} = 2$ $a = 2$	• correctly determines a [1 mark]
$\therefore b = -2$	• determines b [1 mark] • shows logical organisation communicating key steps [1 mark]

Marking Guide – Paper 2 Section 1

2022 Paper 2 Section 1 Question 8 The logarithmic function 2	Determine the equation of the asymptote of the function $f(x) = \log_9(x - 3) - 4$. (A) $x = -4$ (B) $x = -3$ **(C) $x = 3$ – Answer** (D) $x = 4$

2021 Paper 2 Section 1 Question 5 The logarithmic function 2	Solve for x given that $\log_3(x - 1) = 2$. (A) 7 (B) 8 (C) 9 **(D) 10 – Answer**

2020 Paper 2 Section 1 Question 2 The logarithmic function 2	The pH of a substance is a measure of its acidity and is given by the formula $\text{pH} = -\log_{10}[H^+]$ where $[H^+]$ is the concentration of hydrogen ions in moles per litre. If a solution has a pH equal to 0.2, the concentration of hydrogen ions in moles per litre is closest to (A) 0.32 **(B) 0.63 – Answer** (C) 0.70 (D) 1.58

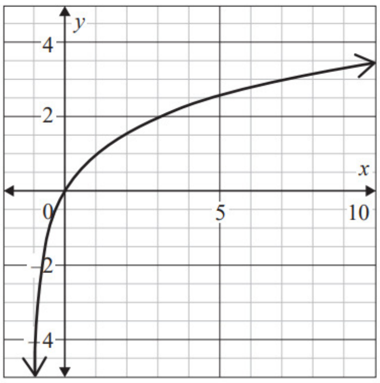
2020 Paper 2 Section 1 Question 4

The logarithmic function 2

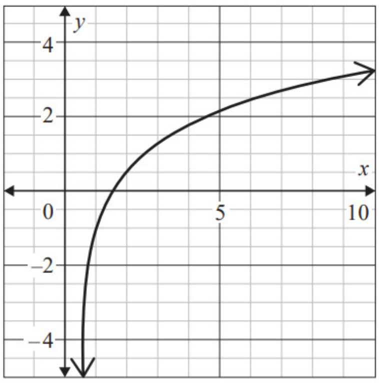
Consider the function $f(x) = \log_p(x + q)$ where $p > 1$ and $0 < q < 1$.

Which of the following could be the graph of $f(x)$?

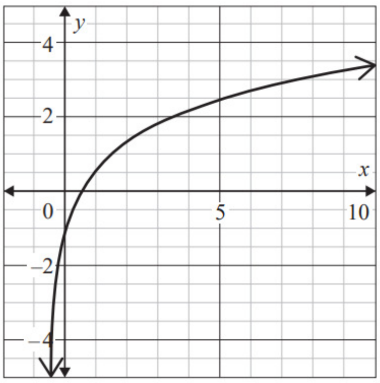
(A)



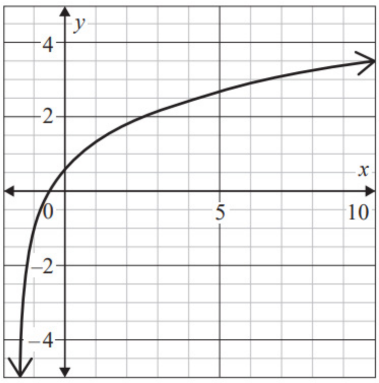
(B)



(C)



(D)



Answer is C.

Marking Guide – Paper 2 Section 2

<table>
<tr>
<td>2021
Paper 2
Section 2
Question 15

The logarithmic function 2</td>
<td>
A new internet search engine gives a ranking R to each website based on the function $R = \log_{10}(50h^2)$, where h is the number of hits (visits) the website has received.

If a website currently has 100 hits, determine how many more hits they need to increase their ranking by 1. [4 marks]

<table>
<tr><th>Sample Response</th><th>The response</th></tr>
<tr><td>Method 1
Current ranking
$= \log_{10}(50 \times 100^2)$
$= 5.69897$
∴ increased ranking is 6.69897</td><td>• correctly determines the website's increased ranking [1 mark]</td></tr>
<tr><td>$6.69897 = \log_{10}(50h^2)$
$h = 316.228$</td><td>• determines number of hits for increased ranking [1 mark]</td></tr>
<tr><td>∴ the website requires an additional 217 hits to increase their ranking by 1.</td><td>• provides reasonable solution for number of additional hits [1 mark]
• shows logical organisation communicating key steps [1 mark]</td></tr>
<tr><td>Method 2
$R_{old} = \log_{10}(50h^2)$
$R_{old} = \log_{10}(50) + 2\log_{10}(h)$

Let k = number of additional hits
$R_{new} = \log_{10}(50) + 2\log_{10}(h+k)$
$R_{new} = R_{old} + 1$</td><td>• correctly determines the equation using increased ranking of 1 [1 mark]</td></tr>
<tr><td>$\log_{10}(50) + 2\log_{10}(h+k) = \log_{10}(50) + 2\log_{10}(h) + 1$
$2\log_{10}(h+k) = 2\log_{10}(h) + 1$
$2\log_{10}(h+k) - 2\log_{10}(h) = 1$
$\log_{10}\frac{(h+k)}{h} = \frac{1}{2}$
$\frac{(h+k)}{h} = \sqrt{10}$
Currently $h = 100$
$k = 216.228$</td><td>• determines number of additional hits for increased ranking [1 mark]</td></tr>
<tr><td>∴ the website requires an additional 217 hits to increase their ranking by 1.</td><td>• provides reasonable solution for number of additional hits [1 mark]
• shows logical organisation communicating key steps [1 mark]</td></tr>
</table>
</td>
</tr>
</table>

Unit 3 – Topic 2: Further differentiation and applications 2

Paper 1 Section 1

2023 Paper 1 Section 1 Question 1 **Further differentiation and applications 2**	$e^{\ln(x)}$ is equivalent to (A) 0 (B) 1 (C) x (D) $\frac{1}{x}$

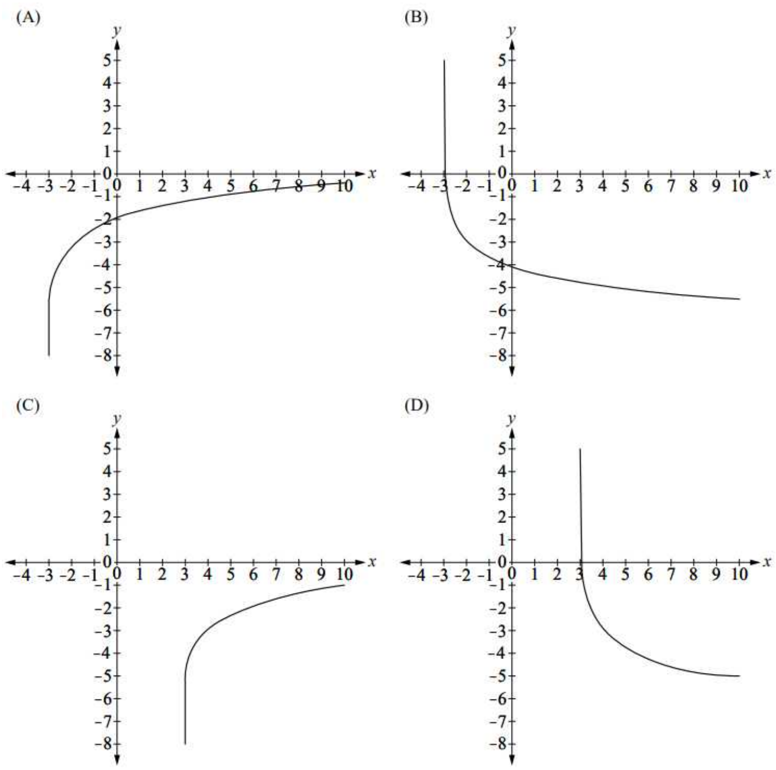
2023 Paper 1 Section 1 Question 4 **Further differentiation and applications 2**	If the gradient of the function $f(x)$ is given by $\frac{20}{x^3}$, then $f(x)$ is equal to (A) $-\frac{60}{x^4}+c$ (B) $-\frac{5}{x^4}+c$ (C) $-\frac{10}{x^2}+c$ (D) $-\frac{40}{x^2}+c$

2023 Paper 1 Section 1 Question 6 **Further differentiation and applications 2**	Substitutions for h are used to estimate the limit of $\frac{a^h-1}{h}$ as $h \to 0$. Which sequence is the most appropriate? (A) −4, −2, −1, −0.5, −0.25, −0.125 … (B) −0.05, −0.1, −0.2, −0.4, −0.8 … (C) 2, 1, 0, −1, −2, −3 … (D) 1, 2, 3, 4, 5, 6 …

2022
Paper 1
Section 1
Question 6

Further differentiation and applications 2

Which graph represents the function $f(x) =- -3 - \ln(x+3)$?



2020
Paper 1
Section 1
Question 9

Further differentiation and applications 2

Determine $\int \frac{x+1}{x^2+2x}dx$

(A) $\ln\left(\frac{1}{2x+2}\right)+c$

(B) $\ln(2x+2)+c$

(C) $\frac{1}{2}\ln(x^2+2x)+c$

(D) $2\ln(x^2+2x)+c$

Paper 1 Section 2

2023 Paper 1 Section 2 Question 16 **Further differentiation and applications 2**	Solve for x in the equation $4+7e^{-2x}=3e^{2x}$. (5 marks)

2023
Paper 1
Section 2
Question 19

Further differentiation and applications 2

Jaxon and Shari each own a shop and have recorded the number of customers entering their shop on two consecutive days.

	Day 1	Day 2
Jaxon's customers	40	30
Shari's customers	10	20
Total	50	50

The number of daily customers for each shop can be modelled by the equation $y = A\ln(Bx)$, where x is the day and y is the number of customers. The constants A and B are different for each shop.

Determine algebraically whether the total number of customers for Jaxon and Shari's shops will be the same every day in the future.

2022 Paper 1 Section 2 Question 11 **Further differentiation and applications 2**	Solve for x in the following. a) $\ln(2x) = 5$ [2 marks] b) $\log_4(4x + 16) - \log_4(x^2 - 2) = 1$ [3 marks]

2022 Paper 1 Section 2 Question 13 **Further differentiation and applications 2**	a) Determine the derivative of $f(x) = 3e^{2x+1}$ [1 mark] b) Given that $g(x) = \frac{\ln(x)}{x}$, determine the simplest value of $g'(e)$. [3 marks] c) Determine the second derivative of $h(x) = x \sin(x)$. (Give your answer in simplest form.) [5 marks]

2021 Paper 1 Section 2 Question 11 **Further differentiation and applications 2**	Determine the derivative with respect to x of the following functions. a) $y = (e^x + 1)^3$ [2 marks] b) $y = \frac{\sin(x)}{x^2}$ (Give your answer in simplest form.) [3 marks]

2021 Paper 1 Section 2 Question 20 **Further differentiation and applications 2**	The population of rabbits (P) on an island, in hundreds, is given by $P(t) = t^2\ln(3t) + 6$, $t > 0$, where t is time in years. Determine the intervals of time when the population is increasing and the intervals when it is decreasing. [7 marks]

2020 Paper 1 Section 2 Question 11 **Further differentiation and applications 2**	Determine the derivative of each of the following with respect to *x*. a) $y = \frac{1}{\sin{(x)}}$ [1 mark] b) $y = x^2 \times e^{-x}$ Express your answer in factorised form. [2 marks]

2020 Paper 1 Section 2 Question 17 Further differentiation and applications 2	The volume of water in a tank is represented by a function of the form $V(t) = Ae^{kt}$, where V is in litres and t is in minutes. Initially, the volume is 100 litres and it is decreasing by 50 litres per minute. Determine the time at which the volume is decreasing at the rate of $\frac{50}{7}$ litres per minute. Express your answer in the form $\ln(a)$.

Paper 2 Section 1

2023 Paper 2 Section 1 Question 1 **Further differentiation and applications 2**	If $f(x) = \sin(3x)$, determine the value of $f'\left(\frac{\pi}{8}\right)$. (A) 2.772 (B) 1.148 (C) 0.929 (D) 0.383

2023 Paper 2 Section 1 Question 5 **Further differentiation and applications 2**	Solve $\ln(x) + \ln(3.70) = \ln(9.25)$ for x. (A) 0.92 (B) 1.71 (C) 2.50 (D) 5.55

2023 Paper 2 Section 1 Question 9 **Further differentiation and applications 2**	If $f(x) = e^{3x}(x+1)^2$ and $f'(x) = ae^{3x}(x+1)$, determine the expression for a. (A) $3x+5$ (B) $3x+3$ (C) $5x+5$ (D) $5x+3$

2022 Paper 2 Section 1 Question 10 **Further differentiation and applications 2**	The solution of $e^{2x-3} = 42$ is (A) 1.48 (B) 2.31 (C) 3.37 (D) 4.54

2021 Paper 2 Section 1 Question 7 **Further differentiation and applications 2**	Determine $f(x)$, given $f'(x) = 6x^2 + \frac{1}{x^2} + \frac{1}{x}$ and $f(1) = 5$. (A) $f(x) = 2x^3 + \frac{3}{x^3} + \ln(x) - 1$ (B) $f(x) = 2x^3 - \frac{1}{x} + \ln(x) + 4$ (C) $f(x) = 2x^3 - \frac{1}{x} + \frac{2}{x^2} + 2$ (D) $f(x) = 2x^3 + \frac{3}{x^3} + \frac{2}{x^2} - 2$

2021 Paper 2 Section 1 Question 9 **Further differentiation and applications 2**	The graphs of the functions $f(x) = 2e^x + 5$ and $g(x) = \frac{3}{e^x}$ intersect at point A. Determine the coordinates of point A. (A) (1.609, 15) (B) (1.099, 1) (C) (0.4065, 2) (D) (–0.693, 6)

2020 Paper 2 Section 1 Question 1 **Further differentiation and applications 2**	The limit of $\frac{12^h-1}{h}$ as h approaches 0 is closest to (A) 0.0 (B) 1.0 (C) 2.5 (D) 3.0

2023 Paper 2 Section 2 Question 19 Further differentiation and applications 2	Over a suitable domain, a hill has a cross-sectional area given by $\int h(x)\,dx = \frac{a}{b}e^{bx} + c$, where: • a, b and c are constants, $b \neq 0$ • $h(x)$ represents vertical distance (m), x represents horizontal distance (m). It is known that $h(0) = 1.22$ and $h(40) = 25$. Where the gradient of the hill is 0.86 there is a tree stump. A second tree stump is located further up the hill. The difference in hill gradient between the two tree stumps is 0.44. A surveyor predicts that the vertical distance separating the two tree stumps is between 7.5 m and 8.5 m. Evaluate the reasonableness of this prediction. (6 marks)

Marking Guide – Paper 1 Section 1

2023 Paper 1 Section 1 Question 1 **Further differentiation and applications 2**	$e^{\ln(x)}$ is equivalent to (A) 0 (B) 1 (C) x (D) $\frac{1}{x}$ **Answer is C.**

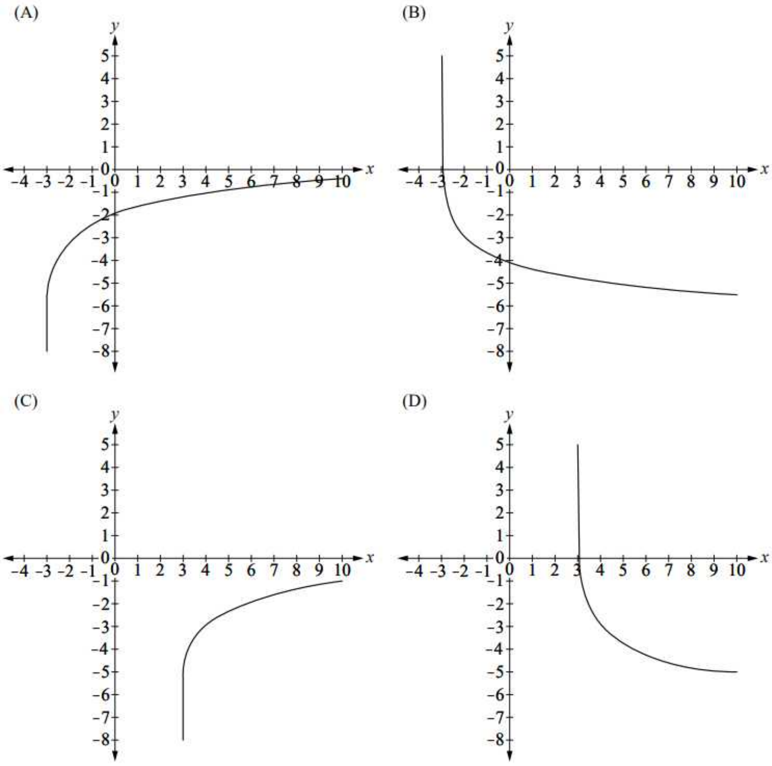
2023 Paper 1 Section 1 Question 4 **Further differentiation and applications 2**	If the gradient of the function $f(x)$ is given by $\frac{20}{x^3}$, then $f(x)$ is equal to (A) $-\frac{60}{x^4}+c$ (B) $-\frac{5}{x^4}+c$ (C) $-\frac{10}{x^2}+c$ (D) $-\frac{40}{x^2}+c$ **Answer is C.**

2023 Paper 1 Section 1 Question 6 **Further differentiation and applications 2**	Substitutions for h are used to estimate the limit of $\frac{a^h-1}{h}$ as $h \to 0$. Which sequence is the most appropriate? (A) $-4, -2, -1, -0.5, -0.25, -0.125 \ldots$ (B) $-0.05, -0.1, -0.2, -0.4, -0.8 \ldots$ (C) $2, 1, 0, -1, -2, -3 \ldots$ (D) $1, 2, 3, 4, 5, 6 \ldots$ **Answer is A.**

2022
Paper 1
Section 1
Question 6

Further differentiation and applications 2

Which graph represents the function $f(x) =- -3 - \ln(x + 3)$?



Answer is B.

2020
Paper 1
Section 1
Question 9

Further differentiation and applications 2

Determine $\int \frac{x+1}{x^2+2x}\,dx$

(A) $\ln\left(\frac{1}{2x+2}\right)+c$

(B) $\ln(2x+2)+c$

(C) $\frac{1}{2}\ln(x^2+2x)+c$

(D) $2\ln(x^2+2x)+c$

Answer is C.

2023 Paper 1 Section 2 Question 16

Further differentiation and applications 2

Solve for x in the equation $4+7e^{-2x}=3e^{2x}$. (5 marks)

Sample response	The response
$4+7e^{-2x}=3e^{2x}$ $4e^{2x}+7=3e^{4x}$ $3e^{4x}-4e^{2x}-7=0$	• correctly removes the negative index in the equation **[1 mark]** • rearranges equation to equate to zero **[1 mark]**
$(3e^{2x}-7)(e^{2x}+1)=0$	• factorises the equation **[1 mark]**
$3e^{2x}-7=0$ $\quad$ $e^{2x}+1=0$ $3e^{2x}=7$ $\quad$ $e^{2x}=-1$ $e^{2x}=\frac{7}{3}$ $\quad$ not possible $\ln\left(\frac{7}{3}\right)=2x$ $x=\frac{1}{2}\ln\left(\frac{7}{3}\right)$	• rejects the non-feasible solution **[1 mark]** • determines a feasible solution for x **[1 mark]**

2023 Paper 1 Section 2 Question 19

Further differentiation and applications 2

Jaxon and Shari each own a shop and have recorded the number of customers entering their shop on two consecutive days.

	Day 1	Day 2
Jaxon's customers	40	30
Shari's customers	10	20
Total	50	50

The number of daily customers for each shop can be modelled by the equation $y = A\ \ln(Bx)$, where x is the day and y is the number of customers. The constants A and B are different for each shop.

Determine algebraically whether the total number of customers for Jaxon and Shari's shops will be the same every day in the future.

Sample response	The response
For each shop use $y = A\ln(Bx)$ as the model. Find the constants A and B for each shop. Shari's shop $10 = A\ln B$ .... (1) $20 = A\ln(2B)$ .... (2) $20 = A\ln 2 + A\ln B$ $20 = A\ln 2 + 10$ $\quad$ from (1) $A\ln 2 = 10$ $A=\frac{10}{\ln 2}$	• correctly determines A (or B) for Shari's shop **[1 mark]**
Substitute into (1) $10=\frac{10}{\ln 2}\ln B$ $\ln B=\frac{10\ln 2}{10}$ $\ln B = \ln 2$ $B = 2$ $y = A\ln Bx$ $y=\left(\frac{10}{\ln 2}\right)\ln(2x)$ … Shari's shop model	• determines the model for Shari's shop **[1 mark]**

	Jaxon's shop $40 = A\ln B$ .... (1) $30 = A\ln(2B)$ .... (2) $30 = A\ln 2 + A\ln B$ $30 = A\ln 2 + 40$ from (1) $A\ln 2 = -10$ $A = \frac{-10}{\ln 2}$ Substitute into (1) $40 = \frac{-10}{\ln 2}\ln B$ $\ln B = \frac{40\ln 2}{-10}$	• correctly determines A (or B) for Jaxon's shop **[1 mark]**
	$\ln B = \left(\frac{40}{-10}\right)\ln 2$ $\ln B = -4\ln 2$ $\ln B = \ln 2^{(-4)}$ $\therefore B = 2^{-4} = \frac{1}{16}$ $y = A\ln(Bx)$ $y = \left(\frac{-10}{\ln 2}\right)\ln(\frac{1}{16}x)$ ... Jaxon's shop model	• determines the model for Jaxon's shop **[1 mark]**
	On any day, x, the sum of the two models is: $= \left(\frac{10}{\ln 2}\right)\ln(2x) + \left(\frac{-10}{\ln 2}\right)\ln(\frac{1}{16}x)$ $= \left(\frac{10}{\ln 2}\right)\ln(2x) - \left(\frac{10}{\ln 2}\right)\ln(\frac{1}{16}x)$ $= \left(\frac{10}{\ln 2}\right)[\ln(2x) - \ln(\frac{1}{16}x)]$ $= \left(\frac{10}{\ln 2}\right)\ln(\frac{2x}{\frac{1}{16}x})$ $= \left(\frac{10}{\ln 2}\right)\ln(\frac{2}{\frac{1}{16}})$ $= \left(\frac{10}{\ln 2}\right)\ln(32)$ $= \frac{10\ln 32}{\ln 2}$ $= \frac{10\ln(2^5)}{\ln 2}$ $= \frac{10\times 5\ln(2)}{\ln 2}$ = 50	• determines an expression for the total number of daily customers in both shops **[1 mark]** • justifies the sum obtained by explaining mathematical reasoning **[1 mark]**
	The sum of the models does result in the same number of customers, i.e. 50 on any day.	• provides an appraisal by interpreting the result of the analysis of the sum of the two models performed **[1 mark]**

2022 Paper 1 Section 2 Question 11

Further differentiation and applications 2

Solve for x in the following.

a) $\ln(2x) = 5$ [2 marks]

Sample Response	The response
Change from log to index form and rearrange $2x = e^5$	• correctly rearranges equation to remove ln [1 mark]
$x = \frac{e^5}{2}$	• correctly determines x [1 mark]

b) $\log_4(4x + 16) - \log_4(x^2 - 2) = 1$ [3 marks]

Sample Response	The response
Using log laws $\log_4\left(\frac{4x+16}{x^2-2}\right) = 1$	• correctly applies the log law [1 mark]
$(4x + 16) = 4(x^2 - 2)$ $x^2 - x - 6 = 0$	• correctly determines quadratic equation to solve [1 mark]
$(x - 3)(x + 2) = 0$ $x = 3, -2$	• determines possible values for x [1 mark]

2022 Paper 1 Section 2 Question 13

Further differentiation and applications 2

a) Determine the derivative of $f(x) = 3e^{2x+1}$ [1 mark]

Sample Response	The response
$f'(x) = 6e^{2x+1}$	• correctly determines the derivative [1 mark]

b) Given that $g(x) = \frac{\ln(x)}{x}$, determine the simplest value of $g'(e)$. [3 marks]

Sample Response	The response
$g(x) = \frac{\ln(x)}{x}$ Let $u = \ln(x)$ and $v = x$ $\therefore \frac{du}{dx} = \frac{1}{x}$ and $\frac{dv}{dx} = 1$ $g'(x) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$	• correctly identifies the use of the product or quotient rule [1 mark]
$= \frac{x \times \frac{1}{x} - \ln(x) \times 1}{x^2}$ $= \frac{1 - \ln(x)}{x^2}$	• correctly determines the derivative [1 mark]
$g'(e) = \frac{1 - \ln(e)}{e^2}$	• determines the derivative at the given value [1 mark]

c) Determine the second derivative of $h(x) = x\sin(x)$. (Give your answer in simplest form.) [5 marks]

Sample Response	The response
$h(x) = x\sin(x)$ Let $u = x$ and $v = \sin(x)$ $\therefore \frac{du}{dx} = 1$ and $\frac{dv}{dx} = \cos(x)$ $h'(x) = u\frac{dv}{dx} + v\frac{du}{dx}$	• correctly identifies the use of the product rule [1 mark]
$= x \times \cos(x) + \sin(x) \times 1$ $x\cos(x) + \sin(x)$	• correctly determines the derivative [1 mark]
Let $u = x$ and $v = \cos(x)$	• correctly identifies the use of the product rule and that

	$\therefore \frac{\mathrm{du}}{\mathrm{dx}} = 1$ and $\frac{dv}{dx} = -\sin(x)$ $h''(x) = u\frac{dv}{dx} + v\frac{du}{dx} + \cos(x)$	$\frac{d}{dx}\big(h(x) + g(x)\big) = \frac{d}{dx}h(x) + \frac{d}{dx}g(x)$
	$= x \times -\sin(x) + \cos(x) \times 1 + \cos(x)$	• determines the second derivative [1 mark]
	$= 2\cos(x) - x\sin(x)$	• simplifies the second derivative [1 mark]

2021
Paper 1
Section 2
Question 11

Further differentiation and applications 2

Determine the derivative with respect to x of the following functions.

a) $y = (e^x + 1)^3$ [2 marks]

Sample Response	The response
Let $u = e^x + 1$ Using the chain rule	• correctly identifies the use of the chain rule [1 mark]
$\frac{dy}{dx} = 3e^x(e^x + 1)^2$	• correctly determines the derivative [1 mark]

b) $y = \frac{\sin(x)}{x^2}$ (Give your answer in simplest form.) [3 marks]

Sample Response	The response
Using the quotient rule	• correctly identifies the use of the quotient rule [1 mark]
$\frac{dy}{dx} = \frac{x^2\cos(x) - \sin(x)2x}{(x^2)^2}$ $\frac{dy}{dx} = \frac{x\cos(x) - 2\sin(x)}{x^3}$	• correctly determines the derivative [1 mark]
	• provides derivative in simplest form [1 mark]

2021
Paper 1
Section 2
Question 20

Further differentiation and applications 2

The population of rabbits (P) on an island, in hundreds, is given by $P(t) = t^2\ln(3t) + 6$, $t > 0$, where t is time in years.

Determine the intervals of time when the population is increasing and the intervals when it is decreasing. [7 marks]

Sample Response	The response
Method 1 To determine intervals $P'(t) = t + 2t \times \ln(3t)$	• correctly determines $P'(t)$ [1 mark]
Critical points $P'(t) = 0$ $0 = t(1 + 2\ln(3t))$ $t = 0$ (reject)	• correctly determines the rejected solution for t [1 mark]
$1 + 2\ln(3t) = 0 \rightarrow t = \dfrac{1}{3\sqrt{e}}$	• determines t-ordinate of critical point [1 mark]
To determine the nature of $t = \dfrac{1}{3\sqrt{e}}$ $P''(t) = 3 + 2\ln(3t)$ $P''\left(\dfrac{1}{3\sqrt{e}}\right) = 3 + 2\ln\left(\dfrac{1}{\sqrt{e}}\right)$ $P''\left(\dfrac{1}{3\sqrt{e}}\right) = 2$	• determines value of $P''(t)$ at critical point [1 mark]
$\therefore$ the point is a minimum.	• determines nature of critical point [1 mark]
Therefore the population decreases for $0 < t <= \dfrac{1}{3\sqrt{e}}$ but increases for $t > \dfrac{1}{3\sqrt{e}}$	• communicates when the population is increasing and when it is decreasing [1 mark] • shows logical organisation communicating key steps [1 mark]
Method 2 To determine intervals $P'(t) = t + 2t \times \ln(3t)$ $P'(t) = t(1 + 2\ln(3t))$	• correctly determines $P'(t)$ [1 mark]
Population is increasing when $P'(t) > 0$ $t(1 + 2\ln(3t)) > 0$	• correctly identifies the time interval required to determine increasing population [1 mark]
Given $t > 0$	• identifies relevance of domain [1 mark]
$\therefore (1 + 2\ln(3t)) > 0$	• establishes inequality in t [1 mark]
$\ln(3t) > \dfrac{-1}{2}$ $t > \dfrac{1}{3\sqrt{e}}$	• determines interval when population is increasing [1 mark]
$\therefore$ the population decreases for $0 > t > \dfrac{1}{3\sqrt{e}}$	• determines interval when population is decreasing [1 mark] • shows logical organisation communicating key steps [1 mark]

2020 Paper 1 Section 2 Question 11 **Further differentiation and applications 2**	Determine the derivative of each of the following with respect to *x*. a) $y = \frac{1}{\sin(x)}$ [1 mark]

Sample Response	The response
$f'(x) = \frac{-\cos(x)}{(\sin(x))^2}$	• correctly determines the derivative [1 mark]

b) $y = x^2 \times e^{-x}$
Express your answer in factorised form. [2 marks]

Sample Response	The response
$f'(x) = -x^2e^{-x} - 2xe^{-x}$	• correctly determines the derivative in expanded form [1 mark]
$= xe^{-x}(-x+2)$	• determines factorised form of derivative [1 mark]

2020 Paper 1 Section 2 Question 17 **Further differentiation and applications 2**	The volume of water in a tank is represented by a function of the form

$V(t) = Ae^{kt}$, where V is in litres and t is in minutes.

Initially, the volume is 100 litres and it is decreasing by 50 litres per minute.

Determine the time at which the volume is decreasing at the rate of $\frac{50}{7}$ litres per minute.

Express your answer in the form ln(*a*).

Sample Response	The response
At $t = 0$, $V = 100$ $V(0) = 100 = Ae^{k\times 0}$ $A = 100$	• correctly determines A [1 mark]
$\therefore V(t) = 100e^{kt}$ $\therefore V'(t) = 100ke^{kt}$ At $t = 0$, $V'(t) = -50$ $-50 = 100k$ $k = \frac{-1}{2}$	• determines k [1 mark]
So $V(t) = 100\,e^{-0.5t}$ and $V'(t) = -50e^{-0.5t}$ Determine t when $V'(t) = \frac{-50}{7}$ $\frac{1}{7} = e^{-0.5t}$	• establishes equation in t [1 mark]
$-0.5t = \ln\left(\frac{1}{7}\right)$ $t = -2\ln\left(\frac{1}{7}\right)$	• determines t [1 mark]
$t = \ln(49)$	• determines t in required form [1 mark] • shows logical organisation communicating key steps [1 mark]

Marking Guide – Paper 2 Section 1

2023 Paper 2 Section 1 Question 1 **Further differentiation and applications 2**	If $f(x) = \sin(3x)$, determine the value of $f'\left(\frac{\pi}{8}\right)$. (A) 2.772 (B) 1.148 (C) 0.929 (D) 0.383 **Answer is B.**

2023 Paper 2 Section 1 Question 5 **Further differentiation and applications 2**	Solve $\ln(x) + \ln(3.70) = \ln(9.25)$ for x. (A) 0.92 (B) 1.71 (C) 2.50 (D) 5.55 **Answer is C.**

2023 Paper 2 Section 1 Question 9 **Further differentiation and applications 2**	If $f(x) = e^{3x}(x+1)^2$ and $f'(x) = ae^{3x}(x+1)$, determine the expression for a. (A) $3x+5$ (B) $3x+3$ (C) $5x+5$ (D) $5x+3$ **Answer is A.**

2022 Paper 2 Section 1 Question 10 **Further differentiation and applications 2**	The solution of $e^{2x-3} = 42$ is (A) 1.48 (B) 2.31 **(C) 3.37 – Answer** (D) 4.54

2021 Paper 2 Section 1 Question 7 **Further differentiation and applications 2**	Determine $f(x)$, given $f'(x)=6x^2+\frac{1}{x^2}+\frac{1}{x}$ and $f(1)=5$. (A) $f(x)=2x^3+\frac{3}{x^3}+\ln(x)-1$ (B) $f(x)=2x^3-\frac{1}{x}+\ln(x)+4$ (C) $f(x)=2x^3-\frac{1}{x}+\frac{2}{x^2}+2$ (D) $f(x)=2x^3+\frac{3}{x^3}+\frac{2}{x^2}-2$ **Answer is B.**

2021 Paper 2 Section 1 Question 9 **Further differentiation and applications 2**	The graphs of the functions $f(x) = 2e^x + 5$ and $g(x) = \frac{3}{e^x}$ intersect at point A. Determine the coordinates of point A. (A) (1.609, 15) (B) (1.099, 1) (C) (0.4065, 2) **(D) (–0.693, 6) – Answer**

2020 Paper 2 Section 1 Question 1 **Further differentiation and applications 2**	The limit of $\frac{12^h-1}{h}$ as h approaches 0 is closest to (A) 0.0 (B) 1.0 **(C) 2.5 – Answer** (D) 3.0

Marking Guide – Paper 2 Section 2

<table>
<tr>
<td>2023
Paper 2
Section 2
Question 19

Further differentiation and applications 2</td>
<td>
Over a suitable domain, a hill has a cross-sectional area given by $\int h(x)\,dx = \frac{a}{b}e^{bx} + c$, where:
• a, b and c are constants, $b \neq 0$
• $h(x)$ represents vertical distance (m), x represents horizontal distance (m).

It is known that $h(0) = 1.22$ and $h(40) = 25$.

Where the gradient of the hill is 0.86 there is a tree stump. A second tree stump is located further up the hill. The difference in hill gradient between the two tree stumps is 0.44.

A surveyor predicts that the vertical distance separating the two tree stumps is between 7.5 m and 8.5 m. Evaluate the reasonableness of this prediction.

(6 marks)

<table>
<tr><th>Sample response</th><th>The response</th></tr>
<tr>
<td>The hill:
$\int h(x)dx = \frac{a}{b}e^{bx} + c$
Differentiating wrt x:
$h(x) = ae^{bx}$
Using $(0, 1.22)$
$1.22 = a \times e^0$
$a = 1.22$
$\therefore h(x) = 1.22e^{bx}$
Using $(40, 25)$
$25 = 1.22e^{40b}$
$\frac{25}{1.22} = e^{40b}$
$40b = \ln\left(\frac{25}{1.22}\right)$
$b = \frac{\ln\left(\frac{25}{1.22}\right)}{40}$
$b = 0.0755$
$\therefore h(x) = 1.22e^{0.0755x}$

The gradient of the hill:
$h'(x) = 0.09211e^{0.0755x}$</td>
<td>• correctly determines the model for the hill with constants a and b found [1 mark]

• differentiates h(x) to determine the gradient of the hill formula [1 mark]</td>
</tr>
<tr>
<td>Determine the location of first tree stump:
Using $h'(x) = 0.86$
$0.86 = 0.09211e^{0.0755x}$
Solving for x:
$x = 29.5887$
$h(29.5887) = 11.3907$</td>
<td>• determines the y-coordinate location of the first tree stump where the hill gradient is 0.86 [1 mark]</td>
</tr>
</table>
</td>
</tr>
</table>

	Determine the location of the second tree stump: The gradient is 0.86+0.44=1.3 Using $h'(x)=1.3$ $1.3=0.09211e^{0.0755x}$ Solving for *x*: $x=35.0614$ $h(35.0614)=17.2185$	• **determines the *y*-coordinate of the second tree stump [1 mark]**
	Vertical distance between the tree stumps: $=h(35.0614)-h(29.5887)$ $=17.2185-11.3907$ $=5.8278\text{ m}$ Evaluation of the prediction: The vertical distance of 5.8278 m is NOT between 7.5 m and 8.5 m, **so the prediction is NOT reasonable.**	• **determines the vertical distance between the tree stumps [1 mark]** • **provides appropriate statement of reasonableness [1 mark]**

Unit 3 – Topic 3: Integrals

Paper 1 Section 1

2023 Paper 1 Section 1 Question 5 **Integrals**	Determine $\int_1^3 \frac{1}{2x} dx$. (A) $\frac{1}{2}\ln 6$ (B) $\frac{1}{2}\ln 5$ (C) $\frac{1}{2}\ln 4$ (D) $\frac{1}{2}\ln 3$

2023 Paper 1 Section 1 Question 9 **Integrals**	Determine $\int_0^3 \pi \sin\left(\frac{\pi}{3}x\right) dx$. (A) 3 (B) 6 (C) −3 (D) −6

2022 Paper 1 Section 1 Question 3 **Integrals**	The area between the curve $y = 9 - x^2$ and the x-axis is (A) 12 units^2 (B) 18 units^2 (C) 36 units^2 (D) 54 units^2

2022 Paper 1 Section 1 Question 9 **Integrals**	The approximate area under the curve $f(x) = \sqrt{2x+1}$ between $x = 0$ and $x = 4$ using the trapezoidal rule with four strips is (A) $2+\sqrt{3}+\sqrt{5}+\sqrt{7}$ (B) $2+2\left(\sqrt{3}+\sqrt{5}+\sqrt{7}\right)$ (C) $4+2\left(\sqrt{3}+\sqrt{5}+\sqrt{7}\right)$ (D) $4+\sqrt{3}+\sqrt{5}+\sqrt{7}$

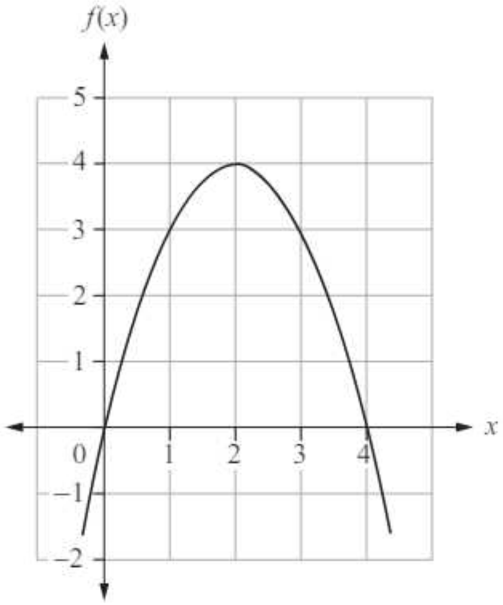
2021 **Paper 1** **Section 1** **Question 3** **Integrals**	Determine $\int 10e^{4x}dx$ (A) $\frac{10e^{4x+1}}{4x+1}+c$ (B) $40e^{4x}+c$ (C) $\frac{5}{2}e^{4x}+c$ (D) $2e^{5x}+c$

2021 **Paper 1** **Section 1** **Question 7** **Integrals**	Determine $\int_1^3 (2x+3)dx$ (A) 2 (B) 4 (C) 14 (D) 16

2020 **Paper 1** **Section 1** **Question 1** **Integrals**	The graphs of $f(x) = e^x$ and $g(x) = x^2 - 1$ are shown. The area of the shaded section bounded by these graphs between the lines $x = 0$ and $x = 1$ is (A) $1-e$ (B) $e-2$ (C) $e-\frac{5}{3}$ (D) $e-\frac{1}{3}$

2020 **Paper 1** **Section 1** **Question 2** **Integrals**	Determine $\int \frac{e^x+1}{e^x}dx$ (A) $x-e^{-x}+c$ (B) $x+e^{-x}+c$ (C) $1+xe^{-x}+c$ (D) $x+xe^{-x}+c$

2020 **Paper 1** **Section 1** **Question 3** **Integrals**	Determine $2\int(4x+6)^3\,dx$ (A) $16(4x+6)^4+c$ (B) $8(4x+6)^4+c$ (C) $\frac{(4x+6)^4}{2}+c$ (D) $\frac{(4x+6)^4}{8}+c$

2023 Paper 1 Section 2 Question 12 **Integrals**	The region bounded by the x-axis and the curve of $f(x) = x(4-x)$ represents the plan of a garden bed. All measurements are in metres.  a) Estimate the area of the garden bed using sums of the form $\sum_i f(x_i)\delta x_i$ where $x_1 = 1$, $x_2 = 2$, $x_3 = 3$ and $x_4 = 4$. *[1 mark]* b) Use a definite integral to determine the area of the garden bed. [3 marks]

2023 Paper 1 Section 2 Question 14 Integrals	The rate of change in the number of bacteria in a science experiment is represented by $\frac{dP}{dt} = e^{2t}$, $t \geq 0$, where t represents the time (hours) since starting the experiment and P represents the number of bacteria present (thousands). Initially there are 60 000 bacteria present, i.e. $P(0) = 60$. a) Determine the equation for $P(t)$. *[2 marks]*

b) Determine the change in the number of bacteria during the third hour. Express your answer in terms of e. [2 marks]

c) Determine how long it will take for the number of bacteria present to double after starting the experiment. [2 marks]

2022 Paper 1 Section 2 Question 14 Integrals	The rate that water fills an empty vessel is given by $\frac{dV}{dt} = 0.25e^{0.25t}$ (in litres per hour), $0 \leq t \leq 8\ln$, where t is time (in hours). a) Determine the function that represents the volume of water in the vessel (in litres). [2 marks] The vessel is full when t = 8ln(6). b) Determine the volume of water, to the nearest litre, the vessel can hold when full. [2 marks] The table shows the approximate rate the water flows into the vessel at certain times.

t	$\frac{dV}{dt}$
0	0.25
1	0.32
2	0.41
3	0.53

c) Use information from the table and the trapezoidal rule to determine the approximate volume of water in the vessel after three hours. [2 marks]

2022 Paper 1 Section 2 Question 17 Integrals	Determine the value of b given $\int_a^b 3x^2 dx = 117$ and $\int_a^{b-1} 3x^2 dx = 56$ for $b > 1$.

2022 Paper 1 Section 2 Question 19 **Integrals**	Two triangles are said to be similar if their corresponding angles are congruent and the corresponding sides are in proportion, e.g. if ΔUVW is similar to ΔXYZ then $$\angle U = \angle X,\ \angle V = \angle Y \text{ and } \angle W = \angle Z \text{ and } \frac{UV}{XY} = \frac{VW}{YZ} = \frac{UW}{XZ}$$ Two parallel walls AB and CD, where the northern ends are A and C respectively, are joined by a fence from B to C. The wall AB is 20 metres long, the angle ABC = 30° and the fence BC is 10 metres long. A new fence is being built from A to a point P somewhere along CD. The new fence AP will cross the original fence BC at O. Let OB = x metres, where $0 < x < 10$. Determine the value of x that minimises the total area enclosed by ΔOBA and ΔOCP. Verify that this total area is a minimum.

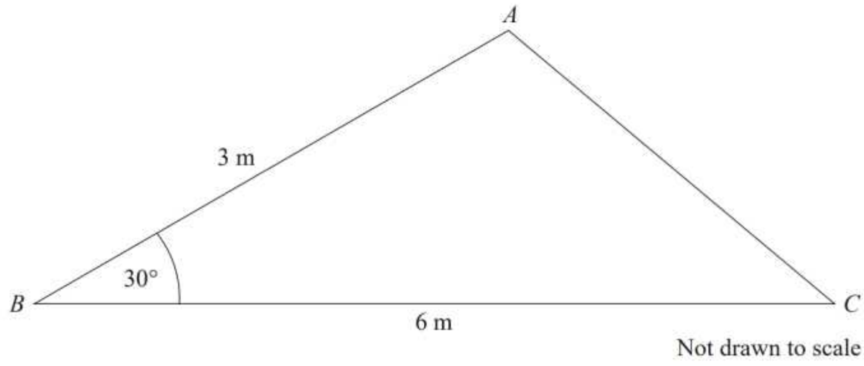
2021 Paper 1 Section 2 Question 13 **Integrals**	Consider the functions $f(x) = x^2$ and $g(x) = 4x$. a) Determine the x-coordinates of the points of intersection of the graphs of the two functions. [2 marks] b) Use the results from Question 13a) to calculate the area enclosed by the graphs of $f(x)$ and $g(x)$. [3 marks]

2021 Paper 1 Section 2 Question 15 **Integrals**	In the isosceles triangle *ABC*, angle *C* is 120° and side a is 4 cm. a) Draw the triangle, indicating all given information. [1 mark] Note: If you make a mistake in the diagram, cancel it by ruling a single diagonal line through your work and use the additional diagram on page 17 of this question and response book. b) Calculate the area of the triangle in cm^2. (Give your answer in simplest form.) [3 marks]

2021 Paper 1 Section 2 Question 18 **Integrals**	The graph of $y = f(x)$, where $f(x)$ is the quadratic function $f(x) = ax^2 + bx + 4$, is shown. Two regions of the area between the graph of $y = f(x)$ and the x-axis are shaded. Region P has an area of $\frac{13}{6}$ units^2 and Region Q has an area of $\frac{43}{6}$ units^2. Determine the values of a and b. [4 marks]

2020 Paper 1 Section 2 Question 12 **Integrals**	An object is moving in a straight line from a fixed point. The object is at the origin initially. The acceleration a (in m s^{-2}) of the object is given by $a(t) = \pi \cos(\pi t) \qquad t \geq 0$, where t is time in seconds. The velocity at $t = 1$ is 0.5 m s^{-1} a) Determine the initial acceleration. [1 mark] b) Determine the initial velocity. [2 marks]

	c) Determine the displacement after one second. [2 marks]

2020 Paper 1 Section 2 Question 14 Integrals	Determine the area of the triangle shown. 

2020 Paper 1 Section 2 Question 20 **Integrals**	At the end of the first stage of its growth cycle, a species of tree has a height of 5 metres and a trunk radius of 15 cm. In the second stage of its growth cycle, the tree stays at this height for the next 10 years. However, the growth rate of the trunk radius (in cm per year) varies over the 10 years and is given by the function $$r(t)=1.5+\sin\left(\frac{\pi t}{5}\right)$$ Assume the density (mass per unit volume) of the tree trunk is approximately 1 g/cm and the tree trunk is in the shape of a cylinder. Determine the ratio of the trunk's mass at the end of the second stage to its mass at the end of the first stage.

Paper 2 Section 1

2023 Paper 2 Section 1 Question 4 **Integrals**	The displacement (m) of a moving particle is given by $d = e^{0.5t} - 1$, where t is time (s). The acceleration (ms^{-2}) of the particle when $t = 4$ is (A) 7.3891 (B) 6.3891 (C) 3.6945 (D) 1.8473

2023 Paper 2 Section 1 Question 6 **Integrals**	$\int_a^{5a} \frac{1}{x+a} dx,\ a \neq 0$ is (A) 1.7918 (B) 1.6094 (C) 1.3863 (D) 1.0986

2023 Paper 2 Section 1 Question 8 **Integrals**	The number of koalas in a conservation park is modelled by $N = 15\ln(7t+1)$, $t \geq 1$, where t represents the time (years) since the park opened. There were 20 koalas in the park when it opened. Determine the approximate rate of change in the number of koalas when $t = 3$. (A) 46 (B) 26 (C) 25 (D) 5

2022 Paper 2 Section 1 Question 1 **Integrals**	The position (in cm) of a particle is given by $x = \cos(4t)$, where t is time (in seconds). The velocity of the particle when t = 5 is (A) 1.6323 cm s^{-1} (B) 0.4081 cm s^{-1} (C) -0.9129 cm s^{-1} (D) -3.6518 cm s^{-1}

2022 Paper 2 Section 1 Question 7 **Integrals**	A marble moves in one direction in a straight line with velocity v = 2ln(t + 1) (in metres per second) where t is time (in seconds) since the marble passed through the origin. Determine the distance from the origin the marble has rolled after 4 seconds. (A) 0.40 m (B) 1.60 m (C) 3.22 m (D) 8.09 m

2021 **Paper 2** **Section 1** **Question 2** **Integrals**	A substance is being heated such that its temperature T in °C after t minutes is given by the function $T = 2e^{0.5t}$ The first integer value of t for which the instantaneous rate of change of temperature is greater than 100°C per minute is (A) $t = 10$ (B) $t = 9$ (C) $t = 8$ (D) $t = 7$

2021 **Paper 2** **Section 1** **Question 4** **Integrals**	Using the trapezoidal rule with an interval size of 1, the approximate value of the integral $\int_0^3 0.5^x dx$ is (A) 1.25 (B) 1.26 (C) 1.31 (D) 1.88

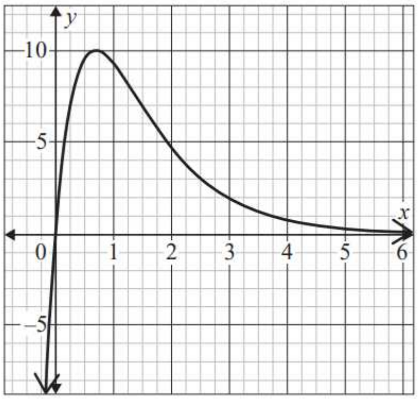
2021 **Paper 2** **Section 1** **Question 8** **Integrals**	The displacement (in metres) of a particle is given by $s(t) = -3\cos(t) + 2\sin(t)$, where t is in seconds. The instantaneous velocity of the particle at time $t = \frac{\pi}{2}$ seconds is (A) –3m s^{-1} (B) –2m s^{-1} (C) 2 m s^{-1} (D) 3 m s^{-1}

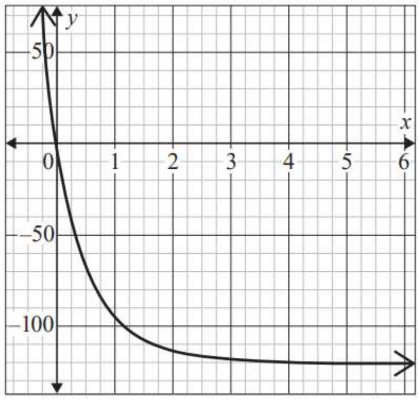
2021 **Paper 2** **Section 1** **Question 10** **Integrals**	An object travels in a straight line so that its velocity at time t seconds is given by $v(t) = 2t + \sin(2t)$. Determine the expression for acceleration as a function of time. (A) $a(t) = 2 + 2\cos(2t)$ (B) $a(t) = 2 - \frac{1}{2}\cos(2t)$ (C) $a(t) = t^2 + 2\cos(2t)$ (D) $a(t) = t^2 - \frac{1}{2}\cos(2t)$

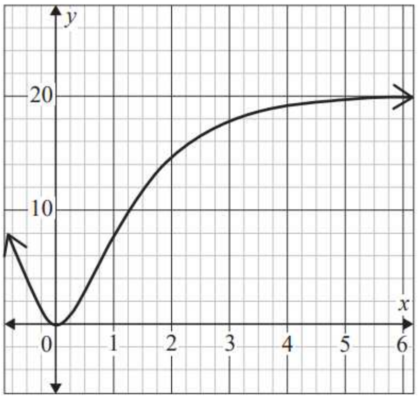
2020 **Paper 2** **Section 1** **Question 3** **Integrals**	Let R be the region enclosed by the graph of $y = xe^x$, the x-axis, and the lines $x = -1$ and $x = 1$. The area of R is closest to (A) 0.74 (B) 1.26 (C) 2.35 (D) 3.09

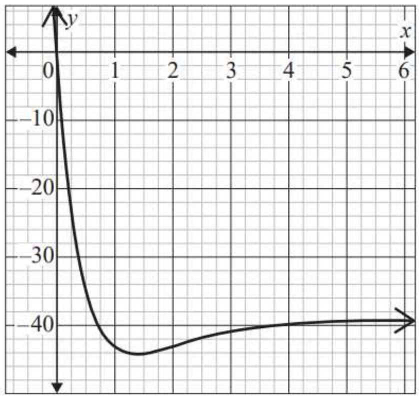
<table>
<tr><td>2020
Paper 2
Section 1
Question 5

Integrals</td><td>An object moves in a straight line with a velocity v given by
$$v(t) = 40(e^{-t} - e^{-2t}) \text{ m s}^{-1} \text{ where } t \geq 0$$
The object is at the origin initially. The displacement–time graph in the first 6 seconds is

(A)


(B)


(C)


(D)

</td></tr>
</table>

<table>
<tr><td>2020
Paper 2
Section 1
Question 6

Integrals</td><td>Oil is leaking from a tanker at the rate of $r(t) = 9000e^{-0.2t}$ litres per hour, where t is in hours.

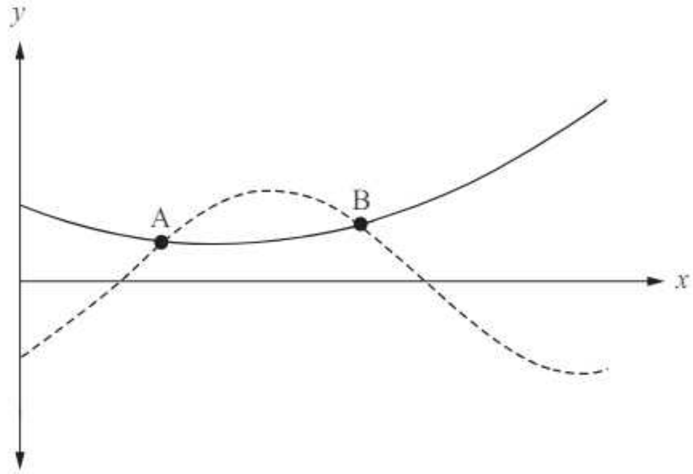
Determine how much oil leaks from the tanker (to the nearest litre) from time $t = 0$ to time $t = 10$.

(A) 38 910 litres
(B) 8756 litres
(C) 7782 litres
(D) 1556 litres</td></tr>
</table>

<table>
<tr><td>2020
Paper 2
Section 1
Question 9

Integrals</td><td>The displacement of a particle (in metres) at time t (in seconds) is represented by the function
$$s(t) = t\ln(t) - t,\ 0 < t < 4$$
Determine the approximate acceleration of the particle at time $t = 3$.

(A) 0.66 m s^{-2}
(B) 0.33 m s^{-2}
(C) –0.33 m s^{-2}
(D) –0.66 m s^{-2}</td></tr>
</table>

2023 Paper 2 Section 2 Question 13 **Integrals**	The curved lines represent graphs of the equations $y = x^2 - 4x + 8$ and $y = 10\cos(x+10)$.  a) Determine the coordinates of the points of intersection A and B. [1 mark] b) State an integral expression representing the area enclosed by the two graphs. [2 marks] c) Determine the area enclosed by the two graphs. [1 mark]

2023 Paper 2 Section 2 Question 16 **Integrals**	A particle is moving in a straight line. The velocity (ms^{-1}) of the particle is given by $v(t)=\dfrac{20\sin(2t)}{6-5\cos(2t)}$, $t \geq 0$, where t is time (s) after moving from its initial position. The initial position of the particle is +6.0 m from the origin. a) Use calculus methods to determine an equation for the position of the particle from the origin at any time t. [3 marks] b) Determine the position of the particle relative to the origin when it first reaches maximum velocity. [3 marks]

2022 Paper 2 Section 2 Question 13 **Integrals**	A sandy beach has a fence on one side and ocean on the other. The width of the beach is the distance (in metres) from the fence to the water's edge. The width, $w(t)$, at a certain point is given by $$w(t) = a + b\sin\left(\frac{\pi}{6}t - \frac{\pi}{3}\right),\ 0 \le t \le 24$$ where t is time (in hours) since 6 am. The width of the beach is 8 metres at 8 am and 3 metres at 5 pm. a) Determine a and b. [2 marks] b) Determine the rate of change of the width of the beach at 8 am and the first time after this when this rate of change is repeated. [2 marks]

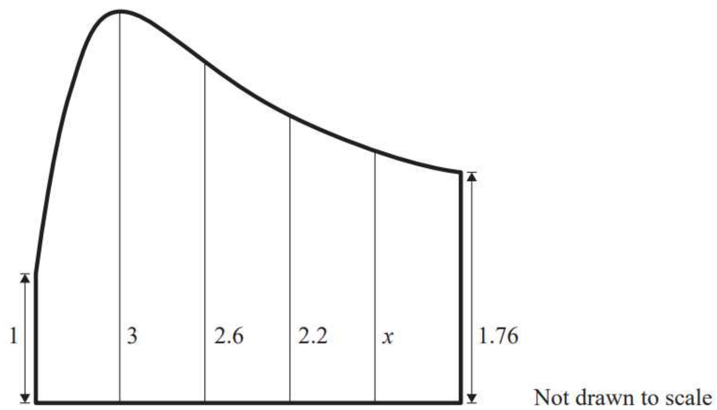
2022 Paper 2 Section 2 Question 17 **Integrals**	A snail is travelling along a straight path from point A. The snail's velocity (cm min^{-1}) is modelled by $v(t) = 1.4\ln(1 + t^2)$, where t is time (in minutes) for $0 \leq t \leq 15$. An ant passes point A 12 minutes after the snail and follows the snail's path. The ant moves with a constant acceleration of 2 cm min^{-2} and passes the snail at t =15 minutes. Determine the ant's velocity at point A. [4 marks]

2022 Paper 2 Section 2 Question 19 Integrals	Flying foxes enter and leave a fruit-growing region every evening. The rate at which the flying foxes enter the region is modelled by the function $$A(t) = 42\sin\left(0.03t - \frac{\pi}{3}\right) + 71,\ 0 \le t \le 240$$ The rate at which the flying foxes leave the region is modelled by the function $$L(t) = 42\sin\left(0.04t - \frac{\pi}{3}\right) + 42,\ 0 \le t \le 240$$ Both *A* (*t*) and *L* (*t*) are measured in animals per minute and t is measured in minutes after 7 pm. There are 100 flying foxes in the region at 7 pm. Determine the maximum number of flying foxes in the region and the time that this occurs. [4 marks]

2021 Paper 2 Section 2 Question 11 Integrals	Consider the function $f(x) = e^x \sin(x), 0 \le x \le 2\pi$ a) State the exact values of the x-intercepts of the graph of $f(x)$. [2 marks] b) Write an expression for the area enclosed between the graph of $f(x)$ and the x-axis. [2 marks] c) Determine the area enclosed between the graph of $f(x)$ and the x-axis to the nearest square unit. [1 mark]

2021 Paper 2 Section 2 Question 12 **Integrals**	The velocity function of an object in m s^{-1} is given by $v(t) = \cos\left(6t + \frac{\pi}{2}\right) + 2, 0 \le t \le 5$. Initially, the object is at the origin. a) Determine the displacement function. [2 marks] b) What is the displacement of the object from the origin, in metres (m), after three seconds? [2 marks]

2020 Paper 2 Section 2 Question 12 **Integrals**	The rates of change in population for two cities are given by City A: $A'(t) = \dfrac{45}{t+1}$ City B: $B'(t) = 105e^{0.03t}$ where t is the number of years since 2018 and both $A'(t)$ and $B'(t)$ are measured in people per year. At the beginning of 2018, City A had a population of 5000, and City B had a population of 3500. a) Determine the population models for both cities. [3 marks] b) Use the information in 12a) to predict the population of City B at the beginning of 2028. [1 mark] c) Use the information in 12a) to predict the year in which the population of both cities will be the same. [3 marks]

2020 **Paper 2** **Section 2** **Question 15** **Integrals**	A field is divided into five sections as shown. The width of each section is 1 metre. The perpendicular height, in metres, of each section is given in the diagram. The area of the field was approximated using the trapezoidal rule and found to be 11.12 m^2.  a) Determine the height marked x on the diagram. [2 marks] b) Determine the area of the field, given the shape of the field is modelled by the function [1 mark] $f(x) = \frac{4x}{x^2+1} + 1, \quad 0 \le x \le 5$

2020 Paper 2 Section 2 Question 19 Integrals	Consider the following information when completing this question.

Consider the following information when completing this question.

$$\text{The length of a curve } y = f(x) \text{ over the interval } [a,b] = \int_a^b \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$$

You are driving along a road with a vertical distance above sea level D (in metres) given by the function

$$D(x) = 300 + \ln(x^2 - 3x + e)$$

where x is the horizontal distance from an initial point of measurement (in kilometres) at sea level.

Assume that if x is positive you are east of the initial point of measurement and if x is negative you are west of the initial point of measurement.

You start your drive along the road at a horizontal distance of 10 kilometres west of the initial point of measurement and drive until you are at a horizontal distance of 10 kilometres east of the initial point.

Determine the time you spend driving downhill, if you drive downhill at an average speed of 40 km/h. [7 marks]

Marking Guide – Paper 1 Section 1

2023 Paper 1 Section 1 Question 5 Integrals	Determine $\int_1^3 \frac{1}{2x} dx$. (A) $\frac{1}{2}\ln 6$ (B) $\frac{1}{2}\ln 5$ (C) $\frac{1}{2}\ln 4$ (D) $\frac{1}{2}\ln 3$ **Answer is D.**

2023 Paper 1 Section 1 Question 9 Integrals	Determine $\int_0^3 \pi \sin\left(\frac{\pi}{3}x\right) dx$. (A) 3 (B) 6 (C) −3 (D) −6 **Answer is B.**

2022 Paper 1 Section 1 Question 3 Integrals	The area between the curve $y = 9 - x^2$ and the x-axis is (A) 12 units^2 (B) 18 units^2 **(C) 36 units^2 – Answer** (D) 54 units^2

2022 Paper 1 Section 1 Question 9 Integrals	The approximate area under the curve $f(x) = \sqrt{2x+1}$ between $x = 0$ and $x = 4$ using the trapezoidal rule with four strips is (A) $2+\sqrt{3}+\sqrt{5}+\sqrt{7}$ (B) $2+2\left(\sqrt{3}+\sqrt{5}+\sqrt{7}\right)$ (C) $4+2\left(\sqrt{3}+\sqrt{5}+\sqrt{7}\right)$ (D) $4+\sqrt{3}+\sqrt{5}+\sqrt{7}$ **Answer is A.**

2021 Paper 1 Section 1 Question 3 Integrals	Determine $\int 10e^{4x}dx$ (A) $\frac{10e^{4x+1}}{4x+1}+c$ (B) $40e^{4x}+c$ (C) $\frac{5}{2}e^{4x}+c$ (D) $2e^{5x}+c$ **Answer is C.**

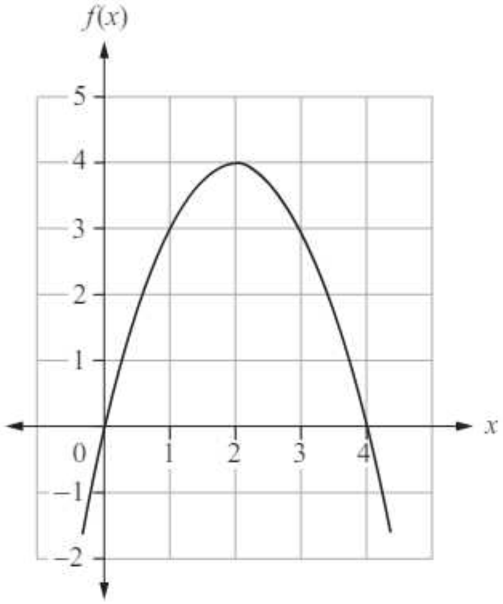
2021 Paper 1 Section 1 Question 7 Integrals	Determine $\int_1^3 (2x+3)dx$ (A) 2 (B) 4 **(C) 14 – Answer** (D) 16

2020 Paper 1 Section 1 Question 1 Integrals	The graphs of $f(x) = e^x$ and $g(x) = x^2 - 1$ are shown. The area of the shaded section bounded by these graphs between the lines $x = 0$ and $x = 1$ is (A) $1-e$ (B) $e-2$ (C) $e-\frac{5}{3}$ (D) $e-\frac{1}{3}$ **Answer is D.**

2020 Paper 1 Section 1 Question 2 Integrals	Determine $\int \frac{e^x+1}{e^x}dx$ (A) $x-e^{-x}+c$ (B) $x+e^{-x}+c$ (C) $1+xe^{-x}+c$ (D) $x+xe^{-x}+c$ **Answer is A.**

2020 Paper 1 Section 1 Question 3 **Integrals**	Determine $2\int (4x + 6)^3\, dx$ (A) $16(4x+6)^4 + c$ (B) $8(4x+6)^4 + c$ (C) $\dfrac{(4x+6)^4}{2} + c$ (D) $\dfrac{(4x+6)^4}{8} + c$ **Answer is D.**

2023 Paper 1 Section 2 Question 12 Integrals	The region bounded by the x-axis and the curve of $f(x) = x(4-x)$ represents the plan of a garden bed. All measurements are in metres.



a) Estimate the area of the garden bed using sums of the form $\sum_i f(x_i)\delta x_i$ where $x_1 = 1$, $x_2 = 2$, $x_3 = 3$ and $x_4 = 4$. *[1 mark]*

Sample response	The response
Let $f(x) = x(4-x)$ $f(1) = 1(4-1) = 3$ $f(2) = 2(4-2) = 4$ $f(3) = 3(4-3) = 3$ $f(4) = 4(4-4) = 0$ $\sum f(x_i)\delta x_i = (3 \times 1) + (4 \times 1) + (3 \times 1) + (0 \times 1)$ $= 10$ metres^2	• correctly calculates the described estimation **[1 mark]**

	b) Use a definite integral to determine the area of the garden bed. [3 marks]

Sample response	The response
Area $= \int_0^4 x(4-x)\,dx$ $= \int_0^4 (-x^2 + 4x)\,dx$	• correctly states the definite integral required **[1 mark]**
$= \left[-\frac{1}{3}x^3 + 2x^2\right]_0^4$	• determines the integration of the function **[1 mark]**
$= -\frac{1}{3}(4)^3 + 2(4)^2$ $= -\frac{64}{3} + 32$ $= \frac{32}{3}$ $= 10\frac{2}{3}\,\text{m}^2$	• calculates the area under the curve by integration **[1 mark]**

2023 Paper 1 Section 2 Question 14

Integrals

The rate of change in the number of bacteria in a science experiment is represented by $\frac{dP}{dt} = e^{2t},\ t \geq 0$, where t represents the time (hours) since starting the experiment and P represents the number of bacteria present (thousands). Initially there are 60 000 bacteria present, i.e. $P(0) = 60$.

a) Determine the equation for $P(t)$. *[2 marks]*

Sample response	The response
$P(t) = \int e^{2t}dt$ $= \frac{1}{2}e^{2t} + c$ $P(0) = 60$ $60 = \frac{1}{2}e^{2(0)} + c$ $c = 59.5$ $P(t) = \frac{1}{2}e^{2t} + 59.5$	• correctly determines the integral from the rate **[1 mark]** • determines the value of *c* **[1 mark]**

b) Determine the change in the number of bacteria during the third hour. Express your answer in terms of e. [2 marks]

Sample response	The response
The total change in the number of bacteria during the third hour ($t = 2$ to $t = 3$) is: $P(3) - P(2)$ $= \frac{1}{2}e^{2\times3} + 59.5 - \left(\frac{1}{2}e^{2\times2} + 59.5\right)$ $= \frac{1}{2}e^6 - \frac{1}{2}e^4$ bacteria	• identifies the method required for the third hour calculation **[1 mark]** • determines an expression for the total change **[1 mark]**

c) Determine how long it will take for the number of bacteria present to double after starting the experiment. [2 marks]

Sample response	The response
$60 \times 2 = 120$	• correctly determines the doubled population **[1 mark]**
$120 = \frac{1}{2}e^{2t} + 59.5$ $60.5 = \frac{1}{2}e^{2t}$ $121 = e^{2t}$ $\ln 121 = 2t$ $t = \frac{1}{2}\ln 121$	• determines expression for the time required **[1 mark]**

2022
Paper 1
Section 2
Question 14

Integrals

The rate that water fills an empty vessel is given by $\frac{dV}{dt} = 0.25e^{0.25t}$ (in litres per hour), $0 \le t \le$ 8ln, where t is time (in hours).

a) Determine the function that represents the volume of water in the vessel (in litres). [2 marks]

Sample Response	The response
$V = \int 0.25e^{0.25t}dt$ $= e^{0.25t} + c$ When $t = 0$, $V = 0$ $\therefore 0 = e^{0.25\times 0} + c$	• correctly determines the integral of the function $VV(tt)$ [1 mark]
$\therefore c = -1$ $\therefore V = e^{0.25t} - 1$	• determines the value of c [1 mark]

The vessel is full when t = 8ln(6).
b) Determine the volume of water, to the nearest litre, the vessel can hold when full. [2 marks]

Sample Response	The response
$V(8\ln(6)) = e^{0.25\times 8\ln(6)} - 1$ $= 36 - 1$	• determines the simplified exponential term [1 mark]
$= 35$	• determines number of litres [1 mark]

The table shows the approximate rate the water flows into the vessel at certain times.

t	$\frac{dV}{dt}$
0	0.25
1	0.32
2	0.41
3	0.53

	c) Use information from the table and the trapezoidal rule to determine the approximate volume of water in the vessel after three hours. [2 marks]

Sample Response	The response
Using trapezoidal rule Volume after 3 hours $= \frac{1}{2}\left(0.25 + 0.53 + 2(0.32 + 0.41)\right)$	• establishes expression for approximate number of litres of water in vessel after 3 hours [1 mark]
Volume after 3 hours = 1.12 litres	• determines approximate number of litres [1 mark]

2022 Paper 1 Section 2 Question 17

Integrals

Determine the value of b given $\int_a^b 3x^2dx = 117$ and $\int_a^{b-1} 3x^2dx = 56$ for $b > 1$.

Sample Response	The response
Using first integral $F(b) - F(a) = 117$ $b^3 - a^3 = 117$ … Equation I	• correctly establishes a formula for one of the integrals [1 mark]
Using second integral $(b-1)^3 - a^3 = 56$ … Equation II Equation I – Equation II $b^3 - (b-1)^3 = 61$	• determines equation in b [1 mark]
$b^3 - (b^3 - 3b^2 + 3b - 1) = 61$ $3b^2 - 3b - 60 = 0$ $b^2 - b - 20 = 0$ $(b-5)(b+4) = 0$ $b = -4, 5$	• determines values of b [1 mark]
Given $b > 1$ $\therefore b = 5$	• evaluates the reasonableness of solutions [1 mark]

2022
Paper 1
Section 2
Question 19

Integrals

Two triangles are said to be similar if their corresponding angles are congruent and the corresponding sides are in proportion, e.g. if ΔUVW is similar to ΔXYZ then

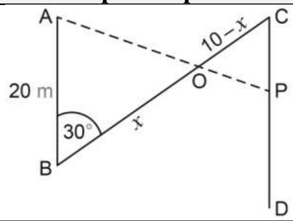
$$\angle U = \angle X,\ \angle V = \angle Y \text{ and } \angle W = \angle Z \text{ and } \frac{UV}{XY} = \frac{VW}{YZ} = \frac{UW}{XZ}$$

Two parallel walls AB and CD, where the northern ends are A and C respectively, are joined by a fence from B to C. The wall AB is 20 metres long, the angle ABC = 30° and the fence BC is 10 metres long.

A new fence is being built from A to a point P somewhere along CD. The new fence AP will cross the original fence BC at O.

Let OB = x metres, where $0 < x < 10$.

Determine the value of x that minimises the total area enclosed by ΔOBA and ΔOCP. Verify that this total area is a minimum.

Sample Response	**The response**
	• correctly uses all of the given information to draw a labelled diagram [1 mark]
Total area *OBA* and *OCP* (A_T)(using area rule) $= \frac{1}{2} \times 20 \times x \times \sin 30° + \frac{1}{2} \times (10 - x) \times CP \times \sin 30°$	• correctly establishes a formula for the total area [1 mark]
Given $\frac{CP}{AB} = \frac{OC}{OB}$ $\therefore \frac{CP}{20} = \frac{10 - x}{x}$ $\therefore CP = \frac{20(10 - x)}{x}$	• correctly determines an expression for CP in terms of xx [1 mark]
$A_T = \frac{1}{2} \times 20 \times x \times \sin 30°$ $+ \frac{1}{2} \times (10 - x) \times \frac{20(10 - x)}{x} \times \sin 30°$ $= 5x + \frac{5}{x}(10 - x)^2$ $A_T = \frac{10x^2 - 100x + 500}{x}$	• determines simplified version of the formula for total area [1 mark]
Differentiate A_T $A_T = 10x + 500x^{-1} - 100$ $A'_T = 10 - 500x^{-2}$ Let $A'_T = 0$ $\therefore 0 = 10 - 500x^{-2}$	• determines an equation to solve for stationary points [1 mark]
$\therefore \frac{500}{x^2} = 10$ $\therefore x = \pm\sqrt{50}$ x is a positive length $\therefore x = \sqrt{50}$	• evaluates the reasonableness of solutions [1 mark]
Verifying using $f''(x)$ $f'(x) = 10 - 500x^{-2}$ $f''(x) = \frac{1000}{x^3}$ $f''(\sqrt{50}) > 0$ ∴ minimum Therefore when $x = \sqrt{50}$ the total area is minimised.	• verifies solution [1 mark]

2021 Paper 1 Section 2 Question 13

Integrals

Consider the functions $f(x) = x^2$ and $g(x) = 4x$.

a) Determine the x-coordinates of the points of intersection of the graphs of the two functions. [2 marks]

Sample Response	The response
Solving simultaneously $x^2 = 4x$	• correctly uses the simultaneous procedure [1 mark]
Rearranging and factorising $x(x-4) = 0$ $\therefore x = 0$ and $x = 4$	• correctly determines both the x-intercept ordinates [1 mark]

b) Use the results from Question 13a) to calculate the area enclosed by the graphs of $f(x)$ and $g(x)$. [3 marks]

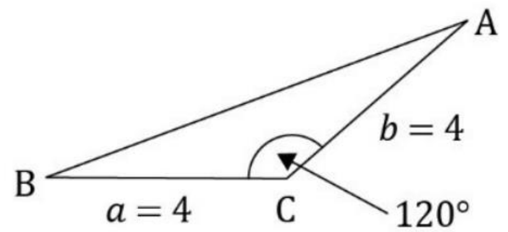
Sample Response	The response
Area $= \int_0^4 (4x - x^2)dx$ $= 2x^2 - \frac{x^3}{3}\Big\|_0^4$	• correctly determines the integral [1 mark]
$= \left(2 \times 4^2 - \frac{4^3}{3}\right) - 0$	• substitutes limits into integral [1 mark]
$= \frac{32}{3}$ square units	• determines area [1 mark]

2021 Paper 1 Section 2 Question 15

Integrals

In the isosceles triangle ABC, angle C is 120° and side a is 4 cm.

a) Draw the triangle, indicating all given information. [1 mark]

Sample Response	The response
	• correctly labels the given angle and sides in the isosceles triangle [1 mark]

b) Calculate the area of the triangle in cm^2. (Give your answer in simplest form.) [3 marks]

Sample Response	The response
Area $= \frac{1}{2} \times 4 \times 4 \times \sin 120°$	• establishes expression for the area [1 mark]
$= \frac{1}{2} \times 4 \times 4 \times \frac{\sqrt{3}}{2}$	• correctly determines the exact value of sine [1 mark]
Area $= 4\sqrt{3}\text{cm}^2$	• determines area in simplest form [1 mark]

2021
Paper 1
Section 2
Question 18

Integrals

The graph of $y = f(x)$, where $f(x)$ is the quadratic function $f(x) = ax^2 + bx + 4$, is shown.

Two regions of the area between the graph of $y = f(x)$ and the x-axis are shaded.

Region P has an area of $\frac{13}{6}$ units^2 and Region Q has an area of $\frac{43}{6}$ units^2.

Determine the values of a and b. [4 marks]

Sample Response	The response
$\int_{-1}^{0} f(x)dx = \frac{13}{6}$ $\int_{0}^{1} f(x)dx = \frac{43}{6}$ $\frac{ax^3}{3} + \frac{bx^2}{2} + 4x + c \Big\vert_{-1}^{0} = \frac{13}{6}$ (i) $\frac{ax^3}{3} + \frac{bx^2}{2} + 4x + c \Big\vert_{-1}^{0} = \frac{43}{6}$ (ii)	• correctly identifies the use of integrals [1 mark]
From (i) $0 - \left(\frac{-a}{3} + \frac{b}{2} - 4\right) = \frac{13}{6}$ $2a - 3b = -11$ (A) From (ii) $2a + 3b = 19$ (B)	• correctly determines the two equations in the two unknowns a and b [1 mark]
(A)−(B) $-6b = -30$ $b = 5$	• determines b [1 mark]
Sub into (A) $2a = 4$ $a = 2$ $\therefore f(x) = 2x^2 + 5x + 4$	• determines a [1 mark]

2020
Paper 1
Section 2
Question 12

Integrals

An object is moving in a straight line from a fixed point. The object is at the origin initially.

The acceleration a (in m s^{-2}) of the object is given by

$$a(t) = \pi \cos(\pi t) \qquad t \geq 0, \text{ where } t \text{ is time in seconds.}$$

The velocity at $t = 1$ is 0.5 m s^{-1}

a) Determine the initial acceleration. [1 mark]

Sample Response	The response
$a(0) = \pi \cos(\pi \times 0)$ $a(0) = \pi$ m s^{-2}	• correctly determines the initial acceleration [1 mark]

b) Determine the initial velocity. [2 marks]

Sample Response	The response
$\int a(t)dt = v(t)$ $v(t) = \sin(\pi t) + c$	• correctly determines the general function $v(t)$ [1 mark]
Given $v = 0.5$ when $t = 1$ $0.5 = \sin(\pi) + c$ $c = 0.5$ $v(0) = \sin(0) + 0.5$ Initial velocity is 0.5 m s^{-1}	• determines initial velocity [1 mark]

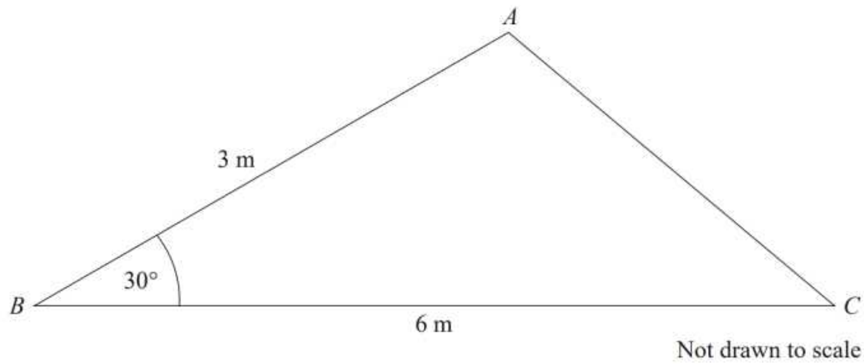
	c) Determine the displacement after one second. [2 marks]

Sample Response	The response
$\int v(t)dt = s(t)$ $s(t) = -\frac{1}{\pi}\cos(\pi t) + 0.5t + c$	• determines general function $s(t)$ [1 mark]
Given $s = 0$ when $t = 0$ $0 = \frac{-1}{\pi}\cos(0) + 0.5 \times 0 + c$ $c = \frac{1}{\pi}$ $s(1) = \frac{-1}{\pi}\cos(\pi) + 0.5 + \frac{1}{\pi}$ $s(1) = 0.5 + \frac{2}{\pi}\text{ m}$	•determines displacement after one second [1 mark]

2020
Paper 1
Section 2
Question 14

Integrals

Determine the area of the triangle shown.



Sample Response	The response
$\angle ABC = 30°$ $\text{Area} = \frac{1}{2} \times a \times c \times \sin B$ $\text{Area} = \frac{1}{2} \times 6 \times 3 \times \sin 30°$ $\text{Area} = \frac{9}{2}\text{m}^2$	• correctly substitutes into the area equation [1 mark] • correctly determines the area [1 mark] correctly • communicates the units [1 mark]

2020 Paper 1 Section 2 Question 20 **Integrals**	At the end of the first stage of its growth cycle, a species of tree has a height of 5 metres and a trunk radius of 15 cm. In the second stage of its growth cycle, the tree stays at this height for the next 10 years. However, the growth rate of the trunk radius (in cm per year) varies over the 10 years and is given by the function $$r(t) = 1.5 + \sin\left(\frac{\pi t}{5}\right)$$ Assume the density (mass per unit volume) of the tree trunk is approximately 1 g/cm and the tree trunk is in the shape of a cylinder. Determine the ratio of the trunk's mass at the end of the second stage to its mass at the end of the first stage.

Sample Response	The response
Let $R(t)$ = radius of the tree trunk $R(t) = \int 1.5 + \sin\left(\frac{\pi t}{5}\right) dt$ $= 1.5t - \frac{5}{\pi}\cos\left(\frac{\pi t}{5}\right) + c$	• correctly establishes an integrated expression for the radius of the tree [1 mark]
Given initial radius is 15 cm (end of first stage/beginning of second stage) $15 = -\frac{5}{\pi} + c$ $c = 15 + \frac{5}{\pi}$ $R(t) = 1.5t - \frac{5}{\pi}\cos\left(\frac{\pi t}{5}\right) + 15 + \frac{5}{\pi}$	• determines radius of the tree [1 mark]
Volume of tree trunk $= 500\pi(R(t))^2$	• identifies use of formula and radius to determine volume of tree trunk [1 mark]
Volume of tree trunk at $t = 10$ $V_{10} = \pi \times (R(10))^2 \times 500$ Volume of tree trunk at $t = 0$ $V_0 = \pi \times (R(0))^2 \times 500$ $\rightarrow V_{10} = 450\,000\pi \text{ cm}^3$ and $V_0 = 112\,500\pi \text{ cm}^3$	• determines volumes of tree trunk initially and at 10 years [1 mark]
The mass of the tree is the density times the volume, and since the density is 1 g/cm^3 ∴ ratio of the tree trunk at $t = 0$ and $t = 10$ years $= \frac{450\,000\pi}{112\,500\pi}$ $= 4$ The tree has quadrupled in mass over this time.	• determines ratio of mass [1 mark] • shows logical organisation communicating key steps [1 mark]

Marking Guide – Paper 2 Section 1

2023 Paper 2 Section 1 Question 4 **Integrals**	The displacement (m) of a moving particle is given by $d = e^{0.5t} - 1$, where t is time (s). The acceleration (ms^{-2}) of the particle when $t = 4$ is (A) 7.3891 (B) 6.3891 (C) 3.6945 (D) 1.8473 **Answer is D.**

2023 Paper 2 Section 1 Question 6 **Integrals**	$\int_a^{5a} \frac{1}{x+a} dx,\ a \neq 0$ is (A) 1.7918 (B) 1.6094 (C) 1.3863 (D) 1.0986 **Answer is D.**

2023 Paper 2 Section 1 Question 8 **Integrals**	The number of koalas in a conservation park is modelled by $N = 15\ln(7t+1),\ t \geq 1$, where t represents the time (years) since the park opened. There were 20 koalas in the park when it opened. Determine the approximate rate of change in the number of koalas when $t = 3$. (A) 46 (B) 26 (C) 25 (D) 5 **Answer is D.**

2022 Paper 2 Section 1 Question 1 **Integrals**	The position (in cm) of a particle is given by $x = \cos(4t)$, where t is time (in seconds). The velocity of the particle when t = 5 is (A) 1.6323 cm s^{-1} (B) 0.4081 cm s^{-1} (C) -0.9129 cm s^{-1} **(D) -3.6518 cm s^{-1} – Answer**

2022 Paper 2 Section 1 Question 7 **Integrals**	A marble moves in one direction in a straight line with velocity v = 2ln(t + 1) (in metres per second) where t is time (in seconds) since the marble passed through the origin. Determine the distance from the origin the marble has rolled after 4 seconds. (A) 0.40 m (B) 1.60 m (C) 3.22 m **(D) 8.09 m – Answer**

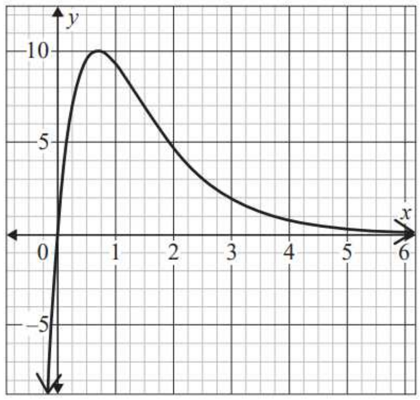
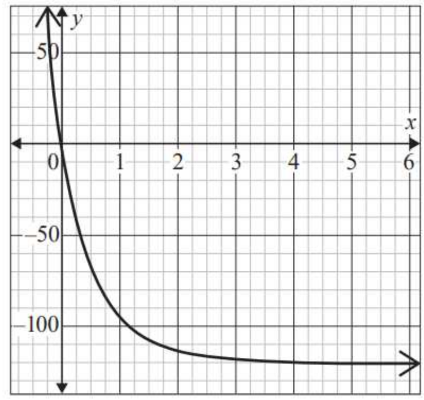
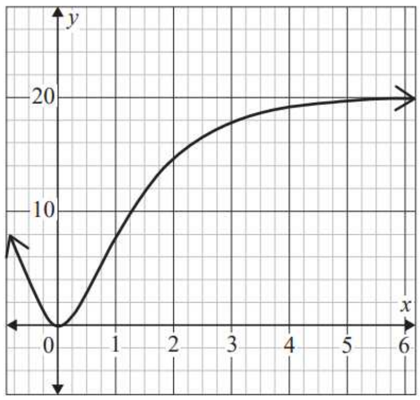
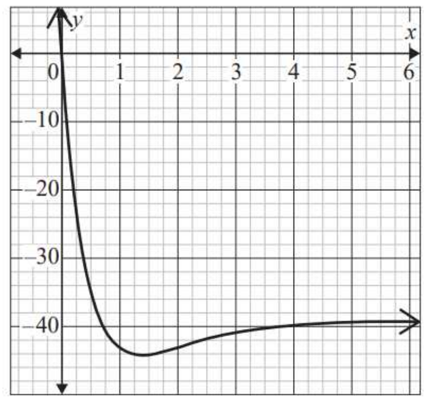
2021 Paper 2 Section 1 Question 2 Integrals	A substance is being heated such that its temperature T in °C after t minutes is given by the function $T = 2e^{0.5t}$ The first integer value of t for which the instantaneous rate of change of temperature is greater than 100°C per minute is **(A) $t = 10$ – Answer** (B) $t = 9$ (C) $t = 8$ (D) $t = 7$

2021 Paper 2 Section 1 Question 4 Integrals	Using the trapezoidal rule with an interval size of 1, the approximate value of the integral $\int_0^3 0.5^x dx$ is (A) 1.25 (B) 1.26 **(C) 1.31 – Answer** (D) 1.88

2021 Paper 2 Section 1 Question 8 Integrals	The displacement (in metres) of a particle is given by $s(t) = -3\cos(t) + 2\sin(t)$, where t is in seconds. The instantaneous velocity of the particle at time $t = \frac{\pi}{2}$ seconds is (A) –3m s^{-1} (B) –2m s^{-1} (C) 2 m s^{-1} **(D) 3 m s^{-1} – Answer**

2021 Paper 2 Section 1 Question 10 Integrals	An object travels in a straight line so that its velocity at time t seconds is given by $v(t) = 2t + \sin(2t)$. Determine the expression for acceleration as a function of time. (A) $a(t) = 2 + 2\cos(2t)$ (B) $a(t) = 2 - \frac{1}{2}\cos(2t)$ (C) $a(t) = t^2 + 2\cos(2t)$ (D) $a(t) = t^2 - \frac{1}{2}\cos(2t)$ **Answer is A.**

2020 Paper 2 Section 1 Question 3 Integrals	Let R be the region enclosed by the graph of $y = xe^x$, the x-axis, and the lines $x = -1$ and $x = 1$. The area of R is closest to (A) 0.74 **(B) 1.26 – Answer** (C) 2.35 (D) 3.09

2020 **Paper 2** **Section 1** **Question 5** **Integrals**	An object moves in a straight line with a velocity *v* given by $$v(t) = 40(e^{-t} - e^{-2t}) \text{ m s}^{-1} \text{ where } t \geq 0$$ The object is at the origin initially. The displacement–time graph in the first 6 seconds is (A)  (B)  (C)  (D)  **Answer is C.**

2020 **Paper 2** **Section 1** **Question 6** **Integrals**	Oil is leaking from a tanker at the rate of $r(t) = 9000e^{-0.2t}$ litres per hour, where *t* is in hours. Determine how much oil leaks from the tanker (to the nearest litre) from time $t = 0$ to time $t = 10$. **(A) 38 910 litres – Answer** (B) 8756 litres (C) 7782 litres (D) 1556 litres

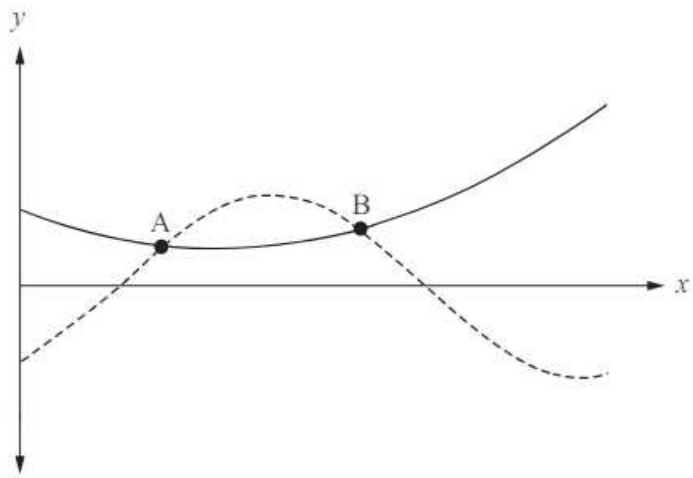
2020 **Paper 2** **Section 1** **Question 9** **Integrals**	The displacement of a particle (in metres) at time *t* (in seconds) is represented by the function $$s(t) = t\ln(t) - t,\ 0 < t < 4$$ Determine the approximate acceleration of the particle at time $t = 3$. (A) 0.66 m s^{-2} **(B) 0.33 m s^{-2} – Answer** (C) –0.33 m s^{-2} (D) –0.66 m s^{-2}

2023 Paper 2 Section 2 Question 13 Integrals	

2023
Paper 2
Section 2
Question 13

Integrals

The curved lines represent graphs of the equations $y = x^2 - 4x + 8$ and $y = 10\cos(x + 10)$.



a) Determine the coordinates of the points of intersection A and B. [1 mark]

Sample response	The response
Using GDC: $A(1.44140, 4.31203)$ $B(3.47247, 6.16818)$	• correctly determines A and B **[1 mark]**

b) State an integral expression representing the area enclosed by the two graphs. [2 marks]

Sample response	The response
Area enclosed: $\int_{1.44140}^{3.47247} (10\cos(x+10) - (x^2 - 4x + 8))dx$	• states the difference in functions required for area, in the correct order (to obtain a positive value) **[1 mark]** • uses the x-coordinates from Q13a) in the definite integral **[1 mark]**

c) Determine the area enclosed by the two graphs. [1 mark]

Sample response	The response
Using GDC: Area $= 7.64702$ units^2	• determines the area value **[1 mark]**

2023 Paper 2 Section 2 Question 16 **Integrals**	A particle is moving in a straight line. The velocity (ms^{-1}) of the particle is given by $v(t) = \frac{20\sin(2t)}{6-5\cos(2t)}$, $t \geq 0$, where t is time (s) after moving from its initial position. The initial position of the particle is +6.0 m from the origin.

a) Use calculus methods to determine an equation for the position of the particle from the origin at any time t. [3 marks]

Sample response	The response
Position is given by: $\int v(t)\,dt$ $= \int \frac{20\sin(2t)}{6-5\cos(2t)}dt$ $= 2\int \frac{10\sin(2t)}{6-5\cos(2t)}dt$	• correctly manipulates the integrand to obtain a numerator that is the derivative of the denominator **[1 mark]**
Let $f(t) = 6 - 5\cos(2t)$ $f'(t) = 10\sin(2t)$ $= 2\int \frac{f'(t)}{f(t)}\,dt$ $= 2\ln f(t)$ $= 2\ln(6-5\cos(2t)) + c$	• correctly determines position formula **[1 mark]**
But $s(0) = 6$ $6 = 2\ln(6-5\cos(0)) + c$ $c = 6$ $s(t) = 2\ln(6-5\cos(2t)) + 6$	• determines the position formula relative to the origin, i.e. allows for starting at +6 m **[1 mark]**

b) Determine the position of the particle relative to the origin when it first reaches maximum velocity. [3 marks]

Sample response	The response
Using a GDC to graph $v(t)$ for $t \geq 0$ Initially $v(t)$ is positive and so heading to the right of the initial position v_{max} first occurs when $t = 0.292843$ s	• correctly determines first time when velocity is a maximum **[1 mark]**
Using a GDC, the position of the particle at this time is given by: $\int_0^{0.292843} v(t)\,dt$ $= 1.21227$ m But $s(0) = 6$	• identifies the required integral (or the formula established in 16a) and the max velocity time is substituted **[1 mark]**
Therefore, the position relative to the origin at this time is given by: $1.21227 + 6 = 7.21227$ m	• determines the position of the particle **[1 mark]**

2022
Paper 2
Section 2
Question 13

Integrals

A sandy beach has a fence on one side and ocean on the other. The width of the beach is the distance (in metres) from the fence to the water's edge. The width, $w(t)$, at a certain point is given by

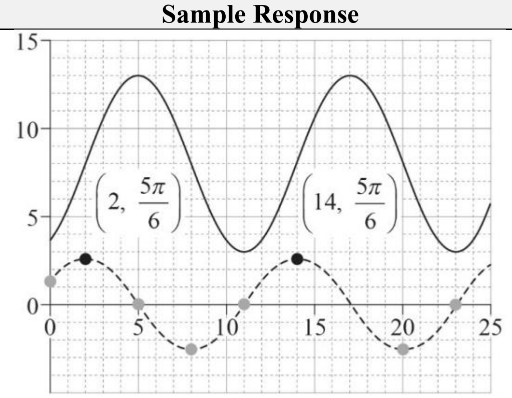
$$w(t) = a + b\sin\left(\frac{\pi}{6}t - \frac{\pi}{3}\right),\ 0 \le t \le 24$$

where t is time (in hours) since 6 am. The width of the beach is 8 metres at 8 am and 3 metres at 5 pm.

a) Determine a and b. [2 marks]

Sample Response	The response
$w(2) = 8$ $\therefore a + b\sin(0) = 8$ $\therefore a = 8$	• correctly determines a [1 mark]
$w(11) = 3$ $\therefore 8 + b\sin\left(\frac{3\pi}{2}\right) = 3$ $\therefore b \times -1 = -5$ $\therefore b = 5$	• correctly determines b [1 mark]

b) Determine the rate of change of the width of the beach at 8 am and the first time after this when this rate of change is repeated. [2 marks]

Sample Response	The response
 The rate of change at 8am is $\frac{5\pi}{6}$ Using sketch $t = 14$	• determines rate when $t = 2$ [1 mark]
At 8 pm the rate is the same (for the first time).	• determines first time when rate is the same as $t = 2$ [1 mark]

<table>
<tr>
<td>2022
Paper 2
Section 2
Question 17

Integrals</td>
<td>
A snail is travelling along a straight path from point A. The snail's velocity (cm min^{-1}) is modelled by $v(t) = 1.4\ln(1 + t^2)$, where t is time (in minutes) for $0 \le t \le 15$.

An ant passes point A 12 minutes after the snail and follows the snail's path. The ant moves with a constant acceleration of 2 cm min^{-2} and passes the snail at t =15 minutes.

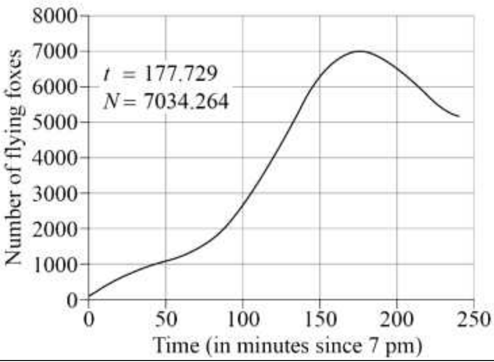
Determine the ant's velocity at point A. [4 marks]

<table>
<tr><th>Sample Response</th><th>The response</th></tr>
<tr><td>Total displacement of the snail
$\int_0^{15} 1.4\ln(1+t^2)dt = 76.0431$ cm

Velocity of the ant = $\int 2dt$
$= 2t + c$</td><td>• correctly determines the total displacement of the snail [1 mark]</td></tr>
<tr><td>Displacement $_{\text{ant from 12 to 15 min}}$
Displacement $_{\text{snail from 0 to 15 min}}$
$\therefore \int_{12}^{15} 2t + c = 76.0431$</td><td>• establishes an equation linking the ant and the snail [1 mark]</td></tr>
<tr><td>Solving numerically on GDC
$c = -1.6523$</td><td>• determines constant [1 mark]</td></tr>
<tr><td>Therefore, velocity of ant at $t = 12$
$= 2 \times 12 - 16523$
$= 22.3477$ cm min^{-1} along the ant's path.</td><td>• determines velocity of ant [1 mark]</td></tr>
</table>
</td>
</tr>
</table>

2022 Paper 2 Section 2 Question 19 **Integrals**	Flying foxes enter and leave a fruit-growing region every evening. The rate at which the flying foxes enter the region is modelled by the function $$A(t) = 42\sin\left(0.03t - \frac{\pi}{3}\right) + 71,\ 0 \le t \le 240$$ The rate at which the flying foxes leave the region is modelled by the function $$L(t) = 42\sin\left(0.04t - \frac{\pi}{3}\right) + 42,\ 0 \le t \le 240$$ Both *A* (*t*) and *L* (*t*) are measured in animals per minute and t is measured in minutes after 7 pm. There are 100 flying foxes in the region at 7 pm. Determine the maximum number of flying foxes in the region and the time that this occurs. [4 marks]

Sample Response	The response
Number of flying foxes entering the region $= \int 42\sin(0.03t - \frac{\pi}{3}) + 71$ $= -1400\cos(0.03t - \frac{\pi}{3}) + 71t + c_1$ Number of flying foxes leaving the region $L(t)$ $= \int 42\sin(0.04t - \frac{\pi}{3}) + 42$ $= -\frac{42}{0.04}\cos\left(0.04t - \frac{\pi}{3}\right) + 42t + c_2$ Number of flying foxes in region $N(t)$ $= -1400\cos\left(0.03t - \frac{\pi}{3}\right) + 71t + c_1$ $-(-1050\cos\left(0.04t - \frac{\pi}{3}\right) + 42t + c_2$ Given number of flying foxes in the region at time $t =$ 0 is 100 $N(t) = -1400\cos\left(0.03t - \frac{\pi}{3}\right)$ $+1050\cos\left(0.04t - \frac{\pi}{3}\right) + 29t + 275$	• correctly determines the function to model the total number of flying foxes in the region [1 mark]

Using GDC to sketch $N(t)$ 	• uses an appropriate mathematical method to identify the maximum values [1 mark]
Using a GDC, the maximum point is identified (177.729, 7034.264).	• determines the maximum number of flying foxes and time, i.e. the coordinates of the turning point [1 mark]
Maximum of 7034 flying foxes in the region at 9:58pm.	• states the number of flying foxes as a whole number and the time after 7 pm it occurs [1 mark]

2021
Paper 2
Section 2
Question 11

Integrals

Consider the function $f(x) = e^x \sin(x), 0 \le x \le 2\pi$

a) State the exact values of the x-intercepts of the graph of $f(x)$. [2 marks]

Sample Response	The response
x-intercepts ($y = 0$) $0 = e^x \sin(x)$	• correctly generates the equation required [1 mark]
Using null factor rule $e^x \neq 0$ $\sin(x) = 0$ $\therefore x = 0, \pi, 2\pi$	• correctly determines the three x-intercepts [1 mark]

b) Write an expression for the area enclosed between the graph of $f(x)$ and the x-axis. [2 marks]

Sample Response	The response
$\int_0^{\pi} e^x \sin(x)dx + \left\lvert \int_{\pi}^{2\pi} e^x \sin(x)dx \right\rvert$	• determines expression for integral above the x-axis [1 mark] • determines expression for integral below the x-axis (including absolute value brackets) [1 mark]

c) Determine the area enclosed between the graph of $f(x)$ and the x-axis to the nearest square unit. [1 mark]

Sample Response	The response
Area enclosed (using GDC) 291 square units	• determines area to the nearest unit [1 mark]

2021
Paper 2
Section 2
Question 12

Integrals

The velocity function of an object in m s^{-1} is given by $v(t) = \cos\left(6t + \frac{\pi}{2}\right) + 2, 0 \le t \le 5.$

Initially, the object is at the origin.

a) Determine the displacement function. [2 marks]

Sample Response	The response
$s(t) = \frac{1}{6}\sin\left(6t + \frac{\pi}{2}\right) + 2t + c$	• correctly determines the indefinite integral $ss(tt)$ [1 mark]
Substituting (0, 0) $0 = \frac{1}{6}\sin\left(\frac{\pi}{2}\right) + c$ $c = \frac{-1}{6}$ $s(t) = \frac{1}{6}\sin\left(6t + \frac{\pi}{2}\right) + 2t + \frac{-1}{6}$	• correctly determines the displacement function [1 mark]

b) What is the displacement of the object from the origin, in metres (m), after three seconds? [2 marks]

Sample Response	The response
distance = $s(3) - s(0)$ $= \frac{1}{6}\sin\left(18 + \frac{\pi}{2}\right) + 6 - \frac{1}{6} - \left(\frac{1}{6}\sin\left(\frac{\pi}{2}\right) - \frac{1}{6}\right)$	• establishes an expression for the distance travelled [1 mark]
= 5.943 m	• determines distance travelled [1 mark]

2020 Paper 2 Section 2 Question 12 **Integrals**	The rates of change in population for two cities are given by

The rates of change in population for two cities are given by

$$\text{City A: } A'(t) = \frac{45}{t+1}$$

$$\text{City B: } B'(t) = 105e^{0.03t}$$

where t is the number of years since 2018 and both $A'(t)$ and $B'(t)$ are measured in people per year. At the beginning of 2018, City A had a population of 5000, and City B had a population of 3500.

a) Determine the population models for both cities. [3 marks]

Sample Response	The response
$A(t) = 45\ln(t+1) + c$	• correctly determines the general population functions for City A [1 mark]
$B(t) = 3500^{e^{0.03t}} + c$	• correctly determines the general population functions for City B [1 mark]
Given $A(0) = 5000$, $B(0) = 3500$ $A(t) = 45\ln(t+1) + 5000$ $B(t) = 3500e^{0.03t}$	• determines population functions for the two cities [1 mark]

b) Use the information in 12a) to predict the population of City B at the beginning of 2028. [1 mark]

Sample Response	The response
$B(10) = 3500e^{0.3}$ $B(10) = 4724$	• determines population of City B in 2028 [1 mark]

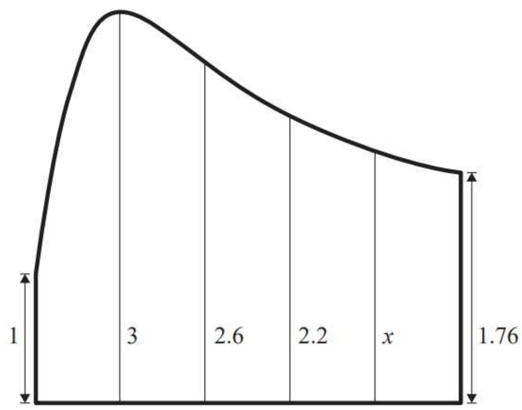
c) Use the information in 12a) to predict the year in which the population of both cities will be the same. [3 marks]

Sample Response	The response
$45\ln(t+1) + 5000 = 3500e^{0.03t}$ Using GDC	• uses an appropriate mathematical representation to communicate approach [1 mark]
$t = 12.66$	• determines time [1 mark]
∴ during the year 2030 the populations will be the same.	• determines the required time [1 mark]

2020
Paper 2
Section 2
Question 15

Integrals

A field is divided into five sections as shown. The width of each section is 1 metre. The perpendicular height, in metres, of each section is given in the diagram. The area of the field was approximated using the trapezoidal rule and found to be 11.12 m^2.



Not drawn to scale

a) Determine the height marked x on the diagram. [2 marks]

Sample Response	The response
Using trapezoidal rule:	• correctly establishes equation in x [1 mark]
$11.12 = \frac{1+3}{2} + \frac{3+2.6}{2} + \frac{2.6+2.2}{2} + \frac{2.2+x}{2} + \frac{x+1.76}{2}$ $3.92 = 1.98 + x$ $x = 1.94$ m	• determines x [1 mark]

b) Determine the area of the field, given the shape of the field is modelled by the function [1 mark]

$$f(x) = \frac{4x}{x^2+1} + 1, \quad 0 \le x \le 5$$

Sample Response	The response
$\int_0^5 \frac{4x}{x^2+1} + 1\, dx$ $= 11.5162 \text{ m}^2$	• correctly determines the area [1 mark]

2020
Paper 2
Section 2
Question 19

Integrals

Consider the following information when completing this question.

$$\text{The length of a curve } y = f(x) \text{ over the interval } [a,b] = \int_a^b \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$$

You are driving along a road with a vertical distance above sea level D (in metres) given by the function

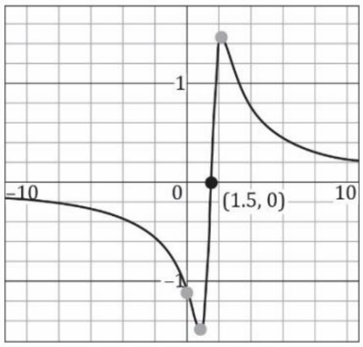
$$D(x) = 300 + \ln(x^2 - 3x + e)$$

where x is the horizontal distance from an initial point of measurement (in kilometres) at sea level.

Assume that if x is positive you are east of the initial point of measurement and if x is negative you are west of the initial point of measurement.

You start your drive along the road at a horizontal distance of 10 kilometres west of the initial point of measurement and drive until you are at a horizontal distance of 10 kilometres east of the initial point.

Determine the time you spend driving downhill, if you drive downhill at an average speed of 40 km/h. [7 marks]

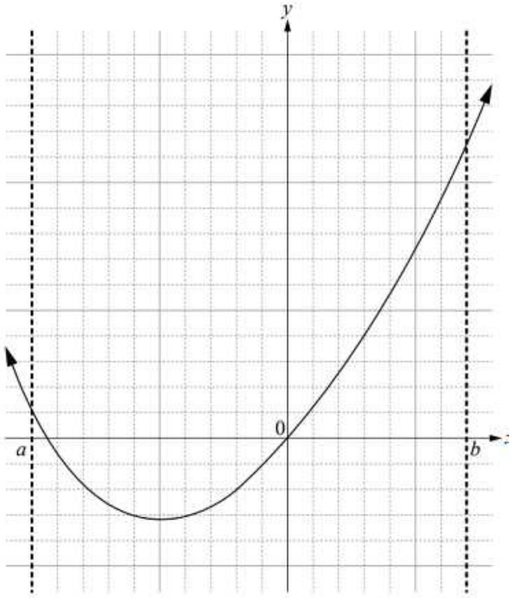
Sample Response	The response
Downhill drive will correspond to domain where derivative is negative.	• correctly identifies values of x associated with downhill drive [1 mark]
Graphing $D'(x)$ 	• correctly uses an appropriate mathematical representation [1 mark]
The function is decreasing $-10 \le x < 1.5$	• determines decreasing interval [1 mark]
The distance driving downhill $\int_{-10}^{1.5} \sqrt{1 + \left(D'(x)\right)^2}\,dx$	• establishes integral expression for the total distance travelled downhill [1 mark]
Using GDC = 11.5 kilometres*	• determines the distance travelled downhill [1 mark]
Time to travel 11.5 kilometres driving at 40 km/hour $t = \frac{11.5}{40} = 0.29$ hours* * Note: An erroneous answer of 0.33 hours, corresponding to a distance of 13.25 km, was obtained when students failed to ensure that x and D(x) were measured in the same units. This was awarded full marks as the length of a curve formula was unfamiliar to students.	• determines time travelling downhill [1 mark] • shows logical organisation communicating key steps [1 mark]

Unit 4: Further functions and statistics

Unit 4 – Topic 1: Further differentiation and applications 3

Paper 1 Section 1

2023 Paper 1 Section 1 Question 2 Further differentiation and applications 3	If $f(x) = e^{6-2x}$, determine the value of $f'(2)$. (A) e^2 (B) $2e^2$ (C) $-e^2$ (D) $-2e^2$

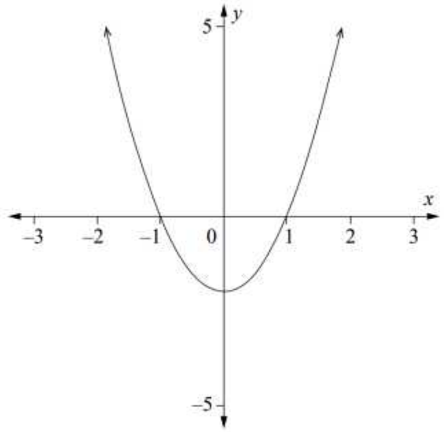
2022 Paper 1 Section 1 Question 1 Further differentiation and applications 3	Consider the graph of $f'(x)$ for a $\leq x \leq b$.  Which statement describes all the local maxima and minima of the graph of $f(x)$ over a $\leq x \leq b$? (A) one local minimum and one local maximum (B) one local minimum and two local maxima (C) one local minimum only (D) one local maximum only

2021 Paper 1 Section 1 Question 4 Further differentiation and applications 3	The second derivative of the function $f(x)$ is given by $f''(x) = \dfrac{2x}{1+x^2}$ The interval on which the graph of $f(x)$ is concave up is (A) $x < 0$ (B) $x \leq 0$ (C) $x > 0$ (D) $x \geq 0$

2021
Paper 1
Section 1
Question 5

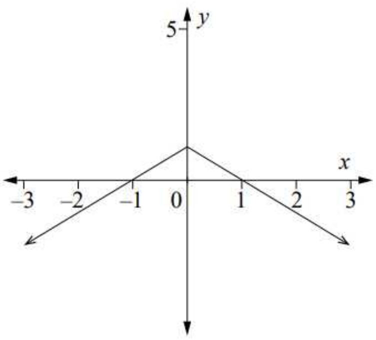
Further differentiation and applications 3

The graph of $f''(x)$ is shown.

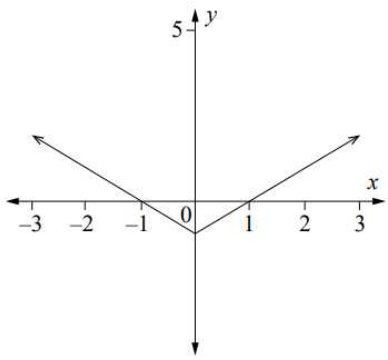


Which of the following could be the graph of $f'(x)$?

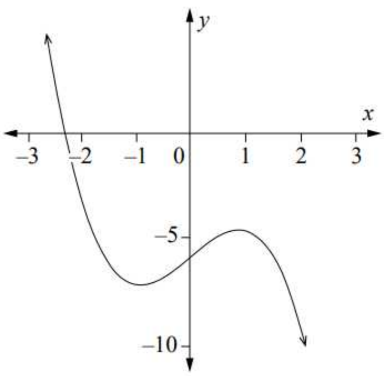
(A)



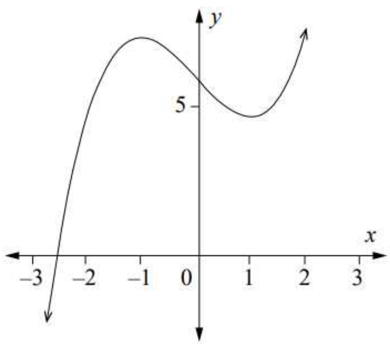
(B)

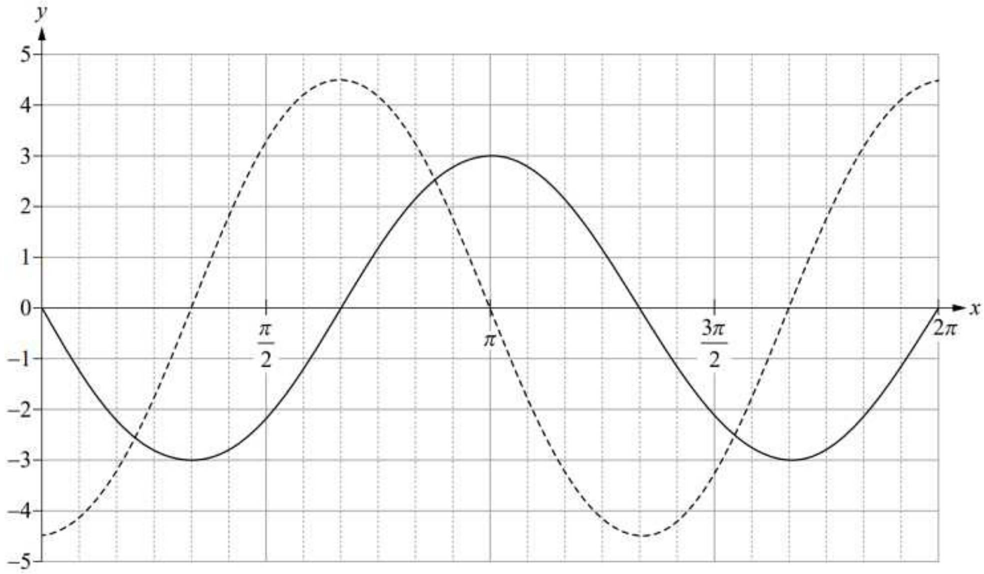


(C)



(D)

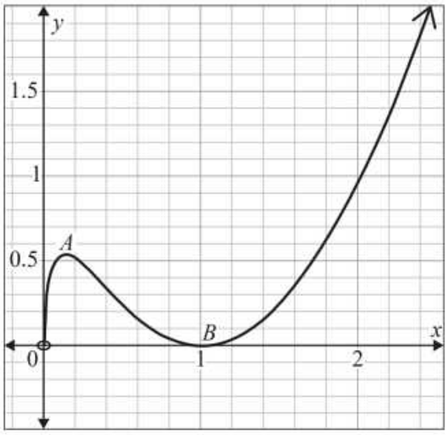


2022 Paper 1 Section 2 Question 16 **Further differentiation and applications 3**	A section of the graphs of the first and second derivatives of a function are shown. Sketch a possible graph of the function on the same axes over the domain $0 \le x \le 2\pi$. Explain all reasoning used to produce the sketch.  Note: If you make a mistake in the graph, cancel it by ruling a single diagonal line through your work and use the additional response space on page 17 of this question and response book.

2023 Paper 1 Section 2 Question 17 **Further differentiation and applications 3**	A chemical is added to the water in a swimming pool at 10:00 am to prevent algae. The amount of chemical absorbed into the water over time t (hours) is represented by $$A = 10t^2 - 4t^3, \quad 0 \le t \le 1\frac{2}{3}$$ Determine the time of day when the rate of absorption of the chemical is at its maximum. Use calculus techniques to verify that your time corresponds to a maximum rate.

2023 Paper 1 Section 2 Question 18 **Further differentiation and applications 3**	A person enters the lowest carriage of a miniature Ferris wheel with a six-metre diameter. The bottom carriage is one metre off the ground. When top speed is reached, it takes three seconds for a carriage to travel from the lowest to the highest point of the ride. It is claimed that: *The vertical motion of the Ferris wheel produces a maximum vertical acceleration on each rider that is more than half the acceleration of free fall.* Free fall occurs when gravity is the only force acting, resulting in an acceleration of 9.8 ms^{-2}. Evaluate the reasonableness of the claim. (4 marks)

2022 **Paper 1** **Section 2** **Question 15** **Further differentiation and applications 3**	The derivative of a function is given by $f'(x) = e^x(x-4)$. Determine the interval on which the graph of $f(x)$ is both decreasing and concave up.

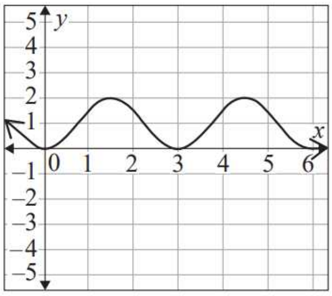
2020 Paper 1 Section 2 Question 13 Further differentiation and applications 3	A function is defined as $f(x) = x(\ln(x))^2$, $x > 0$. The graph of the function is shown and has a local maximum at point A and a global minimum at point B. The derivative of the function is given by $f'(x) = 2\ln(x) + (\ln(x))^2$, $x > 0$.  a) Verify that there is a stationary point at $x = 1$. [2 marks] b) Determine the coordinates of A. [3 marks]

The graph of the function has a point of inflection at $x = e^p$
c) Determine p [2 marks]

2020
Paper 1
Section 2
Question 16

Further differentiation and applications 3

Consider the following graph of $f(x)$.



Identify the graph of the second derivative $f''(x)$ from the graphs in Diagram 1, Diagram 2 and Diagram 3.

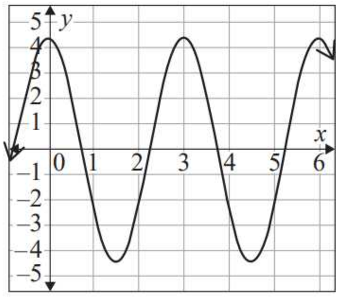


Diagram 1

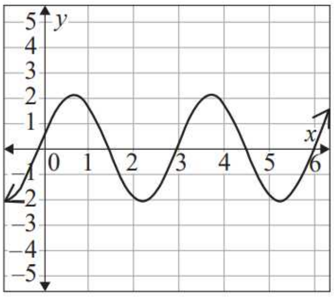


Diagram 2

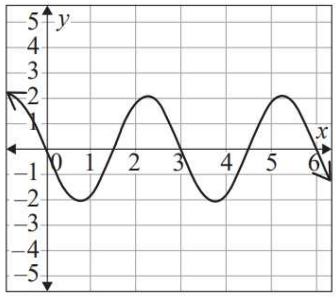


Diagram 3

Justify your decisions using mathematical reasoning. [4 marks]

2020 Paper 1 Section 2 Question 19 **Further differentiation and applications 3**	A horizontal point of inflection is a point of inflection that is also a stationary point. Determine the value/s of k for which the graph of $f(x) = \frac{\ln(x)}{k} - \frac{kx}{x+1}$ has only one horizontal point of inflection.

Paper 2 Section 1

2022 Paper 2 Section 1 Question 2 **Further differentiation and applications 3**	Identify the correct features of the function $f(x) = xe^x$ (A) $f'(-1) = 0, f''(-1) < 0$ (B) $f'(-1) = 0, f''(-1) > 0$ (C) $f'(-1) < 0, f''(-1) < 0$ (D) $f'(-1) < 0, f''(-1) > 0$

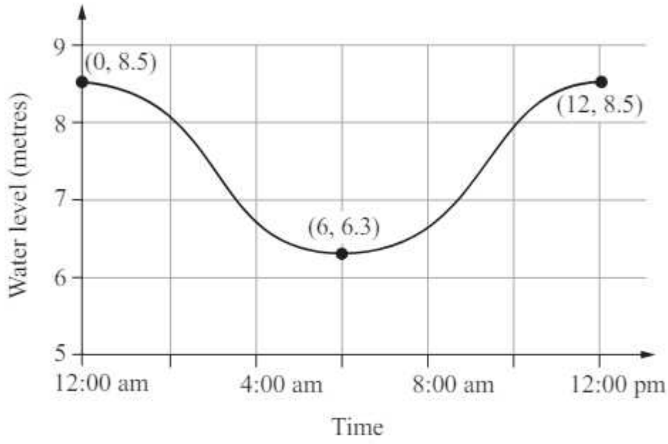
2022 Paper 2 Section 1 Question 3 **Further differentiation and applications 3**	The derivative of the function $f(x)$ is given by $f'(x) = \sin(x^3)$ for the domain $-1.8 < x < 1.8$. The number of points of inflection that the graph of $f(x)$ has on this interval is (A) 1 (B) 3 (C) 4 (D) 5

2020 Paper 2 Section 1 Question 10 **Further differentiation and applications 3**	The approximate value of x where the graph of the function $y = x^3 + 6x^2 + 7x - 2\cos(x)$ changes concavity is (A) –3.26 (B) –2.85 (C) –2.20 (D) –1.89

2023 Paper 2 Section 2 Question 12

Further differentiation and applications 3

The graph shows the water level under a bridge over a 12-hour period.



a) Determine the equation of the cosine function that models the water level as a function of time after 12:00 am. [1 mark]

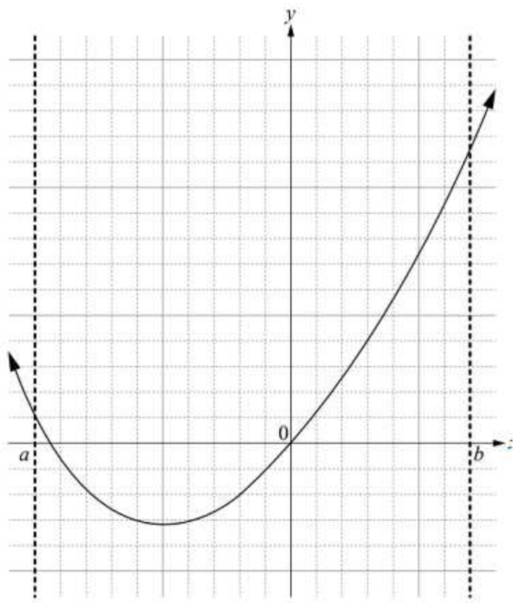
b) How long in the 12-hour period shown is the rate of change of water level more than 0.55 metres per hour? [3 marks]

2023 Paper 2 Section 2 Question 15 **Further differentiation and applications 3**	Determine the derivative of $f(x) = \ln x^2 + \ln(x-5)^3$. Express the derivative as a single fraction in its simplest and factorised form. (4 marks)

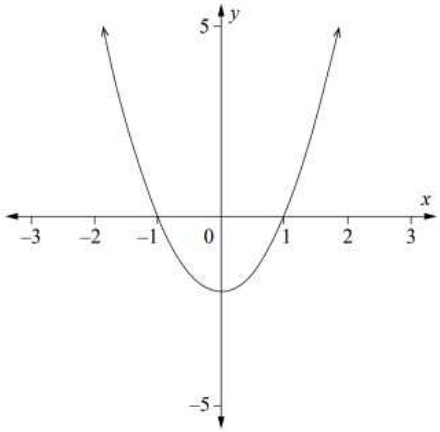
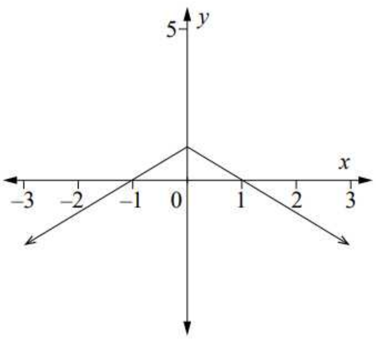
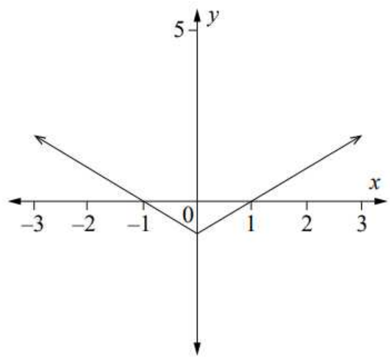
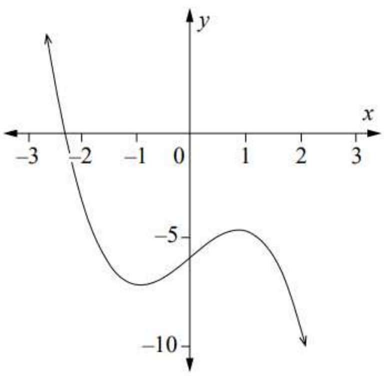
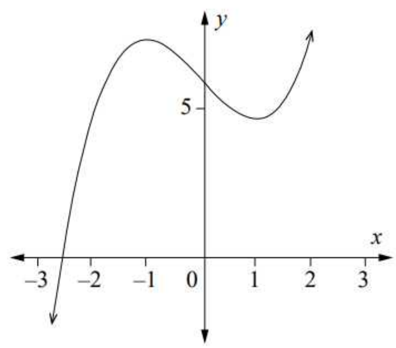
2021 Paper 2 Section 2 Question 18 **Further differentiation and applications 3**	The number of animals in a population (in thousands) is modelled by the function P such that $P(t) = \dfrac{100}{1 + 4e^{-t}}$, where t is in years. Determine the number of animals in the population when the population is growing the fastest. [3 marks]

Marking Guide – Paper 1 Section 1

2023 Paper 1 Section 1 Question 2 Further differentiation and applications 3	If $f(x) = e^{6-2x}$, determine the value of $f'(2)$. (A) e^2 (B) $2e^2$ (C) $-e^2$ (D) $-2e^2$ **Answer is D.**

2022 Paper 1 Section 1 Question 1 Further differentiation and applications 3	Consider the graph of $f'(x)$ for $a \le x \le b$.  Which statement describes all the local maxima and minima of the graph of $f(x)$ over $a \le x \le b$? **(A) one local minimum and one local maximum – Answer** (B) one local minimum and two local maxima (C) one local minimum only (D) one local maximum only

2021 Paper 1 Section 1 Question 4 Further differentiation and applications 3	The second derivative of the function $f(x)$ is given by $f''(x) = \dfrac{2x}{1+x^2}$ The interval on which the graph of $f(x)$ is concave up is (A) $x < 0$ (B) $x \le 0$ **(C) $x > 0$ – Answer** (D) $x \ge 0$

2021 **Paper 1** **Section 1** **Question 5** **Further differentiation and applications 3**	The graph of $f''(x)$ is shown.  Which of the following could be the graph of $f'(x)$? (A)  (B)  (C)  (D)  **Answer is D.**

2023 Paper 1 Section 2 Question 18 Further differentiation and applications 3	A person enters the lowest carriage of a miniature Ferris wheel with a six-metre diameter. The bottom carriage is one metre off the ground. When top speed is reached, it takes three seconds for a carriage to travel from the lowest to the highest point of the ride. It is claimed that: *The vertical motion of the Ferris wheel produces a maximum vertical acceleration on each rider that is more than half the acceleration of free fall.* Free fall occurs when gravity is the only force acting, resulting in an acceleration of 9.8 ms^{-2}. Evaluate the reasonableness of the claim. (4 marks)

Sample response	The response
Because of the repetitive nature of the motion, use a cosine graph (cosine since when t = 0, the graph is a minimum) to model the motion. $h(t)=a\cos(b(t+c))+d$ Amplitude = 3 $\therefore a=3$ Period = 6 seconds $\therefore T=\frac{2\pi}{b}$ $b=\frac{2\pi}{6}=\frac{\pi}{3}$ No phase shift $\therefore c=0$ The axis of the Ferris wheel is 4 m above ground $\therefore d=4$	• correctly recognises a periodic model is to be used for the vertical position of the carriage **[1 mark]**
$h(t)=-3\cos\left(\frac{\pi}{3}t\right)+4$ $v(t)=h'(t)=3\frac{\pi}{3}\sin\left(\frac{\pi}{3}t\right)=\pi\sin\left(\frac{\pi}{3}t\right)$ $a(t)=h''(t)=\pi\frac{\pi}{3}\cos\left(\frac{\pi}{3}t\right)=\frac{\pi^2}{3}\cos\left(\frac{\pi}{3}t\right)$ The amplitude of the acceleration model is $\frac{\pi^2}{3}$. This also corresponds to the maximum value of acceleration.	• correctly determines the model for the vertical position of a carriage on the Ferris wheel **[1 mark]**
$\frac{\pi^2}{3}\approx\frac{3^2}{3}=3$ **Therefore, it is not reasonable** to expect the acceleration of the Ferris wheel alone to be more than half of 9.8 (4.9).	• determines an approximation of the maximum acceleration produced based on the second derivative equation obtained **[1 mark]** • provides appropriate statement of reasonableness **[1 mark]**

2023 Paper 1 Section 2 Question 17 Further differentiation and applications 3	A chemical is added to the water in a swimming pool at 10:00 am to prevent algae. The amount of chemical absorbed into the water over time t (hours) is represented by $A = 10t^2 - 4t^3, \quad 0 \le t \le 1\frac{2}{3}$ Determine the time of day when the rate of absorption of the chemical is at its maximum. Use calculus techniques to verify that your time corresponds to a maximum rate. (6 marks)

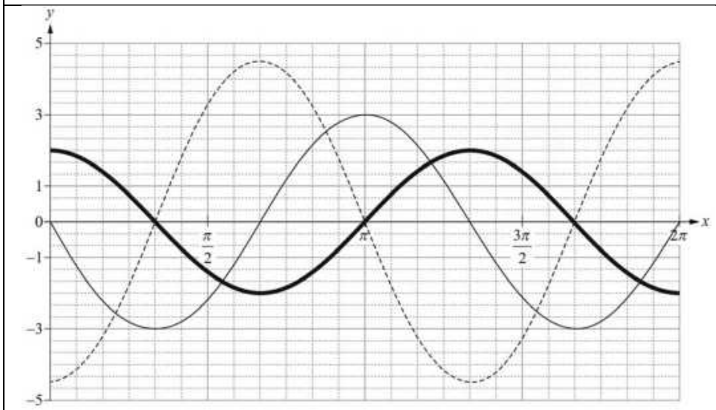
Sample response	The response
The rate of absorption is given by: $\frac{dA}{dt} = 20t - 12t^2$ $\frac{d^2A}{dt^2} = 20 - 24t$ $\therefore 20 - 24t = 0$ $t = \frac{20}{24} = \frac{5}{6}$ hours	• correctly determines the first derivative **[1 mark]** • determines the second derivative **[1 mark]** • equates the second derivative to zero **[1 mark]** • determines time when second derivative is zero **[1 mark]**
Verify this corresponds to a maximum rate. Using the second derivative test, we investigate the sign of the derivative of $\frac{d^2A}{dt^2}$, i.e. $\frac{d^3A}{dt^3}$ $\frac{d^3A}{dt^3} = -24$ This is negative, therefore the rate of absorption is a maximum. The time the chemical is increasing most rapidly since delivery is $\frac{5}{6}$ hours. $= \frac{5}{6} \times 60$ $= 50$ minutes The required time is 10:50 am.	• performs a calculus test to confirm the time corresponds to a maximum for $\frac{dA}{dt}$ **[1 mark]** • determines the time for maximum rate of absorption in minutes **[1 mark]**

2022
Paper 1
Section 2
Question 16

Further differentiation and applications 3

A section of the graphs of the first and second derivatives of a function are shown.

Sketch a possible graph of the function on the same axes over the domain $0 \le x \le 2\pi$. Explain all reasoning used to produce the sketch.

Sample Response	The response
The provided functions are a sine and a cosine function, both with negative amplitudes and the same period. Therefore, the smooth line is $y' = -a\sin(bx)$, and the dotted line is	• correctly identifies the smooth curve as the first derivative [1 mark]
$y'' = -A\cos(bx)$, with $a = 3$ and $A = 4.5$. Therefore, the primary function must be a cosine function with the same period i.e. $y = 2\cos\frac{3}{2}x$	• provides mathematical reasoning for sketch [1 mark]
	• sketches the function [1 mark]

2022
Paper 1
Section 2
Question 15

Further differentiation and applications 3

The derivative of a function is given by $f'(x) = e^x(x - 4)$.

Determine the interval on which the graph of $f(x)$ is both decreasing and concave up.

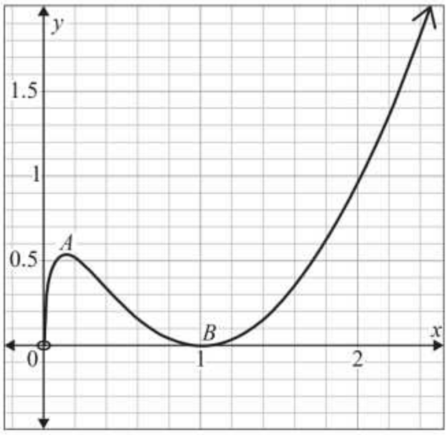
Sample Response	The response
The function is decreasing when $f'(x) < 0$ and concave up when $f''(x) > 0$	• correctly describes conditions when the function is decreasing and concave up [1 mark]
$f'(x) = (x - 4)e^x < 0$ when $x < 4$	• correctly determines the interval where $f(x)$ is decreasing [1 mark]
$f''(x) = (x - 4)e^x + e^x = e^x(x - 3) > 0$ when $x > 3$	• correctly determines the interval where $f(x)$ is concave up [1 mark]
Therefore, the function is decreasing and concave up when $3 < x < 4$	• determines interval when function is decreasing and concave up [1 mark]

2020 Paper 1 Section 2 Question 13 **Further differentiation and applications 3**	

A function is defined as $f(x) = x(\ln(x))^2$, $x > 0$.

The graph of the function is shown and has a local maximum at point A and a global minimum at point B.

The derivative of the function is given by $f'(x) = 2\ln(x) + (\ln(x))^2$, $x > 0$.



a) Verify that there is a stationary point at $x = 1$. [2 marks]

Sample Response	The response
Given $f'(x) = 2\ln(x) + (\ln(x))^2$ Stationary point $f'(x) = 0$	• correctly identifies that the derivative equals 0 [1 mark]
$0 = 2\ln(x) + (\ln(x))^2$ $0 = \ln(x)(2 + \ln(x))$ $\ln(x) = 0 \therefore x = 1$	• correctly shows there is a stationary point at $x = 1$ [1 mark]

b) Determine the coordinates of A. [3 marks]

Sample Response	The response
Stationary point $f'(x) = 0$ From a) $2 + \ln(x) = 0$	• correctly establishes the equation in x [1 mark]
$\ln(x) = -2$ $x = e^{-2}$	• correctly determines the x-ordinate of A [1 mark]
$\therefore y = e^{-2} \times (\ln e^{-2})^2 = 4e^{-2}$ so $A(e^{-2}, 4e^{-2})$	• determines y-ordinate of A [1 mark]

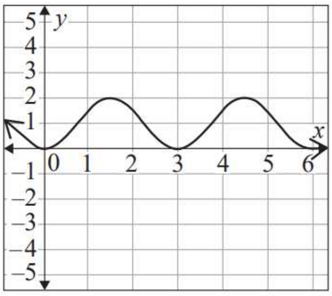
The graph of the function has a point of inflection at $x = e^p$
c) Determine p [2 marks]

Sample Response	The response
Using chain rule $f''(x) = \frac{2}{x} + \frac{2}{x}\ln(x)$ Point of inflection $f''(x) = 0$ $0 = \frac{2}{x} + \frac{2}{x}\ln(x)$	• correctly establishes the equation in x equals 0 [1 mark]
$= \frac{2}{x}(1 + \ln(x))$ $0 = 1 + \ln(x) \rightarrow x = e^{-1}$ $\therefore p = -1$	• determines p [1 mark]

2020
Paper 1
Section 2
Question 16

Further differentiation and applications 3

Consider the following graph of $f(x)$.



Identify the graph of the second derivative $f''(x)$ from the graphs in Diagram 1, Diagram 2 and Diagram 3.

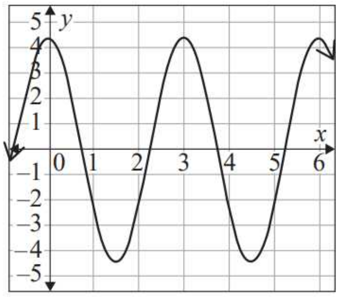


Diagram 1

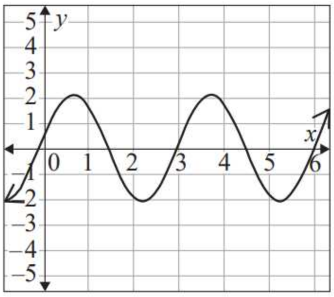


Diagram 2

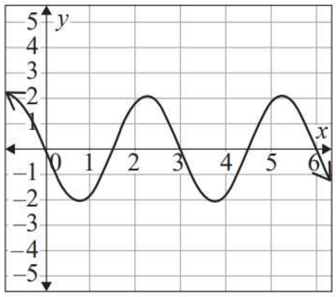
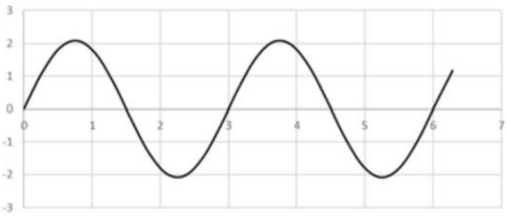


Diagram 3

Justify your decisions using mathematical reasoning. [4 marks]

Sample Response	The response
$f(x)$ changes concavity at certain points and is increasing and decreasing between $x = 0$ and $x = 3$ and between $x = 3$ and $x = 6$	• correctly identifies an appropriate method to determine $f''(x)$ graph [1 mark]
Therefore, the sketch of $f'(x)$ must be positive and negative (above and below the xx-axis) for these intervals.	• correctly determines a relevant feature of the graph of $f(x)$ [1 mark]
 $f'(x)$ has maximum and minimum points at approximately $x = 0.8$ and $x = 2.4$. Therefore, the sketch of $f''(x)$ will cross the x-axis at these points.	• correctly determines a relevant feature for $f''(x)$ [1 mark]
Diagram 1 is $f''(x)$	• correctly identifies Diagram 1 [1 mark]

2020
Paper 1
Section 2
Question 19

Further differentiation and applications 3

A horizontal point of inflection is a point of inflection that is also a stationary point.

Determine the value/s of k for which the graph of $f(x) = \frac{\ln(x)}{k} - \frac{kx}{x+1}$ has only one horizontal point of inflection.

Sample Response	The response
Method 1 $f'(x) = \frac{1}{kx} - \left(\frac{k(x+1) - kx}{(x+1)^2}\right)$ Stationery points $f'(x) = 0$	• correctly determines the first derivative [1 mark]
$0 = \frac{1}{kx} - \left(\frac{k(x+1) - kx}{(x+1)^2}\right)$ $0 = \frac{1}{kx} - \frac{k}{(x+1)^2}$ $0 = x^2 + (2 - k2)x + 1$ (i) The quadratic has real roots when discriminant ≥ 0 $(2 - k^2)^2 - 4 \geq 0$ $-k^2 \geq \pm 2$	• correctly determines the quadratic equation to identify the stationary point/s [1 mark]
There is only ONE phi ∴ $2 - k^2 = \pm 2$ $k = 0$ (not valid) and $k^2 = 4$ so $k = 2, -2$	• determines valid and non-valid solutions of k [1 mark]
Sub into (i) to determine the x-ordinate of the stationary point → $x = 1$	• determines x-ordinate of stationary point [1 mark]
For $k = 2$ ∴ $f''(x) = \frac{-1}{2x^2} + \frac{4}{(x+1)^3}$ $f''(1) = \frac{-1}{2} + \frac{4}{8}$ $f''(1) = 0$ For $k = -2$ $f''(x) = \frac{1}{2x^2} + \frac{4}{(x+1)^3}$ $f''(x) = \frac{1}{2} - \frac{4}{8}$ $f''(1) = 0$ For each k value, $x = 1$ is the x-ordinate of both a stationary point ($f'(x) = 0$) and a point of inflection ($f''(x) = 0$) There is a point of horizontal inflection at $x = 1$ when $k = \pm 2$	• determines values of second derivative for both values of k [1 mark] • shows logical organisation communicating key steps [1 mark]
Method 2 $f'(x) = \frac{1}{kx} - \left(\frac{k(x+1) - kx}{(x+1)^2}\right)$ Stationary points $f'(x) = 0$	• correctly determines the first derivative [1 mark]
$0 = \frac{1}{kx} - \left(\frac{k(x+1) - kx}{(x+1)^2}\right)$	• correctly establishes expression for k^2 in terms of x [1 mark]

	$k = \pm\sqrt{\dfrac{(x+1)^2}{x}}$ (ii)	
	Point of inflection $f''(x) = 0$ $\therefore = \dfrac{-1}{kx^2} + \dfrac{2k}{(x+1)^3}$ $k^2 = \dfrac{(x+1)^3}{2x^2}$ Sub into (i) $$\frac{(x+1)^2}{x} = \frac{(x+1)^3}{2x^2}$$ $2x^2(x+1)2 - x(x+1)^3 = 0$ $x(x+1)^2\big(2x - (x+1)\big) = 0$ $x(x+1)^2(x-1) = 0$ $x = 0, -1, 1$	• determines x values [1 mark]
	Sub into (ii) $x = 0, -1$ non-valid solutions	• determines valid and non-valid solutions of x [1 mark]
	Using $x = 1$ $k = \pm\sqrt{\dfrac{(1+1)^2}{1}} = \pm 2$ For each k value $x = 1$ is the x-ordinate of both a stationary point ($f'(x) = 0$) and a point of inflection ($f''(x) = 0$) There is a point of horizontal inflection at $x = 1$ when $k = \pm 2$	• determines k values [1 mark] • shows logical organisation communicating key steps [1 mark]
	Let $R(t)$ = radius of the tree trunk $R(t) = \int 1.5 + \sin\left(\dfrac{\pi t}{5}\right) dt$ $= 1.5\,t - \dfrac{5}{\pi} cos\left(\dfrac{\pi t}{5}\right) + c$ Given initial radius is 15cm (end of first stage/beginning of second stage)	• correctly establishes an integrated expression for the radius of the tree [1 mark]
	$15 = -\dfrac{5}{\pi} + c$ $c = 15 + \dfrac{5}{\pi}$ $R(t) = 1.5t - \dfrac{5}{\pi}\cos\left(\dfrac{\pi t}{5}\right) + 15 + \dfrac{5}{\pi}$	• determines radius of the tree [1 mark]
	Volume of tree trunk = $500\pi(R(t))^2$	• identifies use of formula and radius to determine volume of tree trunk [1 mark]
	Volume of tree trunk at $t = 10$ $V_{10}\pi \times \big(R(10)\big)^2 \times 500$ Volume of tree trunk at $t = 0$ $V_0 = \pi \times \big(R(0)\big)^2 \times 500$ $\rightarrow V_{10} = 450\,000\pi$ cm^3 and $V_0 = 112\,500\pi$ cm^3	• determines volumes of tree trunk initially and at 10 years [1 mark]
	The mass of the tree is the density times the volume, and since the density is 1 g/cm^3 ∴ ratio of the tree trunk at $t = 0$ and $t = 10$ years $= \dfrac{450\,000\pi}{112\,500\pi}$ $= 4$ The tree has quadrupled in mass over this time.	• determines ratio of mass [1 mark] • shows logical organisation communicating key steps [1 mark]

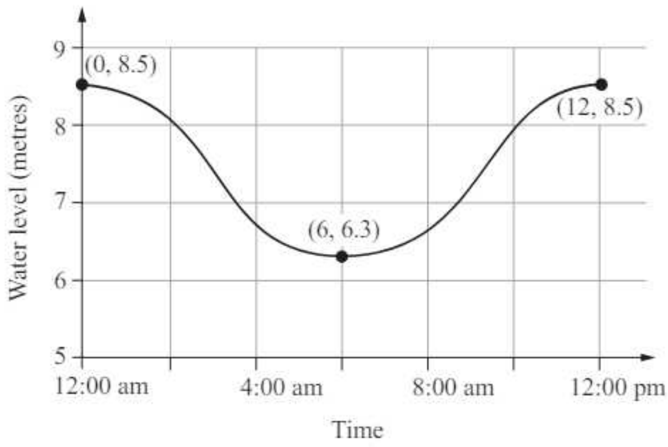
Marking Guide – Paper 2 Section 1

2022 Paper 2 Section 1 Question 2 **Further differentiation and applications 3**	Identify the correct features of the function $f(x) = xe^x$ (A) $f'(-1) = 0, f''(-1) < 0$ (B) $f'(-1) = 0, f''(-1) > 0$ (C) $f'(-1) < 0, f''(-1) < 0$ (D) $f'(-1) < 0, f''(-1) > 0$ **Answer is B.**

2022 Paper 2 Section 1 Question 3 **Further differentiation and applications 3**	The derivative of the function $f(x)$ is given by $f'(x) = \sin(x^3)$ for the domain $-1.8 < x < 1.8$. The number of points of inflection that the graph of $f(x)$ has on this interval is (A) 1 (B) 3 **(C) 4 – Answer** (D) 5

2020 Paper 2 Section 1 Question 10 **Further differentiation and applications 3**	The approximate value of x where the graph of the function $y = x^3 + 6x^2 + 7x - 2\cos(x)$ changes concavity is (A) –3.26 (B) –2.85 (C) –2.20 **(D) –1.89 – Answer**

2023 Paper 2 Section 2 Question 12 **Further differentiation and applications 3**	The graph shows the water level under a bridge over a 12-hour period.



a) Determine the equation of the cosine function that models the water level as a function of time after 12:00 am. [1 mark]

Sample response	The response
The graph is cosine with a period of 12, amplitude 1.1 and equilibrium height 7.4 with no phase shift. $\therefore w(t) = 1.1\cos\left(\frac{\pi}{6}t\right) + 7.4$	• correctly determines the equation of the graph **[1 mark]**

b) How long in the 12-hour period shown is the rate of change of water level more than 0.55 metres per hour? [3 marks]

Sample response	The response
$w'(t) = \frac{-1.1}{6}\pi\sin\left(\frac{\pi}{6}t\right)$	• determines the derivative equation **[1 mark]**
Using GDC: solving $w'(t) = 0.55$ Interval is $(8.4244, 9.5756)$	• determines time interval where rate of change is more than 0.55 metres per hour **[1 mark]**
i.e. 1.1512 hours = **1.1512** x 60 = 69.072 = 69 minutes	• determines the time interval **[1 mark]**

2023 Paper 2 Section 2 Question 15 **Further differentiation and applications 3**	Determine the derivative of $f(x) = \ln x^2 + \ln(x-5)^3$. Express the derivative as a single fraction in its simplest and factorised form. (4 marks)

Sample response	**The response**
$f(x) = \ln x^2 + \ln(x-5)^3$ $f(x) = 2\ln x + 3\ln(x-5)$	
$f'(x) = 2 \times \frac{1}{x} + 3 \times \frac{1}{(x-5)}$ $f'(x) = \frac{2}{x} + \frac{3}{(x-5)}$	• correctly determines first term derivative **[1 mark]** • correctly determines second term derivative **[1 mark]**
$f'(x) = \frac{2(x-5) + 3x}{x(x-5)}$ $f'(x) = \frac{2x - 10 + 3x}{x(x-5)}$ $f'(x) = \frac{5x - 10}{x(x-5)}$	• combines the fractions to one fraction **[1 mark]**
$f'(x) = \frac{5(x-2)}{x(x-5)}$	• determines the simplest and factorised form **[1 mark]**

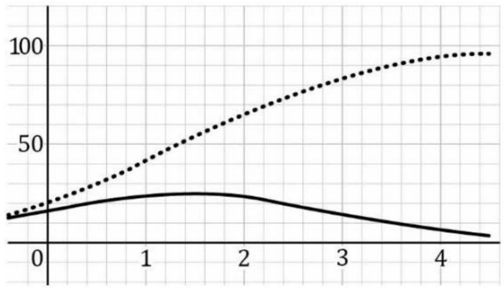
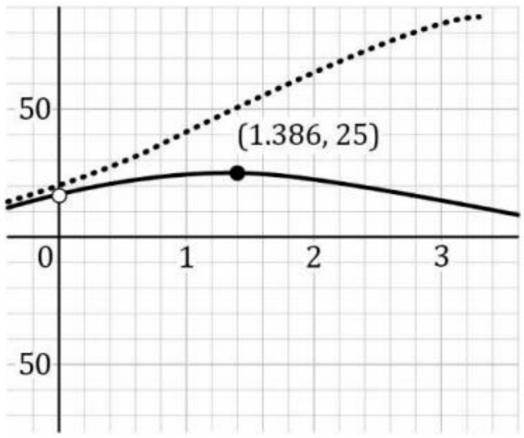
2021
Paper 2
Section 2
Question 18

Further differentiation and applications 3

The number of animals in a population (in thousands) is modelled by the function P such that

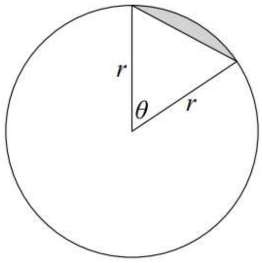
$P(t) = \dfrac{100}{1 + 4e^{-t}}$, where t is in years.

Determine the number of animals in the population when the population is growing the fastest. [3 marks]

Sample Response	The response
Graphing $P(t)$ (dotted line) and $P'(t)$ (solid line)  	• correctly identifies the conditions for the most rapid increase [1 mark]
The population is increasing most rapidly at the maximum value of $P'(t)$  → t = 1.386	• determines when population growing the fastest [1 mark]
∴ P(1.386)~49.993 There are approximately 50 000.	• determines population at this time [1 mark]

Unit 4 – Topic 2: Trigonometric functions 2

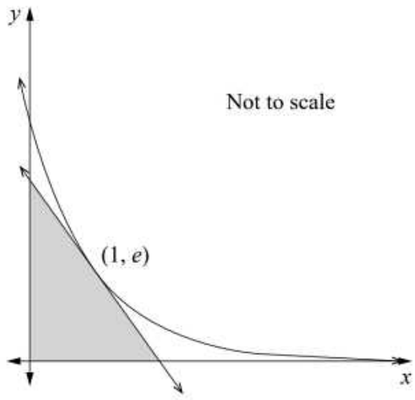
Paper 1 Section 1

2022 Paper 1 Section 1 Question 7 Trigonometric functions 2	A circle with radius r and internal angle θ has a shaded segment as shown.  If θ is in radians, the area of the shaded segment is (A) $\frac{r^2}{2}\left(\frac{\theta\pi}{180}-\sin(\theta)\right)$ (B) $\frac{r^2}{2}\left(\theta-\sin(\theta)\right)$ (C) $\frac{r^2}{4}\left(\frac{\theta\pi}{90}-1\right)$ (D) $\frac{r^2}{2}(\theta-1)$

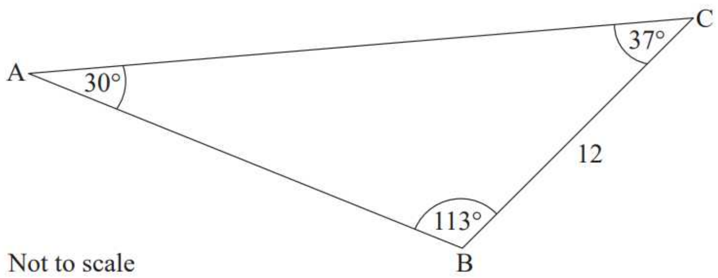
2020 Paper 1 Section 1 Question 5 Trigonometric functions 2	The equation of the tangent to the curve $f(t) = te^t$ at $t = 1$ is (A) $y = et$ (B) $y = 2et - e$ (C) $y = et - e^2 + 1$ (D) $y = 2et - 2e^2 + 1$

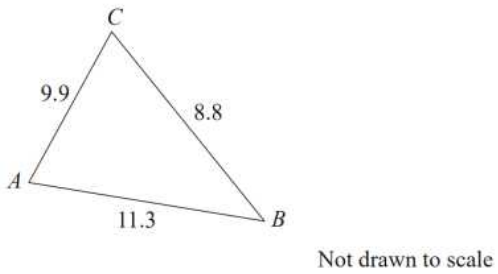
Paper 1 Section 2

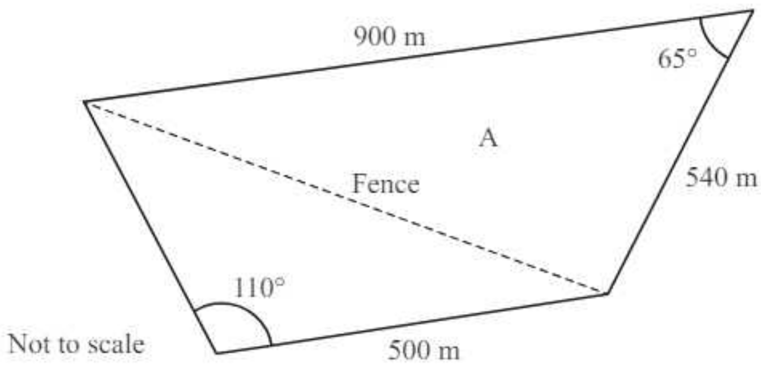
2021 Paper 1 Section 2 Question 14 **Trigonometric functions 2**	Consider the function $f\ (x) = \ln(3x + 4)\,, for\ x > \frac{-4}{3}$ a) Determine $f'(x)$. [1 mark] b) Determine the x-intercept of the graph of $f(x)$. [2 marks] c) Determine the gradient of the tangent to the graph of $f(x)$ at the x-intercept. [1 mark]

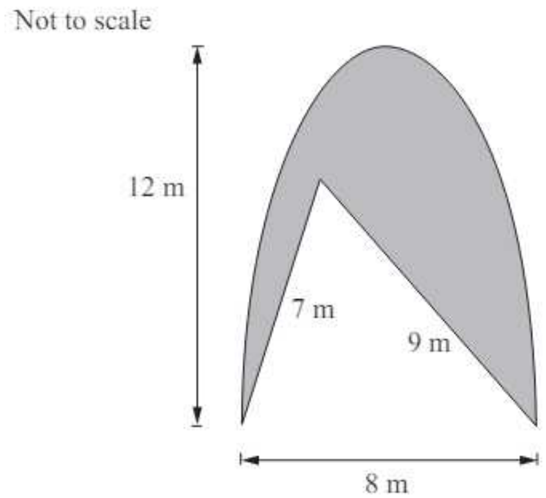
2021 **Paper 1** **Section 2** **Question 16** **Trigonometric functions 2**	A tangent is drawn at the point (1, *e*) on the graph of the function $y = e^{2-x}$ as shown.  Determine the area of the shaded triangle. [4 marks]

Paper 2 Section 1

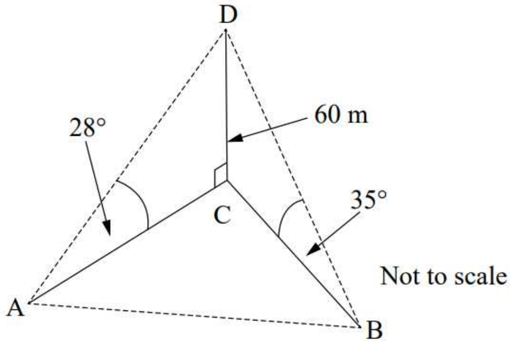
2022 Paper 2 Section 1 Question 9 **Trigonometric functions 2**	Determine the length of side AB in triangle ABC.  (A) 22.13 (B) 14.44 (C) 9.97 (D) 7.82

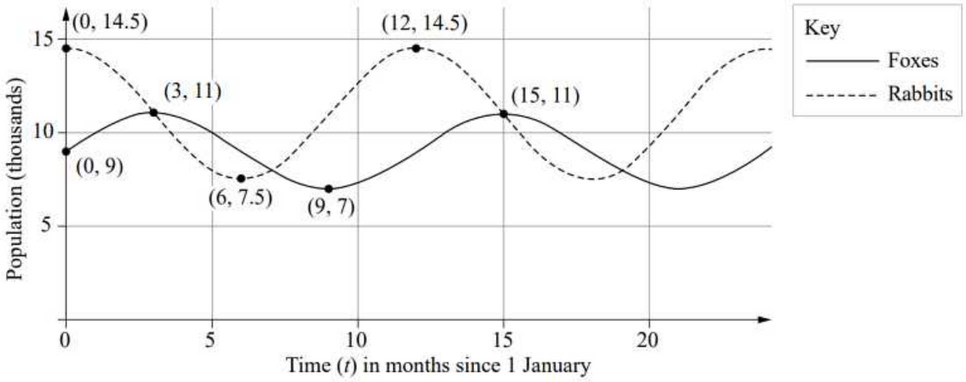
2020 Paper 2 Section 1 Question 8 **Trigonometric functions 2**	Determine the size of angle A in the triangle.  (A) 48.5° (B) 61.4° (C) 118.6° (D) 131.5°

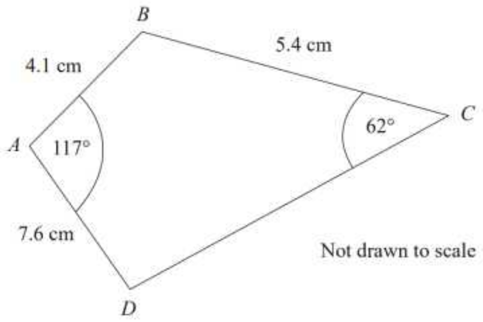
2023 Paper 2 Section 2 Question 14 Trigonometric functions 2	A fence divides a paddock into two triangular sections as shown.  a) Determine the length of the fence. [1 mark] b) Calculate the area of triangular section A. [1 mark] c) Determine the total area of the paddock. [5 marks]

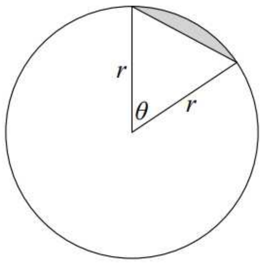
2023 Paper 2 Section 2 Question 18 **Trigonometric functions 2**	A company makes windows using glass that has a mass of 5.6 kg per square metre. A customer orders an unusual window in a partial parabolic shape, as shown.  Determine the mass of the window. (5 marks)

2022 **Paper 2** **Section 2** **Question 15** **Trigonometric functions 2**	A hiker begins her journey at a youth hostel (*H*) and walks for 8 km on a bearing of 052°T to her lunch stop (*L*). She then walks on a bearing of 210°T for 5.2 km until she reaches a campsite (*C*). Determine the direction she would need to walk in a straight line to return directly to the youth hostel. [7 marks]

2021 Paper 2 Section 2 Question 16	In the diagram DC represents a 60 metre vertical tower.
Trigonometric functions 2	A and B are two points in the same horizontal plane as the foot C of the tower.
	The angle above the horizontal from A to D is 28° and the angle above the horizontal from B to D is 35°.
	
	The bearing of C from A is 050°T and the bearing of C from B is 300°T.
	Determine the distance between A and B to the nearest metre. [4 marks]

2021 Paper 2 Section 2 Question 17 **Trigonometric functions 2**	Rabbits and foxes are among two species of mammals that live on an isolated island. Rabbits represent a significant food source for the foxes. The populations of rabbits and foxes were monitored each month for two years. The graph shows the population of foxes (in thousands) and the population of rabbits (in thousands), at any time *t* (in months) over the two years. The two populations can be modelled using trigonometric functions.  Jane believes that there were periods of time over the two years when the total population of foxes and rabbits on the island exceeded 25 000. Evaluate the reasonableness of Jane's claim. [4 marks]

2020 **Paper 2** **Section 2** **Question 18** **Trigonometric functions 2**	The diagram shows the quadrilateral ABCD.  Determine the perimeter of the quadrilateral. [4 marks]

2022 Paper 1 Section 1 Question 7 Trigonometric functions 2	A circle with radius r and internal angle θ has a shaded segment as shown.  If θ is in radians, the area of the shaded segment is (A) $\frac{r^2}{2}\left(\frac{\theta\pi}{180}-\sin(\theta)\right)$ (B) $\frac{r^2}{2}\left(\theta-\sin(\theta)\right)$ (C) $\frac{r^2}{4}\left(\frac{\theta\pi}{90}-1\right)$ (D) $\frac{r^2}{2}(\theta-1)$ **Answer is B.**

2020 Paper 1 Section 1 Question 5 Trigonometric functions 2	The equation of the tangent to the curve $f(t) = te^t$ at $t = 1$ is (A) $y = et$ (B) $y = 2et - e$ (C) $y = et - e^2 + 1$ (D) $y = 2et - 2e^2 + 1$ **Answer is B.**

2021
Paper 1
Section 2
Question 14

Trigonometric functions 2

Consider the function $f\ (x) = \ln(3x + 4)\,, for\ x > \dfrac{-4}{3}$

a) Determine $f'\ (x)$. [1 mark]

Sample Response	The response
$f'\ (x) = \frac{3}{(3x+4)}$	• correctly determines the derivative [1 mark]

b) Determine the x-intercept of the graph of $f\ (x)$. [2 marks]

Sample Response	The response
x-intercept ($y = 0$) $0 = \ln(3x + 4)$ $3x + 4 = 1$	• correctly determines the linear equation [1 mark]
$x = -1$	• correctly determines the x-intercept [1 mark]

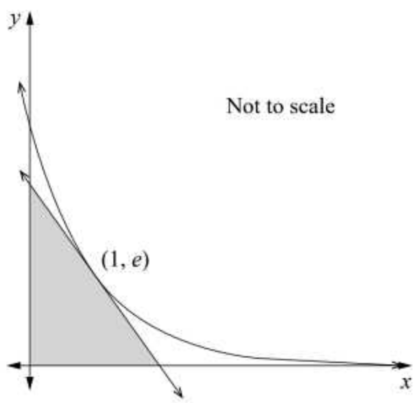
c) Determine the gradient of the tangent to the graph of $f\ (x)$ at the x-intercept. [1 mark]

Sample Response	The response
$f'(-1)$ $= \dfrac{3}{-3 + 4}$ $= 3$	• determines the gradient at the x-intercept [1 mark]

2021
Paper 1
Section 2
Question 16

Trigonometric functions 2

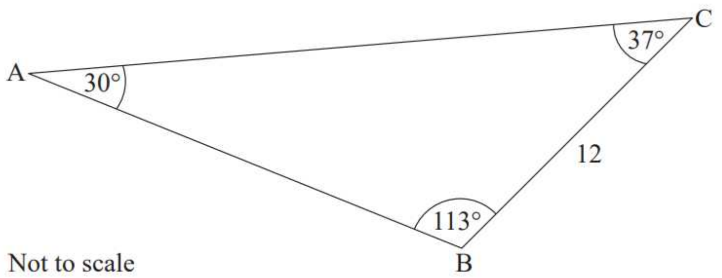
A tangent is drawn at the point (1, e) on the graph of the function $y = e^{2-x}$ as shown.

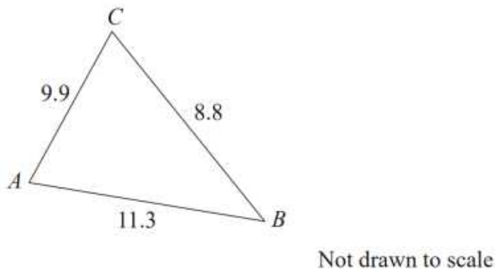


Determine the area of the shaded triangle. [4 marks]

Sample Response	The response
$y' = -e^{2-x}$ Gradient of tangent at (1, e) $y'(1) = -e$	• correctly determines the derivative [1 mark]
Equation of the tangent $(y - e) = -e(x - 1)$	• determines equation of tangent [1 mark]
x-intercept (2,0) y-intercept (0,2e)	• determines x- and y-intercepts [1 mark]
Area of triangle $= \dfrac{1}{2} \times 2 \times 2e$ Area of triangle $= 2e$ units^2	• determines area of triangle [1 mark]

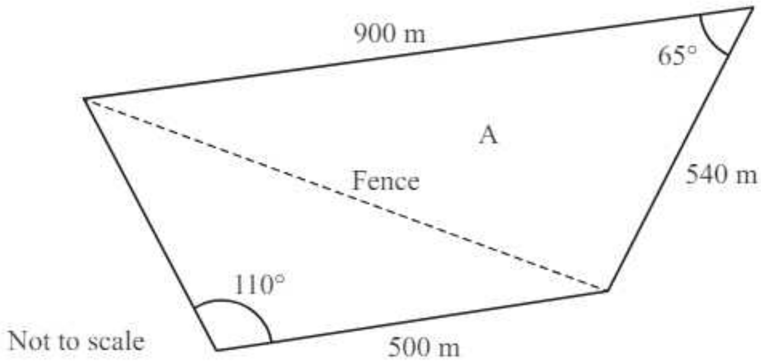
Marking Guide – Paper 2 Section 1

2022 **Paper 2** **Section 1** **Question 9** **Trigonometric functions 2**	Determine the length of side AB in triangle ABC.  (A) 22.13 **(B) 14.44 – Answer** (C) 9.97 (D) 7.82

2020 **Paper 2** **Section 1** **Question 8** **Trigonometric functions 2**	Determine the size of angle *A* in the triangle.  **(A) 48.5° – Answer** (B) 61.4° (C) 118.6° (D) 131.5°

Marking Guide – Paper 2 Section 2

2023 Paper 2 Section 2 Question 14 **Trigonometric functions 2**	A fence divides a paddock into two triangular sections as shown.



a) Determine the length of the fence. [1 mark]

Sample response	The response
Use the cosine rule $c^2 = a^2 + b^2 - 2ab\cos C$ $c = \sqrt{900^2 + 540^2 - 2 \times 900 \times 540 \times \cos 65}$ $= 831.1528$ m	• correctly determines the length of the internal fence **[1 mark]**

b) Calculate the area of triangular section A. [1 mark]

Sample response	The response
Use $A = \frac{1}{2}ab\sin C$ $A = \frac{1}{2} \times 900 \times 540 \times \sin 65°$ $= 220\,232.7922$ m^2	• correctly determines the triangle area **[1 mark]**

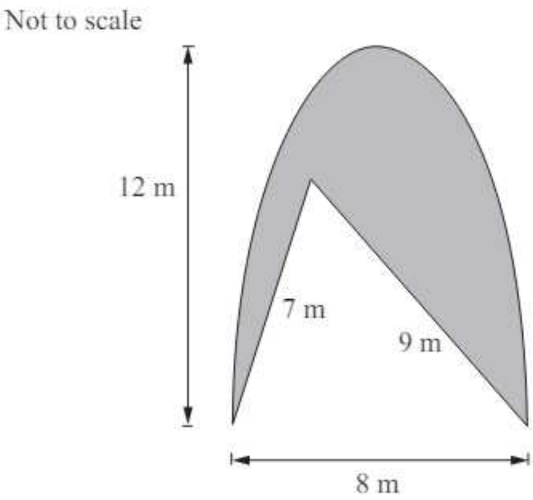
c) Determine the total area of the paddock. [5 marks]

Sample response	The response
Method 1 Use the sine rule to find angle opposite the 500 m side of the second triangle. $\frac{\sin D}{500} = \frac{\sin 110}{831.1528}$	• uses the sine rule with appropriate substitutions evident **[1 mark]**
$\sin D = 500 \times \frac{\sin 110}{831.1528}$ $D = \sin^{-1}\left(500 \times \frac{\sin 110}{831.1528}\right)$ $= 34.4228°$ Determine remaining angle in the triangle $180 - (110 + 34.4228)$ $= 35.5772°$ Determines area of the 110° triangle $A = \frac{1}{2} \times 831.1528 \times 500 \times \sin 35.5772$ $= 120\,891.0424 \text{ m}^2$ Determines the total area Total area $= 220\,232.7922 + 120\,891.0424$ $= 341\,123.8346 \text{ m}^2$	• determines the required angle to determine the area of the second triangle **[1 mark]** • determines the area of the second triangle **[1 mark]** • determines the total area of the paddock **[1 mark]** • shows logical organisation communicating key steps **[1 mark]**
Method 2 Use the cosine rule to find the missing side length of the second triangle. $c^2 = a^2 + b^2 - 2ab\cos(C)$ $831.1528^2 = a^2 + 500^2 - 2 \times a \times 500 \times \cos(110)$	• uses the cosine rule with appropriate substitutions evident **[1 mark]**
$a = 514.599 \text{ m}$ Determines area of the 110° triangle $A = \frac{1}{2} \times 500 \times 514.599 \times \sin(110)$ $= 120\,891.2207 \text{ m}^2$ Determines the total area Total area $= 220\,232.7922 + 120\,891.2207$ $= 341\,124.0129 \text{ m}^2$	• determines the unknown side length in the second triangle **[1 mark]** • determines the area of the second triangle **[1 mark]** • determines the total area of the paddock **[1 mark]** • shows logical organisation communicating key steps **[1 mark]**

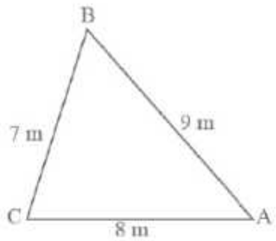
2023
Paper 2
Section 2
Question 18

Trigonometric functions 2

A company makes windows using glass that has a mass of 5.6 kg per square metre. A customer orders an unusual window in a partial parabolic shape, as shown.

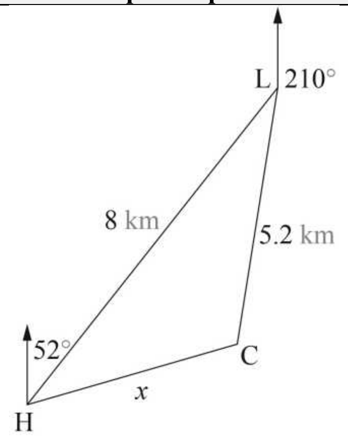


Determine the mass of the window. (5 marks)

Sample response	The response
Determine the area of the triangle removed: TRIANGLE  **Find an angle** (use the cosine rule): $c^2 = a^2 + b^2 - 2ab\cos C$ $C = cos^{-1}\left(\frac{c^2 - a^2 - b^2}{-2ab}\right)$ $C = 73.39845°$ The **area of this triangle**: $area = \frac{1}{2} \times 7 \times 8 \times \sin(73.39845)$ = **26.8328** m^2 Determine the equation of the parabola: Consider an inverted **parabola located on a set of axes with x-intercepts (0, 0) and (8, 0) and the vertex (4, 12)** **The parabola equation is** given by: $y = a(x-h)^2 + k$ Where (*h*, *k*) is the vertex (4, 12) $\therefore y = a(x-4)^2 + 12$ Substitute one point on the parabola e.g. (8, 0) $0 = a(8-4)^2 + 12$ $0 = a(4)^2 + 12$	• correctly determines area of the 'removed' triangle **[1 mark]**

	$-12 = a(4)^2$ $a = \frac{-12}{16}$ $a = \frac{-3}{4}$ $\therefore y = \frac{-3}{4}(x-4)^2 + 12$	• correctly determines the parabola equation **[1 mark]**
	Determine the **area enclosed by this parabola** and the x-axis: area = $\int_0^8 \frac{-3}{4}(x-4)^2 + 12\,dx$ = **64** m^2 (using GDC) Determine the **area of the glass** in the window: area = $64\ \mathrm{m}^2 - 26.8328\ \mathrm{m}^2$ $= 37.1672\ \mathrm{m}^2$ Determine the **mass of the glass** in the window: mass = 37.1672×5.6 = **208.1363** kg	• determines the area between the parabola and the *x*-axis **[1 mark]** • determines the mass of the window glass **[1 mark]** • shows logical organisation, communicating key steps **[1 mark]**

2022 Paper 2 Section 2 Question 15 **Trigonometric functions 2**	A hiker begins her journey at a youth hostel (H) and walks for 8 km on a bearing of 052°T to her lunch stop (L). She then walks on a bearing of 210°T for 5.2 km until she reaches a campsite (C). Determine the direction she would need to walk in a straight line to return directly to the youth hostel. [7 marks]

Sample Response	**The response**
	• correctly establishes the diagram using bearing and distance information [1 mark]
H = $h = 5.2$ L = 22° $l = x$ C = $c = 8$	• correctly determines angle L [1 mark]
Using cosine rule $x^2 = 8^2 + 5.2^2 - 2 \times 8 \times 5.2 \times \cos 22°$ $x = 3.7280$ km	• determines unknown distance [1 mark]
Using sine rule $\frac{3.7280}{\sin 22°} = \frac{8}{\sin C}$	• substitutes into appropriate rule [1 mark]
$C = 53.5019°$ or $C = 126.4981°$	• determine obtuse angle at C [1 mark]
Bearing = 360 – 126.4981° + 30 ∴ walk back at a bearing of 263°30′ T	• determines direction needed to return to youth hostel using the obtuse angle for C [1 mark] • shows logical organisation communicating key steps [1 mark]

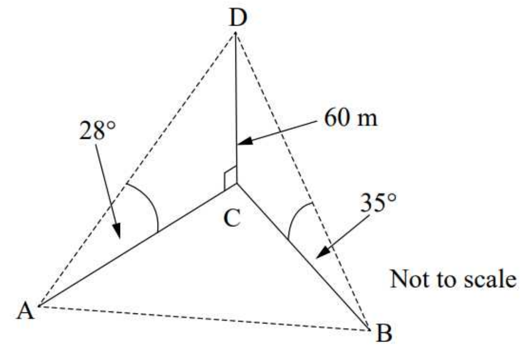
2021
Paper 2
Section 2
Question 16

Trigonometric functions 2

In the diagram DC represents a 60 metre vertical tower.

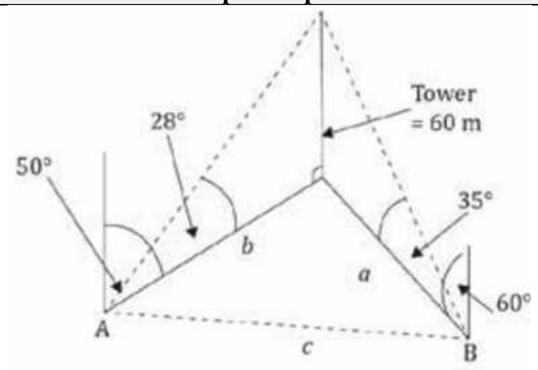
A and B are two points in the same horizontal plane as the foot C of the tower.

The angle above the horizontal from A to D is 28° and the angle above the horizontal from B to D is 35°.



The bearing of C from A is 050°T and the bearing of C from B is 300°T.

Determine the distance between A and B to the nearest metre. [4 marks]

Sample Response	The response
	• correctly establishes the angles using given bearings [1 mark]
Distance from A to the tower (b): $\tan 28° = \frac{60}{b}$ Distance from B to the tower (a): $\tan 35° = \frac{60}{a}$ b = 112.84 m and a = 85.69 m	• correctly determines the distance a and b [1 mark]
Distance between A and B (c) $c^2 = 85.69^2 + 112.84^2 - 2 \times 85.69 \times 112.84 \times \cos 110°$ $c = 163.37$ m The distance between A and B is 163 m.	• determines distance between A and B [1 mark] • shows logical organisation communicating key steps [1 mark]

2021
Paper 2
Section 2
Question 17

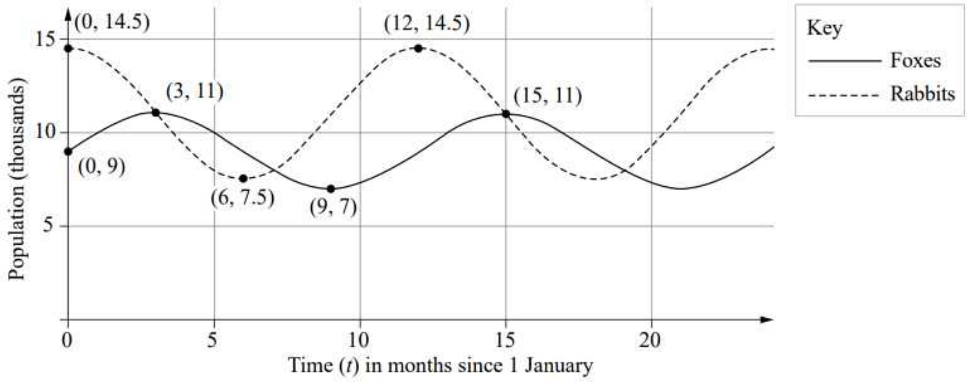
Trigonometric functions 2

Rabbits and foxes are among two species of mammals that live on an isolated island.

Rabbits represent a significant food source for the foxes.

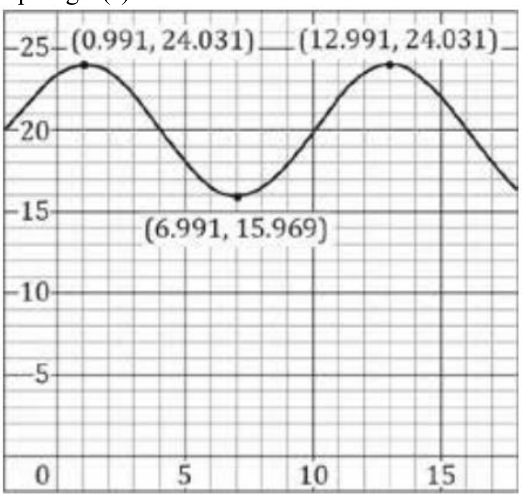
The populations of rabbits and foxes were monitored each month for two years.

The graph shows the population of foxes (in thousands) and the population of rabbits (in thousands), at any time t (in months) over the two years. The two populations can be modelled using trigonometric functions.



Jane believes that there were periods of time over the two years when the total population of foxes and rabbits on the island exceeded 25 000.

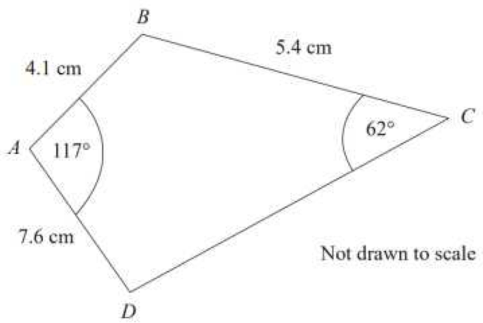
Evaluate the reasonableness of Jane's claim. [4 marks]

Sample Response	The response
Models are periodic. Rabbits: $R(t) = 11 + 3.5\cos\left(\frac{\pi}{6}t\right)$ Foxes: $F(t) = 9 + 2\sin\left(\frac{\pi}{6}t\right)$	• correctly determines the models for the two populations [1 mark]
Total population of foxes and rabbits $T(t) = 20 + 3.5\cos\left(\frac{\pi}{6}t\right) + 2\sin\left(\frac{\pi}{6}t\right)$	• determines model for total population of foxes and rabbits [1 mark]
Graphing $T(t)$ 	• uses an appropriate mathematical representation [1 mark]
Greatest total population of foxes and rabbits is 24 031. Jane's claim is not correct. The maximum total population occurs on one occasion in the year, but does not exceed 25 000 (maximum is 24 031).	• evaluates reasonableness of the claim [1 mark]

2020
Paper 2
Section 2
Question 18

Trigonometric functions 2

The diagram shows the quadrilateral ABCD.



Determine the perimeter of the quadrilateral. [4 marks]

Sample Response	The response
Constructing the line *BD* Using the cosine rule $BD^2 = 4.1^2 + 7.6^2 - 2 \times 4.1 \times 7.6 \times \cos 117°$ $BD = 10.142$ cm	• correctly determines *BD* [1 mark]
Using the cosine rule $10.142^2 = 5.4^2 + DC^2 - 2 \times 5.4 \times DC \cos 62°$	• establishes equation in *DC* [1 mark]
Using solve application GDC *DC* = 11.4865	• determines *DC* [1 mark]
Perimeter of *ABCD* = 11.4865 + 5.4 + 4.1 + 7.6 = 28.5865	• determines perimeter [1 mark]

Unit 4 – Topic 3: Discrete random variables 2

Paper 1 Section 1

2023 Paper 1 Section 1 Question 3 **Discrete random variables 2**	A bag contains 10 buttons of the same shape and size in different colours: 5 blue, 3 green and 2 red. If 3 buttons are randomly drawn from the bag, which probability can be calculated using the binomial distribution? (A) P(3 green) with replacement (B) P(3 blue) without replacement (C) P(2 green and 1 red) with replacement (D) P(2 red and 1 blue) without replacement

2022 Paper 1 Section 1 Question 2 **Discrete random variables 2**	A binomial random variable arises from the number of successes in n independent Bernoulli trials. A context **not** suitable for modelling using a binomial random variable is recording the number of (A) heads when a coin is tossed 12 times. (B) left-handed people in a sample of 100 people. (C) times a player hits a target from 20 shots where each shot is independent of all other shots. (D) red marbles selected when three marbles are drawn without replacement from a bag containing four blue and five red marbles.

2021 Paper 1 Section 1 Question 6

Discrete random variables 2

A random variable X is the number of successes in a Bernoulli experiment with n trials, each with a probability of success p and a probability of failure q. The probability distribution table of X is shown.

k	$P(X=k)$
0	$\frac{1}{81}$
1	$\frac{8}{81}$
2	$\frac{24}{81}$
3	$\frac{32}{81}$
4	$\frac{16}{81}$

Which values of n, p and q will generate this probability distribution?

(A) $n=4,\ p=\frac{2}{3},\ q=\frac{1}{3}$

(B) $n=4,\ p=\frac{1}{3},\ q=\frac{2}{3}$

(C) $n=5,\ p=\frac{2}{3},\ q=\frac{1}{3}$

(D) $n=5,\ p=\frac{1}{3},\ q=\frac{2}{3}$

2020 Paper 1 Section 1 Question 6 **Discrete random variables 2**	If the probability of success in a Bernoulli trial is 0.30, the variance is (A) 0.70 (B) 0.46 (C) 0.30 (D) 0.21

Paper 1 Section 2

2023 Paper 1 Section 2 Question 11 **Discrete random variables 2**	Two random samples (A and B) were obtained using two different Bernoulli experiments. Each Bernoulli trial in the random samples was recorded as 1 (for success) or 0 (for failure). The results are shown.

A	1	1	1	1	0	1	1	0	1	1
B	0	0	1	1	1	0	1	1	0	0

In sample A, for each trial the mean is 0.8 and the variance is 0.16.

a) Use the sample B results to determine the mean and variance for each trial in sample B. [2 marks]

b) Compare the variability about the means of samples A and B. [2 marks]

2023 Paper 1 Section 2 Question 13 **Discrete random variables 2**	At a certain airport, the departure of one in five international flights is delayed every day. The status of any flight is independent of other flights. One international flight is selected at random each day for three days. Each selection is recorded as either ‘delayed’ or ‘not delayed’. a) State two conditions that make this context suitable for modelling using a binomial random variable. [2 marks] b) Calculate the probability that at least two of the selected flights were delayed. [3 marks]

2023 Paper 1 Section 2 Question 15 Discrete random variables 2	In a certain game, players throw one water balloon at a target. There is a one in four chance of hitting the target. a) State the probabilities of all the possible outcomes for one throw at the target. [2 marks] b) Let *H* be the discrete random variable for one of the possible outcomes. Determine the mean and variance of the distribution of random variable *H* when 20 players throw a water balloon at the target. [2 marks]

Paper 2 Section 1

2023 Paper 2 Section 1 Question 2 **Discrete random variables 2**	The probability of hitting a bullseye on a standard dartboard is 1 in 1250. What is the probability of hitting a bullseye exactly once in 10 attempts? (A) $\binom{9}{1}\left(\frac{1}{1250}\right)^1 \times \left(\frac{1249}{1250}\right)^9$ (B) $\binom{9}{1}\left(\frac{1}{1250}\right)^9 \times \left(\frac{1249}{1250}\right)^1$ (C) $\binom{10}{1}\left(\frac{1}{1250}\right)^1 \times \left(\frac{1249}{1250}\right)^9$ (D) $\binom{10}{1}\left(\frac{1}{1250}\right)^9 \times \left(\frac{1249}{1250}\right)^1$

Paper 2 Section 2

2020 Paper 2 Section 2 Question 11 **Discrete random variables 2**	A sugar company samples the packets of sugar it produces and finds that 5% of packets are underweight. Consider a batch of 20 packets. a) Determine how many packets of sugar the company can expect to be underweight in a batch of 20 packets. [1 mark] b) Determine the variance of the batch. [1 mark] c) Determine the probability that at most one of the packets of sugar in the batch is underweight. [2 marks]

2021 Paper 2 Section 2 Question 13 **Discrete random variables 2**	The amount of gravel (in tonnes) sold by a construction company in a given week is a continuous random variable *X* and has a probability density function defined by: $$f(x) = \begin{cases} c\left(1 - x^2\right), & 0 \le x \le 1 \\ 0, & \text{otherwise} \end{cases}$$ a) Show that $c = \frac{3}{2}$ [1 mark] b) Determine $P(X < 0.25)$. [2 marks] c) Determine the variance of *X*. [4 marks]

Marking Guide – Paper 1 Section 1

2023 Paper 1 Section 1 Question 3 **Discrete random variables 2**	A bag contains 10 buttons of the same shape and size in different colours: 5 blue, 3 green and 2 red. If 3 buttons are randomly drawn from the bag, which probability can be calculated using the binomial distribution? (A) P(3 green) with replacement **(B) *P*(3 blue) without replacement – Answer** (C) P(2 green and 1 red) with replacement (D) P(2 red and 1 blue) without replacement

2022 Paper 1 Section 1 Question 2 **Discrete random variables 2**	A binomial random variable arises from the number of successes in n independent Bernoulli trials. A context **not** suitable for modelling using a binomial random variable is recording the number of (A) heads when a coin is tossed 12 times. (B) left-handed people in a sample of 100 people. (C) times a player hits a target from 20 shots where each shot is independent of all other shots. **(D) red marbles selected when three marbles are drawn without replacement from a bag containing four blue and five red marbles. – Answer**

2021 Paper 1 Section 1 Question 6

Discrete random variables 2

A random variable X is the number of successes in a Bernoulli experiment with n trials, each with a probability of success p and a probability of failure q. The probability distribution table of X is shown.

k	$P(X=k)$
0	$\frac{1}{81}$
1	$\frac{8}{81}$
2	$\frac{24}{81}$
3	$\frac{32}{81}$
4	$\frac{16}{81}$

Which values of n, p and q will generate this probability distribution?

(A) $n=4,\ p=\frac{2}{3}, q=\frac{1}{3}$

(B) $n=4,\ p=\frac{1}{3}, q=\frac{2}{3}$

(C) $n=5,\ p=\frac{2}{3}, q=\frac{1}{3}$

(D) $n=5,\ p=\frac{1}{3}, q=\frac{2}{3}$

Answer is A.

2020 Paper 1 Section 1 Question 6 **Discrete random variables 2**	If the probability of success in a Bernoulli trial is 0.30, the variance is (A) 0.70 (B) 0.46 (C) 0.30 **(D) 0.21 – Answer**

Marking Guide – Paper 1 Section 2

2023 Paper 1 Section 2 Question 11

Discrete random variables 2

Two random samples (A and B) were obtained using two different Bernoulli experiments. Each Bernoulli trial in the random samples was recorded as 1 (for success) or 0 (for failure). The results are shown.

A	1	1	1	1	0	1	1	0	1	1
B	0	0	1	1	1	0	1	1	0	0

In sample A, for each trial the mean is 0.8 and the variance is 0.16.

a) Use the sample B results to determine the mean and variance for each trial in sample B. [2 marks]

Sample response	The response
Sample B Mean $= p = \frac{5}{10} = 0.5$	• correctly determines the mean for sample B **[1 mark]**
Variance $= p(1-p) = 0.5 \times 0.5$ $= 0.25$	• determines the variance for sample B **[1 mark]**

b) Compare the variability about the means of samples A and B. [2 marks]

Sample response	The response
Sample B has a larger variance than sample A. Because sample B has the larger variance, it has more variability about the mean compared to sample A.	• identifies that sample B has the larger variance **[1 mark]** • provides reasoning that explains that larger variance indicates larger variability **[1 mark]**

2023
Paper 1
Section 2
Question 13

Discrete random variables 2

At a certain airport, the departure of one in five international flights is delayed every day. The status of any flight is independent of other flights.

One international flight is selected at random each day for three days. Each selection is recorded as either 'delayed' or 'not delayed'.

a) State two conditions that make this context suitable for modelling using a binomial random variable. [2 marks]

Sample response	The response
One condition is that **there are only two outcomes for each selection**: 'delayed' (success) or 'not delayed' (failure), i.e. Bernoulli trials.	• correctly states one condition for binomial probability **[1 mark]**
Another condition that makes this context suitable is that the **probabilities of each outcome do not change in each trial**, i.e. $p = \frac{1}{5}$ and $q = \frac{4}{5}$	• correctly states a second condition for binomial probability **[1 mark]**

b) Calculate the probability that at least two of the selected flights were delayed. [3 marks]

Sample response	The response
$n = 3,\ p = \frac{1}{5},\ 1 - p = \frac{4}{5}$	• correctly determines the number of trials and the probability of a flight being delayed **[1 mark]**
$P(\text{at least } 2)$ $= P(X = 2 \text{ or } X = 3))$ $= C_2^3 p^2(1-p)^1 + C_3^3 p^3 (1-p)^0$ $= 3\left(\frac{1}{5}\right)^2 \times \left(\frac{4}{5}\right) + \left(\frac{1}{5}\right)^3$ $= 3 \times \frac{1}{25} \times \frac{4}{5} + \frac{1}{5^3}$ $= \frac{12}{125} + \frac{1}{125}$	• determines a suitable method **[1 mark]**
$= \frac{13}{125}$	• calculates the probability **[1 mark]**

2023 Paper 1 Section 2 Question 15 **Discrete random variables 2**	In a certain game, players throw one water balloon at a target. There is a one in four chance of hitting the target. a) State the probabilities of all the possible outcomes for one throw at the target. [2 marks]

Sample response	The response
There are two outcomes: hit or miss. Hit $P(\text{hit}) = \frac{1}{4}$ Miss $P(\text{miss}) = \frac{3}{4}$	• correctly states the probability of a 'hit' **[1 mark]** • correctly states the probability of a 'miss' **[1 mark]**

b) Let H be the discrete random variable for one of the possible outcomes. Determine the mean and variance of the distribution of random variable H when 20 players throw a water balloon at the target. [2 marks]

Sample response	The response
Using H to represent the number of 'hits'. The mean of a binomial distribution is np. $np = 20 \times \frac{1}{4} = 5$ The variance of a binomial distribution is $np(1-p)$. $np(1-p) = 5 \times \frac{3}{4}$ $= \frac{15}{4} = 3\frac{3}{4}$	• determines the mean **[1 mark]** • determines the variance **[1 mark]**

Marking Guide – Paper 2 Section 1

2023 Paper 2 Section 1 Question 2 **Discrete random variables 2**	The probability of hitting a bullseye on a standard dartboard is 1 in 1250. What is the probability of hitting a bullseye exactly once in 10 attempts? (A) $\binom{9}{1}\left(\frac{1}{1250}\right)^1 \times \left(\frac{1249}{1250}\right)^9$ (B) $\binom{9}{1}\left(\frac{1}{1250}\right)^9 \times \left(\frac{1249}{1250}\right)^1$ (C) $\binom{10}{1}\left(\frac{1}{1250}\right)^1 \times \left(\frac{1249}{1250}\right)^9$ (D) $\binom{10}{1}\left(\frac{1}{1250}\right)^9 \times \left(\frac{1249}{1250}\right)^1$ **Answer is C.**

Marking Guide – Paper 2 Section 2

2020 Paper 2 Section 2 Question 11

Discrete random variables 2

A sugar company samples the packets of sugar it produces and finds that 5% of packets are underweight. Consider a batch of 20 packets.

a) Determine how many packets of sugar the company can expect to be underweight in a batch of 20 packets. [1 mark]

Sample Response	The response
$5\% \times 20 = 1$	• correctly determines the expected number of the underweight packets [1 mark]

b) Determine the variance of the batch. [1 mark]

Sample Response	The response
Variance $= np\,(1-p)$ $= 0.95$	• correctly determines the variance of the underweight packets [1 mark]

c) Determine the probability that at most one of the packets of sugar in the batch is underweight. [2 marks]

Sample Response	The response
$n = 20$ success $= p = 0.05$ Number of successes = 0 or 1	• correctly uses an appropriate mathematical representation to communicate approach [1 mark]
Using GDC P(at most 1 are underweight) = 0.7358	• correctly determines the probability [1 mark]

2021 Paper 2 Section 2 Question 13

Discrete random variables 2

The amount of gravel (in tonnes) sold by a construction company in a given week is a continuous random variable X and has a probability density function defined by:

$$f(x)=\begin{cases} c\left(1-x^2\right), & 0 \le x \le 1 \\ 0, & \text{otherwise} \end{cases}$$

a) Show that $c = \frac{3}{2}$ [1 mark]

Sample Response	The response
Using GDC $\int_0^1 \frac{3}{2}(1-x^2)dx = 1$	• correctly establishes the definite integral equated to 1 [1 mark]

b) Determine $P\ (X < 0.25)$. [2 marks]

Sample Response	The response
$P(X < 0.25)$ $\int_0^{0.25} \frac{3}{2}(1-x^2)dx$	• correctly establishes the definite integral [1 mark]
$= 0.367$	• correctly determines the probability [1 mark]

c) Determine the variance of X. [4 marks]

Sample Response	The response
Mean $= \int_0^1 \frac{3}{2}x(1-x^2)dx$	• correctly establishes the definite integral [1 mark]
= 0.375	• correctly establishes the mean [1 mark]
Variance $= \int_0^1 \frac{3}{2}(x-0.375)^2(1-x^2)dx$	• establishes definite integral [1 mark]
$= 0.059$ tonnes^2	• determines variance [1 mark]

Unit 4 – Topic 4: Continuous random variables and the normal distribution

Paper 1 Section 1

2023 Paper 1 Section 1 Question 7 **Continuous random variables and the normal distribution**	Determine the mean of the continuous random variable X with the probability density function $$f(x)=\begin{cases}\frac{1}{8}x, & 0\le x\le 4\\ 0, & \text{otherwise}\end{cases}$$ (A) $\frac{1}{8}$ (B) $\frac{3}{8}$ (C) $\frac{1}{2}$ (D) $\frac{8}{3}$

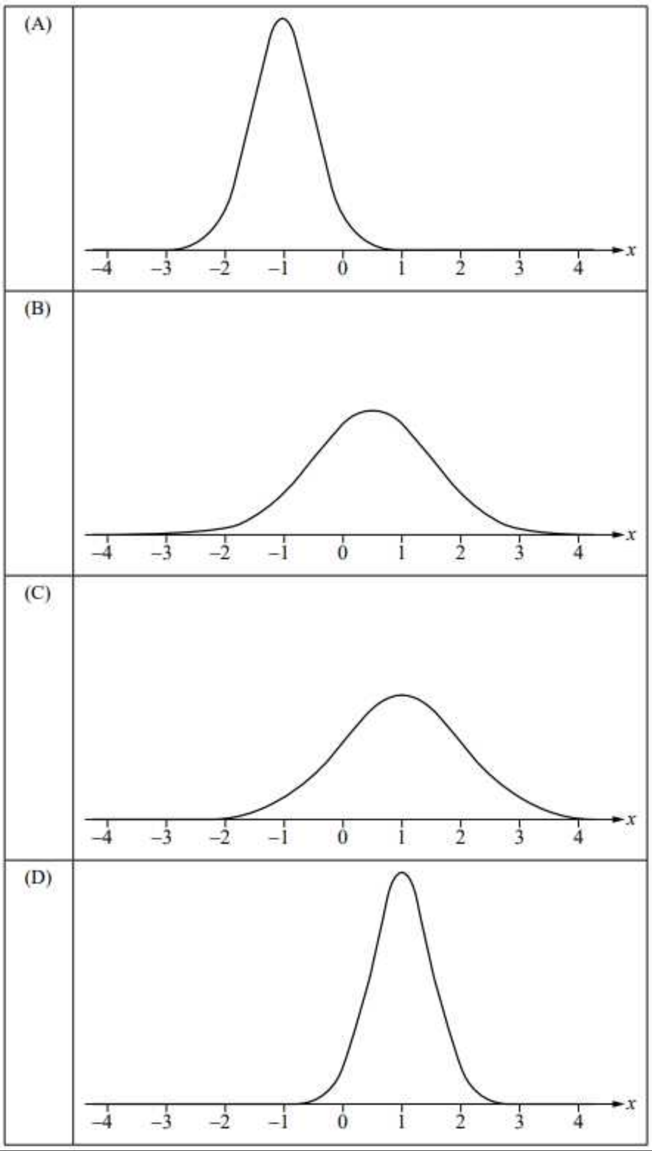
2023 Paper 1 Section 1 Question 10 **Continuous random variables and the normal distribution**	The continuous random variable Y has the probability density function $$f(y)=\begin{cases}1+y, & 0\le y\le \sqrt{3}-1\\ 0, & \text{otherwise}\end{cases}$$ Determine $P(0\le y\le \frac{1}{2})$. (A) $\frac{1}{5}$ (B) $\frac{3}{8}$ (C) $\frac{5}{8}$ (D) $\frac{3}{4}$

2022 Paper 1 Section 1 Question 4 **Continuous random variables and the normal distribution**	The weekly amount of money a company spends on repairs is normally distributed, with a mean of \$1200 and a standard deviation of \$100. Given that $P(Z\le -2.5)=0.0062$ and $P(Z>1)=0.1587$, where Z is a standard normal random variable, determine the probability that the weekly repair costs will be between \$950 and \$1300. (A) 0.6525 (B) 0.6587 (C) 0.8351 (D) 0.8413

2022 Paper 1 Section 1 Question 5

Continuous random variables and the normal distribution

Which normal distribution curve best represents a normal distribution with a mean of 1 and a standard deviation of 0.5?



2021 Paper 1 Section 1 Question 2

Continuous random variables and the normal distribution

The table shows the time a technician has spent servicing photocopiers.

Time (in minutes)	Frequency
$0 \le t < 5$	10
$5 \le t < 10$	20
$10 \le t < 15$	30
$15 \le t < 20$	40
$20 \le t < 25$	100

What is the probability that a given service required at least 10 minutes but less than 20 minutes?

(A) 0.15
(B) 0.35
(C) 0.70
(D) 0.85

2021 Paper 1 Section 1 Question 8 Continuous random variables and the normal distribution	The continuous random variable X has the probability density function $$f(x)=\begin{cases}\frac{3}{x^2}, & 1\le x\le \frac{3}{2}\\ 0, & \text{otherwise}\end{cases}$$ The mean of X is (A) $\ln\left(\frac{3}{2}\right)$ (B) $\ln\left(\frac{27}{8}\right)$ (C) $\ln\left(\frac{9}{2}\right)$ (D) 1

2021 Paper 1 Section 1 Question 9 Continuous random variables and the normal distribution	A basket contains 10 green apples and 30 red apples. Three apples are drawn at random from the basket with replacement. Determine the probability that exactly two of the three apples are green. (A) $\frac{3}{64}$ (B) $\frac{9}{64}$ (C) $\frac{10}{64}$ (D) $\frac{27}{64}$

2021 Paper 1 Section 1 Question 10 Continuous random variables and the normal distribution	Handspans of teenagers are approximately normally distributed, with a mean of 15 cm and a standard deviation of 2 cm. Which of the following groups is expected to be the largest? (A) teenagers with handspans that are between 7 cm and 11 cm (B) teenagers with handspans that are between 11 cm and 15 cm (C) teenagers with handspans that are between 13 cm and 17 cm (D) teenagers with handspans that are between 17 cm and 21 cm

2020 Paper 1 Section 1 Question 4 Continuous random variables and the normal distribution	Pulse rates of adult men are approximately normally distributed with a mean of 70 and a standard deviation of 8. Which of the following choices correctly describes how to determine the proportion of men that have a pulse rate greater than 78? (A) Determine the area to the left of $z = 1$ under the standard normal curve. (B) Determine the area to the right of $z = 1$ under the standard normal curve. (C) Determine the area to the right of $z = -1$ under the standard normal curve. (D) Determine the area between $z = -1$ and $z = 1$ under the standard normal curve.

2020 Paper 1 Section 1 Question 7 **Continuous random variables and the normal distribution**	The life expectancy (in years) of an electronic component can be represented by the probability density function $p(x)=\begin{cases}\dfrac{1}{x^2}, & x\geq 1\\ 0, & \text{otherwise}\end{cases}$ The probability that the component lasts between 1 and 10 years is (A) 0.010 (B) 0.100 (C) 0.900 (D) 0.990

2020 Paper 1 Section 1 Question 8 **Continuous random variables and the normal distribution**	A test includes six multiple choice questions. Each question has four options for the answer. If the answers are guessed, the probability of getting at most two questions correct is represented by (A) $\binom{6}{0}0.25^0\times 0.75^6+\binom{6}{1}0.25^1\times 0.75^5$ (B) $\binom{6}{0}0.25^0\times 0.75^6+\binom{6}{1}0.25^1\times 0.75^5+\binom{6}{2}0.25^2\times 0.75^4$ (C) $1-\left(\binom{6}{0}0.25^0\times 0.75^6+\binom{6}{1}0.25^1\times 0.75^5\right)$ (D) $1-\left(\binom{6}{0}0.25^0\times 0.75^6+\binom{6}{1}0.25^1\times 0.75^5+\binom{6}{2}0.25^2\times 0.75^4\right)$

Paper 1 Section 2

2022 Paper 1 Section 2 Question 12 **Continuous random variables and the normal distribution**	The probability that a debating team wins a debate can be modelled as a Bernoulli distribution. Given that the probability of winning a debate is $\frac{4}{5}$ a) Determine the mean of this distribution. [1 mark] b) Determine the variance of this distribution. [1 mark] c) Determine the standard deviation of this distribution. [1 mark]

2021 Paper 1 Section 2 Question 17 **Continuous random variables and the normal distribution**	In any five-day working week Leonardo either catches a bus to work or uses another form of transportation. On average, he catches the bus to work on three of the five days. His decision on any given day is independent of his decision on any other day. Determine the probability that Leonardo catches a bus to work on exactly one day in a given five-day working week. [3 marks]

2022 Paper 1 Section 2 Question 18 **Continuous random variables and the normal distribution**	A percentile is a measure in statistics showing the value below which a given percentage of observations occur. The continuous random variable X has the probability density function. $f(x)=\begin{cases}2x-2, & 1\le x\le 2\\ 0, & \text{otherwise}\end{cases}$ Determine the 36th percentile of X.

Paper 2 Section 1

2023 Paper 2 Section 1 Question 3 **Continuous random variables and the normal distribution**	In a certain normal distribution curve, 95% of the area lies between the values 50.32 and 113.68. The mean of this distribution is 82. Determine the standard deviation. (A) 16.16 (B) 21.12 (C) 31.68 (D) 63.36

2023 Paper 2 Section 1 Question 10 **Continuous random variables and the normal distribution**	A student is trying to determine which subject they performed best in compared to other students. Results from recent tests in four subjects (A to D) are shown. Assume student results in each subject are normally distributed. In which subject did the student perform best compared to other students?

	Class mean	Class standard deviation	Student's result
(A)	62	22	77
(B)	55	25	74
(C)	61	15	70
(D)	73	20	82

2022 Paper 2 Section 1 Question 4 **Continuous random variables and the normal distribution**	The distribution for a sample proportion p̂ has a mean of 0.15 and a standard deviation of 0.0345. The sample size is (A) 10 (B) 14 (C) 107 (D) 116

2022 Paper 2 Section 1 Question 5 **Continuous random variables and the normal distribution**	The continuous random variable X has the probability density function $f(x)=\begin{cases}\frac{\cos(x)}{2}, & \frac{-\pi}{2}\le x\le\frac{\pi}{2}\\ 0\;, & \text{otherwise}\end{cases}$ The standard deviation of X is (A) 0.467 (B) 0.684 (C) 1.211 (D) 1.467

2022 Paper 2 Section 1 Question 6 Continuous random variables and the normal distribution	A stall at the school fete sells cups of lemonade. Assuming the amount of lemonade in a cup is normally distributed with a mean of 60 mL and a standard deviation of 3 mL, 80% of the cups contain more than (A) 52.4 mL (B) 57.5 mL (C) 61.6 mL (D) 62.5 mL

2021 Paper 2 Section 1 Question 1 Continuous random variables and the normal distribution	The scores obtained on a test can be assumed to be normally distributed with a mean of 102 and a standard deviation of 19. What proportion of scores are over 113? (A) 0.2813 (B) 0.5789 (C) 0.7187 (D) 0.8216

2021 Paper 2 Section 1 Question 6 Continuous random variables and the normal distribution	When seeds of a certain variety of flower are planted, the probability of each seed germinating is 0.8. If eight seeds are planted, what is the probability that at least six seeds will germinate? (A) 0.797 (B) 0.503 (C) 0.294 (D) 0.001

2020 Paper 2 Section 1 Question 7 Continuous random variables and the normal distribution	The records of a shoe manufacturer show that 10% of shoes made are defective. Assuming independence, the probability of getting 2 defective shoes in a batch of 20 is (A) 0.1937 (B) 0.2852 (C) 0.3917 (D) 0.6083

2023 Paper 2 Section 2 Question 17 **Continuous random variables and the normal distribution**	Model bridges were constructed for a competition. The models that could support the heaviest loads before collapsing were given awards. The load results of the competition were normally distributed, with a mean of 1.36 kg and a standard deviation of 0.12 kg. Three award categories were used: honours for the top 15% of load results; distinction for the next 15%; and commendation for the next 15%. The model bridge constructed by Finley only just missed out on a commendation. Kirby's model bridge only just qualified for honours. Determine the difference, to the nearest gram, between the loads supported by Finley and Kirby's models. (5 marks)

2022 Paper 2 Section 2 Question 11 **Continuous random variables and the normal distribution**	A salesperson has a 20% probability of making a sale to each customer who enters the store. Each sale is independent of all other sales. a) Determine the mean number of sales on a day where 25 customers enter the store. [2 marks] b) Determine the standard deviation of the number of sales on a day where 25 customers enter the store. [2 marks] c) Determine the minimum number of customers who would have to enter the store to have an 88% chance or more of making at least one sale. [3 marks]

2022 Paper 2 Section 2 Question 12 **Continuous random variables and the normal distribution**	Suppose that the distance travelled by vehicles in a year can be modelled by a normal distribution. In 2021, vehicles travelled a mean of 13 700 km with a standard deviation of 3400 km. a) Determine the probability that a vehicle chosen at random travelled less than 12 000 km in 2021. [2 marks] b) Determine the value of x where 60% of vehicles travelled less than x km in 2021. [2 marks]

2022 Paper 2 Section 2 Question 16 **Continuous random variables and the normal distribution**	The time spent waiting in a queue at a certain supermarket is given by $(X+11)$ minutes, where X is a random variable with the probability density function $f(x)=\begin{cases}\dfrac{a(4-x^2)}{32}, & -2\le x\le 2\\ 0\quad, & \text{otherwise}\end{cases}$ Determine the probability of waiting between 10 and 12 minutes in a queue at this supermarket. [4 marks]
2022 Paper 2 Section 2 Question 18 **Continuous random variables and the normal distribution**	The intelligence quotient (IQ) of individuals in a population is normally distributed, with a mean of 100 and a standard deviation of 16. Nine individuals are chosen at random from the population. Determine the probability that no more than two of the individuals have an IQ of at least 120. [3 marks]

2021 Paper 2 Section 2 Question 14 Continuous random variables and the normal distribution	The heights of students at School A are normally distributed with a mean of 165 cm and a standard deviation of 15 cm. a) Determine the probability that a student chosen at random from School A is shorter than 180 cm. [1 mark] b) Determine the minimum integer value of the height of a student who is in the top 2% of this distribution. [3 marks] The heights of students at School B are also normally distributed. A student at School B has the same height as the height determined in Question 14b) but their corresponding *z*-score is 3. c) Determine which student's height ranks higher in terms of percentile for their school. [3 marks]

2021 Paper 2 Section 2 Question 19 **Continuous random variables and the normal distribution**	A random variable X, defined over the interval , is uniformly distributed if its probability density function is defined by: $$f(x)=\begin{cases}\frac{1}{(b-a)}, & a \le x \le b \\ 0, & \text{otherwise}\end{cases}$$ The expected value and variance of a uniform random variable X are $$E(X)=\frac{(a+b)}{2} \qquad Var(X)=\frac{(b-a)^2}{12}$$ A manufacturer has observed that the time that elapses between placing an order with a supplier and the delivery of the order is uniformly distributed between 100 and 180 minutes. Determine the probability that the time between placing an order and delivery of the order will be within one standard deviation of the expected time.

2021 Paper 2 Section 2 Question 20 Continuous random variables and the normal distribution	The random variable B is normally distributed with a mean of 0 and a standard deviation of 1. Determine the probability that the quadratic equation $x^2 + 3x + 2B = 0$ has real roots. [3 marks]

2020 Paper 2 Section 2 Question 13 **Continuous random variables and the normal distribution**	An online retailer claims that 90% of all orders are shipped within 12 hours of being received. On a particular day, 121 orders were received and 102 orders were shipped within 12 hours. a) State the sample proportion of orders shipped within 12 hours. [1 mark] The distribution of the sample proportion of all orders that are shipped within 12 hours of being received on any day is approximately normal. b) Assuming the online retailer's claim is true, find the probability that, in a random sample of 121, less than 85% of all orders are shipped within 12 hours. [3 marks] c) Use the result from 13b) to evaluate the reasonableness of the online retailer's claim. [2 marks]

2020 Paper 2 Section 2 Question 14 Continuous random variables and the normal distribution	Let X denote the time in minutes between the arrival of trains at a station. The cumulative distribution function of X is defined by $$F(x)=\begin{cases}2-\dfrac{10}{x}, & 5\le x\le 10\\ 0, & \text{otherwise}\end{cases}$$ a) Determine the probability density function of X. [3 marks] b) Determine the probability that $5 < X < 7$. [1 mark] c) Determine the mean time between the arrival of trains at the station. [2 marks]

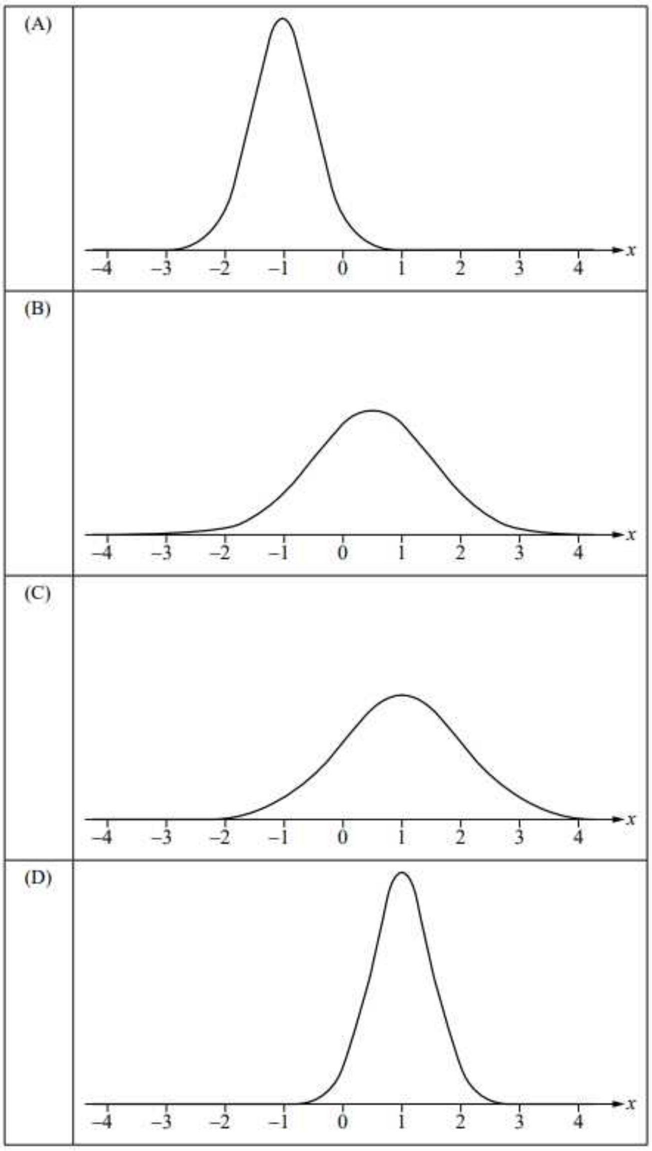
2020 Paper 2 Section 2 Question 16 **Continuous random variables and the normal distribution**	Bottles of soft drink should contain a volume with a mean of 591 mL, but some variation is expected. Any bottle at or below the 20th percentile of the volume distribution is rejected. A percentile is a measure in statistics that shows the values below which a given percentage of observations occur. Thirty-five per cent of the bottles contain 593 mL or more of soft drink. Assuming the volumes are normally distributed, determine the smallest volume (in mL) that will be accepted. [4 marks]

Marking Guide – Paper 1 Section 1

2023 Paper 1 Section 1 Question 7 **Continuous random variables and the normal distribution**	Determine the mean of the continuous random variable X with the probability density function $$f(x)=\begin{cases}\frac{1}{8}x, & 0\le x\le 4\\ 0, & \text{otherwise}\end{cases}$$ (A) $\frac{1}{8}$ (B) $\frac{3}{8}$ (C) $\frac{1}{2}$ (D) $\frac{8}{3}$ **Answer is D.**

2023 Paper 1 Section 1 Question 10 **Continuous random variables and the normal distribution**	The continuous random variable Y has the probability density function $$f(y)=\begin{cases}1+y, & 0\le y\le \sqrt{3}-1\\ 0, & \text{otherwise}\end{cases}$$ Determine $P(0\le y\le \frac{1}{2})$. (A) $\frac{1}{5}$ (B) $\frac{3}{8}$ (C) $\frac{5}{8}$ (D) $\frac{3}{4}$ **Answer is C.**

2022 Paper 1 Section 1 Question 4 Continuous random variables and the normal distribution	The weekly amount of money a company spends on repairs is normally distributed, with a mean of \$1200 and a standard deviation of \$100. Given that $P(Z \leq -2.5) = 0.0062$ and $P(Z > 1) = 0.1587$, where Z is a standard normal random variable, determine the probability that the weekly repair costs will be between \$950 and \$1300. (A) 0.6525 (B) 0.6587 **(C) 0.8351 – Answer** (D) 0.8413

2022 Paper 1 Section 1 Question 5 Continuous random variables and the normal distribution	Which normal distribution curve best represents a normal distribution with a mean of 1 and a standard deviation of 0.5?  **Answer is D.**

2021 Paper 1 Section 1 Question 2

Continuous random variables and the normal distribution

The table shows the time a technician has spent servicing photocopiers.

Time (in minutes)	Frequency
$0 \le t < 5$	10
$5 \le t < 10$	20
$10 \le t < 15$	30
$15 \le t < 20$	40
$20 \le t < 25$	100

What is the probability that a given service required at least 10 minutes but less than 20 minutes?

(A) 0.15
(B) 0.35 – Answer
(C) 0.70
(D) 0.85

2021 Paper 1 Section 1 Question 8

Continuous random variables and the normal distribution

The continuous random variable X has the probability density function

$$f(x) = \begin{cases} \frac{3}{x^2}, & 1 \le x \le \frac{3}{2} \\ 0, & \text{otherwise} \end{cases}$$

The mean of *X* is

(A) $\ln\left(\frac{3}{2}\right)$

(B) $\ln\left(\frac{27}{8}\right)$

(C) $\ln\left(\frac{9}{2}\right)$

(D) 1

Answer is B.

2021 Paper 1 Section 1 Question 9

Continuous random variables and the normal distribution

A basket contains 10 green apples and 30 red apples. Three apples are drawn at random from the basket with replacement. Determine the probability that exactly two of the three apples are green.

(A) $\frac{3}{64}$

(B) $\frac{9}{64}$

(C) $\frac{10}{64}$

(D) $\frac{27}{64}$

Answer is B.

2021 Paper 1 Section 1 Question 10

Continuous random variables and the normal distribution

Handspans of teenagers are approximately normally distributed, with a mean of 15 cm and a standard deviation of 2 cm.

Which of the following groups is expected to be the largest?

(A) teenagers with handspans that are between 7 cm and 11 cm
(B) teenagers with handspans that are between 11 cm and 15 cm
(C) teenagers with handspans that are between 13 cm and 17 cm – Answer
(D) teenagers with handspans that are between 17 cm and 21 cm

---

2020 Paper 1 Section 1 Question 4

Continuous random variables and the normal distribution

Pulse rates of adult men are approximately normally distributed with a mean of 70 and a standard deviation of 8.

Which of the following choices correctly describes how to determine the proportion of men that have a pulse rate greater than 78?

(A) Determine the area to the left of $z = 1$ under the standard normal curve.
(B) Determine the area to the right of $z = 1$ under the standard normal curve. – Answer
(C) Determine the area to the right of $z = -1$ under the standard normal curve.
(D) Determine the area between $z = -1$ and $z = 1$ under the standard normal curve.

2020 Paper 1 Section 1 Question 7

Continuous random variables and the normal distribution

The life expectancy (in years) of an electronic component can be represented by the probability density function

$$p(x) = \begin{cases} \dfrac{1}{x^2}, & x \geq 1 \\ 0, & \text{otherwise} \end{cases}$$

The probability that the component lasts between 1 and 10 years is

(A) 0.010
(B) 0.100
(C) 0.900 – Answer
(D) 0.990

2020 Paper 1 Section 1 Question 8

Continuous random variables and the normal distribution

A test includes six multiple choice questions. Each question has four options for the answer.

If the answers are guessed, the probability of getting at most two questions correct is represented by

(A) $\binom{6}{0}0.25^0 \times 0.75^6 + \binom{6}{1}0.25^1 \times 0.75^5$

(B) $\binom{6}{0}0.25^0 \times 0.75^6 + \binom{6}{1}0.25^1 \times 0.75^5 + \binom{6}{2}0.25^2 \times 0.75^4$

(C) $1 - \left(\binom{6}{0}0.25^0 \times 0.75^6 + \binom{6}{1}0.25^1 \times 0.75^5\right)$

(D) $1 - \left(\binom{6}{0}0.25^0 \times 0.75^6 + \binom{6}{1}0.25^1 \times 0.75^5 + \binom{6}{2}0.25^2 \times 0.75^4\right)$

Answer is B.

2022 Paper 1 Section 2 Question 12

Continuous random variables and the normal distribution

The probability that a debating team wins a debate can be modelled as a Bernoulli distribution. Given that the probability of winning a debate is $\frac{4}{5}$

a) Determine the mean of this distribution. [1 mark]

Sample Response	The response
<table><tr><td>x_i</td><td>0</td><td>1</td></tr><tr><td>p_i</td><td>$\frac{1}{5}$</td><td>$\frac{4}{5}$</td></tr></table> $E\ (X) = \sum p_i x_i$ $= \frac{1}{5} \times 0 + \frac{4}{5} \times 1$ $= \frac{4}{5}$	• correctly determines the mean [1 mark]

b) Determine the variance of this distribution. [1 mark]

Sample Response	The response
$Var(X) = \sum p_i (x_i - \mu)^2$ $= \frac{1}{5} \times \left(\frac{-4}{5}\right)^2 + \frac{4}{5} \times \left(\frac{1}{5}\right)^2$ $= \frac{4}{25}$	• correctly determines the variance [1 mark]

c) Determine the standard deviation of this distribution. [1 mark]

Sample Response	The response
$Standard\ deviation = \sqrt{Variance}$ $= \frac{2}{5}$	• determines the standard deviation [1 mark]

2021 Paper 1 Section 2 Question 17

Continuous random variables and the normal distribution

In any five-day working week Leonardo either catches a bus to work or uses another form of transportation. On average, he catches the bus to work on three of the five days. His decision on any given day is independent of his decision on any other day.

Determine the probability that Leonardo catches a bus to work on exactly one day in a given five-day working week. [3 marks]

Sample Response	The response
Using binomial distribution $n = 5$ $p = \frac{3}{5}$ and $q = \frac{2}{5}$	• correctly determines the values for p and n [1 mark]
P (catches the bus on 1 day) $= \binom{5}{1}\left(\frac{3}{5}\right)^1\left(\frac{2}{5}\right)^4$	• establishes expression for the required probability [1 mark]
$= 3 \times \frac{16}{5^4} = \frac{48}{625}$	• determines probability [1 mark]

2022 Paper 1 Section 2 Question 18 **Continuous random variables and the normal distribution**	A percentile is a measure in statistics showing the value below which a given percentage of observations occur. The continuous random variable X has the probability density function. $f(x) = \begin{cases} 2x - 2, & 1 \leq x \leq 2 \\ 0, & \text{otherwise} \end{cases}$ Determine the 36th percentile of X.

Sample Response	The response
$\int_1^a 2x - 2\ dx = 0.36$ $x^2 - 2x \Big\|_1^a = 0.36$	• correctly determines the definite integral [1 mark]
$(a^2 - 2a) - (1 - 2) = 0.36$ $a^2 - 2a + 1 = 0.36$ $a^2 - 2a + 0.64 = 0$	• determines the quadratic equation [1 mark]
$\therefore a = \frac{2 \pm \sqrt{4 - 4 \times 1 \times 0.64}}{2}$ $\therefore a = \frac{2 \pm \sqrt{1.44}}{2}$ $\therefore a = 1.6$ or 0.4	• determines values of a [1 mark]
Given $1 \leq x \leq 2$ $\therefore a = 1.6$	• evaluates the reasonableness of solutions [1 mark]

Marking Guide – Paper 2 Section 1

2023 Paper 2 Section 1 Question 3 **Continuous random variables and the normal distribution**	In a certain normal distribution curve, 95% of the area lies between the values 50.32 and 113.68. The mean of this distribution is 82. Determine the standard deviation. **(A) 16.16 – Answer** (B) 21.12 (C) 31.68 (D) 63.36

2023 Paper 2 Section 1 Question 10 **Continuous random variables and the normal distribution**	A student is trying to determine which subject they performed best in compared to other students. Results from recent tests in four subjects (A to D) are shown. Assume student results in each subject are normally distributed. In which subject did the student perform best compared to other students?

	Class mean	Class standard deviation	Student's result
(A)	62	22	77
(B)	55	25	74
(C)	61	15	70
(D)	73	20	82

Answer is B.

2022 Paper 2 Section 1 Question 4 **Continuous random variables and the normal distribution**	The distribution for a sample proportion pˆ has a mean of 0.15 and a standard deviation of 0.0345. The sample size is (A) 10 (B) 14 **(C) 107 – Answer** (D) 116

2022 Paper 2 Section 1 Question 5 **Continuous random variables and the normal distribution**	The continuous random variable X has the probability density function $f(x)=\begin{cases}\frac{\cos(x)}{2}, & \frac{-\pi}{2}\le x\le\frac{\pi}{2}\\ 0, & \text{otherwise}\end{cases}$ The standard deviation of X is (A) 0.467 **(B) 0.684 – Answer** (C) 1.211 (D) 1.467

2022 Paper 2 Section 1 Question 6 **Continuous random variables and the normal distribution**	A stall at the school fete sells cups of lemonade. Assuming the amount of lemonade in a cup is normally distributed with a mean of 60 mL and a standard deviation of 3 mL, 80% of the cups contain more than (A) 52.4 mL **(B) 57.5 mL – Answer** (C) 61.6 mL (D) 62.5 mL

2021 Paper 2 Section 1 Question 1 **Continuous random variables and the normal distribution**	The scores obtained on a test can be assumed to be normally distributed with a mean of 102 and a standard deviation of 19. What proportion of scores are over 113? **(A) 0.2813 – Answer** (B) 0.5789 (C) 0.7187 (D) 0.8216

2021 Paper 2 Section 1 Question 6 **Continuous random variables and the normal distribution**	When seeds of a certain variety of flower are planted, the probability of each seed germinating is 0.8. If eight seeds are planted, what is the probability that at least six seeds will germinate? **(A) 0.797 – Answer** (B) 0.503 (C) 0.294 (D) 0.001

2020 Paper 2 Section 1 Question 7 **Continuous random variables and the normal distribution**	The records of a shoe manufacturer show that 10% of shoes made are defective. Assuming independence, the probability of getting 2 defective shoes in a batch of 20 is (A) 0.1937 **(B) 0.2852 – Answer** (C) 0.3917 (D) 0.6083

Marking Guide – Paper 2 Section 2

2023 Paper 2 Section 2 Question 17

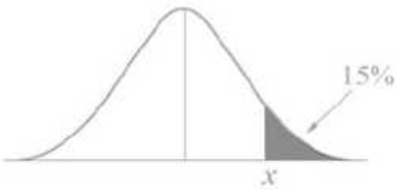
Continuous random variables and the normal distribution

Model bridges were constructed for a competition. The models that could support the heaviest loads before collapsing were given awards.

The load results of the competition were normally distributed, with a mean of 1.36 kg and a standard deviation of 0.12 kg.

Three award categories were used: honours for the top 15% of load results; distinction for the next 15%; and commendation for the next 15%.

The model bridge constructed by Finley only just missed out on a commendation. Kirby's model bridge only just qualified for honours. Determine the difference, to the nearest gram, between the loads supported by Finley and Kirby's models. (5 marks)

Sample response	The response
Mean = 1.36 kg and std dev = 0.12 kg Diagram of top 15% using tail right. 	
Using GDC: InvN with $\mu = 1.36, \sigma = 0.12$ Area 1 = **0.15** Area 2 = **0.45** using 'tail right' to determine the lower cut-off values for the awards.	• correctly identifies both required areas **[1 mark]**
for honours cut-off (top 15%) either: $P(X > x) = 0.15$ (tail right) $P(X < x) = 0.85$ (tail left) $x =$ **1.48437** kg	• correctly determines honours lowest load **[1 mark]**
for commended cut-off (top 45%) either: $P(X > x) = 0.45$ (tail right) $P(X < x) = 0.55$ (tail left) $x =$ **1.37508** kg	• correctly determines commended lowest load **[1 mark]**
$1.48437 - 1.37508$ $=$ **0.10929** kg	• determines the difference in loads **[1 mark]** • converts answer to the nearest gram **[1 mark]**
$\approx$ **109 grams**	

2022 Paper 2 Section 2 Question 11

Continuous random variables and the normal distribution

A salesperson has a 20% probability of making a sale to each customer who enters the store. Each sale is independent of all other sales.

a) Determine the mean number of sales on a day where 25 customers enter the store. [2 marks]

Sample Response	The response
Mean number of sales $= np$ $= 25 \times 0.2$ $= 5$	• correctly substitutes into formula for mean [1 mark] • correctly determines the mean [1 mark]

b) Determine the standard deviation of the number of sales on a day where 25 customers enter the store. [2 marks]

Sample Response	The response
Standard deviation of number of sales $= \sqrt{np(1-p)}$ $= \sqrt{25 \times 0.2 \times (1-0.2)}$	• correctly substitutes into formula for standard deviation [1 mark]
$= 2$	• correctly determines the standard deviation [1 mark]

c) Determine the minimum number of customers who would have to enter the store to have an 88% chance or more of making at least one sale. [3 marks]

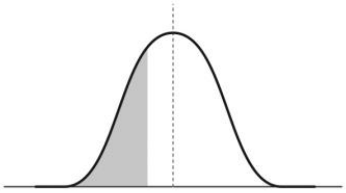
Sample Response	The response
$1 - \binom{n}{0} 0.2^0 0.8^n \geq 0.88$	• correctly determines the required inequation [1 mark]
$\therefore 1 - 0.8^n \geq 0.88$ $\therefore 0.8^n \leq 0.12$ $\therefore n \geq \log_{0.8}(0.12)$ $\therefore n \geq 9.50179$	• correctly determines the unknown in the inequation [1 mark]
$\therefore$ minimum number of customers would be 10.	• determines the minimum number of customers [1 mark]

2022 Paper 2 Section 2 Question 12

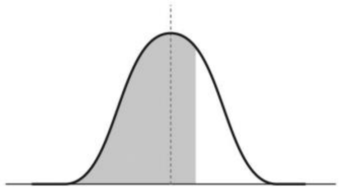
Continuous random variables and the normal distribution

Suppose that the distance travelled by vehicles in a year can be modelled by a normal distribution. In 2021, vehicles travelled a mean of 13 700 km with a standard deviation of 3400 km.

a) Determine the probability that a vehicle chosen at random travelled less than 12 000 km in 2021. [2 marks]

Sample Response	The response
 $P(x < 12000) = 0.3085$	• correctly uses an appropriate mathematical representation [1 mark] • correctly determines the probability [1 mark]

b) Determine the value of x where 60% of vehicles travelled less than x km in 2021. [2 marks]

Sample Response	The response
 $x = 14\,561.38$ kilometres	• correctly uses an appropriate mathematical representation [1 mark] • correctly determines the value of x [1 mark]

2022 Paper 2 Section 2 Question 16

Continuous random variables and the normal distribution

The time spent waiting in a queue at a certain supermarket is given by $(X+11)$ minutes, where X is a random variable with the probability density function

$$f(x)=\begin{cases}\dfrac{a\left(4-x^2\right)}{32}, & -2\le x\le 2\\ 0\ , & \text{otherwise}\end{cases}$$

Determine the probability of waiting between 10 and 12 minutes in a queue at this supermarket. [4 marks]

Sample Response	The response
$\int_{-2}^{2}\frac{a(4-x^2)}{32}dx=1$ Using GDC (solving for equation)	• correctly identifies required integral equation to solve [1 mark]
$a=3$	• correctly determines the value of a [1 mark]
$P(-1\le X\le 1)$ $=\int_{-1}^{1}\frac{a(4-x^2)}{32}dx$	• correctly identifies interval [1 mark]
= 0.6875	• determines probability [1 mark]

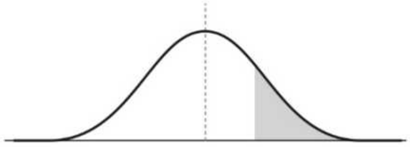
2022 Paper 2 Section 2 Question 18

Continuous random variables and the normal distribution

The intelligence quotient (IQ) of individuals in a population is normally distributed, with a mean of 100 and a standard deviation of 16.

Nine individuals are chosen at random from the population.

Determine the probability that no more than two of the individuals have an IQ of at least 120. [3 marks]

Sample Response	The response
 $P(IQ\ge 120)=0.1057$	• correctly determines the probability of IQ ≥ 120 [1 mark]
Using binomial distribution $n=9$, $p=0.1057$	• correctly recognises context is suitable for modelling as a binomial [1 mark]
Using GDC $P(x\le 2)=0.9391$	• determines the required probability [1 mark]

2021 Paper 2 Section 2 Question 14

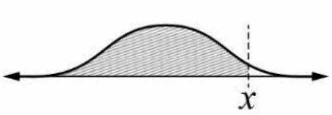
Continuous random variables and the normal distribution

The heights of students at School A are normally distributed with a mean of 165 cm and a standard deviation of 15 cm.

a) Determine the probability that a student chosen at random from School A is shorter than 180 cm. [1 mark]

Sample Response	The response
Using GDC P(student height under 180 cm) = 0.841	• correctly determines the probability [1 mark]

b) Determine the minimum integer value of the height of a student who is in the top 2% of this distribution. [3 marks]

Sample Response	The response
 	• correctly uses an appropriate mathematical representation [1 mark]
x = 195.806	• correctly determines the lowest decimal height [1 mark]
∴ minimum height is 196 cm	• determines lowest whole height [1 mark]

The heights of students at School B are also normally distributed. A student at School B has the same height as the height determined in Question 14b) but their corresponding z-score is 3.

c) Determine which student's height ranks higher in terms of percentile for their school. [3 marks]

Sample Response	The response
$z_{\text{School A}} = \frac{196-165}{15}$ $z_{\text{School A}} = \frac{31}{15}$	• determines z-score for student in school A [1 mark]
$\frac{31}{15} < 3$	• provides statement to justify decision [1 mark]
∴ student in School B ranked higher	• determines higher ranked student [1 mark]

2021
Paper 2
Section 2
Question 19

Continuous random variables and the normal distribution

A random variable X, defined over the interval , is uniformly distributed if its probability density function is defined by:

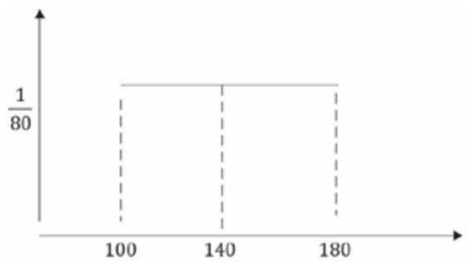
$$f(x)=\begin{cases}\dfrac{1}{(b-a)}, & a\le x\le b\\ 0, & \text{otherwise}\end{cases}$$

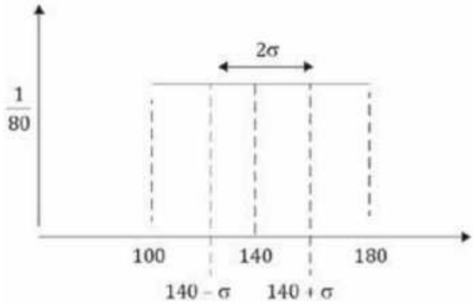
The expected value and variance of a uniform random variable X are

$$E(X)=\frac{(a+b)}{2} \qquad Var(X)=\frac{(b-a)^2}{12}$$

A manufacturer has observed that the time that elapses between placing an order with a supplier and the delivery of the order is uniformly distributed between 100 and 180 minutes.

Determine the probability that the time between placing an order and delivery of the order will be within one standard deviation of the expected time.

Sample Response	The response
Method 1 If X denotes the number of minutes that elapse between the placement and delivery of the order, then X can take any value in the interval $100 \le X \le 180$. Since the width is 80, the height of the density function must be $\frac{1}{80}$ for the total area under the density function to equal 1. The probability density function is: $f(x)=\frac{1}{80}, 100 \le x \le 180$	• correctly determines the probability density function [1 mark]
Using the given rules $E(X)=\frac{180+100}{2}$ $\text{Var}(X)=\frac{80^2}{12}=\frac{1600}{3}$ ∴ Mean = 140 and standard deviation = $\frac{40}{\sqrt{3}}$ Required probability $P(140-\frac{40}{\sqrt{3}}<X<140+\frac{40}{\sqrt{3}})$	• correctly determines the mean and the standard deviation [1 mark]
$\int_{140-\frac{40}{\sqrt{3}}}^{140+\frac{40}{\sqrt{3}}} \frac{1}{80}\,dx$	• establishes definite integral to represent probability [1 mark]
=0.57735	• determines probability [1 mark]
Method 2 Graphing the probability density function: 	• correctly identifies the probability density function graphically [1 mark]

Using the given rules $E(X) = \frac{180 + 100}{2}$ $\text{Var}(X) = \frac{80^2}{12} = \frac{1600}{3}$ ∴ Mean = 140 and standard deviation $\frac{40}{\sqrt{3}}$	• correctly determines the mean and the standard deviation [1 mark]
Identifying required probability as an area 	• uses appropriate graphical representation [1 mark]
Required probability $= \frac{1}{80} \times 2 \times \frac{40}{\sqrt{3}}$ Required probability $= \frac{1}{\sqrt{3}}$	• determines probability [1 mark]

2021
Paper 2
Section 2
Question 20

Continuous random variables and the normal distribution

The random variable B is normally distributed with a mean of 0 and a standard deviation of 1.

Determine the probability that the quadratic equation $x^2 + 3x + 2B = 0$ has real roots. [3 marks]

Sample Response	The response
The quadratic has real roots when $b^2 - 4ac \geq 0$	• correctly identifies the need to use the discriminant [1 mark]
$\therefore 9 - 8B \geq 0$ $\therefore 9 \geq 8B$ $\therefore \frac{9}{8} \geq B$	• correctly determines the range of values for B [1 mark]
Using the standard normal distribution (the given distribution) $P\left(B \leq \frac{9}{8}\right)$ $= 0.8697$	• determines probability [1 mark]

2020 Paper 2 Section 2 Question 13

Continuous random variables and the normal distribution

An online retailer claims that 90% of all orders are shipped within 12 hours of being received. On a particular day, 121 orders were received and 102 orders were shipped within 12 hours.

a) State the sample proportion of orders shipped within 12 hours. [1 mark]

Sample Response	The response
$\hat{p} = \frac{102}{121} = 0.84$	•correctly determines the sample proportion [1 mark]

The distribution of the sample proportion of all orders that are shipped within 12 hours of being received on any day is approximately normal.

b) Assuming the online retailer's claim is true, find the probability that, in a random sample of 121, less than 85% of all orders are shipped within 12 hours. [3 marks]

Sample Response	The response
Using normal distribution	• correctly identifies the mean of the population [1 mark]
$\mu = 0.9$ $\sigma = \sqrt{\frac{0.9 \times 0.1}{121}}$	• determines expression for standard deviation of the normal distribution [1 mark]
Using GDC $P(\hat{p} < 0.85)$ $= 0.0334$	• determines probability of producing sample proportion or less [1 mark]

c) Use the result from 13b) to evaluate the reasonableness of the online retailer's claim. [2 marks]

Sample Response	The response
There is 3.34% chance of shipping less than 85% of orders when the population parameter is 0.90.	• identifies meaning of the probability of producing the sample proportion [1 mark]
Observing a sample proportion of 0.843 (or even lower) would have occurred by chance less than 3.34% of the time if the retailer's claim is true. Therefore, we suspect that the retailer's claim is dubious.	• evaluates reasonableness of the retailer's claim [1 mark]

2020 Paper 2 Section 2 Question 14

Continuous random variables and the normal distribution

Let X denote the time in minutes between the arrival of trains at a station. The cumulative distribution function of X is defined by

$$F(x) = \begin{cases} 2 - \frac{10}{x}, & 5 \le x \le 10 \\ 0, & \text{otherwise} \end{cases}$$

a) Determine the probability density function of X. [3 marks]

Sample Response	The response
$F(x) = \int_{-\infty}^{x} f(t)dt$ $\int_{5}^{x} 2 - \frac{10}{t} dt$ for $5 \le x \le 10$	• correctly identifies the use of the derivative of the cumulative distribution function [1 mark]
$\int_{5}^{x} 2 - \frac{10}{t} dt = [2t - 10\ln(t)]_{5}^{x}$ $= 2(x - 5\ln\left(\frac{x}{5}\right) - 5)$	• correctly determines the derivative of the cumulative distribution function (pdf) [1 mark]
$\therefore F(x) = \begin{cases} 0 & x < 5 \\ 2\left(x - 5\ln\left(\frac{x}{5}\right) - 5\right) & 5 \le x \le 10 \\ 1 & x > 10 \end{cases}$	• uses appropriate convention to communicate the cumulative distribution function as a piecewise function with given domains [1 mark]

b) Determine the probability that $5 < X < 7$. [1 mark]

Sample Response	The response
$F(7) - F(5) = 2\left(7 - 5\ln\left(\frac{7}{5}\right) - 5\right) - 2\left(5 - 5\ln\left(\frac{5}{5}\right) - 5\right)$ $= 0.6353$	• correctly determines the probability that there are 5 to 7 minutes between train arrivals [1 mark]

c) Determine the mean time between the arrival of trains at the station. [2 marks]

Sample Response	The response
Mean $= \int_{5}^{10} 2x - 10dx$	• provides a statement identifying the use of expected value for a continuous random variable [1 mark]
Using GDC Mean = 25 minutes	• determines mean time between arrivals [1 mark]

2020 Paper 2 Section 2 Question 14

Continuous random variables and the normal distribution

Let X denote the time in minutes between the arrival of trains at a station. The cumulative distribution function of X is defined by

$$F(x) = \begin{cases} 2 - \frac{10}{x}, & 5 \le x \le 10 \\ 0, & \text{otherwise} \end{cases}$$

a) Determine the probability density function of X. [3 marks]

Sample Response	The response
$F(x) = \int_{-\infty}^{x} f(t)dt$ $\int_{5}^{x} 2 - \frac{10}{t}dt$ for $5 \le x \le 10$	• correctly identifies the use of the derivative of the cumulative distribution function [1 mark]
$\int_{5}^{x} 2 - \frac{10}{t}dt = [2t - 10\ln(t)]_{5}^{x}$ $= 2(x - 5\ln\left(\frac{x}{5}\right) - 5)$	• correctly determines the derivative of the cumulative distribution function (pdf) [1 mark]
$\therefore F(x) = \begin{cases} 0 & x < 5 \\ 2\left(x - 5\ln\left(\frac{x}{5}\right) - 5\right) & 5 \le x \le 10 \\ 1 & x > 10 \end{cases}$	• uses appropriate convention to communicate the cumulative distribution function as a piecewise function with given domains [1 mark]

b) Determine the probability that $5 < X < 7$. [1 mark]

Sample Response	The response
$F(7) - F(5) = 2\left(7 - 5\ln\left(\frac{7}{5}\right) - 5\right) - 2\left(5 - 5\ln\left(\frac{5}{5}\right) - 5\right)$ $= 0.6353$	• correctly determines the probability that there are 5 to 7 minutes between train arrivals [1 mark]

c) Determine the mean time between the arrival of trains at the station. [2 marks]

Sample Response	The response
Mean $= \int_{5}^{10} 2x - 10dx$	• provides a statement identifying the use of expected value for a continuous random variable [1 mark]
Using GDC Mean = 25 minutes	• determines mean time between arrivals [1 mark]

2020 Paper 2 Section 2 Question 16 **Continuous random variables and the normal distribution**	Bottles of soft drink should contain a volume with a mean of 591 mL, but some variation is expected. Any bottle at or below the 20th percentile of the volume distribution is rejected. A percentile is a measure in statistics that shows the values below which a given percentage of observations occur. Thirty-five per cent of the bottles contain 593 mL or more of soft drink. Assuming the volumes are normally distributed, determine the smallest volume (in mL) that will be accepted. [4 marks]

Sample Response	The response
Given $\mu = 591$ Using GDC z-score associated with 65th percentile = 0.38532	• correctly determines the z-score associated with the 65th percentile [1 mark]
$0.38532 = \frac{593 - 591}{\sigma}$ $\sigma = 5.1905$	• determines σ [1 mark]
Using GDC z-score associated with 20% rejection region = −0.841621	• correctly determines the z-score associated with the 20% rejection region [1 mark]
To determine the smallest volume that will be accepted (x) $0.841\,621 = \frac{x - 591}{5.1905}$ $x = 587$ mL	• determines the smallest volume [1 mark]

Unit 4 – Topic 5: Interval for estimates for proportions

Paper 1 Section 1

2023 Paper 1 Section 1 Question 8 **Interval estimates for proportions**	A sample of size n was used to estimate a population proportion. An approximate margin of error of 3% was calculated using $z = 1.96$. Given the sample proportion was 0.6, determine n. (A) $n = \dfrac{\left(\dfrac{0.03}{1.96}\right)^2}{0.24}$ (B) $n = \dfrac{0.24}{\left(\dfrac{0.03}{1.96}\right)^2}$ (C) $n = \dfrac{\left(\dfrac{0.03}{1.96}\right)^2}{2.4}$ (D) $n = \dfrac{2.4}{\left(\dfrac{0.03}{1.96}\right)^2}$

2022 Paper 1 Section 1 Question 8 **Interval for estimates for proportions**	In a survey, 80 respondents exercised daily, while 120 did not. When calculating the approximate 95% confidence interval for the proportion of people who exercise daily, the margin of error is (A) $1.96\sqrt{\dfrac{0.4(1-0.4)}{200}}$ (B) $0.95\sqrt{\dfrac{0.4(1-0.4)}{200}}$ (C) $1.96\sqrt{\dfrac{0.67(1-0.67)}{120}}$ (D) $0.95\sqrt{\dfrac{0.67(1-0.67)}{120}}$

2022 Paper 1 Section 1 Question 10 **Interval for estimates for proportions**	A survey plans to draw conclusions based on a random sample of 1% of Queensland's adult population. To be regarded as a random sample, every (A) adult in the population will be placed in an alphabetical list and every 100th person will be selected for the sample. (B) adult in the population can choose to participate until the sample size has been reached. (C) subgroup within the population will be represented in a similar proportion in the sample. (D) adult in the population will have an equal chance of being selected for the sample.

2020 Paper 1 Section 1 Question 10 **Interval for estimates for proportions**	Two types of material (A and B) are being tested for their ability to withstand different temperatures. A random selection of both materials was subjected to extreme temperature changes and then classified according to their condition after they were removed from the testing facility. The results are shown in the table.

	Material		
	A	**B**	**Total**
Broke completely	25	43	68
Showed defects	35	38	73
Remained intact	35	24	59
Total	**95**	**105**	**200**

An approximate 95% confidence interval for the probability that material A will break completely or show defects is given by

$$\left(c - 1.96\sqrt{\frac{c(1-c)}{n}}, c + 1.96\sqrt{\frac{c(1-c)}{n}}\right)$$

The values of c and n are

(A) $\frac{60}{95}$ and 95

(B) $\frac{60}{200}$ and 95

(C) $\frac{140}{200}$ and 95

(D) $\frac{60}{200}$ and 200

2021 Paper 1 Section 2 Question 19 **Interval for estimates for proportions**	A firm aims to have 95% confidence in estimating the proportion of office workers who respond to an email in less than an hour to within ± 0.05. A survey has never been undertaken before, so no past data is available. The firm believes that if the proportion is 0.5, then this will result in the largest variability in the sample proportion. Based on this, determine the sample size needed using the approximate value of $z = 2$ for the 95% confidence interval. Justify the choice of 0.5 for the proportion. [4 marks]

Paper 2 Section 1

2023 Paper 2 Section 1 Question 7 **Interval estimates for proportions**	The distribution of a certain sample proportion has a mean of 0.70 and a standard deviation of 0.02. Determine the sample size. (A) 525 (B) 750 (C) 1750 (D) 2500

2021 Paper 2 Section 1 Question 3 **Interval for estimates for proportions**	A random sample of people were surveyed about the most important factor when deciding where to shop. The results appear in the table. **Factor** / **Percentage (%)** Price: 40 Quality of merchandise: 30 Service: 15 Shopping environment: 15 If the sample size was 1200, the approximate 95% confidence interval for the proportion of people who identified price as the most important factor is (A) (0.395, 0.405) (B) (0.386, 0.414) (C) (0.377, 0.423) (D) (0.372, 0.428)

Factor	Percentage (%)
Price	40
Quality of merchandise	30
Service	15
Shopping environment	15

Paper 2 Section 2

2023 Paper 2 Section 2 Question 11 **Interval estimates for proportions**	A researcher found that 17 out of 50 randomly selected people had used public transport in the past week. a) Determine the sample proportion of people who had used public transport in the past week. [1 mark] b) Determine an approximate 95% confidence interval for the proportion of people who had used public transport in the past week. [2 marks] c) Someone claims that: *50% of people use public transport each week.* Use your answer from Question 11b) to explain whether the data can or cannot support this claim. [1 mark]

2022 Paper 2 Section 2 Question 14 Interval for estimates for proportions	Ravi randomly sampled 200 different pet owners in Brisbane and found that 50 celebrate their pet's birthday. a) Determine an approximate 95% confidence interval for the proportion of Brisbane pet owners who celebrate their pet's birthday. [2 marks]

Two of Ravi's friends also randomly sampled Brisbane pet owners. The results are shown in the table.

Friend's name	Number sampled	Number who celebrate their pet's birthday
Khadija	100	26
Tim	150	34

Khadija suggested a more precise estimate for the proportion of Brisbane pet owners who celebrate their pet's birthday could be obtained by combining their results.

b) Using all available data, determine an approximate 95% confidence interval for the proportion of Brisbane pet owners who celebrate their pet's birthday. [2 marks]

c) Use the results from Questions 14a) and 14b) to evaluate the reasonableness of Khadija's suggestion. [2 marks]

The proportion of all Brisbane pet owners who celebrate their pet's birthday is 0.24.

d) Using the normal approximation, determine the probability that in a randomly selected sample of size 200, more than 30% of pet owners celebrate their pet's birthday. [2 marks]

2020 Paper 2 Section 2 Question 17 **Interval for estimates for proportions**	In a survey of 326 lecturers, 303 said that on at least one occasion a mobile phone had rung in a lecture they were giving. Determine the sample size required to conduct a follow-up survey that provides 95% confidence that this one-point estimate is correct to within ± 0.02 of the population proportion. [4 marks]

2020 Paper 2 Section 2 Question 20 **Interval for estimates for proportions**	Assuming the approximate normality of sample proportions ($\hat{p}_1$ and $\hat{p}_2$) and based on two independent samples, the approximate confidence interval for the difference of two proportions is given by $$\left(\hat{p}_1 - \hat{p}_2 - z\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}, \hat{p}_1 - \hat{p}_2 + z\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}\right)$$ If the approximate confidence interval for the difference between two proportions does not contain 0, this provides evidence that the two proportions are not equal. The data in the table shows the observed frequencies of two drink preferences for independent samples of people who live in Town A and Town B. **Town** / **Tea** / **Coffee** / **Total** A / 111 / 105 / 216 B / 150 / 107 / 257 Using the approximate 99% confidence interval for the difference of two proportions, determine if there is evidence to conclude that drink preference is associated with the town where the person lives.

Marking Guide – Paper 1 Section 1

2023 Paper 1 Section 1 Question 8 **Interval estimates for proportions**	A sample of size n was used to estimate a population proportion. An approximate margin of error of 3% was calculated using $z = 1.96$. Given the sample proportion was 0.6, determine n. (A) $n = \dfrac{\left(\frac{0.03}{1.96}\right)^2}{0.24}$ (B) $n = \dfrac{0.24}{\left(\frac{0.03}{1.96}\right)^2}$ (C) $n = \dfrac{\left(\frac{0.03}{1.96}\right)^2}{2.4}$ (D) $n = \dfrac{2.4}{\left(\frac{0.03}{1.96}\right)^2}$ **Answer is B.**

2022 Paper 1 Section 1 Question 8 **Interval for estimates for proportions**	In a survey, 80 respondents exercised daily, while 120 did not. When calculating the approximate 95% confidence interval for the proportion of people who exercise daily, the margin of error is (A) $1.96\sqrt{\dfrac{0.4(1-0.4)}{200}}$ (B) $0.95\sqrt{\dfrac{0.4(1-0.4)}{200}}$ (C) $1.96\sqrt{\dfrac{0.67(1-0.67)}{120}}$ (D) $0.95\sqrt{\dfrac{0.67(1-0.67)}{120}}$ **Answer is A.**

2022 Paper 1 Section 1 Question 10 **Interval for estimates for proportions**	A survey plans to draw conclusions based on a random sample of 1% of Queensland's adult population. To be regarded as a random sample, every (A) adult in the population will be placed in an alphabetical list and every 100th person will be selected for the sample. (B) adult in the population can choose to participate until the sample size has been reached. (C) subgroup within the population will be represented in a similar proportion in the sample. **(D) adult in the population will have an equal chance of being selected for the sample. – Answer**

2020 Paper 1 Section 1 Question 10 **Interval for estimates for proportions**	Two types of material (A and B) are being tested for their ability to withstand different temperatures. A random selection of both materials was subjected to extreme temperature changes and then classified according to their condition after they were removed from the testing facility. The results are shown in the table.

	Material		
	A	**B**	**Total**
Broke completely	25	43	68
Showed defects	35	38	73
Remained intact	35	24	59
Total	**95**	**105**	**200**

An approximate 95% confidence interval for the probability that material A will break completely or show defects is given by

$$\left(c-1.96\sqrt{\frac{c(1-c)}{n}}, c+1.96\sqrt{\frac{c(1-c)}{n}}\right)$$

The values of c and n are

(A) $\frac{60}{95}$ and 95

(B) $\frac{60}{200}$ and 95

(C) $\frac{140}{200}$ and 95

(D) $\frac{60}{200}$ and 200

Answer is A.

2021
Paper 1
Section 2
Question 19

Interval for estimates for proportions

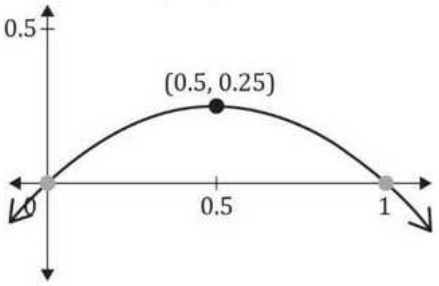
A firm aims to have 95% confidence in estimating the proportion of office workers who respond to an email in less than an hour to within ± 0.05.

A survey has never been undertaken before, so no past data is available.

The firm believes that if the proportion is 0.5, then this will result in the largest variability in the sample proportion.

Based on this, determine the sample size needed using the approximate value of z = 2 for the 95% confidence interval.

Justify the choice of 0.5 for the proportion. [4 marks]

Sample Response	The response
$interval\ margin = z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$	• correctly selects the interval margin formula [1 mark]
$0.05 = 2\sqrt{\frac{0.5(1-0.5)}{n}}$	• substitutes values into the formula [1 mark]
rearranging $n = \frac{2^2 \times 0.5(0.5)}{(0.05)^2}$ $n = 400$	• determines sample size n [1 mark]
The largest sample size will result when $\hat{p}(1-\hat{p})$ is maximised in the numerator, therefore generating largest n value.  Maximum occurs at $\hat{p} = 0.5$	• verifies the firm's decision to use $\hat{p} = 0.5$ using mathematical reasoning [1 mark]

Marking Guide – Paper 2 Section 1

<table>
<tr><td>2023
Paper 2
Section 1
Question 7

Interval estimates for proportions</td><td>The distribution of a certain sample proportion has a mean of 0.70 and a standard deviation of 0.02.

Determine the sample size.

(A) 525 – Answer
(B) 750
(C) 1750
(D) 2500</td></tr>
</table>

<table>
<tr><td>2021
Paper 2
Section 1
Question 3

Interval for estimates for proportions</td><td>A random sample of people were surveyed about the most important factor when deciding where to shop. The results appear in the table.

<table>
<tr><th>Factor</th><th>Percentage (%)</th></tr>
<tr><td>Price</td><td>40</td></tr>
<tr><td>Quality of merchandise</td><td>30</td></tr>
<tr><td>Service</td><td>15</td></tr>
<tr><td>Shopping environment</td><td>15</td></tr>
</table>

If the sample size was 1200, the approximate 95% confidence interval for the proportion of people who identified price as the most important factor is

(A) (0.395, 0.405)
(B) (0.386, 0.414)
(C) (0.377, 0.423)
(D) (0.372, 0.428) – Answer</td></tr>
</table>

2023 Paper 2 Section 2 Question 11

Interval estimates for proportions

A researcher found that 17 out of 50 randomly selected people had used public transport in the past week.

a) Determine the sample proportion of people who had used public transport in the past week. [1 mark]

Sample response	The response
$\hat{p} = \frac{17}{50}$ $= 0.34$	• correctly determines the sample proportion **[1 mark]**

b) Determine an approximate 95% confidence interval for the proportion of people who had used public transport in the past week. [2 marks]

Sample response	The response
Using the formula: Variance= $\frac{\hat{p}(1-\hat{p})}{n}$ to estimate variance $= \frac{0.34(1-0.34)}{50}$ = 0.004488 Standard deviation = 0.066993 Using GDC InvN Area= 0.95 Std dev = $\sqrt{0.004488}$ μ = 0.34 CI = (0.20869, 0.47130)	• identifies all the information required to establish the confidence interval **[1 mark]** • determines an approximate 95% confidence interval **[1 mark]**

c) Someone claims that: *50% of people use public transport each week.*

Use your answer from Question 11b) to explain whether the data can or cannot support this claim. [1 mark]

Sample response	The response
From Q11b) we are 95% confident that the proportion of people using public transport is approximately between 21% and 47%. 50% is outside of this range. The claim is not supported.	• provides a valid evaluation of the claim that references their answer from 11b) **[1 mark]**

2022 Paper 2 Section 2 Question 14 **Interval for estimates for proportions**	Ravi randomly sampled 200 different pet owners in Brisbane and found that 50 celebrate their pet's birthday.

Ravi randomly sampled 200 different pet owners in Brisbane and found that 50 celebrate their pet's birthday.

a) Determine an approximate 95% confidence interval for the proportion of Brisbane pet owners who celebrate their pet's birthday. [2 marks]

Sample Response	The response
Using GDC to determine confidence interval associated with $n = 200$, $\hat{p} = 0.25$, $z = 1.96$	• correctly identifies all of the information required to establish the confidence interval [1 mark]
(0.19, 0.31)	• correctly determines the confidence interval [1 mark]

Two of Ravi's friends also randomly sampled Brisbane pet owners. The results are shown in the table.

Friend's name	Number sampled	Number who celebrate their pet's birthday
Khadija	100	26
Tim	150	34

Khadija suggested a more precise estimate for the proportion of Brisbane pet owners who celebrate their pet's birthday could be obtained by combining their results.

b) Using all available data, determine an approximate 95% confidence interval for the proportion of Brisbane pet owners who celebrate their pet's birthday. [2 marks]

Sample Response	The response
Combining results $n = 450, \hat{p} = \frac{11}{45}$	• correctly determines n and $\hat{p}$ for the combined sample [1 mark]
Using GDC (0.2047, 0.2842)	• determines confidence interval [1 mark]

c) Use the results from Questions 14a) and 14b) to evaluate the reasonableness of Khadija's suggestion. [2 marks]

Sample Response	The response
By combining the results, the sample size is increased and the confidence interval width is reduced.	• identifies changed width of confidence interval [1 mark]
The new sample statistic provides a better estimate for the population parameter.	• evaluates the reasonableness of Khadija's suggestion [1 mark]

The proportion of all Brisbane pet owners who celebrate their pet's birthday is 0.24.

d) Using the normal approximation, determine the probability that in a randomly selected sample of size 200, more than 30% of pet owners celebrate their pet's birthday. [2 marks]

Sample Response	The response
Using approximation to the normal distribution Mean = 0.24 $\text{Standard deviation} = \sqrt{\frac{0.24 \times 0.76}{200}} = 0.0302$	• correctly determines the mean and standard deviation of the normal distribution [1 mark]
Using GDC $P(\hat{p} > 0.30)$	• determines the probability [1 mark]

2020 Paper 2 Section 2 Question 17

Interval for estimates for proportions

In a survey of 326 lecturers, 303 said that on at least one occasion a mobile phone had rung in a lecture they were giving.

Determine the sample size required to conduct a follow-up survey that provides 95% confidence that this one-point estimate is correct to within ± 0.02 of the population proportion. [4 marks]

Sample Response	The response
$\hat{p} = \dfrac{303}{326}$	• correctly determines the sample proportion [1 mark]
Using confidence interval formula (margin of error) $0.02 = 1.96 \times \sqrt{\dfrac{\frac{303}{326} \times \frac{23}{326}}{n}}$	• establishes equation in nn [1 mark]
Using GDC $n = 629.778$	• determines n [1 mark]
∴ a sample of 630 lecturers would be required.	• determines reasonable value of n [1 mark]

2020 Paper 2 Section 2 Question 20

Interval for estimates for proportions

Assuming the approximate normality of sample proportions ($\hat{p}_1$ and $\hat{p}_2$) and based on two independent samples, the approximate confidence interval for the difference of two proportions is given by

$$\left(\hat{p}_1 - \hat{p}_2 - z\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}, \hat{p}_1 - \hat{p}_2 + z\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}\right)$$

If the approximate confidence interval for the difference between two proportions does not contain 0, this provides evidence that the two proportions are not equal.

The data in the table shows the observed frequencies of two drink preferences for independent samples of people who live in Town A and Town B.

Town	Tea	Coffee	Total
A	111	105	216
B	150	107	257

Using the approximate 99% confidence interval for the difference of two proportions, determine if there is evidence to conclude that drink preference is associated with the town where the person lives.

Sample Response	The response
Method 1 $p1$ = proportion of Town A who prefer to drink tea $p2$ = proportion of Town B who prefer to drink tea The sample proportions are: $\hat{p}1 = \dfrac{111}{216}$ $\hat{p}2 = \dfrac{150}{257}$	• correctly determines the sample proportions [1 mark]
Using the 99% confidence interval for the difference of two proportions $(\dfrac{111}{216} - \dfrac{150}{257} - 2.576\sqrt{\dfrac{\frac{111}{216}\left(1-\frac{111}{216}\right)}{216} + \dfrac{\frac{150}{257}\left(1-\frac{150}{257}\right)}{257}}, \dfrac{111}{216} - \dfrac{150}{257}$	• establishes confidence interval for the difference of two proportions [1 mark]

	$+2.576\sqrt{\frac{\frac{111}{216}\left(1-\frac{111}{216}\right)}{216}+\frac{\frac{150}{257}\left(1-\frac{150}{257}\right)}{257}})$	
	= (–0.188, 0.048)	• determines 99% confidence interval [1 mark]
	This interval contains zero; therefore, there is no evidence in the data to say that the two proportions are different, i.e. preference to drink tea does not depend on where the person lives.	• interprets 99% confidence interval to determine equality of proportions [1 mark] • shows logical organisation communicating key steps [1 mark]
	This interval contains zero; therefore, there is no evidence in the data to say that the two proportions are different, i.e. preference to drink tea does not depend on where the person lives.	• shows logical organisation communicating key steps [1 mark]
	Method 2 $\hat{p}_1 = \frac{105}{216}$ $\hat{p}_2 = \frac{107}{257}$	• correctly determines the sample proportions
	Using the 99% confidence interval for the difference of two proportions $(\frac{105}{216}-\frac{107}{257}-2.576\sqrt{\frac{\frac{105}{216}\left(1-\frac{105}{216}\right)}{216}+\frac{\frac{107}{257}\left(1-\frac{107}{257}\right)}{257}},\frac{105}{216}-\frac{107}{257}$ $2.576\sqrt{\frac{\frac{105}{216}\left(1-\frac{105}{216}\right)}{216}+107/257}$	• establishes confidence interval for the difference of two proportions [1 mark]
	= (–0.048, 0.188)	• determines 99% confidence interval [1 mark]
	This interval contains zero; therefore, there is no evidence in the data to say that the two proportions are different, i.e. the preference to drink coffee does not depend on where the person lives.	• interprets 99% confidence interval to determine equality of proportions [1 mark] • shows logical organisation communicating key steps [1 mark]