CAMBRIDGE

AMSI AUSTRALIAN MATHEMATICAL SCIENCES INSTITUTE

ICE-EM MATHEMATICS 6

THIRD EDITION

Colin Becker
Howard Cole
Andy Edwards
Garth Gaudry
Janine McIntosh
Jacqui Ramagge

INTERNATIONAL CENTRE OF EXCELLENCE FOR EDUCATION IN MATHEMATICS

INCLUDES INTERACTIVE TEXTBOOK POWERED BY CAMBRIDGE HOTMATHS

AMSI
AUSTRALIAN
MATHEMATICAL
SCIENCES
INSTITUTE

ICE-EM MATHEMATICS 6

THIRD EDITION

Colin Becker
Howard Cole
Andy Edwards
Garth Gaudry
Janine McIntosh
Jacqui Ramagge

INTERNATIONAL CENTRE
OF EXCELLENCE FOR
EDUCATION IN
MATHEMATICS

CAMBRIDGE
UNIVERSITY PRESS

CAMBRIDGE
UNIVERSITY PRESS

University Printing House, Cambridge CB2 8BS, United Kingdom

One Liberty Plaza, 20th Floor, New York, NY 10006, USA

477 Williamstown Road, Port Melbourne, VIC 3207, Australia

4843/24, 2nd Floor, Ansari Road, Daryaganj, Delhi – 110002, India

79 Anson Road, #06–04/06, Singapore 079906

Cambridge University Press is part of the University of Cambridge.

It furthers the University's mission by disseminating knowledge in the pursuit of education, learning and research at the highest international levels of excellence.

www.cambridge.org
Information on this title: www.cambridge.org/9781108400497

© The University of Melbourne on behalf of the Australian Mathematical Sciences Institute (AMSI) 2017

This publication is in copyright. Subject to statutory exception
and to the provisions of relevant collective licensing agreements,
no reproduction of any part may take place without the written
permission of Cambridge University Press.

First published 2017
20 19 18 17 16 15 14 13 12 11 10 9 8 7 6 5 4 3 2 1

Cover designed by Loupe Studio
Typeset by diacriTech
Printed in China by C & C Offset Printing Co. Ltd.

A Catalogue-in-Publication entry is available from the catalogue of the National Library of Australia at www.nla.gov.au

ISBN 978-1-108-40049-7 Paperback

Additional resources for this publication at www.cambridge.edu.au/GO

Reproduction and communication for educational purposes
The Australian *Copyright Act 1968* (the Act) allows a maximum of one chapter or 10% of the pages of this publication, whichever is the greater, to be reproduced and/or communicated by any educational institution for its educational purposes provided that the educational institution (or the body that administers it) has given a remuneration notice to Copyright Agency Limited (CAL) under the Act.

For details of the CAL licence for educational institutions contact:
Copyright Agency Limited
Level 15, 233 Castlereagh Street
Sydney NSW 2000
Telephone: (02) 9394 7600
Facsimile: (02) 9394 7601
Email: info@copyright.com.au

Reproduction and communication for other purposes
Except as permitted under the Act (for example a fair dealing for the purposes of study, research, criticism or review) no part of this publication may be reproduced, stored in a retrieval system, communicated or transmitted in any form or by any means without prior written permission. All inquiries should be made to the publisher at the address above.

Cambridge University Press has no responsibility for the persistence or accuracy of URLs for external or third-party internet websites referred to in this publication, and does not guarantee that any content on such websites is, or will remain, accurate or appropriate. Information regarding prices, travel timetables and other factual information given in this work is correct at the time of first printing but Cambridge University Press does not guarantee the accuracy of such information thereafter.

Contents

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

Preface

ICE-EM Mathematics Third Edition is a series of textbooks for students in years 5 to 10 throughout Australia who study the Australian Curriculum and its state variations.

The program and textbooks were developed in recognition of the importance of mathematics in modern society and the need to enhance the mathematical capabilities of Australian students. Students who use the series will have a strong foundation for work or further study.

Background

The International Centre of Excellence for Education in Mathematics (ICE-EM) was established in 2004 with the assistance of the Australian Government and is managed by the Australian Mathematical Sciences Institute (AMSI). The Centre originally published the series as part of a program to improve mathematics teaching and learning in Australia. In 2012, AMSI and Cambridge University Press collaborated to publish the Second Edition of the series to coincide with the introduction of the Australian Curriculum, and we now bring you the Third Edition.

Features

ICE-EM Mathematics Third Edition provides a progressive development from upper primary to middle secondary school. The writers of the series are some of Australia's most outstanding mathematics teachers and subject experts. The textbooks are clearly and carefully written, and contain background information, examples and worked problems.

The series places a strong emphasis on understanding basic ideas, along with mastering essential technical skills. Mental arithmetic and other mental processes are major focuses, as is the development of spatial intuition, logical reasoning and understanding of the concepts.

For the Third Edition, the series has been carefully edited to present the content in a more streamlined way without compromising quality. There is now one book per year level and the flow of topics from chapter to chapter and from each year level to the next has been improved.

For the Third Edition, *ICE-EM Mathematics* now comes with an Interactive Textbook: a cutting-edge digital resource where all textbook material can be answered online (with students' working-out), additional quizzes and features are included at no extra cost. See "The Interactive Textbook and Online Teaching Suite" on page xi for more information.

Author biographies

Colin Becker

Colin Becker works as a Mathematics and ITLT Specialist at an independent boys' school in Adelaide. Colin has written for professional publications, presented at conferences and schools and is actively involved in mathematics education.

Howard Cole

Howard Cole was Senior Mathematics Master at Sydney Grammar School Edgecliff Preparatory for many years. He outlined the primary curriculum during that time, as well as writing and producing in-school workbooks for Years 5 and 6. Now retired from teaching, he still maintains a keen interest in mathematics and curriculum development.

Andy Edwards

Andy Edwards taught in secondary mathematics classrooms for 31 years in Victoria, Canada and Queensland. Since 2004, he has worked for the Queensland Curriculum and Assessment Authority writing materials for their assessment programs from Years 3 to 12 and as a test item developer for WA's OLNA program since 2013. He has written non-routine problems for the Australian Mathematics Trust since 1991.

Garth Gaudry

Garth Gaudry was head of mathematics at Flinders University before moving to the University of New South Wales (UNSW), where he became Head of School. He was the inaugural Director of the Australian Mathematical Sciences Institute before becoming Director of AMSI's International Centre of Excellence for Education in Mathematics (ICE-EM). Previous positions include membership of the South Australian Mathematics Subject Committee and the Eltis Committee appointed by the NSW Government to inquire into Outcomes and Profiles. He was a life member of the Australian Mathematical Society and Emeritus Professor of Mathematics, UNSW.

Janine McIntosh

Janine McIntosh works at The Australian Mathematical Sciences Institute, where she manages AMSI Schools. Janine leads a professional development and schools visit program for teachers across the country. Through clusters of schools supported by industry and government partners, Janine's aim is to encourage more Australians to enjoy and study mathematics. Janine has developed a suite of online and careers materials in her time at AMSI and was one of the writers for the Australian Curriculum: Mathematics F – 10. She is an experienced primary teacher, who has worked as a lecturer in mathematics education at the University of Melbourne and serves on the Maths Challenge and AMOC Committees of the Australian Mathematics Trust.

Jacqui Ramagge

Jacqui Ramagge is currently Head of the School of Mathematics and Statistics at the University of Sydney. After graduating in 1993 with a PhD in Mathematics from the University of Warwick (UK) she worked at the University of Newcastle (Australia) until 2007 and then at the University of Wollongong until 2015 when she moved to the University of Sydney. She has served on the Australian Research Council College of Experts, including as Chair of Australian Laureate Fellowships Selection Advisory Committee. She teaches mathematics at all levels from primary school to PhD courses and has won a teaching award. She contributed to the Vermont Mathematics Initiative (USA) and is a founding member of the Australian Mathematics Trust Primary Problems Committee. In 2013 she received a BH Neumann Award from the Australian Mathematics Trust for her significant contribution to the enrichment of mathematics learning in Australia.

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

Acknowledgements

We are grateful to Professor Peter Taylor OA, former Director of the Australian Mathematical Trust, for his support and guidance as chairman of the Australian Mathematical Sciences Institute Education Advisory Committee, 2003–2011.

We gratefully acknowledge the assistance of:

Richard Barker
Philip Broadbridge
Jan Chantry
Carmel Cribbes
Kevin McAvaney
John Mighton
Mark Mudge
Marian Neal
Michael Shaw
Jan Thomas
James Wan
Geoffrey Wemyss
Hung-His Wu

We also gratefully acknowledge the hard work and professionalism of Fiona Olney-Fraser in the development and editing of this Third Edition.

The author and publisher wish to thank the following sources for permission to reproduce material:

Images: © Getty Images / Lalocracio, **p.11(t)** / deepblu4you, **p.11(b)** / ZargonDesign, **pp.13, 194** / Echo, **p.14** / havseen, **p.15** / Jamie Garbutt, **p.21(t)** / Gail Shotlander, **p.21(b)** / Nina_Piankova, **p.41** / Nenov, **pp.43, 45** / Boarding1Now, **p.47(t-l)** / serts, **p.47(b-r)** / MirasWonderland, **p.53** / Dimitris66, **p.53** / Floorje, **p.54 (b)** / Serj_Velusceac, **p.59(t)** / stockcam, **p.59(b)** / momentimages, **p.70(t-r)** / Halfdark, **p.71** / Image Source, **p.72** / Plume Creative, **pp.76, 77** / JoseGirarte, **p.91** / stockcam, **p.92** / Sezeryadigar, **pp. 95, 97, 132** / Steve Gorton and Alex Wilson, **p.93(1)** / Oeter Dazeket, **p.93(2)** / uatp2, **p.93(3)** / Ghislain & Marie David de Lossy, **p.135(t-l)** / Planet Observer, **p.135(b-r)** / Paul Bricknell, **p.157** / anaa1311, **p.166** / mkos83, **p.167(t)** / Lew Robertson, **p.167 (b)** / Lane Oatey / Blue Jean, **p.185** / londoneye, **p.195** / NickyBlade, **p.201** / Maica, **p.223** / Fuhito Kanayama, **p.235** / KTS Design /Science Photo Library (red dice), **p.241, 242** / Dorling Kingsley (yellow dice), **p.241, 242** / bagi1998, **p.250(c-t)** / Westend614, **p.250(c-r)** / clintsholz, **p.271(l)** / eugenesergeev, **p.271(c)** / ollo, **p.271(r)** / pius99, **p.303** / Creative Crop, **p.311(t)** / Visual Field, **p.311(c-l)** / Science Photo Library, **pp.311(b-r), 322** / ikonacolor, **p.314** / vikif, **p.315** / dianazh, **p.316 (l)** / Creativ Studio Heinemann, **p.316(r)** / pjonson1, **p.317(t-1)** / lenscap67, **p.317(t-2)** / Fridholm, Jakob, **p.317(t-3)** / Atw Photography, **p.317(b-1)**, ylacinsonat1, **p.317(b-2)** / Harry Bischof, **p.317(b-3)** / ChubarovY, **p.326** / Tetra images, **p.327** / 172208211, **p.328(t)** / Stock montage Contributor, **p.331** / Chris Bradley, **p.371** / D'Franc Photograpy, **p.387**.

Every effort has been made to trace and acknowledge copyright. The publisher apologises for any accidental infringement and welcomes information that would redress this situation.

How to use this resource

The textbook

The Year 5 and Year 6 books are written in the style of a 'conversation'. That conversation is meant to take a variety of forms: conversations between the teacher and students about the ideas and methods as they are developed; conversations among the students themselves about what they have done and learnt, and the different ways they have solved problems; and conversations with others at home. Each chapter addresses a specific Australian Curriculum content strand and set of sub-strands. The exercises within chapters take an integrated approach to the concept of proficiency strands, rather than separating them out. Students are encouraged to develop and apply Understanding, Fluency, Problem-solving and Reasoning skills in every exercise.

Question tags

The questions in each chapter are tagged. The tags are intended as a guide to teachers. They should be regarded as a way of encouraging student progress.

🟡 These give students practice using the basic ideas and methods of the section. They should give students confidence to go on successfully to the next level.

🔵 These build on the previous level and help students acquire a more complete grasp of the main ideas and techniques. Some questions require interpretation, using a reading ability appropriate to the age group.

🟢 For these questions, students may need to apply concepts from outside the section or chapter. Problem-solving skills and a higher reading ability are needed, and these questions should help develop those attributes.

Challenge exercises

The Challenge exercises, which can be downloaded via the Interactive Textbook, are a vital part of the *ICE-EM Mathematics* resource. These are intended for students with above-average mathematical and reading ability. However, the questions vary considerably in their level of difficulty. Students who have managed the harder questions in the exercises reasonably well should be encouraged to try them.

The Interactive Textbook and Online Teaching Suite

The Interactive Textbook is the online version of the textbook and is accessed using the 16-character code on the inside cover of this book. The Online Teaching Suite is the teacher version of the Interactive Textbook and contains all the support material for the series, including tests, curriculum documentation and more. For more information on the Interactive Textbook and Online Teaching Suite, see page xi.

The Interactive Textbook and Online Teaching Suite are delivered on the *Cambridge HOTmaths* platform, providing access to a world-class Learning Management System for testing, task management and reporting. They do not provide access to the *Cambridge HOTmaths* stand-alone resource that you or your school may have used previously. For more information on this resource, contact Cambridge University Press.

AMSI's TIMES and SAM modules

The TIMES and SAM web resources were developed by the *ICE-EM Mathematics* author team at AMSI and are written around the structure of the Australian Curriculum. These resources have been mapped against your *ICE-EM Mathematics* book and are available to teachers and students via the AMSI icon on the dashboard of the Interactive Textbook and Online Teaching Suite.

The Interactive Textbook and Online Teaching Suite

Interactive Textbook

The Interactive Textbook is the online version of the print textbook and comes included with purchase of the print textbook. It is accessed by first activating the code on the inside cover. It is easy to navigate and is a valuable accompaniment to the print textbook.

Students can show their working

All textbook questions can be answered online within the Interactive Textbook. Students can show their working for each question using either the Draw tool for handwriting (if they are using a device with a touch-screen), the Type tool for using their keyboard in conjunction with the pop-up symbol palette, or by importing a file using the Import tool.

Once a student has completed an exercise they can save their work and submit it to the teacher, who can then view the student's working and give feedback to the student, as they see appropriate.

Auto-marked quizzes

The Interactive Textbook also contains material not included in the textbook, such as a short auto-marked quiz for each section. The quiz contains 10 questions which increase in difficulty from question 1 to 10 and cover all proficiency strands. There is also space for the student to do their working underneath each quiz question. The auto-marked quizzes are a great way for students to track their progress through the course.

Additional material for Year 5 and 6

For Years 5 and 6, the end-of-chapter Challenge activities as well as a set of Blackline Masters are now located in the Interactive Textbook. These can be found in the 'More resources' section, accessed via the Dashboard, and can then easily be downloaded and printed.

Online Teaching Suite

The Online Teaching Suite is the teacher's version of the Interactive Textbook. Much more than a 'Teacher Edition', the Online Teaching Suite features the following:

- The ability to view students' working and give feedback – When a student has submitted their work online for an exercise, the teacher can view the student's work and can give feedback on each question.
- For Years 5 and 6, access to Chapter tests, Blackline Masters, Challenge exercises, curriculum support material, and more.
- For Years 7 to 10, access to Pre-tests, Chapter tests, Skillsheets, Homework sheets, curriculum support material, and more.
- A Learning Management System that combines task-management tools, a powerful test generator, and comprehensive student and whole-class reporting tools.

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

CHAPTER 1

Preparation

Useful skills for this chapter:

- understanding place value of numbers to 1000
- the ability to ‘pull apart’ numbers into their place-value components
- an understanding of the relative size of numbers and their placement on a number line.

KICK OFF

1 Write the number that is 100 more than:

a 300 **b** 324 **c** 936 **d** 1013 **e** 19 079

2 Write the number that is 100 less than:

a 300 **b** 324 **c** 936 **d** 1013 **e** 19 079

3 Write the number that is 1000 more than:

a 5000 **b** 702 **c** 31 **d** 3058 **e** 19 892

4 Write the number that is 1000 less than:

a 8000 **b** 1003 **c** 11 035 **d** 10 403 **e** 20 202

Show what you know

1 Draw a number line from 0 to 1000. Mark in 25, 100, 400, 750 and 888.

2 Write the value of the ‘5’ digit in these numbers.

a 57 903 **b** 957 **c** 1225 **d** 15 092 **e** 13 582

CHAPTER 1

Number and Algebra

Positive and negative whole numbers

This chapter begins by looking at whole numbers; then we look at negative whole numbers.

The whole numbers are sometimes called the ‘counting numbers’. The whole numbers are the numbers 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, and so on. We can always add 1 to any whole number and get another whole number. That is why we say the list of whole numbers is infinite – it never ends.

We use numbers to record countable quantities where we can ask ‘How many?’

For example:

- *How many* buses are in the car park?
- *How many* children are at school today?
- *How many* salad rolls did you eat for lunch?

We also use whole numbers for quantities that we measure, rather than count. When we are talking about quantities, we ask ‘How much?’

For example:

- *How much* rice is needed for dinner?
- *How much* money do you need to save for your holiday?
- *How much* orange juice is in the jug?

Negative whole numbers are used to describe numbers below zero, such as temperatures. For example, the temperature at the top of Mt Kosciusko in winter can be as low as –18°C.

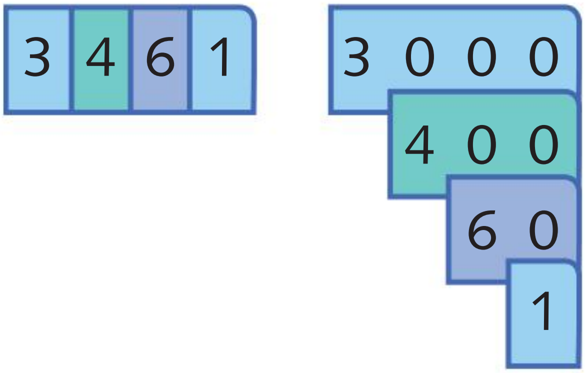
1A Place value

The **digits** 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9 can be used to write:

- a 5-digit number such as 89 371
- a 6-digit number such as 450 672
- a 1-digit number such as 2
- any whole number.

We can use place-value cards to help break a number into its place-value parts.

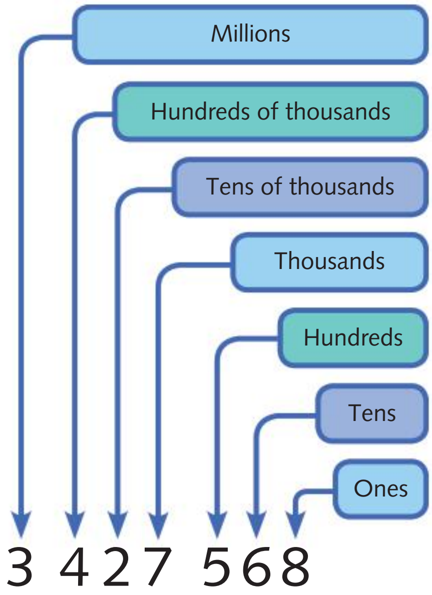
These place-value cards show the value of each digit in the number 3461.



Each place in a number has a special value.

For example, in the number 3 427 568:

- the ‘3’ means 3 millions
- the ‘4’ means 4 hundreds of thousands
- the ‘2’ means 2 tens of thousands
- the ‘7’ means 7 thousands
- the ‘5’ means 5 hundreds
- the ‘6’ means 6 tens
- the ‘8’ means 8 ones.

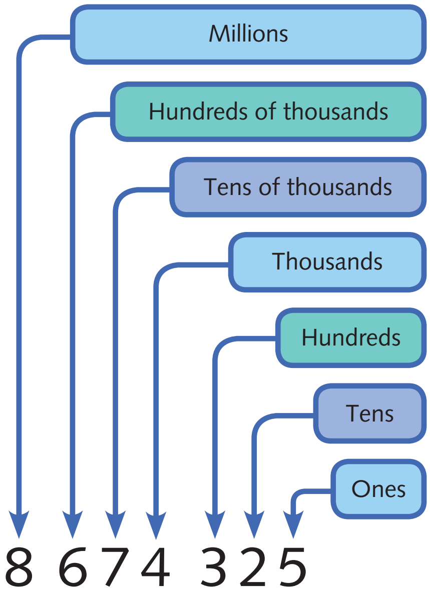


ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7

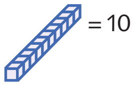
Cambridge University Press

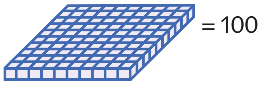
The value of a digit changes if it is in a different place. If we take the number 3 427 568 (on the previous page) and change the position of the digits to make 8 674 325:

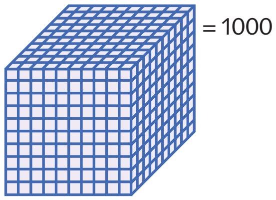
- the ‘8’ means 8 millions
- the ‘6’ means 6 hundreds of thousands
- the ‘7’ means 7 tens of thousands
- the ‘4’ means 4 thousands
- the ‘3’ means 3 hundreds
- the ‘2’ means 2 tens
- the ‘5’ means 5 ones.



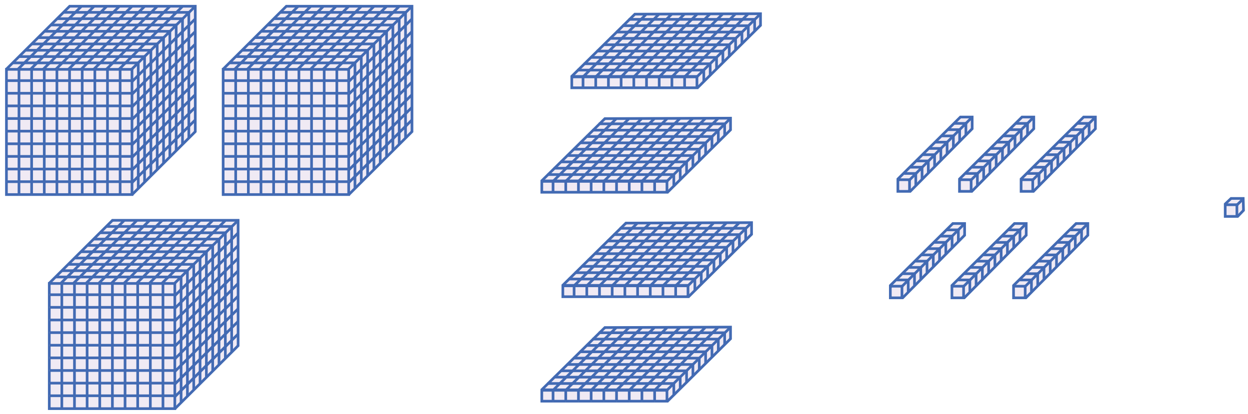
We can use base-ten materials to help us understand place value.







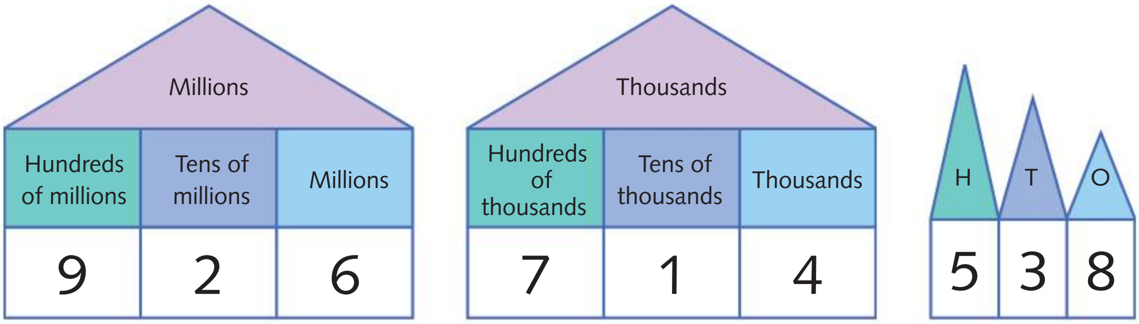
This is the number 3461 shown with base-ten materials:



3461 means 3 thousands + 4 hundreds + 6 tens + 1 one.

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7

Knowing the value of the places in a number helps us to read it.



We read 926 714 538 as nine hundred and twenty-six million, seven hundred and fourteen thousand, five hundred and thirty-eight.

Example 1

Write the value of the 8 in each number.

a 28 774 **b** 289 **c** 18 693 002 **d** 781 721

Solution

a In 28 774, the 8 is in the thousands place, so it has the value of 8 thousands or 8000.

b In 289, the 8 digit is in the tens place, so it has the value of 8 tens or 80.

c In 18 693 002, the 8 digit is in the millions place, so it has the value of 8 millions or 8 000 000.

d In 781 721, the 8 digit is in the tens of thousands place, so it has the value of 8 tens of thousands or 80 000.

1A Whole class CONNECT, APPLY AND BUILD

1 Download **BLM 1** 'Place-value cards' from the Interactive Textbook.

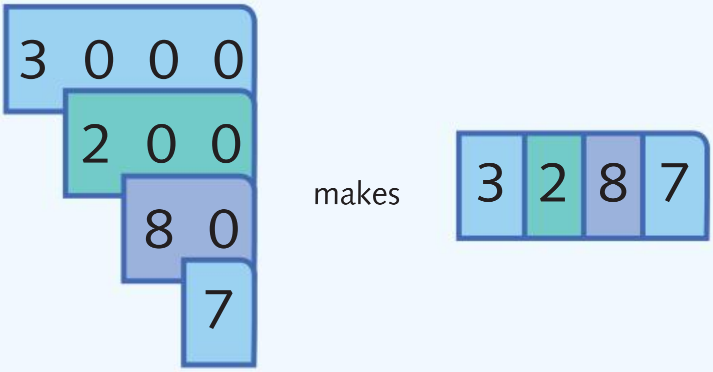
Use your place-value cards to make these numbers.

a 8912

b 4938

c 389

d 9072



ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

2 Write these numbers in words.

a 5038 **b** 28 445 **c** 46 102 **d** 298 577

3 Write the number.

a 85 tens and 2 ones
b 300 ones
c 23 hundreds, 5 tens and 3 ones
d 4 thousands, 5 hundreds and 18 ones
e 23 hundreds and 45 ones
f 128 thousands, 43 tens and 9 ones
g 34 ones, 18 hundreds
h 17 ones, 1 ten thousand, 23 hundreds

1A Individual

1 Write each number.

a Eight thousand, two hundred and twenty-three
b One hundred and five thousand, two hundred and forty-nine
c Thirteen million, seven hundred and ninety-eight thousand, five hundred and sixty-two

2 Write each number in words.

a 91 **b** 48 **c** 392 **d** 6789
e 18 002 **f** 21 309 **g** 589 902 **h** 6 893 407

3 Write the value of each highlighted digit.

a 802 **b** 3196 **c** 54 795 **d** 832 489
e 1 904 684 **f** 589 444 239 **g** 603 540 002 **h** 347 862 919

4 Copy and complete this place-value chart.

Number	Place-value parts							
	Tens of millions	Millions	Hundreds of thousands	Tens of thousands	Thousands	Hundreds	Tens	Ones
9045								
21 947								
101 010								
800 641								
1 794 376								

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

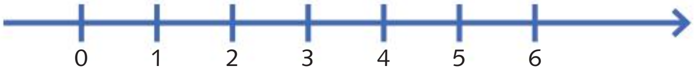
1B The number line

A number line helps us make sense of numbers. To make a number line, draw a line on the page and mark ‘0’ for zero. The arrow shows that the line continues in the same way forever.

Mark equally spaced points to the right side of the zero.

Label the first marker to the right of the zero as ‘1’.

Label the next marker ‘2’, then keep going.

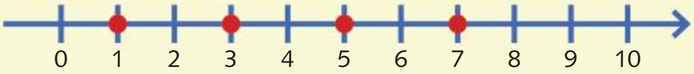


You can make number lines out of string or tape, or draw them on paper. They can be used to show any number, from the very smallest number to the largest number you can think of.

Example 2

Draw a number line from 0 to 10. Use large dots to mark the numbers 1, 3, 5 and 7.

Solution



A number ladder is like a vertical number line with equally spaced rungs. Georgie has her left foot on 2, her right foot on 3 and her left hand on 7.

We usually mark 0 on the number line and then mark 1, 2, 3, … evenly spaced as far as we need to go.

Sometimes we mark the multiples of 10 or the multiples of 100, depending on what we need.

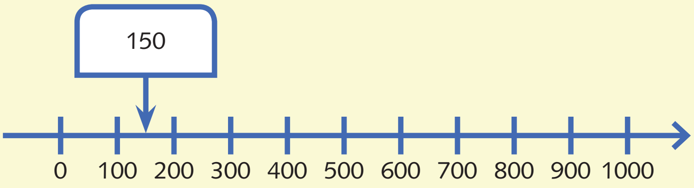
ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

Example 3

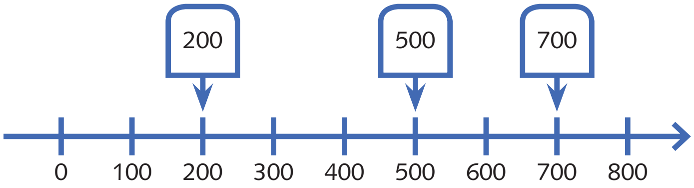
Show where 150 would be on this number line.

Solution

Mark in steps of 100. The number 150 is halfway between 100 and 200.



We can also use number lines to compare numbers.



Numbers get larger as we go to the right on the number line. So 700 is larger than 200 because it lies further to the right.

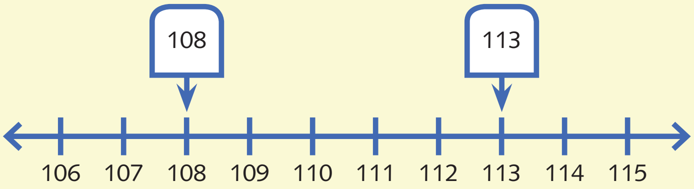
Numbers get smaller as we go to the left on the number line. So 200 is smaller than 500 because it lies further to the left.

Example 4

Use a number line to show that 113 is larger than 108.

Solution

Place both numbers on the number line. 113 is to the right of 108, so 113 is the larger number.



ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

1B Whole class CONNECT, APPLY AND BUILD

1 **Chalk on the playground**

You will need chalk. Draw these number lines on the concrete or bitumen surface of the school yard.

a Mark in 0 and 10, then mark 2, 4, 6 and 8.

b Mark in 0 and 100, then mark 25, 50 and 75.

c Mark in 0 and 1000, then mark 100, 250 and 777.

1B Individual

1 Draw a number line, then mark 20 and 40 on it.
Use a large dot to mark each number from 21 to 35.

2 Draw a number line, then mark 0 and 100 on it.
Use large dots to mark 30, 50 and 75.

3 Draw a number line, then mark 0 and 1000 on it.
Use large dots to mark 100, 400, 825 and 940.

4 Draw a number line, then mark 0 and 10 000 on it.
Use large dots to mark 1000, 2500 and 9999.

5 What numbers do the dots on these number lines show?

a

b

c

d

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

1C Counting

We use counting to find the answer to the question 'How many?' When we count, we mentally attach a number to each object, taking care not to skip any.

For example, there are 87 cars in the car park.

It is good to be able to count forwards and backwards by ones from any number.

Sometimes it is easier to skip-count than to count every item.

Example 5

7 buses took people to the school picnic. There were 15 people on each bus.

How many people were there in total?

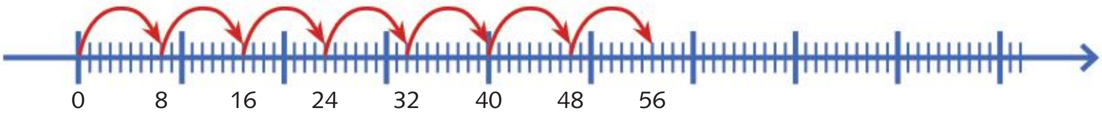
Solution

Count by fifteens.

There were 105 people at the picnic.

We can skip-count forwards and backwards by any number from any starting point. We can show skip-counting on a number line.

This number line shows counting forwards by eights, starting at 0.

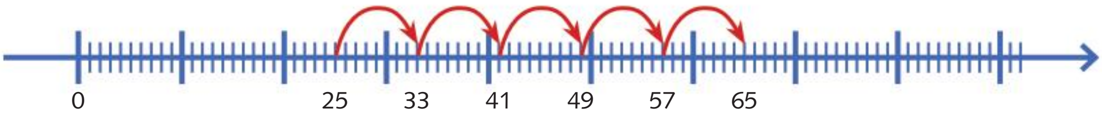


ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

When we count forwards by eights starting at 8, we say:

8, 16, 24, 32, 40, 48, 56, 64 …

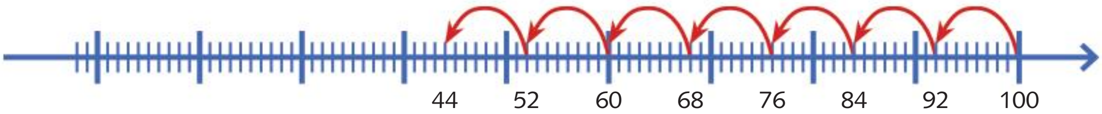
This number line shows counting forwards by eights, starting at 25.



When we count forwards by eights starting at 25, we say:

25, 33, 41, 49, 57, 65 …

This number line shows counting backwards by eights, starting at 100.



When we count backwards by eights starting at 100, we say:

100, 92, 84, 76, 68, 60 …

The ability to skip-count forwards and backwards from various starting points – and by different amounts – is an important skill.

1C Whole class CONNECT, APPLY AND BUILD

These activities may be used as whole-class or small-group activities.

1 Buzz forwards

Start counting by ones. When you get to a number that is in the counting pattern listed below, substitute the word 'buzz' for the number word. Your teacher can make the game trickier by randomly pointing to the student who is to say the next number.

a Count by sixes from 0.

b Count by nines from 0.

c Count by sixes from 15.

d Count by nines from 121.

2 Buzz backwards

Start counting backwards by ones. When you get to a number that is in the counting pattern listed below, substitute the word 'buzz' for the number word. Your teacher can make the game trickier by randomly pointing to the student who is to say the next number.

a Count backwards by fours from 100.

b Count backwards by elevens from 100.

c Count backwards by sevens from 1000.

d Count backwards by twelves from 170.

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

3  **Beachball**

Pass a beachball around the room. Whoever catches the ball says the next number in the sequence.

a Start at 0 and count forwards by threes.
b Start at 3 and count forwards by sevens.
c Start at 1000 and count backwards by fours.
d Start at 889 and count backwards by eights.
e Start at 56 and count forwards by thirteens.

1C Individual

1 Write the first 10 numbers in each counting sequence.

a Count forwards by ones, starting at 897.
b Count forwards by threes, starting at 98.
c Count backwards by ones, starting at 56.
d Count backwards by fours, starting at 213.

2 For each counting sequence, write as many numbers as you can in 2 minutes.

a Count forwards by sevens, starting at 99.
b Count forwards by nines, starting at 346.
c Count backwards by eights, starting at 1777.

3 You will need a long strip of paper.

List the first 20 numbers in the counting sequence for each of the numbers in the pair, starting with the number itself. Then highlight the numbers that are the same in both lists. The first one has been done for you.

a 2 and 3

2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 32, 34, 36, 38, 40
3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, 39, 42, 45, 48, 51, 54, 57, 60

b 3 and 4 **c** 4 and 5 **d** 3 and 8 **e** 8 and 9

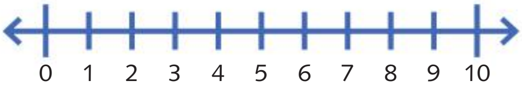
4 What did you notice about the numbers that were the same in both lists in the question above? Write the first 5 numbers that will be the same in the counting sequences for 4 and 9 without writing the list for each number.

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

1D The integers

If you have ever been to the snow or used a freezer to keep food, then you will already be familiar with negative numbers being used to describe temperatures below zero. You might not have thought about negative numbers in a mathematical sense. In this section we are going to extend the number line to include negative numbers.

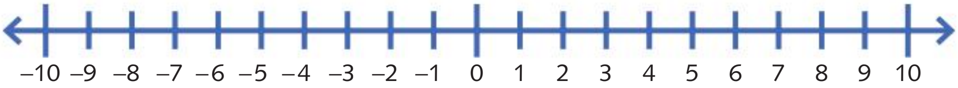
This number line shows positive numbers. The numbers continue to the right of the number line indefinitely.



But what happens to the left of 0 on the number line?

The numbers to the left of the zero are known as the **negative** numbers. All of the negative and positive whole numbers, together with zero, are called the **integers**.

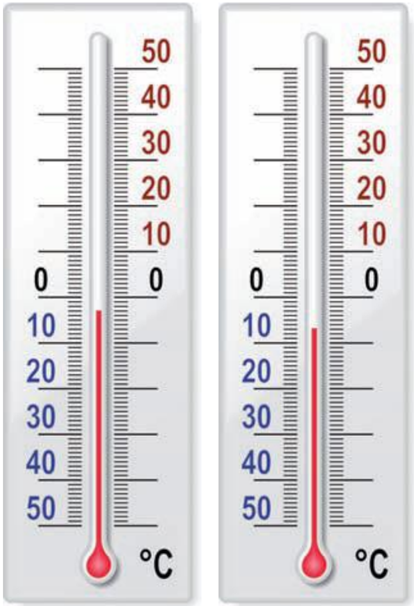
This number line shows negative and positive whole numbers and zero.



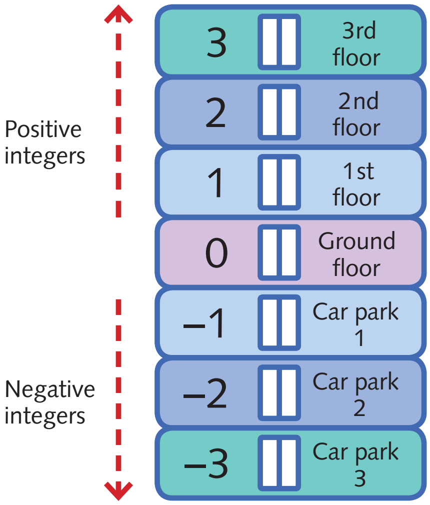
Every integer except zero has a symbol to show if it is positive (+) or negative (–). Since -0 is equal to $+0$, we usually write 0 without a symbol. A positive integer can be written with or without the plus sign, so +3 is the same as 3.

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

We can use integers to describe temperatures. For example, Mick checked the thermometer at the back door of the ski chalet. It showed 5°C. Overnight, the temperature dropped 8 degrees. In the morning the temperature was –3°C. The next night, the temperature was –7°C, which was even colder.



The diagram below shows a building that has 3 floors above the ground floor and 3 car parks below the ground. It is like a vertical number line.



The ground floor is zero. Imagine that you are in a lift going down from the first floor to car park 3.

This sequence of integers going from 1 to –3 is: 1, 0, –1, –2, –3.

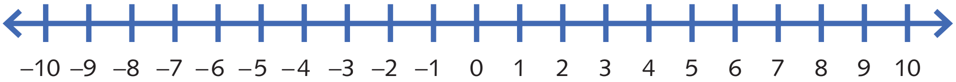
If you park your car in car park 3 and go up to the third floor, you go through this sequence: –3, –2, –1, 0, 1, 2, 3.

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

We can decide whether an integer is larger than another just as we did with whole numbers.

The same rules apply.

- Numbers get larger as we move to the right on a number line.
- Numbers get smaller as we move to the left on a number line.



Using these rules, we see that:

3 is less than 5

–1 is less than 0

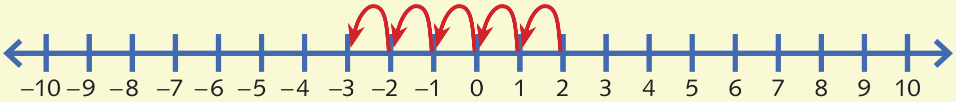
–8 is less than –4

Example 6

Write the number that is 5 less than 2. Use a number line to help you.

Solution

Start from 2 and take 5 steps along the number line to the left.



5 less than 2 is –3.

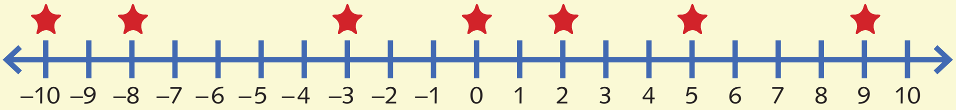
Example 7

Use a number line to place these integers in order, from smallest to largest.

–10, 9, –8, 5, –3, 0, 2

Solution

Mark the integers on the number line.



In order, the integers are: –10, –8, –3, 0, 2, 5, 9.

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

1D Whole class CONNECT, APPLY AND BUILD

1 Draw a ladder with each rung labelled from 9 to –9, as shown.

The rungs going up from zero are positive integers, and the rungs going down from zero are negative integers. Use the ladder to act out these situations.

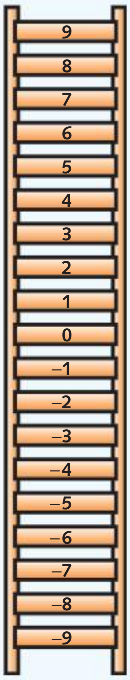
a Start from 5. Move up 4 rungs.
Which rung are you on?

b Start from 0. Move down 3 rungs.
Which rung are you on?

c Start from 3. Move down 10 rungs.
Which rung are you on?

d Start from –2. Move down 3 rungs.
Which rung are you on?

e Start from –3. Move up 5 rungs.
Which rung are you on?



2 Make a number ladder that goes from –100 to 100. Use the ladder and some clothes pegs to show how many steps it is from:

a 2 to –3 **b** 6 to 4 **c** –1 to 3 **d** 5 to –5

e –6 to –10 **f** –13 to –20 **g** –1 to –37 **h** 40 to –40

1D Individual

1 Draw a number line and mark these integers on it.

a –2, 4, –6 **b** 0, 3, 10, –10, –3

2 List the integers that are:

a smaller than 2 and larger than –3 **b** larger than –10 and smaller than –2

3 Put these integers in order, smallest to largest.

a –1, 3, –5, 2, 6 **b** 18, –23, –34, 47, 99 **c** 434, –23, –57, 21, 0

4 Put these integers in order, largest to smallest.

a 9, –3, 4, –2, 0 **b** –111, 99, –56, –99, 56 **c** –77, 136, 0, 3, –2

5 This sequence is going down by twos: 4, 2, 0, –2, –4, –6. Write the next 5 integers in the sequence.

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

1E Review questions

1 Write these in numbers.

a Forty-two thousand, seven hundred and eighteen

b Two hundred and ten thousand, five hundred and sixty-three

c Eleven million, nine hundred and sixty-seven thousand, three hundred and twenty-four

2 Write these numbers in words.

a 75 **b** 102 **c** 4962

d 23 178 **e** 914 207 **f** 4 769 052

3 Write the value of the highlighted digit.

a 604 **b** 2009 **c** 79 417

d 359 267 **e** 15 081 925 **f** 641 283 996

4 Copy and complete this place-value chart.

Number	Place-value parts							
	Tens of millions	Millions	Hundreds of thousands	Tens of thousands	Thousands	Hundreds	Tens	Ones
3208								
56 849								
213 054								
609 170								
75 451 821								

5 Draw a number line with 10, 20 and 30 marked on it. Use large dots to mark each multiple of 5 from 15 to 25.

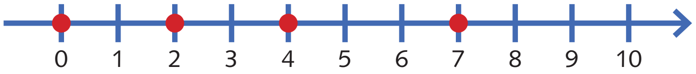
6 Draw a number line with 0 and 100 marked on it. Use large dots to mark 20, 75 and 95.

7 Draw a number line with 0 and 1000 marked on it. Use large dots to mark 300, 700, 860 and 975.

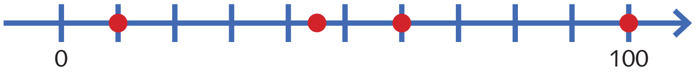
8 Draw a number line with 0 and 10 000 marked on it. Use a large dot to mark 1111, 3500 and 9000.

9 Write the numbers shown by the red dots on these number lines.

a

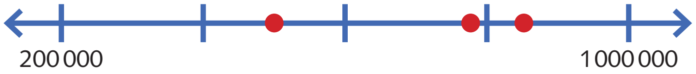


b



c

d



10 Write the first 10 numbers in each counting sequence.

a Count forwards by ones, starting at 691.

b Count forwards by threes, starting at 89.

c Count forwards by fours, starting at 1013.

d Count backwards by twos, starting at 79.

e Count backwards by fives, starting at 421.

f Count backwards by sixes, starting at 5347.

g Count forwards by eights, starting at 87.

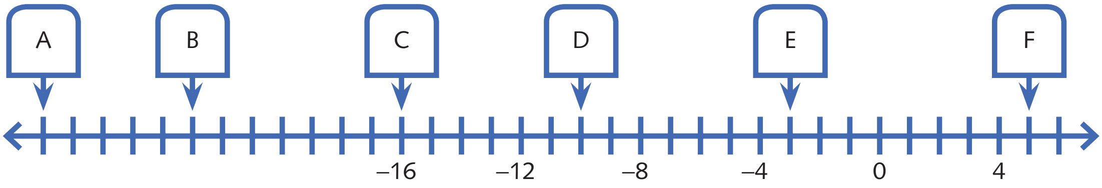
h Count forwards by nines, starting at 572.

i Count forwards by elevens, starting at 634.

j Count backwards by sevens, starting at 2701.

k Count backwards by twelves, starting at 3659.

11 Each letter on this number line represents a number. Write the numbers.



12 Draw a number line and mark these integers on it.
–17, 5, –2, –21, 7, –3, 2

13 List the integers that are:

a between 0 and –5

b smaller than 3 but larger than –10

c smaller than –22 but larger than –33

14 Write these integers in order, smallest to largest.

3, –6, 8, –9, 0, –33, 133, –132

15 Write these integers in order, largest to smallest.

89, – 2, – 66, – 101, – 4, 45, 6

CHAPTER 2 Preparation

Useful skills for this topic:

- an understanding of the place value of numbers to 1000 000
- knowledge of single-digit addition and subtraction facts
- the ability to use a standard algorithm for addition and for subtraction.

KICK OFF

Write the number that is:

a 16 more than 31

b 16 less than 48

c 17 more than 12

d 27 less than 99

e 14 more than 27

f 24 less than 53

g 25 more than 28

h 16 less than 43

i 35 more than 27

j 48 less than 91

Show what you know

1 Calculate these mentally.

a 16 + 9

b 24 + 8

c 38 + 6

d 41 + 17

e 34 + 23

f 32 + 28

g 47 + 19

h 65 + 28

i 18 – 12

j 27 – 13

k 28 – 17

l 45 – 26

m 31 – 14

n 32 – 27

o 43 – 25

CHAPTER

2

Number and Algebra

Addition and subtraction

Addition and subtraction are two of the most frequently used mathematical operations. We use addition and subtraction every day, and not just at school.

When we use addition to work out the **sum** of two or more numbers, this tells us the **total** amount or number of things we have.

For example, during a soccer game, Handel scores four goals, Uyen scores two goals, and Brodie scores nine goals. The sum of the goals is 4 + 2 + 9 = 15 goals in total.

We can use subtraction to find out how many items are left when we **take away** one number of items from another.

For example, if 44 cars were parked in a supermarket parking lot, and 14 of them were driven away, there would be a total of 44 – 14 = 30. We are left with 30, which is the **difference** between 14 and 44.

We can also use subtraction to find differences such as how much taller one person is than another, or **how much more** money we need to buy a $10 item if we only have $7.

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

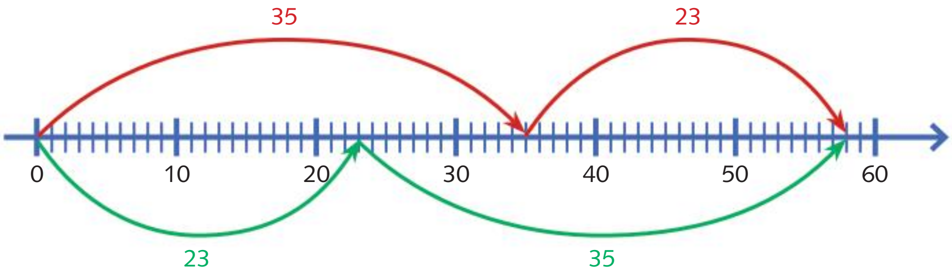
2A Mental strategies for addition

A well-chosen mental strategy allows you to do addition more quickly in your head than using pencil and paper.

Order does not matter for addition

The order in which we do addition does not matter. The answer will be the same no matter which order we use. For example:

$35 + 23 = 58$



$23 + 35 = 58$

This strategy works well for three or more numbers. Carefully choosing the order in which you add the numbers can save a lot of time.

For example:

$$\begin{aligned} 19 + 26 + 11 &= 19 + 11 + 26 \\ &= 30 + 26 \\ &= 56 \end{aligned}$$

Make up to a ten

Take some of one number and add it to the other number to make up a ten or a multiple of 10.

Example 1

Add 34 and 26.

Solution

$$\begin{aligned} 34 + 26 &= 34 + 6 + 20 \\ &= 40 + 20 \\ &= 60 \end{aligned}$$

or

$$\begin{aligned} 34 + 26 &= 30 + 4 + 26 \\ &= 30 + 30 \\ &= 60 \end{aligned}$$

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

Add from the left

When we add from the left, we start with the largest parts first. Add the digits with the same place value, starting from the left.

Example 2

a Peter has 87 marbles. Ali has 55 marbles. How many marbles do they have in total?

b Lisa has 3428 cards in her collection. Her brother Michael has 2236 cards. Find the total number of cards.

Solution

a $87 + 55 = 80 + 50 + 7 + 5$ (Add tens, then ones)

$$\begin{aligned} 87 + 55 &= 80 + 50 + 7 + 5 \\ &= 130 + 7 + 5 \\ &= 142 \end{aligned}$$

b $3428 + 2236 = 3000 + 2000 + 400 + 200 + 20 + 30 + 8 + 6$

(Add thousands, then hundreds, then tens, then ones)

$$\begin{aligned} &= 5000 + 400 + 200 + 20 + 30 + 8 + 6 \\ &= 5600 + 20 + 30 + 8 + 6 \\ &= 5650 + 8 + 6 \\ &= 5664 \end{aligned}$$

Compensation

Add more than is needed, then subtract the extra you added on.

Example 3

Add 35 and 28.

Solution

$$\begin{aligned} 35 + 28 &= 35 + 30 - 2 \\ &= 65 - 2 \\ &= 63 \end{aligned}$$

(Adding 28 is the same as adding 30 and subtracting 2.)

Practise your mental strategies to find out which strategies work best for you. You will need to vary your strategies to suit the numbers you are working with.

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

2A Whole class CONNECT, APPLY AND BUILD

1 Add 19 to each number by adding 20, then taking 1 away.

a 16 **b** 51 **c** 31 **d** 47 **e** 68 **f** 89

2 Complete each addition mentally. Discuss the different strategies for each addition. Is there a best one? Why is it best?

a 26 + 85 **b** 17 + 64 **c** 58 + 29 **d** 343 + 57
e 645 + 327 **f** 17 + 88 **g** 75 + 38 **h** 54 + 66

3 Work in pairs. You will need 2 dice. Player 1 rolls the dice to create a 2-digit number. Player 2 then mentally adds a number from the table on the right to the number rolled. For example:

5 and 3
That makes 53.
Add 5 from the table on the right.
53 + 5 = 58

5	31
4	27
6	48
8	123
7	446
9	592

2A Individual

1 Complete these additions.

a 23 + 17 **b** 46 + 104 **c** 18 + 92 **d** 107 + 23
e 28 + 55 **f** 29 + 43 **g** 96 + 96 **h** 123 + 88

2 What number do you need to add to each of these to make a total of 50?

a 15 **b** 18 **c** 13 **d** 9
e 0 **f** 26 **g** 34 **h** 29

3 Georgia needs to save $100 for a DVD boxed set. How much more does she need to save if she already has:

a $36? **b** $18? **c** $74? **d** $35?
e $65? **f** $88? **g** $43? **h** $59?

4 Complete these additions mentally. Choose the best strategy.

a 16 + 35 **b** 24 + 27 **c** 15 + 73 **d** 41 + 18

e 26 + 74 **f** 38 + 25 **g** 54 + 27 **h** 35 + 37

i 64 + 18 **j** 26 + 57 **k** 38 + 55 **l** 64 + 27

m 45 + 77 **n** 66 + 68 **o** 86 + 97 **p** 1043 + 87

5 There are 68 boys and 77 girls in Year 6 at Happy Hills School. How many children are there in Year 6?

6 The Village Belle Pie Shop cooked 87 meat pies and 79 chicken pies. How many pies did it cook altogether?

7 Farmer Kim looks after 93 sheep, 75 cows and 57 goats. How many animals does she look after?

8 Simon's grandfather loves gardening. He has 65 pots of roses, 37 pots of dahlias and 43 pots of petunias. How many plants does he have in total?

9 **a** How many days are there in September, October and November combined?

b Is your answer to part **a** more or less than the total number of days in March, April and May? By how much?

10 This is called a 'magic square'. Each row, each column and each diagonal should add up to the same total.

19		17
	16	
		13

To complete this magic square, first add the diagonal. It totals 48, so every row, column and diagonal must total 48. Now look at the right column. 17 and 13 makes 30, so the missing number must be 18. The top row currently adds to 36, so the missing number is 12. Now work out the other numbers.

Copy these magic squares, then write the missing numbers.

a

		23
	22	
21		19

b

23		
19	24	17

c

28		24
	22	
20		

d

30		26
	24	
		18

e

52	24	44
	40	

f

130	123	128
126		

11 Make four magic squares of your own. See if your partner can find the solution to each of your magic squares.

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

2B The standard addition algorithm

The addition algorithm is like a recipe for doing addition. An algorithm works most efficiently if it uses a small number of steps that apply in all situations.

If we want to add 37, 48 and 56, we can use the addition algorithm.

	Hundreds	Tens	Ones
		3	7
		4	8
+		5	6

Set out the numbers one under the other, according to their place value.

	Hundreds	Tens	Ones
		3	7
		4	8
+		5_2	6
			1

Start with the ones digits.

We say, '7 ones plus 8 ones plus 6 ones is 21 ones. That is the same as 2 tens and 1 one'.

Write the '1' in the ones column and carry the 2 tens to the tens column.

	Hundreds	Tens	Ones
		3	7
		4	8
+		5_2	6
	1	4	1

Now add the tens digits.

We say, '3 tens plus 4 tens plus 5 tens, plus the 2 tens carried from before, is 14 tens. That is the same as 1 hundred and 4 tens'.

Write the '4' in the tens column and, as there are no hundreds to add, write the '1' in the hundreds column.

$37 + 48 + 56 = 141$

The sum of 37, 48 and 56 is 141.

The addition algorithm can be extended to add numbers of any size. All you need to do is add the columns, starting from the right and carrying when needed.

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

Example 4

a Find the sum of 345 and 267.
b Find the sum of 3526, 988, 469 and 85.

Solution

a

$$\begin{array}{cccc} & 3 & 4 & 5 \\ + & 2_1 & 6_1 & 7 \\ \hline & 6 & 1 & 2 \\ \hline \end{array}$$

(Remember to put the digits in the correct place-value columns.)

The sum of 345 and 267 is 612.

b

$$\begin{array}{ccccc} & 3 & 5 & 2 & 6 \\ & & 9 & 8 & 8 \\ & & 4 & 6 & 9 \\ + & {}_2 & {}_2 & 8_2 & 5 \\ \hline & 5 & 0 & 6 & 8 \\ \hline \end{array}$$

(Add the ones, carrying 2 tens into the tens column. Add the tens, including the carried tens from before. Add the hundreds, carrying where necessary. Then add the thousands.)

The sum is 5068.

Remember

The addition algorithm can be used to accurately find the sum of two or more numbers.

2B Individual

1 Use the addition algorithm to do these calculations.

These involve adding a single-digit number to another number:

a 187 + 9 **b** 568 + 7 **c** 896 + 8 **d** 1029 + 6

These have no carrying:

e 133 + 145 **f** 712 + 235 **g** 316 + 583 **h** 444 + 333

These involve carrying from the ones to the tens:

i 547 + 126 **j** 825 + 149 **k** 607 + 357 **l** 715 + 138

These carry from the ones to the tens, and from the tens to the hundreds:

m 887 + 356 **n** 485 + 376 **o** 272 + 388 **p** 647 + 767

2 In their first three matches, the Everton Eagles basketball team scored 87 points, 67 points and 74 points.

What was the total number of points they scored?

3 In their last four matches, the Smithton Rangers netball team scored 47 goals, 38 goals, 52 goals and 49 goals.

What was the total number of goals scored?

4 Hamish bought a 1964 Mini Minor for $2775. He spent $875 on repairs and $388 on new tyres. How much did he spend altogether?

5 The Riverton Newsagents sold 357 newspapers on Monday, 289 newspapers on Tuesday, 336 newspapers on Wednesday and 427 newspapers on Thursday. What was the total number of newspapers sold?

6 Andrew helped with the stocktake at the Washer Company. He had to count the number of washers in five different boxes. He counted 1455, 1327, 1604, 1298 and 1576 washers in the five boxes.

He then added them together for a grand total of 7030.

Was Andrew's addition correct or not? If not, what should it be?

7 **a** Which number is 3775 more than 8370?

b What is the sum of 2613 and 3621?

c Add 23 to 6985.

d Write the number that is 1805 more than 99.

e Which number is added to 6937 to make 10 000?

f What is the sum of 693, 271, 596 and 703?

8 Complete these additions.

a 2289 + 358 + 673 + 88

b 522 + 988 + 3766 + 55

c 2069 + 544 + 3088 + 776

d 2289 + 405 + 38 + 966

e 27 + 655 + 4328 + 998

f 622 + 87 + 6558 + 99

9 The missing digits are marked with a ★.

Write the missing digits to make each addition correct.

a
$$\begin{array}{rccc} & 3 & 4 & \bigstar \\ & 3 & \bigstar & 8 \\ + & \bigstar & 5 & 5 \\ \hline & 9 & 8 & 9 \end{array}$$

b
$$\begin{array}{rccc} & 2 & \bigstar & 9 \\ & 1 & 5 & \bigstar \\ + & 3 & 8 & 3 \\ \hline & \bigstar & 1 & 6 \end{array}$$

c
$$\begin{array}{rcccc} & 2 & 3 & \bigstar & 8 \\ & 5 & \bigstar & 9 & 4 \\ + & \bigstar & 7 & 7 & \bigstar \\ \hline & 9 & 3 & 3 & 8 \end{array}$$

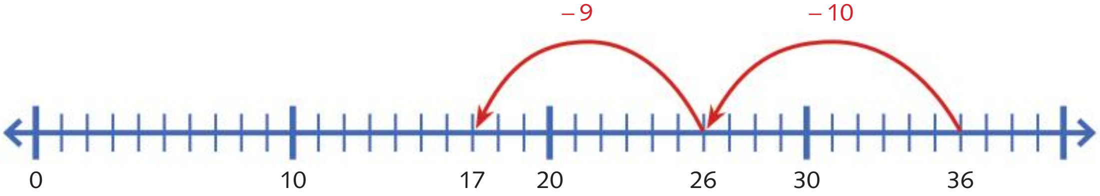
d
$$\begin{array}{rcccc} & 2 & 7 & \bigstar & 5 \\ & \bigstar & 6 & 9 & \bigstar \\ + & 2 & \bigstar & 8 & 7 \\ \hline & 9 & 2 & 9 & 1 \end{array}$$

2C Mental strategies for subtraction

When we subtract, we are either 'taking away' one number from another, or 'adding on' to get from one number to another.

Taking away

A subtraction such as 36 – 19 means starting at 36 and going back 19 steps. On a number line it looks like this.

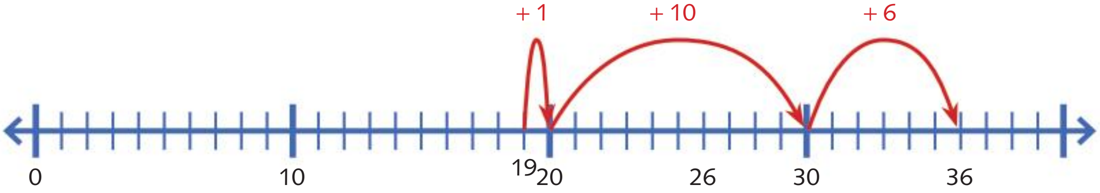


We go back 10 steps to get to 26, then take a further 9 steps and arrive at 17:

36 – 19 = 17

Adding on

Using the adding-on strategy to calculate 36 – 19 means asking 'what do I add to 19 to get to 36?'



One step gets us to 20, then 10 more to 30 and 6 more to 36.

1 + 10 + 6 = 17, so we have to add on 17.

Subtract a bit at a time

Break the number you are taking away into separate pieces and take away one piece at a time.

Example 5

Subtract 38 from 95.

Solution

$$\begin{aligned} 95 - 38 &= 95 - 30 - 8 \\ &= 65 - 8 \\ &= 57 \end{aligned}$$

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

Build up to the larger number

Add on to the smaller number to build up to the larger number. Keep track of what you have added.

Example 6

a 82 subtract 35
b 643 – 285

Solution

a 82 – 35
35 + 5 = 40 (5 has been added.)
40 + 40 = 80 (A total of 45 has been added.)
80 + 2 = 82 (A total of 47 has been added.)
82 – 35 = 47

b 643 – 285
285 + 15 = 300 (15 has been added.)
300 + 300 = 600 (A total of 315 has been added.)
600 + 43 = 643 (A total of 358 has been added.)
643 – 285 = 358

Add the same to both numbers

Adding the same number to both numbers does not change the difference between them.

Example 7

Subtract 47 from 93 by adding the same amount to both numbers.

Solution

93 – 47 = 96 – 50 (Add 3 to both numbers.)
= 46

2C Individual

1 Calculate the difference between:

a 64 and 27

b 45 and 28

c 39 and 83

d 322 and 128

2 Calculate these mentally.

a 72 – 38

b 80 – 29

c 87 – 49

d 344 – 229

e 401 – 396

3 Use the 'build up to a larger number' strategy to calculate these subtractions.

a 63 – 37

b 82 – 45

c 71 – 38

d 462 – 178

e 444 – 199

f 279 – 86

4 Flora's Flowers started the day with 125 bunches of flowers.

Flora sold 87 bunches. How many bunches were left?

5 Mario's family set off from Townsville on a journey of 265 kilometres.

They drove 148 kilometres before stopping.

How far did they have left to drive?

6 Kate the electrician had 227 metres of cable left on a roll. Kate used 89 metres on her next job. How much cable was there left on the roll?

7 Harcourt Electrical was advertising a television for $425. If it cut the price by $57, what would be the new price?

8 **a** What is the difference between 644 and 397?

b What is 2047 take away 1804?

c Calculate 1743 less than 2962.

9 **a** Write these numbers: 98, 97, 96, 95, 94, 93, 92, 91.

Now reverse the digits of each number and subtract the smaller number from the larger number; for example, 98 – 89 = 9.

Do this for each number.

b What do you notice about all of your answers to **a**?

c Try this with other 2-digit numbers. Do you get the same results?

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

2D Subtraction algorithms

In the subtraction algorithms, we work from right to left. We subtract the digits one column at a time. Here are two different subtraction algorithms.

Trading or decomposition

'Trading' is based on the idea that 10 ones is the same as 1 ten; that 10 tens is the same as 1 hundred; and so on.

	Hundreds	Tens	Ones
	3	$^{3}\cancel{4}$	$^{1}2$
–	1	5	9
			3

Start in the ones column. There are not enough ones to take 9 away.

Trade 1 ten for 10 ones in the top number. Cross out the 4 and write a 3 to show that there are 3 tens left. Write a 1 to the left of the 2 to show that there are now 12 ones.

12 ones take away 9 ones = 3 ones.

	Hundreds	Tens	Ones
	$^{2}\cancel{3}$	$^{13}\cancel{4}$	$^{1}2$
–	1	5	9
		8	3

Now work in the tens column.

There are not enough tens to take 5 tens away. Trade 1 hundred for 10 tens. Cross out the 3 and write a 2 to show that there are 2 hundreds left. Now write a 1 in the tens column to show that there are now 13 tens.

13 tens take away 5 tens = 8 tens.

	Hundreds	Tens	Ones
	$^{2}\cancel{3}$	$^{13}\cancel{4}$	$^{1}2$
–	1	5	9
	1	8	3

Now for the hundreds column.

2 hundreds take away
1 hundred = 1 hundred.

342 – 159 = 183

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

Equal addition (borrow and pay back)

This uses the idea that if you add the same amount to two numbers, the difference between them the numbers remains the same.

For example, find the difference between 342 and 159.

	Hundreds	Tens	Ones
	3	4	$^{1}2$
–	1	5_1	9
			3

Start with the ones digits. There are not enough ones to take 9 away.

Add 10 to both numbers by adding 10 ones to the 2 in the ones column and 1 ten to the 5 in the tens column.

12 ones take away 9 ones = 3 ones.

	Hundreds	Tens	Ones
	3	$^{1}4$	$^{1}2$
–	1_1	5_1	9
		8	3

Now work in the tens column.
There are not enough tens to take
5 + 1 = 6 tens away.

Add 100 to both numbers by adding 10 tens to the 4 in the tens column and 1 hundred to the 1 in the hundreds column.

14 tens take away 6 tens = 8 tens.

	Hundreds	Tens	Ones
	3	$^{1}4$	$^{1}2$
–	1_1	5_1	9
	1	8	3

Now work in the hundreds column.

3 hundreds take away 2 hundreds
(1 + the 1 added before)
= 1 hundred.

342 – 159 = 183

Example 8

Find the difference between 6043 and 2796.

Solution

Trading

6043 – 2796

$$\begin{array}{rcccc} & {}^{5}\cancel{6} & {}^{9}\cancel{0} & {}^{13}\cancel{4} & {}^{1}3 \\ - & 2 & 7 & 9 & 6 \\ \hline & 3 & 2 & 4 & 7 \\ \hline \end{array}$$

Equal addition

6043 – 2796

$$\begin{array}{rcccc} & 6 & {}^{1}0 & {}^{1}4 & {}^{1}3 \\ - & 2_1 & 7_1 & 9_1 & 6 \\ \hline & 3 & 2 & 4 & 7 \\ \hline \end{array}$$

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7

2D Whole class CONNECT, APPLY AND BUILD

1 Work in pairs. Person 1 does subtractions **a–d** and Person 2 checks the answers. Then Person 2 does subtractions **e–h** and Person 1 checks the answers.

a 7383 – 4269 **b** 3801 – 2567 **c** 5321 – 3784 **d** 7004 – 3855
e 8733 – 4629 **f** 8310 – 5627 **g** 5132 – 4873 **h** 4007 – 3585

2 In groups, take turns to roll four dice to get a 4-digit number. For example, if you rolled 2, 4, 6 and 3, two of the numbers you could make are 6342 and 2634.

Each person starts with 100 000 points and takes turns to subtract the number from their total. The first person below 50 000 points wins the game.

2D Individual

1 Calculate these subtractions.

a 834 – 475 **b** 711 – 487 **c** 904 – 266 **d** 632 – 348

2 Work out the answers to these.

a 8021 – 2583 **b** 3666 – 2257 **c** 5444 – 2748
d 5007 – 2398 **e** 7002 – 3206 **f** 8020 – 6434

3 Calculate these subtractions.

a 7024 – 4222 **b** 4831 – 2875 **c** 2222 – 1999

4 The Orange Company took 8435 cases of oranges to the market. It sold 3768 cases of oranges. How many cases were left?

5 A library has 5043 fiction books and 2706 non-fiction books. How many more fiction books are there than non-fiction books?

6 The Lovely Egg Company normally sells 9750 eggs each week. It has already sold 7995 eggs this week. How many more eggs does it need to sell?

7 Mr McDuff earned $56 044 last year. He saved $8675. How much did he spend?

8 There are 7746 sheep on Helen's farm. Helen sells 4975 sheep. How many are left?

9 Adams' Apples has a contract to supply 9250 apples to the market. It has already picked 3878 apples. How many more does it need to pick?

10 The Fabulous Fish Farm had 12 125 small fish in a large pond. It sold 5850 fish. How many were left?

11 Download **BLM 2** 'Addition and subtraction grids' from the Interactive Textbook and complete.

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

2E Working with larger numbers

Both the addition and subtraction algorithms can be extended to larger numbers. You are only ever dealing with one column of single digits at a time.

Addition

Start at the right-hand side and add each column in turn, moving from the right to the left. Remember to record any carry numbers in the next column.

Example 9

Find the sum of 53 482, 48 677, 21 953 and 30 945.

Solution

$$
\begin{array}{cccccc}
 & 5 & 3 & 4 & 8 & 2 \\
 & 4 & 8 & 6 & 7 & 7 \\
 & 2 & 1 & 9 & 5 & 3 \\
+ & 3_1 & 0_3 & 9_2 & 4_1 & 5 \\
\hline
1 & 5 & 5 & 0 & 5 & 7 \\
\hline
\end{array}
$$

To add whole numbers of different lengths, line them up according to their place value. The order you write them in does not matter. As long as the digits and the carry numbers are in the correct column, and your addition is accurate, you will get the right answer.

Example 10

Find the sum of 7, 43 468, 62, 6504 and 793.

Solution

$$
\begin{array}{cccccc}
 & 4 & 3 & 4 & 6 & 8 \\
 & & 6 & 5 & 0 & 4 \\
 & & & 7 & 9 & 3 \\
 & & & & 6 & 2 \\
+ & {}_1 & {}_1 & {}_2 & {}_2 & 7 \\
\hline
 & 5 & 0 & 8 & 3 & 4 \\
\hline
\end{array}
$$

Subtraction

Write the numbers one under the other, according to their place value. The number to be subtracted goes underneath. Start at the right-hand side and subtract each column in turn, moving from the right to the left. Trade wherever needed.

Example 11

Find the difference between 70 204 and 31 627.

Solution

Use trading or equal addition to find 70 204 – 31 627 .

Trading

70 204 – 31 627

$$\begin{array}{cccccc} & {}^{6}\cancel{7} & {}^{9}\cancel{0} & {}^{11}\cancel{2} & {}^{9}\cancel{0} & {}^{1}4 \\ - & 3 & 1 & 6 & 2 & 7 \\ \hline & 3 & 8 & 5 & 7 & 7 \\ \hline \end{array}$$

Equal addition

70 204 – 31 627

$$\begin{array}{cccccc} & 7 & {}^{1}0 & {}^{1}2 & {}^{1}0 & {}^{1}4 \\ - & 3_{1} & 1_{1} & 6_{1} & 2_{1} & 7 \\ \hline & 3 & 8 & 5 & 7 & 7 \\ \hline \end{array}$$

The difference between 70 204 and 31 627 is 38 577.

2E Individual

1 Calculate:

a 32 264 + 25 308 **b** 17 755 + 26 426 **c** 29 216 + 13 278

d 45 623 – 32 458 **e** 66 009 – 35 228 **f** 91 334 – 48 675

2 Complete these additions.

a 43 600 + 65 + 6897 + 378 **b** 3801 + 66 224 + 89

c 55 214 + 899 **d** 276 + 88 + 46 354 + 4683

e 3009 + 41 355 + 274 + 67

3 Calculate these subtractions.

a 36 221 – 18 365 **b** 40 832 – 26 338 **c** 54 361 – 28 979

d 17 055 – 9648 **e** 29 342 – 4366

4 Use the algorithms to calculate these money amounts.

a \$3266.75 + \$2845.65 **b** \$5504.30 + \$4385.85

c \$4423.20 – \$1758.95 **d** \$8000.00 – \$3365.55

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

5 In the 2006 census, the population of these Australian cities was as follows.

City	State or territory	Population
Albany	WA	31 981
Alice Springs	NT	28 178
Bendigo	Vic	93 073
Dubbo	NSW	39 277
Launceston	TAS	60 833
Mount Isa	QLD	21 755
Whyalla	SA	24 152

a How many more people lived in Launceston than Alice Springs?

b What was the difference in population between Mount Isa and Bendigo?

c What was the total number of people living in Albany, Dubbo and Whyalla in 2006?

d How many fewer people lived in Whyalla than in Dubbo?

e What was the total population of all the towns listed in the table in 2006?

f How many short of 300 000 is your answer to part **e**?

6 **a** Take 19 748 from 33 925.

b Subtract 45 968 from 100 000.

c Find the difference between 1 795 936 and 3 857 339.

d Write the number that is 142 587 937 more than 372 959 475.

7 Add ten thousand four hundred and fifty-six to twenty-three thousand and eighty-eight.

8 Find the sum of 28 726, 365, 39 248 and 5867.

9 Tamara's brother has exactly fifty thousand dollars in his bank account. If he buys a new car for $36 895 and pays for it out of his bank account, how much will be left in his account?

10 In 1975, George's father paid $28 895 for a flat. In 2000, the flat was valued at $397 000. How much did the flat go up in value during the 25 years?

11 The Mathsy Theatre has 13 seats in the first row, 15 seats in the second row, 17 seats in the third row, and so on. How many seats are in the theatre if there are 15 rows in all?

12 A spider caught 175 flies in her web in one week. Each day she caught 7 more flies than she did the day before. How many flies did the spider catch on each individual day?

13 For which numbers between 1 and 209 do the digits of the number add to 8?

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

2F Review questions

1 Copy and complete these addition and subtraction tables.

+	15	35	29	17	53
5					
9					
2					
7					
3					
4					
6					
8					

–	65	72	53	81	64
14					
26					
18					
35					
19					
22					
17					
23					

2 What number makes a total of 50 when added to:

a 14? **b** 17? **c** 11? **d** 8?

3 Tom needs to save $100. How much more does he need to save if he has:

a $16? **b** $27? **c** $61? **d** $85?

4 Calculate these additions.

a 27 + 14 **b** 13 + 39 **c** 62 + 29 **d** 73 + 19

e 46 + 84 **f** 54 + 47 **g** 26 + 37 **h** 43 + 59

i 66 + 19 **j** 36 + 57 **k** 58 + 35 **l** 76 + 78

5 There are 47 boys and 38 girls who want to play Newcomb ball for interschool sport. How many children, in total, want to play Newcomb ball?

6 In a magic square, each row, each column and each diagonal should add up to the same total. Work out the numbers missing from each magic square.

a

23	17	26
		21

b

	20	
17	22	21

c

		15
	16	
17		19

d

29		21
	24	
27		

e

55		
	40	
39		25

f

129		
130	126	125

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

7 What number is 4967 more than 7084?

8 Complete these additions.

a 4375 + 619 + 241 + 98 **b** 643 + 879 + 4277 + 67

9 The missing digits are marked below with a ★.

Write the missing digits to make each addition correct.

a
$$\begin{array}{cccc} & 3 & 4 & \star \\ & 2 & \star & 7 \\ + & \star & 6 & 5 \\ \hline & 6 & 9 & 8 \\ \hline \end{array}$$

b
$$\begin{array}{cccc} & 1 & \star & 9 \\ & 2 & 8 & \star \\ + & 4 & 7 & 4 \\ \hline & \star & 3 & 7 \\ \hline \end{array}$$

c
$$\begin{array}{ccccc} & 2 & 3 & \star & 9 \\ & 4 & \star & 9 & 7 \\ + & \star & 7 & 7 & \star \\ \hline & 9 & 0 & 4 & 6 \\ \hline \end{array}$$

d
$$\begin{array}{ccccc} & 4 & 0 & 8 & \star \\ & 1 & \star & 7 & 6 \\ + & \star & 9 & \star & 7 \\ \hline & 8 & 6 & 7 & 3 \\ \hline \end{array}$$

10 Find the difference between these pairs of numbers.

a 54 and 17 **b** 65 and 39 **c** 77 and 28 **d** 334 and 136

11 Mentally calculate these subtractions.

a 83 – 28 **b** 71 – 39 **c** 95 – 59 **d** 682 – 327

12 Use the 'adding on' method to do these subtractions.

a 85 – 49 **b** 54 – 37 **c** 81 – 19 **d** 524 – 287 **e** 655 – 488

13 The Newtown newsagents had 237 newspapers delivered in the morning. They sold 98 newspapers. How many newspapers did they have left?

14 Helen wants to drive the 873 kilometres from Melbourne to Sydney in one day. She stopped for lunch after driving 467 kilometres. How far does she have left to drive?

15 A mobile phone was priced at $187. If the price was cut by $29, what was the new sale price?

16 What is the difference between 691 and 487?

17 Stella has a job picking oranges. Last week, she picked 9387 oranges. So far this week she has picked 4629 oranges. How many oranges does she need to pick to equal last week's number?

18 Trevor earned $7279 from his wheat harvest in the first week and $6825 in the second week. How much less did Trevor earn in the second week?

19 Calculate:

a 47 132 + 31 619 **b** 28 693 + 52 728 **c** 15 846 + 23 375

d 38 157 – 22 968 **e** 71 070 – 43 181 **f** 105 634 – 73 745

20 Use the algorithms to calculate these amounts of money.

a $5379.45 + $1947.75 **b** $6207.60 + $2782.95

c $4526.20 – $3764.85 **d** $7000.00 – $5279.15

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

CHAPTER 3 Preparation

Useful skills for this topic:

- quick recall of multiplication facts to 12 × 12.

KICK OFF

Number ladder

You will need long strips of paper, such as cash register rolls. Mark the numbers from 0 to 100 along the strip, making sure the marks are equally spaced.

Use pegs, stickers or coloured dots to mark the counting pattern for 3. Use stickers of a different colour to mark the counting pattern for 4. Which numbers are in both counting patterns? Why?

Show what you know

1 Write the first 10 numbers you say when you count from zero by:

a fives
b threes
c fours
d sevens
e eights
f nines
g twelves
h thirteens

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7

Cambridge University Press

CHAPTER

3

Number and Algebra

Multiples and factors

If you look at a group of items, you often see that they can be arranged in a number of different arrays. Even though they are arranged using a different number of rows and columns, the number of items stays the same.

For instance, 24 students are going to a class dress-up party. They line up at the classroom door in four rows, each with six students. This shows that 4 × 6 = 24.

The students could also line up in other ways. If they line up in three rows, each will have eight students, and so 3 × 8 = 24.

If they line up in two rows, each will have 12 students, and so 2 × 12 = 24.

They could even just stand in one line of 24 students, because 1 × 24 = 24.

All of these different arrangements come to the same total when we multiply the number of rows by the number of columns.

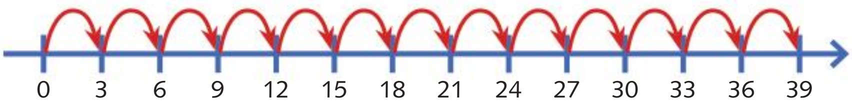
In this chapter we investigate different ways of thinking about the multiplication of whole numbers.

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

3A Multiples and products

Skip-counting in threes

If we skip-count in threes, we can mark the results on a number line.

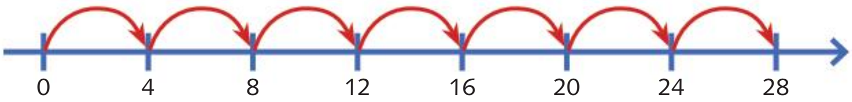


The numbers we get when we skip-count in threes are called the **multiples** of 3. Here are the multiples of 3 up to 45.

3 = 1 × 3	12 = 4 × 3	21 = 7 × 3	30 = 10 × 3	39 = 13 × 3
6 = 2 × 3	15 = 5 × 3	24 = 8 × 3	33 = 11 × 3	42 = 14 × 3
9 = 3 × 3	18 = 6 × 3	27 = 9 × 3	36 = 12 × 3	45 = 15 × 3

Multiples of 4

If we skip-count in fours, the numbers we get are called the multiples of 4.



Here are the multiples of 4 up to 36.

4 = 1 × 4	16 = 4 × 4	28 = 7 × 4
8 = 2 × 4	20 = 5 × 4	32 = 8 × 4
12 = 3 × 4	24 = 6 × 4	36 = 9 × 4

x	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10	11	12
2	0	2	4	6	8	10	12	14	16	18	20	22	24
3	0	3	6	9	12	15	18	21	24	27	30	33	36
4	0	4	8	12	16	20	24	28	32	36	42	44	48
5	0	5	10	15	20	25	30	35	40	45	50	55	60
6	0	6	12	18	24	30	36	42	48	54	60	66	72
7	0	7	14	21	28	35	42	49	56	63	70	77	84
8	0	8	16	24	32	40	48	56	64	72	80	88	96
9	0	9	18	27	36	45	54	63	72	81	90	99	108
10	0	10	20	30	40	50	60	70	80	90	100	110	120
11	0	11	22	33	44	55	66	77	88	99	110	121	132
12	0	12	24	36	48	60	72	84	96	108	120	132	144

The numbers shaded in green show the multiples of 4 up to 48.

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

If you know your multiplication tables, you will be able to write the first 12 multiples of any number between 1 and 12.

Example 1

Write the first 12 multiples of 6.

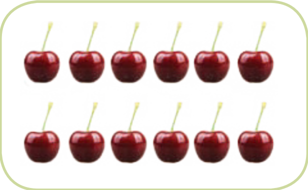
Solution

The multiples of 6 are the numbers we get when we skip-count in sixes:
6, 12, 18, 24, 30, 36, 42, 48, 54, 60, 66, 72

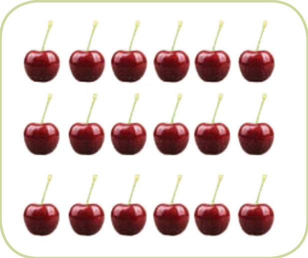
Multiples and arrays

We can also show multiples by drawing rectangular arrays.

$1 \times 6 = 6$ cherries, because there is 1 row of 6 cherries.



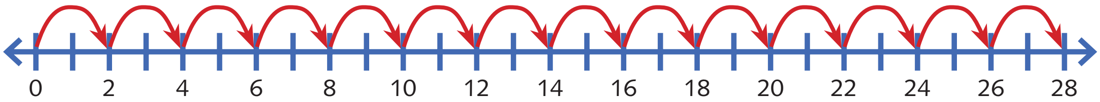
$2 \times 6 = 12$ cherries, because there are 2 rows, each with 6 cherries.



$3 \times 6 = 18$ cherries, because there are 3 rows, each with 6 cherries.

Odd and even numbers

When we count in twos on the number line, starting at 0, this is what we get.



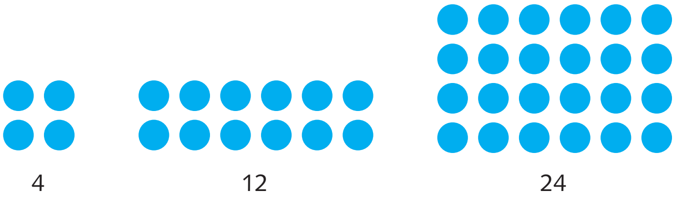
Look at the last digit of each number in the twos counting pattern.
The 0, 2, 4, 6, 8 pattern repeats in each group of five numbers.

Every multiple always of 2 ends in 0, 2, 4, 6 or 8.

The multiples of 2, including 0, are called **even numbers**.
Every even number ends in 0, 2, 4, 6 or 8.
Also, every number that ends in 0, 2, 4, 6 or 8 is even.

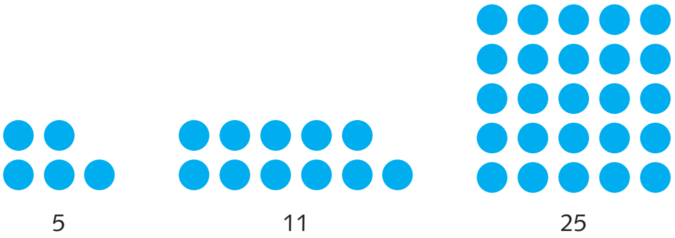
ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

Even numbers always make complete rectangular arrays.



A number that is not a multiple of 2 is called an **odd number**.
Every odd number ends in 1, 3, 5, 7 or 9.
Also, every number that ends in 1, 3, 5, 7 or 9 is an odd number.

Odd numbers do not make a complete rectangular array unless they are square numbers.



Multiples of 10

Starting at 0, the first few multiples of 10 are:

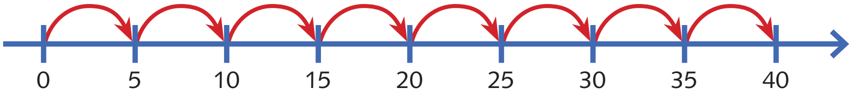
0, 10, 20, 30, 40, 50, 60, 70, 80, 90, 100, 110

They all end in 0.

Every multiple of 10 ends in 0.
Every number that ends in 0 is a multiple of 10.

Multiples of 5

When we count in fives on the number line, this is what we get.



Look at the last digit in each number. Can you see the pattern?
It repeats 0, 5, 0, 5, and so on.

Every multiple of 5 ends in 0 or 5.
Every number that ends in 0 or 5 is a multiple of 5.

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7

Cambridge University Press

Products

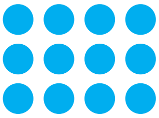
When we multiply 3 by 4, we get:

$$3 \times 4 = 12$$

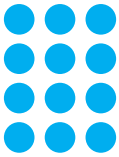
This is called the **product** of 3 and 4.

We say ‘the product of 3 and 4 is 12’.

When you are writing the product, you can use either 3×4 or 4×3. They both have the same result.



$3 \times 4 = 12$

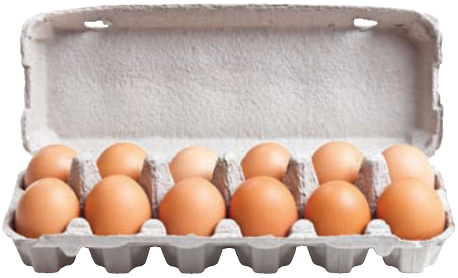


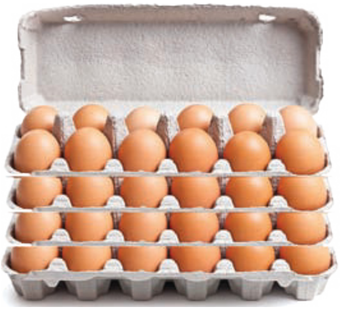
$4 \times 3 = 12$

Products of three numbers

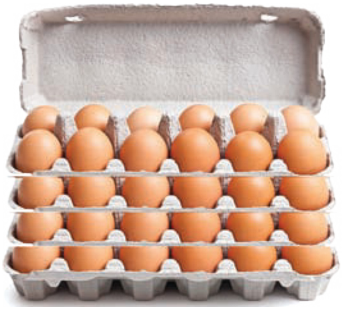
Eggs are usually sold in cartons of 12.

In the shop, or at the market, you sometimes see stacks of egg cartons.





4×12 eggs



4×12 eggs

Each stack has 4×12 eggs and there are 2 stacks.

The total number of eggs is ‘2 lots of 4×12’, or:

$$2 \times 4 \times 12 = 96 \text{ eggs}$$

We could stack the same egg cartons in another way.

We now have 4 lots of 2×12 eggs.

$$4 \times 2 \times 12 = 96 \text{ eggs}$$

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

3A Whole class CONNECT, APPLY AND BUILD

1 **Buzz**

Take turns to count by ones, going around the class. When you get to a number that is a multiple of the given number, say 'buzz'.

a Start at 0, buzz on multiples of 3.

b Start at 0, buzz on multiples of 7.

c Start at 0, buzz on multiples of 4.

d Start at 0, buzz on multiples of 11.

2 Take turns making up fun multiples of 2, 10 or 5. For example:

- the number nine eight seven six five four three two one zero (9 876 543 210) is a multiple of 10
- the number one two one two one two one two (12 121 212) is a multiple of 2
- the number four four four four four four four five (44 444 445) is a multiple of 5

3 Copy these sentences and write the missing words or numbers.

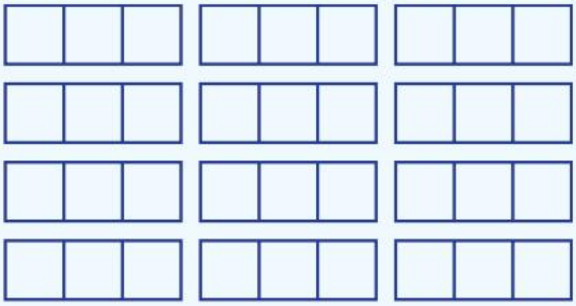
a The product of 3 and 8 is ________.

b $4 \times 7 = 28$. The product of ______ and ______ is 28.

c The product of 2 and ______ is 28.

d $6 \times 6 = 36$. The product of 6 and ______ is 36.

4 There are 36 squares in this diagram.



a Discuss how the diagram shows that:

$36 = 3 \times 12 = 3 \times 4 \times 3$

$36 = 4 \times 9 = 4 \times 3 \times 3$

$36 = 6 \times 6 = 6 \times 2 \times 3$

b Copy and shade the diagram to show 2 lots of 2×9.

c Copy and shade the diagram to show 2 lots of '2 lots of 3×3'.

5 Arrange 24 counters in an array like this.

Rearrange your counters to make an array that shows:

a 2 lots of 3×4 counters

b 3 lots of 2×4 counters

c 4 lots of 3×2 counters

d 2 lots of 2 lots of 2 lots of 3 counters

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

3A Individual

1 Write the first 14 multiples of 2.

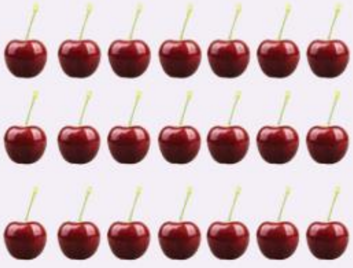
2 Copy and complete the first 12 multiples of 5.

Say '1 five is 5', '2 fives are 10', and so on, as you complete each one.
Check your answers by counting in fives.

a $1 \times 5 =$ ____ **b** $2 \times 5 =$ ____ **c** $3 \times 5 =$ ____ **d** $4 \times 5 =$ ____
e $5 \times 5 =$ ____ **f** $6 \times 5 =$ ____ **g** $7 \times 5 =$ ____ **h** $8 \times 5 =$ ____
i $9 \times 5 =$ ____ **j** $10 \times 5 =$ ____ **k** $11 \times 5 =$ ____ **l** $12 \times 5 =$ ____

3 Write the first 12 multiples of 7, without looking at a multiplication table. Skip-count by seven to check your answers.

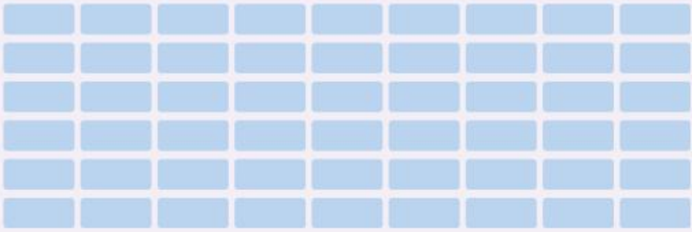
4 Copy and complete the sentence, without counting the fruits one by one.



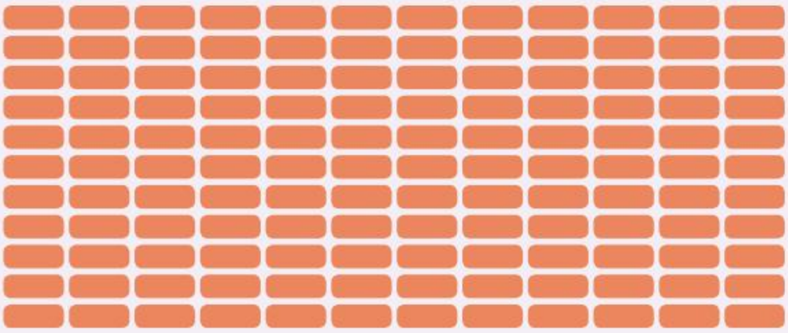
____ × ____ = ____ cherries

____ × ____ = ____ bananas

5 This is a stack of shipping containers. What is the total number of containers? Work this out mentally without counting one by one.



6 Michael built this wall in his garden. What is the total number of bricks Michael used? Work this out mentally without counting one brick at a time.



7 Solve these problems mentally.

a Yasmin buys Yummy Sourdrops in large jars of 582 sourdrops. She packs them into bags of 5 to sell in her milk bar. Can Yasmin pack all of her sourdrops in bags of 5 without having any left over? Explain your answer.

b The Terrific Toy Shop buys tennis balls in large sacks of 4760 tennis balls. It sells them in boxes of 5. Can the toy shop make up an even number of boxes of 5 with none left over? Explain your answer.

8 Copy and complete each statement by using the word 'even' or 'odd'.

a 332 is _______. **b** 1296 is _______. **c** 2 493 767 is _______.

9 Which of these numbers are multiples of 2?

23 35 38 70 81

10 **a** Write seven 9-digit numbers that are multiples of 10.

b Write seven 9-digit numbers that are not multiples of 10.

11 Which of these numbers are multiples of 5?

10 21 135 74 16 8 88 2395 4 297 770 877 795

12 Which of these numbers are *not* multiples of 5?

7 290 84 90 19 2000 954 691

13 Which of these numbers are multiples of 5 but not 10?

2 25 100 93 95 50 35 1005

14 Copy and complete each sentence by writing 'is' or 'is not'.

a 12 _____ a multiple of 3. **b** 15 _____ a multiple of 2.

c 18 _____ a multiple of 2. **d** 18 _____ a multiple of 3.

15 Tyrone went to the market and bought 21 bananas. There are 3 people in Tyrone's family, and each person eats 1 banana per day. Did Tyrone buy the right number of bananas for a week? Explain why.

16 Copy and complete each sentence by writing 'is' or 'is not'.

a 15 _______ a multiple of 3. **b** 15 _______ a multiple of 7.

c 15 _______ a multiple of 6. **d** 18 _______ a multiple of 6.

e 18 _______ a multiple of 9. **f** 21 _______ a multiple of 3.

g 21 _______ a multiple of 7. **h** 21 _______ a multiple of 6.

17 The Marvellous Mango Company pack 6 mangoes in every box. Petra's family bought 12 boxes of mangoes. How many mangoes did they buy?

18 Use the digits 5, 6, 2, 5, 0, 1, 0, 4 to make:

a a 5-digit number that is even and a multiple of 5

b an 8-digit number that is odd and not a multiple of 5

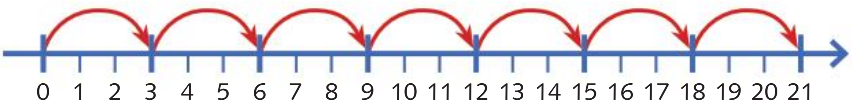
c a number that is a multiple of 5 and 100

d three different numbers that have more than 5 digits and are multiples of 2, 5 and 10

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

3B Factors and primes

This skip-counting pattern shows the first few multiples of 3 on a number line.



When we skip-count in threes, the numbers we say all have 3 as a factor.

That means 3 is a **factor** of these numbers.

3 6 9 12 15 18 21 24

We say that 3 is a factor of a number if that number is a multiple of 3. For example:

3 is a factor of 3 because $1 \times 3 = 3$

3 is a factor of 6 because $2 \times 3 = 6$

3 is a factor of 9 because $3 \times 3 = 9$

3 is a factor of 12 because $4 \times 3 = 12$

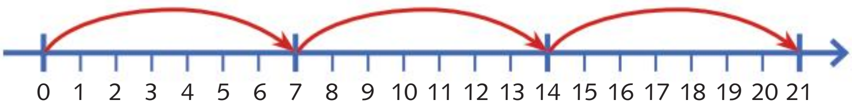
3 is a factor of 15 because $5 \times 3 = 15$

Factors and multiples are related.

21 is a multiple of 3, because when we take 7 steps of 3 we get 21.

$7 \times 3 = 21$

We also know, as shown on this number line, that $3 \times 7 = 21$.



This means that:

21 is a *multiple* of 7 and 7 is a *factor* of 21

21 is a *multiple* of 3 and 3 is a *factor* of 21

Example 2

Copy this sentence and fill in the blank.

$2 \times 6 = 12$ So, 2 is a factor of 12 and _____ is a factor of 12.

Solution

$2 \times 6 = 12$ So, 2 is a factor of 12 and 6 is a factor of 12.

Prime numbers

Some numbers have only two factors, 1 and themselves. For example:

2 has only two factors: 1 and 2 3 has only two factors: 1 and 3

7 has only two factors: 1 and 7

These are called **prime numbers**.

A prime number is a number with only two factors, itself and 1.

Sometimes numbers have more than two factors. For example:

4 has three factors: 1, 2 and 4 10 has four factors: 1, 2, 5 and 10

Numbers with more than two factors are called **composite numbers**.

The number 1 is neither prime nor composite.

Prime factorisation of whole numbers

We can always write a composite number as a product of two numbers other than 1 and itself. We can never do this for prime numbers.

We know that the numbers 4, 9 and 21 are composite because:

$4 = 2 \times 2$ $9 = 3 \times 3$ $21 = 3 \times 7$

We can keep decomposing composite numbers until we have written them as a product of primes.

For example, $36 = 4 \times 9$. We can find factors for 4 and 9:

$4 = 2 \times 2$ and $9 = 3 \times 3$

So we can write 36 as:

$36 = 2 \times 2 \times 3 \times 3$

We cannot find any more factors because all of the factors 2, 2, 3 and 3 are prime. $2 \times 2 \times 3 \times 3$ is called the **prime factorisation** of 36.

The standard way to write the prime factorisation of a number is to put the prime factors in increasing order from left to right.

Example 3

Find the prime factorisation of 45.

Solution

First write 45 as a product: $45 = 9 \times 5$

5 is a prime number. 9 is not a prime number: $9 = 3 \times 3$

So, $45 = 3 \times 3 \times 5$ 3 and 5 are prime.

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

Sometimes the prime factors are not so obvious, especially when the numbers are large.

Example 4

Find the prime factorisation of 54.

Solution

We know that:

$54 = 6 \times 9$

We can write 6 as 2×3 and 9 as 3×3.

So the prime factorisation is $54 = 2 \times 3 \times 3 \times 3$.

Example 5

Find the prime factorisation of 117.

Solution

117 is an odd number, so 2 is not a factor.

Divide 117 by 3.

$$\begin{array}{r} 3\ 9 \\ 3\overline{)1\,1\,{}^{2}7} \end{array} \qquad 117 = 3 \times 39$$

39 is divisible by 3. $(39 = 3 \times 13)$

So, $117 = 3 \times 3 \times 13$ 3 and 13 are prime numbers.

The prime factorisation is: $117 = 3 \times 3 \times 13$

Remember

When we write a number as a product of two numbers, those two numbers are factors of the first number. For example, $7 \times 8 = 56$, so 7 and 8 are factors of 56.

A prime number is a number larger than 1 that has only two factors, itself and 1.

Composite numbers are larger than 1 and have more than two factors.

To find the prime factorisation of a number, write it as a product of primes.

The standard way to write the prime factorisation is to put the prime factors in increasing order from left to right.

3B Whole class CONNECT, APPLY AND BUILD

1 Use counters to make arrays showing that:

a 3, 6 and 12 are factors of 12

b 4, 8 and 16 are factors of 16

c 7, 14 and 28 are factors of 28

d 10, 20 and 40 are factors of 40

2 Download **BLM 3** 'Looking for primes' from the Interactive Textbook and complete.

3 **100 chart prime search**

a This activity involves working out which numbers between 1 and 100 are prime numbers. Remember: 1 is not a prime number. Any other number larger than 1 is prime if it has only two factors: itself and 1.

Download a copy of **BLM 4** '100 chart prime search' from the Interactive Textbook. Cross out 1, as it is not a prime number or a composite number.

b The first prime number is 2. Circle it, then colour all the multiples of 2 because they are composite numbers. Your chart should look like this.

~~1~~	②	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

c Go to the first number after 2 that has not been coloured in. It is 3. Circle it, because 3 is prime. Now colour all the multiples of 3 that are not already coloured.

d Now go to the next number that has not already been coloured in. Circle it. It is prime. Colour its multiples. Repeat this step and continue until you cannot go any further.

e When you have finished, the numbers that have circles around them are the prime numbers less than 100. List these prime numbers.

f Find the next prime number after 97.

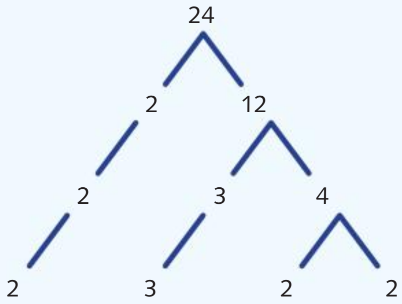
ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

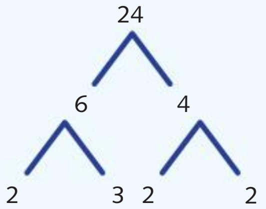
4 The numbers 12, 18, 33, 98, 196 and 333 are composite. Show this by writing each number as a product of two numbers in which no factor is 1.

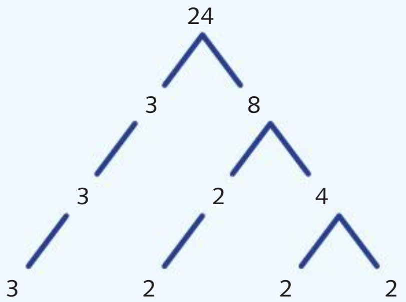
5 One way to find the prime factorisation of a number is to draw a factor tree.

In this example, 24 is first split into its largest and smallest factors, 12 and 2 (leaving out 24 and 1). Then each of these is split into its factors until they cannot be split any further.

It is not always necessary to start with the smallest and largest factor other than the number itself and 1. Sometimes it is easier to start with a factor you know.







Even though all three factor trees are different, they all give us the prime factorisation for 24, that is:

$24 = 2 \times 2 \times 2 \times 3$

Draw factor trees for these numbers. Write down the prime factorisations.

a 36 **b** 100 **c** 520

6 Find the prime factorisation of 280.

3B Individual

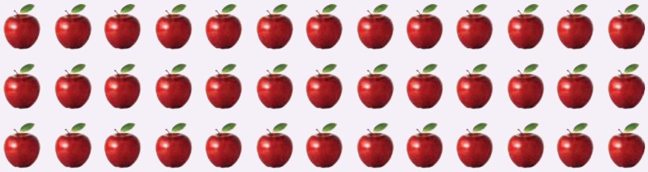
1 Copy the sentence that describes the array, then write the missing numbers and/or words.

a

This array shows that _____ × _____ = 24. So _____ and _____ are factors of 24.

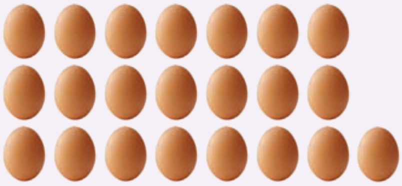
ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

b



This array shows that 3 × ______ = 39. So ______ and ______ are factors of ______.

c



This array shows that 7 is *not* a ________ of 22.

2 Copy these sentences, then fill in the blanks.

a 8 × 3 = 24, so 8 is a factor of 24 and ______ is a factor of 24.

b 6 × 8 = 48, so ______ and ______ are factors of 48.

c 9 × 6 = 54, so ______ and ______ are factors of 54.

3 Which of these are prime numbers?

2 3 11 14 23 37 45 57 62 90 99

4 **a** List the prime numbers between 40 and 50.

b Write the prime numbers between 10 and 20.

c Write the prime numbers between 40 and 60.

5 **a** List the prime numbers less than 100 that contain the digit 3.

b Why is every prime number odd except for 2?

6 Copy and complete these sentences by writing ‘factor’ or ‘multiple’.

a 3 is a ______ of 27 because 27 is a ______ of 3.

b 3 is a ______ of 36 because 36 is a ______ of 3.

c 4 is a ______ of 8 because 8 is a ______ of 4.

d 4 is a ______ of 16 because 16 is a ______ of 4.

e 3 is *not* a ______ of 7 because 7 is not a ______ of 3.

f 5 is a ______ of 10 because 10 is a ______ of 5.

g 16 is *not* a ______ of 5, so 5 is not a ______ of 16.

h 8 is a ______ of 4, so 4 is a ______ of 8.

i 19 is *not* a ________ of 2, so 2 is not a ________ of 19.

7 Copy these sentences. Fill in the blanks by writing ‘factor’, ‘multiple’ or the appropriate number.

a 4 × 3 = 12, so 12 is a ______ of 3 and 3 is a ______ of 12.

______ and ______ are factors of 12.

b 6 × 3 = 18, so 18 is a ______ of 3 and 3 is a ______ of 18.

______ and ______ are factors of 18.

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

c $4 \times 2 = 8$, so 8 is a ______ of 2 and 2 is a ______ of 8.

______ and ______ are factors of 8.

d $7 \times 9 = 63$, so 63 is a ______ of 9 and 9 is a ______ of 63.

______ and ______ are factors of 63.

8 Copy these sentences and write the missing numbers.

a $4 \times 11 = 44$ and $2 \times$ ______ $= 44$

This means that ______, ______, ______ and ______ are factors of 44.

b $5 \times 12 = 60$ and $3 \times$ ______ $= 60$

This means that _____, _____, _____ and _____ are factors of 60.

9 Find the prime factorisation of these numbers.

a 30 **b** 84 **c** 63 **d** 44 **e** 120

10 Find the prime factorisation of these numbers.

a 444 **b** 396 **c** 176 **d** 261 **e** 7000

11 Which number has the prime factorisation:

a $2 \times 2 \times 3$? **b** $2 \times 5 \times 19$?

c $3 \times 3 \times 3$? **d** $2 \times 2 \times 2 \times 2 \times 2$?

e $5 \times 7 \times 11$? **f** $2 \times 3 \times 5 \times 7$?

12 a If it costs $6 for 3 bananas, how much does each banana cost?

b Tammy eats 2 apples every day. Each apple costs $2. How many apples does Tammy need for a week, and how much would they cost?

13 a If 7 pizzas cost $63, how much do 11 pizzas cost?

b If James pays $42 for 14 rides at the show, how much does he pay for 27 rides?

14 a It is interschool sport day at Echidna Ridge School. 30 students from Echidna Ridge need to arrive at their sports venue at 10 a.m., but the school minibus can only hold 6 passengers. How many minibuses in total will be needed to get the students to the venue?

b The Echidna Ridge School can hire larger buses that hold 15 passengers. How many larger buses will be needed to get the 30 students to interschool sport?

c Copy and complete: $6 \times$ __ $= 30$ and __$\times 15 = 30$

This means that _____, _____, _____ and _____ are factors of 30.

15 True or false? Sam wrote this statement in his maths book.

The prime factorisation of 1140 is $1140 = 2 \times 2 \times 2 \times 5 \times 57$

Is this correct? If not, what corrections does Sam need to make?

16 Write six composite numbers greater than 30 and less than 60 that do *not* have any even digits.

17 The sum of two prime numbers is 60. What might the numbers be?

3C Review questions

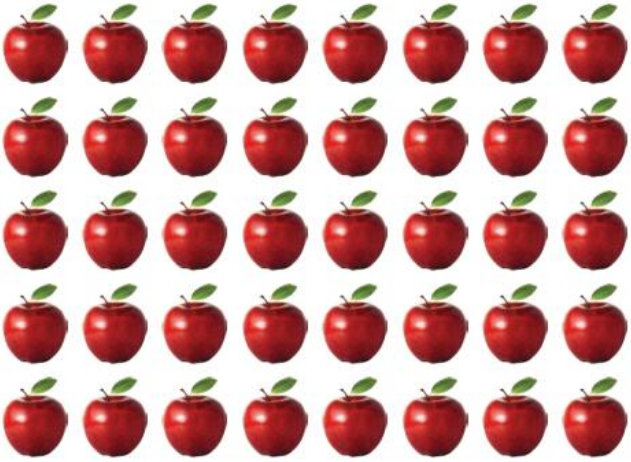
1 Write down the first 12 multiples of 4. Then complete **a** to **l**. To fill in the blanks, say '1 four is 4', '2 fours are 8', and so on. Count in fours to check your answers.

a 1 × 4 = ___
b 2 × 4 = ___
c 3 × 4 = ___
d 4 × 4 = ___
e 5 × 4 = ___
f 6 × 4 = ___
g 7 × 4 = ___
h 8 × 4 = ___
i 9 × 4 = ___
j 10 × 4 = ___
k 11 × 4 = ___
l 12 × 4 = ___

2 Every bag of Peter's Potatoes contains 7 potatoes. Copy and complete these statements.

a 2 bags contain ____ potatoes.
b 3 bags contain ____ potatoes.
c 5 bags contain ____ potatoes.
d 8 bags contain ____ potatoes.

3 Complete this statement *without* counting the apples one by one.



____ × ____ = ____ apples

4 **a** Draw 3 rows of 7 oranges.

b Complete this statement about your array *without* counting the oranges one by one.

There are 3 × ____ = ____ oranges.

5 Copy these statements and complete them by writing 'is' or 'is not'.

a 24 ________ a multiple of 3.
b 18 ________ a multiple of 7.
c 18 ________ a multiple of 9.
d 24 ________ a multiple of 4.
e 24 ________ a multiple of 5.
f 18 ________ a multiple of 2.
g 16 ________ a multiple of 6.
h 28 ________ a multiple of 8.

6 On a fruit stand, 12 boxes each contain 6 mangoes. How many mangoes are there altogether?

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

7 The Hayes family bought 42 bananas to share among 6 people. Each person eats 1 banana each day for 1 week. Explain why they bought exactly the right number of bananas.

8 Copy these statements and fill in the blanks.

a 7 rows of beads with 5 beads in each row gives a multiple of ______.

The total is ____ × ____ = ____ beads.

b Edna Sharp teaches 8 different music classes. Each class has 3 people in it.

This gives a multiple of _____.

The total is ____ × ____ = ____ people.

9 Copy these statements and fill in the blanks.

a 28 is the product of ___ and ___.

b 15 is the product of ___ and ___.

c 27 is the product of 3 and ___.

d The product of 6 and ___ is 48.

e The product of 8 and ___ is 56.

f 24 is the product of 4 and ___.

g 24 is the product of 2 and ___.

h 24 is the product of 8 and ___.

i 60 is the product of 2 and ___.

j 60 is the product of 4 and ___.

k 60 is the product of 5 and ___.

l 60 is the product of 6 and ___.

m 72 is the product of 12 and ___.

n 72 is the product of 6 and ___.

10 Copy these statements and fill in the blanks by writing 'factor' or 'multiple'.

a 3 is a ________ of 24 because 24 is a ________ of 3.

b 3 is a ________ of 21 because 21 is a ________ of 3.

c 4 is a ________ of 12 because 12 is a ________ of 4.

d 4 is a ________ of 20 because 20 is a ________ of 4.

e 3 is not a ________ of 10 because 10 is not a ________of 3.

f 5 is a ________ of 15 because 15 is a ________ of 5.

g 16 is not a ________ of 6, so 6 is not a ________ of 16.

h 9 is a ________ of 3, so 3 is a ________ of 9.

i 17 is not a ________ of 2, so 2 is not a ________ of 17.

11 Draw factor trees to find the prime factorisation of these numbers.

a 28 **b** 76 **c** 95 **d** 52 **e** 135

12 Find the prime factorisation of these numbers.

a 555 **b** 472 **c** 381 **d** 154 **e** 7020

13 Write the number that is the product of each prime factorisation.

a $2 \times 3 \times 5$ **b** $3 \times 5 \times 17$ **c** $2 \times 3 \times 3$

d $3 \times 3 \times 5 \times 5 \times 5$ **e** $7 \times 11 \times 13$ **f** $2 \times 3 \times 11 \times 13$

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

CHAPTER 4 Preparation

Useful skills for this chapter:

- quick recall of multiplication facts to 12 × 12.

KICK OFF

There are 7 people in the Dingle family. The Dingles make sure they buy packets of food that they can share equally, so that each family member gets the same amount with none left over. Circle the foods that the Dingles will buy, then write the number of items each family member will receive from each packet or container.

Show what you know

1 Write the answers.

a 6 × 7

b 9 × 3

c 5 × 9

d 8 × 6

e 6 × 8

f 9 × 6

g 7 × 9

h 7 × 4

i 8 × 9

j 9 × 12

k 11 × 12

l 12 × 8

CHAPTER

4

Number and Algebra

Multiplication and division

Multiplication is used to calculate the product of two numbers. It is especially useful when repeated addition becomes unmanageable or when large numbers are involved. You can think of multiplication as 'lots of' a number.

For example, if you have 3 nests, each containing 3 Easter eggs, you have 3 lots of 3. This is written as 3 × 3 and you have a total of 9 Easter eggs.

If you have 1427 nests each with 2384 Easter eggs, you have 1427 lots of 2384 eggs, or $1427 \times 2384 = 3\,401\,968$ Easter eggs.

Division, on the other hand, is a way of splitting a number into equal parts. If we have eight tennis balls, and we split them into groups of four, we have two groups. In other words, $8 \div 4 = 2$.

In this chapter we look at the connection between multiplication and division. We will see how multiplication can be used to check a division calculation, and how division is the inverse of multiplication.

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

4A Mental strategies for multiplication

The multiplication tables up to 12×12 form the basis of many of the multiplication mental strategies that we look at in this section. It is important to be able to recall your 'tables' very quickly.

4A Whole class CONNECT, APPLY AND BUILD

Discuss each strategy as a class, then complete the related activities.

1 Multiplying by 4

To multiply a number by 4, double it, then double the result.

This works because $4 = 2 \times 2$.

For example, to calculate 45×4 mentally:

double 45 to get 90, then double 90 to get 180.

$45 \times 4 = 180$.

Multiply these numbers by 4.

a 15 **b** 21 **c** 42 **d** 36
e 77 **f** 102 **g** 255 **h** 220
i 500 **j** 1026 **k** 1050 **l** 2222

2 Multiplying by 10, 100 and 1000

Numbers that are multiples of 10 always end in zero. For example:

$11 \times 10 = 110 \qquad 12 \times 10 = 120 \qquad 13 \times 10 = 130$

To multiply a whole number by 10, write a zero at the end of the number.

$23 \times 10 = 230 \qquad 9898 \times 10 = 98\,980 \qquad 123\,456 \times 10 = 1\,234\,560$

To multiply a whole number by 100, write two zeroes at the end of the number. This works because $100 = 10 \times 10$. For example:

$23 \times 100 = 2300 \qquad 9898 \times 100 = 989\,800 \qquad 123\,456 \times 100 = 12\,345\,600$

When multiplying by 1000, we write three zeroes at the end; for 10 000 we write four zeroes; and so on.

Multiply these numbers by 10, then by 100 and then by 1000.

a 23 **b** 85 **c** 121 **d** 934
e 700 **f** 1001 **g** 8594 **h** 10 101
i 15 462 **j** 273 912 **k** 848 084 **l** 295 034 957

3 Multiplying by 8

To multiply a number by 8, double, then double again, then double for a third time. This works because $8 = 2 \times 2 \times 2$.

For example, to calculate 15×8 mentally:

double 15 to get 30, then double again to get 60, then double again to get 120.

$15 \times 8 = 120$

Use the 'double three times' strategy to multiply these numbers by 8.

a 3 **b** 5 **c** 11 **d** 8
e 15 **f** 9 **g** 16 **h** 33
i 24 **j** 51 **k** 101 **l** 94

4 Multiplying by 9

If we want to get 9 lots of something, it is easier to find 10 lots and then take 1 lot away. For example:

$$\begin{aligned} 9 \times 17 &= \text{10 lots of 17 take away 1 lot of 17} \\ &= 170 - 17 \\ &= 153 \end{aligned}$$

Multiply each number by 9.

a 14 **b** 21 **c** 34 **d** 52
e 63 **f** 81 **g** 107 **h** 113
i 150 **j** 122 **k** 354 **l** 750

5 Multiplying by 11

If we want to get 11 lots of something, find 10 lots and then add 1 lot more.

Multiply these numbers by 11.

a 14 **b** 21 **c** 34 **d** 36
e 37 **f** 51 **g** 56 **h** 83
i 97 **j** 122 **k** 354 **l** 750

6 Multiplying by 5

There are two ways to multiply by 5.

- The first way is to multiply by 10, then halve the result. This works because $5 = 10 \div 2$.

Use this strategy to multiply these odd numbers by 5.

a 21 **b** 13 **c** 45 **d** 99 **e** 111

f Write five odd numbers of your own. Multiply each number by 5.

- The second way is to halve the number, then multiply the result by 10.

Use this strategy to multiply these even numbers by 5.

g 18 **h** 26 **i** 54 **j** 102 **k** 450

l Write five even numbers of your own. Multiply each number by 5.

m Which strategy did you find better for multiplying even numbers by 5?

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7

Cambridge University Press

7 Multiplying by 20

When you want to multiply by 20, double the number, then multiply it by 10. This works because $20 = 2 \times 10$.

Complete these multiplications.

a 8×20 **b** 23×20 **c** 42×20 **d** 36×20

8 Multiplying by 6

A shortcut to multiplying a number by 6 is to first multiply the number by 3, then multiply it by 2 (or the other way around). This works because $6 = 3 \times 2$.

Use this strategy to multiply these numbers by 6.

a 13 **b** 21 **c** 33 **d** 52

e 54 **f** 72 **g** 80 **h** 100

9 Multiplying by 25

You can multiply numbers by 25 by skip-counting in twenty-fives. Use your fingers to keep track of how many you have counted; later on, you can keep track of the skip-counting ‘in your head’.

For example, for 25×7:

①	②	③	④	⑤	⑥	⑦
25	50	75	100	125	150	175

Skip-count to multiply these numbers by 25.

a 5 **b** 3 **c** 6 **d** 8 **e** 9

Now think up a variation on the same strategy to multiply these numbers by 25.

f 50 **g** 30 **h** 60 **i** 80 **j** 90

10 Multiplying by multiples of 10

a Calculate 30×9 by thinking $3 \times 9 = 27$ and putting a zero on the end.

b Copy and complete this table. Use a strategy similar to the strategy in part **a** to multiply each number in the left-hand column by each number in the top row.

×	30	60	50	90
4				
8				
7				
9				

11 Use mental strategies to solve these word problems.

a 25 children each ate 11 bananas. How many bananas were eaten?

b At the zoo Mary fed 11 monkeys 23 peanuts each. Mary ate 2 bags with 19 peanuts in each herself. How many peanuts were eaten altogether.

12 a Make up your own strategy for multiplying by 19. (Hint: Do something like the ‘multiplying by 9’ strategy and the ‘multiplying by 20’ strategy.)

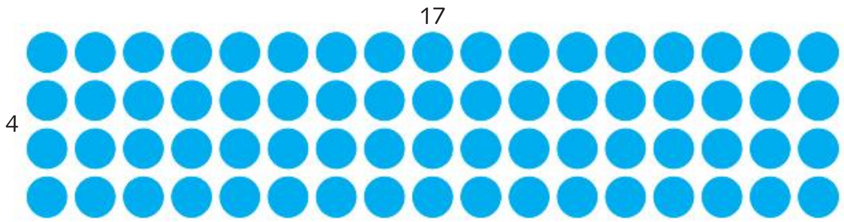
b Make up your own strategy for multiplying by 21. (Hint: Do something like the ‘multiplying by 11’ strategy and the ‘multiplying by 20’ strategy.)

c Write a strategy for multiplying by 30. Test your strategy on five different numbers.

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

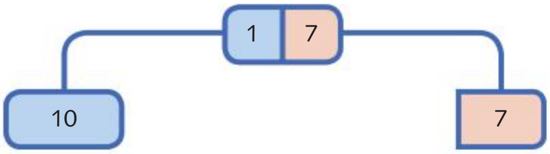
4B Breaking a multiplication apart

This is the rectangular array for 4×17.

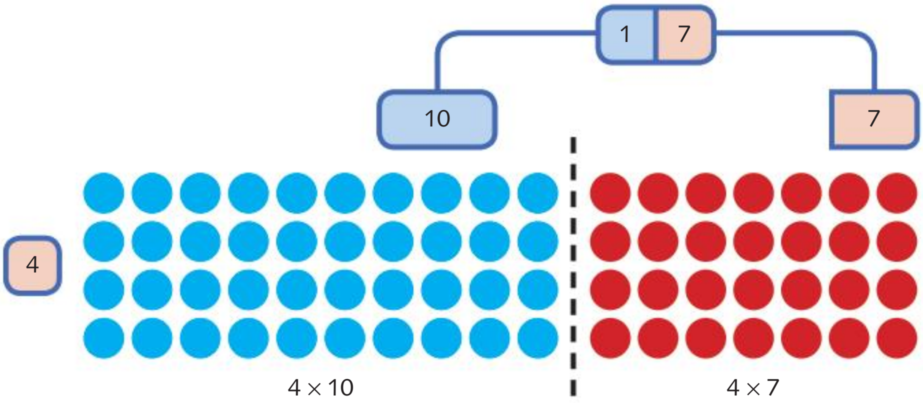


If we count all of the dots in the array, we will find that $4 \times 17 = 68$.

We can split 17 into 1 ten and 7 ones.



This breaks the array apart to show **multiplication chunks**.

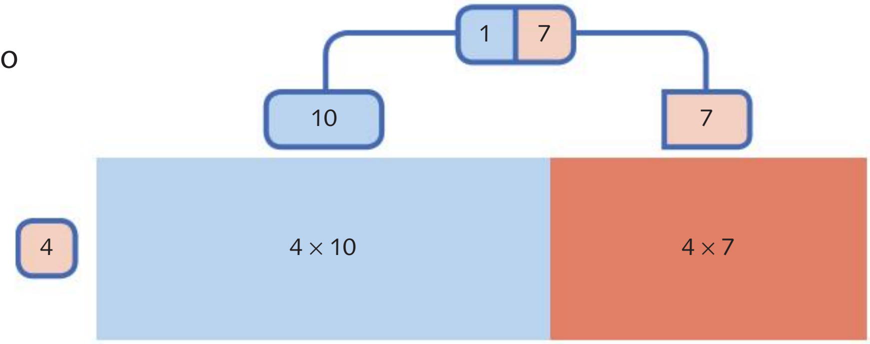


We can do the multiplication in each chunk first, then add to find the product of 4 and 17.

$$\begin{aligned} 4 \times 17 &= 4 \times 10 + 4 \times 7 \\ &= 40 + 28 \\ &= 68 \end{aligned}$$

Instead of drawing arrays, you can draw multiplication diagrams to help you 'see' the multiplication. This diagram uses the chunks 4×10 and 4×7 to show 4×17.

You can use multiplication diagrams to explain how to multiply large numbers.

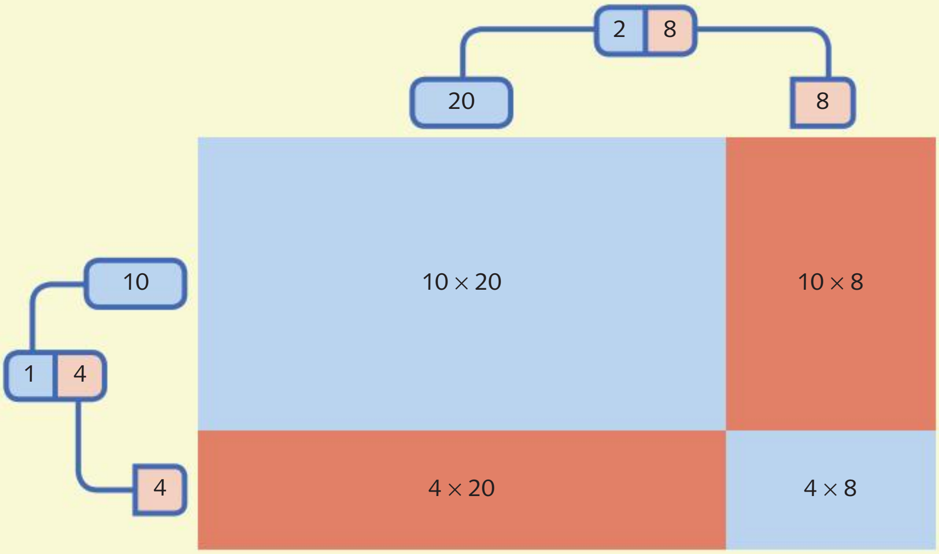


ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

Example 1

Draw a multiplication diagram for 14 × 28, then use it to calculate the answer.

Solution



$$\begin{aligned}14 \times 28 &= 10 \times 20 + 10 \times 8 + 4 \times 20 + 4 \times 8\\ &= 200 + 80 + 80 + 32\\ &= 392\end{aligned}$$

4B Individual

1 Draw a multiplication diagram and complete the calculation for each of these.

a 4 × 17 **b** 8 × 16 **c** 5 × 18 **d** 12 × 13

2 Complete these multiplications. Try to solve them 'in your head'. The first multiplication has been done for you. Remember to do all the multiplications before the additions.

a $12 \times 7 = 10 \times 7 + 2 \times 7$
$= \underline{70} + \underline{14}$
$= \underline{84}$

b $13 \times 5 = 10 \times 5 + 3 \times 5$
= __+ __
= __

c $27 \times 8 = 20 \times 8 + 7 \times 8$
= __+ __
= __

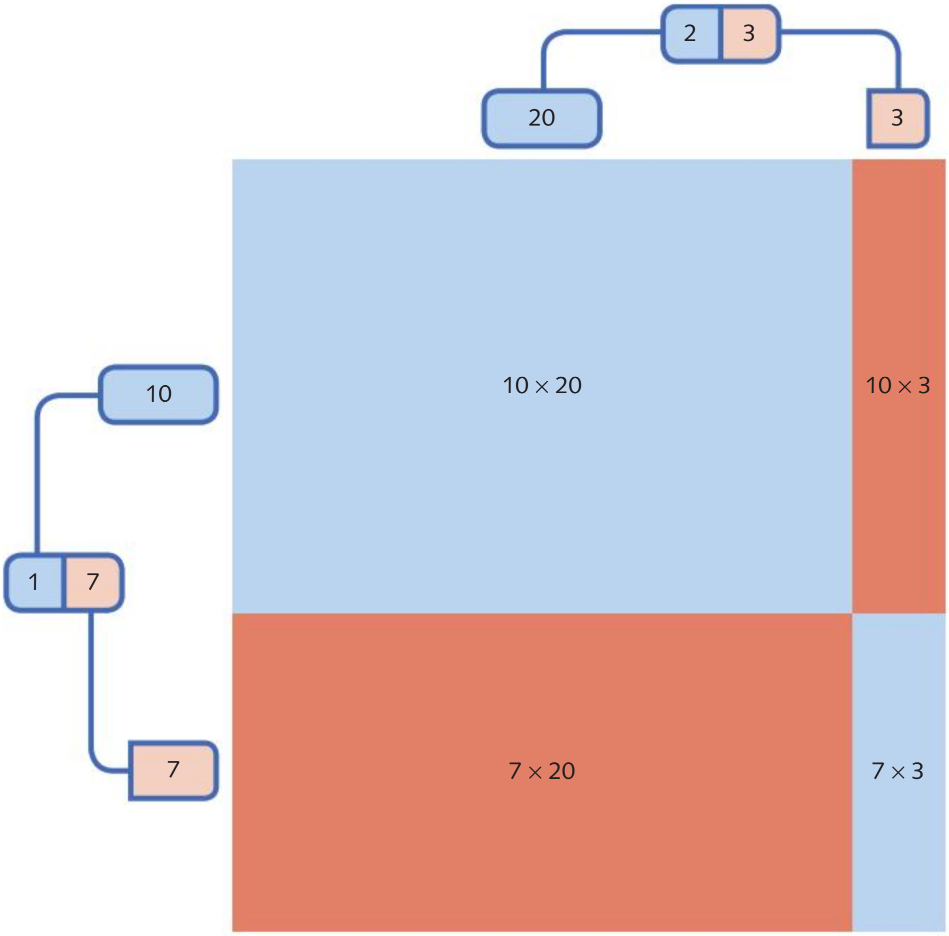
d $15 \times 23 = 10 \times 20 + 10 \times 3 + 5 \times 20 + 5 \times 3$
$= 200 + 30 + 100 + 15$
= __

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

4C The multiplication algorithm

Multiplication diagrams are connected to the **algorithm** for multiplication. An algorithm is like a recipe that gives you steps to follow. The diagrams help explain how the algorithm works.

This multiplication diagram for 17 × 23 gives the products for each chunk.



We can find the product of 17 and 23 by multiplying the chunks and adding them together.

$$\begin{aligned} 17 \times 23 &= (10 \times 20) + (10 \times 3) + (7 \times 20) + (7 \times 3) \\ &= 200 + 30 + 140 + 21 \\ &= 391 \end{aligned}$$

There is a more efficient way to find the product of 17 and 23, and that is by using the multiplication algorithm.

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

This is how the multiplication algorithm works for the multiplication 23 × 17.

	Hundreds	Tens	Ones
		2	3
×		1	7

Set out the numbers so the digits line up according to their place value.

	Hundreds	Tens	Ones
		2_2	3
×		1	7
			1

Start with the ones. Multiply the ones digit in 17 by the ones digit in 23.

Say '7 times 3 is 21'. Write '1' in the ones column and carry '2' to the tens column.

	Hundreds	Tens	Ones
		2_2	3
×		1	7
	1	6	1

Next work with the tens. Multiply the ones digit in 17 by the tens digit in 23.

Say '7 times 2 is 14'. Add the 2 tens carried from before to make 16. Write '6' in the tens column and '1' in the hundreds column.

	Hundreds	Tens	Ones
		2_2	3
×		1	7
	1	6	1
		3	0

Now multiply the tens digit in 17 by the ones digit in 23. This will give a certain number of tens, so start by writing '0' in the ones column.

Say '1 times 3 is 3'. Write '3' in the tens column.

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

	Hundreds	Tens	Ones
		2_2	3
×		1	7
	1	6	1
	2	3	0

Next, multiply the tens digit in 17 by the tens digit in 23. Say ‘1 times 2 is 2’.
Write ‘2’ in the hundreds column.

	Hundreds	Tens	Ones
		2_2	3
×		1	7
	1	6	1
	2	3	0
	3	9	1

The final step is to add 161 to 230.

The product of 23 × 17 is 391.

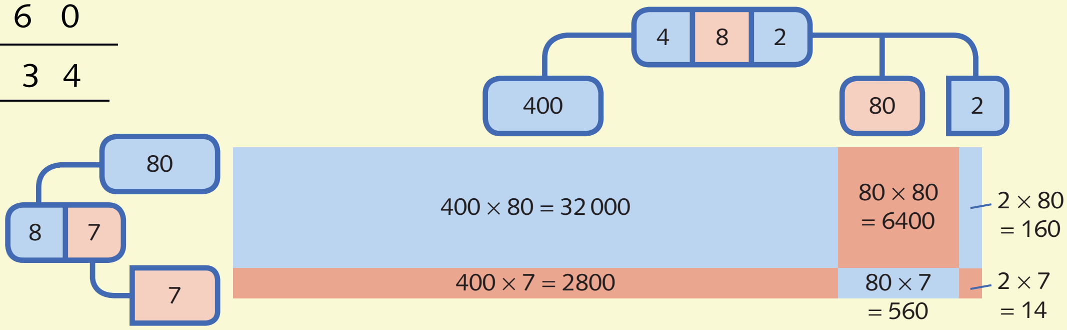
Example 2

Calculate 482 × 87 using the multiplication algorithm.

Explain each step in the algorithm using a multiplication diagram.

Solution

$$\begin{array}{rrrrrr} & & 4^{6}_{5} & 8^{1}_{1} & {}^{1}2 \\ \times & & & 8 & 7 \\ \hline & 3 & 3 & 7 & 4 \\ 3_1 & 8 & 5_1 & 6 & 0 \\ \hline 4 & 1 & 9 & 3 & 4 \\ \hline \end{array}$$



3374 = 2800 + 560 + 14 is the sum of the chunks in the bottom row of the multiplication diagram.

38 560 = 32 000 + 6400 + 160 is the sum of the chunks in the top row of the multiplication diagram.

4C Individual

1 Use the multiplication algorithm to calculate these.

a 18 × 17 **b** 24 × 28 **c** 31 × 37

d 49 × 43 **e** 83 × 78 **f** 103 × 32

g 480 × 64 **h** 638 × 485 **i** 978 × 498

j 3333 × 687 **k** 2031 × 1002 **l** 4395 × 3982

2 **a** The canteen at Pine Hills School sold 38 pies every day for 31 days. How many pies were sold in total?

b There are 34 pairs of shoes in the cupboard. How many shoes is this?

c Steven has 6 pencil cases, each with 28 pens and pencils in them. How many pens and pencils is this?

3 Simon's car can travel 13 kilometres on 1 litre of petrol if he drives at the same speed.

a How far can Simon's car travel with 14 litres of petrol in the tank?

b How far can Simon's car travel with 27 litres of petrol in the tank?

c How far can Simon's car travel with 68 litres of petrol in the tank?

4 Jesse's car releases 119 grams of carbon into the atmosphere for each kilometre it travels (as long as Jesse drives at a constant speed). How much carbon is released if Jesse travels:

a 12 kilometres? **b** 46 kilometres? **c** 108 kilometres?

5 A jet uses 12 788 litres of fuel each hour. How much fuel would it use in:

a 2 hours? **b** 3 hours? **c** 7 hours?

6 **a** Hannah's school has 14 classrooms. Each classroom has 12 tables. If there are 4 children to each table, how many children are at the school?

b A hotel has 3 beds in each room. There are 9 rooms on each level of the hotel, which is 13 storeys high. How many beds are there?

c There are 3 windows on each of 3 walls of a room in Janie's house. If the house has 6 rooms like this, how many windows are there all together?

d In the supermarket near Katie's house there are 23 cartons of eggs on 6 shelves and 14 cartons of eggs on 8 shelves. Each carton holds 12 eggs. How many eggs are there in total?

e Ali is moving house. Three of the rooms have 16 boxes each and 6 rooms have 19 boxes each. How many boxes is that in total?

f Lauren has 6 drawers with 32 items of clothing in each of them. In each of her 3 cupboards, there are 83 items of clothing. How many items of clothing does Lauren have?

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

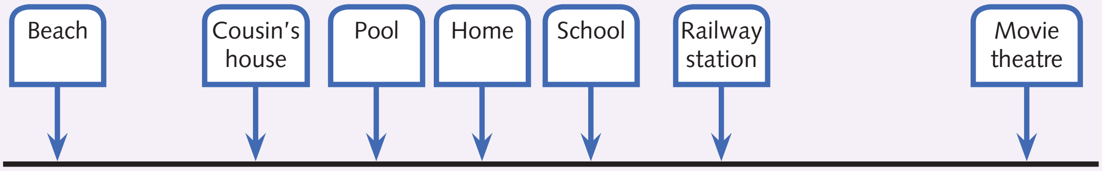
7 Use the multiplication algorithm to calculate **a** to **i**.

- **a** 1×1
- **b** 11×11
- **c** 111×111
- **d** 1111×1111
- **e** $11\,111 \times 11\,111$
- **f** $111\,111 \times 111\,111$
- **g** $1\,111\,111 \times 1\,111\,111$
- **h** $11\,111\,111 \times 11\,111\,111$
- **i** $111\,111\,111 \times 111\,111\,111$
- **j** What do you notice?

8 Use the multiplication algorithm to calculate **a** to **h**.

- **a** $9 \times 9 + 7$
- **b** $98 \times 9 + 6$
- **c** $987 \times 9 + 5$
- **d** $9876 \times 9 + 4$
- **e** $98\,765 \times 9 + 3$
- **f** $987\,654 \times 9 + 2$
- **g** $9\,876\,543 \times 9 + 1$
- **h** $98\,765\,432 \times 9 + 0$
- **i** What do you notice?

9 Daniel drew a map of the highway that runs past his home in the country.



School is 13 kilometres from home. From home, the nearest railway station is 2 times this distance and the nearest movie theatre is 5 times the distance. The nearest swimming pool is 17 kilometres from home. From home, his cousin's house is 2 times this distance and the beach is 4 times the distance.

What is the distance Daniel travels:

- **a** from home to school and back?
- **b** from his home to the railway station?
- **c** from his home to visit his cousin?
- **d** from the swimming pool to see a movie?
- **e** from his home to the beach and back home?
- **f** from his home to the movie theatre and then return home via his cousin's house?

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7

4D Connecting multiplication and division

Division is about splitting or sharing quantities equally.

If we have 24 balloons to share equally, there are two ways we can do this.

The first way is by asking 'How many groups?'

For example, if we have 24 balloons and we give 8 balloons each to a number of children, how many children get 8 balloons?

If we split 24 balloons into groups of 8, then 3 children get 8 balloons each.

We say '24 divided by 8 is 3'. This is written as $24 \div 8 = 3$.

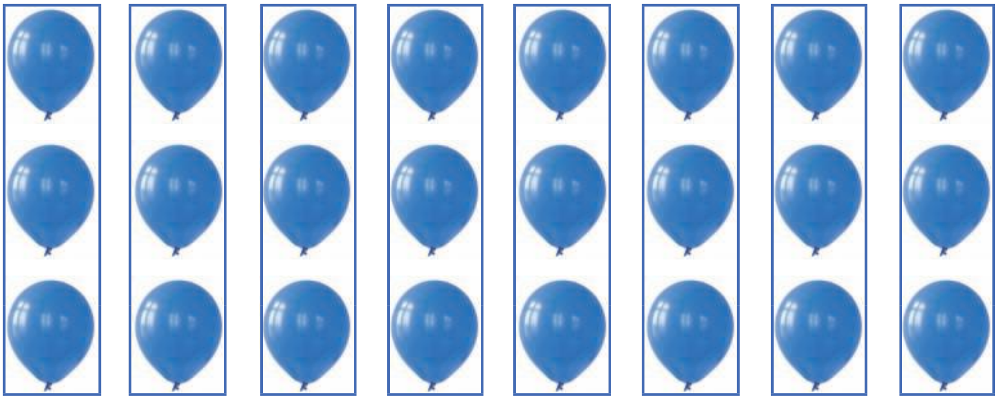
The second way is by asking 'How many in each group?'

For example, if we share 24 balloons among 8 children, how many balloons does each child receive?

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

We want to make 8 equal groups. We do this by handing out one balloon to each child. This uses 8 balloons. Then we do the same again until there are no balloons left. We can do this 3 times, so each child gets 3 balloons.

We can see this from the multiplication array:



8 lots of 3 make 24. 24 ÷ 8 = 3

So dividing 24 by 8 is the same as asking 'Which number do I multiply 8 by to get 24?'

Division is the inverse operation to multiplication. When we know one multiplication fact, we know two division facts.

We can see this on the multiplication table.

×	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10	11	12
2	0	2	4	6	8	10	12	14	16	18	20	22	24
3	0	3	6	9	12	15	18	21	24	27	30	33	36
4	0	4	8	12	16	20	24	28	32	36	42	44	48
5	0	5	10	15	20	25	30	35	40	45	50	55	60
6	0	6	12	18	24	30	36	42	48	54	60	66	72
7	0	7	14	21	28	35	42	49	56	63	70	77	84
8	0	8	16	24	32	40	48	56	64	72	80	88	96
9	0	9	18	27	36	45	54	63	72	81	90	99	108
10	0	10	20	30	40	50	60	70	80	90	100	110	120
11	0	11	22	33	44	55	66	77	88	99	110	121	132
12	0	12	24	36	48	60	72	84	96	108	120	132	144

The table shows two ways of multiplying to get 54:

9 × 6 = 54 and
6 × 9 = 54

If we reverse both multiplications, we find that:

54 ÷ 6 = 9 and
54 ÷ 9 = 6

This means that we can use the multiplication table in reverse to do calculations involving division.

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

Example 3

Copy and complete these sentences by filling in the gaps.

a If $5 \times 9 = 45$, then $45 \div 9 =$____ and $45 \div 5 =$____.
b If $12 \times 4 = 48$, then $48 \div$ ___ $= 4$ and $48 \div$ ___ $= 12$.

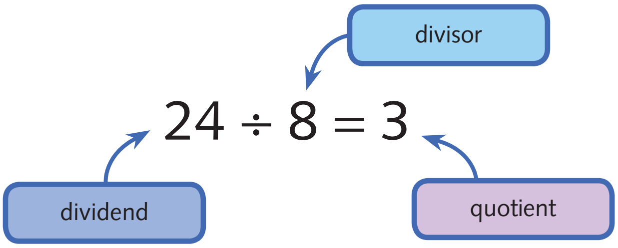
Solution

a If $5 \times 9 = 45$, then $45 \div 9 = 5$ and $45 \div 5 = 9$.
b If $12 \times 4 = 48$, then $48 \div 12 = 4$ and $48 \div 4 = 12$.

We can also use the word **divisible**: 24 is divisible by 8 because $24 \div 8 = 3$ with none left over.

Some new language

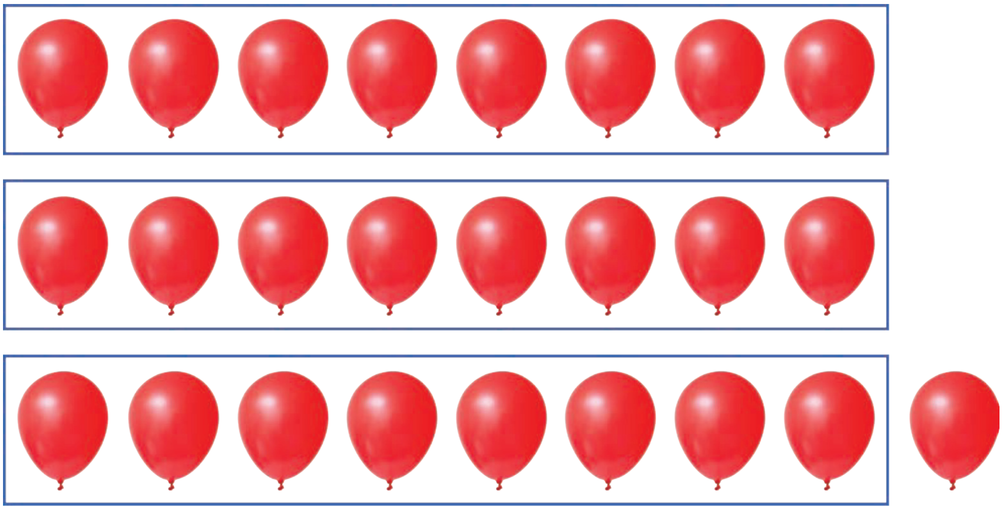
In the division $24 \div 8 = 3$, 24 is the **dividend**, 8 is the **divisor** and 3 is the **quotient**.



The divisor is the number that you divide by.

Division with remainder

Sometimes the divisor does not divide exactly into the dividend and there is a **remainder** left over. This can be seen by drawing an array. For example, if we try to find out $25 \div 8$, this is the closest array we can draw.



ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

The array uses only 24 of the balloons, with one balloon left over.

So, we can make 3 groups of 8 with 1 remainder.

We write this as:

$$25 \div 8 = 3 \text{ remainder } 1$$

We say division is **exact** if the remainder is 0.

Sharing often involves remainders. Here are two division stories to show division with remainder.

Division story 1

Alex bought a box of 30 chocolates to share with her 3 friends. Can Alex and her friends share the chocolates equally?

There are 4 people, including Alex. The multiples of 4 are 4, 8, 12, 16, 20, 24, 28, 32 …

$4 \times 7 = 28$, so 28 is the largest number of chocolates that can be eaten if each person gets the same number. There will be 2 chocolates left over.

We write this as:

$$30 = 4 \times 7 + 2 \qquad \text{or} \qquad 30 \div 4 = 7 \text{ remainder } 2$$

Each of the 4 people will get 7 chocolates. There will be 2 chocolates left over.

Division story 2

Nick has 22 skateboard wheels. He wants to use them to build as many skateboards as he can.

How many skateboard decks will Nick need? How many wheels will he have left over?

Divide 22 by 4.

We get 5 lots of 4, with 2 left over.

We write this as:

$$5 \times 4 + 2 = 22 \qquad \text{or} \qquad 22 \div 4 = 5 \text{ remainder } 2$$

Nick can build 5 skateboards. He will have 2 wheels left over.

In this example, 22 is the dividend, 4 is the divisor (the number you divide by), 5 is the quotient and 2 is the remainder.

$$\underset{\text{dividend}}{22} \div \overset{\text{divisor}}{4} = \underset{\text{quotient}}{5} \text{ remainder } \underset{\text{remainder}}{2}$$

The remainder is always smaller than the divisor. We can see that 22 is not divisible by 4 because when we try to do the division there is a remainder.

4D Whole class CONNECT, APPLY AND BUILD

1 **Beachball**

Write these numbers on stickers and place them randomly on a beachball.

36	18	28	44	72	24	32
48	56	121	27	100	81	35

Pass the beachball around the classroom. Whoever catches the ball says a division statement that has the number nearest their right thumb as its dividend.

2 Mentally divide each number by 2 by halving it. Then double the result to check your answer.

a 8 **b** 12 **c** 28 **d** 44 **e** 32 **f** 102

3 Numbers that are multiples of 10 end in zero. Divide each of the following numbers by 10 by mentally 'chopping off' the last zero.

a 70 **b** 130 **c** 240 **d** 1290 **e** 4000 **f** 1 000 000

4 Mentally divide each number by 100 by 'chopping off' two zeroes at the end of the number.

a 200 **b** 800 **c** 3400 **d** 8000 **e** 25 000

5 Mentally divide each number by 1000 by 'chopping off' three zeroes at the end of the number.

a 4000 **b** 7000 **c** 27 000 **d** 20 000 **e** 680 000

6 Mentally divide each number by 4 by halving and halving again. Check your answer by doubling, then doubling again.

a 8 **b** 12 **c** 28 **d** 44 **e** 32 **f** 1000

4D Individual

1 Copy and complete. The first one has been done for you.

a If 4 × 10 = 40, then 40 ÷ 10 = 4 and 40 ÷ 4 = 10.

b If 3 × 9 = 27, then 27 ÷ 9 = ___ and 27 ÷ 3 = ___.

c If 6 × 7 = 42, then 42 ÷ 6 = ___ and 42 ÷ 7 = ___.

d If 12 × 8 = 96, then 96 × 8 = ___ and 96 ÷ 12 = ___.

e If 144 × 72 = 10 368, then 10 368 ÷ 144 = ___ and 10 368 ÷ 72 = ___.

2 Use the corresponding multiplication to check that each calculation is correct.

a $121 \div 11 = 11$ **b** $162 \div 9 = 18$ **c** $504 \div 63 = 8$

d $1170 \div 45 = 26$ **e** $7154 \div 73 = 98$ **f** $5814 \div 57 = 102$

3 Use the multiplication table in reverse to write two division statements for each number.

a 32 **b** 36 **c** 84 **d** 96 **e** 132 **f** 90

g 25 **h** 63 **i** 144 **j** 72 **k** 108 **l** 120

4 Which of these numbers is divisible by 6?

12 73 18 55 71

5 Which of these numbers is divisible by 8?

24 18 36 56 94 96

6 Draw arrays to calculate each division and the remainder (if there is one).

a $12 \div 4 =$ **b** $18 \div 4 =$ **c** $29 \div 10 =$

d $56 \div 8 =$ **e** $100 \div 8 =$ **f** $120 \div 9 =$

7 Mentally divide each number by 20 by skip-counting in twenties. Keep count of each time you say a multiple.

a 80 **b** 120 **c** 160 **d** 220 **e** 300 **f** 320

8 Copy and complete the following.

a $49 = 7 \times 7$, so $55 = 7 \times 7 + \underline{\qquad}$ and $55 \div 7 = 7$ remainder $\underline{\qquad}$

b $94 = 7 \times 12 + \underline{\qquad}$, so $94 \div 12 = \underline{\qquad}$ remainder $\underline{\qquad}$

9 Tammy fills paper bags from a sack containing 20 kilograms of sugar. Each full bag of sugar weighs 3 kilograms. How many bags can Tammy fill? How much sugar will she have left over?

10 Copy these and fill in the missing numbers. The missing number is either a divisor, a quotient or a remainder.

a $15 = 4 \times \underline{\qquad} + 3$ **b** $29 = 4 \times 7 + \underline{\qquad}$ **c** $30 = 12 \times \underline{\qquad} + \underline{\qquad}$

d $157 = 15 \times \underline{\qquad} + \underline{\qquad}$ **e** $12 = 4 \times \underline{\qquad} + \underline{\qquad}$

f $14 \div 5 = 2$ remainder $\underline{\qquad}$ **g** $26 \div 5 = \underline{\qquad}$ remainder $\underline{\qquad}$

h $191 \div 10 = \underline{\qquad}$ remainder $\underline{\qquad}$ **i** $192 \div 3 = \underline{\qquad}$ remainder $\underline{\qquad}$

Homework

1 Use the multiplication table in reverse to write two division statements for each number.

a 48 **b** 64 **c** 110 **d** 84

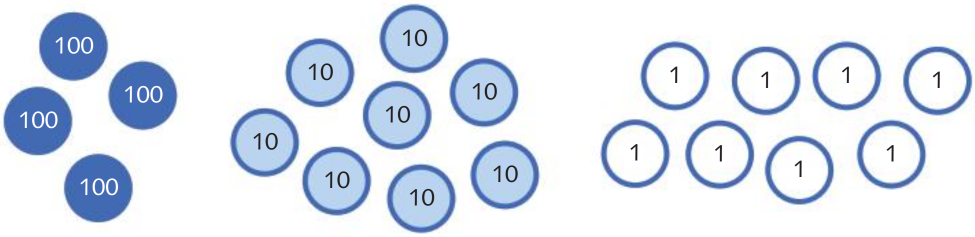
2 Draw arrays to calculate each division and the remainder (if there is one).

a $18 \div 3$ **b** $39 \div 4$ **c** $121 \div 5$ **d** $86 \div 7$

4E The long division algorithm

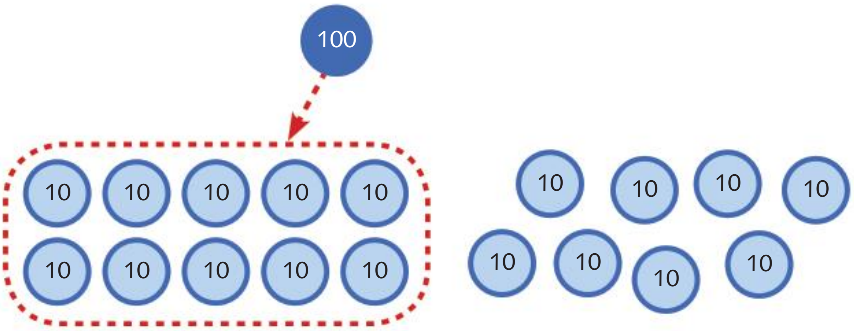
The division algorithm is unusual, as it starts on the left of the number and shares out the big pieces first. We can see this if we divide 488 by 3.

This picture shows the number 488.



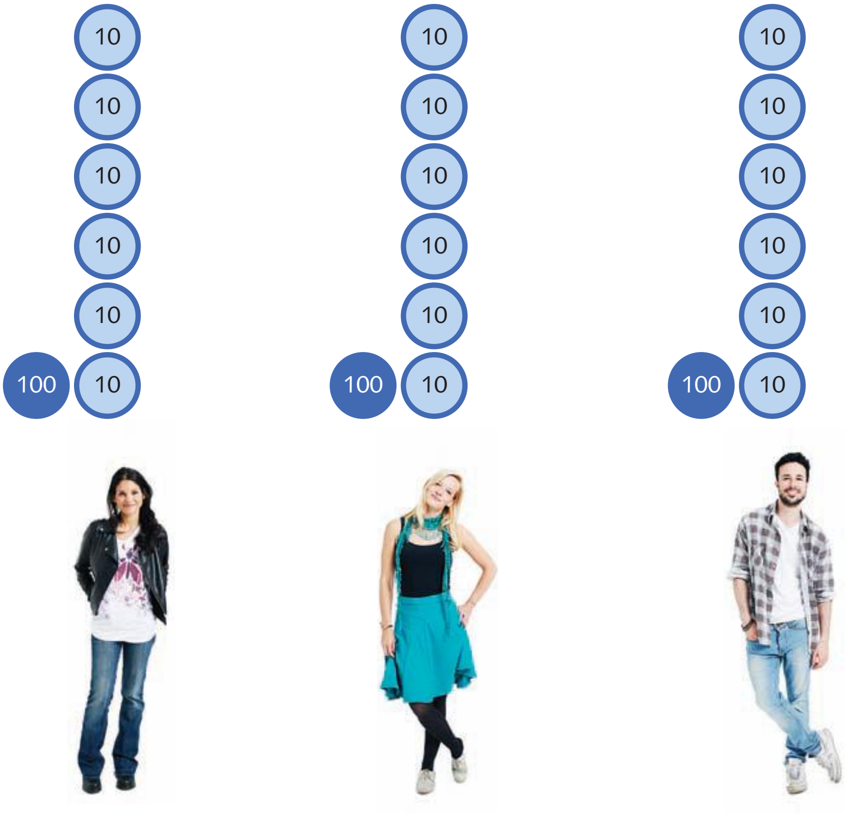
To divide 488 by 3 we try to make 3 equal groups. We begin with the hundreds. There are 4 hundreds. When we share 4 hundreds between 3 people, each person's share is 1 hundred and there is 1 hundred left over.

Now we deal with the 1 hundred and the 8 tens. Convert the 1 hundred into 10 tens and try to make 3 equal groups.

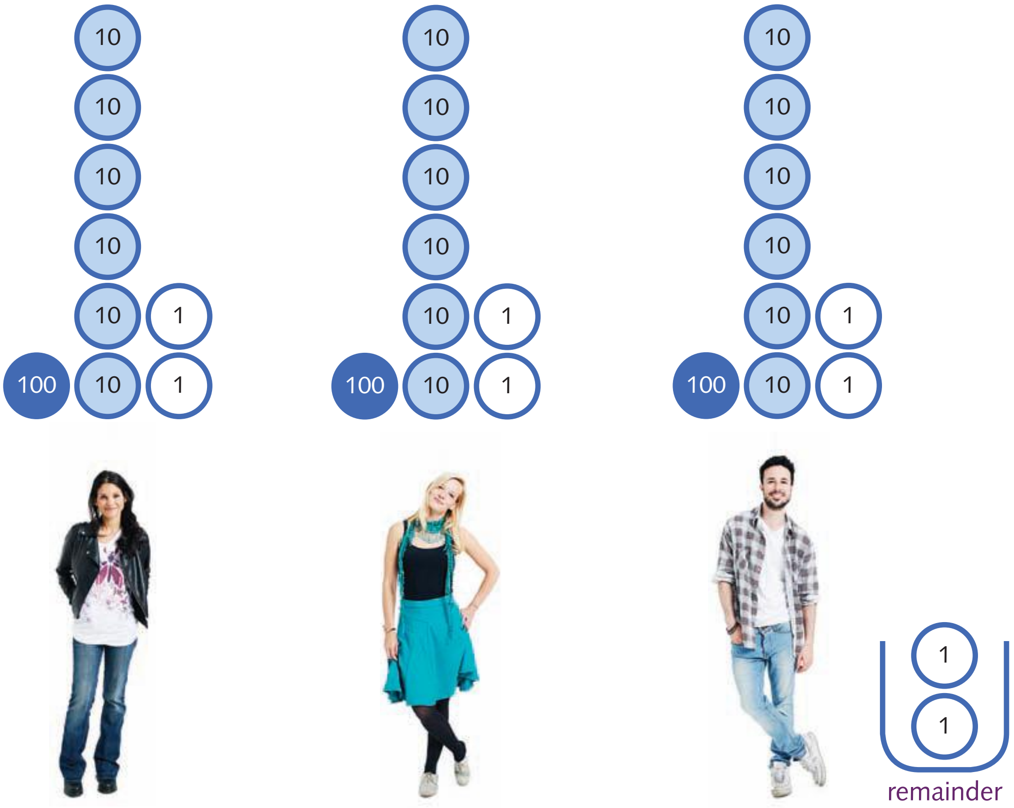


There are 18 tens. When we share 18 tens between 3 people, each person's share is 6 tens.

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press



There are 8 ones to share. When we share 8 ones between 3 people, each person gets 2, with 2 left over.



ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

When we share 488 among 3 people, each person gets 1 hundred, 6 tens and 2 ones and there are 2 ones left over. This is the same as 162 remainder 2.

$$\begin{aligned} 488 \div 3 &= 1 \text{ hundred} + 6 \text{ tens} + 2 \text{ ones, remainder } 2 \\ &= 162 \text{ remainder } 2 \end{aligned}$$

We record this using the algorithm. Set out the dividend and the divisor as shown.

	Hundreds	Tens	Ones
	1		
3)	4	8	8
	3	0	0
	1	8	1

Start with the hundreds. There are 4 hundreds. This is 3 lots of 1 hundred with 1 hundred left over. We shorten that and say 3 goes into 4 once. Write '1' in the hundreds column.

Say 1 hundred × 3 is 3 hundreds.

Write '300' on the next line. Subtract 300 from 488 and write '188'.

	Hundreds	Tens	Ones
	1	6	
3)	4	8	8
	3	0	0
	1	8	8
	1	8	0
			8

Next, deal with the tens. There are 18 tens. This is 3 lots of 6 tens. We shorten that and say 3 goes into 18 six times.

Write '6' above the line in the tens place.

Say 6 tens × 3 is 18 tens.

Subtract 180 from 188 and write '8' on the next line.

	Hundreds	Tens	Ones	
	1	6	2	r2
3)	4	8	8	
	3	0	0	
	1	8	8	
	1	8	0	
			8	
			6	
			2	

Now work with the ones. There are 8 ones. This is 3 lots of 2 ones. We say 3 goes into 8 two times with 2 left over.

Write '2' above the line in the ones place.

Say 2 ones × 3 is 6 ones. Subtract 6 ones from 8 ones.

Write '2' on the next line.

Write remainder 2 ('r2') on the top line. The calculation is complete.

488 ÷ 3 = 162 remainder 2

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

We can check the division by using the multiplication algorithm and adding the remainder.

$$\begin{array}{rrrr} & 1_1 & 6 & 2 \\ \times & & & 3 \\ \hline & 4 & 8 & 6 \\ \hline \end{array}$$

486 + 2 = 488, so the calculation is correct.

Long division can be used to divide by numbers that have more than one digit.

The example below shows a different way to set out long division.

Example 4

Calculate 438 ÷ 17 using long division. Use multiplication to check your work.

Solution

It is a good idea to calculate all of the multiples of 17 up to 9 × 17 first. You should be able to do this mentally using the strategies discussed earlier in 4A. For example, $6 \times 17 = 6 \times 10 + 6 \times 7 = 60 + 42 = 102$.

$$\begin{array}{rrrrl} & \text{H} & \text{T} & \text{O} & \\ & & 2 & 5 & \text{r}13 \\ \cline{2-5} 17) & 4 & 3 & 8 & \\ & 3 & 4 & \downarrow & \\ \cline{2-4} & & 9 & 8 & \\ & & 8 & 5 & \\ \cline{3-4} & & 1 & 3 & \\ \cline{3-4} \end{array}$$

17 into 4 won't go.

How many 'lots of 17' in 43?
At least 2, but not 3.

2 lots of 17 is 34. Subtract 34 from 43.

Write '9' and 'bring down' the 8.

How many 'lots of 17' in 98?
At least 5, but not 6.

5 lots of 17 is 85. Subtract 85 from 98.
13 remains.

We can make no more groups of 17.

$1 \times 17 = 17$
$2 \times 17 = 34$
$3 \times 17 = 51$
$4 \times 17 = 68$
$5 \times 17 = 85$
$6 \times 17 = 102$
$7 \times 17 = 119$
$8 \times 17 = 136$
$9 \times 17 = 153$

438 ÷ 17 = 25 remainder 13

Check that this is correct by multiplying 25 × 17 and then adding the remainder.

$$\begin{array}{rrrr} & & 2_3 & 5 \\ \times & & 1 & 7 \\ \hline & 1 & 7 & 5 \\ & 2_1 & 5 & 0 \\ \hline & 4 & 2 & 5 \\ \hline \end{array}$$

425 + 13 is 438, so the calculation is correct.

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

4E Whole class CONNECT, APPLY AND BUILD

1 Use place-value blocks to calculate these.

a 127 ÷ 18
b 2634 ÷ 23
c 4081 ÷ 39
d 8392 ÷ 97

2 How many would each student in your class receive if they had an equal share of:

a 298 balloons?
b 325 lollies?
c 1046 books?
d 2834 marbles?

4E Individual

1 Calculate these using the long division algorithm. Use multiplication to check your work.

a 848 ÷ 4
b 435 ÷ 5
c 912 ÷ 8
d 111 ÷ 37
e 1728 ÷ 32
f 10 488 ÷ 12
g 1189 ÷ 29
h 41 608 ÷ 56

2 Use the long division algorithm to calculate these divisions. Use multiplication to check your work.

a 848 ÷ 3
b 435 ÷ 6
c 912 ÷ 7
d 111 ÷ 21
e 294 ÷ 43
f 1084 ÷ 67
g 24 842 ÷ 72
h 38 583 ÷ 101

3 Divide 20 000 by:

a 4
b 13
c 26
d 125

4 Bernadette is a kindergarten teacher. She teaches 92 kindergarten children in total. Bernadette divides her children into 4 groups. How many children are in each group?

5 Lorraine cuts the hair of 102 clients each week. She works 6 days per week and sees the same number of clients each day. How many haircuts does Lorraine give in a day?

6 Trevor works in a hospital with 984 beds. There are 8 beds in a ward. How many wards are there?

7 Larry's company sold 1005 fridges during a 5-day working week. They sold the same number of fridges each day. How many fridges did Larry's company sell in one day?

8 There are 435 light globes in a crate. How many boxes of 72 light globes is this?

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

4F The short division algorithm

You can use the short division algorithm when you are dividing by a 1-digit number. Short division involves the same amount of work as long division, but you do most of it in your head.

If we want to divide 392 by 8, we can use long division.

$$
\begin{array}{r@{\,}r@{\;\;}r@{\;\;}r}
 & & 4 & 9 \\ \cline{2-4}
8\big) & 3 & 9 & 2 \\
 & 3 & 2 & \downarrow \\ \cline{2-4}
 & & 7 & 2 \\
 & & 7 & 2 \\ \cline{3-4}
 & & & 0 \\ \cline{3-4}
\end{array}
$$

Or we can use short division.

$$
\begin{array}{r@{\,}r@{\;\;}r@{\;\;}r}
 & & 4 & 9 \\ \cline{2-4}
8\big) & 3 & 9 & {}^{7}2
\end{array}
$$

8 into 3 hundreds.

We do not have enough hundreds.
Convert 3 hundreds and 9 tens to 39 tens.
8 into 39 tens is 4 tens with 7 tens left over.

Write '4' (tens) above the line and carry the 7.
7 tens and 2 ones is 72 ones.

Now divide 8 into 72.
72 ÷ 8 = 9, so write '9' (ones).

392 ÷ 8 = 49

Example 5

Calculate 228 ÷ 7. Use multiplication to check your work.

Solution

Do the division. Check by multiplying. Then add the remainder.

$$
\begin{array}{r@{\,}r@{\;\;}r@{\;\;}r@{\;}l}
 & & 3 & 2 & \text{r}\,4 \\ \cline{2-4}
7\big) & 2 & 2 & {}^{1}8 &
\end{array}
\qquad\qquad
\begin{array}{r@{\;\;}r@{\;\;}r}
 & 3^{1} & 2 \\
\times & & 7 \\ \hline
2 & 2 & 4 \\ \hline
\end{array}
\qquad\qquad
224 + 4 = 228
$$

Factors and division

The multiplication statement:

$4 \times 3 = 12$

is equivalent to the division statement:

$12 \div 3 = 4$

When we divide 12 by 3, we get the answer 4, with zero remainder.

4 and 3 are factors of 12 because they divide into 12 exactly.

We can use short division to test whether we have a factor of a larger number. For example, a number is divisible by 7 if it is a multiple of 7. So a number is divisible by 7 if the remainder is 0 when it is divided by 7.

If you want to find some factors of a number, start by dividing that number by small prime numbers. For example, divide 441 by 3 using the short division algorithm.

$$\begin{array}{r} 1\ \ 4\ \ 7 \\ 3\overline{)4\ {}^{1}4\ {}^{2}1} \end{array}$$

The remainder is 0 and the division is exact.

$441 \div 3 = 147$

The equivalent multiplication statement is:

$147 \times 3 = 441$

This tells us that 3 is a factor of 441 and 147 is also a factor of 441.

Example 6

a Divide 322 by 7 to find out whether 7 is a factor of 322.

b Divide 274 by 7 to find out whether 7 is a factor of 274.

Solution

If a number is divisible by 7, then 7 is a factor of that number.

If a number is *not* divisible by 7, then 7 is *not* a factor of that number.

a

$$\begin{array}{r} 4\ \ 6 \\ 7\overline{)3\ \ 2\ {}^{4}2} \end{array}$$

7 is a factor of 322.

$322 \div 7 = 46$ (remainder 0)

So, $46 \times 7 = 322$

and 322 is divisible by 7.

b

$$\begin{array}{r} 3\ \ 9\ \ \text{r}1 \\ 7\overline{)2\ 7\ {}^{6}4} \end{array}$$

7 is not a factor of 274.

$274 \div 7 = 39$ remainder 1

274 is not a multiple of 7.

7 is not a factor of 274.

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7

4F Individual

○ **1** Use the short division algorithm to calculate these. Use multiplication to check your answers.

a $848 \div 4$ **b** $435 \div 5$ **c** $912 \div 8$ **d** $20\,124 \div 9$

● **2** Which of these numbers have 9 as a factor?

19 29 36 81 73 98 108

● **3** Copy these statements and fill in the blanks.
The first two have been done for you.
You will need to do the short division in each statement.

a I divided 127 by 6 and got remainder 1.

So 6 is not a factor of 127.

b I divided 216 by 6 and got remainder 0.

So 6 is a factor of 216.

c I divided 313 by 3 and got remainder ___.

So 3 ___ a factor of 313.

d I divided 414 by 3 and got remainder ___.

So 3 ___ a factor of 414.

e I divided 413 by 3 and got remainder ___.

So 3 ___ a factor of 413.

f I divided 414 by 9 and got remainder ___.

So 9 ___ a factor of 414.

4G Tests for divisibility

Divisibility by 10 and 5

We have already seen that a number that ends in 0 is a multiple of 10.

For example:

$$2 \times 10 = 20 \text{ and } 20 \div 10 = 2$$

$$127 \times 10 = 1270 \text{ and } 1270 \div 10 = 127$$

Numbers that end in 0 can be divided exactly by 10. The number 10 divides these numbers with zero remainder.

A number is divisible by 10 if it ends in 0.

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

A number that ends in 5 or 0 is a multiple of 5.

$2 \times 5 = 10$ and $10 \div 5 = 2$

$125 \times 5 = 625$ and $625 \div 5 = 125$

All numbers that end in 0 or 5 are divisible by 5. The number 5 divides these numbers with 0 remainder.

A number is divisible by 5 if it ends in 5 or 0.

Divisibility by 2, 4 and 8

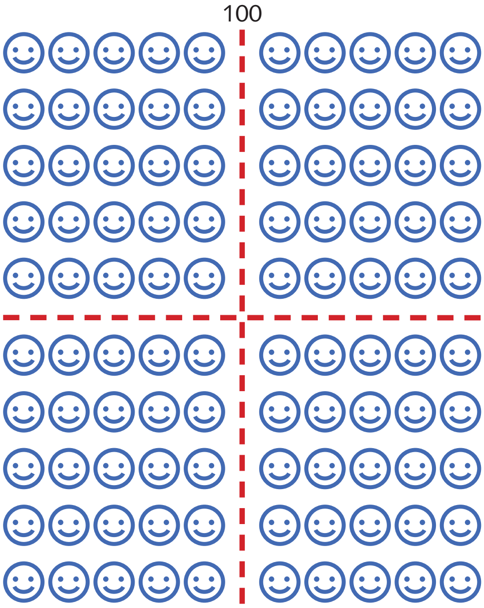
Even numbers are multiples of 2. Even numbers end in 0, 2, 4, 6 or 8. Because they are multiples of 2, even numbers are also divisible by 2.

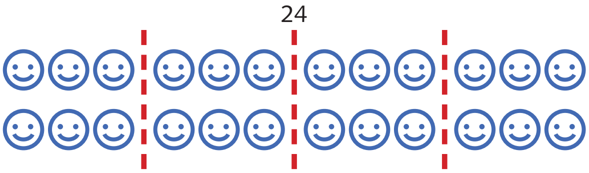
$13 \times 2 = 26$ and $26 \div 2 = 13$

$148 \times 2 = 296$ and $296 \div 2 = 148$

A number is divisible by 2 if it ends in 0, 2, 4, 6, or 8.

If you draw an array to show 100, it shows that 100 is divisible by 4. If you then draw another array to show 24, you can see that 124 is divisible by 4.





Any number of hundreds is divisible by 4, because 100 is divisible by 4.

$100 = 4 \times 25$ $\quad 200 = 4 \times 50$ $\quad 300 = 4 \times 75$ $\quad 400 = 4 \times 100$

So, you only need to think about the last two digits to find if a number is divisible by 4.

A number is divisible by 4 if its last two digits make a number that is divisible by 4.

Divide 1000 by 8:

$$\begin{array}{r} 1\ \ \ 2\ \ \ 5 \\ 8\overline{)1\ 0\ {}^{2}0\ {}^{4}0} \end{array}$$

Any number of thousands is divisible by 8, because 1000 is divisible by 8. So, you only need to think about the last three digits to find if a number is divisible by 8. Use the division algorithm to do this if necessary.

> A number is divisible by 8 if the last three digits make a number divisible by 8.

Divisibility by 3, 6 and 9

Look at the first few multiples of 3 after 9. What happens if you add the digits in each of these numbers?

Number	Sum of its digits
12	$1 + 2 = 3$
15	$1 + 5 = 6$
18	$1 + 8 = 9$
21	$2 + 1 = 3$
24	$2 + 4 = 6$
27	$2 + 7 = 9$
30	$3 + 0 = 3$
33	$3 + 3 = 6$

The answers are all divisible by 3.

If the sum of its digits is divisible by 3, the number is divisible by 3.

Why is this rule true? The key to the rule is that:

$$100 = 99 + 1 \text{ and } 10 = 9 + 1$$

Any multiple of 3 is divisible by 3, so 9 and 99 are divisible by 3.

Look at the number 132.

$$\begin{aligned} 132 &= 100 + 30 + 2 \\ &= 99 + 1 + (3 \times 9) + 3 + 2 \\ &= 99 + (3 \times 9) + 1 + 3 + 2 \end{aligned}$$

We know that 99 is divisible by 3 and 9 is divisible by 3.

If $1 + 3 + 2$ is divisible by 3 then 132 is divisible by 3. $1 + 3 + 2 = 6$, and 6 is divisible by 3, so 132 is divisible by 3.

> A number is divisible by 3 if the sum of its digits is divisible by 3.

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

Example 7

Is 189 divisible by 3? Use these two methods to find out.

a Divide 189 by 3.

b Use the divisibility test.

Solution

a

$$\begin{array}{r} 6\ 3 \text{ r}0 \\ 3\overline{)1\,8\,9} \end{array}$$

So 189 is divisible by 3.

b Add the digits.

$1 + 8 + 9 = 18$

18 is divisible by 3.

So 189 is divisible by 3.

If a number is divisible by 2 and 3 then it must be divisible by 6, since 2 × 3 = 6.

> A number is divisible by 6 if it is even and divisible by 3.

Example 8

Test these numbers for divisibility by 6.

a 324

b 106

c 163

Solution

a 324 is an even number, so it is divisible by 2. The sum of its digits is $3 + 2 + 4 = 9$, and 9 is divisible by 3. So 324 is divisible by 2 and 3, and this tells us it is divisible by 6.

b 106 is an even number, so it is divisible by 2. The sum of its digits is $1 + 0 + 6 = 7$, so 106 is not divisible by 3. 106 is not divisible by 6.

c 163 is an odd number and not divisible by 2. So 163 is not divisible by 6.

The test for divisibility by 9 is similar to the test for divisibility by 3. We add the digits together and check if the sum is divisible by 9.

Number	Sum of its digits
573	$5 + 7 + 3 = 15$
201 006	$2 + 0 + 1 + 0 + 0 + 6 = 9$

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

The sum of the digits for 573 is not divisible by 9, so 573 is not divisible by 9.

The sum of the digits for 201 006 is divisible by 9, so 201 006 is divisible by 9.

A number is divisible by 9 if the sum of its digits is divisible by 9.

Divisibility by 7

There is no easy test for divisibility by 7. So if we want to check to see whether a number is divisible by 7, we do a short division.

Divisibility to find prime numbers

We can use divisibility tests to help us find prime numbers.

Example 9

Find the prime factorisation of 999.

Solution

9 is a factor, as $999 = 9 \times 111$

$9 = 3 \times 3$

111 is divisible by 3 because $1 + 1 + 1 = 3$

Divide 111 by 3:

$$\begin{array}{r} 3\ \ 7 \\ 3\overline{)1\ \ 1\ \ {}^{2}1} \end{array}$$

$111 = 3 \times 37$ and 37 is a prime number.

So the prime factorisation is: $999 = 3 \times 3 \times 3 \times 37$

Divisibility by a composite number

If we know that a number is divisible by a composite number, then it follows that the number is also divisible by the prime factors of that composite number.

We can test this idea using the number 234 and one of its factors, 6.
We know that 6 is a composite number, because it has factors 1, 2, 3 and 6.
First, we establish that 6 is a factor of 234, using division.

$$\begin{array}{r} 3\ \ 9 \\ 2\overline{)2\ \ 3\ \ {}^{5}4} \end{array}$$

$234 \div 6 = 39$, so 6 is a factor of 234.

Now we test to see if 2 and 3 are factors of 234 using division.

$$\begin{array}{r} 1\ 1\ \ 7 \\ 2\overline{)2\ \ 3\ \ {}^{1}4} \end{array} \qquad \begin{array}{r} 7\ \ 8 \\ 3\overline{)2\ \ 3\ \ {}^{2}4} \end{array}$$

The number 234 is divisible by 6 and by 2 and 3. The connection is that $2 \times 3 = 6$.

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

So, a number that is divisible by a composite number is also divisible by the factors of that composite number. Sometimes these factors are prime factors as in the case of 2 and 3 above.

It follows that if a number is divisible by two numbers, it is also divisible by the product of those two numbers. For example, 100 is divisible by 2 and 5. It is also divisible by the product of 2 and 5, which is 10.

4G Whole class CONNECT, APPLY AND BUILD

1 As a class, write statements that demonstrate the following.

A number that is divisible by a composite number is also divisible by the factors of that composite number.

For example, '64 is divisible by 2 and 8, so it is divisible by 16' and '3 and 13 are prime factors of 156, so 39 is also a factor of 156'.

2 Copy the 4-digit number 4__2__ and fill in the blanks to make a number that is divisible by:

a 3
b 2
c 4
d 5
e 10
f 8
g 2 and 5
h 3, but not 4

4G Individual

1 Use the short division algorithm to calculate these. Use multiplication to check your answers.

a 49 ÷ 3
b 89 ÷ 3
c 111 ÷ 3
d 313 ÷ 3
e 891 ÷ 3

2 Use the divisibility test to work out whether these numbers are divisible by 3.

49 89 111 313 891

3 Test these numbers for divisibility by 5.

25 40 556 18 200 25 387

4 Write the prime factorisation for each number.

a 138
b 986
c 1118
d 20 790

5 For each of the numbers at the top of the next page, copy and complete the two statements about factors and divisibility. The first number has been done for you.

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

The number ___ is divisible by the prime numbers ___ and ___, so it is also divisible by___.

Also, ___ and ___ are prime factors of ___, so ___ is also a factor of ___.

a 63 The number 63 is divisible by the prime numbers 7 and 3, so it is also divisible by 21. Also, 7 and 3 are prime factors of 63, so 21 is also a factor of 63.

b 96 **c** 72 **d** 200 **e** 1155

4H Review questions

1 **a** Multiply these numbers by 4.

13 25 43 32 76 128

b Multiply these numbers by 8.

14 61 28 73 82 107

c Multiply these numbers by 10.

31 93 476 1028 6420 572 179 863

d Multiply these numbers by 9.

15 23 257 891 1379 32 971

e Multiply these numbers by 11.

16 27 74 265 539 897

f Multiply these numbers by 5.

(Hint: Use different strategies for odd and even numbers.)

18 29 34 87 122 143 395 476

g Multiply these numbers by 20.

7 26 48 33 170 239

h Multiply these numbers by 19.

6 22 36 44 123 340

i Multiply these numbers by 21.

8 21 28 35 382 413

j Multiply these numbers by 30.

9 29 57 43 194 715

k Multiply these numbers by 6.

17 31 56 130 309 600

l Multiply these numbers by 25.

5 9 12 40 90 120

2 Complete this multiplication table.

×	30	40	70	80
2				
6				
7				
9				

3 Draw a multiplication diagram and complete the calculation for each of these.

a 5×13 **b** 12×18 **c** 26×19 **d** 17×14

4 Calculate these multiplications.

a 13×16 **b** 34×38 **c** 41×47 **d** 69×63

e 73×98 **f** 136×52 **g** 390×68 **h** 519×763

i 956×689 **j** 7777×243 **k** 4075×3006 **l** 6173×5748

5 The population of Winton in the year 2000 was 2476 people. If the population increases by 37 people each year, what will the population be in:

a 2015? **b** 2020? **c** 2027?

6 A car park can hold 150 cars. If each driver pays $6.00 per day and the car park is full every day, how much money will the car park owner collect in:

a 5 days? **b** 1 week? **c** 3 weeks?

7 Copy and complete these statements.

a $5 \times 10 = 50$, so $50 \div 10 = \underline{\qquad}$ and $50 \div 5 = \underline{\qquad}$

b $3 \times 7 = 21$, so $21 \div 3 = \underline{\quad}$ and $21 \div 7 = \underline{\qquad}$

c $9 \times 6 = 54$, so $54 \times 9 = \underline{\qquad}$ and $54 \div 6 = \underline{\qquad}$

d $11 \times 8 = 88$, so $88 \div 11 = \underline{\qquad}$ and $88 \div 8 = \underline{\qquad}$

e $256 \times 49 = 12544$, so $12544 \div 256 = \underline{\qquad}$ and $12544 \div 49 = \underline{\qquad}$

8 Do the corresponding multiplication to check if each division calculation is correct.

a $276 \div 12 = 23$ **b** $132 \div 11 = 12$ **c** $3240 \div 72 = 44$

d $945 \div 35 = 27$ **e** $6942 \div 78 = 88$ **f** $7182 \div 63 = 114$

9 Draw arrays to calculate each division and the remainder (if there is one).

a $14 \div 4$ **b** $33 \div 10$ **c** $47 \div 7$ **d** $102 \div 8$ **e** $113 \div 9$

10 Callum is making up 17 party bags. Find the number of each item per bag and the remainder if there are:

a 89 snakes **b** 36 jelly beans **c** 112 sour glow worms **d** 224 Smarties

11 Copy these statements and fill in the missing numbers. (The missing numbers are a divisor, a quotient or a remainder.)

a $17 = 3 \times \underline{\qquad} + 2$ **b** $37 = 4 \times 9 + \underline{\qquad}$

c $45 = 12 \times \underline{\qquad} + \underline{\qquad}$ **d** $169 = 16 \times \underline{\qquad} + \underline{\qquad}$

e $16 \div 5 = 3$ remainder $\underline{\qquad}$ **f** $43 \div 5 = \underline{\qquad}$ remainder $\underline{\qquad}$

g $274 \div 10 = \underline{\qquad}$ remainder $\underline{\qquad}$ **h** $175 \div 3 = \underline{\qquad}$ remainder $\underline{\qquad}$

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

12 Calculate these divisions using the long division algorithm. Check your work by multiplying.

a 735 ÷ 3 **b** 582 ÷ 6 **c** 399 ÷ 7 **d** 216 ÷ 27

e 2751 ÷ 49 **f** 11535 ÷ 14 **g** 9801 ÷ 23 **h** 31009 ÷ 89

13 Divide 15000 by:

a 25 **b** 17 **c** 42 **d** 150

Check your work by multiplying.

14 Elsie teaches a total of 96 recorder students. She has 16 groups of students altogether. How many students are in each group?

15 Lauren swims 1500 metres every time she goes to swimming training. If Lauren swam the same distance each day for 6 days, how far would she swim?

16 Use the short division algorithm to calculate these.

a 628 ÷ 4 **b** 795 ÷ 5

c 928 ÷ 8 **d** 26 019 ÷ 9

e 628 ÷ 3 **f** 795 ÷ 6

g 928 ÷ 7 **h** 26 019 ÷ 8

17 **a** Which of these numbers has 9 as a factor? Why?

12 26 45 72 89 99 123

b Which of these numbers has 6 as a factor? Why?

21 42 76 132 197 438 891

18 Use a divisibility test for each of the following.

a Which of these numbers are divisible by 3?

79 211 303 222222 222300 147368 389012

b Which of these numbers are divisible by 5?

27 60 436 72150 37979

19 Which of the numbers in the box below are divisible by:

a 2? **b** 3? **c** 4? **d** 5? **e** 9?

33 48 76 80 30 399 200 563 927

20 List the prime numbers that are:

a between 25 and 45

b less than 100 and contain the digit 7

c between 10 and 30

d less than 100 and contain the digit 1

21 Why isn't 219 a prime number?

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

CHAPTER 5 Preparation

Useful skills for this chapter:

- being able to find the sums and products of numbers
- quick recall of multiplication facts to 12 × 12
- the ability to measure length and distances in millimetres, centimetres and metres using rulers and tape measures
- the ability to convert a measurement in centimetres to metres, or to metres and centimetres
- experience with reading scales on tape measures
- experience with measuring containers
- previous experience drawing three-dimensional objects.

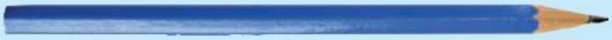
Show what you know

1 Which pencil has a length of 9 cm?

A

B

C



2 Which pencil has a length of 65 mm?

A

B

C

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

Measurement and Geometry

Length, perimeter, area and volume

If we want to know how long or wide or high something is, we can measure it with a measurement tool.

We use a ruler to measure short lengths, such as the width of your hand.

A tape measure can be used to measure longer lengths, such as your height.

Surveyors use a tool called a theodolite to measure long distances, like the length of a road.

No matter what tool we use, we can find out something's size and then use maths to work out other information about it.

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

5A Length

In Australia we use the **metric system** of measurement.

The units of length in the metric system are based on the metre.

In 1790, the French Academy of Sciences decided that one metre would be defined as $\frac{1}{10\,000\,000}$ of the distance from the equator to the north pole, measured along the meridian that runs through Paris.

Today we use the International Bureau of Weights and Measures definition of one metre: it is the distance travelled by light in $\frac{1}{299\,792\,458}$ of a second. You will often see the word 'metre' abbreviated as the letter **m**. As well as the metre, we also use the millimetre (mm), the centimetre (cm) and the kilometre (km) to measure length. The prefix before 'metre' tells you about the size of the unit being used.

milli	means $\frac{1}{1000}$	so 1 millimetre is $\frac{1}{1000}$ of a metre. When using decimals: 1 mm = 0.001 m
centi	means $\frac{1}{100}$	so 1 centimetre is $\frac{1}{100}$ of a metre. When using decimals: 1 cm = 0.01 m
kilo	means 1000	so 1 kilometre is 1000 metres.

Choosing units

There are two important things to remember when measuring length. First, you need to select the most suitable unit of measurement for the object you want to measure. Second, you need to select the right measuring instrument.

For example, if you want to knit a pair of gloves, it would not be very sensible to measure the width of your hand in whole metres. It would be more appropriate to use centimetres or millimetres.

Builders, furniture makers, architects and electricians nearly always use millimetres, even for very large measurements. Most tape measures are marked out in millimetres.

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

Example 1

Measure the length of this pencil in millimetres.

Solution

The pencil is 97 millimetres long.

When making any measurement, we can only be as accurate as our measuring instrument allows. The pencil in **Example 1** might actually be 96.769 38 mm long. But if the smallest unit on the ruler we are using is millimetres, then we must make our measurement to the nearest millimetre.

Example 2

Measure the length of this pencil to the nearest centimetre.

Solution

The pencil is 10 centimetres long.

Notice that the measurement has been rounded up to whole centimetres because it is closer to 10 cm than 9 cm.

When you are measuring and want to compare measurements or add or subtract them, always use the same units.

5A Whole class CONNECT, APPLY AND BUILD

1 **a** Estimate the height of your teacher in centimetres.

b Convert this measurement to millimetres.

c Now write your estimate in metres and centimetres.

d Compare your answers to parts **a** to **c** with the estimates of three of your classmates.

e Estimate the difference between your height and your teacher's height.

f Measure your teacher's height. Which unit will give you the most accurate measurement?

g Measure your own height. How close was your estimate?

2 Estimate the length of each item, then measure it as accurately as you can.

a The height of this book in millimetres

b The width of your classroom in metres

3 **a** Without using a ruler, estimate and cut a piece of paper streamer 10 cm in length.

b If you knew that the width of your hand was about 10 cm, would you change your estimate?

c Measure your streamer to check your estimate.

4 Without using a ruler, estimate and cut pieces of paper streamer to these lengths. Write your estimate on each streamer.

a 30 mm **b** 50 mm **c** 20 cm **d** 60 cm **e** 1 m

f Now ask a friend to use a ruler or tape measure to check your estimates, and to write the new measurement on each streamer.

g How close were your estimates?

5A Individual

1 Write the unit you would use to measure each of these items. Then write the name of the measuring instrument you would use to make the measurement.

a The length of the playground

b The length of your shoe

c The distance from Australia to New Zealand

d The length of a grain of rice

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

2 List three objects in your room that have a length between:

a 1 metre and 2.5 metres

b 25 centimetres and 100 centimetres

c 75 millimetres and 150 millimetres

d 3.5 metres and 4 metres

3 Write the length of each pencil in centimetres.

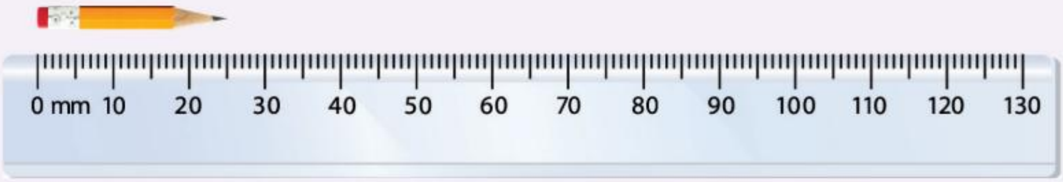
a

b

c

4 Write the length of each pencil in millimetres.

a



b

5 **a** Jenny cut a 70-centimetre piece of string from a length of 10 metres. How much string was left?

b Then Ali cut a piece of string 125 centimetres long from the string left over after Jenny cut her piece. How much of the original string was left?

6 Tina is making a frame for a portrait she painted at school. She needs 2 pieces of timber 240 mm in length and 2 pieces of timber 180 mm in length.

a What is the total length of timber frame Tina needs in millimetres?

b If Tina cut the pieces from a 1-metre length of frame, how much would be left over?

5B Converting length measurements

We use different units to measure the length of different kinds of objects. In many cases, we could use two different units to measure the same object, so it is important to be able to change from one unit to another.

Metres and centimetres

We convert *metres to centimetres* by changing each metre into 100 centimetres. This is the same as multiplying by 100.

Example 3

Convert 6.75 metres to centimetres.

Solution

$$\begin{aligned} 6.75\text{ m} &= (6.75 \times 100)\text{ cm} \\ &= 675\text{ cm} \end{aligned}$$

We know that 1 metre is equal to 100 centimetres, so to convert from *centimetres to metres* we make ‘lots’ of 100 centimetres. For example:

$$\begin{aligned} 456\text{ cm} &= 400\text{ cm} + 56\text{ cm} \\ &= 4\text{ lots of }100\text{ cm} + 56\text{ cm} \\ &= 4\text{ m }56\text{ cm} \\ &= 4.56\text{ m} \end{aligned}$$

This is the same as dividing by 100.

$$\begin{aligned} 456\text{ cm} &= \frac{456}{100}\text{ m} \\ &= 4.56\text{ m} \end{aligned}$$

Example 4

Convert 675 centimetres to metres.

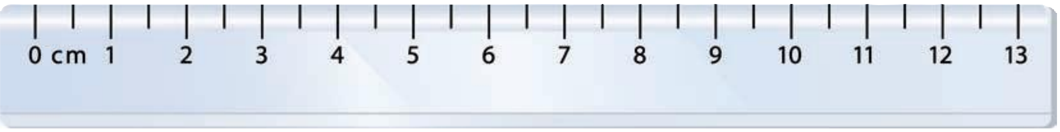
Solution

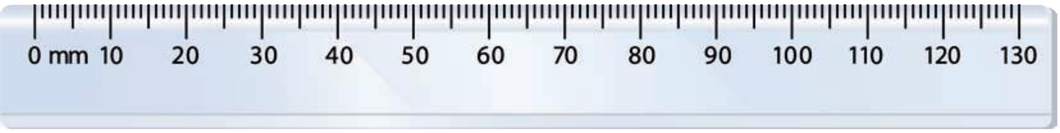
$$\begin{aligned} 675\text{ cm} &= \frac{675}{100}\text{ m} \\ &= 6.75\text{ m} \end{aligned}$$

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

Metres, centimetres and millimetres

There are 100 centimetres in 1 metre and 1000 millimetres in 1 metre, so we know that there are 10 millimetres in 1 centimetre.





To convert *from centimetres to millimetres*, we make 'lots' of 10 millimetres. This is the same as dividing by 10. The amount left over is written as a decimal part of a centimetre.

To convert from *centimetres* to *millimetres* we multiply by 10.

Example 5

a Convert 75 millimetres to centimetres.

b Convert 105 centimetres to millimetres.

Solution

a

$$\begin{aligned} 75\text{ mm} &= \frac{75}{10}\text{ cm} \\ &= 7.5\text{ cm} \end{aligned}$$

b

$$\begin{aligned} 105\text{ cm} &= (105 \times 10)\text{ mm} \\ &= 1050\text{ mm} \end{aligned}$$

To convert from *millimetres to metres*, we make 'lots' of 1000 millimetres. This is the same as dividing by 1000. The amount left over is written as a decimal part of a metre.

Example 6

Convert 2350 millimetres to metres.

Solution

$$\begin{aligned} 2350\text{ mm} &= \frac{2350}{1000}\text{ m} \\ &= 2.350\text{ m or } 2.35\text{ m} \end{aligned}$$

Metres and kilometres

To convert *from metres to kilometres*, we make 'lots' of 1000 metres. This is the same as dividing by 1000. The amount left over is written as a decimal part of a kilometre.

Example 7

Convert 6850 metres to kilometres.

Solution

$$\begin{aligned} 6850\text{ m} &= \frac{6850}{1000}\text{ km} \\ &= 6.850\text{ km or } 6.85\text{ km} \end{aligned}$$

To convert *kilometres to metres*, multiply by 1000.

Example 8

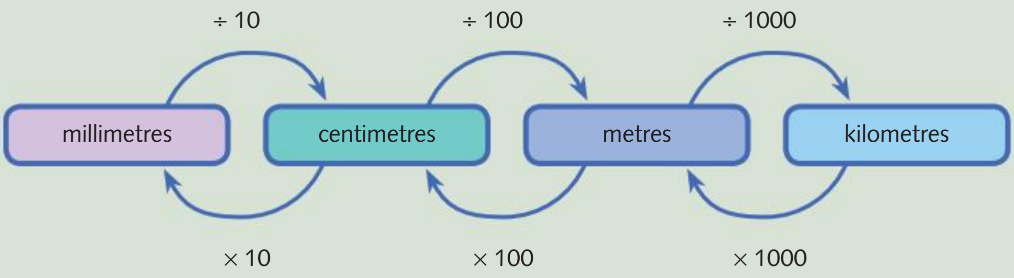
Convert 1.2 kilometres to metres.

Solution

$$\begin{aligned} 1.2\text{ km} &= (1.2 \times 1000)\text{ m} \\ &= 1200\text{ m} \end{aligned}$$

Remember

To convert from one unit to another we multiply or divide by 10, 100 or 1000:



ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

5B Whole class CONNECT, APPLY AND BUILD

1 **a** Use a tape measure to measure the height of your classroom door in centimetres.

b Convert your measurement to metres. Use the tape measure to check your answer.

2 Measure the width of your desk in millimetres.

3 Sylvia's classroom has 10 desks. Each desk measures 125 centimetres in width. The desks in Sylvia's classroom are placed side by side. Calculate their total width in:

a millimetres
b metres

5B Individual

1 Convert each measurement to metres.

Express your answer as a decimal if it is not a whole number.

a 200 cm
b 700 cm
c 425 cm
d 650 cm
e 75 cm
f 7 cm

2 Convert each measurement to centimetres.

Express your answer as a decimal if it is not a whole number.

a 30 mm
b 100 mm
c 387 mm
d 229 mm
e 5 mm
f 4080 mm

3 Convert each measurement to metres.

Express your answer as a decimal if it is not a whole number.

a 4000 mm
b 6000 mm
c 1750 mm
d 2535 mm
e 5 mm
f 635 mm

4 Convert each measurement to kilometres.

Express your answer as a decimal if it is not a whole number.

a 5000 m
b 9000 m
c 2475 m
d 6005 m
e 95 m
f 421 m

5 Convert each measurement to centimetres.

a 6 m
b 2 m
c 4.5 m
d 6.75 m
e 1.09 m
f 0.3 m

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

6 Convert each measurement to millimetres.

a 8 cm
b 11 cm
c 2.8 cm
d 0.2 cm
e 13.6 cm
f 0.8 cm

7 Convert each measurement to millimetres.

a 3 m
b 9 m
c 4.3 m
d 12.75 m
e 6.2 m
f 0.2 m

8 Convert each measurement to metres.

a 7 km
b 4 km
c 8.75 km
d 11.625 km
e 15.005 km
f 832.64 km

9 Martin bought two pieces of wood from the timber yard. One piece was 3 m 65 cm long. The other piece was 4 m 45 cm long. If both pieces of wood were placed end to end, what would their total length be?

10 Freda is 1 m 47 cm tall. Bill is 129 cm tall. Find their height difference in:

a metres
b centimetres
c millimetres

11 Kim bought a cable that was 4.25 metres long. She cut 6 pieces of cable 40 centimetres in length from it. What length of cable was left?

12 The top of Harry's house is 11.45 metres high. The jacaranda tree in his front yard is 4 m 70 cm higher than the top of his house. How high is the tree?

13 Troy is painting a weatherboard before fitting it to his house. The weatherboard is 5.1 metres long. Troy still has 85 centimetres to paint. What length has he painted so far?

14 Jenny is walking from Fishville to Codtown, a distance of 6.75 kilometres. She still has 1320 metres left to walk. How far has she walked already? Give your answer in kilometres.

15 Evan cut 3 lengths of decking measuring 2.5 metres, 1.65 metres and 895 millimetres. What was the total length in metres?

16 Winston's pace is 65 centimetres in length. How many metres would he travel in 15 paces?

17 Lulu cut 20 pieces of string from a ball of string. Each piece she cut was 1.55 metres in length. There are still 19 metres of string left on the ball of string, so how much did Lulu have to begin with?

18 Brittany placed 25 pencils in a row, end to end. Each pencil was 145 millimetres in length.

a What is the length of the row of pencils in millimetres?
b How much short of 5 metres is this?

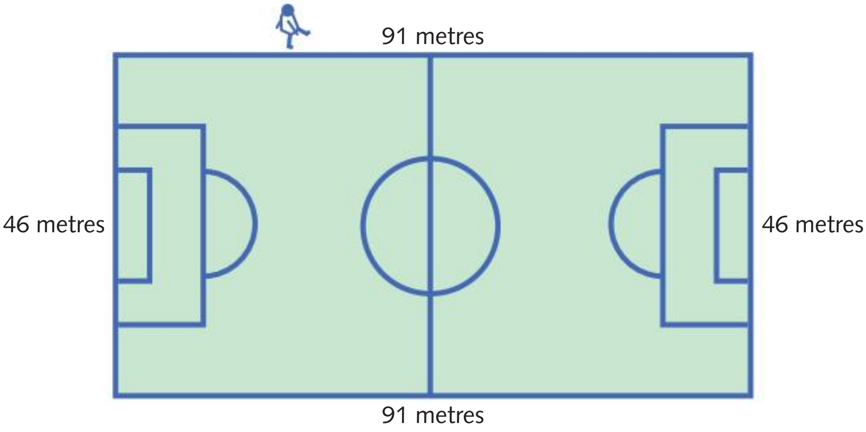
19 Ian swims 2.75 metres with each overarm stroke. How far does he travel in 200 strokes?

20 How many centimetres do you need to add to 16.25 metres to make 20 metres?

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

5C Calculating perimeter

The word ‘perimeter’ comes from two Greek words: *peri*, meaning ‘around’ and *metron*, meaning ‘measure’. So ‘perimeter’ means the measure or distance around something. It is the total length around the edge.

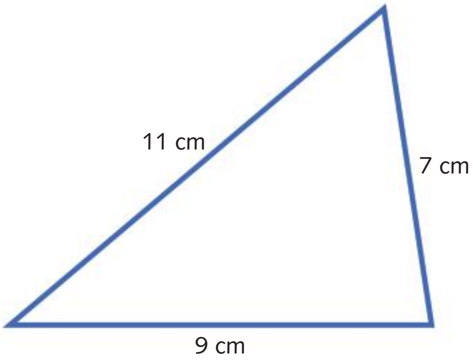


Imagine walking around the edge of a soccer pitch. You would walk 91 metres, 46 metres, 91 metres then 46 metres again. (These measurements are the minimum size for an international soccer pitch.)

The perimeter of the soccer pitch is the sum of these lengths.

$$\begin{aligned}\text{Perimeter} &= 91 + 46 + 91 + 46\\ &= 274 \text{ metres}\end{aligned}$$

The perimeter of any straight-sided shape is the sum of the lengths of its sides. Shapes with three or more straight sides, such as squares, triangles and hexagons, are also known as **polygons**.

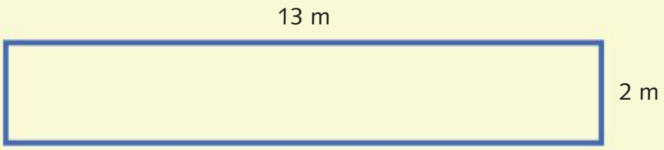


The perimeter of this triangle is calculated as follows.

$$\begin{aligned}\text{Perimeter} &= 7 + 9 + 11\\ &= 27 \text{ centimetres}\end{aligned}$$

Example 9

Calculate the perimeter of this rectangle.



ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

Solution

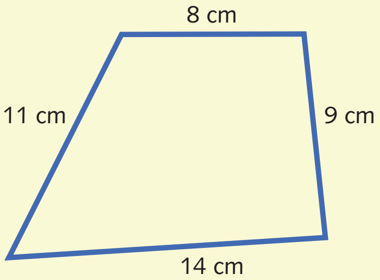
The perimeter of the rectangle is the sum of the length of its sides.

$$\begin{aligned}\text{Perimeter} &= 13 + 2 + 13 + 2\\ &= 30 \text{ m}\end{aligned}$$

The perimeter of the rectangle is 30 metres.

Example 10

Calculate the perimeter of this quadrilateral.



Solution

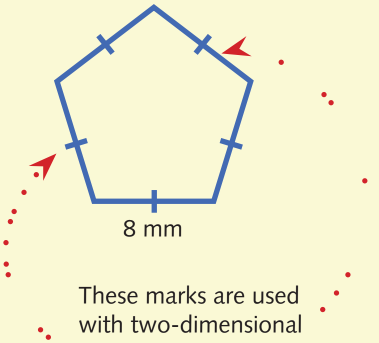
The perimeter of the quadrilateral is the sum of the lengths of its sides.

$$\begin{aligned}\text{Perimeter} &= 14 + 11 + 8 + 9\\ &= 42 \text{ cm}\end{aligned}$$

The perimeter of the quadrilateral is 42 centimetres.

Example 11

Calculate the perimeter of this pentagon.



These marks are used with two-dimensional shapes to indicate sides of equal length.

Solution

The perimeter of a pentagon is the sum of the lengths of its sides.

$$\begin{aligned}\text{Perimeter} &= 8 + 8 + 8 + 8 + 8\\ &= 5 \times 8\\ &= 40 \text{ mm}\end{aligned}$$

The perimeter of the pentagon is 40 millimetres.

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

5C Whole class CONNECT, APPLY AND BUILD

1 Work in groups of three.

- Player 1 chooses an object and estimates its perimeter.
- Player 2 measures the sides of the object.
- Player 3 uses the measurements to calculate the perimeter of the object.
- Do this for three different objects, changing roles each time.
- The player with the closest estimate to the actual perimeter measurement scores one point.

Continue until one player has scored three points.

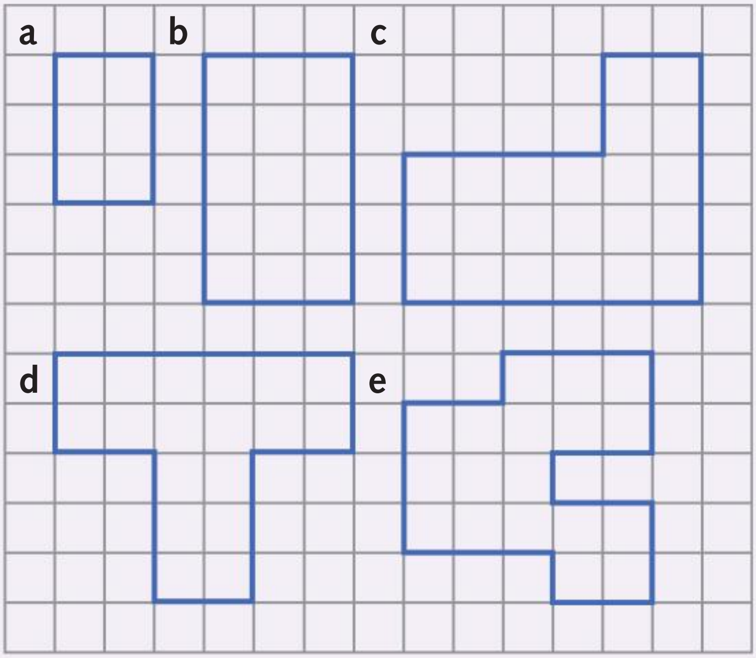
2 Work in pairs to estimate then measure the perimeter of:

a the cover of this book
b the top of your desk
c the door of a classroom cupboard
d the classroom door

Check each other's measurements.

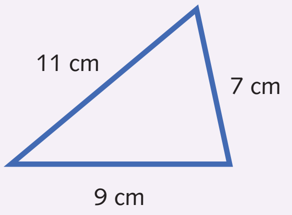
5C Individual

1 These polygons are drawn on 1-centimetre grid paper. Calculate the perimeter of each shape.

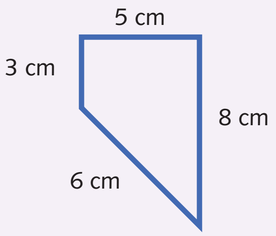


2 Calculate the perimeter of each polygon. All measurements are in centimetres, so remember to put cm after each answer. The polygons are not drawn to scale.

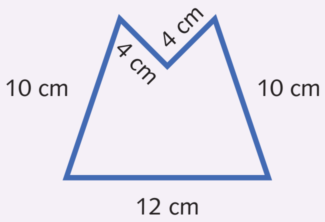
a



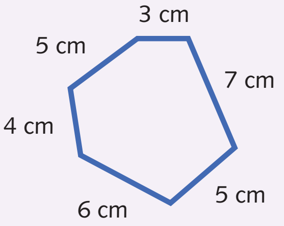
b



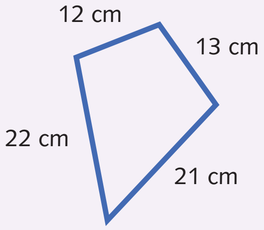
c



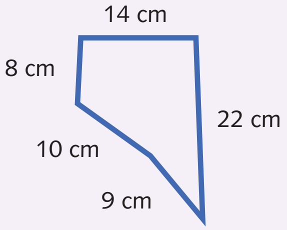
d



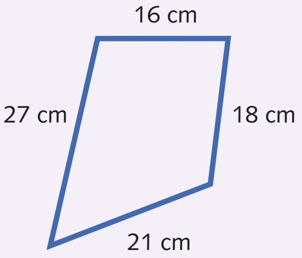
e



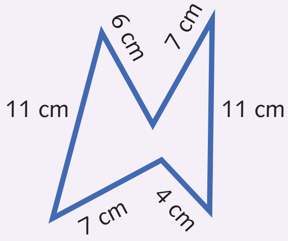
f



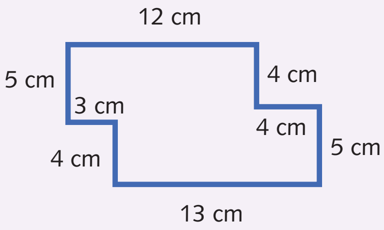
g



h

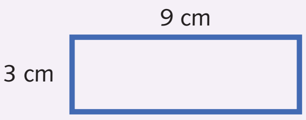


i

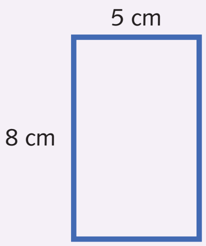


3 You can calculate the perimeter of a rectangle by doubling the two side measurements and adding them together. Calculate the perimeter of these rectangles.

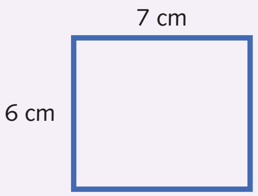
a



b

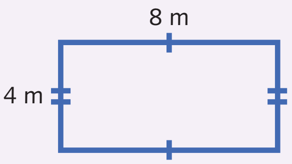


c

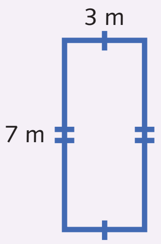


4 The – and = marks on the rectangles below indicate which sides are equal in length. Calculate the perimeter of each rectangle by adding the two measurements given, then doubling the result.

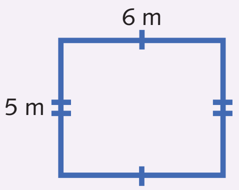
a



b



c



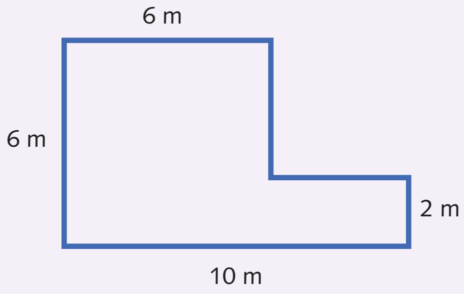
ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

5 Copy and complete this table. Use the length and width of each rectangle to calculate its perimeter.

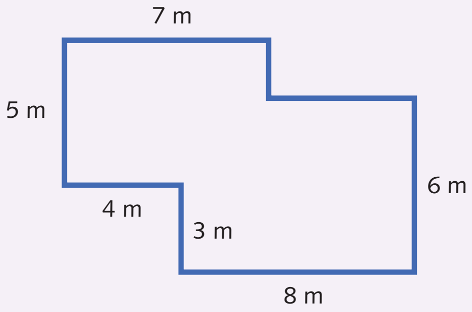
Rectangle	Length	Width	Perimeter
a	8 cm	6 cm	____ cm
b	12 m	4 m	____ m
c	8.5 m	3 m	____ m
d	20 cm	7.5 cm	____ cm
e	16 cm	15 mm	____ cm

6 Only some of the measurements are given for these polygons. Use the given measurements to work out the measurements you do not know. Then calculate the perimeter of each polygon. All measurements are in metres.

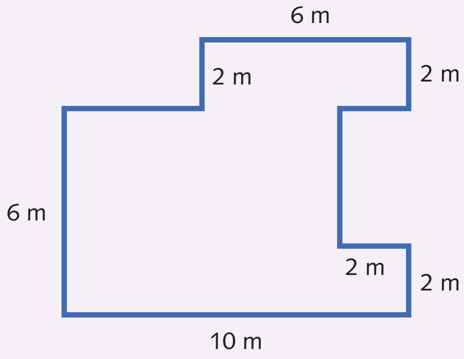
a



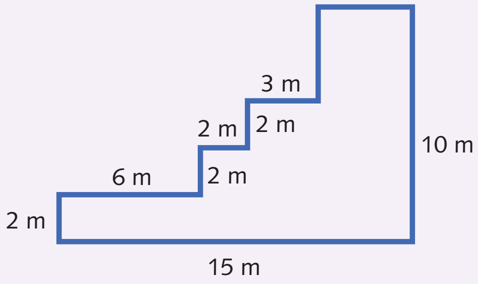
b



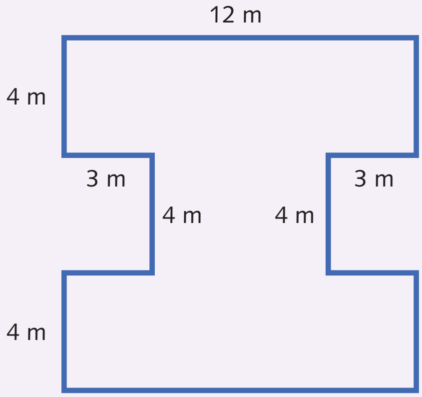
c



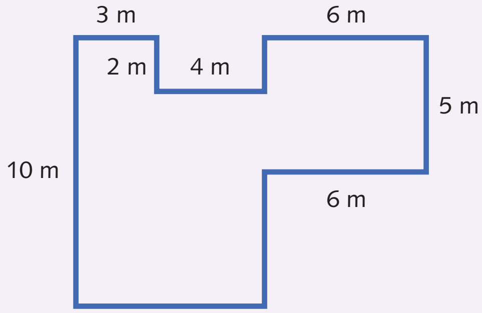
d



e



f



ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

5D Area

The area of a rectangle tells us 'how large' the inside of that rectangle is. It is the amount of material needed to 'cover' the rectangle completely, without any gaps or overlaps.

Area is measured in **square units**: square millimetres, square centimetres, square metres and square kilometres.

A **square millimetre** is an area equal to a square with side length 1 millimetre.

The symbol for square millimeters is mm^2. (The small '2' means 'squared'.)

A **square centimetre** is an area equal to a square with side length 1 centimetre.

The symbol is cm^2.

A **square metre** is an area equal to a square with side length 1 metre.

The symbol is m^2.

Australia has the sixth largest area in the world, after Russia, Canada, China, USA and Brazil.

Australia has an area of 7 692 024 km^2.

The formula for calculating area of a rectangle

To calculate the area of a postage stamp, we could cover it with a square millimetre grid and count each square millimetre. This would take a while, but we would eventually discover that the area of the stamp is 750 mm^2.

There is a quicker way of finding the area of a rectangle than counting little squares. We can find the area of a rectangle by finding the product of its length and width.

This is the formula for calculating the area of a rectangle. It works for all rectangles.

$$\text{Area} = \text{length} \times \text{width}$$

The length and width must be in the same unit of measurement.

If we use the formula, the area of the stamp is:

$$\begin{aligned}\text{Area} &= 25\text{ mm} \times 30\text{ mm}\\ &= 750\text{ mm}^2\end{aligned}$$

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

Example 12

Calculate the area of a rectangle that measures 16 cm by 8 cm.

Solution

$$\begin{aligned}\text{Area} &= \text{length} \times \text{width} \\ &= 16\text{ cm} \times 8\text{ cm} \\ &= 128\text{ cm}^2\end{aligned}$$

A square is a special type of rectangle. Its width and its length are equal. So, the area of a square can be calculated by modifying the formula for the area of a rectangle.

$$\begin{aligned}\text{Area} &= \text{length} \times \text{length} \\ &= \text{length}^2\end{aligned}$$

We read this as 'length squared'.

Example 13

Calculate the area of a square with side length 9 cm.

Solution

$$\begin{aligned}\text{Area} &= \text{length}^2 \\ &= (9\text{ cm})^2 \\ &= 9\text{ cm} \times 9\text{ cm} \\ &= 81\text{ cm}^2\end{aligned}$$

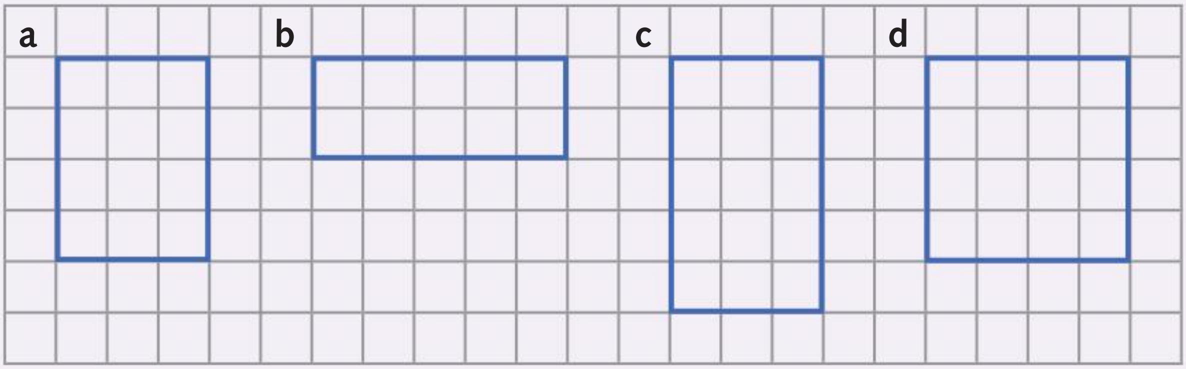
5D Whole class CONNECT, APPLY AND BUILD

1 Work in groups. You will need 1-centimetre grid paper. Draw four pair of rectangles that have the same area but different perimeters. Discuss your results with the class.

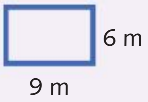
ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

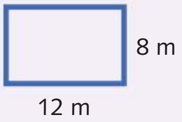
5D Individual

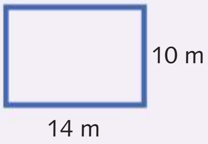
1 These rectangles are drawn on centimetre grid paper.
Calculate the area of each rectangle.



2 Calculate the area of each rectangle. (The rectangles are not drawn to scale.)

a


b


c


d

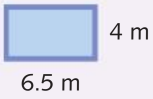

3 Find the area of a square that has a side length of:

a 6 m **b** 15 cm **c** 20 m **d** 90 cm

4 Copy this table, then calculate the area and perimeter of each rectangle.

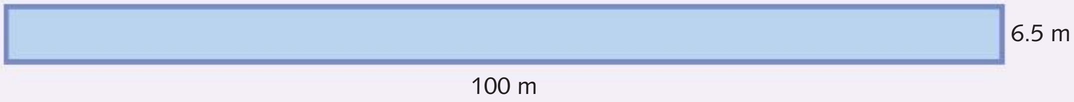
	Length	Width	Area	Perimeter
a	12 mm	9 mm	____ mm^2	____ mm
b	25 cm	8 cm	____ cm^2	____ cm
c	20 m	7 m	____ m^2	____ m
d	9 km	8 km	____ km^2	____ km

5 Calculate the area of each rectangle.

a


b


c


d


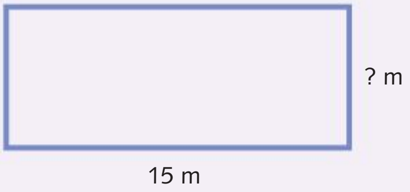
ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

6 Which is the correct area for each rectangle?

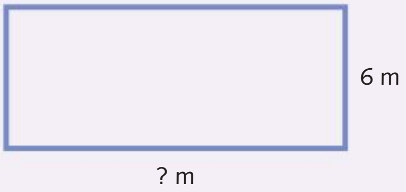
	Length	Width	Which is the area?		
a	8.5 m	6 m	14.5 m^2	51 m^2	29 m^2
b	10 m	3.25 m	32.5 m^2	26.5 m^2	13.25 m^2
c	$4\frac{1}{4}$ cm	2 cm	$6\frac{1}{4}$ cm^2	$12\frac{1}{2}$ cm^2	$8\frac{1}{2}$ cm^2
d	$20\frac{1}{2}$ cm	10 cm	205 cm^2	$30\frac{1}{2}$ cm^2	61 cm^2
e	3 m	50 cm	150 cm^2	1.5 m^2	15 m^2
f	1.5 cm	800 mm	120 cm^2	1200 mm^2	81.5 cm^2
g	1250 mm	30 cm	1280 cm^2	37 500 m^2	0.375 m^2

7 Find the unknown side, then calculate the area of each rectangle.

a Perimeter = 52 m

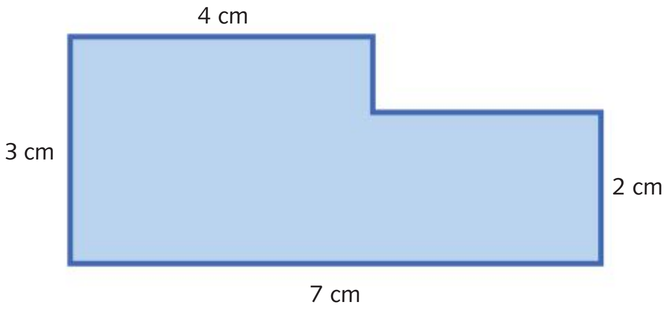


b Perimeter = 40 m

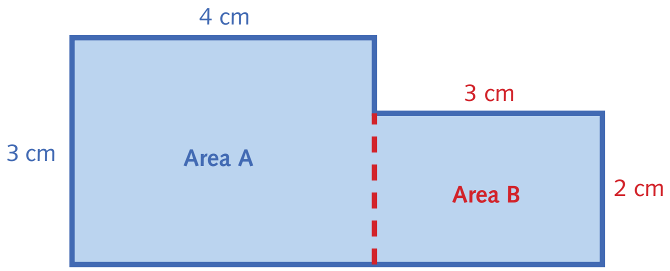


5E Area of composite shapes

Sometimes we need to calculate the area of a shape that is *not* a rectangle or a square. A shape made up from two (or more) shapes is called a **composite shape**.

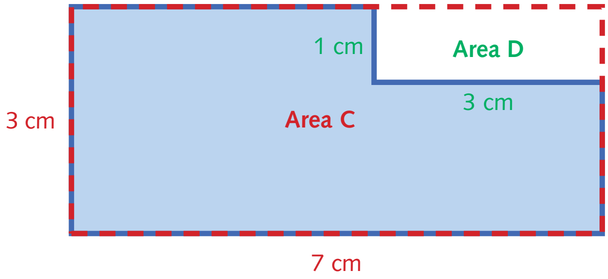


To find the area of a composite shape, we can split the shape into pieces, find the area of each piece, and then add the areas together.



$$\begin{aligned} \text{Area} &= \text{Area A} + \text{Area B} \\ &= 3\text{ cm} \times 4\text{ cm} + 2\text{ cm} \times 3\text{ cm} \\ &= 12\text{ cm}^2 + 6\text{ cm}^2 \\ &= 18\text{ cm}^2 \end{aligned}$$

Another way is to find the area of a larger rectangle, then 'take away' the bit that is not included.

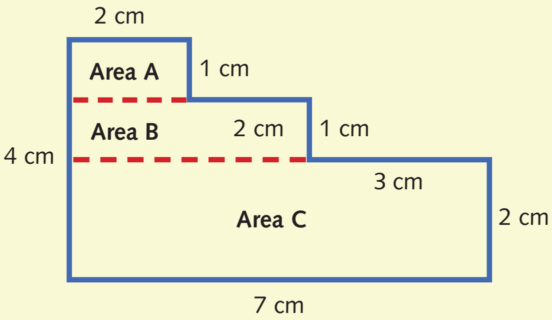


$$\begin{aligned} \text{Area} &= \text{Area C} - \text{Area D} \\ &= 7\text{ cm} \times 3\text{ cm} - 3\text{ cm} \times 1\text{ cm} \\ &= 21\text{ cm}^2 - 3\text{ cm}^2 \\ &= 18\text{ cm}^2 \end{aligned}$$

Example 14

A composite shape has been split in two different ways. Calculate the area for each.

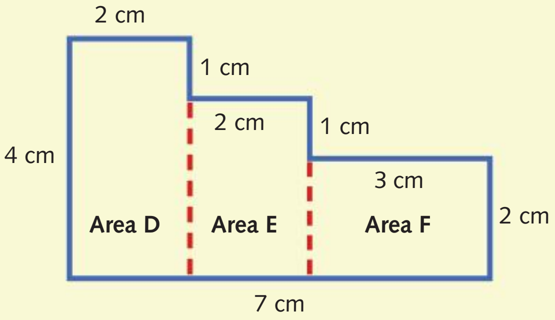
a



continued over page

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

b



Solution

a Area $=$ Area A $+$ Area B $+$ Area C

$= 2\text{ cm} \times 1\text{ cm} + 4\text{ cm} \times 1\text{ cm} + 7\text{ cm} \times 2\text{ cm}$

$= 2\text{ cm}^2 + 4\text{ cm}^2 + 14\text{ cm}^2$

$= 20\text{ cm}^2$

b Area $=$ Area D $+$ Area E $+$ Area F

$= 4\text{ cm} \times 2\text{ cm} + 3\text{ cm} \times 2\text{ cm} + 2\text{ cm} \times 3\text{ cm}$

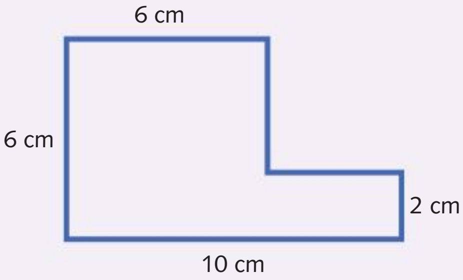
$= 8\text{ cm}^2 + 6\text{ cm}^2 + 6\text{ cm}^2$

$= 20\text{ cm}^2$

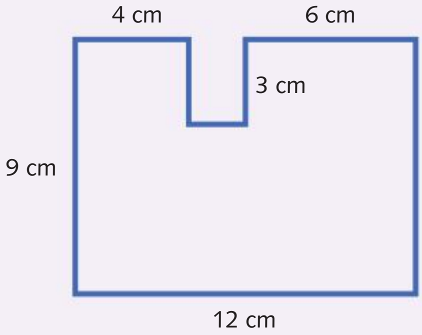
5E Individual

1 Calculate the area of these composite shapes.

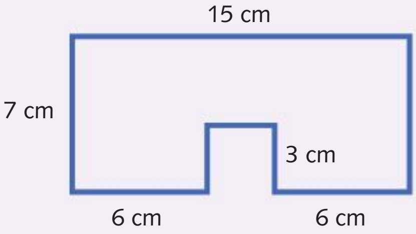
a



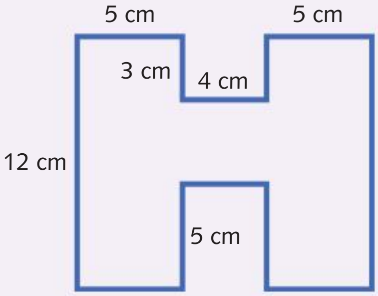
b



c

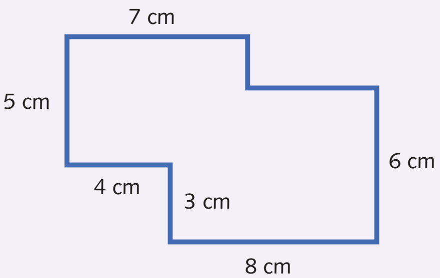


d

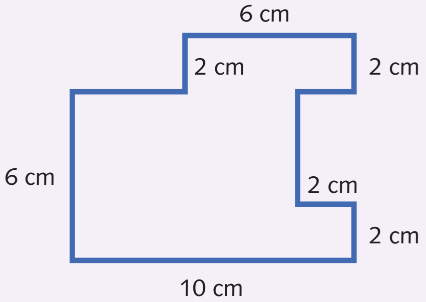


ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

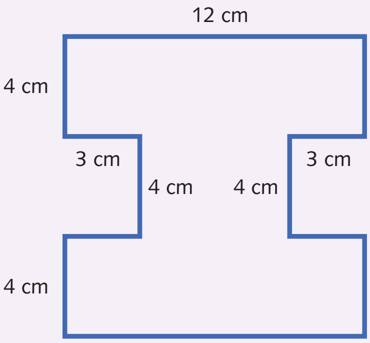
e



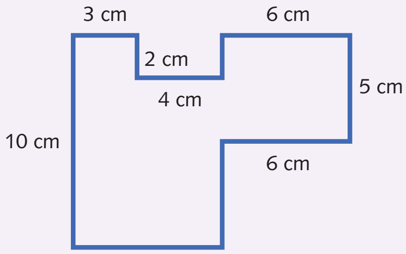
f



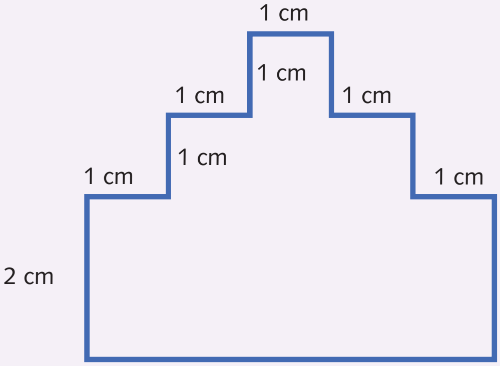
g



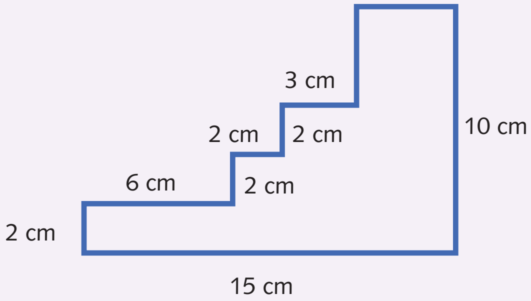
h



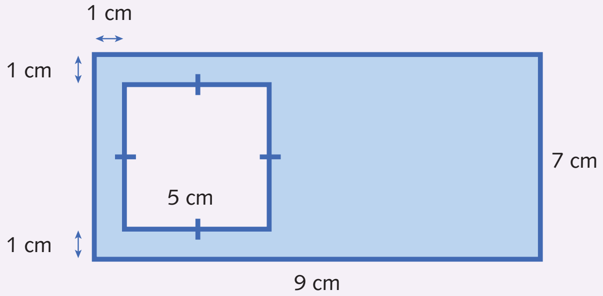
i



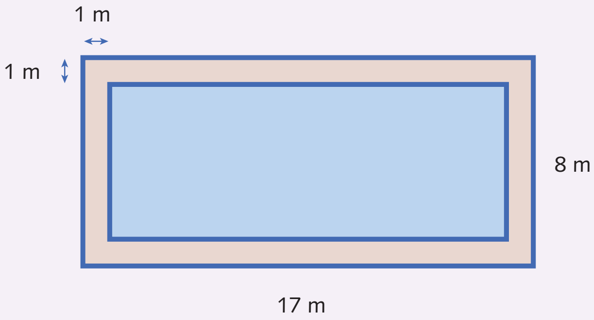
j



2 **a** Calculate the area of the blue region.



b Tim's lounge room measures 17 metres × 8 metres. He bought a rectangular carpet and placed it on the middle of the lounge floor, leaving a 1-metre margin all around.



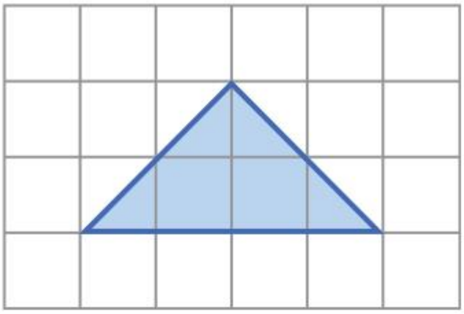
Calculate the perimeter and area of Tim's new carpet.

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

5F Area of triangles

You now know how to work out the area of a rectangle. Now we are going to look at how to calculate the area of a triangle.

There are many ways to find the area of a triangle. We want to work towards the most efficient way.



This triangle has been drawn on 1-centimetre grid paper.

How many whole centimetre squares are there in the triangle?
There are 2 whole centimetre squares.

How many half-centimetre squares are there?
There are 4 half-centimetre squares.

What is the area of the triangle?

$$\begin{aligned}\text{Area} &= 2 \times 1\ \text{cm}^2 + 4 \times \frac{1}{2}\text{cm}^2\\ &= 2\ \text{cm}^2 + 2\ \text{cm}^2\\ &= 4\ \text{cm}^2\end{aligned}$$

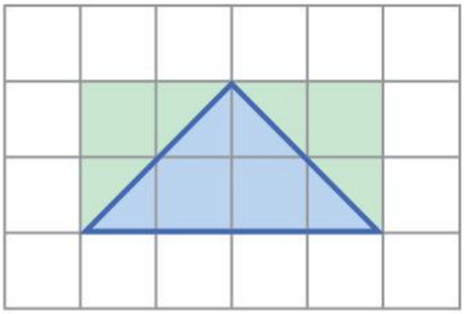
The area of the triangle is 4 cm^2.

Another method for finding the area of a triangle is to add the squares needed to complete a rectangle.

The completed rectangle has the same height as the triangle, and it is as wide as the base of the triangle.

The base of this triangle is 4 cm and its height is 2 cm.

The area of the rectangle is calculated as follows.



$$\begin{aligned}\text{Area of rectangle} &= 4\ \text{cm} \times 2\ \text{cm}\\ &= 8\ \text{cm}^2\\ \text{Area of green triangles} &= 4\ \text{cm}^2\\ \text{Area of blue triangle} &= 4\ \text{cm}^2\end{aligned}$$

So, the area of the blue triangle is equal to half the area of a rectangle with the same base and height.

We can write this another way.

$$\begin{aligned}\text{Area of a triangle} &= \frac{1}{2} \times \text{base} \times \text{height}\\ &= \frac{1}{2} \times 4\ \text{cm} \times 2\ \text{cm}\\ &= 4\ \text{cm}^2\end{aligned}$$

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7

Cambridge University Press

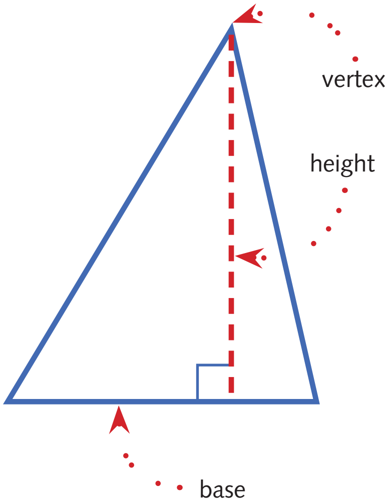
This is the formula for calculating the area of a triangle. It works for all triangles.

$$\text{Area of a triangle} = \frac{1}{2} \times \text{base} \times \text{height}$$

This is often shortened to:

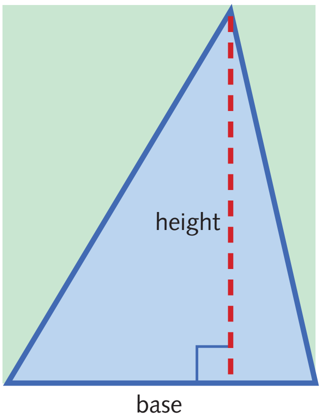
$$\text{Area of a triangle} = \frac{1}{2} \times b \times h$$

The height of a triangle is the measurement from one vertex to the base opposite. This line is at right angles to the base of the triangle.



This diagram shows that the area of each smaller triangle is equal to half the area of the rectangle that encloses it. So, the area of the triangle is equal to half the area of the large rectangle.

$$\text{Area of triangle} = \frac{1}{2} \times b \times h$$



Remember

The formula for calculating the area of a triangle is:

$$\text{Area} = \frac{1}{2} \times \text{base} \times \text{height}$$

The height of a triangle is the measurement from one vertex to the base opposite along a line perpendicular to the base.

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

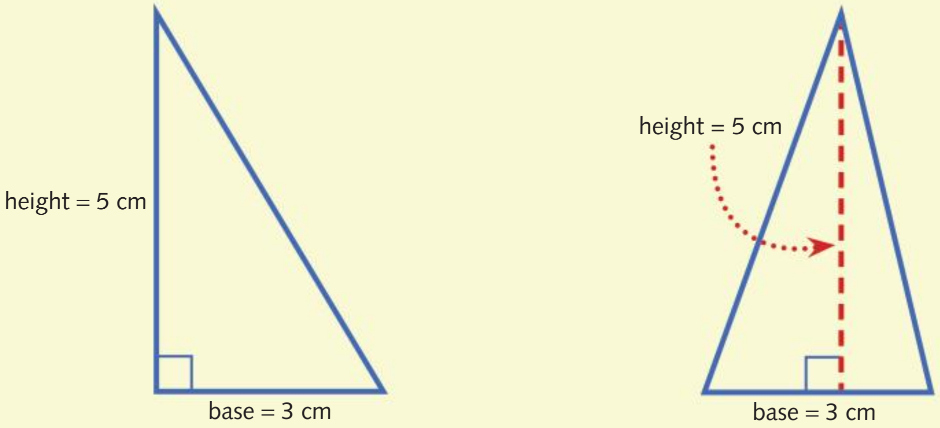
Example 15

a Draw a triangle with a height of 5 cm and a base of 3 cm.

b Calculate the area of the triangle.

Solution

a There are many triangles with these measurements. Here are two of them.



It does not matter which triangle we draw, we can apply the same formula to calculate its area.

b Area of triangle $= \frac{1}{2} \times \text{base} \times \text{height}$

$$= \frac{1}{2} \times 3 \text{ cm} \times 5 \text{ cm}$$

$$= 7\tfrac{1}{2} \text{ cm}^2$$

5F Whole class CONNECT, APPLY AND BUILD

You will need scissors, several sheets of coloured paper, rulers and pencils.

1 **a** Cut a rectangle from a sheet of coloured paper. Record the length and width of your rectangle in centimetres, then calculate its area.

b Fold your rectangle from corner to corner to make a diagonal fold. Cut along the fold. What shapes have you made?

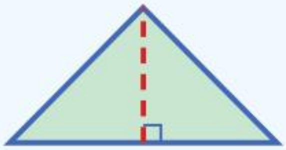
c Rotate one of your triangles and place it on top of the other to show the triangles are identical. What is the area of one triangle?

d Paste the triangles in your book, then write the steps you followed to find out the area of one of your triangles.

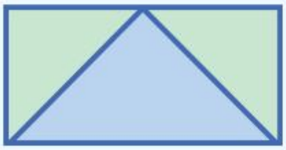
ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

2 **a** Start with two sheets of coloured paper. Use a ruler to draw an isosceles triangle on one sheet. Cut out your triangle, then carefully trace it onto the other sheet of paper to make an indentical triangle. Cut this out.

b Measure the base of one triangle and mark the middle. Draw a perpendicular line from the middle of the base to the vertex, as shown. Cut along this line.



c Place the two cut pieces on each side of your second triangle to make a rectangle, as shown. Calculate the area of your rectangle. (Hint: You combined two triangles that had the same area.)



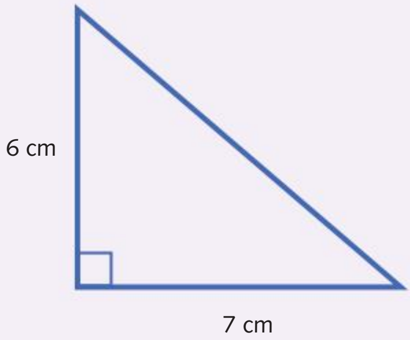
d What is the area of one of the triangles that you started with?

e Paste the triangles in your book. Write the steps you followed to find out the area of one triangle.

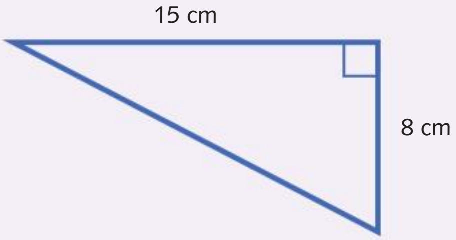
5F Individual

1 Copy and complete these measurements in your workbook. Write the length of the base and the height of each triangle.

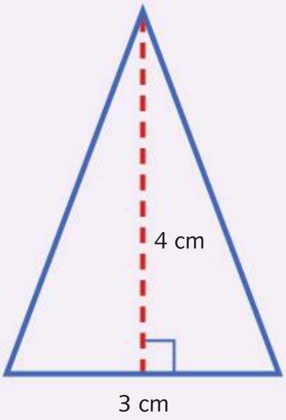
a base = ___ cm, height = ___ cm



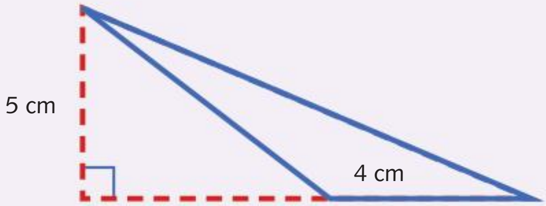
b base = ___ cm, height = ___ cm



c base = ___ cm, height = ___ cm

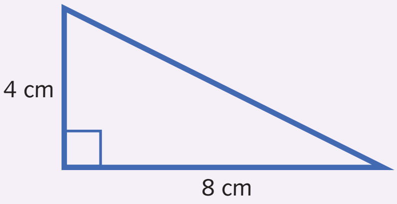


d base = ___ cm, height = ___ cm



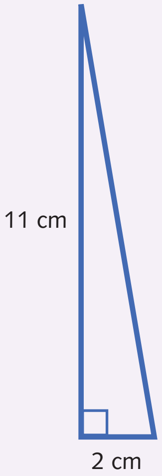
2 Find the area of each triangle. The first one has been done for you.

a



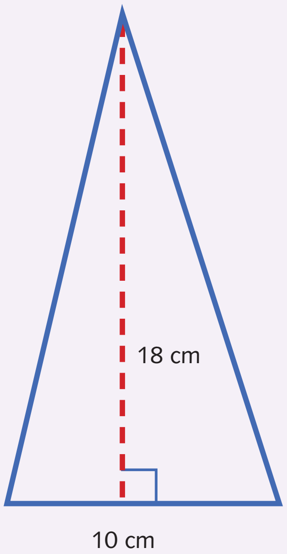
$$\text{Area} = \frac{1}{2} \times \text{base} \times \text{height}$$
$$= \frac{1}{2} \times 8 \times 4$$
$$= \underline{16}\ \text{cm}^2$$

b



$$\text{Area} = \frac{1}{2} \times \text{base} \times \text{height}$$
$$= \frac{1}{2} \times 2 \times 11$$
$$= \_\_\_\_\ \text{cm}^2$$

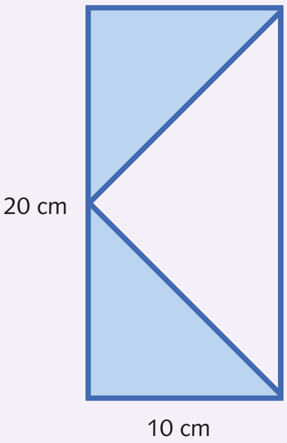
c



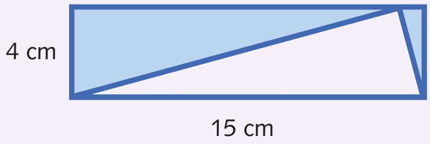
$$\text{Area} = \frac{1}{2} \times \text{base} \times \text{height}$$
$$= \frac{1}{2} \times 10 \times 18$$
$$= \_\_\_\_\ \text{cm}^2$$

3 Find the area of the blue section of each rectangle.

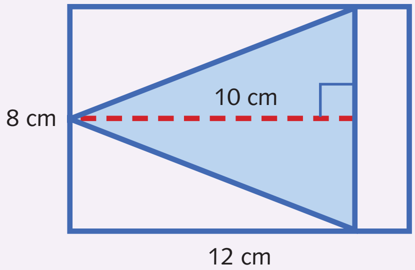
a



b



c



ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

5G Hectares

Square centimetres are useful for measuring and calculating small areas.

We use square metres to measure larger areas, such as a soccer pitch.

We use square kilometres to measure the areas of cities, states and countries.

Areas of things like farms, that are bigger than soccer pitches but smaller than cities, are measured in **hectares**. A hectare is 10 000 m^2, which is the area of a 100 m × 100 m square. The abbreviation for hectare is **ha**.

If a large rectangular field is 700 m long and 500 m wide, its area in hectares can be calculated like this:

$$\begin{aligned}\text{Area in square metres} &= 700\text{ m} \times 500\text{ m}\\ &= 350\,000\text{ m}^2\end{aligned}$$

Each 'lot' of 10 000 m^2 is 1 hectare, so divide by 10 000.

$$\begin{aligned}\text{Area in hectares} &= 350\,000 \div 10\,000\\ &= 35\text{ ha}\end{aligned}$$

The field has an area of 35 hectares.

Example 16

Convert these measurements from square metres to hectares.

a 250 000 m^2

b 118 000 m^2

Solution

Each 10 000 m^2 is 1 hectare, so divide by 10 000.

a $$\begin{aligned}250\,000\text{ m}^2 &= 250\,000 \div 10\,000\text{ ha}\\ &= 25\text{ ha}\end{aligned}$$

b $$\begin{aligned}118\,000\text{ m}^2 &= 118\,000 \div 10\,000\text{ ha}\\ &= 11.8\text{ ha}\end{aligned}$$

Example 17

Convert these measurements from hectares to square metres.

a 58 ha

b 21.8 ha

Solution

Each hectare is 10 000 m^2, so multiply by 10 000.

a 58 ha $= 58 \times 10\,000\ m^2$

$= 580\,000\ m^2$

b 21.8 ha $= 21.8 \times 10\,000\ m^2$

$= 218\,000\ m^2$

5G Whole class CONNECT, APPLY AND BUILD

1 Work in a group of six to make a square with side length of 100 metres (which has an area of 1 hectare). Use a trundle wheel. Four students stand at the corners of your square as 'markers'. The other two students use the trundle wheel to measure the side lengths exactly.

How accurate was your square? The formula for area of a rectangle only works for squares and rectangles, so how could you be sure that the shape you mapped out was a square or a rectangle?

5G Individual

1 Convert these measurements from square metres to hectares. If your answer is not a whole number, write it as a decimal.

a 10 000 m^2 **b** 50 000 m^2 **c** 85 000 m^2 **d** 12 000 m^2

e 36 000 m^2 **f** 125 000 m^2 **g** 43 250 m^2 **h** 74 585 m^2

2 Convert these measurements from hectares to square metres.

a 4 ha **b** 7 ha **c** 11 ha **d** 27 ha

e 3.5 ha **f** 6.9 ha **g** 6.57 ha **h** $5\frac{1}{2}$ ha

3 A rectangular field is 900 m long and 460 m wide. What is its area in hectares?

4 Mr Bull wants to put all of his cows in one paddock. The paddock has an area of 9.75 ha. Each cow needs 300 m^2 of grass. How many cows can Mr Bull put in his paddock?

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

5H Volume

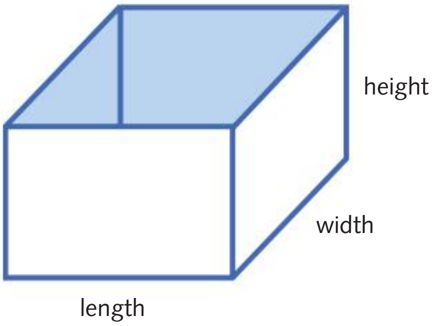
Volume is a measurement of 'how large' the inside of a container is, or how much space something takes up.

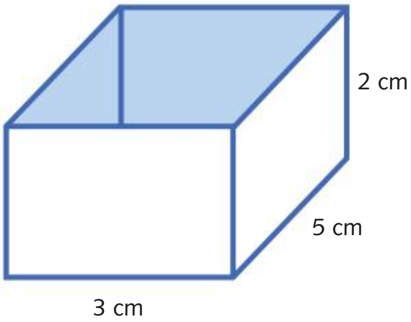
It does not matter if you are measuring the volume of a solid or the volume of a liquid, as both measurements use the same ideas.

The main units for measuring volume are:

- the **cubic centimetre** (cm^3) for small objects
- the **cubic metre** (m^3) for large objects.

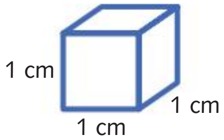
To calculate the volume of a rectangular prism, we measure its length, width and height.





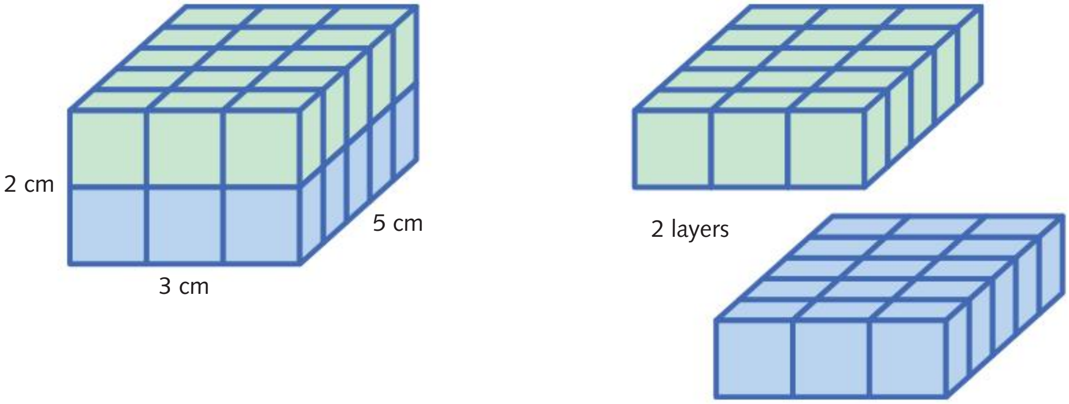
This rectangular prism has length 3 cm, width 5 cm and height 2 cm. It does not matter which measurements we call the length, width and height. If we turn the prism around, its measurements are the same.

The volume of the rectangular prism in cubic centimetres is the number of centimetre cubes that fit inside it. A base-ten one and a centicube are both good examples of a centimetre cube.



volume = 1 cm^3

This diagram shows the rectangular prism (from above) filled with centimetre cubes. It has two layers. Each layer is shown in a different colour.



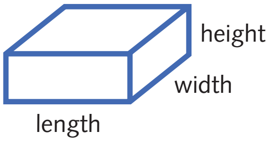
ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

We can count the cubes to find its volume. Each layer contains 3 cubes in its width and 5 cubes in its length, making 15 cubes. There are 2 layers of cubes, so the volume of the rectangular box is 30 cm^3.

Since the area of each layer is length × width and the number of layers is the height, we have calculated the volume as length × width × height.

The volume of a rectangular prism is given by the formula:

$$\text{Volume} = \text{length} \times \text{width} \times \text{height}$$



Using the formula, you can work out the volume of the rectangular prism like this.

$$\begin{aligned}\text{Volume} &= \text{length} \times \text{width} \times \text{height}\\ &= 3\text{ cm} \times 5\text{ cm} \times 2\text{ cm}\\ &= 30\text{ cm}^3\end{aligned}$$

We use the symbol cm^3 to show that volume is calculated by multiplying the three dimensions of length, width and height.

Example 18

Find the volume of a rectangular prism 8 cm long, 3 cm wide and 2 cm high.

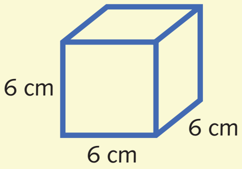
Solution

$$\begin{aligned}\text{Volume} &= \text{length} \times \text{width} \times \text{height}\\ &= 8\text{ cm} \times 3\text{ cm} \times 2\text{ cm}\\ &= 48\text{ cm}^3\end{aligned}$$

The volume of the prism is 48 cm^3.

Example 19

Calculate the volume of this cube.



ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

Solution

Because all the dimensions of a cube are the same, the formula becomes:

$$\begin{aligned}\text{Volume} &= \text{length} \times \text{length} \times \text{length} \ (\text{or length}^3)\\ &= 6\text{ cm} \times 6\text{ cm} \times 6\text{ cm}\\ &= 216\text{ cm}^3\end{aligned}$$

The volume of the cube is 216 cm^3.

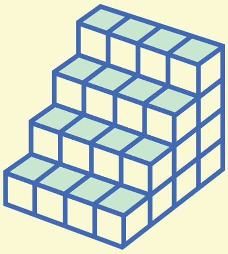
We can also find the volume of objects that are not prisms.

Example 20

a How many cubes of 1 cm^3 were used to make this staircase?

b What is the volume of the staircase?

c How many more centimetre cubes would you need to make a $4\text{ cm} \times 4\text{ cm} \times 4\text{ cm}$ cube?



Solution

a Number of cubes in layer 1: $4 \times 4 = 16$

layer 2: $4 \times 3 = 12$

layer 3: $4 \times 2 = 8$

layer 4: $4 \times 1 = 4$

Total number of cubes:

$16 + 12 + 8 + 4 = 40$

b There are 40 cubes, so the volume is 40 cm^3.

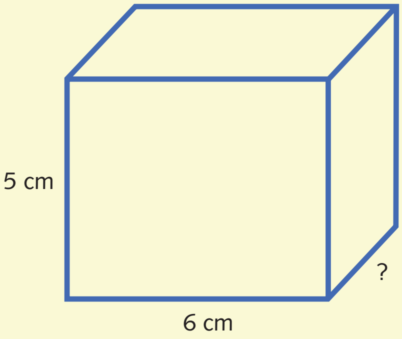
c To make a $4\text{ cm} \times 4\text{ cm} \times 4\text{ cm}$ cube, you need 64 centimetre cubes.

The number of cubes needed is $64 - 40 = 24$ cubes.

If you are given the volume of a prism and the measurement of two of its sides, you can work out the length of the third side.

Example 21

This rectangular prism has a volume of 120 cm^3.
Find its width.



ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

Solution

Remember: it doesn't matter which of the measurements you call the length, width or height. In this case, the width is the unknown edge.

To find the width, you need to divide the volume (120 cm^3) by the length and the height of the prism. The formula looks like this.

$$\text{Width} = \frac{\text{volume}}{\text{length} \times \text{height}}$$

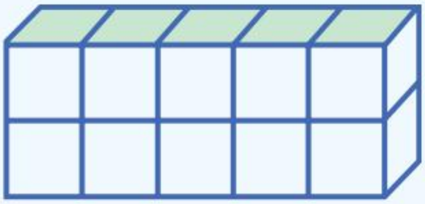
So, for this rectangular prism:

$$\begin{aligned}\text{Width} &= \frac{120}{6 \times 5}\\ &= \frac{120}{30}\\ &= 4 \text{ cm}\end{aligned}$$

The prism has a width of 4 cm.

5H Whole class CONNECT, APPLY AND BUILD

1 Use centimetre cubes to build this rectangular prism.

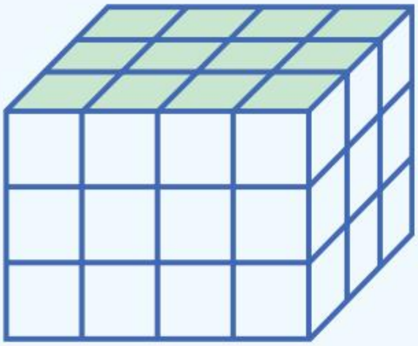


a Find the volume of the prism by counting the number of cubes.

b Use the formula to calculate the volume of the prism.

c If the whole prism was painted red, how many 1 cm × 1 cm faces would be red?

2 Use centimetre cubes to build this rectangular prism.



a Find its volume by counting the number of cubes used.

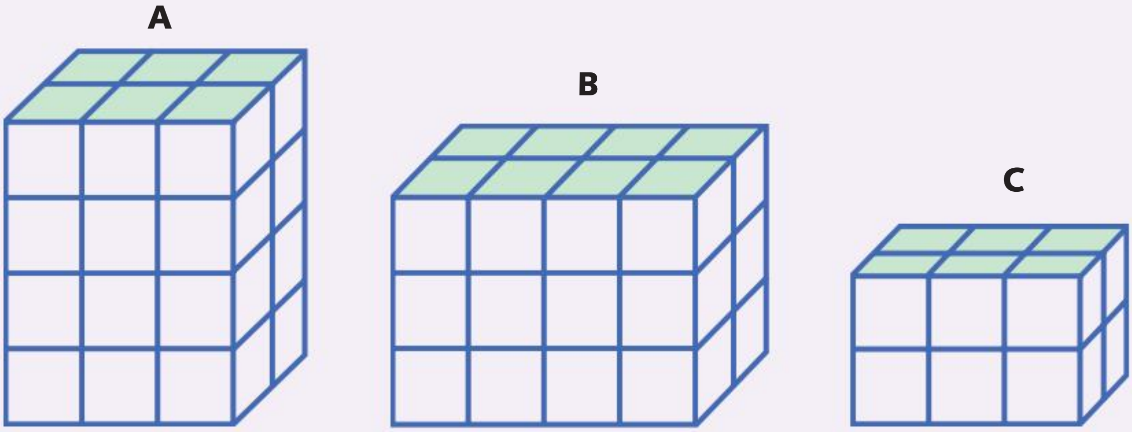
b Use the formula to calculate the volume of the prism.

c If the whole prism was painted blue, how many 1 cm × 1 cm faces would be blue?

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

5H Individual

1 **a** Use centimetre cubes to build these rectangular prisms.



b For each prism, count the number of blocks you used in each layer, then add them together to find the volume of the prism.

c List the prisms in order of volume, smallest to largest.

d Use the formula to calculate the volume of each prism.

2 Find the volume of a rectangular prism that has the dimensions:

a 5 cm × 4 cm × 2 cm

b 4 m × 2 m × 3 m

c 3 m × 4 m × 3 m

d 10 cm × 6 cm × 6 cm

e 8 m × 2.5 m × 5 m

f 10 m × 5 m × 2.8 m

3 Juliet collects small rectangular boxes that measure 4 cm × 3 cm × 2 cm. How many of these small boxes would fit inside a larger box that measures 60 cm × 30 cm × 24 cm?

4 Calculate the volume of a cube that has side length:

a 3 cm **b** 10 cm **c** 12 cm **d** 17 cm **e** 25 cm

5 Copy this table, then calculate the volume of each rectangular prism.

Prism	Length	Width	Height	Volume
a	5 cm	3 cm	2 cm	___ cm^3
b	6 cm	3 cm	2 cm	___ cm^3
c	4 cm	2 cm	5 cm	___ cm^3
d	4 cm	4 cm	2 cm	___ cm^3

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

6 Frankie wants to dig a hole so that he can install a swimming pool. If the pool is 7 metres long, 4 metres wide and 1.5 metres deep, what volume of soil does Frankie need to remove?

7 Bahira wants to concrete her driveway. Her driveway is 12 metres long and 3 metres wide. If the concrete needs to be 0.2 metres deep, what volume of soil does Bahira need to remove?

8 Find the missing length, width or height of these prisms.

a $V = 60\ \text{m}^3$ $L = 5\ \text{m},\ H = 4\ \text{m},\ W = ?$
b $V = 36\ \text{cm}^3$ $L = 6\ \text{cm},\ W = 3\ \text{cm},\ H = ?$
c $V = 36\ \text{m}^3$ $W = 3\ \text{m},\ H = 3\ \text{m},\ L = ?$
d $V = 100\ \text{cm}^3$ $L = 8\ \text{cm},\ H = 2.5\ \text{cm},\ W = ?$
e $V = 84\ \text{m}^3$ $L = 12\ \text{m},\ W = 3.5\ \text{m},\ H = ?$
f $V = 72\ \text{cm}^3$ $W = 8\ \text{cm},\ H = 6\ \text{cm},\ L = ?$

9 The Terrific Tea Company imported a box of tea that was 12 cm long and 8 cm wide. After drinking one-third of the tea, there was still 576 cm^3 of tea left in the box. Calculate the height of the box.

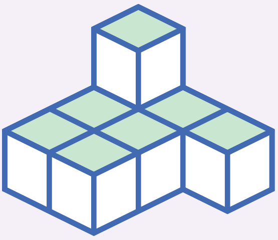
10 Tara built a cube with side length 7 cm. How many more centimetre cubes does she need to make a cube with side length:

a 8 cm? **b** 9 cm? **c** 20 cm? **d** 100 cm?

11 **a** How many centimetre cubes have been used to make this model?

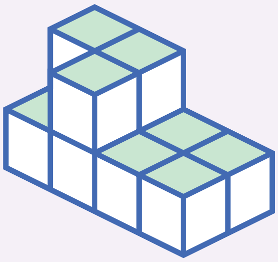
b What is the volume of the model?

c How many *more* cubes are needed to make a cube of side length 3 cm?



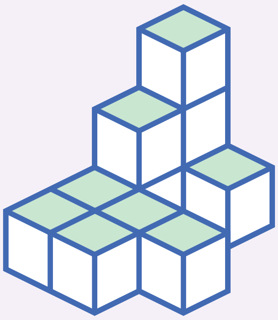
12 **a** How many centimetre cubes have been used to make this model?

b How many *more* cubes are needed to make a 4 cm × 3 cm × 2 cm prism?



13 **a** How many centimetre cubes have been used to make this model?

b How many *more* cubes are needed to make a 5 cm × 4 cm × 3 cm prism?



14 Joe is making a cake. His cake tin is 12 cm long and 6 cm wide. When it is full, his cake tin contains 648 cm^3 of cake mix. Find the height of Joe's cake tin.

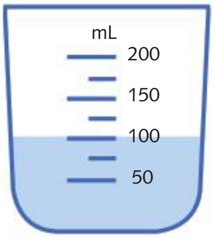
ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

51 Liquid measure

We can measure liquids using measures of volume such as cubic centimetres. There is also another measure called **capacity** that we only use for liquids and gases.

The word ‘capacity’ is used to describe how much liquid a container can hold. A jug that holds 1 litre has a capacity of 1 litre, even if it does not actually have any liquid in it.

Measuring jugs and containers have scales on their sides that are marked with lines. These lines are called **calibrations** or **graduated scales**, and they allow us to measure liquids accurately.



Millilitres and litres

We use millilitres (mL) or litres (L) to measure the volume of a liquid.

1000 millilitres = 1 litre

1 millilitre of water fills a space equal to one centimetre cube and has a mass of 1 gram.

1 millilitre = 1 cm^3 and 1000 cm^3 = 1 litre

To convert *millilitres to litres*, we make ‘lots’ of 1000 millilitres. This is the same as dividing by 1000.

Example 22

Convert 17 000 millilitres to litres.

Solution

$$\begin{aligned} 17\,000 \text{ mL} &= \frac{17\,000}{1000} \text{ L} \\ &= 17 \text{ L} \end{aligned}$$

To convert *litres to millilitres*, we multiply by 1000.

Example 23

Convert 8 litres to millilitres.

Solution

$$\begin{aligned} 8 \text{ L} &= (8 \times 1000) \text{ mL} \\ &= 8000 \text{ mL} \end{aligned}$$

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

Kilolitres and megalitres

One cubic metre of water is known as a **kilolitre** and is equivalent to 1000 litres.

$$1000 \text{ litres} = 1 \text{ kilolitre} \quad \text{and} \quad 1 \text{ kilolitre} = 1 \text{ m}^3$$

There are approximately 750 kilolitres of water in a 50-metre swimming pool. In 2005, the average Australian used 154 kilolitres of water.

To convert litres *to kilolitres*, we make 'lots' of 1000 litres. This is the same as dividing by 1000.

Example 24

The 25-metre pool at Henderson Secondary College holds 375 000 litres of water. What is this in kilolitres?

Solution

$$\begin{aligned} 375\,000 \text{ L} &= \frac{375\,000}{1000} \text{kL} \\ &= 375 \text{ kL} \end{aligned}$$

To convert from *kilolitres to litres*, we multiply by 1000.

Example 25

Simi used 982 kilolitres of water last year. Convert this to litres.

Solution

$$\begin{aligned} 982 \text{ kL} &= (982 \times 1000) \text{ L} \\ &= 982\,000 \text{ L} \end{aligned}$$

A unit for measuring extremely large quantities of liquids is the **megalitre**. A megalitre is 1000 000 litres. Often you will see the capacity of water storage dams measured in megalitres.

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

5I Whole class CONNECT, APPLY AND BUILD

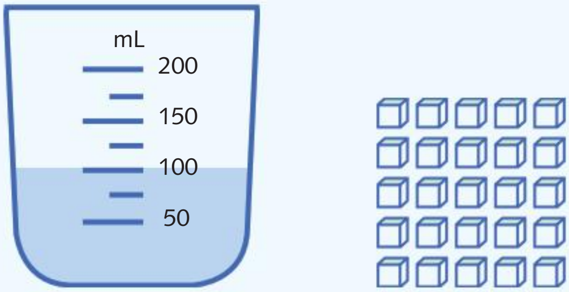
1 Make your own calibrated measuring bottle.

You will need a small clear plastic bottle that holds about half a litre, a calibrated measuring cup (or a medicine cup) with 25 mL measurements, water and a permanent maker.

- Fill the measuring jug with water to the 25 mL mark and tip it carefully into the bottle.
- Let the water settle, then put a mark on the water line and label it 25. Repeat these steps and label the next line 50, and the one after it 75, until you get to the tenth line, which you should label **250 mL**.
- There should now be 250 millilitres of water in your bottle. Tip the water out.

Use your measuring bottle to measure 150 mL of water. Check that your measuring bottle is accurate by pouring the 150 mL into the measuring jug.

2 Fill a measuring jug to the 100 mL mark. Put in 25 centicubes (or plastic base-ten ones). What is the water level now?



Now add another 25 centicubes to the jug. How much did the water rise?

Discuss your answer with the class and see if you can explain what happened.

3 Estimate the capacity in millilitres of:

a a teacup

b a yoghurt container

c a small food container

Use a measuring jug to check your estimate. How accurate were you?

4 Pour water into a measuring jug to the 50 mL mark. Put 50 centicubes (or base-ten ones) into the jug. Estimate what will happen to the water level when you add another 50 cubes to the jug. Mark your estimate on the jug, then add the cubes and mark the height of the water. Repeat this step for another 50 cubes. Were your estimates correct?

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

5I Individual

1 Convert these measurements in litres to millilitres by multiplying by 1000.

a 7 litres **b** 12 litres **c** 342 litres **d** 1000 litres

2 Convert these measurements in millilitres to litres by dividing by 1000.

a 1000 millilitres **b** 13 000 millilitres **c** 3420 millilitres

3 Convert these measurements in litres to kilolitres by dividing by 1000.

a 4000 litres **b** 18 000 litres **c** 39 870 litres

4 Convert these measurements in kilolitres to millilitres by multiplying by 1 000 000. (That is, multiply by 1000 and then multiply by 1000 again.)

a 4 kilolitres **b** 23 kilolitres **c** 815 kilolitres

5 Rosie's shed measures 3 m × 8 m. When it rains, 4 mm of rain falls on the shed roof each hour. What is the total amount of water collected:

a in 3 hours? **b** in 12 hours?

6 Sharon bought a rectangular water tank measuring 8 m × 9 m × 300 mm.

a How many litres of water can Sharon's tank hold?

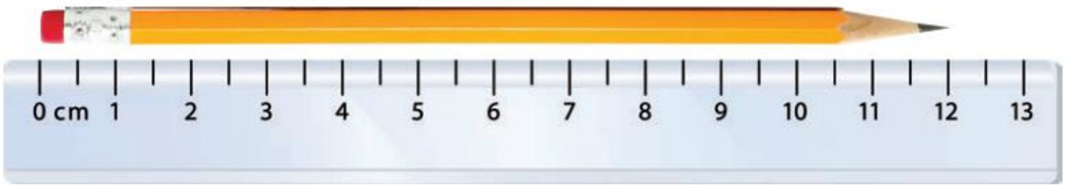
b Sharon collects water from a roof measuring 6 m × 2 m. If rain falls at 5 mm per hour, how long will it take to fill her tank?

5J Review questions

1 Write the length of each pencil.

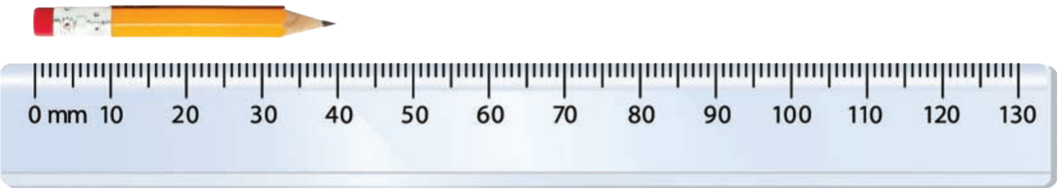
a

b

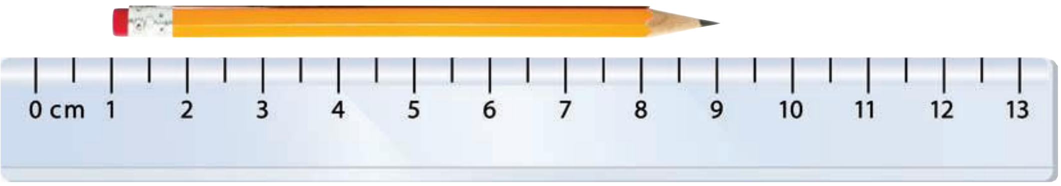


ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

c



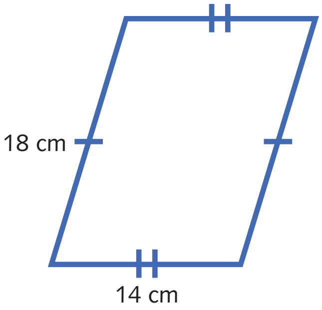
d



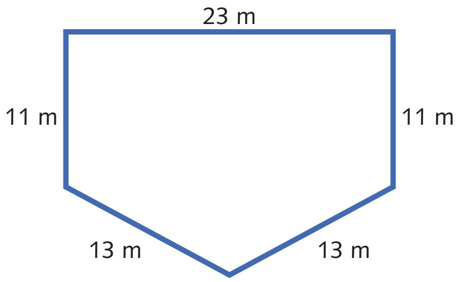
2 A skipping rope is 5.83 metres long. How many skipping ropes can be made from a rope of length 55 metres? How much is left over?

3 Calculate the perimeter of each polygon.

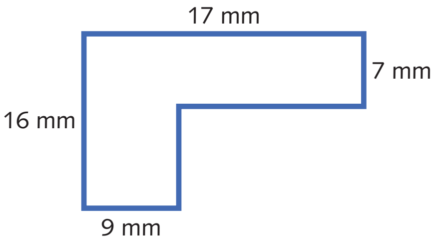
a



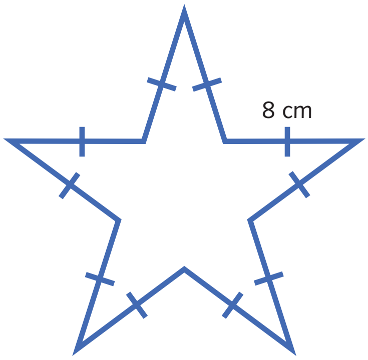
b



c



d



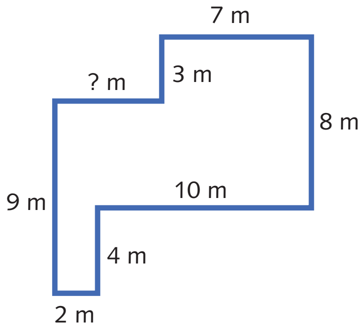
4 Copy and complete this table. Use the lengths and widths to calculate the perimeter and area for each rectangle.

Rectangle	Length	Width	Perimeter	Area
a	4 cm	5 cm	___ cm	___ cm^2
b	10 m	40 m	___ cm	___ cm^2
c	12 mm	20 mm	___ cm	___ cm^2
d	18 cm	0.5 cm	___ cm	___ cm^2
e	52 m	25 cm	___ cm	___ cm^2

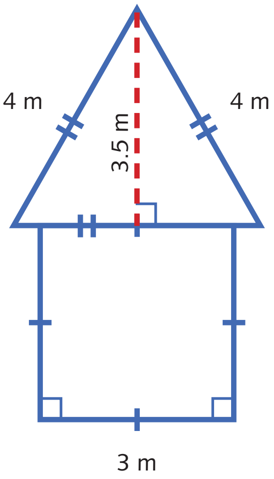
ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

5 Calculate the area and perimeter of each shape.

a



b



6 Convert these measurements to hectares. If your answer is not a whole number, write it as a decimal.

a 10 000 m^2 **b** 60 000 m^2 **c** 25 000 m^2 **d** 19 000 m^2

7 Convert these measurements to square metres.

a 1 ha **b** 3 ha **c** 15 ha **d** 1.2 ha

8 Each sheep on Mr B. A. Lamb's farm requires 15 m^2 of grass. How many sheep can Mr Lamb put in a paddock of:

a 1 hectare? **b** 800 m^2? **c** 1.5 hectares?

9 Copy this table. Calculate the volume of each rectangular prism.

Prism	Length	Width	Height	Calculation	Volume
a	1	1	1	$1 \times 1 \times 1$	___ cm^3
b	3	6	3		___ cm^3
c	5	8	7		___ cm^3
d	13	2	2		___ cm^3

10 Find the missing length, width or height of these prisms.

a $V = 35\ m^3$, $L = 5\ m$, $H = 1\ m$, $W = ?\ cm$

b $V = 48\ cm^3$, $L = 2\ cm$, $W = 6\ cm$, $H = ?\ cm$

c $V = 108\ cm^3$, $W = 4\ m$, $H = 9\ m$, $L = ?\ m$

11 Convert these measurements in litres to millilitres by multiplying by 1000.

a 4 litres **b** 19 litres **c** 603 litres **d** 4294 litres

12 Convert these measurements in millilitres to litres by dividing by 1000.

a 5000 millilitres **b** 78 000 millilitres **c** 1003 millilitres **d** 789 millilitres

13 Convert these measurements in litres to kilolitres by dividing by 1000.

a 6000 litres **b** 46 000 litres **c** 88 000 litres **d** 1 111 001 litres

CHAPTER 6 Preparation

Useful skills for this chapter:

- quick recall of multiplication facts to 12 × 12
- representing a fraction as part of a collection
- expressing the shaded part of a figure that has been divided into equal parts as a fraction of the whole
- representing whole numbers and fractions on a number line
- finding equivalent fractions
- comparing fractions
- an understanding of factors and multiples.

KICK OFF

1 List the factors of 12, then list the factors of 8. Circle the numbers that are in both lists.

2 List the multiples of 3 up to 40, then list the multiples of 4 up to 40. Circle the numbers that are on both lists.

Show what you know

1 Draw a rectangle to show fifths.

Shade $\frac{1}{5}$ of your rectangle.

Show three different ways to shade $\frac{1}{5}$ of a rectangle.

2 Use coloured blocks to explain the fraction $\frac{5}{8}$ to your teacher.

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

CHAPTER

6

Number and Algebra

Fractions

Fractions are used to describe parts of a collection or parts of an object.

The word 'fraction' comes from the Latin word *frango*, which means 'I break'.

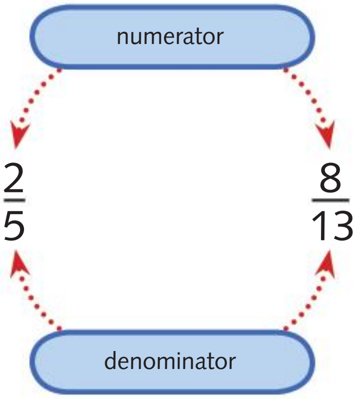
We use fractions when we cook. For example, a cake recipe might contain $\frac{1}{2}$ a cup of sugar and $1\frac{1}{2}$ cups of flour.

We use fractions when we share things. For example, if 8 friends bought a pizza and shared it equally, each person would get $\frac{1}{8}$ of the pizza.

We use fractions when we compare distances. For example, Bunbury in Western Australia is $\frac{2}{3}$ of the distance along the South Western Highway from Albany to Perth.

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

6A What is a fraction?



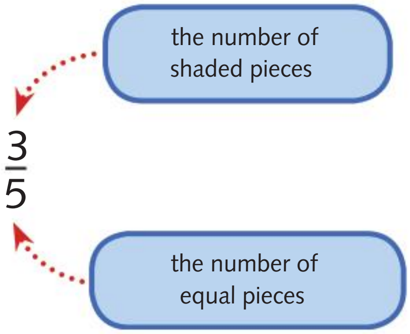
A fraction has a number on the top and a number down below. These numbers have special names.

The top number is called the **numerator**. The number down below is called the **denominator**. One way to remember where the denominator goes is to say 'D for denominator, D for down'.

The line between the numerator and the denominator is called the **vinculum**.

We use fractions to describe part of a whole. Here is a rectangle that has been cut into 5 equal pieces.

Three of the pieces are shaded. We say $\frac{3}{5}$ of the rectangle is shaded.



The numerator represents the number of pieces shaded, and the denominator represents the total number of pieces.

To help you remember the difference, think of them like this:

N is for *numerator*; it means the *number* of equal parts that we are interested in.

D is for *denominator*; it means *divided* into this many equal parts.

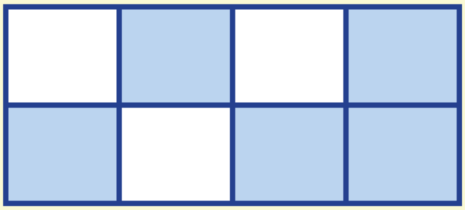
ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

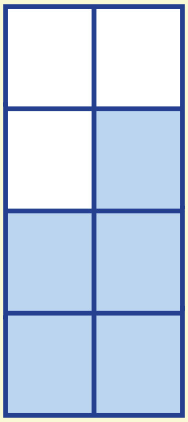
Example 1

Fold and shade a sheet of paper to show the fraction $\frac{5}{8}$.

Solution

Fold a piece of paper into 8 equal pieces, then shade 5 of those pieces. The shaded pieces are $\frac{5}{8}$ of the whole sheet of paper. Here are two ways of doing that.



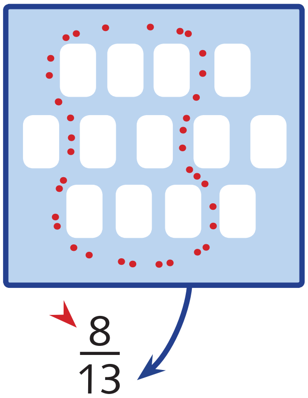


We can also use fractions to describe part of a collection of objects.

We can draw 13 blocks and show the fraction $\frac{8}{13}$.

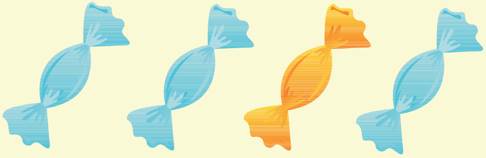
The circled group of blocks is $\frac{8}{13}$ of the total collection of blocks.

The numerator is the number of blocks circled.
The denominator is the total number of blocks.



Example 2

What fraction of the lollies are blue?



Solution

The numerator represents the number of lollies that are blue.
The denominator represents the number in the whole collection.
There are 3 blue lollies, so the numerator is 3.
There are 4 lollies, so the denominator is 4.
So we write the fraction as $\frac{3}{4}$.

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7

6A Whole class CONNECT, APPLY AND BUILD

1 **a** Tina collected 12 eggs from her chickens. 3 of the eggs were cracked. What fraction of the eggs were cracked?

b There were 3 litres of orange juice in Susan's fridge. She drank 1 litre. What fraction of the orange juice did Susan drink?

c Ahmed has 19 green marbles and 37 red marbles. First calculate the total numbers of marbles. Then work out the fraction of marbles that are green.

d Think of the numbers from 1 to 19. What fraction of these are odd?

e Veronica's kitchen table top has an area of 300 000 mm^2. Veronica's school diary covers an area of 43 000 mm^2. Veronica puts her diary in the middle of the table. What fraction of the table does it cover?

2 Look at a 6-sided die. Discuss and write down the fraction of sides of the die that show:

a the number 1 **b** the number 4

c an odd number **d** a prime number

6A Individual

1 What does the star (★) stand for in these fractions: numerator or denominator?

a $\frac{6}{\star}$ **b** $\frac{99}{\star}$ **c** $\frac{\star}{23}$ **d** $\frac{\star}{100}$

2 Write each fraction in words.

a $\frac{1}{2}$ **b** $\frac{11}{12}$ **c** $\frac{27}{37}$ **d** $\frac{89}{100}$

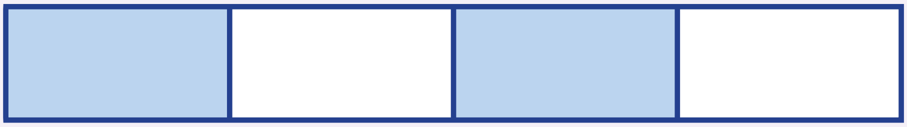
3 These are football scarves. Write the fraction that corresponds to the blue part of each scarf.

a

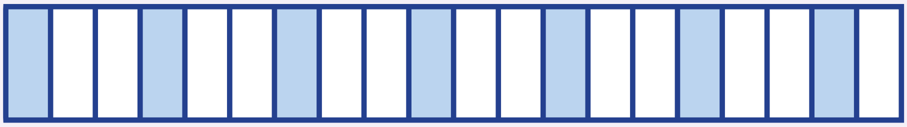
b

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

c



d



4 Draw a football scarf to show each fraction.

a $\frac{1}{2}$ **b** $\frac{7}{8}$ **c** $\frac{9}{10}$

5 Some of these juice bottles are empty. Write the number of bottles that are empty as a fraction of each group.

a

b

c

d

e

6 Draw juice bottles to show these fractions.

a $\frac{1}{4}$ **b** $\frac{2}{6}$ **c** $\frac{4}{5}$ **d** $\frac{2}{3}$

7 Edith has started a stamp collection. She has 8 stamps from Peru, 3 stamps from Australia and 1 stamp from Sweden.

What fraction of Edith's stamps are from:

a Sweden? **b** Peru? **c** Australia?

8 Vince bought a rectangular chocolate bar divided into 16 equal squares. He ate 7 squares of chocolate at recess. What fraction of the chocolate bar is left for Vince to eat at lunchtime?

9 Kerri brought a box of 12 doughnuts to share for morning tea. $\frac{2}{3}$ of the doughnuts were strawberry-filled. The rest were chocolate-iced. How many of Kerri's doughnuts were chocolate-iced?

6B Fractions on the number line

The first four whole numbers and zero are marked on this number line. The whole numbers are equally spaced.

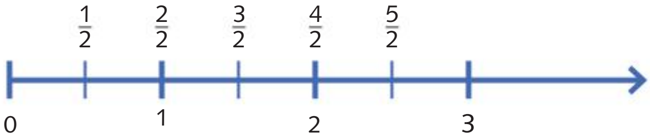
Fractions can be marked on a number line, too.

Halves

This is how to mark the fractions $\frac{1}{2}$, $\frac{2}{2}$, $\frac{3}{2}$, $\frac{4}{2}$ and $\frac{5}{2}$ on a number line.
First mark in 0 and 1.

Now break the line between 0 and 1 into 2 equal pieces. Each piece is one-half.

Copy halves across the number line and label the markers: $\frac{1}{2}$, $\frac{2}{2}$, $\frac{3}{2}$, $\frac{4}{2}$, $\frac{5}{2}$ …



We can see that:

$\frac{2}{2}$ is the same as 1

$\frac{3}{2}$ is halfway between 1 and 2

$\frac{4}{2}$ is the same as 2

The numbers $\frac{1}{2}$, $\frac{2}{2}$, $\frac{3}{2}$, $\frac{4}{2}$, $\frac{5}{2}$ and so on, are called the **multiples** of $\frac{1}{2}$.

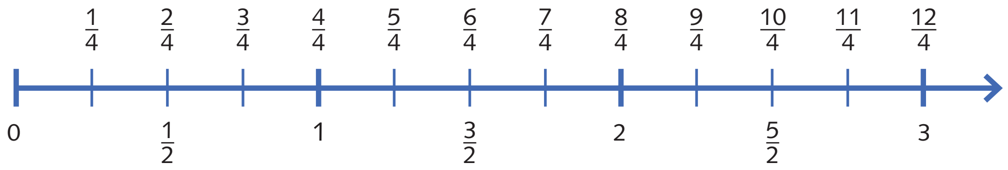
We read these as ‘one-half’, ‘two-halves’, ‘three-halves’, ‘four-halves’, and so on.

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

Quarters

Draw a number line from 1 to 3. Divide the number line between 0 and 1 into 4 equal pieces. Each piece is one-quarter.

Copy quarters across the number line and label the markers $\frac{1}{4}, \frac{2}{4}, \frac{3}{4}, \frac{4}{4}$, and so on.



We can see that:

$\frac{2}{4}$ is the same as $\frac{1}{2}$

$\frac{4}{4}$ is the same as 1

$\frac{6}{4}$ is the same as $\frac{3}{2}$

$\frac{8}{4}$ is the same as 2

$\frac{10}{4}$ is the same as $\frac{5}{2}$

$\frac{12}{4}$ is the same as 3

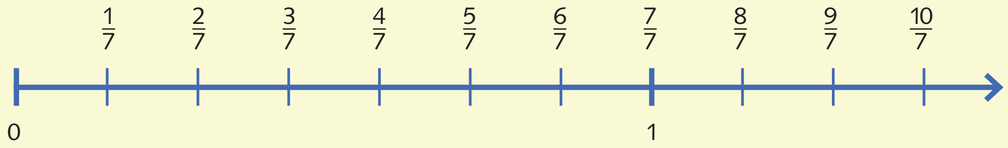
The numbers $\frac{1}{4}, \frac{2}{4}, \frac{3}{4}, \frac{4}{4}, \frac{5}{4}$, and so on, are called the multiples of $\frac{1}{4}$.

We read these as ‘one-quarter’, ‘two-quarters’, ‘three-quarters’, and so on.

Example 3

Represent $\frac{1}{7}, \frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{5}{7}, \frac{6}{7}, \frac{7}{7}, \frac{8}{7}, \frac{9}{7}$ and $\frac{10}{7}$ on a number line.

Solution



6B Whole class CONNECT, APPLY AND BUILD

1 Chalk on the playground

You will need chalk. Draw a new number line on the playground for each question.

a Mark the whole numbers 0, 1, 2, 3 and 4 on your number line. Now mark the multiples of $\frac{1}{3}$ between 0 and 4.

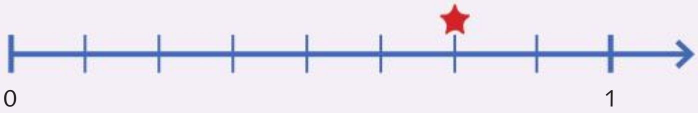
b Mark the whole numbers 0, 1, 2, 3 and 4 on your number line. Now mark the multiples of $\frac{1}{4}$ between 0 and 4.

6B Individual

1 Write the fraction shown by the star on each number line.

a

b



c

2 Draw a number line from 0 to 1. Mark these fractions on it.

$\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}$ and $\frac{1}{6}$

What do you notice about the fractions?

3 Draw a number line from 0 to 1. Mark these fractions on it.

$\frac{1}{2}, \frac{1}{4}, \frac{1}{8}$ and $\frac{1}{16}$

What do you notice about the fractions?

4 Draw a number line from 0 to 1. Mark these fractions on it.

$\frac{1}{3}, \frac{1}{6}$ and $\frac{1}{12}$

What do you notice about the fractions?

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

5 Draw three number lines, one under the other, on an A3 sheet of paper. Make the first number line 10 cm long, the second number line 20 cm long, and the third number line 30 cm long.

Mark each number line 0 at one end and 1 at the other.

Mark these fractions on each number line: $\frac{1}{2}, \frac{1}{3}$ and $\frac{1}{5}$.

What do you notice about the different length number lines?

6C Area model for fractions

Rectangles

We can use rectangles to make fraction pictures.

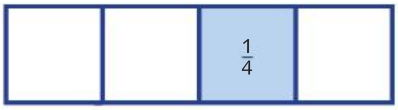
This is a rectangle. We think of it as 'the whole'. It has the value of 1.

The whole

If the rectangle is the whole, then this is one-half.

$\frac{1}{2}$ of the whole

This is one-quarter.



$\frac{1}{4}$ of the whole

This is one-third.

$\frac{1}{3}$ of the whole

The top number, or numerator, tells us how many parts of the rectangle are shaded.

The bottom number, or denominator, tells us how many equal parts the rectangle is divided into.

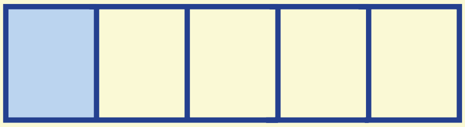
ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

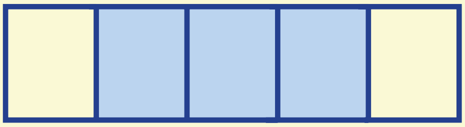
Example 4

Draw two rectangles. Shade $\frac{1}{5}$ of the first rectangle. Shade $\frac{3}{5}$ of the second rectangle.

Solution

Shading $\frac{1}{5}$ and $\frac{3}{5}$ of a rectangle could be done in several different ways. Here is one way for each fraction.

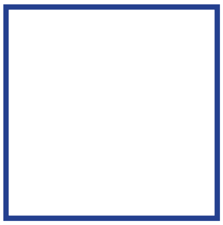




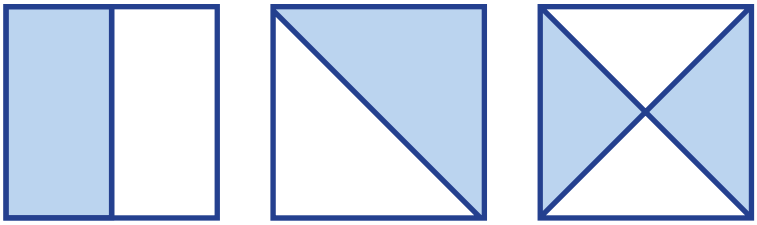
Squares

We can also use squares to make fraction pictures.

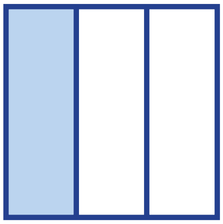
Think of this square as 'the whole'.



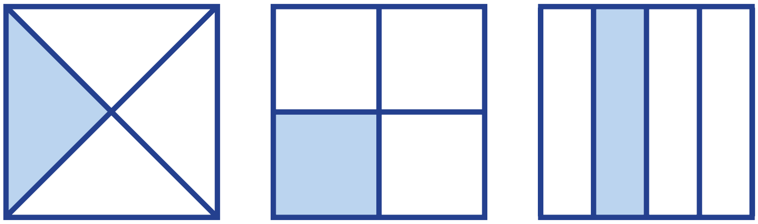
These squares have been coloured to show $\frac{1}{2}$.



This square has been coloured to show $\frac{1}{3}$.



These squares have been coloured to show $\frac{1}{4}$.

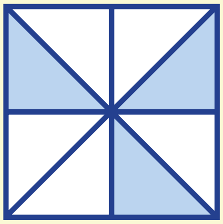


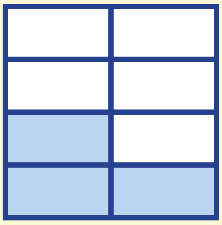
ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7

Example 5

Draw a square. Draw lines on the square to show eighths, then shade $\frac{3}{8}$.
Draw another square. Draw lines to show eighths in a different way, then shade $\frac{3}{8}$.

Solution





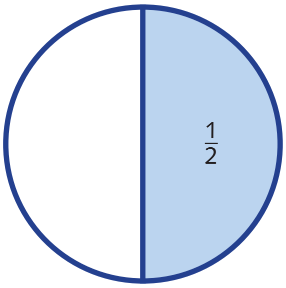
The square could be shaded in other ways to show $\frac{3}{8}$.

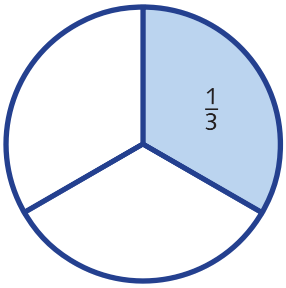
How else could the squares be shaded?

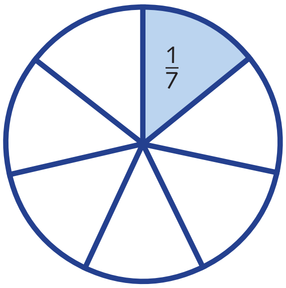
Circles

We can use circles to make fraction pictures, too. Always make sure that the circle is divided into a number of equal pieces. The best way to do this is to draw lines from the centre of the circle.

These circles have been shaded to show $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{7}$.







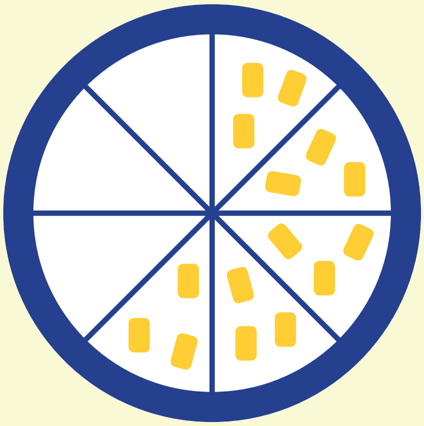
Example 6

This pizza has been cut into equal 8 pieces.

5 pieces of pizza have pineapple on them.

3 pieces of pizza do not have any pineapple.

Write the fraction for the part of the pizza that has pineapple on it.



Solution

5 parts out of a total of 8 pieces have pineapple on them, so $\frac{5}{8}$ of the pizza has pineapple on it. The answer is $\frac{5}{8}$.

6C Whole class CONNECT, APPLY AND BUILD

1 Work in groups of three or four to make one of the posters described below. Each group can use squares, rectangles or circles cut from coloured paper to show how the fractions on their poster are connected.

a	Poster 1	Halves, quarters and eighths
b	Poster 2	Thirds and ninths
c	Poster 3	Halves, fifths and tenths
d	Poster 4	Halves, thirds and sixths
e	Poster 5	Halves, thirds, quarters and twelfths

2 Use chalk to draw rectangles, squares and circles on the playground, then draw lines on the shapes to represent fractions that have different denominators.

3 Download **BLM 6** 'Fraction blocks' from the Interactive Textbook and complete.

6C Individual

1 Draw a square, then draw lines on it to divide it into quarters. Shade your square to show these fractions.

a $\frac{1}{4}$ **b** $\frac{2}{4}$ **c** $\frac{3}{4}$ **d** $\frac{4}{4}$

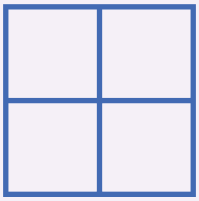
2 Draw a rectangle, then draw lines on it to divide it into sevenths. Shade your rectangle to show these fractions.

a $\frac{1}{7}$ **b** $\frac{6}{7}$ **c** $\frac{4}{7}$ **d** $\frac{7}{7}$

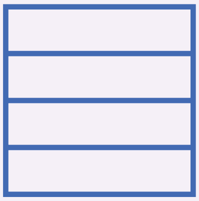
ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

3 Copy these squares, then shade $\frac{3}{4}$ of each square.

a

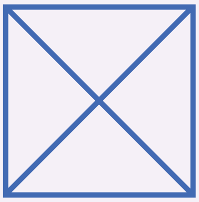


b

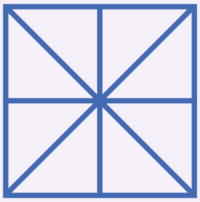


c

d

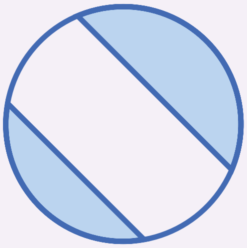


4 Make five copies of this square.
Use shading to represent
$\frac{3}{8}$ in five different ways.



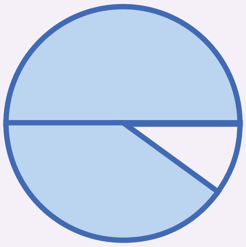
5 Which of these pictures represents the fraction $\frac{2}{3}$? Explain why the other pictures do *not* show $\frac{2}{3}$.

A



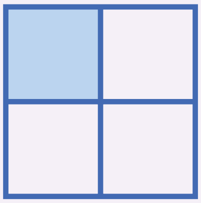
B

C

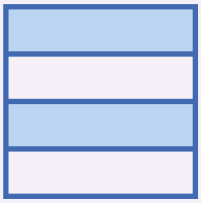


6 What fraction of each square has been shaded?

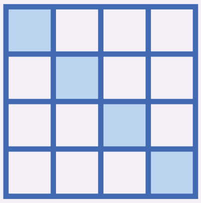
a



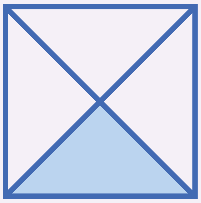
b



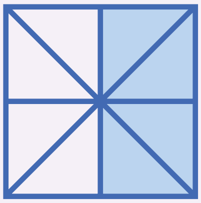
c



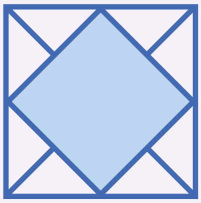
d



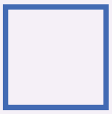
e



f



7 This is $\frac{1}{3}$ of a whole.



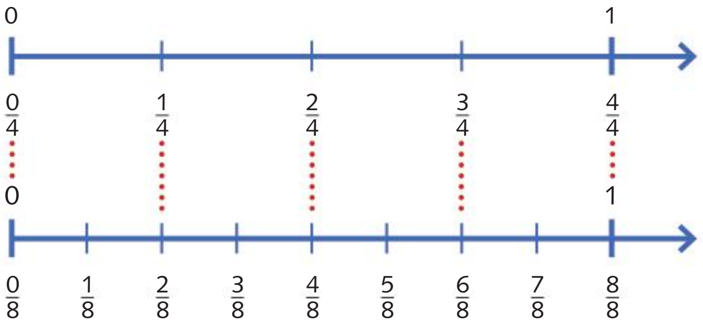
Draw what the whole might look like.

6D Equivalent fractions and simplest form

Equivalent fractions

Two fractions can mark the same point on the number line. For example, we can show $\frac{1}{2}$ and $\frac{2}{4}$ on number lines as follows.

- Draw a number line. Label 0 and 1 as shown.
- Divide the line between 0 and 1 into 4 equal pieces to get quarters. Each length is $\frac{1}{4}$.
- Draw a second number line directly below the first number line.
- Divide the second number line into 8 equal pieces to get eighths. Each length is $\frac{1}{8}$.

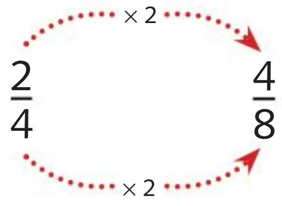


Notice that two lengths of $\frac{1}{8}$ is the same as one length of $\frac{1}{4}$. This means that $\frac{2}{8}$ and $\frac{1}{4}$ mark the same point on the number line.

Fractions that mark the same point on the number line, such as $\frac{2}{8}$ and $\frac{1}{4}$, are called **equivalent fractions**. If you look at both number lines, you will be able to find other pairs of equivalent fractions, such as $\frac{3}{4}$ and $\frac{6}{8}$, $\frac{2}{4}$ and $\frac{4}{8}$, $\frac{4}{4}$ and $\frac{8}{8}$.

We can find equivalent fractions without drawing two number lines.

Think about what happened to the numerator and the denominator of $\frac{2}{4}$.

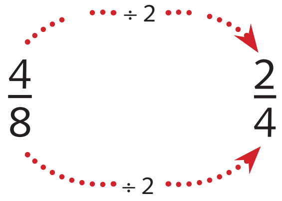


The numerator (2) and the denominator (4) were both multiplied by the same whole number (2).

We get an equivalent fraction if we multiply the numerator and the denominator by the same whole number.

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

This also works in reverse.



If we divide the numerator and the denominator in $\frac{4}{8}$ by 2, we get $\frac{2}{4}$.

We get an equivalent fraction if we divide the numerator and the denominator by the same whole number.

Example 7

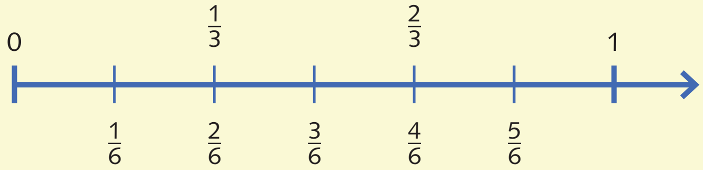
a Use a number line to show that the fractions $\frac{2}{3}$ and $\frac{4}{6}$ are equivalent.

b What whole number were the numerator and denominator of $\frac{2}{3}$ multiplied by to get the equivalent fraction $\frac{4}{6}$?

c Give another equivalent fraction for $\frac{2}{3}$.

Solution

a Mark a number line in thirds, and label them. Then cut each third into two equal pieces to get sixths.



$\frac{2}{3}$ and $\frac{4}{6}$ mark the same point on the number line.

b The numerator and denominator were both multiplied by 2.

c Multiply the numerator and denominator of $\frac{2}{3}$ by 5.

This gives $\frac{10}{15}$, which is equivalent to $\frac{2}{3}$.

You can also multiply by any other number. For example, you can multiply the numerator and denominator of $\frac{2}{3}$ by 6 to get $\frac{12}{18}$.

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

Simplest form

A fraction is in simplest form if the only common factor of the numerator and the denominator is 1.

For the equivalent fractions $\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{4}{8}$ and $\frac{50}{100}$, the simplest form is $\frac{1}{2}$.

$\frac{1}{2}$ is in simplest form because 1 is the only number that is a factor of both the numerator and denominator.

$\frac{3}{17}$ is in simplest form because 1 is the only number that is a factor of both the numerator and denominator.

$\frac{4}{6}$ is not in simplest form, as 2 is a factor of both the numerator and denominator. Dividing the numerator and the denominator by 2, we find the simplest form of $\frac{4}{6}$, which is $\frac{2}{3}$.

$$\frac{4}{6} = \frac{4 \div 2}{6 \div 2} = \frac{2}{3}$$

Example 8

Simplify each fraction.

a $\frac{10}{15}$ **b** $\frac{8}{10}$ **c** $\frac{25}{100}$ **d** $\frac{27}{45}$ **e** $\frac{75}{65}$

Solution

a $\frac{10}{15} = \frac{10 \div 5}{15 \div 5} = \frac{2}{3}$

b $\frac{8}{10} = \frac{8 \div 2}{10 \div 2} = \frac{4}{5}$

c $\frac{25}{100} = \frac{25 \div 25}{100 \div 25} = \frac{1}{4}$

d $\frac{27}{45} = \frac{27 \div 9}{45 \div 9} = \frac{3}{5}$

e $\frac{75}{65} = \frac{75 \div 5}{65 \div 5} = \frac{15}{13}$

Sometimes you may need to do the divisions in two or more steps.

Example 9

Reduce the fraction $\frac{84}{126}$ to its simplest form.

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

Solution

$$\frac{84}{126} = \frac{84 \div 2}{126 \div 2} = \frac{42}{63}$$
$$= \frac{42 \div 3}{63 \div 3} = \frac{14}{21}$$
$$= \frac{14 \div 7}{21 \div 7} = \frac{2}{3}$$

Simplest form and equivalence

The fractions $\frac{16}{24}$ and $\frac{14}{21}$ both have $\frac{2}{3}$ as their simplest form.

$$\frac{16}{24} = \frac{16 \div 8}{24 \div 8} = \frac{2}{3}$$

$$\frac{14}{21} = \frac{14 \div 7}{21 \div 7} = \frac{2}{3}$$

They both mark the same point on the number line. They are equivalent fractions.

But we cannot go from $\frac{16}{24}$ to $\frac{14}{21}$ by multiplying or dividing by the same whole number.

To test whether two fractions are equivalent, we check to see if they have the same simplest form.

Example 10

Show that $\frac{40}{24}$ is equivalent to $\frac{15}{9}$ by reducing each to its simplest form.

Solution

$$\frac{40}{24} = \frac{40 \div 8}{24 \div 8} = \frac{5}{3} \qquad \frac{15}{9} = \frac{15 \div 3}{9 \div 3} = \frac{5}{3}$$

Both fractions $\frac{40}{24}$ and $\frac{15}{9}$ simplify to $\frac{5}{3}$, so they are equivalent to each other.

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

6D Whole class CONNECT, APPLY AND BUILD

1 **Who am I?**

Someone reads each ‘who am I’ to the class, one clue at a time.

a My numerator is 16.
I am closer to 1 than to 0.
I am equivalent to $\frac{4}{5}$. Who am I?

b Three of me is equivalent to $\frac{12}{16}$.
I am equivalent to $\frac{25}{100}$.
My denominator is 4. Who am I?

2 Set a 3-minute limit. Your challenge is to:

a use multiplication to find as many fractions as you can that are equivalent to $\frac{3}{4}$

b use division to find as many fractions as you can that are equivalent to $\frac{180}{700}$

3 **Paper bag sort**

Draw three paper bags and label them ‘Smaller than $\frac{3}{5}$’, ‘Equivalent to $\frac{3}{5}$’, and ‘Larger than $\frac{3}{5}$’. Sort each fraction into the correct bag.

$\frac{1}{4}$ $\frac{1}{2}$ $\frac{1}{8}$ $\frac{3}{4}$ $\frac{5}{8}$ $\frac{6}{10}$ $\frac{30}{50}$ $\frac{2}{9}$ $\frac{5}{10}$ $\frac{5}{3}$ $\frac{1}{10}$

6D Individual

1 Use a number line to show that the two fractions are equivalent.

a $\frac{1}{2}$ and $\frac{2}{4}$ **b** $\frac{1}{6}$ and $\frac{2}{12}$ **c** $\frac{5}{8}$ and $\frac{10}{16}$ **d** $\frac{2}{3}$ and $\frac{6}{9}$

2 Write three equivalent fractions for each of these.

a $\frac{1}{2}$ **b** $\frac{2}{5}$ **c** $\frac{5}{6}$ **d** $\frac{2}{7}$

3 Copy these fractions, then fill in the missing numerators and denominators to make equivalent fractions.

a $\frac{1}{\square} = \frac{2}{4} = \frac{\square}{8} = \frac{\square}{100}$

b $\frac{2}{\square} = \frac{4}{6} = \frac{\square}{60} = \frac{6}{\square}$

c $\frac{6}{\square} = \frac{3}{5} = \frac{60}{\square} = \frac{\square}{35}$

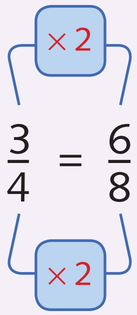
d $\frac{4}{\square} = \frac{80}{60} = \frac{16}{\square} = \frac{\square}{15}$

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

4 Copy each of these. Fill in the boxes to show which number the numerator and denominator were *multiplied* by to arrive at the equivalent fraction.

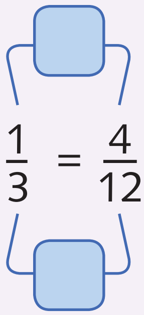
The first one has been done for you.

a



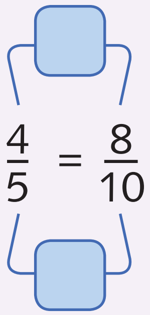
$\frac{3}{4} = \frac{6}{8}$

b



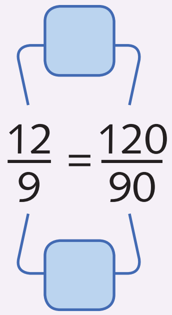
$\frac{1}{3} = \frac{4}{12}$

c



$\frac{4}{5} = \frac{8}{10}$

d

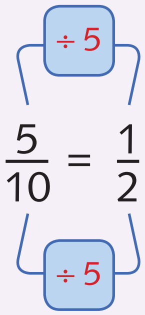


$\frac{12}{9} = \frac{120}{90}$

5 Copy each of these. Fill in the boxes to show which number the numerator and denominator were *divided* by to arrive at the equivalent fraction.

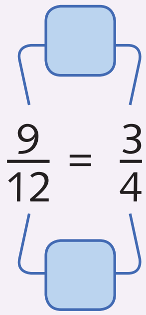
The first one has been done for you.

a



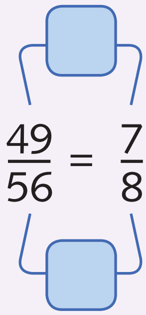
$\frac{5}{10} = \frac{1}{2}$

b



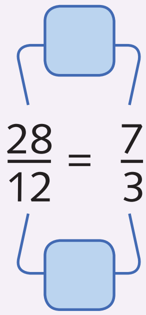
$\frac{9}{12} = \frac{3}{4}$

c



$\frac{49}{56} = \frac{7}{8}$

d



$\frac{28}{12} = \frac{7}{3}$

6 **Paper bag sort**

Draw three paper bags and label them 'Smaller than $\frac{1}{3}$', 'Equivalent to $\frac{1}{3}$', and 'Larger than $\frac{1}{3}$'. Sort each fraction into the correct bag.

$\frac{1}{4}$ $\frac{1}{2}$ $\frac{1}{8}$ $\frac{12}{16}$ $\frac{5}{8}$ $\frac{6}{8}$ $\frac{75}{100}$ $\frac{2}{9}$ $\frac{5}{10}$ $\frac{1}{3}$ $\frac{2}{10}$ $\frac{9}{11}$

7 **a** How many thirds are equivalent to $\frac{6}{9}$?

b How many fifteenths are equivalent to $\frac{2}{3}$?

c How many quarters are equivalent to $\frac{15}{12}$?

d How many twelfths are equivalent to $\frac{1}{3}$?

8 Simplify each fraction by dividing the numerator and the denominator by the same whole number.

a $\frac{12}{18}$ **b** $\frac{16}{20}$ **c** $\frac{70}{100}$ **d** $\frac{88}{24}$ **e** $\frac{450}{700}$

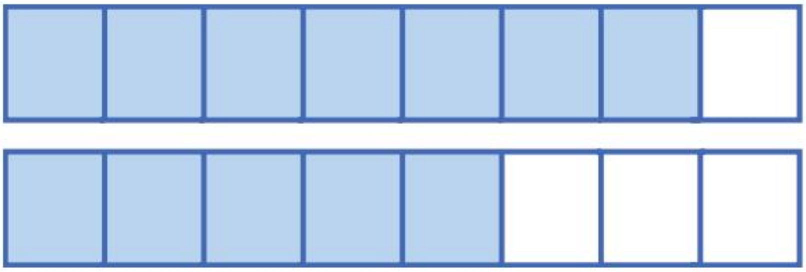
ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

6E Comparing fractions

When the denominators of two fractions are the same, the one with the *larger* numerator is the larger fraction.

These two rectangles show $\frac{7}{8}$ and $\frac{5}{8}$.

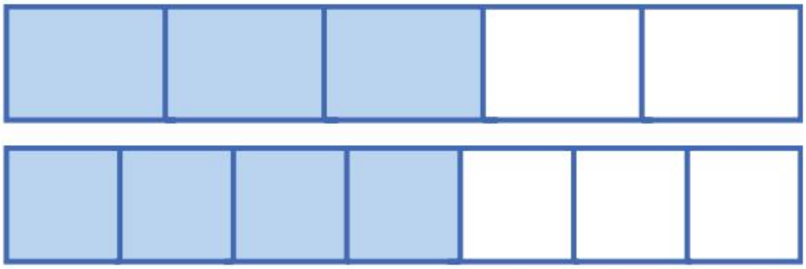
You can see that $\frac{7}{8}$ is larger than $\frac{5}{8}$.



It is not as simple to compare fractions when the denominators are different.

These two rectangles show $\frac{3}{5}$ and $\frac{4}{7}$.

You can see that $\frac{3}{5}$ is larger than $\frac{4}{7}$.



But sometimes it is difficult to draw diagrams that show clearly which of a pair of fractions is larger.

The best way to compare fractions is to find an equivalent fraction with the same denominator for each fraction.

Example 11

Is $\frac{3}{4}$ larger or smaller than $\frac{13}{16}$?

Solution

Convert $\frac{3}{4}$ into sixteenths.

$$\frac{3}{4} = \frac{3 \times 4}{4 \times 4} = \frac{12}{16}$$

$\frac{12}{16}$ is smaller than $\frac{13}{16}$, so $\frac{3}{4}$ is smaller than $\frac{13}{16}$.

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

Example 12

Is $\frac{3}{2}$ larger or smaller than $\frac{4}{3}$?

Solution

6 is a common multiple of both denominators, so convert $\frac{3}{2}$ and $\frac{4}{3}$ into sixths.

$\frac{3}{2} = \frac{3 \times 3}{2 \times 3} = \frac{9}{6}$ $\qquad$ $\frac{4}{3} = \frac{4 \times 2}{3 \times 2} = \frac{8}{6}$

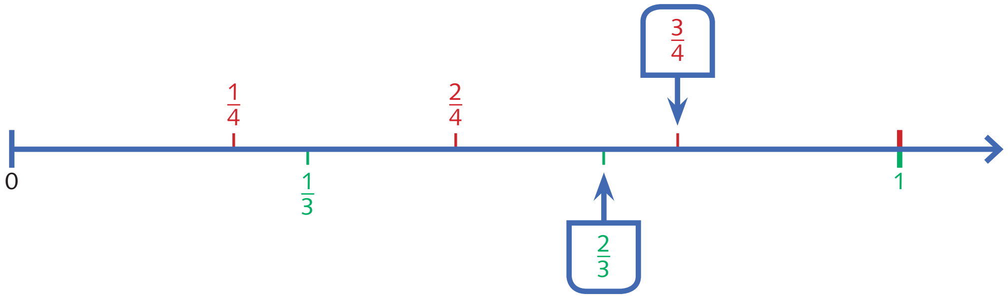
$\frac{9}{6}$ is larger than $\frac{8}{6}$, so $\frac{3}{2}$ is larger than $\frac{4}{3}$.

Larger, smaller or equivalent

When we compare any two fractions, one of three things can happen:

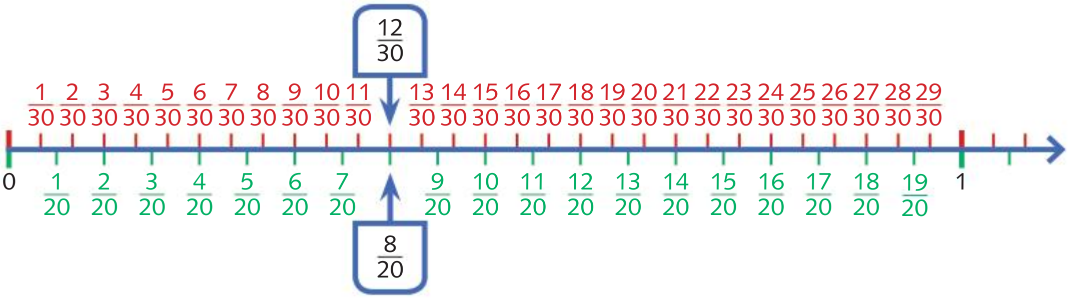
- the first fraction is *smaller than* the second fraction, or
- the first fraction is *equivalent to* the second fraction, or
- the first fraction is *larger than* the second fraction.

A number line can be used to compare two fractions. For example, to compare $\frac{2}{3}$ and $\frac{3}{4}$ we locate them on the number line. We know that numbers to the left on the number line are smaller, so $\frac{2}{3}$ is smaller than $\frac{3}{4}$.



ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7

If we want to compare $\frac{8}{20}$ and $\frac{12}{30}$ we can mark them on the number line.



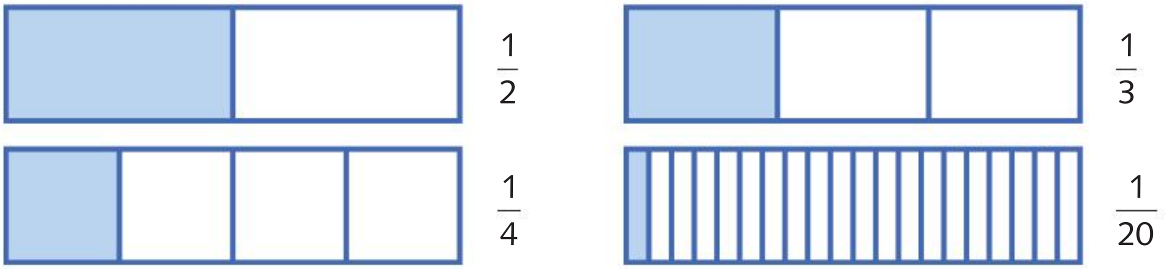
We see that they mark the same place on the number line, so they are equivalent.

Comparing unit fractions

Fractions that have 1 as the numerator are called **unit fractions**.

For example, $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{10}, \frac{1}{100}, \frac{1}{837}$ and $\frac{1}{1000\,000}$ are unit fractions.

Here are some rectangles that show unit fractions.



You can see that the more parts the whole is divided into, the *smaller* each part is.

This means that 1 part out of 4 is larger than 1 part out of 20. So $\frac{1}{4}$ is larger than $\frac{1}{20}$.

When we compare unit fractions, the one with the larger denominator is the *smaller* fraction.

6E Whole class

CONNECT, APPLY AND BUILD

1 Set up a piece of string as a number line. Make these fraction cards.

$\frac{3}{4}$ $\frac{1}{2}$ $\frac{5}{6}$ $\frac{2}{9}$ $\frac{1}{10}$ $\frac{3}{10}$ $\frac{1}{3}$ $\frac{7}{10}$ $\frac{2}{3}$ $\frac{1}{5}$

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

Each student pegs their fraction on the number line in order from smallest to largest. Discuss where cards were placed and the strategy used.

2 Repeat the activity on the previous page using these fraction cards.

$\frac{4}{3}$ $\frac{1}{2}$ $\frac{5}{4}$ $\frac{13}{3}$ $\frac{1}{6}$ $\frac{3}{4}$ $\frac{1}{3}$ $\frac{6}{4}$ $\frac{2}{3}$ $\frac{8}{6}$

3 A domino standing on its end can be read as a fraction. For example, this domino could be read as $\frac{3}{5}$ or $\frac{5}{3}$.

You will need a class set of dominoes (after removing the dominoes with blank spaces).

Work in small groups. Each group needs a handful of dominoes. Order your 'domino fractions' from smallest to largest. The fractions will vary depending on which way you stand them up.

6E Individual

1 Which is larger:

a $\frac{1}{6}$ or $\frac{2}{3}$? **b** $\frac{5}{10}$ or $\frac{3}{5}$? **c** $\frac{3}{8}$ or $\frac{5}{24}$? **d** $\frac{25}{4}$ or $\frac{5}{1}$?

2 Order these fractions from smallest to largest.

a $\frac{3}{7}$ $\frac{1}{7}$ $\frac{6}{7}$ $\frac{5}{7}$ $\frac{8}{7}$

b $\frac{5}{10}$ $\frac{2}{10}$ $\frac{10}{10}$ $\frac{3}{10}$ $\frac{9}{10}$

c $\frac{1}{4}$ $\frac{3}{8}$ $\frac{7}{12}$ $\frac{1}{8}$ $\frac{1}{12}$

d $\frac{7}{10}$ $\frac{3}{5}$ $\frac{1}{20}$ $\frac{1}{5}$ $\frac{18}{20}$

e $\frac{1}{4}$ $\frac{1}{8}$ $\frac{23}{24}$ $\frac{1}{2}$ $\frac{5}{6}$

f $\frac{5}{12}$ $\frac{1}{8}$ $\frac{37}{80}$ $\frac{9}{11}$ $\frac{66}{67}$

3 Which is larger:

a $\frac{2}{3}$ or $\frac{7}{10}$? **b** $\frac{3}{8}$ or $\frac{5}{12}$? **c** $\frac{5}{9}$ or $\frac{5}{12}$? **d** $\frac{7}{8}$ or $\frac{8}{9}$?

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

4 Copy and complete the table by choosing the response ('is smaller than', 'is equivalent to' or 'is larger than') that makes the statement true. The first one has been done for you.

	Fraction	• is smaller than • is equivalent to • is larger than	Fraction
a	$\frac{1}{2}$	is larger than	$\frac{1}{4}$
b	$\frac{1}{2}$		$\frac{4}{8}$
c	$\frac{3}{8}$		$\frac{3}{4}$
d	$\frac{5}{6}$		$\frac{2}{3}$
e	$\frac{3}{4}$		$\frac{4}{3}$
f	$\frac{6}{8}$		$\frac{9}{12}$
g	$\frac{3}{20}$		$\frac{1}{15}$

5 Name a fraction that is:

a between $\frac{7}{10}$ and $\frac{1}{4}$

b greater than $\frac{4}{5}$ but less than 1

c less than $\frac{2}{3}$ but greater than $\frac{4}{12}$

d near 0

e almost 1

f less than $\frac{1}{900}$

6 A milkshake recipe needs $\frac{7}{8}$ of a cup of milk.

A thickshake recipe needs $\frac{3}{4}$ of a cup of milk.

Which shake needs more milk?

7 Swansea Soccer Club has two teams: the A team and the B team. Both teams have the same number of players. The A team had $\frac{3}{4}$ of its players turn up to training. The B team had $\frac{7}{12}$ of its players turn up.

Which team had more players turn up to training?

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

6F Improper fractions and mixed numbers

Proper and improper fractions

We call a fraction a **proper fraction** if the numerator is less than the denominator. For example, $\frac{1}{4}$ and $\frac{3}{4}$ are proper fractions.

If the numerator is greater than the denominator, or equal to the denominator, then the fraction is called an **improper fraction**.

For example, $\frac{5}{4}$ and $\frac{4}{4}$ are improper fractions.

Example 13

Label each fraction as 'proper' or 'improper'.

a $\frac{2}{3}$ **b** $\frac{5}{4}$ **c** $\frac{6}{6}$ **d** $\frac{17}{18}$

Solution

a $\frac{2}{3}$ is a proper fraction.

b $\frac{5}{4}$ is an improper fraction.

c $\frac{6}{6}$ is an improper fraction.

d $\frac{17}{18}$ is a proper fraction.

Whole numbers as fractions

All whole numbers can be written as fractions. For example, $1 = \frac{4}{4}$ and $2 = \frac{8}{4}$.

If the numerator and the denominator are the same number, we get a fraction that is equivalent to 1. For example, $\frac{4}{4} = 1$ and $\frac{100}{100} = 1$.

If the numerator is a multiple of the denominator, the fraction is equivalent to a whole number.

For example, $\frac{12}{4} = 3$ and $\frac{49}{7} = 7$.

Every whole number is also a fraction.

For example, $3 = \frac{3}{1}$ and $227 = \frac{227}{1}$.

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7

Example 14

Write the whole number equivalent to each improper fraction.

a $\frac{6}{3}$ **b** $\frac{20}{4}$ **c** $\frac{100}{100}$ **d** $\frac{288}{144}$ **e** $\frac{64}{1}$

Solution

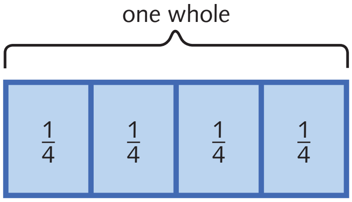
a $\frac{6}{3} = 2$ **b** $\frac{20}{4} = 5$ **c** $\frac{100}{100} = 1$ **d** $\frac{288}{144} = 2$ **e** $\frac{64}{1} = 64$

Mixed numbers

A mixed number is a whole number plus a fraction smaller than 1. For example, $1\frac{1}{6}$ is a mixed number. It means 1 plus $\frac{1}{6}$ more.

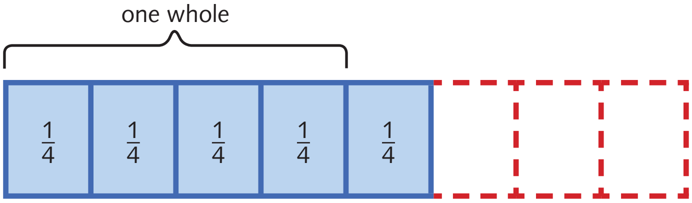
Improper fractions are either whole numbers or can be written as mixed numbers.

If we divide a rectangle into 4 equal pieces, each piece is $\frac{1}{4}$ of the whole.



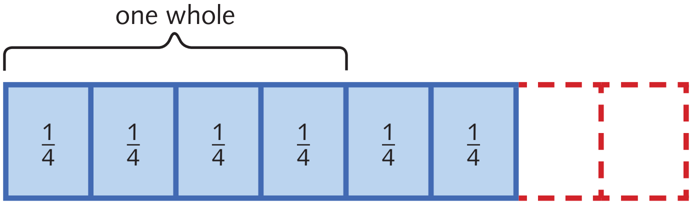
$\frac{4}{4}$ or four-quarters is the same as 1.

Now we extend the drawing by adding on pieces of size $\frac{1}{4}$.

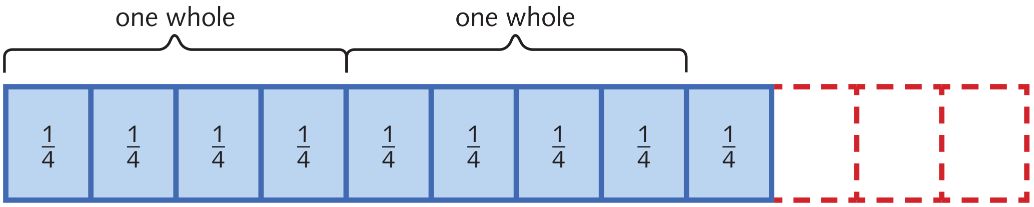


$\frac{5}{4}$ is the same as $1\frac{1}{4}$.

$$\begin{aligned}\frac{5}{4} &= \frac{4}{4} + \frac{1}{4}\\ &= 1 + \frac{1}{4}\\ &= 1\frac{1}{4}\end{aligned}$$

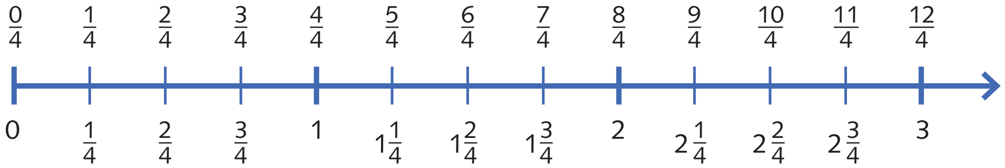


$\frac{6}{4}$ is the same as $1\frac{2}{4}$ or $1\frac{1}{2}$.



$\frac{9}{4}$ is the same as $2\frac{1}{4}$.

We can see the same result using a number line. This number line is marked using quarters and mixed numbers.



We can see that: $\frac{4}{4} = 1$, $\frac{8}{4} = 2$ and $\frac{12}{4} = 3$.

Also, $\frac{5}{4} = 1\frac{1}{4}$, $\frac{6}{4} = 1\frac{2}{4}$ and $\frac{9}{4} = 2\frac{1}{4}$.

Example 15

Write these fractions as mixed numbers in simplest form. Make the whole number part as large as possible and write the fraction part in simplest form.

a $\frac{6}{4}$

b $\frac{99}{50}$

Solution

a
$$\begin{aligned}\frac{6}{4} &= \frac{4}{4} + \frac{2}{4}\\ &= 1 + \frac{2}{4}\\ &= 1\frac{1}{2}\end{aligned}$$

b
$$\begin{aligned}\frac{99}{50} &= \frac{50}{50} + \frac{49}{50}\\ &= 1 + \frac{49}{50}\\ &= 1\frac{49}{50}\end{aligned}$$

6F Individual

1 Copy these fractions, then label each one ‘proper fraction’, ‘improper fraction’ or ‘mixed number’.

a $\frac{1}{2}$ **b** $\frac{19}{2}$ **c** $\frac{8}{8}$ **d** $\frac{3}{5}$ **e** $2\frac{2}{11}$ **f** $100\frac{18}{100}$ **g** $\frac{1}{1000}$

2 Write these fractions as whole numbers.

a $\frac{6}{2}$ **b** $\frac{21}{3}$ **c** $\frac{40}{4}$ **d** $\frac{35}{5}$ **e** $\frac{36}{6}$ **f** $\frac{900}{100}$ **g** $\frac{1000}{500}$

3 Convert these fractions to mixed numbers. Write the fraction part in simplest form.

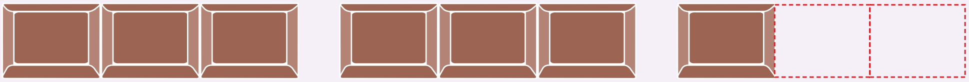
a $\frac{5}{2}$ **b** $\frac{7}{3}$ **c** $\frac{12}{10}$ **d** $\frac{101}{100}$ **e** $\frac{28}{3}$ **f** $\frac{189}{100}$ **g** $\frac{102}{8}$

4 Convert these mixed numbers to improper fractions.

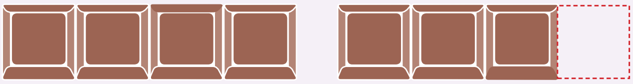
a $3\frac{2}{3}$ **b** $2\frac{1}{2}$ **c** $5\frac{3}{8}$ **d** $1\frac{14}{15}$ **e** $4\frac{7}{9}$ **f** $11\frac{2}{4}$ **g** $100\frac{4}{7}$

5 Write the mixed number that describes how many chocolate bars there are.

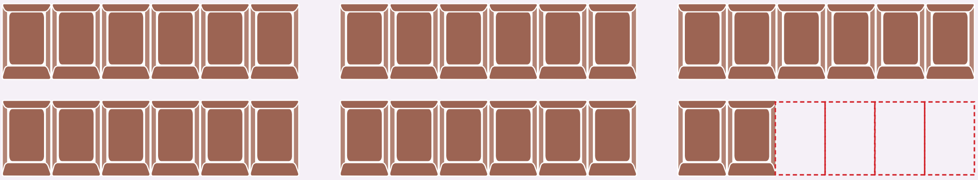
a



b



c



6 Con is planning a party. He is allowing $\frac{1}{2}$ a bottle of soft drink and $\frac{3}{8}$ of a pizza for each person who comes to his party.

a If 13 people attend Con’s party, exactly how much soft drink will he need?

b If 13 people attend Con’s party, exactly how many pizzas will he need?

c If 21 people attend Con’s party, how much soft drink and pizza will he need?

7 **a** How many pieces of rope of length $\frac{1}{8}$ metre can be cut from a piece of rope of length $6\frac{1}{2}$ metres?

b How many lengths of $\frac{1}{6}$ metre can be cut from a rope $8\frac{1}{3}$ metres long?

c How many lengths of $\frac{3}{4}$ metre can be cut from a rope $7\frac{3}{12}$ metres long?

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

6G Review questions

1 Write each fraction in words.

a $\frac{1}{3}$ **b** $\frac{9}{11}$ **c** $\frac{16}{39}$ **d** $\frac{65}{1000}$

2 Write the fraction that matches the shaded part of each football scarf.

a

b

3 Draw a football scarf to show each fraction.

a $\frac{1}{2}$ **b** $\frac{3}{7}$ **c** $\frac{9}{10}$

4 Jim has started a stamp collection. He has 12 stamps from Hong Kong, 23 stamps from the USA and 5 from the United Kingdom. What fraction of Jim's stamp collection is:

a from Hong Kong? **b** from the USA? **c** from the UK?

5 The Better Bakery makes a mixed dozen selection of bread rolls. Of the 12 rolls, 3 are poppy seed, 4 are wholemeal, 2 are white, 1 is multigrain and 2 are sunflower seed.

a What fraction of the rolls are white?

b What fraction of the rolls are wholemeal?

c What fraction of the rolls are *not* multigrain or sunflower seed?

6 Draw a number line starting at 0 and ending at 1. Mark these fractions on it.

$\frac{1}{2}, \frac{1}{4}, \frac{1}{7}, \frac{1}{8}$ and $\frac{1}{9}$

What do you notice?

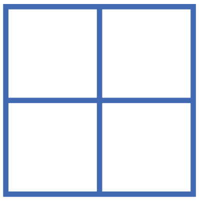
7 Draw a number line starting at 0 and ending at 1. Mark these fractions on it.

$\frac{1}{3}, \frac{1}{6}, \frac{1}{9}$ and $\frac{1}{18}$

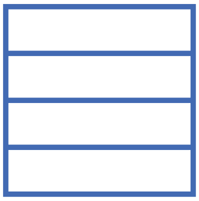
What do you notice?

8 Copy and shade $\frac{1}{4}$ of each square.

a

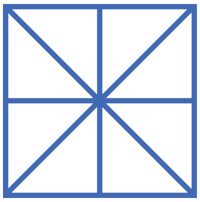


b

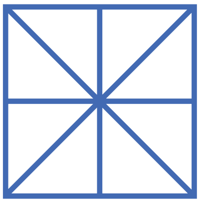


c

d



9 Make four copies of this square. Shade each square to show $\frac{5}{8}$ in four different ways.



10 Write three equivalent fractions for each of these.

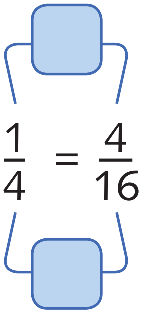
a $\frac{1}{3}$ **b** $\frac{4}{7}$ **c** $\frac{3}{4}$ **d** $\frac{4}{8}$

11 Copy these fractions and write the missing numerators and denominators.

a $\frac{1}{\square} = \frac{2}{6} = \frac{\square}{9} = \frac{\square}{54}$ **b** $\frac{2}{\square} = \frac{4}{10} = \frac{\square}{50} = \frac{10}{\square}$ **c** $\frac{4}{\square} = \frac{20}{15} = \frac{16}{\square} = \frac{\square}{18}$

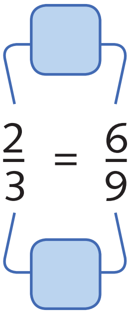
12 Copy these diagrams. Fill in the boxes to show what the numerator and denominator were *multiplied* by to arrive at the equivalent fraction.

a



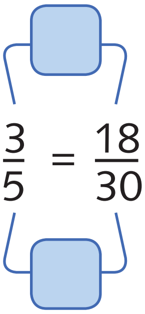
$\frac{1}{4} = \frac{4}{16}$

b



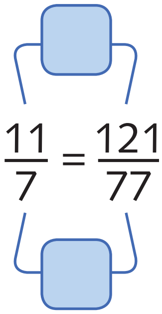
$\frac{2}{3} = \frac{6}{9}$

c



$\frac{3}{5} = \frac{18}{30}$

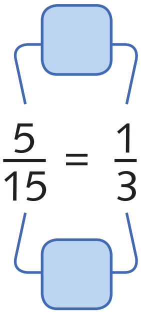
d



$\frac{11}{7} = \frac{121}{77}$

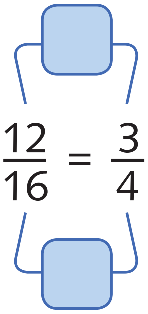
13 Copy these diagrams. Fill in the boxes to show what the numerator and denominator were *divided* by to arrive at the equivalent fraction.

a



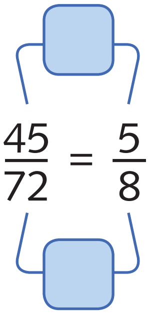
$\frac{5}{15} = \frac{1}{3}$

b



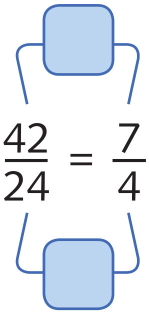
$\frac{12}{16} = \frac{3}{4}$

c



$\frac{45}{72} = \frac{5}{8}$

d



$\frac{42}{24} = \frac{7}{4}$

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

14 **a** How many tenths are equivalent to $\frac{6}{20}$?

b How many quarters are equivalent to $\frac{6}{8}$?

c How many thirds are equivalent to $\frac{12}{9}$?

d How many sixteenths are equivalent to $\frac{3}{12}$?

15 Reduce the fractions in each pair to their simplest form to show that they are equivalent.

a $\frac{6}{16}$ $\frac{9}{24}$ **b** $\frac{18}{27}$ $\frac{2}{3}$ **c** $\frac{9}{12}$ $\frac{21}{28}$ **d** $\frac{36}{28}$ $\frac{18}{14}$

16 Write these fractions in order, smallest to largest.

a $\frac{3}{5}$ $\frac{8}{5}$ $\frac{1}{5}$ $\frac{4}{5}$

b $\frac{5}{8}$ $\frac{2}{8}$ $\frac{10}{8}$ $\frac{3}{8}$ $\frac{9}{8}$

c $\frac{1}{10}$ $\frac{3}{10}$ $\frac{7}{20}$ $\frac{1}{20}$ $\frac{1}{30}$

d $\frac{3}{6}$ $\frac{1}{3}$ $\frac{8}{15}$ $\frac{1}{9}$

17 Name a fraction that is:

a between $\frac{5}{9}$ and $\frac{1}{5}$

b greater than $\frac{9}{10}$ but less than 1

c less than $\frac{3}{4}$ but greater than $\frac{8}{16}$

d near 0

18 Bernice bought two identical boxes of oranges and labelled them Box A and Box B. Last week, she ate $\frac{1}{3}$ of the oranges from Box A and $\frac{4}{9}$ of the oranges from Box B. Which box has more oranges left in it?

19 A cake recipe needs $\frac{5}{8}$ of a kilogram of flour. A scone recipe needs $\frac{10}{12}$ of a kilogram of flour. Which recipe needs more flour?

20 Copy these fractions and label them 'proper fraction', 'improper fraction' or 'mixed number'.

a $\frac{3}{17}$ **b** $\frac{4}{9}$ **c** $\frac{12}{12}$ **d** $\frac{4}{13}$ **e** $\frac{14}{5}$ **f** $\frac{3}{2}$ **g** $52\frac{17}{10}$

21 Convert these fractions to mixed numbers. Write the fraction part in its simplest form.

a $\frac{20}{3}$ **b** $\frac{5}{2}$ **c** $\frac{6}{5}$ **d** $\frac{147}{120}$ **e** $\frac{36}{4}$ **f** $\frac{73}{12}$ **g** $\frac{110}{9}$

22 Convert these mixed numbers to improper fractions.

a $3\frac{2}{3}$ **b** $2\frac{1}{2}$ **c** $5\frac{3}{8}$ **d** $1\frac{14}{15}$ **e** $4\frac{7}{9}$ **f** $10\frac{6}{11}$ **g** $100\frac{4}{7}$

CHAPTER 7 Preparation

Useful skills for this chapter:

- understanding fractions on a number line.

KICK OFF

Oranges

You will need a bag of 16 oranges (or apples).
Cut 4 oranges into halves,
4 oranges into quarters,
4 oranges into eighths and
4 oranges into thirds.
Use the oranges to model these additions,
then write the answers in simplest form.

a $\frac{1}{2} + \frac{1}{2}$ **b** $\frac{1}{4} + \frac{1}{4}$ **c** $\frac{1}{3} + \frac{1}{3}$ **d** $\frac{1}{8} + \frac{1}{8}$

e $\frac{2}{4} + \frac{1}{2}$ **f** $\frac{5}{8} + \frac{2}{8}$ **g** $\frac{1}{2} + \frac{1}{4}$ **h** $\frac{7}{8} + \frac{5}{8}$

Show what you know

1 Use the fraction blocks from **BLM 6** 'Fraction blocks' (available for download in the Interactive Textbook).

Suppose this shape is one whole.

What fractions, mixed numbers or whole numbers do these shapes show?

a

b

c

CHAPTER

7

Number and Algebra

Fraction arithmetic

Fractions can be treated like ordinary numbers. We can add one fraction to another, or multiply two fractions together.

If you look around your home, you might see situations where you add fractions.

For instance, think of a loaf of bread being cut into slices. Each slice is a fraction of the whole loaf. If you cut the loaf into ten equal slices, then each slice is $\frac{1}{10}$ of the loaf. When you use two slices to make a sandwich, you have $\frac{1}{10} + \frac{1}{10} = \frac{2}{10}$ of the loaf. If you make another sandwich for a friend, you've used $2 \times \frac{2}{10} = \frac{4}{10}$ of the loaf.

In this chapter we will be adding, subtracting, multiplying and dividing fractions.

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

7A Adding fractions with the same denominator

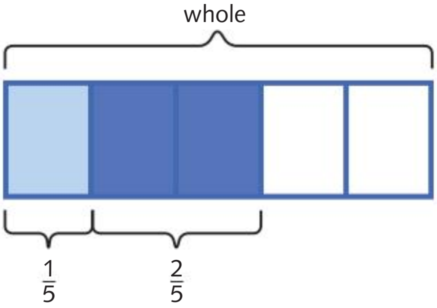
Adding fractions with the same denominator is the same as any other addition.

$3\ \boxed{\text{chocolates}} + 5\ \boxed{\text{chocolates}} = 8\ \boxed{\text{chocolates}}$

$3\ \boxed{\text{eighths}} + 5\ \boxed{\text{eighths}} = 8\ \boxed{\text{eighths}}$

If we want to add $\frac{1}{5}$ and $\frac{2}{5}$, we can draw a diagram like this.

We count the total number of pieces of size $\frac{1}{5}$ and write: $\frac{1}{5} + \frac{2}{5} = \frac{3}{5}$

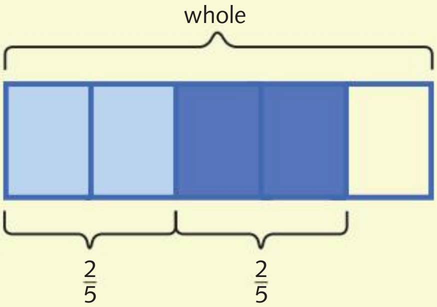


1 fifth + 2 fifths = 3 fifths

Example 1

Calculate $\frac{2}{5} + \frac{2}{5}$.

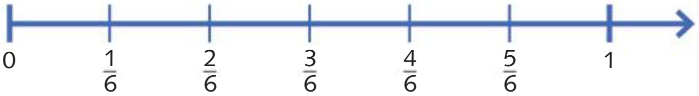
Solution



2 fifths + 2 fifths = 4 fifths

$$\frac{2}{5} + \frac{2}{5} = \frac{4}{5}$$

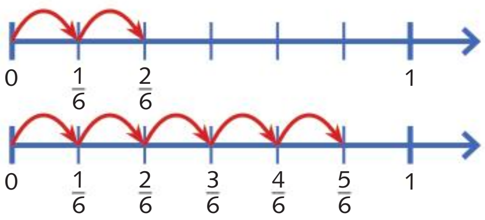
We can add two fractions on the number line. For example, $\frac{2}{6} + \frac{3}{6}$. First, divide the number line from 0 to 1 into sixths.



Then show $\frac{2}{6}$ as jumps on the number line.

Add $\frac{3}{6}$ by making 3 more jumps of $\frac{1}{6}$.

We can see that: $\frac{2}{6} + \frac{3}{6} = \frac{5}{6}$



ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

7A Whole class CONNECT, APPLY AND BUILD

○ **1** Draw rectangle pictures to show these additions.

a $\frac{1}{4} + \frac{2}{4}$ **b** $\frac{4}{6} + \frac{1}{6}$ **c** $\frac{1}{3} + \frac{1}{3}$ **d** $\frac{4}{5} + \frac{3}{5}$

○ **2** Draw number lines to show these additions. Write the answers.

a $\frac{3}{4} + \frac{1}{4}$ **b** $\frac{2}{8} + \frac{11}{8}$ **c** $\frac{8}{10} + \frac{7}{10}$ **d** $\frac{8}{5} + \frac{3}{5}$

7A Individual

○ **1** Calculate:

a $\frac{1}{3} + \frac{2}{3}$ **b** $\frac{1}{5} + \frac{3}{5}$ **c** $\frac{2}{7} + \frac{1}{7}$

d $\frac{5}{16} + \frac{8}{16}$ **e** $\frac{4}{11} + \frac{6}{11}$ **f** $\frac{1}{3} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3}$

g $\frac{1}{5} + \frac{1}{5} + \frac{2}{5}$ **h** $\frac{6}{17} + \frac{4}{17} + \frac{1}{17} + \frac{3}{17}$

○ **2** Draw a number line to calculate each addition.

a $\frac{2}{5} + \frac{2}{5}$ **b** $\frac{4}{7} + \frac{1}{7}$ **c** $\frac{2}{7} + \frac{1}{7}$

d $\frac{5}{16} + \frac{8}{16}$ **e** $\frac{4}{11} + \frac{6}{11}$ **f** $\frac{27}{100} + \frac{63}{100}$

○ **3** Murray recorded how much bread his family ate each day for 1 week.

	Sat	Sun	Mon	Tue	Wed	Thu	Fri
Fraction of a loaf of bread eaten	$\frac{3}{8}$	$\frac{2}{8}$	$\frac{7}{8}$	$\frac{9}{8}$	$\frac{11}{8}$	$\frac{10}{8}$	$\frac{4}{8}$

a On which days did Murray's family eat more than 1 loaf of bread?

b How much bread did Murray's family eat from Monday to Friday?

c How much bread did Murray's family eat on the weekend?

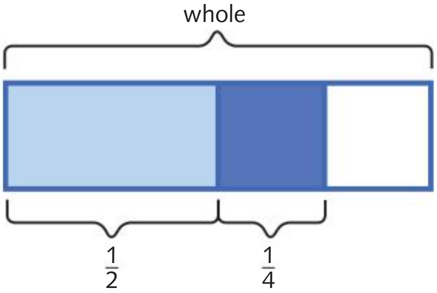
d How much bread did Murray's family eat for the whole week?

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

7B Adding fractions with different denominators

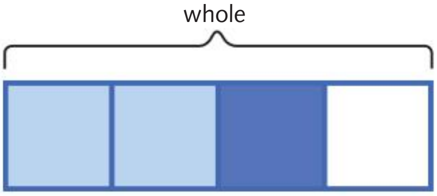
We use equivalent fractions to add fractions with different denominators.

We can show $\frac{1}{2} + \frac{1}{4}$ using a rectangle.



We can draw the same diagram differently to show that one-half plus one-quarter equals three-quarters.

This works because $\frac{1}{2}$ is equivalent to $\frac{2}{4}$.



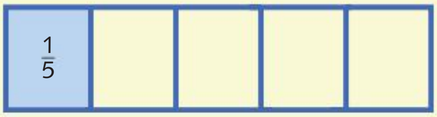
$$\frac{1}{2} + \frac{1}{4} = \frac{2}{4} + \frac{1}{4}$$
$$= \frac{3}{4}$$

Example 2

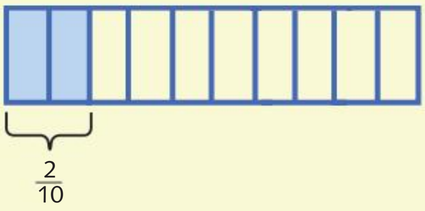
Use shaded parts of rectangles to add $\frac{1}{5}$ and $\frac{3}{10}$.

Solution

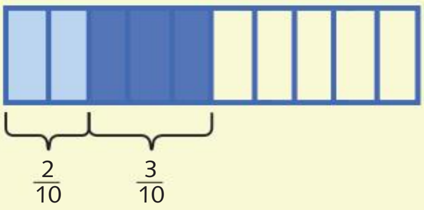
Draw a rectangle and divide it into 5 equal pieces. Shade $\frac{1}{5}$.



If we cut $\frac{1}{5}$ into two equal pieces we get two-tenths. Cut each fifth into two. Now the rectangle is divided into tenths.



Shade another $\frac{3}{10}$ to show the addition.



$$\frac{1}{5} + \frac{3}{10} = \frac{2}{10} + \frac{3}{10}$$
$$= \frac{5}{10} = \frac{1}{2}$$

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

We can use equivalent fractions to make the denominators of both fractions being added the same. Then adding the fractions is straightforward.

Example 3

Add $\frac{1}{3}$ and $\frac{1}{6}$.

Solution

These fractions do not have the same denominator. We can change $\frac{1}{3}$ to an equivalent fraction with a denominator of 6.

$\frac{1}{3} = \frac{1 \times 2}{3 \times 2} = \frac{2}{6}$ (Multiply the numerator and the denominator by 2.)

This means that:

$$\begin{aligned} \frac{1}{3} + \frac{1}{6} &= \frac{2}{6} + \frac{1}{6} \\ &= \frac{3}{6} \end{aligned}$$

We can write $\frac{3}{6}$ as a simpler, equivalent fraction: $\frac{3}{6} = \frac{3 \div 3}{6 \div 3} = \frac{1}{2}$

So, $\frac{1}{3} + \frac{1}{6} = \frac{1}{2}$

Lowest common multiple

Sometimes you will need to convert *both* fractions to equivalent fractions that have the same denominator.

For example, if we want to work out $\frac{1}{2} + \frac{1}{3}$, we need to find equivalent fractions for both $\frac{1}{2}$ and $\frac{1}{3}$.

We do this by finding the lowest common multiple of both denominators.

The multiples of 2 are 2, 4, 6, 8, 10, …

The multiples of 3 are 3, 6, 9, 12, 18, …

The lowest common multiple of 2 and 3 is 6.

Now we find equivalent fractions for $\frac{1}{2}$ and $\frac{1}{3}$ that have 6 as the denominator.

$\frac{1}{2} = \frac{1 \times 3}{2 \times 3} = \frac{3}{6}$ and $\frac{1}{3} = \frac{1 \times 2}{3 \times 2} = \frac{2}{6}$

Using equivalent fractions with the same denominator, the addition becomes as shown on the right.

$$\begin{aligned} \frac{1}{2} + \frac{1}{3} &= \frac{3}{6} + \frac{2}{6} \\ &= \frac{5}{6} \end{aligned}$$

Example 4

Add $\frac{3}{4}$ and $\frac{1}{3}$.

Solution

The fractions do not have the same denominator.

The denominators of the two fractions are 3 and 4. The lowest common multiple of 3 and 4 is 12. So now we change $\frac{3}{4}$ into a fraction that has 12 as denominator.

$$\frac{3}{4} = \frac{3 \times 3}{4 \times 3} = \frac{9}{12} \qquad (12 = 4 \times 3)$$

We also change $\frac{1}{3}$ into a fraction that has 12 as denominator.

$$\frac{1}{3} = \frac{1 \times 4}{3 \times 4} = \frac{4}{12} \qquad (12 = 3 \times 4)$$

We can now add $\frac{9}{12}$ and $\frac{4}{12}$ because they have the same denominator.

$$\frac{9}{12} + \frac{4}{12} = \frac{13}{12}$$

We convert $\frac{13}{12}$ to a mixed number.

$$\frac{13}{12} = \frac{12}{12} + \frac{1}{12}$$

$$= 1\frac{1}{12}$$

This means that our addition has become: $\frac{3}{4} + \frac{1}{3} = 1\frac{1}{12}$

7B Whole class CONNECT, APPLY AND BUILD

1 Use oranges cut into halves, quarters and eighths to help you calculate each addition.

a $\frac{1}{4} + \frac{1}{4}$

b $\frac{1}{2} + \frac{1}{4}$

c $\frac{3}{8} + \frac{1}{2}$

d $2\frac{1}{2} + \frac{3}{4}$

e $\frac{3}{8} + \frac{3}{4}$

f $1\frac{1}{2} + 2\frac{1}{8}$

g $\frac{1}{2} + \frac{1}{4} + \frac{1}{8}$

h $\frac{3}{4} + \frac{3}{4} + \frac{3}{4}$

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

2 Calculate each addition together as a class.

a $\frac{2}{3}+\frac{1}{12}$ **b** $\frac{1}{4}+\frac{1}{6}$ **c** $\frac{1}{2}+\frac{2}{3}$ **d** $\frac{3}{4}+\frac{1}{6}$

3 **a** Write the first six fractions equivalent to $\frac{1}{2}$, starting from $\frac{1\times 2}{2\times 2}=\frac{2}{4}$.

b Write the first six fractions equivalent to $\frac{1}{5}$, starting from $\frac{1\times 2}{5\times 2}=\frac{2}{10}$.

c Find a fraction in your answer to part **a** and a fraction in your answer to part **b** that have the same denominator.

d Copy this addition and fill in the blanks. $\frac{1}{2}+\frac{1}{5}=\frac{5}{\square}+\frac{\square}{10}$

$=\frac{\square}{10}$

4 **a** Write down the first six fractions equivalent to $\frac{1}{4}$, starting from $\frac{1\times 2}{4\times 2}=\frac{2}{8}$.

b Write down the first six fractions equivalent to $\frac{3}{5}$, starting from $\frac{3\times 2}{5\times 2}=\frac{6}{10}$.

c Find a fraction in your answer to part **a** and a fraction in your answer to part **b** that have the same denominator.

d Copy this addition and fill in the blanks. $\frac{1}{4}+\frac{3}{5}=\frac{\square}{\square}+\frac{\square}{20}$

$=\frac{\square}{20}$

5 Draw circles cut into halves, thirds and sixths to show each addition and its solution.

a $\frac{1}{2}+\frac{1}{3}$ **b** $\frac{4}{6}+\frac{1}{2}$ **c** $\frac{5}{6}+\frac{2}{3}$ **d** $2\frac{1}{3}+1\frac{1}{2}$

7B Individual

1 Copy these, write the missing numerators and denominators, and then solve.

a $\frac{1}{8}+\frac{1}{4}=\frac{1}{8}+\frac{\square}{8}$

$=\frac{\square}{8}$

b $\frac{2}{5}+\frac{3}{10}=\frac{\square}{10}+\frac{3}{10}$

$=\frac{\square}{10}$

c $\frac{1}{5}+\frac{3}{10}=\frac{\square}{10}+\frac{3}{10}$

$=\frac{\square}{10}$

$=\frac{\square}{2}$

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

2 Add these fractions.

a $\frac{1}{4}+\frac{1}{8}$ **b** $\frac{2}{6}+\frac{3}{12}$ **c** $\frac{4}{10}+\frac{1}{5}$ **d** $\frac{1}{100}+\frac{2}{50}$ **e** $\frac{2}{5}+\frac{2}{15}$

3 Copy and complete each addition.

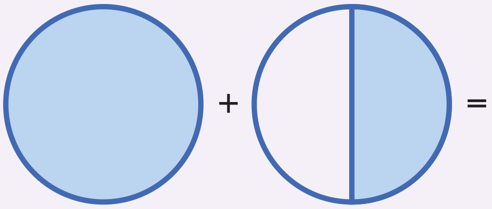
a $\frac{3}{8}+\frac{1}{2}=\frac{\square}{8}$ **b** $\frac{1}{5}+\frac{7}{10}=\frac{9}{\square}$ **c** $\frac{1}{6}+\frac{7}{12}=\frac{\square}{12}$
$=\frac{\square}{4}$

d $\frac{1}{9}+\frac{2}{3}=\frac{\square}{9}$ **e** $\frac{1}{9}+\frac{1}{3}=\frac{4}{\square}$ **f** $\frac{2}{9}+\frac{1}{3}=\frac{5}{\square}$

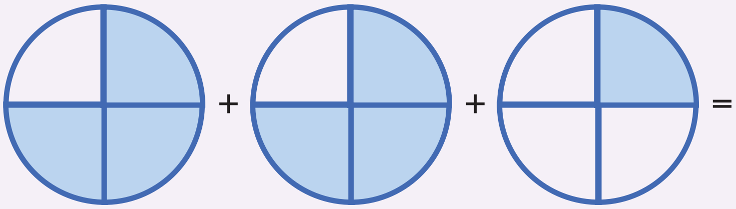
4 Janine ate $\frac{1}{8}$ kilogram of grapes. Her sister Yvonne ate $\frac{1}{4}$ kilogram of grapes. What fraction of a kilogram of grapes did they eat altogether?

5 Write the addition for each picture. Use the picture to help you calculate each addition.

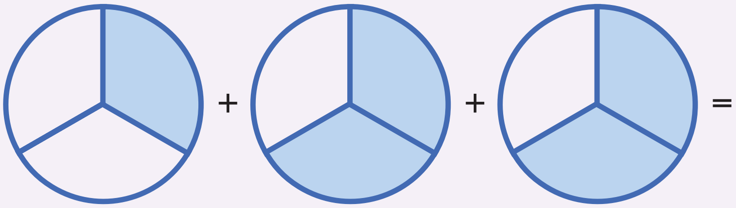
a



b



c



6 Add these fractions, then write each answer as a mixed number.

a $\frac{3}{5}+\frac{9}{10}$ **b** $\frac{3}{8}+\frac{11}{12}$ **c** $\frac{5}{6}+\frac{3}{4}$ **d** $\frac{14}{20}+\frac{4}{5}$

7 Convert the fractions in each addition to equivalent fractions with the same denominator, then add them.

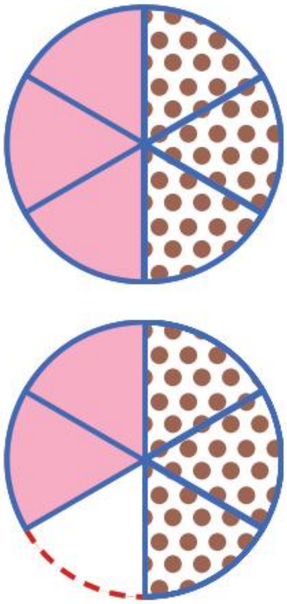
a $\frac{1}{4}+\frac{2}{12}+\frac{1}{6}$ **b** $\frac{2}{5}+\frac{1}{10}+\frac{11}{20}+\frac{2}{10}$ **c** $\frac{2}{7}+\frac{1}{14}+\frac{1}{7}+\frac{5}{14}+\frac{3}{7}$

d $\frac{5}{16}+\frac{1}{8}+\frac{1}{4}+\frac{1}{16}$ **e** $\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}$ **f** $\frac{2}{3}+\frac{8}{20}+\frac{4}{30}+\frac{7}{15}$

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

7C Subtraction of fractions

Fractions with the same denominator



This cake has been divided into 6 equal pieces.

The shaded part of the cake has icing on it, and the the spotted part has chocolate sprinkles. The iced part of the cake makes up $\frac{3}{6}$ of the total cake.

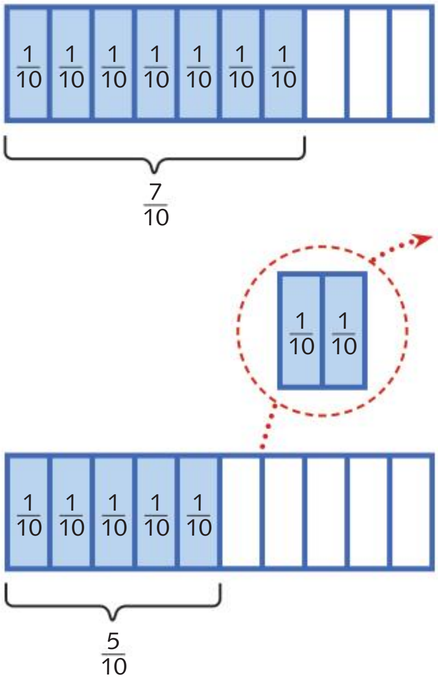
If someone eats an iced piece of cake, it is the same as subtracting $\frac{1}{6}$ of the cake.

Only 2 pieces of cake with icing are left.

We can write this as a subtraction:

$$\frac{3}{6} - \frac{1}{6} = \frac{2}{6}$$

Using rectangles



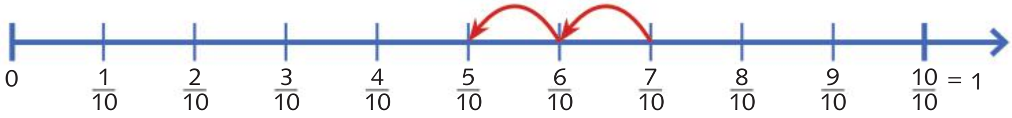
We can use rectangles cut into equal pieces to help explain subtraction. These diagrams show $\frac{2}{10}$ taken away from $\frac{7}{10}$.

Taking $\frac{2}{10}$ away leaves $\frac{5}{10}$.

$$\frac{7}{10} - \frac{2}{10} = \frac{5}{10}$$

Subtraction on the number line

We can also use a number line to show the subtraction $\frac{7}{10} - \frac{2}{10}$.



Subtracting $\frac{2}{10}$ from $\frac{7}{10}$ means starting at $\frac{7}{10}$ and moving left two steps of $\frac{1}{10}$.

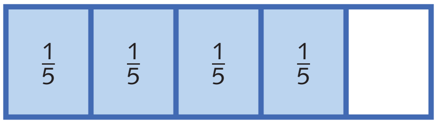
$$\frac{7}{10} - \frac{2}{10} = \frac{5}{10}$$

Subtracting fractions with different denominators

We can use rectangles to subtract fractions that have different denominators.

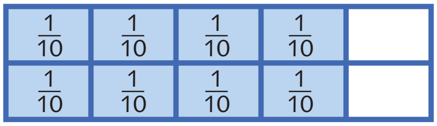
For example, if we want to take $\frac{1}{10}$ from $\frac{4}{5}$, we can draw rectangles to show what happens.

This rectangle has been cut into 5 equal pieces. Then 4 of the pieces have been shaded to show the fraction $\frac{4}{5}$.



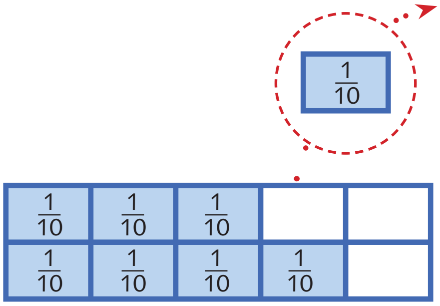
$\frac{4}{5}$

We start by cutting each $\frac{1}{5}$ piece into two equal pieces to make tenths. Each small piece is $\frac{1}{10}$.



$\frac{4}{5} = \frac{8}{10}$

Take $\frac{1}{10}$ away.



$\frac{8}{10} - \frac{1}{10} = \frac{7}{10}$

So, $\frac{4}{5} - \frac{1}{10} = \frac{7}{10}$

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

We use *equivalent fractions* when we subtract fractions that have different denominators.

Example 5

Take $\frac{1}{4}$ from $\frac{3}{8}$.

Solution

Write $\frac{1}{4}$ as an equivalent fraction with a denominator of 8.

$$\begin{aligned}\frac{1}{4} &= \frac{1 \times 2}{4 \times 2}\\ &= \frac{2}{8}\end{aligned}$$

$$\begin{aligned}\text{So, } \frac{3}{8} - \frac{1}{4} &= \frac{3}{8} - \frac{2}{8}\\ &= \frac{1}{8}\end{aligned}$$

Example 6

Calculate $\frac{4}{5} - \frac{3}{10}$.

Solution

Change $\frac{4}{5}$ to an equivalent fraction with a denominator of 10.

$$\begin{aligned}\frac{4}{5} &= \frac{4 \times 2}{5 \times 2}\\ &= \frac{8}{10}\end{aligned}$$

$$\begin{aligned}\text{So, } \frac{4}{5} - \frac{3}{10} &= \frac{8}{10} - \frac{3}{10}\\ &= \frac{5}{10}\\ &= \frac{1}{2}\end{aligned}$$

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

7C Individual

1 Work out these subtractions. Draw a rectangle to show your working.

a $\frac{3}{4} - \frac{1}{4}$ **b** $\frac{4}{5} - \frac{2}{5}$ **c** $\frac{6}{7} - \frac{2}{7}$ **d** $\frac{3}{4} - \frac{1}{8}$

e $\frac{4}{5} - \frac{1}{10}$ **f** $\frac{2}{3} - \frac{1}{6}$ **g** $\frac{3}{4} - \frac{8}{8}$ **h** $\frac{9}{10} - \frac{1}{5}$

2 Work out each subtraction, then sort the answers into these three groups:

Answer is between 0 and $\frac{1}{2}$,

Answer is equivalent to $\frac{1}{2}$, and

Answer is between $\frac{1}{2}$ and 1.

a $\frac{9}{10} - \frac{2}{10}$ **b** $\frac{1}{4} - \frac{1}{12}$ **c** $\frac{4}{5} - \frac{2}{10}$ **d** $\frac{11}{12} - \frac{3}{4}$

e $\frac{7}{8} - \frac{3}{8}$ **f** $\frac{9}{10} - \frac{2}{5}$ **g** $\frac{17}{18} - \frac{4}{9}$ **h** $\frac{5}{8} - \frac{1}{4}$

i $\frac{7}{8} - \frac{3}{12}$ **j** $\frac{9}{10} - \frac{1}{7}$ **k** $\frac{2}{3} - \frac{8}{15}$ **l** $\frac{4}{5} - \frac{3}{12}$

3 Mara made a cake and cut it into 8 equal pieces. Her brother ate 1 piece. Mara took the remaining cake to a party, where her friends ate 5 pieces. She took home what was left. What fraction of the whole cake did Mara bring home?

4 Matthew is an apprentice electrician. He started the day with $\frac{7}{8}$ of a whole roll of cable. He used $\frac{1}{3}$ of a whole roll in the morning, and $\frac{1}{4}$ of a whole roll in the afternoon. How much cable did Matthew have left at the end of the day?

5 There are two strategies you can use when subtracting mixed numbers:

- you can subtract the whole number first, then deal with the fractions, or
- you can convert both fractions to improper fractions, then deal with them as usual.

Use the most suitable strategy to subtract these mixed numbers.

a $2\frac{1}{4} - \frac{3}{4}$ **b** $4\frac{1}{3} - 2\frac{2}{3}$ **c** $6\frac{1}{10} - 4\frac{2}{10}$ **d** $5\frac{1}{12} - 3\frac{11}{12}$

e $2\frac{7}{8} - 1\frac{3}{4}$ **f** $4\frac{3}{10} - 2\frac{4}{5}$ **g** $5\frac{2}{18} - 4\frac{3}{9}$ **h** $2\frac{5}{6} - 1\frac{2}{3}$

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

7D Multiplying fractions: Taking a fraction of a number

Gary has 9 apples. He wants to take $\frac{1}{3}$ of the apples to school. How many apples is that?

We can find out by dividing the whole group into 3 equal groups and taking 1 of those groups.

Gary takes 3 apples to school.

We can write this as:

$$\frac{1}{3} \text{ of } 9 \text{ apples} = 3 \text{ apples}$$

What is two-thirds of 9 apples?

If one-third of the apples is 3 apples, then two-thirds is twice as many.

$$\begin{aligned}\text{Two-thirds of the apples} &= 2 \times 3 \text{ apples}\\ &= 6 \text{ apples}\end{aligned}$$

We can use the multiplication sign × instead of the word 'of'. We write 9 as $\frac{9}{1}$.

$$\begin{aligned}\frac{2}{3} \text{ of } 9 &= \frac{2}{3} \times \frac{9}{1}\\ &= \frac{2 \times 9}{3 \times 1}\\ &= \frac{18}{3}\\ &= 6\end{aligned}$$

Two-thirds of 9 apples is 6 apples.

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7

Example 7

a What is one-fifth of 20 chocolate bars?
b What is three-fifths of 20 chocolate bars?

Solution

a One-fifth of 20 chocolate bars

$$= \frac{1}{5} \times \frac{20}{1}$$
$$= \frac{1 \times 20}{5 \times 1}$$
$$= \frac{20}{5}$$
$= 4$ chocolate bars

b Three-fifths of 20 chocolate bars

$$= \frac{3}{5} \times \frac{20}{1}$$
$$= \frac{3 \times 20}{5 \times 1}$$
$$= \frac{60}{5}$$
$= 12$ chocolate bars

7D Whole class CONNECT, APPLY AND BUILD

1 Use counters to find the fraction of each collection.

a $\frac{3}{4}$ of 12 counters
b $\frac{3}{4}$ of 20 counters
c $\frac{3}{4}$ of 60 counters
d $\frac{1}{8}$ of 56 counters
e $\frac{7}{8}$ of 56 counters
f $\frac{1}{6}$ of 48 counters
g $\frac{4}{6}$ of 48 counters
h $\frac{1}{5}$ of 100 counters
i $\frac{3}{5}$ of 50 counters

7D Individual

1 Calculate each of these. Draw a picture to help.

a $\frac{3}{8}$ of 16
b $\frac{1}{4}$ of 12
c $\frac{3}{4}$ of 8
d $\frac{5}{4}$ of 12
e $\frac{6}{7}$ of 28
f $\frac{2}{9}$ of 162

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

2 There are 5 people in the O'Brien family: Mum, Dad and three children. Dad bought 15 potatoes for dinner. These were to be shared equally, but then Mum and one of the children went out to have dinner with a friend.

a How many people are home for dinner?

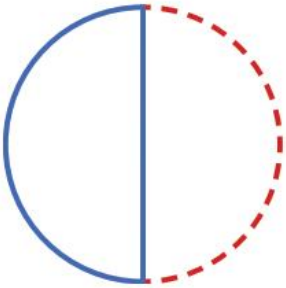
b Copy these statements and fill in the blanks.

$\frac{\square}{\square}$ of the family is home for dinner.

So Mr O'Brien should cook $\frac{\square}{\square}$ of the 15 potatoes.

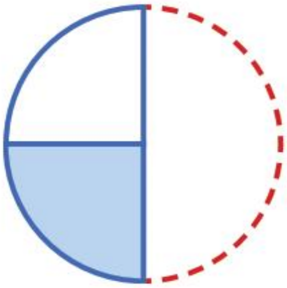
c How many of the potatoes should he cook?

7E Multiplying fractions: A fraction of a fraction



This is half a cake. It was left over from Tina's party.

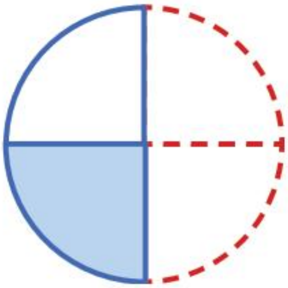
Tina's brother eats half of this piece of cake. How much of the whole cake is that?



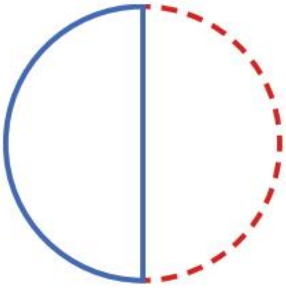
Half of half the cake is the same as one-quarter of the whole cake.

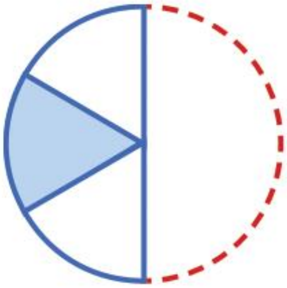
$\frac{1}{2}$ of $\frac{1}{2}$ the cake is $\frac{1}{4}$ of the cake.

$$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$



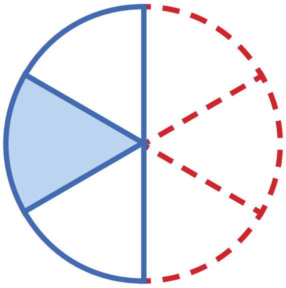
Here is another example. What is one-third of one-half of a cake?





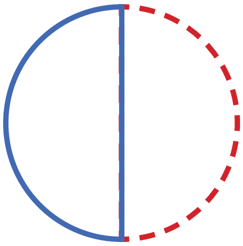
ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

Cutting $\frac{1}{2}$ of the cake into 3 equal pieces gives you the same fraction as cutting the whole cake into 6 equal pieces. Each piece is $\frac{1}{6}$ of the cake.

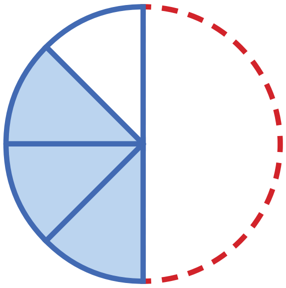


So $\frac{1}{3}$ of $\frac{1}{2}$ of the cake is the same as $\frac{1}{6}$ of the whole cake.

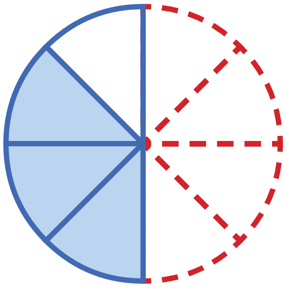
$$\frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$$



What is $\frac{3}{4}$ of $\frac{1}{2}$ a cake?



If $\frac{1}{2}$ the cake is divided into 4 equal pieces, then each piece is $\frac{1}{8}$ of the whole cake.



$\frac{3}{4}$ of $\frac{1}{2}$ the cake is $\frac{3}{8}$ of the whole cake.

$$\frac{3}{4} \times \frac{1}{2} = \frac{3}{8}$$

You might have noticed a pattern in the cake fractions.

$\frac{1}{2}$ of $\frac{1}{2}$ the cake $= \frac{1}{4}$ of the cake

$$\frac{1}{2} \text{ of } \frac{1}{2} = \frac{1 \times 1}{2 \times 2} = \frac{1}{4}$$

$\frac{1}{3}$ of $\frac{1}{2}$ the cake $= \frac{1}{6}$ of the cake

$$\frac{1}{3} \text{ of } \frac{1}{2} = \frac{1 \times 1}{3 \times 2} = \frac{1}{6}$$

$\frac{3}{4}$ of $\frac{1}{2}$ the cake $= \frac{3}{8}$ of the cake

$$\frac{3}{4} \text{ of } \frac{1}{2} = \frac{3 \times 1}{4 \times 2} = \frac{3}{8}$$

Did you see the pattern? The numerators and the denominators were multiplied to find the new fraction.

Remember

To multiply two fractions, multiply the numerators and then multiply the denominators.

For example: $\frac{3}{4}$ of $\frac{2}{3} = \frac{3}{4} \times \frac{2}{3}$
$$= \frac{6}{12}$$
$$= \frac{1}{2}$$

7E Whole class CONNECT, APPLY AND BUILD

1 Use circles of coloured paper. Cut then into equal pieces to show each ‘fraction of a fraction’.

a $\frac{1}{2}$ of $\frac{1}{2}$ **b** $\frac{1}{4}$ of $\frac{1}{3}$ **c** $\frac{1}{3}$ of $\frac{1}{2}$ **d** $\frac{1}{8}$ of $\frac{1}{4}$

2 **a** Copy this rectangle.

b Divide the rectangle into 3 equal pieces and shade $\frac{1}{3}$.

c Divide each $\frac{1}{3}$ into 2 equal pieces.

d How many pieces are shaded now?

e One of the shaded pieces is equal to $\frac{1}{2} \times \frac{1}{3}$.

Complete this statement: $\frac{1}{2} \times \frac{1}{3} = \frac{\square}{\square}$

7E Individual

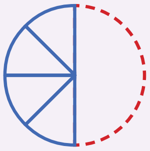
1 Copy these statements and fill in the blanks.

a This cake is divided into $\square$ equal pieces.

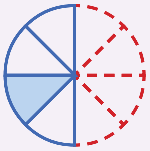
b This is $\frac{1}{\square}$ of the whole cake.

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

c This is $\frac{1}{2}$ of the cake divided into $\square$ equal pieces.



d This is $\frac{1}{\square}$ of $\frac{1}{2}$ of the cake or $\frac{1}{\square}$ of the whole cake.



e $\frac{1}{4}$ of $\frac{1}{2}$ the cake $= \frac{1 \times 1}{\square \times \square}$ of the whole cake $= \frac{1}{\square}$ of the cake.

2 Follow the instructions to find a fraction of a fraction. The first one has been done for you (see the blue solution below).

a This rectangle has been cut into halves and $\frac{1}{2}$ has been shaded.

Copy and colour the rectangle to show of $\frac{3}{4}$ of $\frac{1}{2}$.

Copy and complete this statement:

$\frac{3}{4}$ of $\frac{1}{2}$ the rectangle $= \frac{\square}{\square}$ of the whole rectangle.

a Solution: Cut the shaded half of the rectangle into 4 pieces.

Each piece is $\frac{1}{4}$ of $\frac{1}{2}$.

Shade 3 of these pieces in a different colour.

$\frac{3}{4}$ of $\frac{1}{2}$ the rectangle $= \frac{3}{8}$ of the whole rectangle

b This rectangle has been cut into thirds and $\frac{1}{3}$ has been shaded.

Copy and colour the rectangle to show $\frac{1}{2}$ of $\frac{1}{3}$.

Copy and complete this statement:

$\frac{1}{2}$ of $\frac{1}{3}$ of the rectangle $= \frac{\square}{\square}$ of the whole rectangle

3 Calculate $\frac{3}{4}$ of each of these fractions.

a $\frac{1}{2}$ **b** $\frac{1}{4}$ **c** $\frac{3}{4}$ **d** $\frac{1}{9}$

4 Craig spent $\frac{1}{2}$ his money at the show. Of the money he spent, $\frac{1}{4}$ was spent on showbags and the rest was spent on rides. Craig spent \$36 on rides.

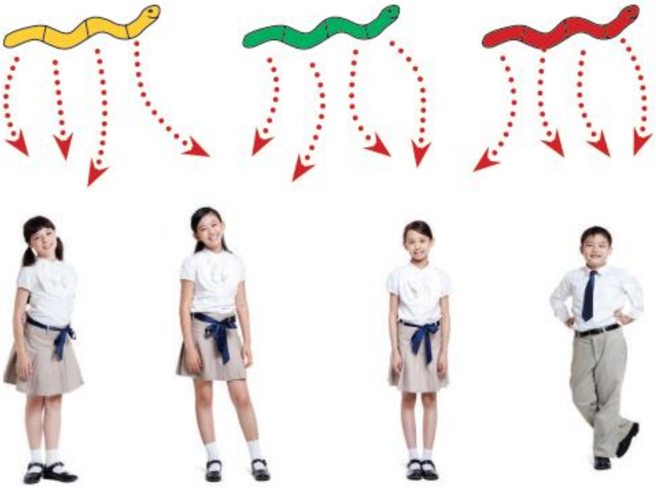
a How much money did he spend on showbags?

b How much money did Craig begin with?

7F Fractions and division

In this section we look at what happens when division by a whole number results in a fraction.

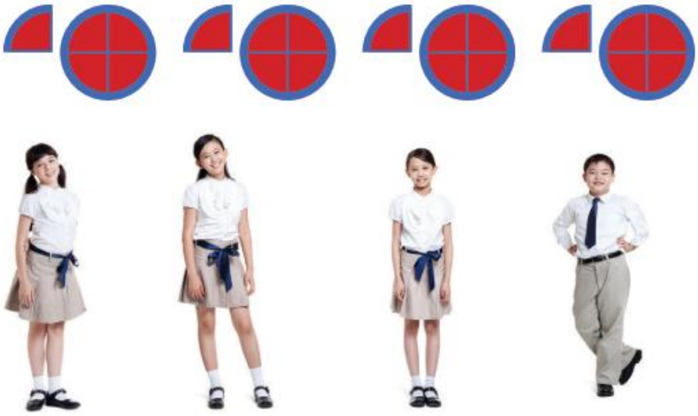
Suppose 4 children share 3 giant lolly snakes equally. Each receives $\frac{3}{4}$ of a snake.



$$3 \div 4 = \frac{3}{4}$$

Suppose 4 children share 5 jam tarts equally.

Each receives $1\frac{1}{4}$, which is the same as $\frac{5}{4}$.



$$5 \div 4 = \frac{5}{4}$$

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

Example 8

7 children share a box of 29 mangoes equally. How many mangoes does each child get?

Solution

$$\begin{aligned} 29 \div 7 &= \frac{29}{7} \\ &= \frac{28}{7} + \frac{1}{7} \qquad \text{(There are 4 lots of 7 in 29, with 1 left over.)} \\ &= 4\tfrac{1}{7} \end{aligned}$$

Each child gets $4\frac{1}{7}$ mangoes.

7F Individual

1 Find the answer to each division. Write your answer as a fraction or a mixed number in simplest form.

a $6 \div 7$
b $12 \div 13$
c $4 \div 8$
d $21 \div 5$
e $45 \div 12$
f $25 \div 100$
g $200 \div 1000$
h $41 \div 6$
i $123 \div 11$
j $140 \div 12$
k $107 \div 9$
l $444 \div 3$

2 A truckload of 63 tyres was delivered to Tony's Tyre Mart. How many sets of 4 tyres is this? Give the answer as a mixed number.

3 Nancy's science project should take her 33 hours to complete. She has 6 days to complete her project. She works on it for the same amount of time each day. How many hours will Nancy need to work on her project each day until it is finished?

4 Give the answers to these problems in their simplest form.

a 8 children share 6 giant lolly snakes. How much will each receive?

b 12 friends share 8 pizzas. How much will each receive?

c 10 litres of water are to be shared between 8 people. How much will each person receive?

d 25 oranges are shared by 10 people. How many did each receive?

e 150 potatoes are put into bags of 12. How many bags of potatoes is this?

f 24 students share 18 hours of internet time equally. How much time do they get each?

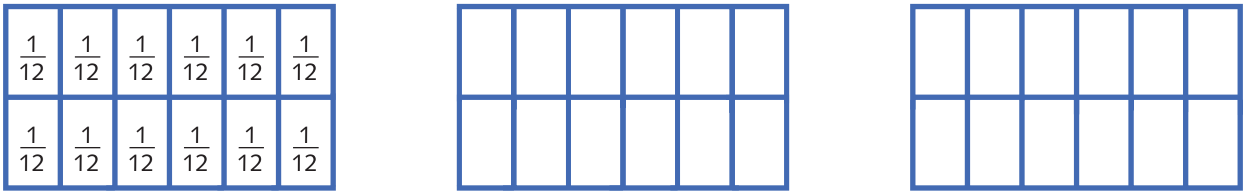
ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7

7G Dividing by a fraction

Dividing a whole number by a fraction

Ginny takes 3 identical cakes and cuts each one into 12 equal pieces. She has exactly the right number of pieces to give everyone at her party $\frac{1}{12}$ of a cake. How many people are at her party?

Here are Ginny's 3 cakes, each divided into twelfths.



There are enough pieces for 36 people to have $\frac{1}{12}$ each. So there are 36 people at Ginny's party.

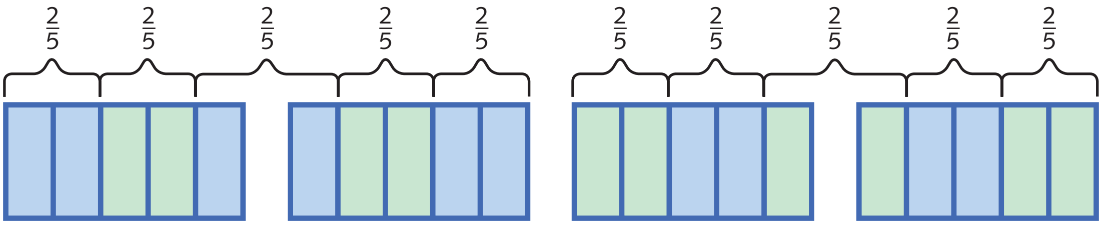
$$3 \div \frac{1}{12} = 36$$

Dividing 3 by 12 is asking how many lots of $\frac{1}{12}$ there are in 3.

There are 12 lots of $\frac{1}{12}$ in 1. So there are $3 \times 12 = 36$ lots of $\frac{1}{12}$ in 3.

What happens when we have 4 cakes and we give each person $\frac{2}{5}$ of a cake?

Divide each cake into fifths.



We get 10 lots of $\frac{2}{5}$ of a cake.

So $4 \div \frac{2}{5} = 10$ and $4 \times \frac{5}{2} = 10$

You might have noticed a pattern.

$$3 \div \frac{1}{12} = 36 \text{ and } 3 \times 12 = 36 \qquad 4 \div \frac{2}{5} = 10 \text{ and } 4 \times \frac{5}{2} = 10$$

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7

This gives us the rule:

To divide by a fraction, invert the fraction and multiply.

If we invert $\frac{1}{12}$ we get $\frac{12}{1}$ which is the same as 12.

If we invert $\frac{2}{5}$ we get $\frac{5}{2}$.

Example 9

Calculate:

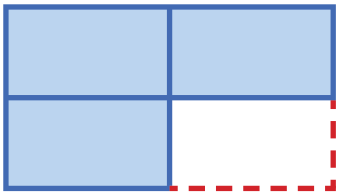
a $4 \div \frac{2}{3}$ **b** $7 \div \frac{3}{5}$

Solution

a
$$\begin{aligned} 4 \div \frac{2}{3} &= \frac{4}{1} \times \frac{3}{2} \\ &= \frac{12}{2} \\ &= 6 \end{aligned}$$

b
$$\begin{aligned} 7 \div \frac{3}{5} &= \frac{7}{1} \times \frac{5}{3} \\ &= \frac{35}{3} \\ &= 11\tfrac{2}{3} \end{aligned}$$

Dividing a fraction by a fraction



Here is a cake divided into quarters. $\frac{3}{4}$ of it is left.

To divide $\frac{3}{4}$ by $\frac{1}{4}$, ask the question:

'How many quarters are there in $\frac{3}{4}$?'

There are three $\frac{1}{4}$ in $\frac{3}{4}$:

$$\frac{3}{4} \div \frac{1}{4} = 3$$

Notice that the answer is the same as:

$$\begin{aligned} \frac{3}{4} \div \frac{1}{4} &= \frac{3}{4} \times \frac{4}{1} \\ &= \frac{12}{4} \\ &= 3 \end{aligned}$$

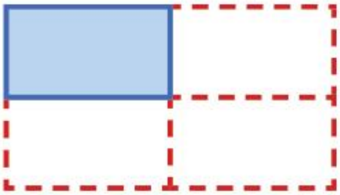
ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

Let's try another example.

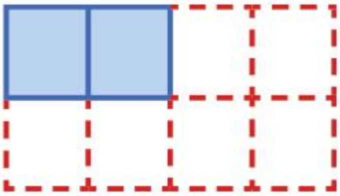
What is $\frac{1}{4} \div \frac{1}{8}$?

Here is a cake divided into quarters.

$\frac{1}{4}$ of it is left.



To see how many eighths there are in one-quarter, we need to cut the same cake into eighths.



You can see that $\frac{1}{4}$ is the same as $\frac{2}{8}$. So:

$$\frac{1}{4} \div \frac{1}{8} = 2$$

This answer also agrees with the rule:

To divide by a fraction, invert the fraction and multiply.

$$\begin{aligned}\frac{1}{4} \div \frac{1}{8} &= \frac{1}{4} \times \frac{8}{1} \\ &= \frac{8}{4} \\ &= 2\end{aligned}$$

Example 10

a Solve $\frac{3}{4} \div \frac{1}{2}$.

b Solve $\frac{7}{8} \div \frac{1}{6}$.

Solution

a $$\begin{aligned}\frac{3}{4} \div \frac{1}{2} &= \frac{3}{4} \times \frac{2}{1} \\ &= \frac{6}{4} \\ &= 1\tfrac{2}{4} \\ &= 1\tfrac{1}{2}\end{aligned}$$

b $$\begin{aligned}\frac{7}{8} \div \frac{1}{6} &= \frac{7}{8} \times \frac{6}{1} \\ &= \frac{42}{8} \\ &= 5\tfrac{2}{8} \\ &= 5\tfrac{1}{4}\end{aligned}$$

Remember

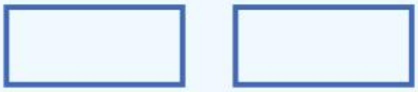
To divide by a fraction, invert the fraction and multiply.

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7

7G Whole class CONNECT, APPLY AND BUILD

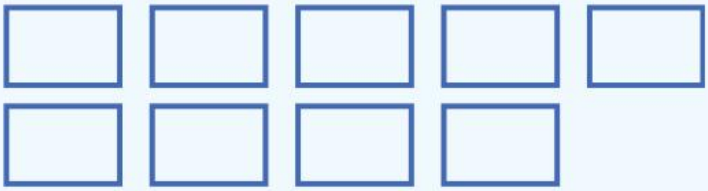
1 Copy the rectangles and shade to show each division. Then give the answer.

a $2 \div \frac{1}{4}$



b $3 \div \frac{1}{3}$

c $9 \div \frac{3}{8}$



d $\frac{1}{2} \div \frac{1}{4}$

e $\frac{2}{3} \div \frac{1}{6}$

2 Calculate:

a $\frac{1}{2} \div 2$ **b** $\frac{1}{2} \div 1$ **c** $\frac{1}{2} \div \frac{1}{2}$

d $\frac{1}{2} \div \frac{1}{4}$ **e** $\frac{1}{2} \div \frac{1}{8}$ **f** $\frac{1}{2} \div \frac{1}{16}$

7G Individual

1 How many thirds in:

a 3? **b** 4? **c** 10? **d** 5?

2 How many $\frac{2}{3}$ in:

a 4? **b** 10? **c** 3? **d** 5?

3 How many quarters in:

a 9? **b** 15? **c** 25? **d** 100?

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7

4 Complete these divisions using the rule 'to divide by a fraction, invert the fraction and multiply'.

a $9 \div \frac{1}{2}$ **b** $6 \div \frac{1}{3}$ **c** $12 \div \frac{1}{5}$ **d** $6 \div \frac{3}{4}$

e $14 \div \frac{2}{5}$ **f** $18 \div \frac{3}{8}$ **g** $12 \div \frac{3}{7}$ **h** $20 \div \frac{4}{5}$

i $28 \div \frac{2}{3}$ **j** $21 \div \frac{3}{5}$ **k** $30 \div \frac{5}{7}$ **l** $88 \div \frac{8}{11}$

5 Paul has 24 pies to share equally with a group of friends. How many people can have some pie if each person's share is:

a $\frac{3}{4}$? **b** $\frac{2}{3}$? **c** $1\frac{1}{2}$? **d** $2\frac{2}{5}$?

6 Roy has 35 litres of water. He needs to water his plants. How many plants can he water if each plant is given:

a $\frac{7}{8}$ litre? **b** $1\frac{2}{3}$ litres? **c** $2\frac{1}{2}$ litres?

7 Use the 'invert the fraction and multiply' rule to work out:

a $\frac{3}{5} \div \frac{3}{10}$ **b** $\frac{14}{7} \div \frac{2}{7}$ **c** $\frac{3}{9} \div \frac{1}{27}$ **d** $\frac{21}{9} \div \frac{1}{3}$

e $5\frac{1}{3} \div \frac{2}{3}$ **f** $22\frac{1}{2} \div \frac{3}{4}$ **g** $9\frac{3}{4} \div \frac{3}{8}$ **h** $14\frac{2}{5} \div \frac{3}{10}$

8 Graham has $10\frac{1}{8}$ metres of rope. How many pieces of rope can he cut that are $4\frac{1}{2}$ metres long?

9 Jenny walked $3\frac{1}{4}$ laps of the oval in $21\frac{1}{8}$ minutes. How long did it take her to walk each lap?

7H Review questions

1 Write the answers to these additions as proper fractions or mixed numbers.

a $\frac{1}{4} + \frac{2}{4}$ **b** $\frac{2}{5} + \frac{3}{5}$ **c** $\frac{1}{9} + \frac{4}{9}$ **d** $\frac{9}{13} + \frac{2}{13}$

e $\frac{8}{17} + \frac{7}{17}$ **f** $\frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5}$ **g** $\frac{1}{9} + \frac{3}{9} + \frac{4}{9}$

2 Draw a number line to calculate each addition.

a $\frac{2}{7} + \frac{2}{7}$ **b** $\frac{3}{5} + \frac{1}{5}$ **c** $\frac{4}{8} + \frac{3}{8}$ **d** $\frac{5}{18} + \frac{11}{18}$

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

3 Add these fractions.

a $\frac{1}{3}+\frac{3}{4}$ **b** $\frac{2}{5}+\frac{3}{10}$ **c** $\frac{1}{2}+\frac{7}{8}$

d $\frac{1}{75}+\frac{6}{25}$ **e** $\frac{3}{7}+\frac{5}{21}$ **f** $\frac{12}{15}+\frac{5}{20}$

4 Add the fractions, then write the answer as a mixed number.

a $\frac{3}{7}+\frac{2}{3}$ **b** $\frac{5}{8}+\frac{4}{6}$ **c** $\frac{9}{15}+\frac{4}{5}$

d $\frac{75}{100}+\frac{30}{50}$ **e** $\frac{5}{12}+\frac{15}{18}$ **f** $\frac{5}{9}+\frac{23}{45}$

5 Copy and complete these additions.

a $\frac{2}{5}+\frac{1}{2}=\frac{\boxed{}}{10}$ **b** $\frac{5}{6}+\frac{1}{8}=\frac{23}{\boxed{}}$ **c** $\frac{1}{7}+\frac{1}{3}=\frac{10}{\boxed{}}$

d $\frac{3}{4}+\frac{7}{12}=\frac{\boxed{}}{12}$
$=\frac{\boxed{}}{3}$

e $\frac{1}{9}+\frac{1}{6}=\frac{\boxed{}}{18}$ **f** $\frac{3}{4}+\frac{3}{5}=\frac{\boxed{}}{20}$

6 Convert the fractions in each set to equivalent fractions with the same denominator, then add them.

a $\frac{1}{3}+\frac{1}{12}+\frac{1}{8}$ **b** $\frac{1}{5}+\frac{3}{10}+\frac{7}{20}+\frac{7}{10}$ **c** $\frac{2}{3}+\frac{5}{6}+\frac{4}{9}+\frac{5}{12}$

d $\frac{3}{16}+\frac{3}{8}+\frac{3}{4}+\frac{5}{16}$ **e** $\frac{7}{10}+\frac{1}{15}+\frac{11}{30}+\frac{1}{5}$ **f** $\frac{1}{14}+\frac{6}{7}+\frac{4}{7}+\frac{9}{14}+\frac{1}{7}$

7 At the end of their holiday, the Russell family drove back home from Fingal Bay to Brunswick. On the first day, Mrs Russell drove $\frac{1}{4}$ of the distance. On the second day, Mr Russell drove $\frac{3}{8}$ of the distance and Mrs Russell drove $\frac{5}{16}$ of the distance. How much of the distance home had the Russells driven in those two days?

8 Work out these subtractions.

a $\frac{2}{3}-\frac{1}{3}$ **b** $\frac{3}{8}-\frac{2}{8}$ **c** $\frac{5}{9}-\frac{2}{9}$ **d** $\frac{3}{5}-\frac{3}{10}$

e $\frac{5}{8}-\frac{1}{4}$ **f** $\frac{1}{7}-\frac{1}{14}$ **g** $\frac{5}{6}-\frac{7}{12}$ **h** $\frac{7}{10}-\frac{2}{5}$

9 Shark Bay School bought 15 kilograms of potting mix to make a vegetable garden. They used 2 kilograms for cherry tomato pots, 3 kilograms for cucumber pots and 4 kilograms for the capsicum pots. What fraction of the potting mix was left for the garden?

10 **a** Calculate $\frac{5}{6}$ of 18. **b** Calculate $\frac{2}{3}$ of 9. **c** Calculate $\frac{2}{5}$ of 9.

11 Find the value of each division. Write your answer as a fraction or mixed number in its simplest form.

a $3 \div 5$ **b** $9 \div 11$ **c** $3 \div 12$

d $19 \div 5$ **e** $53 \div 4$ **f** $24 \div 72$

g $300 \div 7000$ **h** $62 \div 8$ **i** $212 \div 30$

12 Jason's Seafood received a delivery of 325 kilograms of fresh prawns. Jason wants to sell the prawns in bulk lots of 6 kilograms. How many 6-kilogram bulk lots can Jason sell?

13 Amrita's homework should take her 90 minutes to complete. She does homework for the same amount of time each day, and she has 6 days to complete the homework. How many hours will Amrita need to work on her homework each day until it is finished?

14 Calculate each division and show your answer as a mixed number, where possible.

a $20 \div 7$ **b** $20 \div 3$ **c** $20 \div 9$

d $20 \div 11$ **e** $20 \div 13$ **f** $20 \div 17$

g $20 \div 19$ **h** $20 \div 21$

15 Calculate, and simplify where you can.

a $\frac{1}{2} \times \frac{1}{4}$ **b** $\frac{1}{2} \times \frac{1}{2}$ **c** $\frac{1}{3} \times \frac{2}{3}$

d $\frac{1}{2} \times \frac{2}{3}$ **e** $\frac{4}{5} \times \frac{9}{20}$ **f** $\frac{3}{4} \times \frac{4}{7}$

g $\frac{6}{7} \times \frac{7}{8}$ **h** $\frac{5}{9} \times \frac{4}{5}$

16 Calculate, and simplify where you can.

a $21 \div \frac{1}{3}$ **b** $42 \div \frac{5}{6}$ **c** $36 \div \frac{1}{3}$

d $\frac{1}{4} \div \frac{1}{2}$ **e** $\frac{1}{2} \div \frac{1}{4}$ **f** $\frac{3}{4} \div \frac{1}{2}$

g $\frac{1}{2} \div \frac{3}{4}$ **h** $\frac{3}{5} \div \frac{1}{15}$ **i** $\frac{19}{20} \div \frac{1}{4}$

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

CHAPTER 8 Preparation

Useful skills for this topic:

- an understanding of the base-ten system of hundreds, 10 and ones
- knowledge about fractions being part of a whole
- using number lines to represent numbers, including fractions and mixed numbers
- some understanding of decimal numbers to thousandths.

KICK OFF

Beachball

Write these numbers on stickers and place them randomly on a beachball.

0.3 0.7 1.4 3.6 0.04 0.07 1.33 4.82

Throw the beachball around the room. The person who catches the ball says the number nearest their right thumb.

Show what you know

1 Which decimal is the same as:

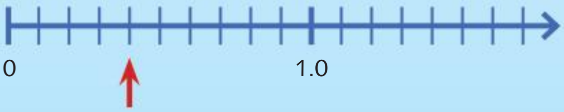
a $\frac{4}{10}$? **b** $\frac{8}{10}$? **c** $\frac{9}{10}$?

2 Write two fractions which are the same as:

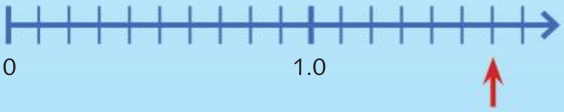
a 0.2 **b** 0.5 **c** 0.9

3 Which decimal numbers are these arrows pointing to?

a



b



CHAPTER

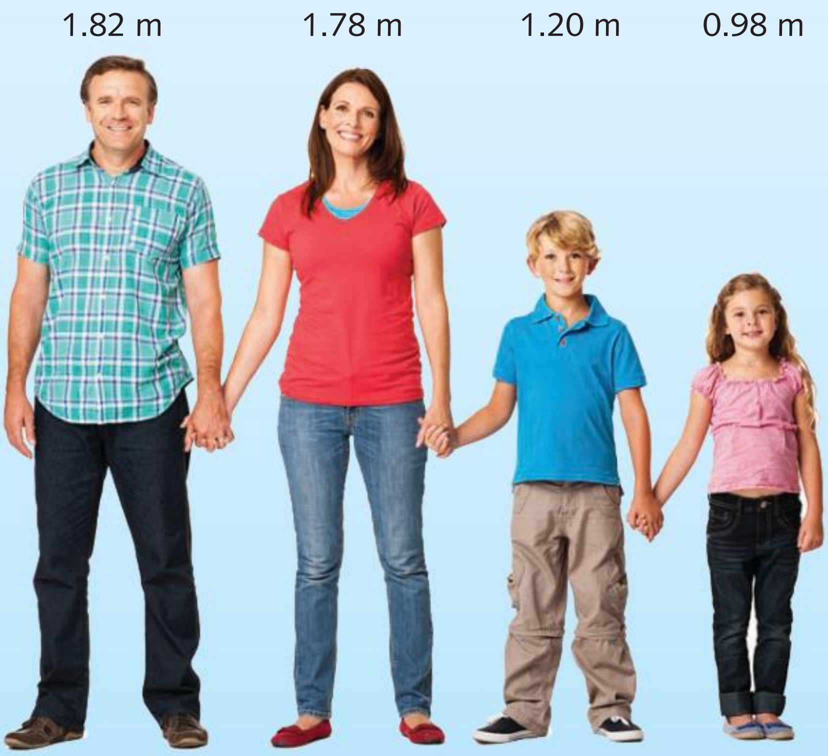
8

Number and Algebra

Decimals

Decimal numbers are built up from whole numbers and fractions such as $\frac{1}{10}, \frac{1}{100}, \frac{1}{1000}$, and so on.

Decimal numbers are used for measurements in the metric system, such as measuring height. For example, here are the heights of the Maney family (in metres).



We also use decimal numbers in our currency system. For example, 185 cents is written as $1.85.

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7

Cambridge University Press

8A Decimal numbers

The word ‘decimal’ comes from the Latin word *decem*, which means ‘ten’. Our number system is based on ‘lots of’ ten.

The whole number 3946 means: 3 thousands + 9 hundreds + 4 tens + 6 ones

So $3946 = 3 \times 1000 + 9 \times 100 + 4 \times 10 + 6 \times 1$

Thousands	Hundreds	Tens	Ones
3	9	4	6

Decimal numbers use the fractions $\frac{1}{10}, \frac{1}{100}, \frac{1}{1000}$, and so on, as well as whole numbers. The decimal point is used to indicate where the fraction parts start.

For example, the number 3946.572 means:

3 thousands + 9 hundreds + 4 tens + 6 ones + 5 tenths + 7 hundredths + 2 thousandths

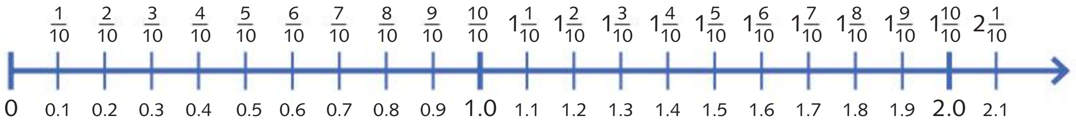
So $3946.572 = 3 \times 1000 + 9 \times 100 + 4 \times 10 + 6 \times 1 + 5 \times \frac{1}{10} + 7 \times \frac{1}{100} + 2 \times \frac{1}{1000}$

Thousands	Hundreds	Tens	Ones	tenths	hundredths	thousandths
3	9	4	6 •	5	7	2

Tenths

Decimal numbers can be shown on a number line. This number line shows 0, 1 and 2.

Cut the line between 0 and 1 into 10 equal pieces. Each piece has a length of $\frac{1}{10}$. Label the first marker $\frac{1}{10}$, then continue to label across the number line. Remember that $\frac{10}{10}$ is the same as 1. After this, the number line shows whole numbers and tenths: $1\frac{1}{10}$, $1\frac{2}{10}$, and so on.

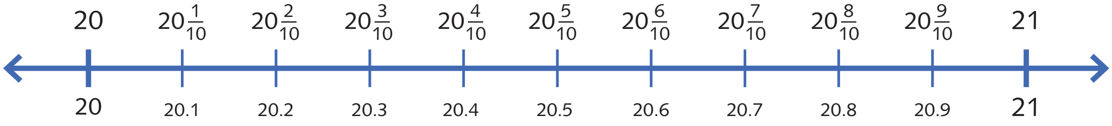


The number line shows fractions and decimals that are equivalent.

$0.1 = \frac{1}{10}$ $\quad 0.2 = \frac{2}{10}$ $\quad 0.3 = \frac{3}{10}$ $\quad 1.4 = 1\frac{4}{10}$ $\quad 2.3 = 2\frac{3}{10}$

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

Between every whole number and the next, the number line can be marked in tenths. Here is a number line marked in tenths between 20 and 21.



We can see that:

$20.1 = 20\frac{1}{10}$

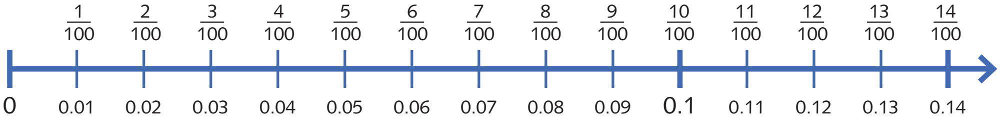
$20.8 = 20\frac{8}{10}$, and so on

Hundredths

If we cut the number line between 0 and $\frac{1}{10}$ into 10 equal pieces, we get hundredths. Each piece has length $\frac{1}{100}$. Using decimals that becomes:

$$\frac{1}{100} = 0.01$$

We can mark across the number line in hundredths, starting at 0.



The second decimal place is for hundredths. We write:

$$\frac{1}{100} = 0.01 \qquad \frac{2}{100} = 0.02 \qquad \frac{3}{100} = 0.03 \qquad \frac{4}{100} = 0.04$$

$$\frac{5}{100} = 0.05 \qquad \frac{6}{100} = 0.06 \qquad \frac{7}{100} = 0.07 \qquad \frac{8}{100} = 0.08 \qquad \frac{9}{100} = 0.09$$

$$\frac{10}{100} = 0.1 \text{ because } \qquad \frac{10}{100} = \frac{1}{10}$$

$$\frac{11}{100} = 0.11 \text{ because } \qquad \frac{11}{100} = \frac{10}{100} + \frac{1}{100} = \frac{1}{10} + \frac{1}{100}$$

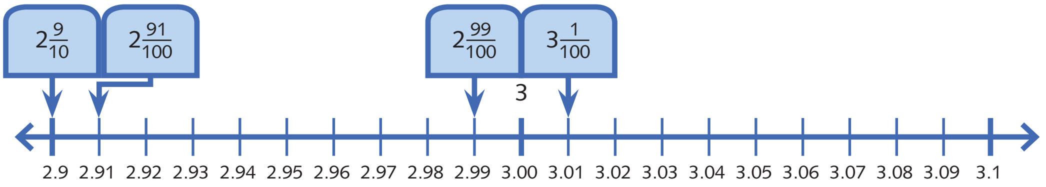
$$\frac{14}{100} = 0.14 \text{ because } \qquad \frac{14}{100} = \frac{1}{10} + \frac{4}{100}, \text{ and so on.}$$

When we get to 1, we continue in steps of one-hundredth.

$$1.01 = 1\frac{1}{100} \qquad 1.02 = 1\frac{2}{100} \qquad 1.09 = 1\frac{9}{100}$$

$$1.10 = 1\frac{10}{100} \qquad 1.11 = 1\frac{11}{100}, \text{ and so on}$$

This number line shows hundredths between 2.9 and 3.1.



Look at the number line. You can see that:

$2.9 = 2\frac{9}{10}$ $\quad 2.91 = 2\frac{91}{100}$ $\quad 2.99 = 2\frac{99}{100}$ $\quad 3.01 = 3\frac{1}{100}$

Example 1

a Write the decimal number 87.46 on a place-value chart.

b Write the decimal number 87.46 as a sum of tens, ones, tenths and hundredths.

c Mark 87, 87.46 and 88 on a number line.

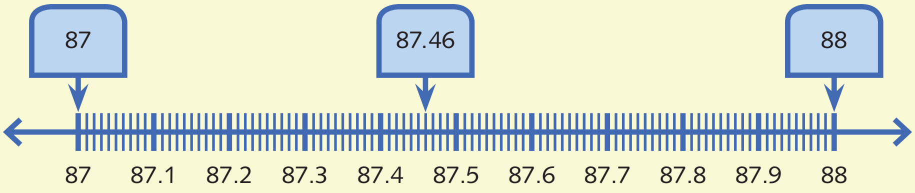
Solution

a

Tens	Ones	tenths	hundredths
8	7 ●	4	6

b $87.46 = 80 + 7 + \frac{4}{10} + \frac{6}{100}$

c 87.46 is between 87.4 and 87.5



Thousandths

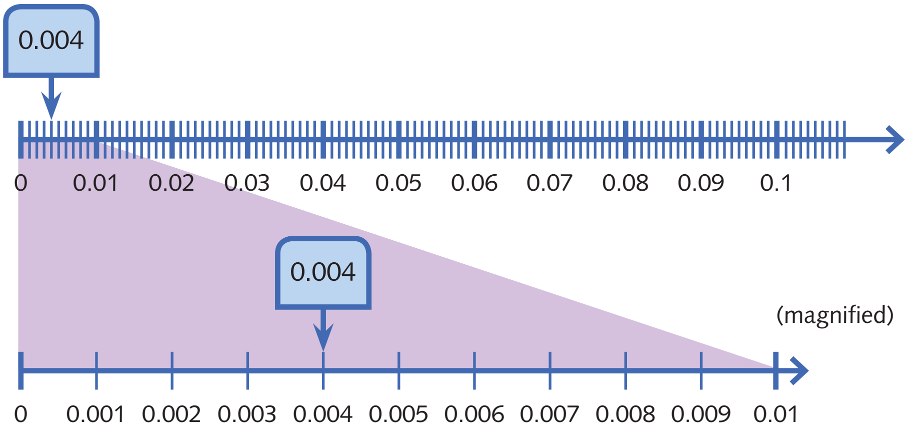
If we cut the part of the number line between 0 and $\frac{1}{100}$ into 10 equal pieces, we get thousandths.

$$\frac{1}{1000} = 0.001$$

The third decimal place is used for thousandths, so 4 thousandths is written as 0.004.

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

When we magnify the number line between 0 and 0.01, we can see more clearly where 0.004 is placed.



The following number line shows thousandths further along the number line, between 19.09 and 19.1.

Example 2

Write the number 29.487 in a place-value chart.

Solution

Tens	Ones	tenths	hundredths	thousandths
2	9 •	4	8	7

Decimals as mixed numbers or fractions

We know that 1.2 is the same as $1\frac{2}{10}$.

$$
\begin{aligned}
1.2 &= 1 + \frac{2}{10} \\
&= 1\tfrac{2}{10} \\
&= 1\tfrac{1}{5}
\end{aligned}
$$

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

If we want to write 2.45 as a mixed number, we do it like this.

$$\begin{aligned}2.45 &= 2 + \frac{4}{10} + \frac{5}{100}\\ &= 2 + \frac{40}{100} + \frac{5}{100}\\ &= 2\tfrac{45}{100}\\ &= 2\tfrac{9}{20}\end{aligned}$$

However, there is a quicker way to write fractions as mixed numbers. As we've seen in the previous example, 2.45 goes as far as the hundredths place. So we write 100 as the denominator and 45 as the numerator.

$$2.45 = 2\tfrac{45}{100}$$

The same shortcut works for thousandths. The decimal number 12.765 goes as far as thousandths and it has 765 after the decimal point, so:

$$\begin{aligned}12.765 &= 12\tfrac{765}{1000}\\ &= 12\tfrac{153}{200}\end{aligned}$$

The decimal 0.08 is less than 1. We can write it as a fraction.

$$\begin{aligned}0.08 &= \frac{8}{100}\\ &= \frac{2}{25}\end{aligned}$$

8A Whole Class CONNECT, APPLY AND BUILD

1 **a** Draw a number line between 13 and 14 and include these markers.

13 13.1 13.2 13.3 13.4 13.5 13.6 13.7 13.8 13.9 14

b Draw arrows on your number line to show where these decimal numbers are located.

13.2 13.22 13.08 13.97

2 **What number could I be?**

Draw this place-value chart.

Hundreds	Tens	Ones	tenths	hundredths	thousandth

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

Students throw a 10-sided die four times and choose a place for each throw. For example:

First throw Someone throws a 5 and says 'I have a 5 in the hundreds place. What number could I be?'

Students suggest possible numbers, which could be any number from 500 to 599.999.

Second throw Someone throws a 6 and says 'I have a 5 in the hundreds place and I also have a 6 in the tenths place. What number could I be?'

Suggestions could include 500.6 or 500.62 or any other number with a 5 in the hundreds place and a 6 in the tenths place. For each suggested number, the class should decide if this is a suitable number.

Continue for four or five throws. You can then fill in any blank spots with digits of your choice. For example, after 4 throws, the class might have:

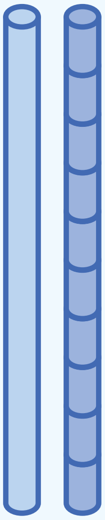
5	6	3	. 6		

You then fill in the remaining spaces with digits of your choice.

Work with a small group (or the whole class) to order your numbers from smallest to largest.

3 Decimal sticks

a Draw on the board a vertical line about 50 centimetres long. Call its length 1. How long do you think 1 tenth might be?



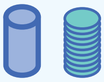
b Draw 10 tenths next to the 1, as shown (left).

Copy and complete:

____ tenths is the same as 1.

c How long do you think 1 hundredth might be?

d Draw 10 hundredths next to 1 tenth:

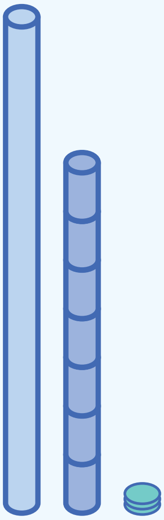


Copy and complete:

____ hundredths is the same as 1 tenth.

We can use decimal stick pictures to model decimal numbers.

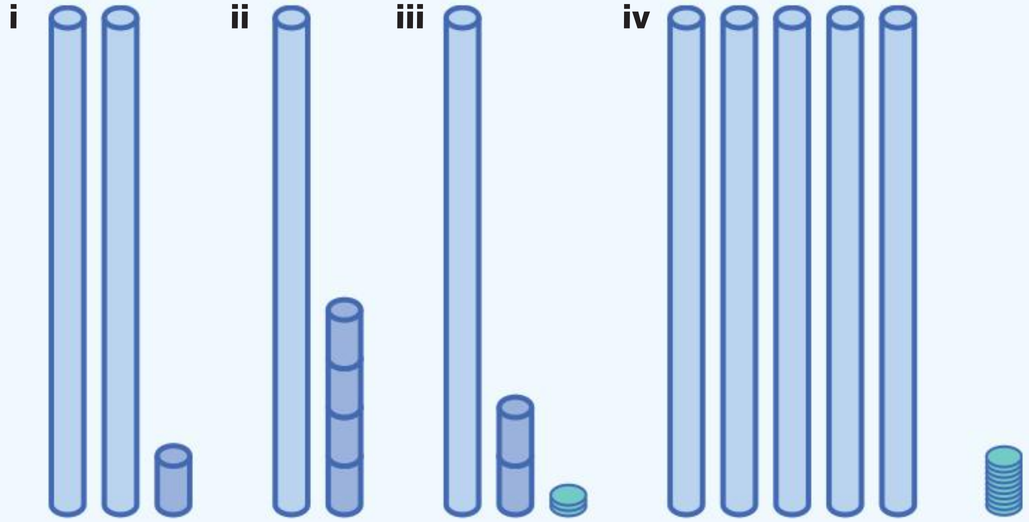
This is what 1.73 looks like using a decimal stick picture (right).



1.73 is the same as 1 one, 7 tenths and 3 hundredths.

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7

e Which decimal numbers are shown by these decimal stick pictures?



f Use decimal stick pictures to model these numbers.

0.6 2.8 1.9 1.11 3.47 3.75 4.99 10.08

g Make decimal stick pictures for some decimal numbers of your own.

4 Convert these decimals into fractions or mixed numbers with denominators of 100.

a 19.29 **b** 20.02 **c** 256.23 **d** 0.05 **e** 0.84

8A Individual

1 Copy these statements and fill in the blanks with digits 0 to 9. The first one has been done for you.

a $$46.39 = \boxed{4}\boxed{6} + \frac{\boxed{3}}{10} + \frac{\boxed{9}}{100} = \boxed{4}\boxed{6} + \frac{\boxed{39}}{100}$$

b $$17.04 = \boxed{}\boxed{} + \frac{\boxed{}}{10} + \frac{\boxed{}}{100} = \boxed{}\boxed{} + \frac{\boxed{}}{100}$$

c $$20.09 = \boxed{}\boxed{} + \frac{\boxed{}}{10} + \frac{\boxed{}}{100} = \boxed{}\boxed{} + \frac{\boxed{}}{100}$$

d $$99.99 = \boxed{}\boxed{} + \frac{\boxed{}}{10} + \frac{\boxed{}}{100} = \boxed{}\boxed{} + \frac{\boxed{}}{100}$$

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

2 Copy and complete these statements.

a $27.39 = 27 + \frac{\square}{10} + \frac{\square}{100}$

$= 27 + \frac{\square}{100}$

b $80.08 = 80 + \frac{\square}{10} + \frac{\square}{100}$

$= 80 + \frac{\square}{100}$

c $72.01 = 72 + \frac{\square}{10} + \frac{\square}{100}$

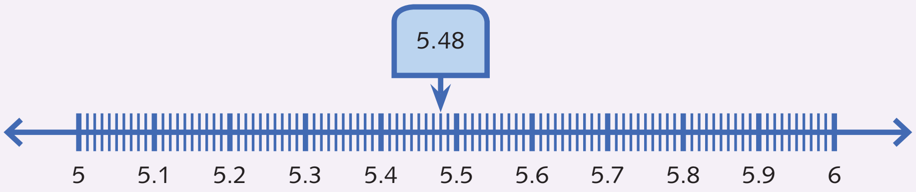
$= 72\frac{\square}{100}$

d $38.24 = 38 + \frac{\square}{10} + \frac{\square}{100}$

$= 38\frac{\square}{100}$

3 Mark each number on a number line, then copy the statement and fill in the blanks. The first one has been done for you.

a 5.48 is between 5.$\boxed{4}$ and 5.$\boxed{5}$



b 9.37 is between 9.☐ and 9.☐

c 29.91 is between 29.☐ and 30.☐

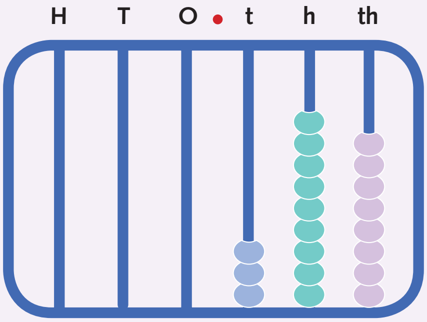
d 19.98 is between 19.☐ and 20.☐

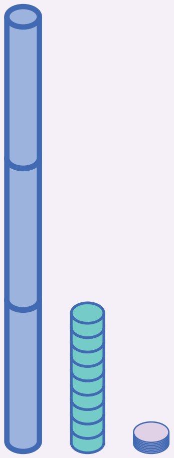
e 19.99 is between 19.9 and ☐☐.☐

4 Decimals can be shown in several ways.
For example, we can show 0.398 on a place-value chart . . .

Hundreds	Tens	Ones	tenths	hundredths	thousandths
		0 •	3	9	8

or on an abacus . . .





or by using decimal stick pictures (right).

Show each decimal in two different ways.

a 0.06 **b** 8.14 **c** 5.49 **d** 0.752

e 6.057 **f** 3.561 **g** 1.952

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

5 Convert these numbers into fractions or mixed numbers the long way. Then do it the short way. The first one has been done for you.

a 296.74

Long way:

$$\begin{aligned} 296.74 &= 296 + \frac{7}{10} + \frac{4}{100} \\ &= 296 + \frac{70}{100} + \frac{4}{100} \\ &= 296\tfrac{74}{100} \end{aligned}$$

Short way: 296.74 goes as far as hundredths.
We have 74 after the decimal point. So:

$$296.74 = 296\tfrac{74}{100}$$

b 24.37 **c** 0.09 **d** 39.99 **e** 25.06 **f** 99.08

6 Write these fractions and mixed numbers as decimals.

a $12\frac{2}{10}$ **b** $13\frac{14}{100}$ **c** $\dfrac{9}{10}$ **d** $24\frac{9}{10}$

7 Convert these mixed numbers into decimals.

a $2\frac{7}{10}$ **b** $21\frac{19}{100}$ **c** $33\frac{7}{100}$ **d** $99\frac{9}{10}$

e $89\frac{9}{100}$ **f** $721\frac{313}{1000}$ **g** $8\frac{456}{1000}$ **h** $77\frac{72}{100}$

i $19\frac{57}{1000}$ **j** $37\frac{1}{1000}$ **k** $4\frac{9}{100}$ **l** $12\frac{2}{1000}$

8 Write these as decimals.

a $7 + \dfrac{7}{100}$ **b** $6 + \dfrac{3}{10} + \dfrac{9}{100}$ **c** $987 + \dfrac{4}{100}$

9 Write these decimals as fractions.

a 2.3 **b** 3.9 **c** 1.99 **d** 2.05 **e** 4.02 **f** 121.98

10 Convert each decimal into a mixed number or fraction.

a 9.96 **b** 0.091 **c** 134.007 **d** 120.079

e 44.04 **f** 76.012 **g** 295.007 **h** 303.303

11 a Write an equivalent fraction with a denominator of 10 for each fraction.

$\dfrac{1}{2}$ $\dfrac{1}{5}$ $\dfrac{3}{5}$ $\dfrac{2}{5}$

b Now write each fraction in part **a** as a decimal.

12 a Write each fraction as an equivalent fraction with a denominator of 100.

$\dfrac{1}{4}$ $\dfrac{3}{4}$ $\dfrac{1}{25}$ $\dfrac{7}{25}$ $\dfrac{3}{25}$ $\dfrac{1}{50}$ $\dfrac{9}{50}$ $\dfrac{1}{20}$ $\dfrac{18}{20}$ $\dfrac{13}{20}$

b Now write each fraction in part **a** as a decimal.

13 Write these mixed numbers as decimals.

$1\frac{1}{2}$ $2\frac{1}{4}$ $17\frac{3}{4}$ $8\frac{1}{5}$ $6\frac{3}{5}$ $3\frac{1}{50}$ $2\frac{7}{50}$

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7

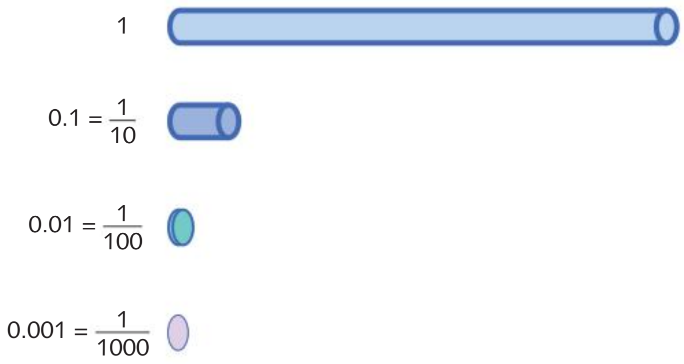
14 Anne bought $1\frac{1}{2}$ kg of sugar. Anne's mum had already bought 2.5 kg of sugar. How much sugar did they have in total?

15 Tom used 4.3 litres of paint when he painted his bedroom. His brother Jake used $4\frac{1}{4}$ litres to paint his bedroom. How much paint did Tom and Jake use in total?

8B Comparing decimal numbers

We can use decimal stick pictures to help us understand more about the relative size of decimal numbers.

Here are the pictures we use for 1, 0.1, 0.01 and 0.001.



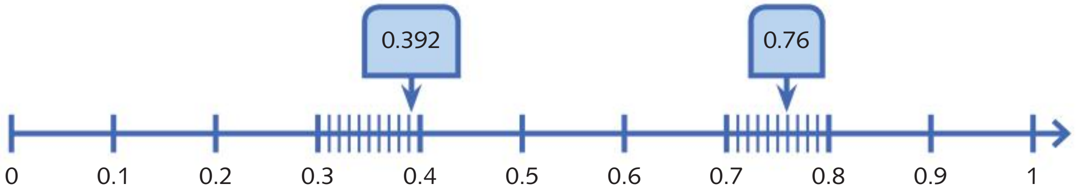
So 0.76 is shown as: 

and 0.392 is shown as: 

0.76 has 7 tenths and 0.392 has 3 tenths.

The decimal stick picture for 0.76 is longer than the picture for 0.392. So 0.76 is the larger decimal number.

We can see this on a number line.



0.76 is to the right of 0.392 on the number line, so 0.76 is larger than 0.392.

Example 3

Put these numbers in order, smallest to largest.

7.426 7.845 7.417

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

Solution

Line up the numbers using the decimal point as a marker, so that digits with the same place value are lined up under each other. All of the numbers have the same whole number part, so we compare the tenths.

7.426
7.845
7.417

Two of the numbers have 4 tenths and one number has 8 tenths. 8 tenths is larger than 4 tenths, so 7.845 is the largest number. Adjust the order to show this.

7.426
7.417
7.845

Because two numbers have 4 tenths, we need to compare the hundredths. 2 hundredths is larger than 1 hundredth, so 7.426 is larger than 7.417.

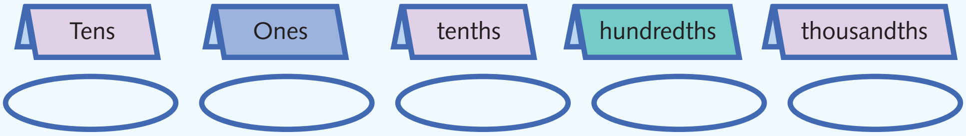
The order, smallest to largest, is:

7.417
7.426
7.845

8B Whole Class CONNECT, APPLY AND BUILD

1 Hoops

You will need five hoops labelled Tens, Ones, tenths, hundredths and thousandths. Place the hoops on the floor.



Each student will need one card from **BLM 7** 'Hoops', which can be downloaded from the Interactive Textbook. (The numbers have been chosen to highlight important ideas.)

a Students take turns asking questions related to place value. For example: Raise your hand if your number has a digit with a number '2' in it. Who has 2 ones? Who has 2 tenths? Who has more than one '2' in their number?

b All the students who have a number with a 7 in the ones place stand in the 'Ones' hoop and arrange themselves in order, from smallest to largest number. Then they hold up their numbers for the rest of the class to see. If the order is incorrect, the class asks the 'hoop' students to move or swap places until everyone agrees that the numbers are in the correct order.

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

c Repeat the steps in part **b** for numbers with a 5 in the tenths place.

d Repeat the steps in part **b** for numbers with a 3 in the hundredths place.

e Repeat the steps in part **b** for numbers with a 2 or a 9 in the thousandths place.

f Repeat the steps in part **b** for numbers with a 3 in the tens place.

8B Individual

1 Draw the number line and mark each pair of numbers on it.

Then circle the larger number in each pair.

a 2.4 and 3.2

b 4.1 and 3.9

c 8.8 and 7.9

d 2.24 and 2.42

e 5.59 and 5.89

2 Write each set of numbers in order, smallest to largest.

a 3.1 3.4 3.8 3.2 3.6 3.3 3.5 3.7

b 6.1 4.6 10 1.6 6.4 0.6 10.1 6.0

c 12.74 12.47 12.64 12.46 12.50 12.05

d 0.3 0.4 0.34 0.43 0.034 0.043 0.403 0.004

e 5.6 2.923 27.464 5.467 3.921 2.001

f 0.000 196 0.2 0.9934 0.036 0.82 0.4009

g 1.1 11.1 10.111 0.0101 11.01 11.011 111 11

3 Write the next four numbers in these counting sequences.

a 8.4 8.5 8.6 ____ ____ ____ ____

b 10.8 10.9 11.0 ____ ____ ____ ____

c 26.7 26.8 26.9 ____ ____ ____ ____

d 82.46 82.47 82.48 ____ ____ ____ ____

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

8C Adding decimals

We can add 0.2 to 0.6 as follows.

$$\begin{aligned} 0.2 + 0.6 &= \frac{2}{10} + \frac{6}{10} \\ &= \frac{8}{10} \\ &= 0.8 \end{aligned}$$

We can follow the same steps as an addition algorithm for whole numbers. Start by lining up the numbers with the decimal points and the places aligned.

$$\begin{array}{r} 0\,.\,2 \\ +\ 0\,.\,6 \\ \hline 0\,.\,8 \\ \hline \end{array}$$

Say '2 tenths plus 6 tenths is 8 tenths'.

Write '8' in the tenths column.

Example 4

Complete these additions.

a $\begin{array}{r} 12\,.\,4 \\ +\ \ 7\,.\,8 \\ \hline \\ \hline \end{array}$

b $\begin{array}{r} 8\,.\,23 \\ +9\,.\,02 \\ \hline \\ \hline \end{array}$

c $\begin{array}{r} 3\,.\,496 \\ +18\,.\,999 \\ \hline \\ \hline \end{array}$

Solution

a $\begin{array}{r} 1_1 2.4 \\ +\ \ 7_1.8 \\ \hline 20.2 \\ \hline \end{array}$

b $\begin{array}{r} 8.23 \\ +\ 9.02 \\ \hline 17.25 \\ \hline \end{array}$

c $\begin{array}{r} 3\,.4\ 9\ 6 \\ +\ 1_1 8_1 .9_1 9_1\ 9 \\ \hline 2\ 2.4\ 9\ 5 \\ \hline \end{array}$

We can use the addition algorithm to add two, three or more decimal numbers.

	T	O	•	t	h	th
		8	•	9	3	5
	1	9	•	0	6	7
+	2	7	•	8	8_1	4
						6

First deal with the thousandths.

5 thousandths plus 7 thousandths makes 12 thousandths, plus 4 thousandths makes 16 thousandths.

Write '6' in the thousandths column, and carry 1 into the hundredths.

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

	T	O	•	t	h	th
		8	•	9	3	5
	1	9	•	0	6	7
+	2	7	•	8_1	8_1	4
					8	6

Now work with the hundredths.

3 plus 6 makes 9 hundredths, plus 8 plus 1 makes 18 hundredths.

Write ‘8’ hundredths and carry 1 into the tenths.

	T	O	•	t	h	th
		8	•	9	3	5
	1	9	•	0	6	7
+	2_2	7_1	•	8_1	8_1	4
	5	5	•	8	8	6

Now add the tenths, the ones and finally the tens.

Example 5

Flynn went to the supermarket. He bought a packet of corn chips for $3.85, an energy drink for $1.90 and a bag of avocadoes for $4.35. How much did Flynn spend?

Solution

$$\begin{array}{rr} & 3.85 \\ & 1.90 \\ + & 4_2.3_15 \\ \hline & 10.10 \\ \hline \end{array}$$

Flynn spent $10.10.

8C Whole class CONNECT, APPLY AND BUILD

The Whole Class activity ‘Delightful decimal numbers’ is available in the Interactive Textbook.

8C Individual

1 Use the addition algorithm to calculate these.

a 12.5 + 1.2 **b** 33.8 + 21.1 **c** 1.11 + 3.74

d 3.08 + 3.62 **e** 4.567 + 5.432 **f** 0.3442 + 1.2537

2 Nancy went to the paint store and bought a tin of paint for $32.85, a paintbrush for $12.75 and some tiles for $86.05. How much did she spend in total?

3 Tani has five different bank accounts. Here are their balances.

Bank account	A	B	C	D	E
Balance	$38.45	$33.87	$8.13	$8.45	$1027.46

a How much money does Tani have in total?

b If Tani put the balance of account A and account E into account D, how much would be in account D?

c Tani owes her mum $42.00. From which two accounts should she withdraw all the money to get closest to $42.00?

4 Two trucks delivered gravel to Grumpy's Gravel Yard. The first load was 3.478 tonnes. The second load was 4.963 tonnes. What was the total amount of gravel delivered?

5 Felisia went to the local supermarket and bought items costing $2.95, $5.65 and $8.85. How much did Felisia spend?

6 Richard measured the length of three shelves in metres.

Shelf	Blue	Black	White
Length	1.986 m	2.012 m	1.884 m

a What is the length of the blue and the white shelves together?

b What is the length of the black and the white shelves together?

c What is the total length of the three shelves?

d Which two shelves would fit best in a 4-metre space, leaving the smallest gap?

7 Add these decimal numbers. Remember to line up the decimal points.

a 12.5 + 1.2 + 21.1 **b** 114.577 + 5.472 + 77.2

c 1.11 + 7.74 + 77.7 **d** 53.02 + 190.7 + 7.702

e 7.245 + 0.702 + 37.74 + 588.3 **f** 1.0025 + 104.7442 + 1.27 + 0.2

8 Pat walked $8\frac{1}{5}$ kilometres and Bill walked 2.4 kilometres.

What is the total distance that they walked?

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

9 Mary had two mats in her hallway. One was $7\frac{6}{10}$ metres long and the other was 3.25 metres. What was their total length?

10 Todd and Carmel were playing golf. Todds's chip shot went $14\frac{3}{4}$ metres and Carmel's went 2.75 metres further. What was the distance of Carmel's shot?

8D Subtracting decimals

We use the subtraction algorithm to subtract decimals. Be sure to align the decimal points and the places.

Example 6

Calculate these subtractions.

a 2.4 – 1.3 **b** 12.26 – 9.74

Solution

a

$$\begin{array}{rccc} & 2 & . & 4 \\ - & 1 & . & 3 \\ \hline & 1 & . & 1 \\ \hline \end{array}$$

b **Trading**

$$\begin{array}{rcccccc} & \cancel{1} & {}^{11}\cancel{2} & . & {}^{1}2 & 6 \\ - & & 9 & . & 7 & 4 \\ \hline & & 2 & . & 5 & 2 \\ \hline \end{array}$$

Equal addition

$$\begin{array}{rccccc} & 1 & {}^{1}2 & . & {}^{1}2 & 6 \\ - & {}_{1} & 9_{1} & . & 7 & 4 \\ \hline & & 2 & . & 5 & 2 \\ \hline \end{array}$$

When the decimal numbers are different lengths, align the decimal point and the places. We write a zero on the end of the shorter number to show that place is empty.

Example 7

Calculate 12.583 – 3.9.

Solution

Trading

$$\begin{array}{rccccccc} & \cancel{1} & {}^{11}\cancel{2} & . & {}^{1}5 & 8 & 3 \\ - & & 3 & . & 9 & 0 & 0 \\ \hline & & 8 & . & 6 & 8 & 3 \\ \hline \end{array}$$

Equal addition

$$\begin{array}{rccccccc} & 1 & {}^{1}2 & . & {}^{1}5 & 8 & 3 \\ - & 1 & 3_{1} & . & 9 & 0 & 0 \\ \hline & & 8 & . & 6 & 8 & 3 \\ \hline \end{array}$$

Example 8

Calculate these subtractions.

a 6.8 – 3.94 **b** 19.4 – 9.567

Solution

a Write a zero at the end of 6.8 to match the 4 in the hundredths place of 3.94.

Trading

$$\begin{array}{rcccc} & {}^{5}\cancel{6} & . & {}^{17}\cancel{8} & {}^{1}0 \\ - & 3 & . & 9 & 4 \\ \hline & 2 & . & 8 & 6 \\ \hline \end{array}$$

Equal addition

$$\begin{array}{rcccc} & 6 & . & {}^{1}8 & {}^{1}0 \\ - & 3_{1} & . & 9_{1} & 4 \\ \hline & 2 & . & 8 & 6 \\ \hline \end{array}$$

b Write two zeros at the end of 19.4 to match the 6 in the hundredths place and the 7 in the thousandths place of 9.567.

Trading

$$\begin{array}{rcccccc} & \cancel{1} & {}^{18}\cancel{9} & . & {}^{13}\cancel{4} & {}^{9}\cancel{0} & {}^{1}0 \\ - & & 9 & . & 5 & 6 & 7 \\ \hline & & 9 & . & 8 & 3 & 3 \\ \hline \end{array}$$

Equal addition

$$\begin{array}{rcccccc} & 1 & {}^{1}9 & . & {}^{1}4 & {}^{1}0 & {}^{1}0 \\ - & {}_{1} & 9_{1} & . & 5_{1} & 6_{1} & 7 \\ \hline & & 9 & . & 8 & 3 & 3 \\ \hline \end{array}$$

8D Individual

1 Use one of the subtraction algorithms to calculate these.

a 3.8 – 1.2 **b** 8.4 – 7.3 **c** 10.63 – 3.32

d 7.06 – 4.02 **e** 9.592 – 5.331 **f** 0.4267 – 0.4035

g 18.4 – 2.6 **h** 56.8 – 27.9 **i** 14.22 – 8.94

j 8.27 – 3.35 **k** 19.493 – 12.716 **l** 1.048 – 0.395

2 Subtract 4.5 from:

a 18.8 **b** 5.04 **c** 10.214 **d** 15 893.06

3 Subtract 122.095 from:

a 185.898 **b** 2345.678 **c** 4306.03 **d** 1000

4 Moira bought 0.85 m of coloured ribbon. She cut off 0.29 m. How much ribbon was left?

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

8E Multiplying decimals by 10, 100 or 1000

Multiplying decimals by 10

What happens when we multiply 0.2 by 10?

$0.2 = \frac{2}{10}$ If we multiply $\frac{2}{10}$ by 10 we get 2.

We can see this using decimal stick pictures.

This is 1:  And this is 0.2: 

This is what 10 lots of 0.2 looks like.

It is the same as 2 ones.

$$0.2 \times 10 = 2$$

$$\begin{aligned} 0.2 \times 10 &= \frac{2}{10} \times 10 \\ &= \frac{20}{10} \\ &= 2 \end{aligned}$$

Let's try multiplying 0.12 by 10.

$$0.12 = \frac{1}{10} + \frac{2}{100}$$

$$\begin{aligned} 0.12 \times 10 &= \left(\frac{1}{10} \times 10\right) + \left(\frac{2}{100} \times 10\right) \\ &= \frac{10}{10} + \frac{20}{100} \\ &= 1 + \frac{2}{10} \qquad \left(10 \text{ tenths } = 1 \qquad 20 \text{ hundredths } = \frac{2}{10}\right) \\ &= 1.2 \end{aligned}$$

Now let's try a multiplication that involves thousandths.

$$0.349 = \frac{3}{10} + \frac{4}{100} + \frac{9}{1000}$$

$$\begin{aligned} 0.349 \times 10 &= \left(\frac{3}{10} \times 10\right) + \left(\frac{4}{100} \times 10\right) + \left(\frac{9}{1000} \times 10\right) \\ &= \frac{30}{10} + \frac{40}{100} + \frac{90}{1000} \\ &= 3 + \frac{4}{10} + \frac{9}{100} \\ &= 3.49 \end{aligned}$$

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

Have you noticed a pattern when multiplying by 10?

	Ones	tenths	hundrredths	thousandths
0.2	0 •	2		
0.2 × 10	2			
0.12	0 •	1	2	
0.12 × 10	1 •	2		
0.349	0 •	3	4	9
0.349 × 10	3 •	4	9	

When we multiply by 10, each digit moves one place to the left in the place-value chart.

This is the same as moving the decimal point one place to the right.

Start with	*Move the point*	*Gives*
0.2 × 10		02. which we write as 2
0.12 × 10		1.2
0.349 × 10		3.49

Multiplying decimals by 100

Multiplying decimals by 100 is the same as multiplying by 10 and then multiplying by 10 again.

When we multiply by 100, each digit moves two places to the left in the place-value chart. This is the same as moving the decimal point two places to the right.

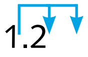
To multiply a decimal by 100, move the decimal point two places to the right and insert zeroes if necessary.

Let's see why this works.

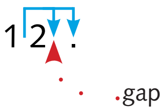
1.2 is the same as $1\frac{2}{10}$

$$\begin{aligned} 1.2 \times 100 &= (1 \times 100) + \left(\frac{2}{10} \times 100\right) \\ &= 100 + \frac{200}{10} \\ &= 100 + 20 \\ &= 120 \end{aligned}$$

Using the shortcut, the decimal point moves two places to the right, like this:



which gives



Fill the gap with zero to get:

1 2 0.

As this is a whole number, we do not need the decimal point.

1.2 × 100 = 120

Example 9

a Multiply 2.347 by 100.

b Multiply 19.4 by 1000.

Solution

a 2.347 × 100 = 2.3 4 7
= 234.7

Move the decimal point two places to the right.

b 19.4 × 1000 = 19.4
= 19 400

Move the decimal point three places to the right. Fill the gaps with zeroes.

Remember

Multiplying a decimal number by 10 is the same as moving the decimal point *one* place to the right.

Multiplying a decimal number by 100 is the same as moving the decimal point *two* places to the right.

Multiplying a decimal number by 1000 is the same as moving the decimal point *three* places to the right.

8E Whole class CONNECT, APPLY AND BUILD

1 Multiply each decimal by 10.

a 2.3 **b** 0.5 **c** 1.8 **d** 0.03 **e** 0.077 **f** 0.009

2 Multiply each decimal by 100.

a 1.83 **b** 4.01 **c** 1.9

d 0.08 **e** 1.101 01 **f** 4.074

3 Multiply each decimal by 1000.

a 1.832 **b** 4.25 **c** 0.210

d 1.4 **e** 100.41 **f** 76.8

8E Individual

1 Copy the place-value chart for each number into your workbook. Show each number on the chart. Multiply each number by 10 and show it on the place-value chart. Then complete each multiplication. (The first one has been done for you.)

a 34.25 × 10

	Hundreds	Tens	Ones	tenths	hundredths	thousandths
Starting number		3	4 •	2	5	
×**10**	3	4	2 •	5		

34.25 × 10 = 342.5

b 6.83 × 10

	Hundreds	Tens	Ones	tenths	hundredths	thousandths
Starting number			•			
× **10**			•			

6.83 × 10 = ______

c 80.09 × 10

	Hundreds	Tens	Ones	tenths	hundredths	thousandths
Starting number			•			
× **10**			•			

80.09 × 10 = ______

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

2 Multiply each number first by 10, then by 100.

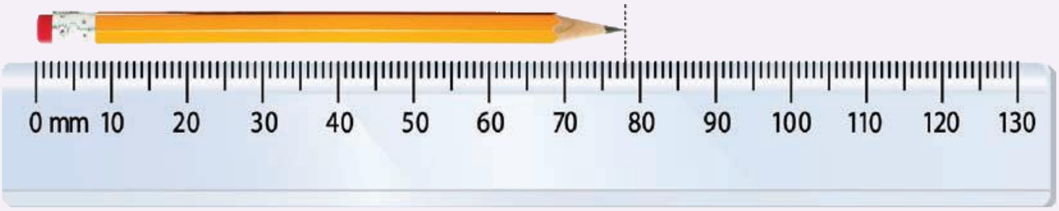
a 8.79 **b** 0.005 **c** 6.085 **d** 17.2

e 240.09 **f** 0.003 **g** 12.4

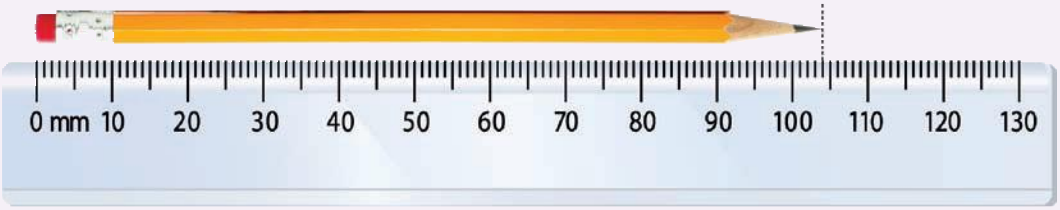
3 Dessie measured her shoe. It was 21.7 cm long. If Dessie placed 10 of her shoes end to end, how long would the line be?

4 Stephen made 2.5 L of cordial and poured out 10 glasses for his friends. Each glass contained 0.125 L of cordial. How much cordial was left in the jug?

5 **a** Hamish measured the length of one of his pencils, as shown. If he laid 10 of these pencils end to end, how long would the line of pencils be?



b Jamie measured her pencil.



If Jamie laid 10 of her pencils end to end, how long would the line of pencils be?

6 Multiply each fraction or mixed number by 10 , then by 100, then by 1000. Write each answer as a decimal.

a $\frac{5}{100}$ **b** $\frac{3}{10}$ **c** $\frac{47}{100}$ **d** $\frac{9}{100}$ **e** $\frac{281}{1000}$ **f** $43\frac{86}{100}$

7 **a** Chloe is 1.67 m tall. How many centimetres tall is she? (There are 100 centimetres in 1 metre.)

b Oscar weighs 32.45 kg. How many grams is that? (There are 1000 grams in 1 kilogram.)

8 Copy this table. Complete it by multiplying each number by 10, 100 and 1000 .

	Number	**×10**	**×100**	**×1000**
a	0.492			
b	83.06			
c	507.08			
d	9.23			
e	99.999			

9 Convert these weights and measures.

a 5.62 kg = _____ g   **b** 12.4 m = ____ cm   **c** 0.8 km = ____ m

d 2.745 t = ____ kg   **e** 7.6 cm = ____ mm   **f** 0.007 kg = ____ g

10 Copy this table. Convert each fraction or mixed number to a decimal, then multiply by 10, 100 and 1000. Write your answer as a decimal.

	Starting number	**×10**	**×100**	**×1000**
a	$\frac{7}{10}$			
b	$\frac{7}{100}$			
c	$\frac{7}{1000}$			
d	$\frac{107}{1000}$			
e	$5\frac{38}{100}$			

8F Dividing decimals by 10, 100 or 1000

When we divide $\frac{1}{10}$ by 10 we get $\frac{1}{100}$. This is because 10 hundredths make $\frac{1}{10}$.
We can write this using decimals.

$$0.1 \div 10 = 0.01$$

When we divide by 10, each digit moves one place to the right in the place-value chart.

	Ones	tenths	thousandths
0.1	0 •	1	
0.1 ÷ 10	0 •	0	1

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

We can write the number 12.4 like this:

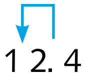
$$\begin{aligned} 12.4 &= 12 + \frac{4}{10} \\ &= 10 + 2 + \frac{4}{10} \end{aligned}$$

Dividing by 10 gives:

$$\begin{aligned} 12.4 \div 10 &= (10 \div 10) + (2 \div 10) + \left(\frac{4}{10} \div 10\right) \\ &= 1 + \frac{2}{10} + \frac{4}{100} \\ &= 1.24 \end{aligned}$$

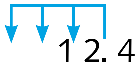
	Tens	Ones	tenths	hundredths
12.4	1	2 ●	4	
12.4 ÷ 10		1 ●	2	4

Dividing by ten is the same as moving the decimal point one place to the left.



$12.4 \div 10 = 1.24$

When necessary, fill the spaces with zeroes.



$12.4 \div 1000 = 0.0124$

Example 10

Divide 12.3 by 10, then by 100, then by 1000.

Solution

$12.3 \div 10 = 1.23$

$12.3 \div 100 = 0.123$

$12.3 \div 1000 = 0.0123$

Remember

Dividing by 10 is the same as moving the decimal point *one* place to the left.

Dividing by 100 is the same as moving the decimal point *two* places to the left.

Dividing by 1000 is the same as moving the decimal point *three* places to the left.

8F Whole class CONNECT, APPLY AND BUILD

● **1** Put these numbers on a place-value chart, then draw a new place-value chart to show each number divided by 10.

a 324 **b** 32.4 **c** 3.24

● **2** Use an abacus to show these numbers. Then draw an abacus to show each number divided by 100.

a 2185 **b** 218.5 **c** 21.85

● **3** This table shows the nutrition information, in milligrams, from a packet of lolly snakes. Copy the table and convert the amounts to grams. (Remember: 1 g = 1000 mg.) Record your answers in the third column.

Nutrient	Per 100 g	Per 100 g
Protein	4000 mg	___ g
Fats	1050 mg	___ g
Carbohydrates	82 200 mg	___ g
Sugars	42 300 mg	___ g
Sodium	61 mg	___ g

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

4 **Beachball**

Write these numbers on stickers and place them randomly on a beachball.

13.6 22.4 89.75 129.46 76.43 2.6

3.4 7.9 5.8 1.5 9.9 0.4

Whoever catches the ball divides the number nearest their right thumb by:

a 10 **b** 100 **c** 1000

8F Individual

1 Divide each number by 10.

a 2.3 **b** 256 **c** 128.47 **d** 0.12 **e** 6.08 **f** 999

2 Mary and her 9 friends love licorice. Mary bought 2.5 m of licorice and cut it into equal pieces for herself and her 9 friends. What length of licorice did they each get?

3 Steve placed ten blocks of wood the same length end to end and measured the total length. The length was 1.4 m. How long was each block?

4 **a** Sue divided a number by 10 and got 22.3. What was the number she started with?

b Zoe divided a number by 10 and got 0.065. What was the starting number?

c Minh divided a number by 20 and got 0.03. What was the starting number?

5 Jan bought a packet of 10 Crunchy Biscuits. The biscuits weighed 0.6 kg altogether. Jan's brother ate one of the biscuits. What was the weight of the remaining biscuits?

8G Multiplying decimals by a whole number

When we want to multiply a decimal number by a whole number, we can use the multiplication algorithm.

For example, for 0.098 × 9, the algorithm is:

$$\begin{array}{r} 0.09_{7}8 \\ \times \qquad 9 \\ \hline 0.88\ 2 \\ \hline \end{array}$$

Notice that there are a total of three decimal places in the two numbers being multiplied and three decimal places in the solution.

Example 11

Multiply 27.389 by 6.

Solution

$$\begin{array}{r} 2_4\,7_2.\,3_5\,8_5\,9 \\ \times \qquad\qquad 6 \\ \hline 1\,6\,\ 4.\ 3\ \ 3\ 4 \\ \hline \end{array}$$

The multiplication algorithm can also be used to multiply a decimal by a 2-digit number.

Example 12

Multiply 27.389 by 26.

Solution

$$\begin{array}{r} 2_4\,7_2.\,3_5\,8_5\,9 \\ \times \qquad\qquad 2\ \ 6 \\ \hline 1\ \ 6\ \ 4\ \ 3\ \ 3\ \ 4 \\ 5_1\,4_1\,7_1\ 7_1\,8\ \ 0 \\ \hline 7\ \ 1\ \ 2.\ 1\ \ 1\ \ 4 \\ \hline \end{array}$$

8G Individual

1 Use the multiplication algorithm to calculate these products.

a 1.2 × 3 **b** 2.1 × 4 **c** 1.6 × 2 **d** 4.3 × 5

e 11.3 × 6 **f** 23.7 × 8 **g** 18.7 × 6 **h** 134.3 × 7

2 Garth made a pile of 6 bricks, one on top of the other. Each brick was 10.4 cm high. How high was Garth's pile of bricks?

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

3 Sarah bought 8 packets of loose beads from the Mighty Bead Shop. Each packet weighed 0.56 kg.

- **a** What was the total weight of Sarah's beads?

The Mighty Bead Shop sell their beads for $10 per kilogram.

- **b** How much did Sarah pay for her beads?

4 Nella lives 1.38 kilometres from her school. This week, she rode her bike to school four times. Her target is to ride 10 kilometres each week. Did Nella reach her target? How far did she ride?

5 Zak uses 0.3 metres of dental floss each time he brushes his teeth.

- **a** How much dental floss does Zak use if he brushes his teeth:
  - **i** 3 times in one day?
  - **ii** 2 times per day for 5 days?
  - **iii** 3 times per day for 7 days?
- **b** How much is left from a 50-metre packet of dental floss after Zak has brushed his teeth 2 times a day for the whole month of March?
- **c** Zak continued to use the same packet of dental floss. He brushed his teeth 3 times a day for 13 days in April and 2 times a day for the rest of the month. How much dental floss was left at the end of April?

6 Chloe needed a curtain to cover the shelves on the wall in her room. Each shelf is 37.5 cm high and there are 5 of them. How long does her curtain need to be?

8H Division and decimals

When we divide one number by another, sometimes the division comes out evenly with no remainder.

For example, if we divide 24 by 4 we get 6 as the solution.

$$\begin{array}{r} 6 \\ 4\overline{)24} \end{array}$$

If we divide 508 by 4, the answer is 127.

$$\begin{array}{r} 1\ 2\ 7 \\ 4\overline{)5^{1}0^{2}8} \end{array}$$

If we divide 431 by 4 we get a remainder.

The answer to 431 ÷ 4 is 107 remainder 3.

$$\begin{array}{r@{}l} & \underline{107}\ \text{remainder } 3 \\ 4\big) & 431 \end{array}$$

When there is a remainder, we can use our understanding of decimals to continue the division. After the whole number, we write a decimal point followed by as many zeroes as we need.

Since 431 is equal to 431.00 we are not changing the number.

Now we can continue the division to the right of the decimal point. Notice the decimal points in the number and in the answer are lined up one under the other.

The 3 remainder becomes 30 tenths, and 30 tenths divided by 4 is 7 tenths, with a remainder of 2 tenths.

Then the 2 tenths remainder becomes 20 hundredths, and 20 hundredths divided by 4 is 5 hundredths.

$$\begin{array}{r@{}l} & \underline{107.\ 7\ \ 5} \\ 4\big) & 431.{}^{3}0{}^{2}0 \end{array}$$

In the case of 431 ÷ 4 we get an exact answer.

$$431 \div 4 = 107.75$$

Sometimes a division does not work out exactly, and we get a string of digits that repeats because the division goes on forever.

$$\begin{array}{r@{}l} & \underline{\ \ 3.\ 3\ 3\ 3\ 3\ 3\ 3\ldots} \\ 3\big) & 10.{}^{1}0{}^{1}0{}^{1}0\ldots \end{array}$$

Example 13

Tyler shares 3 oranges among 4 footballers. Use the division algorithm to show how much orange each player will receive.

Solution

$$\begin{array}{r@{}l} & \underline{0.7\ \ 5} \\ 4\big) & 3.0{}^{2}0 \end{array}$$

Each player receives 0.75 of an orange.

When we want to divide a decimal number by a whole number, we can use either the short division algorithm or the long division algorithm.

For example, if you want to divide 25.62 by 4, set out the working in the same way as you would for whole numbers. Be sure to line up the decimal point and the places in the answer with the decimal point and places in the number being divided.

$$\begin{array}{r} 6.405 \\ 4\overline{)25.{}^{1}620} \end{array}$$

We place a zero on the end because we cannot divide 2 hundredths by 4. But we can divide 20 thousandths by 4.

Example 14

Mary-Ellen has 7.44 metres of licorice to share equally with 5 friends. How much licorice will each person receive?

Solution

$$\begin{array}{r} 1.\,2\,4 \\ 6\overline{)7.{}^{1}4{}^{2}4} \end{array}$$

Each person will receive 1.24 metres of licorice.

Sometimes we have to write a zero before the decimal point in the answer because we cannot divide into the whole number part. In the division 2.76 ÷ 3 below, 3 does not go into the whole number, 2, so a zero has to be inserted.

$$\begin{array}{r} 0.92 \\ 3\overline{)2.{}^{2}76} \end{array}$$

Example 15

Mary-Ellen has a lolly snake 1.8 metres in length. She wants to share it among 6 people. How much will each person receive?

Solution

$$\begin{array}{r} 0.3 \\ 6\overline{)1.{}^{1}8} \end{array}$$

Each person will receive 0.3 metres of lolly snake.

We can use our understanding of multiplication tables to help us do divisions in our head. For example, $6 \times 3 = 18$ could have been used to solve example 15. Further examples are:

$0.16 \div 4 = 0.04$, because $4 \times 4 = 16$

$2.4 \div 6 = 0.4$, because $6 \times 4 = 24$

Example 16

Bruce won 7.2 kilograms of chocolate freckles at the school fete. He wants to share them equally among himself and 5 friends. How much will each person receive?

Solution

$12 \times 6 = 72$, so $1.2 \times 6 = 7.2$

Each person will receive 1.2 kilograms of chocolate freckles.

8H Whole class CONNECT, APPLY AND BUILD

1 Calculate the result when your height in metres is divided by:

a 4 **b** 3 **c** 8 **d** 7

2 An average is calculated by adding the values, then dividing by the number of values. For example:

The average of 1, 2, 3, 4 and 5 is $\frac{1 + 2 + 3 + 4 + 5}{5} = \frac{15}{5} = 3$

a Calculate the average height of 5 students.

b Now include one person who is very tall. What is the average of the 6 people now?

8H Individual

1 Calculate these divisions. Show your answer as a remainder and as a decimal.

a $10 \div 4$ **b** $12 \div 5$ **c** $20 \div 8$ **d** $27 \div 6$

e $15 \div 4$ **f** $102 \div 8$ **g** $43 \div 8$ **h** $123 \div 4$

2 Calculate these divisions.

a $1.2 \div 3$ **b** $2.8 \div 4$ **c** $1.6 \div 2$ **d** $4.5 \div 5$

e $5.34 \div 6$ **f** $728.022 \div 3$ **g** $49.35 \div 5$ **h** $7.02 \div 9$

3 Divide 36.288 by:

a 2
b 3
c 4
d 5
e 6
f 7
g 8
h 9

4 Peter makes a fence using 132.5 metres of fencing wire. How long is the fence if there are 5 rows of wire in the fence?

5 Billy's grandma is making square cushions. Each cushion has 2.8 metres of ribbon around its edges.

a What is the side length of the square cushion?
b If Billy's grandma makes 8 cushions, how many metres of ribbon will she need?
c If Grandma makes 4 cushions, how much ribbon will be left over from a 14-metre roll?
d How many cushions can Grandma make from 27 metres of ribbon? How much ribbon will be left over?

8I Multiplying whole numbers by decimals

We can calculate $\frac{3}{8}$ of 100 cm like this.

$$\begin{aligned} \frac{3}{8} \text{ of } 100\text{ cm} &= \frac{3}{8} \times 100 \\ &= \frac{300}{8} \\ &= 37\tfrac{4}{8} \\ &= 37\tfrac{1}{2}\text{ cm} \end{aligned}$$

We can do the same for decimal parts of a number. For example:

0.25 of \$100 means 0.25 × 100 dollars.

Moving the decimal point two places gives the answer: \$25

Try to remember the fraction equivalents for decimals, as they allow you to take shortcuts.

$$0.1 = \frac{1}{10} \qquad 0.2 = \frac{2}{10} = \frac{1}{5} \qquad 0.25 = \frac{1}{4} \qquad 0.5 = \frac{1}{2} \qquad 0.75 = \frac{3}{4}$$

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

Example 17

Calculate 0.75 of 160.

Solution

$$\begin{array}{r} 0.7^{3}5 \\ \times \quad 1\,6\,0 \\ \hline 4\,5\,0\,0 \\ 7_{1}5\,0\,0 \\ \hline 1\,2\,0.0\,0 \end{array}$$

We can also do this mentally.

0.75 is the same as $\frac{3}{4}$

$\frac{3}{4}$ of $160 = 3 \times 40 = 120$

There are two decimal places in the numbers being multiplied. So there must be two decimal places in the answer.

Example 18

Calculate 0.6 of \$350.

Solution

$$0.6 = \frac{6}{10} = \frac{3}{5}$$

$$\begin{aligned} \frac{3}{5} \text{ of } \$350 &= \frac{3}{5} \times \frac{\$350}{1} \\ &= \frac{\$1050}{5} \\ &= \$210 \end{aligned}$$

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

Example 19

Calculate 0.73 of 18 metres.

Solution

$$\begin{array}{r} 0.7\,3 \\ \times \quad 1\,8 \\ \hline 5\;8\,4 \\ 7_{1}3\,0 \\ \hline 1\,3.1\,4 \end{array}$$

0.73 of 18 metres is 13.14 metres.

Remember

To multiply a whole number by a decimal, we can either use a fraction equivalent or use the algorithm.

8I Whole class CONNECT, APPLY AND BUILD

1 Suppose that part-time workers pay 0.29 of their earnings in tax. Calculate the tax paid and the amount left over if:

a Peter earns $300 **b** Sandra earns $1000 **c** Arlene earns $879

8I Individual

1 Calculate 0.8 of:

a $10 **b** 12 grams **c** 24 metres

d $98 **e** 1000 cm **f** 84 kilometres

g $267.95 **h** $1 000 000

2 Calculate 1.25 of:

a \$10	**b** 12 grams
c 24 metres	**d** \$98
e 1000 centimetres	**f** 84 kilometres
g \$267.95	**h** \$1 000 000

Homework

1 Jenny is 0.7 of her father's height. If Jenny's father is 2 metres tall, how tall is Jenny?

2 Martin earns 0.6 of the wage earned by his sister, Fiona. If Fiona earns \$560 each week, how much does Martin earn per week?

3 Marlene had \$280 in her account. She withdrew 0.25 of it for her holiday.

- **a** How much did she withdraw?
- **b** How much is left in her account?

8J Rounding decimals

If we want to make an estimate of the sum of two numbers, sometimes it is useful to round the numbers to the nearest ten or hundred. For example, suppose we calculate $31 + 68 = 99$ and want to check that our answer is about right. We round down 31 to 30 and round up 68 to 70.

So we find that $31 + 68$ is approximately equal to $30 + 70 = 100$.

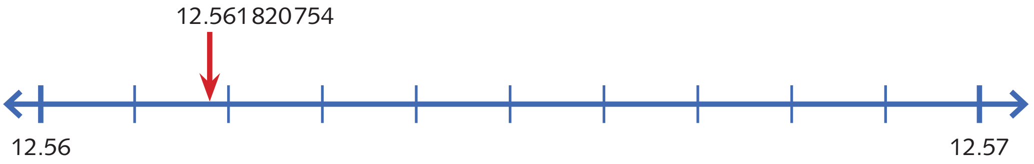
This is a useful technique to check that our answer is reasonable or 'within the ball park'.

Rounding can also be used with decimal numbers.

In the number 12.561 820 754 there are nine digits after the decimal point, but to make it easier to use we can round it to two places. This would give us an approximate value of the number.

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

On a number line, 12.561 820 754 would be about here:



We can see that 12.561 820 754 is larger than 12.56 and smaller than 12.57.

Because it is closer to 12.56 we would round 12.561 820 754 to 12.56.

What if we want to round 12.561 820 754 to one decimal place? That is, what do we do if we need to know whether 12.561 820 754 is closer to 12.5 or 12.6?

There are some rules that can help us round whole numbers and decimal numbers that do not require the use of a number line.

If we want to round to one decimal place, the first digit after the decimal point is our *rounding digit*. Then we follow these steps.

Look at the very next digit to the right of the rounding digit.

- If the next digit is 0, 1, 2, 3 or 4, then the rounding digit stays the same.
- If the next digit is 5, 6, 7, 8 or 9, then the rounding digit gets larger by 1.

In 12.561 820 754, our rounding digit is '5' and the digit after it is a 6, so the rounding digit gets larger by 1. Our rounding digit becomes '6'.

We then discard all the other digits after the rounding digit.

So 12.561 820 754 rounded to one decimal place is 12.6.

Example 20

a Round 235 to the nearest ten.

b Round 428 to the nearest hundred.

c Round 9.876 543 21 to the nearest hundredth.

Solution

a When we round 235 to the nearest ten, our rounding digit is 3.

235

The number to the right of the 3 is 5, so the rounding digit gets larger by 1. The other digits are discarded and replaced by a zero to hold the place value.

240

Rounding 235 to the nearest ten gives 240.

continued over page

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

b When we round 428 to the nearest hundred, our rounding digit is 4.

428

The digit to the right of the 4 is 2, so the rounding digit stays the same.
The digits after it are discarded and replaced by zeroes to hold the place value.

400

Rounding 428 to the nearest hundred gives 400.

c When we round 9.876 543 21 to the nearest hundredth, our rounding digit is 7.

9.876 543 21

The digit to the right of the 7 is 6, so the rounding digit increases by 1 and the digits after it are discarded.

9.88

Rounding 9.876 543 21 to the nearest hundredth gives 9.88.

8J Individual

1 Round these numbers to the nearest ten.

a 43 **b** 52 **c** 65 **d** 86 **e** 999

2 Round these numbers to the nearest hundred.

a 99 **b** 101 **c** 224 **d** 897 **e** 2893

3 Round these numbers to the nearest tenth.

a 0.38 **b** 2.12 **c** 0.444 **d** 0.555

4 Round these numbers to the nearest whole number.

a 23.7 **b** 42.1 **c** 76.5 **d** 99.9 **e** 23.0873

5 Round these numbers to the nearest hundredth.

a 0.123 **b** 0.789 **c** 0.415 **d** 888.888

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

8K Review questions

1 Write these fractions and mixed numbers as decimals.

a $\frac{3}{10}$ **b** $\frac{23}{100}$

c $16\frac{1}{1000}$ **d** $214\frac{13}{100}$

2 Write these decimals as mixed numbers and fractions.

a 0.8 **b** 14.2

c 27.75 **d** 39.04

3 The table shows the length of four lines. Each of these lines needs to be made longer. Copy and complete the table.

Line	Length	$\frac{1}{10}$ cm longer	Write as a fraction
a	8.4 cm		
b	19.6 cm		
c	8.48 cm		
d	28.51 cm		

4 Write each set of numbers in order from smallest to largest.

a 7.4 7.2 7.8 7.3 7.7 7.5

b 43.4 45.3 44.3 43.5 40.3 40.4 43.0

c 72.89 72.92 72.87 72.72 72.08 72.07

5 Convert each decimal into a mixed number.

a 8.49 **b** 21.08

c 76.407 **d** 219.091

6 Convert these mixed numbers into decimals.

a $11\frac{5}{10}$ **b** $35\frac{53}{100}$

c $17\frac{7}{100}$ **d** $29\frac{18}{1000}$

7 Use the addition algorithm to calculate:

a $8.03 + 3.08$ **b** $19.85 + 17.58$

c $71.849 + 14.984$ **d** $28.025 + 82.98$

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

CHAPTER

9 Preparation

Useful skills for this chapter:

- understanding of fractions and decimals
- the ability to convert fractions and mixed numbers to decimals, and to convert decimals to fractions or mixed numbers.

KICK OFF

Ian's Electrical Store is having a sale. For every $100 a customer spends, Ian is giving a $20 discount. So a $100 toaster will now cost $80 and a $300 camera will now cost $240.

What is the sale price of a:

a $200 stereo?

b $900 computer?

c $1200 television?

Show what you know

1 Write a fraction for each statement.

a One of two hands is up in the air.

b One out of four tyres on Harry's car is flat.

c Three out of five windows in Kathie's house are open.

d Eleven out of a group of 12 children are sleeping.

2 Convert these fractions and mixed numbers to decimals.

a $\frac{3}{10}$ **b** $\frac{23}{100}$ **c** $\frac{1}{2}$ **d** $2\frac{3}{100}$ **e** $\frac{3}{4}$ **f** $6\frac{3}{5}$

3 Convert these decimals to fractions or mixed numbers.

a 0.5 **b** 0.75 **c** 0.1 **d** 2.6 **e** 3.4

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

CHAPTER

9

Number and Algebra

Percentages

Percentages are another way of writing fraction or decimal quantities.

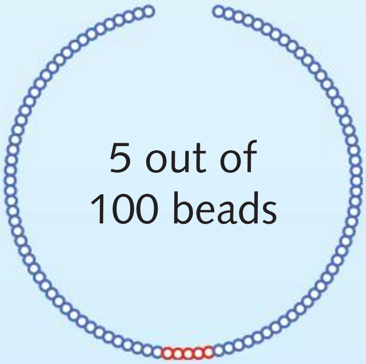
There are 100 beads on Freya's necklace. Five of the beads are red.

We can write this as a fraction.

5 out of 100 beads are red or $\frac{5}{100}$ beads are red.

We can also write it as a decimal.

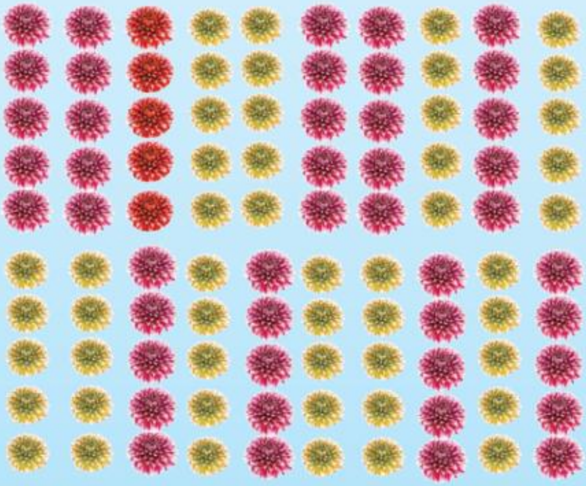
5 out of 100 beads are red or 0.05 of the beads are red.



Or we can say that '5 per cent' of the beads are red. 'Per cent' comes from the Latin words *per centum*, which mean 'out of one hundred'. So '5 per cent' is a quick way of saying '5 out of one hundred'.

There are 100 flowers in Bree's garden. Five of the flowers are red.

The symbol for per cent is %, so 5 per cent is written as 5%. In Bree's garden 5% of the flowers are red.



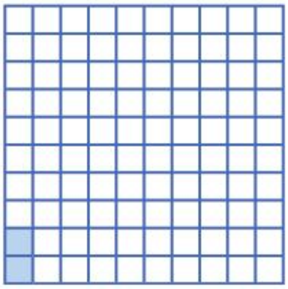
ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

9A Converting fractions to percentages

A percentage is another way of writing a fraction with a denominator of 100.

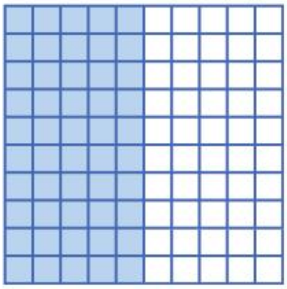
$\frac{2}{100} = 2\%$ $\frac{50}{100} = 50\%$ $\frac{100}{100} = 100\%$

We can understand percentage by drawing a picture of a square divided into 100 equal parts, then shading some of them.



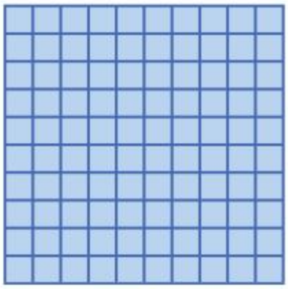
2 out of 100 parts are shaded.

2% of the square is shaded.



50 out of 100 parts are shaded.

50% of the square is shaded.

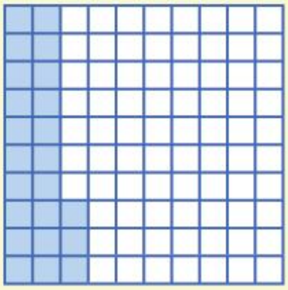


100 out of 100 parts are shaded.

100% of the square is shaded.

Example 1

This square is divided into 100 equal parts.
What percentage is shaded?



Solution

23 out of the 100 parts are shaded.

So 23% of the square is shaded.

Example 2

The Broome Cinema has 100 seats. On Saturday night, 75 of the 100 seats were filled. Write this as a percentage of the number of cinema seats, then as a fraction of the number of cinema seats.

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

Solution

75 out of 100 = 75% So 75% of the cinema seats were filled.

75 out of 100 $= \frac{75}{100} = \frac{3}{4}$ So $\frac{3}{4}$ of the cinema seats were filled.

We can also write fractions that do not have 100 as a denominator as percentages. First convert the fraction to an equivalent fraction with a denominator of 100, then write the fraction as a percentage. For example:

$$\frac{3}{10} = \frac{30}{100} = 30\% \quad \text{and} \quad \frac{1}{5} = \frac{20}{100} = 20\%$$

You can also multiply the fraction by 100%.

Example 3

Convert these fractions into percentages by multiplying by 100%.

a $\frac{1}{5}$ **b** $\frac{1}{4}$ **c** $\frac{3}{8}$

Solution

a $\frac{1}{5} \times 100\% = \frac{100\%}{5} = 20\%$ So $\frac{1}{5}$ is 20%

b $\frac{1}{4} \times 100\% = \frac{100\%}{4} = 25\%$ So $\frac{1}{4}$ is 25%

c $\frac{3}{8} \times 100\% = \frac{300\%}{8} = 37.5\%$ So $\frac{3}{8}$ is 37.5%

Whole numbers can be converted to percentages.

We have seen that $1 = \frac{100}{100} = 100\%$

So $2 = \frac{200}{100} = 200\%$, $3 = \frac{300}{100} = 300\%$, $4 = \frac{400}{100} = 400\%$ and $18 = \frac{1800}{100} = 1800\%$

To convert a percentage to an equivalent fraction or mixed number, first write it as a fraction with a denominator of 100, then simplify it. For example:

$$20\% = \frac{20}{100} = \frac{1}{5} \quad \text{and} \quad 130\% = \frac{130}{100} = 1\frac{30}{100} = 1\frac{3}{10}$$

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7

Example 4

Write these percentages as fractions or mixed numbers.

a 80% **b** 40% **c** 75% **d** 12% **e** 140%

Solution

a $80\% = \frac{80}{100} = \frac{4}{5}$ **b** $40\% = \frac{40}{100} = \frac{2}{5}$ **c** $75\% = \frac{75}{100} = \frac{3}{4}$

d $12\% = \frac{12}{100} = \frac{3}{25}$ **e** $140\% = \frac{140}{100} = 1\frac{40}{100} = 1\frac{2}{5}$

We often use 100% to describe 'all' of something. For example, '100% of the audience enjoyed the movie' means that all of the audience liked the movie.

Percentages less than 100% describe something less than the whole. For example, 'only 86% of people voted' means not everyone voted. Also, 'Teresa is not feeling 100% today' means that Teresa does not feel completely well.

We also use percentages greater than 100% to indicate that something was more than expected. For example, three glasses of orange juice give 120% of the recommended daily intake of vitamin C, which is more than required.

9A Whole class CONNECT, APPLY AND BUILD

1 Create each situation in your classroom, then say what the percentage is.
- **a** There are 10 children standing; 8 of them have one hand in the air.
- **b** There are 5 children standing; 3 of them are smiling.
- **c** There are 2 people standing; 1 of them is older than 12.

9A Individual

1 Write each of these as a percentage.

a 10 out of 100 **b** 50 out of 100 **c** 78 out of 100

2 Write the percentage for each situation.

a 87 out of 100 children at Queenstown School like cricket.

b 62 out of 100 plants in Graeme's garden are native plants.

c In Victoria, 8 litres out of every 100 litres of water consumed is used by private homes.

3 Write these fractions as percentages.

a $\frac{70}{100}$ **b** $\frac{20}{100}$ **c** $\frac{33}{100}$ **d** $\frac{10}{25}$ **e** $\frac{40}{50}$ **f** $\frac{8}{10}$

g $\frac{20}{50}$ **h** $\frac{3}{4}$ **i** $\frac{3}{25}$ **j** $\frac{4}{5}$ **k** $\frac{1}{8}$ **l** $\frac{19}{20}$

4 Write these percentages as fractions.

a 7% **b** 90% **c** 75% **d** 80% **e** 72%

5 Write these mixed numbers as percentages.

a $2\frac{20}{100}$ **b** $1\frac{3}{50}$ **c** $7\frac{3}{4}$ **d** $1\frac{3}{25}$ **e** $1\frac{40}{100}$

6 100 people ran in a City to Surf fun run. 56 of them were women. What percentage of the runners were men?

7 At Kangaroo Flat School, 34 out of 50 children catch the bus each day. What percentage is this?

8 $\frac{3}{5}$ of the houses in Helen's street are double-storey houses. What percentage of the houses are single storey?

9 75% of the children in Kim's family are boys. What fraction are girls?

10 15 out of the 45 students who attend Riley Regional School are over 10 years of age. What percentage is this?

11 In January, the Foster dam was at 42% of capacity. By December, it was only 33% full. By what percentage of capacity had the water level dropped?

12 Liam bought 40 lollies. 10 of the 40 lollies were green. What percentage of the lollies were not green?

13 Students at Burnie School were surveyed about their favourite ice-cream flavour. 25% of students liked strawberry, 40% liked chocolate, and the rest liked vanilla. What percentage of students liked vanilla ice-cream the best?

14 There are 12 Year 6 students at the Alice Springs School of the Air. 9 of the 12 students live within 1000 km of Alice Springs. What percentage is this?

15 100 children enrolled for Little Athletics. 13% enrolled on Monday, 29% on Tuesday, 8% on Wednesday and 16% on Thursday. If enrolments were not taken at the weekend, what fraction of the children enrolled on Friday?

16 In our street there are 25 houses. 16 have a double garage, 6 have a single garage and 3 have a carport. Work out the percentage for each type of garage.

9B Decimals and percentages

A decimal can also be written as a percentage. To convert a decimal to a percentage, first write it as a fraction with a denominator of 100.

$$0.02 = \frac{2}{100} = 2\% \qquad 0.5 = \frac{50}{100} = 50\% \qquad 1 = \frac{100}{100} = 100\%$$

Example 5

Write these decimals as percentages.

a 0.85 **b** 1.23 **c** 0.03 **d** 5.8

Solution

a $0.85 = \frac{85}{100} = 85\%$

b $1.23 = 1\frac{23}{100} = \frac{123}{100} = 123\%$

c $0.03 = \frac{3}{100} = 3\%$

d $5.8 = 5\frac{80}{100} = \frac{580}{100} = 580\%$

A decimal that has hundredths as the last place converts easily to a percentage.

Example 6

Write these decimals as percentages.

a 0.12 **b** 1.38 **c** 0.8

Solution

a $0.12 = 12\%$

b $1.38 = 138\%$

c $0.8 = 0.80$
$= 80\%$

A percentage can be converted to a decimal by first writing it as a fraction with a denominator of 100, then converting it to a decimal. For example:

$$34\% = \frac{34}{100} = 0.34$$

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

Example 7

Write these percentages as decimals.

a 24%

b 50%

c 3%

d 343%

Solution

a $$\begin{aligned} 24\% &= \frac{24}{100} \\ &= 0.24 \end{aligned}$$

b $$\begin{aligned} 50\% &= \frac{50}{100} \\ &= 0.05 \end{aligned}$$

c $$\begin{aligned} 3\% &= \frac{3}{100} \\ &= 0.03 \end{aligned}$$

d $$\begin{aligned} 343\% &= \frac{343}{100} \\ &= 3\frac{43}{100} \\ &= 3.43 \end{aligned}$$

9B Whole class CONNECT, APPLY AND BUILD

1 Select dice of two different colours and roll them to give a decimal number. One dice is the tenths digit, and the other dice is the hundredths digit.

For example, using blue for tenths and red for hundredths:

gives the number 0.46

Roll the dice 10 times and convert each decimal number into a percentage.

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

2 Select dice of three different colours and roll them to give a decimal number. One dice is the ones digit, another dice is the tenths digit and the third dice is the hundredths digit.

For example:

could give the number 2.46

Roll the dice 10 times and convert each decimal number into a percentage.

9B Individual

1 Write these decimals as percentages.

a 0.37	**b** 0.25	**c** 0.94	**d** 0.62
e 0.02	**f** 0.08	**g** 0.01	**h** 0.04
i 0.9	**j** 0.6	**k** 0.8	**l** 0.2
m 7.25	**n** 1.33	**o** 7.04	**p** 11.09
q 1.5	**r** 5.8	**s** 2.07	**t** 6.4

2 Write these percentages as decimals.

a 75%	**b** 86%	**c** 37%	**d** 72%
e 80%	**f** 90%	**g** 60%	**h** 70%
i 7%	**j** 1%	**k** 3%	**l** 5%

3 Write these percentages as decimals.

a 265%	**b** 526%	**c** 652%	**d** 375%

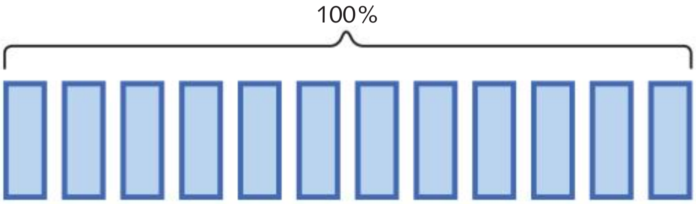
ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

9C Percentage 'of' a quantity

We often need to calculate a percentage of a quantity.

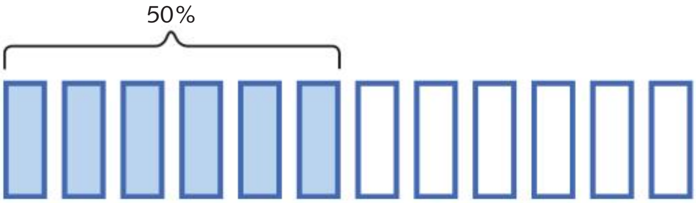
For example:

100% of 12 is 12



$$\begin{aligned}100\% \text{ of } 12 &= \frac{100}{100} \times 12\\ &= 1 \times 12\\ &= 12\end{aligned}$$

50% of 12 is 6

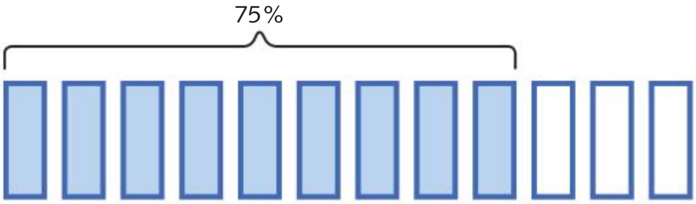


$$\begin{aligned}50\% \text{ of } 12 &= \frac{50}{100} \times 12\\ &= \frac{1}{2} \times \frac{12}{1}\\ &= \frac{12}{2}\\ &= 6\end{aligned}$$

25% of 12 is 3

$$\begin{aligned}25\% \text{ of } 12 &= \frac{25}{100} \times 12\\ &= \frac{1}{4} \times \frac{12}{1}\\ &= \frac{12}{4}\\ &= 3\end{aligned}$$

75% of 12 is 9



$$\begin{aligned}75\% \text{ of } 12 &= \frac{75}{100} \times 12\\ &= \frac{3}{4} \times \frac{12}{1}\\ &= \frac{36}{4}\\ &= 9\end{aligned}$$

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7

To calculate a percentage of another number, convert the percentage to a fraction, then multiply. The word ‘of’ tells us that we need to multiply.

Example 8

Calculate 20% of each number.

a 80

b 170

Solution

a

$$\begin{aligned} 20\% \text{ of } 80 &= \frac{20}{100} \times \frac{80}{1} \\ &= \frac{1}{5} \times \frac{80}{1} \\ &= \frac{80}{5} \\ &= 16 \end{aligned}$$

b

$$\begin{aligned} 20\% \text{ of } 170 &= \frac{20}{100} \times \frac{170}{1} \\ &= \frac{1}{5} \times \frac{170}{1} \\ &= \frac{170}{5} \\ &= 34 \end{aligned}$$

(Use short division.)

$$\begin{array}{r} 3\,4 \\ 5\overline{)1\,7\,0} \end{array}$$

Unitary method

There is sometimes an easier way to calculate percentages. It is called the **unitary method**. First work out 10% by dividing the number by 10 in your head, then work in multiples or fractions of 10.

Example 9

a Calculate 20% of 80.

b Calculate 5% of 70.

Solution

a

$$\begin{aligned} 100\% \text{ of } 80 &= 80, \text{ so} \\ 10\% \text{ of } 80 &= 80 \div 10 = 8 \\ \text{So } 20\% \text{ of } 80 &= 2 \times 8 \\ &= 16 \end{aligned}$$

b

$$\begin{aligned} 100\% \text{ of } 70 &= 70, \text{ so} \\ 10\% \text{ of } 70 &= 70 \div 10 = 7 \\ \text{So } 5\% \text{ of } 70 &= \frac{1}{2} \times 7 \\ &= 3.5 \end{aligned}$$

Percentage increase

We use percentage increase when we talk about an increase in wages, when a fee is added to a bill and when interest is paid on a bank account. To calculate a percentage increase, first calculate the percentage ‘of’ the number. To get the new number, add the percentage increase to the original number.

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

Example 10

Simon leaves a 5% tip for good table service when he goes out to eat in a restaurant. He adds this percentage onto each food bill. Calculate the total cost of a meal for Simon, including the 5% tip, if the food bill comes to a total of:

a \$100 **b** \$60 **c** \$300

Solution

a

$$\begin{aligned} 5\% \text{ of } \$100 &= \frac{5}{100} \times 100 \\ &= \frac{1}{20} \text{ of } 100 \\ &= \$5 \end{aligned}$$

$$\begin{aligned} \text{The restaurant bill, plus} & \\ \text{Simon's tip} &= \$100 + \$5 \\ &= \$105 \end{aligned}$$

b

$$\begin{aligned} 5\% \text{ of } \$60 &= \frac{5}{100} \times 60 \\ &= \frac{1}{20} \times \frac{60}{1} \\ &= \frac{60}{20} \\ &= \$3 \end{aligned}$$

$$\begin{aligned} \text{The restaurant bill, plus} & \\ \text{Simon's tip} &= \$60 + \$3 \\ &= \$63 \end{aligned}$$

c

$$\begin{aligned} 5\% \text{ of } \$300 &= \frac{5}{100} \times 300 \\ &= \frac{1}{20} \times \frac{300}{1} \\ &= \frac{300}{20} \\ &= \$15 \end{aligned}$$

$$\begin{aligned} \text{The restaurant bill, plus} & \\ \text{Simon's tip} &= \$300 + \$15 \\ &= \$315 \end{aligned}$$

Percentage decrease

Percentage decrease is used to describe a situation such as a drop in the number of people attending football matches this year compared to last year, or when an item is discounted because it is on sale. To calculate a percentage decrease, calculate the percentage 'of' the number. To get the new number, *subtract* the percentage decrease from the original number.

Example 11

Katrina went to a sale and bought a jumper that usually sells for \$100 at 40% discount. What price did Katrina pay for the jumper?

Solution

$$
\begin{aligned}
40\% \text{ of } \$100 &= \frac{40}{100} \times 100 \\
&= \$40
\end{aligned}
$$

Original price – discount = sale price

$100 – $40 = $60

Katrina paid $60 for her new jumper.

9C Individual

1 Calculate these percentages.

a 50% of 100 **b** 20% of 120 **c** 20% of 50
d 10% of 450 **e** 5% of 100 **f** 5% of 200
g 10% of 10 **h** 10% of 1000 **i** 200% of 100

2 25% of the water used in Australian households is used for washing clothes. What is the amount of water one household would use in one day for washing clothes if its daily water usage is:

a 100 litres? **b** 150 litres? **c** 300 litres?

3 Calculate the total amount after the percentage increase for each of these examples.

a Hamish earned 5% interest on a bank balance of $20.

b Last year, 300 children attended basketball clinics at Port Haven. This year, the number of children attending basketball clinics has risen by 10%.

c Stella’s bean plant was 30 centimetres tall at the start of January. By the start of February, her bean plant had grown 20% taller.

4 Jake and Thomas Smith earned 5% on the balance of their bank account last year. Jake saved $340 and Thomas saved $715. How much money did each brother have in the bank at the end of the year, including interest?

5 Calculate the total amount after the percentage decrease for each of these.

a Toni lost 8% of her weight of 50 kilograms.

b 12% of the 150 trees in Trevor’s garden died because of the drought.

c Sales at Bright Books are down 50% on last week’s figure of 250 books.

6 Stephanie saves 25% of her pocket money each week. How much does she have left to spend if she receives:

a $10? **b** $12? **c** $7?

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

9D Review questions

1 Write each of these as a percentage.

a 75 out of 100 **b** 23 out of 100 **c** 99 out of 100

2 Write each number as a percentage.

a $\frac{50}{100}$ **b** $\frac{15}{100}$ **c** 0.5 **d** $1\frac{9}{10}$

e $\frac{48}{100}$ **f** $\frac{23}{50}$ **g** 0.98 **h** 0.03

3 Write the missing fractions (or mixed numbers), decimals and percentages.

Decimal	Fraction or mixed number	Percentage
0.1		
	$\frac{25}{100}$	
0.66		
		40%
	$3\frac{1}{2}$	

4 Calculate these percentages.

a 20% of 100 **b** 50% of 120 **c** 80% of 50

d 10% of 900 **e** 5% of 20 **f** 8% of 200

g 100% of 99 **h** 1% of 1000

5 **a** Write down a number that is more than 30% of 50.

b Write down a number that is 30% more than 50.

6 For each of these, calculate the total amount after the percentage *increase*.

a A bank balance of $130 earned 8% interest.

b Last year, 200 children attended the Dapto hearing clinic. This year the number of children attending the clinic has risen by 18%.

c Francesca was 130 centimetres tall at the start of January. By the end of December she had grown 20% taller.

7 For each of these, calculate the total amount after the percentage *decrease*.

a Cilla had 180 trees in her garden. Ten per cent of them died because of the drought.

b Chocolate sales are 50% down on last year's figure of 1000.

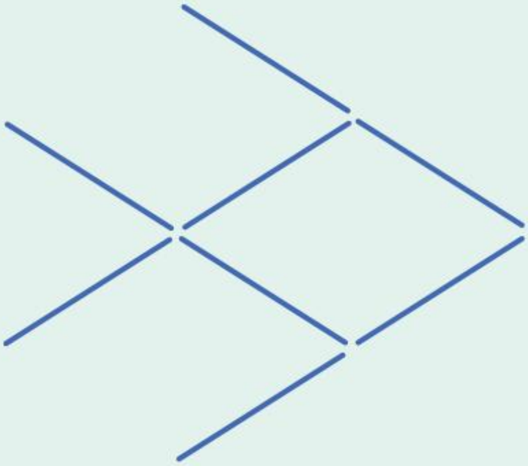
CHAPTER 10 Preparation

Useful skills for this chapter:

- previous experience in identifying lines and angles
- the ability to draw lines and angles using a ruler
- the ability to identify and draw vertical, horizontal and oblique lines.

KICK OFF

Use 8 craft sticks to make this fish. Show the fish swimming to the left by moving only 3 craft sticks.



Show what you know

Look around your classroom to find an object that has 2 lines that are parallel. Draw the object. Now find and draw another object that has 2 perpendicular lines.

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

CHAPTER

10

Measurement and Geometry

Lines and angles

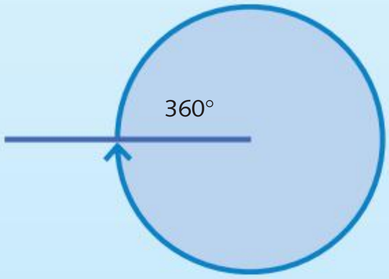
Lines and angles are all around us. We use them when we draw, we use them when we build and we even use them in games. Architects and builders use lines and angles in their work.

In mathematics, a line is always a straight line. It does not include curves such as circles or squiggles.

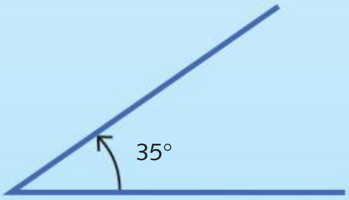
Lines go on forever in both directions. It is impossible to draw a line that goes on forever because we eventually run out of paper. So we usually draw part of a line and imagine that it goes on forever. Sometimes we add arrows to show this.

An angle is the measurement of a turn. If you turn through one revolution, you have turned 360°.

There are 360° of turn in a circle.



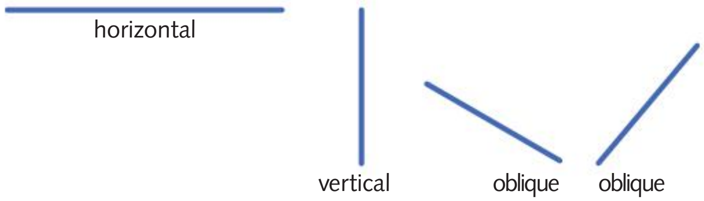
When two lines meet (or intersect) at a point, we measure the number of degrees you would need to turn from one line to the other. For example, the angle between these two lines is 35°.



ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

10A Lines and angles

Lines can be horizontal, vertical or oblique.



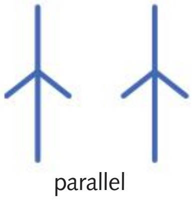
Do you know how to check whether an edge is horizontal? People often use a tool called a spirit level. Have you ever seen or used a spirit level? How does it work?

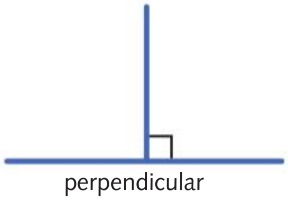
Do you know how to make a vertical line? Builders sometimes use a tool called a 'plumb bob'. A plumb bob is a string with a pointed weight on one end. The weight used to be made of a heavy metal called lead. The Latin name for lead is *plumbum* and this is where plumb bob gets its name.

To use a plumb bob, start by finding a mark on the wall, such as a nail hole. Then hold one end of the string level with that mark. When the plumb bob stops moving, have a friend mark the floor beside the pointed end of the plumb bob. Then rule a line from the mark on the wall to the mark on the floor and you have a vertical line.

Parallel and perpendicular lines

Pairs of lines can be related. Two important relationships are parallel lines and perpendicular lines.



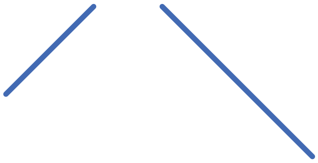


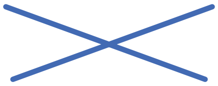
Two lines are parallel if they will not cross no matter how far they are extended.

When we draw parallel lines, we draw two small arrows to show that the lines are parallel. For example, these pairs of lines are parallel.

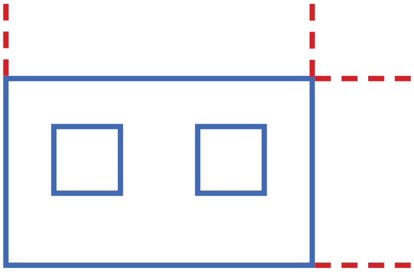
ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

These pairs of lines are not parallel.



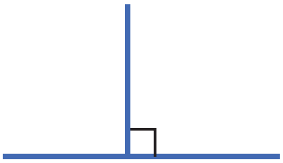


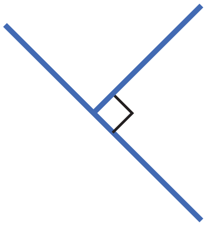
You might be able to see pairs of parallel lines in your classroom. Choose a wall and look at its side edges. Now imagine extending those two edges into the air. The edges are parallel and will not meet.



Two lines are perpendicular if they are at right angles (90°) to each other.

When one line is perpendicular to the other, we draw a small right angle where the lines intersect to show that the lines are at a 90° angle to each other.





A builder makes sure that the walls are perpendicular to the floor and adjacent walls are perpendicular to each other.

Line segments

We have seen that we can draw part of a line to mean a line that goes on forever. However, sometimes we draw part of a line and really mean only the piece instead of the whole line. A piece of a line is called a **line segment**. The word 'segment' means part.

line segment

Sometimes we draw part of a line and really mean half of a line. A half line is called a **ray**. A ray starts at a point and then goes on forever in one direction. Think of rays of sunshine.

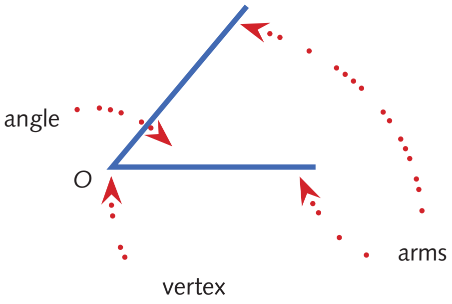
ray

The arrows on the ends of rays and lines are not always used, but sometimes it is helpful to use them.

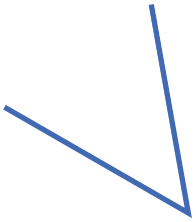
ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

Angles

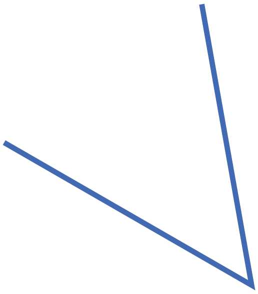
To make an angle we need two rays meeting at a point. We draw this by showing two line segments meeting at a point. The rays or segments are called the arms of the angle. The point where the arms of the angle intersect is called the **vertex**. It is sometimes labelled *O*.



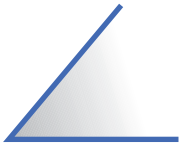
Turning the whole picture around doesn't change the angle.



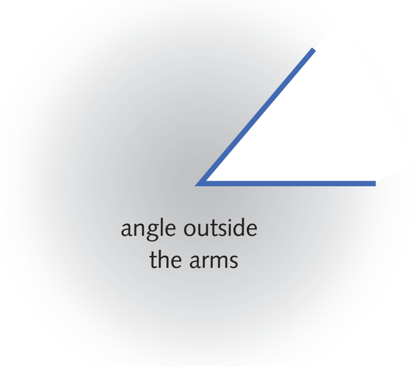
Changing the length of the arms doesn't change the angle.



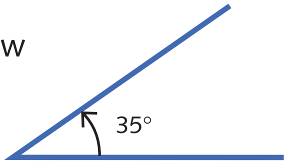
When we cut the first slice from a cake, we make two cuts. Those two cuts form the **arms** of an angle. In fact, they form the arms of two angles; a smaller one between the arms and a larger one outside the arms.

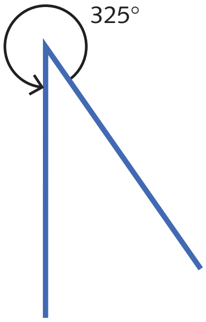


angle between
the arms

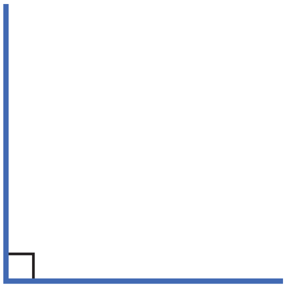


Any two arms will produce two angles. We draw a small curved arrow to mark the angle we are talking about.

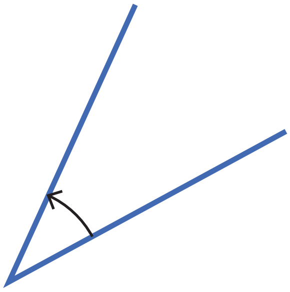


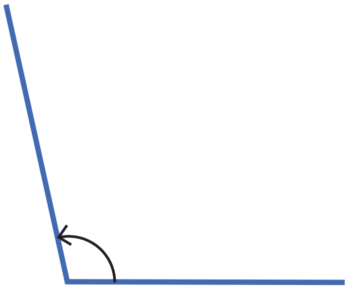


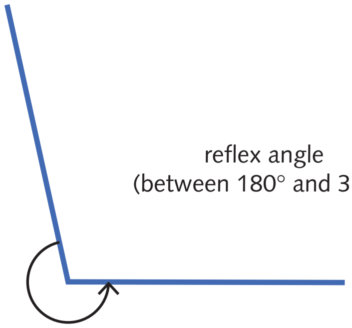
There are many types of angles.


right angle (90°)


straight angle (180°)


acute angle
(less than 90°)

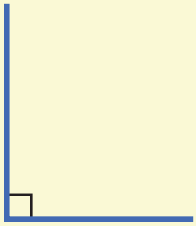

obtuse angle
(between 90° and 180°)


reflex angle
(between 180° and 360°)

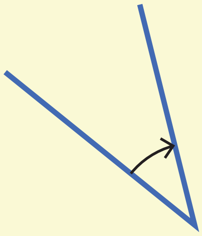
Example 1

Which angle is a right angle?

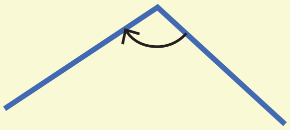
A



B



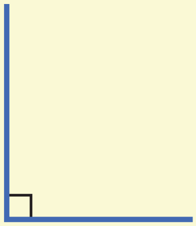
C



Solution

Look at angle A. The intersecting lines are perpendicular, so angle A is a right angle. Now look at the other angles.
Angle B is less than a right angle.
Angle C is greater than a right angle.

A



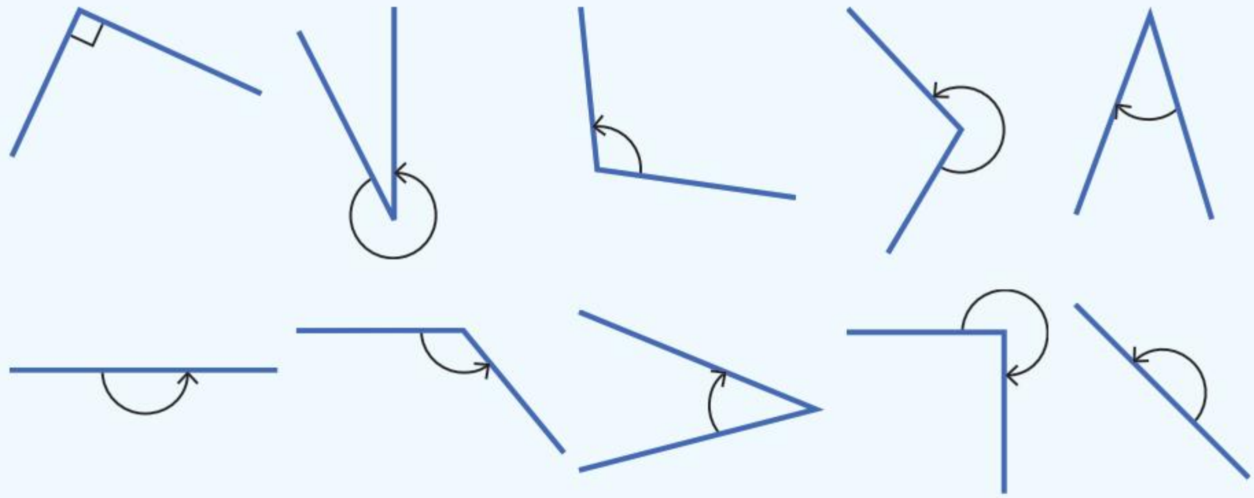
ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

10A Whole class CONNECT, APPLY AND BUILD

1 Draw lines to show angles **a** to **f**.

a Right angle	**b** Straight angle
c Acute angle	**d** Obtuse angle
e Reflex angle	**f** Look around the room to find an example of each angle

2 Draw these angles, then group them according to their type.



3 You can use the hands of an analogue clock to show angles. What type of angle do the hands of a clock make when the clock shows:

a 6:00 am?	**b** 9:00 pm?	**c** 5:00 pm?
d 11:00 am?	**e** 2:00 pm?	**f** 8:00 am?

4 Write the following compass directions on cards and place them on the appropriate walls of the classroom.

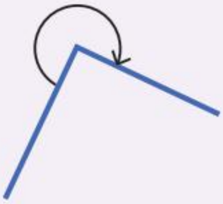
- North
- North-east
- North-west
- South
- South-east
- South-west
- East
- West

Face north, then turn to face east. What turn have you made? (A right-angle turn.) Repeat with other compass directions to show turns that are a straight angle, a reflex angle, an acute angle and an obtuse angle.

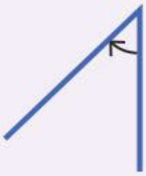
10A Individual

1 Name each type of angle.

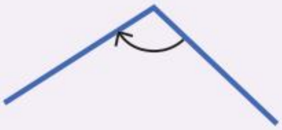
a



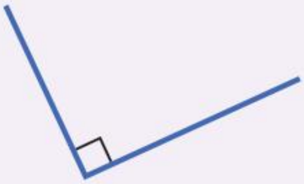
b



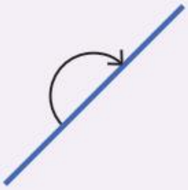
c



d



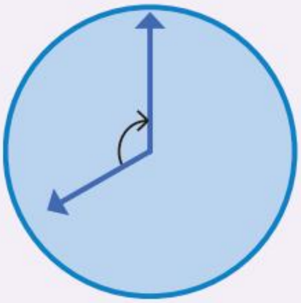
e



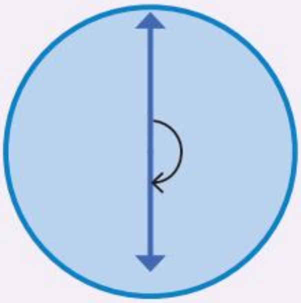
f

2 Name the type of angle made by the hands of each clock.

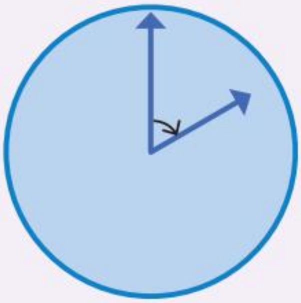
a



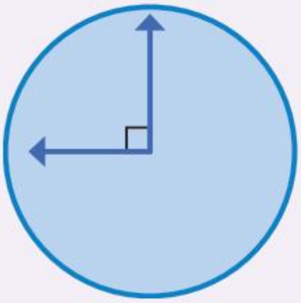
b



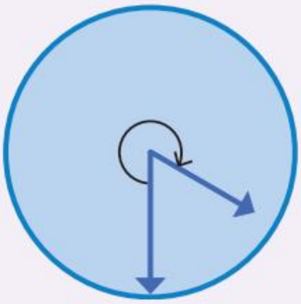
c



d



e



3 Name the angle that matches each clue. What type of angle am I?

a I am half a right angle.

b I am twice as big as a right angle.

c I am three times the size of a right angle.

d I am half a straight angle.

e I am half a revolution.

f I am more than a right angle added to a straight angle.

4 Draw an example of:

a a straight angle

b an obtuse angle

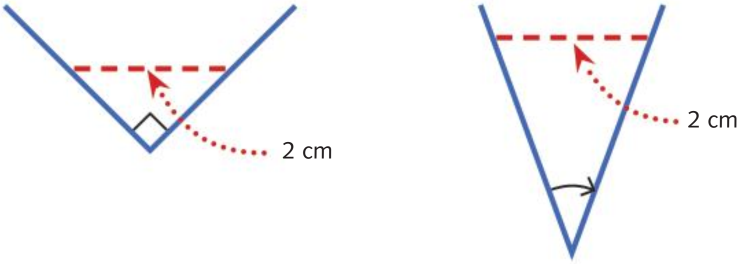
c a reflex angle

d a right angle

e an acute angle

10B Using a protractor

How do we measure the angle made when two lines intersect?



Look at these two angles. We cannot use a ruler to measure the distance between the arms; the measurement could be the same, but we know that one angle is 90° and the other is an acute angle, which is less than 90°.

Also, if you move the ruler up or down the angle, the length changes.

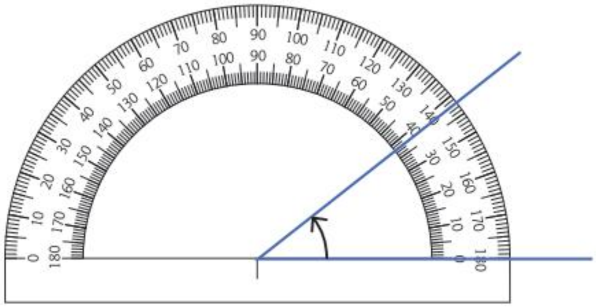
The best way to measure an angle is to use a protractor.

Measuring acute angles

A protractor has two sets of numbers: one set on the inside edge and one set on the outside. The inside numbers measure angles from the right.

To measure an angle, we put the 0° line along one arm and the centrepoint of the 0° line on the vertex of the angle.

For example, this angle measures 38°.

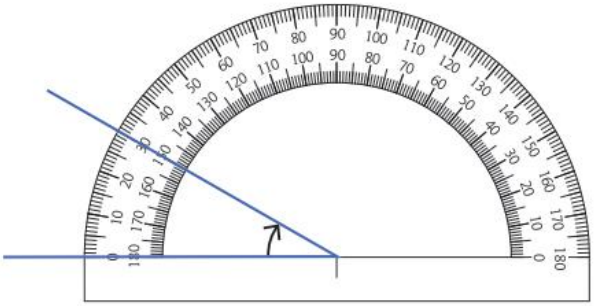


The angle is 38°.

The outside numbers are for measuring angles from the left.

One arm is on the 0° line. We measure the angle from the left using the outside numbers.

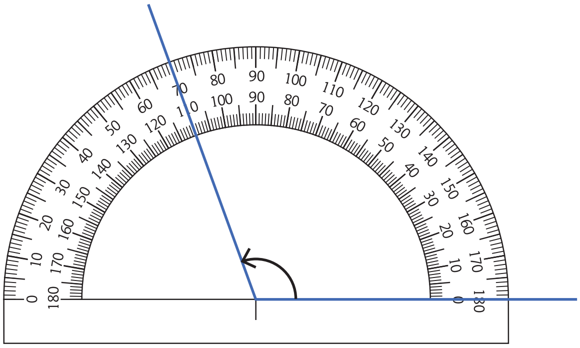
For example, this angle measures 30°.



The angle is 30°.

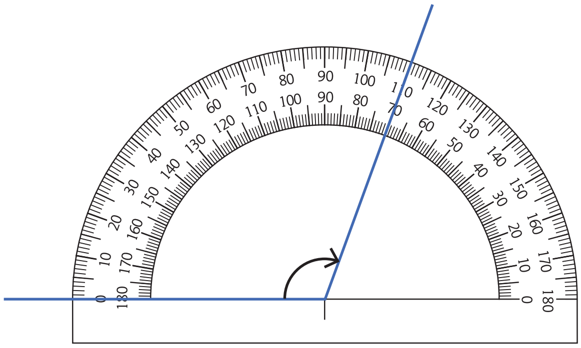
ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

Measuring obtuse angles



The angle is 110°.

Both of these angles are greater than a right angle.



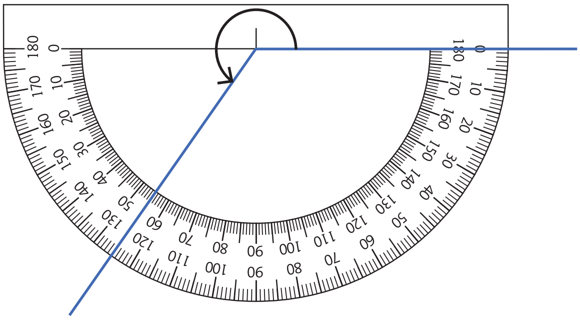
The angle is 110°.

Measuring reflex angles

To measure a reflex angle, you may need to rotate the protractor.

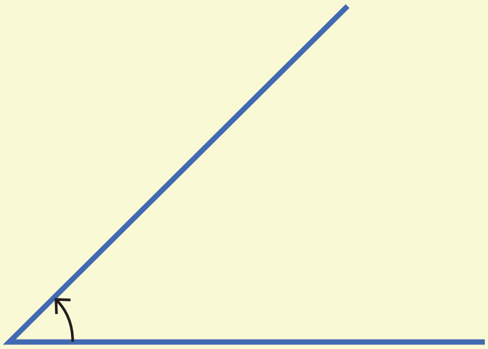
This gives you part of the angle (55°). To find the full size of the angle, you now need to add 180° to the number of degrees shown on the protractor.

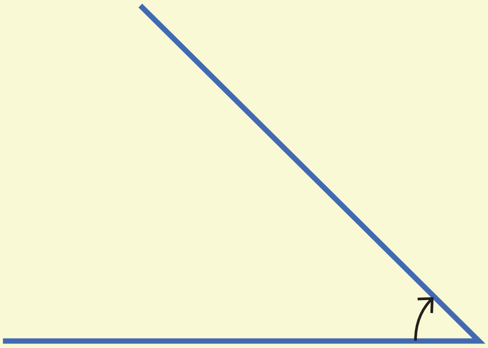
$$180^\circ + 55^\circ = 235^\circ$$



Example 2

What is the size of these angles?

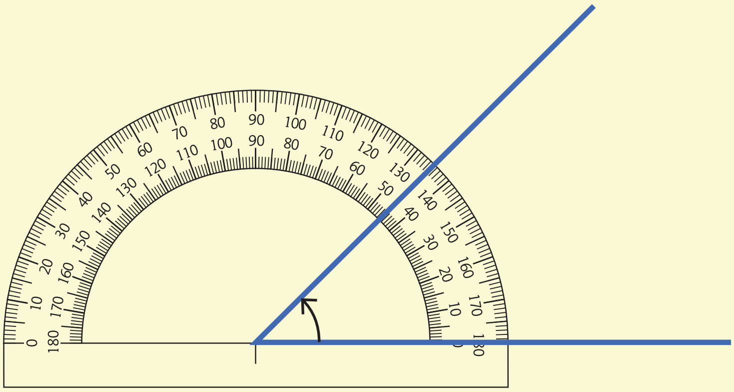


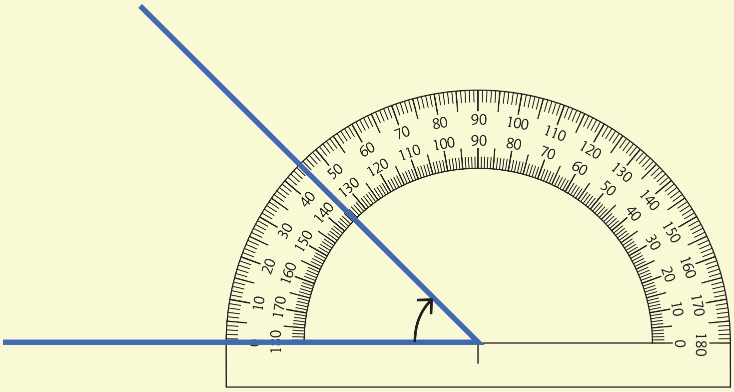


ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7

Solution

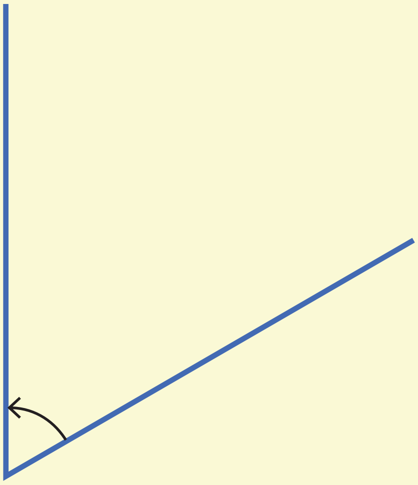
Use a protractor to measure the angle. Place the 0° line of the protractor on the horizontal arm of the angle. Make sure that the centrepoint of the 0° line is at the vertex of the angle. Read the number where the other arm of the angle is pointing. Both of these show 45° angles.





Example 3

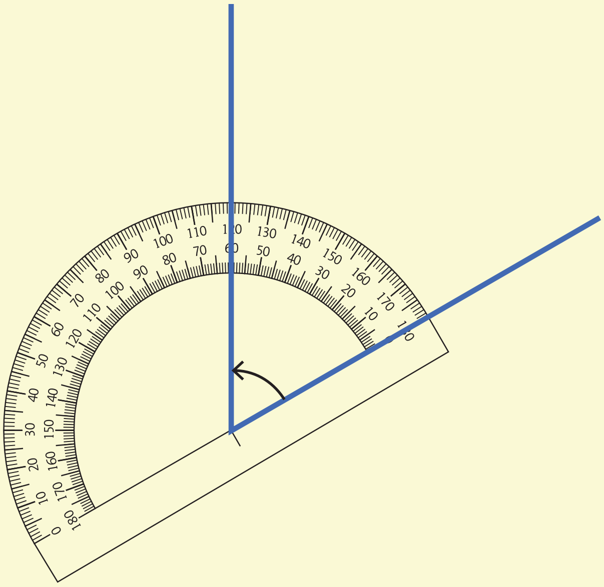
What is the size of this angle?



ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

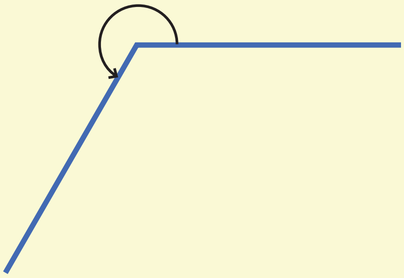
Solution

Rotate the protractor so the $0°$ line on the protractor is on the oblique arm of the angle. Make sure the centre of the $0°$ line is at the vertex. Read the number where the other arm of the angle is pointing. The angle turns from the right, so we use the inside scale. This is a $60°$ angle.



Example 4

What is the size of this angle?

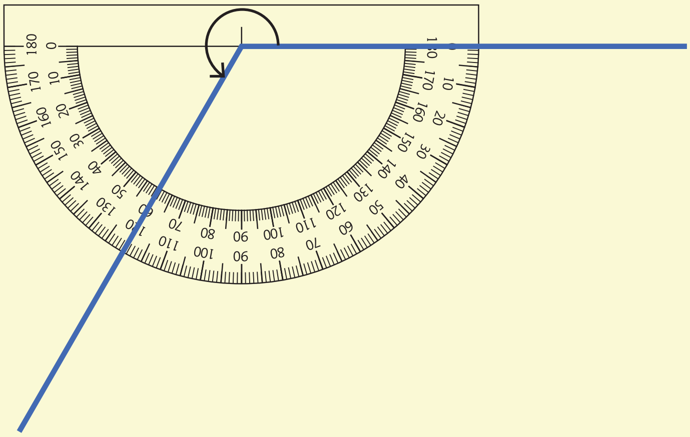


Solution

Turn the protractor upside down so that the $0°$ line on the protractor is on the horizontal arm of the angle, with the arc below the line. Make sure the centre of the $0°$ line is at the vertex. Measure how much bigger than $180°$ the angle is by reading the inside scale. The arm is pointing to a $60°$ angle. Now add $180°$ to $60°$.

$$180° + 60° = 240°$$

This is a $240°$ angle.



ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7

10B Whole class CONNECT, APPLY AND BUILD

1 Work in pairs. One student draws an acute angle and their partner draws an obtuse angle. Swap angles with your partner and use a protractor to measure each other's angles.

2 Work in pairs. Draw a reflex angle each. Swap angles with your partner and use a protractor to measure each other's angles.

3 Draw a sketch of these angles then use a protractor to measure them. How close was your sketch to the correct angle?

a 60° **b** 180° **c** 120° **d** 270° **e** 210°

f 45° **g** 300° **h** 155° **i** 20° **j** 100°

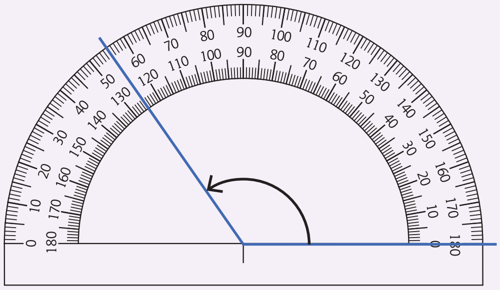
4 **Make your own protractor**

- Take a paper semicircle. The base is a straight angle. Write 0° at one end of the straight angle and 180° at the other.
- Fold the semicircle in half, mark the middle and draw a perpendicular line to show a right angle. What will this angle be? Write 90°.
- Find the mark halfway between 0° and 90°. What will this angle be? Write 45°.
- Repeat this step for the mark halfway between 90° and 180° to show 135°.
- Then divide the arc into increments of 10° and write the number of degrees.

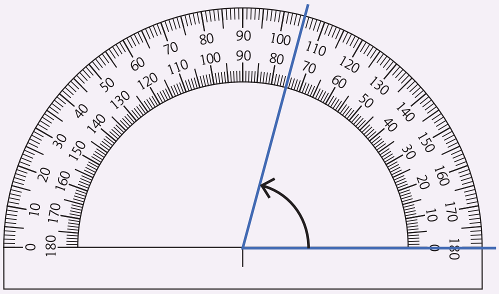
10B Individual

1 Write the size of each angle, then name the type of angle.

a

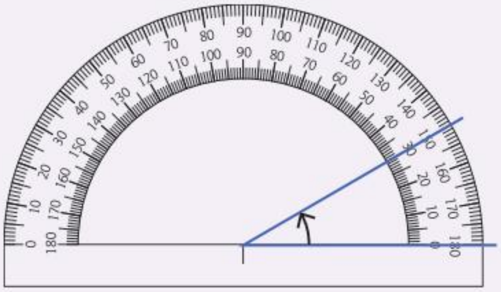


b

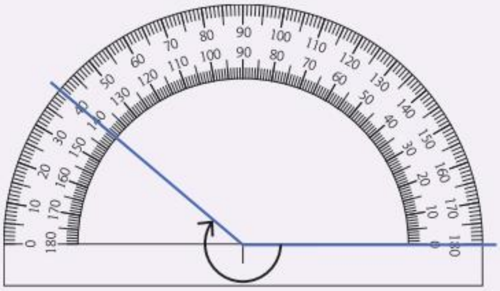


ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

c

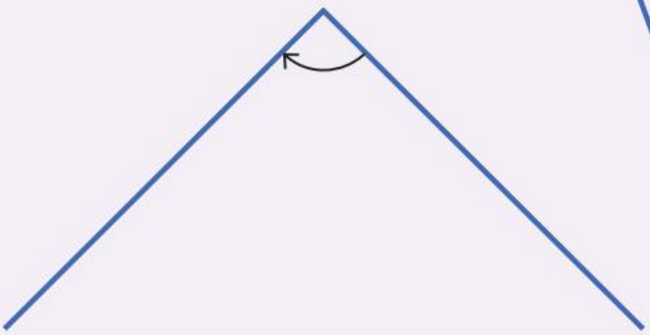


d

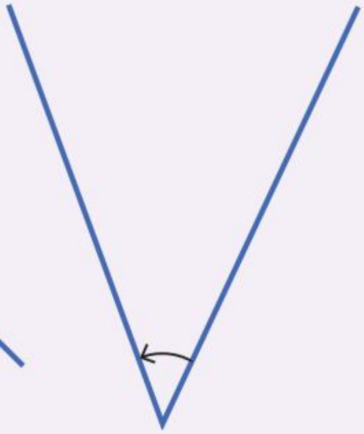


2 Estimate the size of these marked angles, then use a protractor to measure each one.

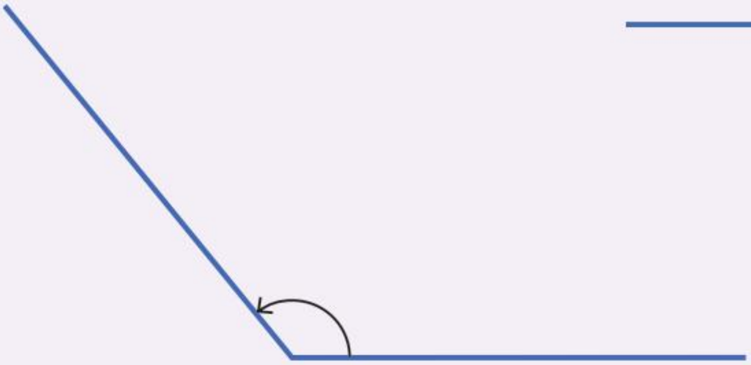
a



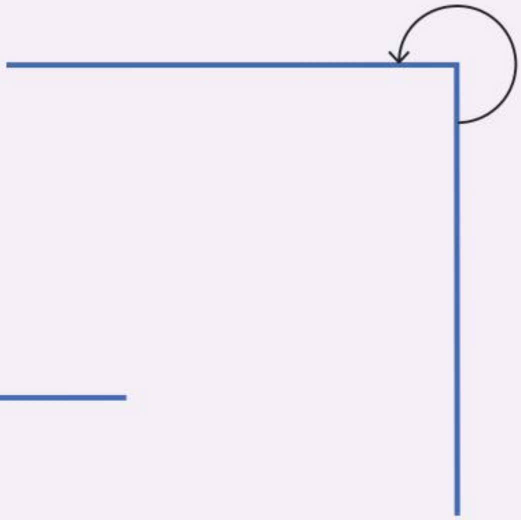
b



c



d



3 Use a protractor to draw two intersecting lines that make these angles. Mark the angle with a curved arrow to show clearly which angle is the answer.

a 80° **b** 135° **c** 25° **d** 230° **e** 190°

Homework

1 **a** Use your angle marker and a protractor to find a right angle, an acute angle, an obtuse angle and a reflex angle in your home.

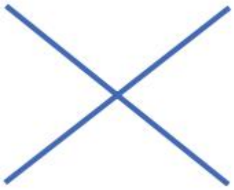
b Draw each object that has those angles, then write the size of the angle.

10C Finding unknown angles

You do not need to use a protractor to find the size of every angle.

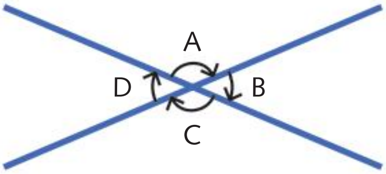
Sometimes, the angle you need to measure is related to one you already know about.

When two lines cross, we can see four angles at a point.



We will investigate how angles at a point are related.

In mathematics we use letters of the alphabet to label angles because it helps us know which angle we are talking about.

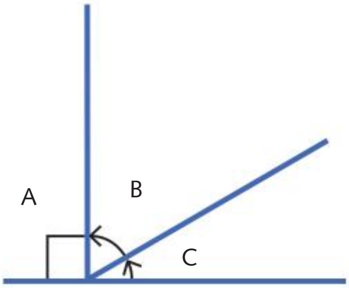


Complementary angles

The three lines on the right intersect and we can see three angles, marked A, B, and C.

We can see that angle A is a right angle (90°).

Angles B and C make up another right angle (90°). They are called **complementary angles** because the two angles together make a right angle.



If we know the size of angle C, then we can work out the size of angle B.

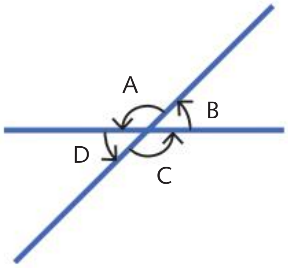
If angle C is 35°, angle B is 90° – 35° = 55°.

Supplementary angles

When two lines cross, we can see four angles.

Look at the horizontal line.

We can see that angles A and B make a straight angle (180°).



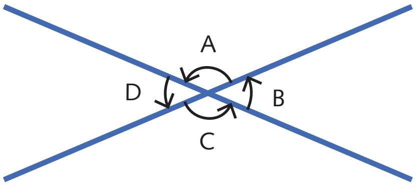
Angles A and B are called **supplementary angles** because the two angles add up to 180°. If angle A is 135°, then angle B is 180° – 135° = 45°. Angles C and D are also supplementary angles.

Now look at the other (oblique) line. The angles A and D are supplementary because they make a straight angle. So are the angles B and C.

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7

Cambridge University Press

Opposite angles



When two lines intersect, the two opposite angles are equal.

Angles A and C are the same size.

This is because:

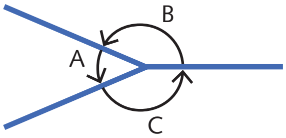
angle A + angle D = 180° and angle C + angle D = 180°

Angles B and D are also the same size.

If angle A is 145°, then angle C is 145°.

If angle D is 35°, then angle B is 35°.

Angles about a point



When three rays meet at one point, we get three angles. The three angles add up to 360°.

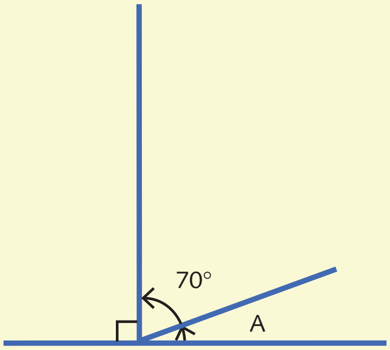
angle A + angle B + angle C = 360°

If angle A is 60° and angle B is 160°, then angle C is:

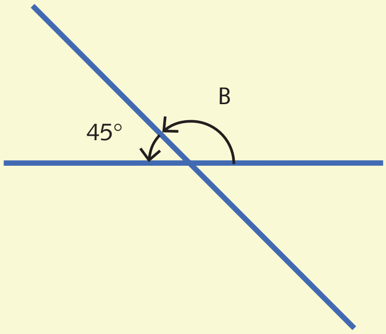
360° – 60° – 160° = 140°

Example 5

a What is the size of angle A?



b What is the size of angle B?



Solution

a Angle A + 70° = 90°

So angle A = 90° – 70° = 20°

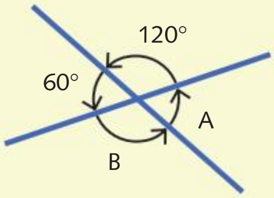
b Angle B + 45° = 180°

So angle B = 180° – 45° = 135°

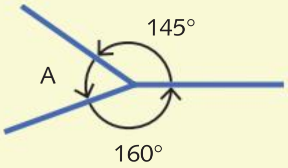
ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

Example 6

a What is the size of angle A and angle B?



b What is the size of angle A?



Solution

a Angle A is opposite an angle that measures 60°, so angle A = 60°.
Angle B is opposite an angle that measures 120°, so angle B = 120°.

b Angle A + 145° + 160° = 360°
So angle A = 360° – 160° – 145° = 55°

Remember

Two angles that add up to 90° are called complementary angles.

Two angles that add up to 180° are called supplementary angles.

When two lines cross each other, the opposite angles are equal.

When three lines meet at one point, the angles they make add up to 360°.

10C Whole class CONNECT, APPLY AND BUILD

1 Work in pairs.

- The first student draws a right angle.
- Their partner draws a line from the vertex cutting the right angle into two angles and measures one of the new angles.
- The first student then works out the size of the complementary angle by subtracting from 90°.

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

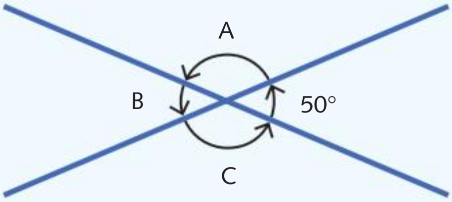
2 Work in pairs.

- The first student draws a straight angle and marks a vertex on it.
- Their partner draws a line out from the vertex and measures one of the two angles.
- The first student then works out the supplementary angle by taking away from 180°.

3 Work in pairs.

- The first student draws two lines crossing each other.
- Their partner measures one of the angles.
- The first student then works out which angle is the same size, using the principle that opposite angles are equal.
- How many pairs of opposite angles can you find?
- Measure the angles to check that they are equal.

4 As a class, discuss how you can find the size of all the angles if you know the size of one of the angles. Draw this diagram and mark the angles.



a How many degrees are there when all of the angles are added together?

b Find angle A.

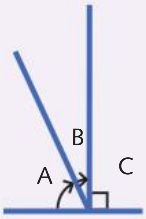
c Find angle C.

d Find angle B. Can you do this in more than one way?

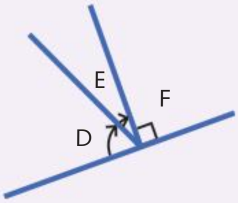
10C Individual

1 Name the complementary angles.

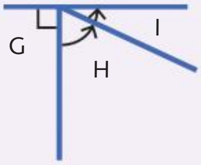
a



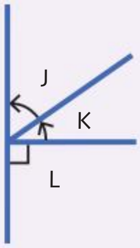
b



c

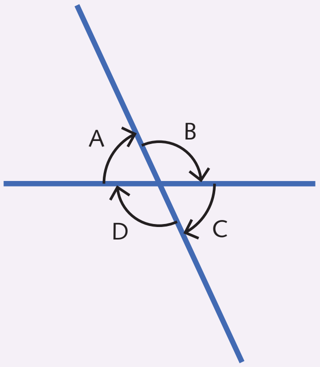


d

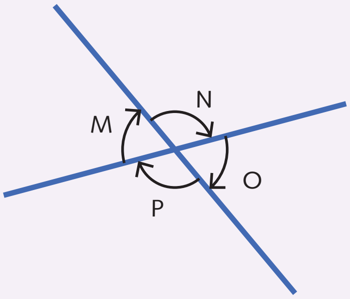


2 Name the supplementary angles.

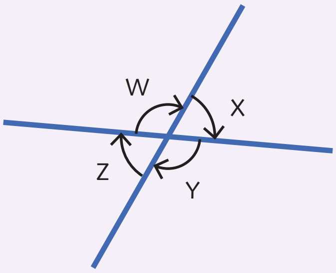
a



b

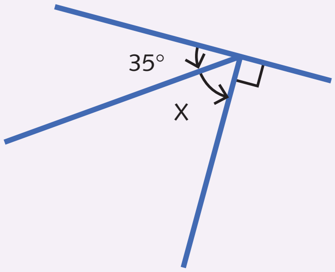


c

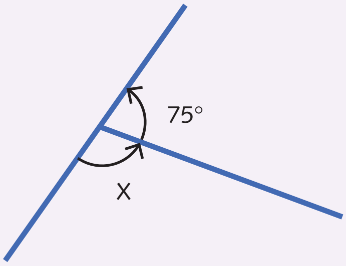


3 These angles are not drawn to scale. Find the unknown angle without using a protractor.

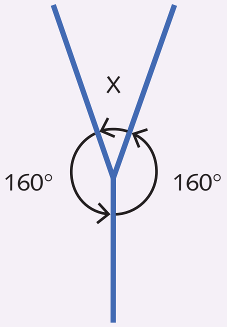
a



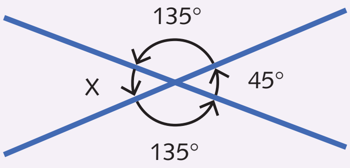
b



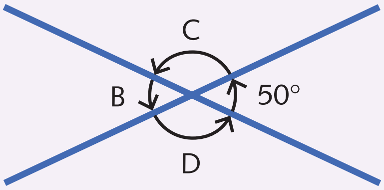
c



d



4 Without using a protractor, write the size of the angle B, then work out the size of angles C and D.



10D Review questions

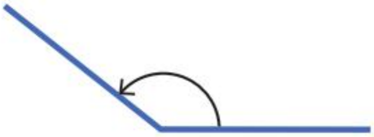
1 Draw a simple line drawing of a house. Use one or more of these words to describe the lines in your drawing.

horizontal vertical parallel perpendicular

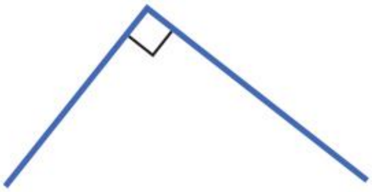
2 Choose from the following words to describe the angles **a** to **f** below.

right angle obtuse angle acute angle
straight angle reflex angle

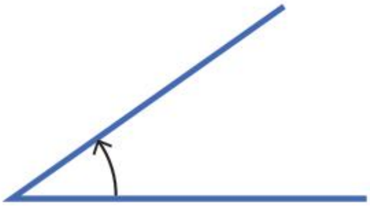
a



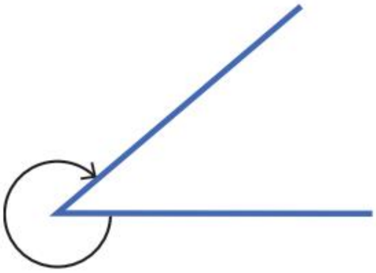
b



c



d



e

f

3 Draw two rays to make these kinds of angles. Mark the angle with a curved arrow.

a Acute angle
b Straight angle
c Right angle
d Reflex angle
e Obtuse angle

4 Use a protractor to measure these angles.

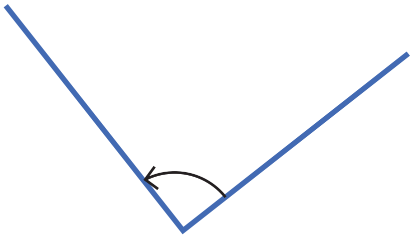
a

b

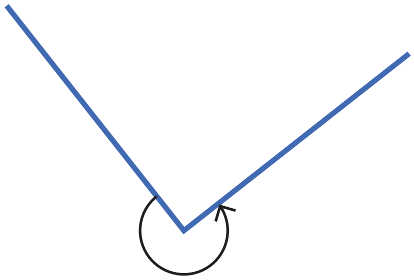
c

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

d



e

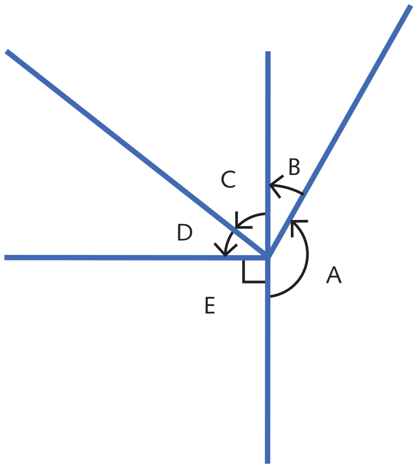


5 Use a protractor to draw these angles.

a 90°	**b** 120°	**c** 45°	**d** 285°	**e** 310°
f 20°	**g** 25°	**h** 300°	**i** 340°	**j** 205°

6 **a** Name the complementary angles.

b Name the supplementary angles.

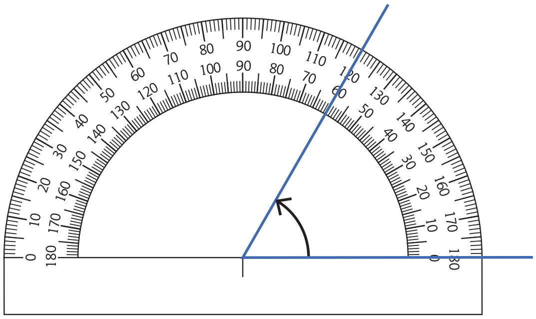


c If angle C is equal to 45°, what size is angle D?

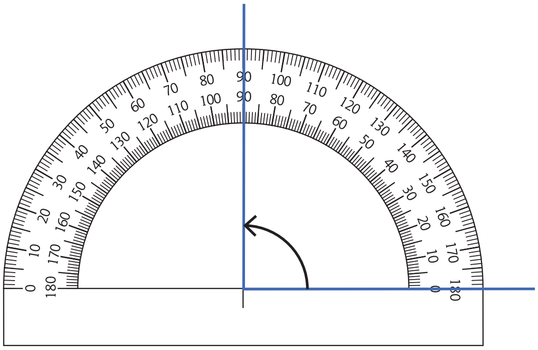
d If angle B is equal to 38°, what size is angle A?

7 For **a** to **f**, describe the kind of angle shown and write down its size.

a

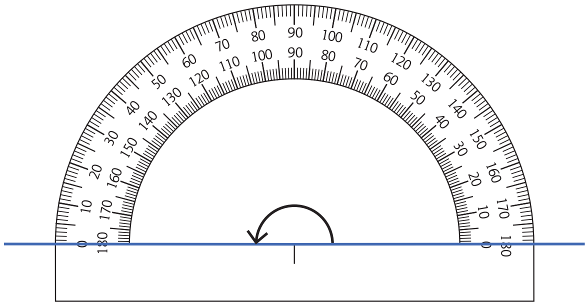


b

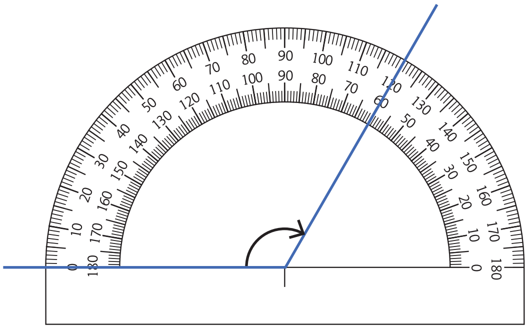


ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7

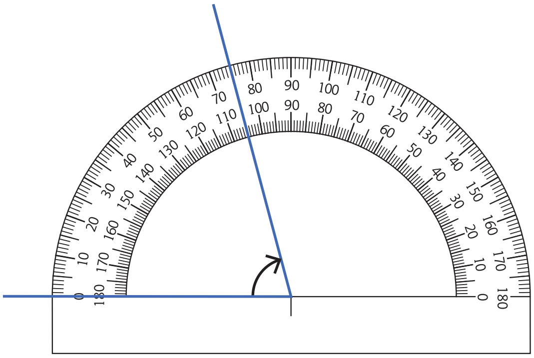
c



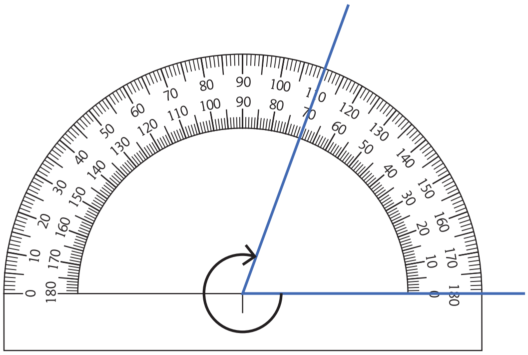
d



e



f

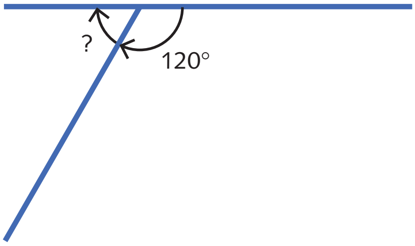


g Which is the smallest angle?

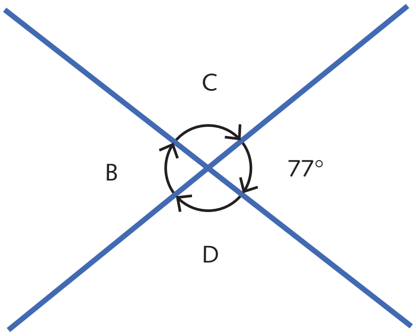
h Which is the largest angle?

i What is the difference between the largest and smallest angles?

8 Find the unknown angle *without* using a protractor.



9 Write the size of angle B, then work out the size of angle C and angle D.



ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

CHAPTER 11

Preparation

Useful skills for this chapter:

- understanding of triangles, rectangles, squares and circles
- the ability to use a protractor to measure angles and draw angles.

KICK OFF

The four prefixes below are often used to describe shapes. Find out what each prefix means, then list as many words as you can based on each prefix.

How does the meaning of the prefix connect to the meaning of the word?

a tri **b** quad **c** tetra **d** penta

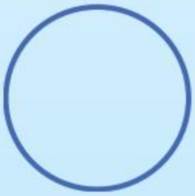
Show what you know

1 Name each shape.

a

b

c



d

e

f

g

h

i

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

CHAPTER

11

Measurement and Geometry

Two-dimensional shapes

In this chapter we look at two-dimensional shapes. We begin by looking at shapes called **polygons**. A polygon is a two-dimensional shape enclosed by three or more line segments called sides. Exactly two sides meet at each vertex and the sides do not cross.

One common place where we see polygons is on street and traffic signs.

The names for polygons vary, depending on how many sides they have, or how many angles they have, or both.

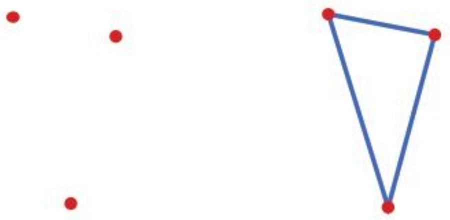
ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

11A Triangles

Think of some words that start with the prefix ‘tri’. For example, a tricycle has three wheels, and a trilogy is the name for a series of three books.

The prefix ‘tri’ means ‘three’. So a triangle has three angles. It also has three straight sides.

We can draw a triangle by using line segments to join three dots that are not in the same line.

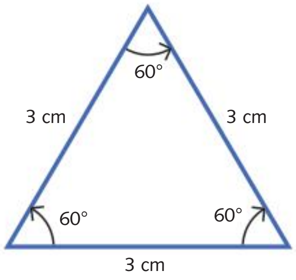


Equilateral triangles

Some triangles have all of their sides the same length. We call these equilateral triangles.

‘Equilateral’ is from the Latin words *equi* and *latus*, meaning ‘equal sides’.

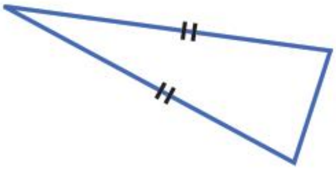
An equilateral triangle has equal angles. Equilateral triangles are also called ‘regular triangles’.



Isosceles triangles

‘Isosceles’ comes from the Greek words *isos* and *skelos*, which mean ‘equal legs’.

So an isosceles triangle has two equal ‘legs’ or sides. Every equilateral triangle is also isosceles. The two small marks on the sides indicate that they are the same length.



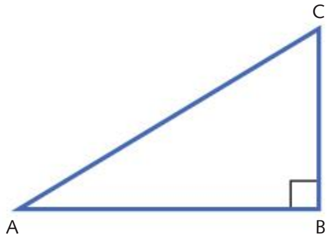
Scalene triangles

A scalene triangle is one in which the sides have different lengths. The word *scalene* comes from Latin and means ‘mixed up’.

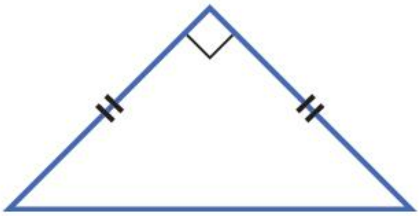
Right-angled triangles

As its name suggests, a right-angled triangle has a right angle as one of its angles.

The little square at vertex B means that the angle is 90°.



A right-angled triangle can be isosceles.

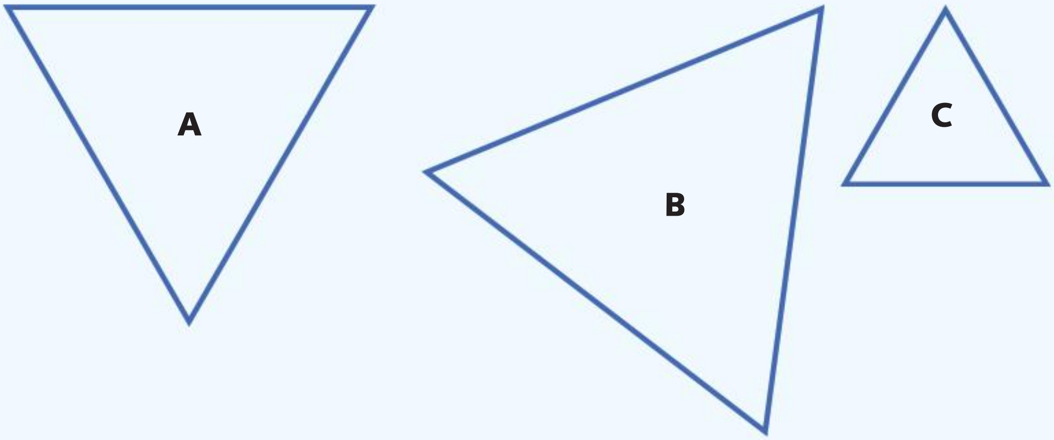


A triangle cannot be both equilateral and right-angled.

11A Whole class CONNECT, APPLY AND BUILD

Equilateral triangles

○ **1** **a** Use a protractor to measure the three angles of each equilateral triangle as accurately as you can. Write down the measurements.

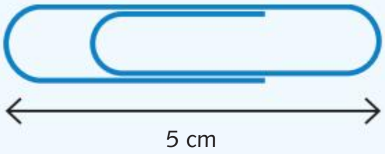


b What do you notice about the size of each angle?

c How do you your answers compare with a classmate's answers.

d What do the three angles in each triangle add up to?

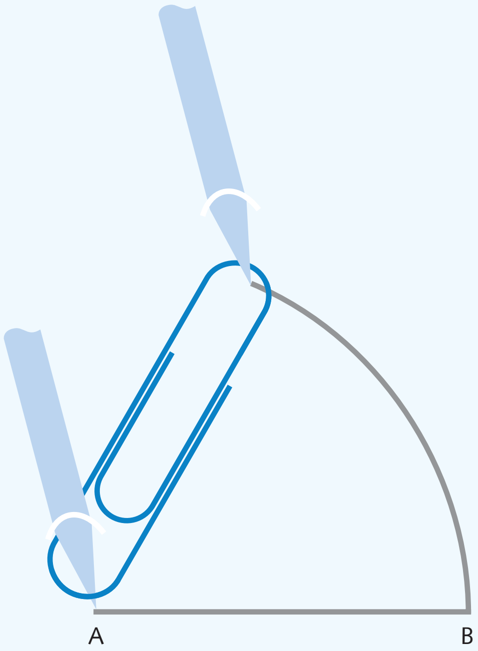
● **2** You can use a 5 cm paper clip to draw an equilateral triangle.



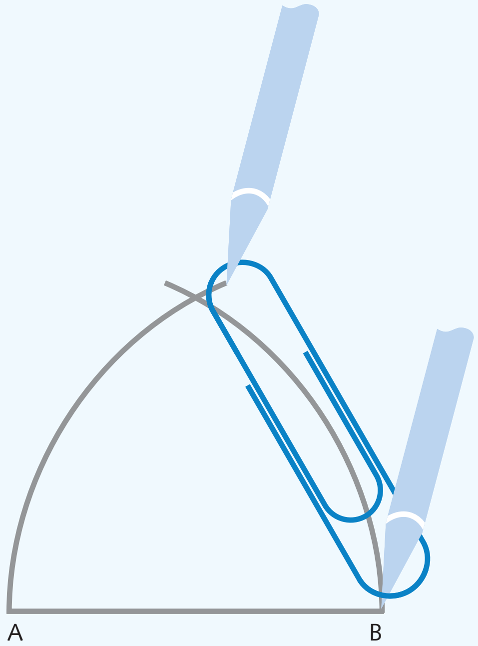
a Use a ruler to draw a line that is 5 cm in length. Label one end A. Label the other end B.

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

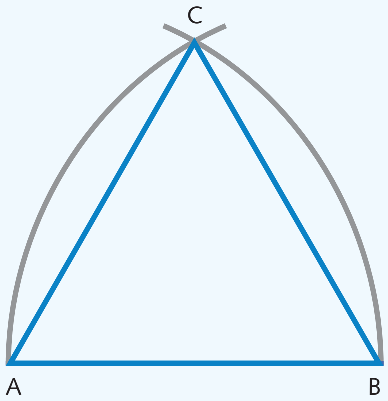
b Now put one pencil through the paper clip and place the point on A. Put the other pencil through the paper clip at point B.
Hold the pencil still at point A and move the pencil anti-clockwise from point B to draw an arc. Keep both pencils vertical while you do this.



c Now hold the pencil still at point B.
Move the pencil clockwise from A until you cross your first arc.



d Find the point where the two arcs cross. Draw a dot there and label it C.
Use a ruler and a pencil to join up points A, B and C.



ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

e Why is your triangle an equilateral triangle?

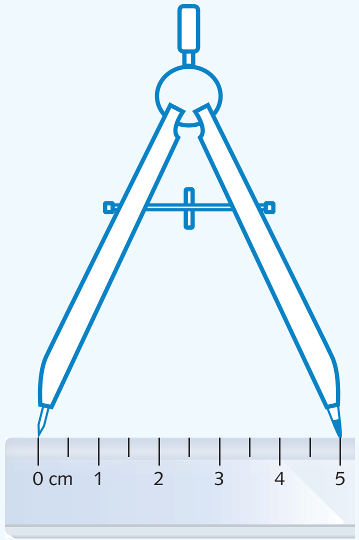
f Measure the lengths of lines AB, AC and BC. How accurate were you in using the paper clip and pencil?

g Measure angles A, B and C (inside the triangle). What did you find?

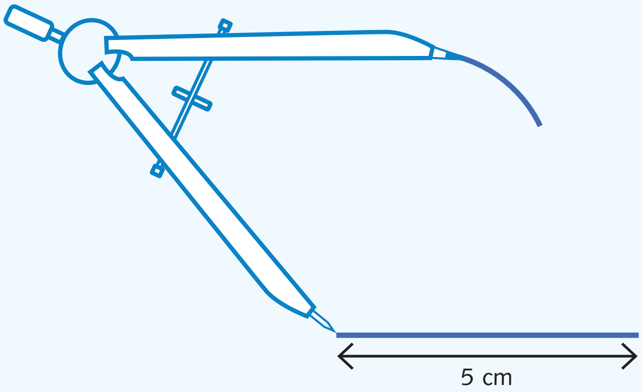
Try this exercise a few times. See how close you can get to making an accurate equilateral triangle.

3 You can also use a pair of compasses to draw an equilateral triangle.

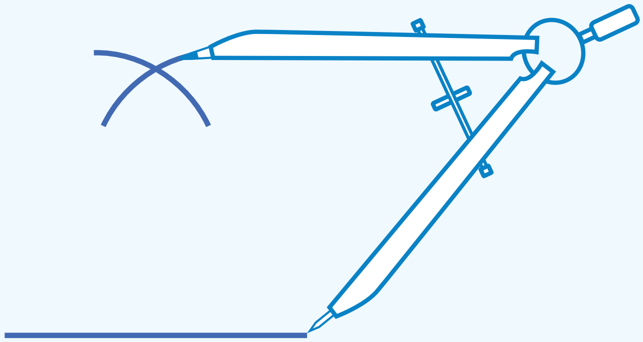
a Place the compasses against a ruler. Move the point and the pencil 5 cm apart.



b Draw a line 5 cm in length. Put the point of the compasses on one end of the line and use the pencil to draw an arc.



c Now place the point of the compasses on the other end of the 5 cm line and draw another arc, as shown.



d Use a ruler and pencil to join the ends of the 5 cm line to the point where the two arcs cross. What have you made?

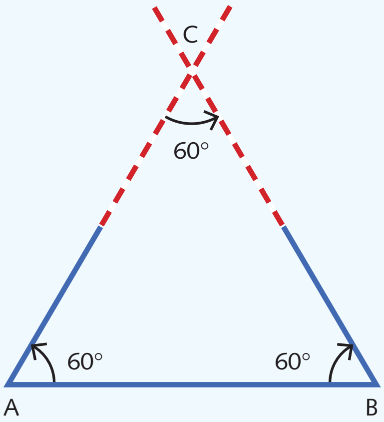
e Use a ruler to measure the length of each side. Use a protractor to measure the angles to make sure you have an equilateral triangle.

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

4 You can create equilateral triangles by drawing 60° angles.

a Draw a line of length 6 cm. Label the ends A and B. Now use your protractor to mark 60° angles at A and B.

b Use your ruler to extend the upper arms in a straight line. Label the point where they meet C.



c Measure lines AB, AC and BC to see if you have made an equilateral triangle.

d Measure the angle of the triangle at C. How close to 60° did you get?

Isosceles triangles

5 Use a ruler and a pencil to draw an isosceles triangle with two sides of length 4 cm.

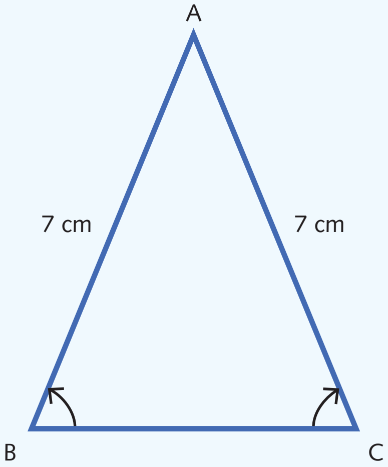
6 **a** Draw an isosceles triangle with two sides of length 6.5 cm.

b How many different isosceles triangles can you draw that have two sides of length 6.5 cm?

c Discuss how the isosceles triangles you have drawn are 'different' from each other.

7 **a** Draw an isosceles triangle with two sides of length 7 cm. Use letters to name the vertices, like this.

b Measure angle B and angle C. We call these 'the angles opposite the two equal sides'. What do you notice about these angles? Compare your angle measurements with a friend's angle measurements.



c Draw an isosceles triangle with two sides of length 7 cm and with the angles opposite those sides very small.

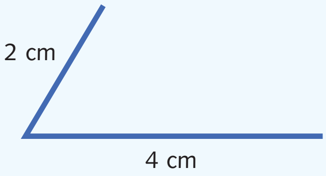
d Now draw an isosceles triangle with two sides of length 7 cm and the two angles as big as you can make them.

e Is there anything to stop you making both angles in an isosceles triangle 100°? Is there a limit?

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

Scalene triangles

8 **a** Draw two lines meeting at a vertex. Make one line 2 cm and the other 4 cm, as shown below.



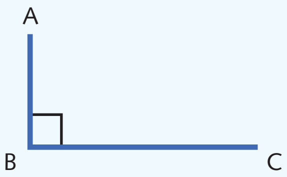
Do this three times, changing the angle between the lines each time.

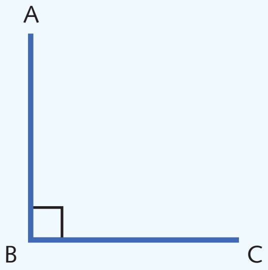
b Now answer this question: Akram has one line of length 2 cm and one line of length 4 cm, which he calls 'arms'. He joined the ends of the arms with a third line to form a triangle. If Akram increases the angle between the arms, will the length of the third side become larger or smaller?

c Use your answer to part **b** to draw four different scalene triangles. (Or see if you can think of another way to make scalene triangles.)

Right-angled triangles

9 **a** Use a ruler and a protractor to draw three right angles with different arm lengths, as shown. The little square at vertex B shows that the angle is 90°.





b Now use a ruler and pencil and join points A and C. You have just made three right-angled triangles.

10 Without using a protractor, draw four right-angled triangles in your workbook. (Hint: Look for right angles already there on the lined page.)

11 **a** Use your ruler and a protractor to draw an isosceles right-angled triangle. Mark the right angle.

b Measure the other two angles of your triangle, then compare your answers with a classmate's.

c What do you think the other two angles should have been, and why?

Angles in a triangle

12 **a** Draw four different triangles. Make sure you include at least one right-angled triangle and at least one triangle with an obtuse angle. Label the vertices of each triangle A, B and C.

b Copy this table.

	Angle A	Angle B	Angle C	Total
Triangle 1				
Triangle 2				
Triangle 3				
Triangle 4				

Carefully measure the inside angle at A, B and C of each of your triangles from part **a** on the previous page. Write the angles in the table, then find the sum of the three angles in each triangle.

c Now fill in the blank in this statement.

If we add up the angles at the vertices of a triangle, the sum is ____.

13 Can a triangle have two right angles? Explain your answer.

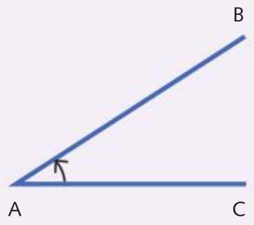
14 Can a triangle have a reflex angle? Explain your answer.

11A Individual

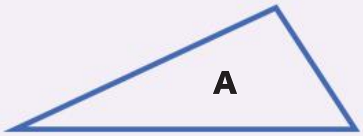
1 List as many words as you can that begin with the prefix 'tri'. Find out the meaning of each word.

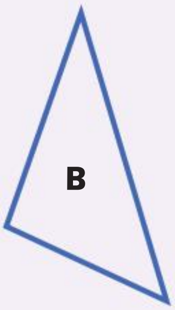
Acute-angled triangles

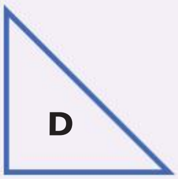
2 An acute angle is an angle that is less than 90°.
This is an acute angle.



Use a protractor to measure the angle to check that it is less than 90°.
An acute-angled triangle is a triangle that has all acute angles. Which of these are acute-angled triangles?

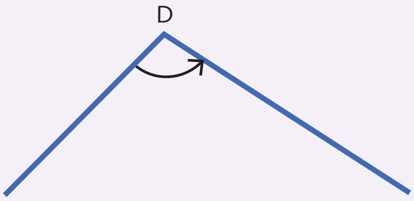






Obtuse-angled triangles

3 **a** An obtuse angle is an angle that is larger than 90°. Angle D below is an obtuse angle.

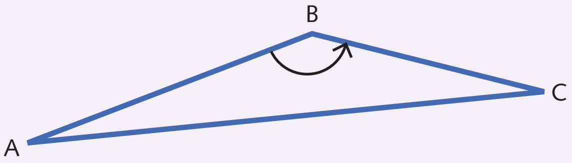


Use a ruler and protractor to draw the angles:

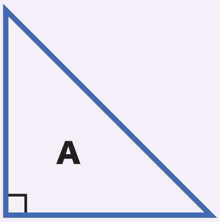
80°, 110°, 150°, 179°, 88°.

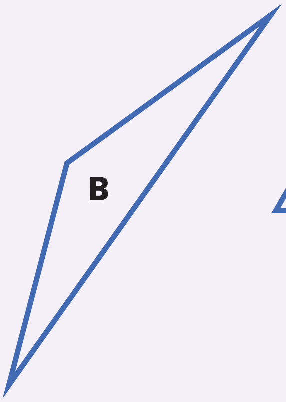
Which angles are obtuse? Label the obtuse angles.

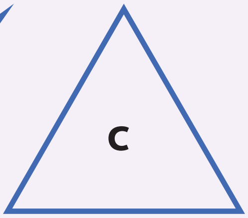
b An obtuse-angled triangle has one angle larger than 90°. In this obtuse-angled triangle, angle B is greater than 90°.

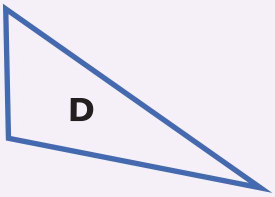


Which of these triangles are obtuse-angled triangles?









4 **a** Can a triangle be acute-angled and isosceles? If you think the answer is 'yes', draw one.

b Can a triangle be obtuse-angled and isosceles? If you think the answer is 'yes', draw one.

c Can a triangle be obtuse-angled *and* equilateral? Explain your answer.

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

11B Quadrilaterals

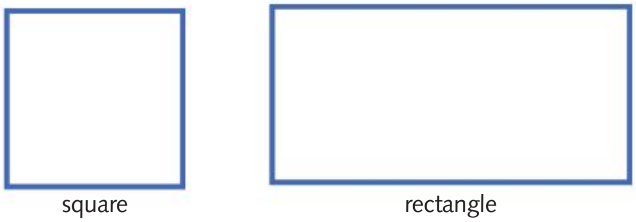
What is a **quadrilateral**? You probably already know some words that start with the prefix ‘quad’. For example:

> Quadruplets are four children born to the same mother at the same time.
>
> An all-terrain vehicle, sometimes called a quad bike, is a motor bike with four wheels.

Can you think of some other words that start with ‘quad’?

The word ‘lateral’ is from the Latin word *latus*, which means ‘side’. So a shape called a **quadrilateral** has four sides. The sides must not cross over. It also has four corners or **vertices**.

Both a square and a rectangle have four vertices and four sides.



Mark four points (or dots) on your page, as shown. Don't spread your points out too far.

Use a pencil. Start at one point and draw a line to a second point. Continue the line to the third point, then to the point you started from. Here are two possible outcomes.

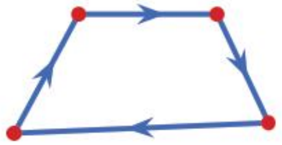


Figure 1

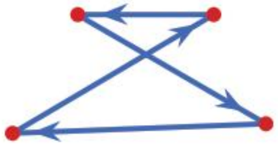


Figure 2

Shapes such as figure 2 are *not* quadrilaterals because their sides cross over. From now we will look only at shapes that are quadrilaterals.

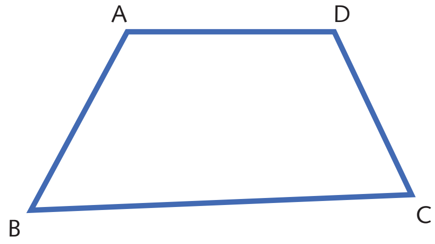
Like all polygons, quadrilaterals have several important features. No side may contain more than two vertices, or cross another side. There must not be any gaps in the sides of a shape.

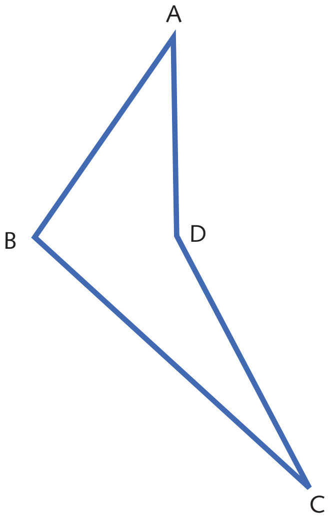
There are many different types of quadrilaterals, and some of them have special names.

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

Convex and non-convex quadrilaterals

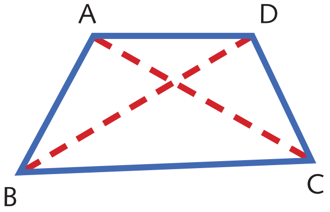
Here are two quadrilaterals.





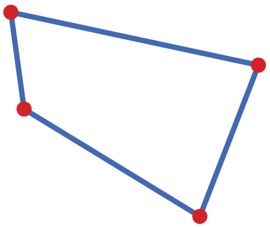
The first quadrilateral bulges outwards. This is called a **convex** shape. The second quadrilateral is pushed in on one side. It is called **non-convex**.

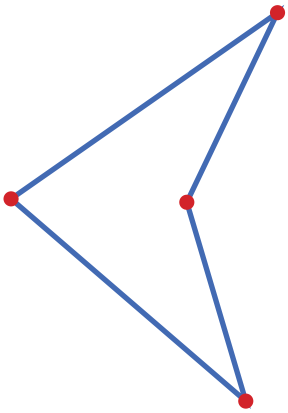
We can show the **diagonals** of a quadrilateral by drawing the lines that join any two vertices that are *not* next to each other.



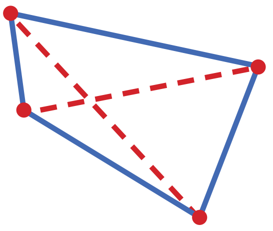
The dotted lines AC and BD are the diagonals of the quadrilateral ABCD.

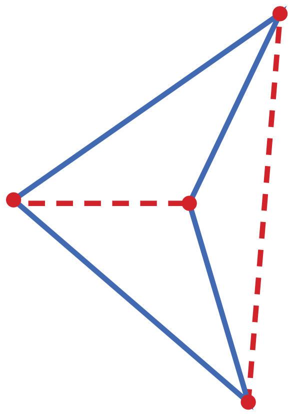
Start with four points with no three of them in the same line.
We can make a quadrilateral that uses those points as vertices.
Here are two different ways of doing this.





We can then draw the diagonals of each quadrilateral.





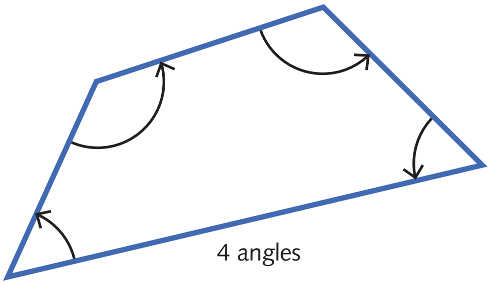
Look closely at the diagonals for each shape. What do you notice? What can you call a quadrilateral that has its diagonals *inside* the shape? A quadrilateral is **convex** if both diagonals are inside the shape.

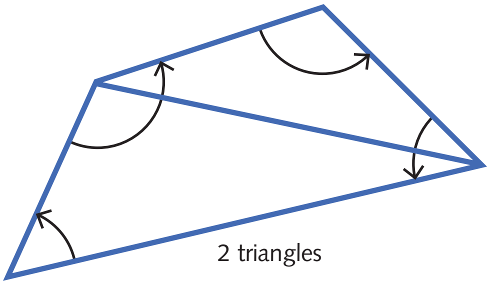
What can you call a quadrilateral that has one diagonal *outside* the shape? A quadrilateral is **non-convex** if one of its diagonals is outside the shape.

Angles in a quadrilateral

A quadrilateral has four angles. What do they add up to?

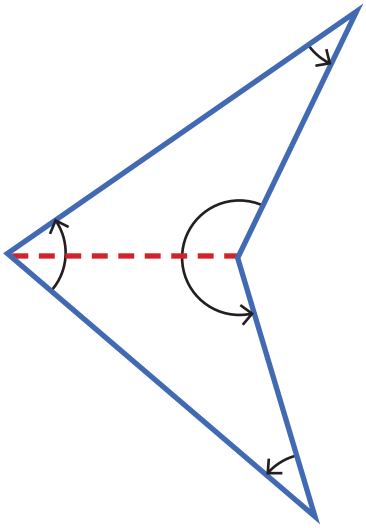
You can find the answer to this by drawing a quadrilateral and separating it into two triangles. Remember: the angles of a triangle always sum to 180°.





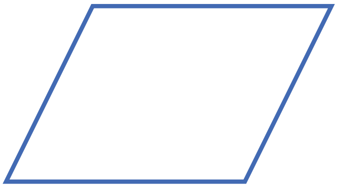
Even if you cut a non-convex quadrilateral into two triangles, the result is the same.

Given that the three angles of a triangle always sum to 180°, then the sum of the four angles in a quadrilateral must always be 360°.



ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

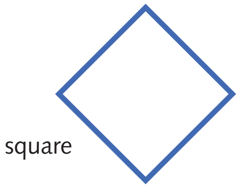
Parallelogram

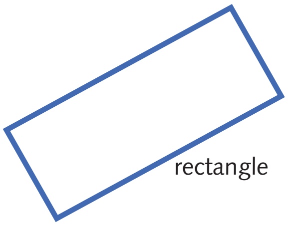


A parallelogram is a quadrilateral with opposite sides parallel.

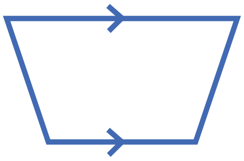
It looks like a 'pushed over' rectangle.

Rectangles and squares are special kinds of parallelograms. They have four right angles as well as opposite sides parallel.



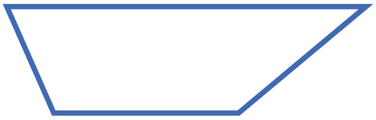


Trapezium



A trapezium has two sides that are parallel. You might have seen a table at school with this shape. It is a trapezium.

The non-parallel sides in a trapezium can be different lengths.

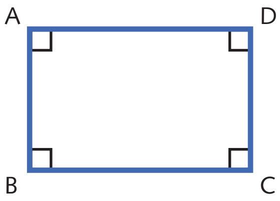


Rectangle

A rectangle is a quadrilateral in which all the angles are right angles.

The opposite sides of a rectangle have the same length. These sides are also parallel to each other.

Properties of a rectangle



1 All angles are right angles.

2 Opposite sides are parallel:

AB is parallel to CD.

BC is parallel to AD.

3 Opposite sides have the same length:

$AB = CD$

$BC = AD$

Square

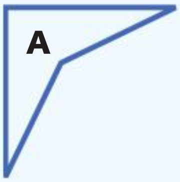
A square is a quadrilateral with all its angles equal and all its sides the same length. So a square is a special type of rectangle, with all sides the same length.

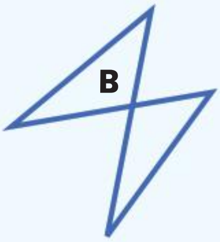
ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

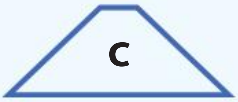
11B Whole class CONNECT, APPLY AND BUILD

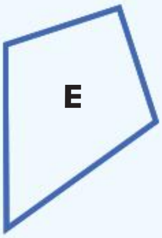
1 **a** Which of these shapes are quadrilaterals?

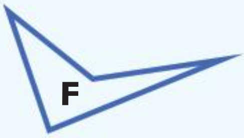
b Which of the quadrilaterals are convex?

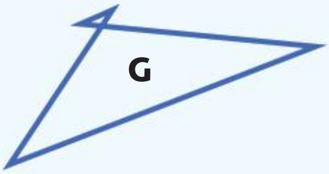












2 **Who am I?**

I am a regular quadrilateral. That means my sides are the same length and my angles are equal.

I am a shape you already know.

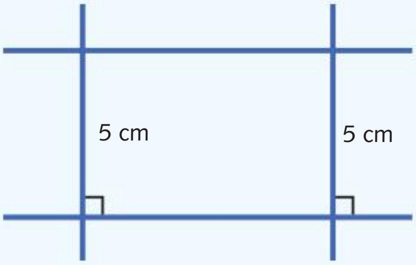
I have equal angles and equal length sides.

All my angles are 90°.

Who am I?

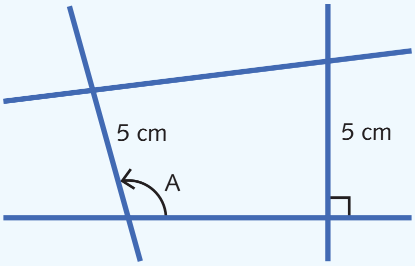
3 **Drawing a rectangle**

a Use your ruler to draw a line. Mark two points on the line and use your protractor to construct two right angles.



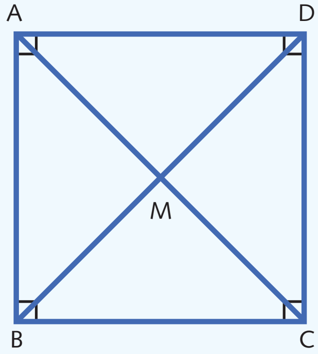
b Mark off two lengths of 5 cm, as shown. Use a ruler to join the two new points. What shape have you made?

c Helen made an error with her protractor when she was measuring a right angle at A. Discuss the shape Helen made. Why isn't it a rectangle?



4 **a** What happens if you rotate a square 90°? Select two square shapes that are the same size but different colours. Place one shape on top of the other to cover it completely. Rotate the top square 90° and see if it fits the shape underneath exactly. Now rotate the top shape another 90°. Describe what happened.

b What happens if you draw the diagonals? Think about what happened when you rotated the square 90°. The square looks the same and the diagonals look the same.



c Copy and complete this statement.

The lengths AM, MC, BM and MD are all ________.

d Copy and complete this statement.

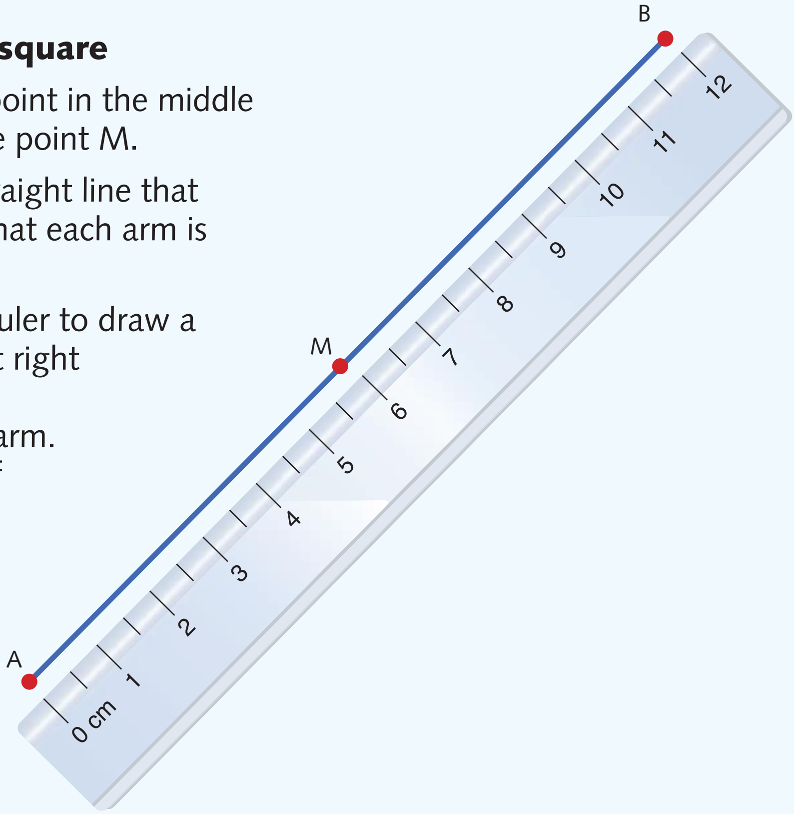
The four angles at M are all ________ angles.

5 **A smart way to draw a square**

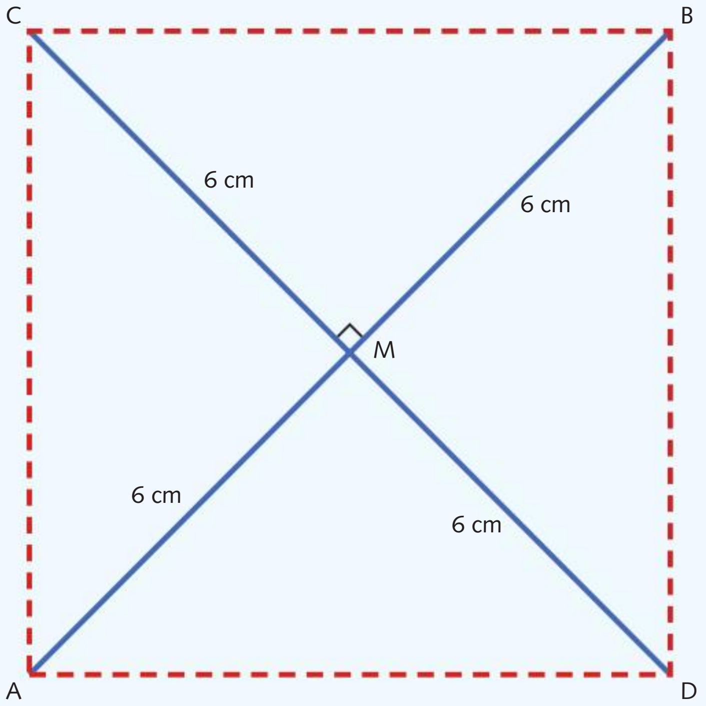
a Use a pencil to mark a point in the middle of a blank page. Call the point M.

b Use a ruler to draw a straight line that has its centre at M, so that each arm is 6 cm long.

c Use a protractor and a ruler to draw a line through M that is at right angles to your first line. Mark off 6 cm on each arm. Now join up the ends of the arms as shown on following page.



ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press



d Have you created a square? Explain your answer.

11B Individual

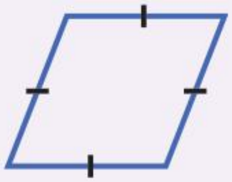
1 Write the labels that match each shape. Some shapes have more than one label, and labels can be used more than once.

rectangle, square, has four sides of equal length, quadrilateral, not a quadrilateral, convex, has four right angles, non-convex

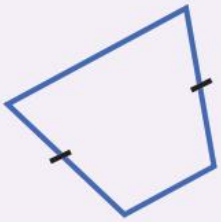
a

b

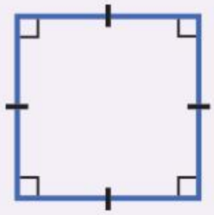
c



d



e

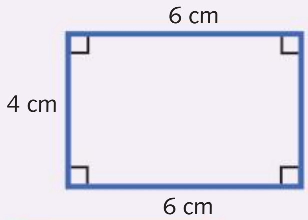


f

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

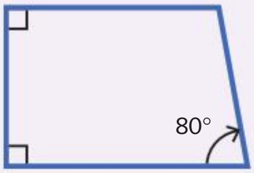
2 Which of these shapes are rectangles?

A



B

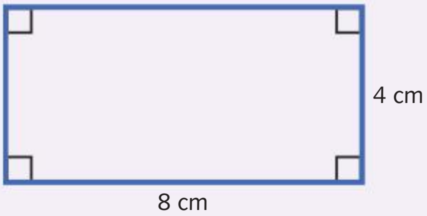
C



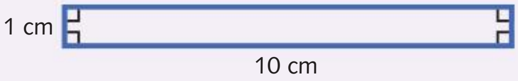
D

3 What are the lengths of the other sides of these rectangles?

a

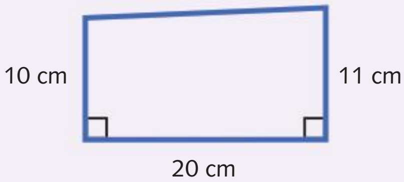


b



4 Which shape is *not* a rectangle? Why?

A



B

5 Use a ruler, a pencil and a protractor to construct a rectangle with side lengths 4 cm and 7 cm. Check that all angles are right angles, and that opposite sides are the same length.

6 Use a ruler, pencil and a protractor to construct a square with side lengths 6 cm.

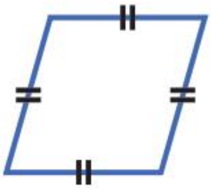
11C The rhombus

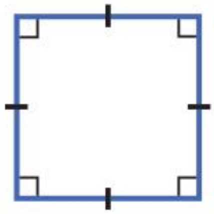
A good way to think of a rhombus is to imagine a square pushed sideways.

A rhombus is a parallelogram with four equal sides.

A square is a special kind of rhombus – that is, a rhombus with four right angles.

Although a square is also a rhombus, a rhombus need not be a square.

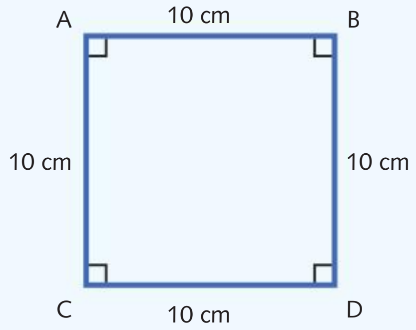


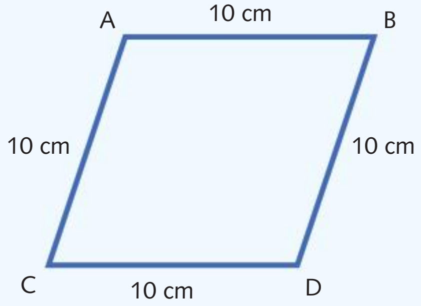


11C Whole class CONNECT, APPLY AND BUILD

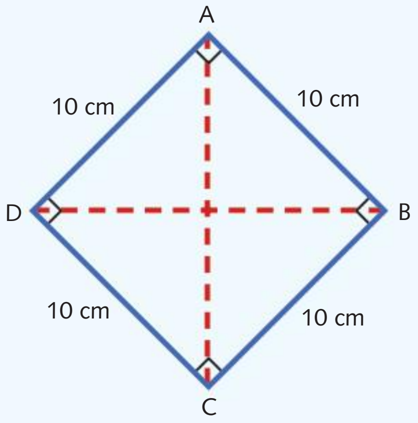
For these activities, it will be useful to have a square made from cardboard strips with the joints held by split pins or paper fasteners. If you do not have a moveable square, imagine that the corners of a square have screws in them. The screws are not tight.

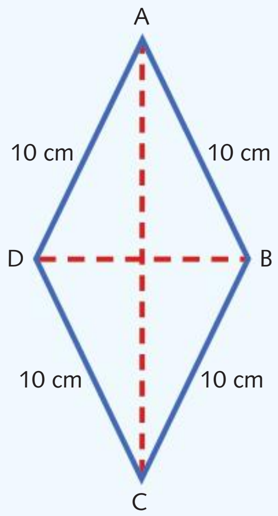
1 Discuss what happens if the square is pushed sideways. What happened to the angles? Which angles got bigger? Which angles got smaller?





2 Squeeze two opposite corners of a square in towards each other so that it looks like a diamond.





What happened to the diagonals? Which diagonal got longer? Which diagonal got shorter?

3 Make or draw a rhombus and measure the angles around the point where its diagonals cross. What do you notice about the diagonals?

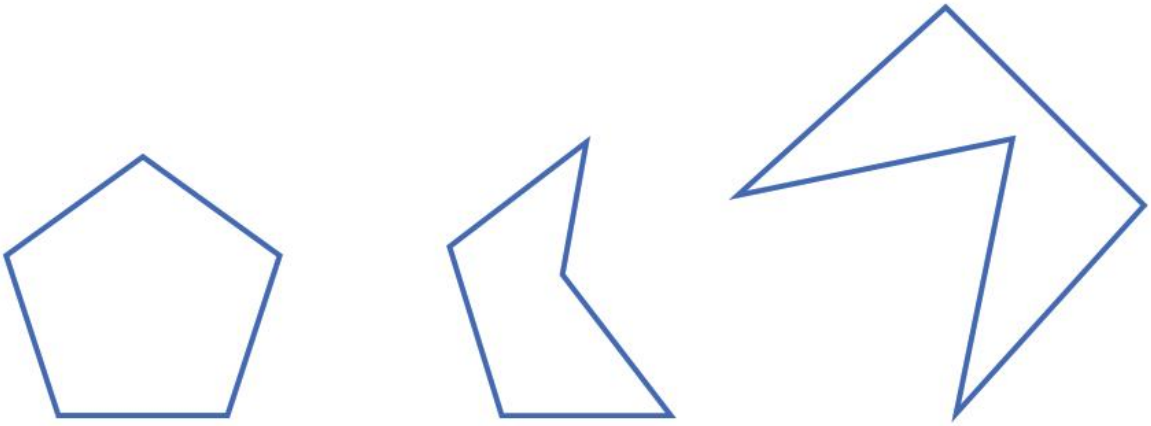
4 What happens if you flip the rhombus over its vertical diagonal? What happens if you flip it over its horizontal diagonal?

5 What happens if you rotate a rhombus 90°? What if you rotate it 180°? When does it remain the same?

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

11D Pentagons

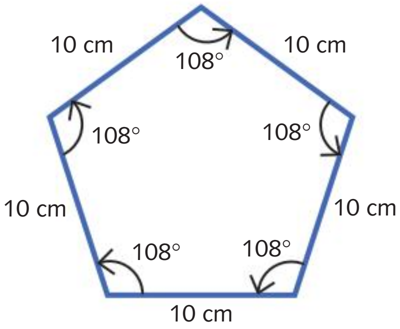
The word pentagon comes from the Greek words *penta* meaning ‘five’ and *gon* meaning ‘angle’. So a pentagon has five angles, five vertices and five sides. Its sides must not intersect except where they meet at the vertices.



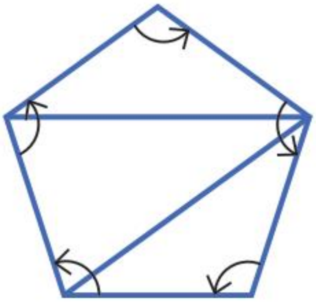
All of the shapes above are pentagons. The first is a regular pentagon. It is convex. The second one is an irregular non-convex pentagon. The third is not regular because its angles are not all equal, even though the sides are all the same length.

Regular pentagons

A regular pentagon has five equal sides and five equal angles. The angles in a pentagon add up to 540°, so in a regular pentagon, each angle is 108°.



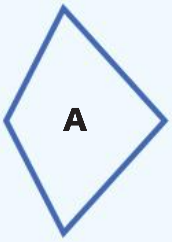
To see that the sum of the angles is 540°, rule lines from one vertex to each of the others to get three triangles. We already know that the sum of angles in a triangle is 180°, so if there are three triangles in a pentagon, then the sum is three lots of 180°, which is 540°.

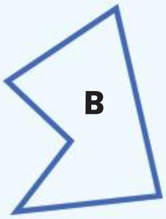


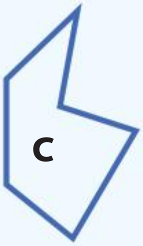
ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

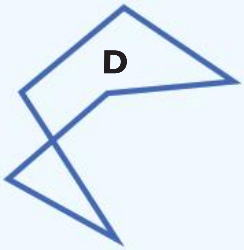
11D Whole class CONNECT, APPLY AND BUILD

1 Which of these shapes are pentagons? Why?



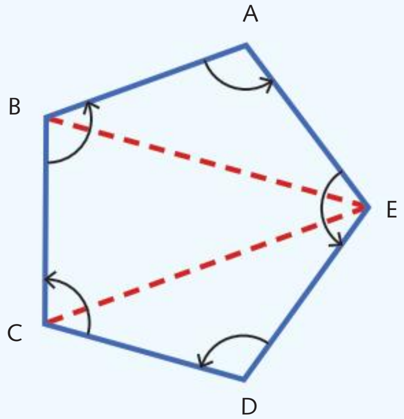


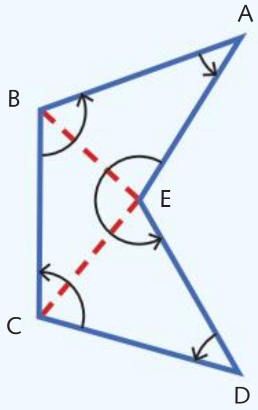




2 **Investigation: Angles in a pentagon**

Copy these two pentagons and draw the dashed diagonals. How many triangles are there in each pentagon? Now work out the sum of the angles in each pentagon.



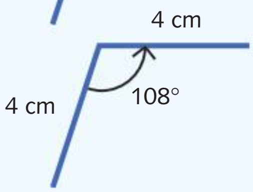


3 Draw two pentagons of your own. Divide each pentagon into triangles and work out the sum of its angles. (Don't measure the angles; use the number of triangles.)

4 **Make your own regular pentagon**

a Start near the middle of a blank page. Draw a line of length 4 cm on an angle, like this.

b Use your protractor to mark an angle of 108°. Now draw the second 4 cm-long arm.

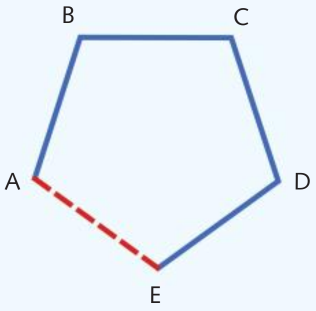


c Continue these steps until you have drawn four sides of length 4 cm and four angles of 108°. Label the vertices, as shown.

d Now draw side AE to complete your pentagon.

e Measure the length of side AE. Is it close to 4 cm?

f Measure the angles at A and E. Are they close to 108°?



ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

11D Individual

1 A clever way to draw a pentagon

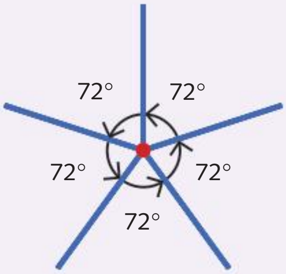
Have you ever seen a regular pentagon that has five triangles shaded or coloured differently, like this?

Spinners used in games are often pentagons shaded in 5 segments.

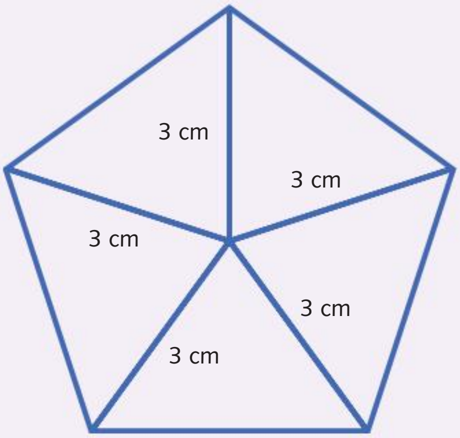
The 5 triangles all look the same.

We have seen that the sum of the angles about a point is 360°. In the pentagon, we have 5 equal angles meeting at the centre to make a complete revolution of 360°. This gives $\frac{360°}{5} = 72°$ for each one, so adjacent arms are 72° apart.

a Take your ruler and protractor and draw five angles of 72° at a point in the middle of your page, like this.



b Now take your ruler and mark points on the arms 3 cm from the centre and join them up. You have made a regular pentagon.

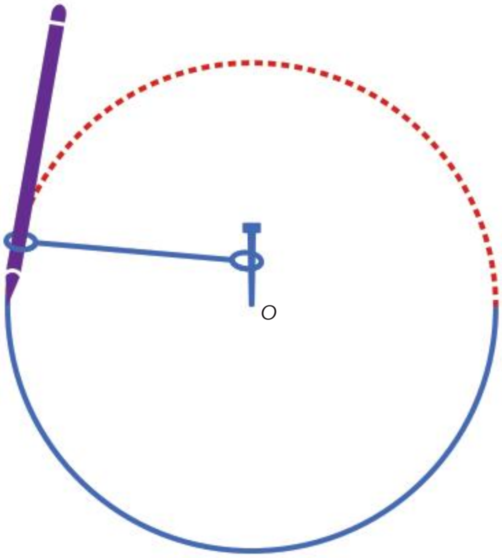


2 Draw a pentagon with equal angles but unequal sides

See if you can draw a pentagon that has all of its angles 108°, but has sides of different lengths. (Hint: Start with a line segment, make a turn of 108° and then draw another line segment.)

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

11E Circles

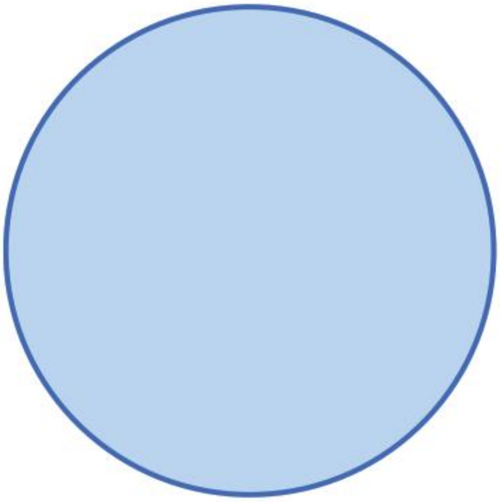


Imagine a flat piece of wood with a nail hammered halfway into it. The nail has a piece of string tied to it. Now imagine tying a pencil to the other end of the string, pulling the string tight and using the pencil to draw a path all the way around the nail.

The path that you have drawn is called a **circle**. The nail marks the **centre** of the circle and every point on the circle is always the same distance from the centre, *O*.

When the circle has been completed and the nail removed, the nail hole shows the centre of the circle. A line drawn from the centre to any point on the circle is called a **radius** of the circle. Each radius is the same length. The plural of radius is **radii**.

The word 'circle' is also used to refer to the two-dimensional shape created by the path you have drawn.



Using a pair of compasses to draw a circle

You have probably already used compasses to draw a circle. When you use a compass to draw a circle, the distance between the point of the compass and the point of the pencil is the radius of the circle.

Always hold the compass at the top with one hand and carefully make a full circle. Sometimes it is easier to make two half-circles. Try not to use two hands to control the compass, as it usually makes the arms move further apart as you draw, and you won't get a circle.

The correct name for the tool we use to draw circles is 'a pair of compasses' but you will also see 'compasses' or 'compass' used.

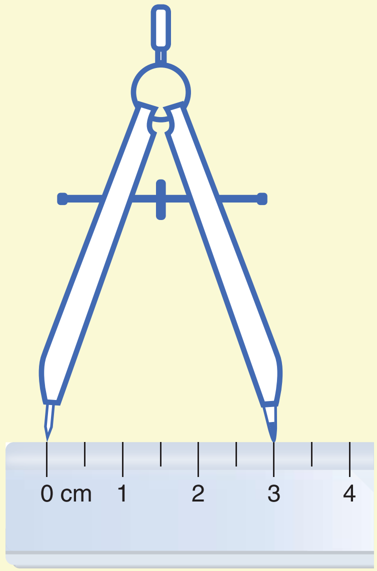
ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

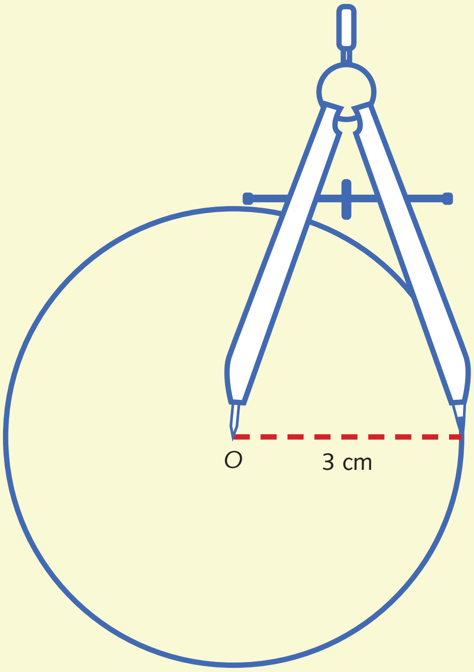
Example 1

Use compasses to draw a circle with a radius of 3 cm.

Solution

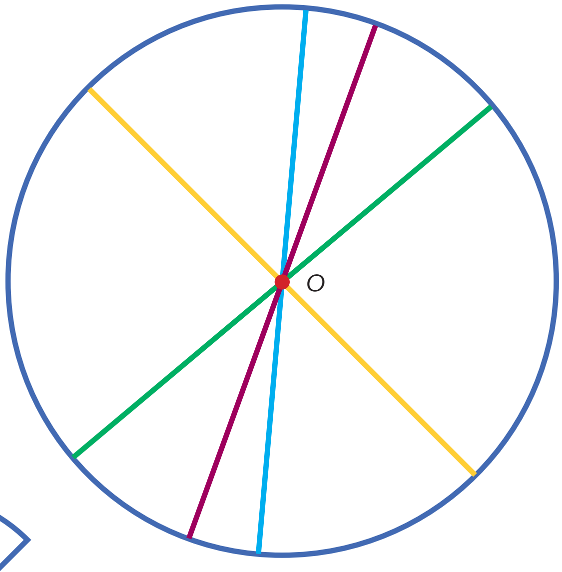
If you want a circle with a radius of 3 cm, use a ruler to make sure that your compasses are set to 3 cm, as shown below.

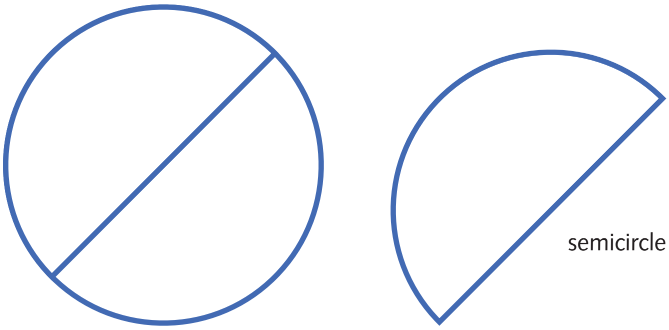




A straight line drawn from any point on the circle through the centre to the opposite point on the circle is called a **diameter**. A diameter is the largest possible chord of a circle. The four coloured lines you can see on the right are all diameters of the circle.

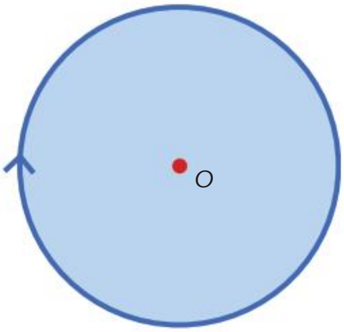
When we draw a diameter, we get two half-circles. Each half-circle is called a **semicircle**. 'Semi' means 'half'.





ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

Sometimes we want to know about the distance around a circle. The distance around a circle is called its **circumference**.



The word 'circumference' comes from the Latin words *circum*, which means 'around', and *ferre*, which means to move or carry. So circumference means to move around.

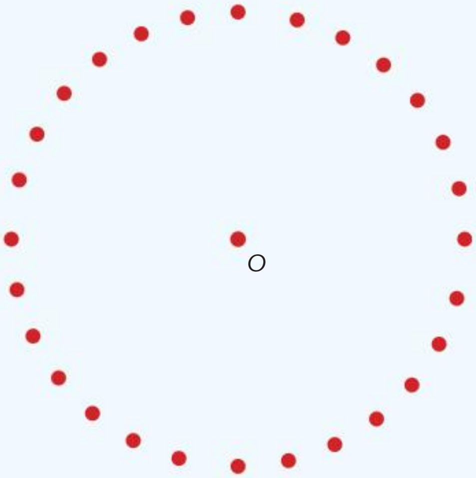
11E Whole class CONNECT, APPLY AND BUILD

1 Using a ruler to draw a circle

Every point on a circle is the same distance from the centre. You can use this idea to use your ruler to draw a rough circle. Mark a point in the middle of your page, and use it for the centre of your circle. Place your ruler with the 0 on the centre point and mark a dot at the 3 cm point.

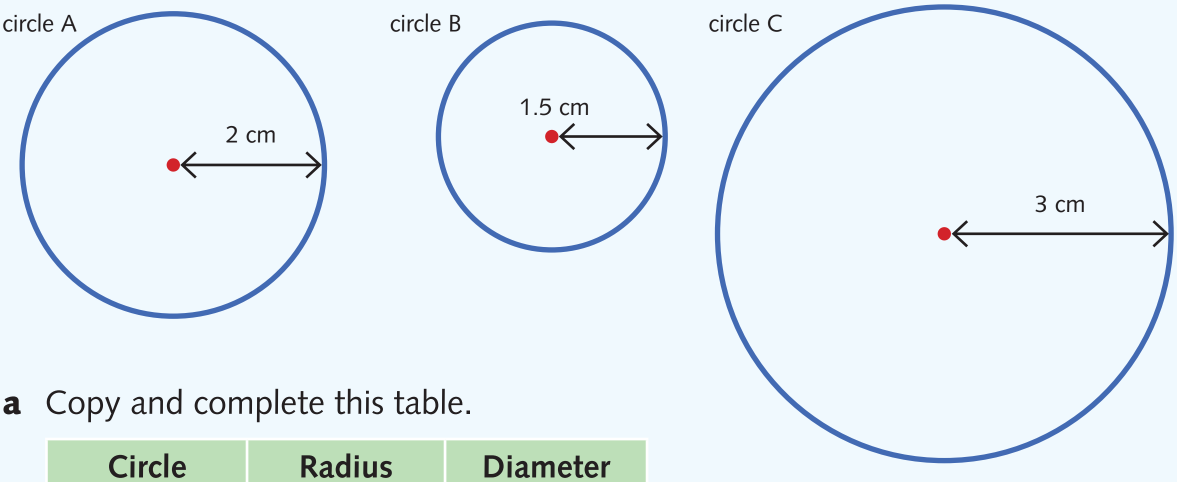
Rotate the ruler slightly, making sure to keep the 0 mark on the ruler at the centre of the circle. Mark another dot. Do this at least 10 times. Try to move your ruler through a complete rotation, like on the right.

Now draw a smooth curve through the points you have marked. This gives an approximate drawing of a circle.



ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

2 Use your ruler to find the diameter of circles A, B and C.



a Copy and complete this table.

Circle	Radius	Diameter
A	2 cm	
B	1.5 cm	
C	3 cm	

b What is the relationship between the length of the radius and the length of the diameter? Copy and complete these statements.

The radius is _______ the diameter.

The diameter is _______ times the radius.

3 **a** Use compasses to draw a circle.

b Work out how to cut the circle into 8 equal sectors. (Hint: Divide 360 by 8 to get the angle at the centre for each sector.) What size will the angle at the centre be?

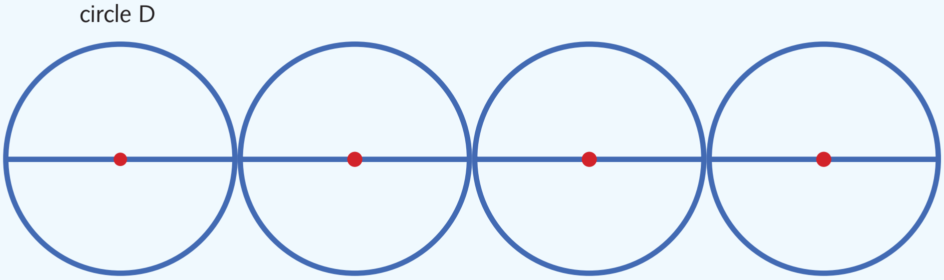
c Draw a radius.

d Use a protractor to measure the angle you calculated in part **b**, then draw another radius.

e Keep going around the circle until you have 8 sectors.

4 Is there a relationship between the length of the diameter and the length of the circumference of a circle?

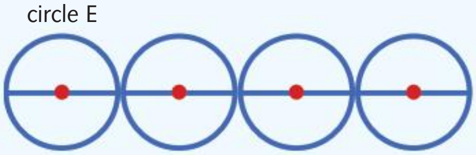
a Take a piece of string and lay it as carefully as you can around the circumference of circle D below. Mark the length of the circumference of circle D with a marker.



ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7

b Straighten out the string and fix one end to the blue line running through the centres of the four circle Ds. Compare the lengths.

c Follow the same steps for circle E.



d Then copy and complete this statement.

The circumference of a circle is slightly longer than _______ of its diameters.

11E Individual

1 Draw a circle with a radius of 5 cm.

2 Draw a circle with a diameter of 5 cm.

3 Kim mixed up these labels for her circles.

Copy the table below and write the measurements in the correct places.

Radius = 5 cm

Diameter = 6 cm

Circumference = 125.6 cm

Radius = 3 cm

Diameter = 40 cm

Circumference = 31.4 cm

Radius = 20 cm

Diameter = 10 cm

Circumference = 25.1 cm

Radius = 4 cm

Diameter = 8 cm

Circumference = 18.8 cm

	Radius	Diameter	Circumference
a	3 cm		
b	4 cm		
c	5 cm		
d	20 cm		

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

11F Review questions

1 Use your ruler to draw an isosceles triangle that has longest sides of length 6 cm.

2 Use compasses or a loop of string to draw two small equilateral triangles. Make each triangle a different size.

3 Use a protractor to draw an equilateral triangle with angles of 60°. You can have side lengths of up to 8 cm.

4 Peter has 36 metres of string. He needs to use all of the string to mark out six equilateral triangles on the ground. How long will the sides of each triangle be? Draw a diagram to show your answer.

5 Draw these scalene triangles.

a A scalene triangle that has a right angle

b A scalene triangle that has a 40° angle

c A scalene triangle that has one side half as long as one of the other sides

6 Are the angles marked with a smiley face ☺ acute or obtuse?

a

b

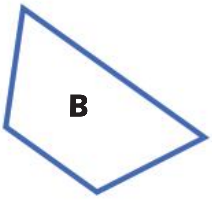
c

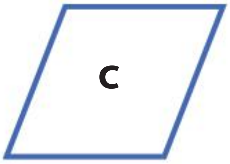
7 Unscramble each word, then match each name to the correct shape (**A** to **C**).

a burhsom

b tanepong

c arzptemiu





8 Draw a circle. Now use a protractor to draw a regular pentagon inside the circle.

9 This table lists some measurements for five circles. Copy and complete the table.

	Circle A	Circle B	Circle C	Circle D	Circle E
Radius	8 cm		7 cm	1.5 cm	
Diameter		22 cm			11 cm

10 How many circles with a radius of 3 cm will fit with their centres along an 18 cm line?

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

CHAPTER

12 Preparation

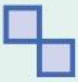
Useful skills for this chapter:

- understanding of two-dimensional shapes
- the ability to draw, identify and name: cubes, rectangular prisms and some other polyhedra
- the ability to calculate the volume of a cube and a rectangular prism
- some experience constructing three-dimensional objects from construction kits.

KICK OFF

Pentominoes

A pentomino is an arrangement of five squares. The squares must be arranged so that they have a complete side in common.

This means that they can touch like this  or this , but not like this  or this .

The pentomino below has been flipped or rotated, but it can only be counted as one pentomino.

How many different pentominoes can you make? Use grid paper to draw your pentominoes.

Show what you know

1 Name these solid objects.

a 

b 

c 

d 

CHAPTER

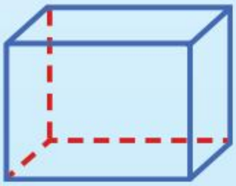
12

Measurement and Geometry

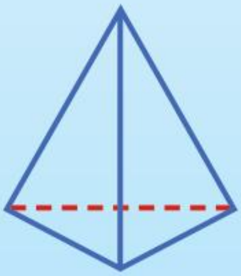
Three-dimensional objects

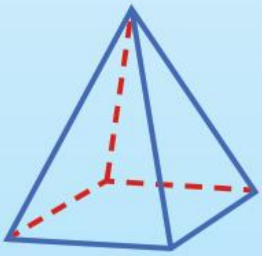
In this chapter we look at three-dimensional objects. Everything around us exists in three dimensions. So you are a three-dimensional object, and a chair is a three-dimensional object too. In mathematics, we refer to three-dimensional objects as **solids**.

We see a lot of solids when we go shopping at the supermarket. People usually call them by other names, such as *cylinder*, *triangular box*, *rectangular box*, and so on.



Look at the shapes above. In what way are they similar to the shapes below? In what way are they different from the shapes below?





Think of some things you buy at the supermarket that look like these objects, or that are sold in boxes or packets shaped like these.

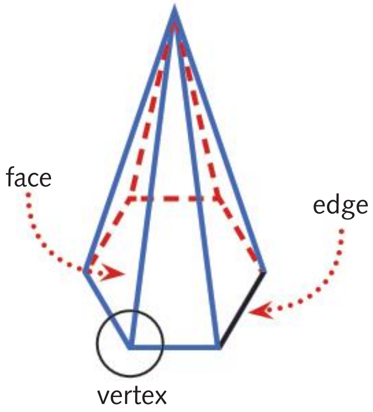
12A Polyhedra, prisms, pyramids and cylinders

The word **polyhedron** is made up from the Greek words *poly*, which means 'many', and *hedron*, which means 'face'. So polyhedron means 'many faces'. A polyhedron is a three-dimensional object with flat faces and straight edges. The faces are polygons. They are joined at their edges. The plural of polyhedron is **polyhedra**, so we can have one polyhedron, and two or more polyhedra.

When we describe a polyhedron, we are interested in its **properties**, or features.

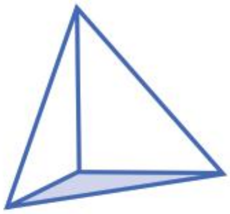
Each flat side of a polyhedron is called a **face**. Where two faces meet, you get an edge. An edge is a line segment. The sharp corner on a polyhedron where two or more edges meet is called a **vertex**. The plural of vertex is **vertices**.

We give polyhedra special names according to how many faces they have. The smallest number of faces that a polyhedron can have is four.



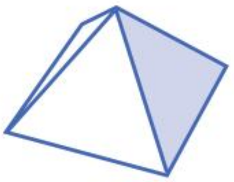
Tetrahedra

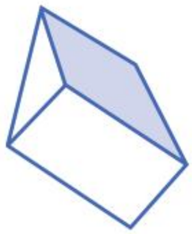
Tetrahedron is from the Greek words *tetra*, which means 'four', and *hedron*, which means 'face'. Tetrahedrons have four faces.



Pentahedra

Here are two different pentahedra. *Penta* means 'five', so these objects have five faces.



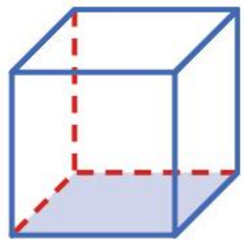


Regular polyhedra

For a polyhedron, the word 'regular' means that all of its faces are identical regular polygons and the same number of faces meet at each vertex. There are five regular polyhedra also called platonic solids. A cube is a regular polyhedron.

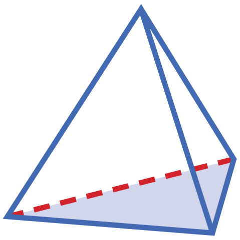
Cube

A cube has 6 faces, all identical squares. Three faces meet at each vertex.



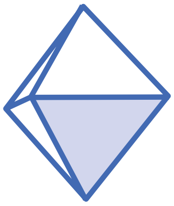
ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

Regular tetrahedron

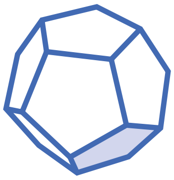


A regular tetrahedron has 4 faces. *Tetra* means four. They are identical regular triangles (equilateral triangles) and 3 faces meet at each vertex.

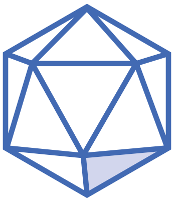
The three remaining platonic solids are:



octahedron



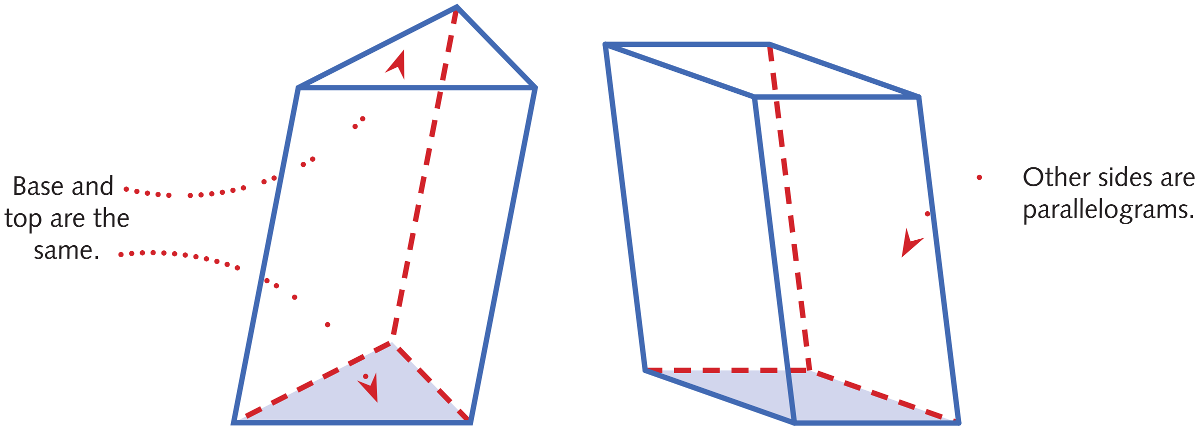
dodecahedron



icosahedron

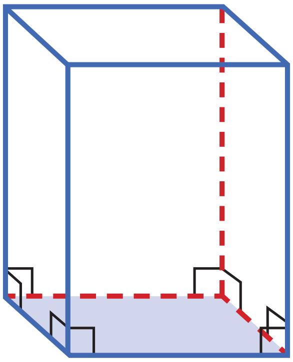
Prisms

A **prism** is a polyhedron with a base and a top that are the same and whose other faces are all parallelograms.



When the parallelograms are all rectangles, this is known as a right prism.

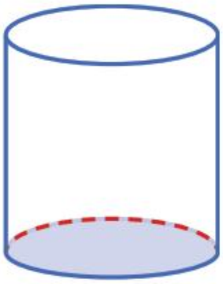
A prism gets its name from the shape of its base. The prism below has a rectangular base, so it is called a **rectangular prism**.



Because a rectangular prism has 6 faces, it is a hexahedron.

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

Cylinders

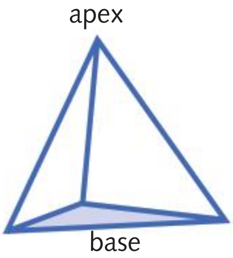


This solid is called a **cylinder**. It has a circular base and top.

Cylinders are not prisms, as they do not have rectangular faces.

Pyramids

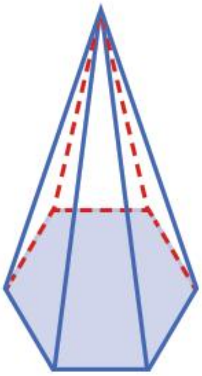
A pyramid is a polyhedron that has a polygon for its base and all of its other faces are triangles that meet at one vertex. The point at which these faces meet is called the **apex**.



Cones are not pyramids.

The solid on the right is a triangular-based pyramid because it has a triangle for the base. It is also a tetrahedron because it has 4 faces.

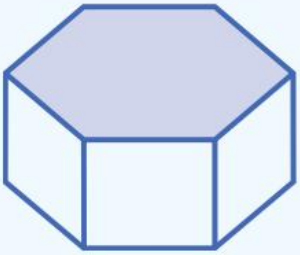
This is a hexagonal-based pyramid. It is another solid that has two names: one name for the shape of its base and one name for the number of faces. It has 7 faces, so it can also be called a heptahedron.



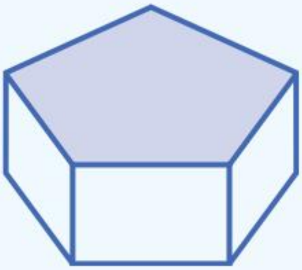
12A Whole class CONNECT, APPLY AND BUILD

1 Look at the solids **a** to **c** below. They all sit nicely on a flat surface. Collect classroom solids like those shown and place them on a flat surface. Sketch each solid, then draw its base. Name the shape of the base.

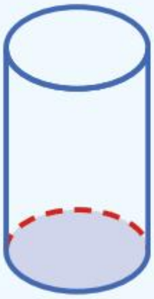
a



b



c



ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

2 Work in pairs using a classroom set of solids. Copy and complete this table for at least three of the polyhedra, making sure you have at least one pyramid and one prism. (Remember: spheres, cones and cylinders are not polyhedra.) Share your results with the class.

Sketch the polyhedron	Is it a prism or a pyramid?	Number of faces	Name the polyhedron

3 Use construction equipment or toothpicks and plasticine to construct three-dimensional objects. Name them.

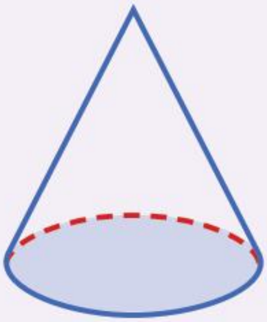
4 **a** Find examples of three-dimensional objects in real life. Describe the objects using their mathematical names.

b Investigate interesting three-dimensional objects such as the pyramids of Egypt, China, Korea and Indonesia. Prepare a short report for the class about the mathematics involved.

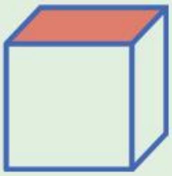
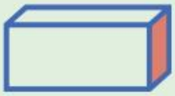
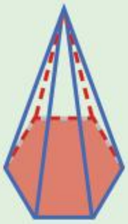
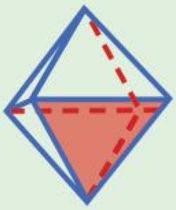
ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

12A Individual

1 This 3D shape is a **cone**. In what way is it similar to a cylinder?



2 Copy and complete this table for each object.

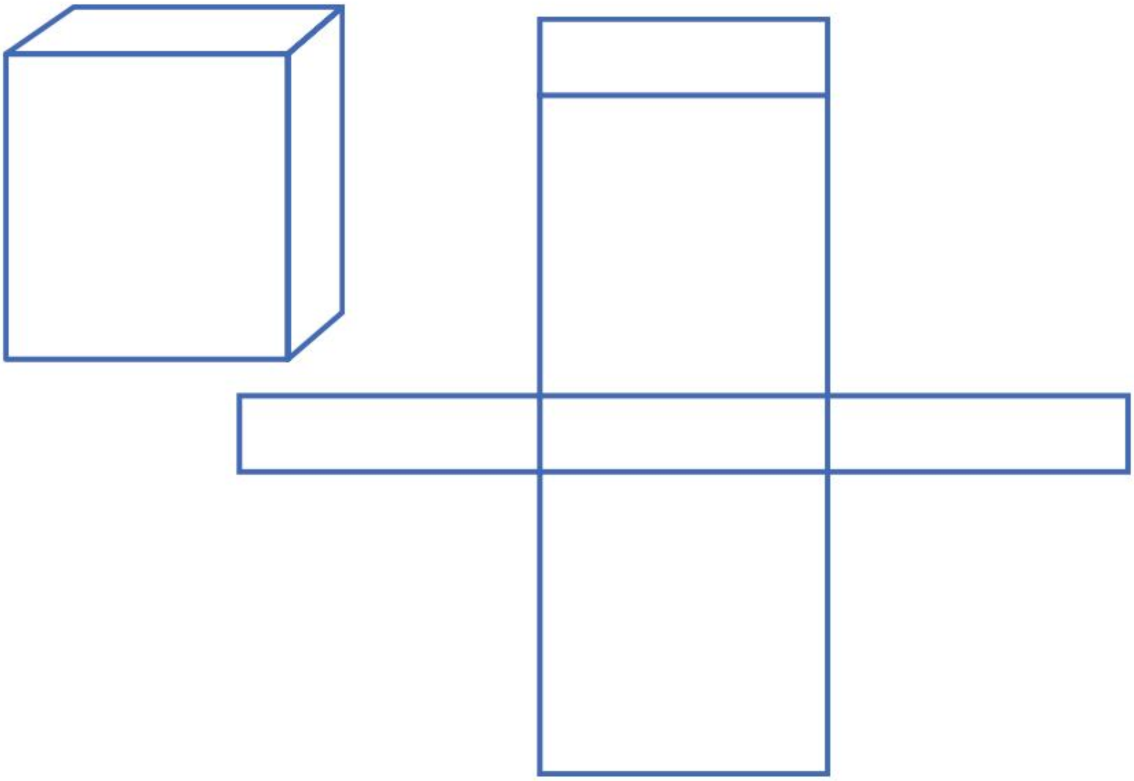
	3D object	Shape of shaded face	Shape of other faces	Number of faces
a		Square		
b		Square		
c				
d				
e				

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

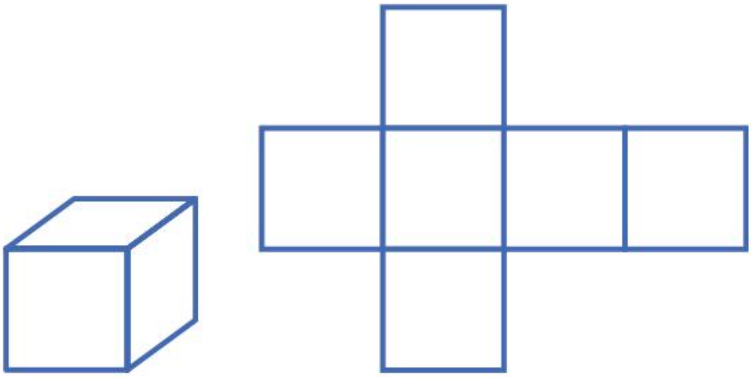
12B Nets of three-dimensional objects

A net is a flat shape that can be folded up to make a three-dimensional object. Every polyhedron can be cut into a net.

If we take an empty breakfast cereal box and cut around three of the top edges and down the four vertical edges we can fold it down flat. This gives us a net of the box.



If you took a cube and 'unfolded' it, you would have 6 squares joined in a net.

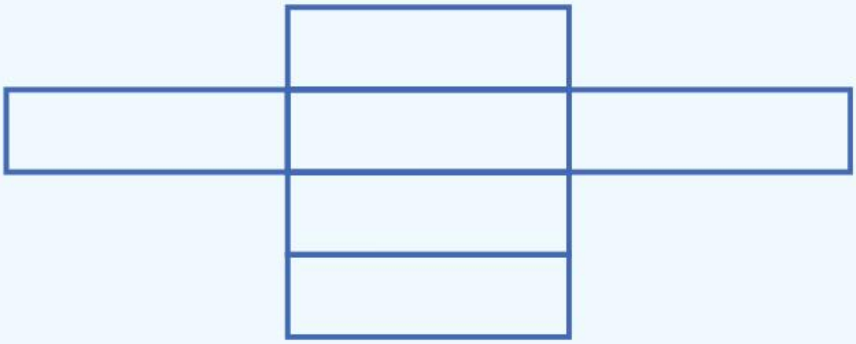


Because the cube has 6 square faces, the net must have 6 squares. There are many possible nets for a cube.

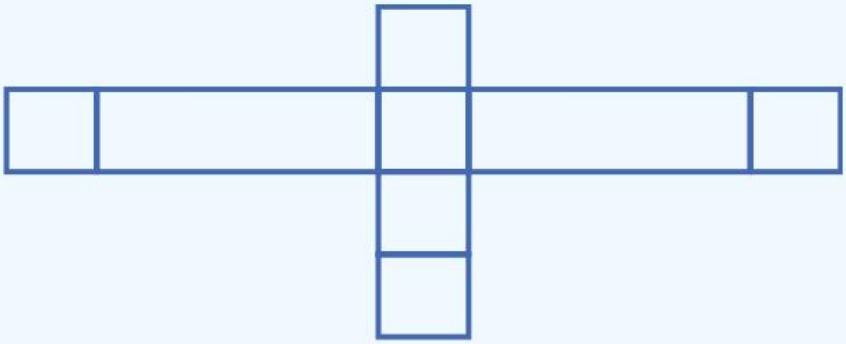
12B Whole class CONNECT, APPLY AND BUILD

1 Which net (**A** to **D**) matches this polyhedron?

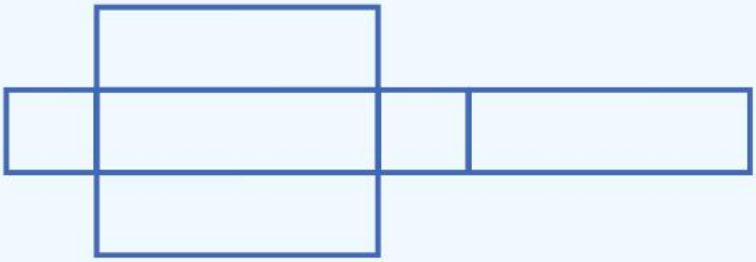
A



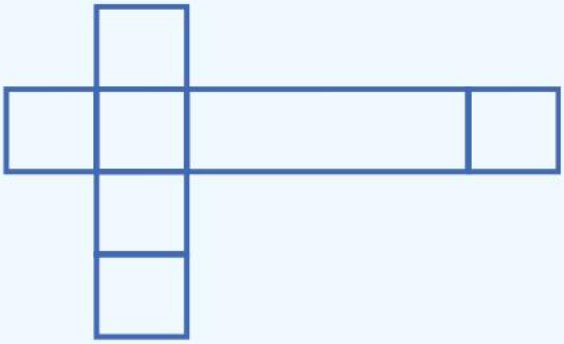
B



C

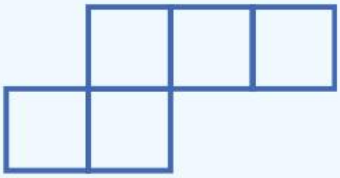


D

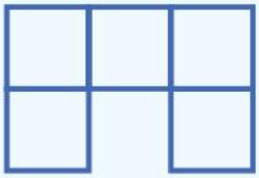


2 Which of these pentominoes (**A** to **I**) is also a net for an open box? (An open box is like a rectangular prism with one face missing.) There is more than one correct answer.

A

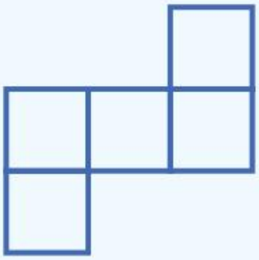


B

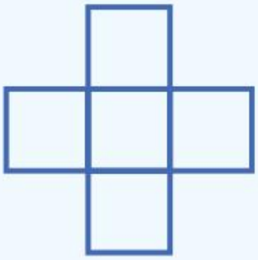


C

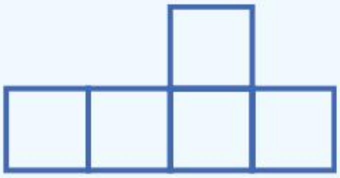
D



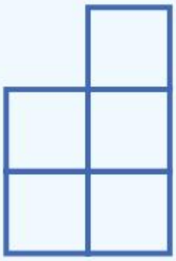
E



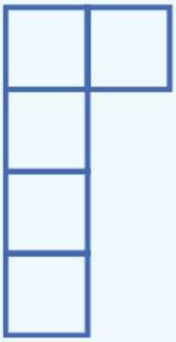
F



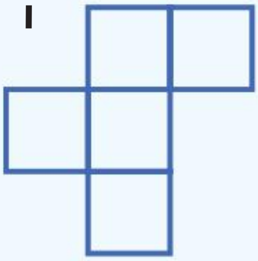
G



H



I



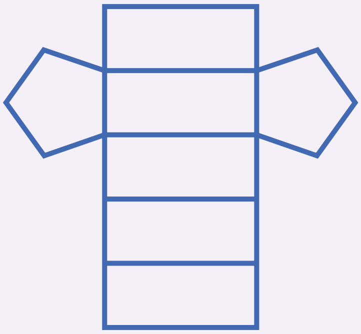
12B Individual

1 Label each net on the following page using one of these names:

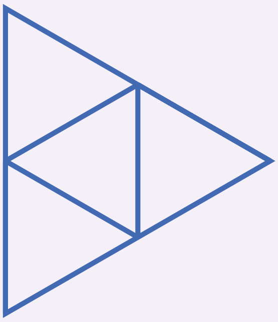
square-based pyramid
dodecahedron
octahedron
pentagonal prism
pentagonal-based pyramid
cube
tetrahedron

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

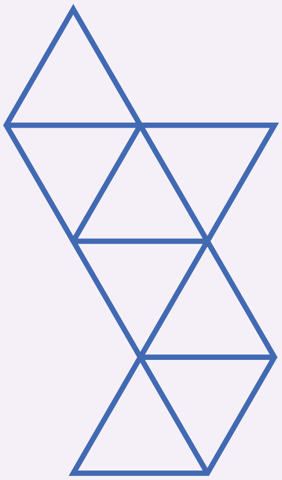
a



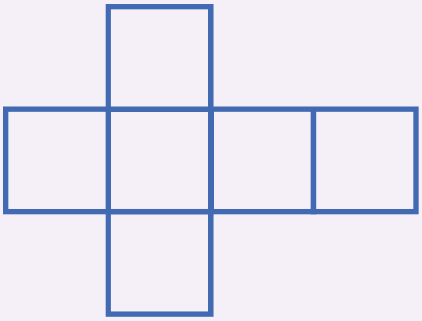
b



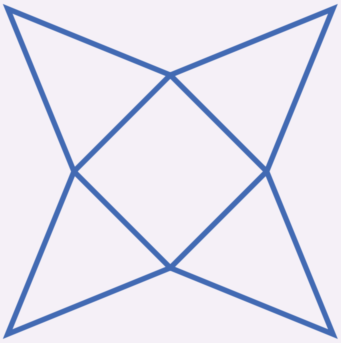
c



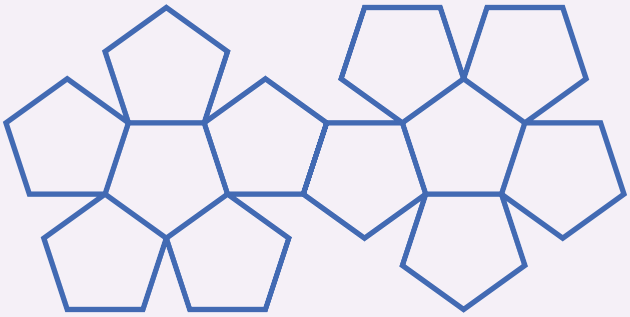
d



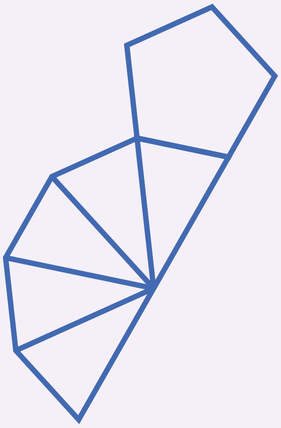
e



f

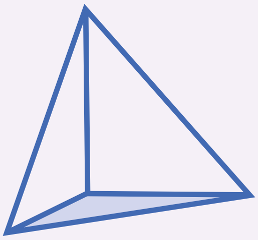


g

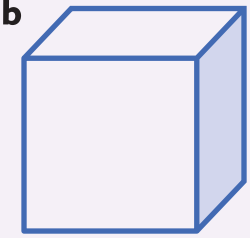


2 Draw a net for each solid.

a

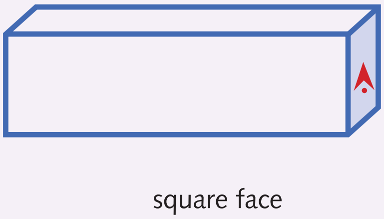


b



cube

c



ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

12C Review questions

1 Match each net with the correct label. Some nets have more than one label.

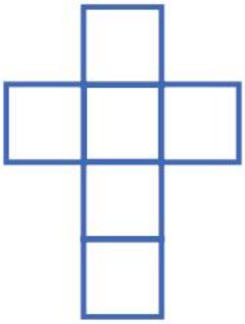
rectangular prism
cube
triangular-based pyramid
octahedron

hexagonal-based pyramid
pentagonal-based pyramid
heptahedron

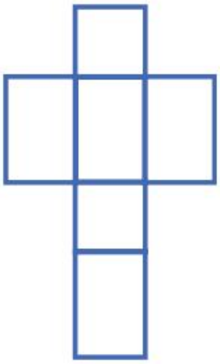
octagonal prism
hexahedron
tetrahedron

a

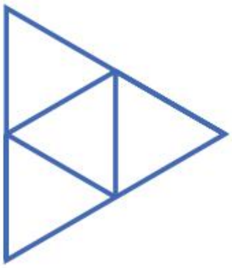
b



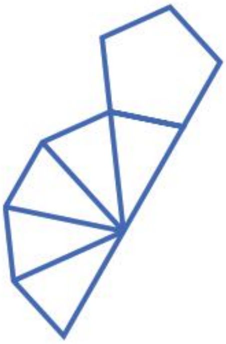
c



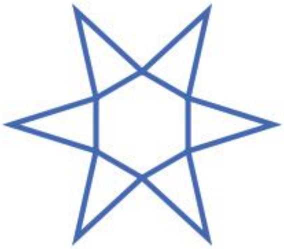
d



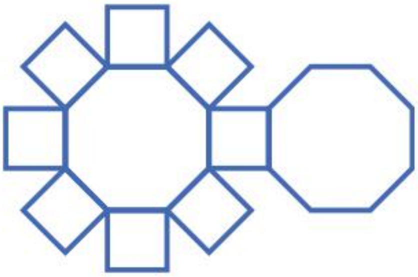
e



f



g



Homework

1 Become an architect by using 3D objects in a house design.

- **a** Sketch a house that uses three-dimensional objects in its design. Include at least one of each of these: triangular prism, rectangular prism, cube, hexagonal-based pyramid, cylinder. You can use more than one of each object if you wish. You can also use other 3D objects.
- **b** Use paper or cardboard to construct the 3D objects you need for your house design, then use the objects to construct your house. Label all the 3D objects you used to construct your house.
- **c** Sketch your house design. Think about how to draw a three-dimensional shape on paper. Which angle will you draw it from? How will you show perspective?

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

CHAPTER 13 Preparation

Useful skills for sections 13A and 13B:

- previous experience in comparing masses of different objects
- knowledge of the relationship between tonnes, kilograms, grams and milligrams
- the ability to multiply and divide decimals by 10, 100 and 1000.

KICK OFF

Organise four teams sitting in lines. Ask the first player in each team to stand. Ask questions like those in the list below. Only the standing players can answer the question; the first to answer earns one point for their team. The first players sit and the second player in each team stands. Continue until all players have had three turns.

The team with the most points wins.

300 × 10
8 × 1000
3.4 × 10
1.5 × 1000

125 × 100
270 × 10
2.84 × 100
40.2 × 10

12 × 100
2.5 × 10
1.234 × 10
100.2 × 100

43 × 100
1.2 × 100
0.2 × 100
1030.30 × 1000

Show what you know

1 Draw something that has a mass that would be measured in:

a grams **b** kilograms **c** tonnes

2 Given that there are 1000 grams in 1 kilogram:

a how many kilograms are there in 5000 grams?

b how many grams are there in 4 kilograms?

3 Given that there are 1000 kilograms in 1 tonne:

a how many kilograms are there in 11 tonnes?

b how many tonnes are there in 4200 kilograms?

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

CHAPTER

13

Measurement and Geometry

Measurement

In this chapter, we are going to investigate measuring mass, time and temperature.

Measuring is something that we do every day. We might need to find out how much something weighs, how long it takes us to do things or what the temperature is.

Scales can be used to measure weight.

A stopwatch can be used to measure time.

A thermometer can be used to measure temperature.

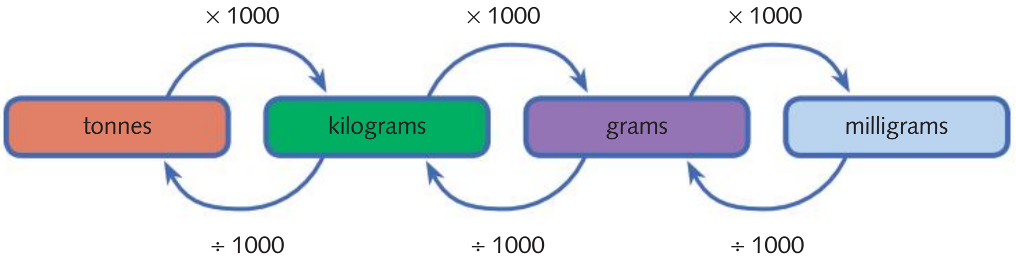
ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7

Cambridge University Press

13A Mass

The units of measurement we use for measuring mass are **milligrams**, **grams**, **kilograms** and **tonnes**. The basic unit for measuring mass is the kilogram, which is abbreviated **kg**.

Each unit for mass is related to the other units. We can convert from one unit to another by multiplying or dividing, as this diagram shows.



The prefix 'kilo' means 1000. If you combine 'kilo' and 'gram' it tells you that there are 1000 grams in 1 kilogram. The letter **g** is used as an abbreviation for grams.

$$1000 \text{ grams (g)} = 1 \text{ kilogram (kg)}$$

If we have 4 kilograms, we can work out how many grams we have.

$$\begin{aligned} 1 \text{ kg} &= 1000 \text{ g} \\ \text{So } 4 \text{ kg} &= 4 \times 1000 \text{ g} \\ &= 4000 \text{ g} \end{aligned}$$

There are 1000 kilograms in 1 tonne. The letter **t** is used as an abbreviation for tonnes.

If we have 3000 kilograms, we can work out how many tonnes we have.

1000 kilograms is the same as 1 tonne.

$$\begin{aligned} \text{So } 3000 \text{ kg} &= (3000 \div 1000) \text{ t} \\ &= 3 \text{ t} \end{aligned}$$

Example 1

A bag of potatoes weighs 3.5 kg. How many grams is that?

Solution

We need to *multiply* the kilograms by 1000 to find the number of grams.

$3.5 \times 1000 = 3500$

There are 3500 grams of potatoes in 3.5 kilograms.

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7

Example 2

Patsy bought 250 grams of beads. How many kilograms of beads did she buy?

Solution

We need to *divide* the number of grams by 1000 to convert it to kilograms.

$$\frac{250}{1000} = 0.25$$

Patsy bought 0.25 kg of beads.

Example 3

John's car has a mass of 1.35 t. How many kilograms is that?

Solution

To convert tonnes to kilograms, we need to *multiply* by 1000:

$$\begin{aligned} 1\text{ t} &= 1000\text{ kg} \\ \text{So, } 1.35\text{ t} &= 1.35 \times 1000\text{ kg} \\ &= 1350\text{ kg} \end{aligned}$$

John's car has a mass of 1350 kilograms.

We need a smaller unit than grams to measure the mass of very small objects.

A letter in an envelope weighs about 5 grams. If we want to measure the mass of the postage stamp on the envelope, we need to use a smaller unit called milligrams.

Milligrams are used for measuring the mass of objects that are very light, such as a stamp, a rose petal or a leaf.

The prefix 'milli' means one-thousandth. There are 1000 milligrams in 1 gram. Milligrams are abbreviated **mg**.

$$1000\text{ milligrams (mg)} = 1\text{ gram (g)}$$

$$1\text{ milligram (mg)} = \frac{1}{1000}\text{ of a gram (g)}$$

You have probably heard of small animals called millipedes. Despite their name, millipedes have about 60 legs, not 1000 legs. Long ago, people thought these little animals looked as if they had at least 1000 legs, and that is how they got their name.

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

Example 4

A can of tomatoes contains 40 mg of salt. Convert this measurement to grams.

Solution

To convert milligrams to grams, we need to *divide* by 1000:

$$\begin{aligned} 1000 \text{ mg} &= 1 \text{ g} \\ \text{So, } 40 \text{ mg} &= (40 \div 1000) \text{ g} \\ &= 0.04 \text{ g} \end{aligned}$$

Example 5

A pharmacist weighs out 1.125 g of powder for each capsule. How many milligrams of powder does he use?

Solution

To convert grams to milligrams, we need to *multiply* by 1000.

$$\begin{aligned} 1 \text{ g} &= 1000 \text{ mg} \\ \text{So, } 1.125 \text{ g} &= 1.125 \times 1000 \text{ mg} \\ &= 1125 \text{ mg} \end{aligned}$$

13A Whole class CONNECT, APPLY AND BUILD

○ **1** You will need bathroom scales or kitchen scales. Select five different objects from around the room, and use the scales to measure the mass of each object. Then draw up a table and convert the measurement of each object into milligrams, grams, kilograms and tonnes.

● **2** A standard chicken egg weighs 70 grams. Calculate the mass of these characters in eggs.

a Billy the baby bilby weighs 280 grams.

b Wally the weightlifter weighs 175 kilograms.

c Carl the clownfish weighs 35 000 milligrams.

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

13A Individual

1 **a** Jessica bought 350 g of grapes. How many kilograms of grapes did she buy?

b James bought 4 kilograms of apples. How many grams of apples did he buy?

c Jordan bought 1.5 tonnes of grapes. How many kilograms of grapes did he buy?

2 **a** How many tonnes are there in 3875 kg?

b How many kilograms are there in 1.074 t?

c How many kilograms are there in 2855 g?

d How many grams are there in 0.045 kg?

e How many grams are there in 4455 mg?

f How many milligrams are there in 4.072 g?

3 Convert each mass from tonnes to kilograms.

a 4.5 t **b** 2.08 t **c** 3.005 t **d** 0.85 t **e** 0.035 t

4 Convert each mass from grams to kilograms.

a 2857 g **b** 3050 g **c** 1007 g **d** 756 g

5 Convert each mass from grams to milligrams.

a 4.8 g **b** 0.4 g **c** 8.75 g **d** 36.04 g

6 **a** Carly's grocery bag weighs 5.75 kg. How many grams is that?

b Paul loaded his truck with 3.04 t of furniture. How many kilograms did he load?

c Annabelle bought 805 g of apples. Write this in kilograms.

d Winnie Witch used 4.25 g of powdered bark in her potion. How many milligrams of powdered bark did she use?

e Rebecca needs 1.35 kg of flour to make some cakes. How many grams of flour does she need?

f A packet of potato chips contained 1256 mg of salt. How many grams of salt was that?

13B Calculating different masses

When we want to find the total mass of a number of objects, we need to convert them all to the same unit before we can add to find the total.

Also, if we need to find the difference between two masses that are measured in different units, we need to convert the masses to the same unit before we can subtract.

Example 6

What is the total mass in kilograms of 3.5 kg of potatoes and 875 g of tomatoes?

Solution

Convert the mass of the tomatoes to kilograms, then add the masses together.

Potatoes 3.5 kg = 3.5 kg
Tomatoes 875 g = 0.875 kg
Total = 4.375 kg

13B Whole class CONNECT, APPLY AND BUILD

1 Look at supermarket catalogues to find items that have their mass in kilograms and items that have their mass in grams. Select three different items and find the total mass. Convert items to the same unit of mass before adding.

2 Labels on packaged foods show the mass of the product inside. This is called the **net weight**. If we add the mass of the packaging we have the **gross weight**. You can find out the mass of the packaging by subtracting the net weight from the gross weight. Use this information to solve the problem below.

Grandma made 12 identical jars of jam and posted them to Norah for her birthday. The gross weight of the parcel was 4.8 kilograms. Each jar and its lid had a mass of 150 grams. Grandma used 180 grams of cardboard, bubble wrap and sticky tape for packaging. Calculate the net weight in grams of the jam in each jar.

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

13B Individual

2.5 kg

1.2 kg

850 g

595 g

65 g

3.125 kg

1 What is the total mass of:

- **a** the box of oranges and the tin of fruit?
- **b** the packet of chips and the box of cereal?
- **c** the carrots and the watermelon?
- **d** the box of oranges, the packet of chips and the carrots?
- **e** all of the items?

2 Copy this table and write the missing values.

Mass	Washing powder	Can of paint	Bacon	Biscuits
Gross	4100 g	10 kg		
Net	4 kg		1.5 kg	1 kg
Packaging		300 g	4 g	12 g

3 One small tin of spaghetti has a net weight of 125 g. How many small tins should Stella buy if she needs a total mass of:

- **a** 500 g of spaghetti?
- **b** 1.25 kg of spaghetti?

4 A large tin contains 425 g of tuna. Helen needs 1.7 kg of tuna. How many large tins of tuna does she need to buy?

5 One table-tennis ball weighs 450 mg. What would be the mass in grams of six table-tennis balls?

6 The net weight of a bag of polystyrene balls is 5 g. If each ball weighs 500 mg, how many balls are in the bag?

13C Reading and recording time

The basic unit of measurement for time is the **second**.

- There are 60 seconds in 1 minute.
- There are 60 minutes in 1 hour.
- There are 24 hours in 1 day.
- There are 7 days in 1 week.

There are two ways of recording the time of day. You can use either a 12-hour clock or a 24-hour clock.

Using a 12-hour clock

When we use a 12-hour clock, the day is broken up into two blocks of 12 hours. A time such as 11:30 am means half-past eleven in the morning (eleven-thirty), and 11:30 pm means half-past eleven in the evening.

We use the abbreviation **am** to show that we mean the morning. The letters 'am' come from the Latin *ante meridiem*, which means 'before noon'.

We use the abbreviation **pm** to show that we mean the afternoon or evening. The letters 'pm' come from the Latin *post meridiem*, which means 'after noon'.

On a digital clock, midnight is shown as 12:00 am and midday (12 noon) is shown as 12:00 pm.

Using a 24-hour clock

When we use a 24 hour clock, the day is measured in one block of 24 hours. All times are measured from midnight on one day until midnight the next day. So 11:30 am is written as 1130 and 11:30 pm is written as 2330.

It is incorrect to use am or pm with 24-hour times. Midnight is written as 0000 (or sometimes 2400) and midday is written as 1200.

Adding time

Jim went around to his friend Aman's place after school. It took Jim 35 minutes to ride his bike to Aman's. Jim and Aman watched a movie for 1 hour and 30 minutes. To find out how long Jim has been away from home, we need to add the two time periods.

	Riding his bike	35 minutes	
+	Watching a movie	1 hour 30 minutes	
	Total time away	1 hour 30 minutes	or 2 hours and 5 minutes

There are 60 minutes in one hour, so we convert 65 minutes to 1 hour 5 minutes.

The total amount of time that Jim has been away from home is 2 hours and 5 minutes.

Example 7

Justine was doing a daily timed maths test. On Monday it took her 2 minutes and 50 seconds to complete the test. On Tuesday it took her 2 minutes and 40 seconds. How long did Justine spend on the tests over the two days?

Solution

Monday	2 minutes	50 seconds
Tuesday	2 minutes	40 seconds
	4 minutes	90 seconds

Total time = 4 minutes + 1 minute + 30 seconds

= 5 minutes + 30 seconds

Elapsed time

Sometimes we need to calculate how long it is between two times. This is called calculating **elapsed time**. The word 'elapsed' means 'gone by', so elapsed time means the amount of time that has gone by, or passed.

We calculate elapsed time by building up to whole minutes, hours or days, keeping track of the amounts of time as we go. For example, if Helen's train left Maryborough at 2:55 pm and arrived at Daisy Hill at 3:45 pm, how long did it take? We can work out the elapsed time by building up from 2:55 to 3:00. Then there are 45 minutes more. Helen's train trip took 50 minutes.

Example 8

Rani started her homework at 4:45 pm and finished it at 6:10 pm. How long did she spend doing her homework?

Solution

		Hours	Minutes
Start time	4:45 pm		
Build up to	5:00 pm		5 minutes
Build up to	6:00 pm	1 hour	
Build up to finish time	6:10 pm		10 minutes
Total time taken to do homework		1 hour	25 minutes

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

13C Whole class CONNECT, APPLY AND BUILD

1 Draw a 24-hour timeline showing the amount of time you spend sleeping, working, playing, eating and travelling in a normal school day.

2 Use a torch and a globe of the world to discuss world time zones. Hold the torch in a fixed position so that it acts as the sun, then rotate the globe from west to east. Use the torch beam to show that when it is night in Europe, it is daytime here in Australia.

13C Individual

1 Caitlin is training for a triathlon. On Monday she trained for 1 hour and 25 minutes. On Tuesday she trained for 2 hours and 10 minutes. On Wednesday she trained for 1 hour and 40 minutes. On Thursday she trained for 2 hours and 5 minutes. On Friday she trained for 1 hour and 55 minutes. What is the total amount of time Caitlin spent training?

2 Lachlan practises his guitar every morning and every evening. Each day he records how long he practised. Calculate the total time that Lachlan practised each day (**a** to **g**).

a Monday: 2 minutes 30 seconds + 1 minute 15 seconds
b Tuesday: 4 minutes 45 seconds + 3 minutes 25 seconds
c Wednesday: 5 minutes 27 seconds + 2 minutes 45 seconds
d Thursday: 2 hours 25 minutes + 3 hours 10 minutes
e Friday: 1 hour 30 minutes + 2 hours 55 minutes
f Saturday: 4 hours 23 minutes + 2 hours 55 minutes
g Sunday: 5 hours 8 minutes + 3 hours 27 minutes

3 The following times are all on the same day. How much time has elapsed between:

a 6:35 am and 8:55 am?
b 11:33 am and 3:45 pm?
c 1535 and 1755?
d 1047 and 1326?

4 Jeremy has a weekend job delivering supermarket flyers. He started his delivery run at 9:20 am and finished it at 3:50 pm. How long did it take Jeremy to deliver the flyers?

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

5 The Term 1 timetable for Ms Chantry's Year 6 class is shown below.

- **a** At what time do the students start school?
- **b** How long is the school day?
- **c** How many hours per week are spent doing Mathematics?
- **d** Which topic takes the most hours in the school week?
- **e** What is the total time spent at lunch and recess, in hours and minutes?
- **f** What is the total time spent in class at school, in minutes? … in seconds?

	Monday	Tuesday	Wednesday	Thursday	Friday
8:45	Assembly	Roll/Notes/ Messages	Roll/Notes/ Messages	Roll/Notes/ Messages	Roll/Notes/ Messages
9:00	Mathematics	English – Reading	English – Reading	Mathematics	Integrated Sci/Geog/ Hist
10:00	English – Reading	Mathematics	English – Writing	English – Speaking & Listening	Mathematics
11:00			Recess		
11:30	English – Writing	Library	Mathematics	English – Writing	English – Reading
12:30	English – Writing	The Arts	Mathematics	Integrated Sci/Geog/ Hist	English – Writing
1:30			Lunch		
2:15	Language	Physical Education	Integrated Sci/Geog/ Hist	The Arts	Integrated Sci/Geog/ Hist
3:30			School finishes		

13D Temperature

The basic unit of measurement for temperature is the **degree Celsius**.

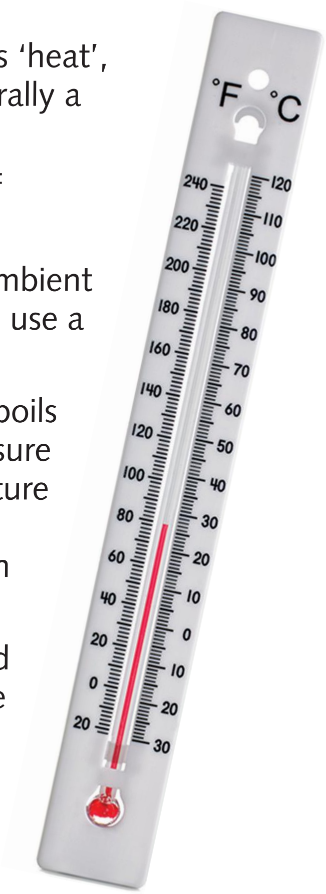
Degrees are shown using the degree symbol °. Celsius is abbreviated as **C**. The symbol and the abbreviation are combined into °**C**, so twelve degrees Celsius is written 12°C.

When we measure temperature using degrees Celsius:

- 0°C is the freezing point of water
- 100°C is the boiling point of water.

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

We use a **thermometer** to measure temperature. The word 'thermometer' is from the Greek words *thermos*, which means 'heat', and *metron*, which means 'measure'. So a thermometer is literally a 'heat measure'.



There are many different types of thermometers. Each type of thermometer has different scales.

To measure the temperature of the air around us we use an ambient temperature thermometer. To measure body temperatures we use a digital thermometer.

Some thermometers measure very high temperatures. Water boils and becomes steam at 100°C, so a thermometer used to measure body temperature could not be used to measure the temperature of steam. Sugar boils at 130°C, so to make jam we need a special jam thermometer that measures temperatures between 40°C and 200°C.

Each thermometer has a scale on it with **graduations** marked on it. The steps between could be 1°C at a time, 2°C at a time or 30°C at a time, but the number of degrees between one marking and the next is always the same.

When the temperature is below freezing (which is 0°C), the temperature is written as a negative number, such as –4°C. The word 'minus' is often used in weather reports. For example, –5°C might be read out as 'minus five degrees Celsius'.

When the temperature is above zero, we do not usually include the + symbol. For example, twenty-two degrees is written 22°C.

Calculating temperature change

Sometimes we need to calculate how much a temperature has changed.

Example 9

It was 5°C when Irene woke up this morning. It is now 17°C. By how many degrees has the temperature risen since this morning?

Solution

17°C is above 5°C so the change is $17 - 5 = 12$°C

The temperature has risen by 12°C since this morning.

Example 10

It was 4°C when Winston went to bed on Thursday night. It was –3°C when he woke up on Friday morning. On Friday afternoon the temperature was 9°C.

a By how many degrees did the temperature drop overnight?

b What was the temperature increase between Friday morning and Friday afternoon?

Solution

a Start at 4°C, then count back to 0°C = 4°C
From 0°C, count back to –3°C = 3°C
4°C + 3°C = 7°C
The temperature dropped by 7°C overnight.

b Start at –3°C, then count up to 0°C = 3°C
From 0°C, count to 9°C = 9°C
3°C + 9°C = 12°C
The temperature increased by 12°C on Friday.

13D Whole class CONNECT, APPLY AND BUILD

1 Discuss what the current temperature in your classroom might be. Make estimates, then check the temperature with a thermometer. Suggest places in Australia where the temperature would be hotter (or colder) than your classroom.

2 Draw a 0 to 100 number line on the board. Mark 0°C at one end and 100°C at the other. Decide what graduations should be marked on the number line to show 20°C, 50°C and 75°C.

3 You will need water, water containers, ice cubes, ambient temperature thermometers and clinical thermometers. Work through these steps in groups.

- **a** Measure the temperature of a container of water.
- **b** Discuss any difficulties encountered when using the clinical thermometer to measure water temperature.
- **c** Drop one ice cube into the water, then stir the water and wait one minute. Use an ambient temperature thermometer to measure the water temperature. Repeat these steps several times, adding an ice cube, stirring, waiting one minute, then measuring the water temperature.
- **d** Record your times and temperatures in a table. Can you graph your results using a line graph?

13D Individual

1 Draw a thermometer with the most appropriate scale for measuring these temperatures.

- **a** A person's body temperature
- **b** The temperature in an oven
- **c** The outside temperature in the shade
- **d** The temperature of a supermarket freezer

2 Draw a thermometer ranging from –10°C to 10°C, using 1°C graduations. Use your thermometer to help you answer the following questions.

- **a** Start at 4°C. Add on 5°C. What is the temperature?
- **b** Start at 2°C. Take off 4°C. What is the temperature?
- **c** Start at –3°C. Add on 3°C. What is the temperature?
- **d** Start at 0°C. Take off 6°C. What is the temperature?
- **e** Start at –8°C. Add on 6°C. What is the temperature?
- **f** Start at 7°C. Take off 11°C. What is the temperature?

3 The temperature at Mt Thredbo was –14°C. It dropped 7°C. What was the temperature then?

4 At 3:00 am, the temperature at Mt Macedon was –2°C. By 6:30 am the temperature had risen by 3°C. By 10:15 am, the temperature had risen a further 6°C. What was the temperature at 10:15 am?

5 When Patrick woke at 7:00 am, the temperature was 12°C. It rose by 1 degree in the first hour, 2 degrees in the second hour and 3 degrees in the third hour. The temperature continued to rise in the same pattern until 1:00 pm, when a cool change arrived. The temperature had dropped by 17 degrees by the time Patrick came home from school. What was the temperature when Patrick arrived home?

Homework

1 Find a thermometer (or several thermometers) in your home. Draw the thermometer and show its scale.

2 Find out today's maximum and minimum temperatures in your area. Calculate the difference between the maximum and minimum temperatures.

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

13E Review questions

1 **a** How many kilograms are there in 3395 g?

b Convert 6.093 45 g to kilograms.

c How many grams are there in 3.02 kg?

d How many grams are there in 0.003 kg?

e How many milligrams are there in 32.09 g?

f How many grams are there in 12 005 mg?

2 Convert these measurements to kilograms.

a 2.29 t **b** 4 000 000 g **c** 80 000 mg

3 Tina bought 3.5 kg of apples, 2.08 kg of pears, 200 g of spices and 850 g of coffee from her local shop. What was the total weight of Tina's purchases in kilograms?

4 Murray is studying for his first-aid certificate.

On Monday he went to his first-aid course for 3 hours and 40 minutes.

On Tuesday he studied the first-aid manual for 1 hour and 15 minutes.

On Wednesday he studied the manual for 1 hour and 48 minutes.

On Thursday he went to the course for 3 hours and 5 minutes.

On Friday he took his first-aid exam. It went for 2 hours and 57 minutes.

What was the total amount of time Murray spent getting his first-aid certificate?

5 How much time has elapsed between:

a 4:05 am and 7:35 am on the same day?

b 7:55 pm and 8:03 pm on the same day?

c 10:08 pm and 3:45 am the next day?

d 6:28 am and 3:45 pm that afternoon?

e 0545 and 1355 the same day?

f 1256 and 1326 the next day?

6 Draw a thermometer that ranges from –30°C to 15°C, with 1°C increments. Use your thermometer to find the difference between these temperatures.

a 12°C and 3°C **b** –15°C and 3°C **c** –28°C and –2°C

7 Calculate the new temperature after these temperature changes.

a Start at 8°C. Take off 15°C. What is the temperature?

b Start at –6°C. Take off 12°C. What is the temperature?

c Start at –18°C. Add on 19°C. What is the temperature?

CHAPTER 14 Preparation

Useful skills for this chapter:

- some prior experience using maps.

KICK OFF

Carmel is facing north. In which direction will she be looking if she turns:

a 45 degrees to her right?
b 90 degrees to her left?
c 135 degrees to her left?
d 135 degrees to her right?
e 45 degrees to her left?
f 180 degrees?
g 90 degrees to her right?

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

CHAPTER

14

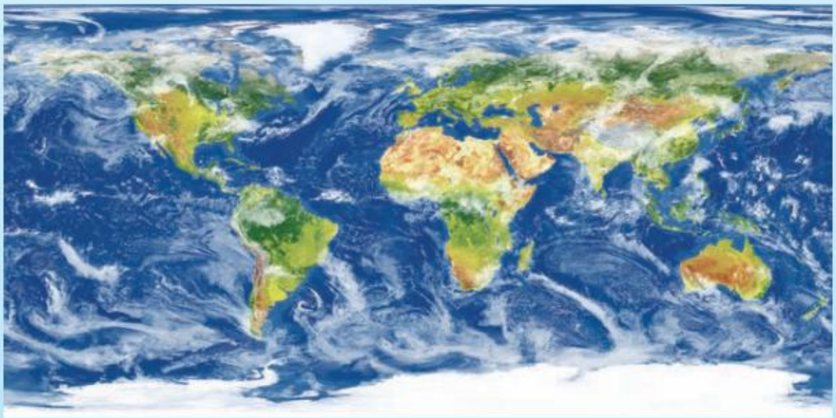
Measurement and Geometry

Maps and coordinates

For thousands of years, people have made maps of their surroundings.

Early maps relied on what people could see and measure, and sometimes they weren't very accurate.

Today's maps are based on satellite images. They are very accurate and are based on photos taken from space.

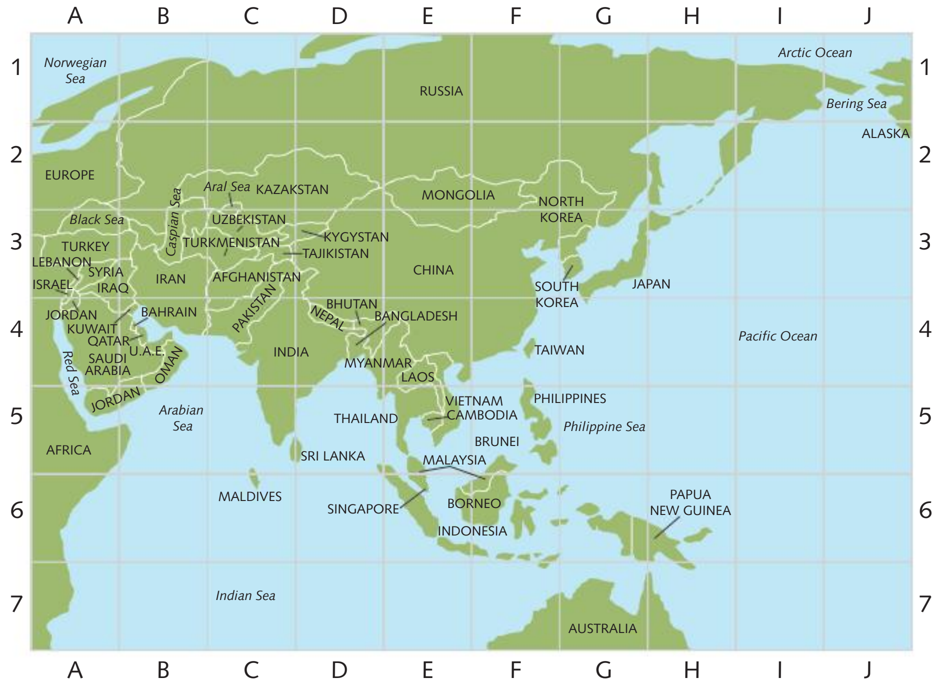


ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

14A Maps and grid references

The map of Asia below is divided into columns and rows and uses letters and numbers to locate places on the map.

The combination of a letter and a number to describe a position on a map is called a **grid reference**. Each square on the map has its own grid reference. For example, B3 is the square in column B, row 3. The grid reference for the middle of northern Australia is G7. Which countries are located at G3?



Example 1

There are several grid references that could be used to locate parts of Australia on the map above. What are they?

Solution

Australia crosses columns F, G and H, and is in row 7. The grid reference for Australia could F7, G7 and H7.

Example 2

Which countries would you find at E2?

Solution

Trace your finger up column E until it meets row 2. Mongolia and Russia are at E2.

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

14A Whole class CONNECT, APPLY AND BUILD

Use the map on page 328 for these questions.

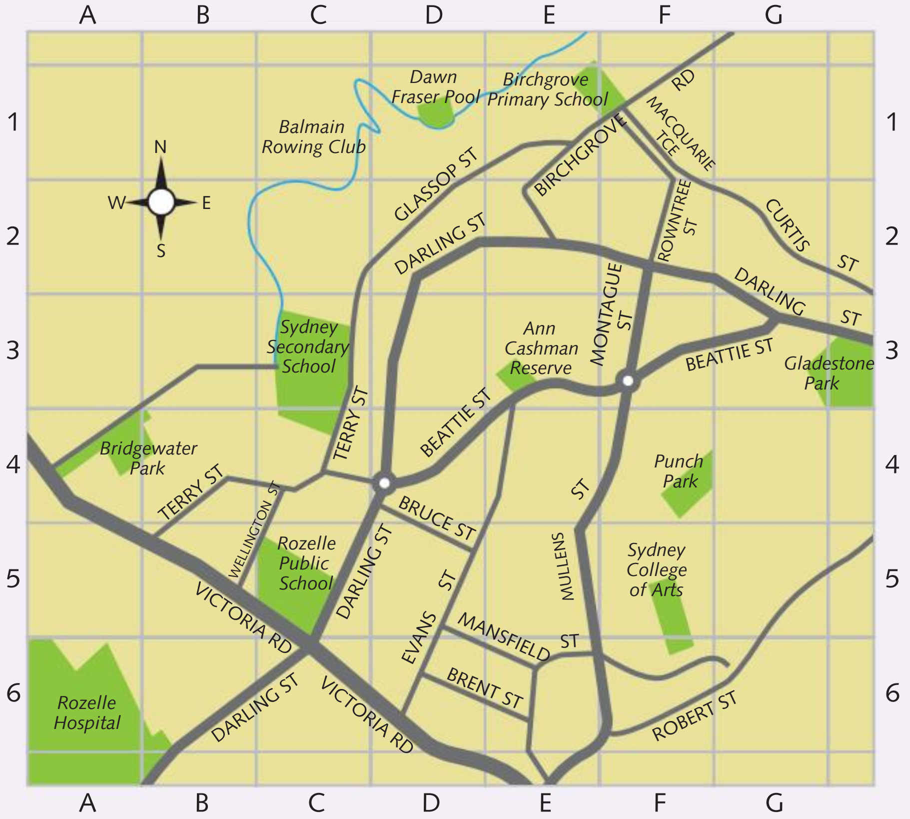
1 Which country has the grid reference:

a C4? **b** E1? **c** E3? **d** E4? **e** H6?

2 Write the grid reference that could be used to find:

a Pakistan **b** Nepal **c** Kazakstan **d** South Korea

14A Individual



1 On the map of Manly above, what would you find at:

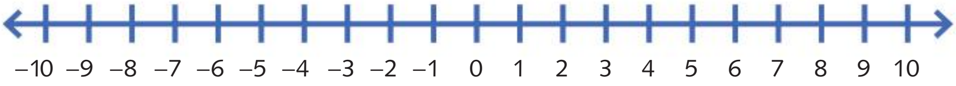
a F5? **b** E3? **c** A4? **d** F4? **e** A6?

2 Write the grid reference(s) that would locate:

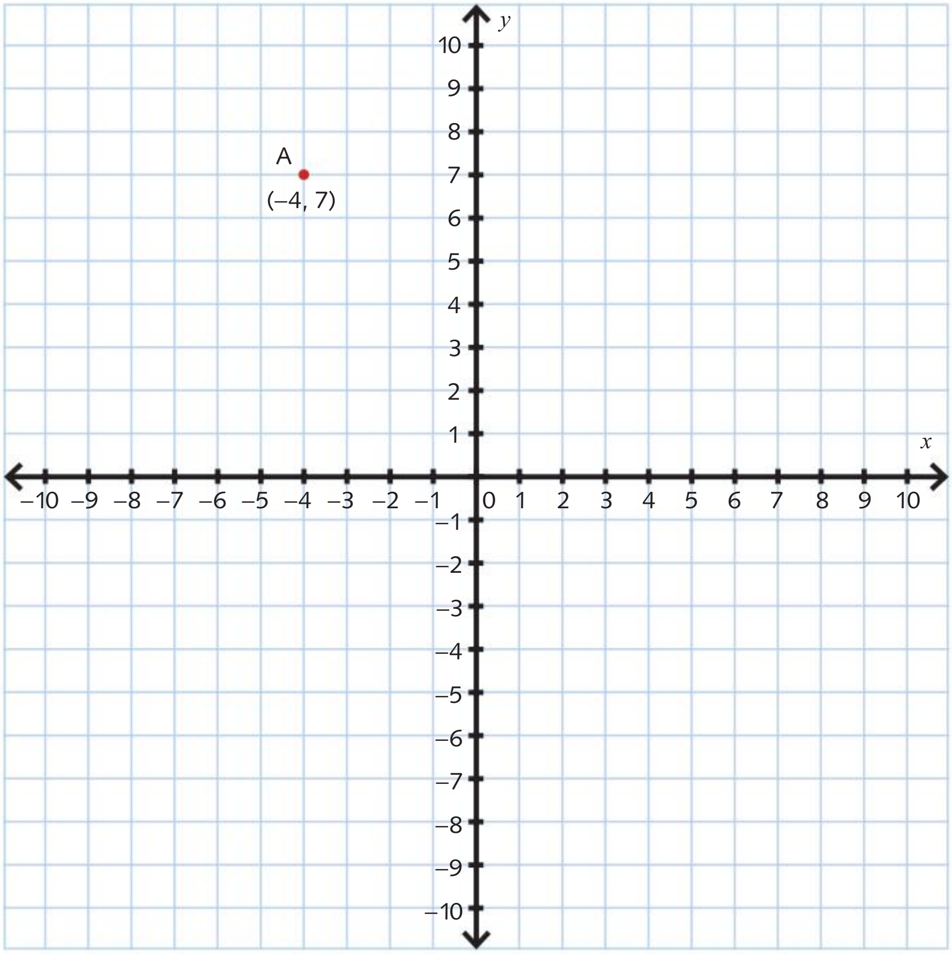
a Macquarie Terrace **b** Balmain Rowing Club **c** Dawn Fraser Pool

14B The Cartesian plane

A number line can be used to show both positive and negative integers.



If we take a sheet of grid paper and draw two number lines on it, at right angles to each other, we have a **Cartesian plane**.



René Descartes

The axes are called the **coordinate axes**. They are named after the French mathematician and philosopher René Descartes (1596–1650). Descartes introduced coordinate axes to show how algebra could be used to solve geometric problems.

The horizontal axis of the Cartesian plane is called the **x-axis**. The vertical axis is called the **y-axis**. The **axes** intersect at zero.

Axes, the plural of axis, is pronounced 'axees', not like the plural of axe.

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

Any point on the Cartesian plane can be described using two numbers. The first number tells us which vertical line we are on. The second one tells us which horizontal line we are on. These are called the coordinates of the point. On the Cartesian plane on the previous page, the point A is located at (–4, 7).

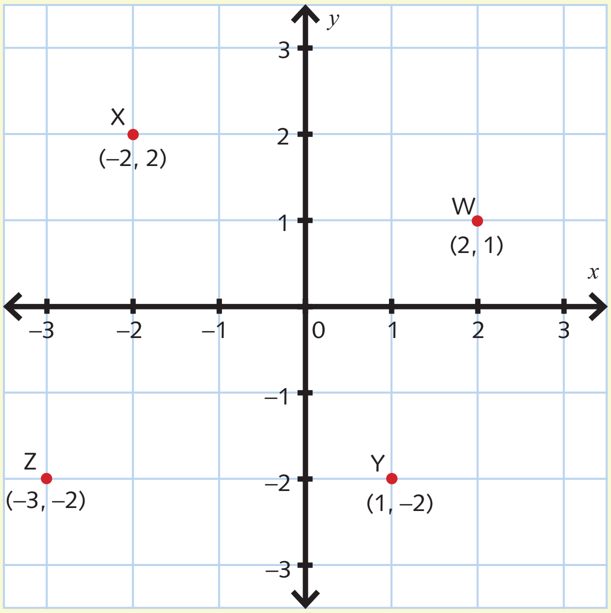
The coordinates are an **ordered pair**. We call it an ordered pair because it is a pair of numbers and the order in which they are given makes a difference. An ordered pair is written in brackets. The x-coordinate is always written first, then a comma and then the y-coordinate. For example, (–4, 7).

Example 3

Rule up and label a Cartesian plane, then plot and name these points.

W = (2, 1)
X = (–2, 2)
Y = (1, –2)
Z = (–3, –2)

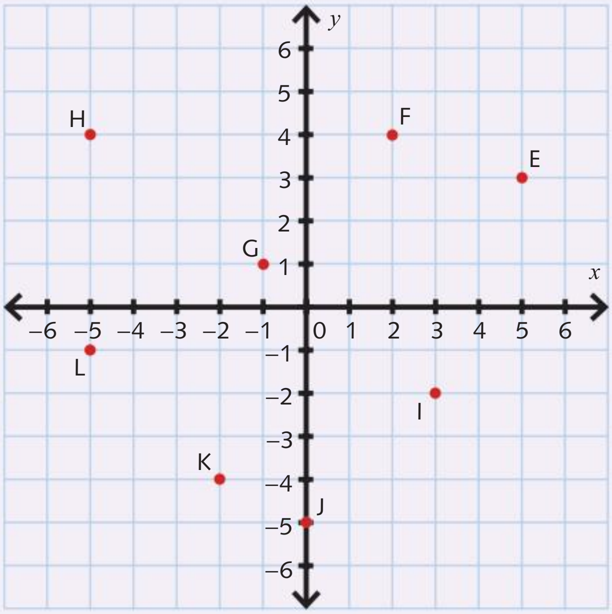
Solution



ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7

14B Individual

1 Write the coordinates of the points E, F, G, H, I, J, K and L.



2 Draw your own Cartesian plane with axes marked from 6 to –6, as shown above. Mark these points on the Cartesian plane with the letter and a dot.

L = (3,2)
M = (–2,0)
N = (4,–5)
P = (–5,–4)
Q = (5,0)
R = (0,3)
S = (1,–3)
T = (0,–4)
U= (0,0)
V= (–4,–2)

3 Draw another Cartesian plane and number the axes from 6 to –6. Plot each set of ordered pairs, and draw a dot for each point. Use your ruler to join the dots, then name the geometrical shape that you have created.

a (3,2) (5,3) (3,4) (1, 3)
b (–3,1) (–3,5) (–5,5) (–5,1)
c (–5, –2) (–5, 2) (–1, 6) (4, 2) (5, –3) (0, –6)
d (5,–1) (5,1) (3,1) (3,–1)
e (1,3) (1,5) (–2,4)
f (2,–1) (1,2) (–2,2) (–3,–1)
g (4,–5) (4,–2) (–1,–2)
h At what point do the diagonals intersect in parts **a**, **b** and **d**?

4 Look at the Cartesian plane you made for question **3**.

How far is it along the *x*-axis from:

a 5 to 1?
b 4 to 0?
c 1 to 4?
d 2 to 5?
e 0 to 3?
f 3 to –2?
g –2 to –5?
h –3 to 4?

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

14C Review questions

1 Here is a map of Albany, Western Australia.

Write what you would find at:

a H4 **b** G1 **c** F5 **d** A2 **e** H3 **f** H1

2 Write the grid references for these locations.

a Semaphore Point **b** Dog Rock **c** Town Jetty

d Princess Royal Fortress **e** Brig Amity **f** Reservoir

g The roundabout where Middleton Road and Campbell Road intersect

h Albany Primary School

3 Follow these directions. Start at A1 on the Albany Highway. Travel south-east to York Street. Turn right into York Street, go through the roundabout, then turn left on Princess Royal Drive. As you go along Princess Royal Drive, what will you see on your right?

4 Select a location of your own and write a set of directions to get there. Swap your directions with a friend and see if you can find each other's locations.

5 Draw a Cartesian plane with each axis numbered from –10 to 10. Plot the following ordered pairs. Now use your ruler to join the pairs in the order they are written. Name the geometrical shape that you have made.

(–2, 7) (2, 3) (2, –1) (–2, –5)

(–6, –5) (–10, –1) (–10, 3) (–6, 7)

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

CHAPTER

15 Preparation

Useful skills for this chapter:

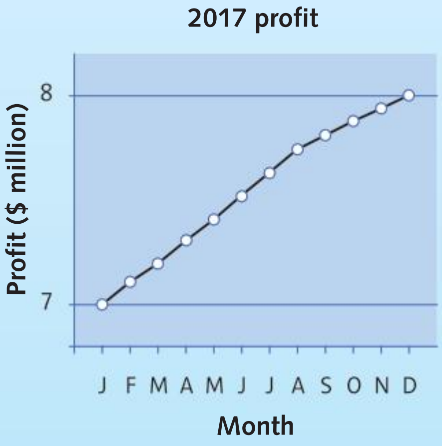
- previous experience drawing and interpreting data tables, pictographs and bar graphs.

KICK OFF

Collect articles in newspapers, magazines and leaflets that contain tables, graphs and charts. Discuss how the information is presented. What does it tell you? Is it accurate?

Show what you know

In a newspaper article, the graphs below were accompanied by this text: ‘Company profits look much better in 2017 when compared with 2010’.



What is wrong with this statement? Discuss.

CHAPTER

15

Statistics and Probability

Data

How can we find out about the world around us? One way is to collect information, organise it, then study the results. Collecting and studying information in this way is called **statistics**. People who gather and analyse statistics are called **statisticians**.

Most people use the word **population** to describe the number of people living in one place. Statisticians use the word population to describe a group that they are interested in studying. For example:

- people who live in Alice Springs
- iPhones purchased in Tasmania in 2017
- cane toads in the Northern Territory.

How would you find out about cane toads in the Northern Territory? It would not be possible to count every cane toad. So a statistician would collect information about a smaller group within the population, such as the number of cane toads in one square kilometre of land near Maningrida, in Arnhem Land. This smaller group is known as a **sample**.

When the statistician has some information from the sample, they then try to make predictions about the entire population of cane toads.

The information that we gather about a population is called **data**. If we have many questions to answer, we need to gather data about different aspects of our population (or sample). The data can be organised and presented in tables, charts and graphs, and we interpret that information in order to make some conclusions and recommendations.

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

The statistical process

When we plan a statistical data investigation, we need to decide on the problem we are going to investigate and then pose some questions that we might like answers for. For example, if the school canteen wants to start selling frozen yoghurt we might ask ‘What flavours of frozen yoghurt do Year 6 students like?’ and ‘Who are the people who will buy frozen yoghurt?’.

Next we think about what data we need to collect so that we can answer those questions. There are lots of ways to collect, organise and present the information – so there are many choices to be made.

Finally we look at the data that has been organised and presented in tables, charts and graphs, and we interpret that information in order to make some conclusions and recommendations. In our frozen yoghurt example above, we might use the information we have collected to make some suggestions to the canteen staff about the flavours of frozen yoghurt that Year 6 students like.

The statistical data investigation process will be explained in more detail in section 15G where you will use it to carry out your own data investigations.

Types of data

There are several different types of data. For each type, there are different things to consider when collecting and recording the data, and different ways to present it.

One type of data is data that we can *count*. We get count data when we investigate situations such as:

- the number of trees in different backyards
- the number of goals scored in a netball match
- the number of jelly beans in a packet.

Another type of data is data that we can *measure*. You might collect measurement data such as:

- the height of students in your class
- the age of students when they first rode a bike without training wheels
- the amount of water left in students’ drink bottles after lunch.

Then there is data that belongs in *categories*. Sometimes there is a choice to be made about which category the data belongs to. Categorical data includes:

- types of houses
- colours of cars
- hairstyles.

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

15A Using tables

Tables are used to record and present data. The information in a table is organised so that the important ideas can be understood easily and quickly.

James interviewed his classmates and wrote down the different ways each person came to school.

James wanted to draw some conclusions about the information in his list, but the patterns were not easy to see. He did not find his list very useful.

So James organised his information into a table, using tally marks to record his data.

James' list

HOW DID YOU COME TO SCHOOL?

car, car, bike, walked, bus, car, bus, bus, bike, walked, bike, car, car, car, bike, bike, bike, walked, bus, car, bus, bus, bike, car, car, bike, bike, bus, car, bus

Travelling to school

	Tally	Frequency
Bike	𝍸 ||||	9
Bus	𝍸 |||	8
Car	𝍸 𝍸	10
Walked	|||	3

Each stroke in a tally stands for one item. The fifth stroke is made across a group of four. This makes it easy to count by fives to work out how many are in a tally.

James then counted the number of times that he had received the same answer. This is called the **frequency**.

Now James can use the data to answer many different questions.

For example:

- How many students were interviewed?

  30 students were interviewed.

  (Add the number of students in each category to find this.)

- How many students caught the bus to school?

  8 children caught the bus to school.

- What fraction of the class travelled by car?

  $\frac{10}{30}$ or $\frac{1}{3}$ of the class travelled to school by car.

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

- What percentage of the class walked to school?

$$\text{Percentage} = \frac{3}{30} \times \frac{100}{1}\% = 10\%$$

10% of the class walked to school.

Sometimes we need to compare the data from two different groups of people. We can show the data for each group in the same table by using a two-way table.

This two-way table shows how ICE-EM staff travelled to work on a Friday in May.

Travelling to work		
	Women	**Men**
By car	2	2
Walked	2	3
By bike	1	1
By train	1	3
By tram	1	0

Example 1

Eliza asked the students in her class how they travelled to school.
Then Eliza and James both showed their data in the same two-way table:

Travelling to school		
	James's class	**Eliza's class**
Bike	9	3
Bus	8	7
Car	10	3
Walked	3	12

a How many students did Eliza interview?

b How many students travelled to school by bike in each class?

c What fraction of each class came to school by bus?

d Which class had the larger percentage of children coming to school by bus?

Solution

a By adding the numbers in each category, we get a total of 25 students for Eliza's class.

b In James's class, 9 students travelled to school by bike. In Eliza's class, 3 students travelled to school by bike.

c $\frac{8}{30}$ or $\frac{4}{15}$ of James' class travelled by bus. $\frac{7}{25}$ of Eliza's class travelled to school by bus.

continued over page

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

d We know that 8 students in James's class came by bus. First, we convert this to a percentage.

$$\begin{aligned}\text{Percentage} &= \frac{8}{30} \times \frac{100}{1}\% \\ &= 26.7\%\end{aligned}$$

26.7% of the students in James's class came to school by bus.

In Eliza's class, 7 students came by bus.

$$\begin{aligned}\text{Percentage} &= \frac{7}{25} \times \frac{100}{1}\% \\ &= 28\%\end{aligned}$$

28% of the students in Eliza's class came to school by bus.

So even though a greater number of people in James's class came to school by bus, a greater percentage of Eliza's whole class came by bus. Percentage is a useful tool for comparing the relative size of groups within groups.

15A Whole class CONNECT, APPLY AND BUILD

1 **Class colour favourites**

Decide on four or five colours, then vote for the colour you like best. Use tally marks (𝍸) to collect the data, then make a table to show the results.

2 Work in pairs to collect the data for this scenario. The school canteen is going to introduce frozen yoghurt treats. You have to find out which frozen yoghurt flavours should be stocked.

- **a** Write the question that you will ask your classmates.
- **b** Draw up a table that will let you use tally marks to collect your data. Make sure that you have space on your table for a total.
- **c** Survey each member of your class to gather the data.
- **d** Make five statements about your data. What recommendations will you make to the canteen?

3 Measure and record the length of the left foot of each member of your class. Your measurement should be correct to the nearest centimetre.

- **a** Draw up a table for the results, including a column for tallies and a column for the frequency.
- **b** Use the data in your table to answer these questions.
  - What is the shortest foot length?
  - What is the longest foot length?
  - Which foot length occurred the most often?

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

15A Individual

1 Imagine you want to find accurate data for questions **a** to **e** below. For each question, select the most appropriate group of people to survey from this list.

Pre-school children	*Primary-school student*	*Mums*
Plumbers	*Librarians*	*Adults over 18*

a What time of day do people use their cars?

b What is the best type of pipe to use for drains?

c What is your favourite song by The Wiggles?

d Which school subject is the most fun?

e What is the most popular book for teenagers?

2 Write a question that you could ask to get data about these topics. Who could you ask?

a The cost of weekly groceries

b The different types of library books borrowed in one week

c The best food for dogs

d The most-watched TV news service

e The favourite breakfast cereal

3 The Year 6 class at Mt Botanic School counted the number of Australian native trees in their two local parks. This table shows their results.

Type of tree	Joseph Banks Park		Celia Rosser Park	
	Tally	Frequency	Tally	Frequency
Rose mallee	|||		||	
Apple myrtle	||||		𝍸 ||||	
Golden wattle	𝍸 𝍸 |		||	
Bottlebrush	𝍸 𝍸 𝍸		|||	
Quandong	𝍸 𝍸 ||		𝍸	

a Copy and complete the table by filling in the frequency column for each type of tree.

b How many trees are there in each park?

c For each park, write each type of tree as a fraction of the total number of trees.

d Which park has the greater percentage of bottlebrush trees?

15B Mode, median and mean

When we make statements about data that we have collected, we often want to say which item is the most popular, which item is in the middle and which item is the average. There are mathematical words to describe these three ideas. They are mode, median and mean.

Mode

How do we find out which item is the most popular? Or the most common? Or the favourite? All of these questions are asking the same thing. They want to know which value occurs the most often, or has the highest frequency. This value is called the **mode**.

For example, Ivan surveyed a group of people about their hair colour and wrote down the results.

Ivan's list

WHAT COLOUR IS YOUR HAIR?

brown
blonde
black
black
blonde
brown

brown
red
blonde
black
brown
brown

Ivan arranged his data into a frequency table.

Hair colour	Tally	Frequency
Black	𝍷𝍷𝍷	3
Blonde	𝍷𝍷𝍷	3
Brown	𝍸	5
Red	𝍷	1

In Ivan's survey, the hair colour with the highest frequency is brown, so brown is the mode for his survey.

There is an easy way to remember this. *Mode* is the French word for 'fashion', and it is also the most fashionable (or most popular) value in a set of data.

Sometimes two values are equally popular and all the others are less popular. In this case, we take both values to be the mode.

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

Example 2

A group of Year 6 students at York School recorded their shoe size.

4, 5, 6, 4, 5, 3, 4, 2, 2, 3, 2, 5, 3, 6, 4, 3, 2, 5, 4, 7, 4, 5, 6, 2, 5, 4, 3

Use the data to calculate the mode.

Solution

Put the data into a frequency table.

Shoe size	Tally	Frequency
2	𝍸	5
3	𝍸	5
4	𝍸 𝍷𝍷	7
5	𝍸 𝍷	6
6	𝍷𝍷𝍷	3
7	𝍷	1

The mode is size 4, because size 4 shoes occur most often in this class.

Median

When a set of values is arranged in order of their size, the ‘middle value’ is the **median**. Here are the ages of a group of children in order, from youngest to oldest.

10, 10, 11, 11, 11, 11, 11, 11, 12, 12, 12, 12, 13

This set of data has 13 values. The seventh value is the middle value, as it has 6 values on either side. The median age is 11.

Ten students at Snapper Point School decided to work out their median age. Their ages were:

9, 9, 10, 10, 10, 12, 12, 13, 13, 14

This set of data has an even number of values, with two middle values: 10 and 12.

To calculate the median, we need to find the average of 10 and 12.

$$\begin{aligned}\text{Median} &= \frac{10 + 12}{2}\\ &= 11\end{aligned}$$

So the median age is 11, even though 11 does not occur in this set of data.

There is an easy way to remember what ‘median’ means. Think of the median strip that runs along the middle of a road. The median is always in the middle, with an equal number of values on either side.

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

Mean

You might have already heard of the **mean**, and know that it is also called the **average**. We use the mean when we want to make a simple statement about the average value in a population or sample. To calculate the mean, we add up (or sum) the values, then divide by the number of values.

$$\text{Mean} = \frac{\text{sum of values}}{\text{number of values}}$$

So the mean of the numbers 2, 3, 4, 5, 5, 5 is $\dfrac{2+3+4+5+5+5}{6} = 4$

Sometimes you will see reports like this.

THE AVERAGE FAMILY
HAS 2.4 CHILDREN

This does not mean that Australian families have 2 whole children and another 0.4 of a child.

It means that many families were surveyed. Some of those families had no children, some had 1, 2 or 3 children and others had perhaps 5 or 9 children. There might have been families with other numbers of children, too.

For example, the families of Kid Street have 1, 7, 1, 0 and 3 children. The mean (or average) number of children in each house in Kid Street is:

$$\begin{aligned}\text{Mean} &= \frac{\text{sum of values}}{\text{number of values}}\\ &= \frac{1+7+1+0+3}{5}\\ &= \frac{12}{5}\\ &= 2.4\end{aligned}$$

So the mean number of children in each family in Kid Street is 2.4 children.

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

Example 3

Twenty-six Year 4 students recorded the number of children in their families.

1, 2, 1, 4, 5, 3, 2, 3, 2, 6, 7, 2, 1, 3, 2, 3, 4, 1, 4, 2, 2, 3, 8, 2, 1, 3

Use this data to calculate:

a the mode

b the median number of children in a family

c the mean number of children in a family (rounded to two decimal places)

Solution

a Sort the data into a frequency table.

Number of children	Tally	Frequency
1	𝍸	5
2	𝍸 |||	8
3	𝍸 |	6
4	|||	3
5	|	1
6	|	1
7	|	1
8	|	1

The value that occurs the most often is 2. So the mode is 2.

b Arrange the data in order from smallest to largest, then find the middle value.

1, 1, 1, 1, 1, 2, 2, 2, 2, 2, 2, 2, 2, 3, 3, 3, 3, 3, 3, 4, 4, 4, 5, 6, 7, 8

There is an even number of values, so we take the average of the middle two values.

$$\text{Median} = \frac{2 + 3}{2} = 2.5$$

The median number of children in a family is 2.5.

continued over page

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

c $$\begin{aligned}\text{Sum of values} &= 1+1+1+1+1+2+2+2+2+2+2+2+2+3+3+3\\ &\quad +3+3+3+4+4+4+5+6+7+8\\ &= 77\end{aligned}$$

$$\begin{aligned}\text{Mean} &= \frac{\text{sum of values}}{\text{number of values}}\\ &= \frac{77}{26}\\ &= 2.96\end{aligned}$$

The mean number of children in a family is 2.96.

15B Whole class CONNECT, APPLY AND BUILD

1 The canteen at Riverina School sells frozen oranges. This table shows the sales of frozen oranges for one week.

Frozen orange sales

Day	**Tally**	**Frequency**
Monday	|||	
Tuesday	𝍸 ||||	
Wednesday	|||	
Thursday	𝍸 𝍸 𝍸 |	
Friday	𝍸 |||	

a Copy the table and complete it as a class.

b Calculate the mode.

c Calculate the mean.

d What is the median?

e Discuss these questions as a whole class.

- How many oranges do you think the canteen staff should have in the freezer at the beginning of the day?
- Think about the calculations you have made. Would the mode be a good value to rely upon?
- Do you think the canteen should have a ‘safety margin’ to avoid running out?

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

15B Individual

1 Calculate the mode, mean and median for each set of data.

a 1, 2, 2, 3, 4, 5, 5, 5, 6, 7, 9

b 2, 7, 4, 3, 5, 6, 2, 3, 4, 3

c 10, 100, 100, 10, 1000, 10, 1, 10, 1000

d 23, 12, 45, 12, 34, 33, 67, 59, 72, 43

2 Shelley and Darren kept a record of how much they spent each day on petrol, food and other expenses during their holiday.

Monday: $43.00 Tuesday: $138.00 Wednesday: $560.00

Thursday: $120.00 Friday: $43.00

a What is the mode?

b Calculate the median.

c Calculate the mean.

d How much per day should Shelley and Darren budget for their next holiday? Should they rely on the mode, mean or median? Explain your answer.

3 The students in Ms Madison's class each measured their hand span in centimetres. Here are the results.

14, 15, 13, 18, 20, 16, 17, 14

Write 'True' or 'False' to answer these statements about the hand spans.

a The mode is 20 cm.

b There is no mode.

c The mode is 14 cm.

d The median is 15.5 cm.

e The mean is 15.875 cm.

f The average is 127 cm.

4 **a** The mean of a set of numbers is 12. The sum of the numbers is 72. How many numbers are in the set?

b The mean of a set of four numbers is 10. The numbers are all different. What might the numbers be? Give at least two possibilities.

c Tahlia had five tests. Each test was marked out of 50. In the first four tests her marks were 45, 42, 43 and 48. If Tahlia's average test mark was 44, what was the mark on her fifth test?

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

15C Dot plots

A dot plot is used for count data, where one dot drawn above a base line represents each time a particular value occurs in the data.

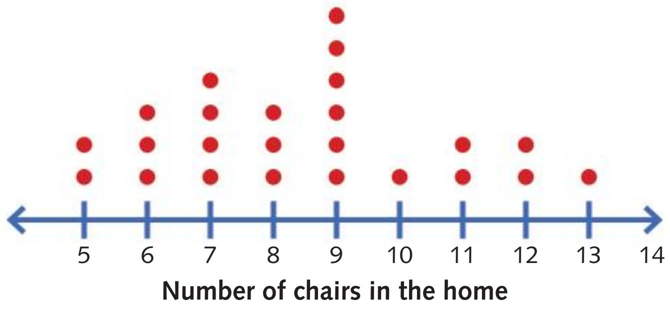
Mrs Beri's Year 6 students collected data about the number of chairs in their homes. At first, Mrs Beri wrote the numbers on the board as a list.

5, 9, 12, 4, 6, 8, 6, 5, 7, 9, 10, 9, 9, 7, 11, 8, 7, 9, 6, 11, 13, 12, 7, 8, 9

The list did not tell the students very much, so they tallied the number of times each value occurred and organised the data into a frequency table.

Value (number of chairs)	Tally	Frequency
5	||	2
6	|||	3
7	||||	4
8	|||	3
9	𝍸 |	6
10	|	1
11	|	2
12	||	2
13	|	1

The students then organised the data into a dot plot. For each number of chairs, they drew a dot above the line each time that number occurred in the data.



From the dot plot, we can see that the most frequently occurring value was 9. This means that the most frequently occurring number of chairs was 9.

Dot plots are useful when we want to see at a glance what the data shows.

We can also use the dot plot to find out how many chairs there were in total.

$$\begin{aligned}\text{Number of chairs} &= 2 \times 5 + 3 \times 6 + 4 \times 7 + 3 \times 8 + 6 \times 9 + 10 + 2 \times 11 + 2 \times 12 + 13\\ &= 10 + 18 + 28 + 24 + 54 + 10 + 22 + 24 + 13\\ &= 203\end{aligned}$$

There were 203 chairs in total.

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7

Example 4

Mr Mudge's Year 6 students recorded how many of the seven books in the *Wizard* series they had read.

7, 6, 1, 6, 0, 7, 3, 4, 5, 2, 6, 7, 7, 1, 4, 0, 2, 5, 7, 7, 7, 7, 7, 2, 0, 3

a Organise the data into a frequency table.

b Create a dot plot for the data.

Solution

a

Value (number of books)	Tally	Frequency
0	III	3
1	II	2
2	III	3
3	II	2
4	II	2
5	II	2
6	III	3
7	~~IIII~~ IIII	9

b

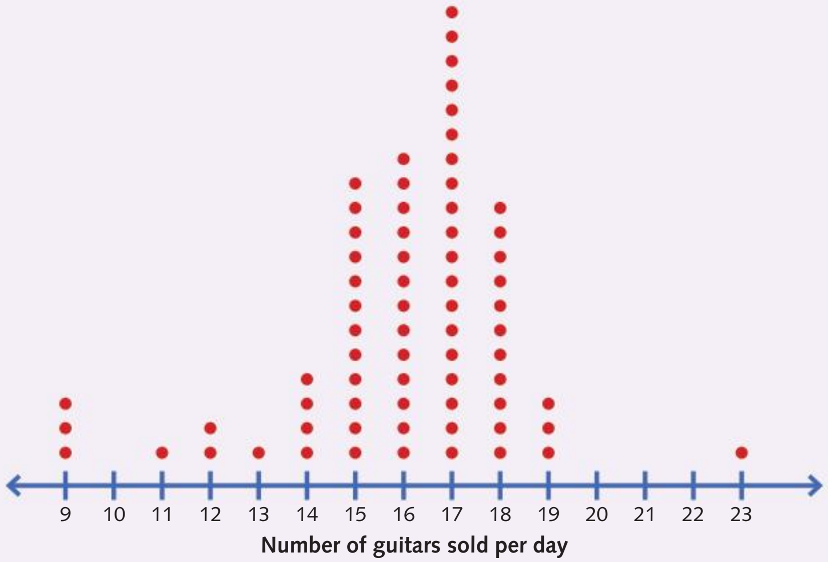
Remember

Dot plots are used for count data, where one dot represents each time the value occurs in the data.

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

15C Individual

1 Nellie sold guitars. She made a dot plot to record the number sold each day.



a What was the greatest number of guitars sold on one day?

b What was the most frequently occurring number for guitars sold in one day?

c On how many days did Nellie collect data?

d How many guitars were sold in total?

e What was the mean number of guitars sold?

2 Jake recorded the number of lolly snakes in each packet he opened.

6, 7, 8, 6, 7, 5, 7, 8, 9, 7, 8, 6, 7, 8, 6, 7, 7, 7, 8, 7, 7, 8, 7, 6, 7, 7, 6, 7, 8, 9, 8, 7, 10, 7

a Create a dot plot for this data.

b What is the most frequently occurring number of lolly snakes in a packet?

c What is the smallest number of lolly snakes found in a packet?

3 Tom counted the number of young born to some of the female nail-tail wallabies in Taunton National Park in 2001 and in 2011.

2001	2011
0, 1, 0, 2, 2, 3, 2, 4, 4, 1, 2, 1, 1, 2, 1, 2, 3, 1, 2, 0, 0, 2, 0, 2, 1, 2, 3, 1, 2, 2, 1, 1, 3, 2, 1, 1	0, 0, 0, 0, 1, 1, 2, 1, 1, 1, 1, 0, 0, 0, 0, 0, 1, 0, 0, 1, 0, 0, 1, 2, 1, 1, 0, 3, 1, 1, 0, 0, 1, 0, 1, 3

a Create a dot plot for each set of data.

b What do you notice about the number of young born in 2011 compared to 2001?

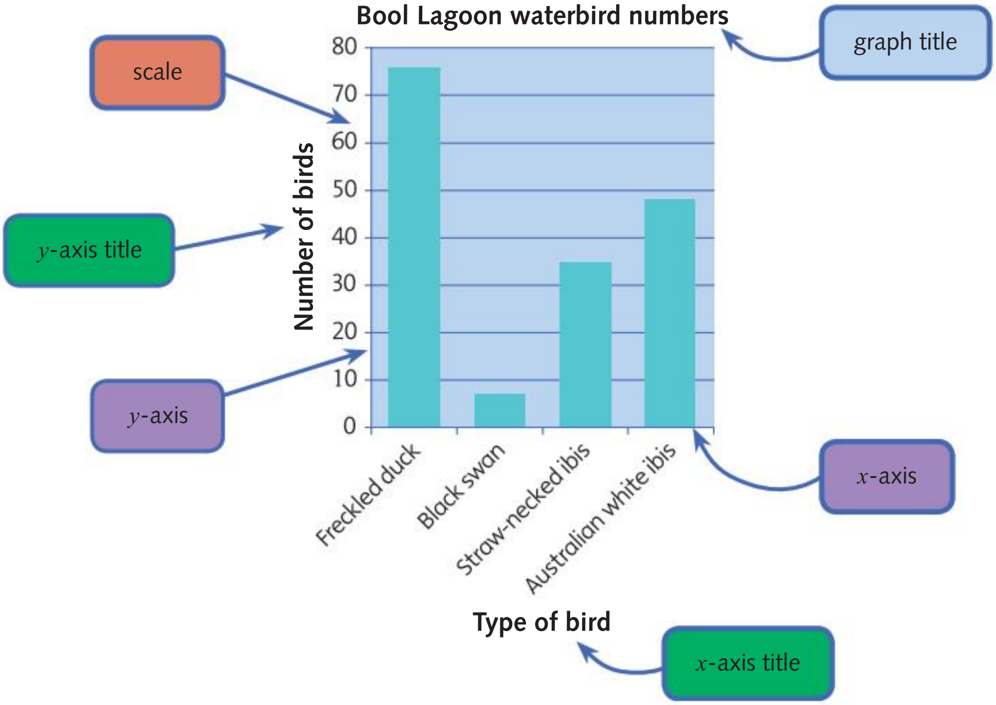
15D Column graphs

A column graph uses columns of different lengths to compare different quantities. The columns can be either vertical or horizontal, and they make it easy to compare different values. Column graphs are also known as bar graphs or bar charts.

This table shows data collected about the number of different types of birds found in a 'sample' site at Bool Lagoon, South Australia, on Christmas Day, 2006.

Bool Lagoon waterbirds	Number
Freckled duck	76
Black swan	7
Straw-necked ibis	35
Australian white ibis	48

This column graph shows the same information as the table.



There is a title at the top of the column graph so that people will know what it shows. The column graph has a horizontal axis (the x-axis) and a vertical axis (the y-axis). Each axis has a title so that you know which data are being compared. The y-axis on this chart has a scale in multiples of 10.

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

Side-by-side column graphs

Often, more than one piece of information is collected for each category. Column graphs can be used to show data from two categories on the same graph. This is useful when we want to compare data from different categories.

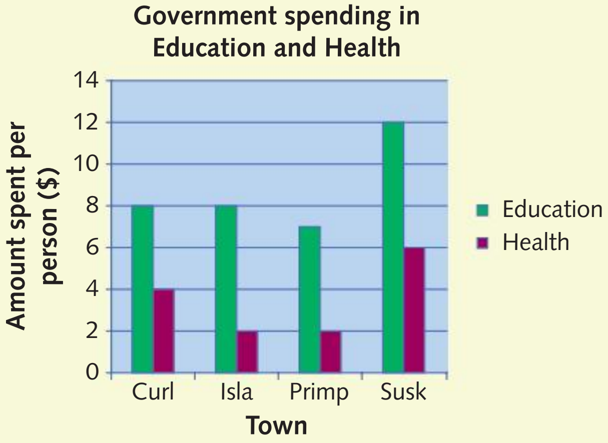
Example 5

This table gives the amount of government spending (in Australian dollars) per person on education and health in four towns for one year.

Town	Education	Health
Curl	8	4
Isla	8	2
Primp	7	2
Susk	12	6

Present this information in a side-by-side column graph.

Solution



15D Whole class CONNECT, APPLY AND BUILD

1 Collect information about the eye colour of each member of your class. Organise this information into a frequency table and display your data in a column graph.

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

2 Discuss the number of siblings that each student has. Make a 'human graph' to show the results, grouping students from left to right in an arc across the front of the room. Each group should hold a different position, as shown. Students with no siblings lie on the floor, students with one sibling sit, students with two siblings kneel, and so on.

Ask questions about your human graph, such as:

- How many children have three siblings?
- How many children have one sibling?
- How many students are there in total?
- What percentage of students are the only child in their family?

15D Individual

1 **a** Four students were surveyed about the number of hours of TV they watched each day. This table shows the data. Copy and complete the table.

Student	Tally	Frequency
Marco	||	
Anelle	𝍸 ||	
Felicity	|	
Trent	|||	

b Draw a column graph to represent the data.

2 Meagan is learning to drive. She keeps a log of the number of hours she drives the family car.

Monday: 1.5 hours Tuesday: 1 hour Wednesday: 0 hours

Thursday: 0.5 hours Friday: 1 hour

a Draw a bar chart to represent the data.

b Calculate the mean number of hours that Meagan spent driving over the week.

c If learner drivers should drive an average of 1 hour per day, how much extra time should Meagan spend driving on Saturday and Sunday to make her average equal to 1?

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

15E Line graphs

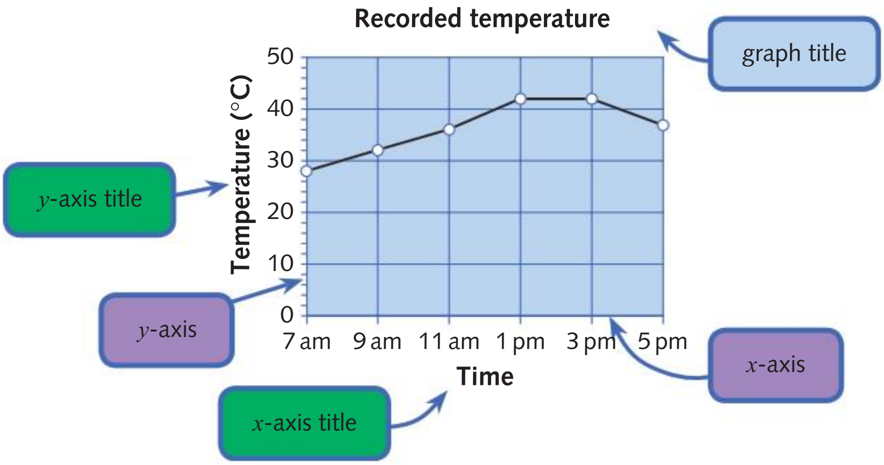
Line graphs are created by plotting points, then drawing line segments to join the points together. Line graphs are often used to display data such as temperature, where the data goes up and down (or fluctuates) over a period of time. The line joining the points gives us an idea of up and down changes.

This table shows temperatures recorded in Portland on one day in January.

Time	Recorded temperature
7 am	28°C
9 am	32°C
11 am	36°C
1 pm	42°C
3 pm	42°C
5 pm	37°C

Before we can plot any data, we need to create the axes. In this example, the x-axis will show the times, and the y-axis will show the temperatures. To plot the first piece of data, we need to draw a point where 7 am and 28°C meet on the graph. Repeat this step for 9 am and 32°C, then for the rest of the data.

If we draw a dot for each piece of data we get a series of dots like this:



Remember to name the x- and y-axes to show what is being recorded.

Joining the points to make a line graph is useful in this example, as it gives an idea of the likely temperature between the times that the data was measured.

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

Example 6

These temperatures were recorded in Mildura on one July day.

Time	0600	0700	0800	0900	1000	1100	1200	1300	1400	1500
Temperature	3°C	4°C	6°C	8°C	11°C	14°C	17°C	13°C	12°C	11°C

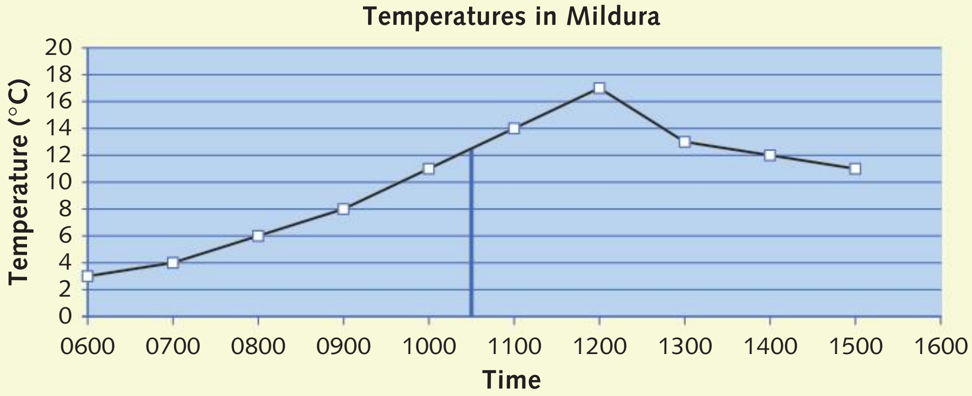
a Display the data in a line graph.

b What is the difference between the temperatures at 0600 and 1500?

c Describe what happened to the temperature over the day.

Solution

a



b At 0600 the temperature was 3°C. At 1500 the temperature was 11°C. The difference in temperature is 8°C.

c The temperature rose gradually until 1200, then it dropped by 4°C in one hour. It then continued to drop gradually, but not as fast.

15E Whole class CONNECT, APPLY AND BUILD

1 You will need to download **BLM 8** 'Graphing temperature' from the Interactive Textbook. Without looking at the information on the BLM, fold it back along the dashed line so that only the graph is visible. Discuss what the graph might be displaying.

a What elements would we need to add to understand the graph?

b Reveal the data table.

c Suggest a title for the graph, and titles for the x- and y-axes, then label them.

d Where should the days be written?

e Suggest the temperature scale.

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

15E Individual

1 **a** Harriet drew up the following data table for the money she earned from her holiday job.

Draw a line graph to show the data. Write a title for the graph, and label the x- and y-axes.

Money earned						
Hours	1	2	3	4	5	6
Wages ($)	14	28	42	56	70	84

b How much money did Harriet earn in 4 hours?

c How much money did Harriet earn in 10 hours?

d How much money did Harriet earn in 3.5 hours?

2 The Lancaster family have water tanks as their only source of water.

At 4 am, one of their tanks had 300 litres in it. After it rained from 5 am until 6 am, the volume of water in the tank was 400 litres.

Between 7 am and 8 am the family woke and got ready for the day. They used 90 litres for showers, 35 litres for flushing the toilet, 4 litres for washing the dishes and 1 litre for cooking breakfast.

At 8 am the Lancasters went to work and school.

It rained from 2 pm to 3 pm and the tank received 70 litres of water.

At 5 pm the Lancasters returned home. They used 1 litre to make coffee and tea.

At 6 pm they watered their garden, using 74 litres of water.

a Copy and complete the following table.

Time	4 am	5 am	6 am	7 am	8 am	2 pm	3 pm	5 pm	6 pm
Amount									

b Present this data in a line graph by plotting the points and joining them with line segments.

c What was the volume of water in the tank at 8 am?

d How much water was in the tank at 6 pm?

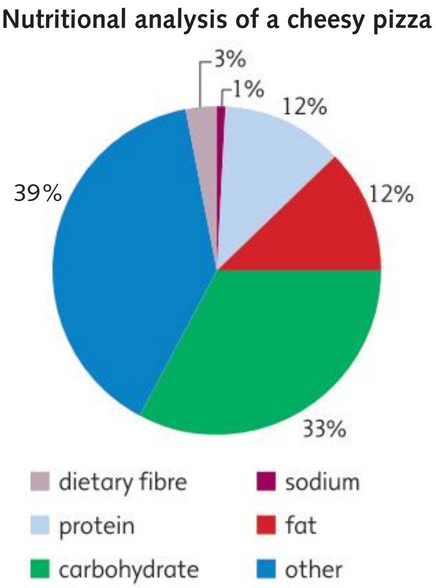
ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

15F Pie charts

Pie charts are another way to represent data. Just as the name suggests, a pie chart is like a whole pie that has been cut into different portions, or pieces.

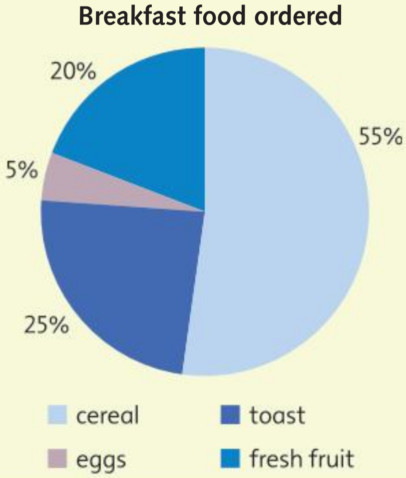
Pie charts are often constructed using percentages. The size of each slice of pie is in proportion to its percentage of the whole pie. For example, 25% will be represented by a slice that is 25% of the whole pie, and 50% will be represented by a slice that is 50% of the whole pie.

This pie chart represents the nutritional value of a pizza. From the pie chart we can see that 33% of a cheesy pizza is carbohydrate and 12% of it is fat.



Example 7

This pie chart shows the main breakfast dish ordered by students from Karratha School at their school camp. What percentage of students did *not* eat cereal for breakfast?



Solution

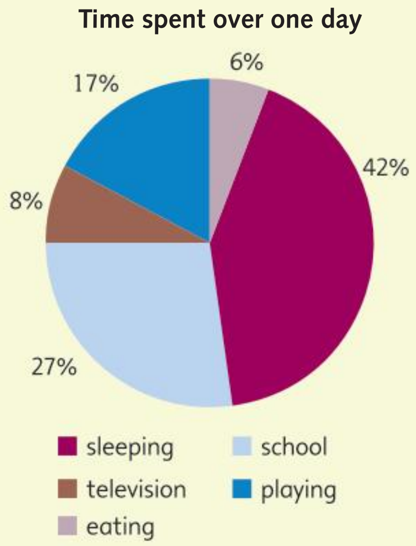
We can see that 55% of students ate mainly cereal. The percentage that did not eat cereal is 100% – 55% = 45%. So 45% of students did not eat cereal.

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

Example 8

Patrick wanted to know how much time he spent on different activities over the course of one day. He timed each activity, then made a pie chart to show the percentage of time he spent on each activity over one day.

How much more time did Patrick spend sleeping than at school?



Solution

Patrick spent 42% of his time sleeping and 27% of his time at school.

42% – 27% = 15%

Patrick spent 15% more of his time sleeping than at school.

15F Whole class CONNECT, APPLY AND BUILD

1 This pie chart shows sales of pasta at the Top Value Supermarket.

Use the information in the pie chart to answer the questions.

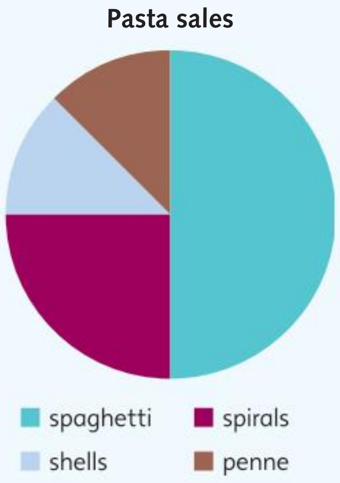
a Which was the best-selling pasta?

b Half of the pasta sold was spaghetti. This is 50% of the total sales. What percentage of sales was spiral pasta?

c Which two types of pasta sold the same amount?

d What percentage of sales was shells?

e Top Value Supermarket sold 24 packets of spaghetti. How many packets of spirals, shells and penne did they sell?



ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

15F Individual

1 Hannah recorded how much time she spent on different activities over 24 hours. She presented her data as a pie chart, recording whole hours instead of percentages.

a How many hours were spent *not* sleeping?

b Which two activities used the same amount of time?

c How many hours did Hannah spend reading and relaxing?

d Write the amount of time Hannah spent relaxing as a percentage of her whole day.

2 This pie chart shows data about animals cared for at the Jabiru animal shelter.

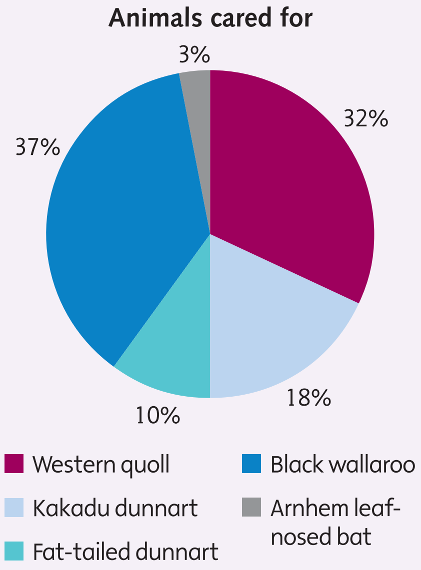
a If there are 100 animals, how many fat-tailed dunnarts are there?

b If there are 100 animals, how many Arnhem leaf-nosed bats are there?

c If there are 100 animals, how many black wallaroos are there?

d If there are 200 animals, how many Kakadu dunnarts are there?

e If there are 50 animals, how many western quolls are there?



3 Create a pie chart to represent this data.

Sources of home entertainment

Television	Computer	MP3 player	Radio	Books
50%	10%	10%	5%	25%

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

15G The statistical data investigation process

Sometimes, the way data is presented in the media can give us the wrong impression about the information being presented. When a media report says ‘Statistics have shown …’ you should be asking questions such as the following.

- Which statistics are being quoted?
- How was the data collected?
- How large was the sample?
- Are the results being reported accurately?

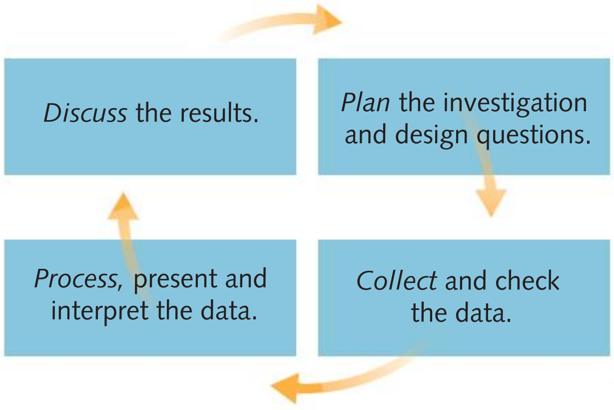
It is important to develop good skills with statistics and data so that you can understand how statistics are used in newspapers and on television. Using and understanding statistics plays an important part in many careers, from football statistician to biologist. Statistics are used to understand the world we live in.

Earlier in this chapter we described three types of data. There is:

- data that we can *count*, such as the number of jelly beans in a packet
- data that we can *measure*, such as the height of students in your class
- data that belongs in *categories*, such as hairstyles or the colour of cars.

The best way to understand how data is collected and presented is to carry out some data collection and presentation activities yourself. Some of the different ways we can organise, present and discuss data have been explained throughout this chapter. Now we are going to put everything together so that we can plan data collection, investigation and interpretation activities in an organised way. We will call this the **statistical data investigation process**. Though there are many ways to collect and organise data, we will use the following steps.

- *Plan:* We plan the investigation, design questions to start the investigation and identify the types of data that could be involved.
- *Collect:* We collect and check the data.
- *Process:* We present and interpret the data.
- *Discuss:* We discuss the results.



ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

Plan: Identifying issues and planning the investigation

In the first part of the data investigation process, we decide on the topic we want to investigate and the issues related to that topic. Then we design questions that help us find out about the issues.

We ask the following questions.

- What do we want to find out about?
- What data can we get?
- How do we get the data?

Let's look at this step through an example.

Ms Draper's class 6D read a newspaper report about the longest strip of paper torn from a lolly wrapper. The students wonder if they could collect their own data.

Their investigation will involve measurement data, as the length is the important idea here. To collect the data, they will ask each student in the class to tear a piece of paper into the longest strip they can, and measure the strip.

They decide to give each student a 5 cm × 5 cm sheet of paper.

Before they collect the data, the students identify several issues.

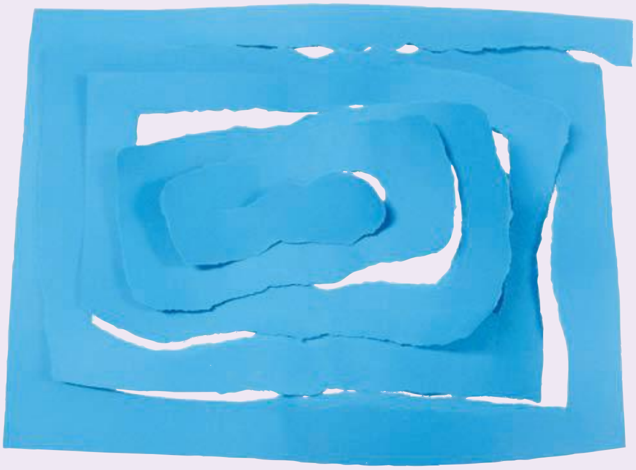
- There might be different ways to tear the strip.
- They may need to allow a certain number of attempts.
- Who will check the measurements?
- What tools, if any, can be used?

The students decide on the following rules.

- The strip can be torn in any way the person decides.
- No tools other than the hands can be used to tear the strip.
- Each person can have three attempts. If the strip breaks, then that measurement is recorded as 0. Data for up to three attempts may be submitted.
- One other person must also measure the strip to check the measurement.

The class create their questions.

- What is the longest continuous strip of paper than can be torn from a 5 cm × 5 cm piece of paper by people in our class?
- What is the mode and mean for this data?
- How can we use this investigation to practise what we have learnt about data collection and investigation?



ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

Next, the students decide what data is to be collected. The name of each person, the length of their measurement to the nearest centimetre and the name of the 'checker' will be recorded in a table.

Collect: Collecting, handling and checking data

Once we have decided what to collect and how it will be collected, we can proceed with the data collection. Tables with tallies are a useful way to collect data.

The data collected by 6D looked like this

Name	Length of strip (cm)	Checker
Amy	37, 0, 35	Kylie
Arthur	29, 45, 35	Ben
Ben	28, 34, 29	Amy
Buddy	35, 40, 29	Eric
Carlos	35, 32, 35	Buddy
Carly	0, 36, 40	Joe
Eric	30, 28, 33	Tony
Joan	32, 35, 27	Paul
Joe	28, 33, 35	Arthur
Kasey	33, 30, 0	Sarah
Kylie	38, 30, 39	Carlos
Marcia	34, 0, 28	Renee
Nick	37, 35, 36	Pete
Nina	0, 37, 35	Carly
Patsy	37, 35, 30	Nina
Paul	32, 30, 34	Nick
Pete	35, 34, 37	Marcia
Renee	30, 38, 32	Kasey
Sarah	0, 0, 33	Joan
Tony	35, 36, 30	Patsy

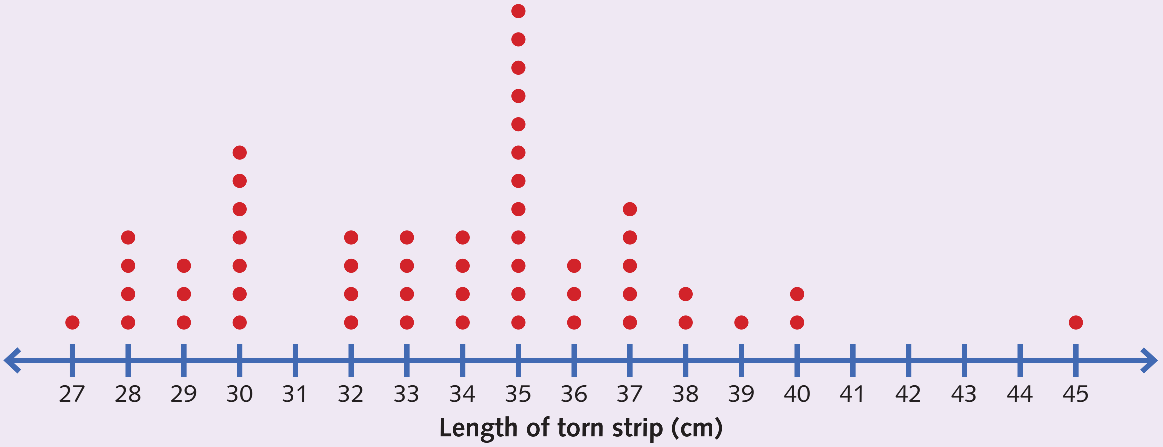
Process: Exploring and interpreting data

We need to organise the data using frequency tables and present it in a way that helps us understand it. We can work out the mode, median and mean, and make statements from the data that help us to understand it.

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

The students in 6D created a frequency table and dot plot for their data.

Length of strip	Tally	Frequency
0	~~||||~~ ||	7
27	|	1
28	||||	4
29	|||	3
30	~~||||~~ ||	7
31		0
32	||||	4
33	||||	4
34	||||	4
35	~~||||~~ ~~||||~~ ||	12
36	|||	3
37	~~||||~~	5
38	||	2
39	|	1
40	||	2
41		0
42		0
43		0
44		0
45	|	1



The students saw from the dot plot that the mode was 35. This means that the most frequently occurring value for the length of the torn paper strip was 35 cm.

They also observed that the longest length of paper strip torn from a 5 cm × 5 cm square was 45 cm, and congratulated Arthur on his effort.

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

The mean for the data from Ms Draper's class was calculated by adding each value and dividing by the number of values. The zero values were not included.

$$\begin{aligned}\text{Sum of values} &= 37 + 35 + 29 + 45 + 35 + 28 + 34 + 29 + 35 + 40 + 29 + 35\\ &\quad + 35 + 35 + 36 + 40 + 30 + 28 + 33 + 32 + 35 + 27 + 28 + 33\\ &\quad + 35 + 33 + 30 + 38 + 30 + 39 + 34 + 28 + 37 + 35 + 36 + 37\\ &\quad + 35 + 37 + 35 + 30 + 32 + 30 + 34 + 35 + 34 + 37 + 30 + 38\\ &\quad + 32 + 33 + 35 + 36 + 30\\ &= 1785\end{aligned}$$

$$\begin{aligned}\text{Mean} &= \frac{\text{sum of values}}{\text{number of values}}\\ &= \frac{1785}{53}\\ &= 33.679\,24\ \ldots\ \text{cm or } 34\text{ cm}\ \ (\text{rounded to the nearest centimetre})\end{aligned}$$

Discuss: Discuss the results and pose new questions that arise from them

Once we have made some conclusions from the data, we might realise that there are further questions to be answered. We might organise the existing data in a different way or collect more data. At this point, it is important to discuss the use of graphs: were the graphs chosen the best for this purpose?

The students in 6D realised that including the zero values (obtained when a paper strip broke) would make their mean lower. They knew that pie graphs and line graphs were not appropriate choices for presenting this data.

15G Whole class CONNECT, APPLY AND BUILD

These activities are suggestions for whole-class investigations using the statistical data investigation process. Investigations that are designed by the participants from real situations in their immediate surroundings might have more meaning and are encouraged.

1 Favourite television programs

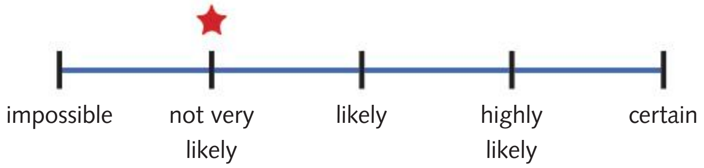
Do boys and girls have different preferences?
Do young children watch different programs to older children?

2 Hours spent doing after-school activities

Is there a particular day of the week where more activities are done by more people?

15H Probability

What is the chance that the Prime Minister will walk into your classroom in the next 5 minutes? You might say that there is 'not a very good chance'. Or you might rate the chance 'not very likely' on a scale like this:



In mathematics, we use the word **probability** to describe the chance of an event occurring (or taking place).

Writing probabilities

You can draw a **probability scale** (like the one shown above) using numbers instead of words. The probability of an event happening is expressed on a scale from 0 to 1.

An event that is rare has a probability close to 0, while an event that is very common has a probability close to 1.

Events that will not happen have a probability of 0. Events that are certain to happen have a probability of 1.

Events that happen sometimes have a probability between 0 and 1, and are often written as fractions. The numerator is the number of ways the event can happen, and the denominator is the total number of possibilities:

$$\text{Probability of an event} = \frac{\text{number of ways the event may happen}}{\text{total number of possibilities}}$$

For example, the probability of rolling a 3 on a dice is 'one in six' or $\frac{1}{6}$. This is because there is only 1 way to roll a 3, but 6 different numbers could show up.

The probability of rolling an odd number on a dice is 'three in six' or $\frac{3}{6}$, as there are 3 ways to roll an odd number out of 6 different numbers that could come up.

Example 9

If we roll a 6-sided dice numbered from 1 to 6, what is the probability of rolling:

a the number 7?

b number from 1 to 6?

c the number 2?

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

Solution

a $\dfrac{\text{number of ways event may happen}}{\text{total number of possibilities}} = \dfrac{0}{6}$ (7 is not on the dice) (6 numbers on the dice)

The probability of a 7 appearing = 0

b $\dfrac{\text{number of ways event may happen}}{\text{total number of possibilities}} = \dfrac{6}{6}$ (1 to 6 are all on the dice) (6 numbers on the dice)

The probability of a number from 1 to 6 appearing = $\dfrac{6}{6} = 1$

c $\dfrac{\text{number of ways event may happen}}{\text{total number of possibilities}} = \dfrac{1}{6}$ (1 number 2 on the dice) (6 numbers on the dice)

The probability of a 2 being rolled = $\dfrac{1}{6}$

Probabilities can also be written as decimals, as the next example shows.

Example 10

There are 10 marbles in a bag. The marbles are different colours. There are 4 blue marbles, 3 green marbles, 2 red marbles and 1 white marble. Suppose that you select one of the marbles without looking. What is the probability of selecting:

a a blue marble? Write your answer as a decimal.

b a white marble? Write your answer as a decimal.

Solution

a $\dfrac{\text{number of ways event may happen}}{\text{total number of possibilities}} = \dfrac{4}{10}$ (4 blue marbles) (10 marbles in the bag)

$= 0.4$

So the probability of picking a blue marble is 0.4.

b $\dfrac{\text{number of ways event may happen}}{\text{total number of possibilities}} = \dfrac{1}{10}$ (1 white marble) (10 marbles in the bag)

$= 0.1$

So the probability of picking a white marble is 0.1.

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

Expected outcomes

Sometimes the probabilities we predict are not what we see when we do the actual experiment.

For example, if we were to conduct the marble experiment as in **Example 10** (on the previous page) we might *expect* to select a blue marble 4 out of 10 times, but we might observe a different result.

15H Individual

1 Rolling one normal die

Predict how many times you would expect each number on a 6-sided die to come up if you rolled the die 30 times. Write down your predicted probability as a fraction between 0 and 1.

Now carry out the experiment, working in groups of four. Roll the die 30 times and record the results in a table. Each time a number is rolled, record it by drawing a tally mark in the table. After 30 rolls, record the total number of times each number was rolled.

Number	Predicted probability	Tally	Total	Total as fraction out of 30
1				
2				
3				
4				
5				
6				

Discuss your results. Did your prediction match your results? What happens if you roll the die 60 times?

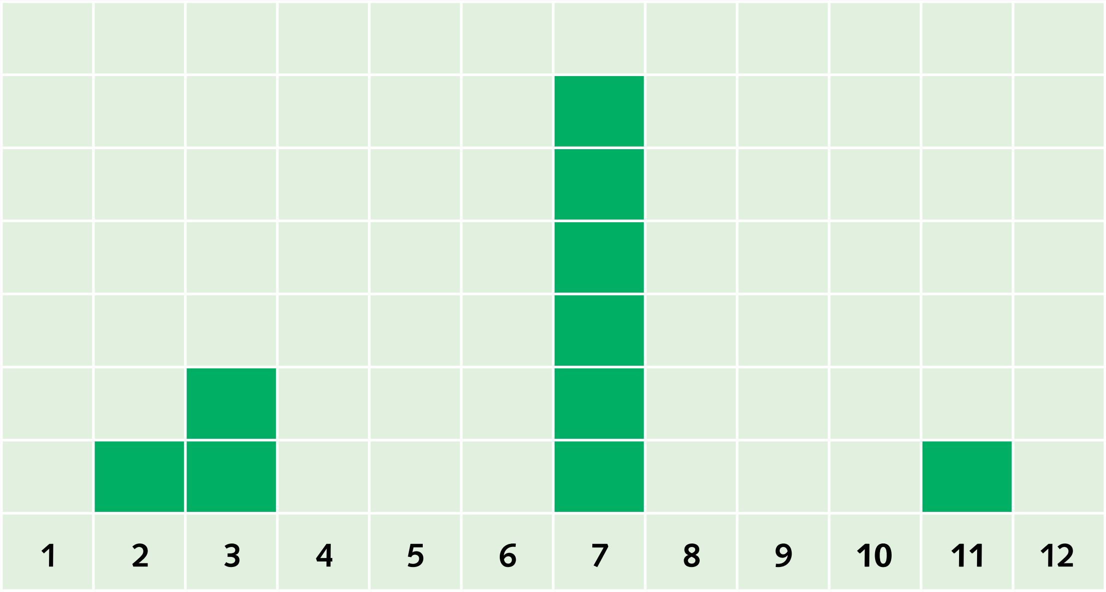
2 The sum of two dice

a Jenni is trying a dice experiment. She is rolling two dice then finding their sum. She says that 6 is the most common sum from rolling two dice. Make a prediction. Do you think Jenni is right? If not, which sum do you think will be the most common?

b Do the experiment. Roll two dice 30 times, recording their sum in a chart like the one on the following page. Colour a box on the chart each time the sum is rolled.

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

This chart shows that for the first 10 throws: 2 was rolled once, 3 was rolled twice, 7 was rolled six times, and 11 was rolled once.



c Do the maths: in how many different ways is it possible to make each number from 2 to 12 by adding the numbers on the faces of two 6-sided dice? Copy and complete this chart. The different ways to make 2, 6 and 10 have been done for you.

				5 + 1						
				4 + 2						
				3 + 3						
				2 + 4						
1 + 1				1 + 5						
2	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**

d How does the chart in part **c** compare to your experiment in part **b**? Why do you think that some people believe that 7 is a lucky number?

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

3 Tossing coins

a Make a prediction. If you tossed two coins 40 times, how often might you throw 2 tails?

b Toss two coins 40 times. Keep a tally, then write down the total number of times you tossed each combination.

	Tally	Total
2 heads		
1 tail, 1 head		
2 tails		

c Compare your results with your classmates' results.

d Do the maths: How many different combinations are possible when tossing two coins?

Copy this two-way table and use it to help you.

		Coin 1	
		Head	*Tail*
Coin 2	*Head*	Head, Head	
	Tail	Tail, Head	

15I Review questions

1 Mike collected golf balls at the local golf course and sold them back to the players for $1 each. This table shows the number of golf balls he collected each day for two weeks.

	Sun	Mon	Tue	Wed	Thu	Fri	Sat
Week 1	1	2	3	4	5	6	7
Week 2	1	2	1	2	0	1	28

For each week, calculate:

a the median number of balls collected

b the mean number of balls collected

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

2 Year 6 students at Barry Street School took part in a survey. Each student was asked two questions:

- How many hours of TV did you watch last week?
- How many hours did you use a computer last week?

Here are the results.

TV hours																				
4	7	21	5	8	3	43	13	14	7	5	8	19	4	11	10	13	12	14	14	17
Computer hours																				
7	6	9	3	5	12	14	32	22	12	13	9	7	8	4	11	10	12	14	12	9

a How many children were interviewed?

b What is the mode for TV watching?

c What is the mode for computer use?

d Calculate the mean number of hours for each data set.

e What is the mean number of hours spent either watching TV or using a computer?

f Calculate the median number of hours of television watched.

g Calculate the median number of hours children used a computer.

3 Mika wanted to sell a doll on the internet. She watched as the price of the doll increased, and recorded the price each half hour until the auction closed at 7 pm.

Time	Price
3.00 pm	$2.00
3.30 pm	$2.00
4.00 pm	$3.00
4.30 pm	$3.00
5.00 pm	$18.00
5.30 pm	$19.00
6.00 pm	$21.00
6.30 pm	$37.00
7.00 pm	$42.00

a Show this data on a line graph.

b What was the final price for the doll?

c What happens to the line of the graph when the price remains constant?

d What was the price at 4:30 pm?

e If Mika checked the price at 6:15 pm, what price might she have seen?

4 **a** Find out how many children are in each class at your school.

b Draw up a table to show your data.

c Draw a bar graph to represent your data.

CHAPTER 16 Preparation

Useful skills for this chapter:

- a solid grasp of arithmetic
- an understanding of inverse operations.

KICK OFF

Daniel and Mark have some pets.

- They each have the same number of pets.
- The two boys own 9 parrots between them. The rest of their animals are lizards.
- Each parrot has 2 legs. Each lizard has 4 legs.
- Mark's animals have 2 more legs in total than Daniel's.
- Daniel owns the same number of parrots as lizards.
- If Mark gave Daniel a lizard, Daniel's animals would have 28 legs in all.
- How many parrots and how many lizards does each boy have?

Show what you know

Work these out 'in your head'.

1 I am a number. Twice me is 24. Who am I?

2 I am a number. Three times me is 39. Who am I?

3 I am a number. Six added to me gives 13. Who am I?

4 I am a number. Double me and add 2 gives 18. Who am I?

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

CHAPTER

16

Number and Algebra

Algebra

In this chapter we look at an important part of mathematics called **algebra**. Algebra uses letters to stand for numbers. It helps us to write about mathematical ideas simply, and it helps us to solve problems that would otherwise be very difficult.

Many of the ideas in algebra are based on what you already know about numbers. Learning algebra is just like learning any new language: there are new words to learn and you will need to practise different ways of putting those words together.

The first book about algebra was written by Al-Khwarizmi in about 830 AD. Al-Khwarizmi was a scholar at the House of Wisdom, in Baghdad, and his work was influential in introducing algebra into Europe in the 13th century. Scholars at the House of Wisdom translated Greek scientific manuscripts and wrote about algebra, geometry and astronomy. Al-Khwarizmi's algebra book, *Hisab al-jabr w'al-muqabala*, was the most famous and most important of all of his works. Our word 'algebra' comes from the title of his book. We get the word 'algorithm' from his name.

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

16A Finding an unknown

When we are told that $\square + 6 = 18$, we usually read this as ‘Something plus 6 is 18.’ The $\square$ stands for a number, but what is it? You can probably solve this in your head by now because you know that $12 + 6 = 18$.

Or you could start with something that you are sure of, such as $10 + 6 = 16$, then go up 2 more: $12 + 6 = 18$. You can probably think of other ways of finding out what the $\square$ represents.

Example 1

$\square - 2 = 14$ What is $\square$?

Solution

Taking away 2 from $\square$ gives 14. This means that $\square$ must be 2 more than 14.

So $\square = 16$.

We can often use mathematical symbols to write what we have been told. This is an important skill.

For example: Martine said, ‘If you add 3 to my age, you get 12’. What is Martine’s age?

You can probably solve this in your head, but we will practise using $\square$ to represent Martine’s age instead.

Martine has already said that adding 3 to $\square$ gives 12. If we use the + sign, this becomes:

$3 + \square$ is 12

Now replace ‘is’ with the = sign. You get:

$3 + \square = 12$

From this you can work out that $\square = 9$.

Example 2

I am a number. Four times me is 28. What am I?

Solution

You can do this in your head by remembering your tables:

4 × 7 = 28. So the number is 7.

We can also write this information in another way, using □ to represent the unknown number:

4 × □ = 28 What number is □?

We can then answer this by using our tables:

4 × 7 = 28, so □ = 7.

Another way is to use division:

4 × □ = 28, so □ = 7 because 28 ÷ 4 = 7.

The number is 7.

Remember: division is the inverse of multiplication, so you can use division to solve multiplication problems like this one.

16A Whole class CONNECT, APPLY AND BUILD

1 Find the number that □ represents in each question. Then discuss the different methods people used to work out □.

a □ + 6 = 12

b □ + 8 = 19

c □ + 3 = 37

d □ − 2 = 8

e □ − 8 = 10

f 2 × □ = 10

g 3 × □ = 18

h □ ÷ 2 = 7

i □ ÷ 4 = 12

j □ + 12 = 52

k □ + 14 = 64

l □ × 2 + 18 = 68

2 Use □ to represent the number you need to find. Use +, −, ×, or ÷ and the = sign to write down what you are told about □, then find out what □ represents.

a Ian's age plus 4 is 14.

b Laura's sister's age plus 6 is 12.

c 3 added to David's weight in kilograms is 30.

d Celia has a box with a number of chocolates in it. Her friend Tammy ate 2 chocolates and there were 10 left.

3 A group of children was on a bus. Three children got off the bus at the first stop. At the next stop, 12 children got on the bus. There are now twice as many children on the bus as there were at the start. How many children are on the bus?

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

16B Pronumerals

Finding the value of a pronumeral

Until now, we have been using □ to represent the missing number. In algebra, instead of using □, we use a letter like x, y, a or b to represent the number we want to find.

When we use a letter in place of a number, that letter is called a **pronumeral**. The word 'pronumeral' is made up from the words *pro*, which means 'for' and *numeral*, which is another word for 'number'. So pronumeral means 'stands for a number'. Sometimes the word **variable** is used.

Example 3

x is a number and $x + 2 = 5$. Find x.

Solution

We can do this in our head: $x = 3$. Or we can count back 2 from 5 to get $x = 3$.

Example 4

I am a number. When 6 is added to me, the answer is 14. Who am I?

Solution

We usually begin by saying, 'Let x be the unknown number' or 'Let x be the number we have to find'. Then we write down what we are told about the number x. Here we are told that $x + 6$ is 14. So we write:

$$x + 6 = 14$$

Now we have to find x.

We need to think: What number plus 6 equals 14?

So $x = 8$

We check our solution by using 8 in the 'Who am I?'

I am the number 8.

When 6 is added to me, the answer is 14.

This statement is true, so 8 is correct.

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

Substituting a value for a pronumeral

The word *substitute* means 'put in the place of another'. When we use substitution in algebra, we are putting a number in place of a pronumeral.

This is like football or soccer players sitting on the bench until they are asked to take the place of another player. We call these players substitutes.

Let's think about these statements. Mark has five bags of potatoes.

If each bag has 20 potatoes, then Mark has $5 \times 20 = 100$ potatoes.

If each bag has 25 potatoes, then Mark has $5 \times 25 = 125$ potatoes.

If each bag has 12 potatoes, then Mark has $5 \times 12 = 60$ potatoes.

We can use some algebra as a shorthand way of describing the answer in general for the total number of potatoes.

If Mark has 5 bags, each containing y potatoes, then the total number of potatoes is $5 \times y$. Isn't that neat?

Once we know the total number of potatoes is $5 \times y$ when there are y potatoes in each bag, we can work out the total number in any particular case.

Example 5

If Mark has 5 bags of potatoes we can use algebra to write the total number of potatoes as $5 \times y$. How many potatoes does Mark have if:

a $y = 7$?

b $y = 142$?

Solution

a We put 7 in place of y.

If $y = 7$, then $5 \times y = 5 \times 7$

$= 35$

b We put 142 in place of y.

If $y = 142$, then $5 \times y = 5 \times 142$

$= 710$

Example 6

Find the value of $2 \times y$ when y is equal to 6.

Solution

Substitute 6 for y.

$$\begin{aligned} 2 \times y &= 2 \times 6 \\ &= 12 \end{aligned}$$

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

We can also use algebra to describe situations that involve more than one operation. When we are dealing with pronumerals in algebra, we need to use the same rules of arithmetic as we use for whole numbers.

Example 7

Terri had 17 football cards. Then she bought 5 more packets. Each packet had the same number of football cards. Now Terri has 217 football cards.

a How many football cards were in each packet?

b Use algebra to write a statement about the total number of cards.

Solution

a This situation involves addition and multiplication, so we can use subtraction and division to 'undo' these operations. We can do this mentally.

Terri ended up with 217 cards. She started with 17. If we subtract 17 from 217 we will find out how many cards Terri bought:

$217 - 17 = 200$

Terri bought 200 cards in 5 packets.

$200 \div 5 = 40$, so there are 40 cards in each packet.

b $17 + (5 \times x) = 217$

16B Whole class CONNECT, APPLY AND BUILD

1 Find the value of each pronumeral.

a $x + 3 = 9$ **b** $x + 4 = 14$ **c** $q + 1 = 7$ **d** $z + 8 = 18$

e $y - 2 = 7$ **f** $y - 1 = 4$ **g** $a - 2 = 13$ **h** $2 \times b = 14$

i $2 \times a = 16$ **j** $3 \times r = 24$ **k** $x \div 2 = 2$ **l** $x \div 2 = 4$

2 Use pencil cases and pencils to act out this story in class. There are 4 pencil cases, each with the same number of pencils inside. We can write $4 \times k$ to describe the number of pencils.

a If there are 5 pencils inside each pencil case, this means that k is equal to 5. How many pencils do we have?

b If there are 7 pencils inside each pencil case, how many pencils do we have?

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

3 At Thomas Street Primary School there are 4 classes. Each class has the same number of children in it. There are 6 staff members who work at the school. The total number of children and adults in the school is 106.

a How many children are in each class?

b Use algebra to write a statement about the number of children and adults at the school.

16 B Individual

1 Write these statements using algebra. Use +, −, × or ÷ and the = sign, and use a pronumeral to replace the word ‘something’.

a Something plus 4 is 11.

b Something take away 5 is 9.

c Two lots of something is 12.

d Something shared between two is 4.

2 Write these statements in words.

a $12 - x = 6$

b $4 + p = 5$

c $21 \div z = 3$

3 Find the value of the pronumeral.

a $12 + d = 20$

b $w - 5 = 15$

c $2 \times f = 20$

d $8 - u = 5$

e $47 + t = 50$

f $100 \div m = 4$

g $5 \times s = 50$

h $1000 - x = 1$

i $200 - b = 125$

4 Kaitlyn’s favourite biscuits are sold in packets of 4.

a Copy and complete the table on the right.

b If Kaitlyn buys 2 packets of biscuits, how many biscuits is this in total?

c If Kaitlyn buys 3 packets of biscuits, how many biscuits is this in total?

d If Kaitlyn buys 8 packets of biscuits, how many biscuits is this in total?

e If Kaitlyn buys 30 packets of biscuits, how many biscuits is this in total?

f If Kaitlyn buys 200 packets of biscuits, how many biscuits is this in total?

g If Kaitlyn buys n packets of biscuits, how many biscuits is this in total?

Number of packets	Total number of biscuits
1	4
2	8
3	
4	16
5	
10	40
20	
100	
n	$4 \times n$

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

5 Sonja is 9 and her brother Jared is 16. Sonja drew up a table to compare their ages as they get older.

Sonja's age	Jared's age
9	16
10	17
11	18
12	19
13	20

a When Sonja is 10, how old will Jared be?

b When Sonja is 13, how old will Jared be?

c When Sonja is 14, how old will Jared be?

d When Sonja is 18, how old will Jared be?

e When Jared is 24, how old will Sonja be?

f When Sonja is n years old, how old will Jared be?

g When Jared is p years old, how old will Sonja be?

h **Challenge**: When Jared was twice Sonja's age, how old was Sonja?

6 Tommy is 11 years old.

a How old will he be in 5 years' time?

b How old will he be in x years' time?

7 Cheryl needs to save $50 over 12 months. She has already saved $14.

a How much does she need to save each month?

b Use algebra to write a statement you could use for this situation.

8 Scott saved the same amount of money each week for 6 weeks. Then his grandfather gave him $34. Now Scott has $100.

a How much money did Scott save each week?

b Use algebra to write a statement you could use for this situation.

9 The Mitchell family use rainwater from their tank to water their garden. At the start of June, their tank had 2800 litres in it. After 13 days, the tank held 6700 litres. During this time, it rained regularly and the Mitchells did not need to water the garden. Each day, the tank collected the same amount of water. How much water was collected each day?

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

16C Number patterns

Think of two numbers and then add them. Now add them in reverse order. What do you notice? We can do this for other pairs of numbers and observe that, whatever order we add the two numbers, the result is the same.

$$26 + 4 = 4 + 26$$

$$100 + 3 = 3 + 100$$

$$27 + 96 = 96 + 27$$

We know that this is true for any pair of whole numbers. This is the any-order property for addition.

We can use the language of algebra to describe situations like this in mathematics. Algebra describes – in very few letters, words and symbols – situations when something is always true.

To describe the any-order property for addition, we use letters in the place of whole numbers and write:

$a + b = b + a$, where a and b are any whole number

This means that when two numbers are added, the order in which they are added does not matter – we get the same result.

Notice that the order does matter for subtraction. For example, 4 – 2 is not the same as 2 – 4.

Patterns often occur in mathematics and they can be described using algebra. For example, the next number in the sequence:

2, 4, 6, 8, 10, …

is given by $s + 2$, where s stands for any number already in the sequence.

We could also form the sequence 2, 4, 6, 8, … by doubling 1, 2, 3, 4, and so on. That is:

- the 1st term is $2 \times 1 = 2$
- the 2nd term is $2 \times 2 = 4$
- the 3rd term is $2 \times 3 = 6$
- the 4th term is $2 \times 4 = 8$, and so on.

The nth term of the sequence 2, 4, 6, 8, … would be $2 \times n$.

Example 8

Sanjit makes a pattern by multiplying by a number and adding.

The first term in Sanjit's pattern is the number 5, the second term is the number 7, and the third term is 9. Complete the table and write the rule for the nth term.

	Number
1st term	5
2nd term	7
3rd term	9
4th term	11
5th term	
6th term	
7th term	17
8th term	
nth term	

Solution

The 1st term is $2 \times 1 + 3 = 5$
The 2nd term is $2 \times 2 + 3 = 7$
The 3rd term is $2 \times 3 + 3 = 9$
So the 5th term is $2 \times 5 + 3 = 13$
The 6th term is $2 \times 6 + 3 = 15$
The 8th term is $2 \times 8 + 3 = 19$
The nth term is $2 \times n + 3$

So the rule that lets you work out any number in Sanjit's pattern is:

To find the nth term, double n and add three.

	Number
1st term	5
2nd term	7
3rd term	9
4th term	11
5th term	13
6th term	15
7th term	17
8th term	19
nth term	$2 \times n + 3$

16C Individual

1 Write the next five numbers in each sequence.

a 1, 3, 5, 7, 9, 11, 13, …

b 10, 20, 30, 40, 50, …

c 10, 9, 8, 7, 6, 5, 4, 3, 2, 1, …

d 2, 4, 8, 16, 32, …

e 130, 115, 100, 85, 70, …

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

2 A teacher creates a pattern for her class. The pattern begins:

1, 5, 9, 13, 17, 21

Which number is the next in the pattern?

A 23 **B** 24 **C** 25 **D** 26

3 A pattern begins: 1, 3, 5, 7, 9, 11.

What is the sum of the first 8 numbers in this pattern?

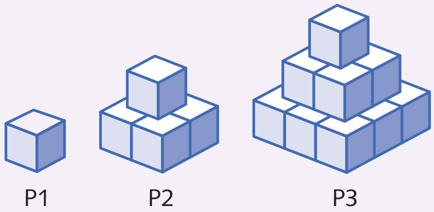
A 64 **B** 36 **C** 72 **D** 32

4 A pattern begins: 1,–2,–5, 8, 11, 14 – 17.

Which number is the next in the pattern?

A 14 **B** 20 **C** –14 **D** –20

5 Danny is building pyramids from blocks. So far he has built 3 pyramids.



To build the pyramid that is 3 blocks high (P3), he first made a 3 × 3 square of 9 blocks on the table. Then he put a 2 × 2 square of 4 blocks on top of the 3 × 3 square. Finally, he put a 1 × 1 square of 1 block on the top.

Danny wants to build pyramid P4. He had only 50 blocks when he started and he can't use the blocks from pyramids P1, P2 and P3 because he glued them together. Which of the following is correct?

A Danny does not have enough blocks left to build P4.

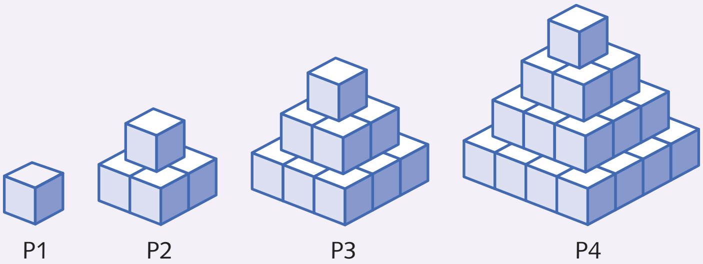
B Danny has exactly enough blocks left to build P4, with no blocks left over.

C Danny has enough blocks left to build P4, with 1 block left over.

D Danny has exactly enough blocks left to build P4, with 20 blocks left over.

6 Anthea is building pyramids, using the same method as Danny in question **5**. So far, she has built 4 pyramids (see below). She has exactly the right number of blocks left to build another P4. How many extra blocks will Anthea need to borrow if she wants to build P5 instead of another P4?

A 9 blocks **B** 16 blocks **C** 25 blocks **D** 30 blocks



ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

CHAPTER 17 Preparation

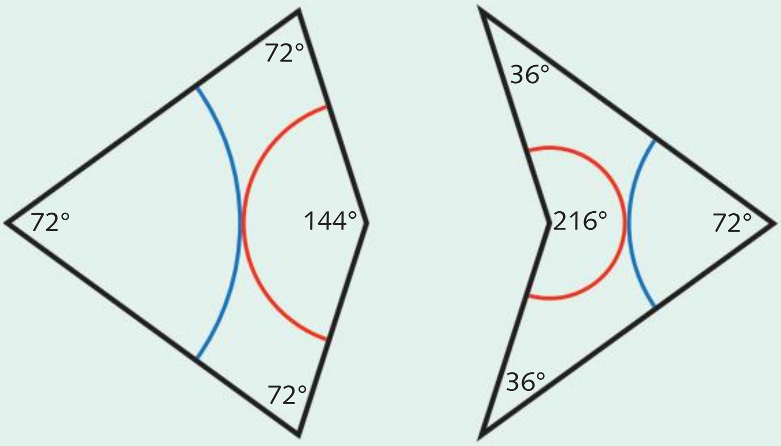
Useful skills for this chapter:

- understanding of two-dimensional shapes, including the naming of attributes
- the ability to name angles and measure them using a protractor.

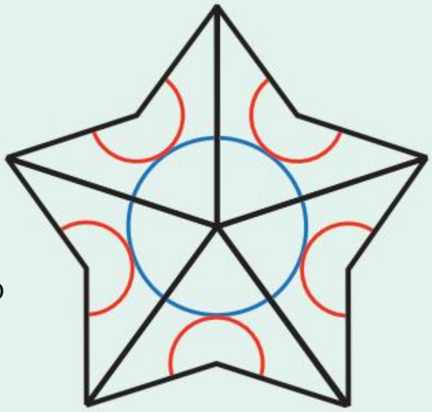
KICK OFF

Kites and darts

Sir Roger Penrose is a mathematician who uses shapes like these kites and darts to make tiling patterns.



Trace and cut out six kites and six darts. Use combinations of kites and/or darts *meeting at one vertex* to make other interesting shapes. The rule for joining the kites and darts is that when two shapes share an edge, the red or blue patterns must match on these edges.



There are seven shapes that you can make from kites and darts. The first one has been done for you. Can you find the other six?

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

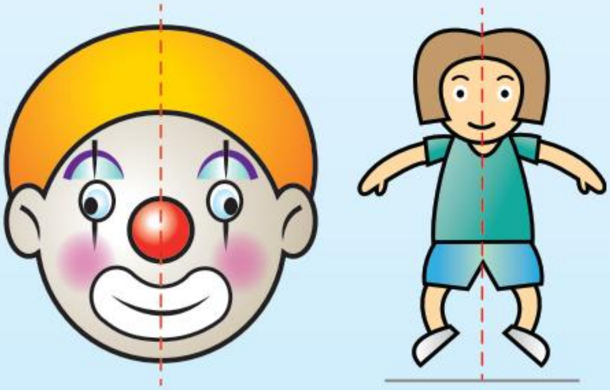
CHAPTER

17

Measurement and Geometry

Symmetry and transformation

If you drew a line down the middle of your face, you would see that the two halves match up exactly across the line. This is called **symmetry**. If you place a small mirror along the dotted lines below, you will find that the image in the mirror completes the picture.



We see patterns all around us. Many patterns are made by shapes fitting together.

Rotation, **reflection** and **translation** are some of the different ways we can transform a two-dimensional shape.

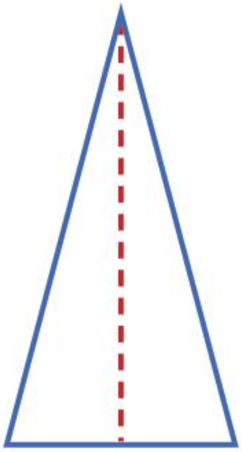
In this chapter we continue to discover more about the properties of two- and three-dimensional shapes. We look at symmetry in geometry and think of how this might apply in nature. We investigate the effect of moving two-dimensional shapes and visualise these transformations.

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

17A Symmetry of two-dimensional shapes

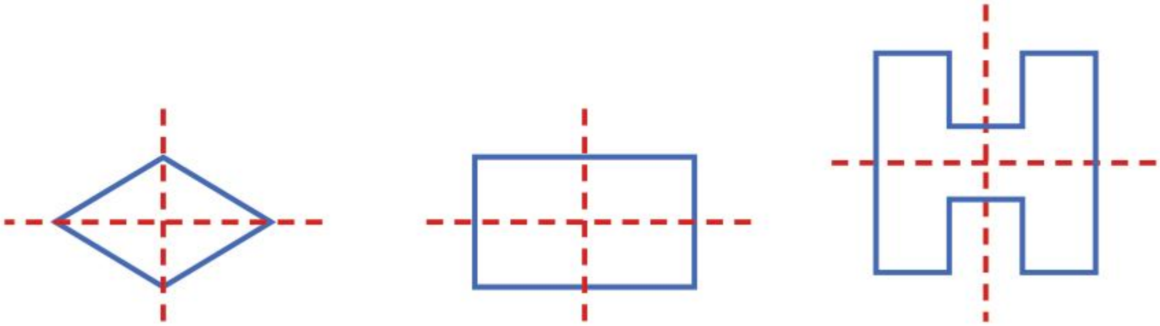
If we stand up straight, the vertical line down the centre of our face or body divides us into two almost identical pieces. In mathematics, when the pieces of a two-dimensional shape match up exactly across a straight line, we say the shape is ‘symmetrical about the line’.

Draw an isosceles triangle on a piece of paper, then cut it out. Fold the right-hand side of the triangle over so it lies exactly on the left-hand side of the triangle and make a crease down the centre of the triangle. The halves of the triangle on either side of the fold line should match exactly. The fold line is called the **axis of symmetry** or the **line of symmetry**. Use a protractor to measure the angles where the fold line meets the base. They should be 90°.

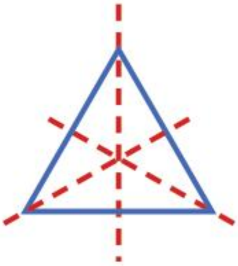


The isosceles triangle is symmetrical about the fold line.

Some shapes have more than one line of symmetry.
These shapes have two lines of symmetry.



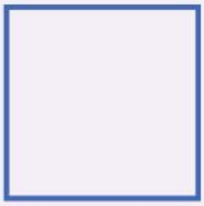
This shape has three lines of symmetry.



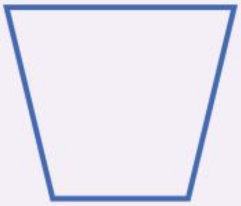
17A Individual

1 How many lines of symmetry does each quadrilateral have?

a



b



c

d

2 **a** Draw 5 regular polygons of different sizes.

b Mark dotted lines on the polygons to show all the lines of symmetry.

3 Write the alphabet in capital letters. List the letters that have at least one line of symmetry.

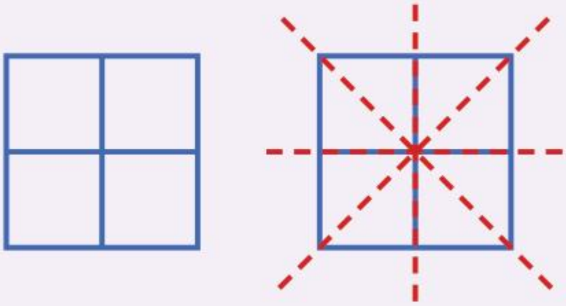
4 **a** Draw a right-angled triangle with a short side equal to 5 cm and a long side perpendicular to it equal to 7 cm. Mark in the right angle and draw in the third side.

b Draw the shape you would get if the 7 cm line is the axis of symmetry for a new shape and your triangle is one half of it.

c Draw the shape you would get if the 5 cm line is the axis of symmetry for a new shape.

5 This shape is made from 4 identical small squares.

It has 4 lines of symmetry.



Use 4 identical squares to make a shape that has:

a 1 line of symmetry **b** 2 lines of symmetry

Use 5 identical squares to make a shape that has:

c 1 line of symmetry **d** 4 lines of symmetry **e** 0 lines of symmetry

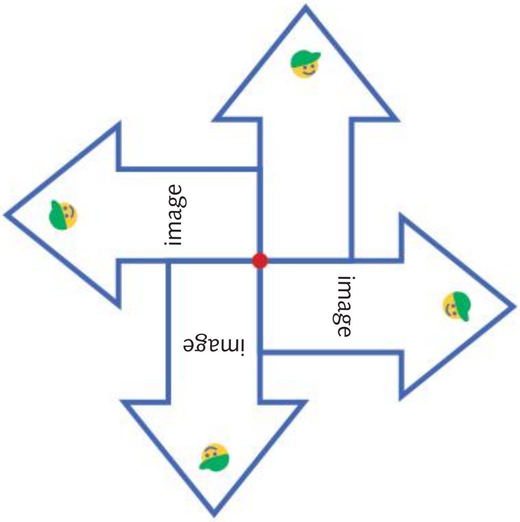
17B Transformation and tessellation

Rotation, reflection and translation are some of the different ways we can transform a two-dimensional shape.

Rotation

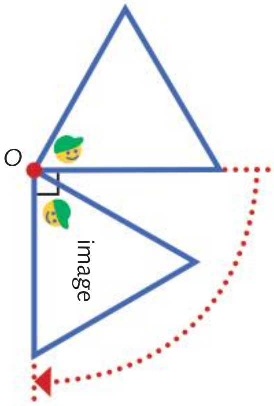
We rotate a shape when we turn it through an angle.

The diagram at the right shows an arrow shape pointing upwards. The shape has been rotated clockwise around the red dot three times, each time by 90°. The word 'image' has been used to label the shape in each new position.

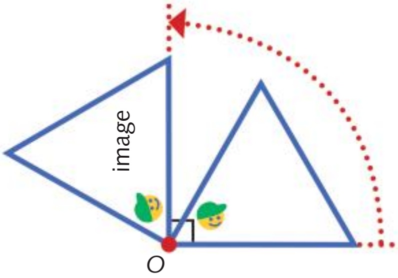


We can rotate clockwise or anticlockwise about a point.

This triangle has been turned 90° in a clockwise direction about O.



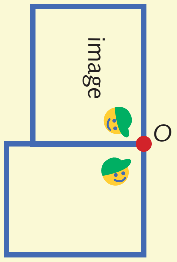
This triangle has been rotated 90° anticlockwise.



ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

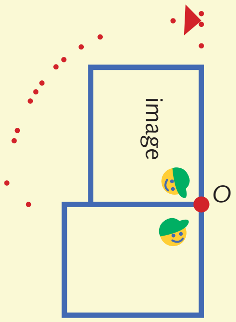
Example 1

How has this shape been moved?



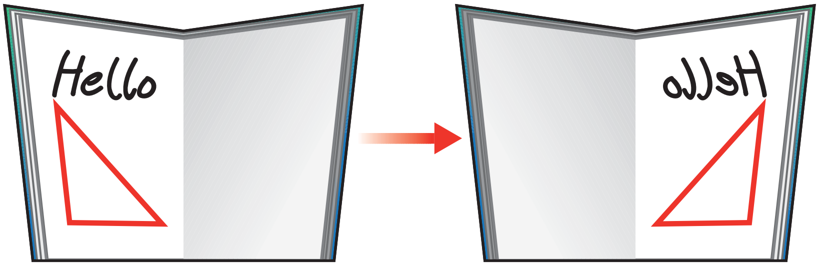
Solution

The rectangle has been rotated 90° in a clockwise direction.

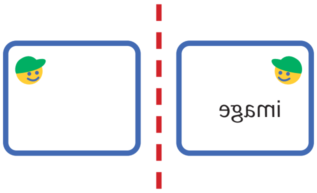


Reflection

A reflection is a transformation that flips a figure about a line. This line is called the axis of reflection. A good way to understand this is to suppose that you have a book with clear plastic pages and a triangle drawn on one page, as in the first diagram below. If the page is turned, the triangle is flipped over. We say it has been reflected; in this case the axis of reflection is the binding of the book.



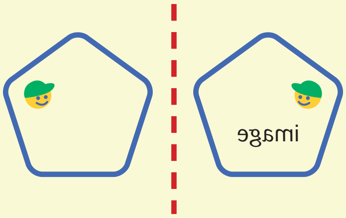
This shape has been reflected in the vertical line.



ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

Example 2

Which transformation has been used to produce the image?



Solution

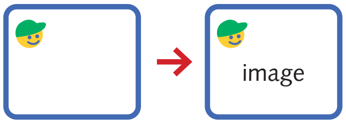
The image is the mirror image of the shape on the left. It has been reflected in the vertical line.

Translation

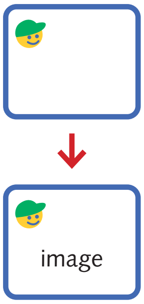
When we translate a shape, we slide it. We can slide it left or right, up or down.

Transformations move the shape without rotating it.

This shape has been translated horizontally.

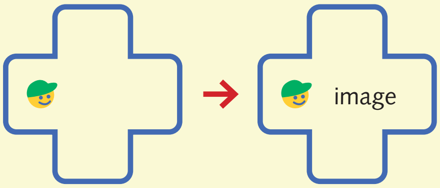


This shape has been translated vertically.



Example 3

How has this shape been moved?



Solution

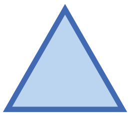
The shape has been translated horizontally.

Tessellation

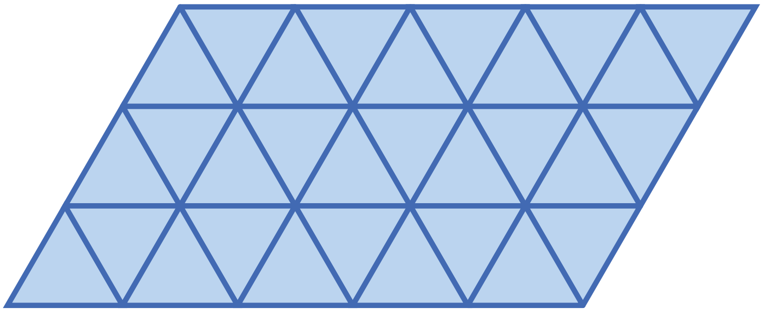
A **tessellation** is a tiling pattern made by fitting together transformations of a two-dimensional shape with no gaps or overlaps. The tessellation can continue in all directions.

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

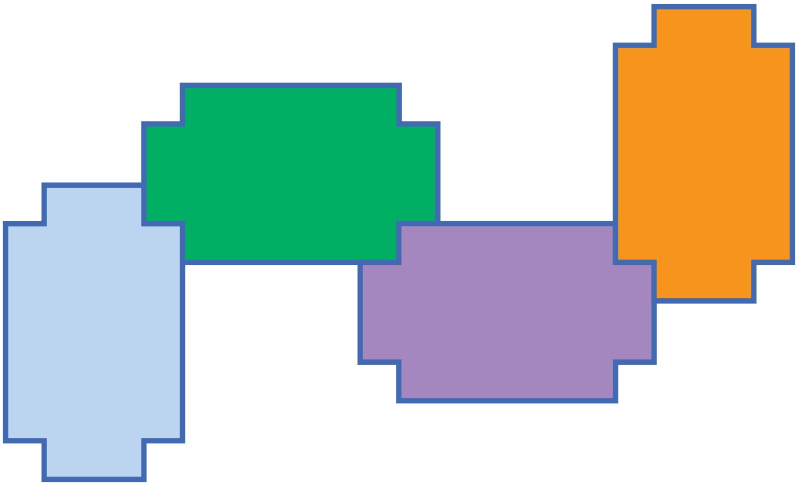
Start with an equilateral triangle.



We can rotate it 180° and translate it so the triangles fit together perfectly. The tiling goes on forever. We say that the equilateral triangle **tessellates**.



The shape used in the pattern below is not a tessellating shape because we cannot rotate and translate it to fill up the whole space without gaps or overlaps.

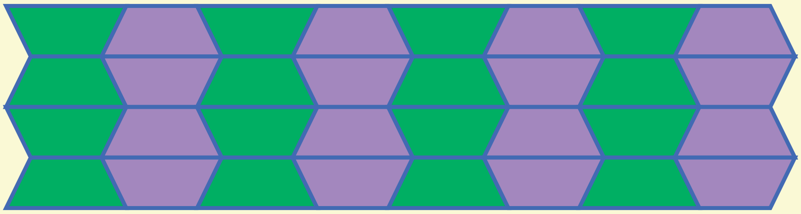


Example 4

Will this shape tessellate?

Solution

Yes, this shape will tessellate. It can be rotated 180° and translated so the pieces fit together without any gaps or overlaps.



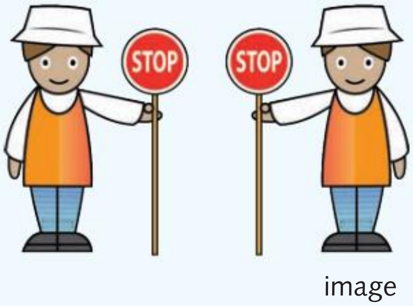
The pattern can be continued horizontally and vertically as far as you wish.

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

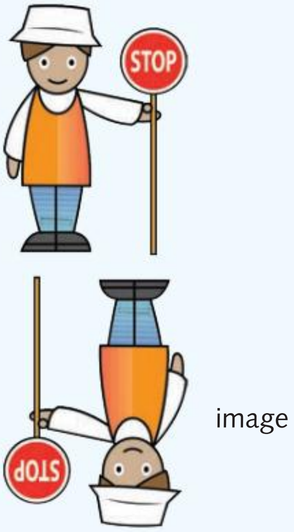
17B Whole class CONNECT, APPLY AND BUILD

1 Describe the transformations.

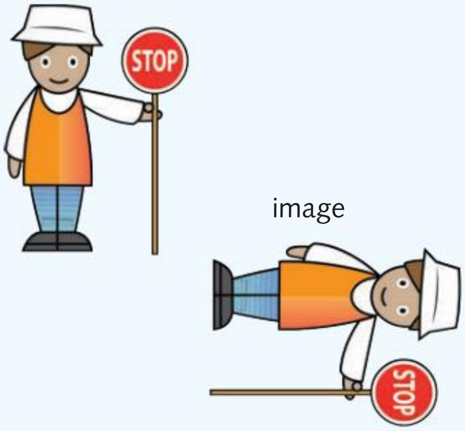
a



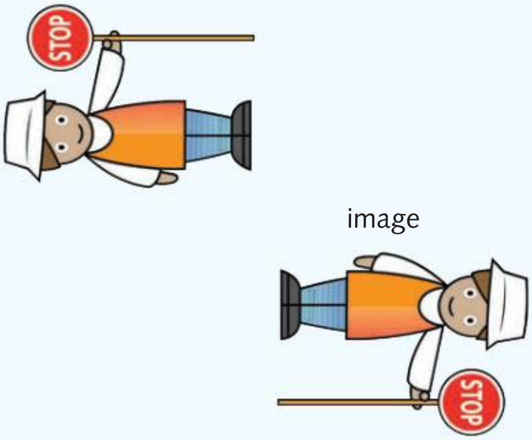
b



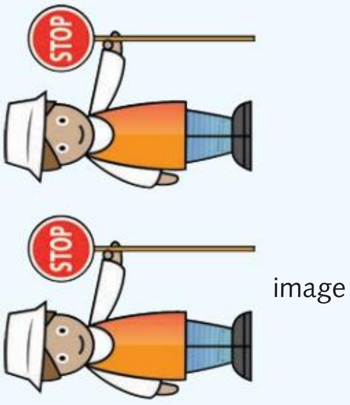
c



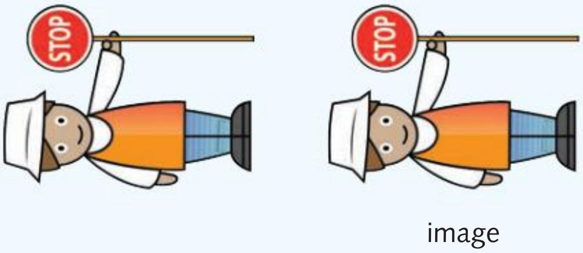
d



e



f



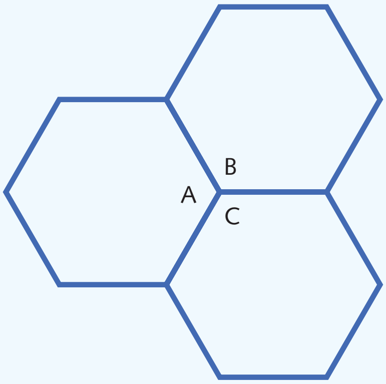
ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

2 Look around the school for tessellating patterns such as brickwork, floor tiles, wall tiles, wallpaper, carpets and mats. Draw a sketch of three interesting tessellating patterns.

3 **a** Trace a hexagon from your class set of shapes.

b Measure the internal angles of the hexagon and label your drawing.

c Draw two more hexagons in a tessellating pattern as shown below and mark in the size of the angles A, B and C.



d What is the sum of the angles A, B and C?

e What does this tell you about tessellating patterns? Complete this sentence:

In a tessellation the angles about a point sum to ________ degrees.

4 Use attribute blocks or pattern blocks.

a Choose a shape (not a hexagon) that you think will tessellate. Show that the shape tessellates by putting together at least 10 tracings of the shape.

b Measure or calculate the angles about a point within your tessellating pattern.

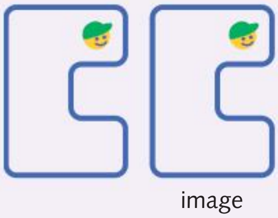
5 Use blocks to make a tiling pattern with two or more different-shaped tiles. Which shapes did you use? Measure the angles about a point within your tessellating pattern.

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

17B Individual

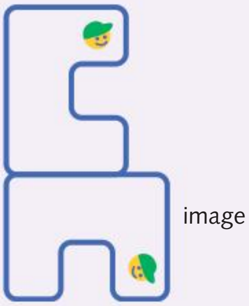
1 Describe the transformation of these shapes.

a



b

c



2 Draw these shapes, then draw what they look like after they have been reflected in the line.

a

b

3 Draw these shapes, then draw what they look like after they have been translated.

a Translate horizontally

b Translate vertically

4 For each of these shapes:

- rotate the shape 90° clockwise
- draw the image
- repeat the above steps twice.

a

b

c

5 Select one shape from your class set of shapes that will tessellate. Draw a tessellation using the shape. Colour the shapes to show the pattern you have made.

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

Answers

Chapter 1: Positive and negative whole numbers

Kick off

1 **a** 400 **b** 424 **c** 1036 **d** 1113 **e** 19 179

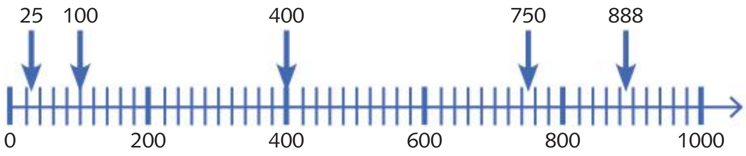
2 **a** 200 **b** 224 **c** 836 **d** 913 **e** 18 979

3 **a** 6000 **b** 1702 **c** 1031 **d** 4058 **e** 20 892

4 **a** 7000 **b** 3 **c** 10 035 **d** 9403 **e** 19 202

Show what you know

1



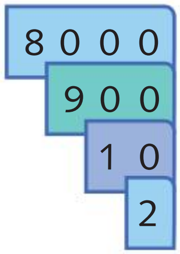
2 **a** 5 tens of thousands, or 50 000 **b** 5 tens, or 50

c 5 ones, or 5 **d** 5 thousands, or 5000

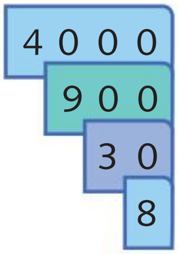
e 5 hundreds, or 500

1A WHOLE CLASS

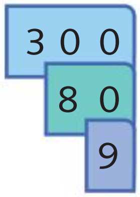
1 **a**



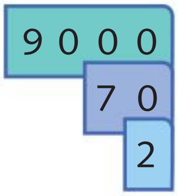
b



c



d



2 **a** five thousand and thirty-eight

b twenty-eight thousand, four hundred and forty-five

c forty-six thousand, one hundred and two

d two hundred and ninety-eight thousand, five hundred and seventy-seven

3 **a** 852 **b** 300

c 2353 **d** 4518

e 2345 **f** 128 439

g 1834 **h** 12 317

1A INDIVIDUAL

1 **a** 8223 **b** 105 249

c 13 798 562

2 **a** ninety-one

b forty-eight

c three hundred and ninety-two

d six thousand, seven hundred and eighty-nine

e eighteen thousand and two

f twenty-one thousand, three hundred and nine

g five hundred and eighty-nine thousand, nine hundred and two

h six million, eight hundred and ninety-three thousand, four hundred and seven

3 **a** 8 hundreds

b 9 tens

c 5 ones

d 3 tens of thousands

e 1 million

f 8 tens of millions

g 0 tens of millions, 0 thousands, 0 hundreds, 0 tens

h 3 hundred millions, 7 millions, 2 thousands, 9 ones

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

4

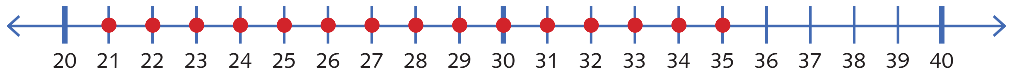
Number	Place-value parts							
	Tens of millions	Millions	Hundreds of thousands	Tens of thousands	Thousands	Hundreds	Tens	Ones
9045					9	0	4	5
21947				2	1	9	4	7
101010			1	0	1	0	1	0
800641			8	0	0	6	4	1
1794376		1	7	9	4	3	7	6

1B WHOLE CLASS

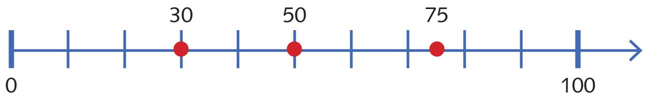
Teacher check

1B INDIVIDUAL

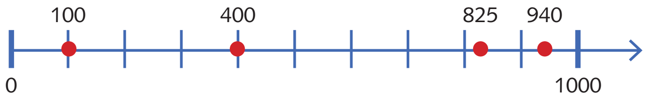
1



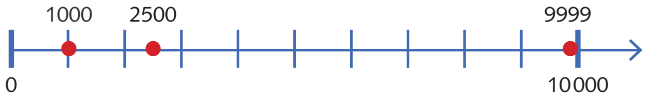
2



3



4



5 **a** 0, 1, 6 and 9 **b** 30, 40, 70 and 90 **c** 700, 800 and 960

d 600 000, 680 000, 800 000, 950 000

1C WHOLE CLASS

Teacher check

1C INDIVIDUAL

1 **a** 897, 898, 899, 900, 901, 902, 903, 904, 905, 906 **b** 98, 101, 104, 107, 110, 113, 116, 119, 122, 125

c 56, 55, 54, 53, 52, 51, 50, 49, 48, 47 **d** 213, 209, 205, 201, 197, 193, 189, 185, 181, 177

2 **a** 99, 106, 113, 120, 127, 134, 141, 148, 155, 162, 169, 176, 183, 190…

b 346, 355, 364, 373, 382, 391, 400, 409, 418, 427, 436, 445, 454, 463…

c 1777, 1769, 1761, 1753, 1745, 1737, 1729, 1721, 1713, 1705, 1697…

3 **a** 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 32, 34, 36, 38, 40 and 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, 39, 42, 45, 48, 51, 54, 57, 60. The numbers in both lists are 6, 12, 18, 24, 30, 36.

b 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, 39, 42, 45, 48, 51, 54, 57, 60 and 4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44, 48, 52, 56, 60, 64, 68, 72, 76, 80. The numbers in both lists are 12, 24, 36, 48, 60.

c 4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44, 48, 52, 56, 60, 64, 68, 72, 76, 80 and 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70, 75, 80, 85, 90, 95, 100. The numbers in both lists are 20, 40, 60, 80.

d 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, 39, 42, 45, 48, 51, 54, 57, 60 and 8, 16, 24, 32, 40, 48, 56, 64, 72, 80, 88, 96, 104, 112, 120, 128, 136, 144, 152, 160. The numbers in both lists are 24 and 48.

e 8, 16, 24, 32, 40, 48, 56, 64, 72, 80, 88, 96, 104, 112, 120, 128, 136, 144, 152, 160 and
9, 18, 27, 36, 45, 54, 63, 72, 81, 90, 99, 108, 117, 126, 135, 144, 153, 162, 171, 180.
The numbers in both lists are 72 and 144.

4 The numbers that were the same in both lists are the multiples of the product of the two numbers. So the first 5 numbers that are the same in the counting sequences for 4 and 9 will be multiples of 36: 36, 72, 108, 144 and 180.

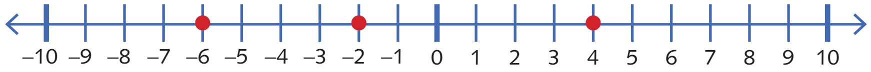
1D WHOLE CLASS

1 **a** 5 + 4 = 9 **b** 0 – 3 = –3 **c** 3 – 10 = –7 **d** –2 – 3 = –5 **e** –3 + 5 = 2

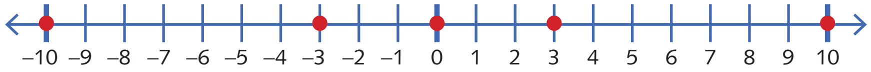
2 **a** 5 steps **b** 2 steps **c** 4 steps **d** 10 steps
e 4 steps **f** 7 steps **g** 36 steps **h** 80 steps

1D INDIVIDUAL

1 **a**



b



2 **a** 1, 0, –1, –2 **b** –9, –8, –7, – 6, –5, – 4, –3

3 **a** –5, –1, 2, 3, 6 **b** –34, –23, 18, 47, 99 **c** –57, –23, 0, 21, 434

4 **a** 9, 4, 0, –2, –3 **b** 99, 56, –56, –99, –111 **c** 136, 3, 0, –2, –77

5 –8, –10, –12, –14, –16

1E REVIEW QUESTIONS

1 **a** 42 718 **b** 210 563 **c** 11 967 324

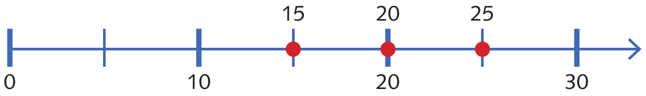
2 **a** seventy-five
b one hundred and two
c four thousand, nine hundred and sixty-two
d twenty-three thousand, one hundred and seventy-eight
e nine hundred and fourteen thousand, two hundred and seven
f four million, seven hundred and sixty-nine thousand and fifty-two

3 **a** 6 hundreds **b** 0 tens **c** 7 ones
d 5 tens of thousands **e** 5 millions **f** 4 tens of millions

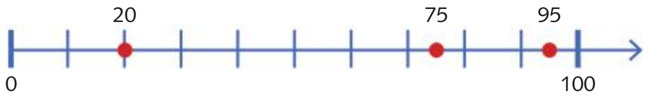
4

Number	Place-value parts							
	Tens of millions	Millions	Hundreds of thousands	Tens of thousands	Thousands	Hundreds	Tens	Ones
3208					3	2	0	8
56 849				5	6	8	4	9
213 054			2	1	3	0	5	4
609 170			6	0	9	1	7	0
75 451 821	7	5	4	5	1	8	2	1

5



6



(Because of the size of the number lines, the placement is approximate.)

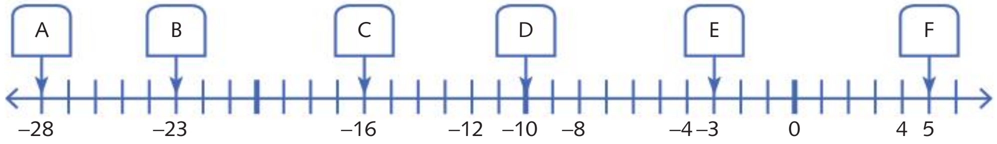
7

8

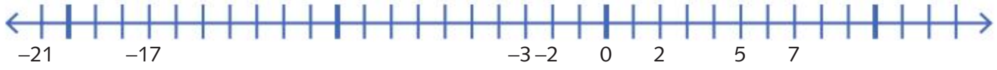
9 **a** 0, 2, 4, 7 **b** 10, 45, 60, 100
c 600, 900, 975
d 500 000, 780 000, 850 000

10 **a** 691, 692, 693, 694, 695, 696, 697, 698, 699, 700
b 89, 92, 95, 98, 101, 104, 107, 110, 113, 116
c 1013, 1017, 1021, 1025, 1029, 1033, 1037, 1041, 1045, 1049
d 79, 77, 75, 73, 71, 69, 67, 65, 63, 61
e 421, 416, 411, 406, 401, 396, 391, 386, 381, 376
f 5347, 5341, 5335, 5329, 5323, 5317, 5311, 5305, 5299, 5293
g 87, 95, 103, 111, 119, 127, 135, 143, 151, 159
h 572, 581, 590, 599, 608, 617, 626, 635, 644, 653
i 634, 645, 656, 667, 678, 689, 700, 711, 722, 733
j 2701, 2694, 2687, 2680, 2673, 2666, 2659, 2652, 2645, 2638
k 3659, 3647, 3635, 3623, 3611, 3599, 3587, 3575, 3563, 3551

11



12



13 **a** –1, –2, –3, –4
b 2, 1, 0, –1, –2, –3, –4, –5, –6, –7, –8, –9
c –23, –24, –25, –26, –27, –28, –29, –30, –31, –32

14 –132, –33, –9, –6, 0, 3, 8, 133

15 89, 45, 6, –2, –4, –66, –101

Chapter 2: Addition and subtraction

Kick off

a 47 **b** 32 **c** 29 **d** 72
e 41 **f** 29 **g** 53 **h** 27
i 62 **j** 43

Show what you know

1 **a** 25 **b** 32 **c** 44 **d** 58
e 57 **f** 60 **g** 66 **h** 93
i 6 **j** 14 **k** 11 **l** 19
m 17 **n** 5 **o** 18

2A WHOLE CLASS

1 **a** 35 **b** 70 **c** 50 **d** 66
e 87 **f** 108

2 **a** 111 **b** 81 **c** 87 **d** 400
e 972 **f** 105 **g** 113 **h** 120

2A INDIVIDUAL

1 **a** 40 **b** 150 **c** 110 **d** 130
e 83 **f** 72 **g** 192 **h** 211

2 **a** 35 **b** 32 **c** 37 **d** 41
e 50 **f** 24 **g** 16 **h** 21

3 **a** \$64 **b** \$82 **c** \$26 **d** \$65
e \$35 **f** \$12 **g** \$57 **h** \$41

4 **a** 51 **b** 51 **c** 88 **d** 59
e 100 **f** 63 **g** 81 **h** 72
i 82 **j** 83 **k** 93 **l** 91
m 122 **n** 134 **o** 183 **p** 1130

5 145 children **6** 166 pies

7 225 animals **8** 145 plants

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

9 **a** 91 days **b** less, by 1 day

10

19	12	17
14	16	18
15	20	13

a

25	18	23
20	22	24
21	26	19

b

23	16	21
18	20	22
19	24	17

c

28	14	24
18	22	26
20	30	16

d

30	16	26
20	24	28
22	32	18

e

52	24	44
32	40	48
36	56	28

f

130	123	128
125	127	129
126	131	124

11 Teacher to check

2B INDIVIDUAL

1 **a** 196 **b** 575 **c** 904 **d** 1035
e 278 **f** 947 **g** 899 **h** 777
i 673 **j** 974 **k** 964 **l** 853
m 1243 **n** 861 **o** 660 **p** 1414

2 228 points **3** 186 goals **4** $4038

5 1409 newspapers **6** No; 7260 washers

7 **a** 12 145 **b** 6234 **c** 7008
d 1904 **e** 3063 **f** 2263

8 **a** 3408 **b** 5331 **c** 6477
d 3698 **e** 6008 **f** 7366

9 **a**
346
388
+255
989

b
279
154
+383
816

c
2368
5194
+1776
9338

d
2705
3699
+2887
9291

2C INDIVIDUAL

1 **a** 37 **b** 17 **c** 44 **d** 194

2 **a** 34 **b** 51 **c** 38
d 115 **e** 5

3 **a** 26 **b** 37 **c** 33
d 284 **e** 245 **f** 193

4 38 bunches **5** 117 km

6 138 m **7** $368

8 **a** 247 **b** 243 **c** 1219

9 **a** 97 – 79 = 18, 96 – 69 = 27 … 91 – 19 = 72
b All the answers are multiples of 9.
c Yes

2D WHOLE CLASS

1 **a** 3114 **b** 1234 **c** 1537
d 3149 **e** 4104 **f** 2683
g 259 **h** 422

2D INDIVIDUAL

1 **a** 359 **b** 224 **c** 638 **d** 284

2 **a** 5438 **b** 1409 **c** 2696
d 2609 **e** 3796 **f** 1586

3 **a** 2802 **b** 1956 **c** 223

4 4667 cases **5** 2337 books

6 1755 eggs **7** $47 369

8 2771 sheep **9** 5372 apples

10 6275 fish

11 See **BLM 2** answers in the Interactive Textbook.

2E INDIVIDUAL

1 **a** 57 572 **b** 44 181 **c** 42 494
d 13 165 **e** 30 781 **f** 42 659

2 **a** 50 940 **b** 70 114 **c** 56 113
d 51 401 **e** 44 705

3 **a** 17 856 **b** 14 494 **c** 25 382
d 7407 **e** 24 976

4 **a** $6112.40 **b** $9890.15 **c** $2664.25
d $4634.45

5 **a** 32 655 people **b** 71 318 people
c 95 410 people **d** 15 125 people
e 299 249 people **f** 751

6 **a** 14 177 **b** 54 032
c 2 061 403 **d** 515 547 412

7 thirty-three thousand, five hundred and forty-four, or 33 544

8 74 206 **9** $13 105

10 $368 105 **11** 405 seats

12 4, 11, 18, 25, 32, 39 and 46 flies

13 8, 17, 26, 35, 44, 53, 62, 71, 80, 107, 116, 125, 134, 143, 152, 161, 170, 206

2F REVIEW QUESTIONS

1

+	15	35	29	17	53
5	20	40	34	22	58
9	24	44	38	26	62
2	17	37	31	19	55
7	22	42	36	24	60
3	18	38	32	20	56
4	19	39	33	21	57
6	21	41	35	23	59
8	23	43	37	25	61

−	65	72	53	81	64
14	51	58	39	67	50
26	39	46	27	55	38
18	47	54	35	63	46
35	30	37	18	46	29
19	46	53	34	62	45
22	43	50	31	59	42
17	48	55	36	64	47
23	42	49	30	58	41

2 **a** 36 **b** 33 **c** 39 **d** 42
3 **a** $84 **b** $73 **c** $39 **d** $15
4 **a** 41 **b** 52 **c** 91 **d** 92
e 130 **f** 101 **g** 63 **h** 102
i 85 **j** 93 **k** 93 **l** 154
5 85 children

6 **a**

23	17	26
25	22	19
18	27	21

b

19	18	23
24	20	16
17	22	21

c

13	20	15
18	16	14
17	12	19

d

29	22	21
16	24	32
27	26	19

e

55	24	41
26	40	54
39	56	25

f

129	128	124
122	127	132
130	126	125

7 12 051
8 **a** 5333 **b** 5866
9 **a** 6, 8, 0 **b** 7, 4, 9
c 7, 8, 1, 0 **d** 0, 6, 2, 1
10 **a** 37 **b** 26
c 49 **d** 198
11 **a** 55 **b** 32 **c** 36 **d** 355
12 **a** 36 **b** 17 **c** 62
d 237 **e** 167
13 139 newspapers **14** 406 kilometres
15 $158 **16** 204
17 4758 oranges **18** He earned $454 less.
19 **a** 78 751 **b** 81 421 **c** 39 221
d 15 189 **e** 27 889 **f** 31 889
20 **a** $7327.20 **b** $8990.55
c $761.35 **d** $1720.85

Chapter 3: Multiples and factors

Kick off

12, 24, 36, 48, 60, 72, 84, 96
The numbers in both counting patterns are the multiples of 12 because 3 × 4 = 12.

Show what you know

1 **a** 5, 10, 15, 20, 25, 30, 35, 40, 45, 50
b 3, 6, 9, 12, 15, 18, 21, 24, 27, 30
c 4, 8, 12, 16, 20, 24, 28, 32, 36, 40
d 7, 14, 21, 28, 35, 42, 49, 56, 63, 70
e 8, 16, 24, 32, 40, 48, 56, 64, 72, 80
f 9, 18, 27, 36, 45, 54, 63, 72, 81, 90
g 12, 24, 36, 48, 60, 72, 84, 96, 108, 120
h 13, 26, 39, 52, 65, 78, 91, 104, 117, 130

3A WHOLE CLASS

3 **a** 24 **b** 7, 4 **c** 14 **d** 6

3A INDIVIDUAL

1 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28
2 **a** 1 × 5 = 5 **b** 2 × 5 = 10
c 3 × 5 = 15 **d** 4 × 5 = 20
e 5 × 5 = 25 **f** 6 × 5 = 30
g 7 × 5 = 35 **h** 8 × 5 = 40
i 9 × 5 = 45 **j** 10 × 5 = 50
k 11 × 5 = 55 **l** 12 × 5 = 60
3 7, 14, 21, 28, 35, 42, 49, 56, 63, 70, 77, 84
4 3 × 7 = 21 cherries
5 6 × 9 = 54 shipping containers
6 11 × 12 = 132 bricks
7 **a** No, 582 is not a multiple of 5, as it doesn't end in 0 or 5.
b Yes, 4760 is a multiple of 10 which is an even multiple of 5.

8 **a** 332 is even **b** 1296 is even

c 2 493 767 is odd

9 38 and 70

10 **a** Possible answers:

100 000 000, 100 000 100, 100 000 010, 100 001 000, 100 001 110, 100 000 020, 999 999 990

b Possible answers:

100 000 001, 100 000 103, 100 000 015, 100 001 007, 100 001 111, 100 000 023, 999 999 999

11 10, 135, 2395, 4 297 770, 877 795

12 7, 84, 19, 954, 691 **13** 25, 95, 35, 1005

14 **a** 12 is a multiple of 3.

b 15 is not a multiple of 2.

c 18 is a multiple of 2.

d 18 is a multiple of 3.

15 Yes. There are 7 days in a week and 3 people in the family. Tyrone needs 7 × 3 = 21 bananas so each person can have 1 banana each day.

16 **a** 15 is a multiple of 3.

b 18 is a multiple of 9.

c 15 is not a multiple of 7.

d 21 is a multiple of 3.

e 15 is not a multiple of 6.

f 21 is a multiple of 7.

g 18 is a multiple of 6.

h 21 is not a multiple of 6.

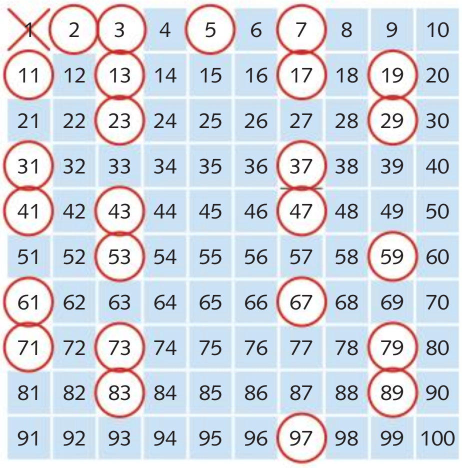
17 72 mangoes **18** Teacher check

3B WHOLE CLASS

1 Teacher check

2 See **BLM 3** answers in the Interactive Textbook.

3 **a–d** Your 100 chart should look like this.



1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

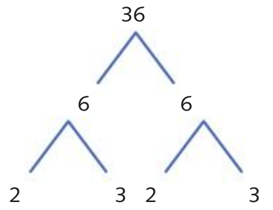
e 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

f The next prime number after 97 is 101.

4 Possible answers:
12 = 3 × 4, 18 = 6 × 3, 33 = 3 × 11,
98 = 2 × 49, 196 = 4 × 49, 333 = 3 × 111

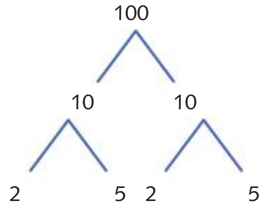
5 Possible answers:

a



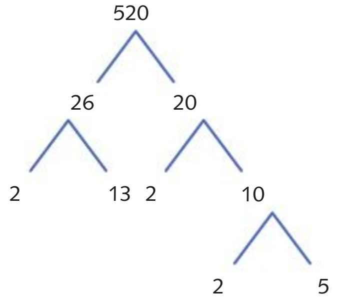
36 = 2 × 2 × 3 × 3

b



100 = 2 × 2 × 5 × 5

c



520 = 2 × 2 × 2 × 5 × 13

6 280 = 2 × 2 × 2 × 5 × 7

3B INDIVIDUAL

1 **a** 4 × 6 = 24; 4 and 6 are factors of 24.

b 3 × 13 = 39; 3 and 13 are factors of 39.

c 7 is not a factor of 22.

2 **a** 3 is a factor of 24.

b 6 and 8 are factors of 48.

c 9 and 6 are factors of 54.

3 2, 3, 11, 23, 37

4 **a** 41, 43, 47

b 11, 13, 17, 19

c 41, 43, 47, 53, 59

5 **a** 3, 13, 23, 31, 37, 43, 53, 73, 83

b Every prime number is odd except for 2 because every even number is divisible by 2.

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

6 **a–f** factor; multiple **g–i** multiple; factor

7 **a** multiple, factor; 4 and 3 are factors of 12.

b multiple, factor; 6 and 3 are factors of 18.

c multiple, factor; 4 and 2 are factors of 8.

d multiple, factor; 7 and 9 are factors of 63.

8 **a** $4 \times 11 = 44$ and $2 \times 22 = 44$; 4, 11, 2 and 22 are factors of 44.

b $5 \times 12 = 60$ and $3 \times 20 = 60$; 5, 12, 3 and 20 are factors of 60.

9 **a** $30 = 2 \times 3 \times 5$ **b** $84 = 2 \times 2 \times 3 \times 7$

c $63 = 3 \times 3 \times 7$ **d** $44 = 2 \times 2 \times 11$

e $120 = 2 \times 2 \times 2 \times 3 \times 5$

10 **a** $444 = 2 \times 2 \times 3 \times 37$

b $396 = 2 \times 2 \times 3 \times 3 \times 11$

c $176 = 2 \times 2 \times 2 \times 2 \times 11$

d $261 = 3 \times 3 \times 29$

e $7000 = 2 \times 2 \times 2 \times 5 \times 5 \times 5 \times 7$

11 **a** 12 **b** 190 **c** 27

d 32 **e** 385 **f** 210

12 **a** \$2

b 14 apples costing \$28

13 **a** \$99 **b** \$81

14 **a** 5 minibuses **b** 2 buses

c $6 \times 5 = 30$ and $2 \times 15 = 30$; 6, 5, 2 and 15 are factors of 30.

15 No. $1140 = 2 \times 2 \times 3 \times 5 \times 19$

16 33, 35, 39, 51, 55, 57

17 $53 + 7 = 60$, $47 + 13 = 60$, $43 + 17 = 60$, $41 + 19 = 60$, $37 + 23 = 60$, $31 + 29 = 60$

3C REVIEW QUESTIONS

1 4, 8, 16, 20, 24, 28, 32, 36, 40, 44, 48

a 4 **b** 8 **c** 12 **d** 16

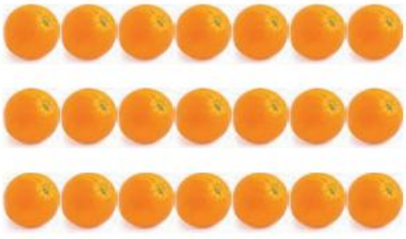
e 20 **f** 24 **g** 28 **h** 32

i 36 **j** 40 **k** 44 **l** 48

2 **a** 14 **b** 21 **c** 35 **d** 56

3 $5 \times 8 = 40$ apples

4 **a**



b There are $3 \times 7 = 21$ oranges.

5 **a** 24 is a multiple of 3

b 18 is not a multiple of 7

c 18 is a multiple of 9

d 24 is a multiple of 4

e 24 is not a multiple of 5

f 18 is a multiple of 2

g 16 is not a multiple of 6

h 28 is not a multiple of 8

6 72 mangoes

7 42 bananas shared between 6 people is 7 each; 1 per person each day for a week.

8 **a** Multiple of 7; the total is $7 \times 5 = 35$ beads.

b Multiple of 3; the total is $8 \times 3 = 24$ people.

9 **a** 4 and 7 **b** 5 and 3 **c** 9

d 8 **e** 7 **f** 6

g 12 **h** 3 **i** 30

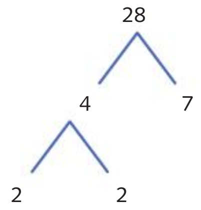
j 15 **k** 12 **l** 10

m 6 **n** 12

10 **a–f** factor; multiple **g–i** multiple; factor

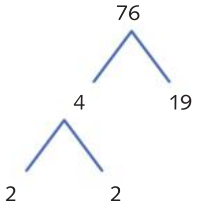
11 Possible answers:

a



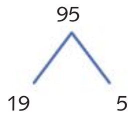
$28 = 2 \times 2 \times 7$

b



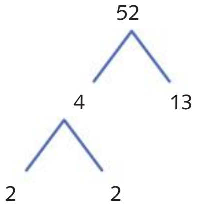
$76 = 2 \times 2 \times 19$

c



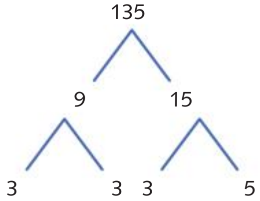
$95 = 5 \times 19$

d



$52 = 2 \times 2 \times 13$

e



$135 = 3 \times 3 \times 3 \times 5$

12 **a** $5 \times 3 \times 37 = 555$ **b** $2 \times 4 \times 59 = 472$

c $3 \times 127 = 381$ **d** $2 \times 7 \times 11 = 154$

e $2 \times 2 \times 3 \times 3 \times 3 \times 5 \times 13 = 7020$

13 **a** 30 **b** 255 **c** 18

d 1125 **e** 1001 **f** 858

Chapter 4: Multiplication and division

Kick off

a 5 each **b** can't share **c** 6 each
d 11 each **e** 3 each **f** 7 each

Show what you know

1 a 42 **b** 27 **c** 45 **d** 48
e 48 **f** 54 **g** 63 **h** 28
i 72 **j** 108 **k** 132 **l** 96

4A WHOLE CLASS

1 a 60 **b** 84 **c** 168 **d** 144
e 308 **f** 408 **g** 1020 **h** 880
i 2000 **j** 4104 **k** 4200 **l** 8888

2 a 230, 2300, 23 000 **b** 850, 8500, 85 000
c 1210, 12 100, 121 000
d 9340, 93 400, 934 000
e 7000, 70 000, 700 000
f 10 010, 100 100, 1 001 000
g 85 940, 859 400, 8 594 000
h 101 010, 1 010 100, 10 101 000
i 154 620, 1 546 200, 15 462 000
j 2 739 120, 27 391 200, 273 912 000
k 8 480 840, 84 808 400, 848 084 000
l 2 950 349 570, 29 503 495 700, 295 034 957 000

3 a 24 **b** 40 **c** 88 **d** 64
e 120 **f** 72 **g** 128 **h** 264
i 192 **j** 408 **k** 808 **l** 752

4 a 126 **b** 189 **c** 306 **d** 468
e 567 **f** 729 **g** 963 **h** 1017
i 1350 **j** 1098 **k** 3186 **l** 6750

5 a 154 **b** 231 **c** 374 **d** 396
e 407 **f** 561 **g** 616 **h** 913
i 1067 **j** 1342 **k** 3894 **l** 8250

6 a 105 **b** 65
c 225 **d** 495
e 555 **f** Teacher check
g 90 **h** 130
i 270 **j** 510
k 2250 **l–m** Teacher check

7 a 160 **b** 460 **c** 840 **d** 720

8 a 78 **b** 126 **c** 198 **d** 312
e 324 **f** 432 **g** 480 **h** 600

9 a 125 **b** 75 **c** 150 **d** 200
e 225 **f** 1250 **g** 750 **h** 1500
i 2000 **j** 2250

10 a 270

b

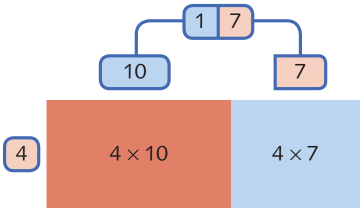
×	30	60	50	90
4	120	240	200	360
8	240	480	400	720
7	210	420	350	630
9	270	540	450	810

11 a 275 bananas
b 253 + 38 = 291 peanuts

12 Teacher check

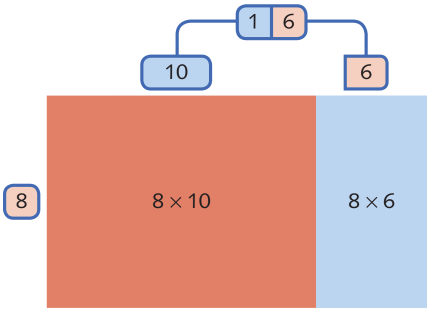
4B INDIVIDUAL

1 a 4 × 17



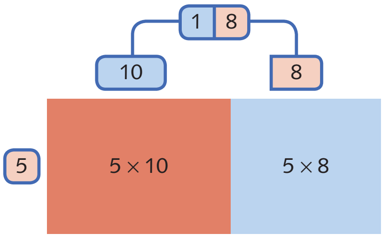
$$\begin{aligned} 4 \times 17 &= 4 \times 10 + 4 \times 7 \\ &= 40 + 28 \\ &= 68 \end{aligned}$$

b 8 × 16



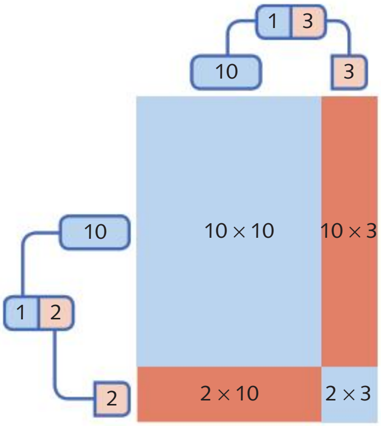
$$\begin{aligned} 8 \times 16 &= 8 \times +10 + 8 \times 6 \\ &= 80 + 48 \\ &= 128 \end{aligned}$$

c 5 × 18



$$\begin{aligned} 5 \times 18 &= 5 \times 10 + 5 \times 8 \\ &= 50 + 40 \\ &= 90 \end{aligned}$$

d 12×13



$$\begin{aligned} 12 \times 13 &= 10 \times 10 + 10 \times 3 + 2 \times 10 + 2 \times 3 \\ &= 100 + 30 + 20 + 6 \\ &= 156 \end{aligned}$$

2 b $$\begin{aligned} 13 \times 5 &= 10 \times 5 + 3 \times 5 \\ &= 50 + 15 \\ &= 65 \end{aligned}$$

c $$\begin{aligned} 27 \times 8 &= 20 \times 8 + 7 \times 8 \\ &= 160 + 56 \\ &= 216 \end{aligned}$$

d $$\begin{aligned} 15 \times 23 &= 10 \times 20 + 10 \times 3 + 5 \times 20 + 5 \times 3 \\ &= 200 + 30 + 100 + 15 \\ &= 345 \end{aligned}$$

4C INDIVIDUAL

1 a 306 **b** 672 **c** 1147
d 2107 **e** 6474 **f** 3296
g 30 720 **h** 309 430 **i** 487 044
j 2 289 771 **k** 2 035 062 **l** 17 500 890
2 a 1178 pies **b** 68 shoes
c 168 pens and pencils
3 a 182 km **b** 351 km **c** 884 km
4 a 1428 grams **b** 5474 grams **c** 12 852 grams
5 a 25 576 litres **b** 38 364 litres **c** 89 516 litres
6 a 672 children
b 351 beds
c 54 windows
d $23 \times 6 = 138, 14 \times 8 = 112, 138 + 112 = 250$, $250 \times 12 = 3000$ eggs
e $3 \times 16 = 48, 6 \times 19 = 114, 48 + 114 = 162$ boxes
f $6 \times 32 = 192, 3 \times 83 = 249, 192 + 249 = 441$ items
7 a 1 **b** 121 **c** 12 321
d 1 234 321
e 123 454 321
f 12 345 654 321
g 1 234 567 654 321
h 123 456 787 654 321
i 12 345 678 987 654 321
j The number of ones multiplied is the middle digit in a string of digits that go up in order and then down again.
8 a 88 **b** 888
c 8888 **d** 88 888
e 888 888 **f** 8 888 888
g 88 888 888 **h** 888 888 888
i Each answer has one more 8 digit than the last.
9 a 26 km **b** 26 km **c** 34 km
d 82 km **e** 136 km **f** 198 km

4D WHOLE CLASS

2 a 4 **b** 6 **c** 14
d 22 **e** 16 **f** 51
3 a 7 **b** 13 **c** 24
d 129 **e** 400 **f** 100 000
4 a 2 **b** 8 **c** 34
d 80 **e** 250
5 a 4 **b** 7 **c** 27
d 20 **e** 680
6 a 2 **b** 3 **c** 7
d 11 **e** 8 **f** 250

4D INDIVIDUAL

1 b 3, 9 **c** 7, 6
d 12, 8 **e** 72, 144
2 a $11 \times 11 = 121$ **b** $9 \times 18 = 162$
c $63 \times 8 = 504$ **d** $45 \times 26 = 1170$
e $73 \times 98 = 7154$ **f** $57 \times 102 = 5814$
3 Possible answers:
a $32 \div 8 = 4$ and $32 \div 4 = 8$
b $36 \div 3 = 12$ and $36 \div 12 = 3$
c $84 \div 7 = 12$ and $84 \div 12 = 7$
d $96 \div 4 = 24$ and $96 \div 24 = 4$
e $132 \div 11 = 12$ and $132 \div 12 = 11$

f $90 \div 9 = 10$ and $90 \div 10 = 9$

g $25 \div 5 = 5$ and $25 \div 1 = 25$

h $63 \div 7 = 9$ and $63 \div 9 = 7$

i $144 \div 12 = 12$ and $144 \div 2 = 72$

j $72 \div 6 = 12$ and $72 \div 12 = 6$

k $108 \div 9 = 12$ and $108 \div 12 = 9$

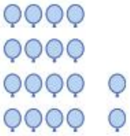
l $120 \div 12 = 10$ and $120 \div 10 = 12$

4 12 and 18

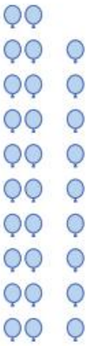
5 24, 56 and 96

6 **a** $12 \div 4 = 3$

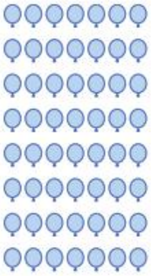
b $18 \div 4 = 4$ remainder 2



c $29 \div 10 = 2$ remainder 9



d $56 \div 8 = 7$



e $100 \div 8 = 12$ remainder 4

f $120 \div 9 = 13$ remainder 3

7 **a** 4 **b** 6 **c** 8

d 11 **e** 15 **f** 16

8 **a** $49 = 7 \times 7$ so $55 = 7 \times 7 + 6$; $55 \div 7 = 7$ remainder 6

b $94 = 7 \times 12 + 10$; $94 \div 12 = 7$ remainder 10

9 6 bags of 3 kg with 2 kg of sugar left over

10 **a** $15 = 4 \times 3 + 3$

b $29 = 4 \times 7 + 1$

c $30 = 12 \times 2 + 6$

d $157 = 15 \times 10 + 7$

e $12 = 4 \times 3 + 0$

f $14 \div 5 = 2$ remainder 4

g $26 \div 5 = 5$ remainder 1

h $191 \div 10 = 19$ remainder 1

i $192 \div 3 = 64$ remainder 0

Homework

1 Possible statements:

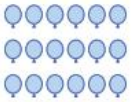
a $48 \div 8 = 6$ and $48 \div 6 = 8$

b $64 \div 8 = 8$ and $64 \div 4 = 16$

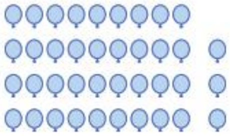
c $110 \div 10 = 11$ and $110 \div 11 = 10$

d $84 \div 12 = 7$ and $84 \div 7 = 12$

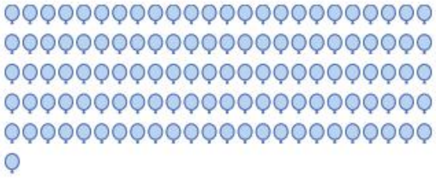
2 **a** $18 \div 3 = 6$



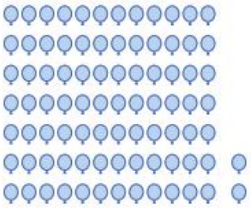
b $39 \div 4 = 9$ remainder 3



c $121 \div 5 = 24$ remainder 1



d 86 ÷ 7 = 12 remainder 2



4E WHOLE CLASS

1 a 7 remainder 1

b 114 remainder 12

c 104 remainder 25

d 86 remainder 50

4E INDIVIDUAL

1 a

$$\begin{array}{r}212\\ 4\overline{)848}\\ 8\\ \hline 4\\ 4\\ \hline 8\\ 8\\ \hline 0\\ \hline\end{array}$$

Check:

$$\begin{array}{r}212\\ \times\ \ 4\\ \hline 848\\ \hline\end{array}$$

b

$$\begin{array}{r}87\\ 5\overline{)435}\\ 40\\ \hline 35\\ 35\\ \hline 0\\ \hline\end{array}$$

Check:

$$\begin{array}{r}87\\ \times\ 5\\ \hline 435\\ \hline\end{array}$$

c

$$\begin{array}{r}114\\ 8\overline{)912}\\ 8\\ \hline 11\\ 8\\ \hline 32\\ 32\\ \hline 0\\ \hline\end{array}$$

Check:

$$\begin{array}{r}114\\ \times\ 8\\ \hline\hline 912\end{array}$$

d

$$\begin{array}{r}3\\ 37\overline{)111}\\ 111\\ \hline 0\\ \hline\end{array}$$

Check:

$$\begin{array}{r}37\\ \times\ 3\\ \hline 111\\ \hline\end{array}$$

e

$$\begin{array}{r}54\\ 32\overline{)1728}\\ 160\\ \hline 128\\ 128\\ \hline 0\\ \hline\end{array}$$

Check:

$$\begin{array}{r}54\\ \times\ \ 32\\ \hline 108\\ +\ 1620\\ \hline 1728\\ \hline\end{array}$$

f

$$\begin{array}{r}874\\ 12\overline{)10\,488}\\ 96\\ \hline 88\\ 84\\ \hline 48\\ 48\\ \hline 0\\ \hline\end{array}$$

Check:

$$\begin{array}{r}874\\ \times\ \ 12\\ \hline 1748\\ +\ 8740\\ \hline 10\,488\\ \hline\end{array}$$

g

$$\begin{array}{r}41\\ 29\overline{)1189}\\ 116\\ \hline 29\\ 29\\ \hline 0\\ \hline\end{array}$$

Check:

$$\begin{array}{r}41\\ \times\ \ 29\\ \hline 369\\ +\ 820\\ \hline 1189\\ \hline\end{array}$$

h

$$\begin{array}{r}743\\ 56\overline{)41608}\\ 392\\ \hline 240\\ 224\\ \hline 168\\ 168\\ \hline 0\\ \hline\end{array}$$

Check:

$$\begin{array}{r}743\\ \times\ \ \ 56\\ \hline 4458\\ +\ 37\,150\\ \hline 41\,608\\ \hline\end{array}$$

2 a

$$\begin{array}{r}282\ \ \text{r}2\\ 3\overline{)848}\phantom{\ \ \text{r}2}\\ 6\phantom{\ \ \text{r}2}\\ \hline 24\phantom{\ \ \text{r}2}\\ 24\phantom{\ \ \text{r}2}\\ \hline 8\phantom{\ \ \text{r}2}\\ 6\phantom{\ \ \text{r}2}\\ \hline 2\phantom{\ \ \text{r}2}\\ \hline\end{array}$$

Check:

$$\begin{array}{r}282\\ \times\ \ 3\\ \hline 846\\ +\ \ 2\\ \hline 848\\ \hline\end{array}$$

b $$\begin{array}{r} 72\ \text{r}3 \\ 6\overline{)435} \\ 42 \\ \hline 15 \\ 12 \\ \hline 3 \\ \hline \end{array} \qquad \begin{array}{r} \textit{Check:} \\ 72 \\ \times\ \ 6 \\ \hline 432 \\ +\ \ 3 \\ \hline \underline{435} \end{array}$$

c $$\begin{array}{r} 130\ \ \text{r}2 \\ 7\overline{)912} \\ 7\ \ \ \ \\ \hline 21\ \ \\ 21\ \ \\ \hline 2 \\ \hline \end{array} \qquad \begin{array}{r} \textit{Check:} \\ 130 \\ \times\ \ 7 \\ \hline 910 \\ +\ \ 2 \\ \hline \underline{912} \end{array}$$

d $$\begin{array}{r} 5\ \text{r}6 \\ 21\overline{)111} \\ 105 \\ \hline 6 \\ \hline \end{array} \qquad \begin{array}{r} \textit{Check:} \\ 21 \\ \times\ \ 5 \\ \hline 105 \\ +\ \ 6 \\ \hline \underline{111} \end{array}$$

e $$\begin{array}{r} 6\ \text{r}36 \\ 43\overline{)294} \\ 258 \\ \hline 36 \\ \hline \end{array} \qquad \begin{array}{r} \textit{Check:} \\ 43 \\ \times\ \ 6 \\ \hline 258 \\ +\ 36 \\ \hline \underline{294} \end{array}$$

f $$\begin{array}{r} 16\ \ \ \text{r}12 \\ 67\overline{)1084} \\ 67\ \ \\ \hline 414 \\ 402 \\ \hline 12 \\ \hline \end{array} \qquad \begin{array}{r} \textit{Check:} \\ 67 \\ \times\ \ 16 \\ \hline 402 \\ +\ 670 \\ \hline 1072 \\ +\ \ 12 \\ \hline \underline{1084} \end{array}$$

g $$\begin{array}{r} 345\ \ \text{r}2 \\ 72\overline{)24\,842} \\ 216\ \ \ \ \\ \hline 324\ \ \\ 288\ \ \\ \hline 362 \\ 360 \\ \hline 2 \\ \hline \end{array} \qquad \begin{array}{r} \textit{Check:} \\ 345 \\ \times\ \ 72 \\ \hline 690 \\ +\ 24\,150 \\ \hline 24\,840 \\ +\ \ 2 \\ \hline \underline{24\,842} \end{array}$$

h $$\begin{array}{r} 382\ \ \text{r}1 \\ 101\overline{)38\,583} \\ 303\ \ \ \ \\ \hline 828\ \ \\ 828\ \ \\ \hline 203 \\ 202 \\ \hline 1 \\ \hline \end{array} \qquad \begin{array}{r} \textit{Check:} \\ 382 \\ \times\ \ 101 \\ \hline 382 \\ +\ 38\,200 \\ \hline 38\,582 \\ +\ \ 1 \\ \hline \underline{38\,583} \end{array}$$

3 **a** 5000 **b** 1538 remainder 6

c 769 remainder 6 **d** 160

4 23 children **5** 17 haircuts

6 123 wards **7** 201 fridges

8 6 boxes of 72 light globes (remainder 3)

4F INDIVIDUAL

1 **a** $$\begin{array}{r} 212 \\ 4\overline{)848} \end{array} \qquad \begin{array}{r} \textit{Check:} \\ 212 \\ \times\ \ 4 \\ \hline \underline{848} \end{array}$$

b $$\begin{array}{r} 87 \\ 5\overline{)43^{3}5} \end{array} \qquad \begin{array}{r} \textit{Check:} \\ 87 \\ \times\ \ 5 \\ \hline \underline{435} \end{array}$$

c $$\begin{array}{r} 114 \\ 8\overline{)9^{1}1^{3}2} \end{array} \qquad \begin{array}{r} \textit{Check:} \\ 114 \\ \times\ \ 8 \\ \hline \underline{912} \end{array}$$

d $$\begin{array}{r} 2236 \\ 9\overline{)20^{2}1^{3}2^{5}4} \end{array} \qquad \begin{array}{r} \textit{Check:} \\ 2236 \\ \times\ \ \ 9 \\ \hline \underline{20\,124} \end{array}$$

2 36, 81 and 108

3 **c** I divided 313 by 3 and got remainder 1. So 3 is not a factor of 313.

d I divided 414 by 3 and got remainder 0. So 3 is a factor of 414.

e I divided 413 by 3 and got remainder 2. So 3 is not a factor of 413.

f I divided 414 by 9 and got remainder 0. So 9 is a factor of 414.

4G WHOLE CLASS

1 Teacher check

2 Possible solutions:

a 4020 **b** 4026 **c** 4020 **d** 4025

e 4120 **f** 4024 **g** 4220 **h** 4023

4G INDIVIDUAL

1 **a**

$$\begin{array}{r} 1\ 6\ \text{r}1 \\ 3\overline{)\,4\ {}^{1}9} \end{array}$$

Check:

$$\begin{array}{r} 16 \\ \times\ \ 3 \\ \hline 48 \\ +\ \ 1 \\ \hline 49 \end{array}$$

b

$$\begin{array}{r} 2\ 9\ \text{r}2 \\ 3\overline{)\,8\ {}^{2}9} \end{array}$$

Check:

$$\begin{array}{r} 29 \\ \times\ \ 3 \\ \hline 87 \\ +\ \ 2 \\ \hline 89 \end{array}$$

c

$$\begin{array}{r} 3\ 7 \\ 3\overline{)\,11\ {}^{2}1} \end{array}$$

Check:

$$\begin{array}{r} 37 \\ \times\ \ 3 \\ \hline 111 \end{array}$$

d

$$\begin{array}{r} 10\ 4\ \ \text{r}1 \\ 3\overline{)\,3\ 1\ {}^{1}3} \end{array}$$

Check:

$$\begin{array}{r} 104 \\ \times\ \ \ 3 \\ \hline 312 \\ +\ \ \ 1 \\ \hline 313 \end{array}$$

e

$$\begin{array}{r} 2\ 9\ 7 \\ 3\overline{)\,8\ {}^{2}9\ {}^{2}1} \end{array}$$

Check:

$$\begin{array}{r} 297 \\ \times\ \ 3 \\ \hline 891 \end{array}$$

2 No: 49, 89, 313. Yes: 111, 891

3 25, 40 and 18 200 are divisible by 5.

4 **a** 138 = 2 × 3 × 23 **b** 986 = 2 × 493

c 1118 = 2 × 13 × 43

d 20 790 = 2 × 3 × 3 × 3 × 5 × 7 × 11

5 Teacher check

4H REVIEW QUESTIONS

1 **a** 52, 100, 172, 128, 304, 512

b 112, 488, 224, 584, 656, 856

c 310, 930, 4760, 10 280, 64 200, 5720, 1 798 630

d 135, 207, 2313, 8019, 12 411, 296 739

e 176, 297, 814, 2915, 5929, 9867

f 90, 145, 170, 435, 610, 715, 1975, 2380

g 140, 520, 960, 660, 3400, 4780

h 114, 418, 684, 836, 2337, 6460

i 168, 441, 588, 735, 8022, 8673

j 270, 870, 1710, 1290, 5820, 21 450

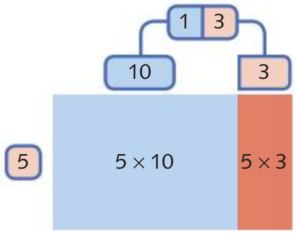
k 102, 186, 336, 780, 1854, 3600

l 125, 225, 300, 1000, 2250, 3000

2

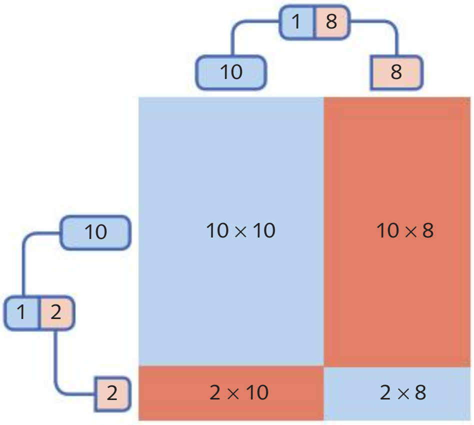
×	30	40	70	80
2	60	80	140	160
6	180	240	420	480
7	210	280	490	560
9	270	360	630	720

3 **a** 5 × 13



$$\begin{aligned} 5 \times 13 &= (5 \times 10) + (5 \times 3) \\ &= 50 + 15 \\ &= 65 \end{aligned}$$

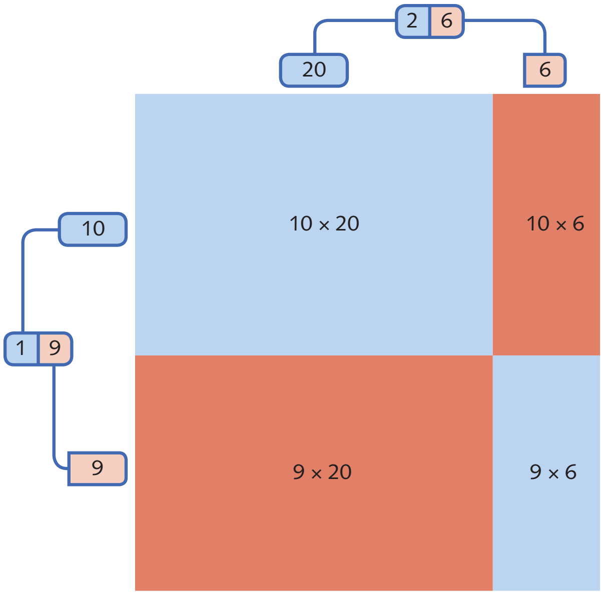
b 12 × 18



$$\begin{aligned} 12 \times 18 &= (10 \times 10) + (10 \times 8) + (2 \times 10) + (2 \times 8) \\ &= 100 + 80 + 20 + 16 \\ &= 216 \end{aligned}$$

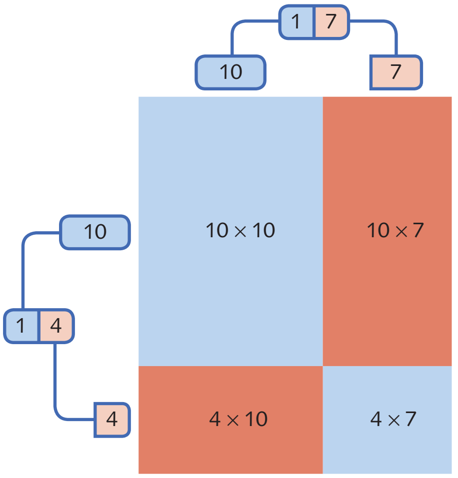
ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

c 26×19



$$\begin{aligned} 19 \times 26 &= (10 \times 20) + (10 \times 6) + (9 \times 20) + (9 \times 6) \\ &= 200 + 60 + 180 + 54 \\ &= 494 \end{aligned}$$

d 17×14



$$\begin{aligned} 17 \times 14 &= (10 \times 10) + (10 \times 4) + (7 \times 10) + (7 \times 4) \\ &= 100 + 40 + 70 + 28 \\ &= 238 \end{aligned}$$

4 **a** 208 **b** 1292 **c** 1927
d 4347 **e** 7154 **f** 7072
g 26 520 **h** 395 997 **i** 658 684
j 1 889 811 **k** 12 249 450 **l** 35 482 404

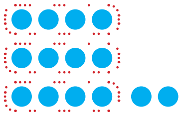
5 **a** 3031 **b** 3216 **c** 3475

6 **a** \$4500 **b** \$6300 **c** \$18 900

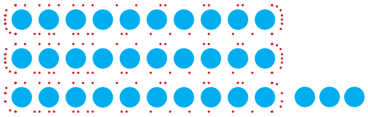
7 **a** 5, 10 **b** 7, 3 **c** 6, 9
d 8, 11 **e** 49, 256

8 **a** $23 \times 12 = 276$; correct
b $12 \times 11 = 132$; correct
c $45 \times 72 = 3168$; incorrect
d $27 \times 35 = 945$; correct
e $88 \times 78 = 6864$; incorrect
f $114 \times 63 = 7182$; correct

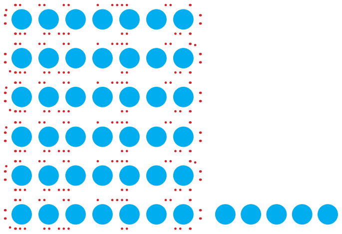
9 **a** $14 \div 4 = 3$ remainder 2



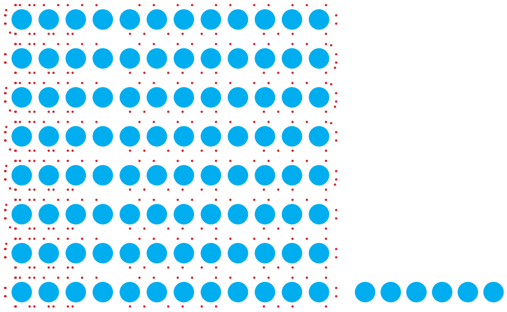
b $33 \div 10 = 3$ remainder 3



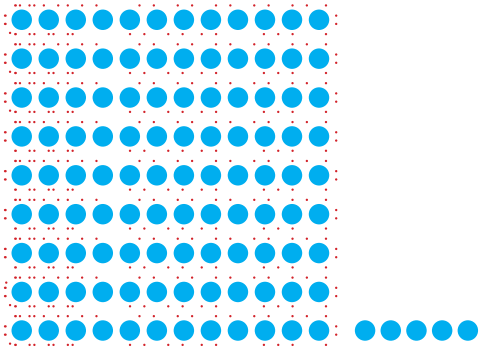
c $47 \div 7 = 6$ remainder 5



d $102 \div 8 = 12$ remainder 6



e $113 \div 9 = 12$ remainder 5



10 **a** 5 remainder 4 snakes
b 2 remainder 2 jelly beans
c 6 remainder 10 sour glow worms
d 13 remainder 3 Smarties

11 **a** 5 **b** 1
c 3, 9 **d** 10, 9
e 1 **f** 8 remainder 3
g 27 remainder 4 **h** 58 remainder 1

12 **a** 245; $245 \times 3 = 735$
b 97; $97 \times 6 = 582$
c 57; $57 \times 7 = 399$
d 8; $27 \times 8 = 216$
e 56 remainder 7; $56 \times 49 + 7 = 2751$
f 823 remainder 13; $823 \times 14 + 13 = 11\,535$
g 426 remainder 3; $426 \times 23 + 3 = 9801$
h 348 remainder 37; $348 \times 89 + 37 = 31\,009$

13 **a** 600; $600 \times 25 = 15\,000$
b 882 remainder 6; $882 \times 17 + 6 = 15\,000$
c 357 remainder 6; $357 \times 42 + 6 = 15\,000$
d 100; $150 \times 100 = 15\,000$

14 6 students

15 9000 metres

16 **a** 157 **b** 159
c 116 **d** 2891
e 209 remainder 1 **f** 132 remainder 3
g 132 remainder 4 **h** 3252 remainder 3

17 **a** 45, 72, 99 have 9 as a factor because their digits sum to a number divisible by 9.
b 42, 132, 438 have 6 as a factor because their digits sum to a number divisible by 3 and they are even.

18 **a** 303, 222 222 and 222 300
b 60 and 72 150

19 **a** 48, 76, 80, 30 and 200
b 33, 48, 30, 399 and 927
c 48, 76, 80 and 200
d 80, 30 and 200 **e** 927

20 **a** 29, 31, 37, 41, 43
b 7, 17, 37, 47, 67, 71, 73, 79, 97
c 11, 13, 17, 19, 23, 29
d 11, 13, 17, 19, 31, 41, 61, 71

21 219 is composite. Its factors are 1, 3, 73 and 219.

Chapter 5: Length, perimeter, area and volume

Show what you know

1 pencil B **2** pencil C

5A WHOLE CLASS

Teacher check

5A INDIVIDUAL

1 **a** metres; tape measure or metre wheel
b centimetres; ruler
c kilometres; atlas
d millimetres; ruler

2 Teacher check

3 **a** 5 cm **b** 13 cm **c** 9 cm

4 **a** 25 mm **b** 84 mm

5 **a** 9.3 m or 9 m 30 cm
b 8.05 m or 8 m 5 cm

6 **a** 840 mm
b 160 mm

5B WHOLE CLASS

1–2 Teacher check

3 **a** 12 500 mm **b** 12.5 m

5B INDIVIDUAL

1 **a** 2 m **b** 7 m **c** 4.25 m
d 6.5 m **e** 0.75 m **f** 0.07 m

2 **a** 3 cm **b** 10 cm **c** 38.7 cm
d 22.9 cm **e** 0.5 cm **f** 408 cm

3 **a** 4 m **b** 6 m **c** 1.75 m
d 2.535 m **e** 0.005 m **f** 0.635 m

4 **a** 5 km **b** 9 km **c** 2.475 km
d 6.005 km **e** 0.095 km **f** 0.421 km

5 **a** 600 cm **b** 200 cm **c** 450 cm
d 675 cm **e** 109 cm **f** 30 cm

6 **a** 80 mm **b** 110 mm **c** 28 mm
d 2 mm **e** 136 mm **f** 8 mm

7 **a** 3000 mm **b** 9000 mm **c** 4300 mm
d 12 750 mm **e** 6200 mm **f** 200 mm

8 **a** 7000 m **b** 4000 m **c** 8750 m
d 11 625 m **e** 15 005 m **f** 832 640 m

9 8.1 m in length

10 **a** 0.18 m **b** 18 cm **c** 180 mm

11 1.85 m of cable

12 16.15 m **13** 4.25 m **14** 5.43 km

15 5.045 m **16** 9.75 m

17 50 m of string

18 **a** 3625 mm **b** 1375 mm

19 550 m **20** 375 cm

5C INDIVIDUAL

1 **a** 10 cm **b** 16 cm **c** 22 cm
d 22 cm **e** 24 cm

2 **a** 27 cm **b** 22 cm **c** 40 cm
d 30 cm **e** 68 cm **f** 63 cm
g 82 cm **h** 46 cm **i** 50 cm

3 **a** 24 cm **b** 26 cm **c** 26 cm
4 **a** 24 m **b** 20 m **c** 22 m
5 **a** 28 cm **b** 32 m **c** 23 m
d 55 cm **e** 35 cm
6 **a** 32 m **b** 40 m **c** 40 m
d 50 m **e** 60 m **f** 50 m

5D INDIVIDUAL

1 **a** 12 cm^2 **b** 10 cm^2
c 15 cm^2 **d** 16 cm^2
2 **a** 54 m^2 **b** 96 cm^2
c 140 m^2 **d** 225 cm^2
3 **a** 36 m^2 **b** 225 cm^2
c 400 m^2 **d** 8100 cm^2
4 **a** area = 108 mm^2; perimeter = 42 mm
b area = 200 cm^2; perimeter = 66 cm
c area = 140 m^2; perimeter = 54 m
d area = 72 km^2; perimeter = 34 km
5 **a** 26 m^2 **b** 72.5 cm^2
c 35 m^2 **d** 650 m^2
6 **a** 51 m^2 **b** 32.5 m^2 **c** $8\frac{1}{2}$ cm^2
d 205 cm^2 **e** 1.5 m^2 **f** 120 cm^2
g 0.375 m^2
7 **a** 11 m, 165 m^2 **b** 14 m, 84 m^2

5E INDIVIDUAL

1 **a** 44 cm^2 **b** 102 cm^2 **c** 96 cm^2
d 136 cm^2 **e** 74 cm^2 **f** 64 cm^2
g 120 cm^2 **h** 92 cm^2 **i** 14 cm^2
j 78 cm^2
2 **a** 38 cm^2 **b** 42 m; 90 m^2

5F INDIVIDUAL

1 **a** base = 7 cm; height = 6 cm
b base = 15 cm; height = 8 cm
c base = 3 cm; height = 4 cm
d base = 4 cm; height = 5 cm
2 **b** 11 cm^2 **c** 90 cm^2
3 **a** 100 cm^2 **b** 30 cm^2 **c** 40 cm^2

5G INDIVIDUAL

1 **a** 1 ha **b** 5 ha **c** 8.5 ha
d 1.2 ha **e** 3.6 ha **f** 12.5 ha
g 4.325 ha **h** 7.4585 ha
2 **a** 40 000 m^2 **b** 70 000 m^2
c 110 000 m^2 **d** 270 000 m^2
e 35 000 m^2 **f** 69 000 m^2
g 65 700 m^2 **h** 55 000 m^2
3 41.4 ha
4 325 cows

5H WHOLE CLASS

1 **a** 10 cm^3
b $5 \times 2 \times 1 = 10$ cm^3
c 34 faces
2 **a** 36 cm^3
b $4 \times 3 \times 3 = 36$ cm^3
c 66 faces

5H INDIVIDUAL

1 **a** Teacher check
b A = 24 cm^3 B = 24 cm^3 C = 12 cm^3
c Prism C is the smallest; prisms A and B are equal in volume.
d $A = 3 \times 2 \times 4 = 24$ cm^3
$B = 4 \times 2 \times 3 = 24$ cm^3
$C = 3 \times 2 \times 2 = 12$ cm^3
2 **a** 40 cm^3 **b** 24 cm^3 **c** 36 m^3
d 360 cm^3 **e** 100 m^3 **f** 140 m^3
3 1800 small boxes
4 **a** 27 cm^3 **b** 1000 cm^3 **c** 1728 cm^3
d 4913 cm^3 **e** 15 625 cm^3
5 **a** 30 cm^3 **b** 36 cm^3
c 40 cm^3 **d** 32 cm^3
6 42 m^3 of soil **7** 7.2 m^3 of soil
8 **a** 3 m **b** 2 cm **c** 4 m
d 5 cm **e** 2 m **f** 1.5 cm
9 9 cm high
10 **a** 169 cubes **b** 386 cubes
c 7657 cubes **d** 999 657 cubes
11 **a** 8 cubes **b** 8 cm^3 **c** 19 cubes
12 **a** 11 cubes **b** 13 cubes
13 **a** 11 cubes **b** 49 cubes
14 9 cm

5I INDIVIDUAL

1 **a** 7000 mL **b** 12 000 mL
c 342 000 mL **d** 1 000 000 mL
2 **a** 1 L **b** 13 L **c** 3.42 L
3 **a** 4 kL **b** 18 kL **c** 39.870 kL
4 **a** 4 000 000 mL **b** 23 000 000 mL
c 815 000 000 mL

5 **a** 288 L of water **b** 1.152 kL of water
6 **a** 21 600 L **b** 360 hours

5J REVIEW QUESTIONS

1 **a** 8 cm **b** 12 cm **c** 40 mm **d** 8 cm
2 9 ropes, 2.53 m left over
3 **a** 64 cm **b** 71 m **c** 66 mm **d** 80 cm
4

Rectangle	Length	Width	Perimeter	Area
a	4 cm	5 cm	18 cm	20 m^2
b	10 m	40 m	100 m	400 m^2
c	12 mm	20 mm	64 mm	240 mm^2
d	18 cm	0.5 cm	37 cm	9 cm^2
e	52 m	25 cm	104.5 m	13 m^2

5 **a** unknown side = 5 m; area = 89 m^2; perimeter = 48 m
b area = 16 m^2; perimeter = 18 m
6 **a** 1 ha **b** 6 ha **c** 2.5 ha **d** 1.9 ha
7 **a** 10 000 m^2 **b** 30 000 m^2 **c** 150 000 m^2 **d** 12 000 m^2
8 **a** 666 sheep **b** 53 sheep **c** 1000 sheep
9

Prism	Length	Width	Height	Calculation	Volume
a	1	1	1	$1 \times 1 \times 1$	1 cm^3
b	3	6	3	$3 \times 6 \times 3$	54 cm^3
c	5	8	7	$5 \times 8 \times 7$	280 cm^3
d	13	2	2	$13 \times 2 \times 2$	52 cm^3

10 **a** W = 700 cm **b** H = 4 cm **c** L = 3 m
11 **a** 4000 mL **b** 19 000 mL **c** 60 300 mL **d** 4 294 000 mL
12 **a** 5 L **b** 78 L **c** 1.003 L **d** 0.789 L
13 **a** 6 kL **b** 46 kL **c** 88 kL **d** 1111.001 kL

Chapter 6: Fractions

Kick off

1 Factors of 12: 1, 2, 3, 4, 6 and 12. Factors of 8: 1, 2, 4 and 8. The shared factors are 2 and 4.
2 Multiples of 3: 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, 39
Multiples of 4: 4, 8, 12, 16, 20, 24, 28, 32, 36, 40. The shared multiples are 12, 24, 36.

6A WHOLE CLASS

1 **a** $\frac{1}{4}$ of the eggs are cracked. **b** Susan drank $\frac{1}{3}$ of the orange juice.
c $\frac{19}{56}$ of the marbles are green.
d $\frac{10}{19}$ of the numbers from 1 to 19 are odd: 1, 3, 5, 7, 9, 11, 13, 15, 17, 19
e $\frac{43}{300}$ of the table is covered by Veronica's diary.

2 **a** $\frac{1}{6}$ of the sides show the number 1.

b $\frac{1}{6}$ of the sides show the number 4.

c $\frac{1}{2}$ of the sides show odd numbers.

d $\frac{1}{2}$ of the sides show a prime number (2, 3 and 5, as 1 is not a prime number).

6A INDIVIDUAL

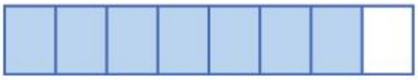
1 **a** denominator **b** denominator
c numerator **d** numerator

2 **a** one-half **b** eleven-twelfths
c twenty-seven thirty-sevenths
d eighty-nine hundredths

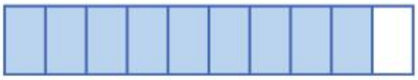
3 **a** $\frac{2}{3}$ **b** $\frac{7}{7} = 1$ **c** $\frac{1}{2}$ **d** $\frac{7}{20}$

4 **a**

b



c



5 **a** $\frac{2}{5}$ **b** $\frac{3}{8}$ **c** $\frac{5}{7}$ **d** $\frac{1}{3}$ **e** $\frac{5}{8}$

6 **a**

b

c

d

7 **a** $\frac{1}{12}$ **b** $\frac{2}{3}$ **c** $\frac{1}{4}$

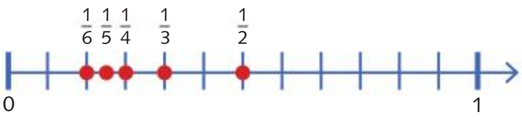
8 $\frac{9}{16}$

9 4 doughnuts were chocolate-iced.

6B INDIVIDUAL

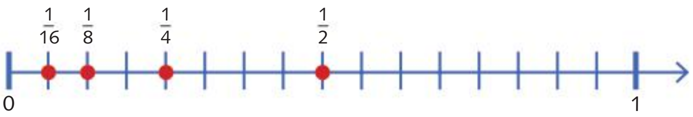
1 **a** $\frac{3}{5}$ **b** $\frac{6}{8}$ or $\frac{3}{4}$ **c** $\frac{2}{4}$ or $\frac{1}{2}$

2



Each fraction is smaller that the previous one. As the numbers in the denominator get bigger, the fractions get smaller.

3



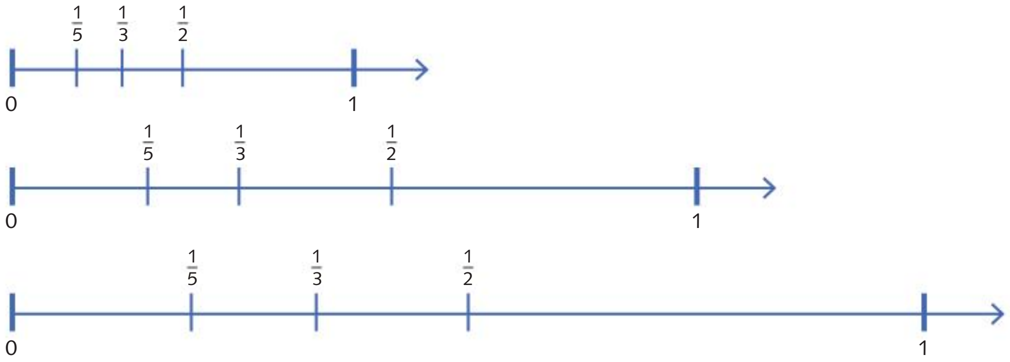
Each fraction is half the size of the previous one. As the denominator doubles, the fraction halves.

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

4

Each fraction is half the size of the previous one. As the denominator doubles, the fraction halves.

5



(The number lines are not to scale.) The $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{5}$ marks on the 20 cm number line are twice the distance of the marks on the 10 cm number line. The $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{5}$ marks on the 30 cm number line are three times the distance of the marks on 10 cm number line.

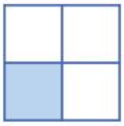
6C WHOLE CLASS

See **BLM 6** answers in the Interactive Textbook.

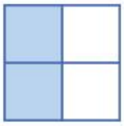
6C INDIVIDUAL

1 Possible answers:

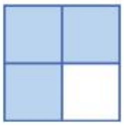
a



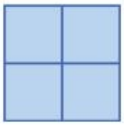
b



c

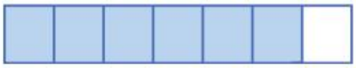


d



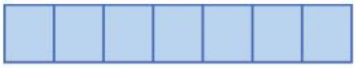
2 **a**

b

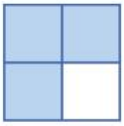


c

d

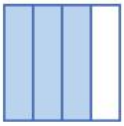


3 **a**



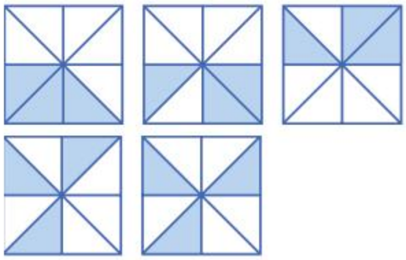
b

c



d

4



5 Picture B represents $\frac{2}{3}$. The other pictures do not show equal parts.

6 **a** $\frac{1}{4}$ **b** $\frac{1}{2}$ **c** $\frac{1}{4}$

d $\frac{1}{4}$ **e** $\frac{1}{2}$ **f** $\frac{1}{2}$

7

6D WHOLE CLASS

1 **a** $\frac{16}{20}$ **b** $\frac{1}{4}$

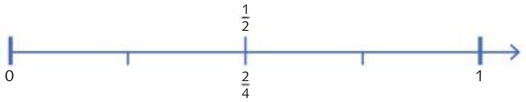
3 smaller than $\frac{3}{5}$: $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{10}, \frac{2}{9}, \frac{5}{10}$

equivalent to $\frac{3}{5}$: $\frac{6}{10}, \frac{30}{50}$

larger than $\frac{3}{5}$: $\frac{5}{8}, \frac{5}{3}, \frac{3}{4}$

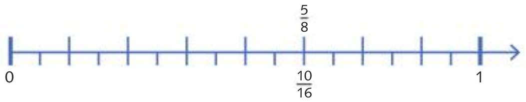
6D INDIVIDUAL

1 a



b

c



d

2 Possible answers:

a $\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8}$ **b** $\frac{2}{5} = \frac{4}{10} = \frac{6}{15} = \frac{8}{20}$

c $\frac{5}{6} = \frac{10}{12} = \frac{15}{18} = \frac{20}{24}$ **d** $\frac{2}{7} = \frac{4}{14} = \frac{6}{21} = \frac{8}{28}$

3 a 2, 4, 50 **b** 3, 40, 9

c 10, 100, 21 **d** 3, 12, 20

4 b
$$\overbrace{\underbrace{\frac{1}{3} = \frac{4}{12}}_{\times 4}}^{\times 4}$$

c
$$\overbrace{\underbrace{\frac{4}{5} = \frac{8}{10}}_{\times 2}}^{\times 2}$$

d
$$\overbrace{\underbrace{\frac{12}{9} = \frac{120}{90}}_{\times 10}}^{\times 10}$$

5 b
$$\overbrace{\underbrace{\frac{9}{12} = \frac{3}{4}}_{\div 3}}^{\div 3}$$

c
$$\overbrace{\underbrace{\frac{49}{56} = \frac{7}{8}}_{\div 7}}^{\div 7}$$

d
$$\overbrace{\underbrace{\frac{28}{12} = \frac{7}{3}}_{\div 4}}^{\div 4}$$

6 smaller than $\frac{3}{4}$: $\frac{1}{8}, \frac{1}{4}, \frac{1}{2}, \frac{5}{8}, \frac{2}{9}, \frac{5}{10}, \frac{1}{3}, \frac{2}{10}$

equivalent to $\frac{3}{4}$: $\frac{12}{16}, \frac{6}{8}, \frac{75}{100}$

larger than $\frac{3}{4}$: $\frac{9}{11}$

7 a 2 **b** 10 **c** 5 **d** 4

8 a $\frac{2}{3}$ **b** $\frac{4}{5}$ **c** $\frac{7}{10}$

d $\frac{11}{3}$ **e** $\frac{9}{14}$

6E WHOLE CLASS

1 The order should be:
$\frac{1}{10}, \frac{1}{5}, \frac{2}{9}, \frac{3}{10}, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \frac{7}{10}, \frac{3}{4}, \frac{5}{6}$

2 The order should be:
$\frac{1}{6}, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{5}{4}, \frac{4}{3}$ and $\frac{8}{6}$ on the same place, $\frac{6}{4}, \frac{13}{3}$

6E INDIVIDUAL

1 a $\frac{2}{3}$ **b** $\frac{3}{5}$ **c** $\frac{3}{8}$ **d** $\frac{25}{4}$

2 a $\frac{1}{7}, \frac{3}{7}, \frac{5}{7}, \frac{6}{7}, \frac{8}{7}$ **b** $\frac{2}{10}, \frac{3}{10}, \frac{5}{10}, \frac{9}{10}, \frac{10}{10}$

c $\frac{1}{12}, \frac{1}{8}, \frac{1}{4}, \frac{3}{8}, \frac{7}{12}$ **d** $\frac{1}{20}, \frac{1}{5}, \frac{3}{5}, \frac{7}{10}, \frac{18}{20}$

e $\frac{1}{8}, \frac{1}{4}, \frac{1}{2}, \frac{5}{6}, \frac{23}{24}$ **f** $\frac{1}{8}, \frac{5}{12}, \frac{37}{80}, \frac{9}{11}, \frac{66}{67}$

3 a $\frac{7}{10}$ **b** $\frac{5}{12}$ **c** $\frac{5}{9}$ **d** $\frac{8}{9}$

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

4

	Fraction	• is smaller than • is equivalent to • is larger than	Fraction
a	$\frac{1}{2}$	is larger than	$\frac{1}{4}$
b	$\frac{1}{2}$	is equivalent to	$\frac{4}{8}$
c	$\frac{3}{8}$	is smaller than	$\frac{3}{4}$
d	$\frac{5}{6}$	is larger than	$\frac{2}{3}$
e	$\frac{3}{4}$	is smaller than	$\frac{4}{3}$
f	$\frac{6}{8}$	is equivalent to	$\frac{9}{12}$
g	$\frac{3}{20}$	is larger than	$\frac{1}{15}$

5 Possible answers:

a $\frac{3}{8}$ **b** $\frac{9}{10}$ **c** $\frac{5}{12}$

d $\frac{1}{100}$ **e** $\frac{99}{100}$ **f** $\frac{1}{905}$

6 The milkshake recipe needs more milk, as $\frac{3}{4} = \frac{6}{8}$ and $\frac{7}{8}$ is greater than $\frac{6}{8}$.

7 Team A has more players at training, as $\frac{3}{4} = \frac{9}{12}$ and $\frac{9}{12}$ is greater than $\frac{7}{12}$.

6F INDIVIDUAL

1 **a** $\frac{1}{2}$, proper **b** $\frac{19}{2}$, improper

c $\frac{8}{8}$, improper **d** $\frac{3}{5}$, proper

e $2\frac{2}{11}$, mixed number

f $100\frac{18}{100}$, mixed number

g $\frac{1}{1000}$, proper

2 **a** 3 **b** 7 **c** 10 **d** 7

e 6 **f** 9 **g** 2

3 **a** $2\frac{1}{2}$ **b** $2\frac{1}{3}$ **c** $1\frac{1}{5}$ **d** $1\frac{1}{100}$

e $9\frac{1}{3}$ **f** $1\frac{89}{100}$ **g** $12\frac{3}{4}$

4 **a** $\frac{11}{3}$ **b** $\frac{5}{2}$ **c** $\frac{43}{8}$ **d** $\frac{29}{15}$

e $\frac{43}{9}$ **f** $\frac{46}{4}$ **g** $\frac{704}{7}$

5 **a** $2\frac{1}{3}$ **b** $1\frac{3}{4}$ **c** $5\frac{1}{3}$

6 **a** $6\frac{1}{2}$ bottles of soft drink

b $4\frac{7}{8}$ pizzas

c $10\frac{1}{2}$ bottles of soft drink and $7\frac{7}{8}$ pizzas

7 **a** 52 pieces of rope of length $\frac{1}{8}$ metres

b 50 pieces of rope of length $\frac{1}{6}$ metres

c 9 lengths of $\frac{3}{4}$ metres with $\frac{1}{2}$ metre piece left over

6G REVIEW QUESTIONS

1 **a** one-third **b** nine-elevenths

c sixteen thirty-ninths

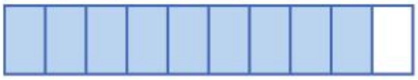
d sixty-five thousandths

2 **a** $\frac{5}{9}$ **b** $\frac{8}{23}$

3 **a**

b

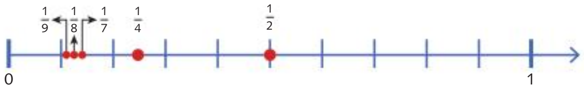
c



4 **a** $\frac{12}{40} = \frac{3}{10}$ **b** $\frac{23}{40}$ **c** $\frac{5}{40} = \frac{1}{8}$

5 **a** $\frac{1}{6}$ **b** $\frac{1}{3}$ **c** $\frac{3}{4}$

6



As the denominator gets bigger, the fraction gets smaller.

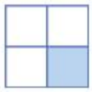
7

As the denominator gets bigger, the fraction gets smaller.

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press
Photocopying is restricted under law and this material must not be transferred to another party.

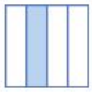
8 Possible answers:

a



b

c



d

9 Possible answers:

10 Possible answers:

a $\frac{2}{6}$ $\frac{3}{9}$ $\frac{10}{30}$

b $\frac{8}{14}$ $\frac{12}{21}$ $\frac{44}{77}$

c $\frac{9}{12}$ $\frac{6}{8}$ $\frac{12}{16}$

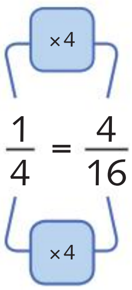
d $\frac{1}{2}$ $\frac{3}{6}$ $\frac{25}{50}$

11 **a** $\frac{1}{3} = \frac{2}{6} = \frac{3}{9} = \frac{18}{54}$

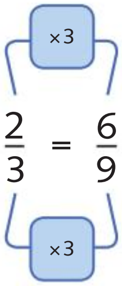
b $\frac{2}{5} = \frac{4}{10} = \frac{20}{50} = \frac{10}{25}$

c $\frac{4}{3} = \frac{20}{15} = \frac{16}{12} = \frac{24}{18}$

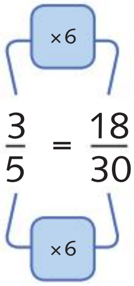
12 **a**



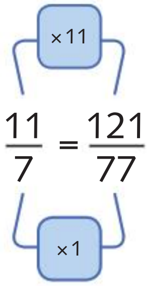
b



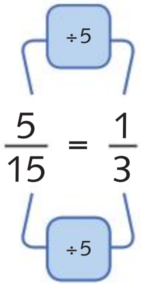
c



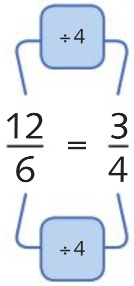
d



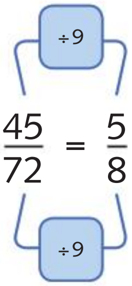
13 **a**



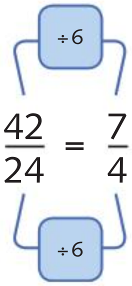
b



c



d



14 **a** 3 **b** 3 **c** 4 **d** 4

15 **a** $\frac{3}{8}, \frac{3}{8}$ **b** $\frac{2}{3}, \frac{2}{3}$ **c** $\frac{3}{4}, \frac{3}{4}$ **d** $\frac{9}{7}, \frac{9}{7}$

16 **a** $\frac{1}{5}, \frac{3}{5}, \frac{4}{5}, \frac{6}{5}, \frac{8}{5}$ **b** $\frac{2}{8}, \frac{3}{8}, \frac{5}{8}, \frac{9}{8}, \frac{10}{8}$

c $\frac{1}{30}, \frac{1}{20}, \frac{1}{10}, \frac{3}{10}, \frac{7}{20}$

d $\frac{1}{9}, \frac{1}{3}, \frac{3}{6}, \frac{8}{15}, \frac{18}{21}$

17 Possible answers:

a $\frac{2}{5}$ **b** $\frac{19}{20}$ **c** $\frac{10}{16}$ **d** $\frac{1}{1\,000\,000}$

18 Box A has $\frac{2}{3} = \frac{6}{9}$ remaining; box B has $\frac{5}{9}$ remaining. Box A has more oranges left.

19 The cake needs $\frac{5}{8} = \frac{15}{24}$ of a kilogram; the scone recipe needs $\frac{10}{12} = \frac{20}{24}$ of a kilogram. So the scone recipe needs more flour.

20 **a** proper fraction **b** proper fraction
c improper fraction **d** proper fraction
e improper fraction **f** improper fraction
g mixed number

21 **a** $6\frac{2}{3}$ **b** $2\frac{1}{2}$ **c** $1\frac{1}{5}$ **d** $1\frac{27}{120} = 1\frac{9}{40}$

e 9 **f** $6\frac{1}{12}$ **g** $12\frac{2}{9}$

22 **a** $\frac{11}{3}$ **b** $\frac{5}{2}$ **c** $\frac{43}{8}$ **d** $\frac{29}{15}$

e $\frac{43}{9}$ **f** $\frac{116}{11}$ **g** $\frac{704}{7}$

Chapter 7: Fraction arithmetic

Kick off

a 1 **b** $\frac{1}{2}$ **c** $\frac{2}{3}$ **d** $\frac{1}{4}$

e 1 **f** $\frac{7}{8}$ **g** $\frac{3}{4}$ **h** $1\frac{1}{2}$

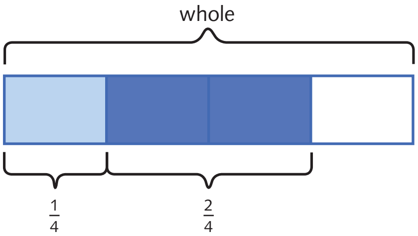
Show what you know

1 **a** 3 **b** $\frac{1}{2}$ **c** $1\frac{1}{2}$

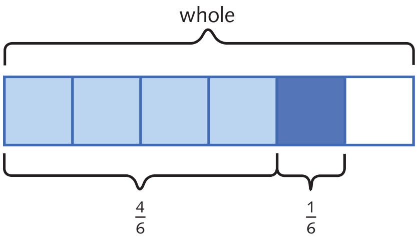
ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

7A WHOLE CLASS

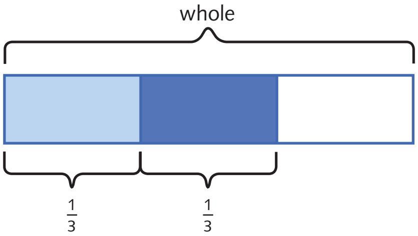
1 **a** $\frac{1}{4}+\frac{2}{4}=\frac{3}{4}$



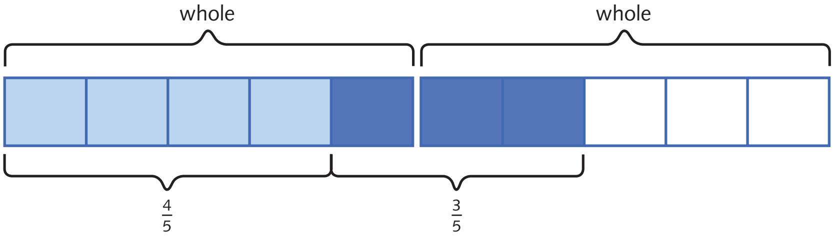
b $\frac{4}{6}+\frac{1}{6}=\frac{5}{6}$



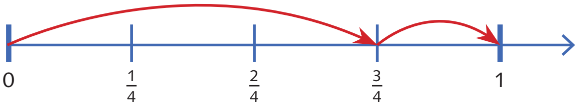
c $\frac{1}{3}+\frac{1}{3}=\frac{2}{3}$



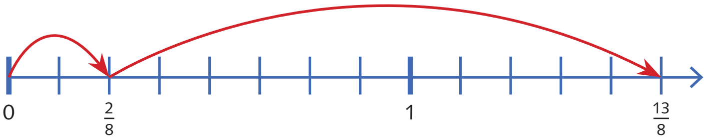
d $\frac{4}{5}+\frac{3}{5}=1\frac{2}{5}$



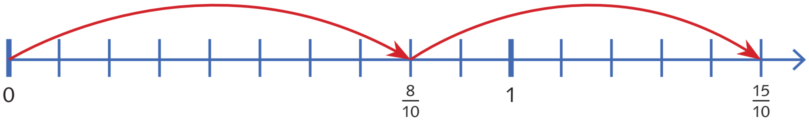
2 **a** $\frac{3}{4}+\frac{1}{4}=1$



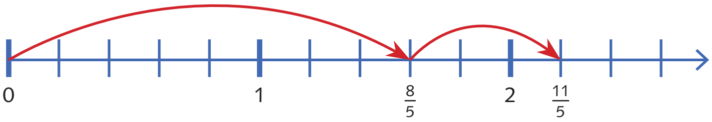
b $\frac{2}{8}+\frac{11}{8}=\frac{13}{8}+1\frac{5}{8}$



c $\frac{8}{10}+\frac{7}{10}=\frac{15}{10}=1\frac{1}{2}$



d $\frac{8}{5}+\frac{3}{5}=\frac{11}{5}=2\frac{1}{5}$



7A INDIVIDUAL

1 **a** 1

b $\frac{4}{5}$

c $\frac{3}{7}$

d $\frac{13}{16}$

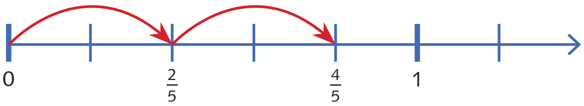
e $\frac{10}{11}$

f $1\frac{1}{3}$

g $\frac{4}{5}$

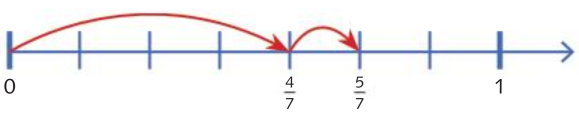
h $\frac{14}{17}$

2 **a** $\frac{2}{5}+\frac{2}{5}=\frac{4}{5}$



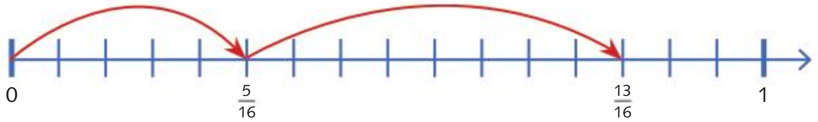
ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

b $\frac{4}{7} + \frac{1}{7} = \frac{5}{7}$

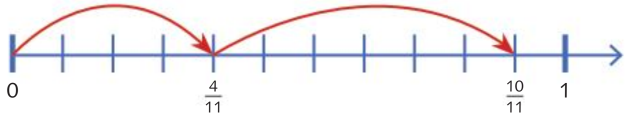


c $\frac{2}{7} + \frac{1}{7} = \frac{3}{7}$

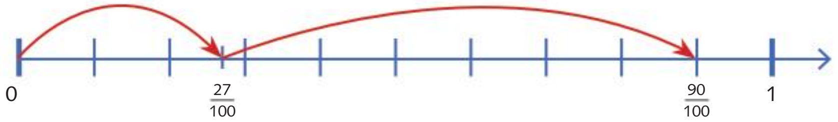
d $\frac{5}{16} + \frac{8}{16} = \frac{13}{16}$



e $\frac{4}{11} + \frac{6}{11} = \frac{10}{11}$



f $\frac{27}{100} + \frac{63}{100} = \frac{90}{100}$



3 **a** Tuesday, Wednesday and Thursday

b $\frac{41}{8}$ or $5\frac{1}{8}$ loaves of bread

c $\frac{5}{8}$ of a loaf of bread

d $\frac{46}{8} = 5\frac{3}{4}$ loaves of bread

7B WHOLE CLASS

1 **a** $\frac{1}{2}$ **b** $\frac{3}{4}$ **c** $\frac{7}{8}$ **d** $3\frac{1}{4}$

e $1\frac{1}{8}$ **f** $\frac{5}{8}$ **g** $\frac{7}{8}$ **h** $2\frac{1}{4}$

2 **a** $\frac{3}{4}$ **b** $\frac{5}{12}$ **c** $1\frac{1}{6}$ **d** $\frac{11}{12}$

3 **a** $\frac{2}{4}, \frac{3}{6}, \frac{4}{8}, \frac{5}{10}, \frac{6}{12}, \frac{7}{14}$

b $\frac{2}{10}, \frac{3}{15}, \frac{4}{20}, \frac{5}{25}, \frac{6}{30}, \frac{7}{35}$

c $\frac{5}{10}$ and $\frac{2}{10}$

d $\frac{1}{2} + \frac{1}{5} = \frac{5}{10} + \frac{2}{10} = \frac{7}{10}$

4 **a** $\frac{2}{8}, \frac{3}{12}, \frac{4}{16}, \frac{5}{20}, \frac{6}{24}, \frac{7}{28}$

b $\frac{6}{10}, \frac{9}{15}, \frac{12}{20}, \frac{15}{25}, \frac{18}{30}, \frac{21}{35}$

c $\frac{5}{20}$ and $\frac{12}{20}$

d $\frac{1}{4} + \frac{3}{5} = \frac{5}{20} + \frac{12}{20} = \frac{17}{20}$

5 Teacher to check drawings.

a $\frac{5}{6}$ **b** $1\frac{1}{6}$ **c** $1\frac{1}{2}$ **d** $3\frac{5}{6}$

7B INDIVIDUAL

1 **a** $\frac{1}{8} + \frac{1}{4} = \frac{1}{8} + \frac{2}{8} = \frac{3}{8}$

b $\frac{2}{5} + \frac{3}{10} = \frac{4}{10} + \frac{3}{10} = \frac{7}{10}$

c $\frac{1}{5} + \frac{3}{10} = \frac{2}{10} + \frac{3}{10} = \frac{5}{10} = \frac{1}{2}$

2 **a** $\frac{1}{4} + \frac{1}{8} = \frac{2}{8} + \frac{1}{8} = \frac{3}{8}$

b $\frac{2}{6} + \frac{3}{12} = \frac{4}{12} + \frac{3}{12} = \frac{7}{12}$

c $\frac{4}{10} + \frac{1}{5} = \frac{4}{10} + \frac{2}{10} = \frac{6}{10} = \frac{3}{5}$

d $\frac{1}{100} + \frac{1}{50} = \frac{1}{100} + \frac{4}{100} = \frac{5}{100} = \frac{1}{20}$

e $\frac{2}{5} + \frac{2}{15} = \frac{6}{15} + \frac{2}{15} = \frac{8}{15}$

3 **a** $\frac{3}{8} + \frac{1}{2} = \frac{7}{8}$ **b** $\frac{1}{5} + \frac{7}{10} = \frac{9}{10}$

c $\frac{1}{6} + \frac{7}{12} = \frac{9}{12} = \frac{3}{4}$ **d** $\frac{1}{9} + \frac{2}{3} = \frac{7}{9}$

e $\frac{1}{9} + \frac{1}{3} = \frac{4}{9}$ **f** $\frac{2}{9} + \frac{1}{3} = \frac{5}{9}$

4 They ate $\frac{1}{8}+\frac{1}{4}=\frac{3}{8}$ of a kilogram of grapes.

5 **a** $1+\frac{1}{2}=1\frac{1}{2}$

b $\frac{3}{4}+\frac{3}{4}+\frac{1}{4}=1\frac{1}{4}$

c $\frac{1}{3}+\frac{2}{3}+\frac{2}{3}=1\frac{2}{3}$

6 **a** $\frac{3}{5}+\frac{9}{10}=\frac{6}{10}+\frac{9}{10}=\frac{15}{10}=\frac{10}{10}+\frac{5}{10}=1\frac{1}{2}$

b $\frac{3}{8}+\frac{11}{12}=\frac{9}{24}+\frac{22}{24}=\frac{31}{24}=\frac{24}{24}+\frac{7}{24}=1\frac{7}{24}$

c $\frac{5}{6}+\frac{3}{4}=\frac{10}{12}+\frac{9}{12}=\frac{19}{12}=\frac{12}{12}+\frac{7}{12}=1\frac{7}{12}$

d $\frac{14}{20}+\frac{4}{5}=\frac{14}{20}+\frac{16}{20}=\frac{30}{20}=\frac{20}{20}+\frac{10}{20}=1\frac{1}{2}$

7 **a** $\frac{1}{4}+\frac{2}{12}+\frac{1}{6}=\frac{3}{12}+\frac{2}{12}+\frac{2}{12}=\frac{7}{12}$

b $\frac{2}{5}+\frac{1}{10}+\frac{11}{20}+\frac{2}{10}=\frac{8}{20}+\frac{2}{20}+\frac{11}{20}+\frac{4}{20}=\frac{25}{20}$
$=1\frac{5}{20}=1\frac{1}{4}$

c $\frac{2}{7}+\frac{1}{14}+\frac{1}{7}+\frac{5}{14}+\frac{3}{7}=\frac{4}{14}+\frac{1}{14}+\frac{2}{14}+\frac{5}{14}$
$+\frac{6}{14}=\frac{18}{14}=1\frac{4}{14}=1\frac{2}{7}$

d $\frac{5}{16}+\frac{1}{8}+\frac{1}{4}+\frac{1}{16}=\frac{5}{16}+\frac{2}{16}+\frac{4}{16}+\frac{1}{16}$
$=\frac{12}{16}=\frac{3}{4}$

e $\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}=\frac{8}{16}+\frac{4}{16}+\frac{2}{16}+\frac{1}{16}=\frac{15}{16}$

f $\frac{2}{3}+\frac{8}{20}+\frac{4}{30}+\frac{7}{15}=\frac{20}{30}+\frac{12}{30}+\frac{4}{30}+\frac{14}{30}=\frac{50}{30}$
$=\frac{5}{3}=1\frac{2}{3}$

7C INDIVIDUAL

1 **a** $\frac{3}{4}-\frac{1}{4}=\frac{2}{4}=\frac{1}{2}$

b $\frac{4}{5}-\frac{2}{5}=\frac{2}{5}$

c $\frac{6}{7}-\frac{2}{7}=\frac{4}{7}$

d $\frac{3}{4}-\frac{1}{8}=\frac{5}{8}$

e $\frac{4}{5}-\frac{1}{10}=\frac{7}{10}$

f $\frac{2}{3}-\frac{1}{6}=\frac{3}{6}=\frac{1}{2}$

g $\frac{3}{4}-\frac{3}{8}=\frac{3}{8}$

h $\frac{9}{10}-\frac{1}{5}=\frac{7}{10}$

2 **a** $\frac{9}{10}-\frac{2}{10}=\frac{7}{10}$

b $\frac{1}{4}-\frac{1}{12}=\frac{1}{6}$

c $\frac{4}{5}-\frac{2}{10}=\frac{3}{5}$

d $\frac{11}{12}-\frac{3}{4}=\frac{1}{6}$

e $\frac{7}{8}-\frac{3}{8}=\frac{1}{2}$

f $\frac{9}{10}-\frac{2}{5}=\frac{1}{2}$

g $\frac{17}{18}-\frac{4}{9}=\frac{1}{2}$

h $\frac{5}{8}-\frac{1}{4}=\frac{3}{8}$

i $\frac{7}{8}-\frac{3}{12}=\frac{5}{8}$

j $\frac{9}{10}-\frac{1}{7}=\frac{53}{70}$

k $\frac{2}{3}-\frac{8}{15}=\frac{2}{15}$

l $\frac{4}{5}-\frac{3}{12}=\frac{11}{20}$

Answer is between 0 and $\frac{1}{2}$:

$\frac{1}{4}-\frac{1}{12}, \frac{11}{12}-\frac{3}{4}, \frac{5}{8}-\frac{1}{4}, \frac{2}{3}-\frac{8}{15}$

Answer is equivalent to $\frac{1}{2}$:

$\frac{7}{8}-\frac{3}{8}, \frac{17}{18}-\frac{4}{9}, \frac{9}{10}-\frac{2}{5}$

Answer is between $\frac{1}{2}$ and 1:

$\frac{9}{10}-\frac{2}{10}, \frac{4}{5}-\frac{2}{10}, \frac{7}{8}-\frac{3}{12}, \frac{9}{10}-\frac{1}{7}, \frac{4}{5}-\frac{3}{12}$

3 Mara brought home $\frac{8}{8}-\frac{1}{8}-\frac{5}{8}=\frac{2}{8}=\frac{1}{4}$ of the whole cake.

4 Matthew had $\frac{7}{8}-\frac{1}{3}-\frac{1}{4}=\frac{21}{24}-\frac{8}{24}-\frac{6}{24}=\frac{7}{24}$ of the cable.

5 **a** $2\frac{1}{4}-\frac{3}{4}=\frac{9}{4}-\frac{3}{4}=\frac{6}{4}=1\frac{2}{4}=1\frac{1}{2}$

b $4\frac{1}{3}-2\frac{2}{3}=2\frac{1}{3}-\frac{2}{3}=\frac{7}{3}-\frac{2}{3}=\frac{5}{3}=1\frac{2}{3}$

c $6\frac{1}{10}-4\frac{2}{10}=2\frac{1}{10}-\frac{2}{10}=\frac{21}{10}-\frac{2}{10}=\frac{19}{10}=1\frac{9}{10}$

d $5\frac{1}{12}-3\frac{11}{12}=2\frac{1}{12}-\frac{11}{12}=\frac{25}{12}-\frac{11}{12}=\frac{14}{12}=1\frac{1}{6}$

e $2\frac{7}{8}-1\frac{3}{4}=1\frac{7}{8}-\frac{3}{4}=\frac{15}{8}-\frac{6}{8}=\frac{9}{8}=1\frac{1}{8}$

f $4\frac{3}{10}-2\frac{4}{5}=2\frac{3}{10}-\frac{4}{5}=\frac{23}{10}-\frac{8}{10}=\frac{15}{10}=1\frac{1}{2}$

g $5\frac{2}{18}-4\frac{3}{9}=1\frac{2}{18}-\frac{3}{9}=\frac{20}{18}-\frac{6}{18}=\frac{14}{18}=\frac{7}{9}$

h $2\frac{5}{6}-1\frac{2}{3}=1\frac{5}{6}-\frac{2}{3}=\frac{11}{6}-\frac{4}{6}=\frac{7}{6}=1\frac{1}{6}$

7D WHOLE CLASS

a 9

b 15

c 45

d 7

e 49

f 8

g 32

h 20

i 30

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

7D INDIVIDUAL

1 **a** $\frac{3}{8}$ of $16 = 6$ **b** $\frac{1}{4}$ of $12 = 3$

c $\frac{3}{4}$ of $8 = 6$ **d** $\frac{5}{4}$ of $12 = 15$

e $\frac{6}{7}$ of $28 = 24$ **f** $\frac{2}{9}$ of $162 = 36$

2 **a** 3 people

b $\frac{3}{5}$ of the family, $\frac{3}{5}$ of the 15 potatoes

c 9 potatoes

7E WHOLE CLASS

1 **a** $\frac{1}{2}$ of $\frac{1}{2} = \frac{1}{4}$ **b** $\frac{1}{4}$ of $\frac{1}{3} = \frac{1}{12}$

c $\frac{1}{3}$ of $\frac{1}{2} = \frac{1}{6}$ **d** $\frac{1}{8}$ of $\frac{1}{4} = \frac{1}{32}$

2 **d** 2 pieces **e** $\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$

7E INDIVIDUAL

1 **a** 2 equal pieces **b** $\frac{1}{2}$ of the cake

c 4 equal pieces

d $\frac{1}{4}$ of $\frac{1}{2}$ the cake or $\frac{1}{8}$ of the whole cake

e $\frac{1}{4}$ of $\frac{1}{2}$ the cake $= \frac{1 \times 1}{4 \times 2}$ of the whole cake $= \frac{1}{8}$

2 **b**

$\frac{1}{2}$ of $\frac{1}{3}$ of the rectangle $= \frac{1}{6}$ of the whole rectangle

3 **a** $\frac{3}{8}$ **b** $\frac{3}{16}$ **c** $\frac{9}{16}$ **d** $\frac{1}{12}$

4 **a** \$12 **b** \$96

7F INDIVIDUAL

1 **a** $6 \div 7 = \frac{6}{7}$ **b** $12 \div 13 = \frac{12}{13}$

c $4 \div 8 = \frac{4}{8} = \frac{1}{2}$ **d** $21 \div 5 = \frac{21}{5} = 4\frac{1}{5}$

e $45 \div 12 = \frac{45}{12} = 3\frac{9}{12} = 3\frac{3}{4}$

f $25 \div 100 = \frac{25}{100} = \frac{1}{4}$

g $200 \div 1000 = \frac{200}{1000} = \frac{2}{10} = \frac{1}{5}$

h $41 \div 6 = \frac{41}{6} = 6\frac{5}{6}$

i $123 \div 11 = 11\frac{2}{11}$

j $140 \div 12 = 11\frac{8}{12} = 11\frac{2}{3}$

k $107 \div 9 = 11\frac{8}{9}$ **l** $444 \div 3 = 148$

2 $63 \div 4 = 15$ sets of tyres with 3 tyres left over: $15\frac{3}{4}$ sets

3 Nancy will work $33 \div 6 = 5\frac{1}{2}$ hours each day for 6 days.

4 **a** $\frac{3}{4}$ snakes **b** $\frac{2}{3}$ pizzas

c $1\frac{1}{4}$ litres of water **d** $2\frac{1}{2}$ oranges

e $12\frac{1}{2}$ bags **f** $\frac{3}{4}$ hour

7G WHOLE CLASS

1 **a** 8 **b** 9 **c** 24 **d** 2 **e** 4

2 **a** $\frac{1}{4}$ **b** $\frac{1}{2}$ **c** 1

d 2 **e** 4 **f** 8

7G INDIVIDUAL

1 **a** 9 **b** 12 **c** 30 **d** 15

2 **a** 6 **b** 15 **c** $4\frac{1}{2}$ **d** $7\frac{1}{2}$

3 **a** 36 **b** 60 **c** 100 **d** 400

4 **a** 18 **b** 18 **c** 60 **d** 8

e 35 **f** 48 **g** 28 **h** 25

i 42 **j** 35 **k** 42 **l** 121

5 **a** 32 people **b** 36 people

c 16 people **d** 10 people

6 **a** 40 plants **b** 21 plants

c 14 plants

7 **a** 2 **b** 7 **c** 9 **d** 7

e 8 **f** 30 **g** 26 **h** 48

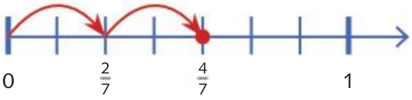
8 $2\frac{1}{4}$ pieces of rope **9** $6\frac{1}{2}$ minutes

7H REVIEW QUESTIONS

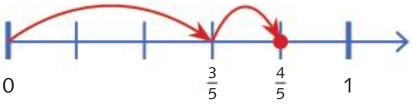
1 **a** $\frac{3}{4}$ **b** 1 **c** $\frac{5}{9}$ **d** $\frac{11}{13}$

e $\frac{15}{17}$ **f** $1\frac{1}{5}$ **g** $\frac{8}{9}$

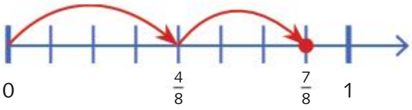
2 **a** $\frac{2}{7}+\frac{2}{7}=\frac{4}{7}$



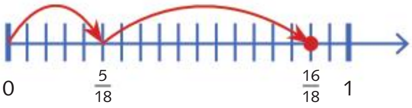
b $\frac{3}{5}+\frac{1}{5}=\frac{4}{5}$



c $\frac{4}{8}+\frac{3}{8}=\frac{7}{8}$



d $\frac{5}{18}+\frac{11}{18}=\frac{16}{18}=\frac{8}{9}$



3 **a** $\frac{4}{12}+\frac{9}{12}=\frac{13}{12}=1\frac{1}{12}$ **b** $\frac{4}{10}+\frac{3}{10}=\frac{7}{10}$

c $\frac{4}{8}+\frac{7}{8}=\frac{11}{8}=1\frac{3}{8}$ **d** $\frac{1}{75}+\frac{18}{75}=\frac{19}{75}$

e $\frac{9}{21}+\frac{5}{21}=\frac{14}{21}=\frac{2}{3}$ **f** $\frac{48}{60}+\frac{15}{60}=\frac{63}{60}=1\frac{1}{20}$

4 **a** $\frac{3}{7}+\frac{2}{3}=\frac{9}{21}+\frac{14}{21}=\frac{23}{21}=1\frac{2}{21}$

b $\frac{5}{8}+\frac{4}{6}=\frac{15}{24}+\frac{16}{24}=\frac{31}{24}=1\frac{7}{24}$

c $\frac{9}{15}+\frac{4}{5}=\frac{9}{15}+\frac{12}{15}=\frac{21}{15}=1\frac{2}{5}$

d $\frac{75}{100}+\frac{30}{50}=\frac{75}{100}+\frac{60}{100}=\frac{135}{100}=1\frac{7}{20}$

e $\frac{5}{12}+\frac{15}{18}=\frac{5}{12}+\frac{10}{12}=\frac{15}{12}=1\frac{1}{4}$

f $\frac{5}{9}+\frac{23}{45}=\frac{25}{45}+\frac{23}{45}=\frac{48}{45}=1\frac{1}{15}$

5 **a** $\frac{9}{10}$ **b** $\frac{23}{24}$ **c** $\frac{10}{21}$

d $\frac{16}{12}=\frac{4}{3}=1\frac{1}{3}$ **e** $\frac{5}{18}$ **f** $\frac{27}{20}=1\frac{7}{20}$

6 **a** $\frac{1}{3}+\frac{1}{12}+\frac{1}{8}=\frac{8}{24}+\frac{2}{24}+\frac{3}{24}=\frac{13}{24}$

b $\frac{1}{5}+\frac{3}{10}+\frac{7}{20}+\frac{7}{10}=\frac{4}{20}+\frac{6}{20}+\frac{7}{20}+\frac{14}{20}$

$=\frac{31}{20}=1\frac{11}{20}$

c $\frac{2}{3}+\frac{5}{6}+\frac{4}{9}+\frac{5}{12}=\frac{24}{36}+\frac{30}{36}+\frac{16}{36}+\frac{15}{36}$

$=\frac{85}{36}=2\frac{13}{36}$

d $\frac{3}{16}+\frac{3}{8}+\frac{3}{4}+\frac{5}{16}=\frac{3}{16}+\frac{6}{16}+\frac{12}{16}+\frac{5}{16}$

$=\frac{26}{16}=1\frac{5}{8}$

e $\frac{7}{10}+\frac{1}{15}+\frac{11}{30}+\frac{1}{5}=\frac{21}{30}+\frac{2}{30}+\frac{11}{30}+\frac{6}{30}$

$=\frac{40}{30}=1\frac{1}{3}$

f $\frac{1}{14}+\frac{6}{7}+\frac{4}{7}+\frac{9}{14}+\frac{1}{7}=\frac{1}{14}+\frac{12}{14}+\frac{8}{14}+\frac{9}{14}$

$+\frac{2}{14}=\frac{32}{14}=2\frac{3}{7}$

7 $\frac{1}{4}+\frac{3}{8}+\frac{5}{16}=\frac{4}{16}+\frac{6}{16}+\frac{5}{16}=\frac{15}{16}$ of the distance

8 **a** $\frac{2}{3}-\frac{1}{3}=\frac{1}{3}$ **b** $\frac{3}{8}-\frac{2}{8}=\frac{1}{8}$

c $\frac{5}{9}-\frac{2}{9}=\frac{3}{9}=\frac{1}{3}$

d $\frac{3}{5}-\frac{3}{10}=\frac{6}{10}-\frac{3}{10}=\frac{3}{10}$

e $\frac{5}{8}-\frac{1}{4}=\frac{5}{8}-\frac{2}{8}=\frac{3}{8}$

f $\frac{1}{7}-\frac{1}{14}=\frac{2}{14}-\frac{1}{14}=\frac{1}{14}$

g $\frac{5}{6}-\frac{7}{12}=\frac{10}{12}-\frac{7}{12}=\frac{1}{4}$

h $\frac{7}{10}-\frac{2}{5}=\frac{7}{10}-\frac{4}{10}=\frac{3}{10}$

9 $\frac{9}{15}=\frac{3}{5}$ was used, so $\frac{2}{5}$ was left for the garden.

10 **a** 15 **b** 6 **c** $\frac{18}{5}=3\frac{3}{5}$

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

11 **a** $\frac{3}{5}$ **b** $\frac{9}{11}$ **c** $\frac{1}{4}$

d $\frac{19}{5} = 3\frac{4}{5}$ **e** $\frac{53}{4} = 13\frac{1}{4}$ **f** $\frac{1}{3}$

g $\frac{3}{70}$ **h** $\frac{62}{8} = 7\frac{3}{4}$ **i** $\frac{212}{30} = 7\frac{1}{15}$

12 $\frac{325}{6} = 54\frac{1}{6}$. Jason can sell 54 six-kilogram bulk lots with 1 kilogram left over.

13 $\frac{90}{6}$ = 15 minutes per day = $\frac{1}{4}$ hour per day

14 **a** $2\frac{6}{7}$ **b** $6\frac{2}{3}$ **c** $2\frac{2}{9}$ **d** $1\frac{9}{11}$

e $1\frac{7}{13}$ **f** $1\frac{3}{17}$ **g** $1\frac{1}{19}$ **h** $\frac{20}{21}$

15 **a** $\frac{1}{8}$ **b** $\frac{1}{4}$ **c** $\frac{2}{9}$ **d** $\frac{1}{3}$

e $\frac{9}{25}$ **f** $\frac{3}{7}$ **g** $\frac{3}{4}$ **h** $\frac{4}{9}$

16 **a** 63 **b** $50\frac{2}{5}$ **c** 108

d $\frac{1}{2}$ **e** 2 **f** $1\frac{1}{2}$

g $\frac{2}{3}$ **h** 9 **i** $3\frac{4}{5}$

Chapter 8: Decimals

Show what you know

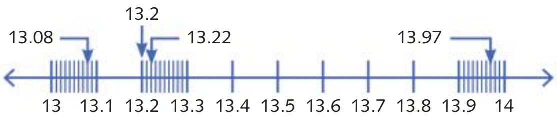
1 **a** 0.4 **b** 0.8 **c** 0.9

2 **a** $\frac{2}{10}$ or $\frac{1}{5}$ **b** $\frac{5}{10}$ or $\frac{1}{2}$ **c** $\frac{9}{10}$ or $\frac{18}{20}$

3 **a** 0.4 **b** 1.6

8A WHOLE CLASS

1



3 **b** 10 **d** 10

e **i** 2.1 **ii** 1.4

iii 1.23 **iv** 5.09

4 **a** $19\frac{29}{100}$ **b** $20\frac{2}{100}$ **c** $256\frac{23}{100}$

d $\frac{5}{100}$ **e** $\frac{84}{100}$

8A INDIVIDUAL

1 **b** $17.04 = 17 + \frac{0}{10} + \frac{4}{100} = 17 + \frac{4}{100}$

c $20.09 = 20 + \frac{0}{10} + \frac{9}{100} = 20 + \frac{9}{100}$

d $99.99 = 99 + \frac{9}{10} + \frac{9}{100} = 99 + \frac{99}{100}$

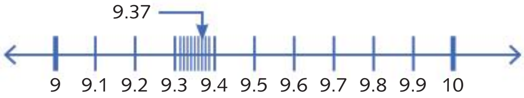
2 **a** $27.39 = 27 + \frac{3}{10} + \frac{9}{100} = 27 + \frac{39}{100}$

b $80.08 = 80 + \frac{0}{10} + \frac{8}{100} = 80 + \frac{8}{100}$

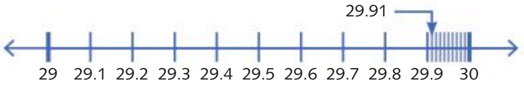
c $72.01 = 72 + \frac{0}{10} + \frac{1}{100} = 72\frac{1}{100}$

d $38.24 = 38 + \frac{2}{10} + \frac{4}{100} = 38\frac{24}{100}$

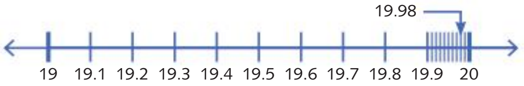
3 **b** 9.37 is between 9.3 and 9.4.



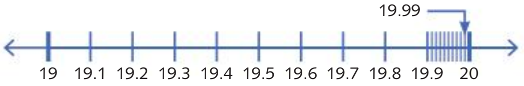
c 29.91 is between 29.9 and 30.0.



d 19.98 is between 19.9 and 20.0.

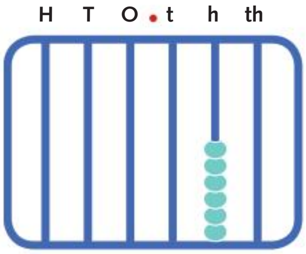


e 19.99 is between 19.9 and 20.0.

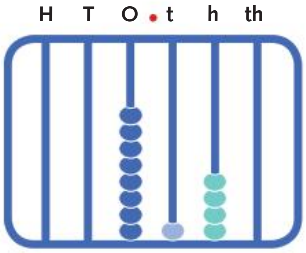


4 Numbers are shown on an abacus. Teacher check for place-value chart and/or decimal sticks

a 0.06

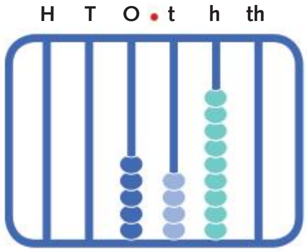


b 8.14

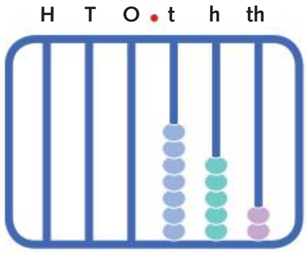


ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

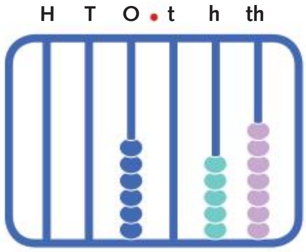
c 5.49



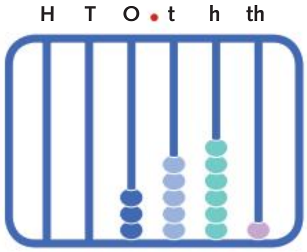
d 0.752



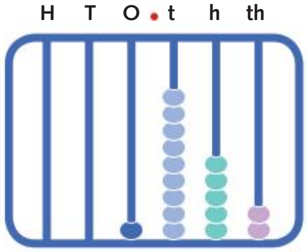
e 6.057



f 3.561



g 1.952



5 **b** $24.37 = 24 + \frac{3}{10} + \frac{7}{100}$

$= 24 + \frac{30}{100} + \frac{7}{100} = 24\frac{37}{100}$

c $0.09 = 0 + \frac{0}{10} + \frac{9}{100} = \frac{9}{100}$

d $39.99 = 39 + \frac{9}{10} + \frac{9}{100}$

$= 39 + \frac{90}{100} + \frac{9}{100} = 39\frac{99}{100}$

e $25.06 = 25 + \frac{0}{10} + \frac{6}{100} = 25\frac{6}{100}$

f $99.08 = 99 + \frac{0}{10} + \frac{8}{100} = 99\frac{8}{100}$

6 **a** 12.2 **b** 13.14 **c** 0.9 **d** 24.9

7 **a** 2.7 **b** 21.19 **c** 33.07 **d** 99.9
e 89.09 **f** 721.313 **g** 8.456 **h** 77.72
i 19.057 **j** 37.001 **k** 4.09 **l** 12.002

8 **a** 7.07 **b** 6.39 **c** 987.04

9 **a** $2\frac{3}{10}$ **b** $3\frac{9}{10}$ **c** $1\frac{99}{100}$
d $2\frac{5}{100}$ **e** $4\frac{2}{100}$ **f** $121\frac{98}{100}$

10 **a** $9\frac{96}{100}$ **b** $\frac{91}{1000}$ **c** $134\frac{7}{1000}$
d $120\frac{79}{1000}$ **e** $44\frac{4}{100}$ **f** $76\frac{12}{1000}$
g $295\frac{7}{1000}$ **h** $303\frac{303}{1000}$

11 **a** $\frac{5}{10}$ $\frac{2}{10}$ $\frac{6}{10}$ $\frac{4}{10}$ **b** 0.5 0.2 0.6 0.4

12 **a** $\frac{25}{100}$ $\frac{75}{100}$ $\frac{4}{100}$ $\frac{28}{100}$ $\frac{12}{100}$
$\frac{2}{100}$ $\frac{18}{100}$ $\frac{5}{100}$ $\frac{90}{100}$ $\frac{65}{100}$

b 0.25 0.75 0.04 0.28 0.12
0.02 0.18 0.05 0.9 0.65

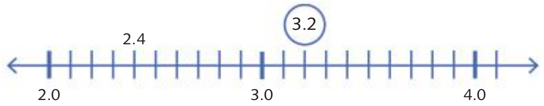
13 1.5 2.25 17.75 8.2 6.6 3.02 2.14

14 1.5 + 2.5 = 4 kg of sugar

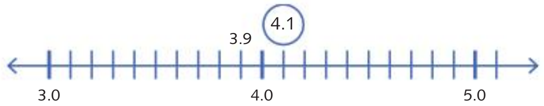
15 4.3 + 4.25 = 8.55 L of paint

8B INDIVIDUAL

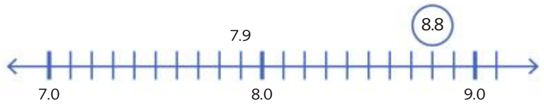
1 **a** 3.2 is larger.



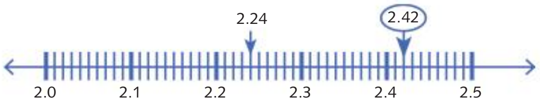
b 4.1 is larger.



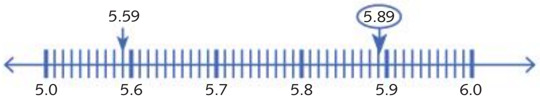
c 8.8 is larger.



d 2.42 is larger.



e 5.89 is larger.



2 **a** 3.1, 3.2, 3.3, 3.4, 3.5, 3.6, 3.7, 3.8

b 0.6, 1.6, 4.6, 6.0, 6.1, 6.4, 10, 10.1

c 12.05, 12.46, 12.47, 12.50, 12.64, 12.74

d 0.004, 0.034, 0.043, 0.3, 0.34, 0.4, 0.403, 0.43

e 2.001, 2.923, 3.921, 5.467, 5.6, 27.464

f 0.000196, 0.036, 0.2, 0.4009, 0.82, 0.9934

g 0.0101, 1.1, 10.111, 11.01, 11.0111111, 11.1

3	**a**	8.7	8.8	8.9	9.0
	b	11.1	11.2	11.3	11.4
	c	27.0	27.1	27.2	27.3
	d	82.49	82.50	82.51	82.52

8C INDIVIDUAL

1 **a** 13.7 **b** 54.9 **c** 4.85
d 3.70 **e** 9.999 **f** 1.5979

2 $131.65

3 **a** $1116.36 **b** $1074.36
c $42.00 in accounts B and C

4 8.441 tonnes of gravel

5 $17.45

6 **a** 3.87 m **b** 3.896 m **c** 5.882 m
d the blue and black shelves; 3.998 m

7 **a** 34.8 **b** 197.249 **c** 86.55
d 251.422 **e** 633.987 **f** 107.2167

8 10.6 kilometres

9 10.85 metres

10 17.50 metres

8D INDIVIDUAL

1 **a** 2.6 **b** 1.1 **c** 7.31
d 3.04 **e** 4.261 **f** 0.0232
g 15.8 **h** 28.9 **i** 5.28
j 4.92 **k** 6.777 **l** 0.653

2 **a** 14.3 **b** 0.54
c 5.714 **d** 15 888.56

3 **a** 63.803 **b** 2223.583
c 4183.935 **d** 877.905

4 0.56 m of ribbon

8E WHOLE CLASS

1 **a** 23 **b** 5 **c** 18
d 0.3 **e** 0.77 **f** 0.09

2 **a** 183 **b** 401 **c** 190
d 8 **e** 110.101 **f** 407.4

3 **a** 1832 **b** 4250 **c** 210
d 1400 **e** 100.410 **f** 76 800

8E INDIVIDUAL

1 **b** $6.83 \times 10 = 68.3$
c $80.09 \times 10 = 800.9$

2 **a** 87.9, 879 **b** 0.05, 0.5
c 60.85, 608.5 **d** 172, 1720
e 2400.9, 24 009 **f** 0.03, 0.3
g 124, 1240

3 217 cm **4** 1.25 L

5 **a** 78 cm **b** 104 cm

6 **a** $\frac{50}{100} = 0.5,\ \frac{500}{100} = 5,\ \frac{5000}{100} = 50$

b $\frac{30}{10} = 3,\ \frac{300}{10} = 30,\ \frac{3000}{10} = 300$

c $\frac{470}{100} = 4.7,\ \frac{4700}{100} = 47,\ \frac{47\,000}{100} = 470$

d $\frac{90}{100} = 0.9,\ \frac{900}{100} = 9,\ \frac{9000}{100} = 90$

e $\frac{2810}{1000} = 2.81,\ \frac{28\,100}{100} = 28.1,\ \frac{281\,000}{1000} = 281$

f $430\frac{860}{100} = 438.6,\ 4300\frac{8600}{100} = 4386,$
$43\,000\frac{86\,000}{100} = 43\,860$

7 **a** Chloe is 167 cm tall.
b Oscar has a mass of 32 450 g.

8

	Number	×10	×100	×1000
a	0.492	4.92	49.2	492
b	83.06	830.6	8306	83 060
c	507.08	5070.8	50 708	507 080
d	9.23	92.3	923	9230
e	99.999	999.99	9999.9	99.999

9 **a** 5620 g **b** 1240 cm **c** 800 m
d 2745 kg **e** 76 mm **f** 7 g

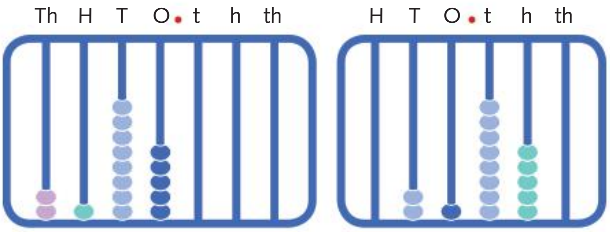
10

	Starting number	×10	×100	×1000
a	$\frac{7}{10}$	7	70	700
b	$\frac{7}{100}$	0.7	7	70
c	$\frac{7}{1000}$	0.07	0.7	7
d	$\frac{107}{1000}$	1.07	10.7	107
e	$5\frac{38}{100}$	53.8	538	5380

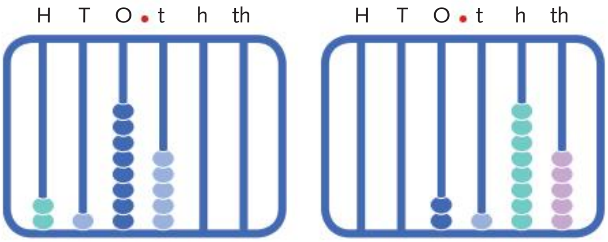
8F WHOLE CLASS

1 **a** 32.4 **b** 3.24 **c** 0.324

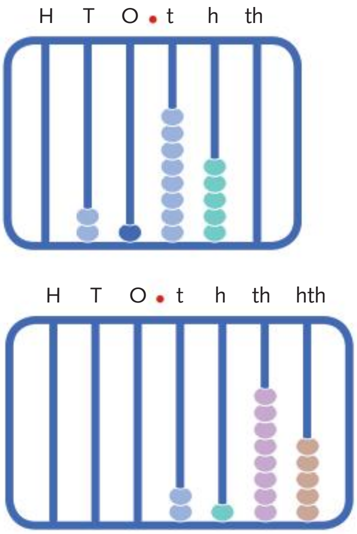
2 **a** 2185 and 21.85



b 218.5 and 2.185



c 21.85 and 0.2185



3

Nutrient	Per 100 g	Per 100 g
Protein	4000 mg	4 g
Fats	1050 mg	1.05 g
Carbohydrates	82 200 mg	82.2 g
Sugars	42 300 mg	42.3 g
Sodium	61 mg	0.061 g

8F INDIVIDUAL

1 **a** 0.23 **b** 25.6 **c** 12.847
d 0.012 **e** 0.608 **f** 99.9

2 0.25 m of licorice

3 0.14 m long

4 **a** 223 **b** 0.65
c $0.03 \times 2 \times 10 = 0.06 \times 10 = 0.6$

5 The remaining biscuits weighed $0.6 \times \frac{9}{10} = 0.54$ kg.

8G INDIVIDUAL

1 **a** 3.6 **b** 8.4 **c** 3.2 **d** 21.5
e 67.8 **f** 189.6 **g** 112.2 **h** 940.1

2 Garth's pile of bricks was 62.4 cm high.

3 **a** The total weight of Sarah's beads was 4.48 kg.
b She paid $44.80.

4 Nella exceeded her target; 11.04 km

5 **a** **i** 0.9 metres **ii** 3.0 metres
iii 6.3 metres
b 31.4 metres **c** 9.5 metres

6 187.5 cm long

8H INDIVIDUAL

1 **a** 2 r2 or 2.5 **b** 2 r4 or 2.4
c 2 r4 or 2.5 **d** 4 r3 or 4.5
e 3 r3 or 3.75 **f** 12 r6 or 12.75
g 5 r3 or 5.375 **h** 30 r3 or 30.75

2 **a** 0.4 **b** 0.7 **c** 0.8 **d** 0.9
e 0.89 **f** 242.674 **g** 9.87 **h** 0.78

3 **a** 18.144 **b** 12.096 **c** 9.072 **d** 7.2576
e 6.048 **f** 5.184 **g** 4.536 **h** 4.032

4 26.5 metres of fence

5 **a** 0.7 metres
b 22.4 metres

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7

c 2.8 metres

d 9 cushions with 1.8 metres left over

8I WHOLE CLASS

1 **a** $87 tax paid, with $213 left over
b $290 tax paid, with $710 left over
c $254.91 tax paid, with $624.09 left over

8I INDIVIDUAL

1 **a** $8 **b** 9.6 grams
c 19.2 metres **d** $78.40
e 800 cm **f** 67.2 km
g $214.36 **h** $800 000
2 **a** $12.50 **b** 15 grams
c 30 metres **d** $122.50
e 1250 cm **f** 105 kilometres
g $334.94 **h** $1 250 000

Homework

1 Jenny is 0.7 × 2 = 1.4 metres tall.

2 Martin earns 0.6 × 560 = 6 × 56 = $336 per week.

3 **a** $70 **b** $210

8J INDIVIDUAL

1 **a** 40 **b** 50 **c** 70
d 90 **e** 1000
2 **a** 100 **b** 100 **c** 200
d 900 **e** 2900
3 **a** 0.4 **b** 2.1 **c** 0.4
d 0.6
4 **a** 24 **b** 42 **c** 77
d 100 **e** 23
5 **a** 0.12 **b** 0.79 **c** 0.42
d 888.89

8K REVIEW QUESTIONS

1 **a** 0.3 **b** 0.23 **c** 16.001
d 214.13

2 **a** $\frac{8}{10}$ **b** $14\frac{2}{10}$ **c** $27\frac{75}{100}$

d $39\frac{4}{100}$

3

Line	Length	$\frac{1}{10}$ cm longer	Write as a fraction
a	8.4 cm	8.5 cm	$8\frac{5}{10}$
b	19.6 cm	19.7 cm	$19\frac{7}{10}$
c	8.48 cm	8.58 cm	$8\frac{58}{100}$
d	28.51 cm	28.61 cm	$28\frac{61}{100}$

4 **a** 7.2, 7.3, 7.4, 7.5, 7.7, 7.8

b 40.3, 40.4, 43.0, 43.4, 43.5, 44.3, 45.3

c 72.07, 72.08, 72.72, 72.87, 72.89, 72.92

5 **a** $8\frac{49}{100}$ **b** $21\frac{8}{100}$ **c** $76\frac{407}{1000}$

d $219\frac{91}{1000}$

6 **a** 11.5 **b** 35.53 **c** 17.07 **d** 29.018
7 **a** 11.11 **b** 37.43 **c** 86.833 **d** 111.005

Chapter 9: Percentages

Kick off

a $160 **b** $720 **c** $960

Show what you know

1 **a** $\frac{1}{2}$ **b** $\frac{1}{4}$ **c** $\frac{3}{5}$ **d** $\frac{11}{12}$
2 **a** 0.3 **b** 0.23 **c** 0.5
d 2.03 **e** 0.75 **f** 6.6
3 **a** $\frac{1}{2}$ **b** $\frac{3}{4}$ **c** $\frac{1}{10}$ **d** $2\frac{3}{5}$ **e** $3\frac{2}{5}$

9A WHOLE CLASS

1 **a** 80% **b** 60% **c** 50%

9A INDIVIDUAL

1 **a** 10% **b** 50% **c** 78%
2 **a** 87% **b** 62% **c** 8%
3 **a** 70% **b** 20% **c** 33%
d 40% **e** 80% **f** 80%
g 40% **h** 75% **i** 12%
j 80% **k** 12.5% **l** 95%
4 **a** $\frac{7}{100}$ **b** $\frac{9}{10}$ **c** $\frac{3}{4}$ **d** $\frac{4}{5}$ **e** $\frac{18}{25}$

5 **a** 220% **b** 106% **c** 775%
d 112% **e** 140%
6 44% **7** 68% **8** 40%
9 $\frac{1}{4}$ **10** $33\frac{1}{3}\%$ **11** 9%
12 75% **13** 35% **14** 75%
15 $\frac{34}{100} = \frac{17}{50}$
16 64% double garage, 24% single garage, 12% carport

9B INDIVIDUAL

1 **a** 37% **b** 25% **c** 94%
d 62% **e** 2% **f** 8%
g 1% **h** 4% **i** 90%
j 60% **k** 80% **l** 20%
m 725% **n** 133% **o** 704%
p 1109% **q** 150% **r** 580%
s 207% **t** 640%
2 **a** 0.75 **b** 0.86 **c** 0.37
d 0.72 **e** 0.8 **f** 0.9
g 0.6 **h** 0.7 **i** 0.07
j 0.01 **k** 0.03 **l** 0.05
3 **a** 2.65 **b** 5.26 **c** 6.52
d 3.75

9C INDIVIDUAL

1 **a** 50 **b** 24 **c** 10
d 45 **e** 5 **f** 10
g 1 **h** 100 **i** 200
2 **a** 25 litres **b** 37.5 litres
c 75 litres
3 **a** $21
b 330 children
c 36 centimetres
4 Jake: $357; Thomas: $750.75
5 **a** 46 kilograms **b** 132 trees
c 125 books
6 **a** $7.50 **b** $9 **c** $5.25

9D REVIEW QUESTIONS

1 **a** 75% **b** 23% **c** 99%
2 **a** 50% **b** 15%
c 50% **d** 190%
e 48% **f** 46%
g 98% **h** 3%

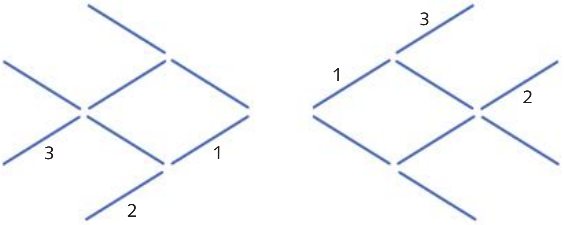
3

Decimal	Fraction or mixed number	Percentage
0.1	$\frac{1}{10} = \frac{10}{100}$	10%
0.25	$\frac{25}{100}$	25%
0.66	$\frac{66}{100} = \frac{33}{50}$	66%
0.4	$\frac{2}{5} = \frac{4}{10} = \frac{40}{100}$	40%
3.5	$3\frac{1}{2} = 3\frac{5}{10} = \frac{35}{10} = \frac{350}{100}$	350%

4 **a** 20 **b** 60 **c** 40
d 90 **e** 1 **f** 16
g 99 **h** 10
5 **a** any number greater than 15
b 65
6 **a** $140.40 **b** 246 children
c 156 cm
7 **a** 162 trees **b** 500 chocolates

Chapter 10: Lines and angles

Kick off



10A WHOLE CLASS

3 **a** 180° **b** 90° and 270°
c 150° and 210° **d** 30° and 330°
e 60° and 300° **f** 120° and 240°

10A INDIVIDUAL

1 **a** reflex **b** acute **c** obtuse
d right **e** straight **f** reflex
2 **a** obtuse **b** straight **c** acute
d right **e** reflex
3 **a** acute **b** straight **c** reflex
d right **e** obtuse **f** acute
4 Teacher check

10B INDIVIDUAL

1 **a** 125° (obtuse) **b** 75° (acute)
c 30° (acute) **d** 220° (reflex)
2 **a** 90° **b** 45°
c 130° **d** 270°
3 Teacher check

10C WHOLE CLASS

4 **a** 360° **b** 130° **c** 130°
d 50°. There are three ways to find B. Either supplementary to angle A or angle C so 130° + B = 180° or opposite angle equal to 50°.

10C INDIVIDUAL

1 **a** A and B **b** D and E
c H and I **d** J and K
2 **a** A and B, B and C, C and D, D and A
b M and N, N and O, O and P, P and M
c W and X, X and Y, Y and Z, Z and W
3 **a** 55° **b** 105°
c 40° **d** 45°
4 B = 50°, C = 130°, D = 130°

10D REVIEW QUESTIONS

1 Teacher check
2 **a** obtuse angle **b** right angle
c acute angle **d** reflex angle
e obtuse angle **f** straight angle
3 Teacher check
4 **a** 170° **b** 10° **c** 350°
d 90° **e** 270°
5 Teacher check
6 **a** angles D and C are complementary.
b Angles A and B are supplementary. Angle E is supplementary to angles D and C.
c 45°
d 142°
7 **a** acute angle, 60° **b** right angle, 90°
c straight angle,180° **d** obtuse angle, 120°
e acute angle, 75° **f** reflex, 290°
g angle A **h** angle F
i 230°
8 60°
9 Angle B is 77°. Angles C and D are 103°.

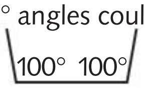
Chapter 11: Two-dimensional shapes

Show what you know

1 **a** square **b** rectangle
c circle **d** triangle
e hexagon **f** pentagon
g octagon **h** trapezium
i rhombus

11A WHOLE CLASS

1 **a** 60° each
b The angles are equal for each triangle.
d 180°
2 **e** All of the sides and angles are equal.
3–6 Teacher check
7 **e** Two 100° angles could not form a triangle:



8 **b** If the angle between the arms increases, the third side gets longer.
9–11 Teacher check
12 **b** Answers will vary; the total for each triangle should be 180°.
c If we add up the angles at the vertices of a triangle, the sum is 180°.
13 No. If a triangle had two right angles it would sum to more than 180° by the time the third angle was added.
14 No. A reflex angle is greater than 180° but the sum of the angles in a triangle is 180°.

11A INDIVIDUAL

1 Possible words: tricycle, tripod, triathlon, trident …
2 Triangle C is an acute-angled triangle.
3 **a** Obtuse angles: 110°, 150°, 179°
b B and D are obtuse-angled triangles.
4 **a** Yes. (Teacher to check triangle.)
b Yes. (Teacher to check triangle.)
c No. All the angles in an equilateral triangle are 60° and 60° is not obtuse.

11B WHOLE CLASS

1 **a** All of the shapes are quadrilaterals except B and G.
b Quadrilaterals C, D and E are convex.
2 I am a square.
3 **b** It is a rectangle.
c It is not a rectangle because one of the four angles is larger than 90°.

4 **a** When the square is rotated, it fits exactly.

c The lengths AM, MC, BM and MD are all equal.

d The four angles at M are all 90° angles or right angles.

5 **d** This creates a square because the diagonals are equal in length and there are four right angles in the middle, or because the side lengths are equal and the angles are equal.

11B INDIVIDUAL

1 **a** non-convex, quadrilateral

b rectangle, quadrilateral, has four right angles

c four sides of equal length, quadrilateral, convex

d convex, quadrilateral

e has four right angles, square, has four sides of equal length, quadrilateral

f not a quadrilateral

2 A is a rectangle.

3 **a** 4 cm and 8 cm

b 10 cm and 1 cm

4 Shape A is not a rectangle because the opposite sides are not equal in length.

5 Teacher to check

6 Teacher to check

11C WHOLE CLASS

1 The opposite angles stay equal. Angles A and D get bigger; angles B and C get smaller.

2 Diagonal AC got longer; diagonal DB got shorter

3 The diagonals are perpendicular.

4 The rhombus will look the same.

5 If you rotate a rhombus 90°, it won't look the same unless it was a square to start with. If you rotate it 180°, it remains the same.

11D WHOLE CLASS

1 Shape B is a pentagon because it has 5 sides. Shape A is a quadrilateral, C is a hexagon and D is not a polygon.

2 There are 3 triangles.

The sum of the angles in a pentagon is 180° + 180° + 180° = 540°

3–4 Teacher to check

11D INDIVIDUAL

1–2 Teacher to check

11E WHOLE CLASS

2 **a**

Circle	Radius	Diameter
A	2 cm	4 cm
B	1.5 cm	3 cm
C	3 cm	6 cm

b The radius is half of the diameter.

The diameter is two times the radius.

3 **b** The angle at the centre will be 45°.

4 **d** The circumference of a circle is slightly longer than 3 of its diameters.

11E INDIVIDUAL

1–2 Teacher to check

3

	Radius	Diameter	Circumference
a	3 cm	6 cm	18.8 cm
b	4 cm	8 cm	25.1 cm
c	5 cm	10 cm	31.4 cm
d	20 cm	40 cm	125.6 cm

11F REVIEW QUESTIONS

1–3 Teacher to check

4 36 ÷ 6 = 6, so each triangle uses 6 metres of string.

The triangles are equilateral so each side must be 2 metres.

5 Teacher to check

6 **a** obtuse **b** acute **c** obtuse

7 **a** rhombus, C **b** pentagon, A

c trapezium, B

8 Draw a circle, then draw in five radii each $\frac{1}{5}$ of 360° = 72° apart at the centre.

9

	Circle A	Circle B	Circle C	Circle D	Circle E
Radius	8 cm	11 cm	7 cm	1.5 cm	5.5 cm
Diameter	16 cm	22 cm	14 cm	3 cm	11 cm

10 3 circles

Chapter 12: Three-dimensional objects

Show what you know

1 **a** cube **b** square pyramid

c cylinder **d** rectangular prism

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

12A WHOLE CLASS

1 **a** The base of this octahedron is a hexagon.

  b The base of this heptahedron is a pentagon.

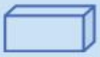
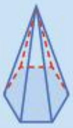
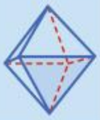
  c The base of a cylinder is a circle.

2-4 Teacher to check

12A INDIVIDUAL

1 Both a cone and a cylinder have a circle for a base.

2

	3D Shape	Shape of grey shaded face	Shape of other faces	Number of faces
a		Square	Squares	6
b		Square	A square and rectangles	6
c		Hexagon	Triangles	7
d		Triangle	Triangles	8
e		Hexagon	A hexagon and rectangles	8

12B WHOLE CLASS

1 net C

2 nets A, D, E, F, H and I

12B INDIVIDUAL

1 **a** pentagonal prism

  b tetrahedron

  c octahedron

  d cube

  e square-based pyramid

  f dodecahedron

  g pentagonal-based pyramid

2 Teacher to check

12C REVIEW QUESTIONS

1 **a** octahedron

  b cube; hexahedron; rectangular prism

  c rectangular prism; hexahedron

  d tetrahedron; triangular-based pyramid

  e pentagonal-based pyramid; hexahedron

  f hexagonal-based pyramid; heptahedron

  g octagonal prism

Chapter 13: Measurement

Show what you know

2 **a** 5 kilograms **b** 4000 grams

3 **a** 11 000 kilograms **b** 4.2 tonnes

13A WHOLE CLASS

2 **a** 4 eggs **b** 2500 eggs

  c 0.5 eggs

13A INDIVIDUAL

1 **a** 0.35 kg **b** 4000 g **c** 1500 kg

2 **a** 3.875 t **b** 1074 kg **c** 2.855 kg

  d 45 g **e** 4.455 g **f** 4072 mg

3 **a** 4500 kg **b** 2080 kg **c** 3005 kg

  d 850 kg **e** 35 kg

4 **a** 2.857 kg **b** 3.05 kg

  c 1.007 kg **d** 0.756 kg

5 **a** 4800 mg **b** 400 mg

  c 8750 mg **d** 36 040 mg

6 **a** 5750 g **b** 3040 kg **c** 0.805 kg

  d 4250 mg **e** 1350 g **f** 1.256 g

13B WHOLE CLASS

2 235 g

13B INDIVIDUAL

1 **a** 3.095 kg **b** 1.265 kg **c** 3.975 kg

  d 3.415 kg **e** 8.335 kg

2

Mass	Washing powder	Can of paint	Bacon	Biscuits
Gross	4100 g	10 kg	1.504 kg	1.012 kg
Net	4 kg	9.7 kg	1.5 kg	1 kg
Packaging	100 g	300 g	4 g	12 g

3 **a** 4 **b** 10

4 4

5 2.7 g

6 10

13C INDIVIDUAL

1 9 hours 15 minutes

2 a 3 minutes 45 seconds
  b 8 minutes 10 seconds
  c 8 minutes 12 seconds
  d 5 hours 35 minutes
  e 4 hours 25 minutes
  f 7 hours 18 minutes
  g 8 hours 35 minutes

3 a 2 hours 20 minutes
  b 4 hours 12 minutes
  c 2 hours 20 minutes
  d 2 hours 39 minutes

4 6 hours 30 minutes

5 a 8:45 am
  b 8:45 to 3:30 is 8 hours 45 minutes
  c 6 hours
  d English
  e 6 hours 15 minutes
  f 1650 minutes or 99 000 seconds

13D WHOLE CLASS

1–3 Teacher to check

13D INDIVIDUAL

1 Teacher to check

2 a 4°C + 5°C = 9°C
  b 2°C – 4°C = –2°C
  c –3°C + 3°C = 0°C
  d 0°C – 6°C = –6°C
  e –8°C + 6°C = –2°C
  f 7°C – 11°C = –4°C

3 –14°C – 7°C = –21°C

4 –2°C + 3°C + 6°C = 7°C at 10:15 am

5 12°C + 1°C + 2°C + 3°C + 4°C + 5°C + 6°C
  = 33°C; 33°C – 17°C = 16°C

13E REVIEW QUESTIONS

1 a 3.395 kg b 0.006 093 45 kg
  c 3020 g d 3 g
  e 32 090 mg f 32 090 mg

2 a 2290 kg b 4000 kg c 0.08 kg

3 Tina carried 6.630 kg.

4 12 hours 45 minutes

5 a 3 hours 30 minutes
  b 8 minutes
  c 5 hours 37 minutes
  d 9 hours 17 minutes
  e 8 hours 10 minutes
  f 24 hours 30 minutes

6 a 9°C b 18°C c 26°C

7 a –7°C b –18°C c 1°C

Chapter 14: Maps and coordinates

Kick off

a NE b W c SW d SE
e NW f S g E

14A WHOLE CLASS

1 a India or Pakistan
  b Russia
  c China or Mongolia
  d China, Vietnam, Myanmar, Laos or Butan
  e Papua New Guinea

2 a C4 or C3 b D4
  c C2 or D2 d G3

14A INDIVIDUAL

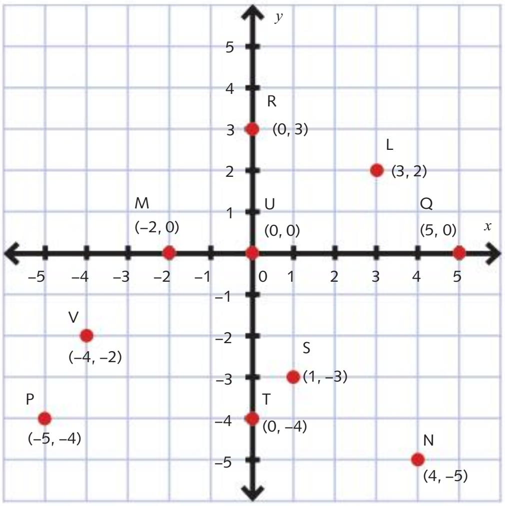
1 a Sydney College of the Arts
  b Ann Cashman Reserve
  c Bridgewater Park
  d Punch Park
  e Rozelle Hospital

2 a F1 b C1 c D1

14B INDIVIDUAL

1 E = (5,3) F = (2,4) G = (–1,1) H = (–5,4)
  I = (3,– 2) J = (0,– 5) K = (–2,– 4) L = (–5,– 1)

2



ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

3 **a** a rhombus **b** a rectangle **c** an irregular hexagon
d a square **e** an isosceles triangle **f** a trapezium
g a right-angled triangle **h** (3, 3) (–4, 3) (4, 0)

4 **a** 4 **b** 4 **c** 3 **d** 3 **e** 3 **f** 5 **g** 3 **h** 7

14C REVIEW QUESTIONS

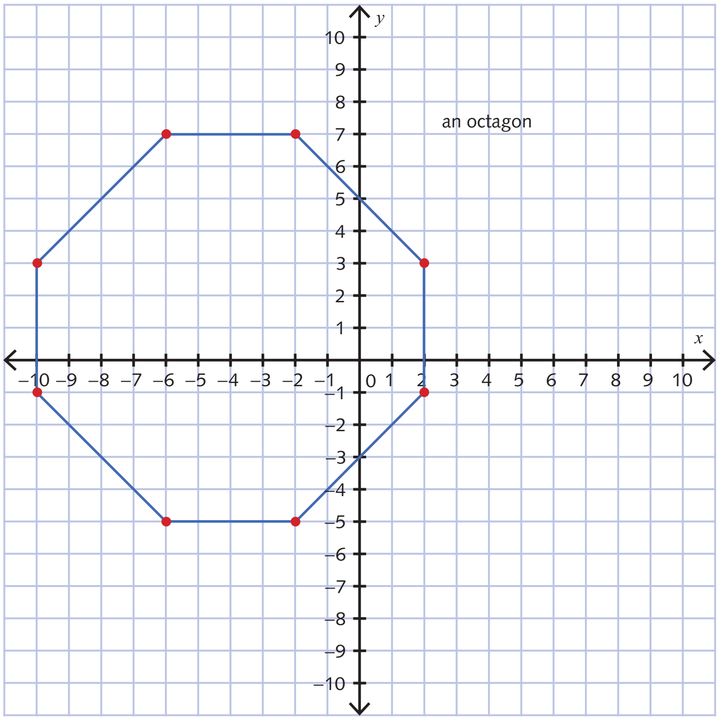
1 **a** Wooding Point **b** Lake Seppings **c** Woodchipping Mill
d Mt Melville **e** Middleton Beach, Ellen Cove and Jetty **f** Golf Links Road

2 **a** G6 **b** C2 **c** C4 **d** G4 **e** B4 **f** F4 **g** D2 **h** D2

3 Town Jetty, Wharf, Princess Royal Harbour

4 Teacher to check

5



Chapter 15: Data

15A INDIVIDUAL

1 **a** adults over 18 **b** plumbers **c** pre-school children
d primary school students **e** librarians

2 Teacher to check

3 **a**

Type of tree	Joseph Banks Park		Celia Rosser Park	
	Tally	Frequency	Tally	Frequency
Rose mallee	|||	3	||	2
Apple myrtle	||||	4	𝍸 ||||	9
Golden wattle	𝍸 𝍸 |	11	||	2
Bottlebrush	𝍸 𝍸 𝍸	15	|||	3
Quandong	𝍸 𝍸 ||	12	𝍸	5

b Joseph Banks Park has 45 trees. Celia Rosser Park has 21 trees.

c Joseph Banks Park:
rose mallee, $\frac{1}{15}$ apple myrtle $\frac{4}{45}$, golden wattle $\frac{11}{45}$, bottlebrush $\frac{1}{3}$, quandong $\frac{4}{15}$
Celia Rosser Park:
rose mallee $\frac{2}{21}$, apple myrtle $\frac{3}{7}$, golden wattle $\frac{2}{21}$, bottlebrush $\frac{1}{7}$, quandong $\frac{5}{21}$

d Bottlebrush at Joseph Banks Park: $\frac{15}{45} = 33.3\%$

Bottlebrush at Celia Rosser Park: $\frac{3}{21} = 14.3\%$

So Joseph Banks Park has the highest percentage of bottlebrush trees.

15B WHOLE CLASS

1

	Frozen orange sales	
Day	**Tally**	**Frequency**
Monday	|||	3
Tuesday	𝍸 ||||	9
Wednesday	|||	3
Thursday	𝍸 𝍸 𝍸 ||	17
Friday	𝍸 |||	8

b mode: 17 oranges
c mean: 8(40 ÷ 5) oranges
d median: 8 oranges

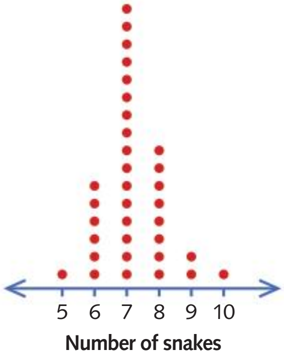
15B INDIVIDUAL

1 a mode = 5, mean = 4.45, median = 5
b mode = 3, mean = 3.9, median = 3.5
c mode = 10, mean = 249, median = 10
d mode = 12, mean = 40, median = 38.5
2 a mode = $43.00
b median = $120.00
c mean = $180.80
d Discuss answers.
3 a False **b** False **c** True
d True **e** True **f** False
4 a 6 (6 × 12 = 72)
b Answers will vary. The sum of the four numbers must be 40.
c Tahlia's fifth test mark was 42. (44 × 5 = 220) Tahlia's total for 4 tests is 178: 220 – 178 = 42

15C INDIVIDUAL

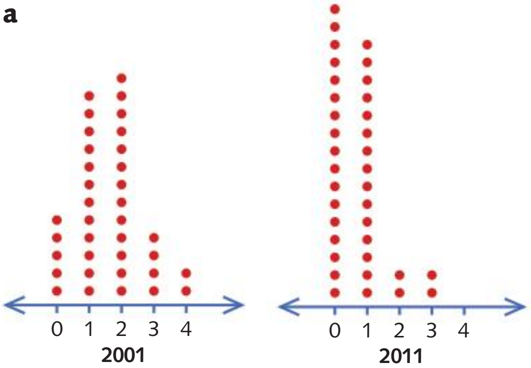
1 a 23 guitars **b** 17 guitars
c 70 days **d** 1120 guitars
e 16 guitars

2 a



b 7 snakes **c** 5 snakes

3 a



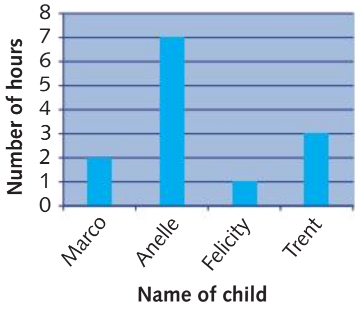
b Answers will vary. In 2001 there were more than twice the number of young than in 2011. In 2001, more wallabies had 2, 3 or 4 young than in 2011. In 2011, more wallabies had no young at all than in 2001.

15D INDIVIDUAL

1 a

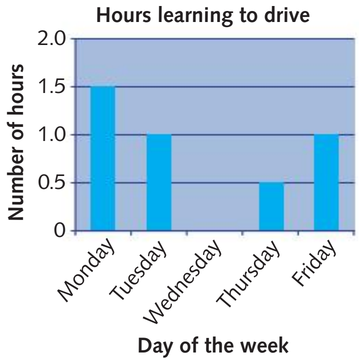
	Hours of television watched per day	
	Tally	**Frequency**
Marco	||	2
Anelle	𝍸 ||	7
Felicity			1
Trent	|||	3

b



ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 Cambridge University Press

2 **a**

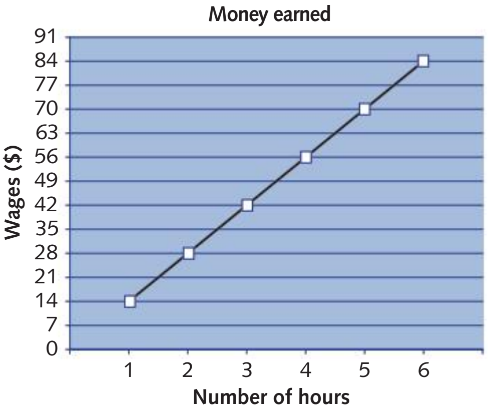


b mean = 0.8 hours (4 hours ÷ 5)

c 3 hours more over the weekend

15E INDIVIDUAL

1 **a**

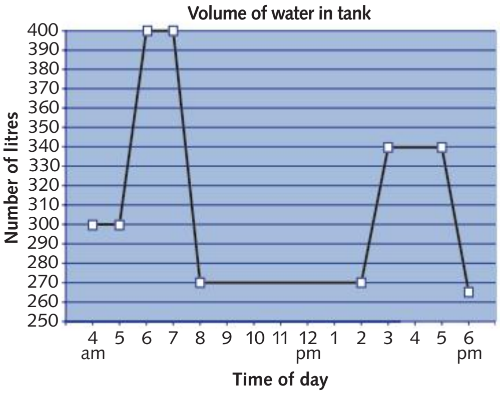


b $56 **c** $140 **d** $49

2 **a**

Time	Amount
4 am	300 L
5 am	300 L
6 am	400 L
7 am	400 L
8 am	270 L
2 pm	270 L
3 pm	339 L
5 pm	339 L
6 pm	265 L

b



c 400 – 130 = 270 litres

d 270 + 70 – 75 = 265 litres

15F WHOLE CLASS

1 **a** spaghetti **b** 25%

c shells and penne **d** 12.5%

e 12 packets of spirals; 6 packets of shells; 6 packets of penne

15F INDIVIDUAL

1 **a** 15 hours **b** eating and reading

c 5 hours **d** $\frac{3}{24} = 12.5\%$

2 **a** 10 fat-tailed dunnarts

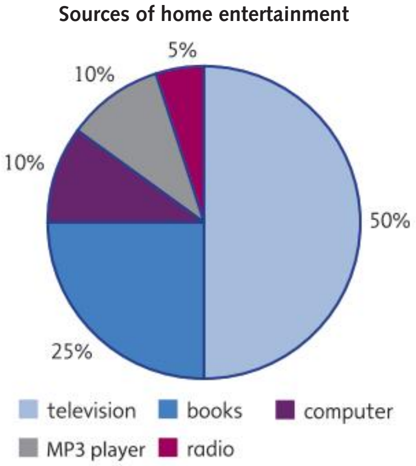
b 3 Arnhem leaf-nosed bats

c 37 black wallaroos

d 36 Kakadu dunnarts

e 16 western quolls

3



15H INDIVIDUAL

1 Teacher to check

2 **a–b** Teacher to check

c

					6 + 1					
				5 + 1	5 + 2	6 + 2				
			4 + 1	4 + 2	4 + 3	5 + 3	6 + 3			
		3 + 1	3 + 2	3 + 3	3 + 4	4 + 4	5 + 4	6 + 4		
	1 + 2	2 + 2	2 + 3	2 + 4	2 + 5	3 + 5	4 + 5	5 + 5	6 + 5	
1 + 1	2 + 1	1 + 3	1 + 4	1 + 5	1 + 6	2 + 6	3 + 6	4 + 6	5 + 6	6 + 6
2	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**

d Teacher to check

3 **a** $\frac{1}{4}$ of the time

b–c Teacher to check

d

		Coin 1	
		Head	*Tail*
Coin 2	*Head*	Head, Head	Head, Tail
	Tail	Tail, Head	Tail, Tail

15I REVIEW QUESTIONS

1 **a** week 1: median = 4 golf balls; week 2: median = 1 golf ball

b week 1: mean = 4 golf balls; week 2: mean = 5 golf balls

2 **a** 21 children

b 14 hours

c 12 hours

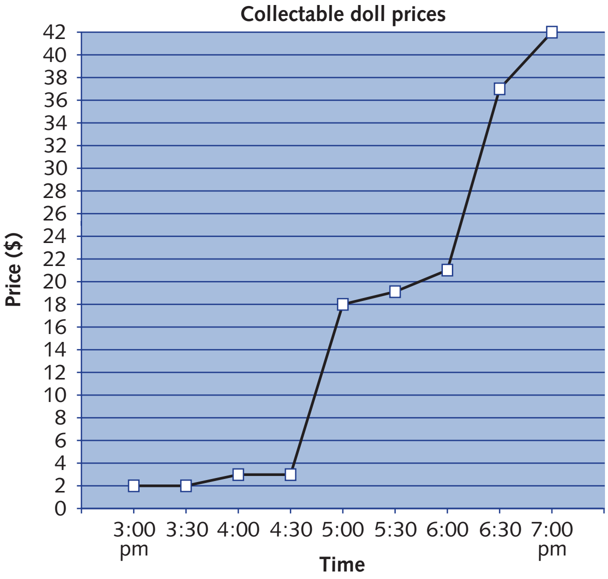
d TV: 12 hours; computer: 11 hours

e 11.5 hours

f 3, 4, 4, 5, 5, 7, 7, 8, 8, 10, 11, 12, 13, 13, 14, 14, 14, 17, 19, 21, 43 The median is 11 hours.

g 3, 4, 5, 6, 7, 7, 8, 9, 9, 9, 10, 11, 12, 12, 12, 12, 13, 14, 14, 22, 32 The median is 10 hours.

3 **a**



ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press

b $42.00
c The line is horizontal when the price is constant.
d $3.00
e At 6:15 pm the price might have been $29.00.
4 Teacher to check

Chapter 16: Algebra

Kick off

Daniel: 4 parrots = 8 legs, 4 lizards = 16 legs

Mark: 5 parrots = 10 legs, 3 lizards = 12 legs

Show what you know

1 12 **2** 13 **3** 7 **4** 8

16A WHOLE CLASS

1 **a** 6 **b** 11 **c** 34 **d** 10
e 18 **f** 5 **g** 6 **h** 14
i 48 **j** 40 **k** 50 **l** 25
2 **a** ☐ + 4 = 14, ☐ = 10
b ☐ + 6 = 12, ☐ = 6
c ☐ + 6 = 12, ☐ = 6
d ☐ – 2 = 10, ☐ = 12
3 18 children

16B WHOLE CLASS

1 **a** $x = 6$ **b** $x = 10$
c $q = 6$ **d** $z = 10$
e $y = 9$ **f** $y = 5$
g $a = 15$ **h** $b = 7$
i $a = 8$ **j** $r = 8$
k $x = 4$ **l** $x = 8$
2 **a** 20 pencils **b** 35 pencils
3 **a** 25 children are in each class.
b $4 \times c + 6 = 106$

16B INDIVIDUAL

1 **a** $p + 4 = 11$ **b** $n - 5 = 9$
c $2 \times t = 12$ **d** $r \div 2 = 4$
2 **a** Twelve take-away something is six.
b Four plus something is five.
c Twenty-one divided by a number is three.
3 **a** $d = 8$ **b** $w = 20$
c $f = 10$ **d** $u = 3$
e $t = 3$ **f** $m = 25$
g $s = 10$ **h** $x = 999$
i $b = 75$

4 **a**

Number of packets	Total number of biscuits
1	4
2	8
3	12
4	16
5	20
10	40
20	80
100	400
n	$4 \times n$

b 8 biscuits **c** 12 biscuits
d 32 biscuits **e** 120 biscuits
f 800 biscuits **g** $4 \times n$ biscuits
5 **a** 17 **b** 20 **c** 21 **d** 25
e 17 **f** $n + 7$ **g** $p - 7$
h When Jared was 14, he was twice Sonja's age of 7.
6 **a** 16 **b** $11 + x$
7 **a** $3 **b** $12 \times s + 14 = 50$
8 **a** $11 **b** $6 \times n + 34 = 100$
9 300 litres

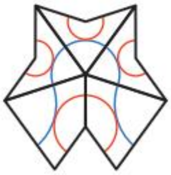
16C INDIVIDUAL

1 **a** 15, 17, 19, 21, 23
b 60, 70, 80, 90, 100
c 0, – 1, – 2, – 3, – 4
d 64, 128, 256, 512, 1024
e 55, 40, 25, 10, –5
2 C **3** A **4** D
5 B **6** C

Chapter 17: Symmetry and transformation

Kick off



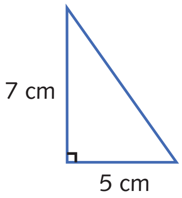
17A INDIVIDUAL

1 **a** 4 **b** 1 **c** 2 **d** 2

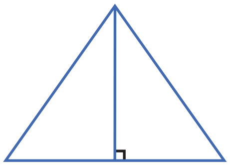
2 Teacher to check

3 Letters with at least one line of symmetry:
A B C D E H I K M O T U V W X Y

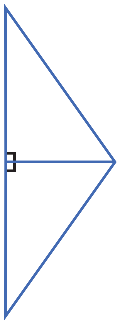
4 **a**



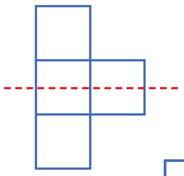
b



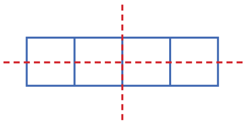
c



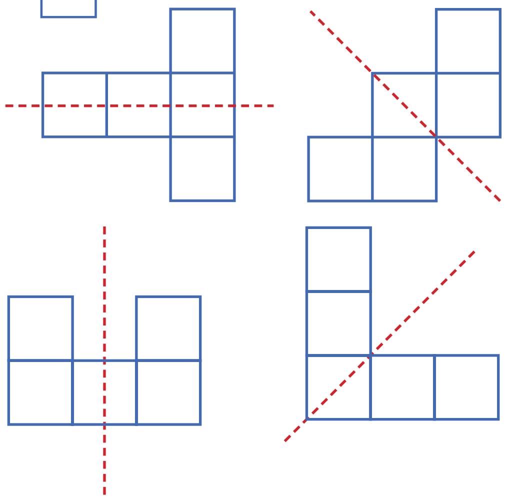
5 **a**



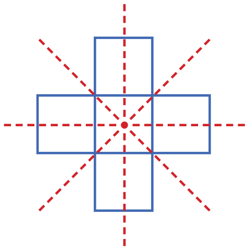
b



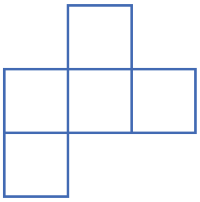
c



d



e



(There are 5 other possible answers.)

17B WHOLE CLASS

1 **a** reflection **b** rotation 180°

c rotation 90° **d** rotation 180°

e vertical translation

f horizontal translation

2 Teacher to check

3 **a–c** Teacher to check

d 360°

e A shape will tessellate if the angles about a point sum to 360 degrees.

4 **a–b** Teacher to check

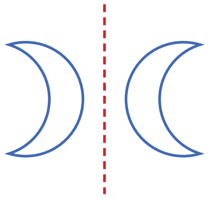
5 Teacher to check

17B INDIVIDUAL

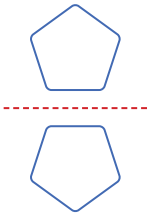
1 **a** horizontal translation

b reflection **c** rotation 90°

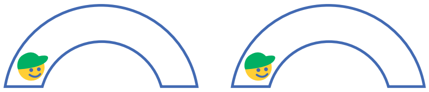
2 **a**



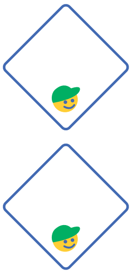
b



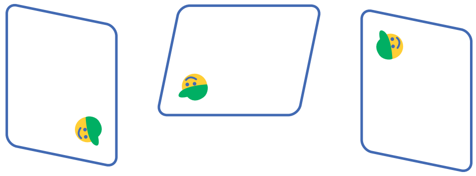
3 **a**



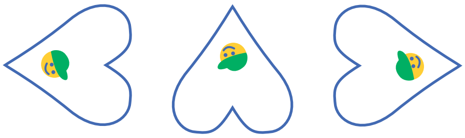
b



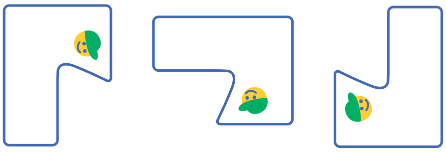
4 **a**



b



c



5 Teacher to check

ICE-EM Mathematics 6 3ed ISBN 978-1-108-40049-7 © The University of Melbourne / AMSI 2017 Cambridge University Press