nelson maths.

7

Rachel Theunissen
Rashmi Bhagwati
Judy Binns
Gaspare Carrozza
Series editor
Robert Yen

WA
Australian Curriculum

CENGAGE Learning™

COPYRIGHT NOTICE

Copyright in this work is owned by Cengage Learning Australia ("the work"). A condition of purchase of this electronic version of the work is that you agree to respect the copyright in the work, abide by the Copyright Act 1968 and specifically agree not to transfer, sell, assign, misuse, copy or transmit an electronic or other version of the work to any third party.

Please note: This product is accompanied by a licence (single user, network or adoption) governing the terms and conditions of its use.

This is a legal agreement between the you, (the "Customer") and Cengage Learning Australia Pty Limited (ABN 14 058 280 149) (the "Licensor") which provides the terms and conditions of this non-exclusive licence and the limited warranty for the Product. Use of the Product indicates an acknowledgement that the Customer has read and agreed to be bound by the terms and conditions of this Agreement. If you do not agree to these terms and conditions, return the Product to the place of purchase within 15 days of the date of purchase (with proof of purchase) for a full refund

1. **Licence Grant**
   You do not receive title to the Product. Copyright in the Product (which includes all images, photographs, video, animations, audio, music and text incorporated in the Product, including all of the accompanying printed material) is owned by the Licensor and/or its suppliers and is protected by Australian copyright laws. The Licensor grants you a non-exclusive licence to use the Product subject to the restrictions and terms set out in this Agreement.

2. **A Licence allows you to:**
   Use the Product on your computer. The Customer represents that they shall in no way place the Product in the public domain or in any way compromise our copyright in the Material. You agree to take reasonable steps to protect our copyright.

3. **You may not:**
   Alter, modify, translate, reverse engineer, decompile, or adapt the software or create derivative works based on the Product. Make further copies by any means technological, electronic, digital whatsoever without the written permission of the Licensor. Rent or transfer all or any part of your rights under this Agreement. Remove or alter any copyright or other proprietary notice or label attached to the software.

4. **Termination**
   Any failure to comply with the terms and conditions of this agreement will result in the automatic termination of this licence. Upon termination of this licence for any reason, the Customer must destroy or return to the Licensor all copies of the software and accompanying documentation.

5. **Warranties**
   To the extent permitted by law, the Licensor's liability for any breach of the warranty or any term implied by law into this licence is limited to the lowest cost of replacing the goods, acquiring equivalent goods or having the goods repaired.

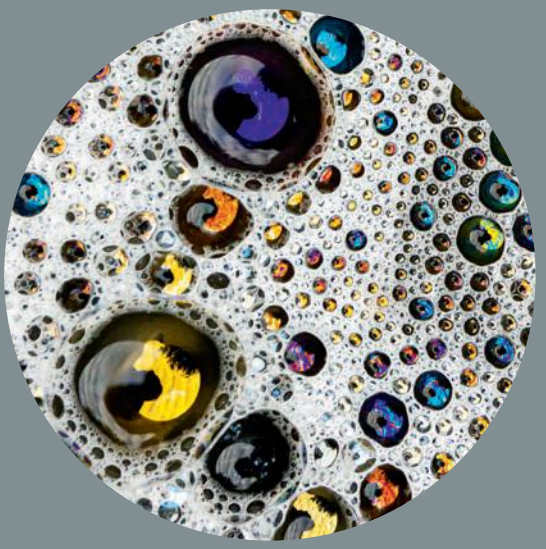
nelson maths.

7

Rachel Theunissen
Rashmi Bhagwati
Judy Binns
Gaspare Carrozza
Series editor
Robert Yen

LEARNING DISCOVERY

SOAP BUBBLES



A bubble is created when air is blown into liquid soap. The soap becomes a thin surface and tries to form a solid shape that holds together and doesn't disintegrate. The tension (tightness) in the surface causes the soap to become a structure that has the minimum possible surface area. This structure is the ball shape called the sphere.

WA
Australian Curriculum

Nelson Maths 7 for the Australian Curriculum WA
1st Edition
Rachel Theunissen
Rashmi Bhagwati
Klaas Bootsma
David Badger
Sarah Hamper
ISBN 9780170465557

Publisher: Robert Yen
Project editor: Alan Stewart
Series cover design: Leigh Ashforth (Watershed Art & Design) and Cengage Creative Studio
Series text design: Leigh Ashforth (Watershed Art & Design)
Series designer: Danielle Maccarone
Permissions researcher: Catherine Kerstjens
Production controller: Karen Young
Typeset by: KGL
Proofreader: MPS Limited
Cover image: iStock.com/Alphotographic

Any URLs contained in this publication were checked for currency during the production process. Note, however, that the publisher cannot vouch for the ongoing currency of URLs.

ACARA Copyright Notice

All material identified by AC is material subject to copyright under the Copyright Act 1968 (Cth) and is owned by the Australian Curriculum, Assessment and Reporting Authority 2023.

For all Australian Curriculum material except elaborations: This is an extract from the Australian Curriculum.

Elaborations: This may be a modified extract from the Australian Curriculum and may include the work of other authors.

Disclaimer: ACARA neither endorses nor verifies the accuracy of the information provided and accepts no responsibility for incomplete or inaccurate information. In particular, ACARA does not endorse or verify that:
- The content descriptions are solely for a particular year and subject;
- All the content descriptions for that year and subject have been used; and
- The author's material aligns with the Australian Curriculum content descriptions for the relevant year and subject.

You can find the unaltered and most up to date version of this material at **http://www.australiancurriculum.edu.au**. This material is reproduced with the permission of ACARA.

© 2022 Cengage Learning Australia Pty Limited

Copyright Notice
This Work is copyright. No part of this Work may be reproduced, stored in a retrieval system, or transmitted in any form or by any means without prior written permission of the Publisher. Except as permitted under the *Copyright Act 1968,* for example any fair dealing for the purposes of private study, research, criticism or review, subject to certain limitations. These limitations include: Restricting the copying to a maximum of one chapter or 10% of this book, whichever is greater; providing an appropriate notice and warning with the copies of the Work disseminated; taking all reasonable steps to limit access to these copies to people authorised to receive these copies; ensuring you hold the appropriate Licences issued by the Copyright Agency Limited ("CAL"), supply a remuneration notice to CAL and pay any required fees. For details of CAL licences and remuneration notices please contact CAL at Level 11, 66 Goulburn Street, Sydney NSW 2000,
Tel: (02) 9394 7600, Fax: (02) 9394 7601
Email: info@copyright.com.au
Website: www.copyright.com.au

For product information and technology assistance,
in Australia call **1300 790 853**;
in New Zealand call **0800 449 725**

For permission to use material from this text or product, please email **aust.permissions@cengage.com**

ISBN 978 0 17 046555 7

Cengage Learning Australia
Level 7, 80 Dorcas Street
South Melbourne, Victoria Australia 3205

Cengage Learning New Zealand
Unit 4B Rosedale Office Park
331 Rosedale Road, Albany, North Shore 0632, NZ

For learning solutions, visit **cengage.com.au**

Printed in China by 1010 Printing International Limited.
1 2 3 4 5 6 7 26 25 24 23 22

PHOTOCOPYING OF BOOKS IS RESTRICTED UNDER LAW

ACKNOWLEDGEMENT OF COUNTRY

Nelson acknowledges the Traditional Owners and Custodians of the lands of all First Nations Peoples of Australia. We pay respect to their Elders past and present.

We recognise the continuing connection of First Nations Peoples to the land, air and waters, and thank them for protecting these lands, waters and ecosystems since time immemorial.

Warning – First Nations Australians are advised that this book and associated learning materials may contain images, videos or voices of deceased persons.

Preface

Nelson Maths 7–10 has been designed for the 2020s classroom, focusing on core skills with explicit grading of exercise questions, 'flipped classroom' video tutorials, online interactivity, more applications and problem-solving questions, and worked solutions to every question.

This new series is carefully mapped to the Australian Curriculum (Version 9, 2022) and built on solid pedagogical foundations that integrate into every chapter practical classroom activities, engaging investigations, problem-solving, reasoning, communicating, reflecting, summarising, extension, revision, mental calculation, technology, numeracy and literacy.

This book, *Nelson Maths 7,* has been designed for Year 7 students, but includes Year 6 revision and Year 8 extension work.

The *MindTap* online learning platform contains print and multimedia content: worksheets, videos, quizzes, interactives, topic tests, worked solutions and much more. We have provided an abundance of resources for teachers to plan and teach for a variety of pathways. *Nelson Maths* is clear, concise, fresh and smart. We have designed this series to be user-friendly and uncomplicated so that teachers and students everywhere can pick it up and use straight away. So let's get started.

About the authors

Rachel Theunissen is Head of Mathematics at Lesmurdie Senior High School in Perth, where she also leads the Literacy and Numeracy team. She is a Curriculum Support Teacher with the Department of Education and a volunteer of MAWA and the Australian Mathematics Trust (AMT). Rachel has been teaching for over 25 years and presents regularly at conferences on mathematics and problem solving.

Rashmi Bhagwati teaches at a high school in southeast Brisbane and has been a mathematics educator for over 20 years. She has been involved in external exam marking and assessment processes, and was a panellist for Mathematics A (senior General Mathematics). Rashmi wrote topic tests for the *Nelson Senior Maths Essential Mathematics* series for Queensland and WA.

Judy Binns was head teacher of mathematics at Mulwaree High School in Goulburn, NSW. She has an interest in motivating students with learning difficulties and wide experience in teaching senior practical mathematics courses.

Gaspare Carrozza was head teacher of mathematics at Homebush Boys High School in Sydney. He has conducted training workshops in both mathematics and computing..

Series editor **Robert Yen** taught at Hurlstone Agricultural High School in Sydney and works for Cengage as the mathematics publisher.

Contributing authors

Sarah Hamper wrote the technology sections.

Kahlia Dreyer, Lachlan Grierson, Brook Johns, Tracey Johnson, Kuldip Khehra, Judy Binns, Gaspare Carrozza and **Robert Yen** wrote the worksheets and topic tests.

Rashmi Bhagwati, Jade Lori, Deborah Da Cruz, Monique Ellement, John Drake, Katie Jackson, Joanne Magner, Scott Smith and **Robert Yen** created the video tutorials.

Table of contents

	Preface	v
	About the authors	v
	Syllabus grids	xi
	About this book	xiv
	Symbols and abbreviations	xviii
	Mathematical verbs	xix

* = EXTENSION

1	**INTEGERS**	**2**
	SkillCheck	4
6N01	**1.01** Numbers above and below zero	5
6N01	**1.02** Integers on a number line	8
	Technology: Introducing spreadsheets	11
7N07	**1.03** Ordering integers	12
7N07	**1.04** Adding integers	15
	Mental skills 1A: Adding 8 or 9	17
7N07	**1.05** Subtracting integers	18
	Investigation: Multiplying integers	21
8N04	**1.06** Extension: Multiplying integers (Year 8)*	22
	Technology: Daily temperatures in Thredbo	24
	Investigation: Dividing integers	26
8N04	**1.07** Extension: Dividing integers (Year 8)*	26
8N04	**1.08** Extension: Order of operations (Year 8)*	28
	Mental skills 1B: Subtracting 8 or 9	30
7N07	**1.09** Integers and the calculator	31
	Technology: Sorting numbers	32
7N07	**1.10** Integer problems	33
	Investigation: Above and below sea level	35
	Power plus	36
	Test yourself 1	38
2	**ANGLES**	**40**
	SkillCheck	43
6M04	**2.01** Naming angles	44
6M04	**2.02** Measuring and drawing angles	47
6M04	**2.03** Types of angles	54
	Technology: Constructing angles 1	56
	Investigation: Complementary and supplementary angles	57
7M04	**2.04** Complementary and supplementary angles	58
	Mental skills 2: Adding or multiplying in any order	61
	Investigation: Angles at a point and vertically opposite angles	62
7M04	**2.05** Angles at a point and vertically opposite angles	62
7M04	**2.06** Parallel and perpendicular lines	66
7M04	**2.07** Corresponding angles on parallel lines	72
7M04	**2.08** Alternate angles on parallel lines	74
7M04	**2.09** Co-interior angles on parallel lines	77
7M04	**2.10** Angles on parallel lines	80
	Technology: Constructing angles 2	83
7M04	**2.11** Proving parallel lines	84
	Power plus	86
	Test yourself 2	89
3	**WHOLE NUMBERS**	**94**
	SkillCheck	96
5N08	**3.01** Rounding and estimating	97
5N06	**3.02** Multiplying numbers	100
5N07	**3.03** Dividing numbers	103
5N02	**3.04** Divisibility tests	105
	Mental skills 3A: Multiplying by a multiple of 10	108
	3.05 Extension: Roman numerals*	109
	Investigation: Aboriginal numbers	111
7N02, 3	**3.06** Index and expanded notations	112

	Technology: Powers	115
	Investigation: Perfect squares	116
7N01	**3.07** Square root and cube root	117
	Mental skills 3B: Dividing by a multiple of 10	122
6N02	**3.08** Prime and composite numbers	122
7N02	**3.09** Factor trees	125
	Technology: Prime factors calculator	127
	Technology: Spreadsheet formulas	127
7N02	**3.10** Highest common factor	129
	Investigation: Common factor puzzle	131
	Investigation: HCF by repeated division	131
7N02	**3.11** Lowest common multiple	132
	Technology: Lowest common multiple	134
	Power plus	135
	Test yourself 3	137
Practice set 1		**139**
4	**FRACTIONS AND PERCENTAGES**	142
	SkillCheck	145
6N03	**4.01** Fractions	146
7N04	**4.02** Ordering fractions	151
	Investigation: What fraction is shaded?	154
7N06	**4.03** Adding and subtracting fractions	155
7N06	**4.04** Adding and subtracting mixed numerals	158
7N06	**4.05** Fraction of a quantity	160
	Mental skills 4A: Fraction of a quantity	163
	Investigation: Netball scores	164
7N06	**4.06** Multiplying fractions	164
	Investigation: The Aboriginal flag	168
	Technology: More spreadsheet practice	168
7N06	**4.07** Dividing fractions	169
	Investigation: Thanh's garden	172
7N09	**4.08** Fraction problems	173
7N06	**4.09** Fractions and the calculator	175
	Investigation: Fraction puzzles	178
7N04	**4.10** Percentages, fractions and decimals	178
7N06	**4.11** Percentage of a quantity	181
	Mental skills 4B: Percentage of a quantity	183
7N06	**4.12** Expressing quantities as fractions and percentages	184
	Technology: Calculating test marks as percentages	186
	Power plus	188
	Test yourself 4	190
5	**ALGEBRA AND EQUATIONS**	194
	SkillCheck	197
7A02	**5.01** The laws of arithmetic	198
7A02	**5.02** The distributive law	201
7A01	**5.03** Variables	204
	Investigation: The laws of arithmetic	208
7A02	**5.04** Creating algebraic expressions	209
	Mental skills 5A: Multiplying an even number by 5	211
	Technology: Using variables and values	211
7A01	**5.05** Substitution	213
	Technology: Substitution	216
	Investigation: Equivalent expressions	217
7A02	**5.06** Collecting variables	217
7A02	**5.07** Adding and subtracting terms	221
7A02	**5.08** Multiplying terms	224
7A02	**5.09** Dividing terms	226
7A03	**5.10** Equations	228
7A03	**5.11** One-step equations	229
7A03	**5.12** Two-step equations	234
	Mental skills 5B: Multiplying by 9, 11 and 12	237
7A03	**5.13** Equation problems	239
	Investigation: School concert	243
	Power plus	243
	Test yourself 5	246

6	**GEOMETRICAL FIGURES**	248
	SkillCheck	250
6SP03	**6.01** Transformations	251
	Technology: Transformations	256
6SP03	**6.02** Composite transformations	257
	Technology: Composite transformations	260
7SP02, 4	**6.03** Classifying triangles	262
	Investigation: Angle sum of a triangle	266
	Technology: Angle sum of a triangle	267
7M05	**6.04** Angle sum of a triangle	267
	Mental skills 6: Estimating answers	269
	Investigation: Exterior angle of a triangle	270
7M05	**6.05** Exterior angle of a triangle	271
7SP02, 4	**6.06** Classifying quadrilaterals	274
	Technology: Measuring quadrilaterals	275
	Investigation: Angle sum of a quadrilateral	277
7M05	**6.07** The angle sum of a quadrilateral	277
7SP02, 4	**6.08** Properties of quadrilaterals	279
	Investigation: Shape puzzles	282
7SP01, 4	**6.09** Polygons and polyhedra	283
7SP01	**6.10** Drawing solids	292
	Power plus	299
	Test yourself 6	301
	Practice set 2	**306**
7	**DECIMALS**	312
	SkillCheck	315
7N04	**7.01** Ordering decimals	316
	Investigation: Comparing heights	319
7N04	**7.02** Decimals and fractions	320
7N05	**7.03** Adding and subtracting decimals	322
	Investigation: Multiplying and dividing decimals by powers of 10	324
7N05	**7.04** Multiplying and dividing decimals by powers of 10	325
	Mental skills 7: Calculating differences and making change	326
7N05	**7.05** Multiplying decimals by estimating	327
	Investigation: Decimal places in multiplication answers	329
7N05	**7.06** Multiplying decimals	330
7N05	**7.07** Dividing decimals by whole numbers	332
7N05	**7.08** Dividing decimals	333
	Investigation: Back-to-front problems	335
7N04, 8N03	**7.09** Terminating and recurring decimals	336
	Investigation: Prime factors and recurring decimals	338
7N05	**7.10** Rounding decimals	340
	Technology: Rounding decimals	344
7N09	**7.11** Decimal problems	345
	Technology: Fruit and vegetables	347
	Power plus	347
	Test yourself 7	350
8	**AREA AND VOLUME**	352
	SkillCheck	355
6M01	**8.01** The metric system	355
5M02	**8.02** Perimeter	359
7M03	**8.03** Parts of a circle	363
	Technology: Circle parts	365
	Investigation: Measuring the circumference of a circle	367
	Technology: The circumference and diameter of a circle	368
7M03	**8.04** Circumference of a circle	368
7M01, 8M01	**8.05** Metric units for area	372
	Investigation: The human lagoon	376
	Investigation: Standing room only!	377
	Investigation: Student space in a classroom	377
	Investigation: Area of a rectangle	378
7M01	**8.06** Area of a rectangle	379
	Mental skills 8A: Doubling and halving numbers	382
	Technology: Perimeter and area of a chicken pen	383
	Investigation: Area of a triangle	384

7M01	**8.07** Area of a triangle	385
	Investigation: Area of a parallelogram	387
7M01	**8.08** Area of a parallelogram	388
8M01	**8.09** Extension: Areas of composite shapes (Year 8)*	390
	Investigation: Area puzzles	395
	Investigation: Local areas	396
	Mental skills 8B: Multiplying and dividing by 4 or 8	396
7M02, 8M02	**8.10** Metric units for volume	397
	Investigation: Volume of a rectangular prism	400
7M02	**8.11** Volume of a rectangular prism	401
	Investigation: What is your volume?	404
7M02	**8.12** Volume of a prism	405
	Power plus	408
	Test yourself 8	411
9	**FORMULAS AND GRAPHS**	416
	SkillCheck	418
6SP02	**9.01** Location using coordinates	419
6SP02	**9.02** The number plane	423
6SP02	**9.03** Quadrants of the number plane	426
	Investigation: Number plane picture puzzles	428
7SP03	**9.04** Transformations on the number plane	429
	Technology: Transformations on the number plane	435
7A05	**9.05** Graphing tables of values	436
	Mental skills 9: Simplifying multiplication by factorising	438
	Investigation: Height vs head circumference	439
7A05, 6	**9.06** Formulas and tables of values	439
7A05	**9.07** Graphing linear functions	443
	Technology: Graphing linear functions	446
	Power plus	447
	Test yourself 9	451
	Practice set 3	**455**
10	**ANALYSING DATA**	458
	SkillCheck	460
7ST02	**10.01** Interpreting graphs	461
	Technology: Column graphs	466
7ST02	**10.02** Misleading graphs	466
7ST02	**10.03** Dot plots	469
	Technology: Olympic winning times	473
7ST02	**10.04** Stem-and-leaf plots	474
	Mental skills 10: Reading linear scales	477
7ST01	**10.05** The mean and mode	478
	Investigation: Finding the middle data values	481
7ST01	**10.06** The median and range	482
	Investigation: Michael's family	485
	Technology: Mean, median, mode and range	486
7ST01, 2	**10.07** Analysing dot plots and stem-and-leaf plots	487
7ST01, 2	**10.08** Comparing data sets	490
7ST02	**10.09** The shape of a distribution	494
7ST03	**10.10** Statistical investigations	498
	Power plus	499
	Test yourself 10	501
11	**PROBABILITY**	506
	SkillCheck	508
7P01	**11.01** Sample spaces	510
7P01	**11.02** Probability	513
6P01	**11.03** The range of probability	519
	Investigation: The Greedy Pig game	522
7P01, 2	**11.04** Observed and expected frequencies	522
	Technology: Rolling a die	526
	Investigation: Spinning chance	527
	Mental skills 11: Adding 90c and 95c	528
	Investigation: Complementary events	529
8P01	**11.05** Extension: Complementary events (Year 8)*	530
	Power plus	533
	Test yourself 11	535

12

	RATIOS AND TIME	538
	SkillCheck	540
7N08	**12.01** Ratios	541
7N08	**12.02** Simplifying ratios	544
	Investigation: Scale diagrams	548
7N08, 7M06	**12.03** Ratio problems	549
	Mental skills 12A: Multiplying by 5, 15, 25 and 50	551
7A04	**12.04** Travel graphs	552
7A04	**12.05** Graphs of change	556
6M03	**12.06** Time calculations	559
5M03	**12.07** 24-hour time	562
	Mental skills 12B: Dividing by 5, 15, 25 and 50	565
8M04	**12.08** Extension: Time differences (Year 8)*	566
6M03	**12.09** Timetables	570
8M04	**12.10** Extension: World time zones (Year 8)*	574
	Investigation: World trip	577
	Power plus	577
	Test yourself 12	579
Practice set 4		**581**
General practice		**585**
Calculator skills		**592**
Answers		**598**
Glossary and Index		**654**

Curriculum grids

Australian Curriculum

Strand	*Nelson Maths 7* chapter		*Nelson Maths 8* chapter	
NUMBER	1	Integers	1	Pythagoras' theorem
	3	Whole numbers	2	Working with numbers
	4	Fractions and percentages	5	Area and volume
	7	Decimals	6	Fractions and percentages
	9	Formulas and graphs		
	12	Ratios and time		
ALGEBRA	5	Algebra and equations	3	Algebra
	9	Formulas and graphs	10	Equations
	12	Ratios and time	11	Ratios, rates and time
			12	Graphing linear functions
MEASUREMENT	2	Angles	1	Pythagoras' theorem
	6	Geometrical figures	5	Area and volume
	8	Area and volume	11	Ratios, rates and time
	12	Ratios and time		
SPACE	6	Geometrical figures	4	Geometry
	9	Formulas and graphs	8	Congruent and similar figures
			12	Graphing linear functions
STATISTICS	10	Analysing data	7	Investigating data
PROBABILITY	11	Probability	9	Probability

Year 7 content descriptions

Australian Curriculum descriptions (© ACARA 2022).

Content description	*Nelson Maths 7* chapter	
NUMBER (N)		
AC9M7N01: describe the relationship between perfect square numbers and square roots, and use squares of numbers and square roots of perfect square numbers to solve problems	3	Whole numbers
AC9M7N02: represent natural numbers as products of powers of prime numbers using exponent notation	3	Whole numbers
AC9M7N03: represent natural numbers in expanded notation using place value and powers of 10	3	Whole numbers
AC9M7N04: find equivalent representations of rational numbers and represent rational numbers on a number line	4	Fractions and percentages
	7	Decimals
AC9M7N05: round decimals to a given accuracy appropriate to the context and use appropriate rounding and estimation to check the reasonableness of solutions	7	Decimals

Content description	*Nelson Maths 7* chapter	
(EXTENSION) YEAR 8 AC9M8N03: recognise terminating and recurring decimals, using digital tools as appropriate	7	Decimals
AC9M7N06: use the 4 operations with positive rational numbers including fractions, decimals and percentages to solve problems using efficient calculation strategies	4	Fractions and percentages
	7	Decimals
AC9M7N07: compare, order and solve problems involving addition and subtraction of integers	1	Integers
(EXTENSION) YEAR 8 AC9M8N04: use the 4 operations with integers and with rational numbers, choosing and using efficient strategies and digital tools where appropriate	1	Integers
AC9M7N08: recognise, represent and solve problems involving ratios	12	Ratios and time
AC9M7N09: use mathematical modelling to solve practical problems involving rational numbers and percentages, including financial contexts; formulate problems, choosing representations and efficient calculation strategies, using digital tools as appropriate; interpret and communicate solutions in terms of the situation, justifying choices made about the representation	4	Fractions and percentages
ALGEBRA (A)		
AC9M7A01: recognise and use variables to represent everyday formulas algebraically and substitute values into formulas to determine an unknown	5	Algebra and equations
AC9M7A02: formulate algebraic expressions using constants, variables, operations and brackets	5	Algebra and equations
AC9M7A03: solve one-variable linear equations with natural number solutions; verify the solution by substitution	5	Algebra and equations
AC9M7A04: describe relationships between variables represented in graphs of functions from authentic data	9	Formulas and graphs
	12	Ratios and time
AC9M7A05: generate tables of values from visually growing patterns or the rule of a function; describe and plot these relationships on the Cartesian plane	9	Formulas and graphs
AC9M7A06: manipulate formulas involving several variables using digital tools, and describe the effect of systematic variation in the values of the variables	9	Formulas and graphs
MEASUREMENT (M)		
AC9M7M01: solve problems involving the area of triangles and parallelograms using established formulas and appropriate units	8	Area and volume
AC9M7M02: solve problems involving the volume of right prisms including rectangular and triangular prisms, using established formulas and appropriate units	8	Area and volume
AC9M7M03: describe the relationship between π and the features of circles including the circumference, radius and diameter	8	Area and volume
(EXTENSION) YEAR 8 AC9M8M01: solve problems involving the area and perimeter of irregular and composite shapes using appropriate units	8	Area and volume
AC9M7M04: identify corresponding, alternate and co-interior relationships between angles formed when parallel lines are crossed by a transversal; use them to solve problems and explain reasons	2	Angles
AC9M7M05: demonstrate that the interior angle sum of a triangle in the plane is 180° and apply this to determine the interior angle sum of other shapes and the size of unknown angles	6	Geometrical figures

Content description	*Nelson Maths 7* chapter	
AC9M7M06: use mathematical modelling to solve practical problems involving ratios; formulate problems, interpret and communicate solutions in terms of the situation, justifying choices made about the representation	12	Ratios and time
(EXTENSION) YEAR 8 AC9M8M04: solve problems involving duration, including using 12- and 24-hour time across multiple time zones	12	Ratios and time
SPACE (SP)		
AC9M7SP01: represent objects in 2 dimensions; discuss and reason about the advantages and disadvantages of different representations	6	Geometrical figures
AC9M7SP02: classify triangles, quadrilaterals and other polygons according to their side and angle properties; identify and reason about relationships	6	Geometrical figures
AC9M7SP03: describe transformations of a set of points using coordinates in the Cartesian plane, translations and reflections on an axis, and rotations about a given point	9	Formulas and graphs
AC9M7SP04: design and create algorithms involving a sequence of steps and decisions that will sort and classify sets of shapes according to their attributes, and describe how the algorithms work	6	Geometrical figures
STATISTICS (ST)		
AC9M7ST01: acquire data sets for discrete and continuous numerical variables and calculate the range, median, mean and mode; make and justify decisions about which measures of central tendency provide useful insights into the nature of the distribution of data	10	Analysing data
AC9M7ST02: create different types of numerical data displays including stem-and-leaf plots using software where appropriate; describe and compare the distribution of data, commenting on the shape, centre and spread including outliers and determining the range, median, mean and mode	10	Analysing data
AC9M7ST03: plan and conduct statistical investigations involving data for discrete and continuous numerical variables; analyse and interpret distributions of data and report findings in terms of shape and summary statistics	10	Analysing data
PROBABILITY (P)		
AC9M7P01: identify the sample space for single-stage events; assign probabilities to the outcomes of these events and predict relative frequencies for related events	11	Probability
AC9M7P02: conduct repeated chance experiments and run simulations with a large number of trials using digital tools; compare predictions about outcomes with observed results, explaining the differences	11	Probability
(EXTENSION) YEAR 8 AC9M8P01: recognise that complementary events have a combined probability of one; use this relationship to calculate probabilities in applied contexts	11	Probability

About this book

Coverage of the Australian Curriculum

- *Nelson Maths 7* covers the Australian curriculum, as shown by the table of contents and curriculum grid on the previous pages.
- This book contains Year 7 content. It also contains revision of some Year 6 content, and some Year 8 or extension work marked by *.
- Each chapter begins with a **chapter outline** that includes the 4 curriculum proficiencies covered in each section.

U = UNDERSTANDING **Understanding** is 'knowing and relating' maths. It is more than just learning facts. It's deep understanding, seeing how mathematical content is interconnected, knowing 'why' as well as 'how'.	**F = FLUENCY** **Fluency** is 'applying' maths. It is being able to use mathematics competently and effectively. When you are fluent in a language, you have mastered it so that you can improvise and confidently use the correct word or phrase. Fluency in maths is choosing an appropriate skill, method or formula to use at the right place and time.
PS = PROBLEM SOLVING **Problem solving** is 'modelling and investigating' with maths. It involves interpreting a rich, elaborate problem, selecting an appropriate strategy or model, solving the problem, then evaluating, communicating and justifying the solution.	**R = REASONING** **Reasoning** is 'generalising and proving' with maths, using higher-order thinking to connect specific facts to general principles, using algebra, logic, proof and justification.

To these proficiencies, we have added **C = COMMUNICATING.**

Communicating is 'describing and explaining' maths, representing mathematical theory and solutions in words, algebraic symbols, special notations, diagrams, graphs and tables.

Understanding and **Fluency** can be found in every exercise and activity, while **Problem solving, Reasoning** and **Communicating** are found in the **Investigations, Technology, Language of Maths** and **Topic summary** activities, and explicitly labelled in every exercise (see below).

EXERCISE 1.03 ANSWERS ON P. 598

Ordering integers

U F R C

1 Copy and complete each statement using a > or < symbol.

R **a** 4 ____ 1 **b** 0 ____ 9 **c** 7 ____ –7 **d** 0 ____ –3

C **e** 5 ____ –11 **f** –1 ____ –6 **g** –12 ____ 8 **h** –10 ____ –2

i –6 ____ 5 **j** –6 ____ –2 **k** 17 ____ 23 **l** 24 ____ –4

m –15 ____ –35 **n** –27 ____ 29 **o** –8 ____ 47 **p** –12 ____ 0

At the beginning of each chapter

A listing of **Nelson MindTap** chapter resources

SkillCheck reviews prerequisite skills and knowledge for the chapter

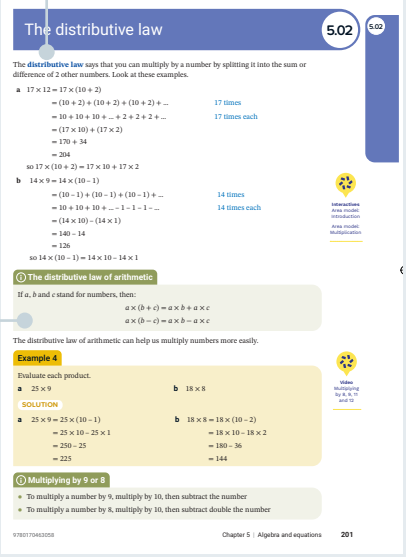
A Chapter outline and a **Wordbank** chapter glossary (a full glossary appears at the back of the book)

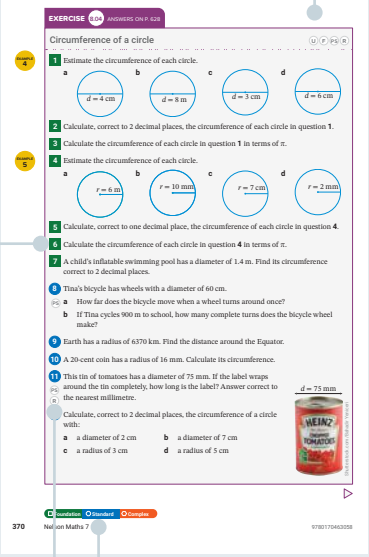
In each chapter

Important facts and formulas are highlighted in a shaded box.

Glossary terms are printed in **blue**.

Graded exercises are linked to worked examples and include multiple-choice questions, exam-style problems and realistic applications.





The exercise questions are colour-coded for level of difficulty:

Foundation | Standard | Complex

Specific questions are also labeled PS for **Problem solving**, R for **Reasoning** and C for **Communicating**.

MENTAL SKILLS

Mental skills reinforce mental calculation strategies ('calculator-free maths')

TECHNOLOGY

Technology includes spreadsheets, dynamic geometry software and the Internet.

INVESTIGATION

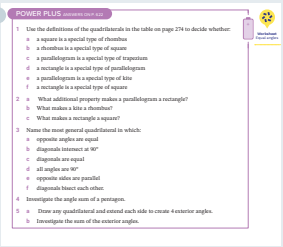
Investigations explore the syllabus in more detail, through group work, discovery and modelling activities.

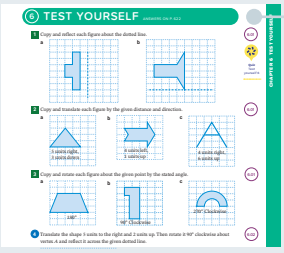
DID YOU KNOW?

Did you know? contains interesting facts and applications of the mathematics learned in the chapter.

At the end of each chapter

Power plus is an extension/challenge exercise.

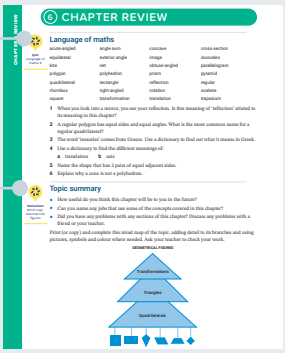


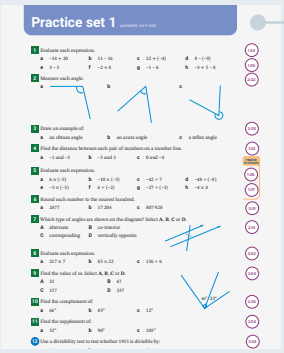


Test Yourself contains chapter revision linked to the relevant exercise set.

Language of maths has a chapter word list and literacy questions.

Topic summary has a mind map activity with downloadable solutions.



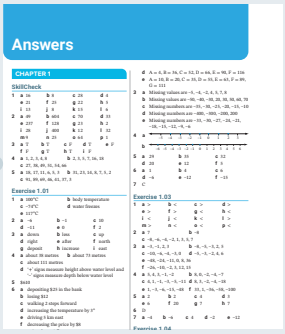


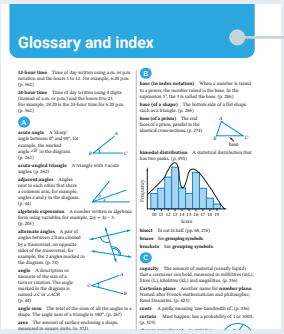
Practice sets after every 3 chapters.

At the end of the book

General practice exercise

Answers (worked solutions available for teachers).





Glossary and **index**

Nelson MindTap

An online learning space that provides students with tailored learning experiences.

- Access tools and content that make learning simpler yet smarter to help you achieve maths mastery.
- Includes an eText with integrated interactives and online assessment.
- Margin links in the student book signpost multimedia student resources found on MindTap.

Video
Equation problems

For students

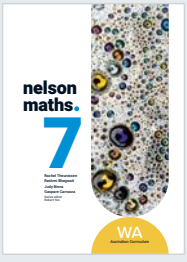
- **Watch** video tutorials featuring expert teacher advice to unpack new concepts and develop your understanding.
- **Revise** using quizzes, worksheets and skillsheets to practise your skills and build your confidence.
- **Navigate** your own path, accessing the content, analytics and support as you need it.
- **Do the Maths** animations: on problem-solving and explaining real-life applications of maths
- ***Twig* mini-documentaries** showing the background or context of mathematics
- ***PhET* interactives**: online maths widgets

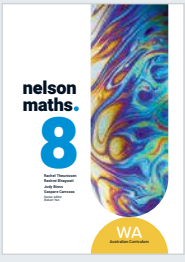
For teachers*

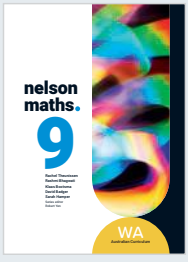
- Tailor content to different learning needs – assign directly to the student, or the whole class.
- Monitor progress using assessment tools like Gradebook and Reports.
- Integrate content and assessment directly within your school's LMS for ease of access.
- Access topic tests, teaching plans and worked solutions to each exercise set.

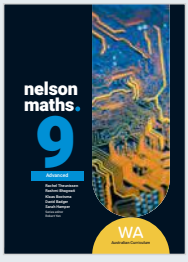
* Complimentary access to these resources is only available to teachers who use this book as part of a class set, book hire or booklist. Contact your Cengage Education Consultant for information about access codes and conditions

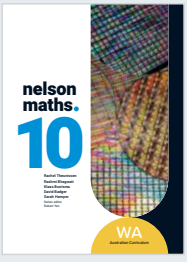
Nelson Maths 7–10 series

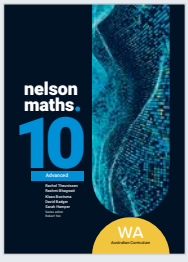












Symbols and abbreviations

$=$	is equal to
$\neq$	is not equal to
$\approx$	is approximately equal to
$<$	is less than
$>$	is greater than
$\leq$	is less than or equal to
$\geq$	is greater than or equal to
( )	parentheses, round brackets
[ ]	(square) brackets
{ }	braces
$\pm$	plus or minus
-3	negative 3
$0.\dot{1}5\dot{2}$	the recurring decimal 0.152152 ...
$^\circ$	degree
$\angle A$	angle A
$\triangle ABC$	triangle ABC
$\parallel$	is parallel to
$\perp$	is perpendicular to
$\therefore$	therefore
x^2	x squared, $x \times x$
x^3	x cubed, $x \times x \times x$
$\sqrt{\ }$	square root, radical sign
$\sqrt[3]{\ }$	cube root
$P(E)$	the probability of event E
LHS	left-hand side
RHS	right-hand side
%	percentage
p.a.	per annum (per year)
$\bar{x}$	the mean (average)
μ	micro-, mu

Mathematical verbs

A glossary of 'doing words' commonly found in mathematics problems

analyse: study in detail the parts of a situation

bisect: cut in half

calculate: *see* **evaluate**

classify, identify: state the type, category or feature of an item or situation

comment: express an observation or opinion about a result

complete: fill in detail to make a statement, diagram or table correct or finished

compare: show how 2 or more things are similar or different

construct: draw an accurate diagram

convert: change from one form to another, for example, from a fraction to a decimal, or from kilograms to grams

decrease: make smaller

describe: state the features of a situation

estimate: make an educated guess for a number, measurement or solution, to find roughly or approximately.

evaluate, calculate: find the value of a numerical expression, for example, 3×8^2 or $4x + 1$ when $x = 5$

expand: remove brackets in an algebraic expression by multiplying, for example, expanding $3(2y + 1)$ gives $6y + 3$

explain: describe why or how

factorise: take out the highest common factor (HCF) of an expression and insert brackets, for example, factorising $5x - 20$ gives $5(x - 4)$. The opposite of **expand**.

give reasons: show the rules or thinking used when solving a problem. See also **justify**.

graph: display on a number line, number plane or statistical graph.

hence find/prove: find an answer or prove a result using previous answers or information supplied

identify: *see* **classify**.

increase: make larger

interpret: find meaning in an answer or result

justify: give reasons or evidence to support your argument or conclusion. See also **give reasons**.

measure: use an instrument to find the size of something, for example, use a ruler to determine the length of a pen.

prove that: *see* **show that**

recall: remember and state.

reduce (a fraction) to its lowest terms: see **simplify (a fraction)**.

round (a number): find the nearest approximation of a number. For example, 4.3 rounded to the nearest whole number is 4, \$12.9598 rounded to the nearest cent is \$12.96, 0.166 66 rounded to 3 decimal places is 0.167.

show that, prove that: (in questions where the answer is given) use calculation, procedure or reasoning to prove that an answer or result is true

show working: show the steps you used to find the answer

simplify: give a result in its most basic, shortest, neatest form, for example, simplifying a ratio or algebraic expression

simplify (a fraction): reduce the numerator and denominator of a fraction by dividing by their highest common factor (HCF), for example, $\frac{16}{20}$ simplified is $\frac{4}{5}$.

simplify (a ratio or rate): reduce the terms or units of a ratio or rate by dividing by their highest common factor (HCF), for example, 10 : 4 simplified is 5 : 2.

sketch: draw a rough diagram that shows the general shape or idea, less accurate than **construct**

solve: find the value(s) of an unknown variable in an equation or inequality

state: *see* **write**.

substitute: replace a variable by a number and evaluate

verify: check that a solution or result is correct, usually by substituting back into the equation or referring back to the problem

write correct to: See **round (a number)**.

write, state: give the answer, formula or result without showing any working or explanation (this usually means that the answer can be found mentally, or in one step)

1

NUMBER

Integers

Antarctica is the coldest continent on Earth as it is at the South Pole. During its winter, from March to August, the Sun does not rise, the sky stays dark for 6 months and inland temperatures are from −66°C to −57°C. During its summer, from September to February, the Sun does not set, the sky stays bright for 6 months, inland temperatures are from −45°C to −25°C and this is the only safe time to visit. Antarctica is permanently covered with snow and ice so it reflects the Sun's rays rather than absorbs them, and its dry air and high altitudes contribute to its sub-zero temperatures.

Alamy Stock Photo/Danita Delimont

Chapter outline

		Proficiencies				
1.01	Numbers above and below zero	U	F	PS	R	C
1.02	Integers on a number line	U	F		R	
1.03	Ordering integers	U	F		R	C
1.04	Adding integers	U	F		R	C
1.05	Subtracting integers	U	F		R	C
1.06	Extension: Multiplying integers*	U	F		R	C
1.07	Extension: Dividing integers*	U	F		R	C
1.08	Extension: Order of operations*	U	F		R	C
1.09	Integers and the calculator	U	F		R	C
1.10	Integer problems	U	F	PS	R	

*YEAR 8 EXTENSION

U = Understanding
F = Fluency
PS = Problem solving
R = Reasoning
C = Communication

*Note: Although multiplying and dividing integers is Year 8 content, teaching it in Year 7 allows more problems in number and algebra to be examined.

Wordbank

ascending Moving upward, increasing from smallest to largest

descending Moving downward, decreasing from largest to smallest

evaluate To find the value of an expression

integer A positive or negative whole number or zero

minus To subtract or 'take-away' a number

negative number A number that is less than zero; the opposite of a positive number

number line A line that shows the position and order of numbers

Videos (5):

1.04 Adding integers
1.05 Subtracting integers
1.07 Multiplying and dividing integers
1.08 Order of operations • BODMAS

Twig video (1):

1.01 India and negative numbers

PhET interactives (3):

1.02 Number line: Integers
1.04 Number line: Operations
1.05 Number line: Operations • Number line: Distance

Quizzes (6):

- Wordbank 1
- SkillCheck 1
- Mental skills 1A
- Mental skills 1B
- Language of Maths 1
- Test Yourself 1

Skillsheets (4):

1.02 Spreadsheets
1.04 Integers using coloured squares
1.05 Integers • Integers using coloured squares
1.06, 1.07 Integers
1.08 Order of operations

Worksheets (4):

1.02 A page of number lines
1.06 Integers writing activity
1.07 What is the question?
Mind map: Integers

Puzzles (6):

1.02 Integer Snap
1.04 Integers game
1.06 Multiplying integers game
1.08 Order of operations puzzle • Directed numbers
1.10 Integers group clues

Presentations (2):

1.04 Adding directed numbers
1.05 Subtracting directed numbers

Spreadsheet (1):

1.06 Integer quiz: Multiplication

Nelson MindTap

To access resources above, visit **cengage.com.au/nelsonmindtap**

1 In this chapter you will:

- ✓ understand and apply positive and negative whole numbers and zero, and represent them on a number line
- ✓ compare and order integers
- ✓ add and subtract integers, with and without technology
- ✓ (EXTENSION) multiply and divide integers, with and without technology
- ✓ (EXTENSION) apply 'order of operations' rules with integers
- ✓ solve problems involving operations with integers

SkillCheck

ANSWERS ON P. 598

Quiz SkillCheck 1

1 Simplify each expression.

a $7 + 9$	**b** $12 - 4$	**c** 7×4	**d** $20 \div 5$
e $13 + 8$	**f** $30 - 5$	**g** 2×11	**h** $15 \div 3$
i $6 + 7$	**j** $14 - 6$	**k** 3×5	**l** $24 \div 4$

2 Simplify each **expression**.

a $12 + 37$ **b** $652 - 48$ **c** 14×5 **d** $66 \div 2$
e 79×3 **f** $384 \div 3$ **g** $3 + 5 \times 4$ **h** $18 \div 2 - 7$
i $30 - 12 \div 6$ **j** $20 \times (3 + 17)$ **k** $12 - 3 \times 2 + 6$ **l** $43 - [5 \times 2 + 1]$
m 3^2 **n** 5^2 **o** 8^2 **p** 1^2

3 State whether each statement is true (T) or false (F).

a $15 > 12$ **b** $8 < 11$ **c** $6 < 3$
d $4 + 3 < 10$ **e** $12 - 3 > 11$ **f** $7 < 3 \times 2$
g $15 \div 3 > 12 \div 4$ **h** $8 + 3 + 2 > 6 \times 1$ **i** $18 \div 3 < 2 + 2 + 2$

4 Write each set of numbers in ascending order (smallest to largest).

a 3, 8, 2, 1, 4 **b** 2, 18, 5, 7, 3, 16 **c** 38, 51, 49, 27, 66, 54

5 Write each set of numbers in descending order (largest to smallest).

a 6, 11, 18, 17, 5, 3 **b** 8, 7, 14, 23, 31, 5, 2 **c** 89, 91, 37, 69, 41, 3, 46

Numbers above and below zero 1.01

'The temperature in Canberra was 3°C yesterday but overnight it dropped 5 degrees to –2°C'
'When Lisa withdrew \$400 from her bank account, her bank balance of \$340 went down to –\$60'
Both of the above situations involve numbers above and below zero.

Video India and negative numbers

Numbers greater than zero, such as 3, 400, $8\frac{1}{2}$ and 14.37, are called **positive numbers**.

Numbers less than zero, such as –2, –60, $-10\frac{3}{4}$ and –21.6, are called **negative numbers**.

The number –2 is the number that is 2 below 0. It is called 'negative 2', not 'minus 2'.
The number 3 is the number that is 3 above 0. It can also be called 'positive 3' and be written as '+3'.
The negative numbers are the opposites of the positive numbers. The opposite of 5 is –5.
The opposite of –9 is 9.

EXERCISE 1.01 ANSWERS ON P. 598

Numbers above and below zero

U F PS R C

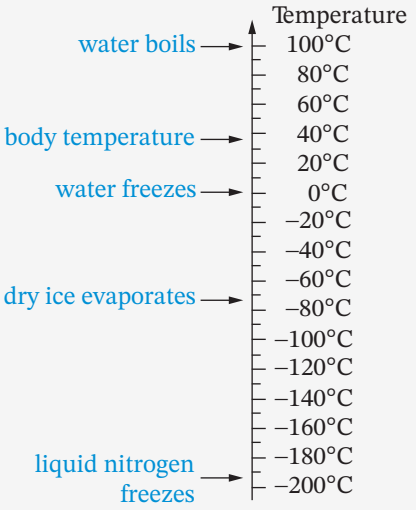
1 This scale shows temperatures in degrees Celsius (°C) above and below 0°C.

R **a** At what temperature does water boil?
C **b** What is measured at 37°C?
c Estimate the temperature at which dry ice evaporates.

Foundation Standard Complex

d What is measured at 0°C?

e Last week, it was 48°C in Algeria, Africa and –69°C at the South Pole. What was the difference between the 2 temperatures?



2 For each number, write its opposite number.

C **a** 6 **b** 1 **c** –10

d 11 **e** 0 **f** –2

3 Write a word that is the opposite of:

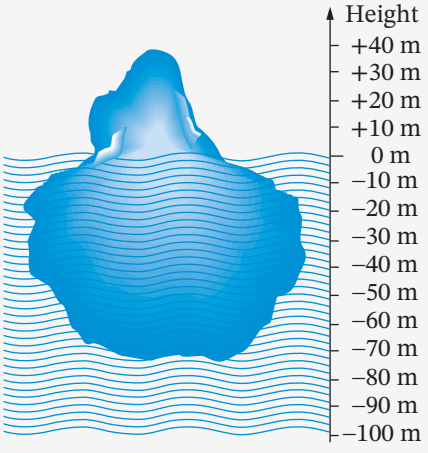
C **a** up **b** more **c** down

d left **e** before **f** south

g withdraw **h** decrease **i** west

4 Icebergs can be dangerous to ships because about $\frac{5}{6}$ of their mass lies under water.

R C The following diagram shows the position of an iceberg above and below sea level.



Foundation Standard Complex

a What is the height of the iceberg above sea level?

b What is the depth of the iceberg below sea level?

c What is the total height of the iceberg?

d Why are '+' and '–' signs useful in these measurements?

5 PS Dilshan's bank account has money coming in and going out over one day. Withdrawals have a negative sign.

Balance at the start:	$ 720
	–$ 50
	$ 60
	–$ 40
	$100
	$ 80
	–$260
Balance at the end of the day:	$

Calculate the balance at the end of the day.

6 R C Write the opposite of:

a withdrawing $25 from the bank

b winning $12

c walking 2 steps backwards

d lowering the temperature by 3°

e driving 5 km west

f increasing the price by $8

g going up 3 floors

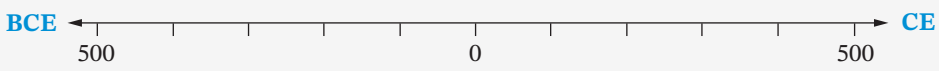
h 5 seconds before take-off

7 R What number's opposite is itself?

8 R C This timeline starts at 500 BCE and ends at 500 CE.

CE (or AD) stands for Common Era while BCE (or BC) stands for Before the Common Era.

PS



Copy the timeline and place each event below in the correct place.

A 100 CE — The first Chinese dictionary was compiled.

B 400 BCE — The first temple dedicated to the Greek god Zeus was built.

C 490 BCE — Greek runner Phidippides inspired the first marathon with a 26-mile run.

D 55 BCE — Roman forces under Julius Caesar invaded Britain.

E 190 CE — The abacus was invented.

F 370 CE — Hypatia, a female mathematician, was born in Alexandria, Egypt.

Foundation Standard Complex

G	410 CE	Roman forces left Britain.
H	330 BCE	Greek mathematician Euclid showed that an infinite number of prime numbers exist.
I	260 CE	The 2-year war between Rome and Persia ended.
J	240 BCE	Greek mathematician Eratosthenes estimated the circumference of the Earth using 2 sticks.

9 In the Jewish calendar, the year of Creation, 3760 BCE, is Year Zero, while in the Indian calendar, 78 CE is Year Zero.

a In what year were you born, according to the:

i Jewish calendar? **ii** Indian calendar?

b What is this year, according to the:

i Jewish calendar? **ii** Indian calendar?

DID YOU KNOW?

Tutankhamen, King of Egypt

Alamy Stock Photo/Prisma by Dukas Presseagentur GmbH

In 1922, English archaeologist Howard Carter discovered the tomb of a pharaoh (king) in the Valley of the Kings in Egypt. The pharaoh's name, Tutankhamen, meant 'living image of Amun,' an ancient Egyptian god. Tutankhamen became pharaoh at the age of 9 after the death of his parents, and he died aged 18 in 1323 BCE. The cause of his death remains a mystery – many possibilities have been suggested, including murder, but none can be proved. His tomb contained over 3000 treasures, many of them covered in gold, including his coffin. Carter's discovery was significant because Tutankhamen's tomb was so well-preserved. It still exists in the Valley of the Kings today.

In what year did Tutankhamen become king?

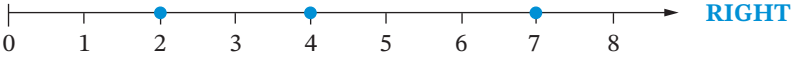
How many years after his death was King Tutankhamen's tomb discovered?

1.02 Integers on a number line

Worksheet A page of number lines

Puzzle Integer Snap

A **number line** shows the position and order of numbers. The number line below shows the positions of 2, 4 and 7 marked in blue.



As we move further to the right on the number line, the numbers become bigger. The arrow on the right end of the number line shows that numbers can increase infinitely ('forever').

Foundation Standard Complex

We can extend our number line to the left to include the negative numbers below zero.

As we move further to the left on the number line, the numbers become smaller. The arrow on the left end of the number line shows that numbers can decrease infinitely.

Positive and negative numbers have **size** and **direction**. For example, 5 (or +5) means 5 units in the positive direction, while –2 means 2 units in the negative direction.

Positive and negative **whole** numbers are called **integers**, for example, –3, 4 and –20. Zero is also an integer.

Interactive
Number line: Integers

1.02

Integers

Integers are the positive and negative whole numbers and zero.

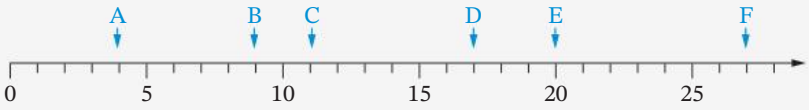
Integers are **whole** numbers, not fractions or decimals, so $8\frac{1}{2}$, 14.37, $-10\frac{3}{4}$ and –21.6 are not integers. The general name for **all** positive or negative numbers is the **directed numbers**.

EXERCISE 1.02 ANSWERS ON P. 598

Integers on a number line

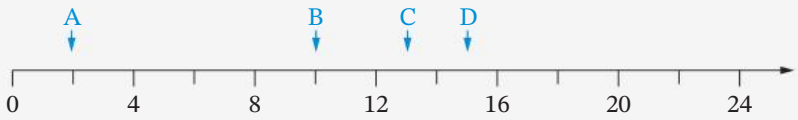
U F R

1 Write the positive integer indicated by each arrow on this number line.

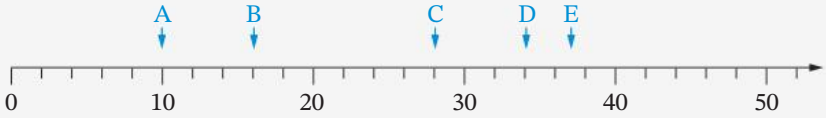


2 Estimate the value of the positive integer indicated by each arrow on the number lines below.

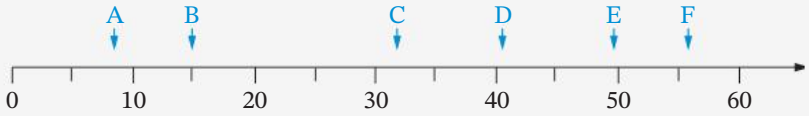
a



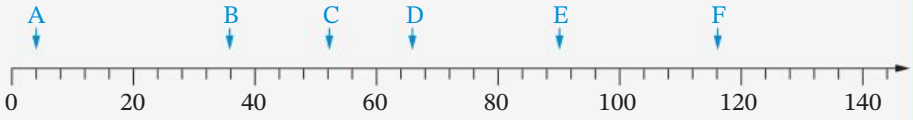
b



c

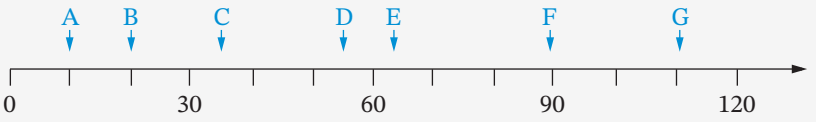


d



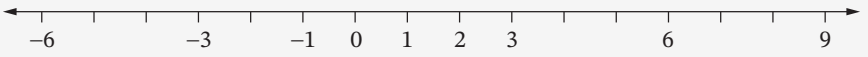
Foundation Standard Complex

e

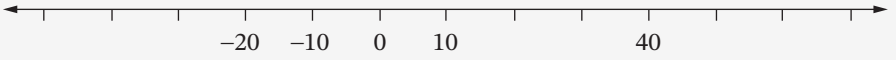


3 Copy and complete each number line, using a ruler to mark the positions evenly.

a

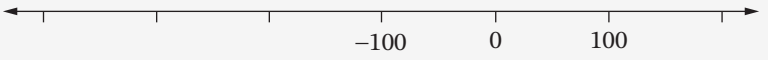


b

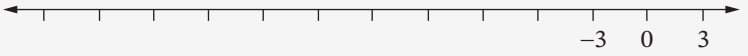


c

d



e

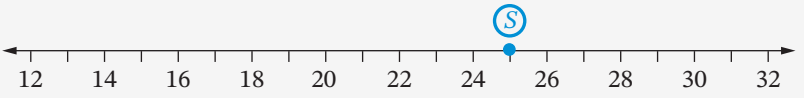


4 **a** Draw a number line and show all the integers between –5 and 3.

b Draw a number line marked from –6 to 6 and show all the integers less than 1.

5 On this number line, where will *S* be if it moves:

R



a 4 places to the right?

b 10 places to the right?

c 7 places to the right?

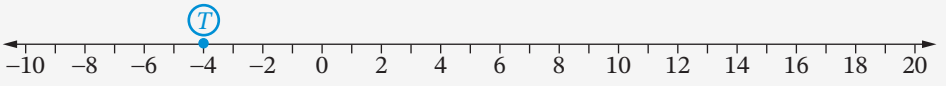
d 5 places to the left?

e 13 places to the left?

f 20 places to the left?

6 On this number line, where will *T* be if it moves:

R



a 5 places to the right?

b 8 places to the right?

c 10 places to the right?

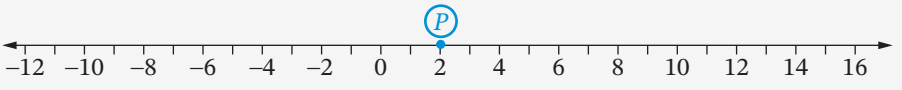
d 2 places to the left?

e 8 places to the left?

f 11 places to the left?

7 On this number line, where will *P* be if it moves 4 places to the left and then 9 places to the right? Select the correct answer **A**, **B**, **C** or **D**.

R



A –11 **B** –3 **C** 7 **D** 15

Foundation Standard Complex

TECHNOLOGY

Introducing spreadsheets

Skillsheet
Spreadsheets

1.02

A spreadsheet is a **calculator** in the form of a grid made up of many cells where we can enter labels and numbers and display answers to calculations. The **columns** of a spreadsheet are ordered A, B, C, D, ... while the **rows** are ordered 1, 2, 3, 4, ...

	A	B	C	D	E	F	G	H
1								
2								
3								
4								
5								
6								
7								
8								

To make a calculation, we need to write a **formula** (rule) in a cell. A spreadsheet formula uses special symbols and always starts with an equal sign '='. Here are some examples of formulas:

Formula	Meaning
=A1+A2+A3 or **=sum(A1:A3)**	**add** the values in cells A1, A2 and A3
=A5−A4	**subtract** the value in cell A4 from the value in cell A5
=A1*10	**multiply** the value in cell A1 by 10 (* is used instead of ×)
=A1/A2	**divide** the value in cell A1 by the value in cell A2 (/ is used instead of ÷)
=average(A1:A5)	find the **average** of all values from cells A1 to A5

1 Open a new spreadsheet and enter the 5 numbers shown in column A at cells A1, A2, A3, A4 and A5.

2 In cell B1, enter the formula '=A4-7' to calculate the value in A4, minus 7. You should get the answer 37, because 44 – 7 = 37.

	A	B
1	4	=A4-7
2	10	
3	6	
4	44	
5	25	
6		

3 For the rest of the cells in column B, enter the appropriate formula for each expression shown below and check that the spreadsheet calculates the correct answer. Remember to start each formula with an '=' sign.

a at B2, A2 + A1

b at B3, 10 × A5 – 6

c at B4, A4 – A1 × 2

d at B5, A1 × A2 + A3

e at B6, A1 × (A2 + A3)

f at B7, 3 × A2 ÷ 2

g at B8, $\frac{A5-A3}{3}$

h at B9, sum of A1, A2, A3, A4, A5

i at B10, average of A1, A2, A3, A4, A5

j at B11, $\frac{A4}{A1}-\frac{A5}{A3}$

4 Now enter **new** values in cells A1 to A5. Using the formulas entered from question **3**, check the new answers calculated in column B.

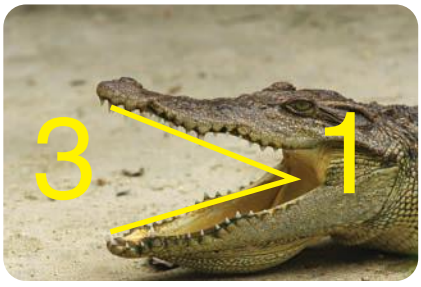
1.03 Ordering integers

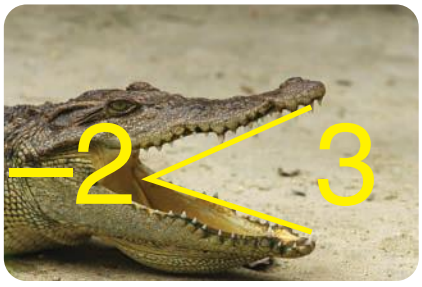
Greater than and less than

- > means 'is greater than'
- < means 'is less than'

Think of the > and < signs as being like the mouths of crocodiles. they always open towards the **bigger** number.

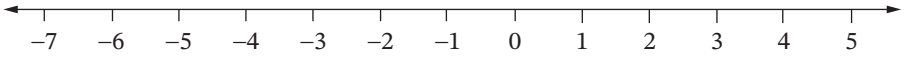
'3 is greater than 1' is written as '3 > 1'. '–2 is less than 3 is written as '–2 < 3.





Shutterstock.com/Andreas Rose

Some more examples are shown in the table below. Remember that on a number line, the numbers on the right are larger than the numbers on the left.



Expression	Meaning	On a number line ...
3 > –4	3 is **greater than** –4	3 is to the **right** of –4
–5 < –3	–5 is **less than** –3	–5 is to the **left** of –3
–6 > –7	–6 is **greater than** –7	–6 is to the **right** of –7
–5 < 2	–5 is **less than** 2	–5 is to the **left** of 2

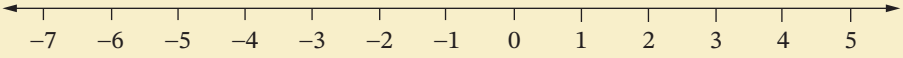
Example 1

Use the symbols > or < to compare each pair of integers.

a 7 ___ –4 **b** –9 ___ –3

SOLUTION

Think about the position of the values on a number line.

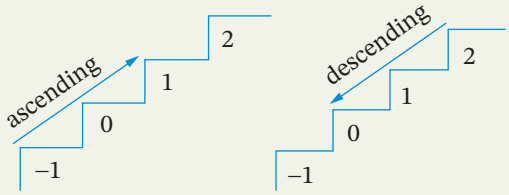


a Reading left to right, 7 is greater than –4, so write: 7 > –4

b Reading left to right, –9 is less than –3, so write: –9 < –3

Ascending and descending

- **Ascending** means moving UP, from smallest to largest: think of 'A' as a ladder and going up a ladder
- **Descending** means moving DOWN, from largest to smallest: think of 'D' for 'down'



Example 2

Write these integers in ascending order: –5, 7, 0, 6, 3, –2, 4, –1.

SOLUTION

Ascending order means from smallest to largest.

–5, –2, –1, 0, 3, 4, 6, 7.

EXERCISE 1.03 ANSWERS ON P. 598

Ordering integers

U F R C

1 Copy and complete each statement using a > or < symbol. EXAMPLE 1

R C

a 4 ____ 1 **b** 0 ____ 9 **c** 7 ____ –7 **d** 0 ____ –3

e 5 ____ –11 **f** –1 ____ –6 **g** –12 ____ 8 **h** –10 ____ –2

i –6 ____ 5 **j** –6 ____ –2 **k** 17 ____ 23 **l** 24 ____ –4

m –15 ____ –35 **n** –27 ____ 29 **o** –8 ____ 47 **p** –12 ____ 0

2 Here is a list of integers: 1, –2, 3, –4, 5, –6, 7, –8. EXAMPLE 2

R C

a Which is the biggest integer?

b Which is the smallest integer?

c Rewrite the integers in order from smallest to biggest.

3 Rewrite each set of integers in ascending order.

R C

a –3, 2, –1, 3 **b** 5, –5, 2, –8, –3

c –4, –6, –3, –10, 0 **d** 6, –3, 4, –2, –5

e –48, 36, –24, 8, 0, –11 **f** 15, 12, –10, –26, 3, –2

Foundation Standard Complex

4 Rewrite each set of integers in descending order.

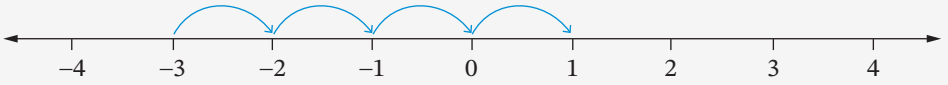
R C

a 4, 3, –1, 5, –2 **b** –4, 8, –7, –2, 0

c 1, –1, 4, –5, –11, –3 **d** 8, –4, –18, 3, –2

e –6, –15, –48, –3, 1 **f** 33, 1, –100, –58, –36

5 The distance between –3 and 1 on a number line is 4.



Find the distance between each pair of integers.

a 2 and 4 **b** –2 and –4 **c** 0 and 4 **d** 0 and –3

e –3 and 3 **f** –10 and 10 **g** –6 and 1 **h** –1 and 6

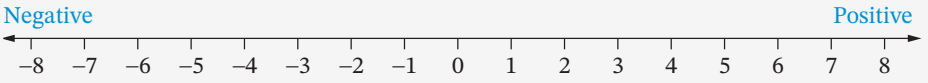
6 Which of the following statements is true? Select the correct answer **A**, **B**, **C** or **D**.

R C

A $-6 > 2$ **B** $4 < -5$ **c** $0 < -3$ **d** $12 > -2$

7 In this question, POS means positive and NEG means negative.

R C



a Start at –3. Move 4 steps in the NEG direction. Move 9 steps in the POS direction. Move 6 steps in the NEG direction. Where are you now?

b Start at 8. Move 12 steps in the NEG direction. Move 5 steps in the POS direction. Move 7 steps in the NEG direction. Where are you now?

c Start at –5. Move 6 steps in the POS direction. Move 4 steps in the NEG direction. Move 7 steps in the POS direction. Where are you now?

d Start at 3. Move 5 steps in the POS direction. Move 8 steps in the NEG direction. Move 2 steps in the NEG direction. Where are you now?

e Start at –1. Move 10 steps in the NEG direction. Move 4 steps in the POS direction. Move 5 steps in the NEG direction. Where are you now?

Foundation Standard Complex

Adding integers

1.04

Integers can be added using a number line.

Video Adding integers

Presentation Adding directed numbers

Puzzle Integers game

Skillsheet Integers using coloured squares

Interactive Number line: Operations

Example 3

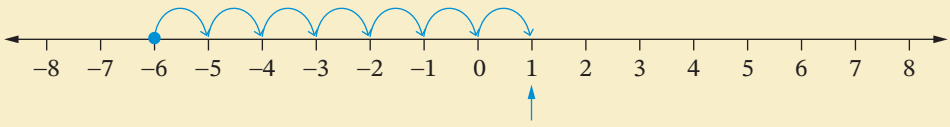
Evaluate each sum.

a $-6 + 7$ **b** $8 + (-6)$ **c** $2 + (-5)$

Evaluate means 'Find the value of'.
Here it means 'to work out the answer'.
A **sum** is the answer to an addition.

SOLUTION

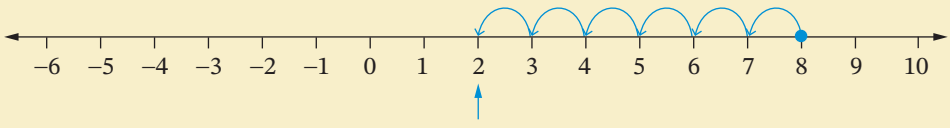
a $-6 + 7$



- -6 is where you start
- $+$ direction is right
- 7 is how far you move

$-6 + 7 = 1$

b $8 + (-6)$



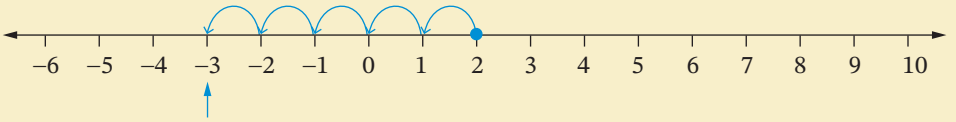
- 8 is where you start
- $+$ direction is right
- -6 is how far you move in the *opposite* direction (that is, left)

$8 + (-6) = 2$

Adding a negative integer is like subtracting its opposite

$8 + (-6)$ is the same as $8 - 6$.

c $2 + (-5)$



- 2 is where you start
- + direction is right
- –5 is how far you move in the *opposite* direction (that is, left)

$2 + (-5) = -3$ $2 + (-5)$ is the same as $2 - 5$.

Adding integers

When adding integers on a number line:

- adding a positive integer means moving to the right
- adding a negative integer is the same as subtracting its opposite, so it means moving to the left: $2 + (-5) = 2 - 5$

EXERCISE 1.04 ANSWERS ON P. 598

Adding integers

U F R C

EXAMPLE 3

1 Evaluate each sum.

a $-6 + 10$	**b** $8 + (-3)$	**c** $4 + (-5)$	**d** $3 + (-1)$
e $-2 + 2$	**f** $-1 + 5$	**g** $-7 + 4$	**h** $-2 + (-2)$
i $-8 + (-12)$	**j** $8 + (-12)$	**k** $-11 + 3$	**l** $10 + (-10)$
m $7 + (-9)$	**n** $-13 + 11$	**o** $-16 + 5$	**p** $-7 + 13$

2 What is the correct answer for $-2 + 7$? Select the correct answer **A**, **B**, **C** or **D**.

A 9 **B** –9 **C** 5 **D** –5

3 **a** Is the answer to $-5 + 9$ the same as the answer to $9 + (-5)$?

R C **b** Can integers be added in any order? Give an example to support your answer.

4 Evaluate each expression.

a $-6 + (-6)$	**b** $-11 + 11$	**c** $-5 + 25$	**d** $-14 + (-9)$
e $23 + (-8)$	**f** $17 + (-17)$	**g** $0 + (-3)$	**h** $-27 + 36$
i $-4 + (-15)$	**j** $9 + (-19)$	**k** $-32 + 0$	**l** $-13 + 4$
m $3 + (-4) + 1$	**n** $-1 + 9 + (-3)$		

5 What happens when you add an integer and its opposite?

R C

Foundation Standard Complex

6 For each sum, state whether it is always positive (+), always negative (–), or could be either positive or negative (E).

R C

a The sum of 2 positive integers

b The sum of 2 negative integers

c The sum of a positive integer and a negative integer

7 Copy and complete the blank for each equation.

R

a $7 + ___ = 3$ **b** $-3 + ___ = 4$ **c** $-3 + ___ = -6$

d $___ + (-8) = -1$ **e** $___ + 5 = 2$ **f** $___ + (-5) = 2$

8 Copy and complete the blanks for each equation, where at least one of the numbers must be negative.

R

a $___ + ___ = 7$ **b** $___ + ___ = -6$ **c** $___ + ___ = 0$

d $___ + ___ = 1$ **e** $___ + ___ = -4$ **f** $___ + ___ = -9$

9 Copy and complete the blanks for each equation (0s not allowed!).

R

a $___ + ___ + ___ = 2$ **b** $___ + ___ + ___ = -1$

c $___ + ___ + ___ = -3$ **d** $___ + ___ + ___ = 0$

☆ MENTAL SKILLS 1A ANSWERS ON P. 599 Maths without calculators

Adding 8 or 9

A quick way to mentally add 9 or 8 to a number is to add 10 and count back 1 or 2 respectively.

Quiz Mental skills 1A

1 Study each example.

a $17 + 9 = 17 + 10 - 1$
$= 27 - 1$
$= 26$

Count: '17, 27, 26'

b $44 + 8 = 44 + 10 - 2$
$= 54 - 2$
$= 52$

Count: '44, 54, 52'

c $128 + 19 = 128 + 20 - 1$
$= 148 - 1$
$= 147$

Count: '128, 148, 147'

d $256 + 38 = 256 + 40 - 2$
$= 296 - 2$
$= 294$

Count: '256, 296, 294'

2 Now evaluate each sum by counting.

a $146 + 9$ **b** $212 + 9$ **c** $308 + 9$ **d** $1755 + 9$

e $29 + 19$ **f** $687 + 39$ **g** $254 + 29$ **h** $933 + 19$

i $714 + 8$ **j** $623 + 8$ **k** $207 + 8$ **l** $155 + 8$

m $386 + 8$ **n** $418 + 48$ **o** $909 + 28$ **p** $277 + 18$

Foundation Standard Complex

1.05 Subtracting integers

Video
Subtracting integers

Presentation
Subtracting directed numbers

Skillsheets
Integers

Integers using coloured squares

Interactives
Number line: Operations

Number line: Distance

Integers can also be subtracted using a number line.

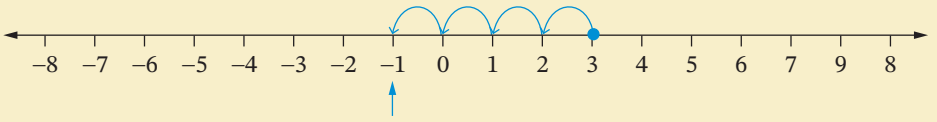
Example 4

Evaluate each **difference**: A difference is the answer to a subtraction.

a 3 – 4 **b** –3 – 4 **c** 4 – (–3) **d** –3 – (–1)

SOLUTION

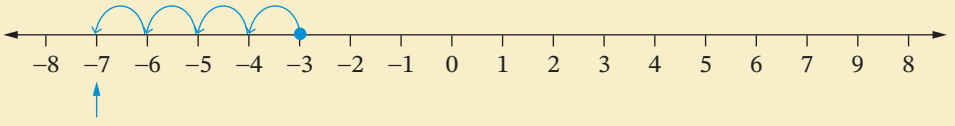
a 3 – 4



- 3 is where you start
- – direction is is left
- 4 is how far you move

3 – 4 = –1

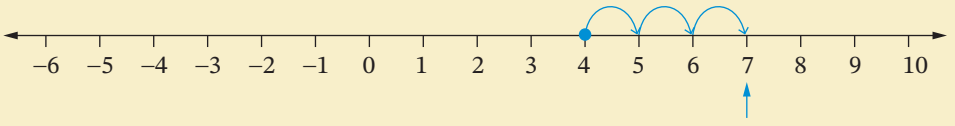
b –3 – 4



- –3 is where you start
- – direction is left
- 4 is how far you move

–3 – 4 = –7

c 4 – (–3)

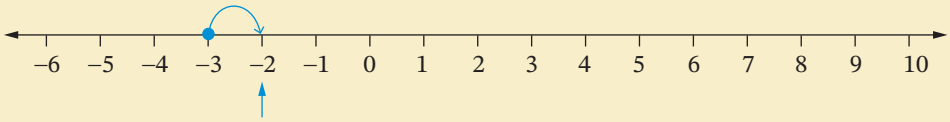


- 4 is where you start
- – direction is left
- –3 is how far you move in the *opposite* direction (that is, right)

4 – (–3) = 7 4 – (–3) is the same as 4 + 3

Subtracting a negative integer is like adding its opposite

d $-3 - (-1)$



- –3 is where you start
- – direction is left
- –1 is how far you move in the *opposite* direction (that is, right)

$-3 - (-1) = -2$ $-3 - (-1)$ is the same as $-3 + 1$

Subtracting integers

When subtracting integers on a number line:

- subtracting a positive integer means moving to the left
- subtracting a **negative** integer is the same as **adding its opposite**, so it means moving to the right: $4 - (-3) = 4 + 3$

Note: '4 – (–3)' is read as '4 minus negative 1', not '4 minus minus 1'.

EXERCISE 1.05 ANSWERS ON P. 599

Subtracting integers

U F R C

EXAMPLE 4

1 Evaluate each difference.

a $4 - 2$	**b** $2 - 4$	**c** $3 - 13$	**d** $5 - (-1)$
e $7 - (-6)$	**f** $-3 - (-2)$	**g** $-5 - (-5)$	**h** $-5 - 5$
i $-6 - 3$	**j** $-4 - (-5)$	**k** $2 - 9$	**l** $4 - (-2)$
m $-6 - (-1)$	**n** $12 - 18$	**o** $9 - (-3)$	**p** $-2 - 7$

2 What is the correct answer for $7 - 12$? Select the correct answer **A**, **B**, **C** or **D**.

A 19 **B** –5 **C** –19 **D** 5

3 **a** Is the answer to $10 - 4$ the same as the answer to $4 - 10$?

R **b** Is $-5 - 9$ the same as $9 - (-5)$?

C **c** Can integers be subtracted in any order?

4 Evaluate each expression.

a $20 - 9$	**b** $9 - 20$	**c** $20 - (-9)$	**d** $-9 - 20$
e $-4 - 27$	**f** $-4 - (-27)$	**g** $13 - (-3)$	**h** $-13 - (-3)$
i $0 - 50$	**j** $0 - (-50)$	**k** $17 - (-25)$	**l** $-17 - 25$
m $16 - 8 - 20$	**n** $-4 - (-10) - 1$		

Foundation Standard Complex

5 What happens when you subtract an integer from its opposite?

R

C

6 For each difference, state whether it is always positive (+), always negative (–), or could be either positive or negative (E).

R

C

a A larger integer minus a smaller integer

b A smaller integer minus a larger integer

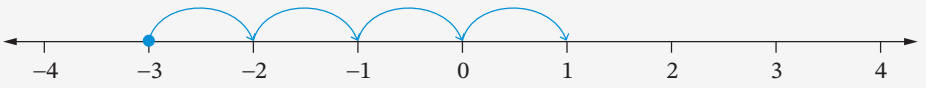
c A positive integer minus a positive integer

d A positive integer minus a negative integer

e A negative integer minus a negative integer

7 The difference between 1 and –3 is also how much 1 is more than –3 on the number line.

R



a Find the difference between 1 and –3.

b Evaluate 1 – (–3).

c Find the difference between 4 and 1.

d Evaluate 4 – 1.

e Find the difference between –1 and –2.

f Evaluate –1 – (–2).

g Find the difference between –1 and 3.

h Evaluate –1 – 3.

i Find the difference between –4 and –2.

j Evaluate –4 – (–2).

8 Copy and complete the blank for each equation.

R

a 4 – ___ = –3

b –1 – ___ = –4

c –2 – ___ = –10

d ___ – (–1) = 9

e ___ – 6 = –5

f ___ – (–4) = –2

9 Copy and complete the blanks for each equation, where at least one of the numbers must be negative.

R

a ___ – ___ = 3

b ___ – ___ = –2

c ___ – ___ = 0

d ___ – ___ = –5

e ___ – ___ = 6

f ___ – ___ = –11

Foundation Standard Complex

INVESTIGATION

Multiplying integers

1 Copy and complete each pattern.

a 9, 6, 3, 0, –3, ___, ___, ___

b 4, 2, 0, ___, ___, ___

c 15, 10, 5, ___, ___, ___

2 Copy and complete each multiplication.

a $3 \times (-7) = (-7) + (-7) + (-7) =$ ____

b $2 \times (-5) = (-5) + (-5) =$ ____

c $3 \times (-4) = (-4) + (-4) + (-4) =$ ____

d $4 \times (-2) = (-2) + (-2) + (-2) + (-2) =$ ____

e $2 \times (-6) =$ ____

f $3 \times (-3) =$ ____

3 **a** In pairs, or as a group activity, copy the multiplication grid below onto a large piece of paper and complete the shaded section.

×	5	4	3	2	1	0	–1	–2	–3	–4	–5
5											
4											
3											
2											
1											
0											
–1											
–2											
–3											
–4											
–5											

b Continue the pattern for the first row: 25, 20, 15, 10, 5, 0, –5, ...

c Continue the pattern for the second row: 20, 16, 12, 8, 4, 0, –4, ...

d Complete the next 4 rows.

e Continue the pattern for each **column**.

4 Use your completed table to evaluate each product.

a $4 \times (-3)$ **b** -3×5 **c** $-4 \times (-2)$ **d** $5 \times (-1)$

e $0 \times (-4)$ **f** -3×3 **g** $2 \times (-4)$ **h** $-5 \times (-5)$

5 How do the signs (positive or negative) of the integers in the question affect the sign of the product (answer)? Try to put the rules for multiplying integers into words and write them down.

1.06 Extension: Multiplying integers

YEAR 8 EXTENSION

Puzzle Multiplying integers game

Worksheet Integers writing activity

Skillsheet Integers

Spreadsheet Integer quiz: Multiplication

Video Multiplying and dividing integers

Multiplying integers

When multiplying integers:

⊗	+	−
+	+	−
−	−	+

positive × positive = positive

positive × negative = negative

negative × positive = negative

negative × negative = positive

or

- If both numbers have the **same** sign, the answer is **positive**.
- If both numbers have **different** signs, the answer is **negative**.

Example 5

Evaluate each product: A **product** is the answer to a multiplication

a -3×2 **b** $-6 \times (-7)$ **c** $4 \times (-4)$ **d** $(-6)^2$

SOLUTION

a $-3 \times 2 = -6$ negative × positive = negative

b $-6 \times (-7) = 42$ negative × negative = positive

c $4 \times (-4) = -16$ positive × negative = negative

d $(-6)^2 = (-6) \times (-6) = 36$ negative × negative = positive

$(-6)^2$ is '−6 squared' and means multiplying −6 by itself

EXERCISE 1.06 ANSWERS ON P. 599

Multiplying integers

U F R C

EXAMPLE 5

1 Evaluate each product.

a -3×6 **b** $6 \times (-4)$ **c** $-3 \times (-7)$

d $4 \times (-8)$ **e** $-7 \times (-9)$ **f** -9×5

g 7×6 **h** $-7 \times (-6)$ **i** $3 \times (-8)$

Foundation Standard Complex

j -6×1 **k** $-1 \times (-1)$ **l** -9×6
m $5 \times (-2)$ **n** -7×6 **o** $-11 \times (-4)$
p -9×9 **q** $-12 \times (-5)$ **r** $20 \times (-3)$
s $-10 \times (-10)$ **t** $-7 \times (-8)$

2 For each expression, select the correct answer **A**, **B**, **C** or **D**.

a $8 \times (-5)$
A 13 **B** −13 **C** −40 **D** 40

b $-7 \times (-3)$
A 21 **B** −21 **C** −10 **D** 10

c $-2 \times 3 \times 7$
A 1 **B** 19 **C** −35 **D** −42

3 Copy and complete each multiplication grid.

a

⊗	−3	5
−3		
5		

b

⊗	−5	5
5		
−5		

c

⊗	6	−4
−4		
6		

d

⊗	−1	1
−1		
1		

e

⊗	7	−3
4		
−6		

f

⊗	−3	−6
8		
2		

4 **a** Is the answer to $6 \times (-4)$ the same as the answer to $(-4) \times 6$?
R **b** Can integers be multiplied in any order?
C

5 What happens when you multiply an integer by its opposite?
R
C

6 Evaluate:
a 3^2 **b** $(-3)^2$ **c** 7^2 **d** $(-7)^2$
e 5^2 **f** $(-5)^2$ **g** 1^2 **h** $(-1)^2$

7 For each product, state whether it is always positive (+), always negative (−), or could be either positive or negative (E).
R
C
a The product of 2 positive integers
b The product of 2 negative integers
c The product of a positive integer and a negative integer
d A positive integer squared
e A negative integer squared

Foundation Standard Complex

YEAR 8 EXTENSION

8 Copy and complete the blank for each equation.

a $2 \times ___ = -10$ **b** $-4 \times ___ = 16$ **c** $9 \times ___ = -27$

d $___ \times (-5) = 35$ **e** $___ \times 8 = -40$ **f** $___ \times (-11) = 22$

9 Copy and complete the blanks for each equation, where at least one of the numbers must be negative.

a $___ \times ___ = 18$ **b** $___ \times ___ = -28$ **c** $___ \times ___ = 30$

d $___ \times ___ = -2$ **e** $___ \times ___ = 6$ **f** $___ \times ___ = -45$

TECHNOLOGY

Daily temperatures in Thredbo

Shutterstock.com/littlree

This spreadsheet shows the minimum and maximum daily temperatures for a ski resort in Thredbo, NSW, for the last 2 weeks of July.

1 Enter the data into a spreadsheet.

	A	B	C	D
1	Date	Min Temp (deg C)	Max Temp (deg C)	Difference
2	Sun 18	-5	4.8	9.8
3	Mon 19	0	4	
4	Tue 20	-8.5	5	
5	Wed 21	-9.5	5.3	
6	Thu 22	-5	4.9	
7	Fri 23	-1.2	7	
8	Sat 24	-4.5	8	
9	Sun 25	-4	6.8	
10	Mon 26	-6.4	7	
11	Tue 27	-8	8.5	
12	Wed 28	-6	7.5	
13	Thu 29	-3.5	7.4	
14	Fri 30	1.6	8.5	
15	Sat 31	3	6	
16				

Foundation Standard Complex

2 In cell D1, enter the label 'Difference'. To find the difference between the maximum and minimum temperatures for Sunday 18, enter the formula '=C2-B2' in cell D2.

	A	B	C	D
1	Date	Min Temp (deg C)	Max Temp (deg C)	Difference
2	Sun 18	-5	4.8	9.8
3	Mon 19	0	4	
4	Tue 20	-8.5	5	
5	Wed 21	-9.5	5.3	
6	Thu 22	-5	4.9	
7	Fri 23	-1.2	7	
8	Sat 24	-4.5	8	
9	Sun 25	-4	6.8	
10	Mon 26	-6.4	7	
11	Tue 27	-8	8.5	
12	Wed 28	-6	7.5	
13	Thu 29	-3.5	7.4	
14	Fri 30	1.6	8.5	
15	Sat 31	3	6	
16				

3 To copy D2's formula into cells D3 to D15, click on D2 and position the cursor over the bottom right-hand corner of this cell so that it becomes a '+' sign. Drag down to cell D15 to highlight cells D3 to D15, as shown on the previous page. This is called **Fill Down** and you should notice that the temperature differences for each day have been calculated. Fill Down can also be selected by highlighting cells D2 to D15 and either selecting Fill Down from the Editing toolbar in the Home menu or by pressing Ctrl-D.

4 a On which day was the largest difference between the maximum and minimum temperatures recorded? Enter your answer in cell E1.

b On which day was the smallest difference recorded? Enter your answer in cell E2.

5 Enter the following labels: 'Lowest Minimum' in cell A17, 'Highest Minimum' in cell A18, 'Lowest Maximum' in D17, 'Highest Maximum' in D18, 'Difference' in A19.

6 a To find the lowest minimum, in cell B17 enter the formula '**=min(B2:B15)**', which finds the lowest number from cells B2 to B15.

b To find the highest minimum, in cell B18 enter '**=max(B2:B15)**'.

c Now, enter appropriate formulas into cells C17 and C18 for the lowest and highest maximums.

d What do you notice about the value in cell B17, compared to the other 3 values?

7 In cell B19 enter '**=B18-B17**' for the difference between the highest and lowest minimums. From cell B19, use **Fill Right** to copy this formula into cell C19. Alternatively, you can highlight cells B19 and C19 and either select Fill Right from the Editing toolbar in the Home menu or press Ctrl-R.

8 a Which column (B or C) had the greatest difference in temperature? Enter your answer in cell E3.

b In cell E4, enter a formula to find the difference between the lowest minimum temperature and the highest maximum temperature.

YEAR 8 EXTENSION

INVESTIGATION

Dividing integers

1 In pairs or as a group activity, copy and complete each question.

a $5 \times ___ = -20$

b $-6 \times ___ = 12$

c $-3 \times ___ = 27$

d $___ \times (-4) = -16$

e $___ \times 2 = -10$

f $___ \times (-5) = 15$

g $-20 \div 5 = ___$

h $27 \div (-3) = ___$

i $-16 \div (-4) = ___$

j $-14 \div 2 = ___$

k $-12 \div (-12) = ___$

l $21 \div (-3) = ___$

2 Write down the rules for dividing integers in words.

1.07 Extension: Dividing integers

Because division is the opposite of multiplication, the rules for dividing integers are the same as those for multiplying them.

Worksheet What is the question?

Skillsheet Integers

Dividing integers

When dividing integers:

÷	+	−
+	+	−
−	−	+

- positive ÷ positive = positive
- positive ÷ negative = negative
- negative ÷ positive = negative
- negative ÷ negative = positive

or

- If both numbers have the **same** sign, the answer is **positive**.
- If both numbers have **different** signs, the answer is **negative**.

Example 6

Evaluate each quotient:

A **quotient** is the answer to a division

a $-15 \div 3$ **b** $18 \div (-2)$ **c** $-24 \div (-4)$ **d** $\frac{14}{-7}$

SOLUTION

a $-15 \div 3 = -5$ negative ÷ positive = negative

b $18 \div (-2) = -9$ positive ÷ negative = negative

c $24 \div (-4) = 6$ negative ÷ negative = positive

d $\frac{14}{-7} = 14 \div (-7) = -2$ positive ÷ negative = negative

YEAR 8 EXTENSION

1.07

Video Multiplying and dividing integers

EXERCISE 1.07 ANSWERS ON P. 599

Dividing integers

U F R C

EXAMPLE 6

1 Evaluate each quotient.

a $20 \div (-5)$ **b** $-8 \div 4$ **c** $-12 \div (-3)$ **d** $6 \div (-2)$

e $27 \div (-3)$ **f** $-18 \div (-6)$ **g** $24 \div (-8)$ **h** $40 \div (-4)$

i $-20 \div 5$ **j** $-21 \div (-3)$ **k** $45 \div (-5)$ **l** $-36 \div 6$

m $-45 \div (-5)$ **n** $-16 \div 2$ **o** $28 \div (-7)$ **p** $48 \div (-8)$

2 For each expression, select the correct answer **A**, **B**, **C** or **D**.

a $-48 \div (-2)$

A 24 **B** −24 **C** 46 **D** −46

b $\frac{16}{-4}$

A 12 **B** −4 **C** 4 **D** −12

3 Evaluate:

a $\frac{12}{-2}$ **b** $\frac{100}{-50}$ **c** $\frac{-18}{-3}$ **d** $\frac{-32}{-2}$

e $\frac{-42}{6}$ **f** $\frac{45}{-5}$ **g** $\frac{100}{5}$ **h** $\frac{-81}{-9}$

4 Divide the top row by the left-hand column to complete each division grid.

a

÷	−6	−3	3	6
−6				
−3				

b

÷	8	−8	32	−32
−2				
4				

Foundation Standard Complex

YEAR 8 EXTENSION

5 What happens when you divide an integer by its opposite?

R

C

6 For each quotient, state whether it is always positive (+), always negative (–), or could be either positive or negative (E).

R

C

a The quotient of 2 positive integers

b The quotient of 2 negative integers

c The quotient of a positive integer and a negative integer

7 Copy and complete the blank for each equation.

a $16 \div ___ = -2$ **b** $-24 \div ___ = -8$ **c** $35 \div ___ = -5$

d $___ \div (-11) = 4$ **e** $___ \div 5 = -10$ **f** $___ \div (-8) = 4$

8 Copy and fill in the blanks for each equation, where at least one of the numbers must be negative.

a $___ \div ___ = 10$ **b** $___ \div ___ = -4$ **c** $___ \div ___ = 5$

d $___ \div ___ = -8$ **e** $___ \div ___ = 6$ **f** $___ \div ___ = -3$

1.08 Extension: Order of operations

Now that you can add, subtract, multiply and divide with integers, you can evaluate **mixed expressions** involving integers. Remember to follow the '**order of operations**' rules.

Videos Order of operations

BODMAS

Skillsheet Order of operations

Puzzles Order of operations

Directed numbers

Order of operations

1. Evaluate any expression inside **grouping symbols** (brackets)
2. Evaluate any multiplication (×) and division (÷), from left to right
3. Evaluate any addition (+) and subtraction (–), from left to right

Example 7

Evaluate each expression.

a $3 \times (-4 - 6)$ **b** $8 + (-7) \times 4 - (-2)$

SOLUTION

a $3 \times (-4 - 6) = 3 \times (-10)$ brackets first: $-4 - 6 = -10$

$= -30$

b $8 + (-7) \times 4 - (-2) = 8 + (-28) - (-2)$ multiplication first: $(-7) \times 4 = -28$

$= -20 - (-2)$ left to right: $8 + (-28) = -20$

$= -18$ $-20 - (-2) = -20 + 2$

Foundation Standard Complex

EXERCISE 1.08 ANSWERS ON P. 599

Order of operations

U F R C

1 Evaluate each expression.

a $9 - 12 + 10$ **b** $-13 - 10 - (-1)$ **c** $4 - 6 + 2 + (-1)$
d $1 - 6 + 7 - 9$ **e** $6 + (-3) - 9$ **f** $7 - 5 - (-4) + (-2)$
g $5 \times (-10) \div 2$ **h** $6 \times (-4) \div (-3)$ **i** $7 \times 6 \div (-2)$
j $-3 \div (-1) \times 4$ **k** $24 \div (-6) \times 50$ **l** $-56 \div (-8) \times 4$
m $-100 \div (-20) \times 3$ **n** $-5 \times (-9) \div 9$ **o** $2 \times 12 \div (-4) \div (-2)$

2 What is the value of $6 + 4 \times (-2)$? Select the correct answer **A**, **B**, **C** or **D**.

A 14 **B** −20 **C** −2 **D** −14

3 Evaluate each expression.

a $-5 \times 2 + 3$ **b** $-5 \times (2 + 3)$ **c** $3 \times 6 \times (-2)$
d $12 - 8 \div 4$ **e** $12 - 8 \div (-4)$ **f** $-3 \times (9 - 10)$
g $21 \div (-7) + 8$ **h** $[-6 - (-2)] \times (-4)$ **i** $(-6 - 2) \div (-4)$
j $-3 \div 3 + 9$ **k** $12 \times (6 - 7) + 1$ **l** $-4 \times 7 \div (-7 - 7)$

4 Copy and complete the blank for each equation.

R **a** $-5 \times 6 - ___ = -38$ **b** $___ \times (6 - 8) = -10$
c $[8 - ___] \times 4 = 44$ **d** $5 + ___ \times (-7) = 19$
e $8 - (-3 \times ___) = 17$ **f** $___ \times (-5 + 3) = -24$
g $6 \times ___ + (-4) = -16$ **h** $15 \div (-3) \times ___ = 10$
i $___ + 4 \times 6 = 5$ **j** $[-6 - ___] \times (-2) = -6$
k $(-6 - 3) \div ___ = -3$ **l** $___ \div (-5) + 6 = 1$

5 Nuraan completed these 10 questions. Mark her work and correct any mistakes.

a $6 - 9 + 5 - 11 = -9$ **b** $-32 \div (-4) \times (-2) = 4$
c $-6 \times (-2) - 8 = -4$ **d** $-11 + 11 \div (-1) = -22$
e $24 \div (-4) \div (-2) = 3$ **f** $(-2 + 6) \times 2 = -8$
g $-7 - 6 + 10 = 3$ **h** $(-12 - 3) \div (-3) = 3$
i $12 - 3 \times 5 = 45$ **j** $2 - 9 \times (-3) - (-18) = 39$

6 Copy each equation and insert grouping symbols to make it true.

R **a** $-4 \times 7 - 2 + (-3) = -8$ **b** $-4 \times 7 - 2 + (-3) = -27$
C **c** $-4 \times 7 - 2 + (-3) = -33$ **d** $40 \div 10 \times (-2) + 15 = 13$
e $40 \div 10 \times (-2) + 15 = 52$ **f** $40 \div 10 \times (-2) + 15 = -8$

Foundation Standard Complex

YEAR 8 EXTENSION

7 Evaluate $-24 \div 3 + (-4 + 7) \times (-2)$. Select **A**, **B**, **C** or **D**.

A 10 **B** 8 **C** −14 **D** −26

8 Copy each equation and use all the integers 4, −8 and 3 and grouping symbols to complete the equation.

R C

a _____ = −20 **b** _____ = 15

c _____ = 8

Quiz Mental skills 1B

☆ MENTAL SKILLS 1B ANSWERS ON P. 600 Maths without calculators

Subtracting 8 or 9

A quick way to mentally subtract 9 or 8 from a number is to subtract 10 and count forward 1 or 2 respectively.

1 Study each example.

a $66 - 9 = 66 - 10 + 1$
$= 56 + 1$
$= 57$

Count: '66, 56, 57'

b $83 - 8 = 83 - 10 + 2$
$= 73 + 2$
$= 75$

Count: '83, 73, 75'

c $72 - 49 = 72 - 50 + 1$
$= 22 + 1$
$= 23$

Count: '72, 22, 23'

d $141 - 28 = 141 - 30 + 2$
$= 111 + 2$
$= 113$

Count: '141, 111, 113'

2 Now evaluate each difference by counting.

a 26 − 9 **b** 44 − 9 **c** 123 − 9 **d** 270 − 9

e 161 − 29 **f** 187 − 59 **g** 75 − 19 **h** 457 − 39

i 82 − 8 **j** 131 − 8 **k** 96 − 8 **l** 120 − 8

m 44 − 8 **n** 577 − 28 **o** 203 − 18 **p** 365 − 48

Foundation Standard Complex

Integers and the calculator

1.09

Negative numbers can be entered into a calculator using the sign change key (–) or +/–.
For example, to enter '–3', type (–) 3 or +/– 3.

Example 8

Use your calculator to evaluate:

a $798 \div (-7) + 135$ **b** $23 \times [10 - (-6)]$ **c** $(-18)^2$

SOLUTION

a $798 \div (-7) + 135 = 21$ On a calculator, enter 798 ÷ (–) 7 + 135 =

b $23 \times [10 - (-6)] = 368$ Enter 23 × (10 – (–) 6) =

c $(-18)^2 = 324$ Enter ((–) 18) x^2 =

There are more examples in Calculator Skills at the back of this book on page 592.

EXERCISE 1.09 ANSWERS ON P. 600

Integers and the calculator

U F R C

1 Evaluate each expression *without* using a calculator.

a $6 - 13$ **b** $4 - (-2)$ **c** $-7 + 3$ **d** $-5 - (-6)$
e $3 + (-8)$ **f** $11 - (-3)$ **g** $11 - (-15)$ **h** $-13 + (-15)$
i $12 - (-18)$ **j** -8×6 **k** $-11 \times (-4)$ **l** $-5 \times (-6)$
m $13 \times (-5)$ **n** $88 \div (-11)$ **o** $-30 \div (-2)$ **p** $-36 \div 9$
q $(-5)^2$ **r** $(-8)^2$ **s** $-3 \times (-4)^2$ **t** $18 \div (-3)^2$

2 Check your answers to question **1** using a calculator.

3 When using your calculator, why are there different answers for $(-5)^2$ and -5^2? Explain the difference in meaning between $(-5)^2$ and -5^2.

R C

4 Use a calculator to evaluate each expression.

a $13 + (-22) - 9$ **b** $64 + 23 + (-16)$ **c** $-200 \div (-100) \div 2$
d $-40 \times (-3) \div (-20)$ **e** $-44 \times (-15) \times (-6)$ **f** $48 \div 3 \times (-12)$
g $125 \div (-5) \div (-5)$ **h** $[12 - (-10)] \times (-3) + 6$ **i** $-3 \times (-8) \div (-4) + (-11)$
j $9 - 19 - 8 \times (-6)$ **k** $-12 - 30 \div (-2) + 58$ **l** $6 + 4 \times (-5) \div (-2)$

YEAR 8 EXTENSION

EXAMPLE 8

YEAR 8 EXTENSION

Foundation Standard Complex

YEAR 8 EXTENSION

5 Copy and complete the blank for each equation.

R **a** $6 \times (___ - 27) = -84$ **b** $___ - 8 - (-14) = -4$ **c** $45 \div (-9) \times ___ = -10$

C **d** $-2 \times ___ + 11 = -19$ **e** $___ \times (-4) \div 2 = 112$ **f** $-14 - 6 \times ___ = 34$

6 Copy each equation and use all the integers 75, –18 and –15 and grouping symbols to complete the equation.

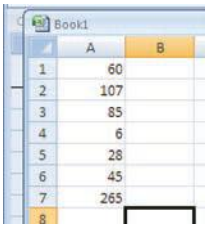
a _____ = –1395 **b** _____ = –13 **c** _____ = –855

TECHNOLOGY

Sorting numbers

In this activity, we will sort this set of numbers {60, 107, 85, 6, 28, 45, 265} by using a spreadsheet.

1 **a** Enter the 7 numbers shown below, in the given order, into column A of a spreadsheet.



b Highlight cells A1 to A7 and select **Sort & Filter** from the Editing menu on the top right-hand side. Choose the option shown for ascending order: **Smallest to Largest**.

c The data should now be sorted from the smallest number, 6 (in cell A1), to the largest number, 265 (in cell A7).

d A set of numbers can also be sorted in descending order: choose **Largest to Smallest**.

2 Now enter each of the following sets of numbers in the columns given, and sort in ascending order.

a {55, 89, 36, 21, 19, 4, 95} in column B

b {–17, 14, –90, 58, 0, 392, –6} in column C

c {4987, –4200, 8740, –5176, 2601, 713, –4810} in column D

d {16 101, 12 167, 10 010, 11 412, 10 107, 10 761, 11 214} in column E

3 Sort each column of data from question **2** in descending order.

Foundation Standard Complex

Integer problems

1.10

Example 9

The average temperature on the planet Mars is –63°C, while the average temperature on Jupiter is –150°C. What is the difference between these temperatures?

SOLUTION

Write a number sentence to calculate the answer.

$-63 - (-150) = -63 + 150$

$= 87$

The difference in temperature is 87°C.

Puzzle
Integers group clues

EXERCISE 1.10 ANSWERS ON P. 600

Integer problems

U F PS R

For each question, write a number sentence to calculate the answer.

EXAMPLE 9

1 PS In Canberra, the temperature reached a maximum of 11°C. Overnight, it dropped to a minimum of –2°C. How many degrees did it drop?

2 PS Craig walked 16 kilometres east and then 20 kilometres west. How far is he from his starting point?

3 PS The heights above and below sea level of some places around the world are shown:

The Dead Sea (Jordan)	–397 metres
Death Valley (USA)	–86 metres
Mt Kosciuszko (Australia)	2230 metres
Mt Everest (Nepal)	8840 metres

a How much higher is Mt Everest than Mt Kosciuszko?

b How much higher is Mt Kosciuszko than Death Valley?

c How much lower is the Dead Sea than Death Valley?

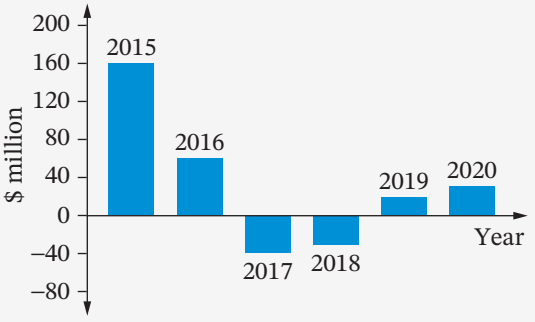
d The highest point on Earth is Mt Everest and the lowest point is the Dead Sea. What is the difference in height between these two points?

Foundation Standard Complex

4 This graph shows the yearly profits made by the Integral company over 6 years.

R

Integral yearly profits



a In which years did Integral make a profit?

b In which years did Integral make a loss?

c What was the decrease in profit between 2016 and 2017?

d Did the company make an overall profit or loss over the 6 years shown? How much profit or loss did they make?

5 Anya left home and walked 3 kilometres south to Brock's home. Together, they then walked north for 4 kilometres. What distance and direction are they from Anya's home now?

PS

6 Karina had $74 in her bank account yesterday but was allowed to withdraw $121. Today, she deposited $40 into the account. What is her account balance now?

PS

7 At Mt Hotham, the temperature overnight dropped to −6°C. During the day it rose by 13°C, but then dropped 5°C after sunset. Which expression describes the temperature in Cooma after sunset? Select the correct answer **A**, **B**, **C** or **D**.

A −6 + 13 − 5 **B** −6 − 13 − 5

C −6 + 13 + 5 **D** −5 − 13 − 6

8 In indoor cricket, each time a batting player gets out, the team's score is reduced by 5 (that is, −5 is added to the total). Liam's cricket team is midway through a match. The first 5 batters have each scored 23, 16, 19, 0 and 16 before getting out. Liam is still batting and has scored 10 so far. What is the current score for Liam's team?

PS

9 Mt Etna is an active volcano in Italy, 3200 m high. The lava starts 652 m below the Earth's surface and shoots to a height of 750 m above the top of Mt Etna. How far does the lava travel?

PS

Shutterstock.com/Tomarchio Francesco

Foundation Standard Complex

10 A bird dives into a river from a height of 320 cm above the water, travelling at an average speed of 60 cm per second. PS

a How far does the bird travel in 8 seconds?

b How far below the water will it be after 8 seconds?

11 The submarine Nemesis descends from the surface to a depth of 955 metres to inspect a shipwreck. It then ascends 800 metres to send a message. How far below the surface is it when it sends the message? PS

12 Find the closing balance of this bank account after all deposits and withdrawals. PS R

Starting balance	$37.85
Deposit	$18.20
Deposit	$120.00
Withdraw	$45.00
Deposit	$8.15
Withdraw	$248.20

13 A scuba diver was swimming at a depth of 18 metres. Her diving friend was at a depth of 7 metres. What was the vertical distance between them? PS

14 Jayden's bank balance is –$120, but he deposits $9 each day for the next 20 days.

PS **a** On which day will his bank balance become positive?

R **b** What will be his bank balance after 20 days?

INVESTIGATION

Above and below sea level

1. What is sea level?
2. Where is the highest point in Australia? What is its height above sea level?
3. Where is the lowest point in Australia? What is its height below sea level?
4. What is the difference in height between the highest and lowest points in Australia? Is it more than a kilometre?
5. What is the height of your town or city above sea level?
6. Find 5 places in the world that are above sea level and five that are below sea level. Write four questions about this information for other students to answer. Swap your work with other students and answer their questions.

Note: you must know the answers to your questions so you can mark other students' work!

Foundation | Standard | Complex

POWER PLUS ANSWERS ON P. 600

1 Copy and complete each pattern.

a $(-1)^2 = (-1) \times (-1) =$ ___

b $(-1)^3 = (-1) \times (-1) \times (-1) =$ ___

c $(-1)^4 = (-1) \times (-1) \times (-1) \times (-1) =$ ___

d $(-1)^5 =$ _____ = ____

e $(-1)^6 =$ _____ = ____

f $(-1)^7 =$ _____ = ____

2 Explain how $(-1)^n$, where n is a whole number, is equal to only one of 2 possible numbers and which one it is depends on the type of number n is.

3 Find the number (or numbers) that make each of these expressions true.

a ____$^2 = 16$ **b** ____$^2 = 100$ **c** ____$^3 = -8$

4 Dylan says there is no answer to ____$^2 = -25$. Is he correct?

5 **a** Find 3 integers that have a sum of –7.

b Find 2 negative integers that have a sum of –1.

c Find 3 integers that have a product of 36.

d Find 3 integers that have a product of –24.

6 Find the average of:

a –8 and 12 **b** –1 and –17 **c** 21 and –19

7 If a and b are integers, determine whether the following equations are true or false.

a $a + b = b + a$ **b** $a - b = b - a$ **c** $a \times b = b \times a$

d $a \div b = b \div a$ **e** $a + (-a) = 0$ **f** $(-a) \times (-a) = a \times a$

g $a - (-a) = 0$ **h** $a \div (-a) = -1$ **i** $(-a) + (-a) = -2 \times a$

8 Find 2 integers that have:

a a sum of –3 and a product of –10

b a sum of –8 and a product of 15

c a sum of –2 and a product of –8

d a sum of –1 and a product of –6

e a sum of 7 and a product of –30

1 CHAPTER REVIEW

Language of maths

Quiz
Language of maths 1

ascending	degrees (°)	deposit	descending
difference	evaluate	greater than (>)	integer
less than (<)	minus	negative	number line
opposite	order	positive	sign
sum	whole number	withdraw	zero

1 Find pairs of words that are opposites.

2 What number is neither positive nor negative?

3 Write sentences using the words 'ascending' and 'descending' to show their meanings.

4 Write '$-5 < -1$' in words.

5 Is 0 an integer?

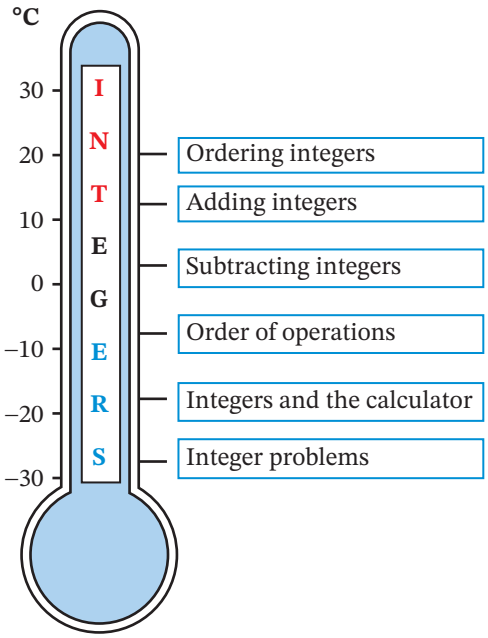
6 What is the difference between –4 ('negative 4') and – 4 ('minus 4')?

Topic summary

Worksheet
Mind map: Integers

- Using your own words, write down what integers are.
- Write any rules you have learnt about integers.
- What parts of this topic did you find difficult? What parts didn't you understand? Discuss them with a friend or your teacher.
- Give some examples of where negative numbers are used.

Print (or copy) and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.



1 TEST YOURSELF

ANSWERS ON P. 600

1.01

Quiz
Test yourself 1

1 Write the opposite of each integer.

a −2 **b** −23 **c** 56
d −10 **e** 8 **f** 0

1.02

2 Use a separate number line to mark each set of numbers.

a 6, −3, 5, 1, −1, 4, 7
b integers between −2 and 5
c integers less than 1 and greater than −6

1.03

3 Write true (T) or false (F) for each statement.

a $-4 > 2$ **b** $-2 < -3$ **c** $-1 > 1$ **d** $2 > -1$
e $-11 < -10$ **f** $-3 > -4$ **g** $-8 < 11$ **h** $-4 > -3$
i $5 < -4$ **j** $-16 < 0$ **k** $1 > -5$ **l** $-9 > -2$

1.04

4 Write this set of integers in descending order:
−9, 14, 3, 0, −5, −10, −1.

1.04

5 Evaluate each expression.

a $3 + (-4)$ **b** $-8 + (-2)$ **c** $10 + (-13)$
d $-4 + 8$ **e** $-5 + (-5)$ **f** $-3 + 6$
g $-5 + (-4) + (-2)$ **h** $-8 + 5 + 4$ **i** $-7 + 6 + (-7)$

1.05

6 Evaluate each expression.

a $3 - (-4)$ **b** $-5 - (-9)$ **c** $7 - 11$
d $-4 - 8$ **e** $-5 - (-5)$ **f** $-3 - 4 - 5$
g $-8 + 5 - 4$ **h** $8 - 6 - (-4)$ **i** $3 - 8 + (-6)$

YEAR 8 EXTENSION

1.06

7 Evaluate each product.

a $6 \times (-3)$ **b** $-8 \times (-6)$ **c** -2×5
d -11×4 **e** $7 \times (-9)$ **f** 6×8
g $-6 \times (-4) \times (-1)$ **h** $-2 \times 3 \times 7$ **i** $-4 \times 2 \times (-5)$

Foundation Standard Complex

9780170465557

YEAR 8 EXTENSION

8 Evaluate each quotient. 1.07

a $-12 \div (-4)$ **b** $15 \div (-5)$ **c** $-6 \div (-6)$

d $16 \div (-4)$ **e** $-24 \div 12$ **f** $-1000 \div 10$

g $24 \div (-6) \div (-2)$ **h** $-36 \div 3 \div 4$ **i** $-64 \div (-4) \div 2$

9 Evaluate each expression. 1.08

a $-6 + (-3) \times 2$ **b** $4 - 3 \times 6$ **c** $-12 \div 4 - 10$

d $-3 \times 5 + 6 \div (-2)$ **e** $30 \div (-6) + 5$ **f** $-2 \times 7 + 15 \div (-3)$

g $-12 + (-3) \times (-7) - 2$ **h** $14 + 7 \times (-3) + 5$ **i** $-6 + (-6) \div (-2) \times 2$

10 Use your calculator to evaluate each expression. 1.09

a $28 - 37$ **b** $-17 + 11$

c $18 - 15 \times 2$ **d** $100 - 120 \div 4 + 17$

e $-16 \times 3 + (-4)$ **f** $[5 + (-20)] \div (-3)$

11 The minimum overnight temperature in Stanthorpe, QLD, was –3°C. During the day, it rose to 13°C. By how much did the temperature rise? 1.10

12 Harrison starts a new job and opens a new bank account. Each week he saves \$80 to put into the account. Each month, however, \$210 is withdrawn from the account to pay off a car loan. Calculate how much will be in Harrison's account after 6 months if there are 26 weeks in 6 months. 1.10

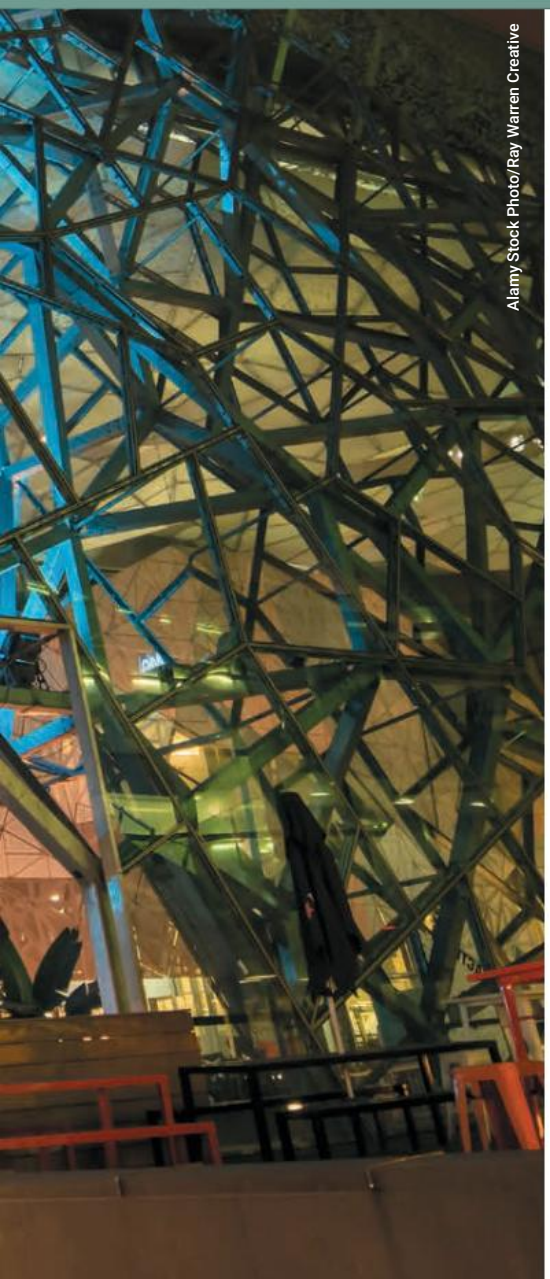
Foundation Standard Complex

2

MEASUREMENT

Angles

Angles are used in all aspects of life. Builders and engineers use angles for buildings, roads and bridges. Sportspeople use angles when playing snooker, when shooting for goals and even when skiing. Artists use their knowledge of angles to draw portraits and paintings.



Alamy Stock Photo/Ray Warren Creative

Chapter outline

		Proficiencies				
2.01	Naming angles	U	F		R	C
2.02	Measuring and drawing angles	U	F		R	C
2.03	Types of angles	U	F		R	C
2.04	Complementary and supplementary angles	U	F		R	C
2.05	Angles at a point and vertically opposite angles	U	F		R	C
2.06	Parallel and perpendicular lines	U	F		R	C
2.07	Corresponding angles on parallel lines	U	F		R	C
2.08	Alternate angles on parallel lines	U	F	PS	R	C
2.09	Co-interior angles on parallel lines	U	F	PS	R	
2.10	Angles on parallel lines	U	F	PS	R	
2.11	Proving parallel lines	U	F	PS	R	

U = Understanding
F = Fluency
PS = Problem solving
R = Reasoning
C = Communication

Wordbank

alternate angles Pairs of angles on opposite sides of the transversal crossing 2 lines, between the lines

co-interior angles Pairs of angles on the same side of the transversal crossing 2 lines, between the lines

complementary angles 2 angles that add to 90°

corresponding angles Pairs of 'matching' angles formed when a transversal crosses 2 lines

obtuse angle An angle whose size is between 90° and 180°

parallel lines Lines that point in the same direction and do not intersect

perpendicular lines Lines that intersect at right angles

supplementary angles 2 angles that add to 180°

transversal A line that cuts across 2 or more other lines

Quiz
Wordbank 2

Videos (5):

2.02 Naming angles
2.04 Measuring angles
2.05 Angles at a point and vertically opposite angles
2.07, 2.08, 2.09 Angle relationships
2.10 Angles on parallel lines

Quizzes (5):

- Wordbank 2
- SkillCheck 2
- Mental skills 2
- Language of Maths 2
- Test Yourself 2

Skillsheets (3):

2.02 Measuring angles
2.03 Types of angles
2.07, 2.08, 2.09 Angles and parallel lines

Worksheets (9):

2.01 A page of angles
2.02 A page of protractors • Using protractors • A page of angles
2.03 A page of angles
2.05 Straight angles • Right angles and revolutions
2.06 A page of intervals
2.07, 2.08, 2.09 Investigating angles on parallel lines
2.10 What is the diagram? • Matching angles
Mind map: Angles

Puzzles (3):

2.01, 2.02, 2.03 Angle cards
2.05 Angles: a dog day • Angles

Presentation (1):

2.02 Measuring angles with a protractor

Nelson MindTap

To access resources above, visit **cengage.com.au/nelsonmindtap**

2 In this chapter you will:

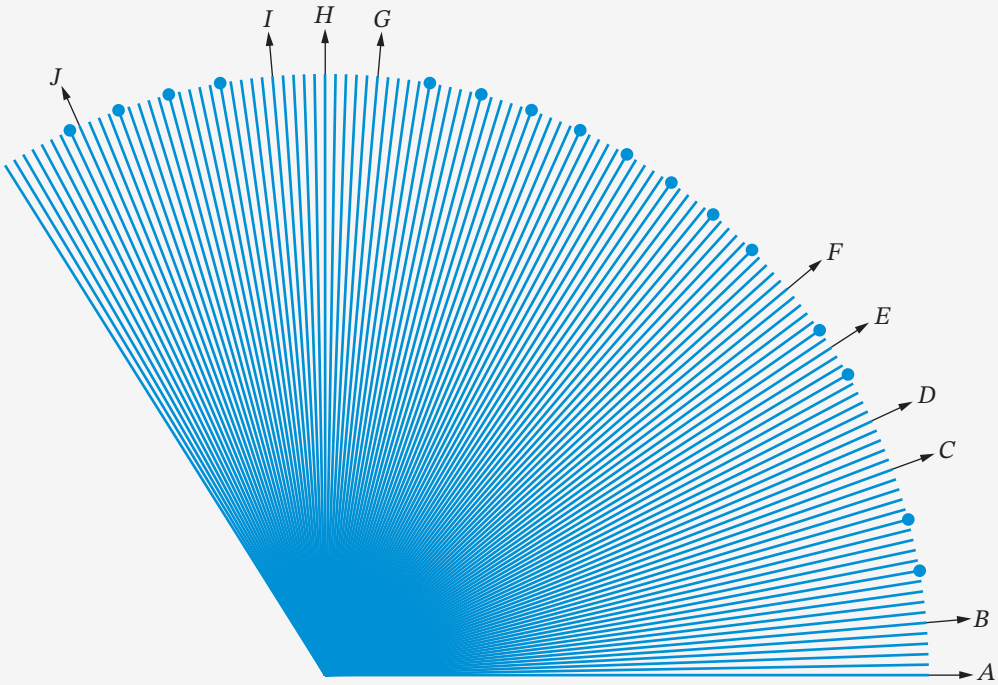
- ✓ use 1 or 3 letters to name and label an angle
- ✓ estimate and measure the size of an angle
- ✓ classify angles such as acute, right, obtuse, straight, reflex and revolution
- ✓ name pairs of angles that are adjacent, complementary, supplementary and vertically opposite
- ✓ solve problems involving angles on a straight line, angles at a point and vertically opposite angles
- ✓ identify corresponding, alternate and co-interior angles when parallel lines are crossed by a transversal, and their properties
- ✓ use the above properties to solve geometry problems and to prove that 2 lines are parallel
- ✓ identify and draw parallel and perpendicular lines

SkillCheck

ANSWERS ON P. 600

Quiz
SkillCheck 2

All questions refer to this angle diagram, where each gap represents 1° of angle size.



1 What is the angle size between the lines *I* and *J*? Select the correct answer **A**, **B**, **C** or **D**.

A 19° **B** 29° **C** 34° **D** 39°

2 What is the angle size between the lines labelled:

a *A* and *C*? **b** *A* and *D*? **c** *B* and *C*?
d *C* and *F*? **e** *A* and *F*? **f** *B* and *G*?
g *D* and *G*? **h** *E* and *H*? **i** *D* and *I*?
j *C* and *J*? **k** *B* and *E*? **l** *E* and *J*?

3 Find one pair of labelled lines that have a 45° angle between them.

4 Find 2 pairs of labelled lines that have a 90° angle between them.

5 Find the pair of labelled lines that have each angle size between them.

a 7° **b** 8° **c** 13° **d** 28°
e 50° **f** 89° **g** 95° **h** 114°

2.01 Naming angles

Worksheet
A page of angles

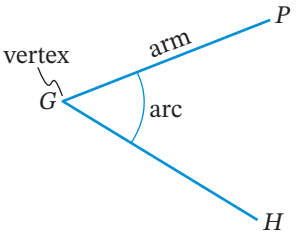
Puzzle
Angle cards

Video
Naming angles

An **angle** describes the size of a turn or rotation. It is drawn with 2 **arms** that meet at a **vertex**. The size of the turn is normally marked with a curve called an **arc**. The angle marked in this diagram can be written as:

$\angle G$, $\angle PGH$ or $\angle HGP$.

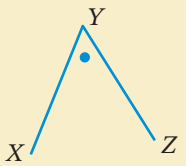
The middle letter always labels the vertex of the angle.



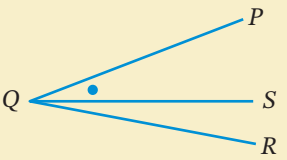
Example 1

Name the angle marked with • in each diagram.

a



b



SOLUTION

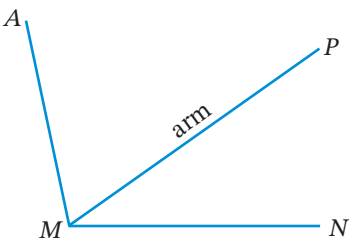
a $\angle Y$ or $\angle XYZ$ or $\angle ZYX$

b $\angle PQS$ or $\angle SQP$

Note: We cannot use $\angle Q$ in part **b** because it is not clear which angle that means. There are 3 different angles whose vertex is $\angle Q$. They are $\angle PQS$, $\angle SQR$ and $\angle PQR$.

Adjacent angles

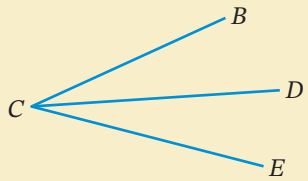
In this diagram, $\angle AMP$ and $\angle PMN$ share a common arm, PM. Angles that are next to each other in this way are called **adjacent angles**.



9780170465557

Example 2

What angle is adjacent to $\angle DCE$?



SOLUTION

$\angle BCD$ and $\angle DCE$ share a common arm CD. So $\angle BCD$ is adjacent to $\angle DCE$.

Video Naming angles

2.01

EXERCISE 2.01 ANSWERS ON P. 601

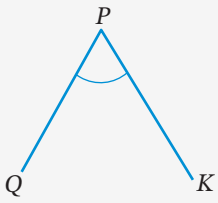
Naming angles

U F R C

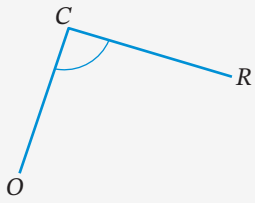
EXAMPLE 1

1 Name each angle in 2 different ways.

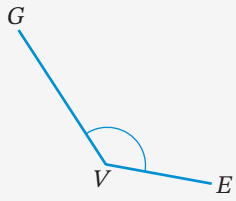
C **a**



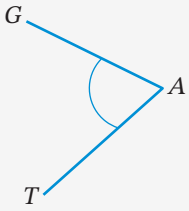
b



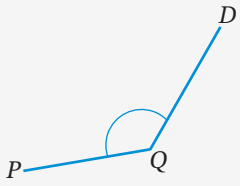
c



d



e



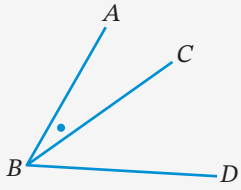
f

2 How can the angle marked • be named?

C Select the correct answer **A**, **B**, **C** or **D**.

A $\angle ABD$ **B** $\angle CBD$

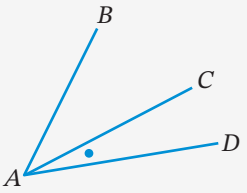
C $\angle ABC$ **D** $\angle BCA$



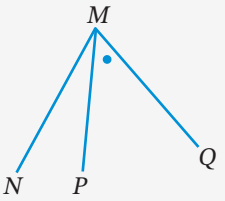
Foundation Standard Complex

3 Name the angle marked • in each diagram.

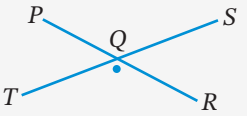
C **a**



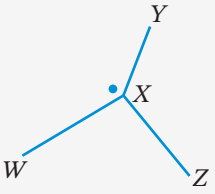
b



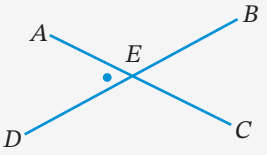
c



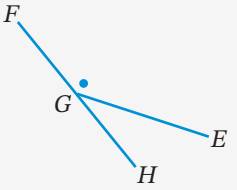
d



e



f



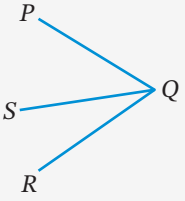
4 Draw each angle named, labelling it correctly.

a $\angle POT$ **b** $\angle TAF$ **c** $\angle AFE$ **d** $\angle H$

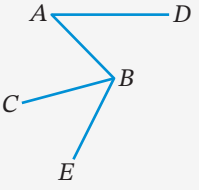
EXAMPLE 2

5 Name the angle adjacent to:

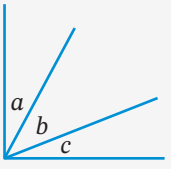
C **a** $\angle RQS$



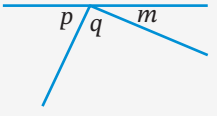
b $\angle ABC$



c the angle marked c



d the angle marked p



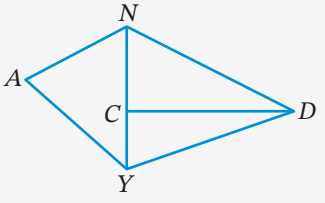
6 **a** There are 13 different angles on this diagram. Name them all.

R

C **b** Name all 3 angles that have C as their vertex.

c What type of angle is $\angle NCY$?

d Name the angle that is adjacent to $\angle YND$.

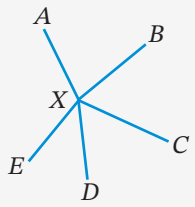


Foundation Standard Complex

7 Which angle is adjacent to $\angle AXB$? Select the correct answer **A**, **B**, **C** or **D**.

A $\angle BXC$ **B** $\angle DXE$

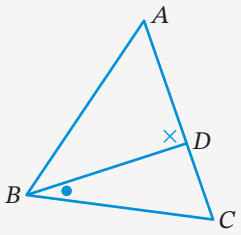
C $\angle DXC$ **D** $\angle CXE$



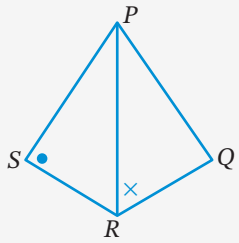
8 Name the angles marked • and × in each diagram.

C

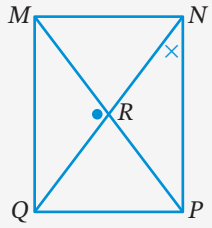
a



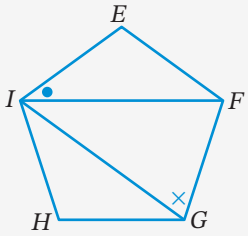
b



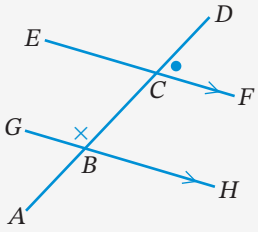
c



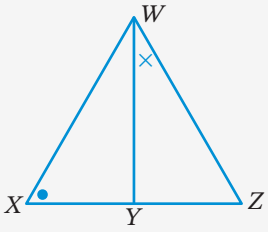
d



e



f



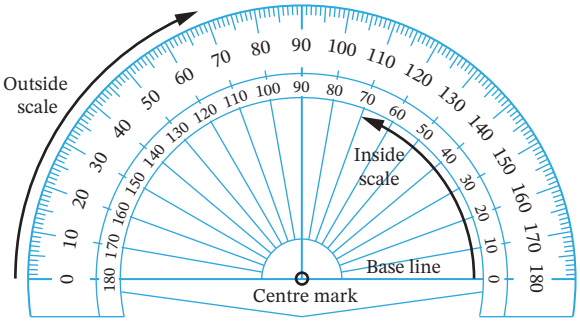
9 For each diagram in question **8**, name a pair of adjacent angles.

R

C

Measuring and drawing angles 2.02

A **protractor** is an instrument used for measuring and drawing angles.



Video Measuring angles

Skillsheet Measuring angles

Presentation Measuring angles with a protractor

Foundation Standard Complex

Worksheets
A page of protractors

Using a protractor

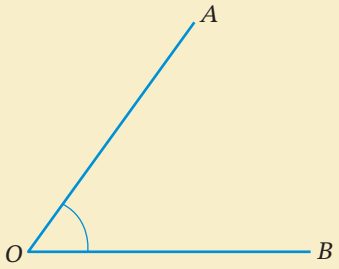
A page of angles

Puzzle
Angle cards

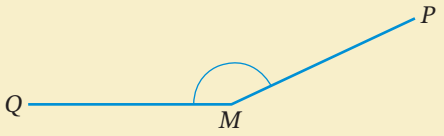
Example 3

Use a protractor to measure the size of each marked angle.

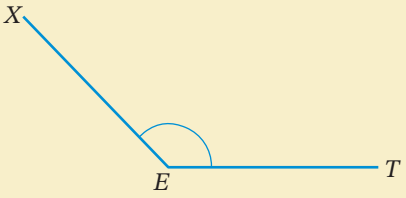
a



b



c

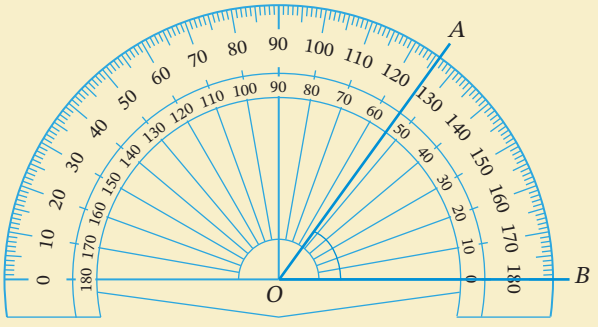


SOLUTION

a Measuring $\angle AOB$:

- Line up OB with the base line of the protractor.
- Place the centre mark over the vertex, O.
- The angle is smaller than 90°.
- Use the inside scale, counting from 0°.

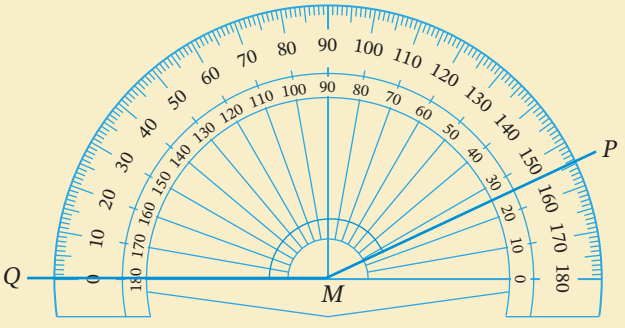
$\angle AOB = 54°$



b Measuring $\angle PMQ$:

- Line up QM with the base line of the protractor.
- Place the centre mark over the vertex, M.
- The angle is greater than 90°.
- Use the outside scale, counting from 0°.

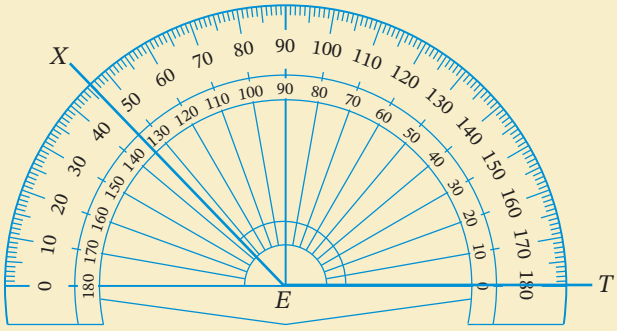
$\angle PMQ = 155°$



c Measuring $\angle TEX$:

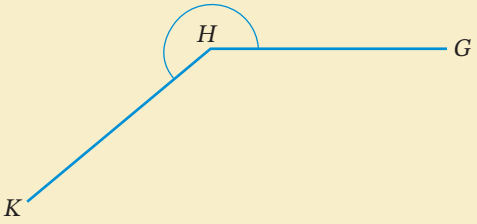
- Line up TE with the base line of the protractor.
- Place the centre mark over the vertex, E.
- $\angle TEX$ is bigger than 90°.
- Use the inside scale.

$\angle TEX = 134°$



Example 4

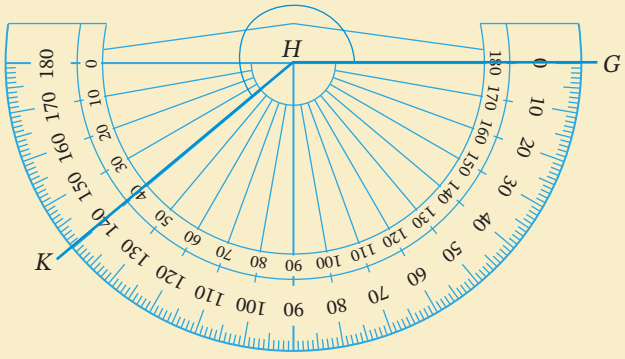
Measure the **reflex angle** GHK.



SOLUTION

As the protractor only measures up to 180°, turn it upside down to measure the smaller adjacent angle underneath first.

Obtuse $\angle GHK = 140°$



To calculate the size of reflex $\angle GHK$, subtract 140° from 360° as there are 360° in a revolution.

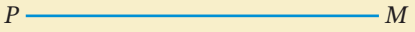
Reflex $\angle GHK = 360° - 140° = 220°$

Example 5

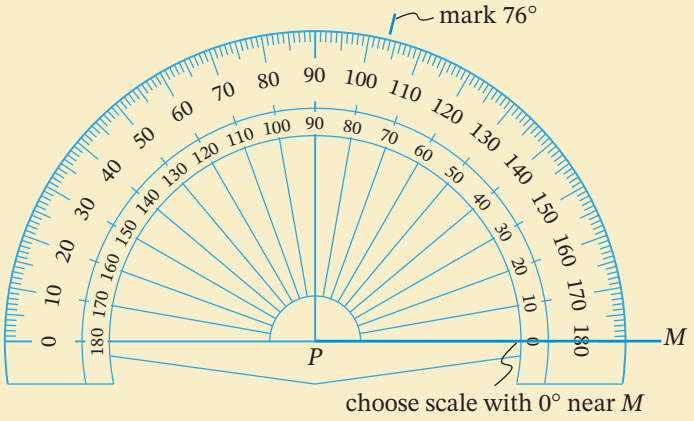
Construct $\angle KPM$ of size 76°.

SOLUTION

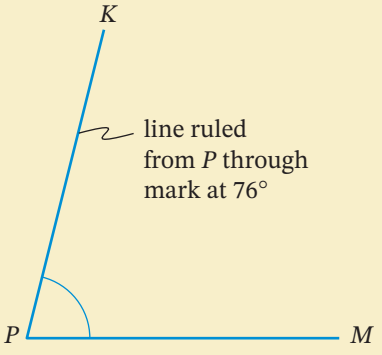
- Draw a line with endpoints P and M.



- Line up the base line of the protractor over PM with the centre mark on P.
- Follow the inside scale around on the protractor, from 0° to 76°.
- Mark this point.



- Draw a line from P through this mark and label the end of this line K.



- This creates $\angle KPM$, measuring 76°.

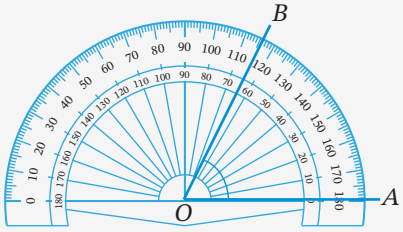
EXERCISE 2.02 ANSWERS ON P. 601

Measuring and drawing angles

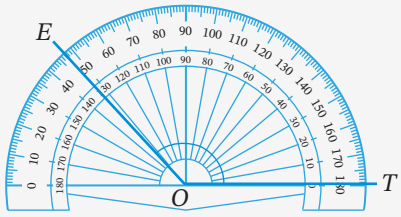
U F R C

1 Find the size of each marked angle.

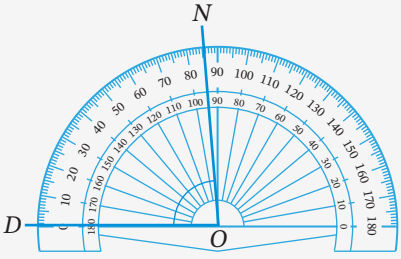
a



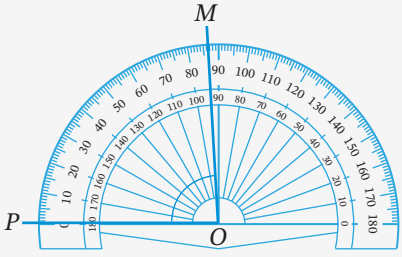
b



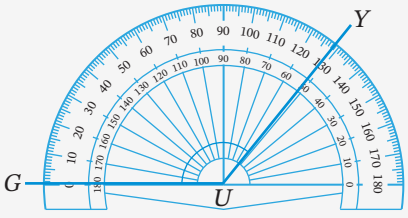
c



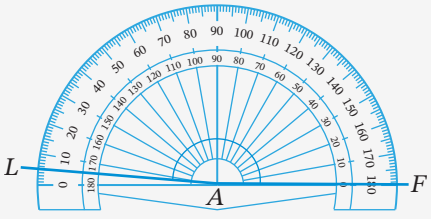
d



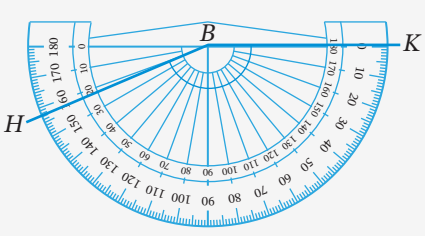
e



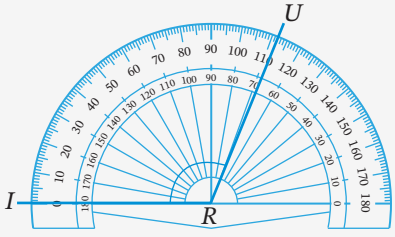
f



g



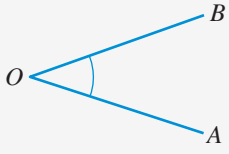
h



2 Estimate the size of each angle, then check your estimate by measuring with a protractor.

EXAMPLE 3

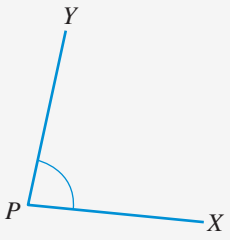
a



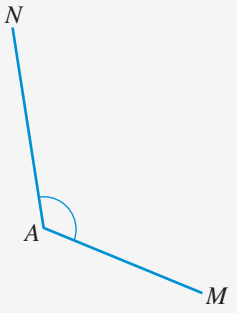
b

Foundation | Standard | Complex

c

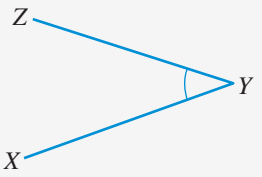


d

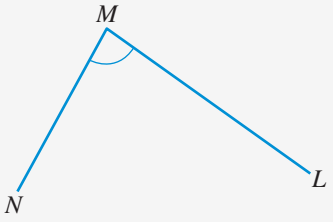


e

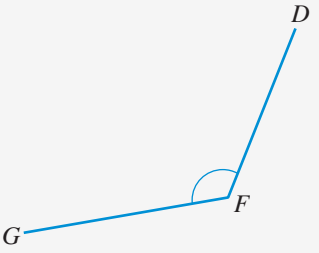
f



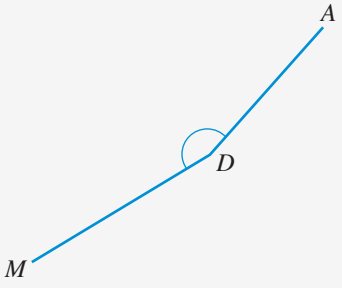
g



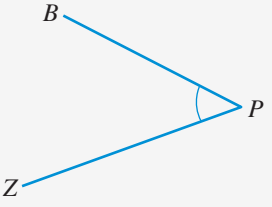
h



i

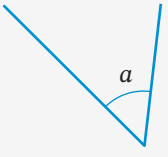


j

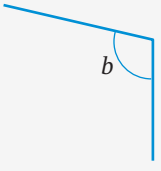


3 Sort the following angles from smallest to largest.

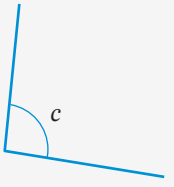
R **a**



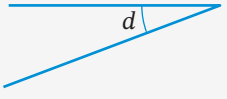
b



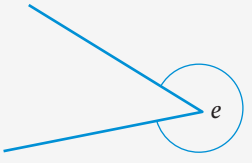
c



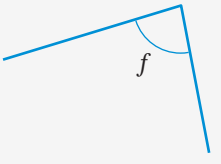
d



e



f



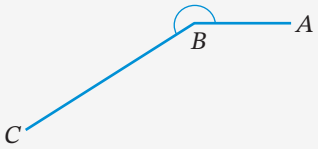
Foundation Standard Complex

EXAMPLE 4

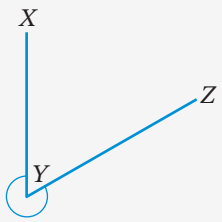
2.02

4 Estimate the size of each angle, then check by measuring with a protractor.

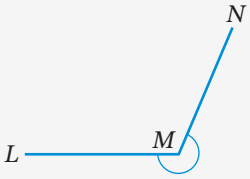
R **a**



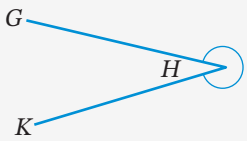
b



c



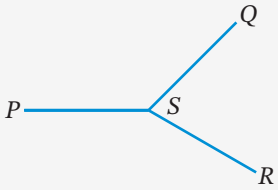
d



5 Measure the size of $\angle QSR$. Select **A**, **B**, **C** or **D**.

A 135° **B** 105°

C 75° **D** 70°



6 This photo shows Daniel shooting for goal in a game of football. His shooting angle is shown on the photo. Estimate the size of this angle. Select **A**, **B**, **C** or **D**.

R

iStock.com/AzmanJaka

A 60° **B** 80° **C** 100° **D** 120°

7 The word **'degree'** has many meanings.

C Find 4 non-mathematical meanings for 'degree'.

EXAMPLE 5

8 Use a protractor to accurately construct an angle of size:

a 35° **b** 115° **c** 150° **d** 63°

e 17° **f** 170° **g** 204° **h** 319°

9 Use a protractor to accurately draw and label each angle.

a $\angle DRE = 65°$ **b** $\angle BGH = 145°$ **c** $\angle GRT = 32°$

d $\angle ABC = 45°$ **e** $\angle SAQ = 110°$ **f** $\angle NMH = 265°$

g $\angle KLY = 28°$ **h** $\angle LMN = 180°$ **i** $\angle LKY = 99°$

Foundation Standard Complex

DID YOU KNOW?

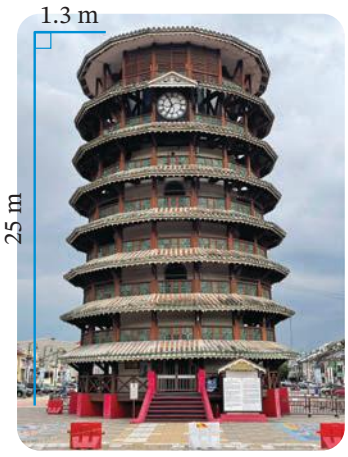
The Leaning Tower of Teluk Intan

The Leaning Tower of Pisa is not the only tower that leans.

The Clock Tower of Teluk Intan in Malaysia was built in 1885 as a water tower but became a watch tower during World War II. Although from the outside it looks like a 8-storey building, on the inside it has only 3 storeys.

The tower is 25 metres high and began leaning due to the soft soil foundation and the weight of the water tank inside. It now has a tilt of 3° from the vertical.

Draw a scale diagram of this tower. Research what the inside of the building looks like.



Getty Images/Azmir Dan/EyeEm

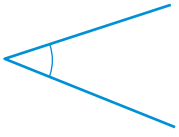
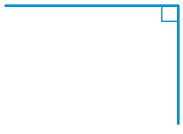
2.03 Types of angles

Skillsheet Types of angles

Worksheet A page of angles

Puzzle Angle cards

Angles are classified into types according to their size.

Angle	Type	Description
	acute	less than 90°
	right	90° (quarter turn) Note that a right angle is marked with a box symbol.
	obtuse	greater than 90° but less than 180°
	straight	180° (half turn)
	reflex	greater than 180° but less than 360°
	revolution	360° (complete turn)

EXERCISE 2.03 ANSWERS ON P. 601

Types of angles

U F R C

1 State whether each angle is acute (A), obtuse (O) or reflex (R).

R C

a

b

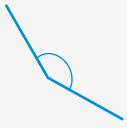
c

d

e

f

g

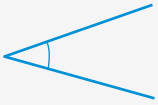
h


i

j

k

l

m


n

o

2 Draw 2 different:

a	acute angles	**b**	obtuse angles	**c**	right angles
d	reflex angles	**e**	straight angles	**f**	revolutions

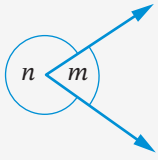
3 **Classify** each angle into one of the 6 types.

R C

a	37°	**b**	107°	**c**	252°	**d**	195°	**e**	79°
f	180°	**g**	163°	**h**	179°	**i**	360°	**j**	5°
k	345°	**l**	91°	**m**	14°	**n**	299°	**o**	90°
p	205°	**q**	126°	**r**	44°				

4 What types of angles are m and n respectively?
Select **A**, **B**, **C** or **D**.

R C



Foundation Standard Complex

5 Decide whether each marked angle is acute, obtuse or reflex.

R C

a

Shutterstock.com/EQRoy

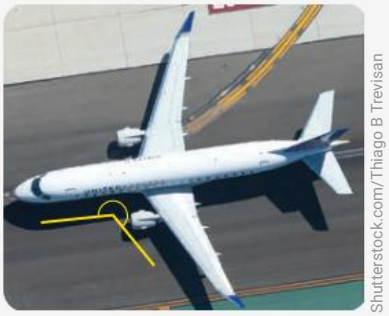
b

Shutterstock.com/photosync

c

Shutterstock.com/Mehmet Cetin

d



Shutterstock.com/Thiago B Trevisan

TECHNOLOGY

Constructing angles 1

Using dynamic geometry software, construct and label angles of various sizes.

1 Draw, label and measure:

a an acute angle **b** a right angle

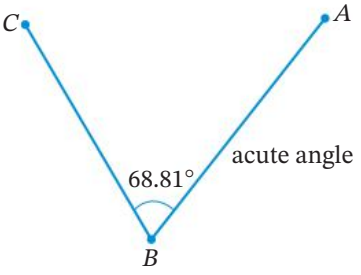
c an obtuse angle **d** a reflex angle

2 Start a new sketch and accurately construct separate angles of size:

a 72° **b** 310° **c** 165°

d 98° **e** 236° **f** 90°

3 Using **Insert Text**, classify each angle according to its type, for example, acute, reflex.



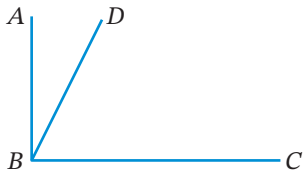
Foundation Standard Complex

INVESTIGATION

Complementary and supplementary angles

1 For each diagram, measure the angles named, then copy and complete the equations.

a

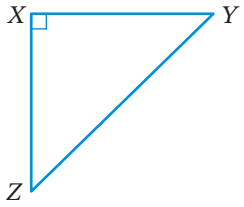


$\angle ABD =$ _____

$\angle CBD =$ _____

$\angle ABD + \angle CBD =$ _____

b



$\angle Y =$ _____

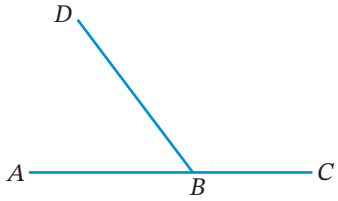
$\angle Z =$ _____

$\angle Y + \angle Z =$ _____

2 The 2 angles you measured in each diagram of question **1** are called **complementary angles**. What do complementary angles add up to?

3 For each diagram, measure the angles named, then copy and complete the equations.

a

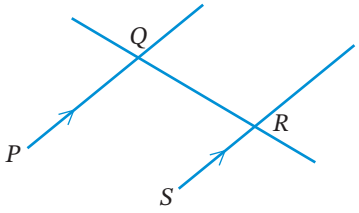


$\angle ABD =$ _____

$\angle CBD =$ _____

$\angle ABD + \angle CBD =$ _____

b



$\angle PQR =$ _____

$\angle SRQ =$ _____

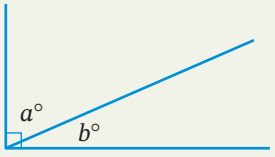
$\angle PQR + \angle SRQ =$ _____

4 The 2 angles you measured in each diagram of question **3** are called **supplementary angles**. What do supplementary angles add up to?

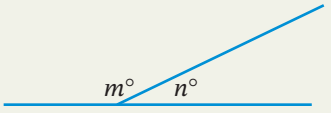
2.04 Complementary and supplementary angles

Complementary and supplementary angles

- **Complementary angles** add to 90°
- **Supplementary angles** add to 180°
- **Angles in a right angle** are complementary (add to up 90°)
- **Angles on a straight line** are supplementary (add to 180°).



$a + b = 90$

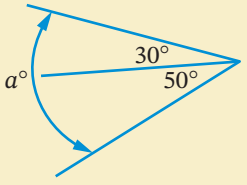


$m + n = 180$

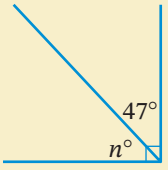
Example 6

In each diagram, there is an angle whose size is represented by a letter called a **variable** or **pronumeral**. A variable stands for a number. Find the value of each variable.

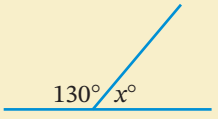
a



b



c



SOLUTION

a $a = 30 + 50$
$= 80$

Adding 2 adjacent angles.

b $n + 47 = 90$
$n = 90 - 47$
$= 43$

Angles in a right angle are complementary.

c $x + 130 = 180$
$x = 180 - 130$
$= 50$

Angles on a straight line are supplementary.

EXERCISE 2.04 ANSWERS ON P. 601

Complementary and supplementary angles

U F R C

1 **a** Why is 57° the complement of 33°?

R **b** Why is 147° the supplement of 33°?

C

2 Find the complement of:

a 30°	**b** 70°	**c** 25°	**d** 38°
e 89°	**f** 57°	**g** 42°	**h** 66°

3 Look up 'complement' and 'supplement' in a dictionary. Find one non-mathematical meaning of each word.

C

4 Find the supplement of:

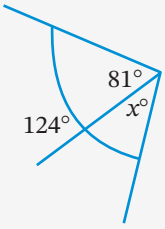
a 18°	**b** 150°	**c** 35°	**d** 125°
e 62°	**f** 87°	**g** 111°	**h** 4°

EXAMPLE 6

5 Find the value of x. Select the correct answer **A**, **B**, **C** or **D**.

R **A** 9 **B** 43

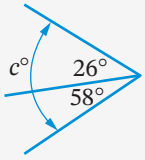
C 56 **D** 205



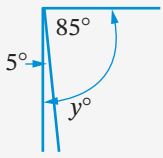
6 Find the value of the variable in each diagram.

R

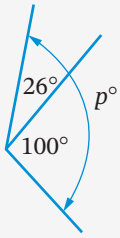
a



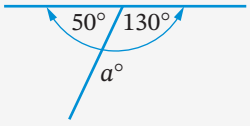
b



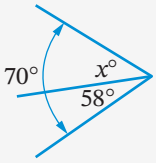
c



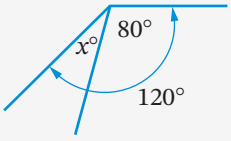
d



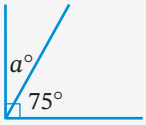
e



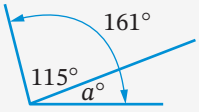
f



g



h



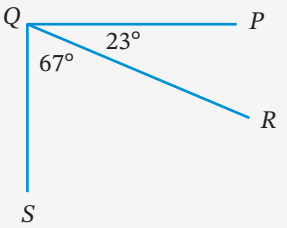
Foundation Standard Complex

7 **a** Name the pair of complementary angles in this diagram.

R **b** How do you know that the angles you named are complementary?

C

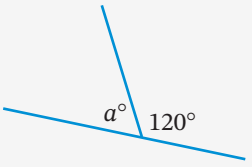
c What type of angle is $\angle PQS$?



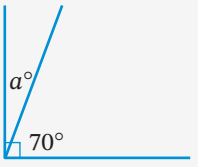
8 Find the value of the each variable.

R

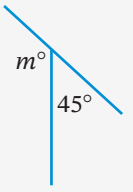
a



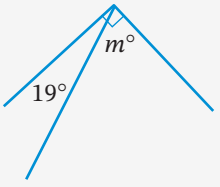
b



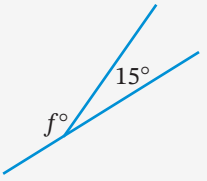
c



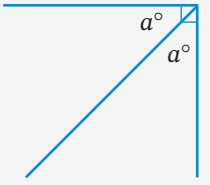
d



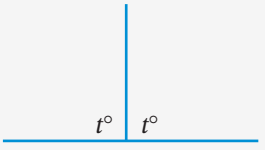
e



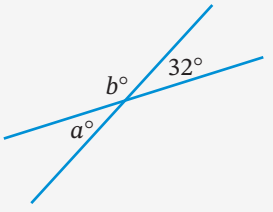
f



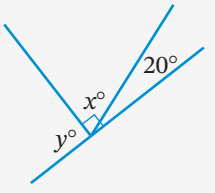
g



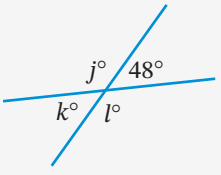
h



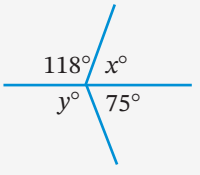
i



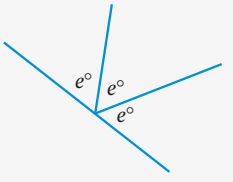
j



k



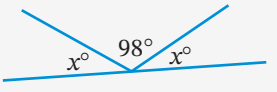
l



9 Find the value of x. Select **A**, **B**, **C** or **D**.

A 41 **B** 42

C 82 **D** 131



Foundation Standard Complex

☆ **MENTAL SKILLS** 2 ANSWERS ON P. 602

Maths without calculators

Quiz
Mental skills 2

Adding or multiplying in any order

Have you noticed that $4 + 7 = 7 + 4$?

Have you noticed that $3 \times 5 = 5 \times 3$?

Numbers can be added or multiplied in any order. We can use this property to make our calculations simpler.

1 Study each example.

a $19 + 5 + 5 + 1 = (19 + 1) + (5 + 5)$
$= 20 + 10$
$= 30$

b $13 + 8 + 20 + 27 + 80 = (13 + 27) + (20 + 80) + 8$
$= 40 + 100 + 8$
$= 148$

c $2 \times 36 \times 5 = (2 \times 5) \times 36$
$= 10 \times 36$
$= 360$

d $25 \times 11 \times 4 \times 7 = (25 \times 4) \times (11 \times 7)$
$= 100 \times 77$
$= 7700$

2 Now evaluate each sum.

a $45 + 16 + 45 + 4 + 7$
b $38 + 600 + 50 + 12 + 40$
c $18 + 91 + 9 + 20$
d $75 + 33 + 7 + 25$
e $24 + 16 + 80 + 44 + 10$
f $56 + 5 + 20 + 15 + 4$
g $100 + 36 + 200 + 10 + 90$
h $54 + 27 + 9 + 16 + 3$
i $70 + 50 + 30 + 25 + 25$
j $32 + 120 + 40 + 80 + 40$

3 Now evaluate each product.

a $8 \times 4 \times 5$
b $50 \times 7 \times 2$
c $3 \times 5 \times 6$
d $5 \times 11 \times 40$
e $12 \times 2 \times 3$
f $2 \times 4 \times 25 \times 8$
g $3 \times 20 \times 7 \times 5$
h $6 \times 8 \times 5 \times 2$
i $2 \times 3 \times 2 \times 11$

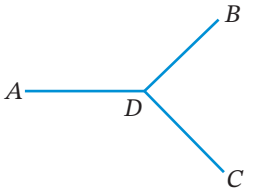
INVESTIGATION

Angles at a point and vertically opposite angles

1 In each diagram, there are adjacent angles that meet at a central point.

Measure all angles, then copy and complete the equations.

a



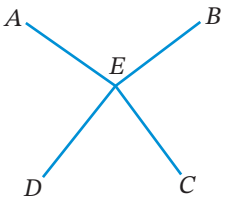
$\angle ADB =$ _____

$\angle ADC =$ _____

$\angle BDC =$ _____

$\angle ADB + \angle ADC + \angle BDC =$ _____

b



$\angle AEB =$ _____

$\angle BEC =$ _____

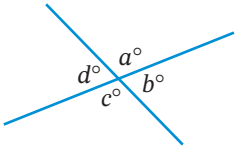
$\angle CED =$ _____

$\angle DEA =$ _____

$\angle AEB + \angle BEC + \angle CED + \angle DEA =$ _____

2 What do the angles that meet at a point add up to? Why?

3 When 2 lines cross, 4 angles are created.



a Which of these angles are equal?

b Can you prove it using supplementary angles?

2.05 Angles at a point and vertically opposite angles

Puzzles
Angles: a dog day

Angles

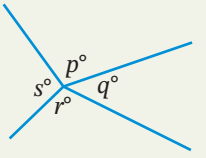
Worksheets
Straight angles

Right angles and revolutions

Angles at a point

Angles at a point (in a revolution) add to 360°.

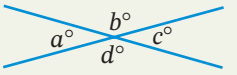
$$p + q + r + s = 360$$



Vertically opposite angles

Vertically opposite angles are equal.

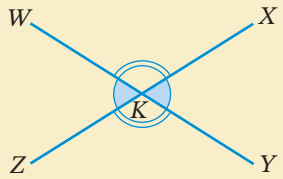
$$a = c \text{ and } b = d$$



Example 7

Name the angle that is vertically opposite:

a $\angle WKZ$ **b** $\angle ZKY$



SOLUTION

a $\angle XKY$ is vertically opposite $\angle WKZ$

b $\angle WKX$ is vertically opposite $\angle ZKY$

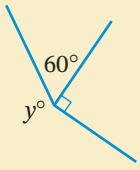
Video
Angles at a point and vertically opposite angles

2.05

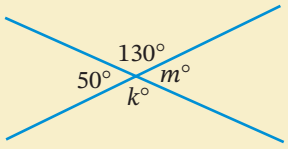
Example 8

Find the value of the variable in each diagram.

a



b



Video
Angles at a point and vertically opposite angles

SOLUTION

a $y + 60 + 90 = 360$ Angles at a point add to 360°

$y = 360 - 60 - 90$

$= 210$

b $k = 130$ Vertically opposite angles are equal

$m = 50$ Vertically opposite angles are equal

EXERCISE 2.05 ANSWERS ON P. 602

Angles at a point and vertically opposite angles

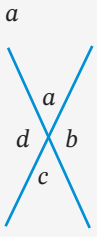
U F R C

1 Write the angle that is vertically opposite each named angle.

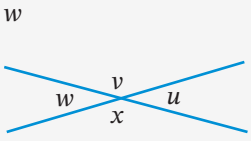
EXAMPLE 7

R C

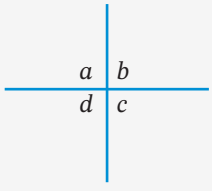
a



b

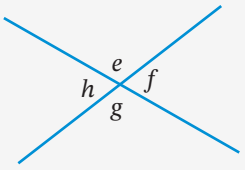


c

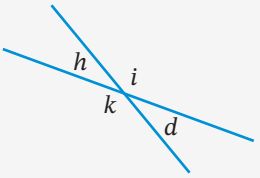


Foundation Standard Complex

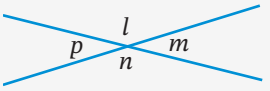
d h



e k

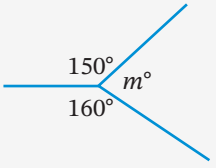


f m

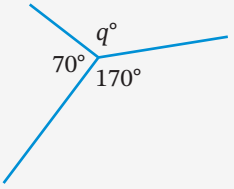


2 Find the value of each variable.

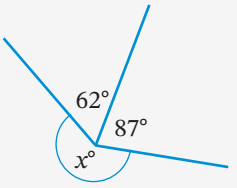
R **a**



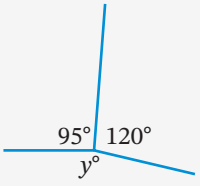
b



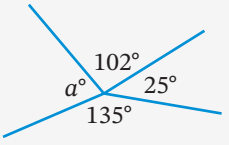
c



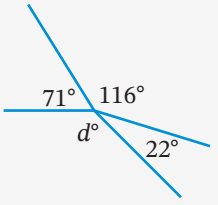
d



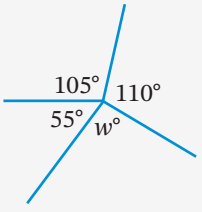
e



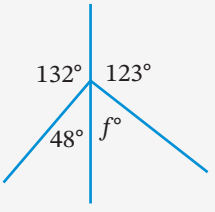
f



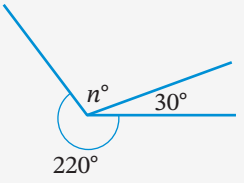
g



h



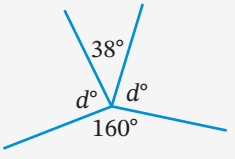
i



3 Find the value of d. Select the correct answer **A**, **B**, **C** or **D**.

R **A** 122 **B** 61

C 142 **D** 81

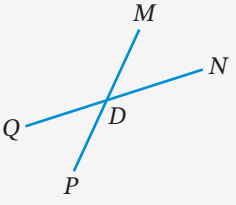


4 **a** Which angle is vertically opposite $\angle NDP$?

R **b** Which angle is equal to $\angle MDQ$?

C **c** Name 2 straight angles.

d Name 2 different pairs of supplementary angles.

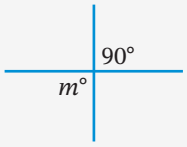


Foundation Standard Complex

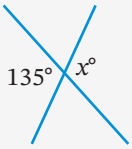
9780170465557

5 Find the value of each variable.

R **a**

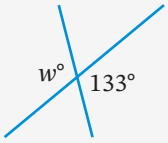


b

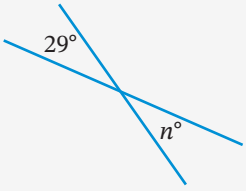


c

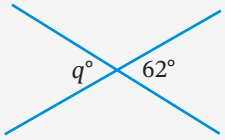
d



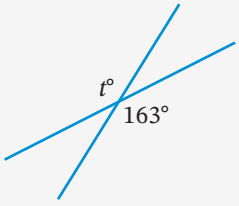
e



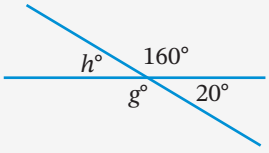
f



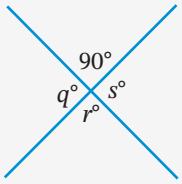
g



h

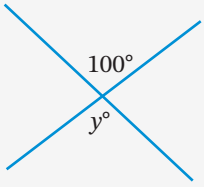


i

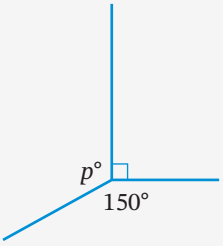


6 Find the value of each variable.

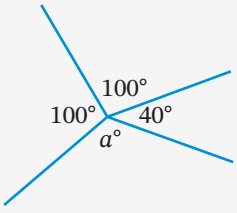
R **a**



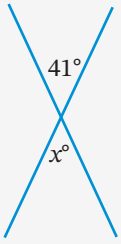
b



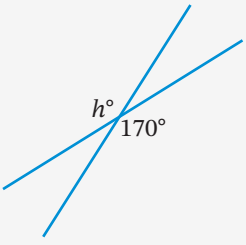
c



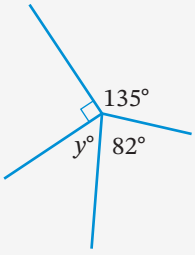
d



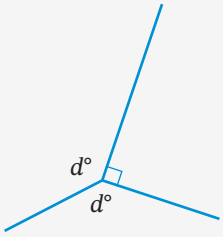
e



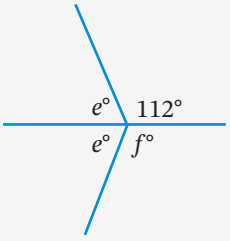
f



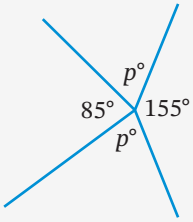
g



h



i

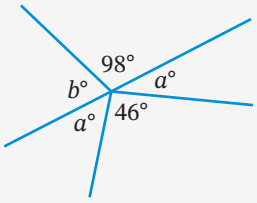


Foundation | Standard | Complex

7 Find the value of a and b. Select **A**, **B**, **C** or **D**.

R **A** $a = 46, b = 82$ **B** $a = 54, b = 80$

C $a = 67, b = 46$ **D** $a = 67, b = 82$



DID YOU KNOW?

Why 360 degrees?

Why are there 90° in a right angle and 360° in a revolution? Why do we use such unusual numbers instead of more conventional numbers like 10 and 100?

Over 4000 years ago, in 2000 BCE, the ancient Babylonians used a base 60 system of numbers. They used a base 60 number system because 60 has more factors than 10. You can divide 60 by 2, 3, 4, 5, 6, 10, 12, 15, 20 and 30.

Furthermore, $6 \times 60 = 360$, which was the Babylonian approximation of the number of days in a year. They defined a revolution as being 360° so that each day the Earth would travel 1° around the Sun. A right angle, being a quarter-revolution, thus became $360° \div 4 = 90°$.

Some people who prefer a base 10 system of measuring angles use **grads** instead of degrees. With this system, a right angle is 100 grads and a revolution is 400 grads.

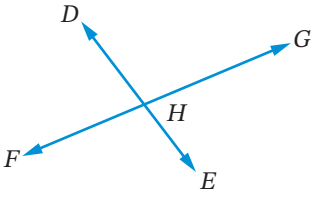
Find out more information about grads, including the exact relationship between degrees and grads.

2.06 Parallel and perpendicular lines

Worksheet
A page of intervals

A line is named using 2 points on the line. For example, this is the line AB.

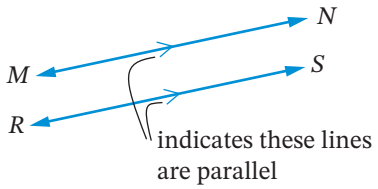
When 2 lines cross, we say that they **intersect** at a point. For example, in this diagram, line DE intersects line FG at the point H.



Foundation Standard Complex

Parallel lines

Lines that point in the same direction and never intersect are called **parallel** lines. Parallel lines are marked with identical arrowheads and are always the same distance apart. For example, in this diagram, *MN* is parallel to *RS*.

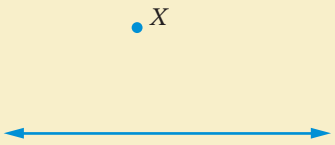


This is written as '*MN* || *RS*', where the symbol || stands for 'is parallel to'.

EXTENSION

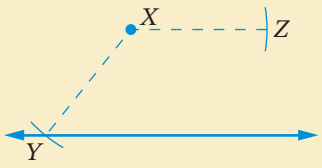
Example 9

Use compasses to construct a line through *X* that is parallel to the given line.



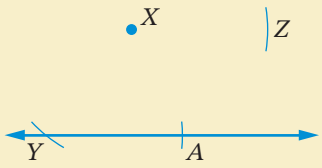
SOLUTION

Step 1



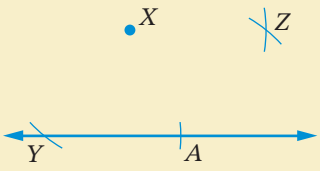
Use compasses from *X* to mark 2 large arcs at *Y* and *Z*.

Step 2



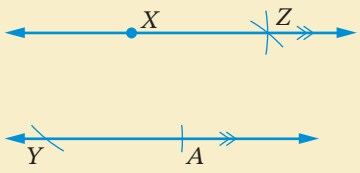
Use compasses from *Y* to mark an arc with the same radius at *A* on the line.

Step 3



Use compasses from *A* to mark an arc with the same radius to cross the arc at *Z*.

Step 4

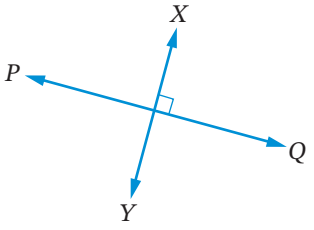


Join *XZ* to construct a line parallel to *AY*.

Perpendicular lines

Lines that intersect at right angles (90°) are called **perpendicular** lines. For example, in this diagram, PQ is perpendicular to XY.

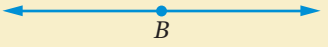
This is written as '$PQ \perp XY$', where the $\perp$ symbol stands for 'is perpendicular to'.



EXTENSION

Example 10

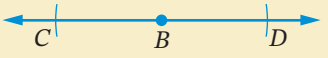
Use compasses to construct a perpendicular through point B on the line.



SOLUTION

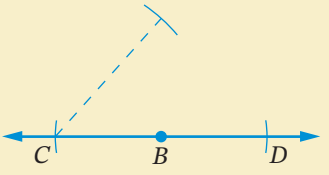
Constructing a perpendicular to a point on a line is the same as **bisecting** a straight angle (180°).

Step 1



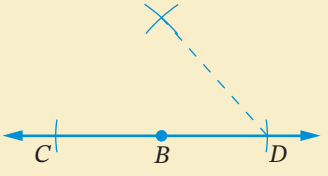
Use compasses to draw 2 arcs from B.

Step 2



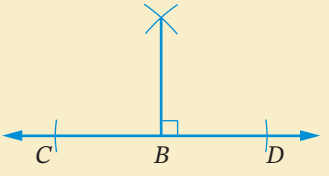
Open compasses wider to draw an arc from C.

Step 3



Use compasses to draw an arc with the same distance from D.

Step 4

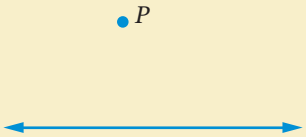


Join B to where the 2 arcs cross.

Use a protractor or set square to check that the line is perpendicular (at 90°) to CD.

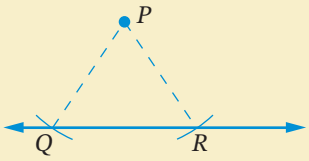
Example 11

Use compasses to draw a perpendicular through point P above the line.



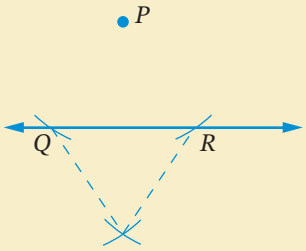
SOLUTION

Step 1



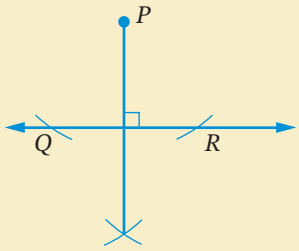
Use compasses from P to mark 2 arcs with the same radius on the line.

Step 2



Use compasses from Q and R to mark 2 intersecting arcs with the same radius below the line.

Step 3



Join P to where the 2 arcs cross.

Use a protractor or set square to check that the line is perpendicular to QR.

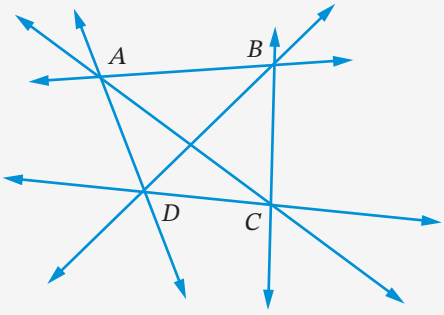
EXERCISE 2.06 ANSWERS ON P. 602

Parallel and perpendicular lines

U F R C

1 Name the 6 different lines in this diagram.

C



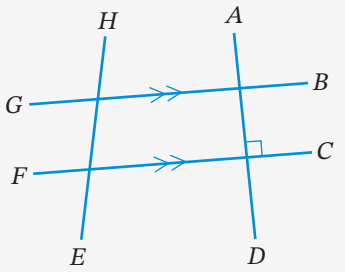
Foundation Standard Complex

2 In this diagram, name 2 lines that:

C **a** are perpendicular

b are parallel

c intersect



3 Rewrite your answers to question **2** parts **a** and **b** using the symbols for 'is perpendicular to' and 'is parallel to'.

C

4 Draw and label correctly:

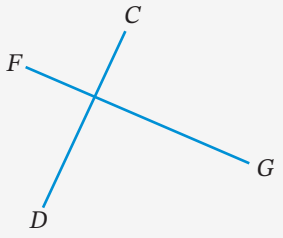
C **a** line *FG*

b line *AB* intersecting line *CD* at point *E*

c line *PQ* parallel to line *YZ*

d line *JK* perpendicular to line *LM*.

5 An **interval** is a section of a line, with a definite length. Which interval is parallel to *FG*? Select the correct answer **A**, **B**, **C** or **D**.



A *CD*

B *LM*

C *AB*

D *PQ*

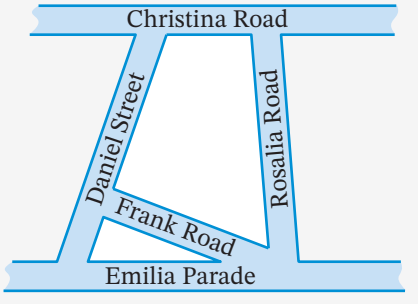
6 On the map, what is perpendicular to Frank Road? Select **A**, **B**, **C** or **D**.

C **A** Emilia Parade

B Rosalia Road

C Daniel Street

D Christina Road



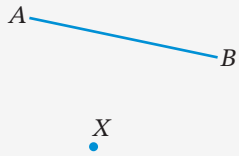
Foundation Standard Complex

EXAMPLE 9

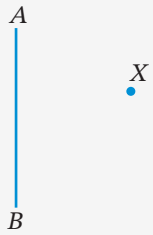
2.06

7 Copy each diagram and construct or draw a line parallel to AB through X.

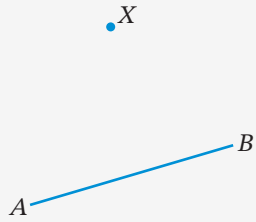
a



b



c



EXAMPLE 10

8 Draw a line and mark a point, L, on it. Construct a perpendicular line through L using compasses or a protractor.

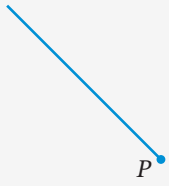
9 Draw a line and mark a point, X, above it. Construct or draw a perpendicular to the line that passes through X.

10 Copy each diagram and construct or draw a perpendicular line through P.

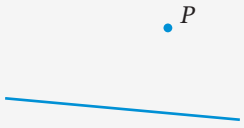
R C

a

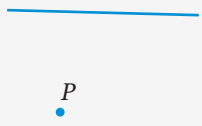
b



c



d



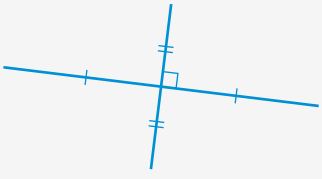
11 **a** Draw 2 different-sized intervals that are parallel.

R **b** Join their ends to make a quadrilateral.

C **c** What type of quadrilateral have you constructed?

12 **a** Draw an interval and mark its midpoint.

R C



b Draw a different-sized interval through the midpoint of the first interval, perpendicular to it and with the same midpoint (as shown).

c Join the ends of both intervals to make a quadrilateral.

d What type of quadrilateral have you constructed?

Foundation Standard Complex

2.07 Corresponding angles on parallel lines

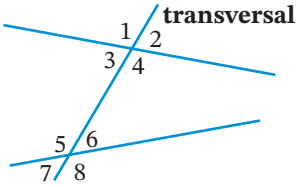
Worksheet Investigating angles on parallel lines

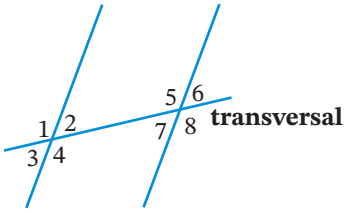
Skillsheet Angles and parallel lines

Video Angle relationships

A line that crosses 2 or more other lines is called a **transversal**. Transverse means 'crossing'.

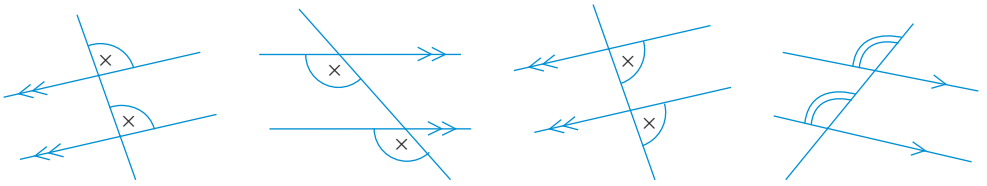
If a transversal crosses 2 lines, then 8 angles are created.





These 8 angles have special properties if the transversal crosses *parallel* lines.

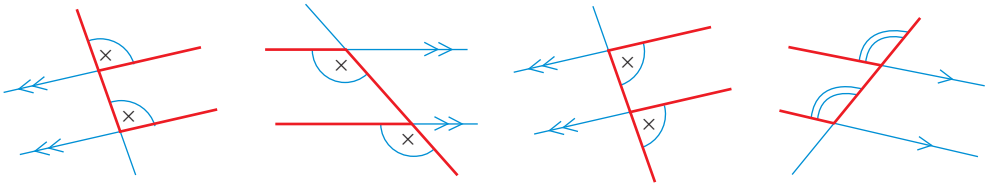
Corresponding angles on parallel lines are any 2 angles in matching positions according to the transversal and a parallel line, as shown below.



'Corresponding' means 'matching', and corresponding angles on parallel lines are equal in size.

The positions of corresponding angles will often form the capital letter 'F'.

You may need to flip or turn the F in order to see it.



EXERCISE 2.07 ANSWERS ON P. 602

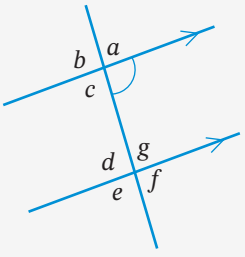
Corresponding angles on parallel lines

U F R C

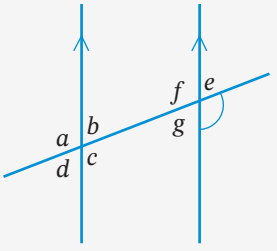
1 Write the corresponding angle to each marked angle.

R C

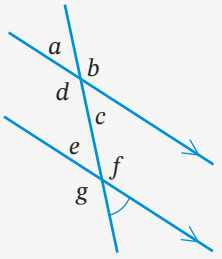
a



b



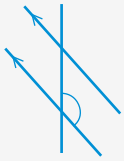
c



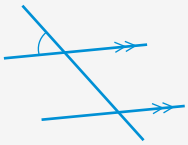
Foundation Standard Complex

2 Copy each diagram and mark the corresponding angle to the marked angle.

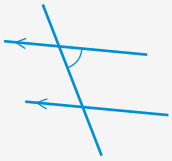
R **a**



b

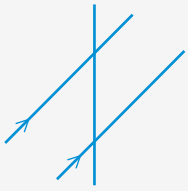


c

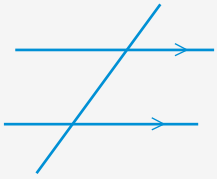


3 Copy each diagram and mark any pair of corresponding angles.

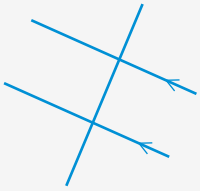
a



b

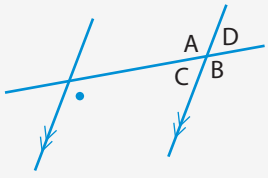


c



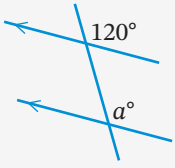
4 Which angle is corresponding to the angle marked • ?

R Select the correct answer **A**, **B**, **C** or **D**.

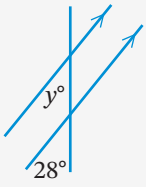


5 Find the value of each variable.

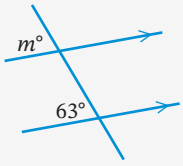
a



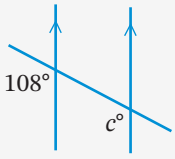
b



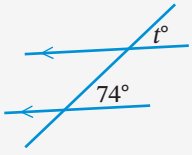
c



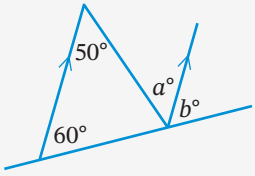
d



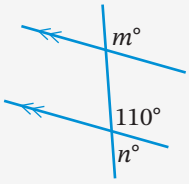
e



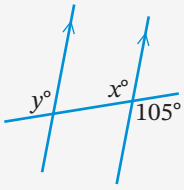
f



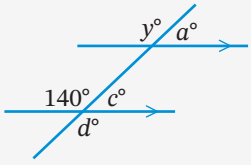
g



h



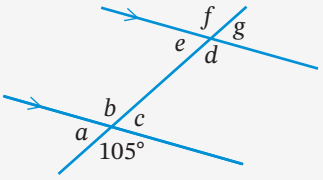
i



Foundation | Standard | Complex

6 **a** How many pairs of corresponding angles are in this diagram?

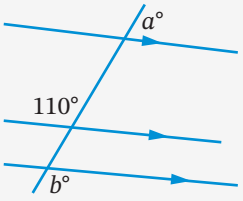
(R) **b** Find the sizes of angles a to g.



7 Find the values of a and b. Select **A**, **B**, **C** or **D**.

(R) **A** $a = 110, b = 110$ **B** $a = 70, b = 70$

C $a = 70, b = 110$ **D** $a = 110, b = 70$



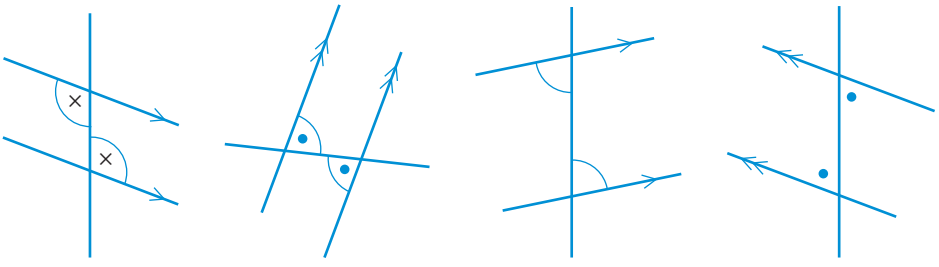
2.08 Alternate angles on parallel lines

Worksheet Investigating angles on parallel lines

Skillsheet Angles and parallel lines

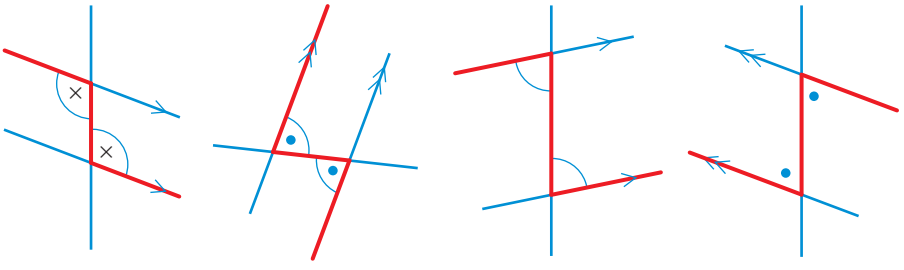
Video Angle relationships

Alternate angles on parallel lines are any 2 angles *between* the lines and on *opposite* sides of the transversal, as shown below.



'Alternate' means 'going back and forth between 2 sides'.

The positions of alternate angles on parallel lines will often form the capital letter 'Z'. You may need to flip or turn the 'Z' in order to see it.



Foundation Standard Complex

EXERCISE 2.08 ANSWERS ON P. 602

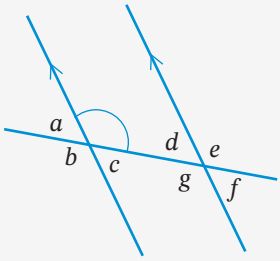
Alternate angles on parallel lines

U F R C

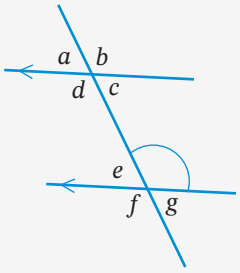
1 Write the alternate angle to each marked angle.

R C

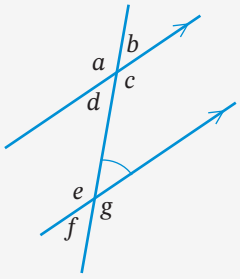
a



b



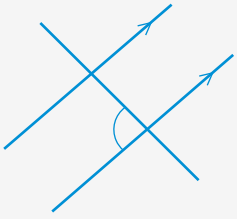
c



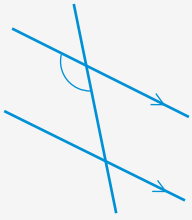
2 Copy each diagram and mark the alternate angle to the marked angle.

R

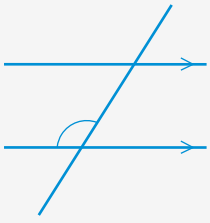
a



b



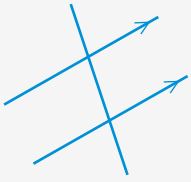
c



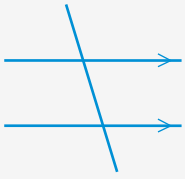
3 Copy each diagram and mark any pair of alternate angles.

R

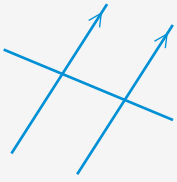
a



b

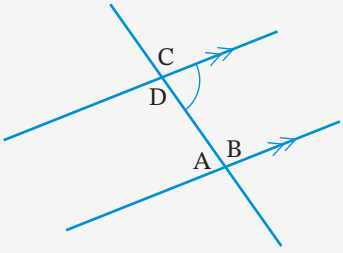


c



4 Which angle is alternate to the marked angle? Select the correct answer **A**, **B**, **C** or **D**.

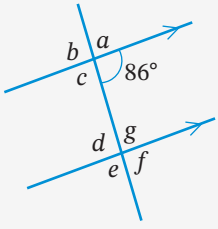
R



5 **a** How many pairs of alternate angles are there in this diagram?

R **b** Find the sizes of the angles a to g.

C **c** What is the relationship between the alternate angles on parallel lines?



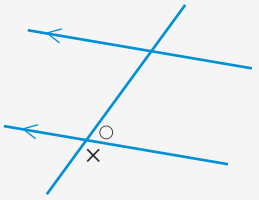
Foundation Standard Complex

6 Two angles have been marked on this diagram. The other 6 angles are either ○ or ×.

R C

a Copy and complete the diagram with ○s or ×s.

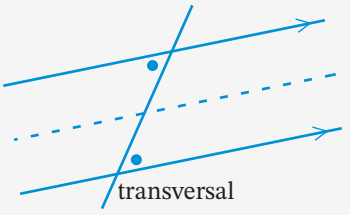
b What do you notice about the pairs of alternate angles?



7 **a** Copy this diagram on a sheet of paper and spin it upside down to show that it has rotational symmetry. What does that mean about the alternate angles?

R C

b Cut out your diagram and then cut along the dotted line. Spin one of the alternate angles to fit exactly on top of the other. Are they the same size?



8 Examine this diagram.

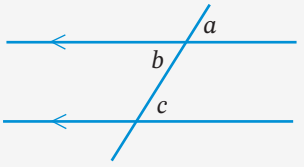
R C

a Why is $b = a$?

b Why is $c = a$?

c So what are the values of b and c?

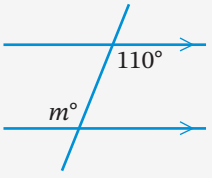
d What does this prove about alternate angles on parallel lines?



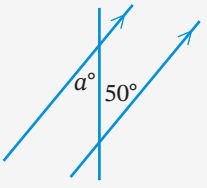
9 Find the value of each variable.

R

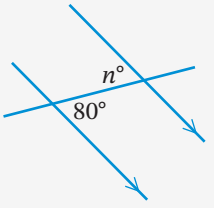
a



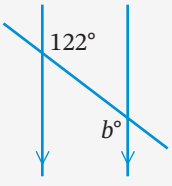
b



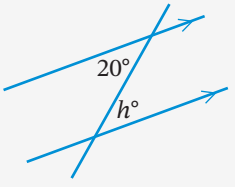
c



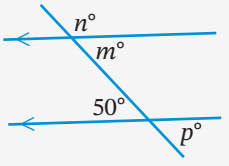
d



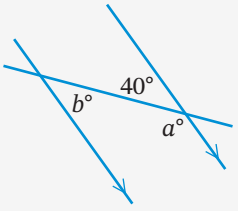
e



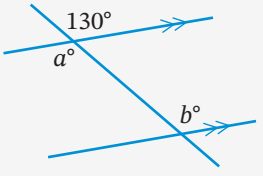
f



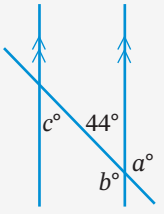
g



h



i



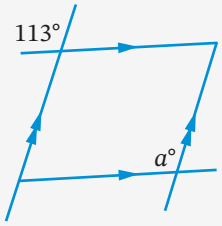
Foundation Standard Complex

9780170465557

10 Find the value of a. Select **A**, **B**, **C** or **D**.

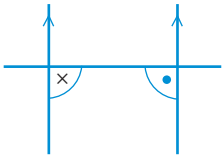
R **A** 113 **B** 67

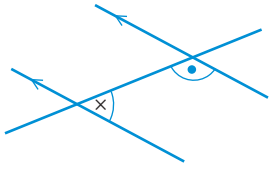
C 117 **D** 247



Co-interior angles on parallel lines 2.09

Co-interior angles on parallel lines are any 2 angles *between* the lines and on the *same* side of the transversal, as shown below.

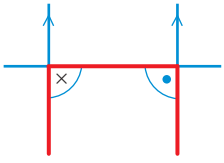


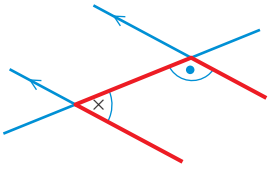


'Co-interior' means 'together inside'.

The positions of co-interior angles on parallel lines will often form the capital letter 'C'.

You may need to flip or turn the 'C' in order to see it.





Worksheet Investigating angles on parallel lines

Skillsheet Angles and parallel lines

Video Angle relationships

EXERCISE 2.09 ANSWERS ON P. 603

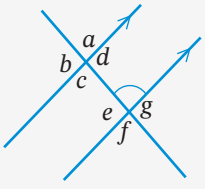
Co-interior angles on parallel lines

U F R C

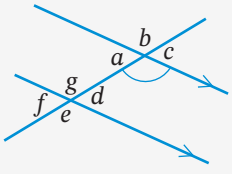
1 Write the co-interior angle to each marked angle.

R C

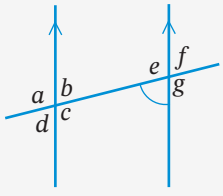
a



b



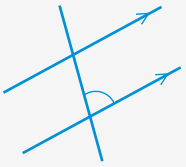
c



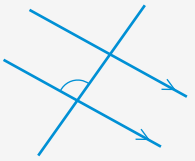
Foundation Standard Complex

2 Copy each diagram and mark the co-interior angle to the marked angle.

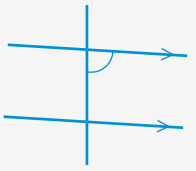
R **a**



b

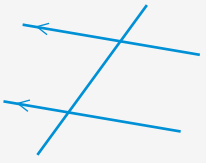


c

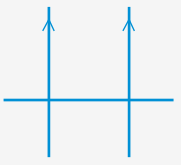


3 Copy each diagram and mark any pair of co-interior angles.

R **a**



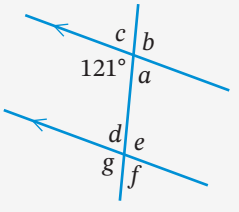
b



c

4 **a** How many pairs of co-interior angles are on this diagram?

R C **b** Which angle is co-interior to the 121° angle? Select the correct answer **A**, **B**, **C** or **D**.



A d **B** b

C e **D** g

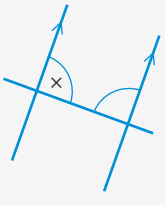
c Find the sizes of -angles a to g.

d What is the relationship between the co-interior angles on parallel lines?

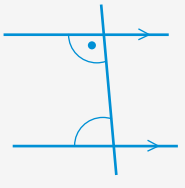
5 Copy each diagram, measure the marked angles and calculate their sum.

C What do you notice about your answer?

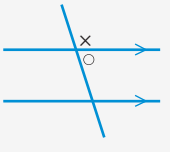
a



b



6 Two angles have been marked on this diagram.

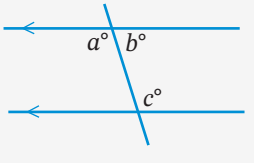


R **a** How many degrees is ○ + ×? Why?

C **b** The other 6 angles are either ○ or ×. Copy and complete the diagram with ○s or ×s.

c What do you notice about the pairs of co-interior angles?

7 Examine this diagram.



R **a** What is the value of $a + b$? Why?

C **b** Why is $a = c$?

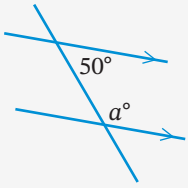
c So what is the value of $c + b$?

d What does this prove about co-interior angles on parallel lines?

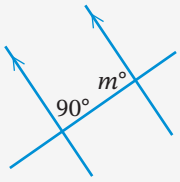
Foundation Standard Complex

8 Find the value of each variable.

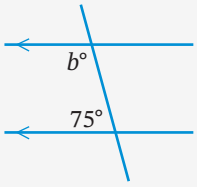
C **a**



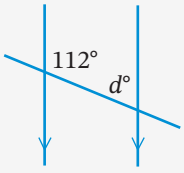
b



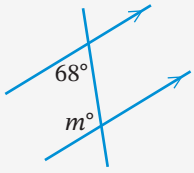
c



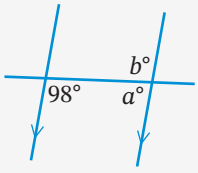
d



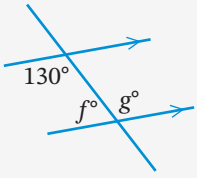
e



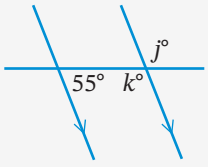
f



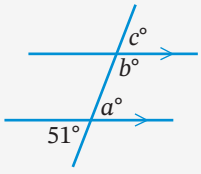
g



h



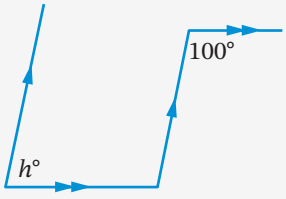
i



9 Find the value of h. Select **A**, **B**, **C** or **D**.

R **A** 20 **B** 80

C 100 **D** 260



Foundation Standard Complex

2.10 Angles on parallel lines

Worksheets
What is the diagram?

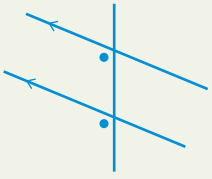
Matching angles

Skillsheet
Angles and parallel lines

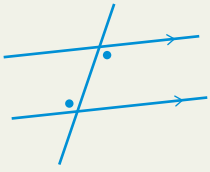
Angles on parallel lines

When parallel lines are crossed by a transversal:

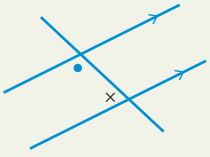
- corresponding angles are equal



- alternate angles are equal



- co-interior angles are supplementary (add to 180°).

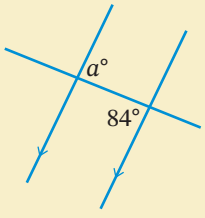


Video
Angles on parallel lines

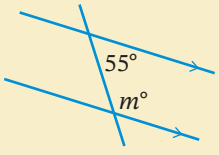
Example 12

Find the value of each variable, giving a reason.

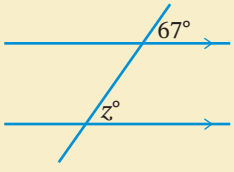
a



b



c



SOLUTION

a $a = 84$ (Alternate angles on parallel lines)

b $m + 55 = 180$ (Co-interior angles on parallel lines)

$m = 180 - 55$

$= 125$

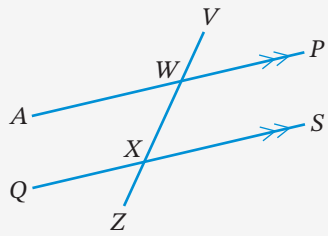
c $z = 67$ (Corresponding angles on parallel lines)

EXERCISE 2.10 ANSWERS ON P. 603

Angles on parallel lines

U F R C

1 For this diagram, name the angle that is:



R **a** corresponding to $\angle VWA$

C **b** alternate to $\angle QXW$

c co-interior with $\angle PWX$

d co-interior with $\angle AWX$

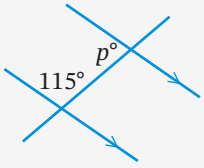
e alternate to $\angle SXV$

f corresponding to $\angle ZXS$.

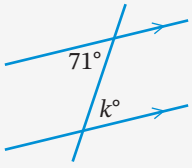
2 Find the value of each variable, giving a reason.

EXAMPLE 12

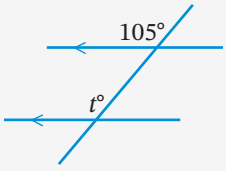
R C **a**



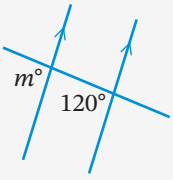
b



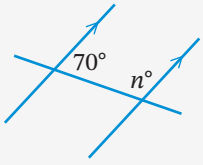
c



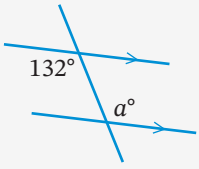
d



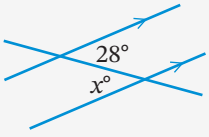
e



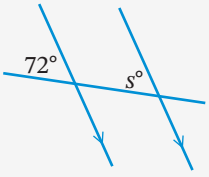
f



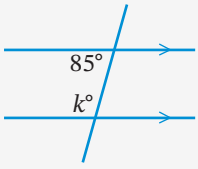
g



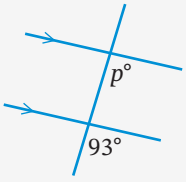
h



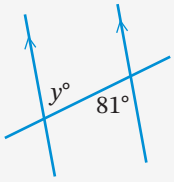
i



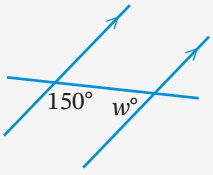
j



k

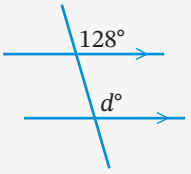


l

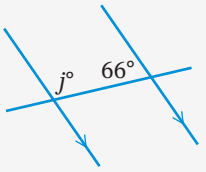


Foundation Standard Complex

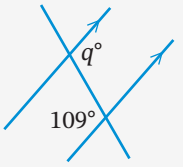
m



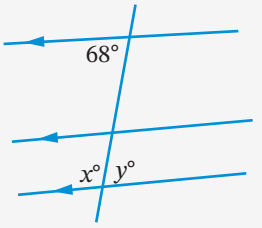
n



o



3 Find the value of x and y.



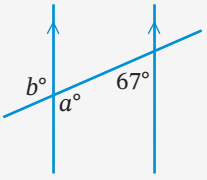
R Select the correct answer **A**, **B**, **C** or **D**.

A $x = 22, y = 68$ **B** $x = 68, y = 112$

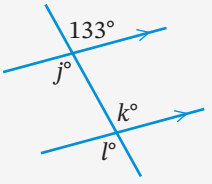
C $x = 112, y = 68$ **D** $x = 68, y = 22$

4 Find the value of each variable.

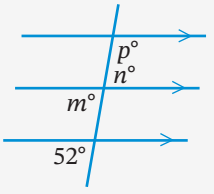
a



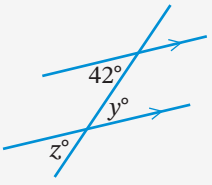
b



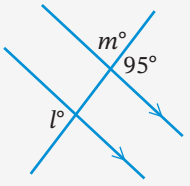
c



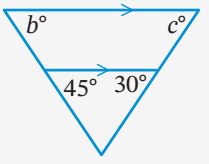
d



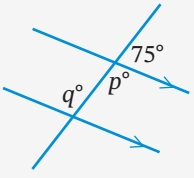
e



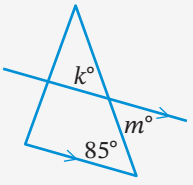
f



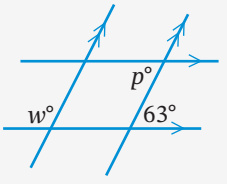
g



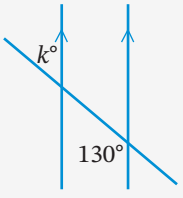
h



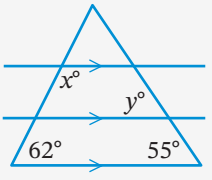
i



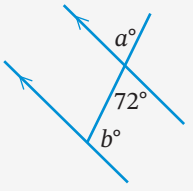
j



k

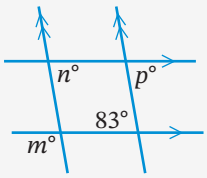


l

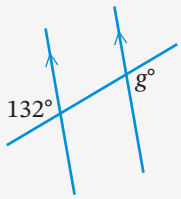


Foundation Standard Complex

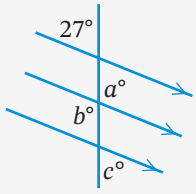
m



n



o

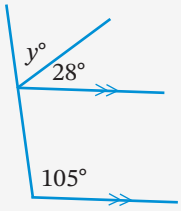


5 Which is the value of y? Select **A**, **B**, **C** or **D**.

R

A 28 **B** 47

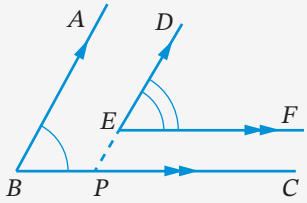
C 77 **D** 152



6 Prove that $\angle ABC = \angle DEF$.

R

C

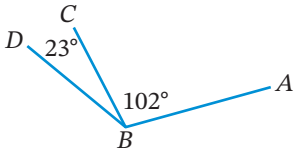


TECHNOLOGY

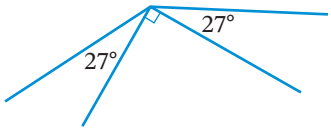
Constructing angles 2

1 Use dynamic geometry software to construct the following angles accurately.

a



b



c 2 complementary angles

d 2 supplementary angles

2 Construct each pair of angles on parallel lines.

a Corresponding angles of 28°

b Alternate angles of 65°

c Co-interior angles, where one of the angles is 130°

Foundation Standard Complex

2.11 Proving parallel lines

We can use what we know about angles and parallel lines to prove that 2 lines are parallel.

Proving parallel lines

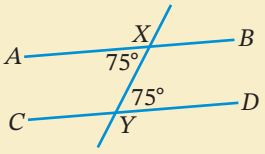
Two lines are parallel if:

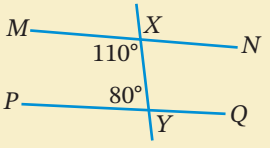
- alternate angles are equal, or
- corresponding angles are equal, or
- co-interior angles are supplementary (add up to 180°).

Example 13

Prove whether each pair of lines is parallel.

a $AB \parallel CD$ **b** $MN \parallel PQ$





SOLUTION

a $\angle AXY$ is alternate to $\angle DYX$.

$\angle AXY = \angle DYX = 75°$

$\therefore AB \parallel CD$ (alternate angles are equal)

$\therefore$ means 'therefore'

b $\angle MXY$ is co-interior to $\angle PYX$

$\angle MXY + \angle PYX = 110° + 80°$

$= 190°$

$\neq 180°$

$\therefore MN$ is *not* parallel to PQ (co-interior angles *do not* add to 180°)

EXERCISE 2.11 ANSWERS ON P. 603

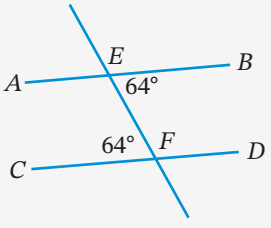
Proving parallel lines

U F R C

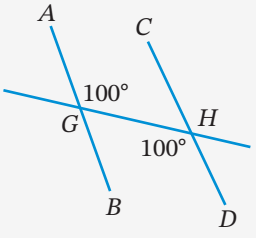
EXAMPLE 13

1 For each diagram, prove whether $AB \parallel CD$.

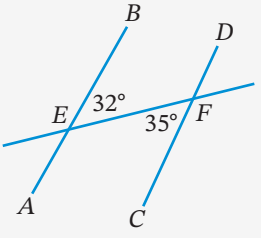
R C **a**



b

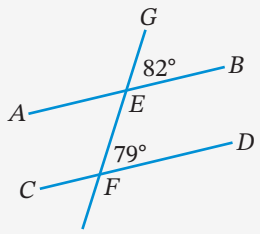


c

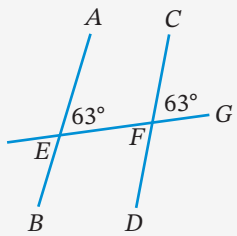


Foundation Standard Complex

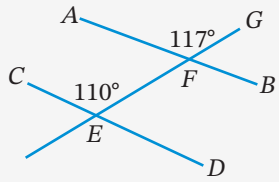
d



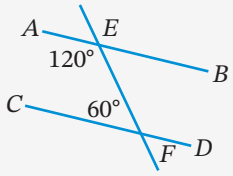
e



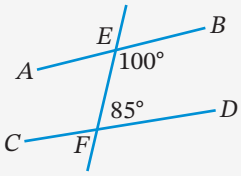
f



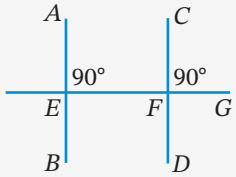
g



h

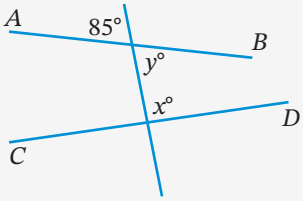


i



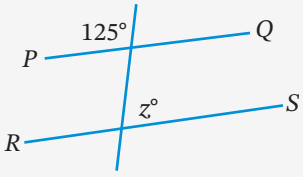
2 **a** Find the value of y, giving a reason.

R **b** Which value of x would prove that $AB \parallel CD$?



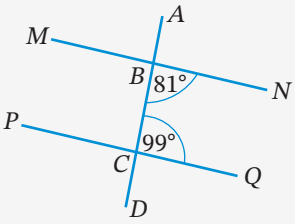
3 Which value of z would prove that $PQ \parallel RS$?

R

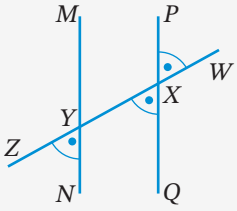


4 For each diagram, prove whether $PQ \parallel MN$, giving a reason.

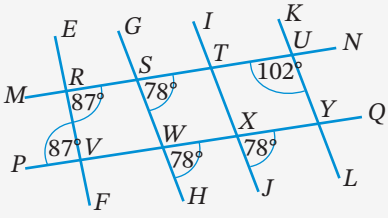
R C **a**



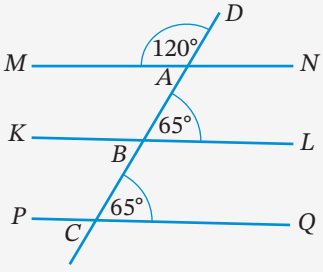
b



c

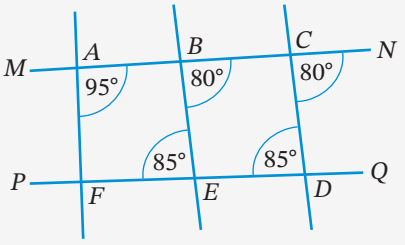


d

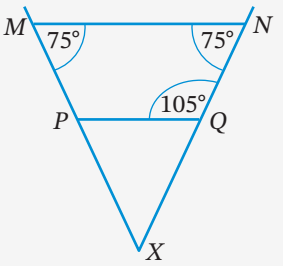


Foundation Standard Complex

e

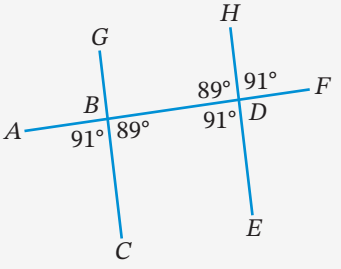


f



5 What reason proves that $GC \parallel HE$? Select the correct answer **A**, **B**, **C** or **D**.

R



A $\angle ABC = \angle HDF$ (alternate angles)

B $\angle CBD = \angle BDH$ (alternate angles)

C $\angle ADE = 91°$ (corresponding angles)

D $\angle BDE = \angle FDH$ (vertically opposite angles)

POWER PLUS ANSWERS ON P. 604

1 How many degrees does the Earth spin on its axis in:

a one day? **b** one hour?

c 8 hours? **d** 10 minutes?

2 Work out which direction (left, right, front or back) you would be facing after making each series of turns.

a Right 80°, right 240°, left 90°, right 40°

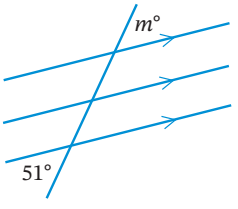
b Left 140°, left 140°, left 140°, right 60°

c Right 200°, left 70°, right 40°, right 10°

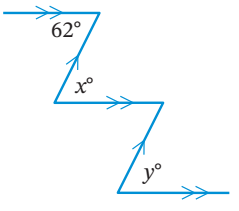
d Left 240°, right 190°, right 100°, left 50°

3 Find the value of each variable, giving reasons.

a

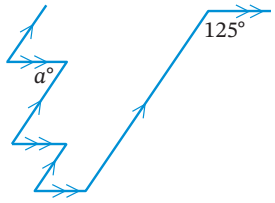


b

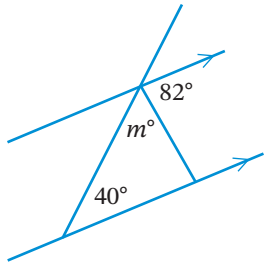


Foundation Standard Complex

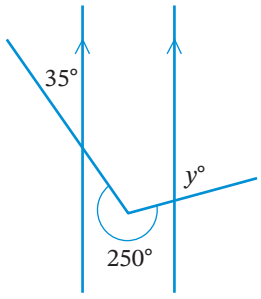
c



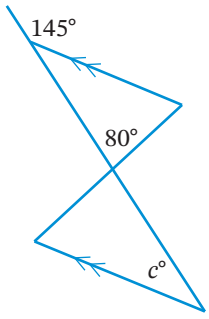
d



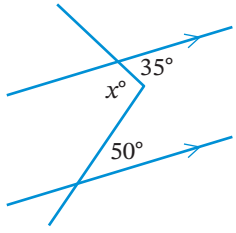
e



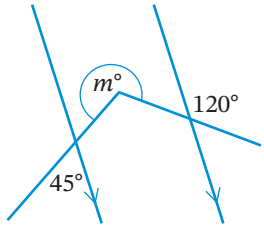
f



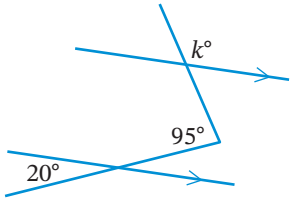
g



h



i



4 Draw an interval *DE* 6 cm in length. At *D* draw a line at any angle. On this line, mark a point *G*, also 6 cm from *D*. Construct a line through *G* parallel to *DE*. Construct a line through *E* parallel to *DG*. What shape have you drawn?

2 CHAPTER REVIEW

Language of maths

Quiz
Language of maths 2

acute	adjacent	alternate	angles at a point
co-interior	complementary	construct	corresponding
degrees	intersect	interval	obtuse
parallel	perpendicular	pronumeral	protractor
revolution	right	straight	supplementary
transversal	variable	vertex	vertically opposite

1 What is the name given to a line that crosses 2 or more other lines?

2 Find the meaning of 'acute' when referring to a disease, for example, acute appendicitis.

3 What is the difference between 'complementary' and 'complimentary'?

4 When something happens that dramatically changes the way we think or do things, it is said to be 'revolutionary'. Why do you think this word is used?

5 Draw 'angles at a point' and describe their property.

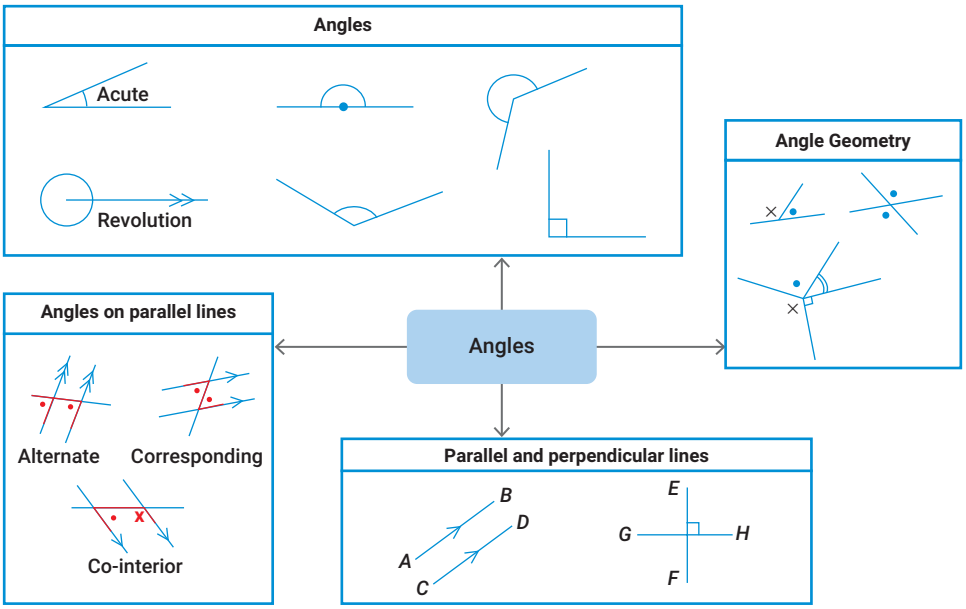
6 Mr Transversal visits his parents on **alternate** days. What does this mean? How is it similar to the mathematical meaning of 'alternate'?

Topic summary

Worksheet
Mind map: Angles

- Give 3 examples of where angles are used.
- How confident do you feel about working with angles?
- Is there anything you did not understand? Ask a friend or your teacher for help.

Print (or copy) and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.



2 TEST YOURSELF

ANSWERS ON P. 604

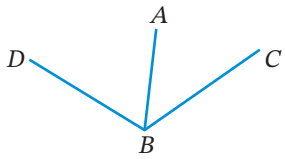
1 Draw and label each angle. 2.01

a $\angle BKT$ **b** $\angle FPR$ **c** $\angle MZQ$

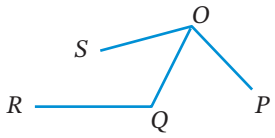
2 Use a protractor to measure each angle you drew in question **1**. 2.02
Name the smallest angle and the largest angle.

3 Name the angle adjacent to: 2.01

a $\angle ABC$



b $\angle POQ$

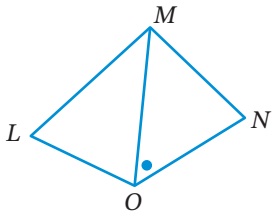


Quiz
Test yourself 2

4 Name the angle marked with a • . Select **A**, **B**, **C** or **D**. 2.01

A $\angle NMO$ **B** $\angle LON$

C $\angle OMN$ **D** $\angle MON$



5 Use a protractor to draw each angle. 2.02

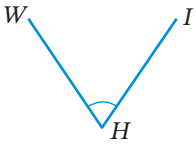
a $\angle JUG = 84°$ **b** $\angle QRA = 117°$ **c** $\angle POT = 41°$

d $\angle DGE = 150°$ **e** $\angle SAR = 96°$ **f** $\angle XDW = 210°$

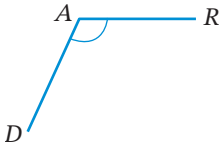
g $\angle MNB = 195°$ **h** $\angle PLO = 270°$ **i** $\angle AMP = 300°$

6 Name each angle using 3 letters, then classify as acute, obtuse, right, reflex or straight. 2.03

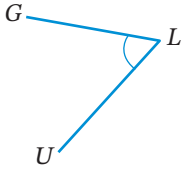
a



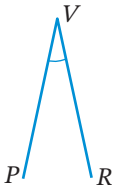
b



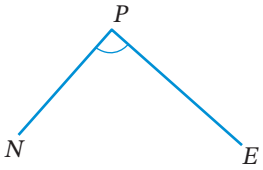
c



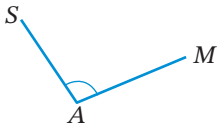
d



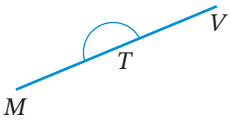
e



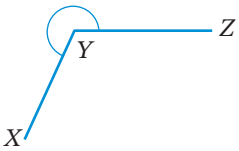
f



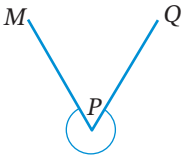
g



h



i



Foundation Standard Complex

2.04

7 **a** Write the complement of:

i 35° **ii** 78° **iii** 4°

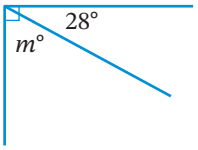
b Write the supplement of:

i 45° **ii** 100° **iii** 178°

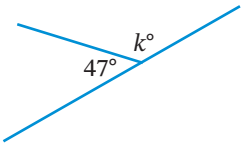
2.04

8 Find the value of each variable.

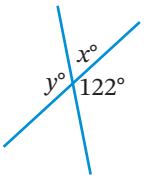
a



b



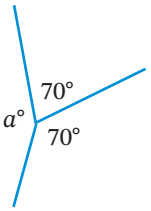
c



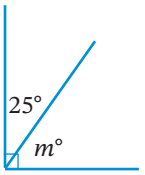
2.05

9 Find the value of each variable.

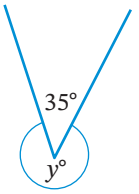
a



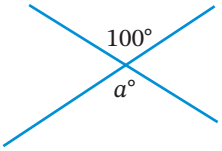
b



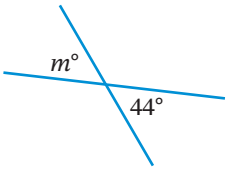
c



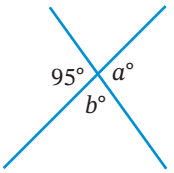
d



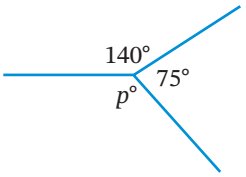
e



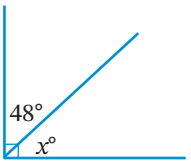
f



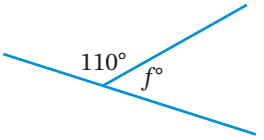
g



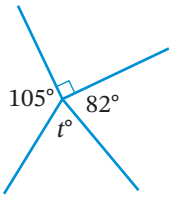
h



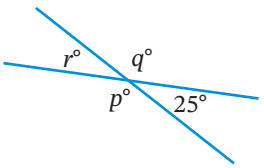
i



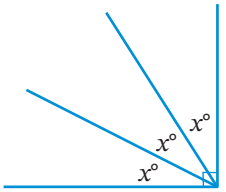
j



k



l



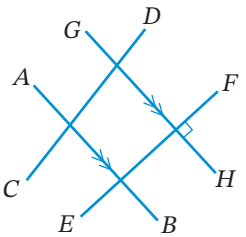
2.06

10 In this diagram, name 2 lines that:

a are parallel

b are perpendicular

c intersect.

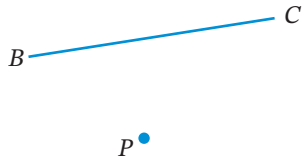


Foundation Standard Complex

9780170465557

11 Copy each diagram and use compasses to construct a line parallel to *BC* going through *P*. 2.06

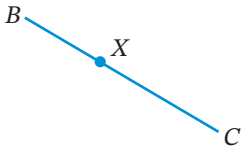
a



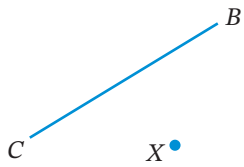
b

12 Copy each diagram and use compasses to construct a line perpendicular to *BC* going through *X*. 2.06

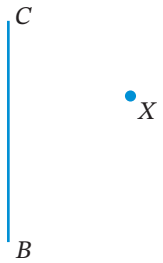
a



b

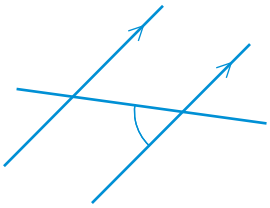


c

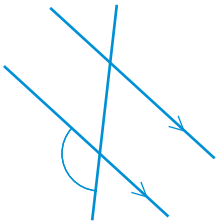


13 Copy each diagram and mark in the corresponding angle to the one shown. 2.07

a

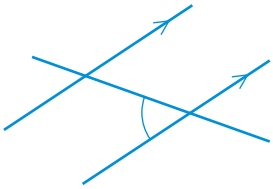


b

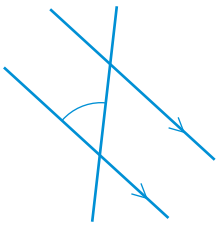


14 Copy each diagram and mark in the alternate angle to the one shown. 2.08

a

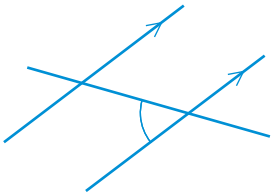


b

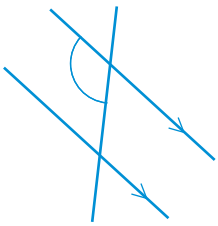


15 Copy each diagram and mark in the co-interior angle to the one shown. 2.09

a



b

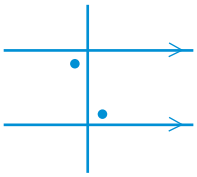


Foundation | Standard | Complex

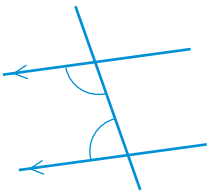
2.10

16 Label the marked pairs of angles as corresponding, alternate or co-interior.

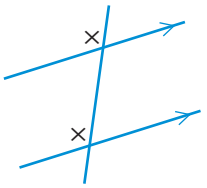
a



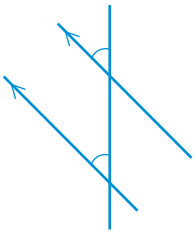
b



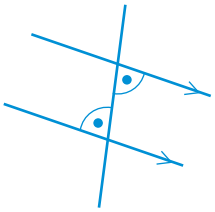
c



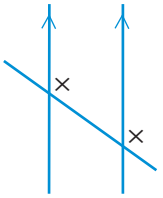
d



e



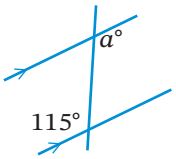
f



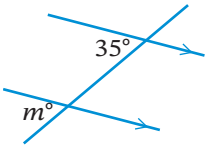
2.10

17 Find the value of each variable.

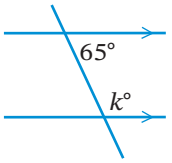
a



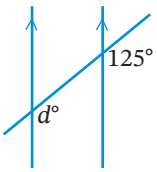
b



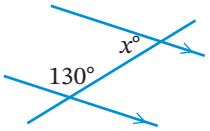
c



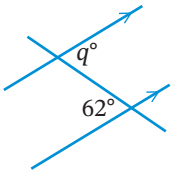
d



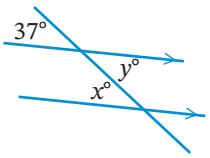
e



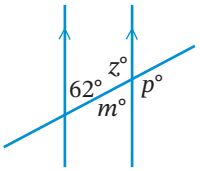
f



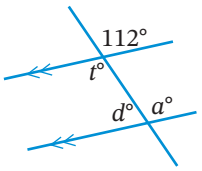
g



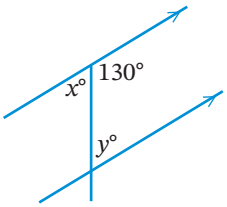
h



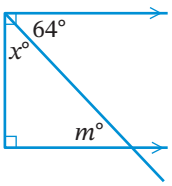
i



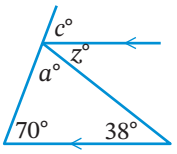
j



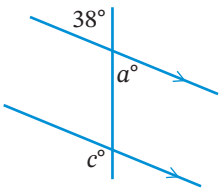
k



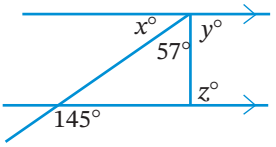
l



m



n

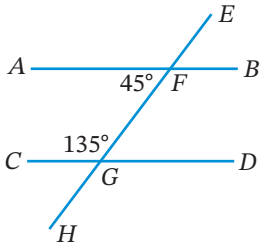


Foundation Standard Complex

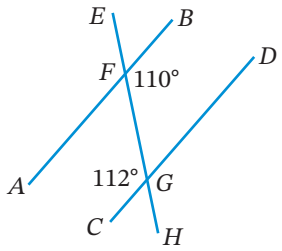
9780170465557

18 In each diagram below, determine whether $AB \parallel CD$, giving a reason. 2.11

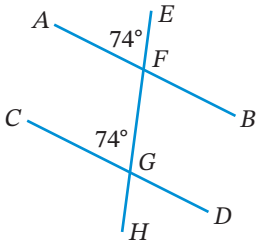
a



b



c



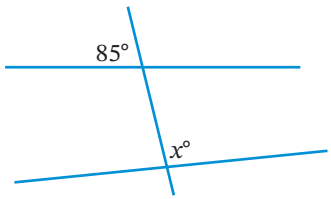
19 Which value of x would prove that $AB \parallel CD$? Select **A**, **B**, **C** or **D**. 2.11

A 95

B 85

C 275

D 5



Foundation Standard Complex

3

NUMBER

Whole numbers

We use numbers all the time: for measuring fabric to make a quilt or an item of clothing, measuring the temperature, rainfall or time, budgeting how we will spend our money, setting up training at the gym or keeping score in all different sports.

Chapter outline

		Proficiencies				
3.01	Rounding and estimating	U	F	PS	R	C
3.02	Multiplying numbers	U	F	PS	R	
3.03	Dividing numbers	U	F	PS	R	
3.04	Divisibility tests	U	F		R	
3.05	Extension: Roman numerals*	U	F			C
3.06	Index and expanded notations	U	F	PS	R	C
3.07	Square root and cube root	U	F		R	C
3.08	Prime and composite numbers	U	F		R	C
3.09	Factor trees	U	F		R	
3.10	Highest common factor	U	F		R	C
3.11	Lowest common multiple	U	F		R	C

*Extension

U = Understanding
F = Fluency
PS = Problem solving
R = Reasoning
C = Communication

Wordbank

composite number A number with more than 2 factors

divisibility test A rule for testing whether a number is divisible by a specific value, for example, divisible by 3

expanded notation A way of representing natural numbers to show the place value of its digits, for example, $7219 = 7 \times 10^3 + 2 \times 10^2 + 1 \times 10 + 9 \times 1$

factor A value that divides evenly into a given number, for example, 3 is a factor of 15

factor tree A diagram that lists the prime factors of a number

highest common factor (HCF) The largest factor shared by 2 or more numbers, also called the **greatest common divisor (GCD)**

index notation A way of writing powers for the repeated multiplication of a number, for example, 3^5

prime number A number with only 2 factors, 1 and itself

square root (of a number, symbol $\sqrt{\ }$) The positive value which, if squared, gives the number

Quiz
Wordbank 3

3 In this chapter you will:

Videos (9):

3.01 Estimating answers
3.02 Long multiplication • Mental multiplication
3.03 Dividing numbers • Mental division
3.04 Divisibility tests
3.09 Factor trees • Using factor trees to find square roots
3.10 Highest common factor

Twig videos (5):

3.01 Rounding: snails vs rockets
3.05 The Romans and numerals • Numbers: Life without numbers
3.06 The biggest number ever
3.08 The prime number code

PhET interactives (3):

3.02 Arithmetic • Area model Introduction • Area model Multiplication

Quizzes (6):

- Wordbank 3
- SkillCheck 3
- Mental skills 3A
- Mental skills 3B
- Language of Maths 3
- Test Yourself 3

Skillsheets (8):

3.01 Place value • Reading and writing large numbers • Rounding whole numbers
3.04 Factors and divisibility
3.05 Roman numerals
3.07 Square roots and cube roots
3.08 Prime and composite numbers
3.09 Prime factors by repeated division

Worksheets (9):

SkillCheck Calculation aids • Number grids
3.02 Whole numbers 6 • Calculation aids • Number grids
3.03 Whole numbers 7
3.04 Divisibility tests
3.06 Big numbers
3.08 Sieve of Eratosthenes
3.11 Perfect and amicable numbers

- Mind map: Whole numbers

Puzzles (5):

- SkillCheck Magic squares

3.02 Magic squares • Find the quote 2
3.03 The accidental detective
3.05 Roman numerals dominoes
3.07 Square root Snap

Presentation (1):

3.10 Highest common factors

Nelson MindTap

To access resources above, visit **cengage.com.au/nelsonmindtap**

- ✓ round whole numbers, estimate answers and check whether answers sound reasonable
- ✓ solve problems involving multiplying and dividing whole numbers, without using calculators
- ✓ understand and use divisibility tests
- ✓ (EXTENSION) interpret Roman numerals
- ✓ recognise index notation and powers of numbers
- ✓ use powers of 10 to write whole numbers in expanded notation
- ✓ recognise square roots and cube roots of numbers
- ✓ recognise prime and composite numbers
- ✓ use a factor tree to write a whole number as a product of its prime factors
- ✓ find lowest common multiples (LCM) and highest common factors (HCF) of 2 or more numbers

SkillCheck

ANSWERS ON P. 605

1 Is 691 odd or even? How can you tell?

2 Is 270 divisible by 5? How can you tell?

3 Evaluate each product.

a 7×3	**b** 6×5	**c** 4×8	**d** 3×6
e 9×4	**f** 5×8	**g** 7×7	**h** 6×9
i 8×8	**j** 9×7	**k** 4×6	**l** 7×8
m $-3 \times (-3)$	**n** 9×9	**o** 5×5	**p** $4 \times (-4)$

4 Write all the factors of:

a 20 **b** 12 **c** 19

5 Is 166 divisible by 2? How can you tell?

6 Evaluate each quotient.

a $24 \div 3$	**b** $35 \div 7$	**c** $24 \div 4$	**d** $36 \div 6$
e $20 \div 5$	**f** $72 \div 9$	**g** $48 \div 8$	**h** $32 \div 4$
i $28 \div 7$	**j** $45 \div 5$	**k** $54 \div 9$	**l** $63 \div 7$

7 Write the first 5 **multiples** of:

a 8 **b** 7 **c** 12

8 Is 34 divisible by 10? How can you tell?

Quiz SkillCheck 3

Worksheets Calculation aids

Number grids

Puzzle Magic squares

Rounding and estimating 3.01

Rounding numbers

When calculating with numbers, it is often useful to have a rough idea of the answer before actually working it out. Rounded numbers are easier to work with and to compare. For example, a town's population of 18 256 can be rounded to 18 300 (to the nearest hundred) because 18 256 is closer to 18 300 than 18 200.

Skillsheets Place value

Reading and writing large numbers

Rounding whole numbers

Video Rounding: Snails vs rockets

Rounding whole numbers

To round a number, 'cut' it at the required place and look at the **digit** in the next place:

- if the digit is less than 5 (that is 0, 1, 2, 3 or 4), round down
- if the digit is 5 or more (that is 5, 6, 7, 8 or 9), round up

Example 1

Round 8470 to the nearest hundred.

SOLUTION

Counting by hundreds, 8470 is between 8400 and 8500.

- In 8**4**70, the hundreds digit is **4**
- The digit in the next (tens) place, 7, is more than 5, so **round up** to 8500.

$8470 \approx 8500$ (rounded to the nearest hundred).

The symbol '≈' means 'approximately equal to'.

Example 2

Round 247 182 to the nearest thousand.

SOLUTION

In 247 182, the thousands digit is 7 and the next digit is 1.

$1 < 5$, so round down to 247 000. The thousands digit, 7, stays the same.

$247\,182 \approx 247\,000$ (rounded to the nearest thousand).

Estimating answers

A quick way of estimating an answer is to round each number in the calculation.

Video Estimating answers

Example 3

Estimate the answer to each expression.

a $631 + 280 + 51 + 43 + 96$ **b** 67×12 **c** $55 + 132 - 34 + 17 - 78$ **d** $510 \div 24$

SOLUTION

a $631 + 280 + 51 + 43 + 96 \approx 600 + 300 + 50 + 40 + 100$ Estimating

$\approx (600 + 300 + 100) + (50 + 40)$

$= 1000 + 90$

$= 1090$ (Exact answer = 1101)

b $67 \times 12 \approx 70 \times 10$ Estimating

$= 700$ (Exact answer = 804)

c $55 + 132 - 34 + 17 - 78 \approx 60 + 130 - 30 + 20 - 80$ Estimating

$= (60 + 20 - 80) + (130 - 30)$

$= 0 + 100$

$= 100$ (Exact answer = 92)

d $510 \div 24 \approx 500 \div 20$ Estimating

$= 50 \div 2$

$= 25$ (Exact answer = 21.25)

EXERCISE 3.01 ANSWERS ON P. 605

Rounding and estimating

U F PS R C

1 Round each number to the nearest hundred. EXAMPLE 1

C **a** 3148 **b** 49 028 **c** 2597 **d** 4 934 277

2 Round each number to the nearest thousand. EXAMPLE 2

C **a** 23 538 **b** 45 370 **c** 62 941 **d** 47 929

3 Round each number to the nearest ten.

C **a** 45 819 **b** 1699 **c** 8314 **d** 71 262

4 Round 64 218 to the nearest:

C **a** thousand **b** hundred **c** ten **d** ten thousand

5 Round 1 327 509 to the nearest:

C **a** thousand **b** ten thousand **c** ten **d** million

6 Estimate the answer for each expression. EXAMPLE 3

R **a** $27 + 11 + 87 + 142 + 64$ **b** $55 + 34 - 22 - 46 + 136$ **c** $684 + 903$

d $35 + 81 + 110 + 22 + 7$ **e** $517 - 96$ **f** $210 - 38 - 71 + 151 - 49$

g $766 - 353$ **h** 367×2 **i** 83×81

j 984×16 **k** $828 \div 3$ **l** $507 \div 7$

7 Over the holidays, 27 792 people visited a museum. Write this figure correct to the nearest hundred.

C

8 The extensions on Nina's house are quoted as costing \$17 464. Write this amount correct to the nearest \$100.

C

9 The crowd at the football grand final was 104 427. Round this figure to the nearest thousand.

10 Max buys bags of washers to use in his plumbing business. Each bag contains between 48 and 51 washers. Max has 11 bags in his supply cupboard. Approximately how many washers does he have? Select **A**, **B**, **C** or **D**.

PS R

A 50 **B** 550 **C** 1100 **D** 2500

11 The distance between Brisbane and Sydney is 998 km. Round this distance to the nearest:

C **a** 10 km **b** 100 km **c** 1000 km

12 The population of Australia is 25 921 840. Round this figure to the nearest thousand.

C

13 Write a number that can be rounded to:

R C **a** 370 **b** 5400 **c** 12 900 **d** 6000

Foundation Standard Complex

3.02 Multiplying numbers

Video
Long multiplication

Worksheets
Whole numbers 6
Calculation aids
Number grids

Puzzles
Magic squares
Find the quote 2

Interactives
Arithmetic
Area model Introduction
Area model Multiplication

Example 4

Evaluate each product.

a 243×6 **b** 573×36

SOLUTION

a

$$\begin{array}{r} 243 \times \\ 6 \\ \hline 1458 \end{array}$$

$243 \times 6 \approx 200 \times 6$ Check by estimating

$= 1200$

b

$$\begin{array}{r} 573 \times \\ 36 \\ \hline 3438 \\ 17\,190 \\ \hline 20\,628 \end{array}$$

$573 \times 36 \approx 600 \times 40$ Check by estimating

$= 24\,000$

Mental multiplication

Multiplying by:	Mental strategy
2, 4 or 8	Double once, 2 times or 3 times respectively
5	Halve, then multiply by 10 (because $\frac{1}{2} \times 10 = 5$)
9	Multiply by 10, then subtract the number
10	Insert 0 at the end of the number
100	Insert 00 at the end of the number

Video
Mental multiplication

Example 5

Evaluate each product mentally.

a 68×4 **b** 36×5 **c** 12×9 **d** 14×8

SOLUTION

a $68 \times 4 = 68 \times 2 \times 2$ Double twice

$68 \times 2 = (60 + 8) \times 2$ Double 68

$= 60 \times 2 + 8 \times 2$

$= 120 + 16$

$= 136$

$136 \times 2 = (130 + 6) \times 2$ Double 136

$= 130 \times 2 + 6 \times 2$

$= 260 + 12$

$= 272$

$\therefore 68 \times 4 = 272$ Estimate: $68 \times 4 \approx 70 \times 4 = 280$

b $36 \times 5 = 36 \times \frac{1}{2} \times 10$ Because $\frac{1}{2} \times 10 = 5$

$= 18 \times 10$ Insert a 0 at the end

$= 180$ Estimate: $36 \times 5 \approx 40 \times 5 = 200$

c $12 \times 9 = 12 \times 10 - 12$ Multiply by 10, then subtract the number

$= 120 - 12$ Estimate: $12 \times 9 \approx 12 \times 10 = 120$

$= 108$

d $14 \times 8 = 14 \times 2 \times 2 \times 2$ Double 3 times

$14 \times 2 = (10 + 4) \times 2$ Double 14

$= 10 \times 2 + 4 \times 2$

$= 20 + 8$

$= 28$

$28 \times 2 = (20 + 8) \times 2$ Double 28

$= 20 \times 2 + 8 \times 2$

$= 40 + 16$

$= 56$

$56 \times 2 = (50 + 6) \times 2$ Double 56

$= 50 \times 2 + 6 \times 2$

$= 100 + 12$

$= 112$

$\therefore 14 \times 8 = 112$ Estimate: $14 \times 8 \approx 14 \times 10 = 140$

EXERCISE 3.02 ANSWERS ON P. 605

Multiplying numbers

U F PS R

EXAMPLE 4

1 Copy and complete each multiplication.

a 34×7 **b** 219×5 **c** 28×16 **d** 325×21

2 Jesinta pays \$15 each time she plays tennis. How much does Jesinta pay to play 42 times a year?

Foundation Standard Complex

3 Evaluate each product.

a 14×8 **b** 238×3 **c** 344×9 **d** 506×7

e 1084×3 **f** 45×14 **g** 64×25 **h** 107×32

4 A bus route is 46 km long. A bus makes 12 trips in one day. Find the total distance travelled each day.

PS

5 Ned can type 76 words per minute. How many words can he type in 15 minutes?

PS

EXAMPLE 5

6 Estimate and then evaluate each product mentally.

R **a** 85×2 **b** 39×2 **c** 64×2 **d** 57×4

e 28×4 **f** 44×4 **g** 16×8 **h** 33×8

7 Estimate and then evaluate each product mentally.

R **a** 22×5 **b** 14×5 **c** 28×5 **d** 36×5

e 54×5 **f** 82×5 **g** 16×5 **h** 48×5

8 Estimate and then evaluate each product mentally.

R **a** 34×9 **b** 51×9 **c** 27×9 **d** 19×9

e 45×10 **f** 18×100 **g** 36×10 **h** 41×100

9 How many hours are there in one week? Select the correct answer **A**, **B**, **C** or **D**.

PS **A** 84 **B** 151 **C** 168 **D** 240

R

10 Mala ordered 4 pallets of oranges to sell. Each pallet holds 8 boxes of oranges. Each box contains 124 oranges.

PS

R **a** How many boxes of oranges did Mala order?

b How many oranges is this?

c The oranges are sold for $52 per box. How much money will Mala receive if she sells all boxes?

Foundation Standard Complex

Dividing numbers

3.03

Division is the opposite of multiplication.

Worksheet Whole numbers 7

Video Dividing numbers

Puzzle The accidental detective

Example 6

Evaluate each quotient.

a $318 \div 6$ **b** $1964 \div 5$

SOLUTION

a

$$\begin{array}{r} 53 \\ 6\overline{)3\,1^{1}8} \end{array}$$

$318 \div 6 = 53$

b

$$\begin{array}{r} 3\ \ 9\ \ 2\ \text{r}4 \\ 5\overline{)\,19^{4}6^{1}4} \end{array}$$

Write the remainder, 4, as a fraction over the divisor, 5:

$1964 \div 5 = 392\frac{4}{5}$

Mental division

Dividing by:	Mental strategy
2, 4 or 8	Halve once, 2 times or 3 times respectively
Dividing a multiple of 10 by:	
5	Divide by 10, then double
10	Delete 0 from the end of the number
100	Delete 00 from the end of the number

Video Mental division

Example 7

Evaluate each quotient mentally.

a $520 \div 8$ **b** $400 \div 100$ **c** $260 \div 5$ **d** $316 \div 4$

SOLUTION

a $520 \div 8 = 520 \div 2 \div 2 \div 2$ Halve 3 times

$520 \div 2 = (500 + 20) \div 2$ Halve 520

$= 500 \div 2 + 20 \div 2$

$= 250 + 10$

$= 260$

$260 \div 2 = (200 + 60) \div 2$ Halve 260

$= 200 \div 2 + 60 \div 2$

$= 100 + 30$

$= 130$

	$130 \div 2 = (100 + 30) \div 2$	Halve 130
	$= 100 \div 2 + 30 \div 2$	
	$= 50 + 15$	
	$= 65$	
	$\therefore 520 \div 8 = 65$	Estimate:
		$520 \div 8 \approx 520 \div 10 = 52$
b	$400 \div 100 = 4 \div 1 = 4$	Delete 00 from the end of the number
c	$260 \div 5 = 260 \div 10 \times 2$	Divide by 10, then double
	$= 26 \times 2$	Estimate:
	$= 52$	$260 \div 5 \approx 300 \div 5 = 60$
d	$316 \div 4 = 316 \div 2 \div 2$	Halve 2 times
	$316 \div 2 = (300 + 16) \div 2$	Halve 316
	$= 300 \div 2 + 16 \div 2$	
	$= 150 + 8$	
	$= 158$	
	$158 \div 2 = (150 + 8) \div 2$	Halve 158
	$= 150 \div 2 + 8 \div 2$	
	$= 75 + 4$	
	$= 79$	
	$\therefore 316 \div 4 = 79$	Estimate:
		$316 \div 4 \approx 320 \div 4 = 80$

EXERCISE 3.03 ANSWERS ON P. 605

Dividing numbers

U F PS R

EXAMPLE 6

1 Evaluate each quotient.

a $44 \div 4$ **b** $68 \div 2$ **c** $84 \div 7$ **d** $210 \div 3$

e $105 \div 7$ **f** $390 \div 5$ **g** $441 \div 3$ **h** $861 \div 7$

2 Evaluate each quotient, expressing the remainder as a fraction.

a $49 \div 4$ **b** $688 \div 3$ **c** $952 \div 6$ **d** $1815 \div 7$

3 At Westvale Catholic College, there are 135 students in Year 7. If they are placed evenly into 5 classes, how many students are in each class? Select the correct answer **A**, **B**, **C** or **D**.

PS R

A 21 **B** 25 **C** 27 **D** 29

4 Divide a restaurant bill of \$204 evenly among 6 people.

Foundation Standard Complex

5 Evaluate each quotient mentally.

R **a** 320 ÷ 10 **b** 8000 ÷ 100 **c** 60 000 ÷ 100 **d** 1800 ÷ 10

e 410 ÷ 5 **f** 90 ÷ 5 **g** 230 ÷ 5 **h** 600 ÷ 5

6 Evaluate each quotient mentally.

R **a** 144 ÷ 2 **b** 232 ÷ 2 **c** 648 ÷ 2 **d** 970 ÷ 2

e 216 ÷ 4 **f** 488 ÷ 4 **g** 872 ÷ 8 **h** 208 ÷ 8

7 A factory makes 1232 boxes of cereal. These are packed into cartons that each hold 8 boxes of cereal. How many cartons are needed to pack these boxes?

PS R

EXAMPLE 7

Divisibility tests 3.04

Is 840 divisible by 3? How do you know?

It is often useful to know whether a number is divisible by another number or not. Numbers that divide evenly (without a remainder) into a bigger number are called the **factors** or **divisors** of the number. **Divisibility tests** are rules for finding whether a number is divisible by specific values.

Worksheet Divisibility tests

Skillsheet Factors and divisibility

Divisibility tests

Divisible by:	Test
2	Last digit 0, 2, 4, 6 or 8
3	Sum of digits divisible by 3
4	Last 2 digits form a number divisible by 4 Or: Sum of double the tens digit and the units digit divisible by 4
5	Last digit 0 or 5
6	Divisible by 2 and 3
8	Last 3 digits form a number divisible by 8
9	Sum of digits divisible by 9
10	Last digit 0

Foundation Standard Complex

Video
Divisibility tests

Example 8

Test whether 2016 is divisible by:

a 3 **b** 4 **c** 6 **d** 8

SOLUTION

a Sum of digits $= 2 + 0 + 1 + 6 = 9$, which is divisible by 3.

So 2016 is divisible by 3. (Check: $2016 \div 3 = 672$)

b The last 2 digits of 2016 are 16.

16 is divisible by 4.

So 2016 is divisible by 4.

OR: Sum of double the tens digit and the units digit $= 2 \times 1 + 6 = 8$.

8 is divisible by 4.

So 2016 is divisible by 4. (Check: $2016 \div 4 = 504$)

c 2016 ends in 6, so it is divisible by 2 (even).

2016 is divisible by 3 (from part **a**).

So 2016 is divisible by 6 (because it is divisible by 2 and 3). (Check: $2016 \div 6 = 336$)

d The last 3 digits of 2016 are $016 = 16$.

16 is divisible by 8.

So 2016 is divisible by 8. (Check: $2016 \div 8 = 252$)

Note:

- If a number is divisible by 6, it must also be divisible by 2 and 3 (because $2 \times 3 = 6$)
- If a number is divisible by 10, it must also be divisible by 2 and 5 (because $2 \times 5 = 10$)
- If a number is divisible by 4, it must also be divisible by 2 (because $2 \times 2 = 4$)
- If a number is divisible by 9, it must also be divisible by 3 (because $3 \times 3 = 9$)
- If a number is divisible by 8, it must also be divisible by 2 and 4 (because $2 \times 4 = 8$)
- If a number is divisible by another number, it must also be divisible by the **factors** of that number

EXERCISE 3.04 ANSWERS ON P. 605

Divisibility tests

U F R C

EXAMPLE 8

1 Test whether each number is divisible by 3.

a 140 **b** 612 **c** 315 **d** 928
e 209 **f** 525 **g** 132 **h** 1652

2 Test whether each number is divisible by 2.

a 117 **b** 205 **c** 6196 **d** 340

3 Test whether each number is divisible by 6. Note that the numbers in parts **a** to **d** have already been tested for divisibility by 3 in question **1**.

a 140 **b** 612 **c** 315 **d** 928
e 475 **f** 303 **g** 864 **h** 1278

4 Test whether each number is divisible by 4.

a 2040 **b** 518 **c** 365 **d** 242
e 356 **f** 728 **g** 4176 **h** 817

5 Test whether each number is divisible by 9.

a 812 **b** 309 **c** 567 **d** 243
e 837 **f** 462 **g** 6444 **h** 3111

6 Test whether 1964 is divisible by 8.

7 Explain by just looking at the last digit why:

R **a** 409 is not divisible by 4 **b** 316 is not divisible by 5
C **c** 2015 is not divisible by 8 **d** 343 is not divisible by 6
e 472 is not divisible by 10 **f** 511 is not divisible by 12

8 Which number is divisible by both 4 and 5? Select the correct answer **A**, **B**, **C** or **D**.

R **A** 10 **B** 15 **C** 20 **D** 25

9 If a number is divisible by 3 and 4, what other number must it also be divisible by?

R

10 Write a number between 50 and 100 that is divisible by:

R **a** 3 and 5 **b** 4 and 5 **c** 6 and 7 **d** 2 and 6

11 Which number is divisible by 6, 8 and 9? Select **A**, **B**, **C** or **D**.

R **A** 48 **B** 102 **C** 126 **D** 144

12 There are several complicated divisibility tests for 7. Here is the most popular one:

R
- Remove the last digit of the number and double it
- Subtract it from the rest of the number
- If the difference is 0 or divisible by 7, the number is divisible by 7

Foundation Standard Complex

For example, to test 112:

Last digit = 2, double 2 = 4

11 – 4 = 7, which is divisible by 7

So 112 is divisible by 7.

Test whether each number is divisible by 7.

a 357 **b** 1013 **c** 6258 **d** 901

13 Here is a divisibility test for 11.

- Add the digits in the odd places of the number
- Add the digits in the even places of the number
- Subtract the 2 sums
- If the difference is 0 or divisible by 11, the number is divisible by 11

For example, to test 5808:

Add digits in odd places: 5 + 0 = 5

Add digits in even places: 8 + 8 = 16

16 – 5 = 11, which is divisible by 11

So 5808 is divisible by 11.

R Test whether each number is divisible by 11.

a 814 **b** 2728 **c** 4051 **d** 6105

Quiz
Mental skills 3A

☆ MENTAL SKILLS 3A ANSWERS ON P. 605 Maths without calculators

Multiplying by a multiple of 10

Place value allows us to simply add 0s to the end of a number whenever we multiply by a power of 10.

1 Consider these examples.

a $37 \times 10 = 370$ **b** $45 \times 100 = 4500$

c $16 \times 1000 = 16\,000$ **d** $100 \times 1000 = 100\,000$

e $7 \times 90 = 7 \times 9 \times 10 = 63 \times 10 = 630$

f $5 \times 400 = 5 \times 4 \times 100 = 20 \times 100 = 2000$

g $12 \times 300 = 12 \times 3 \times 100 = 36 \times 100 = 3600$

h $40 \times 800 = 4 \times 10 \times 8 \times 100 = 4 \times 8 \times 10 \times 100 = 32 \times 100 = 32\,000$

Foundation Standard Complex

2 Now evaluate each product.

a	18×100	**b**	26×1000	**c**	$77 \times 10\,000$	**d**	10×100
e	315×1000	**f**	1000×1000	**g**	294×10	**h**	475×100
i	3×80	**j**	8×200	**k**	6×50	**l**	7×30
m	2×6000	**n**	11×900	**o**	4×400	**p**	5×700
q	5×80	**r**	25×20	**s**	300×60	**t**	900×4000

Extension: Roman numerals 3.05

EXTENSION

Skillsheet Roman numerals

Puzzle Roman numerals dominoes

Video The Romans and numerals

In 117 CE, the Roman Empire ruled all of Europe, north Africa and the Middle East, and Roman numerals were widely used in western civilisation until the end of the 16th century, when they were gradually replaced by our modern Hindu-Arabic numerals. Roman numerals use letters of the alphabet and are still often seen today.

The ancient Romans used the following numerals:

1	2	3	4	5	6	7	8	9	10
I	II	III	IV	V	VI	VII	VIII	IX	X

20	30	40	50	60	90	100	400	500	1000
XX	XXX	XL	L	LX	XC	C	CD	D	M

The Romans had an unusual method of writing numbers involving 4 or 9, to avoid writing the same letter more than 3 times in a row. A smaller number written before a larger number meant a subtraction ('minus').

- Instead of writing 4 as IIII, they wrote **IV** meaning V – I (that is $5 - 1 = 4$).
- Instead of writing 9 as VIIII they wrote **IX** meaning X – I (that is $10 - 1 = 9$).
- For 40, they wrote **XL** (that is $50 - 10 = 40$).
- For 90, they wrote **XC** (that is $100 - 10 = 90$).

Example 9

Write each number in Roman numerals.

a 23 **b** 46 **c** 279

SOLUTION

a $23 = 20 + 3$
$= \text{XX} + \text{III}$
$= \text{XXIII}$

b $46 = 40 + 6$
$= \text{XL} + \text{VI}$
$= \text{XLVI}$

c $279 = 200 + 70 + 9$
$= \text{CC} + \text{LXX} + \text{IX}$
$= \text{CCLXXIX}$

EXTENSION

Example 10

Write each Roman numeral as a number.

a LXXIII **b** CI **c** CDXXVIII

SOLUTION

a LXXIII = LXX + III
= 70 + 3
= 73

b CI = C + I
= 100 + 1
= 101

c CDXXVIII = CD + XX + VIII
= 400 + 20 + 8
= 428

EXERCISE 3.05 ANSWERS ON P. 606

Roman numerals

U F

EXAMPLE 9

1 Write each number using Roman numerals.

C **a** 27 **b** 52 **c** 79 **d** 93
e 2600 **f** 314 **g** 928 **h** 3476
i 165 **j** 230 **k** 2482 **l** 109

EXAMPLE 10

2 Titus, a student in ancient Rome, wrote these numerals. Rewrite them as modern numerals.

C **a** XXVI **b** XL **c** CCLXIV **d** LIV
e LXXVII **f** XLV **g** CCCXXXIX **h** DXXVIII
i MMCLXII **j** MCMXC **k** XCVIII **l** MDVII

3 **a** Where do you see Roman numerals being used today?

C **b** Why do you think Roman numerals are no longer widely used?

4 The Roman word for hundred was 'centum', which is why C stands for 100. List some words beginning with 'cent' that mean one hundred of something.

C

5 **a** Write this year in Roman numerals.

C **b** Which year of the 21st century will use the most letters when written in Roman numerals?

6 Find out what the Roman numerals $\overline{\text{V}}$ and $\overline{\text{X}}$ mean.

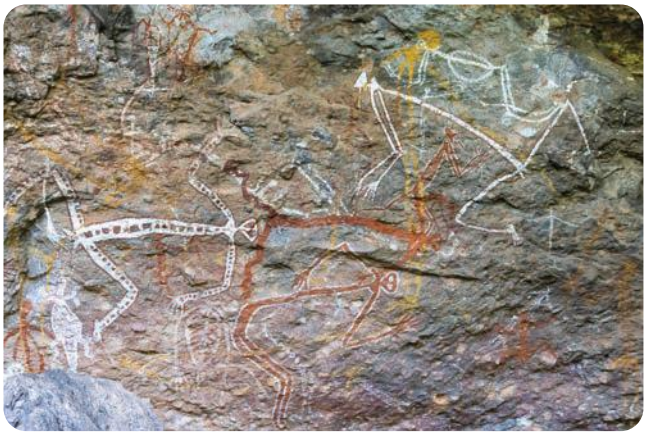
C

Foundation Standard Complex

INVESTIGATION

Aboriginal numbers

First Nations Australians relied on story-telling and using the spoken language rather than writing, so they used words rather than symbols to describe numbers. Different regions had their own language and counting systems, and different names for numbers.



Alamy Stock Photo/Moshe Einhorn

The **Belyando River people** of central Queensland use the following words to name their numbers:

1 = wogin 2 = booleroo

3 = booleroo wogin 4 = booleroo booleroo

The **Kamilaroi people** live in northern New South Wales, including the regions surrounding Moree and Tamworth. They use 3 words to name their numbers.

1 = mal 2 = bularr 3 = guliba

4 = bularr bularr 5 = bularr guliba 6 = guliba guliba

Body parts such as fingers, arms, shoulders, neck and head are also used by some First Nations peoples to represent numbers when counting.

1 How do the Belyando River people form words for the numbers 3 and 4?

2 How do the Kamilaroi people form words for 4, 5 and 6?

3 Write the answer to each expression, using the correct Aboriginal words:

a wogin + booleroo wogin **b** guliba × bularr

c bularr + bularr + mal **d** booleroo × booleroo

e guliba guliba – guliba **f** bularr bularr – mal

4 Find out more about how body parts are used to represent numbers by some First Nations Australians.

3.05

Video Numbers: Life without numbers

3.06 Index and expanded notations

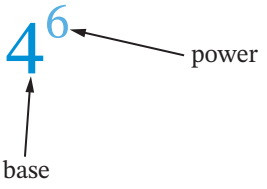
Index notation

Video
The biggest number ever

The use of powers allows us to write repeated multiplication in a shorter way.

$4^2 = 4 \times 4$	'4 squared' or '4 to the power of 2'
$4^3 = 4 \times 4 \times 4$	'4 cubed' or '4 to the power of 3'
$4^4 = 4 \times 4 \times 4 \times 4$	'4 to the power of 4'
$4^5 = 4 \times 4 \times 4 \times 4 \times 4$	'4 to the power of 5'
$4^6 = 4 \times 4 \times 4 \times 4 \times 4 \times 4$	'4 to the power of 6'

In 4^6, the number 4 is called the **base** and is the number that is repeated in the multiplication. The small raised number 6 is called the **power** or **index**.



The power tells us how many times the base appears in the repeated multiplication.
The power of 6 in 4^6 tells us that 4 is in the repeated multiplication 6 times.

Example 11

Write each expression using **index notation**.

a $6 \times 6 \times 6 \times 6 \times 6$

b $7 \times 7 \times 7 \times (-2) \times (-2) \times (-2) \times (-2)$

SOLUTION

a $6 \times 6 \times 6 \times 6 \times 6 = 6^5$

b $7 \times 7 \times 7 \times (-2) \times (-2) \times (-2) \times (-2) = 7^3 \times (-2)^4$

Example 12

Evaluate each expression.

a 11^2 **b** $(-3)^3$ **c** 2^6

SOLUTION

a $11^2 = 11 \times 11$
$= 121$

b $(-3)^3 = (-3) \times (-3) \times (-3)$
$= -27$

c $2^6 = 2 \times 2 \times 2 \times 2 \times 2 \times 2$
$= 64$

Powers of 10

Notice this pattern when we raise 10 to a power.

$10^2 = 10 \times 10$	$= 100$	2 0s	hundred
$10^3 = 10 \times 10 \times 10$	$= 1000$	3 0s	thousand
$10^4 = 10 \times 10 \times 10 \times 10$	$= 10\,000$	4 0s	10 thousand
$10^5 = 10 \times 10 \times 10 \times 10 \times 10$	$= 100\,000$	5 0s	100 thousand
$10^6 = 10 \times 10 \times 10 \times 10 \times 10 \times 10$	$= 1\,000\,000$	6 0s	million
$10^7 = 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10$	$= 10\,000\,000$	7 0s	10 million

The power of 10 tells us how many 0s follow the 1 in the numeral.

Expanded notation

8752 written in **expanded notation** is $8 \times 1000 + 7 \times 100 + 5 \times 10 + 2 \times 1$.

This shows 8752 being 'expanded' to show the value of its individual digits added together, using place value.

We can also write expanded notation using powers of 10.

$8752 = 8 \times 10^3 + 7 \times 10^2 + 5 \times 10 + 2 \times 1$.

Expanded notation is a way of representing a whole number using each of its digits being multiplied by a power of 10, to show its place value.

Example 13

Write each whole number using expanded notation.

a 893 **b** 16 795 **c** 2019

SOLUTION

a $893 = (8 \times 100) + (9 \times 10) + (3 \times 1)$

$= 8 \times 10^2 + 9 \times 10 + 3 \times 1$

b $16\,795 = (1 \times 10\,000) + (6 \times 1000) + (7 \times 100) + (9 \times 10) + (5 \times 1)$

$= 1 \times 10^4 + 6 \times 10^3 + 7 \times 10^2 + 9 \times 10 + 5 \times 1$

c $2019 = (2 \times 1000) + (1 \times 10) + (9 \times 1)$

$= 2 \times 10^3 + 1 \times 10 + 9 \times 1$

We leave out the '0×10^2' in the hundreds place here as this 0 is a place holder.

EXERCISE 3.06 ANSWERS ON P. 606

Index and expanded notations

U F PS R C

1 Write each expression using index notation. EXAMPLE 11

C **a** $8 \times 8 \times 8 \times 8$ **b** $7 \times 7 \times 7 \times 7 \times 7 \times 7$

c $-5 \times (-5) \times (-5)$ **d** $1 \times 1 \times 1 \times 1 \times 1$

e $13 \times 13 \times 13 \times 13$ **f** 10

Foundation Standard Complex

g $(-4) \times (-4) \times 9 \times 9 \times 9$ **h** $2 \times 2 \times 12 \times 12$

i $3 \times 3 \times 11 \times 11 \times 11 \times 11$ **j** $15 \times (-6) \times (-6) \times 15$

k $2 \times 8 \times 2 \times 8 \times 2$ **l** $10 \times 10 \times 20$

2 Write each expression in expanded form (for example, $2^4 = 2 \times 2 \times 2 \times 2$).

C **a** 3^5 **b** 5^2 **c** 7^4

d $(-6)^3$ **e** $2^4 \times 4^2$ **f** $12^3 \times 11^6$

EXAMPLE 12

3 Evaluate each expression.

C **a** 4^2 **b** 5^3 **c** 2^5 **d** 10^6

e 3^5 **f** 7^4 **g** 1^3 **h** $(-2)^4$

i 3^3 **j** $(-1)^2$ **k** $(-9)^5$ **l** 4^6

EXAMPLE 13

4 Write each number using expanded notation.

R **a** 234 **b** 7040 **c** 28 675 **d** 92

C **e** 7361 **f** 63 928 **g** 876 **h** 174 922

i 94 631 j 3692 **k** 149 **l** 8 706 945

5 Simplify each number represented in expanded notation.

R **a** $7 \times 10 + 3 \times 1$

C **b** $4 \times 10^5 + 9 \times 10^4 + 5 \times 10^2 + 3 \times 10 + 7 \times 1$

c $1 \times 10^4 + 8 \times 10^3 + 3 \times 10^2 + 4 \times 10 + 5 \times 1$

d $6 \times 10^3 + 8 \times 10^2 + 1 \times 1$

e $3 \times 10^6 + 6 \times 10^5 + 7 \times 10^4 + 2 \times 10^3 + 5 \times 10 + 9 \times 1$

f $4 \times 10^2 + 6 \times 10 + 2 \times 1$

Worksheet
Big numbers

6 Copy and complete this table of powers of 10.

R
C

Power of 10	Number	Name
10^1	10	ten
10^2	100	hundred
10^3	1000	
10^6	1 000 000	
10^9		billion
10^{12}	1 000 000 000 000	
10^{15}		quadrillion
	1 000 000 000 000 000 000	quintillion
10^{21}		sextillion
10^{24}		septillion
	1 000 000 000 000 000 000 000 000 000	octillion
10^{30}		nonillion
10^{33}	1 000 000 000 000 000 000 000 000 000 000 000	decillion

Foundation Standard Complex

7 Anton has just finished writing a sequel to a novel he wrote 2 years ago. The first book had $7 \times 10^5 + 7 \times 10^4 + 3 \times 10^3 + 6 \times 10^2 + 9 \times 10 + 2 \times 1$ words in it while the sequel has 3 556 480 words. Anton says that the sequel is about 6 times bigger that the original novel. Is he correct? Give reasons for your answer.

PS R C

TECHNOLOGY

Powers

1 Set up a spreadsheet to calculate powers by first entering the information shown below.

	A	B	C	D	E
1	**Number:**	**2**	**3**	**5**	**7**
2	1				
3	2				
4	3				
5	4				
6	5				
7	6				
8	7				
9	8				
10	9				
11	10				
12	11				
13	12				

2 In cell B2, enter **=$B$1^A2** to calculate 2^1, where ^ in a spreadsheet formula means 'to the power of'. We also use **$B$1** instead of **B1** because when we copy this formula, we want it to **always** refer to the **2** in cell B1, not another cell. This is called **absolute cell referencing**. We use this technique to maintain a particular value in a cell without changing it when writing a formula. However, we use **A2** instead of **$A$2** because when we copy this formula, we want it to refer to the different powers shown in column A to calculate 2^1, 2^2, 2^3, and so on. This is called **relative cell referencing**.

3 Click on cell B2 and Fill Down to cell B13. Your spreadsheet will now show the first 12 powers of 2, up to $2^{12} = 4096$.

4 Enter similar formulas for cells C2, D2 and E2, and **Fill Down** for columns C, D and E to show the first 12 powers of 3, 5 and 7.

5 If a number is too long to show in a cell, it will either look something like **2·44E + 08** or **########**. For cells that show something like **2·44E + 08**, right-click, select **Format cells**, **Number** and **0** for decimal places. **########** means the column is not wide enough to show all digits of the number. You need to widen the column until you can see all digits.

Foundation Standard Complex

DID YOU KNOW?

Googol vs Google

The number 10^{100}, the **googol**, is 1 followed by one hundred zeros. The name 'googol' was created by the 9-year-old nephew of American mathematician Dr Edward Kasner. The number 10^{googol}, that is 1 followed by a googol zeros, is called the **googolplex**. The googol is a very big number but it is rarely used for practical purposes. Even the number of particles in the observable universe, estimated at being between 10^{72} and 10^{87}, is less than a googol!

The Internet search engine Google was named after the googol, to reflect the huge size of the World Wide Web. It was created in 1996 by two Stanford University students, Larry Page and Sergey Brin. They even named Google's global headquarters in California the Googleplex. Google is a powerful search engine because it can find information from over a trillion (a million million) web pages in less than 1 second.

How many googols are there in a googolplex?

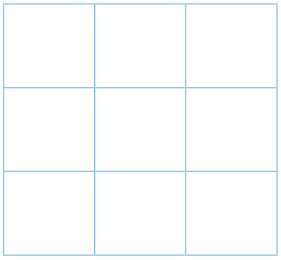
Shutterstock.com/Ken Wolter

INVESTIGATION

Perfect squares

You need: 100 square tiles

Here is an example of using square tiles to build a perfect square.



This perfect square requires 9 square tiles.

1 Make the smallest perfect square you can using as few tiles as possible. Draw a diagram to show how many tiles you used as well as the length and width of your perfect square.

2 Copy and complete the table for Square 1.

Square	1	2	3	4	5	6	7	8	9	10
Tiles used										
Length										
Width										

3 Make the *next* smallest perfect square you can. Draw a diagram to show how many tiles you used as well as the length and width of the square.

4 Make the *next* smallest perfect square you can, draw a diagram and write down the length and width of your square.

5 Continue making perfect squares using the tiles and complete the table as you proceed.

Questions

1 What does it mean for a number to be a perfect square (or a square number)? Can any number be considered a perfect square? Explain why or why not.

2 What does it mean to find the square root of a number? What part of your perfect square diagram would represent the square root?

3 What is the relationship between squaring a number and finding the square root of a number? Use a diagram to help with your explanation.

4 Find the square root of the square numbers in the table below. Use diagrams to help if required.

Square number	16	25	36	100	169	225	289	400
Square root								

Square root and cube root 3.07

Square and square root

Skillsheet Square roots and cube roots

Puzzle Square root Snap

Squaring a number means raising it to the **power of 2**.

For example, 'squaring 7' or 'the square of 7' means $7^2 = 7 \times 7 = 49$, read '7 squared'. Also, 49 is called a **square number** or **perfect square** because it is the square of a whole number.

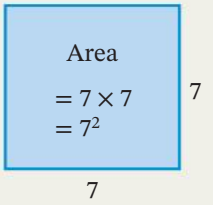
It is called '7 squared' because it is the area of a square of side 7 units.

The opposite of squaring is the **square root** (symbol $\sqrt{\ }$).

Square root

The **square root** $\sqrt{\ }$ of a number is the positive value which, when squared, gives that number.

For example, $\sqrt{49} = 7$ is read 'the square root 49', because $7^2 = 7 \times 7 = 49$.



Example 14

Evaluate:

a the square root of 64 **b** $\sqrt{9}$ **c** $\sqrt{25}$

SOLUTION

a The square root of 64 $= \sqrt{64} = 8$ because $8^2 = 8 \times 8 = 64$

b $\sqrt{9} = 3$ because $3^2 = 3 \times 3 = 9$

c $\sqrt{25} = 5$ because $5^2 = 5 \times 5 = 25$

Cube and cube root

Cubing a number means raising it to the **power of 3**.

For example, 'cubing 7' or 'the cube of 7' means $7^3 = 7 \times 7 \times 7 = 343$, read '7 cubed'. Also, 343 is called a **cube number** or **perfect cube** because it is the cube of a whole number.

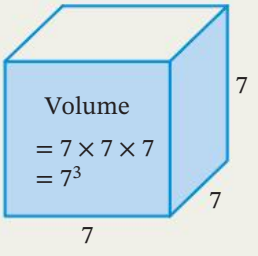
It is called '7 cubed' because it is the volume of a cube of side 7 units.

The opposite of cubing is the **cube root** (symbol $\sqrt[3]{\ }$).

Cube root

The **cube root** $\sqrt[3]{\ }$ of a number is the value which, when cubed, gives that number.

For example, $\sqrt[3]{343} = 7$ is read 'the cube root of 343', because $7^3 = 7 \times 7 \times 7 = 343$.



Example 15

Evaluate:

a the cube root of 125 **b** $\sqrt[3]{8}$ **c** $\sqrt[3]{-64}$

SOLUTION

a The cube root of 125 = $\sqrt[3]{125} = 5$ because $5^3 = 5 \times 5 \times 5 = 125$

b $\sqrt[3]{8} = 2$ because $2^3 = 2 \times 2 \times 2 = 8$

c $\sqrt[3]{-64} = -4$ because $(-4)^3 = -4 \times (-4) \times (-4) = -64$

Note: The square root of a number is positive but a cube root can be positive or negative. However, the square root and cube root of 0 are both 0.

Example 16

Estimate the value of $\sqrt{40}$.

SOLUTION

There is no *exact* answer for the square root of 40, because there isn't a number which, if squared, equals 40 exactly. However, we can find a decimal whose square is close to 40.

Noting that:

$5^2 = 25$

$6^2 = 36$

$7^2 = 49$

we can tell that $\sqrt{40}$ must lie somewhere between 6 and 7, and closer to 6 because 40 is closer to 36 than 49.

So we can estimate that $\sqrt{40} \approx 6.3$.

(In fact, $6.3^2 = 39.69$).

EXERCISE 3.07 ANSWERS ON P. 606

Square root and cube root

U F R C

1 Evaluate each square number.

C **a** 9^2 **b** 17^2 **c** 10^2

2 The x^2 key on a calculator can be used to square a number. For example, to find 9^2, press 9 x^2 = . Use your calculator to evaluate each square number.

a 7^2 **b** 13^2 **c** 18^2

d 1^2 **e** 5^2 **f** 20^2

g 16^2 **h** 34^2 **i** 15^2

j $(-3)^2$ **k** $(-7)^2$ **l** $(-13)^2$

Foundation Standard Complex

EXAMPLE 14

3 Use your answers from question **2** to find:

R **a** $\sqrt{400}$ **b** $\sqrt{225}$ **c** $\sqrt{324}$
d $\sqrt{256}$ **e** $\sqrt{169}$ **f** $\sqrt{1156}$

4 Copy and complete this table.

Number, x	1	2	3	4	5	6	7	8	9	10	11	12
Number squared, x^2												

5 Which number below is a square number? Select the correct answer **A**, **B**, **C** or **D**.

A 32 **B** 36 **C** 40 **D** 45

6 Use your answers from the table in question **4** to find:

a the square root of 100 **b** $\sqrt{36}$ **c** $\sqrt{121}$
d $\sqrt{16}$ **e** $\sqrt{1}$ **f** the square root of 81
g $\sqrt{64}$ **h** the square root of 144 **i** $\sqrt{25}$

7 The [√] key on a calculator can be used to find the square root of a number. For example, to find $\sqrt{64}$, press [√] 64 [=]. Use your calculator to find each square root.

a $\sqrt{49}$ **b** $\sqrt{81}$ **c** $\sqrt{625}$
d $\sqrt{484}$ **e** $\sqrt{1764}$ **f** $\sqrt{361}$
g $\sqrt{900}$ **h** $\sqrt{784}$ **i** $\sqrt{256}$
j $\sqrt{196}$ **k** $\sqrt{400}$ **l** $\sqrt{3136}$

8 There are 2 numbers whose square root is itself, that is, $\sqrt{x} = x$.

R What are the 2 numbers?

9 Evaluate each cube number.

C **a** 4^3 **b** 11^3 **c** 1^3

10 The [x^3] key on a calculator can be used to cube a number. For example, to find 4^3, press 4 [x^3] [=]. Use your calculator to find each cube number.

a 7^3 **b** 16^3 **c** 5^3
d 10^3 **e** 20^3 **f** 17^3
g 35^3 **h** $(-8)^3$ **i** $(-5)^3$

11 Use your answers from question **10** to find:

R **a** $\sqrt[3]{-512}$ **b** $\sqrt[3]{8000}$ **c** $\sqrt[3]{42\,875}$
d $\sqrt[3]{-125}$ **e** $\sqrt[3]{4913}$ **f** $\sqrt[3]{4096}$

Foundation Standard Complex

12 Copy and complete this table.

Number, x	1	2	3	4	5	6	7	8	9	10	11	12
Number cubed, x^3												

EXAMPLE 15

13 Which number below is a cube number? Select **A**, **B**, **C** or **D**.

A 192 **B** 512 **C** 625 **D** 800

14 Use your answers from question 12 to find:

R **a** the cube root of 512 **b** $\sqrt[3]{729}$ **c** $\sqrt[3]{125}$
d $\sqrt[3]{1}$ **e** $\sqrt[3]{27}$ **f** the cube root of 1728
g $\sqrt[3]{64}$ **h** the cube root of 1331 **i** $\sqrt[3]{1000}$

15 The $\sqrt[3]{\ }$ key on a calculator can be used to find the cube root of a number. For example, to find $\sqrt[3]{64}$, press $\sqrt[3]{\ }$ 64 = . Use your calculator to find each cube root.

a $\sqrt[3]{4096}$ **b** $\sqrt[3]{2744}$ **c** $\sqrt[3]{10\,648}$
d $\sqrt[3]{64\,000}$ **e** $\sqrt[3]{19\,683}$ **f** $\sqrt[3]{9261}$

EXAMPLE 16

16 Use your answers from the table in question **4** to determine between which 2 consecutive whole numbers $\sqrt{27}$ must lie. Select **A**, **B**, **C** or **D**.

R **A** 26 and 28 **B** 13 and 14 **C** 4 and 5 **D** 5 and 6

17 Determine between which 2 consecutive numbers $\sqrt{15}$ must lie. Select **A**, **B**, **C** or **D**.

R **A** 7 and 8 **B** 3 and 4 **C** 196 and 197 **D** 4 and 5

18 Estimate the value of each root, then use your calculator to check.

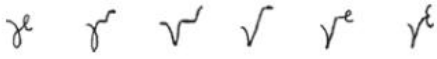
R **a** $\sqrt{50}$ **b** $\sqrt{32}$ **c** $\sqrt{96}$
d $\sqrt[3]{71}$ **e** $\sqrt[3]{900}$ **f** $\sqrt[3]{184}$

DID YOU KNOW?

The radical symbol

The formal name for the square root symbol ($\sqrt{\ }$) is the **radical symbol.** This symbol was created by the German mathematician Christoff Rudolff in 1525, but it was not until the end of the 17th century that the symbol was widely accepted. Before this, the most commonly-used symbol for square root was ρ_x. It was first used in 1220 and was an abbreviation for the word **radix**, which means 'root' in Latin. One theory says that Rudolff invented the symbol $\sqrt{\ }$ to look like the letter 'r'.

Other early symbols for square root were



The symbols on the right all represent the same thing.

What do you think they stand for?

Foundation Standard Complex

Quiz
Mental skills 3B

☆ **MENTAL SKILLS** 3B ANSWERS ON P. 607 **Maths without calculators**

Dividing by a multiple of 10

Place value allows us to remove 0s from the end of a number when we divide by a power of 10.

1 Study each example.

a $2000 \div 10 = 200\cancel{0} \div 1\cancel{0} = 200$

b $1800 \div 100 = 18\cancel{00} \div 1\cancel{00} = 18$

c $37\,000 \div 100 = 37\,0\cancel{00} \div 1\cancel{00} = 370$

d $5\,000\,000 \div 1000 = 5\,000\,\cancel{000} \div 1\cancel{000} = 5000$

e $6000 \div 200 = 60\cancel{00} \div 1\cancel{00} \div 2 = 60 \div 2 = 30$

f $350 \div 70 = 35\cancel{0} \div 1\cancel{0} \div 7 = 35 \div 7 = 5$

g $2800 \div 40 = 280\cancel{0} \div 1\cancel{0} \div 4 = 280 \div 4 = 70$

h $40\,000 \div 5000 = 40\,\cancel{000} \div 1\cancel{000} \div 5 = 40 \div 5 = 8$

2 Now evaluate each quotient.

a	$200 \div 10$	**b**	$6000 \div 100$	**c**	$45\,000 \div 100$	**d**	$30\,000 \div 1000$
e	$1900 \div 10$	**f**	$2600 \div 100$	**g**	$530 \div 10$	**h**	$720\,000 \div 1000$
i	$180 \div 30$	**j**	$300 \div 50$	**k**	$1600 \div 400$	**l**	$45\,000 \div 5000$
m	$4200 \div 60$	**n**	$21\,000 \div 700$	**o**	$44\,000 \div 2000$	**p**	$1600 \div 200$
q	$24\,000 \div 600$	**r**	$15\,000 \div 3000$	**s**	$64\,000 \div 80$	**t**	$5400 \div 900$

3.08 Prime and composite numbers

Skillsheet
Prime and composite numbers

Worksheet
Sieve of Eratosthenes

Video
The prime number code

The **factors** of a number are those whole numbers that divide exactly into it (without a remainder). For example:

- The factors of 16 are 1, 2, 4, 8 and 16
- The factors of 7 are 1 and 7

A number is **prime** if it has only **2** factors: 1 and itself. Some examples of prime numbers are 7, 3, 19, 2 and 11.

A number is **composite** if it has **more than 2** factors. Some examples of composite numbers are 16, 10, 9, 15 and 4.

Prime and composite numbers

- A **prime number** has only 2 factors: 1 and itself
- A **composite number** has more than 2 factors

Note: The number 1 has only one factor so 1 is neither prime nor composite.

EXERCISE 3.08 ANSWERS ON P. 607

Prime and composite numbers

U F R C

1 List all the factors of each number.

a 17 **b** 21 **c** 24 **d** 11
e 35 **f** 4 **g** 18 **h** 23
i 25 **j** 9 **k** 3 **l** 19

2 List all the numbers from question **1** that are:

a prime **b** composite

3 Which one of these numbers is not a factor of 45? Select the correct answer **A**, **B**, **C** or **D**.

A 9 **B** 5 **C** 7 **D** 3

4 R The ancient Greek mathematician, Eratosthenes, discovered an easy way to sort out the prime numbers from a list of numbers. It is called the Sieve of Eratosthenes (pronounced 'Siv of Era-tos-the-nees'), and works by crossing out multiples of numbers (the composite numbers).

a Copy the grid below or print out the Worksheet 'Sieve of Eratosthenes'.

1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20	21	22	23	24
25	26	27	28	29	30
31	32	33	34	35	36
37	38	39	40	41	42
43	44	45	46	47	48
49	50	51	52	53	54
55	56	57	58	59	60
61	62	63	64	65	66
67	68	69	70	71	72
73	74	75	76	77	78
79	80	81	82	83	84
85	86	87	88	89	90
91	92	93	94	95	96
97	98	99	100	101	102
103	104	105	106	107	108
109	110	111	112	113	114
115	116	117	118	119	120

Worksheet Sieve of Eratosthenes

Foundation Standard Complex

b Cross out 1. It is neither prime nor composite.

c Except for 2, colour all the multiples of 2 red and notice the pattern.

d Except for 3, colour all the multiples of 3 green and notice the pattern.

e Continue to colour multiples of other numbers with different colours, until there are no more multiples.

f What do you notice about the 30 numbers left that are not coloured?

5 Sort the following numbers into primes and composites.

10 2064 101 129 33 77 967 504 999 472 17 27 41 71 62 59

6 List all the:

a prime numbers between 36 and 50

b composite numbers between 65 and 80

c prime numbers less than 20

d composite numbers between 30 and 47

7 Look up other meanings for the word 'composite'. Suggest why this word is used the way it is in mathematics.

C

8 Which number below is divisible only by prime numbers, itself and 1?
Select **A**, **B**, **C** or **D**.

A 12 **B** 14 **C** 16 **D** 18

9 **a** Find a number that has an odd number of factors.

R

b What type of numbers have an odd number of factors?

C

Foundation Standard Complex

Factor trees

3.09

Every number can be written as a product of its prime factors. The prime factors can be found by using a **factor tree**.

Skillsheet Prime factors by repeated division

Video Factor trees

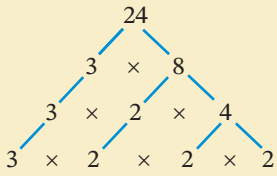
Example 17

Write 24 as a product of its prime factors.

SOLUTION

Draw a factor tree for 24.

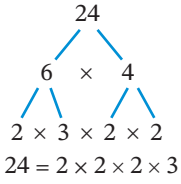
- 8 is composite, so write 8 as a product of 2 factors.
- 3 is prime, so keep writing 3 along the branches.



So 24 as a product of prime factors is:

$24 = 2 \times 2 \times 2 \times 3$.

Note: It is possible to draw different factor trees using different factors for the same number, but the final list of prime factors should still be the same. Here is another factor tree for 24:



$24 = 2 \times 2 \times 2 \times 3$

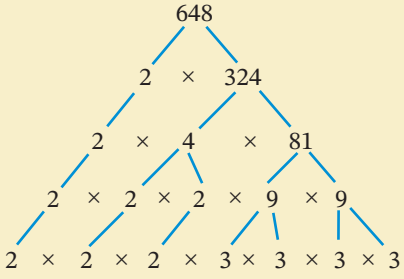
Example 18

Write 648 as a product of its prime factors, using index notation.

SOLUTION

Draw a factor tree for 648.

- 2 is prime, so keep writing 2 along the branches.
- 324 is composite, so write 324 as a product of 2 factors, 4 and 81.
- Keep going until there are no composite factors left.



So 648 as a product of prime factors is:

$648 = 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3$

$= 2^3 \times 3^4$

Video
Using factor trees to find square roots

Example 19

Use a factor tree to find the value of:

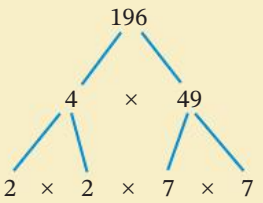
a $\sqrt{196}$ **b** $\sqrt[3]{216}$

SOLUTION

a Draw a factor tree for 196.

So $196 = 2 \times 2 \times 7 \times 7$

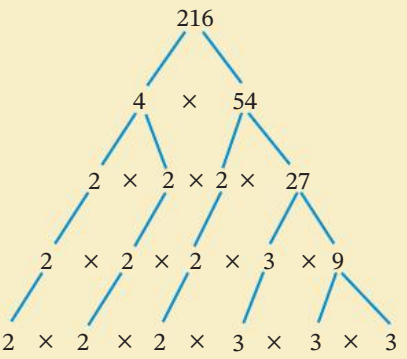
$$\begin{aligned} \therefore \sqrt{196} &= \sqrt{2 \times 2 \times 7 \times 7} \\ &= \sqrt{2 \times 2} \times \sqrt{7 \times 7} \\ &= 2 \times 7 \\ &= 14 \end{aligned}$$



b Draw a factor tree for 216.

So $216 = 2 \times 2 \times 2 \times 3 \times 3 \times 3$

$$\begin{aligned} \therefore \sqrt[3]{216} &= \sqrt[3]{2 \times 2 \times 2 \times 3 \times 3 \times 3} \\ &= \sqrt[3]{2 \times 2 \times 2} \times \sqrt[3]{3 \times 3 \times 3} \\ &= 2 \times 3 \\ &= 6 \end{aligned}$$



EXERCISE 3.09 ANSWERS ON P. 607

Factor trees

U F R

EXAMPLE 17

1 Use a factor tree to write each number as a product of its prime factors.

a 88	**b** 63	**c** 45	**d** 51
e 132	**f** 270	**g** 396	**h** 218
i 630	**j** 520	**k** 275	**l** 342

2 What is 1260 expressed as a product of its prime factors?
Select the correct answer **A**, **B**, **C** or **D**.

A $2 \times 3 \times 3 \times 3 \times 5 \times 7$ **B** $2 \times 2 \times 2 \times 3 \times 5 \times 7$

C $2 \times 2 \times 3 \times 3 \times 3 \times 7$ **D** $2 \times 2 \times 3 \times 3 \times 5 \times 7$

3 Draw 2 different factor trees for 280 and show that both give the same prime factors.

EXAMPLE 18

4 Use a factor tree to write each number as a product of its prime factors, in index notation.

a 48	**b** 200	**c** 460	**d** 712
e 98	**f** 144	**g** 325	**h** 135

Foundation Standard Complex

EXAMPLE 19

3.09

5 484 is a perfect square.

R **a** Use a factor tree to write 484 as a product of its prime factors.

b Note that because 484 is a square number, the prime factors are in pairs. Use the factor tree to find $\sqrt{484}$.

6 Use a factor tree to evaluate each root.

R **a** $\sqrt{625}$ **b** $\sqrt{900}$ **c** $\sqrt{225}$ **d** $\sqrt{784}$

e $\sqrt[3]{9261}$ **f** $\sqrt[3]{3375}$ **g** $\sqrt[3]{1728}$ **h** $\sqrt[3]{4096}$

TECHNOLOGY

Prime factors calculator

Search the Internet for an online prime factors calculator. This is a calculator that will take a number that you type in and write it as a product of its prime factors, perhaps even drawing its factor tree. Use it to show that $648 = 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3 = 2^3 \times 3^4$, then use it to check some of your answers from the previous exercise.

TECHNOLOGY

Spreadsheet formulas

Remember that a spreadsheet is like a calculator. We use special symbols to make calculations and a spreadsheet formula always begins with an equals (=) sign.

Formula	Meaning
=A1+A2+A3 or =sum(A1:A3)	**add** the values in cells A1, A2 and A3
=A5-A4	**subtract** the value in cell A4 from the value in cell A5
=A1*A3	**multiply** together the values in cells A1 and A3 (* is used instead of ×)
=A1/A2	**divide** the value in cell A1 by the value in cell A2 (/ is used instead of ÷)
=A2^2	**square** the value in cell A2 (^2 is used instead of (A2)2)
=A5^3	**cube** the value in cell A5, that is, (A5)3
=sqrt(A3)	find the **square root** of the value in cell A3, that is, $\sqrt{A3}$
=average(A1:A5)	find the **average** of all values from cells A1 to A5
=max(A1:A8)	find the **maximum (highest)** of all values from cells A1 to A8
=min(A1:A8)	find the **minimum (lowest)** of all values from cells A1 to A8

Foundation Standard Complex

1 Open a new spreadsheet and enter the 5 numbers shown in column A. To enter the fraction **1/2** in cell A4, right-click on B4 to select **Format Cells**, then choose **Fraction** and select **Up to three digits**.

	A	B
1	4	=SQRT(A1)
2	10	
3	-6	
4	1/2	
5	18	

2 In cell B1, enter the formula **=sqrt(A1)** to calculate the square root of the value in cell A1. You should get the answer 2, because $\sqrt{4} = 2$.

3 For the rest of the cells in column B, enter the appropriate formula for each expression shown below and check that the spreadsheet calculates the correct answer. Remember to start each formula with an '=' sign.

a at B2, A2 ÷ A1

b at B3, $(A3)^2$

c at B4, $(A3)^3$

d at B5, A1 × A2 + A3 × A4

e at B6, A5 – A4

f at B7, the sum of A1 to A5

g at B8, the product of A1 to A5

h at B9, $(A4)^2$

i at B10, $(A5)^3$

j at B11, $\sqrt{\frac{A5}{A4}}$

4 For the following cells in column C, enter the appropriate formula for each expression and check that the spreadsheet calculates the correct answer.

a at C1, the average of column A

b at C2, the maximum value of column A

c at C3, the minimum value of column A

d at C4, $(A1 + A5)^2$

e at C5, the average of A1, A2 and A3

f at C6, $\sqrt{A2 - A3}$

5 Now enter **new** values in cells A1 to A5 (including a new fraction for A4) and notice the new answers calculated in columns B and C.

6 Create your own values and formulas. Write at least five formulas and use up to five different values. Enter the new values in column D and the new formulas in column E of your spreadsheet.

Highest common factor

3.10

Divisor is just another name for **factor**.

Highest common factor

The **highest common factor** (HCF) or **greatest common divisor** (GCD) of 2 (or more) numbers is the largest number that is a factor of **both** (**or all**) of these numbers.

Presentation Highest common factors

Video Highest common factor

Example 20

Find the highest common factor (HCF) of:

a 24 and 30

b 40 and 15

SOLUTION

a Factors of 24 = **1**, **2**, **3**, 4, **6**, 8, 12, 24

Factors of 30 = **1**, **2**, **3**, 5, **6**, 10, 15, 30

The common factors are 1, 2, 3, 6

The highest common factor is 6.

b Factors of 40 = **1**, 2, 4, **5**, 8, 10, 20, 40

Factors of 15 = **1**, 3, **5**, 15

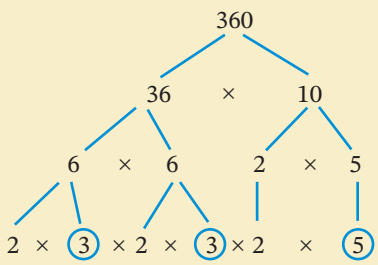
The highest common factor is 5.

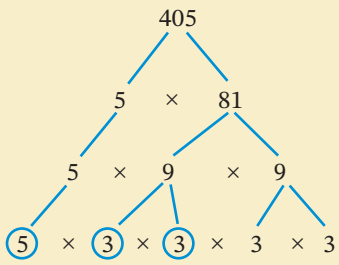
Using prime factors to find the HCF

Example 21

Use factor trees to find the HCF of 360 and 405.

SOLUTION





- Draw factor trees for 360 and 405
- Circle common prime factors: 3, 3 and 5
- Multiply 3, 3 and 5 to calculate the HCF

HCF = $3 \times 3 \times 5 = 45$

45 is the highest factor of both 360 and 405

Using prime factors to find the HCF

To find the highest common factor of 2 numbers using their prime factors:

1 Circle common prime factors

2 Multiply them together

EXERCISE 3.10 ANSWERS ON P. 607

Highest common factor

U F R C

1 Which one of these numbers is **not** a factor of 36? Select the correct answer **A**, **B**, **C** or **D**.

A 4 **B** 6 **C** 8 **D** 9

2 Find all common factors for each pair of numbers.

a 27, 45 **b** 60, 32 **c** 75, 45 **d** 18, 28

EXAMPLE 20

3 Find the highest common factor of each pair of numbers.

a 32, 28 **b** 9, 6 **c** 6, 14 **d** 8, 12

e 14, 70 **f** 44, 64 **g** 10, 15 **h** 12, 6

i 75, 125 **j** 56, 40 **k** 60, 90 **l** 39, 26

m 132, 60 **n** 36, 84 **o** 27, 63 **p** 350, 210

4 On a spreadsheet, the highest common factor can be found using the function **=gcd()**.

a Enter **=gcd(27, 45)** into a spreadsheet cell to find the HCF of 27 and 45.

b Check your answers to question **3** using a spreadsheet.

EXAMPLE 21

5 Use factor trees to find the highest common factor of each pair of numbers.

R **a** 324, 486 **b** 144, 60

c 35, 210 **d** 1404, 1900

6 Explain why the 'factor trees' method for finding the HCF works, using 324 and 486 from question **5a** as an example.

R C

Foundation Standard Complex

INVESTIGATION

Common factor puzzle

1 Copy this grid, and try to move from the 200 square in the top left corner to the 100 square at the bottom right corner. You can move one step horizontally or vertically (but not diagonally), but only if both numbers have a common factor (not 1). For example, you can move from 65 to 91 (common factor 13), but not to 96 (no common factor).

200	80	65	91	143	156	195
175	32	96	71	110	77	121
35	28	15	209	87	90	21
39	169	117	95	57	37	81
63	11	29	72	76	75	51
14	98	56	132	48	78	85
105	45	44	187	112	221	100

2 Now try to move from the 105 square in the bottom left corner to the 195 square at the top right corner, using the same rules.

3 Choose different starting and finishing positions. Do they all have connecting factor paths?

INVESTIGATION

HCF by repeated division

The highest common factor can be found by repeated division with prime factors.
For example, to find the HCF of 24 and 60:

- both numbers are even so divide by 2
- both numbers are even so divide by 2 again

$$\begin{array}{rr} \rightarrow 2)\underline{24} & 2)\underline{60} \\ \rightarrow 2)\underline{12} & 2)\underline{30} \\ \rightarrow 3)\underline{\ \ 6} & 2)\underline{30} \\ 2 & 5 \end{array}$$

- both numbers are divisible by 3 so divide by 3
- Since it is not possible to divide any more, stop.
- Multiply the common prime factors to find the HCF.
- HCF of 24 and 60 $= 2 \times 2 \times 3 = 12$

Use this method to find the HCF of each set of numbers.

a 45, 120 **b** 18, 48 **c** 20, 28

d 96, 144 **e** 16, 48 **f** 15, 21, 45

3.11 Lowest common multiple

- The multiples of 3 are 3, 6, 9, 12, 15, 18, 21, 24, ...
- The multiples of 7 are 7, 14, 21, 28, 35, 42, 49, 56, ...

The **multiples** of a number are found by multiplying that number by the whole numbers 1, 2, 3, 4, ...

Lowest common multiple

The **lowest common multiple** (LCM) of 2 (or more) numbers is the smallest number that is a multiple of **both** (**or all**) of these numbers.

Example 22

Find the lowest common multiple (LCM) of:

a 3 and 4 **b** 6 and 9

SOLUTION

a Multiples of 4 are 4, **12**, 16, 20, **24**, 28, 32, ...

Multiples of 3 are 3, 6, 9, **12**, 15, 18, 21, **24**, ...

The common multiples are **12**, **24**, ...

The lowest common multiple of 3 and 4 is **12**.

b Multiples of 6 are 6, 12, **18**, 24, 30, **36**, 42, 48, **54**, ...

Multiples of 9 are 9, **18**, 27, **36**, 45, **54**, 63, 72, 81, ...

The lowest common multiple of 6 and 9 is **18**.

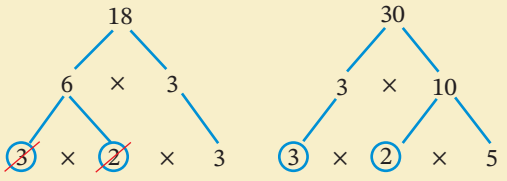
Using prime factors to find the LCM

Example 23

Use factor trees to find the LCM of 18 and 30.

SOLUTION

- Draw factor trees for 18 and 30.
- Circle common prime factors: 3 and 2.
- Cross out one of the 3s and one of the 2s.
- Multiply the remaining factors to calculate the LCM.



LCM of 18 and 30 $= 3 \times 3 \times 2 \times 5$

$= 90$

Notice that a common multiple of 18 and 30 is $18 \times 30 = 540$, but to find the lowest common multiple, you can divide that product by their HCF $= 3 \times 2 = 6$, giving $540 \div 6 = 90$. This means that:

$$\text{LCM of } a \text{ and } b = \frac{a \times b}{\text{HCF of } a \text{ and } b}.$$

Using prime factors to find the LCM

To find the lowest common multiple of 2 numbers using their prime factors:

1. Circle common prime factors
2. Cross out one of each pair of common factors
3. Multiply the remaining factors

EXERCISE 3.11 ANSWERS ON P. 607

Lowest common multiple

U F R C

1 Write the first 5 multiples of 7.

2 Which number is NOT a multiple of 8? Select the correct answer **A**, **B**, **C** or **D**.

A 24 **B** 36 **C** 56 **D** 72

3 List the multiples of 4 between 13 and 45.

4 How many multiples of 13 are less than 220?

5 Find the lowest common multiple of each set of numbers. EXAMPLE 22

a 3, 5 **b** 6, 7 **c** 4, 6 **d** 15, 10
e 5, 8 **f** 4, 10 **g** 10, 5 **h** 2, 8
i 9, 6 **j** 3, 7 **k** 3, 4, 5 **l** 4, 12, 10

6 On a spreadsheet, the lowest common multiple can be found using the function **=lcm()**.

a Enter **=lcm(8, 12)** into a spreadsheet cell to find the LCM of 8 and 12.

b Check your answers to question **5** using a spreadsheet.

7 Why are some lowest common multiples easier to find? Write down your strategy for finding LCMs. R C

8 Use factor trees to find the lowest common multiple of each pair of numbers. EXAMPLE 23

R **a** 28, 15 **b** 16, 25 **c** 44, 20 **d** 18, 60

9 Explain why the 'factor trees' method for finding the LCM works, using 28 and 15 from question **8a** as an example. R C

a Find the HCF of 95 and 76.

b Use the formula (rule)

$$\text{LCM of } a \text{ and } b = \frac{a \times b}{\text{HCF of } a \text{ and } b}.$$

to find the LCM of 95 and 76.

Foundation Standard Complex

TECHNOLOGY

Lowest common multiple

This activity searches for the lowest common multiple of 8, 12 and 16 by listing their multiples first. Start a new spreadsheet and enter the information shown below.

	A	B	C	D
1	**Number:**	**8**	**12**	**16**
2	1			
3	2			
4	3			
5	4			
6	5			
7	6			
8	7			
9	8			
10	9			
11	10			
12	11			
13	12			
14	13			
15	14			
16	15			

1 In cell B2, enter the formula **=$B$1*A2**. Use **Fill Down** to find the first 15 multiples of 8.

2 In cells C2 and D2, enter similar formulas and **Fill Down** to find the first 15 multiples of 12 and 16. [Hint: Only change the absolute cell reference.]

3 By examining the columns of multiples, find the lowest common multiple of 8, 12 and 16.

4 By using the formula **=lcm(8, 12, 16)**, find the lowest common multiple of 8, 12 and 16.

5 Modify your spreadsheet to find the LCM of each of the following sets of numbers.

Note: You may need to extend beyond the first 15 multiples.

a 6 and 15

b 12 and 18

c 3, 7 and 15

d 48, 60 and 75

POWER PLUS ANSWERS ON P. 607

Worksheet Perfect and amicable numbers

1 For each set of numbers, evaluate and write in ascending order.

a $2^3, 3^2, 3^5, 5^3, 2^5, 5^2$

b $4^4, 7^3, 3^5, 8^2, 5^2, 6^3$

c $100^2, 11^4, 2^7, 3^4, 5^3$

2 Evaluate each square number.

a 1^2 **b** 11^2

c 111^2 **d** 1111^2

3 Based on the patterns in your answers to question **2**, evaluate each square number.

a $11\ 111^2$ **b** $1\ 111\ 111^2$

c $111\ 111\ 111^2$ **d** $1\ 111.1111^2$

4 Find the square root of:

a $3 \times 3 \times 2 \times 2$ **b** $5 \times 5 \times 4 \times 4 \times 3 \times 3$

5 Evaluate each square root without using a calculator.

a $\sqrt{9^2}$ **b** $\sqrt{3^4}$ **c** $\sqrt{5^4 \times 2^6}$

d $\sqrt[3]{2^9}$ **e** $\sqrt[3]{3^3 \times 5^6}$ **f** $\sqrt[4]{16}$

6 Find the square root of each number without using a calculator.

a 2500 **b** 8100 **c** 10 000

d 1 000 000 **e** 1 210 000 **f** 100 000 000

g 640 000 **h** 176 400 **i** 10 000 000 000

7 Two prime numbers that differ by 2 are called **twin primes**. For example, 11 and 13 are twin primes, but 23 and 29 are not. Find all 8 pairs of twin primes below 100.

3 CHAPTER REVIEW

Language of maths

Quiz Language of maths 3

composite	cube	divisibility test
estimate	expanded notation	factor
factor tree	highest common factor (HCF)	index notation
lowest common multiple (LCM)	multiple	power
prime	prime factor	round down
round up	square	square root

1 Which positive integer is neither prime nor composite?

2 What notation is used as an abbreviation for repeated multiplication?

3 What type of numbers have more than 2 factors?

4 Describe the divisibility test for 3.

5 Find a non-mathematical meaning of:

a prime **b** index

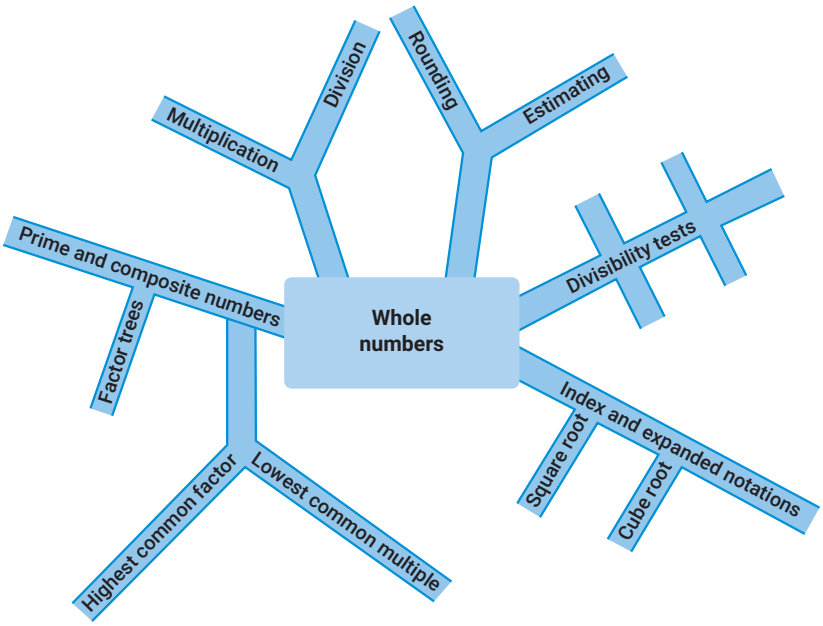
6 What helps us to write a number as a product of its prime factors?

Topic summary

- Using your own words, write down what you have learnt about whole numbers.
- What was your favourite part of this topic?
- What parts of this topic didn't you understand? Talk to your teacher or a friend about them.

Worksheet Mind map: Whole numbers

Copy and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.



 9780170465557

3 TEST YOURSELF

ANSWERS ON P. 607

1 Round 87 531 to the nearest: 3.01

a thousand **b** hundred **c** ten **d** ten thousand

2 Estimate the value of each expression. 3.01

a $425 + 81 + 940 + 22$ **b** $864 + 250 - 417$

c 93×19 **d** $2402 \div 32$

3 Evaluate each product. 3.02

a 12×4 **b** 34×5 **c** 33×9

d 25×8 **e** 126×7 **f** 565×18

Quiz Test yourself 3

4 Evaluate each quotient. 3.03

a $2400 \div 10$ **b** $620 \div 5$ **c** $1296 \div 8$

d $528 \div 4$ **e** $144 \div 9$ **f** $847 \div 7$

5 Test whether 1208 is divisible by: 3.04

a 2 **b** 3 **c** 4 **d** 5

e 6 **f** 8 **g** 9 **h** 10

6 Which number is divisible by both 3 and 8? Select the correct answer **A**, **B**, **C** or **D**. 3.04

A 30 **B** 24 **C** 90 **D** 94

7 Write each expression using index notation. 3.06

a $3 \times 3 \times 3 \times 3$

b $6 \times 6 \times 6 \times 6 \times 6$

c $11 \times 11 \times 11 \times 11 \times 11 \times 11 \times 11$

8 Evaluate each power. 3.06

a 6^2 **b** 5^3 **c** 10^5 **d** 2^8

9 Write each whole number using expanded notation. 3.06

a 4321 **b** 605 **c** 74 201 **d** 9015

10 Write each number represented in expanded notation. 3.06

a $4 \times 10 + 7 \times 1$

b $3 \times 10^4 + 6 \times 103 + 3 \times 10$

c $8 \times 10^3 + 5 \times 10 + 2 \times 1$

11 Evaluate each root. 3.07

a $\sqrt{49}$ **b** $\sqrt{144}$ **c** $\sqrt[3]{8000}$ **d** $\sqrt[3]{729}$

Foundation Standard Complex

3.08 **12** **a** List the prime numbers between 26 and 50.

b List the composite numbers between 45 and 70.

c List the prime numbers less than 30.

d List the composite numbers larger than 40 but less than 59.

3.09 **13** Use a factor tree to express each number as a product of its prime factors in index notation.

a 60 **b** 200 **c** 144

3.09 **14** Use a factor tree to evaluate each root.

a $\sqrt{256}$ **b** $\sqrt[3]{5832}$

3.10 **15** Find the highest common factor of each pair of numbers.

a 20 and 48 **b** 36 and 84

3.10 **16** Use factor trees to find the highest common factor of 243 and 162.

3.11 **17** Find the lowest common multiple of each pair of numbers.

a 6 and 8 **b** 18 and 10

3.11 **18** Use factor trees to find the lowest common multiple of 25 and 16.

Foundation Standard Complex

Practice set 1

ANSWERS ON P. 608

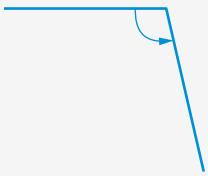
1 Evaluate each expression. 1.04 1.05

a $-14 + 20$ **b** $11 - 16$ **c** $22 + (-4)$ **d** $8 - (-9)$

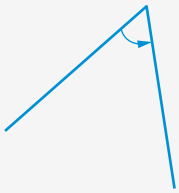
e $3 - 5$ **f** $-2 + 8$ **g** $-1 - 6$ **h** $-3 + 5 - 8$

2 Measure each angle. 2.02

a



b



c

3 Draw an example of: 2.03

a an obtuse angle **b** an acute angle **c** a reflex angle

4 Find the distance between each pair of numbers on a number line. 1.02

a -1 and -3 **b** -5 and 3 **c** 0 and -4

YEAR 8 EXTENSION

5 Evaluate each expression. 1.06 1.07

a $6 \times (-5)$ **b** $-10 \times (-3)$ **c** $-42 \div 7$ **d** $-48 \div (-8)$

e $-5 \times (-3)$ **f** $6 \div (-2)$ **g** $-27 \div (-3)$ **h** -4×4

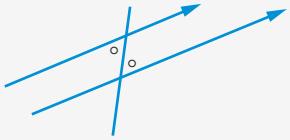
6 Round each number to the nearest hundred. 3.01

a 2477 **b** 17 204 **c** 807 928

7 Which type of angles are shown on the diagram? Select **A**, **B**, **C** or **D**. 2.10

A alternate **B** co-interior

C corresponding **D** vertically opposite



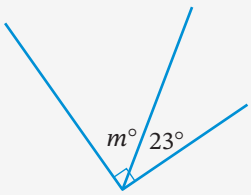
8 Evaluate each expression. 3.02

a 217×7 **b** 85×22 **c** $156 \div 6$

9 Find the value of m. Select **A**, **B**, **C** or **D**. 2.04

A 23 **B** 67

C 157 **D** 337



10 Find the complement of: 2.04

a 66° **b** 85° **c** 12°

11 Find the supplement of: 2.04

a 32° **b** 90° **c** 105°

12 Use a divisibility test to test whether 1955 is divisible by: 3.04

a 3 **b** 4 **c** 5

Foundation Standard Complex

1.03

13 Which statement is true? Select **A**, **B**, **C** or **D**.

A $-7 > 3$ **B** $8 < -11$ **C** $0 < -15$ **D** $5 > -9$

2.05

14 Draw 2 intersecting lines and mark in a pair of vertically opposite angles.

2.10

15 **a** Draw a pair of parallel lines and cut them with a transversal.

b Mark a pair of alternate angles with (•).

c Mark a pair of corresponding angles with (*).

d Mark a pair of co-interior angles with (+).

YEAR 8 EXTENSION

1.08

16 Evaluate each expression.

a $-3 \times 2 - 5$ **b** $-2 - 3 \times 8$ **c** $12 - 14 \div 2 + 5$

d $5 - [2 + (-2 \times 6)]$ **e** $15 - 3 \times 5$ **f** $7 \times 3 + 2 \times 5$

g $26 \div 2 - 14 \div 7$ **h** $7 + 5 \times (12 - 3)$ **i** $[(3 + 5) \times 2 - (20 \div 5)] \times 5$

3.08

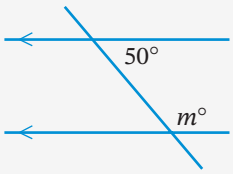
17 Which number is NOT a factor of 66? Select **A**, **B**, **C** or **D**.

A 3 **B** 6 **C** 9 **D** 11

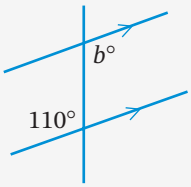
2.04 2.05 2.10

18 Find the value of each variable.

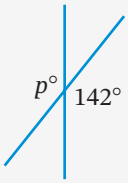
a



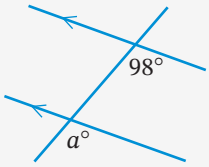
b



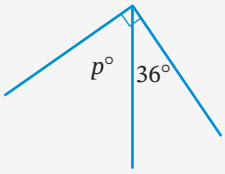
c



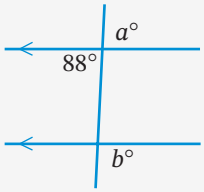
d



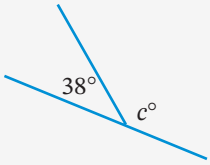
e



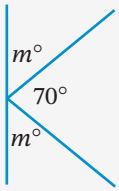
f



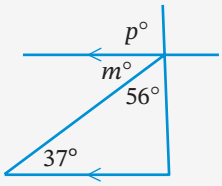
g



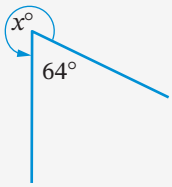
h



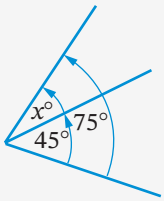
i



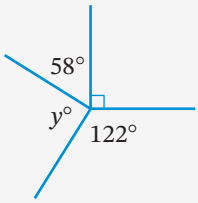
j



k



l



Foundation Standard Complex

9780170465557

19 Evaluate each expression. 3.06 3.07

a 6^2 **b** 3^3 **c** $\sqrt{25}$ **d** $\sqrt{121}$

20 The average daytime temperature in winter is 13°C and the average night-time temperature is –5°C. Find the difference between these 2 temperatures. 1.10

21 Write each number using expanded notation with powers of 10. 3.06

a 963 **b** 8042 **c** 52 301

22 Simplify each number written in expanded notation. 3.06

a $4 \times 10 + 9 \times 1$

b $7 \times 10^3 + 3 \times 10^2 + 5 \times 10$

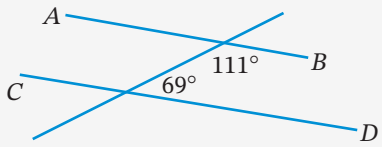
c $2 \times 10^5 + 2 \times 10^4 + 5 \times 10^3 + 9 \times 10^2 + 5 \times 1$

23 Use a factor tree to write each number as a product of its prime factors. 3.09

Write your answers using index notation.

a 18 **b** 45 **c** 360

24 Is $AB \parallel CD$? Give a reason for your answer. 2.11



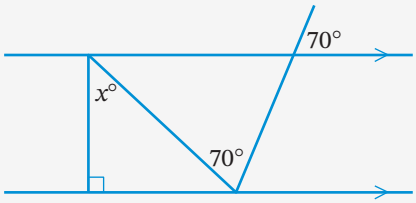
25 Find the highest common factor of: 3.10

a 24 and 36 **b** 45 and 81

26 Find the lowest common multiple of: 3.11

a 7 and 11 **b** 6 and 21

27 Hannah was asked to find the value of x in the diagram below. She states that this angle is 40°. Use angle relationships to determine if Hannah is correct and provide mathematical evidence to support your decision. 2.10



28 Liam estimates the value of $\sqrt{23}$ to be approximately 4.3. 3.07
Use mathematical reasoning to determine if Liam is correct.

Foundation Standard Complex

4

NUMBER

Fractions and percentages

We use fractions whenever we want to find part of a whole: when we share pizza or cake, when we measure ingredients in cooking or when we say "it's quarter past 10" to tell the time.

The word **fraction** comes from the Latin *fractio*, meaning 'to break'. Chemists use fractional distillation to break chemical compounds in liquid form into their component parts.

iStock.com/juliannafunk

Chapter outline

		Proficiencies				
4.01	Fractions	U	F		R	C
4.02	Ordering fractions	U			R	C
4.03	Adding and subtracting fractions	U	F		R	
4.04	Adding and subtracting mixed numerals	U	F	PS	R	
4.05	Fraction of a quantity	U	F	PS	R	C
4.06	Multiplying fractions	U	F	PS	R	C
4.07	Dividing fractions	U	F	PS	R	C
4.08	Fraction problems	U	F	PS	R	
4.09	Fractions and the calculator	U	F	PS	R	
4.10	Percentages, fractions and decimals	U	F			C
4.11	Percentage of a quantity	U	F			
4.12	Expressing quantities as fractions and percentages	U	F			

U = Understanding
F = Fluency
PS = Problem solving
R = Reasoning
C = Communication

Wordbank

denominator The number at the bottom in a fraction

improper fraction A fraction such as $\frac{7}{3}$, in which the numerator is larger than or equal to the denominator

mixed numeral A number such as $3\frac{2}{5}$, made up of a whole number and a fraction

numerator The number at the top in a fraction

percentage A special type of fraction in which the denominator is 100

proper fraction A fraction such as $\frac{4}{10}$ in which the numerator is smaller than the denominator

reciprocal (of a fraction) The fraction 'turned upside down', for example, the reciprocal of $\frac{3}{8}$ is $\frac{8}{3}$

simplify (a fraction) To reduce the numerator and the denominator of a fraction to their lowest form by dividing by the same factor

Quiz
Wordbank 4

Videos (10):

SkillCheck Using fractions

4.01 Improper fractions and mixed numerals • Equivalent fractions

4.03 Working out fractions • Adding and subtracting fractions

4.05 Fraction of a quantity

4.06 Multiplying fractions

4.07 Dividing fractions

4.10 Converting fractions and decimals to percentages

4.11 Percentages of an amount

4.12 Expressing quantities as fractions and percentages

Twig video (1):

4.05 Fractions: Slow motion

PhET interactives (5):

4.01 Fractions: Intro • Fraction matcher • Build a fraction • Mixed numbers

4.02 Fractions: Equality

Quizzes (6):

- Wordbank 4
- SkillCheck 4
- Mental skills 4A
- Mental skills 4B
- Language of Maths 4
- Test Yourself 4

Skillsheets (4):

SkillCheck Fractions

4.01 Fractions • Improper fractions and mixed numerals • Equivalent fractions • Simplifying fractions

Worksheets (18):

SkillCheck Fraction diagrams

4.01 Classifying fractions • Classifying fractions • Fractions wall • Pop stick calculator • Fraction strips

4.02 Fractions wall • Pop stick calculator

4.03 Pop stick calculator • Magic squares

4.06 Multiplying fractions using rectangles

4.07 What's the fraction question? • Multiplying and dividing fractions number grids

4.10 Percentages wall • Percentage shapes • Fractions, decimals and percentages

4.11 Percentages of an amount

4.12 Calculating percentages • Money and percentages review

Mind map: Fractions and percentages

Fractions and decimals review

Puzzles (9):

4.01 Mixed numbers to improper fractions • Equivalent fractions 1

4.03 Adding and subtracting fractions

4.05 Fractions of words

4.10 Converting percentages to fractions • Converting fractions to percentages • Converting decimals to percentages

4.11 Percentages

4.12 Calculating percentages

Presentations (2):

4.01 Improper fractions

4.10 Percentages, fractions and decimals

Nelson MindTap

To access resources above, visit **cengage.com.au/nelsonmindtap**

4 In this chapter you will:

- ✓ compare and order fractions, including mixed numerals and negative fractions
- ✓ solve problems involving adding and subtracting fractions
- ✓ find a fraction of a quantity
- ✓ solve problems involving multiplying and dividing fractions
- ✓ use the fraction key on a calculator
- ✓ convert between fractions, decimals and percentages
- ✓ find a percentage of a quantity
- ✓ express one quantity as a fraction of another
- ✓ express one quantity as a percentage of another

SkillCheck

ANSWERS ON P. 608

Quiz SkillCheck 4

Video Using fractions

Worksheet Fraction diagrams

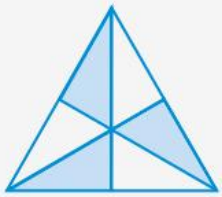
Skillsheet Fractions

1 For each diagram, state what fraction of the whole:

i is shaded **ii** is unshaded

a

b

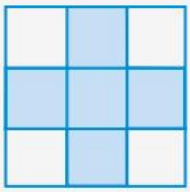


c

d

e

f



2 What fraction of the month shown on this calendar has been crossed out?

March 2023						
Sun	Mon	Tue	Wed	Thu	Fri	Sat
			~~1~~	~~2~~	~~3~~	~~4~~
~~5~~	~~6~~	~~7~~	~~8~~	~~9~~	~~10~~	~~11~~
~~12~~	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30	31	

3 Draw a square and divide it into quarters.

4 Draw a rectangle and divide it into thirds.

5 Approximately how full is this glass?

6 Find:

a $\frac{1}{2}$ of 24 **b** $\frac{1}{2}$ of 38 **c** $\frac{1}{3}$ of 21

d $\frac{1}{10}$ of 70 **e** $\frac{1}{4}$ of 20 **f** $\frac{1}{5}$ of 15

7 Draw a circle and divide it into eighths.

8 Find the highest common factor (HCF) of each pair of numbers.

a 15 and 18 **b** 9 and 21 **c** 16 and 40

9 Find the lowest common multiple (LCM) of each pair of numbers.

a 3 and 4 **b** 5 and 10 **c** 8 and 6

10 Draw any shape and divide it into fifths.

11 What fraction of one hour is:

a 13 minutes? **b** 45 minutes? **c** 10 minutes?

12 Which fraction is larger: $\frac{1}{3}$ or $\frac{1}{4}$?

4.01 Fractions

Skillsheet
Fractions

Interactives
Fractions: Intro

Fraction matcher

Build a fraction

Fractions: Mixed numbers

A **fraction** is called a **rational number** because it can be written as $\frac{a}{b}$, where a and b are integers (but b cannot be zero). Its more correct name is **common fraction** because decimals and percentages are also types of fractions.

- The top number (a) is the **numerator**
- The bottom number (b) is the **denominator**

numerator

$$\frac{4}{9}$$

denominator

The line between the top and bottom is called the **vinculum**.

Types of fractions

A **proper fraction** has a numerator that is less than the denominator, for example, $\frac{4}{9}, \frac{11}{13}, \frac{37}{60}, \frac{80}{100}$.

An **improper fraction** has a numerator that is more than or equal to the denominator, for example, $\frac{9}{4}, \frac{17}{3}, \frac{15}{15}, \frac{12}{10}$.

A **mixed numeral** is made up of a whole number and a proper fraction, for example, $4\frac{1}{2}, 5\frac{2}{3}, 3\frac{1}{7}, 4\frac{3}{10}$.

Improper fractions and mixed numerals

An improper fraction is a 'top-heavy' fraction so its value is greater than or equal to 1, and it can be converted to a mixed numeral or whole number.

Converting improper fractions to mixed numerals

To convert an improper fraction to a mixed numeral, divide the numerator by the denominator and write the remainder as a proper fraction.

Example 1

Convert $\frac{38}{3}$ to a mixed numeral.

SOLUTION

$\frac{38}{3} = 38 \div 3$ — To find how many 'wholes' in 38 thirds, divide 38 by 3.

$= 12$ remainder 2

$= 12\frac{2}{3}$ — Write the remainder in the numerator of a fraction.

Example 2

Convert $5\frac{3}{7}$ to an improper fraction.

SOLUTION

$$5\frac{3}{7} = 5 + \frac{3}{7}$$

$$= \frac{5 \times 7}{7} + \frac{3}{7}$$

$$= \frac{35}{7} + \frac{3}{7}$$

$$= \frac{38}{7}$$

Converting mixed numerals to improper fractions

To convert a mixed numeral to an improper fraction, multiply the denominator by the whole number, then add the numerator terms and write the total as the new numerator of the fraction.

$$5\frac{3}{7} = \frac{7 \times 5 + 3}{7} = \frac{38}{7}$$

+

×

4.01

Skillsheet Improper fractions and mixed numerals

Worksheets Classifying fractions 1

Classifying fractions 2

Video Improper fractions and mixed numerals

Presentation Improper fractions

Puzzle Mixed numbers to improper fractions

Equivalent fractions

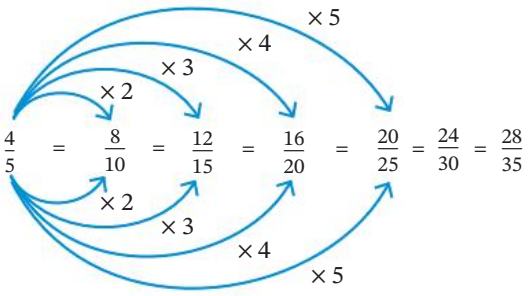
Skillsheet
Equivalent fractions

Worksheets
Fractions wall

Pop stick calculator

Fraction strips

This diagram shows fractions that are **equivalent** (equal) to $\frac{4}{5}$.



Equivalent fractions

Equivalent fractions are found by multiplying or dividing the numerator and the denominator by the same number.

Video
Equivalent fractions

Example 3

For each pair of equivalent fractions, find the missing number.

a $\frac{3}{4} = \frac{?}{20}$ **b** $\frac{5}{9} = \frac{35}{?}$ **c** $\frac{16}{24} = \frac{?}{6}$

SOLUTION

a To find the missing numerator, look at the 2 denominators, 4 and 20.

$\frac{3}{4} = \frac{?}{20}$

4 is multiplied by 5 to give 20, so do the same thing to the numerator 3.

$\frac{3}{4} = \frac{3 \times 5}{4 \times 5} = \frac{15}{20}$

The missing number is 15.

b To find the missing denominator, look at the numerators 5 and 35.

$\frac{5}{9} = \frac{35}{?}$

5 is multiplied by 7 to give 35, so do the same to the 9.

$\frac{5}{9} = \frac{5 \times 7}{9 \times 7} = \frac{35}{63}$

The missing number is 63.

c To find the missing numerator, look at the denominators 24 and 6.

$\frac{16}{24} = \frac{?}{6}$

24 is **divided** by 4 to give 6, so do the same to 16.

$\frac{16}{24} = \frac{16 \div 4}{24 \div 4} = \frac{4}{6}$

The missing number is 4.

Simplifying fractions

We have seen the equivalent fractions $\frac{4}{5}=\frac{8}{10}=\frac{12}{15}$ described above. The most basic fraction, $\frac{4}{5}$, is called the **simplified fraction** or the fraction **reduced** to its **lowest form**. A simplified fraction has the smallest numerator and denominator possible.

Skillsheet Simplifying fractions

Worksheet Pop stick calculator

Puzzle Equivalent fractions 1

Simplifying fractions

- To simplify a fraction, keep dividing the numerator and denominator by the same number (preferably a large number such as their **highest common factor (HCF)**)
- If the numerator and denominator are both even, divide by 2 or perhaps 4
- Otherwise, try dividing by the odd numbers 3, 5 or 7

Video Simplifying fractions

Example 4

Simplify each fraction.

a $\frac{20}{30}$ **b** $\frac{16}{48}$ **c** $\frac{36}{27}$

SOLUTION

a $\frac{20}{30}=\frac{20\div10}{30\div10}=\frac{2}{3}$

b $\frac{16}{48}=\frac{16\div2}{48\div2}=\frac{8}{24}=\frac{8\div8}{24\div8}=\frac{1}{3}$ or $\frac{16}{48}=\frac{16\div16}{48\div16}=\frac{1}{3}$

c $\frac{36}{27}=\frac{36\div9}{27\div9}=\frac{4}{3}=1\frac{1}{3}$

EXERCISE 4.01 ANSWERS ON P. 609

Fractions

U F R C

1 Classify each fraction as being proper (P), improper (I) or a mixed numeral (M).

C **a** $1\frac{1}{2}$ **b** $\frac{3}{100}$ **c** $3\frac{2}{5}$ **d** $20\frac{1}{4}$

e $\frac{2}{3}$ **f** $\frac{100}{45}$ **g** $\frac{150}{250}$ **h** $\frac{86}{87}$

i $\frac{3}{2}$ **j** $\frac{3}{5}$ **k** $4\frac{1}{20}$ **l** $\frac{1000}{25}$

EXAMPLE 1

2 Convert each improper fraction to a mixed numeral or whole number.

a $\frac{13}{3}$ **b** $\frac{13}{4}$ **c** $\frac{13}{10}$ **d** $\frac{21}{5}$

e $\frac{38}{35}$ **f** $\frac{35}{8}$ **g** $\frac{30}{6}$ **h** $\frac{36}{5}$

i $\frac{28}{7}$ **j** $\frac{120}{11}$ **k** $\frac{72}{19}$ **l** $\frac{101}{10}$

Foundation Standard Complex

EXAMPLE 2

3 Convert each mixed numeral to an improper fraction.

a $1\frac{1}{2}$ **b** $1\frac{5}{8}$ **c** $1\frac{2}{7}$ **d** $3\frac{1}{3}$

e $4\frac{1}{2}$ **f** $3\frac{7}{10}$ **g** $6\frac{1}{6}$ **h** $7\frac{3}{5}$

EXAMPLE 3

4 Complete each set of equivalent fractions.

a $\frac{1}{4}=\frac{3}{?}$ **b** $\frac{5}{6}=\frac{?}{30}$ **c** $\frac{2}{3}=\frac{?}{12}$

d $\frac{7}{10}=\frac{?}{80}$ **e** $\frac{7}{8}=\frac{63}{?}$ **f** $\frac{3}{4}=\frac{?}{100}$

g $\frac{20}{24}=\frac{5}{?}$ **h** $\frac{15}{40}=\frac{?}{8}$ **i** $\frac{18}{32}=\frac{?}{16}$

5 Write 3 equivalent fractions for each fraction.

a $\frac{3}{5}$ **b** $\frac{2}{3}$ **c** $\frac{1}{8}$

6 What is the missing numerator in $\frac{12}{18}=\frac{?}{3}$? Select the correct answer **A**, **B**, **C** or **D**.

A 6 **B** 1 **C** 5 **D** 2

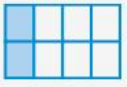
EXAMPLE 4

7 Simplify each fraction.

a $\frac{6}{16}$ **b** $\frac{6}{24}$ **c** $\frac{40}{50}$ **d** $\frac{24}{32}$

e $\frac{18}{42}$ **f** $\frac{60}{75}$ **g** $\frac{5}{20}$ **h** $\frac{14}{21}$

8 What fraction of each shape is shaded, in simplest form? Select **A**, **B**, **C** or **D**.

a 

A $\frac{2}{8}$ **B** $\frac{6}{8}$ **C** $\frac{1}{4}$ **D** $\frac{1}{8}$

b

A $\frac{1}{3}$ **B** $\frac{4}{6}$ **C** $\frac{3}{2}$ **D** $\frac{2}{3}$

c

A $\frac{4}{8}$ **B** $\frac{2}{4}$ **C** $\frac{1}{2}$ **D** $\frac{8}{16}$

9 Convert each improper fraction to a mixed numeral in simplest form.

a $\frac{18}{8}$ **b** $\frac{30}{14}$ **c** $\frac{50}{25}$ **d** $\frac{90}{54}$

e $\frac{28}{12}$ **f** $\frac{75}{60}$ **g** $\frac{32}{24}$ **h** $\frac{48}{36}$

10 Write each mixed numeral in simplest form.

a $1\frac{4}{8}$ **b** $3\frac{5}{30}$ **c** $4\frac{5}{20}$ **d** $6\frac{30}{40}$

e $17\frac{35}{100}$ **f** $100\frac{44}{64}$ **g** $1\frac{54}{100}$ **h** $5\frac{4}{12}$

11 Which fraction is not equivalent to $\frac{3}{4}$? Select **A**, **B**, **C** or **D**.

A $\frac{12}{16}$ **B** $\frac{6}{8}$ **C** $\frac{24}{32}$ **D** $\frac{10}{18}$

12 Which is larger: a proper fraction or a mixed numeral? Explain your answer.

R

C

Foundation Standard Complex

Ordering fractions

4.02

- Is $\frac{4}{10}$ larger than $\frac{3}{8}$?
- Is $1\frac{2}{3}$ smaller than $1\frac{5}{9}$?

The best way to **compare** fractions with different denominators is to convert them to equivalent fractions with the same denominator first. A common denominator can be found by multiplying the denominators of both fractions together or by using the lowest common multiple (LCM) of the denominators, also called the **lowest common denominator** (LCD).

Worksheets
Fractions wall

Pop stick calculator

Interactive
Fractions: Equality

Example 5

a Which fraction is larger, $\frac{4}{10}$ or $\frac{3}{8}$?

b Which fraction is smaller, $1\frac{2}{3}$ or $1\frac{5}{9}$?

SOLUTION

a **Method 1**

Convert both fractions so that they share a common denominator of $10 \times 8 = 80$.

$\frac{4}{10} = \frac{4 \times 8}{10 \times 8} = \frac{32}{80}$ and $\frac{3}{8} = \frac{3 \times 10}{8 \times 10} = \frac{30}{80}$

Note that we multiply the numerator and denominator of each fraction by the denominator of the other fraction.

By comparing numerators, $\frac{32}{80} > \frac{30}{80}$.

$\therefore \frac{4}{10}$ is larger.

Method 2

Another suitable denominator is the lowest common denominator (LCD) of both denominators.

The LCD of 10 and 8 is 40.

Convert both fractions.

$\frac{4}{10} = \frac{4 \times 4}{10 \times 4} = \frac{16}{40}$ and $\frac{3}{8} = \frac{3 \times 5}{8 \times 5} = \frac{15}{40}$

By comparing numerators, $\frac{16}{40} > \frac{15}{40}$.

$\therefore \frac{4}{10}$ is larger.

b Since both whole numbers are 1, we only compare the fraction parts.

Method 1

Common denominator = **3 × 9 = 27.**

Convert both fractions so that they share a common denominator.

$\frac{2}{3}=\frac{2\times 9}{3\times 9}=\frac{18}{27}$ and $\frac{5}{9}=\frac{5\times 3}{9\times 3}=\frac{15}{27}$

By comparing numerators, $\frac{15}{27}<\frac{18}{27}$.

$\therefore$ $1\frac{5}{9}$ is smaller.

Method 2

We only need to convert the first fraction because the second fraction already has 9 as a denominator.

$\frac{2}{3}=\frac{2\times 3}{3\times 3}=\frac{6}{9}$ and $\frac{5}{9}$

By comparing numerators, $\frac{5}{9}<\frac{6}{9}$.

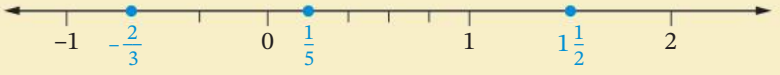
$\therefore$ $1\frac{5}{9}$ is smaller.

Example 6

Draw a number line and mark in the fractions $\frac{1}{5}$, $-\frac{2}{3}$ and $1\frac{1}{2}$.

SOLUTION

- Divide the interval between 0 and 1 into fifths and mark $\frac{1}{5}$
- Divide the interval between 0 and –1 into thirds and mark $-\frac{2}{3}$
- Divide the interval between 1 and 2 into halves and mark $1\frac{1}{2}$



EXERCISE 4.02 ANSWERS ON P. 609

Ordering fractions

U F R C

EXAMPLE 5

1 For each pair of fractions, find the larger fraction.

a $\frac{7}{10}, \frac{3}{10}$ **b** $\frac{5}{3}, \frac{2}{3}$ **c** $\frac{3}{4}, \frac{7}{8}$

d $\frac{2}{3}, \frac{3}{10}$ **e** $2\frac{1}{5}, 2\frac{1}{6}$ **f** $3\frac{4}{7}, 2\frac{5}{6}$

Foundation Standard Complex

2 For each pair of fractions, find the smaller fraction.

a $\frac{4}{5}, \frac{3}{5}$ **b** $\frac{11}{8}, \frac{9}{8}$ **c** $\frac{1}{4}, \frac{3}{10}$

d $\frac{1}{6}, \frac{2}{3}$ **e** $4\frac{3}{4}, 4\frac{2}{3}$ **f** $1\frac{2}{7}, 1\frac{1}{5}$

3 Which set of fractions shows $\frac{19}{16}, \frac{7}{6}, \frac{5}{4}, \frac{33}{32}$ in ascending order? Select the correct answer **A**, **B**, **C** or **D**.

A $\frac{7}{6}, \frac{5}{4}, \frac{19}{16}, \frac{33}{32}$ **B** $\frac{5}{4}, \frac{7}{6}, \frac{19}{16}, \frac{33}{32}$ **C** $\frac{33}{32}, \frac{7}{6}, \frac{19}{16}, \frac{5}{4}$ **D** $\frac{19}{16}, \frac{33}{32}, \frac{7}{6}, \frac{5}{4}$

4 Copy and complete each statement with a < or > symbol.

C **a** $\frac{1}{2}$ ____ $\frac{1}{3}$ **b** $\frac{2}{3}$ ____ $\frac{3}{4}$ **c** $\frac{3}{8}$ ____ $\frac{1}{2}$

d $2\frac{3}{5}$ ____ $2\frac{7}{10}$ **e** $\frac{5}{6}$ ____ $\frac{1}{2}$ **f** $\frac{5}{12}$ ____ $\frac{1}{3}$

g $\frac{17}{20}$ ____ $\frac{6}{10}$ **h** $\frac{11}{15}$ ____ $\frac{2}{3}$ **i** $2\frac{3}{7}$ ____ $5\frac{2}{5}$

5 Find a proper fraction that is greater than $\frac{3}{4}$:

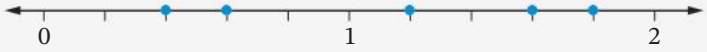
a with a denominator of 10

b with any other denominator

6 For each number line, find the value of each fraction marked by a dot.

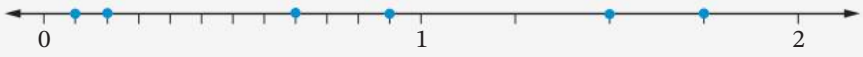
a

b



c

d



e

7 Plot each set of fractions on a separate number line.

EXAMPLE 6

a $\frac{2}{3}$ $\frac{3}{3}$ $1\frac{1}{3}$ $\frac{7}{3}$ **b** $\frac{1}{5}$ $1\frac{3}{5}$ $\frac{5}{5}$ $2\frac{2}{5}$ $\frac{3}{5}$ $\frac{9}{5}$

c $\frac{5}{6}$ $-\frac{1}{3}$ $\frac{1}{2}$ $-1\frac{1}{6}$ **d** $-\frac{7}{10}$ $\frac{2}{5}$ $-\frac{1}{2}$ $-\frac{1}{4}$ $\frac{9}{10}$

Foundation Standard Complex

8 Which one of these fractions is closest to $\frac{3}{4}$? Select **A**, **B** or **C**.

R **A** $\frac{1}{2}$ **B** $\frac{7}{8}$ **C** $\frac{13}{16}$

9 Find a fraction that is between:

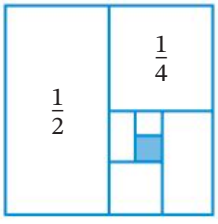
R **a** $\frac{3}{4}$ and $\frac{4}{5}$ **b** $\frac{5}{8}$ and $\frac{2}{3}$

INVESTIGATION

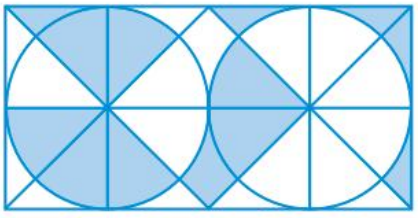
What fraction is shaded?

In groups or individually, decide which fraction of each diagram has been shaded. Explain how you solved each puzzle.

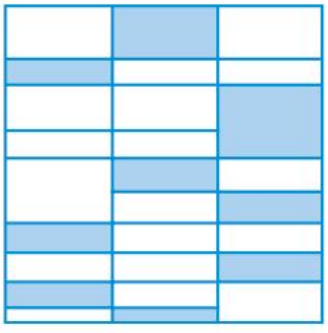
1



2



3



Foundation Standard Complex

Adding and subtracting fractions

4.03

Adding and subtracting fractions with the same denominator

To add or subtract fractions with the same denominator, simply add or subtract the numerators.

Video Working out fractions

Worksheets Pop stick calculator

Magic squares

Puzzle Adding and subtracting fractions

Example 7

Find:

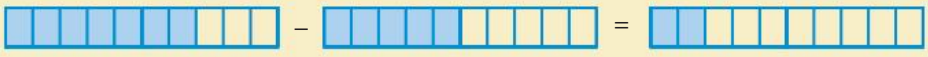
a $\frac{1}{5}+\frac{2}{5}$ **b** $\frac{7}{10}-\frac{5}{10}$

SOLUTION

a

$\frac{1}{5} + \frac{2}{5} = \frac{3}{5}$

b



$\frac{7}{10} - \frac{5}{10} = \frac{2}{10} = \frac{1}{5}$

Fractions with different denominators

Adding and subtracting fractions with different denominator

To add or subtract fractions with different denominators, we must first convert them to equivalent fractions with the same denominator.

Video Adding and subtracting fractions

Presentation Adding and subtracting fractions

Example 8

Find:

a $\frac{1}{4}+\frac{5}{6}$ **b** $\frac{2}{5}-\frac{3}{20}$

SOLUTION

a **Method 1**

Common denominator $= 6 \times 4 = 24$.

$\frac{1}{4}=\frac{1\times 6}{4\times 6}=\frac{6}{24}$ and $\frac{5}{6}=\frac{5\times 4}{6\times 4}=\frac{20}{24}$

$$\begin{aligned}\frac{1}{4}+\frac{5}{6}&=\frac{6}{24}+\frac{20}{24}\\&=\frac{26\div 2}{24\div 2}\\&=\frac{13}{12}\\&=1\frac{1}{12}\end{aligned}$$

Method 2

The LCD of 4 and 6 is 12.

$\frac{1}{4}=\frac{1\times 3}{4\times 3}=\frac{3}{12}$ and $\frac{5}{6}=\frac{5\times 2}{6\times 2}=\frac{10}{12}$

$$\begin{aligned}\frac{1}{4}+\frac{5}{6}&=\frac{3}{12}+\frac{10}{12}\\&=\frac{13}{12}\\&=1\frac{1}{12}\end{aligned}$$

b **Method 1**

Common denominator $= 5 \times 20 = 100$

$\frac{2}{5} = \frac{2 \times 20}{5 \times 20} = \frac{40}{100}$ and $\frac{3}{20} = \frac{3 \times 5}{20 \times 5} = \frac{15}{100}$

$$\frac{2}{5} - \frac{3}{20} = \frac{40}{100} - \frac{15}{100}$$
$$= \frac{25 \div 25}{100 \div 25}$$
$$= \frac{1}{4}$$

Method 2

The LCD of 5 and 20 is 20.

$\frac{2}{5} = \frac{2 \times 4}{5 \times 4} = \frac{8}{20}$ and $\frac{3}{20}$

$$\frac{2}{5} - \frac{3}{20} = \frac{8}{20} - \frac{3}{20}$$
$$= \frac{5 \div 5}{20 \div 5}$$
$$= \frac{1}{4}$$

EXERCISE 4.03 ANSWERS ON P. 609

Adding and subtracting fractions

U F R

EXAMPLE 7

1 Evaluate each expression.

a $\frac{3}{6} + \frac{1}{6}$ **b** $\frac{7}{10} + \frac{2}{10}$ **c** $\frac{1}{2} + \frac{1}{2}$ **d** $\frac{2}{3} + \frac{2}{3}$

e $\frac{3}{5} - \frac{1}{5}$ **f** $\frac{5}{4} - \frac{3}{4}$ **g** $\frac{9}{10} - \frac{5}{10}$ **h** $\frac{7}{12} - \frac{4}{12}$

EXAMPLE 8

2 Evaluate each expression.

a $\frac{1}{2} + \frac{1}{3}$ **b** $\frac{2}{3} + \frac{1}{5}$ **c** $\frac{3}{4} - \frac{1}{2}$ **d** $\frac{5}{8} - \frac{1}{2}$

e $\frac{2}{6} + \frac{1}{3}$ **f** $\frac{4}{5} - \frac{3}{4}$ **g** $\frac{3}{7} - \frac{1}{9}$ **h** $\frac{7}{12} + \frac{1}{6}$

i $\frac{5}{8} + \frac{1}{4}$ **j** $\frac{9}{10} - \frac{3}{5}$ **k** $\frac{7}{9} - \frac{2}{3}$ **l** $\frac{5}{6} + \frac{3}{8}$

m $\frac{7}{8} - \frac{2}{10}$ **n** $\frac{3}{4} + \frac{4}{5}$ **o** $\frac{5}{8} - \frac{1}{3}$ **p** $\frac{6}{7} + \frac{1}{5}$

3 What fraction goes in the blank? $\frac{7}{8} -$ ____ $= \frac{1}{4}$. Select the correct answer **A**, **B**, **C** or **D**.

R

A $\frac{6}{4}$ **B** $\frac{8}{12}$ **C** $\frac{3}{8}$ **D** $\frac{5}{8}$

4 Copy and complete each equation with the correct fraction.

R **a** $\frac{1}{2} +$ ____ $= \frac{2}{3}$ **b** ____ $- \frac{1}{4} = \frac{2}{3}$

5 A bottle is $\frac{2}{3}$ full. $\frac{1}{4}$ of the bottle is then poured out. What fraction remains in the bottle?

R

Foundation Standard Complex

6 Evaluate each expression.

a $4+\frac{1}{2}$ **b** $3\frac{2}{3}-3$ **c** $4\frac{1}{6}-\frac{1}{6}$ **d** $5\frac{3}{4}-\frac{1}{4}$

e $2\frac{3}{5}+\frac{4}{5}$ **f** $6\frac{5}{8}-\frac{2}{8}$ **g** $\frac{5}{6}+1\frac{5}{6}$ **h** $3\frac{4}{5}+2\frac{1}{5}$

7 A cash prize is shared between 3 girls. Cindy receives $\frac{1}{4}$, Carmen gets $\frac{2}{5}$ and Libby takes the rest. What fraction does Libby take?

R

8 Copy and complete each equation using fractions with different denominators.

R **a** ____ + ____ = $\frac{15}{16}$ **b** ____ − ____ = $\frac{3}{4}$

DID YOU KNOW?

Ancient fractions

The earliest mention of fractions comes from ancient Egypt in 1800 BCE. The Egyptians wrote all their fractions as unit fractions, using 1 as their numerator. The Egyptians drew a mouth above a number to show that it was a fraction. For example, meant $\frac{1}{4}$.

Around 100 CE, the ancient Romans used words to describe fractions. For example:

- $\frac{1}{12}$ was called **uncia** (where the words 'ounce' and 'inch' come from)
- $\frac{6}{12}$ was called **semis**

By 500 CE, Indian mathematicians were writing fractions almost like we do, with a numerator above a denominator, but without a line. The Arabs added the line (vinculum), which we now use to separate the numerator and denominator.

Indian	Arab
5 −7	$\frac{8}{17}$

What does 'semi-' mean?

Why do you think the ancient Romans used the word 'semis' for $\frac{6}{12}$?

Foundation Standard Complex

4.04 Adding and subtracting mixed numerals

Adding and subtracting mixed numerals

To add or subtract mixed numerals, add or subtract the whole numbers first and then add or subtract the fractions.

Example 9

Find:

a $6\frac{1}{3}+4\frac{1}{2}$

b $10\frac{3}{4}-3\frac{2}{5}$

SOLUTION

a
$$\begin{aligned}6\frac{1}{3}+4\frac{1}{2}&=6+4+\frac{1}{3}+\frac{1}{2}\\&=10+\frac{2}{6}+\frac{3}{6}\\&=10\frac{5}{6}\end{aligned}$$

b
$$\begin{aligned}10\frac{3}{4}-3\frac{2}{5}&=10-3+\frac{3}{4}-\frac{2}{5}\\&=7+\frac{15}{20}-\frac{8}{20}\\&=7\frac{7}{20}\end{aligned}$$

Example 10

Find:

a $7-\frac{3}{5}$

b $5\frac{2}{3}-3\frac{3}{4}$

SOLUTION

a
$$\begin{aligned}7-\frac{3}{5}&=6+1-\frac{3}{5}\\&=6+\frac{5}{5}-\frac{3}{5}\\&=6\frac{2}{5}\end{aligned}$$

b
$$\begin{aligned}5\frac{2}{3}-3\frac{3}{4}&=5-3+\frac{2}{3}-\frac{3}{4}\\&=5-3+\frac{2}{3}-\frac{3}{4}\\&=2+\frac{8}{12}-\frac{9}{12}\\&=2-\frac{1}{12}\\&=1+\frac{12}{12}-\frac{1}{12}\\&=1\frac{11}{12}\end{aligned}$$

EXERCISE 4.04 ANSWERS ON P. 610

Adding and subtracting mixed numerals

U F PS C

EXAMPLE 9

1 Simplify each expression.

a $1\frac{1}{2}+1\frac{1}{2}$ **b** $2\frac{1}{2}+1\frac{3}{4}$ **c** $5\frac{1}{3}-2\frac{1}{5}$ **d** $4\frac{3}{8}-1\frac{1}{4}$

e $1\frac{1}{3}+2\frac{3}{4}$ **f** $2\frac{3}{5}+1\frac{1}{2}$ **g** $4\frac{5}{6}-2\frac{1}{3}$ **h** $1\frac{1}{2}+2\frac{1}{3}$

i $3\frac{3}{10}-2\frac{1}{4}$ **j** $2\frac{3}{4}+1\frac{2}{5}$ **k** $3\frac{1}{3}-1\frac{1}{4}$ **l** $3\frac{1}{7}+2\frac{1}{3}$

2 Evaluate $2\frac{3}{5}+1\frac{1}{3}$. Select the correct answer **A**, **B**, **C** or **D**.

A $3\frac{3}{8}$ **B** $3\frac{2}{5}$ **C** $3\frac{14}{15}$ **D** $3\frac{9}{15}$

EXAMPLE 10

3 Evaluate each difference.

a $3-\frac{1}{2}$ **b** $2-\frac{3}{5}$ **c** $1-\frac{4}{7}$ **d** $4-\frac{2}{3}$

e $2-\frac{1}{4}$ **f** $3-\frac{5}{12}$ **g** $14-\frac{7}{9}$ **h** $22-\frac{17}{20}$

4 **a** $3\frac{1}{5}-1\frac{1}{2}$ **b** $3\frac{1}{4}-2\frac{1}{2}$ **c** $6\frac{2}{5}-4\frac{7}{8}$

d $4\frac{3}{4}-2\frac{4}{5}$ **e** $2\frac{1}{3}-1\frac{1}{2}$ **f** $4\frac{3}{5}-2\frac{9}{10}$

g $3\frac{1}{6}-1\frac{2}{3}$ **h** $5\frac{3}{10}-2\frac{1}{2}$ **i** $5\frac{2}{3}-3\frac{3}{4}$

5 Rojesh drives $\frac{3}{4}$ of an hour to the airport, flies on a plane for $2\frac{1}{2}$ hours, then catches a shuttle bus to the hotel for $\frac{2}{3}$ of an hour. How long does the whole trip take? PS

6 Copy and complete each equation with the correct mixed numeral. R

a $3\frac{1}{2}+$ ____ $=5\frac{3}{5}$ **b** $4\frac{3}{4}-$ ____ $=1\frac{1}{8}$

7 Elise participated in a mini-triathlon on the weekend. Her time for the whole event was 6 hours. She swam for $1\frac{3}{4}$ hours and she cycled for $2\frac{2}{5}$ hours. How long did the running section take her? PS

8 Each Year 7 student at Nelson Secondary College must choose one sport from tennis, volleyball and gymnastics. If $\frac{3}{10}$ of students chose tennis and $\frac{2}{5}$ chose volleyball, what fraction of Year 7 chose gymnastics?

Foundation Standard Complex

9 In a magic square, each row, column and diagonal must add to the same total.
R Copy and complete each magic square with appropriate fractions.

a

4	$1\frac{1}{2}$	
$4\frac{1}{2}$		$4\frac{1}{2}$
3		

b

$\frac{1}{6}$		
	$\frac{2}{3}$	
	$\frac{1}{3}$	$1\frac{1}{6}$

4.05 Fraction of a quantity

Puzzle Fractions of words

Video Fractions: slow motion

What is $\frac{2}{3}$ of 12? Here are 12 lollies:

If these lollies were divided into 3 equal piles, each pile would be $\frac{1}{3}$ of 12.

$$\underbrace{\frac{1}{3}\text{ of }12=4 \qquad \frac{1}{3}\text{ of }12=4}_{\frac{2}{3}\times 12=8}$$

Note: The word 'of' usually means to multiply.

Video Fraction of a quantity

Example 11

Find: **a** $\frac{1}{4}\times 28$ **b** $\frac{2}{3}$ of \$30

SOLUTION

a $\frac{1}{4}\times 28 = 28 \div 4$
$= 7$

b $\frac{2}{3}\text{ of }30 = \left(\frac{1}{3}\times 30\right)\times 2$
$= \$10\times 2$
$= \$20$

Foundation Standard Complex

Example 12

Find:

a $\frac{1}{10}$ of 2 km (in metres)

b $\frac{3}{5}$ of 1 minute

SOLUTION

a $\frac{1}{10}$ of 2 km $= \frac{1}{10} \times 2000$ m
$= 200$ m

Convert 2 km to metres

b $\frac{3}{5}$ of 1 min $= \frac{3}{5}$ of 60 s
$= \left(\frac{1}{5} \times 60\text{ s}\right) \times 3$
$= 12\text{ s} \times 3$
$= 36$ s

Convert 1 minute to 60 seconds

Video
Fraction of a quantity

4.05

EXERCISE 4.05 ANSWERS ON P. 610

Fraction of a quantity

U F PS R C

1 Evaluate each product.

EXAMPLE 11

a $\frac{1}{4} \times 36$ **b** $\frac{1}{5} \times 35$ **c** $\frac{2}{5} \times 50$ **d** $\frac{1}{2} \times 16$

e $\frac{5}{12}$ of 12 **f** $\frac{2}{3}$ of 33 **g** $\frac{1}{9}$ of 27 **h** $\frac{6}{9}$ of 45

i $\frac{3}{10} \times 50$ **j** $\frac{5}{6} \times 60$ **k** $\frac{3}{4} \times 100$ **l** $\frac{2}{7} \times 42$

2 Evaluate each product.

a $\frac{1}{2}$ of \$42 **b** $\frac{2}{3}$ of \$12 **c** $\frac{1}{3}$ of \$24 **d** $\frac{1}{3}$ of 60c

e $\frac{1}{4}$ of \$1.20 **f** $\frac{1}{4}$ of \$10 **g** $\frac{3}{8}$ of \$360 **h** $\frac{2}{5}$ of 90c

i $\frac{5}{6}$ of \$3 **j** $\frac{3}{4}$ of \$16 **k** $\frac{4}{5}$ of \$4000 **l** $\frac{3}{10}$ of \$800

3 Evaluate each amount.

EXAMPLE 12

a $\frac{1}{4}$ of 2 hours (in minutes) **b** $\frac{3}{4}$ of 3 hours (in minutes)

c $\frac{3}{4}$ of 1 day (in hours) **d** $\frac{3}{5}$ of half an hour (in minutes)

e $\frac{1}{5}$ of 1 year (in days) **f** $\frac{1}{4}$ of 3 minutes (in seconds)

g $\frac{2}{3}$ of 5 minutes (in seconds) **h** $\frac{3}{10}$ of 1 hour (in minutes)

i $\frac{2}{3}$ of 2 days (in hours) **j** $\frac{1}{3}$ of 2 years (in months)

Foundation Standard Complex

4 In a squad of 45 swimmers, 3 out of every 5 were female.

a How many are female? **b** How many are male?

5 Evaluate each amount.

a $\frac{1}{4}$ of 1 km (in m) **b** $\frac{3}{4}$ of 3 km (in m) **c** $\frac{3}{4}$ of 6 cm (in mm)

d $\frac{1}{20}$ of 2 m (in cm) **e** $\frac{1}{4}$ of 3 kg (in g) **f** $\frac{7}{10}$ of 5 L (in mL)

g $\frac{1}{5}$ of 1.6 km (in m) **h** $\frac{3}{10}$ of 1 cm (in mm) **i** $\frac{3}{8}$ of 7.2 kg (in g)

j $\frac{5}{8}$ of 1.6 kL (in L) **k** $\frac{3}{5}$ of 8 m (in mm) **l** $\frac{2}{3}$ of 60 t (in kg)

6 Mr Arun began to read a 280 page book. On the weekend, he read $\frac{6}{7}$ of the book. How many more pages does he have to read before he finishes the book? Select **A**, **B**, **C** or **D**.

PS R

A 230 **B** 46 **C** 240 **D** 40

7 Rana has a pencil box that is $\frac{1}{2}$ full. She puts 4 extra pencils in and the box is now $\frac{2}{3}$ full. How many pencils will fit into her pencil box?

R

8 Copy and complete each equation.

R **a** $\frac{3}{4} \times$ ____ $= 9$ **b** ____ $\times 24 = 16$

9 Some lollies were placed on a table. Karin takes away half of the pile. Gino then takes one-third of what was left. Heather takes 2 and eats them. Half of those left are red and there are 8 red lollies. How many lollies were there originally?

PS R

10 A farmer died and in his will he left 17 horses to his three children. Half of the horses would go to Ashleigh, the eldest, one-third would go to Bethany, the middle child, and one-ninth would go to Thomas, the youngest. The family was puzzled because you cannot take those fractions of 17 horses without cutting up a horse!

PS R C

The farmer's wife had an idea. She borrowed a horse from a neighbouring farm to make a total of 18 horses. Now it was possible to divide up the horses according to the farmer's will.

a How many horses did each child receive?

b Why was the farmer's wife able to return the extra horse?

c What was mathematically incorrect about the farmer's will? Can you work it out?

Foundation Standard Complex

Quiz
Mental skills 4A

☆ MENTAL SKILLS 4A ANSWERS ON P. 610 Maths without calculators

Fraction of a quantity

Learn these commonly-used fractions and their decimal equivalents.

Fraction	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{3}{4}$	$\frac{1}{5}$	$\frac{1}{10}$	$\frac{1}{20}$	$\frac{2}{5}$
Decimal	0.5	0.25	0.125	0.75	0.2	0.1	0.05	0.4

Now we will use them to find a fraction or decimal of a quantity.

1 Study each example.

a $\frac{1}{4} \times 72 = 72 \div 4$
$= 18$

b $\frac{2}{5} \times 40 = \left(\frac{1}{5} \times 40\right) \times 2$
$= 8 \times 2$
$= 16$

c $\frac{3}{4} \times 32 = \left(\frac{1}{4} \times 32\right) \times 3$
$= 8 \times 3$
$= 24$

d $0.5 \times 66 = \frac{1}{2} \times 66$
$= 33$

e $0.05 \times 80 = \frac{1}{20} \times 80$
$= 4$

f $0.125 \times 56 = \frac{1}{8} \times 56$
$= 7$

2 Now simplify each expression.

a $\frac{1}{2} \times 28$ **b** $\frac{1}{4} \times 36$ **c** $\frac{1}{10} \times 70$ **d** $\frac{1}{8} \times 64$

e $\frac{1}{5} \times 15$ **f** $\frac{1}{10} \times 80$ **g** $\frac{2}{5} \times 25$ **h** $\frac{1}{20} \times 100$

i $\frac{3}{4} \times 44$ **j** $\frac{1}{8} \times 40$ **k** 0.25×60 **l** 0.4×45

m 0.1×260 **n** 0.125×48 **o** 0.75×48 **p** 0.05×120

q 0.2×70 **r** 0.5×320 **s** 0.25×56 **t** 0.125×16

INVESTIGATION

Netball scores

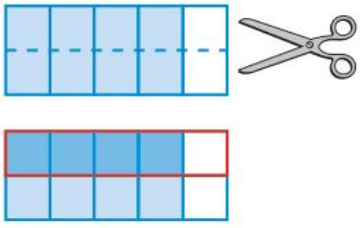
Elvira and Christina play netball for the Jets. A netball game is split into quarters and each goal is worth one point. Last Saturday, Elvira and Christina scored all of the Jets' goals for the first and second quarters.

1. In the first quarter, Elvira shot $\frac{1}{4}$ of the goals and Christina shot 12 goals. How many points did the team score in the first quarter?
2. In the second quarter, Christina scored $\frac{1}{5}$ of the team's 20 points while Elvira scored the rest. How many points did Elvira score?
3. What happened in the last 2 quarters? Finish off the story of the netball game by writing 2 more stories involving fractions.

Alamy Stock Photo/Rob Francis

4.06 Multiplying fractions

Worksheet Multiplying fractions using rectangles



What is $\frac{1}{2}$ of $\frac{4}{5}$?

$\frac{4}{5}$ of this diagram is shaded.

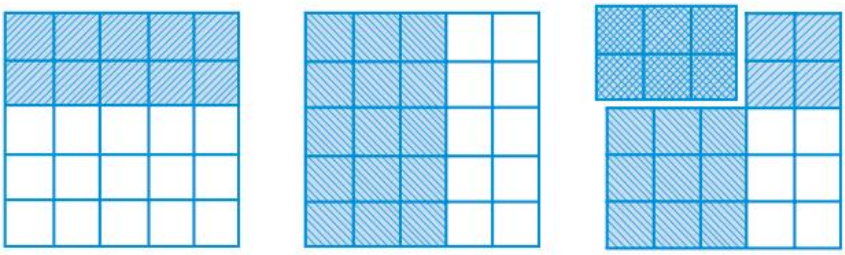
If the diagram were cut in half, each fifth would also be cut in half to make a total of 10 pieces, with 8 of them shaded.

So $\frac{1}{2}$ of $\frac{4}{5}$ (shaded) $= \frac{1}{2}$ of $\frac{8}{10} = \frac{4}{10}$

Another way to work this out is:

$$\frac{1}{2} \text{ of } \frac{4}{5} = \frac{1}{2} \times \frac{4}{5}$$
$$= \frac{1 \times 4}{2 \times 5}$$
$$= \frac{4}{10}$$

You can use diagrams to show other multiplications, for example, $\frac{2}{5} \times \frac{3}{5}$



$\frac{2}{5}$ × $\frac{3}{5}$ = $\frac{6}{25}$

Multiplying fractions

To multiply fractions, multiply the numerators and multiply the denominators separately.

Example 13

Evaluate each product.

a $\frac{3}{5} \times \frac{7}{8}$ **b** $\frac{3}{5} \times \frac{4}{9}$ **c** $\frac{21}{100} \times \frac{8}{15}$

Video Multiplying fractions

SOLUTION

a $\frac{3}{5} \times \frac{7}{8} = \frac{3 \times 7}{5 \times 8}$

$= \frac{21}{40}$

b $\frac{3}{5} \times \frac{4}{9} = \frac{3 \times 4}{5 \times 9}$ because $\frac{3}{9} = \frac{1}{3}$

$= \frac{12 \div 3}{45 \div 3}$

$= \frac{4}{15}$

or, by simplifying first before multiplying:

$\frac{3}{5} \times \frac{4}{9} = \frac{\cancel{3}^{1} \times 4}{5 \times \cancel{9}^{3}}$

$= \frac{4}{15}$

c By simplifying first before multiplying:

$$\frac{21}{100}\times\frac{8}{15}=\frac{\cancel{21}^{7}\times\cancel{8}^{2}}{\cancel{100}^{25}\times\cancel{15}^{5}}$$

$$=\frac{14}{125}$$

because $\frac{21}{15}=\frac{7}{5}$, $\frac{8}{100}=\frac{2}{25}$

Multiplying mixed numerals

To multiply mixed numerals, convert them to improper fractions first.

Example 14

Simplify $3\frac{2}{5}\times1\frac{2}{3}$.

SOLUTION

$$3\frac{2}{5}\times1\frac{2}{3}=\frac{17}{\cancel{5}_{1}}\times\frac{\cancel{5}^{1}}{3}$$

$$=\frac{17}{3}$$

$$=5\frac{2}{3}$$

EXERCISE 4.06 ANSWERS ON P. 610

Multiplying fractions

U F PS R C

EXAMPLE 13

1 Evaluate each product.

a $\frac{1}{3}\times\frac{2}{5}$ **b** $\frac{3}{4}\times\frac{3}{5}$ **c** $\frac{5}{6}\times\frac{7}{8}$ **d** $\frac{1}{8}\times\frac{9}{10}$

e $\frac{3}{8}\times\frac{5}{6}$ **f** $\frac{3}{8}\times\frac{3}{8}$ **g** $\frac{4}{9}\times\frac{5}{7}$ **h** $\frac{7}{8}\times\frac{3}{20}$

2 Evaluate $\frac{2}{9}\times\frac{3}{10}$. Select the correct answer **A**, **B**, **C** or **D**.

A $\frac{1}{8}$ **B** $\frac{1}{15}$ **C** $\frac{6}{19}$ **D** $\frac{1}{18}$

3 Evaluate each product.

a $\frac{1}{5}\times\frac{5}{7}$ **b** $\frac{3}{7}\times\frac{7}{10}$ **c** $\frac{1}{8}\times\frac{4}{9}$

d $\frac{3}{7}\times\frac{7}{3}$ **e** $\frac{3}{8}\times\frac{5}{6}$ **f** $\frac{3}{7}\times\frac{4}{9}$

Foundation Standard Complex

g $\frac{2}{5} \times \frac{3}{8}$ **h** $\frac{3}{8} \times \frac{5}{12}$ **i** $\frac{3}{10} \times \frac{2}{5}$

j $\frac{9}{5} \times \frac{25}{3}$ **k** $\frac{8}{5} \times \frac{10}{7}$ **l** $\frac{1}{6} \times \frac{12}{25}$

m $\frac{5}{6} \times \frac{11}{20}$ **n** $\left(\frac{1}{8}\right)^2$ **o** $\frac{2}{3} \times \frac{9}{10}$

p $\left(\frac{3}{4}\right)^2$

EXAMPLE 14

4 Convert each mixed numeral to an improper fraction before multiplying.

a $1\frac{2}{3} \times \frac{3}{10}$ **b** $4\frac{1}{3} \times \frac{2}{7}$ **c** $5\frac{2}{5} \times 1\frac{1}{8}$

d $3\frac{5}{6} \times 1\frac{3}{4}$ **e** $1\frac{2}{3} \times 1\frac{2}{3}$ **f** $3\frac{1}{9} \times 1\frac{3}{4}$

5 PS Mrs Duschevina works $7\frac{3}{4}$ hours a day. She works a full day each day from Monday to Friday and works half a day on Saturday. How many hours a week does she work?

6 R A cake recipe needs 1 egg, $2\frac{1}{2}$ cups of flour and $\frac{1}{4}$ cup of sugar. What quantities of eggs, flour and sugar will you need to make 8 cakes?

7 Copy and complete each equation.

R C

a $\frac{1}{4} + \frac{1}{4} + \frac{1}{4} = 3 \times \frac{1}{4}$
$=$

b $\frac{3}{5} + \frac{3}{5} = 2 \times \frac{3}{5}$
$=$
$=$

c $\frac{3}{8} + \frac{3}{8} + \frac{3}{8} + \frac{3}{8} + \frac{3}{8} = 5 \times$
$=$
$=$

d $5 \times \frac{1}{7} =$

e $7 \times \frac{3}{20} =$
$=$

f $8 \times \frac{3}{5} =$
$=$

8 PS Noah wants to be a professional skater. He practises for $2\frac{1}{2}$ hours each day. How many hours of practice does he do in 7 days?

9 Copy and complete each equation.

R

a $\frac{3}{4} \times$ ____ $= \frac{21}{32}$ **b** ____ $\times \frac{1}{5} = \frac{2}{15}$

c $\frac{15}{16} \times$ ____ $= \frac{3}{4}$ **d** ____ $\times$ ____ $= \frac{3}{8}$

10 R C True or false: When a number is multiplied by a fraction, the number is decreased. Give reasons for your answer.

Foundation Standard Complex

INVESTIGATION

The Aboriginal flag

The Aboriginal flag was designed in 1970 by artist Harold Thomas, who is descended from the Luritja Peoples of central Australia. It was formally accepted as an Australian flag in 1995. The top half of the flag is black to symbolise Aboriginal people. The red in the lower half stands for the earth. The yellow circle at the centre of the flag represents the sun.

The circle is placed in the middle of the flag. Its diameter (length) is $\frac{1}{3}$ of the length of the flag.

Draw the Aboriginal flag on your page. You can make it 30 cm long and 20 cm wide as shown on the diagram, or if you want to make it smaller, 15 cm by 10 cm. Show the dimensions (length and width) of the rectangles and circle on your drawing, including calculations of how these dimensions were found.

TECHNOLOGY

More spreadsheet practice

	A	B	C
1	10	=A1+A5	
2	-3		
3	0		
4	4		
5	-1		

1 Open a new spreadsheet and enter the 5 numbers shown in column A.

2 In cell B1, enter the formula **=A1+A5** to calculate the sum of the values in cells A1 and A5. You should get the answer 9, because 10 + (–1) = 9.

3 For the rest of the cells in column B, enter the appropriate formula for each expression shown below and check that the spreadsheet calculates the correct answer. Remember to start each formula with an '=' sign.

a at B2, A2 + A4

b at B3, A2 – A4

c at B4, A3 – A4 – A1

d at B5, A3 – A4 + A1

e at B6, A2 – A5

f at B7, A1 × A2

g at B8, A2 × A4 × A5

h at B9, $(A2)^2$

i at B10, $(A5)^3$

j at B11, A4 ÷ A5

4 At C1, enter a formula for $\frac{A5}{A1}$. When you have obtained the answer, right-click on the cell, select **Format Cells** and convert the answer to a fraction.

5 At C2, enter a formula for $\frac{A1}{A2}$. Right-click on the cell, select **Format Cells** and convert the answer to a mixed numeral.

6 For the rest of the cells in column C, enter the appropriate formula for each expression shown below and check that the spreadsheet calculates the correct answer.

a at C3, the sum of column A

b at C4, the average of column A

c at C5, the minimum value of

d at C6, A4 ÷ (–2) column A

e at C7, A4 ÷ A4

f at C8, (A2 + A4) × A1

g at C9, (A2 – A4) ÷ A5

h at C10, A1 ÷ (A5 – A4)

7 At C11, enter a formula for $\frac{A3}{A2}$. At C12, enter a formula for $\frac{A2}{A3}$. Why doesn't the second formula work?

8 Now enter **new** values in cells A1 to A5 and notice the new answers calculated in columns B and C.

Dividing fractions 4.07

If we turn a fraction upside-down and exchange the numerator with the denominator, we get its **reciprocal**.

- The reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$ (or $1\frac{1}{2}$)
- The reciprocal of $\frac{1}{4}$ is 4 Note: $\frac{4}{1} = 4$
- The reciprocal of 3 is $\frac{1}{3}$ Note: $3 = \frac{3}{1}$
- The reciprocal of $\frac{a}{b}$ is $\frac{b}{a}$

Note that dividing by 3 is the same as multiplying by $\frac{1}{3}$.

Similarly, dividing by $\frac{1}{3}$ is the same as multiplying by 3.

When dividing by any fraction, we can use its reciprocal.

Worksheets
What's the fraction question?

Multiplying and dividing fractions number grids

Dividing fractions

To divide by a fraction $\frac{a}{b}$, multiply by its reciprocal $\frac{b}{a}$.

Video
Dividing fractions

Example 15

Evaluate each quotient.

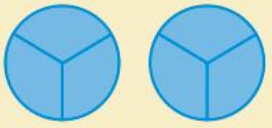
a $2 \div \frac{1}{3}$ **b** $\frac{3}{4} \div \frac{1}{2}$ **c** $\frac{7}{3} \div \frac{5}{6}$ **d** $\frac{5}{7} \div 5$

SOLUTION

a

$$2 \div \frac{1}{3} = 2 \times \frac{3}{1}$$
$$= 6$$

How many times does $\frac{1}{3}$ go into 2?

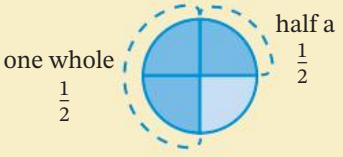


b

$$\frac{3}{4} \div \frac{1}{2} = \frac{3}{4} \times \frac{2}{1}$$
$$= \frac{6}{4}$$
$$= \frac{3}{2}$$
$$= 1\frac{1}{2}$$

$\frac{3}{4}$ is '$1\frac{1}{2}$ halves'

How many times $\frac{1}{2}$ go into $\frac{3}{4}$?



c

$$\frac{7}{3} \div \frac{5}{6} = \frac{7}{\cancel{3}^{1}} \times \frac{\cancel{6}^{2}}{5}$$
$$= \frac{14}{5}$$
$$= 2\frac{4}{5}$$

d

$$\frac{5}{7} \div 5 = \frac{\cancel{5}^{1}}{7} \times \frac{1}{\cancel{5}^{1}}$$
$$= \frac{1}{7}$$

Dividing by 5 is the same as multiplying by $\frac{1}{5}$.

Dividing mixed numerals

To divide mixed numerals, convert them to improper fractions first.

Example 16

Simplify $7\frac{1}{2} \div 5\frac{5}{6}$.

SOLUTION

$$7\frac{1}{2} \div 5\frac{5}{6} = \frac{15}{2} \div \frac{35}{6}$$
$$= \frac{\cancel{15}^{\cancel{3}}}{\cancel{2}_{\cancel{1}}} \times \frac{\cancel{6}^{\cancel{3}}}{\cancel{35}_{\cancel{7}}}$$
$$= \frac{9}{7}$$
$$= 1\frac{2}{7}$$

EXERCISE 4.07 ANSWERS ON P. 611

Dividing fractions

U F PS R C

1 Find the reciprocal of each fraction.

a $\frac{2}{3}$ **b** $\frac{1}{5}$ **c** $\frac{5}{8}$ **d** $\frac{1}{10}$

e 7 **f** $2\frac{1}{2}$ **g** $1\frac{3}{4}$ **h** 9

2 Evaluate each quotient. EXAMPLE 15

a $4 \div \frac{1}{3}$ **b** $5 \div \frac{1}{2}$ **c** $7 \div \frac{1}{3}$ **d** $3 \div \frac{1}{5}$

e $7 \div \frac{1}{6}$ **f** $8 \div \frac{1}{10}$ **g** $4 \div \frac{1}{4}$ **h** $1 \div \frac{1}{8}$

3 Evaluate $\frac{3}{8} \div \frac{1}{2}$. Select the correct answer **A**, **B**, **C** or **D**.

A $\frac{1}{12}$ **B** $1\frac{1}{4}$ **C** $\frac{3}{4}$ **D** $\frac{3}{16}$

4 Water is dripping from a tank at the rate of $\frac{5}{8}$ litre per hour. How long will it take for a 24-litre tank to empty?

PS R

5 Evaluate each quotient.

a $\frac{1}{4} \div \frac{2}{3}$ **b** $\frac{8}{7} \div \frac{2}{7}$ **c** $\frac{3}{5} \div \frac{2}{5}$ **d** $\frac{5}{8} \div \frac{2}{5}$

e $\frac{7}{10} \div \frac{5}{6}$ **f** $\frac{3}{5} \div \frac{5}{8}$ **g** $\frac{11}{20} \div \frac{9}{10}$ **h** $\frac{5}{12} \div \frac{5}{6}$

i $\frac{5}{6} \div \frac{5}{12}$ **j** $\frac{1}{4} \div \frac{1}{5}$ **k** $\frac{1}{10} \div \frac{1}{2}$ **l** $\frac{1}{2} \div \frac{2}{3}$

Foundation Standard Complex

6 **a** What number's reciprocal is itself?

R **b** What number does not have a reciprocal?

c What is the reciprocal of the reciprocal of $\frac{7}{10}$?

7 What happens when you:

R **a** multiply any number by its reciprocal?

C **b** divide any number by its reciprocal?

8 A bag of rice weighing $\frac{4}{5}$ kg was divided among 6 friends. How much rice did each person get? Select **A**, **B**, **C** or **D**.

A 480 g **B** $\frac{2}{15}$ kg **C** 800 g **D** $\frac{4}{15}$ kg

9 Evaluate each quotient.

a $\frac{5}{7} \div 5$ **b** $\frac{5}{7} \div 7$ **c** $\frac{1}{4} \div 3$ **d** $\frac{5}{6} \div 2$

EXAMPLE 16

10 Convert each mixed numeral to an improper fraction before dividing.

a $3\frac{1}{8} \div \frac{5}{6}$ **b** $3\frac{1}{2} \div 1\frac{1}{4}$ **c** $2\frac{3}{4} \div 3\frac{1}{7}$ **d** $2\frac{1}{2} \div 1\frac{7}{8}$

11 How many speakers were there if each speaker was allowed $3\frac{1}{4}$ minutes and the debate went for about 26 minutes?

12 Copy and complete each equation.

R **a** $____ \div \frac{2}{3} = \frac{1}{7}$ **b** $\frac{4}{15} \div ____ = \frac{8}{21}$

13 True or false: When a number is divided by a proper fraction, the number is increased. Give reasons for your answer.

R

C

INVESTIGATION

Thanh's garden

In this activity, you will be using fractions to make a decorative pattern.

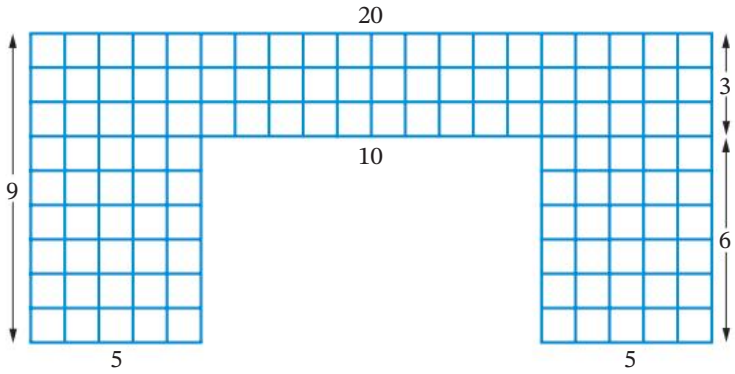
In Thanh's garden there are places for 120 flowers. He wants to plant them in the following proportions:

$\frac{1}{3}$ golden daffodils $\frac{3}{10}$ red tulips $\frac{1}{6}$ white snowdrops $\frac{1}{5}$ blue hyacinths

1 How many of each type of flower should Thanh plant?

2 Copy the plan below or use one provided by your teacher. Colour in squares for each flower. Make up your own pattern.

Foundation Standard Complex



3 Make up different fractions for the same garden plot. Make a list of flowers and the fractions of their proportions and colour them on a new plan (or colour the plan first and list the fractions afterwards).

Fraction problems 4.08

EXERCISE 4.08 ANSWERS ON P. 611

Fraction problems

U F R C

1 A class investigation found that 3 out of every 25 cars passing their school was white. If 2150 cars pass the school in a day, how many would you expect to be white?

2 $\frac{3}{4}$ of every Year 7 class are boys. There are 24 students in 7 Ang, 20 in 7 Binns, 28 in 7 Carrozza and 16 in 7 Drake.

a How many boys are there in Year 7?

b How many girls are there in 7 Carrozza?

3 Anthony ate $\frac{2}{5}$ of a pizza and Lisa ate $\frac{1}{3}$.

a Who ate more?

b How much did they eat together?

c How much is left?

4 Mrs Rizzo's patio measures 5 m × 3 m. How many paving stones does it contain if there are 8 stones per m²?

PS

Foundation Standard Complex

5 A bag holds $\frac{5}{8}$ of a kilogram of oranges. How many bags can $12\frac{1}{2}$ kg of oranges be packed into?

6 PS When Paige walks quickly to school, she takes half an hour. When she walks slowly, she takes an extra $\frac{1}{3}$ of an hour.

a What fraction of an hour does Paige take walking at her slow pace?

b How many minutes is this?

7 Jeff bought 6 books and put them in his bag. His orange juice leaked and ruined $\frac{2}{3}$ of them. How many books were not ruined?

8 Mrs Chan is planning a slide presentation. Each slide is to be shown for $2\frac{1}{4}$ seconds. How many slides can she show in 90 seconds?

9 Each day, Mr Lanouri is at work for $7\frac{1}{2}$ hours, less $\frac{3}{4}$ of an hour for lunch. How long does he work in one day?

10 The school hall has 840 seats. If it is two-thirds full, how many seats are vacant?

11 4 authors share royalties of $\frac{1}{8}$ of the price of a textbook. Darren receives $\frac{4}{15}$ of the author share, so what fraction of the price of each book does he receive?

12 Ben was told, 'When mowing, never cut more than $\frac{1}{3}$ off the grass height'. If the lawn is $12\frac{1}{2}$ cm high, what is the minimum height to which Ben should cut the grass? Select **A**, **B**, **C** or **D**.

A $4\frac{1}{6}$ cm **B** $12\frac{1}{6}$ cm **C** $7\frac{1}{2}$ cm **D** $8\frac{1}{3}$ cm

13 PS R 200 sausages were required for a barbecue. One-tenth of them were donated by a parent, 25 had been left in the freezer from a previous occasion, and a local butcher gave one-quarter of the total free. The rest had to be bought at wholesale price. What fraction had to be bought?

14 R Kylie finds $\frac{1}{3}$ of her birthday cake in the fridge. If she eats $\frac{2}{3}$ of this amount, what fraction of the whole cake will be left?

15 R $\frac{1}{6}$ of a class were a day late with their assignments, while the rest handed them in on time. 20 students handed their assignments in on time.

a How many students were in the class?

b How many students were late in handing in their assignments?

Foundation Standard Complex

Fractions and the calculator

4.09

Scientific calculators have a fraction key for entering fractions: [▭] or [a^b/c]. Casio calculators have 2 ways of entering fractions: MATH mode or LINE mode. MATH mode allows you to enter the numerator and denominator into 2 blank spaces on the calculator's screen, while LINE mode makes the fraction key act like a vinculum (fraction bar). Press [SHIFT] [MODE] to change between MATH and LINE modes. To ensure that mixed numeral answers are not converted to improper fractions, press [SHIFT] [MODE] and the down arrow to choose 'ab/c'.

Example 17

Question	Calculator steps		Answer
	Casio LINE mode	Sharp	
Simlply $\frac{16}{20}$	16 [▭] 20 [=]	16 [a^b/c] 20 [a^b/c]	$\frac{4}{5}$
Change $1\frac{3}{4}$ to an improper fraction	1 [▭] 3 [▭] 4 [=] [SHIFT] [S⇔D]	1 [a^b/c] 3 [a^b/c] 4 [=] [2nd F] [a^b/c]	$\frac{7}{4}$
Change $\frac{19}{4}$ to an improper fraction	19 [▭] 4 [=]	19 [a^b/c] 4 [=]	$4\frac{3}{4}$

Press [SHIFT] [S⇔D] or [2nd F] [a^b/c] to convert an answer between mixed numeral and improper fraction forms.

For Casio MATH mode, press the [▭] key first and use arrow keys to move the cursor to the 2 blank spaces to enter the values. Then press [=].

To enter a mixed numeral, press [SHIFT] and move the cursor to the 3 blank spaces to enter the values. Then press [=].

Example 18

Question	Calculator steps		Answer
	Casio LINE mode	Sharp	
$\frac{1}{2}+\frac{1}{3}$	1 [▭/▭] 2 [+] 1 [▭/▭] 3 [=]	1 [a b/c] 2 [+] 1 [a b/c] 3 [=]	$\frac{5}{6}$
$\frac{5}{7}-\frac{2}{3}$	5 [▭/▭] 7 [−] 2 [▭/▭] 3 [=]	5 [a b/c] 7 [−] 2 [a b/c] 3 [=]	$\frac{1}{21}$
$\frac{3}{5}\times\frac{2}{7}$	3 [▭/▭] 5 [×] 2 [▭/▭] 7 [=]	3 [a b/c] 5 [×] 2 [a b/c] 7 [=]	$\frac{6}{35}$
$\frac{4}{5}\div\frac{2}{3}$	4 [▭/▭] 5 [÷] 2 [▭/▭] 3 [=]	4 [a b/c] 5 [÷] 2 [a b/c] 3 [=]	$1\frac{1}{5}$

EXERCISE 4.09 ANSWERS ON P. 611

Fractions and the calculator

U F PS C

EXAMPLE 17

1 Simplify each fraction.

a $\frac{14}{21}$ **b** $\frac{65}{130}$ **c** $\frac{27}{81}$

d $\frac{52}{65}$ **e** $\frac{168}{196}$ **f** $\frac{260}{325}$

2 Convert each mixed numeral to an improper fraction.

a $2\frac{3}{4}$ **b** $5\frac{1}{3}$ **c** $4\frac{2}{7}$

d $6\frac{3}{5}$ **e** $10\frac{1}{3}$ **f** $8\frac{5}{6}$

3 Convert each improper fraction to a mixed numeral.

a $\frac{18}{5}$ **b** $\frac{23}{7}$ **c** $\frac{45}{8}$

d $\frac{79}{11}$ **e** $\frac{123}{10}$ **f** $\frac{458}{24}$

EXAMPLE 18

4 Evaluate each expression.

a $\frac{3}{5}-\frac{1}{2}$ **b** $\frac{4}{7}+\frac{1}{2}$ **c** $\frac{3}{5}+\frac{2}{3}$ **d** $\frac{1}{4}\times\frac{2}{7}$

e $\frac{5}{7}\div\frac{10}{21}$ **f** $\frac{5}{8}+\frac{5}{6}$ **g** $\frac{9}{15}-\frac{1}{3}$ **h** $\frac{7}{8}\times\frac{3}{4}$

i $\frac{7}{8}\div\frac{3}{4}$ **j** $14\div2\frac{1}{3}$ **k** $2\frac{1}{2}+1\frac{1}{3}$ **l** $3\frac{1}{4}-\frac{5}{8}$

m $2\frac{3}{4}\times1\frac{1}{2}$ **n** $5\frac{2}{3}\div\frac{3}{8}$ **o** $\frac{4}{5}\div8$ **p** $1\frac{1}{2}\times2\frac{3}{10}$

Foundation Standard Complex

5 Find:

a $\frac{2}{5}$ of 170 **b** $\frac{4}{7}$ of 98 **c** $\frac{1}{3}$ of 75

d $\frac{3}{8}$ of \$1360 **e** $\frac{2}{5}$ of \$4200 **f** $\frac{4}{9}$ of \$198

6 If a farm of 78 hectares is subdivided into lots of $4\frac{1}{2}$ hectares, how many lots are there?

7 PS Nikki has a part-time job mowing lawns. This table shows the number of hours she worked in the last month.

Customer	Hours
Green	$2\frac{1}{2}$
Cook	$1\frac{3}{4}$
Verbeek	$3\frac{1}{2}$
Thompson	$2\frac{3}{4}$
Burns	$2\frac{1}{4}$

a Find the total number of hours Nikki worked.

b If Nikki charges \$12.50 per hour, how much did she earn in total?

8 PS Radio station FM 3.14 plays $32\frac{1}{2}$ minutes of music every hour. The average song played on the station lasts $3\frac{3}{4}$ minutes. About how many songs can be played in 24 hours?

9 PS R An echidna has a mass of $1\frac{1}{2}$ kg. A field mouse has a mass of $\frac{1}{20}$ that of the echidna, but 5000 ants are needed to balance the mass of one field mouse. Find the mass of one ant.

Foundation Standard Complex

INVESTIGATION

Fraction puzzles

Find strategies to solve each problem. Write down the thinking you used to work it out.

1 How many jelly beans?

Samantha has a bag of jelly beans to share. Ali takes $\frac{1}{4}$ of them. Duyen takes $\frac{1}{2}$ of what is left. Nami takes $\frac{2}{3}$ of the remainder. Samantha has 7 jelly beans left. How many jelly beans were in the bag to start with?

2 Who won the race?

- Jane finished half a minute behind Bill.
- Bill took less than half the time that Joel took.
- Joel's time was 5 minutes 23 seconds.
- Dimitra took about $3\frac{1}{4}$ minutes.
- Esther finished about $1\frac{1}{2}$ minutes after Bill.
- David finished three-quarters of a minute in front of Joel.

What is the finishing order of the six competitors?

3 How old is Zoe?

When asked her age, Zoe gave this reply:

If to my age there added be
One-half, one-third (of it) and 3 times 3,
Six score and 10 the sum you would see.
Now pray tell me what age I be.

Hint: First find how much a 'score' is.

4.10 Percentages, fractions and decimals

Worksheets
Percentages wall
Percentage shapes
Fractions, decimals and percentages

Percentages and decimals are special types of fractions. A **percentage** is a fraction whose denominator is always 100, for example, $78\% = \frac{78}{100}$. The symbol % is an abbreviation of '/100' and 'per cent', which comes from the Latin *per centum*, meaning 'by the hundred'. A **decimal** is a fraction whose denominator is a **power of 10**, such as 10, 100, 1000; for example, $0.015 = \frac{15}{1000}$. Because fractions, percentages and decimals can be written in the form $\frac{a}{b}$, where a and b are integers but $b \neq 0$, they can all be classified as being **rational numbers**.

- $50\% = \frac{1}{2}$ because $50\% = \frac{50}{100} = \frac{1}{2}$
- $50\% = 0.5$ because $50\% = \frac{50}{100} = 50 \div 100 = 0.5$

Converting percentages to fractions and decimals

- **To convert a percentage to a fraction**, write the percentage with a denominator of 100 and simplify if needed
- **To convert a percentage into a decimal**, divide it by 100

Example 19

Convert each percentage to a fraction in simplest form.

a 3% **b** 45% **c** $5\frac{1}{2}\%$

SOLUTION

a $3\% = \frac{3}{100}$

b $45\% = \frac{45}{100}$

$= \frac{9}{20}$

c $5\frac{1}{2}\% = \frac{5\frac{1}{2}}{100}$

$= \frac{5\frac{1}{2} \times 2}{100 \times 2}$

$= \frac{11}{200}$

Example 20

Convert each percentage to a decimal.

a 39% **b** 2% **c** 16.9%

SOLUTION

a $39\% = 39 \div 100$

$= 0.39$

b $2\% = 2 \div 100$

$= 0.02$

c $16.9\% = 16.9 \div 100$

$= 0.169$

To divide a number by 100, move the decimal point 2 places to the left.

- $\frac{1}{4} = 25\%$ because $\frac{1}{4} \times 100\% = 25\%$
- $0.2 = 20\%$ because $0.2 \times 100\% = 20\%$

Converting fractions and decimals to percentages

To convert a fraction or a decimal to a percentage, multiply it by 100%.

4.10

Puzzles
Converting percentages to fractions

Converting fractions to percentages

Converting decimals to percentages

Presentation
Percentages, fractions and decimals

Video
Converting fractions and decimals to percentages

Example 21

Convert each fraction to a percentage.

a $\frac{3}{4}$ **b** $\frac{1}{8}$ **c** $\frac{34}{40}$

SOLUTION

a $\frac{3}{4} = \frac{3}{4} \times 100\%$

$= 75\%$

b $\frac{1}{8} = \frac{1}{8} \times 100\%$

$= 12\frac{1}{2}\%$

c $\frac{34}{40} = \frac{34}{40} \times 100\%$

$= 85\%$

Video
Converting fractions and decimals to percentages

Example 22

Convert each decimal to a percentage.

a 0.23 **b** 0.147 **c** 0.6

SOLUTION

a $0.23 = 0.23 \times 100\%$

$= 23\%$

b $0.147 = 0.147 \times 100\%$

$= 14.7\%$

c $0.6 = 0.6 \times 100\%$

$= 60\%$

To multiply a number by 100, move the decimal point 2 places to the right

EXERCISE 4.10 ANSWERS ON P. 611

Percentages, fractions and decimals

U F R

EXAMPLE 19

1 Convert each percentage to a fraction in simplest form.

C **a** 5% **b** 14% **c** 8% **d** 90%

e 7% **f** $3\frac{1}{2}\%$ **g** $10\frac{1}{3}\%$ **h** $5\frac{3}{4}\%$

EXAMPLE 20

2 Convert each percentage to a decimal.

C **a** 28% **b** 6% **c** 16.1% **d** 21.3%

e 2% **f** 20% **g** 78.07% **h** $26\frac{1}{2}\%$

EXAMPLE 21

3 Convert each fraction to a percentage.

C **a** $\frac{9}{100}$ **b** $\frac{17}{100}$ **c** $\frac{5}{8}$ **d** $\frac{27}{30}$

e $\frac{11}{50}$ **f** $\frac{3}{10}$ **g** $\frac{1}{5}$ **h** $\frac{22}{40}$

Foundation Standard Complex

i $\frac{3}{4}$	**j** $\frac{12}{25}$	**k** $\frac{17}{40}$	**l** $\frac{41}{50}$
m $\frac{1}{16}$	**n** $\frac{1}{3}$	**o** $1\frac{1}{2}$	**p** $\frac{2}{9}$

4 Convert each decimal to a percentage.

EXAMPLE 22

C

a 0.24	**b** 0.5	**c** 0.86	**d** 0.571
e 0.06	**f** 0.345	**g** 2.4	**h** 0.007
i 1.3	**j** 0.001	**k** 0.315	**l** 0.026

5 Cameron sat for a Geography test and scored 35 out of 40. What is his score as a percentage? Select the correct answer **A**, **B**, **C** or **D**.

C

A 87.5% **B** 80% **C** 35% **D** 14%

6 Which one of the following numbers is the smallest? Select **A**, **B**, **C** or **D**.

A $\frac{4}{5}$ **B** 0.885 **C** $\frac{7}{8}$ **D** 88%

Percentage of a quantity 4.11

Percentage of a quantity

To find a percentage of a quantity, calculate:

$$\frac{\text{percentage}}{100} \times \text{quantity}$$

or percentage ÷ 100 × quantity

Worksheet Percentages of an amount

Puzzle Percentages

Also see **Mental skills 4B Percentage of a quantity** on page 183 to learn how to mentally calculate with percentages.

Video Percentages of an amount

Example 23

Find:

a 12% of 650 mL **b** 4.5% of $2400

SOLUTION

a $12\% \text{ of } 650 \text{ mL} = \frac{12}{100} \times 650 \text{ mL}$ or $12 \div 100 \times 650$ mL

$= 78$ mL

Estimate: $12\% \text{ of } 650 \approx 10\% \times 650 = 65$

Note: If you know that $12\% = 0.12$, then you can also use $0.12 \times 650 \text{ mL} = 78 \text{ mL}$.

Foundation Standard Complex

b 4.5% of \$2400 $= \frac{4.5}{100} \times \$2400$ or $4.5 \div 100 \times \$2400$

$= \$108$

Estimate: 4.5% of \$2400 $\approx 5\% \times \$2400 = \$120$

Note: If you know that $4.5\% = 0.045$, then you can also use $0.045 \times \$2400 = \$108$.

EXERCISE 4.11 ANSWERS ON P. 612

Percentage of a quantity

U F

EXAMPLE 23

1 Find:

a 10% of 60 **b** 85% of \$240 **c** 5% of 150

d 24% of 350 mm **e** 20% of 740 kg **f** 12% of 60 g

g 33% of \$500 **h** 6% of 180 **i** 2% of \$4800

j 80% of 280 mL **k** 20% of \$6.50 **l** 50% of 810 m

m 7% of \$220 **n** 3% of 90 hectares **o** 25% of 60 minutes

2 Find 5% of 74 tonnes. Select the correct answer **A**, **B**, **C** or **D**.

A 370 tonnes **B** 3.7 tonnes **C** 37 tonnes **D** 3700 tonnes

3 Erika achieved 60% in her last mathematics test. If the test was out of 70, how many marks did she receive?

4 In a school of 650 students, 48% are girls. How many girls are there in the school?

5 Farmer Chen owns 775 animals. Of these, 16% are chickens.

a How many chickens does farmer Chen have?

b How many other animals does he have?

6 Jaden planted 168 seedlings in his garden. If 12% died, how many plants survived?

7 In a city of 3 million people, 1% of the population are doctors. How many doctors are there? Select **A**, **B**, **C** or **D**.

A 3 **B** 3000 **C** 300 **D** 30 000

8 Copy and complete:

a $20\% \times$ _____ $= \$30$ **b** $35\% \times$ _____ $= \$56$

Foundation Standard Complex

Quiz
Mental skills 4B

☆ MENTAL SKILLS 4B ANSWERS ON P. 612 Maths without calculators

Percentage of a quantity

Learn these commonly-used fractions and their percentage equivalents.

Fraction	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{3}{4}$	$\frac{1}{5}$	$\frac{1}{10}$	$\frac{1}{20}$	$\frac{2}{5}$
Percentage	50%	25%	12.5%	75%	20%	10%	5%	40%

Now we will use them to find a percentage of a quantity.

1 Study each example.

a $20\% \times 45 = \frac{1}{5} \times 45$
$= 9$

b $50\% \times 28 = \frac{1}{2} \times 28$
$= 14$

c $12.5\% \times 48 = \frac{1}{8} \times 48$
$= 6$

d $5\% \times 120 = \frac{1}{20} \times 120$
$= 6$

e $75\% \times 36 = \frac{3}{4} \times 36$
$= \left(\frac{1}{4} \times 36\right) \times 3$
$= 9 \times 3$
$= 27$

f $40\% \times 35 = \frac{2}{5} \times 35$
$= \left(\frac{1}{5} \times 35\right) \times 2$
$= 7 \times 2$
$= 14$

2 Now evaluate each product.

a $75\% \times 20$ **b** $20\% \times 30$ **c** $12.5\% \times 24$ **d** $5\% \times 40$

e $10\% \times 90$ **f** $50\% \times 38$ **g** $40\% \times 40$ **h** $12.5\% \times 640$

i $25\% \times 64$ **j** $20\% \times 35$ **k** $10\% \times 170$ **l** $75\% \times 72$

m $5\% \times 600$ **n** $25\% \times 28$ **o** $50\% \times 18$ **p** $40\% \times 45$

DID YOU KNOW?

What fraction of Australia is desert?

Large parts of Australia are classified as desert, approximately $\frac{7}{20}$ or 35%.

However, $\frac{7}{10}$ or 70% of Australia is arid (dry), receiving less than 500 mm annual rainfall.

Only $\frac{3}{100}$ or 3% of the population live in these dry regions of Australia.

Find out the names of the 5 largest deserts in Australia.

Which states are they in?

Shutterstock.com/N Mrtgh

4.12 Expressing quantities as fractions and percentages

Worksheets
Calculating percentages

Money and percentages review

Puzzle
Calculating percentages

Natalie scored 34 out of 40 for an English speech. What is this mark as a percentage?

To express one quantity as a percentage of another, we need to write it as a fraction first and then convert it to a percentage by multiplying by 100%.

Expressing quantities as fractions and percentages

To write an amount as a fraction of a whole amount:

- write the amount in the numerator of the fraction
- write the whole amount (total) in the denominator
- Fraction $= \frac{\text{amount}}{\text{whole amount}}$

To write an amount as a percentage of a whole amount:

- write the amount in the numerator of a fraction
- write the whole amount (total) in the denominator
- multiply by 100% and write '%' next to the answer
- Percentage $= \frac{\text{amount}}{\text{whole amount}} \times 100\%$

Example 24

A school has 800 students. If 156 of them are in Year 7, find:

a the fraction of students that are in Year 7

b the percentage of students that are in Year 7.

SOLUTION

a Fraction $= \frac{156}{800} = \frac{39}{200}$

b Percentage $= \frac{156}{800} \times 100\%$ So 156 is 19.5% of 800

$= 19.5\%$

Example 25

Express:

a 34 out of 40 as a percentage

b 45 minutes as a fraction of 1 hour

c 9 hours as a percentage of 1 day

SOLUTION

a Percentage $= \frac{34}{40} \times 100\%$ So 34 is 85% of 40.

$= 85\%$

b Fraction $= \frac{45\,\text{min}}{1\,\text{h}}$ 1 hour = 60 minutes

So 45 min is $\frac{3}{4}$ of 1 hour.

$= \frac{45\,\text{min}}{60\,\text{min}}$

$= \frac{3}{4}$

c Percentage $= \frac{9\,\text{h}}{1\,\text{day}} \times 100\%$ 1 day = 24 hours

So 9 hours is 37.5% of 1 day.

$= \frac{9\,\text{h}}{24\,\text{h}} \times 100\%$

$= 37.5\%$

Video Expressing quantities as fractions and percentages

EXERCISE 4.12 ANSWERS ON P. 612

Expressing quantities as fractions and percentages

U F

1 In a road rules test, Billal answered 20 questions correctly out of 25.

a What is this score as a simplified fraction?

b If the pass mark is 92%, did Billal pass?

EXAMPLE 24

Foundation Standard Complex

2 In one season a soccer team scored 20 goals. If Kirstin scored 4 of them, what is this as a percentage of the team score? Select the correct answer **A**, **B**, **C** or **D**.

A 4% **B** 5% **C** 20% **D** 40%

3 In a class of 30 students, 12 catch the train to school, 6 walk to school and the rest are driven by their parents.

a What fraction catch the train?

b What percentage walk to school?

c How many students are driven to school? What percentage is this?

4 Inga received the following marks in her semester exams.

Maths: 74 out of 100 English: 28 out of 40 Science: 50 out of 60

Geography: 20 out of 32 Health: 13 out of 20 Technology: 69 out of 75

a Convert each test mark to a percentage.

b In which subject did Inga achieve the best result?

EXAMPLE 25

5 Express each quantity as a percentage.

a 24 min of 1 h **b** 250 mL of 1 L **c** 700 m of 1 km

d 50c of $2 **e** 50 g of 1 kg **f** 9 mm of 30 cm

g 6 h of 1 day **h** $1.75 of $7 **i** 84 h of 5 days

j 1800 g of 4 kg **k** 1500 m of 10 km **l** 400 mL of 2 L

TECHNOLOGY

Calculating test marks as percentages

Spreadsheets can easily be used to convert test marks into percentages.
Copy the data shown below into a spreadsheet.

	A	B	C	D	E	F	G	H	I	J
1	Student	Mathematics		Geography		Science		Art		
2		45	%	60	%	40	%	75	%	
3	Bond, Jeremy	33	=B3/45	45		31		60		
4	Coutler, Louise	42		50		32		55		
5	Dunlop, Nash	27		36		26		62		
6	Jones, Pip	28		52		24		70		
7	Shepherd, David	31		57		39		68		
8	Tran, Melissa	44		51		38		54		
9	Tyler, Phoebe	40		55		40		71		
10	Young, Tahlia	37		46		29		72		
11										

Foundation Standard Complex

1. In cell C3, enter the formula **=B3/45** to convert Jeremy Bond's Mathematics mark into a decimal. Click % in the toolbar to convert the decimal into a percentage.
2. Use **Fill Down** to copy this formula into cells C4 to C10. Then highlight cells C4 to C10 and right-click on the column. Choose **Format Cells** and **Percentage**, then set **Decimal Places** to 0. This will round each percentage mark to the nearest whole number.
3. Enter the formula **=D3/60** into cell E3 to convert Jeremy's Geography mark into a decimal, then click %. **Fill Down** to copy this formula into cells E4 to E10. Choose **Format cells**, **Percentage**, then set **Decimal Places** to 0.
4. Complete the table by using appropriate formulas to convert the Science and Art marks into percentages, rounded to the nearest whole number.
5. Now use your spreadsheet to answer each question, writing the answer in the cell shown in [square brackets].
   - **a** What was the highest Mathematics percentage mark? [Answer in cell J1]
   - **b** Who scored a mark of 87%? What subject was this for? [Answer in cell J2]
   - **c** What was the highest mark? Which subject had this mark? [J3]
   - **d** How many students achieved over 80% in Art? [J4]
   - **e** Who scored over 85% in all subjects? [J5]
   - **f** What was Nash's worst subject? [J6]
6. Which subject do you think had the best marks overall? In cell A11, enter the label **Average** and in cell C11, enter the formula **=AVERAGE(C3:C10)** to calculate the average of the Mathematics percentage marks. Click % in the toolbar to convert the decimal to a percentage. Choose **Format cells** and **Percentage**, then set **Decimal Places** to 1 to round the percentage to one decimal place.
7. Select cell C11 to **Copy** the formula, then **Paste** it into cells E11, G11 and I11 separately. This will calculate the averages for the Geography, Science and Art marks.
8. Which of the 4 subjects had the highest average percentage mark? Write your answer in cell J7.

POWER PLUS ANSWERS ON P. 612

1 Evaluate each expression.

a $\left(\frac{2}{5}\right)^2$ **b** $\left(\frac{1}{3}\right)^3$ **c** $\left(\frac{1}{2}\right)^3$

d $\left(\frac{7}{10}\right)^2$ **e** $\left(\frac{3}{4}\right)^3$ **f** $\sqrt{\frac{4}{9}}$

g $\sqrt{\frac{25}{49}}$ **h** $\sqrt{\frac{81}{100}}$ **i** $\sqrt{\frac{9}{16}}$

j $\sqrt[3]{\frac{27}{64}}$

2 Only $\frac{2}{5}$ of an iceberg is visible above the water. If the section above the water is 30 m high, find:

a the total height of the iceberg

b the height of the section below the water.

3 Todd has $\frac{3}{4}$ of a packet of biscuits. He eats half of them and has six biscuits left. How many were in the packet originally?

4 Rukaya sold all of her eggs. She first sold half her eggs plus half an egg to her neighbour, without having to break an egg. Then she sold half her remaining eggs plus half an egg to her uncle, again without breaking an egg. Finally she sold half her remaining eggs plus half an egg to her friend, without breaking an egg. How many eggs did Rukaya begin with?

5 Simplify each fraction.

a $\frac{1}{\left(\frac{1}{2}\right)}$ **b** $\frac{1}{\left(\frac{4}{5}\right)}$ **c** $\frac{1}{1+\frac{1}{1+\frac{1}{2}}}$

6 Convert each fraction to a percentage.

a $\frac{1}{3}$ **b** $\frac{1}{6}$ **c** $\frac{1}{7}$ **d** $\frac{1}{9}$

e $\frac{1}{11}$ **f** $\frac{1}{12}$ **g** $\frac{2}{3}$ **h** $\frac{4}{9}$

7 Find:

a 87.5% of \$300 **b** $2\frac{1}{2}$% of 80 kg **c** $33\frac{1}{3}$% of 36 L

4 CHAPTER REVIEW

Quiz Language of maths 4

Worksheet Fractions and decimals review

Language of maths

common	decimal	denominator	equivalent
fraction	improper	mixed numeral	numerator
order	percentage	proper	reciprocal
reduce	simplify	vinculum	

1 What is the bottom part of a fraction called?

2 What is the difference between a proper and improper fraction?

3 Why do you think a 'mixed numeral' has that name?

4 What does 'simplifying' or 'reducing' a fraction mean?

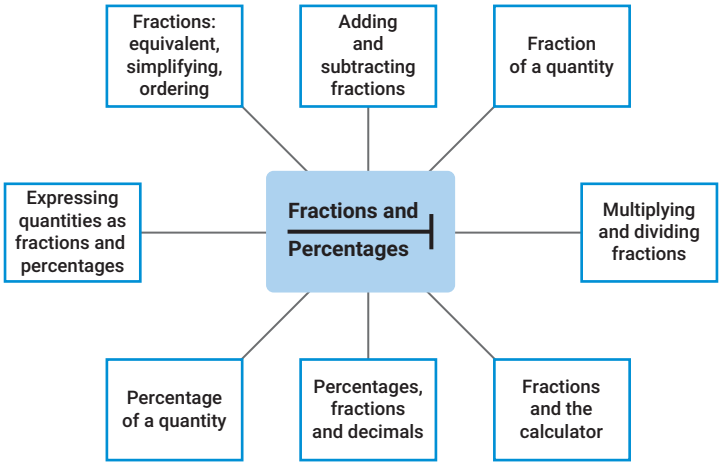
5 Look up the words 'reciprocal' and 'reciprocate' in the dictionary. How does what you find relate to the mathematical meaning of each word?

6 The 'cent' in percentage means 100. Find other words that use 'cent' and are related to 100.

Topic summary

- What have you learnt about fractions and percentages?
- Is there anything you did not understand? Ask a friend or your teacher for help.
- Where are fractions used? Give 3 examples.
- Where are percentages used? Give 3 examples.

Print (or copy) and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.



4 TEST YOURSELF

ANSWERS ON P. 612

4.01

1 Write each mixed numeral as an improper fraction.

a $1\frac{3}{4}$ **b** $2\frac{4}{9}$ **c** $5\frac{3}{5}$ **d** $3\frac{1}{7}$

Quiz
Test yourself 4

4.01

2 Write each improper fraction as a mixed numeral.

a $\frac{7}{4}$ **b** $\frac{26}{3}$ **c** $\frac{23}{7}$ **d** $\frac{36}{24}$

4.01

3 Copy and complete each pair of equivalent fractions.

a $\frac{1}{3} = \frac{?}{15}$ **b** $\frac{3}{4} = \frac{12}{?}$ **c** $\frac{4}{5} = \frac{?}{20}$

d $\frac{5}{12} = \frac{15}{?}$ **e** $\frac{1}{2} = \frac{?}{50}$ **f** $\frac{3}{20} = \frac{?}{100}$

4.01

4 Simplify each fraction.

a $\frac{25}{30}$ **b** $\frac{30}{40}$ **c** $\frac{15}{20}$

d $\frac{20}{15}$ **e** $\frac{24}{48}$ **f** $\frac{24}{42}$

4.02

5 Order each set of fractions, from smallest to largest.

a $\frac{4}{5}, \frac{3}{10}, \frac{2}{5}, \frac{7}{10}$ **b** $\frac{1}{2}, \frac{1}{3}, \frac{7}{12}, \frac{11}{12}$

4.03

6 Evaluate each expression.

a $\frac{3}{4} + \frac{1}{4}$ **b** $\frac{2}{5} + \frac{3}{10}$ **c** $\frac{5}{10} - \frac{1}{3}$

d $\frac{3}{4} - \frac{1}{5}$ **e** $\frac{1}{2} + \frac{1}{3}$ **f** $\frac{1}{2} - \frac{3}{8}$

Foundation Standard Complex

9780170465557

7 Evaluate each expression. 4.04

a $10-\frac{1}{3}$ **b** $2\frac{5}{6}+1\frac{1}{2}$ **c** $3\frac{2}{3}-2\frac{5}{6}$

d $3+2\frac{1}{2}-1\frac{1}{5}$ **e** $6-4\frac{2}{3}$ **f** $2\frac{5}{6}+4\frac{1}{2}-3\frac{2}{3}$

8 Evaluate each expression. (The letters are not in order.) 4.05

A $\frac{3}{7}$ of 21 **B** $\frac{2}{5}$ of 20 **R** $\frac{4}{9}$ of 27

O $\frac{1}{4}$ of 28 **T** $\frac{2}{3}$ of 30 **H** $\frac{1}{10}$ of 35

Order the 6 answers, beginning with the smallest. Replace each answer by its letter. What word do you spell?

9 Evaluate each expression. 4.05

a $\frac{2}{3}$ of 1 day (in hours) **b** $\frac{4}{5}$ of 2 km (in metres) **c** $\frac{3}{4}$ of 3 hours (in min)

d $\frac{7}{10}$ of 15 cm (in mm) **e** $\frac{1}{6}$ of 5 minutes (in seconds) **f** $\frac{5}{8}$ of 2 m (in cm)

10 Evaluate each product. 4.06

a $\frac{2}{3}\times\frac{1}{5}$ **b** $\frac{3}{5}\times 10$ **c** $\frac{4}{7}\times\frac{5}{8}$

d $\frac{1}{3}\times\frac{1}{7}$ **e** $1\frac{1}{2}\times\frac{1}{2}$ **f** $\frac{1}{5}\times 2\frac{1}{2}$

g $1\frac{1}{2}\times 1\frac{1}{2}$ **h** $2\frac{2}{7}\times 3\frac{1}{2}$ **i** $3\frac{1}{5}\times\frac{3}{8}$

11 Evaluate each quotient. 4.07

a $10\div\frac{1}{5}$ **b** $\frac{1}{2}\div 3$ **c** $\frac{1}{2}\div\frac{1}{3}$

d $\frac{3}{4}\div\frac{3}{8}$ **e** $\frac{3}{5}\div\frac{1}{10}$ **f** $\frac{4}{7}\div\frac{5}{8}$

g $1\frac{1}{2}\div\frac{1}{2}$ **h** $2\frac{1}{4}\div 1\frac{1}{8}$ **i** $1\frac{7}{8}\div 1\frac{1}{4}$

Foundation Standard Complex

4.08 **12** The recipe for a fruit punch is 2 litres of ginger ale, $1\frac{1}{2}$ litres of orange juice, $2\frac{3}{4}$ litres of lemonade and $\frac{1}{3}$ of a litre of raspberry juice. How many litres of punch does this recipe make?

4.08 **13** A farmer owns 24 goats. $\frac{1}{4}$ of the goats are black, $\frac{2}{3}$ are white, and the rest are divided equally between brown and spotted. How many of the goats are spotted?

4.09 **14** Use your calculator to evaluate each expression.

a $\frac{13}{18} - \frac{3}{8}$ **b** $\frac{5}{6} + \frac{4}{5}$ **c** $3\frac{1}{4} - 2\frac{3}{4}$

d $\frac{9}{16} \times \frac{2}{45}$ **e** $\frac{17}{20} \div \frac{3}{5}$ **f** $2\frac{1}{3} \times 4\frac{2}{3}$

4.10 **15** Convert each percentage to:

i a fraction **ii** a decimal

a 13% **b** 8% **c** $7\frac{1}{2}\%$

4.10 **16** Convert each to a percentage.

a $\frac{7}{10}$ **b** $\frac{23}{25}$ **c** $\frac{18}{80}$

d 0.8 **e** 0.361 **f** 0.04

4.10 **17** Copy and complete this table.

Fraction	Decimal	Percentage
$\frac{3}{5}$		
	0.8	
		82%

Foundation Standard Complex

18 Find: 4.11

a 40% of 90 **b** 12% of $600 **c** 5% of 120

d 11% of $210 **e** 34% of 150 kg **f** 65% of 360 m

19 In one hour of TV, 22% of the time was devoted to advertising. How many minutes were devoted to advertising? 4.11

20 The netball team scored 26 goals. Jessica scored 8 goals. What fraction of her team's score did Jessica make? 4.12

21 What percentage is 300 m of 1.5 km? 4.12

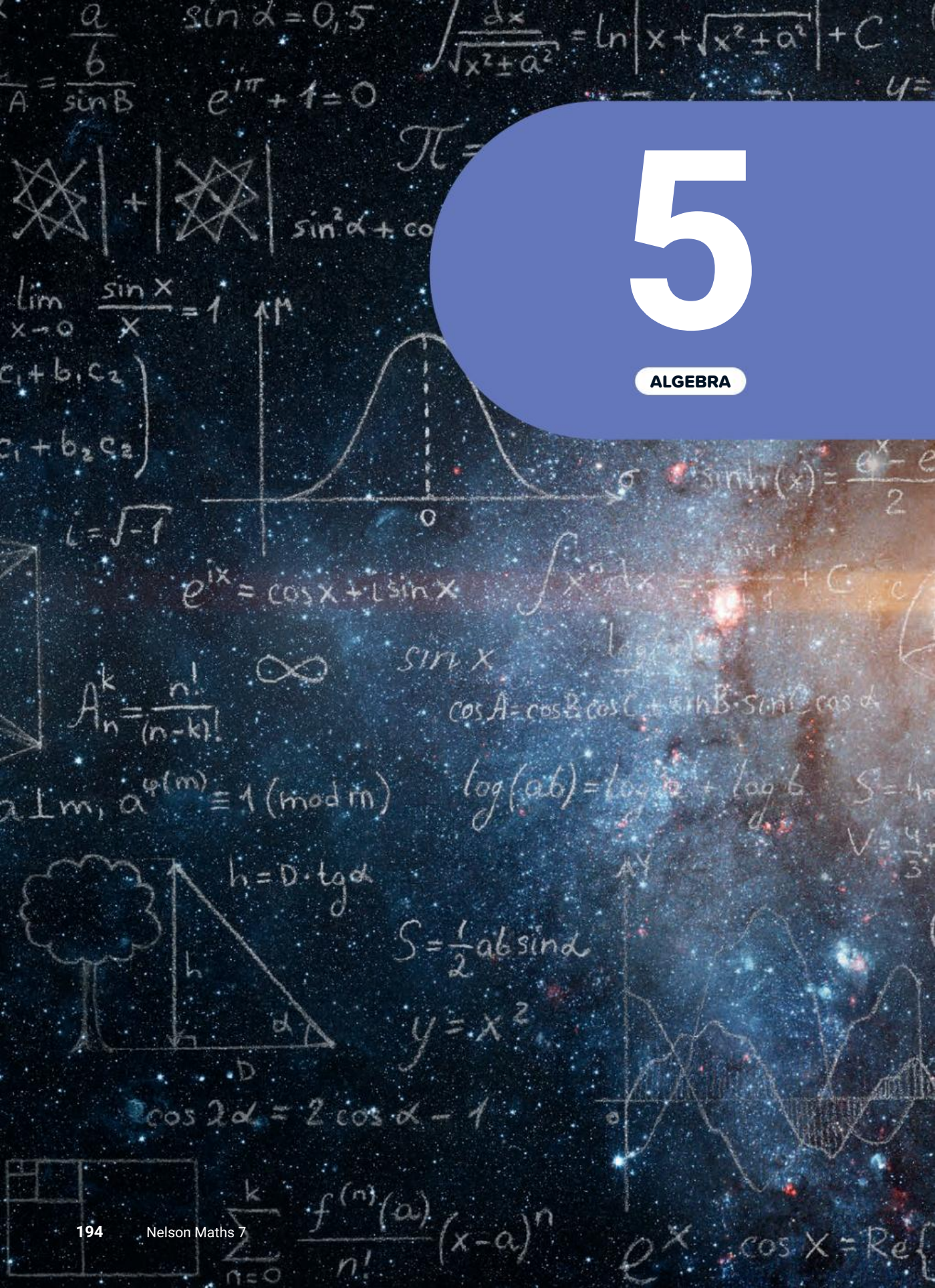
22 Anand scored 51 out of 60 for an Art project. What is his mark: 4.12

a as a simplified fraction? **b** as a percentage?

Foundation Standard Complex

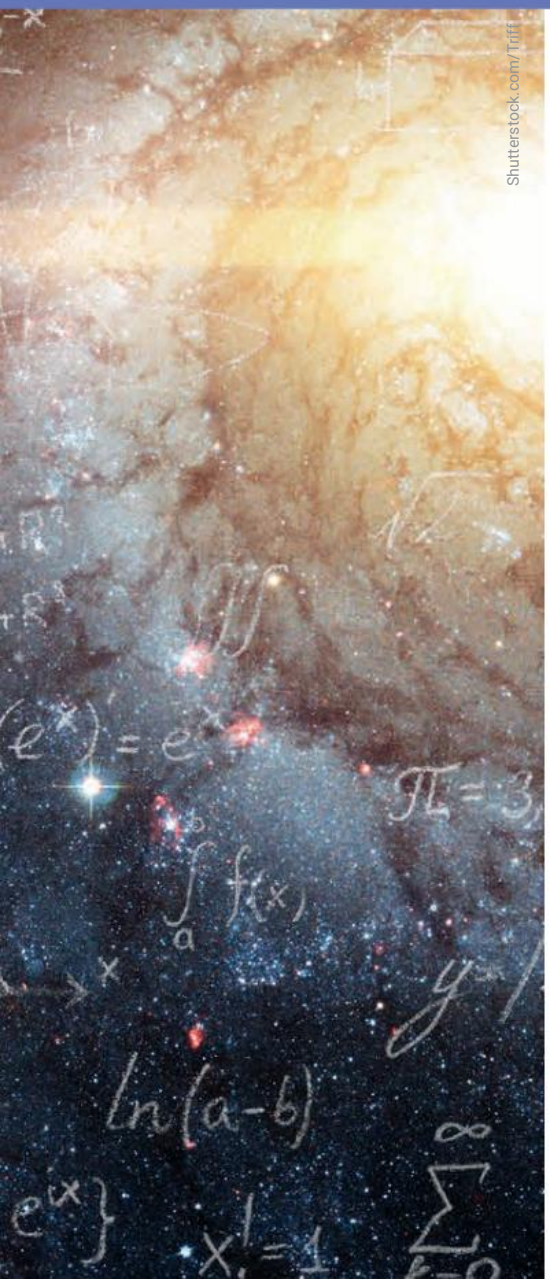
5

ALGEBRA



Algebra and equations

Algebra is the branch of mathematics that uses letters of the alphabet to write general rules called **formulas** or **equations**. Algebra has been a powerful tool for mathematicians for over 4000 years and was first used in ancient Babylon and Egypt in 2000 BCE. It was brought to Europe from India by the Arabs during the 9th century, and the word *algebra* comes from the Arabic *al-jabr*, a method of solving equations. Algebra today has uses in technology-related careers such as science, medicine, astronomy, navigation and finance.



Shutterstock.com/Triff

Chapter outline

		Proficiencies
5.01	The laws of arithmetic	U F R
5.02	The distributive law	U F R C
5.03	Variables	U F R C
5.04	Creating algebraic expressions	U F R C
5.05	Substitution	U F
5.06	Collecting variables	U F R C
5.07	Adding and subtracting terms	U F R C
5.08	Multiplying terms	U F R C
5.09	Dividing terms	U F R C
5.10	Equations	U F
5.11	One-step equations	U F R C
5.12	Two-step equations	U F R C
5.13	Equation problems	U F PS R

U = Understanding
F = Fluency
PS = Problem solving
R = Reasoning
C = Communication

Wordbank

algebraic term A part of an algebraic expression, such as $3x$ in $4x^2 + 3x - 11$

evaluate To find the value of an expression

formula A general mathematical rule written using letters and symbols

like terms Terms that have exactly the same variables, for example, $5p$ and $2p$

inverse operation An opposite operation used in solving an equation, for example, the inverse operation to multiplying is dividing

solve (an equation) To find the value of an unknown variable in an equation

substitute To replace a variable with a number

variable A quantity that can take on different values, represented by a symbol such as a letter of the alphabet

Quiz
Wordbank 5

Videos (11):

5.02 Multiplying by 8, 9, 11 and 12
5.03 Algebraic expressions
5.04 From words to algebraic expressions
5.05 Substitution
5.07 Adding and subtracting like terms
5.08 Multiplying terms
5.09 Dividing terms
5.11 One-step equations
5.12 Two-step equations • Solving equations
5.13 Equation problems

Twig videos (2):

5.05 The heartbeat formula
5.09 The Arabic science of balancing

PhET interactives (5):

5.02 Area model: Introduction
5.02 Area model: Multiplication
5.07 Expression exchange
5.09 Equality explorer: Basics
5.09 Equality explorer

Quizzes (6):

- Wordbank 5
- SkillCheck 5
- Mental skills 5A
- Mental skills 5B
- Language of Maths 5
- Test Yourself 5

Skillsheets (3):

5.03 Order of operations
5.04 Algebraic expressions
5.07 Algebra using diagrams

Worksheets (10):

5.05 Substitution • Why aren't they the same?
5.07 Collecting like terms
5.08 Perimeter and area
5.09 Simplifying algebraic expressions • Why aren't they the same?
5.10 Guess-and-check
5.11 One-step equations
5.12 Equations writing activity • Backtracking 1 • Backtracking 2

Mind map: Algebra and equations

Puzzles (10):

5.03 Algebra bingo game
5.04 What's the expression? • Generalised arithmetic • What does the symbol mean?
5.05 Substitution game • Formula 1 game
5.07 Algebra bingo game • Adding and subtracting like terms
5.11 Equations dominoes
5.12 Solving equations
5.13 Writing and solving equations

Presentation (1):

5.07 Collecting like terms

Nelson MindTap

To access resources above, visit **cengage.com.au/nelsonmindtap**

5 In this chapter you will:

- ✓ use the laws of arithmetic (commutative, associative, distributive) to calculate with numbers mentally and with pen-and-paper
- ✓ use algebra and variables to generalise the laws of arithmetic
- ✓ use variables to write algebraic terms and expressions
- ✓ convert between worded and algebraic expressions
- ✓ evaluate algebraic expressions by substituting values
- ✓ add, subtract, multiply and divide algebraic terms
- ✓ solve one- and two-step linear equations
- ✓ apply equations to solve real-life problems

Quiz
SkillCheck 5

SkillCheck

ANSWERS ON P. 613

1 If ● = 3, then find the value of each expression.

a ● + 5 **b** 7 − ● **c** 2 × ● **d** 12 + ●
e ● − 1 **f** ● × 10 **g** ● + ● **h** 27 ÷ ●
i −6 + ● **j** ● × (−1) **k** −3 ÷ ● **l** −4 − ●

2 Find the value of ■ in each equation.

a ■ + 4 = 10 **b** 6 × ■ = 36 **c** ■ + ■ = 20
d 18 − ■ = 11 **e** 12 ÷ ■ = −3 **f** ■ − 6 = 1

3 Write the number that is:

a 4 more than 7 **b** double 10
c the next odd number after 9 **d** half of 18
e 10 less than 14 **f** the difference between 8 and 1
g 5 multiplied by itself **h** 5 divided by itself
i the amount that 11 is greater than 9 **j** the product of 4 and 8

4 Evaluate each product mentally.

a 15 × 10 **b** 28 × 9 **c** 7 × 4 × 5
d 16 × 5 **e** 13 × 10 **f** 13 × 2

5 Is 13 × 12 the same as 13 × 10 + 13 × 2?

6 Find the value of ▲ in each equation.

a ▲ − 7 = 5 **b** ▲ ÷ 5 = 4 **c** ▲ × ▲ = 81
d 33 ÷ ▲ = 3 **e** −4 + ▲ = 3 **f** ▲ − 8 = −3

7 **a** If ▲ = −7, evaluate 2 × ▲ + 3 **b** If ● = 4, evaluate 3 × ● + 5
c If ● = 15, evaluate ● ÷ 3 − 4 **d** If ▲ = 5, evaluate 4 × ▲ − 6
e If ■ = −2, evaluate 9 × 2 + ■ **f** If ■ = 6, evaluate 4 × 3 − ■
g If ● = −4, evaluate ● × 4 + 3 **h** If ▲ = −1, evaluate 10 × 2 − ▲

8 Evaluate each expression.

a 5 − 8 **b** −8 + 3 **c** 3 × (−4) **d** −6 ÷ 3
e −2 + 5 **f** −2 − 9 **g** −4 × (−9) **h** −12 ÷ (−4)
i −3 − 7 **j** 80 ÷ (−10) **k** 6 − 15 **l** −8 × 9

5.01 The laws of arithmetic

Arithmetic is the branch of mathematics involved in calculating with **numbers**, using the 4 operations of adding, subtracting, multiplying and dividing. We will now look at some general rules of arithmetic and use algebraic symbols to write these rules formally.

The commutative laws

The **commutative law** says that you can operate with 2 numbers in reverse order and still get the same answer. For example:

$4 + 7 = 11$ and $7 + 4 = 11$, so $4 + 7 = 7 + 4$.

$9 \times 5 = 45$ and $5 \times 9 = 45$, so $9 \times 5 = 5 \times 9$.

The commutative law works for **adding** or **multiplying** only. It does not work for subtracting or dividing. For example, 8 – 2 is not equal to 2 – 8 and 14 ÷ 2 is not equal to 2 ÷ 14.

The commutative laws of addition and multiplication

If a and b stand for numbers, then:

$$a + b = b + a$$
$$a \times b = b \times a$$

The associative laws

The **associative law** says that when operating with more than 2 numbers, you can group them in any way and still get the same answer. For example:

$$\begin{aligned} 6 + 7 + 4 &= (6 + 7) + 4 \\ &= 13 + 4 \\ &= 17 \end{aligned}$$

and

$$\begin{aligned} 6 + 7 + 4 &= 6 + (7 + 4) \\ &= 6 + 11 \\ &= 17 \end{aligned}$$

so $(6 + 7) + 4 = 6 + (7 + 4)$.

$$\begin{aligned} 5 \times 3 \times 2 \times 2 &= (5 \times 3) \times 2 \times 2 \\ &= 15 \times 2 \times 2 \\ &= 30 \times 2 \\ &= 60 \end{aligned}$$

and

$$\begin{aligned} 5 \times 3 \times 2 \times 2 &= 5 \times (3 \times 2) \times 2 \\ &= 5 \times 6 \times 2 \\ &= 30 \times 2 \\ &= 60 \end{aligned}$$

so $(5 \times 3) \times 2 \times 2 = 5 \times (3 \times 2) \times 2$.

The associative law also works for **adding** or **multiplying** only, not for subtracting or dividing. For example, 10 – 3 – 1 is not equal to 10 – (3 – 1) and 20 ÷ 5 ÷ 2 is not equal to 20 ÷ (5 ÷ 2).

The associative laws of addition and multiplication

If a, b and c stand for numbers, then:

$$(a + b) + c = a + (b + c)$$
$$(a \times b) \times c = a \times (b \times c)$$

The commutative and associative laws of arithmetic help us calculate mentally with numbers more easily.

Example 1

Evaluate each sum.

a $6 + 16$ **b** $26 + 15 + 4$ **c** $58 + 55 + 45$ **d** $-8 + 22$

SOLUTION

a $6+16=16+6$
$=22$

It is easier to add smaller numbers to larger ones, so swap 6 and 16 around.

b When adding, pair numbers together that add to multiples of 10, 100 and so on.

$26+15+4=(26+4)+15$
$=30+15$
$=45$

26 and 4 add to 30, so put them together.

c $58+55+45=58+(55+45)$
$=58+100$
$=158$

55 and 45 add to 100, so put them together.

d $-8+22=22+(-8)$
$=22-8$
$=14$

It is easier to work with larger and positive numbers first, so swap –8 and 22 around.

Example 2

Evaluate each product.

a $17 \times 5 \times 2$ **b** $25 \times 9 \times 4$

SOLUTION

a $17\times5\times2=17\times(5\times2)$
$=17\times10$
$=170$

$5 \times 2 = 10$, so put 5 and 2 together.

b $25\times9\times4=(25\times4)\times9$
$=100\times9$
$=900$

$25 \times 4 = 100$, so put 25 and 4 together.

Multiplying by a multiple of 10

Example 3

Evaluate each product.

a 9×50 **b** 6×400 **c** 20×300

SOLUTION

You can split multiples of 10, 100, 1000 and so on to make multiplying easier.

a $9 \times 50 = 9 \times 5 \times 10$
$= 45 \times 10$
$= 450$

b $6 \times 400 = 6 \times 4 \times 100$
$= 24 \times 100$
$= 2400$

c $20 \times 300 = 2 \times 10 \times 3 \times 100$
$= 2 \times 3 \times 10 \times 100$
$= 6 \times 1000$
$= 6000$

EXERCISE 5.01 ANSWERS ON P. 613

The laws of arithmetic

U F R

EXAMPLE 1

1 Evaluate each sum.

R **a** $4 + 21$ **b** $19 + 99$ **c** $18 + 23 + 7$
d $75 + 50 + 25$ **e** $16 + 80 + 44 + 10$ **f** $56 + 5 + 15 + 4$
g $-18 + 28$ **h** $16 + 45 + 4$ **i** $-6 + 20$
j $81 + 7 + 19 + 12$ **k** $54 + 27 + 16 + 3$ **l** $44 + 12 + 16 + 12$

EXAMPLE 2

2 Evaluate each product.

R **a** $2 \times 17 \times 5$ **b** $81 \times 2 \times 5$ **c** $50 \times 14 \times 2$
d $10 \times 32 \times 2$ **e** $5 \times 7 \times 4$ **f** $2 \times 5 \times 3 \times 5$
g $8 \times 5 \times 2$ **h** $25 \times 8 \times 2$ **i** $4 \times 4 \times 5$
j $20 \times 6 \times 5$ **k** $3 \times 2 \times 5 \times 3$ **l** $2 \times 14 \times 5 \times 10$

EXAMPLE 3

3 Evaluate each product.

R **a** 7×1000 **b** 100×100 **c** 3×90 **d** 4×700
e 5×80 **f** 2×600 **g** 4×40 **h** 12×2000
i 4×6000 **j** 3×1100 **k** 90×20 **l** 5×500
m 8×40 **n** 20×20 **o** 300×70 **p** 700×300

4 Evaluate $18 \times 4 \times 5$. Select the correct answer **A**, **B**, **C** or **D**.

R **A** 110 **B** 162 **C** 180 **D** 360

5 Evaluate 60×600. Select the correct answer **A**, **B**, **C** or **D**.

R **A** 1200 **B** 3600 **C** 12 000 **D** 36 000

6 Write True (T) or False (F) for each equation.

R **a** $4 - 8 = 8 - 4$ **b** $6 \times 5 = 5 \times 6$ **c** $10 + 11 = 11 + 10$
d $12 \div 4 = 4 \div 12$ **e** $a + 5 = 5 + a$ **f** $3 - x = x - 3$
g $b \div 2 = 2 \div b$ **h** $c \times 2 = 2 \times c$ **i** $x - y = y - x$
j $a \times b = b \times a$ **k** $x + y = y + x$ **l** $c \div d = d \div c$

Foundation Standard Complex

The distributive law

5.02

The **distributive law** says that you can multiply by a number by splitting it into the sum or difference of 2 other numbers. Look at these examples.

a $17 \times 12 = 17 \times (10 + 2)$

$= (10 + 2) + (10 + 2) + (10 + 2) + \ldots$ 17 times

$= 10 + 10 + 10 + \ldots + 2 + 2 + 2 + \ldots$ 17 times each

$= (17 \times 10) + (17 \times 2)$

$= 170 + 34$

$= 204$

so $17 \times (10 + 2) = 17 \times 10 + 17 \times 2$

b $14 \times 9 = 14 \times (10 - 1)$

$= (10 - 1) + (10 - 1) + (10 - 1) + \ldots$ 14 times

$= 10 + 10 + 10 + \ldots - 1 - 1 - 1 - \ldots$ 14 times each

$= (14 \times 10) - (14 \times 1)$

$= 140 - 14$

$= 126$

so $14 \times (10 - 1) = 14 \times 10 - 14 \times 1$

Interactives
Area model: Introduction
Area model: Multiplication

The distributive law of arithmetic

If a, b and c stand for numbers, then:

$$a \times (b + c) = a \times b + a \times c$$
$$a \times (b - c) = a \times b - a \times c$$

The distributive law of arithmetic can help us multiply numbers more easily.

Example 4

Evaluate each product.

a 25×9

b 18×8

SOLUTION

a $25 \times 9 = 25 \times (10 - 1)$

$= 25 \times 10 - 25 \times 1$

$= 250 - 25$

$= 225$

b $18 \times 8 = 18 \times (10 - 2)$

$= 18 \times 10 - 18 \times 2$

$= 180 - 36$

$= 144$

Video
Multiplying by 8, 9, 11 and 12

Multiplying by 9 or 8

- To multiply a number by 9, multiply by 10, then subtract the number
- To multiply a number by 8, multiply by 10, then subtract double the number

Video
Multiplying by 8, 9, 11 and 12

Example 5

Evaluate each product.

a 13×11 **b** 27×12

SOLUTION

a
$$\begin{aligned} 13 \times 11 &= 13 \times (10 + 1) \\ &= 13 \times 10 + 13 \times 1 \\ &= 130 + 13 \\ &= 143 \end{aligned}$$

b
$$\begin{aligned} 27 \times 12 &= 27 \times (10 + 2) \\ &= 27 \times 10 + 27 \times 2 \\ &= 270 + 54 \\ &= 324 \end{aligned}$$

Multiplying by 11 or 12

- To multiply a number by 11, multiply by 10, then add the number
- To multiply a number by 12, multiply by 10, then add double the number

EXERCISE 5.02 ANSWERS ON P. 613

The distributive law

U F R C

1 Copy and complete each multiplication.

R C **a**
$$\begin{aligned} 18 \times 12 &= 18 \times (10 + ___) \\ &= 18 \times ___ + 18 \times ___ \\ &= ____ + 36 \\ &= ____ \end{aligned}$$

b
$$\begin{aligned} 16 \times 9 &= ___ \times (___ - 1) \\ &= ___ \times 10 - ___ \times 1 \\ &= 160 - ___ \\ &= ____ \end{aligned}$$

c
$$\begin{aligned} 21 \times 11 &= 21 \times (___ + ___) \\ &= ___ \times 10 + 21 \times ___ \\ &= 210 + ___ \\ &= ____ \end{aligned}$$

d
$$\begin{aligned} 15 \times 8 &= 15 \times (10 - ___) \\ &= ___ \times 10 - ___ \times ___ \\ &= ___ - ___ \\ &= ____ \end{aligned}$$

EXAMPLE 4

2 Evaluate each product.

a 15×9 **b** 18×9 **c** 26×9 **d** 31×9

e 16×8 **f** 13×8 **g** 22×8 **h** 14×8

i 27×9 **j** 45×8 **k** 38×8 **l** 29×9

Foundation Standard Complex

EXAMPLE 5

5.02

3 Dean tried to use the distributive law to evaluate 56×101 but made an error in his working shown below. At which step did Dean make an error?
Select the correct answer **A**, **B**, **C** or **D**.

R C

$56 \times 101 = 56 \times (100 + 1)$ **A**

$= 56 \times 100 + 1$ **B**

$= 5600 + 1$ **C**

$= 5601$ **D**

4 Evaluate each product.

a 17×11 **b** 24×11 **c** 15×11 **d** 29×11

e 25×12 **f** 14×12 **g** 24×12 **h** 33×12

i 40×12 **j** 12×11 **k** 9×12 **l** 19×11

5 Use the distributive law to copy and complete each multiplication.

R C

a $6 \times 22 = 6 \times (20 + ___)$
$= 6 \times ___ + 6 \times ___$
$= 120 + _____$
$= _____$

b $12 \times 19 = ___ \times (20 - ___)$
$= ___ \times 20 - ___ \times 1$
$= 240 - ____$
$= _____$

c $40 \times 41 = ____ \times (___ + 1)$
$= ___ \times 40 + ____ \times ___$
$= 1600 + ____$
$= _____$

d $25 \times 48 = ____ \times (50 - ___)$
$= ___ \times 50 - ___ \times ___$
$= ____ - ____$
$= _____$

6 Use the distributive law to evaluate each product.

a 5×32 **b** 4×51 **c** 12×99 **d** 8×102

e 6×29 **f** 12×21 **g** 15×18 **h** 11×49

7 Which is the correct distributive law for calculating 14×12?
Select the correct answer **A**, **B**, **C** or **D**.

A $14 \times 10 + 14 \times 2$ **B** $10 + 4 \times 10 + 2$

C $14 \times 10 \times 2$ **D** $14 \times 10 + 2$

Foundation Standard Complex

5.03 Variables

Algebra uses letters or symbols to write general mathematical rules called **formulas** or **equations**. We have used algebra to write the laws of arithmetic we have learned.

The commutative laws:

$a + b = b + a$

$a \times b = b \times a$

The associative laws:

$(a + b) + c = a + (b + c)$

$(a \times b) \times c = a \times (b \times c)$

The distributive laws:

$a \times (b + c) = a \times b + a \times c$

$a \times (b - c) = a \times b - a \times c$

In the formulas above, a, b and c represent any 3 numbers and are called **variables** or **pronumerals**. A variable is a symbol, usually a letter of the alphabet, that represents a number. It is called a **variable** because its value can vary (change), and a **pronumeral** because it stands in place of a numeral.

Example 6

Examine each number pattern, then write the general rule using a variable.

a

$4 \times 1 = 4$
$7 \times 1 = 7$
$9 \times 1 = 9$
$1 \times 1 = 1$
$6 \times 1 = 6$
$b \times 1 =$ ____

b

$9 + 9 + 9 = 3 \times 9$
$4 + 4 + 4 = 3 \times 4$
$11 + 11 + 11 = 3 \times 11$
$2 + 2 + 2 = 3 \times 2$
$5 + 5 + 5 = 3 \times 5$
$x + x + x =$ ____

c

$6 - 6 = 0$
$1 - 1 = 0$
$17 - 17 = 0$
$12 - 12 = 0$
$4 - 4 = 0$

d

$2 \times (-2) = -(2^2)$
$8 \times (-8) = -(8^2)$
$5 \times (-5) = -(5^2)$
$9 \times (-9) = -(9^2)$
$1 \times (-1) = -(1^2)$

SOLUTION

The general rule is:

a $b \times 1 = b$ — Any number multiplied by 1 is itself.

b $x + x + x = 3 \times x$ — The sum of the same number 3 times is 3 times that number.

c $a - a = 0$ — Any number minus itself is 0.

Other variables are possible, for example, $k - k = 0$

d $r \times (-r) = -(r^2)$ Any number multiplied by its opposite is equal to the negative of that number squared.

Other variables are possible, for example, $y \times (-y) = -(y^2)$

Algebraic abbreviations

Mathematicians prefer to write expressions as simply as they can. For example:

$5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 = 9 \times 5$

and $2 \times 2 \times 2 \times 2 \times 2 = 2^5$.

When writing **algebraic expressions**, we use the following abbreviations:

- $3 \times k = 3k$ for multiplication, we leave out the '×' symbol
- $m \div 4 = \frac{m}{4}$ for division, we write in fraction form
- $r \times r = r^2$ for powers
- $1x = x$ for multiplying by 1
- $c \times a \times 6 = 6ac$ write the number first, then the variables in alphabetical order

Example 7

Simplify each expression.

a $d \times 8$ **b** $m + m + m + m + m$ **c** $k \div 2$

d $u \times 9 \times w$ **e** $6x + 2x$ **f** $3 \times p \times 2 \times h$

g $z \times z \times z$ **h** $3n - 2n$ **i** $y \times y \times 4$

SOLUTION

a $d \times 8 = 8d$ Number first

b $m + m + m + m + m = 5m$

c $k \div 2 = \frac{k}{2}$

d $u \times 9 \times w = 9uw$ Number first, then variables in alphabetical order

e $6x + 2x = 8x$ $(x + x + x + x + x + x) + (x + x) = 8$ lots of x

f $3 \times p \times 2 \times h = 3 \times 2 \times p \times h$
$= 6hp$

g $z \times z \times z = z^3$

h $3n - 2n = 1n$ $(n + n + n) - (n + n) =$ One n left
$= n$ Just n: no need to write the '1'

i $y \times y \times 4 = 4y^2$ The '2' (squared) belongs to the y only.

Video Algebraic expressions

Puzzle Algebra bingo game

Skillsheet
Order of operations

Example 8

Use order of operations to simplify each expression.

a $2 \times r + 4$ **b** $15 - d \div 3$ **c** $7 \div (n \times n) + 1$

SOLUTION

a $2 \times r + 4 = 2r + 4$ $\times$ first, then $+$

b $15 - d \div 3 = 15 - (d \div 3)$ $\div$ first, then $-$

$= 15 - \frac{d}{3}$

c $7 \div (n \times n) + 1 = 7 \div n^2 + 1$ () first, then $\div$, then $+$

$= \frac{7}{n^2} + 1$

Example 9

Write each expression in expanded form.

a $3ab$ **b** $-8st^2$ **c** $x^2 - \frac{y}{10}$

SOLUTION

a $3ab = 3 \times a \times b$

Note that expanding is the opposite of simplifying.

b $-8st^2 = -8 \times s \times t \times t$

c $x^2 - \frac{y}{10} = x \times x - y \div 10$

EXERCISE 5.03 ANSWERS ON P. 613

Variables

U F R C

EXAMPLE 6

1 Examine each number pattern, then write the general rule using a variable. A clue has been given in parts **a** and **b**.

R C

a
$8 \div 8 = 1$
$1 \div 1 = 1$
$-2 \div (-2) = 1$
$14 \div 14 = 1$
$-7 \div (-7) = 1$
$u \div u =$ ____

b
$6 + 6 = 2 \times 6$
$10 + 10 = 2 \times 10$
$4 + 4 = 2 \times 4$
$9 + 9 = 2 \times 9$
$-4 + (-4) = 2 \times (-4)$
$p + p =$ ____

Foundation Standard Complex

c $3 + (-3) = 0$
$7 + (-7) = 0$
$20 + (-20) = 0$
$1 + (-1) = 0$
$16 + (-16) = 0$

d $2 \times 0 = 0$
$19 \times 0 = 0$
$-3 \times 0 = 0$
$0 \times 0 = 0$
$-24 \times 0 = 0$

e $7 + 0 = 7$
$-7 + 0 = -7$
$3 + 0 = 3$
$11 + 0 = 11$
$-100 + 0 = -100$

f $-5 \times (-5) = 5^2$
$-7 \times (-7) = 7^2$
$-12 \times (-12) = 12^2$
$-1 \times (-1) = 1^2$
$-8 \times (-8) = 8^2$

2 Simplify each expression. EXAMPLE 7

a $w \times 3$
b $b \times b$
c $9 \times c \times a \times d$
d $a + a + a + a + a + a$
e $e \div 5$
f $f + f$
g $5m + 2m$
h $2 \times w \times 3 \times h$
i $5m - 2m$
j $4 \times f \times f$
k $6d - 5d$
l $a \times c \times b \times 10$
m $16 \div n$
n $g \times 2 \times g$
o $3x + 6x$
p $h + h - h$
q $12q - 4q$
r $2 \times m \times 4 \times n$
s $6 \times p \times r \times (-2) \times r$
t $a + a + a + b + b$
u $a \times a \times a \times b \times b$

3 Which of the following is not equal to $2p$? Select the correct answer **A**, **B**, **C** or **D**. EXAMPLE 8

R **A** $3p - p$ **B** p^2 **C** $p + p$ **D** $p \times 2$

4 Use order of operations to simplify each expression.

R **a** $16 \div (a \times 5)$
b $7 + 3 \times n$
c $(e - 6) \div 2$
d $4 \times s - 8$
e $c \times c + d \times d$
f $k \div 9 + m$
g $k \div (9 + m)$
h $12 - r \div 2$
i $7 + u \times u \times 7$

5 Which of the following is equal to $m \times m \times 3 + 4$? Select the correct answer **A**, **B**, **C** or **D**.

R **A** $3m^2 + 4$ **B** $2m^3 + 4$ **C** $6m + 4$ **D** $m^2 + 7$

6 Explain the meaning of:

R **a** xy
b $\frac{2n}{r}$
c $1x = x$

Foundation Standard Complex

EXAMPLE 9

7 Write each expression in expanded form.

a $12rs$ **b** $-2n^2$ **c** $\frac{8}{f}$ **d** $r^2 + t^2$

e $9a^2b$ **f** $5mn - 2a$ **g** $\frac{x+1}{3}$ **h** q^3

i $14 - 2d^2$ **j** $\frac{-4c}{5}$ **k** x^2y^2 **l** $4j - \frac{8}{k}$

8 What is $4m^3$ in expanded form? Select the correct answer **A**, **B**, **C** or **D**.

A $4m \times 4m \times 4m$ **B** $4m + 4m + 4m$

C $4 \times m \times m \times m$ **D** $4 \times m + m + m$

INVESTIGATION

The laws of arithmetic

We can use algebra and variables to describe general laws about arithmetic.

For example, if we add zero to any number, the answer is still that number.

If we let N stand for any number: $N + 0 = N$

In groups of 2 to 4 students, answer the following questions.

1 Describe in words what each number rule below means.

a $N \times 1 = N$ **b** $N \times 0 = 0$

c $a - b \neq b - a$ **d** $a + b + c = b + a + c$

e $N - 0 = N$ **f** $ab = ba$

2 Write each rule algebraically using variables.

a Any number divided by 1 equals itself.

b Multiplying a number by 8 is the same as doubling it 3 times.

c Any 3 numbers can be multiplied together in any order.

d Any number added to itself is the same as multiplying that number by 2.

e Any number subtracted from itself equals 0.

f Any number multiplied by its reciprocal equals 1.

3 Is each equation true or false? Test your decision by substituting a number for the variable and checking.

a $a \div b = b \div a$ **b** $N \div N = 1$

c $4a - a = 4$ **d** a is a factor of a

e If N is even, then $N + 3$ is odd **f** $N = N - 2$

g $a + (-a) = 2a$ **h** $0 \div N = 0$

i $N \div 0 = 0$ **j** $N \div 1 = N$

4 If k is an odd number, what is an expression for:

a the previous odd number? **b** the next even number?

Foundation Standard Complex

Creating algebraic expressions

5.04

We can use variables to convert worded expressions to algebraic expressions.

Video From words to algebraic expressions

Skillsheet Algebraic expressions

Worksheet Writing algebraic expressions

Puzzles What's the expression?

Generalised arithmetic

What does the symbol mean?

Example 10

If A represents any number, write an algebraic expression for:

a 3 times that number

b 3 less than that number

c the next consecutive number

consecutive means 'following in order': for example, 7, 8, 9 are consecutive numbers

d that number multiplied by itself

e the square root of that number

f one-third of that number

SOLUTION

a $3 \times A = 3A$

b $A - 3$

c $A + 1$

The number after A is $A + 1$

(the number after 4 is $4 + 1 = 5$)

d $A \times A = A^2$

e $\sqrt{A}$

f $\frac{1}{3}A = A \div 3 = \frac{A}{3}$

Example 11

Write an expression for:

a the sum of m and 5

b double k

c u increased by 10

d the difference between p and q

e the product of r and 12

f how many times y divides into 50

SOLUTION

a $m + 5$

b $2k$

c $u + 10$

d $p - q$

e $12r$

f $50 \div y = \frac{50}{y}$

EXERCISE 5.04 ANSWERS ON P. 614

Creating algebraic expressions

U F R C

EXAMPLE 10

1 If N represents any number, write an algebraic expression for:

R C
- **a** the number added to itself
- **b** half the number
- **c** triple the number
- **d** 7 less than the number
- **e** one-tenth of the number
- **f** the previous consecutive number
- **g** 5 times the number
- **h** the sum of the number and 21
- **i** the difference between the number and 1
- **j** the number divided by 8
- **k** the number increased by 3
- **l** the number cubed
- **m** the square root of the number

EXAMPLE 11

2 Suppose that in question **1**, the variable X represents any number instead of N. What difference would this make to your answers? Does it matter which letter of the alphabet you choose to use?

R C

3 Write an expression for:

R C
- **a** the sum of all x, y and z
- **b** the difference between b and c
- **c** the product of u and v
- **d** the product of u, v and v
- **e** the quotient of C and d
- **f** the sum of m and n, divided by 2
- **g** the square root of the product of 10 and a
- **h** 3 less than b
- **i** 5 decreased by c
- **j** c decreased by 5
- **k** how many times 3 divides into e
- **l** t squared
- **m** the product of x, z and 9
- **n** how much k is more than 12
- **o** half of the difference between 20 and g

4 What is an algebraic expression for 'the product of 4 and 3 more than a number?' Select the correct answer **A**, **B**, **C** or **D**.

R C

A $4n + 3$ **B** $4 \times 3 + n$ **C** $4 \times (n + 3)$ **D** $4 \times 3 \times n$

5 Write an expression for:

R C
- **a** the number of students in a class if there are b boys and g girls
- **b** the number of chicken nuggets needed for a party if there are n guests and each guest eats an average of 4 nuggets
- **c** the number of children left in a class of d students if r of them have gone to the zone swimming carnival
- **d** the amount of dollars earned selling s sushi rolls at the canteen, where each roll is sold at \$2
- **e** the cost of one film ticket if it costs \$$a$ for 3 tickets
- **f** the perimeter of a rectangle of length x and width y
- **g** the area of a rectangle of length x and width y
- **h** the total cost of buying m milkshakes and i ice creams, where each milkshake costs \$5 and each ice cream costs \$3

Foundation Standard Complex

6 What is the correct expression for the total admission cost to Water World for c children at \$55 each and a adults at \$77 each? Select the correct answer **A**, **B**, **C** or **D**.

R C

A $55a + 77c$ **B** $132ac$ **C** $77a + 55c$ **D** $a + c + 132$

☆ MENTAL SKILLS 5A ANSWERS ON P. 614 Maths without calculators

Quiz Mental skills 5A

Multiplying an even number by 5

To multiply an even number by 5, it's easier to halve it, then multiply by 10 (by inserting a 0 at the end). This is because $\frac{1}{2} \times 10 = 5$.

1 Study each example.

a $14 \times 5 = 14 \times \frac{1}{2} \times 10 = 7 \times 10 = 70$

b $36 \times 5 = 36 \times \frac{1}{2} \times 10 = 18 \times 10 = 180$

c $22 \times 5 = 11 \times 10 = 110$

d $18 \times 5 = 9 \times 10 = 90$

2 Now simplify each product.

a 32×5 **b** 26×5 **c** 12×5 **d** 28×5

e 42×5 **f** 54×5 **g** 38×5 **h** 44×5

i 60×5 **j** 34×5 **k** 16×5 **l** 58×5

TECHNOLOGY

Using variables and values

1 **a** Create the following spreadsheet to calculate the value of the expression $x + 2$ for different values of x.

	A	B	C	D	E	F	G	H	I	J	K
1	x	1	2	3	4	5	10	20	25	50	100
2	x + 2										
3											

b To evaluate $x + 2$ when $x = 1$ (column B), enter **=B1+2** into cell B2. What value should you get?

	A	B	C	D	E	F	G	H	I	J	K
1	x	1	2	3	4	5	10	20	25	50	100
2	x + 2	=B1+2									
3											

c To complete the table, use **Fill Right** to copy the rule from cell B2 across row 2 until cell K2. **Centre** your answers in each of the cells. Check that the answers look correct.

d Type over some values in row A of the spreadsheet to find the value of $x + 2$ when:

i $x = 6$ **ii** $x = 17$ **iii** $x = 31$

e Predict what value of x will give the answer 96 and check your guess using the spreadsheet.

Foundation Standard Complex

2 **a** Create the following spreadsheet to calculate the value of $x - 3$ for values of x.

	A	B	C	D	E	F	G	H	I	J	K
1	x	1	2	3	4	5	10	20	25	50	100
2	x - 3										

b In cell B2, enter a formula beginning with **=B1** to calculate $x - 3$.

c Use **Fill Right** to copy this formula up to cell L2. **Centre** your answers.

d Type over some values in row A to evaluate $x - 3$ when:

i $x = 9$ **ii** $x = 28$

e Predict what value of x will give the answer:

i 37 **ii** 105

Check your guess using the spreadsheet.

3 **a** Create the following spreadsheet and in cell B2, enter a formula to evaluate $x \div 2$.

	A	B	C	D	E	F	G	H	I	J	K
1	x	-8	-6	-4	-2	0	2	10	20	30	40
2	x ÷ 2										

b Use **Fill Right** to copy this formula up to cell L2. **Centre** your answers.

c Use the spreadsheet to evaluate $x \div 2$ when:

i $x = -10$ **ii** $x = 6$

d Predict what value of x will give the answer 21 and check using the spreadsheet.

4 **a** Create the following spreadsheet and in cell B2, enter a formula to evaluate $10x - 1$.

	A	B	C	D	E	F	G
1	x	-10	-5	0	5	10	15
2	10x - 1						

b Use **Fill Right** to copy this formula up to cell L2. **Centre** your answers.

c Use the spreadsheet to evaluate $x \div 2$ when:

i $x = 8$ **ii** $x = -7$

d Predict what value of x will give the answer 199 and check using the spreadsheet.

5 **a** Create a new spreadsheet to evaluate x^2 for different values of x. In cell A1, enter the label 'x', then along row A enter the following values: –2, –1, 0 and so on until 4 in cell H1.

b In cell A2, enter the label 'x^2' and in cell B2, enter a formula to evaluate x^2.

c **Fill Right** to complete the answers for cells C2 to H2.

Substitution

5.05

The word **substitution** means replacing one thing with another thing. In sport, substitution means to replace one player with another during a game. In algebra, substitution means to replace a variable with a value to evaluate an algebraic expression.

Worksheet
Substitution

Puzzles
Substitution game

Formula 1 game

Videos
Substitution

The heartbeat formula

Example 12

a Evaluate $4k - 9$ when $k = 15$.

b If $r = m^2 + 3m$, evaluate r when $m = 4$.

c If $a = 6$, $b = -4$ and $c = 10$, evaluate $\frac{c-a}{b}$.

SOLUTION

a When $k = 15$,

$4k - 9 = 4 \times 15 - 9$

$= 51$

b When $m = 4$,

$r = 4^2 + 3 \times 4$

$= 28$

c When $a = 6$, $b = -4$ and $c = 10$,

$\frac{c-a}{b} = \frac{10-6}{-4}$

$= \frac{4}{-4}$

$= -1$

Substituting into formulas

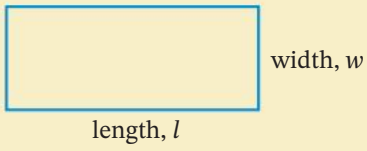
A **formula** is a general rule written as an algebraic equation showing the relationship between variables. Solving mathematical problems often involves substituting values into formulas.

Example 13

The formula for the perimeter of a rectangle is

$$P = 2l + 2w$$

where P is the perimeter, l is the length and w is the width.



Use this formula to calculate the perimeter of a rectangle with length 16 m and width 3 m.

SOLUTION

When length $l = 16$ and width $w = 3$,

$P = 2l + 2w$

$= 2 \times 16 + 2 \times 3$

$= 38$ m

EXERCISE 5.05 ANSWERS ON P. 614

Substitution

U F

EXAMPLE 12

1 Evaluate $k + 3$ when:

a $k = 12$ **b** $k = 48$ **c** $k = 119$ **d** $k = -21$

2 Evaluate $45 - k$ when:

a $k = 5$ **b** $k = -13$ **c** $k = 28$ **d** $k = 45$

3 Evaluate $4k$ when:

a $k = 2$ **b** $k = 11$ **c** $k = -4$ **d** $k = \frac{1}{2}$

4 Find the value of $5k + 1$ when:

a $k = 3$ **b** $k = -4$ **c** $k = 20$ **d** $k = 7$

5 Find the value of $9k - 8$ if:

a $k = 9$ **b** $k = -5$ **c** $k = 0$ **d** $k = 2$

6 Find the value of $\frac{k}{3}$ if:

a $k = 15$ **b** $k = 33$ **c** $k = -9$ **d** $k = 57$

7 If $a = 6$, $b = 14$ and $c = -2$, find:

a $a + b - c$ **b** $2b + c$ **c** $\frac{a-b}{c}$ **d** $ab - 50$

8 If $m = 4$, $n = -2$ and $p = 3$, find:

a m^2 **b** $3n + p$ **c** mnp

9 If $d = 4c - 10$, find d when:

a $c = 5$ **b** $c = 8$ **c** $c = -10$

10 If $b = 3t - 1$, find b when:

a $t = 5$ **b** $t = 12$ **c** $t = -9$

11 If $y = 4 - ax^2$, find the value of y when $a = -2$ and $x = -3$. Select the correct answer **A**, **B**, **C** or **D**.

A -54 **B** 54 **C** 22 **D** -14

EXAMPLE 13

12 The formula for the cost of a party is:

$$C = 45n + 500$$

where C is the cost in dollars and n is the number of guests.

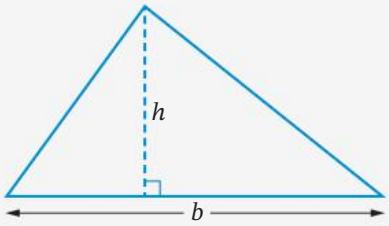
Use this formula to calculate the cost of a party for 72 guests.

Foundation Standard Complex

13 The formula for the area of a triangle is:

$$A = \frac{1}{2}bh$$

where A is the area of the triangle,
b is the length of the base and h is the height.



Find the area of a triangle with base length 9 m and height 6 m.

14 The number of hours of sleep recommended for children has the formula:

$$H = 17 - \frac{A}{2}$$

where H is the number of hours and A is the age of the child.

Find the number of hours sleep recommended for a 13-year-old person.

15 The formula for converting Celsius temperatures to Fahrenheit temperatures is:

$$F = \frac{9}{5}C + 32$$

where F is the temperature in Fahrenheit and C is the temperature in Celsius.

Use the formula to convert 30°C to Fahrenheit.

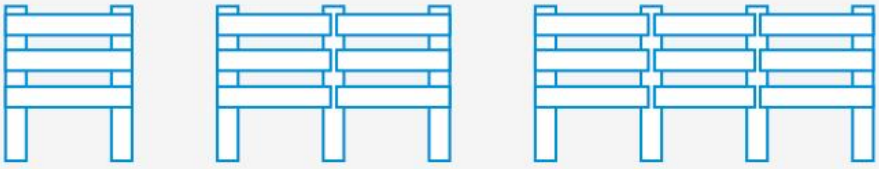
16 A locksmith charges according to this formula:

$$C = 42h + 65$$

where C is the charge in dollars and h is the number of hours worked.

How much does the locksmith charge for working for 2 hours?

17 Krystal is building a timber fence at her farm. The sections of fence are shown below:



1 section, 5 pieces 2 sections, 9 pieces 3 sections, 13 pieces

The formula to calculate the number of pieces (P) to be used for a certain number of sections (S) is $P = 4S + 1$. How many pieces of timber are needed to make 16 sections? Select the correct answer **A**, **B**, **C** or **D**.

A 65 **B** 64 **C** 20 **D** 19

Foundation Standard Complex

TECHNOLOGY

Substitution

1 Create the spreadsheet below for evaluating 5 algebraic expressions involving 2 variables, a and b, whose values are entered in columns A and B.

	A	B	C	D	E	F	G
1	a	b	ab	a + b	2a + b	5(a + b)	3b ÷ 2a
2	2	1	=A2*B2				
3	10	0					
4	6	8					
5	-3	5					
6	0.4	1.7					
7	86	45					

2 Column C is for evaluating ab for the different values of a and b shown in rows 2 to 7. In cell C2, enter the formula for calculating ab using the values in A2 and B2.

3 **Fill Down** to copy this formula down to C7.

4 Enter appropriate formulas for the other 4 expressions in cells D2 to G2. Use brackets where necessary.

5 For each column, D to G, **Fill Down** to row 7.

6 Create the new spreadsheet below that shows a value for each variable, for example, $m = 5$, $n = 2$, $p = -4$.

	A	B	C	D	E	F	G
1	m	n	p	t	u	A	B
2	5	2	-4	12	0	50	31
3							

7 In each cell given, enter an appropriate formula to evaluate the algebraic expression shown.

a In cell A4: $m + p$

b In cell B4: npt

c In cell C4: $n + 2p$

d In cell D4: $B - A$

e In cell A5: $(p - n) \div t$

f In cell B5: $np + tu$

g In cell C5: $\frac{mt}{p}$

h In cell D5: $u - At$

i In cell A6: $A^2 - B^2$

j In cell B6: $6p + t$

k In cell C6: $\frac{m(t-n)}{A}$

l In cell D6: $3n^2 - p$

INVESTIGATION

Equivalent expressions

1 Determine whether each equation below is true or false by substituting a number (of your own choice) for the variable and checking whether the left expression equals the right expression.

a $6 + 4m + 4 = 10 + 4m$ **b** $5x + 3x = 8x^2$ **c** $4k - k = 4$

d $7g + 12 = 19g$ **e** $3 + 7n + 2n = 3 + 9n$ **f** $3p \times 7 = 21p$

g $21m \div 3 = 7m$ **h** $2y \times 4y = 24y$ **i** $2r \times 2 \times 2r = 8r^2$

2 For each of the false statements in question **1**, work out what the correct statement should be.

Worksheet Why aren't they the same?

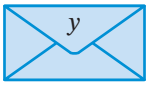
Collecting variables 5.06

Let the variable x stand for an unknown number. Let x be represented by a cup that holds the unknown number of balls.

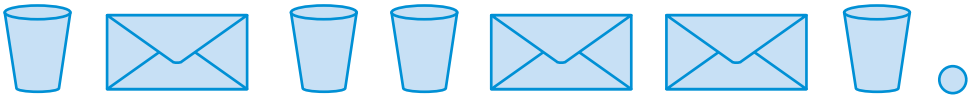
Then the total number of balls represented in the diagram below can be written using the algebraic expression $3 \times x + 5 = 3x + 5$.

Note that even though we don't know how many balls are in each cup, it is the same number of balls, x, and we can still describe the total number of balls algebraically.

Let y stand for an unknown number that is different from x. Let y be represented by an envelope that contains this number of balls.



Then the total number of balls represented in the diagram below can be written using the algebraic expression $4x + 3y + 1$.



EXERCISE 5.06 ANSWERS ON P. 614

Collecting variables

U F R C

1 Write an algebraic expression for the number of balls represented in each diagram, if each cup holds x balls.

R C

a

b

c

d

e

f

g

h

i

j

2 Write an algebraic expression for the number of balls represented in each diagram, if each cup holds x balls and each envelope holds y balls.

R C

a

b

c

d

e

f

g

h

3 Draw cups and balls to represent each algebraic expression, if each cup holds x balls.

C

a $x + 3$

b $x + x + 1$

c $4 + 2x$

d $2 + x + 2$

e $x + 5 + 2x$

f $1 + 2x + 3 + x$

Foundation Standard Complex

9780170465557

4 If N stands for the number of paperclips in each envelope, which diagram represents $3N + 2$ paperclips? Select the correct answer **A**, **B**, **C** or **D**.

R C

A

B

C

D

5 If m stands for the number of coins in each envelope and p stands for the number of coins in each cup, match each expression in the left-hand column with the correct expression in the right-hand column.

R C

a
$m + p + m$

b
$p + m + m + p$

c
$p + m + p + m + p$

d
$p + 2 + p + 1$

e
$2m + 2 + m + 1$

f
$2 + p + 2 + 2p$

A
$2m + 2p$

B
$2p + 3$

C
$3m + 3$

D
$p + 2m$

E
$3p + 4$

F
$3p + 2m$

6 Match each expression in the left column with the correct expression on the right.

R C

a
$m + p - m$

b
$m + 4 - 2$

A
$2m$

B
$2p + 2$

Foundation Standard Complex

c

$2m + p - p$

C

$2p$

d

$2p + 6 - 4$

D

p

e

$2m + 2p - 2m$

E

$2p + 2m$

f

$2p + 3m - m$

F

$m + 2$

7 Simplify each expression.

R C

a

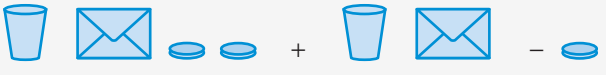
$p + m + p$

b

$m + 2p - p$

c

$2p + 2 + p + 3$

d 

$p + m + 2 + p + m - 1$

e

$3p + 2m - m - p$

8 Simplify $9a + 4b - 5a + 2b$. Select **A**, **B**, **C** or **D**.

R **A** $14a + 6b$ **B** $10ab$ **C** $4a + 6b$ **D** $14a + 11b$

9 Simplify each expression.

R **a** $3d + 2d + 4a$ **b** $6h + 4r + 5r - h$ **c** $e + e - e$

d $2k + 3 + 2k - 6$ **e** $3x + 4z - x + 2z$ **f** $2s - p - s + 3p$

g $6u + 4 - 2u + 8$ **h** $7n - 6 + 5 - 3n$ **i** $20 + 6i - 3i - 20$

Foundation Standard Complex

Adding and subtracting terms

5.07

What is the fastest way to mentally calculate the sum $3 + 3 + 5 + 3 + 5 + 5 + 3 + 3 + 5$?

Do you think of collecting all the 3s together and all the 5s together like this?

$$\begin{aligned} 3 + 3 + 5 + 3 + 5 + 5 + 3 + 3 + 5 &= (3 + 3 + 3 + 3 + 3) + (5 + 5 + 5 + 5) \\ &= (5 \times 3) + (4 \times 5) \\ &= 15 + 20 \\ &= 35 \end{aligned}$$

We can do the same thing algebraically. Suppose an unknown number p is represented by a cup and another unknown number m is represented by an envelope . Now consider these 2 sums:

a

$p + m + p + m = 2p + 2m$

b

$m + p + m + p + m = 3m + 2p$

Note that we can add all the p **terms** together, and we can add all the m terms together, but we cannot add the ps and ms together because p and m represent different numbers. We cannot simplify the answers $2p + 2m$ and $3m + 2p$ because p and m are different.

These are examples of collecting **like terms**, terms that contain exactly the same variable(s), such as $4h$, $5h$ and h. Here are some more examples:

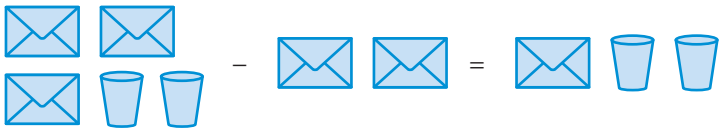
$3x$ and $-5x$	$4mw$ and $2mw$	$12m$ and m
$-6d$ and $10d$	$5ab$ and $2ba$	xyz and $2yzx$

Note: $ab = ba$

Here are some examples of **unlike terms**, terms that have different variables:

$3x$ and $5m$	$4mw$ and $2mn$	$-6d$ and $10d^2$

We can add or subtract like terms because the variables in them represent the same number. We can only add or subtract things that are the same. For example:



$3m + 2p - 2m = m + 2p$

Skillsheet Algebra using diagrams

Worksheet Collecting like terms

Puzzles Algebra bingo game

Adding and subtracting like terms

Interactive Expression exchange

$$(3 \text{ lots of } m) + (2 \text{ lots of } p) - (2 \text{ lots of } m) = (3 \text{ lots of } m - 2 \text{ lots of } m) + (2 \text{ lots of } p)$$
$$= (1 \text{ lot of } m) + (2 \text{ lots of } p)$$

This makes sense because if $m = 5$ and $p = 8$:

$$3m + 2p - 2m = (3 \times 5) + (2 \times 8) - (2 \times 5)$$
$$= (3 \times 5) - (2 \times 5) + (2 \times 8) \quad \text{grouping like terms}$$
$$= (1 \times 5) + (2 \times 8) \quad \text{3 lots of 5 – 2 lots of 5 = 1 lot of 5}$$

Adding and subtracting terms

- When adding and subtracting terms, only like terms can be collected
- Like terms have exactly the same variables

Example 14

Select the like terms from each list below.

a $5x, -x, x^2, xy, \frac{1}{2}x, -8x, \frac{4x}{3}$

b $3ab, 5, a, ba, -2b, 4ab^2, -ab$

SOLUTION

a $5x, -x, \frac{1}{2}x, -8x, \frac{4x}{3}$ are like terms because they each have a single x in them.

b $3ab$, ba and $-ab$ are like terms because each contains ab.

Video Adding and subtracting like terms

Presentation Collecting like terms

Example 15

Simplify each expression by collecting like terms.

a $7d + 3e + 3d + e$

b $6m - 8 + 3m + 6$

c $6xy + 7x^2 - 3yx - 12x^2$

d $14 - 4t - 9t + 2$

SOLUTION

a $7d + 3e + 3d + e = 7d + 3d + 3e + e$ Grouping like terms

$= 10d + 4e$

b $6m - 8 + 3m + 6 = 6m + 3m - 8 + 6$ The + or – sign goes with the term after it.

$= 9m - 2$

c $6xy + 7x^2 - 3yx - 12x^2 = 6xy - 3xy + 7x^2 - 12x^2$ $3yx = 3xy$

$= 3xy - 5x^2$

d $14 - 4t - 9t + 2 = 14 + 2 - 4t - 9t$

$= 16 - 13t$

EXERCISE 5.07 ANSWERS ON P. 614

Adding and subtracting terms

U F R C

EXAMPLE 14

1 Select the like terms from each list.

C **a** $4p, 2y, 2p, 5z, p$ **b** $m^2, m, n, 3m^2, mn$

c $5ac, 4a, 7ca, 3, 5$ **d** $vw, 5v, 9v, w, v^2, 2wv$

e $p, 6pq, 2qp, 3pq, 7q$ **f** $2d, 5, 3d^3, 7d^2, 9, 3d, 2$

g $4mn, 3m, 4, 2nm, mn, 2n$ **h** $x^2y, 2x, 3y, 4x^2y$

i $7, 2a, 4b, 5ba, 9, ab, 2$

2 What are the like terms in $3ab, 2a^2, 2b^2a, 2p^2, 2ba$? Select the correct answer **A**, **B**, **C** or **D**.

C

A $2a^2, 2p^2$ **B** $2a^2, 2b^2a, 2p^2, 2ba$

C $3ab, 2ba$ **D** $2b^2a, 2ba$

3 Simplify each expression by collecting like terms.

a $4k + 7k$ **b** $5mn + 2nm$ **c** $xy + xy$

d $3abc + 4abc + 2bac$ **e** $3x^2 + 2x^2 + x^2$ **f** $ef^2 + 4ef^2$

g $8d - 3d$ **h** $12mk - 7mk$ **i** $12xy - 7yx - xy$

j $8de - 12de$ **k** $6k^2 - 3k^2$ **l** $4w^2 - w^2$

EXAMPLE 15

4 Which expression is $2m + 4p - 5m - 2p$ simplified? Select **A**, **B**, **C** or **D**.

A $-mp$ **B** $-3m + 2p$ **C** $7m + 6p$ **D** $-3p$

5 Simplify each expression.

a $5k - 2j + 6j$ **b** $10ab + 2 - ab$ **c** $12m^2 - 5m^2 + 2m$

d $12s^2 - 7st - 2s^2$ **e** $4mn + 7mn - mn$ **f** $14k + 3 - 5k$

g $y^2 + 2y^2 - y^2$ **h** $9d - 4e + 3d$ **i** $7 - 2x - 3$

j $15gb - 8gb + gb$ **k** $4 - 2q + 5$ **l** $5a + 6c - 4c$

6 Simplify each expression.

a $4x - y - x - 2y$ **b** $7ab + 2bc - 3ab + bc$

c $4k^2 + 3 + 5k^2 + 8$ **d** $4bc - 8a + 2bc - a$

e $x^2 - 6x - x + 7$ **f** $p^2 + q^2 + 3p^2 - q^2$

g $3 + 4y + 8y + 11$ **h** $11r - 3s - 4r + 6s$

i $7g + 8b - b - g$ **j** $6x^2 - 4x - 9x + 6$

k $20a + 11m + 32a - 11m$ **l** $3q - 7 - 8q + 12q$

m $5 - 2y - 5 - 4y$ **n** $s^2 + 4s^2 + 12 - 5s^2$

Foundation Standard Complex

7 Which expression is $a^5 + a^5$ simplified? Select **A**, **B**, **C** or **D**.

A a^{10} **B** a^{25} **C** $2a^5$ **D** $2a^{10}$

8 Write an algebraic expression involving a sum that simplifies to:

R C **a** $2x + 6$ **b** $6m + 12p$ **c** $x^2 - x$

9 Write an algebraic expression involving a difference that simplifies to:

R C **a** $2x + 6$ **b** $6m + 12p$ **c** $x^2 - x$

5.08 Multiplying terms

Worksheet Perimeter and area

What is the fastest way to mentally calculate the product $6 \times 4 \times 5 \times 3$?

Do you think of re-arranging the numbers to pair convenient factors like this?

$$\begin{aligned} 6\times4\times5\times3 &= (4\times5)\times(6\times3) \\ &= 20\times18 \\ &= 360 \end{aligned}$$

We can do the same thing algebraically. As numbers can be multiplied in any order, we can multiply like *and* unlike terms.

Video Multiplying terms

Example 16

Simplify each product.

a $7b \times 3a$ **b** $8x \times (-2y)$ **c** $5d \times 6d \times 4$

d $9m^2 \times 4p$ **e** $4a \times 3b \times (-2a)$

SOLUTION

a $7b \times 3a = 7 \times b \times 3 \times a$ Group numbers and variables separately

$= 7 \times 3 \times a \times b$

$= 21ab$

b $8x\times(-2y) = 8\times(-2)\times x\times y$

$= -16xy$

c $5d\times6d\times4 = 5\times6\times4\times d\times d$

$= 120d^2$

d $9m^2\times4p = 9\times4\times m^2\times p$

$= 36m^2p$

e $4a\times3b\times(-2a) = 4\times3\times(-2)\times a\times a\times b$

$= -24a^2b$

Foundation Standard Complex

Multiplying terms

When **multiplying** **algebraic terms**, multiply the numbers first, then multiply the variables. Write all variables in alphabetical order.

Video
Multiplying terms

Example 17

Use index notation to simplify each expression.

a $k \times k \times k \times k$ **b** $j \times j \times j \times j \times j \times j$

SOLUTION

a $k \times k \times k \times k = k^4$ **b** $j \times j \times j \times j \times j \times j = j^6$

EXERCISE 5.08 ANSWERS ON P. 615

Multiplying terms

U F R C

EXAMPLE 16

1 Simplify $2p \times 5pq$. Select the correct answer **A**, **B**, **C** or **D**.

A $10pq$ **B** $7p^2q$ **C** $7pq$ **D** $10p^2q$

2 Simplify each expression.

a $5 \times 3y$ **b** $8 \times 2m$ **c** $4k \times m$
d $4t \times 5$ **e** $2 \times f \times 6$ **f** $2x \times 8y$
g $10p \times 3m$ **h** $3b \times 6d$ **i** $3d \times 4d$
j $5 \times 4m \times 6n$ **k** $2y \times 3y \times 8$ **l** $3r \times 3 \times 3r$
m $2 \times 4a \times 9b$ **n** $x \times xy \times y$ **o** $2j \times 3k \times 7$
p $5rst \times 2rs$ **q** $3ac \times 6cd$ **r** $hjk \times hj \times jk$
s $4q^2 \times 6p^2$ **t** $2m \times 3n \times mn$

3 Simplify each expression.

a $-4b \times 2d$ **b** $6m \times (-4m)$ **c** $-2 \times 5k \times 9l$
d $20 \times (-3p)$ **e** $9k \times (-7)$ **f** $-15b \times (-5b)$
g $-3m \times mn \times 4n$ **h** $6abc \times (-2ab)$ **i** $5r \times (-2r) \times (-4)$
j $9d \times (-4e) \times 6d$ **k** $-8x \times 9x \times (-2)$ **l** $5m \times 4 \times (-2m)$

4 Which expression simplifies to $14a^2n^2$? Select **A**, **B**, **C** or **D**.

A $2a \times an \times 7$ **B** $2a \times 7n \times an$ **C** $7a^2 \times 2n$ **D** $7a^2n^2 \times 2a^2n^2$

Foundation Standard Complex

EXAMPLE 17

5 Simplify each expression.

a $y \times y \times y \times y$

b $a \times a \times a$

c $2 \times c \times c$

d $t \times t \times t \times t \times t$

e $q \times q \times q \times 7$

f $k \times k \times k \times l \times l$

g $m \times m \times m \times n \times n$

h $-5 \times p \times p \times p \times p \times k \times k \times k$

i $-3 \times y \times y \times y$

j $d \times d \times (-1) \times e \times e \times f$

k $q \times q \times r \times r \times r \times r \times 4$

l $3p + 2 \times 4p$

m $10m \times 4n - 5mn$

n $7a^2 - 2a \times 3a$

o $6r \times 2y - y \times 10r$

p $5p - 4p \times 8$

6 Write an algebraic expression involving a product that simplifies to:

R C

a $18de$

b $-20a^2y$

c $5pr^2$

5.09 Dividing terms

Worksheets
Simplifying algebraic expressions

Why aren't they the same?

Videos
Dividing terms

The Arabic science of balancing

Interactives
Equality explorer: basics

Equality explorer

Dividing terms

When **dividing algebraic terms**, divide the numbers first, then divide the variables.

Write all variables in alphabetical order.

Example 18

Simplify each quotient.

a $20p \div 5$

b $\frac{24y}{8y}$

c $\frac{-2m}{4mn}$

d $12a^2b \div 3a$

SOLUTION

a $20p \div 5 = \left(\frac{20}{5}\right)p$

$= 4p$

Group numbers and variables separately

b $\frac{24y}{8y} = \left(\frac{24}{8}\right)\frac{\cancel{y}}{\cancel{y}}$

$= 3$

c $\frac{-2m}{4mn} = \left(\frac{-2}{4}\right)\frac{\cancel{m}}{\cancel{m}n}$

$= \left(\frac{-1}{2}\right)\frac{1}{n}$

$= -\frac{1}{2n}$

d $12a^2b \div 3a = \left(\frac{12}{3}\right)\frac{\cancel{a}ab}{\cancel{a}}$

$= 4ab$

Foundation Standard Complex

EXERCISE 5.09 ANSWERS ON P. 615

Dividing terms

U F R C

EXAMPLE 18

1 Simplify $\frac{55m^2n}{5m}$. Select the correct answer **A**, **B**, **C** or **D**.

A $11mn$ **B** $11m^2n$ **C** $50mn$ **D** $11n$

2 Simplify each quotient.

a $12m \div 3$ **b** $36e \div 4$ **c** $21p \div (-3)$

d $\frac{81x}{9}$ **e** $\frac{60w}{15}$ **f** $\frac{8m}{m}$

g $-25k \div 5k$ **h** $30y \div 40y$ **i** $28pq \div 14q$

j $\frac{63a}{-9a}$ **k** $\frac{4pq}{20q}$ **l** $\frac{-56mn}{8m^2}$

3 Simplify each quotient.

a $14m^2 \div 7$ **b** $6e^2 \div (-2e)$ **c** $-5p^2 \div p$

d $\frac{14x^2}{2x}$ **e** $\frac{6wm}{-3m}$ **f** $\frac{8m^2n}{2mn}$

g $32k^2y \div (-4k)$ **h** $9jk^2 \div 12jk^2$ **i** $\frac{15d^2e^2}{3de}$

j $\frac{3a^2b^2c}{3ab^2c}$ **k** $\frac{-24acb}{3bc}$ **l** $\frac{-10m^2n}{5mn}$

4 Simplify $12x \div (-3) \times 2x$. Select **A**, **B**, **C** or **D**.

A $-2x$ **B** -2 **C** $-8x$ **D** $-8x^2$

5 Simplify each expression.

a $-18pq \div 6pq$ **b** $48p \div (-8) \div 2p$ **c** $y \times 3y \times (-5y)$

d $9d \times 4e \div 6d$ **e** $8x \times 9x \div 12$ **f** $48 \div 12m \times 2n$

g $\frac{15c}{5c}$ **h** $\frac{a \times 3ab}{ab}$ **i** $\frac{2ab}{10bc}$

j $\frac{3d \times 4e}{6e}$ **k** $\frac{15mn^2p}{3m^2n^2}$ **l** $\frac{-4x^2y \times 3y}{2xy \times (-6xy)}$

m $6x + 4x \div 2$ **n** $14r \div r + 9r$ **o** $24p - \frac{18p^2}{3p}$

6 Write an algebraic expression involving a quotient that simplifies to:

R C

a $7m$ **b** $-3ab$ **c** $5pr^2$

7 Simplify:

a $6w^2 - \frac{10w^5}{5w^3}$ **b** $x^4 - \frac{x^7}{x^3}$

Foundation Standard Complex

5.10 Equations

Worksheet
Guess-and-check

An **equation** is a statement involving a variable (such as x), numbers and an equals (=) sign, for example, $\mathbf{4x + 5 = 57}$. When we find the correct value that makes the equation true, we **solve** the equation. The value is called the **solution** to the equation.

There are different ways of solving equations. The simplest method is the 'guess, check and improve' method.

The 'guess, check and improve' method

The 'guess, check and improve' method involves:

- guessing a number for the solution
- checking the guess by substituting the number into the equation
- improving on the guess by testing better numbers until the correct solution is found

Example 19

Use 'guess, check and improve' method to solve the equation $4x + 5 = 57$ for x.

SOLUTION

Guess	Check	Comment
$x = 3$	$4 \times 3 + 5 = 17$	Smaller than 57. Try a bigger number.
$x = 10$	$4 \times 10 + 5 = 45$	Still smaller than 57. Try a bigger number.
$x = 20$	$4 \times 20 + 5 = 85$	Bigger than 57. Try a number between 10 and 20.
$x = 15$	$4 \times 15 + 5 = 65$	Still bigger than 57. Try a number between 10 and 15.
$x = 13$	$4 \times 13 + 5 = 57$	Correct.

The solution is $x = 13$.

EXERCISE 5.10

ANSWERS ON P. 615

Equations

U F

EXAMPLE 19

1 Solve each equation.

a $x + 3 = 6$ **b** $x - 3 = 6$ **c** $a + 12 = 17$

d $m - 5 = 15$ **e** $b + 1 = 11$ **f** $5c = 20$

g $4k = 24$ **h** $d - 3 = 10$ **i** $10 + m = 25$

j $14 - x = 9$ **k** $\frac{y}{3} = 5$ **l** $\frac{n}{5} = 5$

Foundation Standard Complex

2 What is the solution to $2x - 6 = 24$? Select the correct answer **A**, **B**, **C** or **D**.

A $x = 12$ **B** $x = 15$ **C** $x = 14$ **D** $x = 9$

3 Solve each equation.

a $2x + 4 = 14$ **b** $3p - 5 = 16$ **c** $4k + 6 = 26$

d $5x - 9 = 11$ **e** $7x + 6 = 27$ **f** $8x - 30 = 34$

g $\frac{m}{4} - 3 = 4$ **h** $\frac{a}{2} + 12 = 18$ **i** $10 - 3d = 4$

j $12 - 5n = 7$ **k** $\frac{r-9}{2} = 3$ **l** $\frac{k+1}{3} = 4$

m $2p + 3 = -1$ **n** $\frac{r}{5} + 4 = 6$ **o** $\frac{2y}{3} = 10$

p $2 \times (x + 1) = 6$

4 Solve the equation $6\left(\frac{x}{2} - 1\right) = 12$. Select **A**, **B**, **C** or **D**.

A $x = \frac{13}{3}$ **B** $x = 6$ **C** $x = 3$ **D** $x = 2$

One-step equations

5.11

The 'guess, check and improve' method is simple but slow. There are 2 **algebraic** methods for solving equations that are faster and more efficient:

balancing: performing the same operation on both sides of the equation

backtracking: undoing each operation by performing the inverse (opposite) operation

Shutterstock.com/defpicture

Shutterstock.com/ml

Worksheet One-step equations

Puzzle Equations dominoes

Foundation Standard Complex

Example 20

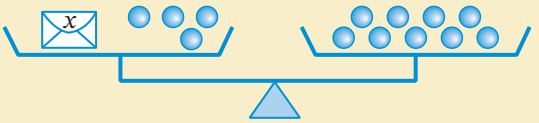
Solve the equation $x + 4 = 9$.

SOLUTION

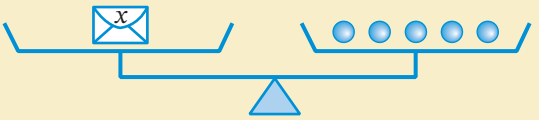
Method 1: The balancing method

Let x represent an unknown number in an envelope, and ● represent 1.

We can represent the equation $x + 4 = 9$ on balance scales like this:



The 2 sides of the balance are equal. To find the value of x, we can remove 4 balls from both sides.



The solution is $x = 5$.

This equation can also be solved algebraically without diagrams.

$x + 4 = 9$

$x + 4 - 4 = 9 - 4$ Subtracting 4 from both sides

$\boldsymbol{x = 5}$ Check: $5 + 4 = 9$

Method 2: The backtracking method

First use a flowchart to show how we get from x to $x + 4$.

$$x \xrightarrow{+4} x + 4$$

In this equation, $x + 4 = 9$. To backtrack (get back) to x, we need to undo the operation 'add 4,' which is 'subtract 4'.

Using a reverse flowchart:

$$\begin{array}{ccc} x & \xrightarrow{+4} & x + 4 \\ = & & = \\ 5 & \xleftarrow[-4]{} & 9 \end{array}$$

Undoing what has been done to x.

The solution is $\boldsymbol{x = 5}$.

This equation can also be solved algebraically without flowcharts.

$x + 4 = 9$

$x = 9 - 4$ Undoing '+ 4' by subtracting 4

$\boldsymbol{x = 5}$ Check: $5 + 4 = 9$

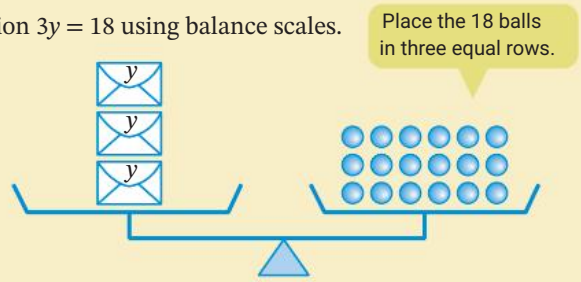
Example 21

Solve $3y = 18$.

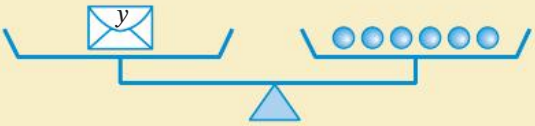
SOLUTION

Method 1: The balancing method

Represent the equation $3y = 18$ using balance scales.



To find the value of y, divide both sides by 3.



The solution is $\boldsymbol{y = 6}$.

Solving this algebraically, we have:

$3y = 18$

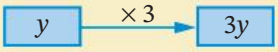
$\frac{3y}{3} = \frac{18}{3}$ Dividing both sides by 3

Check: $3 \times 6 = 18$

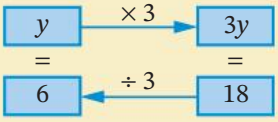
$\boldsymbol{y = 6}$

Method 2: The backtracking method

First use a flowchart to show how we get from y to $3y$.



In this equation, $3y = 18$. To backtrack to y, we need to undo the operation '× 3', which is '÷ 3', using a reverse flowchart:



The solution is $\boldsymbol{y = 6}$.

Solving this algebraically, we have:

$3y = 18$

$y = \frac{18}{3}$ Undoing '× 3' in 3y by dividing by 3

$\boldsymbol{y = 6}$ Check: $3 \times 6 = 18$

Video
One-step equations

5.11

Solving an equation

When solving an equation, aim to have the variable (such as x) on one side of the equation and a number on the other side, in the form:

$$x = _____$$

Check your solution by substituting it back into the equation.

We call these equations 'one-step' equations because they require one step or one inverse operation to solve.

Example 22

Solve $u - 5 = 7$.

SOLUTION

Method 1: The balancing method

$u - 5 = 7$

$u - 5 + 5 = 7 + 5$ Add 5 to both sides

$\boldsymbol{u = 12}$ Check: $12 - 5 = 7$

Method 2: The backtracking method

$u - 5 = 7$

$u = 7 + 5$ Undoing '– 5' by adding 5

$\boldsymbol{u = 12}$ Check: $12 - 5 = 7$

Example 23

Solve $\frac{k}{7} = 2$.

SOLUTION

Method 1: The balancing method

$\frac{k}{7} = 2$ Multiply both sides by 7

$\frac{k}{7} \times 7 = 2 \times 7$ Check: $\frac{14}{7} = 2$

$\boldsymbol{k = 14}$

Method 2: The backtracking method

$\frac{k}{7} = 2$ Undoing '÷ 7' by multiplying by 7

$k = 2 \times 7$

$\boldsymbol{k = 14}$ Check $\frac{14}{7} = 2$

Inverse operations

Operation	Inverse operation
+	−
−	+
×	÷
÷	×

EXERCISE 5.11 ANSWERS ON P. 615

One-step equations

U F R C

1 Solve each equation, showing the working. EXAMPLE 20

R **a** $w + 8 = 15$ **b** $x + 2 = 20$ **c** $m + 3 = 19$

C **d** $p + 12 = 30$ **e** $x + 13 = 22$ **f** $k + 11 = 20$

g $5 + m = 2$ **h** $6 + y = 16$ **i** $15 + d = 6$

2 Solve each equation, showing the working. EXAMPLE 21

R **a** $3m = 18$ **b** $5n = 20$ **c** $4k = 44$

C **d** $2c = 32$ **e** $9x = 81$ **f** $3x = -27$

g $7d = 42$ **h** $6h = -30$ **i** $10a = 70$

3 Solve each equation, showing the working. EXAMPLE 22

R **a** $p - 3 = 8$ **b** $m - 11 = 2$ **c** $x - 5 = 12$

C **d** $y - 20 = 40$ **e** $k - 21 = 3$ **f** $n - 3 = 18$

g $7 = d - 11$ **h** $y - 13 = -4$ **i** $-5 = m - 9$

4 Solve each equation, showing the working. EXAMPLE 23

R C **a** $\frac{m}{3} = 4$ **b** $\frac{d}{2} = 5$ **c** $\frac{x}{5} = 8$ **d** $\frac{k}{4} = 12$

e $\frac{x}{10} = 6$ **f** $\frac{a}{2} = -4$ **g** $15 = \frac{k}{2}$ **h** $-11 = \frac{n}{5}$

5 Solve $\frac{p}{3} = -5$. Select the correct answer **A**, **B**, **C** or **D**.

A $p = 15$ **B** $p = 8$ **C** $p = -8$ **D** $p = -15$

Foundation Standard Complex

6 Solve each equation, showing the working.

R **a** $m - 3 = 16$ **b** $3 + p = 11$ **c** $24 = p + 5$

C **d** $n + 7 = -1$ **e** $3x = 30$ **f** $n - 14 = -5$

g $3x = 21$ **h** $4y = -18$ **i** $\frac{h}{7} = 1$

j $-2x = -6$ **k** $6 + k = -4$ **l** $\frac{n}{4} = 0$

7 Solve $3 - m = -2$, showing all working. Select **A**, **B**, **C** or **D**.

R **A** $m = -5$ **B** $m = 5$ **C** $m = -1$ **D** $m = 1$

C

5.12 Two-step equations

Videos
Two-step equations
Solving equations

Worksheets
Equations writing activity
Backtracking 1
Backtracking 2

Puzzle
Solving equations

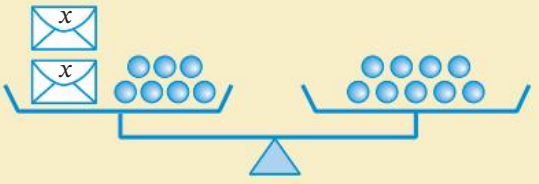
Now we will look at 'two-step' equations that require 2 steps or 2 inverse operations to solve.

Example 24

Solve the equation $2x + 7 = 9$.

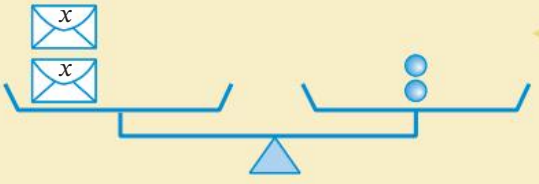
SOLUTION

Method 1: The balancing method



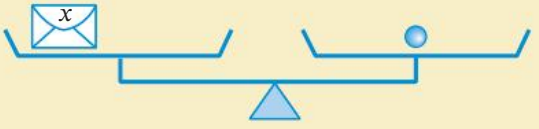
We can represent $2x + 7 = 9$ using balance scales.

First subtract 7 balls from both sides:



Place the remaining balls into 2 equal rows.

Then divide both sides by 2.



The solution is $\boldsymbol{x = 1}$.

Foundation Standard Complex

Using algebra we have:

$2x + 7 = 9$

$2x + 7 - 7 = 9 - 7$ — Step 1: Subtracting 7 from both sides

$2x = 2$

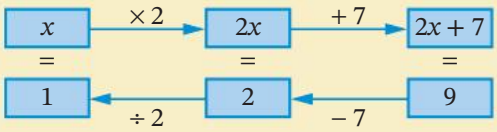
$\frac{2x}{2} = \frac{2}{2}$ — Step 2: Dividing both sides by 2

$\boldsymbol{x = 1}$ — Check: $2 \times 1 + 7 = 9$

Method 2: The backtracking method

Use a flowchart to go from x to $2x + 7$.

$2x + 7 = 9$. To backtrack, we need to undo the operations '× 2'and '+ 7' in reverse order, so '– 7' and '÷ 2' using a reverse flowchart.



The solution to $2x + 7 = 9$ is $x = 1$.

Using algebra, we have:

$2x + 7 = 9$

$2x = 9 - 7$ — Step 1: Undoing '+ 7' by subtracting 7

$2x = 2$

$x = \frac{2}{2}$ — Step 2: Undoing '× 2' in $2x$ by dividing by 2

$\boldsymbol{x = 1}$ — Check: $2 \times 1 + 7 = 9$

Note that inverse operations are performed in **reverse order**. For example, to undo putting on our socks and shoes, we take off our shoes first, then our socks.

Example 25

Solve $\frac{M}{3} + 2 = 5$

SOLUTION

Method 1: The balancing method

$\frac{M}{3} + 2 = 5$

$\frac{M}{2} + 2 - 2 = 5 - 2$ — Step 1: Subtracting 2 from both sides

$\frac{M}{3} = 3$

$\frac{M}{3} \times 3 = 3 \times 3$ — Check: $\frac{9}{3} + 2 = 3 + 2 = 5$

$\boldsymbol{M = 9}$ — Step 2: Multiplying both sides by 3

Method 2: The backtracking method

$\frac{M}{3}+2=5$ Step 1: Undo '+ 2' by subtracting 2

$\frac{M}{3}=5-2$

$\frac{M}{3}=3$

$M=3\times 3$ Step 2: Undo '÷ 3' by multiplying by 3

$\boldsymbol{M=9}$ Check: $\frac{9}{3}+2=3+2=5$

EXERCISE 5.12 ANSWERS ON P. 615

Two-step equations

U F R C

EXAMPLE 24

1 Solve each equation showing all steps. Remember to check your answers.

R **a** $2x+1=11$ **b** $3x+8=17$ **c** $4x+5=13$

C **d** $4x+9=25$ **e** $5x+2=47$ **f** $2x+10=-6$

g $-3x+10=4$ **h** $-4x+9=5$ **i** $-5x+8=-12$

j $-7x+4=-17$ **k** $-2x+7=9$ **l** $-4x+14=22$

2 The following is Liam's incorrect solution for $7x+5=13$.

R $7x+5=13$

$7x=13-5$ **A**

$7x=8$ **B**

$x=\frac{8}{7}$ **C**

$x=1\frac{1}{8}$ **D**

In which line was the error made? Select **A**, **B**, **C** or **D**.

3 Solve each equation, showing all steps.

R **a** $2x-5=9$ **b** $3x-1=23$ **c** $5x-3=12$

C **d** $2x-4=18$ **e** $6x-4=8$ **f** $5x-19=-9$

g $4x-7=5$ **h** $7x-3=32$ **i** $2x-4=-14$

j $5x-2=43$ **k** $3x-4=-10$ **l** $8x-12=-20$

4 What is the solution for $\frac{n}{4}-8=7$? Select **A**, **B**, **C** or **D**.

R **A** $n=4$ **B** $n=20$ **C** $n=36$ **D** $n=60$

Foundation Standard Complex

EXAMPLE 25

5.12

5 Solve each equation.

R C

a $\frac{x}{2}+5=9$ **b** $\frac{m}{3}+6=9$ **c** $\frac{k}{5}+2=6$

d $\frac{a}{4}+4=8$ **e** $\frac{n}{2}+6=14$ **f** $\frac{h}{7}+2=-3$

g $\frac{m}{2}-8=-5$ **h** $\frac{k}{5}-7=-4$ **i** $\frac{x}{3}-6=-10$

j $\frac{a}{3}-1=2$ **k** $\frac{h}{5}-4=-7$ **l** $\frac{x}{2}-3=4$

6 Solve $\frac{k+1}{3}=5$, showing all working. Select **A**, **B**, **C** or **D**.

R C

A $k=19$ **B** $k=18$ **C** $k=17$ **D** $k=14$

☆ MENTAL SKILLS 5B ANSWERS ON P. 615 Maths without calculators

Quiz Mental skills 5B

Multiplying by 9, 11 and 12

To multiply a number by 9, multiply by 10 and then subtract the number.

This is because 10 times a number minus the same number equals 9 times the number.

1 Study each example.

a $14 \times 9 = 14 \times 10 - 14 = 140 - 14 = 126$

b $25 \times 9 = 25 \times 10 - 25 = 250 - 25 = 225$

c $18 \times 9 = 18 \times 10 - 18 = 180 - 18 = 162$

2 Now simplify each product.

a 12×9 **b** 27×9 **c** 46×9

d 19×9 **e** 34×9 **f** 63×9

g 21×9 **h** 15×9

To multiply a number by 11, multiply by 10 and then add the number.

This is because 10 times a number plus the same number equals 11 times the number.

3 Study each example.

a $26 \times 11 = 26 \times 10 + 26 = 260 + 26 = 286$

b $13 \times 11 = 13 \times 10 + 13 = 130 + 13 = 143$

c $35 \times 11 = 35 \times 10 + 35 = 350 + 35 = 385$

Foundation Standard Complex

4 Now simplify each product.

a 17×11 **b** 22×11 **c** 38×11

d 40×11 **e** 25×11 **f** 19×11

g 54×11 **h** 31×11

To multiply a number by 12, multiply by 10, then add double the number.

This is because 10 times a number plus double the same number equals 12 times the number.

5 Study each example.

a $22 \times 12 = 22 \times 10 + 22 \times 2 = 220 + 44 = 264$

b $16 \times 12 = 16 \times 10 + 16 \times 2 = 160 + 32 = 192$

c $70 \times 12 = 70 \times 10 + 70 \times 2 = 700 + 140 = 840$

6 Now simplify each product.

a 44×12 **b** 15×12 **c** 29×12

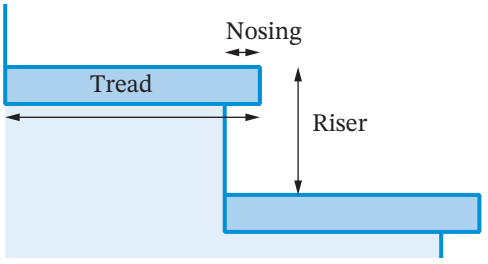
d 31×12 **e** 52×12 **f** 18×12

g 26×12 **h** 37×12

DID YOU KNOW?

Taking safe steps

Staircases must be designed to be safe to use; they must provide enough stepping space and not be too steep. The diagram below shows 2 steps. The **tread** is the length of a step, and the **riser** is the height between the tops of steps. The **nosing** is how much the tread of one step 'overhangs' horizontally with the step below it.



The Australian Building Code recommends that risers must be between 13 cm and 22.5 cm and the tread must be between 21.5 cm and 35.5 cm.

The French architect François Blondel developed an equation to design a comfortable and efficient staircase:

$$2 \times \text{Riser} + \text{Tread} = \text{a number from 63 to 65}$$

For example, suppose we need to design a staircase that is 2.6 m high.

1 Calculate the number of steps that will be needed

Assuming an ideal riser of 18 cm, the number of steps will be:

$260 \text{ cm} \div 18 \text{ cm} = 14.44 \ldots \approx 15$ steps

2 Calculate the height of each riser

Riser = 260 cm ÷ 15 = 17.33 ... ≈ 17 cm

3 Calculate the tread

Apply the Blondel formula:

$2 \times \text{Riser} + \text{Tread} = 64$

$2 \times 17 + \text{Tread} = 64$

Solving the equation:

$34 + \text{Tread} = 64$

Tread = 30 cm

The staircase will have 15 steps, each with a 30 cm tread and 17 cm riser.

Now calculate the measurements for a staircase that is 3.2 m high.

Equation problems 5.13

Many mathematical problems are stated in words. We can solve these problems by translating the problems into algebraic symbols and equations.

Puzzle Writing and solving equations

Video Equation problems

Example 26

When a number is doubled and then 5 is added, the answer is 69. Find the number.

SOLUTION

Let the number be x.

Translating the problem into an equation:

$x \times 2 + 5 = 69$

$2x + 5 = 69$

Solve the equation:

$2x + 5 - 5 = 69 - 5$ Subtracting 5 from both sides

$2x = 64$

$\frac{2x}{2} = \frac{64}{2}$ Dividing both sides by 2

Check: $2 \times 32 + 5 = 69$

$x = 32$

The number is 32.

Video
Equation problems

Example 27

Tom is 8 years older than Susi. If the sum of their ages is 22, find their ages.

SOLUTION

Let Susi's age be n.	
So Tom's age is $n + 8$.	Tom is 8 years older.
$n + n + 8 = 22$	Susi's age + Tom's age = 22
$2n + 8 = 22$	Simplifying $n + n$ to $2n$
$2n + 8 - 8 = 22 - 8$	Subtracting 8 from both sides
$2n = 14$	
$\frac{2n}{2} = \frac{14}{2}$	Dividing both sides by 2
$n = 7$	Check: $2 \times 7 + 8 = 22$
So Susi is 7 and Tom is $7 + 8 = 15$.	Check: $7 + 15 = 22$

Solving word problems using equations

- Choose a variable
- Translate the problem into an equation
- Solve the equation
- Write a sentence that answers the problem

EXERCISE 5.13 ANSWERS ON P. 615

Equation problems

U F PS R

EXAMPLE 26

1 Solve each problem by writing an equation and then solving it. You may use diagrams to help you think about the information.

PS R

a 5 tickets for a film cost \$55. How much does each ticket cost?
(Let t represent the price of a ticket.)

b 10 oranges cost \$4.80. How much does each orange cost?
(Let x represent the cost of one orange.)

c A number is doubled and the result is 110. What is the number?
(Let n represent the unknown number.)

d A number has 4 subtracted from it and the result is 6. Find the number.
(Let y represent the unknown number.)

Foundation Standard Complex

2 For each word problem, select the correct equation **A**, **B**, **C** or **D**. Then solve the equation to solve the problem.

PS R

a Jarrad has collected 1794 beetles. This is 6 times as many beetles as Lisa has in her collection. How many beetles does Lisa have?

A $1794 - N = 6$ **B** $6N = 1794$

C $N = 6 \times 1794$ **D** $N + 6 = 1794$

b Kurt mixed 590 mL of white paint with some blue paint. He mixed 1.73 L of paint altogether. How much blue paint did he use?

A $N + 590 = 1730$ **B** $N - 590 = 1730$

C $\frac{N}{590} = 1730$ **D** $590N = 1730$

c 14 packets of chocolate biscuits are packed in a box. The supermarket sold 546 packets of biscuits. How many boxes were sold?

A $14N = 546$ **B** $546 - N = 14$

C $N + 14 = 546$ **D** $\frac{N}{14} = 546$

d When a certain number is subtracted from 100, the result is 47. What is the number?

A $N - 100 = 47$ **B** $100 + 47 = N$

C $N - 47 = 100$ **D** $100 - N = 47$

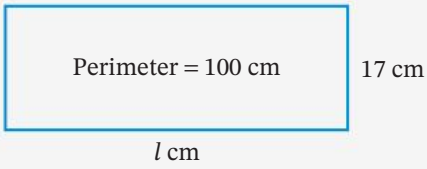
3 Translate each of the following problems into an equation, then solve the equation to solve the problem.

EXAMPLE 27

PS R

a Mr Abdul says, 'If you multiply my age by 4 and add 12, the answer is 240.' How old is Mr Abdul? (Let a stand for his age.)

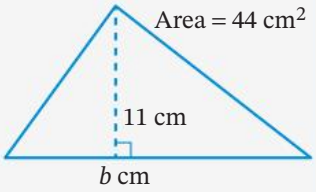
b The perimeter of a rectangle is 100 cm and its width is 17 cm. What is the length? (Let l cm represent the length of the rectangle.)



c I think of a number. If I multiply it by 6 and subtract 13, the answer is 95. What is the number? (Let y represent the number.)

d The sum of 3 consecutive numbers is 48. Find the smallest number. (Let n be the smallest number.)

e The area of a triangle is calculated by multiplying the base by the height and dividing by 2. If a triangle has an area of 44 cm² and its height is 11 cm, what is the length of its base? (Let b cm represent the length of the base.)



Foundation Standard Complex

f The Student Council is holding a school disco to raise money. Each ticket bought by students raises \$7, but the cost of running the disco is \$184. How many tickets must be sold to make a profit of \$2000? (Let n stand for the number of tickets sold.)

g Grace earns \$200 per week plus one-fifth of the value of her gym equipment sales for that week. If she is paid \$750 for one week, what is the value of her sales for that week? (Let x stand for the value of her sales.)

4 A repairer of washing machines charges using the formula:

PS R

$$C = 32h + 45$$

where C is the charge in dollars and h is the number of hours the job takes.

Find the number of hours worked if the charge is \$205.

5 The weekly profit, in dollars, made by a beach hire stall is given by the formula $P = 5u - 900$, where u represents the number of beach umbrellas hired. Find the number of umbrellas hired if the profit is \$1055.

PS R

6 The volume of a rectangular prism is given by the formula $V = lwh$. What is the value of l when $V = 340$, $w = 4$ and $h = 5$? Select **A**, **B**, **C** or **D**.

A 17 **B** 68 **C** 85 **D** 425

7 Julian, Marcus and Aiden compared how many merit awards they received in class. Marcus was awarded 7 more than Julian, but Aiden had as many as Marcus and Julian put together. Altogether they collected 54 merit awards. Let a be the number of awards Julian had. How many awards did Aiden receive? Select **A**, **B**, **C** or **D**.

PS R

A 10 **B** 17 **C** 20 **D** 27

8 There are 50 boys and girls in a club. At the end of the year, each boy contributed \$6 towards the cost of the end of year party. The girls contributed \$5 each. In total, they raised \$270. Let g be the number of girls. How many boys and girls are in the club? Select **A**, **B**, **C** or **D**.

PS R

A 20 boys, 30 girls **B** 34 boys, 16 girls

C 30 boys, 20 girls **D** 25 boys, 25 girls

Foundation Standard Complex

INVESTIGATION

School concert

Year 7 wants to hold a school concert to help subsidise the cost of their camp. They want to raise \$1500 for this.

The concert will cost \$100 for each night it is on. The hall will hold 250 people. If a band is hired for the concert, this would cost an extra \$550 per night.

a If Year 7 decide to stage one concert only without a band, calculate how much each ticket should be to cover the costs and raise the required amount. Use an equation to help you.

b If they decide to have the band, write another equation to find the new ticket price.

c If the SRC decides to have 3 concerts with the band, calculate what the ticket price should be. Consider how many people you think would attend each night. Write another equation and solve it to justify your answer.

POWER PLUS ANSWERS ON P. 616

1 Simplify each expression.

a $2a + b + 3a - 4b$ **b** $m - 3n - 5m - 7n$

c $3k - 2j - 5 - 4k + 6j + 9$ **d** $a^2 + 2b^2 + 3b^2 - a^2$

e $4x^2 + 6x - x^2 - 12x$ **f** $xy + yz + xz + 4xy + 3xz + yz$

2 Evaluate each expression if $p = 4$, $q = -6$, $r = 3$ and $t = -1$.

a $p + q$ **b** pq **c** $r^2 + t^2$

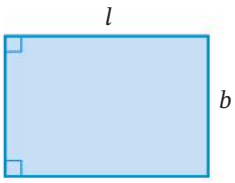
d $q^2 - 6p + rt$ **e** $\frac{pq}{rt}$ **f** $6 - 2r + p$

g $2p + 3q + 4r - 5t$ **h** $\frac{p+4}{2}$

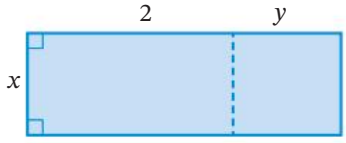
3 For each shape, write an algebraic expression for:

i its perimeter **ii** its area

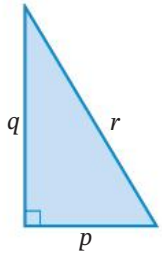
a



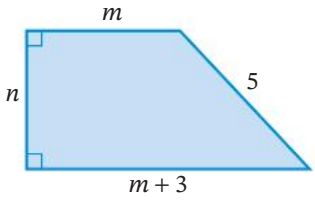
b



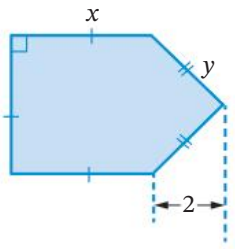
c



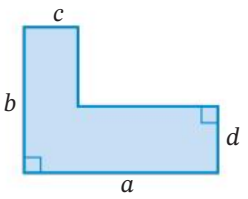
d



e



f



4 If x is any number, then simplify each expression.

a $x + 0$ **b** $x \times x$ **c** $0 \div x$

d $x \div x$ **e** $x - (-x)$ **f** $0 \times x$

g $0 - x$ **h** $(-x)^2$

i $x \times \frac{1}{x}$ **j** $x + \frac{1}{x}$

5 Simplify each expression.

a $-7p - 8q + 3p - 5q + 4q$ **b** $-m - 3n - 5m - 7mn$

c $3k - 2j - 5 - 4k + 6j + 9$ **d** $a^2 + 2b^2 + 3b^2 - a^2$

e $4x^2 + 6x - x^2 - 12x$ **f** $xy + yz + xz + 4xy + 3xz + yz$

g $-d^2 + 7 + 6e^2 - 3d^2 - 2e^2 + 9$ **h** $5f^2 + 6f + 2 + 8f^2 - 1$

6 Solve each equation.

a $6 - 3x = 9$ **b** $9 - 8k = 33$ **c** $7 - 2n = 11$

d $2(r - 4) = 10$ **e** $3(d + 6) = 18$ **f** $4(z + 1) = -6$

5 CHAPTER REVIEW

Language of maths

abbreviation	algebra	associative law	backtracking
balancing	check and improve	collecting	commutative law
consecutive	distributive law	equation	evaluate
expanded form	expression	formula	guess
inverse	like terms	one-step	pronumeral
solution	solve	substitute	two-step
unlike terms	variable		

Quiz Language of maths 5

1 What is another name for **pronumeral**?

2 What is the difference between an **expression** and an **equation**?

3 Which law involves multiplying by a number by splitting that number to make the calculation easier?

4 What are **consecutive numbers**?

5 What are **like terms?**

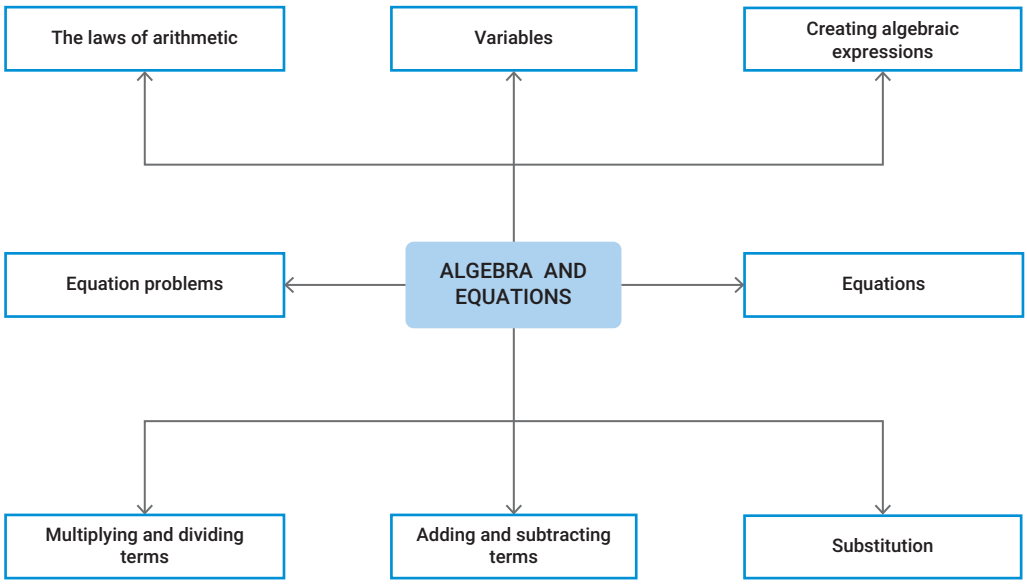
6 What name is given to the value that makes an equation true?

Topic summary

Worksheet Mind map: Algebra and equations

- Which parts of this topic did you find easy? Was there anything you already knew?
- Are there parts of this chapter that you still do not understand? Talk to your teacher.
- Give 2 examples of jobs where algebra or equations might be needed.

Print (or copy) and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.



5 TEST YOURSELF

ANSWERS ON P. 616

Quiz
Test yourself 5

5.01 **1** Use the commutative and associative laws to evaluate each expression.

a $7 + 18 + 3$ **b** $91 + 18 + 9$ **c** $2 \times 17 \times 5$
d $25 \times 31 \times 4$ **e** 13×30 **f** 7×400

5.02 **2** Use the distributive law to evaluate each expression.

a 22×9 **b** 17×8 **c** 27×11 **d** 13×12

5.03 **3** Examine this number pattern, then write the general rule using a variable.

$3 - (-3) = 2 \times 3$
$5 - (-5) = 2 \times 5$
$11 - (-11) = 2 \times 11$
$2 - (-2) = 2 \times 2$
$8 - (-8) = 2 \times 8$

5.03 **4** Simplify each expression.

a $x + x + x + x$ **b** $u \div 5$ **c** $2y - y$
d $5 \times a \times 2$ **e** $8 \times c \times b \times 3 \times a$ **f** $2 \times m \times w \times 10 \times w$

5.03 **5** Write each of these in expanded form.

a $4mnk$ **b** $\frac{6a}{b}$ **c** $9c^2$

5.04 **6** Write an algebraic expression for each statement. Use N to represent any number.

a 3 times the number **b** the difference between the number and 5
c the next consecutive number **d** one-third of the number

5.04 **7** Write an expression for:

a the sum of M and 3 **b** 5 more than B **c** $2H$ decreased by k

5.05 **8** Find the value of these expressions if $a = 2$, $b = 5$ and $c = 6$.

a $a + b$ **b** $3a - c$ **c** $a + 2b - c$ **d** abc

5.05 **9** If $C = 30h + 19$, find C when

a $h = 3$ **b** $h = -1$ **c** $h = 5$ **d** $h = 2.5$

5.07 **10** Simplify each expression.

a $5k + 7k$ **b** $29d - 12d$
c $4m + 18m - 5m$ **d** $13m - 3m + 8m$
e $4ac + 4ac - 5ca$ **f** $11cd + 7cd - 9cd$
g $7x + 3y + 3x + 7y$ **h** $13r + 4s + 11s + 2r$
i $33h + 6g - 5h + g$

Foundation Standard Complex

9780170465557

11 Simplify each expression. 5.08

a $5 \times 2x$ **b** $2b \times 7d$ **c** $11m \times 6$

d $3p \times 8q$ **e** $4 \times 2w \times 4w$ **f** $15j \times 5$

g $3m \times 6m \times 2$ **h** $4p \times 3p \times 5p$

12 Simplify each expression. 5.09

a $50x \div 2$ **b** $28b \div 7d$ **c** $66m \div 6$

d $\frac{32rs}{4}$ **e** $\frac{15p}{3p}$ **f** $14d^2 \div 2$

g $14e^2 \div 2e$ **h** $13b^2 \div b$ **i** $\frac{18x^2}{9x}$

j $\frac{44m^2n^2}{11m}$ **k** $\frac{16e^2g}{4eg}$ **l** $\frac{35a^2e^2}{7ae}$

13 Solve each equation using the 'guess, check and improve' method. 5.10

a $4m - 9 = 7$ **b** $\frac{k+2}{7} = 2$ **c** $\frac{d}{3} - 4 = 2$

14 Solve each equation algebraically, showing all steps. 5.11

a $w + 17 = 52$ **b** $e - 12 = 0$ **c** $5p = 15$ **d** $\frac{x}{2} = 6$

15 Solve each equation algebraically, showing all steps. 5.12

a $3t + 4 = 10$ **b** $2n + 6 = 12$ **c** $4y - 7 = 5$

d $\frac{m}{4} - 8 = 9$ **e** $\frac{x}{3} + 5 = 8$ **f** $\frac{m-4}{2} = 3$

16 **a** When a certain number is multiplied by 17, the product is 1003. Which equation can be used to solve this problem? (Let N represent the number.) Select the correct answer **A**, **B**, **C** or **D**. 5.13

A $N + 17 = 1003$ **B** $1003 - N = 17$

C $17N = 1003$ **D** $1003 + N = 17$

b Solve the equation to solve the problem.

17 Caitlyn organises a charity raffle at her school. The only cost is \$180 for prizes. 5.13

a If she sets the price of raffle tickets at \$5, write an equation to find how many tickets she must sell to just cover costs, and solve the equation. (Let N represent the number of tickets.)

b To make a profit of \$2100, how many tickets must she sell?

Foundation Standard Complex

6

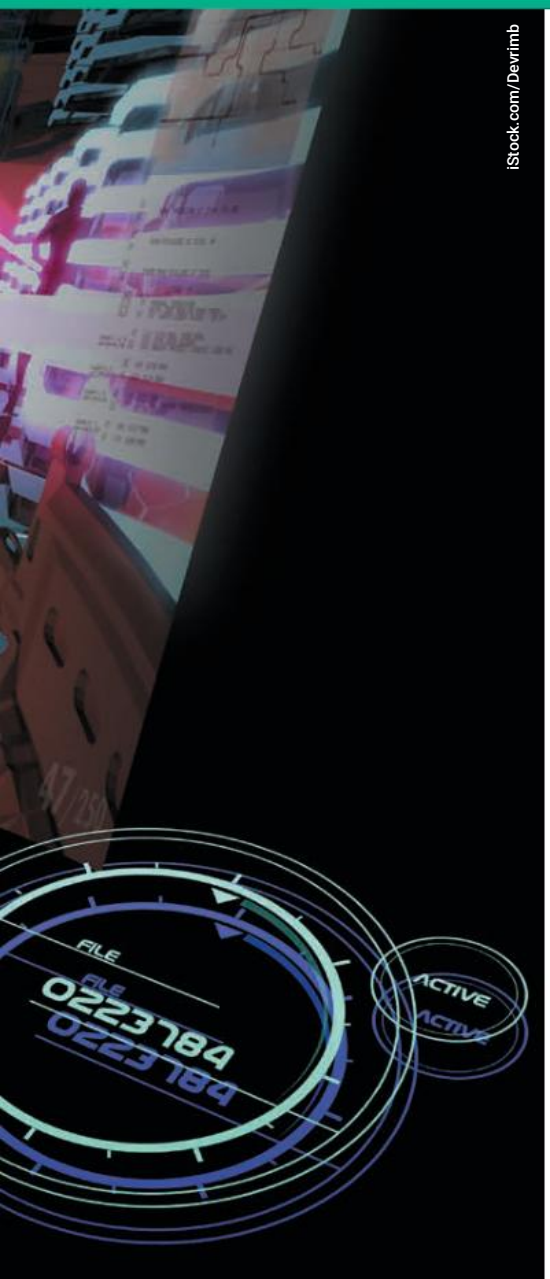
SPACE

CODE 1A
LEVEL 7
Tab
Caps lock
Shift
Ctrl
Alt
Backspace
Enter
AltGr
Shift
Menu
Ctrl

Geometrical figures

Geometry is the study of points, angles, lines, shapes and solids. The word 'geometry' comes from the Greek *ge*, meaning earth or land, and *metra*, meaning 'a measure of'.

It is used by artists, architects, engineers and physicists. It is useful in construction projects, computer imaging and video game design.



iStock.com/Devrimb

Chapter outline

		Proficiencies				
6.01	Transformations	U	F		R	C
6.02	Composite transformations	U	F		R	C
6.03	Classifying triangles	U	F		R	C
6.04	Angle sum of a triangle	U	F	PS	R	C
6.05	Exterior angle of a triangle	U	F		R	C
6.06	Classifying quadrilaterals	U	F		R	C
6.07	Angle sum of a quadrilateral	U	F	PS	R	C
6.08	Properties of quadrilaterals	U	F		R	C
6.09	Polygons and polyhedra	U	F		R	C
6.10	Drawing solids	U	F		R	C

U = Understanding
F = Fluency
PS = Problem solving
R = Reasoning
C = Communication

Wordbank

angle sum The total of the sizes of the angles in a shape, such as a triangle

exterior angle of a triangle An 'outside' angle of a triangle formed after extending one of the sides of the triangle

isosceles triangle A triangle with 2 equal sides

obtuse-angled triangle A triangle with one obtuse angle

polygon A flat (2D) closed shape with straight sides

polyhedron A solid (3D) shape with flat faces

quadrilateral A shape with 4 straight sides

translation The process of 'sliding' a shape a certain distance and direction

trapezium A quadrilateral with one pair of parallel sides

Quiz
Wordbank 6

Videos (4):
6.01 Transformations
6.03 Classifying triangles
6.04 Angle sum of a triangle
6.08 Classifying quadrilaterals

Twig videos (5):
6.01 Transformations: Skateboarding
6.09 Tessellated designs • Polyhedra: Platonic solids
6.10 Escher and the endless staircase • Perspective: Dazzle camouflage

Quizzes (5):
- Wordbank 6
- SkillCheck 6
- Mental skills 6
- Language of Maths 6
- Test Yourself 6

Skillsheets (2):
SkillCheck Naming shapes • Line and rotational symmetry
6.06 Naming shapes

Worksheets (14):
SkillCheck Symmetry • Symmetry of flat shapes
6.01, 6.02 5 mm grid paper
6.03 Properties of triangles
• Constructing triangles and quadrilaterals
6.05 Triangle geometry
6.06 Classifying quadrilaterals
• Constructing triangles and quadrilaterals
6.08 Properties of quadrilaterals
• Diagonal properties of quadrilaterals
• Always, sometimes, never true
6.10 Square dot paper • Isometric dot paper • Equal angles
Mind map: Geometrical figures

Puzzles (10):
6.03 Classifying triangles
6.05 Angles in triangles
6.06 Constructions group clues
• What shape is it? • Singing in the car
6.07 Find the unknown angle
• Mixed angle problems
• Triangles and quadrilaterals
6.08 Geometry crossword
• Shapes puzzle 1

Nelson MindTap

To access resources above, visit **cengage.com.au/nelsonmindtap**

6 In this chapter you will:

- ✓ translate, reflect and rotate geometrical figures, and composite transformations of these
- ✓ classify triangles by sides and by angles
- ✓ solve problems involving the angle sum of a triangle and the exterior angle of a triangle
- ✓ identify and classify the special quadrilaterals and their properties: square, rectangle, parallelogram, rhombus, kite and trapezium
- ✓ solve problems involving the angle sum of a quadrilateral
- ✓ identify and classify polygons and polyhedra, including prisms and pyramids
- ✓ sketch solid shapes, including from different views

Skillsheets
Naming shapes
Line and rotational symmetry

Worksheets
Symmetry
Symmetry of flat shapes

Quiz
SkillCheck 6

SkillCheck
ANSWERS ON P. 616

1 Copy each figure and draw its axes of symmetry.

a

b

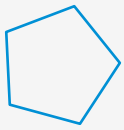
c

9780170465557

2 Test whether each shape has rotational symmetry. If it does, write the order of rotational symmetry.

a

b



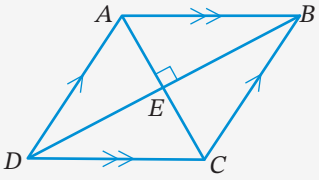
c

3 Draw:

a 2 perpendicular lines **b** 2 parallel lines

4 Draw a rectangle and all its diagonals.

5 For this **quadrilateral**:



a name 2 intervals that are parallel

b are the diagonals equal in length?

c what is the size of $\angle DEC$?

d if $\angle DAB$ and $\angle ABC$ are co-interior and $\angle DAB = 115°$, what is the size of $\angle ABC$?

6 Draw a pair of parallel lines crossed by a transversal and mark a pair of alternate angles.

7 **a** Draw a parallelogram and label it *DEFG*.

b Mark both pairs of parallel sides.

c Name both pairs of parallel sides.

d Mark the equal sides *DG* and *EF*.

8 Draw a **triangle** that has:

a a right angle **b** an obtuse angle.

9 Draw 2 parallel lines crossed by a transversal and mark a pair of corresponding angles.

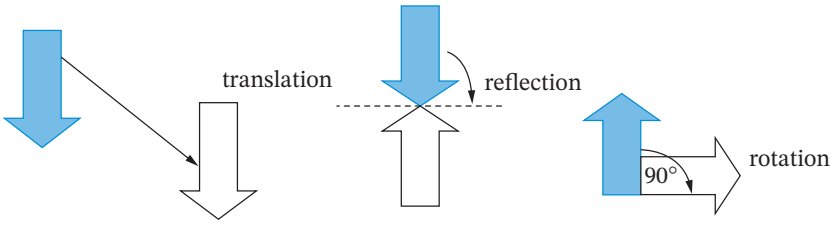
Transformations 6.01

Patterns in tiles, wallpaper and paving are usually made by taking a basic shape and repeating it. A pattern can be created by **sliding**, **flipping** or **spinning** the shape.

Video Transformations: Skateboarding

These movements have special names.

- A 'slide' is called a **translation**.
- A 'flip' is called a **reflection**.
- A 'spin' is called a **rotation**.



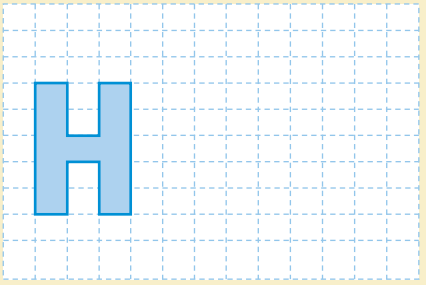
These movements are called **congruence transformations**, where 'congruence' means identical and 'transformation' means change. Even though a shape has changed position (transformed), it still has the same shape and size (congruence).

- The original shape is called the **original**.
- The transformed shape is called the **image**.

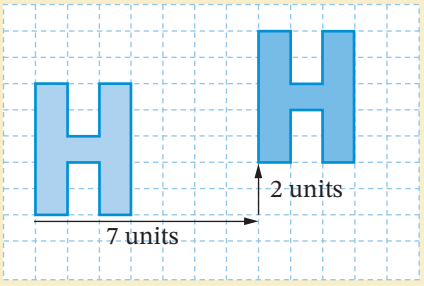
Video
Transformations

Example 1

Translate the H-shape below 7 units right and 2 units up.



SOLUTION

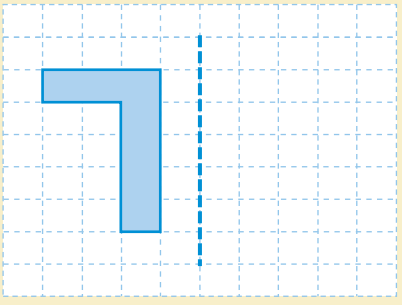


Every point on the original is shifted 7 units right and 2 units up to create the image

Video
Transformations

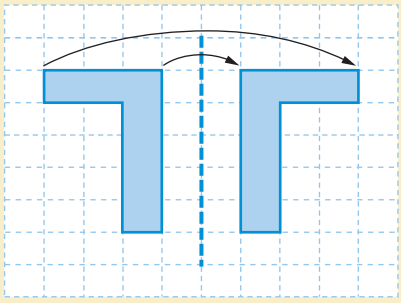
Example 2

Reflect the L-shape across the dotted line.



The dotted line is called the **line of reflection**.

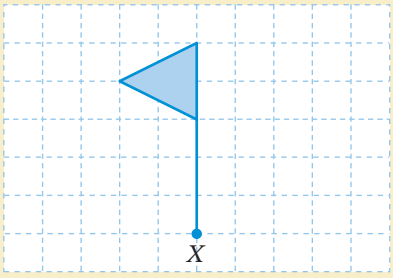
SOLUTION



The original and its mirror-image are the same distance from the line of reflection.

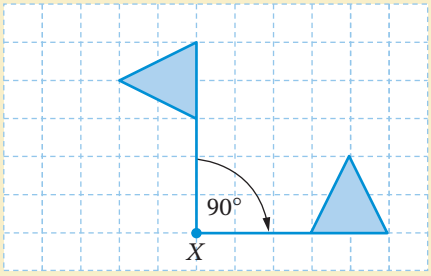
Example 3

Rotate the flag shape 90° in a clockwise direction about the point X.



X is called the **centre of rotation**.

SOLUTION



'Clockwise' is the direction the hands turn (as the numbers increase) on an analog clock, from the top towards the right. The original and image are the same distance from the centre of rotation, so the point X does not move.

Video
Transformations

EXERCISE 6.01 ANSWERS ON P. 617

Transformations

U F R C

Questions **1** to **4** of this exercise can also found in the workbook for students to write in.

Worksheet
Transformations

1 Translation, reflection or rotation?

C **a** To make a shape back-to-front to create a mirror-image

b To slide or shift a shape

Foundation Standard Complex

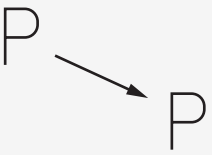
c To flip a shape

d To turn a shape upside-down

e To move a shape up, down or diagonally

f To spin a shape

g



h

i

j

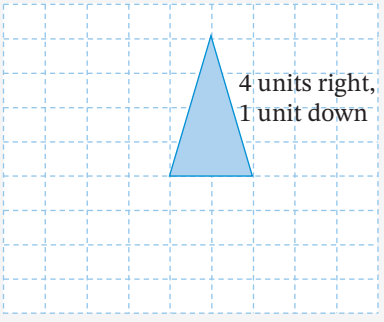
k

l

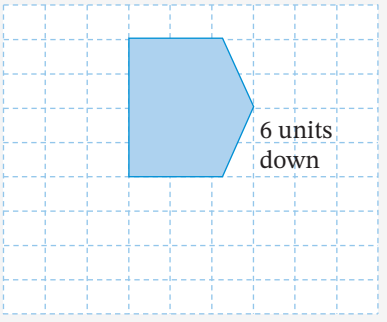
EXAMPLE 1

2 Copy each shape onto grid paper and leave enough space to translate it according to the directions given.

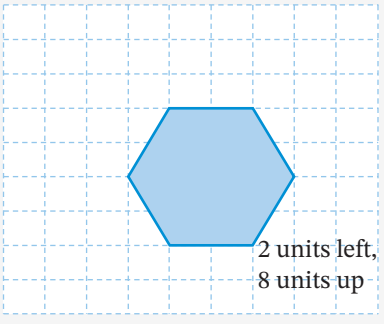
a



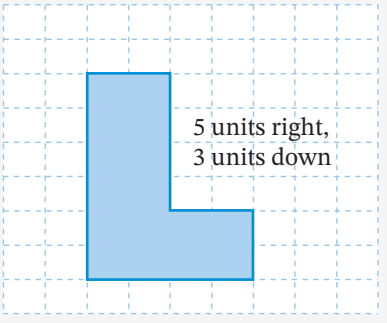
b



c



d

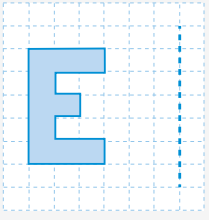


EXAMPLE 2

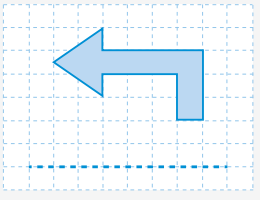
3 Copy each shape onto grid paper and leave enough space to reflect it across the dotted line. Matching points on the original and reflected shape should be the same distance from the line of reflection.

Worksheet
5 mm grid paper

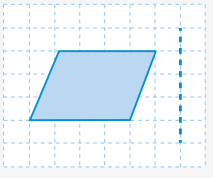
a



b

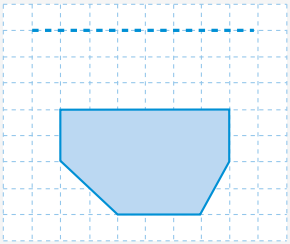


c

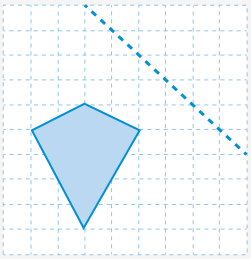


Foundation Standard Complex

d



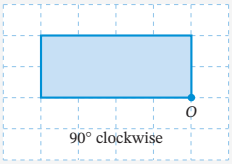
e



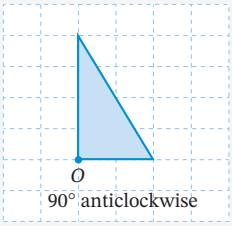
EXAMPLE 3

4 Copy each shape onto grid paper and leave enough space to rotate it by the given angle about the point O.

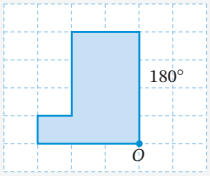
a



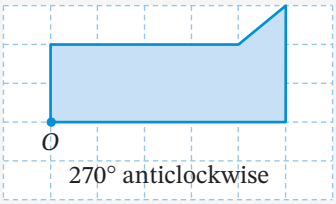
b



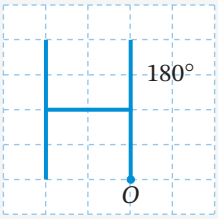
c



d



e



5 Why don't rotations of 180° need the label 'clockwise' or 'anti-clockwise'?

C

6 What clockwise rotation gives the same result as:

R **a** a 90° rotation anti-clockwise?

b a 270° rotation anti-clockwise?

c a 90° rotation clockwise 3 times?

d a 90° rotation anti-clockwise followed by a 270° rotation clockwise?

7 What single translation gives the same result as a translation of:

R **a** 3 units right, 1 unit up followed by 4 units left, 4 units up?

C **b** 2 units left, 8 units down followed by 2 units right, 5 units up?

8 Which single translation gives the same result as 4 units right, 2 units up, 5 units right, 4 units down? Select the correct answer **A**, **B**, **C** or **D**.

R C

A 9 right, 2 down

B 11 right, 2 down

C 6 right, 2 up

D 9 right, 2 up

Foundation Standard Complex

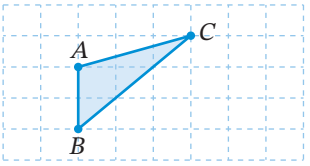
TECHNOLOGY

Transformations

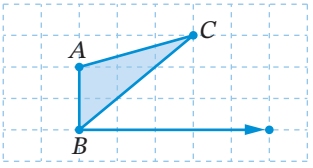
Use dynamic geometry software to explore transformations.

Translations

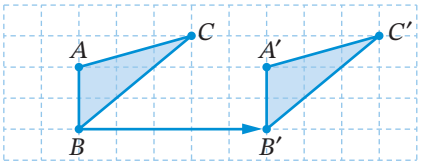
1 Draw and label a triangle $\triangle ACB$.



2 Click point B and draw a vector by clicking a point 5 units to the right as shown.

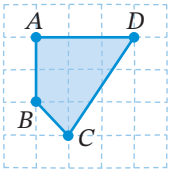


3 Use the vector to translate the entire triangle.



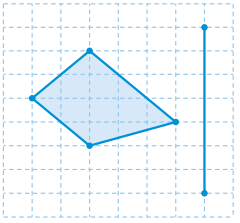
When an original shape is moved, the copy that is made is represented by letters with 'dashes' after each letter. You should now see triangle $A'\ B'\ C'$ on your screen. It is an exact copy of the original triangle ABC and has been translated 5 units to the right of the original triangle ABC.

4 Now draw, label and translate quadrilateral $ABCD$ 3 units down and 4 units to the left of point A.



Reflections

1 Draw this quadrilateral and interval.



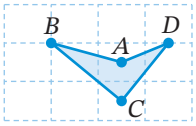
2 Use the software to reflect the quadrilateral in the line.

3 Drag any vertex of the original shape to see its reflection change.

4 Draw some other shapes and reflect each one.

Rotations

1 Draw and label this quadrilateral.



2 Click on the shape and rotate it around point D 180° clockwise.

3 Now rotate the shape around the point using different angles and choose clockwise or anticlockwise.

4 Now rotate the shape around a different point using different angles.

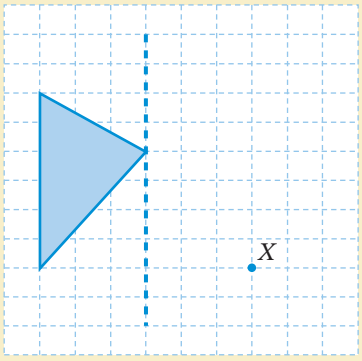
5 Now experiment by creating your own shapes and rotating them around a point using different angles.

Composite transformations 6.02

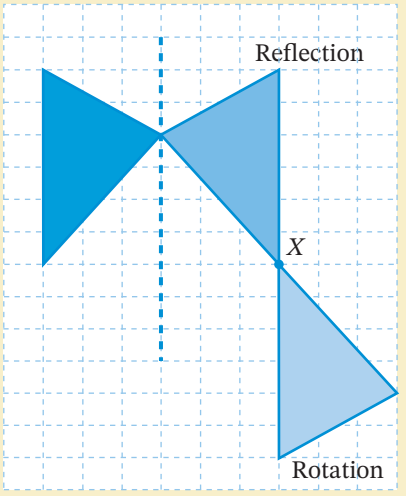
Combinations of translations, reflections and rotations can be applied one after the other. These are called **composite transformations**.

Example 4

Reflect the triangle below across the dotted line and then rotate the image 180° about X.



SOLUTION



EXERCISE 6.02 ANSWERS ON P. 618

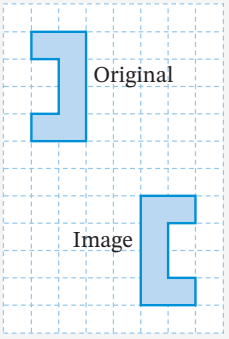
Composite transformations

U F R C

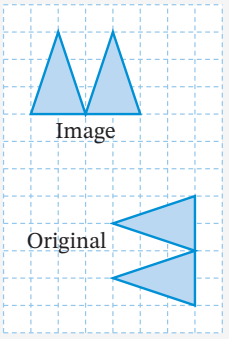
1 For each diagram below, state the combination of 2 transformations used on the original figure to form the image.

R C

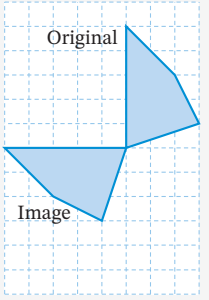
a



b



c

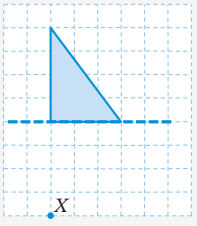


EXAMPLE 4

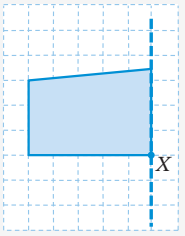
Worksheet
5 mm grid paper

2 Copy each shape on grid paper and leave enough space to draw the final image when it is reflected in the given line and then rotated 90° clockwise about the point X.

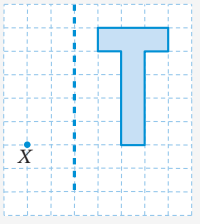
a



b



c

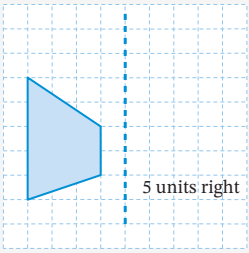


Foundation Standard Complex

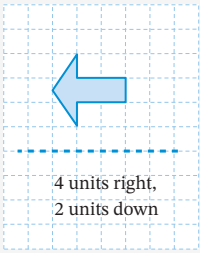
9780170465557

3 Copy each shape on grid paper and leave enough space to draw the final image when it is reflected in the given line and then translated by the given distance and direction.

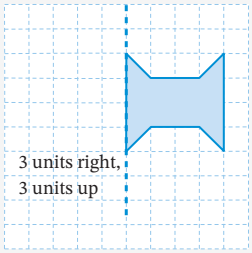
a



b

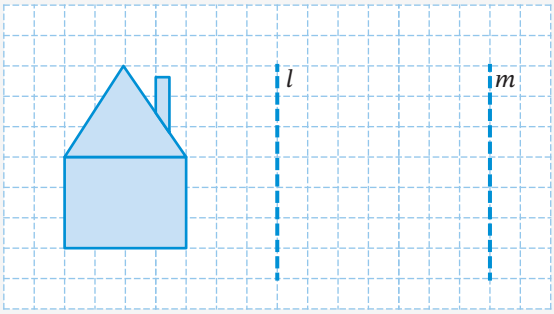


c



4 **a** Copy the house shape on grid paper and leave enough space to draw the final image when it is reflected across line l and then across line m.

R
C



b Which single transformation would give the same result as the 2 reflections in part a?

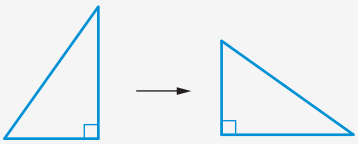
5 Each diagram below has been transformed twice.

R
C

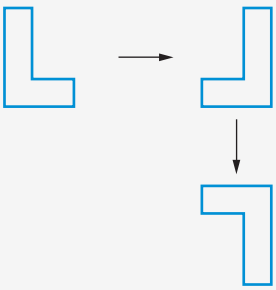
i Name the 2 transformations that have been performed.

ii Name one transformation that would give the same result as the 2 transformations.

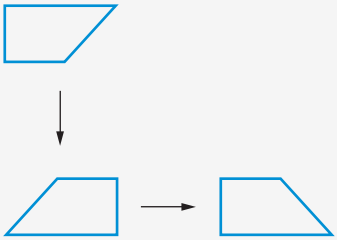
a



b



c



Foundation Standard Complex

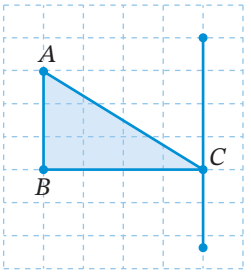
TECHNOLOGY

Composite transformations

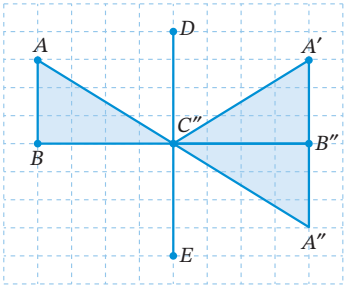
This activity will use dynamic geometry software to explore composite transformations.

Triangle transformation

1 Draw and label the triangle shown. Draw the interval as shown as a line of reflection. Use the software to reflect the triangle across the line.



2 Now reflect the reflected triangle $A'B'C'$ across its base $B'C'$.

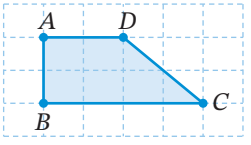


3 There is another way to do this combined transformation. Draw the triangle shown in step **1** again. Rotate the triangle about point C 180° clockwise.

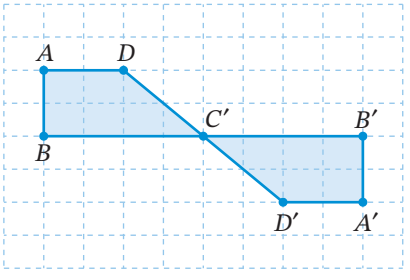
4 Compare this with the method used in steps **2** and **3** above. What do you notice?

Trapezium transformation

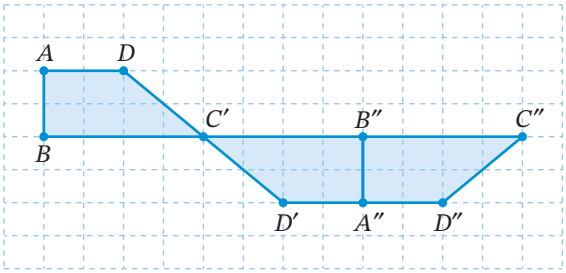
1 Draw and label this trapezium.



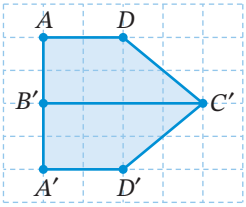
2 Rotate the trapezium clockwise, 180°.



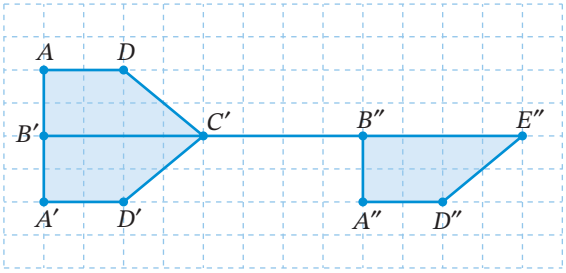
3 Now reflect the rotated trapezium $A|B|C|D|$ across the line $A|B|$.



4 Again, there is another way to do the same composite transformation. Draw the trapezium shown in step **1** again and reflect it across its base BC.



5 Translate the reflected trapezium 8 units to the right.



6 Compare this method of reflection and translation with the rotation and reflection used in steps **2** and **3**. What do you notice?

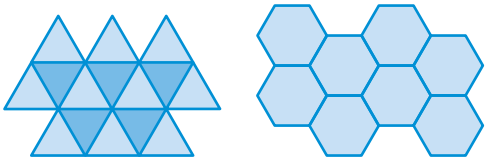
7 Now create your own composite transformation, using any combination of translation, reflection and rotation.

8 Swap your composite transformation with your group members and try to find an alternative method (or more if possible) to the composite transformation that they have created.

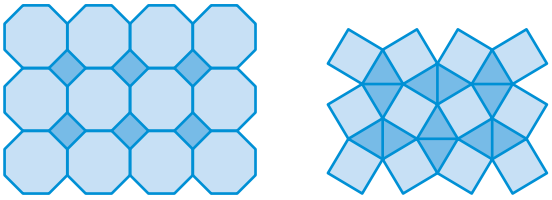
DID YOU KNOW?

Tessellations

A tessellation is a pattern of shapes that fit together perfectly, leaving no gaps between the shapes. It is like tiling an infinite floor with shapes so that there are no gaps or overlaps.



Sometimes it is necessary to use more than one shape to make sure that there are no gaps.



Maurits Cornelis Escher (1898–1972) was a Dutch graphic artist who created some interesting tessellations, such as this one.

M.C. Escher's 'Symmetry Drawing E34B' © 2022 The M.C. Escher Company-The Netherlands. All rights reserved. www.mcescher.com

Which of these shapes tessellate?

kite	**oval**	**parallelogram**	**rhombus**
hexagon	**rectangle**	**octagon**	**pentagon**

6.03 Classifying triangles

Worksheets
Properties of triangles

Constructing triangles and quadrilaterals

Puzzle
Classifying triangles

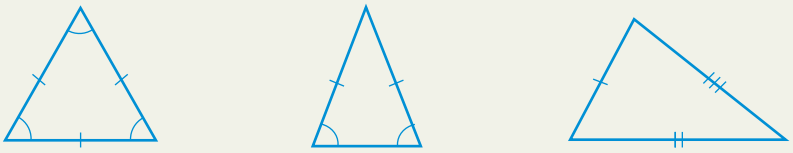
Video
Classifying triangles

Triangles can be classified in 2 ways:

- by their **sides**: **equilateral**, **isosceles** or **scalene**
- by their **angles**: **acute-angled**, **obtuse-angled** or **right-angled**

Types of triangles

Sides

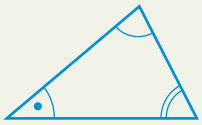


Equilateral
3 equal sides

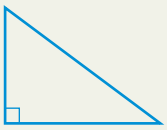
Isosceles
2 equal sides

Scalene
No equal sides

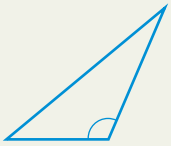
Angles



Acute-angled
3 acute angles



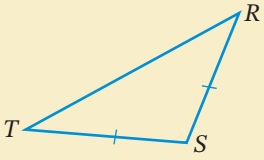
Right-angled
1 right angle



Obtuse-angled
1 obtuse angle

Example 5

Classify this triangle by sides and angles.



SOLUTION

The triangle has 2 equal sides so it is **isosceles**.

The triangle has an obtuse angle, S, so it is **obtuse-angled**.

The triangle is **isosceles** and **obtuse-angled**.

Video
Classifying triangles

EXERCISE 6.03 ANSWERS ON P. 618

Classifying triangles

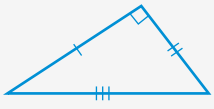
U F R C

EXAMPLE 5

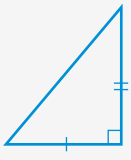
1 Classify each triangle by sides and by angles.

R C

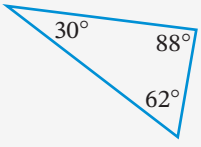
a



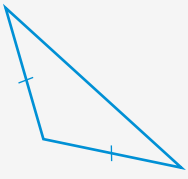
b



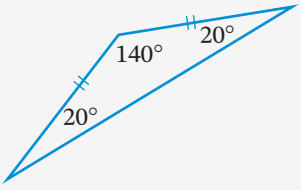
c



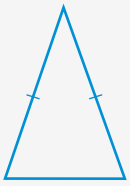
d



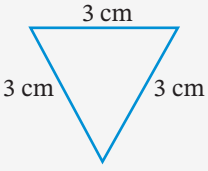
e



f

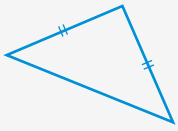


g



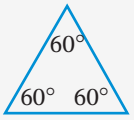
h

i



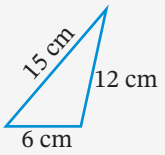
Foundation Standard Complex

j



k

l



2 Draw an example of:

R **a** a scalene triangle

b a right-angled scalene triangle

C **c** an isosceles triangle

d an equilateral triangle

e a right-angled isosceles triangle

f an acute scalene triangle

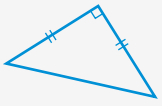
3 Copy each triangle and draw its axes of symmetry.

R **a**

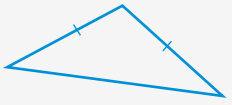
b

c

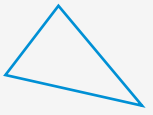
d



e



f



4 Do any triangles have rotational symmetry? Give examples to support your answer.

R

C

5 Is it possible to draw an equilateral right-angled triangle? Why?

R

C

6 The prefix 'tri' means '3'. Find the meaning of these mathematical 'tri' words.

C **a** trisect

b trilateral

c triangulate

7 Is it possible to draw a triangle with 2 obtuse angles? Why?

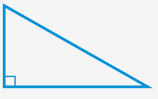
R

C

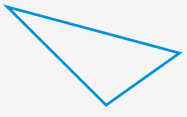
8 Which triangle is both obtuse-angled and scalene? Select **A**, **B**, **C** or **D**.

C **A**

B



C



D

9 Draw and cut out an isosceles triangle.

R **a** How many axes of symmetry does an isosceles triangle have?

C **b** By folding, mark the equal angles in an isosceles triangle.

c Describe in words where the equal angles in an isosceles triangle are.

Foundation Standard Complex

10 Draw and cut out an equilateral triangle.

R **a** How many axes of symmetry does an equilateral triangle have?

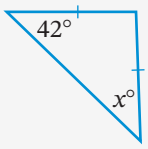
C **b** By folding, mark the equal angles in an equilateral triangle.

c Describe in words where the equal angles in an equilateral triangle are.

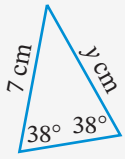
11 Find the value of the variable(s) in each diagram, giving a reason.

R C

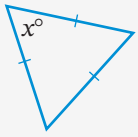
a



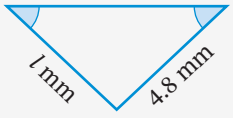
b



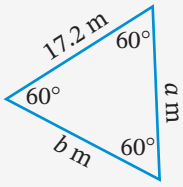
c



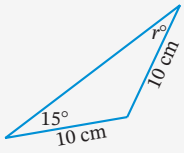
d



e



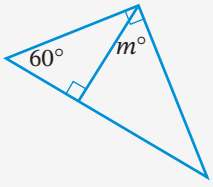
f



12 Find m. Select **A**, **B**, **C** or **D**.

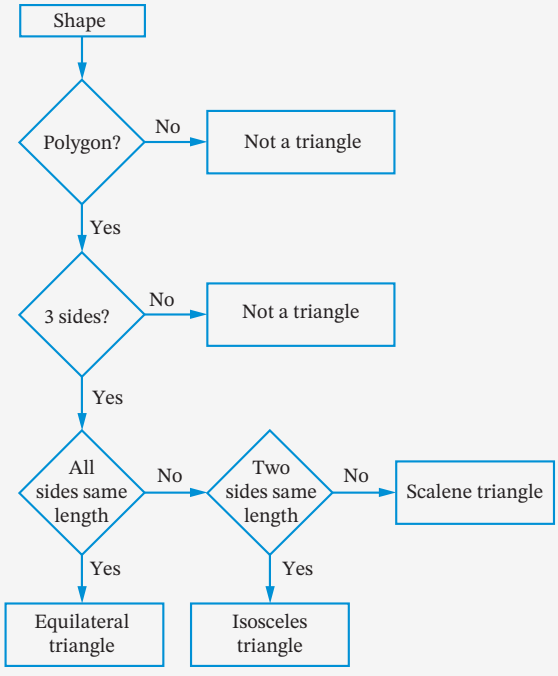
R **A** 45° **B** 60°

C 30° **D** 120°



13 Below is a flow chart that can be used for classifying triangles by length of sides.

Using the same process, design a system for classifying triangles by size of angles.



Foundation Standard Complex

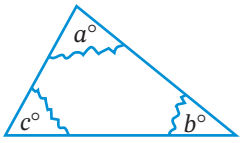
INVESTIGATION

Angle sum of a triangle

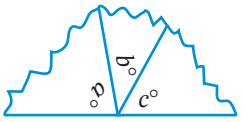
What is the sum of the 3 angles in a triangle? In groups of 2 to 4, complete this activity.

Paper-cutting activity

1 Draw a large triangle on paper, cut it out and label its 3 angles a, b and c.



2 Tear off the 3 angles and arrange them next to each other, so that their points meet.



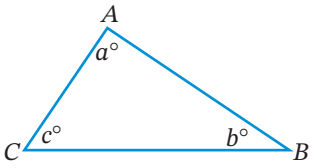
3 What type of angle do they form? How many degrees are in this type of angle?

4 To see if this works for all triangles, repeat the above steps for different triangles.

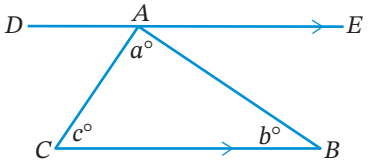
Formal proof

We can use parallel lines to prove that the angle sum of a triangle is 180°.

1 Draw any triangle ABC, with angles of size $a°$, $b°$ and $c°$.



2 Draw a line DE parallel to CB through A.



3 Which angle in $\triangle ABC$ is equal to $\angle DAC$? Why?

4 Which angle in $\triangle ABC$ is equal to $\angle EAB$? Why?

5 What is $\angle DAC + a° + \angle EAB$? Why?

6 What does this show about $a° + b° + c°$?

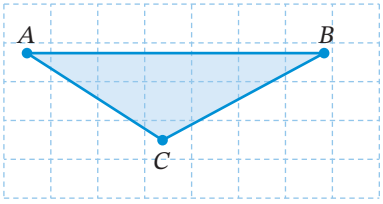
7 What does this show about the angle sum of a triangle?

8 Check by measuring with a protractor.

TECHNOLOGY

Angle sum of a triangle

Use dynamic geometry software to find the angle sum of a triangle.



1 Draw and label any triangle *ABC*.

2 Measure the size of each angle.

3 Find the angle sum of the triangle.

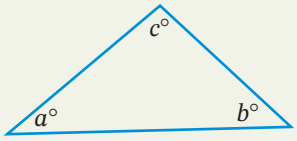
4 Move each vertex in the triangle that you have drawn and watch the size of each angle change. What is the angle sum of any triangle?

Angle sum of a triangle 6.04

Angle sum of a triangle

The **angle sum** of a triangle is 180°

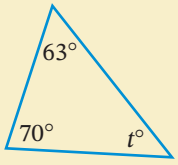
$$a + b + c = 180$$



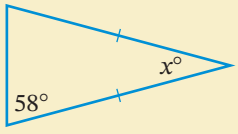
Example 6

Find the value of each variable, giving reasons.

a



b



SOLUTION

a $t + 70 + 63 = 180$ (angle sum of a triangle)

$t + 133 = 180$

$t = 180 - 133$

$= 47$

b $x + 58 + 58 = 180$ (angle sum of an isosceles triangle)

$x + 116 = 180$

$x = 180 - 116$

$= 64$

Video
Angle sum of a triangle

EXERCISE 6.04 ANSWERS ON P. 619

Angle sum of a triangle

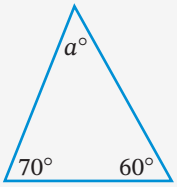
U F PS R C

EXAMPLE 6

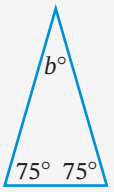
1 Find the value of each variable.

R C

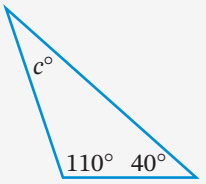
a



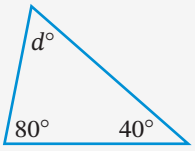
b



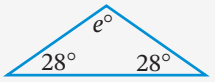
c



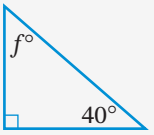
d



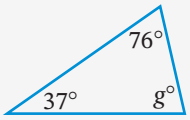
e



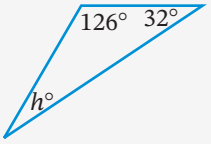
f



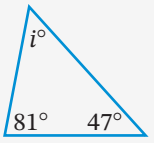
g



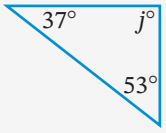
h



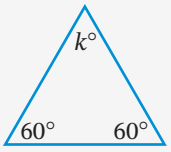
i



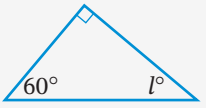
j



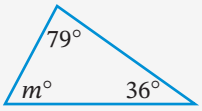
k



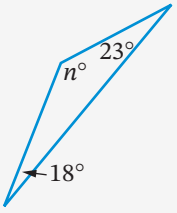
l



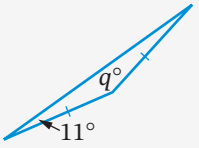
m



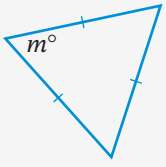
n



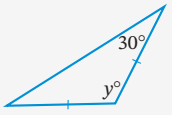
o



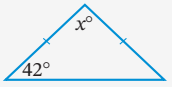
p



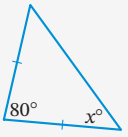
q



r



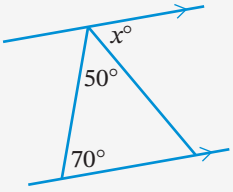
s



2 Using what you know about angles, find the value of each variable, giving reasons.

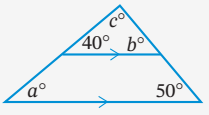
PS R C

a



b

c

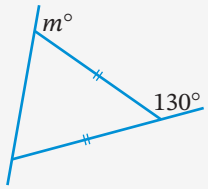


Foundation Standard Complex

9780170465557

3 What is the value of m in this diagram? Select the correct answer **A**, **B**, **C** or **D**.

PS R

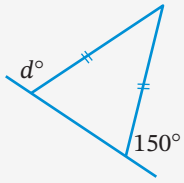


A 50 **B** 100

C 115 **D** 130

4 What is the value of d in this diagram? Select **A**, **B**, **C** or **D**.

PS R

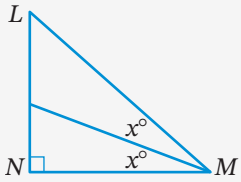


A 30 **B** 120

C 150 **D** 60

5 LMN is an isosceles right-angled triangle. What is the value of x? Select **A**, **B**, **C** or **D**.

PS R

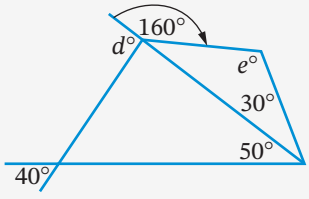


A 22.5 **B** 30

C 45 **D** 60

6 In the diagram, find the value of d and e, giving reasons.

PS R C



☆ MENTAL SKILLS 6 ANSWERS ON P. 619 Maths without calculators

Quiz Mental skills 6

Estimating answers

A quick way of estimating an answer is to round each number in the calculation.

1 Study each example.

a $631 + 280 + 51 + 43 + 96 \approx 600 + 300 + 50 + 40 + 100$

$= (600 + 300 + 100) + (50 + 40)$

$= 1000 + 90$

$= 1090$ (Actual answer = 1101)

b $55 + 132 - 34 + 17 - 78 \approx 60 + 130 - 30 + 20 - 80$

$\approx (60 + 20 - 80) + (130 - 30)$

$= 0 + 100$

≈ 100 (Actual answer = 92)

Foundation Standard Complex

c $67 \times 13 \approx 70 \times 12$

$= 840$ (Actual answer = 871)

d $78 \times 7 \approx 80 \times 7$

$= 560$ (Actual answer = 546)

e $929 \div 5 \approx 1000 \div 5$

$= 200$ (Actual answer = 185.8)

f $510 \div 24 \approx 500 \div 20$

$= 50 \div 2$

$= 25$ (Actual answer = 21.25)

2 Now estimate each answer.

a $27 + 11 + 87 + 142 + 64$

b $55 + 34 - 22 - 46 + 136$

c $684 + 903$

d $35 + 81 + 110 + 22 + 7$

e $517 - 96$

f $210 - 38 - 71 + 151 - 49$

g $766 - 353$

h 367×2

i 83×81

j 984×16

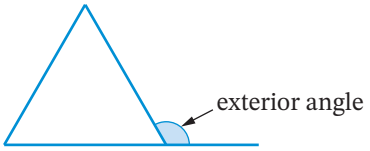
k $828 \div 3$

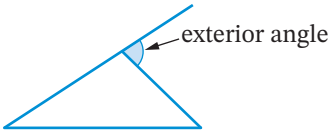
l $507 \div 7$

INVESTIGATION

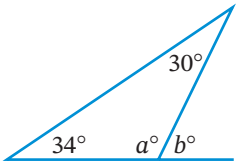
Exterior angle of a triangle

An **exterior angle** of a triangle is created by extending one side of the triangle. 'Exterior' means 'outside', while 'interior' means 'inside'.





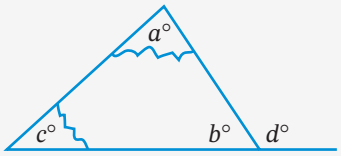
What is the relationship between the exterior angle of a triangle and the interior angles?



1. Find the value of a and b in this triangle.
2. Which is the exterior angle: a or b?
3. How is the exterior angle related to **2** of the (interior) angles of this triangle?
4. Copy and complete: An exterior angle of a triangle is equal to the _____ of the interior opposite _____.

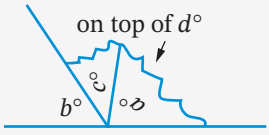
Paper-cutting activity

1 Draw a large triangle on paper and label its 3 angles $a°$, $b°$ and $c°$.



2 Extend one of the sides of the triangle and label the exterior angle $d°$. Cut out the triangle as well as angle $d°$.

3 Tear off the 2 angles $a°$ and $c°$ that are not adjacent to $d°$ and arrange them next to each other on top of angle $d°$.



4 Do the 2 interior angles fit exactly on top of the exterior angle?

5 Check by measuring with a protractor.

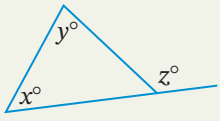
6 To see if this works for all triangles, repeat the above steps for different triangles.

Exterior angle of a triangle 6.05

Exterior angle of a triangle

An **exterior angle** of a triangle is equal to the sum of the 2 interior opposite angles.

$z = x + y$



Worksheet Triangle geometry

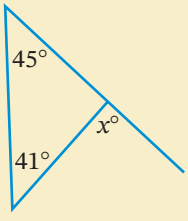
Puzzle Angles in triangles

Video Exterior angle of a triangle

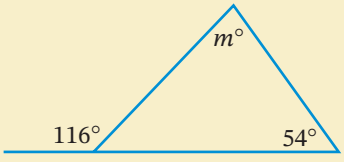
Example 7

Find the value of each variable, giving reasons.

a



b



SOLUTION

a $x = 45 + 41$ (exterior angle of a triangle)

$= 86$

b $m + 54 = 116$ (exterior angle of a triangle)

$m = 116 - 54$

$= 62$

EXERCISE 6.05 ANSWERS ON P. 619

Exterior angle of a triangle

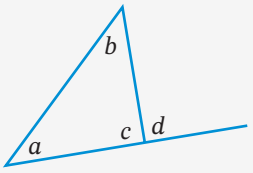
U F R C

1 For each triangle below, name:

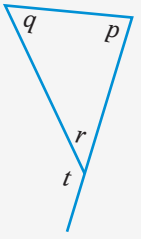
R **i** the exterior angle

C **ii** the 2 interior angles opposite the exterior angle.

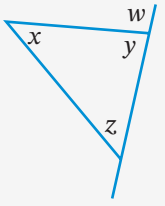
a



b



c

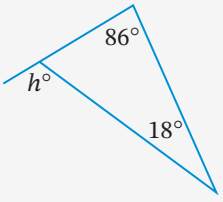


EXAMPLE 7

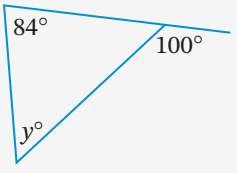
2 Find the value of each variable, giving reasons.

R C

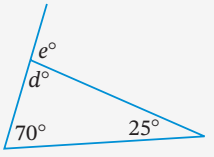
a



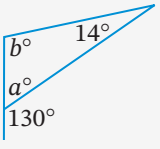
b



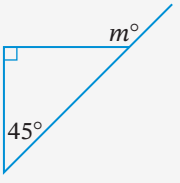
c



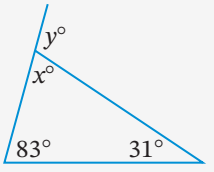
d



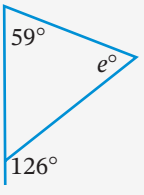
e



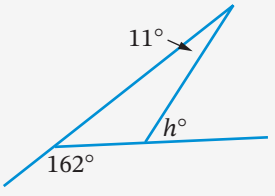
f



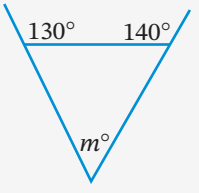
g



h



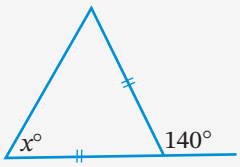
i



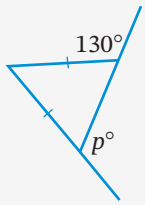
Foundation Standard Complex

9780170465557

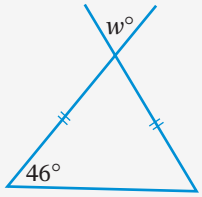
j



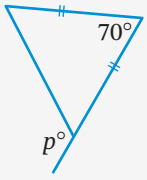
k



l



3 What is the value of p? Select the correct answer **A**, **B**, **C** or **D**.



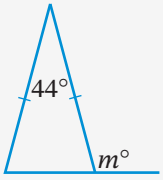
A 110 **B** 125

C 55 **D** 140

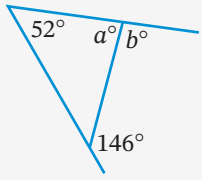
4 Find the value of each variable, giving reasons.

PS R C

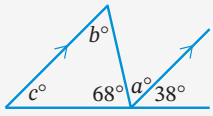
a



b

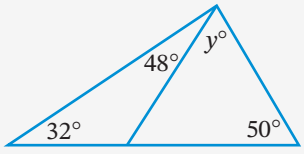


c



5 Find the value of y. Select **A**, **B**, **C** or **D**.

R



A 80 **B** 50

C 130 **D** 230

6 **Another exterior angle proof**

R C

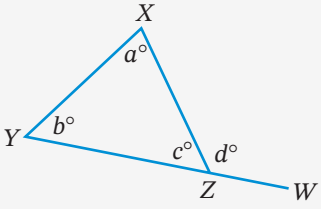
Copy and complete the missing reasons in this proof. Consider any triangle XYZ in which the angles are $a°$, $b°$ and $c°$. Extend the interval YZ to the point W.

$a° + b° + c° = 180°$ because _____.

$\therefore a° + b° = 180° - c°$

But $d° = 180° - c°$ because _____.

$\therefore d° = a° + b°$



Foundation Standard Complex

6.06 Classifying quadrilaterals

Worksheets
Classifying quadrilaterals

Constructing triangles and quadrilaterals

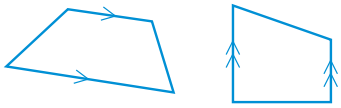
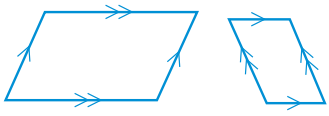
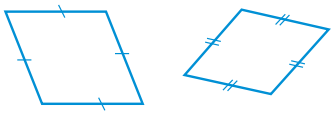
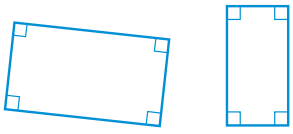
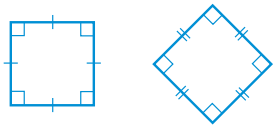
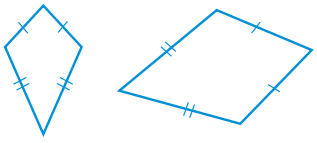
Puzzles
Constructions group clues

What shape is it?'

Singing in the car

Skillsheet
Naming shapes

A **quadrilateral** is any shape with 4 sides, but there are 6 special quadrilaterals described below.

Name	A quadrilateral with:	Diagram
Trapezium	1 pair of opposite sides parallel	
Parallelogram	2 pairs of opposite sides parallel	
Rhombus (or diamond)	4 equal sides	
Rectangle	4 right angles	
Square	4 equal sides and 4 right angles	
Kite	2 pairs of adjacent sides equal	

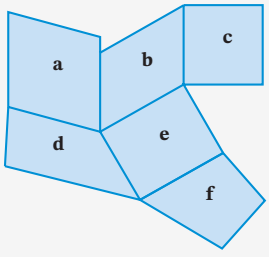
EXERCISE 6.06 ANSWERS ON P. 619

Classifying quadrilaterals

U F R C

1 Name each quadrilateral in this diagram.

C



Foundation Standard Complex

2 Find what the prefix 'quad' means. Find other words beginning with 'quad' and their meanings. (C)

3 Draw each of the special quadrilaterals and mark all axes of symmetry.

a rectangle **b** square **c** parallelogram
d rhombus **e** trapezium **f** kite

4 Copy and complete this table. (C)

Quadrilateral	Number of axes of symmetry	Order of rotational symmetry
	4	
Parallelogram		2
Trapezium		No rotational symmetry
Rhombus		2
	1	
	2	

5 Name all quadrilaterals that have:

(R) **a** all angles equal to 90° **b** opposite sides parallel
(C) **c** opposite sides equal **d** 1 pair of parallel sides
e 4 equal sides **f** 2 pairs of adjacent sides equal
g no axes of symmetry **h** rotational symmetry of order 2

6 Name the quadrilateral that has opposite sides equal and only 2 axes of symmetry. Select the correct answer **A**, **B**, **C** or **D**. (R) (C)

A rectangle **B** square **C** parallelogram **D** kite

TECHNOLOGY

Measuring quadrilaterals

In this activity, we use dynamic geometry software to construct quadrilaterals.

Square

1 Use the Regular polygon function to draw and label a square of length 7.5 cm.

2 Construct the 2 diagonals for the square. Measure the length of each diagonal. What do you notice?

3 Measure each angle of the square. What do you notice?

4 Draw another square with a side length of 9 cm. Repeat steps **2** and **3**. List 2 properties of the square.

Rectangle

1 Use the Polygon function to draw and label a rectangle with sides 6 cm by 3 cm.

Worksheet Constructing triangles and quadrilaterals

Foundation Standard Complex

2 Measure the sides and angles of the rectangle to check your accuracy.

3 Draw another rectangle that is 5 cm by 8.4 cm. Measure the sides and angles.

4 Measure the lengths of the 2 diagonals for each rectangle. What do you notice?

5 Copy and complete this property: The _____ in a rectangle are _____.

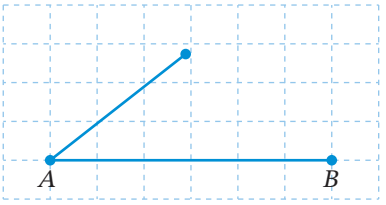
6 For the diagonals of the first rectangle, measure the distances to the point where they cross. Do they **bisect** each other or not? Repeat for the second rectangle.

7 Copy and complete: The _____ of a rectangle _____ each other.

Parallelogram

1 Draw and label an interval *AB* 6 cm long. Label the vertices.

2 From point *A*, draw an interval of 4 cm. Label the new point, *C*.



3 Draw a line parallel to *AB* through point *C*.

4 Create the 4th vertex of the parallelogram. Label the vertex *D*.

5 Measure the length of each side of the parallelogram. What do you notice?

6 Copy and complete: The _____ sides of a parallelogram are _____.

7 Measure the size of $\angle CAB$ and $\angle CDB$. Repeat for $\angle ACD$ and $\angle ABD$. What do you notice?

8 Copy and complete: The _____ angles of a parallelogram are _____.

9 Draw and measure the lengths of the diagonals of the parallelogram. Are they equal?

10 Copy and complete: The diagonals of a parallelogram are _____.

11 Do the diagonals of a parallelogram bisect each other or not?

12 Copy and complete: The diagonals of a parallelogram _____ each other.

13 Drag any vertices of the parallelogram that you can. Is it possible to draw other parallelograms with the same dimensions, 6 cm by 4 cm? What do you notice?

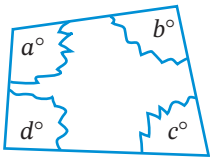
14 Now accurately construct other quadrilaterals such as a rhombus, kite or trapezium.

INVESTIGATION

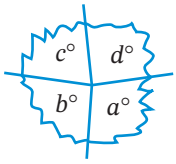
Angle sum of a quadrilateral

What is the sum of the 4 angles in a quadrilateral? In groups of 2 to 4, complete the following activity.

1. Draw any large quadrilateral on paper, cut it out and label its 4 angles **A**, **B**, **C** or **D**.

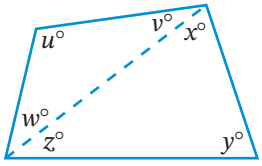


2. Tear off the 4 angles and arrange them so that their points meet.
3. What type of angle do they form? How many degrees are in this angle?
4. To see if this works for all quadrilaterals, repeat the above steps for different quadrilaterals.



The angle sum of a quadrilateral 6.07

Any quadrilateral can be divided into 2 triangles along one of its **diagonals**. Because the angles in each triangle add to 180°, the angles in both triangles add to $2 \times 180° = 360°$.



$u° + v° + w° = 180°$ and $x° + y° + z° = 180°$

$\therefore$ Angle sum of a quadrilateral $= 180° + 180°$

$= 360°$

Puzzles
Find the unknown angle

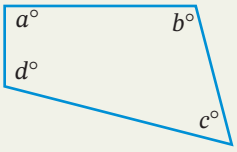
Mixed angle problems

Triangles and quadrilaterals

Angle sum of a quadrilateral

The angle sum of a quadrilateral is 360°.

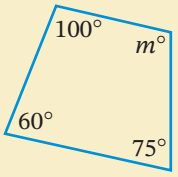
$a + b + c + d = 360$



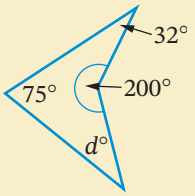
Example 8

Find the value of each variable, giving reasons.

a



b



SOLUTION

a $m + 75 + 60 + 100 = 360$ (angle sum of a quadrilateral)

$m + 235 = 360$

$m = 360 - 235$

$= 125$

b $d + 75 + 32 + 200 = 360$ (angle sum of a quadrilateral)

$d + 307 = 360$

$d = 360 - 307$

$= 53$

EXERCISE 6.07 ANSWERS ON P. 619

Angle sum of a quadrilateral

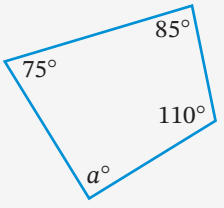
U F PS R C

EXAMPLE 8

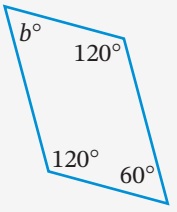
1 Find the value of each variable.

R C

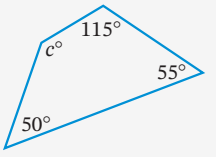
a



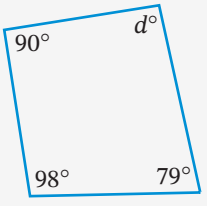
b



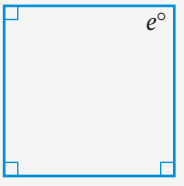
c



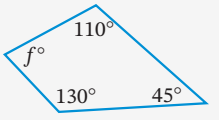
d



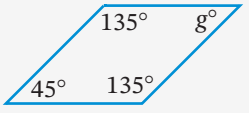
e



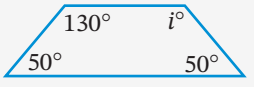
f



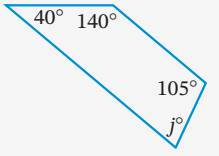
g



h

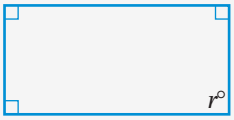


i

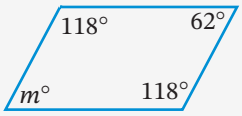


Foundation Standard Complex

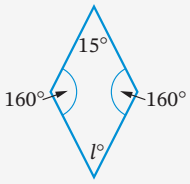
j



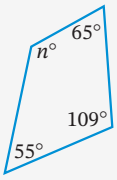
k



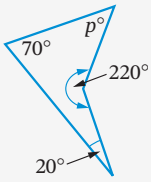
l



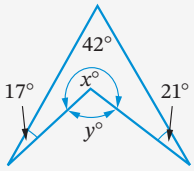
m



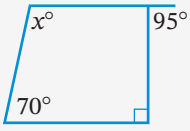
n



o



2 What is the value of x? Select the correct answer **A**, **B**, **C** or **D**.



R

A 85 **B** 95

C 105 **D** 115

3 Sketch a rhombus. If one of the angles is 75°, what are the sizes of the other 3 angles? Select **A**, **B**, **C** or **D**.

R

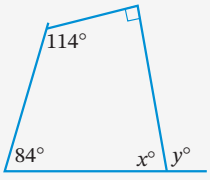
A 75°, 105°, 105° **B** 95°, 95°, 95°

C 75°, 75°, 75° **D** 75°, 15°, 15°

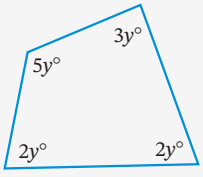
4 Find the value of each variable, giving reasons.

PS R C

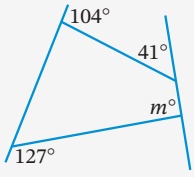
a



b



c



5 Robert's favourite quadrilateral is equiangular, meaning that all of its angles are equal.

R **a** What is the size of each angle?

C **b** What is the most general name for Robert's favourite quadrilateral?

Properties of quadrilaterals 6.08

EXERCISE 6.08 ANSWERS ON P. 619

Properties of quadrilaterals

U F R C

Questions **1** to **5** can also be found on the worksheet *Properties of quadrilaterals.*

1 Draw and cut out a trapezium, parallelogram, rhombus, rectangle, square and kite or use the ones supplied on the worksheet.

Worksheets
Properties of quadrilaterals

Diagonal properties of quadrilaterals

Foundation Standard Complex

Worksheet Always, sometimes, never true

Video Classifying quadrilaterals

Puzzle Geometry crossword

2 **a** By folding and measuring with a ruler, protractor and set square, discover the properties of each quadrilateral from question **1**. Copy the table or use the one supplied on the worksheet.

	Trapezium	Parallelogram	Rhombus	Rectangle	Square	Kite
Opposite sides are equal						
Opposite sides are parallel						
Opposite angles are equal						
All angles are 90°						
Diagonals are equal						
Number of axes of symmetry						
Order of rotational symmetry						

R C Complete the table by placing a tick in the appropriate place if the property is true for that quadrilateral and writing a number for the 'Number of axes of symmetry' and 'Order of rotational symmetry' rows.

b Ask your teacher to check your results.

c You will notice that there were no ticks for the kite. Write 2 properties of the kite (that is, 2 things that are always true about its sides, angles or diagonals).

3 Draw each quadrilateral and mark all axes of symmetry,

4 List the quadrilaterals that have rotational symmetry and mark the centre of symmetry, *O*, for each one.

5 **a** Draw the diagonals of each quadrilateral from question **1**.

b By measuring with a ruler, protractor and set square, discover the properties of the diagonals of each quadrilateral as listed in the table below. Copy this table or use the one on the worksheet. Complete the table by placing ticks for each shape.

	Trapezium	Parallelogram	Rhombus	Rectangle	Square	Kite
Diagonals are equal						
Diagonals bisect each other						
Diagonals intersect at right angles						
Diagonals bisect angles of quadrilateral						

R C **c** Ask your teacher to check your results.

Foundation Standard Complex

9780170465557

6 I am a quadrilateral with equal and parallel opposite sides. My diagonals are equal and I have 4 axes of symmetry. Which quadrilateral am I? Select **A**, **B**, **C** or **D**.

R C

A rectangle **B** square **C** parallelogram **D** rhombus

7 Which quadrilateral am I? (There may be more than one answer.)

R C

a My diagonals are equal.

b My diagonals bisect each other.

c I have 4 right angles.

d I have 2 pairs of parallel opposite sides.

e I have rotational symmetry, but no axes of symmetry.

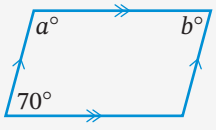
f My diagonals cross each other at right angles.

g My diagonals bisect each other at right angles

h My diagonals are equal and bisect each other.

8 Copy and complete the blanks to find the values of a and b.

R C



$a + 70 = 180$ (_____ angles on _____ lines)

$a =$ _____

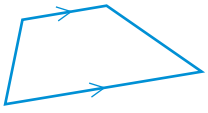
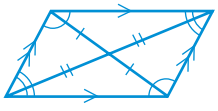
$b =$ _____ (opposite _____ of a parallelogram)

9 **a** Does a square have all the properties of a rectangle? Why?

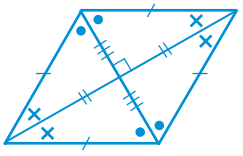
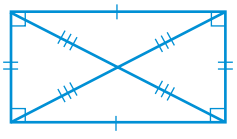
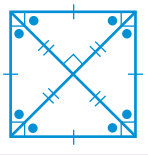
R C

b Does a rhombus have all the properties of a parallelogram? Why?

10 Copy and complete this table of properties of quadrilaterals.

Shape	Properties
Trapezium 	• One pair of _____ sides • No axes of symmetry
Kite	• _____ pairs of adjacent sides are equal • One pair of opposite angles are _____ • Has _____ axis of symmetry • Diagonals intersect at _____ angles
Parallelogram 	• _____ sides are equal and parallel • Opposite angles are _____ • No axes of symmetry • Diagonals _____ each other

Foundation Standard Complex

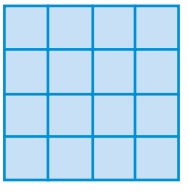
Rhombus 	• All 4 sides are_____ • _____ sides are parallel • Opposite angles are _____ • Has _____ axes of symmetry • _____ bisect at right angles • Diagonals bisect the _____ of the rhombus
Rectangle 	• All 4 angles measure _____ • Opposite sides are _____ and _____ • Has 2 axes of _____ • Diagonals are _____ • _____ bisect each other
Square 	• All 4 sides are _____ • All 4 angles measure _____ • Has _____ axes of symmetry • Diagonals are equal and _____ each other at right angles • _____ bisect the angles of the square

INVESTIGATION

Puzzle
Shapes puzzle 1

Shape puzzles

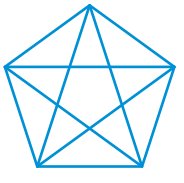
This activity can also be downloaded as a puzzle sheet: 'Shapes puzzle 1'.



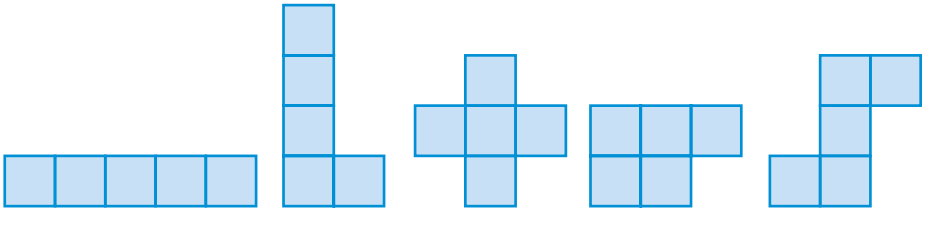
1 **a** How many squares can you find in this diagram? The answer is not 16!

b How many rectangles can you find?

2 Can you trace this shape without going over any line twice and without lifting your pencil from the paper?



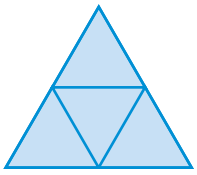
3 There are 12 different ways to join 5 squares edge to edge. These shapes are called pentominoes. Here are 5 of them. Draw the other 7.



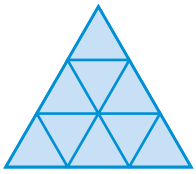
Foundation Standard Complex

4 How many triangles can you find in each diagram?

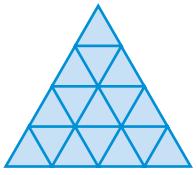
a



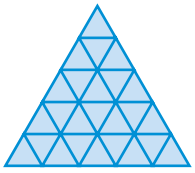
b



c



d



Polygons and polyhedra 6.09

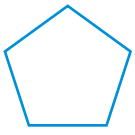
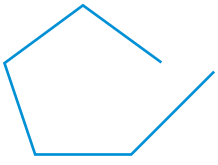
Polygons

Triangles and quadrilaterals are examples of **polygons.**

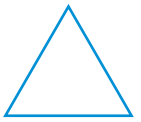
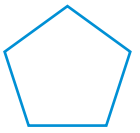
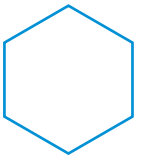
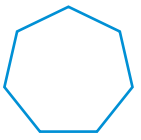
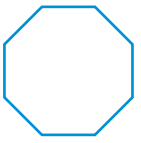
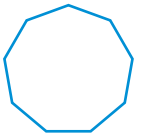
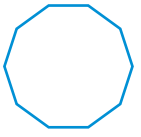
A **polygon** is a flat closed shape with straight sides. A flat shape has **2 dimensions** (2D)**:** length and width.

Videos
Tessellated designs

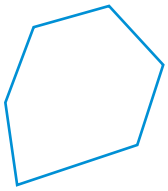
Polyhedra: Platonic solids

	This pentagon is a polygon as it is a closed shape made of 5 straight sides.
	This shape is not a polygon as it is not closed.
	This shape is not a polygon because all sides are not straight. Some are curved. A circle and ellipse (oval) are not polygons.

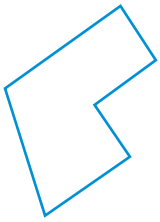
The names of the polygons with 3 to 10 sides are shown in the table below. If the polygon has **equal sides** and **equal angles,** then it is called a **regular polygon**.

Name	Number of sides	Regular polygon
Triangle	3	 equilateral triangle
Quadrilateral	4	square
Pentagon	5	 regular pentagon
Hexagon	6	 regular hexagon
Heptagon or septagon	7	 regular heptagon
Octagon	8	 regular octagon
Nonagon	9	 regular nonagon
Decagon	10	 regular decagon

Polygons can also be **concave** or **convex**.



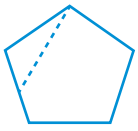
convex hexagon



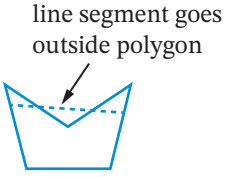
concave hexagon

Both shapes above are hexagons because they have 6 sides, but the convex hexagon has sides and vertices (corners) that all point outwards, while the concave hexagon has some sides and vertices that point inwards.

For a convex polygon, if you draw a line from any side to another side, the entire line will remain inside the polygon.



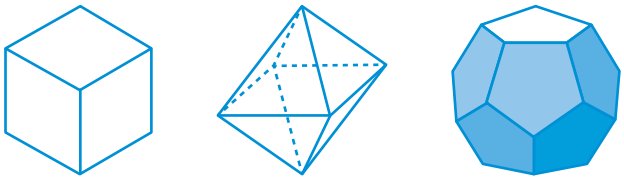
Convex



Concave, not convex

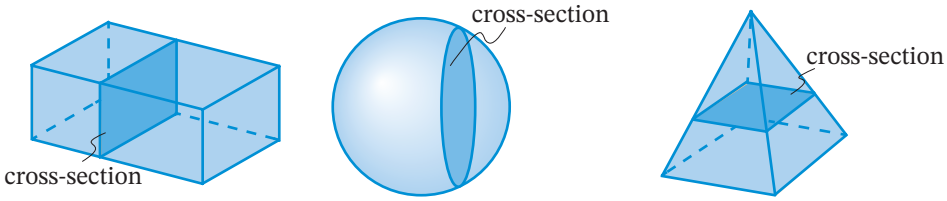
Polyhedra

While a polygon is a flat shape with straight sides, a **polyhedron** is a solid shape with flat faces. Every face is a polygon. A solid shape has **3 dimensions** (3D)**:** length, width and height. The plural of polyhedron is **polyhedra**.



Prisms and **pyramids** are examples of polyhedra. A cylinder is not a polyhedron because it has a curved face.

A **cross-section** of a solid is a 'slice' of the solid, cut *across* it, parallel to its end faces, rather than along it. These diagrams show cross-sections of a rectangular prism, a sphere and a square pyramid.



Rectangular prism **Sphere** **Square pyramid**

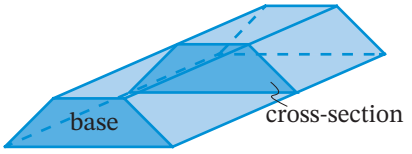
Note that the cross-sections of a rectangular prism are identical rectangles, the cross-sections of a sphere are circles (but of different sizes) and the cross-sections of a square pyramid are squares (again, of different sizes).

Prisms

If a solid has the same (uniform) cross-section along its length, and each cross-section has straight sides (is a polygon), then the solid is called a **prism**. So, from the above diagrams:

- a rectangular prism is a prism (its cross-sections are uniform rectangles)
- a sphere is not a prism (its cross-sections are not uniform)
- a square pyramid is not a prism (its cross-sections are not uniform)

The end faces of a prism are called its **base**. Prisms take their names from their base. For example, this is a **trapezoidal prism** because its cross-sections are all trapeziums, identical and parallel to its base.



Trapezoidal prism

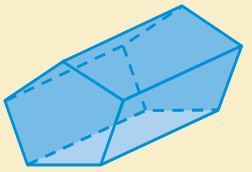
Example 9

a Draw a cross-section of the prism shown.

b What shape is the prism's base?

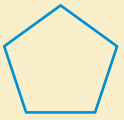
c What shape are the side faces?

d What is the name of this prism?



SOLUTION

a



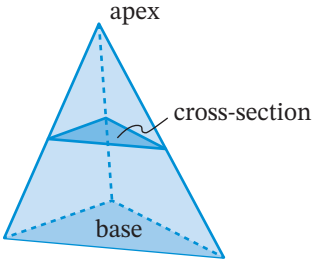
b The base is a pentagon.

c The side faces are all rectangles.

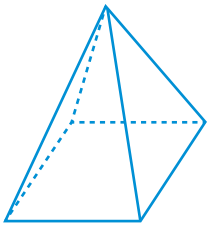
d This is a pentagonal prism.

Pyramids

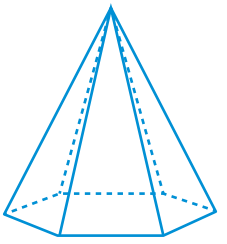
A **pyramid** has a polygon for the base and then the edges from each vertex of the polygon meet at a single point called the pyramid's **apex**. A pyramid's cross-sections have the same shape as the base but are of different sizes. The side faces of a pyramid are all triangles.



Triangular pyramid



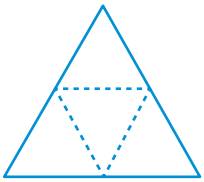
Square pyramid



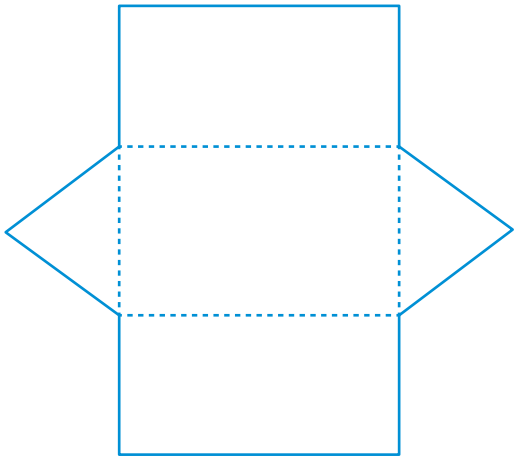
Hexagonal pyramid

Nets of solids

A **net** of a solid shape is a flat arrangement of shapes that can be folded and stuck together to make the solid shape. It is a 2D representation of a 3D shape.



Net of triangular pyramid



Net of triangular prism

EXERCISE 6.09 ANSWERS ON P. 620

Polygons and polyhedra

U F R C

1 Draw each polygon.

a pentagon **b** equilateral triangle **c** hexagon

2 State how many sides each polygon has.

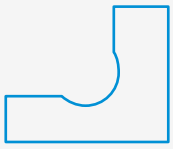
R **a** rectangle **b** octagon **c** triangle
d hexagon **e** quadrilateral **f** pentagon

3 State whether each shape is a polygon.

R C

a

b

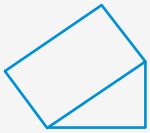


c

d

e

f



Foundation Standard Complex

4 Draw an irregular pentagon.

R

C

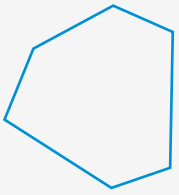
5 Copy each polygon and write its correct name from this list.

R hexagon nonagon heptagon decagon

C octagon triangle quadrilateral pentagon

Also, state if each polygon is **regular** or **irregular**, **convex** or **non-convex**.

a



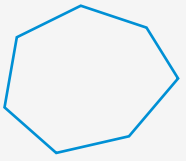
b

c

d

e

f



g

h

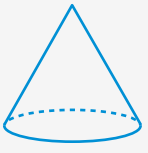
6 State whether each shape is a polyhedron.

R

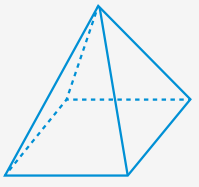
C

a

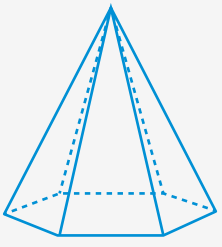
b



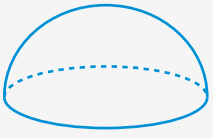
c



d



e



f

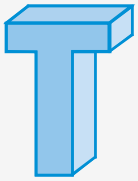
Foundation Standard Complex

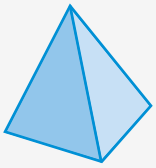
7 State whether each solid is a prism.

R C

a

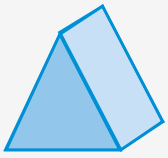
b

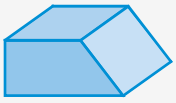
c 

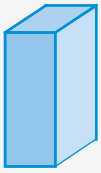
d 

e

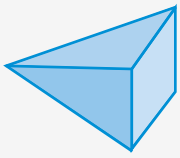
f

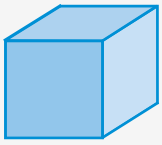
g 

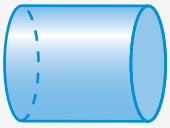
h 

i 

j

k 

l 

m 

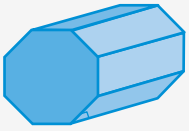
n

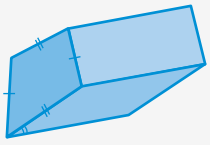
8 For each prism:

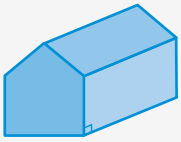
R C

i draw its cross-section **ii** write the name of the prism

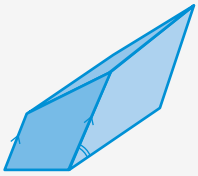
EXAMPLE 9

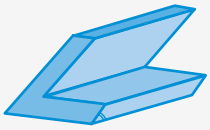
a 

b 

c 

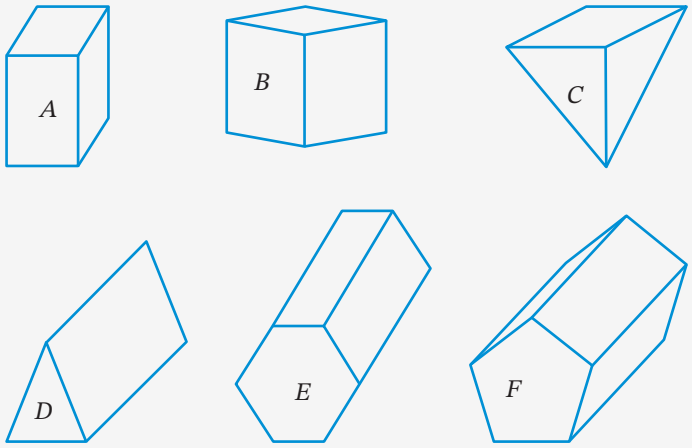
d

e 

f 

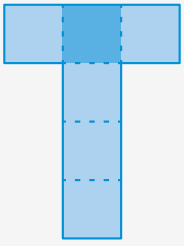
Foundation Standard Complex

9 Name each solid.

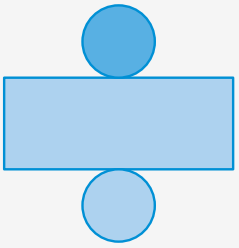


10 **a** Copy or trace the nets below, or print out the worksheet 'Nets of solids 1'. For best results, paste them on cardboard.

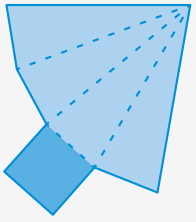
i



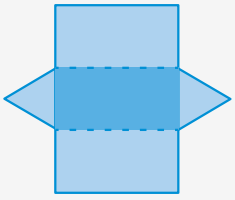
ii



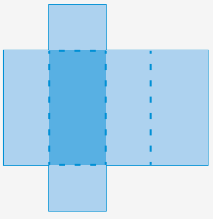
iii



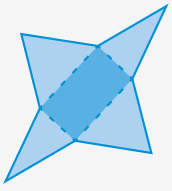
iv



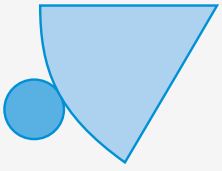
v



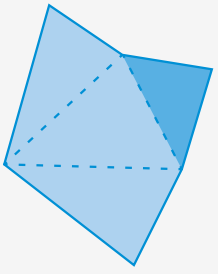
vi



vii



viii



b Cut out each net along its boundary.

c Fold each net along the dotted lines to make a solid.

d Write the name of each solid on one of the paler faces.

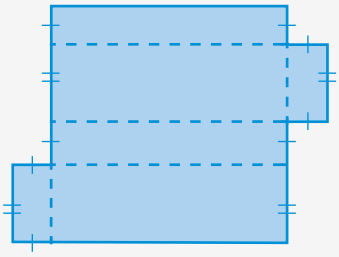
e Unfold each net and paste it into your workbook on its dark face.

Foundation Standard Complex

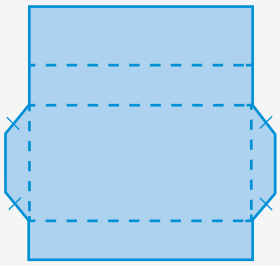
11 Name the solid that can be created from each net. Choose from the list below.

cube | cylinder | rectangular prism
square pyramid | triangular prism | triangular pyramid

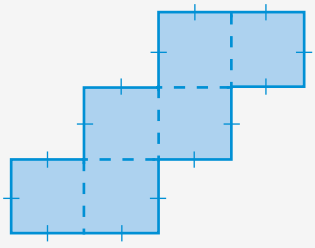
a



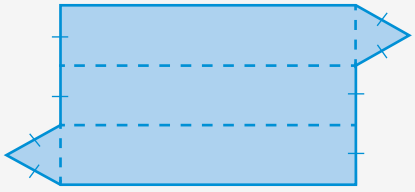
b



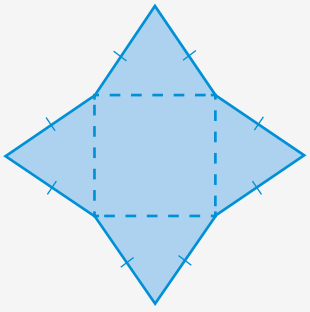
c



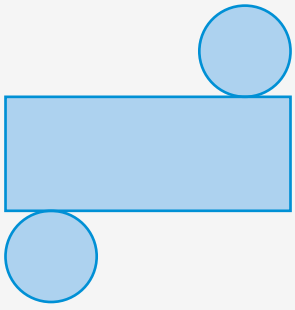
d



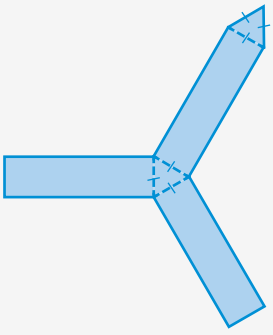
e



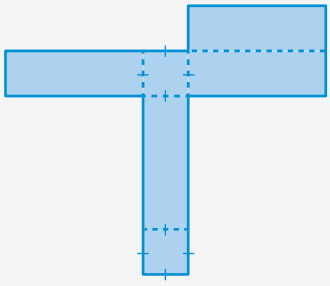
f



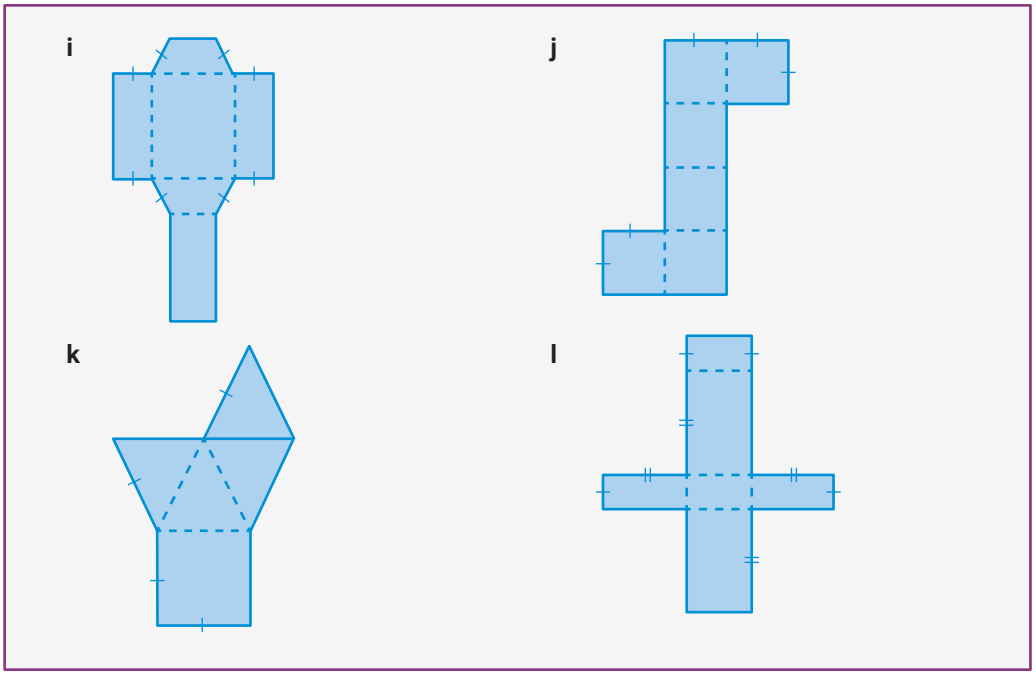
g



h



Foundation | Standard | Complex



6.10 Drawing solids

Videos
Escher and the endless staircase

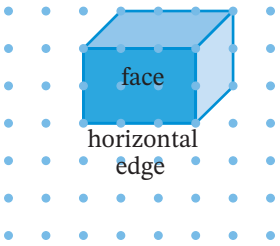
Perspective: Dazzle camouflage

When a 3D (solid) shape is drawn, it is necessary to use dotted lines or shading to show depth. 3D shapes can be drawn using a variety of methods, including isometric drawing and oblique drawing.

In an **oblique drawing**:

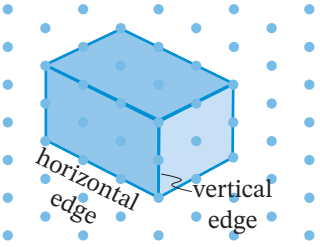
oblique means 'at an angle', neither parallel nor perpendicular

- the front or side view is drawn, where horizontal edges stay horizontal
- vertical edges stay vertical
- horizontal edges drawn backward are at an angle of 45° and are half the length of the original measurement



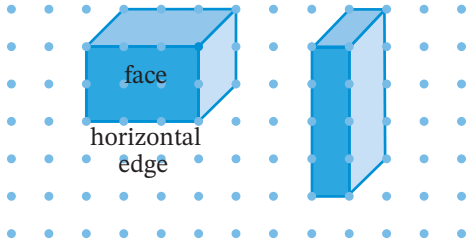
In an **isometric drawing**:

- one vertical edge is shown up front
- vertical edges stay vertical
- horizontal edges are drawn at an angle of 30°

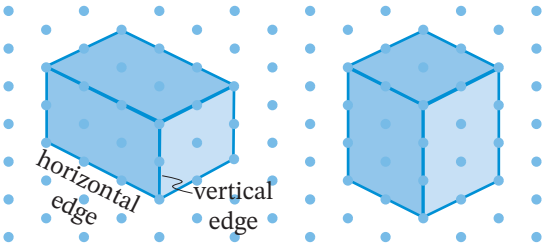


Foundation Standard Complex

These rectangular prisms are drawn on **square dot paper**, a square grid of dots.



These rectangular prisms are drawn on **isometric dot paper**, a '3D' triangular grid of dots arranged as equilateral triangles. A 2D diagram of a 3D figure looks more natural when drawn on isometric paper.



Example 10

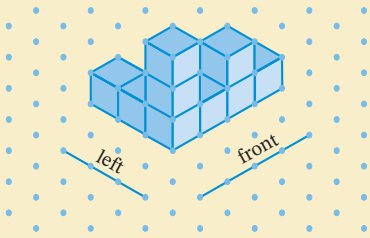
For the solid shown, assume that there are no hidden cubes. Draw:

a the front view

b the left view

c the top view

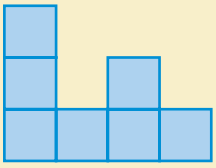
It may help your drawing if you built these solids using centicubes first.



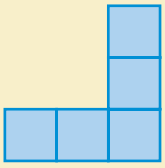
SOLUTION

For each view, there will be some cubes hidden behind the cubes closest to you.

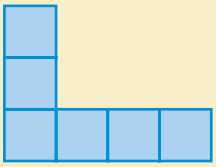
a Viewed from the front



b Viewed from the left



c Viewed from the top

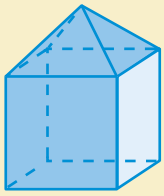


Example 11

For the solid shown, draw:

a the front view

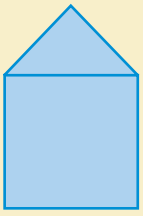
b the top view



front

SOLUTION

a Viewed from the front



b Viewed from the top

EXERCISE 6.10 ANSWERS ON P. 621

Drawing solids

U F R C

Worksheets
Square dot paper

Isometric dot paper

For this exercise you will need square dot paper, isometric dot paper and centicubes. Both types of dot paper can be downloaded. You can also draw shapes using dynamic geometry software, using a square grid or an isometric grid.

1 Draw each prism and shade its base.

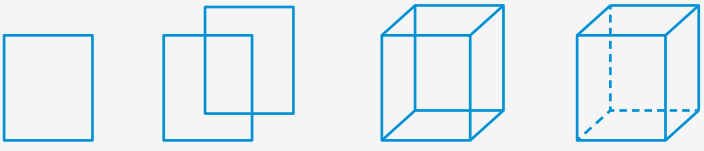
a square prism

b isosceles triangular prism

c trapezoidal prism

d hexagonal prism

2 Make an oblique drawing of a rectangular prism using these diagrams and instructions.



a Draw a rectangle that will be the front face of the prism.

b Draw another rectangle the same size, but position it slightly above and to the right of the first rectangle.

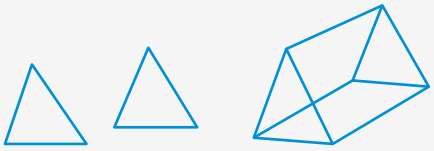
c Join the matching vertices (corners).

d Make the outside edges of the prism darker and use lighter or dotted lines for the inside edges.

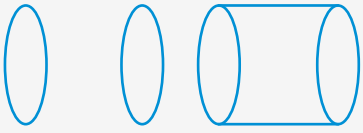
Foundation Standard Complex

3 Make an oblique drawing of each solid using the method shown in question **2**.

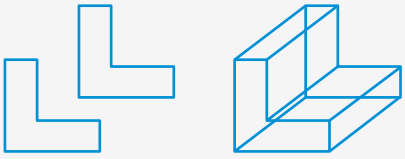
a A triangular prism



b A cylinder



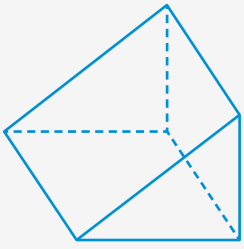
c An L-shaped prism



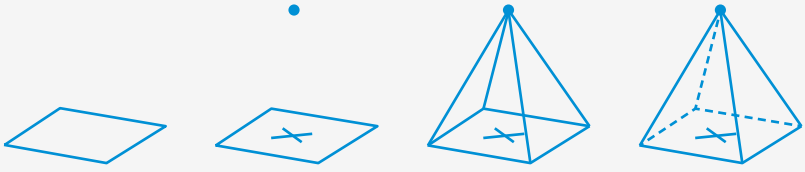
d A half-cylinder

4 What is the purpose of the dashed lines in this oblique drawing?

R
C



5 Sketch a rectangular pyramid using these diagrams and instructions.



a Draw a parallelogram for the rectangular base.

b Determine the centre of the parallelogram (where the diagonals cross) and place a dot above this position: this dot will be the top of the pyramid.

c Join the 4 vertices of the parallelogram to the dot.

d Make the outside edges of the pyramid darker and the inside edges lighter or dotted.

Foundation | Standard | Complex

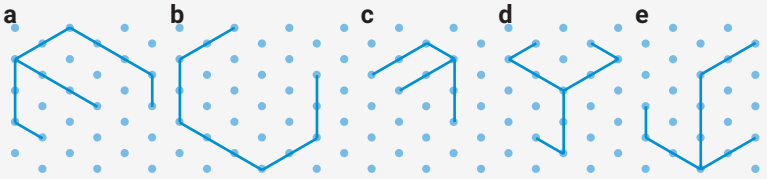
6 Sketch a cone using these diagrams and instructions.

a Draw an oval for the circular base.

b Draw a dot above the centre of the oval.

c Join the dot to the oval.

7 Each diagram is the start of a drawing of a rectangular prism. Copy each drawing onto isometric dot paper and complete it.

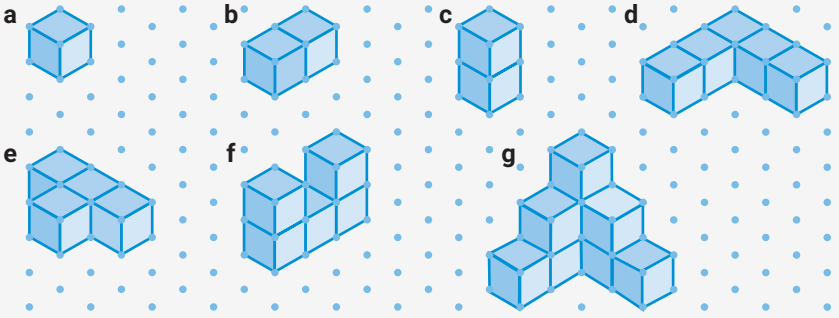


8 Draw a triangular prism:

a on square dot paper

b on isometric dot paper.

9 Use centicubes to make each solid, then draw them on isometric dot paper.

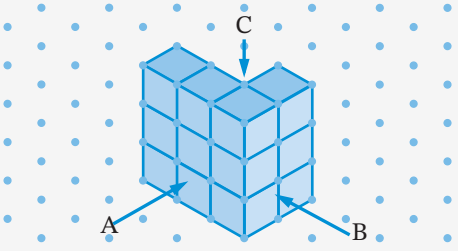


EXAMPLE 10

10 Draw what you would see of this prism if you were looking from A, then from B and then from C.

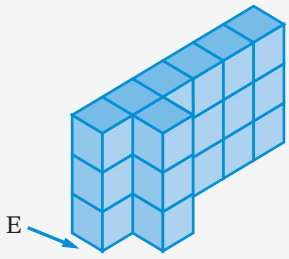
R

C

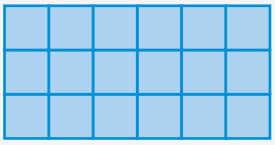


Foundation Standard Complex

11 What would Emma see if she were looking at this prism from point E? Select the correct answer **A**, **B**, **C** or **D**.

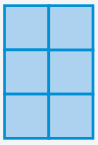


A

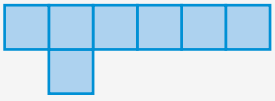


B

C



D

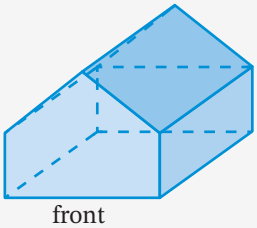


EXAMPLE 11

12 For each prism, draw each view requested.

R C

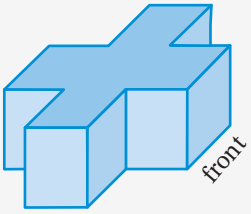
a



i front view

ii right view

b

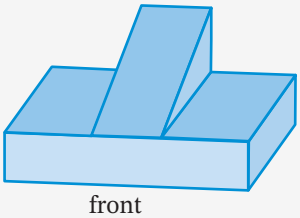


i left view

ii top view

iii front view

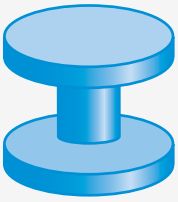
c



i front view

ii left view

d



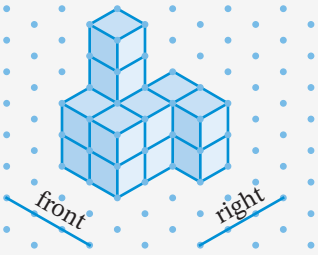
i top view

ii front view

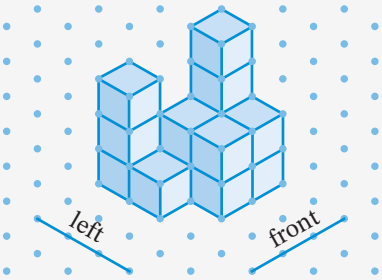
iii right view

iv top view

e



f

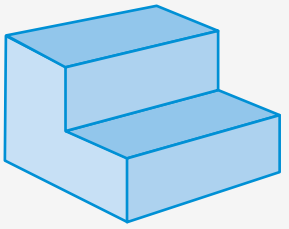


Foundation Standard Complex

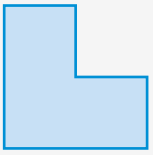
i	front view	**i**	back view
ii	right view	**ii**	left view
iii	top view	**iii**	top view

13 Damien flew over the top of this building.

Which view would he see when directly overhead?
Select **A**, **B**, **C** or **D**.

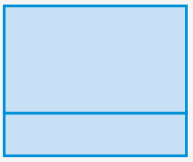


A

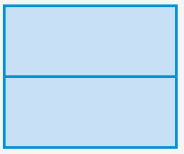


B

C

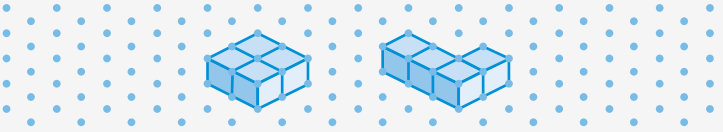


D



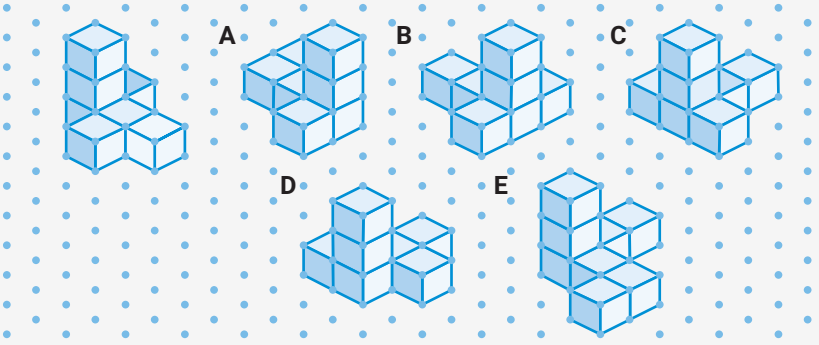
14 On isometric dot paper, draw the different solids that can be made if 4 cubes are joined together. Two have already been drawn for you. How many different solids are there?

R



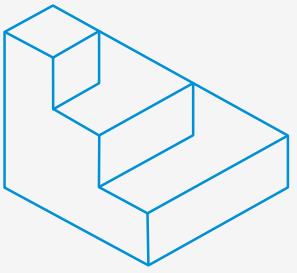
15 Which of the given solids is the same as the solid on the left? Select the correct answer **A**, **B**, **C** or **D**.

R



16 An isometric drawing is shown.

R
C



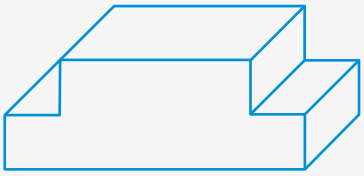
Foundation Standard Complex

a List 3 advantages in drawing the diagram this way.

b List 3 disadvantages in drawing the diagram this way.

17 An oblique drawing is shown.

R C



a List 3 advantages in drawing the diagram this way.

b List 3 disadvantages in drawing the diagram this way.

18 Research other ways of drawing 3D objects in 2 dimensions. Outline one way and consider its advantages and disadvantages.

R C

POWER PLUS ANSWERS ON P. 622

Worksheet
Equal angles

1 Use the definitions of the quadrilaterals in the table on page 274 to decide whether:

- **a** a square is a special type of rhombus
- **b** a rhombus is a special type of square
- **c** a parallelogram is a special type of trapezium
- **d** a rectangle is a special type of parallelogram
- **e** a parallelogram is a special type of kite
- **f** a rectangle is a special type of square

2
- **a** What additional property makes a parallelogram a rectangle?
- **b** What makes a kite a rhombus?
- **c** What makes a rectangle a square?

3 Name the most general quadrilateral in which:

- **a** opposite angles are equal
- **b** diagonals intersect at 90°
- **c** diagonals are equal
- **d** all angles are 90°
- **e** opposite sides are parallel
- **f** diagonals bisect each other.

4 Investigate the angle sum of a pentagon.

5
- **a** Draw any quadrilateral and extend each side to create 4 exterior angles.
- **b** Investigate the sum of the exterior angles.

Foundation Standard Complex

6 CHAPTER REVIEW

Language of maths

Quiz
Language of maths 6

acute-angled	angle sum	concave	cross-section
equilateral	exterior angle	image	isosceles
kite	net	obtuse-angled	parallelogram
polygon	polyhedron	prism	pyramid
quadrilateral	rectangle	reflection	regular
rhombus	right-angled	rotation	scalene
square	transformation	translation	trapezium

1 When you look into a mirror, you see your reflection. Is this meaning of 'reflection' related to its meaning in this chapter?

2 A regular polygon has equal sides and equal angles. What is the more common name for a regular quadrilateral?

3 The word 'isosceles' comes from Greece. Use a dictionary to find out what it means in Greek.

4 Use a dictionary to find the different meanings of:

a translation **b** axis

5 Name the shape that has 2 pairs of equal adjacent sides.

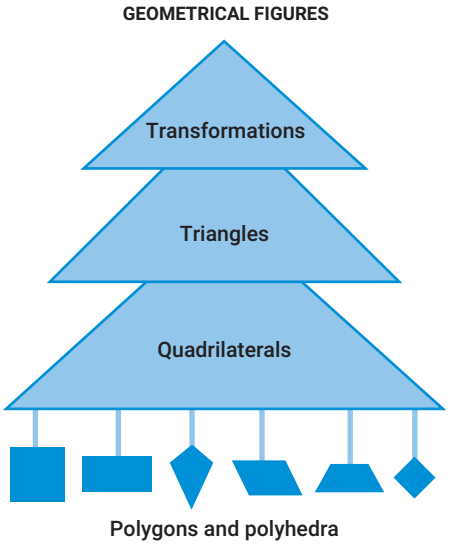
6 Explain why a cone is not a polyhedron.

Topic summary

Worksheet
Mind map: Geometrical figures

- How useful do you think this chapter will be to you in the future?
- Can you name any jobs that use some of the concepts covered in this chapter?
- Did you have any problems with any sections of this chapter? Discuss any problems with a friend or your teacher.

Print (or copy) and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.



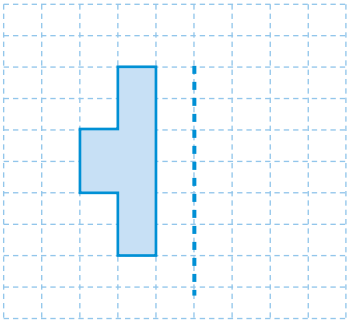
6 TEST YOURSELF

ANSWERS ON P. 622

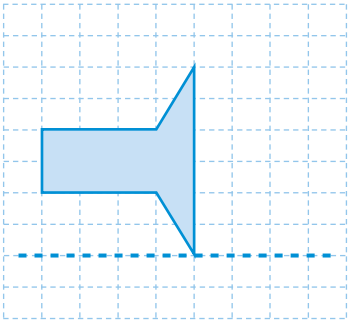
1 Copy and reflect each figure about the dotted line. 6.01

Quiz Test yourself 6

a

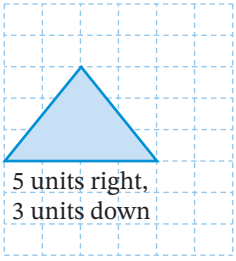


b

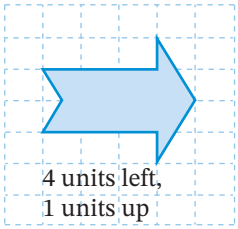


2 Copy and translate each figure by the given distance and direction. 6.01

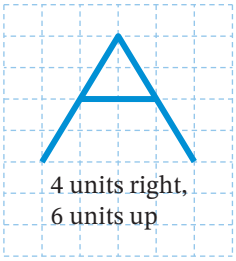
a



b

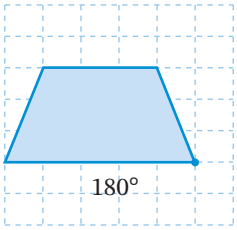


c

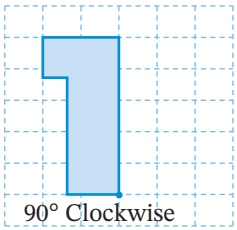


3 Copy and rotate each figure about the given point by the stated angle. 6.01

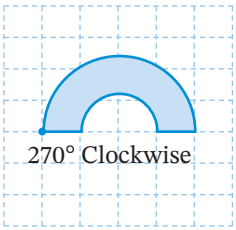
a



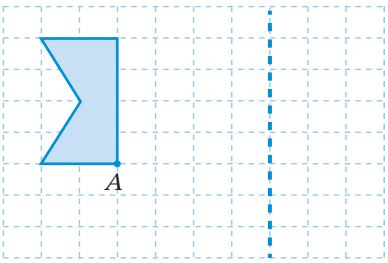
b



c



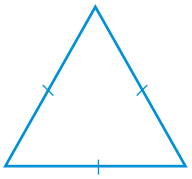
4 Translate the shape 5 units to the right and 2 units up. Then rotate it 90° clockwise about vertex A and reflect it across the given dotted line. 6.02



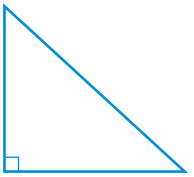
Foundation | Standard | Complex

6.03 **5** Classify each triangle by sides and by angles.

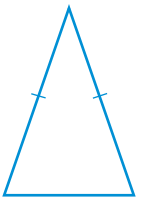
a



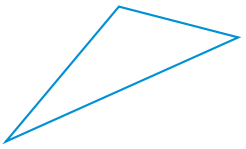
b



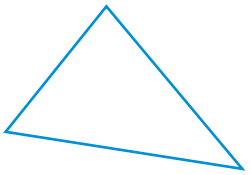
c



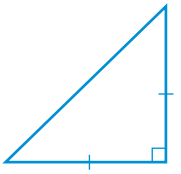
d



e

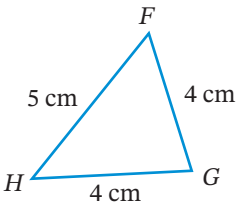


f



6.03 **6** **a** Classify $\triangle FGH$ by sides and by angles.

b Which angles in $\triangle FGH$ are equal?



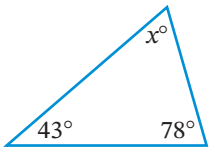
6.03 **7** Classify each described triangle by sides and by angles.

a All of my angles are equal.

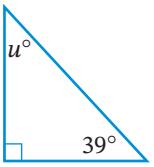
b My angles are 60°, 80° and 40°.

6.04 **8** Find the value of each variable, giving reasons.

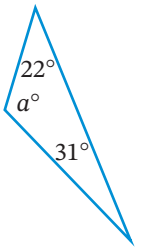
a



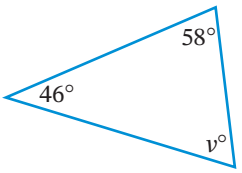
b



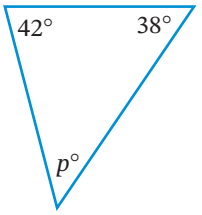
c



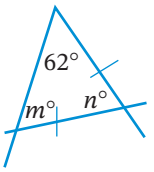
d



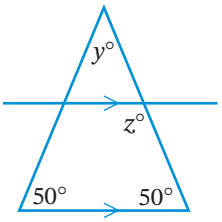
e



f



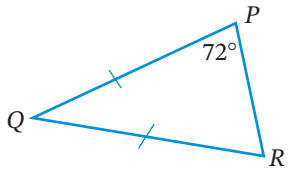
g



Foundation Standard Complex

9780170465557

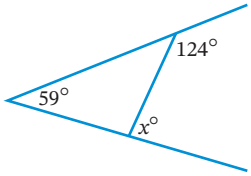
9 Find the size of $\angle PQR$. Select the correct answer **A**, **B**, **C** or **D**. 6.04



A 18° **B** 36°

C 54° **D** 72°

10 Find x. Select **A**, **B**, **C** or **D**. 6.05

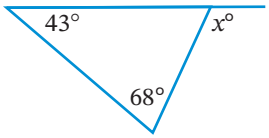


A 59 **B** 65

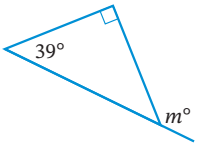
C 115 **D** 124

11 Find the value of each variable, giving reasons. 6.05

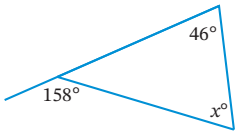
a



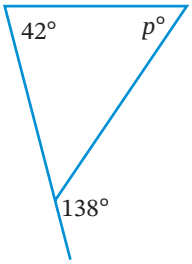
b



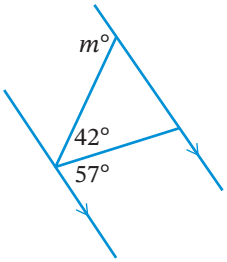
c



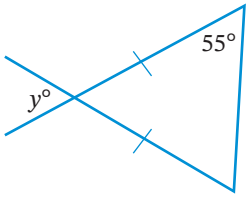
d



e



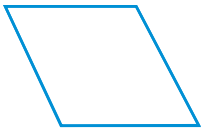
f



12 Name each special quadrilateral. 6.06

a

b

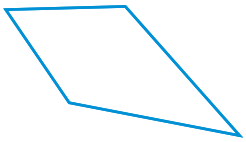


c

d

e

f



13 What is the definition of a rhombus? 6.06

14 What quadrilateral am I? (Answer with the most general quadrilateral.) 6.06

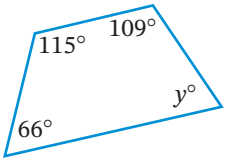
a I have opposite sides parallel.

b I have one pair of parallel sides.

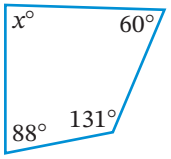
Foundation | Standard | Complex

6.07 **15** Find the value of each variable, giving reasons.

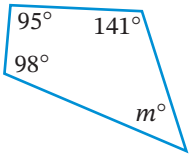
a



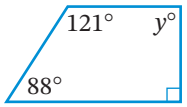
b



c



d



6.08 **16** *EFGH* is a parallelogram. Which statement is NOT true? Select **A**, **B**, **C** or **D**.

A $\angle EFG = \angle EHG$ **B** $EF = HG$

C $EF = FG$ **D** $\angle EFG + \angle FGH = 180°$

6.08 **17** List 2 properties of a parallelogram.

6.08 **18** What quadrilateral am I? (Answer with all possible quadrilaterals.)

a All of my angles are equal.

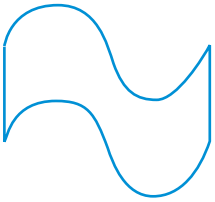
b My diagonals are equal and bisect each other.

c My diagonals bisect each other.

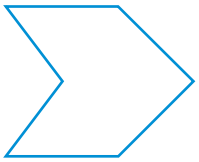
d My opposite angles are equal.

6.09 **19** Which of the following is a polygon? Select **A**, **B** or **C**.

A



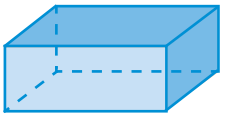
B



C

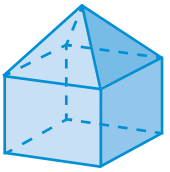
6.09 **20** Draw a cross-section of each solid and state whether it is a prism.

a

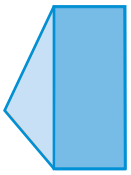


b

c

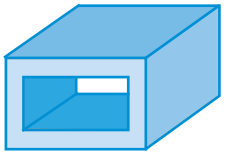


d



e

f

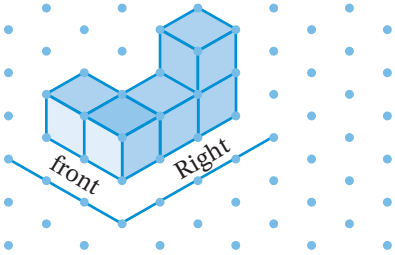


Foundation Standard Complex

6.10

21 For each solid, sketch each view requested.

a

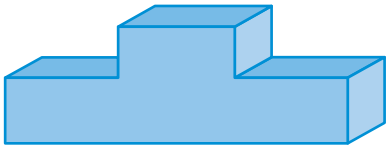


i front view

ii left view

iii top view

b



front

i right view

ii front view

iii top view

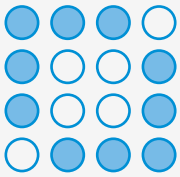
Foundation | Standard | Complex

Practice set 2

ANSWERS ON P. 623

4.01

1 What fraction of these circles are white? Select the correct answer **A**, **B**, **C** or **D**.



A $\frac{1}{3}$ **B** $\frac{3}{5}$ **C** $\frac{3}{8}$ **D** $\frac{5}{8}$

4.03

2 Evaluate each expression.

a $\frac{1}{2}+\frac{1}{3}$ **b** $\frac{1}{3}+\frac{2}{5}$ **c** $\frac{1}{2}-\frac{3}{8}$

4.10

3 Convert each fraction to a percentage.

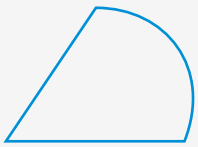
a $\frac{1}{4}$ **b** $\frac{3}{5}$ **c** $\frac{23}{40}$

6.09

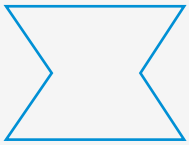
4 State whether each shape is a polygon.

a

b



c



d

5.03

5 Simplify each expression.

a $w \times 2$ **b** $a \times a$ **c** $4 \times a \times b \times k$ **d** $p + p + p + p$

e $m \div 2$ **f** $h - h$ **g** $6k + 2k$ **h** $5 \times m \times 2$

6 Change each improper fraction to a mixed numeral.

a $\frac{12}{5}$ **b** $\frac{7}{2}$ **c** $\frac{5}{3}$ **d** $\frac{11}{6}$

4.01

7 Simplify each fraction.

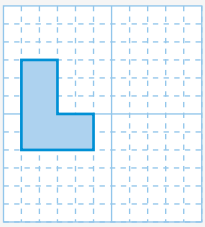
a $\frac{8}{10}$ **b** $\frac{12}{18}$ **c** $\frac{15}{24}$ **d** $\frac{70}{100}$

Foundation Standard Complex

8 Evaluate each expression mentally. 5.01

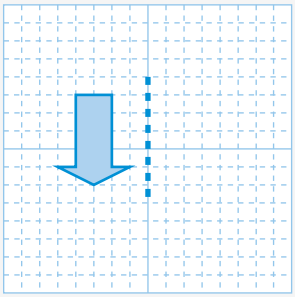
a $32 + 24 + 6$ **b** 2×900 **c** $-12 + 22$ **d** $31 \times 5 \times 4$

9 Copy this L-shape on grid paper and translate it 5 units right and 2 units down.

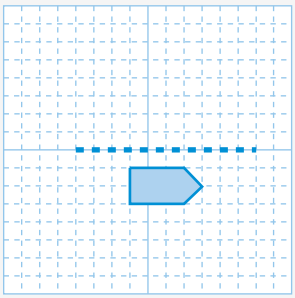


9 Copy each shape on grid paper and reflect it across the line. 6.01

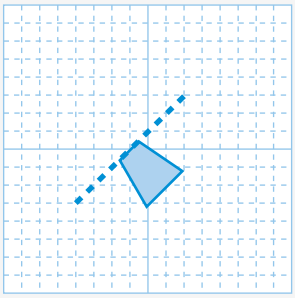
a



b



c



11 Copy and complete each statement with a > or < symbol. 4.02

a $\frac{1}{3}$ ___ $\frac{1}{4}$ **b** $\frac{2}{5}$ ___ $\frac{5}{10}$ **c** $\frac{5}{7}$ ___ $\frac{3}{5}$

Foundation Standard Complex

5.02 **12** Evaluate each product without using a calculator.

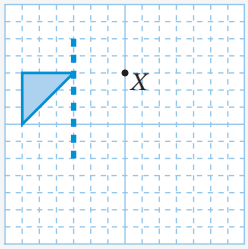
a 18×11 **b** 24×8 **c** 15×9 **d** 22×12

4.04 **13** Evaluate each expression.

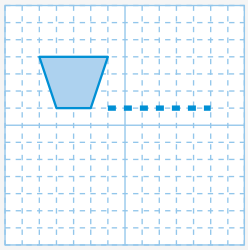
a $5\frac{3}{4} + 1\frac{2}{3}$ **b** $4\frac{3}{5} - 1\frac{1}{2}$ **c** $1\frac{2}{3} - \frac{7}{8}$

6.02 **14** Copy each shape on grid paper and draw the image after the composite transformations.

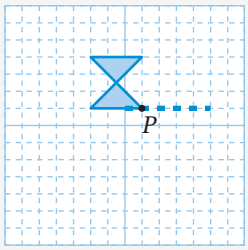
a Reflect across the line, rotate 90° clockwise about *X*.



b Translate 5 units right, then reflect across the line.



c Rotate 90° clockwise about *P*, then reflect across the line, then translate 3 units left.



4.06 **15** Evaluate each product.

a $\frac{3}{5} \times \frac{1}{2}$ **b** $\frac{2}{3} \times \frac{7}{12}$ **c** $3\frac{1}{3} \times 2\frac{1}{2}$

4.10 **16** Convert each decimal to a percentage.

a 0.4 **b** 0.05 **c** 0.12

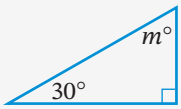
Foundation Standard Complex

17 Find: 4.11

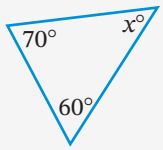
a 5% of \$200 **b** 30% of 1 hour (in minutes)

18 Find the value of the variable in each triangle. 6.04

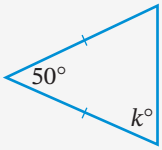
a



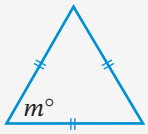
b



c



d



19 Classify each triangle above by sides and by angles. 6.03

20 Evaluate each expression. 5.05

a $m - 3$ if $m = -2$ **b** $2p - 1$ if $p = 10$ **c** $\frac{2d}{3}$ if $d = 12$

21 Solve each equation. 5.11

a $x + 11 = 34$ **b** $3x = -27$ **c** $d - 3 = -9$ **d** $\frac{m}{5} = 1$

22 A class investigation found that 4 out of every 7 cars in the car park was blue. If 280 cars were in the car park, how many were blue? 4.08

23 Find $\frac{3}{8}$ of 2 m (in cm). 4.05

24 Solve the equation $108x = 18$. Select **A, B, C** or **D**. 5.11

a $x = 21$ **b** $x = 15$ **c** $x = 6$ **d** $x = \frac{1}{6}$

25 Chloe owns 40 pairs of shoes. 2 pairs are black. 4.12

a What fraction of Chloe's shoes are black?

b What percentage of Chloe's shoes are black?

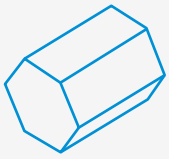
26 Find: 4.09

a $\frac{3}{5}$ of \$4200 **b** $\frac{3}{7}$ of 98 **c** $\frac{5}{9}$ of \$198

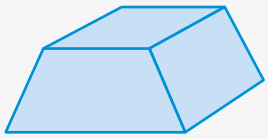
Foundation Standard Complex

609 **27** Draw the cross-section of each prism and then write the name of the prism.

a



b



4.07 **28** Evaluate each quotient.

a $\frac{3}{4} \div \frac{1}{2}$ **b** $\frac{2}{10} \div 5$ **c** $2\frac{1}{2} \div \frac{3}{5}$

5.12 **29** Solve each equation.

a $5m - 3 = 22$ **b** $6n + 3 = 12$ **c** $-3x - 11 = -20$ **d** $\frac{x}{2} + 5 = 9$

5.04 **30** Write an algebraic expression for:

a the product of p and q

b 4 less than x

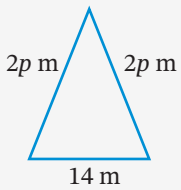
c d decreased by 7

d the number of girls if there are b boys in a class of 28 students

5.13 **31** Use an equation to solve each problem.

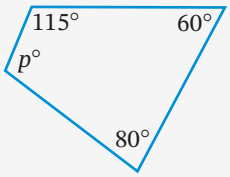
a If you multiply Mr Ha's age by 4 and add 16, you obtain 200. How old is Mr Ha? (Let h be his age.)

b The perimeter of this isosceles triangle is 56 metres. Find the value of p.

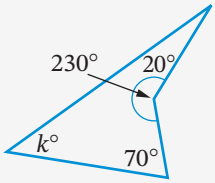


6.07 **32** Find the value of each variable.

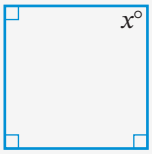
a



b



c



5.06 **33** Simplify each expression.

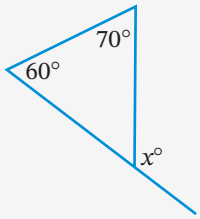
a $5m - 3 + 2m$ **b** $6n + 3m - 4n + 7m$ **c** $-3x - 4x^2 + x + 2x^2$

Foundation Standard Complex

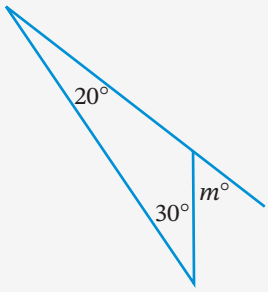
 9780170465557

34 Find the value of each variable. 6.05

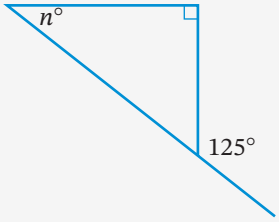
a



b



c



35 Simplify each quotient. 5.07

a $15m^2 \div 3$ **b** $8e^2 \div (-4e)$ **c** $-7p^2 \div p$ **d** $\frac{9x^2}{3x}$

36 Name all quadrilaterals (parallelogram, trapezium, rectangle, kite, rhombus, square) that have: 6.06

a opposite sides parallel

b 4 right angles

c 4 equal sides and 4 equal angles

d 2 pairs of equal adjacent sides

37 Which number could replace the △ in $\frac{\Delta+3}{5}$ to make the fraction between 6 and 7? Select **A**, **B**, **C** or **D**. 4.02

A 26 **B** 32 **C** 31 **D** 36

Foundation Standard Complex

7

NUMBER

Decimals

Before decimal currency was introduced in Australia in 1966, our money was based on the British pound, shilling and penny. 1 pound = 20 shillings and 1 shilling = 12 pence. Before the metric system was adopted in Australia in 1974, we used the imperial system of measurement. For example, 1 mile = 1760 yards, 1 yard = 3 feet, 1 foot = 12 inches. This meant that 50 years ago the calculations and conversions were more complicated.

Today, our currency and measurement units follow a decimal system using powers of 10. For example, 1 L = 1000 mL, 1 m = 100 cm, 1 cm = 10 mm. With the metric system and computer technology, decimals have become more commonly-used than fractions to describe parts of a whole.

iStock.com/wx-bra/wang

Chapter outline

		Proficiencies				
7.01	Ordering decimals	U	F	PS	R	C
7.02	Decimals and fractions	U	F			C
7.03	Adding and subtracting decimals	U	F	PS	R	C
7.04	Multiplying and dividing decimals by powers of 10	U	F		R	
7.05	Multiplying decimals by estimating	U	F			
7.06	Multiplying decimals	U	F	PS		C
7.07	Dividing decimals by whole numbers	U	F		R	
7.08	Dividing decimals	U	F		R	C
7.09	Terminating and recurring decimals	U	F		R	C
7.10	Rounding decimals	U	F	PS	R	C
7.11	Decimal problems	U	F	PS	R	C

U = Understanding
F = Fluency
PS = Problem solving
R = Reasoning
C = Communication

Wordbank

ascending Increasing from smallest to largest

decimal places The places after the decimal point in a number

recurring decimal A decimal that has one or more digits that repeat endlessly

round To write a number to a given number of places

tenth The fraction $\frac{1}{10}$ or the decimal 0.1, the first decimal place

terminating decimal A decimal that is not recurring, but comes to an end

Quiz
Wordbank 7

Videos (5):

7.04 Multiplying and dividing decimals by powers of 10
7.06 Multiplying decimals
7.08 Dividing decimals
7.09 Converting fractions to decimals
7.10 Rounding decimals

Twig video (1):

7.01 Decimal places: Photofinish

PhET interactive (1):

7.06 Area Model Decimals

Quizzes (5):

- Wordbank 7
- SkillCheck 7
- Mental skills 7
- Language of Maths 7
- Test Yourself 7

Skillsheets (3):

SkillCheck Decimals • Multiplying by 10, 100, 1000
7.02, 7.09 Fractions and decimals

Worksheets (11):

7.01 Decimals wall • Dewey decimals • Decimals 1 • Decimals 10
7.06 Shopping and change • Decimal cards
7.08 Decimals writing activity • Decimals 7 • Decimal number grid
7.11 Shopping and change • Calculating change
Mind map: Decimals

Puzzles (4):

7.02 Decimals squaresaw 1
7.06 Which decimals?
7.08 Operations with decimals
7.09 Decimals squaresaw 2

Nelson MindTap

To access resources above, visit **cengage.com.au/nelsonmindtap**

7 **In this chapter you will:**

- ✓ compare and order decimals
- ✓ convert between fractions and decimals
- ✓ solve problems involving adding and subtracting decimals
- ✓ multiply and divide decimals by powers of 10
- ✓ solve problems involving multiplying and dividing decimals
- ✓ convert a fraction to a terminating decimal
- ✓ (EXTENSION) convert a fraction to a recurring decimal
- ✓ round a decimal to a specified number of decimal places
- ✓ solve problems involving decimals

SkillCheck

ANSWERS ON P. 624

1 Copy this place value table so that it has 8 blank rows.

hundreds	tens	units		tenths	hundredths	thousandths
			.			
			.			
			.			

Write each decimal in the table, with the digits in their correct columns.

a 14.82 **b** 6.014 **c** 931.02 **d** 70.8

e 0.375 **f** 0.19 **g** 8.592 **h** 715.3

2 Write the value of the digit 4 in each decimal.

a 431.70 **b** 52.407 **c** 761.84

d 114.65 **e** 2.734 **f** 907.481

3 Simplify each fraction.

a $\frac{18}{100}$ **b** $\frac{4}{10}$ **c** $\frac{350}{1000}$ **d** $\frac{24}{100}$

4 List each set of numbers in ascending order.

a 21, 32, 34, 30, 20 **b** 213, 216, 234, 206, 230

5 Evaluate each expression.

a 32×10 **b** 578×100 **c** 325×1000

d 400×10 **e** $400 \div 10$ **f** $1200 \div 100$

g $1\,000\,000 \div 1000$ **h** $81\,000 \div 100$ **i** $640 \div 10$

6 List each set of numbers in descending order.

a 44, 39, 42, 45, 38, 40 **b** 505, 556, 513, 520, 549, 522

7 Round:

a 27 to the nearest ten **b** 752 to the nearest hundred

c 9079 to the nearest thousand **d** 16 837 to the nearest thousand

8 Estimate the value of each expression.

a $140 + 32 + 381$ **b** $432 + 45 + 341 + 7$ **c** $746 - 29$

d $596 - 338$ **e** $1138 - 374$ **f** 58×3

g 126×25 **h** $5920 \div 8$ **i** $2233 \div 7$

9 Evaluate each expression in question **8** without using a calculator.

Skillsheets
Decimals

Multiplying by 10, 100, 1000

Quiz
SkillCheck 7

7.01 Ordering decimals

Video Decimal places: Photofinish

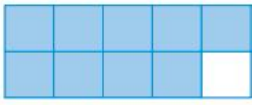
Skillsheet Decimals

Worksheets Decimals wall

Dewey decimals

Decimals 1

Decimals 10



A **decimal** is a special type of fraction where the denominator is a **power of 10** such as 10, 100 or 1000. This shape has been divided into 10 equal parts, of which 9 have been shaded.

- As a fraction, $\frac{9}{10}$ or 'nine-tenths' of the shape is shaded.
- As a decimal, 0.9 or 'zero-point-nine' of the shape is shaded.

You are familiar with numbers that have decimal points, for example:

- money $532.81
- measurements 6.2 km.

A decimal is an example of a **rational number** because it can be written as a fraction $\frac{a}{b}$, where a and b are integers but $b \neq 0$.

The position of a digit in a number shows its size. This is called the **place value**.

In a decimal, the digits after the decimal point indicate a part of a whole. For example, the meaning of the decimal 532.81 is shown on the place value table below:

hundreds 100	tens 10	units 1	decimal point	tenths $\frac{1}{10}$	hundredths $\frac{1}{100}$
5	3	2	.	8	1

$$532.81 = 5 \times 100 + 3 \times 10 + 2 \times 1 + 8 \times \frac{1}{10} + 1 \times \frac{1}{100}.$$

Decimal places

The number of digits after the decimal point tells us the number of **decimal places** in a decimal.

Example 1

How many decimal places are there in:

a 3.6567?

b 15.801?

SOLUTION

a 3.6567 has 4 decimal places.
1234

b 15.801 has 3 decimal places.
123

Example 2

Arrange these decimals in ascending order: 67.41, 67.14, 6.714, 67.04.

SOLUTION

To compare decimals more easily, place them in a column with the decimal points in a vertical column and make them all have the same number of decimal places by inserting 0s at the end.

For example, 67.41 becomes 67.410.

67.410

67.140

6.714

67.040

All numbers now have 3 decimal places.
From smallest to largest: 6.714, 67.040, 67.140, 67.410
In ascending order: **6.714, 67.04, 67.14, 67.41**

Example 3

Arrange these numbers in descending order: 0.5, 0.08, 1.7, 0.85.

SOLUTION

To compare decimals more easily, place them in a column with the decimal points in a vertical column and make them all have the same number of decimal places by inserting 0s at the end.
0.5 becomes 0.50 and 1.7 becomes 1.70.

0.50

0.08

1.70

0.85

All numbers now have 2 decimal places.
From largest to smallest: 1.70, 0.85, 0.50, 0.08
In descending order: **1.7, 0.85, 0.5, 0.08**

EXERCISE 7.01 ANSWERS ON P. 624

Ordering decimals

U F PS R C

1 How many decimal places does each number have? EXAMPLE 1

a 1.65 **b** 3.881 **c** 15.3062
d 0.005 **e** 7.045 73 **f** 814.3
g 9.597 684 **h** 203.602 39 **i** 0.042 470 6

2 Which of these is the smallest decimal? Select the correct answer **A**, **B**, **C** or **D**. EXAMPLE 2

A 1.07 **B** 1.7 **C** 1.077 **D** 1.77

3 Arrange each set of numbers in ascending order.

R **a** 43.89, 56.324, 9.998, 80.879, 400, 23.89, 56.314
b 0.568, 0.684, 0.099, 1.002, 0.586, 5.608, 0.0586
c 1.23, 0.891, 1.814, 0.222, 7.007, 0.89
d 0.5, 0.05, 0.005
e 3.441, 3.404, 3.4, 3.44, 3.004, 3.044
f 0.2, 0.202, 0.22, 0.022

Foundation Standard Complex

EXAMPLE 3

4 Which of these is the largest decimal? Select **A**, **B**, **C** or **D**.

A 2.509 **B** 2.059 **C** 2.59 **D** 2.05

5 Arrange each set of numbers in descending order.

R **a** 570.25, 125.63, 0.9899, 4000.99, 1256.3, 400.099

b 5.37, 6.539, 5.639, 5.367, 3.659, 3.66, 5.369

c 1.6, 1.61, 1.599, 1.601, 1.509

d 6, 0.06, 0.6, 6.6

e 0.7, 0.07, 0.707, 0.77, 0.007, 7.07

f 0.4004, 0.044, 0.404, 0.44

6 Insert < or > to make each statement true.

R **a** 0.2 ___ 0.25 **b** 0.731 ___ 0.73 **c** 0.035 ___ 0.305

C **d** 0.007 ___ 0.070 **e** 1.59 ___ 1.059 **f** 0.099 ___ 0.99

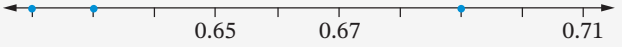
g 44.44 ___ 4.444 **h** 0.7932 ___ 0.7239

7 Copy each number line and write the values of the points marked with dots.

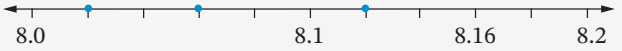
R **a**

b

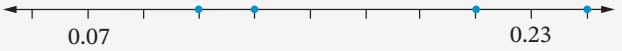
c



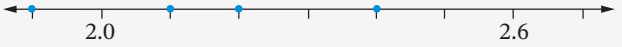
d



e



f



g

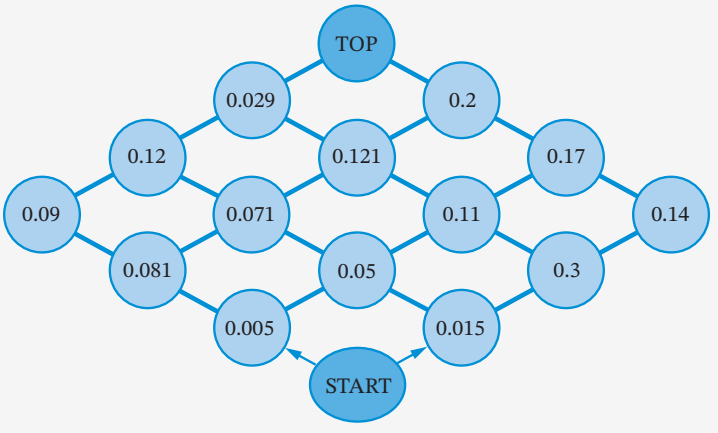
8 Write a decimal that is:

R **a** less than 1.3 **b** greater than 2.07

C **c** between 8.25 and 8.5 **d** between 6.401 and 6.41

9 Find a path from START to TOP. You can make your move from START in any direction, but then you can only move to a larger decimal. Try to find the shortest path, then try to find the longest path.

PS R



Foundation Standard Complex

INVESTIGATION

Comparing heights

1 Use the clues below to find each girl's height.

- Mandy is taller than Sarah.
- Kelly is taller than Sarah but shorter than Mandy.
- Sarah is shorter than Amal.
- Mandy is not the tallest.

The heights of the girls are 168.5 cm, 166.3 cm, 164.2 cm and 160.7 cm.

Shutterstock.com/michaeljung

iStock.com/Dean Mitchell

iStock.com/4x6

Shutterstock.com/all_about_people

2 Use the clues below to find each boy's height.

- Steve is 164.7 cm tall.
- Steve is 3.9 cm shorter than Milof.
- Mike is 14.7 cm taller than Milof.
- Liong is 1.6 cm taller than Mike.

Shutterstock.com/pikselstock

Shutterstock.com/East

Shutterstock.com/Viktor Gladkov

Stock.com/track5

3 Use the clues below to find each student's height.

- Yoko is 15.1 cm taller than Peter.
- Karl is 20.6 cm taller than Jade.
- Jade is 13.7 cm shorter than Yoko.
- Peter is 163 cm tall.

iStock.com/4x6

iStock.com/PeopleImages

Shutterstock.com/A Lot Of People

Shutterstock.com/Di Studio

4 Create your own problem using 4 students from your class. Estimate their heights and height differences. Write a set of clues. (Don't forget to change their names!)

7.02 Decimals and fractions

Skillsheet Fractions and decimals

Puzzle Decimals squaresaw 1

Example 4

Convert each decimal to a simplified fraction.

a 0.09 **b** 0.274 **c** 1.45

SOLUTION

a $0.09 = \frac{9}{100}$ — 2 decimal places means hundredths $\left(\frac{\quad}{100}\right)$

b $0.274 = \frac{274}{1000}$ — 3 decimal places means thousandths $\left(\frac{\quad}{1000}\right)$

$= \frac{137}{500}$ — Simplify the fraction

c $1.45 = 1 + \frac{45}{100}$ — 2 decimal places means hundredths $\left(\frac{\quad}{100}\right)$

$= 1\frac{9}{20}$ — Simplify the fraction

Note: The number of decimal places in the decimal gives the number of 0s in the denominator of the fraction.

Decimal places

- tenths have one decimal place $\left(\frac{1}{10} \text{ has 1 zero}\right)$
- hundredths have 2 decimal places $\left(\frac{1}{100} \text{ has 2 zeros}\right)$
- thousandths have 3 decimal places $\left(\frac{1}{1000} \text{ has 3 zeros}\right)$
- ten thousandths have 4 decimal places $\left(\frac{1}{10000} \text{ has 4 zeros}\right)$

Example 5

Convert each fraction to a decimal.

a $\frac{22}{100}$ **b** $3\frac{7}{10}$ **c** 5 thousandths

SOLUTION

a $\frac{22}{100} = 0.22$ — 2 zeros → 2 decimal places

b $3\frac{7}{10} = 3.7$ — 1 zero → 1 decimal place

c 5 thousandths $= \frac{5}{1000}$ — 3 zeros → 3 decimal places

$= 0.005$

EXERCISE 7.02 ANSWERS ON P. 625

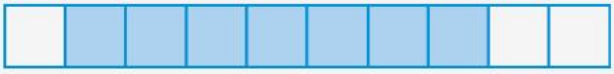
Decimals and fractions

U F C

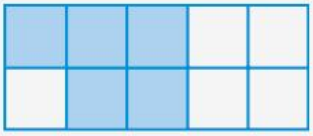
1 What part of each shape has been shaded? Write the answer as:

C **i** a fraction **ii** a decimal

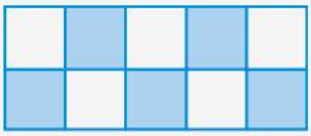
a



b



c



d

EXAMPLE 4

2 Convert 0.028 to a simplified fraction. Select the correct answer **A**, **B**, **C** or **D**.

C **A** $\frac{28}{100}$ **B** $\frac{28}{1000}$ **C** $\frac{14}{500}$ **D** $\frac{7}{250}$

3 Convert each decimal to a simplified fraction.

C

a 0.7	**b** 0.4	**c** 0.39	**d** 0.572
e 0.003	**f** 0.05	**g** 0.11	**h** 0.328
i 0.3	**j** 0.999	**k** 0.013	**l** 0.0004
m 0.0471	**n** 0.321	**o** 0.5005	**p** 0.91
q 0.082	**r** 1.6	**s** 27.25	**t** 2.007

EXAMPLE 5

4 Which decimal is equal to $\frac{23}{1000}$? Select **A**, **B**, **C** or **D**.

C **A** 0.23 **B** 0.023 **C** 0.0023 **D** 0.000 23

5 Convert each fraction to a decimal.

C

a $\frac{9}{10}$	**b** $\frac{15}{100}$	**c** $\frac{79}{100}$	**d** $\frac{60}{100}$
e 4 tenths	**f** $\frac{23}{100}$	**g** $\frac{6}{10}$	**h** $\frac{411}{1000}$
i $\frac{704}{1000}$	**j** 87 hundredths	**k** $\frac{7}{100}$	**l** $\frac{14}{100}$
m $\frac{8}{10}$	**n** $\frac{235}{1000}$	**o** $\frac{247}{1000}$	**p** $\frac{17}{100}$
q $\frac{368}{1000}$	**r** $\frac{9345}{10000}$	**s** $\frac{493}{1000}$	**t** $1\frac{67}{100}$
u $4\frac{47}{100}$	**v** $23\frac{9}{10}$	**w** $6\frac{8}{10}$	**x** 23 thousandths
y 45 hundredths			

Foundation Standard Complex

7.03 Adding and subtracting decimals

Adding and subtracting decimals

- Keep decimal points below one another
- Check your answer by estimating

Example 6

Find the sum of 10.92, 0.89, 32 and 0.6.

SOLUTION

$$\begin{array}{r} 10.92 \\ 0.89 \\ 32.00 \\ +\,0.60 \\ \hline 44.41 \end{array}$$

Fill in any spaces with 0s.

Remember that 32 is the same as 32.00.

Estimate: $10.92 + 0.89 + 32 + 0.6 \approx 11 + 1 + 32 + 1$
$= 45$

Example 7

Evaluate each difference.

a 76.029 – 8.914 **b** 4.31 – 2.183

SOLUTION

a
$$\begin{array}{r} 76.029 \\ -\,8.914 \\ \hline 67.115 \end{array}$$

Estimate: $76.029 - 8.914 \approx 76 - 9$
$= 67$

b
$$\begin{array}{r} 4.310 \\ -\,2.183 \\ \hline 2.127 \end{array}$$

Fill in any spaces with 0s.

Estimate: $4.31 - 2.183 \approx 4 - 2$
$= 2$

EXERCISE 7.03 ANSWERS ON P. 625

Adding and subtracting decimals

U F PS R C

EXAMPLE 6

1 Copy and complete each sum.

a
$$\begin{array}{r} 5.3 \\ 6.2 \\ +\,0.5 \\ \hline \end{array}$$

b
$$\begin{array}{r} 4.723 \\ 0.01 \\ +\,12.2 \\ \hline \end{array}$$

c
$$\begin{array}{r} 43.5 \\ 116.29 \\ 7.3 \\ +\,0.227 \\ \hline \end{array}$$

d
$$\begin{array}{r} 0.0076 \\ 1.23 \\ +\,0.9 \\ \hline \end{array}$$

Foundation Standard Complex

EXAMPLE 7

7.03

2 Copy and complete each difference.

a 57.703 − 16.21 **b** 6.1 − 0.2 **c** 23.57 − 16.88 **d** 22.6 − 13.54

3 Evaluate 0.61 + 12.345. Select the correct answer **A**, **B**, **C** or **D**.

A 73.345 **B** 18.445 **C** 12.955 **D** 12.406

4 PS An electrician needed these lengths of cable to complete a wiring job: 12.3 m, 4.8 m, 18.7 m, 7.98 m, 13.65 m and 23.6 m.

a How many metres of cable did the electrician use?

b If the full spool of cable was 100 m long, how many metres of cable were left on the spool after the electrician completed the job?

5 PS To keep fit, Angela runs each day. Last week she ran 3.8 km, 4.1 km, 2.3 km, 2.6 km, 3 km, 1 km and 1.8 km. How far did she run last week?

6 PS A truck carrying sand had a total mass of 13 248 kg. If the truck alone had a mass of 5210.8 kg, what is the mass of the sand?

7 R C 5 runners in the school's 100 m race recorded the following times: 13.5 s, 13.81 s, 12.7 s, 14.62 s, 12.45 s

a Place these times in order, from fastest to slowest.

b What is the time difference between the fastest and slowest runners?

c If the runner in second place had run 0.3 seconds faster, would she have won the race? Explain your answer.

8 Dionne's expenses for one week are shown in the table.

a How much did he spend?

b How much did he have left out of his weekly pay of $620.80?

Food	$128.80
Clothing	$88.45
Car	$58.35
Rent	$185.00
Entertainment	$78.95
Savings	$66.00

9 A new block of wood is 11.27 cm thick. If it has 0.34 cm shaved off one side and 0.55 cm shaved off the other side, how thick is the block of wood? Select the correct answer **A**, **B**, **C** or **D**.

A 10.38 cm **B** 11.06 cm **C** 11.16 cm **D** 11.48 cm

10 Wendy was making teddy bears. She needed these amounts of material for 5 bears: 2.6 m, 0.8 m, 1.2 m, 0.75 m and 0.88 m. How much material did she need altogether?

11 R Copy and complete each equation with the correct decimals.

a 4.8 + _____ = 7.34 **b** _____ − 6.6 = 2.14 **c** 8.75 − _____ = 3.3

d _____ + _____ = 10.719 **e** _____ − _____ = 3.016

Foundation Standard Complex

INVESTIGATION

Multiplying and dividing decimals by powers of 10

When a number is multiplied by 10, 100, 1000 or other powers of 10, every digit in the number increases in place value by moving to the left. When a number is divided by a power of 10, every digit decreases in place value by moving to the right.

1 Copy the place value table below, then use a calculator to evaluate each expression involving multiplying or dividing by a power of 10. Note the pattern in your answers.

Calculation	Ten thousands	Thousands	Hundreds	Tens	Units	•	tenths	hundredths	thousandths	ten thousandths
36.7				3	6	•	7			
36.7 × 10						•				
36.7 × 100						•				
36.7 × 1000						•				
2.35					2	•	3	5		
2.35 × 10						•				
2.35 × 100						•				
2.35 × 1000						•				
36.7 ÷ 10						•				
36.7 ÷ 100						•				
36.7 ÷ 1000						•				
2.35 ÷ 10						•				
2.35 ÷ 100						•				

2 Copy and complete each statement.

a To multiply by 10, move the decimal point _____ place to the _____.

b To multiply by 100, move the decimal point _____ places to the _____.

c To multiply by 1000, move the decimal point _____ places to the _____.

d To divide by 10, move the decimal point _____ place to the _____.

e To divide by 100, move the decimal point _____ places to the _____.

f To divide by 1000, move the decimal point _____ places to the _____.

Multiplying and dividing decimals by powers of 10

7.04

Multiplying and dividing decimals by powers of 10

To multiply a decimal by:	Move the decimal point:
10	1 place to the right
100	2 places to the right
1000	3 places to the right
To divide a decimal by:	**Move the decimal point:**
10	1 place to the left
100	2 places to the left
1000	3 places to the left

Note that the number of 0s in the power of 10 is how many places the decimal point is moved.

Example 8

Video
Multiplying and dividing decimals by powers of 10

Evaluate each expression.

a 13.69×100 **b** 2.54×1000 **c** $13.69 \div 10$ **d** $2.54 \div 100$

SOLUTION

a $13.69 \times 100 = 13.69$ — Move the decimal point 2 places to the right.

$= 1369$ — Moving right makes the decimal bigger.

b $2.54 \times 1000 = 2.540$ — Move the decimal point 3 places to the right: add a 0 to the end to allow this.

$= 2540$

c $13.69 \div 10 = 13.69$ — Move the decimal point 1 place to the left.

$= 1.369$ — Moving left makes the decimal smaller.

d $2.54 \div 100 = 002.54$ — Move the decimal point 2 places to the left: add 2 0s to the start to allow this.

$= 0.0254$

EXERCISE 7.04 ANSWERS ON P. 625

Multiplying and dividing decimals by powers of 10

U F R

EXAMPLE 8

1 Evaluate each product.

a 2.49×100 **b** 0.81×10 **c** 37.42×1000

d 3.416×100 **e** 7.25×100 **f** 2.196×1000

g 6.043×100 **h** 0.032×100 **i** 0.065×10

Foundation Standard Complex

j	45.213×100	**k**	10.64×1000	**l**	63.04×100
m	5.98×1000	**n**	847.612×100	**o**	0.0592×10
p	36.2×100	**q**	219.4×1000	**r**	40.075×10

2 Evaluate $18.5 \div 1000$. Select the correct answer **A**, **B**, **C** or **D**.

A 0.00 185 **B** 1.85 **C** 0.0185 **D** 0.185

3 Evaluate each quotient.

a	$46.3 \div 10$	**b**	$507 \div 100$	**c**	$1203 \div 1000$
d	$36.4 \div 100$	**e**	$381.5 \div 1000$	**f**	$55.02 \div 10$
g	$11.46 \div 1000$	**h**	$694.3 \div 1000$	**i**	$4.28 \div 1000$
j	$67 \div 100$	**k**	$21.31 \div 1000$	**l**	$5.72 \div 10$
m	$81.348 \div 1000$	**n**	$50.2 \div 100$	**o**	$4.91 \div 100$
p	$0.4 \div 1000$	**q**	$1.73 \div 100$	**r**	$125.3 \div 10\,000$

4 Copy and complete each equation.

R **a**	$14.219 \times$ ____ $= 14\,219$	**b**	$892.5 \div$ ____ $= 8.925$
c	$742 \div$ ____ $= 7.42$	**d**	$205.7 \div$ ____ $= 20.57$
e	____ $\times$ ____ $= 560.1$	**f**	____ $\div$ ____ $= 0.382$

Quiz
Mental skills 7

☆ MENTAL SKILLS 7 ANSWERS ON P. 625 Maths without calculators

Calculating differences and making change

In every subtraction problem, for example 135 – 47, think of finding the 'gap' between the 2 numbers. That is, find the number in this case that must be added to 47 to get 135.

1 Study each example.

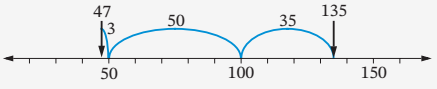
a 135 – 47

Think: 47 + ___ = 135

Count: '47, 50, 100, 135'

Add: 3 + 50 + 35 = 88

Answer: 135 – 47 = 88



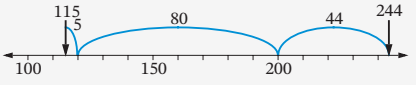
b 244 – 115

Think: 115 + ___ = 244

Count: '115, 120, 200, 244'

Add: 5 + 80 + 44 = 129

Answer: 244 – 115 = 129



c \$60 – \$47.65

Count: '\$47.65, \$48, \$50, \$60'

Add: \$0.35 + \$2 + \$10 = \$12.35

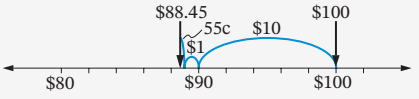
Answer: \$60 – \$47.65 = \$12.35

d \$100 – \$88.45

Count: '\$88.45, \$89, \$90, \$100'

Add: \$0.55 + \$1 + \$10 = \$11.55

Answer: \$100 – \$88.45 = \$11.55



Foundation Standard Complex

9780170465557

2 Now evaluate each difference.

a	176 – 88	**b**	221 – 54	**c**	670 – 356	**d**	425 – 340
e	518 – 389	**f**	199 – 78	**g**	\$70 – \$58.40	**h**	\$80 – \$73.25
i	\$45 – \$40.30	**j**	\$100 – \$69.95	**k**	\$30 – \$22.90	**l**	\$50 – \$17.10

Multiplying decimals by estimating 7.05

If you know the answer to a whole number multiplication, then you can use estimation to find the answer to a related decimal multiplication by correctly positioning the decimal point.

Example 9

a Given that $17 \times 12 = 204$, find:

i 1.7×12 **ii** 1.7×1.2

b Given that $23 \times 47 = 1081$, find:

i 2.3×4.7 **ii** 230×4.7 **iii** 23×0.47

SOLUTION

a $17 \times 12 = 204$

	Multiplication	Estimate	Answer	
i	1.7×12	$\approx 2 \times 10 = 20$	20.4	← use the digits 204 to make a number near 20
ii	1.7×1.2	$\approx 2 \times 1 = 2$	2.04	← use the digits 204 to make a number near 2

b $23 \times 47 = 1081$

	Multiplication	Estimate	Answer	
i	2.3×4.7	$\approx 2 \times 5 = 10$	10.81	← use the digits 1081 to make a number near 10
ii	230×4.7	$\approx 200 \times 5 = 1000$	1081	← use the digits 1081 to make a number near 1000
iii	23×0.47	$\approx 20 \times 0.5 = 10$	10.81	← use the digits 1081 to make a number near 10

EXERCISE 7.05 ANSWERS ON P. 625

Multiplying decimals by estimating

U F

EXAMPLE 9

1 Use the result of the whole number multiplication to copy and complete each table.

a $69 \times 18 = 1242$

Multiplication	Estimate	Answer
0.69 × 18		
6.9 × 180		
6.9 × 1.8		
690 × 1.8		

b $104 \times 42 = 4368$

Multiplication	Estimate	Answer
10.4 × 42		
1.04 × 4.2		
104 × 4.2		
0.104 × 4.2		

c $38 \times 92 = 3496$

Multiplication	Estimate	Answer
3.8 × 92		
0.38 × 92		
38 × 0.92		
380 × 0.92		

2 Given that $63 \times 34 = 2142$, use estimates to find:

a 6.3×3.4 **b** 0.63×3.4 **c** 0.63×3400
d 630×3.4 **e** 6.3×34 **f** 63×0.34

3 Given that $1.7 \times 1.2 = 2.04$, use estimates to find:

a 1.7×12 **b** 17×0.12 **c** 0.17×1.2
d 0.17×12 **e** 17×12 **f** 17×1.2

4 Given that $7.2 \times 3.4 = 24.48$, use estimates to find:

a 7.2×34 **b** 72×3.4 **c** 0.72×3.4
d 72×34 **e** 7.2×0.34 **f** 0.72×34

5 Given that $1.26 \times 6 = 7.56$, use estimates to find:

a 12.6×6 **b** 126×6 **c** 1.26×0.6
d 0.126×6 **e** 0.126×0.6 **f** 126×0.6

Foundation Standard Complex

6 Use the fact that $0.3 \times 0.24 = 0.072$ to find:

a 3×0.24 **b** 0.3×2.4 **c** 0.3×24

d 3×24 **e** 3×2.4 **f** 0.03×24

7 Write an estimate for each product.

a 2.7×3.1 **b** 6.2×0.44 **c** 8.6×19

INVESTIGATION

Decimal places in multiplication answers

1 What happens when you multiply by a number less than one?

a Is the answer to 12×0.8 more or less than 12? Why?

b Estimate the answer to 12×0.8.

c How many decimal places do 12 and 0.8 each have?

d Use a calculator to evaluate 12×0.8. How many decimal places does the answer have?

2 **a** Is the answer to 0.7×0.3 more or less than 0.7? Why?

b Estimate the answer to 0.7×0.3.

c How many decimal places do 0.7 and 0.3 each have?

d Use a calculator to evaluate 0.7×0.3. How many decimal places does the answer have?

3 **a** Is the answer to 2.5×4.1 more or less than 2.5? Why?

b Estimate the answer to 2.5×4.1.

c How many decimal places do 2.5 and 4.1 each have?

d Use a calculator to evaluate 2.5×4.1. How many decimal places does the answer have?

4 What is the relationship between the number of decimal places in the question and the number of decimal places in the answer?

5 **a** If $82 \times 6 = 492$, what do you think is the answer to 82×0.6? Where does the decimal point go?

b If $4 \times 17 = 68$, what do you think is the answer to 0.4×1.7?

c If $367 \times 51 = 18\,717$, what do you think is the answer to 3.67×5.1?

d What is the answer to 0.5×0.9?

Foundation Standard Complex

7.06 Multiplying decimals

Multiplying decimals

When **multiplying decimals**, the number of decimal places in the answer equals the total number of decimal places in the question.

Video Multiplying decimals

Worksheets Shopping and change

Decimal cards

Puzzle Which decimals?

Interactive Area Model Decimals

Example 10

Evaluate each product.

a 0.6×4.1

b 3.06×4.8

SOLUTION

a Complete the multiplication without decimal points.

$$\begin{array}{r} 41 \\ \times\ 6 \\ \hline 246 \end{array}$$

Decide where to place the decimal point by counting decimal places or by estimating.

Method 1

0.6 has 1 decimal place, 4.1 has 1 decimal place.

So the answer has 2 decimal places.

Insert the decimal point so that the answer has 2 decimal places:

$0.6 \times 4.1 = \mathbf{2.46}$

Method 2

Estimate the answer: $0.6 \times 4.1 \approx 1 \times 4 = 4$

Insert the decimal point so that the answer is near 4:

$0.6 \times 4.1 = \mathbf{2.46}$

b Complete the multiplication without decimal points.

$$\begin{array}{r} 306 \\ \times\ 48 \\ \hline 2448 \\ 12240 \\ \hline 14688 \end{array}$$

Method 1

3.06 has 2 decimal places, 4.8 has 1 decimal place.

So we need $2 + 1 = 3$ decimal places.

Insert the decimal point so that the answer has 3 decimal places:

$3.06 \times 4.8 = \mathbf{14.688}$

Method 2

Estimate the answer: $3.06 \times 4.8 \approx 3 \times 5 = 15$

Insert the decimal point so that the answer is near 15.

$3.06 \times 4.8 = \mathbf{14.688}$

EXERCISE 7.06 ANSWERS ON P. 626

Multiplying decimals

U F PS C

1 How many decimal places will each product have?

C **a** 0.25×11 **b** 10.2×4 **c** 0.5×10
d 7×2.193 **e** 0.9×0.75 **f** 8.06×4.1
g 0.11×1.01 **h** 6.3×0.04 **i** 2.95×5.3
j 0.237×1.2 **k** 0.023×0.042 **l** 321.2×8.1

EXAMPLE 10

2 Evaluate 0.4×5.76. Select the correct answer **A**, **B**, **C** or **D**.

A 0.2304 **B** 2.304 **C** 23.04 **D** 230.4

3 Evaluate each product.

a 3.05×4 **b** 1.02×7 **c** 2.001×9 **d** 17.1×2
e 10×2.25 **f** 3×4.2 **g** 6.95×5 **h** 1.004×8
i 0.18×5 **j** 0.4×12 **k** 6×0.002 **l** 8×6.75
m 0.4×0.8 **n** 3.9×0.5 **o** 0.8×0.6 **p** 0.3×0.24

4 Estimate each product, then evaluate the product.

a 47.9×0.3 **b** 6.43×7.2 **c** 83.4×6.3

5 Dress material costs \$31.24 per metre. How much does 5.2 metres of material cost?

6 Sean earns \$18.45 per hour working at a cinema. How much will he earn for working a 39-hour week?

7 A flower bed is 2.6 m long by 0.8 m wide. Calculate its area.

8 Calculate the total of this grocery shopping bill. Select **A**, **B**, **C** or **D**.

PS
- 500 g of butter at \$1.20 per 100 g
- 500 g of cheese at \$8.40 per kilogram
- 2 kg of chicken at \$9.50 per kilogram
- 2 dozen eggs at \$5.95 per dozen.

A \$25.05 **B** \$35.70 **C** \$41.10 **D** \$53.70

Foundation Standard Complex

7.07 Dividing decimals by whole numbers

Dividing decimals by whole numbers

When dividing a decimal by a whole number:

- rewrite the question in 'short division' form
- make the decimal point in the answer line up with the decimal point in the question
- add 0s to the end of the decimal being divided, if needed

Example 11

Evaluate each quotient.

a $10 \div 4$ **b** $0.018 \div 6$ **c** $2.66 \div 8$

SOLUTION

a

$$\begin{array}{r} 2.\ 5 \\ 4\overline{)10.{}^{2}0} \end{array}$$

$10 \div 4 = 2.5$

Write 10 as 10.0 so that you can complete the division.

b

$$\begin{array}{r} 0.00\,3 \\ 6\overline{)0.01{}^{1}8} \end{array}$$

$0.018 \div 6 = 0.003$

c

$$\begin{array}{r} 0.3\,3\,2\,5 \\ 8\overline{)2.6{}^{2}6{}^{2}0{}^{4}0} \end{array}$$

$2.66 \div 8 = 0.3325$

Write 2 zeros after 2.66 so that you can complete the division.

EXERCISE 7.07 ANSWERS ON P. 626

Dividing decimals by whole numbers

U F R

EXAMPLE 11

1 Evaluate each quotient.

a $4.8 \div 2$ **b** $18.6 \div 3$ **c** $20.8 \div 5$

d $32.8 \div 8$ **e** $29.3 \div 2$ **f** $8.79 \div 4$

g $0.056 \div 7$ **h** $10.71 \div 4$ **i** $195.6 \div 8$

j $7.35 \div 2$ **k** $4.15 \div 8$ **l** $0.318 \div 3$

m $12 \div 5$ **n** $13.56 \div 4$ **o** $23 \div 8$

p $256.84 \div 4$ **q** $107.1 \div 9$ **r** $82.5 \div 6$

s $0.732 \div 6$ **t** $2075.6 \div 8$

Foundation Standard Complex

2 Yesterday, Anh earned $137.12 for working 8 hours at the chemist. How much was he paid per hour?

3 Trent took 5 hours to travel 163.5 km on his bike. What was his average speed in km/h? Select the correct answer **A**, **B**, **C** or **D**.

A 30.6 **B** 32.6 **C** 8.175 **D** 32.7

4 At the restaurant, the bill for 6 friends totalled $366.96. How much should each friend pay if the bill is to be split evenly?

5 Nadine ran 100 metres in the following times (in seconds):

11.4 12.1 11.5 11.3 11.6 11.7

a Calculate the sum of Nadine's 6 times.

b Calculate Nadine's average time.

6 Copy and complete each equation with decimals.

R **a** _____ ÷ 8 = 3.1 **b** _____ ÷ 5 = 2.46 **c** _____ ÷ _____ = 6

Dividing decimals 7.08

Look at this pattern:

18 ÷ 3 = 6

180 ÷ 30 = 6

1800 ÷ 300 = 6

This is like writing them as equivalent fractions: $\frac{18}{3} = \frac{180}{30} = \frac{1800}{300}$.

When dividing numbers, if we multiply both numbers by the **same number** first, the answer stays the same. We can use this property to help us divide decimals. For example:

9.8 ÷ 0.08 = 980 ÷ 8 (multiplying both numbers by 100)

= 122.5

980 ÷ 8 is easier to evaluate than 9.8 ÷ 0.08 because 8 is a whole number.

Worksheets
Decimals writing activity
Decimals 7
Decimal number grid

Puzzle
Operations with decimals

Dividing decimals

To divide a decimal by a decimal:

- make the second decimal a whole number by moving the decimal point the required number of places to the right
- move the point in the first decimal the same number of places to the right
- divide the new first number by the whole number

This works because we multiply both decimals by the same power of 10 before dividing.

Foundation Standard Complex

Video
Dividing decimals

Example 12

Evaluate each quotient:

a $0.4 \div 0.2$ **b** $1.75 \div 0.5$ **c** $122.4 \div 0.03$

SOLUTION

a $0.4 \div 0.2 = 0.4 \div 0.2$ — Move both decimal points one place to the right so that 0.2 becomes a whole number

multiplying both decimals by 10

$= 4 \div 2$

$= 2$

b $1.75 \div 0.5 = 1.75 \div 0.5$ — Move both decimal points one place to the right.

$= 17.5 \div 5$

$= 3.5$

c $122.4 \div 0.03 = 122.40 \div 0.03$ — Move both decimal points 2 places to the right so that 0.03 becomes a whole number.

multiplying both decimals by 100

$= 12\,240 \div 3$

$= 4080$

EXERCISE 7.08 ANSWERS ON P. 626

Dividing decimals

U F R C

1 Rewrite each division expression so that the second decimal is a whole number.

a $508.8 \div 0.8$ **b** $17.92 \div 0.07$ **c** $333 \div 0.9$

d $1.725 \div 0.05$ **e** $129.2 \div 0.4$ **f** $49.5 \div 0.06$

g $168 \div 0.7$ **h** $14.823 \div 0.09$ **i** $0.66 \div 0.3$

EXAMPLE 12

2 Evaluate each quotient, and check that your answers seem reasonable by estimating.

a $3.48 \div 0.4$ **b** $7.32 \div 0.2$ **c** $2.94 \div 0.6$

d $16.28 \div 0.08$ **e** $27 \div 0.9$ **f** $10.08 \div 0.8$

g $10.4 \div 0.05$ **h** $5.6 \div 0.07$ **i** $1.71 \div 0.3$

j $40.82 \div 0.02$ **k** $0.532 \div 0.5$ **l** $0.7812 \div 0.006$

3 **a** $18 \div 0.5$ means 'how many times does 0.5 go into 18?'.
R Is the answer more or less than 18? Why?

C **b** Estimate the answer to $18 \div 0.5$.

c Evaluate $18 \div 0.5$.

d Show that $18 \div \frac{1}{2}$ gives the same answer.

4 **a** $20.4 \div 0.3$ means 'how many times does 0.3 go into 20.4?'.
R Is the answer more or less than 20.4?

C **b** Estimate the answer to $20.4 \div 0.3$.

c Find the exact answer to $20.4 \div 0.3$.

Foundation Standard Complex

5 What happens when you divide by a number less than 1? Is the answer more or less than the number? (Check your answers to question **2**).

R C

6 Which of the following is the answer to 13.59 ÷ 0.03? Select **A**, **B**, **C** or **D**.

A 45.3 **B** 453 **C** 4.53 **D** 4530

7 A drink bottle holds 0.8 litres. How many drink bottles can be filled from a container that holds 12.8 litres?

8 A square wall tile has length 0.3 m. If a bathroom wall has length 5.2 m, how many whole tiles are needed for one row along the wall?

R

9 James can cycle 0.7 km in one minute.

a How far will he travel in 20 minutes?

b How long will it take him to travel 26.6 km?

10 Copy and complete each equation with the correct decimals.

R **a** 27.2 ÷ ____ = 8 **b** _____ ÷ 5 = 2.46 **c** ____ ÷ ____ = 6.2

INVESTIGATION

Back-to-front problems

The cards for this set of questions have been printed without any decimal points. Insert the decimal points so that the numbers on the cards fit the clues.

1 The difference between these 2 numbers is 53.7. The sum of the numbers is 58.5.

561 24

2 The difference between the numbers is 178.8. When you divide the greater number by the smaller number, the quotient is between 43 and 44.

183 42

3 The sum of the 3 numbers is 5.36.The product of the numbers is 1.2096.

8 42 36

4 The sum of the 3 numbers is 4.61. The product of the numbers is 0.9135.

15 29 21

5 The sum of the 4 numbers is 2.55.The product of 2 of the numbers is 0.196. The product of the other 2 numbers is 0.217.

7 31 14 14

Foundation Standard Complex

7.09 Terminating and recurring decimals

Skillsheet Fractions and decimals

Puzzle Decimals squaresaw 2

When fractions are converted to decimals, the decimals can be **terminating** or **recurring**.

Terminating and recurring decimals

Terminating decimals have a definite number of decimal places, such as 0.625. The word 'terminate' means 'to stop'.

Recurring decimals are decimals whose digits repeat endlessly, such as 0.272727... The word 'recurring' means 'repeating'.

Terminating decimals

Video Converting fractions to decimals

Example 13

Convert each fraction into a decimal.

a $\frac{3}{5}$ **b** $\frac{5}{8}$

SOLUTION

a $\frac{3}{5}$ means $3 \div 5$

$$\begin{array}{r} 0.6 \\ 5\overline{)3.0} \end{array}$$

Remember to add 0s, if necessary, to complete the division.

$\frac{3}{5} = 0.6$

Note also that $\frac{3}{5} = \frac{6}{10} = 0.6$.

b $\frac{5}{8}$ means $5 \div 8$

$$\begin{array}{r} 0.625 \\ 8\overline{)5.000} \\ {\scriptstyle 2\ 4} \end{array}$$

$\frac{5}{8} = 0.625$

Recurring decimals

YEAR 8 EXTENSION

Video Converting fractions to decimals

Example 14

Convert each fraction to a decimal.

a $\frac{1}{3}$ **b** $\frac{5}{6}$ **c** $\frac{2}{11}$

SOLUTION

a $\frac{1}{3}$ means $1 \div 3$

$$\begin{array}{r} 0.333\ldots \\ 3\overline{)1.000\ldots} \end{array}$$

$\frac{1}{3} = 0.333... = 0.\dot{3}$ or $0.\overline{3}$

b $\frac{5}{6}$ means $5 \div 6$

$$\begin{array}{r} 0.833\ldots \\ 6\overline{)5.0000\ldots} \end{array}$$

$\frac{5}{6} = 0.8333... = 0.8\dot{3}$ or $0.8\overline{3}$

c $\frac{2}{11}$ means $2 \div 11$

$$\begin{array}{r} 0.18181\ldots \\ 11\overline{)2.00000\ldots} \end{array}$$

$\frac{2}{11} = 0.181\,818... = 0.\dot{1}\dot{8}$ or $0.\overline{18}$

The answers in the previous example are all **recurring decimals**, where one or more of the digits in the decimal repeat endlessly. To show this, we use dots or a line to mark the repeating section: for example, 0.259 259 259... = $0.\dot{2}5\dot{9}$ or $0.\overline{259}$.

EXERCISE 7.09 ANSWERS ON P. 626

Terminating and recurring decimals

U F R C

EXAMPLE 13

1 Copy and complete this table. Use a calculator if you need to.

Common fraction	Meaning as division	Decimal
$\frac{3}{5}$	3 ÷ 5	0.6
$\frac{1}{2}$	1 ÷ 2	
$\frac{1}{4}$		0.25
$\frac{4}{5}$		
$\frac{2}{5}$		
	3 ÷ 4	0.75
	1 ÷ 5	
	1 ÷ 8	

2 Use your answers from question **1** to help you convert each fraction into a decimal.

a $\frac{2}{5}$ **b** $\frac{3}{8}$ **c** $\frac{3}{4}$ **d** $\frac{2}{2}$

e $\frac{2}{4}$ **f** $\frac{6}{8}$ **g** $\frac{3}{5}$ **h** $\frac{2}{8}$

i $\frac{5}{8}$ **j** $\frac{7}{8}$ **k** $\frac{4}{8}$ **l** $\frac{5}{5}$

3 Explain why some of the fractions in question **2** have the same decimal value.

R C

4 Write each mixed numeral as a decimal.

a $4\frac{8}{10}$ **b** $23\frac{3}{4}$ **c** $12\frac{5}{8}$ **d** $6\frac{3}{5}$

e $57\frac{2}{5}$ **f** $19\frac{1}{8}$ **g** $110\frac{7}{8}$ **h** $80\frac{1}{4}$

5 Rewrite each recurring decimal using the dot notation.

C **a** 0.41666666 ... **b** 0.27272727...

c 0.111111 ... **d** 0.1027510275 ...

Foundation Standard Complex

YEAR 8 EXTENSION

EXAMPLE 14

6 Write each recurring decimal showing the repeated pattern.

C **a** $0.858\dot{3}$ **b** $0.0\dot{4}\dot{5}$ **c** $0.\dot{7}$ **d** $0.\dot{4}6153\dot{8}$

7 Convert each fraction to a recurring decimal.

C **a** $\frac{1}{9}$ **b** $\frac{1}{6}$ **c** $\frac{5}{6}$ **d** $\frac{1}{7}$ **e** $\frac{2}{3}$

f $\frac{2}{7}$ **g** $\frac{2}{9}$ **h** $\frac{3}{7}$ **i** $\frac{4}{7}$ **j** $\frac{4}{9}$

k $\frac{4}{6}$ **l** $\frac{5}{9}$ **m** $\frac{6}{7}$ **n** $\frac{7}{9}$ **o** $\frac{8}{9}$

8 Copy and complete the table and note the pattern.

R C

Fraction	$\frac{1}{9}$	$\frac{2}{9}$	$\frac{3}{9}$	$\frac{4}{9}$	$\frac{5}{9}$	$\frac{6}{9}$	$\frac{7}{9}$	$\frac{8}{9}$
Decimal								

INVESTIGATION

Prime factors and recurring decimals

What types of fractions convert to recurring decimals rather than terminating decimals? It has something to do with the denominator of the fraction and its prime factors.

1 Copy and complete the table below, using a calculator to convert each fraction to a decimal.

Fraction	Prime factors of denominator	Decimal equivalent	Terminating or recurring?
$\frac{1}{2}$	2		
$\frac{1}{3}$		$0.\dot{3}$	Recurring
$\frac{1}{4}$			
$\frac{1}{5}$			
$\frac{1}{6}$	2 and 3		
$\frac{1}{7}$			
$\frac{1}{8}$		0.125	Terminating
$\frac{1}{9}$			

Foundation Standard Complex

YEAR 8 EXTENSION

7.09

Fraction	Prime factors of denominator	Decimal equivalent	Terminating or recurring?
$\frac{1}{10}$	2 and 5		
$\frac{1}{11}$			
$\frac{1}{12}$			
$\frac{1}{13}$	13		
$\frac{1}{14}$			
$\frac{1}{15}$		$0.0\dot{6}$	Recurring
$\frac{1}{17}$			
$\frac{1}{19}$			
$\frac{1}{20}$	2 and 5		
$\frac{1}{25}$			
$\frac{1}{30}$			
$\frac{1}{45}$	3 and 5		
$\frac{1}{50}$		0.02	Terminating

2 What do you notice about the prime factors of the denominators of fractions that convert to terminating decimals?

3 If the prime factors include a number other than 2 and 5, is the equivalent decimal terminating or recurring?

4 Use your understanding from this investigation to predict whether each fraction will convert to a terminating or recurring decimal.

a $\frac{1}{40}$ **b** $\frac{1}{60}$ **c** $\frac{1}{80}$ **d** $\frac{1}{100}$

e $\frac{1}{110}$ **f** $\frac{1}{140}$ **g** $\frac{1}{200}$ **h** $\frac{1}{225}$

7.10 Rounding decimals

Sometimes, to approximate an answer with many decimal places, we round to fewer decimal places. We need to be able to **round** when working with money, measuring quantities or writing answers to division calculations.

A decimal number line can be used to help you round off to a specified number of decimal places. You will need to cut at the required decimal place to set up the number line and then also indicate the half-way value.

Example 15

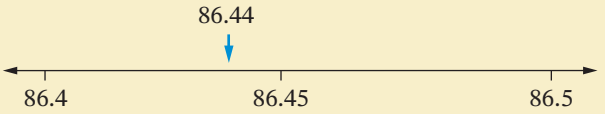
Use a number line to:

a round 86.442 to one decimal place

b round 17.567 to 2 decimal places

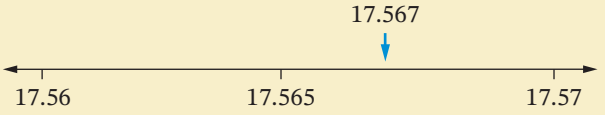
SOLUTION

a
- 86.44 lies somewhere between 86.4 and 86.5 (one decimal place), so draw a number line that has these values at either end
- Halfway between 86.4 and 86.5 is 86.45, so plot this value
- 86.44 is less than 86.45, so plot 86.44 to the left of 86.45



- From the number line, 86.44 is closer to 86.4 than 86.5 (one decimal place), so 86.44 rounds down to 86.4.

b
- 17.567 lies somewhere between 17.56 and 17.57 (2 decimal places), so draw a number line that has these values at either end
- Halfway between 17.56 and 17.57 is 17.565, so plot this value
- 17.567 is more than 17.565, so plot 17.567 to the right of 17.565



- From the number line, 17.567 is closer to 17.57 than 17.56 (2 decimal places), so 17.567 rounds up to 17.57.

Note: If the number is exactly on the halfway mark (17.565), then round up (to 17.57).

A simpler process can be used based on the value of the digit after the required number of decimal places.

7.10

Rounding decimals

To round a decimal, 'cut' it at the required decimal place and look at the digit in the next place:

- if the digit is less than 5 (that is, 0, 1, 2, 3 or 4), **round down**
- if the digit is 5 or more (that is, 5, 6, 7, 8 or 9), **round up**

Example 16

a Round 86.246 to:

i one decimal place **ii** 2 decimal places

b Write 0.087 1245 correct to:

i one decimal place **ii** the nearest thousandth

Video Rounding decimals

SOLUTION

a **i** 86.2 46

cut the next digit is 4 (less than 5), so round **down**

So 86.246 ≈ 86.2 (correct to one decimal place).

ii 86.24 6

cut the next digit is 6 (more than 5), so round **up** to 86.25

So 86.246 ≈ 86.25 (correct to 2 decimal places).

b **i** 0.0 87 1245

cut the next digit is 8 (more than 5), so round **up** to 0.1

0.087 1245 ≈ 0.1

ii 0.087 1245

cut the next digit is 1 (less than 5), so round **down**

'the nearest thousandth' means 3 decimal places 0.087 1245 ≈ 0.087

Rounding to decimal places

Rounding to:

- the nearest tenth = one decimal place
- the nearest hundredth = 2 decimal places
- the nearest thousandth = 3 decimal places

It is important to consider the real-world situation or the circumstances when rounding quantities. You need to think logically about whether you should round up or round down for the situation.

Example 17

For each situation, decide whether to round up or round down. Explain why your rounding was appropriate.

a One litre of paint can cover and area of 16 m^2. How many litres of paint are required to paint an area of 100 m^2?

b The recipe for a vanilla butter cake requires 430 g of self-raising flour. How many cakes can be baked with a 5 kg bag of flour?

SOLUTION

a Amount of paint needed $= 100 \div 16$

$= 6.25$ L

≈ 7 L

Round the answer up to the next whole litre because rounding down to 6 litres would not provide enough paint for the job.

b First convert 5 kg to grams.

$5 \times 1000 = 5000$ g

No. of cakes $= 5000 \div 430$

$= 11.63$

≈ 11

Round the answer down to 11 cakes because there is not enough flour to make the 12th cake.

$11 \times 430 = 4730$ g, with only 270g of flour left, which is not enough to make another cake.

EXERCISE 7.10 ANSWERS ON P. 626

Rounding decimals

U F PS R C

EXAMPLE 15

1 Use a number line to help you round:

C **a** 0.743 to one decimal place **b** 1.767 to 2 decimal places

c 16.564 to 2 decimal places **d** 82.873 to one decimal place

e 478.749 to one decimal place **f** 823.549 to 2 decimal places

EXAMPLE 16

2 Write each decimal correct to one decimal place.

C **a** 0.35 **b** 0.47 **c** 0.81 **d** 0.69

e 2.55 **f** 0.32 **g** 0.90 **h** 2.88

3 Round each decimal to 2 decimal places.

C **a** 0.481 **b** 0.736 **c** 0.069 **d** 0.293

e 0.309 **f** 0.655 **g** 2.096 **h** 3.995

Foundation Standard Complex

4 Copy and complete this table. Use a calculator to help you.

C

	Question	Calculator display	Rounded to the nearest tenth	Rounded to the nearest hundredth
a	12.19 ÷ 3	4.0633333333	4.1	4.06
b	12.32 ÷ 6			
c	19.82 ÷ 9			
d	56.85 ÷ 11			
e	17.13 ÷ 4			
f	12.65 ÷ 12			
g	4.875 ÷ 21			
h	27.45 ÷ 8			
i	17 ÷ 12			
j	254.678 ÷ 32			

5 Write each number correct to 2 decimal places.

C

a 25.3759 **b** 341.6143 **c** 420.8971

d 734.6541 **e** 27.359 94 **f** 1314.210 47

6 Round each number to 4 decimal places.

C

a 10.333 74 **b** 431.543 27 **c** 1.444 95

d 3217.654 061 **e** 4.670 89 **f** 0.888 88

7 Calculate the answer to each problem but round up or down appropriate to the situation. Give the reason why you rounded up or down.

EXAMPLE 17

PS R C

a A box of chocolates with 44 chocolates is shared among a family of 5 people. How many chocolates does each person receive?

b A new bathroom requires 32 square metres of tiles. A box of tiles covers 0.6 square metres of area. How many boxes are needed to tile the bathroom?

c A team of 4 golfers wins 27 new golf balls in a competition. How many does each person receive?

d Some timber comes in 0.9 m lengths. How many lengths are needed to build a chicken house needing 23 m of timber?

e One dress requires 1.3 m of material. How many dresses can be made from a 5 m length of material?

f Hannah is baking 6 batches of brownies for the school bake stall and needs 0.328 kg of sugar for each batch. How much sugar should Hannah buy, correct to the nearest kilogram?

Foundation Standard Complex

g Lachlan is buying all his friends a personalised water bottle for Christmas this year. Each bottle costs $8.47 and Lachlan has $100 to spend. How many water bottles can he buy?

h A pair of pants require 1.46 m of material to be made. How much material should be purchased to make 22 pairs of pants if the material can be purchased to the nearest half-metre?

8 Write a decimal that could be rounded to:

R **a** 2.718 **b** 0.8 **c** 25.6

9 Australia had 1c coins and 2c coins until 1990. After that, supermarket prices had to be rounded to the nearest 5 cents. Find out how prices are rounded this way.

R

C

TECHNOLOGY

Rounding decimals

This activity will show you how to format a cell so that its value is rounded to a specific number of decimal places. Start a new spreadsheet.

1. Enter **748.61** into cell A1, **x** (for multiply) into cell A2 and **3.75** into cell A3.
2. Enter a formula into cell A4 for the product of A1 and A3 and **bold** your answer.
3. To round this answer to 1 decimal place, **right-click** on cell A4, choose **Format Cells**, **Number**, and 1 decimal place.
4. Enter **748.61** into cell B1, '/ (for divide, including the ') into cell B2 and **3.75** into cell B3.
5. Enter a formula into cell B4 for B1 divided by B3 and **bold** your answer.
6. To round this answer to 2 decimal places, **right-click** on cell B4, choose **Format Cells**, **Number**, and 2 decimal places.
7. Complete each calculation using your spreadsheet and creating appropriate formulas.
   - **a** 284.796 ÷ 32.4
   - **b** 1217.9 ÷ 45.6 (round to 3 decimal places)
   - **c** 1.15 ÷ 1.5 (round to 2 decimal places)
   - **d** 1604.12 ÷ 0.02 ÷ 4.578
   - **e** (8756.32 – 9025.198 + 1023.5697) ÷ 1.444 (round to 4 decimal places)
   - **f** (12.3 + 6.59) ÷ (56.4 × 0.04) (round to 5 decimal places)

Foundation Standard Complex

DID YOU KNOW?

Scoring in Olympics gymnastics

How is the score for a gymnast in an Olympic competition calculated? Each gymnast starts with a score of 10.0 plus an allowance for the level of difficulty of the routine. Deductions are made from 10.0 for errors, missteps or falls. The deductions can be as low as 0.01 or multiples of 0.01 for small errors. Higher deductions for a fall or a bigger mistake can be 0.1 or multiples of 0.1.

Shutterstock.com/I T A L O

High-level competitions have 6 judges and each judge gives a score. The highest and lowest scores are removed, and the remaining 4 scores are averaged to give the athlete's score. The difficulty score is then added. Team events combine the scores of individual athletes.

What were the highest scores in the last Olympic gymnastics competition?

In which events are the highest scores found?

Decimal problems 7.11

EXERCISE 7.11 ANSWERS ON P. 627

Decimal problems

U F PS R C

1 A farmer wants to fence a rectangular paddock. The paddock is 35.6 metres long and 20.85 metres wide. How many metres of fencing will be needed? PS

2 A drink bottle holds 0.6 litres. How many drink bottles can be filled from a tub that holds 4.5 litres? PS

3 A car travels 110.3 kilometres on 7 litres of petrol. How many kilometres would the car travel on one litre of petrol? (Give the answer correct to one decimal place.)

4 Anja runs 3.8 kilometres each day of the week. How far does she run in one week?

Worksheets
Shopping and change
Calculating change

Foundation Standard Complex

5 These calculator displays show amounts of money in decimal form. Rewrite each amount in dollars and cents, to the nearest cent.

C

a 15.236 **b** 6.8412 **c** 487.759318 **d** 1234.047126

6 Find the cost of 352 units of electricity at 25.3 cents per unit. Select the correct answer **A**, **B**, **C** or **D**.

A $8905.60 **B** $890.56 **C** $89.06 **D** $8.91

7 Mark buys golf balls for $4.85 each and sells them for $5.15 each. How much money does he make if he sells 30 golf balls?

PS

8 A long distance train is made up of a diesel engine, 2 dining cars and 15 passenger carriages. The engine has a mass of 20.2 tonnes, each dining car has a mass of 14.35 tonnes and each passenger carriage has a mass of 13.96 tonnes. How heavy is the entire train?

PS

9 Samir is cutting shelves from a board that is 4.6 metres long. Each shelf needs to be 0.9 metres long. How many shelves can be cut?

10 This table lists some Brisbane FM radio stations and their allocated frequencies measured in megahertz (MHz).

PS R C

Station	Frequency (MHz)	Station	Frequency (MHz)
SBS	93.3	Classic FM	103.7
River	94.9	Triple M	104.5
96Five	96.5	B105	105.3
97.3 FM	97.3	ABC Classic	106.1
98.9 FM	98.9	Nova	106.9
Bay FM	100.3	Triple J	107.7

a Copy this number line, then locate the stations on it according to their frequencies.

b What is the frequency difference in megahertz between B105 and 98.9 FM?

c Find the smallest frequency difference between adjacent stations. ('Adjacent' means side-by-side.)

d What is the largest difference in frequency between adjacent stations?

11 A sheet of cardboard is 0.3 mm thick. How many sheets would be in a stack that is 3.6 cm high?

PS

12 Holly walks to work and back each day. She works 6 days a week and, in one week, walks 16.8 kilometres. How far is Holly's apartment from work?

PS R

Foundation Standard Complex

TECHNOLOGY

Fruit and vegetables

The spreadsheet below shows the items on Zoe's shopping list. Follow the steps below to complete the 'Cost' column and answer the questions.

1 Zoe has done her fruit and vegetables shopping for the week. Enter the items into a spreadsheet, as shown below. (Centre values, bold headings, include cell borders and $ signs for column B values.)

	A	B	C	D
1	**Item**	**Cost per kg**	**kg bought**	**Cost**
2	Oranges	$3.99	1	
3	Apples	$4.99	0.5	
4	Kiwi fruit	$2.99	0.4	
5	Bananas	$5.60	1.2	
6	Tomatoes	$7.49	1.5	
7	Grapes	$3.50	1	
8	Carrots	$1.55	2	
9	Potatoes	$2.45	5	
10	Beans	$6.95	0.355	
11	Pumpkins	$3.20	0.562	
12	Zucchinis	$2.85	0.38	

2 Write a formula in cell D2 to calculate the cost of the oranges.

3 Use **Fill Down** to calculate the cost of each item purchased.

4 In cell C13, enter the label 'Total cost'.

5 In cell D13, write a **sum** formula to find the cost of Zoe's shopping.

6 In cell C14, enter the label 'Cash'. In cell C15, enter the label 'Change'.

7 If Zoe paid $50 for her shopping, enter this value into cell D14 and in cell D15 write a formula to calculate the amount of change Zoe would have received.

8 Zoe paid cash, but because the change was an irregular amount, she could not be given the amount in cell D15 in coins. In cell C16, enter the label 'Rounded change'. In cell D16, enter the amount of change Zoe actually received.

9 Create a spreadsheet with similar formatting to calculate a different shopping list.

POWER PLUS ANSWERS ON P. 627

1 Copy and complete this table and note the pattern.

Fraction	$\frac{1}{7}$	$\frac{2}{7}$	$\frac{3}{7}$	$\frac{4}{7}$	$\frac{5}{7}$	$\frac{6}{7}$
Decimal						

2 Investigate the value of $0.\dot{9}$.

3 Evaluate each expression.

a $(0.02)^2$ b $(0.02)^3$ c $(1.1)^3$

d $\sqrt{0.04}$ e $\sqrt{0.36}$ f $\sqrt[3]{0.027}$

4 Wolf, the warrior, is trying to break a decimal code that will open the dungeon doors. He has to free the prisoners before midnight so that they will not be turned into frogs by an evil spell. Each castle door is operated by a combination lock. Use the following clues to match the various combinations with the doors of the different rooms in the castle.

Combinations	Rooms
8.262	Queen's chamber
9.24	armoury
9.96	throne room
8.07	banquet hall
8.16	kitchen
8.79	dungeon

Clues

- Combination 9.24 opens a door to a room that deals with food.
- The combination to the armoury has a 6 in the hundredths place.
- The combinations of the throne room and the banquet hall add to 18.03.
- The Queen's chamber has a combination that is smaller than 25.11 ÷ 3.1.
- The kitchen combination is one of the 3 largest combination numbers.
- The doors in the dungeon and the throne room remained locked when Wolf tried 9.96 and 8.262.

5 Evaluate 28.98 ÷ 1.2 without using a calculator.

6 Decide where the decimal points should be so that the numbers in the ovals fit the clues.

a The product of the 2 numbers is 91.02.

The sum of the numbers is 28.3.

37 246

b The sum of the 3 numbers is 14.

The product of the 3 numbers is 78.66.

23 57 6

c The sum of the 4 numbers is 28.32.

The product of 2 of the numbers is 3.672.

51 72

45 18

The product of the other 2 is 81.

7 CHAPTER REVIEW

Language of maths

Quiz
Language of maths 7

ascending	decimal	decimal place	decimal point
descending	estimate	fraction	hundredth
power of 10	recurring decimal	round down	round up
tenth	terminating decimal	thousandth	

1 What is the collective name for numbers such as 10, 100 and 1000?

2 'The bushfire decimated the possum population of the forest.' Look up the meaning of the word '**decimate**'.

3 What is a **recurring** dream or a **recurring** back pain? What does 'recurring' mean?

4 Explain the steps in rounding 3.14159 to 3 decimal places.

5 What is a 'decimetre'?

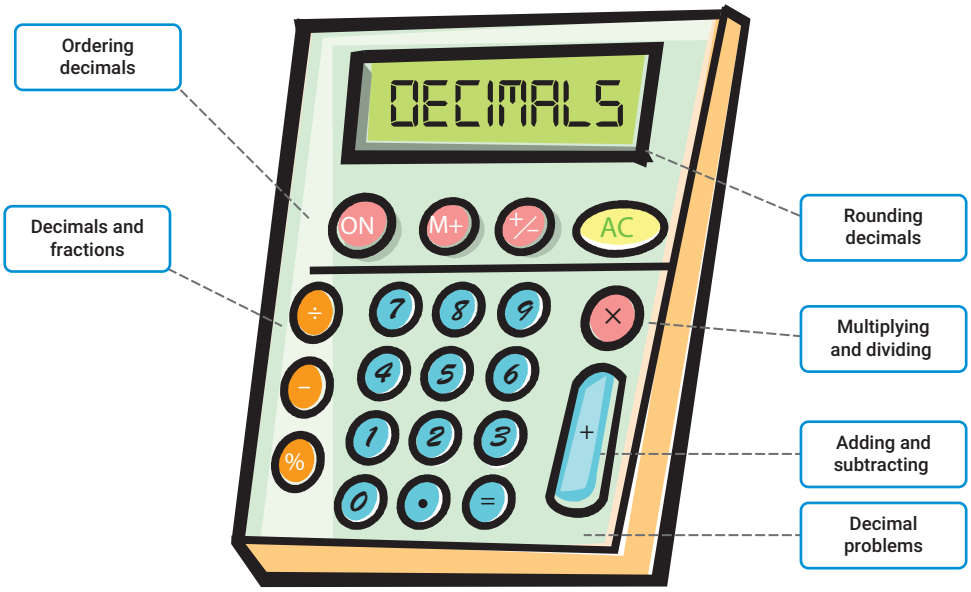
6 How many decimal places do you round to if you are rounding to the nearest hundredth?

Topic summary

Worksheet
Mind map: Decimals

- What parts of this topic were new to you? What parts did you already know?
- Write any rules you have learnt about working with decimals.
- What parts of this topic did you not understand? Be specific. Talk to a friend or your teacher about them.
- Give 3 examples of where decimals are used.

Print (or copy) and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.



7 TEST YOURSELF

ANSWERS ON P. 627

7.01 **1** **a** Arrange 34.98, 36.86, 3.998, 38.141, 340, 34.89 in ascending order.

b Arrange 1.015, 1.293, 1.1015, 1.239, 1.006 in descending order.

Quiz
Test yourself 7

7.02 **2** Convert each decimal to a simplified fraction.

a 0.5 **b** 0.89 **c** 0.09 **d** 0.444

7.02 **3** Convert each fraction to a decimal.

a $\frac{4}{10}$ **b** $\frac{13}{100}$ **c** $\frac{7}{100}$ **d** $\frac{11}{1000}$

7.03 **4** Evaluate each expression.

a $12.35 + 4.53 + 0.56 + 3.125 + 24.7 + 20.09$

b $214.33 - 109.84$

c $0.568 + 23 + 4.027 - 16.28$

d $1600.8 - 562.9$

e $1453.6 + 1287.31 - 2344.4$

f $9.23 - 6.851$

7.04 **5** Evaluate each expression.

a 7.54×10 **b** 7.54×100 **c** 7.54×1000

d $13.9 \div 10$ **e** $13.9 \div 100$ **f** $13.9 \div 1000$

7.05 **6** Given that $42 \times 76 = 3192$, find:

a 4.2×76 **b** 4.2×7.6 **c** 0.42×760

d 4.2×0.76 **e** 42×7.6 **f** 0.42×0.76

7.06 **7** Evaluate each product.

a 2.75×6 **b** 0.5×1.2 **c** 72.23×4

d 6.1×1.2 **e** 0.92×5 **f** 3×9.7

g 3.25×0.41 **h** 0.05×0.02 **i** 4.67×1.1

7.07 **8** Evaluate each quotient.

a $762.4 \div 2$ **b** $97.6 \div 8$ **c** $2.75 \div 4$ **d** $195.6 \div 3$

7.09 **9** Evaluate each quotient.

a $12.5 \div 0.5$ **b** $12.72 \div 0.4$ **c** $6.9 \div 0.03$ **d** $0.508 \div 0.02$

Foundation Standard Complex

9780170465557

10 Convert each fraction to a decimal. YEAR 8 EXTENSION 7.10

a $\frac{4}{5}$ **b** $\frac{3}{8}$

c $\frac{5}{9}$ **d** $\frac{2}{3}$

11 Round: 7.10

a 406.28 to the nearest tenth

b 125.724 to 2 decimal places

c 2345.876 to 1 decimal place

d 3.8967 to the nearest hundredth

e 78 654.056 to 2 decimal places

f 678.4309 to the nearest thousandth

12 Calculate the answer to each problem but round up or down appropriate to the situation. Give the reason why you rounded up or down. 7.10

a Five friends go out for lunch and decide to split the $63.65 bill evenly, correct to the nearest dollar. How much does each person contribute?

b A driveway requires 22 square metres of pavers. A box of pavers covers 0.6 square metres of area. How many boxes are needed to pave the driveway?

c A box of ice-creams has 16 ice-creams in it. There are 3 children in a family who are going to share these ice-creams. How much does each child get?

d Timber comes in 1.2 m lengths. How many lengths are needed to build a fence needing 107 m of timber?

13 The Liverpool Women's Cricket Club is having a pizza night. They order 16 Super Supreme pizzas at $19.70 each and 10 Hawaiian pizzas at $15.50 each. How much will the club spend on pizzas? 7.11

14 Riley bought 825 bricks for $1105.50. How much did one brick cost? 7.11

15 Mia saved $220 to go to a rock concert. Her return fare cost $18.80, her concert ticket cost $116.75, the program cost $25 and food cost $37.70. She did not have enough to buy the band's concert video (priced $59.95) after the concert. How much did she need to borrow from her friend Sam to buy the video? 7.11

Foundation Standard Complex

8

MEASUREMENT

Area and volume

The largest shopping centre in Australia is Chadstone Shopping Centre in Melbourne. It covers an area of 215 574 square metres, which is about the size of 30 soccer fields. The largest city square in Australia is Adelaide's Victoria Square, measuring 322 m by 161 m, with an area of 51 842 square metres, or 7 soccer fields. This means that about 4.2 Victoria Squares will fit inside Chadstone Shopping Centre!

Dreamstime.com/Alexander Cimbal

Chapter outline

		Proficiencies
8.01	The metric system	U F C
8.02	Perimeter	U F PS R
8.03	Parts of a circle	U F R C
8.04	Circumference of a circle	U F PS R
8.05	Metric units for area	U F R C
8.06	Area of a rectangle	U F PS R
8.07	Area of a triangle	U F PS R
8.08	Area of a parallelogram	U F R
8.09	Extension: Areas of composite shapes*	U F PS R
8.10	Metric units for volume	U F C
8.11	Volume of a rectangular prism	U F R
8.12	Volume of a prism	U F R

*YEAR 8 EXTENSION

U = Understanding
F = Fluency
PS = Problem solving
R = Reasoning
C = Communication

Wordbank

area The amount of surface enclosed by a shape

circumference The perimeter of a circle; a circle's outer boundary

cross-section A 'slice' of a solid, taken across the solid rather than along it

cubic metre The volume of a cube that measures 1 m by 1 m by 1 m

perimeter The distance around the outside of a shape

perpendicular height The height of a shape taken at right angles to its base

pi (π) A special irrational number, approximately 3.1416, used in calculating circular measurements

radius The distance from the centre of a circle to the circle's edge

square metre The area of a square that measures 1 m by 1 m

volume The amount of space taken up by a solid

Quiz
Wordbank 8

Videos (4):

8.01 The metric system
8.04 Circumference of a circle
8.09 Areas of composite shapes
8.12 Volume of a prism

Twig video (1):

8.01 How long is a metre?

PhET interactive (1):

8.05 Area builder

Quizzes (6):

- Wordbank 8
- SkillCheck 8
- Mental skills 8A
- Mental skills 8B
- Language of Maths 8
- Test Yourself 8

Skillsheets (4):

SkillCheck Multiplying by 10, 100, 1000
8.05, 8.06 What is area?
8.10 What is volume?
8.12 Solid shapes

Worksheets (14):

8.01 A page of composite shapes
8.03 Parts of a circle • Discovering pi
8.04 A page of circles • Circumference and area
8.05 Australian areas
8.06 Australian areas • 1 cm grid paper • Triangle areas
8.07 Rectangle and triangle areas
8.09 A page of composite shapes • Square dot paper
8.11 Volume
8.12 Measuring shapes review

Mind map: Area and volume

Puzzles (4):

8.04 Circle crossword
8.06 Area group clues
8.09 Areas 1 • Area puzzles

Presentation (1):

8.02 Calculating the perimeter

Nelson MindTap

To access resources above, visit **cengage.com.au/nelsonmindtap**

8 In this chapter you will:

- ✓ use powers of 10 to convert between metric units of length, mass and capacity, and understand the metric prefixes (milli-, centi-, kilo-, mega-) in such units of measurement
- ✓ identify and use the correct operations when converting units, including millimetres, centimetres, metres, kilometres, milligrams, grams, kilograms, tonnes, millilitres, litres, kilolitres and megalitres
- ✓ calculate the perimeter of polygons including squares, rectangles and triangles
- ✓ learn the terminology for parts of a circle
- ✓ calculate the circumference of a circle, given its radius or diameter
- ✓ define and use metric units for area
- ✓ (YEAR 8 EXTENSION) convert between metric units for area
- ✓ solve problems involving the areas of rectangles, triangles and parallelograms
- ✓ (YEAR 8 EXTENSION) solve problems involving the areas of composite shapes
- ✓ define and use metric units for volume
- ✓ (YEAR 8 EXTENSION) convert between metric units for volume
- ✓ find the volume of a rectangular prism and other prisms

SkillCheck

ANSWERS ON P. 627

1 Copy and complete each conversion.

a 1 kg = _____ g **b** 1 min = _____ s **c** 1 L = _____ mL

d 1 cm = _____ mm **e** 1 g = _____ mg **f** 1 day = _____ hours

g 1 kL = _____ L **h** 1 t = _____ kg **i** 1 m = _____ mm

j 1 h = _____ min **k** 1 m = _____ cm **l** 1 km = _____ m

2 Evaluate each expression.

a 10.32×100 **b** 9.45×10 **c** 0.4×1000 **d** 2.7×1000

e 0.9×100 **f** 6.3×10 **g** $2000 \div 100$ **h** $43\,000 \div 1000$

i $650 \div 10$ **j** $3750 \div 100$ **k** $95 \div 10$ **l** $8200 \div 1000$

3 What metric unit would you use to measure:

a the distance from the street to your front door?

b the length of your foot? **c** the length of an ant?

d the length of your bed? **e** the width of a basketball court?

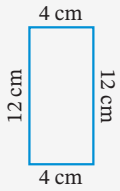
f the depth of a river? **g** the distance around a football field?

h the length of a long-jump pit? **i** the height of a door?

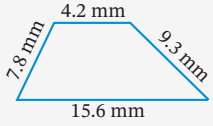
j the length of a fingernail? **k** an athlete's time in a race?

4 Find the perimeter of each shape.

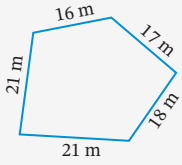
a



b



c



Skillsheet Multiplying by 10, 100, 1000

Quiz SkillCheck 8

The metric system 8.01

In Australia, we measure using the **metric system**. This system began in France in the 1790s after the French Revolution and, because it is based on powers of 10, it is logical and easy to use. The word 'metric' comes from the Greek word *metron* meaning 'to measure'.

In 1970, the Metric Conversion Board was established in Australia, starting the change to metric units, shown in the table.

Video How long is a metre?

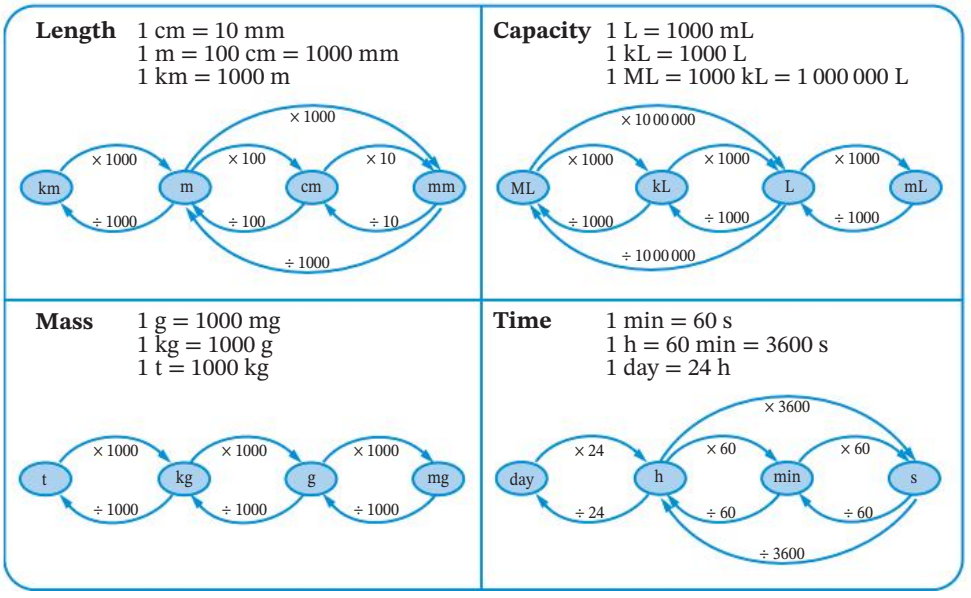
Quantity	Name of unit	Abbreviation
Length	metre	m
	millimetre	mm
	centimetre	cm
	kilometre	km

Quantity	Name of unit	Abbreviation
Mass	kilogram	kg
	gram	g
	milligram	mg
	tonne	t
Time	second	s
	minute	min
	hour	h
	day	d
Area	square metre	m^2
	square centimetre	cm^2
	hectare	ha
	square kilometre	km^2
Volume	cubic metre	m^3
	cubic centimetre	cm^3
Capacity	litre	L
	millilitre	mL
	kilolitre	kL
	megalitre	ML
Temperature	degree Celsius	°C

The metric system uses the following **prefixes** for metric units.

Prefix	Abbreviation	Meaning	Example
micro-	μ	one-millionth: $\frac{1}{1\,000\,000}$	1 μm = $\frac{1}{1\,000\,000}$ m
milli-	m	one-thousandth: $\frac{1}{1000}$	1 mg = $\frac{1}{1000}$ g
centi-	c	one hundredth: $\frac{1}{100}$	1 cm = $\frac{1}{100}$ m
kilo-	k	one thousand times: 1000 ×	1 km = 1000 m
mega-	M	one million times: 1 000 000 ×	1 ML = 1000 kL

This table shows the relationships between commonly-used units of measurement.



Example 1

Convert:

a 13 km to m **b** 5.8 t to kg

c 16 000 mL to L **d** 1500 min to h

SOLUTION

a 13 km = 13 × 1000 m

= 13 000 m

km —× 1000→ m

b 5.8 t = 5.8 × 1000 kg

= 5800 kg

t —× 1000→ kg

To convert large to small units, multiply (×)

c 16 000 mL = 16 000 ÷ 1000 L

= 16 L

L ←÷ 1000— mL

d 1500 min = 1500 ÷ 60 h

= 25 h

h ←÷ 60— min

To convert small to large units, divide (÷)

Video
The metric system

8.01

EXERCISE 8.01 ANSWERS ON P. 628

The metric system

U F R

EXAMPLE 1

1 Copy and complete:

a 1 h = _____ min **b** 1 km = _____ m

c 1 kg = _____ g **d** 1 cm = _____ mm

e 1 day = _____ h **f** 1 kL = _____ L

g 1 t = _____ kg **h** 1 min = _____ s

i 1 L = _____ mL **j** 1 g = _____ mg

k 1 ML = _____ L **l** 1 h = _____ s

2 Convert:

a 6.4 m to cm **b** 4 t to kg

c 3 days to hours **d** 2.4 ML to L

e 0.9 m to mm **f** 12.7 kg to g

g 8 h to min **h** 5.6 L to mL

i 42 cm to mm **j** 72 g to mg

k 3.8 km to cm **l** 4.5 h to s

3 Convert each length to centimetres.

a 2 m **b** 1500 mm **c** 0.5 m **d** 3.5 m

e 0.25 m **f** 2 000 000 μm **g** 15.8 km **h** 0.4 km

Foundation Standard Complex

4 Convert each length to metres.

a 200 cm **b** 3000 mm **c** 850 cm **d** 9800 mm

e 75 cm **f** 325 mm **g** 6 km **h** 18 km

5 Write the metric prefix that means:

C **a** 1000 **b** $\frac{1}{100}$ **c** $\frac{1}{1000}$

6 Find other prefixes that are used in the metric system. Write their meanings.

C

7 Copy and complete each conversion.

a 4.5 kg = _____ g **b** 3 hours = _____ minutes

c 750 mL = _____ L **d** 240 mm = _____ cm

e 12 kL = _____ L **f** 5.2 m = _____ cm

g 420 s = _____ min **h** 2 days = _____ hours

i 8.65 t = _____ kg **j** 72 hours = _____ days

k 65 mm = _____ m **l** 0.96 kL = _____ L

8 Which length is the shortest? Select the correct answer **A**, **B**, **C** or **D**.

A 0.3 m **B** 29 cm **C** 300 mm **D** 160 mm

9 Karla and Cassie measured these objects in the classroom.

C

- length of pencil 12 cm
- width of computer screen 33 cm
- width of door 80 cm
- width of ruler 3.5 cm
- length of room 740 cm

a Joe wrote his measurements in millimetres. What did he write?

b Cassie wrote her measurements in metres. What did she write?

c What unit would you use (cm, mm or m)? Would you use different units for different objects? If so, what would they be?

10 Karla is in Year 7. Select the most reasonable answer for her height in centimetres.

A 14 **B** 50 **C** 150 **D** 1200

11 Copy and complete each conversion.

a 2.2 L = _____ mL **b** 800 mm = _____ m

c 235 cm = _____ m **d** 360 mg = _____ g

e $5\frac{1}{2}$ h = _____ min **f** 4900 L = _____ kL

g 270 kg = _____ t **h** 4 years = _____ months

i 10.8 km = _____ m **j** 16.9 L = _____ mL

k 1 year = _____ days **l** 740 cm = _____ m

Foundation Standard Complex

12 Shahid wishes to fill a 3-litre bucket using a 250-millilitre cup. How many full cups will he need? Select **A**, **B**, **C** or **D**.

A 12 **B** 83 **C** 750 **D** 750 000

Perimeter 8.02

Perimeter

The **perimeter** of a shape is the distance around the shape.

It is the sum of the lengths of the sides of the shape.

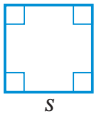
Worksheet A page of composite shapes

Presentation Calculating the perimeter

The **perimeter of a square** can be found by adding all of the sides:

Perimeter $= s + s + s + s$

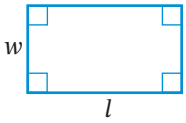
$= 4s$



The **perimeter of a rectangle** can be found by adding all the sides:

Perimeter $= l + w + l + w$

$= 2l + 2w$ or $2(l + w)$



Perimeter of a square and rectangle

Perimeter of a square

$P = 4s$, where s is the length of one side.

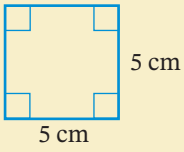
Perimeter of a rectangle

$P = 2l + 2w$ or $2(l + w)$, where l is the length and w is the width (breadth).

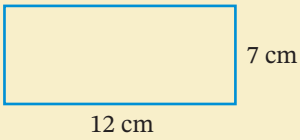
Example 2

Find the perimeter of each shape.

a



b



SOLUTION

a Perimeter of a square $= 4s$

$= 4 \times 5$

$= 20$ cm

b Perimeter of a rectangle $= 2(l + w)$

$= 2 \times (12 + 7)$

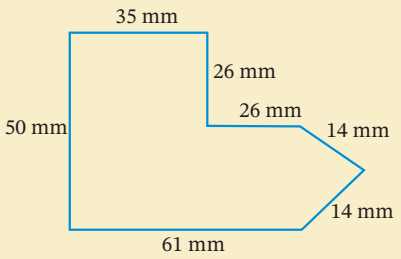
$= 38$ cm

Foundation Standard Complex

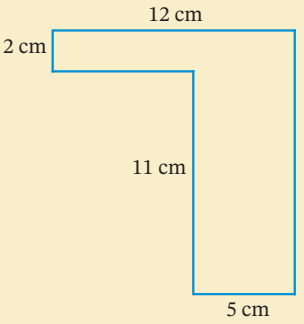
Example 3

Find the perimeter of each shape.

a



b

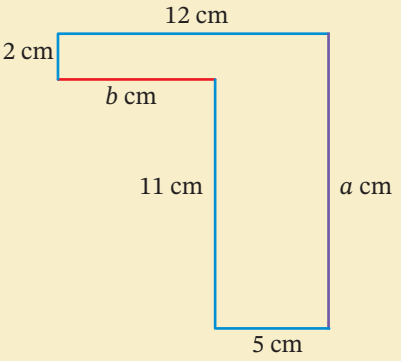


SOLUTION

a Perimeter = 50 + 35 + 26 + 26 + 14 + 14 + 61 — Adding side lengths

= 226 mm

b First, we need to find the lengths of the unknown sides, a and b.



$a = 2 + 11 = 13$ cm — The side shown by the purple line

$b = 12 - 5 = 7$ cm — The side shown by the red line

Perimeter = 12 + 13 + 5 + 11 + 7 + 2

= 50 cm

EXERCISE 8.02 ANSWERS ON P. 628

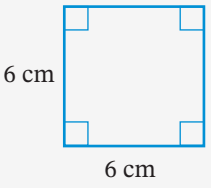
Perimeter

U F PS R

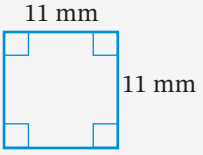
EXAMPLE 2

1 Find the perimeter of each shape.

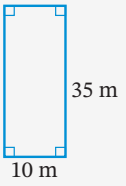
a



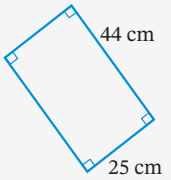
b



c



d



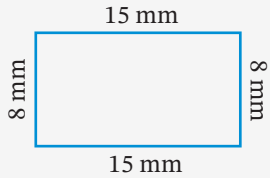
Foundation Standard Complex

EXAMPLE 3

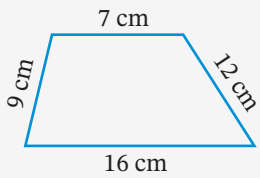
8.02

2 Find the perimeter of each shape.

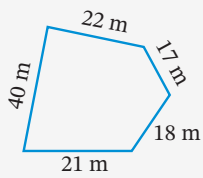
a



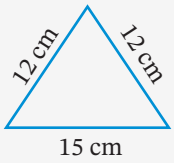
b



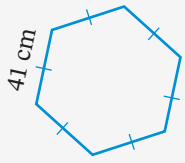
c



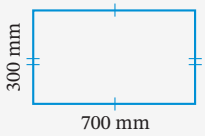
d



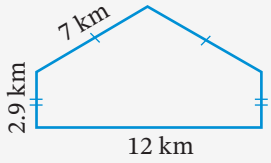
e



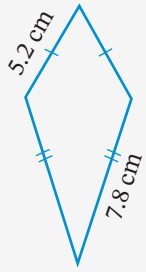
f



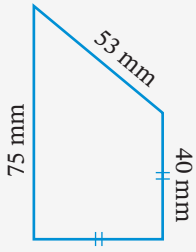
g



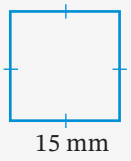
h



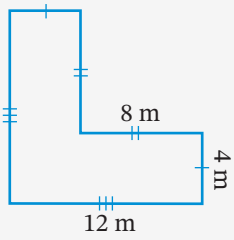
i



j

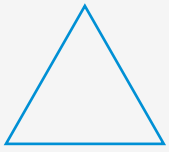


k



3 Measure the sides of each shape in mm and find its perimeter.

a



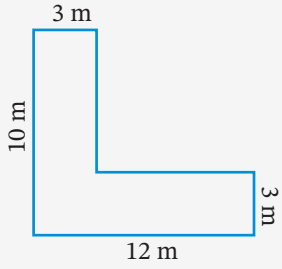
b

c

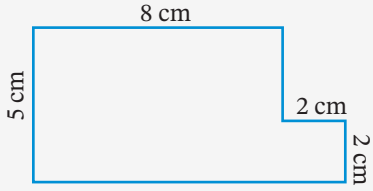
d

4 For each figure, find the missing lengths and then its perimeter.

R **a**

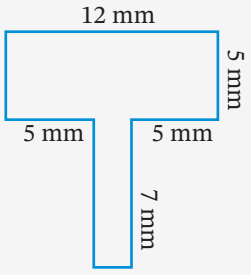


b

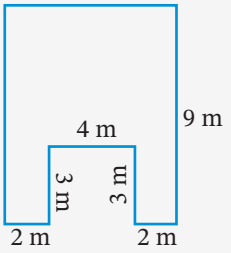


Foundation | Standard | Complex

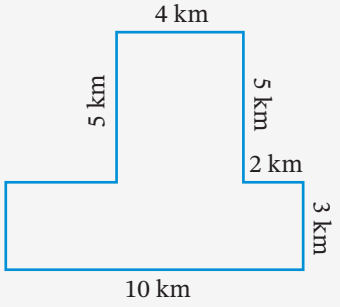
c



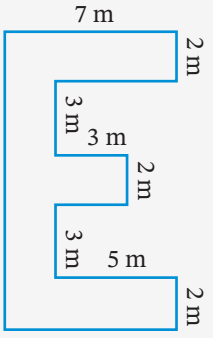
d



e



f



5 A rectangle has a perimeter of 64 m. What could its length and width be (in metres)? Select the correct answer **A**, **B**, **C** or **D**.

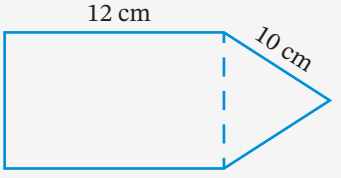
R

A 32, 32 **B** 49, 15 **C** 16, 18 **D** 18, 14

6 A rectangle has a perimeter of 30 cm. What could its length and width be?

R

7 This figure is made up of a rectangle and an equilateral triangle. Find its perimeter. Select **A**, **B**, **C** or **D**.



A 44 cm **B** 64 cm

C 54 cm **D** 22 cm

8 **a** A square has a perimeter of 16 cm. Find the length of one side.

R **b** A rectangle has a perimeter of 40 cm. If its length is 12 cm, find its width.

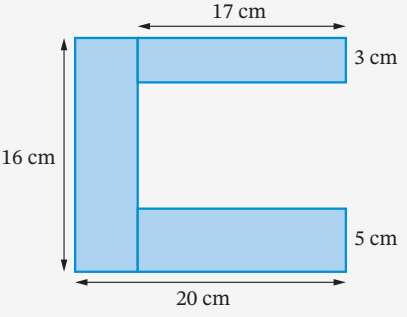
c A square has a perimeter of 51 cm. Find the length of one side.

d A rectangle has a perimeter of 62 cm. If its width is 13 cm, find its length.

9 Dana built the letter C using 3 planks of wood as shown in the figure.

PS

R Find the perimeter of the wooden letter in centimetres. Select **A**, **B**, **C** or **D**.



A 61 **B** 72

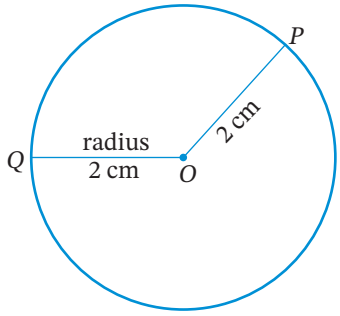
C 122 **D** 106

Foundation Standard Complex

Parts of a circle

8.03

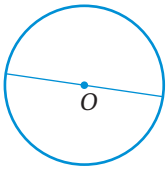
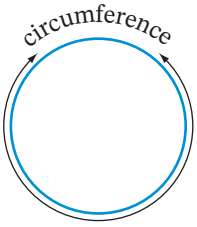
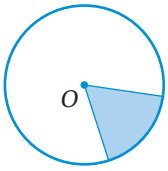
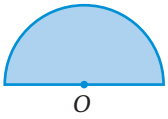
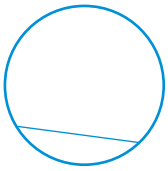
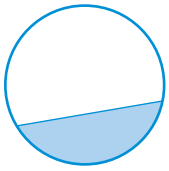
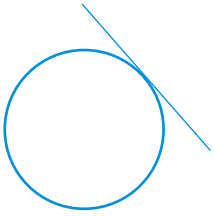
The circle is a completely round shape. Every point on a circle is the same distance from its centre. The circle shown has centre O, and points P and Q are exactly 2 cm from O. This distance, from the centre to the edge of the circle, is called the **radius** of the circle (*plural*: radii).



Worksheet
Parts of a circle

Circle words

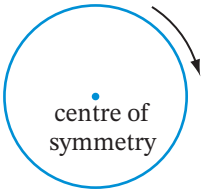
There are special words that describe the parts of a circle or the related lines.

Diameter	Circumference	Arc
An interval running from one side of a circle to the other side and through the circle's centre 	The perimeter of a circle 	Part of the circumference
Sector	**Quadrant**	**Semicircle**
A 'pizza-slice' of a circle cut along 2 radii 	A sector that is a quarter of a circle, with angle 90°	Half a circle 
Chord	**Segment**	**Tangent**
An interval joining any 2 points on a circle 	A region cut off by a chord 	A line outside a circle that touches it at exactly one point 

Circle symmetry

A circle can be folded in half in an infinite number of ways, along any diameter, so it has an *infinite* number of axes of symmetry.

A circle can also be rotated through any angle size and still map onto itself, so it has an *infinite* order of rotational symmetry. The centre of symmetry is the centre of the circle.



EXERCISE 8.03 ANSWERS ON P. 628

Parts of a circle

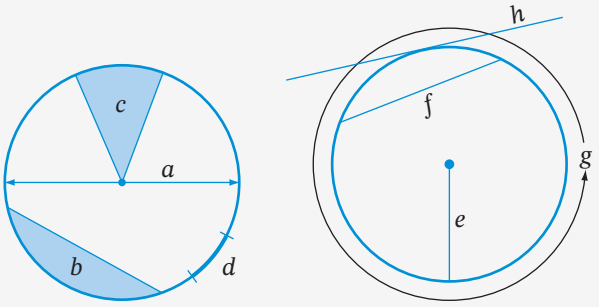
U F R C

1 Use a ruler and compasses to construct a circle of:

a radius 4 cm **b** radius 2.5 cm **c** radius 56 mm

2 Name each part of a circle marked by letters in these diagrams.

C



3 Match each description to the correct word from this list.

C

circumference	radius	sector
tangent	arc	quadrant
segment	diameter	chord

a The distance from the centre of a circle to its side

b A quarter of a circle

c A line that touches the outside of a circle once

d The distance from one side of a circle to the other side, through the circle's centre

e A line segment from one side of the circle to the other side, not through the centre

f Part of the circumference of a circle

g The area inside a circle formed by 2 radii and an arc

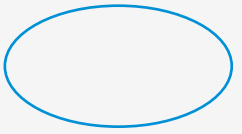
h The distance around a circle

i The area inside a circle formed by a chord and an arc

4 This ellipse (oval) is a 'flattened' circle. How many axes of symmetry has an ellipse? Select the correct answer **A**, **B**, **C** or **D**.

R

C



A 1 **B** 2

C 4 **D** an infinite number

Foundation Standard Complex

9780170465557

5 What is the order of rotational symmetry of an ellipse? Select **A**, **B**, **C** or **D**.

R C

A 1 **B** 2 **C** 4 **D** infinite number

6 Sketch a diagram of each term.

C

a diameter **b** circumference **c** chord
d sector **e** tangent **f** segment
g quadrant **h** arc **i** radius

7 Copy each shape. Then state:

R C

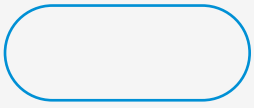
i the number of axes of symmetry in the shape and draw them
ii the order of rotational symmetry of the shape if it has rotational symmetry.

a

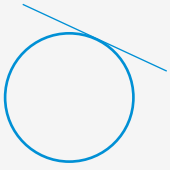
b

c

d



e



f

8 Using a radius of 4 cm, draw:

a a semicircle
b a quadrant
c a sector whose angle at the centre of the circle is 60°
d an arc that is longer than half the circumference of the circle
e a circle with a chord of length 2.5 cm
f a circle with a tangent of length 4 cm

TECHNOLOGY

Circle parts

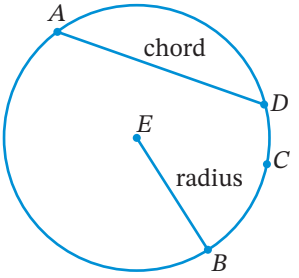
In this activity you will use your dynamic geometry software to construct a circle and display all the circle parts.

1 Construct a large circle, radius 4 cm.

2 Draw a radius and a chord in your circle.

Foundation Standard Complex

3 Label the chord and the radius as shown.



4 Construct each circle part below on your page and label each one.

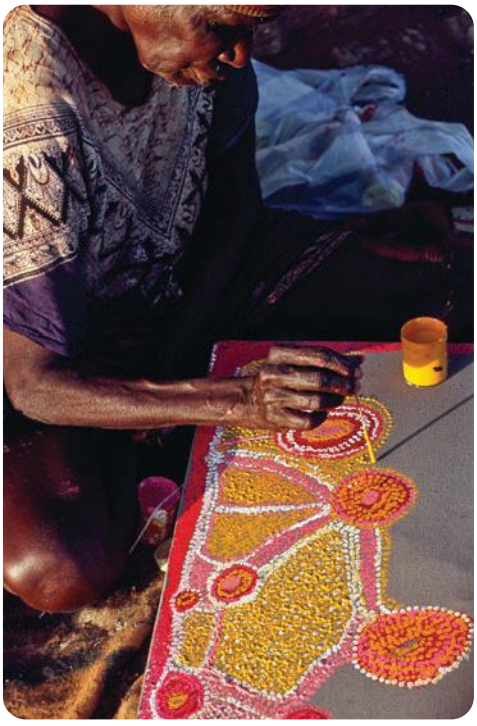
centre	diameter	circumference	sector	arc
segment	semicircle	tangent	quadrant	

5 What are **concentric circles**? Find out and construct a diagram showing 3 or more concentric circles. Remember to clearly label the centre of the circle.

DID YOU KNOW?

Circles and Aboriginal culture

Aboriginal people have a strong connection to circles because they observe that circles resemble many cycles in the natural world and they believe that the power of the world works in circles. Examples of this include the shape of Earth, Sun and Moon, birds' nests, waterholes and the rings in trees that represent their age.


Alamy Stock Photo/Bill Bachman

Yarning (talking) circles are used by First Nations Australians to learn from a collective group of people, in a way that builds respectful relationships, so that Elders can preserve and pass on their cultural knowledge and experience through the telling of stories. Members of the community sit in a circle, introduce and share something about themselves and then share their thoughts and ideas safely without judgement.

Circles are also prominent in Aboriginal artwork where they often signify meeting places. Waterholes often feature frequently in such art, as these places are critical for survival in the desert and are often considered a sacred place for ceremonies.

Research what other meeting places and objects are represented by circles in Aboriginal art.

Worksheet
Discovering pi

INVESTIGATION

Measuring the circumference of a circle

This is a measuring activity to be done in groups of 3.

You will need: A measuring tape, or some string and a ruler; graph paper; 6 round objects such as cans, round cake tins, pipes, coins, drums, bottles and so on.

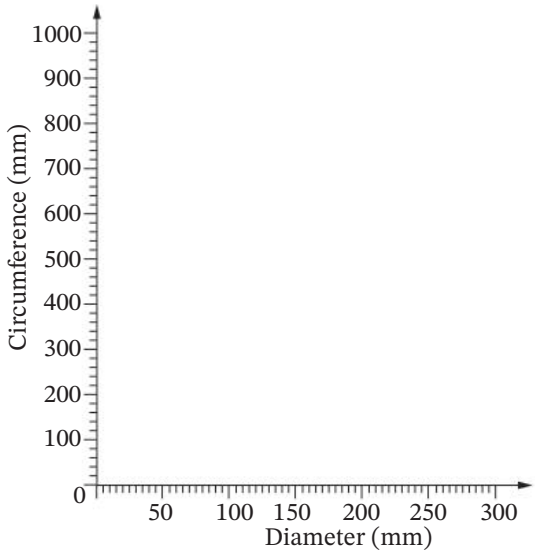
1 Copy this table.

Object	Diameter, d (mm)	Circumference, C (mm)	The ratio $\frac{C}{d}$
a			
b			
c			
d			
e			
f			

2 Measure (in millimetres) to find the diameter and the circumference of 6 different objects. Record your results in the table.

3 Calculate the ratio $\frac{C}{d}$ for each object. Round your answers to 2 decimal places.

4 Copy and complete the graph, using your results from the table. What do you notice about the graph?



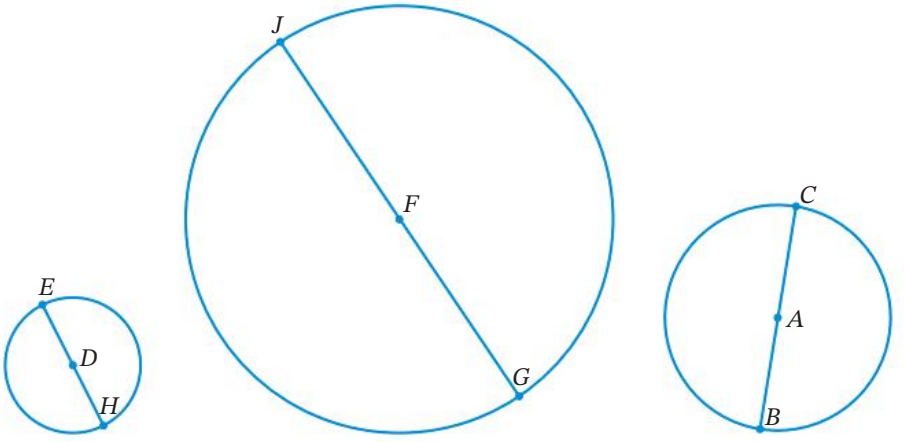
5 There is a formula for finding the circumference of a circle.

Copy and complete the formula: $C = d \times$ _____

TECHNOLOGY

The circumference and diameter of a circle

1 Use your dynamic geometry software to construct 3 different circles. Construct the diameter of each circle.



2 Calculate each circle's circumference and diameter.

3 For each circle, use your calculator to find $\frac{\text{length of circumference}}{\text{length of diameter}}$

4 What do you notice? Discuss this with another student.

8.04 Circumference of a circle

Worksheets
A page of circles

Circumference and area

Puzzle
Circle crossword

The **circumference** of a circle can be found by multiplying its **diameter** by a special number called **pi** (pronounced 'pie'), represented by the Greek letter π. For any circle,

$$\frac{\text{circumference}}{\text{diameter}} = \pi = 3.14\ldots$$

Pi is often estimated as 3.14, but a more accurate value can be found on your calculator when you press the π key (you may need to press SHIFT first). As a decimal, the digits of π continue endlessly without any repeats or patterns:

$\pi = 3.141\ 592\ 653\ 589\ 793\ \ldots$

so, like $\sqrt{2}$, it is called an **irrational number**.

This number was named 'pi' by the Swiss mathematician Leonhard Euler in 1737.

The formula for the circumference of a circle is $C = \pi \times \text{diameter} = \pi d$.

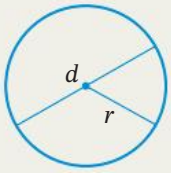
Because the diameter of a circle is double its radius, the circumference, C, of a circle with radius r is $C = \pi \times 2 \times \text{radius} = 2\pi r$.

Circumference of a circle

The **circumference (perimeter) of a circle** is:

$C = \pi \times \text{diameter}$ or $C = 2 \times \pi \times \text{radius}$

$C = \pi d$ $\quad$ $C = 2\pi r$



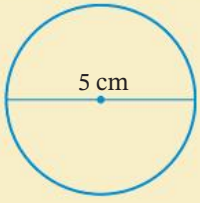
Example 4

a Estimate the circumference of a circle with a diameter of 5 cm.

b Calculate the circumference of the circle:

i correct to 2 decimal places

ii in terms of π



Video Circumference of a circle

SOLUTION

a $C = \pi d$

$= \pi \times 5$

$\approx 3 \times 5$ $\quad$ π is approximately 3.

$= 15$ cm

b **i** $C = \pi d$

$= \pi \times 5$

$= 15.707\,963\ldots$

≈ 15.71cm

On your calculator enter π × 5 =

ii $C = \pi d$

$= \pi \times 5$

$= 5\pi$ cm

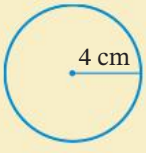
Writing an answer in terms of π is **more exact** as there is no rounding involved. This is also called writing an answer 'in exact form'.

Example 5

Calculate the circumference of the circle:

a correct to one decimal place

b in terms of π



Video Circumference of a circle

SOLUTION

a $C = 2\pi r$

$= 2 \times \pi \times 4$

$= 25.13274\ldots$

≈ 25.1 cm

b $C = 2\pi r$

$= 2 \times \pi \times 4$

$= 8\pi$cm

EXERCISE 8.04 ANSWERS ON P. 628

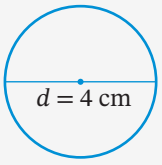
Circumference of a circle

U F PS R

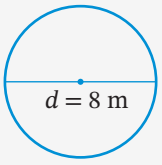
EXAMPLE 4

1 Estimate the circumference of each circle.

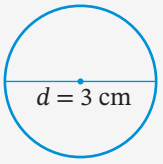
a



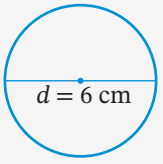
b



c



d



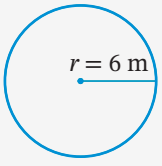
2 Calculate, correct to 2 decimal places, the circumference of each circle in question **1**.

3 Calculate the circumference of each circle in question **1** in terms of π.

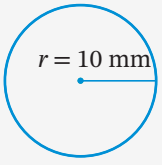
EXAMPLE 5

4 Estimate the circumference of each circle.

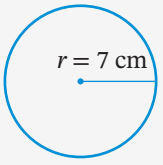
a



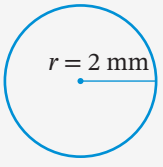
b



c



d



5 Calculate, correct to one decimal place, the circumference of each circle in question **4**.

6 Calculate the circumference of each circle in question **4** in terms of π.

7 A child's inflatable swimming pool has a diameter of 1.4 m. Find its circumference correct to 2 decimal places.

8 Tina's bicycle has wheels with a diameter of 60 cm.

PS **a** How far does the bicycle move when a wheel turns around once?

b If Tina cycles 900 m to school, how many complete turns does the bicycle wheel make?

9 Earth has a radius of 6370 km. Find the distance around the Equator.

10 A 20-cent coin has a radius of 16 mm. Calculate its circumference.

11 This tin of tomatoes has a diameter of 75 mm. If the label wraps around the tin completely, how long is the label? Answer correct to the nearest millimetre.

PS R

Shutterstock.com/Bahadir Yeniceri

12 Calculate, correct to 2 decimal places, the circumference of a circle with:

a a diameter of 2 cm

b a diameter of 7 cm

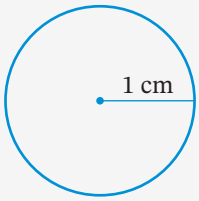
c a radius of 3 cm

d a radius of 5 cm

Foundation Standard Complex

13 Which of the following intervals best shows the circumference of this circle? Select **A**, **B**, **C** or **D**.

R



A _____

B _____

C _____

D _____

14 The International Space Station completes one circular orbit of Earth at a height of 400 km. If the radius of Earth is approximately 6400 km, find the distance travelled by the station. Select the closest answer **A**, **B**, **C** or **D**.

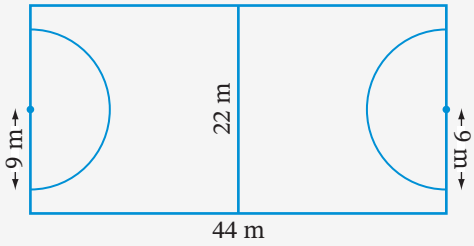
A 2513 km **B** 37 699 km **C** 40 212 km **D** 42 726 km

15 Tape has been placed on all the lines of this indoor hockey pitch. How much tape was used? Express the answer:

PS R

a correct to one decimal place

b in terms of π.

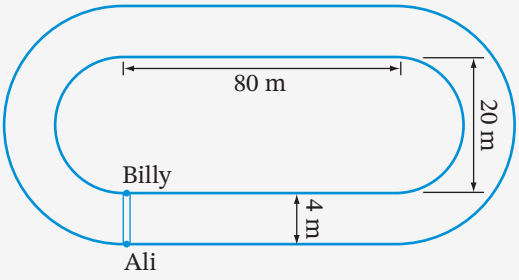


16 A circle has a circumference of 50.265 cm. Find its diameter, correct to the nearest centimetre.

R

17 Ali and Billy raced each other around this athletic track. Ali ran along the outside perimeter while Billy ran along the inside perimeter. After one lap of the track, who ran the longer distance, and by how much? Answer correct to the nearest metre.

PS R



Foundation Standard Complex

DID YOU KNOW?

Never-ending pi!

$\pi = 3.141\ 592\ 653\ 589\ 793\ 238\ 462\ 643\ 383\ 279\ 502\ 884\ 197\ 169\ 399\ 375\ 105\ 820 \ldots$

Ancient civilisations knew about the value of π, estimating it as 3. Over time, the calculations have improved due to better formulas and technology. Since the first computer, the ENIAC, was invented in 1949, much progress has been made. In 2019 Emma Haruka Iwao, a Google employee from Japan, calculated the value of π to 31.4 trillion digits with the help of Google's cloud computing service.

Year	Person/Country	Number of decimal places
1855	Shanks, England	527
1949	ENIAC computer, USA	2037
1973	CDC 7600 computer, France	1 000 000
1988	Kanada, Hitachi S-820 computer, Japan	200 000 000
1989	Chudnovsky brothers, USA	1 000 000 000
1999	Kanada and Takahashi, Hitachi SR8000 computer, Japan	206 158 430 000
2002	Kanada, Hitachi SR8000 computer, Japan	1 241 100 000 000
2010	Shigeru Kondo, Alexander Yee, Japan and USA	5 trillion
2019	Emma Haruka Iwao	31.4 trillion
???	???	???

The Guinness World Record for reciting the most digits of pi belongs to Rajveer Meena of India, who recited pi to 70 000 decimal places (while blindfolded) in 2015. Unofficially, in 2006, Japanese counsellor Akira Haraguchi memorised π to 100 000 decimal places. It took him over 16 hours to recite it. He had to stop after 3 hours as he lost his place, and had to start from the beginning.

On average, how many digits would Haraguchi have recited per minute?

8.05 Metric units for area

Skillsheet What is area?

Worksheet Australian areas

Interactive Area builder

A person estimating the number of tiles needed to cover the floor of an outdoor space needs to know the size of a tile and the size of the floor. In this case, the **area** of the floor gives its 'size'.

Area

The **area** of a shape is the amount of surface that is enclosed by the shape.

For example, the area of this trapezium has been shaded.

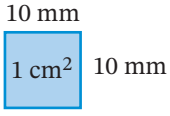
The table shows metric units for area.

Area unit	Abbreviation	The size of a square with each side measuring
square millimetre	mm^2	1 mm (Actual size:)
square centimetre	cm^2	1 cm (Actual size:)
square metre	m^2	1 m
hectare	ha	100 m

A **square metre** is approximately the size of the floor of a large shower recess.

A **square centimetre** is about the area of a fingernail.

A **hectare** is about the area of Sydney Football Stadium, or the grassed area inside an athletics (400 m) track.

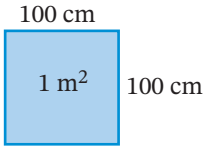


1 cm = 10 mm

$1\ cm^2 = 10 \times 10\ mm^2$

$= 10^2\ mm^2$

$= 100\ mm^2$ (double the number of 0s)

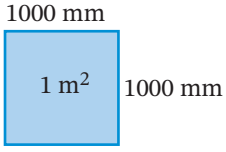


1 m = 100 cm

$1\ m^2 = 100 \times 100\ cm^2$

$= 100^2\ cm^2$

$= 10\ 000\ cm^2$ (double the number of 0s)



Because the side length is squared to get area, the length conversion factor is also squared to get the area conversion factor. This results in doubling the number of 0s in the length conversion factor.

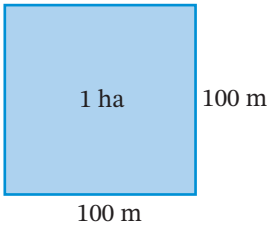
1 m = 1000 mm

$1\ m^2 = 1000^2\ mm^2$

$= 1\ 000\ 000\ mm^2$ (double the number of 0s)

1 ha = 100 m × 100 m

$= 10\ 000\ m^2$ (double the number of 0s)



YEAR 8 EXTENSION

Metric units for area

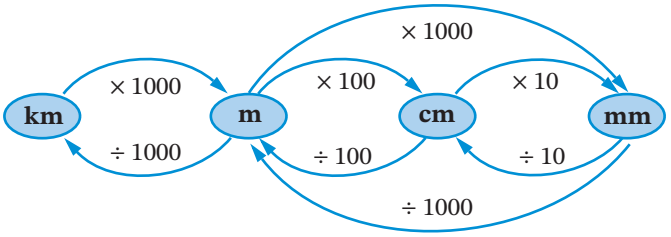
$1\ cm^2 = 100\ mm^2$

$1\ m^2 = 10\ 000\ cm^2 = 1\ 000\ 000\ mm^2$

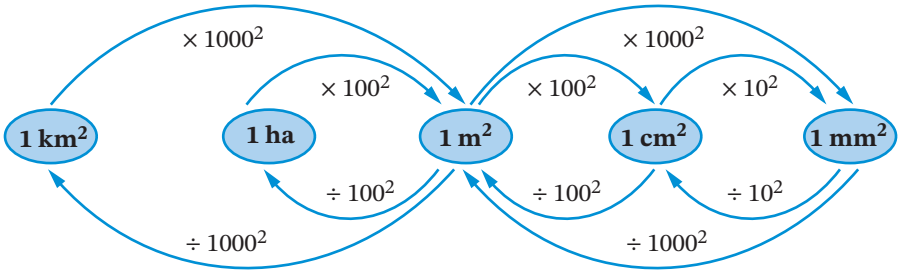
$1\ ha = 10\ 000\ m^2$

YEAR 8 EXTENSION

We have already used a flowchart diagram to help us convert between units of length in this chapter:



Remember that we can square the length conversion factors to find the area conversion factors. For area conversions, we can modify the length conversion flowchart above to produce the area conversion flowchart below (note: the conversion between m^2 and hectares has also been included):



Example 6

Convert

a $3\ cm^2$ to mm^2 **b** $4000\ mm^2$ to m^2

c $81\ 000\ m^2$ to ha **d** $1.2\ km^2$ to cm^2

SOLUTION

a $3\ cm^2 = 3 \times 10^2\ mm^2$ — cm^2 to mm^2 : $\times 10^2$

$= 300\ mm^2$

b $4000\ mm^2 = 4000 \div 1000^2\ m^2$ — mm^2 to m^2 : $\div 1000^2$

$= 0.004\ m^2$

c $81\ 000\ m^2 = 81\ 000 \div 100^2$ ha — m^2 to ha : $\div 100^2$

$= 8.1$ ha

d $1.2\ km^2 = 1.2 \times 1000^2 \times 100^2\ cm^2$ — km^2 to m^2 : $\times 1000^2$

$= 12\ 000\ 000\ 000\ cm^2$ — m^2 to cm^2 : $\times 100^2$

EXERCISE 8.05 ANSWERS ON P. 629

Metric units for area

U F R C

1 What unit of area would you use to measure the area of:

R **a** a farm? **b** a classroom? **c** a shirt?

C **d** the school oval? **e** a sheet of paper? **f** your eardrum?

Foundation Standard Complex

YEAR 8 EXTENSION

EXAMPLE 6

2 What is the approximate area of a calculator? Select the correct answer a **A**, **B**, **C** or **D**.

R C

A 80 cm^2 **B** 120 cm^2 **C** 160 cm^2 **D** 200 cm^2

3 Copy and complete each conversion.

a 15 cm^2 = _____ mm^2
b 1 500 000 mm^2 = _____ m^2
c 690 mm^2 = _____ cm^2
d 6.5 m^2 = _____ cm^2
e 0.5 m^2 = _____ mm^2
f 12 ha = _____ m^2
g 12 200 cm^2 = _____ m^2
h 1250 mm^2 = _____ cm^2
i 0.32 ha = _____ m^2
j 7.9 cm^2 = _____ mm^2
k 0.75 m^2 = _____ mm^2
l 2 450 000 m^2 = _____ ha
m 865 000 cm^2 = _____ m^2
n 51 300 mm^2 = _____ m^2
o 0.47 m^2 = _____ cm^2
p 9000 m^2 = _____ ha

4 A square tile has an area of 1024 cm^2. Write this area in:

a mm^2 **b** m^2

5 **a** Arrange 6.5 m^2, 114 000 cm^2 and 25 050 000 mm^2 in ascending order.

R C

b Arrange 990 mm^2, 54 cm^2, 0.000 032 m^2 in descending order.

6 Sydney's CBD (central business district) is approximately a rectangle bounded by George Street, Circular Quay, Macquarie and College streets and Liverpool Street, an area of 875 000 square metres. What is this area:

a in hectares? **b** in square centimetres?

7 What is the approximate area of a 5-cent coin? Select **A**, **B**, **C** or **D**.

A 25 mm^2 **B** 2.5 cm^2 **C** 25 cm^2 **D** 2.5 m^2

8 Western Australia is Australia's largest state, with an area of 2 529 875 km^2. Tasmania is the smallest state, with an area of 68 401 km^2.

a What is the area of Tasmania in hectares?

b How many 'Tasmanias' would fit into Western Australia?

9 A postage stamp has an area of 8.36 cm^2. What is this area in square millimetres?

10 The theme park, Dreamworld, on Queensland's Gold Coast, has an area of one square kilometre.

R C

a How many square metres are there in a square kilometre?

b How many hectares are there in a square kilometre?

c Name something that might be measured in square kilometres.

Foundation Standard Complex

DID YOU KNOW?

The Roman Colosseum

Shutterstock.com/Dima Moroz

The Colosseum is a famous and impressive ancient building in Rome, Italy, the greatest amphitheatre of the Roman empire. It was built in 80 CE and you can still visit its ruins today. This was the place where huge crowds of Romans watched gladiator fights, animal hunts, executions and other blood-thirsty events.

The Colosseum is oval-shaped, 189 metres long, 16 metres wide and covers an area of about 24 000 square metres. It was surrounded by a wall 48 metres high and could hold over 50 000 spectators.

Find out how the size of the Colosseum compares with the Melbourne Cricket Ground, Brisbane Cricket Ground or the WACA Stadium.

INVESTIGATION

The human lagoon

1 **a** Find a level area of concrete or bitumen. Carefully pour a litre of water over it to form a puddle.

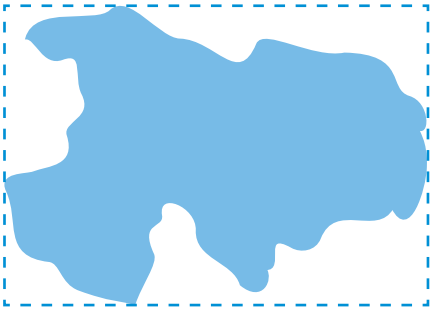
b Use a metre ruler or tape measure to measure the dimensions of the smallest rectangle that surrounds the puddle.

c On 1 cm grid paper, make a scale drawing of the rectangle that surrounds the puddle.

d Sketch the shape of your puddle inside the rectangle on the graph paper.

e Use the method of counting squares to find the area of the puddle formed by the litre of water.

2 **a** About 80% of the human body is made up of water. If 1 litre of water has a mass of 1 kilogram, measure your own mass and use it to calculate the number of litres of water in your body. Use this result, together with those from question **1**, to find the area of the puddle the water in your body would make.



b Go outside and mark out a rectangle that has the same area as the puddle you would make. Record the dimensions of this rectangle.

c Using the scale 1 cm = 1 m, draw a rectangle on graph paper that has the same area as the puddle you would make. Write a brief report of what this rectangle represents and how it was found.

INVESTIGATION

Standing room only!

1. As a group, mark out a square metre in your classroom or school yard.
2. Estimate how many students could stand in the square.
3. Check your estimate by getting as many students as possible to stand in the square.
4. How could you estimate whether all the Year 7 students in your school could stand in your classroom?
5. How many Year 7 students could stand in your room?

INVESTIGATION

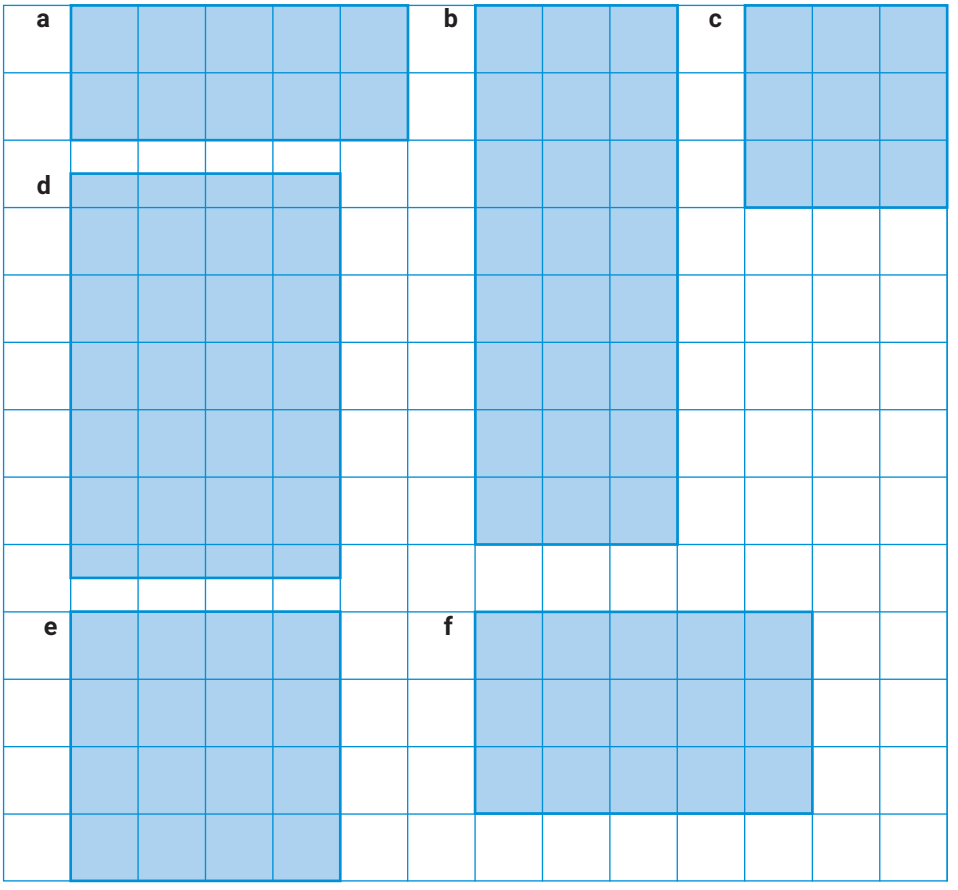
Student space in a classroom

1. As a group, shift all the furniture to the sides of the classroom.
2. Use a set of metre-long sticks and connectors to lay out a grid of square metres on the floor.
3. Calculate or count the area of the floor in metres.
4. Sit 2 people to a square metre. Are you comfortable enough to stay within your square for a whole class?
5. Sit one person to a square metre. How comfortable are you?
6. Replace the furniture over the grid and sit at your desk.
7. How much area do you and your desk occupy?
8. During the COVID-19 pandemic in 2020, social distancing measures were introduced to minimise the spread of the virus. If each student was allocated an area of 4 square metres, how many students could fit inside your classroom?
9. Copy and complete each sentence.
   - **a** My class has _____ students.
   - **b** Today my maths class was held in a room with floor dimensions _____ by _____.
   - **c** The area of the floor is approximately _____.
   - **d** Each student is allocated _____ of floor space.
   - **e** The floor space left over for aisles and the teacher is _____.

INVESTIGATION

Area of a rectangle

1 The rectangles and squares below are drawn on a grid of 1 cm squares. Find the area of each rectangle and square (in square centimetres) by counting the squares in it. Copy and complete the table. For rectangles, we usually call the *shorter* measurement the width.



Rectangle	Length (cm)	Width (cm)	Area (cm^2)
a	5	2	
b	8		
c			
d			
e			
f			

2 Look at your results for question **1**. Write down a rule for finding the area of a rectangle or square from its length and width.

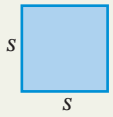
Area of a rectangle

8.06

Area of a square

Area = side × side = $s \times s$

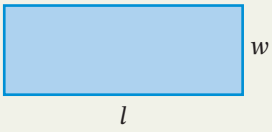
$A = s^2$



Area of a rectangle

Area = length × width

$A = lw$



Skillsheet What is area?

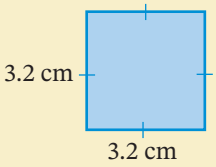
Worksheet Australian areas

Puzzle Area group cluesa

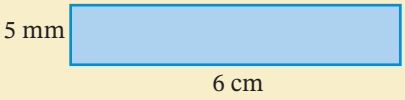
Example 7

Find the area of each shape.

a



b



SOLUTION

a Area $= s^2$

$= 3.2^2$

$= 3.2 \times 3.2$

$= 10.24 \text{ cm}^2$

b Area $= l \times w$

$= 60 \times 5$

$= 300 \text{ mm}^2$

Change 6 cm to 60 mm to make units the same

It is also possible to change 5 mm to 0.5 cm to have an answer of 3 cm^2 instead.

EXERCISE 8.06 ANSWERS ON P. 629

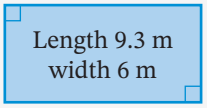
Area of a rectangle

U F PS R

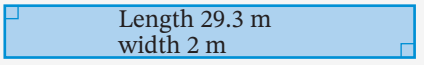
EXAMPLE 7

1 Find the area of each rectangle.

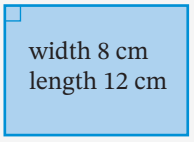
a



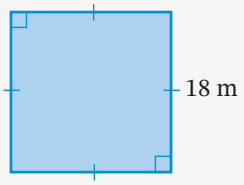
b



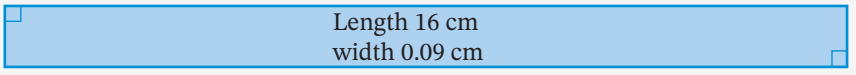
c



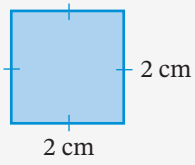
d



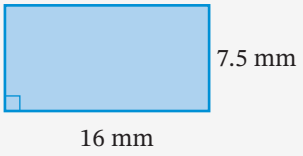
e



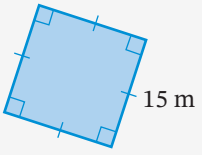
f



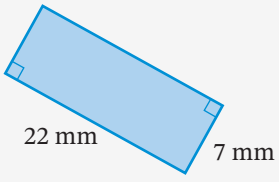
g



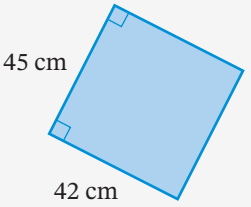
h



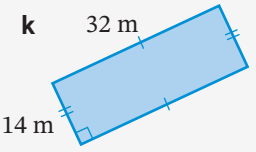
i



j

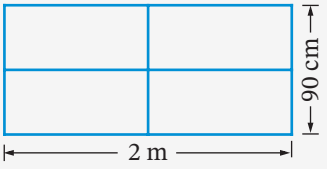


k



2 Find the area of this window in square metres.
Select the correct answer **A**, **B**, **C** or **D**.

A 0.18 m² **B** 1.8 m²
C 18 m² **D** 180 m²



3 Find the area of each item described.

i in m² **ii** in cm²
iii in mm²

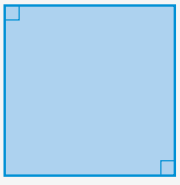
a A square bowling green of length 5 m.

b A desk top measuring 95 cm by 54 cm.

4 Measure the sides of each rectangle and calculate its area:

i in mm² **ii** in cm²

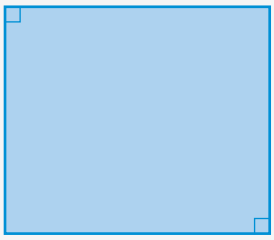
a



b

Foundation Standard Complex

c

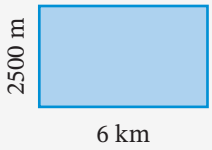


d

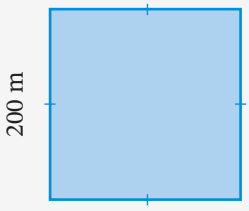
5 A square has an area of 49 cm². What is the length of one side?

6 Find the area of each rectangle in square metres.

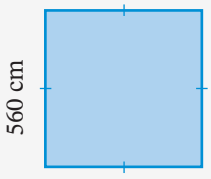
a



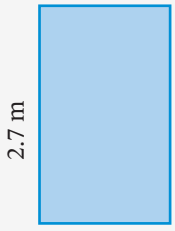
b



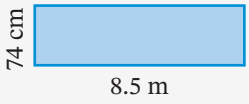
c



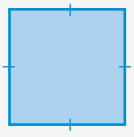
d



e



f



7 The front yard of a house measures 20 m by 8 m while the backyard measures 35 m by 7.5 m. Calculate the cost of covering both yards with grass if it costs $18.60 per square metre.
PS

8 A rectangle has a width of 4 m and an area of 18 m². What is its length?

9 Zara's bedroom floor measures 3.6 m by 2.8 m. She buys carpet squares of length 40 cm.
PS

a What is the area of Zara's bedroom floor?

b What is the area of a carpet square in square metres?

c How many carpet squares are needed to cover Zara's bedroom floor?

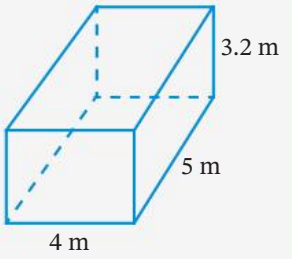
10 A rectangle has an area of 35 cm². What could its length and width be?
R

11 Qiang has a square photo with an area of 81 cm². She needed to enlarge the photo to an area of 729 cm². To achieve this, each side had to be multiplied by what number? Select **A**, **B**, **C** or **D**.
R

A 9 **B** 6 **C** 4 **D** 3

Foundation Standard Complex

12 A 4 m × 5 m basement has a ceiling height of 3.2 m.



PS **a** Find the area of:

- **i** the floor
- **ii** the ceiling
- **iii** one of the smaller walls

b How many square tiles measuring 50 cm on each side are needed to cover the floor of the basement?

c How many litres of paint are needed to cover the 4 walls of the basement if 1 litre of paint covers 16 m^2?

Quiz Mental skills 8A

☆ MENTAL SKILLS 8A ANSWERS ON P. 629 Maths without calculators

Doubling and halving numbers

You can double or halve a number by splitting it up first, then doubling or halving.

1 Study each example.

a	$92 \times 2 = 180 + 4$ $= 184$	*Think*: Double 92 = double 90 + double 2
b	$37 \times 2 = 60 + 14$ $= 74$	*Think*: Double 37 = double 30 + double 7
c	$\frac{1}{2} \times 86 = 40 + 3$ $= 43$	*Think*: Half of 86 = half of 80 + half of 6
d	$\frac{1}{2} \times 244 = 120 + 2$ $= 122$	*Think*: Half of 244 = half of 240 + half of 4
e	$\frac{1}{2} \times 78 = 30 + 9$ $= 39$	If the tens number is **odd**, then: *Think*: Half of 78 = half of 60 + half of 18
f	$\frac{1}{2} \times 132 = 60 + 6$ $= 66$	*Think*: Half of 132 = half of 120 + half of 12

2 Now evaluate each product.

a 54×2	**b** 77×2	**c** 83×2	**d** 105×2
e 26×2	**f** 41×2	**g** 98×2	**h** 162×2
i $\frac{1}{2} \times 182$	**j** $\frac{1}{2} \times 274$	**k** $\frac{1}{2} \times 92$	**l** $\frac{1}{2} \times 138$
m $\frac{1}{2} \times 506$	**n** $\frac{1}{2} \times 76$	**o** $\frac{1}{2} \times 48$	**p** $\frac{1}{2} \times 170$

Foundation Standard Complex

TECHNOLOGY

Perimeter and area of a chicken pen

Farmer Jack has 36 m of fencing to make a rectangular chicken pen.

1 Open a new spreadsheet and enter the labels, values and formula for calculating the perimeter of a rectangle as shown below.

	A	B	C	D
1	Chicken Pens			
2				
3	Length (m)	Width (m)	Perimeter (m)	Area (m)
4	10	8	=2*A4+2*B4	
5				

2 In cell D4, enter a formula for the area of the rectangle.

3 Now use your dynamic geometry software to help you solve this problem. Set up your drawing page, check that the grid is showing and the axes are not showing.

4 Construct a rectangle 10 m by 8 m.

5 Now find the area of this rectangle.

6 Now repeat steps **4** and **5** and find at least 3 pairs of dimensions (whole numbers only) for this rectangle and enter them into your spreadsheet.

7 Return to the spreadsheet and enter the dimensions for length and width that you found. Also, **Fill Down** from cells C4 and D4 to complete the perimeter and area calculations.

8 There are more pairs of whole-number dimensions (lengths and widths) for the rectangular chicken pen. Use your dynamic geometry software and fill in your spreadsheet to find any missing pairs of dimensions.

9 Which dimensions make the largest area for the chicken pen?

iStock.com/twilightproductions

INVESTIGATION

Worksheets
1 cm grid paper

Triangle areas

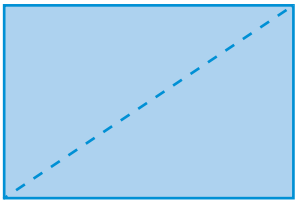
Area of a triangle

You will need 1 cm grid paper and scissors.

Right-angled triangles

1 On your grid paper, draw a rectangle 6 cm by 4 cm.

2 Cut the rectangle in half along a diagonal. What shape have you made?

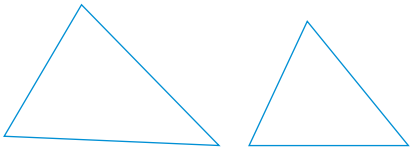


3 Area of rectangle = ____ × ____ = ____cm^2

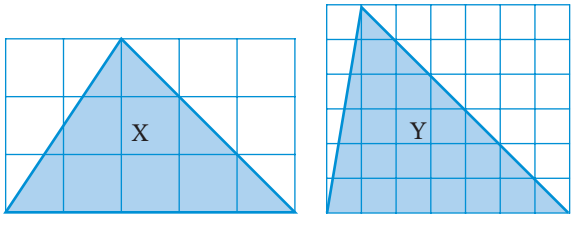
4 So what is the area of each triangle?

5 Draw 2 other rectangles and repeat steps **2** to **4**. What do you notice regarding the area of the triangle and the area of the rectangle?

6 How can we find the area of triangles such as the following?



Non-right-angled triangles



1 On your grid paper, draw triangle X as shown, and cut out the rectangle.

2 Area of rectangle = _____

3 Cut out triangle X.

4 Place the leftover pieces on triangle X. Do they cover triangle X exactly?

5 Area of triangle = _____

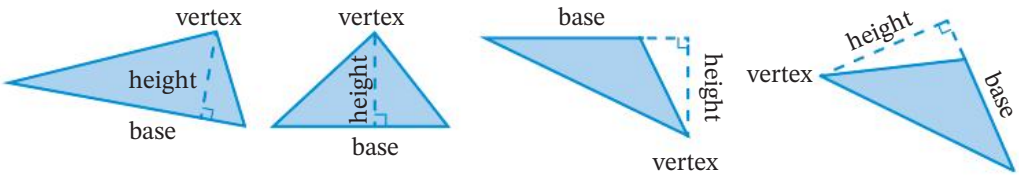
6 Repeat the above steps for triangle Y.

Area of a triangle

8.07

Any side of a triangle can be called its **base**. The **height** is the distance from the base to the opposite vertex. This distance is measured at right angles to the base, so it is also called the **perpendicular height**. Here are some examples:

Worksheet
Rectangle and triangle areas

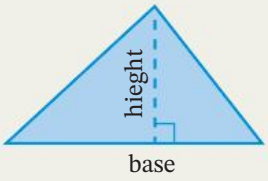


Note that with the last 2 examples involving obtuse-angled triangles, the perpendicular height is **outside** the triangle.

Area of a triangle

$$\text{Area} = \frac{1}{2} \times \text{base} \times \text{height}$$

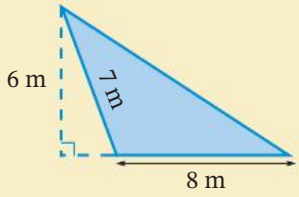
$$A = \frac{1}{2}bh$$



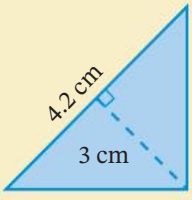
Example 8

Find the area of each triangle.

a



b



SOLUTION

a $\text{Area} = \frac{1}{2}bh$

$= \frac{1}{2} \times 8 \times 6$ Choosing 8 m as the base

$= 24 \text{ m}^2$ The length of 7 m is not required to find this triangle's area.

b $\text{Area} = \frac{1}{2}bh$

$= \frac{1}{2} \times 4.2 \times 3$ Choosing 4.2 cm as the base

$= 6.3 \text{ cm}^2$

EXERCISE 8.07 ANSWERS ON P. 629

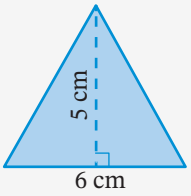
Area of a triangle

U F PS R

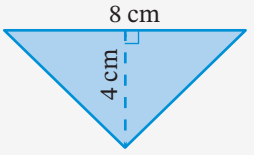
EXAMPLE 8

1 Find the area of each triangle.

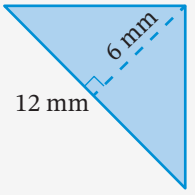
a



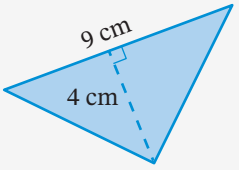
b



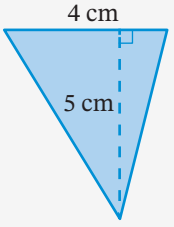
c



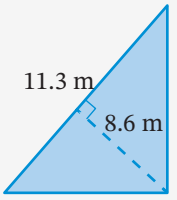
d



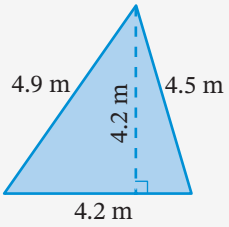
e



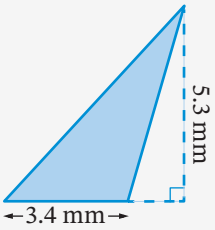
f



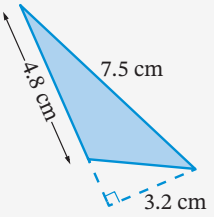
g



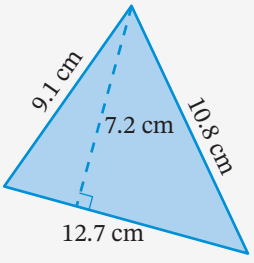
h



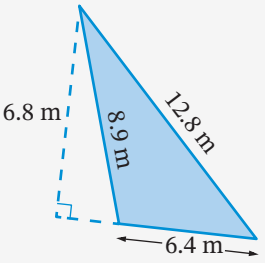
i



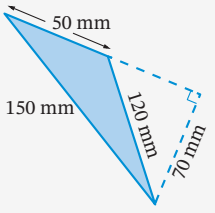
j



k

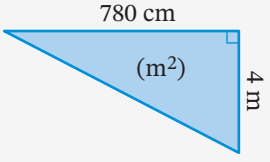


l

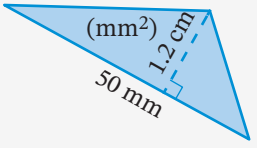


2 Find the area of each triangle, using the units given in brackets.

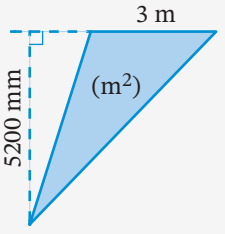
a



b



c

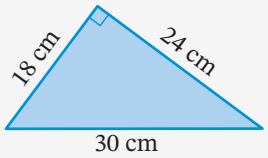


3 A triangle has a base of length 4 m and an area of 40 m². What is its perpendicular height? R

4 A triangle with an area of 27 cm² has a perpendicular height of 18 cm. How long is its base? R

Foundation Standard Complex

5 What is the area of this triangle? Select the correct answer **A**, **B**, **C** or **D**.



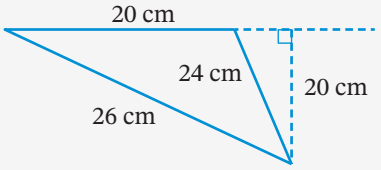
A 360 cm² **B** 432 cm²

C 270 cm² **D** 216 cm²

6 The fly-screen of a tent is to be made into the shape of a triangle 0.8 m long and 1.4 m high. Calculate the cost of the screen if one square metre costs $9.10.

PS

7 Find the area of this triangle. Select **A**, **B**, **C** or **D**.



A 130 cm² **B** 200 cm²

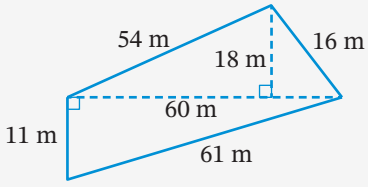
C 260 cm² **D** 240 cm²

8 A triangle has an area of 36 m². What could its base length and perpendicular height be?

R

9 The diagram shows the measurements of a new park.

Find the area of the park.



10 A triangle's perpendicular height is the same as its base length. If its area is 32 cm², what is its perpendicular height?

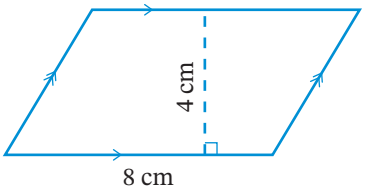
PS

INVESTIGATION

Area of a parallelogram

A **parallelogram** is a quadrilateral with opposite sides parallel and equal in length.

1 On a sheet of paper, draw a parallelogram with a base length of 8 cm and a perpendicular height of 4 cm.



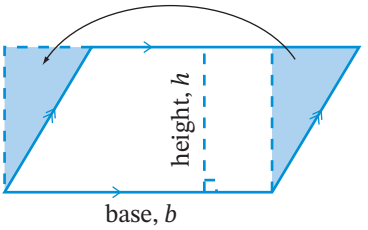
2 Draw an identical parallelogram in your book.

3 Cut out the parallelogram from the sheet of paper.

4 Cut along the dotted line (perpendicular height) and re-arrange the pieces to form a rectangle.

Foundation | Standard | Complex

5 Measure the length and width of the rectangle to the nearest millimetre, and then find its area.



6 Why is the area of the rectangle the same as the area of the parallelogram?

7 Suggest a general formula for finding the area of any parallelogram. (Check your answer with your teacher.)

8 Paste the rectangle in your book under your diagram of the parallelogram.

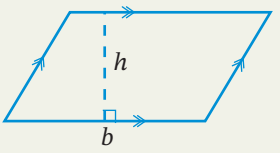
8.08 Area of parallelogram

The area of a **parallelogram** is the same as the area of a rectangle with the same base, b, and perpendicular height, h. It is found by multiplying the base, b, by the height, h.

Area $= b \times h = bh$

Area of a parallelogram

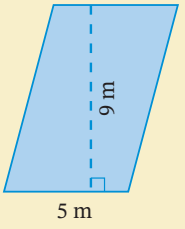
$A = bh$



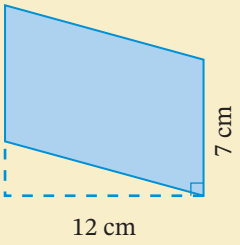
Example 9

Find the area of each parallelogram.

a



b



SOLUTION

a Area $= bh$

$= 5 \times 9$

$= 45\ \text{m}^2$

b Area $= bh$

$= 7 \times 12$

$= 84\ \text{cm}^2$

EXERCISE 8.08 ANSWERS ON P. 629

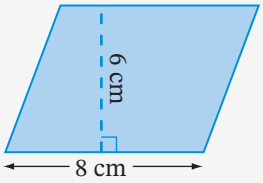
Area of a parallelogram

U F R

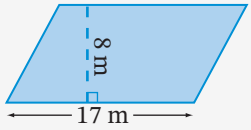
EXAMPLE 9

1 Find the area of each parallelogram.

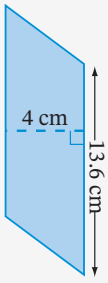
a



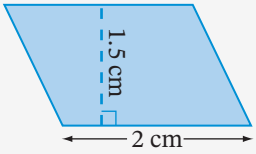
b



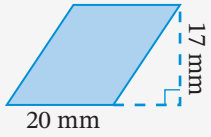
c



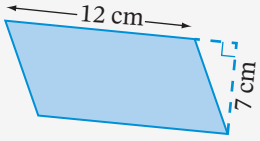
d



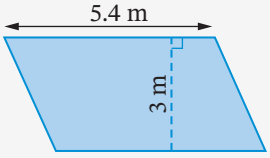
e



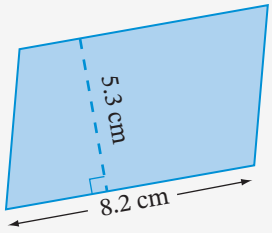
f



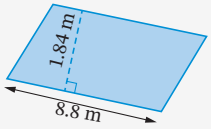
g



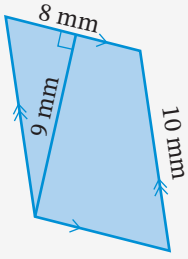
h



i



2 What is the area of this parallelogram? Select the correct answer **A**, **B**, **C** or **D**.

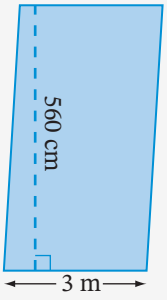


A 72 mm² **B** 36 mm²

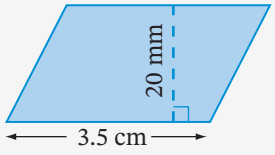
C 45 mm² **D** 80 mm²

3 Find the area of each parallelogram in square centimetres. Make sure you change both lengths to centimetres first.

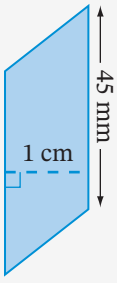
a



b

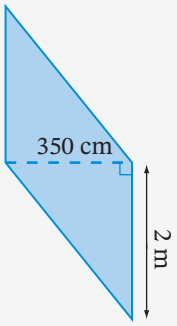


c

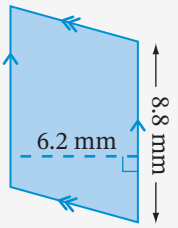


Foundation Standard Complex

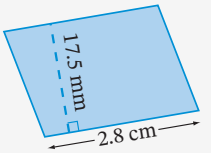
d



e



f



4 The courtyard of a house has the shape of a parallelogram with a base length of 4.3 m and an area of 24.94 m². What is its perpendicular height? Select **A**, **B**, **C** or **D**.

A 5.8 m **B** 10.6 m **C** 53.621 m **D** 107.24 m

5 A parallelogram has an area of 245 cm². What could its base length and perpendicular height be?

6 The area of a parallelogram is the same as the area of a square with side length 30 cm. Find the height of the parallelogram if the base is 36 cm.

8.09 Extension: Areas of composite shapes

YEAR 8 EXTENSION

Worksheet A page of composite shapes

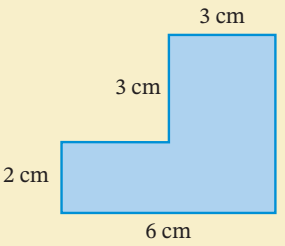
Video Areas of composite shapes

Puzzle Areas 1

A **composite shape** is made up of 2 or more shapes.

Example 10

Find the area of this shape.



SOLUTION

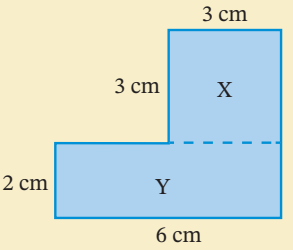
Method 1

Separate the shape into a square (X) and a rectangle (Y).

Area of shape = area of square X + area of rectangle Y

$= 3 \times 3 + 6 \times 2$

$= 21 \text{ cm}^2$



Foundation Standard Complex

Method 2

Separate the shape into 2 rectangles P and Q.

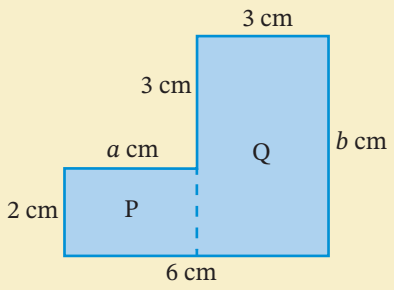
$a = 6 - 3 = 3$

$b = 3 + 2 = 5$

Area of shape = area of P + area of Q

$= 3 \times 2 + 5 \times 3$

$= 21 \text{ cm}^2$



Method 3

The area can also be found by subtracting areas.

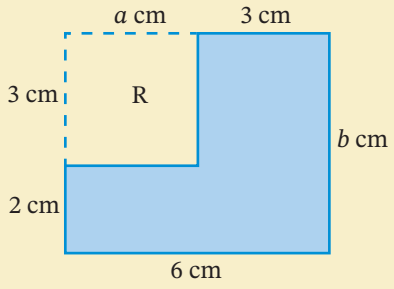
$a = 6 - 3 = 3$

$b = 3 + 2 = 5$

Area of shape = area of big rectangle – area of R

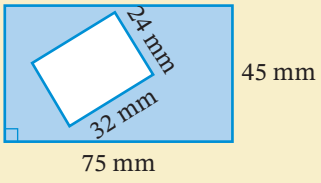
$= 6 \times 5 - 3 \times 3$

$= 21 \text{ cm}^2$



Example 11

Find the shaded area.



SOLUTION

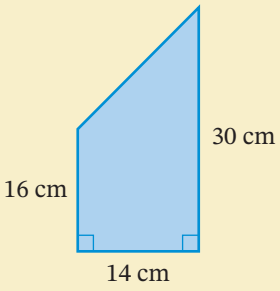
Shaded area = area of big rectangle – area of small rectangle

$= 75 \times 45 - 32 \times 24$

$= 2607 \text{ mm}^2$

Example 12

Find the area of this shape.



YEAR 8 EXTENSION

SOLUTION

Divide the shape into a triangle and a rectangle.

Height of triangle = 30 – 16

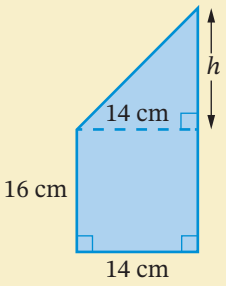
= 14 cm

Area of shape = area of rectangle + area of triangle

$= 16 \times 14 + \frac{1}{2} \times 14 \times 14$

$= 224 + 98$

$= 322 \text{ cm}^2$



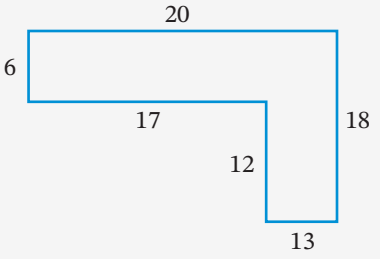
EXERCISE 8.09 ANSWERS ON P. 629

Areas of composite shapes

U F PS R

EXAMPLE 10

1 Which expression can be used to calculate the area of this shape? Select the correct answer **A**, **B**, **C** or **D**.



A $(20 \times 18) - (17 \times 12)$

B $(18 \times 13) + (20 \times 6)$

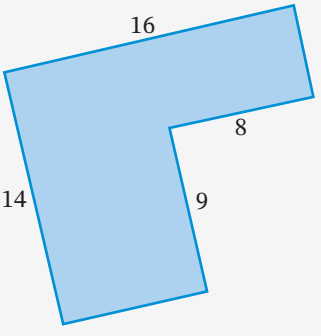
C $(20 \times 18) - (17 \times 6)$

D $(18 \times 13) + (17 \times 13)$

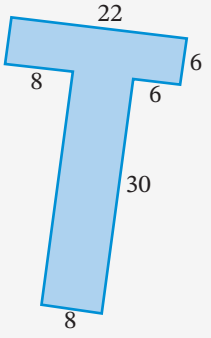
2 Find the area of each shape. All measurements are in centimetres.

PS R

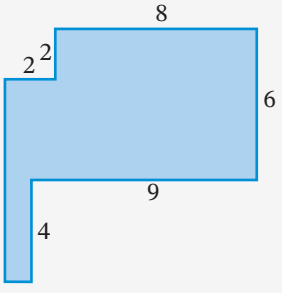
a



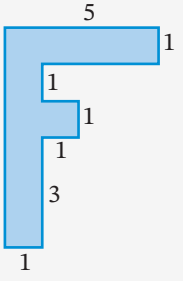
b



c



d



Foundation Standard Complex

EXAMPLE 11

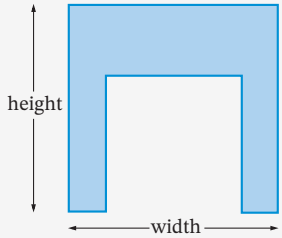
3 Find the area of each shape.

PS R

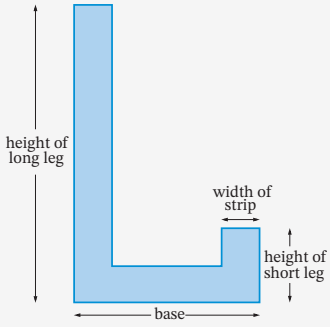
a Outside dimensions: 230 cm × 55 cm. Hole dimensions: 40 cm × 35 cm

b Total height: 30 cm, total width: 30 cm

Dimensions of cut-out: 20 cm × 20 cm

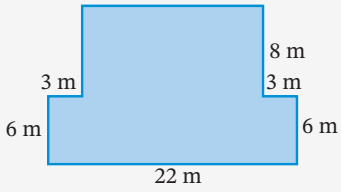


c This shape is formed from strips of metal 10 cm wide. The height of the long leg is 80 cm. The base is 50 cm wide. The height of the short leg is 20 cm.



4 This diagram shows a swimming pool at a holiday resort. What is the area of the floor of the pool? Select **A**, **B**, **C** or **D**.

PS R



A 156 m² **B** 180 m²

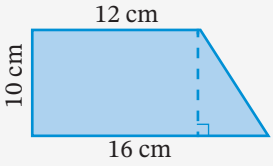
C 72 m² **D** 260 m²

EXAMPLE 12

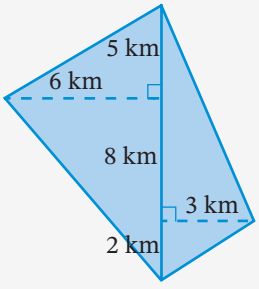
5 Find the area of each shape.

PS R

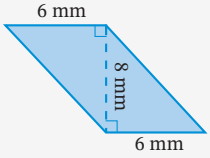
a



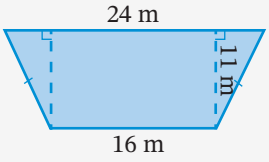
b



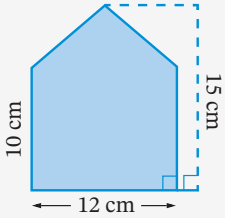
c



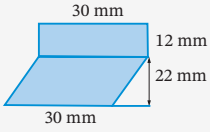
d



e



f



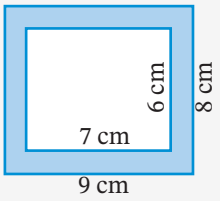
Foundation | Standard | Complex

YEAR 8 EXTENSION

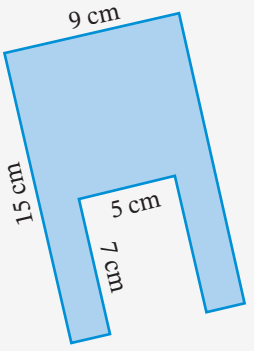
6 Find each shaded area.

PS R

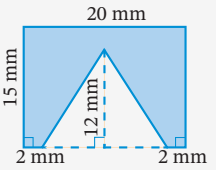
a



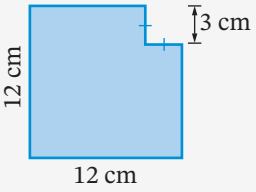
b



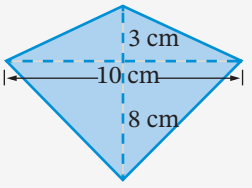
c



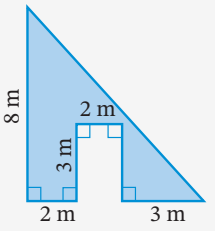
d



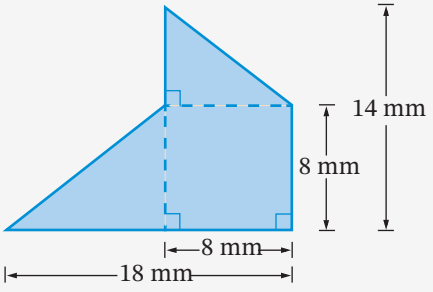
e



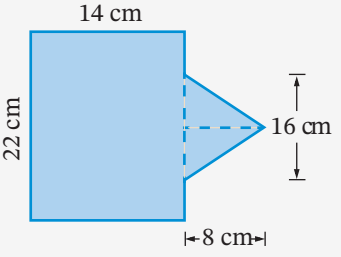
f



g



h

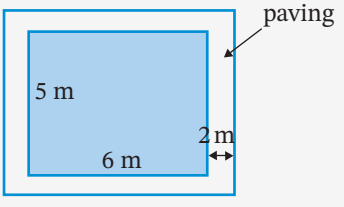


7 A rectangular lawn contains 2 square flower gardens, each measuring 3 m by 3 m. If the lawn measures 9.5 m by 6.4 m, find the area of the grass.

PS R

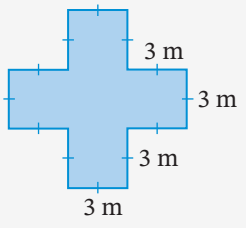
8 A garden measuring 6 m by 5 m has a 2 m wide strip of paving around its border. What is the area of the paving?

PS R



9 The floor of a children's pool is in the shape of a cross as shown. Each side is 300 cm long. How many tiles are required to tile this floor if one tile covers 1600 cm^2?

PS R



Foundation Standard Complex

9780170465557

INVESTIGATION

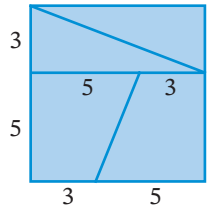
Area puzzles

Puzzle Area puzzles

Worksheet Square dot paper

This investigation can also be printed out as the Puzzle 'Area puzzles' to save you drawing the diagrams.

1 Draw this square on 1 cm dot paper or use the square from the worksheet 'Area puzzles'. All measurements are in centimetres. What is the area of this square in cm^2?

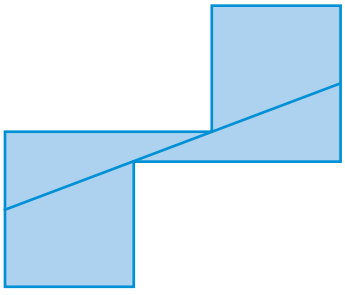


2 **a** Cut out the square, then cut along the marked lines.

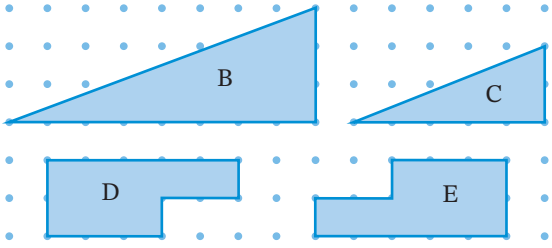
b Arrange the 4 pieces to make a rectangle. Find its length and width, and use these to find its area.

3 Is the calculated area of the rectangle the same as the area of the original square? Should it be? Explain this mystery.

4 Use the same 4 pieces to make this shape. Calculate its area. Is this the same as the area of the original square? Should it be? Why?

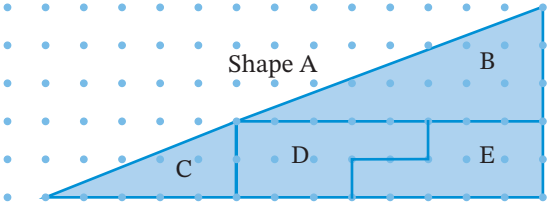


5 **a** Use square dot paper to draw each of the shapes B, C, D and E as shown, or use the shapes on the worksheet 'Area puzzles'. Find the area of each shape in cm^2.



b Calculate the sum of the 4 areas.

c Cut out the 4 shapes and combine them to form Shape A as shown below.



6 a Shape A looks like a right-angled triangle. Find its height and the length of its base, and use them to find its area.

b Does the area of the triangle equal the sum of the areas of B, C, D and E from question **1**? Explain the mystery about the area of the triangle, the area of shape A and the sum of the areas of B, C, D and E.

INVESTIGATION

Local areas

1 a Use a local map to find a park, garden or reserve near your home.

b Find the scale for measuring distance used on your map.

c Estimate the area of the park, garden or reserve.

2 a Rule lines on a sheet of tracing paper to make a grid of squares scaled like your map. Trace your park, garden or reserve from question **1** onto the paper.

b Approximate the area (in m^2 and ha) of your park, garden or reserve. Use the method of counting squares. (It may be more accurate to subdivide each square on the grid into smaller shapes.)

Quiz
Mental skills 8B

☆ MENTAL SKILLS 8B ANSWERS ON P. 629 Maths without calculators

Multiplying and dividing by 4 or 8

Multiplying and dividing by 4 or 8 involves repeated doubling or halving.

1 Study each example.

a 65×4 Think: Double twice

Double 65 = 130

Double 130 = 260

b 27×4 Think: Double twice

Double 27 = 54

Double 54 = 108

c 14×8 Think: Double 3 times

Double 14 = 28

Double 28 = 56

Double 56 = 112

d $236 \div 4$ Think: Halve twice

$\frac{1}{2} \times 236 = 118$

$\frac{1}{2} \times 118 = 59$

e $564 \div 4$ Think: Halve twice

$\frac{1}{2} \times 564 = 282$

$\frac{1}{2} \times 282 = 141$

f $392 \div 8$ Think: Halve 3 times

$\frac{1}{2} \times 392 = 196$

$\frac{1}{2} \times 196 = 98$

$\frac{1}{2} \times 98 = 49$

2 Now evaluate each expression.

a 14×4	**b** 27×4	**c** 16×4	**d** 105×4
e 43×8	**f** 16×8	**g** 28×8	**h** 33×8
i $184 \div 4$	**j** $272 \div 4$	**k** $560 \div 4$	**l** $432 \div 4$
m $624 \div 8$	**n** $312 \div 8$	**o** $256 \div 8$	**p** $152 \div 8$

Metric units for volume 8.10

Volume

The **volume** of a solid is the amount of space occupied by the solid.

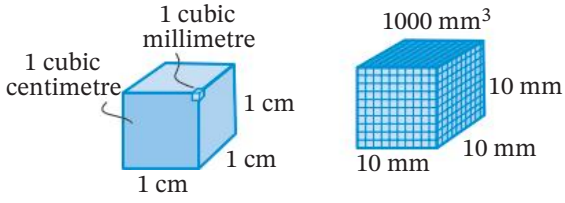
Skillsheet What is volume?

The table shows metric units of volume.

Volume unit	Abbreviation	The size of a cube with each side measuring ...
cubic millimetre	mm^3	1 mm
cubic centimetre	cm^3	1 cm
cubic metre	m^3	1 m

A **cubic millimetre** is about the size of a grain of raw sugar or rock salt.

A **cubic centimetre** is about the size of a person's tooth or a medical pill.



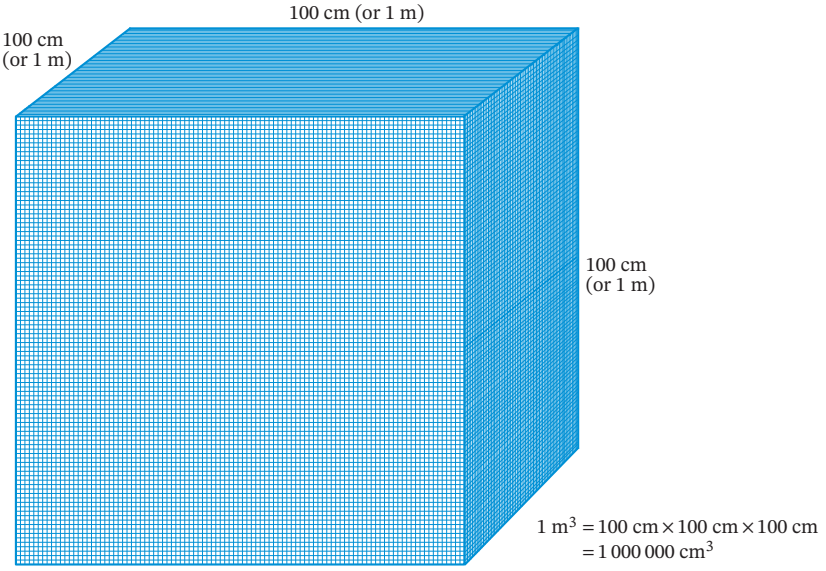
$1\ cm^3 = 10\ mm \times 10\ mm \times 10\ mm$

$= 10^3\ mm^3$

$= 1000\ mm^3$

There are 1000 cubic millimetres in 1 cubic centimetre.

A **cubic metre** is about the size of 2 washing machines. There are 1 000 000 cubic centimetres in 1 cubic metre.



$1\ m^3 = 100\ cm \times 100\ cm \times 100\ cm$

$= 100^3\ cm^3$

$= 1\ 000\ 000\ cm^3$

YEAR 8 EXTENSION

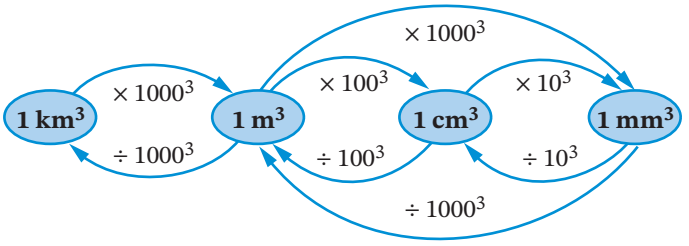
Metric units for volume

$1\ cm^3 = 1000\ mm^3$

$1\ m^3 = 1\ 000\ 000\ cm^3$

$1\ km^3 = 1\ 000\ 000\ 000\ m^3$

Because the side length is cubed to get volume, the length conversion factor is also cubed to get the volume conversion factor. This results in tripling the number of 0s in the length conversion factor.



Example 13

Convert:

a 12 000 mm^3 to cm **b** 48 m^3 to cm^3 **c** 750 000 000 m^3 to km^3

SOLUTION

a 12 000 $mm^3 = (12\,000 \div 10^3)\ cm^3$ — mm^3 to cm^3: $\div 10^3$

$= 12\ cm^3$

b 48 $m^3 = (48 \times 100^3)\ cm^3$ — m^3 to cm^3: $\times 100^3$

$= 48\,000\,000\ cm^3$

c 750 000 000 $m^3 = (750\,000\,000 \div 1000^3)\ cm^3$ — m^3 to km^3: $\div 1000^3$

$= 0.75\ cm^3$

EXERCISE 8.10 ANSWERS ON P. 629

Metric units for volume

U F C

1 What unit of volume would you use to measure the volume of:

C **a** a textbook? **b** a backpack?
c a wardrobe? **d** a large suitcase?
e a matchbox? **f** a room?

2 What is the approximate volume of a brick? Select the correct answer **A**, **B**, **C** or **D**.

A 20 cm^3 **B** 1000 cm^3
C 1600 cm^3 **D** 3000 cm^3

3 Match the correct volume (**A** to **G**) with each of the items (**a** to **g**) listed.

C

a	a bottle of liquid paper	**A**	200 m^3
b	a box of tissues	**B**	3890 m^3
c	a glass of water	**C**	1250 cm^3
d	a bottle of lemonade	**D**	5000 cm^3
e	a classroom	**E**	20 000 mm^3
f	a school hall	**F**	250 cm^3
g	a cereal package	**G**	2200 cm^3
h	a tall carton of milk	**H**	1000 cm^3

4 Copy and complete each conversion.

a 3 cm^3 = _____ mm^3 **b** 5 m^3 = _____ cm^3
c 2.6 cm^3 = _____ mm^3 **d** 4000 mm^3 = _____ cm^3
e 7.2 m^3 = _____ cm^3 **f** 66 000 mm^3 = _____ cm^3

YEAR 8 EXTENSION
EXAMPLE 13

Foundation Standard Complex

YEAR 8 EXTENSION

g $1\ m^3 =$ ____ mm^3

h $2300\ cm^3 =$ ____ m^3

i $126\ 000\ 000\ cm^3 =$ ____ m^3

j $3450\ mm^3 =$ ____ cm^3

k $25\ m^3 =$ ____ mm^3

l $78\ 000\ mm^3 =$ ____ m^3

m $63\ 000\ cm^3 =$ ____ m^3

n $1.4\ mm^3 =$ ____ cm^3

o $5000\ cm^3 =$ ____ m^3

5 The volume of a chest of drawers is $306\ 000\ cm^3$. Convert this to cubic metres.

C

6 The volume of a lunchbox is $2520\ cm^3$. Convert this to cubic millimetres.

C

7 A swimming pool has a volume of $38\ m^3$. Convert this to cubic centimetres.

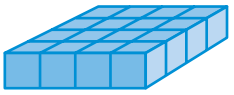
C

INVESTIGATION

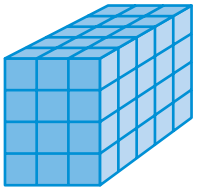
Volume of a rectangular prism

1 These rectangular prisms are made up of 1 cm cubes. Copy and complete the table.

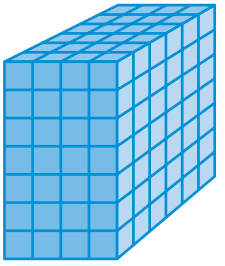
a



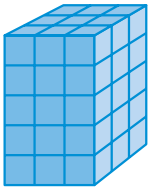
b



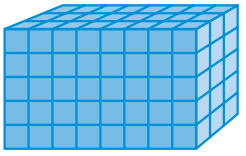
c



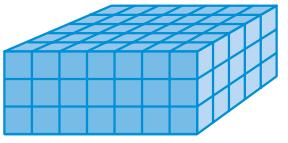
d



e



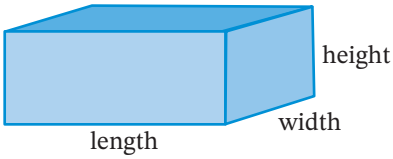
f



Shape	Number of cubes in one layer	Number of layers	Volume (cm^3)
a			
b			
c			
d			
e			
f			

Foundation Standard Complex

2 Copy and complete this table for the rectangular prisms in question **1**.



Shape	Length (cm)	Width (cm)	Height (cm)	Volume (cm^3)
a	4	4	1	16
b				
c				
d				
e				
f				

3 What is the relationship between the length, width and height of a rectangular prism and its volume?

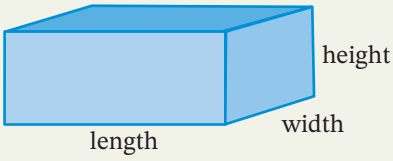
4 Write the relationship as a rule: Volume of a rectangular prism = ____ × ____ × ___

Volume of a rectangular prism 8.11

Volume of a rectangular prism

Volume = length × width × height

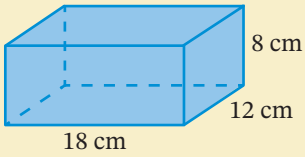
$V = lwh$



Worksheet
Volume

Example 14

Find the volume of the rectangular prism.



SOLUTION

$V = lwh$

$= 18 \times 12 \times 8$

$= 1728$

The volume is 1728 cm^3.

EXERCISE 8.11 ANSWERS ON P. 629

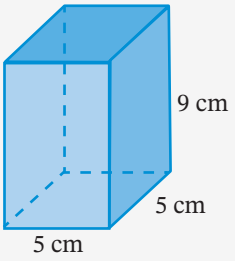
Volume of a rectangular prism

U F R

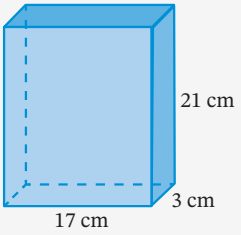
EXAMPLE 14

1 Find the volume of each rectangular prism.

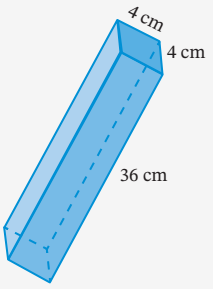
a



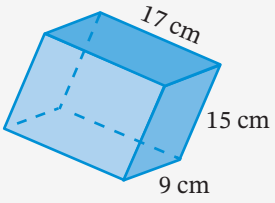
b



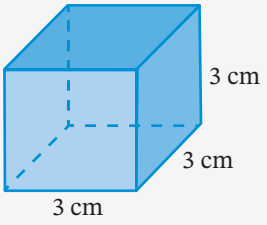
c



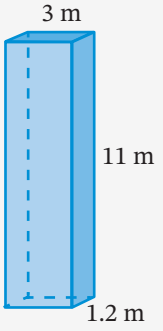
d



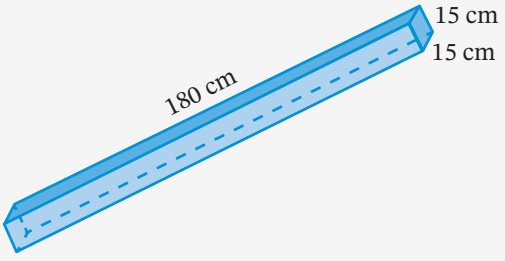
e



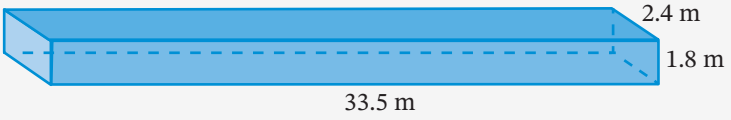
f



g



h



2 Calculate the volume of each rectangular prism described.

	Length	Width	Height
a	50 cm	50 cm	50 cm
b	5 cm	10 cm	18 cm
c	4 m	2.5 m	1.4 m
d	24 mm	16 mm	11 mm

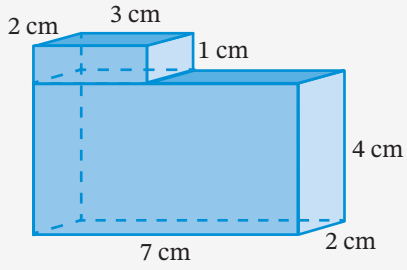
3 A tissue box has dimensions of 21 cm by 9 cm by 12 cm. Calculate its volume:

a in cubic centimetres **b** in cubic millimetres

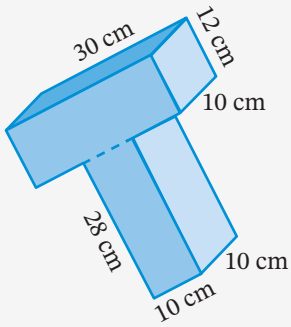
Foundation Standard Complex

4 Find the volume of each composite prism.

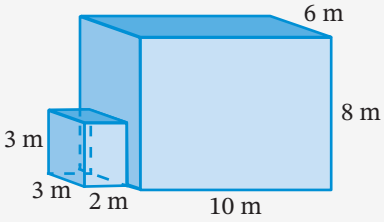
a



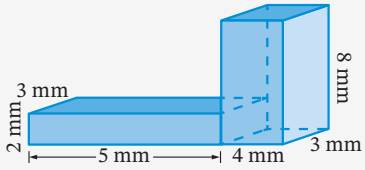
b



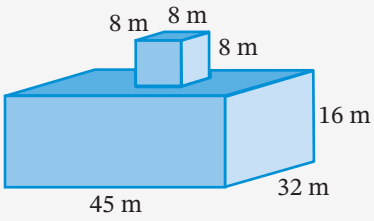
c



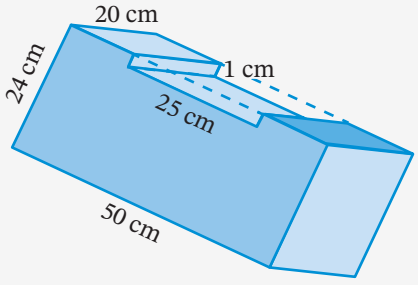
d



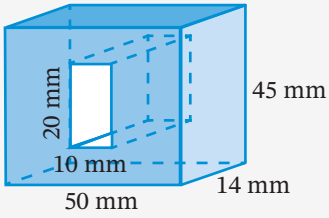
e



f



g

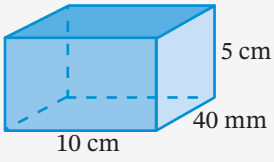


5 A refrigerator has dimensions of 68 cm by 166 cm by 65 cm. Calculate its volume:

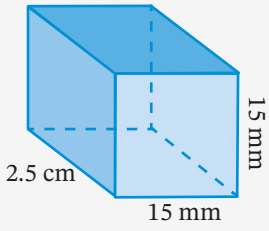
a in cubic centimetres **b** in cubic metres

6 Find the volume of each rectangular prism in cubic centimetres. Make sure you change all measurements to cm first.

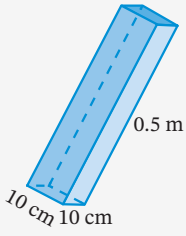
a



b

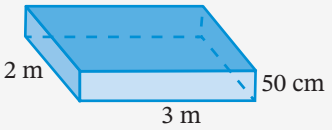


c

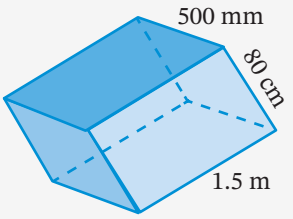


Foundation Standard Complex

d



e



7 A rectangular prism has length 5 mm, width 3 mm and volume 100 mm^3. What is its height as a mixed numeral?

R

8 A rectangular prism has width 1.5 m, height 3 m and volume 27 m^3. What is its length?

R

9 A rectangular prism has a volume of 880 cm^3. What could its length, width and height be?

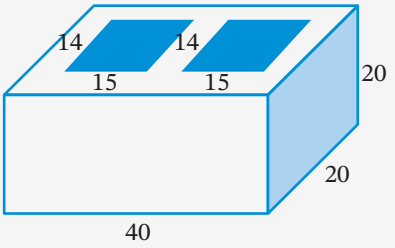
10 A rectangular garden is 12 metres long and 4 metres wide. It is filled with soil to a depth of 15 centimetres. Calculate the volume of the soil. Select the correct answer **A**, **B**, **C** or **D**.

A 0.72 m^3 **B** 7.2 m^3 **C** 72 m^3 **D** 720 m^3

11 Calculate the volume of the hollow cement block. All dimensions are in centimetres. Select **A**, **B**, **C** or **D**.

A 7600 cm^3 **B** 11 800 cm^3

C 15 580 cm^3 **D** 16 000 cm^3



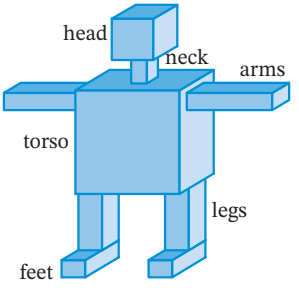
12 A cube has a volume of 125 m^3. What is its side length?

R

INVESTIGATION

What is your volume?

Imagine that you are made up of rectangular prisms.



1. With the help of a partner, make measurements of your body. Use them to find dimensions (to the nearest centimetre) for each of the prism body parts.
2. Sketch each body part prism and label its dimensions.
3. Use the prisms to find your volume in cm^3.
4. Write a report of what you did, showing all diagrams and calculations. Explain how you found the dimensions (length, width and height) for the prisms. Do you believe that you found a good approximation of your volume? Why?

Foundation Standard Complex

Volume of a prism

8.12

Remember from Chapter 6, *Geometrical figures*, that a **prism** is a polyhedron with the same (uniform) cross-section along its length. Because of these identical 'slices', the volume of any prism can be calculated by multiplying the **area of the cross-section** by its **perpendicular height** (the length or depth of the prism).

Skillsheet Solid shapes

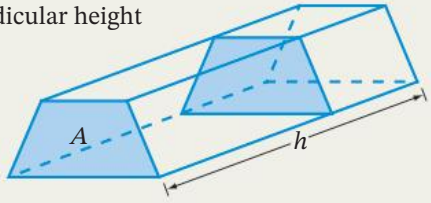
Worksheet Measuring shapes review

Video Volume of a prism

Volume of a prism

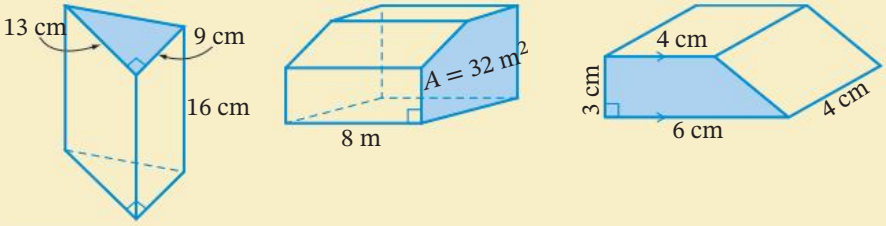
Volume of a prism = area of cross-section × perpendicular height

$= Ah$



Example 15

Find the volume of each prism.



SOLUTION

a $A = \frac{1}{2} \times 9 \times 13$ — Area of shaded triangular cross-section

$= 58.5 \text{ cm}^2$

$V = 58.5 \times 16$ — $V = Ah$, where height $h = 16$ cm

$= 936 \text{ cm}^3$

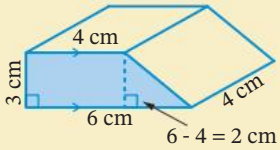
b $A = 32 \text{ m}^2$ — Area of shaded cross-section

$V = 32 \times 8$ — $V = Ah$, where height $h = 8$

$= 256 \text{ m}^3$

c $A = 4 \times 3 + \frac{1}{2} \times 2 \times 3$ — Cross-section is a trapezium, which is made up of a rectangle and a triangle.

$= 15 \text{ cm}^2$



$V = 15 \times 4$ — $V = Ah$, where height $h = 4$

$= 60 \text{ cm}^3$

EXERCISE 8.12 ANSWERS ON P. 630

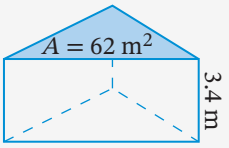
Volume of a prism

U F R

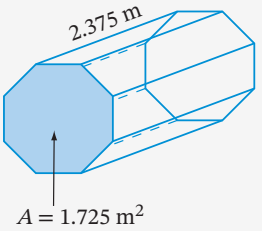
EXAMPLE 15

1 Find the volume of each prism.

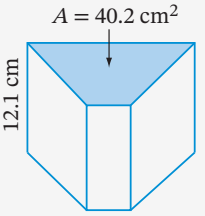
a



b

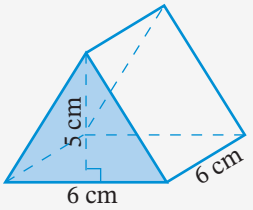


c

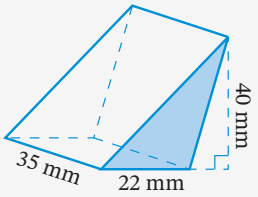


2 Find the volume of each triangular prism.

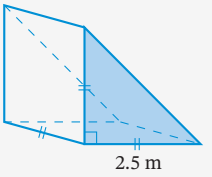
a



b

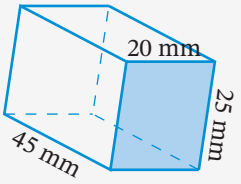


c

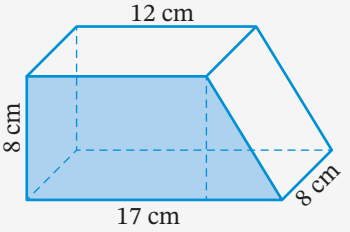


3 Find the volume of each prism.

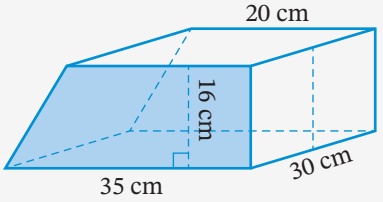
a



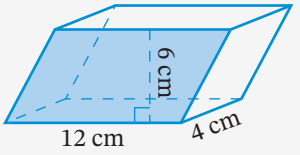
b



c



d

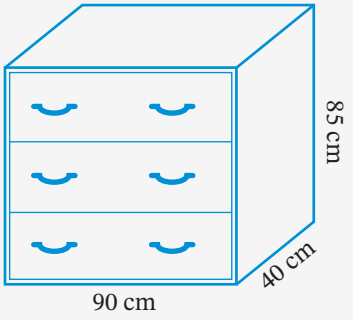


4 Find the volume of this chest of drawers. Give the answer in m^3:

a by calculating the volume in cm^3, then converting to m^3

b by converting each length to metres first, then calculating the volume.

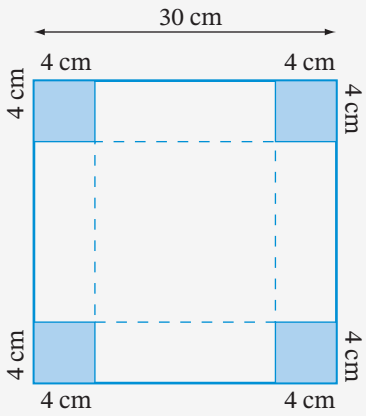
Which method is easier?



Foundation Standard Complex

9780170465557

5 A square sheet of metal has a side length of 30 cm. Four identical squares of length 4 cm are cut away from the corners, as shown. What is the volume of the prism formed when the remaining shape is folded along the dotted lines? Select **A**, **B**, **C** or **D**.



A 1936 cm³ **B** 2704 cm³

C 64 cm³ **D** 3600 cm³

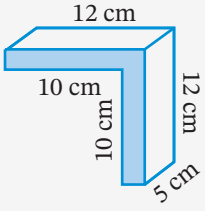
6 A triangular prism has base length 6 m, height 10 m and volume 150 m³. What is its length?

7 A cube has a volume of 343 m³. What is its side length?

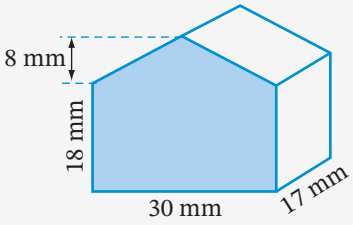
8 A triangular prism has a volume of 36 cm³. What could its length, perpendicular height and length be?

9 Calculate the volume of each prism.

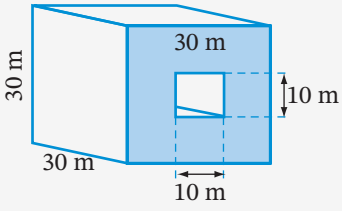
a



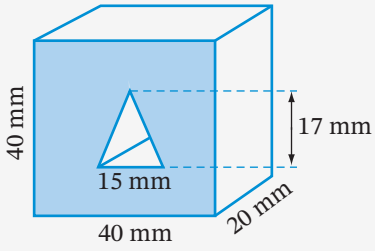
b



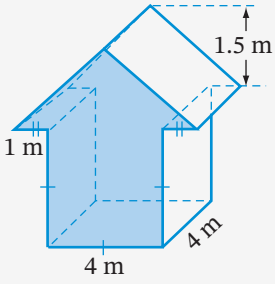
c



d



e

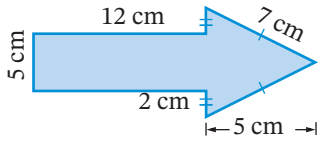


Foundation Standard Complex

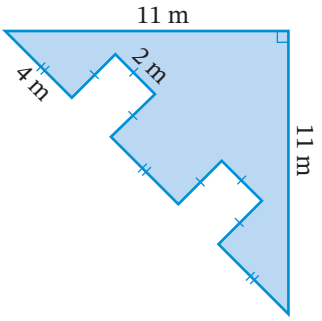
POWER PLUS ANSWERS ON P. 630

1 Find the perimeter and area of each shape.

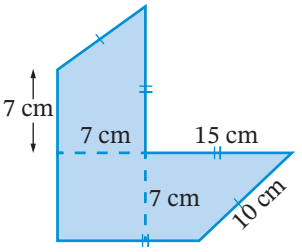
a



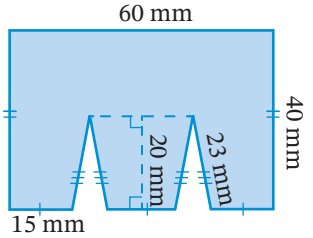
b



c



d



2 A rectangle has a perimeter of 30 cm.

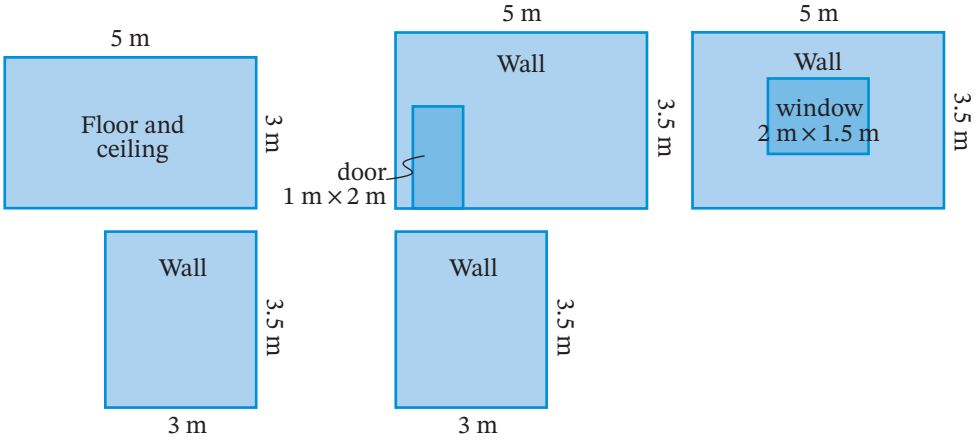
a List some possible dimensions for this rectangle.

b For each pair of dimensions, calculate the area of the rectangle.

c Which dimensions give the greatest area?

3 Find the length and width of a rectangle whose perimeter value is equal to its area value.

4 A room has dimensions as shown.



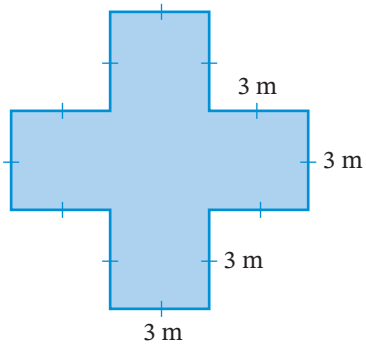
a The 4 walls and the ceiling are to be painted (not including the door and window). Calculate how many square metres are to be painted.

b Each surface requires 2 coats of paint, and 1 L of paint covers approximately 12 m^2. How many litres of paint are needed?

c Paint comes in 4 L cans that cost $45.95. Calculate how many 4 L cans are needed and calculate the cost of the paint.

d The floor is to be carpeted. How many square metres of carpet are needed?

5 A children's pool is in the shape of a cross, as shown on the right. Each side is 3 m long. The pool is filled with water to a depth of 300 mm.



a Find the area of the pool's surface.

b Calculate the volume of water in cubic metres (m^3).

c If water is charged at $0.80 per kL, how much does it cost to fill the pool?

6 In this Aboriginal flag, the yellow circle represents the sun and its diameter is $\frac{1}{3}$ of the length of the flag.

a What is the diameter of the yellow circle?

b The formula for the area of a circle is $A = \pi r^2$, where r is the radius of the circle. Calculate the area of the circle in the flag, correct to one decimal place.

c What percentage of the flag's area is the yellow circle? Answer correct to one decimal place.

d The red region of the flag represents Earth. What percentage of the flag's area is red? Answer correct to one decimal place.

8 CHAPTER REVIEW

Language of maths

Quiz
Language of maths 8

area	centi-	circle	circumference
composite shape	cross-section	cubic	diameter
height	isometric	kilo-	length
mass	metric	milli-	parallelogram
perimeter	perpendicular height	pi (π)	prefix
prism	radius	volume	width

1 Which prefix in metric units means:

a one thousandth? **b** one thousand?

2 In your own words, explain how you would calculate the perimeter of a square.

3 What name is given to the perimeter of a circle?

4 What does perpendicular height mean?

5 Explain: 'A prism has a uniform cross-section'.

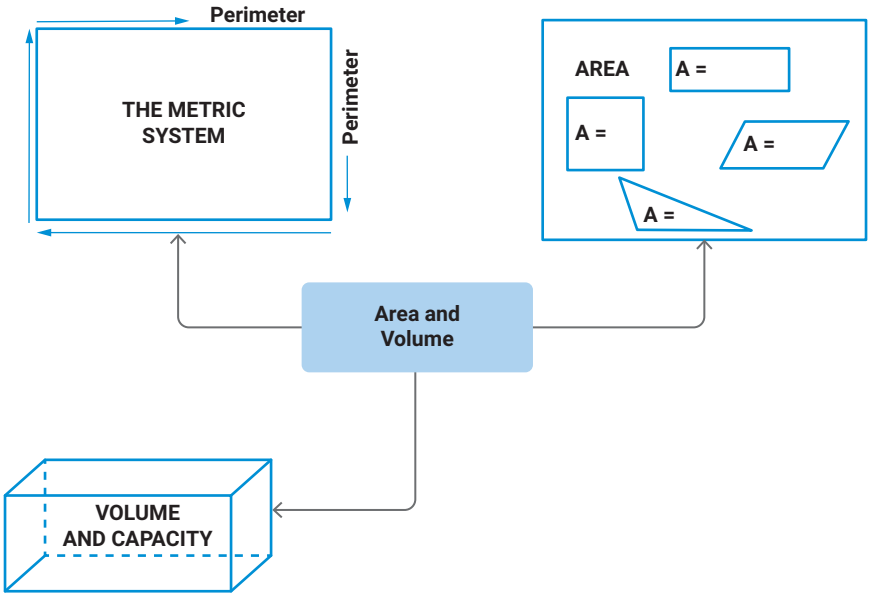
6 What is the difference between area and volume?

Topic summary

Worksheet
Mind map: Area and volume

- What have you learnt about measurement?
- What types of people use area as part of their jobs?
- List 2 measuring situations in which accuracy is important.
- Is there anything that you did not understand about the topic? Ask a friend or your teacher for help.

Print (or copy) and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.



 9780170465557

8 TEST YOURSELF

ANSWERS ON P. 630

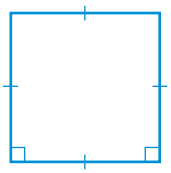
1 Convert: 8.01

Quiz Test yourself 8

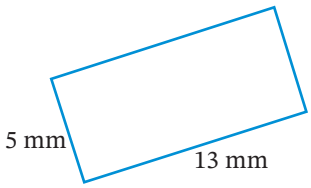
a	240 min to h	**b**	0.042 kg to g	**c**	125 L to kL
d	3200 m to km	**e**	8.3 L to mL	**f**	904 cm to mm
g	3.5 kg to t	**h**	15 h to min	**i**	15 h to s
j	7150 mm to m	**k**	4.2 kL to mL	**l**	17 000 000 mg to kg

2 Find the perimeter of each shape. 8.02

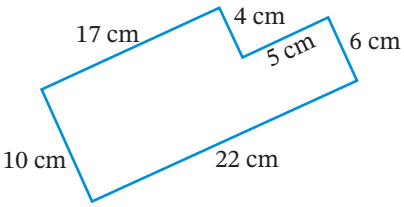
a



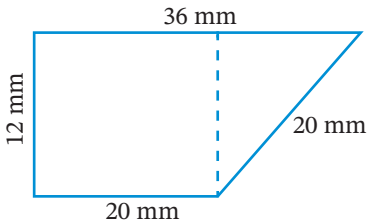
b



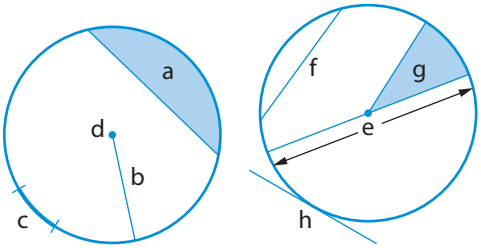
c



d



3 Name each part of the circle labelled. 8.03



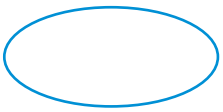
4 For each shape, state: 8.03

i the number of axes of symmetry

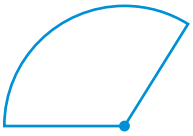
ii the order of rotational symmetry if it has rotational symmetry

a

b



c



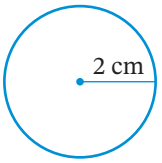
d

Foundation | Standard | Complex

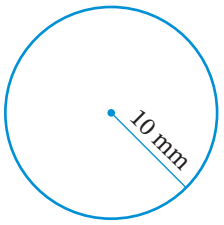
8.04

5 Calculate correct to one decimal place the circumference of each circle.

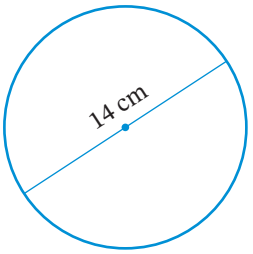
a



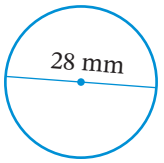
b



c



d



YEAR 8 EXTENSION

8.05

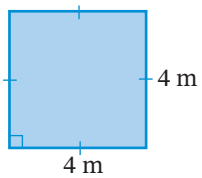
6 How many:

a cm^2 in 9.1 m^2?
b mm^2 in 2.5 cm^2?
c m^2 in 175 000 cm^2?
d m^2 in 240 000 mm^2?
e cm^2 in 240 mm^2?
f mm^2 in 8.6 m^2?

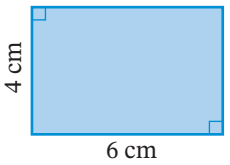
8.06

7 Find the area of each shape.

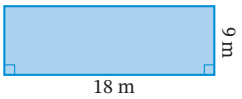
a



b



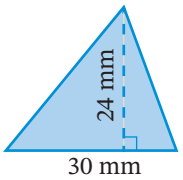
c



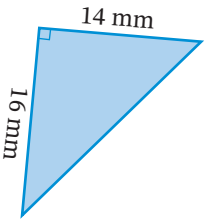
8.07

8 Find the area of each shape.

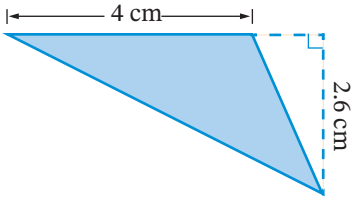
a



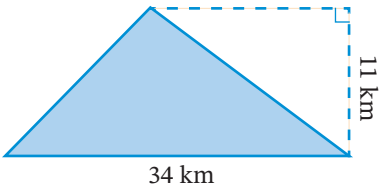
b



c



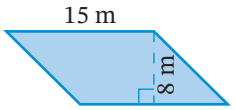
d



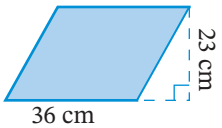
8.08

9 Find the area of each parallelogram.

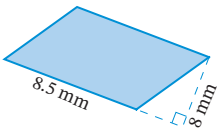
a



b



c



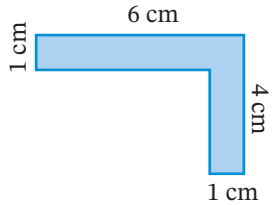
Foundation Standard Complex

9780170465557

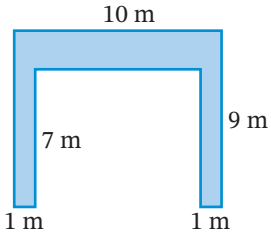
YEAR 8 EXTENSION

10 Find the shaded area in each shape. 8.09

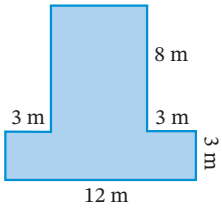
a



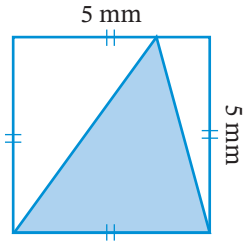
b



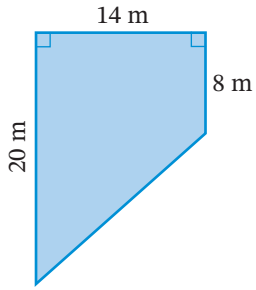
c



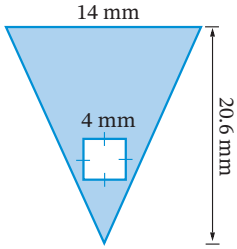
d



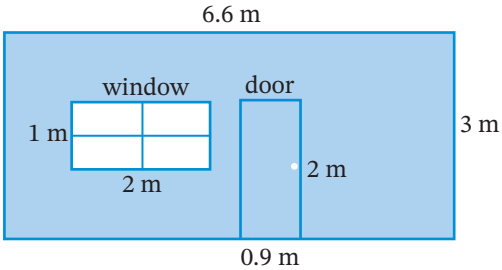
e



f



11 The wall shown needs painting. If 1 litre of paint covers 3 m² of wall, how many whole litres should be bought? 8.09



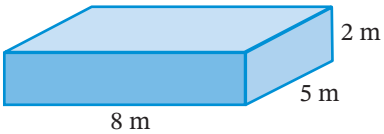
12 Copy and complete each conversion. 8.10

a 20 cm³ = _____ mm³ **b** 0.5 m³ = _____ cm³

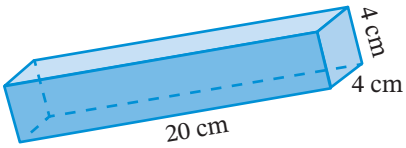
c 7500 mm³ = _____ cm³ **d** 230 000 mm³ = _____ m³

13 Find the volume of each prism. 8.11

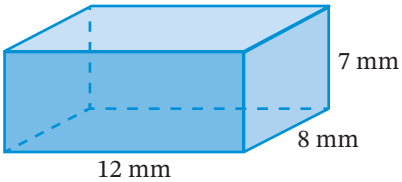
a



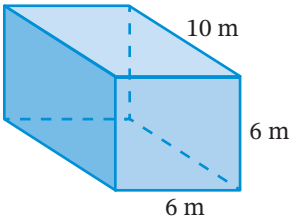
b



c



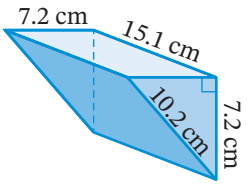
d



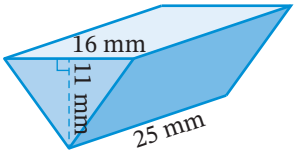
Foundation Standard Complex

8.12 **14** Find the volume of each prism.

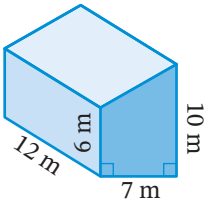
a



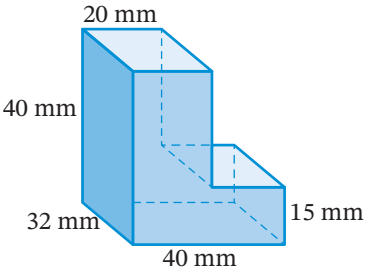
b



c



d



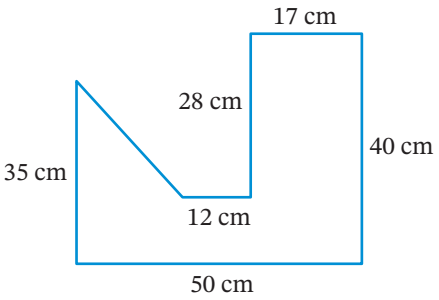
8.01 **15** How many minutes has Isla lived by her 12th birthday? Assume all years have 365 days. Select **A**, **B**, **C** or **D**.

A 525 600 **B** 6 307 200 **C** 5 522 880 **D** 378 432 000

8.09 **16** Find the area of this shape. Select **A**, **B**, **C** or **D**.

A 1034 cm^2 **B** 1047.5 cm^2

C 1317.5 cm^2 **D** 1521.5 cm^2



Foundation Standard Complex

9

SPACE, ALGEBRA

Formulas and graphs

Any place on the Earth's surface can be described using 2 numbers called **coordinates**. We can describe location using lines of **latitude** and **longitude**. Latitude measures the angle north or south from the equator, the imaginary horizontal line that runs across the middle of the Earth. Longitude measures the angle east or west from the Prime Meridian, the imaginary vertical line that runs through Europe and the Greenwich Observatory near London, England. The coordinates of latitude and longitude can be used to describe any point on Earth.

iStock.com/chombosan

Chapter outline

		Proficiencies
9.01	Location using coordinates	U F R C
9.02	The number plane	U F R C
9.03	Quadrants of the number plane	U F R C
9.04	Transformations on the number plane	U F R C
9.05	Graphing tables of values	U F R C
9.06	Formulas and tables of values	U F R C
9.07	Graphing linear functions	U F C

U = Understanding **R** = Reasoning
F = Fluency **C** = Communication
PS = Problem solving

Wordbank

coordinates The 2 values that give the position of a point on a number plane

formula A general rule written as an algebraic equation showing the relationship between variables

function A formula or equation in which an input value (such as x) is substituted to produce an output value (such as y).

number plane A grid that is made up of 2 number lines, meeting at right angles

ordered pair Another name for coordinates

origin The centre point of a number plane, with coordinates (0, 0)

quadrant One quarter of the number plane

***x*-axis** The horizontal number line on the number plane

***y*-axis** The vertical number line on the number plane

Quiz
Wordbank 9

Videos (4):

9.03 The number plane
9.04 Transformations on the number plane
9.07 Graphing linear equations • Linear functions

PhET interactives (2):

9.07 Function builder: Basics • Function builder

Quizzes (5):

- Wordbank 9
- SkillCheck 9
- Mental skills 9
- Language of Maths 9
- Test Yourself 9

Skillsheet (1):

9.02, 9.03 The number plane

Worksheets (9):

9.02 1 cm grid paper
9.03 Plane grid 2 • Number plane writing activity • Number plane review
9.05 Graphing tables of values
9.07 Number plane grid paper • A page of lines • A page of number planes
Power plus Plane grid 2
Mind map: Formulas and graphs

Puzzles (5):

9.03 Girt by sea • Quadrilateral person • Coordinates code puzzle
9.07 Matching linear equations • Graphing functions

Presentation (1):

9.03 Graph coordinates

Technology worksheet (1):

9.07 Graphing straight lines: Finding intercepts

Spreadsheet (1):

9.07 Linear function plotter

Nelson MindTap

To access resources above, visit **cengage.com.au/nelsonmindtap**

9 **In this chapter you will:**

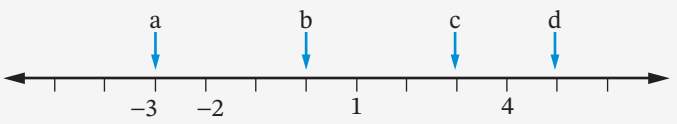
- ✓ use grid coordinates to describe the position of a place
- ✓ learn about the Cartesian (number) plane and coordinate system
- ✓ locate and plot points on the Cartesian plane, including all 4 quadrants
- ✓ translate points and shapes on the Cartesian plane
- ✓ reflect points and shapes on the Cartesian plane across the *x*- or *y*-axis
- ✓ rotate points and shapes on the Cartesian plane by a multiple of 90°
- ✓ graph points from a table of values, including points on a line
- ✓ substitute into a formula to complete a table of values
- ✓ use a table of values to graph a function on the number plane

SkillCheck

ANSWERS ON P. 630

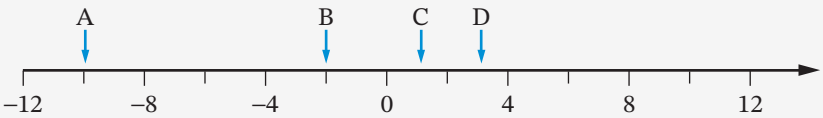
Quiz
SkillCheck 9

1 Write the integer shown by each arrow on this number line.

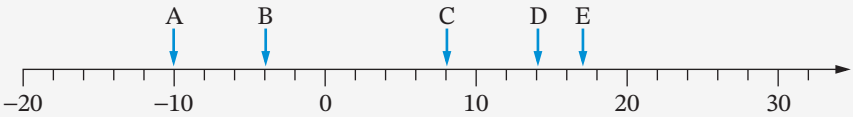


2 Write the integer shown by each arrow on each number line.

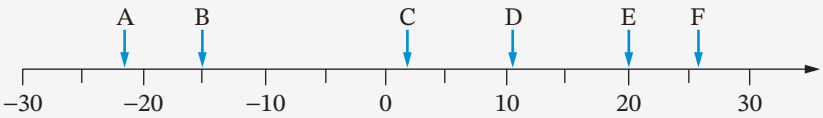
a



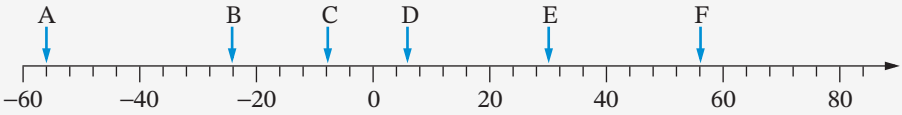
b



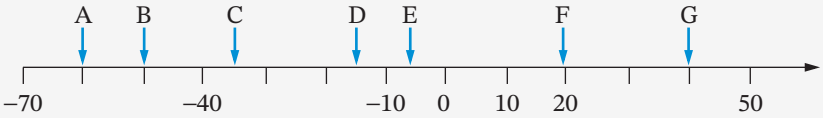
c



d



e



3 Mark each set of numbers as dots on a number line.

a –2, 4, 8, –3, 0

b the integers between –5 and 1

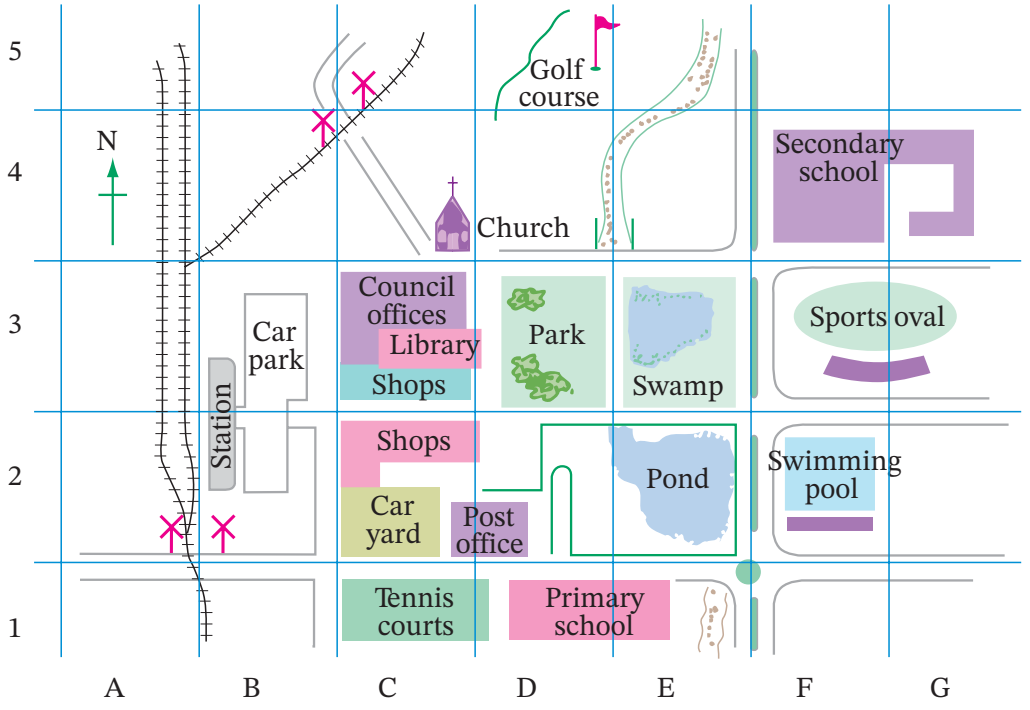
c the negative and positive integers between –10 and 4 that are multiples of 3

d –1.5, 5.5, 2.5, –4.5, –2.5

Location using coordinates 9.01

To locate a position on a map or theatre seating plan, we usually use a grid system of 2 coordinates – one for the horizontal and one for the vertical. On a map this may be a letter and a number, on a theatre plan it may be a row letter and a number.

The map of a country town next page uses a coordinate grid system where every position can be described by a column letter and row number. For example, the position of the golf course is D5, because it is in the square in column D, row 5.



EXERCISE 9.01 ANSWERS ON P. 630

Location using coordinates

U F R C

1 Use the town map above to write the position of:

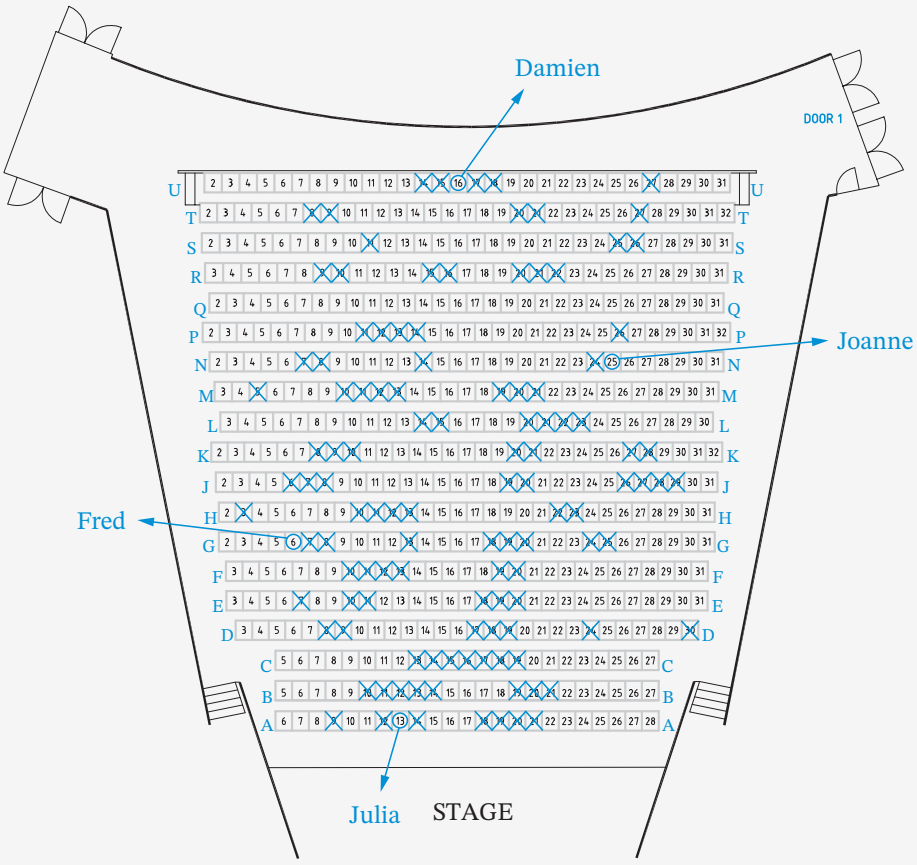
C **a** the swamp **b** the library **c** the church
d the secondary school **e** the sports oval **f** the council offices

2 On the town map, describe what can be found at:

C **a** F2 **b** D1 **c** D3
d E2 **e** B3 **f** C1

3 The map next page shows the seating plan for the Drama Theatre at the Sydney Opera House. Each seat in the theatre has a row number and a position number, for example, seat A17 is in the middle of the front row.

Foundation Standard Complex



Write the seat location of each person.

C **a** Julia **b** Fred **c** Joanne **d** Damien

4 How many rows are there in the Drama Theatre?

5 Are there more seats in the front rows or the back rows?

R

C

6 The seats marked with a cross have already been booked. State whether each seat has been booked:

a D4 **b** H15 **c** R11 **d** T9

7 Tara, Zoe and Asha want to sit together at the Drama Theatre. Give 2 sets of possible locations where they could sit.

R

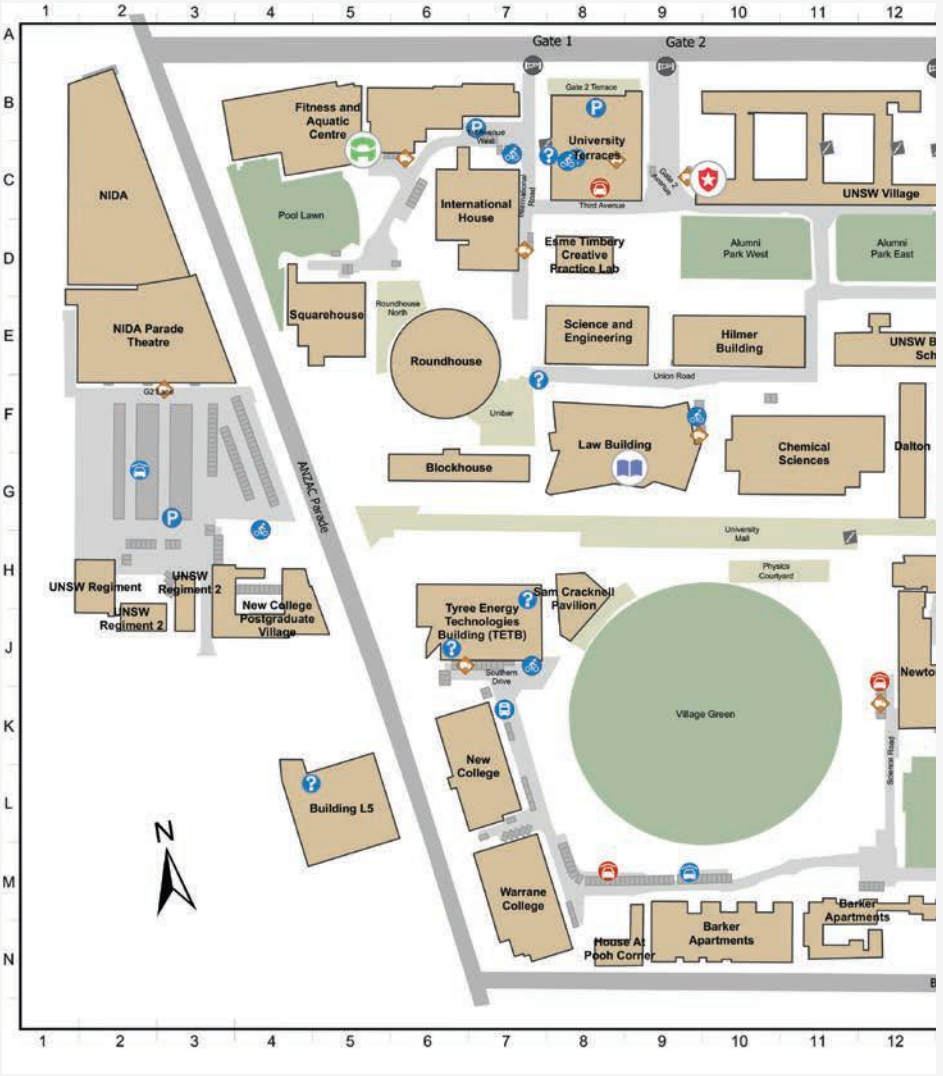
8 What type of pattern would be seen if all the people sitting in:

C **a** a seat with number 15 stood up?

b a seat with letter K stood up?

Foundation Standard Complex

9 This is part of a map of a university campus.



Name the building or feature that can be found at:

(C) **a** E6 **b** L5 **c** E2
d C6 **e** D12 **f** F9

10 From the university map, write the location of:

(C) **a** Warrane College **b** Chemical Sciences
c NIDA Theatre carpark **d** Squarehouse

Foundation Standard Complex

The number plane

9.02

Skillsheet
The number plane

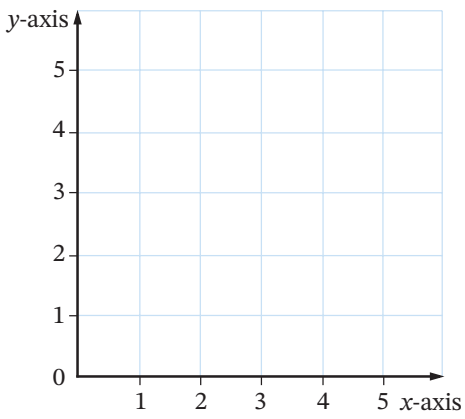
A **number plane** is a grid made from a **horizontal** number line called the ***x*-axis**, and a **vertical** number line called the ***y*-axis**. **Axis** means 'line', and the plural of axis is 'axes'.

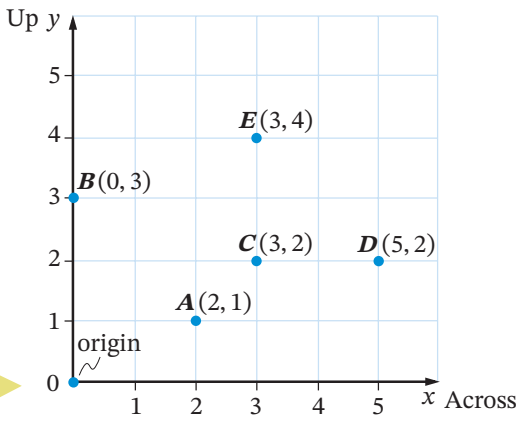
The number plane is also called a **Cartesian plane**, named after **René Descartes** (pronounced 'Ren-ay Day-cart'), a French mathematician and philosopher who developed the idea.

Points on the number plane are located using a pair of **coordinates**, for example (3, 2) means move 3 on the *x*-axis ('3 places right from 0') and move 2 on the *y*-axis ('2 up'). A pair of coordinates is also called an **ordered pair**.

On the number plane shown:

- *A* is the point (2, 1)
- *B* is the point (0, 3).





One way to remember that '*x* is across' is 'X is a cross'!

Coordinates on the number plane

- In any ordered pair, the first number is called the ***x*-coordinate** and the second number is called the ***y*-coordinate**.
- The point (0, 0) is called the **origin**.
- To locate a point, always start from (0, 0): the first coordinate tells you how far to move across, the second tells you how far to move up.

(2, 1) means (2 across, 1 up)

- start from (0, 0)
- move 2 units across
- then 1 unit up

(0, 3) means (0 across, 3 up)

- start from (0, 0)
- do not move across
- move 3 units up

Note that order is important with coordinates: (0, 3) is not the same as (3, 0).

EXERCISE 9.02 ANSWERS ON P. 631

The number plane

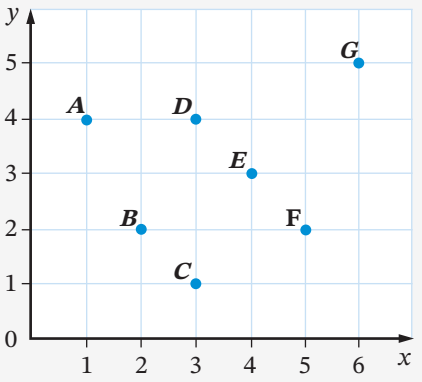
U F R C

Worksheet
1 cm grid paper

Some questions in this exercise require you to draw a number plane. You may want to download the worksheet '1 cm grid paper' to do this.

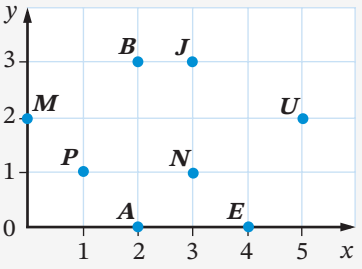
1 Write the coordinates of each point shown on this number plane.

C



2 Write the coordinates of the 8 points shown on this number plane.

C



3 Draw a number plane with the numbers from 0 to 6 on each axis.

C

a Start at the point (1, 1) and draw a line joining it to (4, 1).

b Join (4, 1) to (6, 2), then continue through (3, 2) → (1, 1) → (1, 4) → (3, 5) → (6, 5) → (4, 4) and finish at (1, 4).

c Now join (3, 2) to (3, 5).

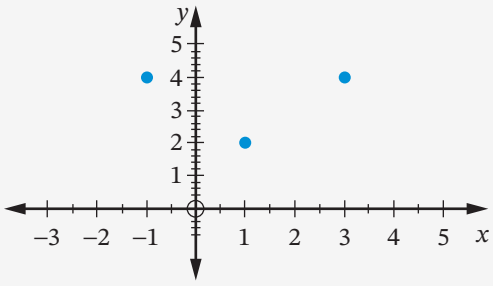
d Join (4, 1) to (4, 4).

e Join (6, 2) to (6, 5).

f What have you drawn?

Foundation Standard Complex

4 Aiden plots these 3 points on the number plane.



He wants to add a 4th point to form a parallelogram when you join the points. Which point could Aiden plot? Select the correct answer **A**, **B**, **C** or **D**.

R **A** (−3, 2) **B** (3, 2) **C** (5, 1) **D** (−1, 2)

5 Write the letter shown at each point on the number plane on the right, to spell some words.

C

a (3, 1), (4, 3), (2, 4), (3, 5), (5, 2), (1, 5)

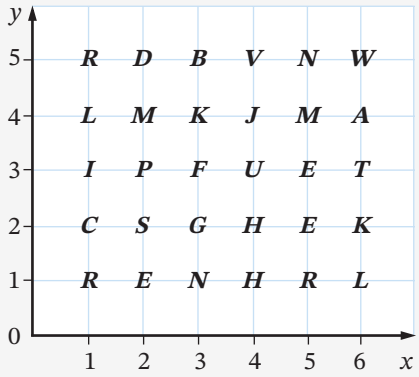
b (2, 3), (6, 1), (6, 4), (5, 5), (2, 1)

c (6, 5), (6, 4), (6, 3), (5, 3), (5, 1)

d (2, 4), (5, 2), (6, 3), (5, 1), (5, 3)

e (3, 1), (4, 3), (5, 1), (2, 2), (2, 1)

f (4, 3), (2, 4), (2, 3), (1, 3), (1, 1), (2, 1)



6 **a** Draw a number plane with the *x*-axis going from 0 to 20 and the *y*-axis going from 0 to 15.

b Plot the following points and connect them as you go. Colour appropriately.

- (0, 9), (8, 11), (10, 14), (15, 10), (16, 12), (17, 9), (19, 7), (19, 5), STOP
- (19, 7), (15, 5), (11, 9), (13, 5), (11, 0), (15, 4), (19, 5), STOP
- (17, 6), (13, 7), STOP
- (12, 7), (10, 7), (11, 3), (7, 7), (3, 7), (5, 8), (2, 8), (0, 9), STOP
- (3, 9), STOP

c What did you draw?

7 **a** Draw a number plane or use the worksheet 'Plane grid 1'. On the *x*-axis, mark 0 to 11. On the *y*-axis mark 0 to 16.

b Plot the following points, joining them as you go.

(1, 13), (3, 14), (4, 15), (5, 15), (6, 14), (6, 13), (4, 10), (7, 11), (9, 10), (10, 9), (11, 6), (11, 4), (8, 8), (9, 2), (9, 1), (7, 1), (8, 2), (7, 6), (6, 6), (7, 4), (7, 1), (5, 1), (6, 2), (6, 4), (5, 6), (3, 7), (2, 9), (2, 10), (4, 13), (1, 13)

c Draw an eye at (4, 14)

d What did you draw?

Foundation Standard Complex

DID YOU KNOW?

Global positioning system GPS

The global positioning system (GPS) originated in the 1960s with the need for the Russians and the United States Navy to track nuclear submarines. It is a space-based radio navigation system owned by the US Government and is freely available for all to use. Every smartphone and new car has a GPS application that allows you to find the position of any place in the world. The system requires a clear line of sight to 4 or more of the 30 GPS satellites circling the world. Other countries are now developing their own GPS that can precisely pinpoint a location to within 30 centimetres.

Research how the GPS receiver knows where you are.

9.03 Quadrants of the number plane

Skillsheet
The number plane

Worksheets
Plane grid 2

Number plane writing activity

Number plane review

Puzzles
Girt by sea

Quadrilateral person

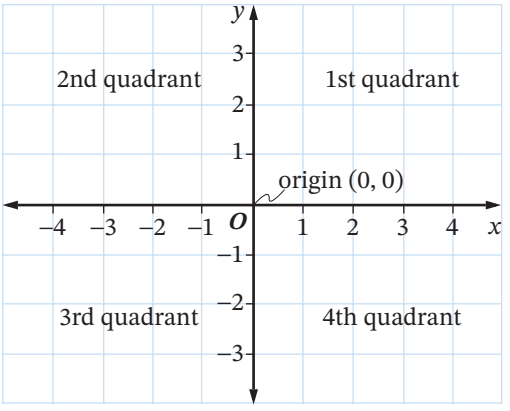
Coordinates code puzzle

Video
The number plane

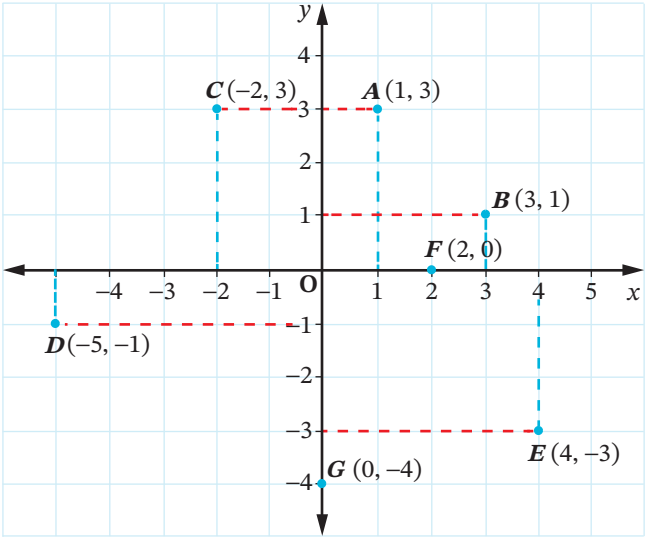
We can extend the number plane to use both positive and negative coordinates, by including negative numbers on the x- and y-axes.

The number plane is divided evenly into 4 regions called **quadrants**.

The centre point (0, 0) is called the **origin**.



A few points are labelled on the number plane below as examples.



Presentation
Graph coordinates

Worksheet
Plane grid 2

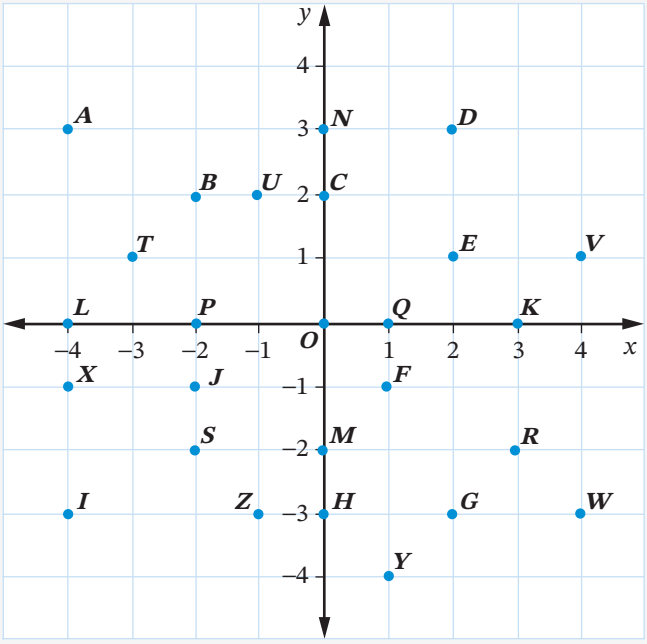
Quadrants of the number plane

U F R C

Some questions in this exercise require you to draw a number plane. You may want to use the 'Plane grid 2' worksheet for this.

1 Write the coordinates of each of the 26 points shown on the number plane.

C



2 **a** Draw a number plane with both axes extending from –6 to 6.

C **b** Plot and label each point on the number plane.

$A(3, 1)$	$B(-4, 3)$	$C(-3, 4)$	$D(-2, -2)$
$E(0, -2)$	$F(1, -5)$	$G(4, -4)$	$H(-3, 0)$
$I(-6, 0)$	$J(-1, -6)$	$K(-3, -4)$	$L(0, 5)$
$M(6, 2)$	$N(-3, 5)$	$P(5, -5)$	$Q(-1, 6)$

3 List all of the points from question **2** that are:

C **a** in the 1st quadrant **b** in the 2nd quadrant

c in the 3rd quadrant **d** in the 4th quadrant

e on the border of 2 quadrants.

4 In which quadrant would you find each point?

C **a** $(3, -5)$ **b** $(-2, -4)$ **c** $(-8, 1)$

Foundation Standard Complex

5 Copy and complete this table by writing + or –.

R

Points in:	*x*-coordinate	*y*-coordinate
1st quadrant	+	
2nd quadrant	–	
3rd quadrant		
4th quadrant		

6 **a** Draw a number plane, with both axes extending from –3 to 3.

b Plot and label each point.

$A(0, 1)$ $B(2, -1)$ $C(-1, 2)$ $D(-2, 3)$ $E(3, -2)$ $F(1, 0)$

c What do you notice about the points? Check this with your ruler.

C **d** Points that lie on the same straight line are called **collinear**.
Write down the coordinates of any 4 collinear points on your number plane.

7 Plot and label the points $P(1, -1)$, $Q(2, 1)$, $R(-1, 2)$, $S(-2,0)$ on a number plane. What shape is *PQRS*? Select the correct answer **A**, **B**, **C** or **D**.

R **A** rectangle **B** square **C** rhombus **D** parallelogram

8 **a** Draw a number plane with both axes extending from –10 to 10.

b Start at the first point and join the points in the order given:

- (4, 0), (5, 1), (8, 1), (9, 0), (9, –3), (8, –4), (5, –4), (4, –3), (4, 0), STOP
- (–2, 0), (–3, 1), (–6, 1), (–7, 0), (–7, –3), (–6, –4), (–3, –4), (–2, –3), (–2, 0), STOP
- (7, –2), (4, 5), (1, –1), (–2, 5), (–5, –2), STOP
- (3, 6), (5, 6), (4, 5), (–2, 5), (–2, 6), (–4, 6), (–4, 4), (–3, 4), STOP
- (1, 1), (1, –3), STOP
- (0.5, 1), (1.5, 1), STOP
- (0.5, –3), (1.5, –3), STOP

C **c** What did you draw?

INVESTIGATION

Number plane picture puzzles

- Draw a number plane from –10 to 10 on both x- and y-axes.
- Create your own number plane drawing using all 4 quadrants.
- Write out the set of points so that someone else could draw it.

Foundation Standard Complex

Transformations on the number plane

9.04

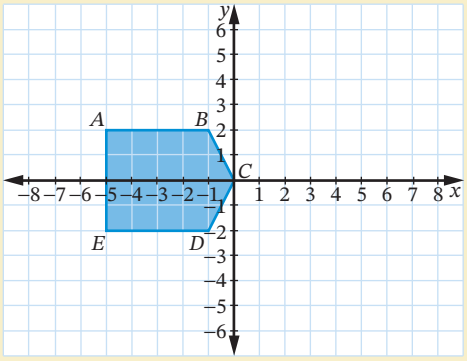
In Chapter 6, *Geometrical figures*, we looked at the 3 congruence transformations: **translation** (slide), **reflection** (flip) and **rotation** (spin). The original shape is called the **original**, while the transformed shape is called the **image**.

When a point or vertex P of the original shape is transformed, the corresponding point or vertex on the image is labelled P', pronounced 'P-dash' or 'P-prime'.

Example 1

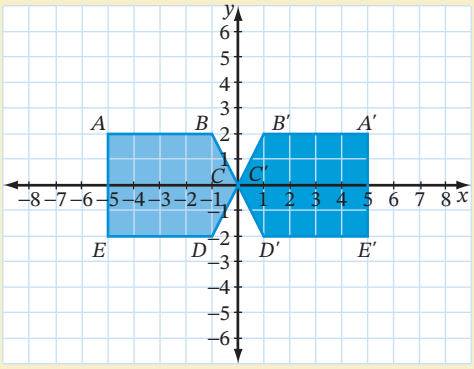
a Reflect the pentagon $ABCDE$ across the y-axis to create the image $A'B'C'D'E'$.

b Compare the coordinates of each vertex of $ABCDE$ to its matching vertex in $A'B'C'D'E'$.



SOLUTION

a



b $A(-5, 2) \rightarrow A'(5, 2)$

$B(-1, 2) \rightarrow B'(1, 2)$

$C(0, 0) \rightarrow C'(0, 0)$

$D(-1, -2) \rightarrow D'(1, -2)$

$E(-5, -2) \rightarrow E'(5, -2)$

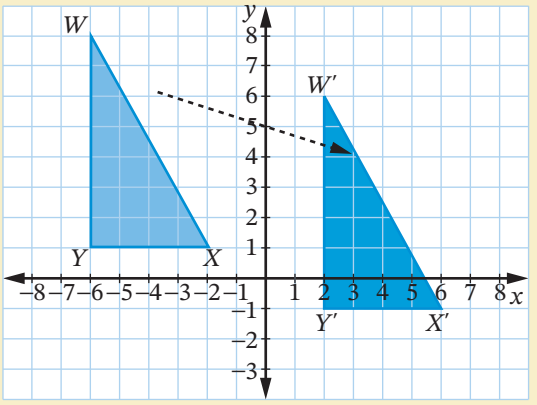
When reflected in the y-axis, the x-coordinate of each vertex changes sign while the y-coordinate stays the same.

Video
Transformations on the number plane

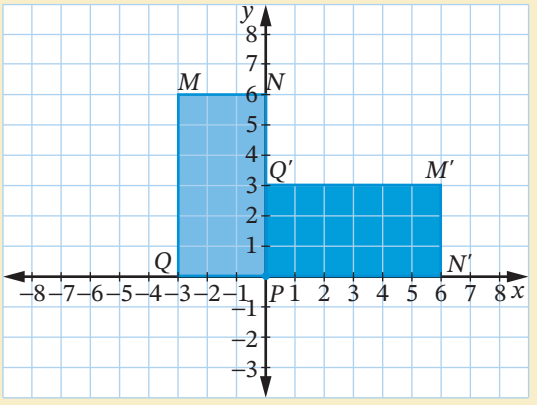
Example 2

Accurately describe the transformation that has been performed on each lighter figure to produce the darker image, and compare the coordinates of the vertices of the original to those of the image.

a



b



SOLUTION

a WXY has been translated 8 units right and 2 units down.

$W(-6, 8) \rightarrow W'(2, 6)$

$X(-2, 1) \rightarrow X'(6, -1)$

$Y(-6, 1) \rightarrow Y'(2, -1)$

The x-coordinate of each vertex increases by 8 units, while the y-coordinate decreases by 2 units.

b Rectangle $MNPQ$ has been rotated 90° clockwise about the point $P(0, 0)$.

$M(-3, 6) \rightarrow M'(6, 3)$

$N(0, 6) \rightarrow N'(6, 0)$

$P(0, 0) \rightarrow P'(0, 0)$

$Q(-3, 0) \rightarrow Q'(0, 3)$

The x-coordinate of each vertex changes sign and becomes the y-coordinate of the image vertex, while the y-coordinate of each vertex becomes the x-coordinate of the image vertex.

EXERCISE 9.04 ANSWERS ON P. 632

Transformations on the number plane

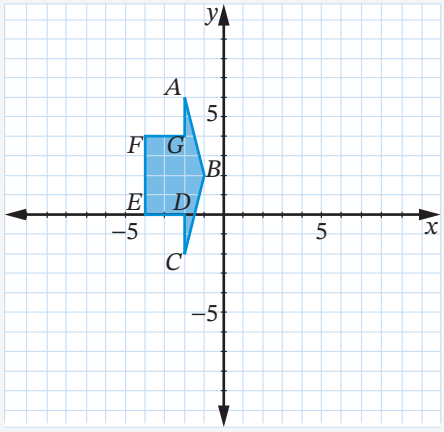
U F R C

EXAMPLE 1

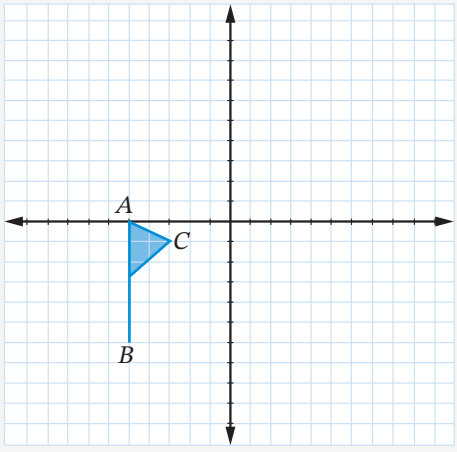
1 **a** Copy and translate arrow *ABCDEFG* 6 units left and 2 units down to create the image $A'B'C'D'E'F'G'$.

R C

b Compare the coordinates of each vertex in the original arrow to its matching vertex in the image arrow.



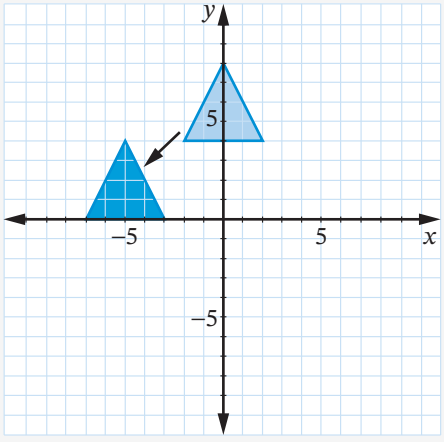
2 Copy and translate this flag 6 units right and 3 units up. Write down the coordinates of the new positions of *A*, *B* and *C*.



EXAMPLE 2

3 Accurately describe the transformation that has been performed on the triangle.

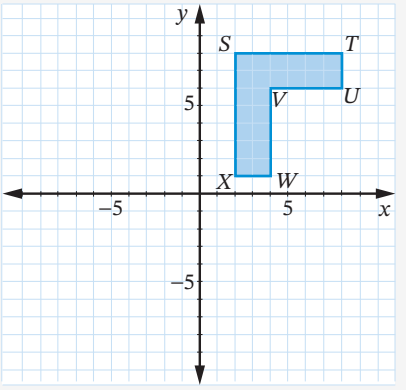
C



Foundation Standard Complex

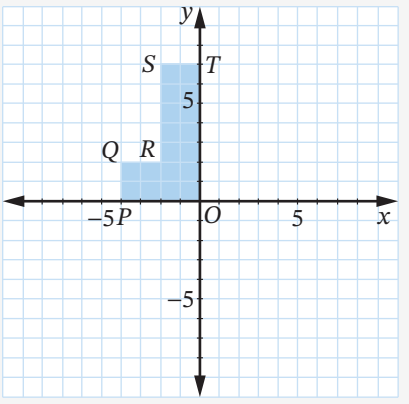
4 **a** Copy and reflect the L-shape across the x-axis to create the image $S'T'U'V'W'X'$.

R C **b** Compare the coordinates of each vertex in the original figure to its matching vertex in the image figure.



5 **a** Copy and rotate $OPQRST$ about O through an angle of 180° to produce the image $O'P'Q'R'S'T'$.

R C **b** Compare the coordinates of each vertex of $OPQRST$ to its matching vertex in $O'P'Q'R'S'T'$.

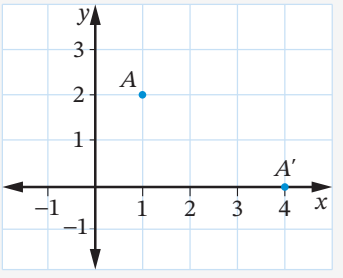


6 The point $A(1, 2)$ is translated to $A'(4, 0)$. If the point $B(-2, 2)$ also undergoes the same translation, what are the coordinates of the image of B? Select the correct answer **A**, **B**, **C** or **D**.

R

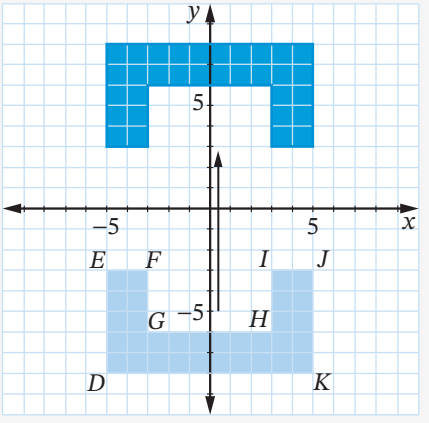
A (3, 0) **B** (1, 0)

C (0, −1) **D** (1, 1)



7 **a** Accurately describe the transformation that has been performed on shape $DEFGHIJK$.

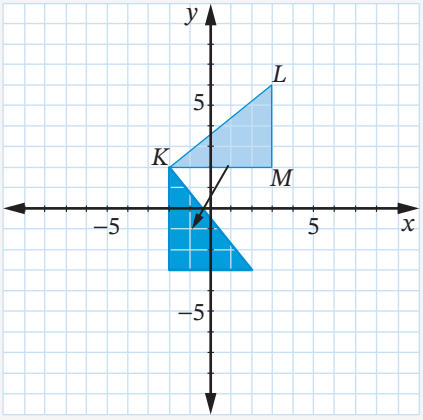
R C **b** Write the coordinates of the new position of D and compare them to its original coordinates.



Foundation Standard Complex

8 **a** Accurately describe the transformation that has been performed on ΔLMK.

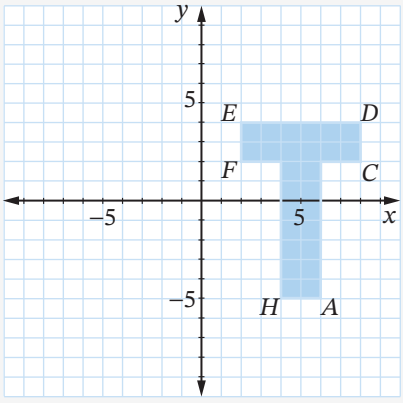
C **b** Write the coordinates of the new position of L.



9 **a** Reflect the T-shape across the y-axis.

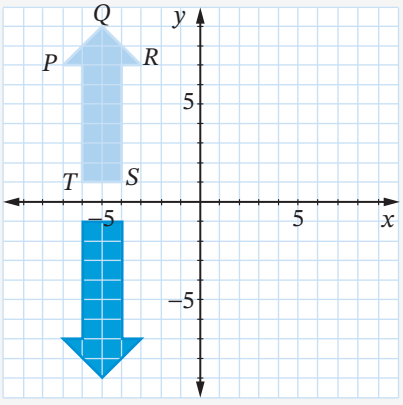
R **b** Write the coordinates of the new position of A and compare them to its original coordinates.

C



10 **a** Accurately describe the transformation that has been performed on this arrow.

C **b** Write the coordinates of the new position of R.

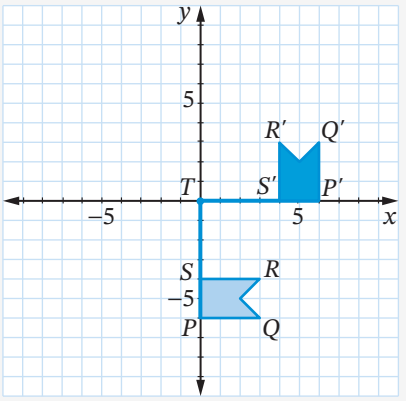


Foundation Standard Complex

11 **a** Accurately describe the transformation that has been performed on this flag.

R

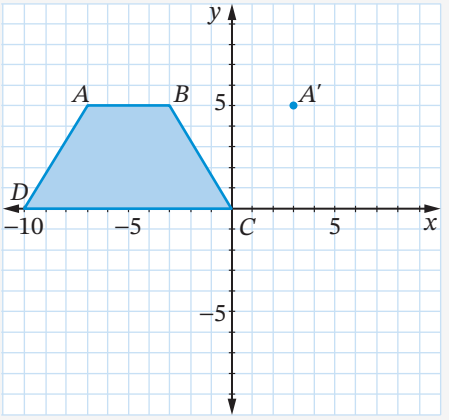
C **b** Write the coordinates of R' and compare them to the coordinates of R.



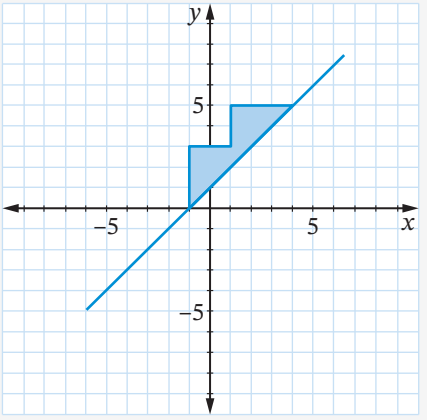
12 Point A of this trapezium is transformed to Point A'. Accurately describe the transformation, write the new coordinates of vertex D and compare them to its original coordinates.

R

C



13 Copy and reflect the shape across the blue line.



Foundation Standard Complex

TECHNOLOGY

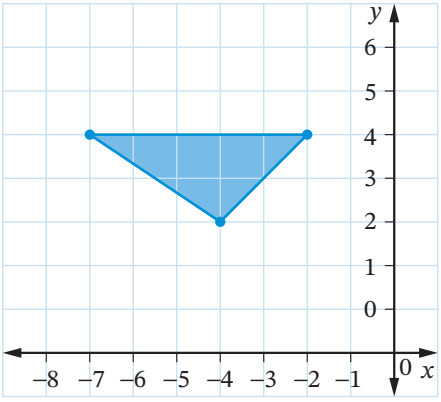
Transformations on the number plane

In this activity you will create reflections, rotations and translations of various shapes using dynamic geometry software. Create a number plane that shows all 4 quadrants and values of x and y from –10 to 10.

Activity 1: Reflection

1 Draw a triangle with vertices at (–7, 5), (–2, 5) and (–4, 3).

2 Draw a **reflection line on the *y*-axis,** through the points (0, 6) and (0, 0).



3 Reflect the triangle across the line.

4 Click a vertex and drag it. What happens?

5 Draw a new **reflection line** through the points (2, 7) and (–5, –2).

6 Reflect the triangle across this line. Click a vertex and drag it. What happens?

Activity 2: Rotation

1 Draw a polygon with vertices at (0, 3), (–2, 3), (–3, 2), (–2, 0) and (0, 3).

2 Select the polygon and rotate it 45° clockwise about the point (0, 3).

3 Repeat step **2**, then rotate the polygon 180° clockwise about (0, 3). What happens?

4 Try other points as the centre point and enter different angles. What happens?

Activity 3: Translation

1 Draw a polygon with vertices at (–3, 4), (–3, –1), (1, –1), (1, 1), (–2, 1), (–2, 4) and (–3, 4).

2 Translate the polygon so that the image of (1, 1) is (4, 3).

3 Now translate the shape in 2 other ways.

Activity 4

1 Construct a **Polygon**.

2 Reflect the polygon about a point.

3 Reflect the polygon about a line.

4 Rotate the polygon by an angle.

5 Translate the polygon by a vector.

9.05 Graphing tables of values

Worksheet
Graphing tables of values

Ordered pairs of coordinates can also be written in a **table of values**.

x	y
−3	−2
−2	−1
0	2
1	0

means
(−3, −2)
(−2, −1)
(0, 2)
(1, 0)

x	0	1	2
y	5	7	3

means
(0, 5)
(1, 7)
(2, 3)

EXERCISE 9.05 ANSWERS ON P. 633

Graphing tables of values

U F R C

1 Write down the ordered pairs shown in each table of values.

C

a

x	y
0	2
2	3
4	4
7	9

b

x	y
−2	−1
−1	3
2	4
5	−2

c

x	0	1	2
y	2	5	8

d

x	−4	−2	1	5
y	−1	−3	0	2

2 Plot the coordinates shown in each table of values on a separate number plane, with both axes extending from −4 to 4. Use a ruler to join each set of points, and describe the pattern of points.

C

a

x	0	1	2	3
y	2	2	2	2

b

x	−1	0	1	2
y	−1	−1	−1	−1

c

x	3	−1	1	−3
y	1	1	1	1

d

x	−3	2	0	4
y	−3	−3	−3	−3

Foundation Standard Complex

3 Graph this table of values on a number plane, with both axes extending from –4 to 4, and describe the pattern of points.
C

x	–2	–1	0	1
y	0	0	0	0

4 Graph each table of values on a number plane. Use a ruler to join each set of points and describe their pattern.
C

a

x	1	1	1	1
y	–2	–1	0	1

b

x	–2	–2	–2	–2
y	–2	–1	0	1

c

x	3	3	3	3
y	–2	1	–1	3

d

x	–1	–1	–1	–1
y	–3	–1	0	2

5 Graph each table of values on a number plane, join each set of points and describe their pattern.
C

a

x	–2	–1	1	2
y	–1	0	2	3

b

x	–2	0	1	3
y	–4	–2	–1	1

c

x	–4	–2	2	4
y	–1	0	2	3

d

x	–3	–1	0	1
y	–3	1	3	5

6 For the tables of values in question **5**, what happens to the y-values as the x-values increase?
R
C

7 A line is drawn through the points (0, 4), (2, 0) and (–1, 6). Which of the following points also lies on the line? Select the correct answer **A**, **B**, **C** or **D**.

A (1, –1) **B** (1, 2) **C** (–2, 7) **D** (3, –3)

8 Graph each table of values on a separate number plane and describe the pattern of the points.

a

x	–3	–2	–1	0	1	2	3
y	9	4	1	0	1	4	9

b

x	–2	–1	0	1	2
y	–8	–1	0	1	8

c

x	–2	–1	0	1	2	3	4	5	6
y	0	–2	0	2	0	–2	0	2	0

Foundation Standard Complex

9 R C This table shows the height of a tree as it ages. Plot each point on a number plane, where the x-coordinate is the age of the tree and the y-coordinate is the height of the tree. After plotting each (age, height) point, explain whether there is a pattern between the age and height of the tree.

Age (years)	5	10	15	20	25
Height of tree (metres)	10	12.5	17.5	21	26

Quiz
Mental skills 9

☆ MENTAL SKILLS 9 ANSWERS ON P. 634 Maths without calculators

Simplifying multiplication by factorising

Sometimes, one big multiplication may be split into many simpler little multiplications if we break each term into 2 'easier' factors. We use the property that numbers can be multiplied in any order.

1 Study each example.

a $15 \times 6 = 3 \times 5 \times 3 \times 2$
$= 3 \times 3 \times 5 \times 2$
$= 9 \times 10$
$= 90$

b $30 \times 22 = 3 \times 10 \times 2 \times 11$
$= 3 \times 2 \times 10 \times 11$
$= 6 \times 11 \times 10$
$= 660$

c $24 \times 8 = 6 \times 4 \times 2 \times 4$
$= 4 \times 4 \times 6 \times 2$
$= 16 \times 6 \times 2$
$= 96 \times 2$
$= 192$

d $14 \times 36 = 7 \times 2 \times 6 \times 6$
$= 6 \times 7 \times 6 \times 2$
$= 42 \times 6 \times 2$
$= 252 \times 2$
$= 504$

Look for different pairs of numbers that you can multiply easily and rearrange.

Note: For each question, there are many different possible ways of arriving at the correct result.

2 Now evaluate each product by factorising first.

a 30×24 **b** 18×27 **c** 25×33 **d** 16×8
e 28×20 **f** 12×18 **g** 21×9 **h** 36×12
i 16×35 **j** 22×28 **k** 9×15 **l** 27×25

Foundation Standard Complex

INVESTIGATION

Height vs head circumference

Is your height related to the circumference of your head?

A number plane can be used to graph different types of information.

Shutterstock.com/Dragon Images

1 Draw a table to collect data about the students from your class:

Name	Head circumference (C)	Height (H)

2 For each student in the class, measure (in cm):
- Height (without shoes)
- Circumference of head

3 Draw a number plane using the first quadrant only. Place Head circumference (C) on the horizontal axis and Height (H) on the vertical axis.

4 For each person, plot a point using your data.

5 By considering your graph, do you think there is a relationship between head circumference and height?

Formulas and tables of values 9.06

We learned in Chapter 5, *Algebra and equations*, that a formula is a general rule written as an algebraic equation showing the relationship between variables. For example, the formula for calculating the volume of a rectangular prism is $V = lwh$, where V represents the volume, l is the length of the prism, w is its width and h is its height.

Example 3

The cost of a taxi trip is given by the formula:

$C = 2.2d + 4.3$

where C is the total cost in dollars and d is the distance travelled in kilometres. Haniya travelled 73 km by taxi from her house to the airport. How much did the taxi ride cost her?

SOLUTION

Haniya travelled 73 km, so substitute $d = 73$ into the formula:

$C = 2.2d + 4.3$

$= 2.2 \times 73 + 4.3$

$= 164.9$

The taxi ride cost Haniya \$164.90.

Formulas can also be used to complete a table of values.

Example 4

Complete this table of values using the formula $v = 2u + 1$.

u	−2	−1	0	1	2	6	9
v							

SOLUTION

Substitute each value of u in the table into the formula $v = 2u + 1$ to find the value of v.

- When $u = -2$, $v = 2 \times (-2) + 1 = -3$
- When $u = -1$, $v = 2 \times (-1) + 1 = -1$
- When $u = 0$, $v = 2 \times 0 + 1 = 1$
- When $u = 1$, $v = 2 \times 1 + 1 = 3$
- When $u = 2$, $v = 2 \times 2 + 1 = 5$
- When $u = 6$, $v = 2 \times 6 + 1 = 13$
- When $u = 9$, $v = 2 \times 9 + 1 = 19$

Completing the table, we have:

u	−2	−1	0	1	2	6	9
v	−3	−1	1	3	5	13	19

Example 5

The formula for converting temperatures from degrees Celsius (°C) to degrees Fahrenheit (°F) is:

$$F = 1.8C + 32$$

where F is the temperature in °F and C is the temperature in °C.

a Use this formula to construct a table of values for F using values of C in intervals of 10 from $C = 0$ to $C = 100$.

b Do the values of F increase or decrease as C increases?

c What happens to the values of F as C increases by 10?

SOLUTION

a Draw up a table with values of C = 0, 10, 20, 30, ..., 100.

c	0	10	20	30	40	50	60	70	80	90	100
f											

Substitute the values of C into the formula to find the corresponding F values.

If $C = 0$, $F = 1.8 \times 0 + 32$

$= 32$

If $C = 10$, $F = 1.8 \times 10 + 32$

$= 50$

Complete the table as you conduct each calculation.

c	0	10	20	30	40	50	60	70	80	90	100
f	32	50	68	86	104	122	140	158	176	194	212

b The values of F increase.

c The values of F increase constantly by 18 as C increases by 10.

EXERCISE 9.06 ANSWERS ON P. 634

Formulas and tables of values

U F R C

EXAMPLE 3

1 The cost of hosting a wedding reception is given by the rule:

C

$$C = 120p + 1800$$

where C is the total cost in dollars and p is the number of guests attending the wedding. What is the cost of hosting a wedding with 325 guests?

2 The average speed at which a vehicle travels over the course of a journey is given by the formula:

C

$$S = \frac{D}{T}$$

where S is the average speed in km/h, D is the total distance travelled in km and T is the time taken for the journey in hours. A car travelled 3600 km from Brisbane to Perth in 37 hours. What was the average speed for this journey? Round your answer to the nearest km/h.

3 A company charges for the hire of a boat using the formula:

C

$$C = 55t + 180$$

where C is the cost of hire and t is the time hired in hours. Lachlan hires a boat for $5\frac{1}{2}$ hours. How much was he charged by the company?

4 The circumference, C, of a circle with radius, r, is given by the formula $C = 2\pi r$. Calculate the circumference of a circle that has a radius of 3 m. Round your answer to one decimal place.

Foundation Standard Complex

5 The formula to convert temperatures from degrees Fahrenheit (°F) to degrees Celsius (°C) is $C = \frac{5}{9}(F - 32)$, where C is the temperature in °C and F is the temperature in °F. Use this formula to convert 77°F to °C.

EXAMPLE 4

6 Copy and complete each table of values given the formula.

a $y = x - 1$

x	3	7	−1	−8	4	5
y					3	

b $q = 3p$

P	8	−3	2	5	10	−4
q				15		

c $k = h \div 2$

h	12	8	−2	−6	0	7
k	6					

d $y = x + 7$

x	−2	11	7	0	13	−9
y		18				

e $b = 2a$

a	8	−1	−5	0	4	2
b			−10			

f $y = x - 3$

x	10	−4	7	5	−8	11
y	7					

7 Given the rule $y = -x - 4$, what is the value of y when $x = -3$? Select the correct answer **A**, **B**, **C** or **D**.

A −1 **B** −7 **C** 7 **D** 12

EXAMPLE 5

8 The school canteen purchases cartons of flavoured milk using the formula

R

$$C = 1.85x$$

C where $\$C$ is the total cost of the milk cartons and x is the number of cartons ordered.

a Copy and complete the table showing the costs of purchasing from 50 to 60 cartons of milk.

x	50	51	52	53	54	55	56	57	58	59	60
$\$C$											

b Do the values of C increase or decrease as x increases?

c What happens to the values of C as x increases by 1?

9 A household's water usage charges per quarter (3 months) can be represented using the rule:

$$C = 56.15 + 3.20k$$

where $\$C$ is the total cost of the quarterly bill and k is kilolitres of water used by the household. Use the rule to complete the table showing the cost from 30 to 40 kL of water used.

k (kL)	30	31	32	33	34	35	36	37	38	39	40
$\$C$											

10 The sum of the angles of a polygon can be found using the formula:

R

$$S = 180n - 360$$

C where S is the angle sum in degrees and n is the number of sides in the polygon.

Foundation Standard Complex

a Use the formula to complete the table of values showing the angle sums of polygons with 3 to 12 sides.

n	3	4	5	6	7	8	9	10	11	12
s										

b What happens to the values of S as n increases by 1?

c Predict the angle sum of a polygon with 13 sides.

11 Jonah's weekly wage can be modelled by the formula:

R

$$W = 28.5t + 135$$

C where $\$W$ is the wage and t is the number of hours worked.

a Use the formula to complete the table for Jonah's total weekly wage when he works between 20 and 40 hours.

t	20	22	24	26	28	30	32	34	36	38	40
$\$W$											

b What happens to the values of W as n increases by 2?

c Predict Jonah's weekly wage if he works:

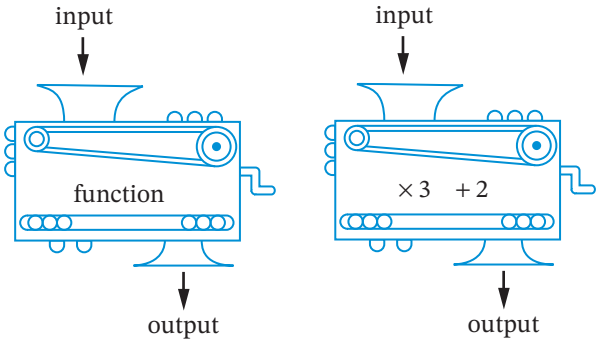
i 18 hours **ii** 41 hours

Graphing linear functions 9.07

A rule that is represented by a formula or table of values is also called a **function.** We can use a 'function machine' to visualise how a function works. With a formula or table, the value that goes into the function machine is called the **input** (usually the x-value), and the value that comes out is called the **output** (the y-value of the function).

The function is the process that operates on the x-value to create the resulting y-value. For example, the function $y = 3x + 2$ takes each x-value inputted, multiplies it by 3, then adds 2 to create the matching y-value.

x	−2	−1	0	1	2
y	−4	−1	2	5	8



Input (x)					
Output (y)					

Input	Opearations	Output
−2	× 3 + 2	−4
−1	× 3 + 2	−1
0	× 3 + 2	2
1	× 3 + 2	5
2	× 3 + 2	8

Worksheets
Number plane grid paper
A page of lines

Puzzles
Matching linear equations
Graphing functions

Interactives
Function builder: Basics
Function builder

Foundation Standard Complex

Graphing functions

- complete a table of values
- graph the table of values on a number plane
- rule a line through the points and label the line with the equation

Example 6

Video Graphing linear equations

Linear functions

Technology Graphing straight lines: Finding intercepts

Spreadsheet Linear function plotter

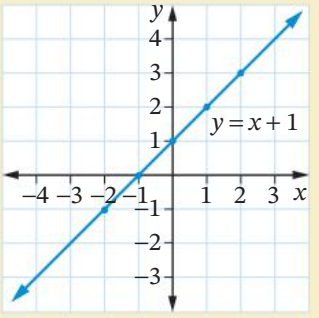
Graph $y = x + 1$ on a number plane.

SOLUTION

Complete a table of values. Choose x-values close to 0 for easy calculation and graphing.

x	−2	−1	0	1	2
y	−1	0	1	2	3

Graph the table of values on a number plane.



Notice that the points lie on a straight line, so we can say that $y = x + 1$ is a **linear equation** or **linear function.** Linear means 'involving a line'.

Rule a straight line through the points, place arrows at each end, and label the line with its equation.

Note:

- Every pair of values that follows the linear function $y = x + 1$ lies on this line, for example, (3, 4), (−4, −3), $(1\frac{1}{2}, 2\frac{1}{2})$, (5, 6), (100, 101), $(-\frac{1}{2}, \frac{1}{2})$.
- Every point on the line follows the linear function $y = x + 1$.
- There are an infinite number of points that satisfy the rule.

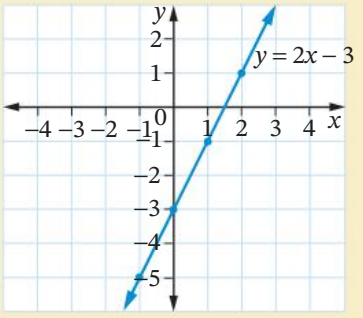
For these reasons, the line is infinite so we must draw arrows at each end.

Example 7

Graph $y = 2x - 3$ on a number plane.

SOLUTION

x	−1	0	1	2
y	−5	−3	−1	1



EXERCISE 9.07 ANSWERS ON P. 634

Graphing linear functions

U F C

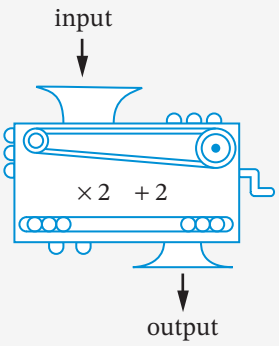
EXAMPLE 6

Worksheets
Number plane grid paper

A page of number planes

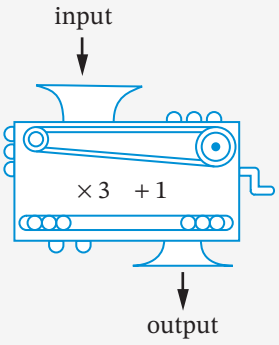
1 Complete each table of values using the operations in the function machine.

a $y = 2x + 2$



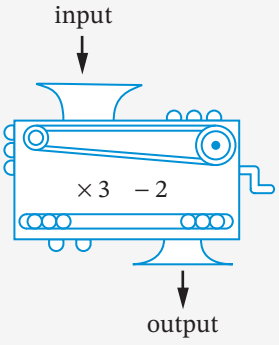
Input (x)	0	1	2	3	4
Output (y)					

b $y = 3x + 1$



Input (x)	−4	−2	0	2	4
Output (y)					

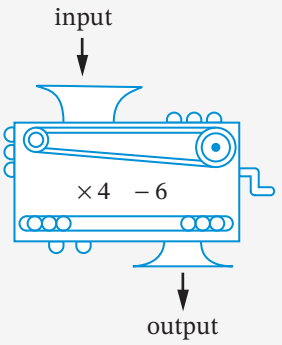
c $y = 3x - 2$



Input (x)	0	3	6	9	12
Output (y)					

Foundation Standard Complex

d $y = 4x - 6$



Input (x)	−10	−5	0	5	10
Output (y)					

EXAMPLE 7

2 For each linear function, copy and complete the table of values and graph the equation on a number plane.

C

a $y = x + 3$

x	−1	0	1	2
y				

b $y = x - 2$

x	0	1	2	3
y				

c $y = 2x$

x	−1	0	1	2
y				

d $y = \frac{x}{2}$

x	−1	0	1	2
y				

e $y = 6 - x$

x	0	1	2	3
y				

f $y = x$

x	−1	0	1	2
y				

EXAMPLE 8

3 Graph each linear function on a number plane.

a $y = 2x + 1$ **b** $y = 2x - 1$ **c** $y = 3x$

d $y = 3x - 2$ **e** $y = 2x + 4$ **f** $y = 3x - 5$

TECHNOLOGY

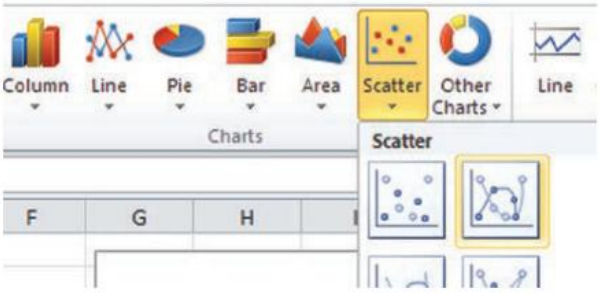
Graphing linear functions

We are going to use a spreadsheet to graph $y = 2x - 3$.

1. Enter these values into a spreadsheet.
2. In cell B2, enter the formula **=2*A2-3** to evaluate $2x - 3$ and use **Fill Down** to copy the formula down to cell B11.
3. Highlight columns A and B, then click Insert and Scatter (with **Smooth Lines and Markers**).

	A	B
1	x	y
2	−4	
3	−3	
4	−2	
5	−1	
6	0	
7	1	
8	2	
9	3	
10	4	
11	5	
12		

Foundation Standard Complex



4 Click the bottom-right corner of the graph border to enlarge the graph.

5 Use your graph to predict the value of y when:

a $x = 5$ **b** $x = -6$ **c** $x = -1.5$

6 Use your graph to predict the value of x when:

a $y = 11$ **b** $y = -9$ **c** $y = 14$

POWER PLUS ANSWERS ON P. 636

1 **a** Draw a number plane extending from –7 to 7 on the x-axis, and from 12 to –8 on the y-axis, or use the worksheet 'Plane grid 2'.

b Start at the first point and join the points in the order given.

- $(3\frac{1}{2}, 0), (4, 2), (4, 4), (3\frac{1}{2}, 7), (4\frac{1}{2}, 8), (4\frac{1}{2}, 9), (5, 10), (4\frac{1}{2}, 11\frac{1}{2}), (3\frac{1}{2}, 10), (2, 10), (\frac{1}{2}, 11\frac{1}{2}), (1, 10), (1, 8), (1\frac{1}{2}, 7), (2, 6\frac{1}{2}), (1\frac{1}{2}, 4\frac{1}{2}), (0, 2\frac{1}{2}), (-2\frac{1}{2}, 1), (-3, 0), (-3\frac{1}{2}, -4), (-4\frac{1}{2}, 6), (-4, -7), (0, -8), (2, -7\frac{1}{2}), (2, -6\frac{1}{2}), (0, -7)$, STOP
- $(-2, -7\frac{1}{2}), (1, -6), (3, -6), (4, -6\frac{1}{2}), (4\frac{1}{2}, -5\frac{1}{2}), (3, -4\frac{1}{2}), (0, -5), (-3, -6\frac{1}{2})$, STOP
- $(\frac{1}{2}, -2), (1, -3), (0, -5)$, STOP
- $(1\frac{1}{2}, 0), (1, -2), (1\frac{1}{2}, -4\frac{1}{2})$, STOP
- $(2, -6), (2\frac{1}{2}, -7\frac{1}{2}), (3\frac{1}{2}, -7\frac{1}{2}), (3\frac{1}{2}, -6\frac{1}{2}), (3, -6)$, STOP
- $(2\frac{1}{2}, -4\frac{1}{2}), (2\frac{1}{2}, -2), (3\frac{1}{2}, 0)$, STOP
- $(1\frac{1}{2}, -3), (1, -5)$, STOP

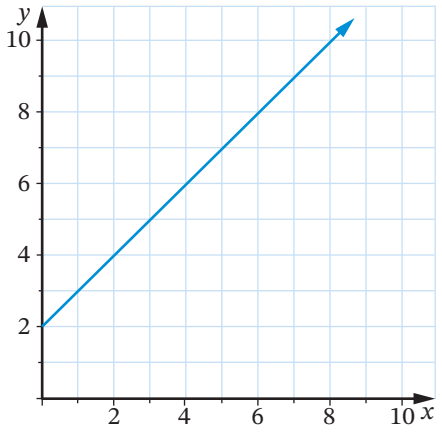
c Complete the picture as you wish.

d What have you drawn?

Worksheet
Plane grid 2

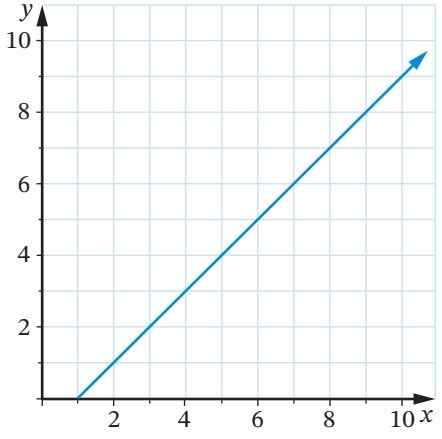
2 Complete each table by reading values from each graph. Find an algebraic formula $y =$ _____ that relates each y-value with the x-value.

a



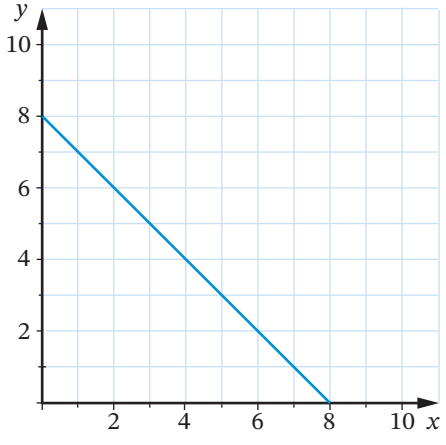
x	y
0	
1	
3	
6	
8	

b



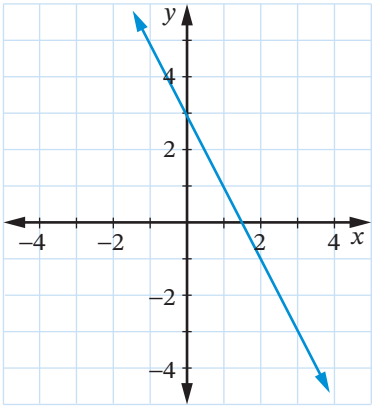
x	y
1	
4	
	6
8	
10	

c



x	y
0	
	6
	4
6	
	0

d



x	y
−1	
0	
	−1
3	
	−5

2 Graph each equation accurately and state whether it is linear or non-linear.

a $y = x^3 - 2$ **b** $x + y = 5$ **c** $y = x^2 - 1$

d $y = 4 - x^2$ **e** $x - y = 4$ **f** $y = \dfrac{4}{x}$

9 CHAPTER REVIEW

Language of maths

Quiz
Language of maths 9

axis/axes	Cartesian	collinear	coordinates
formula	function	number plane	ordered pair
origin	point	quadrant	*x*-axis
y-axis			

1 Look up the word 'coordinate' in the dictionary. Find both its mathematical and non-mathematical meanings.

2 Why do you think 'ordered pair' is another name for coordinates?

3 What is the **origin** and why does it have that name?

4 What word is used to describe a set of points 'lying in a line'?

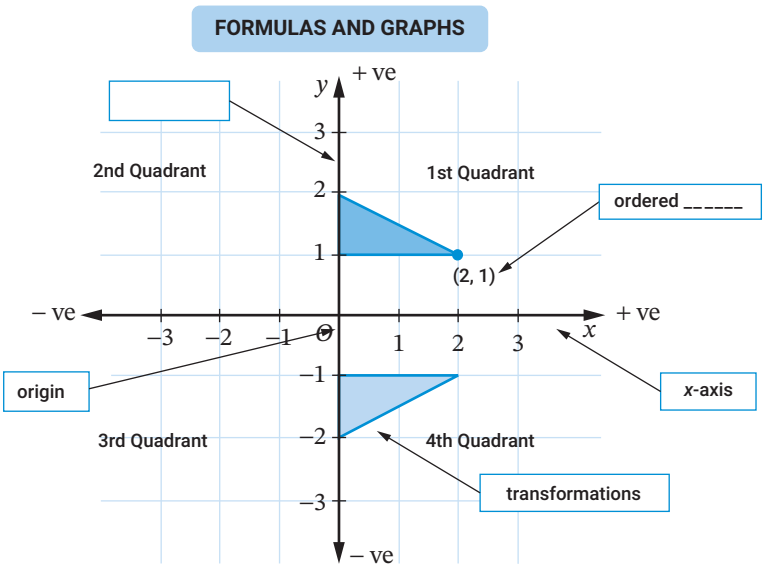
5 What is another name for the number plane?

Topic summary

Worksheet
Mind map: Formulas and graphs

- In your own words, write about how points are plotted on the number plane.
- In your own words, write about how coordinates are used to locate a position on the number plane.
- What parts of this topic did you find difficult? What parts didn't you understand? Discuss them with a friend or your teacher.
- Give some examples of where the number plane is used.

Print (or copy) and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.



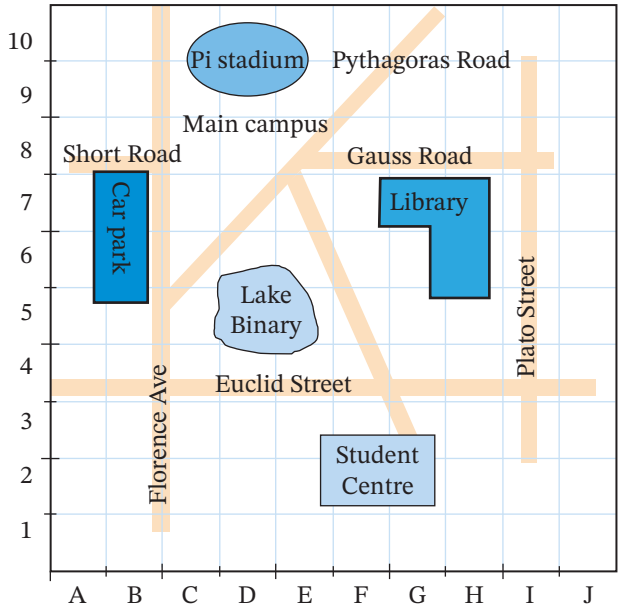
9 TEST YOURSELF

ANSWERS ON P. 637

9.01

Quiz
Test yourself 9

1 This is a map of the Cartesian University campus.



State the feature that appears at:

a F2 **b** D9 **c** A8 **d** H7

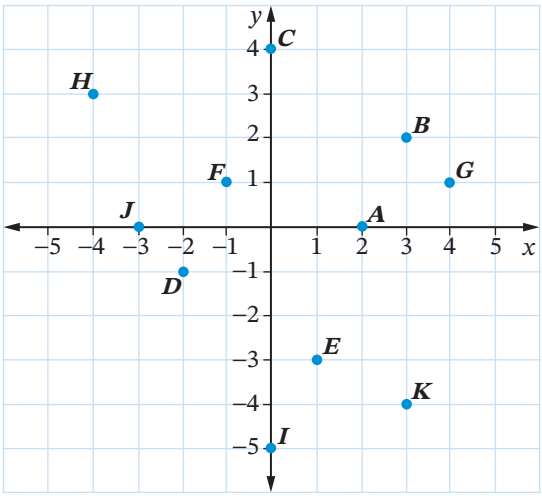
2 Use the map in question **1** to write the location of: 9.01

a Lake Binary **b** the car park **c** Gauss Rd

3 Plot each point on the same number plane. 9.02

$A(2, 3)$, $B(1, 6)$, $C(3, 0)$, $D(0, 4)$, $E(5, 2)$, $F(6, 5)$

4 Write the coordinates of each labelled point on this number plane. 9.03



Foundation Standard Complex

9.03 **5** **a** In which quadrant is the point (–3, –3)?

b Write the coordinates of a point on the *y*-axis.

c In which quadrant is a point with a positive *x*-coordinate and a negative *y*-coordinate?

d Write the coordinates of a point in the 2nd quadrant.

e What are the coordinates of the origin?

9.03 **6** In which quadrant is a point with a negative *x*-coordinate and a positive *y*-coordinate? Select the correct answer **A**, **B**, **C** or **D**.

A 1st **B** 2nd **C** 3rd **D** 4th

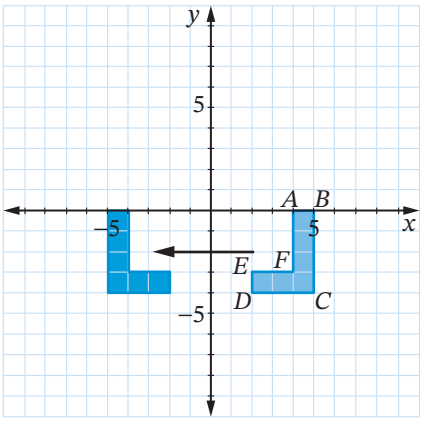
9.03 **7** **a** Plot these points on a number plane, joining them with straight lines.

(1, 3), (2, 3), (2, 2), (3, 1), (2, 0), (2, –1), (1, –1), (0, –2), (–1, –1), (–2, –1), (–2, 0), (–3, 1), (–2, 2), (–2, 3), (–1, 3), (0, 4), (1, 3).

b What shape have you drawn?

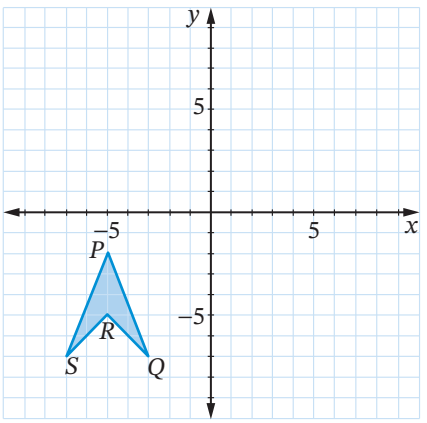
9.04 **8** **a** Accurately describe the transformation that has been performed on the lighter L-shape on the right.

b Write the coordinates of the new position of *F* and compare them to its original coordinates.



9.04 **9** **a** Copy and translate this arrowhead *PQRS* 4 units right and 1 unit up to create the image *P′Q′R′S′*.

b Compare the coordinates of each vertex in *PQRS* to its matching vertex in *P′Q′R′S′*.

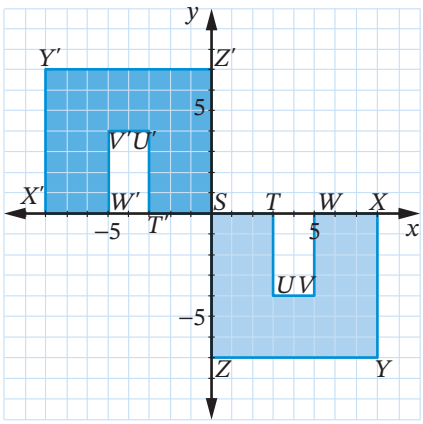


Foundation Standard Complex

9780170465557

10 **a** Accurately describe the transformation that has been performed on the lighter shape. 9.04

b Compare the coordinates of V and V'.



11 Plot the following sets of points on a number plane. Use a ruler to join each set of points. 9.05

a

x	−2	−2	−2	−2
y	−2	−1	0	1

b

x	−2	−1	0	2
y	3	3	3	3

12 Graph each table of values on a separate number plane. Use a ruler to join up the points and describe their pattern. 9.05

a

x	y
0	5
1	4
2	3
5	0

b

x	−1	0	1	2
y	−1	1	3	5

13 The interest earned when money is invested in a term deposit account is given by the formula: 9.06

$$I = Prn$$

where $\$I$ is the total interest earned, $\$P$ is the original amount invested, r is the interest rate per year written as a decimal and n is the number of years the money is invested for.

a Convert the interest rate of 5% to a decimal.

b Calculate the interest earned if \$15 000 is invested at 5% per year for 3 years.

c If the interest rate is doubled, is the interest earned doubled?

d If the original investment is halved, is the interest earned halved?

Foundation Standard Complex

9.06 **14** Copy and complete each table of values.

a $y = x + 3$

x	−1	0	1	2	3
y					

b $q = 2p - 7$

p	−2	−1	0	1	2
q					

c $d = 5 - c$

c	−1	0	1	2	3
d					

9.06 **15** The cost of hiring an electric scooter can be represented by the formula:

$$C = 15.7 + 1.25d$$

where $\$C$ is the cost and d is the number of kilometres travelled on the scooter.

a Copy and complete the table showing the amount of money charged from 10 to 60 kilometres of travel.

d (km)	10	15	20	25	30	35	40	45	50	55	60
$\$C$											

b What is the pattern in the values of C as d increases by 10?

c Predict the hire cost if the scooter travels:

i 5 km **ii** 80 km

d If the distance travelled is doubled, is the hire cost doubled?

9.06 **16** The area of a rectangle has the formula $A = lw$, where l is the length of the rectangle and w is its width.

The area of a rectangle is 108 cm^2 and its length is triple its width.

Find the length and width of the rectangle.

9.07 **17** Graph the linear function $y = 2x - 1$ on a number plane.

Foundation Standard Complex

Practice set 3

ANSWERS ON P. 638

1 Mark and label each point on a number plane. 9.03

A (1, 3) **B** (–2, 2) **C** (3, –4) **D** (–2, –3)
E (0, 7) **F** (–3, 0)

2 Draw a number line labelled from 0 to 3, and mark each decimal on it. 7.01

0.5 2.25 1.3 2.9
0.75 1.6

3 How many metres in 735 centimetres? Select the correct answer **A**, **B**, **C** or **D**. 8.01

A 73 500 **B** 7.35 **C** 73.5 **D** 7350

4 Evaluate each expression. 7.03

a 1.54 + 7.9 **b** 9.2 – 6.1 **c** 22.6 – 13.55
d 6.31 + 0.2 + 15.38 **e** 0.003 + 1.01 + 4.394 **f** 9.803 – 4.52

5 Convert each fraction to a decimal. 7.09

a $\frac{1}{5}$ **b** $\frac{3}{8}$

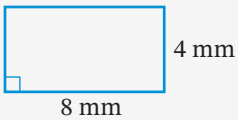
6 Copy and complete each conversion. 8.01

a 2000 mg = _____ g **b** 2 t = _____ kg **c** 56 days = _____ weeks
d 6.5 L = _____ mL **e** 3 days = _____ hours **f** 4 kg = _____ g
g 1500 g = _____ kg **h** 3000 mL = _____ L **i** 480 s = _____ min
j 2500 kg = _____ t **k** 9.5 g = _____ mg **l** 12 000 000 mg = _____ kg

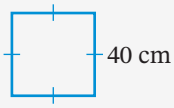
7 A sheet of A4 paper has a width of 21.1 cm and a length of 29.7 cm. Find its perimeter. 8.02

8 Find the perimeter of each shape. 8.02

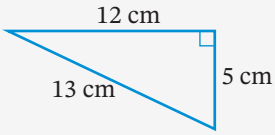
a



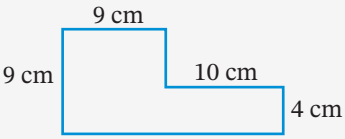
b



c



d



9 Find the area of each shape above. 8.06

Foundation Standard Complex

8.03 **10** Draw each part of the circle.

a chord **b** segment **c** arc **d** quadrant

8.04 **11** Find, correct to 2 decimal places, the circumference of a circle with:

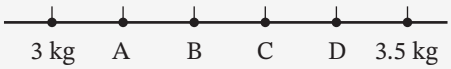
a radius 5 cm **b** diameter 7 cm

7.10 **12** Round:

a 0.753 to one decimal place **b** 12.064 to 2 decimal places

c \$9.8217 to the nearest cent **d** 3.795 to the nearest tenth

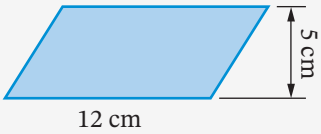
7.01 **13** Which letter represents 3.3 kg on this number line? Select the correct answer **A**, **B**, **C** or **D**.



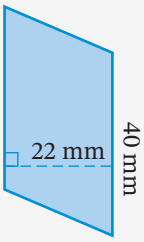
8.01 **14** Convert 7500 L to kL.

8.08 **15** Find the area of each parallelogram.

a



b



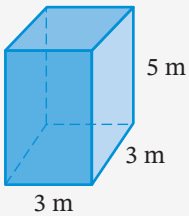
9.05 **16** Graph this table of values on a number plane, with both axes extending from –4 to 4. Use a ruler to join the points.

x	–1	0	2	3
y	–4	–2	2	4

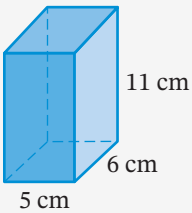
9.04 **17** The point $B(-2, 3)$ on the number plane has been translated 5 units left and 4 units down. Write the coordinates of the image of the point B.

8.11 **18** Find the volume of each rectangular prism.

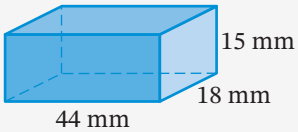
a



b



c



9.07 **19** Graph the linear function $y = 3x - 4$ on a number plane.

Foundation Standard Complex

9780170465557

YEAR 8

20 Convert $\frac{5}{9}$ to a recurring decimal. 7.09

21 **a** If the temperature in Brisbane was 20.6°C at 9 a.m. and was 29.3°C at 3 p.m., by how much did it rise? 7.11

b A piece of cloth 2.5 m long was cut into 10 equal pieces. How long was each piece?

c Libby bought 40 litres of LPG at 79.9c per litre. Linton bought 40 litres at 87.9c per litre. How much more than Libby did Linton pay?

22 Which number is the largest? Select **A**, **B**, **C** or **D**. 7.01

A 0.83 **B** $\frac{4}{5}$ **C** $\frac{81}{100}$ **D** 0.82

23 Evaluate each product. 7.06

a 6.2×0.7 **b** 4.21×0.3 **c** 0.05×1.4

24 Evaluate each quotient. 7.07

a $60.138 \div 6$ **b** $1.672 \div 0.02$ **c** $4.032 \div 0.3$

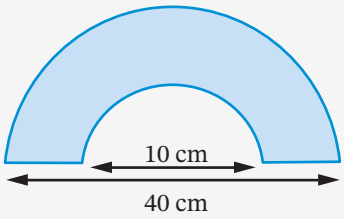
25 The length of the longest side of a right-angled triangle (called the hypotenuse) can be found using the fomula: 9.09

$$c = \sqrt{a^2 + b^2}$$

where c is the length of the hypotenuse and a and b are the lengths of the other 2 sides. Calculate the length of the hypotenuse in a right-angled triangle that has 2 shorter sides of 6 cm and 8 cm.

26 The circumference of a circle is 37.7 m. What is the radius of this circle, correct to the nearest metre? 8.08

27 Find the perimeter of this shape, correct to the nearest cm. Select **A**, **B**, **C** or **D**. 8.06



A 109 cm **B** 77 cm **C** 737 cm **D** 129 cm

28 **a** A rectangle has an area of 20 cm². If its length is 5 cm, how wide is it? 8.06

b A square has an area of 64 cm². Find its perimeter.

29 The surface area of a cube is given by the formula $SA = 6s^2$, where SA is the surface area and s is the side length of the cube. The surface area of a particular cube is 294 cm². What is the side length of this cube? 9.06

Foundation Standard Complex

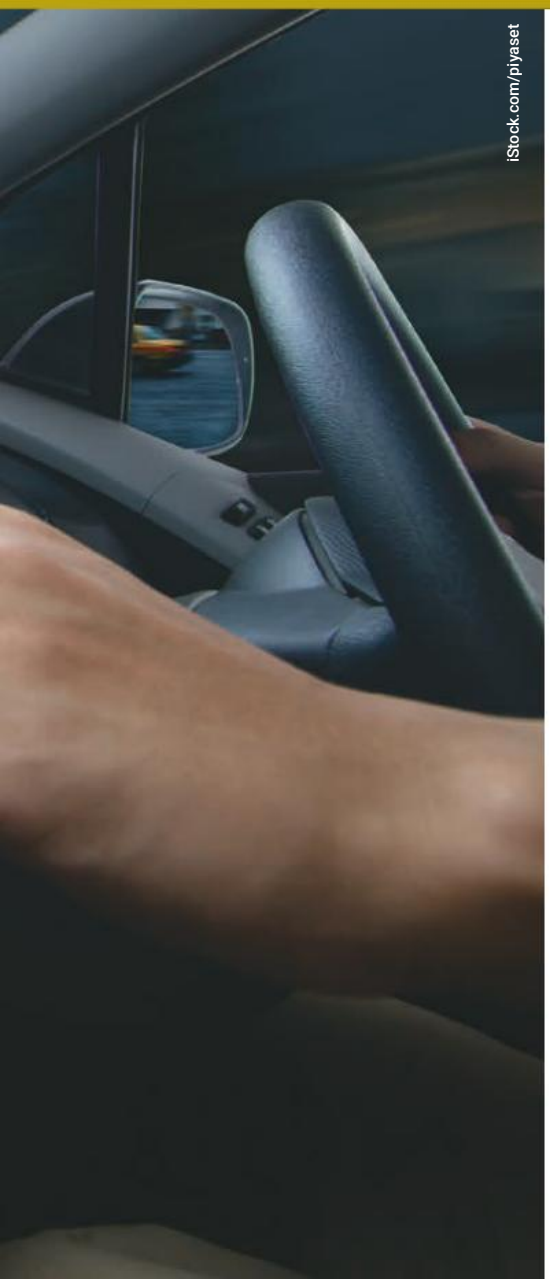
10

STATISTICS

Analysing data

A motor company is releasing a new model of car next year. How do they decide which colours and features to offer? They could conduct a statistical survey to determine which car features are popular.

Statistics is the branch of mathematics concerned with collecting, organising, presenting and analysing data (information). Governments, businesses, research organisations, financial institutions and sporting groups all use statistics in their planning and decision-making.



iStock.com/piyaset

Chapter outline

	Proficiencies				
10.01 Interpreting graphs	U	F	PS	R	C
10.02 Misleading graphs	U	F	PS	R	C
10.03 Dot plots	U	F			C
10.04 Stem-and-leaf plots	U	F			C
10.05 The mean and mode	U	F	PS	R	C
10.06 The median and range	U	F	PS	R	C
10.07 Analysing dot plots and stem-and-leaf plots	U	F	PS	R	C
10.08 Comparing data sets	U	F	PS	R	C
10.09 The shape of a distribution	U	F		R	C
10.10 Statistical investigations	U	F	PS	R	C

U = Understanding
F = Fluency
PS = Problem solving
R = Reasoning
C = Communication

Wordbank

cluster A group of data values that are bunched or close together

data set A collection of data about the same subject

dot plot A special column graph of dots for a small set of numerical data

mean The average value of a data set, found by dividing the sum of values by the number of values

median The middle value of a data set when the values are arranged in order, or the average of the 2 middle values

mode The most common value(s) of a data set

outlier An extreme value that is very different from the other values in a data set

range The difference between the highest value and the lowest value in a set of data

skewness The shape of data when it is not symmetrical but twisted to one side

stem-and-leaf plot A special table where each numerical data value is split into a 'stem' and a 'leaf'

Quiz
Wordbank 10

Videos (8):

10.01 Statistics
10.04 Stem-and-leaf plots 1
10.05 The mean, mode, median and range
10.06 The mean, mode, median and range • The mode, median and mean
10.07 Dot plots • Stem-and-leaf plots 2
10.08 Back-to-back stem-and-leaf plots
10.09 The shape of a frequency distribution

Twig videos (4):

10.01 Nightingale's diagram • Most popular pet
10.02 Distorted graphs: Heat wave
10.06 Average Joe

PhET interactive (1):

Centre and variability

Quizzes (5):

- Wordbank 10
- SkillCheck 10
- Mental skills 10
- Language of Maths 10
- Test Yourself 10

Skillsheets (3):

SkillCheck Reading linear scales
10.01 Displaying data
10.05, 10.06 Statistical measures

Worksheets (9):

10.01 Every picture tells a story • Car survey • Student survey form • Creating a graph
10.02 Misleading graphs
10.04 Stem-and-leaf plots 1
10.06 Students' marks
10.07 What data?
Mind map: Analysing data

Puzzles (5):

10.01 Where all the cars are red
10.06 Mean, median, mode 2 • Data puzzles • Ranges and averages
10.09 Statistical match-up

Presentation (1):

10.04 Stem-and-leaf plot

Technology worksheet (1):

10.08 Excel: Daily rainfall

Nelson MindTap

To access resources above, visit **cengage.com.au/nelsonmindtap**

10 In this chapter you will:

- ✓ interpret a variety of statistical graphs, including divided bar graphs, sector graphs and line graphs
- ✓ identify graphs that are misleading
- ✓ draw stem-and-leaf plots and dot plots
- ✓ calculate the mean, mode, median and range of a set of data, including data presented on graphs and plots, and interpret these statistical measures
- ✓ compare data sets using statistics, including identifying outliers, clusters and skewness
- ✓ describe the shape of a statistical distribution by identifying skewness, symmetry and modality
- ✓ plan and conduct a statistical investigation

Quiz SkillCheck 8

Skillsheet Reading linear scales

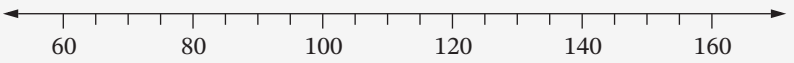
Video Statistics

SkillCheck

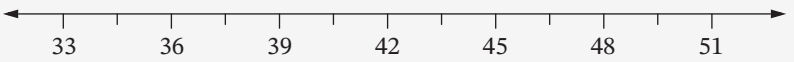
ANSWERS ON P. 639

1 Copy and complete each scale and state the size of one interval (unit).

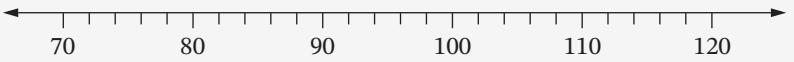
a



b



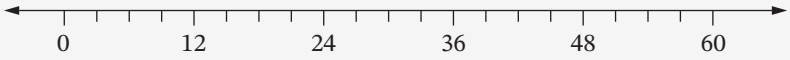
c



9780170465557

d

e



2 Find the average of:

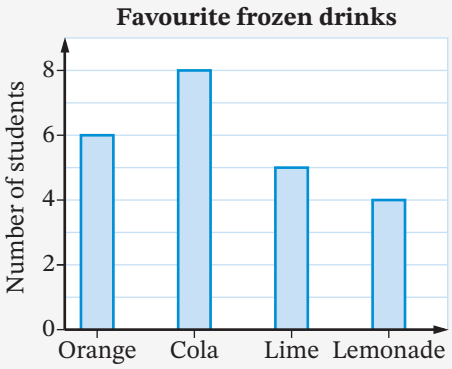
a 42 and 58 **b** 30 and 39

3 Some students were asked to state their favourite frozen drink. The results are displayed in this column graph.

a List the drinks in order of preference, starting with the most popular.

b How many students preferred Lime?

c How many students were in the group?



4 Write each set of numbers in ascending order.

a 31, 28, 22, 11, 24 **b** 21, 18, 15, 17, 13, 16

5 Write each set of numbers in descending order.

a 6, 11, 18, 17, 5 **b** 8, 17, 14, 23, 31, 5, 2

Interpreting graphs 10.01

In statistics, many types of graphs are used to present **data** (information).

- **picture graphs**
- **column graphs**, also called **bar charts**
- **divided bar graphs**
- **sector graphs**, also called **pie charts**
- **line graphs**

Skillsheet Displaying data

Worksheets Every picture tells a story

Car survey

Student survey form

Puzzle Where all the cars are red

EXERCISE 10.01 ANSWERS ON P. 639

Interpreting graphs

U F R C

1 **Picture graphs** are used to show data about things that can be counted.

R C This picture graph shows the number of cars passing a school at different times during a day.

Foundation Standard Complex

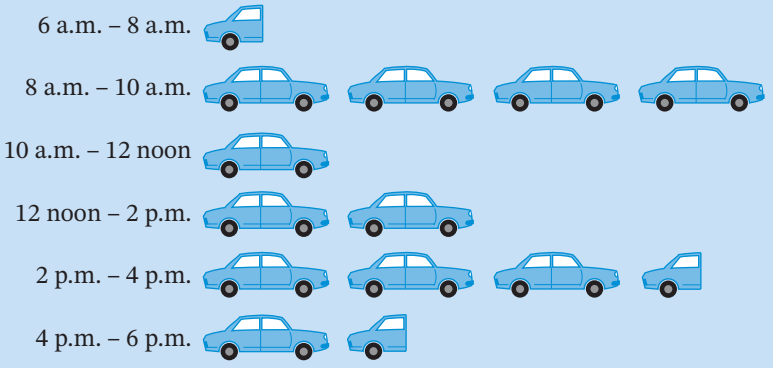
Videos
Statistics

Nightingale's diagram

Most popular pet

Key: Each represents 10 cars.

Number of cars passing the school gate



a How many cars does each represent?

b What does represent? Can you see a disadvantage with this symbol?

c What is the busiest time of day for traffic?

d What is the quietest time?

e List each time period and the number of cars at that time.

f Suggest possible reasons for the flow of traffic at:

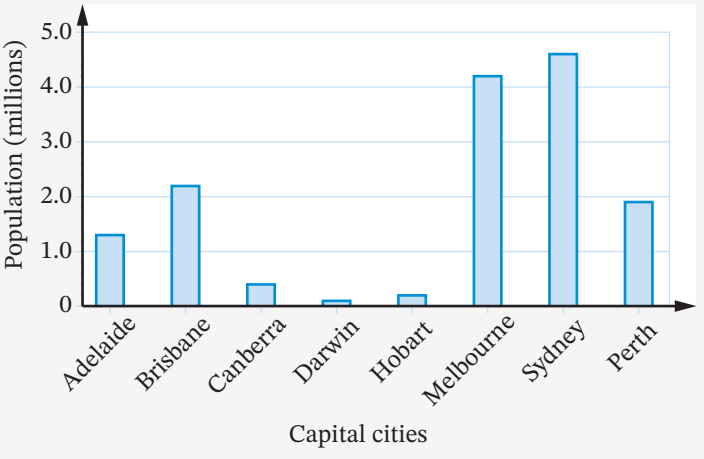
i 8 a.m. – 10 a.m. **ii** 6 a.m. – 8 a.m **iii** 2 p.m. – 4 p.m.

2 **Column graphs** or **bar charts** are mostly used for data that are in categories.

R This column graph shows the populations of the 8 Australian capital cities.

C

Population of Australian cities



a Which city has the biggest population?

b Which city has the smallest population?

Foundation Standard Complex

c What does one interval on the vertical axis (the 'Population' axis) represent? Select the correct answer **A**, **B**, **C** or **D**.

A 0.2 people
B 1 person
C 0.2 million people
D 1 million people

d What is the population of Brisbane?

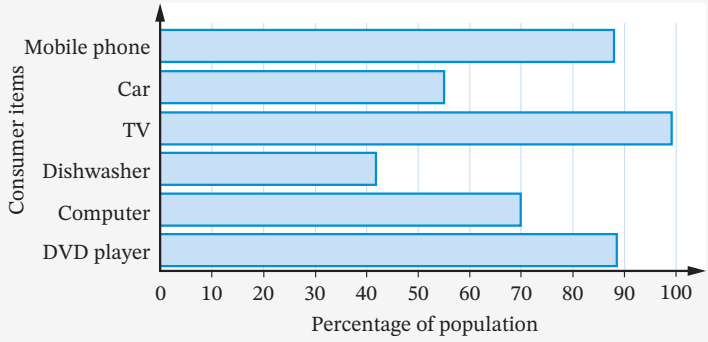
e Which city has a population of 1.3 million?

f How many times Hobart's population is Melbourne's population?

3 Sometimes a column graph is presented sideways. This graph shows the percentage of people that owned various consumer items 10 years ago.

R C

Ownership of consumer goods



a What percentage of the population owned:

i a TV? **ii** a mobile phone? **iii** a DVD player?

b What item was owned by 42% of the population?

c Did more people own mobile phones or computers?

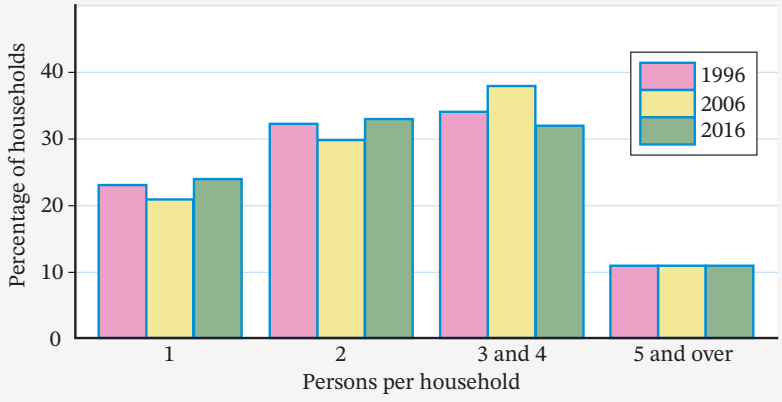
d What was the percentage difference between people owning a car and people owning a computer?

e How might this column graph be different if it described the ownership of consumer items *this* year?

4 This **clustered column graph** compares the number of people living in Australian households in 1996, 2006 and 2016.

R C

Number of people in Australian households



Foundation Standard Complex

a What scale is used on the vertical axis?

b What percentage of households had more than 2 persons in 1996?

c In which year were 30% of households made up of 2 people?

d Generally, which category of persons per household was the:

i most common? **ii** least common?

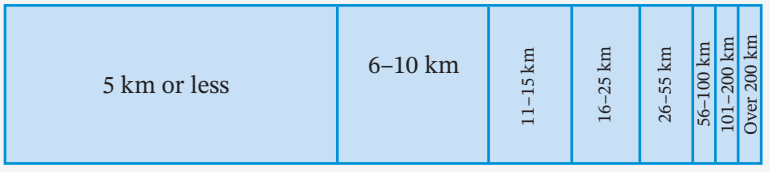
e Which category had the greatest difference between 2006 and 2016?

f True or false? 'There was a higher percentage of one-person households in 2006 than in 1996.'

5 **Divided bar graphs** are rectangular graphs used to compare parts of a whole. This divided bar graph shows the proportions of motor vehicle accidents at various distances from the driver's home.

R C

Proportions of accidents at various distances from home



a What distances from home are accidents most likely to occur?

b What is the total length of this bar graph, in millimetres?

c What fraction of accidents happen within 5 km of home?

d Altogether, what fraction of accidents happen 15 km or less from home?

e What happens as you get farther from home? Suggest a reason why this may be so.

f What fraction of accidents happen more than 100 km from home?

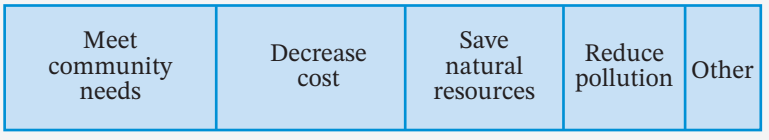
g True or false? 'More than $\frac{3}{4}$ of all accidents occur within 25 km of home.'

h 'Smart Alec' says that to avoid having a car accident he should make all of his trips 100 km or more from home. Why is Alec wrong in saying this?

6 This divided bar graph shows the different reasons that various councils gave for having recycling programs.

R C

Council's reasons for recycling



a What is the total length of this bar graph, in millimetres?

b What is the most common reason given? What fraction of the graph is this?

c What fraction of the graph represents 'Save natural resources'?

d List the reasons in order of popularity.

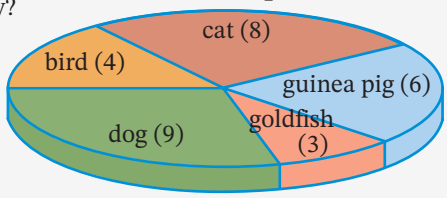
e What are the disadvantages of using a divided bar graph to illustrate this data?

Foundation Standard Complex

7 Like divided bar graphs, **sector graphs** or **pie charts** are also used to compare parts of a whole. This sector graph shows the results of a class survey of 30 students regarding their favourite pet.

R C

Favourite pets



a What is the most popular pet in this survey?

b What is the least popular pet?

c What fraction of students:

i prefer goldfish

ii prefer birds?

d How many degrees are there in a revolution?

e How many degrees are there in the angle of the cat sector?

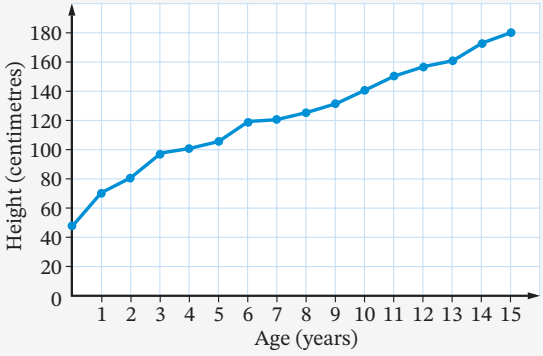
f Do cats and guinea pigs together make up more than half of the preferred pets?

g If the figures from this survey are typical for all students, calculate the number of each type of pet you would expect to have if 90 students were surveyed.

8 **Line graphs** are usually drawn to show data measured over time. This line graph shows Kate's height over her first 15 years. At birth she is 48 cm tall, and at age 10 she is about 140 cm tall.

R C

Kate's height



a What was Kate's height:

i on her first birthday?

ii at age 7?

b At what age did Kate reach:

i 1 metre?

ii 150 cm?

c Between which 2 birthdays did Kate grow the most? Give a possible reason.

d Between which 2 birthdays did Kate grow the least? Give a possible reason.

e How long did it take Kate to double her height from birth?

f How long did it take her to triple her height from birth? Compare this to your answer for the previous question. Why is there a difference?

g What do you think the graph will look like:

i after 15 years?

ii after 20 years?

h How might a graph of a boy's height differ from this one?

Foundation Standard Complex

TECHNOLOGY

Column graphs

Worksheet
Creating a graph

Pet survey

1 Survey the students in your class on their pets. Keep a record of how many of each type of pet students have.

2 Enter your results into a spreadsheet, as shown.

3 Create a column graph. Select **Insert** and **2-D clustered column graph**. Give the graph an appropriate title and label the axes.

	A	B
1	**Pet**	**Number**
2	Dog	
3	Cat	
4	Bird	
5	Fish	
6		

Travel to school

1 Survey the students in your class on how they travel to school. Count the types of transport in your survey.

2 Enter your results into a spreadsheet, as shown.

3 Create a horizontal column graph. Select **Insert** and create a **bar chart**. Give the graph an appropriate title and label the axes.

	A	B
1	**Transport**	**Number**
2	Bus	
3	Train	
4	Walk	
5	Car	
6	Bicycle	
7		

10.02 Misleading graphs

Worksheet
Misleading graphs

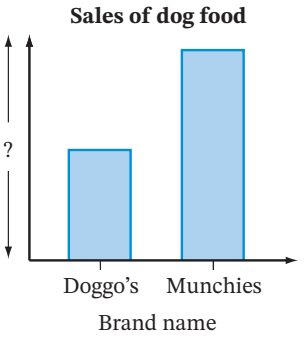
Video
Distorted graphs: Heat wave

When used incorrectly, graphs can give a false or misleading impression.

The following 3 graphs were used by *Munchies* dog food to compare its sales figures to those of *Doggo*'s dog food, but each one is misleading in some way.

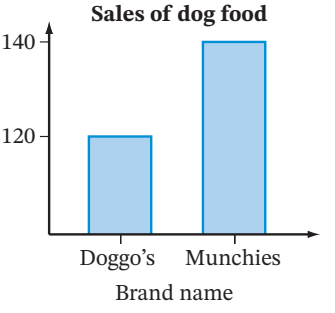
1. No scale

This graph does not have a scale on the vertical axis. You cannot tell how big the difference is between *Munchies*' sales and *Doggo*'s sales.



2. Uneven scale

This graph shows **only part** of a scale, and the scale is not regular and does not start at 0. This makes the difference between *Munchies*' sales and *Doggo*'s sales look much greater.

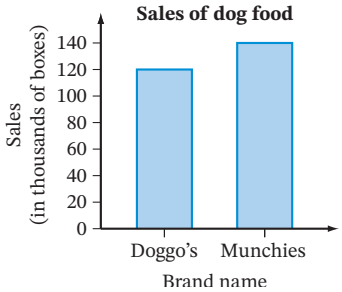


3. Incorrect use of pictures

This graph uses pictures instead of columns. The *Munchies* dog is twice as tall as the *Doggo* dog, but it is also twice as wide, making it seem much bigger (4 times as big in area).

The correct graph

This graph shows the information correctly. The scale is even and begins at 0. Notice that the difference in sales figures is not as large as it seemed in the 3 misleading graphs above.



Misleading graphs

A misleading graph can give a wrong impression by:

- not having a scale
- showing only part of the scale or an irregular scale
- not showing the position of zero on the scale
- using pictures instead of columns to exaggerate the differences

EXERCISE 10.02 ANSWERS ON P. 639

Misleading graphs

U F R C

1 **a** What is the actual difference in sales figures between Munchies and Doggo's in the above example?

R C **b** How does the last graph illustrate this difference correctly?

c Why is it misleading to not show the position of 0 on the vertical axis?

d Why is it misleading to use pictures or diagrams on graphs instead of columns?

2 This line graph shows Lisa's heart rate while she is exercising on a treadmill.

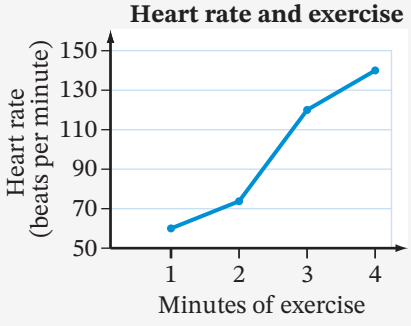
C

a What is Lisa's heart rate after 3 minutes?

b What is the size of one unit on the vertical axis?

c What is misleading about this graph?

d Redraw this graph correctly so that it is not misleading.



Foundation Standard Complex

3 This graph compares the average weekly wages in Malvolia ($700) and Australia ($1400).

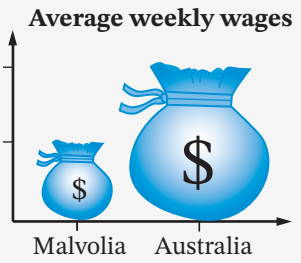
C

a What misleading impression does this graph give?

b Explain 2 things that are wrong about this graph.

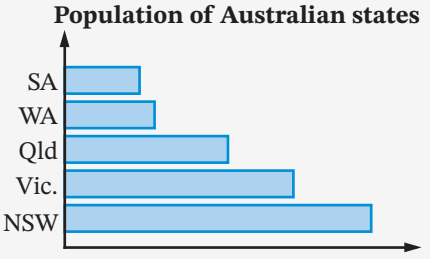
c What should be drawn on the graph instead of pictures?

d Redraw this graph correctly.



4 Why is this graph difficult to interpret?

C



5 **a** Draw a line graph for the profit information in this table. Place 'Year' on the horizontal axis, and use a scale of 1 cm = $2 million on the vertical axis.

C

b Draw another line graph using the same information, this time using a scale of 1 cm = $0.5 million, but show only from $2 million to $4 million on it.

c Which graph looks more impressive? Why?

Year	Profit in $ millions
2016	2.5
2017	2.1
2018	3.2
2019	3.5
2020	3.6
2021	3.9

6 This graph compares the weekly salaries of firefighters in 6 states.

C

Shutterstock.com/PrasongTakham

a Describe 2 ways in which this graph is misleading.

b Redraw this graph correctly.

Foundation Standard Complex

7 Draw 2 graphs using the figures below for school sport, one that is correct and accurate, and one that gives a misleading impression.

C

School sport	Hockey	Netball	Soccer
Number of players	18	20	15

DID YOU KNOW?

Top 10 baby names

This table lists the 10 most popular baby names in New South Wales for boys and girls born in 2010 and 2020:

	Boys				Girls			
	2010		2020		2010		2020	
1	William	726	Oliver	637	Isabella	609	Amelia	514
2	Jack	623	Noah	593	Chloe	604	Olivia	476
3	Oliver	558	William	478	Ruby	600	Charlotte	460
4	Joshua	549	Leo	420	Olivia	586	Mia, Isla	420
5	Thomas	549	Lucas	409	Charlotte	550		
6	Lachlan	546	Liam	383	Mia	523	Ava	391
7	Cooper	536	Jack	376	Lily	485	Chloe	375
8	Noah	536	Henry	374	Emily	480	Grace	323
9	Ethan	535	Elijah	371	Amelia, Sienna	473	Sophia	302
10	Lucas	502	Thomas	351			Zoe	301

Source: State of New South Wales. For current information go to www.nsw.gov.au.

What names were on the list in both 2010 and 2020?

What about in your state (or the whole of Australia) this year?

What is the most common first name for the Year 7 students at your school?

Dot plots 10.03

A **dot plot** is a simple type of column graph that uses dots to display the **frequency** of each data value. It is easy to draw and is useful for small sets of data. A dot plot shows:

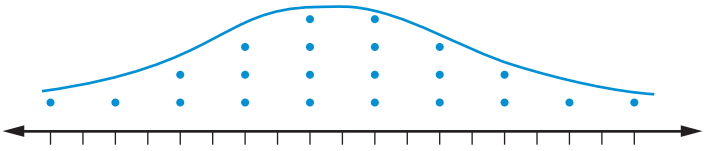
- any gaps in the data
- any **clusters**: where values are grouped or bunched together
- any **outliers**: extremely high or low values that are very different from the other values
- how the values are spread out, the shape of the data.

An **outlier** 'lies outside' the other values in the **data set**. Outlier is pronounced 'out-ly-er'

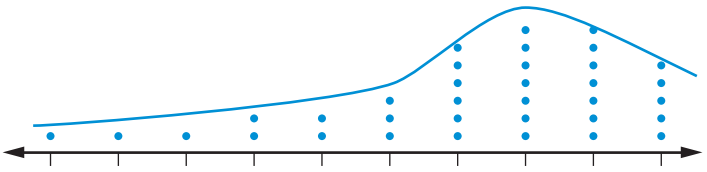
Foundation | Standard | Complex

In a dot plot, it is easy to see the shape of the distribution of the data. Data can be **symmetrical** or **skewed** (twisted to one side). It helps if you draw a curve over the dots to see a pattern.

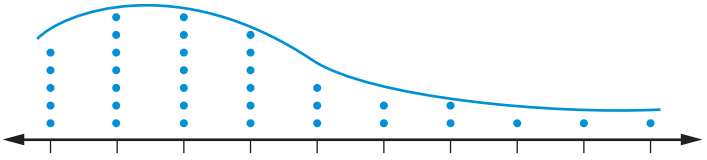
Symmetrical



Negatively skewed: tail points left



Positively skewed: tail points right



Example 1

The data below shows the daily maximum temperatures (in °C) in Tamworth during November.

27	28	26	25	26	27	22	30	28	29
28	26	24	22	28	24	27	29	28	27
19	25	26	29	29	26	28	28	31	25

a Construct a dot plot for the data.

b What was the highest temperature?

c On how many days was the temperature 25°C?

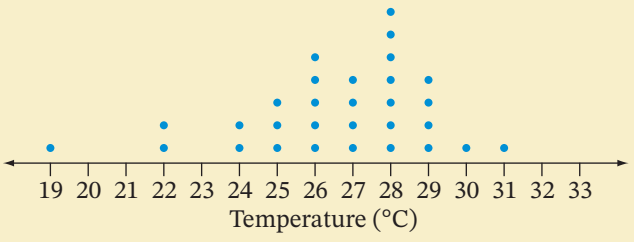
d Where were the temperatures clustered?

e What was the outlier temperature?

f Describe the shape of the distribution.

SOLUTION

a



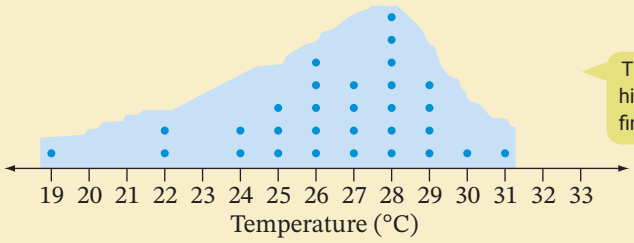
b The highest temperature was 31°C.

c The temperature was 25°C on three days. — 3 dots at 25

d Temperatures were clustered between 26°C and 29°C. — The most dots are bunched here

e The outlier temperature was 19°C. — Extremely low, and different from the rest of the values.

f The distribution is negatively skewed (skewed left).



This means that there are more high temperatures, as you might find in spring or summer.

EXERCISE 10.03 ANSWERS ON P. 640

Dot plots

U F R C

EXAMPLE 1

1 **a** The number of goals that Matias kicked in the soccer games he played last year are listed. Draw a dot plot to represent this data.

4	5	1	2	3	0	3	8	4	6	5
4	1	1	4	4	2	5	3	1	1	0

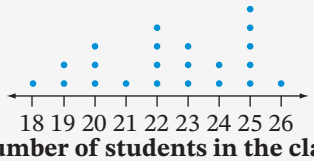
b Are there any outliers in this data? If so, what are they?

c Where is the data clustered?

d How many games did Matias play?

2 This dot plot shows the number of students in each class at Nelson Primary School.

R C



Number of students in the class

a How many students are in the smallest class?

b How many classes have more than 20 students in them?

c Copy and complete: Class sizes range from _____ to _____.

d What is the most common number of students in a class?

e Are there any outliers in this data?

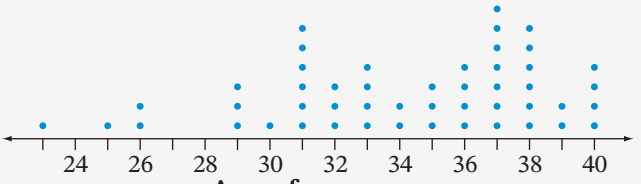
f How many classes are there at Nelson Primary School? Select the correct answer **A**, **B**, **C** or **D**.

A 8 **B** 18 **C** 22 **D** 26

Foundation Standard Complex

3 **a** The ages of the people exercising at a gym one evening are shown in this dot plot. What is the most common age of people at the gym?

R C



Ages of gym users

b Between which ages are the data clustered?

c Are the ages positively or negatively skewed?

d Identify any outliers.

e Copy and complete: Ages range from _____ to _____.

f How many people were at the gym that evening?

4 The number of motor accidents that occurred on the motorway each day was recorded:

C

1	0	0	2	0	3	5
1	0	1	0	2	3	0

a Draw a dot plot for this data.

b Describe the shape of the data.

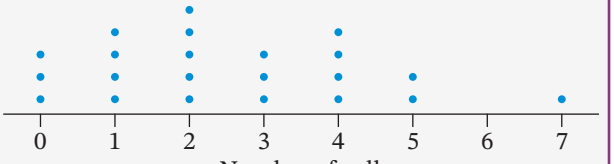
c What is the most common number of daily accidents?

d Calculate, correct to one decimal place, the average number of accidents per day.

e What is the outlier? Why?

5 This dot plot shows the number of phone calls made by a group of students last night.

How many students made phone calls? Select **A**, **B**, **C** or **D**.



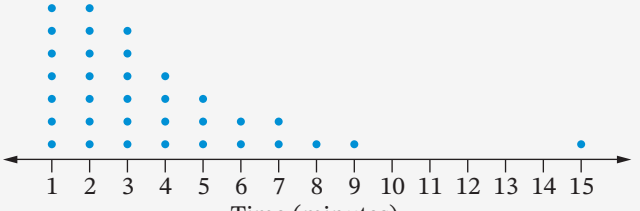
Number of calls

A 7 **B** 19 **C** 22 **D** 8

6 This dot plot shows the time taken for students to walk to school in the morning.

R C

Time taken for students to walk to school



Time (minutes)

a What is the outlier?

b Describe the shape of the data.

c Give one reason why the data might have this distribution.

Foundation Standard Complex

TECHNOLOGY

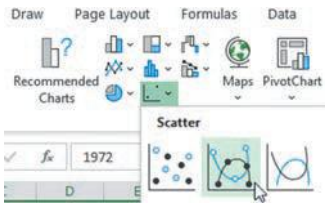
Olympic winning times

This table shows the gold medal winning times of the women's 400 m track event for the Olympic Games from 1972 to 2016.

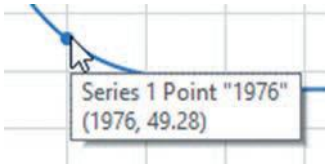
Year	1972	1976	1980	1984	1988	1992	1996	2000	2004	2008	2012	2016
Time (s)	51.08	49.28	48.88	48.83	48.65	48.83	48.25	49.11	49.41	49.62	49.56	49.44

1 Enter the above data into a spreadsheet as one long table (rows 1 and 2, columns A to M).

2 To create a scatter graph choose **Insert** and **Scatter (with smooth lines and markers)**.



3 Give the graph an appropriate title and axes labels. Save the file.



4 On the graph, position the mouse over a data point. (Do not click on it.) You can view the specific details of the Olympic year and winning time. For example (see right), the winning time was 49.28 seconds in 1976.

5 Use your spreadsheet to answer these questions.

a In what year was the fastest gold medal winning time run? In cell A5, enter **=min(B2:M2)**. In cell B5, enter the year that corresponds to this time.

b In cell A6, type the label 'Average'. In cell B6, use the formula **=average(B2:M2)** to calculate the average winning time for this event, from 1972 to 2016.

c In cell A7, enter **=max(B2:M2)** to find the slowest winning time in this event. In cell B7, enter the year that corresponds to this time.

d In cell A8, enter a formula to find the difference between the fastest and slowest winning times.

e Predict the gold medal time at the 2021 Olympic Games for this event.

Justify your answer, and then research the actual time.

f In cell A9, enter a formula to calculate the speed, in metres per second, of the fastest women's 400 m runner, from 1972 to 2016.

g Starting in cell A10, write a paragraph describing the changes in winning times for this event between 1972 and 2016.

h In cell A15, suggest reasons why the pattern of gold medal times has changed between 1972 and 2016.

10.04 Stem-and-leaf plots

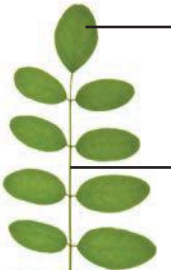
Worksheet
Stem-and-leaf plots 1

A **stem-and-leaf plot** is like a sideways column graph but one that lists the actual data values on the horizontal columns. An example is shown, with the values listed in the leaf column.

Stem	Leaf
5	3 4 7 7 8 9
6	1 1 1 4 8
7	4
8	0 1 3 4
9	2 5 7 8

This leaf means 83.

The name 'stem-and-leaf' comes from the structure of a plant, where the stem is a branch or a vine on which the leaves hang.



iStock.com/fotogal

An **ordered** stem-and-leaf plot shows:

- all the values, listed from smallest to largest
- the most common values
- any clusters or outliers
- how the data are spread out
- the shape of the data

Video
Stem-and-leaf plots 1

Presentation
Stem-and-leaf plot

Example 2

Use a stem-and-leaf plot to show the following masses (in grams) of 30 avocados and describe the shape of the distribution.

85	130	150	137	95	85	142	113	98	103
128	128	105	118	174	113	132	150	137	123
113	137	98	140	115	125	130	162	123	140

SOLUTION

The masses range from 85 to 174.

We write 8, 9, 10 up to 17 down the first column to make the stem.

Stem	Leaf
8	
9	
10	
11	
12	
13	
14	
15	
16	
17	

The leaves are the single digits written next to the stem, for example:

- 85 is shown by writing a 5 next to the 8 stem.
- 130 is shown by writing a 0 next to the 13 stem.
- 150 is shown by writing a 0 next to the 15 stem.
- 137 is shown by writing a 7 next to the 13 stem.

Stem	Leaf
8	5
9	
10	
11	
12	
13	0 7
14	
15	0
16	
17	

Continue in the same manner until all the data has been entered.

Stem	Leaf
8	5 5
9	5 8 8
10	3 5
11	3 8 3 3 5
12	8 8 3 5 3
13	0 7 2 7 7 0
14	2 0 0
15	0 0
16	2
17	4

It is more useful to rearrange the values in ascending order.

This results in an **ordered** stem-and-leaf plot.

When looking at the shape of data in a stem-and-leaf plot, turn the plot sideways so that the stem runs across the bottom.

The shape of this distribution is symmetrical.

Stem	Leaf
8	5 5
9	5 8 8
10	3 5
11	3 3 3 5 8
12	3 3 5 8 8
13	0 2 7 7 7
14	0 0 0
15	0 0
16	2
17	4

Note: The data in a stem-and-leaf plot is negatively skewed if its 'tail' points towards the lower values ('points up'). The data is positively skewed if its 'tail' points towards the higher values ('points down'), as shown in the diagram.

Stem	Leaf
1	8
2	3 5 8 8
3	2 4 6 7 8 8 9
4	0 4 5 5 7 7
5	3 6 7 8
6	0 4 4
7	2 5
8	1 2
9	2

positively skewed

EXERCISE 10.04 ANSWERS ON P. 640

Stem-and-leaf plots

U F R C

EXAMPLE 2

1 This stem-and-leaf plot shows the heights, in centimetres, of 22 women at a gym.

Stem	Leaf
14	7 8 8
15	0 1 2 3 6 6 8
16	1 5 8 9 9 9
17	2 3 3 5
18	1 1

What is the most common height? Select the correct answer **A**, **B**, **C** or **D**.

A 15 cm **B** 156 cm

C 169 cm **D** 181 cm

2 C The daily number of students served at the school canteen over a 3-week period was:

105 76 97 88 114 86 124 101

112 98 95 105 117 81 112

a Show this information in an ordered stem-and-leaf plot.

b What were the lowest and highest numbers of students served over the period?

c On how many days were over 100 students served?

d Find the middle value. Select **A**, **B**, **C** or **D**.

A 95 **B** 101 **C** 105 **D** 112

3 R C The point scores of all AFL teams in their first 4 rounds of the season are shown in this stem-and-leaf plot.

Stem	Leaf
6	4 4 6 6 6 8
7	0 0 2 2 3 3 7 8 8 9
8	0 2 3 4 4 6 9 9
9	3 4 4 4 5 7 7 8 8 9 9 9 9
10	0 3 4 8 9
11	0 1 1 2 3 4 8 9
12	0 7
13	4 4 5 5 7
14	2 3
15	0 0 2 3 4

a How many scores are there altogether?

b How many scores are below 100 points?

c What was the most frequent score?

d Copy and complete: The team scores range from _____ to _____.

e Which stem had the most scores?

f Where are the scores clustered?

g Describe the shape of the distribution.

Foundation Standard Complex

4 The heights (in cm) of the students in a PE class are:

155	153	157	166	163	162	154	175
159	157	137	162	171	140	145	168
158	141	170	166	143	175	157	177

a Make a stem-and-leaf plot of these heights.

b Describe the shape of the data.

c What are the shortest and tallest heights?

d How many students were under 150 cm tall?

e What was the most common height?

f What were the 2 middle heights?

g What fraction of the class had heights in the 160s?

h What percentage of the class had heights in the 170s? Answer correct to one decimal place.

☆ MENTAL SKILLS 10 ANSWERS ON P. 641 Maths without calculators

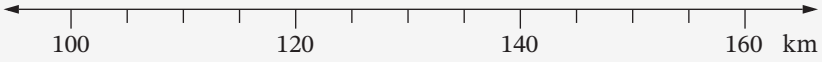
Reading linear scales

Quiz Mental skills 10

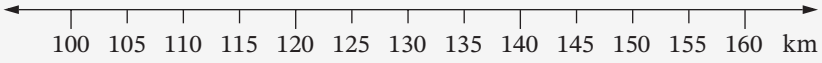
Understanding and reading the scale on a measuring instrument, on a number line or on the axis of a graph is an important mathematical skill.

1 Study each example.

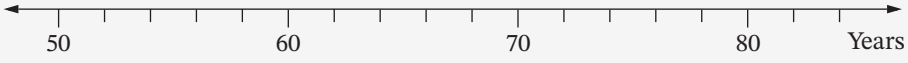
a Complete the missing values on this scale.



- First, choose 2 values on the scale, say 100 and 120.
- Count the number of intervals ('spaces') between the 2 values. There are 4 intervals between 100 and 120.
- To find the size of each interval, divide the difference between the 2 values by the number of intervals:
- Difference = 120 – 100 = 20 km
- Number of intervals = 4
- Size of an interval = 20 ÷ 4 = 5 km
- Use the calculated size of an interval to complete the missing values.

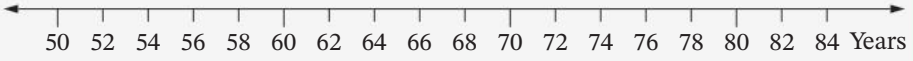


b Complete the values on this scale.



Foundation Standard Complex

- Choose 50 and 60 on the scale.
- Number of intervals (between 50 and 60) = 5
- Difference (between 50 and 60) = 60 – 50 = 10 years
- Size of an interval = 10 ÷ 5 = 2 years.



2 Now copy and complete the following scales.

a

b

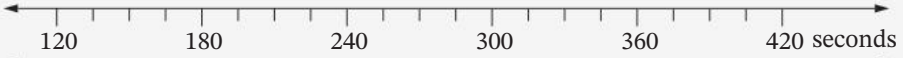
c

d

e

f

g



h

10.05 The mean and mode

Skillsheet Statistical measures

Puzzle Looking for gold

How do you use the word **average**?

- 'That was an average film'
- 'The average person in the street thinks ...'
- 'My average score this year is ...'

In statistics, we use the word 'average' to mean a **typical or central value** of a set of data.

The best-known average is the **mean**. The mean is found by adding all the values and dividing by the number of values.

The mean

The symbol for the mean is $\bar{x}$.

$$\bar{x} = \frac{\text{sum of values}}{\text{number of values}}$$

The **mode** is the most common or frequent value (or values). A set of data may have more than one mode, or no mode at all.

The mode

The **mode** is the value (or values) that occurs the most often; the value with the highest frequency. *Think:* mode = '**m**ost **o**ften'

Example 3

The residents in a street were surveyed about the number of children living in each household. The results were:

2	2	1	2	0	3
2	1	1	4	1	0

a Find the mean, correct to one decimal place.

b Find the mode.

SOLUTION

a Sum of values $= 2 + 2 + 1 + 2 + 0 + 3 + 2 + 1 + 1 + 4 + 1 + 0$

$= 19$

Number of values $= 12$

Mean: $\bar{x} = \dfrac{\text{sum of values}}{\text{number of values}}$

$= \dfrac{19}{12}$

$= 1.58333\ldots$

≈ 1.6 (rounding to one decimal place)

Note that the value of the mean is at the centre of the values.

The mean is about 1.6.

b The modes are 1 and 2. Both occur the most often (with a frequency of 4 each).

Video The mean, mode, median and range

Example 4

Alex's scores in 8 games of ten-pin bowling were:

88 149 153 147 156 168 135 122

a Find the mean.

b Find the mode.

SOLUTION

a Sum of scores $= 88 + 149 + 153 + 147 + 156 + 168 + 135 + 122$

$= 1118$

$\bar{x} = \dfrac{\text{sum of values}}{\text{number of values}}$

$= \dfrac{1118}{8}$

$= 139.75$

The mean is 139.75.

b There is no mode, because every score occurs the same number of times (once).

EXERCISE 10.05 ANSWERS ON P. 641

The mean and mode

U F PS R C

EXAMPLE 3

1 A group of 8 children were surveyed about the amount of pocket money (in dollars) that they received each week. The results were:

20 32 32 40 18 32 18 50

a Find the mean of this set of data.

b Find the mode.

2 For each set of data, find:

i the mean (rounded to 2 decimal places, if needed)

ii the mode(s).

a 1 2 3 3 5 3 2 3 1

b 6 9 2 1 2 9 2

c 67 43 89 65 54 86 45 76 53

d 45.1 45.0 45.4 45.1 45.8 44.6

e 3 3 4 5 5 6 7 9 10

3 **a** Find the mode of this set of data:

C blue, green, yellow, green, blue, red, green, yellow, red, green, red, blue

b Why is it not possible to find the *mean* of this set of data?

EXAMPLE 4

4 In a gymnastics competition, the judges awarded the following scores out of 10:

C 7.0 6.1 8.2 8.8 6.1 9.7 6.1 8.8

a Calculate the mean of these scores.

b Find the mode.

c Which measure (mean or mode) describes this set of scores better? Give a reason for your answer.

5 The ages of the members of the Phuong family are:

R C 19 31 21 3 6 14 19 24 11

The ages of the members of the Arteri family are:

19 31 21 3 6 14 19 24 91

a What is the only difference between these 2 sets of data?

b Which family should have a higher mean age?

c Find the mean age (to one decimal place) for each family.

d What effect does the difference identified in part **a** have on the means?

Foundation Standard Complex

6 R A group of 6 students was surveyed on the number of phones owned in their households. The results were:

3 2 4 3 2 □

where □ represents a missing value.

What is the value of □ if the mean of the results is 2.5?

Select the correct answer **A**, **B**, **C** or **D**.

A 1 **B** 2 **C** 2.5 **D** 3

7 R Tanika scored 68%, 73%, 80% and 75% in 4 maths tests.

a Calculate her mean maths mark.

b Find how much Tanika needs to score in her next maths test to increase her mean to 75%.

8 R **a** Find 5 data values that have a mean of 7 and a mode of 4.

b Find 8 data values that have a mean of 10 and a mode of 12.

9 PS R Jamie, Sam, Karly and Tess all work at the shopping centre on Saturdays. Jamie earns $48 while Tess earns $90. If the mean of the 4 wages is $75, find possible values for Sam's and Karly's wages.

10 R The mean point score of a basketball team for the 30 games they played during the season is 85. What is the total number of points the team scored for the season?

INVESTIGATION

Finding the middle data values

1 **a** Arrange these values in ascending order:

5 4 3 8 7 1 7

b Cross out the first and last values from your sorted list.

c Cross out the second and second-last values.

d Keep crossing out pairs of values from both ends until you find the middle value.

2 **a** Arrange these values in ascending order:

10 3 6 2 6 8 2 10 9 8

b Keep crossing out pairs of values at both ends of your sorted list until you find the 2 middle values.

3 **a** Arrange these values in ascending order:

9 5 5 10 4 6 6 3

b Are there one or 2 middle values? How can you tell?

c Find the middle value(s).

4 **a** Does this set of data have one or 2 middle values? How can you tell?

8 11 15 18 20 24 27 39 44

b Find the middle value(s).

Foundation Standard Complex

5 If a set of data has an odd number of values, how many middle values does it have?

6 a How many values are there in this sorted data set?

2 5 9 12 17 18 27 35 39 41 45

b Is your answer to part **a** an odd number or an even number?

c What number is half of the number of values?

d Is the middle value the 5th, 6th or 7th value?

10.06 The median and range

Like the mean and the mode, the **median** is a measure of the centre of a set of data. It is the middle data value or the average of the 2 middle values.

Skillsheet Statistical measures

Puzzles Mean, median, mode 2

Data puzzles

Ranges and averages

Videos Average Joe

The mean, mode, median and range

The mode, median and mean

Interactive Centre and variability

The median

When data values are ordered, the **median** is:

- the middle value if there is an odd number of values
- the average of the 2 middle values if there is an even number of values.

Think: 'Median' sounds like 'medium', which is halfway between small and large.

The range

The **range** is a measure of the spread of a set of data, and is simply the difference between the highest and lowest data values.

The range

Range = highest value – lowest value

Example 5

Tahir scored the following number of runs in a series of cricket matches:

35 98 17 54 2 22 51 45 86

Find:

a the median **b** the range.

SOLUTION

a First, rewrite the values in order.

2 17 22 35 45 51 54 86 98

4 values 4 values

Median

The median of the data is 45.

Note that the value of the median is at the centre of the data.

b Range = highest value – lowest value

= 98 – 2

= 96

Example 6

The lap times (in seconds) of 6 cyclists were:

13.5 23.1 10.2 18.4 11.9 9.3

Find:

a the median **b** the range

SOLUTION

a Rewrite the times in order.

9.3 10.2 11.9 13.5 18.4 23.1

3 values 3 values

2 middle values

12.7 is halfway between 11.9 and 13.5

$$\text{Median} = \frac{11.9 + 13.5}{2}$$
$$= 12.7$$

b Range = highest value – lowest value

$= 23.1 - 9.3$

$= 13.8$

EXERCISE 10.06 ANSWERS ON P. 641

The median and range

U F R C

EXAMPLE 5

1 For each set of data, find:

i the median **ii** the range.

a	23	20	25	22	20	21	22		
b	5.5	4.5	3.4	5.3	4.9				
c	7	8	3	6	5	3	5	5	4

2 The ages of the members of the Carrozza family are:

R C

7 10 12 42 47

The ages of the members of the Binns family are:

7 10 12 38 47

a What is the only difference between these 2 sets of data?

b Find the median age for each family.

c Find the range of the ages for each family.

d What effect does the difference identified in part **a** have on the medians and ranges?

Foundation Standard Complex

3 The favourite party food for a group of 3-year-old children was recorded:

R

popcorn	fruit	chocolate	fruit	popcorn
chocolate	chocolate	fruit	fruit	fruit

What is the only statistical measure that can be found for this data?
Select the correct answer **A**, **B**, **C** or **D**.

A mean **B** median **C** mode **D** range

4 Which of the following is the median of the scores below? Select **A**, **B**, **C** or **D**.

9 2 4 9 5 3 10

A 9 **B** 6 **C** 5 **D** 4

EXAMPLE 6

5 For each set of data, find:

i the median **ii** the range

a 10 8 6 4

b 36 40 38 37 40 30

c 12 13 11 14 10 15 11 12

6 11 houses were sold in Keswick Street. The selling prices are listed below:

R C

$620 000	$625 000	$700 500	$738 000
$625 000	$1 800 000	$598 000	$612 000
$696 500	$720 000	$705 000	

a Find the median price.

b Find the range.

c Calculate the mean price, correct to the nearest dollar.

d Find the mode.

e Which measure (mean, mode or median) best describes this set of house prices? Give a reason for your answer.

7 Alf's golf scores were (in order):

R C

75 75 75 75 76 76 76 77 77 77

Mike's golf scores were (in order):

73 73 74 75 75 76 76 77 79 79

a Calculate the mean score for each golfer.

b By just looking at the scores, which golfer has the higher range of scores?

c Calculate the range for each golfer.

d Who is the more talented golfer? Explain your answer.

e Who is the more consistent golfer? Explain your answer.

f Find the median score for each golfer.

Foundation Standard Complex

8 8 friends counted the number of letters in their surnames. The results were:

R 4 6 7 5 4 6 9 □

where □ represents a missing value.

Find a possible value of □ if the median is 6 and the range is 5.

9 **a** Find 5 data values that have a median of 7 and a range of 16.

R **b** Find 8 data values that have a median of 12 and a range of 9.

INVESTIGATION

Michael's family

Michael's family decided to have a family photo taken and to record the ages and heights of everyone at that time.

	Age	Height
Father	35	177 cm
Mother	33	170 cm
Big brother	11	150 cm
Big sister	10	145 cm
Michael	6	131 cm
Little brother	4	118 cm

1 Find the mean age of Michael's whole family.

2 Find the mean age of the children in Michael's family.

3 What is the median age of Michael's whole family? Who is closest to this age?

4 What is the median age of the children in Michael's family? Who is closest to this age?

5 Is the mean or the median affected more when the parents' ages are not counted? What is the difference in each case?

6 Predict what would happen to the mean age if Grandpa (aged 75) came to live with the family. Test your prediction.

7 What was the family's mean age 2 years ago? How does this compare to the family's mean age now?

8 Compare the mean height with the median height of the whole family. Are there any outliers?

9 Compare the mean height and the median height of the children in the family. Are there any outliers?

10 Why is the mode not useful in this case?

11 Cousin Lee has come to stay with the family, and the mean height is now 148 cm. What is Lee's height?

Foundation Standard Complex

Worksheet
Students' marks

TECHNOLOGY

Mean, median, mode and range

A spreadsheet can be used to calculate the mean, median and mode of a set of data.

1 Enter into a spreadsheet the following data about the daily maximum temperatures in Alice Springs in one week.

	A	B	C	D	E
1	**Day**	**Temperature (°C)**			
2	Sunday	29		Mean	
3	Monday	31		Mode	
4	Tuesday	30		Median	
5	Wednesday	33			
6	Thursday	29		Maximum	
7	Friday	28		Minimum	
8	Saturday	35		Range	
9					

2 Copy each formula into the given cells.

- Cell E2: =average(B2:B8)
- Cell E3: =mode(B2:B8)
- Cell E4: =median(B2:B8)
- Cell E6: =max(B2:B8)
- Cell E7: =min(B2:B8)
- Cell E8: =E6–E7

Sometimes if you type the first couple of letters of the formula, the spreadsheet will suggest the correct word.

3 Find the same data for the place where you live and enter them into your spreadsheet. Go to the Bureau of Meteorology website www.bom.gov.au to find the data.

4 Survey the students in your class and collect data such as:

- height
- number of hours slept last night
- number of children in family
- number of letters in surname

Use the spreadsheet to calculate the mean, median, mode and range for each set of data.

5 Analyse the data set from your survey.

a Are there any outliers?

b Is the data clustered around specific values?

c What other conclusions can you make for each set of data?

Analysing dot plots and stem-and-leaf plots

10.07

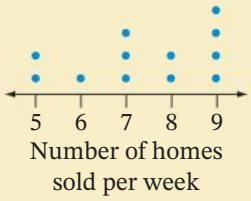
The mean, mode, median and range can be found from data displayed on dot plots and stem-and-leaf plots.

Video
Dot plots

Worksheet
What data?

Example 7

This dot plot shows the number of homes sold per week by a real estate agency over 12 weeks.



Find:

a the range

b the mode

c the median

d the mean, correct to one decimal place.

SOLUTION

a Range = highest value − lowest value

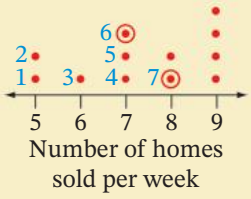
$= 9 - 5$

$= 4$

b The mode is the data value with the most dots.

Mode = 9

c There are 12 data values (12 dots). This is an even number, so there are 2 middle values (the 6th and 7th values). By counting the dots, or by crossing out pairs of dots at each end, we can see that the 6th and 7th values (circled on the right) are 7 and 8 respectively.



$$\text{Median} = \frac{7+8}{2}$$

$$= 7.5$$

d Mean: $\bar{x} = \dfrac{\text{sum of values}}{\text{number of values}}$

$$= \frac{2\times5+6+3\times7+2\times8+4\times9}{12}$$

$$= \frac{89}{12}$$

$$= 7.41666\ldots$$

$$\approx 7.4$$

Note that the mean and the median are close to each other and at the centre of the data.

Example 8

Video
Stem-and-leaf plots 2

This stem-and-leaf plot shows the number of people joining the InstaTik website each day over 20 days.

Stem	Leaf
5	3 4 7 7 8 9
6	1 1 1 4 8
7	4
8	0 1 3 4
9	

Find:

a the range **b** the mode **c** the median **d** the mean

SOLUTION

a Range = highest – lowest

$= 98 - 53$

$= 45$

b Mode = 61 The most frequent value

c There are 20 values, so the median is between the 10th and 11th values (64 and 68).

$\text{Median} = \frac{64+68}{2}$

$= 66$

Stem	Leaf
5	3 4 7 7 8 9
6	1 1 1 4 8
7	4
8	0 1 3 4
9	2 5 7 8

d Mean: $\bar{x} = \frac{\text{sum of values}}{\text{number of values}}$

$= \frac{53+54+57+57+\cdots+95+97+98}{20}$

$= \frac{1437}{20}$

$= 71.85$

Note that the mean and the median are close to each other and at the centre of the data.

EXERCISE 10.07 ANSWERS ON P. 641

Analysing dot plots and stem-and-leaf plots

U F C

In this exercise, round mean values to one decimal place where necessary.

EXAMPLE 7

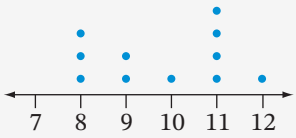
1 For each dot plot, find:

i the range **ii** the mode

iii the median **iv** the mean

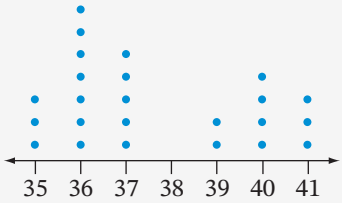
Foundation Standard Complex

a

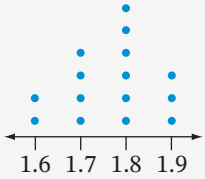


b

c



d



EXAMPLE 8

2 For each stem-and-leaf plot below, find:

i the range **ii** the mode

iii the median **iv** the mean

a

Stem	Leaf
1	0 2 3
2	1 4 4 5 6
3	3 3 3 7
4	1 2 3 5 9

b

Stem	Leaf
7	3 4 5 7 7
8	1 2 2 3 8 9
9	0 4 4 4 4 6 7 9

c

Stem	Leaf
10	0 1 1 2
11	7 8 8 8 9
12	3 6 6 7
13	1 1 1 1 1 8
14	0

d

Stem	Leaf
0	5 5 6
1	4 4 7 7 7 7 7
2	0 3 8 8
3	
4	9 9 9

3 The maximum daily temperatures (in °C) in Cape Arid over a fortnight were:

10	12	10	15	14	15	11
10	19	14	11	10	11	15

Illustrate this data on a dot plot and use it to find:

a the median **b** the mode

c the mean **d** the range

4 The quiz marks out of 10 for 2 Year 8 classes are shown below.

C

8 Huxley: 3 2 0 1 5 8 6 7 6 3
5 4 5 6 7 9 2 5 7

8 Crancher: 7 6 3 7 8 1 9 4 6 7 2
7 2 8 10 9 9 5 7 8 9 10

a Draw a dot plot for the data of each class.

b What is the mode of the marks for 8 Huxley?

Foundation Standard Complex

c What is the median of the marks for 8 Crancher?

d What is the range of the marks for 8 Huxley?

e Calculate the mean for 8 Crancher.

f Which do you think is the 'better' class? Give a reason for your answer.

5 This stem-and-leaf plot shows the number of students buying from the school canteen each day over a 3-week period.

Stem	Leaf
7	6
8	1 6 8
9	5 7 8
10	1 5 5
11	2 2 4 7
12	4

a What is the range for this data?

b Find the median.

c What are the modes?

d Calculate the mean.

6 This stem-and-leaf plot shows the heart rates (in beats per minute) of people riding on a rollercoaster at a theme park.

Stem	Leaf
4	0
5	2 3 5 8 9
6	0 1 1 2 4 4 6 7 7 7 8 8 9
7	0 1 3 3 7 9
8	2 5 6 7
9	1

Find:

a the mode

b the range

c the median

d the mean.

10.08 Comparing data sets

Technology
Excel: Daily rainfall

When we analyse data, we try to describe or summarise the information. This allows us to notice patterns and trends and to draw some conclusions from them. We can use the mean, mode, median and the range to make comparisons between sets of data.

Statistical measure	Features	When is it appropriate to use?
Mean $\bar{x} = \frac{\text{sum of values}}{\text{number of values}}$.	• Depends on all the values in the data set. • Affected by outliers.	When the data set does not have many extreme values (outliers).
Mode Most common value(s)	• There may be more than one mode, or no mode at all. • Not affected by outliers.	When the most common value or category is needed.

Foundation Standard Complex

Statistical measure	Features	When is it appropriate to use?
Median Middle value, or average of the 2 middle values, when values are arranged in order	• Can be one of the values. • Not affected by outliers.	When the data set has extreme values (outliers).
Range Highest value – lowest value	• Depends on the highest and lowest values only.	When a measure of spread is needed.

The mean, mode and median are called **measures of central tendency** while the range is a **measure of spread**.

Example 9

The results of a survey investigating the number of boys and girls who visited a shopping centre each day for 12 days are as shown.

Boys: 105, 76, 97, 88, 114, 86, 124, 102, 111, 97, 96, 81

Girls: 78, 102, 99, 89, 113, 116, 99, 108, 98, 116, 114, 97

a Show this information on a back-to-back stem-and-leaf plot.

b Calculate the range for each set of data.

c Find the median for each set of data.

d Comment on the differences between the data for boys and girls.

Video Back-to-back stem-and-leaf plots

SOLUTION

a A back-to-back stem-and-leaf plot combines 2 stem-and-leaf plots, sharing the same stem.

Boys		Girls
6	7	8
8 6 1	8	9
7 7 6	9	7 8 9 9
5 2	10	2 8
4 1	11	3 4 6 6
4	12	

There are 12 values, so the median is the average of the 6th and 7th values.

b Boys: Range $= 124 - 76$
$= 48$

Girls: Range $= 116 - 78$
$= 38$

c Boys: Median $= \frac{97+97}{2}$
$= 97$

Girls: Median $= \frac{99+102}{2}$
$= 100.5$

d The boys' data is more spread out but the median for the girls is higher.

EXERCISE 10.08 ANSWERS ON P. 642

Comparing data sets

U F R C

EXAMPLE 9

1 Two brands of batteries were tested in the same toy to determine which lasted longer. The back-to-back stem-and-leaf plot shows the 2 sets of data, recorded to the nearest hour.

R C

Dynamo		Energy Plus
4 1	1	3 3 7 7 8 8 9
8 8 6 3	2	5 6 9 9 9 9 9
8 7 7 6 6 5 4 4	3	2 4 6 7 7 8 8 9 9
6 5 5 4 3 2 2	4	0 1
3 2 0 0	5	

a How many batteries of each brand were tested?

b Which brand's data is more symmetrical?

c Find the mean, median and range for each set of data.

d Comment on the differences between Dynamo batteries and Energy Plus batteries.

e Which brand do you think is better? Explain your answer.

2 The 3D-TV MegaStore recorded the weekly number of sales at 2 stores over a 20-week period.

R C

Cannington: 34, 43, 45, 55, 66, 71, 78, 35, 83, 86, 94, 81, 75, 68, 66, 96, 34, 66, 71, 83

Morley: 96, 36, 86, 81, 35, 46, 38, 33, 56, 66, 66, 48, 54, 71, 81, 37, 48, 56, 55, 40

a Display this information on a back-to-back ordered stem-and-leaf plot.

b Calculate the mean, mode, median and range for each store.

c Which store performed better? Explain your answer.

d Which sales figure from the Morley store is an outlier? Which of the measures calculated in part **b** are most affected by this outlier?

3 The marks out of 50 for the same English test, scored by 2 different classes of Year 7 students are listed below.

R C

7 Murray marks: 35, 44, 40, 48, 47, 42, 47, 45, 38, 38, 31, 32, 38, 50, 43, 31, 49, 31, 47, 37, 48, 46, 29

7 Winton marks: 40, 41, 46, 47, 47, 36, 33, 32, 26, 39, 48, 44, 31, 35, 31, 29, 45, 48, 45, 29

Foundation Standard Complex

a Draw a back-to-back stem-and-leaf plot for these 2 data sets.

b What is the highest mark overall?

c What is the lowest mark overall?

d In class 7 Murray where are the marks clustered?

e Copy and complete:

i In 7 Murray, the marks range from _____ to _____.

ii In 7 Winton, the marks range from _____ to _____.

f Find the mean (to one decimal place) and the median for each class.

g Which do you think is the 'better' class? Give a reason why you think this.

h Identify any outliers.

4 2 groups of students achieved these marks out of 10 for their PE project:

R C

Group A: 5 5 5 5 6 6 6 7 7 7 8

Group B: 3 3 4 5 5 6 6 7 9 9 10

a Calculate the mean (to one decimal place) and the median of each group's marks.

b Draw a dot plot for each group and describe the differences between the way each group has its marks spread out.

c Group C achieved these marks, but one student was away.

Group C:

4 5 5 6 6 7 7 7 10

What mark would the absent student need to achieve to give this group the same mean as the other 2 groups? (*Hint*: What total does each group need?)

5 This back-to-back stem-and-leaf plot shows the number of goals scored in each match by 2 basketball teams during last season.

R C

Cobar Cougars		Tilba Tigers
6 6 5 4 3	4	4 9
8 8 3 0	5	2 3 3 6 8
8 8 6 6 3 1 1	6	5 6 8 9
7 4 3 0	7	0 0 1 3 6
6 6 5	8	2 5 7 7 9 9
2 2	9	0 3 4

a How many games did these teams play in one season?

b Find the mean, median, mode and range for each team.

c Comment on the differences between the Cobar Cougars and the Tilba Tigers.

d Which team do you think is better? Explain your answer.

Foundation Standard Complex

6 A basketball coach kept a record of the number of points scored by Maria and Stacey in each match over 12 weeks.

R C

Maria:	41	38	25	19	53	35	30	32	39	45	46	37
Stacey:	35	30	30	24	37	25	37	44	20	29	29	35

a Display this data on a back-to-back ordered stem-and-leaf plot.

b Describe the shape of Maria's scores.

c Calculate the mean (to one decimal place), mode, median and range for each player.

d Who do you think is the more consistent player? Explain.

e Who is the higher scorer?

f Who do you think is the better player? Why?

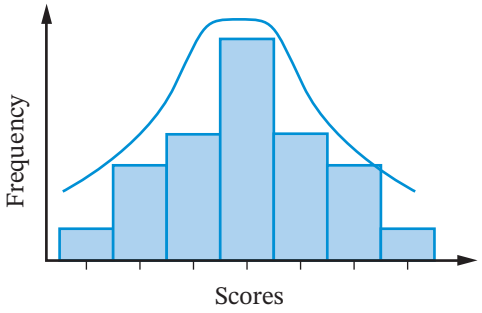
10.09 The shape of a distribution

Puzzle Statistical match-up

A **statistical distribution** is the way the values of a data set are arranged, especially when graphed. When looking at histograms, dot plots and stem-and-leaf plots, an overall pattern can be seen from the **shape of a distribution**.

The shape of a statistical distribution shows how the data is spread and can be seen by drawing a curve around the graph or display.

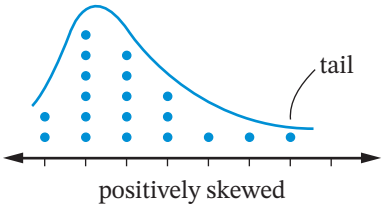
A distribution is **symmetrical** if the data is evenly spread or balanced about the centre. For example, the frequency histogram and stem-and-leaf plot shown below are both symmetrical in shape.



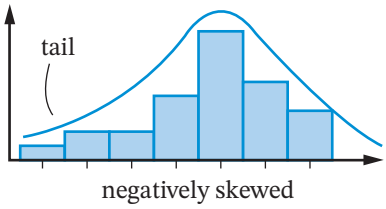
Stem	Leaf
2	6
3	3 7 9
4	5 8
5	0 2 3 3 7 8
6	0 4
7	6 7 8
8	9

A distribution is **skewed** if most of the data is bunched or clustered at one 'end' of the distribution and the other end has a 'tail', as shown on this dot plot and frequency histogram.

Foundation Standard Complex



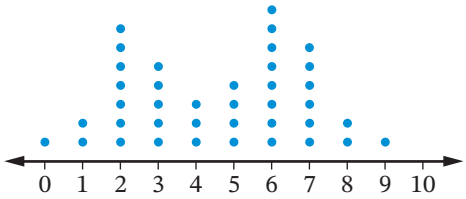
A distribution is **positively skewed** if its tail points to the right.



A distribution is **negatively skewed** if its tail points to the left.

A distribution is **bimodal** if it has two peaks. The higher peak is the mode, while the other peak indicates another data value that has a high frequency.

For example, this dot plot has peaks at 2 and 6 so it is bimodal. The mode, however, is 6.

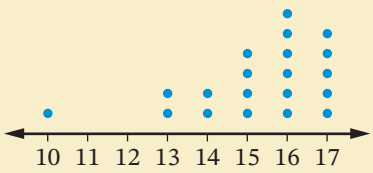


Example 10

For each statistical distribution:

i describe the shape **ii** identify any outliers and clusters.

a



b

Stem	Leaf
3	0 1 2
4	1 3 4 4 5 6
5	0 4 5 7 8
6	3 7 8
7	0 1
8	4
9	8

SOLUTION

a **i** The shape is negatively skewed (tail points towards the lower values).

ii 10 is an outlier and clustering occurs at 15, 16 and 17.

b **i** The shape is positively skewed (tail points towards the higher values).

ii 84 and 98 are outliers and clustering occurs in the 40s and 50s.

Video
The shape of a frequency distribution

EXERCISE 10.09 ANSWERS ON P. 642

The shape of a distribution

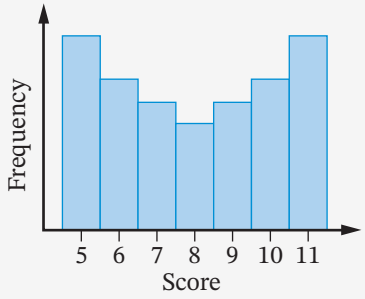
U F R C

EXAMPLE 10

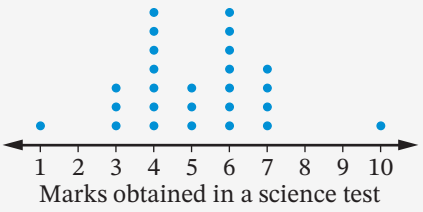
1 For each statistical distribution:

i describe the shape **ii** identify any outliers and clusters.

a



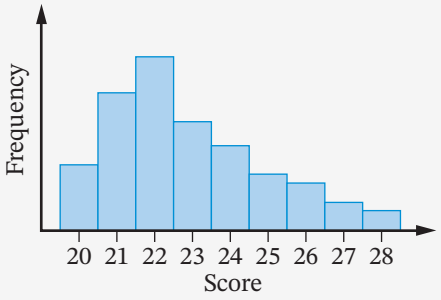
b



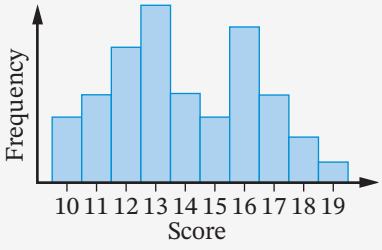
c

Stem	Leaf
2	0 2
3	1 4
4	2 5 6
5	2 3 6 7 8
6	0 4 4 9 9 9
7	1 2 4 5 5 6 7 8 8
8	2 4 5 6 7 7 8
9	0 2 2

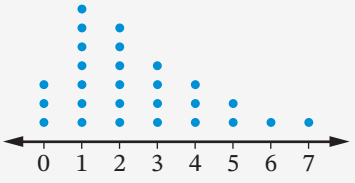
d



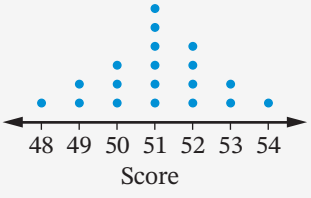
e



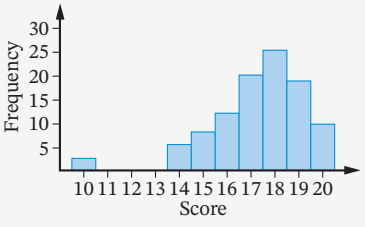
f



g



h



Foundation Standard Complex

2 The results of a Mathematics test for a Year 9 class are shown in the stem-and-leaf plot.

R C

a How many students are in the class?

b Where does the clustering occur?

c Describe the shape of the data.

d Give a possible reason for the shape of the distribution.

e Find the mean, median and mode.

Stem	Leaf
1	8
2	3 5 8 8
3	2 4 6 7 8 8 9
4	0 4 5 5 7 7
5	3 6 7 8
6	0 4 4
7	2 5
8	1 2
9	2

3 The number of children in the families of students at Eastleigh High School are shown below.

4	4	7	3	3	6	8	6	3	3	4	2	9	5	1	2	3	3	4
7	2	7	2	2	3	5	3	1	4	1	4	1	2	5	6	3	5	2

a Arrange the data into a frequency table and construct a frequency histogram.

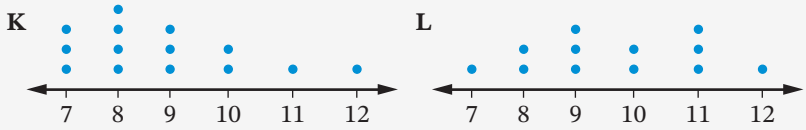
b Are there any outliers?

c Describe the shape of the distribution.

d Where does clustering occur?

e Find the mode, the mean (to one decimal place) and the median and show their position on the histogram.

4 Which statement is true about the 2 sets of data? Select the correct answer **A**, **B**, **C** or **D**.



A K is positively skewed and L is symmetrical.

B K is negatively skewed and L is bimodal.

C L has 2 peaks and K is positively skewed.

D K is positively skewed and the median of L is 9.

5 The results of an entrance exam to select apprentices for an energy supplier are shown in the stem-and-leaf plot.

R C

a Describe the shape of the distribution, excluding the outlier.

b What is the mode?

c Find the median.

d Find the mean, correct to one decimal place.

e Find the range.

f Is the range a good indicator of the spread of scores? Give reasons.

Stem	Leaf
9	2 3 5 6 7 8
8	3 3 6 7
7	2 4 8
6	0 3 4 4
5	3 5 6 6 8 9
4	
3	
2	6

Foundation Standard Complex

6 The average monthly maximum temperatures for Sydney and Melbourne are shown.

R C

Month	J	F	M	A	M	J	J	A	S	O	N	D
Sydney	26	26	25	22	19	17	16	18	20	22	24	26
Melbourne	26	26	24	20	17	14	13	15	17	20	22	24

a **i** Display the data for Sydney and Melbourne on dot plots.

ii For each city, describe the shape of the dot plot.

b Find the mean (to two decimal places), median, mode and range for each city.

c Which city is generally warmer? Give reasons for your answer.

10.10 Statistical investigations

To plan a statistical investigation that will produce a data set you need to consider what data you want to gather and why.

For the students in your class, find out how long it takes them to get to school from home and use this information to decide if students go to your school because they live close to the school.

In order to investigate this, you should apply some of the skills that you have been using in this chapter. You should calculate the mean, median, mode and range for the data you collect and consider if there are any outliers. You should graph the data and examine the distribution (shape) of the data.

EXERCISE 10.10

Statistical investigations

U F PS R C

1 **a** Write down 1 or 2 questions that you will ask the people in your class.

b Write down 1 or 2 assumptions that you have made.

2 Write down the list of steps you will need to do to complete this task and write down what mathematics you will need to use to answer the question and analyse the data.

3 Go through the steps outlined in question **2**.

4 Write a solution to the problem and consider how reasonable the answer is.

5 Write down whether you have made any errors and whether the data you have collected has helped you answer the question asked.

Foundation Standard Complex

6 Write down any strengths or weaknesses that you have noticed in trying to answer the question asked.

7 Write down anything you would do differently if you did try to answer the question again and consider any further questions you could ask to answer a related question.

8 Write a clear answer to the questions asked, using your calculations to help justify your answer.

POWER PLUS ANSWERS ON P. 643

1 The mean of 5 numbers is 24. If a 6th number is added, the mean of the 6 numbers is 21. Find the 6th number.

2 A small class obtained these results in an exam:

66 68 74 76 82 79

a Find the mean of these marks, correct to 2 decimal places.

b The teacher realised there was an error in the marking and added 3 to each mark. Find the mean of the new marks.

c What effect did the extra 3 marks have on the mean?

d What effect does adding or subtracting the same number for all of the data have on the mean?

3 Melissa sat 5 exams. Her average mark was 74%. What mark should Melissa obtain in the 6th exam if she wishes her average mark for the 6 exams to be 77%?

4 5 data values were collected, but the figures were lost. The mean of the data was 8 and the median was 9.

a What was the total of the data?

b What could the data have been?

c If you are now told that the range is 7, what could the set of data have been?

5 4 sisters work at the same bank. One earns $500 per week and another earns $800 per week. The mean weekly wage of the 4 sisters is $2000. Is this possible? How?

Foundation Standard Complex

10 CHAPTER REVIEW

Language of maths

Quiz
Language of maths 10

average	cluster	central	data
dot plot	mean	median	middle
mode	ordered	outlier	range
skewed	spread	stem-and-leaf plot	symmetrical

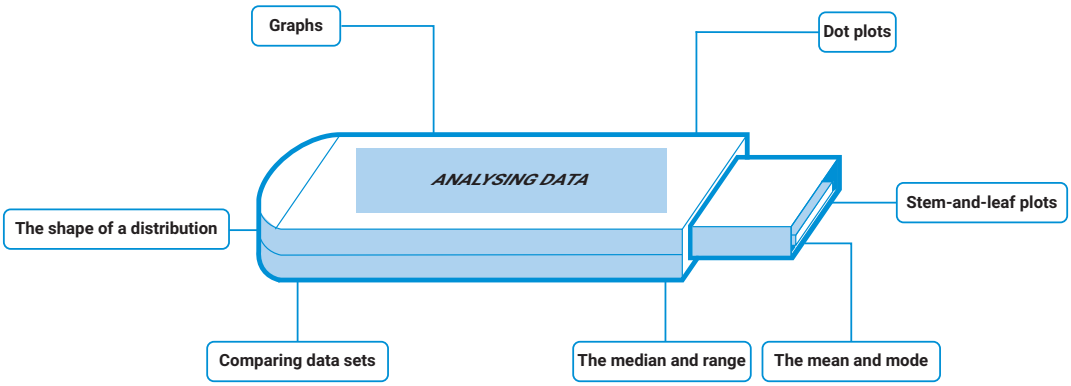
1. Look up the word 'average' in the dictionary. What does it mean?
2. What is another name for an extreme data value?
3. Why are the dot plots and stem-and-leaf plots called 'plots' rather than 'graphs'?
4. Which word is an example of a measure of **spread**?
5. Explain what is meant by a **cluster** of data values.
6. What is a 'median strip'? Why do you think it has this name?

Topic summary

Worksheet
Mind map: Analysing data

- Write about what you have learnt in this chapter.
- Was this work new to you? If not, in what subject have you studied it?
- Did you have any difficulties? Discuss them with a friend or your teacher.

Print (or copy) and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.



9780170465557

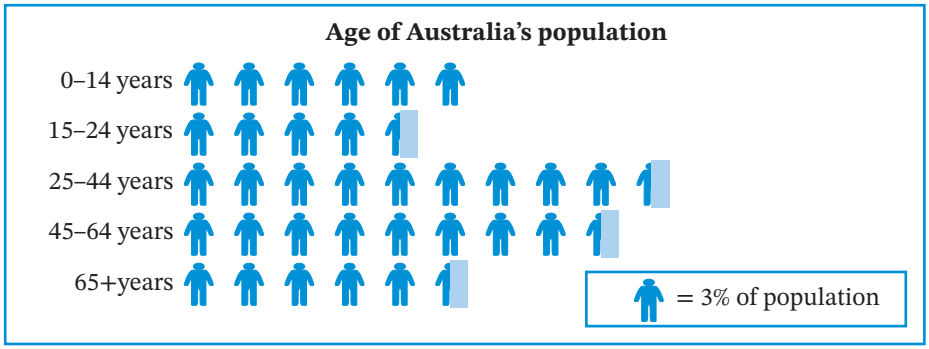
10 TEST YOURSELF

ANSWERS ON P. 643

10.01

Quiz
Test yourself 10

1 **a** What age group has the smallest percentage of the population?



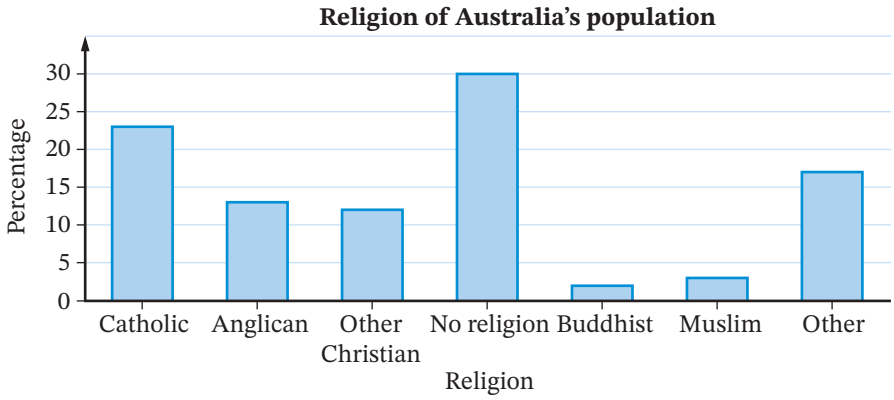
Source: Based on Australian Bureau of Statistics data

b What percentage are in the 15–24 years age group?

c Why might the 25–44 years group be the largest?

10.01

2



Source: Based on Australian Bureau of Statistics data

a What percentage of the population is Catholic?

b Which religion is followed by 13% of the population?

c How might this graph have been different 20 years ago?

10.01

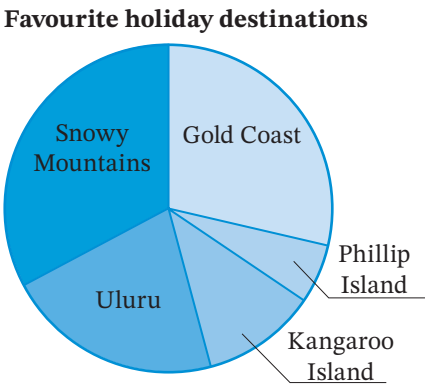
3 This sector graph shows the favourite holiday destinations of 80 people surveyed at a city shopping centre.

a What is the most popular destination?

b Which holiday destination was preferred by 17 people?

c Estimate how many people preferred the Gold Coast.

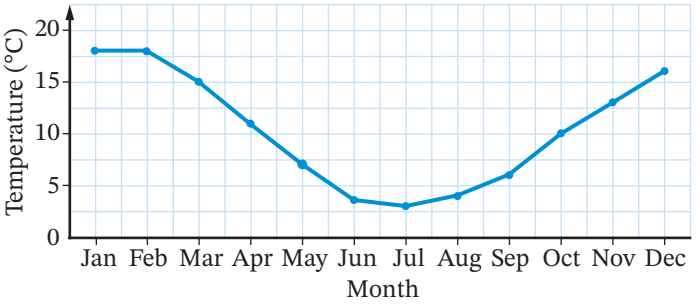
d True or false: Fewer than 10 people preferred Phillip Island.



Foundation | Standard | Complex

10.01 **4** This graph shows temperature data for Rottnest Island.

Mean monthly minimum temperatures for Rottnest Island



a Which months have the highest mean minimum temperature? What is that temperature?

b Name any 2 months that have the same mean minimum temperature.

c Which month has a mean minimum temperature of 10°C?

d Between which 2 consecutive months is the smallest drop in mean minimum temperature?

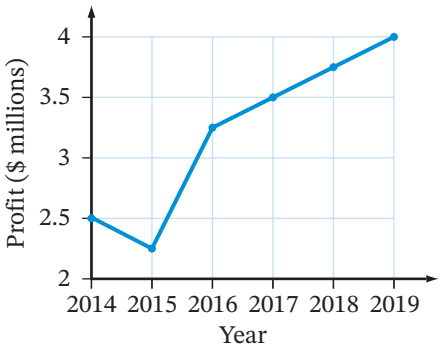
10.02 **5** This line graph shows the yearly profits of a company over a period of 6 years.

a What is incorrect about this graph?

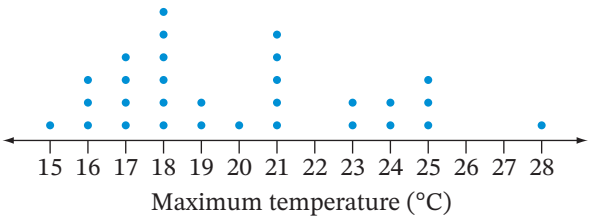
b What misleading impression does this give?

c Redraw the graph correctly.

Company profits



10.03 **6** The daily maximum temperatures (in °C) in Ipswich during April are shown in this dot plot.



a What is the mode?

b What is the outlier?

c On how many days was the temperature 25°C?

d On what fraction of days did the temperature drop below 20°C?

Foundation Standard Complex

7 This unordered stem-and-leaf plot shows the masses of the players of a football team, in kilograms. 10.04

Stem	Leaf
7	9 5 8 1 6 9
8	4 2 5 0 0 8 0 3 2 4 2 8
9	0 8 4 2 2 0 0 8 0 5 6
10	5 4

a How many players are on the team?

b Present this information as an ordered stem-and-leaf plot.

c Which mass occurs most often?

d What is the lowest mass?

For the rest of this exercise, round mean values to 2 decimal places where necessary.

8 For each set of data, find: 10.05

i the mean **ii** the mode.

a 4 3 2 5 6 4 4

b 6 12 11 12 10 6 6 10 6

c 8 4 1 1 4 1 3 6

9 For each set of data in question **8**, find: 10.06

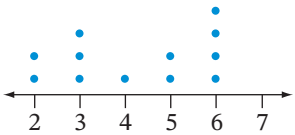
i the median **ii** the range.

10 For each plot below, find: 10.07

i the range **ii** the median

iii the mode **iv** the mean.

a



b

Stem	Leaf
4	0 0 1 2
5	1 3 4
6	6 6 6 7 8
7	4 5

11 The assignment marks for 20 girls and 20 boys are as follows: 10.08, 10.09

Girls:	75	28	37	35	60	73	69	52	94	66
	55	39	48	51	53	18	29	76	59	83
Boys:	88	29	38	72	50	74	73	30	85	10
	28	93	66	17	75	40	55	62	73	58

a Construct a back-to-back ordered stem-and-leaf plot for the data.

b What was the highest mark? Who scored it, a boy or a girl?

c Find the mean, median, mode and range for each group.

Foundation Standard Complex

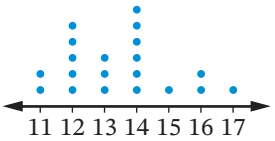
d Which groups' data was negatively-skewed: girls or boys?

e Comment on the differences between the girls and the boys on this assignment.

f Which group of students seemed to be more consistent in the marks they scored, the girls or the boys? Give reasons for your answer.

10.09 **12** For each distribution, describe the shape and identify any outliers or clusters.

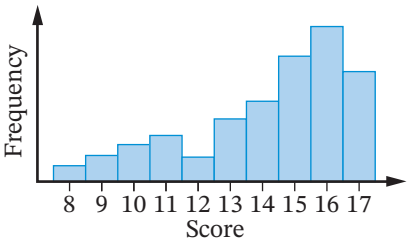
a



b

Stem	Leaf
8	7 8
9	1 3 4 4
10	0 2 7 8 8 9
11	1 2 2
12	0 7

c



Foundation Standard Complex

11

PROBABILITY

Probability

Many people suffer from aviophobia, the fear of flying. However, we are more likely to be involved in a car accident than a plane crash. There is a 1-in-5500 chance of being killed on the road compared to 1-in-3 million in the air. This is amazing when you consider that each day worldwide there are 102 000 planes in the sky carrying over 24 million passengers (that's the population of Australia in the sky!). Why do you think there are so few plane accidents compared to road accidents? And why are people more afraid of flying than travelling in a car?

iStock.com/photovs

Chapter outline

	Proficiencies
11.01 Sample spaces	U F R C
11.02 Probability	U F R C
11.03 The range of probability	U F R C
11.04 Observed and expected frequencies	U F R C
11.05 Extension: Complementary events*	U F R C

*YEAR 8 EXTENSION

U = Understanding
F = Fluency
PS = Problem solving
R = Reasoning
C = Communication

Wordbank

certain Sure to happen; the opposite of impossible

equally likely To have exactly the same chance of occurring; for example, when tossing a coin, heads or tails are equally likely

event A result involving one or more outcomes; for example, the event of rolling an even number on a die contains the 3 outcomes {2, 4, 6}

expected frequency The expected number of times an event will occur over repeated trials

outcome A single result from a situation or experiment; for example, one outcome when rolling a die is to roll a 6

random Describes a situation in which every possible outcome has an equal chance

sample space The set of all possible outcomes of a situation or experiment

Quiz
Wordbank 11

Videos (4):

11.01 Probability of an event

11.04 Experimental probability • Observed and expected frequencies

11.05 Complementary events

***Twig* Video (1):**

11.04 Probability: Irrational fears

***PhET* Interactive (1):**

11.04 Plinko probability

Quizzes (5):

- Wordbank 11
- SkillCheck 11
- Mental skills 11
- Language of Maths 11
- Test Yourself 11

Worksheets (13):

SkillCheck Chance cards • Describing probabilities

11.02 Probability problems

11.04 A page of spinners • Dice probability • Coin toss experiment • Experimental probabilities • Relative frequencies • Spinning chance • A page of spinners

11.05 Theoretical probabilities

Language of maths Probability review

Mind map: Probability

Puzzles (7):

11.02 Spinner game • Theoretical probabilities • Basic probability

11.03 Greedy Pig game

11.04 Experimental probability • Duelling dice • Spinner game

11.05 Complementary events

Presentation (1):

11.04 Theoretical probability

Spreadsheet (2):

11.04 Die rolling simulator • Coin tossing simulator

Nelson MindTap

To access resources above, visit **cengage.com.au/nelsonmindtap**

11 In this chapter you will:

- ✓ list the outcomes of the sample space of a chance experiment
- ✓ use the sample space to calculate the probability of an event
- ✓ recognise that the range of probability is from 0 to 1
- ✓ compare the observed frequency of an event with its expected frequency
- ✓ (EXTENSION) solve problems involving complementary events

SkillCheck

ANSWERS ON P. 644

Quiz SkillCheck 11

Worksheets Chance cards

Describing probabilities

1 Rate each event as being impossible, unlikely, **even chance**, likely or certain.

a Choosing a pink ball from a bag of blue balls

b It will rain next month.

c A coin shows tails when you toss it.

d You obtain a driver's licence tomorrow.

e You will have your driver's licence in 20 years' time.

f Rolling double 6s on a pair of dice

g You are over 10 years old.

h You draw a red card from a deck of playing cards.

i You will use a computer today.

j It will snow in your town tomorrow

2 Simplify each fraction.

a $\frac{2}{10}$ **b** $\frac{8}{18}$ **c** $\frac{27}{30}$ **d** $\frac{15}{80}$

3 Convert each number to a decimal.

a $\frac{7}{8}$ **b** $\frac{3}{5}$ **c** 26% **d** 70%

4 Convert each number to a percentage.

a 0.6 **b** 0.31 **c** $\frac{3}{4}$ **d** $\frac{1}{20}$

5 Is the chance of each event more than or less than $\frac{1}{2}$?

a You having another brother or sister one day

b You going overseas this year

c You being at school next Monday

d You being a parent in 20 years' time

e You getting a good report for English this year

f Your home phone ringing today

6 **a** Draw an interval 15 cm long and use it to make a probability scale for the chances from impossible to certain.

b On your scale, mark the position of each event described below.

A It will be hot tomorrow.

B The Sun will rise tomorrow.

C There will be floods in your town this year.

D There will be no car accidents in Sydney tomorrow.

E There are more than 5 people at your home at the same time today.

F Someone at school has a birthday today.

G It will rain tomorrow.

H The Tigers will win their next game.

I The next person to visit your classroom is male.

J Your favourite song comes on your radio station in the next hour.

7 Which term best describes the chance that the next baby born in Australia is a girl? Select the correct answer **A**, **B**, **C** or **D**.

A certain **B** definite **C** even chance **D** probable

8 Convert each number to a simplified fraction.

a 0.35 **b** 0.2 **c** 48% **d** 6%

9 Evaluate each expression.

a $1-\frac{1}{6}$ **b** $1-\frac{3}{4}$ **c** $1-\frac{4}{10}$

11.01 Sample spaces

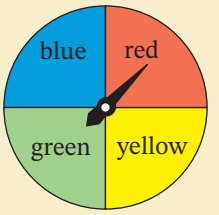
Probability is the branch of mathematics that studies the chances of different events occurring. In any situation, the set of all possible outcomes is called the **sample space**. For example, if a coin is tossed, the sample space is {heads, tails}. If each outcome has an equal chance, then we say that each outcome is **equally likely**.

Example 1

a Write the sample space for this spinner.

b How many outcomes are possible?

c Is each outcome equally likely?



SOLUTION

a The sample space is {red, yellow, green, blue}.

b There are 4 possible outcomes.

c Each coloured region is equal in size ($\frac{1}{4}$ of the circle), so each outcome is equally likely.

The language of probability

This table lists some important terms associated with probability, and how they apply to Example **1**.

Probability term	In Example 1
An **experiment** (or **chance experiment**) is a situation involving chance that leads to results called outcomes.	Spinning a spinner
A **trial** is one go or run of the experiment.	One spin of the spinner
An **outcome** is the result of an experiment.	The arrow landing on green
The **sample space** is the set of all possible outcomes.	{red, yellow, green, blue}
An **event** is one or more outcomes of an experiment.	The arrow landing on a 'traffic light' colour: red, yellow or green
In a **random** experiment, every possible outcome has the same chance of occurring.	All spins on this spinner are random because every colour has the same chance

Example 2

a How many outcomes are in the sample space when a die is rolled?

'Die' is the singular word for 'dice': one 'die', 2 or more 'dice'

b Is each outcome equally likely?

Shutterstock.com/tvioletl

SOLUTION

a The sample space is {1, 2, 3, 4, 5, 6}, so there are 6 possible outcomes.

b Each outcome is equally likely, because each number has the same chance of coming up.

Example 3

A jar contains 5 red, 3 green, 6 yellow and 2 blue lollies.
Taylor selects one lolly from the jar at random.

'At random' means that each lolly has an equal chance of being chosen

a How many lollies are in the jar?

b How many outcomes are in the sample space for the colour of the lolly?

c Is each colour equally likely?

SOLUTION

a Number of lollies $= 5 + 3 + 6 + 2 = 16$

b 4 outcomes {red, green, yellow, blue}

c Each colour is not equally likely because there are different amounts of each colour. Some colours occur more frequently and have higher chances of being chosen.

Video
Probability of an event

11.01

EXERCISE 11.01 ANSWERS ON P. 644

Sample spaces

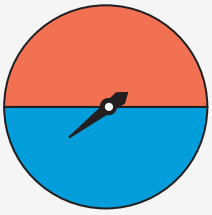
U F R C

1 For each spinner, write down the sample space and count the number of possible outcomes.

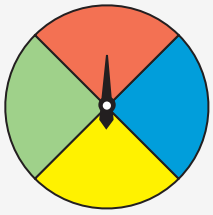
EXAMPLE 1

R C

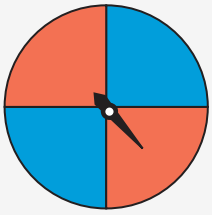
a



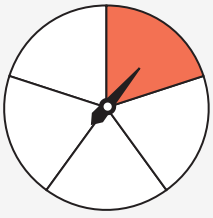
b



c

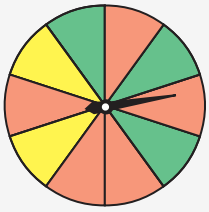


d



e

f



2 For each experiment, count the number of possible outcomes and state whether each outcome is equally likely.

EXAMPLE 2

a Tossing a coin

b The result of a soccer game when Australia plays South Korea

c The first letter of a person's name

d The sex of a baby

e The last digit of a phone number

f The result of a driving test

Foundation Standard Complex

3 List the outcomes in each event.

a Rolling an odd number on a die

b Selecting a vowel from the letters of the alphabet

c Having a house number greater than 4 but less than 10

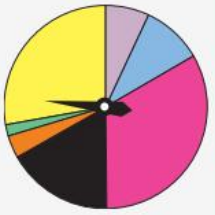
d Having a birthday in a month beginning with M

e Living in a state capital city

f Being in a primary school grade

EXAMPLE 3

4 This spinner is spun.



C **a** How many outcomes are possible?

b Is each outcome equally likely? Explain your answer.

c Which outcome is the most likely to occur?

d Which outcome is the least likely?

5 A money box contains 4 \$2 coins, 3 \$1 coins, 2 50c coins, 6 20c coins and 5 10c coins. It is shaken and one coin falls out at random.

a How many coins are in the money box?

b List the outcomes in the sample space for the type of coin.

c Is each type of coin equally likely?

d Which type of coin is most likely to fall out?

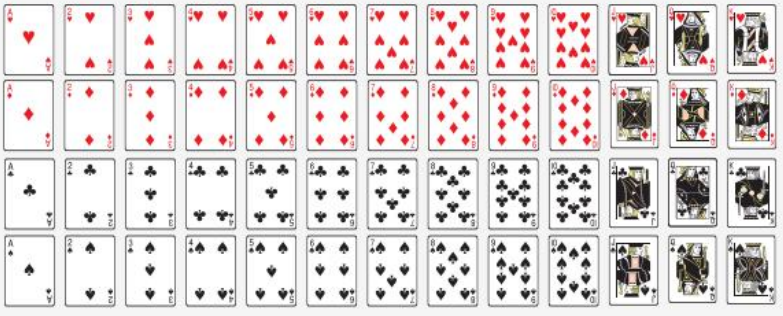
6 The 52 cards in a standard deck of playing cards are shown below, divided evenly into 4 suits: hearts, diamonds, clubs and spades.

Hearts:

Diamonds:

Clubs:

Spades:



The cards are shuffled and one is taken out at random.

R **a** How many outcomes are in the sample space?

b Is each outcome equally likely?

c How many cards are in each suit?

d How many red cards are in the deck?

e How many 7s?

f How many black Kings?

g How many 8 of diamonds?

Foundation Standard Complex

h How many cards with an even number?

i How many picture cards (J, K, Q)?

j How many red Aces?

7 A 3-digit number is to be formed from the digits 1, 2, 5. Which of the following answers shows the sample space? Select the correct answer **A**, **B**, **C** or **D**.

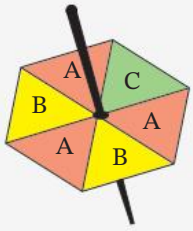
A {1, 2, 5}

B {125}

C {125, 152, 215}

D {125, 152, 215, 251, 512, 521}

8 This spinner is spun.



C **a** Write down the sample space of possible letters.

b Is each letter equally likely?

c Which letter is most likely?

9 Daniel is holding these 12 playing cards. Elvira picks a card without looking.

C **a** How many 5s are held in the 12 cards?

b List the sample space of possible numbers.

c Which number is Elvira most likely to pick?

d List the sample space of possible colours.

e Which colour is Elvira more likely to pick?

f List the sample space of possible suits.

g Which suit is Elvira least likely to pick?

Probability 11.02

We can calculate the chance or probability of an event occurring as a fraction, percentage or a decimal.

Worksheet Probability problems

Puzzles Spinner game

Theoretical probabilities

Basic probability

Probability of an event

$P(E)$ means 'the probability of an event, E (occurring)'. If all possible outcomes are **equally likely**, then:

$$P(E) = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$$

$$\text{or } P(E) = \frac{\text{number of outcomes matching } E}{\text{number of outcomes in the sample space}}$$

A **favourable outcome** is one of the outcomes in the event that you want, whose probability you are calculating.

Foundation Standard Complex

Example 4

A die is rolled. Find the probability of rolling:

a a 5

b a 7

c an odd number

d a number less than 3

SOLUTION

The sample space is {1, 2, 3, 4, 5, 6}. There are 6 possible outcomes and each outcome is equally likely.

a $P(5) = \frac{1}{6}$ — One chance in 6

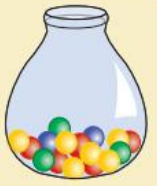
b $P(7) = \frac{0}{6} = 0$ — No chance, because we cannot roll a 7 on a die

c $P(\text{odd}) = \frac{3}{6} = \frac{1}{2}$ — 3 odd numbers out of 6 {1, 3, 5}

d $P(\text{less than } 3) = \frac{2}{6} = \frac{1}{3}$ — 2 numbers out of 6 {1 and 2}

Example 5

A jar of lollies contains 5 red, 3 green, 6 yellow and 2 blue lollies. Taylor selects one lolly from the jar at random and notes its colour. Calculate each probability below.



a $P(\text{yellow})$

b $P(\text{green or blue})$

c $P(\text{traffic light colour})$, as a percentage

d $P(\text{not red})$, as a decimal

SOLUTION

Total number of lollies in the jar = 5 + 3 + 6 + 2 = 16

a $P(\text{yellow}) = \frac{6}{16} = \frac{3}{8}$ — $\frac{\text{number of yellow lollies}}{\text{total number of lollies}}$

b $P(\text{green or blue}) = \frac{3+2}{16} = \frac{5}{16}$ — $\frac{\text{no. of green lollies + no. of blue lollies}}{\text{total number of lollies}}$

c $P(\text{traffic light colour}) = P(\text{red or green or yellow})$ — converting to a percentage

$$= \frac{5+3+6}{16}$$
$$= \frac{14}{16}$$
$$= \frac{14}{16} \times 100\%$$
$$= 87.5\%$$

d $P(\text{not red}) = P(\text{green or yellow or blue})$ — converting to a decimal

$$= \frac{3+6+2}{16}$$
$$= \frac{11}{16}$$
$$= 0.6875$$

EXERCISE 11.02 ANSWERS ON P. 644

Probability

U F R C

EXAMPLE 4

1 **a** List all the possible outcomes when a coin is tossed.

b How many different outcomes are possible?

c What is the probability of getting a tail?

d Write the probability of getting a tail as a percentage.

iStock.com/hueberto

2 A die is rolled. Find the probability of rolling:

a a 4 **b** an even number **c** a 10

d a number greater than 1 **e** a prime number **f** a number less than 7

3 Copy each spinner and shade its regions so that the probability of spinning white is:

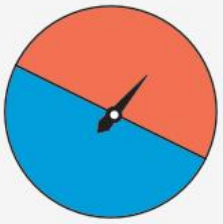
R **a** $\frac{5}{8}$

b $\frac{3}{4}$

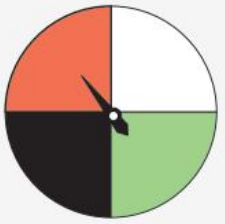
Foundation Standard Complex

4 For each spinner, calculate the probability of the pointer landing on red as a percentage.

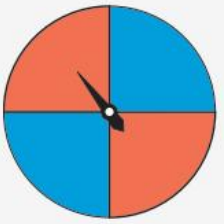
a



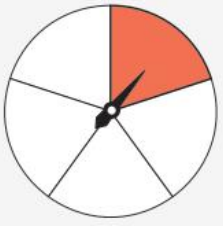
b



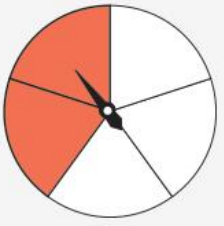
c



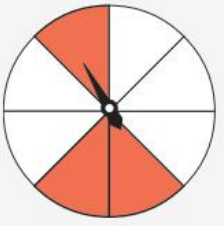
d



e



f



EXAMPLE 5

5 A money box contains 4 \$2 coins, 3 \$1 coins, 2 50c coins, 6 20c coins and 5 10c coins. It is shaken and one coin falls out at random. Calculate each probability:

a P(50c coin)

b P(\$1 coin), as a decimal

c P(10c or 20c coin), as a percentage

d P(not a 10c coin)

e P(gold coin), as a decimal

f P(a coin under \$1), as a percentage

6 A packet of jellybeans has 4 yellow, 3 red, 6 green and 3 black jellybeans remaining. You take out one jellybean at random. What is the probability that it is not a red jellybean? Select the correct answer **A**, **B**, **C** or **D**.

A $\frac{4}{12}$ **B** $\frac{3}{16}$ **C** $\frac{3}{13}$ **D** $\frac{13}{16}$

7 A letter is selected at random from the alphabet. Find the probability that it is:

a M **b** R **c** M or R

d a vowel **e** not a vowel **f** a letter before K in the alphabet

8 The needle on this spinner is spun.

a Which colour has a probability of $\frac{1}{3}$?

b Which colour has a probability closest to $\frac{1}{4}$?

c Which colour has a probability of $\frac{1}{6}$?

d Which 3 colours have a combined probability of 50%?

9 A bag contains 6 white, 9 green and 5 blue marbles. Hannah takes one from the bag at random. Which colour has a probability of:

R

a 0.25? **b** 45%? **c** $\frac{3}{10}$?

10 A computer generates a random number from 1 to 10. Find:

a P(square number)

b P(number less than 4), as a decimal

c P(prime number), as a percentage

d P(factor of 10)

e P(number greater than 6), as a decimal

f P(5 or 7), as a percentage

Foundation Standard Complex

11 What is the probability that a baby is born on a weekday rather than a weekend, if each day is equally likely?

12 Design one spinner that would produce all these probabilities:

R $P(\text{red}) = \frac{1}{8}$, $P(\text{yellow}) = 24\%$, $P(\text{blue}) = \frac{1}{3}$, $P(\text{black}) = 20\%$.

13 Ryan bought 6 raffle tickets at the school fete. If a total of 800 tickets was sold, what is the probability that Ryan wins first prize? Express your answer:

a as a fraction **b** as a decimal

14 Your maths teacher calls out a name randomly from your class roll. What is the probability that it is:

a your name? **b** a girl's name?

c someone aged 12? **d** someone with blond hair?

15 A deck of 52 playing cards is shuffled and one is taken out at random. Find the probability that it is: You may refer to the diagram of playing cards on page 512.

a a red card **b** a club card

c the King of spades **d** a Queen

e a card with an odd number **f** a black picture card

g a red 7 **h** a 4

16 What is the probability that a randomly chosen person has a birthday in a month beginning with the letter J? Select **A**, **B**, **C** or **D**.

A $\frac{1}{12}$ **B** $\frac{1}{6}$ **C** $\frac{1}{4}$ **D** $\frac{1}{3}$

17 There are 16 teams in a football competition. Paula, therefore, believes that her favourite team, The Bulldogs, has a probability of $\frac{1}{16}$ of winning the competition. Discuss with your friends whether Paula is correct and write your answer, giving reasons.

R C

18 Karen had a box of 24 ice blocks. The box had the same number of lemon, orange and strawberry blocks. If she chose one ice block at random, what was the chance that it was either strawberry or orange? Select **A**, **B**, **C** or **D**.

R

A $\frac{1}{12}$ **B** $\frac{1}{24}$ **C** $\frac{1}{3}$ **D** $\frac{2}{3}$

19 A student council is made up of a number of boys and girls. One member of council is selected at random to attend the regional conference. How many boys and girls could there be in the council if the probability of selecting a boy is $\frac{5}{8}$?

R

20 An esky contains 8 cans of lemonade, 5 cans of orange drink and 2 cans of lime drink. How many cans of cola must be added so that the probability of randomly selecting a can of:

R

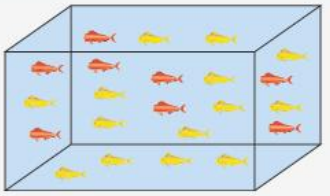
a lemonade is 50%? **b** orange drink is 0.25?

c lime drink is $\frac{1}{12}$? **d** cola is $\frac{2}{5}$?

Foundation Standard Complex

21 A fish tank has 8 red and 14 yellow fish. One fish is randomly selected from the tank.

R



a Write the probability that the fish is yellow as a decimal, correct to 2 decimal places.

b Write the probability that the fish is red as a percentage.

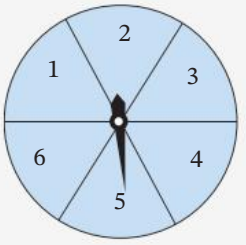
c How many red fish must be removed from the tank to make the probability of selecting a red fish 0.3?

22 **a** What is the lowest probability value? Name an event that could have this value.

R C **b** What is the highest probability value? Name an event that could have this value.

23 This spinner is spun twice and the 2 numbers that come up are added. Which of these totals has the lowest chance of occurring? Select **A**, **B**, **C** or **D**.

R



A 9 **B** 8 **C** 7 **D** 6

24 A box has 3 pens coloured red and blue. Owen randomly chooses a pen and it is blue. There are now 2 remaining pens in the box. What is the probability that the second pen Owen chooses will be red? (Be careful: the answer is not $\frac{1}{2}$. List all the different possible outcomes.)

R C

DID YOU KNOW?

Car insurance

Insurance is a financial scheme that offers money in times of loss or damage. People can insure their cars against damage or theft. You pay an amount each month or year, and if loss or damage happens, the insurance company pays you to repair any damage or replace your car.

Shutterstock.com/somkhuanfoto

Foundation Standard Complex

Insurance was invented in ancient Babylon around 3000 BCE, when voyagers and traders insured their ships against damage or loss. Modern mathematicians working for insurance companies use the statistics of past accidents to predict the chances of future losses. They have found some interesting probabilities involving car accidents.

Event	Chance
Car runs off the road	1 chance in 30
Driver is male	18 chances out of 25
Involves driver fatigue	1 chance in 5
Involves speeding	31 chances out of 100
Involves alcohol	19 chances in 100
Male driver aged 25 to 34	17 chances out of 50

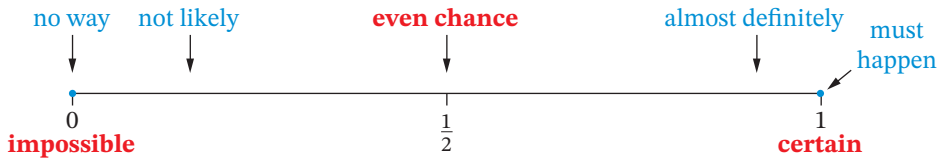
An actuary works in the insurance industry. Find out what an actuary does.

The range of probability 11.03

Because probability is a fraction:

$$P(E) = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$$

its value must range from 0 to 1, or as a percentage, from 0% to 100%.



The range of probability

- The probability of an **impossible** event is 0.
- The probability of a **certain** event is 1.
- The probability of any event ranges from 0 to 1.

A probability close to 1 indicates an event that is very **likely** or probable.

A probability close to zero indicates an event that is very **unlikely** or improbable.

Example 6

5 cards numbered 11, 5, 9, 7 and 3 are shuffled and Jo chooses one at random. Find the probability that the number chosen is:

a divisible by 3 **b** less than 12

c a factor of 45 **d** even

SOLUTION

a $P(\text{divisible by } 3) = \frac{2}{5}$ 2 numbers {9, 3}

b $P(\text{less than } 12) = \frac{5}{5} = 1$ All 5 numbers are less than 12: a certain event.

c $P(\text{a factor of } 45) = \frac{3}{5}$ 3 numbers {5, 9, 3}

d $P(\text{even}) = \frac{0}{5} = 0$ None are even: an impossible event

EXERCISE 11.03 ANSWERS ON P. 645

The range of probability

U F R C

EXAMPLE 6

1 If a die is rolled, find the probability of rolling:

a a 5 **b** an 8 **c** a factor of 12

d a prime number **e** a number less than 7 **f** a multiple of 3

2 One letter is selected at random from the word EVACUATION. Find the percentage probability that it is:

a V **b** A **c** X **d** a vowel

3 Simon is holding these 12 playing cards. Joshua picks a card without looking.

Find the probability that it:

a is red

b shows an even number

c is a picture card

d has the number 7

e is a hearts card

f is not an Ace

4 A basketball team captain is to be chosen randomly from 4 players: Carl, Lee, Su and Daniela. What is the probability that the captain chosen is:

a Daniela? **b** Su? **c** Manjeet?

Foundation Standard Complex

5 Match each probability value to its description.

C **a** $\frac{1}{2}$ **b** 0 **c** 90% **d** 1

e $\frac{3}{4}$ **f** 0.1 **g** 0.6 **h** 2%

A cannot happen **B** better than average chance

C even chance **D** good chance

E very likely **F** almost impossible

G slim chance **H** must happen

6 A coin falls out of a piggy bank containing 4 5c, 5 10c, 6 20c and 3 $1 coins. Find the probability that it is:

a a 5c coin **b** a $1 coin **c** a 50c coin

d a 10c or 20c coin **e** not a $1 coin **f** not a 20c coin

7 A traffic light shows red for 59 seconds, green for 99 seconds and yellow for 2 seconds. Find, as a decimal, the probability that it shows:

a green **b** yellow

c red or green **d** not red, yellow or green

8 A computer generates a random number from 1 to 20. Find, as a percentage, the probability that the number is:

a a multiple of 4 **b** a factor of 20

c a number less than 30 **d** 25

e a prime number **f** divisible by 5

9 A die has 2 blue faces, 1 red face and the other faces are green. It is rolled. Match each event to its correct probability:

a red **b** blue

c yellow **d** a colour that is not red

e blue, red or green **f** a traffic light colour

A $\frac{2}{3}$ **B** 50% **C** $\frac{5}{6}$ **D** $\frac{1}{3}$

E 0 **F** $\frac{1}{6}$ **G** 100%

10 A letter is chosen at random from the words 'NELSON MATHS'. Find each probability.

a P(S) **b** P(M) **c** P(N or E)

d P(a consonant) **e** P(a letter also found in PROBABILITY)

f P(K)

11 Write an event that could have a probability of:

R C **a** $\frac{1}{2}$ **b** 0 **c** 90% **d** 1

e $\frac{3}{4}$ **f** 0.1 **g** 0.6 **h** 2%

Foundation Standard Complex

DID YOU KNOW?

Buckley's chance

When an Australian person says you have 'Buckley's chance' of something happening, it means that you have little chance or no chance at all. This old slang phrase is based on the name of a Melbourne department store in the 1850s called 'Buckley and Nunn'. The original expression was 'You have 2 chances: Buckley's and none'.

English is often a funny and illogical language. For example, 'slim chance' and 'fat chance' sound like opposites, but they both mean the same thing.

Find another slang expression that describes a chance.

INVESTIGATION

The Greedy Pig game

Puzzle
Greedy Pig game

This is a game for 4 or more players. You need a playing die and a score sheet (download and print out the puzzle sheet 'Greedy Pig game').

iStock.com/Byrdyak

1. All players stand at the start of a round. A die is rolled twice and the total is each player's starting score.
2. Before the next roll of the die, each player has the option of sitting down and keeping their score, or remaining standing and 'being greedy' with the chance of increasing their score from the next roll of the die.
3. The die is rolled and the number is added to the score of each player standing, but if 2 comes up, all standing players lose their points.
4. A round ends when a 2 is rolled or all players are seated. Each player then records their score for the round.
5. The round is repeated 4 more times and the overall winner is the player with the highest total score for the 5 rounds.
6. Play this game several times. Is there any strategy to winning?

11.04 Observed and expected frequencies

Worksheet
A page of spinners

Probability that is calculated using the formula:

$$P(E) = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$$

is more specifically called **theoretical probability**.

We can also determine the probability of an event based on the results of an experiment or trial that has been repeated many times, such as the safety testing of different cars, or rely on past statistics, such as the number of rainy days in April. This type of probability is called **experimental probability** or **observed probability**, which is based on **relative frequency**, the number of times an event occurred as a fraction of the total **frequency** of outcomes.

Worksheets
Dice probability
Coin toss experiment
Experimental probabilities
Relative frequencies
Puzzles
Experimental probability
Duelling dice
Presentation
Theoretical probability
Videos
Experimental probability
Probability: Irrational fears
Interactive
Plinko probability

Relative frequency

$$P(E) = \frac{\text{number of times the event happened}}{\text{total number of trials}}$$

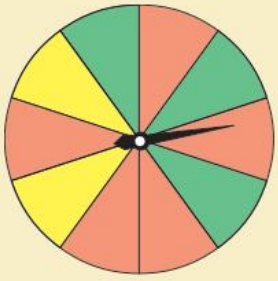
$$\text{or } P(E) = \frac{\text{frequency of } E}{\text{total frequency}}$$

As in statistics, 'frequency' means the number of times something happens

Example 7

Josie spun this spinner 80 times and found the following results:

Outcome	Red	Green	Yellow
No. of times	44	25	11



a What is the theoretical probability of spinning red?

b For 80 spins, what is the expected number of times of spinning red? How does this compare with the actual number of times?

c What is the experimental probability of spinning red?

SOLUTION

a $P(\text{red}) = \frac{5}{10} = \frac{1}{2}$

b Expected number of reds $= \frac{1}{2} \times 80 = 40$.

From the table, the observed number of reds = 44, which is close to the expected number.

c Experimental $P(\text{red}) = \frac{44}{80} = \frac{11}{20}$

Observed and expected frequencies

The expected number of times an event will occur over repeated trials is called the **expected frequency**.

Expected frequency = theoretical probability × number of trials

The actual number of times an event occurs over repeated trials in an experiment is called the **observed frequency**.

Video
Observed and expected frequencies

Example 8

Declan rolled a die 60 times and recorded the results in a table.

Outcome	1	2	3	4	5	6
Frequency	10	11	13	7	8	11

a What is the theoretical probability of rolling a 5 or 6 on a die?

b For 60 rolls, what is the expected frequency of rolling a 5 or 6? How does this compare with the observed frequency?

c What is the experimental probability of rolling a 5 or 6?

SOLUTION

a $P(5 \text{ or } 6) = \frac{2}{6} = \frac{1}{3}$

b Expected frequency of 5s or 6s $= \frac{1}{3} \times 60$ Probability × number of trials

$= 20$

From the table, the observed frequency of 5s and 6s = 8 + 11 = 19, which is close to the expected frequency, 20.

c Experimental $P(5 \text{ or } 6) = \frac{8+11}{60} = \frac{19}{60}$

EXERCISE 11.04 ANSWERS ON P. 645

Observed and expected frequencies

U F R C

EXAMPLE 7

1 For the spinner in Example **7**, find:

a the theoretical probability of spinning yellow

b the expected frequency of spinning yellow and how this compares with the observed number of times

c the experimental probability of spinning yellow

2 A pair of dice was rolled 50 times and their sum calculated each time.

Sum	2	3	4	5	6	7	8	9	10	11	12
Frequency	0	2	4	6	5	5	9	6	8	3	2

a Find, as a decimal, the relative frequency of rolling a sum:

i of 10 **ii** of 7 or 11 **iii** that is even

iv less than 6 **v** greater than 10 **vi** that is a prime number

b Which sum:

i was most likely? **ii** had a probability of $\frac{2}{25}$?

iii was least likely? **iv** had a probability of 10%?

v was second-most likely? **vi** had a probability of $\frac{4}{25}$?

Foundation Standard Complex

EXAMPLE 8

11.04

3 **a** Copy this table.

Outcome	Tally	Frequency
Head		
Tail		
	Total	

b Toss a coin 50 times and record the result for each toss in the table.

c What is the theoretical probability of tossing a tail?

d In 50 tosses, what is the expected frequency of tails? How does this compare with the observed frequency?

e What is the relative frequency of tossing a tail?

R C **f** What do you think would happen to the relative frequency of a tail if the coin was tossed 1000 times rather than 50?

4 The volume of soft drink in 50 cans of drink were measured.

Volume (mL)	373	374	375	376	377
Number of cans	2	6	38	3	1

Using these results, if you bought a can of drink, what is the probability that it will contain exactly 375 mL of drink? Select the correct answer **A**, **B**, **C** or **D**.

A $\frac{375}{50}$ **B** $\frac{19}{25}$ **C** $\frac{6}{25}$ **D** $\frac{3}{50}$

5 **a** Copy this table.

Outcome	Tally	Frequency
1		
2		
3		
4		
5		
6		
	Total:	

b Roll a die 50 times and record the result for each roll in the table.

c Find the theoretical probability and expected frequency of rolling:

i 3 **ii** 6

iii an even number **iv** a number below 6.

d Find the relative frequency of rolling:

i 3 **ii** 6

iii an even number **iv** a number below 6.

R **e** How do the observed frequencies compare with the expected frequencies?

C **f** What would you expect to happen if the die was rolled 1000 times?

Foundation Standard Complex

6 A die was rolled 80 times, with the results shown below.

R
C

Outcome	1	2	3	4	5	6
Frequency	11	13	9	13	12	22

a When rolling a normal die, is each outcome equally likely?

b Do you think this die is biased (unfair)? Give a reason for your answer.

c Write as a percentage the relative frequency of rolling a 1 on this die.

d If this die was rolled 100 times, how many times would you expect 3 to come up?

7 Mila tossed a coin many times and got 140 heads and 110 tails. Calculate the relative frequency of heads with this coin as a percentage.

8 Luna surveyed all the Year 7 students at her school about their favourite sport.

Sport	Number of students
Netball	54
Soccer	75
Hockey	25
Swimming	66

Which sport did 3 out of every 10 students choose?

A Soccer **B** Swimming

C Netball **D** Hockey

TECHNOLOGY

Spreadsheets
Die rolling simulator

Coin tossing simulator

Rolling a die

Scientists and economists often simulate events on a computer, which saves time and money because large amounts of data can be obtained quickly.
In this activity, a scientific calculator or spreadsheet is used to simulate the rolling of a die.

1 Copy this table.

Outcome	Number of times rolled
1	
2	
3	
4	
5	
6	

2 Random numbers from 1 to 6 can be generated on a calculator or spreadsheet using the RanInt, RANDOM or =RAND() functions.

Foundation Standard Complex

Casio scientific calculator	Sharp scientific calculator
Enter the formula RanInt#(1,6) as shown below:	2nd F 7 (RANDOM), select 1 R-DICE, =
ALPHA . 1 Shift) 6) =	

Press = more times to generate more random integers from 1 to 6.
On a spreadsheet, type **=INT(RAND()*6+1)** into a cell and press the Enter key.

3 Run the simulation 20 times and record the results in your table.

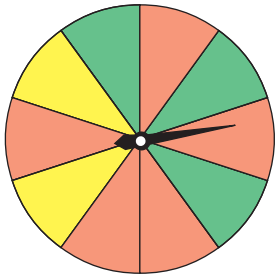
4 Are certain numbers more likely to be rolled than others? (For example, is a 2 more likely to be rolled than a 5?) Do your results reflect this?

5 Compare your results with the simulated results of your class. Are they similar or different? Are they what you and your classmates expected? Discuss.

INVESTIGATION

Spinning chance

You will need: A pencil, a paper clip and an enlarged copy of the spinner used in Example **7** (print out the worksheet 'Spinning chance').



1 Which colour is the most likely to be spun? Why?

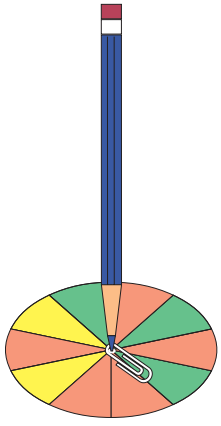
2 Which colour is the least likely?

3 Copy this table.

Outcome	Tally	Frequency
Red		
Yellow		
Green		
	Total:	

4 Work with a partner. Place a pencil on the centre of the spinner and a paper clip around the point. Spin the paper clip around the spinner, and note which colour it lands on.

5 Take turns to spin the spinner 100 times and use the table to record your results.



6 Find the theoretical probability and expected frequency of spinning:

i red ii yellow iii green.

7 How did your results compare with the expected number?

8 Find the relative frequency of spinning:

i red ii yellow iii green.

9 What would happen if 1000 spins were simulated? How many of each colour would be expected?

Worksheets
Spinning chance
A page of spinners

Puzzle
Spinner game

10 Combine the results of your class on a whiteboard or projector to calculate the relative frequency for each colour. For such a large number of trials, are the relative frequencies closer to the theoretical probabilities?

11 The spinner can also be simulated using the random number generator on a calculator, which outputs a random decimal between 0 and 1 with 3 decimal places.

Let 0, 1, 2, 3 and 4 represent red, 5 and 6 represent yellow and 7, 8 and 9 represent green. Now each decimal generated can represent 3 spins of the spinner. For example, 0.561 represents yellow, yellow, red.

12 Draw the blank table from question **8** again and use the 'Random' function on your calculator to simulate 100 spins.

13 Use the table to record your results.

14 How did your results compare with the expected frequencies?

Quiz
Mental skills 11

☆ MENTAL SKILLS 11 ANSWERS ON P. 645 Maths without calculators

Adding 90c and 95c

A quick way to mentally add monetary amounts ending in 90c or 95c is to round up to whole dollars and then subtract 10c or 5c respectively.

1 Study each example.

a $\$8.95 + \$17.95 + \$11.95 = \$9 + \$18 + \$12 - 5c - 5c - 5c$
$= \$39 - 15c$
$= \$38.85$

b $\$14.90 + \$5.95 + \$12.90 = \$15 + \$6 + \$13 - 10c - 5c - 10c$
$= \$34 - 25c$
$= \$33.75$

c $\$18.90 + \$24.90 + \$8.80 = \$19 + \$25 + \$9 - 10c - 10c - 20c$
$= \$53 - 40c$
$= \$52.60$

2 Now simplify each expression.

a $\$12.95 + \$7.95 + \$14.95$

b $\$20.90 + \$4.95 + \$13.95$

c $\$9.95 + \$11.90 + \$13.90$

d $\$11.95 + \$15.90 + \$6.95$

e $\$13.95 + \$19.95 + \$8.95$

f $\$34.90 + \$34.90$

g $\$27.95 + \$16.95 + \$12.90$

h $\$11.90 + \$21.95 + \$19.80$

i $\$24.95 + \$34.95$

j $\$33.90 + \$49.90 + \$49.90$

INVESTIGATION

Complementary events

In everyday life, we use the word 'complementary' to describe things that go together and 'complete the picture' when they are together. For example, when dressing for an occasion:

- a shirt and a matching tie complement each other
- a dress and a matching pair of shoes complement each other

Remember also that 'complementary' angles add to 90°.

In probability, **complementary events** are events that together make up all the possible outcomes.

Event	Complementary event
Tossing a tail on a coin	Tossing a head on a coin
Rolling a 6 on a die	Rolling any of the other numbers, from 1 to 5, on a die
Raining	Not raining
Being born on a Monday	Being born on a day other than Monday

Do not confuse 'complementary' with complimentary, which means speaking highly about someone and paying them a compliment

1 Suppose that there is an equal chance of being born on any day of the week: Monday to Sunday.

a What is $P(\text{Tues})$, the probability of being born on a Tuesday?

b What is $P(\text{not Tues})$, the probability of being born on a day other than Tuesday?

c What do you notice about $P(\text{Tues}) + P(\text{not Tues})$?

2 A fruit bowl contains 7 apples, 4 oranges and 9 bananas. One piece of fruit is selected at random from the bowl.

a Find $P(\text{orange})$

b Find $P(\text{not orange})$

c What do you notice about $P(\text{orange}) + P(\text{not orange})$?

3 A baby is selected at random from the maternity section of a large hospital. There is an equal chance of the baby being a boy or a girl.

a Find $P(\text{boy})$

b Find $P(\text{girl})$

c What do you notice about $P(\text{boy}) + P(\text{girl})$?

4 Copy and complete the following sentence:

The probability of an event _____ the probability of its complementary event must always equal _____.

11.05 Extension: Complementary events

YEAR 8 EXTENSION

Worksheet Theoretical probabilities

Video Complementary events

Puzzle Complementary events

In any situation, the probabilities of all possible outcomes must add to 1.

Complementary events are 2 events that together make up all the possible outcomes, such as a head and a tail when tossing a coin. The **complement** of an event E, are all those outcomes that are *not* E, or that are the 'opposite' of E.

Because an event and its complement covers all possible outcomes, the sum of their probabilities must equal 1.

Complementary events

$P(E) + P(\text{not } E) = 1$

or $P(\text{not } E) = 1 - P(E)$

or $P(\text{complementary event}) = 1 - P(\text{event})$

or $P(\text{event not occurring}) = 1 - P(\text{event occurring})$

If the probability is written in percentage form, then $P(\text{not } E) = 100\% - P(E)$.

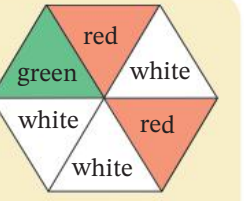
Example 9

On this spinner, what is the probability of spinning:

a red?

b a colour that is not red?

c a colour that is not green?



SOLUTION

a $P(\text{red}) = \frac{2}{6} = \frac{1}{3}$

b $P(\text{not red}) = 1 - P(\text{red})$

$= 1 - \frac{1}{3}$

$= \frac{2}{3}$

The complement of 'red' is 'not red', which is 'white or green'

Note that $P(\text{red}) + P(\text{not red}) = \frac{1}{3} + \frac{2}{3} = 1$, which covers all possible outcomes.

c $P(\text{green}) = \frac{1}{6}$

$P(\text{not green}) = 1 - P(\text{green})$

$= 1 - \frac{1}{6}$

$= \frac{5}{6}$

The complement of 'green' is 'not green', which is 'red or white'

Note that $P(\text{green}) + P(\text{not green}) = \frac{1}{6} + \frac{5}{6} = 1$, which covers all possible outcomes.

EXERCISE 11.05 ANSWERS ON P. 645

Complementary events

U F R C

1 Write the complement for each event.

R **a** The sex of a new baby being female

C **b** Rain tomorrow

c Losing a soccer match

D Being left-handed

e Being over 18 years of age

f Choosing a hearts card from a deck of playing cards

g A traffic light showing green

h A train arriving late

EXAMPLE 9

2 A die is rolled. What is the probability that the result is:

a 3? **b** less than 3? **c** prime?

d not a 3? **e** not less than 3? **f** not prime?

3 Olivia buys a ticket in a raffle in which 1000 tickets are sold and there is only one prize. What is the probability of Olivia:

a winning the prize? **b** not winning the prize?

4 A jar contains jellybeans in the following colours and amounts: red 40, blue 25, black 50, white 15. One jellybean is selected at random. What is the probability that the jellybean is:

a white? **b** not white?

c yellow? **d** not yellow?

e not red? **f** not blue or black?

5 Find the probability of the event that is complementary to each event described.

a The probability of choosing a Jack from a pack of cards is $\frac{1}{13}$.

b The chance of shooting a basketball hoop is 55%.

c The probability of winning a prize is 0.07.

6 A bag of toy cars contains only red, blue and white cars. If you take out a car at random, the chance of it being red is 0.5, and the chance of it being white is 0.2.

a What is the chance of selecting a red or white car?

b What is the chance of selecting a blue car?

c If the bag holds 30 cars, how many of each colour would you expect to find?

d What is the chance of the car you select being pink?

Foundation Standard Complex

YEAR 8 EXTENSION

7 What is the probability that the next person you meet was born in a month:

a beginning with the letter A?

b not beginning with the letter A?

8 4 students, Sue, Liam, Emily and Matt, write their names on cards and place the cards in a bag. A card is chosen, without looking, to select the class captain.

a Find the probability that Emily was not chosen.

b Find the probability that the captain is a boy.

c What is the chance that the captain is not a boy?

d What is the chance that the captain is the teacher?

9 Which of the following is the complementary event to 'winning a race'? Select the correct answer **A**, **B**, **C** or **D**.

R C

A coming last **B** coming second or third

C not winning the race **D** coming second

10 In a netball match, the Swifts have a 43% chance of winning, while the Vixens have a 49% chance of winning.

R C

a What other outcome is possible?

b What is the probability of this outcome?

11 The probability that a person has skin cancer is 0.03. What is the probability that a person doesn't have skin cancer? Select **A**, **B**, **C** or **D**.

A 0.2 **B** 0.7 **C** 0.07 **D** 0.97

12 The letters of the word PROBABILITY are written on separate cards and one is randomly selected. What is the probability that a letter drawn out is:

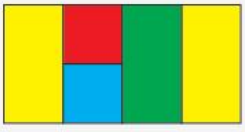
a not P? **b** not a vowel? **c** not I? **d** not A or B?

13 The probability that it will rain this weekend is 85%. What is the probability that it won't rain?

14 A die is rolled. What is the chance that the number rolled is not a factor of 6? Select **A**, **B**, **C** or **D**.

A $\frac{2}{3}$ **B** $\frac{5}{6}$ **C** $\frac{1}{3}$ **D** $\frac{1}{6}$

15 Stef has a rug with this pattern of colours. Ashton drops his ice cream on the rug. What is the probability that the ice cream lands on the section coloured:



a yellow? **b** red?

c blue or green? **d** not green?

16 What is the decimal probability that a mobile phone number selected at random does not end in 0 or 1?

R

Foundation Standard Complex

POWER PLUS ANSWERS ON P. 646

1 2 dice are rolled and the sum of the numbers is calculated.

a Copy and complete this table to show all possible sums.

		1st die					
	+	1	2	3	4	5	6
2nd die	1	2	3	4	5	6	7
	2	3	4				
	3	4					
	4						
	5						
	6						

b How many different sums are possible?

c Why isn't each sum equally likely?

d Which sum is the most likely?

e Which sum is the least likely?

f What is the probability of a sum of 2?

g What is the probability of a sum of 10?

h Which sum has a probability of $\frac{1}{9}$?

2 3 coins are tossed together. One possible outcome is HHT, that is, heads on the first coin, heads on the second coin, tails on the third coin.

a How many other possible outcomes are there? List them.

b Use your answers from part **a** to calculate the probability of:

i 3 heads **ii** 2 heads **iii** one head

iv no heads **v** at least one head **vi** at most one head

3 At Harry's takeaway, you can order a special meal deal for $8 that consists of one main meal, one drink and one dessert.

Harry's Takeaway Menu

Main meals	Drinks	Desserts
Hot dog	*Cola*	*Cake*
Pizza	*Lemonade*	*Ice cream*
Hamburger		

a List all possible meal deal combinations. How many are there?

b If you were to choose a combination at random, what is the probability that it will:

i be pizza, lemonade and cake?

ii not contain a hot dog?

iii contain cola?

11 CHAPTER REVIEW

Language of maths

Quiz Language of maths 11

Worksheet Probability review

certain	chance	complementary event	die/dice
equally likely	event	expected frequency	experiment
favourable	frequency	impossible	improbable
likely	observed	outcome	possible
probability	probable	random	relative frequency
sample space	theoretical probability	trial	unlikely

1 What is the opposite of **impossible** and what is its probability value?

2 What is the difference between **impossible** and **improbable**?

3 What is the name given to the set of all possible outcomes for a situation?

4 What are the highest and lowest probability values if written as percentages?

5 What does it mean when a name is drawn out of a box '**at random**'?

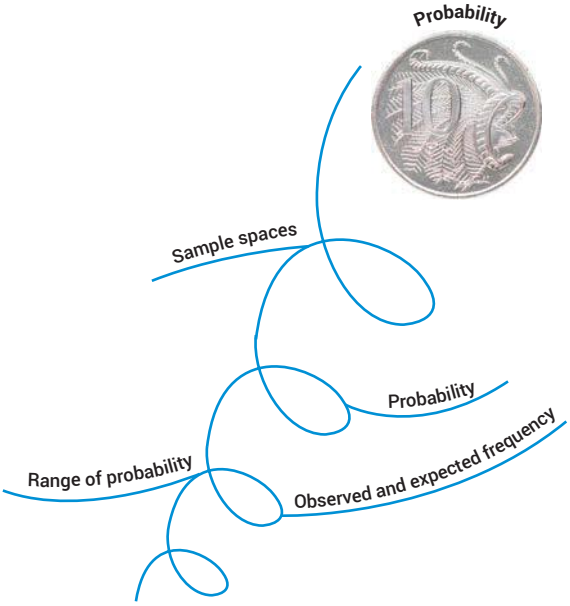
6 What word means the number of times something happens?

Topic summary

Worksheet Mind map: Probability

- Write about what you have learnt in this chapter.
- Write down the parts of the chapter that were new to you.
- Copy and complete:
  - The things I understand about probability are ...
  - The things I am still not confident about in this chapter are ...

Print (or copy) and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.



Shutterstock.com/Andrey Lobachev

11 TEST YOURSELF

ANSWERS ON P. 646

Quiz
Test yourself 11

1 List the outcomes for the sample space in each situation. 11.01

a The result of a driving test

b The classification of a new film (for example, PG)

c The winner of a tennis match when Katrina plays Biljana

d The public transport students use to attend your school

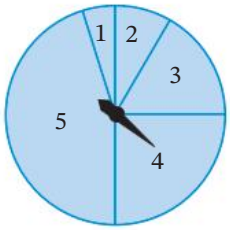
2 **a** For this spinner, how many outcomes are there in the sample space? 11.01

b Why isn't each outcome equally likely?

c Which number is the most likely?

d Which number is the least likely to be spun?

e Which number has a probability of 25%?



3 On a die, what is the probability of rolling: 11.02

a a 1?

b an odd number?

c a number less than 5?

4 A box of chocolates contains 15 dark chocolates, 13 milk chocolates and 12 white chocolates. Markos selects one chocolate at random. What is the probability that he has chosen a white chocolate? Select the correct answer **A**, **B**, **C** or **D**. 11.02

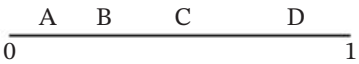
A $\frac{1}{28}$ **B** $\frac{1}{12}$ **C** $\frac{3}{10}$ **D** $\frac{7}{10}$

5 A truck carries 325 boxes of exercise books, 210 boxes of rulers, 360 boxes of paper and 145 boxes of pens. If a box is taken at random from the truck, what is the probability (as a percentage correct to one decimal place) that the box contains: 11.02

a rulers? **b** no pens?

c exercise books or paper? **d** erasers?

6 Where on the probability scale is the chance that the next baby born at the local hospital is a girl? Select **A**, **B**, **C** or **D**. 11.03



7 A jar contains 1 red, 6 yellow, 2 white and 5 black jellybeans. If a jellybean is selected at random, find the probability that it is: 11.03

a yellow **b** white or red **c** blue **d** not green

8 Which word best describes an event that has a probability of 0.29? Select **A**, **B**, **C** or **D**. 11.03

A unlikely **B** highly probable

C almost impossible **D** good chance

Foundation Standard Complex

11.03

9 Which event below could be described as a certain event? Select **A**, **B**, **C** or **D**.

A Choosing a 4 from a hat containing the numbers 1, 2, 3 and 4

B Choosing a red ball from a bag of green balls

C Rolling an odd or even number on a die

D A new baby being a girl

11.04

10 A coin was tossed 80 times and landed heads 24 times.

a What is the theoretical probability of obtaining a head?

b What is the expected frequency of heads from 80 tosses of a coin? How does the observed frequency compare to this?

c What is the relative frequency of obtaining a head?

YEAR 8 EXTENSION

11.05

11 Write the complementary event for each event, and its probability.

a Selecting a queen from a deck of cards.

b Rolling a multiple of 3 on a die.

c Buying the winning ticket out of 550 tickets sold.

11.05

12 The names of 5 players; Ashleigh, Hamish, Sarah, Jake and Hayley are written on separate cards and put into a bag. One name is drawn at random from the bag. What is the probability that the name selected:

a is Hamish? **b** is a girl's name? **c** does not begin with H?

11.05

13 There are 8 brown marbles, 4 black marbles and 3 white marbles in a bag. One marble is taken at random from the bag. What is the probability that the marble selected is not brown?

Foundation Standard Complex

12

NUMBER, MEASUREMENT

Ratios and time

Ancient Greek mathematicians and artists discovered a 'perfect' rectangle whose dimensions were most naturally pleasing to the eye. Try drawing an ideal rectangle yourself, one that is neither too thin nor too tall. Such a rectangle has a length that is $\frac{\sqrt{5}-1}{2}$ times its width. This value, which is approximately 1.618, is called the **golden ratio** and a rectangle with these dimensions is called a **golden rectangle**. Is your rectangle's length 1.6 times its width?

The ancient Greeks used the golden ratio in art and architecture. The Parthenon in Athens was built to the dimensions of the golden rectangle. Even today, the golden rectangle is often used in graphic design and publishing, such as on website panels and sidebars.

Getty Images/Nobilis Bellator

Chapter outline

	Proficiencies				
12.01 Ratios	U	F		R	C
12.02 Simplifying ratios	U	F		R	C
12.03 Ratio problems	U	F	PS	R	
12.04 Travel graphs	U	F	PS	R	C
12.05 Graphs of change	U	F		R	C
12.06 Time calculations	U	F			
12.07 24-hour time	U	F			C
12.08 Extension: Time differences*	U	F	PS	R	C
12.09 Timetables	U	F	PS	R	C
12.10 Extension: World time zones*	U	F	PS	R	C

*YEAR 8 EXTENSION

U = Understanding
F = Fluency
PS = Problem solving
R = Reasoning
C = Communication

Wordbank

24-hour time Time of day written using 4 digits (instead of a.m. and p.m.), for example, 18:20

ratio A relationship between 2 or more quantities measured in the same units;
for example, the ratio of 3 teachers to 40 students is 3 : 40

speed A rate that compares distance travelled with time taken

travel graph A line graph that represents a journey, with distance on the vertical axis and time on the horizontal axis

unitary method A method of finding a quantity by finding one part first

Quiz
Wordbank 12

Videos (9):

12.01 Why do we have ratios?
12.02 Equivalent ratios • Simplifying ratios
12.03 Ratio problems • Ratio problems
12.04 Travel graphs
12.06 Time
12.07 24-hour time
12.08 Time differences

Twig videos (3):

12:02 Ratios: The maths of baking
12:03 Ratios: Currency exchange
12.10 Time zones

PhET interactives (2):

12.01 Ratio and proportion
12.03 Proportion playground

Quizzes (6):

- Wordbank 12
- SkillCheck 12
- Mental skills 12A
- Mental skills 12B
- Language of Maths 12
- Test Yourself 12

Skillsheets (3):

12.07 Units of time • Telling the time • 24-hour time

Worksheets (14):

12.02 Ratios match • Simplifying ratios
12.04 Travel graph stories • The hare and the tortoise
12.06 Converting time units
12.07 TV times • 12- and 24-hour time • 24-hour time on an analogue clock
12.08 Time • Time calculations • Australian times
12.09 Tide chart
12.10 World time zones

Mind map: Ratios and time

Puzzles (5):

12.02 Simplifying ratios
12.04 Jane's diary
12.06 Converting time units
12.07 12- and 24-hour time
12.08, 12.10 Australian times

Presentation (1):

12.08 Time calculations

Nelson MindTap

To access resources above, visit **cengage.com.au/nelsonmindtap**

12 In this chapter you will:

- ✓ compare quantities using ratios
- ✓ simplify ratios and solve problems involving ratios
- ✓ interpret and draw travel graphs, including the slope of the graph
- ✓ interpret and draw informal graphs that describe a situation, such as a curve that represents the temperature over a day
- ✓ add and subtract time mentally and using a calculator
- ✓ convert between 12-hour time and 24-hour time
- ✓ solve problems involving time differences
- ✓ interpret and use timetables
- ✓ (YEAR 8 EXTENSION) interpret and use international time zones

SkillCheck

ANSWERS ON P. 646

Quiz SkillCheck 12

1 Copy and complete:

a 2000 kg = _____t **b** 360 cm = _____m **c** 18 g = _____mg
d 300 s = _____min **e** 6.5 cm = _____mm **f** 480 min = _____h
g 7500 mL = _____L **h** 8.15 km = _____m **i** 850 mm = ____cm

2 Find the highest common factor (HCF) of each pair of numbers.

a 12 and 18 **b** 8 and 36 **c** 20 and 64

3 Complete each pair of equivalent fractions.

a $\frac{7}{10} = \frac{\square}{30}$ **b** $\frac{3}{5} = \frac{12}{\square}$ **c** $\frac{18}{45} = \frac{\square}{5}$

4 Simplify each fraction.

a $\frac{6}{24}$ **b** $\frac{40}{50}$ **c** $\frac{9}{36}$

Ratios 12.01

A **ratio** consists of 2 or more numbers that compare the parts or shares of things of the same type, in the same units. For example, if a cake recipe uses sugar and flour in a ratio of 1 to 2, written '1 : 2,' it means that for every one part of sugar we need 2 parts of flour.

Each number in a ratio is called a **term** of the ratio.

Video Why do we have ratios?

Interactive Ratio and proportion

Ratios

A **ratio** compares quantities of the same type given in the same units.

We say 'a to b' and write $a : b$, where a and b are numbers.

Jackson mixes cordial and water in the ratio 1 : 5, meaning that for every 1 part of cordial there are 5 parts of water.

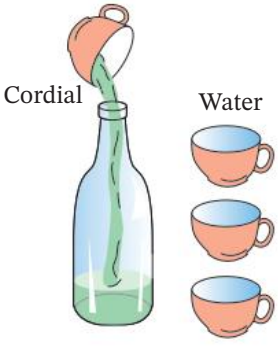
Saba prefers a stronger flavour, so she uses a cordial to water ratio of 1 : 3.

For a picnic, Saba can make a bottle of her drink by mixing one **cup** of cordial to 3 **cups** of water.

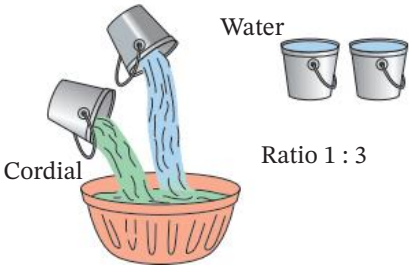
For a party, Saba can make a tub of her drink by mixing 1 **bucket** of cordial to 3 **buckets** of water.

1 : 5 (read as '1 to 5')
1 part cordial to 5 parts water

1 : 3 (read as '1 to 3')
1 part cordial to 3 parts water



Ratio 1 : 3



Ratio 1 : 3

Example 1

For this box of chocolates, write each ratio.

iStock.com/serts

a Dark brown chocolates to light brown chocolates

b Rectangular chocolates to circular chocolates

c White-topped chocolates to all chocolates

SOLUTION

a Dark chocolates to light chocolates = 4 : 9 — 4 dark brown, 9 light brown

b Rectangular chocolates to circular chocolates = 5 : 9 — 5 rectangular, 9 circular

c White-topped chocolates to all chocolates = 4 : 17 — 4 white, 17 in total

Example 2

Express each pair of quantities as a ratio.

a A mass of 69 kg to a mass of 80 kg

b A distance of 37 cm to a distance of 1 metre

SOLUTION

a Since the quantities are in the same units (kg) the ratio is simply 69 : 80.

69 kg : 80 kg = 69 : 80

b Change 1 metre to 100 centimetres so that both quantities are in the same units.

37 cm : 1 m = 37 cm : 100 cm

= 37 : 100

EXERCISE 12.01 ANSWERS ON P. 647

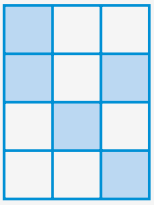
Ratios

U F R C

EXAMPLE 1

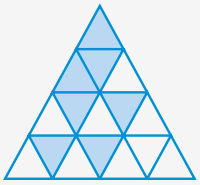
1 For each diagram, write the ratio of shaded to unshaded parts.

C **a**

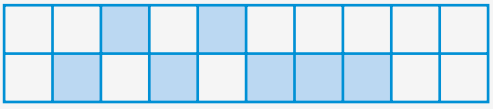


b

c



d



e

2 **a** Joe likes to make a drink with a ratio of 2 parts cordial to 5 parts water (or 2 : 5). Draw a diagram to show this ratio in a glass.

b Melanie made a drink mixed with the ratio shown in the diagram. Estimate the ratio of cordial to water used.

3 In your classroom, find the ratio of:

C **a** teachers to students **b** girls to boys

c chairs to tables **d** students to chairs

4 In a class of 25 students, Hadid found that 13 students have brown eyes, 8 students have blue eyes, 3 students have green eyes and 1 student has hazel eyes. Find the ratios between the student eye colours described.

C **a** blue to brown **b** green to blue

c brown to hazel **d** hazel to the total

5 Out of every 100 people at the football, 39 were men, 23 were women, and the rest were children. Find each ratio.

C **a** men to women **b** women to the total

c children to men **d** women to children

EXAMPLE 2

6 Express each pair of quantities as a ratio.

C **a** 13 g to 25 g **b** \$17 to \$100 **c** 5 litres to 12 litres

d 150 km to 1 km **e** 0.5 kg to 43 g **f** \$3.50 to 83 cents

g 3 kL to 1421 L **h** 27 mm to 4 cm **i** 59 minutes to 3 hours

j 15 years to 7 months **k** 9 days to 7 weeks **l** 77 m to 17 cm

Foundation Standard Complex

7 What is 39 kg to 41 g expressed as a ratio? Select the correct answer **A**, **B**, **C** or **D**.

C **A** 39 : 41 **B** 41 : 39 **C** 39 : 41 000 **D** 39 000 : 41

8 Write each statement as a ratio.

C **a** For each teacher there were 25 students.

b To cook steamed rice, you need 1 cup of rice to 2 cups of water.

c Adventure cycling activities in schools require one teacher for every 4 students.

d There are 11 girls for every 9 boys in the school.

e No matter how much they won on Lotto, Khloe and Kim agreed to share the prize equally.

f She was twice as tall as he was.

9 In the 1950s, the average ratio of teachers to students in classrooms was 1 : 45, in the 1970s it was 1 : 35, and in the 1980s the ratio was set at 1 : 30 for most classes. Today, it is 1 : 24. What does it mean when we say that the teacher to student ratio has improved?

R C

DID YOU KNOW?

Ratios in many careers

Whatever career you follow, the chances are you will need to use ratios.

- A hairdresser mixes hair colour using a ratio of 1 part colour and 1.5 parts developer
- A chef uses ratios to mix the ingredients for a sauce
- In sports analysis, win/loss ratios or improvement ratios are calculated to judge performance
- A plumber uses ratios to calculate water flow and pipe size
- A financial analyst uses ratios involving investment and profit/loss to evaluate a company's performance
- Builders use ratios when mixing mortar or concrete
- Paint colours are mixed with white paint and different ratios of colours

Find out what ratio of metals is used to make Australian coins.

12.02 Simplifying ratios

Worksheets
Ratios match
Simplifying ratios

Video
Ratios: The maths of baking

Equivalent ratios are *equal* ratios and can be found in a similar way to that used for finding **equivalent fractions**.

Equivalent ratios

To find an equivalent ratio, multiply or divide each term by the same number.

Foundation Standard Complex

For example, 2 : 7 and 10 : 35 are equivalent ratios.

$$2 : 7 \overset{\times 5}{\underset{\times 5}{=}} 10 : 35$$

Example 3

Complete each pair of equivalent ratios.

a 6 : 4 = 18 : _____ **b** 3 : 4 = _____ : 24 **c** 12 : 8 = 3 : _____

SOLUTION

a To find the missing term, look at the 2 known matching terms, 6 and 18.

6 is *multiplied* by 3 to give 18, so do the same thing to the 4.

$4 \times 3 = 12$

So 6 : 4 = 18 : 12

$$6 : 4 \overset{\times 3}{\underset{\times 3}{=}} 18 : ?$$

b The 2 known matching terms are 4 and 24.

4 is *multiplied* by 6 to give 24, so do the same thing to the 3.

$3 \times 6 = 18$

So 3 : 4 = 18 : 24

$$3 : 4 \overset{\times 6}{\underset{\times 6}{=}} ? : 24$$

c The 2 known matching terms are 12 and 3.

12 is *divided* by 4 to give 3, so do the same thing to the 8.

$8 \div 4 = 2$

So 12 : 8 = 3 : 2

$$12 : 8 \overset{\times 4}{\underset{\times 4}{=}} 3 : ?$$

Video Equivalent ratios

Simplifying ratios

- To **simplify** a ratio, keep dividing both terms by the same number, preferably a large number such as their highest common factor (HCF), until each term is as small as possible
- If all terms are even, divide by 2 or perhaps 4
- Otherwise, try dividing by the odd numbers 3, 5 or 7

Puzzle Simplifying ratios

Example 4

Simplify each ratio.

a 5 : 10 **b** 27 : 18 **c** 12 : 20 : 16

SOLUTION

a $5 : 10 = \frac{5}{5} : \frac{10}{5}$ Divide both terms by their HCF, 5

$= 1 : 2$

Video Simplifying videos

Or enter 5 : 10 as a fraction $\frac{5}{10}$ on the calculator: 5 [▭/▭] 10 [=]

On some calculators, use the [a b/c] key instead of [▭/▭]

b $27 : 18 = \frac{27}{9} : \frac{18}{9}$ Divide both terms by their HCF, 9

$= 3 : 2$

Or enter 27 : 18 as an improper fraction $\frac{27}{18}$ on the calculator:

27 [▭/▭] 18 [=]

Then to change the mixed numeral to an improper fraction, press: [SHIFT] [S⇔D] or [2nd F] [a b/c]

c $12 : 20 : 16 = \frac{12}{4} : \frac{20}{4} : \frac{16}{4}$ Divide all terms by their HCF, 4

$= 3 : 5 : 4$

Because there are more than 2 terms in this ratio, the calculator cannot be used here.

Example 5

Simplify 10 hours : 1 day.

SOLUTION

10 hours : 1 day = 10 hours : 24 hours Change to the same units

$= 10 : 24$

$= \frac{10}{2} : \frac{24}{2}$

$= 5 : 12$

EXERCISE 12.02 ANSWERS ON P. 647

Simplifying ratios

U F R C

EXAMPLE 3

1 Copy and complete each pair of equivalent ratios.

a 2 : 3 = 8 : _____ **b** 1 : 5 = 2 : _____ **c** 3 : 5 = _____ : 15

d 4 : 7 = _____ : 35 **e** 5 : 8 = 20 : _____ **f** 7 : 12 = 49 : _____

g 5 : 11 = _____ : 66 **h** 3 : 4 = _____ : 100 **i** 2 : 1 = 10 : _____

j _____ : 9 = 20 : 36 **k** 12 : _____ = 3 : 1 **l** _____ : 45 = 6 : 9

m 24 : 12 = 4 : _____ **n** 16 : _____ = 2 : 5 **o** _____ : 20 = 15 : 60

2 Which ratio is not equivalent to 32 : 48? Select the correct answer **A**, **B**, **C** or **D**.

A 16 : 24 **B** 4 : 6 **C** 2 : 3 **D** 6 : 8

Foundation Standard Complex

EXAMPLE 4

3 Simplify each ratio.

a 10 : 100	**b** 8 : 16	**c** 12 : 30	**d** 35 : 49
e 18 : 12	**f** 56 : 24	**g** 1000 : 100	**h** 45 : 99
i 7 : 7	**j** 20 : 48	**k** 56 : 14	**l** 3 : 36
m 8 : 12 : 20	**n** 15 : 20 : 30	**o** 27 : 9 : 36	**p** 9 : 6 : 24 : 12

4 Simplify 8 months : 2 years. Select **A**, **B**, **C** or **D**.

A 8 : 2 **B** 4 : 1 **C** 1 : 3 **D** 2 : 5

EXAMPLE 5

5 Simplify each ratio.

a 80 cm : 2 m	**b** 300 g : 1.2 kg	**c** 6 days : 4 weeks
d 45 min : 2 hours	**e** 70c : $2.10	**f** 2 years : 15 months
g 15 hours : 2 days	**h** 20 mm : 1 m	**i** 4 t : 350 kg
j 250 mL : 3 L	**k** 1800 m : 1 km	**l** 8 kg : 400 g
m 2 min : 8 s	**n** 750 L : 5 kL	**o** $18 : $4.50

6 Simplify 4 m to 25 cm. Select **A**, **B**, **C** or **D**.

A 4 : 25 **B** 400 : 25 **C** 8 : 5 **D** 16 : 1

7 (C) The following amounts of books were sold at a garage sale: 48 crime novels, 20 science fiction novels, 35 non-fiction books, 28 romance novels.

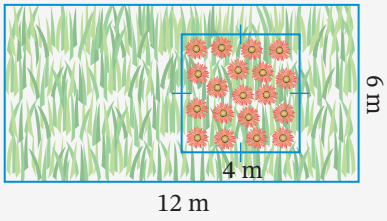
Express each ratio in simplest form:

a crime to romance	**b** sci-fi to non-fiction
c non-fiction to romance	**d** sci-fi to crime
e romance to non-fiction	**f** crime to all books

8 (C) A backyard contains a square garden, as shown. What is the ratio of garden area to backyard area? Select **A**, **B**, **C** or **D**.

A 2 : 7 **B** 2 : 9

C 7 : 2 **D** 9 : 2



9 (R) (C) At Westvale Catholic College there are 72 teachers and 1440 students. Of these, 624 students are girls.

a Find each ratio.

i teachers : students **ii** boys : girls **iii** boys : teachers

b The government states that the teacher to student ratio must be 1 : 25 or higher. Does the college have enough teachers? Give reasons for your answer.

A higher ratio means more teachers compared to students.

Foundation Standard Complex

10 When comparing the rate at which babies are born in different countries (that have different population sizes), we use a base number of 1000. The annual birth rate in Australia is approximately 12 per 1000 people (a ratio of 12 : 1000). The rate in Pakistan is approximately 30 per 1000 people, or 30 : 1000. Express each ratio in simplest form. (C)

11 Tom and Ahmed bought a scooter together, but Tom paid $70 and Ahmed paid $55. Write the ratio of their shares of the scooter in simplest form. (C)

12 To make a 2 L jug of cordial drink, 375 mL of cordial is mixed with water. Find: (C)

a the amount of water used

b the ratio of cordial to water

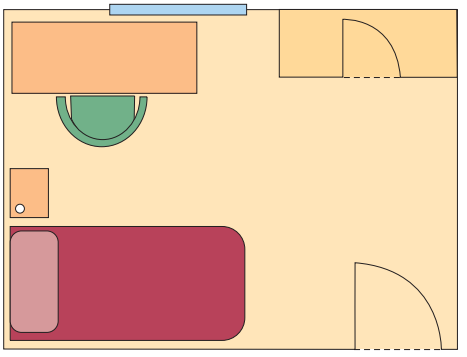
c the ratio of cordial to drink

13 Three Year 7 classes were surveyed to find the ratio of left-handed to right-handed students. In 7N, the ratio was 1 : 5, in 7P the ratio was 2 : 3, and in 7D the ratio was 1 : 6. Which class had the highest proportion of left-handed students? Justify your answer. (R) (C)

INVESTIGATION

Scale diagrams

Here is a scale plan of a bedroom. The scale ratio is 1 : 50, meaning that 1 cm on the plan represents 50 cm on the actual bedroom. The actual bedroom is 50 times larger.



1 By measuring the plan, find in metres:

a the width of the bedroom

b the length of the bedroom

2 Calculate:

a the perimeter of the bedroom in metres

b the area of the bedroom in square metres

3 By measuring the plan, find in centimetres:

a the length of the bed

b the width of the desk

c the width of the door

d the length of the window

Foundation Standard Complex

Ratio problems

12.03

Problems involving ratios can be solved using **equivalent ratios** or the **unitary method**. With the unitary method, we find the size of **one part** first.

Example 6

The ratio of boys to girls in a Year 7 class is 2 : 3. If there are 10 boys in the class, how many girls are there?

SOLUTION

Method 1: Equivalent ratios

Write the problem as a pair of equivalent ratios.

$$\text{Boys : girls} = 2 : 3 \overset{\times 5}{=} 10 : \underline{\quad\quad}$$

2 is multiplied by 5 to give 10, so do the same thing to 3.

Number of girls $= 3 \times 5 = 15$

There are 15 girls in the class.

Method 2: Unitary method

Boys : girls = 2 : 3

2 parts (boys) = 10

1 part $= 10 \div 2 = 5$

'Unitary' means 'one'

Finding one part first

3 parts (girls) $= 3 \times 5 = 15$

There are 15 girls in the class.

Videos
Ratio problems

Ratio problems

Ratios: Currency exchange

Interactive
Proportion playground

Example 7

To make heavy duty concrete, a builder mixes sand and cement in the ratio 5 : 4. If a mix of concrete contains 24 kg of cement, find:

a the amount of sand in the mix

b the total mass of the mix.

SOLUTION

a **Method 1: Equivalent ratios**

4 is multiplied by 6 to give 24, so do the same thing to 5.

$$\text{sand : cement} = 5 : 4 \overset{\times 6}{=} \underline{\quad\quad} : 24$$

Amount of sand $= 5 \times 6 = 30$ kg

Method 2: Unitary method

Sand : cement = 5 : 4

4 parts (cement) = 24 kg

1 part = 24 ÷ 4 = 6 kg

5 parts (sand) = 5 × 6 = 30 kg

b Total mass = 24 kg + 30 kg = 54 kg

So the total mixture was 54 kg.

Cement and sand

EXERCISE 12.03 ANSWERS ON P. 647

Ratio problems

U F PS R

EXAMPLE 6

1 In Year 7, the ratio of boys to girls is 4 : 5. If there are 64 boys in Year 7, find the number of girls.

2 Simon and Joshua's heights are in the ratio 8 : 9. If Joshua is 171 cm tall, how tall is Simon?

3 In a rectangle, the ratio of the width to the length is 5 : 12. If the length of the rectangle is 48 cm, find the width.

4 A car dealer finds that the ratio of sedans to hatchbacks sold last week was 15 : 4. If 105 sedans were sold, how many hatchbacks were sold?

EXAMPLE 7

5 PS A tiler uses 4 green tiles to every 3 white ones. How many tiles are used altogether if 140 green tiles are used?

6 PS When making concrete for the foundations of buildings, sand and cement are mixed together in the ratio 4 : 1. If 120 kg of cement has been delivered, what mass of sand is needed?

7 PS Alison and Elena buy a length of material and divide it between them in the ratio 2 : 3. If Alison has 3.6 m, what length of material does Elena have? Select the correct answer **A**, **B**, **C** or **D**.

A 1.8 m **B** 2.4 m **C** 5.4 m **D** 9 m

8 PS An alloy contains copper and iron in the ratio 2 : 5. A quantity of alloy contains 20 kg of iron. What mass of copper does it contain?

9 PS The ratio of adults to children at the cinema was 3 : 4. If there were 96 children, how many adults are there?

10 PS Enzo's Produce Store buys fruit and vegetables in the ratio 5 : 7. The mass of fruit is 8.5 tonnes. What is the total mass of produce ordered?

Foundation Standard Complex

11 Mai, Bree and Felicity share a Lotto prize in the ratio 10 : 8 : 7. Bree received \$72.

PS **a** How much did Mai and Felicity each receive?

R **b** What was the total prize money shared?

12 In a triangle, the lengths of the sides are in the ratio 3 : 4 : 5. If the longest side is 30 cm long, find the perimeter of the triangle.

PS R

13 In an outback mining town, the ratio of women to men is 2 : 5. If there are 240 women, how many men and women are in the town altogether? Select **A**, **B**, **C** or **D**.

PS R

A 840 **B** 600 **C** 960 **D** 1680

14 To make Superglue, the contents of Tube A and Tube B are mixed in the ratio 3 : 1.

PS **a** If 15 mL of Tube A is used, how much of Tube B is needed?

b How much glue is made altogether if 15 mL of Tube B is used?

15 The heights of 2 buildings are in the ratio 7 : 6. If the shorter building is 150 m tall, how high is the taller building?

16 To make lighter concrete for bricklaying, Jo mixes 1 part cement to 2 parts gravel to 4 parts sand. If she uses 50 kg of gravel, how much cement and sand does she need?

PS R

17 Flour and butter are mixed in the ratio 3 : 1 to make short crust pastry. If Adam makes 800 g of pastry, how much flour does he need?

PS R

18 Sonya and Xanthe share a prize of \$495 in the ratio of 4 : 5. How much prize money does each person receive?

PS R

☆ MENTAL SKILLS 12A ANSWERS ON P. 647 Maths without calculators

Quiz
Mental skills 12A

Multiplying by 5, 15, 25 and 50

It is easier to multiply a number by 10 than by 5, so to multiply by 5, we can halve the number, then multiply by 10. This is because $\frac{1}{2} \times 10 = 5$. We can use a similar strategy to multiply by 15, 25 and 50.

1 Study each example.

a To multiply by 5, halve the number, then multiply by 10.

$18 \times 5 = 18 \times \frac{1}{2} \times 10$ (or factorise 18 into 9×2)

$= 9 \times 10$

$= 90$

Foundation Standard Complex

b To multiply by 50, halve the number, then multiply by 100.

$26 \times 50 = 26 \times \frac{1}{2} \times 100$ (or factorise 26 into 13×2)

$= 13 \times 100$

$= 1300$

c To multiply by 25, quarter the number, then multiply by 100.

$44 \times 25 = 44 \times \frac{1}{4} \times 100$ (or factorise 44 into 11×4)

$= 11 \times 100$

$= 1100$

d To multiply by 15, halve the number, then multiply by 30.

$8 \times 15 = 8 \times \frac{1}{2} \times 30$ (or factorise 8 into 4×2)

$= 4 \times 30$

$= 120$

2 Now simplify each product.

a	32×5	**b**	14×5	**c**	48×5	**d**	18×50
e	52×50	**f**	36×25	**g**	44×50	**h**	12×25
i	12×15	**j**	16×25	**k**	22×50	**l**	14×15
m	28×25	**n**	36×50	**o**	26×15	**p**	22×35

12.04 Travel graphs

Puzzle
Jane's diary

Worksheets
Travel graph stories

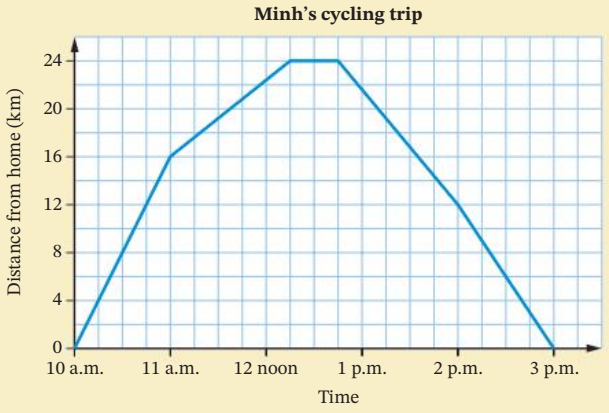
The hare and the tortoise

Video
Travel graphs

A **travel graph** or **distance–time graph** is a line graph that describes a journey, by comparing distance with time (on the vertical and horizontal axes respectively). The slope or steepness of the graph indicates speed (how fast you are travelling).

Example 8

This graph shows Minh's cycling trip.



a At what time did Minh leave home?

b When did Minh stop? How far from home was he?

c Find his speed in km/h:

i in the first hour **ii** in the second hour

d How is a change in speed shown on the graph?

e When did Minh start returning home?

f How long did it take him to return home?

SOLUTION

a Minh left home at 10 a.m. — At the start of the graph, when distance = 0

b He stopped at 12:15 p.m., 24 km from home. — Where the graph is flat

Where the graph is flat, the distance from home does not change, which means that Minh has stopped

c **i** In the first hour, Minh travelled from 0 km to 16 km, so his speed was 16 km/h.

ii In the second hour, Minh travelled from 16 km to 22.5 km.

Distance travelled = 22.5 km – 16 km = 6.5 km.

So his speed was 6.5 km/h.

d A change in speed is shown by a change in the steepness of the graph. The graph is steeper in the first hour than in the second hour because the speed is greater in the first hour.

Where the graph is steeper, the speed is greater: more distance travelled in less time

e Minh started returning home at 12:45 p.m. — Where the graph goes downward

f He reached home at 3 p.m.

Time to return home = 3 p.m. – 12:45 p.m. = $2\frac{1}{4}$ hours

Travel graphs

On a travel graph:

- a horizontal (flat) section on the graph indicates a stop
- the steeper the line, the greater the speed (more distance covered in less time)
- a section going down, towards the right, indicates a change in direction or that the traveller is returning towards the start.

EXERCISE 12.04 ANSWERS ON P. 647

Travel graphs

U F PS R C

EXAMPLE 8

1 This travel graph shows Thorald's return trip to Emil's house.

R C

a At what time did Thorald begin his trip?

b How far is Emil's house from Thorald's house?

c How long did it take Thorald to walk to Emil's house?

d Use the formula

$$\text{Speed} = \frac{\text{Distance travelled}}{\text{Time taken}}$$

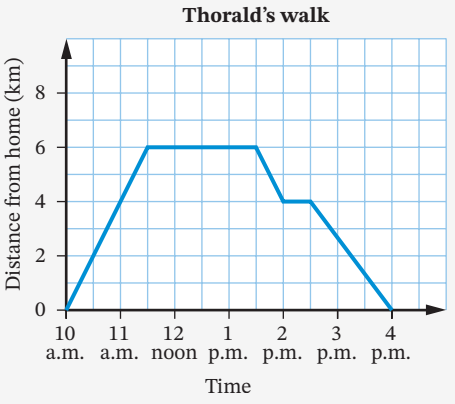
to calculate his speed in km/h during this time.

e How long did Thorald spend at Emil's house?

f What was the total distance that Thorald travelled?

g When and at what distance did Thorald stop on his way home?

h What was the total time of his trip, including rests?



2 This graph shows Shauna's cycling trip.

R C

a When was her first stop? How far from home was she?

b Calculate her speed in the first 2 hours.

c When did Shauna start to return home?

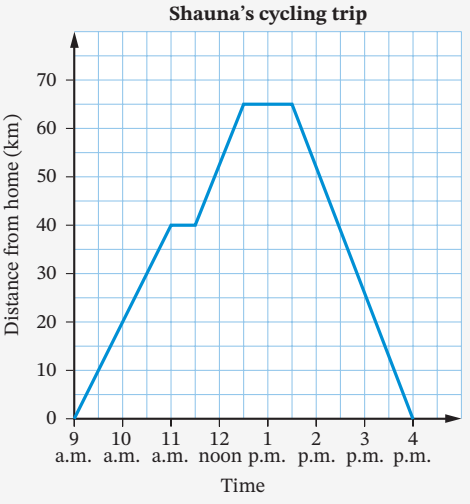
d How long did she take to return home?

e Calculate her speed for her journey home.

f How far from home was Shauna at 3 p.m.?

g How far did Shauna travel altogether on this day?

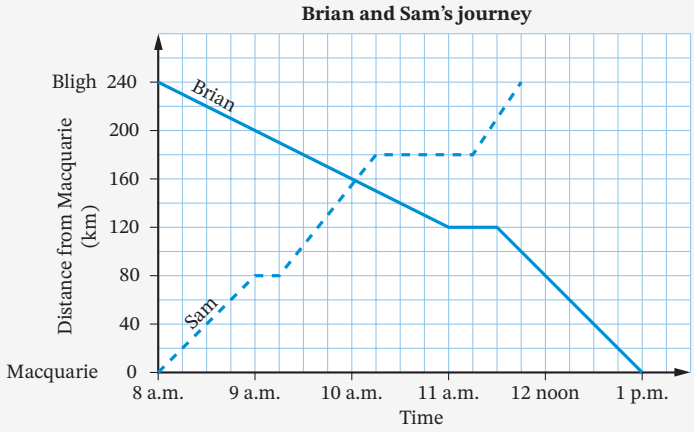
h Did Shauna travel faster between 9 a.m. and 11 a.m., or between 11:30 a.m. and 12:30 a.m.? How does the graph illustrate this?



Foundation Standard Complex

3 Brian travels from Bligh to Macquarie, while Sam travels from Macquarie to Bligh.

R C



a How far is it between the 2 towns?

b How far is Sam from Macquarie at 10:30 a.m.?

c When does Sam reach Bligh?

d At what speed is Brian travelling before he stops?

e When do Brian and Sam pass each other? How far are they from Macquarie when they pass?

f Is Brian or Sam travelling faster during the day? How can you tell?

g Is Brian's speed lower before 11 a.m. or after 11:30 a.m.? How can you tell?

h For how long did Sam stop altogether on the trip? Select **A**, **B**, **C** or **D**.

A 60 minutes **B** 75 minutes **C** 5 minutes **D** 90 minutes

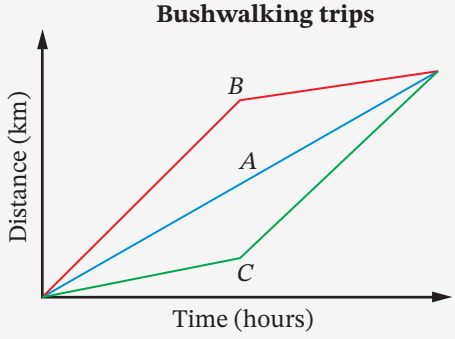
4 Match each bushwalker described below with the correct travel graph.

R C

a Karlie was fast at first, but then got slower after lunch.

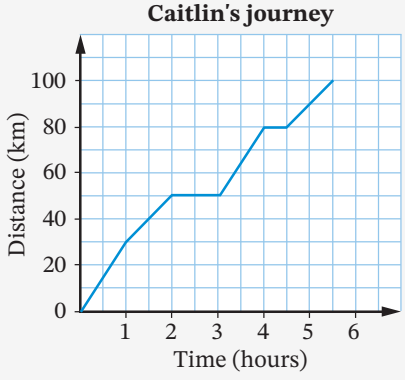
b Luka kept a constant speed all day.

c Tomasi was slow at first, but got faster after lunch.



5 This travel graph shows Caitlin's cycling journey. Write a story about her ride, based on the information in the graph.

C



Foundation Standard Complex

4 Jack goes on a trip to visit his grandparents.

PS R C

a Draw a travel graph to represent Jack's journey:

- Jack starts at 9 a.m.
- He travels 180 km in 2 hours.
- He stops for $\frac{1}{2}$ an hour.
- He travels 120 km in $1\frac{1}{2}$ hours.
- He stops at his grandparents' house for 6 hours.
- It takes Jack 3 hours to travel back home.

b How far from Jack's home do his grandparents live?

c At what time does Jack arrive there?

d In which part of the journey is Jack travelling the fastest?

e How long does the journey home take?

f What is the total time taken for the trip?

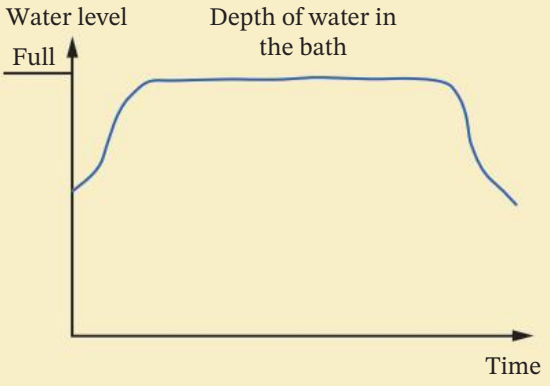
g How far from home is Jack at 9 p.m.?

12.05 Graphs of change

Some graphs are less formal, and show how something changes over time, as a curve rather than a line.

Example 9

This graph shows the level of water in a bathtub from before to after Jayden had a bath.



a Why does the water level not start at zero?

b Why might the water level be lower after Jayden's bath than before he got in?

c Describe the change in water level over time.

Foundation Standard Complex

SOLUTION

a The water level does not start at zero as the bath is already partly filled with water when Jayden gets into it.

b The water level might be lower because some water spilled out or was on Jayden when he got out of the bath.

c The water level is a little over half full at the start. When Jayden gets into the bath, the water level increases until it is close to full. The water level then stays the same while Jayden is in the bath. Finally, it drops rapidly to a level below the starting water level after Jayden gets out of the bath.

EXERCISE 12.05 ANSWERS ON P. 648

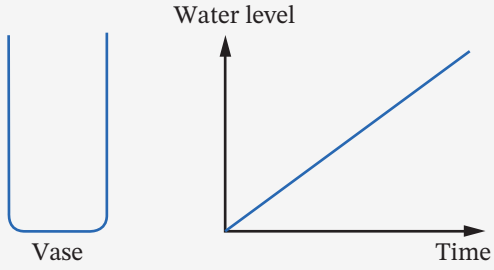
Graphs of change

U F R C

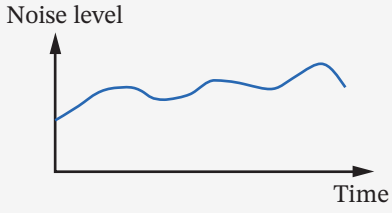
EXAMPLE 9

1 The graph below shows the water level changing over time in the vase on the left. Describe what is happening.

R C



2 This graph shows the noise level in a classroom.



What is the correct description of the noise level in this classroom? Select **A**, **B**, **C** or **D**.

A The class starts off quite noisy and becomes quiet as the lesson goes on

B The class is quiet for the whole lesson

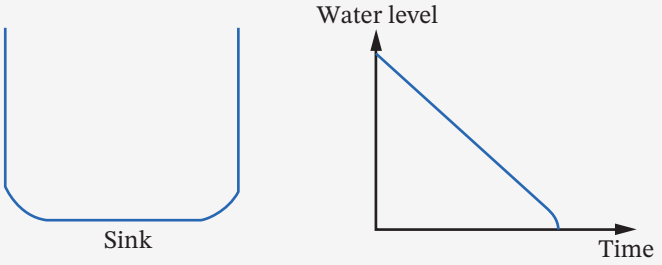
C The class starts off noisy, then becomes louder, then quieter, then louder, then a little quieter, then noisy and quieter again

D The class is noisy for the whole lesson

Foundation Standard Complex

3 The graph below shows the water level changing in the sink on the left.

R
C



a Describe what is happening.

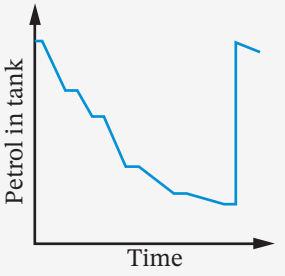
b What does the dip at the end of the graph show about the water level?

4 This graph shows the amount of petrol in Ashwin's car over a 6-hour road trip.

R
C

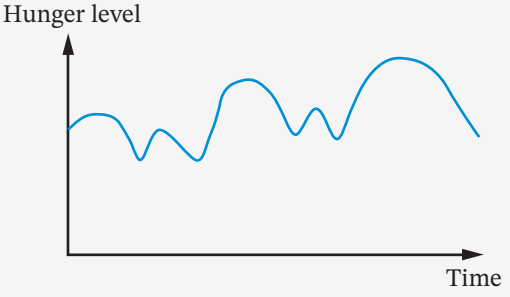
a What do the flat parts of the graph represent?

b Write a story to describe what's happening in the graph.



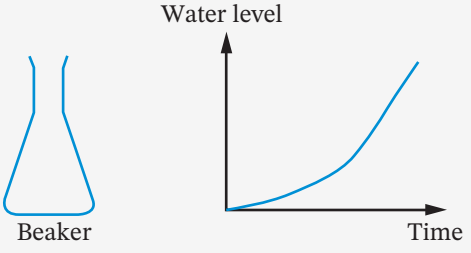
5 This graph describes Hannah's level of hunger over the day. Write a story that describes her hunger level from start to finish.

R
C



6 The graph below shows the water level rising in the beaker on the left as it is being filled. Notice that it is not a straight line but a curve.

R
C

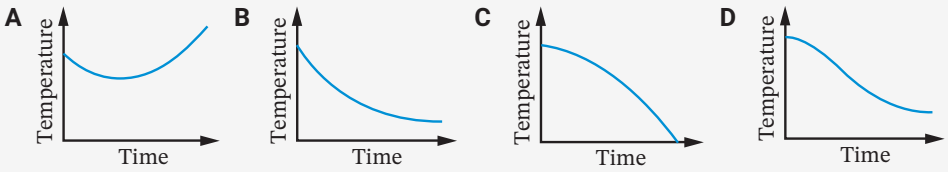


a When is the water level rising the fastest, at the start or at the end? Why?

b When is the water level rising the slowest? Why?

Foundation Standard Complex

7 A bowl of hot soup sits on the dinner table, as time goes by it starts to cool, at first quite quickly and then more slowly until it reaches room temperature. Which of the following graphs best illustrates this?

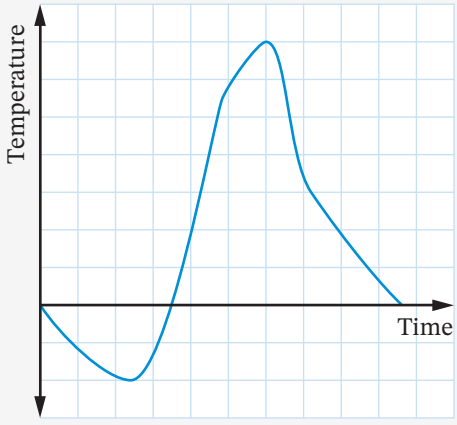


8 This graph shows the temperature in Canberra over a 24-hour period.

R C

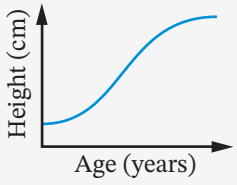
a What was the temperature at the start and end of the period?

b Write a story to describe what's happening in the graph.



9 This graph shows Riley's height from birth to 18 years. Describe how his height has changed over time.

R C



Time calculations 12.06

Example 10

a Round each amount of time to the nearest hour.

i 7.83 h **ii** 12 h 19 min **iii** 2 h 43 min 6 s

b Round each amount of time to the nearest minute.

i 11.4 min **ii** 25 min 37 s **iii** 3 h 6 min 30 s

Worksheet Converting time units

Puzzle Converting time units

Video Time

Foundation Standard Complex

SOLUTION

a **i** 7.83 h ≈ 8 h

ii 12 h 19 min ≈ 12 h — Rounding down because 19 min < 30 min

When rounding to the nearest hour, use 30 minutes as the halfway mark because there are 60 minutes in an hour.

iii 2 h 43 min 6 s ≈ 3 h — Rounding up because 43 min > 30 min

b **i** 11.4 min ≈ 11 min

ii 25 min 37 s ≈ 26 min — Rounding up because 37 s > 30 s

When rounding to the nearest minute, use 30 seconds as the halfway mark because there are 60 seconds in a minute.

iii 3 h 6 min 30 s ≈ 3 h 7 min — Rounding up 30 s

Example 11

Complete each conversion.

a 3 years = _____ months

b 91 days = _____ weeks

c 275 min = _____ h _____ min

SOLUTION

a 3 years = 3 × 12 months — 1 year = 12 months
= 36 months

b 91 days = (91 ÷ 7) weeks — 1 week = 7 days
= 13 weeks

c 275 min = (275 ÷ 60) h
= 4.58333… h (or $4\frac{7}{12}$ h)
= 4 h 35 min — Shown on a calculator as 4°35’ after entering °’’’ or DMS

Most scientific calculators have a degrees-minutes-seconds key (°’’’ or DMS) that is useful for calculations involving minutes and seconds (base 60). This key converts decimal answers for time to hours-and-minutes or minutes-and-seconds.

Example 12

Simplify each expression.

a 9 h 25 min + 6 h 48 min

b 7 h 5 min – 3 h 24 min

SOLUTION

a 9 h 25 min + 6 h 48 min = (9+6) h + (25+48) min
= 15 h + 73 min
= 15 h + 1 h 13 min
= 16 h 13 min

OR On a calculator, enter 9 [°'''] 25 [°'''] [+] 6 [°'''] 48 [°'''] [=]

b 7 h 5 min − 3 h 24 min = (7−3) h + (5−24) min
= 4 h + (−19) min
= 3 h + (60−19) min
= 3 h 41 min

OR On a calculator, enter 7 [°'''] 5 [°'''] [−] 3 [°'''] 24 [°'''] [=]

EXERCISE 12.06 ANSWERS ON P. 648

Time calculations

U F

1 Round each time to the nearest hour. EXAMPLE 10

a 1 h 28 min **b** 20.7 h **c** 5 h 11 min
d 3 h 40 min 18 s **e** 9.15 h **f** 10 h 30 min

2 Round time to the nearest minute.

a 21 min 43 s **b** 7.5 min **c** 8 min 8 s
d 2 h 15 min 30 s **e** 16.31 min **f** 4 h 20 min 39 s

3 Copy and complete each conversion. EXAMPLE 11

a 5 weeks = _____ days **b** $2\frac{1}{2}$ years = _____ months
c 3 days = _____ h **d** 2 years = _____ weeks
e 1 week = _____ h **f** 3 fortnights = _____ days
g 96 h = _____ days **h** 480 s = _____ min
i 84 days = _____ weeks **j** 60 months = _____ years
k 60 h = _____ days **l** 56 days = _____ fortnights

4 Calculate the number of seconds in:

a one hour **b** one day **c** one week

5 Convert each time to hours and minutes.

a 340 min **b** 186 min **c** 227 min
d 405 min **e** 262 min **f** 499 min + 105 min

Foundation Standard Complex

6 Are you over a billion seconds old? Find your age in seconds to answer this question.

7 Convert each time to minutes and seconds.

a 372 s **b** 545 s **c** 238 s

d 422 s **e** 500 s **f** 216 s + 308 s

EXAMPLE 12

8 Simplify each expression.

a 2 h 15 min + 4 h 32 min **b** 9 h 20 min + 3 h 47 min

c 6 h 36 min – 2 h 11 min **d** 10 h 31 min – 7 h 6 min

e 5 h 12 min + 1 h 48 min **f** 3 h 45 min + 3 h 29 min

g 12 h 32 min – 8 h 19 min **h** 4 h 10 min – 42 min

i 7 h 46 min + 2 h 17 min **j** 5 h 33 min + 4 h 18 min

k 8 h 15 min – 6 h 24 min **l** 2 h 29 min – 1 h 40 min

DID YOU KNOW?

Minutes and seconds

In Chapter 2, *Angles*, you learnt that there are 360° in a revolution because the ancient Babylonians used a base 60 number system and believed that a year lasted 360 days. The Babylonians, who lived where Iraq is today, invented the units for measuring angles and time in 2000 BCE. That is why there are 60 minutes in an hour and 60 seconds in a minute.

The word 'minute' has another meaning. When pronounced '*my-newt*', it means tiny, but this meaning is also related to the minute as a unit of time. A minute is a tiny fraction of an hour, and comes from the Latin '*pars minuta prima*', meaning the first division (or part) of an hour.

Find out how the 2 meanings of the word 'second' are similarly related.

12.07 24-hour time

Skillsheets
Units of time
Telling the time
24-hour time

There are 2 ways to show the time of day:

- **12-hour time**: the conventional way using a.m. and p.m. and the hours 1 to 12
- **24-hour time**: a more formal way that uses the hours 0 to 23 and no a.m./p.m.

24-hour time is commonly used around the world, especially in the armed forces, navigation, in transport timetables and hospital records. This table shows the relationship between 12-hour time and 24-hour time.

Foundation Standard Complex

24-hour time	12-hour time
00:00	12 a.m. (midnight)
01:00	1 a.m.
02:00	2 a.m.
03:00	3 a.m.
04:00	4 a.m.
05:00	5 a.m.
06:00	6 a.m.
07:00	7 a.m.
08:00	8 a.m.
09:00	9 a.m.
10:00	10 a.m.
11:00	11 a.m.

24-hour time	12-hour time
12:00	12 p.m. (midday)
13:00	1 p.m.
14:00	2 p.m.
15:00	3 p.m.
16:00	4 p.m.
17:00	5 p.m.
18:00	6 p.m.
19:00	7 p.m.
20:00	8 p.m.
21:00	9 p.m.
22:00	10 p.m.
23:00	11 p.m.

Worksheets
TV times

12- and 24-hour time

24-hour time on an analogue clock

Puzzle
12- and 24-hour time

24-hour time to 12-hour time

- If it begins with '00', then it is the hour after 12:00 a.m. (midnight)
- If it begins with '12', then it is the hour after 12:00 p.m. (midday)
- If it is less than 12:00, then it is a.m. (morning time)
- If it is more than 13:00, then it is p.m. (afternoon/evening) time: subtract 12 from the hour.

Example 13

Convert each 24-hour time to 12-hour time.

a 10:20 **b** 21:30 **c** 00:50 **d** 12:40

SOLUTION

a 10:20 < 12:00, so it is a.m. time.
10:20 = 10:20 a.m.

b 21:30 > 13:00, so it is p.m. time.
Subtract 12 from the hours: 21 – 12 = 9
21:30 = 9:30 p.m.

c 00:50 begins with 00, so it is 12:00 a.m. time.
00:50 = 12:50 a.m.

d 12:40 begins with 12, so it is 12:00 p.m. time.
12:40 = 12:40 p.m.

12-hour time to 24-hour time

- If it is the hour after 12:00 a.m., begin with '00'
- If it is a.m. time or the hour after 12:00 p.m., write as is but make sure the hour has 2 digits (for example, 02, 09)
- If it is after 1:00 p.m., then add 12 to the hour

Video
24-hour time

Example 14

Convert each 12-hour time to 24-hour time.

a 8:15 a.m. **b** 2:40 p.m. **c** 12:20 p.m.

SOLUTION

a 8:15 a.m. is a.m. time, so write as a 4-digit number and rename 8 as 08.

8:15 a.m. = 08:15

b 2:40 p.m. is after 1:00 p.m., so add 12 to the hour: 2 + 12 = 14

2:30 p.m. = 14:30

c 12:20 p.m. is in the hour after 12:00 p.m., so write it as a 4-digit number.

12:20 p.m. = 12:20

EXERCISE 12.07 ANSWERS ON P. 648

24-hour time

U F C

EXAMPLE 13

1 Convert each 24-hour time to 12-hour time.

C **a** 08:45 **b** 13:20 **c** 17:50 **d** 00:17
e 21:05 **f** 18:32 **g** 11:15 **h** 02:38
i 14:40 **j** 03:20 **k** 16:55 **l** 23:31
m 01:08 **n** 10:45 **o** 20:00 **p** 06:43

EXAMPLE 14

2 Convert each 12-hour time to 24-hour time.

C **a** 6:35 p.m. **b** 8:05 a.m. **c** 11:44 a.m. **d** 11:20 p.m.
e 2:51 a.m. **f** 12:30 p.m. **g** 3:38 p.m. **h** 7:11 a.m.
i 9:08 a.m. **j** 9:54 p.m. **k** 12:42 a.m. **l** 7:39 p.m.
m 1:59 a.m. **n** 10:18 p.m. **o** 8:46 a.m. **p** 5:23 p.m.

3 Write the times shown on each clock in 12-hour time.

C **a** 11:23 **b** 17:54 **c** 13:24

4 Write the times shown on each clock in 24-hour time.

C **a** AM PM 5:17 **b** AM PM 10:06 **c** AM PM 2:45

Foundation Standard Complex

5 This plane flight timetable shows flights from eastern Australia to Christchurch in New Zealand.

	Depart	Arrive
Sydney – Christchurch	08:55	13:55
	17:10	22:10
	19:45	00:45
Melbourne – Christchurch	08:45	14:05
	09:05	14:25
	17:55	23:15
Brisbane – Christchurch	07:50	14:15
	16:30	22:55

Write the answers to each question in 12-hour time.

a When does the first flight leave Sydney and when does it arrive in Christchurch?

b When does the last flight leave Melbourne and when does it arrive in Christchurch?

c At what time between 12 midnight and 1 a.m. does a flight arrive in Christchurch?

d What is the earliest departure time?

e What is the latest departure time? Select the correct answer **A**, **B**, **C** or **D**.

A 4:30 p.m. **B** 5:10 p.m.

C 5:55 p.m. **D** 7:45 p.m.

☆ MENTAL SKILLS 12B ANSWERS ON P. 648 Maths without calculators

Quiz
Mental skills 12B

Dividing by 5, 15, 25 and 50

It is easier to divide a number by 10 than by 5, so to divide by 5, we can divide by 10, then double the answer. We can use a similar strategy to divide by 15, 25 and 50.

1 Study each example.

a To divide by 5, divide by 10 and double the answer. We do this because there are 2 5s in every 10.

$140 \div 5 = 140 \div 10 \times 2$

$= 14 \times 2$

$= 28$

b To divide by 50, divide by 100 and double the answer. This is because there are 2 50s in every 100.

$400 \div 50 = 400 \div 100 \times 2$

$= 4 \times 2$

$= 8$

Foundation Standard Complex

c To divide by 25, divide by 100 and multiply the answer by 4. This is because there are 4 25s in every 100.

$600 \div 25 = 600 \div 100 \times 4$
$= 6 \times 4$
$= 24$

d To divide by 15, divide by 30 and double the answer. This is because there are 2 15s in every 30.

$240 \div 15 = 240 \div 30 \times 2$
$= 8 \times 2$
$= 16$

2 Now simplify each quotient.

a	$90 \div 5$	**b**	$170 \div 5$	**c**	$230 \div 5$	**d**	$330 \div 5$
e	$1300 \div 50$	**f**	$800 \div 50$	**g**	$400 \div 50$	**h**	$2200 \div 50$
i	$300 \div 25$	**j**	$1000 \div 25$	**k**	$700 \div 25$	**l**	$600 \div 15$
m	$270 \div 15$	**n**	$360 \div 45$	**o**	$900 \div 25$	**p**	$7000 \div 50$

YEAR 8 EXTENSION

12.08 Extension: Time differences

Video Time differences

Presentation Time calculations

Worksheets Time

Time calculations

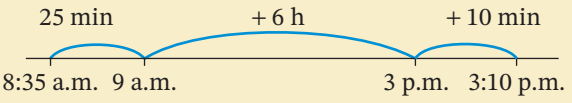
Puzzle Australian times

Example 15

What is the difference in time between 8:35 a.m. and 3:10 p.m.?

SOLUTION

Method 1



From 8:35 a.m. to 9:00 a.m. = 25 minutes

From 9:00 a.m. to 3:00 p.m. = 6 hours

From 3:00 p.m. to 3:10 p.m. = 10 minutes

Total time difference = 6 h + 25 min + 10 min
= 6 h 35 min

Method 2

Convert to 24-hour time first, then use the calculator's [°'"] or [DMS] key to enter hours and minutes, and subtract the times.

Time difference = 3:10 p.m. − 8:35 a.m.
= 15:10 – 08:35 15 [°'"] 10 [°'"] [−] 8 [°'"] 35 [°'"] [=]
= 6 h 35 min

Example 16

What is the difference in time between 10:50 and 19:30?

SOLUTION

Method 1

From 10:50 to 11:00 = 10 min

From 11:00 to 19:00 = 8 h $19 - 11 = 8$

From 19:00 to 19:30 = 30 min

Total time difference = 8 h + 10 min + 30 min

= 8 h 40 min

Method 2

Use the calculator's °''' or DMS key to enter hours and minutes, and subtract the times.

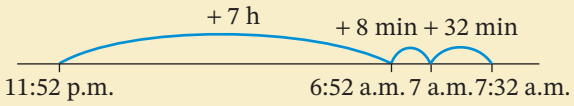
Example 17

What is the time 7 hours and 40 minutes after 11:52 p.m.?

SOLUTION

7 hours after 11:52 p.m. is 6:52 a.m.

40 minutes after 6:52 a.m. is 7:32 a.m.



Video
Time differences

Example 18

a Ms Hossain bought a new car in August 2016. How old is her car in March 2022?

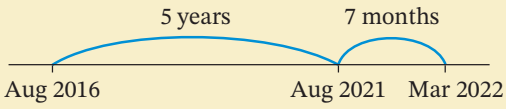
b Boris was born on 14 April 2007. How old is he on 29th September 2021?

SOLUTION

a From August 2016 to August 2021 = 5 years

From August 2021 to March 2022 = 7 months

Age of car = 5 years 7 months

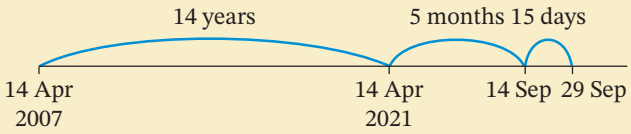


b From 14/4/07 to 14/4/21 = 14 years

From 14/4/21 to 14/9/21 = 5 months

From 14/9/21 to 29/9/21 = 15 days

Boris' age = 14 years 5 months 15 days



YEAR 8 EXTENSION

EXERCISE 12.08 ANSWERS ON P. 648

Time differences

U F PS R C

EXAMPLE 15

1 What is the difference in time between 10:42 a.m. and 2:13 p.m.? Select the correct answer **A**, **B**, **C** or **D**.

A 3 h 31 min
B 4 h 55 min
C 8 h 29 min
D 12 h 55 min

2 Calculate the time difference between:

a 7:15 p.m. and 8:20 p.m.
b 10:16 a.m. and 12:06 p.m.
c 4:09 a.m. and 9:53 a.m.
d 11:15 p.m. and 3:08 a.m.
e 7:27 a.m. and 1:12 p.m.
f 9:36 p.m. and 9:14 a.m.
g 7:45 p.m. and 10:10 p.m.
h 2:24 a.m. and 3:07 a.m.
i 4:15 p.m. and 6:02 p.m.
j 10:25 a.m. and 2:33 p.m.
k 8:40 a.m. and 4:19 p.m.
l 6:45 a.m. and 8:10 p.m.

EXAMPLE 16

3 Calculate the time difference between:

a 08:00 and 11:00
b 05:00 and 15:00
c 04:50 and 19:30
d 06:30 and 10:45
e 09:40 and 14:55
f 11:20 and 20:05
g 05:10 and 12:15
h 07:40 and 13:10
i 12:45 and 21:25
j 13:40 and 21:50
k 10:35 and 16:30
l 09:45 and 18:25

EXAMPLE 17

4 What time will it be:

a 5 hours after 3:00 p.m.?
b 8 hours after 11:00 a.m.?
c 28 minutes after 7:15 p.m.?
d 3 hours 32 minutes after 9:45 a.m.?
e 3 hours 19 minutes after 10:49 p.m.?
f 4 hours after 9:32 a.m.?
g 9 hours after 5:14 p.m.?
h 45 minutes after 3:30 p.m.?
i $2\frac{1}{4}$ hours after 4:02 a.m.?
j 12 hours 40 minutes after 2:45 a.m.?

5 A marathon began at 10:20 a.m. Here are 6 of the competitors and the times they ran. Write the runners in their order of finishing and the time each crossed the finishing line.

R C

Mike 3:11 (3 h 11 min)
Joe 2:23
Anna 2:54
Pathena 3:01
Ken 2:59
Gail 3:42

6 A plane left Perth at 09:00 and arrived in Sydney at 13:45 (Perth time). How long was the journey?

7 At Westvale Islamic College, the school day begins at 8:45 a.m. and ends at 3:10 p.m. How long is the school day?

Foundation Standard Complex

YEAR 8 EXTENSION

12.08

EXAMPLE 18

8 What time will it be:

a 4 hours before 6:15 p.m.?
b 7 hours before 9:30 a.m.?
c 45 minutes before 3:20 p.m.?
d 2 hours 10 minutes before 1:35 p.m.?
e 3 hours 30 minutes before 11:25 a.m.?
f 1 hour 25 minutes before 7:40 p.m.?

9 Jamie bought a computer in June 2019. How old is it in:

R **a** August 2023?
b January 2024?

10 Darika was born on 15th March 2013. How old is she in years, months, days on:

R **a** 20th April 2019?
b 9th September 2028?

11 A movie starts at 1:40 p.m. and ends at 3:55 p.m. What is the length of the movie?

12 A box of chocolates bought in November 2021 has a 'Use by' date of March 2023. How long will it last?

13 Mr Badger started teaching at Westvale High School in February 1998. How long had he taught at the school for in December 2015?

14 PS Karen works at the office from 8:10 a.m. till 4:30 p.m. How much will she earn for the day if her hourly wage is $18.60?

15 PS R South Australia is $1\frac{1}{2}$ hours ahead of Western Australia. Anne is flying from Perth to Port Augusta in South Australia. If the flight takes $2\frac{1}{2}$ hours and it leaves Perth at 10 a.m., what will the local time be in Port Augusta when the plane lands?

16 R Will and Kate were married on 29th April 2011. How long will they have been married in years, months and days on 10th May 2025?

Foundation Standard Complex

12.09 Timetables

Worksheet
Tide chart

Example 19

This part of a timetable is a schedule for a bus travelling between Sydney and Wagga Wagga.

Sydney to Wagga Wagga		Wagga Wagga to Sydney	
Sydney	2:30 p.m.	Wagga Wagga	7:15 a.m.
Strathfield	3:00 p.m.	Gundagai	8:25 a.m.
Yagoona	3:20 p.m.	Jugiong	8:54 a.m.
Liverpool	3:45 p.m.	Yass	9:41 a.m.
Mittagong	4:40 p.m.	Goulburn*	10:41 a.m.
Goulburn*	5:40 p.m.	Mittagong	12:10 p.m.
Yass	7:10 p.m.	Liverpool	1:05 p.m.
Jugiong	7:55 p.m.	Yagoona	1:20 p.m.
Gundagai	8:20 p.m.	Strathfield	1:35 p.m.
Wagga Wagga	9:30 p.m.	Sydney	2:05 p.m.

* 30 minute meal stop at Goulburn

a At what time does the bus from Sydney arrive in Goulburn?

b Where is this bus at 8:20 p.m.?

c How long does it take to travel from Liverpool to Mittagong?

d Tim is waiting at Yass at 9:15 a.m. for the bus to Sydney. How long will he have to wait before the bus arrives?

SOLUTION

a The bus from Sydney arrives in Goulburn at 5:40 p.m.

b At 8:20 p.m., the bus arrives in Gundagai.

c From Liverpool at 3:45 p.m. to Mittagong at 4:40 p.m., the journey takes 55 minutes.

d Tim needs to wait from 9:15 a.m. to 9:41 a.m., a total of 26 minutes.

EXERCISE 12.09 ANSWERS ON P. 649

Timetables

U F PS R C

EXAMPLE 19

1 Refer to the bus timetable from the previous example.

PS **a** How long does the trip from Sydney to Wagga Wagga take?

R **b** How long would the trip take without a meal break?

c At what time does the bus from Sydney arrive in Yagoona?

d Ali joins the return bus at Jugiong and gets off at Liverpool. How long is his trip?

e How long does Cherie have to wait for the bus to Sydney if she arrives in Strathfield at 1:22 p.m.?

f Find the time taken to travel from Liverpool to Sydney and from Sydney to Liverpool. Suggest a reason for the difference.

2 Chris' volleyball team flies from Sydney to Brisbane for a tournament. Chris visited the NelsonAir website and found the following daily flight schedule.

PS R C

Flight no.	Sydney	Brisbane
TH503	09:05	10:30
TH511	09:35	11:00
TH038	10:05	11:30
TH114	10:40	12:10
TH514	11:05	12:30
TH051	11:35	13:00

a When does flight TH511 leave Sydney and how long is the journey?

b What and when is the latest flight before 11 a.m.?

c The team plans to meet at Sydney airport at 9:40 a.m. How long will they need to wait for the next available flight?

d The team needs to be at Sydney airport 40 minutes before a flight takes off. When should they be at the airport to catch the TH038 flight?

e The team needs to be at their hotel in Brisbane by 12:30 p.m. If it takes 30 minutes to drive from the airport to the hotel, what is the latest flight the team can catch from Sydney?

f Which flight takes longer to reach Brisbane than the others? Give one reason why it might take longer.

Foundation Standard Complex

3 This section of a train timetable shows the Goulburn to Sydney run.

PS R C

Goulburn to Sydney — Monday to Friday								
	am	**am**	**am**	**pm**	**pm**	**pm**	**pm**	**pm**
GOULBURN	5:08	7:27	8:17	1:47	2:45	4:26	6:47	7:45
MARULAN	5:26	7:45			3:03			8:03
TALLONG	5:32	7:51	Bookings	Bookings	3:09	Bookings	Bookings	8:09
WINGELLO	5:39	7:58	essential	essential	3:16	essential	essential	8:16
PENROSE	5:44	8:03			3:21			8:21
BUNDANOON	5:50	8:09	8:52	2:22	3:27		7:22	8:27
EXETER	5:55	8:14			3:32			8:32
MOSS VALE	6:05	8:24	9:05	2:35	3:42	5:13	7:35	8:42
BURRADOO	6:10				3:47			8:47
BOWRAL	6:13	8:30	9:11	2:41	3:50		7:41	8:50
MITTAGONG	6:17	8:34	9:16	2:46	3:54		7:46	8:54
YERRINBOOL	6:30				4:07			9:07
BARGO	6:41				4:18			9:18
TAHMOOR	6:48				4:25			9:25
PICTON	6:56	9:08			4:33			9:33
CAMPBELLTOWN	7:23	9:30	10:11	3:41	5:00	6:13	8:42	10:00
STRATHFIELD			10:42	4:17		7:00	9:12	
SYDNEY	8:12	10:12	10:54	4:29	6:20	7:13	9:24	11:04

a Georgina travels from Penrose to Yerrinbool, arriving at 4:07 p.m. How long did the trip take?

b Michael has an interview in Sydney on Tuesday at 10:45 a.m. At what time must he catch the train in Goulburn?

c What is the difference in the time taken to travel from Goulburn to Sydney on the 5:08 a.m. train and the 8:17 a.m. train?

d You have been visiting friends in Moss Vale and are returning to Sydney. Decide which train you would catch and explain why.

e A new train is added to the timetable, leaving Goulburn at 11:12 a.m. Write out a timetable for this train if it stops at the same stations as the 6:47 p.m. train.

Foundation Standard Complex

4 An Explorer Bus is a shuttle bus that takes tourists on a tour of a city and allows them to visit places of interest.

PS R C

Explorer depot	10:00	10:25	10:50	11:15	11:45	12:00	12:25	12:50	1:15
City cathedral	10:08	10:33	10:58	11:23	11:53	12:08	12:33	12:58	1:23
Railway station	10:15	10:40	11:05	11:30	12:00	12:15	12:40	1:05	1:30
Parliament	10:24	10:49	11:14	11:39	12:09	12:24	12:49	1:14	1:39
Museum	10:35	11:00	11:25	11:50	12:20	12:35	1:00	1:25	1:50
City square	10:45	11:10	11:35	12:00	12:30	12:45	1:10	1:35	2:00
Zoo	11:00	11:25	11:50	12:15	12:45	1:00	1:25	1:50	2:15
Town Hall	11:12	11:37	12:02	12:27	12:57	1:12	1:37	2:02	2:27
Arts centre	11:19	11:44	12:09	12:34	1:04	1:19	1:44	2:09	2:34
Water gardens	11:30	11:55	12:20	12:45	1:15	1:30	1:55	2:20	2:45
Hall of fame	11:38	12:03	12:28	12:53	1:23	1:38	2:03	2:28	2:53
Explorer depot	11:50	12:15	12:40	1:05	1:35	1:50	2:15	2:40	3:05

a The bus leaves the railway station at 11:30 a.m. When will it arrive at the Arts centre and how long will the journey take?

b Harrison needs to be at the Hall of fame by 2:30 p.m. What is the latest time he needs to get to Parliament to achieve this?

c Vo, Binh and Vicki came to the city by train, arriving at the station at 11:42 a.m. They caught the Explorer Bus to the zoo. What is the earliest time they could expect to arrive at the zoo?

d What is the smallest number of buses needed to meet this Explorer Bus timetable? Explain how you arrived at your answer.

e Manuel and Sofia arrived at the City cathedral at 10:25 a.m. They arrange to meet their hosts at the Hall of fame at 2:45 p.m. They want to spend at least half an hour at the museum, photograph the City square and have lunch at the Town Hall. Plan a list of times for them to catch the Explorer Bus to do these things and meet their hosts on time.

f In summer, extra Explorer tours leave the depot at 11:30 a.m. and 1:30 p.m. Make a list of departure times that appear in the timetable for the 1:30 p.m. tour.

DID YOU KNOW?

World time zones

During the 19th century, when there was little world travel, cities and countries operated on local time. However, as shipping and business increased between different places around the world, the need for a standard time measurement arose. World time zones were first proposed by a Scottish Canadian engineer, Sir Sandford Fleming, in 1878.

Hourly time zones are based on longitude. 15° difference in longitude across the world equals 1 hour difference in local time. Time is measured in relation to the Greenwich Observatory near London. At the time, Britain was the leading country in shipping and the observatory produced high quality data. Sydney is 10 hours ahead of Greenwich, New York is 5 hours behind. This makes the time difference between Sydney and New York 15 hours.

Foundation Standard Complex

How is time measured at the North and South poles?

Shutterstock.com/Benson HE

12.10 Extension: World time zones

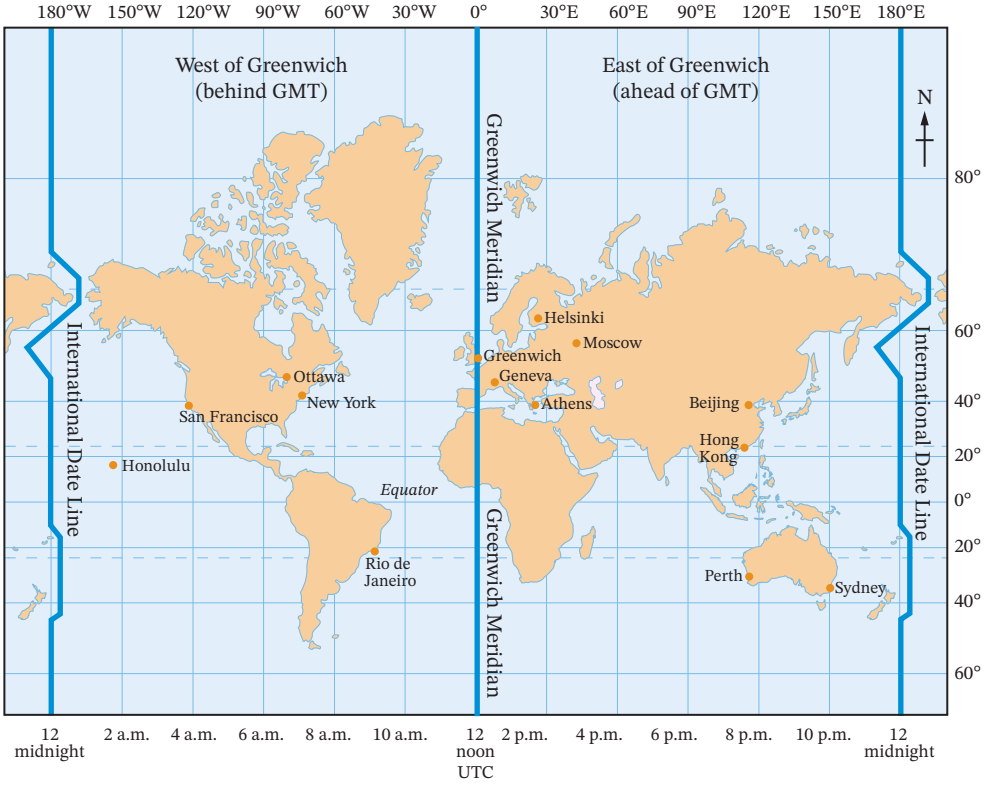
YEAR 8 EXTENSION

video Time zones

Worksheet World time zones

Puzzle Australian times

The world is divided into 24 hourly time zones. Time is the same throughout each zone. The centre of each time zone is a meridian of longitude (an imaginary line running from the North Pole to the South Pole). Each hourly time zone covers 15° of longitude.



The map above shows how times around the world are related. All time is measured in relation to the time at the Greenwich Observatory (near London), either ahead of or behind **UTC (Coordinated Universal Time)**, also known as **Greenwich Mean Time (GMT)**. Greenwich is pronounced 'Grennitch'. Time zones in Australia are ahead of UTC because Australia is east of Greenwich. Time zones in the USA are behind UTC because the USA is west of Greenwich.

EXERCISE 12.10 ANSWERS ON P. 649

World time zones

U F PS R C

1 Use the map above to determine whether each city is ahead of or behind UTC.

R

a Sydney	**b** San Francisco	**c** Rio de Janeiro	**d** Perth
e Beijing	**f** Honolulu	**g** Moscow	**h** Athens
i Hong Kong	**j** Helsinki	**k** New York	**l** Ottawa

2 Find the time in each city when it is 12 midday in Greenwich.

a Sydney	**b** Perth	**c** New York	**d** Beijing
e San Francisco	**f** Honolulu	**g** Moscow	**h** Geneva

3 What is the time difference between:

a Sydney and Perth?	**b** Sydney and Beijing?
c Sydney and Honolulu?	**d** Sydney and Moscow?
e Sydney and New York?	**f** Perth and Beijing?
g San Francisco and New York?	**h** Honolulu and Moscow?
i Geneva and Perth?	**j** San Francisco and Geneva?

4 If it is 2:00 p.m. in Sydney, what is the time in:

a Greenwich?	**b** Perth?	**c** New York?	**d** Beijing?
e San Francisco?	**f** Honolulu?	**g** Moscow?	**h** Geneva?

5 A cricket match being played in India is telecast live at 7 p.m. Sydney time. What is the local time of the cricket match if Sydney's time is $4\frac{1}{2}$ hours ahead of India's?

PS R

6 Simone, in Newcastle (NSW), wants to video call her cousin Zac in Vancouver, Canada. The time in Vancouver is 18 hours behind the time in Newcastle. At what time should Simone call to catch Zac when it is 3 p.m. in Vancouver?

PS R

7 Brisbane is 2 hours behind Auckland. A plane leaves Auckland at midday and takes 3 hours to fly to Brisbane. What is the local time in Brisbane when the plane lands? Select the correct answer **A**, **B**, **C** or **D**.

PS R

A 11 a.m.	**B** 1 p.m.	**C** 3 p.m.	**D** 5 p.m.

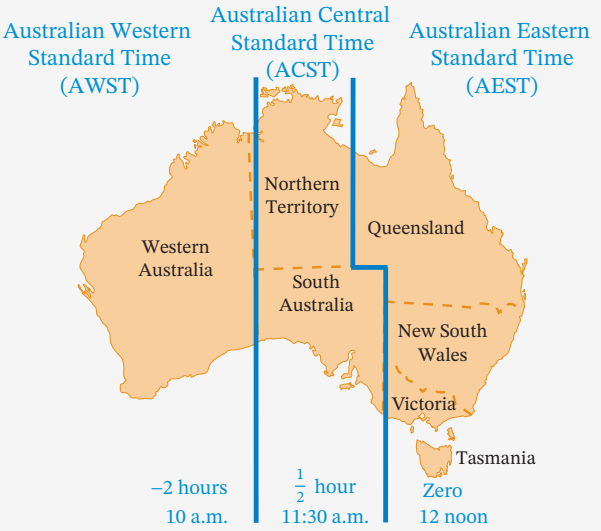
8 Find out what happens if you cross the International Date Line (IDL). Why isn't the IDL straight?

R C

Foundation Standard Complex

YEAR 8 EXTENSION

9 This map shows the 3 time zones for Australia.



State whether each city is ahead of, behind, or has the same time as Adelaide (SA).

a Sydney
b Melbourne
c Darwin
d Perth
e Mt Isa (Qld)
f Geraldton (WA)
g Cobar (NSW)
h Ceduna (SA)
i Cairns (Qld)

10 What is the time difference between:

a Sydney and Adelaide?
b Melbourne and Perth?
c Adelaide and Melbourne?
d Hobart and Darwin?
e Canberra and Perth?
f Brisbane and Canberra?

11 If it is 11 p.m. in Sydney, what time is it in:

a Melbourne?
b Adelaide?
c Perth?
d Darwin?
e Hobart?
f Canberra?

12 If it is 5:30 p.m. in Adelaide, what time is it in:

a Melbourne?
b Sydney?
c Perth?
d Darwin?
e Hobart?
f Brisbane?

13 PS R **a** Jason flies from Sydney to Perth, taking 4 hours. If he leaves Sydney at 2 p.m., at what time does he land in Perth? Give your answer as Perth local time.

b When Jason flies home, he leaves Perth at 9 a.m. At what time does he land in Sydney? Give your answer as Sydney local time.

Foundation Standard Complex

14 **a** Find out when daylight saving begins and ends in those states that practise it.

PS **b** Why do they have daylight saving?

R **c** How does daylight saving affect the different time zones?

C **d** If it is 12:30 p.m. in Western Australia (not on daylight saving), what time is it in New South Wales on Eastern Standard Daylight Saving Time?

INVESTIGATION

World trip

Plan a trip around the world with at least 3 stopovers (for example, Tokyo, Hanoi, Cairo). Use airline timetables so you can give details of departures, arrivals and the length of each flight. Does it matter if you head east or west when you start? What effect does the International Date Line have on your trip?

POWER PLUS ANSWERS ON P. 649

1 Simplify each ratio:

a 0.4 : 0.7 **b** 1.3 : 0.8 **c** 0.5 : 0.05

d 0.49 : 1.4 **e** $\frac{1}{3} : \frac{1}{8}$ **f** $\frac{3}{5} : \frac{1}{2}$

2 Kobe and Hashim renovated their gardens. Kobe planted 19 native plants and 14 non-native plants. Hashim put in 24 native plants and 20 non-native plants. Kobe believes that her garden is more Australian native than Hashim's. Is Kobe right? Explain your answer.

3 Angela goes on a trip from home to the shop.

a Draw a graph to represent Angela's journey:

- Angela starts at 7 a.m.
- She travels 5 km in 30 minutes.
- She stops for 15 minutes.
- She travels a further 2 km in 15 minutes.
- She stops at the shop for 30 minutes.
- She travels back home in 1 hour.

b Suggest a reason for the first stop of 15 minutes.

c When did Angela arrive home?

d Was her travelling speed going to the shop higher or lower than her speed coming back? Suggest a reason why.

4 Mae invents a decimal system of time, where 1 minute = 100 seconds and 1 hour = 100 minutes. One second is the same as our second, but decimal minutes and hours are different to ours.

a How many of our minutes will there be in a decimal hour?

b How many decimal hours will there be in one of our days?

Foundation Standard Complex

12 CHAPTER REVIEW

Language of maths

Quiz Language of maths 12

12-hour time	24-hour time	equivalent ratios
ratio	simplify	speed
steepness	term	timetable
travel graph	unitary method	

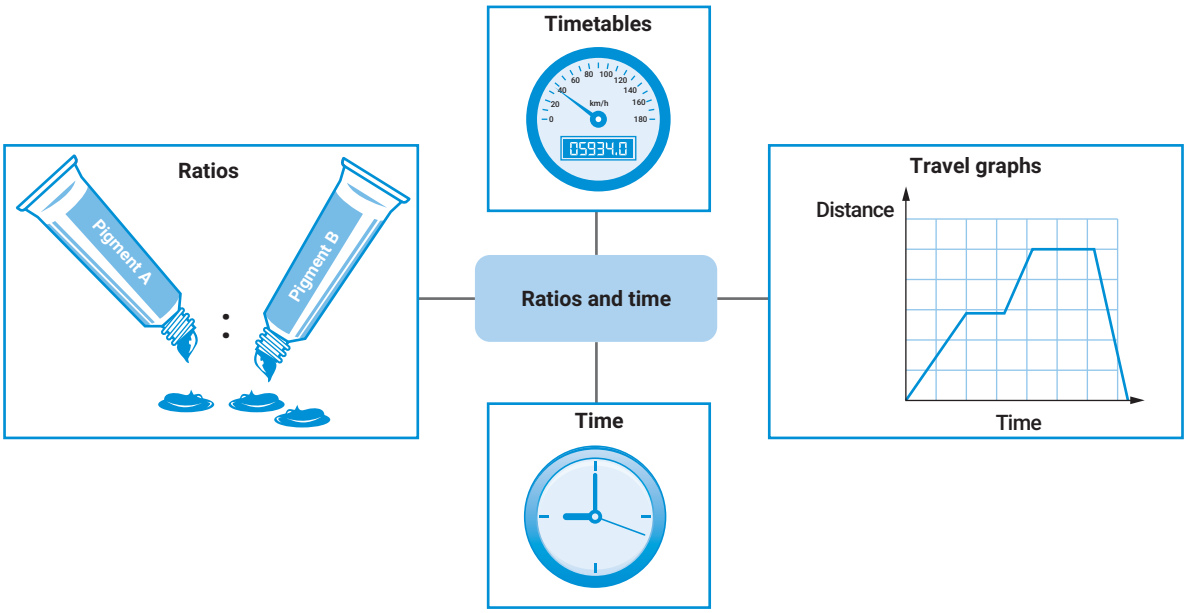
1. What are the numbers in a ratio called?
2. Which feature of a travel graph indicates the speed of the traveller?
3. Explain what **equivalent ratios** are and give an example.
4. How do you simplify a ratio?
5. Why does the 'unitary method' have that name?
6. What are the advantages of using **24-hour time**?

Topic summary

Worksheet Mind map: Ratios and time

- Give examples of places in which ratios or time are used.
- Give some examples of situations where you would use what you know about ratio and time.
- What parts of this chapter were new to you?
- What did you find most difficult about this topic? Discuss any problems with your teacher or with a friend.

Print (or copy) and complete this mind map of the topic, adding detail to its branches and using pictures, symbols and colour where needed. Ask your teacher to check your work.



9780170465557

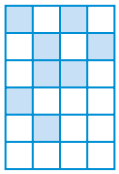
12 TEST YOURSELF

ANSWERS ON P. 649

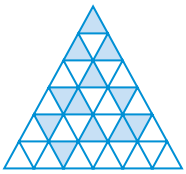
1 In each diagram, find the ratio of shaded parts to unshaded parts. 12.01

Quiz Test yourself 12

a



b



2 In a construction business, there are 17 women and 38 men. Find the ratio of: 12.01

a men to women **b** women to the total

3 Copy and complete each pair of equivalent ratios. 12.02

a 2 : 3 = 8 : ___ **b** 1 : 5 = 2 : ___ **c** 2 : 3 : 4 = 6 : ___ : ___

4 Simplify each ratio. 12.02

a	12 : 21	**b**	25 : 75	**c**	6 : 36
d	25 : 45	**e**	18 : 6 : 24	**f**	5 : 25 : 100
g	2 cm : 10 mm	**h**	5 dozen eggs : 6 eggs	**i**	3 L : 600 mL
j	50 m : 1 km	**k**	20 minutes : 1 hour	**l**	40 cents : $2
m	18 months : 3 years	**n**	12 days : 3 weeks	**o**	4 days : 10 hours

5 **a** The ratio of girls to boys in Year 7 is 4 : 3. If there are 75 boys in Year 7, find how many girls there are. 12.03

b Tyrone and Erin invested in a business in the ratio 5 : 7. If Erin invested $63 000, how much did Tyrone invest?

6 Keith and Kent decided to go bushwalking one day. This travel graph shows their journey. 12.04

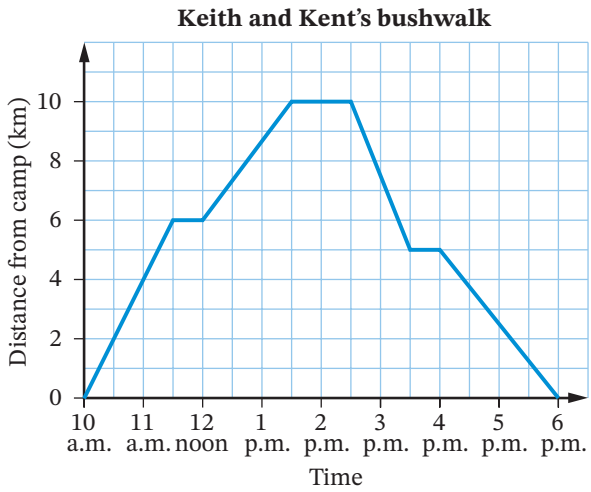
a How far did Keith and Kent walk?

b How many stops did they make?

c At what time did they start back?

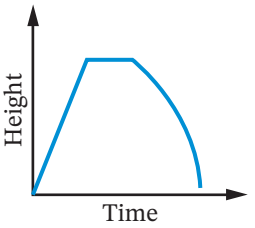
d Between what times were Keith and Kent walking the fastest?

e Calculate their speed in the last 2 hours.



Foundation Standard Complex

12.05 **7** Jess is at the playground and plays on the slide. This graph shows her height above the ground at different times.



a What does the flat part of the graph show?

b Why is the part of the graph where Jess slides down a curve?

c What does the straight line on the left part of the graph describe?

12.06 **8** Round each time to the nearest hour.

a 9 h 50 min **b** 3.2 h **c** 4 h 12 min 49 s

12.06 **9** Convert:

a 316 minutes to hours and minutes

b 208 seconds to minutes and seconds

12.06 **10** Simplify each expression.

a 6 h 45 min + 3 h 20 min **b** 3 h 16 min + 1 h 26 min

c 4 h 33 min – 2 h 24 min **d** 2 h 19 min – 1 h 50 min

12.07 **11** Convert to 12-hour time.

a 01:48 **b** 06:24 **c** 15:50 **d** 12:30

12.07 **12** Convert to 24-hour time.

a 2:10 p.m. **b** 5:33 a.m. **c** 12:19 a.m. **d** 1:50 p.m.

YEAR 8 EXTENSION

12.08 **13** Calculate the time difference between each pair of times.

a 5:25 a.m. and 9:45 a.m. **b** 11:55 p.m. and 7:30 a.m.

c 1:15 p.m. and 8:10 p.m. **d** 07:50 and 14:20

e 23:45 and 00:15 **f** 15:30 and 03:25

12.08 **14** Samantha was born on 3rd June 2004. Calculate her age in years, months and days on 16th August 2021.

12.09 **15** Refer to the train timetable in Question 2 of Exercise 12.09 on page 572.

a How long does the 7:45 p.m. train from Goulburn take to get to Sydney?

b What is the latest time you need to catch the train at Moss Vale to be in Sydney by 11 a.m.?

Foundation Standard Complex

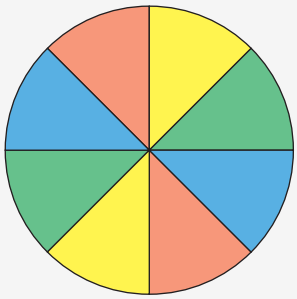
Practice set 4

ANSWERS ON P. 650

1 List the sample space for each experiment. 11.01

a Rolling a die

b Spinning this spinner



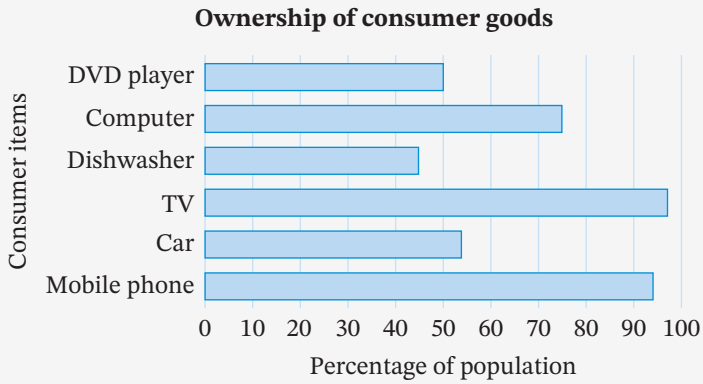
2 There are 18 girls in a class of 30 students. Write in simplest form the ratio of: 12.01

a girls to students

b girls to boys

c students to boys

3 This graph shows the percentage of people who owned various consumer items in the year 2011. 10.01



a What percentage of the population owned:

i a TV? **ii** a mobile phone? **iii** a DVD player?

b What item was owned by 45% of the population?

c Did more people own TVs or DVD players?

d What is the percentage difference between people owning a car and people owning a mobile phone?

YEAR 8 EXTENSION

4 How many hours and minutes between: 12.08

a 4:30 a.m. to 9:55 a.m.? **b** 7:15 p.m. to 8:25 p.m.?

c 9:24 p.m. to 9:05 a.m.? **d** 15:05 to 22:10?

Foundation Standard Complex

12.07 **5** Write in 24-hour time:

a 8:25 a.m. **b** 4:45 p.m.

12.07 **6** Write in 12-hour time:

a 21:15 **b** 04:30

10.03 **7** **a** Construct a dot plot for these marks for a test out of 25:

11	8	12	12	15	13	10	12	13	15	16
22	12	11	11	10	13	16	12	15	12	11

b Where are the marks clustered?

c Identify any outliers.

10.07 **8** Find the range of the marks in question **7**. Select the correct answer **A**, **B**, **C** or **D**.

A 8 **B** 12 **C** 14 **D** 22

12.02 **9** Simplify each ratio.

a 2 : 6 **b** 96 : 36 **c** 24 : 72

d 2 m to 15 cm **e** 10 minutes to 2 hours **f** 12 : 30 : 42

11.02 **10** A die is rolled. Find the probability of rolling:

a an even number **b** a number greater than 2.

12.06 **11** Copy and complete each conversion.

a 3 fortnights = _____ days

b 96 hours = _____ days

c $2\frac{1}{2}$ years = _____ weeks

d 105 days = _____ weeks

e 212 minutes = _____ hours _____ minutes

f 330 seconds = _____ minutes _____ seconds

11.04 **12** At a vehicle checkpoint, the following types of vehicles were counted in a 2-hour period.

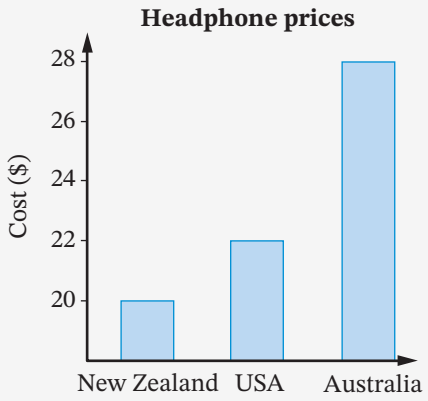
Cars 30 Trucks 7 Motorcycles 11 Buses 2

a How many vehicles passed the checkpoint in this time period?

b Based on this data, what is the probability that the next vehicle to pass the checkpoint will be a car?

10.02 **13** **a** Explain why this graph gives a misleading impression.

b Redraw the graph correctly.



Foundation Standard Complex

9780170465557

14 A packet of mixed lollies has 12 red lollies, 9 green lollies, 4 black lollies and 5 pink lollies. You tip the packet and one lolly rolls out. What is the probability that it is not a green lolly? Select **A**, **B**, **C** or **D**. 11.02

A $\frac{4}{30}$ **B** $\frac{5}{30}$ **C** $\frac{12}{30}$ **D** $\frac{21}{30}$

15 Use the timetable on page 572 to answer this question. Louella catches the train from Bundanoon to Sydney to meet up with friends. Which train gives her the shortest trip? Select **A**, **B**, **C** or **D**. 12.09

A 8:09 a.m. **B** 8:52 a.m. **C** 2:22 p.m. **D** 3:27 p.m.

16 Organise this set of data into an ordered stem-and-leaf plot: 10.04

9	5	15	22	11	14	19	28	18
13	17	6	24	16	15	4	10	21

17 Find the mode of the data above. Select **A**, **B**, **C** or **D**. 10.07

A 5 **B** 15 **C** 18 **D** 24

18 The ratio of boys to girls at the public speaking competition is 4 : 3. If there are 15 girls, find the number of boys. 12.03

19 A pack of 20 cards contains 10 blue, 6 purple and 4 pink cards. One card is drawn from the pack at random. Find the probability that this card is: 11.03

a purple **b** not purple

20 For this set of scores: 10.05

3.3 3.5 3.0 3.0 3.8 3.4 3.5 3.9

find:

a the mean **b** the mode

c the median **d** the range

21 Simon's cycling trip is shown on this travel graph. 12.04

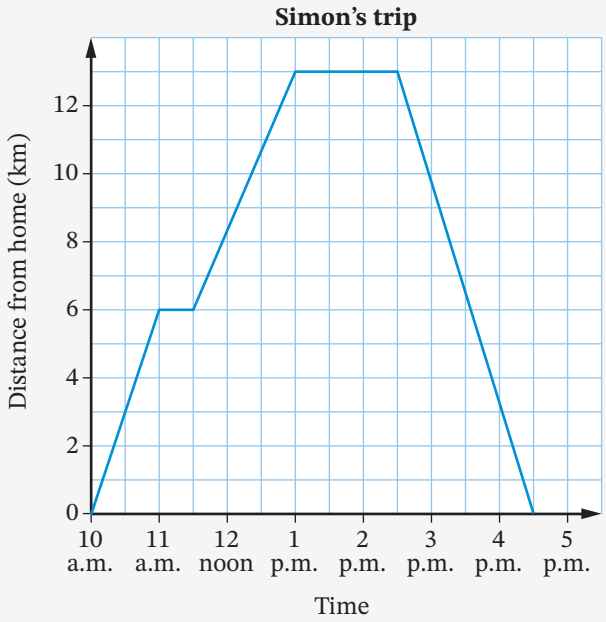
a How far was Simon from home at 11 a.m.?

b When, and for how long, did Simon stop for lunch?

c How far did Simon cycle in total?

d At what times was Simon 3 km from home?

e Was Simon's speed greater in the first hour or between 11:30 a.m. and 1 p.m.?



Foundation Standard Complex

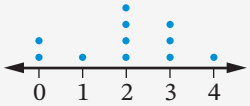
10.07

22 For each set of data, find:

i the range **ii** the mode

iii the median **iv** the mean (to 2 decimal places).

a **Number of goals scored**



b **Ages of soccer players**

Steam	Leaf
1	5 7 9 9
2	1 1 1 2
3	0

11.05

23 The letters of the word MATHEMATICS are written on separate cards and one is randomly selected. What is the probability that the letter drawn out is:

a not S? **b** not a vowel? **c** neither M nor H?

10.06

24 Manal's dad records the number of phone calls she makes each night for one week:

9 3 6 2 13 # 1

where # represents a missing value.

What is # if the mean of the values is 5? Select **A**, **B**, **C** or **D**.

A 1 **B** 2 **C** 5 **D** 6

10.08

25 Jane and Neil were arguing about which local rugby league team is the best. The points scored by their favourite teams in each match over the previous season were:

Eagles: 20, 10, 40, 12, 17, 20, 22, 20, 34, 19, 36, 18, 24, 12, 38, 34, 24, 36, 32, 22, 6, 7, 38, 18

Cougars: 14, 18, 24, 39, 14, 4, 4, 14, 10, 13, 28, 22, 16, 18, 18, 12, 18, 28, 21, 6, 10, 18, 36, 12

a Draw a back-to-back stem-and-leaf plot for these 2 data sets.

b What is the highest number of points scored by either team?

c What is the lowest number of points scored by either team?

d Find the mode for each team.

e Which is the better team based on the data? Justify your answer.

f What other data might you need to know to decide which is the better team?

Foundation Standard Complex

General practice

ANSWERS ON P. 650

1 **a** Use a divisibility test to show why 195 is divisible by 3. CHAPTER 3

b Evaluate $195 \div 3$.

2 Write $8 \times 8 \times 8 \times 8 \times 8$ using index notation (powers). CHAPTER 3

3 Evaluate each expression. CHAPTER 3

a 11^2 **b** $\sqrt{49}$ **c** 10^3

4 Evaluate each expression without using a calculator. CHAPTER 5

a $3 \times 5 \times 4$ **b** 20×18 **c** 45×9 **d** 16×12

5 What are the coordinates of the origin on the number plane? CHAPTER 9

6 What is the remainder when 8025 is divided by 3? Select **A**, **B**, **C** or **D**. CHAPTER 3

A 3 **B** 2 **C** 1 **D** 0

7 Write an algebraic expression for 8 more than double x. CHAPTER 5

8 Simplify each expression. CHAPTER 5

a $2x + x$ **b** $m \times m \times 5$ **c** $7k \div 2$

d $2y \times 3$ **e** $10 - k \times n$ **f** $2 \times p + 4$

9 Copy and complete each conversion. CHAPTER 8

a 7.2 m = _____ mm **b** 425 mL = _____ L

c 96 hours = _____ days **d** 240 kg = _____ t

e 5000 cm = _____ m **f** 6.7 m^3 = _____ L

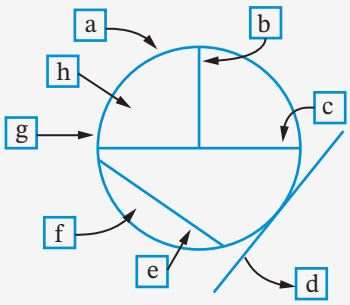
YEAR 8 EXTENSION

10 A movie started at 6:35 p.m. and finished at 8.23 p.m. How long was the movie? Select the correct answer **A**, **B**, **C** or **D**. CHAPTER 12

A 2 h 12 min **B** 1 h 48 min **C** 1 h 12 min **D** 2 h 28 min

11 Name each part of the circle shown using the words from the list. CHAPTER 8

arc, chord, circumference, diameter, quadrant, radius, sector, segment, semicircle, tangent

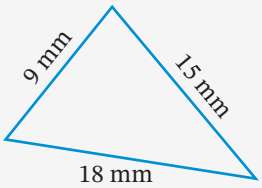


Foundation Standard Complex

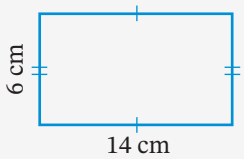
CHAPTER 8

12 Find the perimeter of each shape.

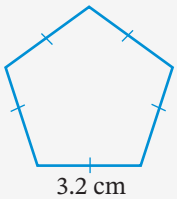
a



b



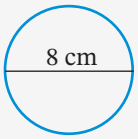
c



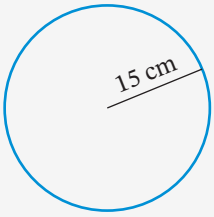
CHAPTER 8

13 Calculate, correct to one decimal place, the circumference of each circle.

a



b



CHAPTER 12

14 Write 6:10 p.m. in 24-hour time.

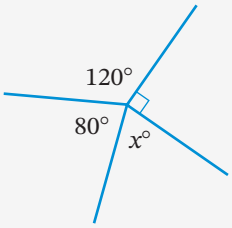
CHAPTER 12

15 The ratio of parents to children at the Wiggles concert was 4 : 5.
If there were 376 parents, how many children were there?

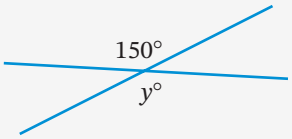
CHAPTER 2

16 Find the value of each variable.

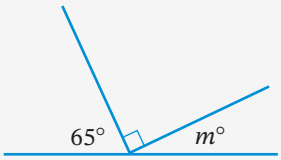
a



b



c



CHAPTER 1

17 Write these integers in ascending order: 6, −5, −3, 2, 4, 11.

YEAR 8 EXTENSION

CHAPTER 1

18 Evaluate each expression.

a $-5 + 3$ **b** $-8 - 4$ **c** $10 - (-3)$ **d** $-7 - (-8)$

e $-12 \times (-10)$ **f** $(-10)^2$ **g** $-\frac{36}{4}$ **h** $10 + 4 \times (-3)$

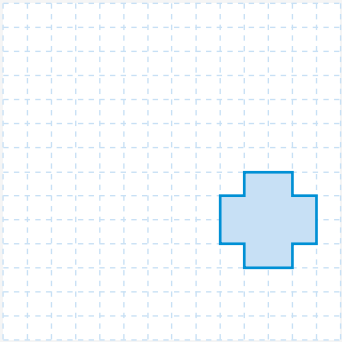
Foundation Standard Complex

19 Write each number in expanded notation using powers of 10.

a 505 **b** 10 723

CHAPTER 3

20 Copy this cross shape on grid paper and translate it 8 units left and 5 units up.



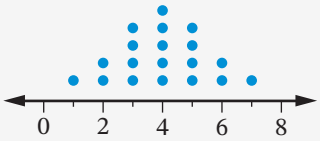
CHAPTER 6

Worksheet
5 mm grid paper

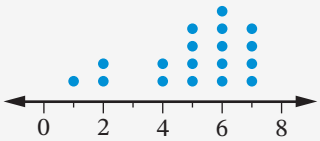
21 Describe the shape of the data shown in each dot plot.

CHAPTER 10

a



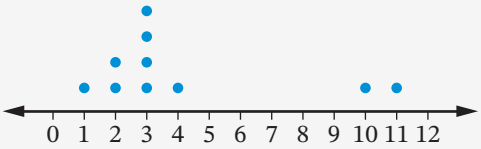
b



c

22 Identify any outliers in this dot plot.

CHAPTER 10



23 If a coin is tossed, what is the probability that heads will come up? Write the answer as:

CHAPTER 11

a a decimal **b** a percentage

24 Write the sample space for the colours of a traffic light.

CHAPTER 11

25 Copy and complete each pair of equivalent fractions.

CHAPTER 4

a $\frac{8}{12}=\frac{\square}{3}$ **b** $\frac{4}{25}=\frac{\square}{100}$ **c** $\frac{15}{20}=\frac{3}{\square}$ **d** $\frac{2}{5}=\frac{10}{\square}$

26 Evaluate each expression.

CHAPTER 4

a $\frac{2}{5}+\frac{1}{2}$ **b** $\frac{2}{3}-\frac{1}{4}$ **c** $\frac{1}{2}\times\frac{3}{4}$ **d** $\frac{3}{4}\div\frac{1}{2}$

Foundation Standard Complex

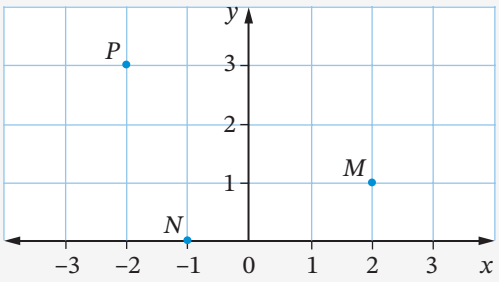
CHAPTER 9

27 For this number plane, write the coordinates of:

a M

b N

c P



CHAPTER 10

28 Represent this set of data on a dot plot:

5 3 2 0 3 2 1 2 4 5

CHAPTER 10

29 For the data above, find:

a the mean **b** the median **c** the mode **d** the range

CHAPTER 12

30 Which ratio is equivalent to 5 : 7? Select **A**, **B**, **C** or **D**.

A 70 : 90 **B** 100 : 140 **C** 35 : 100 **D** 10 : 12

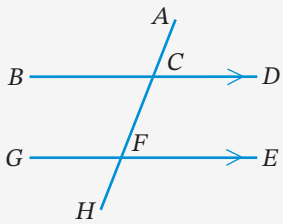
CHAPTER 2

31 For this diagram, name:

a a pair of corresponding angles

b a pair of vertically opposite angles

c the transversal.



CHAPTER 2

32 If $\angle CFE = 75°$ in the diagram above, find the value of:

a $\angle FCD$ **b** $\angle BCF$ **c** $\angle ACD$

CHAPTER 11

33 What is the probability that a person selected at random was born in a month beginning with M?

CHAPTER 11

34 **a** Write an example of an event that is certain.

b What is the probability of an event that is certain?

CHAPTER 12

35 Simplify each ratio.

a 8 : 40 **b** 12 : 9 **c** 45 s : 3 min

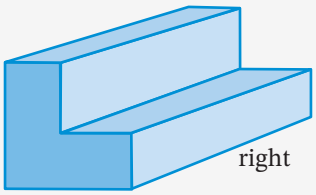
CHAPTER 6

36 For this prism, sketch each view.

a front view

b top view

c right view



CHAPTER 9

37 Plot and label these points on a number plane.

$R(3, 2)$ $S(0, 4)$ $T(-1, 3)$

Foundation Standard Complex

9780170465557

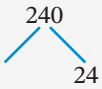
38 Select the prime numbers from this list: 3, 12, 33, 11, 9, 40, 14, 18.

39 Write:

a the factors of 27

b the highest common factor (HCF) of 27 and 45.

40 Copy and complete this factor tree and express 240 as a product of its prime factors.

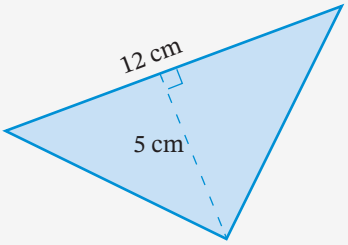


41 **a** Draw a quadrilateral with one pair of parallel sides.

b What type of quadrilateral is it?

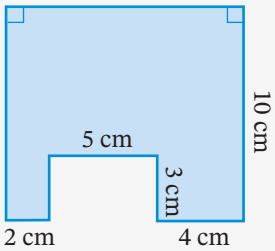
42 Draw a polygon with 8 equal sides and write the full name of this polygon.

43 Find the area of this triangle.



YEAR 8 EXTENSION

44 Find the area of this shape.



45 **a** Draw a rhombus.

b How many axes of symmetry does a rhombus have?

c What order of rotational symmetry does a rhombus have?

46 Convert each decimal to a fraction in simplest form.

a 0.35 **b** 0.1 **c** 0.012

47 Round each decimal to one decimal place.

a 1.736 **b** 0.69 **c** 0.98

48 True or false?

a $5.92 \times 3 = 0.1776$ **b** $\frac{3}{100} = 0.03$ **c** $0.65 = \frac{12}{20}$

49 If $t = -1$, evaluate $3t - 4$.

50 What is the volume of a room that is 7 m by 8 m by 3 m?

Foundation Standard Complex

CHAPTER 6

51 Use isometric dot paper to construct an isometric drawing of a rectangular prism with dimensions 5 cm by 3 cm by 2 cm.

Worksheet
Isometric dot paper

CHAPTER 4

52 Find:

a $\frac{3}{4}$ of \$12 **b** $\frac{2}{3}$ of 24 m **c** $\frac{1}{10}$ of 2 L (in mL)
d 10% of \$200 **e** 25% of 16 kg

CHAPTER 4

53 Convert each number to a percentage.

a $\frac{73}{100}$ **b** $\frac{3}{4}$ **c** 0.6 **d** 0.47

CHAPTER 9

54 The cost of hiring a carpet cleaning machine can be represented by the formula

$$C = 45t + 167.50$$

where \$$C$ is the total cost and t is the number of days the machine is hired for.
Use the formula to complete the table showing the total cost when the machine is hired from 1 to 10 days.

t (days)	1	2	3	4	5	6	7	8	9	10
\$$C$										

CHAPTER 9

55 Graph the linear function $y = 5 - 2x$ on a number plane.

CHAPTER 10

56 This stem-and-leaf plot shows the ages of people waiting at a medical centre.

a How many people were waiting at the centre?
b Find the median age.
c How old was the oldest person?
d Find the mode.

Steam	Leaf
0	3
1	2 6 9
2	4
3	5 5 6 8
4	2 8
5	0 7
6	3 4

CHAPTER 12

57 Write 20:45 in 12-hour time.

CHAPTER 7

58 Evaluate each expression.

a 6.9×4.3 **b** $15.35 \div 5$ **c** $7.3 - 1.986$

CHAPTER 7

59 Convert each fraction to a decimal.

a $\frac{5}{8}$ **b** $\frac{11}{20}$

YEAR 8 EXTENSION

CHAPTER 7

60 Convert $\frac{1}{6}$ to a decimal.

CHAPTER 11

61 Over 30 days, the number of sunny days in Nelson Heights was 18.

a What is the relative frequency of having a sunny day in Nelson Heights?
b Based on this probability, what is the expected frequency of sunny days in a period of 100 days?

Foundation Standard Complex

62 **a** Draw an isosceles right-angled triangle. CHAPTER 6

b Write the size of each angle in the triangle.

63 Masina has a bag of jelly beans. 20% of the jelly beans are blue. She takes one yellow bean from the bag and eats it. She then takes another jelly bean from the bag. What is the probability that this jelly bean is blue? Select **A**, **B**, **C** or **D**. CHAPTER 11

A greater than 20% **B** equal to 20%

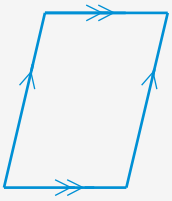
C less than 20% **D** not enough information

YEAR 8 EXTENSION

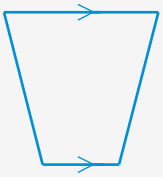
64 How many hours and minutes between 06:35 and 13:10 on the same day? CHAPTER 12

65 Which quadrilateral has diagonals that cross at 90°? Select **A**, **B**, **C** or **D**. CHAPTER 6

A

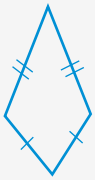


B



C

D

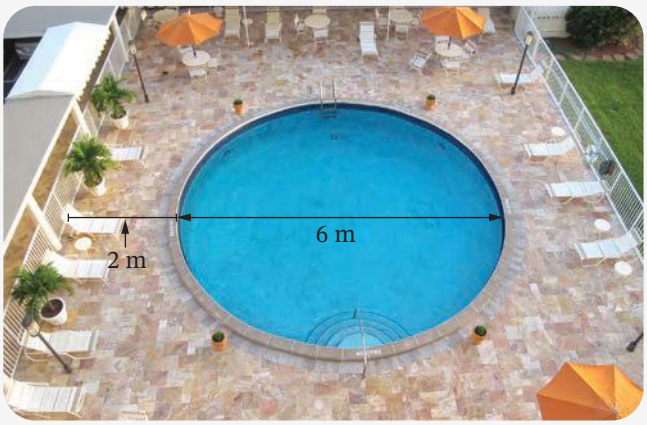


66 Water is filled into each container below at a constant rate. Sketch the graph that shows the height of water in the bottle as it is filled. CHAPTER 12

a

b

67 A fence needs to be installed around the circular pool below. The pool has a diameter of 6 m and the space between the pool edge and fencing must be 2 m wide all the way around the pool. The fencing costs $167.50 per metre, and can only be purchased in whole metre lengths. Dylan has a budget of $5000 for this job. Will he be able to install the pool fence? CHAPTER 8



iStock.com/tiburonstudios

Foundation Standard Complex

Calculator skills

Basic keys

Key	Use	Key	Use
+ , − , × , ÷	Basic operations	(−) or +/−	Enters negative numbers
=	Equals sign, gives the answer	▭/▭ or $a^{b/c}$	Enters fractions
·	Decimal point	( )	Enters parentheses (brackets)
DEL	Deletes previous entry	x^2	Squares a number
Ans	Retrieves previous answer	√	Finds the square root of a number
✥	Moves cursor around the screen	x^3	Cubes a number
MODE or SHIFT or 2ndF	Accesses other operations	∛	Finds the cube root of a number

1 Basic operations

Example 1

PERFORMING A CALCULATION

Question	Calculator steps	Answer
34 + 6 − 16	34 + 6 − 16 =	24
7.4 × 9.1	7.4 × 9.1 =	67.34
234 ÷ 1.5	234 ÷ 1.5 =	156
4569 ÷ (1.7 + 4.3)	4569 ÷ (1.7 + 4.3) =	761.5

Example 2

CORRECTING A WRONG ENTRY

Calculating 34 + 6 − 16

a Wrong entry: 34 + 8 — To delete the 8 and continue, enter DEL 6 − 16 =

b Wrong entry: 34 + 6 + 16 = — To correct this, press ← repeatedly to go back to the second +, press DEL and then −, then press =

EXERCISE 1 ANSWERS ON P. 652

Basic operations

1 Calculate each expression.

a 362 × 14 **b** 26.3 − 11.8

c 810 ÷ 18 **d** 5.3 × 1.08

e $51.1 \div 14.6$

f $19.3 + 8.8 - 1.2$

g $2.4 \div 0.2 + 11$

h $5.6 \times (0.1 + 0.2)$

i $(56 - 24) \div (1.2 - 1)$

j $12.6 \times 3.8 - 18.4$

k $(16.07 + 9.02) - (38.3 - 5.126)$

l $(24.8 + 3.06 + 9.4) \div 3$

2 Bronte purchased a loaf of bread for \$2.75, a packet of cornflakes for \$4.55 and 3 kg of tomatoes at \$3.95 per kg.

a Find the total cost.

b Find the change Bronte will receive from a \$20 note.

3 Mark started the month with \$1650 in his bank account. He made deposits of \$380 and \$725.50 and withdrew \$880. How much was left in Mark's account?

4 An unloaded truck has a mass of 6550 kg. It is loaded with 8 boxes, each weighing 124 kg. What is the total mass of the truck and the boxes?

5 Salma needs to cut a 9.6 m length of rope into 4 equal pieces. Find the length of each piece.

6 Hamish posted 25 Christmas cards to his friends. If the stamps were 75 c each, how much did he spend on stamps?

7 An orchard has 1288 trees arranged in 56 rows. How many trees are in each row?

8 A train takes 15 hours to travel 1128 km. What is its average speed in km/h?

9 Chloe bought 60 L of petrol at 142.9 c per litre. How much did she pay for the petrol?

10 Nelson College has the following numbers of students in each Year level.

Year 7	Year 8	Year 9	Year 10	Year 11	Year 12
122	128	123	131	117	94

The whole school is going on an excursion. If each bus can carry 55 people, how many buses will be needed?

11 The manager of a sports store purchased 32 tennis racquets for \$79.60 each and sold them for \$120.80 each.

a Find the profit made on each racquet.

b Find the total profit.

12 A batter scored the following runs over 5 cricket innings.

27 4 123 66 47

Find the batter's average score.

2 Integers

Example 3

Question	Calculator steps		Answer
	Casio	Sharp	
−4 + 9	(−) 4 + 9 =	+/− 4 + 9 =	5
17 × (−2)	17 × (−) 2 =	17 × +/− 2 =	−34
18 −(−3) × 4	18 − (−) 3 × 4 =	18 − +/− 3 × 4 =	30

EXERCISE 2 ANSWERS ON P. 652

Integers

1 Calculate each expression.

a $-5 + 12$ **b** $-6 - 3$ **c** -3×4

d $16 \div (-2)$ **e** $12 - (-3)$ **f** $-19 + 4$

g $-25 \times (-3)$ **h** $-55 \div (-11)$ **i** $8 - 12 + (-1)$

j $-3 + 7 - 2 - (-5)$ **k** $6 \times (-2) \times 8$ **l** $-18 \div (-2) \times 5$

m $-3 \times 7 - 4$ **n** $(-3 + 9) \times 4$ **o** $80 - (-2) \times 8$

p $12 \times (-3) + 50$ **q** $64 \div (-8) \times 2$ **r** $27 - (-3) \times (-4)$

s $18 \times (-5 + 3)$ **t** $-14 - 3 \times (-8)$

3 Powers and roots

Look for these keys on your calculator: x^2 x^3 $\sqrt{\ }$ $\sqrt[3]{\ }$

You may need to use the SHIFT or key.

Example 4

Question	Calculator steps	Answer
16^2	16 x^2 =	256
11^3	11 x^3 =	1331
$\sqrt{196}$	$\sqrt{\ }$ 196 =	14
$\sqrt[3]{4913}$	$\sqrt[3]{\ }$ 4913 =	17

EXERCISE 3 ANSWERS ON P. 652

Powers and roots

1 Calculate each expression.

a 14^2 **b** 23^2 **c** 6^3 **d** 1.2^3

e 5.13^2 **f** 101^2 **g** 2.04^2 **h** 19^3

i $\sqrt{676}$ **j** $\sqrt{21904}$ **k** $\sqrt[3]{729}$ **l** $\sqrt{6^2 \times 7^2}$

2 Evaluate each expression, correct to one decimal place.

a $\sqrt{24}$ **b** $\sqrt{101}$ **c** $\sqrt{11 \times 2}$ **d** $\sqrt[3]{100}$

3 Calculate each expression.

a $5^2 - 3^2$ **b** $16^2 - 10^2$ **c** $\sqrt{6} \times \sqrt{6}$ **d** $\sqrt{25 \times 4}$

4 Fractions

Casio calculators have 2 ways of entering fractions: MATH mode or LINE mode. MATH mode allows you to enter the numerator and denominator into 2 blank spaces on the calculator's screen, while LINE mode makes the fraction key act like a vinculum (fraction bar). For MATH mode, use the arrow keys to move the cursor to the spaces to enter the numerator and denominator. Press SHIFT MODE to change between MATH I/O and LINE I/O (input/output). To ensure that mixed numeral answers are not converted to improper fractions, press SHIFT MODE and the down arrow to choose $a^{b/c}$.

Example 5

CONVERTING FRACTIONS

Question	Calculator steps		Answer
	Casio LINE mode	Sharp	
Simplify $\frac{16}{20}$	16 ▭ 20 =	16 $a^{b/c}$ 20 =	$\frac{4}{5}$
Change $1\frac{3}{4}$ to an improper fraction	1 ▭ 3 ▭ 4 = SHIFT S⇔D	1 $a^{b/c}$ 3 $a^{b/c}$ 4 = 2ndF $a^{b/c}$	$\frac{7}{4}$
Change $\frac{19}{4}$ to a mixed numeral	19 ▭ 4 =	19 $a^{b/c}$ 4 =	$4\frac{3}{4}$
Change $\frac{3}{4}$ to a decimal	3 ▭ 4 = S⇔D or 3 ÷ 4 =	3 $a^{b/c}$ 4 = $a^{b/c}$ or 3 ÷ 4 =	0.75
Change 1.7 to a fraction	1.7 =	1.7 = $a^{b/c}$	$1\frac{7}{10}$

S⇔D converts an answer between standard fraction and decimal forms.

SHIFT S⇔D or 2ndF $a^{b/c}$ converts an answer between mixed numeral and improper fraction forms.

Example 6

OPERATIONS WITH FRACTIONS

Question	Calculator steps		Answer
	Casio LINE mode	Sharp	
$\frac{1}{2}+\frac{1}{3}$	1 [▭/▭] 2 [+] 1 [▭/▭] 3 [=]	1 [a b/c] 2 [+] 1 [a b/c] 3 [=]	$\frac{5}{6}$
$\frac{5}{7}-\frac{2}{3}$	5 [▭/▭] 7 [−] 2 [▭/▭] 3 [=]	5 [a b/c] 7 [−] 2 [a b/c] 3 [=]	$\frac{1}{21}$
$\frac{3}{5}\times\frac{2}{7}$	3 [▭/▭] 5 [×] 2 [▭/▭] 7 [=]	3 [a b/c] 5 [×] 2 [a b/c] 7 [=]	$\frac{6}{35}$
$\frac{4}{5}\div\frac{2}{3}$	4 [▭/▭] 5 [÷] 2 [▭/▭] 3 [=]	4 [a b/c] 5 [÷] 2 [a b/c] 3 [=]	$1\frac{1}{5}$

Example 7

MIXED OPERATIONS WITH FRACTIONS

Question	Calculator steps	Answer
$\frac{13+7}{9\times 2}$	[(] 13 [+] 7 [)] [÷] [(] 9 [×] 2 [)] [=] or [(] 13 [+] 7 [)] [▭/▭] (or [a b/c]) [(] 9 [×] 2 [)] [=]	1.1111... $1\frac{1}{9}$

EXERCISE 4 ANSWERS ON P. 652

Fractions

1 Simplify each fraction.

a $\frac{20}{32}$ **b** $\frac{40}{85}$ **c** $\frac{45}{72}$

d $\frac{112}{126}$ **e** $\frac{84}{100}$ **f** $\frac{48}{192}$

2 Convert each mixed numeral into an improper fraction.

a $1\frac{7}{8}$ **b** $5\frac{2}{9}$ **c** $10\frac{3}{4}$

3 Convert each improper fraction into a mixed numeral.

a $\frac{20}{9}$ **b** $\frac{44}{5}$ **c** $\frac{18}{8}$

4 Convert each fraction into a decimal.

a $\frac{1}{2}$ **b** $\frac{4}{5}$ **c** $\frac{7}{8}$

d $\frac{9}{25}$ **e** $\frac{34}{200}$ **f** $\frac{5}{9}$

5 Convert each decimal into a fraction in its simplest form.

a 0.35 **b** 2.04 **c** 0.001

d 0.365 **e** 1.06 **f** 0.325

6 Simplify each expression.

a $\frac{3}{5}+\frac{1}{4}$ **b** $\frac{4}{7}+\frac{1}{2}$ **c** $\frac{2}{3}-\frac{3}{5}$ **d** $\frac{1}{3}\times\frac{4}{7}$

e $\frac{5}{7}\div\frac{15}{28}$ **f** $\frac{5}{6}-\frac{5}{8}$ **g** $\frac{9}{10}+\frac{2}{3}$ **h** $\frac{7}{8}\times\frac{5}{12}$

i $\frac{7}{8}\div\frac{1}{2}$ **j** $24\div1\frac{1}{4}$ **k** $5\frac{1}{4}+1\frac{2}{5}$ **l** $9\frac{7}{8}-1\frac{1}{3}$

m $5\frac{1}{4}\times2\frac{3}{7}$ **n** $4\frac{2}{3}\div\frac{2}{3}$ **o** $\frac{8}{9}\div6$ **p** $10\frac{1}{6}\times1\frac{9}{10}$

7 Simplify each expression.

a $\frac{12-4\times2}{18+14}$ **b** $\frac{6^2}{100\div4+23}$ **c** $\frac{9\times9-2}{\sqrt{30-5}}$ **d** $\frac{20\div2+8}{3\times9\times4}$

8 Mr Pettis is at his office for $7\frac{1}{2}$ hours each day. He takes $\frac{3}{4}$ of an hour for lunch each day. How long does he work:

a in one day? **b** in 5 days?

Answers

CHAPTER 1

SkillCheck

1 **a** 16 **b** 8 **c** 28 **d** 4
e 21 **f** 25 **g** 22 **h** 5
i 13 **j** 8 **k** 15 **l** 6

2 **a** 49 **b** 604 **c** 70 **d** 33
e 237 **f** 128 **g** 23 **h** 2
i 28 **j** 400 **k** 12 **l** 32
m 9 **n** 25 **o** 64 **p** 1

3 **a** T **b** T **c** F **d** T **e** F
f F **g** T **h** T **i** F

4 **a** 1, 2, 3, 4, 8 **b** 2, 3, 5, 7, 16, 18
c 27, 38, 49, 51, 54, 66

5 **a** 18, 17, 11, 6, 5, 3 **b** 31, 23, 14, 8, 7, 5, 2
c 91, 89, 69, 46, 41, 37, 3

Exercise 1.01

1 **a** 100°C **b** body temperature
c –74°C **d** water freezes
e 117°C

2 **a** –6 **b** –1 **c** 10
d –11 **e** 0 **f** 2

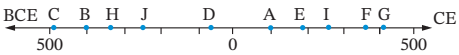
3 **a** down **b** less **c** up
d right **e** after **f** north
g deposit **h** increase **i** east

4 **a** about 38 metres **b** about 73 metres
c about 111 metres
d '+' signs measure height above water level and '–' signs measure depth below water level

5 $610

6 **a** depositing $25 in the bank
b losing $12
c walking 2 steps forward
d increasing the temperature by 3°
e driving 5 km east
f decreasing the price by $8
g going down 3 floors
h 5 seconds after take-off

7 0

8 

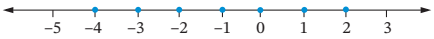
9 Teacher to check.

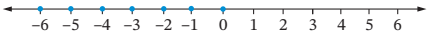
Exercise 1.02

1 A = 4, B = 9, C = 11, D = 17, E = 20, F = 27

2 **a** A = 2, B = 10, C = 13, D = 15
b A = 10, B = 16, C = 28, D = 34, E = 37
c A = 8, B = 15, C = 32, D = 41, E = 49, F = 56
d A = 4, B = 36, C = 52, D = 66, E = 90, F = 116
e A = 10, B = 20, C = 35, D = 55, E = 63, F = 89, G = 111

3 **a** Missing values are –5, –4, –2, 4, 5, 7, 8
b Missing values are –50, –40, –30, 20, 30, 50, 60, 70
c Missing numbers are –35, –30, –25, –20, –15, –10
d Missing numbers are –400, –300, –200, 200
e Missing numbers are –33, –30, –27, –24, –21, –18, –15, –12, –9, –6

4 **a** 

b 

5 **a** 29 **b** 35 **c** 32
d 20 **e** 12 **f** 5

6 **a** 1 **b** 4 **c** 6
d –6 **e** –12 **f** –15

7 C

Exercise 1.03

1 **a** > **b** < **c** > **d** >
e > **f** > **g** < **h** <
i < **j** < **k** < **l** >
m > **n** < **o** < **p** <

2 **a** 7 **b** –8
c –8, –6, –4, –2, 1, 3, 5, 7

3 **a** –3, –1, 2, 3 **b** –8, –5, –3, 2, 5
c –10, –6, –4, –3, 0 **d** –5, –3, –2, 4, 6
e –48, –24, –11, 0, 8, 36
f –26, –10, –2, 3, 12, 15

4 **a** 5, 4, 3, –1, –2 **b** 8, 0, –2, –4, –7
c 4, 1, –1, –3, –5, –11 **d** 8, 3, –2, –4, –18
e 1, –3, –6, –15, –48 **f** 33, 1, –36, –58, –100

5 **a** 2 **b** 2 **c** 4 **d** 3
e 6 **f** 20 **g** 7 **h** 7

6 D

7 **a** –4 **b** –6 **c** 4 **d** –2 **e** –12

Exercise 1.04

1 **a** 4 **b** 5 **c** –1 **d** 2
e 0 **f** 4 **g** –3 **h** –4
i –20 **j** –4 **k** –8 **l** 0
m –2 **n** –2 **o** –11 **p** 6

2 C

3 **a** Yes, 4
b Yes, integers can be added in any order.

4 **a** –12 **b** 0 **c** 20 **d** –23
e 15 **f** 0 **g** –3 **h** 9
i –19 **j** –10 **k** –32 **l** –9
m 0 **n** 5

5 The answer is always 0.

6 **a** + **b** − **c** E

7 **a** −4 **b** 7 **c** −3 **d** 7 **e** −3 **f** 7

8 Teacher to check. **9** Teacher to check.

Mental skills 1A

2 **a** 155 **b** 221 **c** 317 **d** 1764 **e** 48 **f** 726 **g** 283 **h** 952 **i** 722 **j** 631 **k** 215 **l** 163 **m** 394 **n** 466 **o** 937 **p** 295

Exercise 1.05

1 **a** 2 **b** −2 **c** −10 **d** 6 **e** 13 **f** −1 **g** 0 **h** −10 **i** −9 **j** 1 **k** −7 **l** 6 **m** −5 **n** −6 **o** 12 **p** −9

2 B

3 **a** No, 10 − 4 = 6, 4 − 10 = −6
b No, −5 − 9 = −14, 9 − (−5) = 14
c No, they give different answers.

4 **a** 11 **b** −11 **c** 29 **d** −29 **e** −31 **f** 23 **g** 16 **h** −10 **i** −50 **j** 50 **k** 42 **l** −42 **m** −12 **n** 5

5 You get the negative of double the integer.

6 **a** + **b** − **c** E **d** + **e** E

7 **a** 4 **b** 4 **c** 3 **d** 3 **e** 1 **f** 1 **g** 4 **h** −4 **i** 2 **j** −2

8 **a** 7 **b** 3 **c** 8 **d** 8 **e** 1 **f** −6

9 Teacher to check.

Exercise 1.06

1 **a** −18 **b** −24 **c** 21 **d** −32 **e** 63 **f** −45 **g** 42 **h** 42 **i** −24 **j** −6 **k** 1 **l** −54 **m** −10 **n** −42 **o** 44 **p** −81 **q** 60 **r** −60 **s** 100 **t** 56

2 **a** C **b** A **c** D

3 **a**

×	−3	5
−3	9	−15
5	−15	25

b

×	−5	5
5	−25	25
−5	25	−25

c

×	6	−4
−4	−24	16
6	36	−24

d

×	−1	1
−1	1	−1
1	−1	1

e

×	7	−3
4	28	−12
−6	−42	18

f

×	−3	−6
8	−24	−48
2	−6	−12

4 **a** Yes, both give −24. **b** Yes

5 You get the negative of the integer squared.

6 **a** 9 **b** 9 **c** 49 **d** 49 **e** 25 **f** 25 **g** 1 **h** 1

7 **a** + **b** + **c** − **d** + **e** +

8 **a** −5 **b** −4 **c** −3 **d** −7 **e** −5 **f** −2

9 Teacher to check.

Exercise 1.07

1 **a** −4 **b** −2 **c** 4 **d** −3 **e** −9 **f** 3 **g** −3 **h** −10 **i** −4 **j** 7 **k** −9 **l** −6 **m** 9 **n** −8 **o** −4 **p** −6

2 **a** A **b** B

3 **a** −6 **b** −2 **c** 6 **d** 16 **e** −7 **f** −9 **g** 20 **h** 9

4 **a**

+	−6	−3	3	6
−6	1	$\frac{1}{2}$	$-\frac{1}{2}$	−1
−3	2	1	−1	−2

b

+	8	−8	32	−32
−2	−4	4	−16	16
4	2	−2	8	−8

5 You get −1

6 **a** + **b** + **c** −

7 **a** −8 **b** 3 **c** −7 **d** −44 **e** −50 **f** −32

8 Teacher to check.

Exercise 1.08

1 **a** 7 **b** −22 **c** −1 **d** −7 **e** −6 **f** 4 **g** −25 **h** 8 **i** −21 **j** 12 **k** −200 **l** 28 **m** 15 **n** 5 **o** 3

2 C

3 **a** −7 **b** −25 **c** −36 **d** 10 **e** 14 **f** 3 **g** 5 **h** 16 **i** 2 **j** 8 **k** −11 **l** 2

4 **a** 8 **b** 5 **c** −3 **d** −2 **e** 3 **f** 12 **g** −2 **h** −2 **i** −19 **j** −9 **k** 3 **l** 25

5 **a** Correct
b Incorrect, $-32 \div (-4) \times (-2) = -16$
c Incorrect, $-6 \times (-2) - 8 = 4$
d Correct
e Correct
f Incorrect, $(-2 + 6) \times 2 = 8$
g Incorrect, $-7 - 6 + 10 = -3$
h Incorrect, $(-12 - 3) \div (-3) = 5$
i Incorrect, $12 - 3 \times 5 = -3$
j Incorrect, $2 - 9 \times (-3) - (-18) = 47$

6 **a** $-4 \times [7 - 2 + (-3)] = -8$
b $(-4 \times 7) - [2 + (-3)] = -27$
c $(-4 \times 7) - 2 + (-3) = -33$
d $40 \div [10 \times (-2)] + 15 = 13$
e $(40 \div 10) \times [(-2) + 15] = 52$
f $40 \div [10 \times (-2) + 15] = -8$

7 C **8** Teacher to check.

Mental skills 1B

2 **a** 17 **b** 35 **c** 114 **d** 261
e 132 **f** 128 **g** 56 **h** 418
i 74 **j** 123 **k** 88 **l** 112
m 36 **n** 549 **o** 185 **p** 317

Exercise 1.09

1, 2 **a** −7 **b** 6 **c** −4 **d** 1 **e** −5
f 14 **g** 26 **h** −28 **i** 30 **j** −48
k 44 **l** 30 **m** −65 **n** −8 **o** 15
p −4 **q** 25 **r** 64 **s** −48 **t** 2

3 $(-5)^2$ means $-5 \times (-5) = 25$, -5^2 means $-(5^2) = -25$.

4 **a** −18 **b** 71 **c** 1
d −6 **e** −3960 **f** −192
g 5 **h** −60 **i** −17
j 38 **k** 61 **l** 16

5 **a** 13 **b** −10 **c** 2
d 15 **e** −56 **f** −8

6 Teacher to check.

Exercise 1.10

1 13°C **2** 4 km

3 **a** 6610 m **b** 2316 m **c** 311 m **d** 9237 m

4 **a** 2015, 2016, 2019, 2020
b 2017, 2018
c $100 million
d Profit, $200 million

5 1 km north **6** −$7 **7** A

8 59 **9** 4602 m

10 **a** 480 cm **b** 160 cm

11 155 m below the surface

12 −$109.00 **13** 11 m

14 **a** 14th day **b** $60

Power plus

1 **a** 1 **b** −1 **c** 1
d $(-1) \times (-1) \times (-1) \times (-1) \times (-1) = -1$
e $(-1) \times (-1) \times (-1) \times (-1) \times (-1) \times (-1) = 1$
f $(-1) \times (-1) \times (-1) \times (-1) \times (-1) \times (-1) \times (-1) = -1$

2 $(-1)^n = 1$ when n is even, $(-1)^n = -1$ when n is odd

3 **a** 4 or −4 **b** 10 or −10 **c** −2

4 Yes, no number squared gives a negative answer.

5 Teacher to check.

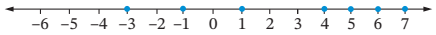
6 **a** 2 **b** −9 **c** 1

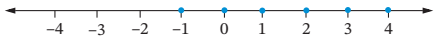
7 **a** True **b** False **c** True
d False **e** True **f** True
g False **h** True **i** True

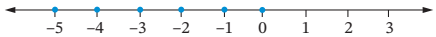
8 **a** −5, 2 **b** −3, −5 **c** −4, 2 **d** −3, 2 **e** −3, 10

Test yourself 1

1 **a** 2 **b** 23 **c** −56
d 10 **e** −8 **f** 0

2 **a**


b


c


3 **a** F **b** F **c** F **d** T
e T **f** T **g** T **h** F
i F **j** T **k** T **l** F

4 14, 3, 0, −1, −5, −9, −10

5 **a** −1 **b** −10 **c** −3
d 4 **e** −10 **f** 3
g −11 **h** 1 **i** −8

6 **a** 7 **b** 4 **c** −4
d −12 **e** 0 **f** −12
g −7 **h** 6 **i** −11

7 **a** −18 **b** 48 **c** −10
d −44 **e** −63 **f** 48
g −24 **h** −42 **i** 40

8 **a** 3 **b** −3 **c** 1
d −4 **e** −2 **f** −100
g 2 **h** −3 **i** 8

9 **a** −12 **b** −14 **c** −13
d −18 **e** 0 **f** −19
g 7 **h** −2 **i** 0

10 **a** −9 **b** −6 **c** −12
d 87 **e** −52 **f** 5

11 16°C **12** $820

CHAPTER 2

SkillCheck

1 A

2 **a** 20° **b** 25° **c** 15° **d** 20°
e 40° **f** 80° **g** 60° **h** 57°
i 70° **j** 94° **k** 28° **l** 81°

3 *G* and *F*

4 *A* and *H*, *B* and *I*

5 **a** *E* and *F* **b** *D* and *E* **c** *C* and *E*
d *B* and *E* **e** *F* and *H* **f** *D* and *J*
g *A* and *I* **h** *A* and *J*

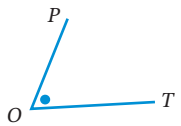
Exercise 2.01

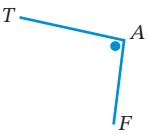
1 **a** $\angle P$, $\angle QPK$, $\angle KPQ$ **b** $\angle C$, $\angle OCR$, $\angle RCO$
c $\angle V$, $\angle GVE$, $\angle EVG$ **d** $\angle A$, $\angle GAT$, $\angle TAG$
e $\angle Q$, $\angle PQD$, $\angle DQP$ **f** $\angle D$, $\angle RDC$, $\angle CDR$

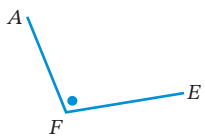
2 D

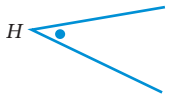
3 **a** $\angle CAD$ or $\angle DAC$ **b** $\angle PMQ$ or $\angle QMP$
c $\angle TQR$ or $\angle RQT$ **d** $\angle WXY$ or $\angle YXW$
e $\angle AED$ or $\angle DEA$ **f** $\angle FGE$ or $\angle EGF$

4 The following are examples, the sizes of the angles will differ.

a


b


c


d


5 **a** $\angle SQP$ **b** $\angle CBE$ **c** angle *b* **d** angle *q*

6 **a** $\angle NAY$, $\angle AYC$, $\angle ANC$, $\angle AND$, $\angle AYD$, $\angle NCD$, $\angle NDY$, $\angle NCY$, $\angle DNC$, $\angle DCY$, $\angle CDY$, $\angle CYD$, $\angle NDC$ (Remember that $\angle NAY$ and $\angle YAN$ are two ways of naming the same angle.)
b $\angle NCD$, $\angle NCY$, $\angle DCY$
c A straight angle
d $\angle ANY$ or $\angle YNA$

7 A

8 **a** • $\angle DBC$, × $\angle ADB$ **b** • $\angle PSR$, × $\angle PRQ$
c • $\angle MRQ$, × $\angle RNP$ **d** • $\angle EIF$, × $\angle FGI$
e • $\angle DCF$, × $\angle GBC$ **f** • $\angle WXY$, × $\angle YWZ$

9 Teacher to check.

Exercise 2.02

1 **a** 64° **b** 133° **c** 85° **d** 87°
e 129° **f** 175° **g** 157° **h** 112°

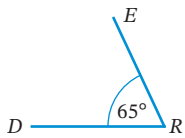
2 **a** $\angle AOB = 38°$ **b** $\angle PDQ = 162°$
c $\angle XPY = 84°$ **d** $\angle MAN = 121°$
e $\angle TQS = 175°$ **f** $\angle XYZ = 38°$
g $\angle LMN = 83°$ **h** $\angle GFD = 121°$
i $\angle ADM = 162°$ **j** $\angle BPZ = 47°$
k $\angle ABC = 122°$ **l** $\angle EFG = 38°$

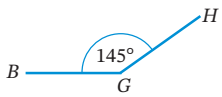
3 *d*, *a*, *h*, *f*, *c*, *b*, *g*, *e*

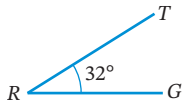
4 **a** $\angle ABC = 212°$ **b** $\angle XYZ = 300°$
c $\angle LMN = 247°$ **d** $\angle GHK = 330°$

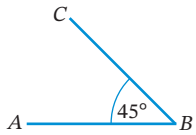
5 C **6** B

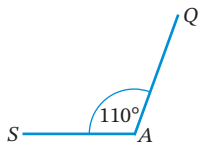
7, 8 Teacher to check.

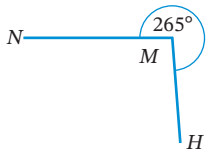
9 **a**


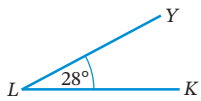
b


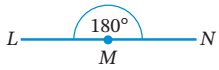
c


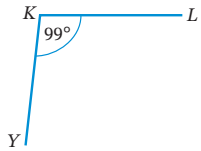
d


e


f


g


h


i


Exercise 2.03

1 **a** acute **b** obtuse **c** obtuse
d acute **e** reflex **f** reflex
g acute **h** obtuse **i** reflex
j obtuse **k** reflex **l** acute
m acute **n** reflex **o** obtuse

2 Teacher to check.

3 **a** acute **b** obtuse **c** reflex
d reflex **e** acute **f** straight
g obtuse **h** obtuse **i** revolution
j acute **k** reflex **l** obtuse
m acute **n** reflex **o** right
p reflex **q** obtuse **r** acute

4 D

5 **a** obtuse **b** acute **c** obtuse **d** reflex

Exercise 2.04

1 **a** 57 + 33 = 90 **b** 147 + 33 = 180

2 **a** 60° **b** 20° **c** 65° **d** 52°
e 1° **f** 33° **g** 48° **h** 24°

3 Teacher to check.

4 **a** 162° **b** 30° **c** 145° **d** 55°
e 118° **f** 93° **g** 69° **h** 176°

5 B

6 **a** $c = 84$ **b** $y = 90$ **c** $p = 126$ **d** $a = 180$
e $x = 12$ **f** $x = 40$ **g** $a = 15$ **h** $a = 46$

7 **a** $\angle PQR$ and $\angle RQS$
b 67 + 23 = 90 **c** right angle

8 **a** $a = 60$ **b** $a = 20$
c $m = 135$ **d** $m = 71$
e $f = 165$ **f** $a = 45$
g $t = 90$ **h** $a = 32, b = 148$
i $x = 70, y = 90$ **j** $j = 132, k = 48, l = 132$
k $x = 62, y = 105$ **l** $c = 60$

9 A

Mental skills 2

2 **a** 117 **b** 740 **c** 138 **d** 140 **e** 174
f 100 **g** 436 **h** 109 **i** 200 **j** 312

3 **a** 160 **b** 700 **c** 90 **d** 2200 **e** 72
f 1600 **g** 2100 **h** 480 **i** 132

Exercise 2.05

1 **a** c **b** u **c** a
d f **e** i **f** p

2 **a** $m = 50$ **b** $q = 120$ **c** $x = 211$
d $y = 145$ **e** $a = 98$ **f** $d = 151$
g $w = 90$ **h** $f = 57$ **i** $n = 110$

3 D

4 **a** $\angle MDQ$ **b** $\angle NDP$
c $\angle NDQ, \angle MDP$
d Teacher to check.

5 **a** $m = 90$ **b** $x = 135$ **c** $f = 25$
d $w = 133$ **e** $n = 29$ **f** $q = 62$
g $t = 163$ **h** $h = 20, g = 160$
i $q = 90, r = 90, s = 90$

6 **a** $y = 100$ **b** $p = 120$ **c** $a = 120$
d $x = 41$ **e** $b = 170$ **f** $y = 53$
g $d = 135$ **h** $e = 68, f = 112$ **i** $p = 60$

7 D

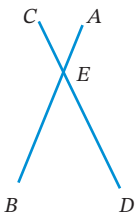
Exercise 2.06

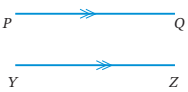
1 AB, AC, AD, BD, BC, CD

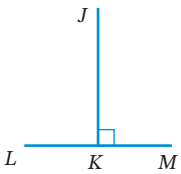
2 **a** AD and CF **b** BG and CF
c Teacher to check.

3 **a** $AD \perp CF$ **b** $BG \parallel CF$

4 **a**

b


c


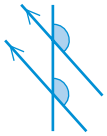
d


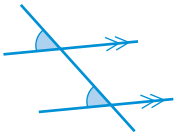
5 D **6** C

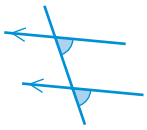
7–12 Teacher to check.

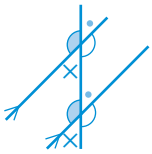
Exercise 2.07

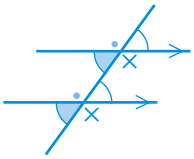
1 **a** f **b** c **c** c

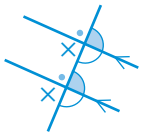
2 **a**


b


c


3 **a**


b


c


4 B

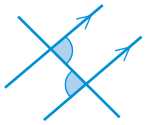
5 **a** $a = 120$ **b** $y = 28$ **c** $m = 63$
d $c = 108$ **e** $t = 74$ **f** $a = 50, b = 60$
g $m = 110, n = 70$
h $x = 105, y = 105$
i $a = 40, c = 40, d = 140, y = 140$

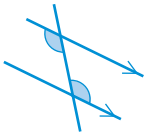
6 **a** 4
b $a = 75, b = 105, c = 75, d = 105, e = 75, f = 105, g = 75$

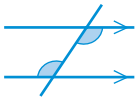
7 C

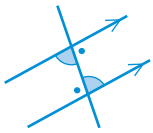
Exercise 2.08

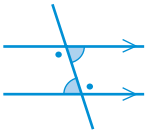
1 **a** g **b** d **c** d

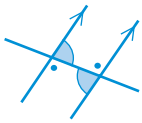
2 **a**


b


c


3 **a**


b


c


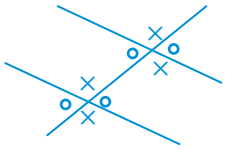
4 A

5 **a** 2

b $a = 94$, $b = 86$, $c = 94$, $d = 86$, $e = 94$, $f = 86$, $g = 94$

c They are equal

6 **a**



b They are equal

7 **a** They are equal **b** Yes

8 **a** Vertically opposite angles

b Corresponding angles

c Both b and c equal a

d They are equal

9 **a** $m = 110$ **b** $a = 50$

c $n = 80$ **d** $b = 122$

e $h = 20$ **f** $m = 50$, $n = 130$, $p = 50$

g $a = 140$, $b = 40$ **h** $a = 130$, $b = 130$

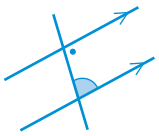
i $a = 136$, $b = 136$, $c = 44$

10 A

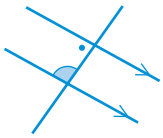
Exercise 2.09

1 **a** d **b** d **c** c

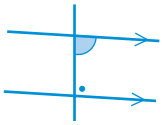
2 **a**



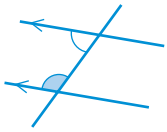
b



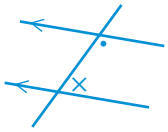
c



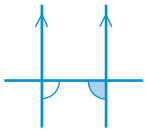
3 **a**



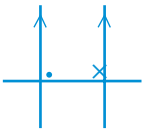
or



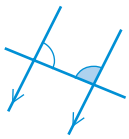
b



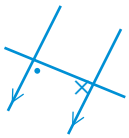
or



c



or



4 **a** 2 **b** A

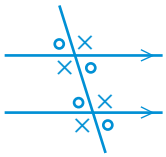
c $a = 59$, $b = 121$, $c = 59$, $d = 59$, $e = 121$, $f = 59$, $g = 121$

d They add up to 180°

5 Both sums are 180°

6 **a** 180°, Angles on a straight line

b



c They add up to 180°

7 **a** 180°, Angles on a straight line

b Alternate angles

c 180°

d They add up to 180°

8 **a** $a = 130$ **b** $m = 90$

c $b = 105$ **d** $d = 68$

e $m = 112$ **f** $a = 82$, $b = 98$

g $f = 50$, $g = 130$ **h** $j = 125$, $k = 125$

i $a = 51$, $b = 129$, $c = 51$

9 B

Exercise 2.10

1 **a** $\angle WXQ$ **b** $\angle XWP$

c $\angle WXS$ **d** $\angle WXQ$

e $\angle XWA$ **f** $\angle XWP$

2 (reasons abbreviated)

a 65 (co-int) **b** 71 (alt) **c** 105 (corr)

d 120 (corr) **e** 110 (co-int) **f** 132 (alt)

g 28 (alt) **h** 72 (corr) **i** 95 (co-int)

j 93 (corr) **k** 81 (alt) **l** 30 (co-int)

m 128 (corr) **n** 114 (co-int) **o** 109 (alt)

3 C

4 **a** $a = 113$, $b = 113$

b $j = 133$, $k = 133$, $l = 133$

c $m = 52$, $n = 52$, $p = 128$

d $y = 42$, $z = 42$ **e** $l = 95$, $m = 85$

f $b = 45$, $c = 30$ **g** $p = 105$, $q = 105$

h $k = 85$, $m = 85$ **i** $p = 63$, $w = 117$

j $k = 50$ **k** $x = 118$, $y = 55$

l $a = 72$, $b = 108$ **m** $m = 97$, $n = 83$, $p = 83$

n $g = 132$ **o** $a = 153$, $b = 153$, $c = 27$

5 C

6 $\angle EPC = \angle ABC$ (corresponding angles, $AB \parallel EP$)

$\angle DEF = \angle EPC$ (corresponding angles, $EF \parallel PC$)

$\therefore \angle ABC = \angle DEF$

Exercise 2.11

1 **a** $AB \parallel CD$, alternate angles are equal

$\angle BEF$ and $\angle CFE$

b $AB \parallel CD$, alternate angles are equal

$\angle AGH$ and $\angle GHD$

c AB is not parallel to CD, alternate angles are not equal

$\angle BEF$ and $\angle EFC$

d AB is not parallel to CD, corresponding angles are not equal

$\angle GEB$ and $\angle DFE$

e $AB \parallel CD$, corresponding angles are equal
$\angle AEF$ and $\angle CFG$

f AB is not parallel to CD, corresponding angles are not equal
$\angle CEF$ and $\angle AFG$

g $AB \parallel CD$, co-interior angles add to 180°
$\angle AEF$ and $\angle EFC$

h AB is not parallel to CD, co-interior angles do not add to 180°.
$\angle BEF$ and $\angle EFD$

i $AB \parallel CD$, corresponding angles are equal
$\angle AEF$ and $\angle CFG$

2 **a** 85 (vertically opposite angles)
b 95

3 55

4 **a** $PQ \parallel MN$, co-interior angles add to 180°
b $PQ \parallel MN$, corresponding angles are equal
c $PQ \parallel MN$, alternate angles are equal, corresponding angles are equal
d PQ is not parallel to MN, corresponding angles are not equal: $\angle PCB = 115°$ (angles on a straight line), $\angle MAD = 120°$
e PQ is not parallel to MN, alternate angles are not equal
f $PQ \parallel MN$, co-interior angles add to 180°

5 B

Power plus

1 **a** 360° **b** 15° **c** 120° **d** 2.5°

2 **a** left **b** front **c** back **d** front

3 **a** $m = 51$, vertically opposite, corresponding angles
b $x = 62$, alternate angles; $y = 62$, alternate angles
c $a = 55$, alternate, co-interior
d $m = 58$, alternate, straight angle, or co-interior
e $y = 75$, Teacher to check. (corresponding)
f $c = 35$, alternate angles, straight angle
g $x = 85$, alternate angles
h $m = 255$, Teacher to check. (corresponding angles)
i $k = 105$, Teacher to check. (corresponding, co-interior, and vertically opposite)

4 A parallelogram

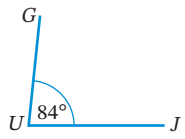
Test yourself 2

1, 2 Teacher to check.

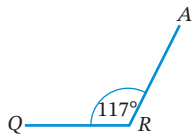
3 **a** $\angle ABD$ **b** $\angle QOS$

4 D

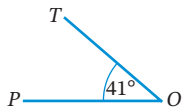
5 **a**



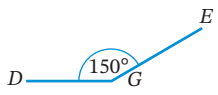
b



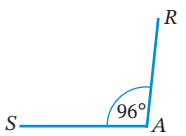
c



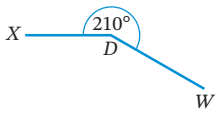
d



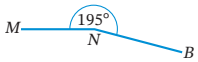
e



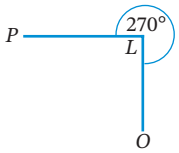
f



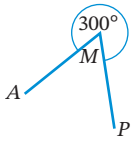
g



h



i



6 **a** $\angle WHI$ or $\angle IHW$, acute angle
b $\angle DAR$ or $\angle RAD$, obtuse angle
c $\angle GLU$ or $\angle ULG$, acute angle
d $\angle PVR$ or $\angle RVP$, acute angle
e $\angle NPE$ or $\angle EPN$, right angle
f $\angle SAM$ or $\angle MAS$, obtuse angle
g $\angle MTV$ or $\angle VTM$, straight angle
h $\angle XYZ$ or $\angle ZYX$, reflex angle
i $\angle MPQ$ or $\angle QPM$, reflex angle

7 **a** **i** 55° **ii** 12° **iii** 86°
b **i** 135° **ii** 80° **iii** 2°

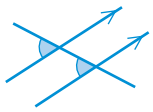
8 **a** $m = 62$ **b** $k = 133$
c $x = 58, y = 122$

9 **a** $a = 220$ **b** $m = 65$
c $y = 325$ **d** $a = 100$
e $m = 44$ **f** $a = 95, b = 85$
g $p = 145$ **h** $x = 42$
i $f = 70$ **j** $t = 83$
k $p = 155, q = 155, r = 25$
l $x = 30$

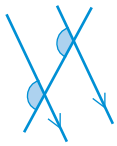
10 **a** AB and GH
b EF and GH, EF and AB
c AB and CD, AB and EF, GH and CD, GH and EF

11, 12 Teacher to check.

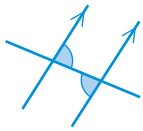
13 **a**



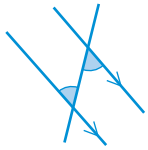
b



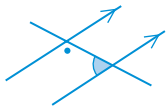
14 **a**



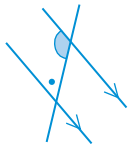
b



15 **a**



b



16 **a** alternate **b** co-interior
c corresponding **d** corresponding
e alternate **f** corresponding

17 **a** $a = 115$ **b** $m = 35$ **c** $k = 115$
d $d = 125$ **e** $x = 50$ **f** $q = 62$
g $x = 37, y = 37$
h $m = 62, p = 118, z = 118$
i $a = 112, d = 68, t = 112$
j $x = 50, y = 50$
k $x = 26, m = 64$
l $a = 72, c = 70, z = 38$
m $a = 38, c = 142$
n $x = 35, y = 88, z = 92$

18 **a** $AB \parallel CD$, co-interior angles add to 180°
b AB is not parallel to CD, alternate angles are not equal
c $AB \parallel CD$, corresponding angles are equal

19 A

CHAPTER 3

SkillCheck

1 Odd, the last digit is an odd number.
2 Yes, the last digit is 0.
3 **a** 21 **b** 30 **c** 32 **d** 18
e 36 **f** 40 **g** 49 **h** 54
i 64 **j** 63 **k** 24 **l** 56
m 9 **n** 81 **o** 25 **p** –16
4 **a** 1, 2, 4, 5, 10, 20 **b** 1, 2, 3, 4, 6, 12
c 1, 19
5 Yes, the last digit is even.
6 **a** 8 **b** 5 **c** 6 **d** 6
e 4 **f** 8 **g** 6 **h** 8
i 4 **j** 9 **k** 6 **l** 9
7 **a** 8, 16, 24, 32, 40 **b** 7, 14, 21, 28, 35
c 12, 24, 36, 48, 60
8 No, the last digit is not a 0.

Exercise 3.01

1 **a** 3100 **b** 49 000 **c** 2600 **d** 4 934 300
2 **a** 24 000 **b** 45 000 **c** 63 000 **d** 48 000
3 **a** 45 820 **b** 1700 **c** 8310 **d** 71 260
4 **a** 64 000 **b** 64 200 **c** 64 220 **d** 60 000
5 **a** 1 328 000 **b** 1 330 000
c 1 327 510 **d** 1 000 000
6 Your answers should be close to the following exact answers.
a 331 **b** 157 **c** 1587 **d** 255
e 421 **f** 203 **g** 413 **h** 734
i 6723 **j** 15 744 **k** 276 **l** 72.43...
7 27 800 **8** $17 500
9 104 000 **10** B
11 **a** 1000 km **b** 1000 km **c** 1000 km
12 25 922 000 **13** Teacher to check.

Exercise 3.02

1 **a** 238 **b** 1095 **c** 448 **d** 6825
2 $630
3 **a** 112 **b** 714 **c** 3096 **d** 3542
e 3252 **f** 630 **g** 1600 **h** 3424
4 552 km **5** 1140
6 **a** 170 **b** 78 **c** 128 **d** 228
e 112 **f** 176 **g** 128 **h** 264
7 **a** 110 **b** 70 **c** 140 **d** 180
e 270 **f** 410 **g** 80 **h** 240
8 **a** 306 **b** 459 **c** 243 **d** 171
e 450 **f** 1800 **g** 360 **h** 4100
9 C
10 **a** 32 **b** 3968 **c** $1664

Exercise 3.03

1 **a** 11 **b** 34 **c** 12 **d** 70
e 15 **f** 78 **g** 147 **h** 123
2 **a** $12\frac{1}{4}$ **b** $229\frac{1}{3}$ **c** $158\frac{2}{3}$ **d** $259\frac{2}{7}$
3 C **4** $34 each.
5 **a** 32 **b** 80 **c** 600 **d** 180
e 82 **f** 18 **g** 46 **h** 120
6 **a** 72 **b** 116 **c** 324 **d** 485
e 54 **f** 122 **g** 109 **h** 26
7 154 cartons

Exercise 3.04

1 **a** No **b** Yes **c** Yes **d** No
e No **f** Yes **g** Yes **h** No
2 **a** No **b** No **c** Yes **d** Yes
3 **a** No **b** Yes **c** No **d** No
e No **f** No **g** Yes **h** Yes
4 **a** Yes **b** No **c** No **d** No
e Yes **f** Yes **g** Yes **h** No
5 **a** No **b** No **c** Yes **d** Yes
e Yes **f** No **g** Yes **h** No
6 No
7 **a** 9 is not divisible by 2. **b** Last digit is not a 0 or 5.
c 5 is not divisible by 2. **d** 3 is not divisible by 2.
e Last digit is not a 0. **f** 1 is not divisible by 2.
8 C **9** 12, 6
10 **a** 60, 75 or 90 **b** 60 or 80 **c** 84
d 54, 60, 66, 72, 78, 84, 90, 96
11 D
12 **a** Yes **b** No **c** Yes **d** No
13 **a** Yes **b** Yes **c** No **d** Yes

Mental skills 3A

2 **a** 1800 **b** 26 000 **c** 770 000 **d** 1000
e 315 000 **f** 1 000 000 **g** 2940 **h** 47 500
i 240 **j** 1600 **k** 300 **l** 210
m 12 000 **n** 9900 **o** 1600 **p** 3500
q 400 **r** 500 **s** 18 000 **t** 3 600 000

Exercise 3.05

1 **a** XXVII **b** LII **c** LXXIX
d XCIII **e** MMDC **f** CCCXIV
g CMXXVIII **h** MMMCDLXXVI **i** CLXV
j CCXXX **k** MMCDLXXXII **l** CIX

2 **a** 26 **b** 40 **c** 264 **d** 54
e 77 **f** 45 **g** 339 **h** 528
i 2162 **j** 1990 **k** 98 **l** 1507

3 **a** Teacher to check.
b They are cumbersome to use, and they take too long to write.

4, 5 Teacher to check.

6 5000 and 10 000

Exercise 3.06

1 **a** 8^4 **b** 7^6 **c** $(-5)^3$
d 1^5 **e** 13^4 **f** 10^1
g $(-4)^2 \times 9^3$ **h** $2^2 \times 12^2$ **i** $3^2 \times 11^4$
j $15^2 \times (-6)^2$ **k** $2^3 \times 8^2$ **l** $10^2 \times 20^1$

2 **a** $3 \times 3 \times 3 \times 3 \times 3$ **b** 5×5
c $7 \times 7 \times 7 \times 7$ **d** $(-6) \times (-6) \times (-6)$
e $2 \times 2 \times 2 \times 2 \times 4 \times 4$
f $12 \times 12 \times 12 \times 11 \times 11 \times 11 \times 11 \times 11 \times 11$

3 **a** 16 **b** 125 **c** 32 **d** 1 000 000
e 243 **f** 2401 **g** 1 **h** 16
i 27 **j** 1 **k** −59 049 **l** 4096

4 **a** $2 \times 10^2 + 3 \times 10 + 4 \times 1$
b $7 \times 10^3 + 4 \times 10$
c $2 \times 10^4 + 8 \times 10^3 + 6 \times 10^2 + 7 \times 10 + 5 \times 1$
d $9 \times 10 + 2 \times 1$
e $7 \times 10^3 + 3 \times 10^2 + 6 \times 10 + 1 \times 1$
f $6 \times 10^4 + 3 \times 10^3 + 9 \times 10^2 + 2 \times 10 + 8 \times 1$
g $8 \times 10^2 + 7 \times 10 + 6 \times 1$
h $1 \times 10^5 + 7 \times 10^4 + 4 \times 10^3 + 9 \times 10^2 + 2 \times 10 + 2 \times 1$
i $9 \times 10^4 + 4 \times 10^3 + 6 \times 10^2 + 3 \times 10 + 1 \times 1$
j $3 \times 10^3 + 6 \times 10^2 + 9 \times 10 + 2 \times 1$
k $1 \times 10^2 + 4 \times 10 + 9 \times 1$
l $8 \times 10^6 + 7 \times 10^5 + 6 \times 10^3 + 9 \times 10^2 + 4 \times 10 + 5 \times 1$

5 **a** 73 **b** 490 537 **c** 18 345
d 6801 **e** 3 672 059 **f** 462

6

Power of 10	Number	Name
10^1	10	ten
10^2	100	hundred
10^3	1000	thousand
10^6	1 000 000	million
10^9	1 000 000 000	billion
10^{12}	1 000 000 000 000	trillion
10^{15}	1 000 000 000 000 000	quadrillion
10^{18}	1 000 000 000 000 000 000	quintillion
10^{21}	1 000 000 000 000 000 000 000	sextillion
10^{24}	1 000 000 000 000 000 000 000 000	septillion
10^{27}	1 000 000 000 000 000 000 000 000 000	octillion
10^{30}	1 000 000 000 000 000 000 000 000 000 000	nonillion
10^{33}	1 000 000 000 000 000 000 000 000 000 000 000	decillion

7 No, Anton is incorrect. The sequel is approximately $4\frac{1}{2}$ the size of the original.

Exercise 3.07

1 **a** 81 **b** 289 **c** 100

2 **a** 49 **b** 169 **c** 324
d 1 **e** 25 **f** 400
g 256 **h** 1156 **i** 225
j 9 **k** 49 **l** 169

3 **a** 20 **b** 15 **c** 18
d 16 **e** 13 **f** 34

4

Number, x	1	2	3	4	5	6	7	8	9	10	11	12
Number squared, x^2	1	4	9	16	25	36	49	64	81	100	121	144

5 B

6 **a** 10 **b** 6 **c** 11
d 4 **e** 1 **f** 9
g 8 **h** 12 **i** 5

7 **a** 7 **b** 9 **c** 25
d 22 **e** 42 **f** 19
g 30 **h** 28 **i** 16
j 14 **k** 20 **l** 56

8 0 and 1

9 **a** 64 **b** 1331 **c** 1

10 **a** 343 **b** 4096 **c** 125
d 1000 **e** 8000 **f** 4913
g 42 875 **h** −512 **i** −125

11 **a** −8 **b** 20 **c** 35
d −5 **e** 17 **f** 16

12

Number, x	1	2	3	4	5	6	7	8	9	10	11	12
Number cubed, x^3	1	8	27	64	125	216	343	512	729	1000	1331	1728

13 B

14 **a** 8 **b** 9 **c** 5 **d** 1 **e** 3
f 12 **g** 4 **h** 11 **i** 10

15 **a** 16 **b** 14 **c** 22
d 40 **e** 27 **f** 21

16 D **17** B

18 **a** 7.1 **b** 5.7 **c** 9.8
d 4.1 **e** 9.7 **f** 5.7

Mental skills 3B

2 **a** 20 **b** 60 **c** 450 **d** 30
e 190 **f** 26 **g** 53 **h** 720
i 6 **j** 6 **k** 4 **l** 9
m 70 **n** 30 **o** 22 **p** 8
q 40 **r** 5 **s** 800 **t** 6

Exercise 3.08

1 **a** 1, 17 **b** 1, 3, 7, 21
c 1, 2, 3, 4, 6, 8, 12, 24 **d** 1, 11
e 1, 5, 7, 35 **f** 1, 2, 4
g 1, 2, 3, 6, 9, 18 **h** 1, 23
i 1, 5, 25 **j** 1, 3, 9
k 1, 3 **l** 1, 19

2 **a** 3, 11, 17, 19, 23 **b** 4, 9, 18, 21, 24, 25, 35

3 C

4 **f** They are all prime numbers.

5 Primes: 17, 41, 59, 71, 101 Composites: 10, 27, 33, 62, 77, 129, 472, 999, 2064, 967 504

6 **a** 37, 41, 43, 47
b 66, 68, 69, 70, 72, 74, 75, 76, 77, 78
c 2, 3, 5, 7, 11, 13, 17, 19
d 32, 33, 34, 35, 36, 38, 39, 40, 42, 44, 45, 46

7 Teacher to check.

8 B

9 **a** Some possible answers: 1, 25, 9, 36, 4.
b Square numbers

Exercise 3.09

1 (Different factor trees are possible.)
a $2 \times 2 \times 2 \times 11$ **b** $3 \times 3 \times 7$
c $3 \times 3 \times 5$ **d** 3×17
e $2 \times 2 \times 3 \times 11$ **f** $2 \times 3 \times 3 \times 3 \times 5$
g $2 \times 2 \times 3 \times 3 \times 11$ **h** 2×109
i $2 \times 3 \times 3 \times 5 \times 7$ **j** $2 \times 2 \times 2 \times 5 \times 13$
k $5 \times 5 \times 11$ **l** $2 \times 3 \times 3 \times 19$

2 D **3** $2 \times 2 \times 2 \times 5 \times 7$

4 (Different factor trees are possible.)
a $2^4 \times 3$ **b** $2^3 \times 5^2$ **c** $2^2 \times 5 \times 23$ **d** $2^3 \times 89$
e 2×7^2 **f** $2^4 \times 3^2$ **g** $5^2 \times 13$ **h** $3^3 \times 5$

5 **a** $2 \times 2 \times 11 \times 11$ **b** 22

6 **a** 25 **b** 30 **c** 15 **d** 28
e 21 **f** 15 **g** 12 **h** 16

Exercise 3.10

1 C

2 **a** 1, 3, 9 **b** 1, 2, 4 **c** 1, 3, 5, 15 **d** 1, 2

3 **a** 4 **b** 3 **c** 2 **d** 4
e 14 **f** 4 **g** 5 **h** 6
i 25 **j** 8 **k** 30 **l** 13
m 12 **n** 12 **o** 9 **p** 70

4 **a** 9

5 **a** 162 **b** 12 **c** 35 **d** 4

6 Teacher to check.

Exercise 3.11

1 7, 14, 21, 28, 35 **2** B

3 16, 20, 24, 28, 32, 36, 40, 44 **4** 16

5 **a** 15 **b** 42 **c** 12 **d** 30
e 40 **f** 20 **g** 10 **h** 8
i 18 **j** 21 **k** 60 **l** 60

6 **a** 24

7 Teacher to check.

8 **a** 420 **b** 400 **c** 220 **d** 180

9 Teacher to check.

10 **a** 19 **b** 380

Power plus

1 **a** 8, 9, 25, 32, 125, 243
b 25, 64, 216, 243, 256, 343
c 81, 125, 128, 10 000, 14 641

2 **a** 1 **b** 121 **c** 12 321 **d** 1 234 321

3 **a** 123 454 321
b 1 234 567 654 321
c 12 345 678 987 654 321
d 1 234 567.876 543 21

4 **a** 6 **b** 60

5 **a** 9 **b** 9 **c** 200
d 8 **e** 75 **f** 2

6 **a** 50 **b** 90 **c** 100
d 1000 **e** 1100 **f** 10 000
g 800 **h** 420 **i** 100 000

7 3 and 5, 5 and 7, 11 and 13, 17 and 19, 29 and 31, 41 and 43, 59 and 61, 71 and 73

Test yourself 3

1 **a** 88 000 **b** 87 500 **c** 87 530 **d** 90 000

2 Estimates should be close to
a 1468 **b** 697 **c** 1767 **d** 75

3 **a** 48 **b** 170 **c** 297
d 200 **e** 882 **f** 10 170

4 **a** 240 **b** 124 **c** 162
d 132 **e** 16 **f** 121

5 **a** Yes **b** No **c** Yes **d** No
e No **f** Yes **g** No **h** No

6 B

7 **a** 3^4 **b** 6^5 **c** 11^7

8 **a** 36 **b** 125 **c** 100 000 **d** 256

9 **a** $4 \times 10^3 + 3 \times 10^2 + 2 \times 10 + 1 \times 1$
b $6 \times 10^2 + 5 \times 1$
c $7 \times 10^4 + 4 \times 10^3 + 2 \times 10^2 + 1 \times 1$
d $9 \times 10^3 + 1 \times 10 + 5 \times 1$

10 **a** 47 **b** 36 030 **c** 8052

11 **a** 7 **b** 12 **c** 20 **d** 9

12 **a** 29, 31, 37, 41, 43, 47
b 46, 48, 49, 50, 51, 52, 54, 55, 56, 57, 58, 60, 62, 63, 64, 65, 66, 68, 69
c 2, 3, 5, 7, 11, 13, 17, 19, 23, 29
d 42, 44, 45, 46, 48, 49, 50, 51, 52, 54, 55, 56, 57, 58

13 **a** $2^2 \times 3 \times 5$ **b** $2^3 \times 5^2$ **c** $2^4 \times 3^2$

14 **a** 16 **b** 18

15 **a** 4 **b** 12

16 81

17 **a** 24 **b** 90

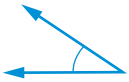
18 400

PRACTICE SET 1

1 **a** 6 **b** −5 **c** 18 **d** 17
e −2 **f** 6 **g** −7 **h** −6

2 **a** 103° **b** 57° **c** 295°

3 **a**

b

c

4 **a** 2 **b** 8 **c** 4

5 **a** −30 **b** 30 **c** −6 **d** 6
e 15 **f** −3 **g** 9 **h** −16

6 **a** 2500 **b** 17 200 **c** 807 900

7 A

8 **a** 1519 **b** 1870 **c** 26

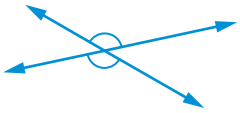
9 B

10 **a** 24° **b** 5° **c** 78°

11 **a** 148° **b** 90° **c** 75°

12 **a** No **b** No **c** Yes

13 D

14


15 Teacher to check.

16 **a** −11 **b** −26 **c** 10 **d** 15 **e** 0
f 31 **g** 11 **h** 52 **i** 60

17 C

18 **a** $m = 130$ **b** $b = 110$
c $p = 142$ **d** $a = 98$
e $p = 54$ **f** $a = 88, b = 92$
g $c = 142$ **h** $m = 55$
i $m = 37, p = 87$ **j** $x = 296$
k $x = 30$ **l** $y = 90$

19 **a** 36 **b** 27 **c** 5 **d** 11

20 18°C

21 **a** $9 \times 10^2 + 6 \times 10 + 3 \times 1$
b $8 \times 10^3 + 4 \times 10 + 2 \times 1$
c $5 \times 10^4 + 2 \times 10^3 + 3 \times 10^2 + 1 \times 1$

22 **a** 49 **b** 7350 **c** 225 905

23 **a** $18 = 2 \times 3^2$ **b** $45 = 3^2 \times 5$
c $360 = 2^3 \times 3^2 \times 5$

24 Yes: 111 + 69 = 180, so co-interior angles add up to 180°, therefore the lines are parallel.

25 **a** 12 **b** 9

26 **a** 77 **b** 42

27 Hannah is incorrect. The angle is 50°.

28 Liam is incorrect. $\sqrt{16} = 4$ and $\sqrt{25} = 5$, so $\sqrt{23}$ should be closer to 5, not 4. The value is approximately 4.8.

CHAPTER 4

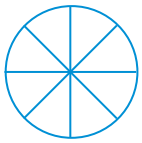
SkillCheck

1 **a** **i** $\frac{1}{4}$ **ii** $\frac{3}{4}$
b **i** $\frac{3}{6}$ **ii** $\frac{3}{6}$
c **i** $\frac{7}{12}$ **ii** $\frac{5}{12}$
d **i** $\frac{15}{20}$ **ii** $\frac{5}{20}$
e **i** $\frac{6}{9}$ **ii** $\frac{3}{9}$
f **i** $\frac{5}{9}$ **ii** $\frac{4}{9}$

2 $\frac{12}{31}$ **3** Teacher to check.

4 Teacher to check. **5** $\frac{1}{3}$

6 **a** 12 **b** 19 **c** 7
d 7 **e** 5 **f** 3

7


8 **a** 3 **b** 3 **c** 8

9 **a** 12 **b** 10 **c** 24

10 Teacher to check.

11 **a** $\frac{13}{60}$ **b** $\frac{3}{4}$ **c** $\frac{1}{6}$

12 $\frac{1}{3}$

Exercise 4.01

1 **a** M **b** P **c** M
d M **e** P **f** I
g P **h** P **i** I
j P **k** M **l** I

2 **a** $4\frac{1}{3}$ **b** $3\frac{1}{4}$ **c** $1\frac{3}{10}$ **d** $4\frac{1}{5}$
e $1\frac{3}{35}$ **f** $4\frac{3}{8}$ **g** 5 **h** $7\frac{1}{5}$
i 4 **j** $10\frac{10}{11}$ **k** $3\frac{15}{19}$ **l** $10\frac{1}{10}$

3 **a** $\frac{3}{2}$ **b** $\frac{13}{8}$ **c** $\frac{9}{7}$ **d** $\frac{10}{3}$
e $\frac{9}{2}$ **f** $\frac{37}{10}$ **g** $\frac{37}{6}$ **h** $\frac{38}{5}$

4 **a** 12 **b** 25 **c** 8
d 56 **e** 72 **f** 75
g 6 **h** 3 **i** 9

5 Teacher to check. **6** D

7 **a** $\frac{3}{8}$ **b** $\frac{1}{4}$ **c** $\frac{4}{5}$ **d** $\frac{3}{4}$
e $\frac{3}{7}$ **f** $\frac{4}{5}$ **g** $\frac{1}{4}$ **h** $\frac{2}{3}$

8 **a** C **b** D **c** C

9 **a** $2\frac{1}{4}$ **b** $2\frac{1}{7}$ **c** 2 **d** $1\frac{2}{3}$
e $2\frac{1}{3}$ **f** $1\frac{1}{4}$ **g** $1\frac{1}{3}$ **h** $1\frac{1}{3}$

10 **a** $1\frac{1}{2}$ **b** $3\frac{1}{6}$ **c** $4\frac{1}{4}$ **d** $6\frac{3}{4}$
e $17\frac{7}{20}$ **f** $100\frac{11}{16}$ **g** $1\frac{27}{50}$ **h** $5\frac{1}{3}$

11 D

12 A mixed numeral, because a proper fraction is less than 1 while a mixed numeral is greater than 1.

Exercise 4.02

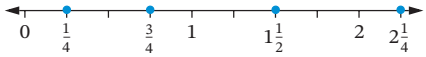
1 **a** $\frac{3}{10}$ **b** $\frac{2}{3}$ **c** $\frac{3}{4}$
d $\frac{2}{3}$ **e** $2\frac{1}{5}$ **f** $3\frac{4}{7}$

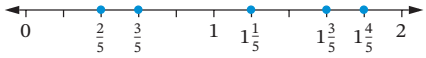
2 **a** $\frac{3}{5}$ **b** $\frac{9}{8}$ **c** $\frac{1}{4}$
d $\frac{1}{6}$ **e** $4\frac{2}{3}$ **f** $1\frac{1}{5}$

3 C

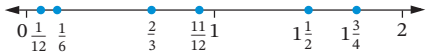
4 **a** $\frac{1}{2} > \frac{1}{3}$ **b** $\frac{2}{3} < \frac{3}{4}$ **c** $\frac{3}{8} < \frac{1}{2}$
d $2\frac{3}{5} < 2\frac{7}{10}$ **e** $\frac{5}{6} > \frac{1}{2}$ **f** $\frac{5}{12} > \frac{1}{3}$
g $\frac{17}{20} > \frac{6}{10}$ **h** $\frac{11}{15} > \frac{2}{3}$ **i** $2\frac{3}{7} < 5\frac{2}{5}$

5 Teacher to check.

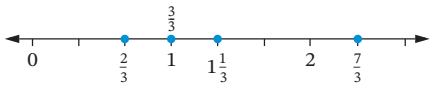
6 **a**


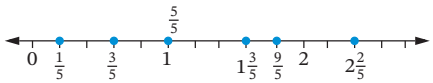
b


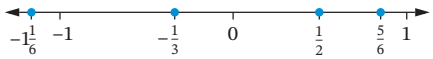
c

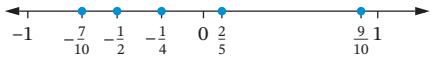
d


e

7 **a**


b


c


d


8 C **9** Teacher to check.

Exercise 4.03

1 **a** $\frac{2}{3}$ **b** $\frac{9}{10}$ **c** 1 **d** $\frac{4}{3}$
e $\frac{2}{5}$ **f** $\frac{1}{2}$ **g** $\frac{2}{5}$ **h** $\frac{1}{4}$

2 **a** $\frac{5}{6}$ **b** $\frac{13}{15}$ **c** $\frac{1}{4}$ **d** $\frac{1}{8}$
e $\frac{2}{3}$ **f** $\frac{1}{20}$ **g** $\frac{20}{63}$ **h** $\frac{3}{4}$
i $\frac{7}{8}$ **j** $\frac{3}{10}$ **k** $\frac{1}{9}$ **l** $1\frac{5}{24}$
m $\frac{27}{40}$ **n** $1\frac{11}{20}$ **o** $\frac{7}{24}$ **p** $1\frac{2}{35}$

3 D

4 **a** $\frac{1}{6}$ **b** $\frac{11}{12}$

5 $\frac{5}{12}$

6 **a** $4\frac{1}{2}$ **b** $\frac{2}{3}$ **c** 4 **d** $5\frac{1}{2}$

e $3\frac{2}{5}$ **f** $6\frac{3}{8}$ **g** $2\frac{2}{3}$ **h** 6

7 $\frac{7}{20}$ **8** Teacher to check.

Exercise 4.04

1 **a** 3 **b** $4\frac{1}{4}$ **c** $3\frac{2}{15}$ **d** $3\frac{1}{8}$

e $4\frac{1}{12}$ **f** $4\frac{1}{10}$ **g** $2\frac{1}{2}$ **h** $3\frac{5}{6}$

i $1\frac{1}{20}$ **j** $4\frac{3}{20}$ **k** $2\frac{1}{12}$ **l** $5\frac{10}{21}$

2 C

3 **a** $2\frac{1}{2}$ **b** $1\frac{2}{5}$ **c** $\frac{3}{7}$ **d** $3\frac{1}{3}$

e $1\frac{3}{4}$ **f** $2\frac{7}{12}$ **g** $13\frac{2}{9}$ **h** $21\frac{3}{20}$

4 **a** $1\frac{7}{10}$ **b** $\frac{3}{4}$ **c** $1\frac{21}{40}$ **d** $1\frac{19}{20}$ **e** $\frac{5}{6}$

f $1\frac{7}{10}$ **g** $1\frac{1}{2}$ **h** $2\frac{4}{5}$ **i** $1\frac{11}{12}$ **j** $\frac{3}{4}$

5 $3\frac{11}{12}$ h or 3 h 55 min

6 **a** $2\frac{1}{10}$ **b** $3\frac{5}{8}$

7 $1\frac{17}{20}$ h or 1 h 51 min **8** $\frac{3}{10}$

9 **a** **i**

4	$1\frac{1}{2}$	2
$\frac{1}{2}$	$2\frac{1}{2}$	$4\frac{1}{2}$
3	$3\frac{1}{2}$	1

ii

$\frac{1}{6}$	1	$\frac{5}{6}$
$1\frac{1}{3}$	$\frac{2}{3}$	0
$\frac{1}{2}$	$\frac{1}{3}$	$1\frac{1}{6}$

Exercise 4.05

1 **a** 9 **b** 7 **c** 20 **d** 8
e 5 **f** 22 **g** 3 **h** 30
i 15 **j** 50 **k** 75 **l** 12

2 **a** \$21 **b** \$8 **c** \$8 **d** 20c
e \$0.30 **f** \$2.50 **g** \$135 **h** 36c
i \$2.50 **j** \$12 **k** \$3200 **l** \$240

3 **a** 30 min **b** 135 min **c** 18 h
d 18 min **e** 73 days **f** 45 s
g 200 s **h** 18 min **i** 32 h
j 8 months

4 **a** 27 **b** 18

5 **a** 250 m **b** 2250 m **c** 45 mm
d 10 cm **e** 750 g **f** 3500 mL
g 320 m **h** 3 mm **i** 2700 g
j 1000 L **k** 4800 mm **l** 40 000 kg

6 D **7** 24

8 **a** 12 **b** $\frac{2}{3}$

9 54

10 **a** Ashleigh 9, Bethany 6, Thomas 2
b Only 17 horses are given to the children
c Teacher to check.

Mental skills 4A

2 **a** 14 **b** 9 **c** 7 **d** 8
e 3 **f** 8 **g** 10 **h** 5
i 33 **j** 5 **k** 15 **l** 18
m 26 **n** 6 **o** 36 **p** 6
q 14 **r** 160 **s** 14 **t** 2

Exercise 4.06

1 **a** $\frac{2}{15}$ **b** $\frac{9}{20}$ **c** $\frac{35}{48}$ **d** $\frac{9}{80}$

e $\frac{5}{16}$ **f** $\frac{9}{64}$ **g** $\frac{20}{63}$ **h** $\frac{21}{160}$

2 B

3 **a** $\frac{1}{7}$ **b** $\frac{3}{10}$ **c** $\frac{1}{18}$ **d** 1

e $\frac{5}{16}$ **f** $\frac{4}{21}$ **g** $\frac{3}{20}$ **h** $\frac{5}{32}$

i $\frac{3}{25}$ **j** 15 **k** $2\frac{2}{7}$ **l** $\frac{2}{25}$

m $\frac{11}{24}$ **n** $\frac{1}{64}$ **o** $\frac{3}{5}$ **p** $\frac{9}{16}$

4 **a** $\frac{1}{2}$ **b** $1\frac{5}{21}$ **c** $6\frac{3}{40}$

d $6\frac{17}{24}$ **e** $2\frac{7}{9}$ **f** $5\frac{4}{9}$

5 $42\frac{5}{8}$ hours

6 8 eggs, 20 cups of flour, 2 cups of sugar

7 **a** $\frac{1}{4}+\frac{1}{4}+\frac{1}{4}=3\times\frac{1}{4}$

$=\frac{3}{4}$

b $\frac{3}{5}+\frac{3}{5}=2\times\frac{3}{5}$

$=\frac{6}{5}=1\frac{1}{5}$

c $\frac{3}{8}+\frac{3}{8}+\frac{3}{8}+\frac{3}{8}+\frac{3}{8}=5\times\frac{3}{8}$

$=\frac{15}{8}=1\frac{7}{8}$

d $5\times\frac{1}{7}=\frac{5}{7}$

e $7 \times \frac{3}{20} = \frac{21}{20}$ $= 1\frac{1}{20}$

f $8 \times \frac{3}{5} = \frac{24}{5}$ $= 4\frac{4}{5}$

8 $17\frac{1}{2}$ hours

9 **a** $\frac{7}{8}$ **b** $\frac{2}{3}$ **c** $\frac{4}{5}$

d Teacher to check.

10 False, Teacher to check.

Exercise 4.07

1 **a** $\frac{3}{2}$ **b** 5 **c** $\frac{8}{5}$ **d** 10

e $\frac{1}{7}$ **f** $\frac{2}{5}$ **g** $\frac{4}{7}$ **h** $\frac{1}{9}$

2 **a** 12 **b** 10 **c** 21 **d** 15

e 42 **f** 80 **g** 16 **h** 8

3 C

4 $38\frac{2}{5}$ hours

5 **a** $\frac{3}{8}$ **b** 4 **c** $1\frac{1}{2}$

d $1\frac{9}{16}$ **e** $\frac{21}{25}$ **f** $\frac{24}{25}$

g $\frac{11}{18}$ **h** $\frac{1}{2}$ **i** 2

j $1\frac{1}{4}$ **k** $\frac{1}{5}$ **l** $\frac{3}{4}$

6 **a** 1 **b** 0 **c** $\frac{7}{10}$

7 **a** It equals 1

b Your answer is the number squared

8 B

9 **a** $\frac{1}{7}$ **b** $\frac{5}{49}$ **c** $\frac{1}{12}$ **d** $\frac{5}{12}$

10 **a** $3\frac{3}{4}$ **b** $2\frac{4}{5}$ **c** $\frac{7}{8}$ **d** $1\frac{1}{3}$

11 8

12 **a** $\frac{2}{21}$ **b** $\frac{7}{10}$

13 True, Teacher to check.

Exercise 4.08

1 258

2 **a** 66 **b** 7

3 **a** Anthony **b** $\frac{11}{15}$ **c** $\frac{4}{15}$

4 $127\frac{1}{2}$

5 20

6 **a** $\frac{5}{6}$ **b** 50 minutes

7 2

8 40

9 $6\frac{3}{4}$ hours

10 280

11 $\frac{1}{30}$

12 D

13 $\frac{21}{40}$

14 $\frac{1}{9}$

15 **a** 24 **b** 4

Exercise 4.09

1 **a** $\frac{2}{3}$ **b** $\frac{1}{2}$ **c** $\frac{1}{3}$

d $\frac{4}{5}$ **e** $\frac{6}{7}$ **f** $\frac{4}{5}$

2 **a** $\frac{11}{4}$ **b** $\frac{16}{3}$ **c** $\frac{30}{7}$

d $\frac{33}{5}$ **e** $\frac{31}{3}$ **f** $\frac{53}{6}$

3 **a** $3\frac{3}{5}$ **b** $3\frac{2}{7}$ **c** $5\frac{5}{8}$

d $7\frac{2}{11}$ **e** $12\frac{3}{10}$ **f** $19\frac{1}{12}$

4 **a** $\frac{1}{10}$ **b** $1\frac{1}{14}$ **c** $1\frac{4}{15}$ **d** $\frac{1}{14}$

e $1\frac{1}{2}$ **f** $1\frac{11}{24}$ **g** $\frac{4}{15}$ **h** $\frac{21}{32}$

i $1\frac{1}{6}$ **j** 6 **k** $3\frac{5}{6}$ **l** $2\frac{5}{8}$

m $4\frac{1}{8}$ **n** $15\frac{1}{9}$ **o** $\frac{1}{10}$ **p** $3\frac{9}{20}$

5 **a** 68 **b** 56 **c** 25

d \$510 **e** \$1680 **f** \$88

6 $17\frac{1}{3}$

7 **a** $12\frac{3}{4}$ h **b** \$159.38

8 208

9 0.015 g

Exercise 4.10

1 **a** $\frac{1}{20}$ **b** $\frac{7}{50}$ **c** $\frac{2}{25}$ **d** $\frac{9}{10}$

e $\frac{7}{100}$ **f** $\frac{7}{200}$ **g** $\frac{31}{300}$ **h** $\frac{23}{400}$

2 **a** 0.28 **b** 0.06 **c** 0.161 **d** 0.213

e 0.02 **f** 0.2 **g** 0.7807 **h** 0.265

3 **a** 9% **b** 17% **c** $62\frac{1}{2}$% **d** 90%

e 22% **f** 30% **g** 20% **h** 55%

i 75% **j** 48% **k** $42\frac{1}{2}$% **l** 82%

m $6\frac{1}{4}$% **n** $33\frac{1}{3}$% **o** 150% **p** $22\frac{2}{9}$%

4 **a** 24% **b** 50% **c** 86% **d** 57.1%
e 6% **f** 34.5% **g** 240% **h** 0.7%
i 130% **j** 0.1% **k** 31.5% **l** 2.6%

5 A **6** A

Exercise 4.11

1 **a** 6 **b** \$204 **c** 7.5 **d** 84 mm
e 148 kg **f** 7.2 g **g** \$1665 **h** 10.8
i \$96 **j** 224 mL **k** \$1.30 **l** 405 m
m \$15.40 **n** 2.7 ha **o** 15 min

2 B **3** 42 **4** 312 girls

5 **a** 124 chickens **b** 651 other animals

6 148 **7** D

8 **a** \$150 **b** \$160

Mental skills 4B

2 **a** 15 **b** 6 **c** 3 **d** 2
e 9 **f** 19 **g** 16 **h** 80
i 16 **j** 7 **k** 17 **l** 54
m 30 **n** 7 **o** 9 **p** 18

Exercise 4.12

1 **a** $\frac{4}{5}$ **b** No (only got 80%)

2 C

3 **a** $\frac{2}{5}$ **b** 20% **c** 12, 40%

4 **a** Maths 74% English 70% Science $83\frac{1}{3}$%
Geography $62\frac{1}{2}$% Health 65% Technology 92%
b Technology

5 **a** 40% **b** 25% **c** 70% **d** 25%
e 5% **f** 3% **g** 25% **h** 25%
i 70% **j** 45% **k** 15% **l** 20%

Power plus

1 **a** $\frac{4}{25}$ **b** $\frac{1}{27}$ **c** $\frac{1}{8}$ **d** $\frac{49}{100}$ **e** $\frac{27}{64}$
f $\frac{2}{3}$ **g** $\frac{5}{7}$ **h** $\frac{9}{10}$ **i** $\frac{3}{4}$ **j** $\frac{3}{4}$

2 **a** 75 m **b** 45 m

3 16 **4** 7 eggs

5 **a** 2 **b** $1\frac{1}{4}$ **c** $\frac{3}{5}$

6 **a** $33\frac{1}{3}$% **b** $16\frac{2}{3}$% **c** $14\frac{2}{7}$% **d** $11\frac{1}{9}$%
e $9\frac{1}{11}$% **f** $8\frac{1}{3}$% **g** $66\frac{2}{3}$% **h** $44\frac{4}{9}$%

7 **a** \$262.50 **b** 2 kg **c** 12 L

Test yourself 4

1 **a** $\frac{7}{4}$ **b** $\frac{22}{9}$ **c** $\frac{28}{5}$ **d** $\frac{22}{7}$

2 **a** $1\frac{3}{4}$ **b** $8\frac{2}{3}$ **c** $3\frac{2}{7}$ **d** $1\frac{12}{24}=1\frac{1}{2}$

3 **a** $\frac{1}{3}=\frac{5}{15}$ **b** $\frac{3}{4}=\frac{12}{16}$ **c** $\frac{4}{5}=\frac{16}{20}$
d $\frac{5}{12}=\frac{15}{36}$ **e** $\frac{1}{2}=\frac{25}{50}$ **f** $\frac{3}{20}=\frac{15}{100}$

4 **a** $\frac{5}{6}$ **b** $\frac{3}{4}$ **c** $\frac{3}{4}$
d $1\frac{1}{3}$ **e** $\frac{1}{2}$ **f** $\frac{4}{7}$

5 **a** $\frac{3}{10}, \frac{2}{5}, \frac{7}{10}, \frac{4}{5}$ **b** $\frac{1}{3}, \frac{1}{2}, \frac{7}{12}, \frac{11}{12}$

6 **a** 1 **b** $\frac{7}{10}$ **c** $\frac{1}{6}$
d $\frac{11}{20}$ **e** $\frac{5}{6}$ **f** $\frac{1}{8}$

7 **a** $9\frac{2}{3}$ **b** $4\frac{1}{3}$ **c** $\frac{5}{6}$
d $4\frac{3}{10}$ **e** $1\frac{1}{3}$ **f** $3\frac{2}{3}$

8 **a** A 9, B 8, R 12, O 7, T 20, H $3\frac{1}{2}$
b $3\frac{1}{2}$, 7, 8, 9, 12, 20; H O B A R T

9 **a** 16 hours **b** 1600 m
c 135 min **d** 105 mm
e 50 seconds **f** 125 cm

10 **a** $\frac{2}{15}$ **b** 6 **c** $\frac{5}{14}$
d $\frac{1}{21}$ **e** $\frac{3}{4}$ **f** $\frac{1}{2}$
g $2\frac{1}{4}$ **h** 8 **i** $1\frac{1}{5}$

11 **a** 50 **b** $\frac{1}{6}$ **c** $1\frac{1}{2}$ **d** 2 **e** 6
f $\frac{32}{35}$ **g** 3 **h** 2 **i** $1\frac{1}{2}$

12 $6\frac{7}{12}$ litres **13** 1 spotted

14 **a** $\frac{25}{72}$ **b** $1\frac{19}{30}$ **c** $\frac{1}{2}$
d $\frac{1}{40}$ **e** $1\frac{5}{12}$ **f** $10\frac{8}{9}$

15 **a** **i** $\frac{13}{100}$ **ii** 0.13
b **i** $\frac{2}{25}$ **ii** 0.08
c **i** $\frac{3}{40}$ **ii** 0.075

16 **a** 70% **b** 92% **c** $22\frac{1}{2}$%
d 80% **e** 36.1% **f** 4%

17 0.6, 60%, $\frac{4}{5}$, 80%, $\frac{41}{100}$, 0.82

18 **a** 36 **b** \$72 **c** 6
d \$23.10 **e** 51 kg **f** 234 m

19 13.2 min **20** $\frac{4}{13}$ **21** 20%

CHAPTER 5

SkillCheck

1 **a** 8 **b** 4 **c** 6 **d** 15
e 2 **f** 30 **g** 6 **h** 9
i −3 **j** −3 **k** −1 **l** −7

2 **a** 6 **b** 6 **c** 10
d 7 **e** −4 **f** 7

3 **a** 11 **b** 20 **c** 11 **d** 9 **e** 4
f 7 **g** 25 **h** 1 **i** 2 **j** 32

4 **a** 150 **b** 252 **c** 140
d 80 **e** 130 **f** 26

5 Yes

6 **a** 12 **b** 20 **c** 9
d 11 **e** 7 **f** 5

7 **a** −11 **b** 17 **c** 1 **d** 14
e 16 **f** 6 **g** −13 **h** 21

8 **a** −3 **b** −5 **c** −12 **d** −2
e 3 **f** −11 **g** 36 **h** 3
i −10 **j** −8 **k** −9 **l** −72

Exercise 5.01

1 **a** 25 **b** 118 **c** 48
d 150 **e** 150 **f** 80
g 10 **h** 65 **i** 14
j 119 **k** 100 **l** 84

2 **a** 170 **b** 810 **c** 1400
d 640 **e** 140 **f** 150
g 80 **h** 400 **i** 80
j 600 **k** 90 **l** 1400

3 **a** 7000 **b** 10 000 **c** 270 **d** 2800
e 400 **f** 1200 **g** 160 **h** 24 000
i 24 000 **j** 3300 **k** 1800 **l** 2500
m 320 **n** 400 **o** 21 000 **p** 210 000

4 D **5** D

6 **a** F **b** T **c** T **d** F
e T **f** F **g** F **h** T
i F **j** T **k** T **l** F

Exercise 5.02

1 **a** $18 \times 12 = 18 \times (10 + 2)$
$= 18 \times 10 + 18 \times 2$
$= 180 + 36$
$= 216$

b $16 \times 9 = 16 \times (10 - 1)$
$= 16 \times 10 - 16 \times 1$
$= 160 - 16$
$= 144$

c $21 \times 11 = 21 \times (10 + 1)$
$= 21 \times 10 + 21 \times 1$
$= 210 + 21$
$= 231$

d $15 \times 8 = 15 \times (10 - 2)$
$= 15 \times 10 - 15 \times 2$
$= 150 - 30$
$= 120$

2 **a** 135 **b** 162 **c** 234 **d** 279
e 128 **f** 104 **g** 176 **h** 112
i 243 **j** 360 **k** 304 **l** 261

3 B

4 **a** 187 **b** 264 **c** 165 **d** 319
e 300 **f** 168 **g** 288 **h** 396
i 480 **j** 132 **k** 108 **l** 209

5 **a** $6 \times 22 = 6 \times (20 + 2)$
$= 6 \times 20 + 6 \times 2$
$= 120 + 12$
$= 132$

b $12 \times 19 = 12 \times (20 - 1)$
$= 12 \times 20 - 12 \times 1$
$= 240 - 12$
$= 228$

c $40 \times 41 = 40 \times (40 + 1)$
$= 40 \times 40 + 40 \times 1$
$= 1600 + 40$
$= 1640$

d $25 \times 48 = 25 \times (50 - 2)$
$= 25 \times 50 - 25 \times 2$
$= 1250 - 50$
$= 1200$

6 **a** 160 **b** 204 **c** 1188 **d** 816
e 174 **f** 252 **g** 270 **h** 539

7 A

Exercise 5.03

1 **a** $u \div u = 1$ **b** $p + p = 2 \times p$ **c** $a + (-a) = 0$
d $p \times 0 = 0$ **e** $u + 0 = u$ **f** $-a \times (-a) = a^2$

2 **a** $3w$ **b** b^2 **c** $9acd$ **d** $6a$
e $\frac{e}{5}$ **f** $2f$ **g** $7m$ **h** $6wh$
i $3m$ **j** $4f^2$ **k** d **l** $10abc$
m $\frac{16}{n}$ **n** $2g^2$ **o** $9x$ **p** h
q $8q$ **r** $8mn$ **s** $-12pr^2$ **t** $3a + 2b$
u a^3b^2

3 B

4 **a** $\frac{16}{5a}$ **b** $7 + 3n$ **c** $\frac{e-6}{2}$
d $4s - 8$ **e** $c^2 + d^2$ **f** $\frac{k}{9} + m$
g $\frac{k}{9+m}$ **h** $12 - \frac{r}{2}$ **i** $7 + 7u^2$
5 A
6 **a** $x \times y$ **b** $2 \times n \div r$ **c** $1 \times x = x$
7 **a** $12 \times r \times s$ **b** $-2 \times n \times n$
c $8 \div f$ **d** $r \times r + t \times t$
e $9 \times a \times a \times b$ **f** $5 \times m \times n - 2 \times a$
g $(x + 1) \div 3$ **h** $q \times q \times q$
i $14 - 2 \times d \times d$ **j** $-4 \times c \div 5$
k $x \times x \times y \times y$ **l** $4 \times j - 8 \div k$
8 C

Exercise 5.04

1 **a** $N + N$ **b** $\frac{N}{2}$ **c** $3N$ **d** $N - 7$
e $\frac{N}{10}$ **f** $N - 1$ **g** $5N$ **h** $N + 21$
i $N - 1$ **j** $\frac{N}{8}$ **k** $N + 3$ **l** N^3
m $\sqrt{N}$
2 No difference except the N would be replaced by the X. The letter used makes no difference to the expression.
3 **a** $x + y + z$ **b** $b - c$ or $c - b$ **c** uv
d uv^2 **e** $\frac{C}{d}$ **f** $\frac{m+n}{2}$
g $\sqrt{10a}$ **h** $b - 3$ **i** $5 - c$
j $c - 5$ **k** $\frac{e}{3}$ **l** t^2
m $9xz$ **n** $k - 12$
o $\frac{1}{2}(20 - g)$ or $\frac{20-g}{2}$
4 C
5 **a** $b + g$ **b** $4n$ **c** $d - r$ **d** $\$2s$
e $\$\frac{a}{3}$ **f** $2x + 2y$ or $2(x + y)$
g xy **h** $\$(5m + 3i)$
6 C

Mental skills 5A

2 **a** 160 **b** 130 **c** 60 **d** 140
e 210 **f** 270 **g** 190 **h** 220
i 300 **j** 170 **k** 80 **l** 290

Exercise 5.05

1 **a** 15 **b** 51 **c** 122 **d** −18
2 **a** 40 **b** 58 **c** 17 **d** 0
3 **a** 8 **b** 44 **c** −16 **d** 2
4 **a** 16 **b** −19 **c** 101 **d** 36
5 **a** 73 **b** −53 **c** −8 **d** 10
6 **a** 5 **b** 11 **c** −3 **d** 19
7 **a** 22 **b** 26 **c** 4 **d** 34
8 **a** 16 **b** −3 **c** −24
9 **a** 10 **b** 22 **c** −50
10 **a** 14 **b** 35 **c** −28
11 C **12** \$3740
13 27 m² **14** $10\frac{1}{2}$
15 86°F **16** \$149 **17** A

Exercise 5.06

1 **a** $x + 2$ **b** $2x + 3$
c $2 + 2x$ **d** $3x + 4$
e $x + 1 + x + 1$ or $2x + 2$
f $2 + x + 2 + 2x$ or $3x + 4$
g $1 + x + 3$ or $x + 4$
h $2x + 2 + 2x + 1$ or $4x + 3$
i $x + 3 + x$ or $2x + 3$
j $4x + 5$
2 **a** $x + y + 2$ **b** $2x + 2y + 1$ **c** $x + 2y$
d $2x + 2y + 3$ **e** $x + 2y + 2$ **f** $2x + 3y + 4$
g $x + y$ **h** $2x + 3y + 1$
3 **a**

b

c

d

e

f

4 B
5 **a** D **b** A **c** F
d B **e** C **f** E
6 **a** D **b** F **c** A
d B **e** C **f** E
7 **a** $m + 2p$ **b** $m + p$ **c** $3p + 5$
d $2m + 2p + 1$ **e** $m + 2p$
8 C
9 **a** $4a + 5d$ **b** $5h + 9r$ **c** e
d $4k - 3$ **e** $2x + 6z$ **f** $2p + s$
g $4u + 12$ **h** $4n - 1$ **i** $3i$

Exercise 5.07

1 **a** $4p, 2p, p$ **b** $m^2, 3m^2$
c $5ac, 7ca$ **d** $vw, 2wv$ and $5v, 9v$
e $6pq, 2qp, 3pq$ **f** $2d, 3d$
g $4mn, 2nm, mn$ **h** $x^2y, 4x^2y$
i $5ba, ab$
2 B
3 **a** $11k$ **b** $7mn$ **c** $2xy$ **d** $9abc$
e $6x^2$ **f** $5ef^2$ **g** $5d$ **h** $5mk$
i $4xy$ **j** $-4de$ **k** $3k^2$ **l** $3w^2$
4 C
5 **a** $5k + 4j$ **b** $9ab + 2$ **c** $7m^2 + 2m$
d $10s^2 - 7st$ **e** $10mn$ **f** $9k + 3$
g $2y^2$ **h** $12d - 4e$ **i** $4 - 2x$
j $8bg$ **k** $9 - 2g$ **l** $5a + 2c$

6 **a** $3x - 3y$ **b** $4ab + 3bc$ **c** $9k^2 + 11$
d $6bc - 9a$ **e** $x^2 - 7x + 7$ **f** $4p^2$
g $12y + 14$ **h** $7r + 3s$ **i** $7b + 6g$
j $6x^2 - 13x + 6$ **k** $52a$ **l** $7q - 7$
m $-6y$ **n** 12

7 C **8, 9** Teacher to check.

Exercise 5.08

1 D
2 **a** $15y$ **b** $16m$ **c** $4km$ **d** $20t$
e $12f$ **f** $16xy$ **g** $30mp$ **h** $18bd$
i $12d^2$ **j** $120mn$ **k** $48y^2$ **l** $27r^2$
m $72ab$ **n** x^2y^2 **o** $42jk$ **p** $10r^2s^2t$
q $18ac^2d$ **r** $h^2j^3k^2$ **s** $24p^2q^2$ **t** $6m^2n^2$
3 **a** $-8bd$ **b** $-24m^2$ **c** $-90kl$ **d** $-60p$
e $-63k$ **f** $75b^2$ **g** $-12m^2n^2$ **h** $-12a^2b^2c$
i $40r^2$ **j** $-216d^2e$ **k** $144x^2$ **l** $-40m^2$
4 B
5 **a** y^4 **b** a^3 **c** $2c^2$ **d** t^5
e $7q^3$ **f** k^3l^2 **g** m^3n^2 **h** $-5p^4k^3$
i $-3y^3$ **j** $-d^2e^2f$ **k** $4q^2r^4$ **l** $11p$
m $35mn$ **n** a^2 **o** $2ry$ **p** $-27p$
6 Teacher to check.

Exercise 5.09

1 A
2 **a** $4m$ **b** $9e$ **c** $-7p$ **d** $9x$
e $4w$ **f** 8 **g** -5 **h** $\frac{3}{4}$
i $2p$ **j** -7 **k** $\frac{p}{5}$ **l** $-\frac{7n}{m}$
3 **a** $2m^2$ **b** $-3e$ **c** $-5p$ **d** $7x$
e $-2w$ **f** $4m$ **g** $-8ky$ **h** $\frac{3}{4}$
i $5de$ **j** a **k** $-8a$ **l** $-2m$
4 D
5 **a** -3 **b** -3 **c** $-15y^3$ **d** $6e$
e $6x^2$ **f** $8mn$ **g** 3 **h** $3a$
i $\frac{a}{5c}$ **j** $2d$ **k** $\frac{5p}{m}$ **l** 1
m $8x$ **n** $9r + 14$ **o** $18p$
6 Teacher to check.
7 **a** $4w^2$ **b** 0

Exercise 5.10

1 **a** $x = 3$ **b** $x = 9$ **c** $a = 5$ **d** $m = 20$
e $b = 10$ **f** $c = 4$ **g** $k = 6$ **h** $d = 13$
i $m = 15$ **j** $x = 5$ **k** $y = 15$ **l** $n = 25$
2 B
3 **a** $x = 5$ **b** $p = 7$ **c** $k = 5$ **d** $x = 4$
e $x = 3$ **f** $x = 8$ **g** $m = 28$ **h** $a = 12$
i $d = 2$ **j** $n = 1$ **k** $r = 15$ **l** $k = 11$
m $p = -2$ **n** $r = 10$ **o** $y = 15$ **p** $x = 2$
4 B

Exercise 5.11

1 **a** $w = 7$ **b** $x = 18$ **c** $m = 16$
d $p = 18$ **e** $x = 9$ **f** $k = 9$
g $m = -3$ **h** $y = 10$ **i** $d = -9$
2 **a** $m = 6$ **b** $n = 4$ **c** $k = 11$
d $c = 16$ **e** $x = 9$ **f** $x = -9$
g $d = 6$ **h** $h = -5$ **i** $a = 7$
3 **a** $p = 11$ **b** $m = 13$ **c** $x = 17$
d $y = 60$ **e** $k = 24$ **f** $n = 21$
g $d = 18$ **h** $y = 9$ **i** $m = 4$
4 **a** $m = 12$ **b** $d = 10$ **c** $x = 40$ **d** $k = 48$
e $x = 60$ **f** $a = -8$ **g** $k = 30$ **h** $n = -55$
5 D
6 **a** $m = 19$ **b** $p = 8$ **c** $p = 19$
d $n = -8$ **e** $x = 10$ **f** $n = 9$
g $x = 7$ **h** $y = -4\frac{1}{2}$ **i** $h = 7$
j $x = 3$ **k** $k = -10$ **l** $n = 0$
7 B

Exercise 5.12

1 **a** $x = 5$ **b** $x = 3$ **c** $x = 2$
d $x = 4$ **e** $x = 9$ **f** $x = -8$
g $x = 2$ **h** $x = 1$ **i** $x = 4$
j $x = 3$ **k** $x = -1$ **l** $x = -2$
2 D
3 **a** $x = 7$ **b** $x = 8$ **c** $x = 3$
d $x = 11$ **e** $x = 2$ **f** $x = 2$
g $x = 3$ **h** $x = 5$ **i** $x = -5$
j $x = 9$ **k** $x = -2$ **l** $x = -1$
4 D
5 **a** $x = 8$ **b** $m = 9$ **c** $k = 20$
d $a = 16$ **e** $n = 16$ **f** $h = -35$
g $m = 6$ **h** $k = 15$ **i** $x = -12$
j $a = 9$ **k** $h = -15$ **l** $x = 14$
6 D

Mental skills 5B

2 **a** 108 **b** 243 **c** 414 **d** 171
e 306 **f** 567 **g** 189 **h** 135
4 **a** 187 **b** 242 **c** 418 **d** 440
e 275 **f** 209 **g** 594 **h** 341
6 **a** 528 **b** 180 **c** 348 **d** 372
e 624 **f** 216 **g** 312 **h** 444

Exercise 5.13

1 **a** $5t = \$55, t = \$11$ **b** $10x = \$4.80, x = \$0.48$
c $2n = 110, n = 55$ **d** $y - 4 = 6, y = 10$
2 **a** B, 299 **b** A, 1140 mL
c A, 39 **d** D, 53
3 **a** $4a + 12 = 240, a = 57$
b $2l + 17 + 17 = 100, l = 33$
c $6y - 13 = 95, y = 18$
d $n + (n + 1) + (n + 2) = 48, n = 15$
e $\frac{11b}{2} = 44, b = 8$

f $7n - 184 = 2000$, $n = 312$

g $200 + \frac{x}{5} = \$750$, $x = \$2750$

4 5 **5** 391 **6** A **7** D **8** A

Power plus

1 **a** $5a - 3b$ **b** $-4m - 10n$
c $4j - k + 4$ **d** $5b^2$
e $3x^2 - 6x$ **f** $5xy + 2yz + 4xz$

2 **a** -2 **b** -24 **c** 10 **d** 9
e 8 **f** 4 **g** 7 **h** 4

3 **a** **i** $P = 2l + 2b$ or $2(l + b)$
ii $A = lb$
b **i** $P = 2 + y + x + y + 2 + x$ or $2(x + 2 + y)$
$= 4 + 2x + 2y$
ii $A = x(2 + y)$
c **i** $P = p + q + r$
ii $A = \frac{pq}{2}$
d **i** $P = m + 5 + m + 3 + n$
$= 8 + 2m + n$
ii $A = mn + \frac{1}{2} \times 3 \times n = mn + \frac{3n}{2}$
e **i** $P = x + y + y + x + x$
$= 3x + 2y$
ii $A = x^2 + \frac{1}{2} \times x \times 2$
$= x^2 + x$
f **i** $P = a + b + c + (b - d) + (a - c) + d$
$= 2a + 2b$
ii $A = ad + c(b - d)$
or $bc + d(a - c)$
or $ab - [(a - c) \times (b - d)]$

4 **a** x **b** x^2 **c** 0 **d** 1 **e** $2x$
f 0 **g** $-x$ **h** x^2 **i** 1 **j** $\frac{x^2 + 1}{x}$

5 **a** $-4p - 9q$ **b** $-6m - 3n - 7mn$
c $4j - k + 4$ **d** $5b^2$
e $3x^2 - 6x$ **f** $5xy + 2yz + 4xz$
g $-4d^2 + 4e^2 + 16$ **h** $13f^2 + 6f + 1$

6 **a** $x = -1$ **b** $k = -3$ **c** $n = -2$
d $r = 9$ **e** $d = 0$ **f** $z = -2\frac{1}{2}$

Test yourself 5

1 **a** 28 **b** 118 **c** 170
d 3100 **e** 390 **f** 2800

2 **a** 198 **b** 136 **c** 297 **d** 156

3 $n - (-n) = 2 \times n$

4 **a** $4x$ **b** $\frac{u}{5}$ **c** y
d $10a$ **e** $24abc$ **f** $20mw^2$

5 **a** $4 \times m \times n \times k$ **b** $6 \times a \div b$ **c** $9 \times c \times c$

6 **a** $3N$ **b** $N - 5$ or $5 - N$ **c** $N + 1$
d $\frac{N}{3}$

7 **a** $M + 3$ **b** $B + 5$ **c** $2H - k$

8 **a** 7 **b** 0 **c** 6 **d** 60

9 **a** 109 **b** -11 **c** 169 **d** 94

10 **a** $12k$ **b** $17d$ **c** $17m$
d $18m$ **e** $3ac$ **f** $9cd$
g $10x + 10y$ **h** $15r + 15s$ **i** $28h + 7g$

11 **a** $10x$ **b** $14bd$ **c** $66m$ **d** $24pq$
e $32w^2$ **f** $75j$ **g** $36m^2$ **h** $60p^3$

12 **a** $25x$ **b** $\frac{4b}{d}$ **c** $11m$ **d** $8rs$
e 5 **f** $7d^2$ **g** $7e$ **h** $13b$
i $2x$ **j** $4mn^2$ **k** $4e$ **l** $5ae$

13 **a** $m = 4$ **b** $k = 12$ **c** $d = 18$

14 **a** $w = 35$ **b** $e = 12$ **c** $p = 3$ **d** $x = 12$

15 **a** $t = 2$ **b** $n = 3$ **c** $y = 3$
d $m = 68$ **e** $x = 9$ **f** $m = 10$

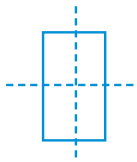
16 **a** C **b** $N = 59$

17 **a** $5N = 180$, $N = 36$ **b** 456

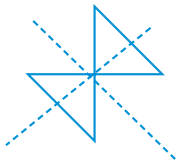
CHAPTER 6

SkillCheck

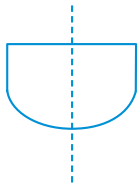
1 **a**



b

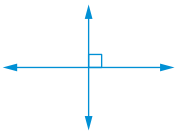


c



2 **a** Yes, order 2 **b** Yes, order 5 **c** No

3 **a**



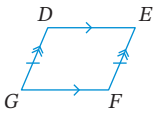
b

4

5 **a** AB and DC, or AD and BC
b No **c** 90° **d** 65°

6 Teacher to check.

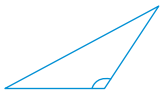
7 **a, b, d**



c DE and FG, DG and EF

8 **a**

b

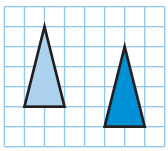


9 Teacher to check.

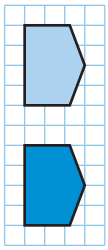
Exercise 6.01

1 **a** reflection **b** translation
c reflection **d** rotation
e translation **f** rotation
g translation **h** rotation
i reflection **j** rotation
k reflection **l** rotation

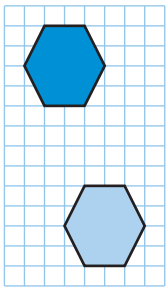
2 **a**



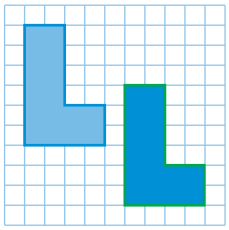
b



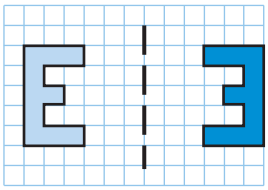
c



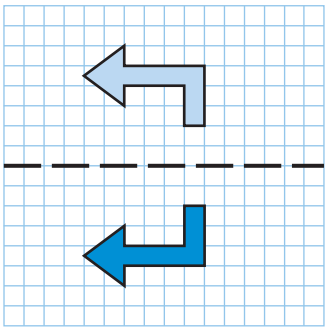
d



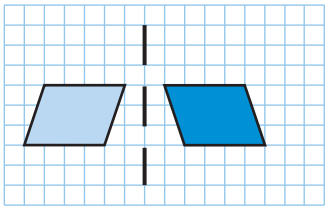
3 **a**



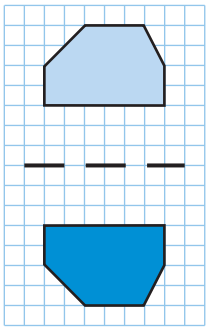
b



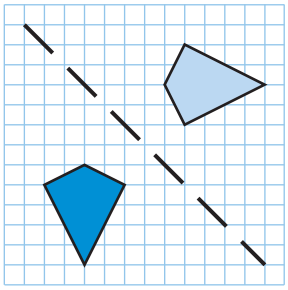
c



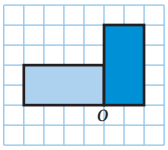
d



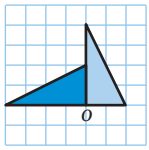
e



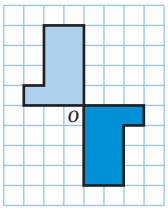
4 **a**



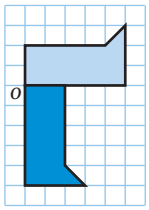
b



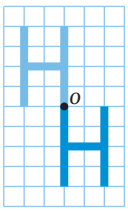
c



d



e



5 The answer is the same.

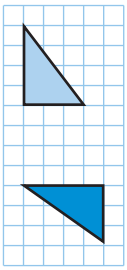
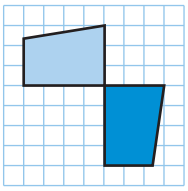
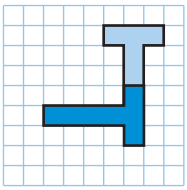
6 **a** 270° **b** 90° **c** 270° **d** 180°

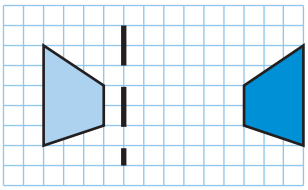
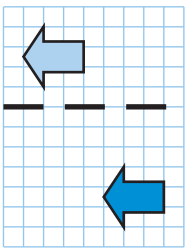
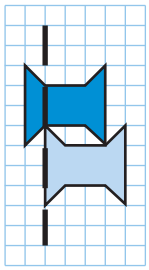
7 **a** 1 left, 5 up **b** 3 down

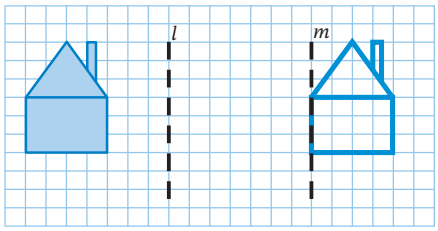
8 A

Exercise 6.02

1 a rotation and translation, or 2 reflections, or reflection and translation
  b rotation and translation
  c rotation and reflection

2 a

b

c


3 a

b

c


4 a


  b Translation

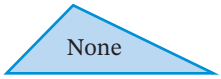
5 a i rotation, rotation
  ii translation
  b i reflection, reflection
  ii rotation
  c i rotation, reflection
  ii reflection

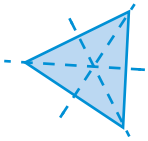
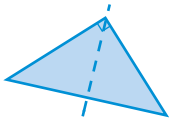
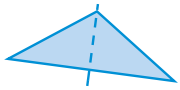
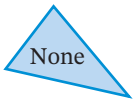
Exercise 6.03

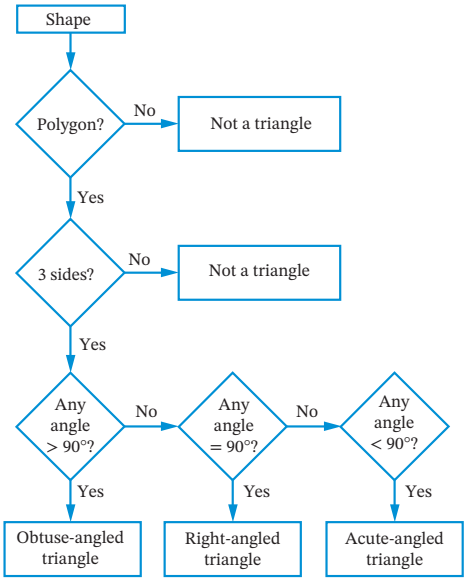
1 a scalene, right b scalene, right
  c scalene, acute d isosceles, obtuse
  e isosceles, obtuse f isosceles, acute
  g equilateral, acute h isosceles, acute
  i isosceles, right j equilateral, acute
  k isosceles, acute l scalene, obtuse

2 Teacher to check.

3 a

b


c

d

e

f


4 Yes, equilateral triangles.
5 No. In an equilateral triangle, all angles are equal.
6 Teacher to check.
7 No, 2 obtuse angles can't be joined to form a triangle.
8 C
9 a 1 b, c Teacher to check.
10 a 3 b, c Teacher to check.
11 a $x = 42$, equal angles of an isosceles triangle
  b $y = 7$, equal sides of an isosceles triangle
  c $x = 60$, angles in an equilateral triangle equal 60°
  d $l = 4.8$, equal sides of an isosceles triangle
  e $a = b = 17.2$, equal sides of an equilateral triangle
  f $r = 15$, equal angles of an isosceles triangle
12 B
13


Exercise 6.04

1 **a** $a = 50$ **b** $b = 30$ **c** $c = 30$ **d** $d = 60$
e $e = 124$ **f** $f = 50$ **g** $g = 67$ **h** $h = 22$
i $i = 52$ **j** $j = 90$ **k** $k = 60$ **l** $l = 30$
m $m = 65$ **n** $n = 139$ **o** $q = 158$ **p** $m = 60$
q $y = 120$ **r** $x = 96$ **s** $x = 50$

2 **a** $x = 60$ **b** $a = 80$
c $a = 40, b = 50, c = 90$

3 C **4** C **5** A

6 $d = 90, e = 130$

Mental skills 6

2 **a** 330 **b** 160 **c** 1580 **d** 260
e 420 **f** 200 **g** 410 **h** 740
i 6600 **j** 16 000 **k** 280 **l** 70

Exercise 6.05

1 **a** **i** d **ii** a and b
b **i** t **ii** q and p
c **i** w **ii** x and z

2 **a** $h = 104$ **b** $y = 16$
c $e = 95, d = 85,$ **d** $b = 116, a = 50$
e $m = 135$ **f** $y = 114, x = 66$
g $e = 67$ **h** $h = 29$
i $m = 90$ **j** $x = 70$
k $p = 130$ **l** $w = 88$

3 B

4 **a** $m = 112$ **b** $a = 94, b = 86$
c $a = 74, b = 74, c = 38$

5 B

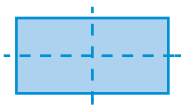
6 angle sum of a triangle, angles on a straight line

Exercise 6.06

1 **a** rhombus **b** parallelogram **c** square
d trapezium **e** rectangle **f** kite

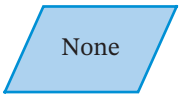
2 Teacher to check.

3 **a**



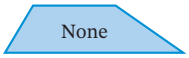
b

c



d

e



f

4

Quadrilateral	No. of axes of symmetry	Order of rotational symmetry
Square	4	4
Parallelogram	0	2
Trapezium	0	No rotational symmetry
Rhombus	2	2
Kite	1	No rotational symmetry
Rectangle	2	2

5 **a** square, rectangle
b parallelogram, rhombus, square, rectangle
c parallelogram, rhombus, square, rectangle
d trapezium
e rhombus, square
f kite, rhombus, square
g parallelogram, trapezium
h parallelogram, rhombus, rectangle

6 A

Exercise 6.07

1 **a** $a = 90$ **b** $b = 60$ **c** $c = 140$
d $d = 93$ **e** $e = 90$ **f** $f = 75$
g $g = 45$ **h** $i = 130$ **i** $j = 75$
j $r = 90$ **k** $m = 62$ **l** $l = 25$
m $n = 131$ **n** $p = 50$ **o** $x = 280, y = 80$

2 D **3** A

4 **a** $x = 72, y = 108$
b $y = 30$
c $m = 92$

5 **a** 90° **b** Rectangle

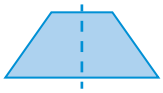
Exercise 6.08

2 **a**

	Trapezium	Parallelogram	Rhombus	Rectangle	Square	Kite
Opposite sides are equal		✓	✓	✓	✓	
Opposite sides are parallel		✓	✓	✓	✓	
Opposite angles are equal		✓	✓	✓	✓	
All angles are 90°				✓	✓	
Diagonals are equal				✓	✓	
No. of axes of symmetry	0	0	2	2	4	1
Order of rotational symmetry	0	2	2	2	4	0

(Note: An isosceles trapezium has line symmetry, but other trapeziums do not.)

c Pairs of adjacent sides equal, diagonals intersect at right angles, one pair of equal and opposite angles, one diagonal bisects the angles of the kite.



3, 4 Teacher to check. See also question **2**.

5 b

	Trapezium	Parallelogram	Rhombus	Rectangle	Square	Kite
Diagonals are equal				✓	✓	
Diagonals bisect each other		✓	✓	✓	✓	
Diagonals intersect at right angles			✓		✓	✓
Diagonals bisect the angles of the quadrilateral			✓		✓	

6 B

7 **a** rectangle, square
b parallelogram, rectangle, square, rhombus
c rectangle, square
d parallelogram, rhombus, rectangle, square
e parallelogram
f kite, rhombus, square
g square, rhombus
h rectangle, square

8 co-interior, parallel, 110, 70, angles

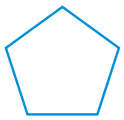
9 **a** Yes, a square is a rectangle
b Yes, a rhombus is a parallelogram.

10

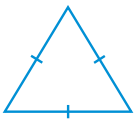
Shape	Properties
Trapezium	• One pair of parallel sides • No axes of symmetry
Kite	• 2 pairs of adjacent sides are equal • One pair of opposite angles are equal • Has one axis of symmetry • Diagonals intersect at right angles
Parallelogram	• Opposite sides are equal and parallel • Opposite angles are equal • No axes of symmetry • Diagonals bisect each other
Rhombus	• All 4 sides are equal • Opposite sides are parallel • Opposite angles are equal • Has 2 axes of symmetry • Diagonals bisect at right angles • Diagonals bisect the angles of the rhombus
Rectangle	• All 4 angles measure 90° • Opposite sides are equal and parallel • Has 2 axes of symmetry • Diagonals are equal • Diagonals bisect each other
Square	• All 4 sides are equal • All 4 angles measure 90° • Has 4 axes of symmetry • Diagonals are equal and bisect each other at right angles • Diagonals bisect the angles of the square

Exercise 6.09

1 **a**



b

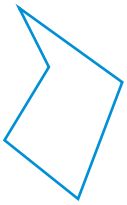


c

2 **a** 4 **b** 8 **c** 3
d 6 **e** 4 **f** 5

3 **a** yes **b** no **c** no
d yes **e** no **f** no

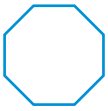
4 Answers may vary.

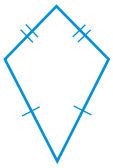


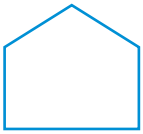
5 **a** hexagon, irregular, convex
b quadrilateral, irregular, convex
c pentagon, irregular, non-convex
d triangle, irregular, convex
e decagon, regular, convex
f heptagon, irregular, convex
g octagon, regular, convex
h nonagon, irregular, non-convex

6 **a** yes **b** no **c** no
d no **e** yes **f** no

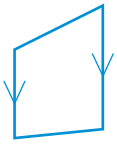
7 **a** yes **b** no **c** yes
d no **e** yes **f** no
g yes **h** yes **i** yes
j no **k** no **l** yes

8 **a i**  **ii** Octagonal prism

b i  **ii** Kite prism

c i  **ii** Pentagonal prism

d i **ii** Octagonal prism or T-prism

e i  **ii** Trapezoidal prism

f i **ii** Hexagonal prism or L-prism

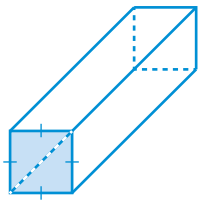
9 *A*. rectangular prism
B. cube
C. square pyramid
D. triangular prism
E. hexagonal prism
F. pentagonal prism

10 **d i** cube **ii** cylinder
iii square pyramid **iv** triangular prism
v rectangular prism **vi** rectangular pyramid
vii cone **viii** triangular pyramid

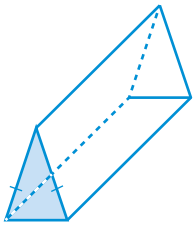
11 **a** rectangular prism **b** trapezoidal prism
c cube **d** triangular prism
e square pyramid **f** cylinder
g triangular prism **h** rectangular prism
l trapezoidal prism **j** cube
k square pyramid **l** rectangular prism

Exercise 6.10

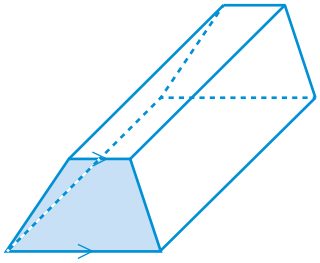
1 **a**



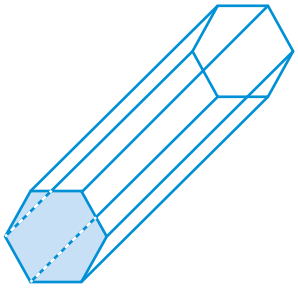
b



c



d



2, 3 Teacher to check.

4 The dashed line shows edges that are hidden from view.

5 Teacher to check.

6 **a**

b

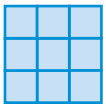
c

d

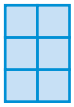
e

5, 6 Teacher to check.

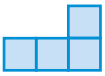
7 A



B



C



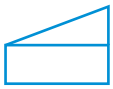
8 C

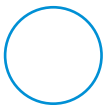
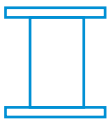
9 **a i**

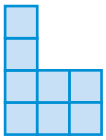
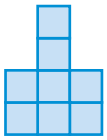
ii

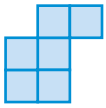
b **i** **ii** **iii**

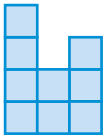
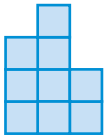
c **i** **ii**

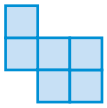
iii  **iv**

d **i**  **ii** 

e **i**  **ii** 

iii 

f **i**  **ii** 

iii 

10 D

11 5 possible solids

12 D

13 **a** Shows the 3D nature of the object
Doesn't need multiple views
Accuracy in measurement
Easy to layout and measure

b Creates a distorted appearance
More useful for rectangular than curved shapes
Distorts shape and depth
Only provides a 2D view, not real view

14 **a** The exact shape of the front face is shown
Better for curved shapes
Comparison of size is easy

b Objects can look distorted
Not realistic looking
Will not show you the back face of the solid

15 Orthogonal (shows three views, all views are the same scale, accurate measurement possible, does not provide sense of 3D)
Perspective drawing (shows depth, does not usually preserve shape of object or scale)

Power plus

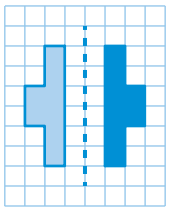
1 **a** Yes **b** No **c** Yes
d Yes **e** No **f** No

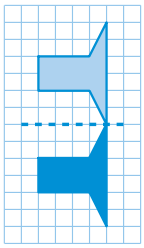
2 **a** all angles equal 90°
b all sides equal
c all sides equal

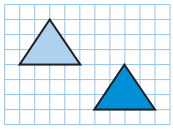
3 **a** parallelogram **b** kite
c rectangle **d** rectangle
e parallelogram **f** parallelogram

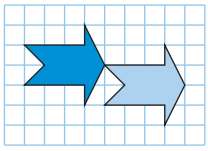
4, 5 Teacher to check.

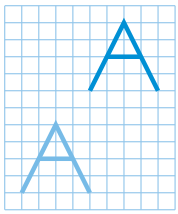
Test yourself 6

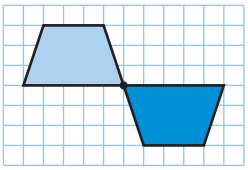
1 **a** 

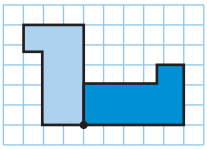
b 

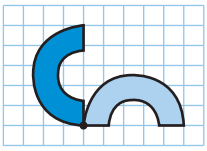
2 **a** 

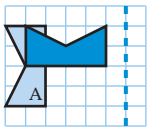
b 

c 

3 **a** 

b 

c 

4 

5 **a** equilateral, acute-angled
b scalene, right-angled
c isosceles, acute-angled
d scalene, obtuse-angled
e scalene, acute-angled
f isosceles, right-angled

6 **a** isosceles, acute-angled
b $\angle F$ and $\angle H$

7 **a** equilateral, acute angled
b scalene, acute angled

8 Reasons include angle sum of a triangle.
a $x = 59$ **b** $u = 51$
c $a = 127$ **d** $v = 76$
e $p = 100$ **f** $m = 62, n = 56$
g $y = 80, z = 130$

9 B **10** C

11 Reasons include exterior angle of a triangle.
a $x = 111$ **b** $m = 129$ **c** $x = 112$
d $p = 96$ **e** $m = 99$ **f** $y = 70$

12 **a** square **b** rhombus
c trapezium **d** parallelogram
e rectangle **f** kite

13 A quadrilateral with all sides equal.

14 **a** parallelogram **b** trapezium

15 **a** $y = 70$ **b** $x = 81$ **c** $m = 26$ **d** $y = 61$

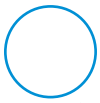
16 C **17** Teacher to check.

18 **a** square, rectangle
b square, rectangle
c square, rectangle, parallelogram, rhombus
d square, rectangle, parallelogram, rhombus

19 B

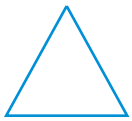
20 **a**

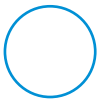
Prism

b

Not a prism

c

Not a prism

d

Prism

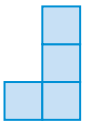
e

Not a prism

f

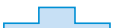
Prism

21 **a** **i**

ii

iii


b **i**

ii

iii

PRACTICE SET 2

1 C

2 **a** $\frac{5}{6}$ **b** $\frac{11}{15}$ **c** $\frac{1}{8}$

3 **a** 25% **b** 60% **c** 57.5%

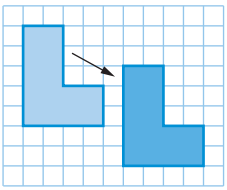
4 **a** yes **b** no **c** yes **d** no

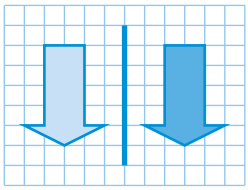
5 **a** $2w$ **b** a^2 **c** $4abk$ **d** $4p$
e $\frac{m}{2}$ **f** 0 **g** $8k$ **h** $10m$

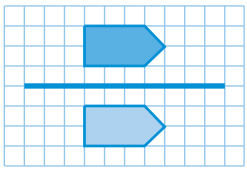
6 **a** $2\frac{2}{5}$ **b** $3\frac{1}{2}$ **c** $1\frac{2}{3}$ **d** $1\frac{5}{6}$

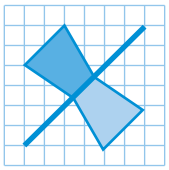
7 **a** $\frac{4}{5}$ **b** $\frac{2}{3}$ **c** $\frac{5}{8}$ **d** $\frac{7}{10}$

8 **a** 62 **b** 1800 **c** 10 **d** 620

9


10 **a**


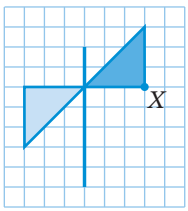
b


c


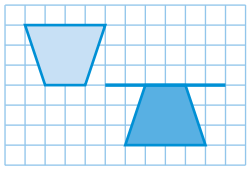
11 **a** > **b** < **c** >

12 **a** 198 **b** 192 **c** 135 **d** 264

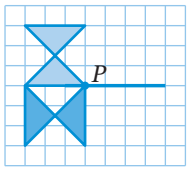
13 **a** $7\frac{5}{12}$ **b** $3\frac{1}{10}$ **c** $\frac{19}{24}$

14 **a**


b



c



15 **a** $\frac{3}{10}$ **b** $\frac{7}{18}$ **c** $8\frac{1}{3}$

16 **a** 40% **b** 5% **c** 12%

17 **a** \$10 **b** 18 min

18 **a** $m = 60$ **b** $x = 50$
c $k = 65$ **d** $m = 60$

19 **a** scalene, right-angled
b scalene, acute-angled
c isosceles, acute-angled
d equilateral, acute-angled

20 **a** –5 **b** 19 **c** 8

21 **a** $x = 23$ **b** $x = -9$ **c** $d = -6$ **d** $m = 5$

22 160

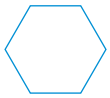
23 75 cm

24 D

25 **a** $\frac{1}{20}$ **b** 5%

26 **a** \$2520 **b** 42 **c** \$110

27 **a**



hexagonal prism

b

trapezoidal prism

28 **a** $1\frac{1}{2}$ **b** $\frac{1}{25}$ **c** $4\frac{1}{6}$

29 **a** $m = 5$ **b** $n = 1\frac{1}{2}$ **c** $x = 3$ **d** $x = 8$

30 **a** pq **b** $x - 4$ **c** $d - 7$ **d** $28 - b$

31 **a** $4h + 16 = 200, h = 46$ **b** $4p + 14 = 56, p = 10\frac{1}{2}$

32 **a** $p = 105$ **b** $k = 40$ **c** $x = 90$

33 **a** $7m - 3$ **b** $10m + 2n$ **c** $-2x^2 - 2x$

34 **a** $x = 130$ **b** $m = 50$ **c** $n = 35$

35 **a** $5m^2$ **b** $-2e$ **c** $-7p$ **d** $3x$

36 **a** parallelogram, rhombus, rectangle, square
b rectangle, square **c** square
d kite

37 C

CHAPTER 7

SkillCheck

1

	Hundreds	Tens	Units	Tenths	Hundredths	Thousandths
a		1	4	8	2	
b			6	0	1	4
c	9	3	1	0	2	
d		7	0	8		
e			0	3	7	5
f			0	1	9	
g			8	5	9	2
h	7	1	5	3		

2 **a** 4 hundreds **b** 4 tenths **c** 4 hundredths
d 4 units **e** 4 thousandths **f** 4 tenths

3 **a** $\frac{9}{50}$ **b** $\frac{2}{5}$ **c** $\frac{7}{20}$ **d** $\frac{6}{25}$

4 **a** 20, 21, 30, 32, 34 **b** 206, 213, 216, 230, 234

5 **a** 320 **b** 57 800 **c** 325 000
d 4000 **e** 40 **f** 12
g 1000 **h** 810 **i** 64

6 **a** 45, 44, 42, 40, 39, 38
b 556, 549, 522, 520, 513, 505

7 **a** 30 **b** 800 **c** 9000 **d** 17 000

8 Teacher to check.

9 **a** 553 **b** 825 **c** 717 **d** 258 **e** 764
f 174 **g** 3150 **h** 740 **i** 319

Exercise 7.01

1 **a** 2 **b** 3 **c** 4 **d** 3 **e** 5
f 1 **g** 6 **h** 5 **i** 7

2 A

3 **a** 9.998, 23.89, 43.89, 56.314, 56.324, 80.879, 400
b 0.0586, 0.099, 0.568, 0.586, 0.684, 1.002, 5.608
c 0.222, 0.89, 0.891, 1.23, 1.814, 7.007
d 0.005, 0.05, 0.5
e 3.004, 3.044, 3.4, 3.404, 3.441
f 0.44, 0.404, 0.4004, 0.044

4 C

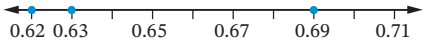
5 **a** 4000.99, 1256.3, 570.25, 400.099, 125.63, 0.9899
b 6.539, 5.639, 5.37, 5.369, 5.367, 3.66, 3.659
c 1.61, 1.601, 1.6, 1.599, 1.509
d 6.6, 6, 0.6, 0.06
e 7.07, 0.77, 0.707, 0.7, 0.07, 0.007
f 0.44, 0.404, 0.4004, 0.044

6 **a** $0.2 < 0.25$ **b** $0.731 > 0.73$
c $0.035 < 0.305$ **d** $0.007 < 0.070$
e $1.59 > 1.059$ **f** $0.099 < 0.99$
g $44.44 > 4.444$ **h** $0.7932 > 0.7239$

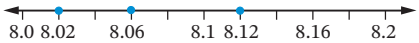
7 **a**

b

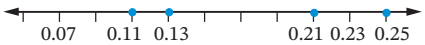
c



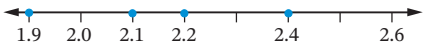
d



e



f



g

8, 9 Teacher to check.

Exercise 7.02

1 **a** **i** $\frac{7}{10}$ **ii** 0.7

b **i** $\frac{5}{10} = \frac{1}{2}$ **ii** 0.5

c **i** $\frac{5}{10} = \frac{1}{2}$ **ii** 0.5

d **i** $\frac{4}{10} = \frac{2}{5}$ **ii** 0.4

2 D

3 **a** $\frac{7}{10}$ **b** $\frac{2}{5}$ **c** $\frac{39}{100}$ **d** $\frac{143}{250}$

e $\frac{3}{1000}$ **f** $\frac{1}{20}$ **g** $\frac{11}{100}$ **h** $\frac{41}{125}$

i $\frac{3}{10}$ **j** $\frac{999}{1000}$ **k** $\frac{13}{1000}$ **l** $\frac{1}{2500}$

m $\frac{471}{10000}$ **n** $\frac{321}{1000}$ **o** $\frac{1001}{2000}$ **p** $\frac{91}{100}$

q $\frac{41}{500}$ **r** $1\frac{3}{5}$ **s** $27\frac{1}{4}$ **t** $2\frac{7}{1000}$

4 B

5 **a** 0.9 **b** 0.15 **c** 0.79 **d** 0.6
e 0.4 **f** 0.23 **g** 0.6 **h** 0.411
i 0.704 **j** 0.87 **k** 0.07 **l** 0.14
m 0.8 **n** 0.235 **o** 0.247 **p** 0.17
q 0.368 **r** 0.9345 **s** 0.493 **t** 1.67
u 4.47 **v** 23.9 **w** 6.8 **x** 0.023
y 0.45

Exercise 7.03

1 **a** 12 **b** 16.933 **c** 167.317 **d** 2.1376
2 **a** 41.493 **b** 5.9 **c** 6.69 **d** 9.06
3 C
4 **a** 81.03 m **b** 18.97 m
5 18.6 km **6** 8037.2 kg
7 **a** 12.45 s, 12.7 s, 13.5 s, 13.81 s, 14.62 s
b 2.17 s
c Yes, 12.7 – 0.3 = 12.4 s, which is faster than 12.45 s.
8 **a** \$605.55 **b** \$15.25
9 A **10** 6.23 m
11 **a** 2.54 **b** 8.74 **c** 5.45
d, e Teacher to check.

Exercise 7.04

1 **a** 249 **b** 8.1 **c** 37 420
d 341.6 **e** 725 **f** 2196
g 604.3 **h** 3.2 **i** 0.65
j 4521.3 **k** 10 640 **l** 6304
m 5980 **n** 84761.2 **o** 0.592
p 3620 **q** 219 400 **r** 400.75
2 C
3 **a** 4.63 **b** 5.07 **c** 1.203
d 0.364 **e** 0.3815 **f** 5.502
g 0.01146 **h** 0.6943 **i** 0.00428
j 0.67 **k** 0.02131 **l** 0.572
m 0.081 348 **n** 0.502 **o** 0.0491
p 0.0004 **q** 0.0173 **r** 0.012 53
4 **a** 1000 **b** 100 **c** 100 **d** 10
e, f Teacher to check.

Mental skills 7

2 **a** 88 **b** 167 **c** 314 **d** 85
e 129 **f** 121 **g** \$11.60 **h** \$6.75
i \$4.70 **j** \$30.05 **k** \$7.10 **l** \$32.90

Exercise 7.05

1 **a**

Multiplication	Estimate	Answer
0.69 × 18	0.7 × 20 = 14	12.42
6.9 × 180	7 × 200 = 1400	1242
6.9 × 1.8	7 × 2 = 14	12.42
690 × 1.8	700 × 2 = 1400	1242

b

Multiplication	Estimate	Answer
10.4 × 42	10 × 40 = 400	436.8
1.04 × 4.2	1 × 4 = 4	4.368
104 × 4.2	100 × 4 = 400	436.8
0.104 × 4.2	0.1 × 4 = 0.4	0.4368

c

Multiplication	Estimate	Answer
3.8 × 92	4 × 100 = 400	349.6
0.38 × 92	0.4 × 100 = 40	34.96
38 × 0.92	40 × 1 = 40	34.96
380 × 0.92	400 × 1 = 400	349.6

2 **a** 21.42 **b** 2.142 **c** 2142
d 2142 **e** 214.2 **f** 21.42
3 **a** 20.4 **b** 2.04 **c** 0.204
d 2.04 **e** 204 **f** 20.4

4 **a** 244.8 **b** 244.8 **c** 2.448
d 2448 **e** 2.448 **f** 24.48
5 **a** 75.6 **b** 756 **c** 0.756
d 0.756 **e** 0.0756 **f** 75.6
6 **a** 0.72 **b** 0.72 **c** 7.2
d 72 **e** 7.2 **f** 0.72
7 Teacher to check.

Exercise 7.06

1 **a** 2 **b** 1 **c** 1 **d** 3
e 3 **f** 3 **g** 4 **h** 3
i 3 **j** 4 **k** 6 **l** 2
2 B
3 **a** 12.2 **b** 7.14 **c** 18.009 **d** 34.2
e 22.5 **f** 12.6 **g** 34.75 **h** 8.032
i 0.9 **j** 4.8 **k** 0.012 **l** 54
m 0.32 **n** 1.95 **o** 0.48 **p** 0.072
4 **a** 14.37 **b** 46.296 **c** 525.42
5 \$162.45 **6** \$719.55
7 2.08 m^2 **8** C

Exercise 7.07

1 **a** 2.4 **b** 6.2 **c** 4.16 **d** 4.1
e 14.65 **f** 2.1975 **g** 0.008 **h** 2.6775
i 24.45 **j** 3.675 **k** 0.51875 **l** 0.106
m 2.4 **n** 3.39 **o** 2.875 **p** 64.21
q 11.9 **r** 13.75 **s** 0.122 **t** 259.45
2 \$17.14 **3** D **4** \$61.16
5 **a** 69.6 s **b** 11.6 s
6 **a** $24.8 \div 8 = 3.1$ **b** $12.3 \div 5 = 2.46$
c Teacher to check.

Exercise 7.08

1 **a** $5088 \div 8$ **b** $1792 \div 7$ **c** $3330 \div 9$
d $172.5 \div 5$ **e** $1292 \div 4$ **f** $4950 \div 6$
g $1680 \div 7$ **h** $1482.3 \div 9$ **i** $6.6 \div 3$
2 **a** 8.7 **b** 36.6 **c** 4.9 **d** 203.5
e 30 **f** 12.6 **g** 208 **h** 80
i 5.7 **j** 2041 **k** 1.064 **l** 130.2
3 **a** More, we are dividing by a number less than 1
b Teacher to check. **c, d** 36
4 **a** more **b** Teacher to check.
c 68
5 The answer is more than the number.
6 B **7** 16 **8** 18
9 **a** 14 km **b** 38 min
10 **a** $27.2 \div 3.4 = 8$ **b** $12.3 \div 5 = 2.46$
c Teacher to check.

Exercise 7.09

1

	Fraction	Meaning as division	Decimal
a	$\frac{3}{5}$	$3 \div 5$	0.6
b	$\frac{1}{2}$	$1 \div 2$	0.5
c	$\frac{1}{4}$	$1 \div 4$	0.25
d	$\frac{4}{5}$	$4 \div 5$	0.8
e	$\frac{2}{5}$	$2 \div 5$	0.4
f	$\frac{3}{4}$	$3 \div 4$	0.75
g	$\frac{1}{5}$	$1 \div 5$	0.2
h	$\frac{1}{8}$	$1 \div 8$	0.125

2 **a** 0.4 **b** 0.375 **c** 0.75 **d** 1
e 0.5 **f** 0.75 **g** 0.6 **h** 0.25
i 0.625 **j** 0.875 **k** 0.5 **l** 1
3 The fractions are equivalent, for example,
$\frac{1}{2} = \frac{2}{4} = \frac{4}{8}$
4 **a** 4.8 **b** 23.75 **c** 12.625 **d** 6.6
e 57.4 **f** 19.125 **g** 110.875 **h** 80.25
5 **a** $0.41\dot{6}$ **b** $0.\dot{2}\dot{7}$ **c** $0.\dot{1}$ **d** $0.\dot{1}027\dot{5}$
6 **a** 0.8583333... **b** 0.0454545...
c 0.777... **d** 0.461538461538...
7 **a** $0.\dot{1}$ **b** $0.1\dot{6}$ **c** $0.8\dot{3}$
d $0.\dot{1}4285\dot{7}$ **e** $0.\dot{6}$ **f** $0.\dot{2}85\ 71\dot{4}$
g $0.\dot{2}$ **h** $0.\dot{4}28\ 57\dot{1}$ **i** $0.\dot{5}71\ 42\dot{8}$
j $0.\dot{4}$ **k** $0.\dot{6}$ **l** $0.\dot{5}$
m $0.\dot{8}57\ 14\dot{2}$ **n** $0.\dot{7}$ **o** $0.\dot{8}$
8 $0.\dot{1}, 0.\dot{2}, 0.\dot{3}, 0.\dot{4}, 0.\dot{5}, 0.\dot{6}, 0.\dot{7}, 0.\dot{8}$

Exercise 7.10

1 **a** 0.7 **b** 1.77 **c** 16.56
d 82.9 **e** 478.7 **f** 823.55
2 **a** 0.4 **b** 0.5 **c** 0.8 **d** 0.7
e 2.6 **f** 0.3 **g** 0.9 **h** 2.9
3 **a** 8.7 **b** 0.74 **c** 0.07 **d** 0.29
e 0.31 **f** 0.66 **g** 2.10 **h** 4.00

4

	Calculator display	Rounded (1 decimal place)	Rounded (2 decimal places)
a	4.063333333	4.1	4.06
b	2.053333333	2.1	2.05
c	2.202222222	2.2	2.20
d	5.168181818	5.2	5.17
e	4.2825	4.3	4.28
f	1.05416666	1.1	1.05
g	0.232142857	0.2	0.23
h	3.43125	3.4	3.43
i	1.416666667	1.4	1.42
j	7.9586875	8.0	7.96

5 **a** 25.38 **b** 341.61 **c** 420.90
d 734.65 **e** 27.36 **f** 1314.21

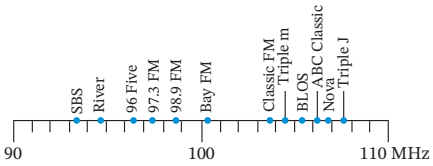
6 **a** 10.3337 **b** 431.5433 **c** 1.4450
d 3217.6541 **e** 4.6709 **f** 0.8889

7 **a** 8 chocolates, rounded down so that each person receives whole chocolates
b 54 boxes, rounded up, otherwise there won't be enough titles (you cannot buy part of a box)
c 6 balls, rounded down so that each golfer receives whole golf balls
d 26 lengths, rounded up because you cannot buy partial timber lengths
e 3 dresses, rounded down since a partial dress cannot be completed
f 2 kg, rounded up, otherwise there will not be enough sugar
g 11 bottles, rounded down for full bottles
h 32.5 m, rounded up, otherwise there will not be enough material

8, 9 Teacher to check.

Exercise 7.11

1 112.9 m **2** 7
3 15.8 km **4** 26.6 km
5 **a** \$15.24 **b** \$6.84 **c** \$487.76 **d** \$1234.05
6 C **7** \$9
8 258.3 t **9** 5
10 **a**



b 6.4 MHz **c** 0.8 MHz **d** 3.4 MHz
11 120 sheets **12** 1.4 km

Power plus

1 $0.\dot{1}4285\dot{7}$, $0.\dot{2}8571\dot{4}$, $0.\dot{4}2857\dot{1}$, $0.\dot{5}7142\dot{8}$, $0.\dot{7}1428\dot{5}$, $0.\dot{8}5714\dot{2}$; the same set of 6 digits are repeated in order, but with a different digit starting each time.

2 $0.\dot{9} = \frac{9}{9} = 1$

3 **a** 0.0004 **b** 0.000 008 **c** 1.331
d 0.2 **e** 0.6 **f** 0.3

4

Queen's chamber	8.07
armoury	8.262
throne room	8.79
banquet hall	9.24
kitchen	9.96
dungeon	8.16

5 24.15

6 **a** 3.7, 24.6 **b** 2.3, 5.7, 6
c 5.1, 0.72, 4.5, 18

Test yourself 7

1 **a** 3.998, 34.89, 34.98, 36.86, 38.141, 340
b 1.293, 1.239, 1.1015, 1.015, 1.006

2 **a** $\frac{1}{2}$ **b** $\frac{89}{100}$ **c** $\frac{9}{100}$ **d** $\frac{111}{250}$

3 **a** 0.4 **b** 0.13 **c** 0.07 **d** 0.011

4 **a** 65.355 **b** 104.49 **c** 11.315
d 1037.9 **e** 396.51 **f** 2.379

5 **a** 75.4 **b** 754 **c** 7540
d 1.39 **e** 0.139 **f** 0.0139

6 **a** 319.2 **b** 31.92 **c** 319.2
d 3.192 **e** 319.2 **f** 0.3192

7 **a** 16.5 **b** 0.6 **c** 288.92
d 7.32 **e** 4.6 **f** 29.1
g 1.3325 **h** 0.001 **i** 5.137

8 **a** 381.2 **b** 12.2 **c** 0.6875 **d** 65.2

9 **a** 25 **b** 31.8 **c** 230 **d** 25.4

10 **a** 0.8 **b** 0.375 **c** $0.\dot{5}$ **d** $0.\dot{6}$

11 **a** 406.3 **b** 125.72 **c** 2345.9
d 3.90 **e** 78 654.06 **f** 678.431

12 **a** \$13 – round up to cover the bill, otherwise it will not be enough
b 37 boxes –round up, otherwise there will not be enough pavers.
c 5 ice creams each – round down so that each person has whole ice creams
d 90 lengths –round up, otherwise there will not be enough timber to build the fence.

13 \$470.20 **14** \$1.34 **15** \$38.20

CHAPTER 8

SkillCheck

1 **a** 1000 **b** 60 **c** 1000 **d** 10
e 1000 **f** 24 **g** 1000 **h** 1000
i 1000 **j** 60 **k** 100 **l** 1000

2 **a** 1032 **b** 94.5 **c** 400 **d** 2700
e 90 **f** 63 **g** 20 **h** 43
i 65 **j** 37.5 **k** 9.5 **l** 8.2

3 **a** metres **b** centimetres **c** millimetres
d metres **e** metres **f** metres
g metres **h** centimetres **i** centimetres
j millimetres **k** seconds

4 **a** 32 cm **b** 36.9 mm **c** 93 m

Exercise 8.01

1 **a** 60 **b** 1000 **c** 1000 **d** 10
e 24 **f** 1000 **g** 1000 **h** 60
i 1000 **j** 1000 **k** 1 000 000 **l** 3600

2 **a** 640 cm **b** 4000 kg **c** 72 h
d 2 400 000 L **e** 900 mm **f** 12 700 g
g 480 mins **h** 5600 mL **i** 420 mm
j 72 000 mg **k** 380 000 cm **l** 16 200 s

3 **a** 200 **b** 150 **c** 50 **d** 350
e 25 **f** 200 **g** 1 580 000 **h** 40 000

4 **a** 2 **b** 3 **c** 8.5 **d** 9.8
e 0.75 **f** 0.325 **g** 1 580 000 **h** 40 000

5 **a** kilo **b** centi **c** milli

6 Teacher to check.

7 **a** 4500 **b** 180 **c** 0.75 **d** 24
e 12 000 **f** 520 **g** 7 **h** 48
i 8650 **j** 3 **k** 0.065 **l** 960

8 D

9 **a** 120, 330, 800, 35, 7400
b 0.12, 0.33, 0.8, 0.035, 7.4
c Teacher to check.

10 C

11 **a** 2200 **b** 0.8 **c** 2.35 **d** 0.36
e 330 **f** 4.9 **g** 0.27 **h** 48
i 10 800 **j** 16900 **k** 365 **l** 7.4

12 A

Exercise 8.02

1 **a** 24 cm **b** 44 mm **c** 90 m **d** 138 cm

2 **a** 46 mm **b** 44 cm **c** 118 m
d 39 cm **e** 246 cm **f** 2000 mm
g 31.8 km **h** 26 cm **i** 208 mm
j 60 mm **k** 48 m

3 **a** 63 mm **b** 114 mm **c** 60 mm **d** 80 mm

4 **a** 44 m **b** 30 cm **c** 48 mm
d 40 m **e** 36 km **f** 54 mm

5 D

6 Any 2 numbers that add up to 15

7 C

8 **a** 4 cm **b** 8 cm **c** 12.75 cm **d** 18 cm

9 D

Exercise 8.03

1 Teacher to check.

2 **a** diameter **b** segment
c sector **d** arc
e radius **f** chord
g circumference **h** tangent

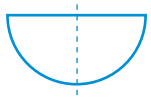
3 **a** radius **b** quadrant
c tangent **d** diameter
e chord **f** arc
g sector **h** circumference
i segment

4 B **5** B

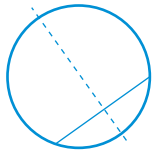
6 Teacher to check.

7 **a** **i** 1 **ii** no rotational symmetry

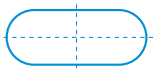
b **i** 1 **ii** no rotational symmetry



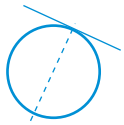
c **i** 1 **ii** no rotational symmetry



d **i** 2 **ii** 2



e **i** 1 **ii** no rotational symmetry



f **i** 1 **ii** no rotational symmetry

8 Teacher to check.

Exercise 8.04

1 **a** 12 cm **b** 24 m **c** 9 cm **d** 18 cm

2 **a** 12.57 cm **b** 25.13 m
c 9.42 cm **d** 18.85 cm

3 **a** 4π cm **b** 8π m **c** 3π cm **d** 6π cm

4 **a** 36 m **b** 60 mm **c** 42 cm **d** 12 mm

5 **a** 37.7 m **b** 62.8 mm **c** 44.0 cm **d** 12.6 mm

6 **a** 12π m **b** 20π mm **c** 14π cm **d** 4π mm

7 4.40 m

8 **a** 188.5 cm **b** 477

9 40 023.9 km **10** 100.5 mm **11** 236 mm

12 **a** 6.28 cm **b** 21.99 cm
c 18.85 cm **d** 31.42 cm

13 C **14** D

15 **a** 210.5 m **b** $(154 + 18\pi)$ m

16 16 cm **17** Ali, 25 m

Exercise 8.05

1 **a** hectare **b** m^2 **c** cm^2
d m^2 **e** cm^2 **f** mm^2
2 B
3 **a** 1500 mm^2 **b** 1.5 m^2 **c** 6.9 cm^2
d 65 000 cm^2 **e** 500 000 mm^2 **f** 120 000 m^2
g 1.22 m^2 **h** 12.5 cm^2 **i** 3200 m^2
j 790 mm^2 **k** 750 000 mm^2 **l** 245 ha
m 86.5 m^2 **n** 0.0513 m^2 **o** 4700 cm^2
p 0.9 ha
4 **a** 102 400 mm^2 **b** 0.1024 m^2
5 **a** 6.5 m^2, 114 000 cm^2, 25 050 000 mm^2
b 54 cm^2, 990 mm^2, 0.000 032 m^2
6 **a** 87.5 ha **b** 8 750 000 000 cm^2
7 B
8 **a** 6 840 100 ha **b** about 37
9 836 mm^2
10 **a** 1 000 000 **b** 100
c Teacher to check.

Exercise 8.06

1 **a** 55.8 m^2 **b** 58.6 m^2 **c** 96 cm^2
d 324 m^2 **e** 1.44 cm^2 **f** 4 cm^2
g 120 mm^2 **h** 225 m^2 **i** 154 mm^2
j 1890 cm^2 **k** 448 m^2
2 B
3 **a** **i** 25 m^2 **ii** 250 000 cm^2
iii 25 000 000 mm^2
b **i** 0.513 m^2 **ii** 5130 cm^2 **iii** 513 000 mm^2
4 **a** **i** 625 mm^2 **ii** 6.25 cm^2
b **i** 1100 mm^2 **ii** 11 cm^2
c **i** 1050 mm^2 **ii** 10.5 cm^2
d **i** 1311 mm^2 **ii** 13.11 cm^2
5 7 cm
6 **a** 15 000 000 m^2 **b** 40 000 m^2
c 31.36 m^2 **d** 3.51 m^2
e 6.29 m^2 **f** 0.25 m^2
7 \$7858.50 **8** 4.5 m
9 **a** 10.08 m^2 **b** 0.16 m^2 **c** 63
10 Teacher to check. **11** D
12 **a** **i** 20 m^2 **ii** 20 m^2 **iii** 12.8 m^2
b 80 tiles **c** 3.6 L

Mental skills 8A

2 **a** 108 **b** 154 **c** 166 **d** 210
e 52 **f** 82 **g** 196 **h** 324
i 91 **j** 137 **k** 46 **l** 69
m 253 **n** 38 **o** 24 **p** 85

Exercise 8.07

1 **a** 15 cm^2 **b** 16 cm^2 **c** 36 mm^2
d 18 cm^2 **e** 10 cm^2 **f** 48.59 m^2
g 8.82 m^2 **h** 9.01 mm^2 **i** 7.68 cm^2
j 45.72 cm^2 **k** 21.76 m^2 **l** 1750 mm^2
2 **a** 15.6 m^2 **b** 300 mm^2 **c** 7.8 m^2
3 20 m **4** 3 cm **5** D
6 \$5.10 **7** B
8 Any 2 numbers (in metres) whose product is 72.
9 870 m^2 **10** 8 cm

Exercise 8.08

1 **a** 48 cm^2 **b** 136 m^2 **c** 54.4 cm^2
d 3 cm^2 **e** 340 mm^2 **f** 84 cm^2
g 16.2 m^2 **h** 43.46 cm^2 **i** 16.192 m^2
2 A
3 **a** 168 000 cm^2 **b** 7 cm^2 **c** 4.5 cm^2
d 70 000 cm^2 **e** 0.5456 cm^2 **f** 4.9 cm^2
4 A
5 Any 2 numbers (in cm) whose product is 245.
6 25 cm

Exercise 8.09

1 A
2 **a** 152 cm^2 **b** 372 cm^2 **c** 60 cm^2 **d** 11 cm^2
3 **a** 11 250 cm^2 **b** 500 cm^2 **c** 1300 cm^2
4 D
5 **a** 140 cm^2 **b** 67.5 km^2 **c** 48 mm^2
d 220 m^2 **e** 150 cm^2 **f** 1020 mm^2
6 **a** 30 cm^2 **b** 100 cm^2 **c** 204 mm^2 **d** 135 cm^2
e 55 cm^2 **f** 22 m^2 **g** 128 mm^2 **h** 372 cm^2
7 42.8 m^2 **8** 60 m^2 **9** 282 tiles

Mental skills 8B

2 **a** 56 **b** 108 **c** 64 **d** 420
e 344 **f** 128 **g** 224 **h** 264
i 46 **j** 68 **k** 140 **l** 108
m 78 **n** 39 **o** 32 **p** 19

Exercise 8.10

1 **a** cm^3 **b** cm^3 **c** m^3
d cm^3 **e** mm^3 **f** m^3
2 D
3 **a** E **b** G **c** F **d** C
e A **f** B **g** D **h** H
4 **a** 3000 mm^3 **b** 5 000 000 cm^3
c 2600 mm^3 **d** 4 cm^3
e 7 200 000 cm^3 **f** 66 cm^3
g 1 000 000 000 mm^3 **h** 0.0023 m^3
i 126 m^3 **j** 3.45 cm^3
k 25 000 000 000 mm^3 **l** 0.000 078 m^3
m 0.063 m^3 **n** 0.0014 cm^3
o 0.005 m^3
5 0.306 m^3
6 2 520 000 mm^3
7 38 000 000 cm^3

Exercise 8.11

1 **a** 225 cm^3 **b** 1071 cm^3 **c** 576 cm^3
d 2295 cm^3 **e** 27 cm^3 **f** 39.6 m^3
g 40500 cm^3 **h** 144.72 m^3

2 **a** 125 000 cm^3 **b** 900 cm^3
c 14 m^3 **d** 4224 mm^3
3 **a** 2268 cm^3 **b** 2 268 000 mm^3
4 **a** 62 cm^3 **b** 6400 cm^3 **c** 498 m^3
d 126 mm^3 **e** 23 552 m^3 **f** 23 500 cm^3
g 28 700 mm^3
5 **a** 733 720 cm^3 **b** 0.73 372 m^3
6 **a** 200 cm^3 **b** 5.625 cm^3
c 5000 cm^3 **d** 3 000 000 cm^3
e 600 000 cm^3
7 $6\frac{2}{3}$ mm **8** 6 m
9 Teacher to check (product = 880).
10 B **11** A **12** 5 m

Exercise 8.12

1 **a** 210.8 m^3 **b** 4.096 875 m^3 **c** 486.42 cm^3
2 **a** 90 cm^3 **b** 15 400 mm^3 **c** 7.8125 m^3
3 **a** 22 500 mm^3 **b** 928 cm^3
c 13 200 cm^3 **d** 288 cm^3
4 **a** 0.306 m^3
b 0.306 m^3, Method **b** is easier
5 A **6** 5 m **7** 7 m
8 Teacher to check (product = 72).
9 **a** 220 cm^3 **b** 11 220 mm^3 **c** 24 000 m^3
d 29 450 mm^3 **e** 82 m^3

Power plus

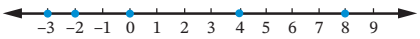
1 **a** $P = 47$ cm, $A = 82.5$ cm^2
b $P = 46$ m, $A = 52.5$ m^2
c $P = 79$ cm, $A = 206.5$ cm^2
d $P = 277$ mm, $A = 2250$ mm^2
2 **a, b** Teacher to check. **c** 7.5 cm × 7.5 cm
3 Different answers possible: for example, $l = w = 4$; $l = 6$, $w = 3$.
4 **a** 66 m^2 **b** 11 L
c 3, \$137.85 **d** 15 m^2
5 **a** 45 m^2 **b** 13.5 m^3 **c** \$10.80
6 **a** 10 cm **b** 78.5 cm^2 **c** 13.1% **d** 43.5%

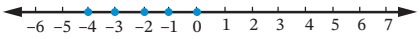
Test yourself 8

1 **a** 4 h **b** 42 g **c** 0.125 kL
d 3.2 km **e** 8300 mL **f** 9040 mm
g 0.0035 t **h** 900 mins **i** 54000 s
j 7.15 m **k** 4 200 000 mL **l** 17 kg
2 **a** 52 m **b** 36 mm **c** 64 cm **d** 88 mm
3 **a** segment **b** radius **c** arc
d centre **e** diameter **f** chord
g sector **h** tangent
4 **a** **i** 1 **ii** no rotational symmetry
b **i** 2 **ii** 2
c **i** 1 **ii** no rotational symmetry
d **i** 1 **ii** no rotational symmetry
5 **a** 12.6 cm **b** 62.8 mm **c** 44.0 cm **d** 88.0 cm
6 **a** 91 000 cm^2 **b** 250 mm^2 **c** 17.5 m^2
d 0.24 m^2 **e** 2.4 cm^2 **f** 8 600 000 mm^2
7 **a** 16 m^2 **b** 24 cm^2 **c** 162 m^2
8 **a** 360 mm^2 **b** 112 mm^2
c 5.2 cm^2 **d** 187 km^2
9 **a** 120 m^2 **b** 828 cm^2 **c** 68 mm^2
10 **a** 9 cm^2 **b** 34 m^2 **c** 84 m^2
d 12.5 mm^2 **e** 196 m^2 **f** 128.2 mm^2
11 6 L
12 **a** 20 000 **b** 500 000 **c** 7.5 **d** 0.00 023
13 **a** 80 m^3 **b** 320 cm^3
c 672 mm^3 **d** 360 m^3
14 **a** 391.392 cm^3 **b** 2200 mm^3
c 672 m^3 **d** 35 200 mm^3
15 B **16** 1317.5 cm^2

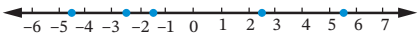
CHAPTER 9

SkillCheck

1 **a** −3 **b** 0 **c** 3 **d** 5
2 **a** A −10 B −2 C 1 D 3
b A −10 B −4 C 8 D 14 E 17
c A −22 B −15 C 2 D 11 E 20 F 26
d A −56 B −24 C −8 D 6 E 30 F 56
e A −60 B −50 C −35 D −15 E −6 F 19 G 40
3 **a**


b


c

d


Exercise 9.01

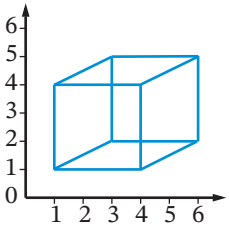
1 **a** E3 **b** C3 **c** C4
d F4 or G4 **e** F3 or G3 **f** C3
2 **a** swimming pool **b** primary school
c park **d** pond
e car park **f** tennis courts
3 **a** A13 **b** G6 **c** N25 **d** U16
4 19 **5** Back rows
6 **a** No **b** No **c** No **d** Yes
7 Teacher to check.
8 **a** Vertical line or column
b Horizontal line or row
9 **a** Roundhouse
b Building L5
c NIDA Parade Theatre
d International House
e Alumni Park East
10 **a** M7 **b** F10
c F2, F3, G2 or G3 **d** E5

Exercise 9.02

1 *A* (1, 4) *B* (2, 2) *C* (3, 1) *D* (3, 4)
E (4, 3) *F* (5, 2) *G* (6, 5)

2 *M* (0, 2) *P* (1, 1) *A* (2, 0) *B* (2, 3)
N (3, 1) *J* (3, 3) *E* (4, 0) *U* (5, 2)

3

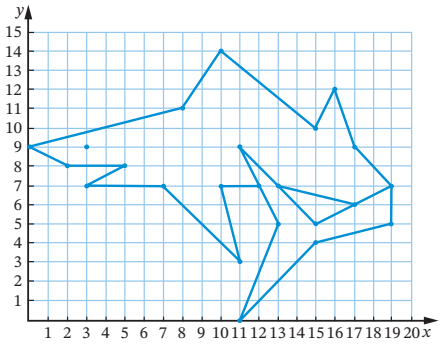


a cube

4 A

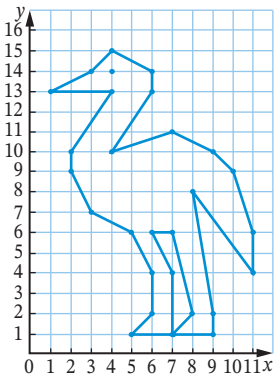
5 **a** NUMBER **b** PLANE **c** WATER
d METRE **e** NURSE **f** UMPIRE

6 **b**



c a shark

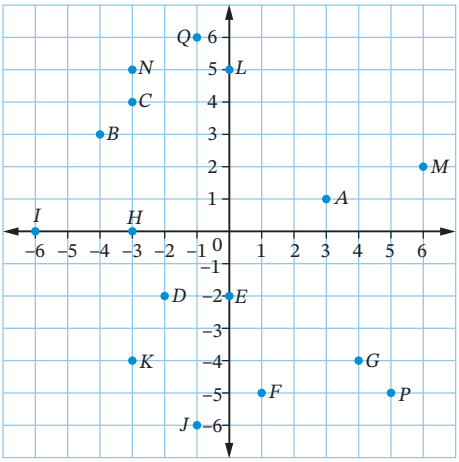
7 **b, d**



d an ostrich or emu

Exercise 9.03

1 *A* (–4, 3) *B* (–2, 2) *C* (0, 2) *D* (2, 3)
E (2, 1) *F* (1, –1) *G* (2, –3) *H* (0, –3)
I (–4, –3) *J* (–2, –1) *K* (3, 0) *L* (–4, 0)
M (0, –2) *N* (0, 3) *O* (0, 0) *P* (–2, 0)
Q (1, 0) *R* (3, –1) *S* (–2, –2) *T* (–3, 1)
U (–1, 2) *V* (4, 1) *W* (4, –3) *X* (–4, –1)
Y (1, –4) *Z* (–1, –3)



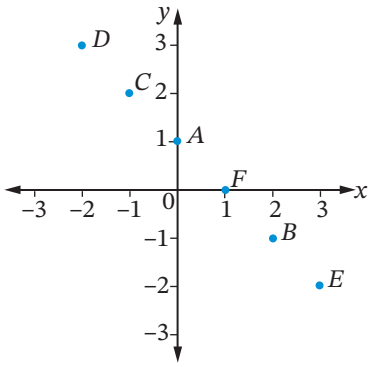
3 **a** *A*, *M* **b** *B*, *C*, *N*, *Q* **c** *D*, *K*, *J*
d *F*, *G*, *P* **e** *E*, *I*, *H*, *L*

4 **a** 4th **b** 3rd **c** 2nd

5

+	+
–	+
–	–
+	–

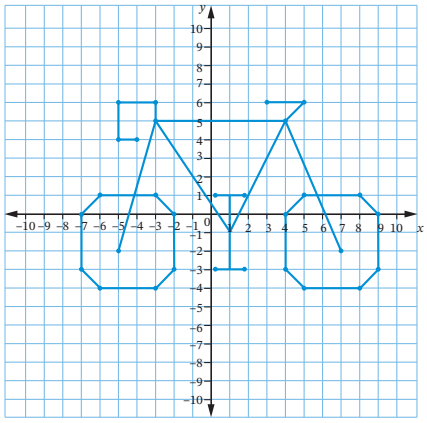
6 **a, b**



c They lie in a straight line.
d Teacher to check.

7 D

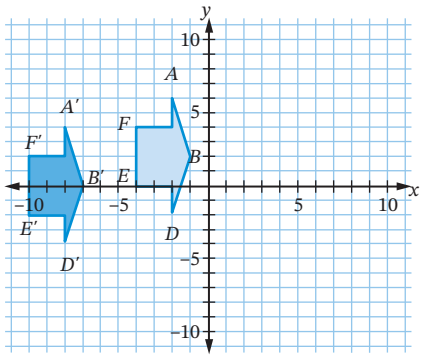
8 **a, b**



c a bicycle

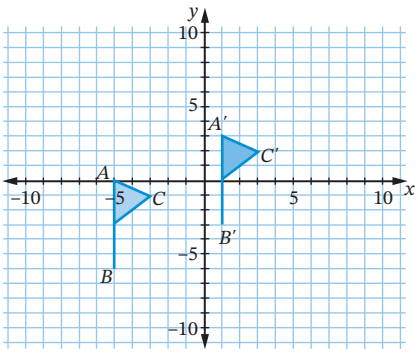
Exercise 9.04

1 **a**



b $A(-2, 6) \rightarrow A'(-8, 4)$
$B(-1, 2) \rightarrow B'(-7, 0)$
$C(-2, -2) \rightarrow C'(-8, -4)$
$D(-2, 0) \rightarrow D'(-8, -2)$
$E(-4, 0) \rightarrow E'(-10, -2)$
$F(-4, 4) \rightarrow F'(-10, 2)$
$G(-2, 4) \rightarrow G'(-8, 2)$
The x-coordinates decrease by 6 and the y-coordinates decrease by 2.

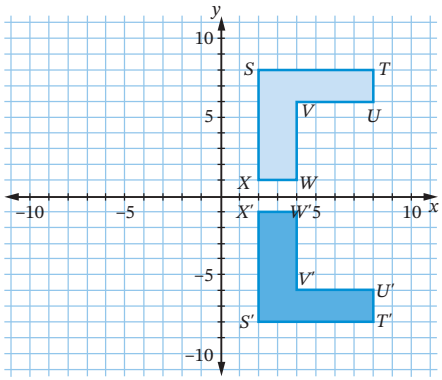
2



A' (1, 3), B' (1, −3), C' (3, 2)

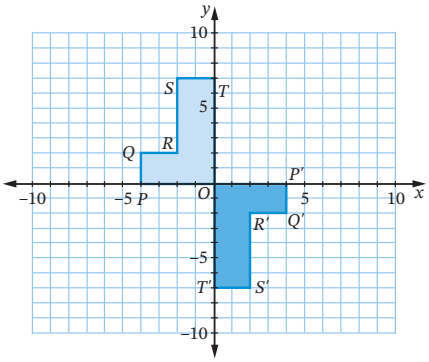
3 Translated 5 units left, 4 units down

4 **a**



b $S(2, 8) \rightarrow S'(2, -8)$
$T(8, 8) \rightarrow T'(8, -8)$
$U(8, 6) \rightarrow U'(8, -6)$
$V(4, 6) \rightarrow V'(4, -6)$
$W(4, 1) \rightarrow W'(4, -1)$
$X(2, 1) \rightarrow X'(2, -1)$
The x-coordinate does not change, the y-coordinate changes sign.

5 **a**



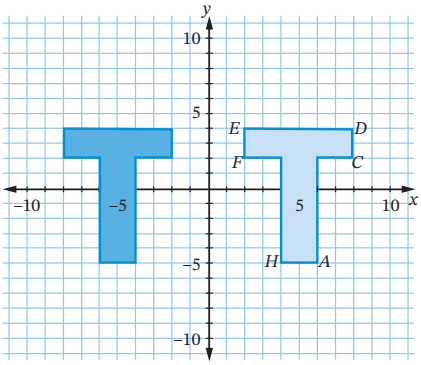
b $O(0, 0) \rightarrow O'(0, 0)$
$P(-4, 0) \rightarrow P'(4, 0)$
$Q(-4, 2) \rightarrow Q'(4, -2)$
$R(-2, 2) \rightarrow R'(2, -2)$
$S(-2, 7) \rightarrow S'(2, -7)$
$T(0, 7) \rightarrow T'(0, -7)$
The x- and y-coordinate of each vertex changes sign.

6 B

7 **a** Reflection across the x-axis
b (−5, 8). The x-coordinate does not change, the y-coordinate changes sign.

8 **a** Rotation 90° clockwise about K
b (2, −3)

9 **a**



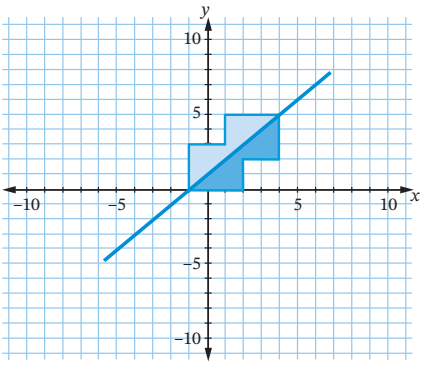
b $A(6, -5) \rightarrow A'(-6, -5)$
The x-coordinate changes sign but the y-coordinate does not change.

10 **a** Reflection across the x-axis
b (−3, −7)

11 **a** Rotation 90° anti-clockwise about T
b (4, 3). The x-coordinate becomes the new y-coordinate, the y-coordinate becomes the new x-coordinate with the sign changed.

12 Translation 10 units to the right, (0, 0)

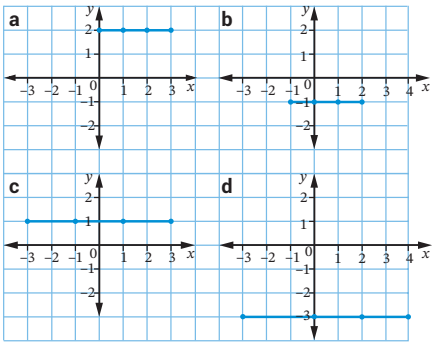
13



Exercise 9.05

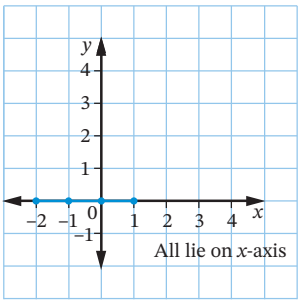
1 **a** (0, 2), (2, 3), (4, 4), (7, 9)
b (−2, −1), (−1, 3), (2, 4), (5, −2)
c (0, 2), (1, 5), (2, 8)
d (−4, −1), (−2, −3), (1, 0), (5, 2)

2

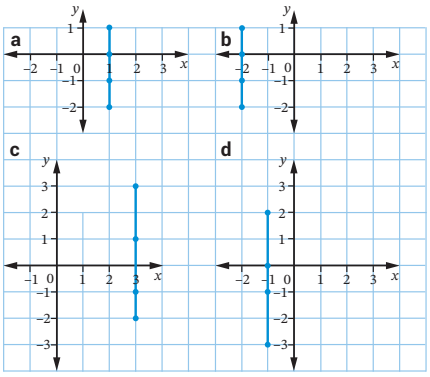


All points lie on a horizontal line.

3

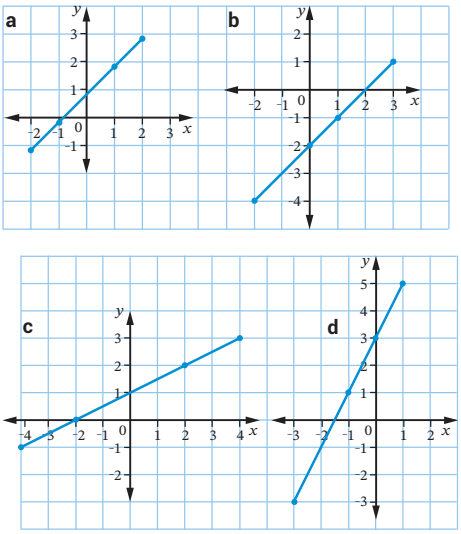


4



All points lie on a vertical line.

5

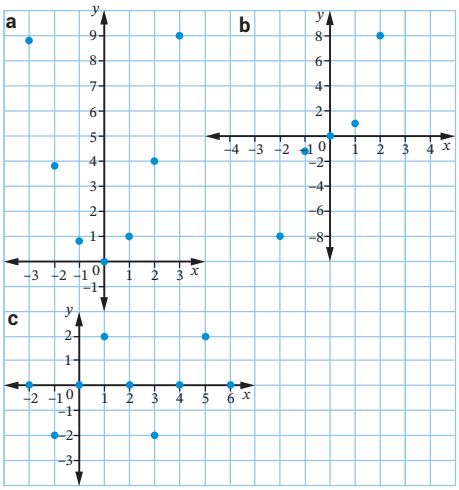


All points lie on a line (are collinear).

6 They increase by the same amount.

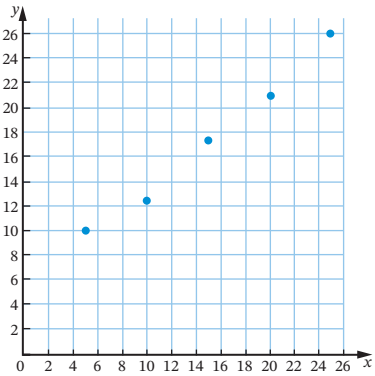
7 B

8



They lie on a curve, not in a line

9 There is a rough pattern; as the age of the tree increases, the height increases at a steady rate.



Mental skills 9

2 **a** 720 **b** 486 **c** 825 **d** 128
e 560 **f** 216 **g** 189 **h** 432
i 560 **j** 616 **k** 135 **l** 675

Exercise 9.06

1 \$40 800
2 97 km/h
3 \$482.50
4 18.8 m
5 25°C
6 **a** 2, 6, −2, −9, 4 **b** 24, −9, 6, 30, −12
c 4, −1, −3, 0, $3\frac{1}{2}$ **d** 5, 14, 7, 20, −2
e 16, −2, 0, 8, 4 **f** −7, 4, 2, −11, 8
7 A

8 **a**

x	50	51	52	53	54	55	56	57	58	59	60
\$*C*	92.50	94.35	96.20	98.05	99.90	101.75	103.60	105.45	107.30	109.15	111.00

b increase
c increases by 1.85

9

k (kL)	30	31	32	33	34	35	36	37	38	39	40
\$*C*	152.15	155.35	158.55	161.75	164.95	168.15	171.35	174.55	177.75	180.95	184.15

10 **a**

n	3	4	5	6	7	8	9	10	11	12
S	180	360	540	720	900	1080	1260	1440	1620	1800

b increases by 180
c 1980°

11 **a**

t	20	22	24	26	28	30	32	34	36	38	40	30
\$*W*	705	762	819	876	933	990	1047	1104	1161	1218	1275	990.00

b increases by 57
c **i** 648 **ii** 1303.5

Exercise 9.07

1 **a**

Input (*x*)	0	1	2	3	4
Output (*y*)	2	4	6	8	10

b

Input (*x*)	−4	−2	0	2	4
Output (*y*)	−11	−5	1	7	13

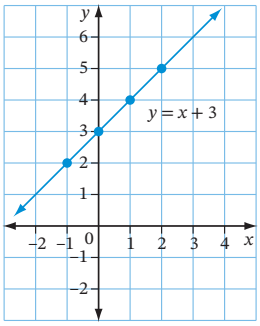
c

Input (*x*)	0	3	6	9	12
Output (*y*)	−2	7	16	25	34

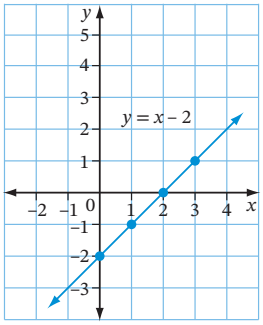
d

Input (*x*)	−10	−5	0	5	10
Output (*y*)	−46	−26	−6	14	34

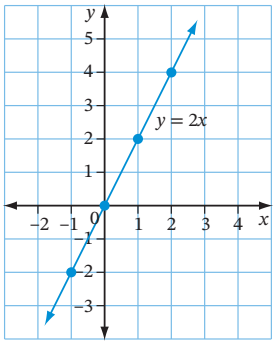
2 **a** 2, 3, 4, 5



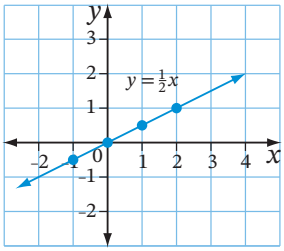
b −2, −1, 0, 1



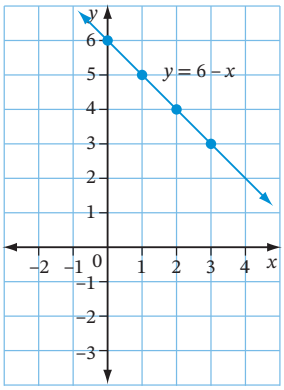
c −2, 0, 2, 4



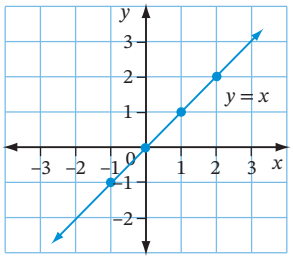
d $-\frac{1}{2}, 0, \frac{1}{2}, 1$



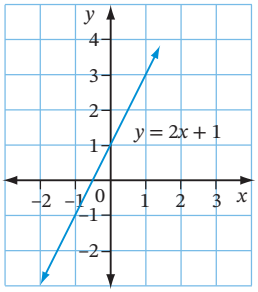
e 6, 5, 4, 3



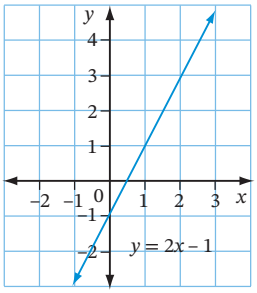
f –1, 0, 1, 2



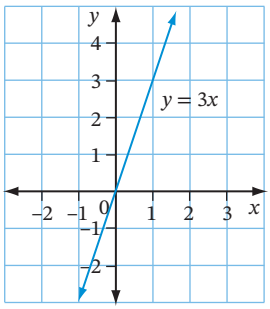
3 a



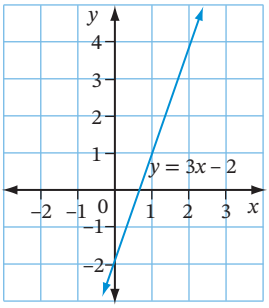
b



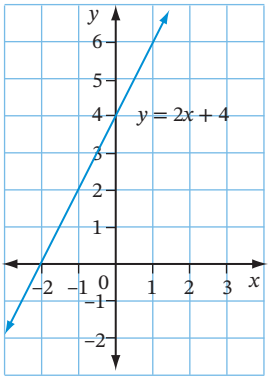
c



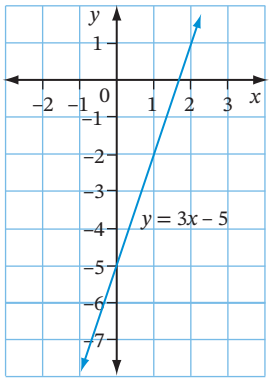
d



e

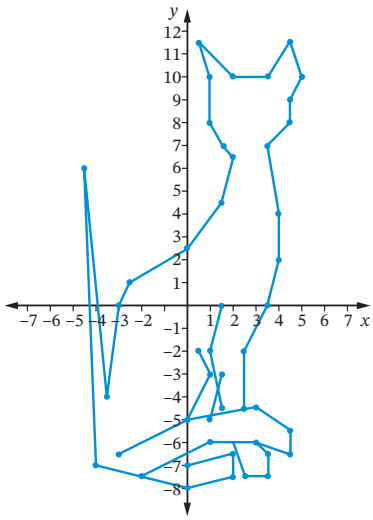


f



Power plus

1 **b**



d a cat

2 **a**

x	y
0	2
1	3
3	5
6	8
8	10

$y = x + 2$

b

x	y
1	0
4	3
7	6
8	7
10	9

$y = x - 1$

c

x	y
0	8
2	6
4	4
6	2
8	0

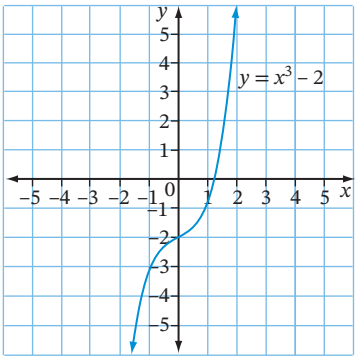
$y = 8 - x$ or $x + y = 8$

d

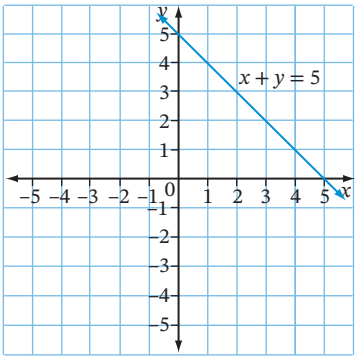
x	y
−1	5
0	3
2	−1
3	−3
4	−5

$y = 3 - 2x$ or $2x + y = 3$

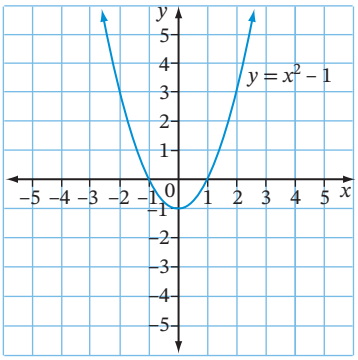
3 **a** non-linear



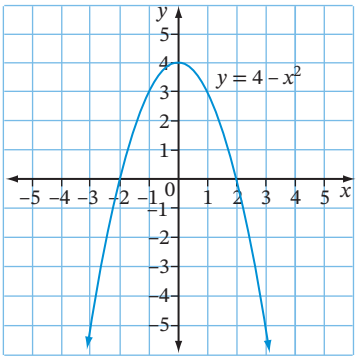
b linear



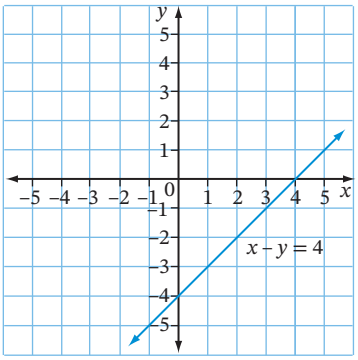
c non-linear



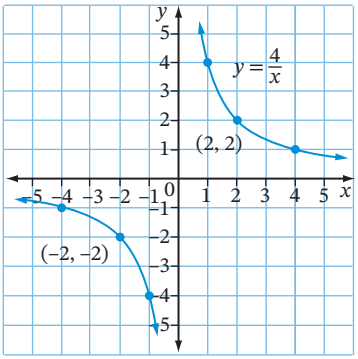
d non-linear



e linear



f non-linear

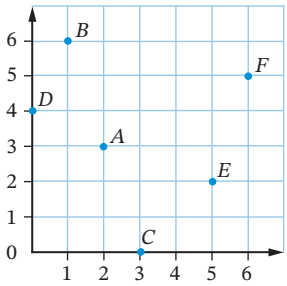


Test yourself 9

1 **a** student centre **b** Pi stadium
c Short Rd **d** library

2 **a** D5 or E5 **b** B6 or B7 **c** E8 to I8

3

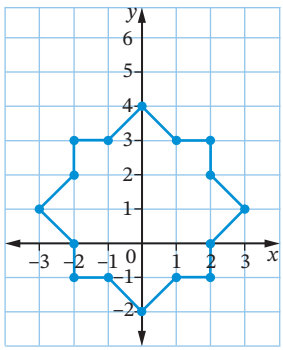


4 *A* (2, 0) *B* (3, 2) *C* (0, 4) *D* (–2, –1)
E (1, –3) *F* (–1, 1) *G* (4, 1) *H* (–4, 3)
I (0, –5) *J* (–3, 0) *K* (3, –4)

5 **a** 3rd
b Any coordinates of the form (0, *n*), where *n* is a number
c 4th
d Any coordinates of the form (–*a*, *b*), where *a* and *b* are positive numbers
e (0, 0)

6 B

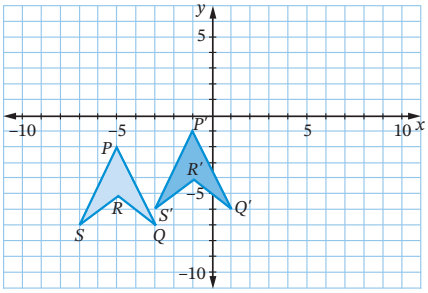
7 **a**



b star

8 **a** Reflection across the *y*-axis
b (–4,–3). The *x*-coordinate changes sign while the *y*-coordinate does not change.

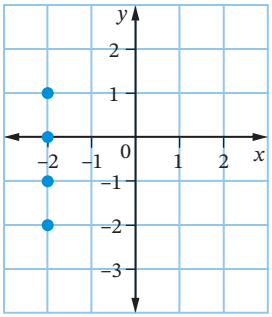
9 **a**



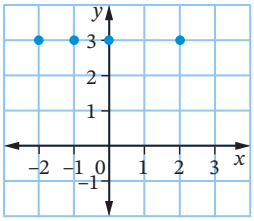
b $P(-5,-2) \rightarrow P'(-1,-1)$
$Q(-3,-7) \rightarrow Q'(1,-6)$
$R(-5,-5) \rightarrow R'(-1,-4)$
$S(-7,-7) \rightarrow S'(-3,-6)$
The *x*-coordinate increased by 4, the *y*-coordinate increased by 1.

10 **a** Rotated 180° about *S* or reflected across the *y*-axis and then across the *x*-axis.
b $V(5,-4) \rightarrow V'(-5,4)$. The *x*- and *y*-coordinates both changed sign.

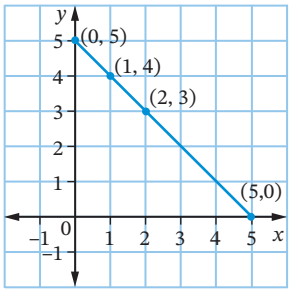
11 **a**



b

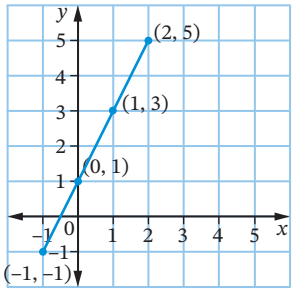


12 a



Collinear, decreasing by same amount

b



Collinear, increasing by same amount

13 **a** 0.05 **b** \$2250 **c** yes **d** yes

14 **a** 2, 3, 4, 5, 6 **b** −11, −9, −7, −5, −3
c 6, 5, 4, 3, 2

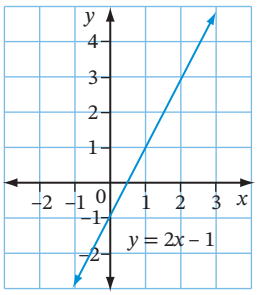
15 a

d (km)	10	15	20	25	30	35	40	45	50	55	60
\$*C*	28.2	34.45	40.7	46.95	53.2	59.45	65.7	71.95	78.2	84.45	90.7

b C increases by 6.25
c **i** \$21.95 **ii** \$115.70

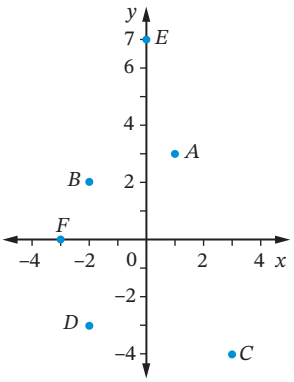
16 length 18 m, width 6 m

17

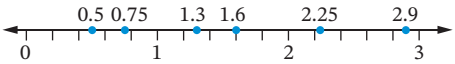


PRACTICE SET 3

1



2



3 7.35

4 **a** 9.44 **b** 3.1 **c** 9.05
d 21.89 **e** 5.407 **f** 5.283

5 **a** 0.2 **b** 0.375

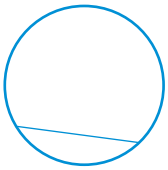
6 **a** 2 g **b** 2000 kg **c** 8 weeks
d 6500 mL **e** 72 hours **f** 4000 g
g 1.5 kg **h** 3 L **i** 8 min
j 2.5 t **k** 9500 mg **l** 12 kg

7 101.6 cm

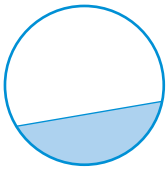
8 **a** 24 mm **b** 160 cm **c** 30 cm **d** 56 cm

9 **a** 32 mm² **b** 1600 cm² **c** 30 cm² **d** 121 cm²

10 **a**



b



c

d

11 **a** 31.42 cm **b** 21.99 cm

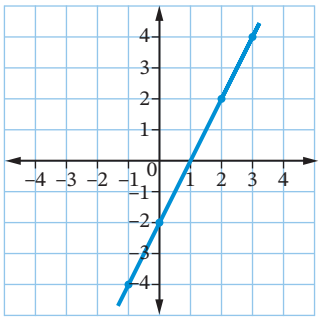
12 **a** 0.8 **b** 12.06 **c** \$9.82 **d** 3.8

13 C

14 **a** 7.5 kL **b** 7.5 m³

15 **a** 60 cm² **b** 880 mm²

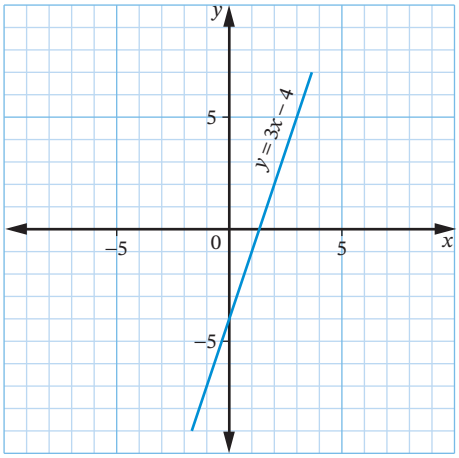
16



17 (–7, –1)

18 **a** 45 m^3 **b** 330 cm^3 **c** 11 880 mm^3

19



20 $0.\dot{5}$

21 **a** 8.7°C **b** 0.25 m or 25 cm
c \$3.20

22 A

23 **a** 4.34 **b** 1.263 **c** 0.07

24 **a** 10.023 **b** 83.6 **c** 13.44

25 10 cm **26** 6 m **27** A

28 **a** 4 cm **b** 32 cm

29 7 cm

CHAPTER 10

SkillCheck

1 Size of interval:
a 5 **b** 1.5 **c** 2 **d** 1 **e** 3
(Teacher to check scale diagrams.)

2 **a** 50 **b** 34.5

3 **a** Cola, Orange, Lime, Lemonade
b 5 **c** 23

4 **a** 11, 22, 24, 28, 31 **b** 13, 15, 16, 27, 18, 21

5 **a** 18, 17, 11, 6, 5 **b** 31, 23, 17, 14, 8, 5, 2

Exercise 10.01

1 **a** 10 cars
b 5 cars. It is difficult to represent the whole range of values accurately.
c 8–10 a.m. **d** 6–8 a.m.
e 6–8 a.m., 5; 8–10 a.m., 40; 10 a.m.–12 noon, 10; 12 noon–2 p.m., 20; 2–4 p.m., 35; 4–6 p.m., 15
f **i** 8–10 a.m. dropping children at school, going to work
ii 6–8 a.m. not many people around yet, a few going to work
iii 2–4 p.m. parents picking up children.

2 **a** Sydney **b** Darwin **c** C
d 2.2 million or 2 200 000
e Adelaide **f** 21 times

3 **a** **i** 99% **ii** 87% **iii** 88%
b Dishwasher **c** Mobile phones
d About 15% **e** Teacher to check.

4 **a** 1 unit = 10% **b** 45% **c** 2006
d **i** 3 and 4 **ii** 5 and over
e 3 and 4 **f** False

5 **a** 5 km or less **b** 100 mm
c $\frac{44}{100} = \frac{11}{25}$ **d** $\frac{75}{100} = \frac{3}{4}$
e Fewer accidents occur. Most travelling is done close to home.
f $\frac{6}{100} = \frac{3}{50}$ **g** True
h Teacher to check.

6 **a** 100 mm
b Meet community needs, $\frac{28}{100} = \frac{7}{25}$
c $\frac{21}{100}$
d Meet community needs, decrease cost, save natural resources, reduce pollution, and other.
e Teacher to check.

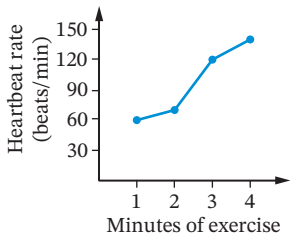
7 **a** dog **b** goldfish
c **i** $\frac{3}{30} = \frac{1}{10}$ **ii** $\frac{4}{30} = \frac{2}{15}$
d 360° **e** 96° **f** No
g Dog 27, Bird 12, Cat 24, Guinea pig 18, Goldfish 9

8 **a** **i** 70 cm **ii** 120 cm
b **i** 4 **ii** 11
c 0 and 1, babies grow quickly
d 6 and 7, children's growth slows down
e 3 years
f 10 years, growth slows down
g **i** fairly flat **ii** possibly horizontal
h Teacher to check.

Exercise 10.02

1 **a** 20 000 boxes
b It has a correct even scale for the sales axis.
c It is difficult to see where the graph starts, it exaggerates the amount of difference.
d Bigger pictures/diagrams can appear larger than the actual amount and exaggerate any difference.

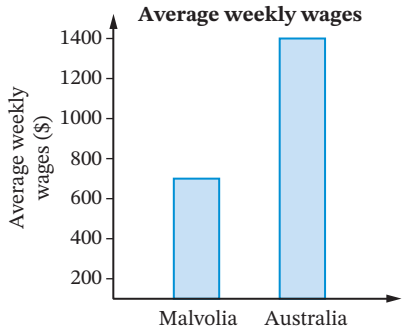
2 **a** 120 beats/min **b** 20 beats/min
c Heart rate scale does not begin at 0.
d Heart rate and exercise



3 **a** Australian weekly wages are 4 times bigger than Malvolia's.

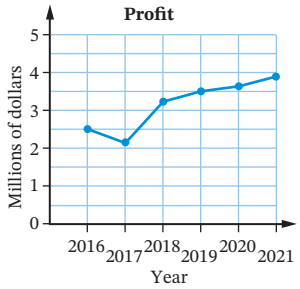
b No scale on vertical axis, use of pictures instead of columns.

c Columns

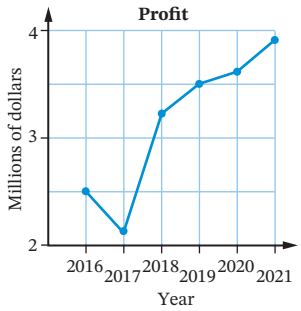


4 No scale on the horizontal population axis.

5 **a**



b



c Graph in part **b** because it is much steeper.

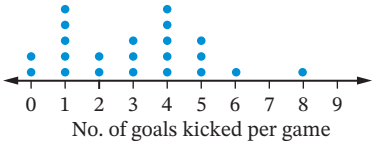
6 **a** The scale does not begin at 0, the scale is not even: increasing by 50, then 25.

b

7 Teacher to check.

Exercise 10.03

1 **a**



b 8 **c** 0 to 5 **d** 22

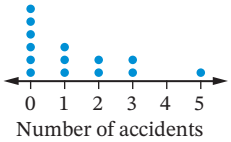
2 **a** 18 **b** 16 **c** 18 to 26

d 25 **e** No **f** C

3 **a** 37 **b** 31 to 40 **c** negatively skewed **d** 23

e 23 to 40 **f** 49

4 **a** Accidents on motorway each day



b positively skewed **c** 0

d 1.3 **e** 5, it's unusually large

5 C

6 **a** 15 is the outlier **b** positively skewed

c Most students live close to their school.

Exercise 10.04

1 C

2 **a**

Stem	Leaf
7	6
8	1 6 8
9	5 7 8
10	1 5 5
11	2 2 4 7
12	4

b 76, 124 **c** 8 **d** B

3 **a** 64 **b** 37 **c** 99

d 64 to 154 **e** 90s **f** 60s to 110s

g positively skewed

4 **a**

Stem	Leaf
13	7
14	0 1 3 5
15	3 4 5 7 7 7 8 9
16	2 2 3 6 6 8
17	0 1 5 5 7

b negatively skewed

c 137 cm, 177 cm **d** 5

e 157 cm **f** 158 cm, 159 cm

g $\frac{1}{4}$ **h** 20.8%

Mental skills 10

2 **a** 36 38 40 42 44 46 48 50 52 54 56 58 60 62 64 °C

b 200 210 220 230 240 250 260 270 280 290 300 310 320 330 340 350 360 370 mL

c 500 504 508 512 516 520 524 528 532 536 540 544 548 552 556 560 564 568 572 576 580 584 588 592 g

d 160 170 180 190 200 210 220 230 240 250 260 270 280 290 300 310 min

e 30 35 40 45 50 55 60 65 70 75 80 85 90 95 100 105 110 115 L

f 200 220 240 260 280 300 320 340 360 380 400 420 440 460 480 500 520 540 560 580 600 620 640 660 680 700 kg

g 120 135 150 165 180 195 210 225 240 255 270 285 300 315 330 345 360 375 390 405 420 seconds

h 100 125 150 175 200 225 250 275 300 325 350 375 400 425 450 475 500 525 550 575 600 625 650 675 700 725 mL

Exercise 10.05

1 **a** 30.25 **b** 32

2 **a** **i** 2.56 **ii** 3
b **i** 4.43 **ii** 2
c **i** 64.22 **ii** no mode
d **i** 45.17 **ii** 45.1
e **i** 5.78 **ii** 3,5

3 **a** green **b** They are not numbers.

4 **a** 7.6 **b** 6.1
c Mean, takes into account all scores.

5 **a** The last age.
b Arteri
c Phuong 16.4, Arteri 25.3
d Arteri family with a higher last age has the higher mean.

6 A

7 **a** 74% **b** 79%

8, 9 Teacher to check. **10** 2550

Exercise 10.06

1 **a** **i** 22 **ii** 5
b **i** 4.9 **ii** 2.1
c **i** 5 **ii** 5

2 **a** The 4th age
b Carrozza 12, Binns 12
c Carrozza 40, Binns 40
d No effect

3 C **4** C

5 **a** **i** 7 **ii** 6
b **i** 37.5 **ii** 10
c **i** 12 **ii** 5

6 **a** $696 500 **b** $1 202 000
c $767 273 **d** $625 000
e Median, not affected by outlier price ($1 800 000).

7 **a** Alf 75.9, Mike 75.7 **b** Mike
c Alf 2, Mike 6 **d** Alf, higher average
e Alf, smaller range **f** Alf 76, Mike 75.5

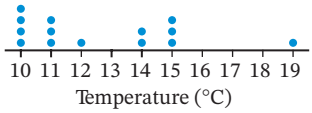
8 6, 7, 8 or 9 **9** Teacher to check.

Exercise 10.07

1 **a** **i** 4 **ii** 11 **iii** 10 **iv** 9.8
b **i** 4 **ii** 24 **iii** 22 **iv** 22.3
c **i** 6 **ii** 36 **iii** 37 **iv** 37.6
d **i** 0.3 **ii** 1.8 **iii** 1.8 **iv** 1.8

2 **a** **i** 39 **ii** 33 **iii** 33 **iv** 30.1
b **i** 26 **ii** 94 **iii** 88 **iv** 86.3
c **i** 40 **ii** 131 **iii** 124.5 **iv** 121.45
d **i** 44 **ii** 17 **iii** 17 **iv** 22.1

3 Maximum daily temperature in Cape Arid

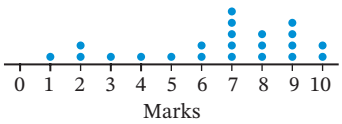


a 11.5°C **b** 10°C **c** 12.6°C **d** 9°C

4 **a**

8 Huxley quiz marks

8 Crancher quiz marks



b 5 **c** 7 **d** 9
e Mean = 6.5
f 8 Crancher, more higher marks

5 **a** 48 **b** 101 **c** 105, 112 **d** 100.7

6 **a** 67 **b** 51 **c** 67 **d** 67.8

Exercise 10.08

1 **a** 25 **b** Dynamo

c Dynamo: mean = 37.16, median = 37, range = 42

Energy Plus: mean = 28.88, median = 29, range = 28

d Dynamo has a greater mean and median and the longest lasting times, but greater range compared to Energy Plus.

e Dynamo is the better brand because it has a higher mean and median compared to Energy Plus, plus the longest lasting times.

2 **a**

Morley		Carrington
8 7 6 5 3	3	4 4 5
8 8 6 0	4	3 5
6 6 5 4	5	5
6 6	6	6 6 6 8
1	7	1 1 5 8
6 1 1	8	1 3 3 6
6	9	4 6

b Cannington: mean = 66.5, mode = 66, median = 69.5, range = 62

Morley: mean = 56.45, modes = 48, 56, 66, 81, median = 54.5, range = 63

c Cannington, scores are clustered in the 60s, 70s and 80s, it has a higher median and mean.

d 96, mean and range

3 **a**

7 Murray		7 Winton
9	2	6 9 9
8 8 8 7 5 2 1 1 1	3	1 1 2 3 5 6 9
9 8 8 7 7 7 6 5 4 3 2 0	4	0 1 4 5 5 6 7 7 8 8
0	5	

b 50 **c** 26

d in the 30s and 40s

e 29 to 50, 26 to 48.

f 7 Murray: mean = 40.7, median = 42,

7 Winton: mean = 38.6, median = 39.5

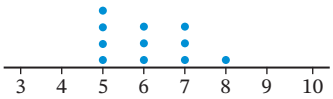
g 7 Murray scored better on the English exam, since it has a higher mean and median, a smaller range, and the highest mark overall.

h No outliers

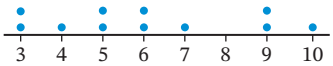
4 **a** Group A: mean = 6.1, median = 6

Group B: mean = 6.1, median = 6

b



Assignment marks for Group A



Assignment marks for Group B

Group B marks are more spread out (from 3 to 10 and spread fairly evenly) than Group A (clustered around 5 to 7).

c 4

5 **a** 25

b Cougars: mean = 65.24, median = 66, modes = 46, 58, 61, 66, 68, 86, 92, range = 49

Tigers: mean = 71.56, median = 70, modes = 53, 70, 87, 89, range = 50

c Cougars are clustered in the 40s to 70s, Tigers are clustered in the 50s to 80s. Tigers have a higher mean and median, but almost the same range compared to the Cougars.

d The Tigers are the better team, they have a higher mean and median than the Cougars while having almost the same range as the Cougars.

6 **a**

Maria		Stacey
9	1	
5	2	0 4 5 9 9
9 8 7 5 2 0	3	0 0 5 5 7 7
6 5 1	4	4
3	5	

b skewed left (negatively skewed)

c Maria: mean = 36.7, median = 37.5, no mode, range = 34

Stacey: Mean 31.3, median = 30, modes = 29, 30, 35, 37, range = 24

d Stacey, smaller range

e Maria

f Maria, because she has a higher mean and median than Stacey, even though she has a greater range.

Exercise 10.09

1 **a** **i** symmetrical and bimodal

ii no outliers, some clustering near 5, 11

b **i** symmetrical and bimodal

ii 10 is an outlier, clustering near 4, 6

c **i** negatively skewed

ii no outliers, clustering in the 70s and 80s

d **i** positively skewed

ii no outlier, clustering near 21, 22

e **i** not symmetrical, bimodal

ii no outliers, some clustering 12–13, 16

f **i** positively skewed

ii no outliers, clustering near 1, 2

g **i** symmetrical

ii no outliers, some clustering near 51

h **i** negatively skewed

ii 10 is an outlier, clustering at 17–19

2 **a** 30 **b** in the 30s and 40s

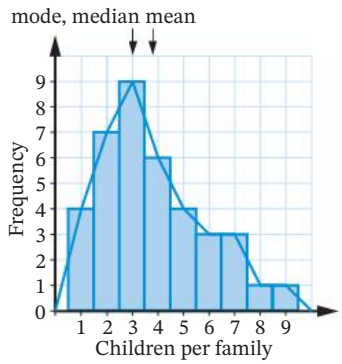
c positively skewed

d Test was too hard or students did not prepare for the test.

e $\bar{x}$ = 48.6, median = 45, mode is 28, 38, 45, 47, 64

3 **a**

Children per family	Frequency
1	4
2	7
3	9
4	6
5	4
6	3
7	3
8	1
9	1

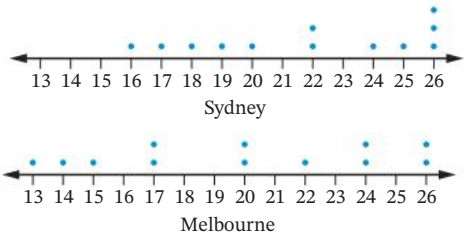


b No **c** positively skewed
d 2 to 4
e mode = 3, median = 3, $\bar{x} = 3.8$

4 C

5 **a** symmetrical **b** 56, 64, 83
c 76 **d** 72.8 **e** 72
f No, because the range has been affected by the outlier of 26. Without the outlier, the range is 45.

6 **a** **i**



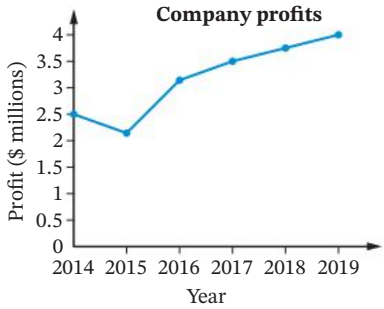
ii Both are slightly negatively-skewed.
b Sydney: $\bar{x} = 21.75$, median = 22, mode = 26, range = 10
Melbourne: $\bar{x} = 19.83$, median = 20, modes = 17, 20, 24, 26, range = 13
c Sydney is generally warmer – higher mean, median and lower range.

Power plus

1 6
2 **a** 74.17 **b** 77.17
c It also went up by 3 marks.
d It changes the mean by the amount added or subtracted.
3 92%
4 **a** 40 **b, c** Teacher to check.
5 Yes, the other 2 sisters earn a total of $6700 per week.

Test yourself 10

1 **a** 15–24 years **b** 13.5%
c It covers more years than each of the other groups (other answers possible).
2 **a** 23% **b** Anglican
c More Catholic, Anglican, Other Christian, fewer No religion, Buddhist, Muslim or Other.
3 **a** Snowy Mountains **b** Uluru
c 23 to 25 **d** True
4 **a** January and February
b January and February
c October
d January and February
5 **a** Profit scale does not begin at 0.
b Sales have increased greatly
c



6 **a** 18 **b** 28 **c** 3 **d** $\frac{8}{15}$
7 **a** 31
b

Stem	Leaf
7	1 5 6 8 9 9
8	0 0 0 2 2 2 3 4 4 5 8 8
9	0 0 0 0 2 2 4 5 6 8 8
10	4 5

c 90 kg **d** 71 kg
8 **a** **i** 4 **ii** 4
b **i** 8.78 **ii** 6
c **i** 3.5 **ii** 1
9 **a** **i** 4 **ii** 4
b **i** 10 **ii** 6
c **i** 3.5 **ii** 7
10 **a** **i** 4 **ii** 4.5 **iii** 6 **iv** 4.25
b **i** 35 **ii** 60 **iii** 66 **iv** 57.36

11 a

Boys		Girls
7 0	1	8
9 8	2	8 9
8 0	3	5 7 9
0	4	8
8 5 0	5	1 2 3 5 9
6 2	6	0 6 9
5 4 3 3 2	7	3 5 6
8 5	8	3
3	9	4

b 94, girl

c Girls: mean = 55, median = 54, no mode, range = 76

Boys: mean = 55.8, median = 60, mode = 73, range = 83

d Boys

e Overall the boys performed better than the girls with a higher median and mean, but they had a greater range of marks.

f The girls seem to be more consistent, clustered from the 50s to 70s, whereas the boys are evenly spread across a larger range.

12 a positively skewed, bimodal, no outliers, clustering at 12, 14.

b symmetrical, no outliers, clustering near 100 – 109.

c negatively skewed, no outliers, clustering near 15 – 17.

CHAPTER 11

SkillCheck

1 **a** impossible **b** Teacher to check.
c even chance **d** impossible
e likely **f** unlikely
g certain **h** even chance
i likely or certain **j** Teacher to check.

2 **a** $\frac{1}{5}$ **b** $\frac{4}{9}$ **c** $\frac{9}{10}$ **d** $\frac{3}{16}$

3 **a** 0.875 **b** 0.6 **c** 0.26 **d** 0.7

4 **a** 60% **b** 31% **c** 75% **d** 5%

5 Teacher to check. **6** Teacher to check.

7 C

8 **a** $\frac{7}{20}$ **b** $\frac{1}{5}$ **c** $\frac{12}{25}$ **d** $\frac{3}{50}$

9 **a** $\frac{5}{6}$ **b** $\frac{1}{4}$ **c** $\frac{3}{5}$

Exercise 11.01

1 **a** {red, blue}, 2
b {red, green, blue, yellow}, 4
c {red, blue}, 2
d {white, red}, 2
e {red, black, white, green}, 4
f {yellow, red, green}, 3

2 **a** 2, equally likely
b 3, not equally a likely
c 26, not equally likely
d 2, equally likely
e 10, equally likely
f 2, not equally likely

3 **a** {1, 3, 5} **b** {a, e, i, o, u} **c** {5, 6, 7, 8, 9}
d {March, May}
e {Adelaide, Brisbane, Hobart, Melbourne, Perth, Sydney}
f {K, 1, 2, 3, 4, 5, 6}

4 **a** 7
b No, some coloured regions are bigger than others.
c pink **d** green

5 **a** 20 **b** {\$2, \$1, 50c, 20c, 10c}
c No **d** 20c

6 **a** 52 **b** Yes **c** 13 **d** 26
e 4 **f** 2 **g** 1 **h** 20
i 12 **j** 2

7 D

8 **a** {A, B, C} **b** No, there are more As
c A

9 **a** 3 **b** {5, 6, 7, 8} **c** 6
d {black, red} **e** red
f {hearts, spades, clubs, diamonds}
g clubs

Exercise 11.02

1 **a** heads, tails **b** 2
c $\frac{1}{2}$ **d** 50%

2 **a** $\frac{1}{6}$ **b** $\frac{1}{2}$ **c** 0
d $\frac{5}{6}$ **e** $\frac{1}{2}$ **f** 1

3 **a** shade any 5 sectors
b shade any 6 sectors

4 **a** 50% **b** 25% **c** 50%
d 20% **e** 40% **f** 37.5%

5 **a** $\frac{1}{10}$ **b** 0.15 **c** 55%
d 75% **e** 0.35 **f** 65%

6 D

7 **a** $\frac{1}{26}$ **b** $\frac{1}{26}$ **c** $\frac{2}{26}=\frac{1}{13}$
d $\frac{5}{26}$ **e** $\frac{21}{26}$ **f** $\frac{10}{26}=\frac{5}{13}$

8 **a** pink **b** yellow
c black **d** purple, blue, pink

9 **a** blue **b** green **c** white

10 **a** $\frac{3}{10}$ **b** 0.3 **c** 40%
d $\frac{2}{5}$ **e** 0.4 **f** 20%

11 $\frac{5}{7}$ **12** Teacher to check.

13 **a** $\frac{6}{800}=\frac{3}{400}$ **b** 0.0075

14 Teacher to check.

15 **a** $\frac{26}{52}=\frac{1}{2}$ **b** $\frac{13}{52}=\frac{1}{4}$ **c** $\frac{1}{52}$
d $\frac{4}{52}=\frac{1}{13}$ **e** $\frac{16}{52}=\frac{4}{13}$ **f** $\frac{6}{52}=\frac{3}{26}$
g $\frac{2}{52}=\frac{1}{26}$ **h** $\frac{4}{52}=\frac{1}{13}$

16 C **17** Teacher to check.
18 D **19** Teacher to check.
20 **a** 1 **b** 5
c 9 **d** 10

21 **a** 0.64 **b** $36\frac{4}{11}$% or $36.\dot{3}\dot{6}$%
c 2

22 **a** 0, Teacher to check.
b 1, Teacher to check.

23 A **24** $\frac{1}{4}$

Exercise 11.03

1 **a** $\frac{1}{6}$ **b** 0 **c** $\frac{5}{6}$
d $\frac{3}{6}=\frac{1}{2}$ **e** 1 **f** $\frac{2}{6}=\frac{1}{3}$

2 **a** $\frac{1}{10}$ **b** $\frac{2}{10}=\frac{1}{5}$ **c** 0 **d** $\frac{6}{10}=\frac{3}{5}$

3 **a** $\frac{7}{12}$ **b** $\frac{7}{12}$ **c** 0
d $\frac{2}{12}=\frac{1}{6}$ **e** $\frac{3}{12}=\frac{1}{4}$ **f** 1

4 **a** $\frac{1}{4}$ **b** $\frac{1}{4}$ **c** 0

5 **a** C **b** A **c** E **d** H
e D **f** G **g** B **h** F

6 **a** $\frac{4}{18}=\frac{2}{9}$ **b** $\frac{3}{18}=\frac{1}{6}$ **c** 0
d $\frac{11}{18}$ **e** $\frac{15}{18}=\frac{5}{6}$ **f** $\frac{12}{18}=\frac{2}{3}$

7 **a** 0.61875 **b** 0.0125 **c** 0.9875 **d** 0

8 **a** $\frac{5}{20}=\frac{1}{4}$ **b** $\frac{1}{20}$ **c** 1
d 0 **e** $\frac{8}{20}=\frac{2}{5}$ **f** $\frac{4}{10}=\frac{2}{5}$

9 **a** F **b** D **c** E
d C **e** G **f** A

10 **a** $\frac{2}{11}$ **b** $\frac{1}{11}$ **c** $\frac{3}{11}$
d $\frac{8}{11}$ **e** $\frac{4}{11}$ **f** 0

11 Teacher to check.

Exercise 11.04

1 **a** $\frac{2}{10}=\frac{1}{5}$
b 16, the expected value is higher than the observed value
c $\frac{11}{80}$

2 **a** **i** 0.16 **ii** 0.16 **iii** 0.56
iv 0.24 **v** 0.1 **vi** 0.32
b **i** 8 **ii** 4 **iii** 2
iv 6, 7 **v** 10 **vi** 10

3 **a, b** Teacher to check.
c $\frac{1}{2}$
d 25, Teacher to check.
e Teacher to check.
f Relative frequency would get closer to $\frac{1}{2}$

4 B

5 **a, b** Teacher to check.
c **i** $\frac{1}{2}$, 25 **ii** $\frac{1}{6}$, 8 **iii** $\frac{1}{2}$, 25 **iv** $\frac{5}{6}$, 41
d, e Teacher to check.
f The relative frequency would get closer to the theoretical probability.

6 **a** Yes
b Yes, 6 appears much more frequently than the other numbers.
c 13.75% **d** 11

7 56% **8** B

Mental skills 11

2 **a** \$35.85 **b** \$39.80 **c** \$35.75 **d** \$34.80
e \$42.85 **f** \$69.80 **g** \$57.80 **h** \$53.65
i \$59.90 **j** \$133.70

Exercise 11.05

1 **a** The sex of a new baby is male
b No rain tomorrow
c Winning or drawing a soccer match
d Being right-handed (or ambidextrous)
e Being 18 years or under
f Choosing a card that is not hearts from a deck of playing cards
g A traffic light showing red or amber
h A train arriving on time or early

2 **a** $\frac{1}{6}$ **b** $\frac{2}{6}=\frac{1}{3}$ **c** $\frac{3}{6}=\frac{1}{2}$
d $\frac{5}{6}$ **e** $\frac{4}{6}=\frac{2}{3}$ **f** $\frac{3}{6}=\frac{1}{2}$

3 **a** $\frac{1}{1000}$ **b** $\frac{999}{1000}$

4 **a** $\frac{15}{130}=\frac{3}{26}$ **b** $\frac{23}{26}$ **c** 0

d 1 **e** $\frac{90}{130}=\frac{9}{13}$ **f** $\frac{55}{130}=\frac{11}{26}$

5 **a** $\frac{12}{13}$ **b** 45% **c** 0.93

6 **a** 0.7 **b** 0.3

c 15 red, 6 white, 9 blue

d 0

7 **a** $\frac{2}{12}=\frac{1}{6}$ **b** $\frac{5}{6}$

8 **a** $\frac{3}{4}$ **b** $\frac{2}{4}=\frac{1}{2}$ **c** $\frac{1}{2}$ **d** 0

9 C

10 **a** draw **b** 8%

11 **a** D

12 **a** $\frac{10}{11}$ **b** $\frac{7}{11}$ **c** $\frac{9}{11}$ **d** $\frac{8}{11}$

13 15% **14** C

15 **a** $\frac{1}{2}$ **b** $\frac{1}{8}$ **c** $\frac{3}{8}$ **d** $\frac{3}{4}$

16 0.8

Power plus

1 **a**

+	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

b 11

c There are more ways of obtaining some sums than others.

d 7 **e** 2, 12 **f** $\frac{1}{36}$

g $\frac{3}{36}=\frac{1}{12}$ **h** 5, 9

2 **a** HHH, HHT, HTH, HTT, TTT, TTH, THT, THH

b **i** $\frac{1}{8}$ **ii** $\frac{3}{8}$ **iii** $\frac{3}{8}$

iv $\frac{1}{8}$ **v** $\frac{7}{8}$ **vi** $\frac{4}{8}=\frac{1}{2}$

3 **a** hot dog, cola, cake
hot dog, cola, ice cream
hot dog, lemonade, cake
hot dog, lemonade, ice cream
pizza, cola, cake
pizza, cola, ice cream
pizza, lemonade, cake
pizza, lemonade, ice cream
hamburger, cola, cake
hamburger, cola, ice cream
hamburger, lemonade, cake
hamburger, lemonade, ice cream
12

b $\frac{1}{12}$ **c** $\frac{8}{12}=\frac{2}{3}$ **d** $\frac{6}{12}=\frac{1}{2}$

Test yourself 11

1 **a** pass, fail **b** G, PG, M, MA15+, R

c Katrina, Biljana **d** train, bus

2 **a** 5

b sections are not of equal size

c 5 **d** 1 **e** 4

3 **a** $\frac{1}{6}$ **b** $\frac{3}{6}=\frac{1}{2}$ **c** $\frac{4}{6}=\frac{2}{3}$

4 C

5 **a** 20.2% **b** 86.1% **c** 65.9% **d** 0%

6 C

7 **a** $\frac{6}{14}=\frac{3}{7}$ **b** $\frac{3}{14}$ **c** 0 **d** 1

8 A **9** C

10 **a** $\frac{1}{2}$

b 40, the observed frequency is lower than the expected frequency

c $\frac{24}{80}=\frac{3}{10}$

11 **a** Choosing a card that is not a queen from a deck of cards, $\frac{48}{52}=\frac{12}{13}$

b Rolling a number that is not a multiple of 3 on a die, $\frac{4}{6}=\frac{2}{3}$

c Buying a ticket that does not win out of 550 tickets sold, $\frac{549}{550}$

12 **a** $\frac{1}{5}$ **b** $\frac{3}{5}$ **c** $\frac{3}{5}$

13 $\frac{7}{15}$

CHAPTER 12

SkillCheck

1 **a** 2 **b** 3.6 **c** 18 000 **d** 5 **e** 65

f 8 **g** 7.5 **h** 8150 **i** 85

2 **a** 6 **b** 4 **c** 4

3 **a** 21 **b** 20 **c** 2

4 **a** $\frac{1}{4}$ **b** $\frac{4}{5}$ **c** $\frac{1}{4}$

Exercise 12.01

1 **a** 5 : 7 **b** 3 : 5 **c** 7 : 9 **d** 7 : 13 **e** 5 : 3

2 **a**

b 1 : 4

3 Teacher to check.

4 **a** 8 : 13 **b** 3 : 8 **c** 13 : 1 **d** 1 : 25

5 **a** 39 : 23 **b** 23 : 100
c 38 : 39 **d** 23 : 38

6 **a** 13 : 25 **b** 17 : 100 **c** 5 : 12
d 150 : 1 **e** 500 : 43 **f** 350 : 83
g 3000 : 1421 **h** 27 : 40 **i** 59 : 180
j 180 : 7 **k** 9 : 49 **l** 7700 : 17

7 D

8 **a** 1 : 25 **b** 1 : 2 **c** 1 : 4
d 11 : 9 **e** 1 : 1 **f** 2 : 1

9 Fewer students for each teacher

Exercise 12.02

1 **a** 12 **b** 10 **c** 9 **d** 20 **e** 32
f 84 **g** 30 **h** 75 **i** 5 **j** 5
k 4 **l** 30 **m** 2 **n** 40 **o** 5

2 D

3 **a** 1 : 10 **b** 1 : 2 **c** 2 : 5 **d** 5 : 7
e 3 : 2 **f** 7 : 3 **g** 10 : 1 **h** 5 : 11
i 1 : 1 **j** 5 : 12 **k** 4 : 1 **l** 1 : 12
m 2 : 3 : 5 **n** 3 : 4 : 6 **o** 3 : 1 : 4 **p** 3 : 2 : 8 : 4

4 C

5 **a** 80 : 200 = 2 : 5 **b** 300 : 1200 = 1 : 4
c 6 : 28 = 3 : 14 **d** 45 : 120 = 3 : 8
e 1 : 3 **f** 24 : 15 = 8 : 5
g 15 : 48 = 5 : 16 **h** 20 : 1000 = 1 : 50
i 4000 : 350 = 80 : 7 **j** 250 : 3000 = 1 : 12
k 9 : 5 **l** 20 : 1
m 30 : 2 = 15 : 1 **n** 3 : 20
o 1800 : 450 = 4 : 1

6 D

7 **a** 48 : 28 = 12 : 7 **b** 20 : 35 = 4 : 7
c 35 : 28 = 5 : 4 **d** 20 : 48 = 5 : 12
e 28 : 35 = 4 : 5 **f** 48 : 131

8 B

9 **a** **i** 72 : 1440 = 1 : 20
ii 816 : 624 = 17 : 13
iii 816 : 72 = 34 : 3
b Yes, teachers : students = 1 : 20 > 1 : 25 so the college has enough teachers, or 1 : 25 = 57.6: 1440 and 72 > 57.6, so the college has enough teachers.

10 Australia 3 : 250, Pakistan 3 : 100

11 14 : 11

12 **a** 1625 mL **b** 375 : 1625 = 3 : 13
c 375 : 2000 = 3 : 16

13 7P

Exercise 12.03

1 80 **2** 152 cm **3** 20 cm
4 28 **5** 245 **6** 480 kg
7 C **8** 8 kg **9** 72 **10** 20.4 t

11 **a** Mai $90, Felicity $63
b $225

12 72 cm **13** A

14 **a** 5 mL **b** 60 mL

15 175 m

16 25 kg cement, 100 kg sand

17 600 g

18 Sonya $220, Xanthe $275

Mental skills 12A

2 **a** 160 **b** 70 **c** 240 **d** 900
e 2600 **f** 900 **g** 2200 **h** 300
i 180 **j** 400 **k** 1100 **l** 210
m 700 **n** 1800 **o** 390 **p** 770

Exercise 12.04

1 **a** 10 a.m. **b** 6 km **c** 1.5 h
d 4 km/h **e** 2 h **f** 12 km
g 2 p.m., 4 km **h** 6 h

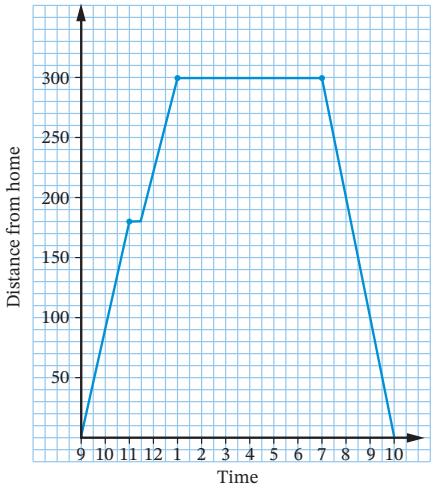
2 **a** 11 a.m., 40 km **b** 20 km/h
c 1:30 p.m. **d** 2.5 h
e 26 km/h **f** 25 km
g 130 km
h 11:30 a.m. – 12:30 p.m., graph is steeper

3 **a** 240 km **b** 180 km **c** 11:45 a.m.
d 40 km/h **e** 10 a.m., 160 km
f Sam, finishes journey earlier, steeper graph
g Before 11 a.m., graph less steep
h B

4 **a** B **b** A **c** C

5 Caitlin travelled 30 km in the first hour, 20 km in the second hour. She stopped for an hour, travelled another 30 km in one hour, stopped for $\frac{1}{2}$ hour, then travelled another 20 km in one hour.

6 a



b 300 km c 1 p.m.

d When he is returning home at 100 km/h from 7 p.m. to 10 p.m.

e 3 hours f 13 h g 100 km

Exercise 12.05

1 The vase is being filled with water and the water level is increasing gradually.

2 C

3 a The water level in the sink is decreasing, so water is being emptied/drained from the sink.

b At the end of the time, the water level decreases more quickly because of the round edges at the bottom of the sink (less water to drain).

4 a When the amount of petrol in the tank is not changing; the car is not moving, perhaps a rest break.

b At first, the car travels a lot. After a break, the car travels a little. Then, it travels a lot again. After another break, not much. Then, the car travels very little. The petrol tank is not yet empty but it is then refilled. Then, the car travels a little.

5 Hannah's hunger level is quite high when she wakes up, then she eats breakfast and is less hungry. She starts to be more hungry until morning tea, then is less hungry. Her hunger level increases until lunchtime. After lunch, Hannah is less hungry, then she becomes more hungry until afternoon tea. After she eats, she is less hungry and then more hungry until dinner time. Then Hannah is less hungry and goes to bed.

6 a The water level rises the fastest at the end, because the beaker is narrow at the top.

b The water level rises the slowest at the start, because the beaker is wider at the bottom.

7 B

8 a 0°C

b The temperature drops below 0°C, then during the day it starts to rise again reaching a peak in the middle of the day, before falling again and returning to 0°C again at the same time as at the start of the period.

9 When Riley was born, he stayed at the same height for a little while before growing more regularly. At age 9, his height increases rapidly before flattening out at around 18 years.

Exercise 12.06

1 a 1 h b 21 h c 5 h
d 4 h e 9 h f 11 h

2 a 22 min b 8 min c 8 min
d 2 h 16 min e 16 min f 4 h 21 min

3 a 35 b 30 c 72 d 104
e 168 f 42 g 4 h 8
i 12 j 5 k 2.5 l 4

4 a 3600 b 86 400 c 604 800

5 a 5 h 40 min b 3 h 6 min c 3 h 47 min
d 6 h 45 min e 4 h 22 min f 10 h 4 min

6 No. A 13-year-old is 410 248 800 seconds old, which is under 1 000 000 000 (a billion) seconds.

7 a 6 min 12 s b 9 min 5 s c 3 min 58 s
d 7 min 2 s e 8 min 20 s f 8 min 44 s

8 a 6 h 47 min b 13 h 7 min c 4 h 25 min
d 3 h 25 min e 7 h f 7 h 14 min
g 4 h 13 min h 3 h 28 min i 10 h 3 min
j 9 h 51 min k 1h 51 min l 49 min

Exercise 12.07

1 a 8:45 a.m. b 1:20 p.m. c 5:50 p.m.
d 12:17 a.m. e 9:05 p.m. f 6:32 p.m.
g 11:15 a.m. h 2:38 a.m. i 2:40 p.m.
j 3:20 a.m. k 4:55 p.m. l 11:31 p.m.
m 1:08 a.m. n 10:45 a.m. o 8:00 p.m.
p 6:43 a.m.

2 a 18:35 b 08:05 c 11:44 d 23:20
e 02:51 f 12:30 g 15:38 h 07:11
i 09:08 j 21:54 k 00:42 l 19:39
m 01:59 n 22:18 o 08:46 p 17:23

3 a 11:23 a.m. b 5:54 p.m. c 1:24 p.m.

4 a 17:17 b 22:06 c 02:45

5 a 8:55 a.m., 1:55 p.m. b 5:55 p.m., 11:15 p.m.
c 12:45 a.m. d 7:50 a.m. e D

Mental skills 12B

2 a 18 b 34 c 46 d 66
e 26 f 16 g 8 h 44
i 12 j 40 k 28 l 40
m 18 n 8 o 36 p 140

Exercise 12.08

1 A

2 a 1 h 15 min b 1 h 50 min c 5 h 44 min
d 3 h 53 min e 5 h 45 min f 11 h 38 min
g 2 h 25 min h 43 min i 1 h 47 min
j 4 h 8 min k 7 h 39 min l 13 h 25 min

3 **a** 3 h **b** 10 h **c** 14 h 40 min
d 4 h 15 min **e** 5 h 15 min **f** 8 h 45 min
g 7 h 5 min **h** 5 h 30 min **i** 8 h 40 min
j 8 h 10 min **k** 5 h 55 min **l** 8 h 40 min

4 **a** 8:00 p.m. **b** 7:00 p.m. **c** 7:43 p.m.
d 1:17 p.m. **e** 2:08 a.m. **f** 1:32 p.m.
g 2:14 a.m. **h** 4:15 p.m. **i** 6:17 a.m.
j 3:25 p.m.

5 Joe 12:43 p.m., Anna 1:14 p.m., Ken 1:19 p.m., Pathena 1:21 p.m., Mike 1:31 p.m., Gail 2:02 p.m.

6 4 h 45 min **7** 6 h 25 min

8 **a** 2:15 p.m. **b** 2:30 a.m. **c** 2:35 p.m.
d 11:25 a.m. **e** 7:55 a.m. **f** 6:15 p.m.

9 **a** 4 years 2 months **b** 4 years 7 months

10 **a** 6 years 1 month 5 days
b 15 years 5 months 25 days

11 2 h 15 min **12** 1 year 4 months

13 17 years 10 months **14** $155

15 2 p.m.

16 14 years 0 months 11 days

Exercise 12.09

1 **a** 7 h **b** 6 h 30 min **c** 3:20 p.m.
d 4 h 11 min **e** 13 min
f 1 h, 1 h 15 min, Liverpool-Sydney trip occurs at the start of peak traffic period

2 **a** 9:35 a.m., 1 h 25 min **b** TH114 (10:40 a.m.)
c 25 min **d** 9:25 a.m.
e TH038, 10:05 a.m.
f TH114, different aircraft or route

3 **a** 46 min
b 7:27 a.m.
c 27 min
d Teacher to check.
e Goulburn 11:12 a.m., Bundanoon 11:47 a.m., Moss Vale 12 noon, Bowral 12:06 p.m., Mittagong 12:11 p.m., Campbelltown 1:07 p.m., Strathfield 1:37 p.m., Sydney 1:49 p.m.

4 **a** 12:34, 1 h 4 min **b** 1:14
c 12:45
d 5, the first bus finishes its run at 11:50, ready to start the 6th run at 12:00
e 10:33 City cathedral, 11:00 Museum, 12:30 City square, 1:12 Town Hall, 2:28 Hall of Fame (other answers possible)
f (p.m.) 1:30, 1:38, 1:45, 1:54, 2:05, 2:15, 2:30, 2:42, 2:49, 3:00, 3:08, 3:20.

Exercise 12.10

1 **a** ahead **b** behind **c** behind **d** ahead
e ahead **f** behind **g** ahead **h** ahead
i ahead **j** ahead **k** behind **l** behind

2 **a** 10 p.m. **b** 8 p.m. **c** 7 a.m. **d** 8 p.m.
e 4 a.m. **f** 2 a.m. **g** 3 p.m. **h** 1 p.m.

3 **a** 2 h **b** 2 h **c** 20 h **d** 7 h
e 15 h **f** 0 h **g** 3 h **h** 13 h
i 7 h **j** 9 h

4 **a** 4 a.m. **b** 12 noon **c** 11 p.m. **d** 12 noon
e 8 p.m. **f** 6 p.m. **g** 7 a.m. **h** 5 a.m.

5 2:30 p.m. **6** 9 a.m. **7** B

8 Teacher to check.

9 **a** ahead **b** ahead **c** same
d behind **e** ahead **f** behind
g ahead **h** same **i** ahead

10 **a** $\frac{1}{2}$ h **b** 2 h **c** $\frac{1}{2}$ h
d $\frac{1}{2}$ h **e** 2 h **f** 0 h

11 **a** 11 p.m. **b** 10:30 p.m. **c** 9 p.m.
d 10:30 p.m. **e** 11 p.m. **f** 11 p.m.

12 **a** 6 p.m. **b** 6 p.m. **c** 4 p.m.
d 5:30 p.m. **e** 6 p.m. **f** 6 p.m.

13 **a** 4 p.m. **b** 3 p.m.

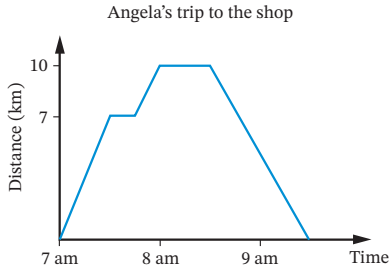
14 **a, b, c** Teacher to check.
d 3:30 p.m.

Power plus

1 **a** 4 : 7 **b** 13 : 8 **c** 10 : 1
d 7 : 20 **e** 8 : 3 **f** 6 : 5

2 Kobe is right (19 : 33 vs 18 : 33)

3 **a**



b Answers may vary, perhaps Angela needed a rest.
c To the shop was 1 hour and coming back was 1 hour, but on the way to the shop she stopped for a 15-minute break. So her travelling speed going to the shop was actually higher than her speed coming back. The reasons may vary, perhaps Angela was slower travelling home as she was tired.

4 **a** $166\frac{2}{3}$ min **b** 8.64 h

Test yourself 12

1 **a** 8 : 16 = 1 : 2 **b** 12 : 24 = 1 : 2

2 **a** 38 : 17 **b** 17 : 55

3 **a** 12 **b** 10 **c** 9 : 12

4 **a** 4 : 7 **b** 1 : 3 **c** 1 : 6 **d** 5 : 9
e 3 : 1 : 4 **f** 1 : 5 : 20 **g** 2 : 1 **h** 10 : 1
i 5 : 1 **j** 1 : 20 **k** 1 : 3 **l** 1 : 5
m 1 : 2 **n** 4 : 7 **o** 48 : 5

5 **a** 100 **b** $45 000

6 **a** 20 km **b** 3 **c** 2:30 p.m.

d 2:30 p.m. – 3:30 p.m. **e** 2.5 km/h

7 **a** When Jess' height is constant, sitting at the top of the slide.
b Because when she is sliding down, her height is decreasing quickly, reaching the ground rapidly.
c Jess climbing the ladder/steps of the slide, at a regular rate.

8 **a** 10 h **b** 3 h **c** 4 h

9 **a** 5 h 16 min **b** 3 min 28 s

10 **a** 10 h 5 min **b** 4 h 42 min
c 2 h 9 min **d** 29 min

11 **a** 1:48 a.m. **b** 6:24 a.m.
c 3:50 p.m. **d** 12:30 p.m.

12 **a** 14:10 **b** 05:33 **c** 00:19 **d** 13:50

13 **a** 4 h 20 min **b** 7 h 35 min **c** 6 h 55 min
d 6 h 30 min **e** 30 min **f** 11 h 55 min

14 17 years 2 months 13 days

15 **a** 3 h 19 min **b** 9:05 a.m.

PRACTICE SET 4

1 **a** {1, 2, 3, 4, 5, 6} **b** {red, blue, green, yellow}

2 **a** 3 : 5 **b** 3 : 2 **c** 5 : 2

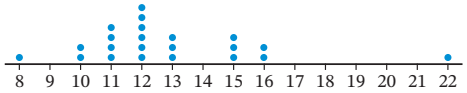
3 **a** **i** 97% **ii** 94% **iii** 50%
b Dishwasher **c** TVs **d** 40

4 **a** 5 h 25 min **b** 1 h 10 min
c 11 h 41 min **d** 7 h 5 min

5 **a** 08:25 **b** 16:45

6 **a** 9:15 p.m. **b** 4:30 a.m.

7



a Between 10 and 13 **b** 22

8 C

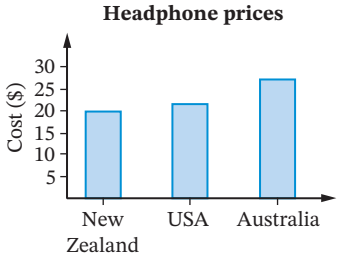
9 **a** 1 : 3 **b** 8 : 3 **c** 1 : 3
d 40 : 3 **e** 1 : 12 **f** 2 : 5 : 7

10 **a** $\frac{1}{2}$ **b** $\frac{2}{3}$

11 **a** 42 **b** 4 **c** 130
d 15 **e** 3, 32 **f** 5, 30

12 **a** 50 **b** $\frac{3}{5}$

13 **a** The cost axis does not begin at 0.
b



14 D **15** B

16

Stem	Leaf
0	4 5 6 9
1	0 1 3 4 5 5 6 7 8 9
2	1 2 4 8

17 B **18** 20

19 **a** $\frac{3}{10}$ **b** $\frac{7}{10}$

20 **a** 3.425 **b** 3, 3.5 **c** 3.45 **d** 0.9

22 **a** 6 km **b** 1 p.m. for 1.5 h
c 26 km **d** 10:30 a.m. and 4 p.m.
e first hour

21 **a** **i** 4 **ii** 2 **iii** 2 **iv** 2
b **i** 15 **ii** 21 **iii** 21 **iv** 20.56

22 **a** $\frac{10}{11}$ **b** $\frac{7}{11}$ **c** $\frac{8}{11}$

23 A

24 **a**

Eagles		Cougars
7 6	0	4 4 6
9 8 8 7 2 2 0	1	0 0 2 2 3 4 4 4 6 8 8 8 8 8
4 4 2 2 0 0 0	2	1 2 4 8 8
8 8 6 6 4 4 2	3	6 9
0	4	

b 40 **c** 4
d Eagles 20, Cougars 18
e Eagles: their scores are spread across the 10s, 20s and 30s, while the Cougars scores are mostly in the 10s.
f number of games won by each team, number of points scored against each team

GENERAL PRACTICE

1 **a** Sum of digits = 1 + 9 = 5 = 15, which is divisible by 3. So 195 is divisible by 3.
b 65

2 8^5

3 **a** 121 **b** 7 **c** 1000

4 **a** 60 **b** 360 **c** 405 **d** 192

5 (0, 0) **6** D **7** $2x + 8$

8 **a** $3x$ **b** $5m^2$ **c** $\frac{7k}{2}$
d $6y$ **e** $10 - kn$ **f** $2p + 4$

9 **a** 7200 **b** 0.425 **c** 4 **d** 0.24
e 50 **f** 6700

10 B

11 **a** arc **b** radius
c diameter **d** tangent
e chord **f** segment
g circumference **h** quadrant

12 **a** 42 mm **b** 40 cm **c** 16 cm

13 **a** 25.1 cm **b** 94.2 cm

14 18:10 **15** 470

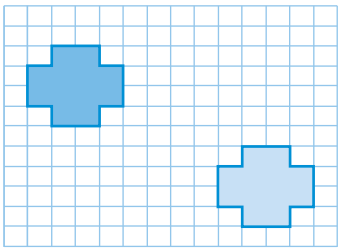
16 **a** $x = 70$ **b** $y = 150$ **c** $m = 25$

17 −5, −3, 2, 4, 6, 11

18 **a** −2 **b** −12 **c** 13 **d** 1
e 120 **f** 100 **g** −9 **h** −2

19 **a** $5 \times 10^2 + 5$
b $1 \times 10^4 + 7 \times 10^3 + 2 \times 10^2 + 3$

20



21 **a** symmetrical **b** negatively skewed (left)
c positively skewed (right)

22 10 and 11

23 **a** 0.5 **b** 50%

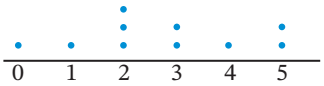
24 {red, amber, green}

25 **a** 2 **b** 16 **c** 4 **d** 25

26 **a** $\frac{9}{10}$ **b** $\frac{5}{12}$ **c** $\frac{3}{8}$ **d** $1\frac{1}{2}$

27 **a** (2, 1) **b** (−1, 0) **c** (−2, 3)

28



29 **a** 2.7 **b** 2.5 **c** 2 **d** 5

30 B

31 **a** $\angle ACD$ and $\angle CFE$ (other answers possible)
b $\angle ACD$ and $\angle BCF$ (other answers possible)
c AH

32 **a** 105° **b** 75° **c** 75°

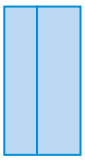
33 $\frac{1}{6}$

34 **a** Teacher to check. **b** 1

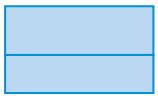
35 **a** 1 : 5 **b** 4 : 3 **c** 1 : 4

36 **a**

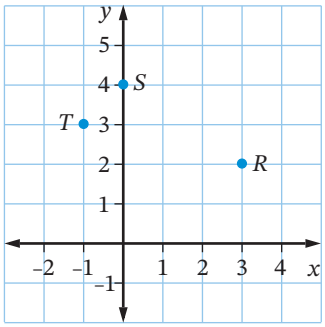
b



c



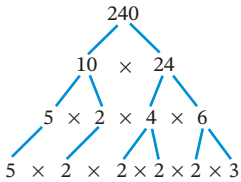
37



38 3, 11

39 **a** 1, 3, 9, 27 **b** 9

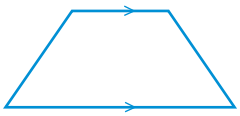
40 **a**



Other factor trees possible.

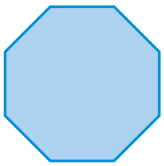
b $240 = 2^4 \times 3 \times 5$

41 **a**



b trapezium

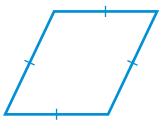
42 regular octagon



43 30 cm²

44 95 cm²

45 **a**



b 2 **c** 2

46 **a** $\frac{7}{20}$ **b** $\frac{1}{10}$ **c** $\frac{3}{250}$

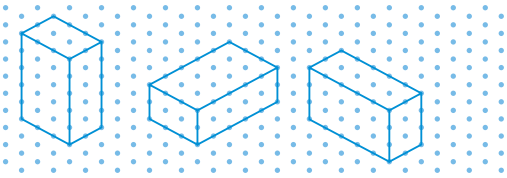
47 **a** 1.7 **b** 0.7 **c** 1.0

48 **a** False **b** True **c** False

49 −7

50 168 m³

51 different answers possible.



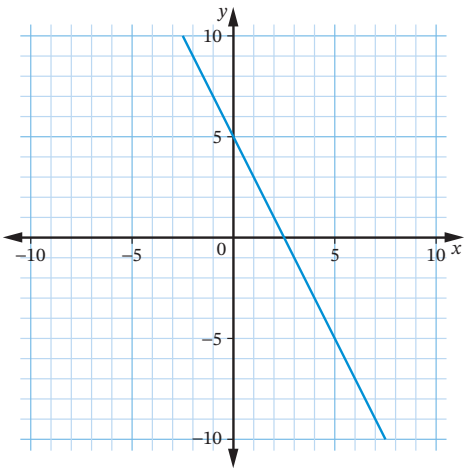
52 **a** \$9 **b** 16 m **c** 200 mL **d** \$20 **e** 4 kg

53 **a** 73% **b** 75% **c** 60% **d** 47%

54

t (days)	1	2	3	4	5	6	7	8	9	10
\$*C*	212.50	257.50	302.50	347.50	392.50	437.50	482.50	527.50	572.50	617.50

55



56 **a** 15 **b** 36 **c** 64 **d** 35

57 8:45 p.m.

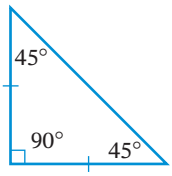
58 **a** 29.67 **b** 3.07 **c** 5.314

59 **a** 0.625 **b** 0.55

60 $0.1\dot{6}$

61 **a** $\frac{3}{5}$ **b** 60

62



63 A **64** 6 h 35 min **65** D

66 **a**

b

66 Need to purchase 32 m of fencing costing \$5360, so Dylan is short by \$360 and will not be able to install the fence.

CALCULATOR SKILLS

Exercise 1

1 **a** 5068 **b** 14.5 **c** 45 **d** 5.724 **e** 3.5 **f** 26.9 **g** 23 **h** 1.68 **i** 160 **j** 29.48 **k** −8.084 **l** 12.42

2 **a** \$19.15 **b** 85 cents

3 \$1875.50 **4** 7542 kg

5 2.4 m **6** \$18.75

7 23 trees **8** 75.2 km/h

9 \$85.74 **10** 13 buses

11 **a** \$41.20 **b** \$1318.40

12 53.4

Exercise 2

1 **a** 7 **b** −9 **c** −12 **d** −8 **e** 15 **f** −15 **g** 75 **h** 5 **i** −5 **j** 7 **k** −96 **l** 45 **m** −25 **n** 24 **o** 96 **p** 14 **q** −16 **r** 15 **s** −36 **t** 10

Exercise 3

1 **a** 196 **b** 529 **c** 216 **d** 1.728 **e** 26.3169 **f** 10 201 **g** 4.1616 **h** 6859 **i** 26 **j** 148 **k** 9 **l** 42

2 **a** 4.9 **b** 10.0 **c** 4.7 **d** 4.6

3 **a** 16 **b** 156 **c** 6 **d** 10

Exercise 4

1 **a** $\frac{5}{8}$ **b** $\frac{8}{17}$ **c** $\frac{5}{8}$ **d** $\frac{8}{9}$ **e** $\frac{21}{25}$ **f** $\frac{1}{4}$

2 **a** $\frac{15}{8}$ **b** $\frac{47}{9}$ **c** $\frac{43}{4}$

3 **a** $2\frac{2}{9}$ **b** $8\frac{4}{5}$ **c** $2\frac{1}{4}$

4 **a** 0.5 **b** 0.8 **c** 0.875
d 0.36 **e** 0.17 **f** $0.\dot{5}$

5 **a** $\frac{7}{20}$ **b** $2\frac{1}{25}$ **c** $\frac{1}{1000}$
d $\frac{73}{200}$ **e** $1\frac{3}{50}$ **f** $\frac{13}{40}$

6 **a** $\frac{17}{20}$ **b** $1\frac{1}{14}$ **c** $\frac{1}{15}$ **d** $\frac{4}{21}$
e $1\frac{1}{3}$ **f** $\frac{5}{24}$ **g** $1\frac{17}{30}$ **h** $\frac{35}{96}$
i $1\frac{3}{4}$ **j** $19\frac{1}{5}$ **k** $6\frac{13}{20}$ **l** $8\frac{13}{24}$
m $12\frac{3}{4}$ **n** 7 **o** $\frac{4}{27}$ **p** $19\frac{19}{60}$

7 **a** $\frac{1}{8}$ **b** $\frac{3}{4}$
c $15\frac{4}{5}$ or 15.8 **d** $\frac{1}{6}$ or $0.1\dot{6}$

8 **a** $6\frac{3}{4}$ hours **b** $33\frac{3}{4}$ hours

ANSWERS

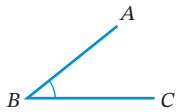
Glossary and index

12-hour time Time of day written using a.m. or p.m. notation and the hours 1 to 12. For example, 6:20 p.m. (p. 562)

24-hour time Time of day written using 4 digits (instead of a.m. or p.m.) and the hours 0 to 23. For example, 18:20 is the 24-hour time for 6:20 p.m. (p. 562)

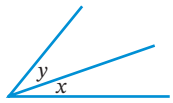
A

acute angle A 'sharp' angle between 0° and 90°, for example, the marked angle $\angle B$ in the diagram. (p. 262)



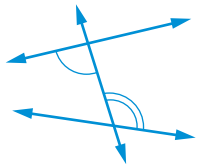
acute-angled triangle A triangle with 3 acute angles. (p. 262)

adjacent angles Angles next to each other that share a common arm, for example, angles x and y in the diagram. (p. 44)

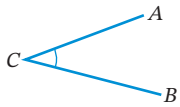


algebraic expression A number written in algebraic form using variables, for example, $2xy + 4y - 5$. (p. 205)

alternate angles A pair of angles between 2 lines crossed by a transversal, on opposite sides of the transversal, for example, the 2 angles marked in the diagram. (p. 74)



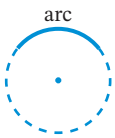
angle A description or measure of the size of a turn or rotation. The angle marked in the diagram is named $\angle C$ or $\angle ACB$. (p. 44)



angle sum The total of the sizes of all the angles in a shape. The angle sum of a triangle is 180°. (p. 267)

area The amount of surface enclosing a shape, measured in square units. (p. 372)

arc Part of the circumference of a circle. (p. 44)



ascending order Going up, increasing, from smallest to largest (1-2-3). The opposite of **descending order**. (p. 13)

associative law A law of arithmetic that says that when operating with more than 2 numbers, you can group them in any way. For example, the associative law of addition says that $(a + b) + c = a + (b + c)$ for any 3 numbers a, b and c. (p. 198)

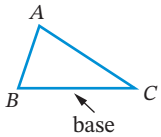
axis (plural: axes) reference line (p. 423)

B

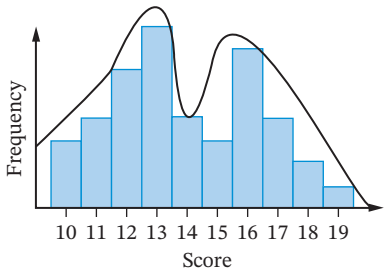
base (in index notation) When a number is raised to a power, the number raised is the base. In the expression 3^5, the 3 is called the base. (p. 286)

base (of a shape) The bottom side of a flat shape such as a triangle. (p. 286)

base (of a prism) The end faces of a prism, parallel to the identical cross-sections. (p. 274)



bimodal distribution A statistical distribution that has two peaks. (p. 495)



bisect To cut in half. (pp. 68, 276)

braces See **grouping symbols**.

brackets See **grouping symbols**.

C

capacity The amount of material (usually liquid) that a container can hold, measured in millilitres (mL), litres (L), kilolitres (kL) and megalitres. (p. 356)

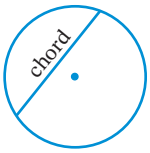
Cartesian plane Another name for **number plane**. Named after French mathematician and philosopher, René Descartes. (p. 423)

centi- A prefix meaning 'one-hundredth of'. (p. 356)

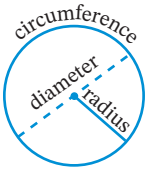
certain Must happen; has a probability of 1 or 100%. (p. 519)

chance experiment (p. 510) See **experiment**.

chord An interval joining 2 points on a circle. (p. 363)



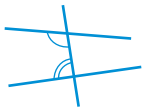
circumference The perimeter of a circle; a circle's outer boundary . (p. 368)



classify To group into categories or types. (p. 55)

cluster A group of data values that are bunched or close together. (p. 469)

co-interior angles A pair of angles between 2 lines crossed by a transversal, on the same side of the transversal (co-interior means 'together inside'), for example, the 2 angles marked in the diagram. (p. 77)



collinear points Points that lie in a line. (p. 428)

commutative law A law of arithmetic that says that you can operate with 2 numbers in any order. For example, the commutative law of multiplication says that $a \times b = b \times a$ for any 2 numbers a and b. (p. 198)

complementary angles 2 angles whose sum is 90°. The angles 30° and 60° are complementary. (p. 57)

complementary event All the possible outcomes that are *not* the event; the 'opposite' event. For example, the complementary event to rolling 1 on a die is rolling a number that is not 1. (p. 530)

composite number A number with more than 2 factors. 12 is a composite number since it has 6 factors: 1, 2, 3, 4, 6 and 12. (p. 122) See also **prime number**.

composite shape A shape made up of 2 or more basic shapes. (p. 378)

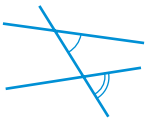
concave polygon A polygon some sides and vertices (corners) that point inwards, not a **convex polygon.** (p. 272)

convex polygon A polygon with sides and vertices (corners) that point outwards. (p. 272) See also **concave polygon.**

consecutive numbers Any series of integers that follow each other in order, for example, 8, 9 and 10. (p. 209)

coordinates An ordered pair of numbers used to locate a point or position, for example, $A(2, 5)$ tells us that the point A is located 2 units to the right and 5 units up from the origin. (p. 423)

corresponding angles A pair of angles in matching positions when 2 lines are crossed by a transversal. They are on the same side of the transversal and the lines, for example, the 2 angles marked on the diagram. (p. 72)



cross-section A 'slice' of a solid cut across it rather than along it, such as the shaded triangle in the triangular prism shown. (p. 285)

cube (of a number) The number raised to the power of 3. For example, 7 cubed $= 7^3 = 7 \times 7 \times 7 = 343$. (p. 118)

cube root (of a number) The value which, if cubed, gives the number. For example, $\sqrt[3]{8} = 2$ because $2^3 = 2 \times 2 \times 2 = 8$. (p. 118)

cubic metre A unit for measuring volume, equal to the volume of a cube with length 1 m. (p. 398)

D

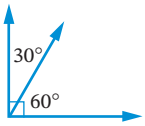
data Information, a collection of facts. (p. 461)

data set A collection of data about the same subject (p. 469)

decimal places The places after the decimal point in a number. For example, 3. 1416 has 4 decimal places. (p. 316)

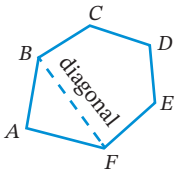
degree (symbol °) A unit for measuring angle size. There are 360° in a complete revolution and 90° in a right angle. (p. 53)

denominator The number below the line in a fraction. The denominator of $\frac{2}{3}$ is 3. (p. 146)



descending order Going down, decreasing, from largest to smallest (3-2-1). The opposite of **ascending order**. (p. 13)

diagonal An interval joining 2 non-adjacent vertices of a shape. (p. 277)



difference The result of a subtraction. The difference between 65 and 10 is 55. (p. 18)

digit A single figure; the whole numbers 0, 1, 2, 3, ..., 9. For example, the number 129 has 3 digits: 1, 2 and 9. (p. 97)

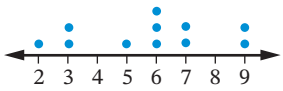
distributive law A law of arithmetic that says that you can multiply by a number by splitting it into the sum or difference of 2 other numbers. For example, $27 \times 12 = 27 \times (10 + 2) = 27 \times 10 + 27 \times 2$. More generally, $a \times (b + c) = a \times b + a \times c$ for any 3 numbers a, b and c. (p. 201)

divided bar graph A rectangular graph that is divided into different-sized sections to represent parts of a whole. (p. 464)

divisibility test A rule for testing whether a number is divisible by a specific value. For example, the divisibility test for 3 is to add the digits of the number and if the answer is a multiple of 3, then the number is divisible by 3. (p. 105)

divisor (p. 105) See **factor**.

dot plot A graph that uses dots above a number line to show frequencies of data values. (p. 469)



E

equation A mathematical statement that 2 quantities are equal. For example, $8 + 2 = 10$ or $3b - 7 = 5$. (p. 228)

equilateral triangle A triangle with 3 equal sides. (p. 262)

equivalent Having the same value. For example, 0.75 and $\frac{3}{4}$ are equivalent. (p. 148)

equivalent fraction An equal fraction created by multiplying or dividing the numerator and the denominator of a fraction by the same number. (p. 148)

equivalent ratio An equal ratio created by multiplying or dividing each term of a ratio by the same number. (p. 544, 549)

even chance Equally likely to happen or not happen, '50-50 chance', having a probability of $\frac{1}{2}$. (p. 508)

event In probability, a result involving one or more outcomes. For example, when rolling a die, the event 'rolling an even number' contains the 3 outcomes {2, 4, 6}. (p. 510)

expanded notation A way of representing natural numbers to show the place value of its digits, for example, $7219 = 7 \times 10^3 + 2 \times 10^2 + 1 \times 10 + 9 \times 1$ (p. 113)

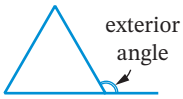
expected frequency The expected number of times an event will occur over repeated trials, found by multiplying the probability of the event by the number of trials. (p. 523)

experiment A situation involving chance that leads to outcomes, for example, rolling a die. (p. 510)

experimental probability An estimate of theoretical probability; the **relative frequency** of an event in repeated trials of an experiment, found using the formula. $P(E) = \frac{\text{frequency of } E}{\text{total frequency}}$ (p. 523)

expression A mathematical description of one or more operations. For example, $2 \times 4 + 5$ and $3b - 7$ are mathematical expressions (but $3b - 7 = 5$ is an equation). (p. 5) See also **algebraic expression**.

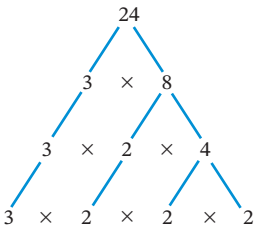
exterior angle An 'outside' angle of a shape created by extending one of the sides of the shape. (Angles inside a shape are *interior angles*.) (p. 270, 271)



F

factor (of a number) A value that divides evenly into a given number. For example, the factors of 15 are 1, 3, 5 and 15. Also called **divisor**. (p. 105, 122)

factor tree A diagram that lists the prime factors of a number. For example, the factor tree for 24 is shown below. (p. 125)



formula (Plural: **formulas** or **formulae**) A rule written as an algebraic equation, using variables. The formula for the area of a triangle is $A = \frac{1}{2}bh$, where A is the area, b is the length of the base and h is the perpendicular height of the triangle. (p. 213)

fraction A number written in the form $\frac{a}{b}$, where a and b are integers and $b \neq 0$. More precisely called a **common fraction**, unlike other types of fractions such as decimals, percentages and ratios. (p. 146)

frequency The number of times an event occurs in repeated trials of a probability experiment, or the number of times a value appears in a set of data. (p. 469, 523)

function Something that connects one 'set' to another 'set' in a particular way. Examples of sets include numbers and variables. (p. 443)

G

greatest common divisor (GCD) (p. 129) See **highest common factor (HCF)**.

grouping symbols The collective name for parentheses ( ), square brackets [ ] and braces {}. (p. 28)

H

hectare A large metric unit of area, equivalent to 10 000 m² or to the area of a square 100 m × 100 m. (p. 373)

height How tall or high something is, the vertical distance between its top and its base. (p. 385)

highest common factor (HCF) or **greatest common divisor (GCD)** The largest factor shared by 2 or more numbers. For example, the factors of 36 are 1, 2, 3, 4, 6, 9, 12, 18 and 36, and the factors of 8 are 1, 2, 4 and 8, so the HCF of 36 and 8 is 4. (p. 149)

horizontal Going across, sideways, flat. (p. 423)

hundredth The fraction $\frac{1}{100}$ or the decimal 0.01, the second decimal place (p. 315)

I

image A transformed shape after it has been translated, reflected or rotated. (p. 252, 429)

impossible Cannot happen, no chance, has a probability of 0. (p. 519)

improbable (p. 501) See **unlikely**.

improper fraction A fraction whose numerator is greater than or equal to its denominator, such as $\frac{7}{4}$. (p. 146)

index (Plural: **indices**, pronounced 'in-de-sees') (p. 112) See **power**.

index notation A way of writing repeated multiplication using indices (powers), in the form a^n. Index notation for $2 \times 2 \times 2 \times 2$ is 2^4. (p. 112)

integer A number that is a positive or negative whole number or zero. The numbers {..., −3, −2, −1, 0, 1, 2, 3, ...} are integers. (p. 9)

intersect To cross. (p. 66)

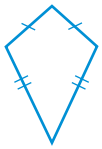
interval A section or segment of a line with a definite length, such as *AB* below. (p. 70)

isosceles triangle A triangle with 2 equal sides. (p. 262)

K

kilo- A prefix meaning one thousand (1000), represented by the symbol k. One kilogram is 1000 grams. (p. 356)

kite A quadrilateral with 2 pairs of equal adjacent sides. (p. 274)

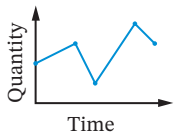


L

like terms Algebraic terms that have exactly the same variables. For example, $5xy$ and $2xy$ are like terms, $3xy$ and $4x^2$ are not like terms. (p. 221)

likely A high chance it will happen, probable. (p. 519)

line graph A graph made up of a line or several line intervals, often showing how a quantity is changing over time. (p. 465)



linear equation (linear function) A formula whose graph is a straight line, or an equation involving a variable that is not raised to a power, such as $2x + 9 = 17$. (p. 444)

lowest common multiple (LCM) The smallest multiple that is shared by 2 or more numbers. For example, the multiples of 4 are 4, 8, 12, 16, 20, ... and the multiples of 10 are 10, 20, 30, 40, ..., so the LCM of 4 and 10 is 20. (p. 132)

M

mean The average of a set of data, represented by $\bar{x}$, calculated by dividing the sum of the values by the number of values. (p. 478)

measure of central tendency An average, middle or typical value of a set of data. The 3 measures of location are the **mean**, **median** and **mode**. (p. 491)

median The middle value when the values of a data set are arranged in order. If the number of values is even, then the median is the average of the 2 middle values. (p. 482)

mega- A prefix meaning one million (1 000 000), represented by the symbol M. A megalitre is one million litres. (p. 356)

mental Using the mind.

micro- A prefix meaning one-millionth, represented by the Greek letter μ. One microsecond is one-millionth of a second. (p. 356)

milli- A prefix meaning one-thousandth $\left(\frac{1}{1000}\right)$, represented by the symbol m. One millimetre is one-thousandth of a metre. (p. 356)

mixed expression An expression with more than one arithmetic operation, such as $[17 - (3 + 1)] \times 2$. (p. 28)

mixed numeral A number written as a whole number and a fraction, for example $5\frac{3}{4}$. (p. 146)

mode The most common or frequent value(s) in a set of data. (p. 478)

multiple (of a number) The product of the number and a whole number. For example, the multiples of 6 are 6, 12, 18, 24, ... (p. 97)

N

negative number A number less than 0, written with a negative sign (−), for example, −3, −10 and −7. (p. 5)

net (of a solid) The faces of a solid shown flat and 'unpacked'. (p. 274)

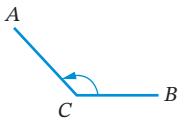
number plane or **Cartesian plane** A coordinate grid system based on 2 number lines that cross at right angles: a horizontal line called the *x*-axis and a vertical line called the *y*-axis. (p. 423)

numerator The number above the line in a fraction. The numerator of $\frac{2}{3}$ is 2. (p. 146)

O

observed frequency The actual number of times an event occurs over repeated trials in an experiment (p. 523)

obtuse angle A 'wide' angle greater than 90° but less than 180°. (p. 250)

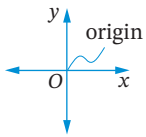


obtuse-angled triangle A triangle with one obtuse angle (between 90° and 180°). (p. 262)

order of operations The order in which a mixed expression such as $[17 - (3 + 1)] \times 2$ is evaluated. Work from left to right, first performing any operations in grouping symbols, then multiplication and division, and finally addition and subtraction. (p. 28)

ordered pair A pair of numbers (x, y) that can be used as coordinates to plot a point on the number plane. (p. 423)

origin The point $O(0, 0)$ at the centre of the number plane, where the *x*-axis and *y*-axis cross. (p. 423, 426)

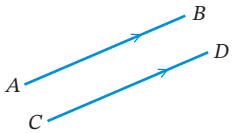


outcome In probability, a result of a situation or experiment. For example, when rolling a die, one possible outcome is rolling a 4. (p. 510)

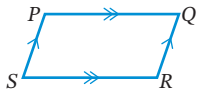
outlier An extreme data value that is very different from the other values in a set. (p. 469)

P

parallel lines Lines that point in the same direction and do not intersect. $AB \parallel CD$ means '*AB* is parallel to *CD*'. (p. 67)



parallelogram A quadrilateral in which the opposite sides are parallel. (p. 274)

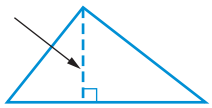


parentheses (Pronounced 'pa-ren-th-sees'.) See **grouping symbols**.

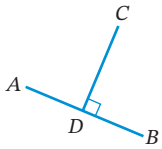
percentage A fraction with a denominator of 100 that is written in a special way. For example, 7% means $\frac{7}{100}$. (p. 178)

perimeter The distance around the outside of a shape. The sum of the lengths of its sides. (p. 359)

perpendicular height The height of a shape, measured at right angles to the base, as shown. (p. 385)



perpendicular lines Lines that intersect to form a right angle. $AB \perp CD$ means AB is perpendicular to CD. (p. 68)



pi (π) A special irrational number, approximately 3.1416, used in calculating circular measurements (p. 368)

place value The way that the position of a digit in a number tells us its value. For example, the value of the 4 in 2417 is 400. (p. 316)

plane A flat surface that extends in all directions.

polygon Any flat (2D) shape with straight sides. (p. 283)

polyhedron (plural: polyhedra) Any solid (3D) shape with flat faces (p. 285)

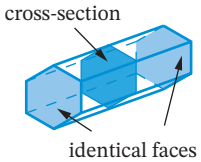
positive number A number greater than 0, for example, 11, 6 and 4. (p. 5)

power (or index) The number of times a base is multiplied by itself. In 2^5, the power is 5. Also called the *exponent*. (p. 112)

power of 10 The numbers 10, 100, 1000 and so on, formed by multiplying 10 by itself repeatedly. (p. 315)

prime number A number that has only 2 factors, 1 and itself. For example, 2 and 7 are prime numbers, but 15 is not. (p. 122) See also **composite number**.

prism A polyhedron with identical cross-sections that have straight sides, such as this hexagonal prism. (p. 286, 405)



probability The chance of an event occurring, measured as a fraction, decimal or percentage between 0 and 1. (p. 509)

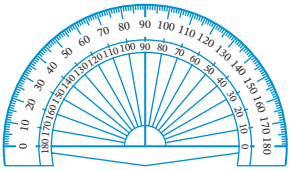
probable (p. 516) See **likely**.

product The result of a multiplication. The product of 7 and 3 is 21. (p. 22)

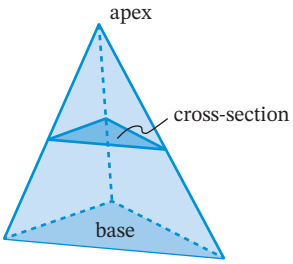
pronumeral (p. 58) See **variable**.

proper fraction A fraction whose numerator is less than its denominator, such as $\frac{3}{8}$. (p. 146)

protractor A geometrical instrument that measures the size of an angle in degrees. (p. 47)



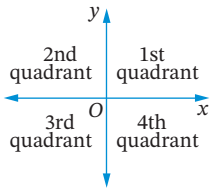
pyramid A polyhedron with a base that is a polygon, where the edges from each vertex of the base meet at the same single point, called the **apex**. (p. 286)



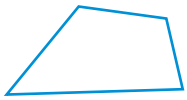
Triangular pyramid

Q

quadrant (of a number plane) A quarter of the number plane created by the x-axis and y-axis crossing at right angles. (p. 426)



quadrilateral Any polygon with 4 sides, for example, rectangle, parallelogram, kite and trapezium. (p. 251, 274)



quotient The result of a division. If 36 is divided by 3, the quotient is 12. (p. 27)

R

radius The distance from the centre of a circle to the circle's edge (plural: radii) (p. 363)

random In probability, describing a situation where every possible outcome has an equal chance, or is equally likely. (p. 510)

range In a set of data, the difference between the highest and lowest values. (p. 482)

ratio A relationship between quantities measured in the same units. For example, the ratio of 3 teachers to 40 students is 3 : 40 (read '3 to 40'). (p. 541)

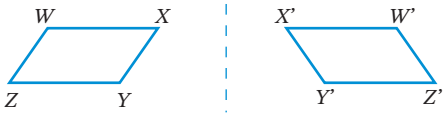
rational number A number that can be written in fraction form $\frac{a}{b}$, where a and b are integers, but b is not zero. Fractions, decimals and percentages are all examples of rational numbers. (p. 146, 178, 316)

reciprocal The reciprocal of any number is found by first writing the number as a fraction and then swapping the numerator with the denominator. The reciprocal of 5 is $\frac{1}{5}$ and the reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$. The product of any number and its reciprocal is 1. (p. 169)

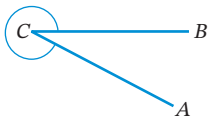
rectangle A quadrilateral with 4 right angles. (p. 274)

recurring (or repeating) decimal A decimal with one or more digits that repeat endlessly. For example 0.1666 ..., abbreviated as $0.1\dot{6}$. (p. 336)

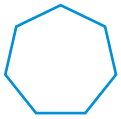
reflection The process of 'flipping' a shape across a line to give a mirror image that is back-to-front. (p. 251, 429)



reflex angle A 'bent-back' angle greater than 180°, but less than 360°. (p. 49)



regular polygon A polygon with equal sides and equal angles. (p. 283)



relative frequency The frequency of an event in repeated trials of an experiment as a fraction of the total frequency. (p. 523)

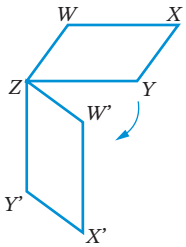
revolution An angle of 360°. (p. 54)

rhombus A quadrilateral with 4 equal sides. (p. 274)

right angle A 90° angle, a 'square corner', a quarter-turn. (p. 262)

right-angled triangle A triangle with a right angle (90°). (p. 262)

rotation The process of 'spinning' or turning a shape about or around a fixed point at a certain angle and direction (clockwise or anti-clockwise). (p. 251, 429)



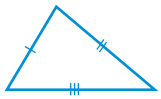
round To give an approximate answer. For example, rounding 3.14159 to 3 decimal places gives 3.142. (p. 340)

S

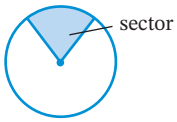
sample space In a probability situation, the set of all possible outcomes. (p. 510)

scale The amount represented by one unit or interval on a measuring instrument or on the axis of a graph.

scalene triangle A triangle with no equal sides. (p. 262)



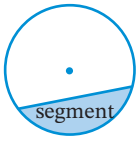
sector A region of a circle cut off by 2 radii, shaped like a slice of pizza. (p. 351)



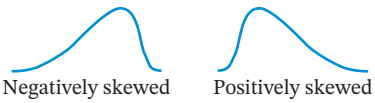
sector graph Also called a pie chart, a circular graph representing parts of a whole by being divided into proportionately-sized sectors. (p. 465)

segment A region of a circle cut off by a chord. (p. 363)

semicircle Half a circle. (p. 363)

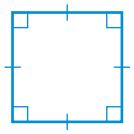


skewed distribution A statistical distribution in which most of the data values are clustered at one end, creating a 'tail' at the other end. The tail determines whether the skew is positive or negative. (p. 494)
See also **symmetrical distribution**.



solution In algebra, the value(s) that make an equation true. (p. 228)

square A quadrilateral with 4 equal sides and 4 right angles. (p. 117, 274)



square (of a number) The number multiplied by itself. For example, 7 squared = $7^2 = 7 \times 7 = 49$. (p. 117)

square metre A unit for measuring area, equal to the area of a square with length 1 m (p. 373)

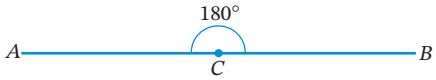
square root (of a number) The positive value which, if squared, gives the number. For example, $\sqrt{25} = 5$ because $5^2 = 5 \times 5 = 25$. (p. 117)

stem-and-leaf plot A 'number graph' that lists all the data values in groups. Each value is split into a 'stem' and a 'leaf'. This stem-and-leaf plot shows 12 test scores, from 42 to 82. (p. 474)

Stem	Leaf
4	2 5
5	2 2 5
6	6 7
7	3 5 7 7
8	2

Key: 5|8 stands for 58

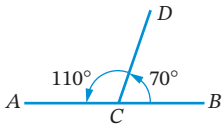
straight angle An angle of 180°. (p. 54)



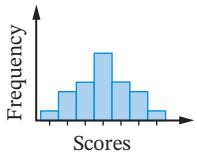
substitution The replacement of a variable with a number to evaluate an expression. For example, substituting $t = 5$ in the expression $t^2 + 6$ gives 31. (p. 213)

sum The result of an addition. The sum of 5 and 7 is 12. (p. 15)

supplementary angles 2 angles whose sum is 180°. The angles 70° and 110° are supplementary. (p. 57)

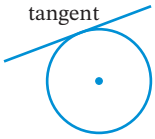


symmetrical distribution A statistical distribution in which all data values are distributed equally on both sides of the centre, its shape having line symmetry. See also **skewed distribution**. (p. 470)



T

tangent A line that touches a circle at one point. (A tangent cannot be inside a circle.) (p. 363)



terminating decimal A decimal whose digits do not run endlessly, for example, 0.125. (p. 336)

tenth The fraction $\frac{1}{10}$ or the decimal 0.1, the first decimal place (p. 303)

terms of an expression The parts of an algebraic expression. For example, $b^2 + 6b - 9$ has 3 terms: b^2, $6b$ and -9. (p. 221)

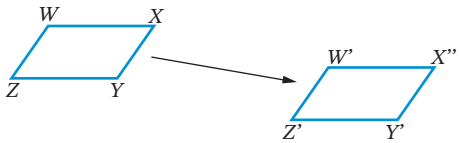
terms of a ratio The numbers in a ratio, for example, in the ratio 14 : 9, the terms are 14 and 9. (p. 541)

thousandth The fraction $\frac{1}{1000}$ or the decimal 0.001, the third decimal place (p. 304)

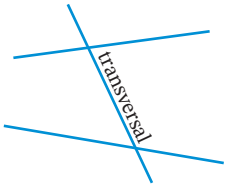
tonne A metric unit of mass for heavy objects, equal to 1000 kg. (p. 344)

transformation In geometry, the process of moving or changing a shape. (p. 251) See also **reflection**, **rotation** and **translation**.

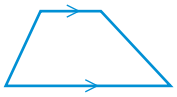
translation The process of 'sliding' a shape a certain distance and direction. (p. 251, 429)



transversal A straight line that crosses 2 or more lines at different points. (p. 72)



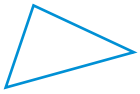
trapezium A quadrilateral with one pair of opposite sides parallel. (p. 274)



travel graph A line graph that describes a journey and shows the distance travelled over time. (p. 552)

trial One go or run of a repeated probability experiment, for example, one roll of a die. (p. 510)

triangle A polygon with 3 sides. (p. 251, 262)



U

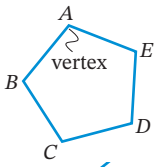
unitary method A method for finding a quantity by finding the size of one part first. (p. 549)

unlikely A low chance it will happen, improbable, probably won't happen. (p. 519)

V

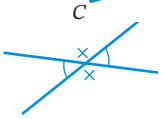
variable or **pronumeral** A symbol, usually a letter of the alphabet, that stands for a number. (p. 58, 204)

vertex (Plural: **vertices**). A corner of a shape or angle. (p. 44)



vertical Going up and down, at a right angle to the **horizontal**. (p. 423)

vertically opposite angles A pair of opposite and equal angles formed when 2 lines cross. (p. 62)

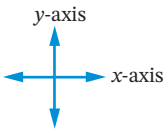


vinculum In a fraction, the horizontal line that separates the numerator from the denominator. (p. 146)

volume The amount of space taken up by a solid object, measured in cubic units. (p. 385)

X

***x*-axis** The horizontal axis of a number plane (running across). (p. 423)



Y

***y*-axis** The vertical axis of a number plane (running up and down, see diagram above). (p. 423)

This is an electronic version of the print textbook. Due to electronic rights restrictions, some third party content may be suppressed. Editorial review has deemed that any suppressed content does not materially affect the overall learning experience. The publisher reserves the right to remove content from this title at any time if subsequent rights restrictions require it. For valuable information on pricing, previous editions, changes to current editions, and alternate formats, please visit www.cengage.com/highered to search by ISBN, author, title, or keyword for materials in your areas of interest.

Important notice: Media content referenced within the product description or the product text may not be available in the eBook version.