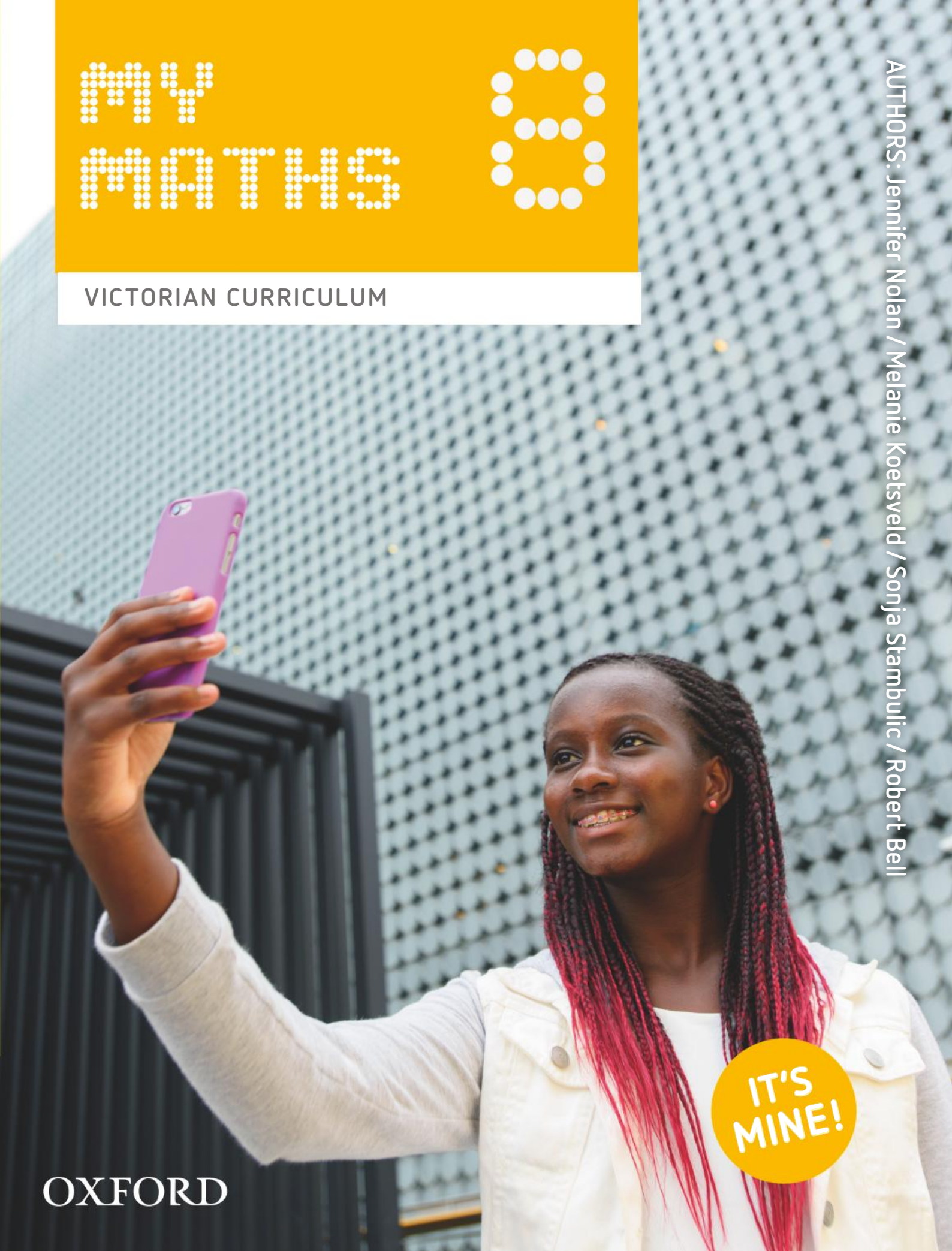
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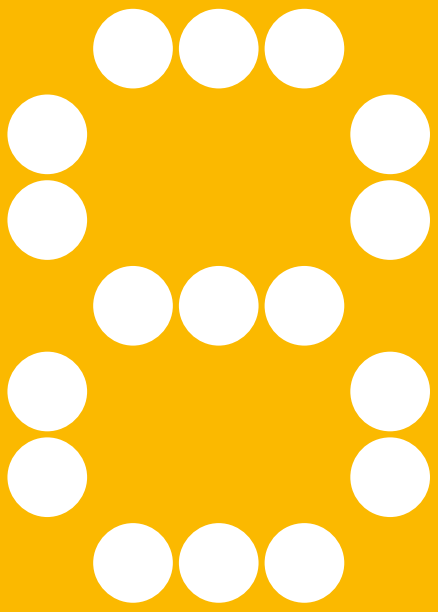
VICTORIAN CURRICULUM

AUTHORS: Jennifer Nolan / Melanie Koetsveld / Sonja Stambulic / Robert Bell



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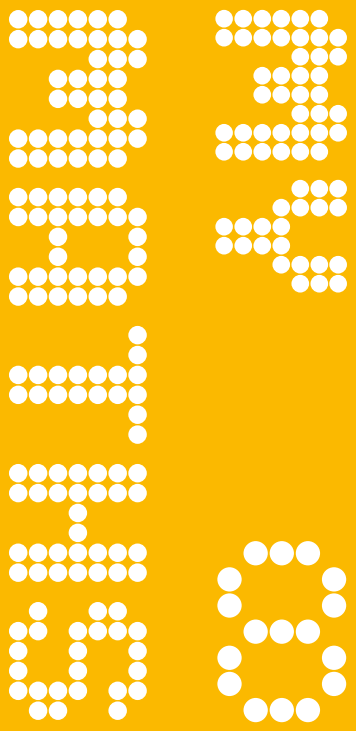
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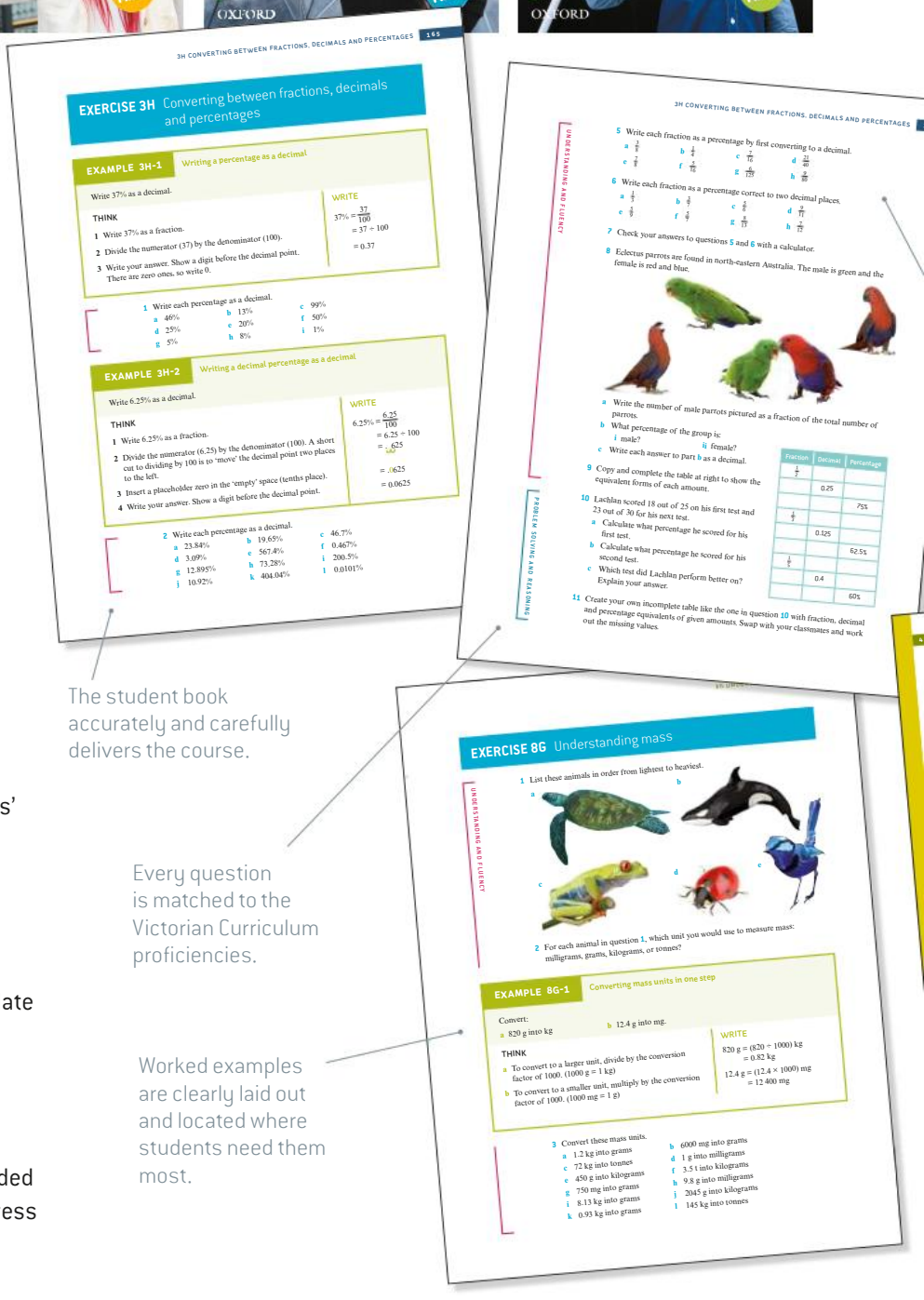
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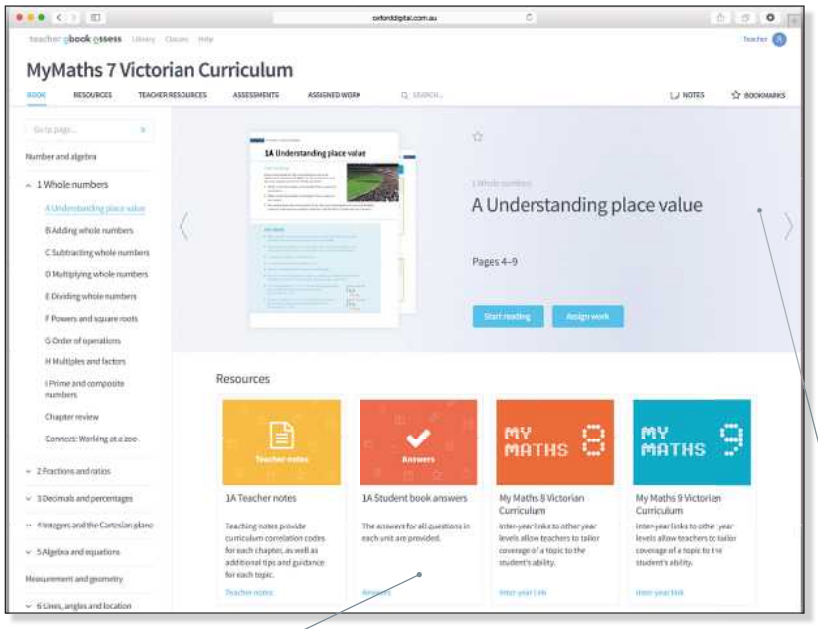


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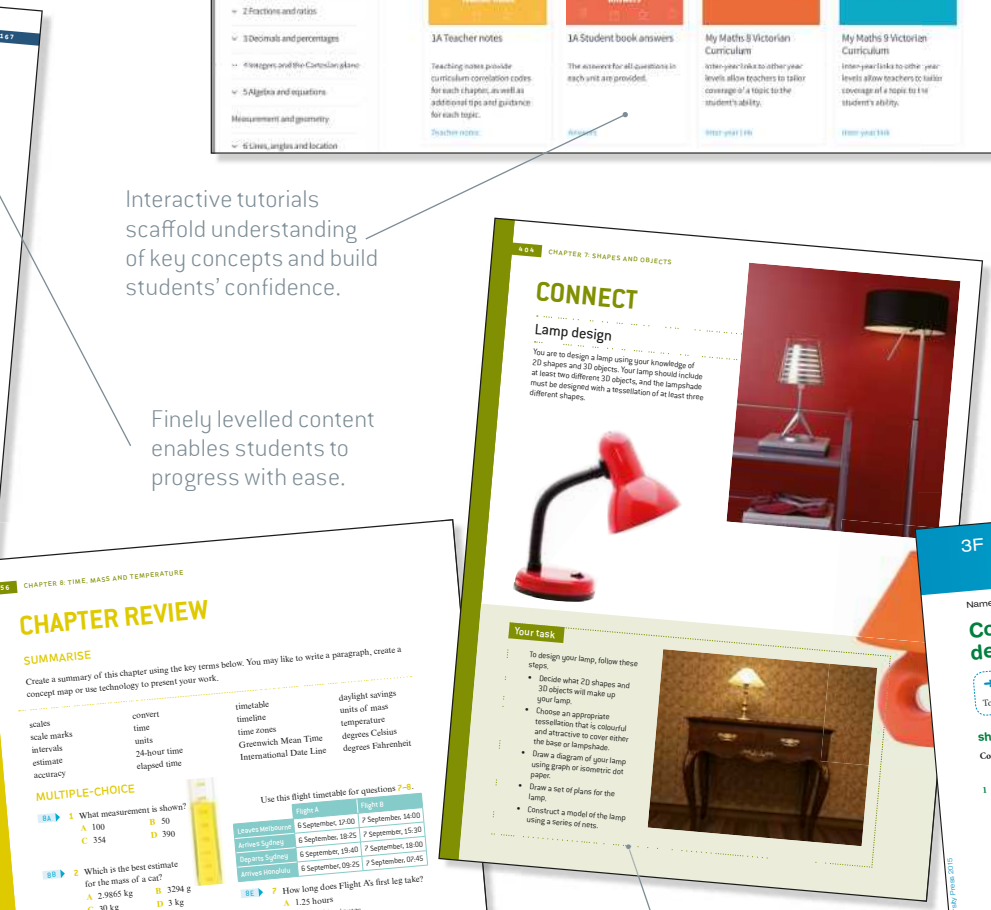
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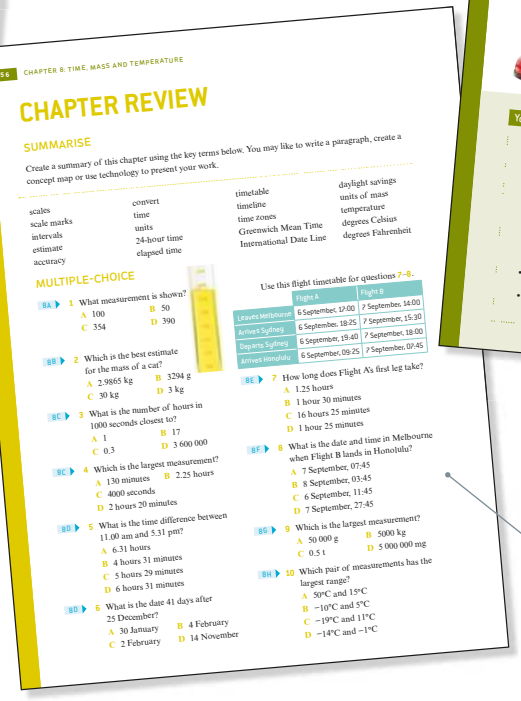
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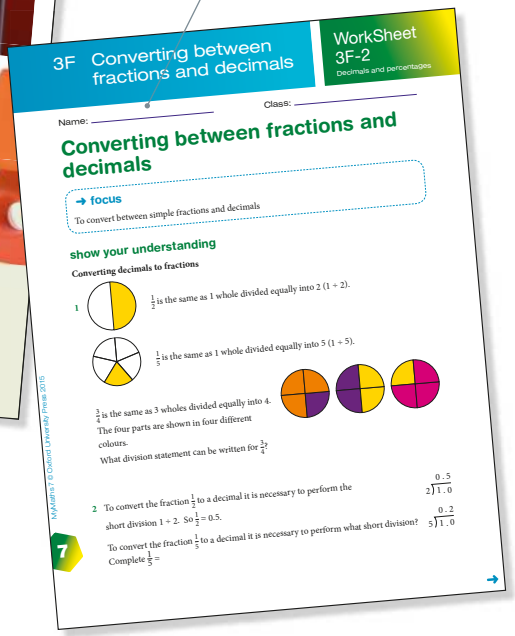
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1 NUMBER SKILLS

1A Estimating and rounding

1B Order of operations

1C Understanding fractions

1D Operations with fractions

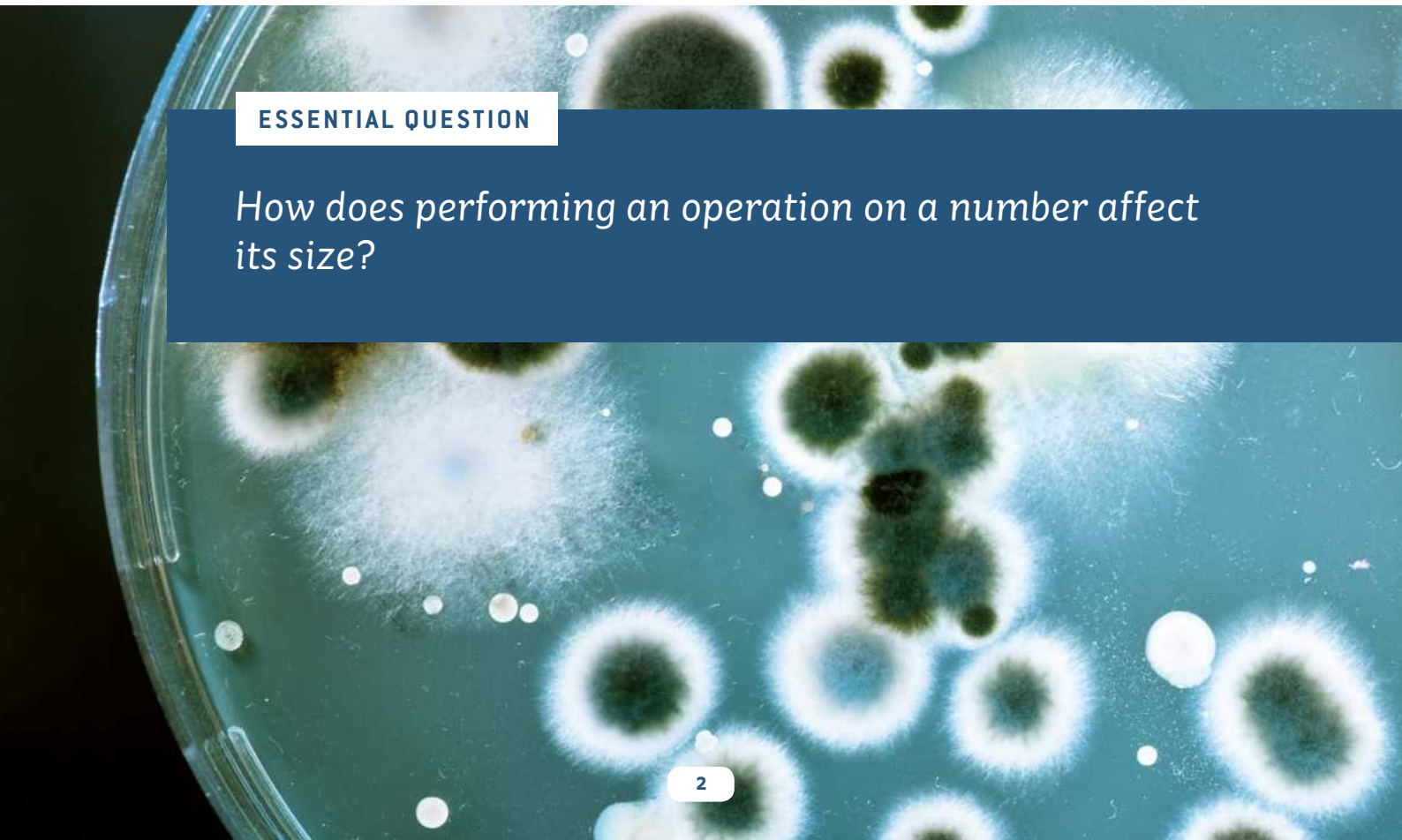
1E Understanding decimals

1F Operations with decimals

1G Terminating, non-terminating and recurring decimals

1H Powers and roots

1I Index laws



ESSENTIAL QUESTION

How does performing an operation on a number affect its size?

Are you ready?

1A **1** a What is 40 × 1000?

A 40 B 400
C 4000 D 40 000

b What is 700 000 ÷ 100?

A 70 B 700
C 7000 D 70 000

1A **2** Calculate:

a 26 × 100 b 51 000 ÷ 10
c 100 000 ÷ 10 000

1A **3** a What is 3000 × 600?

A 18 000 B 180 000
C 900 000 D 1 800 000

b What is 8000 ÷ 40?

A 200 B 760
C 2000 D 320 000

1B **4** a What is 5^2?

A 2 B 5 C 10 D 25

b What is 9^2?

1C **5** a What is the highest common factor of 45 and 72?

A 3 B 9 C 18 D 360

b What is the highest common factor of 24 and 30?

1C **6** a The fraction $\frac{9}{4}$ is equivalent to which mixed number?

A $\frac{1}{4}$ B $2\frac{1}{4}$ C $2\frac{4}{9}$ D $2\frac{1}{2}$

b What is $5\frac{3}{7}$ as an improper fraction?

1D **7** a What is the lowest common multiple of 6 and 8?

A 6 B 8 C 24 D 48

b What is the lowest common multiple of 4 and 10?

1D **8** a What is the lowest common denominator for $\frac{2}{3} - \frac{1}{5}$?

A 3 B 5 C 15 D 35

b What is the lowest common denominator for $\frac{1}{4} + \frac{5}{6}$?

1D **9** a What is $\frac{1}{7} + \frac{2}{3}$?

A $\frac{17}{21}$ B $\frac{3}{10}$ C $\frac{2}{21}$ D $\frac{3}{21}$

b What is $\frac{3}{5} - \frac{1}{4}$?

1E **10** a What is the value of 8 in 21.218 42?

A 8 tenths B 8 hundredths
C 8 thousandths
D 8 ten-thousandths

b What is the value of 3 in 510.347?

A 3 hundreds B 3 hundredths
C 3 tens D 3 tenths

1E **11** a Which number is larger: 0.071 or 0.07?

b Which number is smaller: 4.3 or 4.03?

c What is the largest number in this list?
0.105, 0.152, 0.251, 0.125, 0.05, 0.25

1F **12** a What is 23.56 + 32.47 + 12.98?

A 56.03 B 67.201
C 67.81 D 69.01

b What is 157.62 − 78.57?

A 79.05 B 79.15
C 121.15 D 236.19

1F **13** a What is 3.546 × 4?

b What is 15.42 ÷ 6?

A 9.42 B 2.57 C 2.20 D 2.07

1H **14** a 5^3 is a number written in index form.

i What is the base?

ii What is the power?

b What is the expanded form of 5^3?

A 3 × 3 × 3 × 3 × 3
B 5 + 5 + 5 C 5 × 5 × 5
D 5 × 3

1A Estimating and rounding

Start thinking!



The cost of running a school camp for one year is around $350 000. It is expected 826 students will attend the camp in one year. Does $385 seem a reasonable amount to charge each student to cover the cost?

You don't have a calculator and you really only need a rough estimate.

1 Is 826 closer to 800 or 900? How can you tell?

2 Is 385 closer to 300 or 400? How can you tell?

When you write a number as an approximate value with a first digit of 1, 2, 3 … or 9 followed by zeros, it is called **rounding** to the **leading digit**.

3 Round 826 to its leading digit. (Hint: use your answer to question **1**.)

4 Round 385 to its leading digit. (Hint: use your answer to question **2**.)

5 What is important about the second digit in each number? How does it help you decide what the first digit of the rounded number should be?

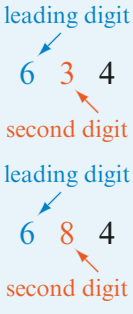
6 Is the third digit important in deciding what the first digit should be in the rounded number? Explain.

7 Multiply the rounded numbers from questions **3** and **4** to find an estimate for 826 × 385. Write your answer as 826 × 385 ≈ _____. (The symbol ≈ means 'approximately equals'.)

8 Is $385 a reasonable amount to cover costs?

KEY IDEAS

- To write an approximate value for a number, round the number to its first or leading digit. To start, look at the second digit in the number.
- If it is 0, 1, 2, 3 or 4, keep the first digit the same and replace the digits that follow with zero. For example, 634 ≈ 600. This is called rounding down.
- If the second digit is 5, 6, 7, 8 or 9, increase the first digit by one and replace the digits that follow with zero. For example, 684 ≈ 700. This is called rounding up.
- To estimate the result of a calculation, round each number to its leading digit before you perform any operations.
- You can also round to the nearest 10, 100, 1000, etc., using a similar method. For example:
  - rounded to the nearest 10, 153 ≈ 150
  - rounded to the nearest 100, 349 ≈ 300
  - rounded to the nearest 1000, 4501 ≈ 5000



EXERCISE 1A Estimating and rounding

1 Decide whether each number is closer to 400 or 500.

a 438 **b** 477 **c** 462
d 455 **e** 433 **f** 449

2 Decide whether each number is closer to 6000 or 7000.

a 6789 **b** 6306 **c** 6010
d 6505 **e** 6880 **f** 6448

EXAMPLE 1A-1 Rounding a number to its leading digit

Round each number to its leading digit.

a 26 **b** 739

THINK

a Since the second digit is 6 (5 or more), 26 is closer to 30 than 20. Increase the leading digit by one and replace the other digit with zero.

b Since the second digit is 3 (4 or less), 739 is closer to 700 than 800. Keep the leading digit the same and replace the other digits with zero.

WRITE

a $26 \approx 30$

b $739 \approx 700$

UNDERSTANDING AND FLUENCY

3 Round each number to its leading digit.

a 77 **b** 42 **c** 81 **d** 347 **e** 160 **f** 555
g 909 **h** 2489 **i** 6902 **j** 22 117 **k** 982 **l** 10 999

4 Round each number to its leading digit.

a 94 **b** 99 **c** 952
d 92 949 **e** 9008 **f** 960

5 Give three examples of numbers that round to these approximations if you round to the leading digit.

a 40 **b** 700 **c** 3000
d 50 000 **e** 800 000 **f** 2 000 000

6 Calculate:

a $900 + 700$ **b** $300 - 10$ **c** 30×20
d $80 \div 4$ **e** 500×40 **f** $800 + 3000$
g $200 \div 8$ **h** $6000 - 500$ **i** $40\,000 \div 20$
j 7000×300 **k** $20\,000 - 9000$ **l** $5000 + 10\,000$

EXAMPLE 1A-2 Estimating the result of a calculation

Estimate the result of each calculation by first rounding each number to its leading digit.

a 1307 + 4875 **b** 576 × 42 **c** 85 229 ÷ 5

THINK	WRITE
a 1 Round each number to its leading digit.	**a** 1307 + 4875 ≈ 1000 + 5000
2 Perform the addition.	= 6000
3 Write your answer.	1307 + 4875 ≈ 6000
b 1 Round each number to its leading digit.	**b** 576 × 42 ≈ 600 × 40
2 It may be easier to rewrite the calculation.	= 6 × 100 × 4 × 10
3 Perform the multiplication.	= 24 × 1000 = 24 000
4 Write your answer.	576 × 42 ≈ 24 000
c 1 Round the first number. Single-digit numbers are not changed.	**c** 85 229 ÷ 5 ≈ 90 000 ÷ 5
2 Perform the division.	= 18 000
3 Write your answer.	85 229 ÷ 5 ≈ 18 000

UNDERSTANDING AND FLUENCY

7 Estimate the result of each calculation by first rounding each number to its leading digit.

a 468 + 731 **b** 92 − 38
c 27 × 49 **d** 83 ÷ 2
e 582 × 17 **f** 245 + 6379
g 4512 ÷ 43 **h** 137 − 51
i 3694 ÷ 442 **j** 187 × 9364
k 7085 − 750 **l** 964 + 5803
m 8277 × 65 234 **n** 13 761 + 8036
o 94 113 ÷ 587 **p** 24 905 − 780

8 Tickets to a concert are $96. Estimate the cost of 32 tickets.

9 Tina thinks 9648 rounded to its leading digit is 10 000. Ruby isn't sure, as she thinks the rounded number should have the same number of digits as the original number. Who is correct? Explain.

EXAMPLE 1A-3 Rounding a number to the nearest ten or hundred

Round 2638 to the nearest:

a ten **b** hundred.

THINK

a Since the digit to the right of the tens digit is 8 (5 or more), 2638 is closer to 2640 than to 2630.

b Since the digit to the right of the hundreds digit is 3 (4 or less), 2638 is closer to 2600 than to 2700.

WRITE

a 2638 ≈ 2640

b 2638 ≈ 2600

UNDERSTANDING AND FLUENCY

10 **a** Round each number to the nearest ten.

i 482 **ii** 6377 **iii** 56 026

iv 738 494 **v** 8075 **vi** 904 507

b Round each number in part **a** to the nearest hundred.

11 **a** Round each number to the nearest thousand.

i 36 428 **ii** 7510 **iii** 183 915

iv 50 703 **v** 6052 **vi** 825

b Round each number in part **a** to the nearest hundred.

12 An office building has 35 782 panes of glass. Write an approximation to this number if you round to:

a the leading digit

b the nearest hundred

c the nearest thousand

d the nearest ten

e the nearest ten thousand.



13 Russia is the largest country in the world, with an area of 17 075 242 km^2. Round this value to:

a its leading digit

b the nearest hundred

c the nearest thousand

d the nearest ten thousand

e the nearest hundred thousand

f the nearest million.

14 Daniel is saving for a quad bike. He has saved $60 per month for the past 28 months.



- **a** Write an approximation for the number of months by rounding the value to its leading digit.
- **b** Using your answer to part **a**, estimate the amount of money Daniel has saved.
- **c** Does he have enough money to buy the quad bike?
- **d** If he doesn't have enough, estimate how many more months he needs to save.

15 Paving tiles cost $7 each. Antonia needs 385 tiles for the back patio.

- **a** Write an approximation of the number of tiles she needs by rounding to the leading digit.
- **b** Estimate the cost of the tiles, using your answer to part **a**.
- **c** Compare your estimate to the exact cost of the tiles.

16 The school fundraising committee wants to raise money by buying boxes of sunscreen from a wholesaler to sell for profit. Each box has a wholesale price of $47.

- **a** Estimate the number of boxes the committee could buy with $2000.
- **b** If the committee used this estimate, without performing the exact calculation decide whether there would be money left over or money owed to the wholesaler. Explain your answer.

17 Australia has an area of 7 692 024 km^2 and Indonesia has an area of 1 919 931 km^2.

- **a** Round each value to its leading digit.
- **b** Use the approximations to estimate:
  - **i** the difference in area between the two countries
  - **ii** the area of Australia compared to the area of Indonesia.
- **c** Check how close your estimations are to the exact answers for part **b**.

PROBLEM SOLVING AND REASONING

18 People attending a State of Origin rugby league match entered the stadium through one of five gates. The number of people passing through each gate is shown.

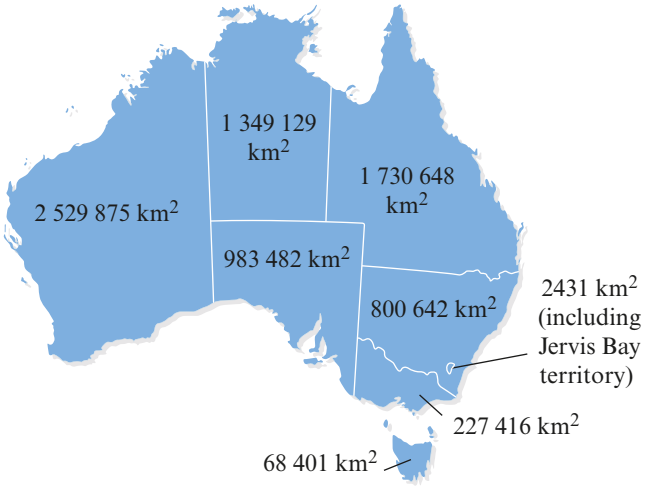
Gate	Number of people
A	9 361
B	10 758
C	12 196
D	8 844
E	11 037

a Estimate the number of people attending the match by rounding the numbers at each gate to the leading digit.

b Calculate the exact number of people attending the match.

c Which answer would a sports commentator be more likely to use when reporting the match?

19 Australia is the sixth largest country in the world. The area of the states and territories (including islands) of Australia is shown on the map.



a Round each area to its leading digit.

b Estimate the area of Australia by adding the approximate values from part **a**.

c The area of NSW, rounded to the nearest thousand, is 801 000 km². Round each of the other areas to the nearest thousand.

d Estimate the area of Australia by adding the approximate values from part **c**.

e Compare the estimates you obtained in parts **b** and **d** with the accepted value for the area of Australia of 7 692 024 km².

CHALLENGE

20 Approximately 600 000 bacteria fit on the head of a pin of diameter 2 mm. Write five different numbers that would round to 600 000.

21 Write three examples of numbers that would:

a round to 5700 when rounded to the nearest hundred

b round to 836 000 when rounded to the nearest thousand.

Reflect

Why is rounding useful in estimating the result of a calculation?

1B Order of operations

Start thinking!

Students in a Year 8 class were asked to calculate $130 + 70 \times 5 - 10^2$ without using a calculator.

1 How many operations need to be performed? List them.

Hanna, Josh and Imogen produced the workings shown at right.

2 Explain how each student produced a different answer.

3 Who do you think is correct?

There is a correct order to follow when you perform a number of operations in one problem. This is the **order of operations**.

Order	Operations
First	**B**rackets (operations inside grouping symbols)
Second	**I**ndices (powers and roots)
Third	**D**ivision and **M**ultiplication
Fourth	**A**ddition and **S**ubtraction

Some people use **BODMAS**, which has 'O' for 'power of' in place of 'I' for 'indices'.

4 To remember this, think **BIDMAS**.
Can you see why?

5 Follow the order of operations to calculate $130 + 70 \times 5 - 10^2$.
Which student produced the correct answer?

Hanna

$130 + 70 \times 5 - 10^2$
$= 200 \times 5 - 10^2$
$= 200 \times 5 - 100$
$= 1000 - 100$
$= 900$

Josh

$130 + 70 \times 5 - 10^2$
$= 200 \times 5 - 10^2$
$= 1000 - 10^2$
$= 990^2$
$= 980\,100$

Imogen

$130 + 70 \times 5 - 10^2$
$= 130 + 70 \times 5 - 100$
$= 130 + 350 - 100$
$= 480 - 100$
$= 380$

KEY IDEAS

- The order of operations is:

first:	**B**rackets (operations inside grouping symbols)
second:	**I**ndices (powers and roots)
third:	**D**ivision and **M**ultiplication (work from left to right)
fourth:	**A**ddition and **S**ubtraction (work from left to right)

- Operations of the same ranking are performed in order from left to right.
- Where there is more than one set of grouping symbols, calculate the operations in the innermost brackets first.

Remember **BIDMAS** (or **BODMAS**).

EXERCISE 1B Order of operations

EXAMPLE 1B-1 Listing the order of operations

List in order the operations to be performed for $7 + 3^2 \times (6 - 2)$.

THINK

$6 - 2$ is performed first (brackets), then 3^2 (indices), then the multiplication and then the addition.

NOTE You may find it easier to think of BIDMAS.

WRITE

Order is: subtraction within brackets, squaring, multiplication, addition

1 For each calculation, list the order of operations.

a $28 - 3 \times 7 + 11$
b $50 + (34 - 19) \div 5$
c $12 + 3 \times 4^2 - 41$
d $20 - 2 \times 6 + 3^2$
e $8 + [9 - (3 + 2)]$
f $24 \div 6 + (5 + 1)^2$

EXAMPLE 1B-2 Using order of operations

Calculate $5^2 \times 3 - 36 \div 4$.

THINK

1 Perform squaring first (indices).

2 × and ÷ come before − so, working from left to right, perform the multiplication first then the division.

3 Perform the subtraction.

WRITE

$5^2 \times 3 - 36 \div 4$
$= 25 \times 3 - 36 \div 4$
$= 75 - 36 \div 4$
$= 75 - 9$
$= 66$

2 Calculate:

a $16 + 28 \div 4$
b $60 - 2 \times 10$
c $27 \div 3 + 6 \times 11$
d $22 + 30 \times 2 \div 15$
e $10 \times 8 - 7 \times 9$
f $6^2 - 8 \times 3 + 35$
g $5 \times 2^2 - 3^2 \times 2 + 8$
h $7 + 7 \times 6 - 7^2$
i $50 - 3 \times 12 + 4^2 \div 4$
j $5^2 + 8 \times 3 - 2^2$

EXAMPLE 1B-3 Using order of operations (with brackets)

Calculate:

a $9 \times (15 - 11) + 24 \div 3$ **b** $42 + [18 \div (12 - 3)]^2 - 17$.

THINK	WRITE
a 1 Perform operations inside brackets first, so calculate $15 - 11$.	**a** $9 \times (15 - 11) + 24 \div 3$ $= 9 \times 4 + 24 \div 3$
2 Perform × and ÷ before +. Working from left to right, calculate 9×4.	$= 36 + 24 \div 3$
3 Perform ÷ before + so calculate $24 \div 3$.	$= 36 + 8$
4 Perform the addition.	$= 44$
b 1 There are two sets of grouping symbols. Perform the operation in the innermost brackets first, so calculate $12 - 3$.	**b** $42 + [18 \div (12 - 3)]^2 - 17$ $= 42 + [18 \div 9]^2 - 17$
2 Perform the operation in the remaining set of brackets.	$= 42 + 2^2 - 17$
3 Perform the operation of squaring as 'indices' comes before + and −.	$= 42 + 4 - 17$
4 Working from left to right, perform the addition then the subtraction.	$= 46 - 17$ $= 29$

UNDERSTANDING AND FLUENCY

3 Calculate:

a $72 \div (7 + 2) - 2 \times 3$ **b** $(27 - 14) \times (19 + 11)$

c $9^2 + 7 \times (34 - 28)$ **d** $10 \times 8 - 4 \times (3^2 + 2)$

e $60 \div [6 + (3 \times 5 - 1)] + 48$ **f** $12 + [22 \div (37 - 26)]^2 - 9$

4 Perform the calculations in question **1** using the order of operation rules.

5 Repeat question **4** using a calculator. Enter each number, each operation and any grouping symbols from left to right. Does your calculator perform the operations in the correct order?

6 The **average** of a set of scores is the total sum of the scores divided by the number of scores. For example, the average of 25, 30 and 23 is $(25 + 30 + 23) \div 3$ or $\frac{25 + 30 + 23}{3}$.

a Use the order of operations to find the average of 25, 30 and 23.

b Find the average of each set of numbers.

i 18, 21, 19, 25, 24, 25 **ii** 344, 363, 351, 346 **iii** 71, 69, 75, 78, 67

iv 2, 4, 7, 0, 1, 5, 3, 2 **v** 3, 2, 1, 1, 2, 1, 4 **vi** 28, 35, 49, 36, 22

vii 1052, 2711, 1949 **viii** 17, 15, 0, 9, 12, 13 **ix** 31, 35, 37, 36, 34, 31

UNDERSTANDING AND FLUENCY

7 **a** The **commutative law** relates to the order of operations. Calculate:
i 3 + 6 **ii** 6 + 3.

b Does the order matter when you add two numbers? Try two more examples.

Your answer to part **b** confirms that addition obeys the commutative law.

c Does subtraction obey the commutative law? Try these calculations.
i 6 − 3 **ii** 3 − 6

d Does multiplication obey the commutative law? Try these.
i 6 × 3 **ii** 3 × 6

e Does division obey the commutative law? Try these.
i 6 ÷ 3 **ii** 3 ÷ 6

8 **a** The **associative law** also relates to the order of operations. Calculate:
i (5 + 6) + 7 **ii** 5 + (6 + 7) **iii** (5 + 7) + 6

b Does it matter in which order you add three numbers? Try two more examples.

Your answer to part **b** confirms that addition obeys the associative law.

c Do subtraction, multiplication and division obey the associative law? Try some examples to help you decide.

9 The commutative and associative laws can make mental calculations easier.

a To calculate 54 + 118 + 16, you can add any two of the numbers together first.
i Which two numbers are easiest to add together first? Explain.
ii Perform the calculation.

b To calculate 87 × 25 × 4, you can multiply any two of the numbers together first.
i Which two numbers are easiest to multiply together first? Explain.
ii Perform the calculation.

c Explain how these laws can make some calculations easier to perform.

EXAMPLE 1B-4 Using mental strategies

Use mental strategies to calculate:
a 37 + 14 + 6 **b** 20 × 63 × 5.

THINK	WRITE
a Decide which two numbers to add together first. 14 and 6 are best as the result of 20 is easy to add to 37.	**a** 37 + 14 + 6 = 37 + (14 + 6) = 37 + 20 = 57
b Decide which two numbers to multiply together first. 20 and 5 are best as the result of 100 is easy to multiply by 63.	**b** 20 × 63 × 5 = (20 × 5) × 63 = 100 × 63 = 6300

UNDERSTANDING AND FLUENCY

10 Use mental strategies to calculate:

a 17 + 28 + 13	**b** 219 + 54 + 16	**c** 39 + 74 + 21
d 166 + 88 + 12	**e** 4 × 56 × 25	**f** 921 × 5 × 20
g 35 × 4 × 5	**h** 29 × 250 × 4	**i** 200 × 186 × 5

11 **a** Calculate these, then compare your answers.

i 4 × (5 + 6) **ii** 4 × 5 + 4 × 6

b Calculate these, then compare your answers.

i 6 × (10 − 2) **ii** 6 × 10 − 6 × 2

c Parts **a** and **b** are examples of the **distributive law** at work. Describe this law.

d Show how the distributive law works by providing two more examples.

12 The distributive law can be used to make some multiplication calculations easier to perform.

a The calculation 16 × 24 can be written as 16 × (20 + 4) or 16 × 20 + 16 × 4.

i Calculate 16 × 20. **ii** Calculate 16 × 4.

iii Add your results and write the answer to 16 × 24.

b The calculation 45 × 998 can be written as 45 × (1000 − 2) or 45 × 1000 − 45 × 2.

i Calculate 45 × 1000. **ii** Calculate 45 × 2.

iii Subtract your results and write the answer to 45 × 998.

c Explain how the distributive law can be used to make some multiplication calculations easier to perform.

EXAMPLE 1B-5 Using the distributive law

Use the distributive law to calculate:

a 25 × 41 **b** 32 × 97.

THINK

a **1** Write 41 as the sum of 40 and 1 as these numbers will be easier to multiply.

2 Use the distributive law to write the calculation as the sum of two products.

3 Calculate the result.

> **NOTE** 25 × 40 could be worked out as 25 × 4 × 10.

b **1** Write 97 as the difference of 100 and 3 as these numbers will be easier to multiply.

2 Use the distributive law to write the calculation as the difference of two products.

3 Calculate the result.

WRITE

a 25 × 41
= 25 × (40 + 1)
= 25 × 40 + 25 × 1
= 1000 + 25
= 1025

b 32 × 97
= 32 × (100 − 3)
= 32 × 100 − 32 × 3
= 3200 − 96
= 3104

13 Use the distributive law to calculate:

a 21×42	**b** 54×102	**c** 32×203	**d** 14×305
e 17×98	**f** 83×99	**g** 24×95	**h** 41×19
i 62×101	**j** 29×1001	**k** 11×97	**l** 15×999

PROBLEM SOLVING AND REASONING

14 Scoring in basketball depends on where a player shoots from. A successful shot from on or inside the three-point line scores two points; from outside the three-point line, it scores three points. Each successful free shot from the foul line scores one point.

The number of successful shots for two teams during a match is shown in the table.



Number of successful shots	Titans	Giants
inside the three-point line	32	35
outside the three-point line	7	5
from a free throw	4	2

a Copy and complete these statements to show the numbers and operations needed to calculate the total points scored by each team.

points scored by Titans = ___ × 2 + ___ × 3 + ___ × 1

points scored by Giants = ___ × ___ + ___ × ___ + ___ × ___

b Calculate the number of points each team scored.

c Which team won and by how much?





15 You have $195 to buy three shirts priced at $58 each.

a Write a calculation to show how to work out how much money you will have left over.

b Estimate how much money is left over by rounding each number to its leading digit then performing the calculation.

c Perform the calculation in part **a** and compare your estimate to the exact amount of money you have left over.



16 Using the images on the right, calculate the cost of buying lunch for the class: 14 salad rolls, 11 samosa packs, 9 flavoured milks, 16 orange juices and 25 apples. Show the numbers and operations you used in the calculation.





CHALLENGE

17 Using 4 as many times as you like and with any of the operations $(+, -, \times, \div)$, write a calculation to obtain every number from 1 to 10. For example, $4 \div 4 = 1$, $(4 + 4) \div 4 = 2$ and so on. You can also square numbers and use brackets. Remember to follow the correct order of operations.

18 Repeat question **17** using:

a 3 **b** 7.

Reflect

How can you remember the order of operations?

1C Understanding fractions

Start thinking!



Paolo has to share this cake with his two sisters.

1 Draw a rectangle to represent the cake and divide it into three equal sections. Shade a section to show Paolo's share.

2 What fraction of the cake is Paolo's share?

3 Visitors have arrived, so now the whole cake has to be divided into 15 slices. Draw a diagram to show this.

4 Paolo still wants to eat his original share. How many slices is this now? Shade Paolo's share on your second rectangle.

5 Write a fraction for Paolo's share of the cake after it has been cut into 15 slices. (Hint: the denominator will be 15.)

6 Do the fractions for questions **2** and **5** represent the same amount of cake? Explain.

7 These fractions are called **equivalent fractions**. Write an equivalent fraction for Paolo's share, if the cake is divided into:

a 12 slices **b** 18 slices.

8 The simplest fraction for Paolo's share was $\frac{1}{3}$. Look at the **numerator** and **denominator** of the equivalent fraction in question **7a** and compare this to the numerator and denominator of $\frac{1}{3}$. What relationship can you see?

9 Repeat question **8** for the equivalent fraction in question **7b**.

KEY IDEAS

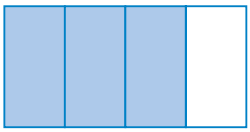
- Equivalent fractions have the same numerical value.
- An equivalent fraction is produced by multiplying or dividing the numerator and the denominator of a fraction by the same value.
- Dividing the numerator and the denominator of a fraction by the **highest common factor (HCF)** produces a fraction in its simplest form. This is called simplifying or cancelling.
- Mixed numbers can be simplified by keeping the whole number the same and simplifying the fraction component.

$$\frac{2}{3} \overset{\times 5}{=} \frac{10}{15}$$

$$\frac{24}{54} \overset{\div 6}{=} \frac{4}{9}$$

EXERCISE 1C Understanding fractions

1 **a** What fraction of the rectangle has been shaded?



b Draw a rectangle that is divided into 12 equal sections. Shade the rectangle so that it represents an equivalent fraction to that in part **a**.

c Write the equivalent fraction.

EXAMPLE 1C-1 Writing equivalent fractions

Fill the gap to produce equivalent fractions. $\frac{3}{7} = \frac{18}{\square}$

THINK

1 Look for a relationship between the two numerators. $3 \times 6 = 18$.

$\frac{3}{7} \overset{\times 6}{=} \frac{18}{\square}$

WRITE

$\frac{3}{7} = \frac{18}{\square}$

2 Use the same relationship (multiply by 6) to obtain the denominator of the second fraction.

$\frac{3}{7} \overset{\times 6}{=} \frac{18}{42}$

$\frac{3}{7} = \frac{18}{42}$

UNDERSTANDING AND FLUENCY

2 Fill the gaps to produce equivalent fractions.

a $\frac{3}{10} = \frac{6}{\square}$ **b** $\frac{2}{7} = \frac{\square}{35}$

c $\frac{18}{33} = \frac{\square}{11}$ **d** $\frac{16}{48} = \frac{2}{\square}$

e $\frac{4}{5} = \frac{\square}{30}$ **f** $\frac{8}{36} = \frac{2}{\square}$

g $1\frac{3}{8} = \frac{\square}{40}$ **h** $3\frac{3}{4} = \frac{75}{\square}$

i $\frac{1}{4} = \frac{2}{\square} = \frac{\square}{20} = \frac{8}{\square} = \frac{20}{\square} = \frac{\square}{100}$

j $\frac{3}{7} = \frac{15}{\square} = \frac{\square}{49} = \frac{\square}{70} = \frac{300}{\square} = \frac{\square}{210}$

k $1\frac{1}{5} = \frac{\square}{10} = \frac{60}{\square} = \frac{\square}{250} = 1\frac{\square}{45} = \frac{72}{\square}$

l $2\frac{1}{3} = \frac{14}{\square} = \frac{\square}{15} = \frac{49}{\square} = 2\frac{3}{9} = \frac{\square}{300}$

3 From this list, select which fractions are equivalent to:

a $\frac{2}{5}$ **b** $\frac{4}{3}$.

$\frac{6}{10}, \frac{20}{15}, \frac{8}{20}, \frac{7}{10}, \frac{28}{21}, \frac{22}{55}, \frac{4}{10}, \frac{16}{9}, 1\frac{4}{12}, \frac{10}{25}, \frac{18}{45}, 2\frac{6}{15}$

EXAMPLE 1C-2 Writing fractions in simplest form

Write each fraction in simplest form.

a $\frac{24}{40}$ **b** $6\frac{25}{35}$

THINK	WRITE
a 1 Find the highest common factor (HCF) of the numerator and the denominator. HCF = 8.	**a** $\frac{24}{40}$
2 Divide both the numerator (24) and the denominator (40) by the HCF (8).	$= \frac{24^{3}}{40^{5}}$
3 Write the answer.	$= \frac{3}{5}$
b 1 Find the HCF of the numerator and the denominator. HCF = 5.	**b** $6\frac{25}{35}$
2 Leave the whole number as it is and divide both the numerator (25) and the denominator (35) by the HCF (5).	$= 6\frac{25^{5}}{35^{7}}$
3 Write the answer.	$= 6\frac{5}{7}$

UNDERSTANDING AND FLUENCY

4 Write each fraction in simplest form.

a $\frac{8}{10}$ **b** $\frac{14}{21}$ **c** $\frac{42}{54}$

d $\frac{45}{72}$ **e** $\frac{40}{12}$ **f** $\frac{55}{30}$

g $3\frac{24}{32}$ **h** $5\frac{35}{63}$ **i** $4\frac{9}{33}$

j $1\frac{24}{42}$ **k** $2\frac{15}{40}$ **l** $7\frac{45}{54}$

5 a Write each fraction in simplest form.

i $\frac{6}{15}$ **ii** $\frac{33}{77}$ **iii** $\frac{18}{45}$

iv $\frac{30}{18}$ **v** $\frac{32}{36}$ **vi** $\frac{18}{42}$

b In part **a**, which are equivalent fractions?

6 Why is a fraction that has been simplified also an equivalent fraction?

7 Create three fractions that are equivalent to each fraction.

a $\frac{3}{5}$ **b** $\frac{7}{4}$ **c** $\frac{5}{6}$ **d** $\frac{8}{12}$

8 Create three mixed numbers that are equivalent to each fraction.

a $1\frac{2}{5}$ **b** $3\frac{18}{24}$ **c** $\frac{10}{7}$ **d** $\frac{21}{9}$

UNDERSTANDING AND FLUENCY

9 Write five fractions that simplify to $\frac{2}{3}$.

10 Decide which fraction is larger in each pair.

a $\frac{5}{9}, \frac{7}{9}$ **b** $\frac{11}{4}, \frac{9}{4}$ **c** $2\frac{3}{5}, \frac{12}{5}$

d $\frac{43}{6}, 7\frac{5}{6}$ **e** $3\frac{7}{11}, \frac{37}{11}$ **f** $\frac{29}{2}, 8\frac{1}{2}$

g $4\frac{2}{3}, \frac{16}{3}$ **h** $\frac{55}{9}, 6\frac{2}{9}$ **i** $3\frac{5}{7}, \frac{25}{7}$

11 Decide which fraction is larger in each pair.
(Hint: write equivalent fractions with the same denominator.)

a $\frac{3}{8}, \frac{1}{2}$ **b** $\frac{4}{7}, \frac{5}{14}$ **c** $\frac{9}{10}, \frac{4}{5}$

d $\frac{5}{12}, \frac{2}{3}$ **e** $1\frac{5}{6}, 1\frac{19}{24}$ **f** $\frac{17}{15}, \frac{7}{3}$

g $\frac{2}{3}, \frac{3}{4}$ **h** $\frac{5}{6}, \frac{4}{5}$ **i** $\frac{11}{8}, \frac{5}{3}$

j $2\frac{3}{4}, \frac{17}{6}$ **k** $\frac{29}{3}, 9\frac{1}{2}$ **l** $3\frac{5}{12}, \frac{25}{8}$

12 Write each list of fractions in **ascending order**.

a $\frac{2}{5}, \frac{7}{5}, \frac{4}{5}, 1\frac{1}{5}, \frac{3}{5}, \frac{1}{5}$ **b** $1\frac{1}{4}, 2\frac{5}{8}, \frac{9}{8}, 2\frac{3}{4}, \frac{3}{8}, \frac{7}{4}$

c $1\frac{1}{2}, \frac{2}{5}, 1\frac{9}{10}, \frac{9}{2}, \frac{3}{10}, \frac{9}{5}$ **d** $\frac{2}{3}, 1\frac{1}{2}, \frac{3}{4}, \frac{1}{2}, \frac{4}{3}, 1\frac{1}{4}$

13 Write each list of fractions in **descending order**.

a $\frac{13}{11}, \frac{5}{11}, \frac{19}{11}, \frac{9}{11}, \frac{3}{11}, 1\frac{5}{11}$ **b** $1\frac{1}{4}, 2\frac{5}{8}, \frac{9}{8}, 2\frac{3}{4}, \frac{3}{8}, \frac{7}{4}$

c $1\frac{1}{2}, \frac{2}{5}, 1\frac{9}{10}, \frac{9}{2}, \frac{3}{10}, \frac{9}{5}$ **d** $\frac{2}{3}, 1\frac{1}{2}, \frac{3}{4}, \frac{1}{2}, \frac{4}{3}, 1\frac{1}{4}$

EXAMPLE 1C-3 Writing an amount as a fraction of a total amount

Write each amount as a fraction of the total amount in simplest form.

a 12 cm out of 28 cm **b** 20 seconds out of 2 minutes

THINK	WRITE
a 1 As each amount is in the same unit, write 12 out of 28 as a fraction.	**a** $\frac{12}{28}$
2 Find the HCF (4) and simplify the fraction.	$= \frac{3}{7}$
b 1 The amounts are not in the same unit so convert 2 minutes to seconds. It is usually easier to convert to the smaller unit.	**b** 2 minutes = 120 seconds
2 Write 20 out of 120 as a fraction.	$\frac{20}{120}$
3 Find the HCF (20) and simplify the fraction.	$= \frac{1}{6}$

UNDERSTANDING AND FLUENCY

14 Write each amount as a fraction of the total amount in simplest form.

a 15 kg out of 25 kg
b 80 m out of 360 m
c \$35 out of \$120
d 18 seconds out of 60 seconds
e 50c out of \$3
f 14 days out of 5 weeks
g 20 mm out of 7 cm
h 300 g out of 2 kg
i 28 seconds out of 1 minute
j 75c out of \$4.25
k 20 cm out of 6 m
l 16 minutes out of 2 hours

15 Write three fractions that are:

a smaller than $\frac{3}{5}$
b larger than $\frac{4}{7}$
c smaller than $2\frac{1}{2}$
d larger than $3\frac{2}{3}$
e between $\frac{1}{12}$ and $\frac{5}{6}$
f between $\frac{2}{3}$ and $\frac{6}{5}$
g between $1\frac{1}{4}$ and $2\frac{3}{8}$
h between $4\frac{1}{2}$ and $4\frac{1}{7}$.

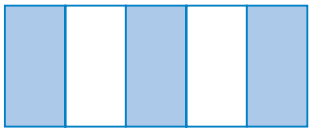
16 Find the fraction that is halfway between each pair of fractions.

a $\frac{3}{7}$ and $\frac{5}{7}$
b $\frac{5}{11}$ and $\frac{9}{11}$
c $\frac{3}{8}$ and $\frac{5}{8}$
d $\frac{1}{3}$ and $\frac{5}{9}$
e $\frac{4}{15}$ and $\frac{2}{5}$
f $\frac{1}{2}$ and $\frac{3}{4}$
g $1\frac{1}{4}$ and $1\frac{1}{3}$
h $2\frac{2}{3}$ and $3\frac{1}{5}$

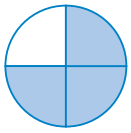
PROBLEM SOLVING AND REASONING

17 Match one of the fractions $\frac{3}{4}$, $\frac{2}{3}$, $\frac{3}{5}$ or $\frac{5}{8}$ to the amount shaded in each diagram.

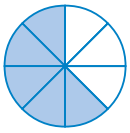
a



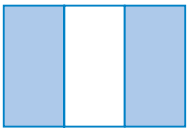
b



c



d



18 **a** Write a fraction to represent the amount eaten from each item.

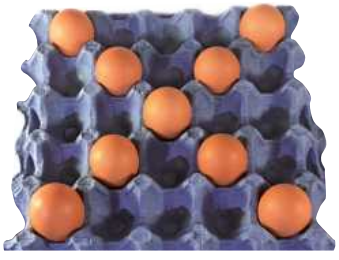
i



ii



iii



iv



b For each fraction in part **a**, write three equivalent fractions.
c For each item, write a fraction for the amount not eaten.
d For each fraction in part **c**, write three equivalent fractions.

PROBLEM SOLVING AND REASONING

19 If you have read 56 out of 224 pages of a novel, what fraction have you read?

20 Brodie scored 25 out of 30 on the first Maths test and 21 out of 28 in the second. On which test did Brodie perform better?

21 *Guinness World Records* states that Stephen Taylor from the UK has the longest tongue of any person. His tongue measures about 10 cm from the tip to the middle of his closed top lip. Typically, the human tongue is about 4 cm long.

a Write a fraction to compare the typical tongue length to that of Stephen Taylor.

b Measure or estimate the length of your tongue to the nearest centimetre.

c Write a fraction, in simplest form, to compare the length of your tongue to the world record.

Gene Simmons (from Kiss) is also known for the length of his tongue, which measures about 8 cm.



d Write a fraction to compare the typical length of a human tongue to that of Gene Simmons.

e Write three different fractions using the information in this question and explain what each fraction represents.

22 Tony and Lisbeth obtained the same score for hitting a target with a ball. Tony hit the target four times in five shots. Lisbeth hit the target 12 times. Use equivalent fractions to work out how many shots Lisbeth had.

CHALLENGE

23 The number of faults out of the total number of serves was recorded for each player in a tennis match.

	Serena	Maria
total number of serves	119	102
number of serving faults	14	10

a Who had the better success rate in serving?

b If Serena served at the same fault rate in her next match, how many faults did she make out of 85 serves?

c If Maria served at the same fault rate in her next match and made 15 faults, what was her total number of serves?

24 What could the denominators of two different fractions be, if a common denominator is 36?

25 What could the denominators of three different fractions be, if a common denominator is 40?

Reflect

How can you compare the size of fractions?

1D Operations with fractions

Start thinking!



True or false: to multiply or divide two fractions, the denominators must be the same.

To answer this, first consider all four of the operations (addition, subtraction, multiplication and division) with fractions. You may like to work with a partner.

Addition	Subtraction	Multiplication	Division
$\frac{3}{4}+\frac{2}{3}$	$\frac{3}{4}-\frac{2}{3}$	$\frac{3}{4}\times\frac{2}{3}$	$\frac{3}{4}\div\frac{2}{3}$
$=\frac{\ }{12}+\frac{8}{\ }$	$=\frac{\ }{12}-\frac{8}{\ }$	$=\frac{\ }{2}\times\frac{1}{\ }$	$=\frac{3}{4}\times\frac{\ }{2}$
$=\frac{\ }{12}$	$=\frac{\ }{12}$	$=\frac{\ }{2}$	$=\frac{\ }{8}$
$=1\frac{\ }{12}$			$=1\frac{\ }{8}$

1 Copy and complete these four calculations involving $\frac{3}{4}$ and $\frac{2}{3}$.

2 Describe the method you used to add the two fractions. Do you need to write each fraction with the same denominator before adding? Explain.

3 Repeat question **2** for:
a subtraction **b** multiplication **c** division.

4 Decide whether the statement at the start of this task is true or false. Provide a reason for your answer.

KEY IDEAS

- To add (or subtract) fractions that have different denominators:
  1. find equivalent fractions with the **lowest common denominator (LCD)**
  2. add (or subtract) the numerators.
- To multiply fractions:
  1. look for any factors common to the numerator and denominator. Cancel a number in the numerator and a number in the denominator by dividing both numbers by the highest common factor (HCF).
  2. multiply the numerators together and the denominators together.
- To divide fractions:
  1. change from a division to a multiplication problem by replacing the division sign with a multiplication sign and turning the fraction that follows upside down
  2. continue as for a multiplication problem.
- Convert mixed numbers into improper fractions before performing any calculation.
- If a result is an improper fraction, convert it to a mixed number.

EXERCISE 1D Operations with fractions

EXAMPLE 1D-1 Adding and subtracting fractions

Calculate:

a $1\frac{3}{8} + \frac{5}{6}$ **b** $2\frac{1}{4} - 1\frac{5}{12}$

THINK	WRITE
a **1** Convert the mixed number to an improper fraction.	**a** $1\frac{3}{8} + \frac{5}{6}$
	$= \frac{11}{8} + \frac{5}{6}$
2 Check if the denominators are the same (no). Identify the LCD (24) and write equivalent fractions with a denominator of 24.	$= \frac{33}{24} + \frac{20}{24}$
3 Add the numerators together. The denominator stays the same.	$= \frac{53}{24}$
4 Convert the improper fraction to a mixed number.	$= 2\frac{5}{24}$
b **1** Convert each mixed number to an improper fraction.	**b** $2\frac{1}{4} - 1\frac{5}{12}$
	$= \frac{9}{4} - \frac{17}{12}$
2 Check if the denominators are the same (no). Identify the LCD (12) and write equivalent fractions with a denominator of 12.	$= \frac{27}{12} - \frac{17}{12}$
3 Subtract the numerators. The denominator stays the same.	$= \frac{10}{12}$
4 Simplify the fraction. HCF is 2.	$= \frac{5}{6}$

UNDERSTANDING AND FLUENCY

1 Calculate:

a $\frac{5}{17} + \frac{8}{17}$ **b** $\frac{3}{4} - \frac{2}{5}$ **c** $\frac{9}{10} + \frac{2}{3}$

d $\frac{3}{8} - \frac{1}{12}$ **e** $\frac{11}{15} - \frac{7}{10}$ **f** $\frac{5}{6} + \frac{3}{4}$

g $1\frac{1}{4} - \frac{7}{8}$ **h** $2\frac{3}{5} + \frac{13}{20}$ **i** $2\frac{2}{3} + 1\frac{4}{5}$

j $3\frac{7}{11} - 1\frac{1}{2}$ **k** $4\frac{3}{4} + 2\frac{2}{3}$ **l** $5\frac{5}{6} - 3\frac{2}{15}$

2 Nine numbers are arranged in a square.

$\frac{4}{5}$	$\frac{1}{10}$	$\frac{3}{5}$
$\frac{3}{10}$	$\frac{1}{2}$	$\frac{7}{10}$
$\frac{2}{5}$	$\frac{9}{10}$	$\frac{1}{5}$

a Add the three numbers in:

i the first row **ii** the second row **iii** the third row.

b Add the three numbers in:

i the first column

ii the second column

iii the third column.

c Add the three numbers in each diagonal.

d What do you notice about your answers to parts **a**, **b** and **c**?

e Your answer to part **d** is the reason why the nine numbers form a magic square. The sum in each row, column and diagonal is called the magic sum. What is the magic sum for this set of numbers?

3 Copy and complete each magic square. (Hint: first work out the magic sum.)

a

	$\frac{1}{6}$	$\frac{1}{4}$
	$\frac{1}{3}$	
	$\frac{1}{2}$	

b

		$\frac{2}{5}$
$\frac{3}{5}$	$\frac{1}{3}$	$\frac{1}{15}$

c

		$\frac{1}{12}$
$\frac{1}{8}$	$\frac{5}{24}$	
$\frac{1}{3}$		

EXAMPLE 1D-2 Multiplying and dividing fractions

Calculate:

a $\frac{8}{9} \times \frac{5}{6}$ **b** $1\frac{7}{8} \div 2\frac{1}{4}$

THINK

a 1 Look for any common factors between numerators and denominators. The numbers 8 and 6 have HCF 2 so cancel by dividing both numbers by 2 (shown in green).

2 Write the result obtained after cancelling.

3 Multiply the numerators together ($4 \times 5 = 20$) and multiply the denominators together ($9 \times 3 = 27$).

b 1 Convert each mixed number to an improper fraction.

2 Change to a multiplication problem. Replace ÷ with × and turn the fraction that follows upside down.

3 Look for any common factors between numerators and denominators. The numbers 15 and 9 have HCF 3 so cancel by dividing both numbers by 3 (shown in green). The numbers 8 and 4 have HCF 4, so cancel by dividing both numbers by 4 (shown in blue).

4 Write the result obtained after cancelling.

5 Multiply the numerators together ($5 \times 1 = 5$) and multiply the denominators together ($2 \times 3 = 6$).

WRITE

a $\frac{8}{9} \times \frac{5}{6}$

$= \frac{{}^{4}\cancel{8}}{9} \times \frac{5}{\cancel{6}_{3}}$

$= \frac{4}{9} \times \frac{5}{3}$

$= \frac{20}{27}$

b $1\frac{7}{8} \div 2\frac{1}{4}$

$= \frac{15}{8} \div \frac{9}{4}$

$= \frac{15}{8} \times \frac{4}{9}$

$= \frac{{}^{5}\cancel{15}}{{}_{2}\cancel{8}} \times \frac{{}^{1}\cancel{4}}{\cancel{9}_{3}}$

$= \frac{5}{2} \times \frac{1}{3}$

$= \frac{5}{6}$

UNDERSTANDING AND FLUENCY

4 Calculate:

a $\frac{3}{8} \times \frac{2}{5}$ **b** $\frac{9}{10} \times \frac{4}{3}$ **c** $\frac{5}{7} \div \frac{3}{2}$ **d** $\frac{3}{14} \div \frac{12}{35}$ **e** $\frac{18}{25} \times \frac{35}{36}$ **f** $\frac{9}{28} \div \frac{6}{7}$

g $1\frac{7}{8} \times \frac{3}{5}$ **h** $2\frac{3}{4} \div \frac{5}{12}$ **i** $3\frac{1}{5} \times 1\frac{1}{4}$ **j** $1\frac{5}{8} \div 2\frac{8}{9}$ **k** $9\frac{1}{6} \times 1\frac{1}{11}$ **l** $2\frac{1}{3} \div 4\frac{2}{3}$

5 Calculate:

a $\frac{5}{6} - \frac{4}{9}$ **b** $\frac{3}{8} + \frac{2}{5}$ **c** $\frac{32}{49} \times \frac{21}{40}$ **d** $\frac{15}{22} \div \frac{5}{33}$ **e** $\frac{5}{18} \times 2\frac{7}{10}$ **f** $3\frac{1}{6} + \frac{7}{8}$

g $\frac{50}{63} \div 3\frac{4}{7}$ **h** $4\frac{3}{5} - \frac{1}{2}$ **i** $2\frac{4}{7} + 3\frac{1}{4}$ **j** $5\frac{1}{3} \times 2\frac{2}{5}$ **k** $3\frac{2}{11} - 1\frac{2}{3}$ **l** $2\frac{1}{3} \div 1\frac{5}{9}$

6 Calculate:

a $\left(\frac{6}{11} + \frac{2}{3}\right) \div \frac{5}{6}$ **b** $3\frac{1}{4} - 2\frac{1}{2} \times \frac{7}{10}$

c $\frac{8}{9} \times \left(\frac{4}{5} - \frac{3}{4}\right) \div 2\frac{2}{3}$ **d** $2\frac{3}{5} \times 3\frac{1}{3} + \frac{2}{11} \times 1\frac{3}{8}$

Remember to think carefully about which operations are performed first.

EXAMPLE 1D-3 Calculating a fraction of an amount

Calculate each amount in the units shown in brackets.

a $\frac{2}{3}$ of 27 m (m) **b** $\frac{1}{2}$ of 3 days (hours)

THINK

a **1** Replace 'of' with '×' and write the whole number as a fraction with a denominator of 1.

2 Cancel factors in the numerator and the denominator. HCF of 27 and 3 is 3.

3 Write the result after cancelling.

4 Complete the multiplication.

5 Write your answer.

b **1** Convert days to hours. (3 × 24 = 72.)

2 Replace 'of' with '×' and write the whole number as a fraction with a denominator of 1.

3 Cancel any factors in the numerator and the denominator. HCF of 72 and 2 is 2.

4 Complete the multiplication.

5 Write your answer.

WRITE

a $\frac{2}{3}$ of 27 m

$= \frac{2}{3} \times \frac{27}{1}$

$= \frac{2}{\cancel{3}_1} \times \frac{\cancel{27}^{9}}{1}$

$= \frac{2}{1} \times \frac{9}{1}$

$= \frac{18}{1}$

$= 18$

$\frac{2}{3}$ of 27 m = 18 m

b $\frac{1}{2}$ of 3 days

$= \frac{1}{2}$ of 72 hours

$= \frac{1}{2} \times \frac{72}{1}$

$= \frac{1}{\cancel{2}_1} \times \frac{\cancel{72}^{36}}{1}$

$= \frac{36}{1}$

$= 36$

$\frac{1}{2}$ of 3 days = 36 hours

7 Calculate each amount in the units shown in brackets.

a $\frac{1}{2}$ of 14 hours (hours) **b** $\frac{3}{4}$ of \$48 (dollars) **c** $\frac{2}{5}$ of 35 kg (kg)

d $\frac{4}{9}$ of 72 cm (cm) **e** $\frac{2}{7}$ of 3 weeks (days) **f** $\frac{3}{5}$ of 2 cm (mm)

g $\frac{3}{5}$ of 1 km (m) **h** $\frac{5}{6}$ of 4 minutes (seconds) **i** $\frac{2}{3}$ of 6 L (mL)

j $\frac{3}{8}$ of \$2 (cents) **k** $\frac{7}{10}$ of 5 kg (g) **l** $\frac{1}{12}$ of 8 hours (minutes)

8 Craig wants to combine two open cartons of milk into one 2-L container. If one carton has $\frac{1}{3}$ L and the other has $1\frac{3}{4}$ L, will the 2-L container be large enough? Explain.

9 On Monday evening, Eleanor spent $1\frac{1}{4}$ hours on the computer researching an assignment for school, Adam spent $2\frac{1}{2}$ hours on the computer catching up with friends on Facebook, and their mother spent $\frac{5}{6}$ of an hour typing up a report for work. How much time was spent on the computer Monday evening?

10 One-fifth of the student population at a local high school ride to school, one-tenth walk to school and two-thirds catch public transport. The remainder is driven to school.

What fraction of students:

a ride or walk to school?

b catch public transport, ride or walk to school?

c are driven to school?



11 The seating capacity of an interstate train carriage is 88 people. If the carriage is three-quarters full, how many people are seated in the train carriage?

12 John had a pizza home delivered and ate half of it before going to soccer training. Jen arrived home and ate half of what was left. Oscar then ate one-third of the remaining pizza. What fraction of the original pizza is now left over?

13 To bake 12 chocolate cupcakes, you need these ingredients.



Chocolate cupcakes

(makes 12)

100 g butter
$\frac{1}{2}$ cup sugar
2 large eggs
$1\frac{1}{2}$ cups flour
1 teaspoon baking powder
$\frac{1}{2}$ cup milk
$\frac{3}{4}$ cup cocoa powder
1 cup boiling water

Chocolate icing

$1\frac{1}{2}$ cups icing sugar
150 g butter
120 g chocolate
$\frac{1}{4}$ cup thickened cream

a Work out the quantities of each ingredient (excluding the icing) if you want to bake:

i 24 cupcakes **ii** 6 cupcakes **iii** 8 cupcakes.

PROBLEM SOLVING AND REASONING

b Finn bakes 12 cupcakes but before he can ice them, one-quarter are eaten. Adjust the ingredient list for the chocolate icing so it makes only enough for the remaining cupcakes.

14 Anna takes $\frac{3}{4}$ hour to ride her bike from home to school.
How many trips could she possibly make between home and school in $2\frac{1}{4}$ hours?



15 Grapes are to be sold in $2\frac{1}{2}$-kg bags.

a How many bags must Tricia pack if she has a 20-kg crate of grapes?

b From another crate, Tricia packs 11 bags and has $1\frac{1}{2}$ kg of grapes left over. How many kilograms of grapes were in the second crate?

16 Metal screws come in different sizes, with diameters often listed in inches.

a Find the difference in size of two screws with diameters $\frac{3}{50}$ inches and $\frac{41}{250}$ inches.

b Wes has drilled a hole in the wall of diameter $\frac{3}{20}$ inches to hang a painting. Which of these two screws would be best for him to use?



CHALLENGE

17 Write three different addition problems that give an answer of $\frac{11}{20}$.

18 Write three different subtraction problems that give an answer of $\frac{5}{6}$.

19 Write three different multiplication problems that give an answer of $\frac{3}{8}$.

20 Write three different division problems that give an answer of $\frac{7}{12}$.

21 The answer to a fraction problem is $2\frac{3}{4}$.

a Write a sample problem that uses each of these operations.

i addition

ii subtraction

iii multiplication

iv division

b Write a sample problem that uses two or more of the operations.

Reflect

What tips would you give a classmate to help them remember how to perform the four operations on fractions?

1E Understanding decimals

Start thinking!



Results in a triple-jump competition are listed in metres as decimal numbers: 12.356, 11.8, 12.64, 11.704, 12.460 and 12.08.

1 The digit 3 is the first digit after the decimal point in 12.356. This is the first **decimal place**.

a What digit is in:

i the second decimal place? **ii** the third decimal place?

b How many decimal places does 12.64 have?

c List the number of decimal places in each of the other measurements.

2 Which measurement is the largest? Which is the smallest? How can you tell? Does it depend on how many decimal places there are?

3 Look at the measurements containing zeros: 11.704, 12.460 and 12.08.

a What would happen if you wrote each number without its zero (for example, if you wrote 11.704 as 11.74 or 12.460 as 12.46)?

b Zeros between non-zero digits are called **placeholder zeros**. Why is it important that these zeros are never left out?

c Zeros at the end of a decimal are called **trailing zeros**. What difference does it make if these zeros are left out?

d Which measurements have:

i placeholder zeros? **ii** trailing zeros?

KEY IDEAS

- Zeros between non-zero digits in a decimal (called placeholder zeros) must never be left out, or the value of the number will be changed.
- Zeros at the end of a decimal (called trailing zeros) do not change the value of the number.
- The number of decimal places in a number is the number of digits after the decimal point.
- A decimal number can be rounded to a given number of decimal places by considering the next digit to the right.
  - If it is 0, 1, 2, 3 or 4, round down. For example, 1.263 rounded to two decimal places is 1.26.
  - If it is 5, 6, 7, 8 or 9, round up. For example, 1.268 rounded to two decimal places is 1.27.

EXERCISE 1E Understanding decimals

UNDERSTANDING AND FLUENCY

1 How many decimal places does each number have?

a 19.511	**b** 0.472 08	**c** 34.8002
d 6.492 177	**e** 0.005	**f** 90.7100
g 5893.2	**h** 802.713 204 15	**i** 7601.004

2 Decide whether each decimal contains placeholder zeros (P), trailing zeros (T) or both (B).

a 4.705	**b** 29.300	**c** 605.9
d 1740.50	**e** 5.3720	**f** 88.302 000

3 How many decimal places does each number in question **2** have?

4 Use trailing zeros to write each number to five decimal places. For example, 4.3 = 4.300 00.

a 5.46	**b** 76.3	**c** 0.306
d 29.7843	**e** 9.0	**f** 15

EXAMPLE 1E-1 Ordering decimals

Write this list of numbers in ascending order: 5.74, 5.7, 5.47, 5.04, 5.4, 5.407.

THINK	WRITE
1 To make them easier to compare, write all the numbers with the same number of decimal places (three) by adding trailing zeros where necessary.	5.740, 5.700, 5.470, 5.040, 5.400, 5.407
2 Write the numbers in order from smallest to largest by comparing the place value of the digits, moving from left to right.	5.040, 5.400, 5.407, 5.470, 5.700, 5.740 In ascending order, the numbers are: 5.04, 5.4, 5.407, 5.47, 5.7, 5.74

5 Write each list of numbers in ascending order.

a 0.214, 0.142, 0.2, 0.41, 0.104, 0.14

b 8.702, 9.87, 8.072, 9.782, 8.2, 8.97

6 Write each list of numbers in descending order.

a 13.7, 13.07, 13.173, 13.31, 13.137, 13.713

b 0.658, 0.6058, 0.6508, 0.685, 0.6085, 0.65

7 Explain why 9.46 is larger than 9.452 even though 9.452 has more digits.

EXAMPLE 1E-2 Rounding decimals to two decimal places

Round each number to two decimal places.

a 5.7381 **b** 12.054 76

THINK	WRITE
a 1 Look at the digit in the third decimal place. Since it is 8, round up.	**a** 5.7381
2 Write the number to two decimal places, with the digit in the second decimal place increased by one. The digits to the left of the second decimal place remain the same and the digits to the right are removed.	≈ 5.74
b 1 Look at the digit in the third decimal place. Since it is 4, round down.	**b** 12.054 76
2 Write the number to two decimal places, with the digit in the second decimal place remaining the same. The digits to the left of the second decimal place also remain the same and the digits to the right are removed.	≈ 12.05

UNDERSTANDING AND FLUENCY

8 Round each number to two decimal places.

a 25.679 **b** 673.8234 **c** 0.784 231
d 101.0157 **e** 8148.255 **f** 54.2618
g 330.406 **h** 4.854 793 **i** 71.006 24

9 Round each decimal to the number of decimal places shown in the brackets.

a 57.234 (2) **b** 8.9137 (3)
c 92.087 12 (4) **d** 0.756 825 (3)
e 135.152 79 (1) **f** 65.071 226 33 (5)
g 7.510 429 (3) **h** 208.009 862 (4)
i 1.727 727 727 (5) **j** 845.845 845 (1)
k 0.123 456 78 (4) **l** 0.123 456 78 (7)

10 Wendy thinks 8.2735 rounded to three decimal places is 8.273. Rosa thinks it is 8.274. Who is correct? Explain.

11 Jayden thinks that 24.7963 rounded to two decimal places is 24.80. Oscar thinks it is 24.71. Who is correct? Explain.

12 Round each number to three decimal places.

a 6.8134 **b** 0.741 52 **c** 21.8306
d 4.0798 **e** 30.2995 **f** 15.9999

EXAMPLE 1E-3 Using place value to write decimals as fractions

Use your knowledge of place value to write each decimal as a fraction.

a 0.3 **b** 0.09

THINK	WRITE
a Consider the place value of 3. The first decimal place shows tenths.	**a** $0.3 = \frac{3}{10}$
b Consider the place value of 9. The second decimal place shows hundredths.	**b** $0.09 = \frac{9}{100}$

UNDERSTANDING AND FLUENCY

13 Use your knowledge of place value to write each decimal as a fraction.

a 0.03 **b** 0.9 **c** 0.007 **d** 0.0001

e 0.7 **f** 0.000 03 **g** 0.000 009 **h** 0.000 000 7

14 Write each fraction as a decimal.

a $\frac{7}{10}$ **b** $\frac{29}{100}$ **c** $\frac{451}{1000}$ **d** $\frac{2047}{10\,000}$ **e** $\frac{4}{100}$ **f** $\frac{9}{1000}$

g $\frac{5}{10\,000}$ **h** $\frac{63}{1000}$ **i** $\frac{19}{10\,000}$ **j** $\frac{368}{10\,000}$ **k** $\frac{20}{100}$ **l** $\frac{70}{1000}$

15 Copy and complete.

a $0.27 = \frac{2}{10} + \frac{}{100}$ or $\frac{27}{100}$

b $0.589 = \frac{5}{10} + \frac{}{100} + \frac{}{1000}$ or $\frac{589}{1000}$

c $0.343 = \frac{}{10} + \frac{4}{100} + \frac{3}{1000}$ or $\frac{}{1000}$

d $0.6251 = \frac{6}{10} + \frac{}{100} + \frac{}{1000} + \frac{}{10\,000}$ or $\frac{6251}{10\,000}$

e $0.109 = \frac{1}{10} + \frac{9}{1000}$ or $\frac{}{1000}$

f $0.0867 = \frac{8}{100} + \frac{}{1000} + \frac{7}{10\,000}$ or $\frac{}{10\,000}$

16 Copy this table and allow enough rows for nine decimal numbers.

Decimal number	Number of decimal places	Value of digit in:				Fractional form
		first decimal place	second decimal place	third decimal place	fourth decimal place	
0.473	3	$\frac{4}{10}$	$\frac{7}{100}$	$\frac{3}{1000}$		$\frac{473}{1000}$
0.8619	4	$\frac{8}{10}$	$\frac{6}{100}$	$\frac{1}{1000}$	$\frac{9}{10\,000}$	$\frac{8619}{10\,000}$

a Complete a row in the table for 0.357.

b How many decimal places does 0.357 have?

c How many zeros are in the denominator when 0.357 is written as a fraction?

d Repeat parts **a**–**c** for each of these numbers.

i 0.7253 **ii** 0.691 **iii** 0.49

iv 0.103 **v** 0.9 **vi** 0.007

e What is the relationship between the number of decimal places in a number and the number of zeros in the denominator of the matching fraction?

EXAMPLE 1E-4 Writing decimals as fractions

Write each number as a fraction in simplest form.

a 0.24 **b** 0.365

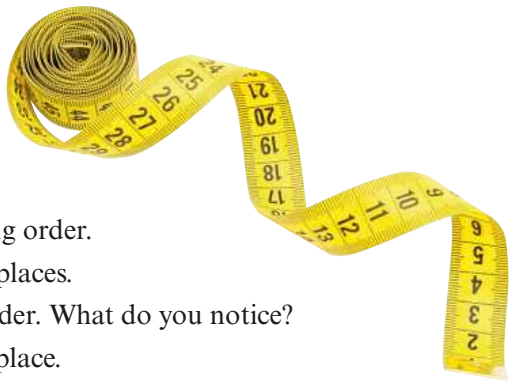
THINK	WRITE
a 1 As 0.24 has two decimal places, the denominator is 100.	**a** $0.24 = \frac{24}{100}$
2 Simplify the fraction. HCF is 4.	$= \frac{6}{25}$
b 1 As 0.365 has three decimal places, the denominator is 1000.	**b** $0.365 = \frac{365}{1000}$
2 Simplify the fraction. HCF is 5.	$= \frac{73}{200}$

17 Write each number as a fraction in simplest form.

a 0.352	**b** 0.5724	**c** 0.803	**d** 0.04
e 0.095	**f** 0.0006	**g** 0.74	**h** 0.252
i 0.0015	**j** 0.0484	**k** 0.000 17	**l** 0.8002

PROBLEM SOLVING AND REASONING

18 Six students' heights were measured as 1.651 m, 1.63 m, 1.568 m, 1.615 m, 1.58 m and 1.657 m.



a Which is the smallest measurement?
b Which is the largest measurement?
c Write the list of measurements in ascending order.
d Round each measurement to two decimal places.
e Write your list from part **d** in ascending order. What do you notice?
f Round each measurement to one decimal place.
g Write your list from part **f** in ascending order. What do you notice?

19 The width of a human hair is measured to be 0.018 35 mm.
Write this value as an approximation to:
a two decimal places **b** three decimal places **c** four decimal places.

20 The diameter of an optical fibre is 0.2845 mm.

a Write this value as an approximation to:
- **i** one decimal place
- **ii** two decimal places
- **iii** three decimal places.

b Write the exact diameter as a fraction.



PROBLEM SOLVING AND REASONING

21 Another way to say that you have an approximate value after rounding is to say that a number is correct to a certain number of decimal places. For example, the length of my fingernail is 1.6 cm, correct to one decimal place.

a Find the length of one of your fingernails correct to one decimal place.

b List some reasons for not quoting the length to lots of decimal places.

22 The length of a mosquito is measured to be 0.68 cm. Write this value correct to one decimal place.



23 Write 2.734 829 954 correct to the following number of decimal places.

a three **b** five

c eight **d** one

e six **f** seven

24 Round each amount to the nearest cent.

a \$3.437 **b** \$28.084 **c** \$117.206

d \$59.115 **e** \$62.7462 **f** \$0.9191

25 Round each amount to the nearest five cents.

a \$42.37 **b** \$207.62 **c** \$7.24

d \$15.03 **e** \$60.71 **f** \$99.99

CHALLENGE

26 Write three examples of numbers that, when rounded to two decimal places, equal 5.74.

27 Write three examples of numbers that, when rounded to one decimal place, equal 0.6.

28 Round 4708.612 95 to:

a the nearest thousand

b the nearest hundred

c the nearest ten

d the nearest one

e the nearest tenth

f the nearest hundredth

g the nearest thousandth

h the nearest ten thousandth.

Reflect

Why are decimals often rounded to a certain number of decimal places?

1F Operations with decimals

Start thinking!

Four friends share the cost of buying food for a holiday. Sasha writes a list.

Item	Unit cost	Amount	Cost of item
Tinned soup	$3.95 each	6	
Packet of pasta	$2.85 each	1	
Mince steak	$13.65 per kg	0.8 kg	
Jar of pasta sauce	$5.79 each	1	
Bread	$3.45 each	1	
Sausages	$8.98 per kg	0.5 kg	
Tomato sauce	$4.70 each	1	
Oranges	$0.85 each	4	
Apples	$4.60 per kg	2 kg	
Box of cereal	$5.99 each	1	
Milk	$2.48 per L	2 L	
Tropical juice	$6.75 each	1	

1 What is the cost of six tins of soup?
2 Copy and complete the list by calculating each item's cost.
3 Sasha has $100. Is this enough? Estimate the total cost by rounding the cost of each item to the nearest dollar.
4 Estimate how much money Sasha would expect to have left over.
5 Calculate the total cost.
6 How much does Sasha actually have left over?
7 The remaining money is shared between the four friends. How much does each person receive?
8 What is important to remember when:
   a multiplying a decimal number by a whole number?
   b adding or subtracting decimals?
   c dividing a decimal number by a whole number?

KEY IDEAS

- To add or subtract decimals:
  - line up the decimal points, and digits with the same place value
  - include trailing zeros to fill blank spaces in columns to minimise mistakes
  - add or subtract digits in each column from right to left.
- To multiply two decimals:
  - multiply them as if they were whole numbers
  - the answer has the same number of decimal places as the total number of decimal places in the original problem.
- To divide a decimal by a whole number:
  - divide into the dividend from left to right
  - line up the decimal point and the digits with the same place value in the quotient with those in the dividend
  - add trailing zeros to the end of the decimal number until you can divide exactly, without a remainder.
- To divide a decimal by another decimal:
  - write an equivalent division problem with a divisor that is a whole number
  - continue as for dividing a decimal by a whole number.

NOTE Estimating an answer before performing the calculation helps you check your result.

$$8\overline{)173.000} = 21.625$$

21.625 ← quotient (the answer)
divisor (the number we are dividing by)
dividend (the number being divided)

Example
$12.328 \div 0.04 = 1232.8 \div 4$

EXERCISE 1F Operations with decimals

UNDERSTANDING AND FLUENCY

1 Use vertical addition to calculate each result.

a 23.82 + 17.13 + 124.06 + 82.95

b 5.613 + 9.27 + 11.3856 + 7.5

c 324.7 + 76.29 + 4.93 + 285.064

d 74.0106 + 2.93 + 28.738 + 0.9167

2 Use vertical subtraction to calculate each result.

a 158.29 − 75.46 **b** 2051.384 − 1265.73

c 0.35 − 0.2156 **d** 70.308 − 44.057

e 9.104 − 6.3728 **f** 265.7 − 88.504

g 5061.03 − 1266.735 **h** 73 412 − 8743.56

3 Use short multiplication to calculate each result.

a 24.35 × 8 **b** 305.6 × 4

c 17 352.7 × 3 **d** 0.327 51 × 9

EXAMPLE 1F-1 Multiplying decimals using long multiplication

Use long multiplication to calculate 24.756 × 3.2.

THINK

1 Estimate the answer. Round each decimal to the nearest whole number.

2 Write the numbers under each other without the decimal points and use long multiplication.

3 Count the number of decimal places in the original calculation (3 + 1 = 4). There will be the same number of decimal places in the answer (four).

4 Write the answer with the decimal point in the correct position. Check that it matches your estimate.

WRITE

Estimate is 25 × 3 = 75.

$$\begin{array}{r} {\scriptstyle 1\,2\,1\,1} \\ {\scriptstyle \,1\,1\,1} \\ 24\,756 \\ \times \quad\;\; 32 \\ \hline 49\,512 \\ 742\,680 \\ \hline 792\,192 \end{array}$$

24.756 × 3.2 = 79.2192

UNDERSTANDING AND FLUENCY

4 Use long multiplication to calculate each result:

a 8.27 × 5.3 **b** 12.86 × 2.4

c 235.2 × 9.7 **d** 62.513 × 1.8

e 9.2 × 0.46 **f** 3.8927 × 0.52

g 56.361 × 0.14 **h** 157.93 × 7.12

EXAMPLE 1F-2 Dividing a decimal by a whole number

Use short division to calculate 37.45 ÷ 4.

THINK	WRITE
1 Estimate the answer. Round each number to its leading digit.	Estimate is 40 ÷ 4 = 10.
2 Divide into 37.45 from left to right. Use trailing zeros (if necessary) to obtain an exact decimal answer without a remainder. Remember to line up the decimal points and the digits that have the same place value.	$4\overline{)37.^{1}4^{2}5^{1}0^{2}0}$ $9.\ 3\ 6\ 2\ 5$
3 Write your answer. Check that it matches your estimate.	37.45 ÷ 4 = 9.3625

5 Use short division to calculate each result.

a 72.596 ÷ 4 **b** 6.724 ÷ 5
c 58.2743 ÷ 7 **d** 65.0955 ÷ 3
e 4.73 ÷ 8 **f** 0.627 ÷ 4
g 0.1536 ÷ 6 **h** 138.0714 ÷ 5

EXAMPLE 1F-3 Converting a fraction to a decimal using short division

Write $\frac{3}{4}$ as a decimal.

THINK	WRITE
1 Write $\frac{3}{4}$ as a division problem.	$\frac{3}{4} = 3 \div 4$
2 Use short division to divide 3 by 4. Include two trailing zeros to obtain an exact decimal answer without a remainder.	$4\overline{)3.0^{2}0}$ $0.7\ 5$
3 Write your answer.	$\frac{3}{4} = 0.75$

6 Write each fraction as a decimal.

a $\frac{4}{5}$ **b** $\frac{3}{8}$ **c** $\frac{1}{2}$ **d** $\frac{5}{4}$
e $\frac{1}{8}$ **f** $\frac{2}{5}$ **g** $\frac{7}{20}$ **h** $\frac{9}{25}$
i $\frac{1}{40}$ **j** $\frac{3}{5}$ **k** $\frac{11}{50}$ **l** $\frac{5}{16}$

7 Dividing a decimal by another decimal is easier if you write an equivalent division problem where the divisor is a whole number.

- **a** In 3.568 ÷ 0.4, the divisor is 0.4. Should you multiply 0.4 by 10, 100 or 1000 to produce 4?
- **b** Copy and complete: 3.568 ÷ 0.4 = _____ ÷ 4
- **c** Why is it important to multiply each decimal in the division problem by the same number?
- **d** Use your answer to part **b** to work out the division.

EXAMPLE 1F-4 Dividing a decimal by another decimal

a Write 9.46 ÷ 0.5 as an equivalent division problem where the divisor is a whole number.
b Calculate 9.46 ÷ 0.5.

THINK	WRITE
a The decimal 0.5 multiplied by 10 becomes a whole number. To keep the calculation equivalent to the original problem, also multiply the dividend by 10.	**a** 9.46 ÷ 0.5 = (9.46 × 10) ÷ (0.5 × 10) = 94.6 ÷ 5
b **1** Divide 94.6 by 5. Include trailing zeros until you can divide exactly without a remainder.	**b** $5\overline{)94.60}$ = 18.92
2 Write the answer.	9.46 ÷ 0.5 = 18.92

8 Write each calculation as an equivalent division problem where the divisor is a whole number.

- **a** 7.24 ÷ 0.2
- **b** 4.902 ÷ 0.6
- **c** 19.764 ÷ 0.03
- **d** 1.728 ÷ 0.08
- **e** 1.4368 ÷ 0.004
- **f** 7.491 ÷ 0.06
- **g** 4.872 ÷ 1.5
- **h** 2.808 ÷ 0.12
- **i** 0.1638 ÷ 0.009

9 Calculate the result of each division problem in question **8**.

10 The circuit of a fun run has five different stages. The lengths of the stages are: 1.435 km, 2.790 km, 1.902 km, 2.285 km, 3.613 km. First estimate the distance around the full circuit by rounding each length to the nearest whole number and then calculate the exact distance.

11 Calculate the cost of each item. Write your answers correct to the nearest cent.

- **a** four punnets of raspberries at $7.35 per punnet
- **b** 3 kg of watermelon at $1.95 per kg
- **c** 0.6 kg of snow peas at $5.62 per kg
- **d** 2.5 kg of potatoes at $3.95 per kg
- **e** 0.55 kg of bok choy at $8.99 per kg
- **f** 1.74 kg of green beans at $6.89 per kg



12 **a** Estimate the total cost of the items listed in question **11** by first rounding the cost of each item to the nearest dollar.

b Find the total cost of these items.
Round your answer to the nearest five cents.

c How much change would you receive if you paid with a $100 note?

13 In each of these calculations, round your answer to the nearest cent.

a Calculate the cost of one item in each case.

i nine bread rolls cost $5.40

ii four cartons of milk cost $11.56

iii five DVDs cost $94.75

b Calculate the cost of 1 kg in each case.

i 3 kg of tomatoes cost $13.95

ii 7 kg of sausages cost $47.95

iii 6 kg of dog food costs $81.66

c Calculate the cost of 100 g in each case.

i 200 g of salami costs $25.90

ii 500 g of cheese costs $7.95

iii 300 g of cashews cost $5.85.

14 A box of 10 coloured pencils costs $7.50. These pencils can also be bought individually for 72 cents each.

a Find the cost per pencil in each case.

b Which option provides better value or is the **best buy**? Explain.

15 William bought a 400-g pack of mushrooms for $3.69. Kate bought 300 g of loose mushrooms for $2.98.

a Calculate the amount paid by each of them for 100 g of mushrooms.

b Who made the best buy? Explain.

c Why does a store offer both options?



16 **a** How many icy-poles could you buy with $24.50?

b How many icy-poles could you buy with $10? What change would you receive?



17 A cup holds 0.25 L of liquid.
How many cups could be filled from a 5-L container of water?

PROBLEM SOLVING AND REASONING

18 Which is the best buy: 0.8 kg of mince steak for $11.20 from the supermarket, or 1.2 kg of mince steak for $16.50 from the butcher?

19 Lee Redmond once held the record for the longest fingernails on both hands (see table), after not cutting them for 30 years.



	Fingernail lengths (cm)	
	right hand	left hand
thumb	76.4	80.0
index finger	72.3	76.4
middle finger	74.1	76.7
ring finger	73.6	76.2
little finger	71.6	73.6

a What is the total length of the fingernails on:
- **i** her right hand?
- **ii** her left hand?

b Which hand has the longer fingernails? Suggest a reason for this.

c What is the total length of the fingernails on both hands?

d What is the average length of her fingernails?

e Measure the length of your fingernails in centimetres to one decimal place.

f Find the average length of your fingernails.

g How many times longer is Lee's average fingernail length than yours? First work out an estimate and then calculate a value correct to one decimal place.

CHALLENGE

20 Calculate:

a $(15.73 + 2.9) \times 1.5 - 26.07$

b $72.816 \div 8 + 4.9 \times 0.5$

c $6.9 + 0.528 \div 0.04 - 9.3$

NOTE Remember to think carefully about which operations are performed first.

21 The answer to a decimal problem is 4.72.

a Write a sample problem that uses each of these operations.
- **i** addition
- **ii** subtraction
- **iii** multiplication
- **iv** division

b Write three different sample problems that use two or more of the operations.

Reflect

Why is it important to estimate your answer before performing calculations with decimals?

1G Terminating, non-terminating and recurring decimals

Start thinking!

In some division problems, you can add trailing zeros to a decimal so that you obtain an exact answer with no remainder. Does this approach work every time?

1 Use short division to calculate: **a** $5 \div 8$ **b** $5 \div 6$.

2 What is different about the answers to parts **a** and **b** of question **1**?

3 The result in question **1a** is a **terminating decimal**.
After how many decimal places does the resulting decimal 'terminate' or stop?

4 Is the result to question **1b** a terminating decimal? Explain.

5 If a decimal does not terminate then it is called a **non-terminating decimal**.
How many decimal places does a non-terminating decimal have?

6 Look at your answer to question **1b**. Is there a pattern to the digits?

7 A special type of non-terminating decimal is a **recurring decimal**, where one or more digits form a repeating or recurring pattern. Explain why the answer to question **1b** is a recurring decimal.

KEY IDEAS

- A terminating decimal 'terminates' or stops after a number of decimal places.
  Some examples are: 24.3, 0.49, 5.768 234.
- A non-terminating decimal has an endless (infinite) number of decimal places.
  Some examples are: 17.326 35…, 0.567 743 …, 0.788 888 ….
- The full stops written after the last decimal place indicate that an endless number of digits follow.
- A recurring decimal is a special type of non-terminating decimal. The digits after the decimal point form a recurring (repeating) pattern. Some examples are: 8.999 999 …, 42.671 671 671…, 0.582 323 232….
- A recurring decimal can be written in an abbreviated form using dots or a horizontal line over repeating digits. Some examples are:
  $8.9999\ldots = 8.\dot{9}$, $42.671\,671\ldots = 42.\overline{671}$, $0.582\,323\,23\ldots = 0.58\dot{2}\dot{3}$.
- **Rational numbers** are numbers that can be written exactly as fractions.
- **Irrational numbers** are numbers that cannot be written exactly as fractions.

EXERCISE 1G Terminating, non-terminating and recurring decimals

UNDERSTANDING AND FLUENCY

1 State whether each decimal is terminating (T) or non-terminating (N).

a 4.572 35…
b 56.803
c 21.577 77…
d 8.561 395 62
e 0.451 730 69…
f 17.565 656 5…
g 0.123 412 341 234…
h 0.787 878 78
i 30.082 174…
j 291.291 291 2
k 0.999…
l 5.043 127

2 Which of the numbers in question **1** are recurring decimals?

3 Explain the difference between terminating, non-terminating and recurring decimals.

EXAMPLE 1G-1 Writing recurring decimals in abbreviated form

Write each recurring decimal in abbreviated form using dots or a horizontal line.

a 7.682 222 222…
b 24.503 150 315 031…

THINK

a Look for a repeating pattern of digits. Since there is only one digit that repeats (2), show a dot above that digit.

b Look for a repeating pattern of digits (5031). Show a horizontal line above those digits (or show a dot above the first and last digit in the pattern).

WRITE

a 7.682 222 222…
$= 7.68\dot{2}$

b 24.503 150 315 031…
$24.\overline{5031}$ (or $24.\dot{5}03\dot{1}$)

4 Write each recurring decimal in abbreviated form using dots or a horizontal line.

a 0.999 999…
b 4.363 636 3…
c 87.255 555…
d 3.476 476 476…
e 0.738 222 222…
f 16.196 666 66…
g 483.888 88…
h 6.729 191 919…
i 0.183 673 673 67…
j 1.356 283 562 835…
k 28.707 070 7…
l 9.230 714 444 44…

5 Write your answers to question **2** in abbreviated form.

EXAMPLE 1G-2 Writing recurring decimals to 12 decimal places

Write each recurring decimal to show 12 decimal places.

a $0.\dot{4}$ **b** $159.8\overline{372}$

THINK

a Identify the repeating digit (4). Write the decimal to 12 decimal places and indicate that more digits follow.

b Identify the repeating digits (372). Write the decimal to 12 decimal places and indicate that more digits follow.

WRITE

a $0.\dot{4}$
$= 0.444\ 444\ 444\ 444\ldots$

b $159.8\overline{372}$
$= 159.837\ 237\ 237\ 237\ldots$

UNDERSTANDING AND FLUENCY

6 Write each recurring decimal to show 12 decimal places.

a $0.\dot{1}$ **b** $5.3\dot{7}$ **c** $14.\overline{82}$
d $21.5\dot{6}\dot{9}$ **e** $8.\overline{104}$ **f** $0.\overline{6513}$
g $232.68\dot{4}$ **h** $44.761\,\overline{359}$ **i** $94.57\dot{0}\dot{2}$
j $1.\overline{73924}$ **k** $11.8121\dot{}$ **l** $6.9\overline{04}$
m $14.24\dot{1}\dot{3}$ **n** $0.5\overline{620}$ **o** $139.\dot{2}74\dot{5}$

7 Convert each fraction to a decimal by dividing the numerator by the denominator.

a $\frac{7}{10}$ **b** $\frac{1}{2}$ **c** $\frac{1}{3}$ **d** $\frac{5}{4}$
e $\frac{2}{11}$ **f** $\frac{5}{8}$ **g** $\frac{8}{3}$ **h** $\frac{3}{4}$
i $\frac{9}{8}$ **j** $\frac{7}{5}$ **k** $\frac{5}{6}$ **l** $\frac{1}{7}$

8 Consider the fractions and your answers to question **7**.

a Which fractions produce:
  i terminating decimals?
  ii non-terminating decimals?
b Which fractions produce recurring decimals?
c Do any of the fractions produce a non-terminating decimal that isn't a recurring decimal?

9 Write each number as a fraction. (Hint: whole numbers can be written with a denominator of 1.)

a 0.5 **b** 4 **c** 6.2
d 0.37 **e** 18 **f** 2.509

UNDERSTANDING AND FLUENCY

10 Numbers that can be written exactly as fractions are rational numbers.

a Use your answers to question **9** to explain why whole numbers and terminating decimals are rational numbers.

b Are recurring decimals rational numbers? Use your answers to questions **7** and **8** to help you decide. Explain your decision.

11 Numbers that cannot be written exactly as fractions are irrational numbers. Non-terminating decimals that are not recurring decimals fit into this category. They are decimals with an endless number of decimal places but no pattern of repeating digits. Write three examples of an irrational number.

EXAMPLE 1G-3 Classifying numbers as rational or irrational

Classify each number as rational or irrational.

a $\frac{5}{6}$ **b** 8.724 **c** 4.333… **d** 0.231 475 6…

THINK

a All fractions are rational.

b All terminating decimals are rational.

c All recurring decimals are rational.

d All non-terminating decimals that are not recurring decimals are irrational.

WRITE

a $\frac{5}{6}$ is a rational number.

b 8.724 is a rational number.

c 4.333… is a rational number.

d 0.231 475 6… is an irrational number.

12 Classify each number as rational or irrational.

a 4.6 **b** 0.121 212 12…

c 0.5871 **d** 2.357 896 21…

e 35 **f** 734.946 518 24…

g $0.872\overline{503}$ **h** 96.734 25

i $\frac{3}{17}$ **j** 500 000

k 0.117 843 26… **l** $10\frac{3}{4}$

13 **a** Convert each fraction to a decimal.

i $\frac{1}{9}$ **ii** $\frac{2}{9}$

iii $\frac{3}{9}$ **iv** $\frac{4}{9}$

b What type of decimal do you get?

c Use the pattern from part **a** to write these as decimals.

i $\frac{5}{9}$ **ii** $\frac{6}{9}$

iii $\frac{7}{9}$ **iv** $\frac{8}{9}$

14 **a** Use a calculator to convert each fraction to a decimal.
Show at least eight decimal places.

i $\frac{1}{9}$ **ii** $\frac{1}{99}$ **iii** $\frac{1}{999}$ **iv** $\frac{1}{9999}$

b Write each recurring decimal in abbreviated form with dots or a horizontal line.

c Use the pattern from part **a** to write these as recurring decimals without using a calculator.

i $\frac{1}{99\,999}$ **ii** $\frac{1}{999\,999}$ **iii** $\frac{1}{9\,999\,999}$ **iv** $\frac{1}{99\,999\,999}$

15 **a** Write $\frac{1}{3}$ and $\frac{2}{3}$ as decimals, showing at least the first five decimal places.

b Add the two decimals you found in part **a**.

c Calculate $\frac{1}{3} + \frac{2}{3}$ and compare your answer to part **b**.

16 An irrational number that is commonly used is **pi** (pronounced 'pie'). It has the symbol π. This number has intrigued mathematicians for thousands of years and is related to measurements on a circle. (You will discover more about this in Chapter 8.)
Since it is a decimal with an endless number of decimal places and no obvious repeating pattern, an estimate or approximation of its value is used in calculations.

NOTE Mathematicians started using the Greek letter π as the symbol for pi from 1706.

a Find the symbol π on your calculator. Press the appropriate keys to obtain a decimal number for pi. Write the number shown on your calculator screen.

b Write an approximation for pi to the following number of decimal places.

i two **ii** four **iii** five **iv** six

17 Another irrational number is $\sqrt{2}$.

a Write the number shown on your calculator screen for $\sqrt{2}$.

b Write an approximation for $\sqrt{2}$ to the following number of decimal places.

i two **ii** four
iii six **iv** seven

c Write the number shown on your calculator screen for $\sqrt{3}$.

d Is $\sqrt{3}$ an irrational number?

e Write an approximation for $\sqrt{3}$ to the following number of decimal places.

i two **ii** four
iii five **iv** six



CHALLENGE

18 Before the use of calculators and computers, many mathematicians used a fraction as an estimate for the value of pi. Some examples are shown in the table that follows.

a Copy this table.

Place, person and era of origin	Fraction estimate	Decimal approximation (to six decimal places)
Ancient Egypt (1700 BCE)	$\frac{256}{81}$	
Ancient Greece Archimedes (287–212 BCE)	$\frac{223}{71}$	
	$\frac{22}{7}$	
China Zu Chongzhi (429–500 CE)	$\frac{355}{113}$	
India Bhaskara II (1114–1185 CE)	$\frac{3927}{1250}$	
Italy Leonardo of Pisa (1180–1250 CE)	$\frac{12\,960}{4126}$	
	$\frac{7200}{2291}$	



Archimedes thought that pi was between $\frac{223}{71}$ and $\frac{22}{7}$.

NOTE Some mathematicians thought that the value of pi was between two fractions.

b Use a calculator to complete the third column of the table.

c Which value is closest to the approximation of pi you wrote for question **16b** part **iv**?

19 Since the 1940s, computers have been used to calculate an approximation for pi to an increasing number of decimal places (from 2000 to over 1 700 000 000 000 decimal places). Here is an approximation for pi written to 120 decimal places.

3.141 592 653 589 793 238 462 643 383 279 502 884 197 169 399 375 105 820 974 944 592 307 816 406 286 208 998 628 034 825 342 117 067 982 148 086 513 282 306 647

a Copy and complete this table to show how many times each digit appears in the approximation for pi shown.

Digit	0	1	2	3	4	5	6	7	8	9
Number of times it appears										

b Which digit appears:

i the most? **ii** the least?

c Does there appear to be any pattern in the digits?

d Comment on your answers.

20 Can you use a computer to write the *exact* value of pi as a fraction or a decimal? Explain.

Reflect

How can you tell the difference between terminating, non-terminating and recurring decimals?

1H Powers and roots

Start thinking!

1 3^5 is written in **index form**.

a What is the **power** or **index**? **b** What is the **base**?

c How is this number read?

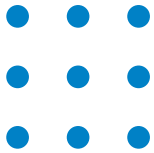
d Write the number in **expanded form** as a repeated multiplication.

e Perform the repeated multiplication to obtain the **basic numeral**.

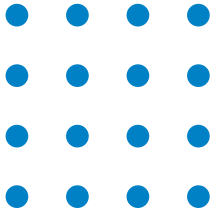
2 The number 3^2 can be read as '3 to the power of 2', 'the **square** of 3' or '3 squared'.

a Use the diagram on the right to explain why the last two expressions are used.

b What is the square of 3?



3 What number, when squared, gives 16? Use the diagram on the right to help you find the answer.



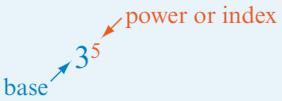
4 Another way of asking 'What number, when squared, gives 16?' is 'Find the **square root** of 16' or $\sqrt{16}$. Copy and complete: $\sqrt{16}$ = ____ as ____^{2} = 16.

5 Use your answer to question **2** to write the value of $\sqrt{9}$.

6 Explain how finding the square of a number and finding the square root of a number are related.

KEY IDEAS

- A repeated multiplication can be written in index form (or index notation), where the base is the number to be repeatedly multiplied and the power or index indicates the number of times the base is written.



- Numbers in index form can be written in expanded form.
  The result of the repeated multiplication produces the basic numeral.

  $3^5 = 3 \times 3 \times 3 \times 3 \times 3 = 243$

  index form — expanded form — basic numeral

NOTE The plural of the word index is **indices**.

- The square of a number is the result of multiplying a number by itself.
  For example, $3^2 = 3 \times 3 = 9$.
- The square root of a number is the number that, when squared, results in the original number.
  For example, $\sqrt{9} = 3$ (as $3^2 = 9$).
- The **cube** of a number is the result of raising a number to the power of 3.
  For example, $2^3 = 2 \times 2 \times 2 = 8$.
- The **cube root** of a number is the number that, when cubed, results in the original number.
  For example, $\sqrt[3]{8} = 2$ (as $2^3 = 8$).

EXERCISE 1H Powers and roots

1 Copy and complete this table.

	Index form	Base	Index or power	Expanded form	Basic numeral
a	3^4	3	4	$3 \times 3 \times 3 \times 3$	
b	2^3	2			8
c			2	8×8	
d				$6 \times 6 \times 6$	

EXAMPLE 1H-1 Calculating the value of a number in index form

Write 5^3 in expanded form and calculate its value.

THINK

Identify the base (5) and the index (3). This means that 5 is written 3 times. Multiply to calculate the basic numeral.

WRITE

$5^3 = 5 \times 5 \times 5$
$= 25 \times 5$
$= 125$

2 Write each number in expanded form and calculate its value.

a 2^5 **b** 4^3 **c** 1^9 **d** 3^3
e 10^7 **f** 2^8 **g** 3^6 **h** 5^4
i 7^3 **j** 6^4 **k** 8^5 **l** 9^4

EXAMPLE 1H-2 Finding the square

Find:

a the square of 9 **b** 6^2.

THINK

a To find the square of a number, multiply the number by itself.

b Change from index form to a repeated multiplication and calculate the result.

WRITE

a $9^2 = 9 \times 9$
$= 81$
The square of 9 is 81.

b $6^2 = 6 \times 6$
$= 36$

3 Find:

a the square of 5	**b** the square of 2	**c** the square of 1	
d 7 squared	**e** 10 squared	**f** 25 squared	
g 8^2	**h** 11^2	**i** 20^2	**j** 33^2
k 17^2	**l** 60^2	**m** 100^2	**n** 500^2

EXAMPLE 1H-3 Finding the square root

Find:

a the square root of 49 **b** $\sqrt{81}$.

THINK

a Think of a number that, when squared, gives 49. (The square of 7 is 49.)

b Think of a number that, when squared, gives 81. ($9^2 = 81$)

WRITE

a The square root of 49 is 7.

b $\sqrt{81} = 9$

UNDERSTANDING AND FLUENCY

4 Find these values.

a the square root of 16	**b** the square root of 9		
c the square root of 25	**d** the square root of 64		
e the square root of 1	**f** the square root of 121		
g $\sqrt{4}$	**h** $\sqrt{100}$	**i** $\sqrt{36}$	**j** $\sqrt{225}$
k $\sqrt{144}$	**l** $\sqrt{400}$	**m** $\sqrt{169}$	**n** $\sqrt{900}$

5 One student incorrectly wrote $4^2 = 8$. Why is 4^2 not the same as 4×2?

6 Another student incorrectly wrote $\sqrt{16} = 8$. Why is $\sqrt{16}$ not the same as $16 \div 2$?

7 Write each in expanded form and calculate its value.

a $2^3 \times 3^4$	**b** $7^2 \times 10^5$	**c** $1^9 \times 5^3$	**d** $4^3 \times 2^2$

8 Write each in expanded form and calculate its value.

a $(0.8)^2$	**b** $(3.4)^2$	**c** $(0.5)^3$	**d** $(0.3)^4$
e $(0.1)^5$	**f** $(0.2)^7$	**g** $\left(\frac{1}{4}\right)^3$	**h** $\left(\frac{7}{9}\right)^2$
i $\left(\frac{5}{6}\right)^3$	**j** $\left(\frac{3}{10}\right)^4$	**k** $\left(\frac{1}{2}\right)^5$	**l** $\left(3\frac{1}{3}\right)^6$

9 Calculate:

a $2^3 \times 7 + 8^2 \div 4$	**b** $5.8 - (1.2)^2 + (0.1)^4$
c $\frac{7}{10} + \left(\frac{2}{5}\right)^3 \times 6\frac{1}{4}$	**d** $\sqrt{81} - 5^2 \times 0.2 + 3^4$

NOTE Remember to use the correct order of operations.

10 With larger numbers, a calculator is useful. Calculate:

a 57^2 **b** 942^2 **c** 101^3

d 13^7 **e** 85^4 **f** 26^5

g $(6.85)^3$ **h** $(1.9)^4$ **i** $(12.4)^5$

j $\left(\frac{18}{23}\right)^3$ **k** $\left(\frac{7}{11}\right)^6$ **l** $\left(\frac{3}{5}\right)^9$

m $\sqrt{196}$ **n** $\sqrt{256}$ **o** $\sqrt{1089}$

p $\sqrt{26\,244}$ **q** $\sqrt{2.25}$ **r** $\sqrt{53.29}$

s $\sqrt{67.24}$ **t** $\sqrt{475.24}$ **u** $\sqrt{25.3009}$

11 **a** Calculate:

i $\sqrt{64}$ **ii** $\sqrt{5.76}$ **iii** $\sqrt{3}$.

b Explain how you can tell whether each result is rational or irrational.

12 For each of the following:

i find the value, using a calculator where necessary

ii decide whether the result is rational or irrational.

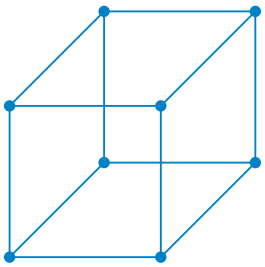
a $\sqrt{144}$ **b** $\sqrt{20}$ **c** $\sqrt{54.76}$

d $\sqrt{331\,776}$ **e** $\sqrt{8.3}$ **f** $\sqrt{0.49}$

g $\sqrt{\frac{1}{4}}$ **h** $\frac{\sqrt{1}}{\sqrt{4}}$ **i** $\frac{\sqrt{9}}{\sqrt{25}}$

j $\frac{\sqrt{81}}{\sqrt{100}}$ **k** $\frac{\sqrt{49}}{\sqrt{36}}$ **l** $\frac{\sqrt{3}}{\sqrt{4}}$

13 The number 2^3 can be read as '2 to the power of 3'.
It can also be read as 'the cube of 2' or '2 cubed'.



a Use this diagram to explain why the last two expressions can be used.

b What is the cube of 2?

c Find:

i 5 cubed **ii** 3 cubed **iii** the cube of 10 **iv** the cube of 7

14 **a** What number, when cubed, gives a result of 8?

b Another way of asking this is to find the cube root of 8 or $\sqrt[3]{8}$.
Copy and complete: $\sqrt[3]{8}$ = ___ as ___^{3} = 8.

c Use your answers to question **13** to write the value of:

i the cube root of 125 **ii** the cube root of 27

iii $\sqrt[3]{1000}$ **iv** $\sqrt[3]{343}$

d Explain how finding the cube of a number and finding the cube root of a number are related.

15 Without using a calculator, find:

a $\sqrt[3]{64}$ **b** $\sqrt[3]{1}$ **c** $\sqrt[3]{216}$

d $\sqrt[3]{8000}$ **e** $\sqrt[3]{729}$ **f** $\sqrt[3]{1331}$

16 **a** Find:

i 1^2 **ii** 11^2 **iii** 111^2 **iv** 1111^2

b Describe the pattern you can see.

c Without using a calculator, use the pattern to write the results for:

i $11\,111^2$ **ii** $111\,111^2$

iii $1\,111\,111^2$ **iv** $11\,111\,111^2$

17 For the six weeks of the school holidays, Sarah can earn $30 per week doing extra jobs around home.



a How much could Sarah earn over the holidays?

Instead, Sarah suggests her parents pay her $3 in the first week and then in each of the other weeks pay her three times as much as the previous week.

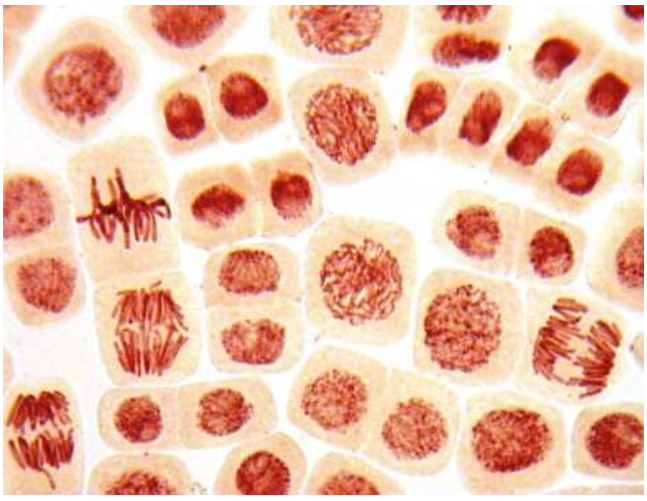
b How much would Sarah earn in each week with this new proposal?

c Compare the two possible amounts Sarah could earn over the school holidays. Which plan do you think Sarah's parents will agree to?

d Explain how you can use index notation to write the amount Sarah could earn in her eleventh week if her proposal was followed over a longer period of time.

e Use a calculator to work out the amount in part **d**.

18 A colony containing 1000 cells doubles every hour.



a How large is the colony after:

i one hour?

ii two hours?

iii three hours?

b Copy this table and complete the second row. (Hint: use your results from part **a** for the first few entries.)

Number of hours	1	2	3	4	5
Size of colony					
Number of times as large as original amount		4		16	

PROBLEM SOLVING AND REASONING

c How many times larger is the new colony compared to the original colony after:
i one hour? **ii** two hours? **iii** three hours?

d Complete the third row of the table. (Hint: use your answers from part **c** for the first few entries.)

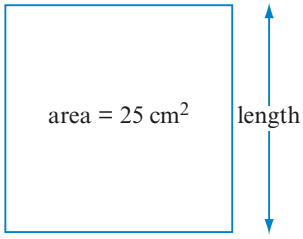
e Add a fourth row to your table and complete it by writing each number in the third row in index form with a base of 2.

f Written in index form, how many times larger is the new colony than the original colony after:
i 6 hours? **ii** 10 hours? **iii** 24 hours?

g Use a calculator to work out how many times larger the colony will be after 24 hours.

h Use your answer to part **g** to write the number of cells in the colony after 24 hours.

19 **a** If a square has area 25 cm^2, explain how you can work out its length.



b Calculate the length of a square with area:
i 169 cm^2
ii 200 m^2
iii 268.96 km^2.

c Which of your answers is irrational?

CHALLENGE

20 In a similar way to finding the square root and cube root of a number, you can find the fourth root, the fifth root and so on.

a If $5^4 = 625$, what is $\sqrt[4]{625}$?

b If $7^5 = 16\,807$, what is $\sqrt[5]{16\,807}$?

c Find each of these without using a calculator.
i $\sqrt[4]{16}$ **ii** $\sqrt[4]{81}$ **iii** $\sqrt[4]{10\,000}$
iv $\sqrt[5]{32}$ **v** $\sqrt[5]{1}$ **vi** $\sqrt[5]{243}$

21 **a** Use a calculator to find these values.
i $\sqrt[3]{3375}$ **ii** $\sqrt[3]{68.921}$ **iii** $\sqrt[3]{99}$
iv $\sqrt[4]{2401}$ **v** $\sqrt[4]{73}$ **vi** $\sqrt[4]{0.1}$
vii $\sqrt[5]{32\,768}$ **viii** $\sqrt[5]{0.000\,32}$ **ix** $\sqrt[5]{200}$
x $\sqrt[6]{729}$ **xi** $\sqrt[6]{496}$ **xii** $\sqrt[6]{15\,625}$

b Classify each result you found as rational or irrational.

22 Calculate $\sqrt{8} + \sqrt[3]{8}$. Is your answer rational or irrational?

Reflect

How are powers and roots related?

1I Index laws

Start thinking!

Index form	Basic numeral
2^1	2
2^2	4
2^3	8
2^4	
2^5	
2^6	
2^7	
2^8	
2^9	
2^{10}	

1 Copy and complete this table.

2 Use the table to answer these questions.

a Write the values for: **i** 2^2 **ii** 2^3 **iii** 2^5.

b Multiply your answers to part **a i** and **ii** to calculate $2^2 \times 2^3$.

c How does this compare to your result for 2^5?

d Write the values for: **i** 2^5 **ii** 2^4 **iii** 2^9.

e Multiply your answers to part **d i** and **ii** to calculate $2^5 \times 2^4$.

f How does this compare to your result for 2^9?

g Describe the pattern or shortcut you can see. This is one of the **index laws**.

h Provide three more examples to show this shortcut works for other values in the table.

3 Use the table to answer these questions.

a Write the values for: **i** 2^6 **ii** 2^4 **iii** 2^2.

b Divide your answers to part **a i** and **ii** to calculate $2^6 \div 2^4$.

c How does this compare to your result for 2^2?

d Write the values for: **i** 2^{10} **ii** 2^7 **iii** 2^3.

e Divide your answers to part **d i** and **ii** to calculate $2^{10} \div 2^7$.

f How does this compare to your result for 2^3?

g Describe the pattern or shortcut you can see. This is another index law.

h Provide three more examples to show this shortcut works for other values in the table.

4 What must be the same for these two index laws to work?
(Hint: do they work for calculations like $2^3 \times 3^4$ and $3^5 \div 2^3$?)

KEY IDEAS

- Index laws make performing some calculations with numbers in index form easier.
- The index laws are shown in these examples:
  - $2^5 \times 2^3 = 2^8$ write the base and add the indices (powers).
  - $2^5 \div 2^3 = 2^2$ write the base and subtract the indices (powers).
  - $(2^5)^3 = 2^{15}$ write the base and multiply the indices
  - $(2 \times 5)^3 = 2^3 \times 5^3$ write each factor of the product to the same power
  - $\left(\frac{2}{5}\right)^3 = \frac{2^3}{5^3}$ write the numerator and the denominator to the same power.
- A number in index form with a power of zero equals one. For example, $2^0 = 1$.
- To simplify a calculation involving indices, write the answer in index form.

NOTE The first two index laws do not work for numbers in index form with different bases.

EXERCISE 1I Index laws

UNDERSTANDING AND FLUENCY

1 Copy and complete these sentences using words from this list:
multiply, *divide*, *same*, *different*, *add*, *subtract*, *indices*, *powers*.

a When multiplying numbers in index form with the ___ base, write the base and ___ the indices (or ___).

b When dividing two numbers in index form with the ___ base, write the base and ___ the ___ (or powers).

2 Use an index law to copy and complete:

a $2^3 \times 2^5 = 2^{3+5} = 2^{-}$

b $2^4 \times 2^2 = 2^{-+2} = 2^6$

c $2^3 \times 2^4 = 2^{-+4} = 2^{-}$

d $2^5 \times 2^5 = 2^{-+-} = 2^{-}$

3 Use an index law to copy and complete:

a $2^5 \div 2^2 = 2^{5-2} = 2^{-}$

b $2^8 \div 2^3 = 2^{--3} = 2^5$

c $2^9 \div 2^5 = 2^{9--} = 2^{-}$

d $2^7 \div 2^4 = 2^{---} = 2^{-}$

4 Use the table on the opposite page to write the result (basic numeral) of each calculation in:

i question **2** **ii** question **3**.

EXAMPLE 1I-1 Multiplying two numbers in index form

Use an index law to simplify

a $7^5 \times 7^3$ **b** $9^4 \times 9$

THINK

a **1** Check the bases are the same. (yes)

2 Write the base and add the indices. (5 + 3 = 8.)

b **1** Check the bases are the same. (yes)

2 Write the base and add the indices. (4 + 1 = 5.)
Note that 9 is the same as 9^1.

WRITE

a $7^5 \times 7^3$
$= 7^8$

b $9^4 \times 9$
$= 9^5$

5 Use an index law to simplify:

a $3^3 \times 3^2$ **b** $4^5 \times 4^3$ **c** $6^2 \times 6^4$

d $5^3 \times 5^6$ **e** $8^2 \times 8^1$ **f** 2×2^4

6 Calculate the basic numeral for each result in question **5**.

EXAMPLE 1I-2 Dividing two numbers in index form

Use an index law to simplify $4^{11} \div 4^6$.

THINK	WRITE
1 Check the bases are the same. (yes)	$4^{11} \div 4^6$
2 Write the base and subtract the indices. $(11 - 6 = 5.)$	$= 4^5$

UNDERSTANDING AND FLUENCY

7 Use an index law to simplify these.

a $7^9 \div 7^6$ **b** $3^8 \div 3^3$ **c** $5^{10} \div 5^8$

d $9^{14} \div 9^{11}$ **e** $4^6 \div 4^5$ **f** $6^4 \div 6^1$

8 Calculate the basic numeral for each result in question **7**.

9 Use an index law to first simplify and then calculate these.

a $2^5 \times 2^6$ **b** $5^2 \times 5^4$ **c** $3^4 \times 3^3$

d $4^6 \times 4^2$ **e** $5^7 \div 5^4$ **f** $3^9 \div 3^3$

g $9^6 \div 9^3$ **h** $2^{12} \div 2^5$ **i** $6^3 \times 6^2$

j $8^5 \div 8^2$ **k** $10^4 \times 10^5$ **l** $7^8 \div 7^6$

10 Use an index law to simplify these. Leave your answer in index form.

a $2^5 \times 2^4 \times 2^3$ **b** $3^3 \times 3^1 \times 3^2$ **c** $10^3 \times 10^2 \times 10^5$

d $4^2 \times 4^4 \times 4^3$ **e** $5^7 \times 5^3 \times 5^6$ **f** $9^1 \times 9^5 \times 9^8$

11 Use an index law to simplify these.

a $(1.5)^3 \times (1.5)^6$ **b** $(0.7)^4 \times (0.7)^2$

c $\left(\frac{1}{2}\right)^3 \times \left(\frac{1}{2}\right)^4$ **d** $\left(\frac{7}{5}\right)^6 \times \left(\frac{7}{5}\right)^8$

e $(9.2)^8 \div (9.2)^5$ **f** $(0.63)^{11} \div (0.63)^4$

g $\left(\frac{3}{4}\right)^6 \div \left(\frac{3}{4}\right)^2$ **h** $\left(\frac{5}{2}\right)^9 \div \left(\frac{5}{2}\right)^7$

NOTE The bases do not need to be whole numbers for the index laws to apply. They just need to be the same.

12 **a** Use an index law to simplify $5^7 \div 5^3$.

b Another way of writing $5^7 \div 5^3$ is $\frac{5^7}{5^3}$.

i Copy and complete: $\frac{5^7}{5^3} = \frac{5 \times 5 \times 5 \times 5 \times 5 \times _ \times _}{5 \times _ \times 5}$.

ii Simplify by cancelling common factors in the numerator and denominator.

c Do you obtain the same result for parts **a** and **b**?

d Explain how an index law can be used to simplify calculations like $\frac{5^7}{5^3}$.

NOTE The **vinculum** in a fraction is equivalent to a division sign.

13 Use an index law to simplify these.

a $\frac{3^6}{3^4}$ **b** $\frac{8^5}{8^2}$ **c** $\frac{2^9}{2^3}$ **d** $\frac{4^7}{4^2}$ **e** $\frac{10^8}{10^5}$ **f** $\frac{6^{11}}{6^7}$

EXAMPLE 1I-3 Using more than one index law

Simplify:

a $\dfrac{3^7 \times 3^2}{3^5}$ **b** $\dfrac{7^4 \times 7^6}{7^5 \times 7^3}$

THINK

a **1** The vinculum groups together the terms in the numerator. Use an index law to multiply them. Write the base and add the indices. (7 + 2 = 9.)

2 Use another index law to divide the terms. Write the base and subtract the indices. (9 − 5 = 4.)

b **1** Multiply the terms in the numerator. Add the indices. (4 + 6 = 10.) Multiply the terms in the denominator. Add the indices. (5 + 3 = 8.)

2 Divide the terms. Subtract the indices. (10 − 8 = 2.)

WRITE

a $\dfrac{3^7 \times 3^2}{3^5}$
$= \dfrac{3^9}{3^5}$
$= 3^4$

a $\dfrac{7^4 \times 7^6}{7^5 \times 7^3}$
$= \dfrac{7^{10}}{7^8}$
$= 7^2$

UNDERSTANDING AND FLUENCY

14 Copy and complete:

a $\dfrac{2^6 \times 2^5}{2^3}$
$= \dfrac{2^{6+-}}{2^3}$
$= \dfrac{2^{-}}{2^3}$
$= 2^{--3}$
$= 2^{-}$

b $\dfrac{5^3 \times 5^7}{5^4}$
$= \dfrac{5^{-+-}}{5^4}$
$= \dfrac{5^{-}}{5^4}$
$= 5^{--4}$
$= 5^{-}$

c $\dfrac{3^7 \times 3^6}{3^5 \times 3^4}$
$= \dfrac{3^{7+-}}{3^{5+-}}$
$= \dfrac{3^{-}}{3^9}$
$= 3^{--9}$
$= 3^{-}$

d $\dfrac{9^8 \times 9^{11}}{9^9 \times 9^3}$
$= \dfrac{9^{-+-}}{9^{-+-}}$
$= \dfrac{9^{-}}{9^{12}}$
$= 9^{---}$
$= 9^{-}$

15 Simplify.

a $\dfrac{4^5 \times 4^3}{4^6}$ **b** $\dfrac{7^6 \times 7^6}{7^{10}}$ **c** $\dfrac{6^4 \times 6^7}{6^8}$ **d** $\dfrac{2^5 \times 2^9}{2^6}$

e $\dfrac{11^8 \times 11^3}{11^5}$ **f** $\dfrac{3^4 \times 3^5}{3^8}$ **g** $\dfrac{5^6 \times 5^{10}}{5^9 \times 5^3}$ **h** $\dfrac{8^7 \times 8^2}{8^1 \times 8^5}$

i $\dfrac{2^8 \times 2^6}{2^3 \times 2^4}$ **j** $\dfrac{9^7 \times 9^{12}}{9^6 \times 9^3}$ **k** $\dfrac{3^5 \times 3^4}{3^3 \times 3^2}$ **l** $\dfrac{4^{11} \times 4^6}{4^{12} \times 4^3}$

16 **a** **i** Copy and complete: $\dfrac{5^7}{5^7} = \dfrac{5 \times 5 \times 5 \times 5 \times 5 \times _ \times _}{5 \times _ \times 5 \times 5 \times _ \times 5 \times 5}$.

ii Simplify by cancelling common factors in the numerator and denominator.

b Use an index law to simplify $\dfrac{5^7}{5^7}$. What power is shown in the result?

c As the answers to parts **a** and **b** should be the same, what does 5^0 equal?

d Repeat parts **a** and **b** for $\dfrac{3^4}{3^4}$. What conclusion do you reach? Will this work for any base?

e Copy and complete: A number in index form with a power of ___ equals ___.

UNDERSTANDING AND FLUENCY

17 Find the value of these.

a 4^0 **b** 6×5^0 **c** $9^3 \div 9^3$

d $7^4 \times 7^0$ **e** $\frac{8^5}{8^5}$ **f** $(3.2)^0$

18 Use the index laws to simplify these.

a $\frac{3^5 \times 3^3}{3^8}$ **b** $\frac{14^8 \times 14^6}{14^{14}}$ **c** $\frac{9^7 \times 9^3}{9^9}$

d $\frac{5^2 \times 5^2 \times 5^4}{5^8}$ **e** $\frac{7^6 \times 7^4}{7^5 \times 7^5}$ **f** $\frac{11^3 \times 11^9}{11^4 \times 11^7}$

g $\frac{3^2 \times 3^2 \times 3^2}{3^3 \times 3^3}$ **h** $\frac{6^4 \times 6^5 \times 6^6}{6^3 \times 6^2}$ **i** $\frac{4^3 \times 4^2 \times 4^3}{4^5 \times 4^1 \times 4^2}$

19 a David and Natalie simplified $(5^2)^3$ in two different ways. However, they both used the power of three to write the contents of the brackets three times as a repeated multiplication.

Can you see another way that is quicker? Describe the shortcut you could use.

David

$(5^2)^3 = 5^2 \times 5^2 \times 5^2$

$= 5^{2+2+2}$

$= 5^6$

Natalie

$(5^2)^3 = (5 \times 5) \times (5 \times 5) \times (5 \times 5)$

$= 5^6$

b Try your shortcut to simplify $(2^3)^4$.

c Check that you obtain the same result using David and Natalie's working.

That is, simplify $2^3 \times 2^3 \times 2^3 \times 2^3$ and $(2 \times 2 \times 2) \times (2 \times 2 \times 2) \times (2 \times 2 \times 2) \times (2 \times 2 \times 2)$.

d This shortcut is another index law. Copy and complete: When raising a number in index form to a power, write the base and ___ the indices (or powers).

EXAMPLE 1I-4 Raising a number in index form to a power

Use an index law to simplify $(3^4)^5$.

THINK

Write the base and multiply the powers. ($4 \times 5 = 20$.)

WRITE

$(3^4)^5 = 3^{20}$

20 Use an index law to simplify these.

a $(6^4)^5$ **b** $(2^6)^2$ **c** $(7^4)^3$

d $(11^2)^4$ **e** $(8^5)^3$ **f** $(3^7)^2$

21 A mistake has been made in each of these.
Explain what the mistake is and write the calculation correctly.

a $5^8 \div 5^2 = 5^4$ **b** $2^3 \times 2^3 = 2^9$

c $(6^5)^2 = 6^7$ **d** $9^{12} \div 9^{12} = 9^1$

PROBLEM SOLVING AND REASONING

22 Find the missing powers in these calculations.

a $7^5 \times 7^- = 7^{15}$ **b** $(4^3)^- = 4^{18}$ **c** $\frac{3^- \times 3^2}{3^6} = 1$ **d** $\frac{6^7 \times 6^7}{6^-} = 6^2$

23 a Use the index laws to simplify:

i $2^5 \times 2^4$ **ii** $2^8 \div 2^2$ **iii** $\frac{2^7 \times 2^6}{2^9}$

iv $\frac{2^9 \times 2^5}{2^4 \times 2^3}$ **v** $(2^2)^5$ **vi** $\frac{2^{13} \times (2^4)^3 \times 2^5}{2^{22}}$

b Use the table of values on page 52 to write each result in part **a** as a basic numeral without using a calculator.

c Use the index laws to simplify these.

i $3^3 \times 3^3$ **ii** $3^7 \div 3^4$ **iii** $\frac{3^{11} \times 3^6}{3^8}$

iv $(3^4)^2$ **v** $\frac{(3^2)^4 \times 3^5}{3^4 \times 3^3}$ **vi** $\frac{3^5 \times 3^6 \times 3^7}{3^{11} \times 3^2}$

d Copy and complete this table.

e Use the table of values in part **d** to write each result in part **c** as a basic numeral without using a calculator.

Index form	Basic numeral
3^1	3
3^2	9
3^3	27
3^4	
3^5	
3^6	
3^7	
3^8	
3^9	
3^{10}	

24 a Use the tables of values above and on page 52 to write the numbers in each calculation in index form.
See the example shown at right.

i 32×16 **ii** $1024 \div 256$ **iii** $\frac{128 \times 32}{16}$

iv $\frac{512 \times 64}{128 \times 16}$ **v** $59\,049 \div 729$ **vi** 243×81

vii $\frac{27 \times 243}{2187}$ **viii** $\frac{2187 \times 729}{6561 \times 27}$ **ix** $\frac{128 \times 1024}{256 \times 64}$

Example

$\frac{64 \times 256}{512}$

$= \frac{2^6 \times 2^8}{2^9}$

b Use the index laws to simplify each calculation.

c Refer to the tables to write the results for part **b** as basic numerals.

d Without using a calculator, find the value of each of these.

i $512 \times 32 \div 64$ **ii** $9 \times 243 \div 27$ **iii** $\frac{32 \times 256}{8 \times 16}$ **iv** $\frac{6561 \times 19\,683}{729 \times 81}$

CHALLENGE

25 Two other index laws involve raising a product to a power and raising a quotient to a power.

a Show that $(2 \times 3)^5$ is the same as $2^5 \times 3^5$.

b Show that $\left(\frac{2}{3}\right)^5$ is the same as $\frac{2^5}{3^5}$.

c Write each of these in index form without brackets.

i $(5 \times 9)^3$ **ii** $\left(\frac{3}{4}\right)^6$ **iii** $\left(\frac{9}{5}\right)^4$

iv $(11 \times 4)^7$ **v** $\left(\frac{7}{10}\right)^3$ **vi** $(3 \times 8)^9$

26 Write a summary of the index laws in this topic and provide an example of each.

27 Write three calculations that simplify to 2^5.

Reflect

Why is it useful to know the index laws?

CHAPTER REVIEW

SUMMARISE

Create a summary of this chapter using the key terms below. You may like to write a paragraph, create a concept map or use technology to present your work.

rounding
leading digit
approximate
estimate
order of operations
equivalent fractions
HCF
simplest form
simplifying
LCD
decimal place
placeholder zero
trailing zero
round up
round down
terminating decimal
non-terminating decimal
recurring decimal
rational number
irrational number
index form
power
base
basic numeral
squaring
square root
cube root
index laws

MULTIPLE-CHOICE

1A **1** After rounding each number to its leading digit, what is the estimate for 526×68?

A 36 000 **B** 35 000
C 33 664 **D** 31 20

1B **2** What operation is performed first in $6 \times (12 - 5) + 8^2 \div 4$?

A $12 - 5$ **B** 8^2
C 6×12 **D** $64 \div 4$

1C **3** What missing value in $\frac{7}{9} = \frac{49}{\square}$ will produce an equivalent fraction?

A 51 **B** 63 **C** 81 **D** 441

1D **4** What is the lowest common denominator for $\frac{3}{8} - \frac{1}{6}$?

A 48 **B** 24 **C** 16 **D** 2

1D **5** What is $\frac{9}{10} \div 1\frac{1}{5}$?

A $\frac{9}{2}$ **B** $\frac{27}{25}$ **C** $\frac{3}{4}$ **D** $\frac{21}{10}$

1E **6** What is 3.125 84 rounded to three decimal places?

A 3.125 **B** 3.13
C 3.128 **D** 3.126

1F **7** How many decimal places are there in the answer to 26.018×15.4?

A 1 **B** 3 **C** 4 **D** 8

1G **8** Which number is an example of a recurring decimal?

A 32.85 **B** 6.666...
C 7.757 575 **D** 3.654 864 321...

1H **9** What is the square root of 36 equivalent to?

A $\sqrt{36}$ **B** $\sqrt{6}$ **C** 36^2 **D** 6^2

1I **10** Which of these is the simplest form of $3^4 \times 3^5$?

A 3^1 **B** 9^9 **C** 3^{20} **D** 3^9

SHORT ANSWER

1A **1** Round each number to its leading digit.
- **a** 236 **b** 67 145
- **c** 3890 **d** 149 046

1A **2** Estimate the result by first rounding each number to its leading digit.
- **a** 12 345 + 3648 **b** 94 501 − 32 566
- **c** 394 × 338 **d** 18 654 ÷ 425

1B **3** Calculate:
- **a** $33 + 40 \times 3 \div 10$
- **b** $75 - 4 \times 11 + 6^2 \div 4$
- **c** $10 \times 7 - 5 \times (4^2 - 4)$
- **d** $120 \div [9 + (4 \times 8 - 11)]$

1C **4** Write each fraction in its simplest form.
- **a** $\frac{35}{60}$ **b** $\frac{30}{16}$ **c** $3\frac{18}{32}$ **d** $5\frac{48}{72}$

1D **5** Calculate:
- **a** $\frac{3}{7} + \frac{2}{5}$ **b** $\frac{11}{15} - \frac{3}{10}$
- **c** $\frac{12}{25} \times \frac{5}{14}$ **d** $\frac{7}{18} \div \frac{7}{9}$

1D **6** Calculate:
- **a** $5\frac{2}{3} + \frac{4}{11}$ **b** $6\frac{2}{5} - 3\frac{4}{7}$
- **c** $6\frac{1}{4} \times 3\frac{2}{5}$ **d** $3\frac{1}{2} \div 1\frac{5}{9}$

1E **7** Write in ascending order.
- **a** 0.123, 0.120, 0.213, 0.0012, 0.231, 0.2
- **b** 8.916, 9.6, 8.16, 8.9, 9.16, 9.916

1E **8** Write in descending order.
- **a** 0.017, 0.7, 0.71, 0.0701, 0.717, 0.107
- **b** 2.127, 2.1, 2.721, 2.27, 2.72, 2.7

1E **9** Round each decimal to the number of decimal places shown in brackets.
- **a** 13.216 51 (2) **b** 0.001 215 (4)
- **c** 54.545 545 (3)
- **d** 181.182 154 136 (1)

1E **10** Calculate:
- **a** 23.56 + 45.98 + 74.13
- **b** 53.15 − 29.67
- **c** 123.5 + 46.12 + 9.0142 + 87.15
- **d** 1284.068 − 294.4687

1F **11** Calculate:
- **a** 6.3128 × 4 **b** 65.589 ÷ 6
- **c** 3.46 × 5.7 **d** 468.268 × 0.89

1G **12** **a** Decide whether each decimal is terminating or non-terminating.
- **i** 0.123 123 123
- **ii** 6.562 562...
- **iii** 5.5
- **iv** 3.645 841 235...

b Which are recurring decimals?

1G **13** Write each decimal in short form using dots or a horizontal line.
- **a** 0.333 333... **b** 5.385 385...
- **c** 45.685 757 575...
- **d** 0.092 451 451...

1G **14** Write each to show 12 decimal places.
- **a** $0.2\dot{3}$ **b** $3.\overline{27}$
- **c** $46.35\dot{8}\dot{7}$ **d** $17.36\overline{485}$

1H **15** Find the value of these.
- **a** 6^3 **b** 2^7
- **c** 8 squared **d** 11^2

1H **16** Find the value of these.
- **a** square root of 4 **b** $\sqrt{81}$
- **c** cube root of 27 **d** $\sqrt[3]{125}$

1H **17** **a** Use a calculator to find these.
- **i** $\sqrt{529}$ **ii** $\sqrt{700}$
- **iii** $\sqrt{156.25}$ **iv** $\sqrt[3]{42}$

b Classify each answer as rational or irrational.

1I **18** Use an index law to simplify these.
- **a** $3^5 \times 3^7$ **b** $2^7 \times 2^3$ **c** $6^8 \div 6^2$
- **d** $5^7 \div 5^7$ **e** $(9^5)^3$ **f** $4^6 \times 4^2 \times 4^3$

1I **19** Simplify these using index laws and then calculate the result.
- **a** $\frac{7^6 \times 7^5}{7^9}$ **b** $\frac{2^8 \times (2^4)^2 \times 2^3}{2^{14}}$

NAPLAN-STYLE PRACTICE

1 There are 26 845 people at a soccer match. What approximate number could be used to describe the crowd if you round to the leading digit?

2 There are approximately 20 000 000 cattle in Australia. Which of these could *not* be the exact number of cattle?

- ◯ 15 674 806
- ◯ 19 058 233
- ◯ 24 947 321
- ◯ 25 004 784

3 After rounding each number to its leading digit, what is the estimate for 33 457 ÷ 167?

150	200	300	400
◯	◯	◯	◯

4 What is $4 \times 15 - 5 \times (3^2 + 2)$?

5 What is the missing number in these equivalent fractions? $\frac{40}{72} = \frac{}{9}$

6 What is $\frac{36}{22}$ written in simplest form?

$\frac{11}{18}$	$\frac{9}{11}$	$1\frac{7}{11}$	$2\frac{4}{7}$
◯	◯	◯	◯

7 What is the lowest common denominator for $\frac{13}{15} + \frac{3}{5}$?

8 What is $3\frac{3}{5} \times 4\frac{4}{9}$?

$3\frac{1}{15}$	$\frac{4}{15}$	$12\frac{4}{15}$	16
◯	◯	◯	◯

9 What is $\frac{5}{21} \div 1\frac{11}{14}$?

$\frac{125}{294}$	$\frac{2}{15}$	$\frac{70}{121}$	$2\frac{1}{42}$
◯	◯	◯	◯

10 How many decimal places does 125.036 04 have?

11 Which list is in descending order?

- ◯ 9.209, 9.82, 8.902, 8.029, 8.29, 8.2
- ◯ 9.82, 9.209, 8.902, 8.29, 8.2, 8.029
- ◯ 9.209, 8.902, 8.029, 9.82, 8.29, 8.2
- ◯ 8.029, 8.2, 8.29, 8.902, 9.209, 9.82

12 What is 39.4951 correct to two decimal places?

- ◯ 39.50
- ◯ 39.40
- ◯ 39.410
- ◯ 39.49

13 What is 26.48 + 348.8 + 31.457?

- ◯ 405.513
- ◯ 406.017
- ◯ 406.737
- ◯ 91.593

Questions **14**–**16** refer to this information.

Chocolate bars cost $2.95. Maria has $30.

14 How many chocolate bars could Maria buy?

8	9	10	11
◯	◯	◯	◯

15 What is the cost of eight chocolate bars?

16 How much change would Maria receive if she bought eight chocolate bars?

17 What is 3.84 × 6.7?

- ◯ 49.92
- ◯ 25.728
- ◯ 4.992
- ◯ 257.28

18 Which is the best buy: a 2-L bottle of soft drink for $5.30 or a 1.25-L bottle for $3.25?

Questions **19**–**22** refer to these numbers:

$\frac{1}{4}$, 0.763 491..., $\frac{2}{3}$, $\frac{3}{5}$, $\frac{1}{8}$, 8.26, $\frac{9}{11}$.

19 Which can be written as terminating decimals?

20 Which can be written as non-terminating decimals?

21 Which can be expressed as recurring decimals?

22 Which are irrational?

23 What is 8 squared?

16 ☐ 2.8 ☐ $\sqrt{8}$ ☐ 64 ☐

24 What is the square root of 144?

☐

25 What is 6 cubed?

6 ☐ 18 ☐ 36 ☐ 216 ☐

26 What is the cube root of 64?

☐

27 If a square piece of paper has area 100 cm^2, how long is the piece of paper?

☐

28 Which of these does *not* simplify to 5^6?

☐ $\frac{5^9 \times 5^5}{5^8}$ ☐ $\frac{5^4 \times 5^8}{5^5 \times 5^7}$

☐ $(5^2)^3$ ☐ $5^2 \times 5^3 \times 5^1$

Questions **29** and **30** refer to this table.

4^2	4^3	4^4	4^5	4^6	4^7
16	64	256	1024	4096	16 384

29 Find 16 384 ÷ 256 without using a calculator.

☐

30 Find $\frac{1024 \times 64}{4096}$ without using a calculator.

☐

31 Which of these is an irrational number?

$\frac{1}{3}$ ☐ $\sqrt{9}$ ☐ $\sqrt{3}$ ☐ 3^7 ☐

ANALYSIS

1 The population figures (rounded to the nearest 100) for each state and territory of Australia at the end of December 2010 are displayed in this table.

State or territory	Population
New South Wales	7 272 200
Victoria	5 585 600
Queensland	4 548 700
Western Australia	2 317 100
South Australia	1 650 400
Tasmania	509 300
Australian Capital Territory	361 900
Northern Territory	229 900

a Determine the actual difference in population between NSW and Victoria.

b Round each population figure to the leading digit.

Use your answers to part **b** for these parts.

c Estimate the difference in population between NSW and Victoria. Compare this to part **a**.

d Estimate the population of Australia.

e Write these populations as fractions in simplest form.

i Victoria as a fraction of NSW

ii Tasmania as a fraction of Queensland

iii NSW as a fraction of WA

iv Queensland as a fraction of Victoria

f Write each answer to part **e** as a decimal number then round to three decimal places.

g Which of your answers to part **f** are:

i terminating decimals?

ii non-terminating decimals?

iii recurring decimals?

2 The population of a town is 10 000 and is expected to double every 5 years.

a The town fits within a square of area 9 km^2. How long is the square?

b Use indices to find the population after 30 years.

c It is predicted the town will fit within a square of area 85 km^2 after 30 years, how far is it along one side of this square?

d Classify each of the answers as rational or irrational. Explain your decision.

CONNECT

Owning a pet

Owning a pet is very rewarding, but owners have the responsibility of caring for the animal. Pet owners need to consider the type and quantity of food their pet needs, the amount of exercise, sleeping arrangements and health care. Your task is to investigate the costs involved.

Dog food prices

2.25 kg	$26.50
7.5 kg	$72.95
13.6 kg	$107.95
16 kg	$114.95



If you don't have a pet, you can refer to the information provided. Choose the type and size of pet you would like. (Even if you do have a pet, you may like to use this information as a starting point for your investigation.)

Here are some recommended food requirements for dogs and cats; however, these can vary depending on how active your pet is.

Weight of dog (kg)	Examples of dog breeds	Amount of dry food per day (g)
0–5	Chihuahua, Yorkshire terrier, Toy poodle	50–100
5–10	Miniature poodle, Scottish terrier	100–200
10–25	Cocker spaniel, Beagle	200–400
25–35	Collie, Boxer, Labrador, Golden Retriever	400–500
over 35	Great Dane, St Bernard	500–850

Your task

To investigate the costs of owning a pet, carry out these steps.

- Work out the appropriate amount and type of food for your pet
- Investigate the cost of feeding your pet
- Explain how you can decide which is the best buy when buying food
- Calculate the size of bedding (or cage or aquarium) required
- Investigate the amount of exercise your pet requires
- Calculate the average amount of exercise your pet has in a week
- Consider the costs in health care for your pet, including preventative measures (such as worm, tick and flea control)
- Estimate the yearly cost of caring for a pet.

Include all necessary working to justify your answers and identify whether you have obtained exact or approximate results.

Weight of cat (kg)	Amount of dry food per day (g)
2	30–45
3	40–60
4	50–70
5	60–85
6	70–100
7	80–100

Dry cat-food prices

1.5 kg	$22.95
6 kg	$64.95
9 kg	$84.95



The number of fleas on a pet can double in about one week!





Complete the 1 CONNECT worksheet to show all your working and answers to this task.

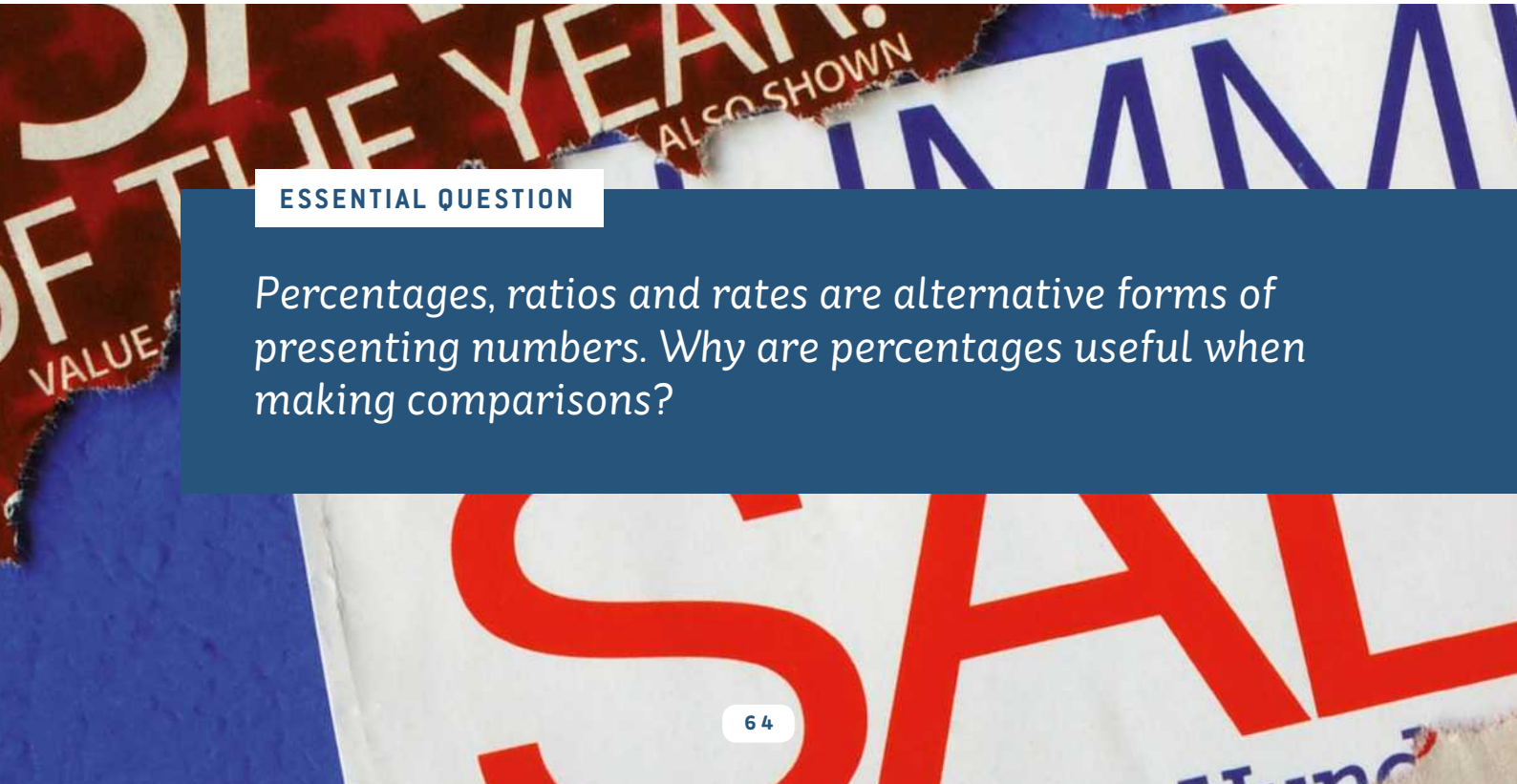
You may like to present your findings as a report. Your report could be in the form of:

- a guide to owning a pet
- a slideshow or video
- a short presentation
- other (check with your teacher).



2 PERCENTAGES, RATIOS AND RATES

2A Understanding percentages
2B Percentages, decimals and fractions
2C Percentage calculations
2D Financial calculations
2E Understanding ratios
2F Working with ratios
2G Dividing a quantity in a given ratio
2H Understanding rates

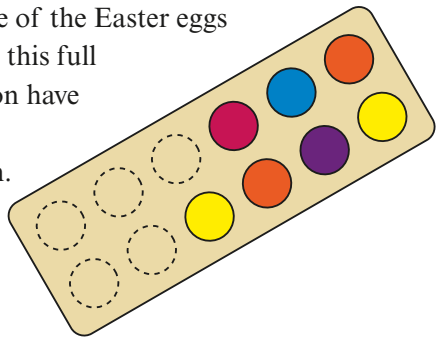


ESSENTIAL QUESTION

Percentages, ratios and rates are alternative forms of presenting numbers. Why are percentages useful when making comparisons?

Are you ready?

2A **1** Some of the Easter eggs from this full carton have been eaten.



a What fraction of the carton contains Easter eggs?

A $\frac{4}{12}$ **B** $\frac{7}{12}$ **C** $\frac{8}{12}$ **D** $\frac{12}{12}$

b What fraction of the Easter eggs are yellow?

2A **2** What is the highest common factor of 40 and 100?

A 2 **B** 4 **C** 10 **D** 20

2A **3** Which fraction is *not* in simplest form?

A $\frac{1}{5}$ **B** $\frac{6}{17}$ **C** $\frac{7}{100}$ **D** $\frac{20}{50}$

2A **4** Write $\frac{45}{100}$ in simplest form.

2A **5** By what number do you need to multiply both the numerator and denominator of $\frac{7}{25}$ to write it as an equivalent fraction with a denominator of 100?

A 4 **B** 5 **C** 25 **D** 100

2A **6** What is the equivalent fraction with a denominator of 100 for each of these?

A $\frac{7}{50}$ **B** $\frac{21}{25}$ **C** $\frac{3}{15}$

2B **7** Calculate:

a 6584×10

b 12.63×100

c 0.0068×1000

d $4870 \div 10$

e $31.72 \div 100$

f $92\,684.8 \div 1000$

2B **8** What is:

a 28.732 178 rounded to four decimal places?

b 518.8394 rounded to one decimal place?

c \$21.08 rounded to the nearest five cents?

d \$1674.37 rounded to the nearest dollar?

2B **9** **a** What is $\frac{3}{4}$ written as a decimal?

A 0.34 **B** 0.43 **C** 0.75 **D** 1.3

b Write $\frac{5}{8}$ as a decimal.

2C **10** What is $\frac{48}{50} \times \frac{100}{1}$?

A 96 **B** 24 **C** $\frac{48}{5}$ **D** $\frac{48}{5000}$

2E **11** Write each quantity in the required unit.

a 28 cm in mm

b 3 kg in g

c 6.5 hours in minutes

d 7.2 L in mL

2E **12** Look at the diagram for question **1**.

a Describe the number of Easter eggs that have been eaten to the number of remaining eggs.

A 5 to 7 **B** 4 to 8
C 12 to 7 **D** 7 to 5

b Describe the number of remaining Easter eggs to the total number of eggs.

A 12 to 7 **B** 7 to 12
C 12 to 5 **D** 7 to 5

2H **13** **a** Which unit relates to distance?

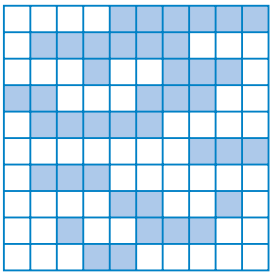
A litres **B** dollars
C kilometre **D** hours

b Which unit does *not* relate to time?

A seconds **B** days
C minutes **D** grams

2A Understanding percentages

Start thinking!



A small theatre can seat 100 people. This grid represents the theatre's seating plan. The shaded squares show the seats that are reserved for Thursday night.

1 Count the number of shaded squares in the grid and then copy and complete this sentence.

On Thursday night, _____ out of the 100 squares have been shaded.

The quantity you found in question **1** is a **percentage**. The term 'per cent' means 'out of 100' and the symbol for percentage is %. Therefore 29% means '29 out of 100'. This can also be written as a fraction with a denominator of 100; that is, $\frac{29}{100}$.

2 What percentage of the grid has been shaded?

3 **a** How many squares have *not* been shaded?

b What percentage of the grid has *not* been shaded?

c Add the percentage of the grid shaded to the percentage of the grid not shaded. What do these add to?

4 Using the results from question **3**, explain what the shaded and unshaded sections of any shape will add to.

5 This grid has a total of 100 parts, which makes it easy to write a fraction and a percentage. What can you do to work in percentages if something does not have 100 parts?

KEY IDEAS

- A percentage represents how part of an amount is related to the total amount. The total of an amount is equal to 100%.
- The term 'per cent' means 'for every one hundred' or 'out of 100'.
- A percentage is a number out of 100.
- A percentage can be written as a fraction with a denominator of 100.
  For example, $27\% = \frac{27}{100}$.
- If a fraction does not have a denominator of 100, find an equivalent fraction that does.
  For example, $\frac{7}{25} = \frac{7}{25} \times \frac{4}{4} = \frac{28}{100} = 28\%$.

EXERCISE 2A Understanding percentages

1 Write each fraction as a percentage.

a $\frac{17}{100}$ **b** $\frac{42}{100}$ **c** $\frac{91}{100}$

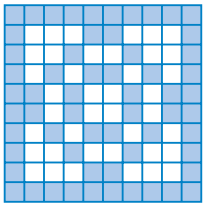
d $\frac{36}{100}$ **e** $\frac{67}{100}$ **f** $\frac{3}{100}$

g $\frac{15}{100}$ **h** $\frac{49}{100}$ **i** $\frac{52}{100}$

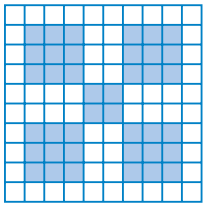
j $\frac{81}{100}$ **k** $\frac{9}{100}$ **l** $\frac{125}{100}$

2 What percentage of each square has been shaded?

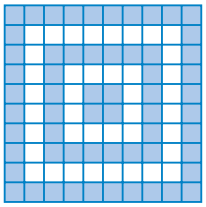
a



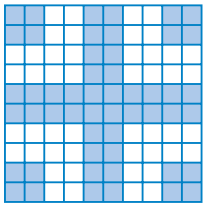
b



c



d



3 Write the percentage of each square in question **2** that is *not* shaded.

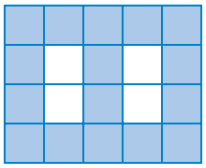
4 For each figure:

i write how many sections are shaded as a fraction of the total number of sections

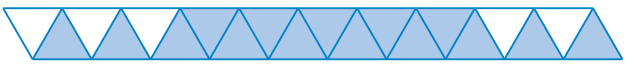
ii write this fraction as an equivalent fraction with a denominator of 100

iii write the percentage of sections that are shaded.

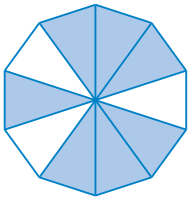
a



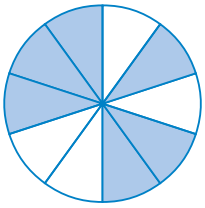
b



c



d

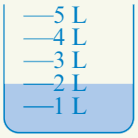


EXAMPLE 2A-1 Calculating percentages

A 5-L can has water poured into it. Calculate the percentage of the can:

a that has been filled

b that is empty.



THINK

a **1** Write the filled proportion of the can as a fraction.

2 Find an equivalent fraction that has a denominator of 100. ($5 \times 20 = 100$.)

3 Write the fraction as a percentage.

b **1** The percentage of a whole amount is 100%, so subtract the percentage that is filled from 100%.

2 Write your answer.

WRITE

a 2 out of 5 L have been filled.

fraction filled $= \frac{2}{5}$

$= \frac{2}{5} \times \frac{20}{20}$

$= \frac{40}{100}$

40% of the can has been filled.

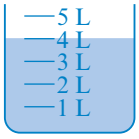
b $100\% - 40\% = 60\%$

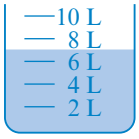
60% of the can is empty.

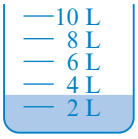
UNDERSTANDING AND FLUENCY

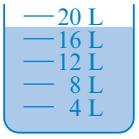
5 For these cans, calculate the percentage of the can:

i that has been filled **ii** that is empty.

a 

b 

c 

d 

6 Find the percentage of balloons that are:

a yellow b not yellow

c blue d not green

e yellow or green f neither blue nor pink.



7 From question **6**, what do you notice about the percentages found when looking at a group of 10 objects?

8 a Huynh has downloaded 76% of a song onto her iPod. What percentage remains to be downloaded?

b If a product is discounted by 23%, what percentage of the original price is to be paid?

c Richmond wins 17% of their games. What percentage do they lose?

d Forty-two per cent of homes were affected by floods. What percentage were not affected?

e Morgan received 93% on her Japanese test. What percentage of marks was lost?

UNDERSTANDING AND FLUENCY

f It is estimated that 54% of Australians do no formal exercise on a regular basis. What percentage of Australians exercise on a regular basis?

9 What percentage of its original value is a car worth if it depreciates (loses value) by these amounts?

a 20% **b** 19% **c** 8% **d** 12% **e** 28.5% **f** $33\frac{1}{3}$

10 A fraction with a denominator of 100 can be written as a percentage; for example, $\frac{29}{100} = 29\%$.

a Write each percentage as a fraction with a denominator of 100.

i 11% **ii** 47% **iii** 27% **iv** 73% **v** 51% **vi** 89%

b Explain how this process is both similar to, and different from, question **1**.

EXAMPLE 2A-2 Using an equivalent fraction to write a percentage

Write each fraction as a percentage by first finding an equivalent fraction that has a denominator of 100.

a $\frac{17}{25}$ **b** $\frac{27}{300}$

THINK

a **1** Find an equivalent fraction that has a denominator of 100.

2 Write the fraction as a percentage.

b **1** Find an equivalent fraction that has a denominator of 100.

2 Write the fraction as a percentage.

WRITE

a $\frac{17}{25} = \frac{17}{25} \times \frac{4}{4}$

$= \frac{68}{100}$

$= 68\%$

b $\frac{27}{300} = \frac{27^{9}}{\cancel{300}_{100}}$

$= \frac{9}{100}$

$= 9\%$

11 Write each fraction as a percentage by first finding an equivalent fraction that has a denominator of 100.

a $\frac{3}{50}$ **b** $\frac{11}{20}$ **c** $\frac{13}{25}$ **d** $\frac{1}{4}$ **e** $\frac{8}{10}$ **f** $\frac{4}{5}$

g $\frac{70}{500}$ **h** $\frac{15}{200}$ **i** $\frac{21}{300}$ **j** $\frac{16}{400}$ **k** $\frac{5}{8}$ **l** $\frac{19}{40}$

12 **a** Explain how you could represent each percentage using a grid of 100 squares.

i 80% **ii** 92% **iii** 25% **iv** 17% **v** 43% **vi** 66%

b Showing examples, explain how you could represent the percentages in part **a** using:

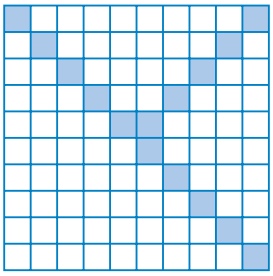
i a rectangle of length 10 cm and width of 2 cm

ii a grid of 200 squares.

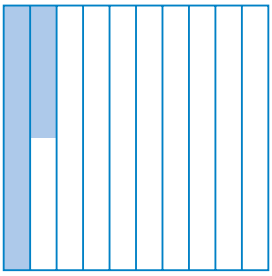
13 Thomas and Matthew were asked to draw a diagram representing 15%. Their answers are shown at right.

a Which diagram is correct? Explain your reasoning.

b Show another way of representing 15%.



Matthew



Thomas

14 Write a percentage to describe these situations by first finding an equivalent fraction that has a denominator of 100.

a Andrew ate three out of five cupcakes.

b Nine people out of twenty went home.

c Seventeen people out of twenty-five voted yes.

d Mark knew 46 people out of 200.

e Steffie obtained a score of 17 out of 20 for the quiz.

f The movie critic rated the movie one out of five stars.

g A dam with capacity 600 megalitres currently holds 144 megalitres.

h Seventy-five of the 250 people surveyed were happy with their mobile phone provider.

15 Remember that a whole amount can be written as 100%.

a What does 200% represent? What about 300%?

b Write 100% as a fraction with a denominator of 100 and hence write 100% as a single number.

c Use your answer to part **b** to write the numbers from 1 to 5 as percentages.

d When might you use a percentage that is more than the whole amount?

16 These containers each contain 50% of their total capacity.

Explain how each container holds a different volume of liquid, but they are the same percentage full.



17 Most mobile phones have an icon that tells you how much battery life is left.

a What percentage of battery life is left for the icons shown in parts **i**–**iii**?

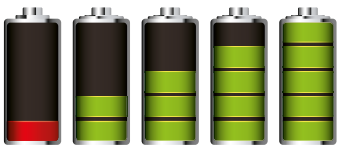
b What percentage of battery life has been used for each?

c Explain how the icon in part **iii** requires a different process compared to those in parts **i** and **ii**.

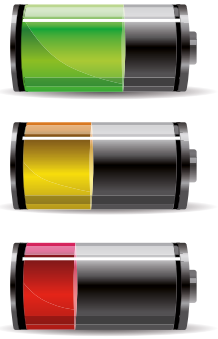
i



ii



iii



PROBLEM SOLVING AND REASONING

18 Percentages can be added and subtracted like other numbers.
Calculate:

a $17\% + 49\%$ **b** $23\% + 32\frac{1}{2}\%$

c $53\frac{1}{2}\% + 16\frac{1}{2}\% + 9\frac{2}{3}\%$ **d** $7.9\% + 28.7\% + 46.5\%$

e $92\% - 68\%$ **f** $16\frac{4}{5}\% - 12\frac{2}{5}\%$

g $37.4\% + 52.9\% - 15.6\%$ **h** $75.95\% + 5.28\% - 32.13\% - 28.66\%$



19 Seventy-seven per cent of reviewers loved *The Avengers* movie, 15% were undecided and the remainder disliked the movie. What percentage of reviewers disliked the movie?

20 At a rock concert, 8% of seats were classed Platinum, 17% Gold, 30% Silver and the remainder were General Admission. What percentage of seats were General Admission?

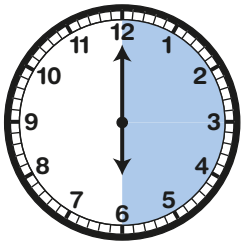
21 Draw this jar in your book and indicate the position described.

a 15% full **b** 80% full **c** 50% full

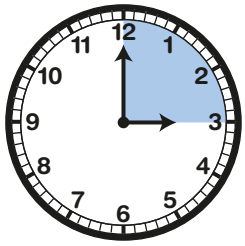
d 20% empty **e** 67% empty

22 Estimate and then calculate the percentage of each clock face that is shaded.

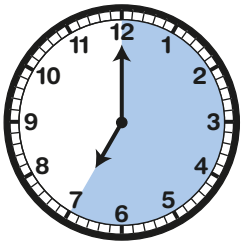
a



b

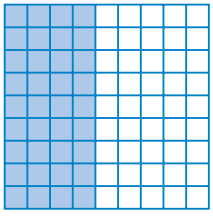


c



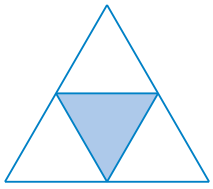
23 Romy has determined the percentage of the shaded area in each shape as follows.

a



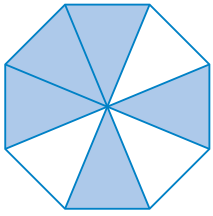
40%

b



25%

c



50%

Which of Romy's calculations are correct? Explain why.

24 Explain the steps you could use to write $\frac{3}{7}$ as a percentage.

CHALLENGE

25 At a party of 20 people, 11 are female. Over the next hour, 8 females and 12 males arrive. By 10 pm, 13 females and 11 males have gone home. What percentage of the party is male at 10 pm?

26 Write your own percentage problem and swap with a classmate.

Reflect

Why is it important to be able to visualise the fraction that a certain percentage represents?

2B Percentages, decimals and fractions

Start thinking!

Percentages, decimals and fractions are often written together and used interchangeably. But how do you compare them? Consider the three numbers 45%, $\frac{9}{20}$ and 0.45.

1 Write 45% as a fraction with a denominator of 100.

2 Write the equivalent fraction for $\frac{9}{20}$ with a denominator of 100.

3 Write 0.45 as a fraction with a denominator of 100.

4 What do you notice about the answers to questions **1**, **2** and **3**?

5 What relationship can you see between percentages, decimals and fractions? (Hint: how is the number 100 important?)



KEY IDEAS

- Percentages, decimals and fractions are all closely related.
- To convert a percentage to a fraction, the number becomes the numerator and the denominator is 100. Simplify the fraction if required by dividing the numerator and the denominator by the highest common factor (HCF).
- To convert a percentage to a decimal, a short cut is to divide by 100.
- To convert a decimal to a percentage, a short cut is to multiply by 100.
- To convert a fraction to a percentage, write the fraction with a denominator of 100 if possible. Otherwise, convert the fraction to a decimal and multiply it by 100.

EXERCISE 2B Percentages, decimals and fractions

1 Write each percentage as a fraction in simplest form.

a 17%	**b** 21%	**c** 35%	**d** 8%
e 15%	**f** 98%	**g** 74%	**h** 57%
i 53%	**j** 87%	**k** 11%	**l** 28%
m 62%	**n** 18%	**o** 48%	**p** 1%

EXAMPLE 2B-1 Writing a percentage as a decimal

Write each percentage as a decimal.

a 82% **b** 3.75%

THINK

a **1** Write 82% as a fraction to show what it means.

2 To write this fraction as a decimal, divide the numerator by the denominator. An easy way to divide by 100 is to 'move' the decimal point two places to the left.

3 Show a digit (0) before the decimal point.

b **1** Write 3.75% as a fraction to show what it means.

2 'Move' the decimal point two places to the left.

3 Insert a place-holding zero in the empty space (tenths column) and a digit (0) before the decimal point.

WRITE

a $82\% = \frac{82}{100}$

$= 82 \div 100$

$= .82$

$= 0.82$

b $3.75\% = \frac{3.75}{100}$

$= 3.75 \div 100$

$= .375$

$= .0375$

$= 0.0375$

UNDERSTANDING AND FLUENCY

2 Write each percentage as a decimal.

a 17%	**b** 21%	**c** 35%	**d** 8%
e 15%	**f** 98%	**g** 74%	**h** 57%
i 53%	**j** 87%	**k** 11%	**l** 28%
m 62%	**n** 18%	**o** 48%	**p** 1%

3 Write each percentage as a decimal.

a 6.78%	**b** 3.1%	**c** 162%	**d** 72.1%
e 347%	**f** 81.09%	**g** 10.57%	**h** 3%
i 472.38%	**j** 215.7%	**k** 0.1155%	**l** 0.04%
m 2.47%	**n** 12.01%	**o** 0.62%	**p** 0.8%

EXAMPLE 2B-2 Writing a decimal as a percentage

Write each decimal as a percentage.

a 0.47 **b** 0.285 **c** 3.4

THINK	WRITE
a 1 Multiply 0.47 by 100%. 'Move' the decimal point two places to the right.	**a** $0.47 = 0.47 \times 100\%$ $= 0.47$
2 Write the answer.	$= 47\%$
b 1 Multiply 0.285 by 100%. 'Move' the decimal point two places to the right.	**b** $0.285 = 0.285 \times 100\%$ $= 0.285$
2 Write the answer.	$= 28.5\%$
c 1 Multiply 3.4 by 100%. 'Move' the decimal point two places to the right and insert a place-holding zero in the empty space.	**c** $3.4 = 3.4 \times 100\%$ $= 3.40$
2 Write the answer.	$= 340\%$

UNDERSTANDING AND FLUENCY

4 Write each decimal as a percentage.

a 0.71	**b** 0.28	**c** 0.05	**d** 0.08
e 0.9	**f** 0.31	**g** 0.54	**h** 0.95
i 0.5	**j** 0.01	**k** 0.23	**l** 0.28
m 0.1	**n** 0.23	**o** 0.67	**p** 0.8

5 Write each decimal as a percentage.

a 0.369	**b** 0.248	**c** 10.06	**d** 0.81
e 7.057	**f** 1.859 12	**g** 12.1	**h** 2.7
i 0.9523	**j** 0.005	**k** 0.244 61	**l** 0.000 504
m 4.17	**n** 0.001 23	**o** 0.000 896	**p** 0.032

6 Write each fraction as a percentage by first converting to a fraction with a denominator of 100.

a $\frac{3}{5}$	**b** $\frac{7}{10}$	**c** $\frac{19}{20}$	**d** $\frac{23}{25}$
e $\frac{47}{100}$	**f** $\frac{3}{2}$	**g** $\frac{9}{10}$	**h** $\frac{7}{2}$
i $\frac{3}{4}$	**j** $\frac{8}{5}$	**k** $\frac{9}{50}$	**l** $\frac{11}{25}$
m $\frac{5}{4}$	**n** $\frac{13}{25}$	**o** $\frac{17}{20}$	**p** $\frac{3}{10}$

EXAMPLE 2B-3 Writing a fraction as a percentage

Write $\frac{7}{15}$ as a percentage.

THINK

1 Multiply $\frac{7}{15}$ by 100%.

2 Simplify by cancelling the highest common factor (HCF). Then multiply the numerators and denominators.

3 Write the improper fraction as a mixed number.

WRITE

a $\frac{7}{15} = \frac{7}{15} \times 100\%$

$= \frac{7}{\cancel{15}_{3}} \times \frac{\cancel{100}^{20}}{1}\%$

$= \frac{7}{3} \times \frac{20}{1}\%$

$= \frac{140}{3}\%$

$= 46\frac{2}{3}\%$

UNDERSTANDING AND FLUENCY

7 Write each fraction as a percentage.

a $\frac{5}{8}$ b $\frac{27}{40}$ c $\frac{7}{80}$ d $\frac{3}{16}$ e $\frac{11}{125}$ f $\frac{3}{8}$

g $\frac{11}{15}$ h $\frac{5}{7}$ i $\frac{7}{9}$ j $\frac{5}{18}$ k $\frac{30}{36}$ l $\frac{40}{45}$

8 Write each mixed number as a percentage. (Hint: change to an improper fraction first.)

a $2\frac{1}{4}$ b $6\frac{1}{2}$ c $7\frac{3}{10}$ d $3\frac{3}{4}$ e $4\frac{7}{25}$ f $5\frac{4}{5}$

g $5\frac{1}{3}$ h $3\frac{2}{9}$ i $6\frac{2}{3}$ j $1\frac{3}{7}$ k $2\frac{1}{6}$ l $3\frac{5}{6}$

9 Check your answers to questions **7** and **8** with a calculator.

10 Write each fraction as a percentage correct to two decimal places. (Hint: change to a decimal first.)

a $\frac{1}{3}$ b $\frac{7}{11}$ c $\frac{4}{9}$ d $\frac{6}{7}$ e $\frac{8}{15}$ f $\frac{11}{12}$ g $\frac{7}{13}$ h $\frac{5}{11}$

EXAMPLE 2B-4 Writing a fractional percentage as a decimal

Write $18\frac{7}{25}\%$ as a decimal.

THINK

1 Convert the fraction part to a decimal.

2 To convert from a percentage to a decimal, divide by 100.

3 Write the answer.

WRITE

$\frac{7}{25} = \frac{28}{100} = 0.28$

$18\frac{7}{25}\% = 18.28 \div 100$

$= 0.1828$

11 Write each percentage as a decimal.

a $17\frac{2}{5}\%$ b $5\frac{9}{10}\%$ c $21\frac{13}{25}\%$ d $62\frac{19}{20}\%$

e $268\frac{23}{50}\%$ f $92\frac{47}{50}\%$ g $38\frac{16}{25}\%$ h $14\frac{11}{20}\%$

EXAMPLE 2B-5 Writing a fractional percentage as a fraction

Write each percentage as a fraction in simplest form. **a** $\frac{1}{5}\%$ **b** $8\frac{2}{3}\%$

THINK

a **1** Divide $\frac{1}{5}\%$ by 100.

2 Write 100 as a fraction.

3 Replace ÷ with × and invert the fraction that follows.

4 Multiply the numerators and denominators.

b **1** Convert the mixed number to an improper fraction and then divide by 100.

2 Simplify by cancelling the highest common factor (HCF). Multiply the numerators and denominators.

WRITE

a $\frac{1}{5}\% = \frac{1}{5} \div 100$

$= \frac{1}{5} \div \frac{100}{1}$

$= \frac{1}{5} \times \frac{1}{100}$

$= \frac{1}{500}$

b $8\frac{2}{3}\% = \frac{26}{3} \div 100$

$= \frac{26}{3} \div \frac{100}{1}$

$= \frac{26}{3} \times \frac{1}{100}$

$= \frac{\cancel{26}^{13}}{3} \times \frac{1}{\cancel{100}_{50}}$

$= \frac{13}{3} \times \frac{1}{50}$

$= \frac{13}{150}$

UNDERSTANDING AND FLUENCY

12 Write each percentage as a fraction in simplest form.

a $\frac{1}{4}\%$ **b** $\frac{5}{8}\%$ **c** $\frac{9}{4}\%$ **d** $\frac{4}{7}\%$

e $\frac{12}{28}\%$ **f** $\frac{11}{3}\%$ **g** $\frac{9}{7}\%$ **h** $\frac{12}{13}\%$

i $8\frac{4}{5}\%$ **j** $15\frac{2}{3}\%$ **k** $3\frac{5}{9}\%$ **l** $13\frac{1}{6}\%$



13 A packet of gumballs is emptied onto a table.

a Write the number of blue gumballs as a fraction of the total.

b Convert this to a decimal and round to four decimal places.

c Convert this decimal to a percentage. What percentage of the gumballs are blue?

d What percentage of the gumballs (correct to two decimal places) is:

i orange? **ii** green? **iii** red or yellow? **iv** not red?

14 Percentages greater than 100% occur when you have more than the whole amount.

a If 200% is twice the original amount, what does 400% mean?

b To write 100% as a number, divide it by 100. For example, $100\% = \frac{100}{100} = 1$. Use this method to write each percentage as a number.

i 400% **ii** 500% **iii** 600% **iv** 250% **v** 375%

15 Many cereals contain more than the recommended daily intake (RDI) of sugar.

a One cereal contains 350% of the RDI of sugar. Write this as a decimal.

b Another cereal has 4.75 times the RDI of sugar. Write this as a percentage.

16 Marigo received a $5\frac{1}{4}\%$ pay rise. What fraction of her salary does this represent?

PROBLEM SOLVING AND REASONING

17 In the 1996 Olympics in Atlanta, Russian gymnast Alexei Nemov won four gold, two silver and six bronze medals.

a What percentage of his medals were bronze?

b What fraction of the medals he won were: **i** silver? **ii** gold?

18 In a piano exam, a student begins with a score of 100% and has marks taken away for mistakes. Georgio received a score of 88% on his piano exam.

a What fraction of marks were lost? Write in simplest form.

b What percentage of marks were lost?

19 Of the 128 tennis players competing in the Australian Open, only 16 will make the finals.

a What fraction of players will not make it through to the finals?

b What percentage will make it through to the finals?



20 On average, a boy will use 12 500 kilojoules of energy per day while a girl will use 10 000 kilojoules.

a Find the difference between these two values.

b Write the difference as a fraction of the boy's energy usage in simplest form.

c Write the difference as a fraction of the girl's energy usage in simplest form.

d Write the value from part **b** as:

i a percentage **ii** a decimal.

e Write the value from part **c** as:

i a percentage **ii** a decimal.

f Calculate the girl's energy usage as a percentage of the boy's energy usage.

g Calculate the boy's energy usage as a percentage of the girl's energy usage.

h Comment on the values obtained in parts **f** and **g**.

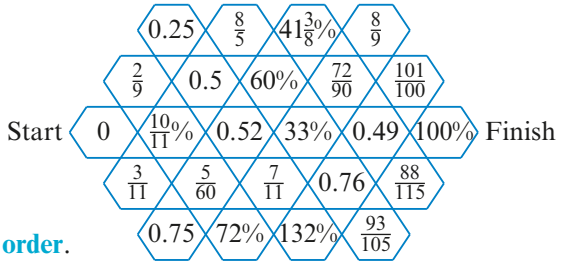
21 For each list, draw a number line and place each number in its correct position.

a $\frac{7}{5}$, 0.89, $\frac{3}{4}$, 200%, $\frac{4}{5}$, 1.3

b 4.9, $\frac{13}{2}$, 560%, 7.2, $4\frac{3}{10}$, $\frac{22}{4}$

c $\frac{16}{5}$, $\frac{3}{2}$, 340%, 0.28, $1\frac{4}{5}$, 65%

22 Starting at zero in this diagram, find the shortest path to 100%, moving in **ascending order**.



23 Sergio tried to explain to his sister Lucy that multiplying (or dividing) by 100% had the same effect as multiplying (or dividing) by 1; however, she was not convinced. Provide calculations supporting Sergio's statement.

CHALLENGE

24 Write three fractions between 0.15 and 16%.

25 Write three decimals between 25% and $\frac{13}{48}$.

26 Write three percentages between $\frac{76}{95}$ and 0.847.

Reflect

Why is it important to be able to convert between fractions, decimals and percentages?

2C Percentage calculations



Start thinking!

This table shows a list of girls trying out for the school basketball team and their efforts at shooting from the three-point line.

Name	Number of attempts	Number of successful shots
Bianca	8	5
Brittney	10	3
Claire	20	11
Evelyn	10	7
Fatima	3	2
Georgina	12	5
Huan	6	4
Joey	15	9
Penelope	11	8
Tamika	5	3

1. Why might the girl who had the most successful shots not be the best shooter?
2. Copy the table and add two columns to the right.
3. In the first of the new columns, write each girl's number of successful shots as a fraction of her number of attempts.
4. Convert each fraction to a percentage to complete the second of the new columns.
5. What do these percentages represent?
6. Which seven girls would you pick to fill the team?

KEY IDEAS

- To write one value as a percentage of another:
  1. write the values as a fraction
  2. convert the fraction to a percentage.
- To calculate the percentage of an amount:
  1. convert the percentage to a fraction
  2. replace 'of' with a multiplication sign
  3. write the amount as a fraction with a denominator of 1
  4. perform the calculation and write the final answer in the correct form.
- Include any units required with your answers to percentage calculations.

$$\text{percentage change} = \frac{\text{change in amount}}{\text{original amount}} \times 100\%$$

- A **percentage increase** occurs when a percentage amount is added to the original amount.
- A **percentage decrease** occurs when a percentage amount is subtracted from the original amount.
- To increase an amount by a percentage:
  1. add the percentage to 100% and rewrite this new value as a decimal number
  2. multiply the amount to be increased by the decimal number.
- To decrease an amount by a percentage:
  1. subtract the percentage from 100% and rewrite this new value as a decimal number
  2. multiply the amount to be decreased by the decimal number.

EXERCISE 2C Percentage calculations

EXAMPLE 2C-1 Writing one value as a percentage of another

Write 60 as a percentage of 150.

THINK

1 Write 60 as a fraction out of 150.

2 To convert to a percentage, multiply the fraction by 100%.

3 Simplify the fraction by dividing the numerator and denominator by the highest common factor (HCF).

4 Perform the division and write your answer.

WRITE

$\frac{60}{150}$

$= \frac{60}{150} \times 100\%$

$= \frac{60}{\cancel{150}_{3}} \times \frac{\cancel{100}^{2}}{1}\%$

$= \frac{\cancel{60}^{20}}{\cancel{3}_{1}} \times \frac{2}{1}\%$

$= 40\%$

1 Write these amounts.

a 20 as a percentage of 80

b 45 as a percentage of 50

c 19 as a percentage of 25

d 90 as a percentage of 120

e 300 as a percentage of 250

f 135 as a percentage of 108

EXAMPLE 2C-2 Writing one value as a percentage of another in fractional or decimal form

Write 99 as a percentage of 500 in:

a fractional form b decimal form.

THINK

a 1 Write 99 as a fraction out of 500 and multiply by 100%.

2 Simplify the fraction by dividing the numerator and denominator by 100 (the HCF).

3 Perform the division. Write the answer as a mixed number.

b 1 Write the improper fraction from step 2 of part a.

2 Perform the division and write your answer.

WRITE

a $\frac{99}{500} = \frac{99}{500} \times 100\%$

$= \frac{99}{\cancel{500}_{5}} \times \frac{\cancel{100}^{1}}{1}\%$

$= \frac{99}{5}\%$

$= 19\frac{4}{5}\%$

b $\frac{99}{5}\%$

$5\overline{)99.40}$

$\frac{99}{5}\% = 19.8\%$

UNDERSTANDING AND FLUENCY

2 Write each percentage in:

i fractional form **ii** decimal form.

a 140 as a percentage of 800
b 72 as a percentage of 36 000
c 46 as a percentage of 125
d 57 as a percentage of 200
e 58 as a percentage of 80
f 282 as a percentage of 48
g 6450 as a percentage of 80
h 25 as a percentage of 3

3 Write each of these as a percentage.

a 72 marks out of 90 marks
b 480 g out of 640 g
c 21 girls out of a class of 70
d 130 m out of 5200 m

4 Write each amount as a percentage of the total.

a $1288 of a $5600 loan is repaid
b 10.55 km of the 42.2 km marathon has been completed
c 42 of the 120 lots of land have been sold
d 63 likes have been received out of 90 views on YouTube
e six of 40 training sessions have been missed
f started late 22 days out of 50 school days in Term 3

EXAMPLE 2C-3 Writing one amount as a percentage of another after converting units

Write 90 cents as a percentage of $12.

THINK

1 Convert $12 to 1200 cents and write 90 as a fraction out of 1200.

2 To convert to a percentage, multiply the fraction by 100%.

3 Simplify the fraction by dividing the numerator and denominator by the HCF.

4 Perform the division and write the answer as a mixed number or in exact decimal form.

WRITE

$\frac{90}{1200}$

$= \frac{90}{1200} \times 100\%$

$= \frac{90}{\cancel{1200}_{12}} \times \frac{\cancel{100}^{1}}{1}\%$

$= \frac{\cancel{90}^{15}}{\cancel{12}_{2}} \times \frac{1}{1}\%$

$= \frac{15}{2}\%$

$= 7\frac{1}{2}\%$ or 7.5%

5 Write each of these as a percentage.

a 50 g out of 4 kg
b 48 minutes out of 8 hours
c 9 months out of 3 years
d 18 hours out of one day
e 35 cents out of $3.50
f 90 minutes out of 24 hours
g 60 mL out of 2 L
h 800 mm out of 3.2 m
i 600 kg out of 12 tonnes
j 1.2 m out of 60 cm

EXAMPLE 2C-4 Calculating a percentage of an amount

Calculate 7% of 220 m.

THINK

1 Convert the percentage to a fraction, replace 'of' with a × sign and write 220 as a fraction with denominator 1.

2 Simplify the fraction by dividing the numerator and denominator by the HCF.

3 Multiply the numerators together and the denominators together. Convert the fraction to a decimal and write the answer with the appropriate unit.

WRITE

7% of 220 m

$= \frac{7}{100} \times \frac{220}{1}$

$= \frac{7}{\cancel{100}_{5}} \times \frac{\cancel{220}^{11}}{1}$

$= \frac{7}{5} \times \frac{11}{1}$

$= \frac{77}{5}$

= 15.4 m

6 Calculate:

a 10% of 840 **b** 42% of 1800 **c** 6% of 150

d 18% of 6400 **e** 125% of 580 **f** 9% of 1250

7 Calculate:

a 63% of 720 m **b** 150% of 90 seconds **c** 35% of 660 cm

d 12% of 840 L **e** 40% of 190 buttons **f** 10.5% of 112 mL

EXAMPLE 2C-5 Increasing or decreasing an amount by a percentage

a Increase \$4500 by 40%. **b** Decrease 240 kg by 15%.

THINK

a 1 Add the percentage to 100% and rewrite this new value as a decimal number.

2 Multiply the amount to be increased by the decimal number. Include the appropriate unit.

b 1 Subtract the percentage from 100% and rewrite this new value as a decimal number.

2 Multiply the amount to be decreased by the decimal number. Include the appropriate unit.

WRITE

a new percentage
= 100% + 40%
= 140%
= 1.4

new amount = \$4500 × 1.4
= \$6300

b new percentage
= 100% − 15%
= 85%
= 0.85

new amount = 240 kg × 0.85
= 204 kg

UNDERSTANDING AND FLUENCY

8 Increase or decrease each amount by the given percentage. Where appropriate, round answers to two decimal places.

a Increase $3700 by 20%.
b Decrease 120 kg by 5%.
c Increase $980 by 5.2%.
d Decrease $850 by 7.5%.
e Increase $87.50 by 1.75%.
f Decrease 75 minutes by 10%
g Increase 5.2 km by 12.4%
h Decrease 516 L by 26%

9 Suehaila created the table below to help her friend Matina understand percentage increase and percentage decrease.

a Copy and complete this table. The first row has been done for you.
b How are the fourth and fifth columns related?
c Calculate:
i a 25% decrease on 200
ii a 20% increase on 150
iii a 7% decrease on 2500
iv a 40% increase on 70.

Percentage	Increase or decrease	Expanded form	Full percentage	Number form
4.5%	increase	100% + 4.5%	104.5%	1.045
7%	decrease	100% – 7%		
25%	decrease			
		100% + 40%	140%	1.40
		100% – 15%	85%	0.85
20%	increase			

10 Explain to a classmate a quick way to calculate percentage increase or decrease of a quantity. Are there any other ways to do this?

11 Calculate the new amount if:

a a score of 400 was increased by 15%
b the 2500 L of water in a tank was decreased by 30%
c a playlist of 60 songs was increased by 75%
d the waiting time of 15 minutes was decreased by 40%
e the crowd at a rugby match was 11 000 one week and increased by 5% the next week.

PROBLEM SOLVING AND REASONING

12 The tallest living horse is a Belgian draft horse named Big Jake, who is 210 cm tall. The smallest living horse is a dwarf miniature horse named Thumbelina who is 44.5 cm tall. As a comparison, an average horse is about 165 cm tall and an average Labrador dog is about 60 cm tall.



a Write Thumbelina's height as a fraction of Big Jake's in simplest form.
b Convert the answer from part **a** to a percentage rounded to two decimal places.
c Find the percentage of:
i Thumbelina's height compared to an average horse's height
ii Thumbelina's height compared to an average Labrador's height
iii Big Jake's height compared to an average horse's height.

PROBLEM SOLVING AND REASONING

The height of a horse (and dog) is measured at its withers (shoulders) because this height does not change when the animal moves, like the height of the head can.

d Estimate how tall these two horses would be measured to the top of their heads.

e How would this change the percentage you calculated in part **b**?

13 **a** What is 20% of 50?

b If a platter originally had 50 cupcakes and 20% were eaten, how many cupcakes would be left?

c By the end of the night, 20% of the remaining cupcakes were taken from the platter. How many cupcakes is this?

d Explain why the amounts obtained in parts **a** and **c** are different.

e Find how many cupcakes are still on the platter at the end of the night.



14 For each situation:

i find the amount of change that has occurred

ii state whether it is an increase or decrease

iii calculate the increase or decrease as a percentage of the original.

a A room with 25 people now has 31 people.

b A shelf that had 40 chocolate bars now has only 5 chocolate bars.

c Stacey moves house and instead of walking 1.5 km to school, she walks 2 km.

d Ahmed was 150 cm tall in Year 7 and is now 165 cm tall.

e A jogger improves her lap time from 20 minutes to 18 minutes.

15 Danni's power bill increased from $445 to $678. Write the increase as a percentage of the previous bill, correct to one decimal place.

16 A basketball team has won 15 of its 24 games and has yet to play 16 games. What is the most games the team can lose and still achieve 70% wins for the whole season?

17 Tom incorrectly measured the length of a table to be 43.6 cm when the actual length is 42.6 cm. Lisa measured the length of a bench to be 85.4 cm when the actual length is 86.4 cm

a What is the size of the error made by each student?

b Calculate the percentage error for each student by finding the size of the error as a percentage of the actual length. Who has the greater percentage error and why?

CHALLENGE

18 **a** 50% of a number is equal to 32. Find the original number.

b 20% of a number is equal to 19.6. Find the original number.

c 70% of a number is equal to 322. Find the original number.

19 **a** The sides of a square increase by 20%. What is the percentage change in area?

b The sides of a triangle increase by 20%. What is the percentage change in area?

c Make a statement about the results you obtained in parts **a** and **b**.

d Investigate another shape and then make a general statement about the percentage change in area when the dimensions of a shape are increased by 20%.

Reflect

What are some examples of situations that involve percentage calculations?

2D Financial calculations

Start thinking!



Rachel makes earrings as a hobby and decides to sell them on eBay. Before she can start, she needs to make some financial decisions.

A standard pair of earrings costs $8 to make. This is called the **cost price**. Rachel needs to decide how much she is going to sell them for. She obviously wants to make money, so she decides to add on 75% of the original cost to make the **selling price**.

1 What is 75% of $8?

2 Add your answer from question **1** to $8 to find the selling price of the earrings.

The amount added onto an original price is called a **mark-up**.
Rachel has added a 75% mark-up to her earrings.

3 A customer buys 10 pairs of earrings. How much money will this make for Rachel?

4 How much will it cost Rachel to make 10 pairs of earrings?

A **profit** occurs when the selling price is greater than the cost price.

5 Does Rachel make a profit, and if so, how much?

KEY IDEAS

- The amount added to the cost price (or original price or wholesale price) is called a mark-up.
- The difference between the regular price and a lower price is called a **discount**.
- A profit occurs when the selling price is greater than the cost price.
- A **loss** occurs when the selling price is less than the cost price.
- The percentage profit of a transaction can be calculated using the formula
  $\text{percentage profit} = \frac{\text{profit}}{\text{original cost}} \times 100\%$.
- The percentage loss of a transaction can be calculated using the formula
  $\text{percentage profit} = \frac{\text{loss}}{\text{original cost}} \times 100\%$.
- A **commission** is a payment to an employee; it is a percentage of value of the sales they make.

EXERCISE 2D Financial calculations

1 Calculate:

a 30% of $1200 **b** 25% of $5000 **c** 40% of $250
d 15% of $10 500 **e** 8% of $200 **f** 21% of $2745
g 13% of $850 **h** 150% of $299 **i** $5\frac{1}{2}$% of $1260
j $10\frac{1}{4}$% of $1260 **k** 20.75% of $199 **l** 450% of $820

EXAMPLE 2D-1 Calculating mark-ups and discounts

Calculate the selling price of:

a a $350 camera with 25% mark-up **b** a $299 pair of shoes discounted 30%.

THINK

a **1** Find 25% of $350.

2 Add this to the original price to find the new marked-up price.

b **1** Find 30% of $299.

2 Subtract this from the original price to find the new discounted price.

WRITE

a $25\% \text{ of } \$350 = 25\% \times 350$

$$= \frac{25}{100} \times \frac{350}{1}$$

$$= \frac{25}{\cancel{100}_{2}} \times \frac{\cancel{350}^{7}}{1}$$

$$= \frac{175}{2}$$

$$= \$87.50$$

$\text{new price} = \$350 + \$87.50$

$= \$437.50$

b $30\% \text{ of } \$299 = 30\% \times 299$

$$= \frac{30}{100} \times \frac{299}{1}$$

$$= \frac{\cancel{30}^{3}}{\cancel{100}_{10}} \times \frac{299}{1}$$

$$= \frac{3}{10} \times \frac{299}{1}$$

$$= \frac{897}{10}$$

$$= \$89.70$$

$\text{new price} = \$299 - \$89.70$

$= \$209.30$

2 Calculate the selling price of these.

a a 30% mark-up on $1200 **b** a 25% discount on $449
c a 40% discount on $2250 **d** a 75% mark-up on $149
e a 47% mark-up on $645 **f** a 38% discount on $750
g a 54% discount on $1200 **h** a 25% discount on $1050
i an 85% mark-up on $175 **j** a 15% mark-up on $399

UNDERSTANDING AND FLUENCY

3 Lakmali works at a clothing store that applies a 45% mark-up on all items. What is the selling price of:

a a pair of jeans with cost price $98?
b a t-shirt with cost price $25?
c a hoodie with cost price $65?
d a shirt with cost price $102?

4 Vesna works as a telemarketer and is paid a percentage of the sales she makes. This type of payment is called a commission. Calculate the amount (rounded to the nearest 5 cents) Vesna receives if she earns a 7% commission on these sales.

a $3680 **b** $6925 **c** $490 **d** $108
e $7390 **f** $621 **g** $10 452 **h** $720

5 A goods and service tax (GST) is added to many items you buy. This means 10% is added to the cost of most goods bought and services provided. For each item, calculate:

i the GST to be added
ii the price it will be sold for (round to the nearest five cents).

a Monopoly board game $64
b watch $54.95
c concert ticket $124.90
d CAS calculator $217
e facial $225
f iPad $874
g car $32 599
h earrings $199
i jeans $259
j smartphone $1245
k headphones $99
l plane tickets $248

EXAMPLE 2D-2 Calculating percentage profit or loss

A music system is sold for $1200 when it originally cost $1500.

a State if a profit or loss has been made.
b Calculate the profit or loss.
c Find the percentage profit or loss.

THINK

a Selling price less than wholesale price, so it's a loss.

b Find the difference between the two prices.

c **1** Write the loss amount as a percentage of the original amount.

2 Write your final answer.

WRITE

a A loss has been made.

b loss = $1500 − $1200
= $300

c percentage loss $= \frac{300}{1500} \times 100\%$
$= \frac{300}{1500} \times \frac{100}{1}\%$
$= \frac{300}{\cancel{1500}_{15}} \times \frac{\cancel{100}^{1}}{1}\%$
$= \frac{\cancel{300}^{20}}{\cancel{15}_{1}} \times \frac{1}{1}\%$
$= 20\%$

The music system sold for a 20% loss.

UNDERSTANDING AND FLUENCY

6 For each situation:

i state if a profit or loss has been made

ii calculate the profit or loss

iii find the percentage profit or loss correct to two decimal places.

a A chair is placed on eBay for $50 and sold for $87.

b A sandwich maker that was originally priced at $59 was sold for $45.

c A camera is bought for $935 and sold for $599.

d A car purchased for $28 899 was sold for $17 580.

e A supermarket buys milk at $0.80 per litre and sells it for $2.24 per litre.

f A toy bought at $8.75 was sold for $2.60.

g Scarves bought at $5.98 each were sold for $13.50 each.

7 Henrietta has been noting changes in prices over time. For each item listed:

i calculate the percentage change correct to two decimal places

ii state whether the percentage change is an increase or decrease.

a paper towels from $4.79 to $5.85

b ice cream from $7.49 to $5

c bread from $5.49 to $4.99

d breakfast cereal from $4.49 to $3.89

e crackers from $2.49 to $1.99

f pasta from $1.88 to $2.15

g bottled water from $2.99 to $3.15

h tuna from $0.99 to $1.36

EXAMPLE 2D-3 Calculating cost price

Calculate the cost price of:

a a mountain bike, if the selling price is $904 and the mark-up is 60%

b a notebook, if the selling price is $1125 and the discount is 25%.

THINK

a **1** Add the percentage mark-up to 100% and rewrite this new value as a decimal number.

2 Write a rule linking selling price and cost price.

3 Rearrange the rule to find the cost price (divide both sides by 1.60).

4 Write your answer with the appropriate unit.

b **1** Subtract the percentage discount from 100% and rewrite this new value as a decimal number.

2 Write a rule linking selling price and cost price.

3 Rearrange the rule to find the cost price (divide both sides by 0.75).

4 Write your answer with the appropriate unit.

WRITE

a 100% + 60% = 160%
= 1.60

1.60 × cost price = 904

cost price = 904 ÷ 1.60
= 565

cost price = $565

b 100% − 25% = 75%
= 0.75

0.75 × cost price = 1125

cost price = 1125 ÷ 0.75
= 1500

cost price = $1500

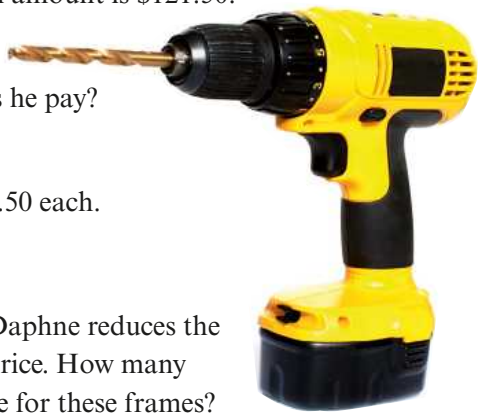


8 Calculate the cost price of:

- **a** a dress with selling price $1260 and mark-up 55%
- **b** a book with selling price $38 and discount 8%
- **c** a loaf of bread with selling price $3.98 and mark-up 60%
- **d** a pair of sunglasses with selling price $560 and discount 7.5%
- **e** a pair of bathers with selling price $88 and mark-up 48%
- **f** a pair of shorts with selling price $36 and discount 15%.

9 Calculate the cost price, if:

- **a** 20% of an amount is $160
- **b** 15% of an amount is $89.25
- **c** 39% of an amount is $6435
- **d** 6% of an amount is $121.50.



10 Tony makes a $60 profit on each power drill sold.

- **a** If Tony sells the drills for $420 each, what price does he pay?
- **b** Calculate the percentage profit Tony made.

11 Daphne purchased 40 picture frames for her store at $8.50 each. She plans to sell them for $12.50.

- **a** What is the expected profit on each frame?
- **b** Fifteen per cent of the frames are damaged and so Daphne reduces the selling price of these to 50% of the planned selling price. How many frames are damaged and what is the new selling price for these frames?
- **c** Does Daphne make a profit or loss on the damaged frames? Calculate how much.
- **d** If all 40 frames are sold, calculate the total profit she has made.
- **e** Find the percentage profit she has made.
- **f** Compare the value you got in part **e** with the percentage profit Daphne would have made if none of the frames were damaged.

12 Thomas and Ivo compared the increase in value of two properties to decide which made the greatest percentage profit. Thomas said the first property had increased most in value, but Ivo thought the second had. Who was correct? Show your workings.

Property 1: valued at $695 000, sold for $963 000.

Property 2: valued at $1 200 000, sold for $1 600 000.

13 Julian accidentally discounted a $699 food mixer by 20% instead of the advertised 30%. Once he realised his mistake he took a further 10% off the discounted price. The customer claimed that this would not be the same as reducing the original price of the mixer by 30%. Who is correct? Show workings to support your answer.

14 David was shown a shortcut for percentage increase and decrease. For example, to find a 15% increase, rather than finding 15% of an amount and adding it to the original amount, you can instead multiply by 1.15.

- **a** Explain to David why this works.
- **b** What would you multiply the original prices by to obtain new prices if there was:
  - **i** a 25% discount?
  - **ii** an increase of 17%?
  - **iii** a reduction of 32%?
  - **iv** a mark-up of 62%?
  - **v** a 200% mark-up?
  - **vi** a discount of 28%?

PROBLEM SOLVING AND REASONING

15 A grocery store applies a 30% mark-up on all its goods. A GST of 10% is then added to this marked-up price.

a Calculate the final price of each item once the mark-up and GST have been added. Where appropriate, round to the nearest five cents.

i toothpaste 160 g originally $3.88 **ii** tuna 95 g originally $1.24

iii cauliflower (half) originally $1.99 **iv** oil 4 L originally $17.99

v rice crackers 90 g originally $2.49 **vi** raisin toast 520 g originally $4.59

b Compare the original price with the final price of each item and calculate the actual percentage increase.

c Explain why increasing a price by 30% then 10% is different from increasing it by 40%.

16 Switched On Electrics is having a 'Scratch and dent' sale. Any damaged stock will be discounted by 35%. A refrigerator is scratched and discounted to $1820.

a What was the original price of the refrigerator?

b How much money is saved because of the scratch on the door?

17 A dishwasher has a dent and is discounted to $700. This price is 70% of its original price. What is the original price of the dishwasher?

18 During its stock-take sale a department store offers a $22\frac{1}{2}\%$ discount on everything. If a pair of designer shoes was discounted by $282.60, what was the original price?



19 Matthew works as a sales consultant and receives 9% commission on the sales he makes.

a If he makes sales totalling $8500, how much will he be paid?

b If he wishes to earn a minimum of $1440, what must his sales total?

20 Consider the two statements: 'All stock discounted by 45%' and 'Stock discounted by up to 45%'. Explain to a friend whether these statements mean the same thing.

CHALLENGE

21 Philip is a real estate agent and is paid a base wage of $780. He also receives a commission on the properties he sells. Philip receives 2.2% on the first $150 000 and $\frac{1}{4}\%$ on the rest.

a If he sells a property for $485 000, how much commission will he earn?

b Write the commission he receives as a percentage of his base pay to the nearest per cent.

22 **a** During the stocktake sales an LCD television is discounted by 15%. Explain what percentage mark-up is required for it to return to its original price.

b Repeat part **a** with a discount of 20% applied.

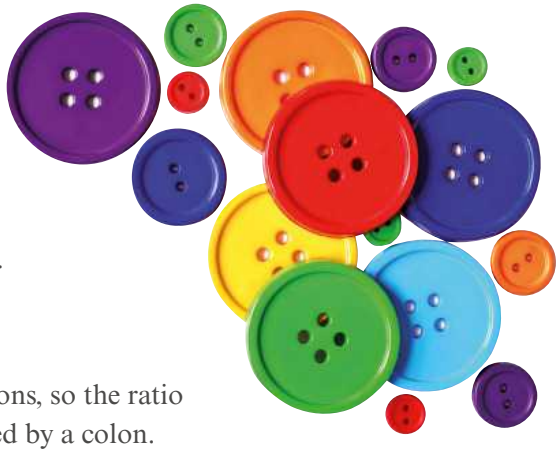
c What do you notice about the relationship between the percentage discount and the percentage mark-up?

Reflect

What are the benefits of using percentages when comparing profit and loss?

2E Understanding ratios

Start thinking!



Four button sizes: jumbo, medium, small and tiny.

Ratios compare two or more quantities of the same kind.
What can you compare in this photo?

1 Copy and complete:

a There are _____ red buttons compared to _____ yellow button.

b There are _____ purple buttons compared to _____ green buttons.

c There are _____ jumbo buttons compared to _____ tiny buttons.

2 Write four more comparisons using the photo provided.

In the photo, there are two orange buttons compared to three red buttons, so the ratio can be written in shorthand as 2:3. The numbers in a ratio are separated by a colon.

3 Rewrite the comparisons from questions **1** and **2** as ratios, using numbers and a colon.

These ratios compare one part to another part (for example, blue to orange). Ratios can also be written to represent part to whole (for example, blue to total number).

4 How do you usually write a comparison of a part out of a whole?

5 Write these comparisons as both a ratio and a fraction.

a number of orange buttons compared to the total number of buttons

b number of small buttons compared to the total number of buttons

c number of buttons with two holes compared to the total number of buttons

6 Why might it be easiest to always write a part-to-part comparison as a ratio and a part-to-whole comparison as a fraction?

KEY IDEAS

- A ratio is a comparison of two or more quantities of the same kind. For example, four parts water compared to one part cordial is written as 4:1.
- The quantities in a ratio must be in the same units.
- Ratios are written as whole numbers with no units shown. They do not contain fractions or decimals.
- A ratio must be written in the correct order. For example, two parts red paint to three parts white paint is written as 2:3.
- Often a part-to-part comparison is written as a ratio and a part-to-whole comparison as a fraction.

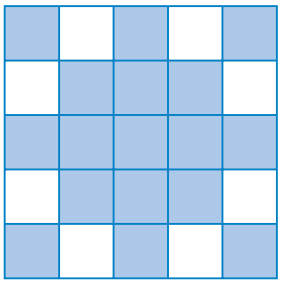
EXERCISE 2E Understanding ratios

UNDERSTANDING AND FLUENCY

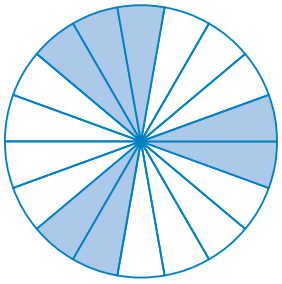
1 For each of these, write:

- **i** the number of shaded sections to the number of non-shaded sections as a ratio
- **ii** the number of non-shaded sections to total number of sections as a fraction.

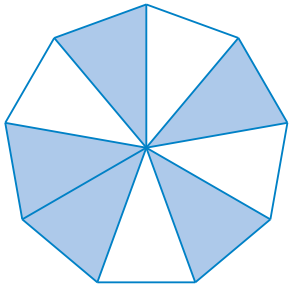
a



b



c



2 In question **1**, why is part **i** written as a ratio and part **ii** as a fraction?

3 Write each comparison as a ratio in the given order.

a 20 seconds to 33 seconds
b 95 kg to 57 kg
c 113 m to 167 m
d 256 mL to 175 mL
e $499 to $575
f 23 days to 2 days
g 4 months to 13 months
h 19 cans to 21 cans
i 42 km to 117 km
j 16 L to 39 L
k 19 wins to 11 losses
l 23 losses to 7 wins to 3 draws
m 15 minutes to 17 minutes to 19 minutes
n 5 red to 7 blue to 11 yellow

EXAMPLE 2E-1 Writing comparisons as ratios

Write the comparison 13 seconds to 3 minutes as a ratio.

THINK	WRITE
1 Ratios must have quantities in the same unit of measurement. Converting the larger unit to the smaller unit is usually simpler. Convert minutes into seconds.	13 seconds to 3 minutes 13 seconds to 180 seconds
2 Write the ratio in the order of the worded description. Do not include units.	13:180

4 Write each comparison as a ratio.

a 13 days to 3 weeks
b 2 hours to 17 minutes
c 27 cents to $5
d 4000 mL to 3 L
e 1.7 km to 793 m
f 7 months to 3 years
g 509 g to 1.3 kg
h 472 kg to 2.371 tonnes
i 61 mm to 7.9 cm
j 38 minutes to 2 hours 11 minutes
k $4.20 to 97 cents
l 0.6 L to 53 mL
m 0.05 km to 23 cm to 6.73 m
n 5631 g to 1.753 kg to 1 tonne

5 Write the number of pins in each comparison as a ratio.



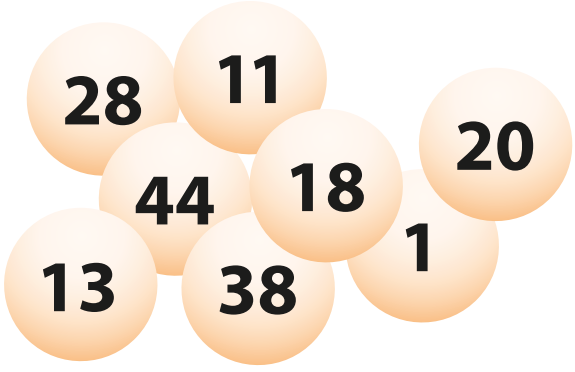
a blue pins compared to yellow pins
b white pins compared to green pins
c yellow pins compared to total number of pins
d red and green pins compared to blue pins

6 Which of the ratios in question **5** could also be written as a fraction? Explain.

7 Write each of these as a ratio.

a Rafael won 11 tournaments in a year compared to Andy, who won 7.
b Fedora kicked 89 goals last season while Sophie kicked 101 goals.
c Mark trains five times a week while Christian trains three times a week.
d Two-stroke petrol fuel is made by mixing 25 parts petrol to 1 part oil.
e A pancake mixture requires two cups of flour to three cups of milk.
f Concrete is made by mixing one part cement, two parts sand and three parts screenings.
g A hair dye solution is made up in the ratio of three parts dye to seven parts water.
h Sarah works 8-hour shifts on the weekend and 3-hour shifts on a weekday.

8 For the winning Lotto numbers shown, write each comparison as a ratio.



a odd numbers to even numbers
b prime numbers to composite numbers
c multiples of 3 to multiples of 2
d numbers less than 20 to numbers greater than 20

EXAMPLE 2E-2 Writing comparisons as ratios, fractions and percentages

A tray contains four pistachio (P), nine choc pudding (Ch), seven caramel (C) and five black forest (B) macarons.

a Write the ratio comparing the number of:
  i pistachio to caramel to black forest macarons
  ii choc pudding macarons to the total number of macarons.

b Write the number of:
  i caramel and black forest macarons as a fraction of the total number of macarons
  ii pistachio and choc pudding macarons as a percentage of the total number of macarons.

THINK	WRITE
a i 1 Write the ratio in the order given.	a i number of P to number of C to number of B macarons = 4:7:5
ii 1 Calculate the total number of macarons.	ii total number of macarons = 4 + 9 + 7 + 5 = 25
2 Write the ratio in the order given.	number of Ch to total number = 9:25
b i 1 Calculate the number of caramel and black forest macarons.	b i number of C and B macarons = 7 + 5 = 12
2 Write the ratio in the order given caramel and black forest : total.	number of C and B to total number = 12:25
3 Write the ratio as a fraction.	$= \frac{12}{25}$
ii 1 Calculate the number of pistachio and choc pudding macarons.	ii number of P and Ch macarons = 4 + 9 = 13
2 Write the ratio as a fraction.	number of P and Ch to total number = 13:25 $= \frac{13}{25}$
3 Convert the fraction to a percentage and simplify.	$= \frac{13}{25} \times 100\%$ $= \frac{13}{\cancel{25}_{1}} \times \frac{\cancel{100}^{4}}{1}\%$ = 52%

9 A showbag contains 11 Cherry Ripes, 9 Flakes, 7 Picnics, 5 Time Outs and 8 Turkish Delights.

a Write the ratio comparing the number of:
  i Cherry Ripes to Turkish Delights to Picnics
  ii Flakes to the total number of chocolate bars.

UNDERSTANDING AND FLUENCY

b Write the number of:

i Time Outs and Turkish Delights as a fraction of the total number of bars

ii Flakes and Picnics as a percentage of the total number of chocolate bars.

10 Samantha's win–loss ratio against Venus is 4:5. What does this mean?

11 Give an example that could illustrate each ratio.

a 5:2	**b** 2:7	**c** 3:1	**d** 4:9	**e** 17:25
f 3:8	**g** 1:4	**h** 8:7	**i** 1:1	**j** 12:19

12 Consider this group of animals.



a What is the ratio of the numbers of:

i dogs to cats?

ii mammals to birds?

iii rabbits to fish?

b What is the fraction of number of lizards to the total number of animals?

c Write the number of mammals and birds as a percentage (to the nearest per cent) of the total number of animals.

d Write the number of dogs and fish as a percentage (to the nearest per cent) of the total number of animals.

e Dane said that the ratio of dogs to cats was 1:2. Explain where Dane went wrong and what he should have written.

PROBLEM SOLVING AND REASONING

13 Eighty-seven per cent of an iceberg is submerged in water.

a What percentage of the iceberg is visible?

b Write as a ratio the percentage of the iceberg submerged to the percentage visible.

14 A manager finds that in a box of 50 nectarines, 18% have blemishes. Write the:

a ratio of the numbers of unblemished nectarines to blemished nectarines

b fraction of number of blemished nectarines out of the total.

15 The mark-up on a pair of jeans purchased for $80 was 315%. What is the ratio of the original price to the selling price?



16 Pete caught three fish on the weekend. The first was 60 cm long, the second was 35% longer than the first and the third was $\frac{8}{9}$ the length of the second fish.

a Work out the lengths of the other two fish.

b Write the lengths of the fish in the order they were caught as a ratio.

PROBLEM SOLVING AND REASONING

17 Consider the height of Melbourne's Rialto Tower (270 m) and Taiwan's Taipei 101 (1670 feet).

- **a** Explain why the ratio for the heights of Rialto Tower and Taipei 101 cannot be written as 270:1670.
- **b** What needs to be done to the measurements before a comparison can be made?
- **c** Write the ratio of the Rialto Tower's height to Taipei 101's height to the nearest metre. (Hint: 1 foot = 0.3048 m.)

18 Whiter Than White bleach recommends three capfuls of bleach for $2\frac{1}{2}$ L of water.

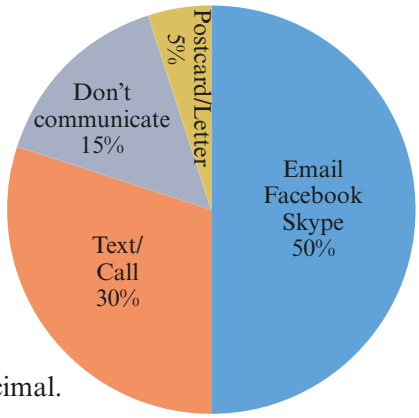
- **a** Can the two quantities be written as a ratio as they are?
- **b** If a capful is 19 mL, what is the ratio of the quantities of bleach to water?
- **c** Sharon thinks the solution of bleach and water is quite strong. Suggest an alternative ratio which would give a weaker solution. Explain your logic.

19 One hundred people attended an auction on the weekend. Seven people expressed interest in bidding but on the day only three actually bid.

- **a** Write a fraction for the number of bidders to the total number of people.
- **b** Write a fraction for the number of non-bidders to the total number of people.
- **c** Write a ratio for the number of bidders to the number of non-bidders.
- **d** Write a sentence explaining when a ratio can be written as a fraction.

CHALLENGE

20 Two hundred and forty travellers returning from holidays were asked how they communicated whilst away. Their responses are shown on the pie chart.



- **a** Work out the number of people in each category.
- **b** Write as a ratio the number of people communicating via email/Facebook/Skype compared to those communicating via postcards/letters.
- **c** Write the proportion of people in each category as a fraction in simplest form.
- **d** Write the proportion of people in each category as a decimal.
- **e** Write a brief paragraph explaining what the information suggests.

21 A rectangular swimming pool of length 6.5 m and width 4 m has a safety fence built around it. The fence is 2 m from the edge of the pool.

- **a** Write the ratio of the area of the pool to the area between the pool and safety fence in simplest form.
- **b** Write the ratio comparing the area between the pool and safety fence to the total area enclosed by the safety fence in simplest form.
- **c** Write the area between the pool and safety fence as a fraction of the area enclosed by the safety fence in simplest form.
- **d** Write the area of the pool as a percentage (to the nearest per cent) of the area enclosed by the safety fence.

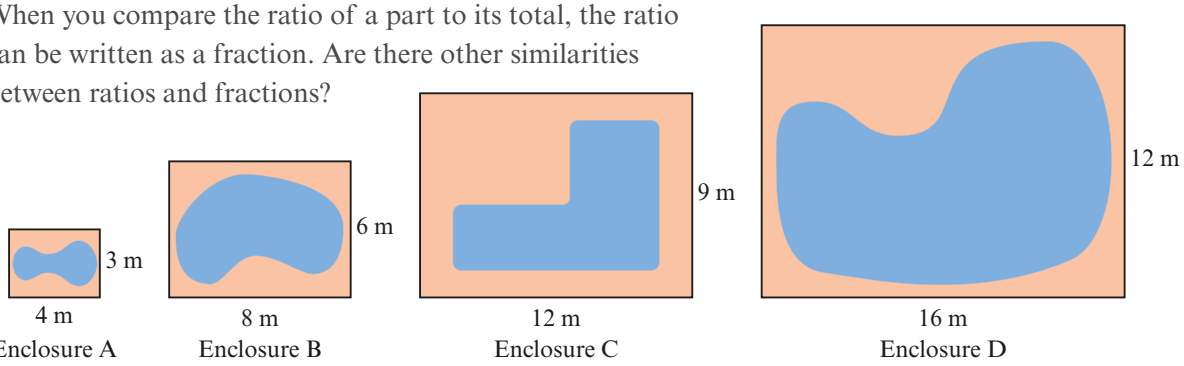
Reflect

What similarities are there between ratios and fractions?

2F Working with ratios

Start thinking!

When you compare the ratio of a part to its total, the ratio can be written as a fraction. Are there other similarities between ratios and fractions?



1 Copy and complete this table for each pool enclosure.

2 For each enclosure, write as a ratio the length of the rectangle compared to the perimeter of the rectangle.

3 Look for a pattern in your answers to question **2**.

4 Use your answers to question **2** to write each ratio as a fraction in simplest form.

5 Compare the fractions you have obtained. What is special about them?

Pool enclosure	Length	Width	Perimeter
A			
B			
C			
D			

KEY IDEAS

- Numbers in a ratio can be multiplied or divided by the same value to create an **equivalent ratio**.
- To write a ratio in simplest form, divide the values in the ratio by the highest common factor (HCF).
- Comparisons containing decimals can be written as ratios by first multiplying each part of the ratio by a power of 10 (that is, 10, 100, 1000, etc.) to form whole numbers.
- Comparisons containing fractions can be written as ratios by first writing equivalent fractions with the lowest common denominator (LCD) and then multiplying each fraction by the value of the LCD to form whole numbers. Mixed numbers must first be converted to improper fractions.
- Equivalent ratios can be used to find an unknown value.

EXERCISE 2F Working with ratios

EXAMPLE 2F-1 Simplifying ratios

Write the ratio 27:45 in simplest form.

THINK	WRITE
1 Write the ratio and identify the highest common factor (HCF) of each number. HCF = 9.	27:45
2 Divide each number in the ratio by the HCF of 9.	= 3:5

1 Write each ratio in simplest form.

a 7:84	**b** 9:12	**c** 106:36
d 22:55	**e** 108:72	**f** 45:95
g 38:34	**h** 17:102	**i** 16:24:42
j 75:50:125	**k** 36:60:84	**l** 18:42:48

EXAMPLE 2F-2 Simplifying ratios involving decimals

Write the comparison 4.5 to 2.75 as a ratio in simplest form.

THINK	WRITE
1 Write the comparison and multiply each number by 100 to obtain whole numbers.	4.5 to 2.75 = 450 to 275
2 Write as a ratio.	= 450:275
3 Divide each number in the ratio by the HCF of 25.	= 18:11

2 Write each comparison as a ratio in simplest form.

a 3.6 to 7.8	**b** 1.25 to 5.5
c 6.8 to 2.48	**d** 3.8 to 11.4
e 2.3 to 5.9	**f** 7.2 to 8.4
g 1.05 to 3.15	**h** 8.1 to 1.75
i 7.8 to 2.4 to 4.2	**j** 2.25 to 0.55 to 1.75
k 7.2 to 9.6 to 8.4	**l** 1.2 to 4.2 to 1.6

EXAMPLE 2F-3 Simplifying ratios involving fractions

Write the comparison $2\frac{1}{6}$ to $\frac{5}{8}$ as a ratio in simplest form.

THINK	WRITE
1 Write the comparison and convert the mixed number to an improper fraction.	$2\frac{1}{6}$ to $\frac{5}{8}$ $= \frac{13}{6}$ to $\frac{5}{8}$
2 Identify the lowest common denominator (LCD) of each fraction. LCD = 24. Write each fraction as an equivalent fraction with a denominator of 24.	$= \frac{52}{24}$ to $\frac{15}{24}$
3 Multiply each fraction by 24 to obtain whole numbers.	= 52 to 15
4 Write as a ratio and check if it can be simplified any further (no).	= 52:15

3 Write each comparison as a ratio in simplest form.

a $\frac{1}{6}$ to $\frac{1}{4}$ **b** $\frac{2}{3}$ to $\frac{3}{4}$ **c** $\frac{1}{3}$ to $\frac{2}{5}$ **d** $\frac{3}{7}$ to $\frac{2}{11}$

e $\frac{9}{22}$ to $\frac{5}{11}$ **f** $\frac{4}{7}$ to $\frac{6}{5}$ **g** $6\frac{1}{4}$ to $12\frac{1}{2}$ **h** $5\frac{2}{3}$ to $7\frac{5}{9}$

i $2\frac{1}{8}$ to $\frac{2}{5}$ **j** $3\frac{1}{5}$ to 4 **k** 0.25 to $\frac{1}{8}$ **l** $\frac{3}{4}$ to 0.125

EXAMPLE 2F-4 Simplifying ratios involving different units

Write the comparison 20 seconds to 2 minutes 15 seconds as a ratio in simplest form.

THINK	WRITE
1 Write the quantities in the order given.	20 seconds to 2 minutes 15 seconds
2 Convert the quantities to the same unit. To keep as whole numbers, write both values in seconds.	= 20 seconds to 135 seconds
3 Write the comparison as a ratio.	= 20:135
4 To simplify, divide each number in the ratio by the HCF. HCF = 5.	= 4:27

4 Write each comparison as a ratio in simplest form.

a 65 mL to 3.2 L **b** 17 mm to 2.34 m **c** 92 cents to $17.28

d $\frac{2}{3}$ hours to 0.8 hours **e** 5 days to 10 weeks **f** 3800 L to 4 kL

g 0.9 km to 550 m **h** 1.28 cm to 5 mm **i** $1\frac{3}{4}$ minutes to 420 seconds

UNDERSTANDING AND FLUENCY

5 Which pairs of ratios are equivalent? Explain.

a 1:2 and 3:6	**b** 8:5 and 2:3	**c** 4:10 and 2:5
d 2:9 and 10:45	**e** 16:25 and 4:5	**f** 63:42 and 9:7
g 2:3 and 22:34	**h** 15:9 and 5:3	**i** 1:5 and 15:30
j 17:51 and 68:204	**k** 49:64 and 7:8	**l** 27:18 and 3:2

6 **a** Using larger numbers, write two ratios that are equivalent to:
i 2:5 **ii** 15:11.
b Using smaller numbers, write two ratios that are equivalent to:
i 12:48 **ii** 80:32.

EXAMPLE 2F-5 Working with equivalent ratios

Find the value of a if $3:5 = 21:a$.

THINK

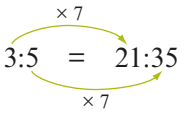
1 Compare the first number in each ratio (3 and 21). What should 3 be multiplied by to give 21? (7, as $3 \times 7 = 21$)

2 As the ratios are equivalent, 5 will also be multiplied by the same number to give a. ($5 \times 7 = a$.)

3 Identify the value for a.

WRITE

$3:5 = 21:a$



$a = 35$

7 Find the value of each pronumeral.

a $5:9 = 25:a$	**b** $15:50 = 3:b$	**c** $9:2 = c:16$	**d** $8:36 = d:9$
e $e:55 = 19:11$	**f** $f:4 = 42:24$	**g** $2:g = 22:66$	**h** $20:h = 5:2$
i $i:23 = 12:92$	**j** $j:6 = 30:60$	**k** $15:k = 3:17$	**l** $18:l = 2:13$
m $7:15 = 49:m$	**n** $72:168 = 6:n$	**o** $68:140 = x:35$	**p** $96:80 = p:5$

8 Fill in the gaps to complete the equivalent ratios.

a 2:5 = ____:20 = 20:____	**b** 6:7 = ____:35 = 54:____
c ____:360 = 70:60 = 7:____	**d** 225:____ = 9:12 = ____:4
e ____:104 = 68:____ = 17:13	**f** ____:48 = 75:____ = 15:4
g 20:____ = 40:96 = ____:240	**h** ____:636 = 72:212 = 18:____

9 Find the value of the letter in each equivalent ratio statement.
(Hint: simplify the complete ratio in the statement first.)

a $a:24 = 20:30$	**b** $b:52 = 20:13$	**c** $13:c = 65:5$
d $25:d = 10:14$	**e** $48:32 = 18:e$	**f** $33:44 = f:12$
g $33:54 = g:72$	**h** $15:20 = 36:h$	**i** $40:36 = i:27$

UNDERSTANDING AND FLUENCY



10 This box of chocolates contains a mix of milk chocolates, white chocolates and dark chocolates (in cones). Write the number of chocolates in each comparison as a ratio in simplest form.

- **a** white chocolates to square chocolates
- **b** chocolate cones to heart chocolates
- **c** milk chocolates to white chocolates
- **d** heart chocolates to round chocolates

11 Write each quantity as a ratio in simplest form.

- **a** The cost price of a new LED television is $890 while its retail price is $3115.
- **b** The new shopping centre will be 3.5 times the size it is now.
- **c** The speed limit in school zones is 40 km/h compared with 100 km/h on the freeway.
- **d** The 2010 AFL grand final between Collingwood and St Kilda ended in a draw.

12 Monday night fitness classes at a local community centre are very popular, with 64 people taking a Zumba class and 38 taking a spin cycle class.

- **a** What is the ratio, in simplest form, of Zumba to spin cycle participants?
- **b** What is the fraction, in simplest form, of Zumba participants to the total number of participants?
- **c** Write the number of spin cycle participants as a percentage (to the nearest per cent) of the total number of participants.

PROBLEM SOLVING AND REASONING

13 Eric paid a deposit of $704.70 for his overseas trip, which cost $4698. Write the ratio, in simplest form, of the deposit paid to the amount owing.

14 Jasmine is helping her mother make pancakes for her class end-of-term breakfast. The ingredients are listed on the right and feed 10 people.

- **a** Which ingredients can be written as a ratio? Clearly explain your choices.
- **b** Write a ratio for the ingredients identified in part **a**.
- **c** Repeat part **b** if 50 pancakes are required.



Pancakes
(serves 10)

500 mL full-cream milk
300 g self-raising flour
2 eggs
60 g butter
2 tablespoons sugar

15 The ratio of rainy days to sunny days in a month was 2:3. If there were 18 sunny days, how many rainy days were there?

16 The ratio of girls to boys in a class is 5:4. If there are 15 girls in the class, how many students are there in total?

17 The ratio of blue pens to black pens to red pens in a classroom is 7:4:1. If there are 24 black pens in the classroom, how many pens are there altogether?



PROBLEM SOLVING AND REASONING

18 For best results, the instructions on a new cleaning product recommend the ratio of detergent to water be 2:7.

- **a** How many litres of water are required if 300 mL of detergent are used?
- **b** How many millilitres of detergent are required if 28 L of water are used?

19 A two-stroke fuel mixture is made by mixing oil and petrol in the ratio of 1:25.

- **a** How many litres of petrol should be added to 850 mL of oil?
- **b** How many millilitres of oil should be added to 15 L of petrol?

20 Deano wants to make concrete in the ratio 1:2:3; that is, one part cement, two parts sand and three parts screenings.

- **a** If he buys four bags of sand, how many bags of cement and screenings are needed?
- **b** If he buys nine bags of screenings, how many bags of cement and sand are needed?
- **c** If he buys five bags of cement, how many bags of screenings and sand are needed?
- **d** If he only uses half a bag of cement, how many bags of sand and screenings are needed?

21 Gordon bought shares in a Telco company for $8.20 and sold them for $40.80. Find the ratio, in simplest form, of:

- **a** the original price to the selling price
- **b** the original price to the profit made.

22 Imelda purchased a pair of shoes that had been discounted by 60%. The shoes were originally priced at $850. Find the ratio, in simplest form, of:

- **a** the original price to the sale price
- **b** the discount compared to the original price
- **c** the discount compared to the sale price.

23 The side lengths of a scalene triangle are in the ratio 7:8:5.

- **a** If the shortest side length is 17.4 cm, find the length of the longest side.
- **b** Find the perimeter of the scalene triangle.

24 Angela wondered whether ratios such as 3:5 and 10:14 could be compared. She also wanted to know whether they could be ranked. Leo knew that each ratio could be written as a fraction then written as an equivalent fraction and compared.

- **a** Discuss in a small group whether Leo's suggestion is valid.
- **b** See if your group can come up with an alternative method for comparing ratios. Share your ideas with the class.

CHALLENGE

25 Compare the ratios 4:5 and 10:13 and state which is larger. Explain why.

26 Write your own ratio problem and swap with a classmate. Discuss any differences in your answers.

27

- **a** Comment on how the ratio 22:27 compares with the ratios 8:9 and 12:15.
- **b** Obtain three ratios that lie between 22:27 and 12:15.

Reflect

How are equivalent ratios useful?

2G Dividing a quantity in a given ratio



Start thinking!

Laura, Sarah and Amelia decide to purchase a $36 Lotto ticket for Saturday's $30 million draw. They decide to pay equal shares for the ticket and share any winnings equally.

1 How much does each person contribute?

2 Write each person's contribution for the ticket as a ratio in simplest form.

3 Add all the numbers (parts) in this simplified ratio. What is the total number of parts in the ratio?

4 The number of parts in the simplified ratio that match Laura's contribution is 1. To write Laura's contribution as a fraction of the total number of parts, you can write $\frac{1}{3}$. In a similar way, write a fraction for:

a Sarah's contribution **b** Amelia's contribution.

5 The girls win a prize of $14 270.40. As they are sharing the winnings equally, how much will each person receive?

6 Explain how the amount each girl contributed and won could have been calculated using ratios? (Hint: use your answers to question **4**.)

KEY IDEAS

- When dividing a quantity in a given ratio, follow these steps.
  1. Find the total number of parts in the ratio.
  2. Write each number in the ratio as a fraction of the total number of parts.
  3. Multiply each fraction by the quantity and simplify.
- Remember to include the same units with your answer.
- One way to check that you have divided the quantity correctly is to add the individual amounts and see that the result is the same as the original quantity.

EXERCISE 2G Dividing a quantity in a given ratio

EXAMPLE 2G-1 Writing numbers in a ratio as fractions

For each ratio:

i calculate the total number of parts
ii write each number in the ratio as a fraction of the total number of parts.

a 2:7 b 3:1:6

THINK

a i Add the numbers in the ratio to find the total number of parts.

ii Write each number in the ratio as a fraction of the total number of parts.

b i Add the numbers in the ratio to find the total number of parts.

ii Write each number in the ratio as a fraction of the total number of parts.

WRITE

a i $2 + 7 = 9$ parts
The ratio 2:7 has 9 parts in total.

ii $\frac{2}{9}$ and $\frac{7}{9}$

b i $3 + 1 + 6 = 10$ parts
The ratio 3:1:6 has 10 parts in total.

ii $\frac{3}{10}$ and $\frac{1}{10}$ and $\frac{6}{10}$
or $\frac{3}{10}$ and $\frac{1}{10}$ and $\frac{3}{5}$

UNDERSTANDING AND FLUENCY

1 For each ratio:

i calculate the total number of parts
ii write each number in the ratio as a fraction of the total number of parts.

a 7:5	**b** 3:1	**c** 5:11
d 14:9	**e** 13:15	**f** 8:3
g 3:4	**h** 2:17	**i** 21:19
j 3:2:9	**k** 6:12:11	**l** 5:7:9

2 Divide \$300 using the ratio 4:6.

a Calculate the total number of parts in the ratio.

b Write each number in the ratio as a fraction of the total number of parts.

c Multiply each fraction from part **b** by the quantity (\$300).

d Do the amounts found in part **c** add to \$300?



EXAMPLE 2G-2 Dividing an amount in a given ratio

Divide $4800 in the ratio 3:1.

THINK	WRITE
1 Add the number of parts in the ratio.	3 + 1 = 4 parts
2 Write each number in the ratio as a fraction of the total number of parts.	$\frac{3}{4}$ and $\frac{1}{4}$
3 Multiply each fraction by the quantity to be divided ($4800) and simplify.	$\frac{3}{4}$ of $4800 and $\frac{1}{4}$ of $4800 $= \frac{3}{\cancel{4}_1} \times \frac{\cancel{4800}^{1200}}{1}$ and $= \frac{1}{\cancel{4}_1} \times \frac{\cancel{4800}^{1200}}{1}$ $= 3 \times 1200$ and $= 1 \times 1200$ $= 3600$ and $= 1200$
4 Answer the question and include the appropriate units. Check: $3600 + $1200 = $4800.	The ratio 3:1 divides $4800 into $3600 and $1200.

UNDERSTANDING AND FLUENCY

3 Divide $9200 in each ratio.

a 3:2 **b** 5:3 **c** 3:1
d 1:1 **e** 21:25 **f** 2:8

4 Divide 36 000 km in each ratio.

a 5:7 **b** 7:1 **c** 10:8
d 4:5 **e** 12:3 **f** 1:1

5 Divide $240 in each ratio.

a 1:2 **b** 9:1 **c** 2:10
d 10:6 **e** 3:1:4 **f** 7:2:1

6 Divide 300 kg in each ratio.

a 4:6 **b** 1:9 **c** 13:2
d 4:11 **e** 4:5:1 **f** 8:5:3:4

7 The ratio of female to male fans at a football training session is 5:2. If there are 224 fans at the training session, how many are male?



8 Morning Sunshine fruit juice drink contains fruit juice and water in the ratio 1:3.

a How much water (in mL) is in a 2-L container of Morning Sunshine? (Hint: 1 L = 1000 mL.)

b How much fruit juice is in 600 mL of Morning Sunshine?

PROBLEM SOLVING AND REASONING

9 The angles in a triangle are in the ratio 3:4:2. What is the size of each angle? (Hint: the sum of angles in any triangle is 180°.)

10 Nadia divides her pocket money for April in the ratio 3:2:1 to cover buying a gift, entertainment expenses and savings. Calculate the amount she has allocated for entertainment expenses if her total pocket money for April is $84.

11 Shane has set himself a goal of completing a 210-km bike ride in three days. Over the three days, the number of kilometres he plans to cycle each day is in the ratio 3:5:2.

a What distance does Shane plan to cycle each day?

b What is the difference between the longest and shortest distance?



12 **a** Stanley's History and Science test results are in the ratio 22:19. If his combined score is 164, what result did he obtain for each subject?

b Miriam's History and Science test results are in the ratio 10:11. If her combined score is 168, did she perform better or worse than Stanley in Science?

13 Katie, Mike and Sam contributed different amounts to a $5 raffle ticket. When they won, they distributed the winnings in the same ratio as they put in. If Katie won $450, Mike won $200 and Sam won $350, find:

a the total amount won

b the ratio of their contribution

c how much they each put towards the ticket.

14 Sarah, Cameron and Hunter bought a $5 raffle ticket. If Sarah contributed $2.50, Cameron contributed $1.50 and Hunter contributed $1.00, find:

a the ratio of their contribution

b their individual winnings, if the ticket won:

i $50 **ii** $150 **iii** $700

iv $2000 **v** $10 000 **vi** $13 000.

15 Gloria is painting and prepares to mix up her favourite shade of green, using yellow and blue paint in a ratio of 2:3.

a If she wants 25 mL of paint, how much yellow and blue paint should she use?

She mixes in 10 mL of yellow paint and then accidentally puts 18 mL of blue paint in.

b What is the total amount of paint she has now?

c What is the ratio of yellow to blue paint in this mixture?

d How much yellow paint should Gloria add in order to get her favourite shade of green? (Hint: 18 mL of blue paint represents 3 parts of the ratio.)

e How much paint does Gloria have now?

f How much excess paint did she make?

16 The fat, protein and carbohydrate in one of Al's specialty hamburgers is in the ratio 8:6:7. For a 420-g hamburger, list the amount of:



- **a** fat
- **b** protein
- **c** carbohydrate.

17 Madeleine gets $50 for her birthday and decides to spend $\frac{1}{10}$ on lollies, $\frac{3}{10}$ on a movie ticket, $\frac{2}{10}$ on a new necklace and the remainder on iTunes.

- **a** What fraction of her birthday money did she spend in the iTunes store?
- **b** Write a ratio representing the money Madeleine spent on each of the lollies, movie ticket, necklace and iTunes.
- **c** Find how much Madeleine spent on each category.

18 Bella wants to organise her time in the holidays. She decides she will spend $\frac{2}{5}$ of her time catching up with friends, $\frac{1}{4}$ of her time doing outdoor activities, $\frac{1}{10}$ of her time doing her homework and chores, and the remainder of her time relaxing at home.

- **a** Find the percentage of time that Bella will catch up with friends, do outdoor activities and homework or chores. (Hint: for each fraction find an equivalent fraction with a denominator of 100.)
- **b** Hence find the percentage of time she will be relaxing at home.
- **c** Write these percentages as a ratio in simplest form.
- **d** Assuming that for the two-week holiday she has 10 'usable' hours a day, find the number of hours Bella spends on each activity.



19 The perimeter of a rectangular computer screen is 228 cm, with a width-to-height ratio of 12:7.

- **a** Calculate the width and height of the screen.
- **b** Calculate the area of the screen.

PROBLEM SOLVING AND REASONING

20 You can use ratios to divide a line into various lengths. Draw a line segment of length 8 cm in your workbook and label it as shown.



Copy and complete these sentences. Part **a** is completed for you.

a Point D divides the line segment AI in the ratio 3:5. The length of AD is $\frac{3}{8}$ of the length of AI.

b Point H divides the line segment AI in the ratio _____.
The length of AH is _____ of the length of AI.

c Point E divides the line segment AI in the ratio _____.
The length of AE is _____ of the length of AI.

d Point G divides the line segment AI in the ratio _____.
The length of AG is _____ of the length of AI.

21 Using question **20** as a guide, draw a line segment of length 15 cm in your workbook. Mark dots at 1-cm intervals and label them A to P. Place a point on the line segment to divide it in each of the given ratios.

a 7:8 **b** 11:4 **c** 4:1
d 2:3 **e** 2:1 **f** 1:2

22 Which point divides the line segment AJ in the ratios given below?



a 5:4 **b** 1:2 **c** 8:1 **d** 4:5
e 2:7 **f** 1:8 **g** 2:1 **h** 7:2

23 How do the questions in this topic highlight the difference between a ratio that shows the relationship of part to part, and a fraction that shows the relationship of part to whole? Why is it important?

CHALLENGE

24 To create two distinct shades of pink paint, the following ratios of white to red paint are required: 4:1 and 7:1. A 4-L can for each shade is purchased.

a Calculate the amount (in mL) of white and red paint required for each shade.

b The two cans are accidently mixed together. What is the new ratio of white to red paint?

c Write the amount of red paint as a fraction, in simplest form, of the mixed paint.

d Write the amount of white paint as a percentage of the mixed paint.



Reflect

What other practical applications of ratios are there?

2H Understanding rates

Start thinking!



Alexis and Kym arrange to meet at the running track to train for an athletics carnival. They cycle to the track, leaving their homes at the same time.

It takes Alexis 8 minutes to travel 2800 m from her home to the track. Kym takes 18 minutes, after travelling a distance of 6.12 km.

1 Alexis thought she was faster, but Kym was not convinced.

a Calculate each girl's speed using the rule: $\frac{\text{distance travelled}}{\text{time taken}}$.

b Who was fastest? How do you know?

c What unit did you use for speed? Explain.

2 A **rate** compares the change in one quantity with respect to another. Speed is an example of a rate. What is speed comparing?

3 Can you list any similarities or differences between rates and ratios?

KEY IDEAS

- A rate compares two quantities of different kinds.
- A rate has units.
- Speed is an example of a rate. It compares distance and time.
- Average speed $= \frac{\text{distance travelled}}{\text{time taken}}$ or $s = \frac{d}{t}$
- The unit of a rate is two units separated by the word 'per' (meaning 'for each') or the symbol /; for example, travelling at a speed of 100 km/h.
- Order is important when writing a rate.
- For a rate to be in simplest form, the second of the two quantities being compared must have a value of 1.

EXERCISE 2H Understanding rates

EXAMPLE 2H-1 Writing statements as rates

Write the statement '100 km in each hour' as a rate with the appropriate unit.

THINK	WRITE
1 Write the statement.	100 km in each hour
2 To write the rate, show the number of the first quantity (100) for one unit of the second quantity. The word 'per' (meaning 'for each') can be replaced by the symbol /.	rate = 100 km in 1 hour = 100 km per hour = 100 km/h

1 Write each statement as a rate with the appropriate unit.

a 60 km in each hour	**b** 591 L in each minute	**c** four quarters in each game
d $16 for each hour	**e** 45 cm each year	**f** five patients each hour
g $1.49 for each day	**h** $28.99 for each month	**i** $5.80 for each box
j 0.5 kg for each week	**k** 290 mm each year	**l** 14 km for each litre

EXAMPLE 2H-2 Writing statements as rates in simplest form

Write each statement as a rate in simplest form.

a 30 m in 5 seconds
b $54 for 40 L of petrol

THINK	WRITE
a 1 Write the two quantities as a rate statement.	**a** rate = 30 m per 5 seconds
2 For the rate to be in simplest form, the second quantity needs be 1. Divide both quantities by 5.	= 30 m per 5 s ÷ 5 ( ) ÷ 5 = 6 m per 1 s = 6 m per second
3 Write your answer. Replace 'per' with /.	The rate is 6 m/s.
b 1 Write the two quantities as a rate statement.	**b** rate = $54 per 40 L
2 For the rate to be in simplest form, the second quantity must be 1. Divide both quantities by 40.	= $54 per 40 L ÷ 40 ( ) ÷ 40 = $1.35 per 1 L = $1.35 per L
3 Write your answer.	The rate is $1.35/L.

2 Write each statement as a rate in simplest form.

a 200 m in 4 minutes
b 6 L in three bottles
c 80 cents for 20 seconds
d 135 students with nine teachers
e 468 seats in 18 rows
f 100 newspapers in four bundles
g 68 runs in 10 overs
h $20 for eight muffins
i 102 goals in 15 games
j 39 996 tickets sold in 9 minutes
k 77 goals over 22 games
l area of 33.6 m^2 requires 6.4 L of paint

3 Copy and complete each sentence so the rate is in simplest form.

a Five metres of fabric cost $214.95 so the rate is $_____ per metre.
b Mani worked a total of 18 hours and earned $283.50 so the rate is $_____ per hour.
c The Australian cricket team made 156 runs in 30 overs so the rate is _____ runs per over.
d A leaky tap dripped a total of 1780 mL over 16 hours so the rate is _____ mL per hour.
e The temperature rose by 18°C over an 8-hour period so the rate is _____°C per hour.
f The most valuable player on the basketball team scored 576 points over 22 games so the rate is _____ points per game.
g AFL record-holder Tony Lockett kicked 1360 goals in 281 games so the rate is _____ goals per game.
h Over three Olympic Games Michael Phelps won 58 gold, 11 silver and 3 bronze medals so the rate is _____ medals per Olympic Games.

4 Write each statement as a rate in simplest form.

a Bobby earns $182.40 working 12 hours.
b The cost of 600 g of breakfast cereal is $9.54.
c Tess typed 3000 words in 65 minutes.
d 56 sheep are sheared in 60 minutes.
e Connie typed her 1800-word essay in 45 minutes.
f Sound travels 23 160 m in 60 seconds

5 Write a rate (in simplest form) to represent each situation.

a A car uses 18 L of petrol to travel 252 km.
b The cost of 15 mangoes is $29.85.
c A bath fills up to 54 L in $4\frac{1}{2}$ minutes.
d A runner takes $2\frac{3}{4}$ hours to complete a 42.2 km marathon.
e Victoria's population is 5 547 527 and its area is 237 629 km^2. (Write your answer correct to two decimal places.)
f Light travels 1080 million km in 60 minutes.



EXAMPLE 2H-3 Solving problems with rates

Ellie worked 15 hours last weekend and received $247.50. At this rate, how much will she earn if she is rostered to work 20 hours next weekend?

THINK

1 Write the two quantities as a rate statement.

2 For the rate to be in simplest form, the second quantity needs be 1. Divide both quantities by 15.

3 Multiply the hourly rate by 20 to work out the new amount.

4 Write your answer.

WRITE

rate = $247.50 per 15 hours

= $247.50 per 15 h

÷ 15 ÷ 15

= $16.50 per 1 h

= $16.50 per hour

The rate is $16.50/h

new amount = $16.50 × 20

= $330

Ellie will receive $330.

6 Fabian worked 22 hours last weekend and received $396. At this rate, how much will he earn if he works $19\frac{3}{4}$ hours next weekend?

7 A selection of 18 pastries costs $63. How much would 30 pastries cost?

EXAMPLE 2H-4 Calculating average speed

Calculate the average speed of a truck that travels 750 km in 13.5 hours, rounding your answer to two decimal places.

THINK

1 Write the formula for average speed.

2 Substitute the values into the formula and calculate s.

3 Write your answer.

WRITE

$s = \frac{d}{t}$

$s = \frac{750 \text{ km}}{13.5 \text{ h}}$

= 55.5555… km/h

Average speed is 55.56 km/h.

8 Calculate the average speed for each situation, rounding your answer to two decimal places where appropriate.

a a bus travels 876 km in 16 hours

b a jet travels 1965 km in 2.3 hours

c a cyclist rides 78 km in 2.4 hours

d an athlete runs 10 km in 26.44 minutes

e James Magnussen took 47.71 seconds to swim 100 m freestyle

f Libby Trickett took 52.88 seconds to swim 100 m freestyle

9 A particular rate has the unit \$/kg, where the quantities represented are money and mass. What quantities are represented by each of these rates?

a words per minute **b** km/h **c** % per annum
d tonnes/year **e** \$/m **f** wickets/over
g m/s **h** \$ per line **i** \$/litre
j sheep/hectare **k** mm per year **l** km/litre

10 What rate unit could be used in each situation?

a a cheetah chasing its prey **b** Usain Bolt sprinting 100 m
c the cost of 10 apples **d** the cost of tiling the bathroom
e filling a bath with water **f** James Pattinson bowling
g downloading songs on to an iPod **h** driving on a freeway

11 The following food was consumed over a two-week period at the Australian Open. Calculate the rate of the amount of food consumed per day for each item listed.

70 000 kg of pasta	79 000 sausages
7000 portions of fish	35 tonnes of chips
80 000 pieces of sushi	9000 schnitzels
10 000 kg of noodles	

12 Max's team scored 230 runs in 40 overs. William's team scored 280 runs in 50 overs.

a Calculate each team's run rate per over.
b Which team had a better run rate?



13 On Thursday, Mauro paid \$65.32 for 46 L of petrol. The next day Joy paid \$61.09 for 41 L.

a How much did each person pay for 1 L of petrol?
b Comment on your answers to part **a**. What do you think happened?

14 Leandro's resting heart rate is 14 beats in 15 seconds.

a Heart rate is often expressed in beats per minute (bpm). Write Leandro's heart rate in bpm.
b How many minutes does it take for Leandro's heart to beat 1512 times?

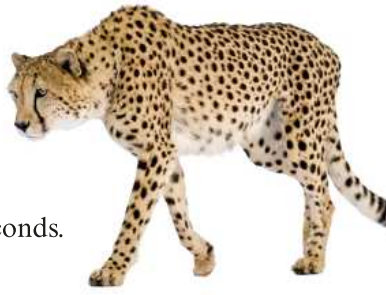
15 Peter is making his famous lemon tarts for a stall. He can make three large lemon tarts in 1.5 hours.

a Write this as a simplified rate in minutes per tart.
b If Peter cooks for 4 hours a day, how many days will it take him to make 30 tarts?

If Peter can buy the pastry readymade, he can increase his tart-making rate to five tarts every two hours.

c Write this new rate in simplified form and find how long it will take Peter to make 30 tarts now.

PROBLEM SOLVING AND REASONING



16 Usain Bolt recorded a world-record time of 9.58 seconds for the 100 m sprint. Compare his rate with that of:

- **a** Michael Johnson's time of 19.32 seconds for the 200 m sprint
- **b** the US 4 × 100 m relay team of 1992, who recorded a time of 37.4 seconds
- **c** a cheetah that was able to cover a distance of 600 m in 37.5 seconds.

17 Victor takes 48 minutes to run 15 km.

- **a** What distance does he cover per minute?
- **b** If Victor is able to maintain this rate, what distance would he cover in these times?
  - **i** 55 min **ii** 60 min **iii** 65 min **iv** 78 min
  - **v** 1 hour 25 min **vi** 1.5 h **vii** 1.75 h **viii** $1\frac{4}{5}$ h
- **c** How long would it take Victor to cover these distances?
  - **i** 625 m **ii** 31.25 km **iii** 25 km **iv** 2 km

18 Calculate each of these.

- **a** If a 15-minute phone call costs $11.70, how much will an 8-minute phone call cost?
- **b** The Bosancic family use 1170 kilowatt hours (kWh) of electricity in 90 days. How much will they use in 58 days?
- **c** If a 5.7 kg parcel costs $14.25 to post, how much will it cost to send a 750 g parcel?

19 Illias' car uses 65 L of petrol to travel 728 km, while Hani's car uses 48 L in 552 km.

- **a** Which car is more economical? Provide calculations to support your answer.
- **b** How much petrol would each car use to travel 830 km?



20 At a convenience store, milk is available in two sizes. How would you decide which size is the better buy?

21 Compare each pair of grocery items and work out which option is the better value for money.

- **a** 5 kg of potatoes for $4.95 or 7.5 kg for $7.15
- **b** a packet of four batteries for $3.99 or a packet of ten for $9.89
- **c** a 500 g tin of coffee for $9 or a 150 g jar for $2.99
- **d** a 440 g tin of peaches for $1.85 or a 650 g tin for $2.56

CHALLENGE

22 A phone company offers its customers two plans.

Plan A: connection fee of 40 cents and then 40 cents per minute or part thereof

Plan B: connection fee of 50 cents and then 38 cents per minute or part thereof

- **a** Calculate the cost of a 1-minute call for each plan. Which offers the best deal?
- **b** Calculate the cost of a 3-minute call for each plan. Which offers the best deal?
- **c** Calculate the cost of a 5-minute call for each plan. Which offers the best deal?
- **d** Calculate the cost of a 6-minute 23-second call for each plan. Which offers the best deal?
- **e** Use your answers from parts **a**–**d** to explain when it would be best to use each plan.

Reflect

What is the difference between a ratio and a rate?

CHAPTER REVIEW

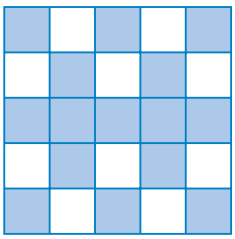
SUMMARISE

Create a summary of this chapter using the key terms below. You may like to write a paragraph, create a concept map or use technology to present your work.

per cent
fraction
percentage
denominator
numerator
equivalent fraction
decimal
percentage of an amount
mark-up
discount
commission
profit
loss
selling price
wholesale price
percentage increase
percentage discount
ratio
equivalent ratio
rate

MULTIPLE-CHOICE

2A **1** What percentage of this figure is shaded?



A 10%
B 15%
C 40%
D 60%

2B **2** What is $\frac{1}{5}$ written as a percentage?

A 10% **B** 15%
C 20% **D** 95%

2B **3** Which value is *not* the same as 75%?

A 0.75 **B** 75
C $\frac{75}{100}$ **D** $\frac{3}{4}$

2C **4** What is 45% of 360?

A 162 **B** 198
C 162% **D** 315

2D **5** After a 32% discount, what is the selling price of an item originally marked $480?

A $153.60 **B** $326.40
C $448 **D** $633.60

2E **6** Written as a ratio, what is 3 hours to 37 minutes?

A 3:37 **B** 37:3
C 37:180 **D** 180:37

2F **7** Which ratio is the same as 16:28:12 written in simplest form?

A 4:7:3 **B** 3:7:4
C 8:14:6 **D** 32:56:24

2F **8** What is the value of a in 3:7 = a:35?

A 7 **B** 14
C 15 **D** 21

2G **9** When $3600 is divided in the ratio 1:3, what are the parts?

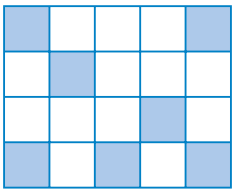
A $1200 and $2400
B $900 and $2700
C $1500 and $2100
D $1200 and $2700

2H **10** Written as a rate, what is 120 m in 24 seconds?

A $\frac{120}{24}$ m/s **B** $\frac{24}{120}$ m/s
C $\frac{1}{5}$ m/s **D** 5 m/s

SHORT ANSWER

2A **1** Consider this figure.



a How many shaded sections are there?

b Write the number of shaded sections as a fraction of the total number of sections.

c Write the fraction in part b as an equivalent fraction with denominator 100.

d Write the shaded amount as a percentage.

2B **2** Write each percentage as a decimal.

a 23% b 62%

c 16.52% d 291.6%

2B **3** Write each decimal as a percentage.

a 0.32 b 7.62

c 31.625 d 0.0003

2B **4** Write each fraction as a percentage.

a $\frac{3}{4}$ b $\frac{7}{10}$ c $\frac{17}{20}$ d $\frac{15}{32}$

2C **5** Write:

a 20 as a percentage of 50

b 24 as a percentage of 150

c 58 as a percentage of 125.

2C **6** Calculate:

a 10% of 340 b 25% of 62

c 150% of 1500 d 37% of 2400.

2D **7** Calculate the price to be paid after each mark-up or discount.

a a 10% mark-up on $950

b a 44% mark-up on $628

c a 28% discount on $1255

2D **8** Calculate the percentage profit or loss.

a a radio bought for $40, sold for $55

b a book bought for $35, sold for $21

c shares bought for $2.98, sold for $4.52

d TV bought for $1400, sold for $958

2E **9** Refer to this coloured grid.



a Write each comparison as a ratio.

i red sections to blue sections

ii yellow sections to green sections

b Write each comparison as a fraction.

i green sections to total sections

ii yellow sections to total sections

2E **10** Write each comparison as a ratio.

a 35 kg to 47 kg

b 7 weeks to 31 weeks

c 8 months to 3 years

d 45 cents to $1.25

2F **11** Simplify each ratio.

a 12:44 b 64:108

c 16:32:96 d 15:75:105

2F **12** Calculate the value of each pronumeral.

a $6:7 = a:49$ b $13:4 = 52:b$

c $c:48 = 3:12$ d $15:d = 3:7$

2G **13** Calculate the total number of parts for each ratio.

a 3:5 b 12:5 c 4:5:8 d 6:10:15

2G **14** Divide $1500 in each ratio.

a 2:3 b 3:7 c 3:4:8 d 4:5:11

2H **15** Write each statement as a rate with the appropriate unit.

a 100 km per hour

b 120 L in 40 seconds

c $140 for 8 hours of work

d 26 books read in 6 months

2H **16** Write each statement as a rate in simplest form.

a the cost of 45 L of petrol was $62.55

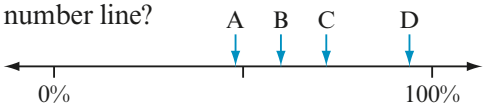
b a 375 mL can of drink costs $1.80

c a football team scored 180 points in 120 minutes of play

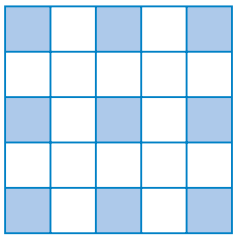
d a worker earned $218.40 for 13 hours of work

NAPLAN-STYLE PRACTICE

1 Which arrow points closest to 70% on this number line?



2 What percentage of the figure has been shaded?



3 What is 42% as a decimal?

0.42 ☐ 4.2 ☐ 42.0 ☐ 4200.0 ☐

4 Which of these is equivalent to $\frac{5}{8}$?

58% ☐ 62.5 ☐ 0.625 ☐ $\frac{5}{800}$% ☐

5 Which list of numbers is ordered from smallest to largest?

☐ 25%, 7.1, $\frac{1}{2}$, 5.2% ☐ 3.6%, 15%, $\frac{7}{8}$, 1.1

☐ 128%, $\frac{7}{15}$, 36%, 95 ☐ $\frac{1}{3}$, $\frac{5}{6}$, 0.905, 15.1%

6 What is $\frac{11}{20}$ as a percentage?

7 Josie scored 35 out of 40 on her Maths test. What percentage is this?

14% ☐ 75% ☐ 87.5% ☐ 114.3% ☐

8 What is 15% of $250?

$265 ☐ $235 ☐ $37.50 ☐ $35 ☐

9 When an item originally priced at $340 has a mark-up of 70%, what is its selling price?

10 Damien notices a sale advertising a discount of 40%. How much would he pay for a coffee machine originally priced at $380?

$532 ☐ $340 ☐ $228 ☐ $152 ☐

11 A car is bought for $22 000 and sold later for $18 500. Which option is correct?

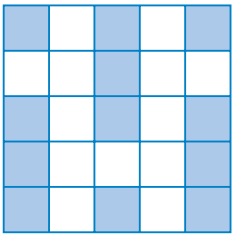
☐ A profit of $3500 has been made.
☐ A loss of 3500% has been made.
☐ A loss of 15.91% has been made.
☐ A loss of 20.45% has been made.

Questions **12** and **13** refer to this information.
Ben runs 10 km in 60 minutes. Hayden completes the same run but does it 1.25 times as fast as Ben.

12 Write how much faster Hayden is than Ben, as a percentage.

13 In how many minutes does Hayden complete the run?

14 What is the ratio of non-shaded squares to shaded squares in this diagram?



12:25 ☐ 13:25 ☐ 13:12 ☐ 12:13 ☐

15 Magan's pace length is 65 cm while Michael's is 1.07 m. What is the ratio of Michael's pace length to Magan's?

☐ 65:107 ☐ 65:1.07
☐ 1.07:65 ☐ 107:65

16 Which ratio is *not* equivalent to 3:5:6?

☐ 9:15:18 ☐ 15:25:30
☐ 30:15:25 ☐ 30:50:60

17 Which ratio represents $\frac{3}{4}$ hour to 90 minutes written in simplest form?

1:2 ☐ 5:6 ☐ 45:90 ☐ 2:1 ☐

18 What value of x completes this ratio statement?

$12{:}x = 132{:}55$

19 When $240 is shared in the ratio 1:3, what fraction is the smallest share?

$\frac{1}{3}$ $\frac{1}{4}$ $\frac{2}{3}$ $\frac{3}{4}$

20 Four students, Mikayla, Morgan, Chelsea and Alexandra, collected money for a charity in the ratio of 5:4:4:2. If the total collected was $330, how much did Mikayla collect?

21 Tess, Alicia and Alexandra contributed money to a lottery ticket. Tess paid $2, Alicia paid $1.50 and Alexandra paid $2.50. If the ticket won $1200, how much money would Tess receive if they distributed their winnings in the same ratio as their contribution?

22 Which option best describes an example of a rate?

- driving 100 km to the beach
- running as fast as you can to the canteen
- earning $15.75 for every hour worked
- observing the second hand on a clock for one minute

23 A car travels 120 km and uses 10 L of petrol. Which option shows the rate in simplest form?

- 120 km/L
- 12 km/L
- 12 L/km
- 110 km/L

24 Which is the best value?

- 8 chocolate bars for $7.00
- 3 chocolate bars for $2.50
- 5 chocolate bars for $4.00
- 13 chocolate bars for $11.00

ANALYSIS

A market stall holder bought a variety of coloured 'hoodies' to sell. Each hoodie cost the stall holder $20 and she intends to sell them to the public at a mark-up of 65%.

a Write the percentage mark-up value as a:
  i fraction in simplest form
  ii decimal.

b Calculate the value of the mark-up on each hoodie.

c What price will the hoodies be?

d If the stall holder sells 35 hoodies on the first day, what is her profit?

To improve the sales of the hoodies for her second day at the market, the stall holder decides to offer a 10% discount on the selling price.

e What is the discount amount for each hoodie?

f What is the new selling price?

g What profit does the stall holder make on the sale of each hoodie?



h The stall holder sells 55 hoodies at the discounted amount. Write the profit from the sale of these as a percentage of the cost she paid for them.

i Of the hoodies sold in part **h**, 20 were blue, 25 were red and the rest were white. Write the number of blue, red and white hoodies as a ratio in simplest form.

On the first day at the market, the stall holder spent 5 hours selling the hoodies. She spent 8 hours selling them on the second day.

j Write the sales for each day as a rate in simplest form.

k Which day had the best rate of sale?

CONNECT



Let's get physical

In 2013, a gym had a total of 860 members. Two-thirds of these were male. In the following year, the percentage of females and males at the gym increased by 20%, and then in 2015 there were 432 males and 252 females.

Your task

For your investigation you will analyse different aspects of what happens at the gym, fitness and nutrition.

To look at **membership**:

- calculate the number of male and female members at the gym
- calculate the total number of members
- calculate the percentage change in male and female members each year
- calculate the percentage change in the total number of members each year
- predict future trends in gym memberships.

To look at **costs**:

- calculate the ratio of current membership costs to proposed costs
- calculate the percentage change in membership costs
- rank membership costs in ascending order
- write a variety of membership costs as monthly rates
- work out the most economical gym membership.

To look at **fitness measures**:

- calculate a person's heart rate per minute
- calculate a person's maximum heart rate
- read a target heart rate chart
- calculate the various zones of a target heart rate chart
- calculate the heart rate for a given zone of the target heart rate chart.

To look at **nutritional information**:

- compare the nutrition panels of two products by converting the amount of nutrients to percentages
- analyse the nutritional information on the panel shown on the right and make comparisons between nutrients found in the cereal and sweet biscuits.



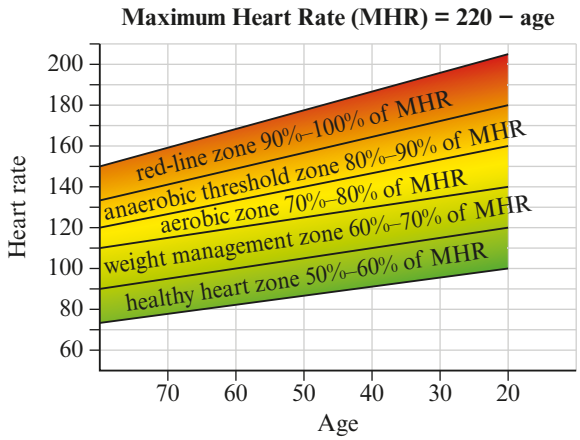
Nutrient	Mass per 40 g serve of cereal	Mass per 22 g serve of sweet biscuits
Protein	4.9	1.7
Fat	2.0	1.2
Carbohydrates (total)	22.4	14.9
Sugars	3.4	6.8
Dietary fibre	7.3	0

This table shows the membership prices for 2015 and the proposed prices for 2016.

	One-month membership	Three-month membership	Six-month membership	Twelve-month membership
2015	$85	$240	$440	$950
2016	$115	$340	$580	$1200

Steven joins the gym after not exercising for many years. He decides to measure his heart rate before using the treadmill. He finds his pulse and counts 23 beats in 15 seconds.

The target heart rate chart shows an estimate of the maximum heart rate (MHR) for various activities based on a person's age.





Complete the **2 CONNECT** worksheet to show all your working and answers to this task.

You may like to present your findings as a report. Your report could be in the form of:

- a poster
- a brochure
- an Excel spreadsheet
- a PowerPoint presentation
- other (check with your teacher).





3 POSITIVE AND NEGATIVE NUMBERS

3A Understanding negative numbers

3B Adding integers

3C Subtracting integers

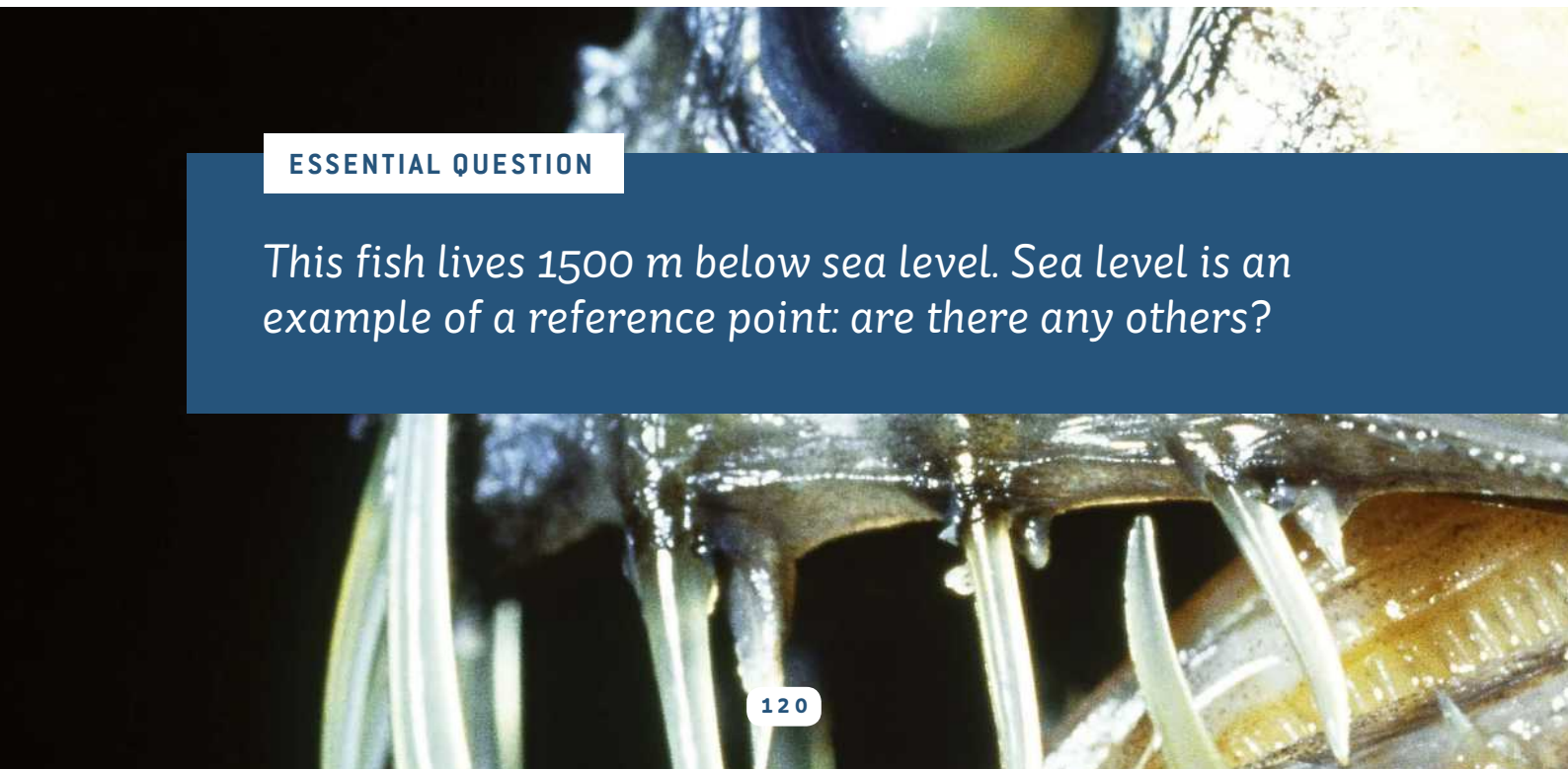
3D Simplifying addition and subtraction of integers

3E Multiplying and dividing integers

3F Operations with directed numbers

3G Powers of directed numbers

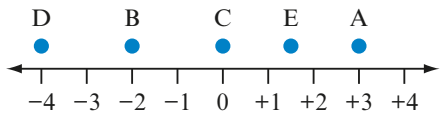
3H The Cartesian plane



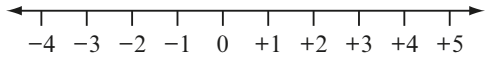
ESSENTIAL QUESTION

This fish lives 1500 m below sea level. Sea level is an example of a reference point: are there any others?

3A **1** List the positions of points A to E.



3B **2** Look at this number line.



a What number is 1 unit to the right of 3?

A 2 **B** 3 **C** 4 **D** 5

b What number is 4 units left of 1?

c What number is 5 units right of −2?

3E **3** **a** What is 30×4?

A 12 **B** 34 **C** 120 **D** 340

b What is $80 \div 4$?

A 20 **B** 76 **C** 84 **D** 320

3E **4** **a** What is 163×5?

b What is $154 \div 7$?

3F **5** **a** What is $\frac{2}{3} + \frac{1}{2}$?

A $\frac{3}{5}$ **B** $\frac{7}{6}$ **C** $\frac{7}{12}$ **D** 1

b What is $\frac{4}{5} - \frac{3}{4}$?

3F **6** **a** What is $\frac{15}{28} \times \frac{8}{9}$?

A $\frac{7}{19}$ **B** $\frac{14}{15}$ **C** $\frac{23}{37}$ **D** $\frac{10}{21}$

b What is $1\frac{5}{6} \div 7\frac{1}{3}$?

3F **7** What is:

a $6.8 + 5.3$? **b** $7.4 - 4.5$?

3F **8** What is:

a 3.6×0.8? **b** $8.4 \div 0.4$?

3F **9** What is the average of 5, 7, 8, 10 and 6?

3G **10** **a** What is the index or power in 2^7?

A 2 **B** 7 **C** 9 **D** 14

b What is the base in 2^7?

A 2 **B** 7 **C** 9 **D** 14

c What is 4^3 equivalent to?

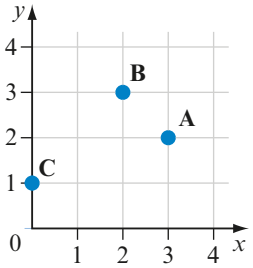
A $4 \times 3 = 12$

B $4 + 4 + 4 = 12$

C $4 \times 4 \times 4 = 64$

D $3 \times 3 \times 3 \times 3 = 81$

3H **11** Look at this Cartesian plane.



a What are the coordinates of point A?

A (1, 0) **B** (2, 3)
C (0, 1) **D** (3, 2)

b What are the coordinates of point B?

A (1, 0) **B** (2, 3)
C (0, 1) **D** (3, 2)

c What are the coordinates of point C?

A (1, 0) **B** (2, 3)
C (0, 1) **D** (3, 2)

3A Understanding negative numbers

Start thinking!



The average temperature in Beijing is −4.5°C in January and −2°C in February. However, the average daily minimum and maximum temperatures (see table) would be more useful if you were visiting Beijing.

Month	Average daily minimum temperature (°C)	Average daily maximum temperature (°C)
January	−9.4	1.6
February	−6.9	4.0

1 Which of the six temperatures are:

a **negative numbers**? **b** **positive numbers**?

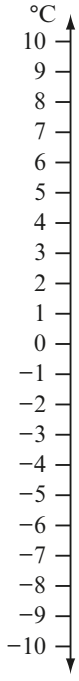
2 Whole numbers that are positive, negative or zero are called **integers**. Which of the six temperatures are integers?

3 A vertical number line can represent the scale on a thermometer, showing both positive and negative temperatures.

a Copy the vertical number line shown and show all six temperatures on it.

b Which temperature is the highest? Which is the lowest?

c A number is positive or negative depending on its position from a reference point. What is the reference point on this number line?



Positive and negative numbers are also called **directed numbers**, as they have both size and direction. The number of units away from the reference point is the size of the number, and the sign − or + (or no sign) is the direction from the reference point. Directed numbers include positive and negative whole numbers, fractions and decimals.

4 Write five examples of directed numbers, including the direction from the reference point.

5 Redraw the number line as a horizontal number line and show all six temperatures on it.

6 How you can tell whether a number is larger or smaller than another number on:

a the vertical number line? **b** the horizontal number line?

7 List the six temperatures in order from lowest to highest.

8 Which month (January or February) is generally colder in Beijing?

KEY IDEAS

- Positive and negative numbers are also called directed numbers as they have both size and direction. They include whole numbers, fractions and decimals.
- Whole numbers that are negative, positive or zero are called integers.
- Number lines can help you compare the size of directed numbers.
- **Opposite integers** are located the same number of units away from zero but on opposite sides of a number line.

EXERCISE 3A Understanding negative numbers

1 Look at the number line opposite.

a Write three examples of:

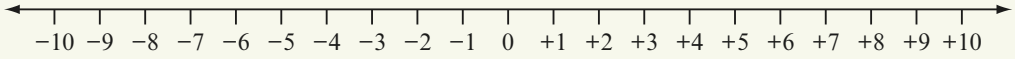
i a negative integer **ii** a positive integer.

b Which integer on the number line is neither positive nor negative?

EXAMPLE 3A-1 Identifying the larger of two integers

Which integer is larger in each pair of numbers? Use the number line to help you.

a −5 and 2 **b** −4 and −7



THINK

a Locate the two numbers on the number line and compare their positions. 2 is to the right of −5 so 2 is larger.

b Locate the two numbers on the number line and compare their positions. −4 is to the right of −7 so −4 is larger.

WRITE

a 2 is larger than −5.

b −4 is larger than −7.

UNDERSTANDING AND FLUENCY

2 Which integer is larger in each pair of numbers?

a 3 and 9 **b** 0 and 5 **c** −7 and 1

d 5 and −8 **e** 0 and −2 **f** −6 and −4

3 Which integer is smaller in each pair of numbers?

a 5 and 6 **b** 0 and 3 **c** −2 and 7

d 4 and −5 **e** 0 and −1 **f** −8 and −9

4 Which number is larger in each pair of numbers?

a 0 and 5.2 **b** $\frac{1}{2}$ and $-1\frac{1}{2}$

c 4.3 and −6.2 **d** $-\frac{3}{7}$ and $\frac{4}{7}$

e −1.5 and −1.7 **f** $-\frac{3}{5}$ and $-\frac{2}{5}$

5 Which number is smaller in each pair of numbers?

a 0 and $-4\frac{1}{2}$ **b** −2.1 and −1.2

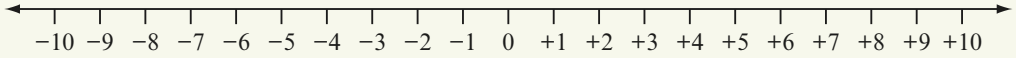
c 5.43 and −6.34 **d** $-8\frac{5}{11}$ and $-8\frac{6}{11}$

e −3.54 and −3.45 **f** $-\frac{2}{3}$ and $-\frac{3}{4}$

EXAMPLE 3A-2 Comparing directed numbers

Copy and complete each number statement by writing < (is less than) or > (is greater than) in the space provided. Use the number line to help you.

a −4 ___ 3 **b** −5.6 ___ −8.1



THINK	WRITE
a 1 Locate the two numbers on the number line and compare their positions. −4 is to the left of 3 so −4 is smaller.	**a** −4 is smaller than 3.
2 Complete the statement with the correct symbol. Remember that mathematical statements are read from left to right (−4 is less than 3).	−4 < 3
b 1 Locate the two numbers on the number line and compare their positions. −5.6 is to the right of −8.1 so −5.6 is larger.	**b** −5.6 is larger than −8.1.
2 Complete the statement with the correct symbol. The statement is read as '−5.6 is greater than −8.1'.	−5.6 > −8.1

UNDERSTANDING AND FLUENCY

6 Copy and complete each number statement by writing < (is less than) or > (is greater than) in the space provided. Use the number line above to help you.

a −3 ___ 2 **b** −8 ___ −4 **c** 0 ___ −1
d 9 ___ −9 **e** −3 ___ −5 **f** −7 ___ −2
g 1.4 ___ −2.6 **h** −1.5 ___ −1.2 **i** 4.3 ___ −4.3
j −3.8 ___ −5.8 **k** −0.6 ___ −0.7 **l** −8.2 ___ 0

7 Which number is larger in each pair of numbers?

a 35 and −45 **b** −58 and −24 **c** 0 and −82
d 0 and 47 **e** $-50\frac{1}{4}$ and $-70\frac{1}{2}$ **f** 113.2 and −225.7
g $-94\frac{2}{3}$ and $-94\frac{1}{3}$ **h** −89.7 and −89.3 **i** $-2000\frac{1}{6}$ and $-2000\frac{1}{7}$

8 Write each list of integers in ascending order (from smallest to largest).

a 14, −20, 10, −7, 8, −11, −12 **b** −33, 42, −19, −41, 0, 6, 29
c −8, −88, 68, 8, −28, −18, −48 **d** 73, −82, 3, −140, 104, 145, −126

9 Write each list of numbers in descending order (from largest to smallest).

a −15.4, 71.9, −27.6, −10.8, 4.3, −9.7 **b** $-\frac{1}{2}$, $-2\frac{1}{2}$, $5\frac{1}{2}$, 10, $-11\frac{1}{2}$, −5
c 0, 5.34, −9.04, −14.2, 21.6, −9.4 **d** $-1\frac{1}{3}$, $5\frac{1}{3}$, $-7\frac{1}{3}$, $-6\frac{2}{3}$, $7\frac{1}{3}$, $1\frac{2}{3}$, $-7\frac{2}{3}$

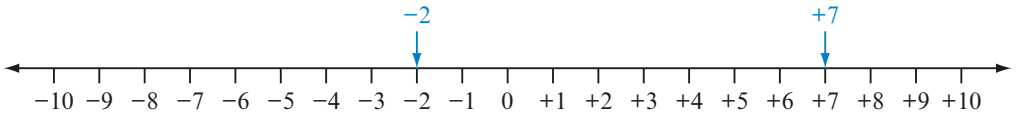
UNDERSTANDING AND FLUENCY

10 List all the integers between −7 and +4.

11 List five fractions that are between −30 and −20.

12 List five decimals that are between −5 and 3.

13 Look at this number line, which shows integers from −10 to +10. The integer −2 is 2 units to the left of 0 and the integer +7 (or just 7) is 7 units to the right of 0.



a What is the reference point for these directed numbers?

b Describe the position of each integer from 0 on the number line.

i −4 **ii** +5 **iii** +3

iv −5 **v** −3 **vi** +9

c Which pairs of integers in part **b** are the same number of units away from 0 but on opposite sides of the number line? These integers are called opposite integers.

d Write the opposite integer to these.

i −8 **ii** +6 **iii** +1

iv −11 **v** −32 **vi** 17

e List three different pairs of opposite integers.

EXAMPLE 3A-3 Representing a situation with a directed number

Write a directed number to represent each situation.

a The temperature is 15 degrees above zero. **b** A fish swims 4 m below the surface of the water.

c Your bank account is overdrawn by $30.

THINK

a **1** Identify the reference point.

2 Decide if the given temperature is above or below the reference point (above).

b **1** Identify the reference point.

2 Decide if the given distance is above or below the reference point (below).

c **1** Identify the reference point.

2 Decide if the overdrawn amount is above or below the reference point. If there is money in the account it is 'above', but if money is owed to the account it is 'below'.

WRITE

a Reference point is 0°.

Directed number is +15.

b Reference point is the water surface.

Directed number is −4.

c Reference point is $0.

Directed number is −30.

UNDERSTANDING AND FLUENCY

14 Write a directed number to represent each situation.



- **a** The lift stops at the third floor below the ground floor.
- **b** You have $2174.30 in the bank.
- **c** The temperature inside a freezer is 18 degrees below zero.
- **d** The shoreline of the Dead Sea, Jordan, is 408 m below sea level.
- **e** The top of Mt Kilimanjaro, Tanzania, is 5895 m above sea level.
- **f** A skydiver is 500 m above the ground.
- **g** Your bank account is overdrawn by $46.55.
- **h** A submarine is 40 m below sea level.

PROBLEM SOLVING AND REASONING

15 Polar bears live close to the edge of sea ice in the Arctic, where in winter the temperatures range from around −45°C to 0°C. In summer, the temperatures range from −10°C to 10°C. The ocean temperature is about −1.5°C.



- **a** Show −45°C, 0°C, −10°C, 10°C and −1.5°C on a number line.
- **b** Is −10°C higher or lower than −45°C?
- **c** Write the five temperatures in order from lowest to highest.
- **d** A polar bear dips her paw into the water to catch a fish. On a day with an air temperature of −18°C, does the water feel warmer or colder?

16 Bank statements usually show a transaction as a positive number if an amount is added or credited to the account or a negative number for withdrawals or debits. Use the bank statement to answer these questions.

Transaction	Balance
	+$51.50
+23.65	+$75.15
−35.00	+$40.15
−51.30	−$11.15
+34.45	+$23.30
+13.40	+$36.70
−49.95	−$13.25

- **a** What is:
  - **i** the largest amount credited to the account?
  - **ii** the smallest amount credited to the account?
- **b** What is:
  - **i** the largest amount debited from the account?
  - **ii** the smallest amount debited from the account?
- **c** What does the final balance indicate about the account?

PROBLEM SOLVING AND REASONING

17 Automatic teller machines (ATMs) allow you to deposit and withdraw money from your bank account.

- **a** Ashwan checks his bank balance and sees he has $86 in his account. Write this as a positive integer.
- **b** The bank allows Ashwan's account to be overdrawn. This means that he can withdraw more money than he actually has in his account. If he withdraws $100 at an ATM, how much does he owe the bank?
- **c** Write his new bank balance as a negative integer.

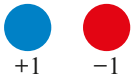


18 Jasmine needs glasses for reading as she is long sighted (hypermetropic). The prescription for the lenses in her glasses is +2.50. Alex wears contact lenses because he is short sighted (myopic). His lenses have prescriptions of −3.50 for his left eye and −4.25 for his weaker right eye.

- **a** If Justin has a prescription of +1.75, is he long or short sighted?
- **b** If Elle has a prescription of −4.75, is she long or short sighted?
- **c** How does Elle's prescription compare to Alex's? Who has weaker vision?
- **d** What do you think the reference of zero means in this situation?

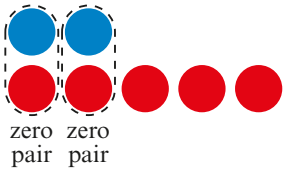
CHALLENGE

19 One way to represent integers is with coloured counters. Use one blue counter to represent +1 and one red counter to represent −1 (or any two colours will do). Collect 10 blue counters and 10 red counters.

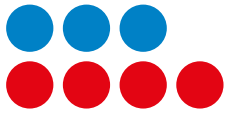
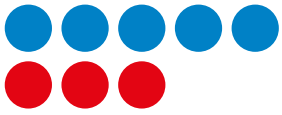


- **a** How many red counters would you use to represent −3?
- **b** Use the counters to show each integer listed below. Describe each arrangement.

  i +2 **ii** −4 **iii** +6
  iv −5 **v** −1 **vi** +8
- **c** Think about how you could use the counters to represent 0. Why does having one blue counter and one red counter together represent 0? This is called a **zero pair**. (Hint: use the idea of opposite integers.)



- **d** How does this arrangement of counters show −3?
- **e** What integer does each arrangement show? Remember that a zero pair can be made by matching one blue and one red counter together.

i 
ii 
iii 

- **f** You can show any integer in many ways. For example, −5 can be shown with five red counters, or six red counters and one blue counter, or nine red counters and four blue counters and so on. Describe three different ways each integer can be shown.

  i +2 **ii** −4 **iii** −7
  iv +3 **v** 0 **vi** −2

Reflect

How are negative numbers useful?

3B Adding integers

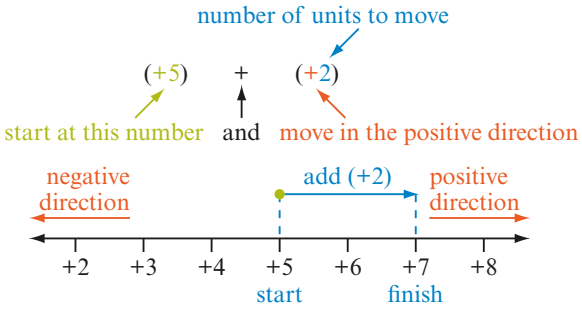
Start thinking!

One way to add integers is with a number line.

1 **a** What is 5 + 2?

b Another way of writing 5 + 2 is (+5) + (+2).
To show this on a number line, start at +5 and move +2 units. That is, move 2 units in the positive direction.
Do you move to the left or right of +5 to add +2?

c What number do you finish at? Does this match your answer to part **a**?



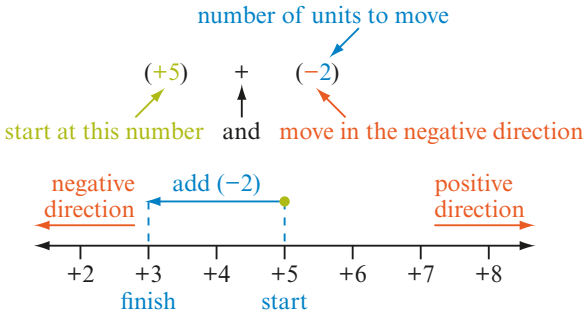
2 **a** Draw a number line for (−7) + (+3). Start at −7 and move +3 units.

b What is (−7) + (+3)?

c Do you move to the left or right of −7 to add +3?

3 **a** To show (+5) + (−2) on a number line, start at +5 and move −2 units. That is, move 2 units in the negative direction. Do you move to the left or right of +5 to add −2?

b What is (+5) + (−2)?



4 **a** Draw a number line for (−7) + (−3). Start at −7 and move −3 units.

b What is (−7) + (−3)?

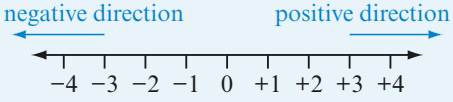
c Do you move to the left or right of −7 to add −3?

5 Explain how to use a number line to:

a add a positive integer **b** add a negative integer.

KEY IDEAS

- On a horizontal number line, moving right is moving in the positive direction and moving left is moving in the negative direction.



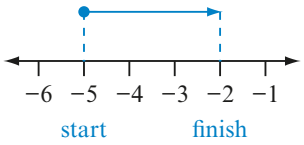
- To add two integers using a number line, start at the position of the first integer and then move left or right the number of units shown by the second integer. The final position is the result.
- To add a positive number, move to the right (or in a positive direction).
- To add a negative number, move to the left (or in a negative direction).

EXERCISE 3B Adding integers

UNDERSTANDING AND FLUENCY

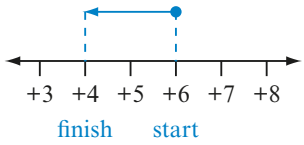
1 Copy and complete each number sentence using the diagram provided.

a



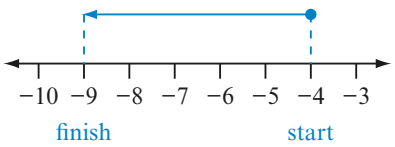
(−5) + (+3) = ___

b



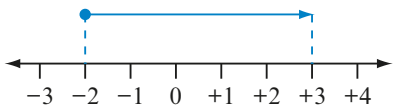
(+6) + (−2) = ___

c



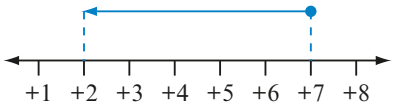
(−4) + (−5) = ___

d



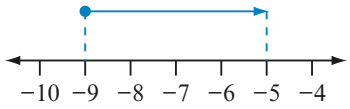
(−2) + (+___) = ___

e



(___) + (−5) = ___

f



(___) + (___) = ___

EXAMPLE 3B-1 Using a number line to add integers

Use a number line to calculate:

a (+2) + (−5) **b** (−4) + (+9).

THINK

a **1** Locate the first number (+2) on the number line. To add −5, move 5 units to the left (negative direction).

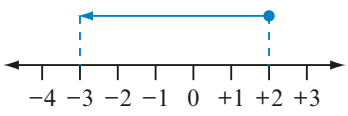
2 Locate the finishing point and write the result.

b **1** Locate −4 on the number line. To add +9, move 9 units to the right (positive direction).

2 Locate the finishing point and write the result.

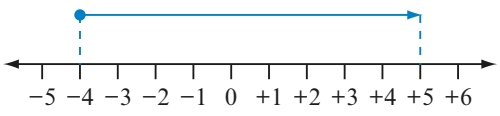
WRITE

a



(+2) + (−5) = −3

b



(−4) + (+9) = +5

2 Use a number line to calculate:

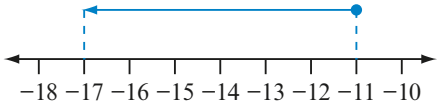
a $(-4) + (+7)$ **b** $(+1) + (+5)$ **c** $(+8) + (-3)$
d $(-2) + (-6)$ **e** $(-9) + (+5)$ **f** $(+3) + (-6)$
g $(-5) + (-1)$ **h** $(+7) + (-2)$ **i** $(+2) + (-6)$
j $(-9) + (+9)$ **k** $(+4) + (-4)$ **l** $(-1) + (+1)$

3 Draw a number line showing integers from -10 to 10 and use it to calculate:

a $(-4) + (+6)$ **b** $(+1) + (+2)$ **c** $(-8) + (+3)$
d $(+5) + (-7)$ **e** $(+9) + (-4)$ **f** $(-3) + (+4)$
g $(+7) + (+2)$ **h** $(-6) + (-3)$ **i** $(-5) + (+5)$
j $(+8) + (-8)$ **k** $(+2) + (-4)$ **l** $(-2) + (+2)$

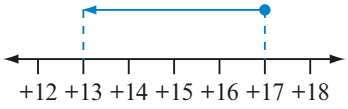
4 Copy and complete each number sentence using the diagram provided.

a



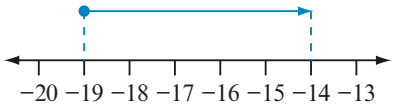
(−11) + (−____) = _____

b



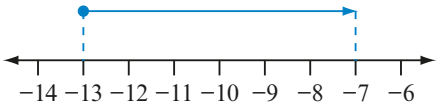
(_____) + (−4) = _____

c



(_____) + (_____) = _____

d



(_____) + (_____) = _____

5 Draw a number line showing integers from -20 to 20 and use it to calculate:

a $(-9) + (-7)$ **b** $(+14) + (-6)$
c $(-11) + (+18)$ **d** $(+5) + (+13)$
e $(+16) + (-16)$ **f** $(-12) + (+4)$
g $(+8) + (-15)$ **h** $(-7) + (-7)$
i $(-13) + (-6)$ **j** $(+10) + (+5)$
k $(-15) + (+17)$ **l** $(+11) + (-20)$
m $(-19) + (+19)$ **n** $(+4) + (-16)$
o $(+10) + (+10)$ **p** $(+14) + (-18)$
q $(-17) + (-3)$ **r** $(+20) + (-15)$

6 Use the words *positive* and *negative* to complete these statements.

a Adding two positive integers gives a _____ result.
b Adding two negative integers gives a _____ result.
c Adding a positive integer and a negative integer gives a result that is either _____ or _____ or zero, depending on the numbers involved.

7 Look at your answer to question **6c**. How can you tell by looking at the numbers to be added whether the answer will be positive, negative or zero?

EXAMPLE 3B-2 Identifying positive, negative or zero results

Decide whether the result of each problem will be positive, negative or zero.

a (+12) + (+15) **b** (−7) + (−11) **c** (−19) + (+19)
d (−13) + (+9) **e** (−12) + (+17)

THINK

a The result of adding two positive numbers is always positive.

b The result of adding two negative numbers is always negative.

c The result of adding opposite numbers is always zero.

d The result of adding a negative number and a positive number could be positive or negative. Since −13 is further from zero than +9 is, the result is negative.

e The result of adding a negative number and a positive number could be positive or negative. Since +17 is further from zero than −12 is, the result is positive.

WRITE

a The result for (+12) + (+15) is positive.

b The result for (−7) + (−11) is negative.

c The result for (−19) + (+19) is zero.

d The result for (−13) + (+9) is negative.

e The result for (−12) + (+17) is positive.

UNDERSTANDING AND FLUENCY

8 Decide whether the result of each problem will be positive, negative or zero.

a (+10) + (+7) **b** (−14) + (−5)
c (+15) + (−15) **d** (+12) + (−19)
e (−8) + (−9) **f** (−12) + (+16)
g (+18) + (−13) **h** (+11) + (+30)
i (−25) + (+25) **j** (−16) + (+11)
k (+22) + (−19) **l** (+14) + (−17)

9 Without using a number line, calculate each problem in question **8**. Explain how you did this.

10 Calculate:

a (+15) + (+13) **b** (−22) + (−7)
c (+19) + (−11) **d** (−20) + (+15)
e (+13) + (−13) **f** (−15) + (−15)
g (+24) + (−17) **h** (−30) + (+12)
i (+20) + (−24) **j** (−32) + (−15)
k (+43) + (−31) **l** (−50) + (+27)
m (+77) + (−94) **n** (−63) + (+82)
o (+111) + (−49) **p** (−124) + (−56)

11 An ice sculpture starts to melt. From a starting temperature of −6°C, the temperature of the surface of the sculpture increases by 8°C.



a Which problem would you use to find the temperature of the sculpture's surface?

A (+6) + (+8)

B (+6) − (+8)

C (−6) + (−8)

D (−6) + (+8)

b Use your answer to part **a** to work out the temperature of the surface.

12 Brandon enters a lift three levels below ground level. He selects a floor and travels seven levels upwards.

a Which problem would you use to work out the level at which the lift stops?

A (+3) + (+7)

B (−3) + (−7)

C (−3) + (+7)

D (+3) − (−7)

b Use your answer to part **a** to work out where the lift stops.

c Which level did you use as the reference level?



PROBLEM SOLVING AND REASONING

13 The air temperature at midnight at the Mawson research base in Antarctica was measured to be −21°C. At 2 pm, the temperature was 12°C higher.



a Write an addition problem to calculate the temperature at 2 pm.

b What is the temperature at 2 pm?

14 A hole 3 m deep is dug for the foot of a pole that is 17 m long.

a Write the depth of the bottom of the hole as an integer if ground level is represented by zero.

b Write an addition problem to work out the height of the pole above the ground once it is secured.

c What is the height of the pole above the ground?

15 Jess owes her brother $60.

a Write this amount as an integer.

b She pays him $35. Write an addition problem to work out the amount she still owes him.

c How much does Jess still owe her brother?

CHALLENGE

16 Calculate:

a (−110) + (−13)
b (+54) + (+126)
c (−120) + (+8)
d (+15) + (−165)
e (−71) + (+100)
f (+62) + (−69)
g (−80) + (−90)
h (+111) + (−98)
i (−122) + (+122)
j (+85) + (+95)
k (+250) + (−125)
l (−324) + (+300)

17 Two samples of frozen nitrogen have a temperature of −212°C. The temperature of one sample is increased by 14°C and it becomes liquid. The other sample has its temperature increased by 19°C and turns to gas.

a Calculate the temperature of the liquid nitrogen. Show the addition problem you used to work out the result.

b Calculate the temperature of the nitrogen gas. Show the addition problem you used to work out the result.

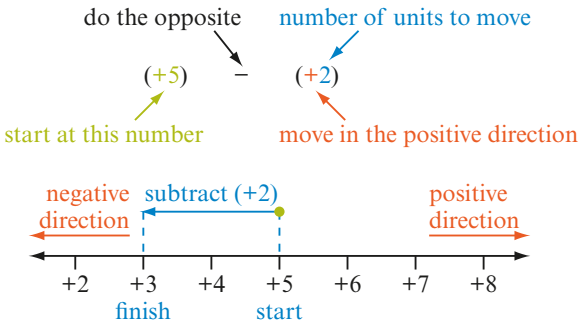
Reflect

How can you predict whether the result of adding two integers will be positive or negative?

3C Subtracting integers

Start thinking!

One way of subtracting integers is to use a number line.



1 **a** What is 5 − 2?

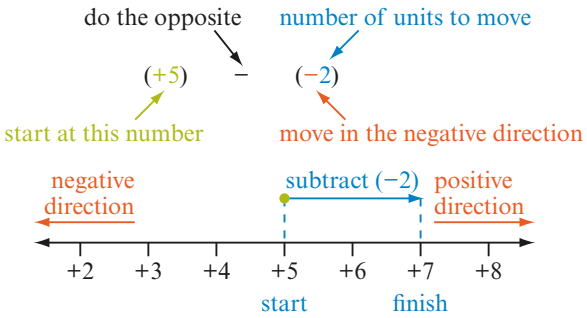
b Another way of writing 5 − 2 is (+5) − (+2).
To show this on a number line, start at +5. The second number (+2) indicates to move 2 units in the positive direction. However, the subtraction sign indicates to do the opposite and move in the negative direction.
Do you move to the left or right of +5 to subtract +2?

c What number do you finish at? Does this match your answer to part **a**?

2 **a** Draw a number line for (−7) − (+3). Start at −7 and instead of moving 3 units in the positive direction, do the opposite and move 3 units in the negative direction.

b What is (−7) − (+3)?

c Do you move to the left or right of −7 to subtract +3?



3 **a** To show (+5) − (−2) on a number line, start at +5.
The number −2 indicates to move 2 units in the negative direction. However, the subtraction sign indicates to do the opposite and move in the positive direction. Do you move to the left or right of +5 to subtract −2?

b What is (+5) − (−2)?

4 **a** Draw a number line for (−7) − (−3). Start at −7 and instead of moving 3 units in the negative direction, do the opposite and move 3 units in the positive direction.

b What is (−7) − (−3)?

c Do you move to the left or right of −7 to subtract −3?

5 Explain how to use a number line to:

a subtract a positive number **b** subtract a negative number.

KEY IDEAS

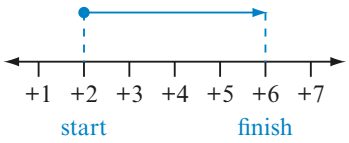
- To subtract one integer from another using a number line, start at the first integer and move left or right the number of units indicated by the second. The final position is the result.
- To subtract a positive number, move to the left (or in a negative direction).
- To subtract a negative number, move to the right (or in a positive direction).

EXERCISE 3C Subtracting integers

UNDERSTANDING AND FLUENCY

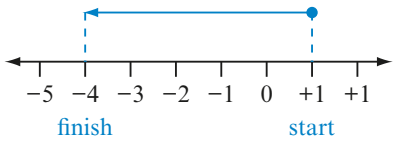
1 Copy and complete each number sentence using the diagram provided.

a



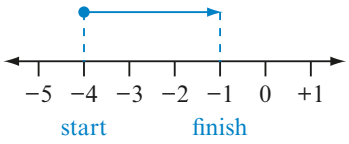
(+2) − (−4) = ___

b



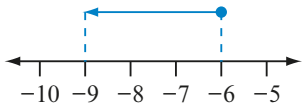
(+1) − (+5) = ___

c



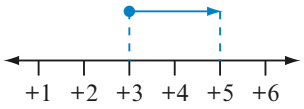
(−4) − (−3) = ___

d



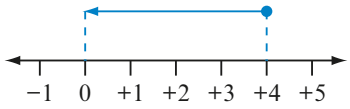
(−6) − (+___) = ___

e



(___) − (−2) = ___

f



(___) − (___) = ___

EXAMPLE 3C-1 Using a number line to subtract integers

Use a number line to calculate:

a (+3) − (+5) **b** (−1) − (−4).

THINK

a **1** Locate the first number (+3) on the number line. To subtract +5, move 5 units to the left (negative direction).

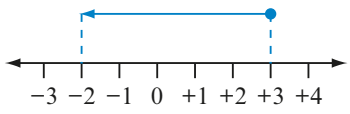
2 Locate the finishing point and write the result.

b **1** Locate the first number (−1) on the number line. To subtract −4, move 4 units to the right (positive direction).

2 Locate the finishing point and write the result.

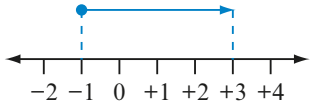
WRITE

a



(+3) − (+5) = −2

b



(−1) − (−4) = +3

2 Use a number line to calculate:

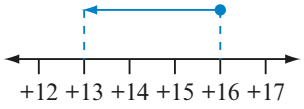
a (+6) − (+4)	**b** (−9) − (−2)	**c** (+3) − (+3)
d (−5) − (−4)	**e** (+7) − (−1)	**f** (−4) − (+2)
g (−1) − (+6)	**h** (+2) − (−8)	**i** (−3) − (−6)
j (+2) − (+7)	**k** (+5) − (+8)	**l** (−1) − (−9)

3 Draw a number line showing integers from −10 to 10 and use it to calculate:

a (+3) − (+8)	**b** (−6) − (+4)	**c** (+5) − (−1)
d (−2) − (−9)	**e** (−7) − (+2)	**f** (−8) − (−8)
g (+1) − (−3)	**h** (−4) − (+5)	**i** (+6) − (+5)
j (−3) − (−7)	**k** (+2) − (−2)	**l** (−5) − (−5)

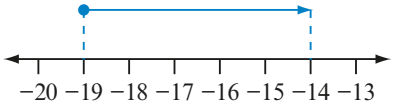
4 Copy and complete each number sentence using the diagram provided.

a



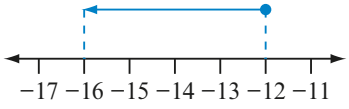
(+16) − (+___) = ___

b



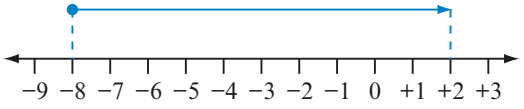
(___) − (−5) = ___

c



(___) − (___) = ___

d



(___) − (___) = ___

5 Draw a number line showing integers from −20 to 20 and use it to calculate:

a (+13) − (+8)	**b** (+18) − (−1)	**c** (+10) − (−10)
d (−7) − (+11)	**e** (+17) − (+17)	**f** (−19) − (+1)
g (+16) − (−2)	**h** (−12) − (+8)	**i** (−5) − (−12)
j (+16) − (+14)	**k** (−15) − (−15)	**l** (+6) − (−10)
m (−18) − (−7)	**n** (+15) − (+12)	**o** (−11) − (−13)
p (+14) − (+20)	**q** (−13) − (+7)	**r** (−16) − (−12)

6 Use the words *positive* and *negative* to complete these statements.

a Subtracting a negative integer from a positive integer gives a result that is _____.

b Subtracting a positive integer from a negative integer gives a result that is _____.

c Subtracting a positive integer from a positive integer, or a negative integer from a negative integer, gives a result that is either _____ or _____ or zero depending on the numbers involved.

7 Look at your answer to question **6c**. How can you tell by looking at the numbers to be subtracted whether the answer will be positive or negative or zero?

EXAMPLE 3C-2 Identifying positive, negative and zero results

Decide whether the result of each problem will be positive, negative or zero.

a (+12) − (−15) **b** (−7) − (+11) **c** (−19) − (−19)
d (−13) − (−9) **e** (+12) − (+17)

THINK

a Subtracting a negative number from a positive number always gives a positive result.

b Subtracting a positive number from a negative number always gives a negative result.

c Subtracting two identical numbers always gives a result of zero.

d Subtracting a negative number from a negative number could give a positive or negative result. Since −13 is further from zero than −9 is, the result is negative.

e Subtracting a positive number from a positive number could give a positive or negative result. Since +17 is further from zero than +12 is, the result is negative.

WRITE

a The result for (+12) − (−15) is positive.

b The result for (−7) − (+11) is negative.

c The result for (−19) − (−19) is zero.

d The result for (−13) − (−9) is negative.

e The result for (+12) − (+17) is negative.

UNDERSTANDING AND FLUENCY

8 Decide whether the result of each problem will be positive, negative or zero.

a (+11) − (−8) **b** (−16) − (+4)
c (+13) − (+13) **d** (+15) − (+18)
e (−10) − (−7) **f** (−17) − (+11)
g (+8) − (−14) **h** (+12) − (+19)
i (−18) − (−18) **j** (−14) − (−13)
k (+16) − (+21) **l** (−20) − (−27)

9 Without using a number line, calculate each problem in question **8**. Explain how you did this.

10 Calculate:

a (−24) − (+8) **b** (−3) − (−28)
c (+30) − (+16) **d** (−29) − (−29)
e (+38) − (+27) **f** (−42) − (−22)
g (−55) − (+14) **h** (+71) − (−69)
i (+86) − (+90) **j** (+65) − (−26)
k (+45) − (−45) **l** (−34) − (+52)
m (−53) − (+17) **n** (+68) − (−24)
o (−94) − (+72) **p** (−102) − (−129)



11 An apple pie is moved from the fridge to the freezer. After 30 minutes, the temperature of the pie has decreased from 4°C to −6°C.

a Which problem would you use to work out the difference in the two temperatures?

A (+6) − (+4)

B (−6) − (+4)

C (−6) − (−4)

D (−6) + (+4)

b Use your answer to part **a** to work out the temperature difference.



12 A captain orders his boat's anchor to be pulled up from a depth of 9 m. At a depth of 4 m it is snagged in seaweed, before finally reaching a position 1 m above the water line.

a Which problem would you use to work out the vertical distance the anchor has moved before becoming snagged?

A (−4) − (−9)

B (+1) − (+4)

C (−4) − (+9)

D (+1) − (−4)

b Use your answer to part **a** to work out this vertical distance.

c Repeat parts **a** and **b** but this time work out the vertical distance moved by the anchor after it is freed from seaweed.

d What did you use as the reference point for zero in each case?

13 What is the difference between a temperature of −18°C and a temperature of 33°C? Show the subtraction problem you used to calculate the result.

PROBLEM SOLVING AND REASONING

14 A helicopter is flying 28 m above the surface of the ocean. The pilot spots a shark swimming at an estimated depth of 3 m and a stingray at an estimated depth of 11 m below the surface of the water.



a If the ocean surface is the reference point for zero, write the position of the helicopter as a positive integer.

b Write the positions of the shark and the stingray as negative integers.

c Write a subtraction problem to calculate the vertical distance between:

i the helicopter and the shark

ii the helicopter and the stingray

iii the shark and the stingray.

(Hint: find the difference between the two positions.)

d Calculate each subtraction problem.

CHALLENGE

15 Calculate:

a $(-100) - (+24)$ **b** $(+60) - (+140)$ **c** $(-150) - (-10)$

d $(+25) - (-145)$ **e** $(-168) - (+200)$ **f** $(+77) - (-58)$

g $(-93) - (-98)$ **h** $(+133) - (-87)$ **i** $(-240) - (+240)$

j $(+176) - (+84)$ **k** $(+360) - (-235)$ **l** $(-415) - (+500)$

16 At the start of March, Tyra's bank account shows a balance of −$310. At the end of the month, it shows a balance of −$247.

a Calculate the difference between the two account balances. Show the subtraction problem you used to calculate the result.

b Interpret the result. (Hint: what does this result mean?)

17 Show three examples of a pair of integers that give a negative result when they are added and when they are subtracted.

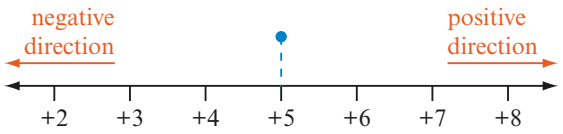
Reflect

How does a number line help when subtracting integers?

3D Simplifying addition and subtraction of integers

Start thinking!

Is there an easier way to add and subtract integers using number lines?



1 **a** Use the number line to calculate:
i (+5) + (+2) **ii** (+5) − (−2).
b Which direction did you move from +5 in each case?
c What does this tell you about adding a positive number and subtracting a negative number?
d (+5) + (+2) is the same as 5 + 2. Why can you also write (+5) − (−2) more simply as 5 + 2?

2 How were you able to make the problems in question **1** easier?

3 **a** Use the number line to calculate:
i (+5) − (+2) **ii** (+5) + (−2).
b Which direction did you move from +5 in each case?
c What does this tell you about subtracting a positive number and adding a negative number?
d (+5) − (+2) is the same as 5 − 2. Why can you also write (+5) + (−2) more simply as 5 − 2?

4 How were you able to make the problems in question **3** easier?

5 Explain how you can remember which equivalent operation to use when simplifying addition and subtraction problems.

KEY IDEAS

- In addition and subtraction problems, you can replace two operation signs that are next to each other with one equivalent sign.
- For example, (−6) − (−2) simplifies to −6 + 2, because − (−) is equivalent to +.
- When there is a + sign between two numbers, move to the right (positive direction) along the number line.
- When there is a − sign between two numbers, move to the left (negative direction) along the number line.

+ (+) is equivalent to +
− (−) is equivalent to +
+ (−) is equivalent to −
− (+) is equivalent to −

EXERCISE 3D Simplifying addition and subtraction of integers

1 Copy and complete this table to show the equivalent operation that simplifies the problem and which direction to move on a number line.

	Operations	Equivalent operation	Move in positive or negative direction?	Move left or right on a number line?
adding a positive number	+ (+)	+	positive	right
adding a negative number	+ (−)			
subtracting a positive number	− (+)			
subtracting a negative number	− (−)			

EXAMPLE 3D-1 Writing a simpler equivalent problem

Write each problem more simply.

a (−6) − (+4) **b** (+5) − (−8)

THINK

a Replace − (+) with −.

b Replace − (−) with + and write +5 as 5.

WRITE

a (−6) − (+4)
= −6 − 4

b (+5) − (−8)
= 5 + 8

UNDERSTANDING AND FLUENCY

2 Copy and complete to write a simpler equivalent problem. Use + or − to fill the gap.

a (−3) − (+7)
= −3 __ 7

b (+1) + (+6)
= 1 __ 6

c (−4) + (−5)
= −4 __ 5

d (+2) − (−4)
= 2 __ 4

e (+5) − (+9)
= 5 __ 9

f (−6) − (−8)
= −6 __ 8

3 Write each problem more simply.

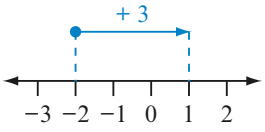
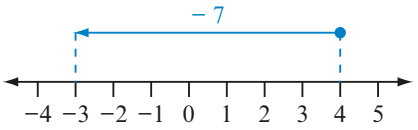
a (−1) − (−3)
b (+8) − (+4)
c (−5) + (−2)
d (+6) − (−1)
e (+4) + (+3)
f (+3) − (+9)
g (−5) − (−8)
h (−3) + (+1)
i (−7) + (−3)
j (−6) − (−7)

EXAMPLE 3D-2

Adding and subtracting integers using simpler equivalent problems

Calculate:

a $(-2) - (-3)$ **b** $(+4) + (-7)$

THINK	WRITE
a 1 Write the problem in a simpler equivalent form by replacing $-(-)$ with $+$.	**a** $(-2) - (-3)$ $= -2 + 3$
2 Locate -2 on the number line. For $+$, move 3 units right (positive direction).	
3 Locate the finishing point and write the result.	$(-2) - (-3) = 1$
b 1 Write the problem in a simpler equivalent form by replacing $+(-)$ with $-$.	**b** $(+4) + (-7)$ $= 4 - 7$
2 Locate 4 on the number line. For $-$, move 7 units left (negative direction).	
3 Locate the finishing point and write the result.	$(+4) + (-7) = -3$

UNDERSTANDING AND FLUENCY

4 Simplify each problem and then calculate the result.

a $(-4) - (-7)$	**b** $(+9) + (-3)$	**c** $(+2) - (+6)$
d $(-5) - (-5)$	**e** $(-8) + (+7)$	**f** $(+6) + (-1)$
g $(-3) - (-4)$	**h** $(+4) - (+9)$	**i** $(+2) - (-7)$

5 Draw a number line from -10 to 10 and use it to calculate:

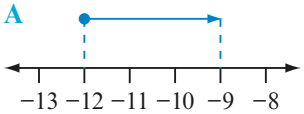
a $9 - 4$	**b** $3 - 5$	**c** $-2 + 8$
d $-6 - 4$	**e** $7 - 10$	**f** $-1 - 6$
g $8 - 4$	**h** $-7 + 9$	**i** $-10 + 10$
j $0 - 9$	**k** $-5 - 4$	**l** $-1 + 8$

6 Calculate each result for question **2**.

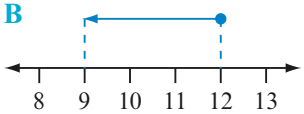
7 Calculate each result for question **3**.

8 Match each diagram (A, B, C) to the appropriate problem (a, b, c).

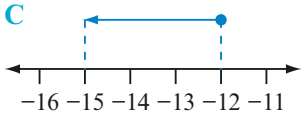
A



B



C



a $-12 - 3$ **b** $-12 + 3$ **c** $12 - 3$

UNDERSTANDING AND FLUENCY

9 Calculate each result for question **8**.

10 Draw a number line from −20 to 20 and use it to calculate:

a 15 − 12	**b** −17 + 11	**c** 9 − 13
d −4 + 20	**e** −2 − 17	**f** 8 − 18
g −14 − 3	**h** −16 + 14	**i** 11 − 24
j −19 + 19	**k** −10 − 10	**l** −18 + 20

11 Decide whether each result will be positive, negative or zero.

a −3 − 7	**b** 16 + 14	**c** 8 − 15
d −13 + 13	**e** −11 + 18	**f** 19 − 12
g −17 + 4	**h** −14 − 10	**i** 17 − 20

12 Without using a number line, calculate each result for question **11**. Explain how you did this.

13 Copy and complete each addition table.

a

+	−5	−3	0	1	4
−3				−2	
−2					
−1			−1		
3					
6	1				

b

+	−10	−14	−7	13	19
11					
14					
−10					
−22					
−18					

14 Calculate:

a −45 + 40	**b** −72 − 27	**c** 56 − 88
d −34 + 43	**e** 130 − 170	**f** −213 − 62
g −158 + 400	**h** 286 − 168	**i** −505 + 505
j 1000 − 3000	**k** −2040 − 940	**l** 1469 − 1500
m − 858 + 885	**n** 3400 − 4040	**o** − 5006 − 679

EXAMPLE 3D-3 Adding and subtracting integers

Calculate −8 + 3 − 6.

THINK	WRITE
1 Work from left to right. Calculate −8 + 3.	−8 + 3 − 6 = −5 − 6
2 Calculate −5 − 6.	= −11

UNDERSTANDING AND FLUENCY

15 Calculate:

a $-2 + 5 + 7$ **b** $3 - 9 + 2$

c $6 - 1 - 8$ **d** $-10 + 4 - 7$

e $-12 + 19 - 7$ **f** $-20 - 11 - 2$

g $33 + 22 - 65$ **h** $70 - 81 - 15$

i $-24 - 18 + 12$ **j** $59 - 93 + 17$

16 Write each as a simpler equivalent problem and then calculate the result.

a $(-5) - (+7) + (+6)$ **b** $(+9) + (-2) - (-8)$

c $(-4) - (-3) + (+1)$ **d** $(+8) + (-6) - (-10)$

e $(-22) - (+5) + (+13)$ **f** $(-14) - (-8) + (-25)$

g $(+34) + (-19) - (+7)$ **h** $(-9) - (+26) - (-37)$

PROBLEM SOLVING AND REASONING

17 A bungee jumper dives from a platform (position A) 50 m above the water. He dips 2 m under the water (position B) before rebounding to a height 18 m below the platform (position C).



a If the reference is the surface of the water, write positions A, B and C as integers.

b Write a problem to find the distance between:

i A and B

ii B and C

iii A and C.

c Calculate each distance.

18 You enter a lift at the 15th floor and travel down 19 floors. Which level do you finish at? Show how you calculated your answer.



19 Tahini has $285 in her bank account.

a Write this as an integer.

b She withdraws $300. Write a subtraction problem to calculate her new account balance.

c What is her new account balance as an integer? Interpret this result. (Hint: what does the integer tell you?)

PROBLEM SOLVING AND REASONING

20 A deep sea submersible is lowered into the ocean from a ship's deck 15 m above sea level. The unmanned vessel is on a mission to take photos of deep sea animals.





For each part, write an addition or subtraction problem and then calculate the answer.

a A deep sea anglerfish is 900 m below sea level. What vertical distance has the submersible travelled from the deck of the ship to reach this fish?

b A deep sea amphipod is 2500 m below sea level. How much further has the submersible travelled vertically to reach it?

c What vertical distance does the submersible now need to travel to return to the deck of the ship?

21 Sarah enters a lift at the third floor and travels down nine floors and then up five floors. What floor does she finish at? Show the problem you used to get your answer.

CHALLENGE

22 Write two different sets of three integers where the sum of the integers is negative.

23 What number should be added to each of these to give 0?

a −24 **b** 312 **c** 5

d −70 **e** −1018 **f** 4218

24 What number should be subtracted from each of these to give 0?

a 100 **b** −21 **c** 366

d −249 **e** −4000 **f** 9007

25 What number should be subtracted from each of these to give 20?

a 65 **b** −10 **c** 1281

d −347 **e** −1000 **f** −3240

26 Find the missing integer to make each number sentence true.

a −54 + 33 + ____ = 12

b 121 − ____ − 50 = −6

c −300 + 225 − ____ = 50

d ____ − 58 + 73 = −25

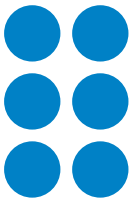
Reflect

How can you simplify addition and subtraction problems?

3E Multiplying and dividing integers

Start thinking!

Collect two sets of coloured counters. Choose one colour (say, blue) to represent positive integers and the other colour (say, red) to represent negative integers.

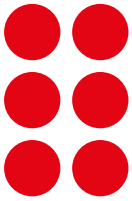


1 The product 3×2 or $+3 \times +2$ is the same as three lots of $(+2)$.
   a Arrange counters to show $(+2) + (+2) + (+2)$. Describe the arrangement.
   b What result does this give for $+3 \times +2$?

2 Show 2×3 or two lots of $(+3)$ with your counters. What is the result for $+2 \times +3$?

3 Compare your results for questions **1** and **2**. Does the order in which you multiply two numbers affect the result?

4 When multiplying two positive numbers, is the result positive or negative?



5 The product 3×-2 or $+3 \times -2$ is the same as three lots of (-2).
   a Arrange counters to show $(-2) + (-2) + (-2)$. Describe the arrangement.
   b What is the result for $+3 \times -2$?

6 When multiplying a positive number by a negative number, is the result positive or negative?

7 What would the result for $-2 \times +3$ be? (Hint: look at question **3**.)

8 When multiplying a negative number by a positive number, is the result positive or negative?

9 What happens when you multiply two negative numbers together?
   To find the result for -2×-3 you can first look at $+2 \times -3$.
   Since -2 is the opposite of $+2$, the result of -2×-3 is the opposite of the result to $+2 \times -3$.
   a What is the result for -2×-3?
   b When multiplying two negative numbers, is the result positive or negative?

KEY IDEAS

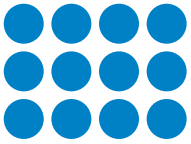
- Multiplying or dividing two integers can give a result that is positive or negative (or zero), according to these rules.
- Remember:
  - if the numbers have the same sign, the result is positive
  - if the numbers have different signs, the result is negative.

$+ \times + = +$	$+ \div + = +$
$+ \times - = -$	$+ \div - = -$
$- \times + = -$	$- \div + = -$
$- \times - = +$	$- \div - = +$

EXERCISE 3E Multiplying and dividing integers

1 Use the diagrams shown to find the result for each problem.

a



(+3) × (+4)

b



(+2) × (−7)

EXAMPLE 3E-1 Multiplying integers

Calculate:

a −4 × +7 **b** −9 × −5.

THINK

a Ignore the signs and calculate 4 × 7. (28)
Decide whether the result will be positive or negative. Signs are different so result is negative.

b Ignore the negative signs and calculate 9 × 5. (45)
Decide whether the result will be positive or negative. Signs are the same so result is positive.

WRITE

a −4 × +7
= −28

b −9 × −5
= +45 (or 45)

UNDERSTANDING AND FLUENCY

2 Calculate:

a +3 × −4	**b** +7 × +5	**c** −6 × + 2	**d** −3 × −9
e −1 × +8	**f** +4 × −6	**g** −8 × −9	**h** +2 × +10
i −4 × +11	**j** +7 × −1	**k** −2 × −4	**l** +9 × +5

3 a What happens when you divide integers? Dividing is the inverse operation to multiplying. Copy and complete each statement.

i +2 × +3 = ____ so ____ ÷ +2 = +3 or ____ ÷ +3 = +2
ii +2 × −3 = ____ so ____ ÷ +2 = −3 or ____ ÷ −3 = +2
iii −2 × +3 = ____ so ____ ÷ −2 = +3 or ____ ÷ +3 = −2
iv −2 × −3 = ____ so ____ ÷ −2 = −3 or ____ ÷ −3 = −2

b Does division follow the same pattern as multiplication to give positive or negative results?

c Is the result positive or negative for each of these division types?

i positive ÷ positive **ii** negative ÷ negative
iii positive ÷ negative **iv** negative ÷ positive

EXAMPLE 3E-2 Dividing integers

Calculate:

a $-30 \div -5$ b $+48 \div -6$.

THINK

a Ignore the signs and calculate $30 \div 5$. (6) Decide whether the result will be positive or negative. Signs are the same so result is positive.

b Ignore the signs and calculate $48 \div 6$. (8) Decide whether the result will be positive or negative. Signs are different so result is negative.

WRITE

a $-30 \div -5$
$= +6$ (or 6)

b $+48 \div -6$
$= -8$

UNDERSTANDING AND FLUENCY

4 Calculate:

a $+15 \div -3$ b $-32 \div -8$ c $+24 \div +4$
d $-63 \div +9$ e $-17 \div -1$ f $+48 \div -6$
g $-56 \div +8$ h $-81 \div -9$ i $+42 \div -7$
j $-12 \div +1$ k $-20 \div -2$ l $+72 \div -8$

5 Calculate:

a 5×-7 b $-9 \div 3$ c $-28 \div -4$
d -10×6 e $36 \div -9$ f -15×-4
g $-100 \div 20$ h 18×-2 i -3×-30
j $25 \div -25$ k -1×38 l $-45 \div -1$

6 Copy and complete each statement.

a $-15 \times$ ____ $= 45$ b $56 \div$ ____ $= -8$
c ____ $\times -4 = 36$ d ____ $\div -25 = -4$
e $-140 \div$ ____ $= 2$ f ____ $\times 16 = -48$

7 Copy and complete each multiplication table.

a

×	−2	−1	0	+1	+2
−4	8			−4	
−2					
0			0		
+2					
+4		−4			
+6					

b

×		−20	−10		20
7				70	
			−50		
0					
−2	50				
		80			
					−120

EXAMPLE 3E-3 Dividing integers written as a fraction

Calculate $\frac{28}{-7}$.

THINK	WRITE
1 Write the fraction as a division problem.	$\frac{28}{-7}$ $= 28 \div -7$
2 Ignore the signs and calculate $28 \div 7$. Decide whether the result will be positive or negative. Signs are different so result is negative.	$= -4$

8 Calculate each of these.

a $\frac{-48}{6}$ **b** $\frac{60}{-5}$ **c** $\frac{-51}{-3}$ **d** $\frac{+150}{+10}$ **e** $\frac{-200}{-25}$ **f** $\frac{57}{-19}$

9 A short way of writing -1×-2 is $-(-2)$. Write each problem as the product of two numbers and then calculate the result.

a $-(-7)$ **b** $-(+12)$ **c** $-(-88)$
d $-(25)$ **e** $-(8 - 5)$ **f** $-(-3 + 2)$

EXAMPLE 3E-4 Multiplying three integers

Calculate $-3 \times -5 \times -2$.

THINK	WRITE
1 Work from left to right. Multiply the first two numbers. $(-3 \times -5 = +15)$	$-3 \times -5 \times -2$ $= +15 \times -2$
2 Multiply the result $(+15)$ by the third number (-2).	$= -30$

10 Calculate:

a $-4 \times +3 \times -5$ **b** $+3 \times -2 \times +7$ **c** $+6 \times -1 \times +9$
d $+8 \times +2 \times -3$ **e** $-5 \times -2 \times -13$ **f** $-7 \times 3 \times -4$
g $4 \times 5 \times -8$ **h** $-11 \times 5 \times -2$ **i** $9 \times -4 \times 5$

11 Calculate:

a $-2 \times 2 \times -3 \times 3$ **b** $20 \div -2 \div -2 \div -1$
c $-4 \times -1 \times 3 \times -2$ **d** $-18 \div -2 \times 4 \div -6$
e $9 \times -2 \div 3 \times -5$ **f** $-7 \times -4 \div 2 \times -3$

12 Copy and complete each set of problems. Describe the pattern you see.

a	**b**	**c**	**d**
$2 \times 5 =$ ____	$-3 \times 5 =$ ____	$-4 \times -5 =$ ____	$5 \times -5 =$ ____
$2 \times 4 =$ ____	$-3 \times 4 =$ ____	$-4 \times -4 =$ ____	$5 \times -4 =$ ____
$2 \times 3 =$ ____	$-3 \times 3 =$ ____	$-4 \times -3 =$ ____	$5 \times -3 =$ ____
$2 \times 2 =$ ____	$-3 \times 2 =$ ____	$-4 \times -2 =$ ____	$5 \times -2 =$ ____
$2 \times 1 =$ ____	$-3 \times 1 =$ ____	$-4 \times -1 =$ ____	$5 \times -1 =$ ____
$2 \times 0 =$ ____	$-3 \times 0 =$ ____	$-4 \times 0 =$ ____	$5 \times 0 =$ ____
$2 \times -1 =$ ____	$-3 \times -1 =$ ____	$-4 \times 1 =$ ____	$5 \times 1 =$ ____
$2 \times -2 =$ ____	$-3 \times -2 =$ ____	$-4 \times 2 =$ ____	$5 \times 2 =$ ____
$2 \times -3 =$ ____	$-3 \times -3 =$ ____	$-4 \times 3 =$ ____	$5 \times 3 =$ ____
$2 \times -4 =$ ____	$-3 \times -4 =$ ____	$-4 \times 4 =$ ____	$5 \times 4 =$ ____
$2 \times -5 =$ ____	$-3 \times -5 =$ ____	$-4 \times 5 =$ ____	$5 \times 5 =$ ____

13 **a** Calculate:

i $+8 \times 0$ **ii** -15×0

iii 0×65 **iv** 0×-72

b What do you notice when you multiply any integer by zero?

c Calculate:

i $0 \div -9$ **ii** $0 \div +26$

iii $-11 \div 0$ **iv** $39 \div 0$

d What do you notice when you divide zero by any integer?

e What happens when you try to divide any number by zero? What does this result mean?

14 **a** List five examples of pairs of integers that multiply to give:

i -24 **ii** 18.

> **NOTE** For some of your examples, include at least one negative integer.

b List five examples of pairs of integers that divide to give:

i -4 **ii** 5.

c List three examples of pairs of integers that multiply to give 0.

d List three examples of pairs of integers that divide to give 0.

e List five examples of a set of three integers that multiply to give:

i -48 **ii** 120.

15 Find two integers that have:

a a sum of -9 and a product of 18

b a sum of 4 and a product of -45

c a sum of -5 and a product of -66

d a sum of 10 and a product of -56

e a difference of -2 and a product of 24

f a difference of -8 and a product of 9

g a difference of 18 and a product of -32.

PROBLEM SOLVING AND REASONING

16 Astro the dog has escaped from his yard. His owners decide to search along the road which runs east–west outside the house. This number line can be used to represent the road outside the family's house, with the house at zero.

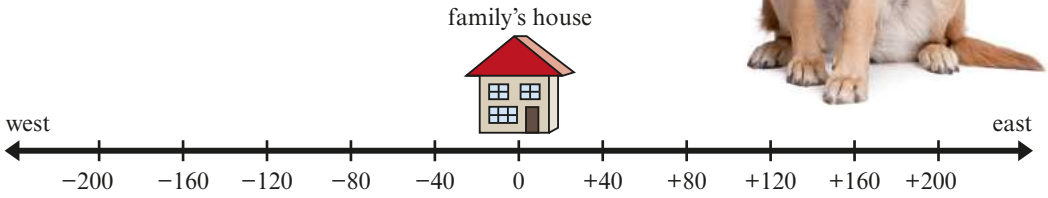


a Tyler and Rhys walk 120 m west from the house and call Astro's name.
What is their position on the number line?

b Sophie and Imogen walk 80 m east from the house and call Astro's name.
What is their position on the number line?

c What is the distance between the pairs of searchers?

d Imogen now walks a further 30 m east.
What is her new position on the number line?



e Rhys now walks 160 m east. What is his new position on the number line?

f What is the distance between:

i Sophie and Rhys?

ii Tyler and Imogen?

g Natalie walks twice as far as Sophie but in the same direction as Tyler.
What is her position on the number line?

h Hayden walks one-third of the distance walked by Tyler and in the same direction.
What is his position on the number line?

i A neighbour has found Astro in her yard, 60 m west of the family's house.
Who is closest to Astro? How far and in which direction does he or she need to walk to collect the dog?

j If each member of the family now walks home, calculate the distance each person walked from the start of the search.

CHALLENGE

17 For each calculation:

i estimate the result by rounding each number to its leading digit.

ii find the exact result.

a $21 \times -3 + 19 \times 2$ **b** $-11 \times -37 - 980 \div 49$

18 Write a problem that contains three integers and two multiplication signs to give a result of −42.

19 Write a problem that contains three integers, one multiplication sign and one division sign to give a result of 8.

20 Write a problem that contains four different integers to give a result of −1000.

Reflect

How do you identify whether the result of multiplying or dividing integers is positive, negative or zero?

3F Operations with directed numbers

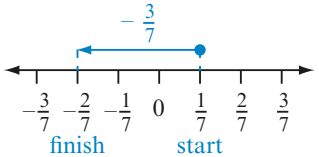
Start thinking!

You can add or subtract positive and negative fractions and decimals in the same way as integers.

1 Look at $\left(+\frac{1}{7}\right) + \left(-\frac{3}{7}\right)$.

a Explain why this can be simplified to $\frac{1}{7} - \frac{3}{7}$.

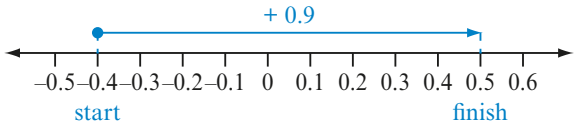
b Use this number line to calculate the result.



2 Look at $(-0.4) - (-0.9)$.

a Write it as a simpler equivalent problem.

b Use this number line to calculate the result.



3 You can also multiply or divide positive or negative fractions and decimals in the same way as integers. Copy these statements and use the words *positive* and *negative* to complete them.
To decide whether the result is positive or negative, remember that:

a positive × positive = _____

b positive ÷ positive = _____

c positive × negative = _____

d positive ÷ negative = _____

e negative × positive = _____

f negative ÷ positive = _____

g negative × negative = _____

h negative ÷ negative = _____

KEY IDEAS

- Directed numbers include integers (positive and negative whole numbers and zero), positive and negative fractions, and positive and negative decimals. They have size and direction.
- You can use a number line to add or subtract directed numbers. Start at the position of the first number and then move left or right the number of units indicated by the second number. The final position is the result.
- To add a positive number or subtract a negative number, move to the right (positive direction) along the number line. Remember that + (+) and − (−) simplify to +.
- To add a negative number or subtract a positive number, move to the left (negative direction) along the number line. Remember that + (−) and − (+) simplify to −.
- When multiplying or dividing two numbers with the same sign, the result is positive.
- When multiplying or dividing two numbers with different signs, the result is negative.

EXERCISE 3F Operations with directed numbers

1 Draw a number line with a scale from −10 to 10 and show the position of each directed number.

a −3 **b** +2.5 **c** $-\frac{1}{2}$ **d** 5 **e** $+7\frac{3}{4}$ **f** −4.2

EXAMPLE 3F-1 Adding or subtracting directed numbers

Calculate:

a (+1.3) + (−3.5) **b** $\left(-\frac{2}{5}\right) - \left(-\frac{3}{10}\right)$.

THINK

a **1** Write as a simpler equivalent problem.

2 Locate 1.3 on a number line. Since the operation is −, move to the left (negative direction).

3 Write the result.

b **1** Write equivalent fractions with the same denominator (10).

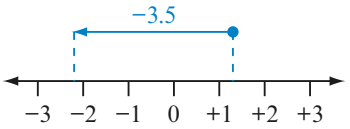
2 Write as a simpler equivalent problem.

3 Locate $-\frac{4}{10}$ on a number line. Since the operation is +, move to the right (positive direction).

4 Write the result.

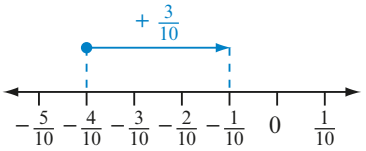
WRITE

a (+1.3) + (−3.5)
= 1.3 − 3.5



(+1.3) + (−3.5) = −2.2

b $\left(-\frac{2}{5}\right) - \left(-\frac{3}{10}\right)$
$= \left(-\frac{4}{10}\right) - \left(-\frac{3}{10}\right)$
$= -\frac{4}{10} + \frac{3}{10}$



$\left(-\frac{2}{5}\right) - \left(-\frac{3}{10}\right) = -\frac{1}{10}$

2 Calculate:

a (+3.1) + (−5.6) **b** (−7.4) + (+1.8) **c** (+2.9) − (−4.3)
d (−6.5) − (+3.8) **e** (−0.7) − (−0.5) **f** (+6.2) − (+4.9)
g (−5.7) + (+9.1) **h** (+4.3) + (−2.6) **i** (+3.9) − (−1.4)

3 Calculate:

a $\left(+\frac{1}{7}\right) - \left(-\frac{3}{7}\right)$ **b** $\left(-\frac{2}{3}\right) + \left(-\frac{5}{3}\right)$ **c** $\left(+\frac{5}{8}\right) - \left(+\frac{7}{8}\right)$
d $\left(-\frac{4}{5}\right) + \left(+\frac{2}{5}\right)$ **e** $\left(-\frac{1}{3}\right) + \left(-\frac{4}{3}\right)$ **f** $\left(-\frac{3}{4}\right) - \left(+\frac{1}{2}\right)$
g $\left(-\frac{1}{3}\right) - \left(-\frac{5}{9}\right)$ **h** $\left(+\frac{2}{5}\right) + \left(-\frac{3}{2}\right)$ **i** $\left(-\frac{1}{2}\right) + \left(-\frac{2}{3}\right)$

EXAMPLE 3F-2 Multiplying or dividing directed fractions

Calculate:

a $-\frac{9}{20} \times -\frac{8}{15}$ b $-\frac{4}{7} \div \frac{3}{14}$.

THINK

a 1 Look for common factors. Divide 9 and 15 by 3 and divide 8 and 20 by 4.

2 Multiply the fractions and decide whether the result will be positive or negative.

b 1 Change to a multiplication problem.

2 Look for common factors. Divide 7 and 14 by 7.

3 Multiply the fractions and decide whether the result will be positive or negative.

4 Write as a mixed number.

WRITE

a $-\frac{9}{20} \times -\frac{8}{15}$

$= -\frac{\cancel{9}^{3}}{\cancel{20}_{5}} \times -\frac{\cancel{8}^{2}}{\cancel{15}_{5}}$

$= -\frac{3}{5} \times -\frac{2}{5}$

$= \frac{6}{25}$ (or $+\frac{6}{25}$)

b $-\frac{4}{7} \div \frac{3}{14}$

$= -\frac{4}{7} \times \frac{14}{3}$

$= -\frac{4}{\cancel{7}_{1}} \times \frac{\cancel{14}^{2}}{3}$

$= -\frac{8}{3}$

$= -2\frac{2}{3}$

4 Calculate:

a $-\frac{6}{11} \times -\frac{2}{3}$ b $+\frac{4}{5} \times -\frac{5}{12}$ c $-\frac{15}{8} \times +\frac{16}{25}$

d $+\frac{12}{35} \times +\frac{7}{9}$ e $-\frac{5}{6} \div +\frac{10}{7}$ f $+\frac{4}{9} \times -\frac{5}{12}$

g $-\frac{15}{8} \times +\frac{16}{25}$ h $+\frac{10}{9} \div -\frac{27}{12}$ i $-\frac{7}{15} \div -\frac{21}{10}$

EXAMPLE 3F-3 Multiplying or dividing directed decimals

Calculate:

a -4.3×-6.1 b $+25.2 \div -7$.

THINK

a 1 Ignore the signs and calculate 4.3×6.1.

2 Decide whether the result will be positive or negative and write your answer.

b 1 Ignore the signs and calculate $25.2 \div 7$.

2 Decide whether the result will be positive or negative and write your answer.

WRITE

a 4.3×6.1

$= 26.23$

$-4.3 \times -6.1 = +26.23$ (or 26.23)

b $7\overline{)25.2}$ 3.6

$+25.2 \div -7 = -3.6$

UNDERSTANDING AND FLUENCY

5 Calculate:

a -5.8×-6 **b** $-46.7 \times +3$ **c** $+18.5 \times -4.3$

d $-34.2 \div +9$ **e** $-4.35 \div -0.5$ **f** $+0.648 \div -0.2$

6 Calculate:

a $-\frac{4}{5} + \frac{3}{5}$ **b** $\frac{2}{7} - \frac{5}{7}$ **c** $-\frac{3}{11} - \frac{6}{11}$ **d** $-\frac{4}{3} + \frac{1}{6}$ **e** $\frac{2}{7} - \frac{1}{2}$ **f** $-\frac{3}{4} + \frac{7}{8}$

7 Calculate:

a $-5.23 + 3.72$ **b** $17.6 - 19.3$ **c** $-53.1 - 25.8$

d $5.104 - 12.6$ **e** $0.378 - 0.783$ **f** $-11.35 + 15.27$

g $5.2 - 8.4 + 7.6$ **h** $-6.9 + 2.3 + 9.1$ **i** $4.55 - 2.15 - 3.8$

j $-15.7 - 12.1 + 9.6$ **k** $\frac{2}{3} - \frac{4}{3} + \frac{1}{3}$ **l** $-\frac{3}{4} + \frac{1}{4} + \frac{1}{2}$

m $\frac{2}{3} - \frac{5}{6} - \frac{1}{2}$ **n** $-\frac{3}{5} - \frac{1}{4} + \frac{7}{10}$ **o** $\frac{5}{9} - \frac{2}{3} + \frac{1}{2}$

8 Copy and complete each magic square.

a

−4	6	−8
−6		2
		0

b

−6.8		
	3.4	
6.8		13.6

c

		$-\frac{1}{3}$
$\frac{1}{6}$	$-\frac{1}{6}$	$-\frac{1}{2}$

NOTE Remember that in a magic square, the sum of the three numbers in each row, column and diagonal is the same.

9 Calculate each problem. Remember to change each mixed number to an improper fraction first.

a $1\frac{2}{3} - 2\frac{1}{3}$ **b** $-4\frac{3}{5} - 1\frac{1}{10}$ **c** $-2\frac{3}{4} + 3\frac{2}{3}$

d $-4\frac{1}{2} \times 3\frac{1}{3}$ **e** $-2\frac{1}{6} \div -3\frac{1}{4}$ **f** $7\frac{2}{3} \div -5\frac{1}{9}$

EXAMPLE 3F-4 Using order of operations with directed numbers

Calculate: **a** $3.2 \times 4 - 7.1 \times 2$ **b** $-\frac{2}{3} - \frac{1}{8} \div \frac{3}{4}$.

THINK

a **1** Work out the multiplication before the subtraction. $3.2 \times 4 = 12.8$ and $7.1 \times 2 = 14.2$.

2 Work out the result.

b **1** Work out the division before the subtraction. Change the division to a multiplication.

2 Look for common factors. Divide 4 and 8 by 4.

3 Work out the multiplication.

4 Subtract fractions using equivalent fractions.

5 Work out the result.

WRITE

b $3.2 \times 4 - 7.1 \times 2$
$= 12.8 - 14.2$
$= -1.4$

c $-\frac{2}{3} - \frac{1}{8} \div \frac{3}{4}$
$-\frac{2}{3} - \frac{1}{8} \times \frac{4}{3}$
$= -\frac{2}{3} - \frac{1}{8_2} \times \frac{4^1}{3}$
$= -\frac{2}{3} - \frac{1}{6}$
$= -\frac{4}{6} - \frac{1}{6}$
$= -\frac{5}{6}$

UNDERSTANDING AND FLUENCY

10 Calculate:

a $7 - 2 \times 8$ **b** $-1 + 16 \div 16$

c $28 - 3 \times (15 - 4)$ **d** $-6 + 5 \times 3 - 2$

e $11 - (4 \times 2 - 19)$ **f** $(-3 \times 7 + 1) \div (8 - 12)$

11 Calculate:

a $-0.5 \times 0.3 + 1.4$ **b** $1.26 \div 3 - 0.68$

c $3.7 + 6.2 \times -0.9$ **d** $4.1 \times 5 - 5.6 \times 4$

e $-6.4 \div 3.2 \times 0.5$ **f** $2.4 \times (15.3 - 18.3)$

12 Calculate:

a $-\frac{5}{9} \times \frac{3}{5} - \frac{2}{3}$ **b** $-\frac{5}{6} \div \frac{10}{9} \times -\frac{4}{7}$

c $\frac{1}{2} + \frac{4}{7} \div -\frac{2}{5}$ **d** $\frac{4}{9} \times \frac{3}{8} - \frac{5}{3} \times \frac{7}{10}$

e $-\frac{5}{2} \times \left(\frac{7}{15} - \frac{3}{5}\right)$ **f** $-\frac{2}{3} \times \frac{6}{7} \div 2\frac{2}{3}$

PROBLEM SOLVING AND REASONING

13 When standing with outstretched arms above her head, the height of an Olympic diver is 2.3 m. She dives from the 10-m platform and touches the bottom of the pool with her outstretched fingers. The depth of the pool is 5.5 m.



a If the pool surface is the reference point for zero, write the maximum height of the diver above the pool surface as a directed number.

b Write the depth of the pool as a directed number.

c Write a subtraction problem to calculate the vertical distance covered by the diver. (Hint: find the difference between the two positions.)

d What vertical distance has the diver covered?

14 The daily maximum and minimum temperatures at Mt Buller were recorded over a week.

	Sun	Mon	Tue	Wed	Thu	Fri	Sat
Maximum temperature (°C)	8.3	4.1	−3.0	−2.4	−1.3	0.4	2.4
Minimum temperature (°C)	1.6	−2.2	−5.3	−5.5	−3.2	−0.9	0.2

a Calculate the difference between the minimum and maximum temperatures for each day.

b Which day had the biggest range of temperatures?

c Calculate the average (or mean) of:

i the maximum temperatures

ii the minimum temperatures.

d What is the difference between the average minimum and average maximum temperature?

NOTE Average (or mean) temperature is calculated by adding all the relevant values and dividing the total by the number of values.

PROBLEM SOLVING AND REASONING

15 For the ice cream and hot chocolate shown, calculate the difference between the two temperatures. Show the problem you used to obtain your answer.



16 This incomplete statement shows some deposits and withdrawals made to Martin's bank account.

Date	Reference	Transaction	Balance
30 April	–	–	+$289.60
2 May	Tony's newsagent	+$132.50	
15 May	Movie place	−$17.45	
21 May	Bicycle city	−$500.00	
24 May	Tony's newsagent	+$230.00	
26 May	DD online music store	−$38.95	
27 May	Sports and stuff Pty Ltd		+$69.80



a Is a transaction of +$132.50 a deposit or a withdrawal?

b Is a transaction of −$17.45 a deposit or a withdrawal?

c What does it mean if the balance in Martin's account is:

i positive?

ii negative?

d Find the account balance after the transaction made on 21 May.

e Find the balance after the transaction made on 24 May.

f What transaction is made on 27 May so that the account balance is +$69.80?

g Was Martin's account overdrawn at any stage? What penalty do banks have for when an account is overdrawn?

CHALLENGE

17 List three examples of pairs of directed numbers where the sum of the numbers is zero.

18 List three examples of pairs of directed numbers that add to:

a −6.5 **b** $-\frac{1}{2}$.

19 List three examples of pairs of directed numbers where:

a the sum of the numbers is negative and the product of the numbers is positive

b the sum and the product of the numbers are negative.

20 If the sum of two numbers is zero, will the product of the numbers be positive or negative? Explain.

Reflect

What do you need to remember when performing operations on directed numbers?

3G Powers of directed numbers

Start thinking!

1 **a** Write 2^3 in expanded form.
b Calculate the basic numeral.

2 **a** Write $(-2)^3$ in expanded form.
b Calculate the basic numeral.
c Is your answer positive or negative? Explain why.

3 Copy and complete each row of this table.

Index form	Base	Index or power	Expanded form	Basic numeral
5^2	5	2	5×5	
$(-5)^2$	-5		-5×-5	
4^3				64
	-4	3		

4 Add a further four rows to your table and complete them using 2^4, $(-2)^4$, 3^5, $(-3)^5$.

5 What do you notice about your results for questions **3** and **4**?

6 Explain why 5^2 and $(-5)^2$ give the same result but 4^3 and $(-4)^3$ do not.

7 Without calculating the result, would you expect 11^8 and $(-11)^8$ to give the same basic numeral? Explain.

KEY IDEAS

- Powers are used to show repeated multiplication. The base is the number that is repeatedly multiplied and the power or index indicates the number of times the base is written.

$$\underset{\text{index form}}{(-2)^3} = \underset{\text{expanded form}}{-2 \times -2 \times -2} = \underset{\text{basic numeral}}{-8}$$

- When multiplying two numbers with the same sign, the result is positive.
- When multiplying two numbers with different signs, the result is negative.
- If the base is negative and the power is an even number, the basic numeral will be positive.
- If the base is negative and the power is an odd number, the basic numeral will be negative.

EXERCISE 3G Powers of directed numbers

UNDERSTANDING AND FLUENCY

1 Calculate each of these.

a $5 \times 5 \times 5$ **b** -7×-7

c $-3 \times -3 \times -3$ **d** $-10 \times -10 \times -10 \times -10$

2 Which of these six options is the expanded form of $(-4)^5$?

A $4 \times 4 \times 4 \times 4 \times 4$ **B** $5 \times 5 \times 5 \times 5$

C $-4 \times -4 \times 4 \times 4 \times 4$ **D** $-4 \times -4 \times -4 \times -4$

E $-4 \times -4 \times -4 \times -4 \times -4$ **F** $-5 \times -5 \times -5 \times -5$

3 Write each part of question **1** in index form.

EXAMPLE 3G-1 Calculating powers of integers

Write each of these in expanded form and calculate its value.

a $(-3)^2$ **b** $(-2)^5$

THINK

a 1 Write as a repeated multiplication (expanded form).

2 Work out the result. Remember that $- \times - \rightarrow +$. The result will be positive since the power is even.

b 1 Write in expanded form.

2 Work out the result. Remember that $- \times - \rightarrow +$ and $+ \times - \rightarrow -$. The result will be negative since the power is odd.

WRITE

a $(-3)^2 = -3 \times -3$

$= 9$

b $(-2)^5 = -2 \times -2 \times -2 \times -2 \times -2$

$= 4 \times -2 \times -2 \times -2$

$= -8 \times -2 \times -2$

$= 16 \times -2$

$= -32$

4 Write each of these in expanded form and calculate its value.

a $(-9)^2$ **b** $(+8)^2$ **c** $(-6)^3$ **d** $(+7)^3$

e $(-5)^4$ **f** $(+1)^4$ **g** $(+4)^5$ **h** $(-10)^5$

i $(+2)^6$ **j** $(-3)^6$ **k** $(-2)^7$ **l** $(-1)^8$

5 Without calculating each result, predict whether the basic numeral is positive or negative.

a $(-2)^9$ **b** $(-15)^6$ **c** $(+6)^7$ **d** $(-100)^4$

EXAMPLE 3G-2 Calculating powers of directed numbers

Write each of these in expanded form and calculate its value.

a $(-0.2)^3$ b $\left(-\frac{2}{3}\right)^4$

THINK

a **1** Write in expanded form.

2 Work out the result. Remember that $- \times - \rightarrow +$ and $+ \times - \rightarrow -$. A negative result is expected since the power is odd.

b **1** Write in expanded form.

2 Work out the result. A positive result is expected since the power is even.

WRITE

a $(-0.2)^3 = -0.2 \times -0.2 \times -0.2$
$= 0.04 \times -0.2$
$= -0.008$

b $\left(-\frac{2}{3}\right)^4 = -\frac{2}{3} \times -\frac{2}{3} \times -\frac{2}{3} \times -\frac{2}{3}$
$= \frac{4}{9} \times -\frac{2}{3} \times -\frac{2}{3}$
$= -\frac{8}{27} \times -\frac{2}{3}$
$= \frac{16}{81}$

UNDERSTANDING AND FLUENCY

6 Write each of these in expanded form and calculate its value.

a $(-1.2)^2$ b $(+0.9)^2$ c $(-0.4)^3$
d $(-0.6)^4$ e $(-0.2)^5$ f $(-0.1)^7$
g $\left(-\frac{1}{2}\right)^3$ h $\left(-\frac{5}{9}\right)^2$ i $\left(-\frac{4}{7}\right)^3$
j $\left(-\frac{1}{3}\right)^4$ k $\left(-\frac{2}{3}\right)^5$ l $\left(-1\frac{1}{2}\right)^6$

7 Would the basic numeral for $\left(-\frac{5}{19}\right)^{13}$ be positive or negative? What about for $(-28.5)^{20}$?

8 Arrange each list of numbers in ascending order.

a $(+5)^4, (-2)^3, (-3)^2, (-4)^5$ b $(-1)^{10}, 10^2, (-7)^2, (-2)^7$
c $(-0.3)^2, (0.2)^3, (-0.2)^2, (-0.1)^3$ d $\left(-\frac{1}{3}\right)^2, \left(-\frac{1}{2}\right)^5, \left(-\frac{1}{2}\right)^4, \left(-\frac{1}{3}\right)^3$

9 Write each of these multiplications in index form.

a $-5 \times -5 \times -5 \times -9 \times -9 \times -9 \times -9$
b $-4 \times -4 \times -4 \times -4 \times -4 \times -4 \times 3 \times 3 \times 3$
c $7 \times 7 \times -6 \times -6 \times -6 \times -6 \times -6 \times -6$
d $-8 \times -8 \times -8 \times -8 \times -10 \times -10$

10 Write each of these in expanded form and calculate its value.

a $(-2)^3 \times (-4)^2$ b $(-5)^2 \times 3^4$ c $(-10)^3 \times (-2)^5$
d $(-3)^5 \times (-1)^2$ e $(-6)^2 \times (-2)^3$ f $(-1)^4 \times 3^2$
g $(-7)^2 \times (-1)^5$ h $(-3)^3 \times (-2)^2$ i $(-1)^3 \times (-2)^5$

11 Calculate each of these by first working out the value of any numbers written in index form.

a $(-3)^4 + (-4)^3$ **b** $(-10)^4 + (-7)^2$ **c** $(-8)^2 - (-2)^5$

d $(-2)^3 - (-1)^9$ **e** $(-1)^{13} \times (-5)^3$ **f** $(-10)^2 \div (-5)^2$

g $6^4 \div 3^3$ **h** $(-0.1)^3 + (-0.4)^2$ **i** $(-0.2)^3 - (-1.1)^2$

j $\left(-\frac{1}{3}\right)^2 + \left(-\frac{1}{2}\right)^3$ **k** $\left(-\frac{1}{2}\right)^5 - \left(-\frac{3}{4}\right)^2$ **l** $(-0.3)^3 \times (-0.1)^4$

m $(-1.2)^2 \div (-0.6)^2$ **n** $\left(-\frac{2}{3}\right)^3 \times \left(-\frac{1}{2}\right)^2$ **o** $\left(-\frac{3}{10}\right)^2 \div \left(-\frac{2}{5}\right)^3$

12 a Calculate each of these by first writing in expanded form.

i $(-1)^1$ **ii** $(-1)^2$ **iii** $(-1)^3$

iv $(-1)^4$ **v** $(-1)^5$ **vi** $(-1)^6$

b Which of these problems give a positive result?

c Which of these problems give a negative result?

d Explain your observations.

e Predict the result for:

i $(-1)^{13}$ **ii** $(-1)^{32}$ **iii** $(-1)^{100}$

iv $(-1)^{203}$ **v** $(-1)^{188}$ **vi** $(-1)^{555}$

13 a Calculate:

i $(-3)^2$ **ii** $(-3)^3$ **iii** $(-3)^5$.

b Multiply your answers to part **a i** and **ii** to calculate $(-3)^2 \times (-3)^3$.

c How does this compare to your result for $(-3)^5$?

d Calculate:

i $(-2)^3$ **ii** $(-2)^5$ **iii** $(-2)^8$.

e Multiply your answers to part **d i** and **ii** to calculate $(-2)^3 \times (-2)^5$.

f How does this compare to your result for $(-2)^8$?

g What pattern can you use to make multiplication problems like this easier to calculate? (Hint: remember the index laws in Chapter 1.)

h Copy and complete the table shown below. The first two rows have been completed.

	Calculation	Simplified calculation	Basic numeral
	$(-3)^2 \times (-3)^3$	$(-3)^5$	-243
	$(-2)^3 \times (-2)^5$	$(-2)^8$	256
i	$(-4)^3 \times (-4)^2$		
ii	$(-2)^4 \times (-2)^3$		
iii	$(-0.1)^5 \times (-0.1)^3$		
iv	$\left(-\frac{1}{2}\right)^3 \times \left(-\frac{1}{2}\right)^2$		

i What must be the same for this pattern (or index law) to work? (Hint: does it work for $(-3)^2 \times (-2)^5$?)

PROBLEM SOLVING AND REASONING

PROBLEM SOLVING AND REASONING

14 **a** Calculate:
i $(-3)^6$ **ii** $(-3)^4$ **iii** $(-3)^2$.
b Divide your answers to part **a i** and **ii** to calculate $(-3)^6 \div (-3)^4$.
c How does this compare to your result for $(-3)^2$?
d Calculate:
i $(-2)^7$ **ii** $(-2)^4$ **iii** $(-2)^3$.
e Divide your answers to part **d i** and **ii** to calculate $(-2)^7 \div (-2)^4$.
f How does this compare to your result for $(-2)^3$?
g What pattern can you use to make division problems like this easier to calculate?
h Copy and complete the table shown at right. The first two rows have been completed.
i What must be the same for this pattern (or index law) to work? (Hint: does it work for $(-3)^6 \div (-2)^4$?)

	Calculation	Simplified calculation	Basic numeral
	$(-3)^6 \div (-3)^4$	$(-3)^2$	9
	$(-2)^7 \div (-2)^4$	$(-2)^3$	−8
i	$(-5)^8 \div (-5)^5$		
ii	$(-6)^9 \div (-6)^7$		
iii	$(-0.2)^{10} \div (-0.2)^6$		
iv	$\left(-\frac{4}{5}\right)^{12} \div \left(-\frac{4}{5}\right)^9$		

15 The basic numeral for a number raised to the power of 0 is 1. For example, $2^0 = 1$.
a Calculate $(-2)^3$ and use this value to calculate $(-2)^3 \div (-2)^3$.
b Subtract the powers to simplify $(-2)^3 \div (-2)^3$. Keep the result in index form.
c Does this relationship work for negative numbers? Explain.

EXAMPLE 3G-3 Using index laws with directed numbers

Use the index laws to first simplify and then calculate each problem.
a $(-2)^8 \div (-2)^6 \times (-2)^3$ **b** $(-5)^7 \times (-5)^3 \div (-5)^8$

THINK

a **1** Work from left to right. To simplify $(-2)^8 \div (-2)^6$, subtract the powers. (8 − 6 = 2)
2 To simplify $(-2)^2 \times (-2)^3$, add the powers. (2 + 3 = 5)
3 Calculate the result.

b **1** Work from left to right. To simplify $(-5)^7 \times (-5)^3$, add the powers. (7 + 3 = 10)
2 To simplify $(-5)^{10} \div (-5)^8$, subtract the powers. (10 − 8 = 2)
3 Calculate the result.

WRITE

a $(-2)^8 \div (-2)^6 \times (-2)^3$
$= (-2)^2 \times (-2)^3$
$= (-2)^5$
$= -32$

b $(-5)^7 \times (-5)^3 \div (-5)^8$
$= (-5)^{10} \div (-5)^8$
$= (-5)^2$
$= 25$

16 Use the index laws to first simplify and then calculate each problem.

a $(-2)^9 \times (-2)^5 \div (-2)^{11}$ **b** $(-6)^5 \div (-6)^4 \times (-6)^2$

c $(-10)^8 \times (-10)^5 \div (-10)^9$ **d** $(-4)^{10} \times (-4)^3 \div (-4)^{13}$

e $(-3)^7 \div (-3)^7 \times (-3)^3$ **f** $(-7)^5 \times (-7)^3 \div (-7)^6$

g $(-5)^4 \times (-5)^3 \div (-5)^7$ **h** $(-3)^4 \times (-3)^2 \div (-3)^5$

CHALLENGE

17 Squaring two different numbers can produce the same result.
For example, $3^2 = 9$ and $(-3)^2 = 9$.

a Which two numbers, when squared (raised to the power of 2), give 25? Why are there two numbers?

b Is there a number that, when squared, gives -25? Explain.

c How many different numbers can be cubed (raised to the power of 3) to give 8?

d How many different numbers can be cubed (raised to the power of 3) to give -8?

e Comment on your results to parts **a**–**d**.

f Predict how many numbers can be raised to the power of 4 to give the same positive result. (Try it for 16.) Repeat for a negative result.

g Similarly, predict what would happen for powers of 5, 6 and so on. Provide examples to support your answer.

18 What number(s), when squared, give(s) each result?

a 49 **b** 81 **c** 1

d 4 **e** 64 **f** 100

19 What number(s), when cubed, give(s) each result?

a 27 **b** -125 **c** 64

d -1 **e** -64 **f** -1000

20 Consider $(+3)^2$, $(-3)^2$, 3^2 and -3^2.

a Which of these produce the same result?

b Explain why $(-3)^2$ is different from -3^2.

c Decide whether each pair produces the same result.

i $(-4)^3$ and -4^3 **ii** $(-2)^4$ and -2^4

iii $(-1)^6$ and -1^6 **iv** $(-10)^5$ and -10^5

d Describe the general pattern you have seen.

21 Calculate each problem. Remember to use the correct order of operations.

a $9^2 \div 3 - 5^2 \times 4$

b $-8.2 - 2^2 + (1.1)^2$

c $\left(-\frac{2}{3}\right)^3 + \frac{4}{9}$

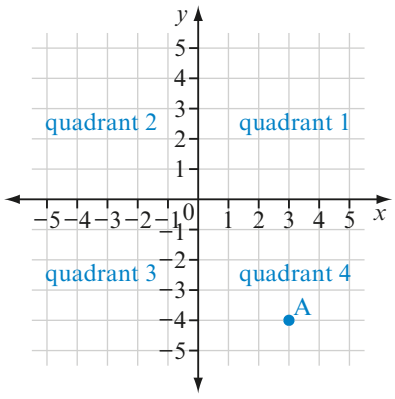
Reflect

What is different about raising a negative number to a power compared to raising a positive number to a power?

3H The Cartesian plane

Start thinking!

1 Copy the diagram of the **Cartesian plane** shown.



- **a** What is the name given to:
  - **i** the horizontal number line?
  - **ii** the vertical number line?
- **b** The reference point for the positive and negative numbers on the scale is called the **origin**. What are the coordinates of the origin?

2 A point A has been drawn on the Cartesian plane.

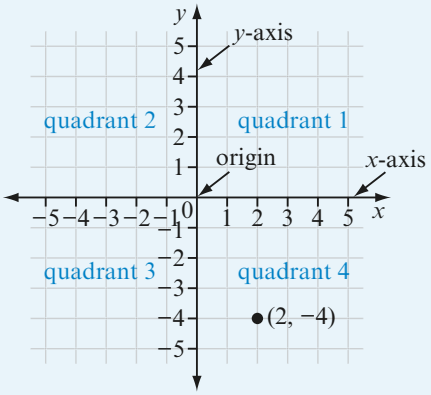
- **a** What is the x-coordinate of this point?
- **b** What is the y-coordinate of this point?
- **c** List the coordinates of point A.
- **d** Which **quadrant** is point A in?

3 Point B has the Cartesian coordinates $(-2, -3)$.

- **a** Write the x-coordinate of point B.
- **b** Write the y-coordinate of point B.
- **c** Plot point B on your Cartesian plane. This means to draw a dot at $(-2, -3)$ and label it as B.
- **d** Which quadrant is point B in?

4 Plot a point in each quadrant of your Cartesian plane and then list the quadrant number and the coordinates of each point.

KEY IDEAS

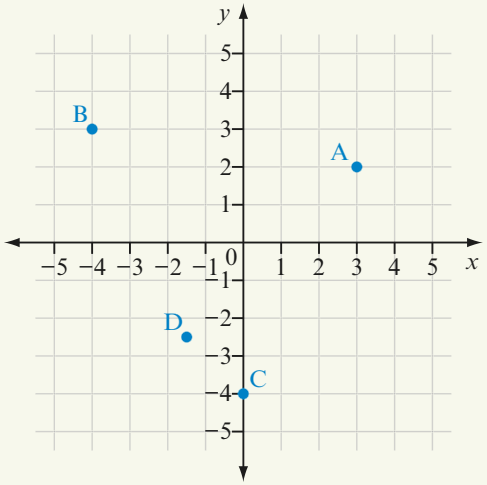


- A Cartesian plane is formed when a horizontal number line (x-axis) and a vertical number line (y-axis) intersect at right angles. The point where they intersect is called the origin.
- The number lines in a Cartesian plane are called axes (singular axis). They divide the plane into four quadrants.
- A pair of Cartesian coordinates describes a point on the Cartesian plane. The horizontal coordinate (x-coordinate) is always listed first, followed by a comma, then the vertical coordinate (y-coordinate).
- The x-coordinate states how many units left or right to move along the x-axis from the origin and the y-coordinate states how many units to move up or down parallel to the y-axis. For example, $(2, -4)$ means to move 2 units right from the origin and 4 units down.

EXERCISE 3H The Cartesian plane

EXAMPLE 3H-1 Writing the Cartesian coordinates of a point

Write the coordinates of points A, B, C and D.



THINK

1 For point A, identify the x-coordinate (3) and the y-coordinate (2). Write the x-coordinate first.

2 Repeat step 1 for points B, C and D. Point C has an x-coordinate of 0, as you don't move any units left or right from the origin to locate it.

WRITE

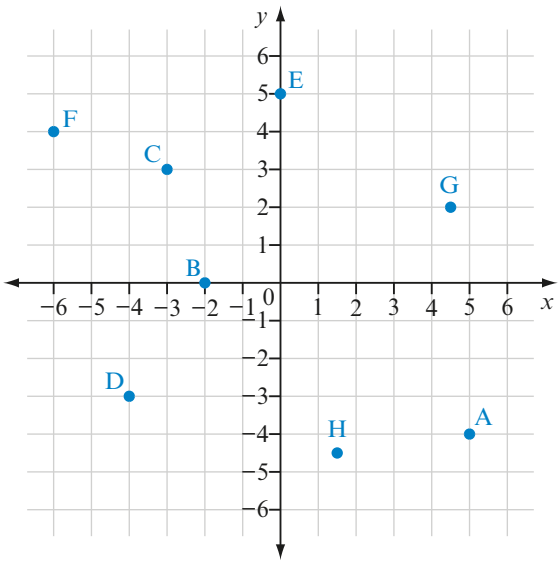
A is at (3, 2).

B is at (−4, 3).
C is at (0, −4).
D is at $(-1\frac{1}{2}, -2\frac{1}{2})$ or (−1.5, −2.5)

UNDERSTANDING AND FLUENCY

1 Write the coordinates of points A to H shown on this Cartesian plane.

2 List the quadrant or the axis of the Cartesian plane on which each point in question 1 is located.



EXAMPLE 3H-2 Plotting a point on the Cartesian plane

Plot these points on a Cartesian plane. Label each point with its letter.

a E(4, −2) **b** F(3, 0)
c G($-3\frac{1}{2}$, $4\frac{1}{2}$) **d** H(−2.25, −3.75)

THINK

a Point E has x-coordinate 4 and y-coordinate −2. Move 4 units right along the x-axis from the origin and 2 units down parallel to the y-axis. Plot the point at this position and label it E.

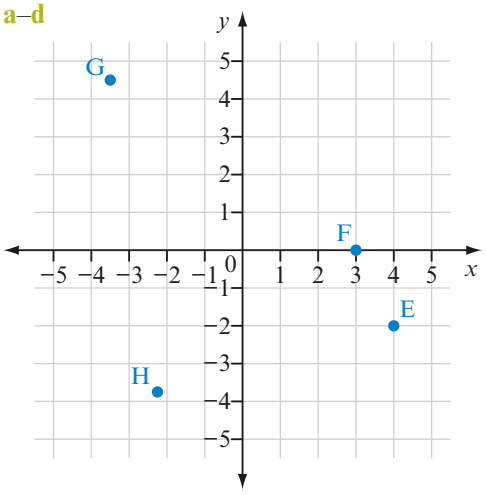
b Repeat this method for point F. (3, 0) is 3 units right and 0 units up from the origin.

c Repeat this method for point G. ($-3\frac{1}{2}$, $4\frac{1}{2}$) is $3\frac{1}{2}$ units left and $4\frac{1}{2}$ units up from the origin.

d Repeat this method for point H. (−2.25, −3.75) is 2.25 units left and 3.75 units down from the origin.

WRITE

a–d



UNDERSTANDING AND FLUENCY

3 Copy the set of x- and y-axes shown in question **1**.
Plot these points on the Cartesian plane. Label each point with its letter.

a I(−2, 5) **b** J(0, −2) **c** K(−4, −5)
d L(6, 0) **e** M(3, −4) **f** N(−4.5, 1.5)
g O(2, 2) **h** P(0, 3) **i** Q(−3.5, −1.5)

4 List the quadrant or the axis of the Cartesian plane on which each point in question **3** is located.

5 Another way of listing the coordinates of a set of points is in a table of values. Copy and complete each table of values.

a

x-coordinate	−6	−4		0	2		
y-coordinate	−3	−2	−1		1		
coordinates	(−6, −3)	(−4, −2)	(−2, −1)	(0, 0)		(4, 2)	(6, 3)

b

x-coordinate	−2	−1		1		3	
y-coordinate	−7			−1	1	3	
coordinates	(−2, −7)	(−1, −5)	(0, −3)		(2, 1)		(4, 5)

EXAMPLE 3H-3 Plotting points from a table of values

For this table of values:

x	−6	−4	−2	0	2	4	6
y	−3	−2	−1	0	1	2	3

a list the coordinates for each point
b plot each point on a Cartesian plane and join them with a smooth line
c describe whether the points lie on a curved or straight line.

THINK

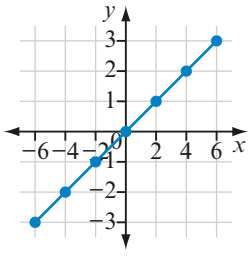
a Write the coordinates for each point. Write the x-coordinate first.

b Plot the points and join them in order.

c Describe the line you have drawn.

WRITE

a (−6, −3), (−4, −2), (−2, −1), (0, 0), (2, 1), (4, 2), (6, 3)

b 

c The points lie on a straight line.

UNDERSTANDING AND FLUENCY

6 For each table of values:
- i list the coordinates for each point
- ii plot each point on a Cartesian plane and join them with a smooth line
- iii describe whether the points lie on a curved or straight line.

a

x	−3	−2	−1	0	1	2	3
y	5	0	−3	−4	−3	0	5

b

x	−2	−1	0	1	2	3	4
y	−5	−4	−3	−2	−1	0	1

c

x	−2	−1	0	1	2	3	4
y	8	6	4	2	0	−2	−4

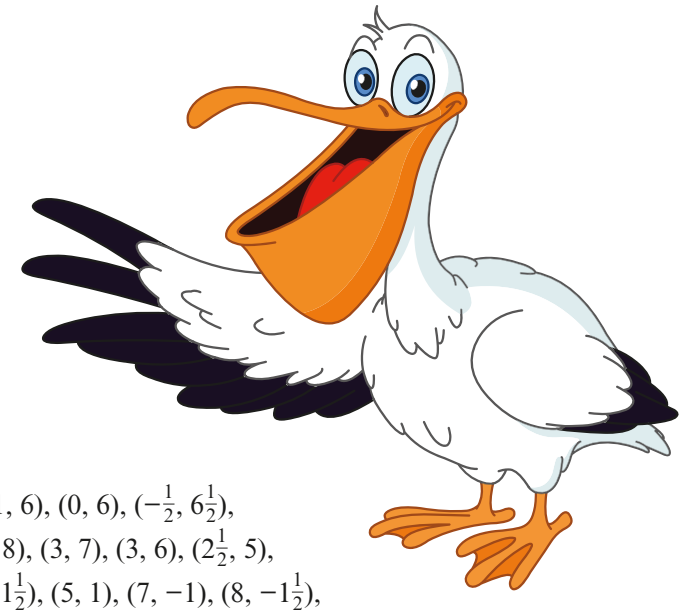
d

x	−6	−5	−1	0	2	3	6
y	−10	$-4\frac{1}{2}$	$7\frac{1}{2}$	8	6	$3\frac{1}{2}$	−10

NOTE Look at the lowest and highest x values to work out the scale needed along the x-axis of the Cartesian plane. Repeat for the y-axis using the lowest and highest y values.

7 Draw a Cartesian plane with axes extending from −5 to 5. Plot each point and join the points together in the order shown. Describe the shape you have created.
(4, 1), (−4, 1), (2.5, −3.5), (0, 4), (−2.5, −3.5), (4, 1)

UNDERSTANDING AND FLUENCY



8 Draw a Cartesian plane with the horizontal axis extending from −10 to 10 and the vertical axis extending from −8 to 9.
Use 1-cm grid paper or graph paper to make plotting easier.
Plot each of these points and join them, in order, with smooth, slightly curved lines.

Join $(-5, 5\frac{1}{2})$, $(-4, 6\frac{1}{2})$, $(-3, 6\frac{1}{2})$, $(-1, 6)$, $(0, 6)$, $(-\frac{1}{2}, 6\frac{1}{2})$,
$(0, 8)$, $(\frac{1}{2}, 7\frac{1}{2})$, $(\frac{1}{4}, 7\frac{3}{4})$, $(1, 8\frac{1}{4})$, $(2, 8)$, $(3, 7)$, $(3, 6)$, $(2\frac{1}{2}, 5)$,
$(2, 3)$, $(2, 2)$, $(2\frac{1}{4}, 1\frac{1}{2})$, $(3, 1\frac{3}{4})$, $(4, 1\frac{1}{2})$, $(5, 1)$, $(7, -1)$, $(8, -1\frac{1}{2})$,
$(9, -3)$, $(8, -3)$, $(9, -4)$, $(7, -3\frac{1}{2})$, $(6, -4)$, $(5\frac{1}{2}, -5)$, $(5\frac{1}{4}, -5)$, $(5\frac{1}{2}, -6)$,
$(6, -5\frac{1}{2})$, $(6\frac{1}{2}, -6)$, $(5\frac{1}{2}, -6\frac{1}{2})$, $(5, -8)$, $(4\frac{1}{2}, -7\frac{1}{2})$, $(3, -8)$, $(3\frac{1}{2}, -7)$, $(3, -7)$, $(5, -6)$,
$(5, -5)$, $(4\frac{1}{2}, -4\frac{1}{2})$, $(4, -5)$, $(4, -5\frac{1}{2})$, $(4\frac{1}{2}, -5\frac{1}{2})$, $(3, -6)$, $(1\frac{1}{2}, -7)$, $(1\frac{1}{2}, -6\frac{1}{2})$, $(\frac{1}{2}, -6\frac{1}{2})$,
$(1, -6)$, $(1, -5\frac{1}{2})$, $(3\frac{1}{2}, -5\frac{1}{2})$, $(3\frac{1}{2}, -5)$, $(2, -4)$, $(\frac{1}{2}, -2\frac{1}{2})$, $(-2, -3)$, $(-2\frac{1}{2}, -2\frac{1}{2})$,
$(-4, -2\frac{3}{4})$, $(-5, -2)$, $(-7, -1\frac{1}{2})$, $(-8, -1)$, $(-9, -\frac{1}{2})$, $(-5, 0)$, $(-9\frac{1}{2}, 1\frac{1}{2})$, $(-6, 1)$,
$(-9\frac{1}{2}, 3)$, $(-9, 3\frac{1}{2})$, $(-3, 1)$, $(-4, 2\frac{1}{2})$, $(0, 5\frac{1}{2})$, $(-3, 6)$, $(-5, 5\frac{1}{2})$.

Use the image of the pelican to finish details like eyes, bill, feathers and so on.

PROBLEM SOLVING AND REASONING



9 You can make a drawing of this helicopter by joining a set of points with straight or slightly curved lines in a given order.

a First trace the outline of the body of the helicopter using tracing paper. Cut out the outline and place it over a Cartesian plane. Choose a starting point and list the coordinates of that point. Decide which points would work best to give you the required outline when they are joined.

Now consider further points needed to draw other features of the helicopter, such as the blades and 'feet'. Produce a list of instructions that anyone could use, similar to question **8**.

b To produce an enlarged drawing of the helicopter, draw a new Cartesian plane with a larger interval between each unit mark on the axes. Follow your instructions to produce the enlarged picture.

10 The temperature at Cradle Mountain in Tasmania was recorded every 3 hours starting at midnight. The results are shown in the table.

Time (hours after midnight)	0	3	6	9	12	15	18	21	24
Temperature (°C)	−3	−5	−2	3	7	5	4	0	−2

PROBLEM SOLVING AND REASONING



a Draw a Cartesian plane and label the horizontal axis Time and the vertical axis Temperature.

b Decide what scale to use for the horizontal axis. Will you need negative numbers? Explain why or why not. Mark your scale on the Time axis.

c Decide what scale to use for the vertical axis. Mark your scale on the Temperature axis.

d Plot the points provided in the table of values.

e What was the coldest recorded temperature? At what time of the day was this temperature recorded?

f What was the warmest recorded temperature? At what time of the day was this temperature recorded?

g What was the range of recorded temperatures? (Hint: find the difference between the highest and lowest temperatures.)

h What was the average recorded temperature?

CHALLENGE

11 A set of points is created by performing an operation on each x-coordinate to produce the y-coordinate.

a Copy and complete this table of values for each operation described below.

x-coordinate	−3	−2	−1	0	1	2	3
y-coordinate							

i Add 2 to each x-coordinate to produce the matching y-coordinate.

ii Double each x-coordinate to produce the matching y-coordinate.

iii Subtract 3 from each x-coordinate to produce the matching y-coordinate.

iv Halve each x-coordinate to produce the matching y-coordinate.

v Square each x-coordinate to produce the matching y-coordinate.

b Draw a Cartesian plane with the horizontal axis extending from −3 to 3 and the vertical axis extending from −6 to 9. For each table of values, plot the points and join them with a smooth line. Which line is different from the others? Can you explain why?

12 Each table of values in question **6** shows a set of points where one or more operations have been performed on each x-coordinate to produce the y-coordinate. What relationship has been used in each case?

Reflect

How are positive and negative numbers used on a Cartesian plane?

CHAPTER REVIEW

SUMMARISE

Create a summary of this chapter using the key terms below. You may like to write a paragraph, create a concept map or use technology to present your work.

positive	opposite integers	basic numeral	quadrants
negative	directed numbers	Cartesian plane	x-coordinate
number line	powers	x-axis	y-coordinate
integers	index form	y-axis	table of values

MULTIPLE-CHOICE

3A **1** Which of these is an integer between -3.5 and 3.5?

A -4 **B** -2.5
C -2 **D** 5

3B **2** What integer when added to $+12$ gives -7?

A -19 **B** -5
C $+5$ **D** $+19$

3C **3** What is $(-24) - (+20)$?

A -4 **B** $+4$
C -44 **D** $+44$

3D **4** What does $(-35) - (-47)$ simplify to?

A $-35 - 47$ **B** $35 - 47$
C $35 + 47$ **D** $-35 + 47$

3D **5** Which of these problems does *not* equal 5?

A $-4 - 11 + 16 - 2 + 6$
B $8 - 13 - 1 + 17 - 6$
C $-7 + 5 - 9 + 22 - 6$
D $5 - 12 + 3 - 7 + 6$

3E **6** What is $8 \div -2$?

A -16 **B** -4 **C** 4 **D** 16

3E **7** What pair of integers gives a sum of -5 and a product of -36?

A -4 and 9 **B** -9 and 4
C 4 and 9 **D** -9 and -4

3F **8** What is $\frac{1}{5} - \frac{4}{5}$?

A -3 **B** $-\frac{3}{5}$ **C** $\frac{3}{5}$ **D** $\frac{5}{5}$

3F **9** Which problem has a positive answer?

A $(-1.2) + (-2.4)$
B $\left(-\frac{2}{5}\right) \times \left(+\frac{1}{2}\right)$
C $(-4.9) - (-1.1)$
D $\left(-\frac{1}{7}\right) \div \left(-\frac{8}{9}\right)$

3G **10** Which gives the largest result?

A $(-2)^6$ **B** $(+6)^2$
C $(-1)^{10}$ **D** $(-4)^3$

3G **11** What does $(-5)^8 \times (-5)^4$ simplify to?

A $(-5)^2$ **B** $(-5)^4$
C $(-5)^{12}$ **D** $(-5)^{32}$

3H **12** The point with coordinates $(-3, -8)$ is found in which quadrant on a Cartesian plane?

A quadrant 1 **B** quadrant 2
C quadrant 3 **D** quadrant 4

SHORT ANSWER

3A **1** Decide whether each statement is true or false.

a $-5 > +2$ b $-10 < -8$

c $0 > -3.5$ d $7\frac{1}{2} < -7\frac{1}{2}$

3A **2** Write each list of numbers in ascending order.

a $-2, 8, 4, -4, 0, -8$

b $-19, 5\frac{1}{3}, 0, -9, -1, 5.5$

3B **3** Calculate:

a $(-5) + (-4)$ b $(+2) + (+7)$

c $(+22) + (-34)$ d $(-50) + (+69)$

3B **4** Tamara's bank account shows a balance of $-\$28$. If she deposits \$150, what is her new account balance?

3C **5** Calculate:

a $(+3) - (+8)$ b $(-1) - (-9)$

c $(-46) - (+35)$ d $(+71) - (+53)$

3C **6** Overnight, the minimum temperature in Alice Springs was $-3°C$. By 2 pm, the temperature had climbed to a maximum of 18°C. Calculate the difference between the minimum and maximum temperatures.

3D **7** Calculate:

a $-8 + 7$ b $-4 - 6$

c $17 - 25$ d $-44 + 34$

e $-66 + 66$ f $-50 - 50$

3E **8** Calculate:

a $+7 \times -8$ b -5×-9

c $-36 \div +4$ d $-100 \div -20$

e -12×0 f 4×-15

g $\frac{18}{-6}$ h $\frac{-42}{-3}$

3E **9** Calculate:

a $-6 \times 3 \times -10$ b $20 \div -4 \times 7$

c $-18 \div -3 \times 2$ d $-5 \times -2 \times -4$

3F **10** Calculate:

a $\left(-\frac{3}{4}\right) - \left(-\frac{1}{2}\right)$ b $\left(+\frac{2}{5}\right) - \left(+\frac{1}{3}\right)$

c $\frac{3}{7} - 1\frac{2}{7}$ d $-2\frac{1}{6} + 6\frac{2}{3}$

e $\frac{1}{2} \div \frac{7}{4} - \frac{2}{5}$ f $-\frac{1}{8} + \frac{2}{3} \times -\frac{9}{8}$

3F **11** Calculate:

a $(+5.7) + (-6.2)$

b $(-0.9) - (-0.64)$

c $7.45 - 9.38$ d $-12.5 - 11.6$

e $-3.2 \div -0.2 - 16$

f $0.7 \times -0.2 - 0.6 \times -0.3$

3F **12** Calculate the average of -4.2, 5.6, 7.1 and -9.3.

3G **13** Calculate:

a $(-2)^5$ b $(-3)^2$

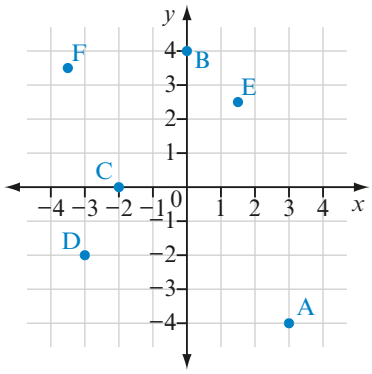
c $(-1)^9$ d $(-10)^4$

3G **14** Use the index laws to first simplify and then calculate:

a $(-9)^8 \times (-9)^5 \div (-9)^{11}$

b $(-6)^7 \times (-6)^4 \div (-6)^{11}$

3H **15** Write the coordinates of points A to F shown on this Cartesian plane.



3H **16** Plots these points on a Cartesian plane and then join them in order.

(0, −1), (1, −1), (1, 1), (−1, 1), (−1, −2), (2, −2), (2, 2), (−2, 2), (−2, −3), (3, −3)

Write the coordinates of the next four points to continue the pattern.

NAPLAN-STYLE PRACTICE

1 Which number is the largest?

3 ◯ −3 ◯ 2 ◯ −2 ◯

2 Which number is smaller than −4.5?

−4.3 ◯ −4.7 ◯ $2\frac{1}{2}$ ◯ $-4\frac{2}{5}$ ◯

3 What is (+5) + (−9)?

−14 ◯ −4 ◯ 4 ◯ 14 ◯

4 What is (−6) − (−7)?

−13 ◯ −1 ◯ 1 ◯ 13 ◯

5 Hayden and Stacy enter a lift at the third level above ground level. They move down five levels. Where does the lift stop?

◯ second level above ground level
◯ second level below ground level
◯ eighth level above ground level
◯ eighth level below ground level

6 Complete this number statement.

17 + ☐ = −25

7 Complete this number statement.

−36 − ☐ = −80

8 A stone falls from a cliff top 50 m above water to the bottom of the sea. The water is 12 m deep. How far does the stone fall?

☐

9 Bread at a temperature of −16°C is taken out of the freezer. After 15 minutes, its temperature rises by 24°C. What is the new temperature of the bread?

−40°C ◯ −8°C ◯ 8°C ◯ 40°C ◯

10 Complete this number statement.

−4 × ☐ = 12

11 A number is multiplied by −5 and the answer is −30.

The original number is ☐

12 Which pair of numbers has a positive sum and a negative product?

−6, 5 ◯ −8, −3 ◯ −4, 7 ◯ 2, 9 ◯

13 A ski resort recorded these temperatures: −5°C, 2°C, 4°C, −3°C, −1°C. What is the average of these temperatures?

☐

14 Transactions for Chloe's bank account are shown below.

Date	Reference	Transaction	Balance
1 May	–	–	+$124.80
4 May	Deposit	+$75.00	
9 May	Music	−$42.75	
15 May	Shoes	−$149.95	
26 May	Movies	−$52.20	
31 May	Deposit	+$150.00	

How much is in Chloe's bank account at the end of the month?

☐

15 What is $-\frac{3}{8} \times \frac{4}{9} \times -\frac{2}{7}$?

☐

16 What is $(-2)^5$?

☐

17 Which of these does *not* give a negative result?

$(-5)^6$ ◯ $(-3)^5$ ◯ $(-2)^7$ ◯ $(-4)^3$ ◯

18 What is $(-3)^{10} \div (-3)^7$?

1 ◯ −9 ◯ 27 ◯ −27 ◯

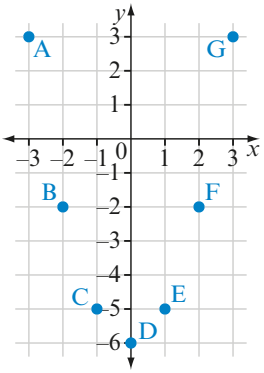
19 A point on the Cartesian plane has the coordinates (−5, 3). What is the x-coordinate?

☐

20 What are the coordinates of the origin on the Cartesian plane?

☐

Questions 21–27 refer to this figure.



21 Which point has the coordinates (2, −2)?

☐ A ☐ B
☐ F ☐ G

22 Which point has the coordinates (−3, 3)?

23 Write the coordinates of point E.

24 Write the coordinates of point D.

25 Which points are in the third quadrant of the Cartesian plane?

26 Which point has the same *y*-coordinate as point A?

27 Which table of values matches the points plotted on the Cartesian plane?

☐ Table 1

x	−3	−2	−1	0	1	2	3
y	−3	−2	−1	0	1	2	3

☐ Table 2

x	3	−2	−5	−6	−5	−2	3
y	−3	−2	−1	0	1	2	3

☐ Table 3

x	−3	−2	−1	0	1	2	3
y	3	2	5	6	5	2	3

☐ Table 4

x	−3	−2	−1	0	1	2	3
y	3	−2	−5	−6	−5	−2	3

ANALYSIS

The daily minimum and maximum temperatures at a ski resort were recorded over a week.

	Mon	Tue	Wed	Thu	Fri	Sat	Sun
Min. temp. (°C)	0.7	−2.2	−0.9	−3.4	−5.1	−1.8	0.2
Max. temp. (°C)	6.5	4.1	7.8	2.5	−0.6	3.9	5.3

a Which day had:
- **i** the highest temperature?
- **ii** the lowest temperature?

b Calculate the difference between the minimum and maximum temperatures for Monday.

c Calculate the difference between the minimum and maximum temperatures for each other day.

d Which day had the biggest range of temperatures?

e Calculate the average of:
- **i** the minimum temperatures (correct to one decimal place).
- **ii** the maximum temperatures.

f What is the difference between the average minimum and average maximum temperatures?

g Draw a Cartesian plane with the horizontal axis labelled Day (show 1, 2, 3, …, 7 on the scale to represent Monday to Sunday) and the vertical axis Temperature (°C). Think about the scale you will use on the vertical axis to represent all the temperatures listed in the table.

h Write the information for the seven daily minimum temperatures as a set of Cartesian coordinates.

i Plot the seven points with these coordinates on your Cartesian plane. To observe a trend, join the points with a smooth line.

j In a different colour, plot and join points on the same Cartesian plane to show the information for the seven maximum temperatures.

k Describe and interpret the trend you see.

CONNECT

Playing golf



The aim in golf is to hit a ball from the tee to the hole in the least number of strokes. Every golf course is rated by the number of strokes taken by a skilled player to complete a round of 18 holes. This number of strokes is called par. A golfer's score is the number of strokes they take that is more or less than par.

Integers can be used to describe these scores. A score of zero means that a golfer achieved par. Scores under par can be described using negative integers and scores over par can be described using positive integers. For example, a score of +5 means that the golfer's total number of strokes is five more than par (five over par).

Leo is a young golfer planning to become a professional golf player. During the 10 rounds of the summer competition season, Leo's scores were:

+2, +1, +4, −3, −2, −1, +2, +1, −4, −1.

The information that follows will help you decide whether he should apply.

Each hole of a golf course also has its own par. This table shows par for each of the 18 holes at Leo's golf course.

Hole	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Par	3	3	4	4	5	4	5	4	4	4	3	4	5	4	3	4	5	4

The total number of strokes for a par round can be worked out from this table. In order to be accepted, Leo needs to have an average number of strokes per round in the summer competition of less than 73.

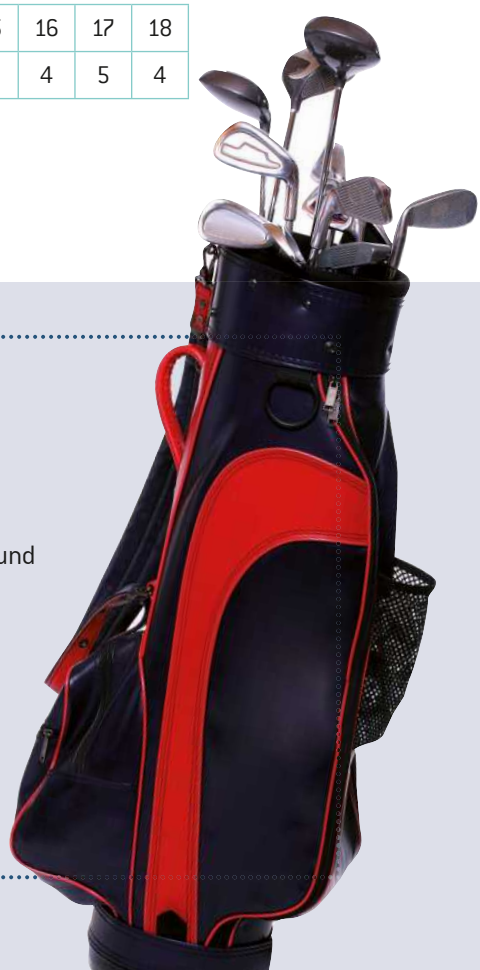
Your task

To investigate Leo's chances of turning professional, carry out these steps:

- decide which round produced Leo's best score
- calculate his average score for the summer competition
- calculate the number of strokes he used to complete the course in each round
- decide whether he can apply to become a professional golfer
- analyse his results to decide where he needs to improve
- work out what his score should be in another round of golf so that his average score for the 11 rounds is close to −0.25

Include all necessary working to justify your answers.

For an extra task, design a 9-hole or 18-hole golf course, showing the tee-off positions and holes on a Cartesian plane.



You can also analyse Leo's result at each hole. For his first two rounds of the summer competition, the number of strokes at each hole is shown below.

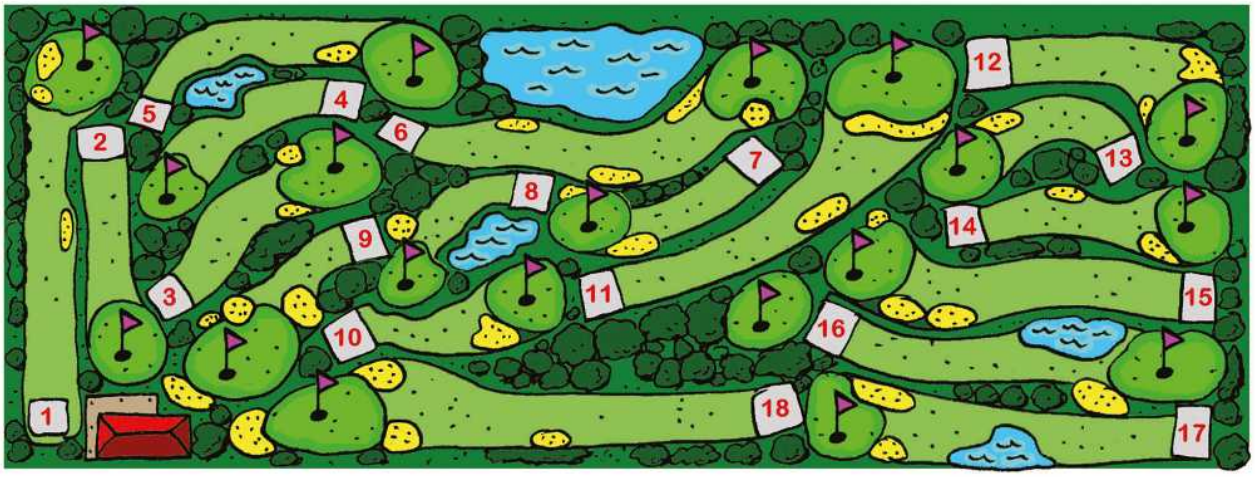
Hole	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Round 1 results	4	3	3	5	8	4	3	6	5	4	4	3	2	5	3	5	4	3
Round 2 results	3	4	4	5	7	5	4	4	4	3	5	2	4	6	3	4	3	3

Golfers often refer to whether the number of strokes at a hole is over or under par for that hole. There are terms to describe this. Can you give a name to each result in Leo's first round?

Golf terms for number of strokes over or under par at a given hole

+3 triple bogey
+2 double bogey
+1 bogey
0 par
−1 birdie
−2 eagle
−3 albatross

As an extension, design a game where you and other players take turns rolling a die to simulate the number of strokes taken at each hole for a round of golf. Describe the rules (including any limitations or assumptions) and then play the game. Who is the best 'golfer' in your group of players? Express your total number of strokes for the round as a score over or under par using the information for Leo's golf course described earlier. Compare your score to those obtained by Leo.





Complete the 3 CONNECT worksheet to show all your working and answers to this task.

You may like to present your findings as a report. Your report could be in the form of:

- a golf brochure
- a player performance report
- a PowerPoint presentation
- other (check with your teacher).

4 ALGEBRA

4A Using pronumerals

4B Evaluating expressions

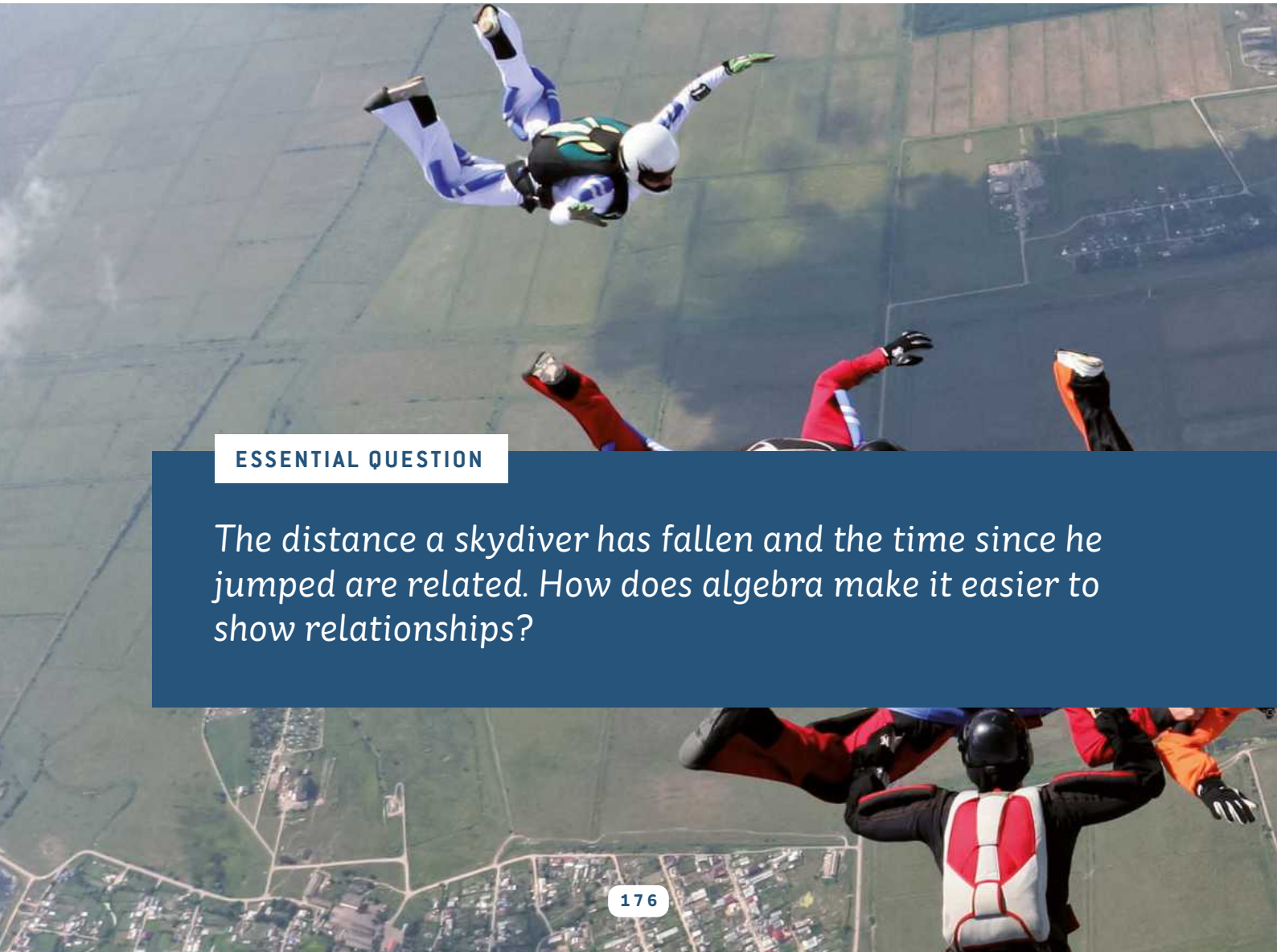
4C Simplifying expressions containing like terms

4D Multiplying algebraic terms

4E Dividing algebraic terms

4F Working with brackets

4G Factorising expressions



ESSENTIAL QUESTION

The distance a skydiver has fallen and the time since he jumped are related. How does algebra make it easier to show relationships?

Are you ready?

4A **1** Look at the algebraic expression $4x + 7$.

a How many terms does it have?

b What is the variable?

A $4x$ **B** 7 **C** x **D** 4

c What is the coefficient of the term containing the pronumeral?

A $4x$ **B** 7 **C** x **D** 4

d What is the constant term?

A $4x$ **B** 7 **C** x **D** 4

4A **2** How many terms does $ab + 3a + abc - 8 + 5ac$ have?

A 3 **B** 4 **C** 5 **D** 6

4A **3** Which of these is an equation?

A $4x + 5y + 7$ **B** $5k - 2p = 12$

C $9a - 5b - 8$ **D** $20 - c^2 + 2cd$

4B **4** What is $-8 + 13$?

A -21 **B** -5 **C** 5 **D** 21

4B **5** Calculate:

a $15 - 19$ **b** $-3 - 5$.

4B **6** What is -6×4?

A -24 **B** -10 **C** 10 **D** 24

4B **7** Calculate:

a -2×-5 **b** $18 \div (-6)$.

4B **8** What is $5 + 3 \times 2$?

A 11 **B** 13 **C** 16 **D** 36

4B **9** Calculate:

a $(10 + 25) \div 5$ b $3(7 - 2)$.

4B **10** **a** What is 8^2?

A 4 **B** 10 **C** 16 **D** 64

b What is 4^3 written in expanded form?

A $4 + 4 + 4$ **B** $3 + 3 + 3 + 3$

C 4×3 **D** $4 \times 4 \times 4$

4B **11** Calculate $7^2 - 5^2$.

4C **12** Calculate $3 + 5 - 9 + 2 - 7$.

A -26 **B** -6 **C** -4 **D** 26

Questions **13** and **14** refer to this rectangle.



4C **13** What is its perimeter?

4D **14** What is its area?

4D **15** **a** How is 5^4 written in expanded form?

A $5 + 5 + 5 + 5$

B $5 \times 5 \times 5 \times 5$

C $4 + 4 + 4 + 4 + 4$

D $4 \times 4 \times 4 \times 4 \times 4$

b Write $3 \times 3 \times 3 \times 3 \times 3$ in index form.

4E **16** **a** What are the factors of 24?

A 1, 2, 3, 4, 6, 8, 12, 24

B 1, 2, 3, 4, 5, 6, 8, 12, 24

C 1, 2, 4, 6, 12, 24

D 1, 3, 6, 8, 10, 12, 14, 16, 24

b Which factors are common to 18 and 48?

A 2, 3, 4, 6, 8, 12, 18

B 1, 2, 3, 4, 6, 8

C 2, 3, 6, 9, 12, 18, 48

D 1, 2, 3, 6

4E **17** **a** What is $\frac{18}{24}$ in simplest form?

A $\frac{18}{24}$ **B** $\frac{9}{12}$ **C** $\frac{6}{8}$ **D** $\frac{3}{4}$

b Write $\frac{45}{72}$ in simplest form.

4A Using pronumerals

Start thinking!



An art teacher orders paint brushes online for her class. There is a delivery charge of $12.

1 **a** What is the total cost if she orders:
i 10 brushes? **ii** 25 brushes?
b Explain how you worked this out.

2 The cost varies depending on how many paint brushes are ordered. How many **variables** are in this relationship? Describe them.

You can use a **pronumeral** to stand for or represent a variable.
A pronumeral is a letter or symbol that takes the place of a number.

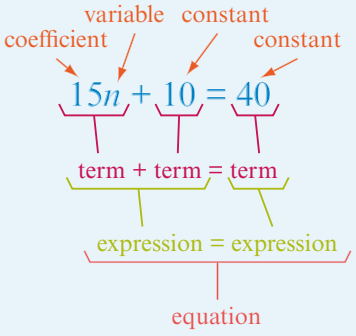
3 The teacher orders n brushes. Why can the cost be written as the algebraic **expression** $8n + 12$?

4 **a** How many terms are in the expression in question **3**?
b What is the **coefficient** of the term containing the variable?
c Is there a **constant term**? Explain.

5 If the teacher spends $140 on paint brushes, the relationship can be written as $8n + 2 = 140$. Is this an expression, an **equation** or a **formula**? Explain.

KEY IDEAS

- A variable is a quantity that can have different values.
- A pronumeral is a letter or symbol used in place of a number. It can represent a variable or an unknown number.
- In algebra, you use pronumerals to make it easier to represent information or a relationship.
- The meanings of the algebraic words expression, equation, formula, term, coefficient and constant are shown in the diagram.



- Equations always have an equals sign. Expressions do not. For example, $5x + 2 = 17$ is an equation and $5x + 2$ is an expression.
- A formula is an equation that has more than one variable; for example, $k = 3m + 7$.

EXERCISE 4A Using pronumerals

UNDERSTANDING AND FLUENCY

1 Answer true or false to each statement. If false, provide your reasoning.

a $5x + y$ is an expression
b $2k + 4$ is an equation
c the constant in $7 + 9p$ is 9
d $a = b + c$ is a formula
e the coefficient of $6m$ is m
f $8y + 3 - x$ has three terms

2 State whether each of these is an expression or an equation.

a $2a + 3$
b $x - 6 = 1$
c $7 = 5 + h$
d $x + y + z = 9$
e $3a + b - 7$
f $k = 4m + 1$

> **NOTE** Pronumerals are printed in italics so they are not confused with abbreviations such as units of measurement. For example, m is for metre but m is a pronumeral representing a variable.

3 Look at the relationships in question **2**.

a Which of the equations is also a formula?
b Which expression contains a term with a coefficient of 2?
c Which expression contains the constant term −7?

EXAMPLE 4A-1 Writing simple expressions

Write an expression for each statement. Use a to represent the unknown number.

a 8 more than a number
b 5 times a number
c a number divided by 9

THINK

a '8 more' means to add 8. Write a plus 8.

b Multiply a by 5. Simplify by writing the number before the pronumeral and leaving out the × sign.

c Write a divided by 9 as a fraction. Remember that the ÷ sign is equivalent to the **vinculum** in a fraction.

WRITE

a Expression is $a + 8$.

b $a \times 5 = 5a$
Expression is $5a$.

c $a \div 9 = \frac{a}{9}$
Expression is $\frac{a}{9}$.

4 Write an expression for each statement. Use a to represent the unknown number.

a 5 more than a number
b a number divided by 3
c 4 less than a number
d 3 times a number
e 8 is subtracted from a number
f 2 is added to a number
g a number is subtracted from 10
h 6 divided by a number
i 50 times a number
j the sum of a number and 19
k twice the value of a number
l the sum of a number, another number x and 8

EXAMPLE 4A-2 Writing more complex expressions

Write an expression for each statement. Use x to represent the unknown number.

a a number is multiplied by 3 and then 5 is added
b a number is divided by 4 and then 7 is subtracted

THINK

a **1** Multiply the number (x) by 3 and simplify.
2 Add 5 to the result and write your answer.

b **1** Divide the number (x) by 4 and simplify.
2 Subtract 7 from the result and write your answer.

WRITE

a $x \times 3 = 3x$
Expression is $3x + 5$.

b $x \div 4 = \frac{x}{4}$
Expression is $\frac{x}{4} - 7$.

5 Write an expression for each statement. Use x to represent the unknown number.
a a number is multiplied by 7 and then 2 is added
b a number is divided by 3 and then 6 is subtracted
c a number is multiplied by 2 and then 5 is subtracted
d a number is divided by 10 and then 1 is added
e a number is multiplied by 15 and then 28 is added
f a number is divided by 20 and then 11 is subtracted

EXAMPLE 4A-3 Simplifying expressions

Write each expression more simply.

a $4 \times (x + 2)$ **b** $(x + 3) \div 5$

THINK

a The × sign can be left out. This is the same as when you write $4 \times a$ more simply as $4a$.

b The brackets and ÷ sign can be replaced with a vinculum.

NOTE Remember that a pair of brackets and a vinculum are examples of grouping symbols, as they group terms together.

WRITE

a $4 \times (x + 2)$
$= 4(x + 2)$

b $(x + 3) \div 5$
$= \frac{x + 3}{5}$

6 Write each expression more simply.

a $3 \times (x + 1)$	**b** $5 \times (x - 9)$	**c** $(x + 2) \times 7$
d $(x - 6) \times 4$	**e** $(x + 8) \times 11$	**f** $(4 - x) \times 9$
g $(x + 5) \div 2$	**h** $(x - 9) \div 8$	**i** $(x + 1) \div 4$
j $(7 + x) \div 12$	**k** $(3 - x) \div 5$	**l** $(x - 10) \div 21$

UNDERSTANDING AND FLUENCY

7 Ingrid starts with an unknown number, x.

- **a** She adds 2 to the number. What expression does she make?
- **b** She wants to multiply the result by 5 so she uses a pair of brackets to group the result before multiplying. Copy and complete: $(x + __) \times 5$ or $__ \times (x + __)$.
- **c** The expression in part **b** can be written more simply. Copy and complete: $__(x + __)$.

8 Jake starts with an unknown number, m.

- **a** He subtracts 3 from the number. What expression does he make?
- **b** He wants to divide the result by 4 so he uses brackets to group the result before dividing. Copy and complete: $(m - __) \div __$.
- **c** This can be written as a fraction. Copy and complete: $\frac{m - __}{__}$.

EXAMPLE 4A-4 Writing expressions using grouping symbols

Write an expression for each statement. Use x to represent the unknown number.

a add 3 to a number and then multiply by 7
b subtract 5 from a number and then divide by 2

THINK	WRITE
a 1 Add 3 to the number (x).	**a** $x + 3$
2 Multiply by 7. Show brackets around $x + 3$ so the multiplication applies to the whole result.	$7 \times (x + 3)$
3 Simplify the expression and write your answer.	Expression is $7(x + 3)$.
b 1 Subtract 5 from the number (x).	**b** $x - 5$
2 Divide by 2. Show brackets around $x - 5$ so the division applies to the whole result.	$(x - 5) \div 2$
3 Simplify the expression using a vinculum and write your answer.	Expression is $\frac{x - 5}{2}$.

9 Write an expression for each statement. Use x to represent the unknown number.

- **a** add 5 to a number and then multiply by 2
- **b** subtract 9 from a number and then multiply by 3
- **c** add 2 to a number and then divide by 4
- **d** subtract 10 from a number and then divide by 7
- **e** subtract a number from 20 and then multiply by 5
- **f** subtract a number from 8 and then divide by 6

10 How is an equation different from an expression?

EXAMPLE 4A-5 Writing equations

Write an equation for each statement. Use a to represent the unknown number.

a 4 more than a number is equal to 15
b the result of multiplying a number by 3 and then subtracting 8 equals 9

THINK	WRITE
a **1** Add 4 to the number (a).	**a** $a + 4$
2 Expression equals 15 so use = to write an equation.	Equation is $a + 4 = 15$.
b **1** Multiply the number (a) by 3 and then subtract 8.	**b** $3a - 8$
2 Expression equals 9 so use = to write an equation.	Equation is $3a - 8 = 9$.

UNDERSTANDING AND FLUENCY

11 Write an equation for each statement. Use a to represent the unknown number.
- **a** 6 more than a number is equal to 18
- **b** 2 less than a number is equal to 9
- **c** a number multiplied by 3 is equal to 24
- **d** a number divided by 5 is equal to 2

12 Write an equation for each statement. Use x for the unknown number.
- **a** the result of multiplying a number by 4 then adding 3 is equal to 11
- **b** the result of dividing a number by 2 then adding 1 is equal to 6
- **c** the result of multiplying a number by 7 then subtracting 5 is equal to 12
- **d** the result of dividing a number by 3 then subtracting 6 is equal to 21
- **e** the result of adding 6 to a number then multiplying by 3 is equal to 24
- **f** the result of adding 11 to a number then dividing by 2 is equal to 9
- **g** the result of subtracting 2 from a number then multiplying by 10 is equal to 60
- **h** the result of subtracting 5 from a number then dividing by 8 is equal to 100

PROBLEM SOLVING AND REASONING

13 Mac owns a bee hive. Use n to represent the number of bees in his hive.
- **a** Write an expression for the number of bees in the hive if 40 young bees hatch.
- **b** What could each expression represent?
  i $n + 300$ **ii** $n - 200$ **iii** $2n$

Mac's friend Kyna also owns a bee hive.
- **c** Kyna estimates she has $n - 500$ bees. Does she have more or fewer bees than Mac?
- **d** Using n, write an expression to represent another hive which has more bees than either Mac's or Kyna's.



PROBLEM SOLVING AND REASONING

14 Hayden is y years old. Write the answer to each of these using algebra.

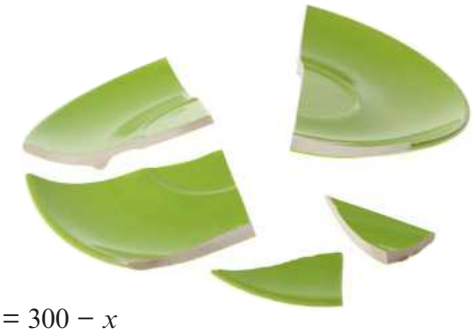
- **a** How old was he:
  - **i** 6 years ago? **ii** 10 years ago?
- **b** How old will he be in:
  - **i** 5 years time? **ii** 23 years time?
- **c** How old is his sister if she is x years younger than him?
- **d** How old is his uncle if he is twice as old?

15 Tickets to a concert cost \$35 for an adult and \$25 for a child. Write an expression for each of these.

- **a** the cost of m adult tickets
- **b** the cost of k child tickets
- **c** the total cost of m adult tickets and k child tickets

16 Shalini is washing up c cups and p plates.

- **a** How many items does she have to wash?
- **b** If she breaks a plate, how many plates are left?
- **c** How many cups and plates does she now have to dry?



17 Kasey earns \$300 each month. The formula $y = 300 - x$ represents a relationship between how much Kasey spends and how much she saves.

- **a** What does x represent?
- **b** What does y represent?
- **c** Write five sets of possible values for x and y that would fit this formula.
- **d** Why are x and y variables?

18 What do expressions, equations and formulas have in common? How are they different? Provide examples to support your answer.

19 What are the advantages of using a pronumeral in a relationship between variables?

CHALLENGE

20 Will has many songs on his iPod. He downloads a further 18 songs. Write a formula to represent the relationship between the number of songs he originally had and the number he now has. Remember to define the pronumerals that represent the two variables.

21 Suggest a real-life situation that could be represented by each expression.

- **a** $a + 6$ **b** $b - 1$ **c** $\frac{c}{4}$
- **d** $2d$ **e** $50 - e$ **f** $3f + 2$

22 If each expression in question **21** is equal to 20, write an equation for each.

Reflect

Why are pronumerals used to represent variables?

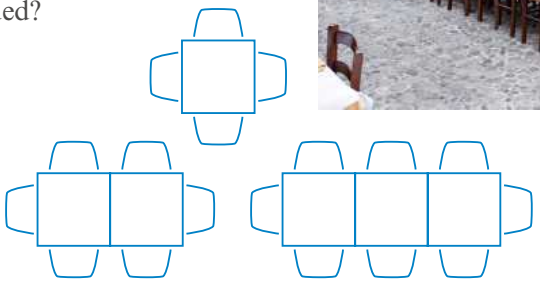
4B Evaluating expressions



Start thinking!

When tables are joined in a line, will four chairs be needed for every table?
What's an easy way to calculate the number of chairs needed?

Consider these table and chair plans.



1 How many chairs fit around two tables?
Why isn't it double the number that fits at one table?

2 How many chairs can be placed around:
a three tables? **b** four tables? **c** five tables?

3 Copy and complete this table.

Number of tables	1	2	3	4	5
Number of chairs	4				

4 If t represents the number of tables and c represents the number of chairs, explain why the formula for the relationship between them is $c = 2t + 2$.

This formula quickly tells you the number of chairs needed if you know the number of tables.

5 If the number of tables is five, replace t with 5 in the formula. This is called **substitution**.
Copy and complete: $c = 2 \times __ + __$.

6 Calculate the value of the expression: multiply 2 by 5 then add 2. This is called **evaluating** the expression.

7 Use the formula to calculate the number of chairs needed for:
a 25 tables **b** 50 tables **c** 117 tables **d** 162 tables.

8 What is the advantage of substituting into a formula rather than drawing a diagram?

KEY IDEAS

- To evaluate an expression, substitute (or replace) each pronumeral with a number and then work out the value.

Remember to use the correct order of operations:
1st **B**rackets (operations inside grouping symbols always calculated first)
2nd **I**ndices (powers and square roots)
3rd **D**ivision and **M**ultiplication (work from left to right)
4th **A**ddition and **S**ubtraction (work from left to right)

- An algebraic term can be written in expanded form, where the multiplication signs are shown, or in simplified form, where the multiplication signs are left out.

$\underbrace{4 \times a \times b}_{\text{expanded form}} = \underbrace{4ab}_{\text{simplified form}}$

EXERCISE 4B Evaluating expressions

EXAMPLE 4B-1 Substituting a value into an expression

Evaluate each expression by substituting 3 for x.

a $x + 4$ **b** $2x - 1$

THINK

a **1** Replace x with 3.

2 Evaluate the expression.

b **1** Remember that $2x$ means $2 \times x$. Replace x with 3.

2 Evaluate the expression. Follow the correct order of operations (multiply then subtract).

WRITE

a $x + 4$

$= 3 + 4$
$= 7$

b $2x - 1$

$= 2 \times 3 - 1$
$= 6 - 1$
$= 5$

UNDERSTANDING AND FLUENCY

1 Evaluate each expression by substituting 8 for x.

a $x + 9$ **b** $x - 1$ **c** $7x$ **d** $\frac{x}{2}$

e $3x + 5$ **f** $2x - 6$ **g** $10 + x$ **h** $12 - x$

2 Evaluate $3a + 5$ by substituting each value for a.

a $a = 2$ **b** $a = 7$ **c** $a = 1$ **d** $a = 0$

e $a = -1$ **f** $a = -3$ **g** $a = \frac{1}{3}$ **h** $a = -\frac{2}{3}$

3 Evaluate $6y - 2$ by substituting each value for y.

a $y = 1$ **b** $y = 3$ **c** $y = -1$ **d** $y = -4$

e $y = 0$ **f** $y = \frac{1}{2}$ **g** $y = \frac{1}{3}$ **h** $y = 2.5$

NOTE Hint: when you substitute into a term that is a product of a number and one or more pronumerals, show the × signs as a reminder to multiply.

EXAMPLE 4B-2 Substituting two values into an expression

Evaluate $5a + 3b$ by substituting $a = 3$ and $b = 2$.

THINK

Replace a with 3 and b with 2. Follow the correct order of operations (multiply then add).

WRITE

$5a + 3b$
$= 5 \times 3 + 3 \times 2$
$= 15 + 6$
$= 21$

EXAMPLE 4B-3 Substituting more than two values into an expression

Evaluate each expression by substituting $a = 3$, $b = 2$ and $c = -4$.

a $2(c - ab)$ **b** $\dfrac{b + 2c}{3}$

THINK

a Replace c with -4, a with 3 and b with 2. Calculate inside the brackets first (multiply then subtract). Remember that $2(-10)$ means $2 \times (-10)$.

b Replace b with 2 and c with -4. Calculate the numerator first and then divide by the denominator.

WRITE

a $2(c - ab)$
$= 2(-4 - 3 \times 2)$
$= 2(-4 - 6)$
$= 2 \times (-10)$
$= -20$

b $\dfrac{b + 2c}{3}$
$= \dfrac{2 + 2 \times (-4)}{3}$
$= \dfrac{2 - 8}{3}$
$= \dfrac{-6}{3}$
$= -2$

4 Evaluate each expression by substituting $a = 5$, $b = 4$ and $c = -2$.

a $a + 4$ **b** $2b + c$ **c** $7a + 2b$ **d** $3b + 6c$

e $4ab$ **f** $\dfrac{b}{c}$ **g** $5b - 2a$ **h** $3(a + bc)$

i $\dfrac{2a + b}{7}$ **j** $8(3a - 4b)$ **k** $\dfrac{abc - 5}{9}$ **l** $\dfrac{b + 2c}{a}$

EXAMPLE 4B-4 Substituting a value into a formula

Find the value of y for each x value, using the formula $y = 3x + 5$.

a $x = -4$ **b** $x = \frac{2}{3}$

THINK

a **1** Replace x with -4.

2 Evaluate the RS of the formula to find y. Take care with negative signs.

b **1** Replace x with $\frac{2}{3}$.

2 Evaluate the RS of the formula to find y.

WRITE

a $y = 3x + 5$
$= 3 \times (-4) + 5$
$= -12 + 5$
$= -7$

b $y = 3x + 5$
$= 3 \times \frac{2}{3} + 5$
$= \dfrac{\cancel{3}^{1}}{1} \times \dfrac{2}{\cancel{3}_{1}} + 5$
$= 2 + 5$
$= 7$

UNDERSTANDING AND FLUENCY

5 Find the value of y for each x value, using the formula $y = 2x + 7$.

a $x = 3$ **b** $x = 8$ **c** $x = -6$ **d** $x = -1$

e $x = 4.5$ **f** $x = 0$ **g** $x = \frac{1}{2}$ **h** $x = -\frac{3}{2}$

6 Find the value of k for each m value, using the formula $k = 4m - 3$.

a $m = 1$ **b** $m = 6$ **c** $m = -2$ **d** $m = 50$

e $m = 1.5$ **f** $m = \frac{1}{4}$ **g** $m = 0$ **h** $m = -\frac{3}{4}$

7 **a** The term 3^2 means 3×3. What does a^2 mean?

b If $a = 5$, evaluate:

i a^2 **ii** $3a^2$ **iii** $a^2 + 7$.

8 **a** The term 2^3 means $2 \times 2 \times 2$. What does p^3 mean?

b If $p = 4$, evaluate:

i p^3 **ii** $2p^3$ **iii** $p^3 - 12$.

EXAMPLE 4B-5 Substituting values into expressions containing powers

Evaluate each expression by substituting $x = 3$ and $y = 2$.

a $x^2 + 2$ **b** $5y^3 - 7$ **c** $\frac{x^2 - y^2}{10}$

THINK

a **1** Replace x with 3.

2 Follow the correct order of operations (square 3 then add 2).

b **1** Replace y with 2.

2 Follow the correct order of operations (raise 2 to the power of 3, multiply by 5 and then subtract 7).

c **1** Replace x with 3 and y with 2.

2 Calculate each power and then subtract. Write the answer as a simplified fraction.

WRITE

a $x^2 + 2$
$= 3^2 + 2$
$= 3 \times 3 + 2$
$= 9 + 2$
$= 11$

b $5y^3 - 7$
$= 5 \times 2^3 - 7$
$= 5 \times (2 \times 2 \times 2) - 7$
$= 5 \times 8 - 7$
$= 40 - 7$
$= 33$

c $\frac{x^2 - y^2}{10}$
$= \frac{3^2 - 2^2}{10}$
$= \frac{9 - 4}{10}$
$= \frac{5}{10}$
$= \frac{1}{2}$

UNDERSTANDING AND FLUENCY

9 Evaluate each expression by substituting $x = 4$ and $y = 3$.

a x^2	**b** y^2	**c** $x^2 + 7$	**d** $y^2 - 5$
e $3x^2$	**f** $2y^2$	**g** $5x^2 - 2$	**h** $8y^2 + 11$
i $2x^3 + 6$	**j** $4y^3 - 1$	**k** $y^4 + 9$	**l** $3x^2 + 5y^2$
m $\frac{x^2}{2}$	**n** $\frac{y^2 - 1}{4}$	**o** $\frac{x^2 + y^2}{5}$	**p** $\frac{x^2 - y^2}{7}$
q $\frac{x^2 + 2y^3}{80}$	**r** $\frac{64}{2x^3}$	**s** $\frac{1}{x^3 + y^3}$	**t** $\frac{3}{2x^2 - y^3}$

10 Evaluate each expression by substituting $a = -6$ and $b = 2$.

a $a - b$	**b** $a^2 + b^2$	**c** $7ab$	**d** $8a + 7b$
e a^2b	**f** ab^2	**g** $5ab - b^2$	**h** $100 - 2a^2b^2$
i $-3ab^2 + 1$	**j** $2(5ab - 3)$	**k** $\frac{4a^2b + b}{10}$	**l** $\frac{9(a - b^2)}{6}$

11 Complete the table of values for each formula.

a $y = 2x + 5$

x	0	1	2	3	4
y					

b $y = 4x - 3$

x	−2	−1	0	1	2
y					

c $y = 3x^2 + 1$

x	0	1	2	3	4
y					

d $y = 2x^3 - 7$

x	−2	−1	0	1	2
y					

PROBLEM SOLVING AND REASONING

12 The number of legs on n grasshoppers can be represented by l.



a Write a formula using l and n to describe the relationship between them.

b Find the value of l for each of these n values.

i 3 **ii** 7

iii 50 **iv** 101

c Use the formula to calculate how many legs in total there would be on 950 grasshoppers.

d If the number of legs on an unknown number of grasshoppers was estimated to be between 500 and 600, give three possible answers for the number of grasshoppers there could be. Explain your reasoning.

e Grasshoppers are unusual as they have five eyes. Write a formula to describe the relationship between the number of grasshoppers and the corresponding number of eyes. Remember to define the pronumeral you use for each variable.

f Use your formula to calculate the number of eyes on:

i 456 grasshoppers

ii 2035 grasshoppers.

PROBLEM SOLVING AND REASONING

13 The relationship between average speed (s), distance (d) and time (t) can be represented by the formula $s = \frac{d}{t}$.

a Calculate the value of s for these d and t values.

i $d = 8$, $t = 2$ **ii** $d = 48$, $t = 6$ **iii** $d = 6.5$, $t = 5$ **iv** $d = 82$, $t = 4$

b Find the average speed (in metres per second) of a bicycle that travels 120 m in 40 seconds.

c Find the average speed (in kilometres per hour) of a motorcyclist who travels 282 km in 3 hours.



14 The distance a skydiver falls through the air in metres (d) after an amount of time in seconds (t) can be calculated using the formula $d = 4.9t^2$.

a How far does the skydiver fall in the first second? (Hint: what value of t will you substitute into the formula?)

b How far does the skydiver fall:

i in the first 3 seconds? **ii** in the first 5 seconds?

c Is the skydiver's speed increasing or decreasing over the first 5 seconds? Explain.

15 A recent 'longest lunch' catered for 900 people. Use the formula in 4B Start thinking (page 184) to work out how many tables and chairs were needed.

CHALLENGE

16 The volume of a sphere can be calculated if you know its radius. If V represents the volume and r represents the radius, use the formula $V = 4.2r^3$ to find the approximate volume of a golf ball with a radius of 2.1 cm.

17 In most countries of the world, temperature is measured in degrees Celsius. Some countries, such as the USA, use degrees Fahrenheit. A formula can be used to convert from one temperature unit to the other. Let C represent the temperature in degrees Celsius and F represent the temperature in degrees Fahrenheit.

a Use the formula $C = \frac{5}{9}(F - 32)$ to convert these temperatures to degrees Celsius. Write each answer to the nearest degree.

i 77°F **ii** −4°F **iii** 41°F **iv** 100°F **v** −1°F **vi** 62°F

b Use the formula $F = \frac{9}{5}C + 32$ to convert these temperatures to degrees Fahrenheit. Write each answer to the nearest degree.

i 30°C **ii** −10°C **iii** 0°C **iv** 27°C **v** −8°C **vi** 42°C

c The temperature in Chicago is 23°F. What is it in degrees Celsius?

d Which temperature is lower: 86°F or 32°C? Justify your answer.

18 Choose a relationship between two (or more) variables and describe it with a formula. Remember to define the pronumeral used for each variable. Now write three questions involving substituting into your formula and give them to a classmate to work out.

Reflect

When is substitution useful in algebra?

4C Simplifying expressions containing like terms



Start thinking!

1 Each bag of Cherry Bites contains a chocolates. Melinda buys three bags of Cherry Bites. Explain why the total number of chocolates in the three bags would be $a + a + a$ or $3 \times a$ or $3a$.

2 Peter buys five bags of Cherry Bites.
Write three expressions for the total number of chocolates in five bags.

3 The term $3a$ is the simplest form of the three expressions in question **1**.
Which expression is the simplest in question **2**?

4 Peter adds his five bags to Melinda's three bags.
- **a** Complete this new expression for the total number of chocolates: $3a + __a$
- **b** Can this expression be simplified further? If so, write your simplified expression.

5 Nick has bought two bags of chocolate bullets. Each bag contains b chocolates. Write a simple expression for the total number of chocolates in the two bags of chocolate bullets.

6 Melinda, Peter and Nick pool all their chocolates together.
- **a** Complete this new expression for the total number of chocolates: $__a + __b$
- **b** Can this expression be simplified further?

7 The terms $3a$ and $5a$ are called **like terms**. Are $8a$ and $2b$ like terms? Explain.

8 Explain when an expression can be simplified.

KEY IDEAS

- Terms containing exactly the same pronumerals are called like terms.
- Expressions can be simplified by adding or subtracting like terms.
- Like terms can be added (or subtracted) by adding (or subtracting) the coefficients of the terms.
- Rearranging an expression to group like terms is called 'collecting like terms'.

NOTE The order of the pronumerals can be different, but pronumerals are usually shown in alphabetical order.

EXERCISE 4C Simplifying expressions containing like terms

UNDERSTANDING AND FLUENCY

1 Answer true or false to each statement. For false statements, provide your reasoning.

a $5p$ and $9p$ are like terms
b $3k$ and k are like terms
c $7a$ and $7d$ are like terms.
d $-6xy$ and $2xy$ are like terms.
e $2a$ and $3ac$ are like terms.
f $8abc$ and $-4acb$ are like terms.

2 Write three more examples of terms that are like terms with each of these.

a $4y$ **b** $7a$ **c** $-9n$ **d** $2ab$
e $-10gh$ **f** $3kmn$ **g** $5a^2b$ **h** x

3 Answer true or false to each statement. For the statements that are false, write the correct simplified expression.

a $8x + 6x$ simplifies to $14x$
b $3p + 6q$ simplifies to $9pq$
c $5m + 2n + m$ simplifies to $6m + 2n$
d $11k - k$ simplifies to 11
e $2ab + 3a + 7ba$ simplifies to $12ab$
f $4a + 3c + 2a + c$ simplifies to $6a + 4c$
g $xy + 2x + y$ cannot be simplified
h $9p + 5q + 2p$ cannot be simplified

EXAMPLE 4C-1 Simplifying an expression with two terms

Simplify each expression.

a $9x + 7x$ **b** $5d - 3d$ **c** $6y - y$ **d** $5ab - 9ab$ **e** $10gh - 8h$

THINK

a The expression contains like terms as the pronumeral is exactly the same. Simplify by adding the terms.

b The expression contains like terms, so simplify by subtracting the terms.

c The expression contains like terms, so the terms can be subtracted. Remember that y is the same as $1 \times y$ or $1y$.

d The expression contains like terms as the two pronumerals in each term are exactly the same. Simplify by subtracting the terms.

e The two terms are not like terms as the pronumerals in each term are not exactly the same. The expression cannot be simplified.

WRITE

a $9x + 7x = 16x$

b $5d - 3d = 2d$

c $6y - y = 5y$

d $5ab - 9ab = -4ab$

e $10gh - 8h$ cannot be simplified.

4 Simplify each expression.

a $5x + 4x$	**b** $8y - 3y$	**c** $6mn + 9mn$
d $10kp - 2kp$	**e** $3a - 7a$	**f** $xy + 4$
g $3c + c$	**h** $5g - g$	**i** $2xy + 4y$
j $6ab - ab$	**k** $m - 8m$	**l** $3abc - 4abc$

EXAMPLE 4C-2 Simplifying expressions with more than two terms

Simplify:

a $4ab + 5ab - 2ab$ **b** $3xy + 4yx + xy$ **c** $2m + 3n + 4m + 5mn$

THINK

a All three terms are like terms as they have exactly the same pronumerals, so the expression can be simplified.

b All three terms are like terms as they have exactly the same pronumerals, even though the order of the pronumerals is different in $4yx$. Simplify by adding the terms. Remember that xy is the same as $1xy$.

c Identify any like terms ($2m$ and $4m$) and group them together. Simplify the expression by adding the like terms.

WRITE

a $4ab + 5ab - 2ab$
$= 9ab - 2ab$
$= 7ab$

b $3xy + 4yx + xy$
$= 7xy + xy$
$= 8xy$

c $2m + 3n + 4m + 5mn$
$= 2m + 4m + 3n + 5mn$
$= 6m + 3n + 5mn$

UNDERSTANDING AND FLUENCY

5 Simplify:

a $2k + 7k + 6k$	**b** $3ab + 5ab - 2ab$	
c $8x + 4x + 3y$	**d** $4m + 3n + 9m$	
e $8d - 6d + 1$	**f** $3x + 2xy + 4yx$	
g $2x + 3y + 8y + 9x$	**h** $a + 7 + a + 2$	
i $5rst + 2rst - 7rst$	**j** $3dy + 5y + yd + 8$	
k $4gh + 3hg - 6gh$	**l** $a + 2ab + 3b + 5a$	

6 Which expression (**A**, **B**, **C** or **D**) is the same as $7a^2b$?

A $7 \times a \times b$	**B** $7 + a + a + b$
C $7 \times a \times a \times b$	**D** $7 \times a \times b \times b$

7 Answer true or false to each statement. For false statements, provide your reasoning.

a $2a^2$ and $7a^2$ are like terms	**b** $8b$ and b^2 are like terms
c $3a^2b$ and $5a^2b$ are like terms	**d** $4ab$ and $6a^2b$ are like terms
e $-2a^2$ and $-7a^2b$ are like terms	**f** ab^2 and $2ab^2$ are like terms
g $8a^2bc$ and $-a^2bd$ are like terms	**h** $5ab^2$ and $-9a^2b$ are like terms

EXAMPLE 4C-3 Simplifying expressions with powers

Simplify:

a $5a^2b + 2a^2b$ **b** $4x^2 - 6x^2$ **c** $2c^2d + 3cd$ **d** $7xy^2 + 4x^2y + xy^2$

THINK

a The expression contains like terms, as the pronumerals are exactly the same ($a \times a \times b$). Simplify by adding the terms.

b The expression contains like terms, so can subtract one from the other.

c The two terms are not like terms as the pronumerals are not exactly the same. $c \times c \times d$ is different from $c \times d$. The expression cannot be simplified.

d Identify any like terms ($7xy^2$ and xy^2) and group them together. Simplify by adding like terms.

WRITE

a $5a^2b + 2a^2b$
$= 7a^2b$

b $4x^2 - 6x^2$
$= -2x^2$

c $2c^2d + 3cd$ cannot be simplified.

d $7xy^2 + 4x^2y + xy^2$
$= 7xy^2 + xy^2 + 4x^2y$
$= 8xy^2 + 4x^2y$

UNDERSTANDING AND FLUENCY

8 Simplify:

a $3x^2y + 11x^2y$ **b** $9ab^2 - 7ab^2$
c $4k^2 + 3k^2$ **d** $4mn + 2m^2n$
e $-5c^2 + 3c^2$ **f** $6pq - 5pq + 8p^2q$
g $ab^2 + 4a^2b + 3ab^2$ **h** $2x^2y + 10yx^2 - x^2y$
i $7g^3h - 9g^3h + 5g^2h$ **j** $4m^2n^4 + 6m^2n - 3m^2n^4$

9 Consider the expression $5m + 4k + 9m + k$.

a Evaluate the expression by substituting $m = 2$ and $k = 7$ into the four terms.

b Simplify the expression by adding like terms.

c Check your answer to part **b** by substituting $m = 2$ and $k = 7$ into the simplified expression. Do you obtain the same answer as part **a**?

d Which expression was easier to evaluate? Explain.

10 Simplify these expressions first, then evaluate each expression by substituting $a = 5$ and $b = 3$.

a $4a + 2a + 6b + 8b$ **b** $2a + 3b + 7a + 11b$
c $9a - 6a + 7b + 2b$ **d** $5ab + 4b + 2a + b$
e $6ab - ba + 3ab + a$ **f** $4b - b + 8a - a$
g $3a^2 + 2a^2 + b^2 - 4b^2$ **h** $7ab^2 + 2a^2b - 5ab^2$

11 Write three different expressions that simplify to $4p + 5t$.

12 Paree is deciding on the size of her garden bed, which is to be rectangular.



a Draw a diagram of a rectangle to represent Paree's garden bed.

b If l stands for the length and w stands for the width, label each of the four sides of the rectangle.

c Write an expression for the perimeter of the garden bed by completing this: $l + w +$ ____ + ____.

d Simplify the expression.

e If P stands for the perimeter, write a formula for the perimeter of a rectangle.

f Use the formula to calculate the perimeter of rectangles with each of these length and width measurements.

i $l = 4$ m, $w = 3$ m **ii** $l = 6.4$ m, $w = 2.8$ m **iii** $l = 15\frac{1}{2}$ m, $w = 7\frac{1}{2}$ m

g Paree decides to plant box hedge around the edge of her garden bed. If she has enough plants to extend around a rectangle with perimeter 20 m, suggest three different sets of length and width measurements for her garden bed.

13 Consider the expression: $7a + 5b - 2a + 3b + 4a - 6b + c$. To simplify this expression, it is easier to rearrange the expression so that like terms are grouped together. This is called collecting like terms.

a Copy and complete the following to show how the expression can be rearranged.

$$\underbrace{7a - __a + 4a}_{\text{like terms}} + \underbrace{5b + __b - __b}_{\text{like terms}} + c$$

b Notice that the minus sign (−) in front of $2a$ has moved with the term. Has this happened with any other term? Explain.

c Check that the addition or subtraction sign in front of each term has moved with that term. (That is, does each term still have the same sign in front of it?)

d Now that like terms have been collected, simplify the expression.

e Show how you can check your simplified expression is equivalent to the original expression. (Hint: substitute a value for each of a, b and c.)

f Which expression is easier to evaluate if you have a, b and c values? Explain.

14 Copy and complete each set of working to simplify the expression.

a $6x + 4y - 3x + 7y + 5x - 2y + y$
$= 6x - __x + __x + 4y + __y - __y + y$
$= __x + __y$

b $2mn + 5m + 8mn - 9m + 4n - 6mn + 2m$
$= 2mn + __mn - __mn + 5m - __m + __m + __$
$= __mn - __m + ___$

15 When rearranging $3x - 2y + 7x - 4y$, Tania wrote $3x - 7x + 2y - 4y$.

a Explain her mistake.

b Show how to get the correct result for simplifying $3x - 2y + 7x - 4y$.

PROBLEM SOLVING AND REASONING

16 Simplify each expression by collecting like terms.

a $5x + 3y + 6y - 2x - 7y + 8x$

b $4y + 6 - 3y - 2 + 5y - y$

c $-2x + 5y + 4x - 8y + 2y - 3$

d $xy - x + 5y - 3xy + 9x - 5y$

e $8x^2 + 3x^2 - 6x + 4 + 5x - 9 + x$

f $3x + 7xy - 2y - 11x - xy + 8y - x$

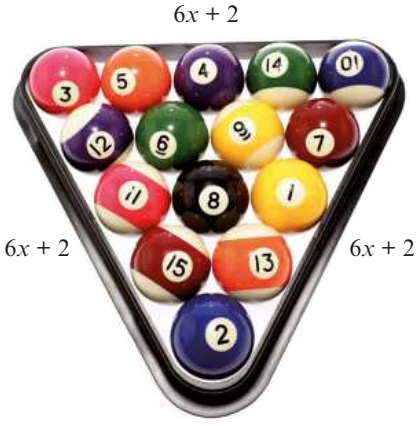
17 Evaluate each expression in question **16** by substituting $x = 2$ and $y = -5$.

18 Write an expression for the perimeter described in each situation.

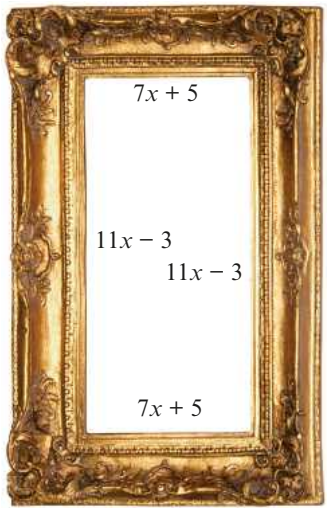
a length of outer edge of frame



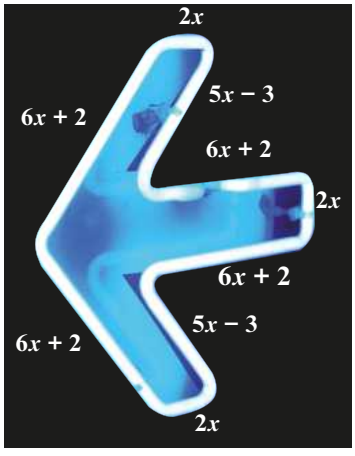
b perimeter of triangle



c perimeter of inner frame



d length of neon used



19 Calculate the perimeter for each shape in question **18** if x is 4 cm.

CHALLENGE

20 For each shape in question **18**, list three possible values of x that would give a perimeter of between 200 and 350 cm.

21 Simplify:

a $8a^2b + 2ab^2 + 3a^2b + 9ab^2$

b $4e^2fg + e^2fh + efgh - 6efgh$

c $x^2y + 3xy^2 + 5xy + xy^2 + 2xy$

Reflect

How can you recognise like terms?

4D Multiplying algebraic terms



Start thinking!

1 During a sale, a store offers DVDs for $15 each.
   - **a** How much would it cost to buy:
     - **i** two DVDs? **ii** seven DVDs?
   - **b** Which operation (+, −, × or ÷) did you use to obtain your answers?
   - **c** How much would it cost to buy k DVDs? Write your answer as an algebraic term.
   - **d** This term can be written in simplified form or expanded form. Copy and complete this statement. Notice that the number goes first in each case.

$$15 \times __ = __k$$

expanded form simplified form

   - **e** How is the simplified form of the term different from the expanded form?

2 Before the sale, the price varied.
   - **a** If the price for each DVD was p dollars, how much would it cost to buy:
     - **i** two DVDs? **ii** seven DVDs?
   - **b** How much would it cost to buy k DVDs?

3 When simplifying an algebraic term, what do you notice when you:
   - **a** multiply numbers?
   - **b** multiply pronumerals?

4 How could you simplify an algebraic term that contains numbers and pronumerals multiplied together?

KEY IDEAS

- An algebraic term can be written in expanded form, where the multiplication signs are shown, or in simplified form, where the multiplication signs are left out.

$$4 \times a \times b = 4ab$$

expanded form simplified form

- The product of any pronumerals that are the same can be simplified using index notation. For example, $a \times a \times a = a^3$.
- When multiplying algebraic terms, multiply the numbers (or coefficients) first and simplify the products of any pronumerals that are the same.
- The final simplified term is written with the number (or coefficient) first and the pronumerals in alphabetical order.
- The **index laws** apply to pronumerals in the same way as they do to numbers.
- To multiply terms in index form with the same base, write the base and add the indices (powers). For example, $a^5 \times a^3 = a^8$.

EXERCISE 4D Multiplying algebraic terms

EXAMPLE 4D-1 Writing algebraic terms in expanded form

Write each term in expanded form.

a $8abc$ **b** $-5m^2n$ **c** $2xy^3z$

THINK

a Write the term with multiplication signs.

b Write the term with multiplication signs.
As the term is negative, the coefficient is negative.
Remember that m^2 means $m \times m$.

c Write the term with multiplication signs.
Remember that y^3 means $y \times y \times y$.

WRITE

a $8abc = 8 \times a \times b \times c$

b $-5m^2n$
$= -5 \times m \times m \times n$

b $2xy^3z$
$= 2 \times x \times y \times y \times y \times z$

1 Write each term in expanded form.

a $5m$ **b** $-2a$ **c** $8xy$ **d** $-6cd$
e $7abc$ **f** $-10gkmn$ **g** $3p^2q$ **h** $-4x^3y^2z$

2 Match each term in simplified form (left column) with the equivalent term in expanded form (right column).

simplified form	expanded form
a $2a^2bc$	**A** $2 \times a \times a \times a \times b \times c$
b $2a^3bc$	**B** $2 \times a \times a \times b \times c$
c $2ab^2c^2$	**C** $2 \times a \times b \times b \times c \times c$

EXAMPLE 4D-2 Multiplying two simple terms

Simplify:

a $7 \times a$ **b** $b \times -6$ **c** $-1 \times -c$

THINK

a Simplify by leaving out the $\times$ sign.

b The product of a positive term and a negative term is negative.
Leave out the $\times$ sign and write the number before the pronumeral.

c The product of two negative terms is positive. Leave out the $\times$ sign.
Remember that if the coefficient is 1, it does not need to be shown.

WRITE

a $7 \times a = 7a$

b $b \times -6 = -6b$

c $-1 \times -c = 1c$
$= c$

3 Simplify:

a $4 \times x$ **b** $6 \times k$ **c** $d \times 7$ **d** $m \times 5$ **e** $1 \times p$ **f** $-3 \times h$
g $a \times -9$ **h** $-y \times 10$ **i** $8 \times -c$ **j** $-1 \times e$ **k** $-2 \times -f$ **l** $-m \times -3$

EXAMPLE 4D-3 Multiplying three simple terms

Simplify:

a $2 \times a \times d$ **b** $b \times -5 \times c$ **c** $3 \times k \times 6$ **d** $4 \times m \times m$

THINK	WRITE
a Simplify by leaving out the × signs.	**a** $2 \times a \times d$ $= 2ad$
b The terms can be multiplied in any order. The product of two positive terms and one negative term is negative. Simplify by leaving out the × signs and writing the number before the pronumerals.	**b** $b \times -5 \times c$ $= -5 \times b \times c$ $= -5bc$
c Multiply the numbers. ($3 \times 6 = 18$) Simplify by leaving out the × sign.	**c** $3 \times k \times 6$ $= 3 \times 6 \times k$ $= 18 \times k$ $= 18k$
d Write the product of any pronumerals that are the same in simplified form. ($m \times m = m^2$) Leave out the × sign.	**d** $4 \times m \times m$ $= 4 \times m^2$ $= 4m^2$

4 Simplify:

a $3 \times a \times b$ **b** $4 \times m \times n$ **c** $p \times 2 \times t$ **d** $d \times -8 \times c$
e $x \times y \times 9$ **f** $k \times f \times 5$ **g** $a \times b \times c$ **h** $h \times y \times e$
i $2 \times q \times 4$ **j** $m \times 3 \times -6$ **k** $7 \times a \times a$ **l** $n \times 2 \times n$

EXAMPLE 4D-4 Multiplying algebraic terms

Simplify $3ab \times 6bc$.

THINK	WRITE
1 Write each term in expanded form.	$3ab \times 6bc$ $= 3 \times a \times b \times 6 \times b \times c$
2 Multiply the numbers (or coefficients). ($3 \times 6 = 18$)	$= 18 \times a \times b \times b \times c$
3 Write the product of any pronumerals that are the same in simplified form. ($b \times b = b^2$)	$= 18 \times a \times b^2 \times c$
4 Simplify by leaving out the × signs.	$= 18ab^2c$

UNDERSTANDING AND FLUENCY

5 Simplify:

a $7a \times 3c$ **b** $-5p \times 6$ **c** $8n \times 3n$

d $2ef \times 5gh$ **e** $9xy \times -2xz$ **f** $3ab \times 2d \times 4f$

g $-2gh \times 5b \times 7k$ **h** $mn \times 6m \times 8q$ **i** $4ab \times 3ac \times 2a$

j $9m \times np \times 2m^2$ **k** $7gh \times -3gh \times fgh$ **l** $-2wx^2 \times 5wy \times -4u^2$

6 Copy and complete these multiplication tables. Multiply each term in the left column by each term in the top row.

a

×	4a	3	6m	2abc
3d				
ab^2				
5mn				
2ac				

b

×	3x	4xyz	−5y	$2x^2$
−2x				
xy^2				
6xy				
−7yz				

7 Copy and complete these multiplication problems. Each term is written in expanded form first.

a $2^3 \times 2^4$
$= 2 \times _ \times _ \times 2 \times _ \times _ \times _$
$= 2^7$

b $a^3 \times a^4$
$= a \times _ \times _ \times a \times _ \times _ \times _$
$= __$

c $m^2 \times m^6$
$= m \times _ \times m \times _ \times _ \times _ \times _ \times _$
$= __$

d $x^5 \times x^2 \times x^3$
$= x \times _ \times _ \times _ \times _ \times x \times _ \times x \times _ \times _$
$= __$

8 Look at your answers to question **7**. Can you see a shortcut that could be used? Explain. This shortcut is known as one of the index laws.

EXAMPLE 4D-5 Multiplying terms in index form using an index law

Use an index law to simplify:

a $a^4 \times a^7$ **b** $b^3 \times b \times b^5$.

THINK

a **1** Check the bases are the same (yes).

2 Write the base and add the indices. (4 + 7 = 11)

b **1** Check the bases are the same (yes).

2 Write the base and add the indices. (3 + 1 + 5 = 9)
Remember that b has an index or power of 1.

WRITE

a $a^4 \times a^7$
$= a^{11}$

b $b^3 \times b \times b^5$
$= b^9$

9 Use an index law to simplify:

a $a^9 \times a^7$ **b** $x^5 \times x^8$ **c** $m^6 \times m^{11}$
d $k^4 \times k$ **e** $y^{10} \times y^{10}$ **f** $e \times e^5$
g $c^4 \times c^5 \times c^6$ **h** $p^2 \times p^7 \times p^3$ **i** $g^8 \times g^4 \times g^2$
j $n^5 \times n^9 \times n$ **k** $x \times x^7 \times x^8$ **l** $h^6 \times h \times h^6$

EXAMPLE 4D-6 Multiplying algebraic terms using an index law

Use an index law to simplify:

a $2a^3 \times a^6$ **b** $4b^3 \times 3b \times 2b^5$.

THINK	WRITE
a 1 Write the first term as the product of a number and a pronumeral in index form.	**a** $2a^3 \times a^6$ $= 2 \times a^3 \times a^6$
2 Simplify the terms in index form. Check that they have the same base (yes), write the base and add the indices. (3 + 6 = 9)	$= 2 \times a^9$
3 Simplify by leaving out the × sign.	$= 2a^9$
b 1 Write each term as the product of a number and a pronumeral in index form.	**b** $4b^3 \times 3b \times 2b^5$ $= 4 \times b^3 \times 3 \times b \times 2 \times b^5$
2 Reorder the terms so the numbers are grouped together and the pronumerals are grouped together.	$= 4 \times 3 \times 2 \times b^3 \times b \times b^5$
3 Multiply the numbers (4 × 3 × 2 = 24) and simplify the terms in index form by adding the indices. (3 + 1 + 5 = 9)	$= 24 \times b^9$
4 Simplify by leaving out the × sign.	$= 24b^9$

UNDERSTANDING AND FLUENCY

10 Use an index law to simplify:

a $3y^3 \times y^6$ **b** $g^2 \times 7g^5$ **c** $2b^8 \times 3b^3$
d $6k^5 \times 2k^8$ **e** $4w^5 \times 2w^3 \times 5w^9$ **f** $5g^5 \times 2g \times 8g^5$
g $3c \times 3c^7 \times 3c^6$ **h** $p^6 \times 3p^2 \times 5p^2$ **i** $7h^7 \times 2h \times h^4$

11 Jacob wrote that $a^6 \times b^5$ simplifies to ab^{11}. Is he correct? Explain.

12 In a few sentences, explain how and when the index law for multiplication of terms can be used.

13 Simplify:

a $a^2 \times h^3$ **b** $6x^4 \times 2y^5$ **c** $4t^3 \times 5b^8$
d $m^6 \times m^2 \times q^7$ **e** $x^2 \times y^5 \times x^6 \times y^2$ **f** $3g^4 \times 5h^3 \times 2g^6$
g $a^5b^4 \times a^3b^2$ **h** $5x^6y^5 \times 3x^2y^5$ **i** $9w^4x^8 \times 6x^5y^4$

PROBLEM SOLVING AND REASONING

14 Give three examples of two terms that can be multiplied to make each statement true.

a ___ × ___ = $abcd$ **b** ___ × ___ = $12x^2yz$ **c** ___ × ___ = $20mn^3p^2$

15 A rectangle has an area given by $24x^2$. One way of writing this is $8x \times 3x$. Describe three other rectangles that have an area of $24x^2$. That is, list the length and width for each of the three different rectangles.

16 The top of this box is rectangular, with a length three times its width. All measurements are in centimetres.



a If the width is x, write an algebraic term that would represent the length in centimetres.

b The area of a rectangle is length × width. Write the area of the top of the box as a multiplication problem using algebra and then simplify.

c Calculate the area of the top of the box if x is 8.

d How can you check your answer to part **c** is correct? Show there are two ways of obtaining the answer.

e Which way would be quicker if you were to calculate the area of the top using many different x values? Explain.

f The height of the box is five times the width. Write an algebraic term to represent the height.

g Draw a diagram of the box with the length, width and height labelled with their respective algebraic terms.

h Copy and complete this table to obtain the simplified algebraic term for the area of each of the six sides (or faces) of the box.

	area
top	$3x \times x = 3x^2$
bottom	__ × __ = __
front	__ × __ = __
back	__ × __ = __
left side	__ × __ = __
right side	__ × __ = __

i Use your answers to part **h** to write an expression for the surface area of the box. Simplify if possible.

j Calculate the surface area of the box if x is 4. Show two different ways of obtaining your answer.

k Calculate the surface area of the box for each x value.

i 5 **ii** 12 **iii** 0.5 **iv** 2.1

l Write an algebraic term to represent the volume of the box. (Hint: volume = length × width × height.)

m Use a calculator to work out the volume of the box for each x value given in part **k**.

CHALLENGE

17 Write two algebraic multiplication problems of your own that need to be simplified. Swap them with a classmate to complete. Check the answers together and discuss how any errors may have been made and how they can be corrected.

18 Simplify $a^m b^x \times a^n b^y$.

Reflect

What is important to remember when multiplying algebraic terms?

4E Dividing algebraic terms

Start thinking!

Dividing algebraic terms is very like dividing numbers written as fractions.

1 What is:

a $3 \div 3$? **b** $a \div a$?

2 These division problems can be written as fractions: $3 \div 3 = \frac{3}{3}$ and $a \div a = \frac{a}{a}$.

Show how dividing the numerator and denominator by the highest common factor (HCF) produces the same results you found in question **1**.

3 The division problem $10 \div 2$ can be written as $\frac{10}{2}$. Show how this simplifies to 5. (Hint: what is the HCF of the numerator and denominator?)

4 Similarly, $10a \div 2$ can be written as $\frac{10a}{2}$ or $\frac{10 \times a}{2}$. Show how $\frac{10a}{2}$ simplifies to $5a$.

5 Now look at simplifying $\frac{10a}{2a}$ in the same way.

a Write $\frac{10a}{2a}$ with both the numerator and denominator written in expanded form.

b Which two factors are common to the numerator and denominator?

c Show that $\frac{10a}{2a}$ simplifies to 5.

6 Repeat question **5** for $\frac{10ab}{2a}$ to show that the fraction simplifies to $5b$.

7 Explain how to divide algebraic terms.

KEY IDEAS

- Division problems that contain algebraic terms should be written as fractions.
- Algebraic fractions can be simplified like any other fractions:
  1. Look for common factors (numbers or pronumerals) in the numerator and denominator.
  2. Cancel (or divide by) the common factors.
  3. Write the remaining terms in simplified form.
- Writing algebraic terms in expanded form can help identify any common factors.
- The index laws apply to pronumerals in the same way as they do to numbers.
- To divide terms in index form that have the same base, write the base and subtract the indices (powers). For example, $a^5 \div a^3 = a^2$.
- A number in index form with a power of zero equals one. For example, $a^0 = 1$.

EXERCISE 4E Dividing algebraic terms

EXAMPLE 4E-1 Writing an algebraic fraction in expanded form then simplifying

For each algebraic fraction: **a** $\frac{8x}{x}$ **b** $\frac{4mn}{10n}$

i write the numerator and denominator in expanded form
ii identify any common factors **iii** simplify.

THINK

a **1** Write the numerator with a multiplication sign.

2 Look for common factors. x is in the numerator and denominator.

3 Cancel (or divide by) x in the numerator and denominator and simplify.

b **1** Write the numerator and denominator with multiplication signs.

2 Look for common factors in the numerator and the denominator. The HCF of 4 and 10 is 2, and n is also in both.

3 Cancel 2 and n in the numerator and denominator and simplify.

WRITE

a **i** $\frac{8x}{x}$
$= \frac{8 \times x}{x}$

ii Common factor is x.

iii $= \frac{8 \times x^{1}}{x_{1}}$
$= \frac{8 \times 1}{1}$
$= 8$

b **i** $\frac{4mn}{10n}$
$= \frac{4 \times m \times n}{10 \times n}$

ii Common factors are 2 and n.

iii $= \frac{{}^{2}\cancel{4} \times m \times \cancel{n}^{1}}{{}_{5}\cancel{10} \times \cancel{n}_{1}}$
$= \frac{2 \times m \times 1}{5 \times 1}$
$= \frac{2m}{5}$

1 For each algebraic fraction:

i write the numerator and denominator in expanded form
ii identify any common factors
iii simplify.

a $\frac{5a}{5}$ **b** $\frac{9d}{d}$ **c** $\frac{4m}{m}$ **d** $\frac{18c}{6}$

e $\frac{10b}{15}$ **f** $\frac{4x}{4y}$ **g** $\frac{6ab}{a}$ **h** $\frac{5m}{mn}$

i $\frac{cd}{3c}$ **j** $\frac{14st}{7t}$ **k** $\frac{8hk}{10k}$ **l** $\frac{15ab}{12a}$

EXAMPLE 4E-2 Dividing algebraic terms

Simplify $7ab \div a$.

THINK

1 Write the division problem as a fraction.

2 Write the numerator in expanded form and look for common factors. a is a common factor, so cancel it in the numerator and denominator.

3 Simplify.

WRITE

$7ab \div a$

$= \frac{7ab}{a}$

$= \frac{7 \times {}^{1}\cancel{a} \times b}{{}_{1}\cancel{a}}$

$= \frac{7 \times 1 \times b}{1}$

$= 7b$

2 Simplify:

a $xy \div y$ **b** $cd \div c$ **c** $8k \div 4$ **d** $5p \div 30$ **e** $14f \div 12$ **f** $20gh \div 5$

g $18ab \div b$ **h** $9xy \div x$ **i** $3mn \div 12$ **j** $abcd \div c$ **k** $7ef \div g$ **l** $4kmn \div m$

EXAMPLE 4E-3 Simplifying an algebraic fraction

Simplify $\frac{15abc}{25bcd}$.

THINK

1 Write the numerator and denominator in expanded form and look for common factors. HCF of 15 and 25 is 5. Common factors are 5, b and c.

2 Cancel (or divide by) 5, b and c in the numerator and denominator.

3 Simplify the numerator and denominator.

WRITE

$\frac{15abc}{25bcd}$

$= \frac{15 \times a \times b \times c}{25 \times b \times c \times d}$

$= \frac{{}^{3}\cancel{15} \times a \times \cancel{b}^{1} \times \cancel{c}^{1}}{{}_{5}\cancel{25} \times \cancel{b}_{1} \times \cancel{c}_{1} \times d}$

$= \frac{3 \times a \times b \times c}{5 \times b \times c \times d}$

$= \frac{3a}{5d}$

3 Simplify:

a $\frac{8a}{2b}$ **b** $\frac{15x}{3y}$ **c** $\frac{24c}{10d}$ **d** $\frac{12w}{4w}$ **e** $\frac{6k}{16k}$ **f** $\frac{3m}{9m}$

g $\frac{4ab}{a}$ **h** $\frac{da}{2a}$ **i** $\frac{18bc}{6c}$ **j** $\frac{6rt}{8r}$ **k** $\frac{20k}{28kn}$ **l** $\frac{9rst}{18s}$

m $\frac{35wx}{21wxy}$ **n** $\frac{7bc}{bc}$ **o** $\frac{25mn}{10mn}$ **p** $\frac{8abc}{2ac}$ **q** $\frac{4mnp}{24mn}$ **r** $\frac{12wxy}{28xy}$

UNDERSTANDING AND FLUENCY

4 Simplify:

a $\frac{a^2b}{ab}$ **b** $\frac{6c^2e}{ce}$ **c** $\frac{4w^2x}{2wx}$ **d** $\frac{3km^2}{9km}$ **e** $\frac{8ef^2}{20ef}$ **f** $\frac{2rs}{r^2s}$

g $\frac{4tx^2}{5xy}$ **h** $\frac{a^2bc^2}{abc}$ **i** $\frac{18b^2d^2}{15bd}$ **j** $\frac{6m^3bc}{9mc}$ **k** $\frac{10k^3mn}{5k^2mn}$ **l** $\frac{7abc^2}{7ab^2c}$

5 Copy and complete these division problems.
Each term is written in expanded form first.

a $a^6 \div a^4$

$= \frac{a^6}{a^4}$

$= \frac{a \times a \times a \times a \times a \times a}{a \times a \times _ \times _}$

$= \frac{{}^{1}\!\not{a} \times {}^{1}\!\not{a} \times {}^{1}\!\not{a} \times {}^{1}\!\not{a} \times a \times a}{{}_{1}\!\not{a} \times {}_{1}\!\not{a} \times {}_{1}\!\not{a} \times {}_{1}\!\not{a}}$

$= \frac{1 \times 1 \times 1 \times 1 \times a \times a}{1 \times _ \times _ \times 1}$

$= \frac{a^2}{1}$

$= _$

b $m^5 \div m^2$

$= \frac{m^5}{m^2}$

$= \frac{m \times m \times m \times m \times m}{_ \times _}$

$= \frac{{}^{1}\!\not{m} \times {}^{1}\!\not{m} \times m \times m \times m}{{}_{1}\!\not{m} \times {}_{1}\!\not{m}}$

$= \frac{1 \times _ \times m \times m \times _}{_ \times _}$

$= \frac{m^-}{1}$

$= _$

6 Look at your answers to question **5**. Can you see a shortcut that could be used? Explain. This shortcut is known as one of the index laws.

EXAMPLE 4E-4 Dividing algebraic terms using an index law

Use an index law to simplify:

a $x^9 \div x^3$ **b** $5k^{11} \div k^7$

THINK

a **1** Check the bases are the same (yes).

2 Write the base and subtract the indices. (9 − 3 = 6)

b **1** Write the first term as the product of a number and a pronumeral in index form.

2 Write the base and subtract the indices. (11 − 7 = 4)

3 Simplify.

WRITE

a $x^9 \div x^3$

$= x^6$

b $5k^{11} \div k^7$

$= 5 \times k^{11} \div k^7$

$= 5 \times k^4$

$= 5k^4$

7 Use an index law to simplify:

a $p^{10} \div p^7$ **b** $a^8 \div a^3$ **c** $n^{14} \div n^{11}$

d $r^9 \div r$ **e** $8x^{17} \div x^6$ **f** $6m^8 \div m^2$

8 In a few sentences, explain how and when the index law for dividing terms is used.

9 **a** Simplify $a^3 \div a^3$ by first writing as a fraction with each term in expanded form.
b Simplify $a^3 \div a^3$ using an index law. Leave your answer in index form.
c Use your answers to parts **a** and **b** to explain why $a^0 = 1$.

10 Simplify each expression.
a $x^5 \div x^5$ **b** $b^8 \div b^8$ **c** $m^{11} \div m^{11}$ **d** $2p^9 \div p^9$ **e** $5y^9 \div y^9$ **f** $8g^2 \div g^2$

EXAMPLE 4E-5 Simplifying an algebraic fraction using index laws

Use index laws to simplify: **a** $\frac{6a^8}{2a^3}$ **b** $\frac{b^4 \times b^6}{b^7}$

THINK

a **1** Write each term in the numerator and denominator as the product of a number and a pronumeral in index form.

2 Cancel 2 in the numerator and denominator. To divide a^8 by a^3, write the base and subtract the indices. $(8 - 3 = 5)$

3 Simplify.

b **1** Simplify the numerator by multiplying b^4 and b^6: write the base and add the indices. $(4 + 6 = 10)$

2 Write the base and subtract the indices. $(10 - 7 = 3)$

WRITE

a $\frac{6a^8}{2a^3}$
$= \frac{6 \times a^8}{2 \times a^3}$
$= 3 \times a^5$
$= 3a^5$

b $\frac{b^4 \times b^6}{b^7}$
$= \frac{b^{10}}{b^7}$
$= b^3$

UNDERSTANDING AND FLUENCY

11 Use index laws to simplify:

a $\frac{7b^6}{7b^4}$ **b** $\frac{2q^7}{2q^3}$ **c** $\frac{10c^7}{2c^3}$ **d** $\frac{15y^{12}}{6y^5}$
e $\frac{ax^{13}}{ax^4}$ **f** $\frac{a^9b}{a^3}$ **g** $\frac{m^5n}{m^5}$ **h** $\frac{b^{20}d}{b^{14}d}$
i $\frac{x^4 \times x^3}{x^2}$ **j** $\frac{m^7 \times m^6}{m^9}$ **k** $\frac{6a^2 \times a^8}{a^4}$ **l** $\frac{n^5 \times n^7}{n^3 \times n^4}$
m $\frac{5d^6 \times d^3}{d^9}$ **n** $\frac{8t^2 \times t^3}{2t^5}$ **o** $\frac{4k \times 3k^9}{6k^{10}}$ **p** $\frac{15e^{13}}{3e^8 \times 5e^5}$

12 Simplify each of these, if possible.

a $\frac{15k}{12kn}$ **b** $\frac{2jk^{11}}{k^4}$ **c** $\frac{3xy^4}{y^4}$ **d** $\frac{m^7}{p^3}$
e $\frac{b^5c^4}{b^3c^2}$ **f** $\frac{4k^8}{2a^5}$ **g** $\frac{18x^4}{22y^4}$ **h** $\frac{6a^3b}{2c^3}$
i $\frac{x^2y}{4x}$ **j** $\frac{8mn^2}{2m}$ **k** $\frac{2m^{13}n^7}{2m^5n^2}$ **l** $\frac{6x^6y^5}{6x^2y^5}$
m $\frac{18w^4x^8}{3x^5y^4}$ **n** $\frac{16pq}{20p^2q}$ **o** $\frac{ab^2cd}{bc^2}$ **p** $\frac{24a^7c^6e^2}{32a^3c^2}$

PROBLEM SOLVING AND REASONING



13 The school canteen has a 10-kg box of apples delivered each week. As the price of apples can vary, the canteen manager must work out how much each apple costs so she knows how to price them for sale. She generally receives about 75 apples in the 10-kg box.

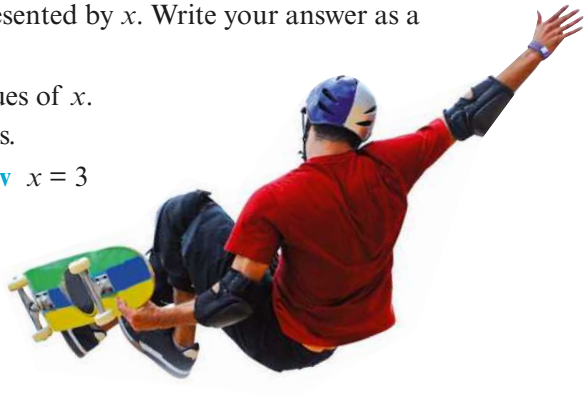
a One week apples cost $4 per kg. Calculate the cost of one apple (to the nearest 5 cents). Clearly show your working.

b The next week apples cost $5 per kg. Calculate the cost of one apple (to the nearest 5 cents). Clearly show your working.

c Write an expression to help the canteen manager calculate the cost of one apple if the cost in dollars per kilogram is represented by x. Write your answer as a fraction in simplest form.

d Calculate the cost per apple for these values of x. Round your answers to the nearest 5 cents.

i $x = 6$ **ii** $x = 8$ **iii** $x = 7$ **iv** $x = 3$



14 Fabian and his father are laying cement pavers to create a smooth surface for skateboarding. The pavers come in different sizes but are rectangular in shape and have a length twice their width.

a Let x represent the width of a paver in centimetres. Write the length in terms of x and hence write an algebraic term for the area of one paver.

b The ground to be paved is square with a length represented by $10x$ cm. Write an algebraic term for the area of the square to be paved.

c Divide the area of the square to be paved by the area of one paver to work out the number of pavers required for this task. Write your answer in simplest form.

d To check whether the answer you found in part **c** is correct, substitute a value for x; for example, let $x = 40$. Evaluate each of these using this x value.

i width of paver **ii** length of paver **iii** area of paver

iv length of square to be paved **v** area of square to be paved

e Use your answers to part **d iii** and **v** to calculate the number of pavers required. How does this compare to your answer to part **c**?

f Will the size of the pavers affect the number of pavers needed? Explain. (Hint: consider different values of x and perform the necessary calculations.)

CHALLENGE

15 Write two algebraic division problems of your own that can be simplified. Swap them with a classmate to complete. Check the answers together and discuss how any errors may have been made and how they can be corrected.

16 Simplify $a^m b^x \div (a^n b^y)$.

Reflect

What is important to remember when dividing algebraic terms?

4F Working with brackets

Start thinking!

A pair of brackets can be used to group particular numbers, pronumerals or operations together. It can change the order of the operations you perform.

1 **a** Calculate:

i $3 \times 2 + 5$ **ii** $3 \times (2 + 5)$.

b Do you obtain the same answer to each problem in part **a**? Explain.

2 The expression $3 \times (2 + 5)$ means 3 lots of $(2 + 5)$.
Copy and complete:

3 lots of $(2 + 5) = (2 + 5) + (2 + 5) + (2 + 5)$
$= 2 + 2 + 2 + 5 + 5 + 5$
$=$ __ lots of 2 + __ lots of 5
so $3 \times (2 + 5) =$ __ $\times 2 +$ __ $\times 5$

You have now written the expression in **expanded form** (that is, without brackets).

3 Use the same method to copy and complete the following working for $3 \times (a + 2)$.

3 lots of $(a + 2) = (a + 2) + (a + 2) + (a + 2)$
$= a + a + a + 2 +$ __ + __
$=$ __ lots of $a +$ __ lots of 2
so $3 \times (a + 2) =$ __ $\times a +$ __ $\times 2$

4 This method is called the **distributive law**. For example, $4 \times (m + n) = 4 \times m + 4 \times n$.
Explain how it works.

5 The example in question **4** can be simplified by leaving out the multiplication signs.
$4 \times (m + n) = 4 \times m + 4 \times n$ is the same as
$4(m + n) = 4m + 4n$
Can $4m + 4n$ be simplified further? Explain.

KEY IDEAS

- A pair of brackets can group numbers, pronumerals and operations together.
- An expression such as $a(b + c)$ can be written in expanded form (that is, without brackets) by using the distributive law.
- Using the distributive law means that each term inside the pair of brackets is multiplied by the term outside the brackets (see example).

Example

$a(b + c) = a \times b + a \times c$
$= ab + ac$

EXERCISE 4F Working with brackets

UNDERSTANDING AND FLUENCY

1 Calculate the value of each expression. What do you notice about each pair?

a i $2(3 + 5)$ **ii** $2 \times 3 + 2 \times 5$ **b i** $6(8 + 1)$ **ii** $6 \times 8 + 6 \times 1$

c i $4(7 - 2)$ **ii** $4 \times 7 + 4 \times (-2)$ **d i** $5(9 - 6)$ **ii** $5 \times 9 + 5 \times (-6)$

e i $-3(5 + 4)$ **ii** $(-3) \times 5 + (-3) \times 4$ **f i** $-2(6 - 5)$ **ii** $(-2) \times 6 + (-2) \times (-5)$

2 Copy and complete each set of working to expand these expressions.

a $4(m + 3)$
$= 4 \times __ + 4 \times __$
$= __ + __$

b $a(c + 5b)$
$= __ \times c + __ \times 5b$
$= __ + __$

c $7(2p - 3)$
$= __ \times 2p + __ \times (-3)$
$= __ - __$

d $-2(3a + 1)$
$= (-2) \times __ + (-2) \times __$
$= __ - __$

EXAMPLE 4F-1 Writing expressions in expanded form

Use the distributive law to write each expression in expanded form and simplify.

a $6(k + 4)$ **b** $2(3x + y)$ **c** $4(a - 5)$

THINK	WRITE
a 1 Multiply each term inside the brackets (k and 4) by the term in front of the brackets (6).	**a** $6(k + 4)$ $= 6 \times k + 6 \times 4$
2 Simplify by performing the multiplications.	$= 6k + 24$
b 1 Multiply each term inside the brackets ($3x$ and y) by the term in front of the brackets (2).	**b** $2(3x + y)$ $= 2 \times 3x + 2 \times y$
2 Simplify by performing the multiplications.	$= 6x + 2y$
c 1 Multiply each term inside the brackets (a and -5) by the term in front of the brackets (4). Note the coefficient of the second term inside the brackets is negative.	**c** $4(a - 5)$ $= 4 \times a + 4 \times (-5)$
2 Simplify by performing the multiplications.	$= 4a - 20$

3 Use the distributive law to write each expression in expanded form and simplify.

a $3(y + 2)$ **b** $5(k + 7)$ **c** $2(5 + a)$ **d** $6(2 + d)$

e $8(a + b)$ **f** $3(p + t)$ **g** $4(2x + 6)$ **h** $2(1 + 8m)$

i $5(4a + b)$ **j** $6(m + 3n)$ **k** $7(5a + 3b)$ **l** $3(2x + 9y)$

m $3(m - 2)$ **n** $2(6 - x)$ **o** $4(1 - p)$ **p** $6(8c - 5d)$

EXAMPLE 4F-2 Expanding and simplifying expressions

Expand and simplify each expression.

a $7p(3q + 4e)$ **b** $2m(5m - 4)$

THINK	WRITE
a 1 Use the distributive law. Multiply each term inside the brackets ($3q$ and $4e$) by the term in front of the brackets ($7p$).	**a** $7p(3q + 4e)$ $= 7p \times 3q + 7p \times 4e$
2 Simplify by performing the multiplications. Write the pronumerals in each term in alphabetical order.	$= 21pq + 28ep$
b 1 Multiply each term inside the brackets ($5m$ and -4) by $2m$.	**b** $2m(5m - 4)$ $= 2m \times 5m + 2m \times (-4)$
2 Simplify the multiplications.	$= 10m^2 - 8m$

4 Expand and simplify each expression.

a $a(b + 4)$	**b** $x(3 + w)$	**c** $d(c - e)$	**d** $p(2m + q)$
e $k(b - 4c)$	**f** $t(3s + 5r)$	**g** $2x(4y + 3w)$	**h** $3a(2b - 4c)$
i $9p(3q + 5k)$	**j** $-8m(2n + 7w)$	**k** $-4t(5a - 2b)$	**l** $-5y(-3x - 4z)$
m $4a(3a + 2)$	**n** $6k(2k - 3)$	**o** $3x(1 - 4x)$	**p** $-2e(4e + 5)$
q $-7n(n - 6)$	**r** $-2b(2a - 3b)$	**s** $5a^2(2a - 3)$	**t** $-11y(x^2 + y^2)$

EXAMPLE 4F-3 Expanding and simplifying more complex expressions

Expand and simplify each expression.

a $4(2p + k - 3m)$ **b** $-3(4xy + 7a - 1)$

THINK	WRITE
a 1 Multiply each term inside the brackets ($2p$, k and $-3m$) by the term in front of the brackets (4). Write negative terms in brackets to see the operations clearly.	**a** $4(2p + k - 3m)$ $= 4 \times 2p + 4 \times k + 4 \times (-3m)$
2 Simplify the multiplications.	$= 8p + 4k - 12m$
b 1 Multiply each term inside the brackets ($4xy$, $7a$ and -1) by -3.	**b** $-3(4xy + 7a - 1)$ $= (-3) \times 4xy + (-3) \times 7a + (-3) \times (-1)$
2 Simplify the multiplications.	$= -12xy - 21a + 3$

5 Expand and simplify each expression.

a $2(3x + y + 7)$	**b** $5(2ab + 4d - 3)$	**c** $7(2c - d + 5ef)$
d $-6(3g - 2h + gh)$	**e** $-4(5k + 2km - 9)$	**f** $-3(2xy + 4 - 7w)$

6 Use the distributive law and then simplify any like terms to copy and complete the following.

a $5(a + 4) + 2(a + 7)$
$= 5 \times _ + 5 \times _ + 2 \times _ + 2 \times _$
$= 5a + _ + 2a + _$
$= _a + _$

b $3b(c + 2) + 2c(a + 5b)$
$= _ \times c + _ \times 2 + 2c \times _ + 2c \times _$
$= 3bc + _ + _ + _$
$= _ + _ + _$

c $d(d + 8) + 6(d - 3)$
$= d \times _ + d \times _ + _ \times d + _ \times (-3)$
$= _ + 8d + _ - _$
$= _ + _ - _$

d $4k(2k + 3) - 2(k - 5)$
$= 4k \times _ + 4k \times _ + (-2) \times _ + (-2) \times (-5)$
$= 8k^2 + _ + _$
$= _ + _ + _$

7 Expand and simplify each expression.

a $2(a + 6) + 5(a + 3)$
b $4(b - 3) + 6(b + 2)$
c $5m(n + 4) + 3n(k + 2m)$
d $7x(3 + y) + 2y(8 - 3x)$
e $k(k - 1) + 5(k + 4)$
f $c(c + 4) - 7(c - 5)$
g $rst(2 + 5t) + 9r(4t + st)$
h $h(h^2 - 2) - 4h(h + 3)$

8 Copy and complete the following to expand and simplify the expressions.
(Hint: $(a + b)$ means $1 \times (a + b)$ and $-(a + b)$ means $-1 \times (a + b)$.)

a $4(m - 2) + (m + 6)$
$= 4(m - 2) + 1(m + 6)$
$= 4 \times _ + 4 \times _ + 1 \times _ + 1 \times _$
$= 4m - _ + m + __$
$= _ - _$

b $2x(y + 3) - (5 + xy)$
$= 2x(y + 3) - 1(5 + xy)$
$= 2x \times _ + 2x \times _ + (-1) \times _ + (-1) \times xy$
$= _ + _ - 5 - _$
$= _ + _ - _$

c $k(k - 1) - (k - 7)$
$= k(k - 1) - 1(k - 7)$
$= k \times _ + k \times _ + _ \times k + _ \times (-7)$
$= _ - _ - _ + _$
$= _ - 2k + _$

d $3w(w - 2) + (4w - 1)$
$= 3w(w - 2) + 1(4w - 1)$
$= 3w \times _ + 3w \times _ + _ \times 4w + 1 \times _$
$= _ - _ + _ - _$
$= _ - _ - _$

9 Expand and simplify each expression.

a $6(p + 3) + (p + 2)$
b $3(d - 2) - (d + 8)$
c $x(x + 2) + (x + 5)$
d $k(k + 4) - (3k + 1)$
e $2e(e + 4) - (e - 3)$
f $4a(b + 1) - (6 + ab)$
g $5w(y - 3) - (wy + 2)$
h $mp(n - m) + (n - 3m^2p)$

10 In three years, Monette will be twice as old as Chanelle.

a Write an expression for Chanelle's age in three years.
(Hint: use a pronumeral to represent Chanelle's current age.)

b Use your answer to part **a** to write an expression for Monette's age.

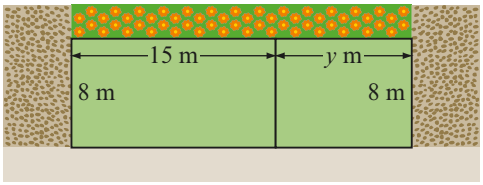
c Use the distributive law to write the expression for Monette's age without brackets.

d Work out Monette's age if Chanelle's current age is: **i** 14 **ii** 18 **iii** 25.

e Which expression did you use for your calculations? Explain your choice.

PROBLEM SOLVING AND REASONING





11 One of Andre's jobs is to mow the front lawn. It is rectangular and measures 15 m by 8 m.

a Calculate the area of lawn to be mown.

A neighbour notices Andre is about to start mowing and asks if he could mow her front lawn as well.

b Since the width of the neighbour's front yard is unknown, represent it with the pronumeral y. Write an expression for the area of lawn in the neighbour's front yard.

c Add your answers to parts **a** and **b** to write an expression for the total area of lawn to be mown.

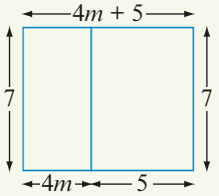
d Now consider both lawns as one large rectangle. Draw a diagram of this rectangle with the length and width measurements shown.

e Explain why the expression for the area of the large rectangle can be written as $8(15 + y)$.

f Explain why the two answers for the total area of lawn to be mown (parts **c** and **e**) are equivalent.

EXAMPLE 4F-4

Demonstrating the distributive law using areas of rectangles



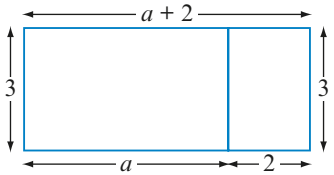
Demonstrate that $7(4m + 5) = 28m + 35$ is true by finding the appropriate areas in the diagram.

THINK	WRITE
1 Write an expression for the area of the large rectangle. Identify 7 as length and $4m + 5$ as width.	area of large rectangle = length × width = $7 \times (4m + 5)$ = $7(4m + 5)$
2 Write an expression for the area of the smaller rectangle on the left (within the large rectangle). Identify 7 as length and $4m$ as width.	area of smaller rectangle on left = length × width = $7 \times 4m$ = $28m$
3 Write an expression for the area of the smaller rectangle on the right (within the large rectangle). Identify 7 as length and 5 as width.	area of smaller rectangle on right = length × width = 7×5 = 35
4 Use the fact that the area of the large rectangle is the same as the sum of the areas of the two smaller rectangles.	area of large rectangle = area of smaller rectangle on left + area of smaller rectangle on right so $7(4m + 5) = 28m + 35$

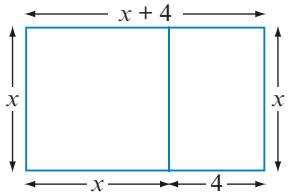
PROBLEM SOLVING AND REASONING

12 Demonstrate that each statement is true by finding the appropriate areas in the diagram.

a $3(a + 2) = 3a + 6$



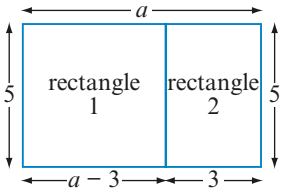
b $x(x + 4) = x^2 + 4x$



13 Draw a diagram to demonstrate that each statement is true.

a $6(k + 8) = 6k + 48$ **b** $4(2d + 9) = 8d + 36$ **c** $2p(p + 3) = 2p^2 + 6p$

14 This diagram can be used to demonstrate that $5(a - 3) = 5a - 15$.



a What is the length and width of rectangle 1?

b Use your answer to part **a** to write an expression for the area of rectangle 1 using brackets.

c Another way to work out the area of rectangle 1 is to subtract the area of rectangle 2 from the total area of the large rectangle.

i Write the length and width of the large rectangle.

ii Use these length and width measurements to write an expression for the area of the large rectangle.

iii Write the length and width of rectangle 2.

iv Use these length and width measurements to write an expression for the area of rectangle 2.

v Use your answers to parts **ii** and **iv** to write an expression for the area of rectangle 1.

d Explain how you were able to show that $5(a - 3) = 5a - 15$.

15 Draw a diagram and find appropriate areas of rectangles to demonstrate why each statement is true.

a $4(x - 2) = 4x - 8$ **b** $c(d - 5) = cd - 5c$ **c** $h(h - 4) = h^2 - 4h$

CHALLENGE

16 Expand and simplify each expression.

a $3x^2(4x^3 - 7) - 5x^4(x + 2)$

b $a^3(ab + c - 1) + a^2(3ac - a^2b + a)$

c $2m(3m^2np^2 + 5mn) - 6n(m^2 + m^2p^2 - 4)$

d $5w(4w^2x - 3y) + 2y(9w - 6xy) - 4x(5w^3 - 3y^2)$

17 Evaluate each expression in question **16** using these values:

$a = -3, b = 7, c = 4$

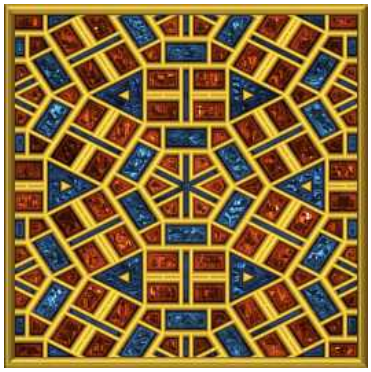
$m = 2, n = -4, p = -1$

$w = -5, x = 2, y = -6$.

Reflect

What is the distributive law and how is it used?

4G Factorising expressions



Start thinking!

Each piece of glass in a leadlight panel must be edged with a strip of lead.

1 **a** Calculate the length of lead needed for this piece of red glass.

b Which one of these three ways did you use to obtain your answer?



$5 + 3 + 5 + 3$	length + width + length + width
or $2 \times (5 + 3)$	2 lots of (length + width)
or $2 \times 5 + 2 \times 3$	2 lots of length + 2 lots of width

c The expression $2 \times (5 + 3)$ is said to be in **factor form**, as it shows the product of factors. In this case, there are two factors. One factor is $(5 + 3)$. What is the other factor?

d The distributive law can also be used in the reverse direction. Explain how you could **factorise** (write in factor form) the expression $2 \times 5 + 2 \times 3$ to produce $2 \times (5 + 3)$. (Hint: which factor is common to 2×5 and 2×3?)

$$\underbrace{2 \times 5 + 2 \times 3}_{\text{expanded form}} = \underbrace{2 \times (5 + 3)}_{\text{factor form}}$$

e Factorise $4 \times 7 + 4 \times 11$. Explain the method you used.

2 **a** Copy and complete this expression for the perimeter of the blue glass in expanded form: $2 \times __ + 2 \times __$.



b For the expression found in part **a**, circle any factors that are common to both terms.

c Copy and complete this expression for the perimeter of the glass in factor form: $__ \times (__ + __)$.

d **i** For the expression you wrote in factor form, list the two factors.

ii Which of these two factors is the same as the common factor you circled in part **b**?

3 Explain the difference between expanding an expression and factorising an expression.

KEY IDEAS

$$4(3a + 2) = 12a + 8 \quad \text{(expanding →, factorising ←)}$$

- Factorising an expression is the opposite to expanding an expression.
- The distributive law can be used to write expressions in factor form. One of the factors is the HCF of each term in the expression. Write the HCF in front of the brackets and put the other factor of each term inside the brackets.

$$\underbrace{a \times b + a \times c}_{\text{expanded form}} = \underbrace{a \times (b + c)}_{\text{factor form}} \text{ or } ab + ac = a(b + c)$$

- To find the HCF of a set of algebraic terms, first identify the common factors (including numbers and pronumerals) of each term and then multiply them.

EXERCISE 4G Factorising expressions

1 Find the missing factor in each of these.

a $2 \times __ = 6$
b $8 \times __ = -16$
c $__ \times k = 6k$
d $3 \times __ = 3x$
e $x \times __ = -7x$
f $a \times __ = ab$
g $__ \times c = cd$
h $4a \times __ = 8a$
i $__ \times 3m = 15m$
j $6e \times __ = -12ef$
k $__ \times 4k = 20kp$
l $5xy \times __ = -5xy$

EXAMPLE 4G-1 Finding the HCF of a pair of terms

Find the highest common factor (HCF) of each pair of terms.

a $3d$ and 15 **b** $10fg$ and $-5gh$

THINK

a **1** Write $3d$ as a product of factors.

2 Write 15 as a product of factors.

3 Look for factors that are common to each term. The only common factor is 3 so this is the HCF.

b **1** Write $10fg$ as a product of factors.

2 Write $-5gh$ as a product of factors. Show -5 as -1×5 since the other term has a factor of 5.

3 Look for common factors.

4 Multiply the common factors to obtain the HCF.

WRITE

a $3d = 3 \times d$

$15 = 3 \times 5$

HCF = 3.

b $10fg = 2 \times 5 \times f \times g$

$-5gh = -5 \times g \times h$
$= -1 \times 5 \times g \times h$

Common factors are 5 and g.

HCF $= 5 \times g$
$= 5g$

UNDERSTANDING AND FLUENCY

2 Find the highest common factor (HCF) of each pair of terms.

a $4x$ and 12
b $10d$ and -5
c 6 and $8k$
d $3a$ and -3
e $9c$ and $9e$
f $7x$ and $14y$
g $6xy$ and $2y$
h $3c$ and $9bc$
i $30gh$ and $-24g$
j $4ab$ and $8bc$
k $2mn$ and $3mp$
l $15rs$ and $-20st$

3 Write each term in question **2** as a product of two factors where one of the factors is the HCF of the pair of terms. For example, for part **a**, $4x = 4 \times x$ and $12 = 4 \times 3$.

4 Copy and complete the following to write each expression in factor form.

a $4x + 12$
$= 4 \times __ + 4 \times __$
$= 4 \times (x + __)$
$= 4(x + __)$

b $10a - 35$
$= 5 \times __ + 5 \times (-7)$
$= 5 \times (__ - 7)$
$= 5(__ - __)$

c $24km + 16k$
$= 8k \times __ + 8k \times __$
$= 8k \times (__ + __)$
$= 8k(__ + __)$

d $18cd - 12de$
$= __ \times 3c + __ \times (-2e)$
$= __ \times (__ - __)$
$= __(__ - __)$

NOTE Remember that you are still using the distributive law but in the opposite direction.

EXAMPLE 4G-2 Factorising expressions

Factorise each expression.

a $8x + 28$ **b** $15mn - 20m$

THINK

a 1 Identify the HCF of $8x$ and 28. (HCF = 4) Write each term as a product of the HCF and its other factor.

2 Write the HCF at the front of a pair of brackets and the other factor for each term inside the brackets.

b 1 Identify the HCF of $15mn$ and $-20m$. (HCF = $5m$) Write each term as a product of the HCF and its other factor.

2 Write the HCF at the front of a pair of brackets and the other factor for each term inside the brackets.

WRITE

a $8x + 28$
$= 4 \times 2x + 4 \times 7$
$= 4 \times (2x + 7)$
$= 4(2x + 7)$

b $15mn - 20m$
$= 5m \times 3n + 5m \times (-4)$
$= 5m \times (3n - 4)$
$= 5m(3n - 4)$

UNDERSTANDING AND FLUENCY

5 Factorise each expression.

a $4a + 8$ **b** $7e + 21$ **c** $45 + 5k$
d $3 + 12y$ **e** $2d - 6$ **f** $27 - 9p$
g $8m + 20$ **h** $6h - 30$ **i** $24b + 18b$
j $12t - 14$ **k** $5ab + 10a$ **l** $9n + 3mn$
m $22xy - 4x$ **n** $3ab + 3bc$ **o** $16mn - 8mp$
p $27xy + 18wx$ **q** $12k + 15m$ **r** $21ef - 24gh$

6 Find the HCF of each pair of terms.

a abc and bcd **b** wxy and x **c** $9mn$ and $9kmn$
d $3bcd$ and $6def$ **e** $8pq$ and $-2pqr$ **f** $12agk$ and $18ak$
g $16gh$ and $-28mn$ **h** $10abcd$ and $14bcde$ **i** $6efgh$ and $-7abfk$

7 Write each term in question **6** as a product of two factors where one of the factors is the HCF of the pair of terms. For example, for part **a**: $abc = bc \times a$ and $bcd = bc \times d$.

8 Copy and complete the following to write each expression in factor form.

a $abc + bcd$
$= bc \times __ + bc \times __$
$= bc \times (a + __)$
$= bc(a + __)$

b $8pq - 2pqr$
$= 2pq \times __ + 2pq \times __$
$= 2pq \times (__ - __)$
$= 2pq(__ - __)$

c $12agk + 18ak$
$= __ \times 2g + __ \times 3$
$= __ \times (__ + 3)$
$= __(__ + 3)$

d $6efgh - 7abfk$
$= __ \times 6egh + f \times __$
$= __ \times (6egh - __)$
$= __(6egh - __)$

9 Factorise each expression.

a $mnp + kmn$ **b** $3ab - 9abc$ **c** $14rst + 21rt$
d $4cde + 5def$ **e** $2wxyz - 2rstx$ **f** $24apy + 40aqy$

10 Explain how to check your answers to question **9** are correct. (Hint: factorising is the opposite process to expanding.) Use this method to check your answers.

11 Find the missing factor in each of these.

a $a \times __ = a^2$ **b** $3m \times __ = 3m^2$ **c** $x \times __ = 7x^2$
d $5b \times __ = 10b^2$ **e** $__ \times 4k = 12k^2$ **f** $__ \times (-6y) = -42y^2$
g $4p \times __ = -16p^2$ **h** $__ \times 8c^2 = 24c^2$ **i** $__ \times -3y = -15y^2$

12 Find the HCF of each pair of terms.

a x^2 and $8x$ **b** $2a^2$ and $3a$ **c** $7k^2$ and $7k$
d $2y$ and $4y^2$ **e** $6m$ and $8m^2$ **f** $12p^2$ and $-15p$
g $28t$ and $35t^2$ **h** $5a^2$ and a **i** $3n^2$ and $-6n$

13 Copy and complete the following to write each expression in factor form.

a $x^2 + 8x$
$= x \times __ + x \times __$
$= x \times (x + __)$
$= x(x + __)$

b $5m^2 - 5m$
$= 5m \times __ + __ \times (-1)$
$= 5m \times (__ - 1)$
$= 5m(__ - __)$

c $3b + 6b^2$
$= __ \times __ + __ \times 2b$
$= __ \times (__ + 2b)$
$= __(__ + __)$

d $18k^2 - 21k$
$= 3k \times __ + 3k \times __$
$= 3k \times (__ - __)$
$= 3k(__ - __)$

14 Factorise each expression.

a $c^2 + 4c$ **b** $6k + k^2$ **c** $a^2 - 2a$
d $3m^2 + 3m$ **e** $10p^2 + 11p$ **f** $4h - 7h^2$
g $6x^2 + 3x$ **h** $5b^2 - 10b$ **i** $8n^2 + 36n$
j $27y^2 - 15y$ **k** $14a + 16a^2$ **l** $e^2 + e$

15 Use the process of expanding to check whether your answers to question **14** are correct.

16 In some cases, it can be best to use an HCF that is a term with a negative coefficient. Find the HCF that has a negative coefficient for each pair of terms.

a $-5a$ and -10 **b** $-2pq$ and $-6p$
c $-9k$ and $-15kx$ **d** $-14b^2$ and $-7b$
e $-4y^2$ and $4y$ **f** $-12m$ and $20mnp$
g $-x^2y - xy$ **h** $-8bc + 28abc$

17 Copy and complete the following to write each expression in factor form. In each case, the HCF is a term with a negative coefficient.

a $-5a - 10$
$= (-5) \times a + (-5) \times 2$
$= -5 \times (a + __)$
$= -5(a + __)$

b $-2pq - 6p$
$= __ \times q + __ \times 3$
$= __ \times (q + 3)$
$= __(q + 3)$

c $-4y^2 + 4y$
$= __ \times y + __ \times (-1)$
$= __ \times (y - 1)$
$= __(y - 1)$

d $-12m + 20mnp$
$= (-4m) \times __ + (-4m) \times __$
$= -4m \times (__ - __)$
$= -4m(__ - __)$

18 Factorise each expression by finding the HCF that has a negative coefficient.

a $-4a - 12$ **b** $-6 - 8k$ **c** $-7m + 14$
d $-15b + 35$ **e** $-3st - 9t$ **f** $-18c - 24cd$
g $-12mx + 21my$ **h** $-h^2 - 2h$ **i** $-16x^2 + 8x$
j $-45b - 10abc$ **k** $-x - x^2$ **l** $-11a^2b + 9ab$

19 Now that you have practised factorising, try writing the expression in factor form directly from the expanded form. Copy and complete the following to factorise each expression in one step.

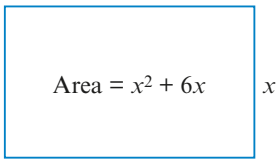
a $2x + 6 = 2(__ + __)$ **b** $8a - 36 = __(2a - 9)$
c $h^2 + 7h = h(__ + __)$ **d** $25mn + 30n = 5n(__ + __)$
e $kmn - mn = __(k - __)$ **f** $12ef - 18df = 6f(__ - __)$
g $-12 - 9y = -3(__ + __)$ **h** $-26b - 24c = -2(__ + __)$
i $-abc + acd = -ac(__ - __)$ **j** $6xy + 4x^2y = 2xy(__ + __)$
k $mn^2 + 5mn = __(__ + 5)$ **l** $24cd - 16abc = __(3d - __)$

20 Factorise each of these. If possible, use just one step to write the expression in factor form. Use the process of expanding to check your answers.

a $6g + 6$ **b** $10 + 18x$ **c** $kp + kq$
d $m^2 - m$ **e** $14 - 7ab$ **f** $3eh + 3gh$
g $8abcd + 4abde$ **h** $-5a^2 - 15a$ **i** $-24k + 12p$
j $3x^2 - xy$ **k** $16cde + 20e^2$ **l** $-ad - 6d$

PROBLEM SOLVING AND REASONING

21 Consider this rectangle, which has all measurements in centimetres. The expression for the width is x and the expression for the area is $x^2 + 6x$.



- **a** Factorise the expression for the area of the rectangle.
- **b** List the two factors that multiply to give the expression for the area.
- **c** What is the expression for the length of this rectangle?
- **d** Demonstrate you have the correct expression for the length of this rectangle by multiplying the length by the width to obtain the expression for its area.
- **e** Another way of showing that you have the correct expression for the length of this rectangle is to use substitution. Try this method with $x = 3$.

22 Write an expression for the missing side length for each rectangle.

a



area of photo is $(8x + 16)$ cm^2

b



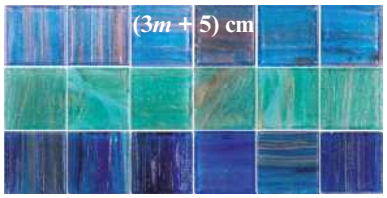
area of oriental rug is $(10x + 14)$ m^2

c



area of television screen is $(k^2 + 20k)$ cm^2

d



area of glass panel is $(6m^2 + 10m)$ cm^2

23 Demonstrate you have obtained the correct expressions in question **22**.

CHALLENGE

24 Write a pair of terms whose HCF is:

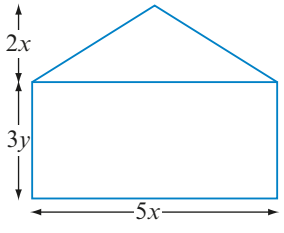
a $2x$ **b** $-3ab$.

25 A rectangle has an area of $8x - x^2$.

- **a** Write this in factor form.
- **b** List the expressions for the length and width of the rectangle.
- **c** Suggest values for the length and width of three different rectangles that would fit these expressions.

26 Write an expression for the area of this shape in:

- **a** expanded form
- **b** factor form.



Reflect

What is the difference between expanding and factorising algebraic expressions?

CHAPTER REVIEW

SUMMARISE

Create a summary of this chapter using the key terms below. You may like to write a paragraph, create a concept map or use technology to present your work.

pronumeral	expression	like terms	factor form
variable	equation	simplify	expand
algebraic term	formula	index laws	factorise
coefficient	substitution	distributive law	
constant term	evaluate	expanded form	

MULTIPLE-CHOICE

4A **1** How many terms does $2x^2 - 3xy + 4y^2 - 1$ have?

A 2 **B** 3 **C** 4 **D** 5

4A **2** What is the coefficient of $-6a^2b$?

A a **B** 6 **C** -6 **D** a^2b

4B **3** What is the result of substituting $x = 4$ and $y = -3$ into $2xy - 9y$?

A 3 **B** -3 **C** 51 **D** -51

4C **4** Which list shows a set of like terms?

A $3a$, ab, $-5a$ **B** $4mn$, mn, $2nm$

C $-8p$, $6p^2$, $5p^3$ **D** $2cd$, de, $-7ce$

4C **5** Simplify $7m + 2n - 5m - 2n$.

A $2m$ **B** $12m + 4n$

C $2mn$ **D** $2m + 4n$

4C **6** What does $3x^2 - 2x^2$ simplify to?

A $5x^2$ **B** 1 **C** $-x^2$ **D** x^2

4D **7** What is the expanded form of $-5a^2bc$?

A $5 \times a \times b \times c$

B $-5 \times a \times a \times b \times c$

C $5 \times a \times a \times b \times c$

D $-5 \times a \times b \times c$

4D **8** Which of these is *not* equivalent to $24a^6b^3c^5$?

A $2a^2b^2 \times 3a^2c^3 \times 4a^2bc^2$

B $3a^4b \times 8b^2c \times a^2c^4$

C $3a^4c^5 \times 4a^2b \times 2ab^2$

D $bc^4 \times 6a^3 \times 4a^3b^2c$

4E **9** What does $\frac{4ab^2}{8ab}$ simplify to?

A $2b$ **B** $\frac{1}{2b}$ **C** $\frac{2}{b}$ **D** $\frac{b}{2}$

4F **10** What is $5m(3m + 2n - 6)$ in expanded form?

A $15m^2 + 10mn - 30m$

B $8m^2 + 7mn - 11m$

C $15m^2 + 10mn + 30m$

D $8m^2 + 10mn - 6$

4F **11** Which expression simplifies to $12mn + 5m$?

A $m(3n - 4) + 2m(5n + 4)$

B $4n(2m - 3) + 2m(2n + 4)$

C $m(2n - 3) + 2m(5n + 4)$

D $3m(3n - 3) + m(3n + 4)$

4G **12** What is the HCF of $24ab^2c$ and $18bc$?

A $2bc$ **B** $6c$ **C** $6bc$ **D** $8ab$

SHORT ANSWER

4A **1** Write an expression for each statement. Use m for the unknown number.

- **a** a number is multiplied by 4 and then 3 is added
- **b** a number is divided by 5 and then 8 is subtracted
- **c** add 3 to a number and then divide the result by 7
- **d** subtract 6 from a number and multiply the result by 2

4A **2** Zoo entry is $25 per adult, $13 per child and $20 per pensioner. Write an expression for each of these (in dollars).

- **a** the cost of entry for x adults
- **b** the cost of entry for y children
- **c** the cost of entry for p pensioners
- **d** the total cost of entry for x adults, y children and p pensioners

4B **3** Evaluate each expression by substituting $a = 2$, $b = 5$ and $c = -3$.

- **a** $7a + 2b$
- **b** $5a^2 + 4c$
- **c** $6(3ab - 2)$
- **d** $\dfrac{b^2 - a^2}{c}$

4B **4** The number of days in a given number of weeks can be written as a formula.

- **a** Choose a pronumeral to represent the number of days and another for the number of weeks.
- **b** Write a formula using the pronumerals for the relationship between the two variables.
- **c** Use your formula to find the number of days in:
  - **i** 13 weeks
  - **ii** 29 weeks
  - **iii** 145 weeks
  - **iv** 7.5 weeks.

4C **5** Simplify:

- **a** $6a + 7a$
- **b** $4b - b$
- **c** $3c - 8c$
- **d** $5de + de + 9de$
- **e** $7g + 2g + 4h$
- **f** $3x^2 + 8x + 2x^2$

4C **6** Simplify:

- **a** $4m + 5n + 8m + 3n$
- **b** $2k^2 + 6 + 9k^2 + 1$
- **c** $3cd + 2c + 5dc + 4$
- **d** $7x + 5y - 2x + y + 3x - 2y$
- **e** $5ab^2 - 2a^2b + 4a^2b - 3ab^2$
- **f** $6pq + 7p + 8q - p^2 - 5q$

4D **7** Simplify:

- **a** $5d \times 4a$
- **b** $3ab \times 8bc$
- **c** $-6gk \times 2gh$
- **d** $7ef \times bf \times 9cf$

4D **8** Simplify:

- **a** $x^6 \times x^8$
- **b** $4a^5 \times 7a^4$
- **c** $h^7 \times 5h \times 2h^3$
- **d** $2m^5n^8 \times 3m^7n^4$

4E **9** Simplify:

- **a** $\dfrac{15a}{20}$
- **b** $\dfrac{6cd}{2d}$
- **c** $\dfrac{4xy}{x}$
- **d** $\dfrac{6abcd}{10bc}$
- **e** $\dfrac{18m^2n}{12mn}$
- **f** $\dfrac{2e^2hk^2}{5ek}$

4E **10** Simplify:

- **a** $k^{11} \div k^7$
- **b** $8c^9 \div c$
- **c** $\dfrac{6a^8b}{a^5}$
- **d** $\dfrac{16x^{13}y^6}{28x^6y}$
- **e** $\dfrac{3w^6 \times 2w^3}{4w^7}$
- **f** $\dfrac{2t^4 \times 4t^5}{8t^9}$

4F **11** Expand:

- **a** $5(a + 3)$
- **b** $4(7b - 2)$
- **c** $c(3d + 10a)$
- **d** $-6m(4m + 1)$

4F **12** Expand and simplify:

- **a** $3(k + 7) + 5(3k - 2)$
- **b** $2y(y - 6) - (y + 4)$

4G **13** Factorise:

- **a** $14h - 18$
- **b** $20ab + 10ac$
- **c** $6a - 4b$
- **d** $8c^2 + 12c$
- **e** $-5ef - 5f$
- **f** $-kmn + 7np$
- **g** $4a^2b - 14ab$
- **h** $xy^2 + wx^2y$

NAPLAN-STYLE PRACTICE

1 What is the coefficient of $-8mnp$?

2 Caitlin buys x pens and y pencils. A pen costs \$2 and a pencil costs \$1. Which expression represents the total cost in dollars?

- ☐ $x + y$
- ☐ $2xy$
- ☐ $x + 2y$
- ☐ $2x + y$

3 Which of these is equivalent to $5ab$?

- ☐ $5 \times a \times b$
- ☐ $5 + a + b$
- ☐ $5 \times a + b$
- ☐ $5 + a \times b$

4 If $a = 4$ and $b = -3$, what is $7a + 2b$?

-13	22	34	97
☐	☐	☐	☐

5 Which of these is equivalent to $-3m^2n$?

- ☐ $3 \times m \times n$
- ☐ $-3 \times m \times n$
- ☐ $3 \times m \times m \times n$
- ☐ $-3 \times m \times m \times n$

6 Simplify $5x^2 - 3x + x^2 + 2x - 4$.

7 What is the product of $4ab$ and $3bc$?

$7abc$	$12ab^2c$	$7ab^2c$	$12abc$
☐	☐	☐	☐

8 Which of these is equivalent to $h^4 \times h \times h^7$?

h^3	h^{11}	h^{12}	h^{28}
☐	☐	☐	☐

9 Which of these shows $\frac{24ef}{20e}$ in simplest form?

$\frac{12f}{10}$	$\frac{6f}{5}$	$\frac{12ef}{10e}$	$\frac{6ef}{5e}$
☐	☐	☐	☐

10 Write $b^8 \div b^2$ in simplest form.

11 Simplify $3mn \times 2m^2n \times n$.

12 Simplify $\frac{2x^3 \times 3x^2y^4}{8x^4y^2}$.

13 A rectangle is three times as long as it is wide. If the width is represented by w, write an expression for the area of the rectangle in terms of w.

w

14 Which expression is equivalent to $a(b + c)$?

- ☐ $a + (b + c)$
- ☐ $a \times b + c$
- ☐ $a \times b + a \times c$
- ☐ $a \times b \times c$

15 Which expression is equivalent to $5x(x - 2y)$?

- ☐ $5x^2 - 2y$
- ☐ $5x - 10xy$
- ☐ $5x^2 - 5xy$
- ☐ $5x^2 - 10xy$

16 If $m = 5$, $n = -2$ and $p = 3$, what is the value of $\frac{2m^2 - 5n}{4mp}$?

17 Which expression becomes $3ab + 6a$ when expanded?

- ☐ $3(ab + 2)$
- ☐ $3a(b + 2)$
- ☐ $3b(a + 2)$
- ☐ $3a(b + 6)$

18 Which expression is equivalent to $d(6 - d) + 2d(3d - 1)$?

$8d$	$4d + 5d^2$	$8d + 7d^2$	$4d - 7d^2$
☐	☐	☐	☐

19 Which expression becomes $4x^2y - 6xy^3$ when expanded?

- ☐ $2xy(2x - 3y^2)$
- ☐ $2xy(2x - 6y^2)$
- ☐ $-xy(4x - 6y^2)$
- ☐ $xy(4x + 6y)$

20 Which expression does *not* simplify to $2x - 3y$?

- ☐ $\frac{8x^2 - 12xy}{4x}$
- ☐ $5x + 3x(y - 1) - 3y(x + 1)$
- ☐ $5(x - y) + 3(2y - x)$
- ☐ $2(x - 3y) + 3y$

21 Which expression is equivalent to $4p^2 - 10pq$?

- ☐ $2p(2p - 5q)$
- ☐ $4p(p - 10q)$
- ☐ $2(2p - 5pq)$
- ☐ $2p^2(2 - 5q)$

22 Which expression is equivalent to $4(m + 3n) - (5m - 2n)$?

- ☐ $5n - m$
- ☐ $10n - m$
- ☐ $12n - m$
- ☐ $14n - m$

23 The area of this rectangle is represented by $c^2 + 5c$. What expression can be used to represent the unknown side length?



☐

Questions **24** and **25** refer to a rug with an area given by $(6x^2 + 12x)$ m^2.

24 Which are possible measurements for the rug?

- ☐ length $(3x + 4)$ m, width $(2x)$ m
- ☐ length $(4x + 6)$ m, width $(2x)$ m
- ☐ length $(6x + 10)$ m, width (x) m
- ☐ length $(2x + 4)$ m, width $(3x)$ m

25 If $x = 2$, what is the length, width and area of the rug?

- ☐ 8 m, 6 m, 28 m^2
- ☐ 12 m, 4 m, 48 m^2
- ☐ 8 m, 6 m, 48 m^2
- ☐ 6 m, 4 m, 24 m^2

ANALYSIS

Jared buys a group of friends some boxes of hot chips to share. He wants a way to quickly calculate the cost for each person. The cost depends on whether he buys large or small boxes.

a Jared buys three large boxes of hot chips.
- **i** What is the total cost?
- **ii** If the cost is shared between five, how much will each person pay?

b Consider when Jared buys x large boxes of chips.
- **i** What is the total cost?
- **ii** If the cost is shared between five, how much will each person pay?

c Write an expression for how much each person pays if x small boxes of chips are bought to be shared between:
- **i** four people
- **ii** five people.

d Jared considers the cost of buying the same number of large and small boxes of chips.
- **i** Write an expression, in simplest form, for the total cost of buying x large boxes and x small boxes of chips.
- **ii** What values could x represent if Jared has \$50 to spend?

e Jared now considers the cost of buying x large boxes of chips and y small boxes of chips. Write an expression for the total cost in:



- **i** expanded form
- **ii** factorised form.

f For the general case of buying x large boxes and y small boxes of chips, write an expression for the cost per person if the chips are shared between:
- **i** six people
- **ii** p people.

g Use your expression from part **f ii** to calculate the cost per person if $x = 3$, $y = 2$ and $p = 6$. Write a sentence to explain this situation.

h For a group of six people, suggest at least three sets of values for x and y so that each person does not spend more than \$5.

i Suggest at least three sets of values for x, y and p so that each person does not spend more than \$8.

CONNECT

The magnificent mind reader!

Have you ever wished to be a mind reader? Here's a mind-reading test to try on your friends and family.

You will be the mind reader and one of your friends or family will be the test subject.

Ask the subject to think of a number and to really concentrate on that number for 5 seconds.

Make a show of closing your eyes to concentrate on 'receiving' the number.

Now get your subject to follow these three steps:

1 Add 5 to the number.
2 Double the result.
3 Subtract 10.

Ask them to say out loud the final result obtained after following these steps.

You can now reveal the original number they chose.
(To do this, just halve the result.)

If your mind-reading skills are doubted, get them to try it again with a different number.

Does this mind-reading test work every time? Are you actually a mind reader or is it just a trick?

You may first like to practise the trick with a classmate. Repeat it a few times with a different starting number. Does the trick work every time?

One way to investigate how this trick works is to use your skills in algebra. Since the starting number can vary, use a pronumeral to represent it. Then follow the steps using this pronumeral. Remember to perform each operation on the result obtained from the previous step. Now can you explain how the trick works?

Trick 1

Think of a number.
1 Add 5 to the number.
2 Double the result.
3 Subtract 10.
What is the result? The result will be double the original number.

Trick 2

Think of a number.
1 Double the number.
2 Add the original number.
3 Divide by 3.
What is the result?

Trick 3

Think of a number.
1 Add 10 to the number.
2 Multiply by 10.
3 Subtract 100.
4 Divide by 10.
What is the result?



Your task

To investigate these mind-reading tricks, follow these steps:

- investigate each trick using algebra
- explain how you think each trick works
- create some of your own tricks
- demonstrate that each trick will work every time.

Include all necessary working to justify your answers.

Trick 4

Think of a number.

1 Add 3 to the number.
2 Multiply by 2.
3 Subtract twice the original number.

The result will be 6.

Trick 5

Think of a number.

1 Multiply the number by 10.
2 Add 10.
3 Divide by 10.
4 Subtract 1.

What is the result?

Trick 6

Think of a number.

1 Multiply the number by 3.
2 Add 6.
3 Double it.
4 Divide by 6.
5 Subtract the original number.

The result will be 2.

3

Complete the 4 CONNECT worksheet to show all your working and answers to this task.

You may like to present your findings as a report. Your report could be in the form of:

- a story about a magnificent mind reader
- a demonstration of your mind-reading skills
- a short presentation to the class of one of your new tricks and how it works
- other (check with your teacher).

Trick 7

This mind-reading trick allows you to discover when a person's birthday is.

Ask the person to think of numbers to represent the month and day of their birthday. For example, if their birthday is June 17, then the month is 6 and the day is 17.

1 Multiply the month number by 10.
2 Add 20.
3 Multiply by 10.
4 Subtract 100.
5 Add the day number.
6 Subtract 100.

Ask the person to say out loud the result obtained.

The last two digits give the day and the other digits give the month of the birthday.

5 LINEAR RELATIONSHIPS

5A Understanding equations

5B Solving equations using tables

5C Building expressions using flowcharts

5D Solving equations using backtracking

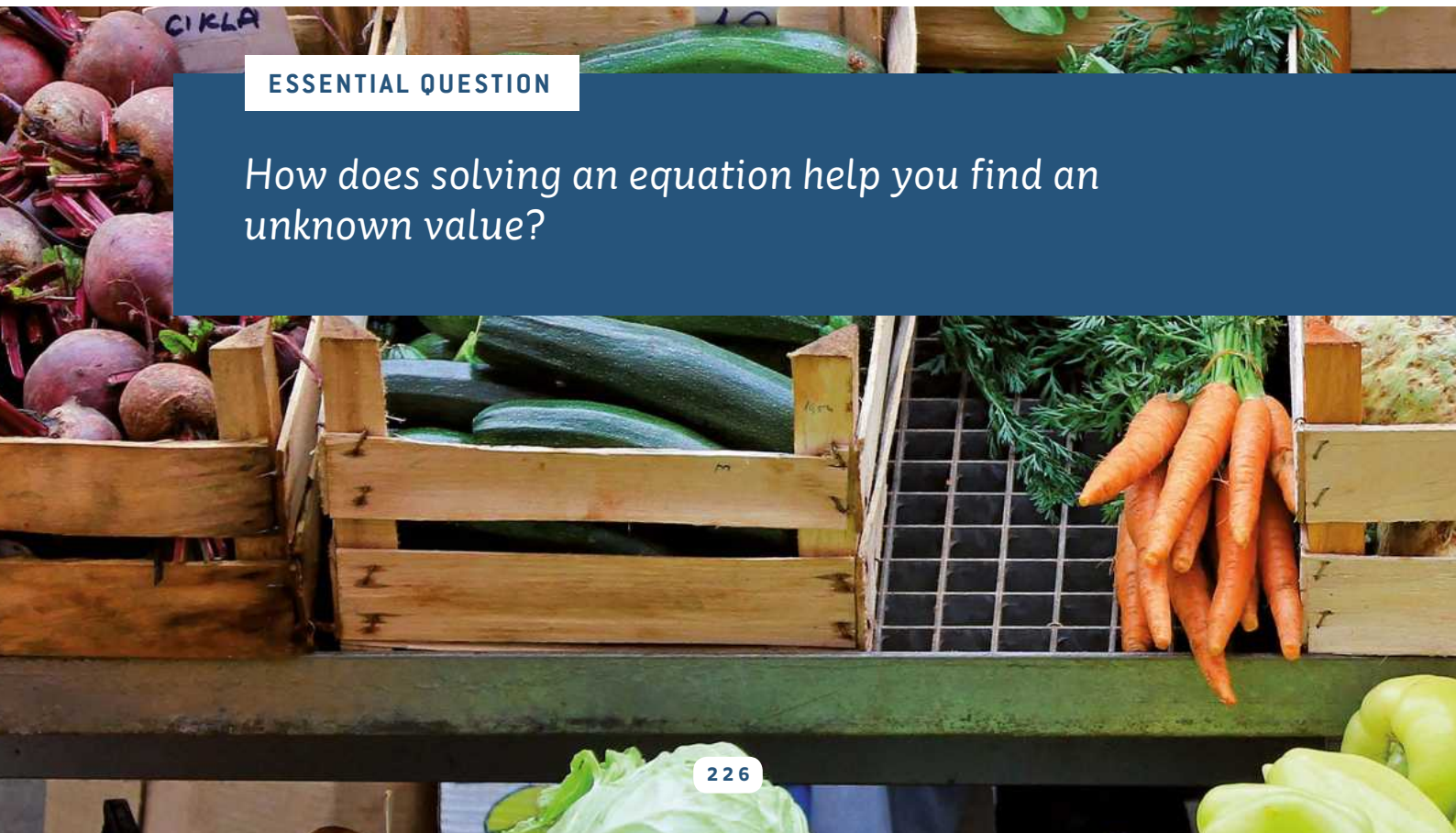
5E The balance model and equivalent equations

5F Solving equations by performing the same operation on both sides

5G Solving equations with the unknown on both sides

5H Plotting graphs of linear relationships

5I Solving linear equations using graphs



ESSENTIAL QUESTION

How does solving an equation help you find an unknown value?

Are you ready?

5A **1** What is (+13) + (−5)?

A −18 **B** −8 **C** 8 **D** 18

5A **2** What is:

a 22 − (−4)? **b** 6 − 8?

c −3 − 9? **d** −15 + 17?

5A **3 a** Evaluate $x + 7$ by substituting 12 for x.

b What is the result when $y = -4$ is substituted into $y - 5$?

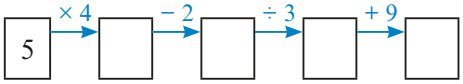
A 9 **B** −1 **C** −9 **D** −20

c Evaluate $3x - 15$ by substituting 4 for x.

5C **4 a** What is the missing number?



b Which number will be in the final box of this flowchart?

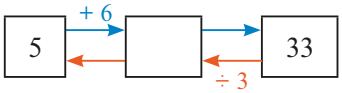


A 20 **B** 18 **C** 15 **D** 6

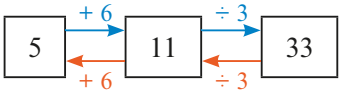
5D **5 a** What is the missing number?



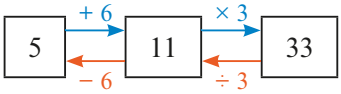
b Which flowchart shows the correct version of this incomplete flowchart?



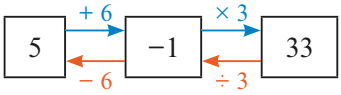
A



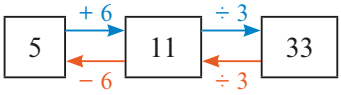
B



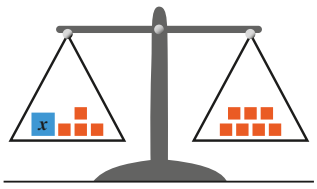
C



D



5E **6** This diagram models the equation $x + 4 = 7$. The left tray shows x g and 4 g while the right tray shows 7 g.



a If 2 g are added to the left tray, what should be done to the right tray to balance the scales?

A add 2 g **B** subtract 2 g

C add $2x$ and 2 g **D** add $2x$

b If 2 g are added to each tray, what equivalent equation does the balanced set of scales now model?

A $x + 4 = 9$ **B** $x + 6 = 9$

C $2x + 6 = 9$ **D** $2x + 8 = 14$

5G **7 a** Simplify $5x + 4 + 2x$.

b What is the simplest form of $4x + 3 - 2x + 6$?

A $2x + 9$ **B** $6x + 9$

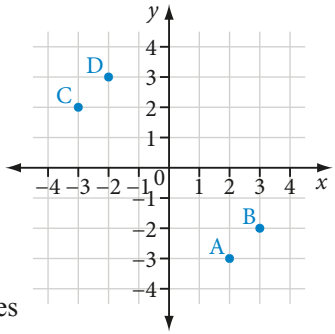
C $2x - 3$ **D** $6x - 3$

5G **8** What does $3(2x + 4)$ expand to?

A $6x + 12$ **B** $6x + 4$

C $5x + 7$ **D** $6x + 7$

5H **9** Look at this Cartesian plane.



a Which point has the coordinates (−3, 2)?

A A **B** B **C** C **D** D

b What are the coordinates of point D?

5A Understanding equations

Start thinking!



A group of friends go ice skating for Martin's birthday. Martin's parents pay $30 for each person's entry and snacks. Snacks cost $8 per person; however, the entry price is not listed separately. Let e represent the cost of entry for one person.

1 Use the **pronumeral** e to write an **expression** for the total cost (in dollars) of entry and snacks per person.

2 Since the total is $30, you can write an **equation** for this relationship. Copy and complete this sentence: The equation is e + ___ = 30.

3 An equation shows two expressions that are equal. What is the expression on the:
a left side of the equation? **b** right side of the equation?

4 The process of finding the value of e is called solving the equation. To **solve** an equation is to find the value of the pronumeral that makes the equation a true statement. This value is called the **solution** to the equation. Is the solution 20? Explain.

5 How can you work out the value of e? You have probably used a mental strategy like 'think of a number that when 8 is added to it gives 30'. This strategy is called **solving by inspection**. It means to inspect or use observation to find the solution. Write the solution to your equation.

6 How can you check whether your solution is correct?

KEY IDEAS

- An equation is a mathematical statement containing an equals sign where one expression is equal to another. That is, the value of the left side of an equation equals the value of the right side of the equation.
- To solve an equation is to find the value of the pronumeral that makes the equation a true statement. This value is called the solution to the equation.
- One strategy to solve an equation is called solving by inspection. This is a mental strategy that means to inspect or use observation to find the solution.
- You can check if a solution is correct by substituting the value for the pronumeral into the equation and seeing whether it makes the equation a true statement.

EXERCISE 5A Understanding equations

UNDERSTANDING AND FLUENCY

1 **a** State whether each of these is an equation or an expression.

i $5x + 4$ **ii** $8k = 24$ **iii** $14 - 3p = 5$

iv $\frac{3k - 1}{2}$ **v** $2 = 9 - x$ **vi** $4(7d + 3) = 8$

b Explain how you can tell the difference between an equation and an expression.

2 Substitute 6 for x in each equation. Does it make the equation a true statement?

a $x + 11 = 16$ **b** $x + 3 = 9$ **c** $x - 9 = -3$ **d** $x - 5 = -1$

e $2x = 8$ **f** $\frac{x}{3} = 2$ **g** $4x + 5 = 19$ **h** $3x - 7 = 11$

3 Substitute -9 for y in each equation. Does it make the equation a true statement?

a $y + 5 = -4$ **b** $y - 1 = -8$ **c** $3y = 27$ **d** $\frac{18}{y} = -2$

e $-5y = 45$ **f** $\frac{y}{3} = 3$ **g** $4y + 1 = -35$ **h** $2y + 6 = 24$

EXAMPLE 5A-1 Checking a solution

Decide whether $x = -3$ is the solution to the equation $2x + 5 = 1$.

THINK	WRITE
1 Write the equation.	$2x + 5 = 1$
2 Substitute -3 for x and evaluate the left side (LS) of the equation.	LS $= 2x + 5$ $= 2 \times -3 + 5$ $= -6 + 5$ $= -1$
3 Write the value of the right side (RS) of the equation.	RS $= 1$
4 Compare the left and right sides.	LS $\neq$ RS
5 Write your decision. Since the two sides of the equation are not equal, $x = -3$ is not the solution to the equation.	$x = -3$ is not the solution.

4 In each case, decide whether the value given for the pronumeral (shown in brackets) is the solution to the equation.

a $a - 5 = 6$ $(a = 11)$ **b** $4k = -16$ $(k = -4)$ **c** $m + 3 = 11$ $(m = 7)$

d $\frac{h}{5} = 2$ $(h = -10)$ **e** $6p - 4 = 2$ $(p = 1)$ **f** $3n + 7 = -1$ $(n = -2)$

g $\frac{b + 2}{3} = 2$ $(b = 7)$ **h** $15 - f = 16$ $(f = -1)$ **i** $\frac{x}{6} - 3 = 5$ $(x = -12)$

EXAMPLE 5A-2 Solving an equation by inspection

Solve each equation by inspection.

a $a + 5 = 12$ **b** $b - 6 = -9$ **c** $7c = -35$ **d** $\frac{d}{4} = 8$

THINK

a **1** Write the equation.

2 Think of a number that when 5 is added to it gives 12. ($7 + 5 = 12$) Write the solution.

b **1** Write the equation.

2 Think of a number that when 6 is subtracted from it gives −9. ($-3 - 6 = -9$) Write the solution.

c **1** Write the equation.

2 Think of a number that when multiplied by 7 gives−35. ($7 \times -5 = -35$) Write the solution.

d **1** Write the equation.

2 Think of a number that when divided by 4 gives 8. ($32 \div 4 = 8$) Write the solution.

WRITE

a $a + 5 = 12$

$a = 7$

b $b - 6 = -9$

$b = -3$

c $7c = -35$

$c = -5$

d $\frac{d}{4} = 8$

$d = 32$

UNDERSTANDING AND FLUENCY

5 Solve each equation by inspection.

a $a + 3 = 10$ **b** $b - 2 = 4$ **c** $3c = 24$ **d** $\frac{d}{4} = 3$

e $-4e = -8$ **f** $\frac{f}{3} = -1$ **g** $g - 5 = -4$ **h** $h + 6 = 6$

i $9 - i = 12$ **j** $8j = -40$ **k** $4 + k = 11$ **l** $l - 15 = 20$

m $\frac{24}{m} = 6$ **n** $-10n = 100$ **o** $\frac{x}{2} = -7$ **p** $p - 4 = -4$

q $8 = q + 2$ **r** $48 = -4r$ **s** $9 = 13 + s$ **t** $5 = \frac{t}{5}$

6 Use substitution to check that your solutions for question **5** are correct.

7 Explain what it means to find the solution to an equation.

8 Which solution will make $2x + 5 = 13$ a true statement?

A $x = 1$ **B** $x = 2$ **C** $x = 3$ **D** $x = 4$

9 Which solution will make $3y - 1 = -7$ a true statement?

A $y = -4$ **B** $y = -2$ **C** $y = 2$ **D** $y = 4$

10 Which solution will make $\frac{m + 3}{8} = 1$ a true statement?

A $m = 5$ **B** $m = 6$ **C** $m = 7$ **D** $m = 8$

UNDERSTANDING AND FLUENCY

11 Which solution will make $\frac{k}{3} - 1 = 0$ a true statement?

A $k = 1$ **B** $k = 2$ **C** $k = 3$ **D** $k = 4$

12 What is the value of the unknown that will make each equation a true statement?

a $3a + 9 = 15$ **b** $5b - 3 = 2$ **c** $7c - 2 = 19$ **d** $2d + 10 = 10$

e $\frac{e + 4}{5} = 2$ **f** $\frac{f - 1}{3} = 3$ **g** $\frac{g}{2} + 6 = 8$ **h** $\frac{h}{7} - 1 = -2$

13 Solve each equation by inspection.

a $4x + 1 = 21$ **b** $3x - 2 = 10$ **c** $7x + 1 = -6$ **d** $2x - 5 = 13$

e $\frac{x}{5} - 6 = -8$ **f** $\frac{x}{3} + 11 = 11$ **g** $\frac{x + 9}{2} = 3$ **h** $\frac{x - 4}{4} = -2$

EXAMPLE 5A-3 Writing and solving an equation

a Write an equation for each of these. Use n for the unknown number.
- **i** 3 more than a number is equal to 8.
- **ii** 5 less than a number is equal to −2.
- **iii** twice a number is equal to 14.

b Solve each equation. **c** Check the solutions you found are correct.

THINK

a Write the equation by identifying the operation performed on n and the result.
'3 more' means 'add 3' to n.
'5 less' means 'subtract 5' from n.
'twice' a number means 'multiply by 2'.

b Solve each equation by thinking of a number that makes the statement true.

c Check the solution is correct by substituting the value into the left side (LS) and seeing that it equals the right side (RS).

WRITE

a **i** $n + 3 = 8$
ii $n - 5 = -2$
iii $n \times 2 = 14$
or $2n = 14$

b **i** Solution to $n + 3 = 8$ is $n = 5$.
ii Solution to $n - 5 = -2$ is $n = 3$.
iii Solution to $2n = 14$ is $n = 7$.

c **i** LS $= n + 3$
$= 5 + 3$ when $n = 5$
$= 8$
$=$ RS
Solution is correct.

ii LS $= n - 5$
$= 3 - 5$ when $n = 3$
$= -2$
$=$ RS
Solution is correct.

iii LS $= 2n$
$= 2 \times 7$ when $n = 7$
$= 14$
$=$ RS
Solution is correct.

UNDERSTANDING AND FLUENCY

14 **a** Write an equation for each of these. Use n for the unknown number.

- **i** 4 added to a number gives 11.
- **ii** 3 subtracted from a number gives −2.
- **iii** 15 more than a number is equal to 21.
- **iv** 8 less than a number is equal to −4.
- **v** Twice a number is equal to 14.
- **vi** One-quarter of a number is 9.
- **vii** The sum of a number and −7 is equal to −20.
- **viii** The product of a number and 11 is equal to 22.
- **ix** When a number is multiplied by 5, the result is 55.
- **x** When a number is divided by 6, the result is −3.

b Solve each equation.

c Check that the solutions you found are correct.

15 **a** Write an equation for each of these. Use x for the unknown number.

- **i** A number is multiplied by 3, then 8 is added to give 11.
- **ii** A number is divided by 6, then 2 is subtracted to give 1.
- **iii** A number has 7 added to it, then the result is divided by 2 to give 5.
- **iv** A number is doubled, then 11 is subtracted to give 31.
- **v** A number has 9 subtracted from it, then the result is divided by 11 to give −1.
- **vi** A number is multiplied by 5, then the result is divided by 3 to give 10.

b Solve each equation.

c Check that the solutions you found are correct.

PROBLEM SOLVING AND REASONING

16 Entry to an aquarium costs $12 per person.

a Write an expression for the cost of a entry tickets.

b Write an equation for finding the number of tickets you could buy with $84.

c Solve the equation by inspection to find the value of a.

d How many tickets can you buy with $84?



17 Evie and her older brother Harry earn pocket money weekly. Harry receives $38.

a Write an expression for the total amount of money they receive if Evie receives x dollars.

b Write an equation for finding how much Evie gets if the total money given is $62.

c Solve the equation by inspection.

d How much does Evie receive?

PROBLEM SOLVING AND REASONING

18 A bag of Fruity Bites is divided amongst four people.

- **a** Write an expression for the number of Fruity Bites each person receives. (Remember to define the pronumeral used.)
- **b** Write an equation for finding the number of Fruity Bites in the bag if each person receives nine Fruity Bites.
- **c** Solve the equation by inspection to find the value of the pronumeral.
- **d** How many Fruity Bites are in the bag?



19 Todd has $200 and buys a pair of runners.

- **a** Write an expression for the amount of change he would receive.
- **b** Write an equation for finding the cost of the runners if he receives $21 change.
- **c** Solve the equation by inspection.
- **d** How much is the pair of runners?

20 These chocolate truffles cost a total of $20.

- **a** Write an equation for working out the cost of each chocolate.
- **b** Work out the cost of each chocolate by solving the equation.



CHALLENGE

21 Lisa finds three odd socks in her sock drawer.

- **a** Write an expression for the total number of socks in the drawer using n for the number of pairs of socks.
- **b** If the total number of socks in the drawer is 25, use your answer to part **a** to form an equation.
- **c** Solve the equation by inspection.
- **d** How many pairs of socks does Lisa have?



22 Kaizan buys three loaves of bread and has $3 left over.

- **a** Write an expression for the amount of money he started with.
- **b** If he had $15 to start with, write an equation for working out the cost of each loaf of bread.
- **c** Solve the equation to find the cost of one loaf of bread.

23 Write an equation that has a solution of:

- **a** $k = 4$ **b** $m = -7$
- **c** $n = 0$ **d** $p = 20$.

Reflect

How can you check that your solution to an equation is correct?

5B Solving equations using tables

Start thinking!



Some equations are not as easy to solve by inspection.
Another strategy you can use is called 'guess, check and improve'.

1 To use this strategy with $x + 9 = 15$, start by guessing a possible value for x. Try $x = 4$.

a When x is 4, what is the value of:

i the left side (LS) of the equation?

ii the right side (RS) of the equation?

b Are the values equal? What does this tell you?

Next, improve your guess. Try $x = 5$.

c Repeat parts **a** and **b** with $x = 5$.

d Improve your guess for x by thinking of another value you can try. Check if this value makes the equation a true statement.

e If necessary, repeat part **d** until the LS of the equation equals the RS. Can you see why this strategy is called 'guess, check and improve'? Explain.

f What is the solution to $x + 9 = 15$?

2 The process you have used to solve $x + 9 = 15$ can be summarised in a table. Copy and complete this table using your answers to question **1**.

Guess x	LS $x + 9$	RS 15	Check Is LS = RS?
4		15	no
5		15	
6		15	

3 Work with a partner to use the 'guess, check and improve' strategy to solve $3x - 7 = 20$. Set up a table so that you can be systematic in recording your guesses.

KEY IDEAS

- One strategy for solving equations is 'guess, check and improve'. It is where you guess a possible value for the solution and check whether this value makes the equation a true statement. If it doesn't, you improve the guess and try again.
- This method is useful when an equation cannot easily be solved by inspection.
- Use a table to be systematic in guessing and checking to find the solution.
- A **linear equation** contains a pronumeral term or terms where the highest power is one. For example, $2x + 3 = 6$.
- A **quadratic equation** contains a pronumeral term or terms where the highest power is two. For example, $x^2 + 3x - 2 = 10$.

EXERCISE 5B Solving equations using tables

1 Copy and complete this table to solve $x - 5 = 12$ using the 'guess, check and improve' strategy. Add more rows to the table if needed.

Guess x	LS $x - 5$	RS 12	Check Is LS = RS?
21	16	12	no
15		12	

EXAMPLE 5B-1 Solving an equation using a table

Use the 'guess, check and improve' strategy to solve $x - 17 = 31$.

THINK

1. Set up a table with rows to show each guess and the checking process.
2. Guess a value for x, say 40.
3. Substitute this value into the LS and RS expressions.
4. If LS does not equal RS, repeat steps 2 and 3 until it does.
5. Write the solution.

WRITE

Guess x	LS $x - 17$	RS 31	Check Is LS = RS?
40	23	31	no
50	33	31	no
49	32	31	no
48	31	31	yes

The solution is $x = 48$.

UNDERSTANDING AND FLUENCY

2 Use the 'guess, check and improve' strategy to solve each equation.

a $x + 13 = 21$ **b** $x - 6 = 38$
c $x - 29 = 32$ **d** $x + 22 = 63$
e $x + 89 = 113$ **f** $x - 65 = 26$
g $84 - x = 9$ **h** $105 - x = 57$

3 Copy and complete this table to solve $2a - 18 = 30$.

Guess x	LS $2a - 18$	RS 30	Check Is LS = RS?
20	22	30	no
30	42	30	no
25		30	

EXAMPLE 5B-2 Solving a more complex equation using a table

Use the 'guess, check and improve' strategy to solve $5x + 4 = 59$.

THINK

1 Set up a table with rows to show each guess and the checking process.

2 Guess a value for x, say 8.

3 Substitute this value into the LS and RS expressions.

4 If LS does not equal RS, repeat steps 2 and 3 until it does.

5 Write the solution.

WRITE

Guess x	LS $5x + 4$	RS 59	Check Is LS = RS?
8	44	59	no
12	64	59	no
10	54	59	no
11	59	59	yes

The solution is $x = 11$.

UNDERSTANDING AND FLUENCY

4 Use the 'guess, check and improve' strategy to solve each equation.

a $3a + 5 = 29$ **b** $5b - 6 = 44$

c $4c - 11 = 1$ **d** $6d + 7 = 73$

e $2e - 17 = 13$ **f** $9f + 4 = 58$

g $11g + 37 = 136$ **h** $8h - 23 = 145$

i $3x - 29 = 55$ **j** $7y + 13 = 300$

5 Copy and complete this table to solve $x + 24 = 19$.
Remember that x could be a negative number.

Guess x	LS $x + 24$	RS 19	Check Is LS = RS?
5	29	19	no
0	24	19	no
−1	23	19	no
−2		19	
−3		19	

6 Use the 'guess, check and improve' strategy to solve each equation.

a $x - 15 = -23$ **b** $x + 39 = 17$

c $2x - 21 = -33$ **d** $3x + 74 = 41$

e $5x + 8 = -7$ **f** $4x - 11 = -51$

g $3x + 62 = 5$ **h** $2x + 43 = -19$

i $4x - 77 = -285$ **j** $6x + 605 = -1$

7 Copy and complete this table to solve $5x + 23 = 35$.
(Hint: the solution is not a whole number.)

Guess x	LS $5x + 23$	RS 35	Check Is LS = RS?
1	28	35	no
2	33	35	no
3		35	
2.5			

8 Use the 'guess, check and improve' strategy to solve each equation.
(Hint: the solutions will not be whole numbers.)

a $4x + 13 = 19$ **b** $3x - 0.4 = 2$

c $10x + 21 = 38$ **d** $2x - 23 = -30$

e $8x + 37 = 17$ **f** $5x - 8 = 13$

g $7x - 6.9 = 54$ **h** $6x + 4.5 = 0$

9 Copy and complete this table to solve $2(x + 11) = 34$.

Guess x	LS $2(x + 11)$	RS 34	Check Is LS = RS?
3	28	34	no
8	38	34	no
7		34	

10 Use the 'guess, check and improve' strategy to solve each equation.

a $3(x - 16) = 15$ **b** $4(x + 12) = 180$

c $\frac{x + 23}{5} = 8$ **d** $\frac{x}{2} - 9 = 4$

e $\frac{x - 5}{8} = 5$ **f** $\frac{x}{7} + 35 = 22$

11 Copy and complete this table to solve $3x - 5 = 2x + 3$.
Notice that x appears on both sides of the equation.

Guess x	LS $3x - 5$	RS $2x + 3$	Check Is LS = RS?
1	−2	5	no
5	10	13	no
10	25	23	no
9	22		

12 Use the 'guess, check and improve' strategy to solve each equation.

a $4x - 1 = x + 5$ **b** $5x + 7 = 4x + 10$

c $2x - 3 = 3x - 13$ **d** $8x + 2 = 7x + 7$

e $x + 6 = 3x - 24$ **f** $3x - 2 = x - 4$

g $6x + 0.4 = 5x + 2$ **h** $4x - 2 = 2x + 3$

i $5x - 7 = 10x + 17$ **j** $24x + 20 = 4x + 27$



13 Angus bought three crayfish and had $14 left over.

a Write an expression for how much money Angus started with. (Hint: let c represent the cost of a crayfish.)

b If he started with $230, write an equation for finding the cost of one crayfish.

c Use the 'guess, check and improve' strategy to solve the equation.
What is the cost of one crayfish?

14 Four friends share the cost of a gift and wrapping paper.
The wrapping paper cost $6.

a Write an expression for the amount each person contributes. (Hint: let g represent the cost of the gift.)

b If each person contributes $15, use your expression to write an equation for finding the cost of the gift.

c Use the 'guess, check and improve' strategy to solve the equation.
What is the cost of the gift?



15 The equations you have worked with so far are called linear equations. They contain a pronumeral with a power of one. How many solutions does a linear equation have? Discuss this with a classmate.

16 Another type of equation is a quadratic equation. Two examples of a quadratic equation are $x^2 - 10x + 25 = 0$ and $x^2 + 7 = 23$. They can have one or two solutions, or even no solution.

a Look at the quadratic equations shown. What is different about a quadratic equation? Compare it to a linear equation.

b Write three more examples of a quadratic equation.

PROBLEM SOLVING AND REASONING

17 The 'guess, check and improve' strategy can be used to solve quadratic equations. Copy and complete this table to solve $x^2 + 25 = 10x$. This equation has only one solution.

Guess x	LS $x^2 + 25$	RS $10x$	Check Is LS = RS?
2	29	20	no
3	34	30	
4			
5			

18 Use the 'guess, check and improve' strategy to solve each equation. In each case, there is only one solution.

a $x^2 + 4 = 4x$ **b** $x^2 + 49 = 14x$

c $x^2 - 12x = -36$ **d** $x^2 + 6x = -9$

e $x^2 + 16 = -8x$ **f** $x^2 + 10x = -25$

g $x^2 - 18x = -81$ **h** $x^2 + 121 = 22x$

19 Show that each of these quadratic equations has two solutions by substituting first one then the other value for x.

a $x^2 + 6 = 5x$ $(x = 2$ and $x = 3)$

b $x^2 - 5 = 4x$ $(x = -1$ and $x = 5)$

c $x^2 + 7 = 23$ $(x = 4$ and $x = -4)$

d $x^2 - 2x = 80$ $(x = -8$ and $x = 10)$

CHALLENGE

20 Use the 'guess, check and improve' strategy to find two solutions for each equation.

a $x^2 + 15 = 8x$ **b** $x^2 + 9 = 10x$

c $x^2 - 4 = 5$ **d** $x^2 - 5x = 14$

e $x^2 + 32 = 12x$ **f** $x^2 + x = 30$

g $x^2 - 3x = 88$ **h** $x^2 + 39 = -16x$

21 Use the 'guess, check and improve' strategy to solve each equation.

a $25 - x = 43$ **b** $0.5x + 0.3 = 1.8$

c $3(x + 17) = 39$ **d** $200 - 9x = 65$

e $4(1 - 3x) = 29.2$ **f** $\frac{2(x + 8)}{5} = 6$

g $7(2x - 5) = 56$ **h** $\frac{2x}{3} + 6 = 0$

22 Tables for this strategy can also be set up using a spreadsheet. Choose any three equations from this section and solve them using a spreadsheet.

Reflect

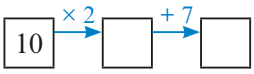
Why is using a table a useful strategy in solving equations?

5C Building expressions using flowcharts



Start thinking!

The number of cars in a car park changes during the day. At 11 am, there are twice as many cars as there were at 9 am. At 4 pm, the number of cars is 7 more than at 11 am.



1 **a** Copy and complete this flowchart to show the number of cars in the car park at the three given times, if there were 10 cars at 9 am.

b Which box of the flowchart shows the number of cars at 11 am?

c Which operation is used to calculate the number of cars at:

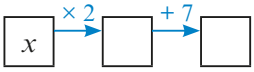
i 11 am? **ii** 4 pm?

d How many cars are in the car park at 4 pm?

e Explain what would happen if you performed the two operations in the opposite order. Does it give you the same result?

2 Draw a flowchart to show the number of cars in the car park if there were 24 cars at 9 am.

3 You can use this flowchart for any starting number.



a If the starting number is x (there are x cars at 9 am), complete the first two boxes of the flowchart. What expression is shown in the second box?

b Complete the third box of the flowchart. What expression is shown in the third box? Remember to perform the operation (add 7) on the entire contents of the second box.

c You have 'built' an expression from x by performing operations in a particular order. What expression have you built?

KEY IDEAS

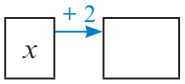
- A flowchart can be used to build an expression starting from the variable (represented by a pronumeral).
- Each operation is performed on the entire expression in the previous box of the flowchart.
- The order in which the operations are performed is important.
- Flowcharts can also show the operations to follow when going from the final expression to the starting variable. Following the flowchart back to the starting number is called **backtracking**.

EXERCISE 5C Building expressions using flowcharts

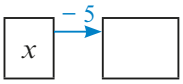
UNDERSTANDING AND FLUENCY

1 Copy and complete each flowchart to build an expression from x.

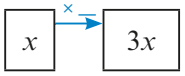
a $x + 2$



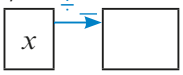
b $x - 5$



c $3x$



d $\frac{x}{7}$



2 Draw a flowchart to build each expression from x.

a $x + 7$ **b** $x - 3$ **c** $6x$ **d** $\frac{x}{2}$ **e** $-10x$ **f** $\frac{x}{-9}$

EXAMPLE 5C-1 Identifying missing operations on a flowchart

Write the operations needed to complete each flowchart.

a $x \rightarrow 5x \rightarrow 5x - 8$ **b** $x \rightarrow x + 5 \rightarrow 8(x + 5)$

THINK

a **1** To get $5x$ from x, you must multiply x by 5. Show $\times 5$ as the first operation.

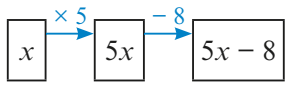
2 To get $5x - 8$ from $5x$, you must subtract 8 from $5x$. Show -8 as the second operation.

b **1** To get $x + 5$ from x, you must add 5 to x. Show $+5$ as the first operation.

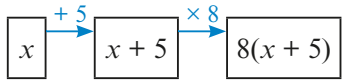
2 To get $8(x + 5)$ from $x + 5$, you must multiply $x + 5$ by 8. Brackets show that all of $x + 5$ is multiplied by 8. Show $\times 8$ as the second operation.

WRITE

a



b



3 Write the operations needed to complete each flowchart.

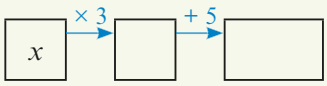
a $x \rightarrow 3x \rightarrow 3x + 4$ **b** $x \rightarrow \frac{x}{8} \rightarrow \frac{x}{8} - 1$

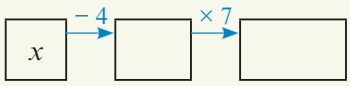
c $x \rightarrow x - 2 \rightarrow 7(x - 2)$ **d** $x \rightarrow 6x \rightarrow 6x - 5$

e $x \rightarrow x + 3 \rightarrow \frac{x + 3}{5}$ **f** $x \rightarrow \frac{x}{4} \rightarrow \frac{x}{4} + 7$

EXAMPLE 5C-2 Completing a flowchart to build an expression

Copy and complete each flowchart to build an expression.

a 

b 

THINK

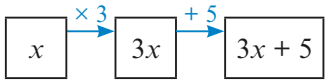
a 1 Perform the first operation (multiply x by 3 to obtain $3x$) and write the result in the second box.

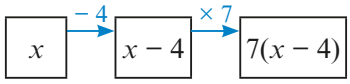
2 Perform the next operation on the contents of the second box (add 5 to $3x$) and write the final result in the last box.

b 1 Perform the first operation (subtract 4 from x to obtain $x - 4$) and write the result in the second box.

2 Perform the next operation on the contents of the second box (multiply $x - 4$ by 7) and write the final result in the last box. Brackets show that all of $x - 4$ is multiplied by 7.

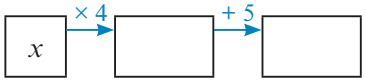
WRITE

a 

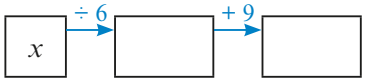
b 

UNDERSTANDING AND FLUENCY

4 Copy and complete each flowchart to build an expression.

a 

b 

c 

d 

e 

f 

5 Which option shows the correct order of operations to be performed to build each expression?

a $5x + 7$

A × 7, + 5 **B** × 5, + 7 **C** + 5, × 7 **D** + 7, × 5

b $\frac{x - 4}{3}$

A ÷ 4, − 3 **B** − 3, ÷ 4 **C** ÷ 3, − 4 **D** − 4, ÷ 3

6 Write the operations to be performed in the correct order to build each expression.

a $3x + 8$ **b** $2x - 6$ **c** $7x + 1$ **d** $5(x - 4)$

e $8(x + 6)$ **f** $4(x - 2)$ **g** $\frac{x}{7} - 3$ **h** $\frac{x}{4} + 9$

i $\frac{x}{5} - 2$ **j** $\frac{x + 1}{6}$ **k** $\frac{x - 8}{3}$ **l** $\frac{x + 5}{2}$

EXAMPLE 5C-3 Drawing a flowchart to build an expression

Draw a flowchart to build each expression.

a $\frac{x}{6} - 11$ **b** $\frac{x+9}{2}$

THINK

a **1** Write the pronumeral in the first box.

2 Work out the first operation to be performed (divide x by 6) and show the operation and result in the flowchart.

3 Perform the next operation on the entire contents of the second box (subtract 11 from $\frac{x}{6}$).

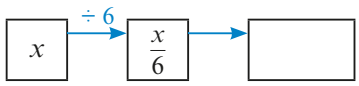
b **1** Write the pronumeral in the first box.

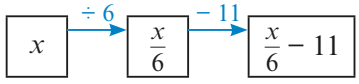
2 Work out the first operation to be performed. The vinculum groups x and 9 together so the first operation is to add 9 to x.

3 Perform the next operation on the entire contents of the second box (divide $x + 9$ by 2).

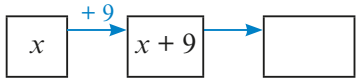
WRITE

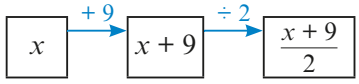
a





b





UNDERSTANDING AND FLUENCY

7 Draw a flowchart to build each expression in question **6**.

8 Draw a flowchart to build each expression.

a $2(x + 5)$	**b** $\frac{x}{6} - 7$	**c** $9x - 4$	**d** $\frac{x+3}{7}$
e $4x + 11$	**f** $\frac{x}{3} + 6$	**g** $\frac{x-2}{8}$	**h** $6(x - 7)$
i $\frac{4x}{5}$	**j** $\frac{x}{2} - 1$	**k** $7(x + 3)$	**l** $\frac{x-9}{4}$

9 Draw a flowchart to build each expression.

a Start with a, multiply by 5 and then add 2.

b Start with b, divide by 8 and then subtract 3.

c Start with c, multiply by 9 and then divide by 7.

d Start with d, add 13 and then divide by 6.

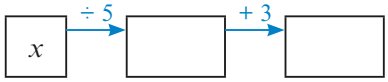
e Start with e, subtract 9 and then multiply by 4.

f Start with f, add 6 and then double the result.

10 This flowchart shows the operations to be used to build an expression from x.

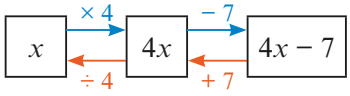
Brandon writes the final expression as $\frac{x+3}{5}$.

Is he correct? Give a reason for your answer.



11 Write an expression and have a classmate draw a flowchart showing how it is built.

12 Flowcharts can also show the operations going from the final expression to the starting variable. Following the flowchart back to the starting number is called backtracking.



a Use the flowchart to write the operations to build the expression from x.

b Write the operations in the correct order to backtrack from the final expression to the starting variable of x.

c What operation is used to undo the step of subtracting 7 from $4x - 7$ to give $4x$?

d What operation is used to undo the step of multiplying by 4 to give x?

e Copy and complete these sentences using the words *add*, *subtract*, *multiply* and *divide*.

i To undo a step where you added, you _____.

ii To undo a step where you subtracted, you _____.

iii To undo a step where you multiplied, you _____.

iv To undo a step where you divided, you _____.

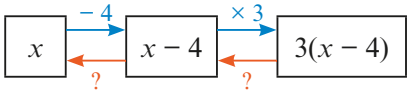
NOTE When you backtrack, you undo the original operation at each step by performing the inverse operation.

13 Write the inverse operation to each of these.

a $+ 6$ **b** $\times 4$ **c** $- 7$ **d** $\div 3$

e $\times -2$ **f** $\div -8$ **g** $- 11$ **h** $\times 20$

14 Copy and complete this flowchart to show the inverse operations needed to backtrack from the final expression to the starting variable.



15 On your flowcharts for question **3**, draw the arrows and inverse operations to backtrack from the final expression to the starting variable.

16 Write the operations needed to complete each flowchart.

a $x \rightarrow x + 3 \rightarrow 2(x + 3) \rightarrow 2(x + 3) - 5$

b $x \rightarrow 4x \rightarrow 4x - 1 \rightarrow \dfrac{4x - 1}{7}$

c $x \rightarrow 3x \rightarrow 3x + 2 \rightarrow 8(3x + 2)$

d $x \rightarrow x - 6 \rightarrow \dfrac{x - 6}{5} \rightarrow \dfrac{x - 6}{5} + 7$

17 Copy and complete each flowchart to build an expression.

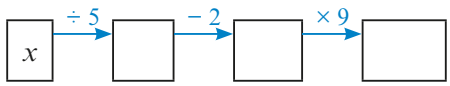
a $x \xrightarrow{-4} \square \xrightarrow{\times 7} \square \xrightarrow{+2} \square$

b $x \xrightarrow{\times 3} \square \xrightarrow{+1} \square \xrightarrow{\times 5} \square$

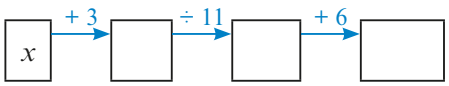
c $x \xrightarrow{\times 2} \square \xrightarrow{-6} \square \xrightarrow{\div 3} \square$

d $x \xrightarrow{\times 4} \square \xrightarrow{\div 7} \square \xrightarrow{-8} \square$

e



f



18 On your flowcharts for question **17**, draw the arrows and inverse operations to backtrack from the final expression to the starting variable.



19 There are x pieces of fruit in a basket. Teagan takes half of the fruit to rehearsal. Her mother then adds six pieces to the basket.

a Represent this situation with a flowchart.

b What expression do you obtain for the final amount of fruit in the basket?

c If x is 8, what is the final amount of fruit in the basket?

d Check you obtain the same result whether you substitute into the final expression or perform the operations shown in the flowchart on the starting value of 8.

20 You have d dollars in your pocket. You spend $42 on a haircut and spend half of what is left on an iTunes card.

a Represent this situation with a flowchart.

b What expression do you obtain for the final amount of money in your pocket?

c If d is 82, what is the final amount of money in your pocket?



21 Zac lights candles with five matches from a box containing m matches. He finds some loose matches in the drawer and places them in the box so there are now three times as many matches in the box.

a Represent this situation with a flowchart.

b What expression do you obtain for the final number of matches in the box?

22 **a** What is the first operation performed on x in $\frac{3x + 5}{7}$?

b Which operation is performed: **i** second? **ii** third?

c Draw a flowchart to build the expression.

d On the same flowchart, draw the arrows and inverse operations to backtrack from the final expression to the starting variable.

23 **a** What is the first operation performed on x in each expression?

i $4(x - 3) + 7$ **ii** $3(2x + 9)$ **iii** $\frac{x + 2}{8} - 5$

iv $\frac{6x - 3}{2}$ **v** $5(\frac{x}{4} + 1)$ **vi** $\frac{7x}{9} + 11$

vii $2(x + 6) - 4$ **viii** $\frac{x - 8}{3} + 6$ **ix** $5(4x - 1)$

b Draw a flowchart to build each expression.

c On each flowchart, draw the arrows and inverse operations to backtrack from the final expression to the starting variable.

24 Write an expression that has three or more operations performed on the variable and have a classmate draw a flowchart to show how this expression is built.

Reflect

Why is the order of the operations important when building an expression using a flowchart?

PROBLEM SOLVING AND REASONING

CHALLENGE

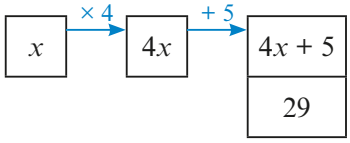


5D Solving equations using backtracking

Start thinking!

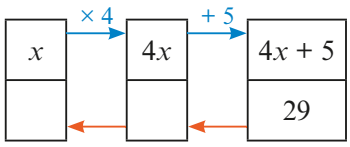
A flowchart shows the order of operations to perform on a variable to produce an expression. You can backtrack from a final expression to the starting variable using inverse operations. Backtracking is another strategy you can use to solve equations.

1 Draw a flowchart to build the expression on the left side of the equation $4x + 5 = 29$. Since the expression is equivalent to 29, you can write 29 under $4x + 5$ on the flowchart.



2 On your flowchart, copy the extra boxes and arrows shown in this diagram.

Write the inverse operations to backtrack from the final expression to the starting variable.

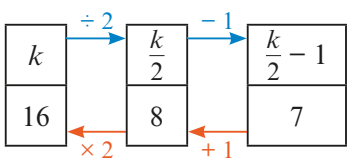


3 Use backtracking to fill in the missing values in the boxes on your flowchart.

4 What is the value of x after backtracking from 29? This is the solution to $4x + 5 = 29$.

5 How can you check that this solution is correct?

6 This flowchart shows how to solve an equation by backtracking.



a What is the variable in the equation?

b What is the equation that is being solved?

c What is the solution to the equation?

7 How can you use backtracking to find the solution to an equation?

KEY IDEAS

- An equation can be solved by drawing a flowchart and backtracking.
- Backtracking is using inverse operations to work backwards from the final expression to the variable.
- You can check if a solution is correct by substituting the value found for the pronumeral into the equation and seeing whether it makes the equation a true statement.

EXERCISE 5D Solving equations using backtracking

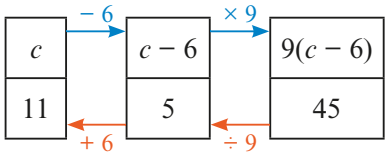
UNDERSTANDING AND FLUENCY

1 This flowchart shows how to solve an equation by backtracking.

a What is the variable in the equation?

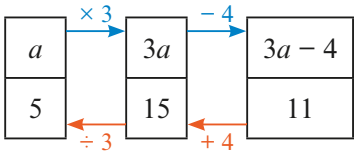
b What is the equation that is being solved?

c What is the solution to the equation?

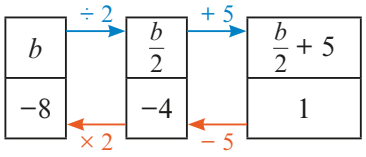


2 For each flowchart, write the equation to be solved and its solution.

a

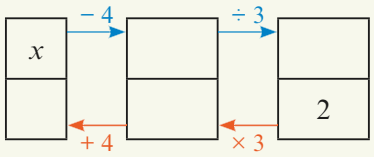


b



EXAMPLE 5D-1 Completing a flowchart to solve an equation

Copy and complete this flowchart.
Write the equation to be solved and its solution.



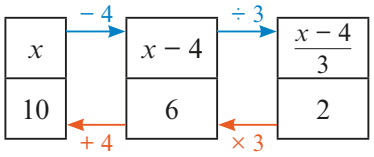
THINK

1 Perform the operations to find the missing values.

2 Write the equation. Use the final expression and the number it is equivalent to, shown below it.

3 Write the solution to the equation. This is the variable (the pronumeral x) and the equivalent value shown below it.

WRITE

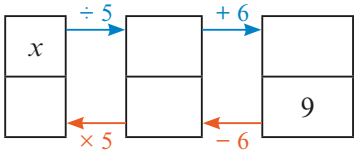


$\frac{x-4}{3} = 2$

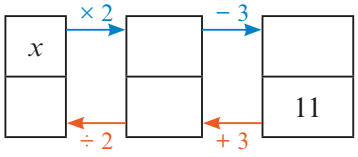
$x = 10$

3 Copy and complete each flowchart. Write the equation to be solved and its solution.

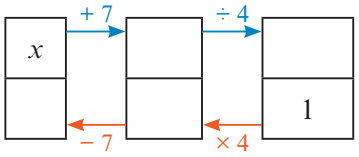
a



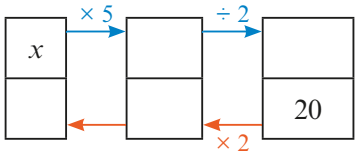
b



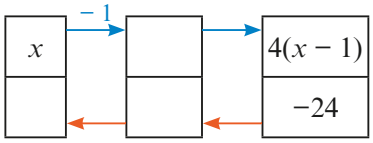
c



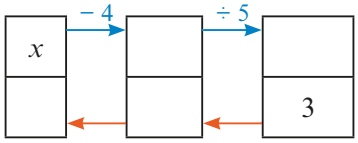
d



e



f



EXAMPLE 5D-2 Solving an equation using backtracking

Solve $5(x + 6) = 20$ using backtracking.

THINK

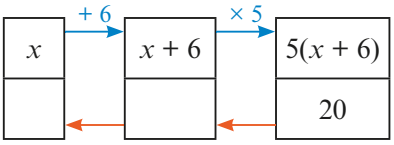
1 Draw a flowchart to build the expression $5(x + 6)$. Write the equivalent value of the expression below.

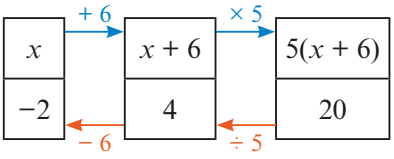
2 Show the inverse operations needed to backtrack from the expression to the variable.

3 Work out the missing values by backtracking.

4 Write the solution.

WRITE





$x = -2$

UNDERSTANDING AND FLUENCY

4 Solve each equation using backtracking.

a $4(x - 3) = 16$ **b** $2x + 1 = 13$ **c** $\frac{x - 1}{6} = 3$

d $\frac{4x}{3} = 12$ **e** $\frac{x}{2} - 9 = -4$ **f** $7x - 9 = 19$

g $\frac{x + 9}{4} = 6$ **h** $2(x + 7) = 8$ **i** $\frac{3x}{2} = 18$

j $\frac{x}{3} + 10 = 6$ **k** $\frac{x - 4}{5} = -2$ **l** $6(x + 3) = 48$

m $\frac{x}{4} - 2 = 0$ **n** $3x + 8 = 5$ **o** $\frac{5x}{8} = -10$

5 Use substitution to check that the solutions you found for question **4** are correct.

6 A number has 3 added to it and the result is multiplied by 5 to give an answer of 45. Let x represent the unknown number.

a Which equation matches this situation?

A $5x + 3 = 45$ **B** $5(x + 3) = 45$ **C** $\frac{x + 3}{5} = 45$ **D** $3(x + 5) = 45$

b Use backtracking to find the unknown number.

c Use substitution to check your solution is correct.

7 **a** Write an equation for each situation.

i A number is multiplied by 4, then 6 is added to give 54.

ii A number is divided by 7, then 3 is subtracted to give 2.

iii A number has 2 added to it, then the result is multiplied by 11 to give 55.

iv A number has 5 subtracted from it, then the result is divided by 3 to give 1.

v A number is multiplied by 3, then the result is divided by 2 to give 6.

vi A number is divided by 5, then 9 is added to give 17.

b Use backtracking to find the unknown number.

NOTE Remember to use a pronumeral for the unknown number.

8 Solve each equation using backtracking.

a $6x + 11 = 5$ **b** $3(x - 2) = 24$

c $\frac{x - 7}{4} = -4$ **d** $\frac{x}{2} + 8 = 8$

e $4(x + 5) = 8$ **f** $8x - 3 = 1$

g $\frac{x}{5} - 2 = -7$ **h** $\frac{2x}{3} = 9$

i $\frac{x + 3}{8} = 4$ **j** $3x - 5.2 = 15.8$

k $\frac{x}{4} + 13 = 11$ **l** $5(x - 1) = -5$

m $\frac{x - 27}{12} = 2$ **n** $4x + 9 = 12$

o $\frac{7x}{6} = 1$ **p** $\frac{x}{10} - 9 = -10$

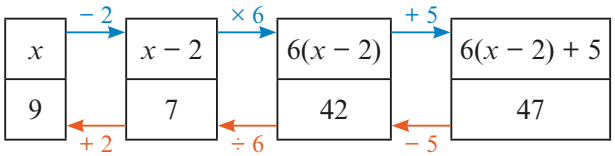
q $2(x + 15) = 14$ **r** $\frac{x + 4}{9} = -3$

NOTE A solution may be negative and may not always be a whole number.

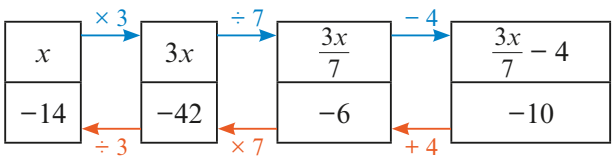
NOTE Use substitution to check your solutions.

9 For each flowchart, write the equation to be solved and its solution.

a



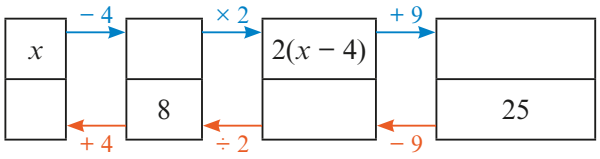
b



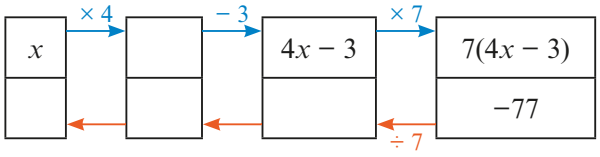
UNDERSTANDING AND FLUENCY

10 Copy and complete each flowchart. Write the equation to be solved and its solution.

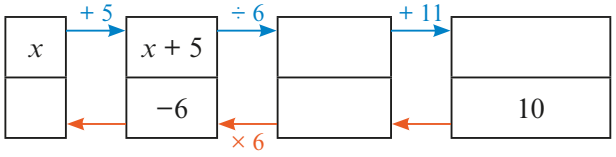
a



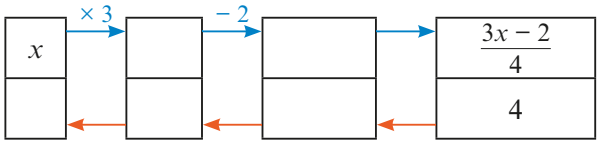
b



c



d



11 Solve each equation using backtracking. Use substitution to check your solutions.

a $3(4x - 1) = 57$ **b** $\frac{x - 8}{5} + 9 = 12$ **c** $2(x + 11) - 6 = 8$

d $\frac{9x - 4}{2} = -2$ **e** $4(\frac{x}{5} + 3) = 36$ **f** $\frac{11x}{4} + 23 = 1$

g $10(x + 3) - 13 = 87$ **h** $\frac{x + 13}{6} - 5 = 0$ **i** $8(3x + 1) = -40$

j $\frac{4x + 11}{3} = 7$ **k** $6(\frac{x}{2} + 5) = 0$ **l** $\frac{4x}{5} + 17 = 18$

PROBLEM SOLVING AND REASONING

12 A rose bush has three times as many roses as last year. Kyra cuts five roses to fill a vase, leaving 13 roses on the bush.



a If n represents the number of roses on the bush last year, which equation matches this situation?

A $5n + 3 = 13$ **B** $5n - 3 = 13$

C $3n + 5 = 13$ **D** $3n - 5 = 13$

b Solve the equation using backtracking.

c How many roses were on the bush last year?

d Use substitution to check that your solution is correct.

13 Todd counts his money, then puts half in his bank account. He then earns $15 for washing the car and finds he has $128 to spend. Work out how much money Todd originally counted by writing an equation and solving it by backtracking.

PROBLEM SOLVING AND REASONING



14 Grace buys two types of sushi, nigiri and maki. She selects nine pieces of nigiri and wants to spend the remainder of her money on maki. Let m represent the number of maki she can buy.

- **a** Write an expression for the total number of sushi she will buy.
- **b** If each sushi costs \$2, use your answer to part **a** to write an expression for the total cost.
- **c** If Grace has \$30 to spend, write an equation to match this situation.
- **d** Solve the equation using backtracking to work out the number of maki she can buy.



15 Leo builds a sand castle with towers. His friends join him and build more towers, so there are three times as many. A freak wave then washes away four of the towers, leaving 11 towers standing.

- **a** Write an equation for this situation. Use a pronumeral for the number of towers Leo built.
- **b** Solve the equation using backtracking to find the number of towers Leo built.

16 Alexis gave three \$10 notes to the shopkeeper to buy six note books and received \$3 in change.

- **a** Write an equation to help you work out the cost of each note book.
- **b** Solve the equation using backtracking. How much did each note book cost?

CHALLENGE

17 A number is multiplied by 2, then 7 is added to the result, then the result is multiplied by 3 and finally 5 is subtracted.

- **a** Write an expression to represent this problem and draw the matching flowchart.
- **b** Ask a classmate to secretly write a number and then tell you the result after performing the operations on it. Use backtracking to work out the original number.

18 Use backtracking to solve:

- **a** $\frac{5(3x + 1)}{4} - 6 = 14$
- **b** $7(\frac{x}{3} - 5) + 56 = 0.$

Reflect

Why are inverse operations important in the backtracking strategy for solving equations?

5E The balance model and equivalent equations

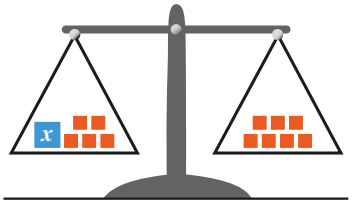


Start thinking!

A set of scales uses gravity to compare the mass of objects in two trays. This set of scales is balanced, as there is the same mass on both sides.

1 An equation can be thought of as a balanced set of scales. Explain why.

2 The diagram on the right models the equation $x + 5 = 7$.
The left side shows a mass of x g and 5 g while the right side shows 7 g.



- **a** Why is the set of scales balanced?
- **b** What is the solution to $x + 5 = 7$?
- **c** What would happen if 3 g were removed from the left side of the scales?
- **d** Describe what you would need to do to the right side to balance the scales.
- **e** What equation does the balanced set of scales now model?
- **f** What is the solution to this equation?

3 Instead of removing 3 g, think about what would happen if you added 4 g to the left side of the scales.

- **a** How could you balance the scales?
- **b** What equation does the balanced set of scales now model?
- **c** What is the solution to this equation?

4 Compare the solutions for the three equations.

- **a** These three equations are said to be **equivalent equations**. Why do you think this is?
- **b** Is $x + 5 = 7$ equivalent to:
  i $x + 4 = 8$? **ii** $x - 3 = -1$?
  Explain.
- **c** Write another five equations that are equivalent to $x + 5 = 7$.

5 How are equivalent equations related to balancing a set of scales?

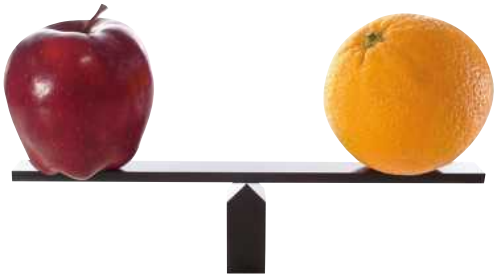
KEY IDEAS

- You can obtain equivalent equations using a **balance model**, such as balancing a set of scales.
- An equivalent equation can be formed by performing the same operation on both sides of an equation.
- Equivalent equations have the same solution.

EXERCISE 5E The balance model and equivalent equations

UNDERSTANDING AND FLUENCY

1 Look at this photo showing an apple and an orange.



a What does this tell you about the mass of the apple compared to the mass of the orange? Explain.

b What would happen if a small bunch of grapes was draped over the apple?

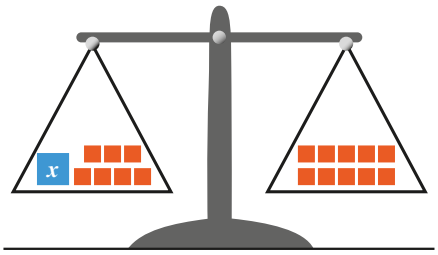
2 This photo shows a set of unbalanced scales.



a Which tray has the heavier mass?

b How could you balance the set of scales?

3 An equation is modelled by this set of scales.



a Is this set of scales balanced? How can you tell?

b What is the equation modelled by this set of scales?

c If 4 g are removed from the left tray, what can you do to the right tray to make the scales balance again?

d What equivalent equation does it now show?

4 **a** Draw a diagram to model the equation $x + 3 = 9$.

b Draw a diagram to show what the model in part **a** looks like after:

i 2 g are removed from both sides

ii 5 g are added to both sides

iii 4 g are added to both sides

iv 3 g are removed from both sides.

c What equivalent equation does each diagram in part **b** represent?

d Solve each equation by inspection. What do you notice?

5 **a** Draw a diagram to model the equation $x + 4 = 6$.

b Draw a diagram to show what the model in part **a** looks like after:

i 3 g are removed from both sides

ii 2 g are added to both sides

iii 3 g are added to both sides

iv 4 g are removed from both sides.

c What equivalent equation does each diagram in part **b** represent?

d Solve each equation by inspection. What do you notice?

EXAMPLE 5E-1 Forming equivalent equations

Perform each operation on both sides of $x + 9 = 14$ to form an equivalent equation.

a add 3 **b** subtract 4 **c** multiply by 2

d divide by 7 **e** subtract 9

THINK	WRITE
a Add 3 to both sides of the equation and simplify. (Showing + 3 in another colour can make the operation easier to see.)	**a** $x + 9 + 3 = 14 + 3$ $x + 12 = 17$
b Subtract 4 from both sides of the equation and simplify.	**b** $x + 9 - 4 = 14 - 4$ $x + 5 = 10$
c Multiply both sides of the equation by 2 and simplify.	**c** $(x + 9) \times 2 = 14 \times 2$ $2(x + 9) = 28$ (or $2x + 18 = 28$)
d Divide both sides of the equation by 7 and simplify.	**d** $(x + 9) \div 7 = 14 \div 7$ $\frac{x + 9}{7} = 2$
e Subtract 9 from both sides of the equation and simplify.	**e** $x + 9 - 9 = 14 - 9$ $x = 5$

UNDERSTANDING AND FLUENCY

6 **a** Perform each operation on both sides of $x + 4 = 20$ to form an equivalent equation.

i add 8 **ii** subtract 1 **iii** multiply by 3

iv divide by 5 **v** add 4 **vi** subtract 4

b Solve each equation by inspection or backtracking. What do you notice?

7 **a** Perform each operation on both sides of $x - 2 = 6$ to form an equivalent equation.

i add 10 **ii** subtract 4 **iii** multiply by 5

iv divide by 3 **v** subtract 2 **vi** add 2

b Solve each equation.

8 What is the solution to each equivalent equation in Example 5E-1?

9 Which of these equations are equivalent to $x + 2 = 5$?

a $x + 5 = 8$ **b** $x + 1 = 2$ **c** $x + 20 = 18$

d $x - 1 = 2$ **e** $x = 3$ **f** $x - 8 = -11$

10 Which of these equations are equivalent to $x - 8 = 3$?

a $x - 7 = 4$ **b** $x = -5$ **c** $x + 2 = 13$

d $x - 2 = 6$ **e** $x - 12 = 1$ **f** $x + 1 = 12$

11 Write three equivalent equations to each of these.

a $x + 6 = 11$ **b** $x - 7 = 2$ **c** $x + 13 = 4$



12 Consider the equation $x + 5 = 8$.

a Erin adds 7 to both sides of the equation.
Which equation matches the result she obtains?

A $x + 5 = 15$ **B** $x + 12 = 15$

C $x + 12 = 8$ **D** $x + 12 = 1$

b Noah subtracts 6 from both sides of the equation.
Which equation matches the result he obtains?

A $x - 1 = 2$ **B** $x + 11 = 14$ **C** $x - 1 = 8$ **D** $x = 2$

c Ashley subtracts 5 from both sides of the equation. Which equation matches her result?

A $x + 5 = 3$ **B** $x + 10 = 13$ **C** $x + 10 = 3$ **D** $x = 3$

d Find the solution to the equivalent equations from parts **a**–**c**.
Compare to the solution of $x + 5 = 8$.

e Which operation produced an equivalent equation that is the same as the solution?

13 **a** Perform each operation on both sides of $x + 11 = 24$ to form an equivalent equation.

i add 9 **ii** subtract 7 **iii** multiply by 2

iv divide by 8 **v** subtract 11 **vi** add 6

vii subtract 15 **viii** add 11 **ix** subtract 10

b What is the solution to each equation?

c Which operation produced an equivalent equation that is the same as the solution?

14 **a** Perform each operation on both sides of $x - 7 = 5$ to form an equivalent equation.

i add 8 **ii** subtract 3 **iii** multiply by 6

iv divide by 5 **v** subtract 5 **vi** add 7

vii subtract 7 **viii** add 2 **ix** multiply by 2

b What is the solution to each equation?

c Which operation produced an equivalent equation that is the same as the solution?

UNDERSTANDING AND FLUENCY

15 In your answers to part **c** of questions **13** and **14**, you found the operation which gave the solution directly. How does this relate to inverse operations? Explain.

16 **a** Perform each operation on both sides of $6x = 12$ to form an equivalent equation.

i add 3	**ii** subtract 3	**iii** multiply by 3
iv divide by 3	**v** subtract 6	**vi** add 6
vii multiply by 6	**viii** divide by 6	**ix** add 12

b What is the solution to each equation?

c Which operation produced an equivalent equation that is the same as the solution?

17 **a** Perform each operation on both sides of $\frac{x}{2} = 8$ to form an equivalent equation.

i add 4	**ii** subtract 4	**iii** multiply by 4
iv divide by 4	**v** subtract 8	**vi** add 8
vii add 2	**viii** multiply by 2	**ix** divide by 2

b What is the solution to each equation?

c Which operation produced an equivalent equation that is the same as the solution?

18 Use your answers to questions **16** and **17** to explain how an inverse operation is used to produce the solution to an equation.

PROBLEM SOLVING AND REASONING

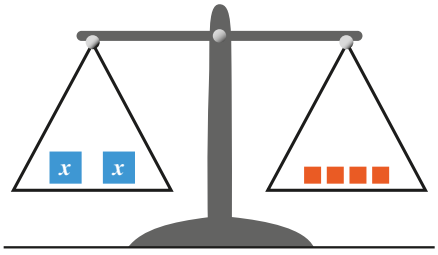
19 A vet weighs these two rabbits.



a If the rabbits are the same size, write an expression for their total mass using x for the mass of one rabbit.

b The total mass is measured to be 4 kg. Use your answer to part **a** to write an equation to represent this information.

c Explain why this diagram can be used to model the equation.



d Mia suggests that a way of obtaining x by itself on the left side of the scales is to subtract x. What would need to happen to the right side of the scales to keep them balanced?

e Does following Mia's suggestion help you to find the value of x? Explain.

f Ryan suggests that dividing by 2 is the correct operation to perform to obtain x by itself on the left side of the scales. What would need to happen to the right side of the scales to keep them balanced?

g Does following Ryan's suggestion help you to find the value of x? Explain.

h What is the mass of one rabbit?

20 Caleb decided that a quick way to find the solution to $x - 37 = 85$ is to subtract 37 from both sides of the equation. Is he correct? Explain.

21 **a** Suggest what operation should be performed on both sides of each equation to obtain the solution.

i $x + 6 = 19$ **ii** $x - 13 = 29$ **iii** $11x = 132$

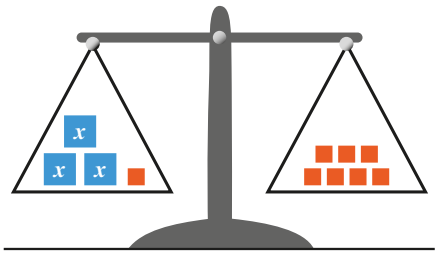
iv $\frac{x}{3} = 9$ **v** $5x = -5$ **vi** $x - 1 = -2$

vii $\frac{x}{5} = -5$ **viii** $x + 2 = -8$ **ix** $x - 7 = 0$

b Find the solution to each equation by performing the operation you have suggested.

CHALLENGE

22 This set of scales models an equation.



a What equation does it show?

b Draw a diagram to show the result of subtracting 1 from each side of the scales. What equation does it show?

c Perform the operation of dividing both sides by 3 on your equation from part **b**. Draw a diagram of what you obtain. What equation does it show?

d How do you know that the three equations you have written are equivalent?

e What is the solution?

f How does understanding inverse operations help in questions like this?

23 **a** Write an equivalent equation to $x = 3$ by performing these operations on both sides of the equation. For example, after multiplying by 2 and then adding 5, the equivalent equation is $2x + 5 = 11$.

i multiply by 4 and then add 7

ii subtract 2 and then multiply by 8

iii add 6 and then multiply by 5

iv divide by 3 and then add 11

b How can you check your answers are correct? Follow your suggestion to check your answers.

Reflect

Why must an operation be performed on both sides of an equation to produce an equivalent equation?

5F Solving equations by performing the same operation on both sides

Start thinking!

You have seen that equivalent equations have the same solution. This fact is used in developing another strategy for solving equations which is particularly useful for more complex equations.

1 Consider $x + 5 = 12$.

a Produce an equivalent equation by subtracting 5 from each side of the equation.

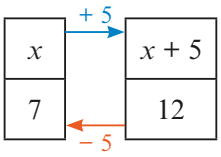
b What is special about the equivalent equation you obtained?

c What is the inverse operation to + 5?

d Copy and complete the steps shown to solve $x + 5 = 12$ using the **balance method**.
Why do you think this strategy is called the balance method?

$x + 5 = 12$
$x + 5 - 5 = 12 - ____$
$x = ____$

e Use this flowchart to explain why you subtract 5 from both sides of the equation to find the solution.



An equivalent equation is a solution to the equation when it contains the pronumeral by itself on one side.

2 Decide what operation needs to be performed on both sides of each equation to obtain x by itself on the left side.

a $x + 6 = 11$ **b** $x - 3 = 7$

c $2x = 16$ **d** $\frac{x}{9} = 4$

> **NOTE** Identify the inverse operation that would undo the operation performed on x.

3 Using the balance method to solve equations is also known as 'doing the same thing to both sides'. In your own words, explain how the balance method works.

KEY IDEAS

- To solve an equation using the balance method, perform operations on both sides of the equation to produce equivalent equations until you obtain the solution equation '$x = \ldots$'.
- An operation can be undone by performing the inverse operation. You can use a simplified flowchart to decide the order of the operations to undo.

EXERCISE 5F Solving equations by performing the same operation on both sides

EXAMPLE 5F-1 Solving an equation using the balance method (one step)

Solve each equation using the balance method.

a $x + 7 = 12$ **b** $x - 4 = 5$ **c** $3x = 18$ **d** $\frac{x}{5} = 8$

THINK

a 1 Consider how the left side was built from x. (+ 7)

2 Undo '+ 7' by performing the inverse operation of '− 7' on both sides and simplify.

b 1 Consider how the left side was built from x. (− 4)

2 Undo '− 4' by performing the inverse operation of '+ 4' on both sides and simplify.

c 1 Consider how the left side was built from x. (× 3)

2 Undo '× 3' by performing the inverse operation of '÷ 3' on both sides and simplify.

d 1 Consider how the left side was built from x. (÷ 5)

2 Undo '÷ 5' by performing the inverse operation of '× 5' on both sides and simplify.

WRITE

a $x + 7 = 12$

$x + 7 - 7 = 12 - 7$

$x = 5$

b $x - 4 = 5$

$x - 4 + 4 = 5 + 4$

$x = 9$

c $3x = 18$

$\frac{3x}{3} = \frac{18}{3}$

$x = 6$

d $\frac{x}{5} = 8$

$\frac{x}{5} \times 5 = 8 \times 5$

$x = 40$

UNDERSTANDING AND FLUENCY

1 Solve each equation using the balance method. Copy and complete the steps shown.

a $x + 6 = 11$

$x + 6 - __ = 11 - __$

$x = __$

b $x - 3 = 7$

$x - 3 + __ = 7 + __$

$x = __$

c $2x = 16$

$\frac{2x}{__} = \frac{16}{__}$

$x = __$

d $\frac{x}{9} = 4$

$\frac{x}{9} \times __ = 4 \times __$

$x = __$

NOTE Refer to your answers for question **2** of Start thinking!.

2 Solve each equation using the balance method. Show all steps of working.

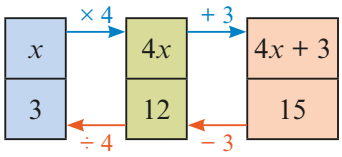
a $x + 4 = 16$ **b** $x - 11 = 3$ **c** $6x = 42$ **d** $\frac{x}{2} = 5$

e $x + 9 = 4$ **f** $x - 8 = 12$ **g** $4x = 24$ **h** $x - 6 = 3$

i $\frac{x}{11} = 2$ **j** $x + 5 = 5$ **k** $9x = -45$ **l** $x - 3 = -7$

UNDERSTANDING AND FLUENCY

3 Consider this flowchart.



a What equation is shown in:

i the orange boxes? **ii** the green boxes?

iii the blue boxes?

b Are these equations equivalent? Explain.

c Which equivalent equation is the solution to $4x + 3 = 15$?

d What operation was used to produce $4x = 12$ from $4x + 3 = 15$?

e What operation was used to produce $x = 3$ from $4x = 12$?

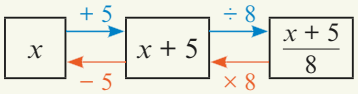
f Copy and complete the steps shown at right to solve $4x + 3 = 15$ using the balance method.

$$4x + 3 = 15$$
$$4x + 3 - __ = 15 - __$$
$$4x = __$$
$$\frac{4x}{4} = \frac{__}{4}$$
$$x = __$$

g How do you know what operation to perform on both sides of the equation? How do you know what order to use if there is more than one operation?

EXAMPLE 5F-2 Identifying the correct order of operations

a Consider this simplified flowchart.



i Which operation is performed first when backtracking?

ii Which operation is performed next to obtain x?

b Copy and complete the steps shown at right to solve $\frac{x+5}{8} = 3$.

$$\frac{x+5}{8} = 3$$
$$\frac{x+5}{8} \times __ = 3 \times __$$
$$x + 5 = __$$
$$x + 5 - __ = __ - __$$
$$x = __$$

THINK

a Look at the operations moving from right to left in the flowchart. First operation is to multiply by 8.

Next operation is to subtract 5.

b **1** Follow the incomplete working provided using the operations identified in part **a**. (× 8 then − 5)

2 First undo '÷ 8' by performing '× 8' on both sides and simplify.

3 Next undo '+ 5' by performing '− 5' on both sides and simplify.

WRITE

a **i** First operation is × 8.

ii Second operation is − 5.

b $\frac{x+5}{8} = 3$

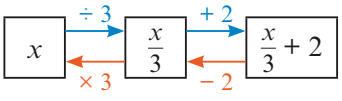
$\frac{x+5}{8} \times \mathbf{8} = 3 \times \mathbf{8}$

$x + 5 = 24$

$x + 5 \mathbf{- 5} = 24 \mathbf{- 5}$

$x = 19$

4 **a** Consider this simplified flowchart.



i Which operation is performed first when backtracking?

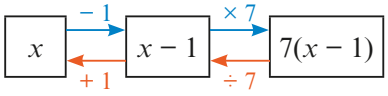
ii Which operation is performed next to obtain x?

b Copy and complete the steps shown at right to solve $\frac{x}{3} + 2 = 7$.

$$\frac{x}{3} + 2 = 7$$
$$\frac{x}{3} + 2 - __ = 7 - __$$
$$\frac{x}{3} = __$$
$$\frac{x}{3} \times __ = 5 \times __$$
$$x = __$$

UNDERSTANDING AND FLUENCY

5 a Consider this simplified flowchart.



i Which operation is performed first when backtracking?

ii Which operation is performed next to obtain x?

b Copy and complete the steps shown at right to solve $7(x - 1) = 14$.

$$7(x - 1) = 14$$
$$\frac{7(x - 1)}{7} = \frac{14}{__}$$
$$x - 1 = __$$
$$x - 1 + __ = __ + 1$$
$$x = __$$

6 These equations must have more than one operation performed on them to obtain the solution. Work with a classmate to list the required operations in the correct order.

a $5x - 2 = 8$ **b** $\frac{x}{4} + 6 = 10$

c $\frac{x - 1}{7} = 2$ **d** $4(x + 3) = 20$

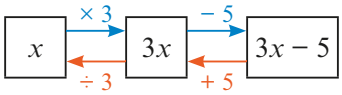
NOTE You may like to draw a flowchart to help you.

EXAMPLE 5F-3 Solving an equation using the balance method (two steps)

Solve $3x - 5 = 22$.

THINK

1 Consider how the expression on the left side was built (× 3 then − 5). If needed show this in a simplified flowchart.



2 First undo '− 5' by performing '+ 5' on both sides and simplify.

3 Undo '× 3' by performing '÷ 3' on both sides and simplify to obtain the solution.

WRITE

$$3x - 5 = 22$$
$$3x - 5 + 5 = 22 + 5$$
$$3x = 27$$
$$\frac{3x}{3} = \frac{27}{3}$$
$$x = 9$$

7 Solve each equation in question **6** using the balance method.

8 Solve each equation.

a $2x + 7 = 23$ **b** $9x - 4 = 5$ **c** $\frac{x}{6} + 3 = 7$ **d** $8(x - 2) = 32$

e $\frac{x + 2}{5} = 4$ **f** $\frac{x}{3} - 9 = 2$ **g** $\frac{x - 8}{2} = 6$ **h** $3(x + 7) = 27$

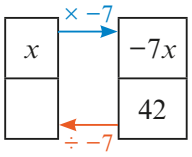
i $7x - 4 = 45$ **j** $\frac{x + 9}{5} = 3$ **k** $10x + 5 = 15$ **l** $2(x - 6) = 14$

9 What is the inverse operation to each of these?

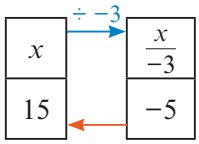
a × 5 **b** × −3 **c** ÷ 8 **d** ÷ −7 **e** × 2.5 **f** × −3.7

10 a Copy and complete each flowchart.

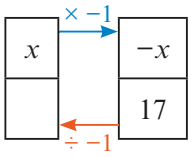
i



ii



iii



b Copy and complete the steps shown to solve each equation.

i $-7x = 42$

$\frac{-7x}{-7} = \frac{\quad}{-7}$

$x = __$

ii $\frac{x}{-3} = -5$

$\frac{x}{-3} \times -3 = -5 \times __$

$x = __$

iii $-x = 17$

$\frac{-x}{-1} = \frac{\quad}{-1}$

$x = __$

11 Solve each equation. Show all steps of working.

a $-3x = 15$ **b** $-8x = -24$ **c** $-11x = 55$ **d** $-x = 25$

e $\frac{x}{-7} = -3$ **f** $\frac{x}{-4} = 12$ **g** $\frac{x}{-2} = 30$ **h** $\frac{x}{-5} = 0$

i $-x = -9$ **j** $-5x = 0$ **k** $-2x = 0$ **l** $-x = 1$

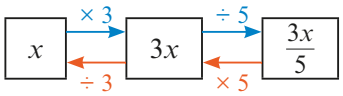
12 Solve each equation.

a $x + 4.5 = 7$ **b** $x - 3.7 = 5$ **c** $x + 1.2 = -7.8$ **d** $x - 0.6 = -5.6$

e $2.5x = 10$ **f** $\frac{x}{1.8} = 4$ **g** $1.2x = -6$ **h** $x - 6.3 = -2.1$

i $-1.4x = 2.8$ **j** $\frac{x}{0.75} = -8$ **k** $-x = -9.6$ **l** $-0.5x = -6.5$

13 a Which operations need to be performed on the expression $\frac{3x}{5}$ to obtain x?
You can use this diagram to help you.



b Copy and complete the steps shown at right to solve $\frac{3x}{5} = 6$.

$\frac{3x}{5} = 6$

$\frac{3x}{5} \times __ = 6 \times __$

$3x = __$

$\frac{3x}{3} = \frac{\quad}{3}$

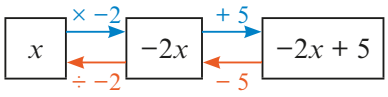
$x = __$

c Solve each equation using the balance method.

i $\frac{4x}{7} = 8$ **ii** $\frac{5x}{2} = 10$

iii $\frac{8x}{3} = -16$ **iv** $\frac{2x}{9} = 0.4$

14 Use this flowchart to complete the steps shown at right to solve $-2x + 5 = 11$.



$-2x + 5 = 11$

$-2x + 5 - __ = 11 - __$

$-2x = __$

$\frac{-2x}{-2} = \frac{\quad}{-2}$

$x = __$

15 Solve each equation.

a $-5x + 8 = 3$	**b** $-4x - 3 = 5$	**c** $11 + 2x = 19$	**d** $7 - 3x = 1$
e $-2(x - 6) = -18$	**f** $-8x + 1 = -15$	**g** $-x + 4 = 13$	**h** $6 - x = -4$
i $-x - 5 = 17$	**j** $-3(x + 8) = 21$	**k** $-(x + 3) = -2$	**l** $-(x - 7) = 1$

PROBLEM SOLVING AND REASONING

16 For each problem, set up an equation and then solve it using the balance method.

a The temperature rose by 4°C in the first hour after daybreak and then doubled by midday. What was the temperature at daybreak if the temperature at midday was 26°C?

b Hiring a party venue costs $280. There is a $15 per person charge for finger food. If Marita has a budget of $1600, how many people can attend?

c Ryan is saving to buy a sound mixing board that costs $649. He can save $56 per month. If he currently has $257, in how many months can he buy the mixing board?

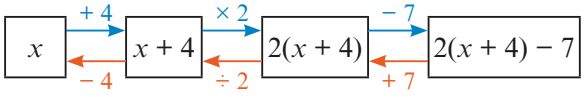


d Tess cooked some donuts. While they were still hot, her friends ate six of them. Half of those left were iced and the other half filled with jam. If there are 11 donuts filled with jam, how many donuts did Tess make?

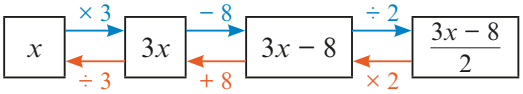
17 You can also use the balance method to solve more complex equations.

a Use each flowchart to list the order of the operations to solve the equation.

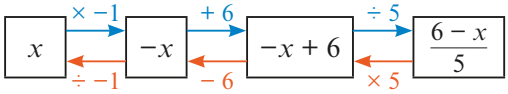
i $2(x + 4) - 7 = 11$



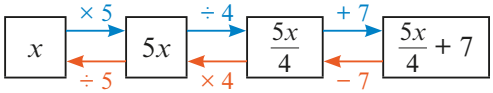
ii $\frac{3x - 8}{2} = -10$



iii $\frac{6 - x}{5} = 3$



iv $\frac{5x}{4} + 7 = 32$



b Solve each equation using the balance method.

18 Solve each equation.

CHALLENGE

a $4(x + 2) - 5 = 15$	**b** $\frac{2x - 6}{5} = 4$
c $\frac{2x}{3} + 8 = 14$	**d** $\frac{5x + 1}{6} = -4$
e $\frac{6 - x}{5} = 2$	**f** $-\frac{x}{4} + 9 = 7$
g $9(x - 4) + 3 = 57$	**h** $\frac{4x}{7} - 3 = 5$

Reflect

When solving an equation, why is it important to perform an operation on both sides of the equation?

5G Solving equations with the unknown on both sides

Start thinking!

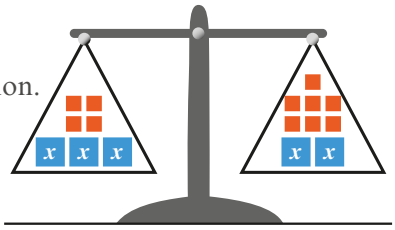
Consider the equation $3x + 4 = 2x + 7$.

1 What is different about this equation compared to most of the ones you have worked with before?

2 Solve this equation using the guess, check and improve strategy.

3 Repeat question **2** using backtracking. What problem do you face in trying to use this strategy?

4 The equation can be modelled with a set of scales like this one, where each orange block weighs 1 g.



a What would happen if you removed $2x$ from the right side of the scales?

b What should you do to the left side to make the scales balanced again?

c Draw a diagram to model the equivalent equation obtained after this operation.

d If you now remove 4 g from the left side, what should you do to the right side to make the scales balanced again? Draw a diagram to model this.

e What is the solution to the equation?

5 Copy and complete the steps shown to solve $3x + 4 = 2x + 7$ using the balance method.

$$3x + 4 = 2x + 7$$
$$3x + 4 - 2x = 2x + 7 - 2x$$
$$x + __ = __$$
$$x + __ - __ = 7 - __$$
$$x = __$$

6 What operation did you perform first on both sides of the equation to remove one of the pronumeral terms?

KEY IDEAS

- Equations can have terms containing the pronumeral or the unknown on both sides.
- The first step in solving equations with a pronumeral term on each side is to remove the pronumeral term from one side.
- A pronumeral term can be removed by performing the inverse operation (adding or subtracting the term) on both sides so that an equivalent equation is formed.
- In some cases, it may be easier to swap the sides of an equation so that the pronumeral term with the larger coefficient is on the left side.

EXERCISE 5G Solving equations with the unknown on both sides

EXAMPLE 5G-1 Solving an equation by subtracting a pronumeral term from both sides (two steps)

Solve $4x - 5 = 3x + 2$ using the balance method.

THINK

1 First, remove one pronumeral term (choose $3x$ from the RS). Undo '$+ 3x$' by performing the inverse operation of '$- 3x$' on both sides.

2 Undo '$- 5$' by performing the inverse operation of '$+ 5$' on both sides and simplify to obtain the solution.

WRITE

$$4x - 5 = 3x + 2$$
$$4x - 5 \mathbf{- 3x} = 3x + 2 \mathbf{- 3x}$$
$$x - 5 = 2$$
$$x - 5 \mathbf{+ 5} = 2 \mathbf{+ 5}$$
$$x = 7$$

1 Solve each equation using the balance method.

a $9x - 2 = 8x + 11$
b $6x + 5 = 5x + 9$
c $2x + 7 = x + 12$
d $7x - 3 = 6x - 1$
e $5x - 3 = 4x - 9$
f $4x + 5 = 3x + 4$
g $3x + 8 = 2x + 5$
h $8x - 13 = 7x + 2$
i $10x - 9 = 9x - 7$
j $2x + 1 = x - 12$
k $11x - 5 = 10x + 3$
l $20x + 3 = 19x + 3$

EXAMPLE 5G-2 Solving an equation by subtracting a pronumeral term from both sides (three steps)

Solve $5x - 1 = 2x + 11$ using the balance method.

THINK

1 Undo '$+ 2x$' by performing the inverse operation of '$- 2x$' on both sides.

2 Undo '$- 1$' by performing the inverse operation of '$+ 1$' on both sides and simplify.

3 Undo '$\times 3$' by performing the inverse operation of '$\div 3$' on both sides and simplify to obtain the solution.

WRITE

$$5x - 1 = 2x + 11$$
$$5x - 1 \mathbf{- 2x} = 2x + 11 \mathbf{- 2x}$$
$$3x - 1 = 11$$
$$3x - 1 \mathbf{+ 1} = 11 \mathbf{+ 1}$$
$$3x = 12$$
$$\frac{3x}{\mathbf{3}} = \frac{12}{\mathbf{3}}$$
$$x = 4$$

UNDERSTANDING AND FLUENCY

2 Copy and complete the steps shown at right to solve $3x + 5 = x + 11$.

$$\begin{aligned} 3x + 5 &= x + 11 \\ 3x + 5 - x &= x + 11 - __ \\ 2x + 5 &= __ \\ 2x + 5 - __ &= __ - __ \\ 2x &= __ \\ \frac{2x}{__} &= \frac{__}{2} \\ x &= __ \end{aligned}$$

3 Solve each equation using the balance method.

a $5x + 4 = 3x + 8$

b $4x - 1 = 2x + 7$

c $7x - 3 = 5x + 9$

d $13x + 6 = 11x - 2$

4 Copy and complete the steps shown at right to solve $2x - 3 = 6x + 1$. The sides of the equation have been swapped so that the larger pronumeral term is on the left side.

$$\begin{aligned} 2x - 3 &= 6x + 1 \\ 6x + 1 &= 2x - 3 \\ 6x + 1 - __ &= 2x - 3 - __ \\ 4x + __ &= -3 \\ 4x + 1 - __ &= -3 - __ \\ 4x &= __ \\ \frac{4x}{__} &= \frac{__}{4} \\ x &= __ \end{aligned}$$

5 Solve each equation.
Swap the sides if it makes it easier.

a $3x + 10 = 7x + 2$

b $2x + 5 = 5x - 4$

c $7x - 2 = 8x - 11$

d $x + 12 = 6x - 13$

EXAMPLE 5G-3 Solving an equation by adding a pronumeral term to both sides

Solve $4x + 7 = -2x - 23$ using the balance method.

THINK

1 Undo '$- 2x$' by performing the inverse operation of '$+ 2x$' on both sides.

2 Undo '$+ 7$' by performing the inverse operation of '$- 7$' on both sides and simplify.

3 Undo '$\times 6$' by performing the inverse operation of '$\div 6$' on both sides and simplify to obtain the solution.

WRITE

$$\begin{aligned} 4x + 7 &= -2x - 23 \\ 4x + 7 \mathbf{+ 2x} &= -2x - 23 \mathbf{+ 2x} \\ 6x + 7 &= -23 \\ 6x + 7 \mathbf{- 7} &= -23 \mathbf{- 7} \\ 6x &= -30 \\ \frac{6x}{\mathbf{6}} &= \frac{-30}{\mathbf{6}} \\ x &= -5 \end{aligned}$$

6 Copy and complete the steps shown at right to solve $7x - 9 = -4x + 13$.

$$\begin{aligned} 7x - 9 &= -4x + 13 \\ 7x - 9 + 4x &= -4x + 13 + __ \\ 11x - 9 &= __ \\ 11x - 9 + __ &= __ + __ \\ 11x &= __ \\ \frac{11x}{__} &= \frac{__}{11} \\ x &= __ \end{aligned}$$

7 Solve each equation.

a $3x - 11 = -2x + 9$

b $2x + 1 = -5x - 13$

c $8x + 3 = -2x - 7$

d $3x - 8 = -6x + 19$

e $5x + 6 = -3x + 22$

f $3x - 5 = -9x + 7$

g $4x + 9 = -x - 11$

h $x - 4 = -x + 6$

UNDERSTANDING AND FLUENCY

8 **a** List the first operation that needs to be performed on both sides to remove the pronumeral term on the right side.

i $5x - 21 = 13 + 3x$ **ii** $3x + 2 = 37 - 4x$ **iii** $7 + 9x = 15 + 8x$

iv $29 - 2x = -6x + 5$ **v** $4x - 25 = 30 - x$ **vi** $x + 1 = 43 - x$

b Solve each equation using the balance method.

c Check your solutions using substitution.

9 Solve each equation.

a $8x - 5 = 5x + 13$ **b** $2x - 5 = 3x - 16$ **c** $4x - 15 = -4x + 17$

d $7x + 5 = 2x + 5$ **e** $12x + 19 = 7x + 4$ **f** $x - 24 = -6x + 11$

g $5x + 8 = 3x - 22$ **h** $6x - 47 = -3x + 16$ **i** $3x - 19 = -x + 13$

10 **a** Simplify $2x + 3 + 4x + 1 = 3x + 7 + x - 5$ by adding like terms on each side.

b Solve the simplified equation.

11 Solve each equation.

a $3x + 2x - 5 = x + 7$ **b** $5x + 2 + 3x + 4 = 13 + 2x + 5$

c $6x + 7 - 3x + 3 = x + 4$ **d** $4x + 5 - 2x + 1 = 26 - 3x$

e $6x + 2 + 4x + 9 = 2x + 3 + 3x - 2$ **f** $9 - 5x + 3x - 7 = 5 - 3x + 10$

PROBLEM SOLVING AND REASONING

12 Three students produce different steps of working to solve $6x + 11 = 2x - 13$.

Andrew	Hayley	Alex
$6x + 11 = 2x - 13$	$6x + 11 = 2x - 13$	$6x + 11 = 2x - 13$
$6x + 11 - 2x = 2x - 13 - 2x$	$6x + 11 = 2x - 13 - 2x$	$6x + 11 - 2x = 2x - 13 - 2x$
$8x + 11 = -13$	$6x + 11 = -13$	$4x + 11 = 13$
$8x + 11 - 11 = -13 - 11$	$6x + 11 - 11 = -13 - 11$	$4x + 11 - 11 = 13 - 11$
$8x = -24$	$6x = -24$	$4x = 2$
$\frac{8x}{8} = \frac{-24}{8}$	$\frac{6x}{6} = \frac{-24}{6}$	$\frac{4x}{4} = \frac{2}{4}$
$x = -3$	$x = -4$	$x = \frac{1}{2}$

a Check each student's work. Has the equation been solved correctly? What mistakes have been made?

b Solve the equation correctly. Show your working.

13 Selena and Hayden have the same amount of money. Selena buys 5 kg of cherries and has \$2 left over. Hayden buys 2 kg of cherries and has \$20 left over.

a If x represents the cost of 1 kg of cherries, which equation best suits this situation?

A $5x + 20 = 2x + 2$

B $5x + 2 = 2x + 20$

C $5x - 2 = 2x - 20$

D $5x - 20 = 2x + 2$

b Solve the equation to find the cost of 1 kg of cherries.



14 The number of biscuits in a box is unknown. However, Kayla can fill the biscuit barrel with five more than the contents of three boxes or seven less than the contents of four boxes.

a If n represents the number of biscuits in a box, which equation best suits this situation?

A $3n - 5 = 4n - 7$

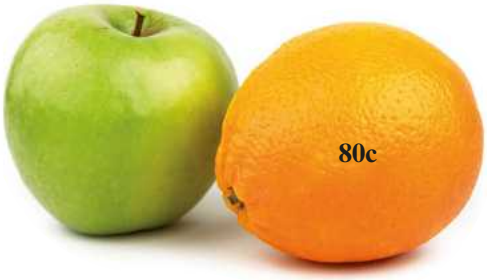
B $3n - 5 = 4n + 7$

C $3n + 5 = 4n + 7$

D $3n + 5 = 4n - 7$

b Solve the equation to find the number of biscuits in a box.

15 At the school canteen, the cost of three apples and five oranges is the same as the cost of seven apples and two oranges. Let a represent the cost of an apple in cents.



a Write an equation to represent this situation.

b Solve the equation to find the cost of an apple.

16 Sean poses this problem to his neighbour: 'In four years, I will be three times as old as my cousin is now. Two years ago, I was twice my cousin's current age. Can you work out how old I am?' Let a represent the current age of Sean's cousin.

a One expression for Sean's current age is $3a - 4$. What is another?

b Use the expressions in part **a** to write an equation.

c Solve the equation to find the current age of Sean's cousin.

d How old is Sean?

17 Jada and Tom have an equal number of marbles.
Jada has four small marbles and
Tom has ten small marbles.
Jada has three times as many large marbles as Tom. Let n represent the number of large marbles Tom has.



a Write an expression for how many marbles Tom has.

b Write an expression for how many marbles Jada has.

c Form an equation using these expressions.

d Solve the equation to find the value of n.

e How many large marbles does Tom have?

f How many large marbles does Jada have?

g If they agree to swap all their marbles, how many marbles is each person swapping?

CHALLENGE

18 An AFL football match ends in a draw. The home team scored twice as many goals as the away team. The home team also scored 3 behinds while the away team scored 27 behinds. (1 goal = 6 points, 1 behind = 1 point.) Let g represent the number of goals scored by the away team.

a Write an expression for the number of points scored by the away team.

b Write an expression for the number of points scored by the home team.

c Form an equation using these expressions.

d Solve the equation to find the value of g and hence list the results for each team.

19 In a rugby league match, the away team scored twice as many points as the home team. The away team scored two penalty goals and three times as many tries as the home team. The home team scored four penalty goals. (1 try = 4 points, 1 penalty goal = 2 points.) Let t represent the number of tries scored by the home team.



a Write an expression for the number of points scored by the home team.

b Write an expression for the number of points scored by the away team.

c Form an equation using these expressions. (Hint: remember that the away team scored twice as many points as the home team.)

d Solve the equation to find the value of t.

e List the number of tries scored by each team.

20 **a** Expand $5(x - 2) = 3(x + 8)$.

b Solve the equation.

21 Solve each equation.

a $2(x - 5) = x + 19$ **b** $7(x + 2) = 3x + 2$

c $4(x - 1) = 2 - 2x$ **d** $3(x + 4) = 2(x + 9)$

e $5(x - 7) = 3(x + 5)$ **f** $6(x + 3) - x + 5 = 2(x + 1)$

g $4(2x - 3) = 3(7 - x)$ **h** $7(3x + 5) = 5(2x + 7)$

i $11(3x + 2) = 4(2x - 7)$ **j** $3(4x - 5) = 2(1 - 3x)$

k $4x(x + 2) = 2(2x^2 + 3)$ **l** $3(x^2 - 4) = 2(x^2 - 6)$

22 Solve $\frac{4(2x - 1)}{3} + 9 = 53$ using each of these three techniques. Show all working.

a using a table

b backtracking

c using the balance method.

Reflect

How can the balance method be used to solve an equation with an unknown on both sides?

5H Plotting graphs of linear relationships



Start thinking!

Two hot air balloons lift off at the same time. Their heights over the first 10 seconds are each recorded in a table.

Red balloon

Time (s)	0	2	4	6	8	10
Height (m)	0	4	8	12	16	20

Blue balloon

Time (s)	0	2	4	6	8	10
Height (m)	0	8	14	18	19	20

1 What are the two variables shown in each table?
Which variable is shown first?

Since one variable 'depends' on the other, one is named the **dependent variable** and the other is the **independent variable**.
Generally, the independent variable is shown first in a table of values.

2 In this relationship, which is the:
- **a** independent variable?
- **b** dependent variable?

3 **a** List the coordinates of each point in the table for the red balloon.
- **b** Plot each point on a Cartesian plane.

4 Repeat question **3** for the blue balloon.

5 In each graph, you plotted six points. Could more points be plotted for each? Explain.

6 List the coordinates of three more points that could be plotted on the graph for the red balloon.

7 To show that there are many points that describe the relationship, join the points with a smooth line.
Do this for each graph and describe the trend you see.

If the line is straight, you have a **linear relationship** between the variables.
Otherwise, it is a **non-linear relationship**.

8 Which of your graphs shows a linear relationship?

KEY IDEAS

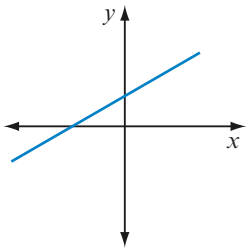
- The relationship between two variables can be graphed on the Cartesian plane with the independent variable shown along the horizontal axis and the dependent variable shown along the vertical axis.
- Creating a table of values helps you work out the coordinates of points to be plotted.
- If the plotted points form a straight line, it shows a linear relationship.
- A **linear graph** shows a linear relationship between two variables.
- The ***x*-intercept** is where the graph crosses the x-axis and the ***y*-intercept** is where the graph crosses the y-axis.

EXERCISE 5H Plotting graphs of linear relationships

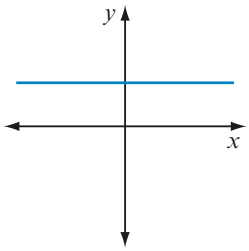
UNDERSTANDING AND FLUENCY

1 Which of these graphs show a linear relationship?

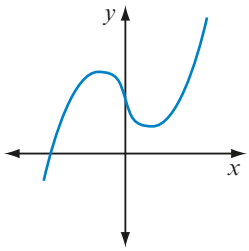
a



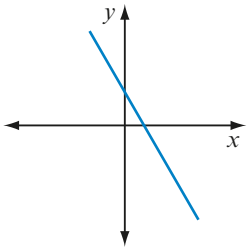
b



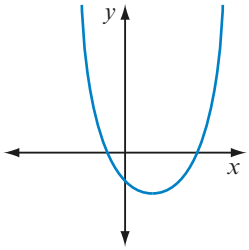
c



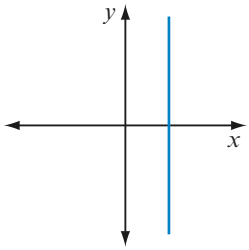
d



e



f



2 Write a sentence to explain what a linear relationship is.

EXAMPLE 5H-1 Using a table of values to plot a graph

Use the table of values to plot the graph of the relationship between x and y.

x	−3	−2	−1	0	1	2	3
y	−6	−4	−2	0	2	4	6

THINK

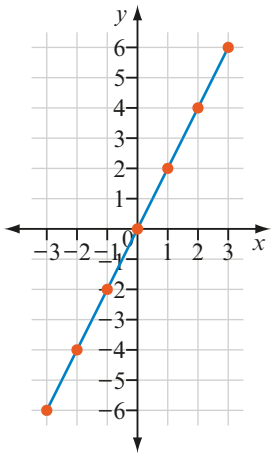
1 Identify the coordinates of the points to be plotted.

2 Use grid paper to draw a Cartesian plane with a scale from −3 to 3 on the horizontal axis and from at least −6 to 6 on the vertical axis. Plot each point.

3 Since there are many points that describe this relationship, join the points with a smooth line.

WRITE

(−3, −6), (−2, −4), (−1, −2), (0, 0), (1, 2), (2, 4), (3, 6)



UNDERSTANDING AND FLUENCY

3 a Use the table of values to plot the graph of each relationship between x and y.

i

x	−3	−2	−1	0	1	2	3
y	−1	0	1	2	3	4	5

ii

x	−3	−2	−1	0	1	2	3
y	−5	0	3	4	3	0	−5

iii

x	−3	−2	−1	0	1	2	3
y	−27	−8	−1	0	1	8	27

iv

x	−3	−2	−1	0	1	2	3
y	4	3	2	1	0	−1	−2

b Which of these graphs show a linear relationship?

EXAMPLE 5H-2 Completing a table of values

Copy and complete this table of values for $y = 2x - 1$.

x	−2	−1	0	1	2
y			−1	1	

THINK

1 Use the equation to work out the missing values. Substitute $x = -2$ into $y = 2x - 1$ to calculate y.

2 Substitute each of the remaining x values into the equation to find the corresponding y value.

3 Copy and complete the table.

WRITE

When $x = -2$,
$y = 2x - 1$
$= 2 \times -2 - 1$
$= -4 - 1$
$= -5$

When $x = -1$,
$y = 2 \times -1 - 1$
$= -2 - 1$
$= -3$

When $x = 2$,
$y = 2 \times 2 - 1$
$= 4 - 1$
$= 3$

x	−2	−1	0	1	2
y	−5	−3	−1	1	3

4 For each linear relationship described by the given equation:
i copy and complete the table **ii** plot the graph.

a $y = x + 1$

x	−2	−1	0	1	2
y	−1		1		3

b $y = 2x + 2$

x	−3	−2	−1	0	1
y		−2			4

c $y = 3 - x$

x	−1	0	1	2	3
y	4			1	

d $y = 3x - 4$

x	−1	0	1	2	3
y			−1		

EXAMPLE 5H-3 Plotting the graph of a linear relationship

Plot the graph of $y = 2x + 1$ by first completing a table for x values from -3 to 3.

THINK

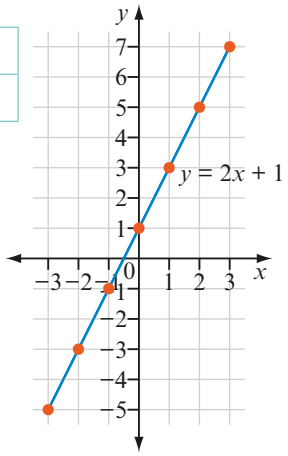
1 To create a table of values, substitute each x value into the equation to find the corresponding y value.
(For $x = -3$, $y = 2 \times -3 + 1 = -5$.)

2 Use grid paper to draw a Cartesian plane with a scale from -3 to 3 on the horizontal axis and from at least -5 to 7 on the vertical axis. Plot each point.

3 Join the points with a smooth line.

WRITE

x	-3	-2	-1	0	1	2	3
y	-5	-3	-1	1	3	5	7

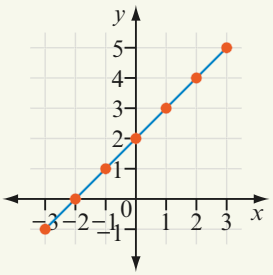


5 Plot the graph of each linear relationship by first completing a table for x values from -3 to 3.

a $y = x + 4$ **b** $y = x - 3$ **c** $y = -x$ **d** $y = 2 - x$
e $y = 5 - x$ **f** $y = -x - 1$ **g** $y = 3x$ **h** $y = 2x + 3$
i $y = 4x - 2$ **j** $y = 3x + 1$ **k** $y = 6 - 3x$ **l** $y = 4 - 2x$

EXAMPLE 5H-4 Identifying the x- and y-intercepts from a graph

List the x-intercept and the y-intercept for this graph.



THINK

1 Identify where the line crosses the x-axis. You can write this in two different ways.

2 Identify where the line crosses the y-axis.

WRITE

The x-intercept is -2.
The coordinates of the x-intercept are $(-2, 0)$.

The y-intercept is 2.
The coordinates of the y-intercept are $(0, 2)$.

UNDERSTANDING AND FLUENCY

6 List the x-intercept and the y-intercept for each graph you have drawn for question **5**.

7 Produce each graph in question **5** using graphing software such as Microsoft Excel or a graphics calculator.

8 **a** Plot the graph of each relationship using the table of values.

i $y = 2$

x	−2	−1	0	1	2
y	2	2	2	2	2

ii $y = -1$

x	−2	−1	0	1	2
y	−1	−1	−1	−1	−1

iii $y = 8$

x	−2	−1	0	1	2
y	8	8	8	8	8

b Are these relationships linear? Explain.

c What do you notice about the equation and its matching graph?

9 **a** Plot the graph of each relationship using the table of values.

i $x = 3$

x	3	3	3	3	3
y	−2	−1	0	1	2

ii $x = 0$

x	0	0	0	0	0
y	−2	−1	0	1	2

iii $x = -6$

x	−6	−6	−6	−6	−6
y	−2	−1	0	1	2

b Are these relationships linear? Explain.

c What do you notice about the equation and its matching graph?

10 Without using a table, draw the graph of each relationship.

a $x = 2$ **b** $y = 4$ **c** $y = -3$

d $x = -1$ **e** $x = 5$ **f** $y = 0$

NOTE use your answers to questions **8** and **9** to guide you.

PROBLEM SOLVING AND REASONING

11 Look at 5H Start thinking! (page 270) again. Use the red balloon graph for parts **a** and **b**.

a Find the height after: **i** 5 seconds **ii** 9 seconds.

b Find the time to reach a height of: **i** 2 m **ii** 7 m.

Use your graph for the blue balloon for parts **c** and **d**.

c Find the height after: **i** 1 second **ii** 7 seconds.

d Find the time to reach a height of: **i** 11 m **ii** 16 m.

e If you use t to represent time and h to represent height, write an equation (or rule) to represent the relationship for the red balloon. (Hint: the equation starts $h = \ldots$.)

12 This table shows the distance Fabio covers in each hour of a road trip in his sports car.

time (h)	0	1	2	3	4	5
distance (km)	0	80	160	240	320	400

PROBLEM SOLVING AND REASONING



a Which is the independent variable?
b Draw a graph of this relationship.
c Is the relationship linear? Explain.
d Use the linear graph to find how far he has travelled in $2\frac{1}{2}$ hours.
e Use the graph to find how long it has taken him to travel 260 km.
f If he travelled for another 2 hours, what total distance would he cover? What assumption do you need to make?
g If t represents time and d represents distance, write an equation using t and d to describe the relationship.
h Use your equation to check your answer to part **f**.

13 The water consumption for a household can be described by the linear equation $w = 900d$, where w represents the amount of water (in L) used for the month and d represents the number of days from the start of the month.
a Plot a graph of this relationship by first creating a table.
b Use your graph to find the amount of water used in:
i the first week **ii** the first three weeks **iii** the first 10 days.
c Use your graph to find when the household has used:
i 4500 L **ii** 12 600 L **iii** 20 000 L.
d Produce the graph of this relationship using technology.

14 Emily is making ribbon streamers for her cheer squad but needs to work to a budget of $40. The length of ribbon she can buy depends on the cost of the ribbon.

cost ($ per m)	0.50	0.80	1.60	2.00	3.60	5.00
length (m)						

a Copy and complete this table.
b Draw a graph of this relationship. **c** Is the relationship linear? Explain.
d Use your graph to find how much ribbon she can buy if the cost of the ribbon is:
i $2.50 per m **ii** $0.75 per m **iii** $4.20 per m.
e Emily needs at least 32 m of ribbon. What price range should she consider?



CHALLENGE

15 Previously you noted whether a graph shows a positive or negative relationship. You can also look at the slope or gradient of a linear graph in the same way. For example, the linear graph drawn in Example 5H-3 on page 273 has a positive gradient. Copy and complete these statements using the words *zero*, *positive* or *negative*.
a A line has a _____ gradient if it slopes upwards from left to right.
b A line has a _____ gradient if it slopes downwards from left to right.
c A line has a _____ gradient if it is horizontal.

16 Decide whether the gradient is positive (P) or negative (N) for each line you drew in:
i question **4** **ii** question **5**.

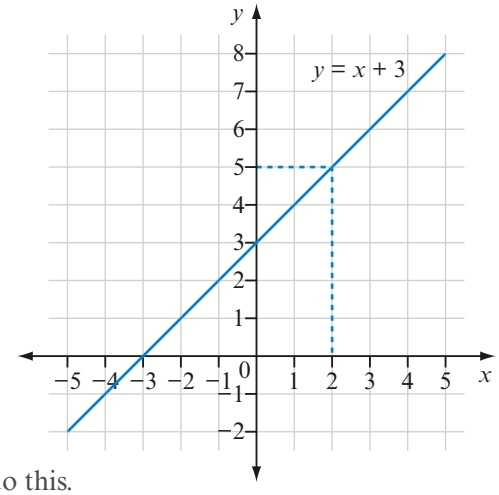
17 What do you think the gradient, x-intercept and y-intercept are for each line in question **10**? Discuss with a classmate.

Reflect

What features does the graph of a linear relationship have?

5I Solving linear equations using graphs

Start thinking!



The graph of $y = x + 3$ is shown at right. You can use this graph to read the corresponding value of y for a value of x and vice versa. For example, the value of y when $x = 2$ is 5.

1 Use the graph to write the y value for each x value.
a $x = 1$ **b** $x = 5$ **c** $x = -4$ **d** $x = 0$

2 Use the graph to write the x value for each y value.
a $y = 4$ **b** $y = 8$ **c** $y = 1$ **d** $y = -2$

3 **a** From the graph, what is the value of x when y is 5?
b Another way to look at this is to replace y with $x + 3$. That is, $y = 5$ becomes $x + 3 = 5$. Explain why you can do this.
c Solve the linear equation $x + 3 = 5$ by inspection.
d Compare your answers to parts **a** and **c**.

4 Replace y with $x + 3$ to write each part of question **2** as a linear equation.

5 Solve each equation in question **4** by inspection. Compare your answers to those obtained in question **2**. Do you obtain the same values for x?

You can find the solution to an equation by reading values from a graph. This strategy is called 'solving equations graphically'.

6 Use the graph to solve each linear equation. (Hint: what is the y value in each case?)
a $x + 3 = 6$ **b** $x + 3 = 2$ **c** $x + 3 = -1$ **d** $x + 3 = 3$

7 Explain how you can solve linear equations using graphs.

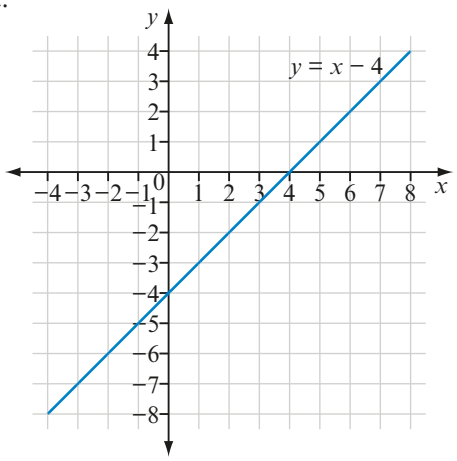
KEY IDEAS

- You can find the solution to an equation by reading values from a graph. This strategy is called 'solving equations graphically'.
- Since you are looking for the x value that makes a linear equation true, you read the x value for a given y value that forms a point on the matching linear graph. For example, to solve $5x - 7 = 3$, you read the x value that matches $y = 3$ on the graph of $y = 5x - 7$.
- Solving equations graphically is a useful strategy when solving a large number of similar equations.

EXERCISE 5I Solving linear equations using graphs

UNDERSTANDING AND FLUENCY

1 Consider the graph of $y = x - 4$ shown at right.



a Use the graph to write the x value for each y value.

i $y = 2$ **ii** $y = -3$

iii $y = -6$ **iv** $y = -2$

b Use your answers to part **a** to solve each equation.

i $x - 4 = 2$ **ii** $x - 4 = -3$

iii $x - 4 = -6$ **iv** $x - 4 = -2$

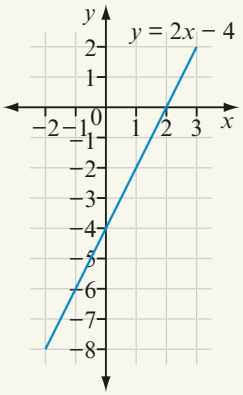
c Use the graph to solve each equation.

i $x - 4 = 3$ **ii** $x - 4 = -1$

iii $x - 4 = 4$ **iv** $x - 4 = 1$

v $x - 4 = -4$ **vi** $x - 4 = -8$

EXAMPLE 5I-1 Using a graph to solve equations



Use the graph of $y = 2x - 4$ to solve each linear equation.

a $2x - 4 = 2$

b $2x - 4 = -8$

c $2x - 4 = 0$

d $2x - 4 = 1$

THINK

a To solve $2x - 4 = 2$, locate 2 on the y-axis and move horizontally until you meet the line for $y = 2x - 4$. Move vertically from this point on the line to read the matching x value on the x-axis (when $y = 2$, $x = 3$).

b Use the graph to find the x value when y is -8.

c Use the graph to find the x value when y is 0.

d Use the graph to find the x value when y is 1.

WRITE

a $2x - 4 = 2$
$x = 3$

b $2x - 4 = -8$
$x = -2$

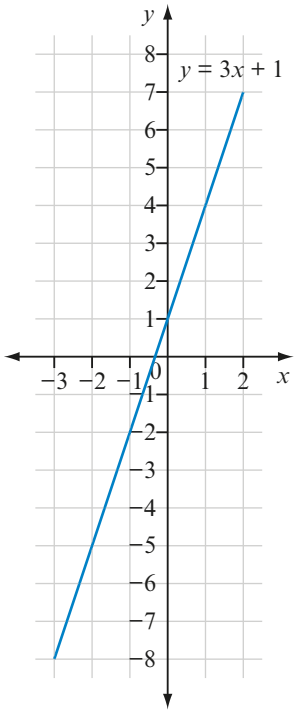
c $2x - 4 = 0$
$x = 2$

d $2x - 4 = 1$
$x = 2.5$ or $2\frac{1}{2}$

UNDERSTANDING AND FLUENCY

2 Use the graph of $y = 3x + 1$ shown at right to solve each linear equation.

- **a** $3x + 1 = 7$
- **b** $3x + 1 = -2$
- **c** $3x + 1 = 4$
- **d** $3x + 1 = 1$
- **e** $3x + 1 = -5$
- **f** $3x + 1 = -8$



EXAMPLE 5I-2 Drawing a graph to solve equations

a Plot the graph of $y = 2x - 1$.
b Use the graph to solve each equation.
i $2x - 1 = 3$ **ii** $2x - 1 = 9$ **iii** $2x - 1 = -7$

THINK

a 1 Create a table of values.

2 Plot the points from the table and rule a straight line through them.

b i Use the graph to find the x value when y is 3.

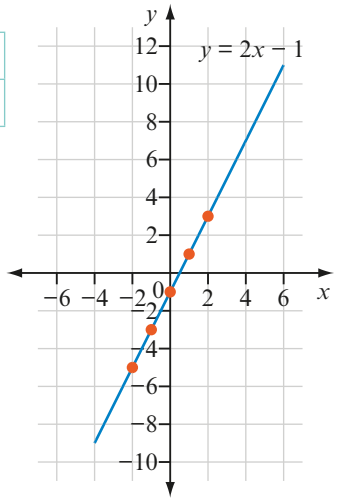
ii Use the graph to find the x value when y is 9.

iii Use the graph to find the x value when y is −7.

WRITE

a

x	−2	−1	0	1	2
y	−5	−3	−1	1	3



b i $2x - 1 = 3$
$x = 2$

ii $2x - 1 = 9$
$x = 5$

iii $2x - 1 = -7$
$x = -3$

3 a Plot the graph of $y = x + 2$.
b Use the graph to solve each equation.
i $x + 2 = 3$ **ii** $x + 2 = -1$ **iii** $x + 2 = 4$
iv $x + 2 = 1$ **v** $x + 2 = -4$ **vi** $x + 2 = -8$

4 **a** Plot the graph of $y = 5 - x$.

b Use the graph to solve each equation.

i $5 - x = 2$ **ii** $5 - x = 7$ **iii** $5 - x = 4$

iv $5 - x = 8$ **v** $5 - x = 6$ **vi** $5 - x = 5$

5 **a** Plot the graph of $y = 2x + 3$.

b Use the graph to solve each equation. Some solutions will not be whole numbers.

i $2x + 3 = 1$ **ii** $2x + 3 = -3$ **iii** $2x + 3 = 6$

iv $2x + 3 = 3$ **v** $2x + 3 = -2$ **vi** $2x + 3 = 8$

EXAMPLE 5I-3 Identifying the graph to draw to solve an equation

Identify the graph to draw to solve each equation graphically.

a $x - 7 = -4$ **b** $5x - 3 = 12$ **c** $6 - 3x = -9$

THINK	WRITE
a From the graph, the solution is the x value when $y = -4$ so graph of $y = x - 7$ is needed.	**a** $y = x - 7$
b From the graph, the solution is the x value when $y = 12$ so graph of $y = 5x - 3$ is needed.	**b** $y = 5x - 3$
c From the graph, the solution is the x value when $y = -9$ so graph of $y = 6 - 3x$ is needed.	**c** $y = 6 - 3x$

UNDERSTANDING AND FLUENCY

6 Identify the graph to draw to solve each equation graphically.

a $x - 3 = 5$ **b** $2x + 5 = 13$ **c** $6x - 1 = -25$

d $7 - x = 4$ **e** $3x - 4 = -11$ **f** $2 - 4x = 0$

7 Consider these linear equations.

i $3x - 2 = 7$ **ii** $3x - 2 = -5$ **iii** $3x - 2 = -20$

a To solve these equations graphically, what linear graph needs to be drawn?

b List the y value that needs to be used to solve each equation.

c Draw the graph and hence find the solution to each equation.

8 Consider these linear equations.

i $4x - 5 = 3$ **ii** $4x - 5 = -13$ **iii** $4x - 5 = -7$

a To solve these equations graphically, what linear graph needs to be drawn?

b List the y value that needs to be used to solve each equation.

c Draw the graph and hence find the solution to each equation.

9 Solve each equation graphically.

a $1 - 2x = 3$ **b** $1 - 2x = -5$ **c** $1 - 2x = 7$

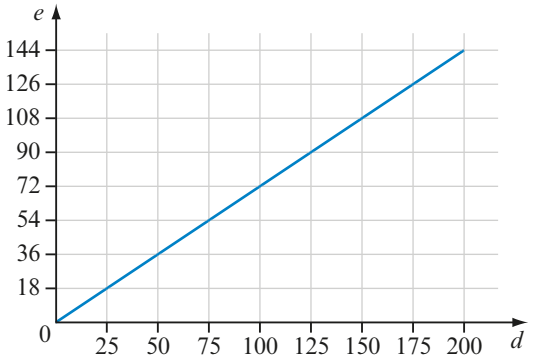
d $1 - 2x = -9$ **e** $1 - 2x = 9$ **f** $1 - 2x = -4$

10 Solve each equation graphically.

a $8x - 3 = 5$ **b** $8x - 3 = 13$ **c** $8x - 3 = 21$

d $8x - 3 = -3$ **e** $8x - 3 = -11$ **f** $8x - 3 = -7$

11 Thomas has been comparing prices of goods online in Italy with those in Australia. To save time, he decides to draw a graph to convert euros to Australian dollars for any items he sees. He looks up the exchange rate and finds that it is A$1 = 0.72 euros.



a If e represents the number of euros and d represents the number of Australian dollars, which equation did he use to produce the graph?

A $e = d + 0.72$ **B** $e = d - 0.72$ **C** $e = 0.72d$

b Use the graph to solve each equation.

i $0.72d = 36$ **ii** $0.72d = 108$ **iii** $0.72d = 144$

c A designer belt costs 50 euros.

i Write an equation so you can work out the equivalent cost in Australian dollars.

ii Use the graph to solve the equation. What is the cost of the belt in Australian dollars (to the nearest dollar)?



d Thomas now lists these items for comparison.

shirt	72 euros
shoes	100 euros
jeans	40 euros
leather jacket	140 euros
mp3 player	29 euros

i Write an equation for each item so you can work out the equivalent cost in Australian dollars.

ii Use the graph to solve each equation to find the value of d. Where necessary, estimate to the nearest dollar.

iii Write the cost of each item in Australian dollars.

12 In most countries temperature is measured in degrees Celsius. Some countries, like the USA, use degrees Fahrenheit. In the formula $F = \frac{9}{5}C + 32$, F represents the temperature in degrees Fahrenheit and C represents the temperature in degrees Celsius. This formula can be used to find F when you know C.

a Draw the graph of this linear relationship by plotting a few points and ruling a straight line through them. Make your graph large enough so that you can easily read values from it. You may like to copy and complete this table first.

C	−25	0	25	50	75
F					

PROBLEM SOLVING AND REASONING

b Use the graph to solve each equation to find the value of C.

i $\frac{9}{5}C + 32 = 40$ **ii** $\frac{9}{5}C + 32 = 104$ **iii** $\frac{9}{5}C + 32 = 14$



Olivia watches an American cooking show on TV and the host gives some tips on how to store food bought from the supermarket. 'Meat to be frozen should be stored in the freezer at −5°F, dairy products and vegetables stored in the fridge at about 37°F and tomatoes kept at room temperature, ideally around 68°F.'

c **i** Write an equation for each of the three examples so you can work out the equivalent temperature in degrees Celsius.

ii Use the graph to solve each equation.

iii Rewrite the three tips for an Australian audience.

d Write an equation and then solve it graphically to find the equivalent temperature in degrees Celsius if the host recommends keeping hot, cooked food above 140°F.

13 Can you solve $4x - 3 = 2x + 1$ graphically? Follow these steps.

a Plot the graph of $y = 4x - 3$ for x values from −1 to 3.

b Plot the graph of $y = 2x + 1$ on the same Cartesian plane.

c For $4x - 3$ to equal $2x + 1$, you need to find the x value where the y value for each is the same. Mark the point where the two lines have the same x and y values. Describe what is special about this point on the graph.

d From the graph, write the solution to $4x - 3 = 2x + 1$. (Hint: what is the x-coordinate of the point you marked on the graph.)

e Check your solution by substituting the value for x into the equation.

CHALLENGE

14 To graphically solve each linear equation shown below:

i write the equations of the linear graphs to be drawn

ii draw the graphs on the same Cartesian plane and hence find the solution to the equation

iii use substitution to check your solution.

a $x - 2 = 7 - 2x$ **b** $2x + 4 = -3x - 6$

15 The number of lamingtons in a packet is unknown. However, Adam can fill a plastic container with two more than the contents of one packet or four less than the contents of two packets.



a If x represents the number of lamingtons in a packet, use the information to write a linear equation.

b What graphs need to be drawn so the equation can be solved graphically?

c Solve the equation graphically to find the value of x.

d How many lamingtons are in a packet?

16 Solve each equation graphically.

a $3x - 4 = x - 1$ **b** $2x - 5 = -x - 5$

Reflect

When is the strategy of solving equations graphically most useful?

CHAPTER REVIEW

SUMMARISE

Create a summary of this chapter using the key terms below. You may like to write a paragraph, create a concept map or use technology to present your work.

pronumeral
expression
equation
substitution
solve
solution
solving by inspection
guess, check and improve
linear equation
quadratic equation
flowchart
order of operations
backtracking
inverse operations
variable
equivalent equations
balance method
dependent variable
independent variable
linear relationship
non-linear relationship
gradient
x-intercept
y-intercept
solving equations graphically

MULTIPLE-CHOICE

5A **1** Which is the solution to $x + 5 = 12$?

A $x = 17$ **B** $x = 12$
C $x = 7$ **D** $x = 5$

5A **2** Which is the solution to $y - 5 = -7$?

A $y = -2$ **B** $y = 2$
C $y = -12$ **D** $y = 12$

5B **3** Using this table, what is the solution to $3x - 2 = x + 6$?

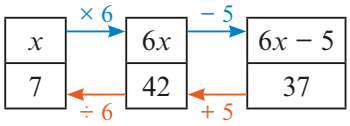
Guess x	LS $3x - 2$	RS $x + 6$
2	4	8
6	16	12
3	7	9
4	10	10

A $x = 2$ **B** $x = 3$
C $x = 4$ **D** $x = 6$

5C **4** What are the operations to build $\frac{x + 3}{7}$?

A add 3 to x and then divide by 7
B subtract 3 from x and then divide by 7
C divide x by 7 and then add 3
D multiply x by 7 and then subtract 3

5D **5** For the equation in the flowchart, the solution is:



A $6x - 5 = 37$ **B** $x = 37$
C $x = 42$ **D** $x = 7$

5E **6** Which is the equivalent equation when 5 is subtracted from both sides of $2x - 6 = 14$?

A $2x - 1 = 9$ **B** $2x - 6 = 9$
C $2x - 11 = 9$ **D** $x = 10$

5G **7** What is the solution to $3x + 7 = 2x + 9$?

A $x = 16$ **B** $x = 2$
C $x = \frac{16}{5}$ **D** $x = -2$

5H **8** Which equation matches the values in the table?

x	−3	−2	−1	0	1	2	3
y	−7	−5	−3	−1	1	3	5

A $y = x - 1$ **B** $y = 2x - 1$
C $y = 2x + 1$ **D** $y = x - 4$

SHORT ANSWER

5A **1** Substitute -3 for x in the expression on the left side of each equation. Does it make each equation a true statement?

a $x + 11 = 8$ **b** $5x = -15$

c $\frac{27}{x} = 9$ **d** $4x + 12 = 0$

5A **2** Solve each equation by inspection.

a $a + 7 = 16$ **b** $b - 6 = 8$

c $\frac{12}{c} = 4$ **d** $4d + 8 = 32$

5A **3** For each statement below:

i write an equation using x for the unknown number

ii solve the equation

iii check that the solution is correct.

a A number is multiplied by 4, then 2 is subtracted to give an answer of 14.

b A number has 5 added to it, then the result is divided by 5 to give an answer of 6.

5B **4** Solve each equation using the guess, check and improve strategy.

a $11a + 12 = 34$ **b** $9b - 14 = 40$

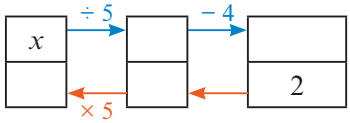
c $\frac{c + 17}{7} = 9$ **d** $5d + 4 = 37$

5C **5** Draw a flowchart to build each expression.

a $5x - 2$ **b** $\frac{x}{3} + 8$

c $7(x + 9)$ **d** $\frac{4x + 5}{2}$

5D **6** **a** Copy and complete this flowchart to show all operations and missing values.



b Write the equation represented by the flowchart and the solution.

5D **7** Solve each equation using backtracking.

a $\frac{a + 5}{6} = 4$ **b** $3(b - 7) = 27$

c $\frac{4c}{7} = 8$ **d** $4(3d + 5) = 44$

5D **8** Solve each equation using backtracking.

a $5x + 4 = -6$ **b** $3(x + 7) = 6$

c $\frac{4x}{5} + 12 = 13$ **d** $\frac{8x + 6}{7} = 10$

5E **9** Perform each operation on both sides of $x - 4 = 8$ to form an equivalent equation.

a add 6 **b** subtract 3

c multiply by 7 **d** divide by 2

5E **10** Perform each operation on both sides of $4x = 20$ to form an equivalent equation.

a add 3 **b** subtract 7

c multiply by 6 **d** divide by 4

5F **11** Solve each equation using the balance method.

a $6x + 2 = 26$ **b** $5(x - 12) = 35$

c $\frac{x + 8}{4} = 3$ **d** $-3x + 6 = 15$

5G **12** Solve each equation using the balance method.

a $5x + 6 = 3x - 4$ **b** $8x + 5 = 7x + 7$

c $4x - 3 = 7x + 15$ **d** $5x + 4 = -x - 2$

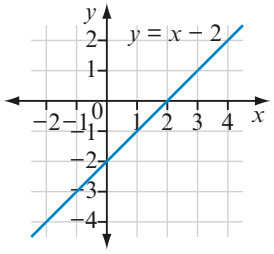
5G **13** With a certain amount of money, you can buy eight fruit bars and get \$4 change or two fruit bars and get \$16 change. Solve an equation to find the cost of one fruit bar.

5H **14** Plot the graph of each linear relationship. Complete a table for x values from -3 to 3.

a $y = x - 4$ **b** $y = 4x + 1$

c $y = 3 - x$ **d** $y = 6 - 3x$

Questions **15** and **16** refer to this graph of $y = x - 2$.



5I **15** Find the value of x for each y value.

a $y = 2$ **b** $y = -3$

5I **16** Use the graph to solve each equation.

a $x - 2 = 2$ **b** $x - 2 = -1$

c $x - 2 = 1$ **d** $x - 2 = -2$

NAPLAN-STYLE PRACTICE

1 Which is the solution to $\frac{x}{5} = 5$?

- ☐ $x = 25$
- ☐ $x = 10$
- ☐ $x = 1$
- ☐ $x = 0$

2 What is the solution to $3x - 2 = 13$?

3 Using n for the unknown number, which statement matches the equation $2n - 5 = 25$?

- ☐ 5 is subtracted from a number and the result is doubled to give an answer of 25.
- ☐ A number is doubled then subtracted from 5 to give an answer of 25.
- ☐ A number is doubled then 5 is subtracted to give an answer of 25.
- ☐ 5 is subtracted from a number to give an answer of 25.

4 Which equation could represent the following statement?
A number is added to 5, then the result is divided by 4 to give an answer of 1.

- ☐ $(n + 5) - 4 = 1$
- ☐ $4(n + 5) = 1$
- ☐ $\frac{n + 4}{5} = 1$
- ☐ $\frac{n + 5}{4} = 1$

5 The table below displays the working using the 'guess, check and improve' strategy to solve $3x + 5 = 44$.

Guess x	LS $3x + 5$	RS 44	Check Is LS = RS?
10	35	44	no
15	50	44	no
12	41	44	no
13	44	44	yes

Which of the following statements is *not* correct?

- ☐ The value of x for the solution is between 10 and 15.
- ☐ The value of x for the solution must be greater than 15.
- ☐ The solution is $x = 13$.
- ☐ $x = 12$ is not the solution to the equation.

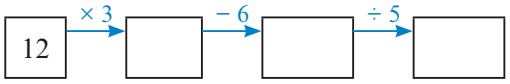
Questions **6** and **7** relate to this information.
Entry to a fun park costs \$15 plus \$3 for every ride. Mikayla takes \$75 to the park.

6 If r represents the number of rides, which equation matches this information?

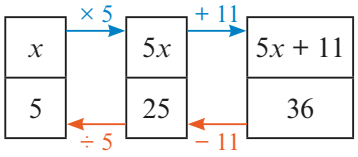
- ☐ $r + 15 = 75$
- ☐ $15r + 3 = 75$
- ☐ $3r + 15 = 75$
- ☐ $75 + 3r = 15$

7 How many rides can Mikayla have?

8 What number will appear in the final box of this flowchart?

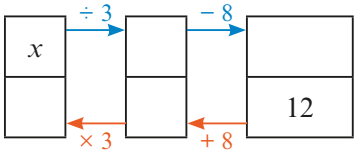


9 This flowchart shows $5x + 11 = 36$.



What is the solution to the equation?

Questions **10** and **11** relate to this flowchart.



10 Which equation does it represent?

- ☐ $\frac{x - 8}{12} = 12$
- ☐ $3x + 8 = 12$
- ☐ $\frac{x}{3} - 8 = 12$
- ☐ $3(x + 8) = 12$

11 Which of these is the solution?

- ☐ $x = 4$
- ☐ $x = 12$
- ☐ $x = 20$
- ☐ $x = 60$

12 Which equation is equivalent to $2x + 3 = 15$?

- ☐ $x = 9$
- ☐ $x = 10$
- ☐ $2x = 12$
- ☐ $2x = 18$

13 Which of these is the equivalent equation formed when 12 is subtracted from both sides of $4x = 32$?

- ☐ $4x - 12 = 20$
- ☐ $-8x = 20$
- ☐ $4x - 12 = 32$
- ☐ $12 - 4x = 20$

14 Which equation is *not* equivalent to $x + 4 = 15$?

- ☐ $x + 9 = 20$
- ☐ $x = 11$
- ☐ $2x + 8 = 30$
- ☐ $x + 8 = 11$

15 What is the solution to $12 - 4x = 24$?

- ☐ $x = 9$
- ☐ $x = 3$
- ☐ $x = -3$
- ☐ $x = -9$

16 Which operation should be performed first on both sides of $5x + 6 = 3x - 10$ to remove one of the pronumeral terms?

$+ 3x$	$- 3x$	$+ 10$	$+ 5x$
☐	☐	☐	☐

17 What is the solution to $6x - 8 = 4x - 12$?

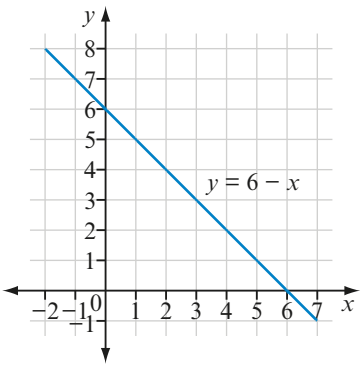
☐

18 Which linear equation matches the values in the table?

x	−2	−1	0	1	2
y	−4	−1	2	5	8

- ☐ $y = x + 2$
- ☐ $y = 3x + 2$
- ☐ $y = 2x + 4$
- ☐ $y = x - 2$

Questions **19** and **20** refer to the graph of $y = 6 - x$ shown at right.



19 What is the x value when y is 7?

☐

20 Using the graph, what is the solution to $6 - x = 4$?

- ☐ $x = 2$
- ☐ $x = 3$
- ☐ $x = 4$
- ☐ $x = 6$

ANALYSIS

A party venue costs $150 to hire, plus $20 per person for games and food.

a If x people attend the party, write an expression for the cost.

b Kacie's mother gives her $450 for venue hire, games and food. Use part **a** to write an equation.

c Solve the equation in part **b** in two different ways to find x. What does this value represent?

d Kacie would like to invite 24 friends. Calculate the total cost.

e Will Kacie be able to afford this many guests at her party? Justify your answer.

Kacie hears of another party venue that offers a similar service. Their costs include a hire fee of $200 plus $10 per person for games and food.

f Write a new expression for the cost of x people attending the party.

g Form an equation and work out how many people Kacie can have at the new venue.

h Can Kacie afford all her guests at the new venue?

i Using the cost expressions for the two venues, how many people could attend the party for the same cost at either venue? (Hint: form an equation from the two expressions and solve for x.)

j Use part **i** to work out the hire cost.



CONNECT

A day at a theme park

Natsuko and her family in Japan are planning a day at a theme park. She is allowed to invite some friends to join them as long as the cost fits within budget. She discusses this with her online friend, Max, who lives in Australia. His family are also planning a day at a theme park in Queensland.

The ticket budget for Natsuko's group is 44 200 yen while Max's group has a ticket budget of $560. Which group has the larger ticket budget?

Although the price of each ticket allows a person unlimited access to all attractions and rides in the theme park, there are other costs. How do the food and drink prices compare?

Natsuko and Max share what they know about average food and drink prices in their respective countries.

Item	Japan	Australia
hamburger combo meal	585 yen	$7.30
375 mL Coke or Pepsi	130 yen	$3.20
375 mL water	107 yen	$2.80
cappuccino	386 yen	$3.90





Your task

For your investigation of the costs of theme parks, carry out these steps:

- work out the number of people who can attend within each budget
- compare the ticket prices in each country by using the current exchange rate
- compare the cost of food and drink purchases
- decide which theme park offers the better value.

Include all necessary working to justify your answers.





One-day ticket prices
Adult $80
Child $50





One-day ticket prices
Adult 6200 yen
Child 5300 yen

Complete the 5 CONNECT worksheet to show all your working and answers to this task.
You may like to present your findings as a report. Your report could be in the form of:

- a poster
- a PowerPoint presentation
- a technology demonstration
- other (check with your teacher).

6 ANGLES AND LOCATION

6A Understanding angles

6B Angles and lines

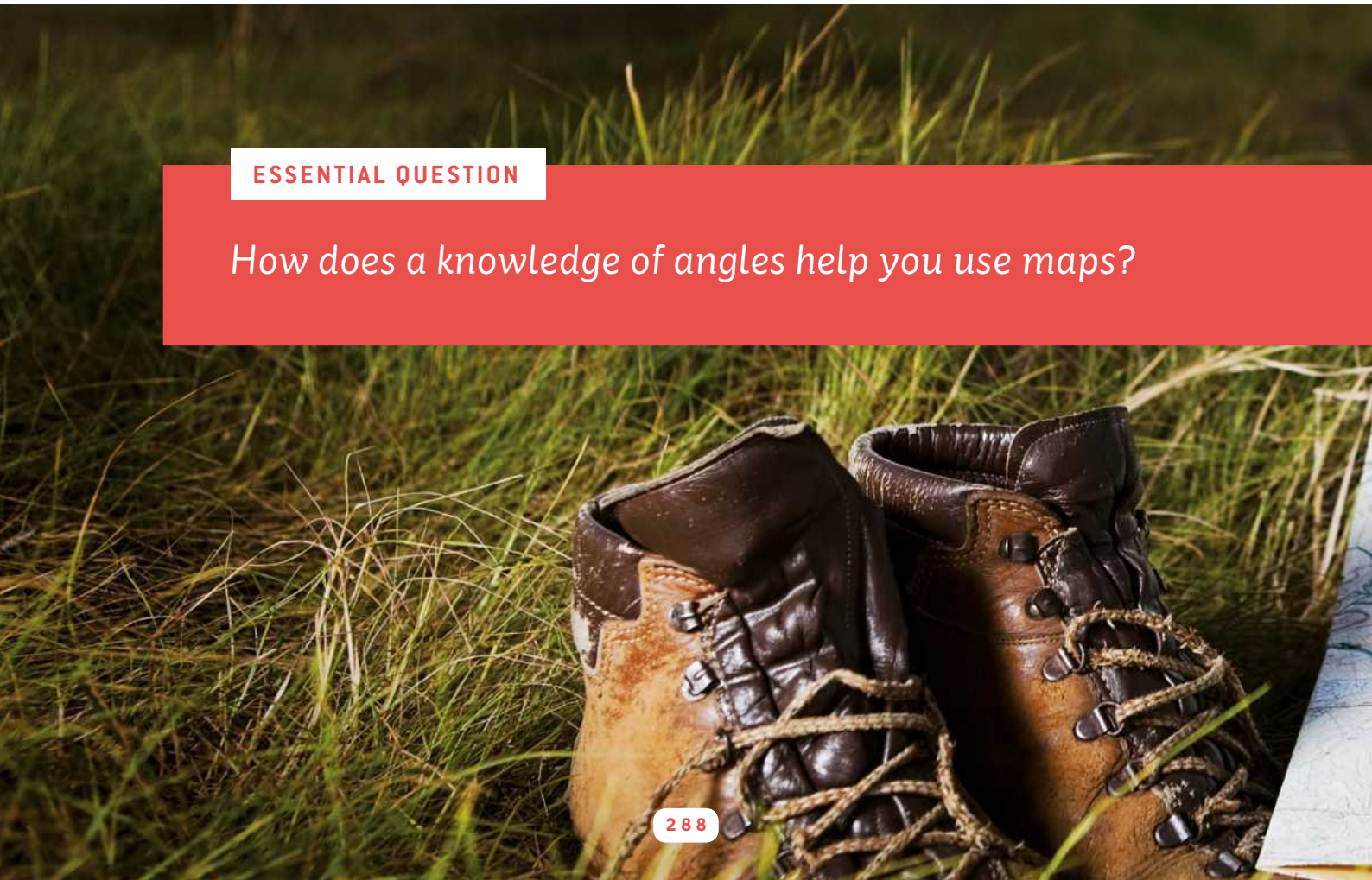
6C Angles and parallel lines

6D Constructions

6E Bearings

6F Angles and time zones

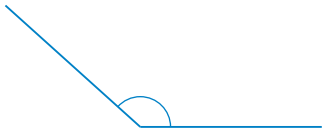
6G Working with time zones



ESSENTIAL QUESTION

How does a knowledge of angles help you use maps?

6A **1** What type of angle is shown in this figure?



A acute angle **B** obtuse angle
C reflex angle **D** right angle

6A **2** What is an acute angle?

A an angle of 90°
B an angle greater than 0°
C an angle greater than 0° but less than 90°
D an angle less than 180°

6A **3** What is an angle of 218° an example of?

A an acute angle
B an obtuse angle
C a straight angle
D a reflex angle

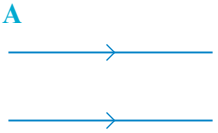
6B **4** How many degrees do supplementary angles add to?

6B **5** What is the complement of 27°?

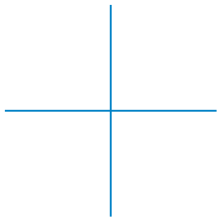
A 27° **B** 63° **C** 73° **D** 153°

6C **6** Which figure displays a pair of parallel lines?

A



B



C

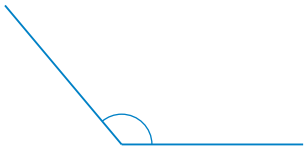


D

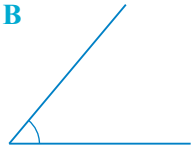
6D **7** How many degrees do the three angles of a triangle add to?

6D **8** Which angle is 50°?

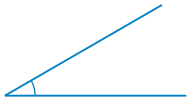
A



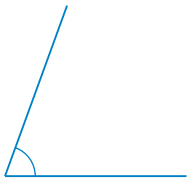
B



C



D



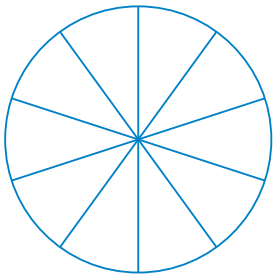
6E **9** How many degrees are there between compass directions north and south?

A 45° **B** 90° **C** 180° **D** 360°

6E **10** Which compass bearing is closest to the direction east?

A N11°E **B** S70°E
C N35°E **D** S21°E

6F **11** The circle shown is divided into 10 equal sectors. What is the angle size of each sector?



A 10° **B** 30° **C** 30.6° **D** 36°

6G **12** What is 7.15 pm in 24-hour time?

A 0715 **B** 1715 **C** 1915 **D** 2115

6G **13** Two movies in a box set have a running time of 2 hours and 27 minutes and 1 hour and 45 minutes. What is their combined running time?

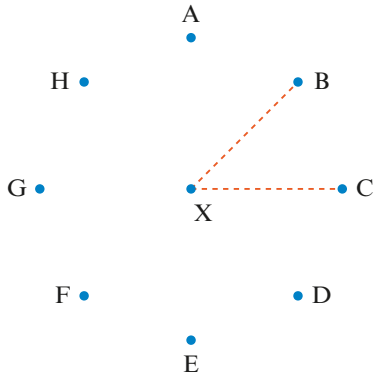
6A Understanding angles

Start thinking!



1 Fill the gaps in these words to complete this list of definitions.
An angle less than 90°: _c_t e
A full circle rotation: re_ol____n
An angle that makes a straight line: _t____ht
An angle that makes a square corner: r___t
More than 90° but less than 180°: __t_s_
More than 180° but less than 360°: r____x

The diagram shows point X surrounded by points A–H.
A dotted line has been drawn from point B to point X and then to point C.



2 What type of angle does this form?

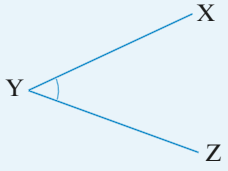
3 What type of angle is formed by drawing a line from:
a point A to point X and then to point C?
b point E to point X and then to point H?
c point F to point X and then to point B?

4 What points could be joined to make:
a a reflex angle starting at D?
b a straight angle starting at C?
c an acute angle starting at G?

5 Points B, X and E make an obtuse angle. What other angle type do they make?

KEY IDEAS

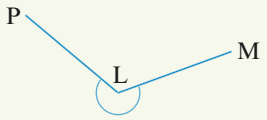
- Angles are measured in degrees and are described as:
  - **acute angle**: less than 90°
  - **right angle**: exactly 90°
  - **reflex angle**: more than 180° but less than 360°
  - **revolution**: exactly 360° (a full rotation)
  - **obtuse angle**: more than 90° but less than 180°
  - **straight angle**: exactly 180°
- Angles can be named using three letters that name points on the line segments that form them. The middle letter is the point, or **vertex**, of the angle. For example, the angle shown at right can be called ∠XYZ or ∠ZYX, where ∠ means angle.



- The line segments that form the angle are called the **arms** of the angle.

EXERCISE 6A Understanding angles

EXAMPLE 6A-1 Naming angles



Name this angle and state what type it is.

THINK

1 The points are labelled P, L and M. L is at the vertex.

2 The arc indicates the position of the angle. It is larger than 180°, so it is a reflex angle.

WRITE

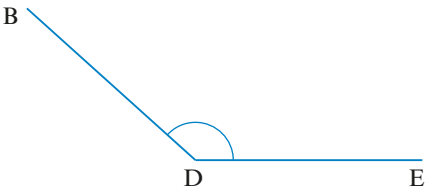
∠PLM or ∠MLP

The angle is a reflex angle.

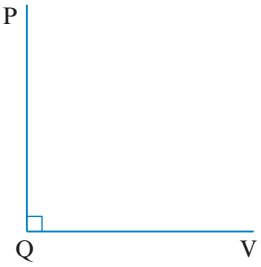
UNDERSTANDING AND FLUENCY

1 Name each angle and state what type it is.

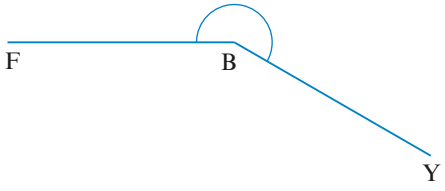
a



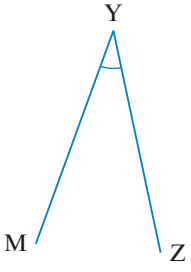
b



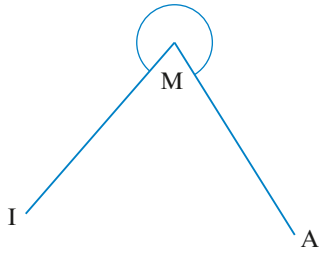
c



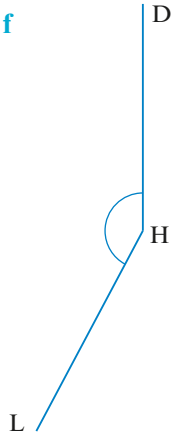
d



e



f



EXAMPLE 6A-2 Drawing angles

Draw acute angle ∠ABC.

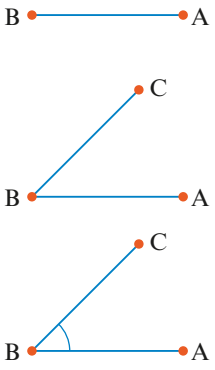
THINK

1 The middle letter, B, is the vertex of the angle. Draw points A and B. Join with a line.

2 An acute angle is less than 90°. Draw point C. Join point C to point B.

3 These lines show both an acute and a reflex angle. Use a small arc to show that the angle ∠ABC is acute.

WRITE



2 Draw each angle.

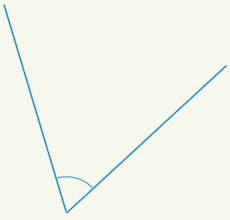
- **a** obtuse angle ∠YBD
- **b** straight angle ∠RMC
- **c** right angle ∠WZB
- **d** reflex angle ∠ALY

3 What types of angles can you see in this photo?



EXAMPLE 6A-3 Measuring angles with a protractor

Use a protractor to measure the size of this angle.



THINK

1 Identify what type of angle it is.

2 Estimate the size of the angle.

3 Place the centre cross on the point of the angle and line up the zero line. Read the scale from 0°.

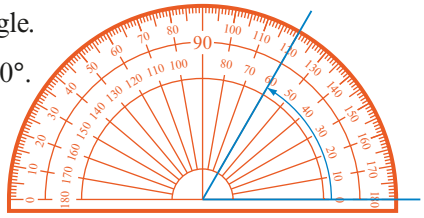
4 Check your answer. Which scale starts at 0°? Which is closer to your estimation?

WRITE

It is an acute angle.

Angle is about 50°.

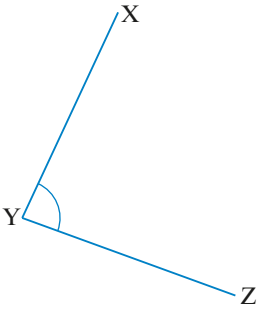
The scale shows 120° but also 60°.



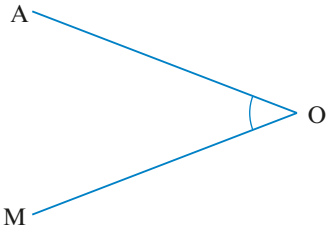
The actual measurement is 60°.

4 Use a protractor to measure each angle. Remember to estimate first. Write the name, size and type of each angle. For example, ∠ABC = 95°, obtuse.

a



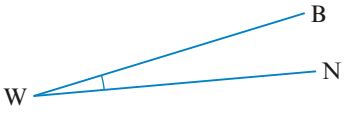
b



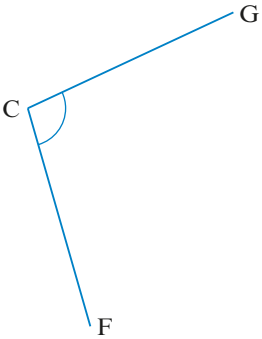
c



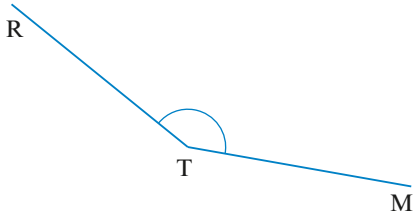
d



e

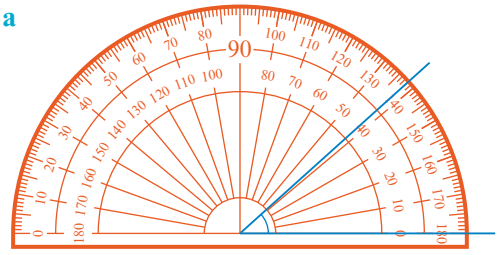


f

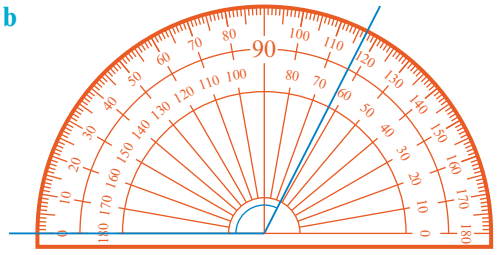


5 What are the two angles that could be read from each protractor? Which is correct?

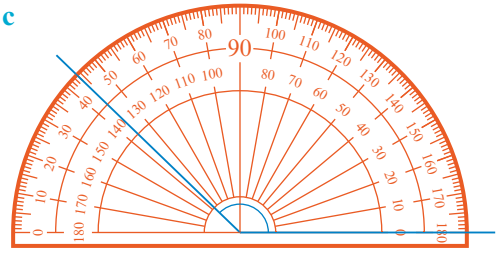
a



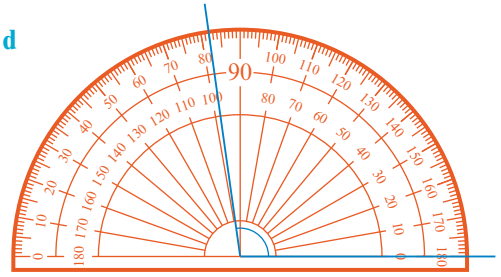
b



c



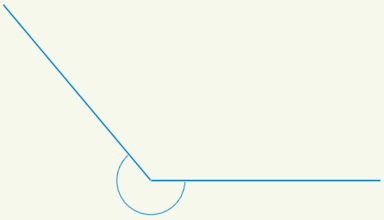
d

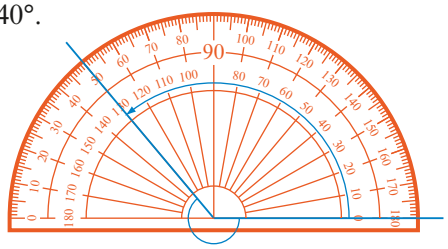


6 How does knowledge of angle types help you estimate and check a final answer?

EXAMPLE 6A-4 Measuring angles greater than 180°

Use a protractor to measure this reflex angle.

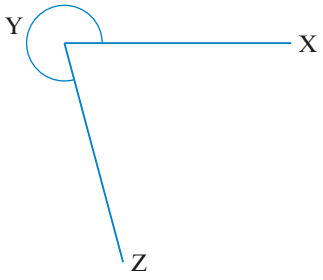


THINK	WRITE
1 Estimate the size of the reflex angle.	Estimate is 240°.
2 Place the protractor over the part of the angle that is *less than* 180° and measure.	The obtuse angle measures 130°. 
3 Subtract 130° from 360° to find the reflex angle.	360° − 130° = 230° The reflex angle is 230°.

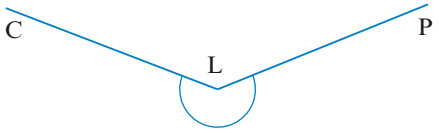
UNDERSTANDING AND FLUENCY

7 Use a protractor to measure each angle. Remember to estimate first. Write the name and size of each angle. For example, ∠ABC = 300°.

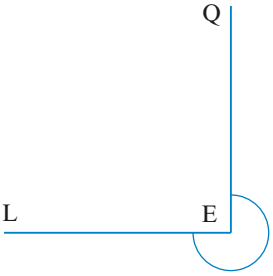
a



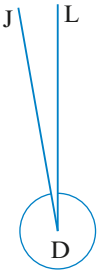
b



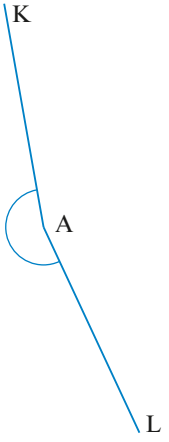
c



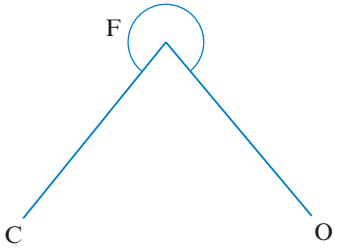
d



e



f



EXAMPLE 6A-5 Drawing angles with a protractor

Use a protractor to draw an angle of 60°.

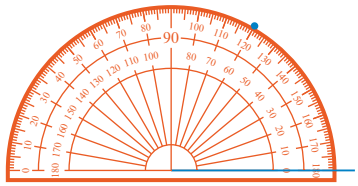
THINK

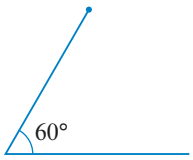
1 Draw a horizontal line and place your protractor over it so that one end is on the centre point.

2 Use the scale that starts at 0°. Place a dot at the 60° mark.

3 Remove the protractor and use a ruler to join the end of your line with the dot

WRITE





8 Use a protractor to draw each angle and label it correctly.

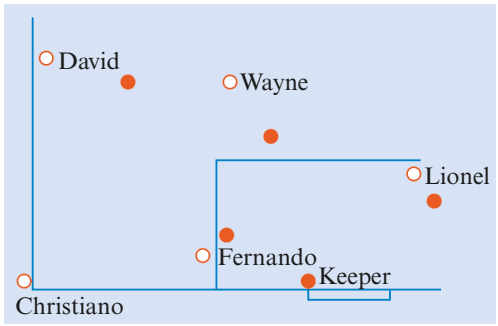
a ∠EHU = 125° **b** ∠OEN = 80° **c** ∠PWB = 173°

d ∠MSK = 23° **e** ∠CPS = 180° **f** ∠RLA = 57°

9 How can a knowledge of drawing acute and obtuse angles help you to draw a reflex angle?

PROBLEM SOLVING AND REASONING

10 Christiano is taking a corner kick for his soccer team. If the line CK from Christiano to the keeper is 0°, what angle should he kick at to reach each of his team mates? Give your answer with CK as one arm and C as the vertex of the angle; for example, ∠KCD = 87°.



11 **a** Use your arms to form:

i an acute angle **ii** an obtuse angle **iii** a right angle

iv a straight angle **v** a reflex angle

b Estimate the size of each angle. Can you think of a way to measure the angle formed between your arms?

CHALLENGE

12 Come up with your own method for drawing reflex angles and try to produce these angles.

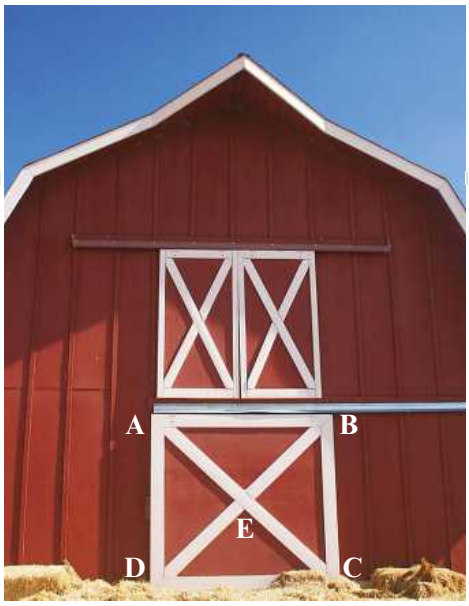
a ∠WJS = 245° **b** ∠EPL = 199°

c ∠XNE = 351° **d** ∠UYC = 202°

Reflect

What should you do before measuring an angle with a protractor?

6B Angles and lines



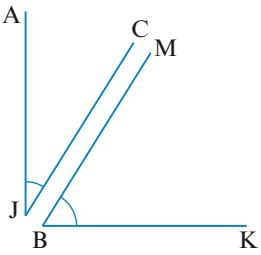
Start thinking!

1 The cross beams on this barn door form angles.

- **a** Which angle is the same size as ∠AEB?
  These are **vertically opposite angles**.
- **b** Which angle is vertically opposite ∠AED?
- **c** Which pairs of angles form a right angle?
  These are **complementary angles**.
- **d** Which pairs of angles form a straight angle?
  These are **supplementary angles**.

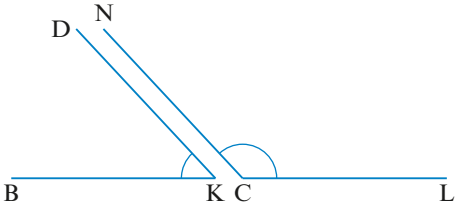
2 **a** Estimate the size of angle ∠AJC in the diagram on the right.

- **b** Use a protractor to measure the size of angle ∠MBK.
- **c** Does this change your estimate of ∠AJC?
  Record your new estimate and then check by measuring the angle.



3 **a** Estimate the size of the two angles in this diagram.

- **b** Measure one of the angles.
- **c** Does this change your estimation of the other angle?
  Record your new estimations and then measure the other angle to check.
- **d** Explain how knowing one angle helped you estimate the size of the other angle.



KEY IDEAS

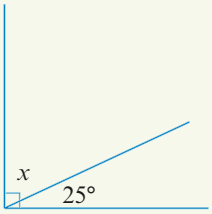
- Angles around a line add to 180°.
- Angles around a point add to 360°.
- Two angles that add to 90° are called complementary angles.
- Two angles that add to 180° are called supplementary angles.
- Vertically opposite angles are equal.

EXERCISE 6B Angles and lines

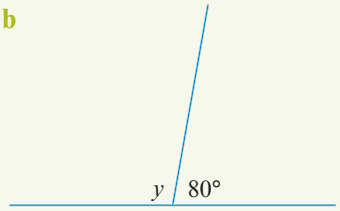
EXAMPLE 6B-1 Complementary and supplementary angles

Find the size of each labelled unknown angle.

a



b



THINK

a **1** Angle x and 25° are complementary so they add to 90°.

2 Subtract 25° from 90° to find x.

b **1** Angle y and 80° are supplementary so they add to 180°.

2 Subtract 80° from 180° to find y.

WRITE

a $x + 25° = 90°$

$x = 90° - 25°$

$x = 65°$

b $y + 80° = 180°$

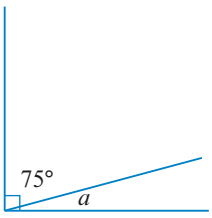
$y = 180° - 80°$

$y = 100°$

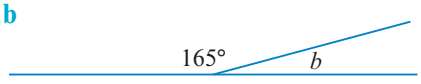
UNDERSTANDING AND FLUENCY

1 Find the size of each labelled unknown angle.

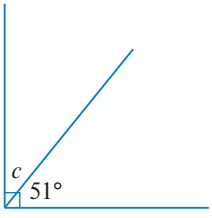
a



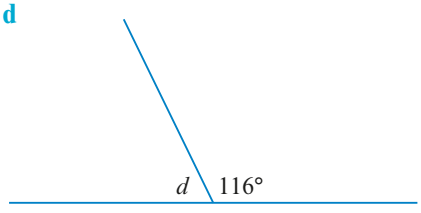
b



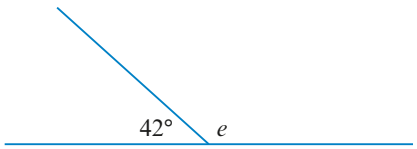
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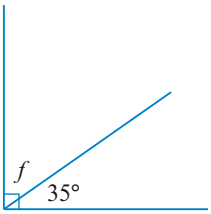
d



e

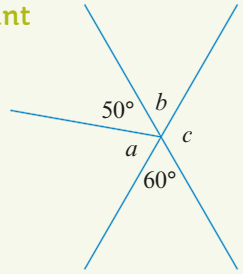


f



EXAMPLE 6B-2 Finding angle size using angles around a point

Find the size of each labelled unknown angle.



THINK

1 Angle b and 60° are vertically opposite, so they are equal.

2 Angles b and c are supplementary so they add to 180°. Angle b = 60°.

3 Angles around a point add to 360°. Subtract the known angles to find angle a.

WRITE

$b = 60°$

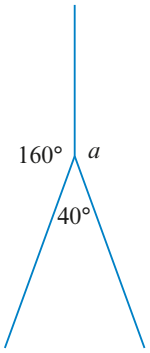
$c + 60° = 180°$
$c = 120°$

$a + b + c + 50° + 60° = 360°$
$a + 60° + 120° + 50° + 60° = 360°$
$a + 290° = 360°$
$a = 70°$

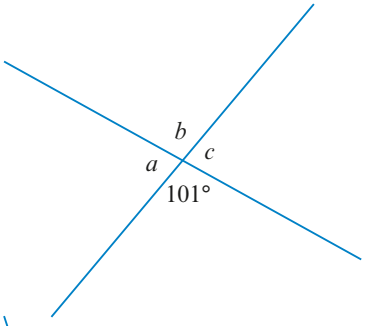
UNDERSTANDING AND FLUENCY

2 Find the size of each labelled unknown angle.

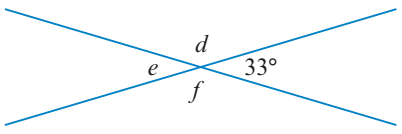
a



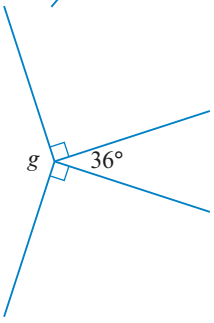
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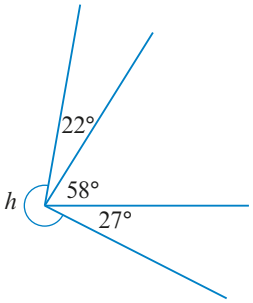
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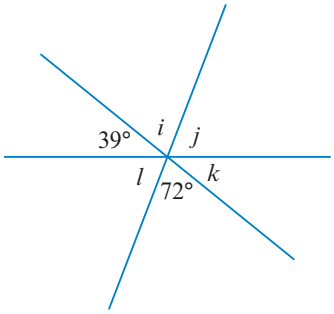
d



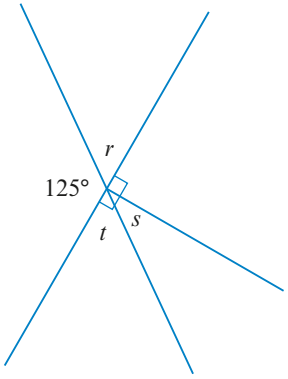
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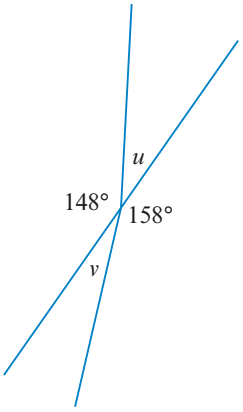
f



g



h

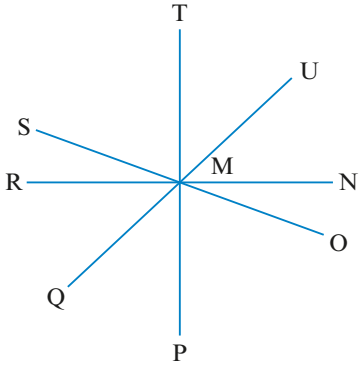


3 **a** Identify the relationship (complementary, supplementary, vertically opposite) between each pair of angles in this diagram.

i ∠UMT and ∠NMU **ii** ∠OMT and ∠PMO

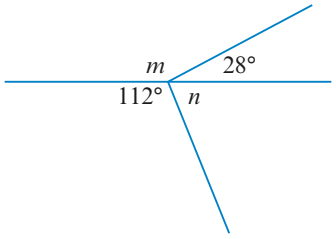
iii ∠SMR and ∠NMO **iv** ∠TMQ and ∠PMQ

b Copy the diagram and mark in another two examples of each type of angle pair (complementary, supplementary and vertically opposite).

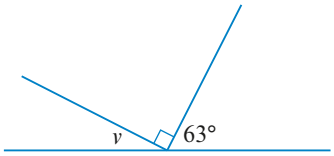


4 Find the value of each pronumeral without using a protractor.

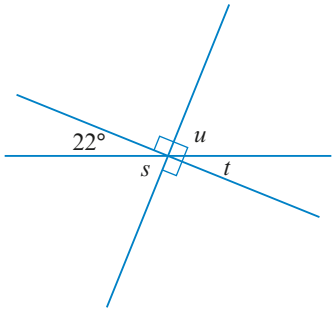
a



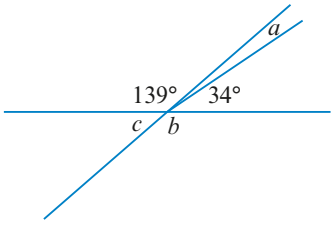
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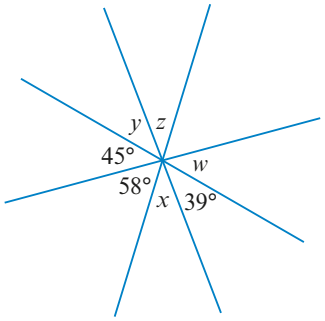
c



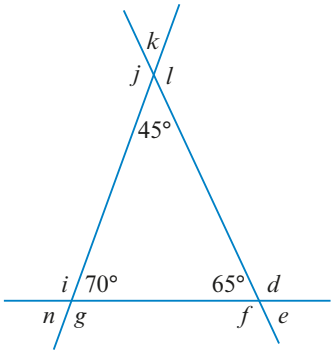
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e



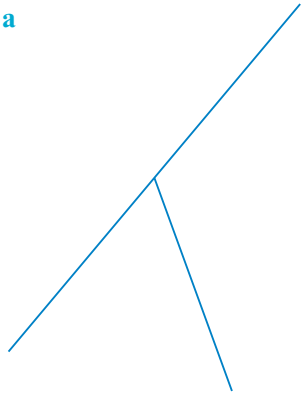
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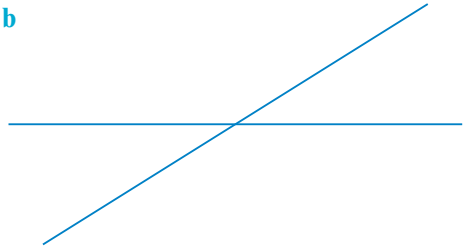
UNDERSTANDING AND FLUENCY

5 Measure one of the angles in each diagram below. Use your knowledge of angles to predict the size of the remaining angles in the diagram. Check your answers by measuring the angles.

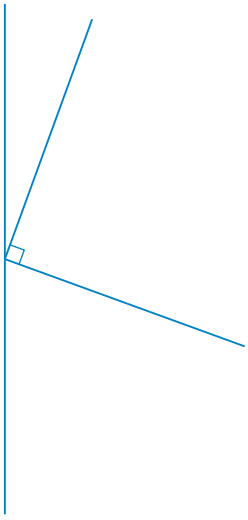
a



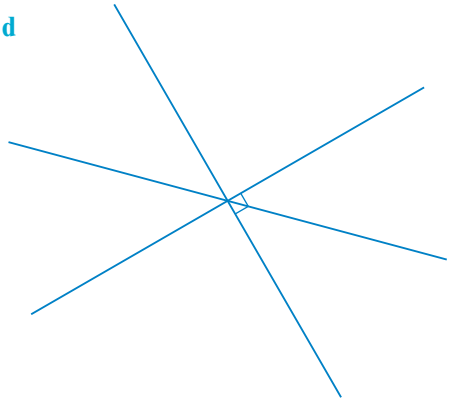
b



c



d



PROBLEM SOLVING AND REASONING

6 Look at this dartboard.



a What is the size of each angle at the centre?

b Copy the dartboard and mark in an example of a pair of:

i complementary angles

ii supplementary angles

iii vertically opposite angles.

7 For each situation, use your understanding of angles to:

i state the angle relationship; for example, complementary

ii find the remaining angle to turn.

a Katie changes direction by 124° as she attempts to turn around in a hockey game.

b Mon starts turning left but only makes it through 28° before being interrupted.

c Jo begins to shut her laptop screen from an upright position and moves it 67°.

d Ben attempts a U-turn but gets stuck after only turning through 88°.

PROBLEM SOLVING AND REASONING

8 An understanding of angles around a point can help you to draw analogue clocks accurately.

a How many degrees in a circle?

b Find the size of the angle indicated on each clock.

i



ii



iii



c What size angle would be between the hands of a clock displaying these times? Start from 12 o'clock and move clockwise to the other hand.

i 4.00 pm **ii** 11.00 am **iii** 8.00 pm **iv** 1.00 am

CHALLENGE

9 What happens when a clock is displaying a time that is not a whole hour?

a What is the size of the angle between two hour-marks?

b What is the size of:

i half this angle?

ii a quarter of this angle?

iii three quarters of this angle?

c Use your answers to parts **a** and **b** to find the size of the angle indicated on these clocks.

i



ii



iii



10 Find the size of the smaller angle between the two hands of a clock displaying these times.

a 10.20 **b** 6.55 **c** 4.35 **d** 2.27 **e** 11.09

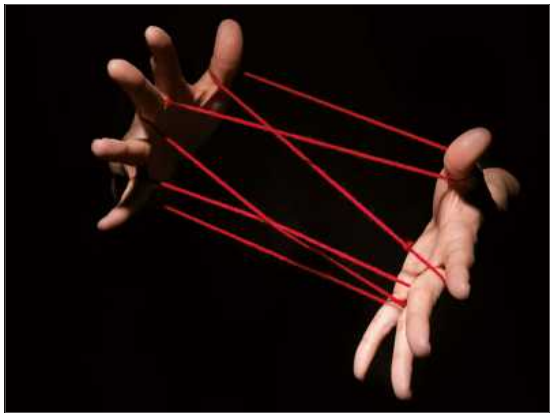
Use these hints to help you.

- How many minute-marks are displayed on a clock?
- What is the size of the angle between two of the minute-marks?
- What is $\frac{1}{60}$ of the size of the angle between two of the minute-marks?
- How far does the small hand move every time the big hand moves 1 minute?

Reflect

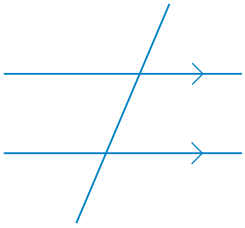
Of all the angle facts you've learned, which do you think is the most important and why?

6C Angles and parallel lines



Start thinking!

When a straight line cuts through a pair of **parallel** lines, it forms some interesting angles. The straight line is called a **transversal**.



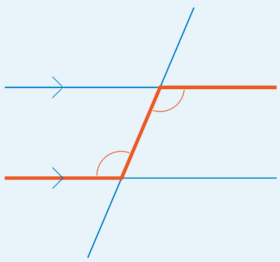
1 Look at this diagram. How can you tell if some angles are the same size?

- **a** Using a ruler and protractor, copy the diagram.
- **b** On a small piece of tracing paper, trace a copy of one of the acute angles in the diagram.
- **c** Use your traced angle to identify which of the other angles are the same size.
- **d** On your diagram, show which acute angles are the same size by shading them in the same colour.
- **e** Repeat this process with one of the obtuse angles. Use a different colour to identify which of the obtuse angles are the same size.
- **f** Explain your findings to a classmate.

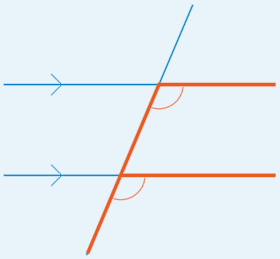
2 Draw your own set of parallel lines cut by a transversal and label all identical angles in a similar way to how you did in question **1**.

KEY IDEAS

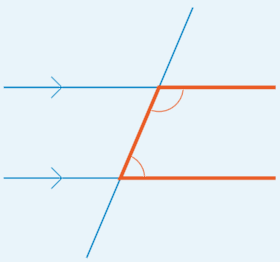
- When two parallel lines are intersected by another line (a transversal) some related angles are formed.
  - **Alternate angles** are equal.



  - **Corresponding angles** are equal.



  - **Co-interior angles** are supplementary.

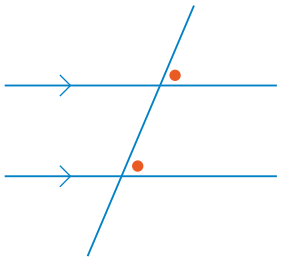


EXERCISE 6C Angles and parallel lines

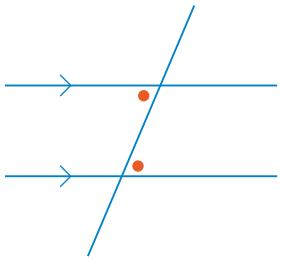
UNDERSTANDING AND FLUENCY

1 Identify the types of angles marked in these diagrams.

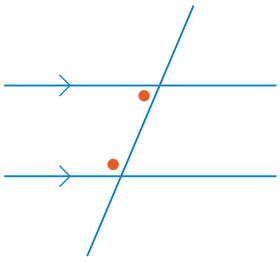
a



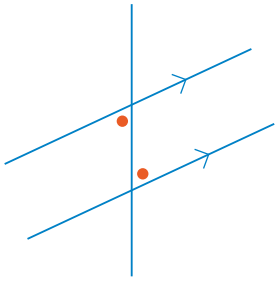
b



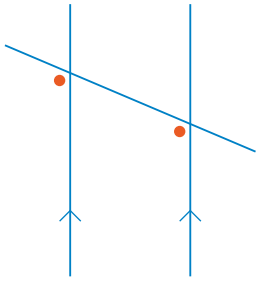
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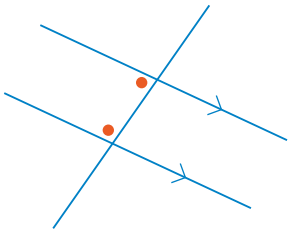
d



e



f



2 Copy and complete these sentences when considering parallel lines cut by a transversal.

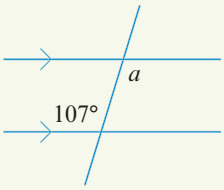
a Alternate angles are _____.

b Co-interior angles add to _____.

c Corresponding angles are _____.

EXAMPLE 6C-1 Finding angle size using angle relationships with parallel lines

Find the size of the angle a.



THINK

1 Look for a relationship between the two angles. They are on opposite sides of the transversal and between the parallel lines. They are alternate angles.

2 With parallel lines, alternate angles are equal, so a is equal to the given angle.

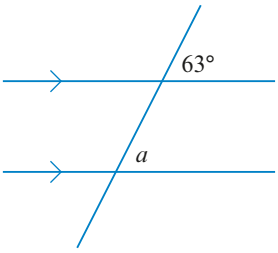
WRITE

a is alternate to 107°.

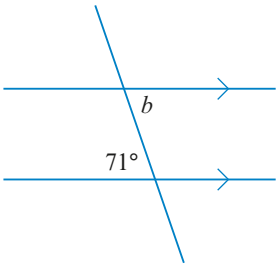
$a = 107°$

3 Find the size of each labelled unknown angle.

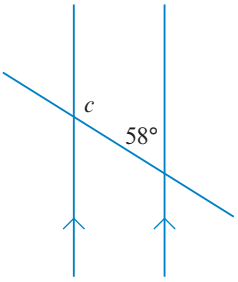
a



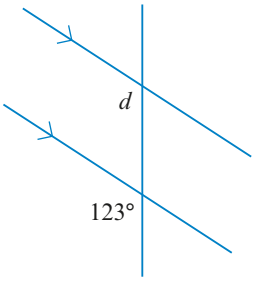
b



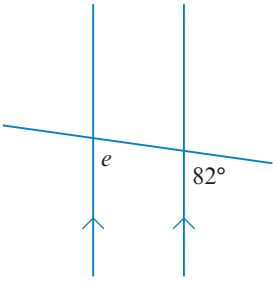
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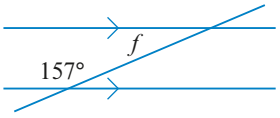
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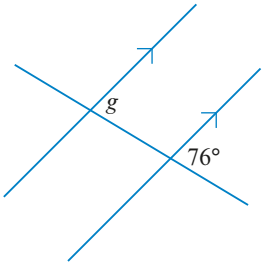
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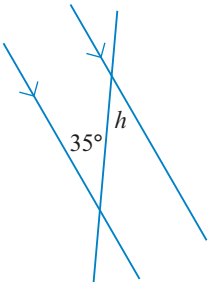
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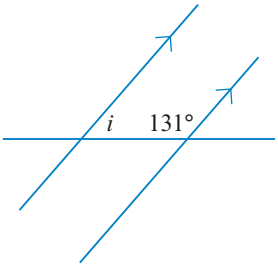
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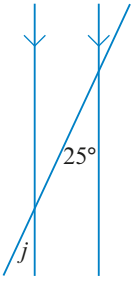
h



i

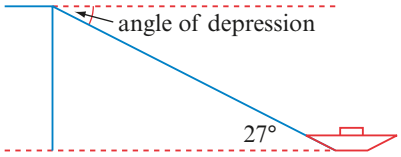


j



4 State whether each angle you found in question **3** was alternate, corresponding or co-interior.

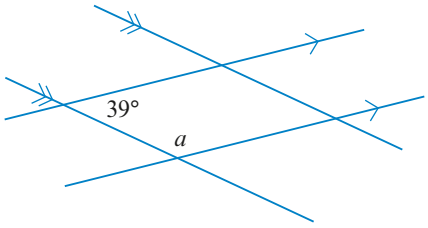
5 **a** The **angle of elevation** (from the boat to the cliff-top) is 27°. What is the **angle of depression** from the cliff-top to the boat?



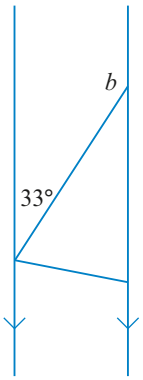
b What key words describe these two angles?

6 Find the size of each labelled unknown angle. In each case, state whether the angle is alternate, corresponding or co-interior to the given angle.

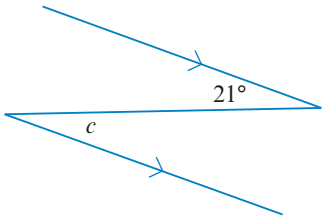
a



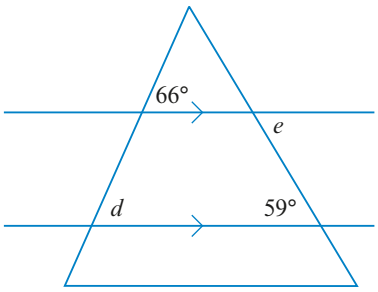
b



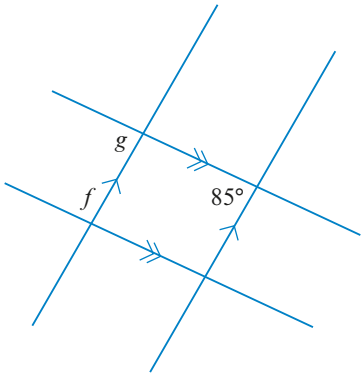
c



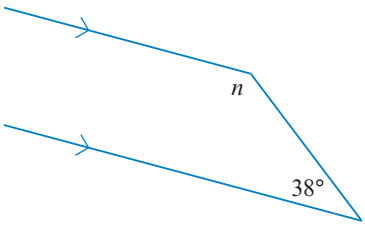
d



e



f





7 Argyle is a fabric pattern made of a series of diamonds and parallel lines. Jesse bought two pairs of socks that apparently had the same pattern, but in different colours. Jesse measured the obtuse angle on one pair of socks to be 135° and the acute angle on the other pair of socks to be 55°. Are the two pairs of socks the same pattern? Explain.

8 Draw two parallel lines with a transversal that makes an angle of:

a 70° **b** 125° **c** less than 20° **d** more than 150°

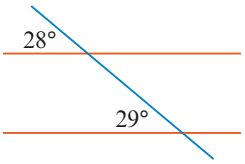
9 For each diagram you drew in question **8**, state the size of the angle that is:

i corresponding **ii** co-interior

iii alternate **iv** vertically opposite to the marked angle.

10 Use your knowledge of co-interior, alternate and corresponding angles to test whether the pairs of orange lines are parallel or not. Give a reason why each one is or is not parallel.

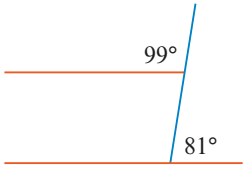
a



b



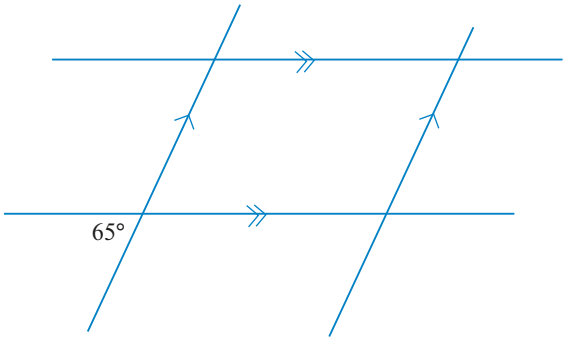
c



11 **a** What shape is formed in the diagram at right?

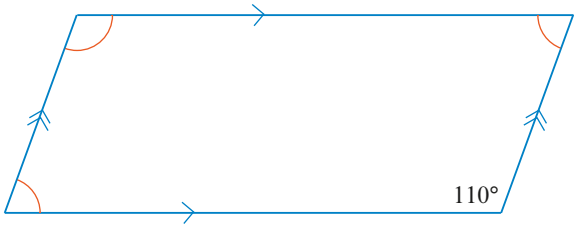
b Use your knowledge of angles and parallel lines to state the size of all the angles.

c Measure the angles with a protractor to check your answers.



12 **a** Use your knowledge of angles and parallel lines to give the size of all the angles in this **parallelogram**.

b Measure with a protractor to check your answers.



13 Draw a parallelogram with interior angles of 48° and 132°.

14 What name is given to a parallelogram with interior angles of 90°?

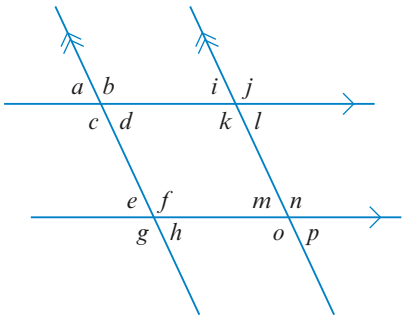
PROBLEM SOLVING AND REASONING

15 In this diagram, identify all pairs of:

a alternate angles

b corresponding angles

c co-interior angles.

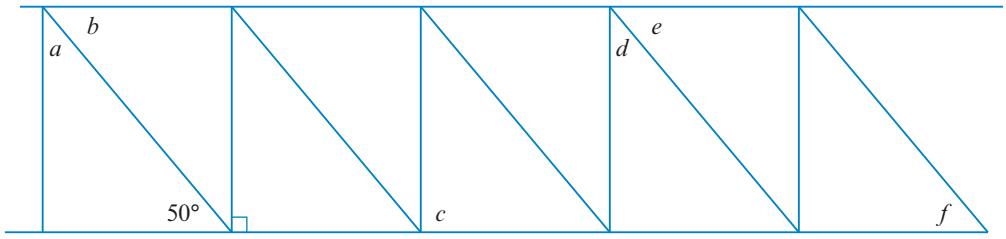


16 Copy and complete this table.
The first column has been started for you.

	32°	85°	45°	12°	60°
Complementary	58°				
Supplementary	148°				
Vertically opposite	32°				
Co-interior					
Alternate					
Corresponding					

CHALLENGE

17 This diagram shows a building's structural support beams. If you know that the main two horizontal beams are parallel, use the information provided to find the size of each angle labelled with a pronumeral.



18 **a** If a pair of parallel lines is cut by a single transversal, explain how knowing the size of one angle in the diagram means that you can find the size of every other angle.

b If a pair of parallel lines is cut by a number of transversals that are parallel to each other, explain how knowing the size of one angle in the diagram still means that you can find the size of every other angle.

Reflect

What do the words 'alternate', 'corresponding' and 'co-interior' mean in everyday language? Why do you think they are used for these angle pairs?

6D Constructions

Start thinking!

1 Figure A shows an incomplete triangle. One side and two angles have been given. This is called an **angle-side-angle (ASA)** description. Copy and complete the triangle.

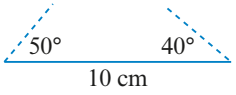


Figure A

2 Figure B shows another incomplete triangle. This time, two sides and the angle between them have been given. This is called a **side-angle-side (SAS)** description. Copy and complete the triangle.

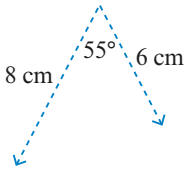


Figure B

3 Figure C shows a third incomplete triangle. This time the lengths of the three sides are given, but no angles. This is called a **side-side-side (SSS)** description. See if you can complete this triangle (it is much harder than it looks).

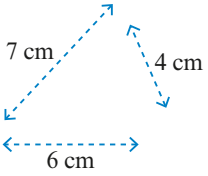
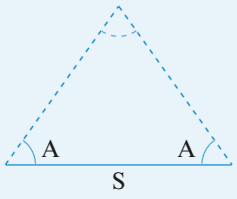


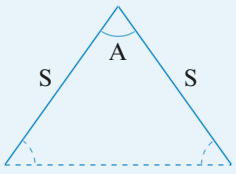
Figure C

KEY IDEAS

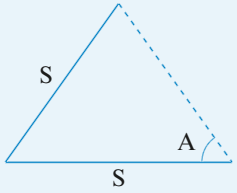
- A triangle can be exactly described in a number of ways, by listing:
  - ASA: two angles and the length of the side between them



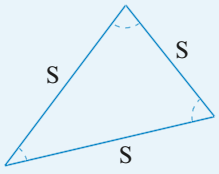
  - SAS: the lengths of two sides and an angle between them



  - SSA: the lengths of two sides and an angle that is not between them



  - SSS: the lengths of all three sides.

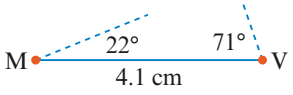


- A triangle can be drawn from any of these sets of information using a protractor, a ruler and a compass.

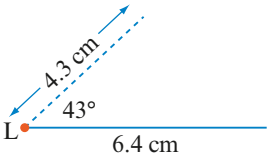
EXERCISE 6D Constructions

1 Copy and complete each triangle. Label each side with its length and mark in each angle.

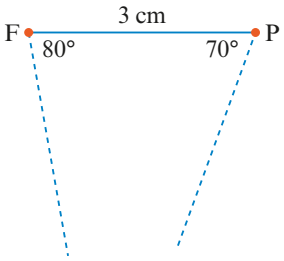
a



b



c



EXAMPLE 6D-1 Drawing an angle-side-angle (ASA) triangle

Draw the triangle EFG, EF = 12 cm, ∠FEG = 40° and ∠EFG = 50°.

THINK

1 Sketch a rough outline of the triangle, marking in the information given.

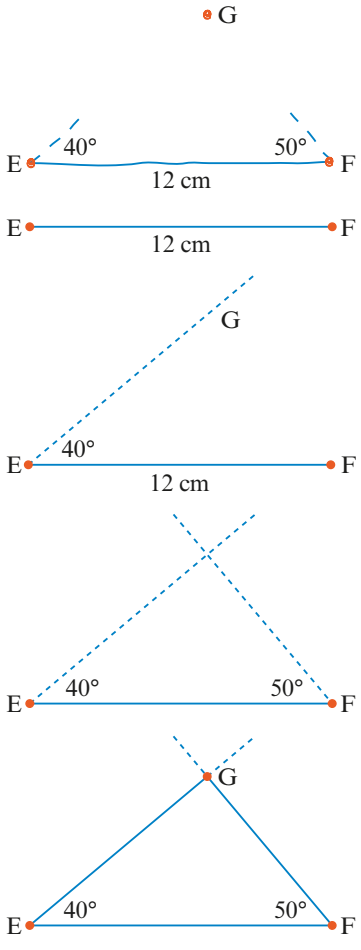
2 Draw a line 12 cm long and label it EF.

3 Make an angle of 40° at point E. Draw the line in pencil so you can erase some of it later.

4 Make an angle of 50° at point F. Draw this line in pencil as well.

5 To complete the triangle, extend the lines to meet if necessary or erase any extra lines.

WRITE



2 Draw each triangle using a ruler and a protractor.

- **a** triangle EFG with EF = 6 cm, ∠GEF = 50° and ∠EFG = 60°
- **b** triangle PSM with PS = 4.1 cm, ∠MPS = 39° and ∠MSP = 51°
- **c** triangle MGZ with GZ = 8 cm, ∠MGZ = 22° and ∠MZG = 34°
- **d** triangle ASW with AS = 6.1 cm, ∠WAS = 80° and ∠WSA = 23°

EXAMPLE 6D-2 Drawing a side-angle-side (SAS) triangle

Draw the triangle PWH with PW = 6.5 cm, PH = 8.2 cm and ∠HPW = 39°.

THINK

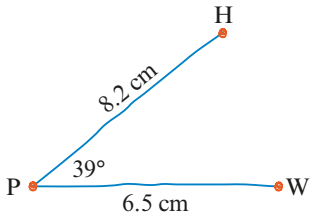
1 Sketch a rough outline of the triangle, marking in the information given.

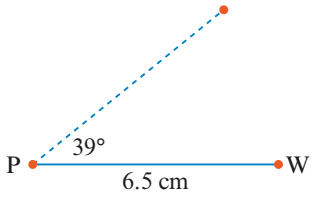
2 Draw a 6.5 cm line and label it PW. Measure the angle 39° with P at the vertex.

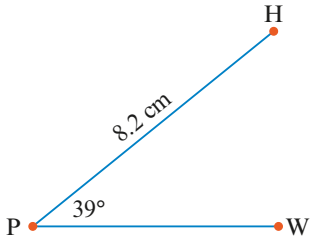
3 Draw an 8.2 cm line PH at the angle of 39°.

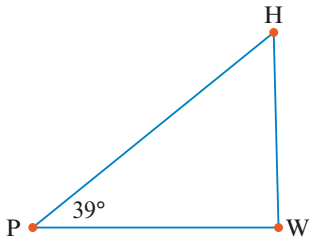
4 Complete the triangle by joining H and W with the line HW and erasing any construction lines.

WRITE









3 Draw each triangle using a ruler and a protractor.

- **a** triangle CGS with CG = 5 cm, CS = 8 cm and ∠SCG = 25°
- **b** triangle NLM with NL = 5 cm, LM = 6.2 cm and ∠NLM = 60°
- **c** triangle QRS with QR = 4.3 cm, SR = 5.7 cm and ∠SRQ = 49°
- **d** triangle TUV with VU = 4 cm, UT = 7.5 cm and ∠TUV = 63°

EXAMPLE 6D-3 Drawing a side-side-angle triangle

Draw the triangle JKL, with JK = 8 cm, LK = 11 cm and ∠KJL = 80°.

THINK

1 Sketch a rough outline of the triangle, marking in the information given. Note: you are given two sides and an angle that is not in between these sides (SSA rather than SAS).

2 Draw the 8 cm line JK.

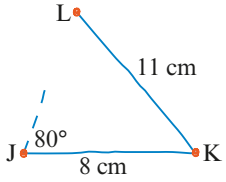
3 Draw the angle ∠KJM = 80°.
(M is a point that will be erased shortly.)

4 Use a compass to draw an arc of radius 11 cm with K at its centre. Make sure the arc crosses the line JM.

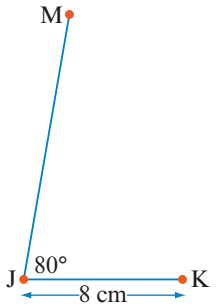
5 Where the arc crosses the line JM, mark the point L.

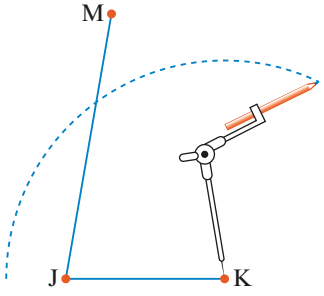
6 Erase the arc and the line LM and complete the triangle.

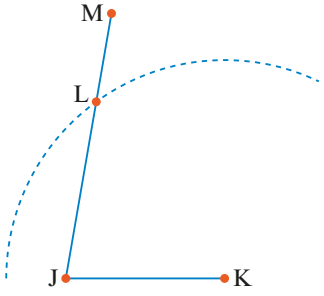
WRITE

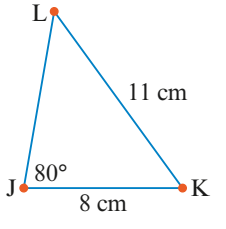












4 Draw each triangle using a ruler, protractor and a compass.

- **a** triangle YDE with YD = 6 cm, DE = 5.5 cm and ∠EYD = 61°
- **b** triangle AFQ with ∠QAF = 38°, AF = 12 cm and FQ = 8.3 cm
- **c** triangle AJM with AM = 5 cm, AJ = 6 cm and ∠AJM = 45°
- **d** triangle MRK with RK = 7 cm, ∠RMK = 125° and MK = 4 cm

EXAMPLE 6D-4 Drawing a side-side-side triangle

Draw the triangle ABC with AB = 6 cm, AC = 7 cm and BC = 8 cm.

THINK

1 Sketch a rough outline of the triangle, marking in the information given.

2 Draw the 6 cm line AB.

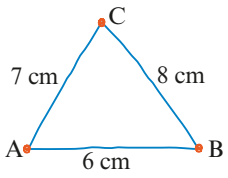
3 Use a compass to draw an arc of radius 7 cm with point A at its centre.

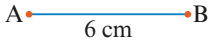
4 Use a compass to draw an arc of radius 8 cm with point B at its centre. Make sure the two arcs intersect.

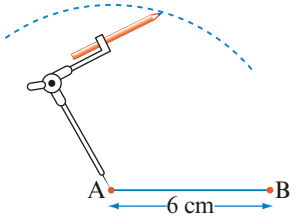
5 Mark the point C where the two arcs intersect.

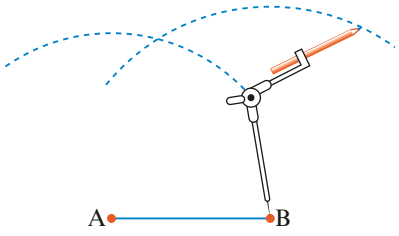
6 Erase the arcs and complete the triangle.

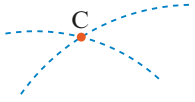
WRITE



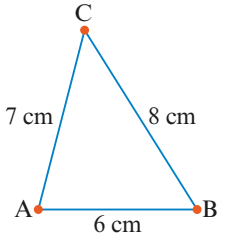












PROBLEM SOLVING AND REASONING

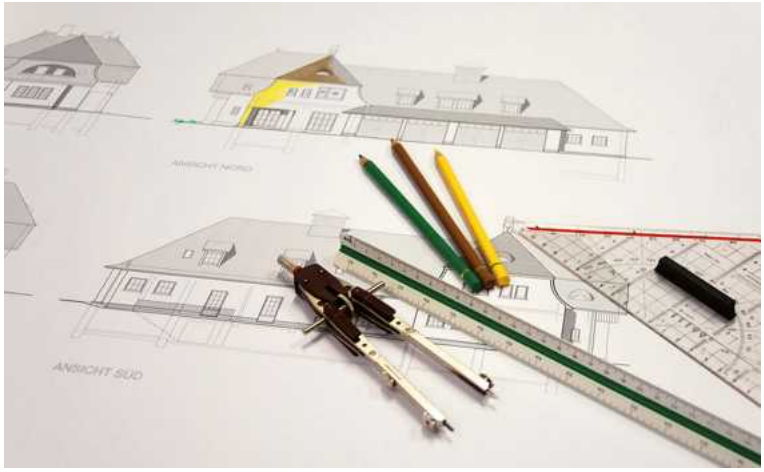
5 Draw each triangle using a ruler and a compass.

- **a** triangle BSP with BP = 8.5 cm, SP = 6.1 cm and BS = 5.2 cm
- **b** triangle EXO with EX = 3 cm, XO = 5 cm and EO = 7 cm
- **c** triangle TAM with TA = 5 cm, AM = 3 cm and MT = 4 cm
- **d** triangle SKA with SK = 6.2 cm, SA = 1.3 cm and KA = 5.4 cm

6 Classify each triangle description as ASA, SAS, SSA or SSS.
Choose an appropriate construction method to draw each triangle.

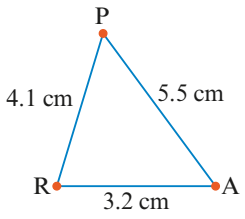
- **a** ΔRPA with RP = 4.1 cm, RA = 3.2 cm and PA = 5.5 cm
- **b** ΔMSZ with MS = 6.6 cm, SZ = 5 cm and ∠MSZ = 53°
- **c** ΔUKW with UK = 6 cm, ∠WUK = 41° and ∠WKU = 33°
- **d** ΔEYS with ES = 7.6 cm, SY = 8.4 cm and ∠ESY = 45°
- **e** ΔGBD with GB = 9 cm, BD = 8 cm and ∠BGD = 50°
- **f** ΔWQE with QE = 9.1 cm, ∠WQE = 37° and ∠WEQ = 55°

NOTE In the same way the symbol ∠ stands for 'angle', you can use Δ instead of writing 'triangle'.



CHALLENGE

7 A triangle can be described in more than one way.
For example, ΔRPA in question **6a** is given as an SSS, but how would you describe it as an SAS or an ASA?
Give an alternative description for each of the triangles you drew in question **6** (you may need to use a ruler and protractor).



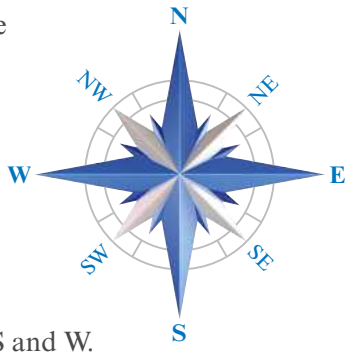
Reflect

Which of the triangle types (SSS, SAS, SSA, ASA) was the easiest to draw? Which was the hardest? Why?

6E Bearings

Start thinking!

Directions from one point to another are often described using **bearings**. Bearings are measured in degrees and are related to compass points. Look at this compass rose.



1 What are the four cardinal points?

2 What are the four intermediate or inter-cardinal points?

3 Draw a copy of the compass rose showing these eight points.

4 Mnemonics are often used to remember the order of these directions. One mnemonic to remember the cardinal points clockwise from the top is Naughty Elephants Squirt Water. Write your own mnemonic to remember N, E, S and W.

5 North is given the bearing 0° and east is 90°. How many degrees are south and west? Add these numbers to your compass rose.

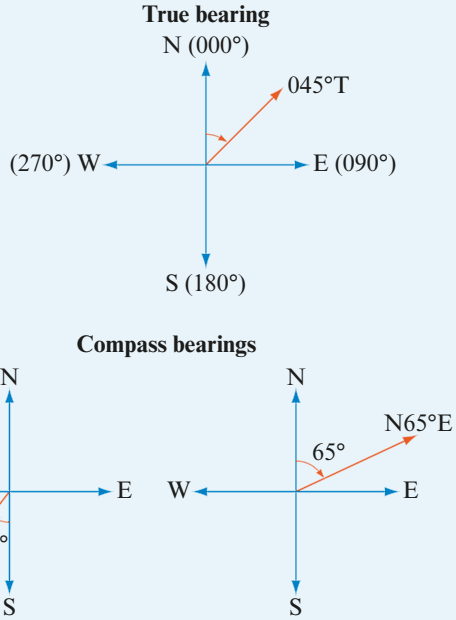
6 Because north-east (NE) is exactly halfway between north (0°) and east (90°), it has a bearing of 45°. Calculate the bearings for south-east (SE), south-west (SW) and north-west (NW) and add them to your compass rose.

7 How many degrees are there between these directions, moving clockwise?

a NE and SE **b** S and NW **c** NW and SE **d** SW and W

KEY IDEAS

- Bearings describe directions around a compass.
- North is 0°, east is 90°, south is 180° and west is 270°.
- There are two main types: **true bearings** and **compass bearings**.
- A true bearing starts at north and goes clockwise. It is a three-digit number with a capital T at the end. For example, north-east has a true bearing of 045°T.
- A compass bearing describes how far from north or south a direction is. It starts with N or S, whichever is closer, and ends with E or W. For example, S38°W or N65°E.
- S38°W means face south, then turn 38° to west.
- N65°E means face north, then turn 65° to east.



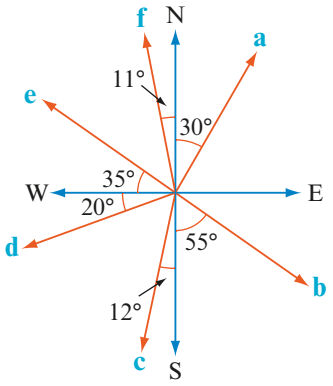
EXERCISE 6E Bearings

UNDERSTANDING AND FLUENCY

1 Write each of the bearings **a**–**f** in this diagram as:

i a compass bearing

ii a true bearing.





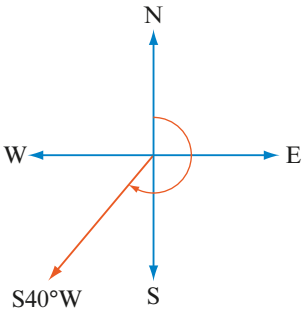
EXAMPLE 6E-1 Converting a compass bearing to a true bearing

Convert S40°W to a true bearing.

THINK

1 Draw the compass bearing on a compass rose.

WRITE



2 South is 180°. This bearing is 40° past south.

180° + 40° = 220°

3 Write the true bearing.

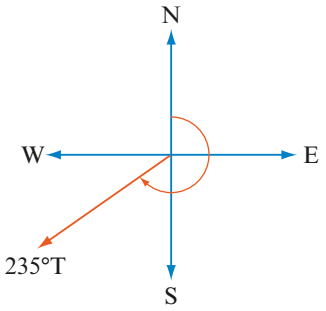
S40°W is the same as 220°T.

2 Convert each compass bearing to a true bearing.

a S21°E	**b** N43°E	**c** N32°W
d S9°W	**e** S45°W	**f** S80°E
g N2°W	**h** N63°E	**i** S67°E

EXAMPLE 6E-2 Converting a true bearing to a compass bearing

Convert 235°T to a compass bearing.

THINK	WRITE
1 Draw the true bearing on a compass rose.	
2 235° is between 180° and 270°, which means it is between south and west.	
3 Find the acute angle between the bearing and south.	235° − 180° = 55°
4 Write the compass bearing.	235°T is the same as S55°W.

3 Convert each true bearing to a compass bearing.

a 150°T **b** 022°T **c** 280°T
d 301°T **e** 097°T **f** 178°T

PROBLEM SOLVING AND REASONING

4 How well do you know Australia? Cover up the map of Australia and, working with a partner, estimate the bearings between each of these cities. When you have both recorded an estimate, uncover the map and check who was closest.

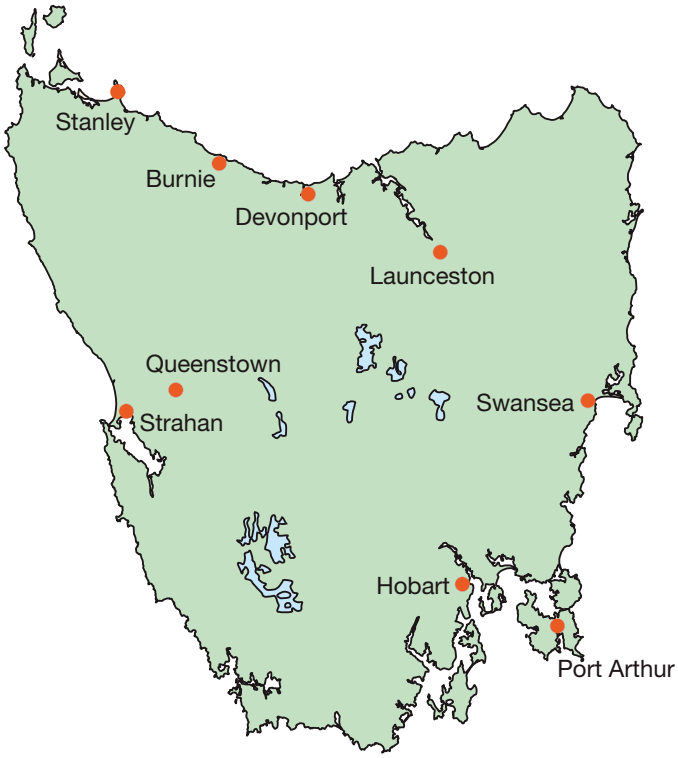


a Perth to Darwin
b Hobart to Sydney
c Brisbane to Melbourne
d Adelaide to Alice Springs

5 Some of these bearings are written incorrectly. Identify which are incorrect, comment on what is wrong with them and write them correctly (where possible).

a	N17°W	**b**	47°T
c	W21°S	**d**	173°T
e	N98°W	**f**	230°T
g	380°T	**h**	S81°W
i	182°	**j**	N44°E

6 Use the map of Tasmania to answer these questions.



a What is the compass bearing from:
- **i** Burnie to Hobart?
- **ii** Launceston to Devonport?

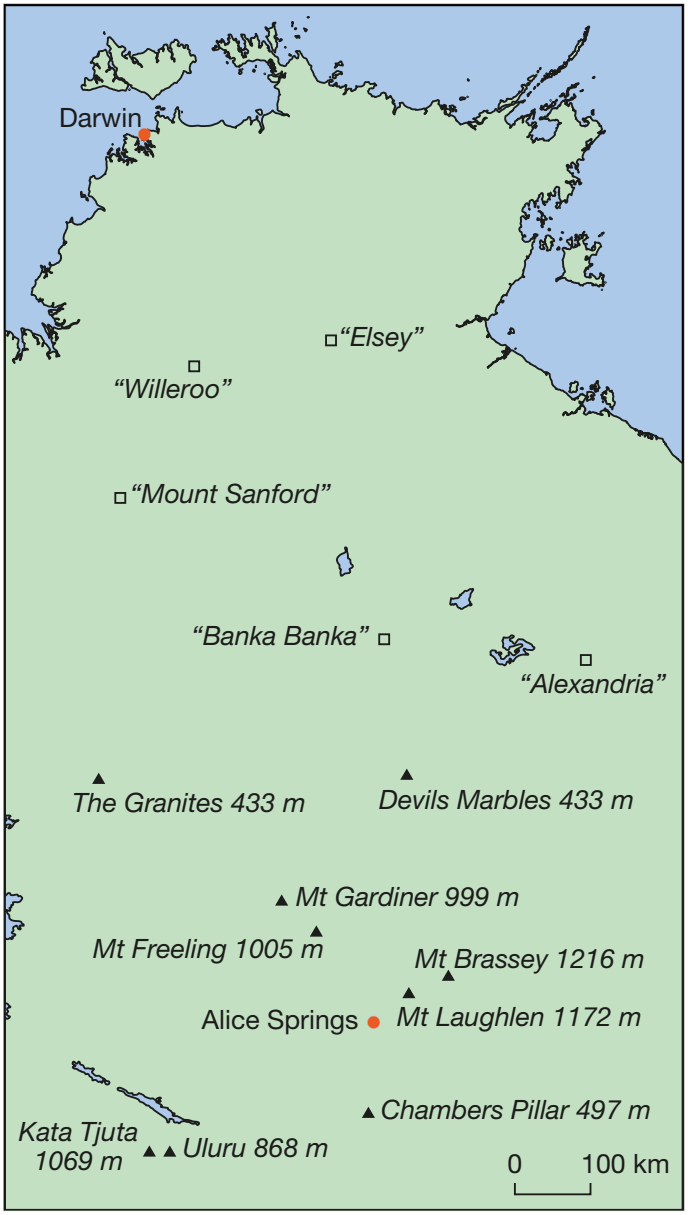
b What is the compass bearing from:
- **i** Hobart to Burnie?
- **ii** Devonport to Launceston?

c What is the difference between your answers to parts **a** and **b**?

d Repeat parts **a**–**c** but this time write your bearings as true bearings.
What do you find?



7 Use this map of the Northern Territory to answer these questions.



a What is the true bearing from Darwin to Alice Springs?

b What is the true bearing from Alice Springs to Darwin?

c What is the compass bearing from Uluru to Chambers Pillar?

d What landmark would you reach if you flew on a bearing of 270°T from the Devil's Marbles?

e What true bearing would you have to take to get back to the Devil's Marbles from this place?

f If you climbed Mt Gardiner and your friend climbed Mt Freeling at the same time, on what true bearing should you look to see your friend?

g On what true bearing should your friend look to see you?

h What is the difference between these two bearings? (Hint: calculate larger bearing − smaller bearing.)

i If you climbed Mt Laughlen and your friend climbed Mt Brassey, what bearings should each of you look to see each other?

j Perform the same calculation as in part **h** to find the difference between the two bearings.

k If the bearing from Kata Tjuta to Uluru is 095°T, what is the bearing from Uluru to Kata Tjuta?

l If I walk from point A to point B on a bearing of S12°W, what bearing will I take when I turn around and walk back the way I came, from point B to point A?

PROBLEM SOLVING AND REASONING



CHALLENGE

8 Jane is a vet in the Northern Territory. In one particularly busy day she needs to treat sick animals at the following locations, not necessarily in this order:

- Alexandria
- Mount Sanford
- Willeroo
- Banka Banka
- Elsey.

(Refer to the map opposite.)

a If Jane starts and ends in Darwin, plan the shortest route she can take for the day. Use the scale on the map to calculate how far each leg of her journey is in kilometres. Use this table to help you.

Start	End	Compass Bearing	True Bearing	Distance
Darwin				
	Darwin			

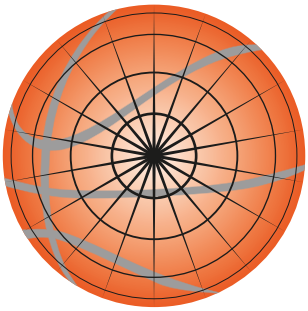
b Add the distance travelled by Jane for the day. Compare your total to your friend's total distance. Find out who in the class planned the shortest route.

Reflect

What do you think is the most common mistake in writing compass bearings? How can you avoid this?

6F Angles and time zones

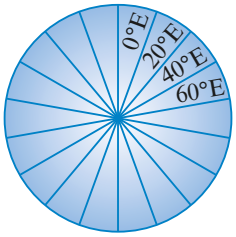
Start thinking!



Grids on maps give useful information about locations, but what if the area shown by the map isn't flat? Can you imagine trying to draw a square grid on a basketball?

1 If there are 18 lines drawn from the point at the centre of the diagram, how many degrees are between each line?

Maps of the Earth are drawn with grids just like this.

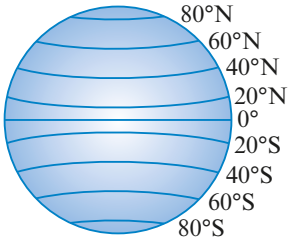


2 Draw a diagram of the Earth with one of its poles in the centre and showing lines similar to the basketball. Label the lines going clockwise (or east) starting with 0° and ending at 180° on the vertical line.

3 Now label the lines going anticlockwise as 20°W, 40°W, etc.

These lines form part of the grid used on maps of the Earth. They are called lines of **longitude** (or **meridians**). They go from 0° at the **prime meridian** to 180° on the other side of the world.

To complete the grid, lines parallel to the **equator** can be drawn. These are called lines of **latitude**. They are numbered with 0° at the equator and 90° at the poles.



4 Draw another diagram of the Earth and draw four lines of latitude in each hemisphere. Label these lines as 20°N, 20°S, etc.

On real globes, lines of longitude are often drawn 15° apart. This is the distance that the Earth rotates in 1 hour. Lines of latitude are also drawn 15° apart. This forms a grid which is square near the equator but almost triangular near the poles.

5 Why do you think the lines of latitude only go from 0° to 90° but the lines of longitude go from 0° to 180°?

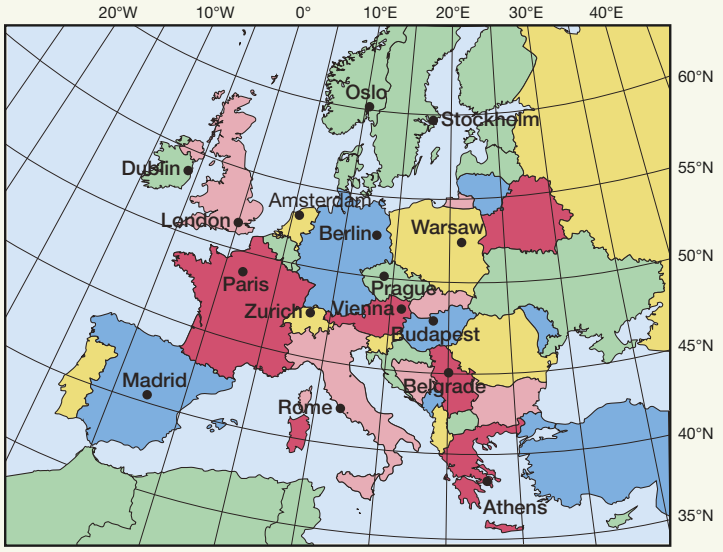
KEY IDEAS

- Maps show the Earth's surface divided into a grid using lines of latitude and longitude.
- Lines of latitude show as horizontal lines on a map. They run around the globe parallel to the equator. They start at 0° at the equator, and go up to 90° at the North or South Pole.
- Lines of latitude are labelled with an N or an S.
- Lines of longitude show as vertical lines on a map. They run from pole to pole. They start at 0° at Greenwich (London) and go to 180° on the other side of the world.
- Lines of longitude are labelled with either an E or a W.
- When giving the coordinates of a location, the latitude coordinate is always listed first (north/south), and the longitude is listed second (east/west).

EXERCISE 6F Angles and time zones

EXAMPLE 6F-1 Finding latitude and longitude

What European capital is located closest to 47°N 19°E?



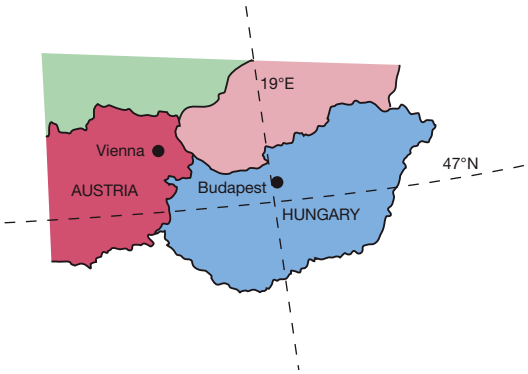
THINK

1 47°N is between 45°N and 50°N but closest to 45°N. Imagine the position of a line at 47°N.

2 19°E is between the lines for 15°E and 20°E, but closest to 20°E. Imagine the position of a line at 19°E.

3 Where the two imagined lines meet is the point 47°N19°E. The closest city is Budapest.

WRITE



Budapest is the capital city closest to 47°N 19°E.

Use the map of Europe above to answer questions **1**–**2**.

1 What European city is near these coordinates?

a 42°N 12°E **b** 53°N 13°E **c** 49°N 2°E **d** 45°N 20°E

2 Give coordinates (to the nearest whole degree) for these cities.

a Warsaw **b** Amsterdam **c** Prague **d** Zurich

Use this map for questions **3**–**7**. It shows Canada and part of the USA. The solid (not dotted) line shows the border between these two countries. Greenland, Iceland and part of Russia are also shown.



3 Which country would you be in if you stood at the following coordinates?

a 40°N, 120°W **b** 50° N, 70°W **c** 70°N, 40°W **d** 68°N, 180°

4 Give coordinates (to the nearest whole degree) for these cities.

a Ottawa (Ontario, Canada)
b Portland (Oregon, USA)
c Reykjavik (Iceland)
d Anchorage (Alaska, USA)

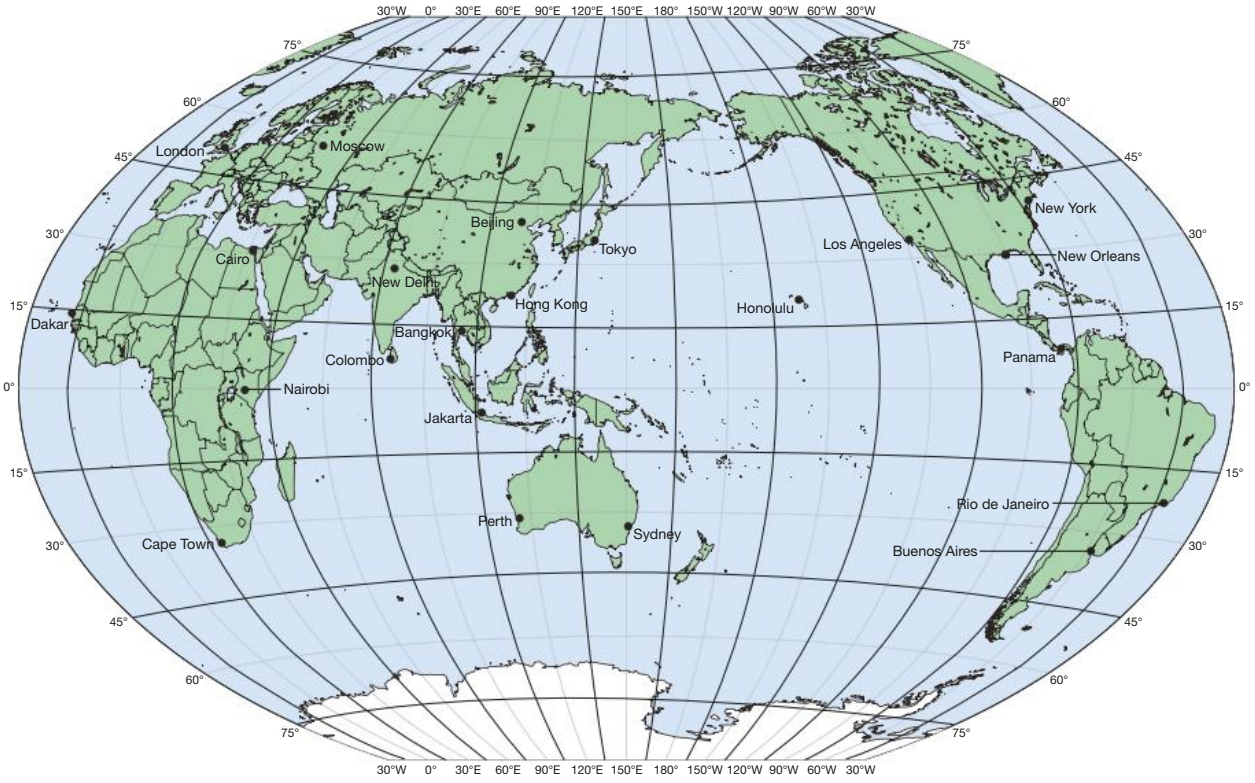
5 How many degrees of latitude is the Arctic Circle?

6 Sometimes lines of latitude and longitude are used to define borders between provinces, states and countries. Describe the lines used for these boundaries.

a Alberta–Northwest Territories
b Alberta–Saskatchewan
c Yukon–Alaska
d Oregon–California

7 Lines of latitude are sometimes called 'parallels'. Could lines of longitude also be described as parallel? Give an example from this map to show why or why not.

Use this world map to answer questions **8**–**10**.



8 A careless student recorded these coordinates for some major cities but left out the north/south and east/west information. Use the world map to complete these coordinates.

a Moscow 55° ___ 37° ___

b Perth 32° ___ 116° ___

c Los Angeles 34° ___ 118° ___

d Dakar 14° ___ 17° ___

e Bangkok 13° ___ 100° ___

f Buenos Aires 35° ___ 58° ___

9 Use the map of the world to match the major cities shown in this table with their latitude and longitude, correct to the nearest degree (the first one is shown).

10 Another careless student has made mistakes writing these coordinates. Explain why each is incorrect and write them correctly.

a 35°E 24°S

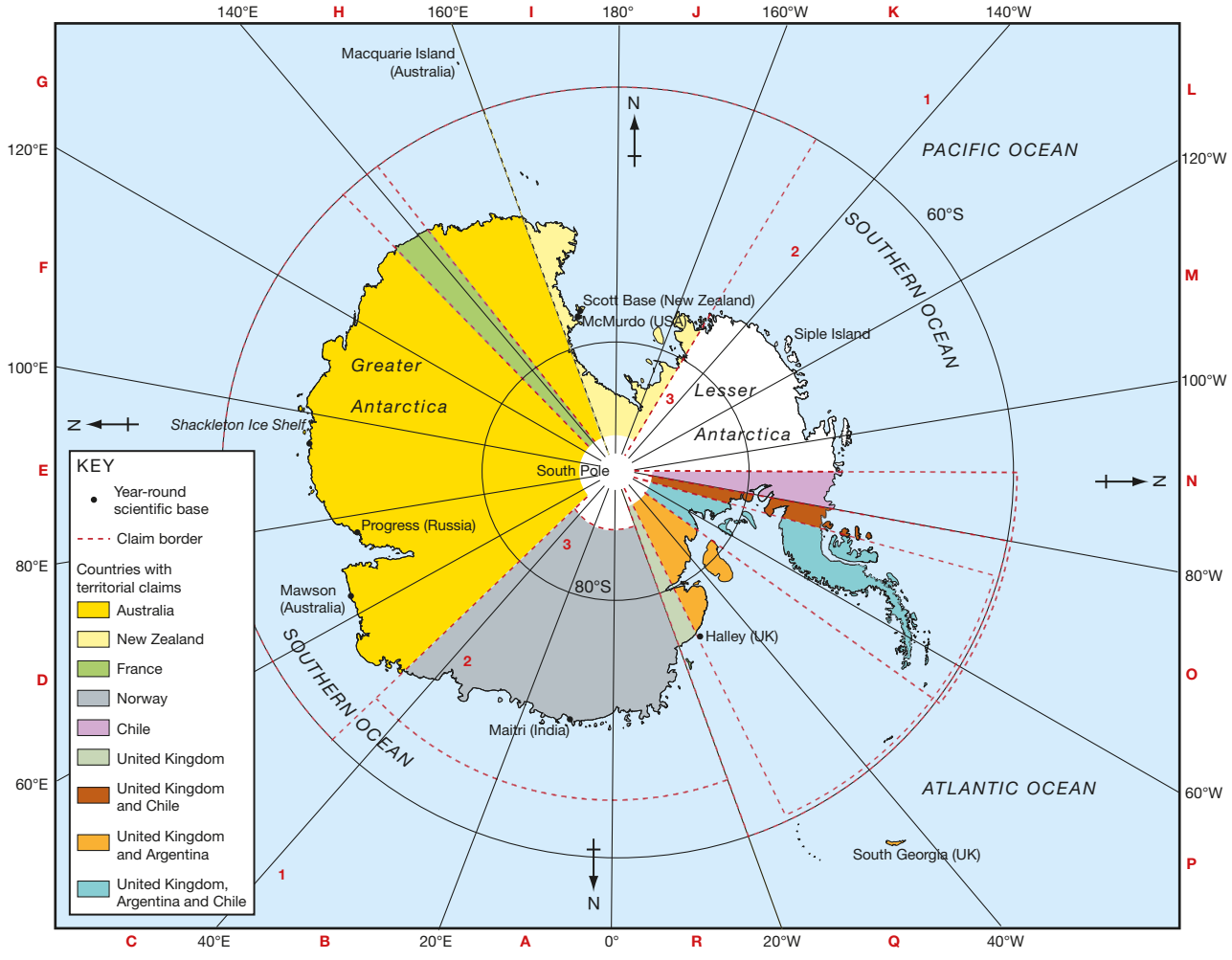
b 42°S 191°W

c 110°N 12°E

City	Latitude and longitude
Cairo	51°N 0°
Rio de Janiero	36°N 140°E
New Delhi	40°N 116°E
Nairobi	23°S 43°W
Honolulu	34°S 18°E
Colombo	22°N 114°E
Tokyo	28°N 77°E
London	30°N 31°E
Cape Town	21°N 158°W
Hong Kong	6°S 107°E
New Orleans	1°S 36°E
Sydney	41°N 74°W
New York	7°N 80°E
Beijing	9°N 80°W
Panama	30°N 90°W
Jakarta	34°S 151°E

11 Explain how knowledge of latitude and longitude can help you identify which 'quarter' of the world a city is in.

12 Look at this map of Antarctica.



a Why are there four North arrows on this map?

b Give the latitude and longitude of these scientific bases.

- **i** Progress (Russia)
- **ii** Mawson (Australia)
- **iii** Halley (UK)
- **iv** McMurdo (USA)

c Make a list of the countries with territorial claims to sections of Antarctica. Use a protractor to measure how many degrees each country has claim to.



d The alphanumeric grid references are quite unusual on this map. Note that the letters go all the way around the map border and the numbers increase moving towards the centre. For example, Siple Island is located at L2. Give the alphanumeric grid references for the following locations.

- **i** Mirny (Russia)
- **ii** Maitri (India)
- **iii** South Georgia (UK)
- **iv** Shackleton Ice-Shelf
- **v** Macquarie Island (Australia)

CHALLENGE

13 Use the map of New Zealand's South Island to answer the following. In this task, latitude and longitude are described to the nearest half a degree; for example, 43.0°S 172.5°E. You may need to use a ruler to help you.



a What would you find at the following latitude and longitude?

- **i** 47.0°S 168.0°E
- **ii** 41.5°S 174.0°E
- **iii** 44.5°S 168.5°E
- **iv** 46.0°S 169.0°E
- **v** 43.5°S 171.5°E
- **vi** 43.5°S 172.0°E

b What is the latitude and longitude of the following locations? Give your answer correct to the nearest half a degree.

- **i** Dunedin
- **ii** Christchurch
- **iii** Queenstown
- **iv** Ranfurly
- **v** Lake Brunner
- **vi** Darfield

14 Use the Internet or a detailed world map to write your own questions about latitude and longitude for a classmate.

Reflect

What mnemonic can you use to remember that lines of latitude run across a map and lines of longitude run up and down a map?

6G Working with time zones

Start thinking!

Refer to the map on the opposite page.



1 How many degrees does the Earth rotate in:
   a 1 day?
   b 1 hour?

Lines of longitude are used as the basis of **time zones**. Just like with lines of longitude, time zones are relative to Greenwich in London. The time in Greenwich is called **UTC** (also known as **Coordinated Universal Time**) or **GMT** (**Greenwich Mean Time**). Parts of the world to the east of Greenwich are ahead of UTC, parts of the world to the west are behind UTC. Sydney is in the UTC +10 zone and Los Angeles is in the UTC −8 zone. This means that when it is midday in London, it is 10 pm in Sydney and 4 am in Los Angeles. Use the map of time zones to answer these questions.

2 When it is night in Europe, where would it be day?

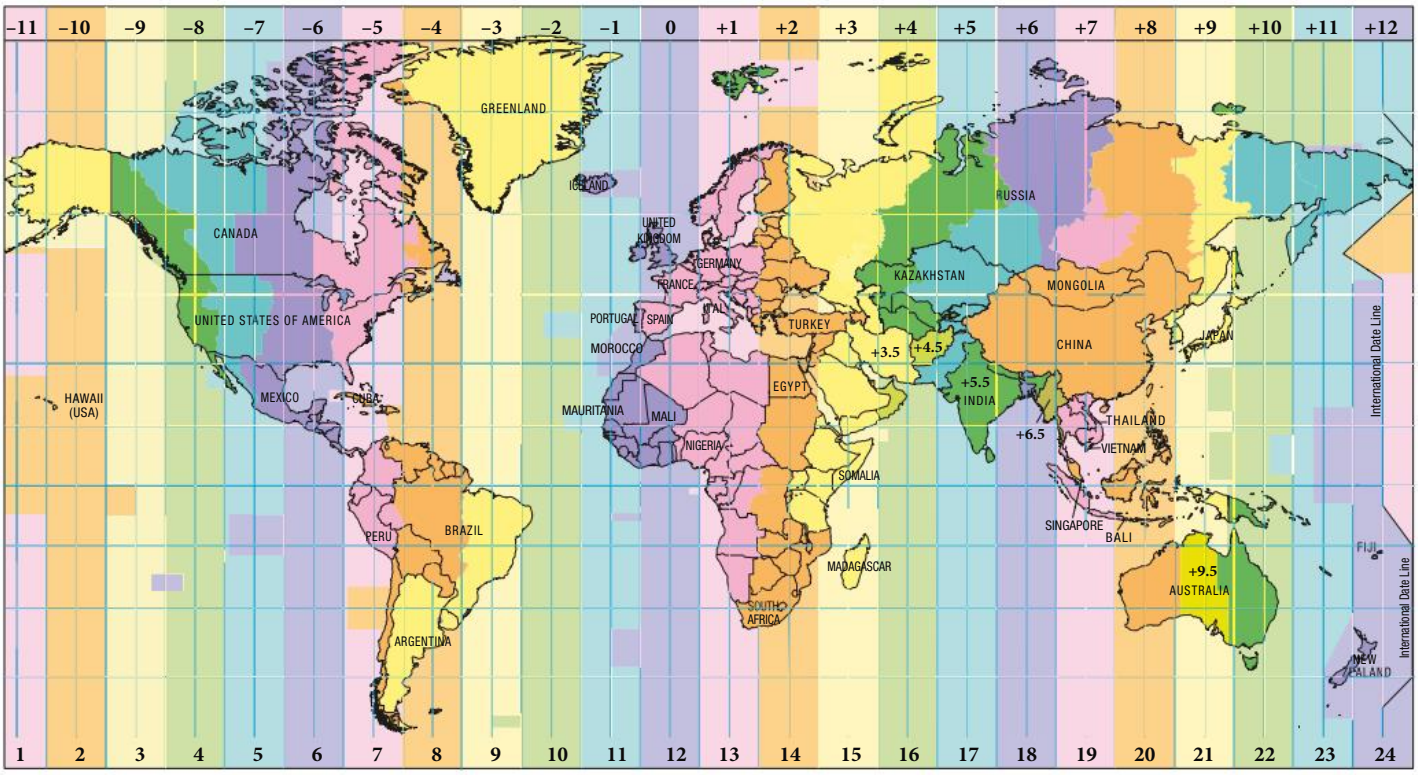
3 When it is midday in London, what time is it in the following countries?
   a Madagascar (GMT +3)
   b Singapore (GMT +7)
   c New Zealand(GMT +12)
   d Argentina (GMT -3)

4 Why do we need time zones?

KEY IDEAS

- There are 24 time zones. Australia covers three. All time zones are relative to the time at 0° of longitude (UTC).
- The **International Date Line** (**IDL**) is the line of longitude that separates UTC+12 and UTC−12. It is where the date changes by one day.
- **24-hour time** counts hours from 0 to 24 once a day instead of from 0 to 12 twice a day.
- To calculate elapsed time, consider hours and minutes separately.

EXERCISE 6G Working with time zones



EXAMPLE 6G-1 Converting between 12-hour and 24-hour time

Convert:

a 3.45 am to 24-hour time **b** 6.29 pm to 24-hour time **c** 1454 to 12-hour time

THINK

a 3.45 am means 3 hours 45 minutes after midnight.
Write it with four digits and no full stop.

b 6.29 pm means 6 hours 29 minutes after noon (12 pm).
Add 12 hours for the time from midnight to noon and write with four digits.

c 1454 means 14 hours 54 minutes after midnight.
Since it is more than 12 hours, it is a pm time.
Subtract 12 hours to find the time after noon.
Include a full stop between hours and minutes.

WRITE

a 3.45 am is 0345.

b 6 + 12 = 18
6.29 pm is 1829.

c 14 − 12 = 2
1454 is 2.54 pm.

1 Complete this table of 24-hour and 12-hour times.

12-hour time	24-hour time
7.30 am	0730
9.12 pm	
	1533
3.57 pm	
	2104
11.45 pm	

2 Calculate the time difference between these times.

- **a** 5.30 pm and 7.30 pm
- **b** 6.13 am and 10.35 am
- **c** 1530 and 2200
- **d** 4.39 pm and 11.22 pm
- **e** 1.49 am and 3.15 pm
- **f** 0700 and 1642

3 **a** How many hours ahead of the UK (time zone '0') is Hobart?

b How many hours ahead of the UK is Turkey?

c Use your answers to parts **a** and **b** to find the time difference between Hobart and Turkey.

4 **a** How many hours ahead of the UK is New Zealand?

b How many hours behind the UK is Peru?

c Use your answers to parts **a** and **b** to find the time difference between New Zealand and Peru.

d How is this calculation different from what you did in question **3c**?

5 What is the time difference between Perth and:

- **a** Madagascar?
- **b** Germany?
- **c** China?
- **d** the UK?
- **e** Fiji?
- **f** Eastern USA?
- **g** Western USA?
- **h** Iceland?

6 Find the time in each place listed in question **5** when it is 2 pm in Perth.

EXAMPLE 6G-2 Working with time zones

A flight from Perth to Melbourne takes 3 hours and 35 minutes. If you leave Perth at 1.50 pm, what time will it be in Melbourne when you arrive?

THINK

1 Add the hours and minutes separately. 1.50 pm plus 3 hours is 4.50 pm.4.50 pm plus 35 minutes is 5.25 pm.

2 Perth is in the UTC +8 time zone and Melbourne is in the UTC +10 time zone. Perth is 2 hours behind Melbourne, so add 2 hours to 5.25 pm.

WRITE

The flight lands at 5.25 pm (Perth time).

The flight lands at 7.25 pm (Melbourne time).

7 What will be the local time when these flights land at their destinations?

- **a** Sydney to Perth, departing Sydney at 10.30 am; flight time 4 hours and 55 minutes
- **b** Brisbane to Adelaide, departing Brisbane at 1315; flight time 2 hours and 45 minutes
- **c** Darwin to Melbourne, departing Darwin at 6.40 pm; flight time 4 hours and 10 minutes



8 At 1 pm UTC, people in London are having lunch. What might they be doing at the same time in?

- **a** Canberra?
- **b** New York?
- **c** New Delhi?

9 Look carefully at the world time zones map (on page 327) to answer these questions.

- **a** How many time zones does Canada have?
- **b** How many time zones does China have?
- **c** Why do you think these countries have made different decisions about time zones?

Countries do not always use the time zone in which they fall. Spain, for example, uses the time zone UTC +1, even though it falls geographically in the UTC 0 time zone.

- **d** Why do you think a country might do this?
- **e** List at least another three countries (or states) that do this.

10 The 2014 Soccer World Cup was in Brazil. If the final is played on Sunday night at 8 pm in Rio De Janeiro (UTC −3), what time was the live broadcast shown in Sydney?



11 The 2014 Soccer World Cup organisers scheduled the Germany vs France match so it was shown at 7 pm in those countries. What time did the game start in Rio?

12 Hugo the movie star finishes filming his latest movie at 5 pm in Paris. He has an appointment to meet the Queen at Buckingham Palace in London at 6 pm. Hugo's helicopter can take him there in exactly 60 minutes but Hugo would like a coffee and baguette first. Remembering that Paris is 1 hour ahead of London, does he have time to eat before he leaves?

13 The Boxing Day Ashes Test starts at 11 am in Melbourne. At what time do cricket fans in the UK have to get up to see the start of the match?

14 A national radio station in Sydney is broadcast live to the whole country at the same time. From 6 pm to 9 pm the station asks listeners to request songs. When should listeners in Western Australia and South Australia call in to make a request?

15 Copy and complete these sentences:

As you move east (to the right) across time zones you _____ the necessary number of hours to find the time.

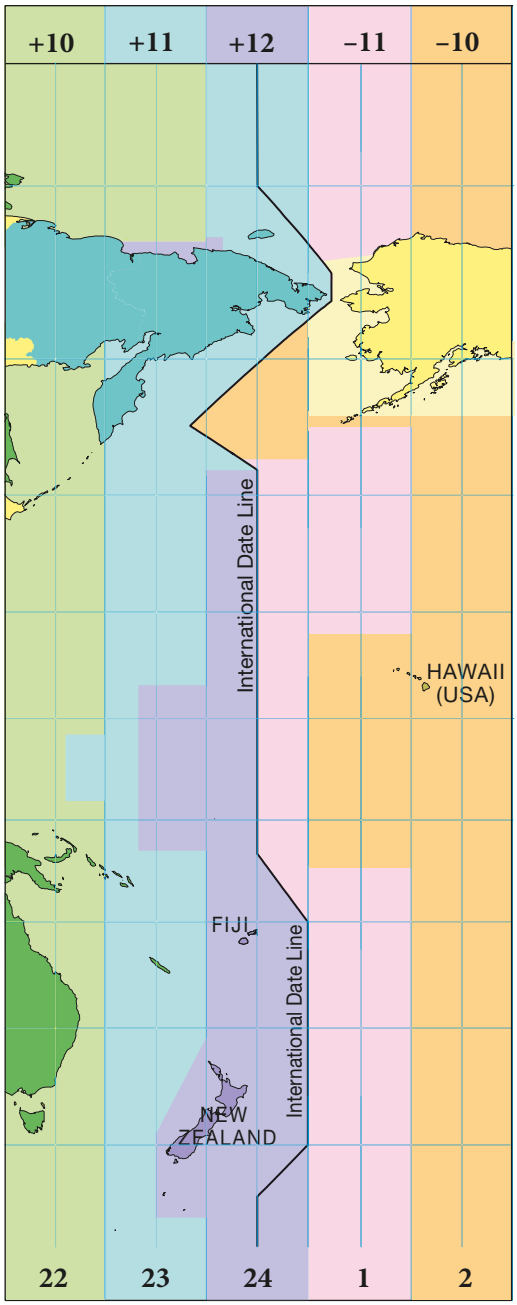
As you move _____ (to the left) across time zones you subtract the necessary number of hours to find the time.

16 Emmanuela is currently travelling in Peru and wants to call her sister, whose birthday is on Tuesday. She knows she needs to call before 8 am Hobart time to catch her sister before school.

- **a** What is the time difference between Peru and Hobart?
- **b** What time will it be in Peru when it is 8 am in Hobart?
- **c** What day will it be in Peru when it is 8 am in Hobart?
- **d** Explain why, if Emmanuela called at 4 pm on Tuesday, it would be too late to wish her sister happy birthday.
- **e** What time will it be in Hobart if Emmanuela calls her sister at 4.45 pm Monday Peru time?

17 The map on the right shows the International Date Line (IDL). It is where the date changes by one day.

- **a** On a globe, the IDL is directly opposite what other important line? (Hint: go back 12 time zones.)
- **b** Ignoring the IDL for the moment, what is the time difference if you move east from Fiji to Hawaii?
- **c** What is the time difference if you move west from Fiji to Hawaii?
- **d** Why do you think it might be important to have the IDL?
- **e** Explain why the IDL means that as you move east from Fiji to Hawaii, you move backwards in time.
- **f** As you move east across the IDL, do you think you add or subtract a day? Explain.
- **g** Write a few sentences explaining why, when you move west across the IDL, you are subtracting and adding time.



CHALLENGE

18 Why do you think that the IDL is not a straight line?

19 Mark is travelling around the world. His mother, in Townsville, Queensland, likes Mark to call at midday every Sunday. What time (and day) should Mark call his mother when he is in these countries?

a Peru **b** Mexico
c Japan **d** Madagascar
e France **f** Scotland (UK)
g China **h** Hawaii (USA)

20 Do you think all the times in question **19** are reasonable for Mark to call? Is there a single time for Mark to always call his mother that might be easier? Discuss.

21 What is the local time when these flights land at their destination?

a London to Moscow, departing London at 3.32 pm Saturday; flight time 3 hours 7 minutes
b Istanbul (Turkey) to Beijing, departing Istanbul at 1605 Monday; flight time 8 hours 47 minutes
c Sydney to Los Angeles (UTC −8), departing Sydney at 11.00 pm Thursday; flight time 13 hours 30 minutes
d Johannesburg (South Africa) to New York (UTC −5), departing Johannesburg at 2035 Wednesday; flight time 16 hours and 5 minutes





22 Calculating time differences can also be affected by Daylight Saving Time (DST). In Australia, only ACT, NSW, Tasmania, South Australia and Victoria use DST. Investigate who uses DST around the world. How would this affect your answers to questions **19** and **21** in:

a summer? **b** winter?

Reflect

Why do time zones usually cover a difference in longitude of 15°?

CHAPTER REVIEW

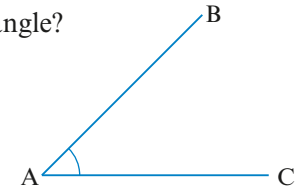
SUMMARISE

Create a summary of this chapter using the key terms below. You may like to write a paragraph, create a concept map or use technology to present your work.

acute angle	supplementary	SSS	prime meridian
obtuse angle	vertically opposite	SAS	equator
reflex angle	parallel	ASA	UTC
right angle	transversal	bearings	GMT
straight angle	alternate	compass rose	time zones
revolution	corresponding	true bearing	24-hour time
vertex	co-interior	compass bearing	IDL
arms	angle of elevation	latitude	
complementary	angle of depression	longitude	

MULTIPLE-CHOICE

6A **1** What is the correct name for this angle?



A ∠ABC
B CAB
C ∠BAC
D ∠ACB

6B **2** Which statement is *not* correct?

A Vertically opposite angles are equal in size.
B Complementary angles add to 180°.
C Two supplementary angles make a straight angle.
D Angles around a point make a revolution.

6C **3** For a pair of parallel lines intersected by a transversal, which pair of angles could be alternate angles?

A 32° and 32° **B** 25° and 65°
C 12° and 192° **D** 55° and 125°

6D **4** Which method would you use to draw triangle ABC with AB = 6 cm, AC = 9 cm and ∠CAB = 30°?

A SAS **B** ASA
C SSS **D** SSA

6E **5** Which is a correct compass bearing?

A E10°N **B** W21°S
C S98°E **D** N37°W

6F **6** What is the most likely latitude and longitude coordinates of Hobart, Tasmania?

A 59°N 132°W **B** 43°S 147°E
C 85°S 19°E **D** 25°N 51°W

6G **7** Which of these is 6.15 pm in 24-hour time?

A 0615 **B** 1815
C 1615 **D** 18.15

SHORT ANSWER

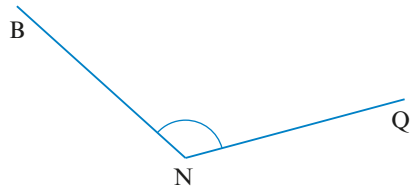
6A **1** Use a ruler and a protractor to draw each angle.

a ∠FLB = 63° **b** ∠UKA = 155°

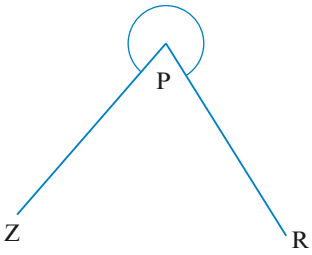
c ∠CHL = 221° **d** ∠XYZ = 300°

6A **2** Measure each angle.

a

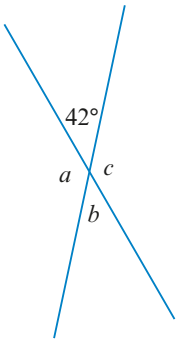


b

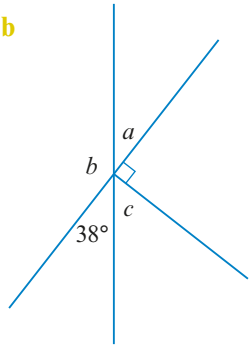


6B **3** Without using a protractor find the size of each unknown angle.

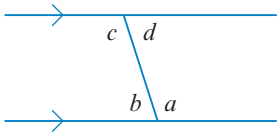
a



b



6C **4** Use this diagram to find:



a the angle co-interior to a

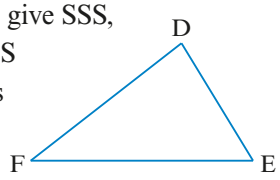
b the angle alternate to b

c the size of c and d if b is 72°.

6D **5** **a** Draw ΔQRS with QR = 7.5 cm, ∠SQR = 32° and ∠SRQ = 44°.

b Draw ΔTUV with TU = 8 cm, TV = 9 cm and UV = 7 cm.

c Use a ruler and a protractor to give SSS, ASA and SAS specifications for this triangle.



6E **6** Use the map on page 316 to give compass and true bearings between each pair of cities.

a Melbourne to Adelaide

b Alice Springs to Perth

c Darwin to Hobart

d Sydney to Brisbane

Question **7** refers to this map.



6F **7** Give latitude and longitude coordinates for each city.

a San Francisco **b** Salt Lake City

c Miami **d** Boston

e Portland **f** Phoenix

6G **8** Refer to the time zone map on page 327.

a If it is midday in London, what time is it in:

i Japan?

ii France?

iii Mexico?

b When it is 4 pm in Melbourne, what time is it in:

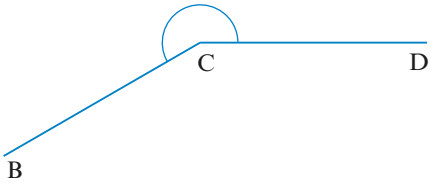
i Mali?

ii India?

iii Madagascar?

NAPLAN-STYLE PRACTICE

Questions 1–3 refer to this angle.



1 What is one name for the angle?

- ☐ ∠CDB
- ☐ ∠DBC
- ☐ ∠CBD
- ☐ ∠DCB

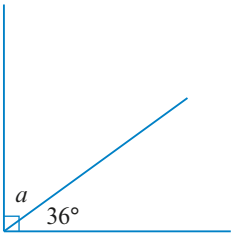
2 What type of angle is shown?

- ☐ an acute angle
- ☐ a reflex angle
- ☐ a right angle
- ☐ an obtuse angle

3 What is the size of the angle?

330° ☐ 230° ☐ 210° ☐ 150° ☐

Questions 4 and 5 refer to this figure.



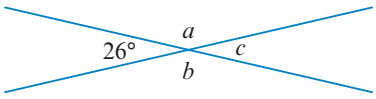
4 What is the value of a?

☐

5 What name could you give these angles?

- ☐ vertically opposite
- ☐ supplementary
- ☐ alternate
- ☐ complementary

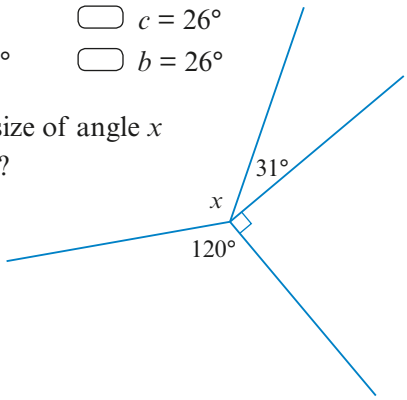
6 A number of angles are created when two lines cross. Which of these is *not* true?



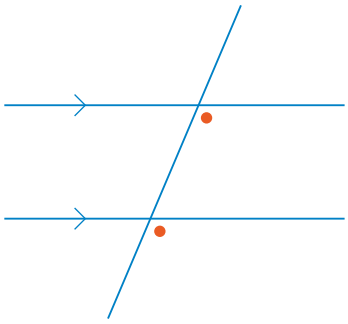
- ☐ $a = b$
- ☐ $c = 26°$
- ☐ $a = 154°$
- ☐ $b = 26°$

7 What is the size of angle x in this figure?

☐



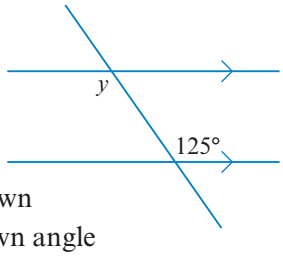
8 Which statement best describes this figure?



- ☐ the angles are corresponding
- ☐ the angles are co-interior
- ☐ the angles are alternate
- ☐ the angles are supplementary

9 What is the size of angle y in this figure?

☐



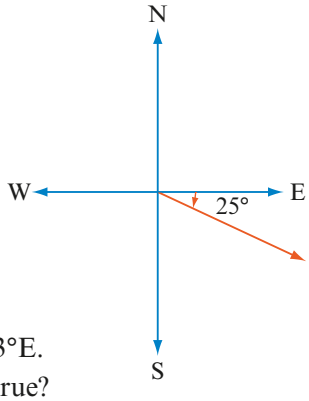
10 A triangle has two known side lengths and a known angle between them. Which description matches this statement?

SSS ☐ ASA ☐ SAS ☐ AAS ☐

11 What is N52°W as a true bearing?

308°T ☐ 352°T ☐ 052°T ☐ 248°T ☐

12 What bearing is shown here?



- ☐ E25°S
- ☐ S65°E
- ☐ S25°E
- ☐ 025°T

13 Brisbane is near the coordinates 27°S 153°E. Which statement is true?

- ☐ the line of longitude is 153°E
- ☐ the line of latitude is 153°E
- ☐ the equator runs through Brisbane
- ☐ the line of longitude is 27°S

14 What are the coordinates for a location with a latitude of 40° north of the equator and a longitude of 14°E?

- ◯ 40°N 14°E
- ◯ 40°E 14°N
- ◯ 40°S 14°E
- ◯ 40°N 40°E

15 How many minutes are there between 11.48 am and 3.15 pm?

Questions **16**–**18** refer to this information.

Mexico City is 6 hours behind UTC. Darwin is 9.5 hours ahead of UTC.

16 How many hours difference is there between Mexico City and Darwin?

17 If it is 3 pm Saturday in Mexico City, what is the time and day in Darwin?

- ◯ 12.30 am Saturday
- ◯ 11.30 pm Friday
- ◯ 11.30 pm Saturday
- ◯ 6.30 am Sunday

18 If it is 3 pm Saturday in Darwin, what is the time and day in Mexico City?

- ◯ 12.30 am Saturday
- ◯ 11.30 pm Friday
- ◯ 11.30 pm Saturday
- ◯ 6.30 am Sunday

19 Auckland (New Zealand) is 2 hours ahead of Sydney (Australia). A plane leaves Sydney at 1 pm and takes 3 hours and 45 minutes to reach Auckland. What is the time in Auckland when the plane lands?

- ◯ 4.45 pm
- ◯ 2.45 pm
- ◯ 6.45 pm
- ◯ 5.45 pm

ANALYSIS

Inspector Claude of Interpol is on the trail of the international art thief known as the Hyena. His pursuit leads him from London to Rome, from Rome to Prague and from Prague back to London. Use the map on page 321 to answer these questions.

a What are the three cities' latitudes and longitudes?

b Using the lines of longitude as north–south lines, what is the true bearing for each of these flights?

- **i** London to Rome
- **ii** Rome to Prague
- **iii** Prague to London

c Inspector Claude's flight from London to Rome takes 2 hours and 25 minutes. If he leaves London at 9 am, what time will it be when he lands in Rome?

d Before Interpol will pay for his flights, Inspector Claude needs to submit a map of where he travelled. Use a ruler and protractor to draw the triangle LRP (London, Rome, Prague). Decide if you will use the SSS, ASA or SAS method. Sketch the European coast line around your triangle and remember to make sure all the bearings are accurate.

e The Inspector spends a few hours in each city before finding the clue that reveals the Hyena's next location. Below are the details of how long he spends in each city and the duration of each flight.

London to Rome	2 h 25 min
Investigating Rome	2 h 17 min
Rome to Prague	1 h 50 min
Investigating Prague	1 h 35 min
Prague to London	1 h 45 min

How much time (in hours and minutes) has passed before Inspector Claude returns to London?

f In Prague, the Inspector learns that the Hyena plans to bring a package to London airport at 7 pm. If Inspector Claude first left London at 9 am, will he make it back there in time to catch the Hyena?

CONNECT

Around the world

A new airline runs a competition in which you have to design a round-the-world ticket. The winner of the most efficient and exciting trip will win plane tickets for their flights around the world. Use your understanding of angle and location to plan your ideal round-the-world trip.



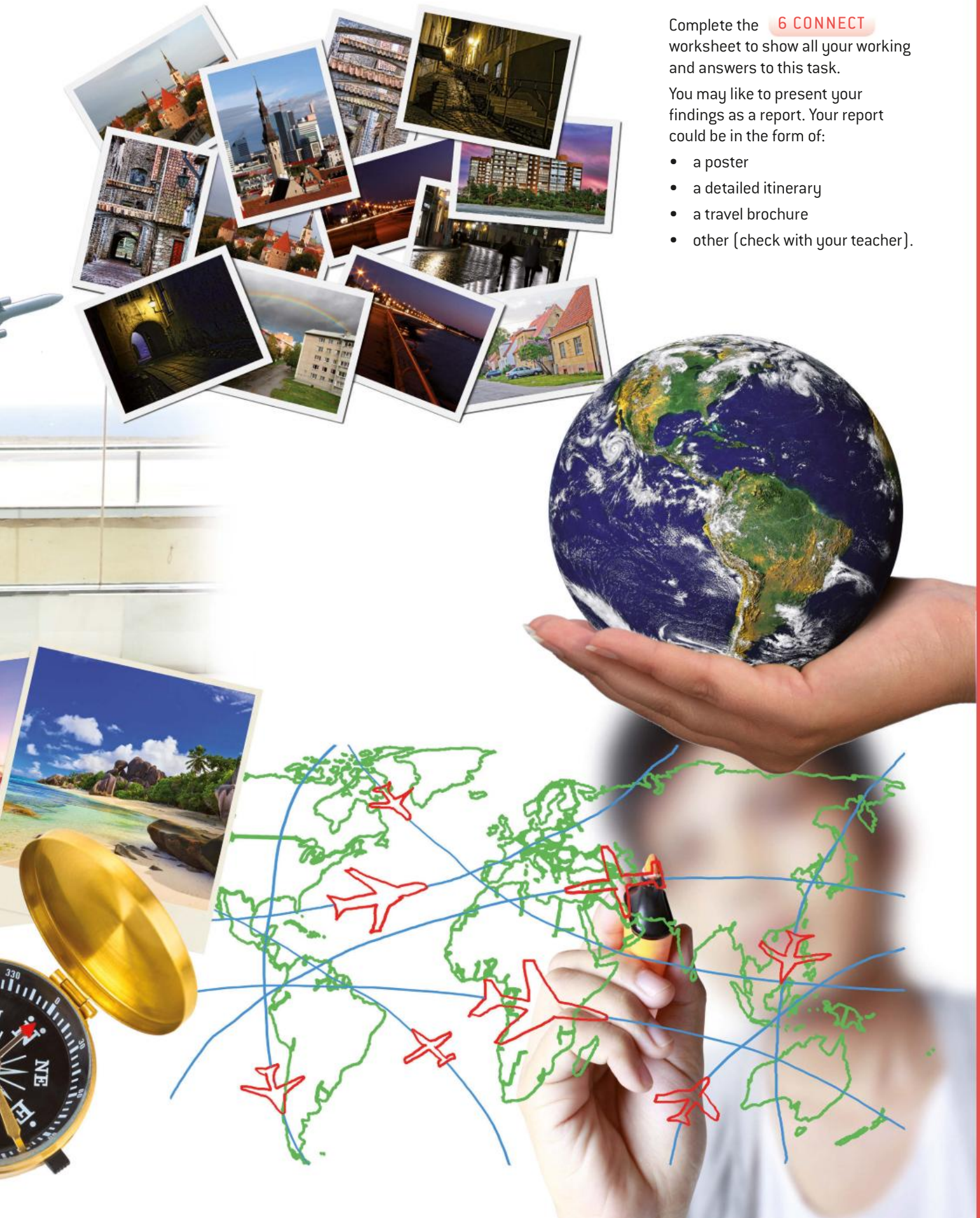


Your task

Create a round-the-world trip with five stops, using what you have learned in this chapter. You will need to consider:

- where you might like to go
- length and number of flights
- direction and bearings of flights
- time zones for making phone calls
- what makes a trip efficient
- what makes a trip interesting.





Complete the 6 CONNECT worksheet to show all your working and answers to this task.

You may like to present your findings as a report. Your report could be in the form of:

- a poster
- a detailed itinerary
- a travel brochure
- other (check with your teacher).

7 SHAPES AND OBJECTS

7A Triangle properties

7B Quadrilateral properties

7C 2D shapes and 3D objects

7D Isometric drawings and plans

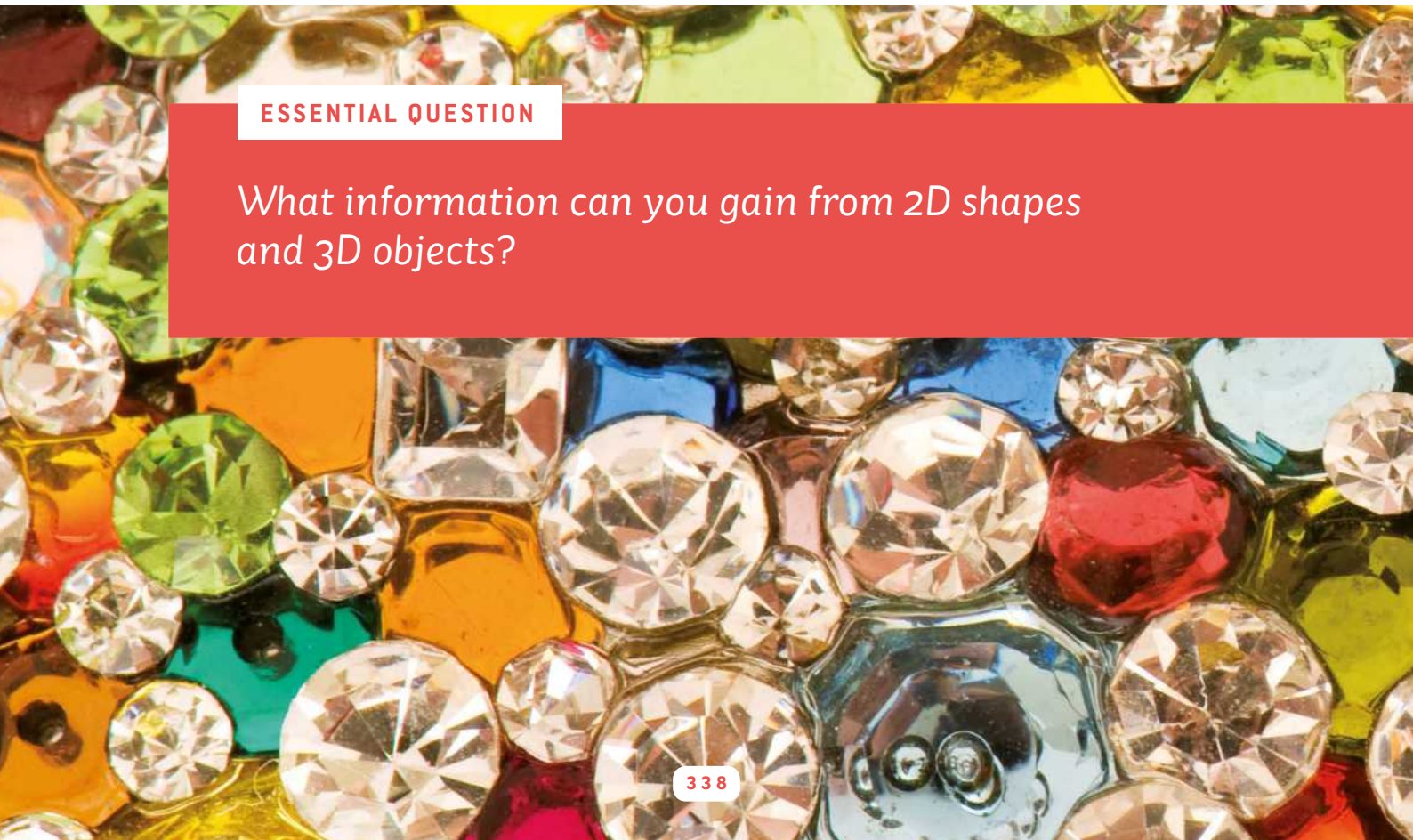
7E Nets and perspective drawings

7F Translations, rotations and reflections

7G Understanding congruence

7H Using congruence

7I Dilations



ESSENTIAL QUESTION

What information can you gain from 2D shapes and 3D objects?

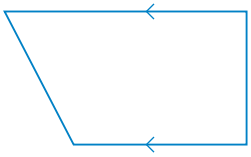
Are you ready?

7A **1** How many degrees in a triangle?

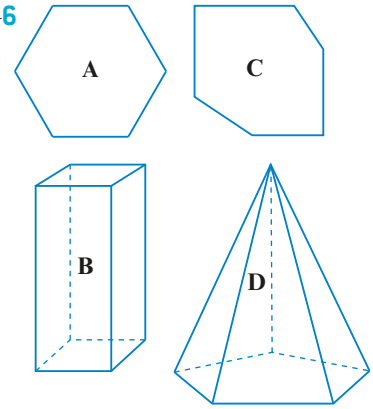
7A **2** How many degrees in a right angle?

7B **3** What is this shape?

A trapezium
B irregular quadrilateral
C rhombus
D kite



Questions **4–6** refer to these shapes and objects.



7C **4** Which of these is a regular polygon?

A A **B** B **C** C **D** D

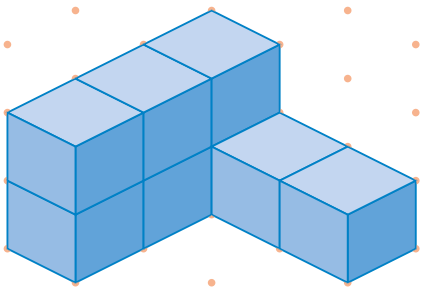
7C **5** What is shape C?

A regular hexagon
B regular heptagon
C irregular hexagon
D irregular heptagon

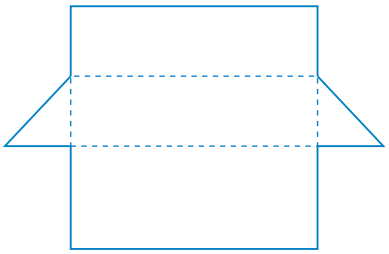
7C **6** Which of these is a pyramid?

A A **B** B **C** C **D** D

7D **7** How many cubes of this object can be seen from the top?

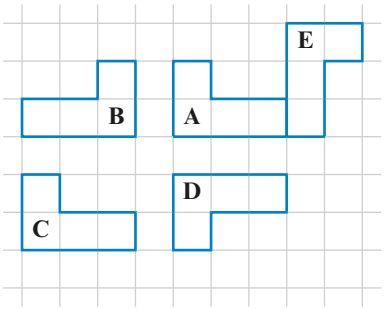


7E **8** Which object does this net represent?



A cube
B rectangular prism
C triangular prism
D pyramid

7F **9** Shapes B, C, D and E are all transformations of shape A.



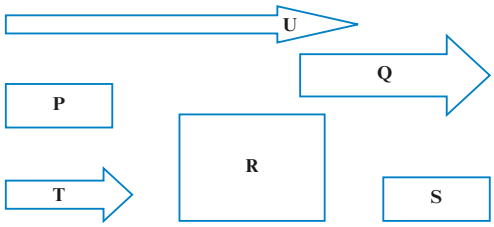
a Which shape is a rotation of A?

A B **B** C **C** D **D** E

b What has been done to produce B?

A translation **B** rotation
C reflection **D** dilation

7G **10** Look at these shapes.



a Which shape is identical to P?

A Q **B** R **C** S **D** T

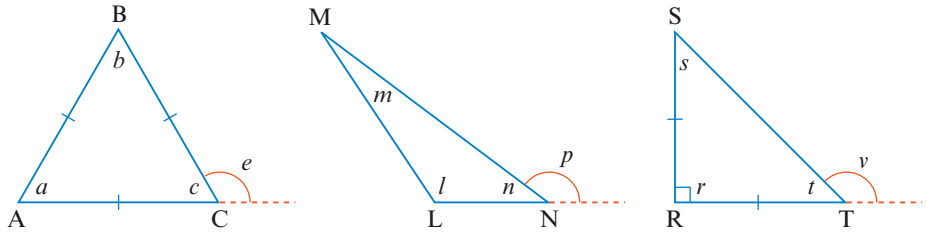
b Which shape is an enlargement of T?

A P **B** Q **C** R **D** S

7A Triangle properties

Start thinking!

A **triangle** is a 2D (two-dimensional) shape with three sides. Look at these three triangles (ignore the orange lines for the moment).



1 What is the angle sum of a triangle?

2 Use a protractor to measure each angle of these triangles to prove your answer to question **1**.

Copy the triangles and mark in these angles.

These triangles have each had one side extended and an **exterior angle** drawn in, shown in orange. Focus on the **scalene triangle**.

3 How many degrees are there in a straight line?

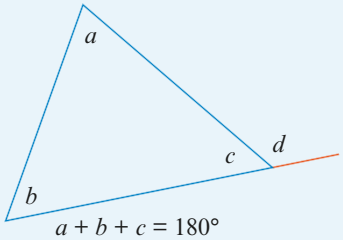
4 What is the size of the angle n? You should have already found this in question **2**.

5 Use your answers from questions **3** and **4** to find the size of the exterior angle, p.

6 Add together the angles l and m. How does this total relate to the size of angle p?

KEY IDEAS

- A triangle is a 2D shape with three sides.
- Triangles can be classified according to their side lengths (**equilateral**, **isosceles** or scalene) or angle size (**acute-angled**, **right-angled** or **obtuse-angled**).
- The internal or interior angles of a triangle add to 180°.
- The exterior angle of a triangle is equal to the sum of the two opposite interior angles.

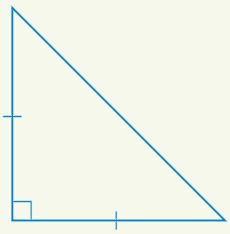


$a + b + c = 180°$
$d = a + b$

EXERCISE 7A Triangle properties

EXAMPLE 7A-1 Naming triangles

Name this triangle according to its sides and angles.



THINK	WRITE
1 The triangle has two sides equal in length, so it is an isosceles triangle.	
2 The triangle has a right angle, so it is a right-angled triangle.	
3 Write your answer.	The triangle is a right-angled isosceles triangle.

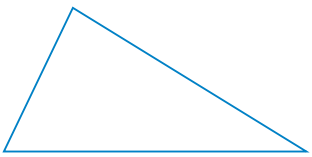
UNDERSTANDING AND FLUENCY

1 Name each triangle according to its sides and angles.

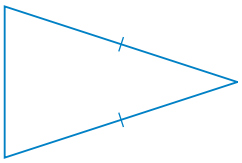
a



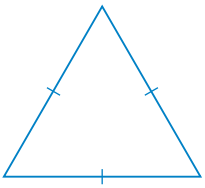
b



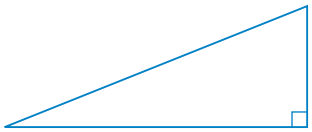
c



d



e

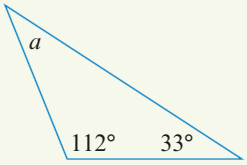


f



EXAMPLE 7A-2 Finding an internal angle in a triangle

Find the size of angle a.

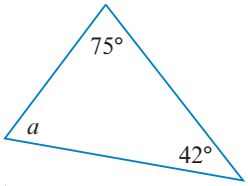


THINK	WRITE
1 Find the sum of the two known angles.	$33° + 112° = 145°$
2 There are 180° in a triangle. Subtract 145° from 180° to find a.	$a = 180° - 145°$ $= 35°$

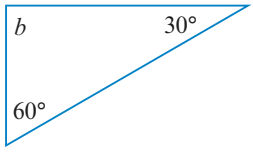
UNDERSTANDING AND FLUENCY

2 Find the size of each unknown angle.

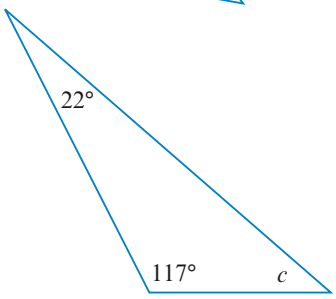
a



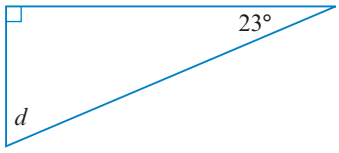
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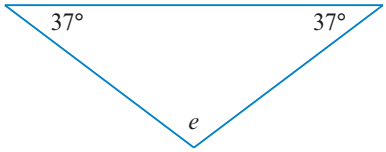
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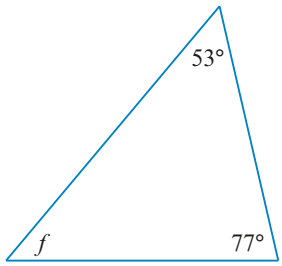
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e

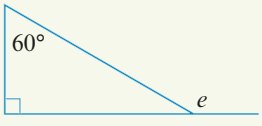


f



EXAMPLE 7A-3 Finding the exterior angle of a triangle

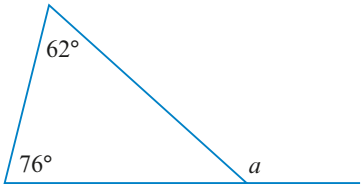
Find the size of angle e.



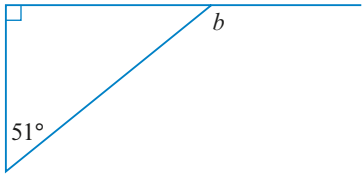
THINK	WRITE
The exterior angle of a triangle is equal to the sum of the two opposite interior angles. Add these together.	$e = 60° + 90°$ $= 150°$

3 Find the size of each labelled unknown angle.

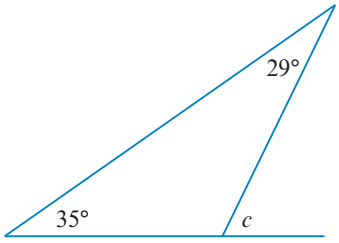
a



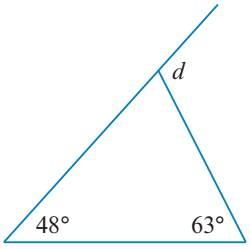
b



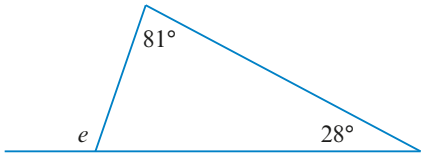
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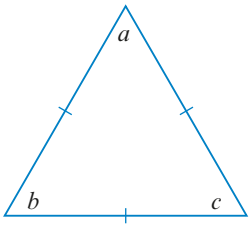


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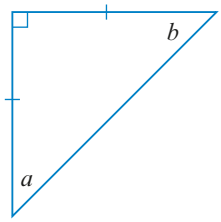


4 Use your knowledge of triangle properties to find the size of each labelled unknown angle.

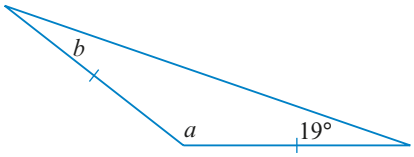
a



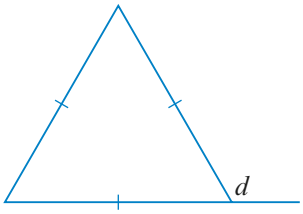
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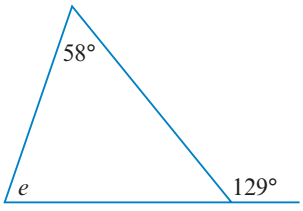
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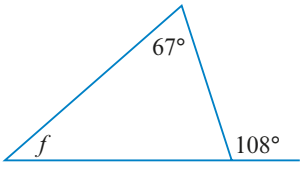
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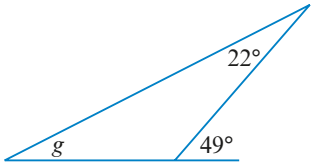
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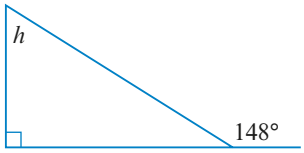
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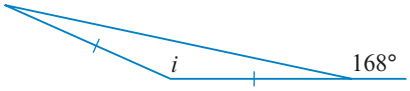
g



h



i



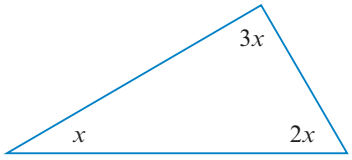


5 One of the best known origami models is the crane, pictured here. What types of triangles can you see in it?

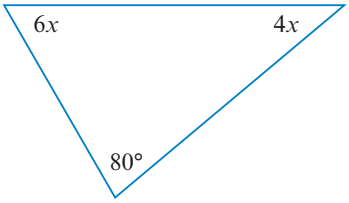
6 Investigate the relationship between the different types of triangles by looking at their side lengths and angles. Can an equilateral triangle have a right angle? Can an isosceles triangle have an obtuse angle? You might like to create a table listing the side types in columns and the angle types in rows to show these relationships.

7 Use your knowledge of triangle properties and basic algebra to find the value of x in each triangle.

a



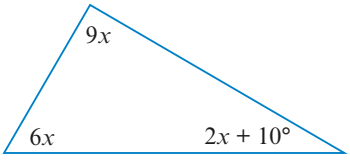
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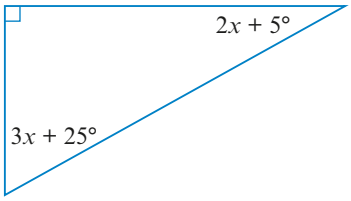
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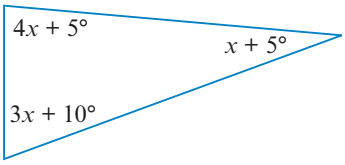
d



e



f



8 Rebecca measures the angles in a triangle and finds they are in a ratio of 1:2:3. That is, angle b is twice as big as angle a and angle c is three times as big as angle a.

- **a** Find the size of each angle in this triangle.
- **b** Classify this triangle.
- **c** Write your own riddle about a triangle and give it to a classmate to solve.



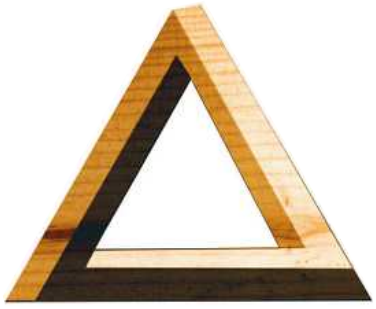
9 Draw three triangles that:

- **a** have at least one angle measuring 40°
- **b** have an obtuse angle
- **c** have two equal sides.

10 Adele bought this pendant for a necklace. The shop assistant promised it was an equilateral triangle, but Adele wasn't sure. Use a ruler and a protractor to find what type of triangle it is.

PROBLEM SOLVING AND REASONING

11 The Penrose triangle is a famous and impossible figure.



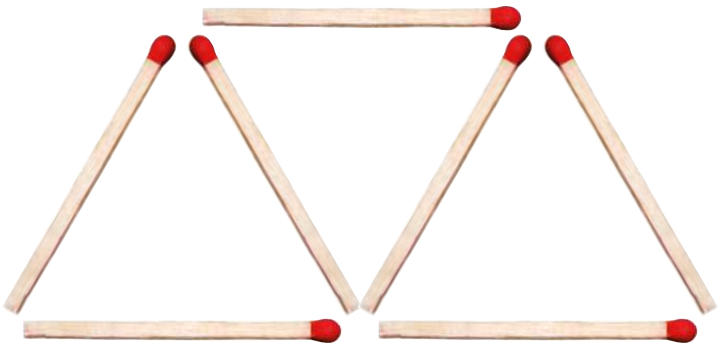
- **a** State why it is impossible.
- **b** A classmate was given a number of triangles to construct, but is having trouble drawing them. For each set of instructions, draw a diagram of the triangle or provide a reason why it cannot be drawn.
  - **i** a triangle with internal angles of 90°, 50° and 60°
  - **ii** a triangle with two internal angles of 40°
  - **iii** a triangle with side lengths of 10 cm, 3 cm and 5 cm
  - **iv** a right-angled isosceles triangle with two side lengths of 4 cm and one of 5 cm
  - **v** a triangle with two obtuse angles
- **c** Describe the conditions you must follow when you draw a triangle.

CHALLENGE

12 The sum of the internal angles of a triangle is 180°, but what can you say about the sum of the exterior angles? Follow these steps to find out.

- **a** Draw three different triangles (one scalene, one isosceles and one equilateral).
- **b** Consider the equilateral triangle.
  - **i** Mark in all internal angles.
  - **ii** Extend each side and mark in each exterior angle.
  - **iii** Find the size of each of the exterior angles and add them together. What do they add to?
- **c** Repeat part **b** for your other two triangles.
- **d** What can you say about the sum of the exterior angles of a triangle?

13 You can construct equilateral triangles using matches, as shown here.



- **a** How many matches were used for these three triangles?
- **b** Use:
  - **i** nine matches to construct four equilateral triangles
  - **ii** nine matches to construct five equilateral triangles
  - **iii** six matches to construct four equilateral triangles.
- **c** Use matches to construct each of the other five triangle types. Draw these in your book, stating how many matches you use for each side length.

Reflect

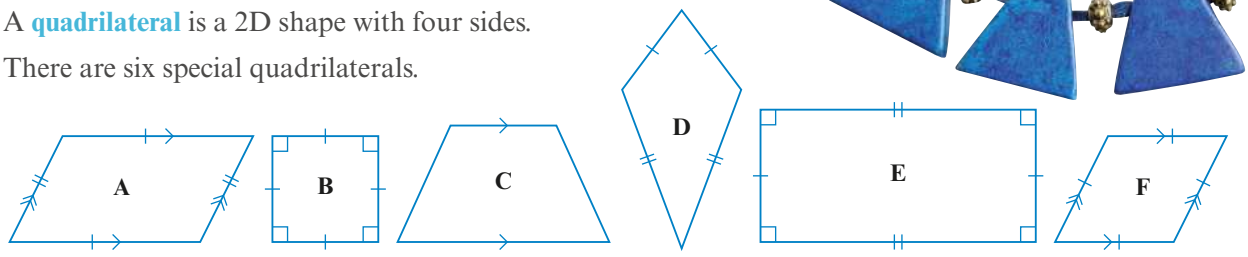
How is knowledge of supplementary angles useful when working with triangles?

7B Quadrilateral properties



Start thinking!

A **quadrilateral** is a 2D shape with four sides.
There are six special quadrilaterals.



1 Draw this table and add another five columns. Label the columns: Rectangle, Square, Rhombus, Trapezium and Kite. Draw the shape in the first row using the diagrams above.

2 For the parallelogram, look at each statement in the first column and tick the ones that apply.

3 Complete the table, looking at one shape at a time.

4 Compare your answers to a classmate's and discuss any differences.

5 What two conditions does a parallelogram meet?

6 What other shapes also meet these conditions?

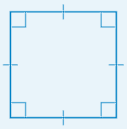
	Parallelogram
Drawing of shape	
All angles are equal	
All sides are equal	
Opposite sides are equal	
Opposite sides are parallel	
One pair of opposite sides are parallel	
Opposite angles are equal	
One pair of angles are equal	
Diagonals meet at right angles	

7 Use your answers to questions **5** and **6** to explain why a square is a type of parallelogram but a parallelogram is not a type of square.

8 Make a list of other special relationships that you can see.

KEY IDEAS

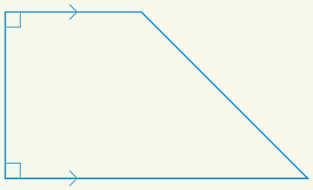
- A quadrilateral is a 2D shape with four sides.
- Some quadrilaterals have special properties involving sides and angles. These are the **parallelogram**, **rectangle**, **square**, **rhombus** (also known as a diamond), **trapezium** and **kite**.
- Quadrilaterals without special properties are known as irregular quadrilaterals.
- The angles in a quadrilateral add to 360°.



EXERCISE 7B Quadrilateral properties

EXAMPLE 7B-1 Classifying quadrilaterals

Classify this quadrilateral.



THINK

Identify the quadrilateral by considering its properties.
The quadrilateral has one pair of parallel sides.

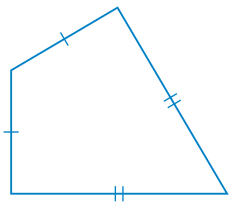
WRITE

The quadrilateral is a trapezium.

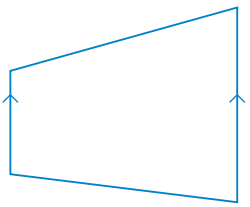
UNDERSTANDING AND FLUENCY

1 Classify each quadrilateral. Give a reason for your answer.

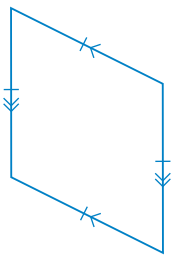
a



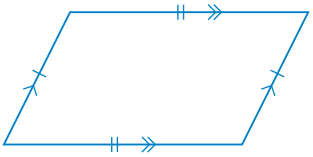
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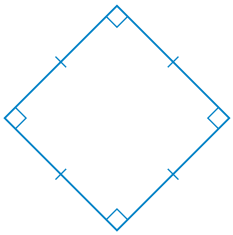
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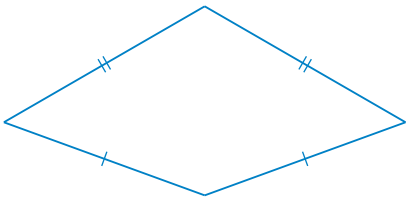
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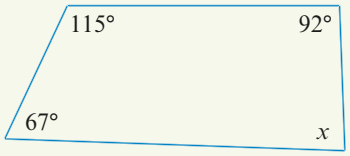


f



EXAMPLE 7B-2 Finding an internal angle in a quadrilateral

Find the size of angle x.



THINK

1 Find the sum of the three known angles.

2 Angles in a quadrilateral add to 360°. Subtract 274° from 360° to obtain x.

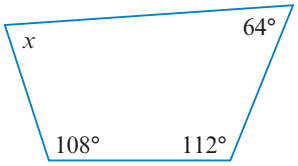
WRITE

$115° + 92° + 67° = 274°$

$x = 360° - 274°$
$= 86°$

2 Find the size of each angle x.

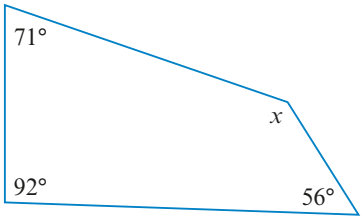
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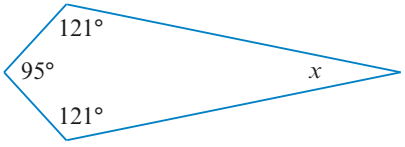
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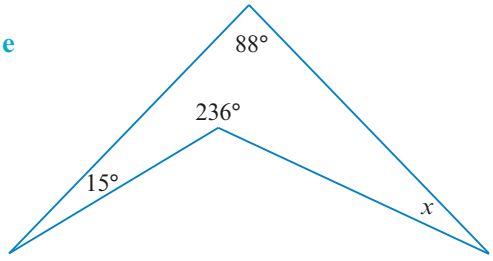
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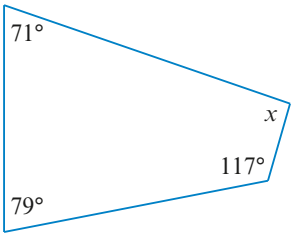
d



e

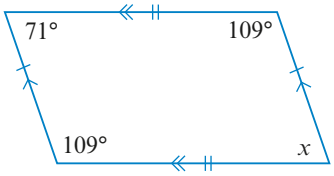


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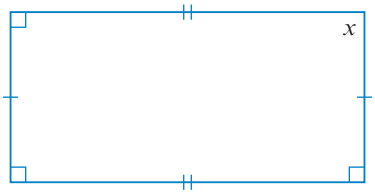


3 Classify each quadrilateral and find the size of angle x.

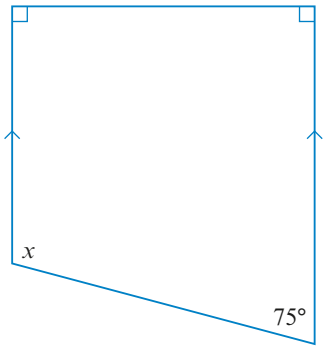
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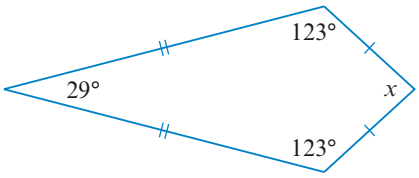
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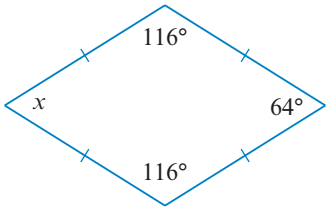
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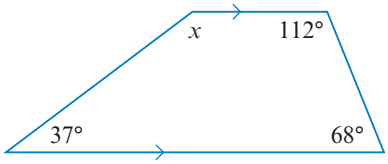
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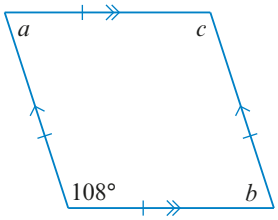


4 Use your knowledge of quadrilaterals to:

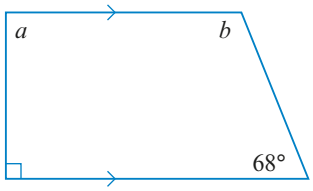
i name each figure

ii find the size of the unknown angles.

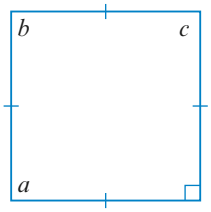
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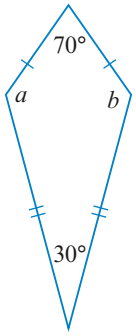
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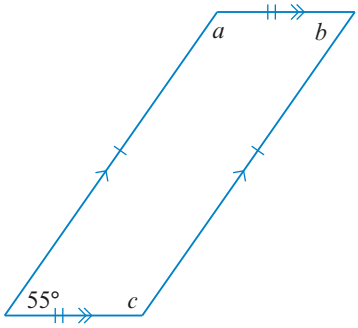
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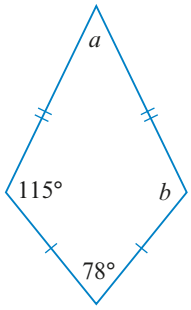
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5 How many different quadrilaterals can you see within this kite? Copy each different quadrilateral and name them.

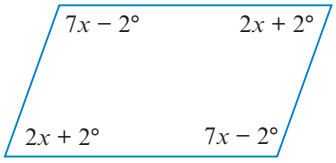
6 Decide if each statement is true or false and rewrite any false statements so they are true.

- **a** A square is a type of parallelogram.
- **b** A kite has a pair of parallel sides.
- **c** A parallelogram is a type of rectangle.
- **d** A trapezium has exactly one pair of parallel sides.
- **e** A rhombus always has four equal sides and four equal angles.
- **f** A rectangle always has four equal sides and four equal angles.

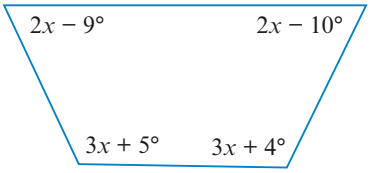
7 Create a concept map showing the connections between all the special quadrilaterals.

8 Use your understanding of quadrilateral properties and basic algebra to find the value of x in each quadrilateral.

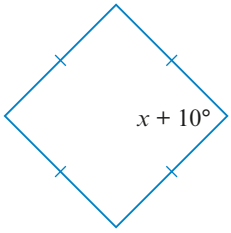
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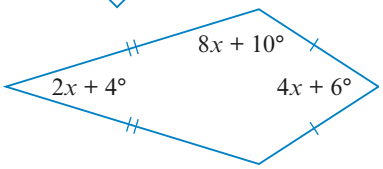
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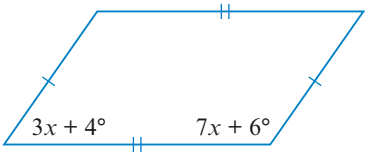
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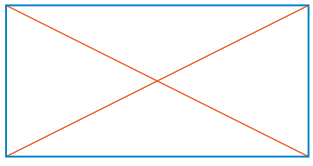


f



9 Draw two identical copies of a quadrilateral on a piece of paper and cut them out.

- **a** Cut one of the quadrilaterals in half to make two triangles.
- **b** How many degrees are there in the angle sum of a triangle?
- **c** Use your answer to part **b** to state the angle sum of a quadrilateral.
- **d** Rip the corners off the second copy of your quadrilateral and place them together in a way that proves your answer to part **c**.
- **e** Repeat parts **a**–**d** with another quadrilateral. What do you find?
- **f** How does this prove that the sum of internal angles of a quadrilateral is 360°?
- **g** Katie divided a rectangle into four triangles as shown. She then states the angle sum of a rectangle is 720°. Explain why Katie's reasoning doesn't work. (Hint: do the angles at the centre of the rectangle count towards the sum total?)



PROBLEM SOLVING AND REASONING

10 Which quadrilaterals display **symmetry**? Create a table showing each quadrilateral with its lines of symmetry marked.

11 A trapezium does not have to have an axis of symmetry. Does this mean that all trapeziums never do? Explain.

12 **a** Draw three different:

i squares **ii** rhombuses

iii rectangles **iv** parallelograms.

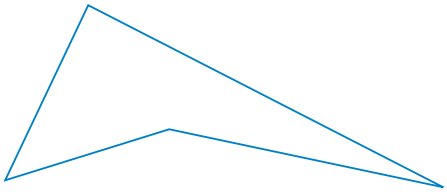
b Why do you have more freedom in what you draw in parts **iii** and **iv** compared to parts **i** and **ii**?

13 **a** Draw three different:

i kites **ii** trapeziums.

b Why do you have more freedom in what you draw in part **ii** compared to part **i** and question **12**?

14 An irregular shape can be **convex** or **concave**. A convex shape has internal angles all smaller than 180°. A concave shape has at least one reflex angle (greater than 180°).



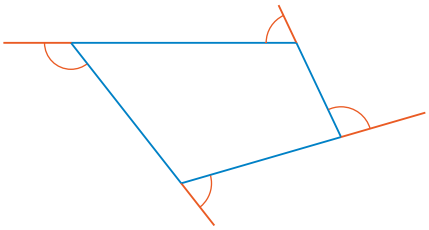
a Explain why a concave quadrilateral can have only one internal reflex angle.

b Find the size of the largest angle if the other three angles measure 20°, 20° and 50°.

c Draw another two examples of a concave quadrilateral.

CHALLENGE

15 Exterior angles can be found for quadrilaterals as well as triangles, as shown here.



a Draw a rectangle and extend each side to mark in the exterior angles.

b What is the size of each exterior angle?

c What is the sum of the exterior angles?

d Repeat parts **a**–**c** with another two quadrilaterals, at least one of them irregular.

e What can you say about the sum of exterior angles for a quadrilateral?

f Use your results from Exercise 7A question **12** or repeat this activity with a triangle and state the sum of the exterior angles of a triangle. How does this compare to your answer to part **e**?

16 Each sentence below gives a clue about a different quadrilateral. Use the clues to:

i find the size of each angle **ii** draw the quadrilateral

iii name the quadrilateral.

a All angles are equal and all sides are equal.

b Two angles are equal but half the other two angles.

c One angle is twice another but one third of the two largest angles.

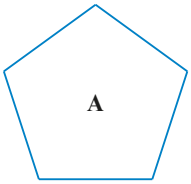
Reflect

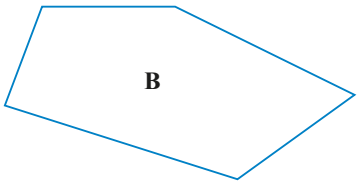
How are the special quadrilaterals related?

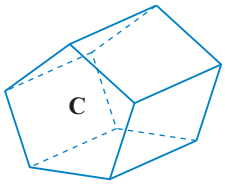
7C 2D shapes and 3D objects

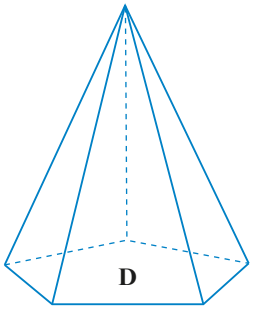
Start thinking!

A **polygon** is a closed 2D shape with straight sides.









1 Which of these figures are polygons?

2 One is a **regular polygon** and one is an **irregular polygon**. Which is which? How do you know?

3 Both polygons are pentagons. How many sides do they have?

4 Look at the other two figures. How do they relate to the polygons?

5 One of these 3D objects is a **prism** and the other is a **pyramid**. Which is which? How do you know?

6 Copy and complete this table for shapes and objects with up to 10 sides. Allow space to show the name and to draw one example of each shape or object.

		Polygon		3D object	
		Regular	Irregular	Prism	Pyramid
Five sides	Name				
	Example				
Six sides	Name				
	Example				

7 Is a circle a polygon? Explain.

8 Add a row for a circle to the table and fill it in.

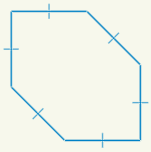
9 Are there other shapes or objects that haven't been considered? List and draw at least three others.

KEY IDEAS

- A polygon is a 2D shape with straight sides.
- Polygons can be regular (all sides and angles the same) or irregular.
- Irregular polygons can be convex (all internal angles less than 180°) or concave (at least one reflex angle).
- 3D objects that contain only polygons are called **polyhedra** (singular = **polyhedron**).
- Many polyhedra can be classified as either prisms (which have ends that are matching polygons) or pyramids (which have one end finishing in a point).
- **Cylinders**, **cones** and **spheres** are examples of 3D objects that are not polyhedra.

EXERCISE 7C 2D shapes and 3D objects

EXAMPLE 7C-1 Describing a shape



Decide if this polygon is:

a regular or irregular b convex or concave.

THINK

a A regular polygon has all sides equal and all angles equal. For this shape, all sides are equal, but not all angles are the same, so it cannot be regular.

b Each angle in a convex shape is less than 180° and a concave shape has at least one angle greater than 180° (a reflex angle). This shape does not have a reflex angle, so it is convex.

WRITE

a The shape is irregular.

b The shape is convex.

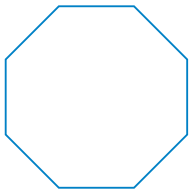
UNDERSTANDING AND FLUENCY

1 Decide if each shape is: **i** regular or irregular **ii** convex or concave.

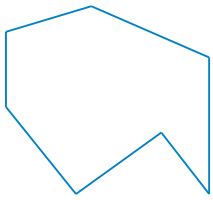
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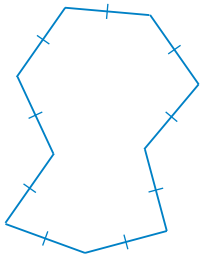
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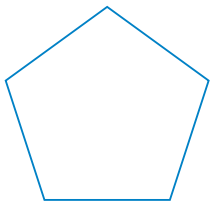
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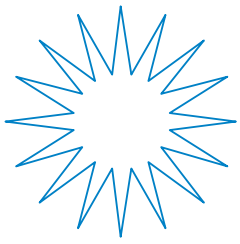
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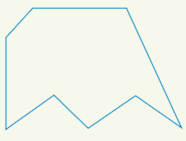
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EXAMPLE 7C-2 Naming a shape



Name this 2D shape.

THINK

All sides are not equal, so it is irregular.
There are two reflex angles, so it is concave.
It has eight sides, so it is an octagon.

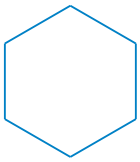
WRITE

The shape is an irregular, concave octagon.

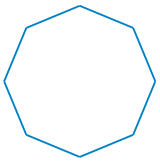
UNDERSTANDING AND FLUENCY

2 Name each 2D shape.

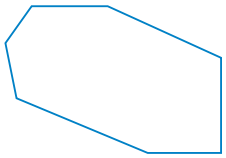
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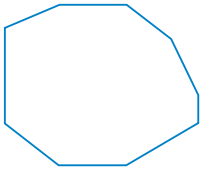
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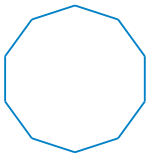
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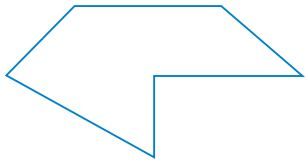
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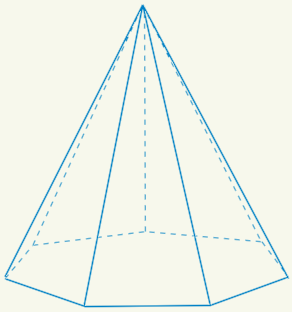


f



EXAMPLE 7C-3 Naming an object

Name this 3D object.



THINK

1 Decide whether the 3D object is a prism or a pyramid. One end finishes in a point, so it is a pyramid.

2 Look at the shape of the base. It is a heptagon because it has seven sides.

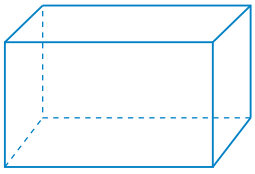
3 Name it using 'heptagon' and 'pyramid'.

WRITE

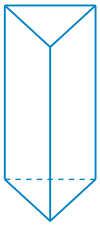
It is a heptagonal pyramid.

3 Name each 3D object.

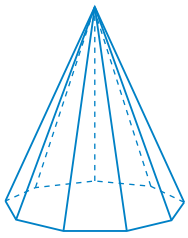
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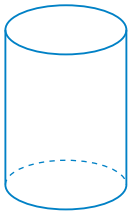
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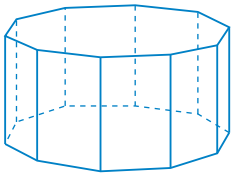
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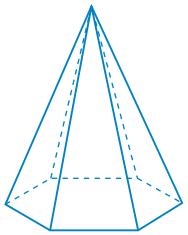
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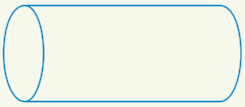
e



f



EXAMPLE 7C-4 Deciding if an object is a polyhedron



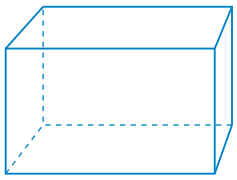
Decide if this 3D object is a polyhedron.

THINK	WRITE
1 A polyhedron is a 3D object that has **faces** that are polygons. It can only have flat faces and straight **edges**.	
2 A cylinder has two faces that are circles, which are not polygons. It is not a polyhedron.	A cylinder is not a polyhedron because it has curved edges.

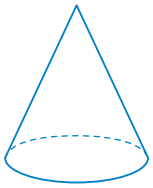
UNDERSTANDING AND FLUENCY

4 Decide if each 3D object is a polyhedron.

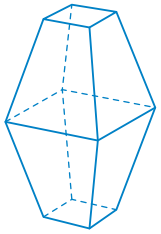
a



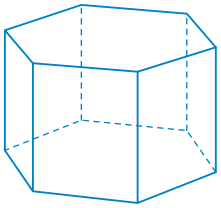
b



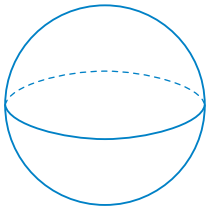
c



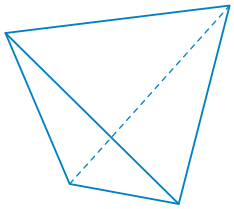
d



e



f



5 Name these shapes and objects.

a



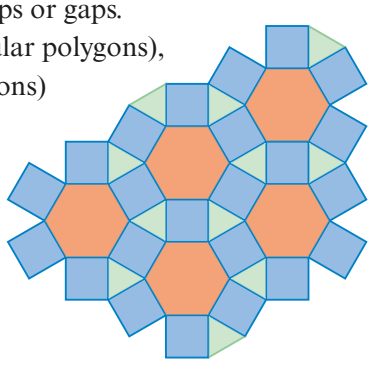
b



c



6 A tessellation is a pattern of shapes that has no overlaps or gaps. Tessellations can be regular (containing identical, regular polygons), semi-regular (containing any number of regular polygons) or irregular (containing any shape).



a What type of tessellation is shown here?

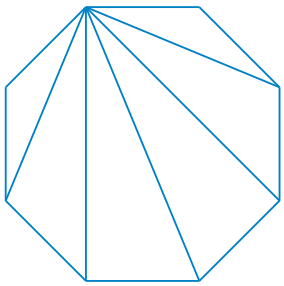
b Create a tessellation of your own, choosing one or more of the shapes shown in this tessellation. State what type of tessellation you have created.

c Create another type of tessellation, choosing any shapes you like.

d What do you have to think about when creating tessellations?

7 Use straws and blu-tac to construct the 3D objects in question **3**.

8 In Exercise 7B question **9**, you used triangles to verify the angle sum of a quadrilateral. You can do the same for any polygon. Consider this octagon.



a How many sides does an octagon have?

b How many triangles is it split into?

c What is the difference between these two numbers?

d Use your knowledge of the internal angle sum of a triangle and your answer to part **b** to calculate the internal angle sum for an octagon.

e Does this work for any octagon? Draw two irregular octagons and find their internal angle sum.

f Why is it important that the triangles all start from one **vertex** (corner)?

9 a Use the steps in question **8** to help you find the internal angle sum of:

i a pentagon **ii** a hexagon **iii** a heptagon

iv a decagon **v** a dodecagon.

b What do you notice about the relationship between the number of triangles in a shape and the number of sides the shape has?

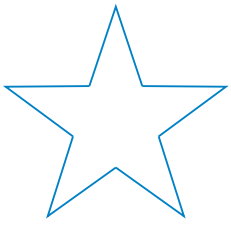
10 A familiar type of concave polygon is the star.

a Name each star according to the number of points it has.

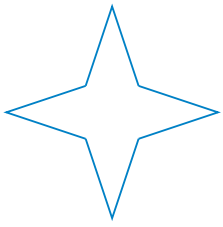
b Name each star according to proper mathematical conventions.

c Find the internal angle sum for each star. Does it match what you found in question **8**? What does this mean for concave polygons?

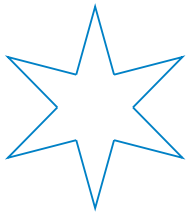
i



ii



iii



11 You can use your findings from questions **8** and **9** to find the size of an individual angle in any regular polygon.

a How many sides does a hexagon have?

b What is the internal angle sum of a hexagon?

c Use your answers to parts **a** and **b** to find the size of one angle in a regular hexagon.

d Use the process in parts **a**–**c** to find the size of one angle in:

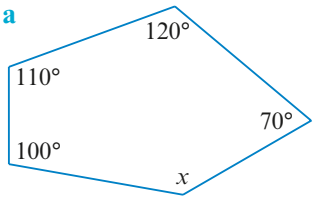
i a regular pentagon **ii** a regular octagon **iii** a regular decagon.

e Why does this only work for regular polygons?

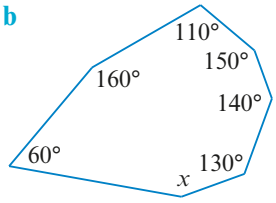
CHALLENGE

12 Use your findings from question **8** to find the size of angle x in each shape.

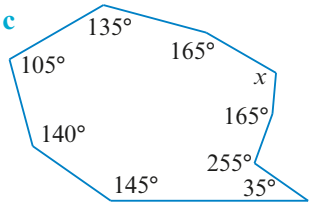
a



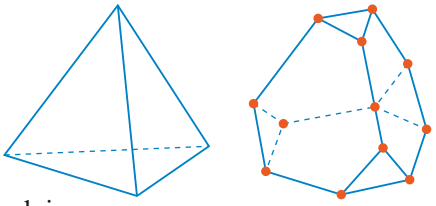
b



c



13 Polyhedra can be made up of a variety of shapes. All polyhedra have flat faces, straight edges and **vertices**. Look at the figures shown.



a What is the difference between the tetrahedron and the truncated tetrahedron?

b Is the truncated tetrahedron a polyhedron? Explain.

c Copy the figure of the truncated tetrahedron and place a dot on each vertex. How many are there? How does this compare to the tetrahedron?

d Colour each edge of the truncated tetrahedron. How many are there? How does this compare to the tetrahedron?

14 Many polyhedra are obtained by truncating (cutting off vertices of) common 3D objects. Draw what you would expect these objects to look like.

a a truncated cube

b a truncated **triangular prism**

c a truncated **square-based pyramid**

15 A 3D object that is not a polyhedron is a torus, the mathematical name for the shape of a donut.



a What other objects can you think of that are in the shape of a torus?

b Explain why a torus is not a polyhedron.

16 A Möbius strip is an object with only one face and one edge.

a Make a Möbius strip by cutting out a thin strip of paper and joining the ends together with a half twist.

b Trace your finger across the surface of the strip. What do you find?

c Write the alphabet from the start to the finish of the strip.

d Undo the strip and look at how the alphabet is written on the strip of paper. What do you find?

e Is a Möbius strip 2D or 3D? Explain your answer.

Reflect

How can you describe 2D shapes and 3D objects?

7D Isometric drawings and plans

Start thinking!

All of the '3D figures' in this book are actually 2D representations of 3D objects. It is important when drawing 3D objects that the lines and sides are drawn correctly to show three dimensions. Isometric dot paper is a useful tool for this.

Consider a basic **rectangular prism** as shown in figure 1. It is 3 units long, 1 unit high and 1 unit wide.

You can use isometric dot paper to help ensure you draw consistent and parallel lines.

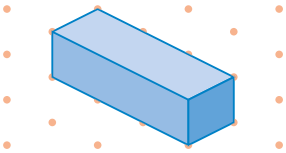


Figure 1

1 How are lines of depth represented using isometric dot paper?

2 Use isometric dot paper to draw a rectangular prism that is:

- **a** 3 units long, 2 units high and 1 unit wide
- **b** 4 units long, 2 units high and 1 unit wide
- **c** 4 units long, 2 units high and 3 units wide.

3 How do the figures in question **2** differ from figure 1?

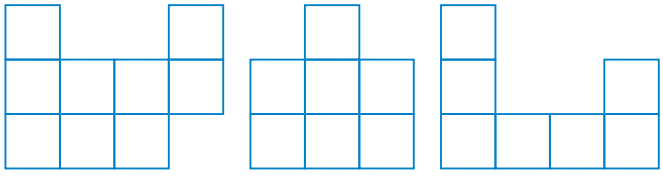
4 Copy the rectangular prism in figure 1 and add lines to represent it broken up into cubes.

5 How many cubes does this rectangular prism contain?

6 Draw three different ways you can add two cubes to this rectangular prism. Think in three dimensions.

Sometimes it is difficult to describe a more complex shape accurately, as parts of it are hidden from view. In this case, you can draw a **set of plans**. A set of plans always consists of the front, right and top view of an object. The front is indicated with an arrow, like in figure 2.

7 Look carefully at the views A, B and C.
Which matches the front view of figure 2?



A B C

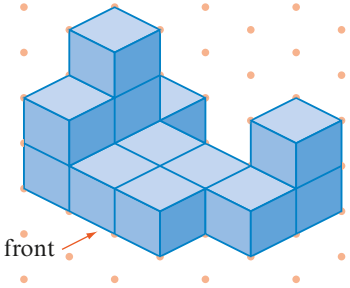


Figure 2

KEY IDEAS

- Isometric dot paper has equally spaced dots to help in drawing 3D objects.
- When 3D objects are complex you can describe them fully with a set of plans.
- A set of plans consists of a front (marked with an arrow), side (usually right side) and top view.

EXERCISE 7D Isometric drawings and plans

EXAMPLE 7D-1 Drawing a prism using isometric dot paper

Draw a pentagonal prism using isometric dot paper.

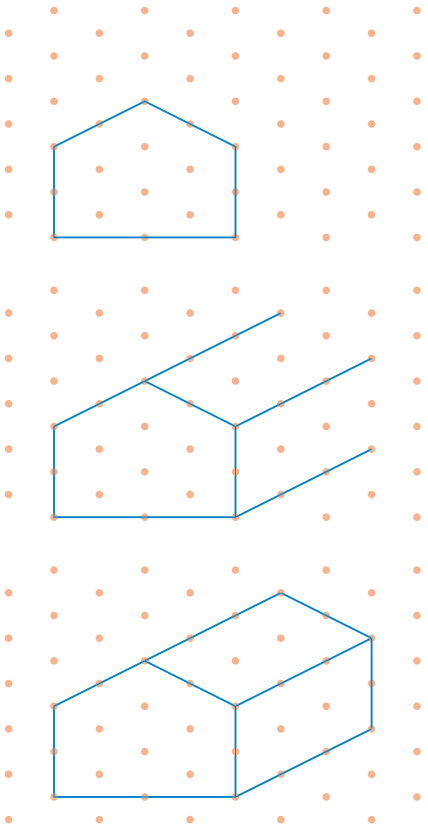
THINK

1 Draw five dots on isometric paper and join them with straight lines to make a pentagon.

2 Draw a line from each of three vertices to the right using the dots as a guide. Make sure each line is the same length and all three are parallel.

3 Complete the prism by joining the ends of the lines you have drawn.

WRITE



UNDERSTANDING AND FLUENCY

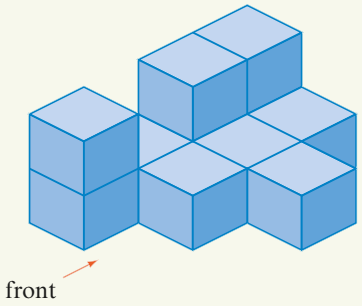
1 Draw each object using isometric dot paper.

- **a** a cube
- **b** a tall rectangular prism
- **c** a hexagonal prism
- **d** a triangular prism
- **e** a rectangular pyramid
- **f** a pentagonal pyramid



EXAMPLE 7D-2 Drawing a set of plans for an object

Draw a set of plans for this object.



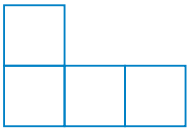
THINK

1 Look at the front of the object. From this angle three cubes are visible in the bottom layer and one on the top. Draw this.

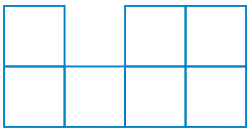
2 Look at the right side of the object. Four cubes are visible in the bottom layer and three on the top. Draw this.

3 Look at the top of the object. Four rows of cubes are visible: two cubes, then three, then two, then one cube. Draw this.

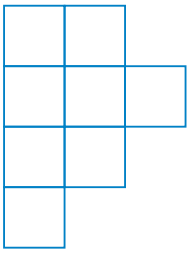
WRITE



front



right

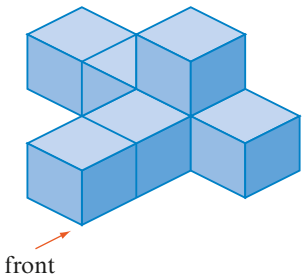


top

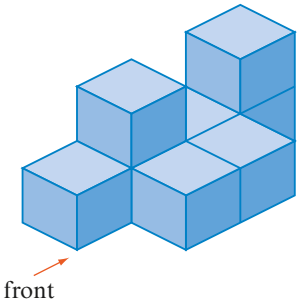
UNDERSTANDING AND FLUENCY

2 Draw a set of plans for each object.

a

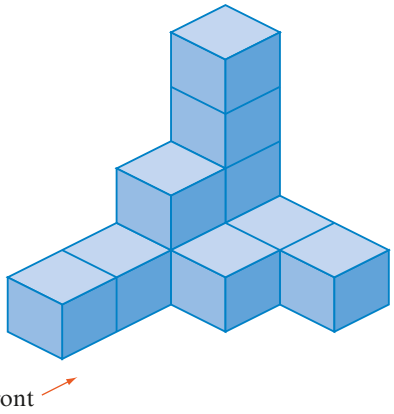


b

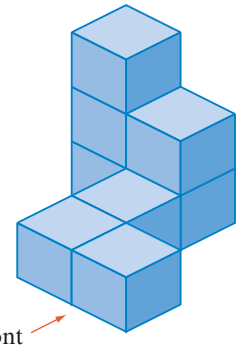


UNDERSTANDING AND FLUENCY

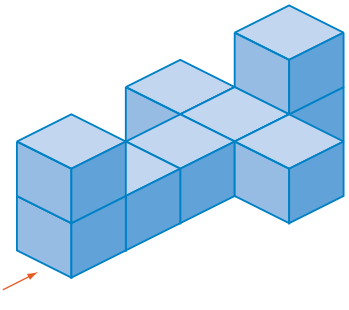
c



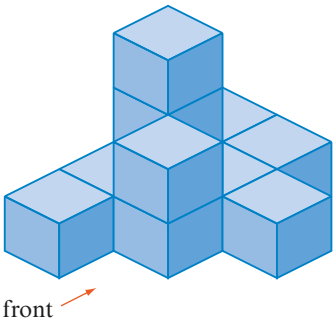
d



e



f

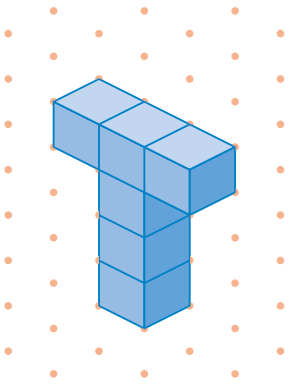


PROBLEM SOLVING AND REASONING

3 Draw a set of plans for the blue bathing box. Think carefully about what shapes you see for the front, right side and top views.



4 You can use isometric dot paper to draw letters in 3D.



a What letter is shown here?

b Could you draw this letter another way? Give an example.

c Draw these letters using isometric dot paper.

i L **ii** E **iii** O **iv** V **v** S

5 Use isometric dot paper to draw your own name in 3D. How tall and how wide is your drawing?

6 Plans don't have to be only for complex objects made from cubes. Draw a set of plans for these objects. Think carefully about the shapes you see from each view.

a



b



c



7 Why is it hard to use isometric dot paper to draw pyramids?

8 A model house is shown.

a Draw this on isometric dot paper.

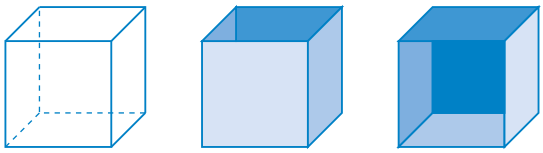
b Draw a set of plans for the house.

c Draw what you imagine a left side view and a back view of the house would look like.



9 Use isometric dot paper to design and draw your own building. Include a set of plans for it.

10 Another strategy to help represent 3D objects in 2D is to use shading. This cube has been shaded in two different ways.

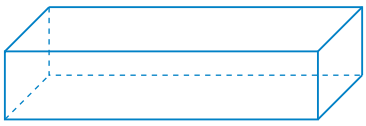


a What is different about the two shaded cubes?

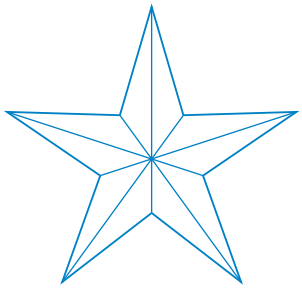
b Copy the cube and use shading to show it in another two ways.

11 Use shading to show each object in at least two different ways.

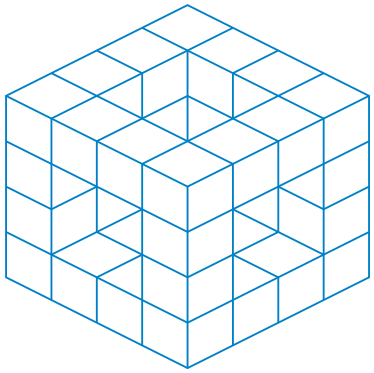
a



b



c



CHALLENGE

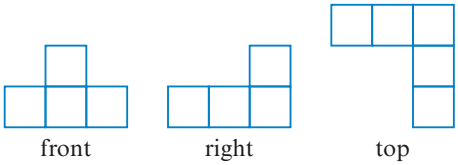
12 Draw a set of plans for:

- **a** your house
- **b** a school building
- **c** your favourite store
- **d** the tallest building in your area
- **e** the construction shown at right.

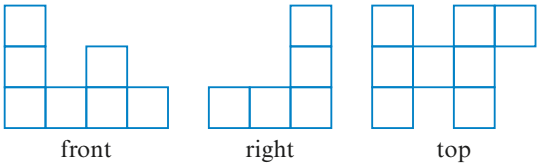


13 Use isometric dot paper to draw the 3D object that belongs to each set of plans.

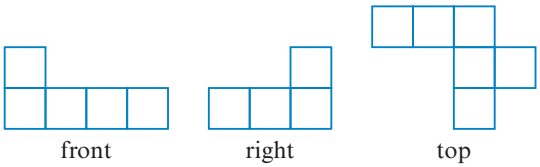
a



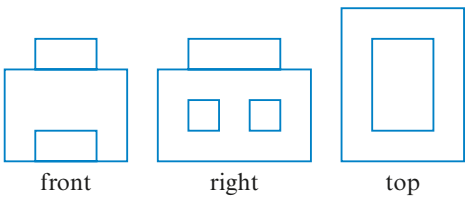
b

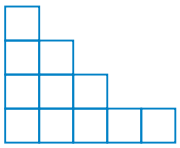


c

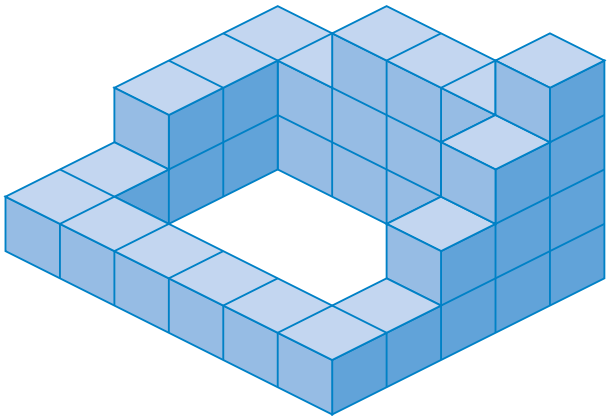


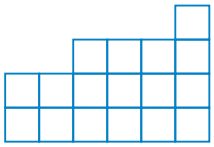
d

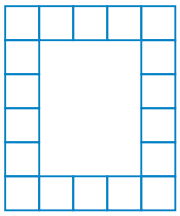




14 Tony drew a set of plans at right to match the object below, but forgot which angle he drew it from. Redraw the object as Tony saw it, labelling the front.







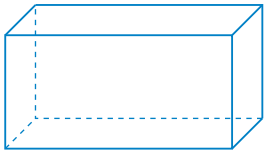
Reflect

How are isometric drawings and plans similar and how are they different?

7E Nets and perspective drawings

Start thinking!

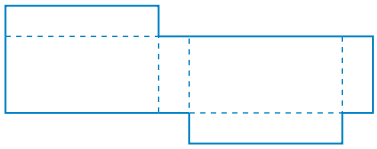
A **net** is a useful 2D representation of a 3D object, because you can fold it up to create the object.



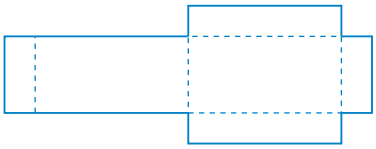
1 How many faces does a rectangular prism have? How many different faces?

2 One of the three nets shown here does not work. Find which one and explain why.

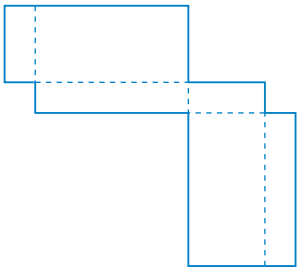
A



B



C

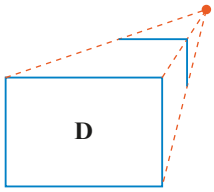


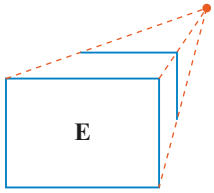
3 Draw a net that is different from the ones above and use it to construct the rectangular prism. Include tabs to make it easier to construct.

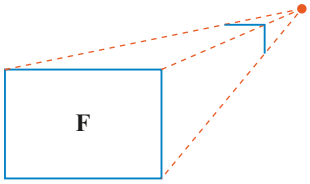
4 Place your constructed prism on the desk and view it at eye level, but on an angle so that it looks roughly like figure E below. Do the lines of depth look parallel to you? Explain.

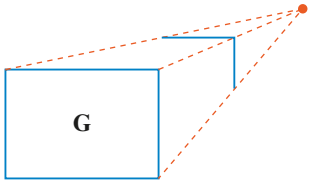
Drawing an object from a view to show what you actually see is called a **perspective drawing**.

Consider these four perspective drawings. To make them, a **vanishing point** is drawn above and to one side of the front face of the prism and lines are drawn back to meet it.









5 What is the difference between: **a** figures D and E? **b** figures E and F?

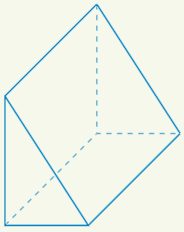
KEY IDEAS

- A net is a 2D plan that can be folded to form a 3D object.
- To draw a net it is easiest to identify the bottom face of a figure and place the other faces around it in a hopscotch arrangement.
- Perspective drawings represent how objects look in real life and make use of a vanishing point.
- For a perspective drawing, always draw the front face first and ensure that the back edges are drawn parallel to the front edges.

EXERCISE 7E Nets and perspective drawings

EXAMPLE 7E-1 Drawing a net for a prism

Draw a net for this triangular prism.

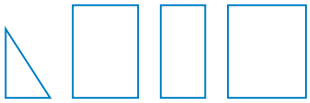


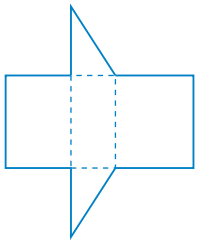
THINK

1 A triangular prism has five faces, made up of four different kinds. Draw each different face.

2 Draw the bottom face first and then arrange the remaining faces in a hopscotch style net.

WRITE

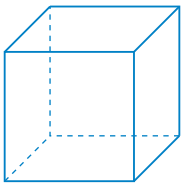




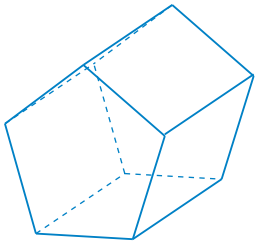
UNDERSTANDING AND FLUENCY

1 Draw a net for each prism.

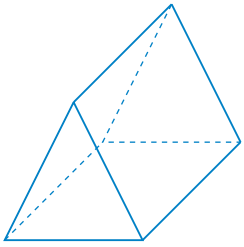
a



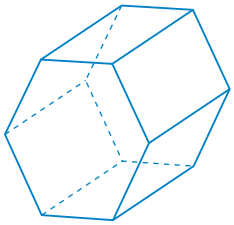
b



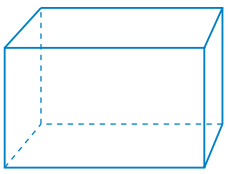
c



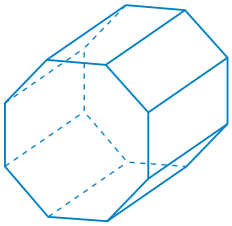
d



e



f



EXAMPLE 7E-2 Creating a perspective drawing of a prism

Create a perspective drawing for the prism shown in Example 7E-1.

THINK

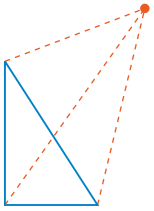
1 Draw the front face of the prism (a triangle).

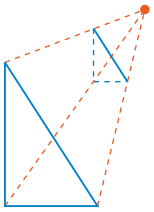
2 Place a vanishing point above and to one side and draw dashed lines from each vertex to the vanishing point.

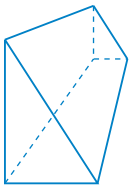
3 Draw in the edges for the back of the prism. Use dashed lines for the hidden edges and a solid line for the visible edge. Make sure all edges are parallel to the front edges of the prism.

4 Complete the prism by making the remaining edges solid, leaving the dashed lines for the hidden edges and erasing any left over dashed lines and the vanishing point.

WRITE





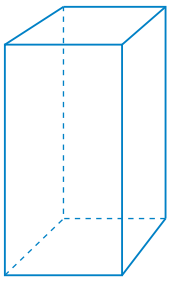


UNDERSTANDING AND FLUENCY

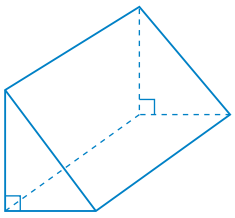
2 Create a perspective drawing for each prism in question **1**.

3 Create a perspective drawing for each object.

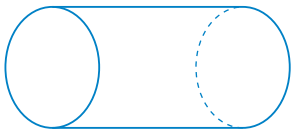
a



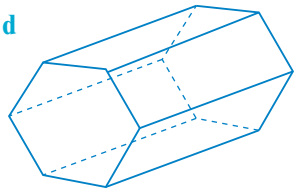
b



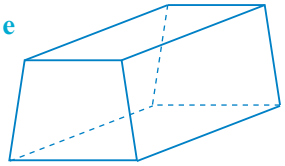
c



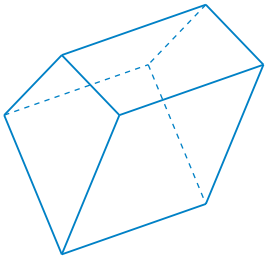
d



e



f



4 Draw a net for each object in question **3**.

5 Draw a net for an envelope. You may wish to find one to help you.

6 There are 11 unique nets for a cube. How many can you find?
Be careful of rotations and mirror reflections!

7 Nets can also be drawn for open objects.
The gift box to the right is actually made of two open boxes – one for the main box and one for the lid.



a How many faces does each part of the gift box have?

b Draw a net for each part of the gift box.

c How do these nets differ?

d How is drawing a net for an open object similar to drawing a net for a pyramid?

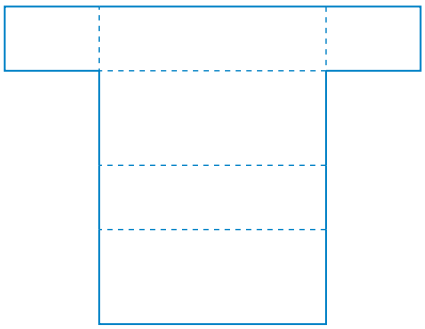
8 For these nets:

i identify which object they make

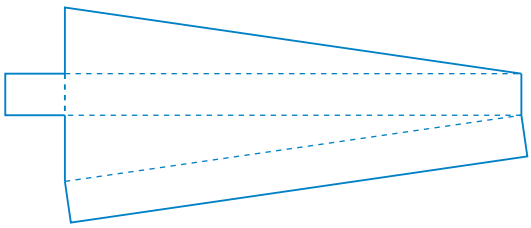
ii draw the object

iii create a perspective drawing of the object.

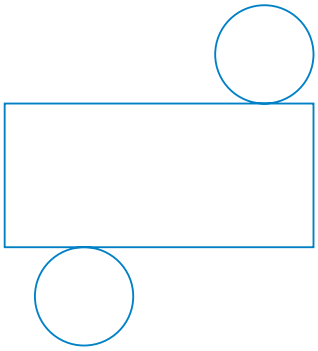
a



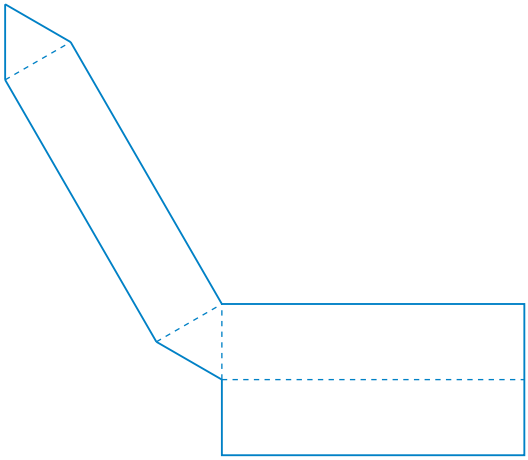
b



c



d



9 Redraw each net in question **8** as if it was an open object.

10 Many people have difficulty seeing that the net in part **c** of question **8** belongs to a cylinder.

- **a** Why do you think this is?
- **b** Use a piece of A4 paper to demonstrate how a rectangle forms the main part of a cylinder.

11 Why is it harder to create a perspective drawing for a pyramid?

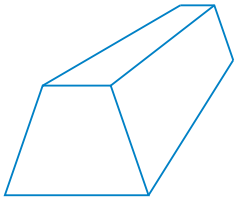
12 Use your understanding of perspective drawings to write your name with a vanishing point, like in this example. Then write it a second time with a different vanishing point.



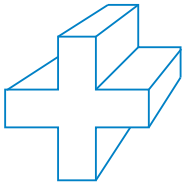
13 For each of these drawings:

- **i** identify the vanishing point
- **ii** redraw the object with a different vanishing point.

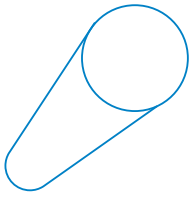
a



b



c



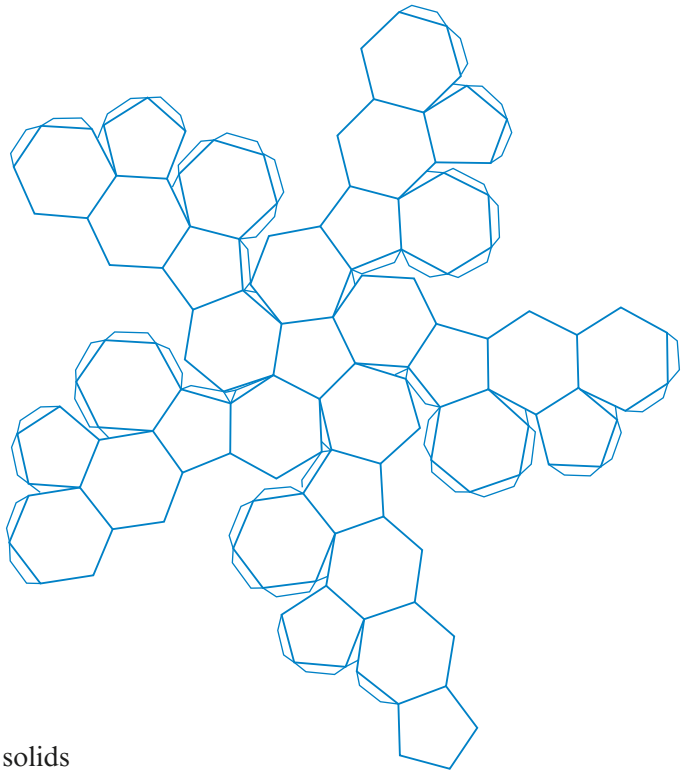
14 Consider the buildings shown in this image.

- **a** How is the perspective different from the perspective drawings you have seen so far?
- **b** Draw a copy of the buildings and identify the vanishing point for each.
- **c** Which building do you think is the tallest? How can you tell?



PROBLEM SOLVING AND REASONING

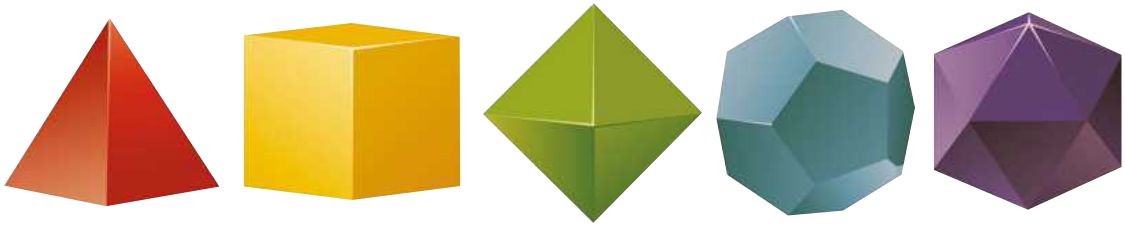
15 This is the net used to create a soccer ball.



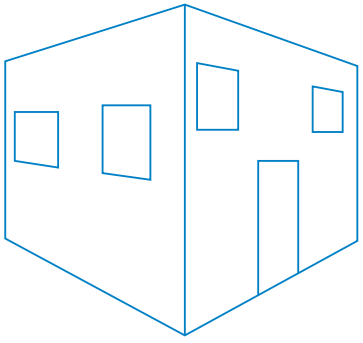
- **a** Which two polygons make up this net?
- **b** What shape are the tabs?
- **c** Which polygons are usually coloured black and which polygons are usually coloured white?
- **d** Obtain a larger copy of the net and colour in the appropriate polygons black and the tabs a different colour.
- **e** Use scissors and glue to cut out the net and construct your own soccer ball.

CHALLENGE

16 Draw a net for each of Euler's solids (tetrahedron, cube, octahedron, dodecahedron and icosahedron) and use them to build the 3D objects.



17 This building is drawn in a style called two-point perspective, rather than the single-point perspective used so far.



- **a** Why is it called this?
- **b** How do you think the process of creating a two-point perspective drawing differs from creating a single-point perspective?
- **c** Identify the vanishing points in this drawing.
- **d** Redraw the objects from parts **a** and **b** of question **13** in two-point perspective.

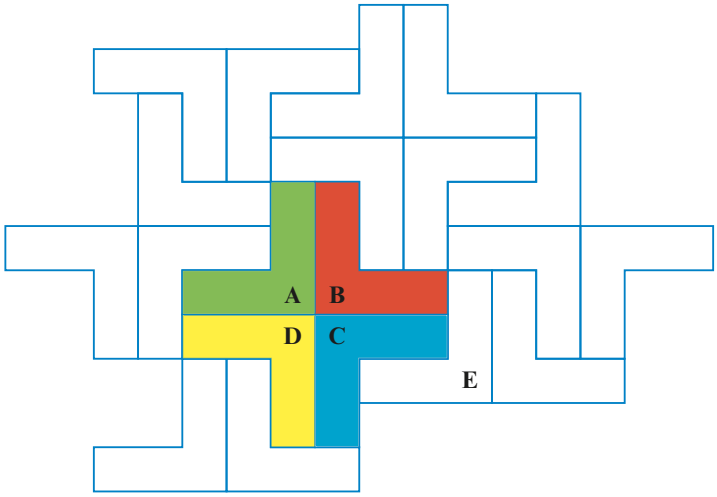
Reflect

Which representation do you think is more useful, a net or a perspective drawing? Explain.

7F Translations, rotations and reflections

Start thinking!

Translations, **rotations** and **reflections** are all **transformations**. In this tessellation, four shapes have been coloured in.



1 Which of the coloured shapes is:
   - **a** a vertical reflection of B?
   - **b** a horizontal reflection of B?

2 Has the remaining shape been rotated or translated from B? How do you know?

3 Copy the tessellation into your book.

4 Find all the shapes that are translations of:
   - **a** B, and colour them in red.
   - **b** A, and colour them in green.

5 Complete the tessellation by colouring the translations of C (blue) and D (yellow).

Each shape in the tessellation can be described in a number of ways. E, for example, can be described as:

- a translation of A
- a rotation then translation of D
- a reflection then translation of B
- a rotation then translation of C.

6 Choose a shape in the tessellation and describe it in a minimum of four ways.

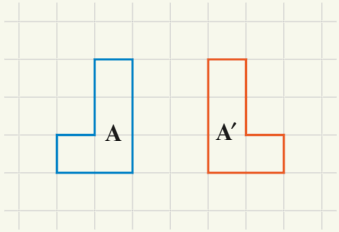
KEY IDEAS

- An **isometric transformation** is one that doesn't change the shape or size of an object.
- Translation, rotation and reflection are all isometric transformations.
- Translation is when a shape or object is moved without turning or changing size.
- Rotation is when a shape or object is turned around a point.
- A reflection of a shape or object is its exact mirror image.
- Transformations should always be described in as much detail as possible.
- The transformation of a shape A is called the image, and is named A′.

EXERCISE 7F Translations, rotations and reflections

EXAMPLE 7F-1 Describing transformations

Describe the transformation shown.



THINK

1 Has the shape been translated, rotated or reflected? (reflected)

2 Has it been reflected in a horizontal mirror line or a vertical mirror line? (vertical)

3 Write your answer.

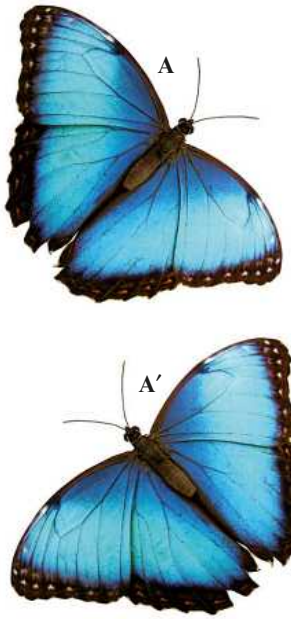
WRITE

The shape has been reflected in a vertical mirror line.

UNDERSTANDING AND FLUENCY

1 Identify each transformation as a translation, rotation or reflection.

a



b



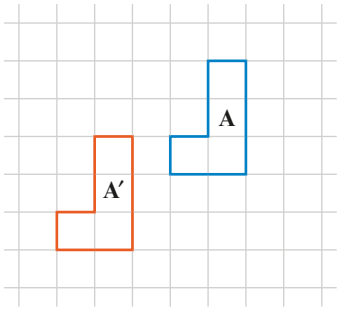
c



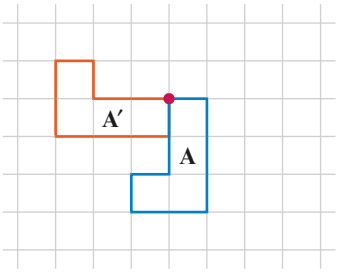
2 Describe in full the transformations performed on each shape in question **1**.

3 Describe each transformation.

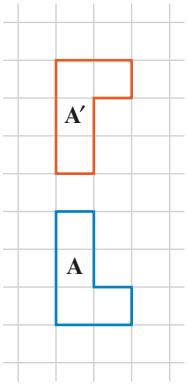
a



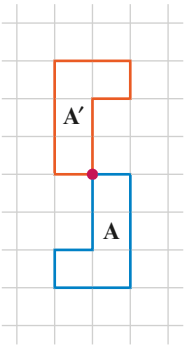
b



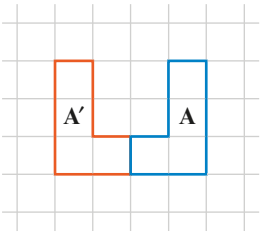
c



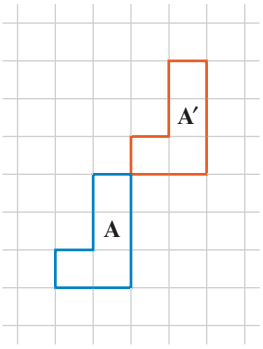
d



e



f



EXAMPLE 7F-2 Performing transformations

Rotate this shape 90° anticlockwise around the point shown.



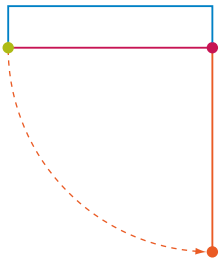
THINK

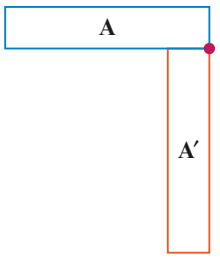
1 Select a point of focus on the shape (green dot). Since the given point of rotation (shown in pink) is on the shape, it is easier to choose a point of focus that has an edge (also pink) in common with it. Rotate this edge 90° anticlockwise around the point of rotation.

2 Draw in the remainder of the image. Check it is the same shape but rotated.

3 Erase the arrow and name the final image.

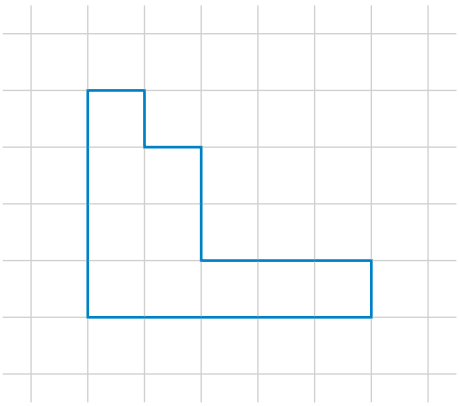
WRITE





UNDERSTANDING AND FLUENCY

4 Perform each transformation on this shape. Start from the original shape each time.



- **a** reflect in a horizontal mirror line
- **b** translate 4 units left and 3 units up
- **c** reflect in a vertical mirror line
- **d** rotate 90° clockwise from the bottom right corner
- **e** translate 1 unit right and 2 units down
- **f** rotate 90° anticlockwise from the bottom left corner

PROBLEM SOLVING AND REASONING

5 Describe the transformations shown in these pinwheels.



6 Draw a shape of your choosing onto graph paper and label its bottom left corner as (5, 5).

- **a** Translate it according to this sequence of steps:
  - **i** 4 right, 2 up
  - **ii** 2 right, 3 down
  - **iii** 3 left, 4 up
  - **iv** 1 left, 6 down.
- **b** What are the coordinates of the bottom left corner of this final image?

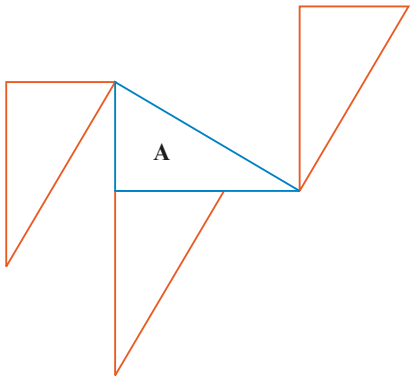
7 Ollie and Sophie are playing a game similar to Pacman. Ollie starts at (4, 2) and Sophie starts at (3, 8). They are each allowed to take five steps per turn. The object of the game is to get as close as you can to the other player at the end of your turn. The winner is the first person to move within one step of the other player.

- **a** Plot each round of the game for each person using the moves listed in this table.

	Round 1	Round 2	Round 3	Round 4	Round 5
Ollie	2 right, 3 up	4 left, 1 down	1 right, 4 up	3 right, 2 up	2 right, 3 down
Sophie	1 right, 4 down	2 left, 3 down	3 right, 2 up	4 right, 1 up	1 left, 4 up

- **b** What is the closest that they get to one another?
- **c** Who was the winner of this game?
- **d** What kind of transformation does this involve?
- **e** You might like to team up with a classmate and play the game yourself. Can you introduce other types of transformations into the rules of the game?

8 When you rotate a shape, the placement of the point of rotation is important in determining the position of the image. Shape A has been rotated in a clockwise direction from all three vertices to produce three different images.



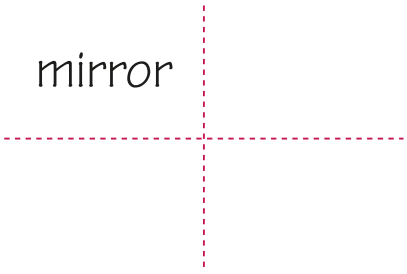
- **a** All of the images have been rotated by the same amount. How much is this?
- **b** Describe how these images differ. How does changing the point of rotation affect the image?
- **c** Copy shape A and place a point of rotation in its centre.
- **d** Rotate A around this point of rotation 90° clockwise to produce a fourth image.
- **e** Choose another two points of rotation that are not part of the shape. Produce another two images by rotating the shape 90° clockwise around these two points.
- **f** How is this different from rotating around a point of rotation that is on the shape?

9 How does changing the distance of the point of rotation from the original shape change the position of the image?

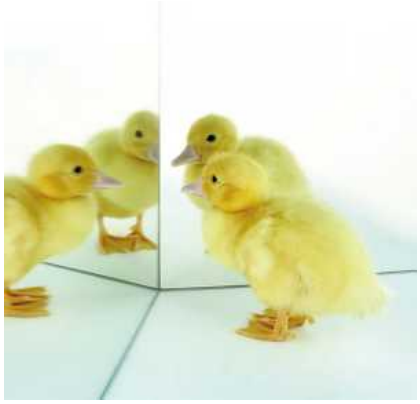


- **a** Copy this shape and the five points shown. Rotate the original shape 90° clockwise around each point of rotation to form five images.
- **b** Describe the difference between the images produced.
- **c** Does the same pattern occur for a vertical line of points of rotation? Copy the image and place five points of rotation in a vertical line. Discuss your results.

10 What happens if you reflect an image in two different mirrors?

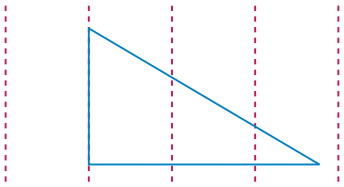


- **a** Write your name (or any word you like) and draw both a horizontal and a vertical mirror line, as shown in the diagram.
- **b** Use these mirrors to help you produce three reflections: one reflected horizontally, one reflected vertically, and one reflected both horizontally and vertically.
- **c** What do you find? Do you notice anything about the reflections?
- **d** Now write your name (or the same word) and draw a diagonal mirror line.
- **e** Reflect in the diagonal mirror line. How does this compare to the image that is produced from both horizontal and vertical reflection?



PROBLEM SOLVING AND REASONING

11 How does the position of a mirror line affect the image? Investigate this using the shape from question **8**.



a Copy the triangle and reflect it in each of the five mirror lines shown.

b What do you notice about the difference between a mirror line that is next to the object as opposed to a mirror line that cuts through the object?

What if you were to consider only one side of the mirror?

c Reflect through each mirror line but this time only consider what is on the right of the mirror line.

d How does this affect the image?

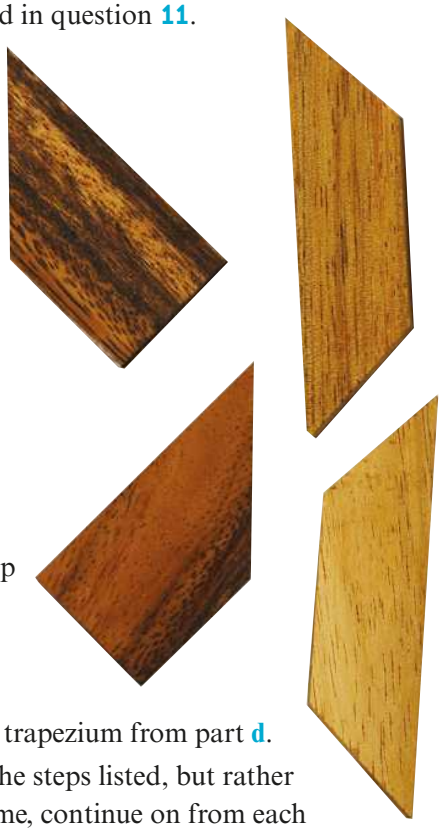
e Repeat part **c** but only consider what is on the left of the mirror line.

f How are the images in part **e** different from those in part **c**?

12 Repeat the investigation from question **11**, but this time draw your own horizontal mirror lines. Write a paragraph stating what you find and how it is different from the results you found in question **11**.

CHALLENGE

13 You can use the Cartesian plane to help accurately describe transformations.



a Draw a Cartesian plane that has a scale from 0 to at least 10 on both the x- and y-axes.

b Draw a trapezium somewhere on the plane. (Hint: place it somewhere near the middle.)

c Label the vertices of the trapezium with coordinates.

d Transform the trapezium according to these steps. Start from the original shape each time.

i translate 3 units to the right and 2 units up

ii reflect in a diagonal mirror line

iii rotate 90° in an anti-clockwise direction from the bottom right corner

e Write the coordinates of each vertex of each trapezium from part **d**.

f Now transform the trapezium according to the steps listed, but rather than starting from the original shape each time, continue on from each image created.

g Write the coordinates of each vertex of the final image in part **f**.

h How might writing the coordinates of the vertices help you when identifying transformations?

Reflect

What common mistakes do you think people make when working with transformations?

7G Understanding congruence

Start thinking!



1 Consider these two sugar gliders. How are they different and how are they the same?

2 Draw a simple figure (for example, a triangle) and cut it out.

3 Perform these transformations on your figure and discuss how it changes the size and shape of your figure.
   - **a** Translate your figure up and to the right.
   - **b** Reflect your figure in a horizontal mirror line.
   - **c** Rotate your figure 90° in a clockwise direction.

All these transformations are isometric. An isometric transformation is one which doesn't change the shape or size of a figure. If two figures are identical in shape and size, they are **congruent**.



4 Are the sugar gliders pictured above congruent? How can you tell?

5 Consider these two other sugar gliders. How are they different from and how are they the same as the first two sugar gliders?

6 Explain why neither of these two sugar gliders are congruent to the sugar gliders above.

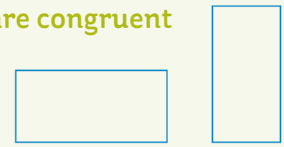
KEY IDEAS

- Isometric transformations produce congruent figures.
- Congruent figures are identical in shape and size but can be in any position or orientation.
- The symbol for congruence is $\cong$.

EXERCISE 7G Understanding congruence

EXAMPLE 7G-1 Deciding if shapes are congruent

Decide if this pair of shapes is congruent.



THINK

1 Look at the shapes to see if they look the same (same basic shape).

2 Check if their corresponding sides are equal (no).

3 Write your answer.

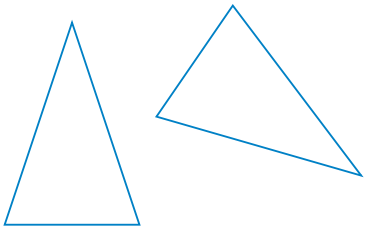
WRITE

The shapes are not congruent because some of their sides are not equal.

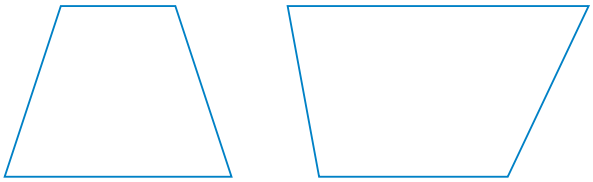
UNDERSTANDING AND FLUENCY

1 Decide whether each pair of shapes is congruent.

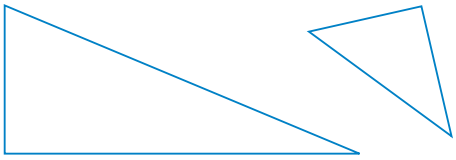
a



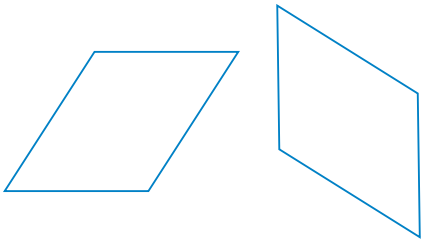
b



c

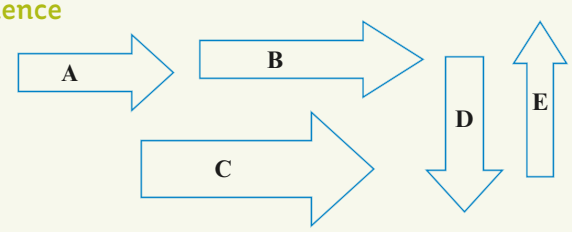


d



EXAMPLE 7G-2 Identifying congruence

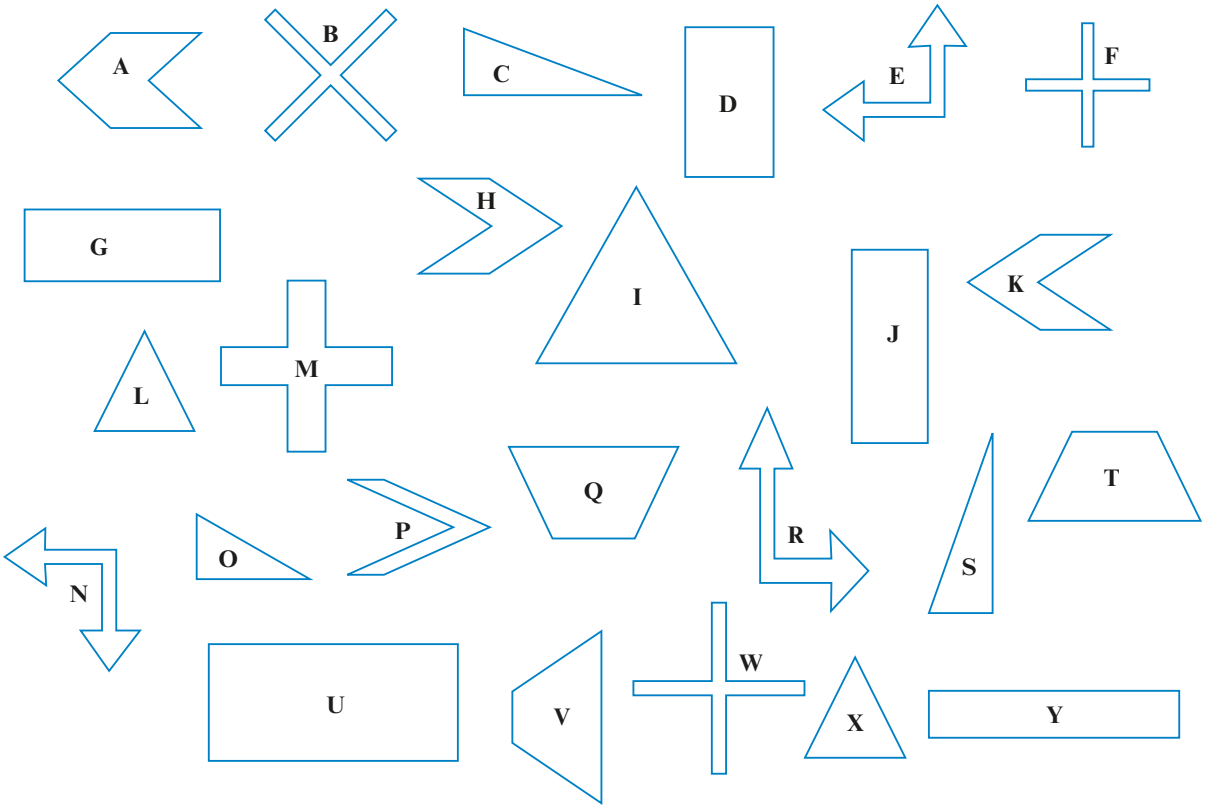
Identify any pairs of congruent shapes.



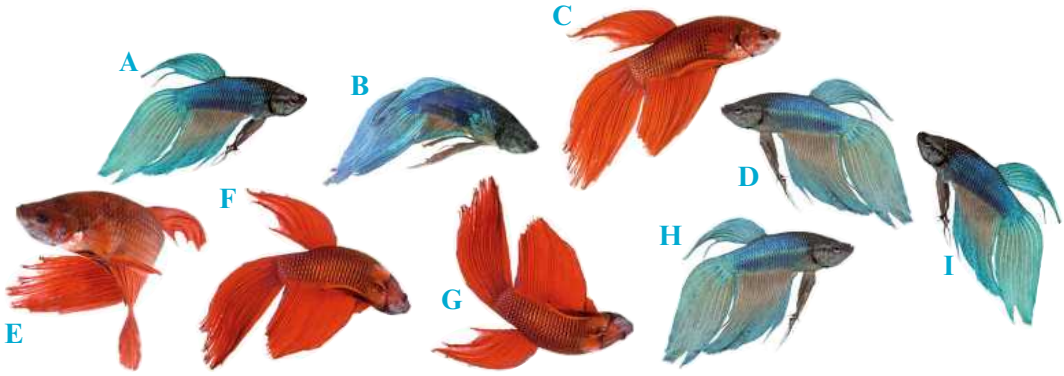
THINK	WRITE
1 Look at the shapes to see if any look the same.	
2 E is thinner than all the rest so it is not congruent to anything.	
3 C is wider than all the rest so it is also not congruent.	
4 B is longer than both A and D so it is not congruent to anything.	
5 A and D look identical. Measure them to check.	A and D are congruent.

UNDERSTANDING AND FLUENCY

2 Identify any pairs of shapes that are congruent.



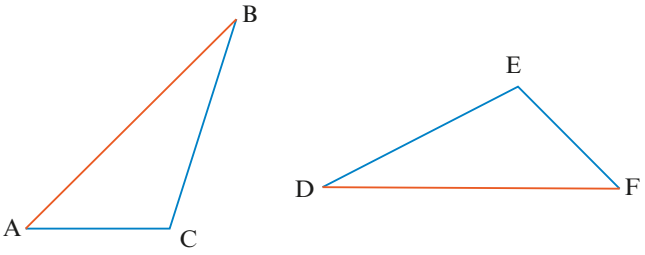
3 Which of these images of fish are congruent?



4 Explain to a classmate the difference between the meanings of the word 'corresponding' when you are talking about:

- **i** a type of angle found within parallel lines and a transversal
- **ii** comparing 'the same but different' sides or angles in a shape. How do these definitions relate to each other?

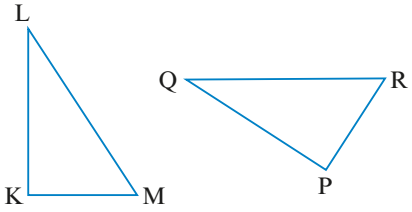
5 Often when a shape has been rotated or reflected, you must match corresponding sides to check if they are congruent. For example, in the congruent figures on the right, side AB matches side FD.



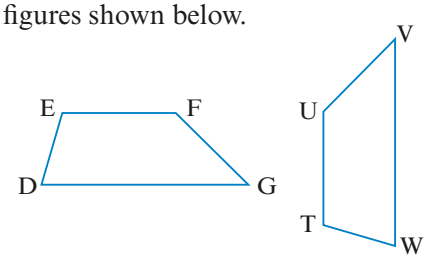
a Match the remaining sides for these figures.

b Match the sides for the pairs of congruent figures shown below.

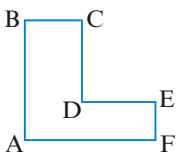
i

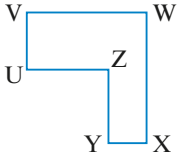


ii

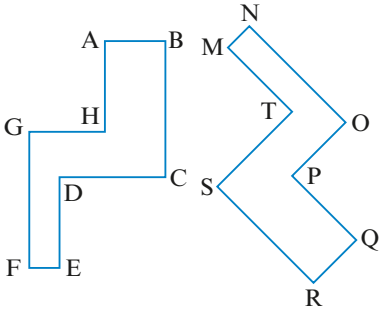


iii



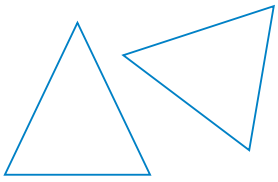


iv

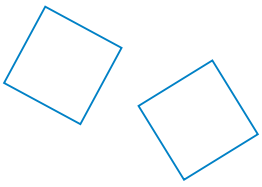


6 These pairs of figures look congruent. Use a ruler to measure their sides and determine if they are congruent or not.

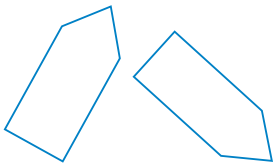
a



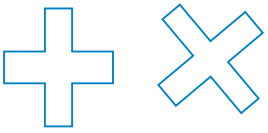
b



c



d

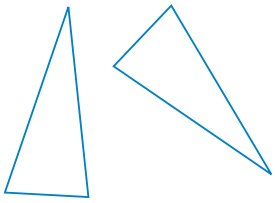


7 For each pair of figures:

i match the corresponding sides **ii** measure all sides

iii state if the figures are congruent.

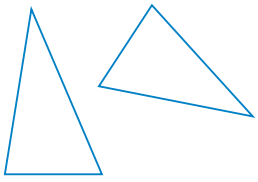
a



b



c



d

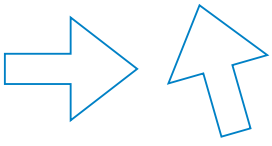


8 For each pair of figures:

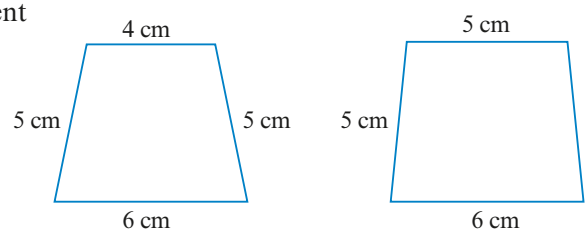
i determine if they are congruent

ii explain your decision.

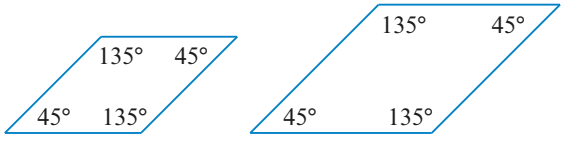
a



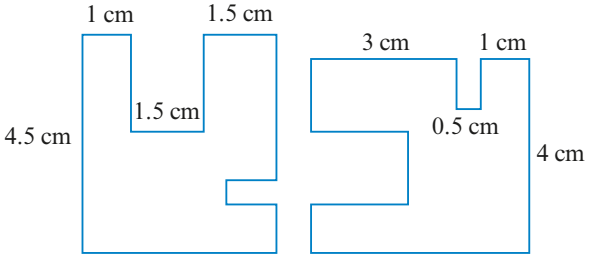
b



c



d

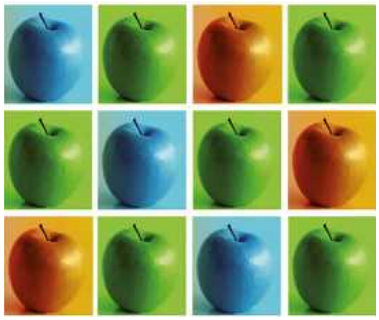


PROBLEM SOLVING AND REASONING

9 Emily buys two pieces of modern art and her friend comments that each piece is made up of congruent figures. Emily disagrees.



Piece A



Piece B

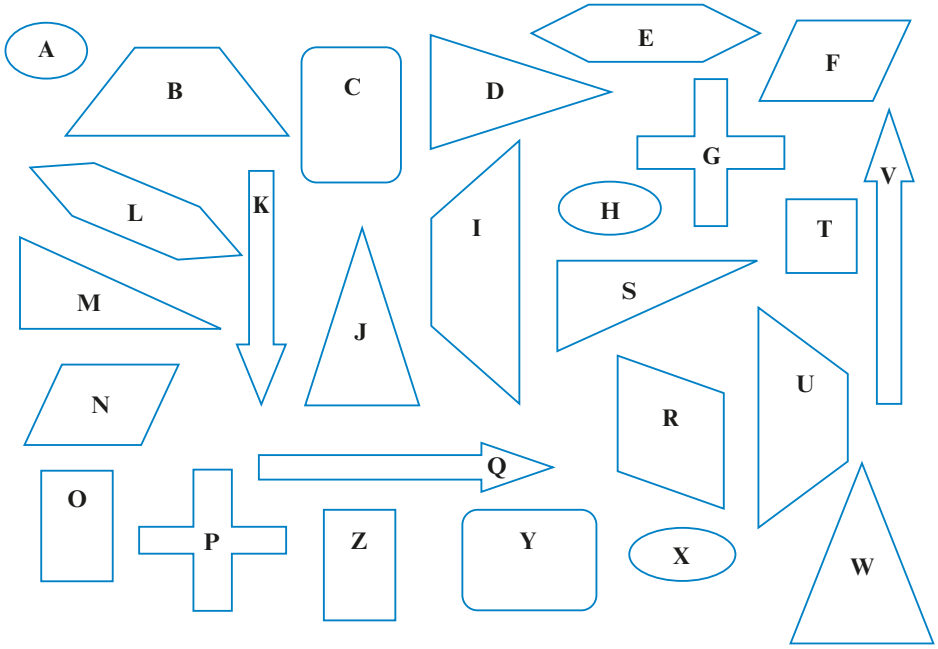
a Determine if each piece is made of congruent figures.

b Provide a reason for your decision.

c Provide a reason why you think someone might come to the wrong conclusion about this.

CHALLENGE

10 Consider these shapes.



a Trace or copy each shape and cut them out, labelling each with its letter.

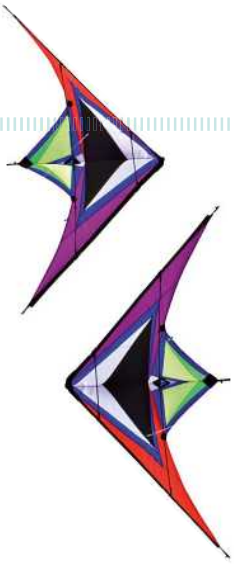
b Find which shapes are congruent.

c Describe each pair of congruent shapes as a transformation.
For example, shape U is the result of a translation and reflection in a horizontal mirror line of shape B.

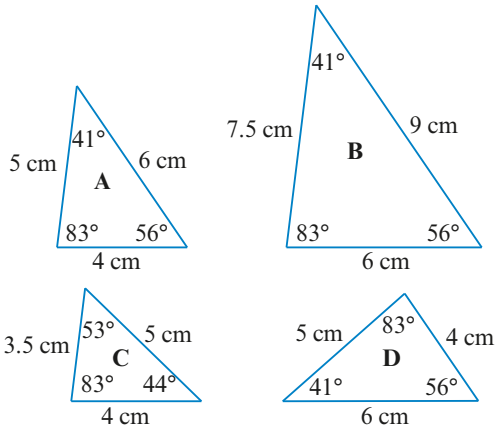
Reflect

How can you determine if two figures are congruent?

7H Using congruence



Start thinking!



1 Which of these four triangles are congruent? How do you know?

2 All three sides of the two congruent triangles are the same length. Do you think this is the case for all congruent triangles?

Triangles that have all three pairs of sides equal are congruent and meet the condition SSS.

3 Why do you think it is named SSS?

There are other conditions that can be used to determine congruence. SAS is one of them.

4 What do you think SAS stands for?

Order is extremely important for these conditions. In SAS, the angle must be in between the two sides.

5 Show that only the pair of congruent triangles meet this condition.

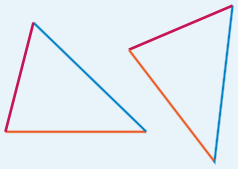
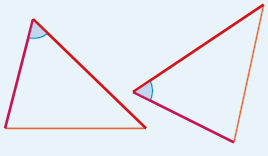
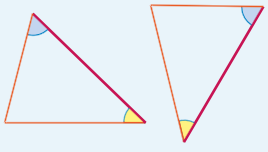
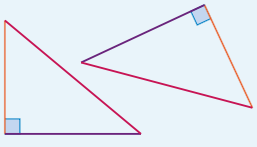
If the angle is not between the two sides, the description becomes SSA.

6 Use triangles A and C to show that if two triangles meet the condition SSA it does not mean that they are congruent.

7 With a classmate, investigate other combinations of angles and sides to see if you can find other conditions for congruence.

KEY IDEAS

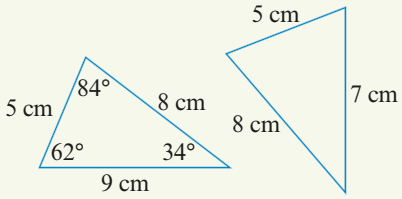
- Two triangles are congruent if all their sides and angles are equal.
- If a pair of triangles meet the conditions shown in the table, they are congruent.
- The specifications AAA (three pairs of angles equal) and SSA (two pairs of sides equal and a pair of non-included angles equal) do not necessarily mean congruence.

SSS	SAS	AAS	RHS
Three pairs of sides are equal.	Two pairs of sides are equal and the pair of angles in between are equal.	Two pairs of angles are equal and a corresponding pair of sides are equal.	The hypotenuses of two right-angled triangles and another pair of sides are equal.
			

EXERCISE 7H Using congruence

EXAMPLE 7H-1 Using SSS

Use the congruence condition SSS to decide if these triangles are congruent.



THINK

1 SSS means the triangles are congruent if all three pairs of sides are equal.

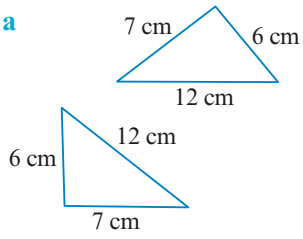
2 Match up each pair of sides and check if they are equal (no). These triangles fail to meet SSS and hence are not congruent.

WRITE

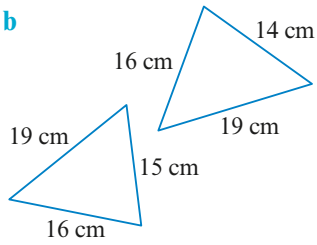
The two triangles are not congruent.

1 Use the congruence condition SSS to decide if these triangles are congruent.

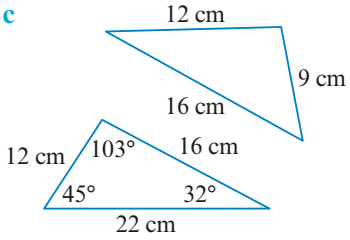
a



b

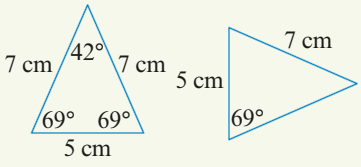


c



EXAMPLE 7H-2 Using SAS

Use the congruence condition SAS to decide if these triangles are congruent.



THINK

1 SAS means two pairs of sides must be equal and the angles in between them must be equal.

2 Match up each pair of sides and check if they are equal.

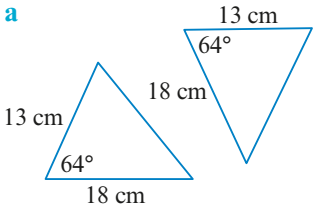
3 Look at the angles between the two pairs of sides. In the second triangle the angle given is not between the sides so you cannot check if the triangle meets SAS.

WRITE

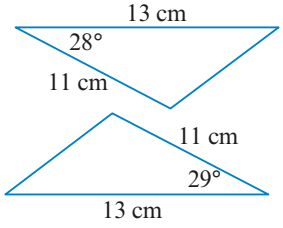
The two triangles may or may not be congruent.

2 Use the congruence condition SAS to decide if these triangles are congruent.

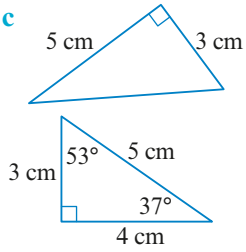
a



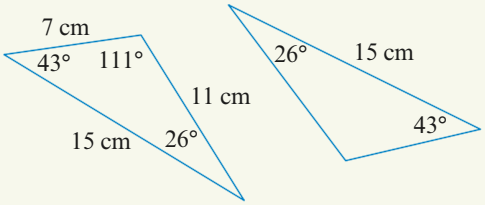
b



c



EXAMPLE 7H-3 Using AAS



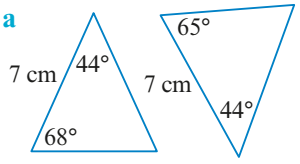
Use the congruence condition AAS to decide if these triangles are congruent.

THINK	WRITE
1 The condition AAS means that if two pairs of angles are equal and a corresponding pair of sides are equal, the two triangles are congruent.	
2 Match up the two pairs of angles and see if they are equal.	
3 Look at a pair of corresponding sides and check if they are equal.	
4 The pairs of angles are equal and the pair of sides between them is equal, so these two triangles meet AAS and hence are congruent.	The two triangles are congruent.

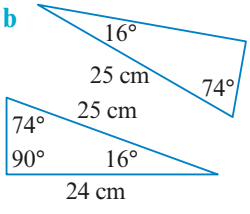
UNDERSTANDING AND FLUENCY

3 Use the congruence condition AAS to decide if these triangles are congruent.

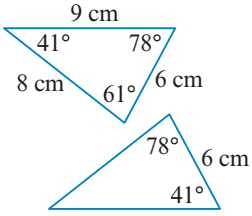
a



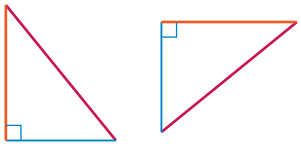
b



c



4 Two triangles that meet the specifications SSA and AAA may or may not be congruent. If the specification SSA is applied to a right-angled triangle it becomes a condition for congruence called RHS. The longest side of a right-angled triangle is called the hypotenuse.



a What colour is the hypotenuse in this triangle?

b If the R stands for right-angled triangle, what do you think the H and S stand for?

c Why must the hypotenuse be one of the sides for this condition? (Hint: if the right angle is between the two sides, what condition does it become?)

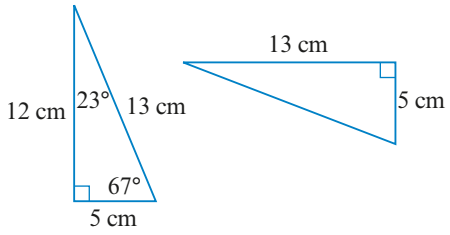
When the triangle is right-angled and you know the length of the hypotenuse, the third side can be calculated using a formula called Pythagoras' Theorem (you will meet this next year).

d If you calculated the length of the third side, which condition would this now be? (Hint: you would know the lengths of all the sides.)

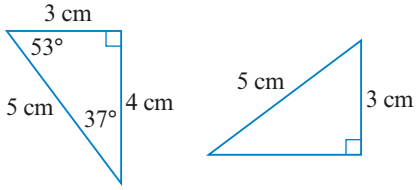
e How does this show that RHS is a condition for congruence?

5 Use the condition RHS to decide if these pairs of triangles are congruent.

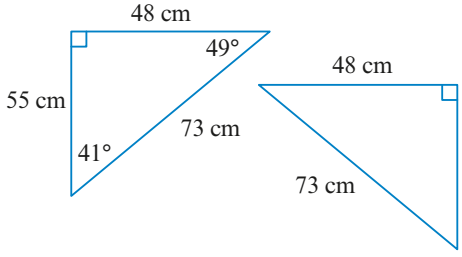
a



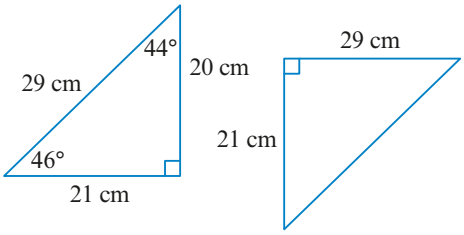
b



c

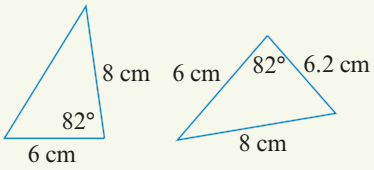


d



EXAMPLE 7H-4 Deciding if triangles are congruent

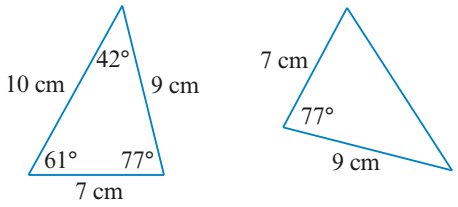
Decide if this pair of triangles is congruent, giving a reason for your answer.



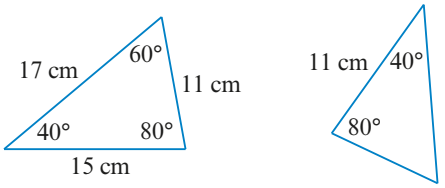
THINK	WRITE
1 Two sides and an angle are given that are the same in both triangles. Check if this meets a condition for congruence.	
2 In the first triangle the angle is between the two sides, which fits the SAS condition for congruence.	
3 The angle in the second triangle is not between the two sides, so it only meets the specification SSA, which is not a condition for congruence.	The two triangles are not necessarily congruent as the only specification they meet (SSA) is not a condition for congruence.

6 Decide if these pairs of triangles are congruent, giving a reason for your answer.

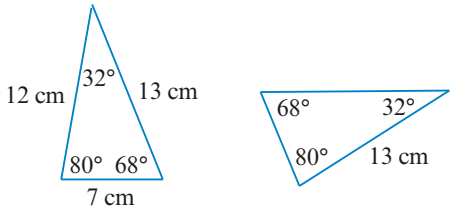
a



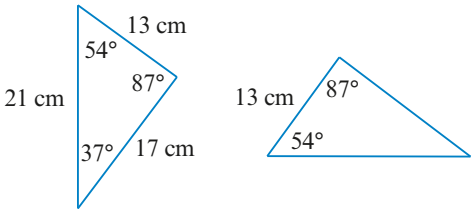
b



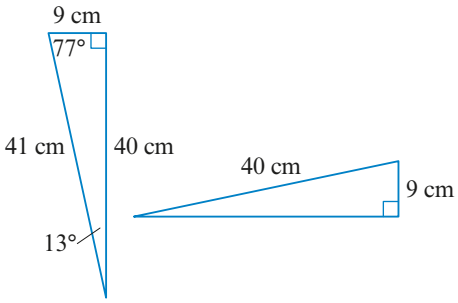
c



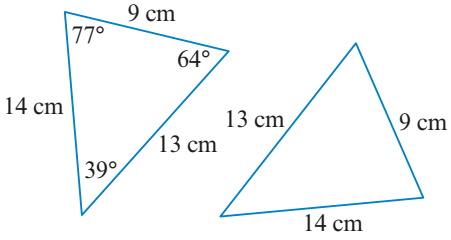
d



e



f



7 **a** Use a compass, ruler and/or protractor to construct a triangle with side lengths 3 cm, 4 cm and 5 cm.

b Construct a congruent triangle by following the condition SAS. That is, draw two sides with an angle in between that match the original triangle.

c Draw in the third side. Does its length match the third side in the original triangle?

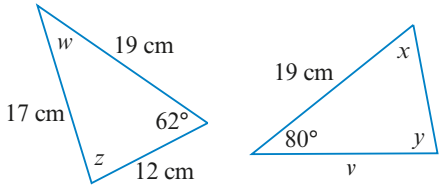
d Do the angles match as well?

e Repeat parts **b–d** and construct more congruent triangles for the other conditions for congruence (AAS, SSS, RHS). What do you find?

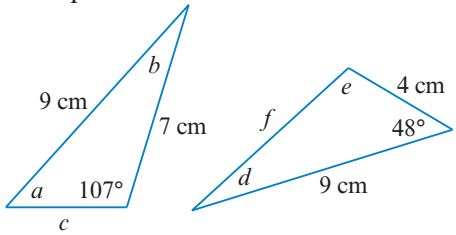
8 **a** Find the unknown side lengths and angles in these triangles, given that each pair is congruent.

b State the condition(s) that you used to solve each problem.

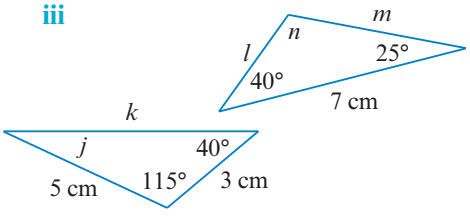
i



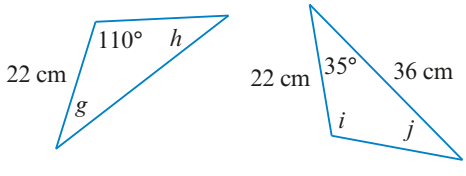
ii



iii



iv



CHALLENGE

9 Elizabeth plays the triangle in her school band. She orders a new triangle online but thinks that the one she receives is not the right one. Use your understanding of congruence to show that these two triangles cannot be congruent even if the dimensions of the photo make the first triangle appear bigger than it is.



10 Another condition for congruence is ASA.

a If you know two of the angles in a triangle, how do you find the third angle?

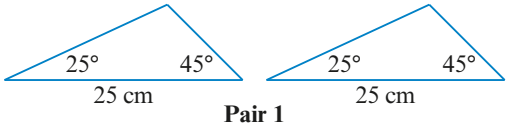
b Using your answer to part **a**, explain how you know that two triangles that have two pairs of equal angles must meet the specification AAA.

ASA refers to two matching pairs of angles and the pair of sides in between.

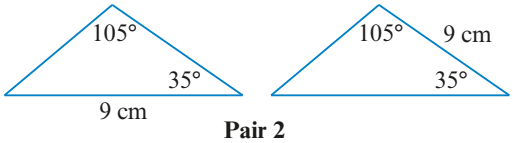
c Explain why pair 1 meets the condition ASA but pair 2 does not.

d Can you explain how, if a pair of triangles meet the condition ASA, then they also meet the condition AAS? (Hint: you need to find the third angle of the triangles.)

e Draw a pair of triangles to demonstrate that the specification ASA is a condition for congruence.



Pair 1



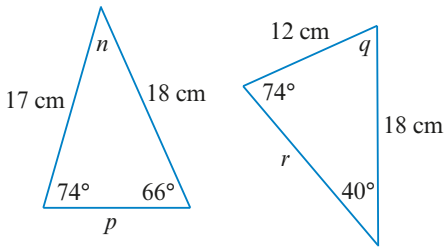
Pair 2

11 For the pairs of triangles below:

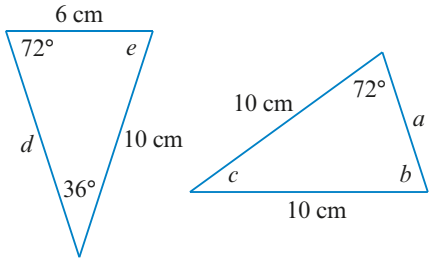
i decide if they are congruent, stating which condition you used

ii find the value of the pronumerals if the pair is congruent.

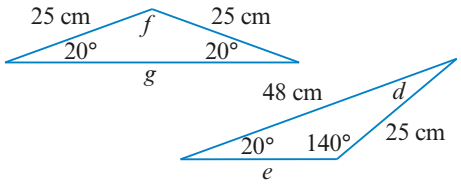
a



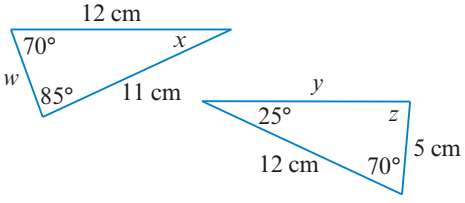
b



c



d

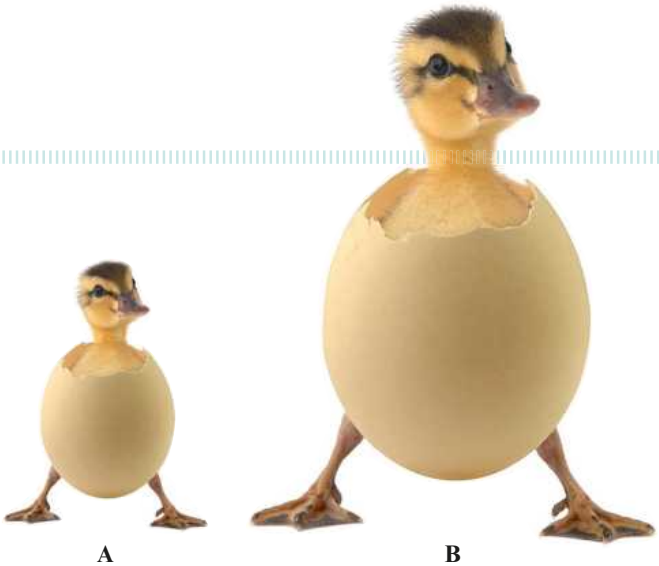


Reflect

What must you remember when using congruence to solve problems?

7I Dilations

Start thinking!



A B

Not all transformations produce identical figures. **Dilation** is one such transformation.

1 Look at figures A and B.

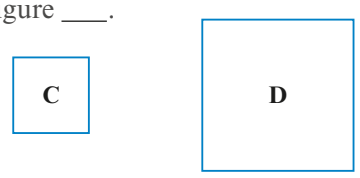
a How are they different from one another?
How are they similar?

b Figure B is an **enlargement** of figure A.
Copy and complete:
This relationship can also be written as: figure ___ is a **reduction** of figure ___.

2 Now look at figures C and D. Figure C is a 1 cm square.

a How would you describe figure D?

b How much bigger is figure D than figure C?



C D

How much bigger or smaller one figure is than another is called the **scale factor**. Figure D is an enlargement of figure C by a scale factor of 2.

> **NOTE** 'Bigger/smaller' refers to how much wider and taller the image is, not to its area.

3 State the scale factor for each enlargement from a 1-cm square.

a 3-cm square **b** 4-cm square **c** 6-cm square **d** 10-cm square

4 State the scale factor for each enlargement from a 2 cm square.

a 4-cm square **b** 6-cm square **c** 8-cm square **d** 12-cm square

5 Naomi gave her answers to question **4** as 4, 6, 8 and 12. What mistake did she make? Explain where she went wrong and what she should have done.

KEY IDEAS

- Dilation is a transformation that does not produce congruent figures.
- Dilations can either be enlargements (the image is bigger) or reductions (the image is smaller).
- Dilations are described using a scale factor, where numbers above 1 indicate an enlargement and numbers between 0 and 1 (usually fractions) indicate a reduction. For example, in Start thinking! above, the scale factor for the dilation of C to D is 2, and the scale factor for the dilation of D to C is $\frac{1}{2}$.

EXERCISE 7I Dilations

EXAMPLE 7I-1 Identifying dilations

Describe this dilation as an enlargement or a reduction.



THINK

1 The original is A and the image is A′.

2 The image (A′) is larger than the original.

WRITE

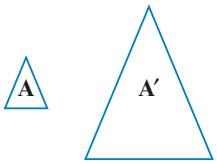
This dilation is an enlargement.

1 Describe these dilations as either enlargements or reductions. Remember that shape A′ is the image.

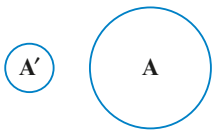
a



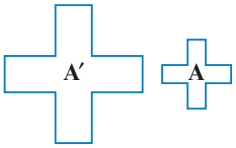
b



c



d



EXAMPLE 7I-2 Finding the scale factor

Shape B is 4 times the size of shape A. What is the scale factor if the original shape is:

a shape A? **b** shape B?

THINK

a **1** Dilation from A to B is an enlargement (because A is smaller than B).

2 An enlargement has a scale factor greater than 1.

b **1** Dilation from B to A is a reduction (because A is smaller than B).

2 A reduction has a scale factor between 0 and 1.

WRITE

a

The scale factor for the dilation of A to B is 4.

b

The scale factor for the dilation of B to A is $\frac{1}{4}$.

UNDERSTANDING AND FLUENCY

2 Copy and complete this table of scale factors.

	Enlargement	Reduction
2 times	scale factor = 2	scale factor = $\frac{1}{2}$
3 times	scale factor = 3	scale factor = $\frac{1}{3}$
4 times		scale factor = $\frac{1}{4}$
5 times		
6 times		
7 times		
8 times		
9 times		
10 times		

3 What is the scale factor for each situation if the original shape is:

- **i** shape A?
- **ii** shape B?

a B is 3 times the size of A
b A is 5 times the size of B
c A is 4 times the size of B
d A is 10 times the size of B
e B is 8 times the size of A
f A is 20 times the size of B

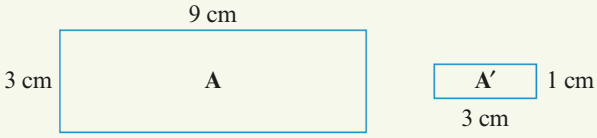
4 Use each scale factor to describe the dilation as an enlargement or a reduction.

a 4 **b** $\frac{1}{4}$
c 9 **d** 3
e $\frac{1}{10}$ **f** $\frac{1}{2}$

5 Which scale factor in question **4** produces the greatest enlargement?

EXAMPLE 7I-3 Describing dilations

Shape A has been dilated and its image, A′, is shown.



a Describe the dilation as an enlargement or a reduction.
b Find the scale factor for the dilation.
c Hence describe the dilation in full.

THINK

a The image (A′) is smaller than the original.

b To find the scale factor, compare corresponding sides. The original measures 3 cm × 9 cm and the image measures 1 cm × 3 cm. The original is three times the size of the image.

c Write your answer.

WRITE

a This dilation is a reduction.

b Scale factor = $\frac{1}{3}$.

c This dilation is a reduction with a scale factor of $\frac{1}{3}$.

UNDERSTANDING AND FLUENCY

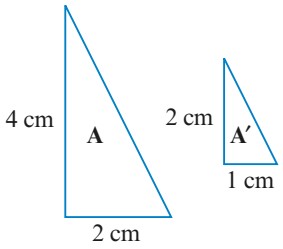
6 For these shapes:

i describe the dilation as an enlargement or a reduction

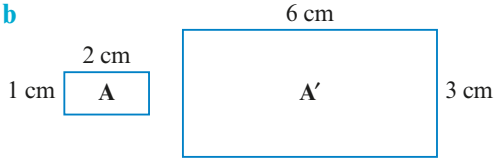
ii find the scale factor for the dilation

iii hence describe the dilation in full.

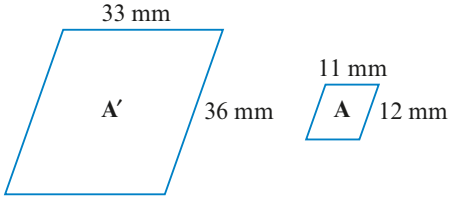
a



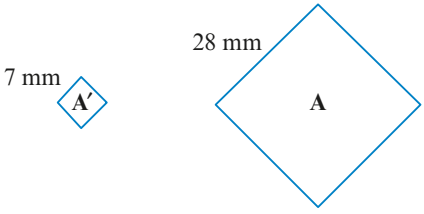
b



c



d

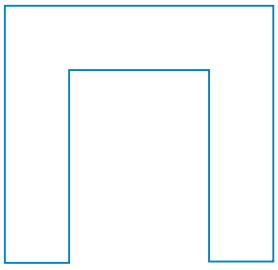


PROBLEM SOLVING AND REASONING

7 Is the small dog a reduction of the big dog? Explain.



8 Dilate this figure by a scale factor of:



a 2 **b** 3 **c** 5 **d** $\frac{1}{4}$ **e** $\frac{1}{3}$ **f** $\frac{1}{2}$

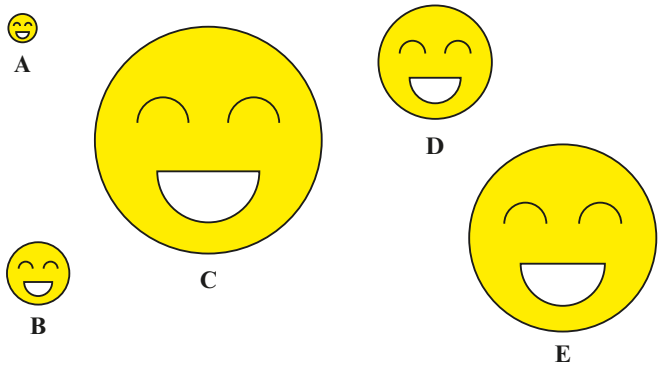
Start from the original each time.

9 Consider this group of shapes. Identify the original and its image if:

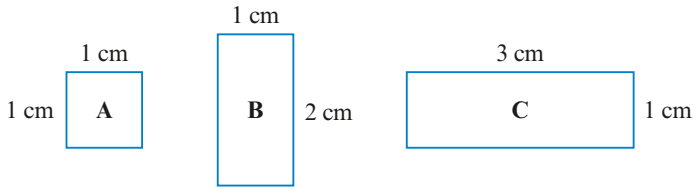
a one is an enlargement of the other by scale factor 3

b one is an enlargement of the other by scale factor 2

c one is a reduction of the other by scale factor $\frac{1}{4}$.



10 Dilations can also be done in only a horizontal direction or only a vertical direction. Consider these figures.



a Figure B is a dilation of figure A. In what direction has it been dilated? By what scale factor?

b Has the horizontal length of figure B changed with respect to figure A?

c Figure C is also a dilation of figure A. In what direction has it been dilated? By what scale factor?

d Has the vertical length of figure C changed with respect to figure A?

e With respect to figure A, draw an image that has been dilated by:

i a vertical scale factor of 3 **ii** a horizontal scale factor of 4

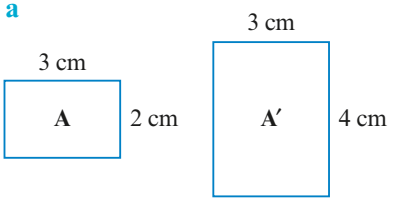
iii a horizontal scale factor of 6 **iv** a vertical scale factor of $\frac{1}{2}$.

11 Describe these dilations:

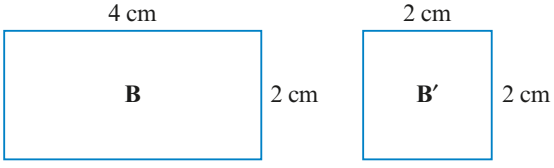
i as enlargements or reductions

ii by stating the scale factor and the direction of dilation.

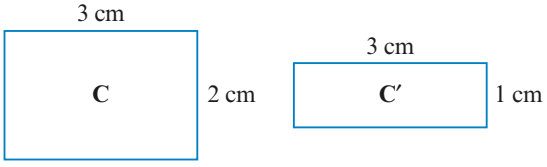
a



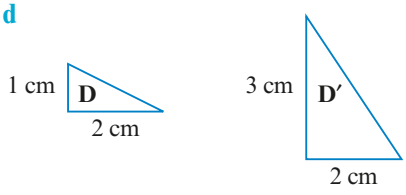
b



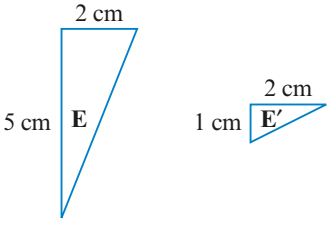
c



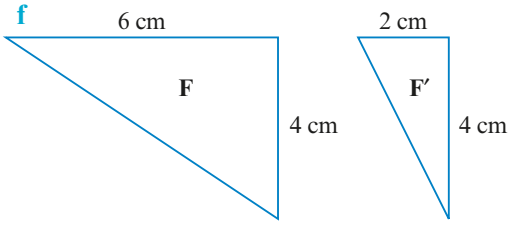
d



e



f



12 Dilate the figure in question **8** by:

a a horizontal scale factor of 2 **b** a vertical scale factor of 3

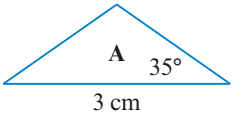
c a horizontal scale factor of 4 **d** a vertical scale factor of $\frac{1}{4}$

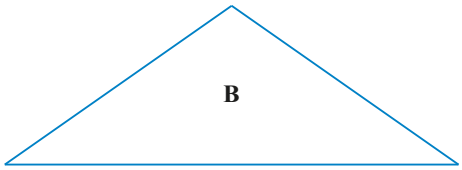
e a vertical scale factor of $\frac{1}{3}$ **f** a horizontal scale factor of $\frac{1}{2}$.

13 An image that changes only in size (not in shape) after a dilation is called a **similar figure**.

PROBLEM SOLVING AND REASONING

Knowledge of similar figures can help you solve problems. Consider these two triangles. Triangle B is a dilation with a scale factor of 2 compared to triangle A.





a What does this mean?

b One side of triangle A is 3 cm long. What is the length of the corresponding side of triangle B?

c One angle in triangle A is 35°. What is the size of the corresponding angle in triangle B?

d A friend said that because triangle B is twice the size of triangle A that the corresponding angle is 70°. Can you explain their mistake?

e Copy and complete: For any pair of similar figures, to find the side length of an image, _____ the corresponding side length of the original figure by the scale factor. Each angle is _____ to the corresponding angle in the original figure.

14 For each pair of similar figures:

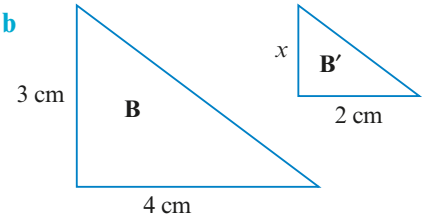
i use one pair of corresponding sides to find the scale factor

ii find the missing side lengths and/or angles for each pair.

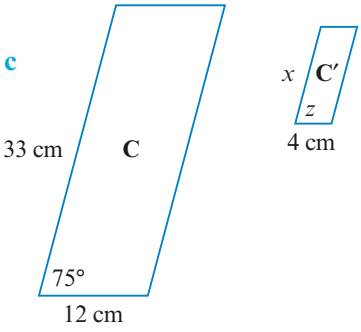
a



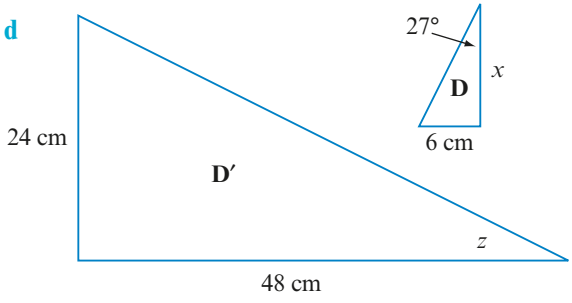
b



c



d



CHALLENGE

15 Dilate the figure in question **8** by:

a scale factor $\frac{2}{3}$ **b** scale factor $\frac{7}{10}$ **c** scale factor $\frac{3}{8}$

d scale factor $\frac{4}{5}$ **e** scale factor $\frac{7}{2}$ **f** scale factor $\frac{3}{2}$

g horizontal scale factor $\frac{1}{2}$ **h** vertical scale factor $\frac{7}{2}$

i horizontal scale factor $\frac{1}{2}$ and vertical scale factor 4.

j horizontal scale factor 3 and vertical scale factor $\frac{2}{3}$.

Start from the original figure each time.

Reflect

What mistakes do you think people commonly make when working with dilations?

CHAPTER REVIEW

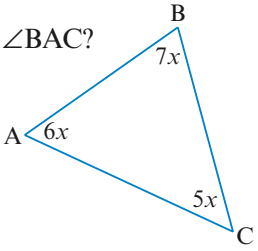
SUMMARISE

Create a summary of this chapter using the key terms below. You may like to write a paragraph, create a concept map or use technology to present your work.

triangle	polygon	edge	translation
equilateral	regular	vertex	reflection
isosceles	irregular	isometric dot paper	rotation
scalene	convex	set of plans	congruent
acute-angled	concave	nets	dilation
right-angled	prism	perspective drawing	enlargement
obtuse-angled	pyramid	vanishing point	reduction
exterior angle	polyhedron	isometric transformation	scale factor
quadrilateral	face		similar figure

MULTIPLE-CHOICE

7A **1** What size is angle ∠BAC?



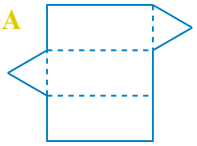
- **A** 6°
- **B** 10°
- **C** 18°
- **D** 60°

7B **2** Which statement is true?

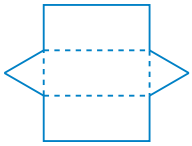
- **A** A parallelogram is a square.
- **B** A square is a rhombus.
- **C** A trapezium cannot have a right angle.
- **D** A kite has parallel sides.

7E **3** Which net does *not* make a triangular prism?

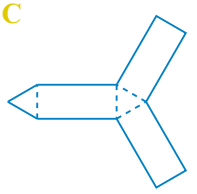
- **A**



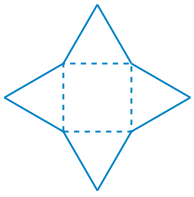
- **B**



- **C**



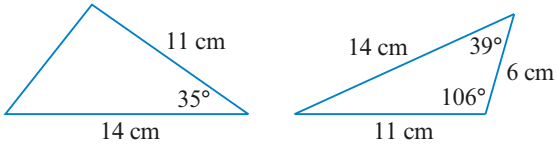
- **D**



7G **4** Which transformation does *not* produce congruent figures?

- **A** translation
- **B** rotation
- **C** reflection
- **D** dilation

7H **5** Which rule could tell you if these triangles are congruent?



- **A** SAS
- **B** SSS
- **C** AAA
- **D** AAS

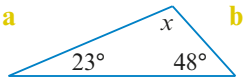
7I **6** Shape G is 5 times the size of shape E. Which statement is *not* true?

- **A** The dilation from E to G has a scale factor of 5.
- **B** The dilation from G to E is a reduction.
- **C** Shape E is one-fifth the size of Shape G.
- **D** If shape E is the original, the dilation has a scale factor of $\frac{1}{5}$.

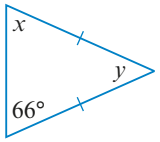
SHORT ANSWER

7A **1** Find the size of angles x and y.

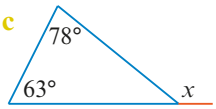
a



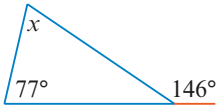
b



c



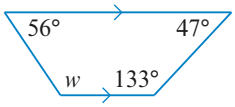
d



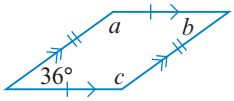
7A **2** Name the triangles shown in question **1**.

7B **3** Find the size of the unknown angles in these quadrilaterals.

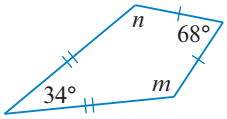
a



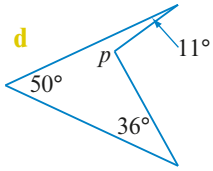
b



c



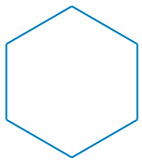
d



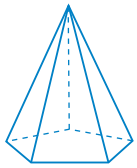
7B **4** Name the quadrilaterals shown in question **3**.

7C **5** Name these shapes and objects.

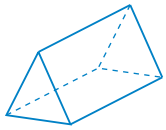
a



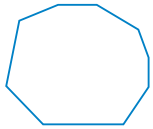
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c



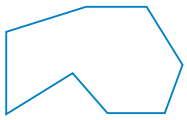
d



e

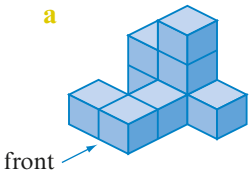


f

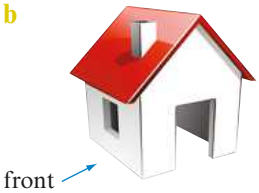


7D **6** Draw a set of plans for each object.

a

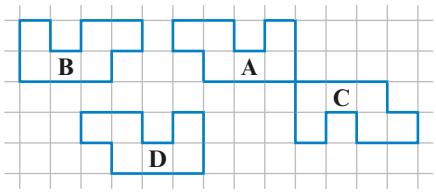


b



7E **7** Draw a perspective drawing for each prism in question **5**.

7F **8** Look at the shapes shown on this grid.

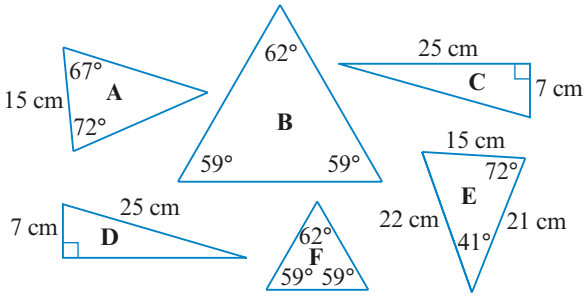


a Describe B, C and D as transformations of A.

b Using C as the original shape each time, draw images that are:

- **i** reflected in a diagonal mirror line
- **ii** rotated 90° anticlockwise from the bottom right corner
- **iii** translated 4 units right and 3 units up.

Use these figures for questions **9**–**11**.



7G **9** Identify any pairs of congruent figures.

7H **10** Explain your answers to question **9** using one of the conditions for congruence (SSS, SAS, AAS, RHS) or specifications that do not necessarily mean congruence (AAA, SSA).

7I **11** Triangle B has been dilated to produce one of the other figures.

a Which triangle is the image?

b Is it an enlargement or a reduction?

c What is the dilation scale factor?

d Describe the dilation in full.

NAPLAN-STYLE PRACTICE

Questions 1 and 2 refer to the following statement.
Devon measured the angles of a triangle and found that angles b and c were twice the size of angle a.

1 What type of triangle is this?

- ☐ right-angled triangle
- ☐ isosceles triangle
- ☐ equilateral triangle
- ☐ scalene triangle

2 What is the size of angle a?

30° ☐ 36° ☐ 60° ☐ 72° ☐

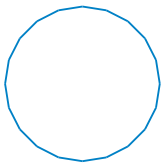
3 Alexandria found that one pair of sides of a quadrilateral were parallel. What type of quadrilateral can it *not* be?

☐ trapezium ☐ kite
☐ rhombus ☐ rectangle

4 A parallelogram has one angle of 23°. What is the size of the other different angle?

☐

5 An icosagon is a polygon with 20 sides. Using the triangle method, what is the internal angle sum of an icosagon?



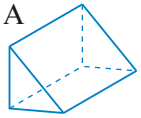
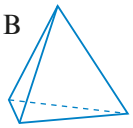
20° ☐ 162° ☐ 360° ☐ 3240° ☐

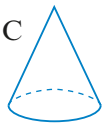
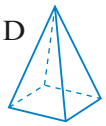
6 Which object is a concave decagon?

☐ A  ☐ B 

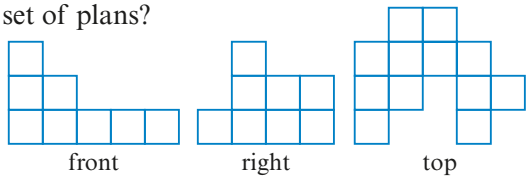
☐ C  ☐ D 

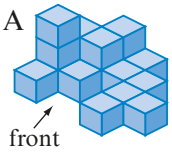
7 Which of the following is *not* a pyramid?

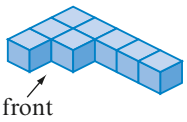
☐ A  ☐ B 

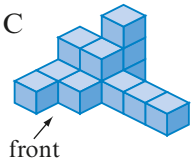
☐ C  ☐ D 

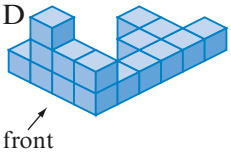
8 Which object matches this set of plans?



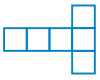
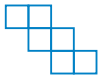
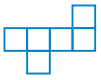
☐ A 

☐ B 

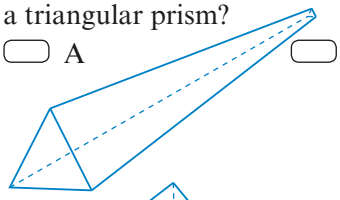
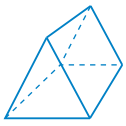
☐ C 

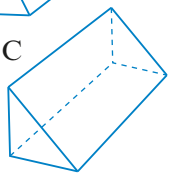
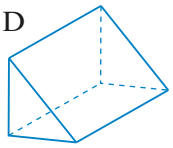
☐ D 

9 Which of these nets does *not* fold up to form a cube?

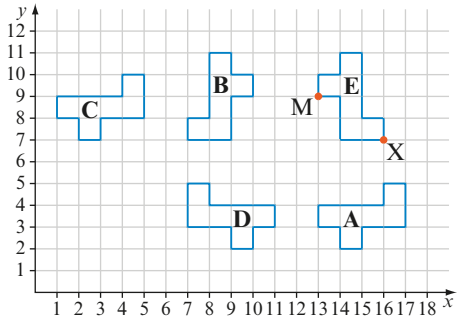
A  ☐
B  ☐
C  ☐
D  ☐

10 Which drawing is *not* a perspective drawing of a triangular prism?

☐ A  ☐ B 

☐ C  ☐ D 

Questions 11–13 refer to the following figure.



11 Which shape is a rotation of B?

A ☐ C ☐ D ☐ E ☐

12 What is the transformation from C to A?

- ☐ reflection in a diagonal mirror line
- ☐ translation of 12 left, 5 up
- ☐ translation of 5 left, 12 down
- ☐ translation of 12 right, 5 down

13 Shape E was reflected in a vertical mirror line at point M and then translated 2 units down and 3 units right. What are the coordinates of corner X after these transformations?

- ☐ (13, 5)
- ☐ (10, 5)
- ☐ (16, 7)
- ☐ (7, 16)

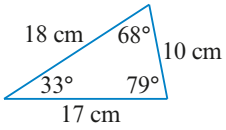
14 Which sequence of transformations does *not* produce congruent figures?

- ☐ reflection, translation, reflection
- ☐ rotation, translation, dilation
- ☐ translation, rotation, translation
- ☐ reflection, rotation, translation

15 Which of these conditions cannot be used to determine congruence?

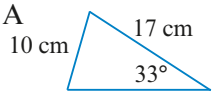
SSA ☐ SAS ☐ SSS ☐ AAS ☐

Questions **16** and **17** refer to this triangle.

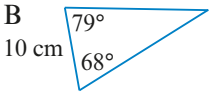


16 Which of these triangles is congruent to the original triangle?

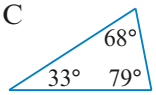
☐ A



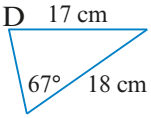
☐ B



☐ C



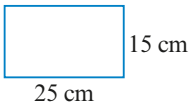
☐ D



17 Which condition for congruence did you use to answer question **16**?

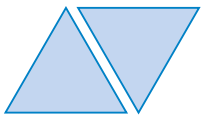
SSS ☐ AAS ☐ SAS ☐ RHS ☐

18 The rectangle shown was dilated so that the shortest side measured 5 cm. What is the scale factor for this dilation?



ANALYSIS

A chocolate company is making a new product and needs your help with the design. Each chocolate is an equilateral triangle with side length 2 cm and they are to be stacked in a prism, as shown in this front view.



a In what ways can you describe one triangle as a transformation of the other?

b What shape is the front view of the prism? Draw this shape and label it with its angles and side lengths.

c Name the prism of this shape.

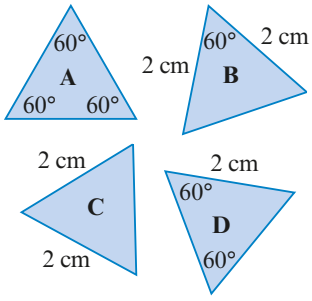
d Use isometric dot paper to draw this prism.

e Draw the following representations of this package.

i a net **ii** a perspective drawing

iii a set of plans

The company produces the chocolates shown here.



f Which chocolate(s) can you be certain are the right dimensions? Explain how you know.

g The company also wants to produce a jumbo size in which the dimensions are doubled. State the scale factor for this dilation and draw the new chocolates and packaging, labelling the angles and side lengths.

CONNECT

Dream home

What would your dream home look like?
How would you arrange the rooms?







Complete the 7 CONNECT worksheet to show all your working and answers to this task.

You may like to present your findings as a report. Your report could be in the form of:

- a set of blueprints and a model
- a coffee-table style book
- a digital presentation
- other (check with your teacher).

Your task

Your task is to use the skills you have learnt in this chapter to design both the inside and outside of your dream house. Follow these steps.

- Decide on the number and types of rooms in your house.
- Decide on the layout of the house (for example, will it have different levels?).
- Produce a basic set of plans for the internal layout of the house.
- Design the outside of the house, including the roof.
- Produce a set of plans for the house.
- Produce detailed plans for at least one room of the house.

Include all necessary working to justify your answers.




échelle 1/20
COUPE B.B'

8 MEASUREMENT

8A Length and perimeter

8B Circumference of a circle

8C Area of rectangles and triangles

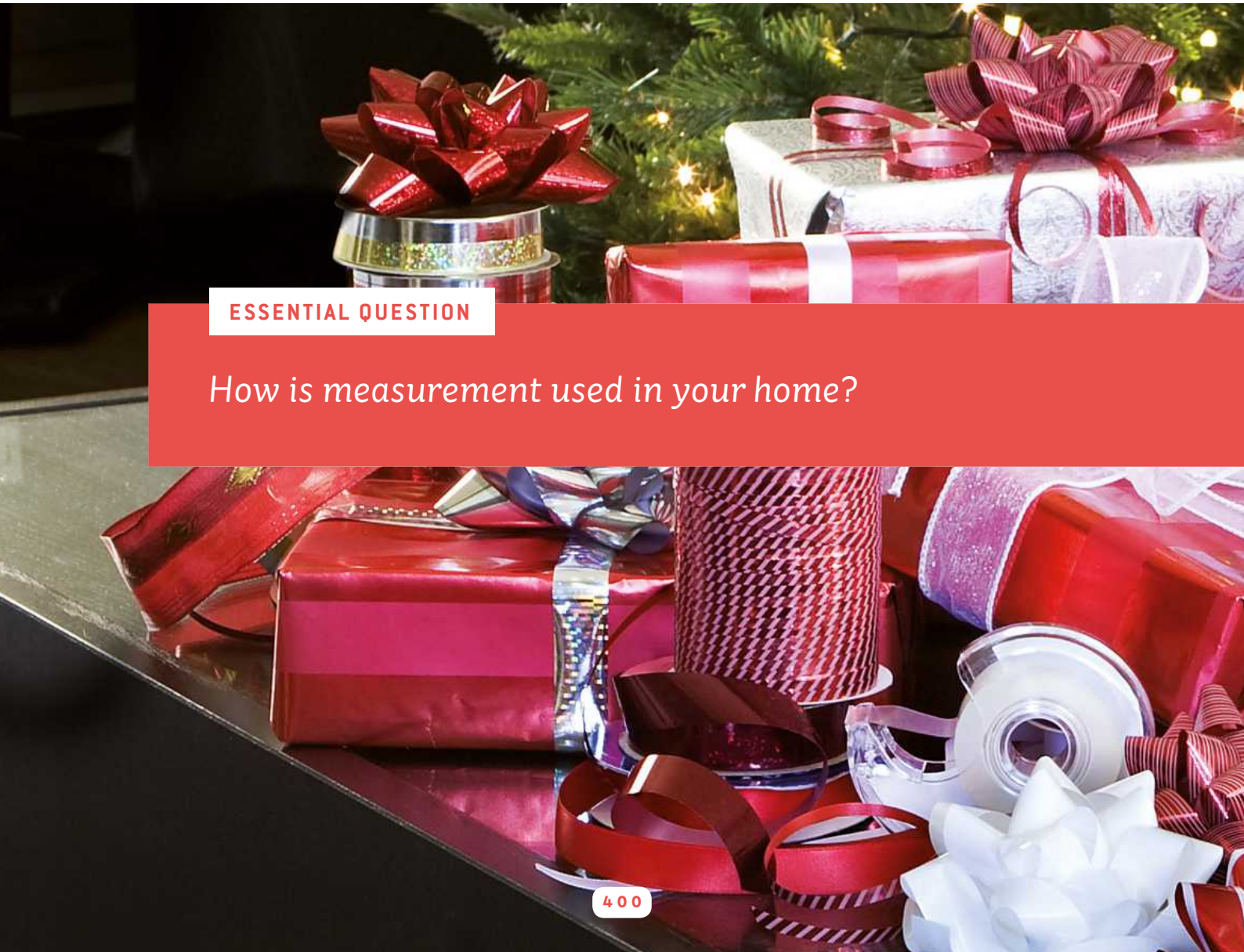
8D Area of other quadrilaterals

8E Area of a circle

8F Surface area

8G Volume of prisms

8H Area and volume conversions



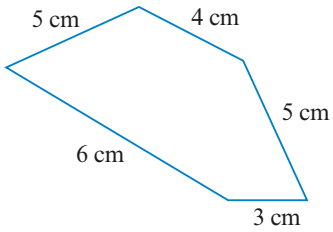
ESSENTIAL QUESTION

How is measurement used in your home?

Are you ready?

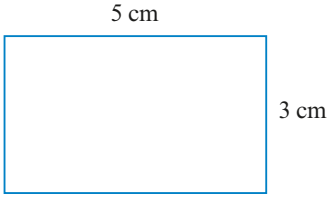
8A **1** How many millimetres long is a 15-cm ruler?

8A **2** What is the perimeter of this shape?

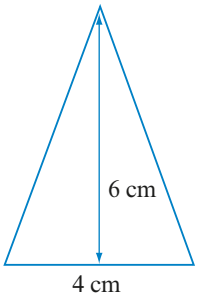


A 18 cm
B 1800 cm
C 20 cm
D 23 cm

8C **3** What is the area of this shape?



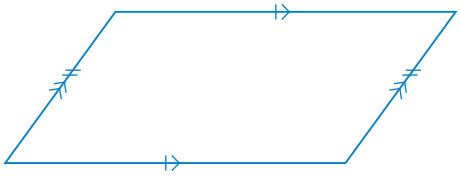
8C **4** What is the area of this triangle?



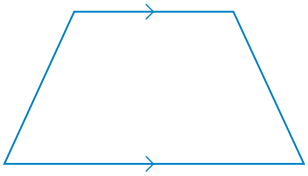
A 24 cm^2
B 12 cm^2
C 10 cm^2
D 2 cm^2

8D **5** Which shape is a rhombus?

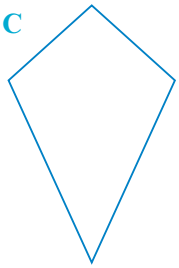
A



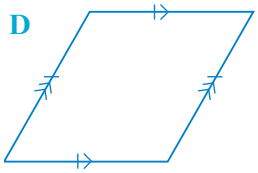
B



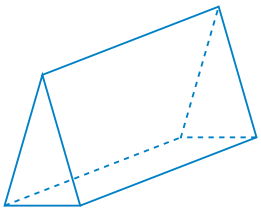
C



D

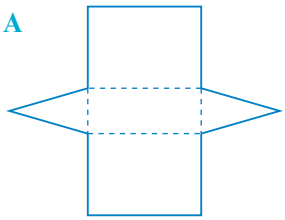


8F **6** How many faces does this object have?

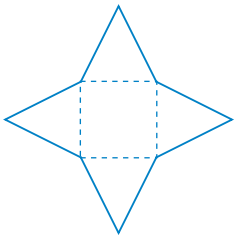


8F **7** Which net matches the object in question **6**?

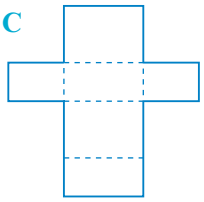
A



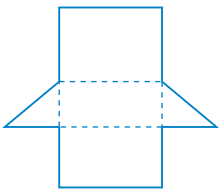
B



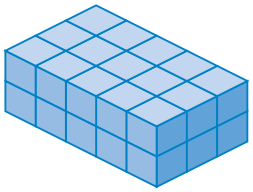
C



D



8G **8** What is the volume of this prism?



A 10 cm^3
B 15 cm^3
C 16 cm^3
D 30 cm^3

8H **9** How can you convert centimetres to metres?

A divide by 100 **B** multiply by 100
C multiply by 10 **D** divide by 10

8H **10** Which unit relates to area?

A mm^3 **B** cm
C m^3 **D** km^2

8A Length and perimeter

Start thinking!

When you think about measurement, two words you may know are **length** and **perimeter**.



1 What is the difference between length and perimeter?

Common units of length are millimetres (mm), centimetres (cm), metres (m) and kilometres (km).

2 Think of an object that you could measure both the length and perimeter of in:
i millimetres **ii** centimetres **iii** metres **iv** kilometres.

3 With a classmate, **estimate**:
a the length **b** the perimeter
of the table that you are sitting at now.

4 Why is it important to estimate?

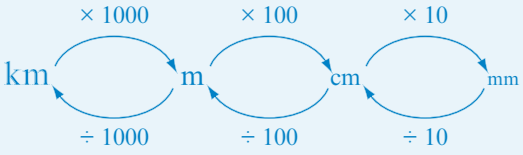
5 Choose a device to measure the table. Explain why your chosen device is better than either a 5-cm ruler or a trundle wheel.

6 Imagine your table is against a wall. How might you find the perimeter of the table without measuring the side that is against the wall?

7 Without moving the table, find both its length and its perimeter. How do these measurements compare to your estimate?

KEY IDEAS

- Common units of length are millimetres (mm), centimetres (cm), metres (m) and kilometres (km).
- You can convert between units of length by multiplying or dividing by the conversion factor.



- Perimeter is the distance around the outside edge of a 2D (two-dimensional) shape.
- To calculate perimeter, add the length measurements of each side together. Make sure that all measurements are in the same unit.
- Always estimate your answer first.

EXERCISE 8A Length and perimeter

EXAMPLE 8A-1 Converting units of length (one step)

Convert 46 cm into metres.

THINK	WRITE
1 The length is being converted to a larger unit (centimetres into metres), so divide by the conversion factor.	
2 The conversion factor from centimetres to metres is 100.	46 cm = (46 ÷ 100) m = 0.46 m

1 Convert:

- **a** 58 m into centimetres
- **b** 2.7 km into metres
- **c** 521 cm into metres
- **d** 398 m into kilometres
- **e** 1.2 cm into millimetres
- **f** 3987 m into kilometres
- **g** 423 mm into centimetres
- **h** 7 mm into centimetres

EXAMPLE 8A-2 Converting units of length (two steps)

Convert 1.23 m into millimetres.

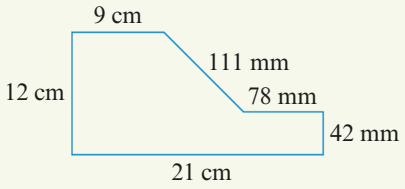
THINK	WRITE
1 First convert to centimetres by multiplying by 100. 1 m = 100 cm.	1.23 m = (1.23 × 100) cm = 123 cm
2 Complete the conversion to millimetres by multiplying by 10. 1 cm = 10 mm.	= (123 × 10) mm = 1230 mm

2 Convert:

- **a** 5.34 m into millimetres
- **b** 3.95 km into centimetres
- **c** 235 000 mm into metres
- **d** 145 000 000 mm into kilometres
- **e** 2368 cm into kilometres
- **f** 287 mm into metres
- **g** 1.934 km into centimetres
- **h** 0.91 km into millimetres

EXAMPLE 8A-3 Calculating perimeter

Calculate the perimeter of this shape in centimetres.



THINK

1. There are six sides to the shape so there are six lengths to be added.
2. Write all measurements in centimetres. Remember that 10 mm = 1 cm.
3. Add all the lengths and include the correct unit.

WRITE

perimeter = 9 cm + 111 mm + 78 mm + 42 mm + 21 cm + 12 cm

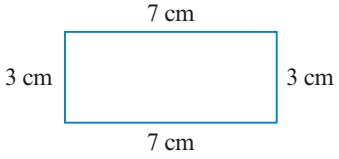
= 9 cm + 11.1 cm + 7.8 cm + 4.2 cm + 21 cm + 12 cm

= 65.1 cm

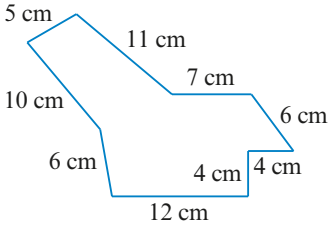
UNDERSTANDING AND FLUENCY

3 Calculate the perimeter of each shape.

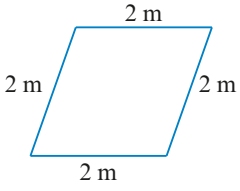
a



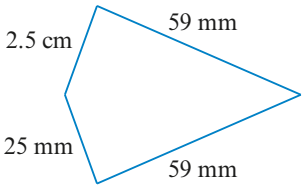
b



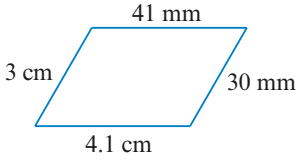
c



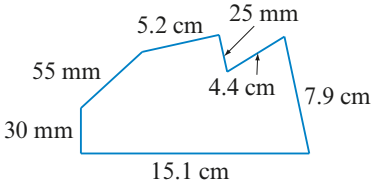
d



e



f



4 For each object, first estimate and then measure:

- **i** the horizontal and vertical lengths
- **ii** the perimeter.

a



b



c



5 Explain how you found the perimeters in question **4**.

UNDERSTANDING AND FLUENCY

6 Name the best unit to use to measure:

a the length of a basketball court

b the perimeter of Australia

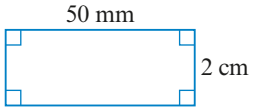
c the perimeter of an iPod

d the length of your textbook.

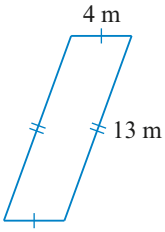
7 Name a device you could use to measure each object in question **6**.

8 Calculate the perimeter of each object.

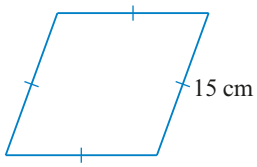
a



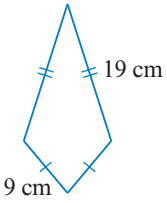
b



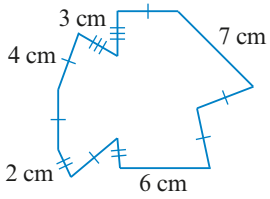
c



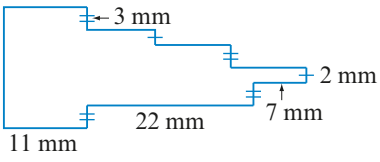
d



e



f



9 Calculate the perimeter of:

a a square with side length 4 cm

b a square with side length 1.2 km

c an equilateral triangle with side length 2.5 cm

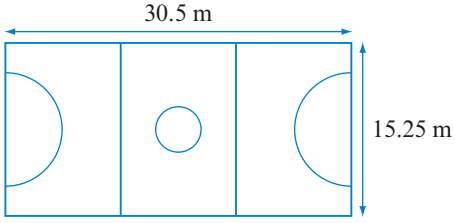
d a rhombus with base 15 mm.

e a rectangle with length 10 mm and width 15 mm

f a rectangle with length 6 m and width 1 m.

PROBLEM SOLVING AND REASONING

10 To warm up before a netball match, Jordan has the option of either running six laps around the netball court or four shuttle runs.



a How far is one lap around the netball court?

b How far is six laps around the netball court?

A shuttle run involves starting at the bottom of the court and running hard to the first third-line, back to the start, running to the second third-line and back to the start and finally up to the end of the court and back.

c How far is the first part of the shuttle run (to the first third-line and back)?

d What distance does a single shuttle run cover?

e How far is four shuttle runs?

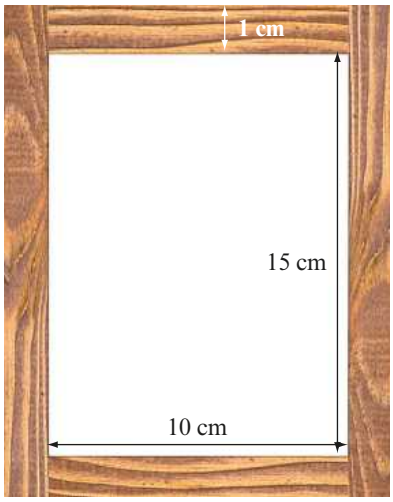
f If you were Jordan, which warm up activity would you choose? Explain.

NOTE The **dimensions** of the court are 30.5 m by 15.25 m or 30.5 m × 15.25 m.

11 Trish makes earrings. One design is a rectangular jewel with wire wrapped around its perimeter twice. If the jewel is 4.5 cm long and 1.5 wide, how much wire (in metres) is used for 20 pairs of earrings?

12 A road has a white line painted on each edge, and a dashed line down the middle. If every 10 m section of the road has eight 650 mm long dashes down its centre, calculate the total length of paint required (in kilometres) for a 10 km road.

13 Alice wants to make some photo frames for a friend. She buys some wood to fit around the outside of a standard-sized photo.



a What is the perimeter of a standard-sized photo?

Alice wants the frame to be 1 cm wide, as shown in the image.

b Why is the length of wood needed greater than the perimeter of the photo? (Hint: are the corner parts of the frame included in the photo perimeter?)

c What length of wood is needed?

d What is the perimeter around the photo frame? Why is this different from part **c**?

e Investigate how these results would change if:

- **i** the frame was 2 cm wide
- **ii** the frame was 3 cm wide
- **iii** the frame was for a 15 cm × 20 cm photo
- **iv** the frame was for a 20 cm × 25 cm photo.

Write a paragraph on your findings.

14 Alice packs the photo frames from question **13** into a gift box with the dimensions shown in the photo and wants to tie a ribbon around the present. Assuming that she will need a minimum of 60 cm of ribbon for the bow, how much ribbon will she need to buy in order to wrap the present as shown?



15 A square has a perimeter of 36 cm. What is its side length?

16 Give three possible sets of dimensions for a rectangle with perimeter 100 cm.

17 A rectangle has perimeter 60 m.

a What might its dimensions be?

b How many answers are possible for part **a**? Assume whole numbers only.

18 Make up your own question like question **17** and give it to a classmate to solve. How many possible whole-number answers are there?

19 A new television screen has a perimeter of 6.1 m. If it has a length of 1.9 m, find its width.

20 A rectangular pool is 12.5 m long and 4.7 m wide.

- **a** What length of paving is required around the pool's edge if the pavers used are 50 cm wide?
- **b** Jordan incorrectly measured the length of the pool as 13.4 m and the width as 5.2 m. What is the size of the error in each measurement?
- **c** Find the percentage error in each measurement by calculating the size of the error as a percentage of the actual measurement.
- **d** Which measurement has a greater: **i** error? **ii** percentage error? Explain.
- **e** Calculate the length of paving required if Jordan's measurements are used.
- **f** Find the percentage error in the length of paving if using Jordan's measurements.

CHALLENGE

21 For each special quadrilateral you met in chapter 7:

- **a** draw a diagram labelling each side with a pronumeral (for example, a, b, h)
- **b** use the diagram to write a formula that can be used to calculate its area.

22 Jackson takes on a scrapbooking project. He decides to cover the perimeter of every page in ribbon that is 2 cm wide. If there are 28 square sheets of paper that measure 25 cm in the book, find the length of ribbon (in metres) required to place ribbon around the edge of each page.



23 A rectangular vegetable garden has a perimeter of 28 m.

- **a** Write down a possible set of dimensions for this garden.

A fence is to be placed 1 m from the edge around the vegetable garden to allow space for more vegetables.

- **b** What length of fencing is required?
- **c** What length of fencing would be required if the distance between the fencing and the edge of the garden was:
  - **i** 2 m? **ii** 3 m?
- **d** Can you see a pattern or rule that you could follow in order to find the length of fencing required, no matter how close or far from the edge it is placed?
- **e** Repeat parts **a**–**d** using a different set of dimensions. Do your findings match the rule you found in part **d**? Explain.

Reflect

How is estimation important when measuring length and perimeter?

8B Circumference of a circle

Start thinking!



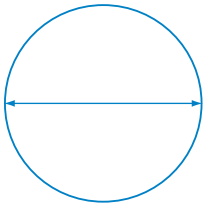
1 How would you measure the perimeter of a circle?

The perimeter of a circle is called its **circumference**.

2 Pair up with a classmate and get a ruler and a long piece of string or ribbon.

3 Copy this table, leaving space for at least five rows.

Object	Diameter (D)	Radius (r)	Circumference (C)	$\frac{C}{D}$



4 Either as a class or in your pairs collect at least five circular objects. Cylinders will work as well.

5 Choose one object. Measure across the width of the circle at its centre as shown here. This is called its **diameter**. Record this in your table. Be as accurate as possible.

The **radius** is the distance from the centre of the circle to its perimeter. It is exactly half the diameter.

6 What is the radius of the circular object you are measuring? Record this in your table.

7 Use your piece of string and ruler to measure the circumference of the circular object and record the measurement in your table.

8 Complete the last cell of your table, correct to two decimal places. What number do you obtain?

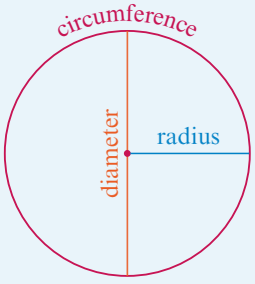
9 Repeat steps **5**–**8** for all your other circular objects. What do you find?

The ratio of the circumference to the diameter $\left(\frac{C}{D}\right)$ is equal to the irrational number **pi** (π), no matter what size a circle is. Remember that π cannot be written as an exact decimal number. Rounded to two decimal places, the value of π is approximately 3.14.

10 Why might the number you get each time not be close to π?

KEY IDEAS

- Circumference (C) is the perimeter of a circle.
- Diameter (D) is the width of a circle through its centre.
- Radius (r) is half the diameter.
- To find the circumference of a circle, multiply the diameter by π.
- $C = \pi D$ or $C = 2\pi r$.



EXERCISE 8B Circumference of a circle

For all questions in this topic, use π on your calculator and then round each answer to two decimal places.

EXAMPLE 8B-1 Finding diameter

Find the length of the diameter if a circle has a radius of 4 cm.

THINK

1 The diameter is twice the length of the radius. Multiply the radius by two.

2 Write your answer.

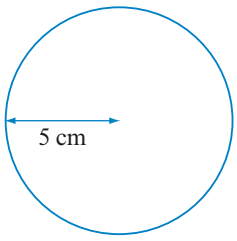
WRITE

$D = 2r$
$= 2 \times 4$ cm
$= 8$ cm

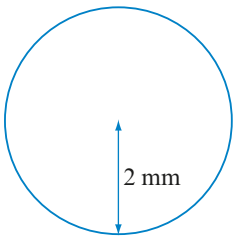
UNDERSTANDING AND FLUENCY

1 Find the length of the diameter of each circle.

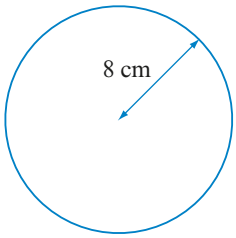
a



b



c



2 Find the length of the diameter if a circle has a radius of:

a 3 cm	b 6 m	c 70 mm
d 1.5 cm	e 111 mm	f 9 cm
g 12 mm	h 2.5 m	i 25 mm

EXAMPLE 8B-2 Finding radius

Find the length of the radius if a circle has a diameter of 12 cm.

THINK

1 The radius is half the size of the diameter. Divide the diameter by two.

2 Write your answer.

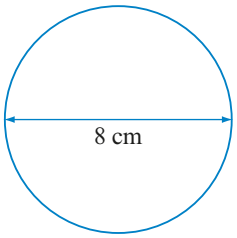
WRITE

$r = \frac{D}{2}$
$= 12 \text{ cm} \div 2$
$= 6$ cm

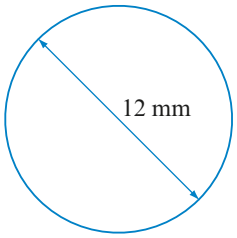
UNDERSTANDING AND FLUENCY

3 Find the length of the radius of each circle.

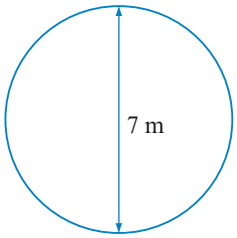
a



b



c

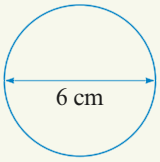


4 Find the length of the radius if a circle has a diameter of:

a 10 cm	**b** 4 m	**c** 6 mm
d 30 cm	**e** 5.6 mm	**f** 9 m
g 16 cm	**h** 5 m	**i** 24 cm

EXAMPLE 8B-3 Using the diameter to calculate circumference

Calculate the circumference of this circle, correct to two decimal places.



THINK

1. Identify which formula to use.
2. Substitute the measurement for diameter.
3. Calculate using π on your calculator.
4. Round to two decimal places and include the appropriate unit.

WRITE

$C = \pi D$

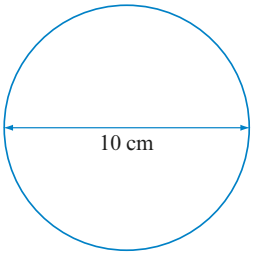
$= \pi \times 6$

$= 18.849\,555\,9\ldots$

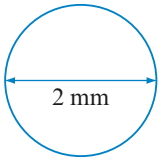
$C = 18.85$ cm

5 Calculate the circumference of each circle.

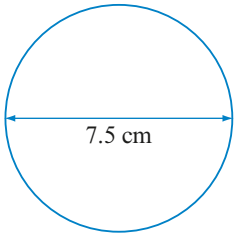
a



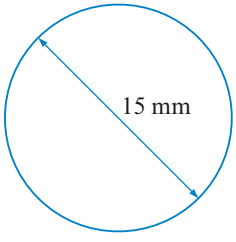
b



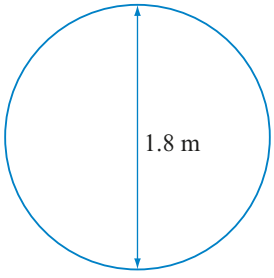
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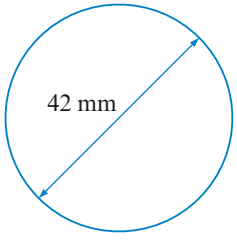
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e

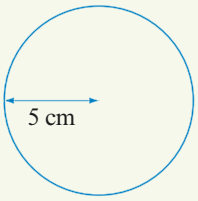


f



EXAMPLE 8B-4 Using the radius to calculate circumference

Calculate the circumference of this circle, correct to two decimal places.



THINK

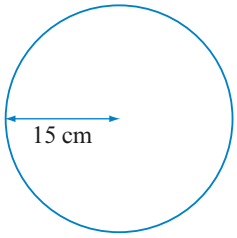
1 Identify which formula to use.
2 Substitute the measurement for radius.
3 Calculate using π on your calculator.
4 Round to two decimal places and include the appropriate unit.

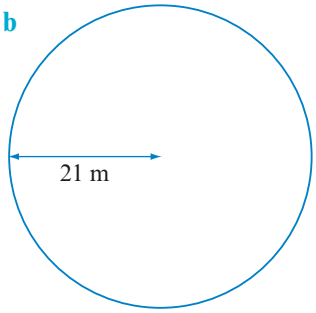
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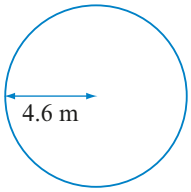
$C = 2\pi r$
$= 2 \times \pi \times 5$
$= 31.415\,926\,5\ldots$
$C = 31.42$ cm

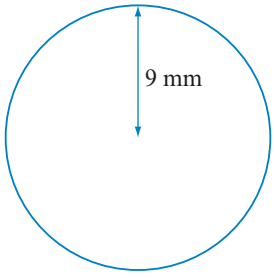
UNDERSTANDING AND FLUENCY

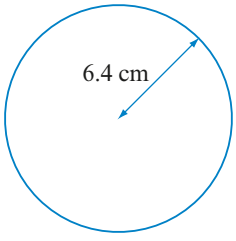
6 Calculate the circumference of each circle.

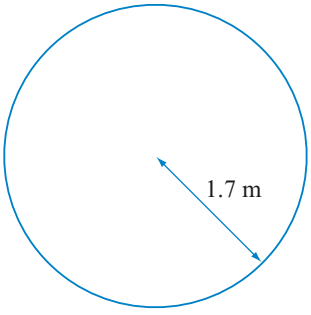
a


b


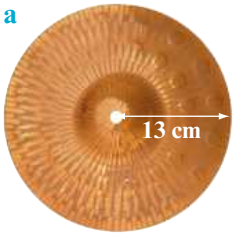
c


d


e


f


7 Calculate the circumference of each object.

a


b


c


d


UNDERSTANDING AND FLUENCY

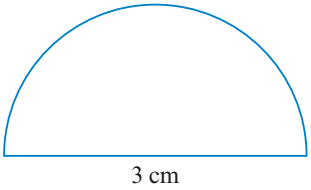
8 Find the circumference of a circle with:

a radius 1.2 m **b** diameter 2.5 cm **c** diameter 6.4 mm

d radius 0.4 cm **e** radius 19 cm **f** radius 13 mm

g diameter 0.75 m **h** diameter 27 cm **i** radius 82 km.

9 Look at this semicircle.



a How much of a circle is it?

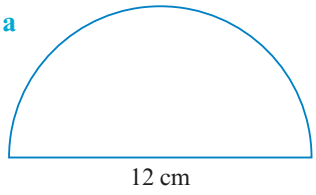
b What would be the circumference if it was a full circle?

c Use your answers to parts **a** and **b** to find the length of the curved section of the semicircle.

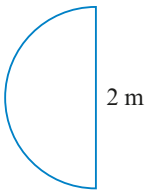
d What is the perimeter of the semicircle?

10 Use your understanding of circumference to find the perimeter of each shape.

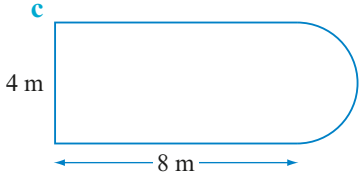
a



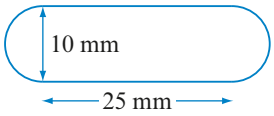
b



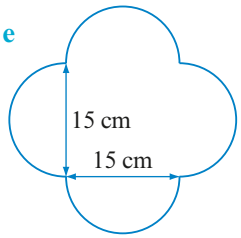
c



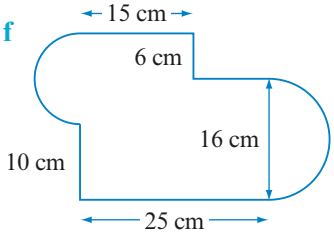
d



e



f



PROBLEM SOLVING AND REASONING

11 Elli bought Thomas a ring. She knows that his ring finger has a circumference of 60 mm. She buys him an S size ring, which has an inner diameter of 19.15 mm.

a Find the inner circumference of this ring. Do you think it will fit?

Elli gives the ring to Thomas and finds that it doesn't fit because it won't go over his knuckle. She tries both a T size ring (diameter of 19.56 mm), which is still slightly too small, and a U size ring (diameter of 19.96 mm), which is slightly too big.

b What size range must the circumference of Thomas' knuckle be in? (Hint: find the circumferences of both the T and U size rings.)

Elli discovers that these rings also come in half sizes.

c What would be the diameter of a ring size $T\frac{1}{2}$?

d If the ring is a perfect fit, find the circumference of Thomas' knuckle.

12 Jack goes for a bike ride with his younger sister Lily, but she has trouble keeping up. Jack gets frustrated but his mother explains that because Lily's bike is smaller, Lily has to work harder to go the same distance. If Jack's bike wheels have a diameter of 55 cm and Lily's have a diameter of 32 cm:



a find the circumference of their wheels

b find how many times each wheel turns to travel 1 km

c discuss whether Jack's mum is correct.

PROBLEM SOLVING AND REASONING

13 Lauren and Matt go for a ride on a merry-go-round. Lauren chooses a horse 5.4 m from the centre and Matt chooses a horse 4.1 m from the centre.

a Do these lengths relate to diameter or radius? Explain.

b Find how far they will each travel in one revolution of the merry-go-round.

c If the merry-go-round rotates six times in a ride, find the difference in the distance Matt and Lauren travel.

d Explain why, although Matt and Lauren are on the same ride, Lauren travels further than Matt.

14 For circles with these circumferences, find:

i the diameter

ii the radius.

a 314 cm **b** 6.28 m **c** 15.7 mm **d** 1.57 cm



15 Imagine you could tie a piece of string around the circumference of the Earth at the equator.

a If Earth has a diameter at the equator of about 12 756 km, how much string would be needed?

b If you added 1 m to the length of the piece of string, would you be able to:

i squeeze your pinkie finger under the string?

ii fit your hand under the string?

iii slide yourself under the piece of string?

iv crawl under the string?

v walk under the string?

Explain your answer.

CHALLENGE

16 A local show has a new Ferris wheel. It has a radius of 54 m.

a What is the diameter of the Ferris wheel?

b If the Ferris wheel stands 5 m off the ground, how high would you be if you were at the top?

c What is the circumference of the Ferris wheel?

d How many carriages could fit on the Ferris wheel if each one was 1 m wide and there has to be at least 3 m between each carriage?

e Use your answer from part **c** to calculate the distance between each carriage on the Ferris wheel.

f If the owner wanted to fit 100 carriages onto the Ferris wheel, assuming a minimum distance of 3 m between each carriage, by how much would the radius of the Ferris wheel have to increase?

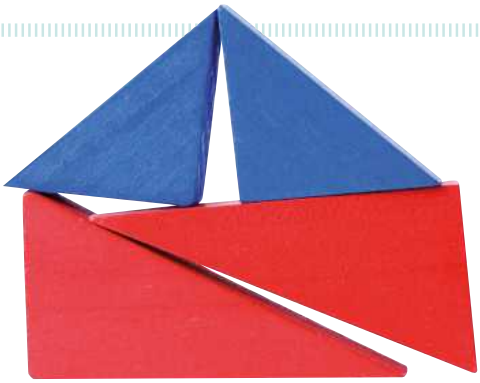
g Are the details in this question realistic? Investigate and write a paragraph on your findings.



Reflect

What is π and how does it relate to radius, diameter and circumference?

8C Area of rectangles and triangles



Start thinking!

1 What is **area** and how is it different from length?

2 For the rectangle in figure A, what is:
- **a** the length (l)?
- **b** the **width** (w)?
- **c** the area (A)?

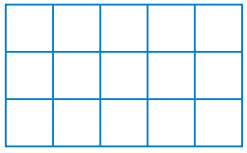


Figure A

3 How does figure A show that the formula for the area of a **rectangle** is $A = l \times w$ or $A = lw$?

Figure B shows the rectangle from figure A split into two identical triangles.

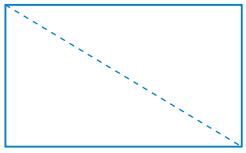


Figure B

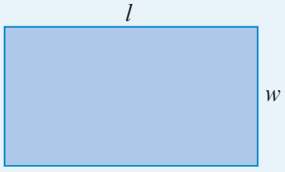
4 How does figure B show that the area of a **triangle** is exactly half that of a rectangle with the same dimensions?

5 How do the **base** (b) and **height** (h) of this triangle relate to the length and width of this rectangle?

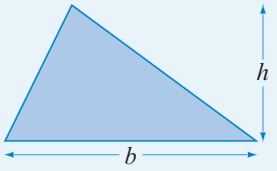
6 The formula for the area of a triangle is $A = \frac{1}{2} \times b \times h$ or $A = \frac{1}{2}bh$. How does this relate to the formula for the area of a rectangle?

KEY IDEAS

- Area (A) is the amount of space enclosed by a 2D shape.
- Common units of area measurement include square millimetres (mm^2), square centimetres (cm^2), square metres (m^2) and square kilometres (km^2).



- To find the area of a rectangle, multiply its length (l) by its width (w).
  $A = lw$
- To find the area of a triangle, multiply its base (b) by its height (h) then divide by 2.
  $A = \frac{1}{2}bh$

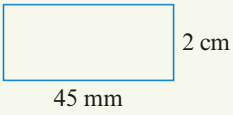


NOTE The base and height of a triangle must be **perpendicular** to each other.

EXERCISE 8C Area of rectangles and triangles

EXAMPLE 8C-1 Calculating area of a rectangle

Calculate the area of this rectangle.



THINK

1. Write the formula.
2. Identify l and w. Write the length in centimetres so that both l and w are in the same units.
3. Substitute the values for l and w into the formula.
4. Calculate the result and include the appropriate unit.

WRITE

$A = l \times w$

$l = 45 \text{ mm} = 4.5 \text{ cm}$
$w = 2 \text{ cm}$

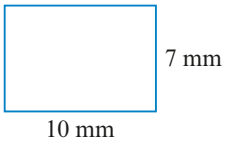
$A = 4.5 \times 2$

$= 9 \text{ cm}^2$

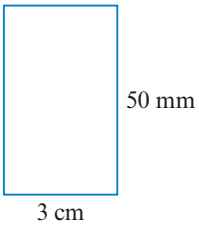
UNDERSTANDING AND FLUENCY

1 Calculate the area of each rectangle.

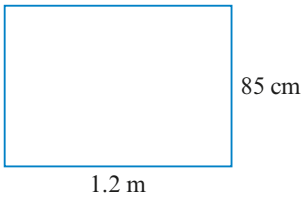
a



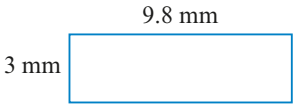
b



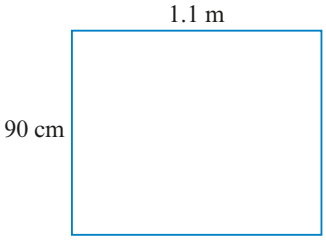
c



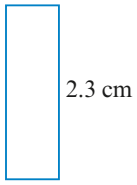
d



e

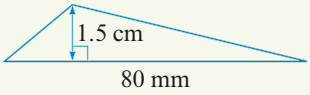


f



EXAMPLE 8C-2 Calculating area of a triangle

Calculate the area of this triangle.

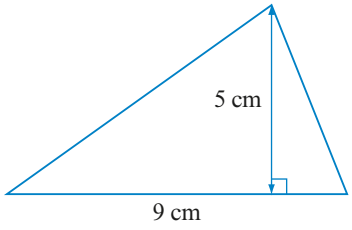


THINK	WRITE
1 Write the formula.	$A = \frac{1}{2}bh$
2 Identify b and h. Write the base in centimetres so that both b and h are in the same units.	$b = 80 \text{ mm} = 8 \text{ cm}$ $h = 1.5 \text{ cm}$
3 Substitute the values for b and h into the formula.	$A = \frac{1}{2} \times 8 \times 1.5$
4 Calculate the result and include the appropriate unit.	$= 6 \text{ cm}^2$

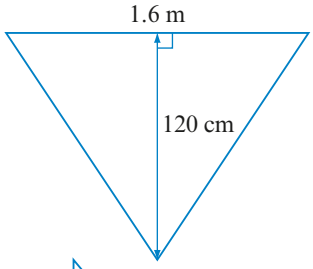
UNDERSTANDING AND FLUENCY

2 Calculate the area of each triangle.

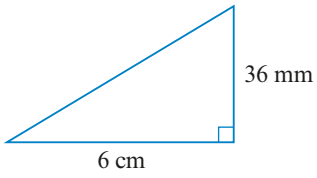
a



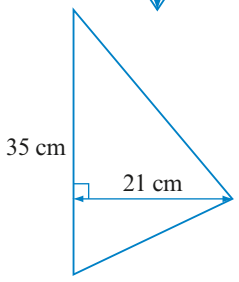
b



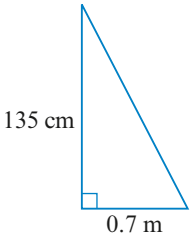
c



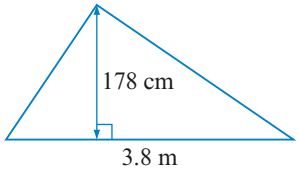
d



e



f



3 Calculate the area of each shape.

- **a** a rectangle with length 35 mm, width 4 cm
- **b** a rectangle with length 12 mm, width 17 cm
- **c** a triangle with base 15 mm, height 2 cm
- **d** a square with length 7.2 mm
- **e** a triangle with base 25 cm, height 10 mm
- **f** a triangle with base 8 cm, height 42 mm
- **g** a square with base 19 cm

h a rectangle with base 4.2 cm, height 11 mm

i a triangle with base 0.7 m, height 49 cm

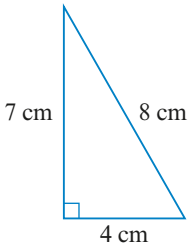
4 How much material is needed to make the multi-coloured umbrella shown?



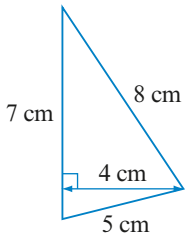
5 A classmate attempted question **4** and after using 68.5 cm as the height of each triangle, obtained an answer of 8768 cm^2. Can you explain where they went wrong?

6 Calculate the area of each shape.

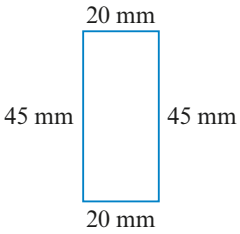
a



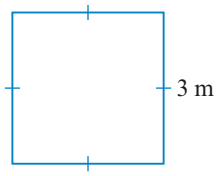
b



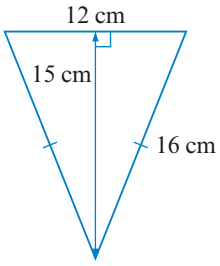
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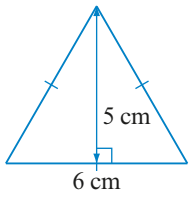
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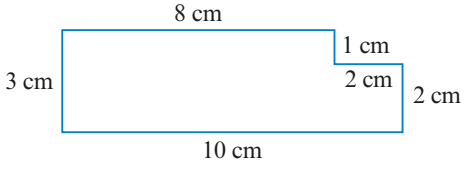


f



7 Two classmates were arguing about the dimensions of the rectangle in question **6c**.
Juan said that the length was the longest side and therefore it was 45 mm.
Erica said it was the horizontal length and therefore it was 20 mm.
Can you explain how they are both wrong and both right?

8 You can use your understanding of areas of rectangles and triangles to find the area of **composite shapes**.



a Copy this figure and draw a line on it splitting it into two rectangles.

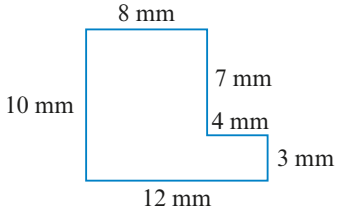
b Find the area of each rectangle and hence the area of the entire shape.

c Is there another way you could have done this?
Draw another diagram to show how it could be done.

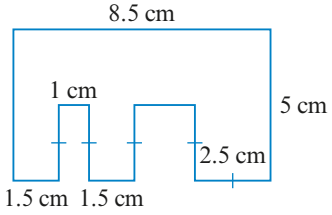
UNDERSTANDING AND FLUENCY

9 Find the area of each shape.

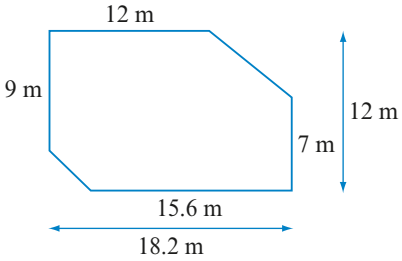
a



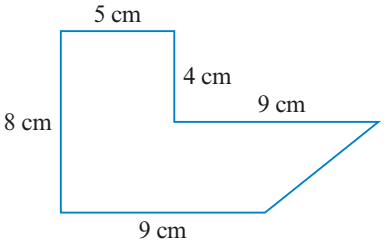
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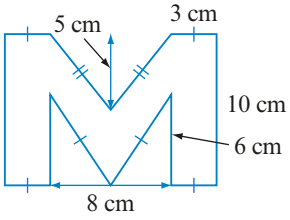
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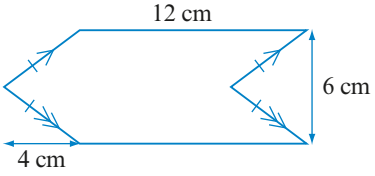
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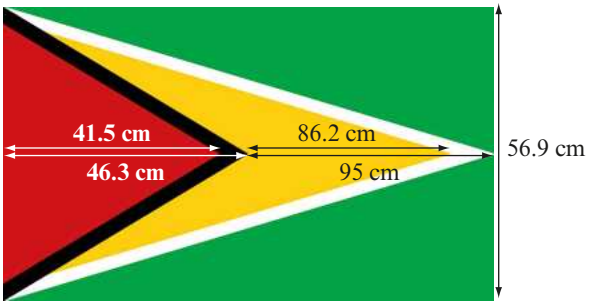


f



PROBLEM SOLVING AND REASONING

10 Guyana's flag consists of numerous coloured triangles. Use the measurements to find the area of each coloured section.



11 This envelope template has an area of 356.49 cm^2.

a Use your understanding of areas of rectangles and triangles to prove this area measurement is correct.

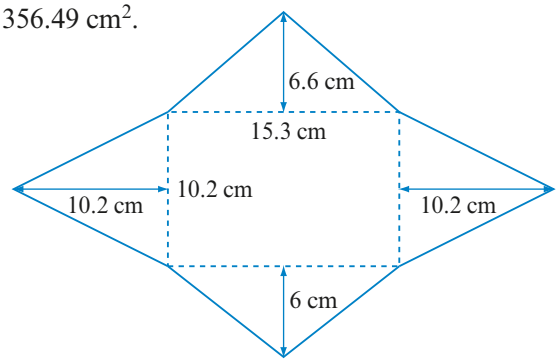
b Create your own envelope (using rectangles and triangles) that has a total area of:

i less than 300 cm^2

ii between 300 cm^2 and 400 cm^2

iii more than 400 cm^2.

What did you change each time?



PROBLEM SOLVING AND REASONING

12 A rectangle has an area of 68 cm^2. Draw three possible rectangles that fit this description, labelling length and width on each one.

13 A rectangle has an area of 20 cm^2 and a length of 4 cm. Find its width and explain why there is only one rectangle that you can draw that fits this description.

14 A triangle has an area of 32 mm^2.

a Draw three possible triangles that fit this description, labelling base and height on each one.

Suppose that this triangle has a base of 8 mm.

b What would its height be?

c Draw two possible triangles that fit this description, labelling base and height on each one.

d Explain why there are many triangles that fit this description.

15 A rectangle has an area of 56 m^2.

a State three possible sets of dimensions.

b Find the perimeter of each possible rectangle.

16 A rectangle has a perimeter of 48 cm. Find three possible areas this rectangle could have.

17 A square has an area of 64 mm^2. Draw this square, stating its dimensions, its perimeter and why there is only one square that matches this description.

18 A wooden frame is to go around a rectangular mirror with an area of 3600 cm^2.

a If the mirror has a length of 90 cm, what is its width?

b What area of wood is required if the frame is to be 5 cm wide?
(Hint: draw a picture to help you with your calculations.)

CHALLENGE

19 Mohammed is making a rectangular vegetable garden and has 24 m of fencing to put around the garden.

a List three possible sets of dimensions and corresponding areas for the garden.

b What is the maximum area that you can find using 24 m of fencing? What shape is this?

Mohammed realises he can enclose a bigger area if he puts the garden against the shed wall and only has to fence three sides of the garden.

c List another three possible sets of dimensions and corresponding areas for the veggie garden if one side is against the shed wall.

d What is the maximum area that you can find now using 24 m of fencing for three sides? What shape is this? How does this relate to your answer to part **b**?

Reflect

How are the dimensions and area of a triangle related to the dimensions and area of a rectangle?

8D Area of other quadrilaterals

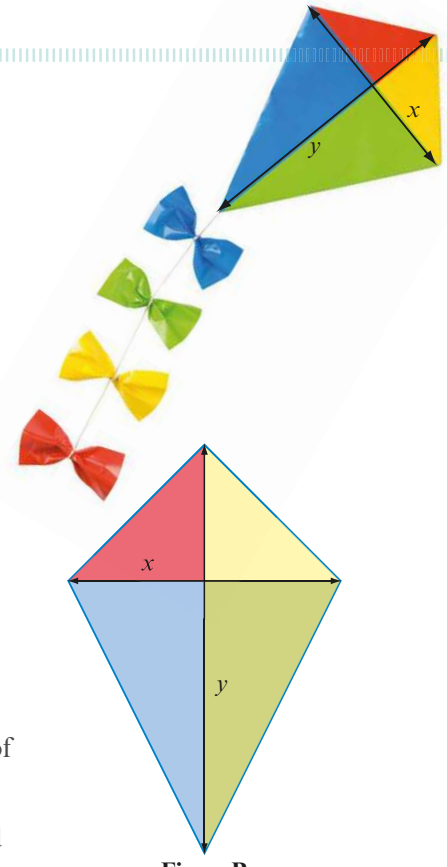


Figure B

Start thinking!

You can use your understanding of the area of a rectangle to find the area of other **quadrilaterals**.

1 Match each description to a **parallelogram**, **rhombus** or **kite**.
   - **a** Opposite sides parallel, all sides equal, opposite angles equal.
   - **b** Two pairs of sides equal in length.
   - **c** Opposite sides parallel and equal in length, opposite angles equal in size.

2 Look at the parallelogram shown in figure A.

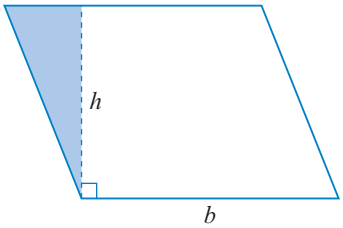


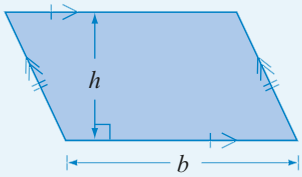
Figure A

   - **a** What do b and h stand for?
   - **b** How might you cut and rearrange this parallelogram so it forms a rectangle?
   - **c** Use the formula for area of a rectangle to show the formula for area of a parallelogram is $A = b \times h$ or $A = bh$.

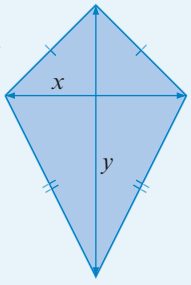
3 Look at the kite in figure B. The diagonals are marked inside the kite and labelled x and y.
   - **a** Copy and cut out the kite twice. Cut one of these copies along its diagonal lines.
   - **b** Rearrange all the parts so that you form a rectangle.
   - **c** How do the diagonals of the kite relate to the length and width of the rectangle?
   - **d** How many copies of the kite make up this rectangle?
   - **e** Use the formula for the area of a rectangle to show that the formula for the area of a kite is $A = \frac{1}{2} \times x \times y$ or $A = \frac{1}{2}xy$.

KEY IDEAS

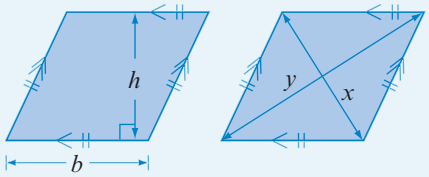
- The formula for the area of a parallelogram is $A = bh$



- The formula for the area of a kite is $A = \frac{1}{2}xy$



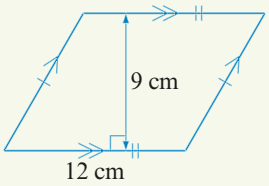
- The formula for the area of a rhombus is $A = bh$ or $A = \frac{1}{2}xy$



EXERCISE 8D Area of other quadrilaterals

EXAMPLE 8D-1 Calculating area of a parallelogram

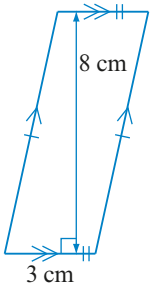
Calculate the area of this parallelogram.



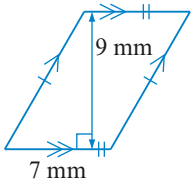
THINK	WRITE
1 Write the formula.	$A = bh$
2 Identify b and h.	$b = 12$ cm, $h = 9$ cm
3 Substitute the values for b and h into the formula.	$A = 12 \times 9$
4 Calculate the result and include the appropriate unit.	$= 108$ cm^2

1 Calculate the area of each parallelogram.

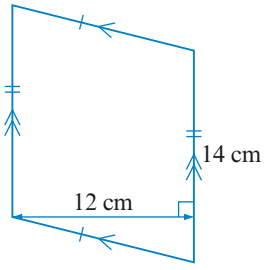
a



b

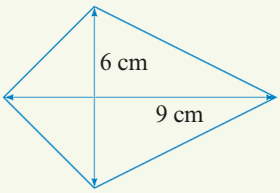


c



EXAMPLE 8D-2 Calculating area of a kite

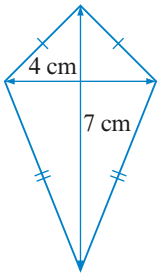
Calculate the area of this kite.



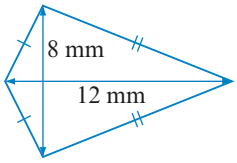
THINK	WRITE
1 Write the formula.	$A = \frac{1}{2}xy$
2 Identify x and y.	$x = 6$ cm, $y = 9$ cm
3 Substitute the values for x and y into the formula.	$A = \frac{1}{2} \times 6 \times 9$
4 Calculate the result and include the appropriate unit.	$= 27$ cm^2

2 Calculate the area of each kite.

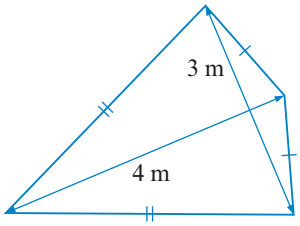
a



b

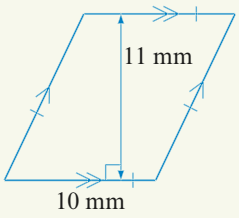


c



EXAMPLE 8D-3 Calculating area of a rhombus

Calculate the area of this rhombus.

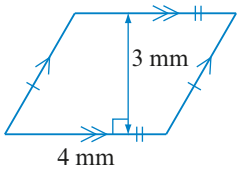


THINK	WRITE
1 A rhombus is both a parallelogram and a kite. As the base and height are given, write the formula for a parallelogram.	$A = bh$
2 Identify b and h.	$b = 10$ mm, $h = 11$ mm
3 Substitute the values for b and h into the formula.	$A = 10 \times 11$
4 Calculate the result and include the appropriate unit.	$= 110$ mm^2

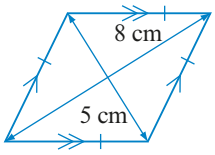
UNDERSTANDING AND FLUENCY

3 Calculate the area of each rhombus.

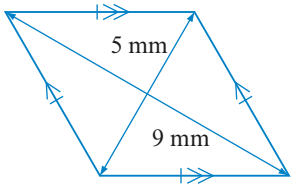
a



b



c



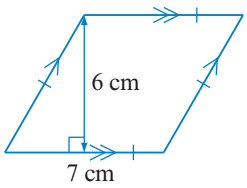
4 For each shape:

i identify if it is a parallelogram, rhombus or kite

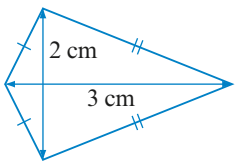
ii write the formula to find its area

iii calculate its area.

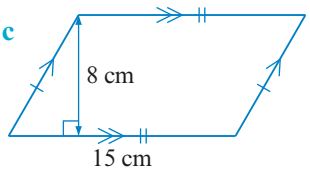
a



b

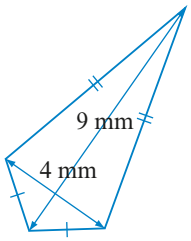


c

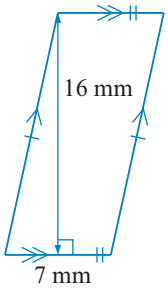


UNDERSTANDING AND FLUENCY

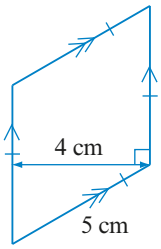
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e

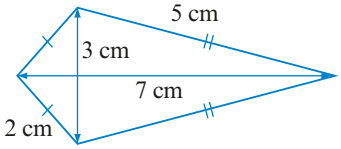


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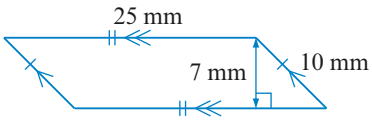


5 Calculate the area of each quadrilateral.

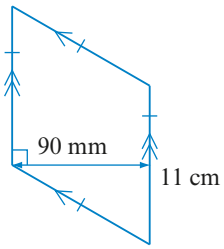
a



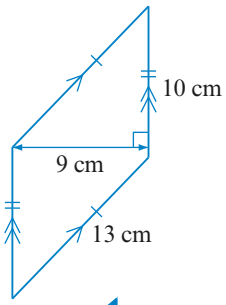
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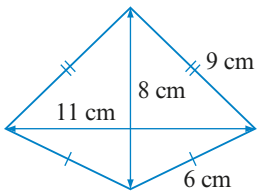
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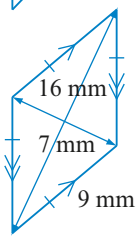
d



e



f



PROBLEM SOLVING AND REASONING

6 Tim's uncle has an old road sign he wants painted on both sides.

a If the sign is 0.7 m tall and 0.7 m wide, find the total area Tim will have to paint if he uses three coats of paint.



Tim's uncle likes the results so much he asks Tim to paint another 20 road signs.

b What is the total area Tim will paint for 20 road signs? Remember they need three coats on both sides.

c How much will it cost to paint the signs if a litre of paint costs $8.99 and covers 15 m^2?

d If Tim is paid $150, find how much money Tim will make after deducting the cost of the paint.

7 Amanda wants to make some kites. She decides to make two types, a large blue kite and a smaller red kite.

- **a** The blue kite needs to be at most 95 cm long and 65 cm wide. What is the largest area it can be?
- **b** The red kite can have a maximum area of 400 cm^2. If the kite is 40 cm long, how wide must it be?

8 The area of a kite can be found by relating it to a parallelogram.

- **a** Draw a kite, using dotted lines for the diagonals.
- **b** Cut the kite in half along one of the diagonals.
- **c** Rearrange these two pieces so that they form a parallelogram.
- **d** What is the formula for the area of a parallelogram?
- **e** One of the diagonals relates to the base of the parallelogram. Which one?
- **f** Half of one of the diagonals relates to the height of the parallelogram. Which one?
- **g** Explain how this method also gives the formula for the area of a kite as $A = \frac{1}{2}xy$.

9 Consider this building in Hamburg.



- **a** What shape is it?
- **b** If the glass section is 21 m tall and 86 m long, what is the area of this entire section?
- **c** Is every window equal in area? Explain.
- **d** How might you calculate the area of every window? Explain.

10 Use the Internet or another resource to investigate the use of quadrilaterals, especially parallelograms, rhombuses and kites, in art and architecture.

11 The area of a parallelogram is 40 cm^2.

- **a** Draw three possible parallelograms that fit this description, labelling base and height.
- **b** Why is this more difficult than if it was a rectangle?

12 A rhombus has an area of 24 mm^2.

- **a** Draw two possible rhombuses that fit this description, labelling base and height.
- **b** Find the perimeter of these two rhombuses.
- **c** Explain why the area of a rhombus can never be a square number unless the shape is a square (remember that a square is a special type of rhombus).

13 Draw three possible kites that have an area of 36 m^2, labelling the appropriate dimensions.

CHALLENGE

14 Write your own open-ended questions about the areas of quadrilaterals and swap them with a classmate. What strategies can you use to solve these problems?

15 The special quadrilateral that has not been covered yet is the **trapezium**. Look at the trapezium in figure A.

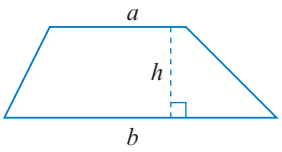


Figure A

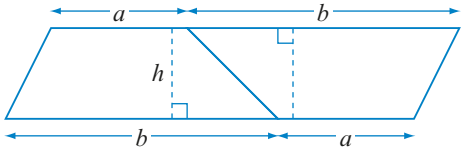


Figure B

a Copy and cut out this trapezium twice. Label the copies as shown in figure A.

b Rotate one of these trapeziums and join it up to the first trapezium as shown in figure B.

c What shape does this now form?

d What is the formula for the area of a parallelogram?

e The height of the trapeziums corresponds to the height of the parallelogram. What part of the parallelogram does the sum of the trapezium lengths $(a + b)$ correspond to?

The formula for the area of this parallelogram is $A = (a + b) \times h$, where $a + b$ is the base.

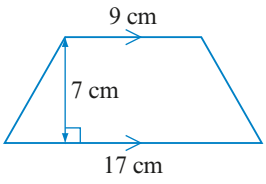
f How many trapeziums make up this parallelogram?

g Explain how this shows that the formula for the area of a trapezium is $A = \frac{1}{2}(a + b) \times h$ or $A = \frac{1}{2}(a + b)h$.

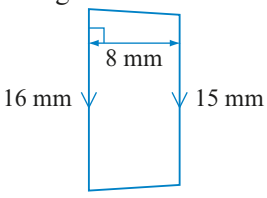
h If $a = 4$, $b = 9$ and $h = 3$, find the area of the trapezium shown in figure A.

16 Find the area of each trapezium using the formula $A = \frac{1}{2}(a + b)h$.

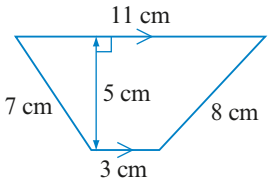
a



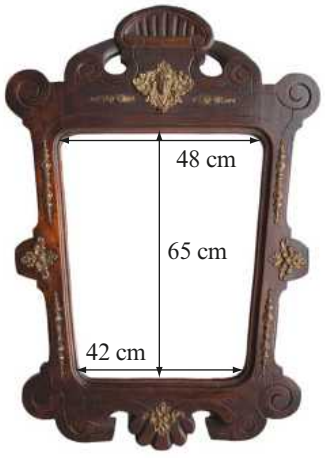
b



c



17 Find the cost of fitting the frame on the right with a mirror if your local hardware store charges \$0.25 per cm^2 for mirror glass.



18 The formula for the area of a trapezium can also be developed by cutting a trapezium in half and joining the two pieces to form another parallelogram. Draw a trapezium and cut it out to show how this might be done.

Reflect

How is the formula for the area of a rectangle useful when finding the formulas for the areas of other quadrilaterals?

8E Area of a circle



Start thinking!

1 If the circumference of a circle is $C = 2\pi r$, what would be the formula for a *half* circumference of a circle?

2 Draw a circle that has a radius of 5 cm and shade half of it in colour, as shown in figure A.

3 How does your answer to question **1** relate to figure A?

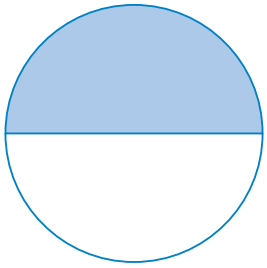


Figure A

A **sector** is a slice of a circle that starts from the centre of the circle.

4 How many sectors does the slice of lime shown above have?

5 Divide your circle from question **2** into 16 equal sectors so you have eight coloured sectors and eight non-coloured sectors.

6 Cut these up and arrange them as shown in figure B.

7 What shape does this form? Draw the quadrilateral and label its base and height.

8 What is the formula for the area of this shape?

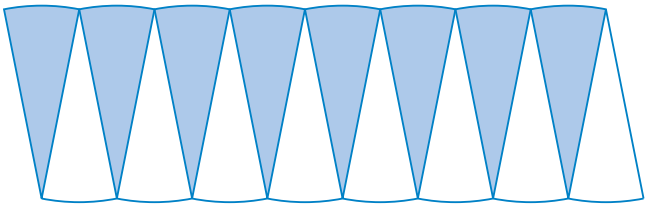


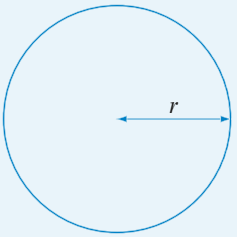
Figure B

9 Measure the base and height of the shape you formed and calculate the area.

10 If the height of your parallelogram is the same as the radius (r) of the circle and the base of the parallelogram is πr, explain why the formula for the area of a circle is $A = \pi r^2$.

KEY IDEAS

- The formula for the area of a circle with radius r is $A = \pi r^2$.
- If you are given the diameter, find the radius before using the formula.

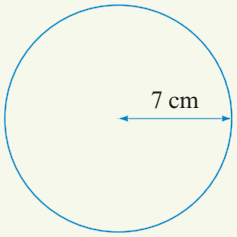


EXERCISE 8E Area of a circle

For all questions in this topic, use π on your calculator and then round each answer to two decimal places.

EXAMPLE 8E-1 Calculating area of a circle, given radius

Calculate the area of this circle.



THINK

1 Write the formula.
2 Identify r and substitute it into the formula.
3 Calculate the result using π on your calculator.
4 Round to two decimal places and include the appropriate unit.

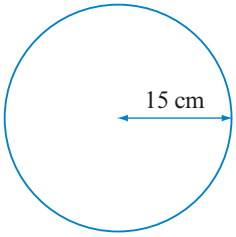
WRITE

$A = \pi r^2$
$= \pi \times 7^2$
$= 153.938\,040\,0\ldots$
$A = 153.94\ \text{cm}^2$

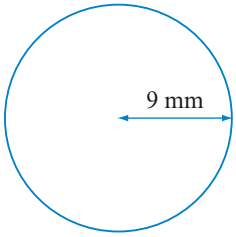
UNDERSTANDING AND FLUENCY

1 Calculate the area of each circle.

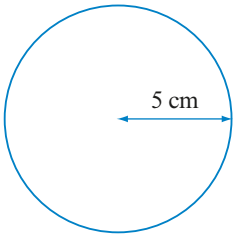
a



b



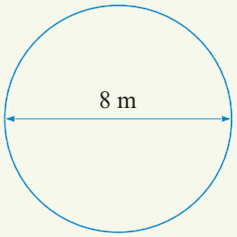
c





EXAMPLE 8E-2 Calculating area of a circle, given diameter

Calculate the area of this circle.

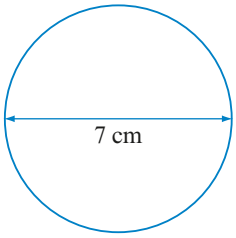


THINK	WRITE
1 Write the formula.	$A = \pi r^2$
2 Identify the radius (half the diameter).	$r = D \div 2$ $= 8 \div 2$ $= 4$ m
3 Substitute the value for r into the formula.	$A = \pi \times 4^2$
4 Calculate the result using π on your calculator.	$= 50.265\,482\ldots$
5 Round to two decimal places and include the appropriate unit.	$A = 50.27$ m^2

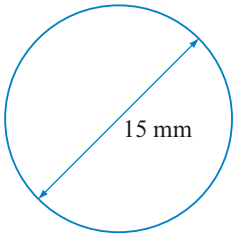
UNDERSTANDING AND FLUENCY

2 Calculate the area of each circle.

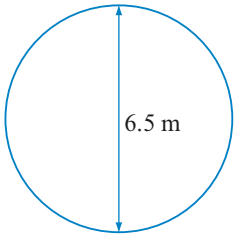
a



b

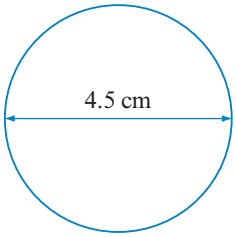


c

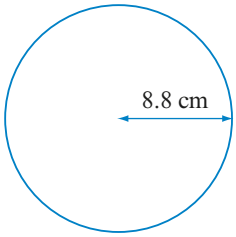


3 Calculate the area of each circle.

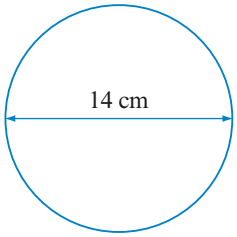
a



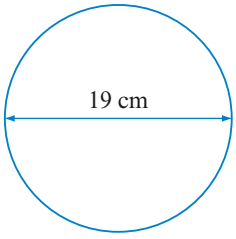
b



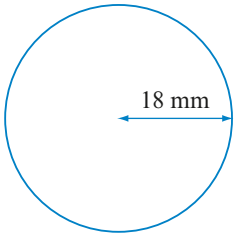
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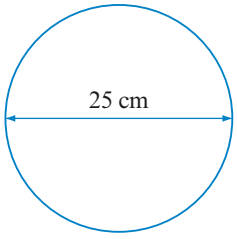
d



e



f

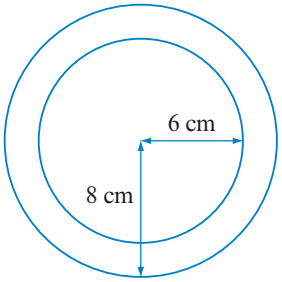


4 Calculate the area of a circle with:

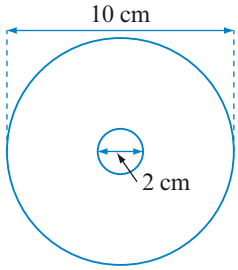
a radius 5 cm
b diameter 7.5 m
c diameter 15 mm
d radius 29 mm
e radius 0.8 m
f diameter 0.42 cm

5 You can subtract the area of one circle from another to find the area of a ring. Another name for a ring is an **annulus**. Find the area for each annulus.

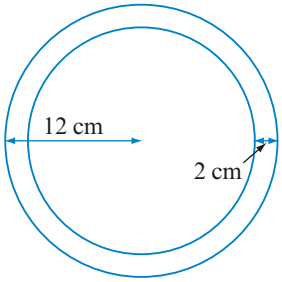
a



b

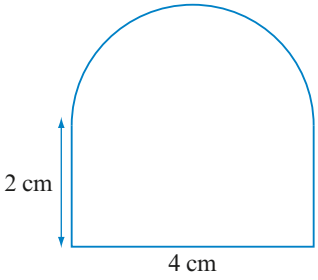


c

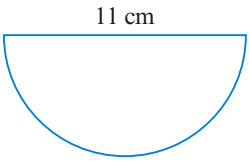


6 Use your understanding of area of a circle to find the area of each composite shape.

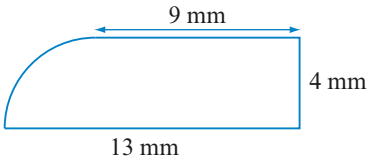
a



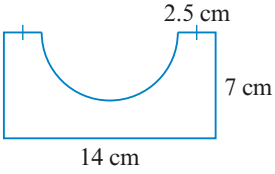
b



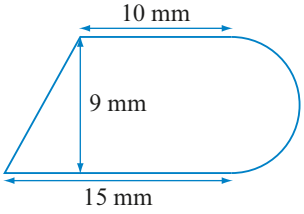
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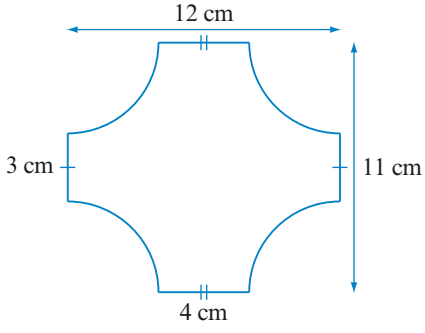
d



e



f



7 Anika wants to make a circular placemat. How much material will she need if the placemat has a diameter of 55 cm?

8 A sprinkler sprays a stream of water 7.5 m long around in a circle. What is the total area covered by the sprinkler?

9 What is the largest pancake that could be cooked in this frying pan, if its base has a diameter of 24 cm?



10 Adrian wants to cook a pancake using the frying pan from question **9** and place it on this plate.



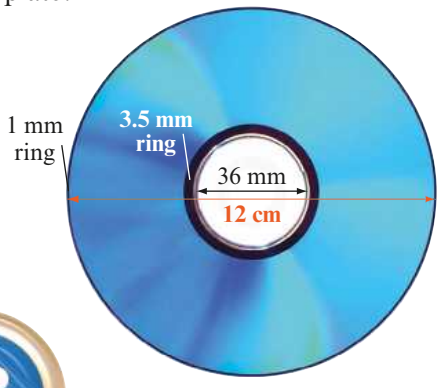
a What is the maximum area a pancake could have if it had to fit on the white section of this plate and the white section has a radius of 10 cm?

b To make a pancake the size you found in part **a**, what distance should be between the edges of the pancake and the side of the frying pan?

c What is the entire area of the plate if it has a diameter of 30 cm?

d What is the area of the painted section of the plate?

11 A DVD has a diameter of 12 cm and an inner (non-recordable) circle of diameter 36 mm. There is a 1 mm gap between the edge of the DVD and where the recording starts, and there is an additional 3.5 mm ring that cannot be recorded onto. What is the recordable area of the DVD?



12 What is the total area that this button covers if it has a diameter of 20 mm and each hole has a radius of 1.5 mm?



13 Design your own button. Draw and label its dimensions, stating the area that it will have.

14 It is possible to find the radius of a circle if you know its area.

a Explain why the formula to find the radius of a circle is $r = \sqrt{\frac{A}{\pi}}$.

b Why might it be easier to find the length of the radius if the area has π in it? Support your answer by finding the radius of a circle that has an area of:

i 16 **ii** 16π.

15 Find the radius of a circle which has an area of:

a 9π cm^2 **b** 25π cm^2 **c** 100 cm^2 **d** 65 cm^2.

16 **a** A circle has an area of 49π cm^2. Find its:

i diameter **ii** circumference.

b A circle has a circumference of 8π m. Find its:

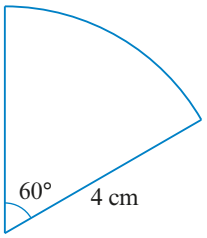
i radius **ii** area.

CHALLENGE

17 What percentage of the top surface of the pieces in a domino set is painted white (i.e. to show the dots)? You will need to research the number and type of tiles found in a domino set. Assume an individual domino piece is 44 mm in length, 22 mm in width and each dot has diameter 4 mm.



18 You can use knowledge of angles to calculate the area of a sector. Look at the sector shown here.



a What is the angle at the vertex of the sector?

b How many degrees in a circle?

c Use your answers to parts **a** and **b** to write the angle for this sector as a fraction of a circle.

d What is the radius of this sector?

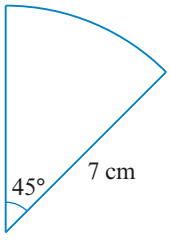
e What would be the area of the circle if it was complete?

f Use your answers to parts **c** and **e** to find the area of the sector.

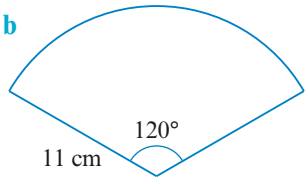
g Explain why this works to a classmate.

19 Calculate the area of each sector.

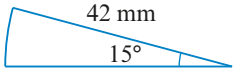
a



b



c



20 How could you use your understanding of sectors to find the areas of the composite shapes in question **6**.

Reflect

Why is π important when working with circles?

8F Surface area

Start thinking!

1 How are figures A and B related?

2 Draw nets for figures C and D.

3 How are the three nets similar and how are they different?

4 A face has been labelled on each object.
What shape is Face 1 for the:

a **rectangular prism**?

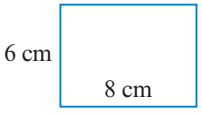
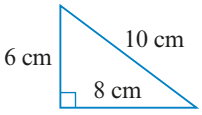
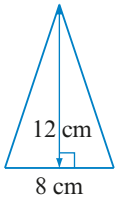
b **triangular prism**?

c **square-based pyramid**?

5 Label this 'Face 1' on each of your nets.

6 Label the remaining faces on each of your nets.

7 Copy and complete this table.
The first face for each object has been done for you.

	Face 1	Face 2	Face 3	Face 4	Face 5	Face 6
Rectangular prism						
Triangular prism						
Square-based pyramid						

8 How would you use the information in the table to calculate the **total surface area** of each object?

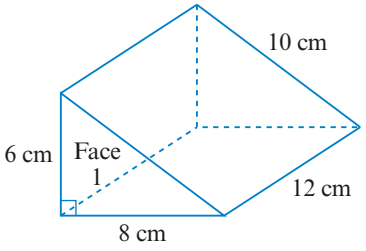


Figure A

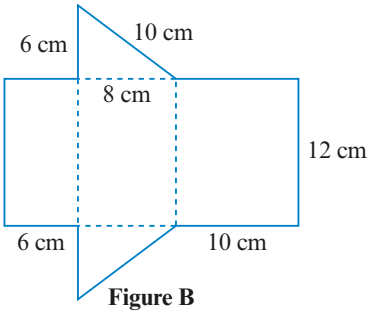


Figure B

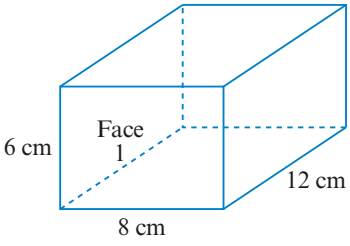


Figure C

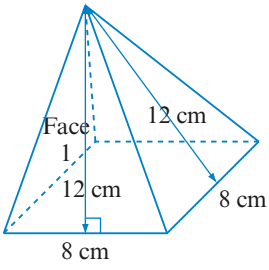


Figure D

KEY IDEAS

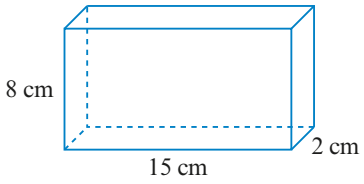
- The total surface area (TSA) of a 3D object is the total area of the outer surface of the object.
- The TSA of a prism is the sum of the areas of each face of the prism.
- Using the net of an object makes the faces easier to identify.

EXERCISE 8F Surface area

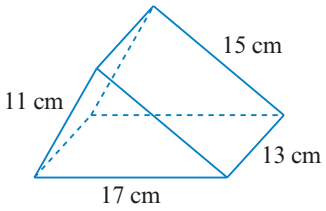
UNDERSTANDING AND FLUENCY

1 Draw each face for these prisms.

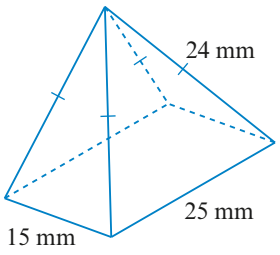
a



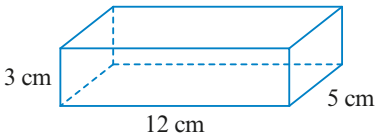
b



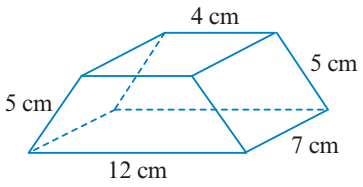
c



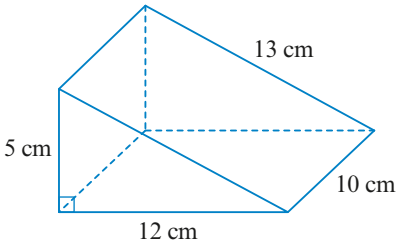
d



e

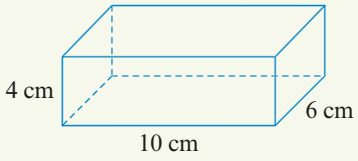


f



EXAMPLE 8F-1 Calculating surface area of a rectangular prism

Calculate the surface area of this rectangular prism.



THINK

1. Identify the faces (three pairs of identical rectangles).
2. Calculate the area of each face.
3. Add the areas together and include the appropriate unit.

WRITE

TSA = 4 cm × 10 cm + 4 cm × 10 cm + 10 cm × 6 cm + 10 cm × 6 cm + 4 cm × 6 cm + 4 cm × 6 cm

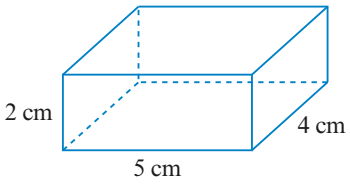
= 40 cm^2 + 40 cm^2 + 60 cm^2 + 60 cm^2 + 24 cm^2 + 24 cm^2

= 248 cm^2

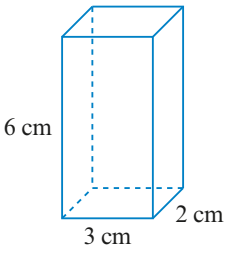
UNDERSTANDING AND FLUENCY

2 Calculate the surface area of each rectangular prism.

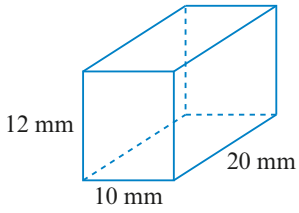
a



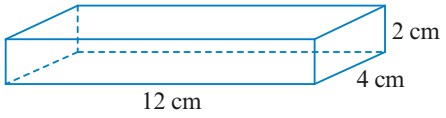
b



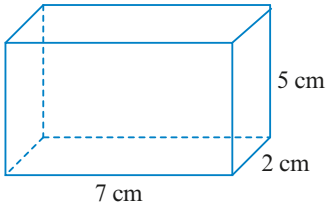
c



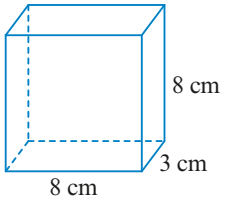
d



e

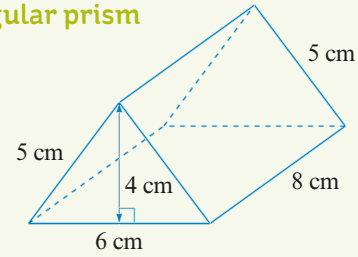


f



EXAMPLE 8F-2 Calculating surface area of a triangular prism

Calculate the surface area of this triangular prism.



THINK

1. Identify the faces (two identical triangles and three rectangles).
2. Calculate the area of each face.
3. Add the areas together and include the appropriate unit.

WRITE

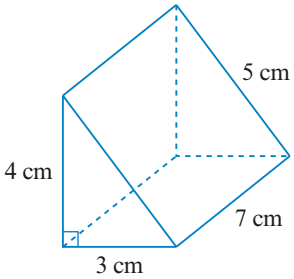
$\text{TSA} = \frac{1}{2} \times 6 \text{ cm} \times 4 \text{ cm} + \frac{1}{2} \times 6 \text{ cm} \times 4 \text{ cm} +$
$5 \text{ cm} \times 8 \text{ cm} + 6 \text{ cm} \times 8 \text{ cm} + 5 \text{ cm} \times 8 \text{ cm}$

$= 12 \text{ cm}^2 + 12 \text{ cm}^2 + 40 \text{ cm}^2 + 48 \text{ cm}^2 + 40 \text{ cm}^2$

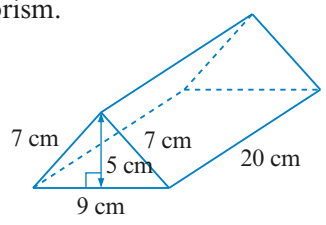
$= 152 \text{ cm}^2$

3 Calculate the surface area of each triangular prism.

a

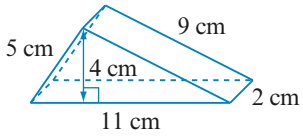


b

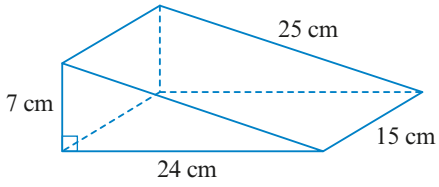


UNDERSTANDING AND FLUENCY

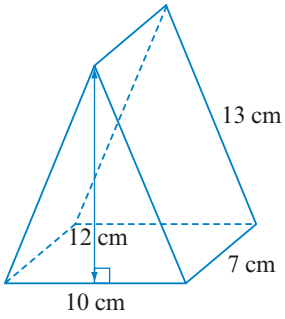
c



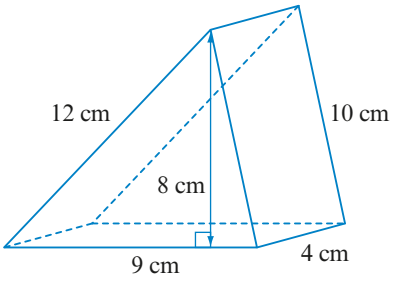
d



e

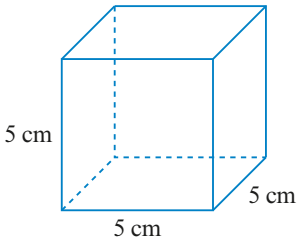


f



4 A friend wanted to calculate the surface area of this cube. She wrote her working as:

TSA = 5 cm × 5 cm + 5 cm × 5 cm + 5 cm × 5 cm + 5 cm × 5 cm + 5 cm × 5 cm + 5 cm × 5 cm.



a What is the surface area of this cube?

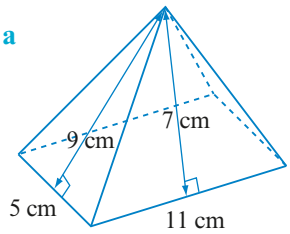
b Is there a shortcut for finding the surface area of a cube? Explain it to a classmate.

5 Calculate the surface area of a cube that has a side length of:

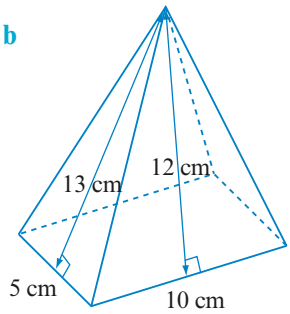
a 4 cm **b** 6 m **c** 12 mm **d** 20 cm.

6 Calculate the surface area of each **pyramid**. Is there a shortcut?

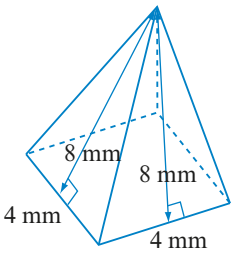
a



b



c



7 Jess wants to make a fitted sheet for the new mattress she bought. Assuming that the mattress is a rectangular prism, what is the minimum amount of material that she will need to buy in order to make the fitted sheet? (Hint: the sheet has to cover the sides and top of the mattress.)



8 Thanh makes a box for his mother in woodwork and needs to cover it in contact inside and out.

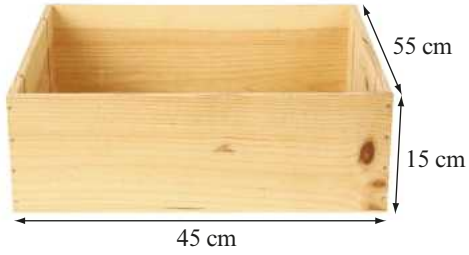
a How many faces will Thanh need to cover?

b Why is there more to cover than if it was just a closed box

Thanh decides to cover it in blue contact on the outer surface and white contact on the inner surface.

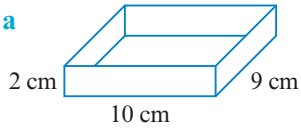
c Calculate how much blue contact and white contact Thanh will need. Ignore the thickness of the wood.

d What is the total surface area that Thanh will cover in contact?

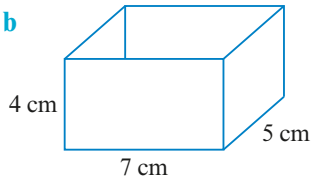


9 Calculate the total surface area for each open box.

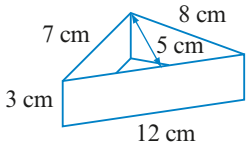
a



b



c



10 Recalculate the surface area for each object in question **9** if you only consider the outer surface area.

11 If a pool measures 18 m by 11 m by 2 m, and it costs $5 per square metre to paint, how much will it cost to paint the interior of the swimming pool?

12 Aaron wants to make his own tent. He decides on a simple triangular prism design. If he wants the tent to be 1 m tall, 2 m wide, 3 m long with a slanted edge of 1.41 m, how much material will he need? (Include material for the floor of the tent.)

13 Find the difference in cardboard needed for a small block of Toblerone compared to a large block of Toblerone. (Hint: assume the ends of each Toblerone are equilateral triangles.)



14 A prism has a surface area of 600 cm^2. Draw an example, labelling its dimensions, if this prism is:

a a cube

b a rectangular prism

c a triangular prism.

PROBLEM SOLVING AND REASONING

15 Why is there only one option for question **14** part **a**?

16 For each special quadrilateral:

- **a** draw a prism that uses the quadrilateral as the base
- **b** label your drawing with appropriate dimensions; for example, height, width, length, etc.
- **c** find the total surface area for your prism
- **d** swap with a classmate, check your answers and discuss any differences.

17 **a** Find the minimum amount of blue paper and red paper to wrap these gifts.

b Why would you need more than the minimum amount of wrapping paper to cover these gifts?



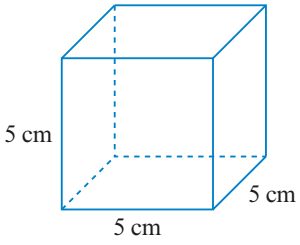
18 How much contact is needed to protect the cover of this textbook? Assume that you cover the spine and both sides of the front and back of the book.

19 How would you calculate the surface area of a tetrahedron?

20 What mistakes are common when finding the surface area of an object?

CHALLENGE

21 Consider the cube from question **4**, shown here.



- **a** What is its surface area?
- **b** What is the surface area if two of these cubes are stacked, one on top of the other?
- **c** What is the surface area if there is a stack of:
  - **i** three cubes?
  - **ii** four cubes?
  - **iii** five cubes?
- **d** Is there a pattern? Describe a shortcut to calculate the surface area of a stack of 17 cubes.
- **e** Would this pattern still work with a different sized cube? Investigate using a cube with different dimensions.

22 How would you calculate the surface area of a cylinder?

Reflect

When could knowledge of surface area be useful?

8G Volume of prisms

Start thinking!

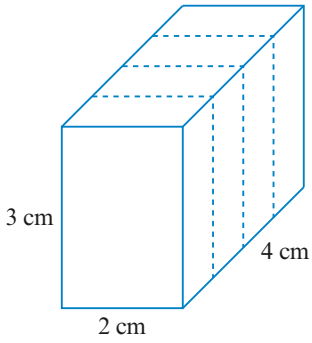


Figure A

1 What type of prism is shown in figure A?

2 What is the shape of its cross section? Does it matter where you 'slice' the prism to obtain the cross section?

This cross section is also known as the **base** of the prism.

3 If the base is the front face, what is the length of the dimension that is at right angles to this face?

This length is called the **height** of the prism.

4 Does it matter that the 'height' is not a vertical length and the base is not a horizontal face of the prism?

The **volume** (V) of a prism is found by multiplying the area of the base (A) by the height (H).

Volume = area of base × height
$V = A \times$ _____
or $V =$ _____

5 Copy and complete the formula for volume of a prism at right.

6 Use the formula to show the volume in figure A is 24 cm^3.

7 Does it matter which face is the base? Show another way of working out the volume using the bottom face as the base this time. Do you obtain the same answer?

8 What type of prism is shown in figure B? (Hint: what shape is its cross section?)

9 What is the area of its base? (Hint: this is the area of the cross section.)

10 What is its height?

11 Use the formula to calculate its volume.

12 Could a different face of the prism be used as the base? Explain.

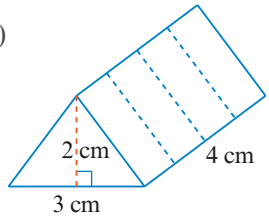


Figure B

KEY IDEAS

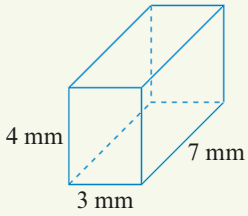
- Volume is the amount of space a 3D object occupies.
- Common units of volume are cubic millimetres (mm^3), cubic centimetres (cm^3) and cubic metres (m^3).
- A rectangular prism has a rectangle for its base.
- A triangular prism has a triangle for its base.
- To find the volume (V) of any prism, use the formula $V = AH$, where A is the area of the base and H is the height of the prism. The base and height of a prism must be perpendicular (at right angles) to each other.
- The volume of other 3D objects such as cylinders can also be found using the formula $V = AH$.

NOTE We use H for height of a prism so that it is not confused with h for height of a triangle or parallelogram.

EXERCISE 8G Volume of prisms

EXAMPLE 8G-1 Calculating volume of a rectangular prism

Calculate the volume of this prism.

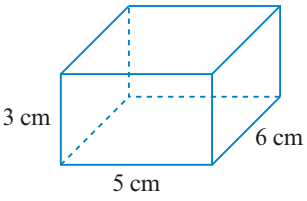


THINK	WRITE
1 Write the formula.	$V = AH$ $= l \times w \times H$
2 Choose the front face as the base. Identify the shape (rectangle) so appropriate area formula is used ($A = lw$).	
3 Identify l, w and H and check they are in the same units.	$l = 3$ mm, $w = 4$ mm, $H = 7$ mm
4 Substitute the values into the formula and calculate the result. Include the appropriate unit.	$V = 3 \times 4 \times 7$ $= 84$ mm^3

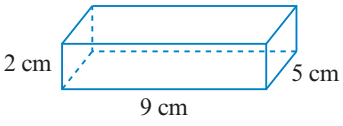
UNDERSTANDING AND FLUENCY

1 Calculate the volume of each rectangular prism.

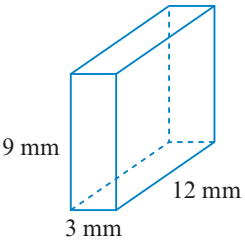
a



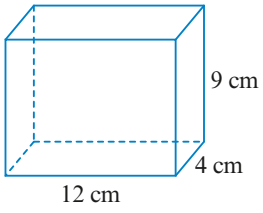
b



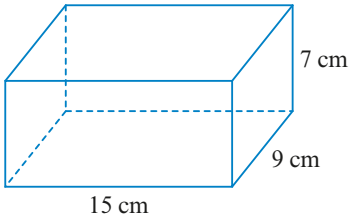
c



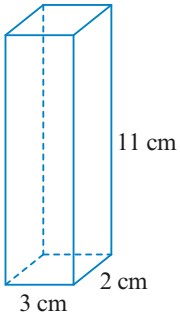
d



e

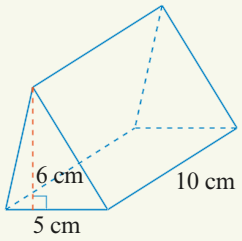


f



EXAMPLE 8G-2 Calculating volume of a triangular prism

Calculate the volume of this prism.

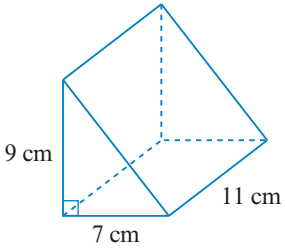


THINK	WRITE
1 Write the formula.	$V = AH$
2 Choose front face as base. Identify the shape (triangle) so appropriate area formula is used ($A = \frac{1}{2}bh$).	$= \frac{1}{2} \times b \times h \times H$
3 Identify b, h and H and check they are in the same units.	$b = 5$ cm, $h = 6$ cm, $H = 10$ cm
4 Substitute the values into the formula and calculate the result. Include the appropriate unit.	$V = \frac{1}{2} \times 5 \times 6 \times 10$ $= 150$ cm^3

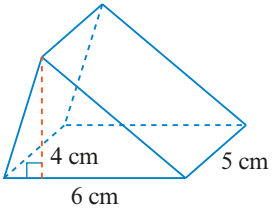
UNDERSTANDING AND FLUENCY

2 Calculate the volume of each triangular prism.

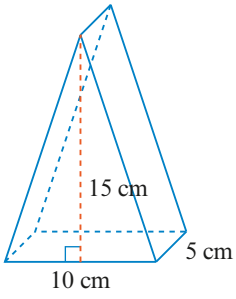
a



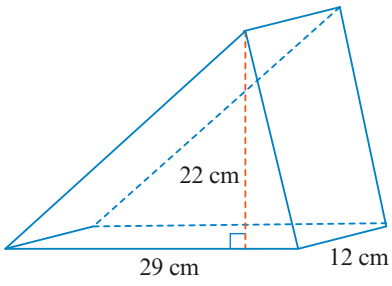
b



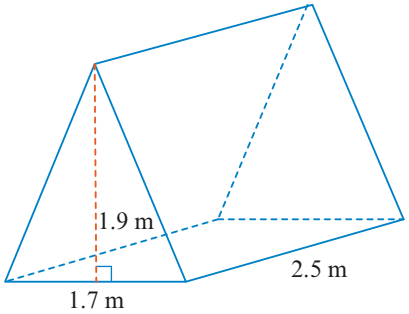
c



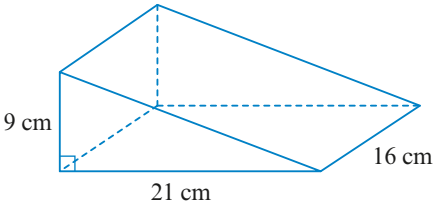
d



e



f



UNDERSTANDING AND FLUENCY

3 Calculate the volume of each prism.

- **a** rectangular prism 2.5 cm long, 4.5 cm wide and 6 cm high
- **b** triangular prism 9 mm high, 3.5 mm wide and with 12 mm base
- **c** rectangular prism 7 m long, 2 m wide and 11 m high
- **d** triangular prism 5.5 mm high, 8 mm wide and with 1.5 mm base
- **e** triangular prism 16 cm high, 5 cm wide and with 12 cm base
- **f** rectangular prism 11.5 mm long, 18 mm wide and 7 mm high

4 Calculate the volume of each object.

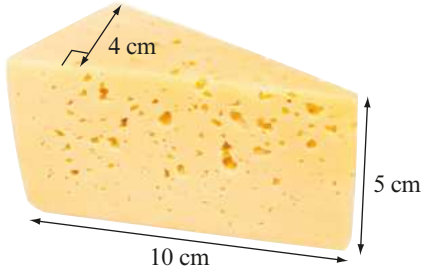
a



b

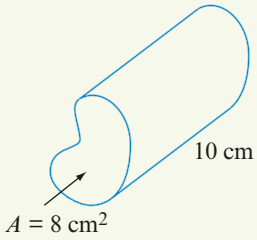


c



EXAMPLE 8G-3 Calculating volume of an object

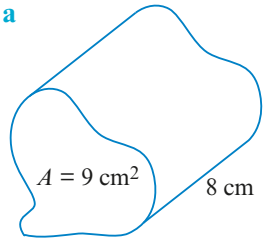
Calculate the volume of this object.



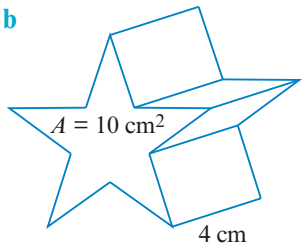
THINK	WRITE
1 Write the formula. This object is not a prism but the same formula can be used.	$V = AH$
2 Identify the shape of the base (irregular). Its area is given.	$A = 8\ \text{cm}^2$
3 Identify H and check its unit is centimetres.	$H = 10\ \text{cm}$
4 Substitute the values into the formula and calculate the result. Include the appropriate unit.	$V = 8 \times 10$ $= 80\ \text{cm}^3$

5 Calculate the volume of each object. The base area is shown.

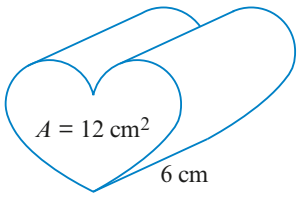
a



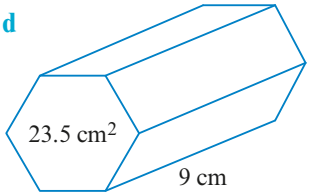
b



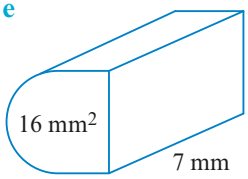
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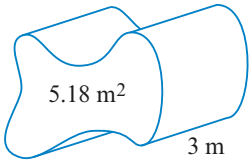
d



e

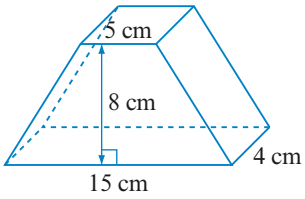


f

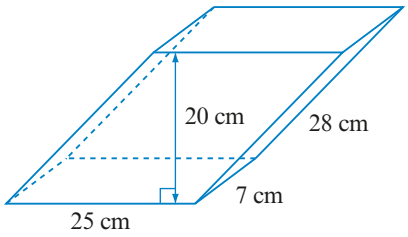


6 Use the formula $V = AH$ to calculate the volume of each object.

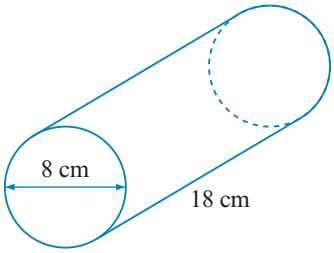
a



b



c



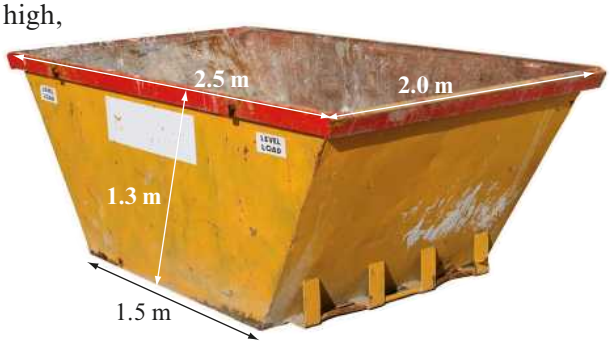
7 A tissue box is 21 cm long, 11.4 cm wide and 12.3 cm tall.

a What is its volume?

b If each tissue laid flat in the box is 1 mm high, how many tissues would it fit?

8 Explain why the volume of any irregular prism can be found by using the formula $V = AH$.

9 A skip has the dimensions shown. Calculate its volume in cubic metres.



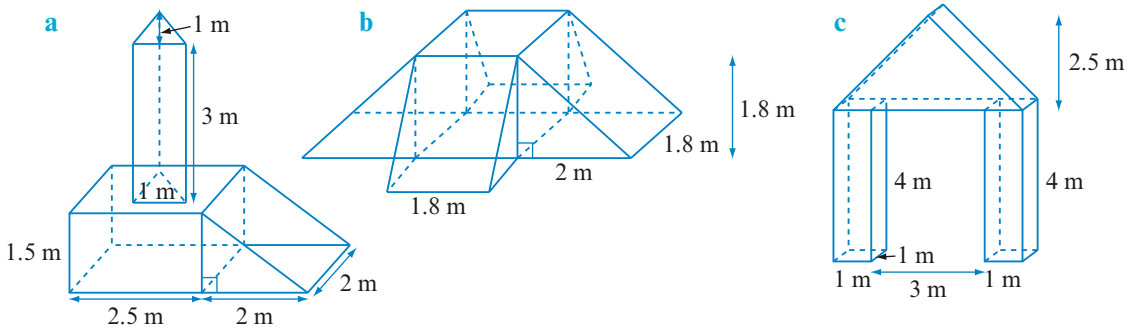
10 A new kind of insulation is to be fitted in the roof of a house. It is measured in cubic metres rather than square metres because it fills the entire roof (normally insulation is just a thick layer). The roof is in the form of a triangular prism. If the roof is 2 m high, 9 m wide and 14 m long, calculate:

a how much insulation is needed (in cubic metres)

b how much it will cost if the insulation costs $30 per cubic metre.

PROBLEM SOLVING AND REASONING

11 Andrew likes to make geometric sculptures. If his materials cost $2 per cubic metre, calculate how much it costs him to produce each sculpture.



12 Mel decides she wants one of the sculptures from question **11**. She can't decide which one, but it needs to be as light as possible.
Use the information at right to recommend which sculpture Mel should buy.

Sculpture a: weighs 3 kg per cubic metre
Sculpture b: weighs 2 kg per cubic metre
Sculpture c: weighs 2.5 kg per cubic metre

13 A rectangular prism has a square base with sides that measure 5 cm. If the prism has a volume of 100 cm^3, what is its length?

14 A triangular prism must be 20 cm long and have a volume of 400 cm^3.

a What area must the triangle face be?

b If the triangle face must be exactly 10 cm high, what is the base measurement?

c If the triangle face must be less than 10 cm high, what are three possible sets of dimensions (base and height) of the triangle face?

15 If you were to fit yourself into a cardboard box, what would be:

a the smallest possible set of dimensions for this box?

b the smallest possible volume it could have?

16 A rectangular prism has a volume of 500 cm^3. List three possible sets of dimensions.

17 A triangular prism has a volume of 500 cm^3.

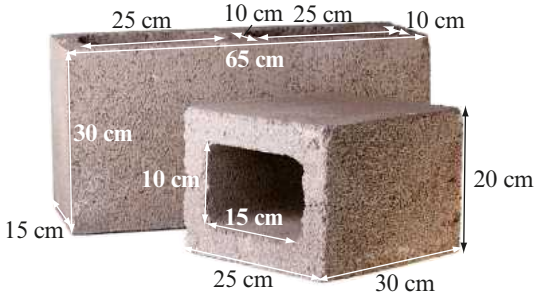
a List three possible sets of dimensions.

b Draw these three examples. Try to use different shaped triangles for each one.

18 What common mistakes do people make when finding the volume of a prism?

CHALLENGE

19 Find the volume of concrete needed to make these two blocks at right.



20 Julia found the answer to question **19** to be 44 250 cm^3. Explain where Julia went wrong and help her find the correct answer.

Reflect

How is the volume of a prism related to its base?

8H Area and volume conversions

Start thinking!

1 **a** What are the dimensions of figure A?
b How many square centimetres in figure A?

2 **a** What are the dimensions of figure B?
b How many square millimetres in figure B?
c Is this area the same or different from figure A?
d Explain why, even though there are 10 mm in 1 cm, there are 100 mm^2 in 1 cm^2.

3 **a** Draw another square and label its sides with '1 m'.
b Use this square to explain why, even though there are 100 cm in 1 m, there are 10 000 cm^2 in 1 m^2.

4 **a** How many cubic centimetres in figure C?
b Copy and relabel figure C in millimetres.
c Explain why, even though there are 10 mm in 1 cm, there are 1000 mm^3 in 1 cm^3.

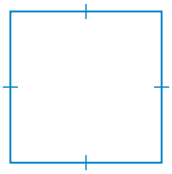


Figure A

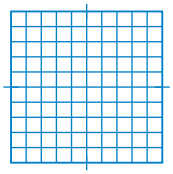


Figure B

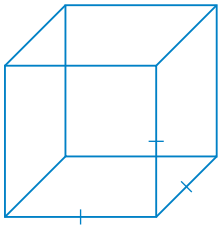
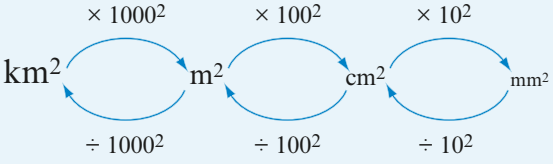


Figure C

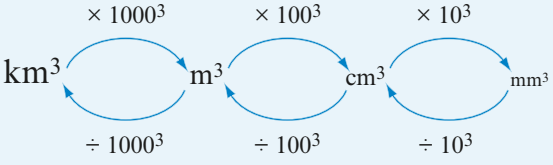
KEY IDEAS

- When converting between units of area, you multiply/divide by the square of the conversion factor.



For example, as 1 cm = 10 mm:
1 cm × 1 cm = 1 cm^2
10 mm × 10 mm = 100 mm^2
so 1 cm^2 = 100 mm^2 or 10^2 mm^2

- When converting between units of volume, you multiply/divide by the cube of the conversion factor.



For example, as 1 m = 100 cm:
1 m × 1 m × 1 m = 1 m^3
100 cm × 100 cm × 100 cm = 1 000 000 cm^3
so 1 m^3 = 1 000 000 cm^3 or 100^3 cm^3

- **Capacity** is the amount of fluid that a container can hold.
- Common units for capacity are millilitres (mL), litres (L), kilolitres (kL) and megalitres (ML).

EXERCISE 8H Area and volume conversions

UNDERSTANDING AND FLUENCY

1 Copy and complete:

a To convert between centimetres and millimetres the conversion factor is 10.
To convert between cm^2 and mm^2 the conversion factor is
$10^2 = 10 \times 10 =$ _____
To convert between cm^3 and mm^3 the conversion factor is
$10^3 = 10 \times 10 \times 10 =$ _____

b To convert between metres and centimetres the conversion factor is 100.
To convert between m^2 and cm^2 the conversion factor is
$100^2 = 100 \times 100 =$ _____
To convert between m^3 and cm^3 the conversion factor is
$100_ =$ ___ × ___ × ___ = _____

c To convert between kilometres and metres the conversion factor is 1000.
To convert between km^2 and m^2 the conversion factor is
$1000_ = 1000 \times 1000 =$ _____
To convert between km^3 and m^3 the conversion factor is
$1000_ =$ ___ × ___ × ___ = _____

2 Copy and complete these conversion statements. The first one has been done for you.

a 6 cm^2 into $mm^2 = (6 \times 10^2)$ $mm^2 = (6 \times 100)$ $mm^2 = 600$ mm^2

b 4 m^2 into $cm^2 = (4 \times 100^2)$ $cm^2 = (4 \times$ _____$)$ $cm^2 =$ _____ cm^2

c 900 mm^2 into $cm^2 = (900 \div 10^2)$ $cm^2 = (900 \div$ _____$)$ $cm^2 =$ _____ cm^2

d 5 cm^3 into $mm^3 = (5 \times 10^3)$ $mm^3 = (5 \times 1000)$ $mm^3 =$ _____ mm^3

e 7 m^3 into $cm^3 = (7 \times 100^3)$ $cm^3 = (7 \times$ _____$)$ $cm^3 =$ _____ cm^3

f 2000 mm^3 into $cm^3 = (2000 \div 10^3)$ $cm^3 = (2000 \div$ _____$)$ $cm^3 =$ _____ cm^3

EXAMPLE 8H-1 Converting to a smaller area unit

Convert 3.8 cm^2 into mm^2.

THINK	WRITE
1 1 cm = 10 mm so 1 $cm^2 = 10^2$ mm^2. Multiply by conversion factor (10^2) to convert to a smaller unit.	3.8 cm^2 $= (3.8 \times 10^2)$ mm^2
2 Calculate 10^2. ($10 \times 10 = 100$)	$= (3.8 \times 100)$ mm^2
3 Complete the multiplication.	$= 380$ mm^2

3 Convert each measurement.

a 3 m^2 into cm^2
b 10 cm^2 into mm^2
c 6.5 m^2 into cm^2
d 19.7 cm^2 into mm^2
e 1.6 km^2 into m^2
f 0.75 km^2 into m^2

EXAMPLE 8H-2 Converting to a larger area unit

Convert 100 000 cm^2 into m^2.

THINK	WRITE
1 1 m = 100 cm so 1 m^2 = 100^2 cm^2. Divide by conversion factor (100^2) to convert to a larger unit.	100 000 cm^2 = (100 000 ÷ 100^2) m^2
2 Calculate 100^2. (100 × 100 = 10 000)	= (100 000 ÷ 10 000) m^2
3 Complete the division.	= 10 m^2

4 Convert each measurement.

a 25 000 cm^2 into m^2
b 750 000 mm^2 into cm^2
c 400 mm^2 into cm^2
d 125 000 m^2 into km^2
e 675 cm^2 into m^2
f 500 000 000 m^2 into km^2

EXAMPLE 8H-3 Converting to a smaller volume unit

Convert 2 m^3 into cm^3.

THINK	WRITE
1 1 m = 100 cm so 1 m^3 = 100^3 cm^3. Multiply by conversion factor (100^3) to convert to a smaller unit.	2 m^3 = (2 × 100^3) cm^3
2 Calculate 100^3. (100 × 100 × 100 = 1 000 000)	= (2 × 1 000 000) cm^3
3 Complete the multiplication.	= 2 000 000 cm^3

5 Convert each measurement.

a 9 cm^3 into mm^3
b 18 m^3 into cm^3
c 4.2 km^3 into m^3
d 4.56 m^3 into cm^3
e 0.15 km^3 into m^3
f 6.1 cm^3 into mm^3

EXAMPLE 8H-4 Converting to a larger volume unit

Convert 216 mm^3 into cm^3.

THINK	WRITE
1 1 cm = 10 mm so 1 cm^3 = 10^3 mm^3. Divide by conversion factor (10^3) to convert to a larger unit.	216 mm^3 = (216 ÷ 10^3) cm^3
2 Calculate 10^3. (10 × 10 × 10 = 1000)	= (216 ÷ 1000) cm^3
3 Complete the division.	= 0.216 cm^3

UNDERSTANDING AND FLUENCY

6 Convert each measurement.

a 16 000 mm^3 into cm^3 **b** 1 750 000 m^3 into km^3
c 4 000 000 cm^3 into m^3 **d** 845 000 cm^3 into m^3
e 28 mm^3 into cm^3 **f** 1 700 000 000 000 m^3 into km^3

7 Convert each measurement.

a 5.6 cm^3 into mm^3 **b** 2.8 m^2 into cm^2
c 3 752 000 m^2 into km^2 **d** 913 458 mm^3 into cm^3
e 186 cm^2 into m^2 **f** 0.075 km^3 into m^3
g 456 cm^2 into mm^2 **h** 75 m^3 into km^3
i 1 cm^3 into m^3 **j** 1.32 km^2 into m^2
k 0.555 m^3 into cm^3 **l** 16 mm^2 into cm^2

PROBLEM SOLVING AND REASONING

8 Ahmed needed to convert 11 cm^2 into mm^2. He said that because there are 10 mm in 1 cm, then there are 110 mm^2 in 11 cm^2. Explain where he went wrong and provide the correct answer.

9 Jess completed a worksheet of conversions, shown below.

i 40 000 cm^2 = (40 000 ÷ 100) m^2
= 400 m^2

ii 200 cm^2 = (200 × 10^2) mm^2
= 20 000 mm^2

iii 7 m^3 = (7 × 100^3) cm^3
= 2100 cm^3

iv 3000 mm^3 = (3000 ÷ 100^3) cm^3
= 30 cm^3

a Mark her work and give her a score out of 4.
b Provide a suggestion for why she made each mistake.
c Provide the correct working for the problems that she got incorrect.

10 A company produces painted wooden toy boxes.



- **a** Calculate the surface area of the outside of the box (all six faces are to be painted).
- **b** If each box needs two coats of paint and one tin of paint covers 15 m^2, how many toy boxes can be painted with one tin?

11 The company would like to know the volume of the box from question **10** in cubic metres, but the dimensions are in cubic centimetres.

- **a** Calculate the volume in cubic centimetres.
- **b** Convert this to cubic metres.
- **c** Another way to do this is to first convert all the dimensions into metres and then find the volume. Show how doing the conversion first gives you the same answer to part **b**.



12 Aaron harvests wheat on his farm and gets a yield of about 3.2 tonnes per hectare. A hectare is 10 000 m^2.

- **a** Write this in:
  - **i** kilograms per hectare
  - **ii** kilograms per square metre
  - **iii** grams per square metre
  - **iv** grams per square centimetre
- **b** Which unit do you think makes the most sense to use? Explain.
- **c** If Aaron is paid 35 cents per kilogram of wheat, find how much money he will earn if he harvests 3000 hectares of wheat.

13 Remember that a hectare is 10 000 m^2.

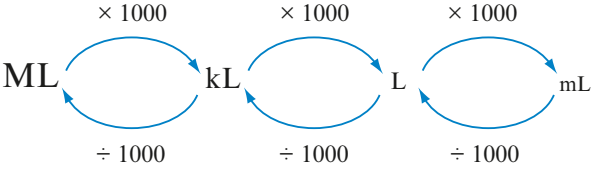
- **a** How many hectares are in 1 km^2?

There are many other units of area that are still used today that are not metric, such as the acre. A hectare is roughly equal to 2.5 acres.

- **b** Approximately how many square metres are in an acre?
- **c** Investigate the exact conversion factor between an acre and metric units such as the square metre and hectare.
- **d** Investigate other units of area; for example, square mile, square yard, and explain how they relate to the metric units of area such as the square metre.

14 Look at this conversion chart for capacity.

- **a** How many millilitres in a litre?
- **b** Convert:
  - **i** 7.5 L into mL
  - **ii** 450 mL into L
  - **iii** 900 kL into ML
  - **iv** 3.75 kL into L.



PROBLEM SOLVING AND REASONING

15 A container of volume 1 cm^3 holds 1 mL of liquid. Similarly, 1 m^3 holds 1 kL of liquid.

a Find the volume of the container (in cm^3) that would hold:

i 15 mL **ii** 1 L **iii** 5 L **iv** 1 kL

b Find the volume of the container (in m^3) that would hold:

i 4 kL **ii** 13 kL **iii** 200 L **iv** 745 L

c Find the capacity held in a container with a volume of:

i 26 cm^3 **ii** 5000 cm^3 **iii** 9 m^3 **iv** 0.4 m^3

16 **a** Calculate the volume of this perfume bottle in cubic centimetres.

b Calculate its capacity in litres.



17 A rectangular bath tub measures 1.5 m long by 30 cm wide, with a depth of 45 cm. How many litres of water could it hold?

18 A car engine is said to be 1200 cc. This means that the cylinders in the engine have a volume of 1200 cm^3. How many litres of petrol can the cylinders hold?

19 A rectangle has an area of 4800 mm^2. Give three possible sets of dimensions in centimetres.

20 A rectangular box has a volume of 36 000 cm^3. Give a possible set of dimensions in:

a millimetres **b** centimetres **c** metres.

21 Write your own open-ended question about area and volume and swap with a classmate. Be careful of conversion errors. Discuss any difference in answers.

CHALLENGE

22 Melbourne's water storages have a capacity of 1 812 175 ML.

a Write this in:

i kilolitres **ii** litres **iii** cubic metres **iv** cubic kilometres.

b Which expression do you think has the most meaning for people? Explain.

c If the water storages are only 47% full, how many kilolitres is this?

d Melbourne Water estimates that each person uses 300 L of water per day.

i How many litres of water is this per year?

ii How many kilolitres of water is this per year?

e If there are about 4 million people living in Melbourne, how many kL of water is this a year?

f If the storages are 47% full, use your answer from part **e** to find how quickly our storages would run out if there was no rain. Answer to the nearest year.

g What other uses of water are there that would take water from storages?

Reflect

What common mistakes might people make when converting area or volume units?

CHAPTER REVIEW

SUMMARISE

Create a summary of this chapter using the key terms below. You may like to write a paragraph, create a concept map or use technology to present your work.

length	total surface area	dimensions	annulus
perimeter	volume	perpendicular	net
estimate	capacity	composite shapes	rectangular prism
circumference	width	quadrilaterals	triangular prism
diameter	rectangle	parallelogram	square-based prism
radius	triangle	rhombus	cube
area	base	kite	pyramids
sector	height	trapezium	conversion factors

MULTIPLE-CHOICE

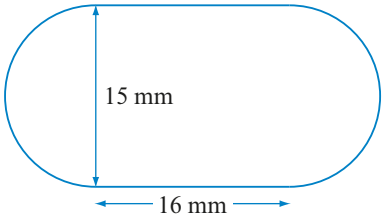
8A 1 What is 7.84 m equal to?

A 0.784 km B 7840 mm
C 78.4 cm D 0.0784 km

8B 2 A circle has a diameter of 6 cm. What is its radius?

A 12 cm B 6π cm
C 36π cm D 3 cm

8B 3 What is the total perimeter of this shape?



A 31 mm B 47 mm
C 63.1 mm D 79.1 mm

8C 4 A triangle has a base length of 10 cm and an area of 20 cm^2. What is its height?

A 2 cm B 4 cm
C 12 cm D 200 cm

8D 5 A rhombus has diagonals of lengths 5 cm and 6 cm. What is its area?

A 30 cm^2 B 11 cm^2
C 56 cm^2 D 15 cm^2

8E 6 What is the area of the shape in question 3?

A 946.9 mm^2 B 754 mm^2
C 416.7 mm^2 D 240 mm^2

8F 7 What is the total surface area of a cube with side length 5 mm?

A 150 mm^2 B 125 mm^2
C 25 mm^2 D 15 mm^2

8G 8 What is the volume of an irregular prism with base area 2.5 cm^2 and length 10 mm?

A 250 mm^3 B 25 mm^3
C 2.5 cm^3 D 25 cm^3

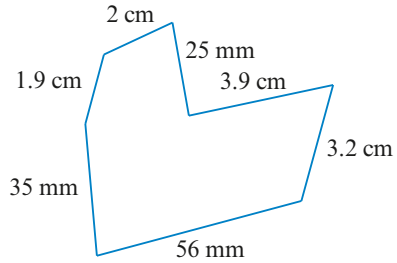
8H 9 What is 3 cm^3 equal to?

A 30 mm^3 B 300 mm^3
C 300 m^3 D 3000 mm^3

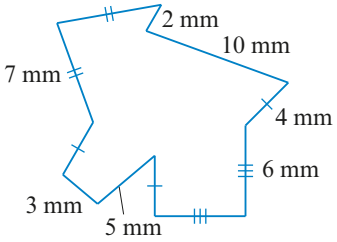
SHORT ANSWER

8A **1** Calculate the perimeter of each shape.

a

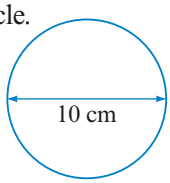


b

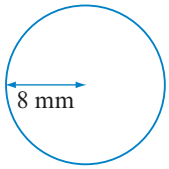


8B **2** Calculate the circumference of each circle.

a

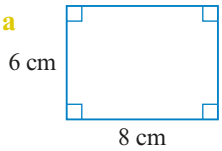


b

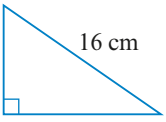


8C **3** Calculate the area of each shape.

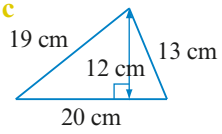
a



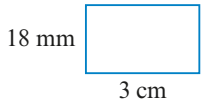
b



c

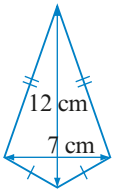


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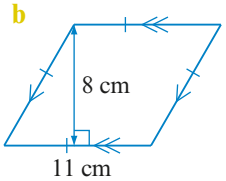


8D **4** Calculate the area of each shape.

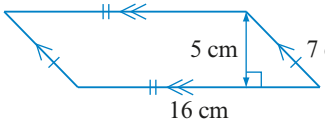
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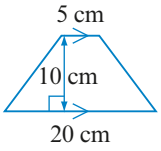
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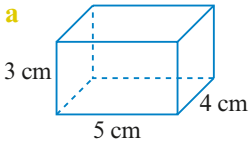
d



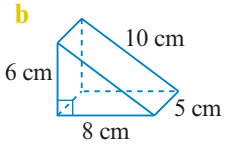
8E **5** Calculate the area of each circle in question **2**.

8F **6** Calculate the total surface area of each object.

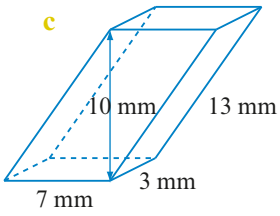
a



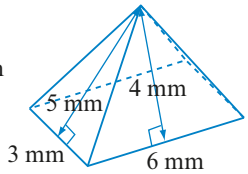
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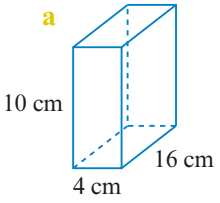


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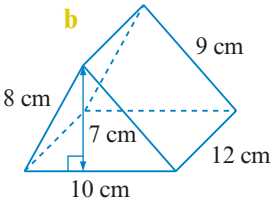


8G **7** Calculate the volume of each object.

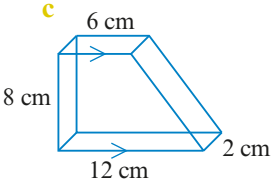
a



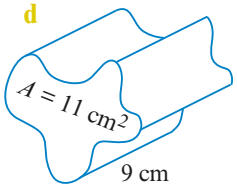
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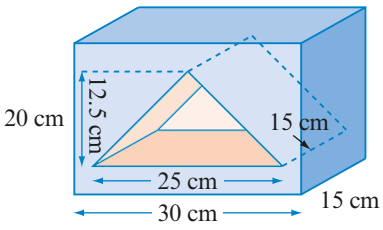
c



d



8G **8** Calculate the volume of clay in this brick.



8H **9** Convert each measurement.

a 5 m^2 into cm^2

b 50 mm^2 into cm^2

c 3.5 cm^3 into mm^3

d 400 cm^3 into m^3

e 7.5 L into mL

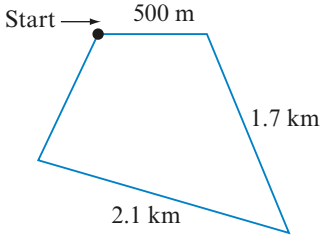
f 845 kL into L.

NAPLAN-STYLE PRACTICE

1 Juan is building a model aeroplane. He needs a plank of wood that is 1150 mm long. If he buys a plank of wood 1.5 m long, how many centimetres will he have to cut off?

1.15 ☐ 350 ☐ 45 ☐ 35 ☐

2 The total length of a cross-country course is 5 km. What is the length of the last leg?



☐ 700 m
☐ 4.3 km
☐ 9.3 km
☐ 430 m

3 A vegetable garden is in the shape of a rectangle. The total length of fencing is 3.4 m and the garden is 1 m long. How wide is the garden?

☐

4 A circular sushi hand roll measures 4.5 cm across its diameter. What length of seaweed wraps around the circumference?

☐

5 A circular hat measures 65 cm around the brim. How wide is it across its diameter?

☐ 204.2 m ☐ 20.69 km
☐ 13 273 cm ☐ 10.35 cm

Questions **6** and **7** refer to a car wheel with a radius of 20 cm.

6 What is the length of one revolution of the wheel?

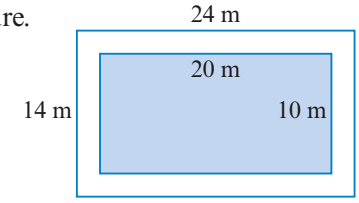
☐ 20 cm ☐ 62.83 cm
☐ 125.66 cm ☐ 1256.64 cm

7 By how much would this radius need to increase if the wheel had to cover 1.5 m in one revolution?

☐ 24.34 cm ☐ 12.17 cm
☐ 7.75 cm ☐ 3.87 cm

Questions **8** and **9** refer to the following information and figure.

A pool has concrete pavers around its edges, as shown in the figure.



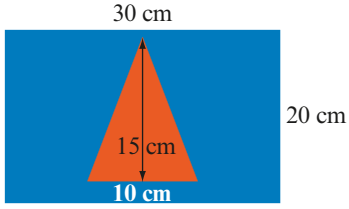
8 What is the area of the swimming pool?

☐

9 What is the area covered by the concrete pavers?

☐ 336 m^2 ☐ 136 m^2
☐ 76 cm^2 ☐ 38 m^2

10 Derek designed this flag for his sports team. What is the area of the blue section?



☐ 75 cm^2 ☐ 450 cm^2
☐ 525 cm^2 ☐ 600 cm^2

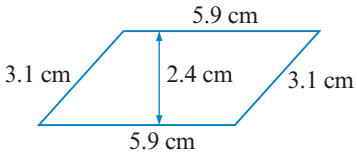
11 A triangle has an area of 40 cm^2. If it is 8 cm high, how long is its base?

☐

12 Brittany has a kite that is 75 cm long and 40 cm wide. What is the area of this kite?

☐

13 What is the area of this shape?



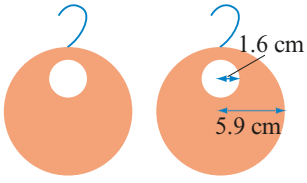
☐

14 A Frisbee has a diameter of 25 cm. What is its area?

☐

15 A pair of earrings is to be made according to the blueprints shown. How much metal is needed to make the pair?

☐ 218.72 cm^2
☐ 202.63 cm^2
☐ 109.36 cm^2
☐ 101.32 cm^2



16 What is the diameter of the largest plate that could be made with 200 cm^2 of porcelain (to the nearest centimetre)?

4 cm ☐ 8 cm ☐ 16 cm ☐ 64 cm ☐

Questions **17**–**19** refer to the following information.

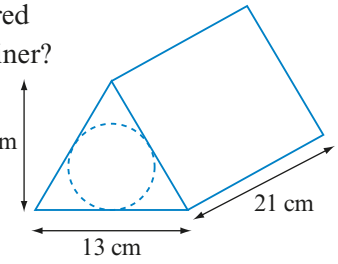
Tennis balls are placed into a container that is a rectangular prism with the dimensions 7 cm by 7 cm by 21 cm.

17 How much cardboard is required to make this container?

☐

18 The tennis balls are instead placed inside a container in the shape of an equilateral triangular prism, as shown. How much cardboard is required to make this container?

☐ 44 cm^2
☐ 949 cm^2
☐ 1079 cm^2
☐ 2730 cm^2



19 What is the difference in volume between these containers?

☐ 263 cm^3 ☐ 336 cm^3
☐ 1029 cm^3 ☐ 1701 cm^3

Questions **20**–**22** refer to the following information.

Katie has a vase in the shape of a rectangular prism. Its base measures 5 cm by 5 cm and it is 20 cm high.

20 What is the volume of the vase?

☐

21 What amount of water will fit inside the vase?

5 L ☐ 50 L ☐ 500 mL ☐ 25 mL ☐

22 How many vases would be required to hold 1 m^3 of water?

2 ☐ 20 ☐ 200 ☐ 2000 ☐

23 A sheet of glass has an area of 3500 mm^2. What is this measurement in cm^2?

☐

ANALYSIS

A friend wants to have a ball pit at their birthday party. They have set aside a rectangular space in their backyard that is 3 m long and 2 m wide.

a What area would this cover?

b How much fencing would be needed to go around the ball pit?

Your friend can only buy the wood that they need for the fencing in planks that are 50 cm wide. The walls of the pit need to stand 1 m tall.

c What length of wood is needed?

d Your friend wants to paint the wooden fencing inside and out with two coats of paint. What is the total surface area to be painted?

They also want to paint a diamond on the front fence of the ball pit. From point to point (that is, its diagonals) it measures 55 cm vertically and 30 cm horizontally.

e What is the area of this diamond?

Around the diamond will be six circles representing the coloured balls in the ball pit. Each ball has a diameter of 6.6 cm.

f What is the circumference of each circle?

g Find the total area to be painted for the six balls.

h What will be the volume within the pit?

i If each ball to go into the pit has a volume of 150 cm^3, how many could theoretically fit into the pit? Be careful with unit conversions!

j Your friend finds that only about 25 000 balls fit into the pit. What explanation can you offer for this difference?

CONNECT

Planning a house

Planning and constructing any building requires a good understanding of measurements and mathematics. What measurements are needed when planning to build a house?





Your task

For your investigation consider the design and cost of constructing a simple house, such as a cubby house, dog kennel, doll house or even a birdhouse. You will need to complete these steps.

- Record the type and purpose of house.
- Prepare diagrams showing the shape, size and dimensions of the house
- Also prepare diagrams to show the shape, size and dimensions of house features (for example, doors, windows, balconies).
- Make lists of the type and cost of building materials (for example, wood, plastic, glass, tiles)
- Work out the cost of painting the house, inside and out, and other decoration.







Complete the 8 CONNECT worksheet to show all your working and answers to this task.

You may like to present your findings as a report. Your report could be in the form of:

- a catalogue
- blueprints
- a brochure
- other (check with your teacher).



9 STATISTICS

9A Sampling data

9B Collecting data

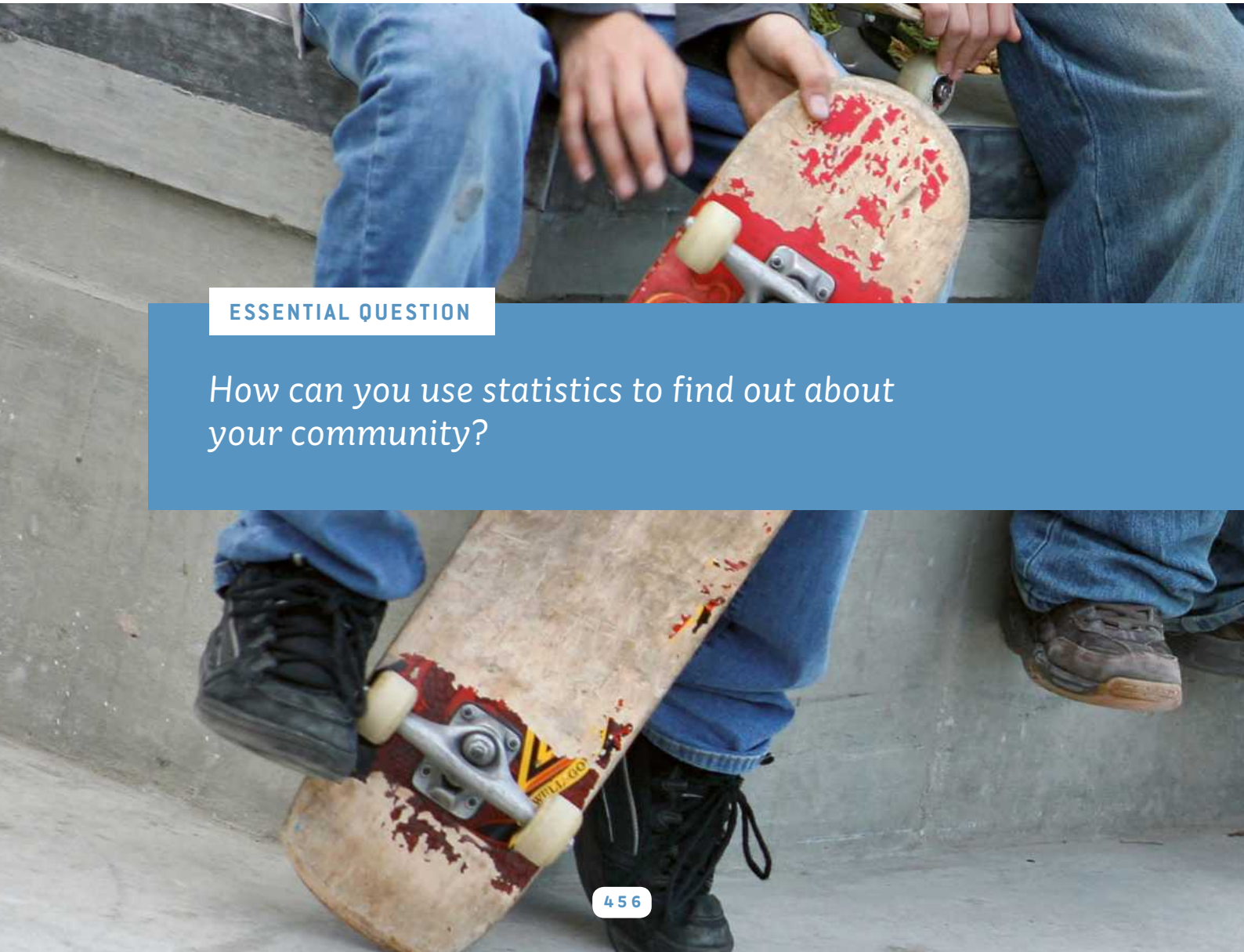
9C Presenting data in graphs

9D Stem-and-leaf plots and dot plots

9E Presenting grouped data

9F Summary statistics

9G Analysing data



ESSENTIAL QUESTION

How can you use statistics to find out about your community?

Are you ready?

9A **1** For a survey on popular pets, which is the best question to use?

A What pet(s) do you have at home?
B What is your favourite pet?
C Do you prefer cats or dogs?
D Do you have a dog?

9B **2** Which is numerical data?

A favourite colour
B opinion of maths
C gender **D** height

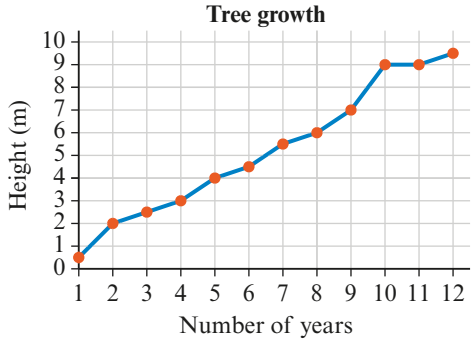
9B **3** Which of these data can be measured?

A number of people in a room
B height of a tree
C number of pencils in a case
D hair colour

9B **4** Complete this table by filling in the missing value.

Pizza flavour	Frequency
Supreme	6
Hawaiian	2
Meat Lovers	3
Vegetarian	4
BBQ Chicken	
Aussie	5
Total	25

Questions **5**–**7** refer to this graph.



9C **5** What type of graph is this?

A line graph **B** pie graph
C column graph **D** scatterplot

9C **6** How tall was this tree when it was 10 years old?

9C **7** How old was the tree when it was 6 m tall?

9D **8** What is the most common score found in this stem-and-leaf plot?

Key: 1 | 0 = 10

Stem	Leaf
1	0 3 5
2	1 1 5 8
3	3 3 5 8 8
4	0 2 2 2 2 8 9
5	8 8

Questions **9** and **10** refer to this table.

Class	Frequency
0–<5	2
5–<10	6
10–<15	19
15–<20	15
20–<25	3

9E **9** What is the size of the class intervals?

A 0 **B** 5
C 10 **D** 20

9E **10** How many people were surveyed?

Questions **11**–**13** refer to this data set.

1 1 2 4 3 2 1 4 1 11

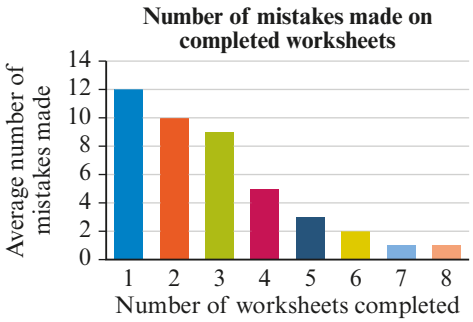
9F **11** What is the mean?

A 1 **B** 2 **C** 3 **D** 10

9F **12** What is the range?

9F **13** What is the most common number?

9G **14** What conclusion could you draw from this graph?



A People make mistakes.
B The more worksheets you attempt, the fewer mistakes you make.
C The more worksheets you attempt, the more mistakes you make.
D People will make 12 mistakes on their first worksheet.

9A Sampling data



Start thinking!

Naomi wants to find out which ice-cream store in Bendigo is the Year 8 students' favourite. She starts to create a list of all the Year 8 students in Bendigo.

1 Does she need to ask every Year 8 student in Bendigo? Explain.

2 List one advantage and one disadvantage of asking every Year 8 student.

Surveying an entire **population** is called a **census**. Rather than taking a census, Naomi decides to take a **sample**. A fair sample would be a smaller number of students that represent the population fairly. For her sample, Naomi goes to the Wendy's store and asks every Year 8 person she knows what their favourite ice-cream store is.

3 What is a disadvantage of sampling this way?

4 Who would not be included in this sample?

A **biased** sample is a sample that does not accurately reflect the population. It may have one or more opinions or outcomes that are favoured over others.

5 Why is Naomi's method of sampling biased?

Instead, Naomi decides to ask every fifth person who comes to Wendy's.

6 Why will this still produce biased results? Suggest an alternative sampling method.

KEY IDEAS

- In statistics, a population is the entire group of people or items being investigated.
- A census is a survey of an entire population.
- A sample is a survey of some members of a population.
- A sample should fairly reflect the population. If it doesn't, it is said to be biased.
- It is important to choose a method of sampling that does not lead to biased data, and where each member of the population has an equal chance of being selected.
- **Random sampling** methods (such as pulling names out of a hat) are commonly used for small samples.
- **Systematic sampling** is selecting an individual at fixed intervals (for example, every 10th person). The starting point should be random.
- **Stratified sampling** involves dividing the population up into separate categories (such as gender and age) and then taking a random sample from each category. The size of each sample is proportional to how large the category is. The results from these samples are then added together.
- For a survey to be successful, the questions must be meaningful and allow all possible answers. Poor questions can lead to biased answers.

EXERCISE 9A Sampling data

EXAMPLE 9A-1 Census or sample?

Zoe asks everybody in her school to find out the favourite park in her town.
Is this a census or a sample?

THINK	WRITE
1 Identify the population.	The population is Zoe's town.
2 Identify who is surveyed.	Zoe's school has been surveyed.
3 A census is an entire population. Is the survey taken the same as the population?	The survey taken is not the same as the population, so this is only a sample.

UNDERSTANDING AND FLUENCY

1 Decide whether a census or a sample has been taken in each situation.

- **a** Nikita surveys her class to find out the favourite chocolate of the entire class.
- **b** Jacqui surveys all her year level to find out the favourite film for the school.
- **c** Travis asks everyone he knows how many pets they have in order to predict the average number of pets in any household in the state.
- **d** Aaron asks everybody in his family what their favourite dessert is to decide what to cook for a family dinner.
- **e** Lisa surveys every 10th person in town to find out the town's average income.
- **f** Harley surveys all his customers to find how he can best serve them.

EXAMPLE 9A-2 Identifying sampling methods

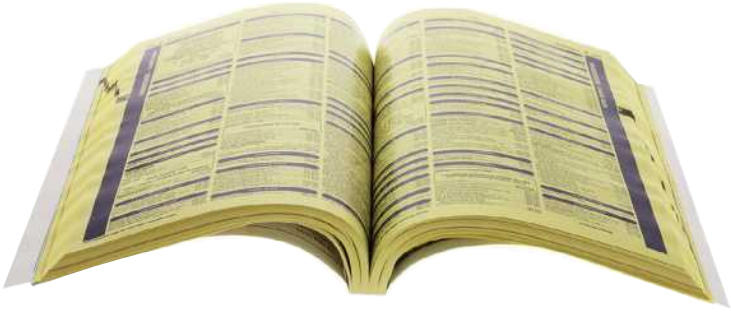
Maria selects a sample of 10 people according to their hair colour, in proportion to how common that hair colour is in the population. Is this sample method random, systematic or stratified?

THINK	WRITE
1 Look at the sampling definitions in the Key ideas. Which method takes a sample that is in proportion to the population?	
2 Write your answer.	This sample is stratified.

UNDERSTANDING AND FLUENCY

2 Describe each sampling method as random, systematic or stratified.

- **a** Jamie asks every fifth person in a store their opinion on a new magazine.
- **b** Helen selects a panel that consists of three girls and two boys out of a group of 15 girls and 10 boys.
- **c** Xavier calls every 100th phone number in the phone book to complete a survey.
- **d** Thanh draws five numbers out of a container.
- **e** Anika draws Bingo numbers from a box to select people to survey.
- **f** Mia surveys 20 students from public and private schools in proportions that reflect the town's population.



EXAMPLE 9A-3 Writing meaningful questions

Alec investigates favourite sports at his school. He asks 50 people if they prefer AFL or NRL. Is this question biased? If so, rewrite it so that it provides fair answers.

THINK	WRITE
1 Does the question provide the opportunity for all favourite sport possibilities? No, it only looks at AFL and NRL, so it will produce biased answers.	This question is biased.
2 Write a question that provides the opportunity for all possibilities.	A better question would be 'What is your favourite sport?'

3 For these topics:

- **i** identify any questions which may lead to biased answers
- **ii** rewrite these questions so that they provide fair answers.

	Topic	Question
a	Finding the most popular animal	Do you like cats or dogs?
b	Deciding which film to see in the holidays	Do you like horror movies?
c	Investigating the most popular chocolate	What is your favourite chocolate?
d	Finding the most common hobby amongst Year 8 students	What do you think the most common hobby might be?
e	Deciding where to go for a holiday	Where are three places you might like to go for the holidays?
f	Investigating voting intentions	Who do you think will win the election?

EXAMPLE 9A-4 Evaluating surveys

Mohamed surveyed his friends and family in order to find out the most popular radio station in town.

a Decide whether a census or sample has been taken.
b Decide whether the survey is fair or biased, giving reasons. If the survey is biased, provide a fair alternative.

THINK	WRITE
a Mohamed wants to know what the entire town thinks but has only asked his friends and family. This is a sample.	a Mohamed has taken a sample.
b **1** His friends and family may share common interests and opinions that do not truly reflect the population. This is a biased sample.	b This is a biased sample.
2 Mohamed needs to ensure that the sample reflects the population fairly. He could use systematic sampling to select people for his survey.	Mohamed could use systematic sampling. For example, he could use a phone book to call every 10th person to ask their opinion.

UNDERSTANDING AND FLUENCY

4 For the following situations:
- i decide whether a census or sample has been taken
- ii decide whether the survey is fair or biased, giving reasons. If the survey is biased, provide a fair alternative.

a Dane asks every 100th person from the local phone book if they prefer skiing or snowboarding to find out the town's favourite winter sport.
b Jacey asks everybody in the school what they think of the uniform to see how popular it is with the students.
c Ryan asks everybody in a gaming store what they think about gaming restrictions to find out the common opinion in Australia.
d Tracey asks every 10th person in Woolworths which supermarket they prefer in order to find the town's favourite.
e Kane asks everyone at his workplace if they like going to the cinema to find out a good option for the office Christmas party.
f Sara asks her friends if they prefer PCs or Macs to find the most popular computer type in her school.



UNDERSTANDING AND FLUENCY



5 Decide whether a census or a sample would be most appropriate for gathering information in each situation.

- **a** deciding who will be the next prime minister
- **b** getting feedback on the latest version of Windows
- **c** finding which store in your town (or suburb) sells PlayStations at the cheapest price
- **d** finding the average number of blue M&Ms in a packet

6 Identify the population in each situation.

- **a** surveying students at a local school to find what the town's Year 8 students think of a local issue
- **b** surveying people at a supermarket to find national attitudes to war
- **c** surveying every fifth person from a Western Australian phone book to find the favourite movie in Western Australia
- **d** surveying people randomly to find the town's attitudes to a new mall being built
- **e** surveying every farmer in the area to find the local area's opinion on supermarket prices
- **f** surveying everybody at school to find the students' favourite canteen item.

PROBLEM SOLVING AND REASONING

7 Lizzie wants to conduct a survey looking at the attitudes of people in town towards graffiti.

- **a** What is the population?
- **b** Somebody suggests Lizzie ask random people that she sees at the local mall. Are there any potential problems with this method?
- **c** Somebody else suggests that Lizzie ask every fifth person who walks past the local post office. What kind of sampling is this?
- **d** Are there any potential problems with this second method?
- **e** Give an example of how she might collect data using stratified sampling.



8 Write two different questions that provide fair answers if you were to investigate:

- **a** favourite soft drinks
- **b** number of pets in a household
- **c** Internet usage per week
- **d** opinions about a new school rule.

PROBLEM SOLVING AND REASONING

9 Look at this group of people. Identify at least three ways they could be split into strata or categories to conduct a survey using stratified sampling (for example, gender).



10 For each way in which you split the group of people in question **9**, form a stratified sample of four people. For example, if you chose to split them by gender, your stratified sample should contain two men and two women because the genders are represented roughly equally.

11 A survey is to be done on a new school uniform.
Provide a method of sampling the school that is:

a random **b** systematic **c** stratified.

12 For each survey:

i identify the population

ii provide an example of how you might collect a sample.

a investigating the top five favourite celebrities of Year 8s at your school

b investigating the town's opinion on a new national law

c investigating the average number of people in an Australian family

13 Choose a topic that you would like to investigate within your class.

a Identify the population concerned.

b Decide if you will take a census or sample.

c Identify the size of the sample (or census) that you will take.

d Form at least three questions that will lead to fair answers.

e Identify the method of sampling that you will use.

f Conduct the survey and write a couple of sentences about what you find.

CHALLENGE

14 Sally wants to find out what people in town think of a local issue. She gets as many email addresses as she can and emails people a survey, asking them to complete it and email the results back to her. How might this method of surveying be biased?

15 Write a short 'How to' guide for somebody planning an investigation that covers sampling and questions.

Reflect

Why is it important to think carefully about questions and methods of sampling before you undertake a survey?

9B Collecting data

Start thinking!



Seth is collecting data on the heights of his classmates and records these measurements in centimetres.
145, 157, 146, 164, 163, 158, 138, 145, 142, 157, 155, 137, 171, 161, 153, 154, 153, 158, 145, 142

1 Arrange the data in a **frequency table** that records a single height in each row.

2 Why might this not be the best way to display such a large **range** of data?

3 Construct another frequency table, but this time use **class intervals** of 5 cm; for example, 140–144 cm, 145–149 cm, 150–154 cm, etc.

4 Name an advantage and a disadvantage of collecting data in groups like this.

Catherine joins Seth's class. Her height is 154.5 cm.

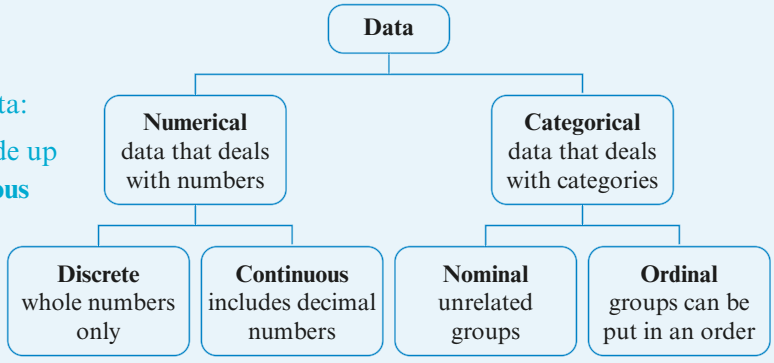
5 Why is it difficult to put Catherine's height into a row of the table?

When using frequency tables, it is important to select class intervals that include all possible data.

6 Construct a new frequency table with updated class intervals of 140–<145 cm, 145–<150 cm, 150–<155 cm, etc., placing Catherine's height into the correct row. How are these class intervals different?

KEY IDEAS

- There are two main types of data:
  - **numerical data**, which is made up of **discrete data** and **continuous data**
  - **categorical data**, which is made up of **nominal data** and **ordinal data**.



- An easy way to display data is in a frequency table.
- For large amounts of numerical data, use class intervals to group the data.
- Class intervals should be chosen so that a table contains 5 to 10 classes or groups.
- Identify the data type before constructing a table. Continuous data need class intervals such as 0–<10, 10–<20, etc. Discrete data are usually shown in class intervals such as 0–9, 10–19, etc., but can be shown in 'continuous' intervals as well.

'Discrete' interval	'Continuous' interval
0–9	0–<10
10–19	10–<20

EXERCISE 9B Collecting data

EXAMPLE 9B-1 Classifying data

Describe the data as discrete, continuous, nominal or ordinal.

a grams of sugar in soft drink **b** clothing size (S, M, L etc.)

THINK	WRITE
a The number of grams of sugar in soft drink is numerical data. Is it in whole numbers or decimal numbers?	**a** continuous
b Clothing size is categorical data. Are they unrelated categories or do they have an order?	**b** ordinal

UNDERSTANDING AND FLUENCY

1 Describe the data as discrete, continuous, nominal or ordinal.

- **a** number of people in your family
- **b** favourite soft drink
- **c** height of a 14-year-old
- **d** mass of puppies at birth
- **e** hair colour
- **f** volume of water in a jug
- **g** opinion on a statement ranging from 'strongly disagree' to 'strongly agree'

2 Identify the data from question **1** that would need to have 'continuous' class intervals. Explain.

EXAMPLE 9B-2 Identifying class intervals

What class interval might you use to display this **set** of data?

15, 19, 23, 38, 9, 12, 7, 22, 29, 31, 9, 11, 34, 33, 19, 10, 20, 34, 37, 26, 21, 11, 30

THINK	WRITE
1 Find the range of numbers.	The data has a range of $38 - 7 = 31$.
2 What class interval would make 5–10 rows in a table, for ease of reading and sorting data?	Class intervals of 5 (e.g. 5 – <10) would suit this data best.

3 What class interval size might you use to display each set of data?

a 11, 4, 16, 82, 34, 22, 25, 36, 32, 12, 6, 45, 63, 25, 42, 5, 17, 33

b 1, 14, 3, 5, 16, 20, 22, 4, 4, 12, 7, 4, 8, 13, 16, 5

c 1.4, 1.2, 1.6, 2.6, 2.35, 2.9, 0.4, 1.3, 1.5, 2.1, 2.2, 2.8, 2.77, 0.57, 0.6, 1.1

d 111, 131, 122, 164, 145, 147, 105, 131, 127, 119, 117, 105, 138, 151

EXAMPLE 9B-3 Arranging grouped data

Megan collected the following data on ages of people at the local swimming pool.

12, 15, 29, 6, 5, 14, 17, 14, 19, 22, 51, 49, 16, 16, 35, 37, 13, 18, 26, 37, 42, 15, 16, 17, 38, 4, 9

a What type of data is this?

b Arrange the data into a frequency table with class intervals of 10.

THINK

a The data gathered contains only numbers, so it is numerical data. There are only whole numbers, so it must be discrete data.

b **1** The smallest number is 4 and the largest number is 67, so draw a frequency table with six rows, each class interval covering 10 years.

2 Tally each age into your frequency table. To ensure you have included each piece of data, add the frequencies to see that you obtain the same total.

WRITE

a numerical, discrete data

b

Age group	Tally	Frequency
0–9	||||	4
10–19	𝍸 𝍸 |||	13
20–29	|||	3
30–39	||||	4
40–49	||	2
50–59	|	1
Total		27

UNDERSTANDING AND FLUENCY

4 Monique collected this data on the number of text messages people in her class sent per day.

6	39	15	26	18	37	48	14	88	9	22	74
42	22	56	33	16	28	31	48	52	17	21	55



a What type of data is this?

b Arrange the data into a frequency table with class intervals of 10.

5 Amal collected this data.

5	9	3	12	17	13	2	0	18	21	14	14
13	19	3	8	11	24	22	19	17	2	11	10

a What type of data is this?

b What class interval size might you use to group this data?

c Construct a frequency table for this data.

UNDERSTANDING AND FLUENCY

6 Use your answers to question **3** to construct a frequency table for each data set.

7 Michelle collected this data on favourite Australian animals.

sugar glider, echidna, kangaroo, koala, koala, sugar glider, red-eyed tree frog, koala, wombat, kangaroo, dingo, wombat, koala, sugar glider, bilby, bilby, bilby, koala, platypus, dingo, kangaroo, sugar glider, wombat, echidna, platypus, koala, wombat



a Arrange the data in a frequency table.

b Which is the most popular animal? How many people chose it?

c What type of data is this?

8 People were asked how many times a day they checked Facebook. These are the results.

1	0	4	6	10	11	3	9	24	19	15	6
7	1	2	9	16	14	20	21	26	8	4	5

a What type of data is this?

b Arrange the data into a frequency table with appropriate class intervals.

PROBLEM SOLVING AND REASONING

9 Households were asked how many TVs they had. The data collected is shown below.

1	3	5	2	3	1	0	3	2	2	4	1
3	4	2	5	1	3	2	4	2	2	1	0
3	1	2	1	1	4	2	5	2	1	1	2

a Construct a frequency table using this data.

b How many households were surveyed?

c What is the most common number of TVs in a household?

d In total, how many TVs are in this survey?

10 Angus collected data and constructed the frequency table on the right.

a What has been surveyed?

b What type of data is this?

c What is the most popular category?

d What is the least popular category?

e How many people have been surveyed?

f Angus realises that the last piece of data he collected should have been rugby, not football. Adjust the frequency table so that it displays the data correctly.

Favourite sport	Frequency
Football	9
Basketball	10
Netball	8
Cricket	12
Soccer	8
Rugby	7
Tennis	6
Badminton	3

11 Consider this frequency table.

Favourite party food	Frequency
Party pies	5
Pizza	8
Cheezels	3
Fairy bread	
Mixed lollies	6
Cocktail frankfurts	3



a What type of data is this?

b The table is incomplete. If 30 people were surveyed, how many people prefer fairy bread?

c If you were to provide one item of food for a party, which would you choose and why?

12 This data was collected at a local school.

5	9	12	24	29	22	1	14
19	24	14	11	8	7	11	17
18	21	21	26	16	15	10	20

a What might the data be?

b Copy and complete this frequency table.

Number of _____	Frequency
0–4	
5–9	
10–14	
15–19	
20–24	
25–29	

13 The weights (in kilograms) of school bags of a Year 8 class were recorded for an investigation.

2.1	6.4	10.7	3.7	1.8	3.6	8.2	5.4
9.3	3.4	4.2	1.9	2.8	4.6	5.5	4.7
1.9	3.5	2.8	4.8	5.2	6.7	10.1	9.9



a What type of data is this?

b Arrange the data into a frequency table with appropriate class intervals.

c What can you say about the weights of these school bags?

PROBLEM SOLVING AND REASONING

14 What is wrong with the class intervals in the following tables? Adjust the class intervals and redistribute the frequencies if possible.

a

Number of pets in household	Frequency
0–9	19
10–19	1

b

Number of hours spent exercising per week	Frequency
0–5	9
5–10	6
10–15	3
15–20	1

c

Pocket money received each week ($)	Frequency
0–<2	2
2–<4	1
4–<6	3
6–<8	2
8–<10	3
10–<12	3
12–<14	2
14–<16	3
16–<18	1
18–<20	3
20–<22	2

d

Height of Year 8 students (cm)	Frequency
140–149	2
150–159	4
160–169	7
170–179	2
180–189	1

15 Position on a sports ladder (1st, 2nd, 3rd, etc.) is often mistaken for numerical, discrete data.

a What type of data is it?

b Why do you think people might make this mistake?

c What strategy can you suggest to help people who might make this mistake?

16 Collect your own data that can be displayed in a frequency table with class intervals. Write a few sentences describing what you find.

CHALLENGE

17 Think of at least two new examples of categorical, ordinal data that could be mistaken for numerical, discrete data.

18 Choose a topic to survey for which you can collect both numerical and categorical data. Collect data on at least two separate questions and present your results in a frequency table. Write a few sentences describing what you find.

Reflect

Why is it important that appropriate class intervals are chosen for frequency tables?

9C Presenting data in graphs

Start thinking!

Data is often displayed in a graph.

1 With a classmate, list as many different graphs as you can think of.

2 Two common graphs are the **column graph** and the **pie graph**.
   - **a** Name at least two differences between them.
   - **b** Which graph would you use to display frequencies (for example, from a frequency table)? Explain.
   - **c** Why couldn't the other option be used?
   - **d** How might your choice change to show the frequencies as percentages? Explain.

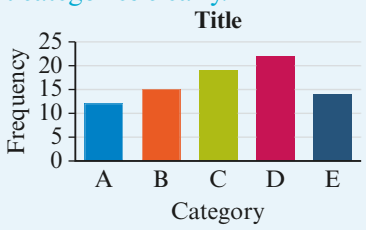
3 A **bar graph** is similar to a column graph.
   - **a** What is the difference between them?
   - **b** When might you use one and not the other?

4 What is the difference between a **line graph** and a **scatterplot**?

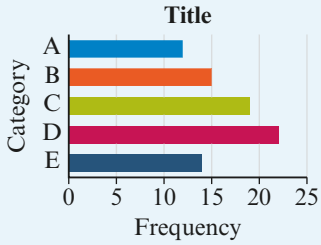
KEY IDEAS

- A good graph displays data clearly and must have:
  - a title
  - axes that are clearly labelled (including any units)
  - an even scale on both axes
  - a legend defining any colours used.

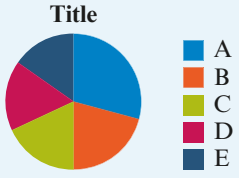
- A column graph shows the frequencies of different categories clearly.



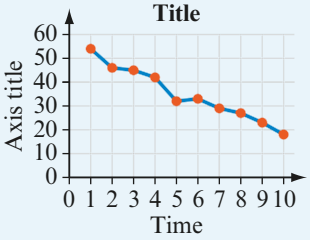
- A bar graph is used when category titles are too long for a column graph.



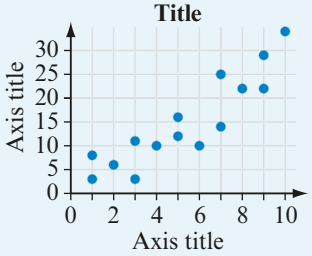
- A pie graph shows the relative size of each category, but does not display frequency.



- A line graph displays how a variable changes with respect to another variable (usually time).



- A scatterplot shows the relationship between two variables (for example, age and height).

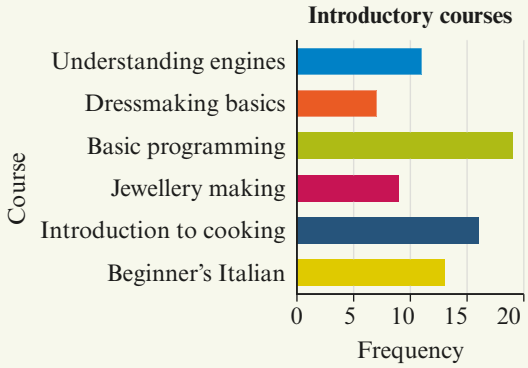


EXERCISE 9C Presenting data in graphs

EXAMPLE 9C-1 Reading graphs

Consider this bar graph.

a What is the most popular course and how many people chose it?
b How many courses are in the survey?
c How many people were surveyed?



THINK

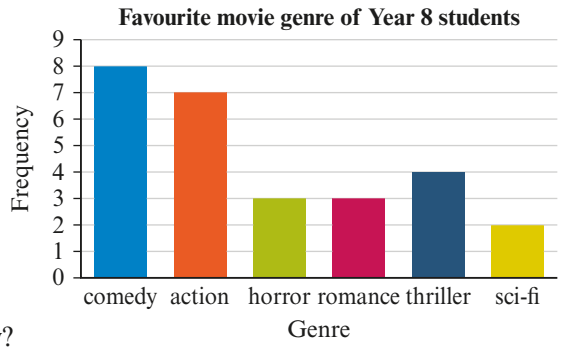
a Look at the longest bar. Look across to the vertical axis to find the category, and trace down from its end to the horizontal axis to find how many people chose it.
b Count the number of categories on the vertical axis.
c Trace down from each bar to find the frequencies and add them together.

WRITE

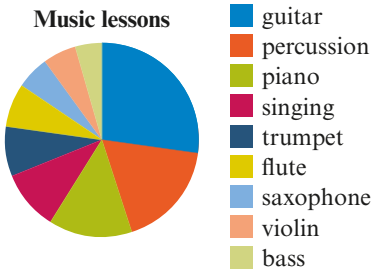
a The most popular course is Basic programming and 19 people chose it.
b There are six courses in the survey.
c 11 + 7 + 19 + 9 + 16 + 13 = 75
75 people were surveyed.

UNDERSTANDING AND FLUENCY

1 This column graph shows the results of a survey of Year 8 students and their favourite movie genre.
   a What is the most popular genre and how many students like it?
   b What is the least popular genre and how many students like it?
   c How many genres are in the survey?
   d How many students were surveyed?



2 The results of a survey are shown in this graph.
   a What type of graph is it?
   b What is it showing?
   c What is the most common category?
   d Can you tell how many people have been surveyed from this graph? Why or why not?



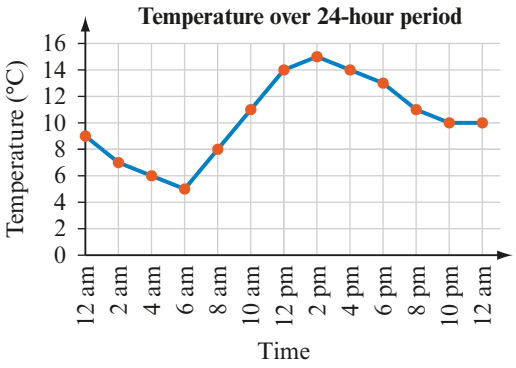
3 Consider this line graph.

a What does the graph show?

b What is the temperature at 10 am?

c When is the temperature 15°C?

d What is the coolest temperature recorded? At what time does it occur?



EXAMPLE 9C-2 Creating an appropriate graph

Create an appropriate graph to represent the data in the frequency table.

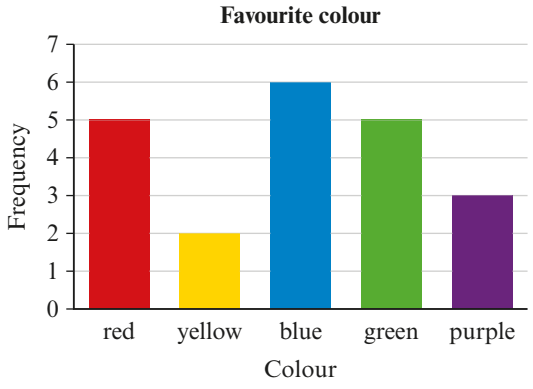
Colour	red	yellow	blue	green	purple
Frequency	5	2	6	5	3

THINK

1 A column, bar or pie graph could be used. A column graph is best as it displays frequency and is a little easier to read than a bar graph.

2 Draw and label a set of axes. The vertical axis has an even scale that covers the minimum and maximum frequencies. Include a title that reflects the data.

WRITE



UNDERSTANDING AND FLUENCY

4 Create an appropriate graph to represent the data in each frequency table.

a

Eye colour	Frequency
Blue	4
Brown	8
Green	6
Hazel	7
Other	3

b

Hobby	Frequency
Reading a book	2
Listening to music	6
Surfing the web	5
Riding a bike or skateboard	4
Watching TV	3
Talking to friends	8

c

Week	Snake length
1	13 cm
2	19 cm
3	26 cm
4	44 cm
5	56 cm
6	71 cm

d

Hours study	2	1	4	5	2	3	3	2	4	1
Maths test score	67	55	87	81	70	75	59	60	93	62

e

Hour	0	2	4	6	8	10	12	14	16	18	20
Candle height	21	20	18	17	15.5	14	13.5	11	8	3	1.5

f

Number of computers in a home	0	1	2	3	4	5
Frequency	2	6	9	14	10	7

5 The data in the table shows the number of people visiting a website over a 10-day period.

Day	1	2	3	4	5	6	7	8	9	10
Number of people	26	35	32	23	22	19	15	17	11	12

a Is this data best shown in a line graph or a scatterplot? Explain why and draw your chosen graph.

b Does the graph show a positive or negative trend? Look at the overall pattern of the graph.

6 This photo shows the contents of a bag of chocolates.



a Draw both a column graph and a pie graph to represent this data.

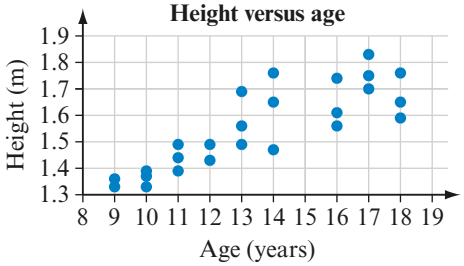
b What is the most common type of chocolate? How many were in this bag?

c What is similar about the two graphs?

d What is different about the two graphs?

e Which graph best represents the data? Explain.

7 Consider the following scatterplot.



a What does it show?

b Does it show a positive or negative trend?

c What is the height of a 13-year-old?

d What is the age of the person that is 174 cm tall?

e Predict the height of somebody who is 15 years old.

f Predict the height of somebody who is 30 years old.

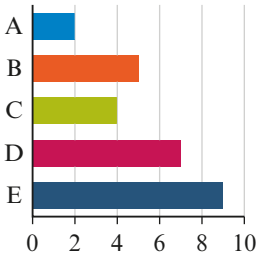
g Why might it not be entirely accurate to use the pattern in the graph to make the prediction for part **f**?

8 Use the data in this table to draw a scatterplot. Remember to label your axes and give the graph a title.

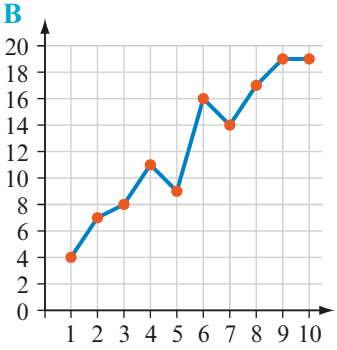
Temperature (°C)	10	15	30	20	5	8	10	17	26	11	29	22	7
Number of jackets sold	40	30	2	10	50	45	36	30	4	25	1	2	43

9 Match each graph to a table shown below.

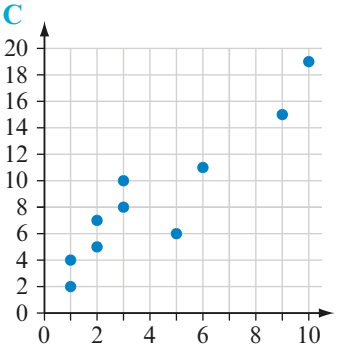
A



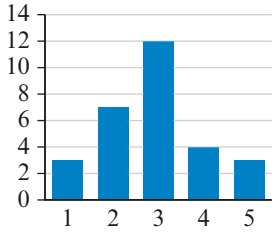
B



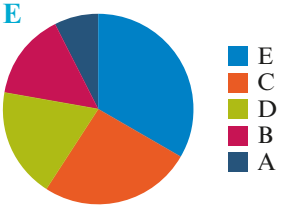
C



D



E



a

1	3
2	7
3	12
4	4
5	3

b

1	2	5	2	6	3	3	1	9	10
4	7	6	5	11	8	10	2	15	19

c

1	2	3	4	5	6	7	8	9	10
4	7	8	11	9	16	14	17	19	19

d

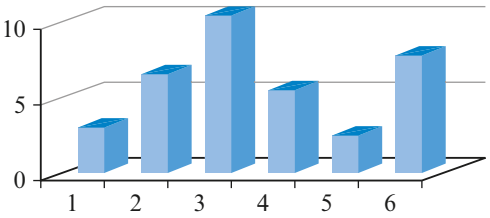
A	2
B	5
C	4
D	7
E	9

e

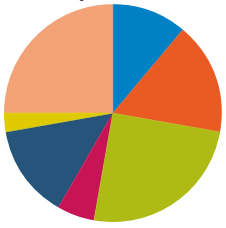
A	2
B	4
C	7
D	5
E	9

10 The graphs that follow are all incomplete, inaccurate or misleading in some way. Identify what needs fixing in each graph and redraw them accurately.

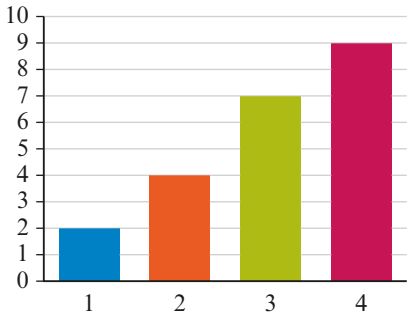
a



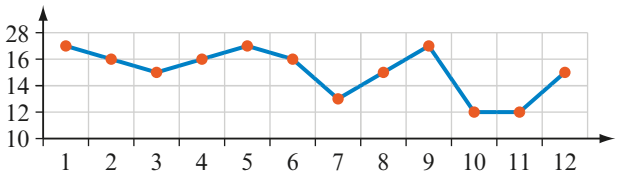
b



c



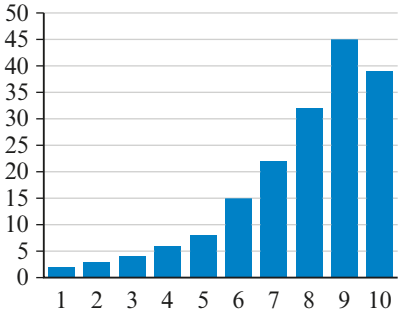
d



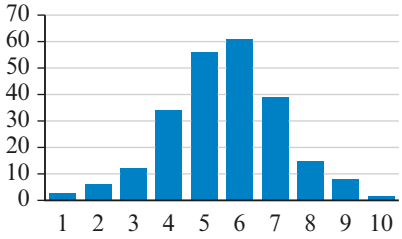
CHALLENGE

11 Graphs can also help you to describe the shape of the data distribution.

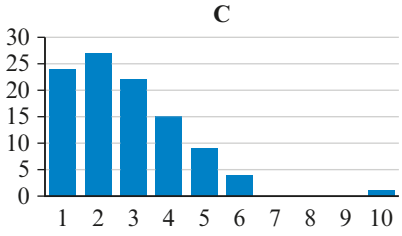
A



B



C



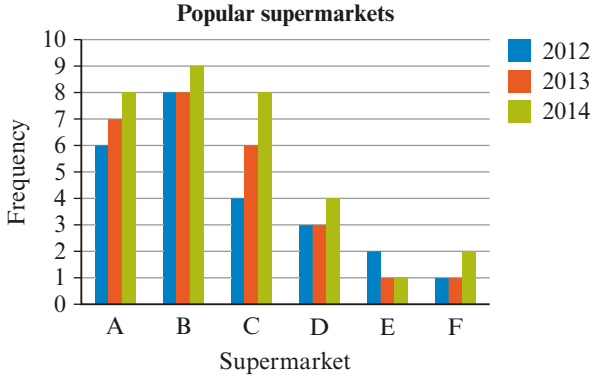
a Which of these three column graphs has an **outlier**? Explain.

b Which one of these graphs can be described as **symmetrical**? Why?

c Why are the other two graphs said to be **skewed**?

d Why could a roughly symmetrical graph still have an outlier?

12 Describe the column graph headed 'Popular supermarkets'.
What conclusions can be drawn from it?



Reflect

Why is it important that a graph be drawn and labelled clearly and accurately?

9D Stem-and-leaf plots and dot plots

Start thinking!

The **stem-and-leaf plot** and the **dot plot** are two types of display that plot every piece of data.

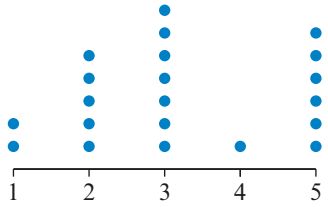


Figure A

1 Which figure is the stem-and-leaf plot? How do you know?

2 Write 20 numbers between 0 and 50 onto individual small pieces of paper. You can repeat numbers if you like.

3 Pair up with a classmate and decide who will create a stem-and-leaf plot and who will create a dot plot.

4 Trade your numbers with your partner and place the pieces of paper in the shape of your selected plot. For the dot plot, scrunch up the individual numbers to form dots, and for the stem-and-leaf plot, tear your numbers into stems and leaves.

5 Compare your plot to your partner's. How are they similar and how are they different?

6 Which plot is better suited to a large range of data? Why?

7 Why is the other plot not suited to a large range of data?

8 List two advantages and two disadvantages of presenting data in a stem-and-leaf plot and in a dot plot.

Key: 1 | 3 = 13

Stem	Leaf
1	3 7
2	2 2 4 6
3	4 6 8 8 8 9
4	1 3 5

Figure B

KEY IDEAS

- Dot plots and stem-and-leaf plots are two forms of presentation which display each piece of data.
- A dot plot has a horizontal axis with dots representing the data arranged vertically. This plot should only be used for data with small frequencies and that has a small range.
- A stem-and-leaf plot displays each piece of data in two parts: a stem and a leaf. The leaf is always a single digit and usually the last digit of the data value. The stem is formed from the remaining digits. This plot is best suited to data sets with large ranges.
- In a stem-and-leaf plot, the stem is written once in a column on the left, and then the leaves are placed on the right, in order from smallest to largest. A key must be included to show how the original numbers are represented.

EXERCISE 9D Stem-and-leaf plots and dot plots

EXAMPLE 9D-1 Creating a dot plot

Consider this data.
apple, orange, strawberry, pear, apple, peach, orange, strawberry, strawberry, orange, apple, banana, banana, apple, strawberry, banana, peach, banana, strawberry, banana, banana, apple, banana, strawberry, apple

a Create a dot plot to represent the data.
b What is the most popular fruit?
c How many people chose strawberry?
d How many people were surveyed?

THINK

a **1** Draw a horizontal axis long enough to fit all six categories and list the categories underneath the axis.

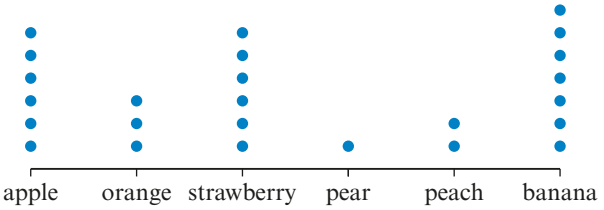
2 For each word in the list, place a dot above the appropriate category.

b Look at the fruit with the most dots.

c Count the number of dots above the category of strawberry.

d Count all the dots on the dot plot.

WRITE



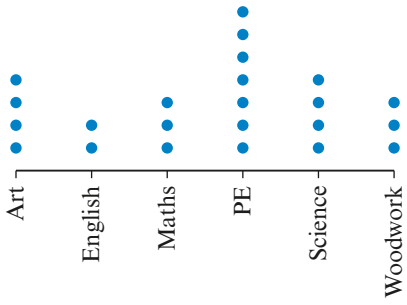
b Banana is the most popular fruit.

c Six people chose strawberry.

d 25 people were surveyed.

UNDERSTANDING AND FLUENCY

1 This dot plot shows the popularity of subjects taken by a Year 8 class.



a What is the most popular subject? How many students voted for this?
b What is the least popular subject? How many students voted for this?
c How many students were surveyed?
d Two new students join the class.
Their favourite subjects are Art and Maths.
Redraw the dot plot so that it shows this additional data.

2 A group of students were asked to name their favourite mythical creature. Use this data to construct a dot plot.

vampire, unicorn, vampire, leprechaun, dragon, dragon, elf, fairy, mermaid, werewolf, fairy, vampire, mermaid, elf, elf, vampire, dragon, fairy, vampire, werewolf, elf, vampire, fairy, mermaid, dragon, vampire, vampire, unicorn, vampire

UNDERSTANDING AND FLUENCY

3 Consider this data.

Fruit Loops, Weet-Bix, Special-K, Cornflakes, Fruit Loops, Fruit Loops, Rice Bubbles, Special K, Coco Pops, Coco Pops, Rice Bubbles, Fruit Loops, Rice Bubbles, Coco Pops, Weet-Bix, Coco Pops, Rice Bubbles, Fruit Loops, Coco Pops, Coco Pops, Rice Bubbles, Special K, Fruit Loops, Coco Pops



a Draw a dot plot to represent this data.
b What does the dot plot show?
c What is the most popular category?
d How many people chose Fruit Loops?
e How many categories are there?
f How many people were surveyed?

EXAMPLE 9D-2 Creating a stem-and-leaf plot

Consider this data.
12, 7, 35, 22, 19, 4, 16, 33, 32, 19, 11, 21, 45, 57, 42, 27, 27, 19, 13, 55, 31, 14, 15, 29

a Create a stem-and-leaf plot.
b What is the most common stem?
c What is the most common value?
d How many data values are there?

THINK

a **1** The minimum value is 4 and the maximum value is 57, so draw stems ranging from 0 to 5.

2 Place each piece of data into the plot. Start with 12, which has a stem of 1 and a leaf of 2. Write the digit 2 in the stem 1 row. Continue until all values have been considered.

3 Rearrange the leaves so they are in order. Include a key.

b Which stem has the most values?

c Which value is repeated most? Remember that a value consists of a stem and a leaf.

d Count the number of data values in the leaf section of the plot.

WRITE

Stem	Leaf
0	7 4
1	2 9 6 9 1 9 3 4 5
2	2 1 7 7 9
3	5 3 2 1
4	5 2
5	7 5

Key: 1 | 2 = 12

Stem	Leaf
0	4 7
1	1 2 3 4 5 6 9 9 9
2	1 2 7 7 9
3	1 2 3 5
4	2 5
5	5 7

b The most common stem is 1.

c The most common value is 19.

d There are 24 values.

UNDERSTANDING AND FLUENCY

4 Consider this stem-and-leaf plot.

Key: 1 | 1 = 11

Stem	Leaf
1	1 5
2	2 2 5 7
3	3 3 4 5 6 8 8
4	1 9 9 9
5	
6	1

- **a** What is the minimum value?
- **b** What is the maximum value?
- **c** What is the most common stem?
- **d** What is the most common value?
- **e** Why is the stem '5' included, when there is no value in that row?
- **f** How many data values are there?

5 Create a stem-and-leaf plot using this data.

11, 34, 23, 17, 46, 22, 29, 46, 51, 29, 13, 54, 37, 27, 42, 36, 71, 28, 25, 35, 48, 19, 17, 33, 38, 41, 11

6 Consider this data.

42, 39, 5, 2, 47, 39, 63, 42, 49, 55, 73, 75, 39, 48, 50, 67, 38, 59, 44, 74, 33, 40, 55, 62, 39, 46, 57, 71

- **a** Draw a stem-and-leaf plot to represent this data.
- **b** What is the most common value?
- **c** What is the range of data?
- **d** How many data values are there?

EXAMPLE 9D-3 Choosing a plot

Would you choose a dot plot or stem-and-leaf plot to represent this data? Explain your choice.
23, 12, 13, 15, 17, 44, 32, 14, 17, 10, 19,
22, 23, 37, 48, 52, 23, 42, 19, 16, 19

THINK

A dot plot best suits data that has low frequencies and a small range. A stem-and-leaf plot best suits data that has a larger range. What is the range of the data set?

WRITE

A stem-and-leaf plot would be most appropriate because there is a large range of numbers.

7 For each data set, choose and create either a stem-and-leaf plot or a dot plot. Explain your choice.

- **a** 45, 46, 39, 41, 46, 46, 42, 41, 43, 44, 47, 42, 42, 45, 42, 39, 42, 44, 42, 46, 41
- **b** 19, 26, 27, 12, 36, 38, 41, 18, 18, 18, 66, 45, 38, 22, 11, 16, 15, 14, 13, 29, 30, 40
- **c** 77, 81, 76, 75, 62, 76, 80, 53, 79, 77, 77, 75, 82, 72, 75, 74, 74, 74
- **d** 55, 46, 6, 55, 46, 32, 32, 33, 39, 41, 58, 42, 47, 50, 49, 53, 56, 39, 30, 46, 40, 55, 55, 42, 41, 43



8 Look at this photo.

- **a** Draw a dot plot to represent the different types of balls shown in the photo.
- **b** What is the most common type of ball?
- **c** What is the least common type of ball?
- **d** How many balls are there in total?
- **e** How else might you categorise the balls?
- **f** Draw another dot plot to represent your answer to part **e**.

9 Emma drew this stem-and-leaf plot.
What advice can you give to improve the plot?

Stem	Leaf
1	4 8 9
2	0 0 4 8
3	4 5 8 3 5 3 1
5	5 8 1 9
6	2 3 2
9	6

10 Consider this stem-and-leaf plot.

Key: 12 | 5 = 125

Stem	Leaf
11	0 0 6 7
12	1 1 2 5 5 8 8
13	0 1 3 5 7 9 9
14	2 5 7 7
15	1

- **a** Use the data to create a frequency table with appropriate class intervals.
- **b** Which is a better representation of the data? Explain.

11 Consider this stem-and-leaf plot.

Key: 3 | 1 = 31

Stem	Leaf
0	1 4 5 5
1	2 3 6 8
2	4 4 4 4 4 7
3	1 5

- **a** Draw a dot plot that represents the data, using each stem as a category.
- **b** Draw a dot plot that represents the data, using the individual pieces of data.
- **c** Which of these three plots is a better representation of the data? Explain.

12 This **back-to-back stem-and-leaf plot** shows the number of visitors per hour to two different stores.

Key: 1 | 4 = 14

Leaf Jetty Surf	Stem	Leaf Jay Jay's
8 4	1	0 2 5 7 8 8
9 9 3 1 0	2	1 1 4 6 7 7 8 9
8 6 6 5	3	0 2 2 2 5 8 9 9 9
9 9 6 4 3 2	4	0 1 5
9 9 7 7 6 5 4 3	5	
2 1	6	
	7	8

- **a** How many sets of data does this plot show?
- **b** How is reading the left side slightly different from reading the right side?
- **c** What is the most visitors to visit Jetty Surf in an hour?
- **d** What is the most visitors to visit either store in an hour?
- **e** Which store had the quietest hour? How many people came in this time?

13 Create a back-to-back stem-and-leaf plot to represent this data, which lists the number of hours spent listening to music per week.

Girls: 17, 35, 3, 11, 18, 22, 30, 5, 7, 14, 22

Boys: 41, 0, 6, 11, 29, 31, 17, 15, 20, 15, 14



PROBLEM SOLVING AND REASONING

14 Consider this split stem-and-leaf plot.

Key: 1 | 0 = 10

Stem	Leaf
1	0 1 1 4
1*	5 5 7 9 9
2	0 0 3 3 4 4
2*	5 6 8

a How is this plot different from the basic stem-and-leaf plots you have seen?

b What is the difference between the stem 1 and the stem 1*? (Hint: look at the values of the leaves.)

c What is the largest value in the plot?

d What is the smallest value in the plot?

e The value 18 needs to be added to the plot. To which stem does this value belong?

f Redraw the stem-and-leaf plot to include 18.

15 Marion collected data and created this stem-and-leaf plot. She notices that it is hard to see a pattern. Redraw this as a split stem-and-leaf plot. What do you notice?

Key: 3 | 0 = 30

Stem	Leaf
3	0 0 1 2 3 3 3 6 6 7 7 8 8 9 9 9 9
4	0 0 0 0 1 2 2 2 2 2 3 4 4 5 6 8

16 **a** Create a split stem-and-leaf plot with five stems to represent this data.

11 14 19 18 15 15 12 12 17 19 12 10 10 14 17 13 13 19
15 17 15 15 14 13 12 10 19 17 13 18 11 12 16 13 14 10

b Redraw the data as a dot plot. Which plot suits the data better? Explain.

17 How do you decide if a data set is better suited to a dot plot or a stem-and-leaf plot? Consider split stem-and-leaf plots in your answer.

18 This stem-and-leaf plot is missing its key.

Stem	Leaf
1	1 4 8
2	2 5 5 6 9
3	1 1 2 4 5 6 6 7
4	4 4 8 9
5	1 2 3
6	0 0 1

For each key listed below:

i write the value of the lowest number

ii write the value of the highest number

iii add a number of your choice to the plot, explaining how it fits in where you place it.

a Key: 1 | 1 = 11 **b** Key: 1 | 1 = 1.1

c Key: 1 | 1 = 1100 **d** Key: 1 | 1 = 110

CHALLENGE

19 Create a back-to-back stem-and-leaf plot with split stems to represent this data.

Ages of students watching school bands

Band A: 16, 13, 15, 17, 13, 14, 12, 14, 13, 14, 15, 13, 13, 13, 14, 12, 15, 17, 12, 13, 15, 13, 14

Band B: 13, 15, 16, 17, 16, 15, 16, 16, 17, 18, 15, 16, 17, 16, 15, 16, 16, 17, 16, 16, 17, 15, 16

Reflect

What are some advantages and disadvantages of using a stem-and-leaf plot rather than a dot plot to display data?

9E Presenting grouped data

Start thinking!

Consider figures A and B.

1 How are they different?

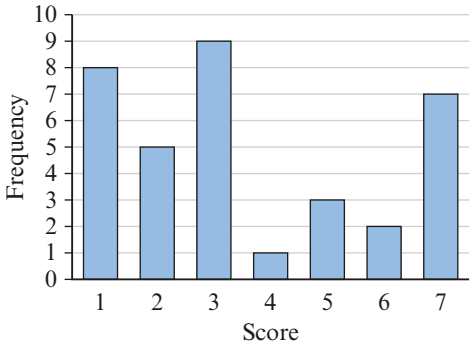


Figure A

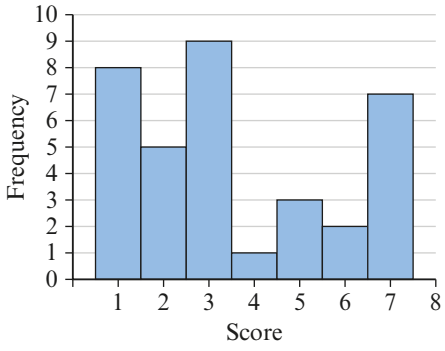


Figure B

A **histogram** is a special kind of column graph that is only used for numerical data.
There are no gaps between columns, but there is a small gap between the vertical axis and the first column.

2 Which figure is a histogram?

A histogram often displays grouped data.
Consider figure C.

3 How is this histogram different from figure B?

4 What is the size of the class interval?

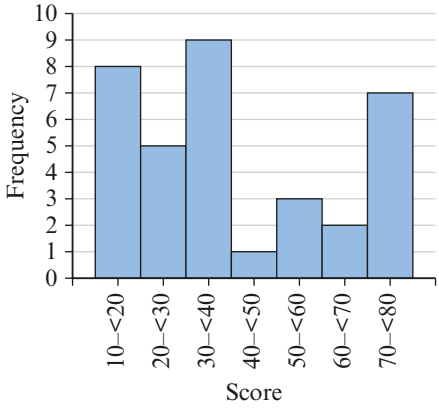
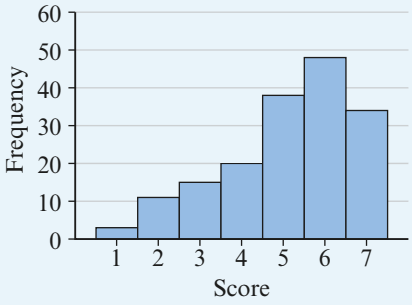


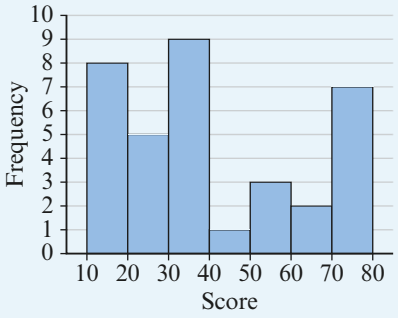
Figure C

KEY IDEAS

- A histogram is a special kind of column graph that is only used to represent numerical data.
- A histogram has no gaps between columns, but there is a small gap between the vertical axis and the first column.
- Histograms are often used to display grouped data.
- Category marks are either in the centre of the column (ungrouped data) or on the edges of the column (grouped data).



Ungrouped data

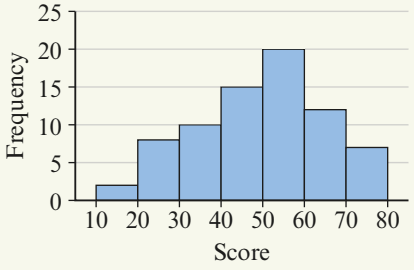


Grouped data

EXERCISE 9E Presenting grouped data

EXAMPLE 9E-1 Understanding histograms

Consider this histogram.



a What is the size of the class interval?
b What is the frequency of class 30–<40?
c What is the most common class? What frequency does it have?
d Which class has a frequency of 15?

THINK

a Look at the horizontal axis. What is the interval between each mark?

b Look at the column in between the marks 30 and 40. What frequency does this reach?

c Look for the tallest column. Use the axes to find the class interval and its frequency.

d Look at the column that has a frequency of 15 and then look at the horizontal axis to find which class it is.

WRITE

a The class interval is 10.

b The class 30–<40 has a frequency of 10.

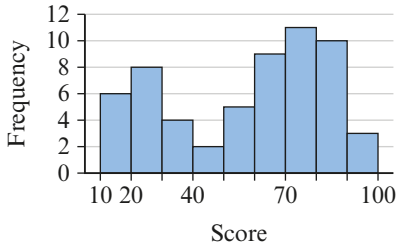
c The most common class in 50–<60 and it has a frequency of 20.

d The class interval with a frequency of 15 is 40–<50.

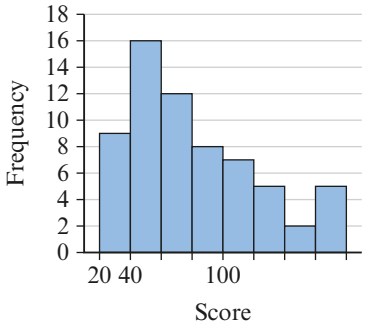
UNDERSTANDING AND FLUENCY

1 Copy these histograms and complete the labels on the category marks.

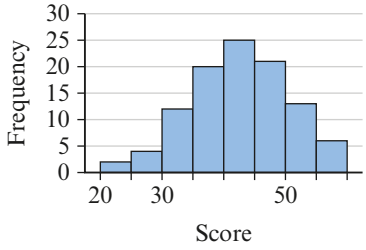
a



b



c



UNDERSTANDING AND FLUENCY

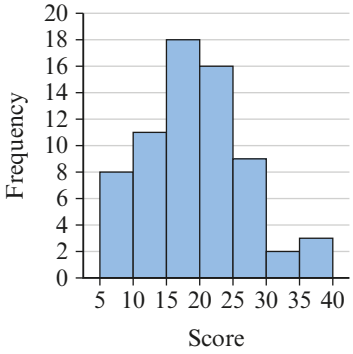
2 Consider this histogram.

a What is the size of the class interval?

b What is the frequency of class 20–<25?

c What is the most common class? What frequency does it have?

d Which class has a frequency of 9?

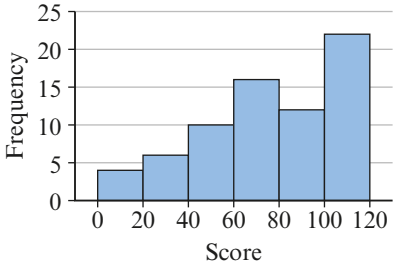


3 Consider this histogram.

a How many classes are there?

b What is the least common class? What frequency does it have?

c How many people were surveyed?



EXAMPLE 9E-2 Drawing histograms

Draw a histogram to represent the data in the table.

Class interval	Frequency
10–19	6
20–29	11
30–39	7
40–49	2

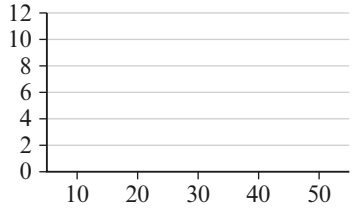
THINK

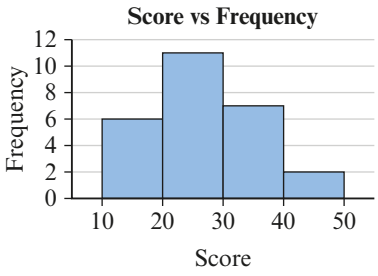
1 Draw a set of axes that allows for four columns horizontally and the maximum frequency (11) vertically.

2 Draw an even scale that includes the minimum and maximum values on the vertical axis and mark in the category values on the horizontal axis. Leave a gap between the first column and the vertical axis.

3 Complete the histogram. Label both axes and give the graph a title.

WRITE





UNDERSTANDING AND FLUENCY

4 Draw a histogram to represent the data in each table.

a

Class interval	0–<5	5–<10	10–<15	15–<20	20–<25
Frequency	2	5	8	10	1

b

Class interval	0–<10	10–<20	20–<30	30–<40	40–<50	50–<60
Frequency	3	9	15	13	11	7

c

Class interval	0–<4	4–<8	8–<12	12–<16	16–<20	20–<24
Frequency	26	37	38	45	54	49

d

Class interval	20–<40	40–<60	60–<80	80–<100
Frequency	18	12	8	10

5 Data was collected on the ages of people on a train carriage one day.

16 22 19 28 25 14 18 19 21
32 27 22 14 15 15 23 22 17
14 18 26 24 39 31 20 15 13
16 16 24 29 30 19 20 21 21

a Arrange this data in a frequency table.

b Draw a histogram to represent this data.



PROBLEM SOLVING AND REASONING

6 Draw a histogram that appropriately represents this data.

12.1 18.4 10.9 13.5 13.2 14.7 15.2 16.9 11.7 11.4 10.4
11.0 17.9 15.5 15.1 14.8 13.7 13.4 17.2 18.9 12.4 15.8
11.9 12.5 12.9 13.4 13.8 14.6 10.8 13.0 16.0 11.8 14.7

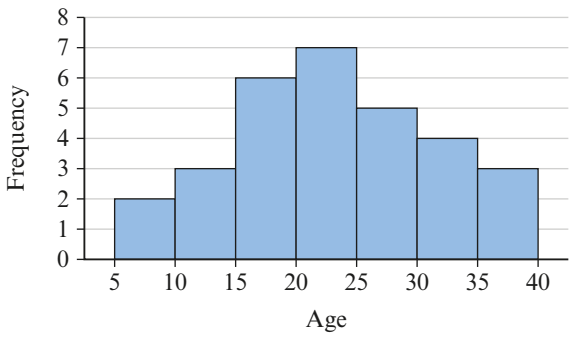
7 Consider this histogram.

a What does it show?

b What is the most common class? What is its frequency?

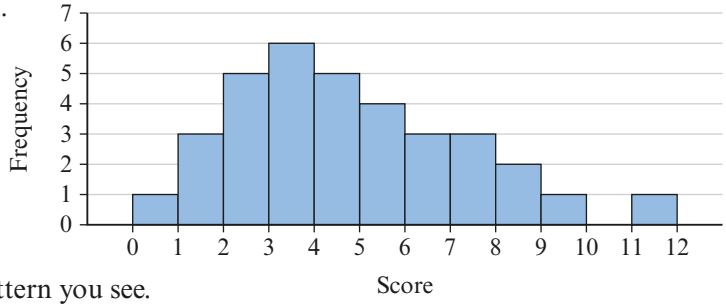
c What is the size of the class interval?

d The piece of data '25' needs to be added to the graph. Which class would you add it to? Explain.





8 Consider this histogram.



a Is the data grouped or ungrouped?

b What is the most common category?

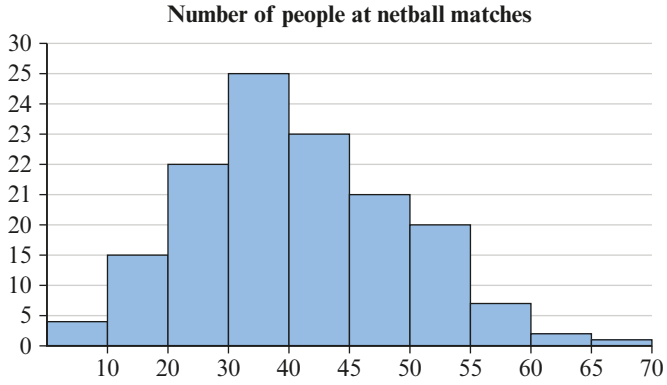
c Comment on any pattern you see.

d Redraw the histogram with class intervals of 3.

e What is the most common class in the redrawn histogram?

f Comment on any pattern you can see. Is it different from the pattern you saw previously?

9 What is wrong with this histogram?



10 Redraw the histogram from question **9** so that it is accurate and use it to answer these questions.

a What is this histogram showing?

b What is the most common class interval? What value does it have?

c What might this maximum value mean?

d How many people were surveyed?

e Describe what you think the histogram represents.

11 This data was collected on how many thousands of people were present at rugby matches at Suncorp Stadium, Brisbane.

10	51	35	22	25	46	34	21	8
28	11	19	22	31	38	42	22	22
48	30	25	26	15	21	18	18	41
39	33	33	38	25	28	10	17	13



a Create a histogram with class intervals of 5 to represent this data.

b Create a histogram with class intervals of 10 to represent this data.

c What is similar and what is different about the graphs you drew for parts **a** and **b**?

PROBLEM SOLVING AND REASONING

12 Why might it be easy to construct a histogram from a stem-and-leaf plot?

13 Follow the steps below to construct a histogram on heights of your classmates.

- **a** Create a frequency table with appropriate class intervals. Remember that you will need to have about 5–10 classes. What size class interval might make this easiest?
- **b** Collect data from your classmates to fill in the frequency table.
- **c** Use your frequency table to construct a histogram.
- **d** What is the most common height class in your class?
- **e** If a new student had a height of 172 cm, which height class would they belong in?
- **f** Would you say they were short, average or tall for your class?

14 Conduct a new survey on your class and construct a histogram to show your results. Write a couple of sentences to describe what you find.

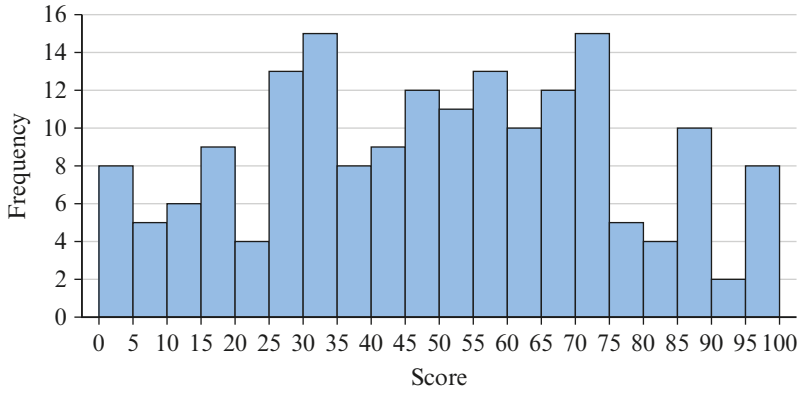
CHALLENGE

15 A friend collected the data in the table on the right. Help them draw a histogram that is easy to read.

Class	Frequency
0–<2	2
2–<4	3
4–<6	9
6–<8	6
8–<10	7
10–<12	3
12–<14	4
14–<16	7
16–<18	8
18–<20	7
20–<22	10
22–<24	10
24–<26	9
26–<28	16
28–<30	16
30–<32	14
32–<34	4
34–<36	6
36–<38	6
38–<40	4

16 Consider the histogram below.

- **a** What is the size of the class interval?
- **b** What can you say about the number of classes? Is it easy to read?
  Can you see a pattern?
- **c** Redraw the histogram with larger class intervals.
- **d** What patterns can you see now?



Reflect

What makes a histogram different from a column graph?

9F Summary statistics



Start thinking!

Alyson asked her classmates how many pets they own. Her results are below.

1 0 1 0 1 2 2 2 1 5 0 1 2 1 2 2 3 1 1 2

1 Arrange the data in order. How many pieces of data are there?

The three measures of centre are the **mean** (average), **median** (middle number) and **mode** (most common number).

2 Find the mean and mode for this data.

To calculate the median of a data set with an even number of pieces, find the average of the two middle numbers or **scores**.

3 Find the median of this data set.

You can calculate the median of any data set with this formula:

median = $\frac{n+1}{2}$th score, where n is the number of data pieces in the set.

In this example, the median is the $\frac{20+1}{2} = 10.5$th score.

This means the median is halfway between the 10th and 11th scores.

To find the measures of centre, it is often easier to put the data into a frequency table first.

4 Create a frequency table to represent the data. Include a 'total' row.

KEY IDEAS

- To find the mean (average), add all the scores together and divide by the number of scores.
- The median is the middle score of the set (when the scores are in order).
  median = $\frac{n+1}{2}$th score, where n is the number of scores in the set.
- The mode is the most common score in the set. There can be more than one mode.
- One measure of spread is the **range**. This is the difference between the largest and smallest scores.
- To calculate the mean from a table, create a column multiplying the score by the frequency. The total of this column is divided by the total of the frequencies.
- To calculate the median from a table, create a column to record the total of the frequencies as they are added (the **cumulative frequency**). The row containing the $\frac{n+1}{2}$th score is then easily identified.
- The mode (the score with the highest frequency) and the range (the different between the highest and lowest scores) can be easily read from a table of data.
- Measures of centre and spread are known as **summary statistics**.

EXERCISE 9F Summary statistics

EXAMPLE 9F-1 Finding summary statistics for raw data

Find the mean, median, mode and range for this data set.
8, 4, 1, 2, 9, 2, 4, 13, 2

THINK	WRITE
1 To find the mean, add all the numbers together and divide them by how many numbers there are.	$\frac{8+4+1+2+9+2+4+13+2}{9} = 5$ The mean is 5.
2 To find the median, rearrange the numbers in order and circle the middle number.	1, 2, 2, 2, (4), 4, 8, 9, 13 The median is 4.
3 To find the mode, look at the ordered number list and state the most common number(s).	The mode is 2.
4 To find the range, subtract the lowest number from the highest number.	13 − 1 = 12 The range is 12.

UNDERSTANDING AND FLUENCY

1 Consider this data set: 5, 8, 7, 11, 4, 9, 5.
- **a** Write the numbers in order from smallest to largest.
- **b** What is the mean (average) of the data set?
- **c** What is the median (middle) of the data set?
- **d** What is the mode (most common) of the data set?
- **e** What is the range of the data set?

2 Find the mean, median, mode and range for each set of data.
- **a** 11, 19, 18, 12, 12
- **b** 2, 8, 4, 9, 7, 6, 2, 4, 1
- **c** 8, 7, 3, 5, 7, 7, 4, 8, 6, 0, 9, 7

3 Students in a Year 8 class were asked the megapixel size of their cameras. The results are:
7.1, 12.1, 10.0, 12.1, 14.1, 8.0, 14.1, 12.1, 10.0, 14.1, 14.1, 14.1, 10.0, 8.0, 10.0, 12.1, 14.1, 14.1, 12.1, 14.1, 15.1, 10.0



- **a** Write the scores in order.
- **b** What is the mean number of megapixels?
- **c** What is the modal number of megapixels?
- **d** What is the median number of megapixels?
- **e** What is the range in number of megapixels?

EXAMPLE 9F-2 Finding the mean from a data table

Find the mean for the data shown in this table.

Score (x)	Frequency (f)
0	4
1	6
2	8
3	7
Total	25

THINK

1 Add a 'score × frequency' column to the table and complete it.

2 Divide the two totals to find the mean.

WRITE

Score (x)	Frequency (f)	Score × frequency
0	4	$0 \times 4 = 0$
1	6	$1 \times 6 = 6$
2	8	$2 \times 8 = 16$
3	7	$3 \times 7 = 21$
Total	25	43

mean = $43 \div 25 = 1.72$

UNDERSTANDING AND FLUENCY

4 Find the mean for the data shown in each table.

a

Score (x)	Frequency (f)
1	9
2	11
3	6
4	4
Total	30

b

Score (x)	Frequency (f)
2	9
3	6
4	5
5	4
Total	24

c

Score (x)	Frequency (f)
10	4
20	8
30	6
40	2
Total	20

d

Score (x)	Frequency (f)
1	4
2	8
3	6
4	2
5	5
Total	25

e

Score (x)	Frequency (f)
13	2
14	4
15	1
16	2
17	1
Total	10

f

Score (x)	Frequency (f)
0	8
10	12
20	20
30	6
40	4
Total	50

EXAMPLE 9F-3 Finding the median from a data table

Find the median for the data in the table in Example 9F-2.

THINK

1 Find the $\frac{n+1}{2}$th score. There are 25 scores, so $n = 25$.

2 Add a cumulative frequency column to the table.

3 The 13th score is in the row containing scores of 2.

WRITE

$\frac{25+1}{2} = \frac{26}{2} = 13$; median = 13th score

x	f	Cumulative frequency
0	4	4
1	6	4 + 6 = 10
2	8	10 + 8 = 18
3	7	18 + 7 = 25
Total	25	

median = 2

5 Find the median for the data shown in each table in question **4**.

EXAMPLE 9F-4 Finding the mode and the range from a data table

For the data in the table in Example 9F-2, find: **a** the mode **b** the range.

THINK

a The mode is the score with the highest frequency (8). Which score has this frequency?

b The range is the difference between the highest score and the lowest score. Read from the table.

WRITE

a The mode is 2.

b 3 − 0 = 3
The range is 3.

6 Find the mode for the data shown in each table in question **4**.

7 Find the range for the data shown in each table in question **4**.

8 A Year 8 class were asked how many Facebook quizzes they did last week. The results were:

0 1 6 3 5 1 2 0 0 2 1 3
2 1 0 0 1 1 2 18 2 3 5 3

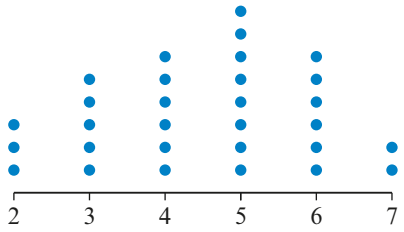
a Create a frequency table to represent the data.

b Find the range. **c** Find the mean, median and mode.

9 Calculate summary statistics for the data shown in this table.

Score (x)	Frequency (f)
11	4
12	8
13	16
14	17
15	13
16	4
17	2
Total	64

10 Describe the dot plot below by calculating its summary statistics.



11 Monica collected this data on the heights (in centimetres) of her classmates.

161 152 148 167 162 149 159 155 161 145 155 156
157 160 157 172 155 160 180 152 162 155 161 169

a Find the mean, median and mode for this data. **b** Find the range of this data.

c How can you explain the large spread of data found in this set?

12 Mario collected data on the heights (in centimetres) of his classmates.

152 176 159 161 148 153 158 169 143 148 149 155
154 157 158 161 162 255 168 147 152 152 153 150

a Find the mean, median and mode for this data. **b** Find the range of this data.

c How can you explain the large spread of data found in this set?

13 Although the mean is usually used as the measure of centre, sometimes it is not a good indicator. The presence of an outlier can affect the mean.

a What is an outlier?

b Explain why an outlier can affect the mean, using the comparison of the heights in Monica's and Mario's classes from questions **11** and **12** to help you.

c Would you use mean or median to describe 'average' height in Mario's class? Explain.

d Identify the outlier in Mario's class.

e Recalculate the mean for Mario's class without the outlier. How does this compare to the median?

f What is the range for Mario's class without the outlier? Compare to your answer for question **12b**.

g Whose class (Monica's or Mario's) would you say is taller on average?

14 What summary statistics can you perform on categorical data? Explain why you can't calculate all measures of centre.

15 This stem-and-leaf plot shows the ages of people at a café on a Saturday afternoon.

Key: 1 | 1 = 11

Stem	Leaf
1	1 5 9
2	2 4 6 8
3	0 1 3 3 8 8 9
4	1 5 5 5 6 9
5	2

a What is the range?

b Find the: **i** mode **i** median **iii** mean.

c How is calculating summary statistics from a stem-and-leaf plot different from calculating them from a raw list of data?

CHALLENGE

16 Georgina asked her classmates the ages of their siblings and gathered this data.

10, 13, 16, 16, 18, 7, 22, 3, 15, 12, 11, 10, 12, 16, 17, 15, 14, 15,
12, 11, 12, 10, 19, 4, 6, 9, 21, 15, 11, 12, 16, 16, 9, 10, 20, 5

a Find the mean, median, mode and range for this data.

A friend tells Georgina that it is much easier to display the data in a frequency table and then calculate the summary statistics. Georgina is worried that calculating statistics on grouped data might not be accurate.

b Construct a frequency table with class intervals of 5; that is, 0–<5, 5–<10, etc.

c What is the modal class? How does this relate to the mode of the raw data?

d What is the median class? How does this relate to the median of the raw data?

e Does the range change? Explain.

To find the mean class you need to find the **midpoint** of the class interval.

f What is the halfway point of the first class interval? (Hint: if you have class intervals of 5, what is half of this?)

g Add a column to the table titled 'midpoint' and enter the class midpoints.

h Add a column to the table titled 'midpoint × frequency' and fill it in.

i Find the sum of the 'midpoint × frequency' column and the 'frequency' column.

j Use these two numbers to find the mean. (Hint: you will need to divide one number by the other.)

k How does this mean relate to the mean you calculated from the raw data?

17 Calculate mean, median, mode and range for the data in this frequency table. You may like to look at parts **f**–**j** of question **16** to help you calculate the mean.

Class interval	Frequency
0–<5	1
5–<10	8
10–<15	16
15–<20	14
20–<25	10

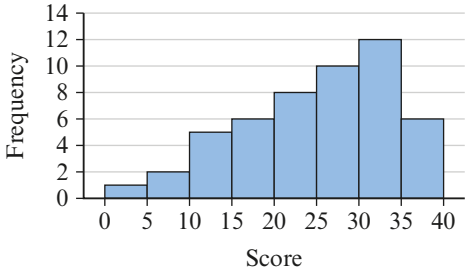
18 Consider this histogram.

a Construct a frequency table to represent the data.

b Find the range.

c Find the mean, median and mode.

d Which statistical measures are easy to find straight from the graph? Are there others you need the frequency table for? Explain.



19 Use the stem-and-leaf plot from question **15** to calculate summary statistics, but this time, treat the plot as having class intervals. What differences do you find? You may like to construct a frequency table to help you.

Reflect

Why might it be useful to calculate summary statistics for a data set?



9G Analysing data

Start thinking!

Laani reads that there are 2.5 people in an average Australian household. She decides to conduct a survey to see if her results match. She asks everybody that she knows at school and displays her results in this table.

Number of people in household	Frequency
2	6
3	19
4	38
5	20
6	14
7	2
8	1

1 Find the mean, median and mode for this data.

2 What is the population of the survey? What is the sample?

3 How many people has Laani surveyed?

4 Why might this sample not truly reflect the population?

5 How could Laani improve her sampling technique?

Laani repeats the survey, but this time she uses an unbiased technique. She surveys 20 people and finds an average of 2.9 people per household.

6 How is this different from her original sample?

7 Why might you still not be confident that this sample reflects the population? (Hint: how might an outlier affect this sample?)

In the end Laani surveys 200 people, finding an average of 2.6 people per household.

8 Why might Laani be more confident of her results with a larger sample?

9 As sample size increases, you can see that the sample mean more closely resembles the population mean. Do you think this would be the same for other summary statistics? Explain.

KEY IDEAS

- A sample can be used to make predictions about a population. To be confident of these predictions, a sample must be:
  - gathered using a fair and unbiased technique
  - large enough to represent the population even if there are outliers.
- If the measures of centre are very similar, this implies that there are no outliers and either the mean or median can be used to describe the centre.
- If the measures of centre vary, it is usually best to use the median to describe the centre. This is because, unlike the mean, it is unaffected by outliers.
- When describing a population or sample in real life, people use words such as:
  - 'average', which usually refers to the mean
  - 'most common', 'most popular', 'majority of' and 'most often', which usually refer to the mode
  - 'varied', 'ranged', and 'spanned', which usually refer to the range.

EXERCISE 9G Analysing data

EXAMPLE 9G-1 Identifying summary statistics

Identify which summary statistics are being used in these sentences.

a The majority of people surveyed disliked Collingwood.
b The levels of a chemical in the water varied by up to 20 g.

THINK

a The word 'majority' means 'most', so this refers to the mode.

b The word 'varied' refers to the spread of the data, which is the range.

WRITE

a mode

b range

UNDERSTANDING AND FLUENCY

1 Identify which summary statistics are being used in these sentences.
   a The average height for a 14-year-old is 162 cm.
   b The most common mp3 player is an iPod.
   c The temperature ranged from 8°C to 18°C.
   d The most popular music in the class was hip hop.
   e The majority of people preferred summer.
   f The times varied by up to 40 minutes.
   g The train is most often 2 minutes late.
   h Andrew is in the middle of his height range.

EXAMPLE 9G-2 Identifying the best sample

Lexi wanted to investigate ancestry in her local town. She takes two samples: one sample contains 200 people at one of the local supermarkets, the other sample contains 100 people randomly selected from the phone book. Which is the better sample?

THINK

1 First look at the sizes of the two samples. The first sample is bigger than the second, which generally leads to a better sample.

2 Next look at the sampling method. The first sample takes people from a local supermarket, which may bias results. The second sample has people surveyed randomly from the phone book, which minimises bias.

3 Write your answer.

WRITE

The second sample is better because although it is smaller, the results should not be biased.

2 Tanja wants to investigate the most popular sports team in South Australia. Which samples do you think would better reflect the population? Explain.

- **a** a sample of 20 people or a sample of 100 people
- **b** a sample of 619 people or a sample of 500 people
- **c** a sample of 20 randomly selected people or a sample of 40 students
- **d** a sample of 100 people in Australia or a sample of 50 people in South Australia

EXAMPLE 9G-3 Identifying the best measure of centre

Identify the best measure of centre for this data. Explain your answer.

Mean	Median	Mode	Range
15	10	11	50

THINK

1. Compare the three measures of centre. The mean is distinctly larger than the median.
2. Look at the range. Is it small or large?
3. The higher mean and the large range suggests that there may be one or more outliers. In this case, the best measure of centre is the median, as it is not affected by outliers.

WRITE

The large range and higher mean suggest an outlier. The best measure of centre is the median, as it is not affected by outliers.

3 Identify the best measure of centre for each data set. Explain your answer.

- **a** mean = 7, median = 8, mode = 7, range = 10
- **b** mean = 14, median = 17, mode = 21, range = 64
- **c** mean = 10, median = 8, mode = 8, range = 70
- **d** mean = 51, median = 50, mode = 50, range = 5

EXAMPLE 9G-4 Identifying population characteristics

Given this data, could you reasonably predict the characteristics of the population? Explain.

Population	Sampling technique	Sample	Mean	Median	Mode	Range
8D students (25)	Draw names out of a hat	20 students from 8D	4.4	4	4	8

THINK

1. Look at the population and sample. Although 20 would usually be a small sample, it is most of this population.
2. Look at the sampling technique. It is random, not biased.
3. Compare the measures of centre. All are similar.
4. Decide if you would be confident enough to use these results to predict the population characteristics.

WRITE

You can be fairly confident in using this sample to predict the population characteristics as the sample size is sufficient, the technique is random and the measures of centre are all similar.

UNDERSTANDING AND FLUENCY

4 Given the data in these tables, state whether you could reasonably predict the characteristics of the population. Explain your answer.

a

Population	Sampling technique	Sample	Mean	Median	Mode	Range
Your school	Asking your friends	30 of your friends	8	7	8	3

b

Population	Sampling technique	Sample	Mean	Median	Mode	Range
Australia	Random stratified sampling	1000 people in Canberra	16	16	14	20

c

Population	Sampling technique	Sample	Mean	Median	Mode	Range
Victoria	Systematic sampling	20 people in Victoria	8	3	3	26

d

Population	Sampling technique	Sample	Mean	Median	Mode	Range
Audience at a circus (200 people)	Stratified sampling	20 people at the circus	4	1	1	10

e

Population	Sampling technique	Sample	Mean	Median	Mode	Range
Your suburb	Systematic sampling	100 people in your suburb	9	8	9	3

PROBLEM SOLVING AND REASONING

5 Andon wants to know how often people go to the cinema in a month. He takes three different samples and records the results shown in the table. How many times on average would you expect a randomly chosen person in the population to go to the cinema in a month?

	Sample 1 (20 people)	Sample 2 (50 people)	Sample 3 (40 people)
Mean	1.5	3.5	2.2
Median	2	2	2
Mode	2	3	2
Minimum	0	0	0
Maximum	4	10	8



6 Chris wants to find the average number of mobile phones in Australian households. He decides to take a sample of 50 people. The results are shown in this frequency table.

Number of mobile phones	Frequency
0	1
1	1
2	11
3	8
4	16
5	9
6	4

a Find the mean, median and mode for this data.

b What might you need to consider before using this data to predict the population average?

c Use either the sample mean or median to estimate the average number of mobile phones in Australian households. Explain your decision.

d How many mobile phones are in your household? Add this to the table.

e How do you think this will affect the sample mean and median?

f Recalculate the sample mean and median. What difference do you notice?

7 Your school wants to hold a mini sports carnival but doesn't know what sports to offer. It decides to offer the three most popular sports. Staff take three different samples from around the school to get an idea of what sports they should offer. Use the results below to make a recommendation to the school, explaining why you chose those three sports.

Sample: 8A students (25 people)	
Sport	Frequency
Football	2
Rugby	3
Soccer	2
Cricket	6
Netball	4
Tennis	2
Badminton	5
Basketball	1

Sample: 20 students at tennis training	
Sport	Frequency
Football	0
Rugby	0
Soccer	2
Cricket	1
Netball	0
Tennis	17
Badminton	0
Basketball	0

Sample: 20 students randomly selected	
Sport	Frequency
Football	2
Rugby	1
Soccer	6
Cricket	4
Netball	4
Tennis	1
Badminton	0
Basketball	2

8 Abdullah wanted to know the average size of a TV screen. He asked a sample of 20 people and found that the average TV screen size was 48 inches (122 cm). He then asked another sample of 50 people and this time found a mean of 41 inches (104 cm).

a Why do you think there was a difference in the two averages?

Abdullah wants to use his findings to make a prediction about the population.

b Which sample mean would you recommend that Abdullah use to predict the population mean? Explain.

c A friend suggests to Abdullah that for such a small sample, he might prefer to use the median to predict the population average. Why is this?

d What advice would you give Abdullah about how sample size affects making predictions about a population?

PROBLEM SOLVING AND REASONING

9 When reporting the value of 'average' house sales, news reports usually use the median. Why do you think this is?

10 The media sometimes use statistics to falsely support their arguments; for example, using the mean as a centre measure when they should use the median, or using poor sampling techniques.

- **a** Research and find two examples of statistics used misleadingly by media.
- **b** Describe why the statistics are misleading.
- **c** Describe what should have been done to make the statistics or reporting more accurate.

11 How do you think culture and society affect what the media presents? Investigate and report on a situation to support your answer. For example, how do you think the Japanese media might present data on whaling compared to how the Australian media present it?

12 In Exercise 9C question **11**, you described the shape of data distributions as either symmetrical or skewed. Describe the data below as either symmetrical or skewed. You may like to imagine each data set drawn as a column graph.

a

Score	1	2	3	4	5	6	7	8	9	10
Frequency	4	6	10	15	24	29	17	11	2	1

b

Score	10	11	12	13	14	15	16	17	18	19
Frequency	25	64	53	32	16	10	4	2	1	1

c

Score	5	10	15	20	25	30	35	40	45	50
Frequency	10	11	15	16	18	20	23	85	89	93

d

Score	10	20	30	40	50	60	70	80	90	100
Frequency	6	8	12	16	11	9	5	0	0	1

CHALLENGE

13 You already know that if a data set has an outlier, the median is a better measure of centre than the mean.

- **a** Why is this?
- **b** Why does it not matter which measure of centre you use to describe a symmetrical data distribution?
- **c** Why is it best to use the median to describe the centre of a skewed data distribution?

14 For each data distribution listed in question **12**, identify which measure (mean, median or mode) is best used to describe the centre of the set.

Reflect

What must you consider before making a prediction about a population from a sample?

CHAPTER REVIEW

SUMMARISE

Create a summary of this chapter using the key terms below. You may like to write a paragraph, create a concept map or use technology to present your work.

population	numerical, discrete data	title	histogram
sample	numerical, continuous data	axes	mean
biased	categorical, nominal data	scale	median
systematic sampling	categorical, ordinal data	legend	mode
stratified sampling	column graph	outlier	range
random sampling	pie graph	symmetrical	score
strata	bar graph	skewed	cumulative frequency
frequency table	line graph	stem-and-leaf plot	midpoint
class intervals	scatterplot	dot plot	

MULTIPLE-CHOICE

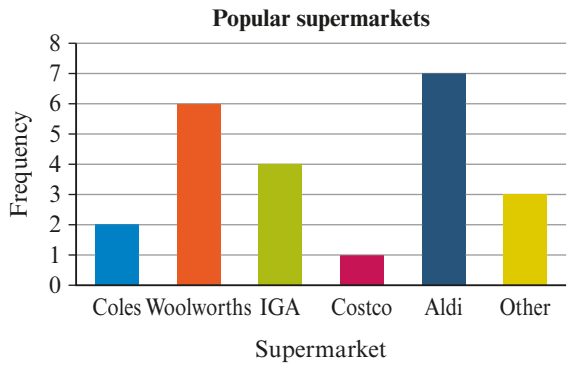
9A **1** Which of these is stratified sampling?

A selecting every third person who walks past
B drawing names out of a hat
C surveying 15 people in a class of 25
D surveying 3 boys and 2 girls out of a group of 12 boys and 8 girls

9B **2** What type of data is the number of movies people see in a month?

A discrete **B** continuous
C nominal **D** ordinal

Questions **3** and **4** refer to this figure.



9C **3** What is the least popular category?

A Coles **B** Woolworths
C Costco **D** Other

9C **4** How many people were surveyed?

A 25 **B** 23 **C** 7 **D** 6

9D 9E **5** A data set contains 50 pieces of data spread over a range of 65. Which display would be best to use?

A histogram **B** dot plot
C stem-and-leaf plot **D** scatterplot

9F **6** For 1, 5, 4, 3, 2, 5, 5, 3, 4, 8, the mean is:

A 5 **B** 4 **C** 7 **D** 8

9G **7** Belinda investigates her local town's opinion on a current issue. Which of these is the best sample to use?

A Asking everybody in the entire town (population 1500)
B Surveying every fourth person at the local mall until 200 are surveyed
C Asking everybody she knows
D Surveying every 10th person in the phone book until 100 are surveyed

SHORT ANSWER

9A **1** For the following surveys, identify:
- i if it is a census or sample
- ii if the sampling technique is fair or biased, providing reasons. If it is biased, provide a fair alternative.

a Liana asks her class members who their favourite celebrity is to find the most popular celebrity in the state.

b Joe asks every fifth person on the school enrolment list how many people are in their family to find the average number of people in a family at his school.

c Gerard asks everybody in his class if they like chocolate to find out the most popular snack in his class.

9B **2** Construct a frequency table with appropriate class intervals to represent this data.

5	16	8	11	1	18	21	23	15
14	14	11	9	3	6	12	18	21
20	10	14	13	15	24	34	7	7
19	18	22	11	5	1	2	9	11

9C **3** Use this data to draw an appropriate graph.

Reebok, Nike, Asics, Nike, Adidas, Puma, Brooks, Brooks, Brooks, Nike, Asics, Nike, Asics, Nike, Adidas, Brooks, Converse, Adidas, Asics, Reebok, Converse, Asics, Nike, Brooks, Puma, Asics, Asics, Converse, Converse, Asics, Converse, Nike, Converse, Asics

9D **4** Draw a dot plot for the data in question **3**.

9D **5** Consider this data.

15	23	28	73	11	9	45	32	11
7	35	22	29	31	26	22	50	54
4	16	14	13	28	32	42	45	24
11	36	16	11	51	46	6	11	26

a Draw a stem-and-leaf plot.

b What is the most common score?

c What is the most common class?

9E **6** Draw a histogram for the data in question **5**.

9F **7** Calculate summary statistics for each data set.

a 1, 4, 3, 2, 5, 2, 4, 1, 1, 6, 4

b 11, 18, 21, 17, 13, 11, 16, 2, 9, 11

c 2, 2, 2, 2, 29, 3

9F **8** Consider this frequency table.

Score	Frequency
12	3
13	6
14	8
15	2
16	0
17	0
18	1

a Calculate summary statistics for this data.

b Which is the best measure of centre to use to describe the data? Explain.

9G **9** Explain why the following sampling methods should *not* be used to find the most popular teacher in school.

a asking everybody in Year 8

b asking 200 people randomly from the phone book

c asking one person from each year level

9G **10** Explain whether you could reasonably predict the population characteristics from this table.

Population	Tasmania
Sampling technique	Stratified sampling
Sample	500 people from Tasmania
Mean	7
Median	10
Mode	11
Range	26

NAPLAN-STYLE PRACTICE

1 What sampling method is surveying every 10th person who walks into a store?

- ◯ systematic sampling
- ◯ random sampling
- ◯ stratified sampling
- ◯ biased sampling

2 Which of these is an example of a census?

- ◯ Asking 1 000 000 Australians if they want to change the Australian flag.
- ◯ Asking everyone in your class if Year 8 students prefer Coke or Pepsi.
- ◯ Asking every person in Victoria if people in Hobart like the prime minister.
- ◯ Asking everyone in your class if the class prefer cats or dogs.

Questions **3**–**5** refer to this data set.

12	35	21	19	18	18	39	11
29	31	28	27	14	31	30	20
19	15	21	22	28	31	37	18

3 What is the best size for class intervals?

30	10	5	1
◯	◯	◯	◯

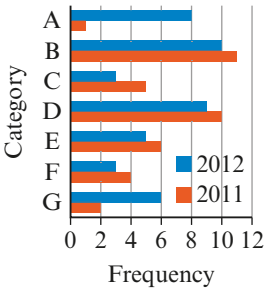
4 What type of data is this?

- ◯ numerical, discrete
- ◯ numerical, continuous
- ◯ categorical, nominal
- ◯ categorical, ordinal

5 If the first class interval is 10–14, what is the most popular class?

_____

Questions **6** and **7** refer to this graph.



6 What is the least popular category overall?

- ◯ A ◯ C
- ◯ F ◯ G

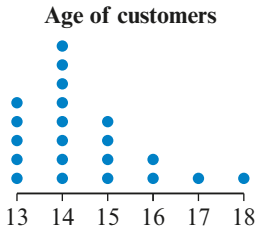
7 How many people were surveyed in 2011?

_____

8 How is the relationship between height and weight best represented?

- ◯ column graph ◯ pie graph
- ◯ line graph ◯ scatterplot

9 Which sentence best describes this dot plot?



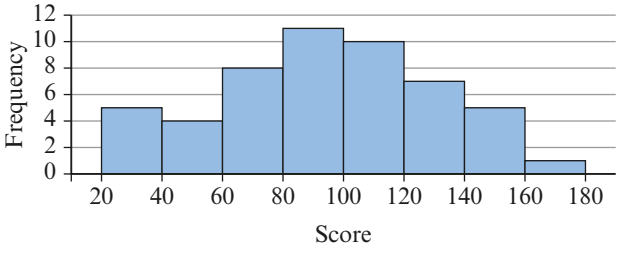
- ◯ Customers are likely to be teenagers.
- ◯ The least common age is 18.
- ◯ The youngest customer is 13.
- ◯ Customers are most likely to be in their early teens (13–15 years).

10 Which key shows the most common value in this stem-and-leaf plot is 1200?

Stem	Leaf
0	7 8 9 9
1	1 1 2 2 2 2 2 8 9
2	0 1 3 5 6
3	0 7
4	
5	1

- ◯ 1 | 1 = 11
- ◯ 1 | 1 = 1100
- ◯ 11 | 00 = 1100
- ◯ 110 | 0 = 1100

Questions **11** and **12** refer to this graph.



11 What is the modal class?

- ◯ 20–40 ◯ 20–180
- ◯ 80–100 ◯ 160–180

12 The shape of the histogram can best be described as:

- ◯ roughly symmetrical
- ◯ roughly symmetrical with an outlier
- ◯ skewed
- ◯ skewed with an outlier

Questions **13**–**15** refer to the data in this table.

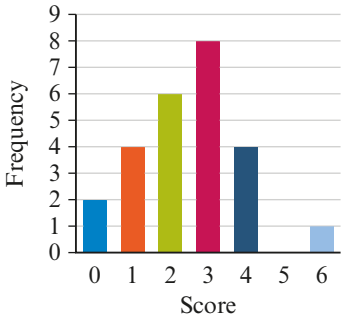
Score	Frequency
0	2
1	7
2	8
3	2
4	1

13 What is the median?

14 What is the mean?

15 What is the range?

16 What is the mean, median, mode and range for this graph?



- ◯ 3, 3, 8, 8
- ◯ 2.48, 3, 3, 6
- ◯ 3.57, 3, 3, 8
- ◯ 2.5, 3, 3, 6

17 There is an issue important to everybody in your postcode. What would be the best sample to survey?

- ◯ asking 50 people you know
- ◯ asking 50 random people at the shopping centre
- ◯ asking 50 random people from the electoral roll
- ◯ asking 200 people in the state

Questions **18** and **19** refer to this situation. Bulaale wants to find the most popular TV show watched by the Year 8s at his school. He surveys two classes of Year 8 students (40 students out of a total of 200) and finds that, out of the 10 TV shows mentioned, the most popular one is *The Simpsons*.

18 Which summary statistic is Bulaale using when he names *The Simpsons*?

19 What might prevent Bulaale from using this sample to predict the most popular TV show for students in Year 8?

- ◯ The sample was not big enough.
- ◯ The surveying technique was biased.
- ◯ *The Simpsons* is probably not the most popular TV show.
- ◯ There are more than 10 shows on TV.

ANALYSIS

Drew wanted to find the average age of people who went to the local beach. One Saturday he went to the beach and asked the age of every fifth person who walked past. The results are shown.

14	17	18	23	21	19	23	45	23	46
61	15	16	17	39	32	9	19	27	26
31	5	21	19	18	16	15	26	22	59
72	13	49	43	16	15	19	23	22	21
63	56	8	24	35	16	19	25		

a What type of data is this?

b Describe the sampling method. Is Drew taking a census or a sample?

c Put the data into a frequency table with appropriate class intervals.

d Draw an appropriate graph of the data.

e Create a stem-and-leaf plot of the data.

f Calculate summary statistics on the data.

g What limitations does this sample have if you want to use it to predict the characteristics of the population?

h Make a prediction about what you would expect to see in the population.

CONNECT

Sampling a population

It is often too difficult to survey an entire population.Usually a sample is taken and then used to predict the characteristics of the population. You are to conduct a survey on a topic of your choice. What do you need to think about to make sure that the sample is a good reflection of the population?



Your task

For your investigation into running a fair survey, complete these steps.

- Decide on a topic and formulate at least three survey questions.
- Decide what your population is and how big your sample should be.
- Decide on an appropriate sampling technique that allows for fair and unbiased results.
- Perform the survey and collect the information.
- Present the data in:
  - frequency tables and
  - at least one of each type of graph mentioned in this unit.
- Calculate summary statistics for the numerical data in the survey.
- Use your results to make a prediction about what you would expect to see in the population.
- Discuss the limitations of your sample and how this affects your confidence in your predictions.











Complete the 9 CONNECT worksheet to show all your working and answers to this task.

You may like to present your findings as a report. Your report could be in the form of:

- a statistical report
- a poster
- a PowerPoint presentation
- other (check with your teacher).



10 PROBABILITY

10A Probability

10B Theoretical probability

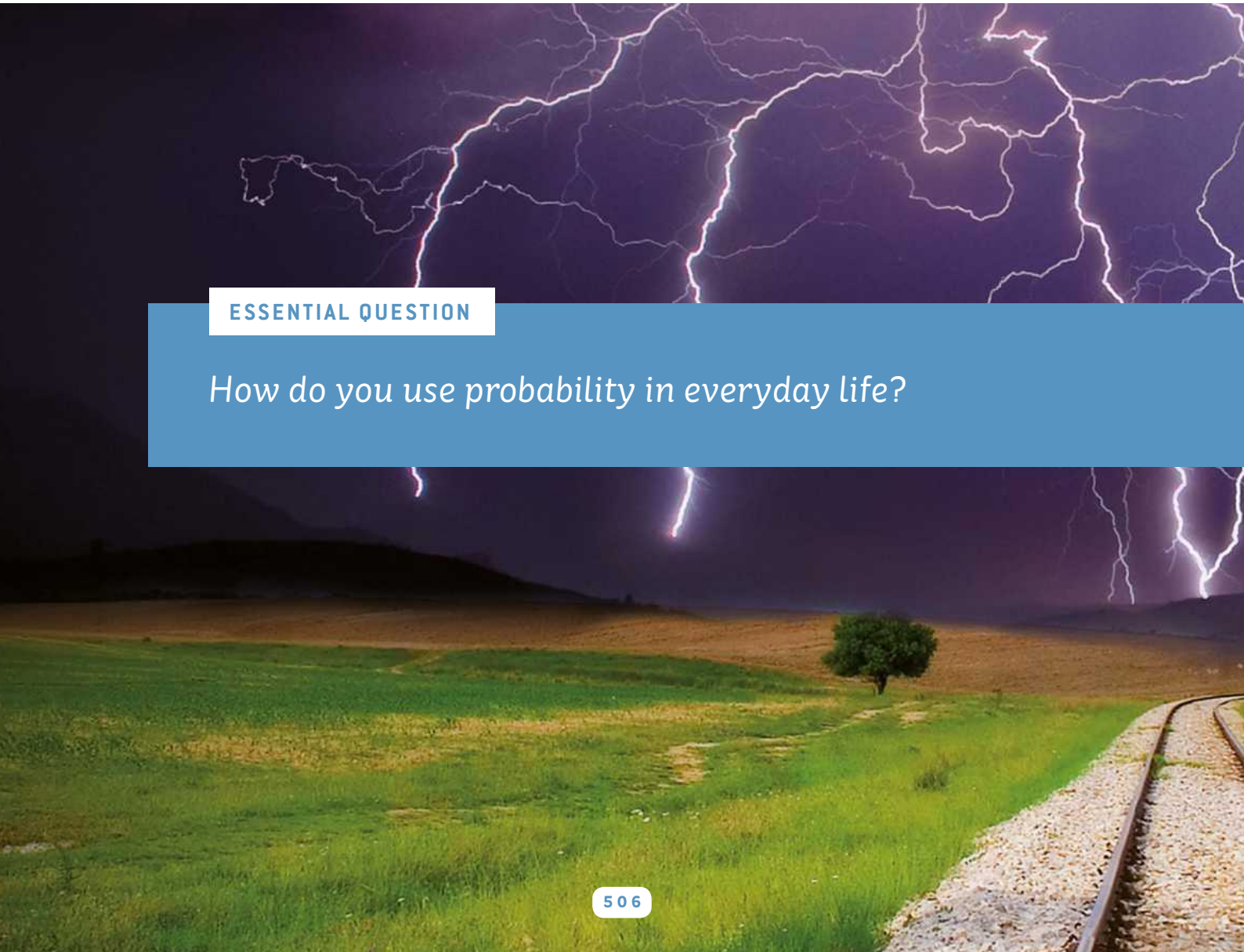
10C Tree diagrams

10D Two-way tables

10E Venn diagrams

10F Experimental probability

10G Simulations and long-term trends



ESSENTIAL QUESTION

How do you use probability in everyday life?

Are you ready?

10A **1** Which of these best describes the probability of a summer day being hot?

A certain **B** very likely
C even chance **D** unlikely

10A **2** What is the probability of an impossible event?

A 1 **B** 0.5 **C** never **D** 0

Questions **3**–**6** refer to this spinner.



10A **3** How many outcomes are there in total?

10A **4** How many different outcomes are there?

10A **5** The sample space for this spinner is:

A different colours
B {blue, green, yellow, red}
C {blue, green, yellow, green, yellow, red}
D six

10B **6** What is the theoretical probability of spinning red?

A $\frac{1}{6}$ **B** $\frac{1}{4}$ **C** $\frac{1}{3}$ **D** 1

10B **7** What is $\frac{1}{4}$ as a decimal number?

A 0.25 **B** 0.4 **C** 0.5 **D** 1.4

10B **8** Which outcome does *not* have a probability of 0.5?

A A has four chances out of eight
B B has six chances out of twelve
C C has three chances out of six
D D has one chance out of ten

10B **9** Which event has a probability of $\frac{1}{2}$?

A rolling a die and getting an even number
B selecting a heart from a deck of cards
C flipping a coin and getting a head or a tail
D rolling a die and getting a 6

10D **10** Look at this table.

	Year 7	Year 8
Male	6	9
Female	8	7

a How many Year 8 students are female?

b How many male students were surveyed?

c How many students were surveyed in total?

A 6 **B** 14 **C** 16 **D** 30

10F **11** David performed an experiment where he rolled a die 30 times. On seven occasions he obtained a 6.

What is the experimental probability of rolling a 6?

A $\frac{1}{6}$ **B** $\frac{7}{30}$ **C** 6 **D** 7

10F **12** If you flipped a coin 20 times, how many times would you expect a head?

A 5 **B** 10 **C** 15 **D** 20

10F **13** Which experiment does *not* have two different outcomes?

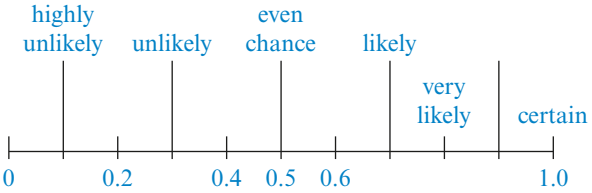
A drawing a card from a deck and recording its colour
B flipping a coin twice and recording the outcome
C rolling a die and recording if the number is odd or even
D flipping a coin and recording the outcome

10F **14** Which statement is true?

A Drawing a card and recording its suit has two possible outcomes.
B Drawing a card and recording if it is a picture card (or not) has two equally likely outcomes.
C Drawing a card and recording its colour has two outcomes that are not equally likely.
D Drawing a card and recording its suit has four equally likely outcomes.

10A Probability

Start thinking!



Probability is how likely something is to happen.

1 You can use a **probability scale** to describe how likely an **event** is to occur.

a Copy the incomplete probability scale and place these words in the correct position.
highly likely, impossible, somewhat likely, very unlikely, somewhat unlikely

b Fill in the missing numbers on the scale using: 0.9, 0.1, 0.7, 0.3, 0.8.

c What is the lowest numerical value a probability can have?
What word do you use to describe an event with this probability?

d What is the highest numerical value a probability can have?
What word do you use to describe an event with this probability?

e Think of four different events to place at different points along the scale.
For example, the sun rising tomorrow is certain to happen and has a probability of 1.

2 Rolling a die is an example of performing a **probability experiment**.



a List the **sample space** for the outcomes of rolling a die.
This is just a list of all the different outcomes within curly brackets {…}.

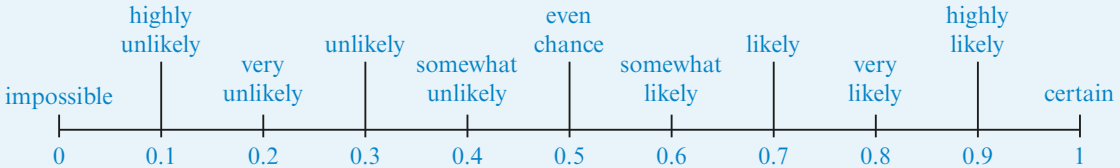
b Would you have an even chance of rolling a 6? Why not?

c Are you equally likely to roll a 6 as you are a 1? Why is this?

d Use your answers to parts **b** and **c** to describe the difference between outcomes being equally likely to occur and an outcome that has an even chance of occurring.

KEY IDEAS

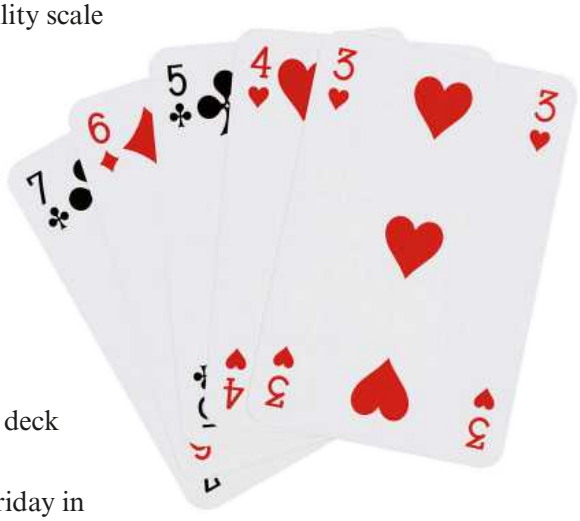
- Probability is how likely an event is to happen. It can be described using words or numbers.
- A probability must have a value in the range 0 (impossible) to 1 (certain).
- The sample space of an experiment is a list of the possible outcomes written in curly brackets.
- An outcome that is 'equally likely to occur' has the same chance of occurring as other outcomes in the sample space. This is different from an outcome 'having an even chance' of occurring, which means it has a probability of 0.5.
- The number of elements in a sample space is the number of objects listed within the sample space.



EXERCISE 10A Probability

UNDERSTANDING AND FLUENCY

1 Describe each event using the probability scale from the Key ideas.



- **a** flipping a coin and obtaining a head
- **b** flipping a coin and obtaining 10 heads in a row
- **c** a random day in summer being warm
- **d** selecting a blue marble out a bag containing yellow and red marbles
- **e** drawing a non-picture card from a deck of cards
- **f** both today and tomorrow being Friday in Brisbane

2 Arrange these events on a probability scale.

- **a** You draw an ace from a deck of cards.
- **b** You roll a die and obtain a 7.
- **c** It will rain at least two days this year.
- **d** You win a game of rock, paper, scissors.
- **e** You flip a coin and get a head.
- **f** An animal selected from the a litter of puppies will be a member of the dog family.
- **g** You roll a nine-sided die (numbered 1–9) and get an odd number.
- **h** A person randomly selected from your class is a girl.

EXAMPLE 10A-1 Listing sample space

List the sample space for each probability experiment.

a randomly selecting a colour from the rainbow
b randomly selecting a letter from the word ENERGETIC

THINK	WRITE
a List each possibility within curly brackets.	**a** {red, orange, yellow, green, blue, indigo, violet}
b List every *different* letter within curly brackets.	**b** {E, N, R, G, T, I, C}

3 List the sample space for each probability experiment.

- **a** rolling a die
- **b** selecting a marble out of a bag containing three blue, two red, one yellow and four green marbles
- **c** drawing a card from a deck and recording its suit
- **d** randomly selecting a letter from the word CHOCOLATE
- **e** flipping a coin
- **f** randomly selecting a person and recording their birth month

EXAMPLE 10A-2 Identifying equally likely outcomes

State whether each experiment has equally likely outcomes.

a rolling a die **b** selecting a letter at random from the word ESCAPE

THINK

a The possible outcomes are {1, 2, 3, 4, 5, 6}. No outcome is more likely to be selected than any other.

b The possible outcomes are {E, S, C, A, P}, where the E is twice as likely to be selected as any other letter.

WRITE

a Rolling a die has six equally likely outcomes.

b Selecting a letter at random from the word ESCAPE does not have equally likely outcomes.

UNDERSTANDING AND FLUENCY

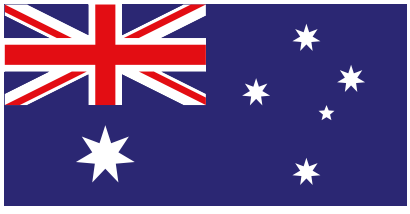
4 State whether each experiment has equally likely outcomes.



- **a** selecting a letter at random from the word DRINK
- **b** drawing a card from a deck and recording its suit
- **c** flipping a coin
- **d** selecting a letter at random from the word CLOCKWORK
- **e** spinning this spinner
- **f** selecting a name from a hat containing your class's names.

5 For each experiment:

- **i** list the sample space
- **ii** state whether the outcomes in the sample space are equally likely.



- **a** randomly pointing at a spot on the Australian flag and recording the colour
- **b** randomly selecting a counter from a bag containing three red, two blue, five yellow, three green and two purple counters
- **c** randomly selecting a letter from the word GRAMMAR
- **d** tossing two coins and recording the outcome.
- **e** randomly selecting a letter from the word EXERCISE
- **f** spinning this spinner



6 For each experiment in question **3**, state whether its outcomes are equally likely. If they are not equally likely, state the most likely outcome and the least likely outcome.

EXAMPLE 10A-3 Identifying the number of elements

State the number of elements in each sample space.

a {red, blue, orange, green} **b** {0, 1, 2, 3, 4, 5}

THINK	WRITE
a Count the number of objects separated by commas.	**a** There are four elements in the sample space {red, blue, orange, green}.
b Count the number of objects separated by commas.	**b** There are six elements in the sample space {0, 1, 2, 3, 4, 5}.

UNDERSTANDING AND FLUENCY

7 State the number of elements in each sample space.

a {head, tail}
b {10, 20, 30, 40, 50, 60}
c {A, E, I, O, U}
d {walk, ride, train, bus, car, tram}
e {13, 14, 15, 16, 17, 18, 19}
f {T, E, W, S, R, O, L, P}

8 State the number of elements in the sample spaces from question **3**.

9 For each sample space from question **3** that contains outcomes that are not equally likely, write all the possible outcomes so that they are equally likely. For example, the sample space from question **5e** would become {E, X, E, R, C, I, S, E}.

PROBLEM SOLVING AND REASONING

10 Provide an example of an event that is:

a unlikely **b** certain
c highly likely **d** somewhat unlikely.

11 A kitten is to be selected at random from this litter. Explain why it is unlikely you will pick the smoky coloured kitten, but also equally as likely as selecting any other kitten.



12 Provide an example of an experiment with outcomes that are:

a equally likely **b** not equally likely.

13 There are many other words that you can use to describe probability. Name at least five and place these on a probability scale.

14 For each spinner:

- **i** list the sample space
- **ii** state the number of elements in the sample space
- **iii** state which colour would be most likely if the spinner was spun
- **iv** explain your choice of colour.

a



b



c



15 Describe an experiment for each sample space where the outcomes are equally likely to occur.

a {A, E, I, O, U}

b {win, lose, draw}

c {Monday, Tuesday, Wednesday, Thursday, Friday}

d {clubs, diamonds, hearts, spades}

e {1, 2, 3,...,9, 10}

f {even, odd}

16 For each sample space listed in question **15**:

- **i** state the number of elements
- **ii** write the probability of each outcome below occurring using both words and numbers.

a E

b win

c Saturday

d red or black card

e 7

f even

17 Describe each experiment in two different ways: one where each outcome is equally likely to occur and another where the outcomes are not equally likely to occur. The first one has been done for you.

	Experiment	Equally likely outcomes	Not equally likely outcomes
a	Selecting a day of the week	randomly selecting any of the seven days	selecting either a weekday or weekend
b	Selecting a letter from the word AMUSING		
c	Selecting a month of the year		
d	Drawing a card from a deck		
e	Selecting a person from your class		

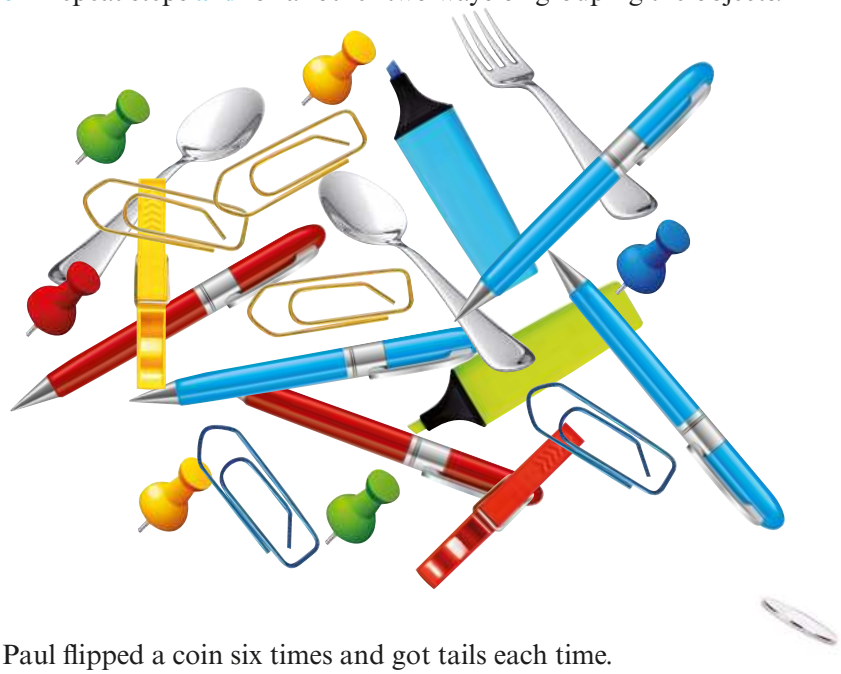
18 Find the sample spaces for both options of each experiment in question **17**.

PROBLEM SOLVING AND REASONING

19 State the number of elements in each sample space you listed in question **18**.

20 Look at these objects.

- **a** Describe one way to group the objects.
- **b** List the sample space for this grouping.
- **c** Are all outcomes equally likely? If not, which outcome is the most likely?
- **d** Pick one outcome and describe the probability of this outcome occurring.
- **e** Repeat steps **a**–**d** for another two ways of grouping the objects.



21 Paul flipped a coin six times and got tails each time. He said that it was highly unlikely that his next flip would be a tails. Can you explain why his reasoning is incorrect?

CHALLENGE

22 Design a spinner that has sectors coloured blue, yellow, green and pink, where the arrow is most likely to land on a yellow sector.

23 Design a spinner that has sectors coloured blue, pink, yellow and green, where the arrow is three times more likely to land on green than either blue, pink or yellow.



Reflect

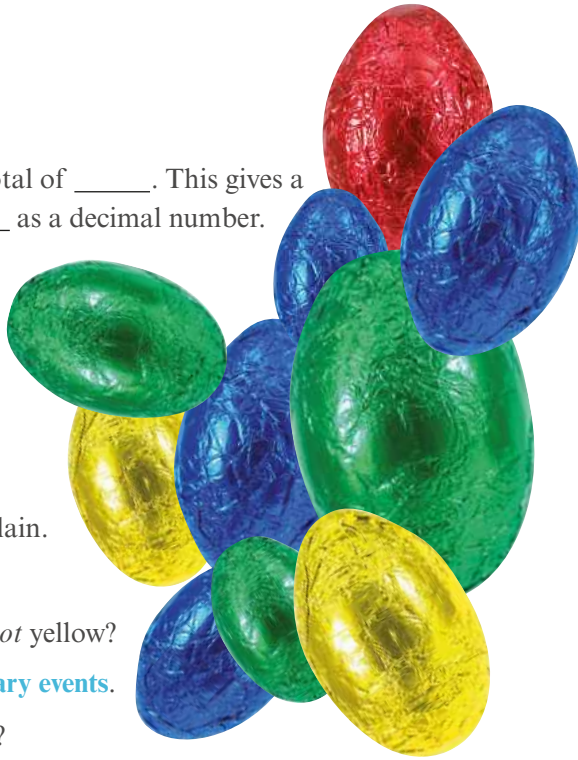
Why is it important to know if outcomes are equally likely to occur when thinking about probability?

10B Theoretical probability

Start thinking!

Rather than just using a probability scale, you can look at the number of outcomes to calculate the **theoretical probability** of an event occurring.

Look at the Easter eggs shown.



1 Copy and complete this sentence.
There are _____ opportunities to select a green Easter egg out of a total of _____. This gives a theoretical probability of _____ as a fraction, which is equal to _____ as a decimal number.

A **favourable outcome** is an outcome that you want.

2 Copy and complete:
probability of an event occurring
= number of _____ outcomes out of _____ outcomes

$$\Pr(\text{event}) = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$$

3 Does this formula work for outcomes that are not equally likely? Explain.

4 What is the theoretical probability of choosing a yellow Easter egg?

5 What is the theoretical probability of choosing an Easter egg that is *not* yellow?

The two events described in questions **4** and **5** are said to be **complementary events**.

6 Add the probabilities for these two events together. What do you find?

KEY IDEAS

- To find the theoretical probability of an event occurring, use the formula:
$$\Pr(\text{event}) = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$$
- To find the total number of outcomes, listing the sample space can help, but it is important that all outcomes in the sample space are equally likely to occur.
- If outcomes within the sample space are not equally likely to occur, you can write them as a list of outcomes that reflects the chance of occurring. For example, the sample space for selecting a letter at random from the word ENERGY is written as {E, N, R, G, Y}, where E is twice as likely to occur as other outcomes. The list can instead be written as {E, N, E, R, G, Y}, where each outcome is equally likely to occur.
- Complementary events are 'opposite' events; for example, rolling a 1 on a die and not rolling a 1 on a die.
- The probabilities of complementary events always add to 1.

EXERCISE 10B Theoretical probability

EXAMPLE 10B-1 Calculating theoretical probability

A letter is selected at random from the word WATCHING.

a State the number of possible outcomes.
b What is the probability of selecting a T?
c What is the probability of selecting a vowel?

THINK	WRITE
a Count the total number of outcomes.	a total number of outcomes = 8
b Count the number of favourable outcomes (1) and substitute into the formula with the total number of outcomes.	b $\Pr(\text{T}) = \frac{1}{8}$
c Count the number of favourable outcomes (2: A and I) and substitute into the formula with the total number of outcomes.	c $\Pr(\text{vowel}) = \frac{2}{8} = \frac{1}{4}$

UNDERSTANDING AND FLUENCY

1 A letter is selected at random from the word BLACKCURRANT.
   a State the number of possible outcomes.
   b What is the probability of selecting a B?
   c What is the probability of selecting a C?

2 Calculate the theoretical probability of:
   a rolling a die and obtaining a 6
   b drawing a club from a deck of cards
   c selecting the letter Z from the word CRAZY
   d selecting a blue marble from a bag containing three red, two blue, one yellow and four green marbles
   e guessing the correct answer to a multiple-choice question with options A–E
   f drawing an ace from a deck of cards.

3 Consider this spinner.
   a State the total number of possible outcomes.
   b What is the probability of spinning blue?
   c What is the probability of spinning yellow or green?
   d What is the probability of spinning anything except red?



UNDERSTANDING AND FLUENCY

4 Calculate the theoretical probability of:

- **a** selecting a consonant from the word CRAZY
- **b** selecting a green or yellow marble from a bag containing four blue, three yellow, two red and six green marbles
- **c** rolling a die and obtaining a number greater than 4
- **d** drawing a picture card from a deck of cards
- **e** selecting a C, E or S from the word SUPERCALIFRAGILISTICEXPIALIDOCIOUS
- **f** rolling a die and obtaining any number except 6.

5 A card is selected from a deck and the suit is recorded.

a What is the sample space? **b** What is the probability of selecting a diamond?
c What is the probability of *not* selecting a diamond?

EXAMPLE 10B-2 Finding complements

Find the **complement** to each probability. **a** 0.3 **b** $\frac{5}{7}$

THINK	WRITE
a Complementary events add to 1. Subtract the given probability from 1.	**a** $1 - 0.3 = 0.7$
b Complementary events add to 1. Subtract the given probability from 1.	**b** $1 - \frac{5}{7} = \frac{2}{7}$

6 Find the complement to each probability.

a 0.4 **b** $\frac{3}{4}$ **c** 0.1 **d** $\frac{1}{2}$ **e** $\frac{1}{8}$ **f** 0.7 **g** $\frac{2}{3}$ **h** 0.25

EXAMPLE 10B-3 Using complementary probability

A six-sided die is rolled.

a List the sample space. **b** What is the probability of rolling a 2 or a 3?
c What is the complementary event to rolling a 2 or a 3?
d What is the probability of this complementary event?

THINK	WRITE
a List all the possible outcomes within curly brackets.	**a** {1, 2, 3, 4, 5, 6}
b You have one chance to roll a 2 and one chance to roll a 3, which gives you two opportunities out of a total of six.	**b** $\Pr(2 \text{ or } 3) = \frac{2}{6} = \frac{1}{3}$
c List all outcomes other than rolling 2 or 3.	**c** Rolling a 1, 4, 5 or 6.
d Subtract the original probability from 1.	**d** $\Pr(1, 4, 5 \text{ or } 6) = 1 - \frac{1}{3} = \frac{2}{3}$

UNDERSTANDING AND FLUENCY

7 A six-sided die is rolled.

a List the sample space.

b What is the probability of rolling a 5 or 6?

c What is the complementary event to rolling a 5 or 6?

d What is the probability of the complementary event?

> **NOTE** In Example 10B-3, the probability of the complementary event could also be found by adding the individual probabilities, as in part **b**.

8 A letter is selected at random from the word EXCITEMENT.

a What is the probability of selecting a consonant?

b What is the complementary event to selecting a consonant?

c What is the probability of the complementary event?

9 Consider this collection of buttons.



a How many buttons are there?

b What is the probability of selecting a blue button?

c State the complementary event to selecting a blue button and find its probability.

d What is the probability of selecting a white button?

e What is the probability of selecting a button with four holes?

f State the complementary event to part **e** and find its probability.

PROBLEM SOLVING AND REASONING

10 Explain why, when calculating theoretical probability, the outcomes listed in the sample space must be equally likely to occur for the number of elements to be used as the total number of possible outcomes.

11 A standard deck of cards has four suits with 13 cards each: ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, jack, queen and king.



a How many cards are there in total?

b What is the probability of selecting an ace?

c What is the probability of the complementary event to selecting an ace?

d What is the probability of selecting a picture card?

e Are you more likely to select a queen of hearts or an ace? Explain.

f What is the probability of selecting a picture card that is not a heart?

g What is the probability of the complementary event to part **f**?

h What is the probability of selecting an even number from the deck?

12 Consider the phrase
YEAR EIGHT IS BETTER THAN YEAR SEVEN.

a How many letters are there in total?

b What is the probability of selecting an E?

c What is the probability of the complementary event to part **b**?

d What is the probability of selecting an A or a B?

e What is the complementary event to part **d**?

Now consider the letters as vowels and consonants.

f How many vowels are in the phrase?

g State the probability of selecting a vowel.

h State the complement of this event and its probability.

Think of another way to group the letters.

i State an outcome and its probability.

j State the complement to the event in part **i** and its probability.

13 Consider this spinner.



a What is the probability that you will spin red or blue?

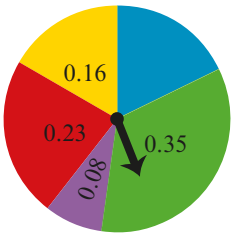
b Explain why the answer to part **a** is *not* $\frac{2}{5}$.

c What is the probability that you will spin yellow?

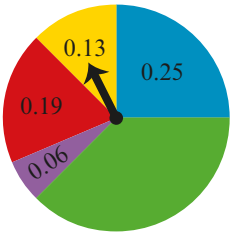
d State the complementary event to spinning yellow and its probability.

14 Use your knowledge of the sum of probabilities to find the probability of landing on the unlabelled sector in each spinner.

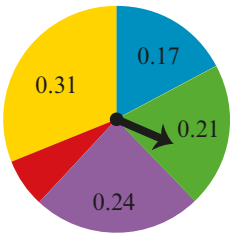
a



b



c



15 Draw a spinner where Pr(blue) = $\frac{1}{2}$, Pr(red) = $\frac{1}{4}$, Pr(green) = $\frac{1}{8}$ and the remaining portion is yellow. State the probability of spinning yellow.

16 At a sporting match there are 200 spectators. Ninety-five people are there to support Team A, eighty-six people are there to support Team B and the remainder don't have a preference.

a Find the probability that a randomly chosen spectator doesn't care who wins.

b If you are there to support Team B, what is the probability that a person who sits next to you will also support Team B?

17 Maynard flipped two coins and got two tails. He said that the complementary event to flipping two tails is to flip two heads. Can you explain where he may have gone wrong?

PROBLEM SOLVING AND REASONING

18 A packet of M&Ms contains 55 individual pieces. You count out 13 blue, 7 brown, 9 green, 11 orange, 7 red and the rest yellow.



- **a** What is the probability of selecting a blue or green M&M?
- **b** What is the probability of selecting a yellow M&M?
- **c** Imagine you select a yellow M&M.
  - **i** How many M&Ms are left in the packet now?
  - **ii** How many of these are yellow?
  - **iii** What is the probability of selecting a yellow M&M from the packet now?
- **d** Imagine that the second M&M you select is blue.
  - **i** How many M&Ms are left in the packet now?
  - **ii** How many of these are blue?
  - **iii** What is the probability of selecting a blue or green M&M now?

19 You purchase a bag of 55 M&Ms that contains 16 blue, 8 brown, 9 green, 5 orange, 6 red and the remainder yellow. Assume that you select M&Ms without replacing them throughout this question.

- **a** What is the probability of selecting a yellow M&M?
- **b** What is the probability of selecting an M&M that is not blue?
- **c** Imagine that you select 3 M&Ms: blue, yellow and then blue. What is the probability that the next M&M you select is:
  - **i** blue? **ii** not yellow?
- **d** Imagine that after those first 3 M&Ms, you select another 6 M&Ms, all blue. What is the probability that the next M&M you select will be blue?
- **e** After selecting those first 9 M&Ms, which colour M&M are you now most likely to pick? What is the probability of selecting this colour?

CHALLENGE

20 Use a protractor and your knowledge of angles in a circle to find the probability of spinning the spinner shown on the right and landing in each of the sectors.



21 A bag contains a number of marbles coloured blue, yellow, green and red. You have a $\frac{1}{4}$ chance of selecting a blue marble, $\frac{1}{5}$ chance of selecting green and $\frac{3}{5}$ chance of not selecting yellow.

- **a** What is the probability of selecting a yellow marble?
- **b** What is the probability of selecting a red marble?
- **c** Hence find the smallest number of marbles that could be in the bag.

Reflect

How is theoretical probability related to sample space?

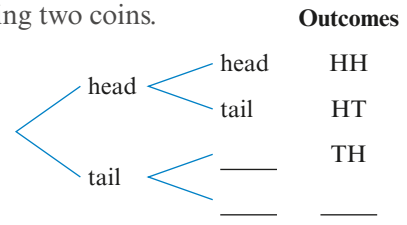
10C Tree diagrams

Start thinking!

A **tree diagram** displays outcomes for a multi-step experiment.
Imagine that you flip a coin twice and record the outcome each time.



1 Complete this tree diagram to show the possible outcomes of flipping two coins.



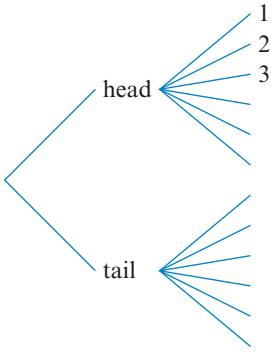
2 How many outcomes are there?

3 What is the probability of:
a flipping two tails? **b** flipping one head and one tail?

4 Why are these two probabilities different?

A tree diagram can also represent unrelated experiments, such as flipping a coin and rolling a die.

5 Complete the second tree diagram shown to display all the possible outcomes for this two-step experiment.



6 What is the probability of:
a flipping a head? **b** rolling a 6?
c flipping a head *and* rolling a 6?

7 Why are these three probabilities different?

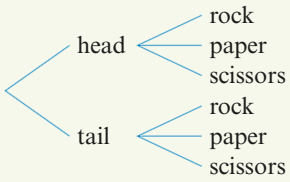
KEY IDEAS

- Tree diagrams display the outcomes of multi-step experiments.
- The branches of the tree represent the possibilities for each step of the experiment.
- The final outcomes are listed at the ends of the branches.

EXERCISE 10C Tree diagrams

EXAMPLE 10C-1 Understanding tree diagrams

Consider this tree diagram.



a What experiment is it showing?
b How many possible outcomes are there?
c How many of these outcomes contain:
i a head? **ii** paper? **iii** a tail or rock?

THINK

a Look at the choices at each branch. The first choices are heads and tails and the second choices are rock, paper and scissors.

b Count the number of final outcomes at the right end of the tree diagram.

c Trace the branches carefully and count the number of outcomes containing the required item/s.
i Three outcomes: head, rock; head, paper; head, scissors.
ii Two outcomes: head, paper; tail, paper.
iii Four outcomes: head, rock; tail, rock; tail, paper; tail, scissors.

WRITE

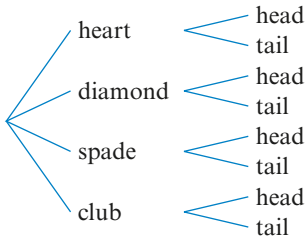
a The experiment is flipping a coin and playing rock, paper, scissors.

b There are six possible outcomes.

c **i** Three outcomes contain a head.
ii Two outcomes contain paper.
iii Four outcomes contain a tail or rock.

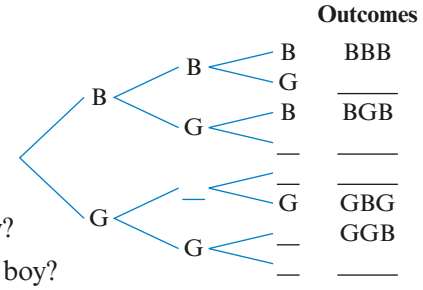
UNDERSTANDING AND FLUENCY

1 Consider this tree diagram.



a What experiment is it showing?
b How many possible outcomes are there?
c How many of these outcomes contain:
i a heart?
ii a tail?
iii a diamond or a head?

2 Consider this incomplete tree diagram, in which B represents a boy and G, a girl.



a Complete the tree diagram so that it fully lists all possible outcomes.
b How many possible outcomes are there?
c How many outcomes contain at least one boy?
d What is the probability of having at least one boy?

EXAMPLE 10C-2 Drawing tree diagrams

Draw a tree diagram to list all the possible outcomes for flipping a coin three times.

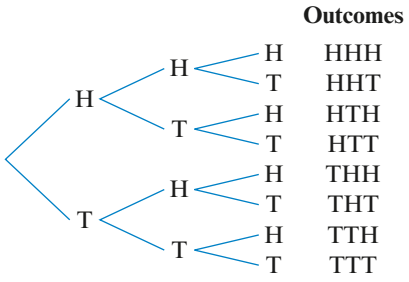
THINK

1 Draw the first two branches to represent the first coin flip. Label the end of these branches with H and T to represent the two different outcomes.

2 From each branch, draw another two branches to represent the next coin flip and label them appropriately. Repeat this one more time so that you are representing the three coin flips.

3 At the end of each of the eight branches, write the complete outcome.

WRITE



UNDERSTANDING AND FLUENCY

3 Draw a tree diagram to list all the possible outcomes for each experiment.

- **a** flipping a coin four times
- **b** drawing a card from a deck, recording its suit and replacing it two times
- **c** playing two rounds of rock, paper, scissors
- **d** the possible combinations of food and drink shown in this photograph. (Hint: start the tree with three food branches.)



EXAMPLE 10C-3 Using tree diagrams to calculate probability

Use the tree diagram from Example 10C-2 to find the probability of flipping three coins and obtaining at least two tails.

THINK

1 Count the number of outcomes that contain at least two tails.

2 Count the total number of possible outcomes.

3 Find the probability of obtaining at least two tails.

WRITE

Four outcomes contain at least two tails (HTT, THT, TTH, TTT).

There are 8 possible outcomes.

Pr(at least two tails) = $\frac{4}{8}$ or $\frac{1}{2}$

UNDERSTANDING AND FLUENCY

4 Use the tree diagram in question **1** to the find the probability of:

a a heart and a tail
b a diamond
c a club or a head
d not a spade
e not a club nor a tail
f a tail but not a heart.

5 Use the tree diagram from Example 10C-2 to find the probability of flipping:

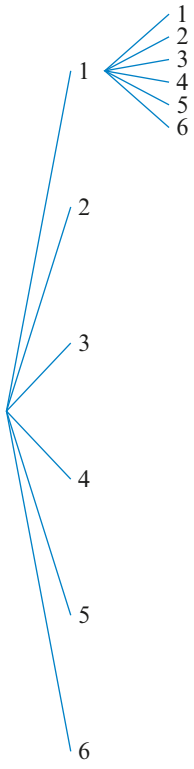
a three tails
b two heads and one tail
c at least two heads
d one head and two tails
e at least one head
f three heads.

6 Use the tree diagram from question **3a** to find the probability of flipping:

a four tails
b two heads and two tails
c at least two tails
d exactly three heads
e three heads or more
f at least one head.

PROBLEM SOLVING AND REASONING

7 The incomplete tree diagram on the right shows the outcomes for rolling two dice.



a Complete the tree diagram so that it fully lists all possible outcomes.
b How many outcomes in total are there?
c How many outcomes involve a double?
d Find the probability of:
  i rolling a double
  ii rolling a double 6.
e A classmate said that because there are 21 different outcomes, the probability of rolling a double 6 is $\frac{1}{21}$. Explain where they may have gone wrong.

8 Kieran drew a card from a deck and recorded its colour before returning it to the deck. He did this a total of five times.

a Draw a tree diagram to represent this five-step experiment.
b How many possible outcomes are there?
c What is the most likely outcome? What is its probability?
d What is the probability of drawing:
  i all red cards?
  ii exactly two black cards?
  iii at least two black cards?
e Explain how parts **d ii** and **iii** are different.
f Add all the probabilities of the final outcomes. What do you find?



9 Alissa did an experiment with a deck of cards like the one in question **8**, but instead recorded the value of the card (ace, 2, 3, etc.) and only performed two trials.

- **a** Draw a tree diagram to represent this two-step experiment.
- **b** What is the probability of drawing an ace both times?
- **c** What is the probability of drawing:
  - **i** a picture card both times?
  - **ii** a picture card and an ace in any order?
- **d** Assume that picture cards have a value of 10 and that an ace can be either 1 or 11. What is the probability of drawing two cards that combine to give a value of 21?

10 Consider this spinner.



- **a** Draw a tree diagram to represent the possible outcomes if it was spun twice.
- **b** What is the probability of spinning:
  - **i** blue both times?
  - **ii** blue first and then red?
  - **iii** a blue and a red?
- **c** Explain how parts **b ii** and **iii** are different.
- **d** What is the probability of spinning green at least once?

11 Huey played a probability game involving the spinner above and a coin.

- **a** Add the flip of a coin to the tree diagram you made for question **10**.
- **b** What is the probability of:
  - **i** flipping a tail?
  - **ii** spinning blue at least once?
  - **iii** spinning blue at least once and flipping a tail?
- **c** Huey was confused because he thought that the answers to parts **b i** and **ii** should add together to give the answer to part **b iii**. Can you explain why they don't?

12 If a family has four children, use a tree diagram to find the probability that they have at least one of each gender.

13 Rebecca's teacher gave her a test she wasn't prepared for. There were three questions on the test.

- **a** Assume all the questions were true/false questions.
  - **i** Draw a tree diagram to represent getting each question right (R) or wrong (W).
  - **ii** What is the probability Rebecca can guess every question correctly?
  - **iii** What is Rebecca's most likely score on the test?
- **b** Now assume all the questions were multiple-choice with four options (A, B, C and D).
  - **i** Draw a tree diagram to represent getting each question right or wrong. (Hint: each step will need *four* branches: one that is correct and the other three that are incorrect.)

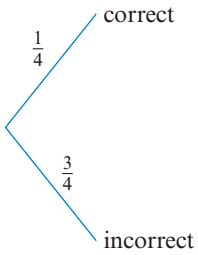
ii What is the probability that Rebecca can guess every question correctly?

iii What is Rebecca's most likely score on the test?

c What does this tell you about guessing answers on a test?

CHALLENGE

14 All the tree diagrams you have looked at so far assume each branch of the tree is equally as likely as any other. However, this is not always the case. Consider part **b** of question **13**, where Rebecca is guessing the answer to a multiple-choice question with four options. You can draw this as a tree diagram with two branches but you need to write the unequal probabilities on each branch.



a Why does the option 'correct' have a probability of $\frac{1}{4}$?

b What do you notice if you add the two probabilities together?

c What do you call these events when their probabilities add together to give this number?

d Extend the tree diagram to represent guessing another two questions, writing the final outcomes at the end of the branches.

To find the probability of each outcome, multiply together the probabilities of the branches that you move along. For example, the probability of Rebecca getting the first question right, the second question wrong and the third question right is $\frac{1}{4} \times \frac{3}{4} \times \frac{1}{4} = \frac{3}{64}$.

e Use this method to find the probability of each outcome.

f What is the probability that Rebecca gets:

i all three questions correct? **ii** exactly one question correct?

iii exactly two questions correct? **iv** at least one question correct?

(Hint: for each section of part **f**, circle the outcomes that you want and add the probabilities together.)

g Add all the probabilities of the final outcomes. What do you find?

h How can you use this answer to check if you have drawn a tree diagram correctly?

15 A bag contains four purple marbles and six green marbles. Imagine you select a marble, record its colour and replace it in the bag.

a Draw a tree diagram to represent three trials of this experiment. (Hint: the outcomes are not equally likely so you will need to draw a tree diagram like that in question **14**.)

b What is the probability of selecting:

i three green marbles?

ii three purple marbles?

iii at least one purple marble?

iv a purple marble, then a green marble, then a purple marble?

v selecting a green marble to begin with? (Hint: it doesn't matter what you select on the second and third trials.)

Reflect

How are tree diagrams useful?

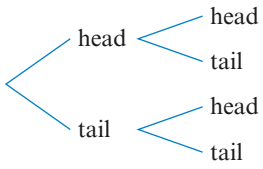
10D Two-way tables

Start thinking!



Another method for displaying outcomes for a two-step experiment is to use a table.

Consider this tree diagram, which is the same as the one used on page 520.



You can display this as a **two-way table** instead.

	H	T
H	H, H	H, T
T	T, H	T, T

1 What is different and what is similar about these two displays?

2 This two-way table shows a survey conducted on a Year 8 class.

	Boys	Girls	Total
Musical instrument	10	4	14
No musical instrument	3	13	16
Total	13	17	30

a The first outcome in the two-way table is boys who play a musical instrument. What are the other three outcomes?

b The two-way table shows there are 10 boys who play a musical instrument. How many girls play a musical instrument?

c How many people in total play a musical instrument?

d If 30 people were surveyed in total, what is the probability a person chosen randomly from the group plays a musical instrument?

e What is the probability a person chosen randomly from the group is a girl who plays a musical instrument?

f A classmate thought that because there are four different outcomes, the probability of a person chosen randomly from the group being a girl who plays a musical instrument is $\frac{1}{4}$. Explain why he is incorrect.

3 What advantage does a two-way table have over a tree diagram?

KEY IDEAS

- A two-way table is another method used to display the sample space of an experiment.
- A two-way table that includes totals is also called a Karnaugh map.
- Two-way tables are used to display outcomes and totals in order to calculate probabilities.

EXERCISE 10D Two-way tables

EXAMPLE 10D-1 Understanding two-way tables

Consider this two-way table.

a How many people were surveyed?
b How many boys prefer toast for breakfast?
c How many people prefer cereal for breakfast?

	Boys	Girls	Total
Cereal	4	9	13
Toast	7	5	12
Total	11	14	25

THINK

a Check the bottom right corner cell for the total number of people surveyed.

b Find the cell that is in the 'boys' column and the 'toast' row.

c Look at the cell that shows the total of the cereal row.

WRITE

a 25 people were surveyed.

b 7 boys prefer toast for breakfast.

c 13 people prefer cereal for breakfast.

UNDERSTANDING AND FLUENCY

1 Consider this two-way table.

	Year 7	Year 8	Total
Play sport	11	17	28
Don't play sport	12	10	22
Total	23	27	50

a What are the four different outcomes?
b How many students were surveyed?
c How many Year 8 students play sport?
d How many Year 7 students are there?
e What does the number 12 in the table represent?
f How many students don't play sport?

2 Consider this two-way table.

	Books	Movies	Total
Children	12	39	51
Adults	29	20	49
Total	41	59	100

a What are the four different outcomes?
b How many people were surveyed?
c How many adults prefer books?
d How many children were surveyed?
e How many children prefer movies?
f How many people prefer books?

EXAMPLE 10D-2 Completing two-way tables

Complete this two-way table.

	Public transport	Private transport	Total
Year 7		19	
Year 8	13		27
Total			50

THINK

1 The numbers in the rows and columns add to give a total, so wherever there are two out of three cells in a row or column, the third can be found using addition or subtraction.

2 The total column has two cells with information. The missing cell must be 50 − 27 = 23.

3 The Year 8 row has two cells with information. The missing cell must be 27 − 13 = 14.

4 The Year 7 row now has two cells with information. The missing cell must be 23 − 19 = 4.

5 The private transport column now has two cells with information. The missing cell must be 19 + 14 = 33.

6 The final cell can be found using information in the public transport column (4 + 13 = 17) or the total row (50 − 33 = 17).

WRITE

	Public transport	Private transport	Total
Year 7		19	**23**
Year 8	13	**14**	27
Total			50

	Public transport	Private transport	Total
Year 7	**4**	19	23
Year 8	13	14	27
Total	**17**	**33**	50

UNDERSTANDING AND FLUENCY

3 Complete each two-way table.

a

	Group A	Group B	Total
Year 7		23	
Year 8	10		18
Total			50

b

	Cat	Dog	Total
Male		48	59
Female	39		
Total			100

c

	Weekday	Weekend	Total
Volleyball	14		31
Tennis			36
Total		48	

d

	Year 8	Year 9	Total
CD	11		26
Digital		19	
Total	29		

4 Use the objects in this photo to complete the two-way table below.



	Lollies	Not lollies	Total
Red			
Not red			
Total			

EXAMPLE 10D-3 Using two-way tables to calculate probability

Consider this two-way table. What is the probability of a person chosen randomly from the group:

a being a boy who prefers toast?
b preferring cereal?
c being a girl who prefers cereal?

	Boys	Girls	Total
Cereal	4	9	13
Toast	7	5	12
Total	11	14	25

THINK

a Identify the number of favourable outcomes (7) out of the total number of possible outcomes (25).

b Identify the number of favourable outcomes (13) out of the total number of possible outcomes (25).

c Identify the number of favourable outcomes (9) out of the total number of possible outcomes (25).

WRITE

a Pr(boy who prefers toast)
$= \frac{7}{25}$
$= 0.28$

b Pr(person who prefers cereal)
$= \frac{13}{25}$
$= 0.52$

c Pr(girl who prefers cereal)
$= \frac{9}{25}$
$= 0.36$

UNDERSTANDING AND FLUENCY

5 Consider this two-way table.

	Boys	Girls	Total
Facebook	27	43	70
MySpace	20	10	30
Total	47	53	100

a How many people were surveyed?
b How many girls prefer Facebook?
c What is the probability of a person chosen randomly from the group being a girl who prefers Facebook?

6 Consider this two-way table.

	DVD	BluRay	Total
Action	5	6	11
Comedy	19	10	29
Total	24	16	40

a How many films were recorded in total?
b How many action films were on BluRay?
c How many DVDs were comedies?
d What is the probability of a film chosen randomly from the group being:
i a BluRay action? **ii** a comedy DVD? **iii** on BluRay?

UNDERSTANDING AND FLUENCY

7 Consider the two-way table shown in question **1**. What is the probability of a person chosen randomly being:

a a Year 8 student who plays sport?
b a Year 7 student who doesn't play sport?
c a Year 7 student?
d a student who plays sport?
e a Year 8 student?
f a Year 7 student who plays sport?

8 Consider the two-way table shown in Example 10D-2. What is the probability of a person chosen randomly being:

a a Year 7 student who takes public transport?
b a student who takes private transport?
c a Year 8 student?
d a Year 8 student who takes private transport?
e a Year 7 student who takes private transport?
f a student who takes public transport?

PROBLEM SOLVING AND REASONING

9 Two classes of students were surveyed on their chocolate preference. In a Year 8 class of 24 students, 16 preferred M&Ms over Smarties. Of the 18 people who preferred Smarties, 10 came from the Year 7 class.

	Year 7	Year 8	Total
M&Ms		16	
Smarties	10		18
Total		24	50

a Copy and complete the two-way table.
b What is the probability that a person chosen randomly from this group:
i prefers Smarties over M&Ms? **ii** is in Year 7?
iii is in Year 8 and prefers M&Ms?
c A person is chosen randomly from the Year 8 class. What is the probability that he or she prefers M&Ms?
d Explain how this is different from part **b iii**.

10 Two-way tables don't always have just two columns or rows. This table has three columns.

	Soccer	AFL	Rugby	Total
Northern	10	12	28	50
Southern	17	30	3	50
Total	27	42	31	100

a What is the table showing?
b What is the probability that a person chosen randomly from the group:
i likes AFL best? **ii** likes Rugby but is not from northern Australia?
iii likes soccer or rugby? **iv** is from northern Australia and likes soccer?
c What can you say about the difference in sport preference between northern and southern Australia?

11 Consider this table.

	Blond hair	Brown hair	Total
Blue eyes	16		
Brown eyes		31	
Total		61	100

a Copy and complete the table.
b Find the probability that a person chosen randomly from the group:
i has blond hair **ii** has blue eyes **iii** has blond hair and blue eyes.

PROBLEM SOLVING AND REASONING

c A person is randomly chosen from the group. You know that they have blond hair.

- **i** How many people have blond hair?
- **ii** How many people with blond hair have blue eyes?
- **iii** What is the probability that somebody with blond hair has blue eyes?
- **iv** Explain why this is different from part **b iii**.

This kind of problem is usually described with the word 'given'. This means instead of considering the whole group, you consider only the limited group in the question. Part **c iii** is better asked as 'What is the probability somebody has blue eyes, given they have blond hair?' This tells you to look at the blond group as the total.

d To answer the question 'What is the probability that somebody has blue eyes, given that they have brown hair?', you can follow these steps.

- **i** The overall group that you are looking at is people with brown hair. How many people are in this group?
- **ii** The specific group that you want is people with brown hair and blue eyes. How many people are in this group?
- **iii** Use your answers to parts **i** and **ii** to find the probability that somebody has blue eyes, given that they have brown hair.

12 Consider the table in question **10**. A person is randomly chosen from the group.

a Find the probability that the person likes AFL, given that they are from:

i southern Australia **ii** northern Australia.

b Find the probability that the person is from northern Australia, given that they like:

i rugby **ii** AFL.

CHALLENGE

13 A group of 100 people were surveyed about what they like on toast. In total, 64 people said they prefer Vegemite. Of these 64 people who like Vegemite, 40 said they have white bread. This is exactly the same as the total number of people who don't have white bread. Construct a two-way table and find the probability that a person chosen randomly from the group has jam on non-white bread.

14 A survey asked people about their phone company. Forty per cent of the people surveyed were from country regions. Telstra was the preferred company for only 35% of those surveyed. Of the people surveyed, 55% lived in the city and did not use Telstra as their phone provider.

	Telstra	Other	Total
Country			0.4
City		0.55	
Total			1

a Copy and complete the two-way table.

b If 200 people were surveyed, redraw the table to represent the numbers of people surveyed.

c Find the probability that somebody has Telstra as a phone provider, given that they are from the:

i country **ii** city.

Reflect

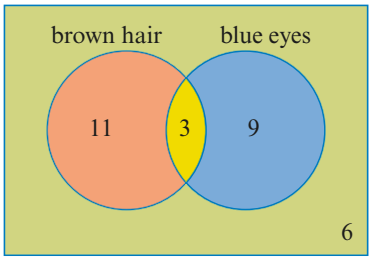
What mistakes do you think people might commonly make when reading and drawing two-way tables?

10E Venn diagrams



Start thinking!

A **Venn diagram** is used to show the relationship between sets of data. This Venn diagram shows two sets of people: people with brown hair and people with blue eyes.



1 The orange section shows people with brown hair and not blue eyes. How many people are in this group?

2 Which section represents people with brown hair and blue eyes? How many people are in this section?

3 Add these two numbers together to find the total number of people in the survey with brown hair.

4 What does the blue section represent? How many people are in this section?

5 How many people have blue eyes? Which two sections does this involve?

6 Which colour represents people who don't have brown hair *or* blue eyes? How many people are in this section?

7 Add all the numbers in the Venn diagram to find how many people were surveyed.

8 How are Venn diagrams related to two-way tables? Can you represent this information in a two-way table?

9 Which display do you find easier to use?

KEY IDEAS

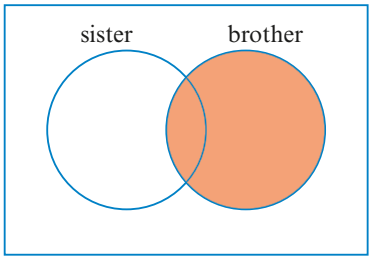
- A Venn diagram is used to display the relationship between different sets of data.
- Venn diagrams consist of a number of circles contained within a rectangle. Each circle represents a set of data.
- Numbers are placed within each section of the diagram to show which set an individual belongs to.
- These numbers can be used to calculate probabilities of individuals belonging to different sets.

EXERCISE 10E Venn diagrams

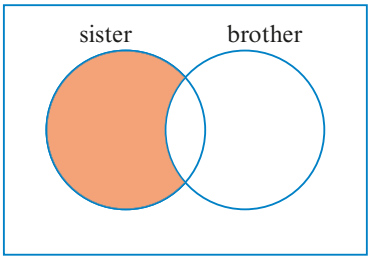
UNDERSTANDING AND FLUENCY

1 These Venn diagrams represent people with sisters and/or brothers. Write what the shaded section in each diagram represents.

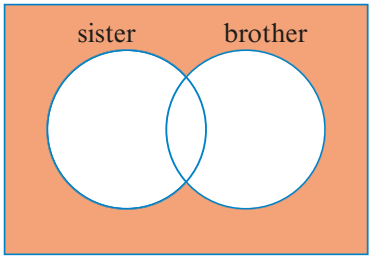
a



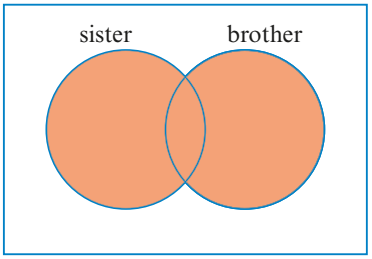
b



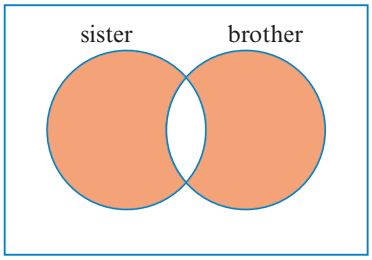
c



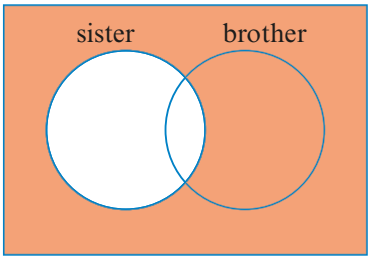
d



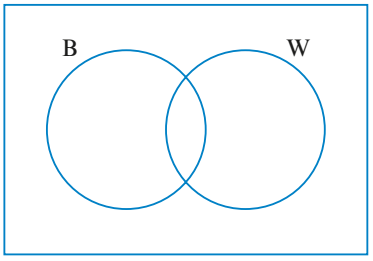
e



f



2 This Venn diagram represents students in a Year 8 class. Section B represents the boys in the class and section W represents students who walk to school.



i Copy the diagram six times.

ii Shade the correct section of the diagram for each of these groups.

a boys who walk to school

b boys who don't walk to school

c students who walk to school

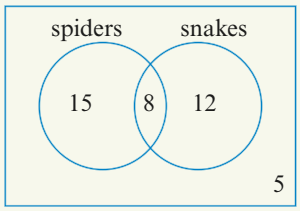
d girls who don't walk to school

e students who don't walk to school

f girls who walk to school

EXAMPLE 10E-1 Understanding Venn diagrams

This Venn diagram shows people who are afraid of spiders and snakes.



a How many people are afraid of both snakes and spiders?
b How many people are afraid of spiders?
c How many people are not afraid of either?
d How many people were surveyed?

THINK

a Look for the section which represents people being afraid of both snakes and spiders. This is the middle section where the two circles overlap, the **intersection** of both sets.

b Add all the numbers that are within the 'spiders' circle.

c People who are not afraid of either belong outside the two circles.

d Add all the numbers within the Venn diagram; that is, within the rectangle.

WRITE

a Eight people are afraid of both snakes and spiders.

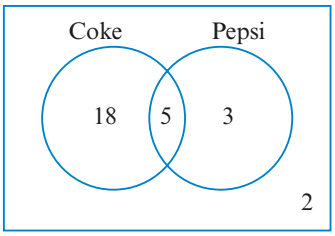
b 15 + 8 = 23
There are 23 people who are afraid of spiders.

c There are five people who are not afraid of either snakes or spiders.

d 15 + 8 + 12 + 5 = 40
There were 40 people surveyed.

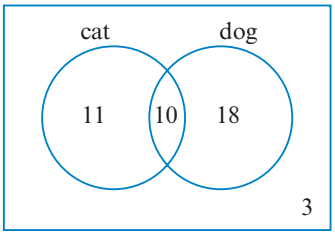
UNDERSTANDING AND FLUENCY

3 Consider this Venn diagram.



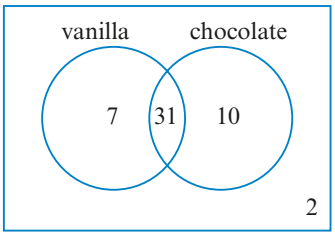
a How many people like only Coke?
b How many people like Coke?
c How many people like both Coke and Pepsi?
d How many people like neither?

4 Consider this Venn diagram.



a How many people own a cat and a dog?
b How many people own a cat?
c How many people do not own either a cat or a dog?
d How many people were surveyed?

5 Consider this Venn diagram.



a How many people:
i like both chocolate and vanilla?
ii like chocolate only?
iii like vanilla? iv do not like either?
v like chocolate or vanilla?
vi do not like chocolate?
b How many people were surveyed?

EXAMPLE 10E-2 Using Venn diagrams to calculate probability

Consider the Venn diagram shown in Example 10E-1. What is the probability that a person chosen at random from the group is afraid of:

a both snakes and spiders? **b** snakes? **c** spiders but not snakes?

THINK

a Identify the number of favourable outcomes (8) and the total number of outcomes (15 + 8 + 12 + 5 = 40).

b Identify the number of favourable outcomes (8 + 12 = 20) and the total number of outcomes (40).

c Identify the number of favourable outcomes (15) and the total number of outcomes (40).

WRITE

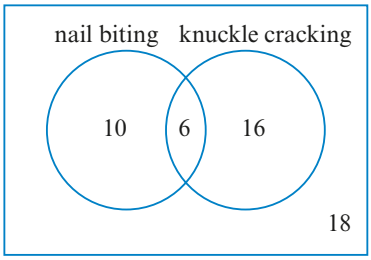
a Pr(snakes and spiders) = $\frac{8}{40}$ or $\frac{1}{5}$

b Pr(snakes) = $\frac{20}{40}$ or $\frac{1}{2}$

c Pr(spiders not snakes) = $\frac{15}{40}$ or $\frac{3}{8}$

UNDERSTANDING AND FLUENCY

6 Consider this Venn diagram, showing the incidence of two bad habits.



a How many people:
- **i** bite their nails and crack their knuckles?
- **ii** crack their knuckles but don't bite their nails?
- **iii** bite their nails?
- **iv** do neither?

b What is the probability that a person chosen at random from the group:
- **i** bites their nails and cracks their knuckles?
- **ii** cracks their knuckles but doesn't bite their nails?
- **iii** bites their nails?
- **iv** doesn't do either?

7 Consider the Venn diagram shown in question **4**. What is the probability that a person chosen at random from the group owns:

a both a cat and a dog? **b** a dog but not a cat?

c a cat? **d** neither a dog nor a cat?

8 Consider the Venn diagram shown in question **5**. What is the probability that a person chosen at random from the group likes:

a vanilla but not chocolate?

b chocolate?

c vanilla and chocolate?

d vanilla or chocolate?



9 Construct a Venn diagram using the objects in this photo, using the categories 'yellow' and 'round fruit'. Add your numbers together when you finish to check that you have completed it correctly.

10 To draw a Venn diagram from given information you need to think carefully. Consider this statement: in a group of 20 people, 14 own a dog and 12 own a cat, and there are two people who don't own either.

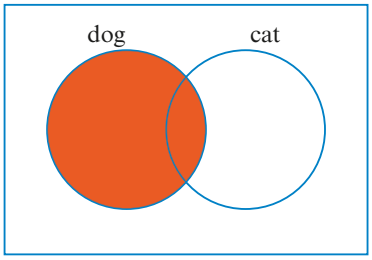
If you just add all the numbers together, you get a total that is more than the number of people surveyed, so there must be some overlap.

a Draw a Venn diagram with two circles that overlap; label them 'dog' and 'cat'.

b How many people don't own either? Place this number outside the circles.

c How many people are left to place in the circles? (Hint: subtract your answer to part **b** from the total number of people.)

The shaded part of the Venn diagram represents the number of people who own dogs (14).



d Subtract this number from your answer to part **c** to find the number to place in the unshaded circle section (people who own cats only).

e If 12 people own cats, find the number of people who own both cats and dogs by subtracting your answer to part **d** from 12. Place this in the overlapping section.

f To find the number of people who own dogs only, subtract your answer to part **e** from the number of people who own dogs (14).

g Place this final number in the Venn diagram to complete your diagram.

h Find the probability that a person chosen randomly:

i owns a dog

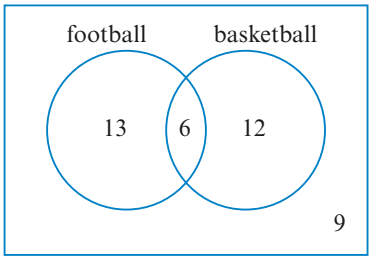
ii owns a cat only

iii owns a dog and a cat

iv does not own either.

11 Construct a Venn diagram from the statement below. Follow the steps shown in question **10** if you need help. In a group of 40 people, 26 people like KFC and 30 people like McDonald's. There are four people who don't like either.

12 Remember that in problems that use the word 'given', you consider only the limited group in question rather than the total group as a whole.



a **i** How many people like football?

ii How many people like both football and basketball?

iii What is the probability that a person chosen randomly from the group likes basketball, given that they like football?

b **i** How many people don't like football?

ii How many people like basketball but not football?

PROBLEM SOLVING AND REASONING

iii What is the probability that a person chosen randomly from the group likes basketball, given that they don't like football?

c i How many people like basketball?

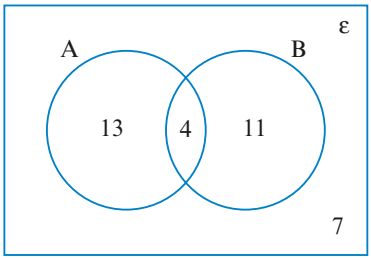
ii How many people like both football and basketball?

iii What is the probability that a person chosen randomly from the group likes football, given that they like basketball?

d Explain the difference between parts **a iii** and **c iii**.

e Explain the difference between parts **a iii** and **b iii**.

13 Consider this Venn diagram. If set A = {people who like white chocolate} and set B = {people who like dark chocolate}, find the probability that a person chosen randomly from the group:



a likes white chocolate

b likes white chocolate but not dark chocolate

c likes both white and dark chocolate

d likes white chocolate, given that they like dark chocolate

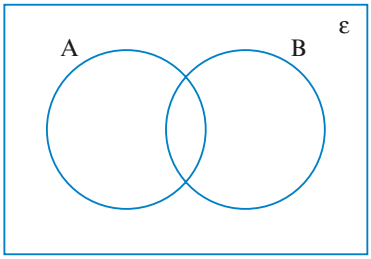
e likes dark chocolate, given that they like white chocolate

f likes white chocolate or dark chocolate

g does not like dark chocolate.

CHALLENGE

14 Set notation can be used to describe sets mathematically. This Venn diagram shows sets A and B. Copy it five times in your book.



a On the first diagram, shade the overlapping section of A and B. In set notation, this is the intersection of sets A and B. Label this $A \cap B$.

b On the second diagram, shade inside the circles. In set notation, this is the **union** of sets A and B. Label this $A \cup B$.

c On the third diagram, shade everything that does not belong to set A; that is, everything outside this circle. This is the complement of set A. Label this A'.

d Everything inside the rectangle is said to belong to the **universal set**. Label this with ε. Shade everything inside the rectangle to represent ε on the fourth diagram.

e Use what you have learnt to shade B' on the fifth diagram.

15 Draw a Venn diagram to represent sets D and E. Copy this diagram to show a separate answer for each part.

a Shade $D \cap E$.

b Shade the union of sets D and E.

c Shade set E.

d Shade D'.

e Shade $(D \cup E)'$. (Hint: this is the complement of your answer to part **b**).

Reflect

How are Venn diagrams similar to two-way tables?

10F Experimental probability

Start thinking!



You can test theoretical probabilities using experiments. **Experimental probability** means calculating the probability of an event using the results of an experiment rather than theory. It is also called **relative frequency**.

1 List the sample space for flipping a single coin.

2 What is the theoretical probability of flipping a head?

You can use theoretical probability to calculate the **expected number** of favourable outcomes.

3 If you flipped a coin 10 times, how many tails would you expect to get?
What about if you flipped it 40 times?

4 Explain how you got your answers to question **3**.

5 Copy and complete these sentences.

To find the expected number of favourable outcomes, you _____ the theoretical probability by the number of trials. You can write this as:

expected number = theoretical probability × number of trials.

This can also be shortened to: $E(x) = Pr(x) \times n$.

6 Which part of this formula stands for theoretical probability? Match each term in the expression to the worded version of the formula.

7 Perform both of the experiments described in question **3** and record your results.

8 Find the experimental probability of flipping a tail. Remember the formula for experimental probability is:

$$Pr(\text{success}) = \frac{\text{number of successful trials}}{\text{total number of trials}}$$

9 How does this compare to the theoretical probability you found in question **2**?

10 Some variation between experimental and theoretical probability is expected in experiments. Is there more variation in a smaller trial or a larger trial? Use your results to support your claims.

KEY IDEAS

- Experimental probability is the probability of an outcome based on the results of an experiment.
- To calculate experimental probability, use the formula: $Pr(\text{success}) = \frac{\text{number of successful trials}}{\text{total number of trials}}$
- In one experiment, experimental probability may not exactly equal theoretical probability. However, as the number of trials increases, experimental probability gets closer to theoretical probability.
- Before conducting an experiment, calculate the expected number of successful outcomes.
- To calculate expected number, multiply theoretical probability by the number of trials. This can be written as $E(x) = Pr(x) \times n$.

EXERCISE 10F Experimental probability

EXAMPLE 10F-1 Calculating experimental probability

Calculate the experimental probability of obtaining a spade if a card is drawn from a deck and replaced 300 times and 81 spades are obtained.

THINK

1 Write the experimental probability formula.

2 Identify the number of successful trials (81) and the total number of trails (300). Substitute into the formula and simplify.

3 Write your answer.

WRITE

$$\text{Pr(success)} = \frac{\text{number of successful trials}}{\text{total number of trials}}$$
$$= \frac{81}{300}$$
$$= \frac{27}{100}$$

The experimental probability of obtaining a spade in this experiment is $\frac{27}{100}$.

1 Find the experimental probability for each of these.

- **a** flipping a tail, if a coin is flipped 1000 times and 481 tails are obtained
- **b** rolling a three, if a die is rolled 60 times and 5 threes are obtained
- **c** flipping two tails, if two coins are flipped 400 times and tails appears on both coins 88 times
- **d** drawing a heart, if a card is drawn from a deck and replaced 200 times and 62 hearts are obtained
- **e** rolling a six, if a die is rolled 300 times and 67 sixes are obtained
- **f** guessing the correct answer, if 25 answers are guessed correctly out of 40

EXAMPLE 10F-2 Finding expected number

Find the expected number of tails if a coin is flipped 250 times.

THINK

1 Write the formula for expected number.

2 Identify Pr(x) (theoretical probability of flipping a coin and obtaining a tail) and n (number of times the coin is flipped). Substitute these values into the formula.

3 Write your answer.

WRITE

$$\text{E}(x) = \text{Pr}(x) \times n$$
$$\text{E}(x) = \frac{1}{2} \times 250$$
$$= 125$$

You would expect to obtain 125 tails if you flipped a coin 250 times.

2 Find the expected number of:
- **a** heads if a coin is flipped 200 times
- **b** hearts if a card is drawn from a deck and replaced 100 times
- **c** twos if a die is rolled 600 times
- **d** heads on both coins if two coins are flipped 200 times
- **e** clubs if a card is drawn from a deck and replaced 300 times
- **f** answers guessed correctly on a quiz of 40 questions with options A–E.

EXAMPLE 10F-3 Working with experimental probability

A die is rolled 300 times.
- **a** What is the expected number of sixes?
- **b** If 45 sixes are obtained, find the experimental probability of rolling a six and compare it to the theoretical probability.
- **c** State what you would expect to happen to the experimental probability in the long term.

THINK

a **1** Write the formula for expected value.

2 Identify the theoretical probability of rolling a six ($\frac{1}{6}$) and the number of trials (300) and substitute into the formula.

b **1** Write the formula for experimental probability.

2 Identify the number of successful trials (45 sixes) out of the total number of trials (300) and substitute into the formula.

3 Compare this to the theoretical probability.

c Over the long term, the experimental probability should get closer to the theoretical probability.

WRITE

a $E(x) = Pr(x) \times n$

$= \frac{1}{6} \times 300$

$= 50$

b $\text{Pr(success)} = \frac{\text{number of successful trials}}{\text{total number of trials}}$

$\text{Pr(six)} = \frac{45}{300}$

$= \frac{3}{20}$

$= 0.15$

The experimental probability of rolling a six is 0.15, which is slightly less than the theoretical probability of $\frac{1}{6} = 0.167$.

c Over the long term, the experimental probability should increase to become closer to the theoretical probability.

3 A die is rolled 1200 times.
- **a** What is the expected number of fours?
- **b** If 250 fours are obtained, find the experimental probability and compare it to the theoretical probability.

4 A card is drawn from a deck and replaced, and this is repeated 100 times.
- **a** What is the expected number of clubs?
- **b** If 17 clubs are obtained, find the experimental probability and compare it to the theoretical probability.

5 According to the Mars company, a packet of M&Ms consists of 24% blue, 20% orange, 16% green, 14% brown, 14% yellow and 13% red M&Ms.



a Find the relative frequency of each colour M&M in the packet shown in the photo.

b Compare it to the theoretical probability given by Mars.

c What do you think you would find if you sampled many packets?

6 A common probability experiment is to flip three coins and record the outcome.

a Draw a tree diagram to represent one trial of this experiment. (Hint: there should be eight outcomes.)

b How many different outcomes are there?

c Find the theoretical probability of each different outcome.

d If this was done 40 times, find the expected number of each different outcome.

Omar performed this experiment and obtained the results in this table.

Outcome	Frequency
three heads	7
two heads, one tail	14
one head, two tails	16
three tails	3

e How do these numbers differ from your expected numbers?

f Find the experimental probability for each different outcome.

g What would you expect to see in the long term?

7 Consider the experiment described in question **6**.

a Perform the 40 trials and record your results.

b Calculate the experimental probability and compare it to:

i the theoretical probability

ii Omar's results.

c Perform another 60 trials (to give a total of 100) and record your results.

d Calculate the experimental probability and compare it to:

i the theoretical probability

ii Omar's results

iii your results from part **a**.

8 Two coins were flipped 20 times. The results are in this two-way table.

		Coin 2		
		H	T	Total
Coin 1	H	3	6	9
	T	9	2	11
	Total	12	8	20

a Find the experimental probability for each outcome. Be sure to use the correct total.

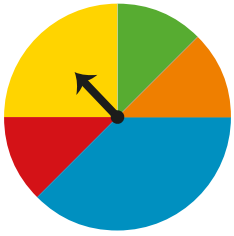
b Find the experimental probability for each different outcome.

c What is the theoretical probability for each different outcome?

d How do your answers to parts **b** and **c** compare?

e Find the expected number for each different outcome and compare it to what is in the table.

9 Consider this spinner.



a Find the theoretical probability of each outcome.

b If the spinner was spun 40 times, calculate the expected value for each outcome.

The spinner was spun 40 times and the results recorded in the table shown.

Outcome	Frequency
Yellow	12
Green	5
Orange	7
Red	3
Blue	13

c How do these results compare to your expected numbers?

d Find the experimental probability for each different outcome.

e What would you expect to see in the long term?

10 Create the spinner and perform the experiment described in question **9**. Record your results and compare them to the theoretical probabilities.

11 Tess and Stu play a game where they select a card from a deck, record if it is a picture card or not, and replace the card.



a State the two possible outcomes and their theoretical probabilities.

b How many picture cards would you expect to get if you did:

i four trials **ii** 10 trials

iii 20 trials **iv** 40 trials.

c Perform the experiment and record your results in the intervals mentioned in part **b**.

d Compare your results from part **c** to your answers to part **b** and discuss the differences.

e If Tess gets 2 points for a picture card and Stu gets 1 point for a non-picture card, who do you think has a better deal? Explain. (Hint: calculate the expected number of points that Tess and Stu will each get for 40 trials.)

f Provide a point system that you think is fair. Use your knowledge of probability to demonstrate that it is.

PROBLEM SOLVING AND REASONING

12 Tess decides the game described in question **11** would be fairer if she gets 10 points for every ace that she draws and 1 point for every other non-picture card, whilst Stu gets 5 points for every picture card that he draws.

- **a** State the three possible outcomes and their theoretical probabilities.
- **b** Draw a table and use it to find the expected number of each outcome for:
  - **i** four trials
  - **ii** 10 trials
  - **iii** 20 trials
  - **iv** 40 trials.
- **c** Calculate the expected number of points that Tess and Stu would each get after 40 trials.
- **d** Perform the experiment and record your results in the intervals mentioned in part **b**.
- **e** Compare your results from part **d** to your answers to part **b** and discuss the differences.
- **f** Discuss whether this is a fair game, using your results to support your answer.

13 Choose a two-step experiment from this section.

- **a** Use either a tree diagram or a table to show the different outcomes, their theoretical probabilities and expected number for 10 trials.
- **b** Perform your experiment and compare your results to your diagram or table from part **a**.
- **c** What would you expect to see if you performed 100 trials? Answer in terms of both expected numbers and the variation you would expect to see between experimental and theoretical probability.
- **d** Perform 100 trials and discuss how your results compare with your answers to part **c**.

CHALLENGE

14 Design a spinner that would give the following expected values when spun 40 times. (Hint: find the theoretical probabilities from the expected numbers first.)
Optional: Create your spinner and check your predictions.

Outcome	Frequency
Yellow	10
Green	5
Blue	10
Red	15

15 Create your own probability experiment.

- **a** Choose what to investigate and how many trials you will do.
- **b** Before you start trials, calculate the theoretical probability and expected number of each outcome for your chosen number of trials. You may like to put this into a table.
- **c** Perform your experiment and record your results.
- **d** Find the experimental probability for each outcome.
- **e** Write a paragraph discussing the variation between experimental and theoretical probability and make a prediction about what you would expect to see in the long term.

Reflect

How can experimental probability be used to determine if a game is fair?

10G Simulations and long-term trends



Start thinking!

The previous topic introduced the idea of long-term trends in experiments.

1 Copy this table.

2 Flip a coin 50 times and record your results at 10-flip intervals in the table.

3 Calculate the experimental probability for each 10-flip interval and enter in the table.

4 Complete the last column of the table by subtracting theoretical probability from experimental probability for each 10-flip interval.

Number of trials	Expected number of tails	Number of tails	Theoretical probability	Experimental probability	Difference
10	5		0.5		
20	10		0.5		
30	15		0.5		
40	20		0.5		
50	25		0.5		

5 What can you say about the relationship about theoretical probability and experimental probability as the number of trials increases? How does the difference change?

6 What would you expect to happen to your experimental probability if you flipped the coin 100 times? What about if you flipped it 1000 times?

It is not practical to flip a coin this many times. You can simulate the experiment to obtain the results needed. A **simulation** uses a device to randomly generate numbers or outcomes. You can use these devices to simulate large numbers of trials for events for which you know the theoretical probability. In this example, technology can easily be used.

7 Use a search engine to find a coin simulator and perform 1000 trials in steps of 100. Add this to your table.

8 What happens to the experimental probability? Is this what you expected?

KEY IDEAS

- A simulation can be used when performing an experiment is impractical, too expensive or time consuming.
- A simulation can be a simple random device (for example, a coin, die or spinner) or technology that generates random outcomes (for example, software or a calculator).
- Before performing a simulation you must know the number of outcomes and that they are equally likely to occur.

EXERCISE 10G Simulations and long-term trends

EXAMPLE 10G-1 Choosing devices for simulations

A quiz contains 25 questions which are all multiple-choice (options A–D). Annabelle wants to simulate guessing the correct answer.

a List the number of outcomes and check that they are equally likely.
b Choose a device to use for a simulation
c Assign each device outcome to an outcome of the situation.

THINK

a Each question has the possible outcomes A, B, C, D. Are these equally likely to occur?

b The situation has four equally likely outcomes (A, B, C, D). What has four outcomes that could be used as a device?

c Randomly assign each device outcome (club, diamond, heart, spade) to a situation outcome (A, B, C, D).

WRITE

a The situation has four equally likely outcomes.

b A deck of cards could be used for a simulation as each card is one of four suits: clubs, diamonds, hearts or spades.

c club = A; diamond = B, heart = C and spade = D.

UNDERSTANDING AND FLUENCY

1 For each situation:
- **i** list the number of outcomes and check that they are equally likely
- **ii** choose a device to use for a simulation
- **iii** assign each device outcome to a situation outcome.

a guessing the answer to a true or false question
b being placed into either Group A, B, C or D
c collecting a set of six toys found randomly in cereal packets
d selecting a Clinker and recording the colour (pink, green or yellow)
e choosing whether to turn left or right in a maze
f a baby being born on a certain day of the week



2 For each situation, choose a device for simulation and assign each device outcome to a situation outcome.

- **a** randomly selecting a Freddo Frog from one of eight available flavours
- **b** being assigned to walk in one of the main compass directions
- **c** a new pet fish being female or male
- **d** getting one of ten different prizes in a Christmas cracker
- **e** a baby being born in a particular month of the year
- **f** getting one of six different-coloured torches for free with an online purchase

EXAMPLE 10G-2 Considering the limitations of simulation

Discuss the limitations of using a spinner with 10 equal sections to simulate collecting a set of 10 cards found randomly in packets of chips.

THINK	WRITE
Consider whether using a spinner will accurately model real life.	Spinner creation may not provide randomness if the sectors are not equal or if the spinner gets stuck. Person spinning the spinner may be able to influence where it points to. Cards may not be randomly distributed throughout chip packets.

UNDERSTANDING AND FLUENCY

3 Discuss the limitations of each simulation.

- **a** Using a coin to simulate getting your preferred meal out of two options.
- **b** Using a deck of cards to simulate being born in one of the four seasons.
- **c** Using a spinner with seven sections to simulate being assigned to one of seven classes.
- **d** Using a standard die to simulate being put in one of six sports teams.
- **e** Using a 10-sided die to simulate winning the grand final in a competition of 10 teams.
- **f** Using a spinner with 25 sections to simulate selecting one person out of a class of 25.

4 Choose a device to simulate each situation and discuss the limitations of using this device.

- **a** randomly selecting a Chupa Chup from six available flavours
- **b** a newborn baby being a boy or girl
- **c** choosing one of four available desserts
- **d** one person being chosen leader out of a group of five people
- **e** randomly selecting an M&M and recording its colour (six available colours)
- **f** guessing the answer to a multiple-choice question with options A–E



5 Boxer dogs come in three main types: plain (either fawn or brindle), flashy (with white markings) or white. When two flashy boxers are bred, the puppies can be either plain (25%), flashy (50%) or white (25%).

- **a** State these theoretical probabilities as fractions.
- **b** What device could you use to simulate boxer puppy types? (Hint: two of these devices combine to give these probabilities.)
- **c** Assign each device outcome to a situation outcome (puppy type).
- **d** Simulate the birth of 50 puppies and put your results in a table.
- **e** Find the experimental probability for each outcome and compare it to the theoretical probability.
- **f** Discuss the limitations of using coins to simulate the types of boxer puppies born.

6 Consider the simulation from question **5**. On average, a litter contains eight puppies.

- **a** Complete 10 experiments with eight trials each and record the number of plain, flashy and white puppies in each litter.
- **b** Calculate the experimental probability and compare it to:
  - **i** the theoretical probability
  - **ii** your results from question **5**.
- **c** Find the average number of each puppy type born in a typical litter using your simulation results.
- **d** A breeder can make $500 from every plain or flashy puppy but only $200 from every white puppy.
  - **i** Calculate how much money could be potentially made over the 10 litters you simulated.
  - **ii** Find the average amount of money made per litter.
  - **iii** Find the average amount of money made by breeding a dog if on average she has five litters.

7 Often there is a promotion where one in every six Mars Bars wrappers wins a free bar.

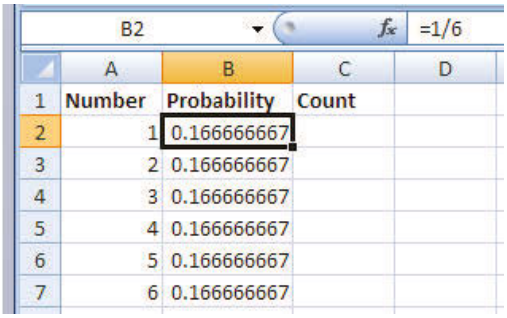
- **a** Explain why even though there are only two outcomes (winning and not winning), you could treat this problem like it has six outcomes.
- **b** What device could you use to simulate winning a free bar? Assign each device outcome to a situation outcome. (Hint: pick one outcome to represent winning and the rest will represent not winning.)
- **c** Find the expected number of Mars Bars that you would win if you bought 30.
- **d** Perform 30 trials and record your results. How do they compare to the expected numbers?
- **e** What would you expect to see in your experiment if you did 3000 trials? Explain.
- **f** What limitations might there be in using a die to model winning free Mars Bars?

8 Running 3000 trials of the experiment from question **7** would take far too long. You could use a random number generator instead to simulate winning free Mars Bars. One way to generate random numbers is to use a spreadsheet.

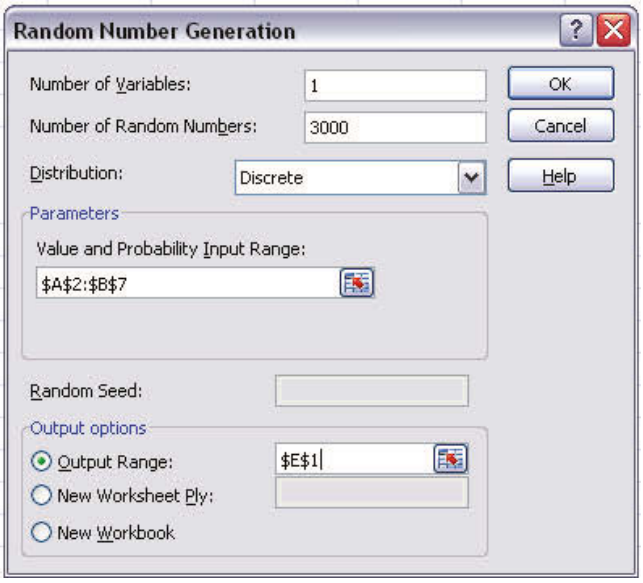
a Open a new file in Excel. To generate individual random numbers, click on a cell and enter **=randbetween(1,6)**. What do you get?

b Do this 30 times and record your results. Do they appear to be random?

c Clear the spreadsheet and set it up as shown.



You want your numbers to be generated into column E. To do this the Analysis Toolpak needs to be loaded. If it is not, try to install it by following the instructions given under Help. Otherwise you may not be able to complete this question.



d Find the Data Analysis button and click Random Number Generation. Complete the dialog box as shown here and then click OK. As you do this, think about what each part means.

e In cell C2, enter **=countif(E1:E3000,1)**. This will count all the 1s in the numbers you have generated.

f In cell C3, enter **=countif(E1:E3000,2)** to count all the 2s. Continue the pattern for C4 to C7.

g Which number did you pick in question **7** to represent winning a Mars Bar? How many times did this number come up in your simulation?

h Find the experimental probability for your results and compare it to the theoretical probability. How does this compare to your results from question **7**?

i There are other devices that generate random numbers, such as CAS calculators and programs on the Internet. Research these and see if you can generate random numbers.

PROBLEM SOLVING AND REASONING

9 The Earth's surface is roughly 70% water.



a If a meteorite was to crash into Earth, state the theoretical probability of it landing:

i in water

ii on land.

b Out of every 100 meteorites to reach Earth, how many would you expect to land on dry land?

c Choose a device to simulate where a meteorite will land on Earth and assign each device outcome to a situation outcome. (Hint: use a device with ten outcomes.)

d Perform 100 trials of simulation and record your results.

e What do you find? Does it match the theoretical probabilities?

f Optional: use the Internet or other means to research the probability of a meteorite crashing to Earth. How is this different from what we have been calculating?

CHALLENGE

10 A couple want to have children, and would like to have at least one boy and one girl.

a What device would you use to simulate gender at birth? Assign each device outcome to a situation outcome.

b Perform as many trials as you need to until you have simulated at least one child of each gender. How many trials did this take?

c Repeat this experiment nine times (to give a total of 10) and put your results into a table.

d Find the average number of children the couple will have in order to have at least one of each gender.

e Are you confident in your predictions? What could you do to be more confident?

f Repeat this experiment another 40 times and tabulate your results. Find the average number of children the couple will have to have based on these results.

g Compare this to your results from part **d**. What do you find?

11 Like in question **7**, a promotion is running where one in six Mars Bars wins a free bar. Follow the steps of question **10** to help you find the average number of Mars Bars you will need to buy in order to win at least:

a one free bar **b** three free bars.

12 A town has 36 births in a year and 29 of these are boys. Is this variation acceptable? Use repeated trials of a coin simulation to support your answer. What other factors might affect gender of newborns?

Reflect

How is simulation related to long-term trends?

CHAPTER REVIEW

SUMMARISE

Create a summary of this chapter using the key terms below. You may like to write a paragraph, create a concept map or use technology to present your work.

probability
probability scale
experiment
event
sample space
theoretical probability
favourable
complementary
tree diagram
two-way tables
Venn diagram
experimental probability
relative frequency
expected number
simulation

MULTIPLE-CHOICE

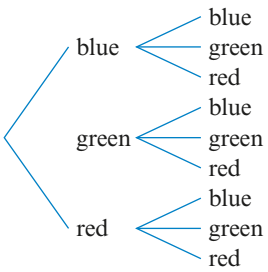
10A **1** Which event could be described as unlikely?

A winning Tattslotto
B a boy being born
C Tuesday following Monday
D selecting a club from a deck of cards

10B **2** What is the complementary probability to rolling a 1 or 2 on a standard die?

A $\frac{1}{6}$ **B** $\frac{2}{6}$ **C** $\frac{3}{6}$ **D** $\frac{4}{6}$

10C **3** Using this tree diagram, what is the probability of selecting at least one blue marble?



A $\frac{1}{9}$ **B** $\frac{3}{9}$
C $\frac{4}{9}$ **D** $\frac{5}{9}$

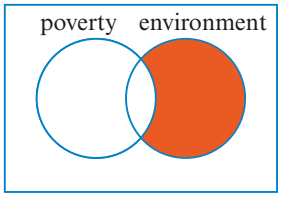
10D **4** Consider this two-way table.

	Brown eyes	Not brown eyes	Total
Blond	3	12	15
Not blond	19	16	35
Total	22	28	50

What is the probability of a person chosen randomly from the group having brown eyes?

A $\frac{3}{22}$ **B** $\frac{3}{50}$ **C** $\frac{19}{50}$ **D** $\frac{11}{25}$

10E **5** This Venn diagram represents students who are concerned about two issues. What does the shaded section represent?



A the environment
B poverty and the environment
C the environment only
D everything except the environment.

10F **6** Two coins were flipped 20 times. What is the expected number of times at least one head would appear?

A 15 **B** 10 **C** 5 **D** 0

10G **7** Which situation could be simulated using a standard die?

A a baby born and its gender recorded
B randomly selecting a marble from a bag containing two blue, four red, five green and three yellow marbles.
C randomly guessing the answer to this question
D a winner randomly selected from six finalists

SHORT ANSWER

10A **1** For each experiment:

i list the sample space

ii state the most likely outcome or if all outcomes are equally likely.

a rolling a die and recording the outcome

b flipping two coins and recording the outcome

c selecting a card from a deck and recording whether it is a picture card or not.



10B **2** Consider this spinner.

a What is the theoretical probability of spinning blue?

b What is the theoretical probability of spinning green or red?

c Find the theoretical probability of the complementary event to part **b**.

10C **3** A die is rolled three times and the outcome is recorded as odd or even.

a Draw a tree diagram to represent this.

b Find the probability of rolling:

i an even number all three times

ii an even number once only

iii at least one even number.

10D **4** Consider this two-way table.

	Like Vegemite	Don't like Vegemite	Total
Like Milo	97	56	153
Don't like Milo	18	29	47
Total	115	85	200

What is the probability that a person chosen randomly from the group:

a likes Vegemite?

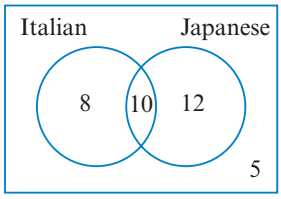
b likes Vegemite but not Milo?

c likes Milo but not Vegemite?

d doesn't like Milo?

e likes Milo, given that they like Vegemite?

10E **5** Consider this Venn diagram.



a How many people were surveyed?

b How many people like Italian food?

c What is the probability that a person chosen randomly from the group:

i likes both food types?

ii likes Italian food only?

iii does not like Japanese food?

iv likes Japanese food, given that they like Italian food?

10F **6** Consider this spinner.



a Construct a table that lists all the outcomes and their theoretical probabilities.

b Calculate the expected number of each outcome if the spinner was spun 300 times.

Sanjay performed the experiment and recorded these results.

Outcome	Frequency
Blue	180
Yellow	30
Red	90

c Write a statement about the experimental probability of each outcome in comparison to the theoretical probability.

10G **7** A chocolate packet is randomly filled with either milk or white, and either solid or caramel-filled chocolates.

a List the possible outcomes and their theoretical probabilities.

b Choose a device to simulate the situation.

c Find how many, on average, you would have to eat if you wanted to eat at least one of each type.

d Discuss the limitations of using your device to simulate this situation.

NAPLAN-STYLE PRACTICE

1 Alicia has 10 highlighters in her pencil case, four yellow, two green, one blue and the rest are pink. What is the sample space of the pencil case?

- ☐ {yellow, green, blue}
- ☐ 10
- ☐ {4 yellow, 2 green, 1 blue, 3 pink}
- ☐ {yellow, green, blue, pink}

2 Peter flips two coins and records the outcome. What is the probability of flipping at least one head?

☐

Questions **3**–**5** refer to this spinner.



3 What is the probability of spinning yellow?

☐ $\frac{1}{4}$ ☐ $\frac{1}{2}$ ☐ $\frac{3}{7}$ ☐ $\frac{3}{8}$

4 What is the probability of spinning green or red?

☐ $\frac{1}{4}$ ☐ $\frac{1}{2}$ ☐ $\frac{2}{7}$ ☐ $\frac{1}{8}$

5 What is the complementary event to spinning green?

- ☐ red
- ☐ yellow, red or blue
- ☐ $\frac{7}{8}$
- ☐ $\frac{1}{8}$

6 An event, A, has a probability of 0.2. What is the probability of its complementary event, A′?

☐

Questions **7**–**9** refer to this two-way table.

	Year 7	Year 8	Total
Female	3	8	11
Male	2	7	9
Total	5	15	20

7 What is the probability that a person chosen at random from the group is a female in Year 8?

☐

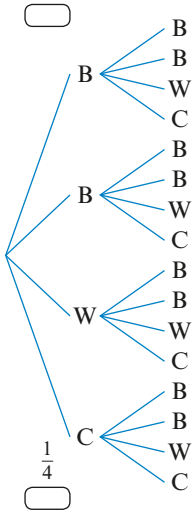
8 What is the probability that a person chosen at random from the group is male?

☐ $\frac{1}{4}$ ☐ $\frac{9}{20}$ ☐ $\frac{1}{10}$ ☐ $\frac{11}{20}$

9 What is the probability that a person chosen at random from the group is in Year 8, given that they are male?

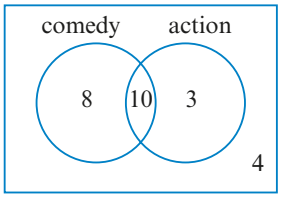
☐ $\frac{7}{9}$ ☐ $\frac{7}{20}$ ☐ $\frac{9}{20}$ ☐ $\frac{7}{15}$

10 A drawer contains loose socks that are black, white or coloured. There are twice as many black socks as there are white or coloured socks. Two socks are selected at random from the drawer, as shown in this tree diagram. What is the probability that the two socks drawn will match?



☐ $\frac{1}{3}$ ☐ $\frac{1}{16}$ ☐ $\frac{3}{8}$ ☐ $\frac{1}{4}$

Questions **11** and **12** refer to this Venn diagram.

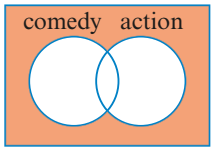


11 What is the probability that a person randomly chosen from the group likes comedy films?

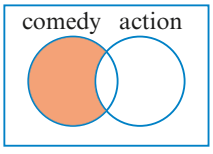
☐ $\frac{18}{25}$ ☐ $\frac{8}{25}$ ☐ $\frac{8}{21}$ ☐ $\frac{18}{21}$

12 Which diagram represents people who do not like action films?

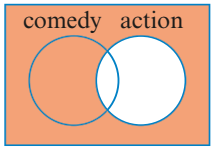
☐ A



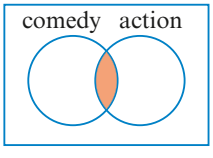
☐ B



☐ C



☐ D



13 Mateo rolled a die 30 times and obtained 11 sixes. He then calculated the experimental probability and made a prediction about what would happen to this experimental probability if he performed 1000 trials. What should his calculation and prediction be?

- ☐ experimental probability = $\frac{11}{30}$ which should increase with further trials.
- ☐ experimental probability = $\frac{11}{30}$ which should decrease with further trials.
- ☐ experimental probability = $\frac{1}{6}$ which should increase with further trials.
- ☐ experimental probability = $\frac{1}{6}$ which should decrease with further trials.

14 Two coins are flipped. If this is performed 20 times, what is the expected number of times that both coins would show tails?

☐

15 A new cereal offers a toy surprise with five different toys to find. One toy is twice as common as the other four.

Which table is the best example of potential results of a simulation involving 600 trials?

☐

Table A	
Toy 1	100
Toy 2	100
Toy 3	100
Toy 4	100
Toy 5	100

☐

Table B	
Toy 1	120
Toy 2	120
Toy 3	120
Toy 4	120
Toy 5	120

☐

Table C	
Toy 1	214
Toy 2	93
Toy 3	101
Toy 4	106
Toy 5	86

☐

Table D	
Toy 1	113
Toy 2	136
Toy 3	125
Toy 4	119
Toy 5	107

ANALYSIS

Eric is researching the popularity of Ford and Holden cars amongst Year 8 boys and girls. He thinks it will be reasonably evenly distributed.

a Draw a tree diagram to represent the situation. (Hint: start the first branches with gender.)

b List the sample space (the possible outcomes) and, assuming all are equally likely, provide the theoretical probability for each outcome.

Eric completes a survey and records these results.

	Ford	Holden
Boys	7	23
Girls	19	9

c Extend this table to make it a two-way table and find how many people were surveyed in total.

d Find the experimental probability of each outcome.

e Find the probability that a person chosen randomly from the group:

- **i** prefers Holdens to Fords
- **ii** is a boy
- **iii** prefers Holdens to Fords, given they are male.

f Do you think each outcome is equally likely? Explain.

Eric decides to simulate the situation by rolling a die. He assigns each outcome as in this table.

Boys who prefer Holden	1 or 2
Boys who prefer Ford	3
Girls who prefer Holden	4
Girls who prefer Ford	5 or 6

g Perform a simulation and comment on how well it represents the experimental data. What limitations exist?

h Challenge! Draw a Venn diagram based on the two-way table.

CONNECT



Real world probability

You often hear people talking about the 'chances' or 'likelihood' of an event or situation occurring. Mathematics can be used to model real-life situations quite accurately if it is used correctly.

Some things to consider when choosing a real-life situation to investigate are:

- whether you can list and describe different outcomes
- whether you can represent theoretical probabilities in a tree diagram and a table
- whether you can represent experimental data in a two-way table and a Venn diagram
- whether you can perform a simulation with a simple device or with technology.

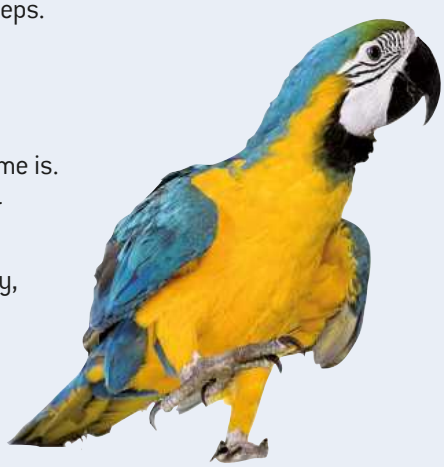
You may choose to investigate something as simple as the probability of hair and eye colour or something more complicated like the probability of developing a medical condition.

You will need to use all the skills you have learnt in this chapter.

Your task

For your investigation of real-world probability, complete these steps.

- Describe the situation you have chosen.
- List each possible outcome and describe the probability of it occurring using a probability scale.
- State what you think the theoretical probability of each outcome is.
- Conduct a survey or research for information to serve as your experimental data.
- Design a simulation using either a simple device or technology, assigning each device outcome to a situation outcome.
- Perform a simulation of a large but feasible number of trials.
- Write a report detailing your predictions and how your experimental and simulation data compare to these.
- Include at least one tree diagram, two-way table and Venn diagram in your report.









Complete the 10 CONNECT worksheet to show all your working and answers to this task.

You may like to present your findings as a report. Your report could be in the form of:

- a PowerPoint slideshow
- a Flash tutorial
- a poster
- other (check with your teacher).





Answers

CHAPTER 1 NUMBER SKILLS

1 Are you ready?

1 **a** D **b** C
2 **a** 2600 **b** 5100 **c** 10
3 **a** D **b** A **4** **a** D **b** 81
5 **a** B **b** 6 **6** **a** B **b** $\frac{38}{7}$
7 **a** C **b** 20 **8** **a** C **b** 12
9 **a** A **b** $\frac{7}{20}$ **10** **a** C **b** D
11 **a** 0.071 **b** 4.03 **c** 0.251
12 **a** D **b** A
13 **a** 14.184 **b** B
14 **a** **i** 5 **ii** 3 **b** C

1A Estimating and rounding

1A Start thinking!

1 800, as 826 is 26 more than 800 but 74 less than 900.
2 400, as 385 is 15 less than 400 but 85 more than 300.
3 800 4 400
5 Shows whether to keep the leading digit the same or increase it by one in the approximate value. If second digit is 0, 1, 2, 3 or 4, first digit stays the same and digits that follow are replaced with zero. If second digit is 5, 6, 7, 8 or 9, first digit increases by one and digits that follow are replaced with zero.
6 no, has no effect on leading digit
7 320 000
8 Not reasonable as the amount collected is approx. $30 000 less than the total amount required.

Exercise 1A Estimating and rounding

1 **a** 400 **b** 500 **c** 500 **d** 500
e 400 **f** 400
2 **a** 7000 **b** 6000 **c** 6000 **d** 7000
e 7000 **f** 6000
3 **a** 80 **b** 40 **c** 80 **d** 300
e 200 **f** 600 **g** 900 **h** 2000
i 7000 **j** 20 000 **k** 1000 **l** 10 000
4 **a** 90 **b** 100 **c** 1000 **d** 90 000
e 9000 **f** 1000
5 Some possible answers are given.
a 37, 42, 35 **b** 692, 705, 744
c 2995, 3125, 3451 **d** 52 168, 46 854, 50 369
e 795 002, 755 036, 827 039
f 1 758 364, 2 498 567, 2 215 000
6 **a** 1600 **b** 290 **c** 600
d 20 **e** 20 000 **f** 3800
g 25 **h** 5500 **i** 2000
j 2 100 000 **k** 11 000 **l** 15 000
7 **a** 1200 **b** 50 **c** 1500
d 40 **e** 12 000 **f** 6200
g 125 **h** 50 **i** 10
j 1 800 000 **k** 6200 **l** 7000
m 560 000 000 **n** 18 000 **o** 150
p 19 200
8 $3000
9 Tina is correct. Since leading digit is 9 and the second digit is 6, first digit increases by one and digits that follow are replaced with zero. Increasing 9 by one gives 10 so there are a total of five digits in the answer.
10 **a** **i** 480 **ii** 6380 **iii** 56 030
iv 738 490 **v** 8080 **vii** 904 510
b **i** 500 **ii** 6400 **iii** 56 000
iv 738 500 **v** 8100 **vii** 904 500
11 **a** **i** 36 000 **ii** 8000 **iii** 184 000
iv 51 000 **v** 6000 **vii** 1000
b **i** 36 400 **ii** 7500 **iii** 183 900
iv 50 700 **v** 6100 **vii** 800
12 **a** 40 000 **b** 35 800 **c** 36 000
d 35 780 **e** 40 000
13 **a** 20 000 000 km^2 **b** 17 075 200 km^2
c 17 075 000 km^2 **d** 17 080 000 km^2
e 17 100 000 km^2 f 17 000 000 km^2
14 **a** 30 **b** $1800 **c** no
d 20 months
15 **a** 400 **b** $2800
c $2695; estimate is $105 more
16 **a** 40
b Estimate of 40 would be lower than actual number of boxes as approximate price ($50) is higher than actual price ($47). There would be money left over.
17 **a** Australia: 8 000 000 km^2
Indonesia 2 000 000 km^2
b **i** 6 000 000 km^2
ii 4 times the area of Indonesia.
c **i** 5 772 093 km^2 **ii** 4.006
18 **a** 48 000 **b** 52 196 **c** approximate
19 **a** WA: 3 000 000 km^2, Qld: 2 000 000 km^2, NT: 1 000 000 km^2, SA: 1 000 000 km^2, NSW: 800 000 km^2, Vic: 200 000 km^2, Tas: 70 000 km^2, ACT: 2000 km^2
b 8 072 000 km^2

c WA: 2 530 000 km^2, Qld: 1 731 000 km^2, NT: 1 349 000 km^2, SA: 983 000 km^2, NSW: 801 000 km^2, Vic: 227 000 km^2, Tas: 68 000 km^2, ACT: 2000 km^2
d 7 691 000 km^2
e Values rounded to nearest thousand produce estimates closer to accepted values.

20 any five numbers from 550 000 to 649 999

21 Some possible answers are given.
a 5706, 5689, 5651, 5726
b 836 487, 836 139, 835 598, 835 760

1B Order of operations

1B Start thinking!

1 The four operations are: squaring 10, multiplication, addition and subtraction.

2, 3 Hanna performed each operation moving from left to right; Josh performed each operation moving from left to right and performed the square of 10 incorrectly; Imogen performed the operations in the correct order.

4 Each letter in BIDMAS stands for an operation given in correct order to follow.

5 380; Imogen

Exercise 1B Order of operations

1 a multiplication, subtraction, addition
b subtraction (within brackets), division, addition
c squaring, multiplication, addition, subtraction
d squaring, multiplication, subtraction, addition
e addition (within inner brackets), subtraction (within brackets), addition
f addition (within brackets), squaring, division, addition

2 a 23 **b** 40 **c** 75 **d** 26 **e** 17 **f** 47
g 10 **h** 0 **i** 18 **j** 45

3 a 2 **b** 390 **c** 123 **d** 36 **e** 51 **f** 7

4 a 18 **b** 53 **c** 19 **d** 17 **e** 12 **f** 40

6 a 26
b **i** 22 **ii** 351 **iii** 72
iv 3 **v** 2 **vi** 34
vii 1904 **viii** 11 **ix** 34

7 a **i** 9 **ii** 9 **b** no
c **i** 3
ii −3 Subtraction does not obey the commutative law.
d **i** 18
ii 18 Multiplication obeys the commutative law.
e **i** 2
ii 0.5 Division does not obey the commutative law.

8 a **i** 18 **ii** 18 **iii** 18 **b** no
c Multiplication obeys the associative law, but subtraction and division do not. For example, multiplication:
$5 \times 4 \times 2 = 4 \times 5 \times 2 = 2 \times 5 \times 4 = 40$;
subtraction: $(7 - 2) - 3 = 2$ and $7 - (2 - 3) = 8$ (does not obey the associative law);
division: $(8 \div 4) \div 2 = 1$ and $8 \div (4 \div 2) = 4$ (does not obey the associative law).

9 a **i** 54 + 16, since the result (70) is a multiple of 10 and easier to add. **ii** 188
b **i** 25 × 4, since the result (100) is easier to multiply by. **ii** 8700
c As order is not important, any two of the three numbers can be added or multiplied first. If this results in a number that is a multiple of 10, it can make the final operation easier.

10 a 58 **b** 289 **c** 134 **d** 266
e 5600 **f** 92 100 **g** 700 **h** 29 000
i 186 000

11 a **i** 44 **ii** 20 + 24 = 44
b **i** 48 **ii** 60 − 12 = 48
c Each number inside brackets is multiplied by number at front of brackets.
d One possible answer is:
$10 \times (5 + 4) = 10 \times 5 + 10 \times 4 = 50 + 40 = 90$
$10 \times (5 + 4) = 10 \times 9 = 90$

12 a **i** 320 **ii** 64 **iii** 384
b **i** 45 000 **ii** 90 **iii** 44 910
c Write one of the numbers to be multiplied as either a sum or difference that involves a multiple of 10 or power of 10 and a single digit number.

13 a 882 **b** 5508 **c** 6496 **d** 4270
e 1666 **f** 8217 **g** 2280 **h** 779
i 6262 **j** 29 029 **k** 1067 **l** 14 985

14 a Titans: $32 \times 2 + 7 \times 3 + 4 \times 1$;
Giants: $35 \times 2 + 5 \times 3 + 2 \times 1$
b Titans 89; Giants 87
c Titans won by 2 points.

15 a $195 - 3 \times 58$ **b** $200 - 3 \times 60 = 20$; \$20
c \$21; estimate is \$1 less.

16 $14 \times 7 + 11 \times 6 + 9 \times 4 + 16 \times 5 + 25 \times 1 = \$305$

17, 18 Some possible answers are given.

Result	Using 4	Using 3	Using 7
1	$4 \div 4$	$3 \div 3$	$7 \div 7$
2	$(4 + 4) \div 4$	$3 - 3 \div 3$	$(7 + 7 \div 7) - (7 - 7 \div 7)$
3	$4 - 4 \div 4$	$3 + 3 - 3$	$(7 + 7 + 7) \div 7$
4	$4 \times 4 \div 4$	$3 + 3 \div 3$	$(7 + 7 + 7 + 7) \div 7$
5	$4 + 4 \div 4$	$(3 - 3 \div 3) + 3$	$(7 + 7 + 7 + 7 + 7) \div 7$
6	$(4 + 4 \div 4) + 4 \div 4$	$3 + 3$	$7 - 7 \div 7$
7	$4 - 4 \div 4 + 4$	$3^2 - (3 - 3 \div 3)$	$7 \div 7 \times 7$
8	$4 + 4$	$3^2 - 3 \div 3$	$7 + 7 \div 7$
9	$(4 + 4) + 4 \div 4$	3^2	$(7 \times 7 + 7 + 7) \div 7$
10	$(4 + 4 \div 4) + (4 + 4 \div 4)$	$3^2 + 3 \div 3$	$[7 \times 7 + (7 + 7 + 7)] \div 7$

1C Understanding fractions

1C Start thinking!

1 

2 $\frac{1}{3}$

3, 4 

five slices

5 $\frac{5}{18}$

6 Yes; the same proportion is shaded in each rectangle.

7 a $\frac{4}{12}$ b $\frac{6}{18}$

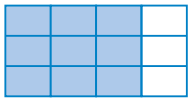
8 Both numerator and denominator have been multiplied by 4.

9 Both numerator and denominator have been multiplied by 6.

Exercise 1C Understanding fractions

1 a $\frac{3}{4}$ b One possible answer is given.

c $\frac{9}{12}$



2 a $\frac{3}{10} = \frac{6}{20}$ b $\frac{2}{7} = \frac{10}{35}$ c $\frac{18}{33} = \frac{6}{11}$ d $\frac{16}{48} = \frac{2}{6}$

e $\frac{4}{5} = \frac{24}{30}$ f $\frac{8}{36} = \frac{2}{9}$ g $1\frac{3}{8} = \frac{55}{40}$ h $3\frac{3}{4} = \frac{75}{20}$

i $\frac{1}{4} = \frac{2}{8} = \frac{5}{20} = \frac{8}{32} = \frac{20}{80} = \frac{25}{100}$

j $\frac{3}{7} = \frac{15}{35} = \frac{21}{49} = \frac{30}{70} = \frac{300}{700} = \frac{90}{210}$

k $1\frac{1}{5} = \frac{12}{10} = \frac{60}{50} = \frac{300}{250} = 1\frac{9}{45} = \frac{72}{60}$

l $2\frac{1}{3} = \frac{14}{6} = \frac{35}{15} = \frac{49}{21} = 2\frac{3}{9} = \frac{700}{300}$

3 a $\frac{8}{20}, \frac{22}{55}, \frac{4}{10}, \frac{10}{25}, \frac{18}{45}$ b $\frac{20}{15}, \frac{28}{21}, 1\frac{4}{12}$

4 a $\frac{4}{5}$ b $\frac{2}{3}$ c $\frac{7}{9}$ d $\frac{5}{8}$ e $\frac{10}{3}$ (or $3\frac{1}{3}$)

f $\frac{11}{6}$ (or $1\frac{5}{6}$) g $3\frac{3}{4}$ h $5\frac{5}{9}$ i $4\frac{3}{11}$

j $1\frac{4}{7}$ k $2\frac{3}{8}$ l $7\frac{5}{6}$

5 a i $\frac{2}{5}$ ii $\frac{3}{7}$ iii $\frac{2}{5}$ iv $\frac{5}{3}$ (or $1\frac{2}{3}$) v $\frac{8}{9}$ vi $\frac{3}{7}$

b $\frac{6}{15}$ and $\frac{17}{45}$, $\frac{33}{77}$ and $\frac{18}{42}$

6 An equivalent fraction is created by dividing (or multiplying) numerator and denominator by same value. When simplifying a fraction divide by HCF.

7 Some possible answers are given.

a $\frac{6}{10}, \frac{9}{15}, \frac{30}{50}$ b $\frac{14}{8}, \frac{21}{12}, \frac{70}{40}$

c $\frac{10}{12}, \frac{20}{24}, \frac{35}{42}$ d $\frac{2}{3}, \frac{4}{6}, \frac{24}{36}$

8 Some possible answers are given.

a $1\frac{4}{10}, 1\frac{10}{25}, 1\frac{20}{50}$ b $3\frac{3}{4}, 3\frac{9}{12}, 3\frac{36}{48}$

c $1\frac{6}{14}, 1\frac{9}{21}, 1\frac{33}{77}$ d $2\frac{1}{3}, 2\frac{6}{18}, 2\frac{12}{36}$

9 Some possible answers are: $\frac{22}{33}, \frac{4}{6}, \frac{20}{30}, \frac{80}{120}$

10 a $\frac{7}{9}$ b $\frac{11}{4}$ c $2\frac{3}{5}$ d $7\frac{5}{6}$ e $3\frac{7}{11}$ f $\frac{29}{2}$

g $\frac{16}{3}$ h $6\frac{2}{9}$ i $3\frac{5}{7}$

11 a $\frac{1}{2}$ b $\frac{4}{7}$ c $\frac{9}{10}$ d $\frac{2}{3}$ e $1\frac{5}{6}$ f $\frac{7}{3}$

g $\frac{3}{4}$ h $\frac{5}{6}$ i $\frac{5}{3}$ j $\frac{17}{6}$ k $\frac{29}{3}$ l $3\frac{5}{12}$

12 a $\frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \frac{4}{5}, 1\frac{1}{5}, \frac{7}{5}$ b $\frac{3}{8}, \frac{9}{8}, 1\frac{1}{4}, \frac{7}{4}, 2\frac{5}{8}, 2\frac{3}{4}$

c $\frac{3}{10}, \frac{2}{5}, 1\frac{1}{2}, \frac{9}{5}, 1\frac{9}{10}, \frac{9}{2}$ d $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, 1\frac{1}{4}, \frac{4}{3}, 1\frac{1}{2}$

13 a $\frac{19}{11}, 1\frac{5}{11}, \frac{13}{11}, \frac{9}{11}, \frac{5}{11}, \frac{3}{11}$ b $2\frac{3}{4}, 2\frac{5}{8}, \frac{7}{4}, 1\frac{1}{4}, \frac{9}{8}, \frac{3}{8}$

c $\frac{9}{2}, 1\frac{9}{10}, \frac{9}{5}, 1\frac{1}{2}, \frac{2}{5}, \frac{3}{10}$ d $1\frac{1}{2}, \frac{4}{3}, 1\frac{1}{4}, \frac{3}{4}, \frac{2}{3}, \frac{1}{2}$

14 a $\frac{3}{5}$ b $\frac{2}{9}$ c $\frac{7}{24}$ d $\frac{3}{10}$ e $\frac{1}{6}$ f $\frac{2}{5}$

g $\frac{2}{7}$ h $\frac{3}{20}$ i $\frac{7}{15}$ j $\frac{3}{17}$ k $\frac{1}{30}$ l $\frac{2}{15}$

15 Some possible answers are given.

a $\frac{1}{4}, \frac{1}{2}, \frac{2}{5}$ b $\frac{5}{7}, \frac{3}{5}, \frac{7}{8}$ c $\frac{3}{4}, \frac{4}{3}, \frac{9}{4}$ d $\frac{17}{4}, \frac{13}{3}, \frac{27}{5}$

e $\frac{3}{4}, \frac{1}{2}, \frac{4}{5}$ f $\frac{4}{5}, \frac{11}{10}, 1\frac{1}{20}$ g $\frac{7}{4}, \frac{11}{5}, 1\frac{3}{4}$ h $\frac{21}{5}, \frac{22}{5}, \frac{30}{7}$

16 a $\frac{4}{7}$ b $\frac{7}{11}$ c $\frac{1}{2}$ d $\frac{4}{9}$ e $\frac{1}{3}$ f $\frac{5}{8}$

g $1\frac{7}{24}$ h $2\frac{14}{15}$

17 a $\frac{3}{5}$ b $\frac{3}{4}$ c $\frac{5}{8}$ d $\frac{2}{3}$

18 a i $\frac{1}{8}$ ii $\frac{1}{4}$ iii $\frac{16}{25}$ iv $\frac{1}{5}$

b, d Equivalent fractions are produced by multiplying or dividing both numerator and denominator of fraction by same value.

c i $\frac{7}{8}$ ii $\frac{3}{4}$ iii $\frac{9}{25}$ iv $\frac{4}{5}$

19 $\frac{1}{4}$ 20 first test

21 a $\frac{4}{10} = \frac{2}{5}$ d $\frac{4}{8} = \frac{1}{2}$

e Some possible answers are: length of Gene Simmons' tongue compared to length of Stephen Taylor's tongue $\frac{8}{10} = \frac{4}{5}$; length of Stephen Taylor's tongue compared to Gene Simmons' tongue $\frac{10}{8} = \frac{5}{4}$ (or $1\frac{1}{4}$); your tongue length compared to Gene Simmons' tongue.

22 15

23 a Maria (lower value is better rate in serving)

b 10 c 153

24 Some possible answers are: 9 and 4; 36 and 3.

25 Some possible answers are: 40, 8 and 5; 20, 8 and 5.

1D Operations with fractions

1D Start thinking!

1 addition: $\frac{3}{4} + \frac{2}{3} = \frac{9}{12} + \frac{8}{12} = \frac{17}{12} = 1\frac{5}{12}$;

subtraction: $\frac{3}{4} - \frac{2}{3} = \frac{9}{12} - \frac{8}{12} = \frac{1}{12}$;

multiplication: $\frac{3}{4} \times \frac{2}{3} = \frac{1}{2} \times \frac{1}{1} = \frac{1}{2}$;

division: $\frac{3}{4} \div \frac{2}{3} = \frac{3}{4} \times \frac{3}{2} = \frac{9}{8} = 1\frac{1}{8}$.

2 To add fractions with different denominators, first find the equivalent fractions with the LCD then add the numerators.

3 a To subtract fractions with different denominators, first find the equivalent fractions with the LCD then subtract the numerators.

b To multiply fractions, look for any factors common to numerator and denominator. Cancel a number in numerator and a number

in denominator by dividing both numbers by HCF. Then multiply numerators together and denominators together.

c To divide fractions, change from a division to a multiplication problem by replacing division sign with a multiplication sign and turning the fraction that follows upside down. Then proceed as for a multiplication problem.

4 The denominators of two fractions do not have to be the same when they are being multiplied or divided. The statement is false.

Exercise 1D Operations with fractions

1 **a** $\frac{13}{17}$ **b** $\frac{7}{20}$ **c** $1\frac{17}{30}$ **d** $\frac{7}{24}$ **e** $\frac{1}{30}$ **f** $1\frac{7}{12}$
g $\frac{3}{8}$ **h** $3\frac{1}{4}$ **i** $4\frac{7}{15}$ **j** $2\frac{3}{22}$ **k** $7\frac{5}{12}$ **l** $2\frac{7}{10}$

2 **a** **i** $1\frac{1}{2}$ **ii** $1\frac{1}{2}$ **iii** $1\frac{1}{2}$
b **i** $1\frac{1}{2}$ **ii** $1\frac{1}{2}$ **iii** $1\frac{1}{2}$
c Each diagonal adds to $1\frac{1}{2}$.
d all the same **e** $1\frac{1}{2}$

2 **a**

$\frac{7}{12}$	$\frac{1}{6}$	$\frac{1}{4}$
0	$\frac{1}{3}$	$\frac{2}{3}$
$\frac{5}{12}$	$\frac{1}{2}$	$\frac{1}{12}$

b

$\frac{2}{15}$	$\frac{7}{15}$	$\frac{2}{5}$
$\frac{3}{5}$	$\frac{1}{3}$	$\frac{1}{15}$
$\frac{4}{15}$	$\frac{1}{5}$	$\frac{8}{15}$

c

$\frac{1}{6}$	$\frac{3}{8}$	$\frac{1}{12}$
$\frac{1}{8}$	$\frac{5}{24}$	$\frac{7}{24}$
$\frac{1}{3}$	$\frac{1}{24}$	$\frac{1}{4}$

4 **a** $\frac{3}{20}$ **b** $\frac{6}{5}$ (or $1\frac{1}{5}$) **c** $\frac{10}{21}$ **d** $\frac{5}{8}$
e $\frac{7}{10}$ **f** $\frac{3}{8}$ **g** $\frac{9}{8}$ (or $1\frac{1}{8}$) **h** $6\frac{3}{5}$
i 4 **j** $\frac{9}{16}$ **k** 10 **l** $\frac{1}{2}$

5 **a** $\frac{7}{18}$ **b** $\frac{31}{40}$ **c** $\frac{12}{35}$ **d** $4\frac{1}{2}$
e $\frac{3}{4}$ **f** $4\frac{1}{24}$ **g** $\frac{2}{9}$ **h** $4\frac{1}{10}$
i $5\frac{23}{28}$ **j** $12\frac{4}{5}$ **k** $1\frac{17}{33}$ **l** $1\frac{1}{2}$

6 **a** $1\frac{5}{11}$ **b** $\frac{3}{2}$ (or $1\frac{1}{2}$) **c** $\frac{1}{60}$ **d** $8\frac{11}{12}$

7 **a** 7 hours **b** \$36 **c** 14 kg
d 32 cm **e** 6 days **f** 12 mm
g 600 m **h** 200 s **i** 4000 mL
j 75 cents **k** 3500 g **l** 40 minutes

8 $\frac{1}{3} + 1\frac{3}{4} = 2\frac{1}{12}$ L; container not large enough

9 $4\frac{7}{12}$ hours

10 **a** $\frac{3}{10}$ **b** $\frac{29}{30}$ **c** $\frac{1}{30}$

11 66 **12** $\frac{1}{6}$

13 **a** **i** 200 g butter, 1 cup sugar, 4 large eggs, 3 cups flour, 2 tspns baking powder, 1 cup milk, $1\frac{1}{2}$ cups cocoa powder, 2 cups boiling water
ii 50 g butter, $\frac{1}{4}$ cup sugar, 1 large egg, $\frac{3}{4}$ cup flour, $\frac{1}{2}$ tspn baking powder, $\frac{1}{4}$ cup milk, $\frac{3}{8}$ cup cocoa powder, $\frac{1}{2}$ cup boiling water
iii $66\frac{2}{3}$ g butter, $\frac{1}{3}$ cup sugar, $1\frac{1}{3}$ large eggs, 1 cup flour, $\frac{2}{3}$ tspn baking powder, $\frac{1}{3}$ cup milk, $\frac{1}{2}$ cup cocoa powder, $\frac{2}{3}$ cup boiling water
b $1\frac{1}{8}$ cups icing sugar, $112\frac{1}{2}$ g butter, 90 g chocolate, $\frac{3}{16}$ cup thickened cream

14 3 **15** **a** 8 **b** 29 kg

16 **a** $\frac{13}{125}$ inches **b** $\frac{41}{250}$ inch screw

For questions **17**–**21**, one possible answer is given.

17 $\frac{1}{5} + \frac{7}{20}$ **18** $1\frac{7}{12} - \frac{3}{4}$ **19** $\frac{3}{5} \times \frac{5}{8}$ **20** $\frac{7}{16} \div \frac{3}{4}$

21 Some possible answers are given.
a **i** $\frac{5}{4} + \frac{3}{2}$ **ii** $\frac{9}{2} - \frac{7}{4}$
iii $1\frac{19}{36} \times \frac{9}{5}$ **iv** $1\frac{23}{32} \div \frac{5}{8}$
b $\left(\frac{5}{7} + \frac{3}{4}\right) \times 1\frac{36}{41}$ or $\frac{3}{4} \times \frac{4}{7} + \frac{13}{7} \times \frac{5}{4}$

1E Understanding decimals

1E Start thinking!

1 **a** **i** 5 **ii** 6
b 2
c 11.704 has 3, 12.460 has 3, 12.08 has 2

2 Largest is 12.64 m, smallest is 11.704 m. Numbers are ordered by comparing corresponding place values, regardless of how many decimal places the number has.

3 **a** It would represent a different number.
b So the value of the decimal number does not change.
c no change to value of number
d **i** 11.704 m, 12.08 m **ii** 12.460 m

Exercise 1E Understanding decimals

1 **a** 3 **b** 5 **c** 4 **d** 6 **e** 3 **f** 4
g 1 **h** 8 **i** 3

2 **a** P **b** T **c** P **d** B **e** T **f** B

3 **a** 3 **b** 3 **c** 1 **d** 2 **e** 4 **f** 6

4 **a** 5.460 00 **b** 76.300 00 **c** 0.306 00
d 29.784 30 **e** 9.000 00 **f** 15.000 00

5 **a** 0.104, 0.14, 0.142, 0.2, 0.214, 0.41
b 8.072, 8.2, 8.702, 8.97, 9.782, 9.87

6 **a** 13.713, 13.7, 13.31, 13.173, 13.137, 13.07
b 0.685, 0.658, 0.6508, 0.65, 0.6085, 0.6058

7 Comparing each place value from left to right, 9.46 contains 6 hundredths whereas 9.452 only contains 5 hundredths. (The first two place values are identical.)

8 **a** 25.68 **b** 673.82 **c** 0.78
d 101.02 **e** 8148.26 **f** 54.26
g 330.41 **h** 4.85 **i** 71.01

9 **a** 57.23 **b** 8.914 **c** 92.0871
d 0.757 **e** 135.2 **f** 65.071 23
g 7.510 **h** 208.0099 **i** 1.727 73
j 845.8 **k** 0.1235 **l** 0.123 456 8

10 Rosa; as digit in fourth decimal place is 5 so you must round up.

11 Jayden; as digit in third decimal place is 6 so you must round up. Adding one to digit in second decimal place gives 10 which means you write 0 in second decimal place and add one to first decimal place (7 + 1 = 8).

12 **a** 6.813 **b** 0.742 **c** 21.831
d 4.080 **e** 30.300 **f** 16.000

13 **a** $\frac{3}{100}$ **b** $\frac{9}{10}$ **c** $\frac{7}{1000}$ **d** $\frac{1}{10\,000}$
e $\frac{7}{10}$ **f** $\frac{3}{100\,000}$ **g** $\frac{9}{1\,000\,000}$ **h** $\frac{7}{10\,000\,000}$

14 **a** 0.7 **b** 0.29 **c** 0.451 **d** 0.2047
e 0.04 **f** 0.009 **g** 0.0005 **h** 0.063
i 0.0019 **j** 0.0368 **k** 0.2 **l** 0.07

15 **a** $0.27 = \frac{2}{10} + \frac{7}{100}$ or $\frac{27}{100}$
b $0.589 = \frac{5}{10} + \frac{8}{100} + \frac{9}{1000}$ or $\frac{589}{1000}$
c $0.343 = \frac{3}{10} + \frac{4}{100} + \frac{3}{1000}$ or $\frac{343}{1000}$
d $0.6251 = \frac{6}{10} + \frac{2}{100} + \frac{5}{1000} + \frac{1}{10\,000}$ or $\frac{6251}{10\,000}$
e $0.109 = \frac{1}{10} + \frac{9}{1000}$ or $\frac{109}{1000}$
f $0.0867 = \frac{8}{100} + \frac{6}{1000} + \frac{7}{10\,000}$ or $\frac{867}{10\,000}$

16 **a** 3, $\frac{3}{10}$, $\frac{5}{100}$, $\frac{7}{1000}$, $\frac{357}{1000}$ **b** 3 **c** 3
d **i** **a** 4, $\frac{7}{10}$, $\frac{2}{100}$, $\frac{5}{1000}$, $\frac{3}{10\,000}$, $\frac{7253}{10\,000}$
b 4 **c** 4
ii **a** 3, $\frac{6}{10}$, $\frac{9}{100}$, $\frac{1}{1000}$, $\frac{691}{1000}$ **b** 3 **c** 3
iii **a** 2, $\frac{4}{10}$, $\frac{9}{100}$, $\frac{49}{100}$ **b** 2 **c** 2
iv **a** 3, $\frac{1}{10}$, $\frac{0}{100}$, $\frac{3}{1000}$, $\frac{103}{1000}$ **b** 3 **c** 3
v **a** 1, $\frac{9}{10}$, $\frac{9}{10}$ **b** 1 **c** 1
vi **a** 3, $\frac{0}{10}$, $\frac{0}{100}$, $\frac{7}{1000}$, $\frac{7}{1000}$ **b** 3 **c** 3
e Number of decimal places in a number equals number of zeros in denominator when decimal is written in fraction form.

17 **a** $\frac{44}{125}$ **b** $\frac{1431}{2500}$ **c** $\frac{803}{1000}$ **d** $\frac{1}{25}$ **e** $\frac{19}{200}$
f $\frac{3}{5000}$ **g** $\frac{37}{50}$ **h** $\frac{63}{250}$ **i** $\frac{3}{2000}$ **j** $\frac{121}{2500}$
k $\frac{17}{100\,000}$ **l** $\frac{4001}{5000}$

18 **a** 1.568 m **b** 1.657 m
c 1.568 m, 1.58 m, 1.615 m, 1.63 m, 1.651 m, 1.657 m
d 1.65 m, 1.63 m, 1.57 m, 1.62 m, 1.58 m, 1.66 m
e 1.57 m, 1.58 m, 1.62 m, 1.63 m, 1.65 m, 1.66 m; measurements are still all different and no change to order.
f 1.7 m, 1.6 m, 1.6 m, 1.6 m, 1.6 m, 1.7 m
g 1.6 m, 1.6 m, 1.6 m, 1.6 m, 1.7 m, 1.7 m; only two different measurements.

19 **a** 0.02 mm **b** 0.018 mm **c** 0.0184 mm

20 **a** **i** 0.3 mm **ii** 0.28 mm **iii** 0.285 mm
b $\frac{569}{2000}$ mm

22 0.7 cm

23 **a** 2.735 **b** 2.734 83 **c** 2.734 829 95
d 2.7 **e** 2.734 830 **f** 2.734 830 0

24 **a** $3.44 **b** $28.08 **c** $117.21
d $59.12 **e** $62.75 **f** $0.92

25 **a** $42.35 **b** $207.60 **c** $7.25
d $15.05 **e** $60.70 **f** $100.00

26 Some possible answers are: 5.735 687, 5.740 236 89, 5.736.

27 Some possible answers are: 0.62, 0.58, 0.6495.

28 **a** 5000 **b** 4700 **c** 4710
d 4709 **e** 4708.6 **f** 4708.61
g 4708.613 **h** 4708.6130

1F Operations with decimals

1F Start thinking!

1 $23.70
2 $23.70, $2.85, $10.92, $5.79, $3.45, $4.49, $4.70, $3.40, $9.20, $5.99, $4.96, $6.75
3 Estimate is $86, so she has enough money.
4 Estimated change is $14.
5 $86.20 **6** $13.80 **7** $3.45
8 **a** Answer must have same number of decimal places as decimal number being multiplied.
b Digits must be positioned in correct place-value columns and decimal points must be lined up.
c Line up decimal points and digits that have same place value in both dividend and quotient.

Exercise 1F Operations with decimals

1 **a** 247.96 **b** 33.7686 **c** 690.984
d 106.5953

2 **a** 82.83 **b** 785.654 **c** 0.1344
d 26.251 **e** 2.7312 **f** 177.196
g 3794.295 **h** 64 668.44

3 **a** 194.80 **b** 1222.4 **c** 52 058.1
d 2.947 59

4 **a** 43.831 **b** 30.864 **c** 2281.44
d 112.5234 **e** 4.232 **f** 2.024 204
g 7.890 54 **h** 1124.4616

5 **a** 18.149 **b** 1.3448 **c** 8.3249
d 21.6985 **e** 0.591 25 **f** 0.156 75
g 0.0256 **h** 27.614 28

6 **a** 0.8 **b** 0.375 **c** 0.5 **d** 1.25
e 0.125 **f** 0.4 **g** 0.35 **h** 0.36
i 0.025 **j** 0.6 **k** 0.22 **l** 0.3125

7 **a** 10 **b** 35.68
c To keep an equivalent division problem.
d 8.92

8 **a** 72.4 ÷ 2 **b** 49.02 ÷ 6 **c** 1976.4 ÷ 3
d 172.8 ÷ 8 **e** 1436.8 ÷ 4 **f** 749.1 ÷ 6
g 48.72 ÷ 15 **h** 280.8 ÷ 12 **i** 163.8 ÷ 9

9 **a** 36.2 **b** 8.17 **c** 658.8
d 21.6 **e** 359.2 **f** 124.85
g 3.248 **h** 23.4 **i** 18.2
10 Estimate: 12 km, exact: 12.025 km
11 **a** \$29.40 **b** \$5.85 **c** \$3.37 **d** \$9.88
e \$4.94 **f** \$11.99
12 **a** \$66 **b** \$65.45 **c** \$34.55
13 **a** **i** \$0.60 **ii** \$2.89 **iii** \$18.95
b **i** \$4.65 **ii** \$6.85 **iii** \$13.61
c **i** \$12.95 **ii** \$1.59 **iii** \$1.95
14 **a** box: \$0.75,individual: \$0.72
b Buying individual pencils, as the cost per pencil is less.
15 **a** 400-g pack: \$0.92, loose: \$0.99
b William, as cost is cheaper per 100 g.
c May need more or less than the 400 g in package.
16 **a** 35 **b** 14; \$0.20
17 20
18 1.2 kg; as 0.8-kg option costs \$14/kg and 1.2-kg option costs \$13.75/kg.
19 **a** **i** 368 cm **ii** 382.9 cm
b Left hand; Lee may be right-handed and use her left hand less, so less wear on the nails.
c 750.9 cm **d** 75.09 cm
20 **a** 1.875 **b** 11.552 **c** 10.8
21 One possible answer is given.
a **i** $2.362 + 2.358$ **ii** $7.08 - 2.36$
iii 5.9×0.8 **iv** $14.16 \div 3$
b $(6.5 + 3.6) \times 0.5 - 0.33$

1G Terminating, non-terminating and recurring decimals

1G Start thinking!

1 **a** 0.625 **b** 0.8333…
2 First result has three decimal places while other has endless number of decimal places.
3 3 **4** No, it doesn't stop.
5 infinite (or endless) amount
6 From second decimal place, digit 3 repeats.
7 repeating pattern

Exercise 1G Terminating, non-terminating and recurring decimals

1 **a** N **b** T **c** N **d** T **e** N **f** N
g N **h** T **i** N **j** T **k** N **l** T
2 c, f, g, k
3 A terminating decimal 'terminates' or stops after a number of decimal places. A non-terminating decimal has an endless (infinite) number of decimal places. A recurring decimal is a special type of non-terminating decimal in which some or all of the digits after the decimal point form a recurring (repeating) pattern.
4 **a** $0.\dot{9}$ **b** $4.\overline{36}$ (or $4.3\dot{6}$)
c $87.2\dot{5}$ **d** $3.\overline{476}$ (or $3.4\dot{7}\dot{6}$)
e $0.738\dot{2}$ **f** $16.1\dot{9}\dot{6}$
g $483.\dot{8}$ **h** $6.729\overline{1}$ (or $6.729\dot{1}$)
i $0.18\overline{367}$ or $(0.18\dot{3}6\dot{7})$ **j** $1.\overline{35628}$ (or $1.\dot{3}5628\dot{}$)
k $28.\overline{70}$ (or $28.7\dot{0}$) **l** $9.23071\dot{4}$
5 $21.5\dot{7}$, $17.\overline{56}$, $0.\overline{1234}$, $0.\dot{9}$
6 **a** 0.111 111 111 111… **b** 5.377 777 777 777…
c 14.828 282 828 282… **d** 21.569 696 969 696…
e 8.104 104 104 104… **f** 0.651 365 136 513…
g 232.684 444 444 444… **h** 44.761 359 359 359…
i 94.570 202 020 202… **j** 1.739 247 392 473…
k 11.812 111 111 111… **l** 6.904 040 404 040…
m 14.241 341 341 341… **n** 0.562 062 062 062…
o 139.274 527 452 745…
7 **a** 0.7 **b** 0.5 **c** $0.\dot{3}$ **d** 1.25
e $0.\overline{18}$ **f** 0.625 **g** $2.\dot{6}$ **h** 0.75
i 1.125 **j** 1.4 **k** $0.8\dot{3}$ **l** $0.\overline{142857}$
8 **a** **i** a, b, d, f, h, i, j **ii** c, e, g, k, l
b c, e, g, k, l **c** no
9 **a** $\frac{1}{2}$ **b** $\frac{4}{1}$ **c** $6\frac{1}{5}$ (or $\frac{31}{5}$)
d $\frac{37}{100}$ **e** $\frac{18}{1}$ **f** $2\frac{509}{1000}$ (or $\frac{2509}{1000}$)
10 **a** They can be expressed as fractions.
b Yes, because they can be expressed as fractions.
11 Some possible answers are: 0.721 725 649…, 0.558 436 154 235 872….
12 **a** rational **b** rational **c** rational
d irrational **e** rational **f** irrational
g rational **h** rational **i** rational
j rational **k** irrational **l** rational
13 **a** **i** $0.\dot{1}$ **ii** $0.\dot{2}$ **iii** $0.\dot{3}$ **iv** $0.\dot{4}$
b recurring decimal
c **i** $0.\dot{5}$ **ii** $0.\dot{6}$ **iii** $0.\dot{7}$ **iv** $0.\dot{8}$
14 **a** **i** 0.111 111 11… **ii** 0.010 101 01…
iii 0.001 001 001… **iv** 0.000 100 01…
b **i** $0.\dot{1}$ **ii** $0.\overline{01}$ **iii** $0.\overline{001}$
iv $0.\overline{0001}$
c **i** $0.\overline{00001}$ **ii** $0.\overline{000001}$
iii $0.\overline{0000001}$ **iv** $0.\overline{00000001}$
15 **a** 0.333 33…, 0.666 66…
b 0.999 99… **c** 1
16 **a** One possible answer is 3.141 592 654.
b **i** 3.14 **ii** 3.1416
iii 3.141 59 **iv** 3.141 593
17 **a** One possible answer is 1.414 213 562.
b **i** 1.41 **ii** 1.4142
iii 1.414 214 **iv** 1.414 213 6
c One possible answer is 1.732 050 807.
d yes
e **i** 1.73 **ii** 1.7321
iii 1.732 05 **iv** 1.732 051
18 **b** 3.160 494, 3.140 845, 3.142 857, 3.141 593, 3.141 600, 3.141 057, 3.142 732
c $\frac{355}{113}$

19 a

Digit	0	1	2	3	4	5	6	7	8	9
Number of times it appears	10	10	15	14	12	9	12	9	16	14

b i 8 **ii** 5 and 7 **c** no

20 It is an irrational number (non-terminating but not recurring decimal), so it cannot be written as an exact fraction or decimal.

1H Powers and roots

1H Start thinking!

1 a 5 **b** 3 **c** 3 to the power of 5
d $3 \times 3 \times 3 \times 3 \times 3$ **e** 243

2 a Dots can be arranged as a square. **b** 9

3 4 **4** $\sqrt{16} = 4$ as $4^2 = 16$ **5** 3

6 The square of a number is the result of multiplying a number by itself; for example, $3^2 = 3 \times 3 = 9$. The square root of a number is the number that, when squared, results in the original number; for example, $\sqrt{9} = 3$ as $3^2 = 9$. So the two operations are closely related (inverse operations).

Exercise 1H Powers and roots

1 a 81 **b** $3, 2 \times 2 \times 2$
c 8^2, 8, 64 **d** 6^3, 6, 3, 216

2 a 32 **b** 64 **c** 1 **d** 27 **e** 10 000 000
f 256 **g** 729 **h** 625 **i** 343 **j** 1296
k 32 768 **l** 6561

3 a 25 **b** 4 **c** 1 **d** 49 **e** 100 **f** 625
g 64 **h** 121 **i** 400 **j** 1089 **k** 289 **l** 3600
m 10 000 **n** 250 000

4 a 4 **b** 3 **c** 5 **d** 8 **e** 1 **f** 11
g 2 **h** 10 **i** 6 **j** 15 **k** 12 **l** 20
m 13 **n** 30

5 4^2 means 4×4.

6 $\sqrt{16}$ is the number that when squared equals 16.

7 a 648 **b** 4 900 000 **c** 125 **d** 256

8 a 0.64 **b** 11.56 **c** 0.125 **d** 0.0081
e 0.000 01 **f** 0.000 012 8
g $\frac{1}{64}$ **h** $\frac{49}{81}$ **i** $\frac{125}{216}$ **j** $\frac{81}{10\,000}$
k $\frac{1}{32}$ **l** $\frac{1\,000\,000}{729}$

9 a 72 **b** 4.3601 **c** $\frac{11}{10}$ (or $1\frac{1}{10}$)
d 85

10 a 3249 **b** 887 364 **c** 1 030 301
d 62 748 517 **e** 52 200 625 **f** 11 881 376
g 321.419 125 **h** 13.0321 **i** 293 162.5062
j $\frac{5832}{12\,167}$ **k** $\frac{117\,649}{1\,771\,561}$ **l** $\frac{19\,683}{1\,953\,125}$ **m** 14
n 16 **o** 33 **p** 162 **q** 1.5
r 7.3 **s** 8.2 **t** 21.8 **u** 5.03

11 a i 8 **ii** 2.4 **iii** 1.732 050 808…
b Whole numbers and terminating decimals are rational numbers as they can be expressed as fractions. Non-terminating decimals that are not recurring are irrational as they can't be written as fractions. $\sqrt{3}$ is irrational.

12 a i 12 **ii** rational
b i 4.472 135 955… **ii** irrational
c i 7.4 **ii** rational **d i** 576 **ii** rational
e i 2.880 972 058… **ii** irrational
f i 0.7 **ii** rational **g i** $\frac{1}{2}$ **ii** rational
h i $\frac{1}{2}$ **ii** rational **i i** $\frac{3}{5}$ **ii** rational
j i $\frac{9}{10}$ **ii** rational
k i $\frac{7}{6}$ (or $1\frac{1}{6}$) **ii** rational
l i 0.866 025 404… **ii** irrational

13 a Dots form a cube. **b** 8
c i 125 **ii** 27 **iii** 1000 **iv** 343

14 a 2 **b** $\sqrt[3]{8} = 2$ as $2^3 = 8$
c i 5 **ii** 3 **iii** 10 **iv** 7
d The cube of a number is the result of a repeated multiplication where the base is written three times. The cube root of a number is the number that, when cubed, results in original number. So the two operations are closely related (inverse operations).

15 a 4 **b** 1 **c** 6 **d** 20 **e** 9 **f** 11

16 a i 1 **ii** 121 **iii** 12 321
iv 1 234 321
b Resulting number displays digits in a pattern with 1 as the first digit, 2 as the second digit, 3 as the third digit, etc, increasing by one each time until it reaches a value that is the same as the number of digits in the base. The digits that follow then decrease by one to finish the pattern at 1. For example, the base of 111^2 contains three digits so the first three digits in the number are 123 followed by 21 to give 12 321.
c i 123 454 321 **ii** 12 345 654 321
iii 1 234 567 654 321
iv 123 456 787 654 321

17 a \$180 **b** \$3, \$9, \$27, \$81, \$243, \$729
c \$30 per week **d** $\$(3^{\text{week number}}) = 3^{11}$
e \$177 147

18 a i 2000 **ii** 4000 **iii** 8000
b 2000, 4000, 8000, 16 000, 32 000
c i 2 **ii** 4 **iii** 8
d 2, 4, 8, 16, 32 **e** $2^1, 2^2, 2^3, 2^4, 2^5$
f i 2^6 **ii** 2^{10} **iii** 2^{24}
g 16 777 216 **h** 16 777 216 000

19 a The area of a square is calculated by squaring the length. For an area of 25 cm^2, the length is the number that, when squared (multiplied by itself), results in 25. As $25 = 5 \times 5$, the length of the square is 5 cm.
b i 13 cm **ii** 14.142 135 62… m
iii 16.4 km
c part ii

20 a 5 **b** 7
c i 2 **ii** 3 **iii** 10 **iv** 2 **v** 1 **vi** 3

21 **a** **i** 15 **ii** 4.1 **iii** 4.626 065 009…
iv 7 **v** 2.923 012 786...
vi 0.562 341 325… **vii** 8
viii 0.2 **ix** 2.885 399 8…
x 3 **xi** 2.813 500 172... **xii** 5
b **i** rational **ii** rational **iii** irrational
iv rational **v** irrational **vi** irrational
vii rational **viii** rational **ix** irrational
x rational **xi** irrational **xii** rational

22 4.828 427…, irrational

1I Index laws

1I Start thinking!

1 16, 32, 64, 128, 256, 512, 1024

2 **a** **i** 4 **ii** 8 **iii** 32
b 32 **c** the same
d **i** 32 **ii** 16 **iii** 512
e 512 **f** the same
g When multiplying numbers in index form with same base, write base and add indices (or powers).

3 **a** **i** 64 **ii** 16 **iii** 4
b 4 **c** the same
d **i** 1024 **ii** 128 **iii** 8
e 8 **f** the same
g When dividing two numbers in index form with same base, write base and subtract indices (or powers).

4 Bases must be same.

Exercise 1I Index laws

1 **a** same, add, powers
b same, subtract, indices

2 **a** $2^3 \times 2^5 = 2^{3+5} = 2^8$
b $2^4 \times 2^2 = 2^{4+2} = 2^6$
c $2^3 \times 2^4 = 2^{3+4} = 2^7$
d $2^5 \times 2^5 = 2^{5+5} = 2^{10}$

3 **a** $2^5 \div 2^2 = 2^{5-2} = 2^3$
b $2^8 \div 2^3 = 2^{8-3} = 2^5$
c $2^9 \div 2^5 = 2^{9-5} = 2^4$
d $2^7 \div 2^4 = 2^{7-4} = 2^3$

4 **i** **a** 256 **b** 64 **c** 128 **d** 1024
ii **a** 8 **b** 32 **c** 16 **d** 8

5 **a** 3^5 **b** 4^8 **c** 6^6 **d** 5^9 **e** 8^3 **f** 2^5

6 **a** 243 **b** 65 536 **c** 46 656
d 1 953 125 **e** 512 **f** 32

7 **a** 7^3 **b** 3^5 **c** 5^2 **d** 9^3 **e** 4^1 **f** 6^3

8 **a** 343 **b** 243 **c** 25 **d** 729 **e** 4 **f** 216

9 **a** $2^{11} = 2048$ **b** $5^6 = 15\,625$
c $3^7 = 2187$ **d** $4^8 = 65\,536$
e $5^3 = 125$ **f** $3^6 = 729$
g $9^3 = 729$ **h** $2^7 = 128$
i $6^5 = 7776$ **j** $8^3 = 512$
k $10^9 = 1\,000\,000\,000$ **l** $7^2 = 49$

10 **a** 2^{12} **b** 3^6 **c** 10^{10} **d** 4^9 **e** 5^{16} **f** 9^{14}

11 **a** $(1.5)^9$ **b** $(0.7)^6$ **c** $\left(\frac{1}{2}\right)^7$ **d** $\left(\frac{7}{15}\right)^{14}$
e $(9.2)^3$ **f** $(0.63)^7$ **g** $\left(\frac{3}{4}\right)^4$ **h** $\left(\frac{5}{2}\right)^2$

12 **a** 5^4
b **i** $\frac{5^7}{5^3} = \frac{5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5}{5 \times 5 \times 5}$
ii $5 \times 5 \times 5 \times 5 = 5^4$
c yes **d** Keep base and subtract powers.

13 **a** 3^2 **b** 8^3 **c** 2^6 **d** 4^5 **e** 10^3 **f** 6^4

14 **a** $\frac{2^6 \times 2^5}{2^3} = \frac{2^{6+5}}{2^3} = \frac{2^{11}}{2^3} = 2^{11-3} = 2^8$
b $\frac{5^3 \times 5^7}{5^4} = \frac{5^{3+7}}{5^4} = \frac{5^{10}}{5^4} = 5^{10-4} = 5^6$
c $\frac{3^7 \times 3^6}{3^5 \times 3^4} = \frac{3^{7+6}}{3^{5+4}} = \frac{3^{13}}{3^9} = 3^{13-9} = 3^4$
d $\frac{9^8 \times 9^{11}}{9^9 \times 9^3} = \frac{9^{8+11}}{9^{9+3}} = \frac{9^{19}}{9^{12}} = 9^{19-12} = 9^7$

15 **a** 4^2 **b** 7^5 **c** 6^3 **d** 2^8 **e** 11^6 **f** 3^1
g 5^4 **h** 8^3 **i** 2^7 **j** 9^{10} **k** 3^4 **l** 4^2

16 **a** **i** $\frac{5^7}{5^7} = \frac{5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5}{5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5}$ **ii** 1
b 0 **c** $5^0 = 1$ **d** $3^0 = 1$ **e** zero, one

17 **a** 1 **b** 6 **c** 1 **d** $7^4 = 2401$
e 1 **f** 1

18 **a** 1 **b** 1 **c** 9 **d** 1 **e** 1 **f** 11
g 1 **h** 6^{10} **i** 1

19 **a** Multiply powers. **b** 2^{12}
c $2^3 \times 2^3 \times 2^3 \times 2^3 = 2^{3+3+3+3} = 2^{12}$,
$(2 \times 2 \times 2) \times (2 \times 2 \times 2) \times (2 \times 2 \times 2) \times (2 \times 2 \times 2) = 2^{12}$
d multiply

20 **a** 6^{20} **b** 2^{12} **c** 7^{12} **d** 11^8 **e** 8^{15} **f** 3^{14}

21 **a** Powers have been divided instead of subtracted; $5^8 \div 5^2 = 5^6$.
b Powers have been multiplied instead of added; $2^3 \times 2^3 = 2^6$.
c Powers have been added instead of multiplied; $(6^5)^2 = 6^{10}$.
d Powers have been divided instead of subtracted; $9^{12} \div 9^{12} = 9^0 = 1$.

22 **a** 10 **b** 6 **c** 4 **d** 12

23 **a** **i** 2^9 **ii** 2^6 **iii** 2^4 **iv** 2^7 **v** 2^{10} **vi** 2^8
b **i** 512 **ii** 64 **iii** 16 **iv** 128
v 1024 **vi** 256
c **i** 3^6 **ii** 3^3 **iii** 3^9 **iv** 3^8 **v** 3^6 **vi** 3^5
d

Index form	Basic numeral
3^1	3
3^2	9
3^3	27
3^4	81
3^5	243
3^6	729
3^7	2187
3^8	6561
3^9	19 683
3^{10}	59 049

e **i** 729 **ii** 27 **iii** 19 683 **iv** 6561
v 729 **vi** 243

24 **a** **i** $2^5 \times 2^4$ **ii** $2^{10} \div 2^8$ **iii** $\frac{2^7 \times 2^5}{2^4}$
iv $\frac{2^9 \times 2^6}{2^7 \times 2^4}$ **v** $3^{10} \div 3^6$ **vi** $3^5 \times 3^4$
vii $\frac{3^3 \times 3^5}{3^7}$ **viii** $\frac{3^7 \times 3^6}{3^8 \times 3^3}$ **ix** $\frac{2^7 \times 2^{10}}{2^8 \times 2^6}$
b **i** 2^9 **ii** 2^2 **iii** 2^8
iv 2^4 **v** 3^4 **vi** 3^9
vii 3^1 **viii** 3^2 **ix** 2^3
c **i** 512 **ii** 4 **iii** 256
iv 16 **v** 81 **vi** 19 683
vii 3 **viii** 9 **ix** 8
d **i** 256 **ii** 81 **iii** 64 **iv** 2187

25 **a** $(2 \times 3)^5 = 6^5 = 7776$; $2^5 \times 3^5 = 32 \times 243 = 7776$
b $\left(\frac{2}{3}\right)^5 = \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} = \frac{32}{243}$; $\frac{2^5}{3^5} = \frac{32}{243}$
c **i** $5^3 \times 9^3$ **ii** $\frac{3^6}{4^6}$ **iii** $\frac{9^4}{5^4}$
iv $11^7 \times 4^7$ **v** $\frac{7^3}{10^3}$ **vi** $3^9 \times 8^9$

27 Some possible answers are: $2^3 \times 2^2$, $2^{10} \div 2^5$, $\frac{2^7 \times 2^2}{2^4}$.

1 Chapter review

MULTIPLE-CHOICE

1 B **2** A **3** B **4** B **5** C
6 D **7** C **8** B **9** A **10** D

SHORT ANSWER

1 **a** 200 **b** 70 000 **c** 4000
d 100 000
2 **a** 14 000 **b** 60 000 **c** 120 000
d 50
3 **a** 45 **b** 40 **c** 10 **d** 4
4 **a** $\frac{7}{12}$ **b** $1\frac{7}{8}$ **c** $3\frac{9}{16}$ **d** $5\frac{2}{3}$
5 **a** $\frac{29}{35}$ **b** $\frac{13}{30}$ **c** $\frac{6}{35}$ **d** $\frac{1}{2}$
6 **a** $6\frac{1}{33}$ **b** $2\frac{29}{35}$ **c** $21\frac{1}{4}$ **d** $2\frac{1}{4}$
7 **a** 0.0012, 0.120, 0.123, 0.2, 0.213, 0.231
b 8.16, 8.9, 8.916, 9.16, 9.6, 9.916
8 **a** 0.717, 0.71, 0.7, 0.107, 0.0701, 0.017
b 2.721, 2.72, 2.7, 2.27, 2.127, 2.1
9 **a** 13.22 **b** 0.0012 **c** 54.546 **d** 181.2
10 **a** 143.67 **b** 23.48 **c** 265.7842
d 989.5993
11 **a** 25.2512 **b** 10.9315 **c** 19.722
d 416.758 52
12 **a** **i** terminating **ii** non-terminating
iii terminating **iv** non-terminating
b part ii 6.562 562…
13 **a** $0.\dot{3}$ **b** $5.\overline{385}$ **c** $45.68\overline{57}$
d $0.092\overline{451}$
14 **a** 0.233 333 333 333… **b** 3.272 727 272 727…
c 463.587 878 787 878…
d 17.364 854 854 854…
15 **a** 216 **b** 128 **c** 64 **d** 121
16 **a** 2 **b** 9 **c** 3 **d** 5
17 **a** **i** 23 **ii** 26.457 513 11…
iii 12.5 **iv** 3.476 026 645…
b **i** rational **ii** irrational
iii rational **iv** irrational
18 **a** 3^{12} **b** 2^{10} **c** 6^6 **d** 1 **e** 9^{15} **f** 4^{11}
19 **a** 49 **b** 32

NAPLAN-STYLE PRACTICE

1 30 000 **2** 25 004 784 **3** 150
4 5 **5** 5 **6** $1\frac{7}{11}$ **7** 15
8 16 **9** $\frac{2}{15}$ **10** 5
11 9.82, 9.209, 8.902, 8.29, 8.2, 8.029
12 39.50 **13** 406.737 **14** 10
15 $23.60 **16** $6.40 **17** 25.728
18 1.25-L bottle for $3.25 ($2.60/L)
19 $\frac{1}{4}, \frac{3}{5}, \frac{1}{8}$, 8.26 **20** 0.763 491…, $\frac{2}{3}, \frac{9}{11}$
21 $\frac{2}{3}, \frac{9}{11}$ **22** 0.763 491… **23** 64
24 12 **25** 216 **26** 4
27 10 cm **28** $\frac{5^4 \times 5^8}{5^5 \times 5^7}$ **29** 64
30 16 **31** $\sqrt{3}$

ANALYSIS

1 **a** 1 686 600
b 7 000 000, 6 000 000, 5 000 000, 2 000 000, 2 000 000, 500 000, 400 000, 200 000
c 1 000 000; difference is 686 600 people
d 23 100 000
e **i** $\frac{6}{7}$ **ii** $\frac{1}{10}$ **iii** $\frac{7}{2}$ or $3\frac{1}{2}$ **iv** $\frac{5}{6}$
f **i** $0.\overline{857142} \approx 0.857$ **ii** 0.1
iii 3.5 **iv** $0.8\dot{3} \approx 0.833$
g **i** parts ii, iii **ii** parts i, iv **iii** parts i, iv
2 **a** 3 km
b population = $10\,000 \times 2^6 = 10\,000 \times 64 = 64\,000$
c approximately 9.22 km
d Answers to parts a and b are rational numbers as they can be expressed exactly. Answer to part c is an irrational number as it cannot be expressed as an exact fraction or decimal.

1 Connect

For feedback on this open-ended task, see your teacher.

CHAPTER 2 PERCENTAGES, RATIOS AND RATES

2 Are you ready?

1 **a** B **b** $\frac{2}{7}$
2 D **3** D **4** $\frac{9}{20}$ **5** A
6 **a** $\frac{14}{100}$ **b** $\frac{84}{100}$ **c** $\frac{20}{100}$
7 **a** 65 840 **b** 1263 **c** 6.8 **d** 487
e 0.3172 **f** 92.6848
8 **a** 28.7322 **b** 518.8 **c** $21.10 **d** $1674
9 **a** C **b** 0.625

10 A

11 **a** 280 mm **b** 3000 g **c** 390 minutes
d 7200 mL

12 **a** A **b** B **13** **a** C **b** D

2A Understanding percentages

2A Start thinking!

1 41 **2** 41%

3 **a** 59 **b** 59% **c** 100%

4 The shaded and unshaded sections of any shape will always add to 100%.

5 Aim to obtain a fraction with denominator of 100. With more than 100 parts, divide by a factor to obtain denominator of 100. With fewer than 100 parts, multiply by a factor to obtain denominator of 100.

Exercise 2A Understanding percentages

1 **a** 17% **b** 42% **c** 91% **d** 36% **e** 67%
f 3% **g** 15% **h** 49% **i** 52% **j** 81%
k 9% **l** 125%

2 **a** 60% **b** 40% **c** 60% **d** 52%

3 **a** 40% **b** 60% **c** 40% **d** 48%

4 **a** **i** $\frac{16}{20}$ **ii** $\frac{80}{100}$ **iii** 80%

b **i** $\frac{15}{20}$ **ii** $\frac{75}{100}$ **iii** 75%

c **i** $\frac{7}{10}$ **ii** $\frac{70}{100}$ **iii** 70%

d **i** $\frac{6}{10}$ **ii** $\frac{60}{100}$ iii 60%

5 **a** **i** 80% **ii** 20% **b** **i** 70% **ii** 30%
c **i** 30% **ii** 70% **d** **i** 90% **ii** 10%

6 **a** 20% **b** 80% **c** 30% **d** 90%
e 30% **f** 30%

7 Percentage is ten times the number of objects.

8 **a** 24% **b** 77% **c** 83% **d** 58%
e 7% **f** 46%

9 **a** 80% **b** 81% **c** 92% **d** 88%
e 71.5% **f** $66\frac{2}{3}\%$

10 **a** **i** $\frac{11}{100}$ **ii** $\frac{47}{100}$ **iii** $\frac{27}{100}$ **iv** $\frac{73}{100}$ **v** $\frac{51}{100}$ **vi** $\frac{89}{100}$

b They are opposite processes. In question 1 the percentage was originally expressed as a fraction with denominator of 100. Here we are expressing a percentage as fraction with denominator of 100. Both rely on the fraction having denominator of 100.

11 **a** $\frac{3}{50} = \frac{6}{100} = 6\%$ **b** $\frac{11}{20} = \frac{55}{100} = 55\%$
c $\frac{13}{25} = \frac{52}{100} = 52\%$ **d** $\frac{1}{4} = \frac{25}{100} = 25\%$
e $\frac{8}{10} = \frac{80}{100} = 80\%$ **f** $\frac{4}{5} = \frac{80}{100} = 80\%$
g $\frac{70}{500} = \frac{14}{100} = 14\%$ **h** $\frac{15}{200} = \frac{7.5}{100} = 7.5\%$
i $\frac{21}{300} = \frac{7}{100} = 7\%$ **j** $\frac{16}{400} = \frac{4}{100} = 4\%$
k $\frac{5}{8} = \frac{62.5}{100} = 62.5\%$ **l** $\frac{19}{40} = \frac{47.5}{100} = 47.5\%$

12 **a** **i** shade 80 squares **ii** shade 92 squares
iii shade 25 squares **iv** shade 17 squares
v shade 43 squares **vi** shade 66 squares

b 10 cm = 100 mm, so a length of 1 mm would represent 1%. For 80%, shade 80 mm (or 8 cm); for 92%, shade 92 mm (or 9.2 cm); for 25%, shade 25 mm (or 2.5 cm); for 17%, shade 17 mm (or 1.7 cm).

c Double the percentage amount. For example, 80% means 80 out of 100 or 160 out of 200. Shade 160 of the 200 squares in the grid to represent 80%.

13 **a** Both are correct.

b Some possible answers are: shade 30 out of 200, shade 60 out of 400.

14 **a** $\frac{3}{5} = \frac{60}{100} = 60\%$ **b** $\frac{9}{20} = \frac{45}{100} = 45\%$
c $\frac{17}{25} = \frac{68}{100} = 68\%$ **d** $\frac{46}{200} = \frac{23}{100} = 23\%$
e $\frac{17}{20} = \frac{85}{100} = 85\%$ **f** $\frac{1}{5} = \frac{20}{100} = 20\%$
g $\frac{144}{600} = \frac{24}{100} = 24\%$ **h** $\frac{75}{250} = \frac{3}{10} = \frac{30}{100} = 30\%$

15 **a** twice an amount; three times an amount

b $\frac{100}{100} = 1$

c 1 = 100%; 2 = 200%; 3 = 300%; 4 = 400%; 5 = 500%

d One possible answer is: when calculating daily intakes.

16 As capacity of each container is different, 50% (or 50 mL out of every 100 mL) would be a different volume of liquid for each.

17 **a** **i** 100%, 75%, 50%, 25%
ii 20%, 40%, 60%, 80%, 100%
iii 60%, 40%, 30%

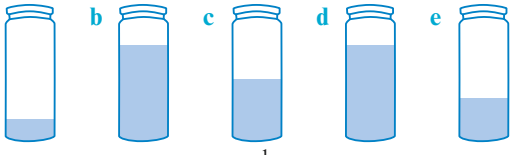
b **i** 0%, 25%, 50%, 75%
ii 80%, 60%, 40%, 20%, 0%
iii 40%, 60%, 70%

c A and B show equal-sized 'blocks' in a panel whereas C is one solid section of a panel. The percentages for A and B are found by first writing the number of blocks as a fraction of the total number of blocks. In C, you need to measure the amount required as a fraction of the total panel length.

18 **a** 66% **b** $55\frac{1}{2}\%$ **c** $79\frac{2}{3}\%$ **d** 83.1%
e 24% **f** $4\frac{2}{5}\%$ **g** 74.7% **h** 20.44%

19 8% **20** 45%

21 **a** **b** **c** **d** **e**



22 **a** 50% **b** 25% **c** $58\frac{1}{3}\%$

23 **a** No, the grid is 9 × 9 (not 10 × 10) so the shaded region represents $\frac{36}{81}$ (not $\frac{40}{100}$).

b Yes, one part out of 4 equal parts has been shaded therefore $\frac{1}{4}$ or 25% is the correct answer.

c No, five parts out of 8 have been shaded which is more than 50%.

24 Calculate 3 ÷ 7 then multiply by 100.
25 62.5%

2B Percentages, decimals and fractions

2B Start thinking!

1 $\frac{45}{100}$ **2** $\frac{45}{100}$ **3** $\frac{45}{100}$
4 All represent the same value.
5 A percentage is a quantity out of 100. This same amount can also be expressed as a fraction with denominator of 100 and a decimal in hundredths.

Exercise 2B Percentages, decimals and fractions

1 **a** $\frac{17}{100}$ **b** $\frac{21}{100}$ **c** $\frac{7}{20}$ **d** $\frac{2}{25}$ **e** $\frac{3}{20}$ **f** $\frac{49}{50}$
g $\frac{74}{100} = \frac{37}{50}$ **h** $\frac{57}{100}$ **i** $\frac{53}{100}$ **j** $\frac{87}{100}$ **k** $\frac{11}{100}$
l $\frac{28}{100} = \frac{14}{50} = \frac{7}{25}$ **m** $\frac{62}{100} = \frac{31}{50}$ **n** $\frac{18}{100} = \frac{9}{50}$
o $\frac{48}{100} = \frac{24}{50} = \frac{12}{25}$ **p** $\frac{1}{100}$

2 **a** 0.17 **b** 0.21 **c** 0.35 **d** 0.08
e 0.15 **f** 0.98 **g** 0.74 **h** 0.57
i 0.53 **j** 0.87 **k** 0.11 **l** 0.28
m 0.62 **n** 0.18 **o** 0.48 **p** 0.01

3 **a** 0.0678 **b** 0.031 **c** 1.62 **d** 0.721
e 3.47 **f** 0.8109 **g** 0.1057 **h** 0.03
i 4.7238 **j** 2.157 **k** 0.001 155
l 0.0004 **m** 0.0247 **n** 0.1201
o 0.0062 **p** 0.008

4 **a** 71% **b** 28% **c** 5% **d** 8%
e 90% **f** 31% **g** 54% **h** 95%
i 50% **j** 1% **k** 23% **l** 28%
m 10% **n** 23% **o** 67% **p** 80%

5 **a** 36.9% **b** 24.8% **c** 1006%
d 81% **e** 705.7% **f** 185.912%
g 1210% **h** 270% **i** 95.23%
j 0.5% **k** 24.461% **l** 0.0504%
m 417% **n** 0.123% **o** 0.0896%
p 3.2%

6 **a** 60% **b** 70% **c** 95% **d** 92%
e 47% **f** 150% **g** 90% **h** 350%
i 75% **j** 160% **k** 18% **l** 44%
m 125% **n** 52% **o** 85% **p** 30%

7 **a** $62\frac{1}{2}$% or 62.5% **b** $67\frac{1}{2}$% or 67.5%
c $8\frac{3}{4}$% or 8.75% **d** $18\frac{3}{4}$% or 18.75%
e $8\frac{4}{5}$% or 8.8% **f** $37\frac{1}{2}$% or 37.5%
g $73\frac{1}{3}$% **h** $71\frac{3}{7}$% **i** $77\frac{7}{9}$% **j** $27\frac{7}{9}$%
k $83\frac{1}{3}$% **l** $88\frac{8}{9}$%

8 **a** 225% **b** 650% **c** 730% **d** 375%
e 428% **f** 580% **g** $533\frac{1}{3}$% **h** $322\frac{2}{9}$%
i $666\frac{2}{3}$% **j** $142\frac{6}{7}$% **k** $216\frac{2}{3}$% **l** $383\frac{1}{3}$%

10 **a** 33.33% **b** 63.64% **c** 44.44% **d** 85.71%
e 53.33% **f** 91.67% **g** 53.85% **h** 45.45%

11 **a** 0.174 **b** 0.059 **c** 0.2152 **d** 0.6295
e 2.6846 **f** 0.9294 **g** 0.3864 **h** 0.1455

12 **a** $\frac{1}{400}$ **b** $\frac{1}{160}$ **c** $\frac{9}{400}$ **d** $\frac{1}{175}$ **e** $\frac{3}{700}$ **f** $\frac{11}{300}$
g $\frac{9}{700}$ **h** $\frac{3}{325}$ **i** $\frac{11}{125}$ **j** $\frac{47}{300}$ **k** $\frac{8}{225}$ **l** $\frac{79}{600}$

13 **a** $\frac{3}{13}$ **b** 0.2308 **c** 23.08%
d **i** 30.77% **ii** 7.69% **iii** 38.46%
iv 84.62%

14 **a** four times the original amount
b **i** 4 **ii** 5 **iii** 6 **iv** 2.5 **v** 3.75

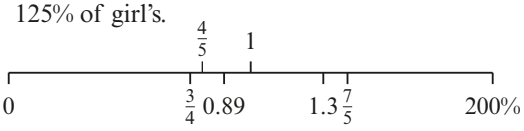
15 **a** 3.5 **b** 475% **16** $\frac{21}{400}$

17 **a** 50% **b** **i** $\frac{1}{6}$ **ii** $\frac{1}{3}$

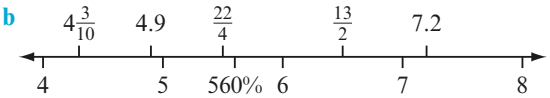
18 **a** $\frac{3}{25}$ **b** 12% **19** **a** $\frac{7}{8}$ **b** 12.5%

20 **a** 2500 kilojoules **b** $\frac{2500}{12\,500} = \frac{1}{5}$
c $\frac{2500}{10\,000} = \frac{1}{4}$
d **i** 20% **ii** 0.2 **e** **i** 25% **ii** 0.25
f $\frac{10\,000}{12\,500} = \frac{4}{5} = 80\%$ **g** $\frac{12\,500}{10\,000} = \frac{5}{4} = 125\%$
h Girl's energy usage is 80% of boy's; boy's is 125% of girl's.

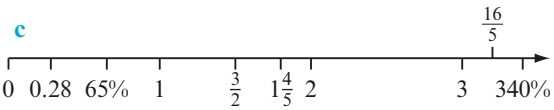
21 **a**



b



c



22 0, $\frac{10}{11}$%, 0.52, $\frac{7}{11}$, 0.76, $\frac{88}{115}$, 100%
23 Sergio is correct. 100% when written as a fraction is equal to $\frac{100}{100}$ or 1. So multiplying or dividing by 100% is the same as multiplying or dividing by 1.
24 Some possible answers are: $\frac{31}{200}$, $\frac{157}{1000}$, $\frac{159}{1000}$.
25 Some possible answers are: 0.251, 0.2655, 0.27.
26 Some possible answers are: 81%, 82%, 84%.

2C Percentage calculations

2C Start thinking!

1 depends on total number of attempts made

2–4

Name	Fraction	Percentage
Bianca	$\frac{5}{8}$	62.5%
Brittney	$\frac{3}{10}$	30%
Claire	$\frac{11}{20}$	55%
Evelyn	$\frac{7}{10}$	70%
Fatima	$\frac{2}{3}$	66.67%
Georgina	$\frac{5}{12}$	41.67%
Huan	$\frac{4}{6}$ or $\frac{2}{3}$	66.67%
Joey	$\frac{9}{15}$ or $\frac{3}{5}$	60%
Penelope	$\frac{8}{11}$	72.73%
Tamika	$\frac{3}{5}$	60%

5 proportion of attempts that produced successful shots; that is, percentage shows how many successful shots expected in 100 attempts
6 the seven girls with the highest percentages: Penelope, Evelyn, Fatima, Huan, Bianca, Joey, Tamika

Exercise 2C Percentage calculations

1 a 25% b 90% c 76% d 75%
e 120% f 125%
2 a i $17\frac{1}{2}\%$ ii 17.5% b i $\frac{1}{5}\%$ ii 0.2%
c i $36\frac{4}{5}\%$ ii 36.8% d i $28\frac{1}{2}\%$ ii 28.5%
e i $72\frac{1}{2}\%$ ii 72.5%
f i $587\frac{1}{2}\%$ ii 587.5%
g i $8062\frac{1}{2}\%$ ii 8062.5%
h i $833\frac{1}{3}\%$ ii $833.\dot{3}\%$
3 a 80% b 75% c 30% d 2.5%
4 a 23% b 25% c 35% d 70%
e 15% f 44%
5 a $1\frac{1}{4}\%$ or 1.25% b 10% c 25%
d 75% e 10% f $6\frac{1}{4}\%$ or 6.25%
g 3% h 25% i 5% j 200%
6 a 84 b 756 c 9 d 1152
e 725 f 112.5
7 a 453.6 m b 135 seconds
c 231 cm d 100.8 L
e 76 buttons f 11.76 mL
8 a \$4400 b 114 kg c \$1030.96
d \$786.25 e \$89.03 f 67.5 min
g 5.84 km h 381.84 L
9 a

Percentage	Increase or decrease	Expanded form	Full percentage	Number form
4.5%	increase	100% + 4.5%	104.5%	1.045
7%	decrease	100% – 7%	93%	0.93
25%	decrease	100% – 25%	75%	0.75
40%	increase	100% + 40%	140%	1.40
15%	decrease	100% – 15%	85%	0.85
20%	increase	100% + 20%	120%	1.2

b The final column is the decimal form of the percentage (fourth column).
c i $200 \times 0.75 = 150$ ii $150 \times 1.2 = 180$
iii $2500 \times 0.93 = 2325$ iv $70 \times 1.4 = 98$
10 For a percentage increase, add the percentage increase to 100% and convert the result to a decimal. Multiply the quantity by this decimal to obtain the increased amount. For a percentage decrease, subtract the percentage decrease amount from 100% and convert the result to a decimal. Multiply the quantity by this decimal to obtain the decreased amount.
Alternatively, calculate the amount of the percentage change and add to (or subtract from) the original amount.
11 a 460 b 1750 L c 105 songs
d 9 minutes e 11 550
12 a $\frac{89}{420}$ b 21.19%
c i 26.97% ii 74.17% iii 127.27%
d One possible answer is: Big Jake may be 252 cm and Thumbelina may be 53 cm.
e Using the heights in part d, the percentage from part b changes to 21.03% (not much difference!).
13 a 10 b 40 c 8
d Part c calculates 20% of the remaining cupcakes not the original amount.
e 32
14 a i six people ii increase iii 24%
b i 35 choc bars ii decrease iii 87.5% or $87\frac{1}{2}\%$
c i 0.5 km ii increase iii $33.\dot{3}\%$ or $33\frac{1}{3}\%$
d i 15 cm ii increase iii 10%
e i 2 minutes ii decrease iii 10%
15 52.4% 16 3
17 a Tom: 1 cm, Lisa: 1 cm
b Tom: 2.3%, Lisa: 1.2%; Tom
18 a 64 b 98 c 460
19 a 44% b 44%
c The percentage increase is the same for both shapes.
d When the side lengths of any shape are increased by 20%, the area will increase by 44%.

2D Financial calculations

2D Start thinking!

1 \$6.00 2 \$14.00 3 \$140.00
4 \$80.00 5 profit of \$60.00

Exercise 2D Financial calculations

1 a \$360 b \$1250 c \$100 d \$1575
e \$16 f \$576.45 g \$110.50 h \$448.50
i \$69.30 j \$129.15 k \$41.29 l \$3690
2 a \$1560 b \$336.75 c \$1350 d \$260.75
e \$948.15 f \$465 g \$552 h \$787.50
i \$323.75 j \$458.85
3 a \$142.10 b \$36.25 c \$94.25 d \$147.90
4 a \$257.60 b \$484.75 c \$34.30 d \$7.55
e \$517.30 f \$43.45 g \$731.65 h \$50.40
5 a i \$6.40 ii \$70.40
b i \$5.50 ii \$60.45
c i \$12.50 ii \$137.40
d i \$21.70 ii \$238.70
e i \$22.50 ii \$247.50
f i \$87.40 ii \$961.40
g i \$3259.90 ii \$35 858.90
h i \$19.90 ii \$218.90
i i \$25.90 ii \$284.90
j i \$124.50 ii \$1369.50
k i \$9.90 ii \$108.90
l i \$24.80 ii \$272.80
6 a i profit ii \$37 iii 74%
b i loss ii \$14 iii 23.73%

c i loss ii $336 iii 35.94%
d i loss ii $11 319 iii 39.17%
e i profit ii $1.44 iii 180%
f i loss ii $6.15 iii 70.29%
g i profit ii $7.52 iii 125.75%

7 a i 22.13% ii increase
b i 33.24% ii decrease
c i 9.11% ii decrease
d i 13.36% ii decrease
e i 20.08% ii decrease
f i 14.36% ii increase
g i 5.35% ii increase
h i 37.37% ii increase

8 a $812.90 b $41.30 c $2.49
d $605.41 e $59.46 f $42.35

9 a $800 b $595 c $16 500 d $2025

10 a $360 b 16\frac{2}{3}$ or 16.$\dot{6}$%

11 a $4 b six frames; $6.25
c loss of $2.25 per frame
d $122.50 e 36.03%
f undamaged 47.06%; damaged 36.03%

12 property 1 increased by 38.56%, property 2 increased by 33.33%

13 30%: $489.30; 20% then 10%: $503.28; customer is correct

14 a A 15% increase is 15% on top of original amount. That is, whole amount (100%) plus increased amount (15%). This is 115%, which as a decimal is 1.15.
b i 0.75 ii 1.17 iii 0.68
iv 1.62 v 3.0 vi 0.72

15 a i $5.55 ii $1.75 iii $2.85
iv $25.75 v $3.55 vi $6.55
b i 43.04% ii 41.13% iii 43.22%
iv 43.14% v 42.57% vi 42.70%
c When you increase price by 40%, you find 40% of original price and add it to original price. When you increase price by 30% then 10%, you first find 30% of original price and add it to original price, then you find 10% of this second price and add this amount to the second price to find final price. Increasing by 30% then 10% will always give a larger amount than just increasing by 40%.

16 a $2800 b $980

17 $1000 18 $1256

19 a $765 b $16 000

20 'All stock discounted by 45%' means all stock will receive a discount of 45%. 'Stock discounted by up to 45%' means the discount can be any amount from 0% to 45%; for example, 10%, 20%, 35%, 45%.

21 a $4137.50 b 530%

22 a 117$\frac{11}{17}$% or $\frac{20}{17}$ b 125% or $\frac{5}{4}$
c In part a the LCD television is 85% $\left(\frac{85}{100}\right)$ or $\frac{17}{20}$ of its original value. The mark-up is 117$\frac{11}{17}$% or $\frac{20}{17}$ of the discounted price.
In part b the LCD television is 80% $\left(\frac{80}{100}\right)$ or $\frac{4}{5}$ of its original value. The mark-up is 125% or $\frac{5}{4}$ of the discounted price.
The discount and new mark-up are reciprocal values.

2E Understanding ratios

2E Start thinking!

1 a 3, 1 b 3, 4 c 7, 4
3 a 3:1 b 3:4 c 7:4
4 a as a fraction
5 a 2:16, $\frac{2}{16}$ b 4:16, $\frac{4}{16}$ c 9:16, $\frac{9}{16}$
6 Denominator of fraction generally represents total number of parts; best to use for part-to-whole comparisons.

Exercise 2E Understanding ratios

1 a i 17:8 ii $\frac{8}{25}$ b i 7:11 ii $\frac{11}{18}$
c i 5:4 ii $\frac{4}{9}$

2 Part-to-part comparisons are best shown as ratios and part-to-whole comparisons are best shown as fractions.

3 a 20:33 b 95:57 c 113:167 d 256:175
e 499:575 f 23:2 g 4:13 h 19:21
i 42:117 j 16:39 k 19:11 l 23:7:3
m 15:17:19 n 5:7:11

4 a 13:21 b 120:17 c 27:500
d 4:3 e 1700:793 f 7:36
g 509:1300 h 472:2371 i 61:79
j 38:131 k 420:97 l 600:53
m 5000:23:673 n 5631:1753:1 000 000

5 a 11:9 b 2:7 c 9:40 d 18:11

6 The ratio 9:40 could be written as the fraction $\frac{9}{40}$ as it represents how a part compares to the whole.

7 a 11:7 b 89:101 c 5:3 d 25:1
e 2:3 f 1:2:3 g 3:7 h 8:3

8 a 3:5 b 2:5 c 1:5 d 4:3

9 a i 11:8:7 ii 9:40 b i $\frac{13}{40}$ ii 40%

10 For every four games she wins, she loses five; or out of every nine games she plays, she wins four and loses five.

12 a i 2:1 ii 5:1 iii 1:2
b $\frac{1}{9}$ c 67% d 44%
e incorrect order; 2:1

13 a 13% b 87:13

14 a 41:9 b $\frac{9}{50}$

15 80:332 (or 20:83)

16 a 81 cm, 72 cm b 60:81:72

17 **a** Measurements must be put into same units.
b convert it to same unit as other measurement
c 1670 feet = (1670 × 0.3048) m = 509 m; 270:509
18 **a** No, units are not the same.
b 57:2500
c More water will result in weaker solution so increase second value in ratio. Or less bleach will also result in weaker solution so decrease first value in ratio. Both options could also apply.
19 **a** $\frac{3}{100}$ **b** $\frac{97}{100}$ **c** 3:97
d when comparing part to whole
20 **a** email/Facebook/Skype: 120; text/call: 72; don't communicate: 36; postcard/letter: 12
b 120:12
c email/Facebook/Skype: $\frac{1}{2}$; text/call: $\frac{3}{10}$; don't communicate: $\frac{3}{20}$; postcard/letter: $\frac{1}{20}$
d email/ Facebook /Skype: 0.5; text/call: 0.3; don't communicate: 0.15; postcard/letter: 0.05
e One possible answer is: half the people surveyed communicate via email/ Facebook /Skype while away. Technological communication makes up 80% of the communication.
21 **a** 26:58 (or 13:29) **b** 58:84 (or 29:42)
c $\frac{29}{42}$ **d** 31% (30.94%)

2F Working with ratios

2F Start thinking!

1

Pool enclosure	Length	Width	Perimeter
A	4 m	3 m	14 m
B	8 m	6 m	28 m
C	12 m	9 m	42 m
D	16 m	12 m	56 m

2 A 4:14, B 8:28, C 12:42, D 16:56
3 The ratios are multiples of each other.
4 Each ratio simplifies to $\frac{2}{7}$.
5 They are the same.

Exercise 2F Working with ratios

1 **a** 1:12 **b** 3:4 **c** 53:18 **d** 2:5
e 3:2 **f** 9:19 **g** 19:17 **h** 1:6
i 8:12:21 **j** 3:2:5 **k** 3:5:7 **l** 3:7:8
2 **a** 6:13 **b** 5:22 **c** 85:31 **d** 1:3
e 23:59 **f** 6:7 **g** 1:3 **h** 162:35
i 13:4:7 **j** 45:11:35 **k** 6:8:7 **l** 6:21:8
3 **a** 2:3 **b** 8:9 **c** 5:6
d 33:14 **e** 9:10 **f** 10:21 (20:42)
g 1:2 **h** 3:4 (51:68) **i** 85:16
j 4:5 (16:20) **k** 2:1 **l** 6:1
4 **a** 13:640 **b** 17:2340 **c** 23:432
d 5:6 **e** 1:14 **f** 19:20
g 18:11 **h** 64:25 **i** 1:4
5 **a** yes, second ratio obtained by multiplying first ratio by 3
b no
c yes, second ratio obtained by dividing first ratio by 2
d yes, second ratio obtained by multiplying first ratio by 5
e no **f** no **g** no
h yes, second ratio obtained by dividing first ratio by 3
i no
j yes, second ratio obtained by multiplying first ratio by 4
k no
l yes, second ratio obtained by dividing first ratio by 9
6 **a** Multiply both numbers by a number greater than 1.
Some possible answers are:
i 4:10, 12:30 **ii** 30:22, 45:33
b Divide both numbers by a number greater than 1.
Some possible answers are:
i 6:24, 1:4 **ii** 10:4, 40:16
7 **a** 45 **b** 10 **c** 72 **d** 2 **e** 95 **f** 7
g 6 **h** 8 **i** 3 **j** 3 **k** 85 **l** 117
m 105 **n** 14 **o** 17 **p** 6
8 **a** 2:5 = 8:20 = 20:50 **b** 6:7 = 30:35 = 54:63
c 420:360 = 70:60 = 7:6
d 225:300 = 9:12 = 3:4
e 136:104 = 68:52 = 17:13
f 180:48 = 75:20 = 15:4
g 20:48 = 40:96 = 100:240
h 216:636 = 72:212 = 18:53
9 **a** 16 **b** 80 **c** 1 **d** 35 **e** 12
f 9 **g** 44 **h** 48 **i** 30
10 **a** 5:2 **b** 3:4 **c** 14:5 **d** 4:13
11 **a** 2:7 **b** 7:2 **c** 2:5 **d** 1:1
12 **a** 32:19 **b** $\frac{32}{51}\left(\frac{64}{102}\right)$ **c** 37% (37.25%)
13 3:17
14 **a** Flour and butter; quantities are in same units.
b Flour to butter is 300:60 = 5:1.
(Butter to flour is 60:300 = 1:5.)
c For 50 pancakes (10 × 5), multiply each number in ratio by 5 to keep in proportion. 300:60 = 1500:300
15 12 **16** 27 **17** 72
18 **a** 1.05 L **b** 8000 mL
19 **a** 21.25 L **b** 600 mL
20 **a** 2, 6 **b** 3, 6 **c** 15, 10 **d** 1, 1.5
21 **a** 41:204 **b** 41:163
22 **a** 5:2 **b** 3:5 **c** 3:2
23 **a** 27.84 cm **b** 69.6 cm
24 Write each ratio as a fraction. Then compare each fraction by rewriting as equivalent fractions with same denominator.
25 4:5 is larger; $\frac{52}{65}$ is larger than $\frac{50}{65}$
27 **a** The ratio 22:27 is larger than 12:15 but smaller than 8:9.
b Some possible answers are: 81:100, 161:200, 101:125.

2G Dividing a quantity in a given ratio

2G Start thinking!

1 $12 2 1:1:1 3 3
4 a $\frac{1}{3}$ b $\frac{1}{3}$
5 $4756.80
6 $\frac{1}{3}$ of $14 270.40 = $4756.80

Exercise 2G Dividing a quantity in a given ratio

1 a i 12 ii $\frac{7}{12}$ and $\frac{5}{12}$
b i 4 ii $\frac{3}{4}$ and $\frac{1}{4}$ c i 16 ii $\frac{5}{16}$ and $\frac{11}{16}$
d i 23 ii $\frac{14}{23}$ and $\frac{9}{23}$ e i 28 ii $\frac{13}{28}$ and $\frac{15}{28}$
f i 11 ii $\frac{8}{11}$ and $\frac{3}{11}$ g i 7 ii $\frac{3}{7}$ and $\frac{4}{7}$
h i 19 ii $\frac{2}{19}$ and $\frac{17}{19}$ i i 40 ii $\frac{21}{40}$ and $\frac{19}{40}$
j i 14 ii $\frac{3}{14}$, $\frac{2}{14}$ and $\frac{9}{14}$ or $\frac{3}{14}$, $\frac{1}{7}$ and $\frac{9}{14}$
k i 29 ii $\frac{6}{29}$, $\frac{12}{29}$ and $\frac{11}{29}$
l i 21 ii $\frac{5}{21}$, $\frac{7}{21}$ and $\frac{9}{21}$ or $\frac{5}{21}$, $\frac{1}{3}$ and $\frac{3}{7}$
2 a 10 b $\frac{4}{10}$ and $\frac{6}{10}$ or $\frac{2}{5}$ and $\frac{3}{5}$
c $120 and $180 d yes
3 a $5520 and $3680 b $5750 and $3450
c $6900 and $2300 d $4600 and $4600
e $4200 and $5000 f $1840 and $7360
4 a 15 000 km and 21 000 km
b 31 500 km and 4500 km
c 20 000 km and 16 000 km
d 16 000 km and 20 000 km
e 28 800 km and 7200 km
f 18 000 km and 18 000 km
5 a $80 and $160 b $216 and $24
c $40 and $200 d $150 and $90
e $90, $30 and $120 f $168, $48 and $24
6 a 120 kg and 180 kg b 30 kg and 270 kg
c 260 kg and 40 kg d 80 kg and 220 kg
e 120 kg, 150 kg and 30 kg
f 120 kg, 75 kg, 45kg and 60 kg
7 64
8 a 1500 mL b 150 mL
9 60°, 80°, 40° 10 $28
11 a 63 km, 105 km, 42 km b 63 km
12 a history 88, science 76
b history 80, science 88; better
13 a $1000 b 9:4:7
c Katie $2.25; Mike $1.00; Sam $1.75
14 a 5:3:2
b i $25, $15, $10 ii $75, $45, $30
iii $350, $210, $140 iv $1000, $600, $400
v $5000, $3000, $2000 vi $6500, $3900, $2600
15 a yellow 10 mL; blue 15 mL
b 28 mL c 10:18 = 5:9
d 2 mL e 30 mL f 5 mL
16 a 160 g b 120 g c 140 g
17 a $\frac{4}{10} = \frac{2}{5}$ b 1:3:2:4
c $5, $15, $10, $20
18 a catching up with friends 40%; outdoor activities 25%; doing homework/chores 10%
b 25% c 8:5:2:5
d 56 hours, 35 hours, 14 hours, 35 hours
19 a width 72 cm, height 42 cm b 3024 cm^2
20 a 3:5, $\frac{3}{8}$ b 7:1, $\frac{7}{8}$ c 1:1, $\frac{1}{2}$ d 3:1, $\frac{3}{4}$
21 a H b L c M d G e K f F
22 a F b D c I d E e C f B
g G h H
23 The ratio that shows relationship of part to part compares one part to another. For example, in 2:3 there is a total of 5 parts. First number represents $\frac{2}{5}$ of total and second number represents $\frac{3}{5}$ of total.
The ratio that shows relationship of a part to a whole is better expressed as a fraction as it compares one number to the total.
24 a Can A has 3200 mL of white paint and 800 mL of red paint.
Can B has 3500 mL of white paint and 500 mL of red paint.
b 67:13 c $\frac{13}{80}$ d $83\frac{3}{4}$% or 83.75%

2H Understanding rates

2H Start thinking!

1 a Alexis 350 m per min, Kym 340 m per min
b Alexis, as she travelled a greater distance per minute
c Speed is distance (m) per one unit of time (min) so speed unit is m per min or m/min.
2 change in distance over time
3 Answers will vary. One possible answer is that a ratio compares quantities of the same kind and a rate describes change in one quantity with respect to another.

Exercise 2H Understanding rates

1 a 60 km/h b 591 L/min
c four quarters/game d $16/h
e 45 cm/year f five patients/h
g $1.49/day h $28.99/month
i $5.80/box j 0.5kg/week
k 290 mm/year l 14 km/L
2 a 50 m/min b 2 L/bottle
c 4 cents/s d 15 students/teacher
e 26 seats/row f 25 newspapers/bundle
g 6.8 runs/over h $2.50/muffin
i 6.8 goals/game j 4444 tickets/min
k 3.5 goals/game l 5.25 m^2/L
3 a 42.99 b 15.75 c 5.2 d 111.25
e 2.25 f 26.18 g 4.84 h 24
4 a $15.20/hour b 1.59 cents/gram
c 46.15 words/min d 0.93 sheep/min
e 40 words/min f 386 m/s
5 a 14 km/L b $1.99/mango
c 12 L/min d 15.3 km/h
e 23.35 people/km^2 f 18 million km/min
6 $355.50 7 $105

8 **a** 54.75 km/h **b** 854.35 km/h **c** 32.5 km/h
d 0.38 km/min **e** 2.10 m/s **f** 1.89 m/s

9 **a** words, time **b** distance, time
c percentage, time **d** mass, time
e money, distance **f** wickets, overs bowled
g distance, time **h** money, line
i money, volume **j** sheep, area
k length, time **l** distance, volume

10 **a** m/s **b** m/s **c** cents/apple or $/kg
d $\$/m^2$ **e** L/min **f** wickets/over or km/h
g KB/s or GB/min **h** km/h

11 pasta: 5000 kg/day, fish: 500 portions/day, sushi: 5714.3 pieces/day, noodles: 714.3 kg/day, sausages: 5642.9 sausages/day, chips: 2.5 tonnes/day, schnitzels: 642.9 schnitzels/day

12 **a** Max 5.75 runs/over, William 5.6 runs/over
b Max's team

13 **a** Mauro $1.42/L, Joy $1.49/L
b Joy paid higher price on Friday. Prices vary across week.

14 **a** 56 bpm **b** 27 min

15 **a** 30 mins/tart **b** 4 days
c 24 mins/tart; 3 days

16 Usain Bolt's rate is 10.4384 m/s.
a 10.35 m/s **b** 10.70 m/s **c** 16 m/s

17 **a** 0.3125 km (312.5 m)
b **i** 17.1875 km **ii** 18.75 km
iii 20.3125 km **iv** 24.375 km
v 26.5625 km **vi** 28.125 km
vii 32.8125 km **viii** 33.75 km
c **i** 2 min **ii** 1 h 40 min
iii 1 h 20 min **iv** 6 min 24 s

18 **a** $6.24 **b** 754 kWh **c** $1.88

19 **a** Hani's car (11.5 km/L compared to 11.2 km/L)
b Illia 74.1 L, Hani 72.2 L

20 the one with cheaper price per litre

21 **a** 7.5 kg of potatoes ($0.95/kg compared to $0.99/kg)
b packet of 10 batteries ($0.989/battery compared to $0.9975/battery)
c 500 g tin of coffee ($0.018/g compared to $0.01993/g)
d 650 g tin of peaches ($0.0039/g compared to $0.0042/g)

22 **a** Plan A: 80 c, Plan B: 88 c; Plan A
b Plan A: 160 c, Plan B: 164 c; Plan A
c Plan A: 240 c, Plan B: 240 c; both plans
d Plan A: 320 c, Plan B: 316 c; Plan B
e Use Plan A for calls shorter than 5 min, use Plan B for calls longer than 5 min, use either plan for calls of exactly 5 min.

2 Chapter review

MULTIPLE-CHOICE

1 D **2** C **3** B **4** A **5** B
6 D **7** A **8** C **9** B **10** D

SHORT ANSWER

1 **a** 7 **b** $\frac{7}{20}$ **c** $\frac{35}{100}$ **d** 35%

2 **a** 0.23 **b** 0.62 **c** 0.1652 **d** 2.916

3 **a** 32% **b** 762% **c** 3162.5% **d** 0.03%

4 **a** 75% **b** 70% **c** 85% **d** 46.875%

5 **a** 40% **b** 16% **c** 46.4%

6 **a** 34 **b** 15.5 **c** 2250 **d** 888

7 **a** $1045 **b** $904.32 **c** $903.60

8 **a** 37.5% profit **b** 40% loss
c 51.68% profit **d** 31.57% loss

9 **a** **i** 5:6 **ii** 2:3 **b** **i** $\frac{3}{16}$ **ii** $\frac{1}{8}$

10 **a** 35:47 **b** 7:31 **c** 2:9 **d** 9:25

11 **a** 3:11 **b** 16:27 **c** 1:2:6 **d** 1:5:7

12 **a** 42 **b** 16 **c** 12 **d** 35

13 **a** 8 **b** 17 **c** 17 **d** 31

14 **a** $600, $900 **b** $450, $1050
c $300, $400, $800 **d** $300, $375, $825

15 **a** 100 km/h **b** 3 L/s **c** $17.50/h
d 4.333 books/month

16 **a** $1.39/L **b** $0.0048/mL or 0.48 c/mL
c 1.5 points/min **d** $16.80/h

NAPLAN-STYLE PRACTICE

1 C **2** 36% **3** 0.42 **4** 0.625
5 3.6%, 15%, $\frac{7}{8}$, 1.1 **6** 55% **7** 87.5%
8 $37.50 **9** $578 **10** $228
11 loss of 15.91% **12** 125%
13 48 minutes **14** 13:12 **15** 107:65
16 30:15:25 **17** 1:2 **18** 5
19 $\frac{1}{4}$ **20** $110 **21** $400
22 earning $15.75 for every hour worked
23 12 km/L
24 five chocolate bars for $4.00

ANALYSIS

a **i** $\frac{13}{20}$ **ii** 0.65 **b** $13 **c** $33
d $455 **e** $3.30 **f** $29.70 **g** $9.70
h 48.5% **i** 4:5:2
j day 1: 7 hoodies/h, day 2: 6.875 hoodies/h
k day 1

2 Connect

For feedback on this open-ended task, see your teacher.

CHAPTER 3 POSITIVE AND NEGATIVE NUMBERS

3 Are you ready?

1 A: +3; B: −2; C: 0; D: −4; E: +1.5

2 **a** C **b** −3 **c** +3

3 **a** C **b** A **4** **a** 815 **b** 22

5 **a** B **b** $\frac{1}{20}$ **6** **a** D **b** $\frac{1}{4}$

7 **a** 12.1 **b** 2.9 **8** **a** 2.88 **b** 21

9 7.2

10 **a** B **b** A **c** C

11 **a** D **b** B **c** C

3A Understanding negative numbers

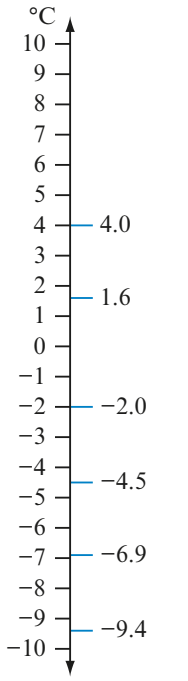
3A Start thinking!

1 **a** −2°C, −4.5°C, −6.9°C, −9.4°C. Negative sign (−) indicates number is negative.
b 1.6°C, 4.0°C

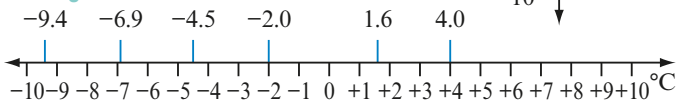
2 −2°C, 4°C

3 **a** see diagram
b highest 4.0°C, lowest −9.4°C
c 0

4 Some possible answers for a vertical number line are: +7 (7 units up from 0), −5 (5 units down from 0), −0.15 (0.15 units down from 0), $+6\frac{2}{5}$ ($6\frac{2}{5}$ units up from 0).



5



6 **a** A number is larger if it is higher on a vertical number line, and smaller if it is lower.
b A number is larger if it is further right on a horizontal number line, and smaller if it is further left.

7 −9.4°C, −6.9°C, −4.5°C, −2°C, 1.6°C, 4.0°C

8 January

Exercise 3A Understanding negative numbers

1 **a** negative integers: −1, −2, −3, −4, −5, −6, −7, −8, −9, −10; positive integers: +1, +2, +3, +4, +5, +6, +7, +8, +9, +10
b 0

2 **a** 9 **b** 5 **a** 1 **d** 5 **e** 0 **f** −4

3 **a** 5 **b** 0 **c** −2 **d** −5 **e** −1 **f** −9

4 **a** 5.2 **b** $\frac{1}{2}$ **c** 4.3 **d** $\frac{4}{7}$ **e** −1.5 **f** $-\frac{2}{5}$

5 **a** $-4\frac{1}{2}$ **b** −2.1 **c** −6.34 **d** $-8\frac{6}{11}$
e −3.54 **f** $-\frac{3}{4}$

6 **a** $-3 < 2$ **b** $-8 < -4$ **c** $0 > 1$
d $9 > -9$ **e** $-3 > -5$ **f** $-7 < -2$
g $1.4 > -2.6$ **h** $-1.5 < -1.2$ **i** $4.3 > -4.3$
j $-3.8 > -5.8$ **k** $-0.6 > -0.7$ **l** $-8.2 < 0$

7 **a** 35 **b** −24 **c** 0 **d** 47
e $-50\frac{1}{4}$ **f** 113.2 **g** $-94\frac{1}{3}$ **h** −89.3
i $-2000\frac{1}{7}$

8 **a** −20, −12, −11, −7, 8, 10, 14
b −41, −33, −19, 0, 6, 29, 42
c −88, −48, −28, −18, −8, 8, 68
d −140, −126, −82, 3, 73, 104, 145

9 **a** 71.9, 4.3, −9.7, −10.8, −15.4, −27.6
b 10, $5\frac{1}{2}$, $-\frac{1}{2}$, $-2\frac{1}{2}$, −5, $-11\frac{1}{2}$
c 21.6, 5.34, 0, −9.04, −9.4, −14.2
d $-7\frac{2}{3}$, $-7\frac{1}{3}$, $-6\frac{2}{3}$, $-1\frac{1}{3}$, $1\frac{2}{3}$, $5\frac{1}{3}$, $7\frac{1}{3}$

10 −6, −5, −4, −3, −2, −1, 0, +1, +2, +3

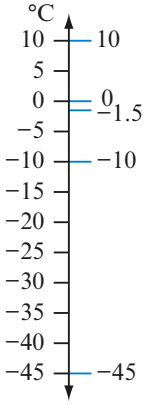
11 Some possible answers are: $-\frac{90}{4}$, $-25\frac{1}{2}$, $-\frac{115}{4}$, $-\frac{593}{20}$

12 Some possible answers are: −4.75, −3.845, 0.1235, 2.9578, −3.1.

13 **a** 0
b **i** 4 units to the left of 0
ii 5 units to the right of 0
iii 3 units right of 0 **iv** 5 units left of 0
v 3 units to the left of 0
vi 9 units to the right of 0
c +5 and −5; +3 and −3
d **i** +8 **ii** −6 **iii** −1
iv +11 **v** +32 **vi** −17
e Some possible answers are: +4 and −4, −10 and +10, −7 and 7.

14 **a** −3 **b** +2174.30 **c** −18
d −408 **e** +5895 **f** +500
g −46.55 **h** −40

15 **a** see diagram
b higher
c −45°C, −10°C, −1.5°C, 0°C, 10°C
d warmer



16 **a** **i** $34.45 **ii** $13.40
b **i** $51.30 **ii** $35.00
c Account is overdrawn; $13.25 owed to bank.

17 **a** +86 **b** $14 **c** −14

18 **a** long sighted
b short sighted
c Elle has weaker vision.
d normal vision; that is, neither long nor short sighted

19 **a** three
b **i** two blue counters **ii** four red counters
iii six blue counters **iv** five red counters
v one red counter **vi** eight blue counters
c The reference point for opposite integers is always zero as both integers are the same number of units away from 0. Moving forward to the positive integer and then moving back as indicated by the opposite integer will always result in zero. When combined, opposite integers 'cancel' each other to result in zero. This can be modelled by equal numbers of blue and red counters.
d Two blue counters and two red counters produce two zero pairs; remaining three red counters represent integer −3.
e **i** +3 **ii** −1 **iii** +2

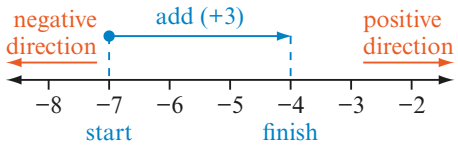
f One possible answer is given.
- **i** five blue counters, three red counters
- **ii** two blue counters, six red counters
- **iii** nine red counters, two blue counters
- **iv** three red counters, six blue counters
- **v** two red counters, two blue counters
- **vi** two blue counters, four red counters

3B Adding integers

3B Start thinking!

1 a 7 **b** right **c** +7; they are the same

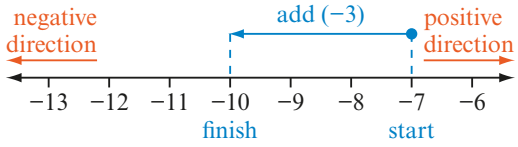
2 a



b −4 **c** right

3 a left **b** +3

4 a



b −10 **c** left

5 a Start at first number and move in positive direction (right) by number of units indicated by second integer.

b Start at first number and move in negative direction (left) by number of units indicated by second integer.

Exercise 3B Adding integers

1 a (−5) + (+3) = −2 **b** (+6) + (−2) = +4
c (−4) + (−5) =−9 **d** (−2) + (+5) = +3
e (+7) + (−5) = +2 **f** (−9) + (+4) = −5

2 a +3 **b** +6 **c** +5 **d** −8 **e** −4 **f** −3
g −6 **h** +5 **i** −4 **j** 0 **k** 0 **l** 0

3 a +2 **b** +3 **c** −5 **d** −2 **e** +5 **f** +1
g +9 **h** −9 **i** 0 **j** 0 **k** −2 **l** 0

4 a (−11) + (−6) = −17 **b** (+17) + (−4) = +13
c (−19) + (+5) = −14 **d** (−13) + (+6) = −7

5 a −16 **b** +8 **c** +7 **d** +18 **e** 0
f −8 **g** −7 **h** −14 **i** −19 **j** +15
k +2 **l** −9 **m** 0 **n** −12 **o** +20
p −4 **q** −20 **r** +5

6 a positive **b** negative
c positive, negative

7 If size (number of units from 0) of positive number is larger than size of negative number, sum will be positive. If size of negative number is larger than size of positive number, sum will be negative. If positive number and negative number have same size (opposite numbers), sum will be zero.

8 a positive **b** negative **c** zero
d negative **e** negative **f** positive
g positive **h** positive **i** zero
j negative **k** positive **l** negative

9 a +17 **b** −19 **c** 0 **d** −7 **e** −17
f +4 **g** +5 **h** +41 **i** 0 **j** −5
k +3 **l** −3

10 a +28 **b** −29 **c** +8 **d** −5 **e** 0
f −30 **g** +7 **h** −18 **i** −4 **j** −47
k +12 **l** −23 **m** −17 **n** 19 **o** 62
p −180

11 a D **b** 2°C

12 a C **b** fourth level **c** ground level

13 a (−21) + (+12) **b** −9°C

14 a −3 **b** (−3) + (+17) **c** 14 m

15 a −60 **b** (−60) + (+35) **c** $25

16 a −123 **b** +180 **c** −112 **d** −150
e +29 **f** −7 **g** −170 **h** +13
i 0 **j** +180 **k** +125 **l** −24

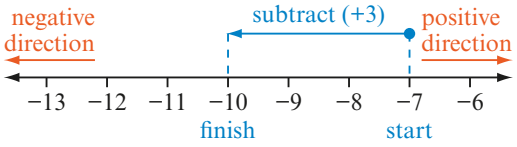
17 a −198°C; (−212) + (+14) = −198
b −193°C; (−212) + (+19) = −193

3C Subtracting integers

3C Start thinking!

1 a 3 **b** left **c** +3, they are the same

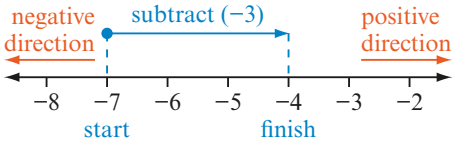
2 a



b −10 **c** left

3 a right **b** +7

4 a



b −4 **c** right

5 a Start at position of first integer and move in negative direction (left) by number of units indicated by second integer.

b Start at position of first integer and move in positive direction (right) by number of units indicated by second integer.

Exercise 3C Subtracting integers

1 a (+2) − (−4) = +6 **b** (+1) − (+5) = −4
c (−4) − (−3) = −1 **d** (−6) − (+3) = −9
e (+3) − (−2) = +5 **f** (+4) − (+4) = 0

2 a +2 **b** −7 **c** 0 **d** −1 **e** +8 **f** −6
g −7 **h** +10 **i** +3 **j** −5 **k** −3 **l** +8

3 a −5 **b** −10 **c** +6 **d** +7 **e** −9 **f** 0
g +4 **h** −9 **i** +1 **j** +4 **k** +4 **l** 0

4 a (+16) − (+3) = +13 **b** (−19) − (−5) = −14
c (−12) − (+4) = −16 **d** (−8) − (−10) = +2

5 a +5 **b** +19 **c** +20 **d** −18 **e** 0 **f** −20
g +18 **h** −20 **i** +7 **j** +2 **k** 0 **l** +16
m −11 **n** +3 **o** +2 **p** −6 **q** −20 **r** −4

6 a positive **b** negative **c** positive, negative

7 If two integers are identical, subtracting them will always result in zero. When subtracting numbers that are both positive, if first number is larger, result will always be positive. If second number is larger, result will always be negative. When subtracting numbers that are both negative, if first number is larger, result will be positive, otherwise result will be negative.

8 a positive b negative c zero
d negative e negative f negative
g positive h negative i zero
j negative k negative l positive

9 a +19 b −20 c 0 d −3 e −3
f −28 g +22 h −7 i 0 j −1
k −5 l +7

10 a −32 b +25 c +14 d 0 e +11
f −20 g −69 h +140 i −4 j +91
k +90 l −86 m −70 n +92 o −166
p +27

11 a B b −10°C

12 a A b 5 m c D, 5 m
d the water level

13 51°C; (+33) − (−18) = 51

14 a +28 b shark −3; sting ray −11
c i (+28) − (−3) ii (+28) − (−11)
iii (−3) − (−11)
d i 31 m ii 39 m iii 8 m

15 a −124 b −80 c −140 d +170 e −368
f +135 g +5 h +220 i −480 j +92
k +595 l −915

16 a $63; (−247) − (−310) = 63
b Initially, account is overdrawn by $310. During the month, $63 is added to the account and overdrawn amount is reduced from $310 to $247.

17 One possible answer is: −7 and −3.
(−7) + (−3) = −10, (−7) − (−3) = −4

3D Simplifying addition and subtraction of integers

3D Start thinking!

1 a i +7 ii +7 b right
c You move in same direction along number line (to right) when adding a positive number or subtracting a negative number.
d They produce same result.

2 The two operation signs next to each other (either + + or − −) can be replaced with one addition operation.

3 a i +3 ii +3 b left
c You move in same direction along number line (to left) when subtracting a positive number or adding a negative number.
d They produce same result.

4 The two operation signs next to each other (either + − or − +) can be replaced with one subtraction operation.

5 For two operations next to each other that are the same, replace with +.
For two operations next to each other that are different, replace with −.

Exercise 3D Simplifying addition and subtraction of integers

1 −, negative, left; −, negative, left;
+, positive, right

2 a (−3) − (+7) = −3 − 7
b (+1) + (+6) = 1 + 6
c (−4) + (−5) = −4 − 5
d (+2) − (−4) = 2 + 4
e (+5) − (+9) = 5 − 9
f (−6) − (−8) = −6 + 8

3 a −1 + 3 b 8 − 4 c −5 − 2 d 6 + 1
e 4 + 3 f 3 − 9 g −5 + 8 h −3 + 1
i −7 − 3 j −6 + 7

4 a 3 b 6 c −4 d 0 e −1 f 5
g 1 h −5 i 9

5 a 5 b −2 c 6 d −10 e −3 f −7
g 4 h 2 i 0 j −9 k −9 l 7

6 a −10 b 7 c −9 d 6 e −4 f 2

7 a 2 b 4 c −7 d 7 e 7 f −6
g 3 h −2 i −10 j 1

8 a C b A c B

9 a −15 b −9 c 9

10 a 3 b −6 c −4 d 16 e −19 f −10
g −17 h −2 i −13 j 0 k −20 l 2

11 a negative b positive c negative
d zero e positive f positive
g negative h negative i negative

12 a −10 b 30 c −7 d 0 e 7 f 7
g −13 h −24 i −3

13 a

+	−5	−3	0	1	4
−3	−8	−6	−3	−2	1
−2	−7	−5	−2	−1	2
−1	−6	−4	−1	0	3
3	−2	0	3	4	7
6	1	3	6	7	10

b

+	−10	−14	−7	13	19
11	1	−3	4	24	30
14	4	0	7	27	33
−10	−20	−24	−17	3	9
−22	−32	−36	−29	−9	−3
−18	−28	−32	−25	−5	1

14 a −5 b −99 c −32 d 9
e −40 f −275 g 242 h 118
i 0 j −2000 k −2980 l −31
m 27 n −640 o −5685

15 a 10 b −4 c −3 d −13 e 0
f −33 g −10 h −26 i −30 j −17

16 a −6 b 15 c 0 d 12 e −14
f −31 g 8 h 2

17 **a** A: +50, B: −2, C: +32
b **i** (+50) − (−2) = 50 + 2
ii (+32) − (−2) = 32 + 2
iii (+50) − (+32) = 50 − 32
c **i** 52 m **ii** 34 m **iii** 18 m
18 four levels below ground; 15 − 19 = −4
19 **a** +285 **b** 285 − 300
c −15; account is overdrawn by $15
20 **a** (−900) − (+15) = −915; 915 m down
b (−2500) − (−900) = −1600; 1600 m down
c (+15) − (−2500) = +2515; 2515 m up
21 first level underground; 3 − 9 + 5 = −1
22 One possible answer is: (+21) + (−15) + (−8).
23 **a** 24 **b** −312 **c** −5 **d** 70
e 1018 **f** −4218
24 **a** 100 **b** −21 **c** 366 **d** −249
e −4000 **f** 9007
25 **a** 45 **b** −30 **c** 1261 **d** −367
e −1020 **f** −3260
26 **a** 33 **b** 77 **c** −125 **d** −40

3E Multiplying and dividing numbers

3E Start thinking!

1 **a** There are six blue counters. **b** +6
2 +6 **3** no **4** positive
5 **a** There are six red counters. **b** −6
6 negative **7** −6 **8** negative
9 **a** +6 **b** positive

Exercise 3E Multiplying and dividing numbers

1 **a** +12 **b** −14
2 **a** −12 **b** +35 **c** −12 **d** +27 **e** −8
f −24 **g** +72 **h** +20 **i** −44 **j** −7
k +8 **l** +45
3 **a** **i** +2 × +3 = +6 so +6 ÷ +2 = +3 or +6 ÷ +3 = +2
ii +2 × −3 = −6 so −6 ÷ +2 = −3 or −6 ÷ −3 = +2
iii −2 × +3 = −6 so −6 ÷ −2 = +3 or −6 ÷ +3 = −2
iv −2 × −3 = +6 so +6 ÷ −2 = −3 or +6 ÷ −3 = −2
b yes, same pattern
c **i** positive **ii** positive
iii negative **iv** negative
4 **a** −5 **b** +4 **c** +6 **d** −7 **e** +17 **f** −8
g −7 **h** +9 **i** −6 **j** −12 **k** +10 **l** −9
5 **a** −35 **b** −3 **c** 7 **d** −60 **e** −4 **f** 60
g −5 **h** −36 **i** 90 **j** −1 **k** −38 **l** 45
6 **a** −3 **b** −7 **c** −9 **d** 100 **e** −70 **f** −3
7 **a**

×	−2	−1	0	+1	+2
−4	8	4	0	−4	−8
−2	4	2	0	−2	−4
0	0	0	0	0	0
+2	−4	−2	0	2	4
+4	−8	−4	0	4	8
+6	−12	−6	0	6	12

b

×	−25	−20	−10	10	20
7	−175	−140	−70	70	140
5	−125	−100	−50	50	100
0	0	0	0	0	0
−2	50	40	20	−20	−40
−4	100	80	40	−40	−80
−6	150	120	60	−60	−120

8 **a** −8 **b** −12 **c** 17 **d** 15 **e** 8 **f** −3
9 **a** −1 × −7 = 7 **b** −1 × +12 = −12
c −1 × −88 = 88 **d** −1 × 25 = −25
e −1 × (8 − 5) = −1 × 3 = −3
f −1 × (−3 + 2) = −1 × −1 = 1
10 **a** 60 **b** −42 **c** −54 **d** −48 **e** −130
f 84 **g** −160 **h** 110 **i** −180
11 **a** 36 **b** −5 **c** −24 **d** −6 **e** 30
f −42
12 **a** 10, 8, 6, 4, 2, 0, −2, −4, −6, −8, −10; decreases by 2, or you get the next number by subtracting 2 (or adding −2)
b −15, −12, −9, −6, −3, 0, 3, 6, 9, 12, 15; increases by 3, or you get the next number by adding 3 (or subtracting −3)
c 20, 16, 12, 8, 4, 0, −4, −8, −12, −16, −20; decreases by 4, or you get the next number by subtracting 4 (or adding −4)
d −25, −20, −15, −10, −5, 0, 5, 10, 15, 20, 25; increases by 5, or you get the next number by adding 5 (or subtracting −5)
13 **a** **i** 0 **ii** 0 **iii** 0 **iv** 0
b Answer is always zero.
c **i** 0 **ii** 0 **iii** undefined **iv** undefined
d Answer is always zero.
e Result is undefined. It is not possible to divide a number by zero.
14 Some possible answers are given.
a **i** −2 × 12, 4 × −6, −3 × 8
ii −2 × −9, 6 × 3, −18 × −1
b **i** −12 ÷ 3, 36 ÷ −9, −8 ÷ 2
ii 10 ÷ 2, −20 ÷ −4, −35 ÷ −7
c −2 × 0, 0 × 6, −200 × 0
d 0 ÷ 3, 0 ÷ −6, 0 ÷ 100
e **i** −2 × 6 × 4, −3 × −8 × −2, 12 × 4 × −1
ii −10 × −2 × 6, 24 × −5 × −1, −12 × 5 × −2
15 **a** −6, −3 **b** 9, −5 **c** −11, 6 **d** −4, 14
e 4, 6 or −6, −4 **f** −9, −1 or 1, 9
g −16, 2 or −2, 16
16 **a** −120 **b** +80 **c** 200 m **d** +110 **e** +40
f **i** 40 m **ii** 230 m **g** −160 **h** −40
i Hayden. He needs to walk 20 m west.
j Natalie: 320 m; Tyler: 240 m; Hayden: 120 m; Rhys: 320 m; Sophie: 160 m; Imogen: 220 m
17 **a** **i** −20 **ii** −25 **b** **i** 380 **ii** 387
18 One possible answer is: −2 × 7 × 3.
19 One possible answer is: −4 × 6 ÷ −3.
20 One possible answer is: 2 × −5 × 25 × 4.

3F Operations with directed numbers

3F Start thinking!

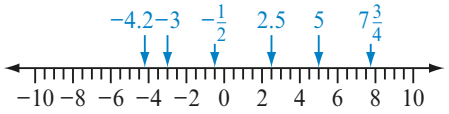
1 a The first number is written more simply as $\frac{1}{7}$ and the operations + (−) can be replaced with −.
b $-\frac{2}{7}$

2 a $-0.4 + 0.9$ b 0.5

3 a positive b positive c negative
d negative e negative f negative
g positive h positive

Exercise 3F Operations with directed numbers

1



2 a −2.5 b −5.6 c 7.2 d −10.3 e −0.2
f 1.3 g 3.4 h 1.7 i 5.3

3 a $\frac{4}{7}$ b $-2\frac{1}{3}$ c $-\frac{2}{8} = -\frac{1}{4}$
d $-\frac{2}{5}$ e $-1\frac{2}{3}$ f $-1\frac{1}{4}$
g $\frac{2}{9}$ h $-1\frac{1}{10}$ i $-1\frac{1}{6}$

4 a $\frac{4}{11}$ b $-\frac{1}{3}$ c $-1\frac{1}{5}$ d $\frac{4}{15}$ e $-\frac{7}{12}$
f $-\frac{5}{27}$ g $-1\frac{1}{5}$ h $-\frac{40}{81}$ i $\frac{2}{9}$

5 a 34.8 b −140.1 c −79.55 d −3.8
e 8.7 f −3.24

6 a $-\frac{1}{5}$ b $-\frac{3}{7}$ c $-\frac{9}{11}$ d $-1\frac{1}{6}$ e $-\frac{3}{14}$ f $\frac{1}{8}$

7 a −1.51 b −1.7 c −78.9 d −7.496
e −0.405 f 3.92 g 4.4 h 4.5
i −1.4 j −18.2 k $-\frac{1}{3}$ l 0
m $-\frac{2}{3}$ n $-\frac{3}{20}$ o $\frac{7}{18}$

8 a

−4	6	−8
−6	−2	2
4	−10	0

b

−6.8	17	0
10.2	3.4	−3.4
6.8	−10.2	13.6

c

$-\frac{2}{3}$	$\frac{1}{2}$	$-\frac{1}{3}$
$\frac{1}{6}$	$-\frac{1}{6}$	$-\frac{1}{2}$
0	$-\frac{5}{6}$	$\frac{1}{3}$

9 a $-\frac{2}{3}$ b $-5\frac{7}{10}$ c $\frac{11}{12}$ d −15 e $\frac{2}{3}$ f $-1\frac{1}{2}$

10 a −9 b 0 c −5 d 7 e 22 f 5

11 a 1.25 b −0.26 c −1.88 d −1.9
e −1 f −7.2

12 a −1 b $\frac{3}{7}$ c $-\frac{13}{14}$ d −1 e $\frac{1}{3}$ f $-\frac{3}{14}$

13 a 12.3 b −5.5
c −5.5 − 12.3 or 12.3 − (−5.5) d 17.8 m

14 a

	Sun	Mon	Tue	Wed	Thu	Fri	Sat
Difference	6.7	6.3	2.3	3.1	1.9	1.3	2.2

b Sunday c 1.2°C d −2.2°C e 3.4°C

15 71.7°C; $(+65.2) - (-6\frac{1}{2})$

16 a deposit b withdrawal
c i He has money in the bank.
ii He owes the bank money.
d −$95.35 e $134.65 f −$25.90
g Yes. A fee is charged.

17 Some examples are: −3.2 and 3.2, $\frac{1}{8}$ and $-\frac{1}{8}$.

18 a Some examples are: 3.86 and −10.36, −3.2 and −3.3.
b Some examples are: $-\frac{1}{6}$ and $-\frac{1}{3}$, $\frac{3}{4}$ and $-\frac{5}{4}$.

19 Some examples are given.
a −5 and −4, −12.1 and −2.1
b −4 and 2, −5 and 3

20 Negative. For sum of two numbers to be zero, they must be opposite numbers. As one number is positive and the other negative, product will be negative (+ × − → −).

3G Powers of directed numbers

3G Start thinking!

1 a $2 \times 2 \times 2$ b 8

2 a $(-2) \times (-2) \times (-2)$ b −8
c negative; product of first pair is positive, then multiplying this by a negative produces a negative result.

3, 4

Index form	Base	Index or power	Expanded form	Basic numeral
5^2	5	2	5×5	25
$(-5)^2$	−5	2	-5×-5	25
4^3	4	3	$4 \times 4 \times 4$	64
$(-4)^3$	−4	3	$-4 \times -4 \times -4$	−64
2^4	2	4	$2 \times 2 \times 2 \times 2$	16
$(-2)^4$	−2	4	$-2 \times -2 \times -2 \times -2$	16
3^5	3	5	$3 \times 3 \times 3 \times 3 \times 3$	243
$(-3)^5$	−3	5	$-3 \times -3 \times -3 \times -3 \times -3$	−243

5 A negative base raised to an even power gives a positive result. A negative base raised to an odd power gives a negative result.

6 Because 5 and −5 are raised to an even power (same positive result) while 4 and −4 are raised to an odd power (one positive result, one negative result).

7 yes, as the power is even

Exercise 3G Powers of directed numbers

1 a 125 b 49 c −27 d 10 000 2 E

3 a 5^3 b $(-7)^2$ c $(-3)^3$ d $(-10)^4$

4 a $-9 \times -9 = 81$ b $8 \times 8 = 64$
c $-6 \times -6 \times -6 = -216$
d $7 \times 7 \times 7 = 343$
e $-5 \times -5 \times -5 \times -5 = 625$
f $1 \times 1 \times 1 \times 1 = 1$
g $4 \times 4 \times 4 \times 4 \times 4 = 1024$
h $-10 \times -10 \times -10 \times -10 \times -10 = -100\,000$
i $2 \times 2 \times 2 \times 2 \times 2 \times 2 = 64$
j $-3 \times -3 \times -3 \times -3 \times -3 \times -3 = 729$
k $-2 \times -2 \times -2 \times -2 \times -2 \times -2 \times -2 \times -2 = -128$
l $-1 \times -1 \times -1 \times -1 \times -1 \times -1 \times -1 \times -1 \times -1 = 1$

5 **a** negative **b** positive **c** positive
d positive

6 **a** $-1.2 \times -1.2 = 1.44$ **b** $0.9 \times 0.9 = 0.81$
c $-0.4 \times -0.4 \times -0.4 = -0.064$
d $-0.6 \times -0.6 \times -0.6 \times -0.6 = 0.1296$
e $-0.2 \times -0.2 \times -0.2 \times -0.2 \times -0.2 = -0.000\,32$
f $-0.1 \times -0.1 \times -0.1 \times -0.1 \times -0.1 \times -0.1 \times -0.1 = -0.000\,000\,1$
g $-\frac{1}{2} \times -\frac{1}{2} \times -\frac{1}{2} = -\frac{1}{8}$ **h** $-\frac{5}{9} \times -\frac{5}{9} = \frac{25}{81}$
i $-\frac{4}{7} \times -\frac{4}{7} \times -\frac{4}{7} = -\frac{64}{343}$
j $-\frac{1}{3} \times -\frac{1}{3} \times -\frac{1}{3} \times -\frac{1}{3} = \frac{1}{81}$
k $-\frac{2}{3} \times -\frac{2}{3} \times -\frac{2}{3} \times -\frac{2}{3} \times -\frac{2}{3} = -\frac{32}{243}$
l $-\frac{3}{2} \times -\frac{3}{2} \times -\frac{3}{2} \times -\frac{3}{2} \times -\frac{3}{2} \times -\frac{3}{2} = \frac{729}{64} = 11\frac{25}{64}$

7 negative, positive

8 **a** $(-4)^5, (-2)^3, (-3)^2, (+5)^4$
b $(-2)^7, (-1)^{10}, (-7)^2, 10^2$
c $(-0.1)^3, (0.2)^3, (-0.2)^2, (-0.3)^2$
d $\left(-\frac{1}{2}\right)^5, \left(-\frac{1}{3}\right)^3, \left(-\frac{1}{2}\right)^4, \left(-\frac{1}{3}\right)^2$

9 **a** $(-5)^3 \times (-9)^4$ **b** $(-4)^6 \times 3^3$
c $7^2 \times (-6)^6$ **d** $(-8)^4 \times (-10)^2$

10 **a** $-2 \times -2 \times -2 \times -4 \times -4 = -128$
b $-5 \times -5 \times 3 \times 3 \times 3 \times 3 = 2025$
c $-10 \times -10 \times -10 \times -2 \times -2 \times -2 \times -2 \times -2 = 32\,000$
d $-3 \times -3 \times -3 \times -3 \times -3 \times -1 \times -1 = -243$
e $-6 \times -6 \times -2 \times -2 \times -2 = -288$
f $-1 \times -1 \times -1 \times -1 \times 3 \times 3 = 9$
g $-7 \times -7 \times -1 \times -1 \times -1 \times -1 \times -1 = -49$
h $-3 \times -3 \times -3 \times -2 \times -2 = -108$
i $-1 \times -1 \times -1 \times -2 \times -2 \times -2 \times -2 \times -2 = 32$

11 **a** 17 **b** 10 049 **c** 96 **d** −7
e 125 **f** 4 **g** 48 **h** 0.159
i −1.218 **j** $-\frac{1}{72}$ **k** $-\frac{19}{32}$
l −0.000 002 7 **m** 4 **n** $-\frac{2}{27}$ **o** $-\frac{45}{32}$

12 **a** **i** −1 **ii** 1 **iii** −1 **iv** 1 **v** −1 **vi** 1
b those with even index or power (ii, iv, vi)
c those with odd index or power (i, iii, v)
d Negative numbers raised to an even power will give a positive result and negative numbers raised to an odd power will give a negative result.
e **i** negative **ii** positive **iii** positive
iv negative **v** positive **vi** negative

13 **a** **i** 9 **ii** −27 **iii** −243
b −243 **c** same result
d **i** −8 **ii** −32 **iii** 256
e 256 **f** same result
g To multiply numbers in index form with the same base, add the powers.
h **i** $(-4)^3 \times (-4)^2, (-4)^5, -1024$
ii $(-2)^4 \times (-2)^3, (-2)^7, -128$
iii $(-0.1)^5 \times (-0.1)^3, (-0.1)^8, 0.000\,000\,01$
iv $\left(-\frac{1}{2}\right)^3 \times \left(-\frac{1}{2}\right)^2, \left(-\frac{1}{2}\right)^5, -\frac{1}{32}$
i Bases must be the same.

14 **a** **i** 729 **ii** 81 **iii** 9
b 9 **c** same result
d **i** −128 **ii** 16 **iii** −8
e −8 **f** same result
g To divide numbers in index form with the same base, subtract the powers.
h **i** $(-5)^8 \div (-5)^5, (-5)^3, -125$
ii $(-6)^9 \div (-6)^7, (-6)^2, 36$
iii $(-0.2)^{10} \div (-0.2)^6, (-0.2)^4, 0.0016$
iv $\left(-\frac{4}{5}\right)^{12} \div \left(-\frac{4}{5}\right)^9, \left(-\frac{4}{5}\right)^3, -\frac{64}{125}$
i Bases must be the same.

15 **a** $-8 \div -8 = 1$ **b** $(-2)^0$ **c** yes; $(-2)^0 = 1$

16 **a** −8 **b** −216 **c** 10 000 **d** 1
e −27 **f** 49 **g** 1 **h** −3

17 **a** 5 and −5; both numbers give the result of 25 when squared.
b No; to square a number is to multiply the number by itself. Either a negative number is squared (− × − → +) or a positive number is squared (+× + → +). In both cases the result is positive.
c only one number (2)
d only one number (−2)
e There will be two numbers that when squared (or raised to an even power) give the same positive result but no number can be squared to give a negative result. There will only be one number that when cubed (or raised to an odd power) results in a given answer.
f Two; raising both 2 and −2 to the power of 4 gives the result of 16. No number can be raised to the power of 4 to give a negative result.
g There will be two numbers that when raised to an even power (such as 2, 4, 6, 8, etc.) give the same positive result. For example, $2^6 = 64$ and $(-2)^6 = 64$. No number raised to an even power gives a negative result. There will only be one number that when raised to an odd power (such as 3, 5, 7, etc.) results in a given answer. For example,
$2^5 = 32$ and $(-2)^5 = -32$.

18 **a** 7 and −7 **b** 9 and −9 **c** 1 and −1
d 2 and −2 **e** 8 and −8 **f** 10 and −10

19 **a** 3 **b** −5 **c** 4 **d** −1 **e** −4 **f** −10

20 **a** $(+3)^2 = 9, (-3)^2 = 9, 3^2 = 9$ and $-3^2 = -9$
b $(-3)^2 = -3 \times -3 = +9$
while $-3^2 = -(3^2) = -(3 \times 3) = -9$
c **i** yes **ii** no **iii** no **iv** yes
d A number of the form $-x^a$ will always produce a negative result regardless of whether the power is odd or even. A negative base raised to an odd power will also produce a negative result.

21 **a** −73 **b** −10.99 **c** $\frac{4}{27}$

3H The Cartesian plane

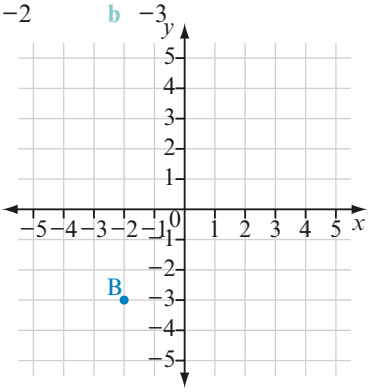
3H Start thinking!

1 **a** **i** *x*-axis **ii** *y*-axis **b** (0, 0)

2 **a** 3 **b** −4 **c** (3, −4) **d** 4

3 **a** −2 **b** −3

c



d 3

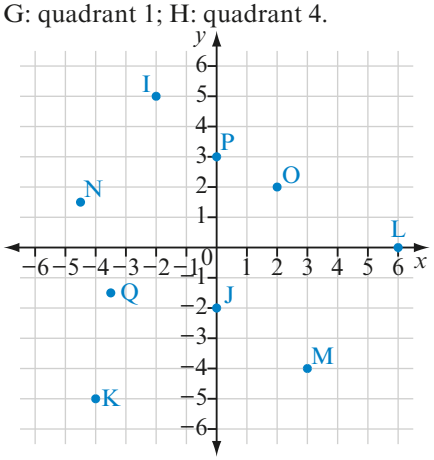
4 One possible answer is: quadrant 1: (1, 2), quadrant 2: (−1, 2); quadrant 3: (−1, −2); quadrant 4: (1, −2).

Exercise 3H The Cartesian plane

1 A(5, −4), B(−2, 0), C(−3, 3), D(−4, −3), E(0, 5), F(−6, 4), G(4.5, 2), H(1.5, −4.5)

2 A: quadrant 4; B: *x*-axis; C: quadrant 2; D: quadrant 3; E: *y*-axis; F: quadrant 2; G: quadrant 1; H: quadrant 4.

3



4 I: quadrant 2; J: *y*-axis; K: quadrant 3; L: *x*-axis; M: quadrant 4; N: quadrant 2; O: quadrant 1; P: *y*-axis; Q: quadrant 3.

5 **a**

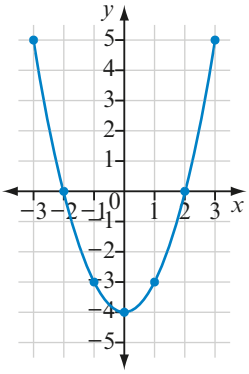
x-coordinate	−6	−4	−2	0	2	4	6
y-coordinate	−3	−2	−1	0	1	2	3
coordinates	(−6, −3)	(−4, −2)	(−2, −1)	(0, 0)	(2, 1)	(4, 2)	(6, 3)

b

x-coordinate	−2	−1	0	1	2	3	4
y-coordinate	−7	−5	−3	−1	1	3	5
coordinates	(−2, −7)	(−1, −5)	(0, −3)	(1, −1)	(2, 1)	(3, 3)	(4, 5)

6 **a** **i** (−3, 5), (−2, 0), (−1, −3), (0, −4), (1, −3), (2, 0), (3, 5)

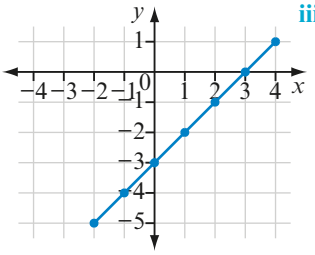
ii



iii curved line

b **i** (−2, −5), (−1, −4), (0, −3), (1, −2), (2, −1), (3, 0), (4, 1)

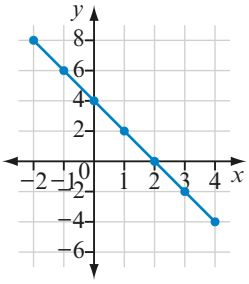
ii



iii straight line

c **i** (−2, 8), (−1, 6), (0, 4), (1, 2), (2, 0), (3, −2), (4, −4)

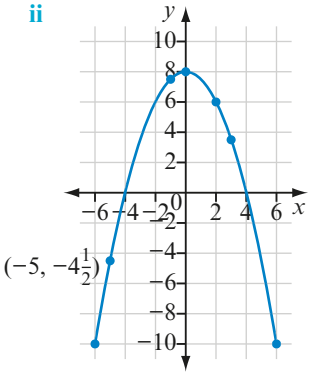
ii



iii straight line

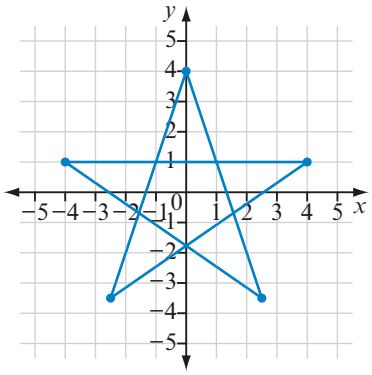
d **i** (−6, −10), (−5, $-4\frac{1}{2}$), (−1, $7\frac{1}{2}$), (0, 8), (2, 6), (3, $3\frac{1}{2}$), (6, −10)

ii

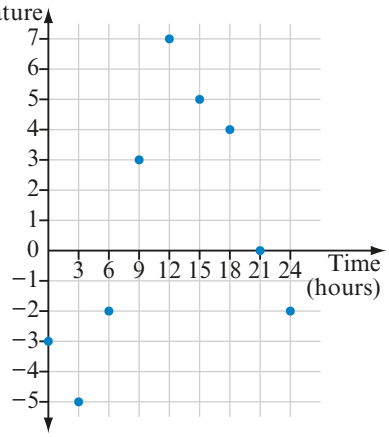


iii curved line

7



10 a–d



e −5°C; at 3 hours after midnight (3.00 am)

f 7°C; at 12 hours after midnight (12.00 pm or midday)

g 12°C **h** 0.78°C

11 a i

x-coordinate	−3	−2	−1	0	1	2	3
y-coordinate	−1	0	1	2	3	4	5

ii

x-coordinate	−3	−2	−1	0	1	2	3
y-coordinate	−6	−4	−2	0	2	4	6

iii

x-coordinate	−3	−2	−1	0	1	2	3
y-coordinate	−6	−5	−4	−3	−2	−1	0

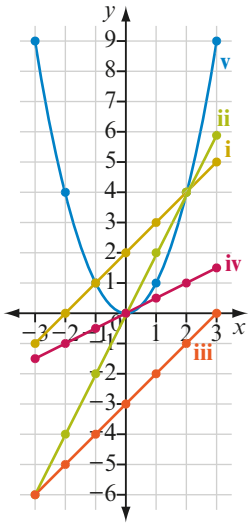
iv

x-coordinate	−3	−2	−1	0	1	2	3
y-coordinate	−1.5	−1	−0.5	0	0.5	1	1.5

v

x-coordinate	−3	−2	−1	0	1	2	3
y-coordinate	9	4	1	0	1	4	9

b



12 a Each x-coordinate is squared and then has 4 subtracted from it to produce the matching y-coordinate.

b Each x-coordinate has 3 subtracted from it to produce the matching y-coordinate.

c Each x-coordinate is multiplied by −2 and then has 4 added to it to produce the matching y-coordinate.

d Each x-coordinate is squared, then divided by −2 and then has 8 added to it to produce the matching y-coordinate.

3 Chapter review

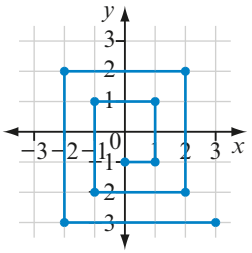
MULTIPLE-CHOICE

1 C **2** A **3** C **4** D **5** D **6** B
7 B **8** B **9** D **10** A **11** C **12** C

SHORT ANSWER

1 a false **b** true **c** true **d** false
2 a −8, −4, −2, 0, 4, 8
b $-19, -9, -1, 0, 5\frac{1}{3}, 5.5$
3 a −9 **b** +9 **c** −12 **d** +19
4 $122
5 a −5 **b** +8 **c** −81 **d** +18
6 21°C
7 a −1 **b** −10 **c** −8 **d** −10
e 0 **f** −100
8 a −56 **b** +45 **c** −9 **d** +5
e 0 **f** −60 **g** −3 **h** +14
9 a 180 **b** −35 **c** 12 **d** −40
10 a $-\frac{1}{4}$ **b** $\frac{1}{15}$ **c** $-\frac{6}{7}$ **d** $4\frac{1}{2}$ **e** $-\frac{4}{35}$ **f** $-\frac{7}{8}$
11 a −0.5 **b** −0.26 **c** −1.93 **d** −24.1
e 0 **f** 0.04
12 −0.2
13 a −32 **b** 9 **c** −1 **d** 10 000
14 a 81 **b** 1
15 A(3, −4), B(0, 4), C(−2, 0), D(−3, −2), E(1.5, 2.5) or $E(1\frac{1}{2}, 2\frac{1}{2})$, F(−3.5, 3.5) or $F(-3\frac{1}{2}, 3\frac{1}{2})$

16



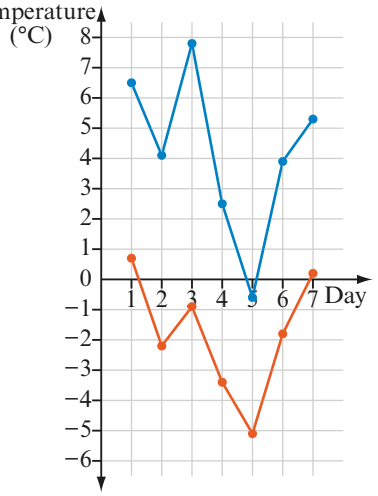
Next four points are: (3, 3), (−3, 3), (−3, −4), (4, −4).

NAPLAN-STYLE PRACTICE

1 3 **2** −4.7 **3** −4 **4** 1
5 second level below ground level
6 −42 **7** 44 **8** 62 m **9** 8°C
10 −3 **11** 6 **12** −4, 7 **13** −0.6°C
14 $104.90 **15** $\frac{1}{21}$ **16** −32 **17** $(-5)^6$
18 −27 **19** −5 **20** (0, 0) **21** F
22 A **23** (1, −5) **24** (0, −6)
25 B and C **26** G **27** Table 4

ANALYSIS

a **i** Wednesday **ii** Friday **b** 5.8°C
c Tuesday: 6.3°C, Wednesday: 8.7°C, Thursday: 5.9°C, Friday: 4.5°C, Saturday: 5.7°C, Sunday: 5.1°C
d Wednesday
e **i** −1.8°C **ii** 4.2°C **f** 6.0°C
g, i, j



h (1, 0.7), (2, −2.2), (3, −0.9), (4, −3.4), (5, −5.1), (6, −1.8), (7, 0.2)
k Maximum daily temperatures show similar trend to minimum daily temperatures. Temperature was coldest on Friday before starting to increase again over next two days.

3 Connect

For feedback on this open-ended task, see your teacher.

CHAPTER 4 ALGEBRA

4 Are you ready?

1 **a** 2 **b** C **c** D **d** B
2 C **3** B **4** C
5 **a** −4 **b** −8 **6** A
7 **a** 10 **b** −3
8 A **9** **a** 7 **b** 15
10 **a** D **b** D **11** 24
12 B **13** 22 cm **14** 28 cm^2
15 **a** B **b** 3^5
16 **a** A **b** D
17 **a** D **b** $\frac{5}{8}$

4A Using pronumerals

4A Start thinking!

1 **a** **i** 10 × 8 + 12 = $92
ii 25 × 8 + 12 = $212
b $8 for each brush and $12 for delivery charge
2 2; number of brushes and total cost
3 Cost of brushes is 8 × n or 8n. Delivery cost is $12. Total cost is cost of brushes plus delivery; that is, 8n + 12.
4 **a** 2 **b** 8
c yes, 12; it is the term without a variable (pronumeral)
5 equation

Exercise 4A Using pronumerals

1 **a** true **b** false; it is an expression
c false; the constant is 7 **d** true
e false; the coefficient is 6 **f** true
2 **a** expression **b** equation **c** equation
d equation **e** expression **f** equation
3 **a** $x + y + z = 9$, $k = 4m + 1$
b $2a + 3$ **c** $3a + b - 7$
4 **a** $a + 5$ **b** $\frac{a}{3}$ **c** $a - 4$ **d** $3a$
e $a - 8$ **f** $a + 2$ **g** $10 - a$ **h** $\frac{6}{a}$
i $50a$ **j** $a + 19$ **k** $2a$ **l** $a + x + 8$
5 **a** $7x + 2$ **b** $\frac{x}{3} - 6$ **c** $2x - 5$
d $\frac{x}{10} + 1$ **e** $15x + 28$ **f** $\frac{x}{20} - 11$
6 **a** $3(x + 1)$ **b** $5(x - 9)$ **c** $7(x + 2)$
d $4(x - 6)$ **e** $11(x + 8)$ **f** $9(4 - x)$
g $\frac{x + 5}{2}$ **h** $\frac{x - 9}{8}$ **i** $\frac{x + 1}{4}$
j $\frac{7 + x}{12}$ **k** $\frac{3 - x}{5}$ **l** $\frac{x - 10}{21}$
7 **a** $x + 2$ **b** $(x + 2) \times 5$ or $5 \times (x + 2)$
c $5(x + 2)$
8 **a** $m - 3$ **b** $(m - 3) \div 4$ **c** $\frac{m - 3}{4}$
9 **a** $2(x + 5)$ **b** $3(x - 9)$ **c** $\frac{x + 2}{4}$
d $\frac{x - 10}{7}$ **e** $5(20 - x)$ **f** $\frac{8 - x}{6}$

10 An equation has an equals sign (=).

11 **a** $a + 6 = 18$ **b** $a - 2 = 9$ **c** $3a = 24$
d $\frac{a}{5} = 2$

12 **a** $4x + 3 = 11$ **b** $\frac{x}{2} + 1 = 6$ **c** $7x - 5 = 12$
d $\frac{x}{3} - 6 = 21$ **e** $3(x + 6) = 24$
f $\frac{x + 11}{2} = 9$ **g** $10(x - 2) = 60$
h $\frac{x - 5}{8} = 100$

13 **a** $n + 40$
b **i** 300 more than initial number of bees
ii 200 less than initial number of bees
iii twice initial number of bees
c less (500 less bees than Mac)
d Some possible answers are:
$n + 1$, $3n$, $n + 100$, $5n + 55$.

14 **a** **i** $y - 6$ **ii** $y - 10$ **b** **i** $y + 5$ **ii** $y + 23$
c $y - x$ **d** $2y$

15 **a** $35m$ **b** $25k$ **c** $35m + 25k$

16 **a** $c + p$ **b** $p - 1$ **c** $c + p - 1$

17 **a** the amount she spends
b the amount she saves
c Some possible answers are:
$x = 50$, $y = 250$; $x = 100$, $y = 200$; $x = 150$, $y = 150$; $x = 250$, $y = 50$; $x = 300$, $y = 0$.
d Both x and y can vary.

18 Expressions, equations and formulas all have pronumerals. Equations and formulas both have equals signs; expressions do not. A formula is an equation that has more than one variable.

19 The relationship can be described more simply and the pronumeral can represent many different values or an unknown value.

20 $t = s + 18$, where s is original number of songs and t is number he has now

22 **a** $a + 6 = 20$ **b** $b - 1 = 20$ **c** $\frac{c}{4} = 20$
d $2d = 20$ **e** $50 - e = 20$ **f** $3f + 2 = 20$

4B Evaluating expressions

4B Start thinking!

1 6; one side of each table is not used as these two sides are joined

2 **a** 8 **b** 10 **c** 12

3

Number of tables	1	2	3	4	5
Number of chairs	4	6	8	10	12

4 For each table there are two chairs along the 'long sides', one opposite the other. This means there are $2t$ chairs along the 'long sides'. There is always a chair at each end, so there are two extra chairs. This means that for t tables there is a total of $(2t + 2)$ chairs, so the formula is $c = 2t + 2$.

5 $c = 2 \times 5 + 2$ **6** 12

7 **a** 52 **b** 102 **c** 236 **d** 326

8 substituting into a formula is much easier and quicker than drawing diagrams

Exercise 4B Evaluating expressions

1 **a** 17 **b** 7 **c** 56 **d** 4 **e** 29 **f** 10
g 18 **h** 4

2 **a** 11 **b** 26 **c** 8 **d** 5 **e** 2 **f** −4
g 6 **h** 3

3 **a** 4 **b** 16 **c** −8 **d** −26 **e** −2 **f** 1
g 0 **h** 13

4 **a** 9 **b** 6 **c** 43 **d** 0 **e** 80 **f** −2
g 10 **h** −9 **i** 2 **j** −8 **k** −5 **l** 0

5 **a** 13 **b** 23 **c** −5 **d** 5 **e** 16 **f** 7
g 8 **h** 4

6 **a** 1 **b** 21 **c** −11 **d** 197 **e** 3 **f** −2
g −3 **h** −6

7 **a** $a \times a$ **b** **i** 25 **ii** 75 **iii** 32

8 **a** $p \times p \times p$ **b** **i** 64 **ii** 128 **iii** 52

9 **a** 16 **b** 9 **c** 23 **d** 4 **e** 48
f 18 **g** 78 **h** 83 **i** 134 **j** 107
k 90 **l** 93 **m** 8 **n** 2 **o** 5
p 1 **q** $\frac{7}{8}$ **r** $\frac{1}{2}$ **s** $\frac{1}{91}$ **t** $\frac{3}{5}$

10 **a** −8 **b** 40 **c** −84 **d** −34
e 72 **f** −24 **g** −64 **b** −188
i 73 **j** −126 **k** 29 **l** −15

11 **a**

x	0	1	2	3	4
y	5	7	9	11	13

b

x	−2	−1	0	1	2
y	−11	−7	−3	1	5

c

x	0	1	2	3	4
y	1	4	13	28	49

d

x	−2	−1	0	1	2
y	−23	−9	−7	−5	9

12 **a** $l = 6n$
b **i** 18 **ii** 42 **iii** 300 **iv** 606
c 5700
d any whole number from 84 to 99;
must be between $\frac{500}{6}$ and $\frac{600}{6}$
e $e = 5n$, where n is number of grasshoppers and e is number of eyes
f **i** 2280 **ii** 10 175

13 **a** **i** 4 **ii** 8 **iii** 1.3 **iv** 20.5
b 3 m/s **c** 94 km/h

14 **a** 4.9 m **b** **i** 44.1 m **ii** 122.5 m
c increasing; average speed in first second is 4.9 m/s, in first 3 seconds is $\frac{44.1}{3}$ or 14.7 m/s, in first 5 seconds is $\frac{122.5}{5}$ or 24.5 m/s

15 449 tables and 900 chairs **16** 38.9 cm^3

17 **a** **i** 25°C **ii** −20°C **iii** 5°C **iv** 38°C
v −18°C **vi** 17°C
b **i** 86°F **ii** 14°F **iii** 32°F **iv** 81°F
v 18°F **vi** 108°F
c −5°C **d** 86°F, as 32°C = 89.6°F

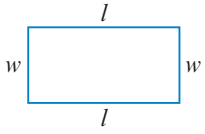
4C Simplifying expressions containing like terms

4C Start thinking!

1 $a + a + a$ means 3 lots of a or $3 \times a$.
This can be simplified to 3a.
2 $a + a + a + a + a$ or $5 \times a$ or $5a$. 3 $5a$
4 a 3a + 5a
b yes; $8a$. In total, this is
$(a + a + a) + (a + a + a + a + a)$ or $8a$.
5 $2b$ 6 a $8a + 2b$ b no
7 No, as the pronumerals are not exactly the same.
8 Expressions containing like terms can be simplified.

Exercise 4C Simplifying expressions containing like terms

1 a true b true
c false; pronumerals not the same in each term
d true
e false; pronumerals not exactly the same in each term
f true
2 Some possible answers are given.
a $5y$, $3y$, y b a, $4a$, $-6a$
c n, $8n$, $-2n$ d $4ab$, $2ba$, $-ab$
e gh, $2gh$, $-5gh$ f $12kmn$, $-5mkn$, $4mnk$
g a^2b, $-7a^2b$, $2ba^2$ h $9x$, $-0.2x$, $5x$
3 a true
b false; $3p + 6q$ (does not simplify further)
c true d false; $10k$
e false; $9ab + 3a$ f true
g true h false; $11p + 5q$
4 a $9x$ b $5y$ c $15mn$ d $8kp$
e $-4a$ f $xy + 4$ g $4c$ h $4g$
i $2xy + 4y$ j $5ab$ k $-7m$ l $-abc$
5 a $15k$ b $6ab$ c $12x + 3y$
d $13m + 3n$ e $2d + 1$ f $3x + 6xy$
g $11x + 11y$ h $2a + 9$ i 0
j 4dy + 5y + 8 k gh l $6a + 2ab + 3b$
6 C
7 a true
b false; pronumerals not exactly the same in each term (first term has one factor of b while second term has two factors of b)
c true
d false; pronumerals not exactly the same in each term (first term has one factor of a while second term has two factors of a)
e false; pronumerals not exactly the same in each term (b only in second term)
f true
g false; pronumerals not exactly the same in each term (c and d not in both terms)
h false; pronumerals not exactly the same in each term (first term has one factor of a and two factors of b, the second term has two factors of a and one factor of b)
8 a $14x^2y$ b $2ab^2$ c $7k^2$
d $4mn + 2m^2n$ e $-2c^2$ f $pq + 8p^2q$
g $4a^2b + 4ab^2$ h $11x^2y$ i $-2g^3h + 5g^2h$
j $m^2n^4 + 6m^2n$
9 a 63 b $14m + 5k$ c yes
d simplified expression; only had to substitute values into two terms
10 a $6a + 14b$; 72 b $9a + 14b$; 87
c $3a + 9b$; 42 d $5ab + 5b + 2a$; 100
e $8ab + a$; 125 f $3b + 7a$; 44
g $5a^2 - 3b^2$; 98 h $2ab^2 + 2a^2b$; 240
11 Some possible answers are:
$3p + 5t + p$; $7p + 3t + 2t - 3p$; $2p + 3p + 5t - p$.
12 a, b



c $l + w + l + w$
d $2l + 2w$
e $P = 2l + 2w$
f i 14 m ii 18.4 m iii 46 m
g Some possible answers are: length 8 m, width 2 m; length 6 m, width 4 m;
length 8.1 m, width 1.9 m.
13 a $7a - 2a + 4a + 5b + 3b - 6b + c$
b $6b$ also has a minus sign that has moved with it.
c yes d $9a + 2b + c$
e For example, if $a = 1$, $b = 2$ and $c = 3$, original expression is
$7 + 10 - 2 + 6 + 4 - 12 + 3 = 16$,
simplified expression is $9 + 4 + 3 = 16$;
so expressions are equivalent.
f simplified expression, as there are fewer terms
14 a $6x - 3x + 5x + 4y + 7y - 2y + y$
$= 8x + 10y$
b $2mn + 8mn - 6mn + 5m - 9m + 2m + 4n$
$= 4mn - 2m + 4n$
15 a She didn't keep minus sign with $2y$ and addition sign with $7x$.
b $3x + 7x - 2y - 4y = 10x - 6y$
16 a $11x + 2y$ b $5y + 4$ c $2x - y - 3$
d $-2xy + 8x$ e $11x^2 - 5$ f $-9x + 6xy + 6y$
17 a 12 b -21 c 6 d 36 e 39 f -108
18 a $8x + 4$ b $18x + 6$ c $36x + 4$ d $40x + 2$
19 a 36 cm b 78 cm c 148 cm d 162 cm
20 a any value between 24.5 and 43.25
b any value between 10.77 and 19.11
c any value between 5.44 and 9.61
d any value between 4.95 and 8.70
21 a $11a^2b + 11ab^2$ b $4e^2fg + e^2fh - 5efgh$
c $x^2y + 4xy^2 + 7xy$

4D Multiplying algebraic terms

4D Start thinking!

1 a i \$30 ii \$105
b multiplied (×) number of DVDs by cost
c $15k$ d $15 \times k = 15k$
e multiplication sign not shown
2 a i $2p$ ii $7p$ b kp

3 a Numbers can be multiplied together to produce a single number.
b Pronumerals are written beside each other without multiplication signs. If any pronumerals are the same, they can be simplified using index notation.

4 Multiply numbers together and write result at start of term (coefficient). List pronumerals after coefficient.

Exercise 4D Multiplying algebraic terms

1 **a** $5 \times m$ **b** $-2 \times a$
c $8 \times x \times y$ **d** $-6 \times c \times d$
e $7 \times a \times b \times c$ **f** $-10 \times g \times k \times m \times n$
g $3 \times p \times p \times q$
h $-4 \times x \times x \times x \times y \times y \times z$

2 a and B; b and A; c and C

3 **a** $4x$ **b** $6k$ **c** $7d$ **d** $5m$
e p **f** $-3h$ **g** $-9a$ **h** $-10y$
i $-8c$ **j** $-e$ **k** $2f$ **l** $3m$

4 **a** $3ab$ **b** $4mn$ **c** $2pt$ **d** $-8cd$
e $9xy$ **f** $5fk$ **g** abc **h** ehy
i $8q$ **j** $-18m$ **k** $7a^2$ **l** $2n^2$

5 **a** $21ac$ **b** $-30p$ **c** $24n^2$
d $10efgh$ **e** $-18x^2yz$ **f** $24abdf$
g $-70bghk$ **h** $48m^2nq$ **i** $24a^3bc$
j $18m^3np$ **k** $-21fg^3h^3$ **l** $40u^2w^2x^2y$

6 **a**

×	$4a$	3	$6m$	$2abc$
$3d$	$12ad$	$9d$	$18dm$	$6abcd$
ab^2	$4a^2b^2$	$3ab^2$	$6ab^2m$	$2a^2b^3c$
$5mn$	$20amn$	$15mn$	$30m^2n$	$10abcmn$
$2ac$	$8a^2c$	$6ac$	$12acm$	$4a^2bc^2$

b

×	$3x$	$4xyz$	$-5y$	$2x^2$
$-2x$	$-6x^2$	$-8x^2yz$	$10xy$	$-4x^3$
xy^2	$3x^2y^2$	$4x^2y^3z$	$-5xy^3$	$2x^3y^2$
$6xy$	$18x^2y$	$24x^2y^2z$	$-30xy^2$	$12x^3y$
$-7yz$	$-21xyz$	$-28xy^2z^2$	$35y^2z$	$-14x^2yz$

7 **a** $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^7$
b $a \times a \times a \times a \times a \times a \times a = a^7$
c $m \times m \times m \times m \times m \times m \times m \times m = m^8$
d $x \times x \times x \times x \times x \times x \times x \times x \times x \times x \times x \times x = x^{10}$

8 When multiplying terms that have the same base, write the base and add the indices (powers).

9 **a** a^{16} **b** x^{13} **c** m^{17} **d** k^5 **e** y^{20} **f** e^6
g c^{15} **h** p^{12} **i** g^{14} **j** n^{15} **k** x^{16} **l** h^{13}

10 **a** $3y^9$ **b** $7g^7$ **c** $6b^{11}$
d $12k^{13}$ **e** $40w^{17}$ **f** $80g^{11}$
g $27c^{14}$ **h** $15p^{10}$ **i** $14h^{12}$

11 No; the base of each term is not the same so the powers cannot be added.

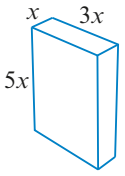
12 This index law only works when multiplying terms that have the same base. Write the base and add the indices (powers). A general rule could be $a^m \times a^n = a^{m+n}$.

13 **a** a^2h^3 **b** $12x^4y^5$ **c** $20t^3b^8$
d m^8q^7 **e** x^8y^7 **f** $30g^{10}h^3$
g a^8b^6 **h** $15x^8y^{10}$ **i** $54w^4x^{13}y^4$

14 One possible answer is given.
a $abc \times d = abcd$ **b** $12x \times xyz = 12x^2yz$
c $5mnp \times 4n^2p = 20mn^3p^2$

15 Some possible answers are:
$4x \times 6x$; $12x \times 2x$; $24x \times x$; $48x \times 0.5x$.

16 **a** $3x$ **b** $3x^2$ **c** 192 cm^2
d When $x = 8$, width = x = 8 cm and length = $3x$ = 24 cm. So area = length × width = $24 \times 8 = 192$ cm^2. This gives the same result as substituting $x = 8$ directly into $3x^2$.
e Quicker and easier to substitute into simplest form of expression ($3x^2$).
f $5x$
g



h

	area
top	$3x \times x = 3x^2$
bottom	$3x \times x = 3x^2$
front	$3x \times 5x = 15x^2$
back	$3x \times 5x = 15x^2$
left side	$x \times 5x = 5x^2$
right side	$x \times 5x = 5x^2$

i $3x^2 + 3x^2 + 15x^2 + 15x^2 + 5x^2 + 5x^2 = 46x^2$
j Use formula ($46x^2$) or calculate area of each face individually and find sum. surface area = 736 cm^2
k **i** 1150 cm^2 **ii** 6624 cm^2 **iii** 11.50 cm^2 **iv** 202.86 cm^2
l $15x^3$
m **i** 1875 cm^3 **ii** 25 920 cm^3 **iii** 1.875 cm^3 **iv** 138.915 cm^3

18 $a^{m+n}b^{x+y}$

4E Dividing algebraic terms

4E Start thinking!

1 **a** 1 **b** 1

2 HCF is 3 so $\frac{3}{3} = \frac{3^{\div 3}}{3_{\div 3}} = \frac{1}{1} = 1$;
HCF is a so $\frac{a}{a} = \frac{a^{\div a}}{a_{\div a}} = \frac{1}{1} = 1$.

3 HCF is 2. Divide both numerator and denominator by 2.
$\frac{10}{2} = \frac{10^{\div 2}}{2_{\div 2}}$ (or $\frac{\cancel{10}^5}{\cancel{2}_1}$) $= \frac{5}{1} = 5$

4 HCF is 2. Divide both numerator and denominator by 2.
$\frac{10a}{2} = \frac{10^{\div 2} \times a}{2_{\div 2}}$ (or $\frac{\cancel{10}^5 \times a}{\cancel{2}_1}$) $= \frac{5 \times a}{1} = \frac{5a}{1} = 5a$

5 **a** $\frac{10 \times a}{2 \times a}$ **b** 2 and a
c $\frac{10^{\div 2} \times a^{\div a}}{2_{\div 2} \times a_{\div a}}$ (or $\frac{\cancel{10}^5 \times \cancel{a}^1}{\cancel{2}_1 \times \cancel{a}_1}$) $= \frac{5 \times 1}{1 \times 1} = \frac{5}{1} = 5$

6 **a** $\frac{10 \times a \times b}{2 \times a}$ **b** 2 and a
c $\frac{10^{\div 2} \times a^{\div a}}{2_{\div 2} \times a_{\div a}} \times b$ (or $\frac{\cancel{10}^5 \times \cancel{a}^1 \times b}{\cancel{2}_1 \times \cancel{a}_1}$) $= \frac{5 \times 1 \times b}{1 \times 1}$
$= \frac{5b}{1} = 5b$

7 Write division problem as a fraction and look for common factors in numerator and denominator. The factors can be numbers or pronumerals. Divide numerator and denominator by common factors and write result in simplified form.

Exercise 4E Dividing algebraic terms

1 **a** **i** $5 \times a$, 5 **ii** 5 **iii** a
b **i** $9 \times d$, d **ii** d **iii** 9
c **i** $4 \times m$, m **ii** m **iii** 4
d **i** $18 \times c$, 6 **ii** 6 **iii** $3c$
e **i** $10 \times b$, 15 **ii** 5 **iii** $\frac{2b}{3}$
f **i** $4 \times x$, $4 \times y$ **ii** 4 **iii** $\frac{x}{y}$
g **i** $6 \times a \times b$, a **ii** a **iii** $6b$
h **i** $5 \times m$, $m \times n$ **ii** m **iii** $\frac{5}{n}$
i **i** $c \times d$, $3 \times c$ **ii** c **iii** $\frac{d}{3}$
j **i** $14 \times s \times t$, $7 \times t$ **ii** 7 and t **iii** $2s$
k **i** $8 \times h \times k$, $10 \times k$ **ii** 2 and k **iii** $\frac{4h}{5}$
l **i** $15 \times a \times b$, $12 \times a$ **ii** 3 and a **iii** $\frac{5b}{4}$

2 **a** x **b** d **c** $2k$ **d** $\frac{p}{6}$
e $\frac{7f}{6}$ **f** $4gh$ **g** $18a$ **h** $9y$
i $\frac{mn}{4}$ **j** abd **k** $\frac{7ef}{g}$ **i** $4kn$

3 **a** $\frac{4a}{b}$ **b** $\frac{5x}{y}$ **c** $\frac{12c}{5d}$ **d** 3
e $\frac{3}{8}$ **f** $\frac{1}{3}$ **g** $4b$ **h** $\frac{d}{2}$
i $3b$ **j** $\frac{3t}{4}$ **k** $\frac{5}{7n}$ **l** $\frac{rt}{2}$
m $\frac{5}{3y}$ **n** 7 **o** $\frac{5}{2}$ **p** $4b$
q $\frac{p}{6}$ **r** $\frac{3w}{7}$

4 **a** a **b** $6c$ **c** $2w$ **d** $\frac{m}{3}$
e $\frac{2f}{5}$ **f** $\frac{2}{r}$ **g** $\frac{4tx}{5y}$ **h** ac
i $\frac{6bd}{5}$ **j** $\frac{2m^2b}{3}$ **k** $2k$ **l** $\frac{c}{b}$

5 **a** $a^6 \div a^4$
$= \frac{a^6}{a^4}$
$= \frac{a \times a \times a \times a \times a \times a}{a \times a \times a \times a}$
$= \frac{{}^{1}\!a \times {}^{1}\!a \times {}^{1}\!a \times {}^{1}\!a \times a \times a}{{}_{1}\!a \times {}_{1}\!a \times {}_{1}\!a \times {}_{1}\!a}$
$= \frac{1 \times 1 \times 1 \times 1 \times a \times a}{1 \times 1 \times 1 \times 1}$
$= \frac{a^2}{1}$
$= a^2$

b $m^5 \div m^2$
$= \frac{m^5}{m^2}$
$= \frac{m \times m \times m \times m \times m}{m \times m}$
$= \frac{{}^{1}\!m \times {}^{1}\!m \times m \times m \times m}{{}_{1}\!m \times {}_{1}\!m}$
$= \frac{1 \times 1 \times m \times m \times m}{1 \times 1}$
$= \frac{m^3}{1}$
$= m^3$

6 When dividing terms that have the same base, write the base and subtract the indices (powers).

7 **a** p^3 **b** a^5 **c** n^3 **d** r^8 **e** $8x^{11}$ **f** $6m^6$

8 When dividing terms that are in index form and have the same base, write the base and subtract the indices (powers). A general rule could be $a^m \div a^n = a^{m-n}$.

9 **a** $a^3 \div a^3 = \frac{a^3}{a^3} = \frac{a \times a \times a}{a \times a \times a} = 1$
b $a^3 \div a^3 = a^{3-3} = a^0$
c The answers to parts a and b should be the same so $a^0 = 1$

10 **a** 1 **b** 1 **c** 1 **d** 2 **e** 5 **f** 8

11 **a** b^2 **b** q^4 **c** $5c^4$ **d** $\frac{5y^7}{2}$ **e** x^9 **f** a^6b
g n **h** b^6 **i** x^5 **j** m^4 **k** $6a^6$ **l** n^5
m 5 **n** 4 **o** 2 **p** 1

12 **a** $\frac{5}{4n}$ **b** $2jk^7$ **c** $3x$ **d** $\frac{m^7}{p^3}$
e b^2c^2 **f** $\frac{2k^8}{a^5}$ **g** $\frac{9x^4}{11y^4}$ **h** $\frac{3a^3b}{c^3}$
i $\frac{xy}{4}$ **j** $4n^2$ **k** m^8n^5 **l** x^4
m $\frac{6w^4x^3}{y^4}$ **n** $\frac{4}{5p}$ **o** $\frac{abd}{c}$ **p** $\frac{3a^4ce^2}{4}$

13 **a** total cost = 10×4 = \$40; cost of one apple = $40 \div 75$ = \$0.55 (or 55 cents)
b total cost = 10×5 = \$50; cost of one apple = $50 \div 75$ = \$0.65 (or 65 cents)
c $10 \times x \div 75 = \frac{2x}{15}$
d **i** \$0.80 **ii** \$1.05 **iii** \$0.95 **iv** \$0.40

14 **a** length = $2x$ cm; area = length × width
$= 2x \times x = 2x^2$ cm^2
b $100x^2$ cm^2 **c** 50
d **i** 40 cm **ii** 80 cm **iii** 3200 cm^2
iv 400 cm **v** 160 000 cm^2
e 50; same
f No; the expression used to calculate the number of pavers simplifies to 50 in each case as the pronumeral x is cancelled in the fraction.

16 $a^{m-n}b^{x-y}$

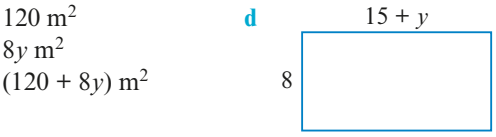
4F Working with brackets

4F Start thinking!

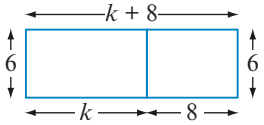
1 a i 11 ii 21
b No, as a different order was required to perform the operations in each problem.
2 3 lots of 2 + 3 lots of 5; $3 \times (2 + 5)$
$= 3 \times 2 + 3 \times 5$
3 3 lots of a + 3 lots of 2; $3 \times (a + 2)$
$= 3 \times a + 3 \times 2$
4 Each term inside the pair of brackets is multiplied by the term outside the brackets.
5 No, as the terms are not like terms.

Exercise 4F Working with brackets

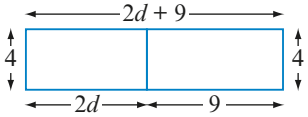
1 Each pair has same value.
a 16 b 54 c 20 d 15 e −27 f −2
2 a $4(m + 3) = 4 \times m + 4 \times 3 = 4m + 12$
b $a(c + 5b) = a \times c + a \times 5b = ac + 5ab$
c $7(2p - 3) = 7 \times 2p + 7 \times (-3) = 14p - 21$
d $-2(3a + 1) = (-2) \times 3a + (-2) \times 1 = -6a - 2$
3 a $3y + 6$ b $5k + 35$ c $10 + 2a$
d $12 + 6d$ e $8a + 8b$ f $3p + 3t$
g $8x + 24$ h $2 + 16m$ i $20a + 5b$
j $6m + 18n$ k $35a + 21b$ l $6x + 27y$
m $3m - 6$ n $12 - 2x$ o $4 - 4p$
p $48c - 30d$
4 a $ab + 4a$ b $3x + wx$ c $cd - de$
d $2mp + pq$ e $bk - 4ck$ f $3st + 5rt$
g $8xy + 6wx$ h $6ab - 12ac$ i $27pq + 45kp$
j $-16mn - 56mw$ k $-20at + 8bt$
l $15xy + 20yz$ m $12a^2 + 8a$
n $12k^2 - 18k$ o $3x - 12x^2$ p $-8e^2 - 10e$
q $-7n^2 + 42n$ r $-4ab + 6b^2$ s $10a^3 - 15a^2$
t $-11x^2y - 11y^3$
5 a $6x + 2y + 14$ b $10ab + 20d - 15$
c $14c - 7d + 35ef$ d $-18g + 12h - 6gh$
e $-20k - 8km + 36$ f $-6xy - 12 + 21w$
6 a $5 \times a + 5 \times 4 + 2 \times a + 2 \times 7$
$= 5a + 20 + 2a + 14 = 7a + 34$
b $3b \times c + 3b \times 2 + 2c \times a + 2c \times 5b$
$= 3bc + 6b + 2ac + 10bc = 13bc + 6b + 2ac$
c $d \times d + d \times 8 + 6 \times d + 6 \times (-3)$
$= d^2 + 8d + 6d - 18 = d^2 + 14d - 18$
d $4k \times 2k + 4k \times 3 + (-2) \times k + (-2) \times (-5)$
$= 8k^2 + 12k - 2k + 10$
$= 8k^2 + 10k + 10$
7 a $7a + 27$ b $10b$
c $11mn + 20m + 3nk$ d $21x + xy + 16y$
e $k^2 + 4k + 20$ f $c^2 - 3c + 35$
g $11rst + 5rst^2 + 36rt$ h $h^3 - 4h^2 - 14h$
8 a $4 \times m + 4 \times (-2) + 1 \times m + 1 \times 6$
$= 4m - 8 + m + 6 = 5m - 2$
b $2x \times y + 2x \times 3 + (-1) \times 5 + (-1) \times xy$
$= 2xy + 6x - 5 - xy = xy + 6x - 5$
c $k \times k + k \times (-1) + (-1) \times k + (-1) \times (-7)$
$= k^2 - k - k + 7 = k^2 - 2k + 7$
d $3w \times w + 3w \times (-2) + 1 \times 4w + 1 \times (-1)$
$= 3w^2 - 6w + 4w - 1$
$= 3w^2 - 2w - 1$
9 a $7p + 20$ b $2d - 14$
c $x^2 + 3x + 5$ d $k^2 + k - 1$
e $2e^2 + 7e + 3$ f $3ab + 4a - 6$
g $4wy - 15w - 2$ h $mnp - 4m^2p + n$
10 a Let Chanelle's current age = c. In three year's time, Chanelle's age = $c + 3$.
b $2(c + 3)$ c $2c + 6$
d i 34 ii 42 iii 56
11 a 120 m^2
b $8y$ m^2
c $(120 + 8y)$ m^2
d



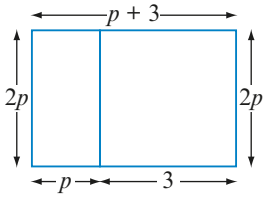
e length = $(15 + y)$ m, width = 8 m,
area = length × width = $(15 + y) \times 8$ or $8 \times (15 + y)$ or $8(15 + y)$.
f The expressions represent same area of lawn and should be equivalent.
12 area of large rectangle = sum of areas of two small rectangles
a area of large rectangle = $(a + 2) \times 3$ or $3(a + 2)$
sum of areas of small rectangles =
$3 \times a + 3 \times 2 = 3a + 6$
b area of large rectangle = $(x + 4) \times x$ or $x(x + 4)$
sum of areas of small rectangles =
$x \times x + x \times 4 = x^2 + 4x$
13 a



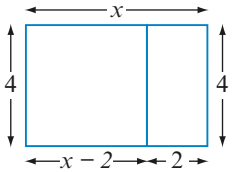
b



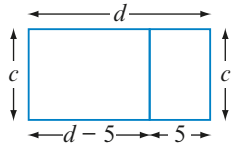
c



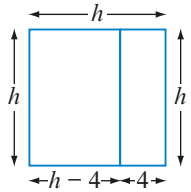
14 a $a - 3, 5$ b $5(a - 3)$
c i a, 5 ii $5a$ iii 3, 5 iv 15
v $5a - 15$
d $5(a - 3)$ = area of rectangle 1. Another way of working out this same area is to subtract area of rectangle 2 from area of large rectangle; that is, $5a - 15$.
15 a



b



c



16 **a** $7x^5 - 10x^4 - 21x^2$ **b** $4a^3c$
c $6m^3np^2 + 4m^2n - 6m^2np^2 + 24n$ **d** $3wy$

17 **a** −20 **b** −432 **c** −256 **d** 90

4G Factorising expressions

4G Start thinking!

1 **a** 16 cm **c** 2
d Common factor is placed at front of brackets. Remaining factors are written inside brackets.
e Write HCF (4) in front of brackets and write remaining factors as a sum inside brackets. That is, $4 \times 7 + 4 \times 11 = 4(7 + 11)$.

2 **a** $2 \times l + 2 \times w$ **b** common factor is 2
c $2 \times (l + w)$ **d** **i** 2 and $l + w$ **ii** 2

3 Expanding is multiplying terms to remove brackets. Factorising is writing the expression in factor form. Expanding and factorising are opposite or reverse processes.

Exercise 4G Factorising expressions

1 **a** 3 **b** −2 **c** 6 **d** x **e** −7 **f** b
g d **h** 2 **i** 5 **j** $-2f$ **k** $5p$ **l** −1

2 **a** 4 **b** 5 **c** 2 **d** 3 **e** 9 **f** 7
g $2y$ **h** $3c$ **i** $6g$ **j** $4b$ **k** m **l** $5s$

3 **a** $4 \times x$, 4×3
b $5 \times 2d$, $5 \times (-1)$
c 2×3, $2 \times 4k$
d $3 \times a$, $3 \times (-1)$
e $9 \times c$, $9 \times e$
f $7 \times x$, $7 \times 2y$
g $2y \times 3x$, $2y \times 1$
h $3c \times 1$, $3c \times 3b$
i $6g \times 5h$, $6g \times (-4)$
j $4b \times a$, $4b \times 2c$
k $m \times 2n$, $m \times 3p$
l $5s \times 3r$, $5s \times (-4t)$

4 **a** $4 \times x + 4 \times 3$
$= 4 \times (x + 3)$
$= 4(x + 3)$
b $5 \times 2a + 5 \times (-7)$
$= 5 \times (2a - 7)$
$= 5(2a - 7)$
c $8k \times 3m + 8k \times 2$
$= 8k \times (3m + 2)$
$= 8k(3m + 2)$
d $6d \times 3c + 6d \times (-2e)$
$= 6d \times (3c - 2e)$
$= 6d(3c - 2e)$

5 **a** $4(a + 2)$ **b** $7(e + 3)$ **c** $5(9 + k)$
d $3(1 + 4y)$ **e** $2(d - 3)$ **f** $9(3 - p)$
g $4(2m + 5)$ **h** $6(h - 5)$ **i** $6(4b + 3)$
j $2(6t - 7)$ **k** $5a(b + 2)$ **l** $3n(3 + m)$
m $2x(11y - 2)$ **n** $3b(a + c)$ **o** $8m(2n - p)$
p $9x(3y + 2w)$ **q** $3(4k + 5m)$ **r** $3(7ef - 8gh)$

6 **a** bc **b** x **c** 9mn **d** $3d$ **e** $2pq$
f $6ak$ **g** 4 **h** $2bcd$ **i** f

7 **a** $bc \times a$, $bc \times d$
b $x \times wy$, $x \times 1$
c $9mn \times 1$, $9mn \times k$
d $3d \times bc$, $3d \times 2ef$
e $2pq \times 4$, $2pq \times (-r)$
f $6ak \times 2g$, $6ak \times 3$
g $4 \times 4gh$, $4 \times (-7mn)$
h $2bcd \times 5a$, $2bcd \times 7e$
i $f \times 6egh$, $f \times (-7abk)$

8 **a** $bc \times a + bc \times d$
$= bc \times (a + d)$
$= bc(a + d)$
b $2pq \times 4 + 2pq \times (-r)$
$= 2pq \times (4 - r)$
$= 2pq(4 - r)$
c $6ak \times 2g + 6ak \times 3$
$= 6ak \times (2g + 3)$
$= 6ak(2g + 3)$
d $f \times 6egh + f \times 7abk$
$= f \times (6egh - 7abk)$
$= f(6egh - 7abk)$

9 **a** $mn(p + k)$ **b** $3ab(1 - 3c)$
c $7rt(2s + 3)$ **d** $de(4c + 5f)$
e $2x(wyz - rst)$ **f** $8ay(3p + 5q)$

10 Expand each answer. Result should match original expression.

11 **a** a **b** m **c** $7x$ **d** $2b$ **e** $3k$
f $7y$ **g** $-4p$ **h** 3 **i** $5y$

12 **a** x **b** a **c** $7k$ **d** $2y$ **e** $2m$
f $3p$ **g** $7t$ **h** a **i** $3n$

13 **a** $x \times x + x \times 8$
$= x \times (x + 8)$
$= x(x + 8)$
b $5m \times m + 5m \times (-1)$
$= 5m \times (m - 1)$
$= 5m(m - 1)$
c $3b \times 1 + 3b \times 2b$
$= 3b \times (1 + 2b)$
$= 3b(1 + 2b)$
d $3k \times 6k + 3k \times (-7)$
$= 3k \times (6k - 7)$
$= 3k(6k - 7)$

14 **a** $c(c + 4)$ **b** $k(6 + k)$ **c** $a(a - 2)$
d $3m(m + 1)$ **e** $p(10p + 11)$ **f** $h(4 - 7h)$
g $3x(2x + 1)$ **h** $5b(b - 2)$ **i** $4n(2n + 9)$
j $3y(9y - 5)$ **k** $2a(7 + 8a)$ **l** $e(e + 1)$

16 **a** −5 **b** $-2p$ **c** $-3k$ **d** $-7b$
e $-4y$ **f** $-4m$ **g** $-xy$ **h** $-4bc$

17 **a** $-5 \times (a + 2)$
$= -5(a + 2)$
b $(-2p) \times q + (-2p) \times 3$
$= -2p \times (q + 3)$
$= -2p(q + 3)$
c $(-4y) \times y + (-4y) \times (-1)$
$= -4y \times (y - 1)$
$= -4y(y - 1)$
d $(-4m) \times 3 + (-4m) \times (-5np)$
$= -4m \times (3 - 5np)$
$= -4m(3 - 5np)$

18 **a** $-4(a + 3)$ **b** $-2(3 + 4k)$ **c** $-7(m - 2)$
d $-5(3b - 7)$ **e** $-3t(s + 3)$ **f** $-6c(3 + 4d)$
g $-3m(4x - 7y)$ **h** $-h(h + 2)$
i $-8x(2x - 1)$ **j** $-5b(9 + 2ac)$
k $-x(1 + x)$ **l** $-ab(11a - 9)$

19 **a** $2(x + 3)$ **b** $4(2a - 9)$ **c** $h(h + 7)$
d $5n(5m + 6)$ **e** $mn(k - 1)$ **f** $6f(2e - 3d)$
g $-3(4 + 3y)$ **h** $-2(13b + 12c)$
i $-ac(b - d)$ **j** $2xy(3 + 2x)$
k $mn(n + 5)$ **l** $8c(3d - 2ab)$

20 **a** $6(g + 1)$ **b** $2(5 + 9x)$ **c** $k(p + q)$
d $m(m - 1)$ **e** $7(2 - ab)$ **f** $3h(e + g)$
g $4abd(2c + e)$ **h** $-5a(a + 3)$ **i** $-12(2k - p)$
j $x(3x - y)$ **k** $4e(4cd + 5e)$ **l** $-d(a + 6)$

21 **a** $x(x + 6)$ **b** x and $x + 6$ **c** $x + 6$
d $(x + 6) \times x = x \times (x + 6) = x \times x + x \times 6$
$= x^2 + 6x$
e using $x + 6$, length $= 3 + 6 = 9$; width $= 3$ so area $=$ length $\times$ width $= 9 \times 3 = 27$
using $x^2 + 6x$, area $= 3^2 + 6 \times 3 = 9 + 18 = 27$

22 **a** $(x + 2)$ cm **b** $(5x + 7)$ m
c $(k + 20)$ cm **d** $2m$ cm

23 Work out area by multiplying length and width together and writing in expanded form.
a $8(x + 2) = 8 \times x + 8 \times 2 = 8x + 16$
b $2(5x + 7) = 2 \times 5x + 2 \times 7 = 10x + 14$
c $k(k + 20) = k \times k + k \times 20 = k^2 + 20k$
d $2m(3m + 5) = 2m \times 3m + 2m \times 5 = 6m^2 + 10m$

24 Some possible answers are given.
a $12xy$ and $14x$, $-2x^2$ and $4x$
b $-3ab$ and $-9a^2b^2$, $-12ab^2$ and $-15a^2b$

25 **a** $x(8 - x)$ **b** length = x, width = $8 - x$
c Some possible answers are: length 5 cm, width 3 cm; length 4.5 cm, width 3.5 cm.

26 **a** $5x \times 3y + \frac{1}{2} \times 5x \times 2x = 15xy + 5x^2$
b $5x(3y + x)$

4 Chapter review

MULTIPLE-CHOICE

1 C **2** C **3** A **4** B **5** A **6** D
7 B **8** C **9** D **10** A **11** C **12** C

SHORT ANSWER

1 **a** $4m + 3$ **b** $\frac{m}{5} - 8$ **c** $\frac{m + 3}{7}$ **d** $2(m - 6)$

2 **a** $25x$ **b** $13y$ **c** $20p$
d $25x + 13y + 20p$

3 **a** 24 **b** 8 **c** 168 **d** -7

4 **a** Let d = number of days and w = number of weeks.
b $d = 7w$
c **i** 91 **ii** 203 **iii** 1015 **iv** 52.5

5 **a** $13a$ **b** $3b$ **c** $-5c$
d $15de$ **e** $9g + 4h$ **f** $5x^2 + 8x$

6 **a** $12m + 8n$ **b** $11k^2 + 7$
c $8cd + 2c + 4$ **d** $8x + 4y$
e $2ab^2 + 2a^2b$ **f** $6pq + 7p + 3q - p^2$

7 **a** $20ad$ **b** $24ab^2c$ **c** $-12g^2hk$ **d** $63bcef^3$

8 **a** x^{14} **b** $28a^9$ **c** $10h^{11}$ **d** $6m^{12}n^{12}$

9 **a** $\frac{3a}{4}$ **b** $3c$ **c** $4y$ **d** $\frac{3ad}{5}$ **e** $\frac{3m}{2}$ **f** $\frac{2ehk}{5}$

10 **a** k^4 **b** $8c^8$ **c** $6a^3b$
d $\frac{4x^7y^5}{7}$ **e** $\frac{3w^2}{2}$ **f** 1

11 **a** $5a + 15$ **b** $28b - 8$
c $3cd + 10ac$ **d** $-24m^2 - 6m$

12 **a** $18k + 11$ **b** $2y^2 - 13y - 4$

13 **a** $2(7h - 9)$ **b** $10a(2b + c)$
c $2(3a - 2b)$ **d** $4c(2c + 3)$
e $-5f(e + 1)$ **f** $-n(km - 7p)$
g $2ab(2a - 7)$ **h** $xy(y + wx)$

NAPLAN-STYLE PRACTICE

1 -8 **2** $2x + y$ **3** $5 \times a \times b$
4 22 **5** $-3 \times m \times m \times n$
6 $6x^2 - x - 4$ **7** $12ab^2c$ **8** h^{12}
9 $\frac{6f}{5}$ **10** b^6 **11** $6m^3n^3$
12 $\frac{3xy^2}{4}$ **13** $3w^2$ **14** $a \times b + a \times c$
15 $5x^2 - 10xy$ **16** 1 **17** $3a(b + 2)$
18 $4d + 5d^2$ **19** $2xy(2x - 3y^2)$
20 $5(x - y) + 3(2y - x)$ **21** $2p(2p - 5q)$
22 $14n - m$ **23** $c + 5$
24 length of $(2x + 4)$ m, width of $(3x)$ m
25 8 m, 6 m, 48 m^2

ANALYSIS

All costs are in dollars.

a **i** 30 **ii** 6 **b** **i** $10x$ **ii** $2x$
c **i** $\frac{3x}{2}$ **ii** $\frac{6x}{5}$ **d** **i** $16x$ **ii** $x = 1, 2$ or 3
e **i** $10x + 6y$ **ii** $2(5x + 3y)$
f **i** $\frac{2(5x + 3y)}{6} = \frac{5x + 3y}{3}$ **ii** $\frac{2(5x + 3y)}{p}$
g Cost of sharing three large boxes and two small boxes of chips between six people is \$7.
h Some possible answers are: $x = 1$, $y = 1$; $x = 1$, $y = 2$; $x = 1$, $y = 3$; $x = 2$, $y = 1$; $x = 3$, $y = 0$; $x = 2$, $y = 0$; $x = 1$, $y = 0$; $x = 0$, $y = 1$; $x = 0$, $y = 2$; $x = 0$, $y = 3$; $x = 0$, $y = 4$; $x = 0$, $y = 5$.
i Some possible answers are: $x = 1$, $y = 1$, $p = 2$; $x = 1$, $y = 1$, $p = 3$; $x = 3$, $y = 3$, $p = 6$; $x = 3$, $y = 3$, $p = 8$; $x = 4$, $y = 5$, $p = 10$.

4 Connect

For feedback on this open-ended task, see your teacher.

CHAPTER 5 LINEAR RELATIONSHIPS

5 Are you ready?

1 C
2 **a** 26 **b** -2 **c** -12 **d** 2
3 **a** 19 **b** C **c** -3
4 **a** 4 **b** C **5** **a** 28 **b** B
6 **a** A **b** B **7** **a** $7x + 4$ **b** A
8 A **9** **a** C **b** $(-2, 3)$

5A Understanding equations

5A Start thinking!

1 $e + 8$ **2** $e + 8 = 30$
3 **a** $e + 8$ **b** 30
4 No; adding 20 to 8 results in 28. This is not the value on right side of equation.
5 $e = 22$
6 Substitute value for e into equation and see whether it makes equation a true statement.

Exercise 5A Understanding equations

1 **a** **i** expression **ii** equation **iii** equation
iv expression **v** equation **vi** equation
b Equation contains an equals sign (=).

2 **a** 17, no **b** 9, yes **c** -3, yes **d** 1, no
e 12, no **f** 2, yes **g** 29, no **h** 11, yes

3 **a** -4, yes **b** -10, no **c** -27, no **d** -2, yes
e 45, yes **f** -3, no **g** -35, yes **h** -12, no

4 **a** yes **b** yes **c** no **d** no
e yes **f** no **g** no **h** yes
i no

5 **a** $a = 7$ **b** $b = 6$ **c** $c = 8$ **d** $d = 12$
e $e = 2$ **f** $f = -3$ **g** $g = 1$ **h** $h = 0$
i $i = -3$ **j** $j = -5$ **k** $k = 7$ **l** $l = 35$
m $m = 4$ **n** $n = -10$ **o** $x = -14$ **p** $p = 0$
q $q = 6$ **r** $r = -12$ **s** $s = -4$ **t** $t = 25$

7 to find the value for the pronumeral that makes the equation a true statement

8 D **9** B **10** A **11** C

12 **a** $a = 2$ **b** $b = 1$ **c** $c = 3$ **d** $d = 0$
e $e = 6$ **f** $f = 10$ **g** $g = 4$ **h** $h = -7$

13 **a** $x = 5$ **b** $x = 4$ **c** $x = -1$ **d** $x = 9$
e $x = -10$ **f** $x = 0$ **g** $x = -3$ **h** $x = -4$

14 **a** **i** $n + 4 = 11$ **ii** $n - 3 = -2$
iii $n + 15 = 21$ **iv** $n - 8 = -4$
v $2n = 14$ **vi** $\frac{n}{4} = 9$
vii $n + (-7) = -20$ or $n - 7 = -20$
viii $11n = 22$ **ix** $5n = 55$
x $\frac{n}{6} = -3$

b **i** $n = 7$ **ii** $n = 1$ **iii** $n = 6$ **iv** $n = 4$
v $n = 7$ **vi** $n = 36$ **vii** $n = -13$
viii $n = 2$ **ix** $n = 11$ **x** $n = -18$

15 **a** **i** $3x + 8 = 11$ **ii** $\frac{x}{6} - 2 = 1$
iii $\frac{x + 7}{2} = 5$ **iv** $2x - 11 = 31$
v $\frac{x - 9}{11} = -1$ **vi** $\frac{5x}{3} = 10$

b **i** $x = 1$ **ii** $x = 18$ **iii** $x = 3$
iv $x = 21$ **v** $x = -2$ **vi** $x = 6$

16 **a** $12a$ **b** $12a = 84$ **c** $a = 7$
d 7 tickets

17 **a** $x + 38$ **b** $x + 38 = 62$
c $x = 24$ **d** \$24

18 **a** $\frac{f}{4}$, where f represents number of Fruity Bites
b $\frac{f}{4} = 9$ **c** $f = 36$ **d** 36 Fruity Bites

19 **a** $200 - r$, where r represents cost of runners
b $200 - r = 21$ **c** $r = 179$ **d** \$179

20 **a** $8t = 20$, where t represents cost of each chocolate
b $t = 2.5$, so cost of each chocolate is \$2.50

21 **a** $2n + 3$ **b** $2n + 3 = 25$
c $n = 11$ **d** 11 pairs

22 **a** $3b + 3$, where b represents cost of one loaf of bread
b $3b + 3 = 15$
c $b = 4$, so cost of one loaf is \$4

23 One possible answer is given.
a $2k = 8$ **b** $3m - 1 = -22$
c $5(n + 2) = 10$ **d** $\frac{p}{10} = 2$

5B Solving equations using tables

5B Start thinking!

1 **a** **i** 13 **ii** 15
b No, so $x = 4$ is not a solution to the equation.
c 14, 15; two sides are not equal so $x = 5$ is not a solution
f $x = 6$

2

Guess x	LS $x + 9$	RS 15	Check Is LS = RS?
4	13	15	no
5	14	15	no
6	15	15	yes

3 One possible answer is given.
Solution is $x = 9$.

Guess x	LS $3x - 7$	RS 20	Check Is LS = RS?
5	8	20	no
11	26	20	no
8	17	20	no
9	20	20	yes

Exercise 5B Solving equations using tables

1

Guess x	LS $x - 5$	RS 12	Check Is LS = RS?
21	16	12	no
15	10	12	no
17	12	12	yes

2 **a** $x = 8$ **b** $x = 44$ **c** $x = 61$ **d** $x = 41$
e $x = 24$ **f** $x = 91$ **g** $x = 75$ **h** $x = 48$

3

Guess a	LS $2a - 18$	RS 30	Check Is LS = RS?
20	22	30	no
30	42	30	no
25	32	30	no
24	30	30	yes

4 **a** $a = 8$ **b** $b = 10$ **c** $c = 3$ **d** $d = 11$
e $e = 15$ **f** $f = 6$ **g** $g = 9$ **h** $h = 21$
i $x = 28$ **j** $y = 41$

5

Guess x	LS $x + 24$	RS 19	Check Is LS = RS?
−2	22	19	no
−3	21	19	no
−5	19	19	yes

6 **a** $x = -8$ **b** $x = -22$ **c** $x = -6$ **d** $x = -11$
e $x = -3$ **f** $x = -10$ **g** $x = -19$ **h** $x = -31$
i $x = -52$ **j** $x = -101$

7

Guess x	LS $5x + 23$	RS 35	Check Is LS = RS?
3	38	35	no
2.5	35.5	35	no
2.4	35	35	yes

8 **a** $x = 1.5$ **b** $x = 0.8$ **c** $x = 1.7$ **d** $x = -3.5$
e $x = -2.5$ **f** $x = 4.2$ **g** $x = 8.7$ **h** $x = -0.75$

9

Guess x	LS $2(x + 11)$	RS 34	Check Is LS = RS?
7	36	34	no
6	34	34	yes

10 **a** $x = 21$ **b** $x = 33$ **c** $x = 17$ **d** $x = 26$
e $x = 45$ **f** $x = -91$

11

Guess x	LS $3x - 5$	RS $2x + 3$	Check Is LS = RS?
9	22	21	no
8	19	19	yes

12 **a** $x = 2$ **b** $x = 3$ **c** $x = 10$ **d** $x = 5$
e $x = 15$ **f** $x = -1$ **g** $x = 1.6$ **h** $x = 2.5$
i $x = -4.8$ **j** $x = 0.35$

13 **a** $3c + 14$ **b** $3c + 14 = 230$
c $c = 72$, so cost of one crayfish is \$72

14 **a** $\frac{g + 6}{4}$ **b** $\frac{g + 6}{4} = 15$
c $g = 54$, so cost of gift is \$54

15 one solution

16 **a** highest power in equation is 2
b Some possible answers are:
$2x^2 + 3x = 5$, $6x^2 = 36$, $x^2 + 5x - 24 = 0$.

17

Guess x	LS $x^2 + 25$	RS $10x$	Check Is LS = RS?
2	29	20	no
3	34	30	no
4	41	40	no
5	50	50	yes

18 **a** $x = 2$ **b** $x = 7$ **c** $x = 6$ **d** $x = -3$
e $x = -4$ **f** $x = -5$ **g** $x = 9$ **h** $x = 11$

19 **a**

x	LS $x^2 + 6$	RS $5x$
2	10	10
3	15	15

b

x	LS $x^2 - 5$	RS $4x$
−1	−4	−4
5	20	20

c

x	LS $x^2 + 7$	RS 23
4	23	23
−4	23	23

d

x	LS $x^2 - 2x$	RS 80
−8	80	80
10	80	80

20 **a** $x = 3$ and $x = 5$ **b** $x = 1$ and $x = 9$
c $x = -3$ and $x = 3$ **d** $x = -2$ and $x = 7$
e $x = 4$ and $x = 8$ **f** $x = -6$ and $x = 5$
g $x = -8$ and $x = 11$ **h** $x = -13$ and $x = -3$

21 **a** $x = -18$ **b** $x = 3$ **c** $x = -4$
d $x = 15$ **e** $x = -2.1$ **f** $x = 7$
g $x = 6.5$ **h** $x = -9$

5C Building expressions using flowcharts

5B Start thinking!

1 **a** 10 → (× 2) → 20 → (+ 7) → 27
b middle box
c **i** × 2 **ii** + 7 **d** 27
e different result; 10 + 7 = 17 then 17 × 2 = 34

2 24 → (× 2) → 48 → (+ 7) → 55

3 **a, b** x → (× 2) → $2x$ → (+ 7) → $2x + 7$
c $2x + 7$

Exercise 5C Building expressions using flowcharts

1 **a** x → (+ 2) → $x + 2$ **b** x → (− 5) → $x - 5$
c x → (× 3) → $3x$ **d** x → (÷ 7) → $\frac{x}{7}$

2 **a** x → (+ 7) → $x + 7$ **b** x → (− 3) → $x - 3$
c x → (× 6) → $6x$ **d** x → (÷ 2) → $\frac{x}{2}$
e x → (× −10) → $-10x$ **f** x → (÷ −9) → $\frac{x}{-9}$

3 The following operations are in order when read from left to right.
a × 3, + 4 **b** ÷ 8, −1 **c** − 2, × 7
d × 6, − 5 **e** + 3, ÷ 5 **f** ÷ 4, + 7

4 Expressions are given for each box of flowchart, from left to right.
a x, $4x$, $4x + 5$ **b** x, $3x$, $3x - 1$
c x, $\frac{x}{6}$, $\frac{x}{6} + 9$ **d** x, $x - 7$, $2(x - 7)$
e x, $x - 3$, $\frac{x - 3}{4}$ **f** x, $8x$, $\frac{8x}{5}$

5 **a** B **b** D

6 **a** × 3, + 8 **b** × 2, − 6 **c** × 7, + 1
d − 4, × 5 **e** + 6, × 8 **f** − 2, × 4
g ÷ 7, − 3 **h** ÷ 4, + 9 **i** ÷ 5, − 2
j + 1, ÷ 6 **k** − 8, ÷ 3 **l** + 5, ÷ 2

7 **a** x → (× 3) → $3x$ → (+ 8) → $3x + 8$
b x → (× 2) → $2x$ → (− 6) → $2x - 6$
c x → (× 7) → $7x$ → (+ 1) → $7x + 1$
d x → (− 4) → $x - 4$ → (× 5) → $5(x - 4)$
e x → (+ 6) → $x + 6$ → (× 8) → $8(x + 6)$
f x → (− 2) → $x - 2$ → (× 4) → $4(x - 2)$
g x → (÷ 7) → $\frac{x}{7}$ → (− 3) → $\frac{x}{7} - 3$
h x → (÷ 4) → $\frac{x}{4}$ → (+ 9) → $\frac{x}{4} + 9$
i x → (÷ 5) → $\frac{x}{5}$ → (− 2) → $\frac{x}{5} - 2$
j x → (+ 1) → $x + 1$ → (÷ 6) → $\frac{x + 1}{6}$
k x → (− 8) → $x - 8$ → (÷ 3) → $\frac{x - 8}{3}$
l x → (+ 5) → $x + 5$ → (÷ 2) → $\frac{x + 5}{2}$

8 **a** x → (+ 5) → $x + 5$ → (× 2) → $2(x + 5)$
b x → (÷ 6) → $\frac{x}{6}$ → (− 7) → $\frac{x}{6} - 7$
c x → (× 9) → $9x$ → (− 4) → $9x - 4$

d $x \xrightarrow{+3} x+3 \xrightarrow{\div 7} \frac{x+3}{7}$

e $x \xrightarrow{\times 4} 4x \xrightarrow{11} 4x+11$

f $x \xrightarrow{\div 3} \frac{x}{3} \xrightarrow{+6} \frac{x}{3}+6$

g $x \xrightarrow{-2} x-2 \xrightarrow{\div 8} \frac{x-2}{8}$

h $x \xrightarrow{-7} x-7 \xrightarrow{\times 6} 6(x-7)$

i $x \xrightarrow{\times 4} 4x \xrightarrow{\div 5} \frac{4x}{5}$

j $x \xrightarrow{\div 2} \frac{x}{2} \xrightarrow{-1} \frac{x}{2}-1$

k $x \xrightarrow{+3} x+3 \xrightarrow{\times 7} 7(x+3)$

l $x \xrightarrow{-9} x-9 \xrightarrow{\div 4} \frac{x-9}{4}$

9 a $a \xrightarrow{\times 5} 5a \xrightarrow{+2} 5a+2$

b $b \xrightarrow{\div 8} \frac{b}{8} \xrightarrow{-3} \frac{b}{8}-3$

c $c \xrightarrow{\times 9} 9c \xrightarrow{\div 7} \frac{9c}{7}$

d $d \xrightarrow{+13} d+13 \xrightarrow{\div 6} \frac{d+13}{6}$

e $e \xrightarrow{-9} e-9 \xrightarrow{\times 4} 4(e-9)$

f $f \xrightarrow{+6} f+6 \xrightarrow{\times 2} 2(f+6)$

10 no, dividing by 5 needs to be performed before adding 3

12 a × 4, − 7 **b** + 7, ÷ 4 **c** + 7 **d** ÷ 4
e i subtract **ii** add **iii** divide **iv** multiply

13 a − 6 **b** ÷ 4 **c** + 7 **d** × 3
e ÷ −2 **f** × −8 **g** + 11 **h** ÷ 20

For questions 14 and 15, inverse operations are shown in order from right to left; that is, moving from final expression to starting variable.

14 ÷ 3, + 4

15 a − 4, ÷ 3 **b** + 1, × 8 **c** ÷ 7, + 2
d + 5, ÷ 6 **e** × 5, − 3 **f** − 7, × 4

16 Following operations are in order from left to right.
a + 3, × 2, − 5 **b** × 4, − 1, ÷ 7
c × 3, + 2, × 8 **d** − 6, ÷ 5, + 7

17 a $7(x-4)+2$ **b** $5(3x+1)$ **c** $\frac{2x-6}{3}$
d $\frac{4x}{7}-8$ **e** $9(\frac{x}{5}-2)$ **f** $\frac{x+3}{11}+6$

18 Inverse operations are shown from right to left; that is, moving from final expression to starting variable.
a − 2, ÷ 7, + 4 **b** ÷ 5, − 1, ÷ 3
c × 3, + 6, ÷ 2 **d** + 8, × 7, ÷ 4
e ÷ 9, + 2, × 5 **f** − 6, × 11, − 3

19 a $x \xrightarrow{\div 2} \frac{x}{2} \xrightarrow{+6} \frac{x}{2}+6$

b $\frac{x}{2}+6$ **c** 10

20 a $d \xrightarrow{-42} d-42 \xrightarrow{\div 2} \frac{d-42}{2}$

b $\frac{d-42}{2}$ **c** $20

21 a $m \xrightarrow{-5} m-5 \xrightarrow{\times 3} 3(m-5)$

b $3(m-5)$

22 a × 3 **b i** + 5 **ii** ÷ 7

c, d $x \underset{\div 3}{\overset{\times 3}{\rightleftarrows}} 3x \underset{-5}{\overset{+5}{\rightleftarrows}} 3x+5 \underset{\times 7}{\overset{\div 7}{\rightleftarrows}} \frac{3x+5}{7}$

23 a i − 3 **ii** × 2 **iii** + 2 **iv** × 6 **v** ÷ 4
vi × 7 **vii** + 6 **viii** − 8 **ix** × 4

b, c
i $x \underset{+3}{\overset{-3}{\rightleftarrows}} x-3 \underset{\div 4}{\overset{\times 4}{\rightleftarrows}} 4(x-3) \underset{-7}{\overset{+7}{\rightleftarrows}} 4(x-3)+7$

ii $x \underset{\div 2}{\overset{\times 2}{\rightleftarrows}} 2x \underset{-9}{\overset{+9}{\rightleftarrows}} 2x+9 \underset{\div 3}{\overset{\times 3}{\rightleftarrows}} 3(2x+9)$

iii $x \underset{\div 2}{\overset{+2}{\rightleftarrows}} x+2 \underset{\times 8}{\overset{\div 8}{\rightleftarrows}} \frac{x+2}{8} \underset{+5}{\overset{-5}{\rightleftarrows}} \frac{x+2}{8}-5$

iv $x \underset{\div 6}{\overset{\times 6}{\rightleftarrows}} 6x \underset{+3}{\overset{-3}{\rightleftarrows}} 6x-3 \underset{\times 2}{\overset{\div 2}{\rightleftarrows}} \frac{6x-3}{2}$

v $x \underset{\times 4}{\overset{\div 4}{\rightleftarrows}} \frac{x}{4} \underset{-1}{\overset{+1}{\rightleftarrows}} \frac{x}{4}+1 \underset{\div 5}{\overset{\times 5}{\rightleftarrows}} 5(\frac{x}{4}+1)$

vi $x \underset{\div 7}{\overset{\times 7}{\rightleftarrows}} 7x \underset{\times 9}{\overset{\div 9}{\rightleftarrows}} \frac{7x}{9} \underset{-11}{\overset{+11}{\rightleftarrows}} \frac{7x}{9}+11$

vii $x \underset{-6}{\overset{+6}{\rightleftarrows}} x+6 \underset{\div 2}{\overset{\times 2}{\rightleftarrows}} 2(x+6) \underset{+4}{\overset{-4}{\rightleftarrows}} 2(x+6)-4$

viii $x \underset{+8}{\overset{-8}{\rightleftarrows}} x-8 \underset{\times 3}{\overset{\div 3}{\rightleftarrows}} \frac{x-8}{3} \underset{-6}{\overset{+6}{\rightleftarrows}} \frac{x-8}{3}+6$

ix $x \underset{\div 4}{\overset{\times 4}{\rightleftarrows}} 4x \underset{+1}{\overset{-1}{\rightleftarrows}} 4x-1 \underset{\div 5}{\overset{\times 5}{\rightleftarrows}} 5(4x-1)$

5D Solving equations using backtracking

5D Start thinking!

1 $x \xrightarrow{\times 4} 4x \xrightarrow{+5} 4x + 5$

2 $-5, \div 4$ 3 $29 - 5 = 24; 24 \div 4 = 6$

4 $x = 6$

5 Substitute 6 into $4x + 5 = 29$ and check that it makes a true statement.

6 a k b $\frac{k}{2} - 1 = 7$ c $k = 16$

7 Work backwards using inverse operations.

Exercise 5D Solving equations using backtracking

1 a c b $9(c - 6) = 45$ c $c = 11$

2 a $3a - 4 = 11; a = 5$ b $\frac{b}{2} + 5 = 1; b = -8$

3 a $\frac{x}{5} + 6 = 9; x = 15$ b $2x - 3 = 11; x = 7$

c $\frac{x + 7}{4} = 1; x = -3$ d $\frac{5x}{2} = 20; x = 8$

e $4(x - 1) = -24; x = -5$

f $\frac{x - 4}{5} = 3; x = 19$

4 a $x = 7$ b $x = 6$ c $x = 19$ d $x = 9$
e $x = 10$ f $x = 4$ g $x = 15$ h $x = -3$
i $x = 12$ j $x = -12$ k $x = -6$ l $x = 5$
m $x = 8$ n $x = -1$ o $x = -16$

6 a B b $x = 6$
c $5 \times (6 + 3) = 5 \times 9 = 45$

7 a i $4x + 6 = 54$ ii $\frac{x}{7} - 3 = 2$
iii $11(x + 2) = 55$ iv $\frac{x - 5}{3} = 1$
v $\frac{3x}{2} = 6$ vi $\frac{x}{5} + 9 = 17$

b i $x = 12$ ii $x = 35$ iii $x = 3$
iv $x = 8$ v $x = 4$ vi $x = 40$

8 a $x = -1$ b $x = 10$ c $x = -9$ d $x = 0$
e $x = -3$ f $x = 0.5$ g $x = -25$ h $x = 13.5$
i $x = 29$ j $x = 7$ k $x = -8$ l $x = 0$
m $x = 51$ n $x = 0.75$ o $x = \frac{6}{7}$ p $x = -10$
q $x = -8$ r $x = -31$

9 a $6(x - 2) + 5 = 47; x = 9$

b $\frac{3x}{7} - 4 = -10; x = -14$

10 a $2(x - 4) + 9 = 25; x = 12$
b $7(4x - 3) = -77; x = -2$

c $\frac{x + 5}{6} + 11 = 10; x = -11$

d $\frac{3x - 2}{4} = 4; x = 6$

11 a $x = 5$ b $x = 23$ c $x = -4$ d $x = 0$
e $x = 30$ f $x = -8$ g $x = 7$ h $x = 17$
i $x = -2$ j $x = 2.5$ k $x = -10$ l $x = 1.25$

12 a D b $n = 6$ c 6
d $3 \times 6 - 5 = 18 - 5 = 13$

13 $\frac{x}{2} + 15 = 128; x = 226;$
original amount saved is \$226

14 a $m + 9$ b $2(m + 9)$
c $2(m + 9) = 30$ d $m = 6$; six maki

15 a $3t - 4 = 11$ b $t = 5$, so Leo built 5 towers

16 a $6n + 3 = 30$ or $6n = 27$
b $n = 4.5$, so each note book cost \$4.50

17 a $3(2x + 7) - 5$

$x \xrightarrow{\times 2} 2x \xrightarrow{+7} 2x + 7 \xrightarrow{\times 3} 3(2x + 7) \xrightarrow{-5} 3(2x + 7) - 5$

18 a $x = 5$ b $x = -9$

5E The balance model and equivalent equations

5E Start thinking!

1 A balanced set of scales has equal mass on each side. Equations also have two sides (expressions) that are equal.

2 a same mass on each side, both trays level
b $x = 2$
c scales not balanced, right side heavier
d remove 3 g e $x + 2 = 4$ f $x = 2$

3 a add 4 g to right side b $x + 9 = 11$
c $x = 2$

4 a All have same solution.
b i $x + 4 = 8$ is not equivalent as it does not have same solution.
ii $x - 3 = -1$ is equivalent as it does have same solution.
c Some possible answers are:
$x + 6 = 8$, $x + 15 = 17$, $2x + 10 = 14$.

5 Performing the same operation on both sides of an equation produces an equivalent equation. This is similar to adding or removing the same mass from both sides of scales.

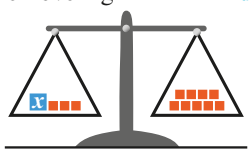
Exercise 5E The balance model and equivalent equations

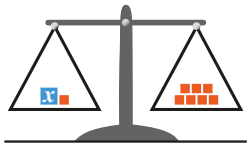
1 a Since scales are balanced, mass of apple is same as mass of orange.
b scales unbalanced with right side higher

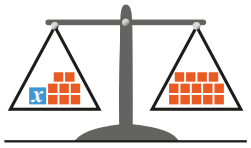
2 a right tray
b add mass to left tray or remove mass from right tray

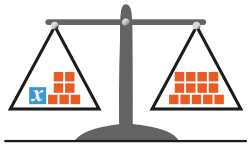
3 a yes, scales are level b $x + 7 = 10$
c remove 4 g d $x + 3 = 6$

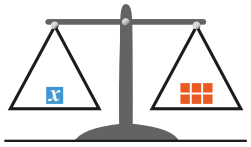
4 a



b i 

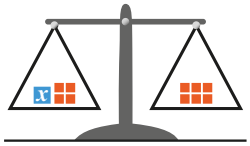
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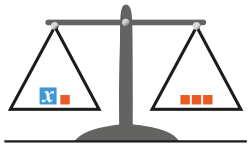
iii 

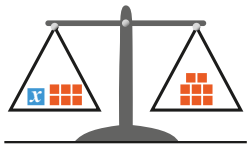
iv 

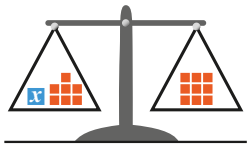
c i $x + 1 = 7$ ii $x + 8 = 14$
iii $x + 7 = 13$ iv $x = 6$

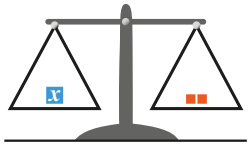
d Each equation has same solution ($x = 6$).

5 a 

b i 

ii 

iii 

iv 

c i $x + 1 = 3$ ii $x + 6 = 8$
iii $x + 7 = 9$ iv $x = 2$

d Each equation has same solution ($x = 2$).

6 a i $x + 12 = 28$ ii $x + 3 = 19$
iii $3x + 12 = 60$ iv $\dfrac{x + 4}{5} = 4$
v $x + 8 = 24$ vi $x = 16$

b Each equation has same solution ($x = 16$).

7 a i $x + 8 = 16$ ii $x - 6 = 2$
iii $5x - 10 = 30$ iv $\dfrac{x - 2}{3} = 2$
v $x - 4 = 4$ vi $x = 8$

b $x = 8$

8 $x = 5$ **9** a, d, e **10** a, c, f

11 Some possible answers are given.

a $x + 7 = 12$, $x + 2 = 7$, $x = 5$

b $x + 1 = 10$, $x - 8 = 1$, $x = 9$

c $x + 15 = 6$, $x + 2 = -7$, $x = -9$

12 a B **b** A **c** D

d Each equation has same solution ($x = 3$).

e subtracting 5 from both sides

13 a i $x + 20 = 33$ ii $x + 4 = 17$
iii $2(x + 11) = 48$ iv $\dfrac{x + 11}{8} = 3$
v $x = 13$ vi $x + 17 = 30$
vii $x - 4 = 9$ viii $x + 22 = 35$
ix $x + 1 = 14$

b $x = 13$ **c** subtracting 11 from both sides

14 a i $x + 1 = 13$ ii $x - 10 = 2$
iii $6(x - 7) = 30$ iv $\dfrac{x - 7}{5} = 1$
v $x - 12 = 0$ vi $x = 12$
vii $x - 14 = -2$ viii $x - 5 = 7$
ix $2(x - 7) = 10$ or $2x - 14 = 10$

b $x = 12$ **c** adding 7 to both sides

15 Performing the inverse operation on both sides of the equation results in the solution. Addition is the inverse operation of subtraction and subtraction is the inverse operation of addition.

16 a i $6x + 3 = 15$ ii $6x - 3 = 9$
iii $18x = 36$ iv $2x = 4$
v $6x - 6 = 6$ vi $6x + 6 = 18$
vii $36x = 72$ viii $x = 2$
ix $6x + 12 = 24$

b $x = 2$ **c** dividing both sides by 6

17 a i $\dfrac{x}{2} + 4 = 12$ ii $\dfrac{x}{2} - 4 = 4$
iii $2x = 32$ iv $\dfrac{x}{8} = 2$ v $\dfrac{x}{2} - 8 = 0$
vi $\dfrac{x}{2} + 8 = 16$ vii $\dfrac{x}{2} + 2 = 10$
viii $x = 16$ ix $\dfrac{x}{4} = 4$

b $x = 16$ **c** multiplying both sides by 2

18 Performing the inverse operation on both sides of the equation results in the solution. Multiplication is the inverse operation of division and division is the inverse operation of multiplication.

19 a $2x$ **b** $2x = 4$

c The balanced scales show that $2x$ has equivalent mass to 4 kg.

d subtract x from right side

e No, as there is not x on the right side to subtract from.

f divide by 2 on the right side

g Operation on left side is × 2, so performing inverse operation of dividing by 2 gives solution.

h $x = 2$, so mass of one rabbit is 2 kg.

20 No, as he has not performed the inverse operation on both sides of equation. He needs to add 37 to obtain solution.

21 **a** **i** -6 **ii** $+13$ **iii** $\div 11$ **iv** $\times 3$ **v** $\div 5$ **vi** $+1$ **vii** $\times 5$ **viii** -2 **ix** $+7$

b **i** $x = 13$ **ii** $x = 42$ **iii** $x = 12$ **iv** $x = 27$ **v** $x = -1$ **vi** $x = -1$ **vii** $x = -25$ **viii** $x = -10$ **ix** $x = 7$

22 **a** $3x + 1 = 7$

b scales have one less on each side; $3x = 6$

c one lot of x on left side of scales and 2 on right side; $x = 2$

d all have same solution **e** $x = 2$

23 **a** **i** $4x + 7 = 19$ **ii** $8(x - 2) = 8$ **iii** $5(x + 6) = 45$ **iv** $\frac{x}{3} + 11 = 12$

b solve each equation; each should have a solution of $x = 3$

5F Solving equations by performing the same operation on both sides

5F Start thinking!

1 **a** $x = 7$ **b** same as the solution **c** -5

d $x + 5 = 12$; $x + 5 - 5 = 12 - 5$; $x = 7$

e Adding 5 is undone by performing inverse operation of subtracting 5 from both sides of equation.

2 **a** -6 **b** $+3$ **c** $\div 2$ **d** $\times 9$

3 Operations are performed on both sides of equation to produce equivalent equations until we obtain the solution '$x = \ldots$'.

Exercise 5F Solving equations by performing the same operation on both sides

1 **a**
$$x + 6 = 11$$
$$x + 6 - 6 = 11 - 6$$
$$x = 5$$

b
$$x - 3 = 7$$
$$x - 3 + 3 = 7 + 3$$
$$x = 10$$

c
$$2x = 16$$
$$\frac{2x}{2} = \frac{16}{2}$$
$$x = 8$$

d
$$\frac{x}{9} = 4$$
$$\frac{x}{9} \times 9 = 4 \times 9$$
$$x = 36$$

2 **a** $x = 12$ **b** $x = 14$ **c** $x = 7$ **d** $x = 10$ **e** $x = -5$ **f** $x = 20$ **g** $x = 6$ **h** $x = 9$ **i** $x = 22$ **j** $x = 0$ **k** $x = -5$ **l** $x = -4$

3 **a** **i** $4x + 3 = 15$ **ii** $4x = 12$ **iii** $x = 3$

b yes, they each have same solution

c $x = 3$ **d** -3 **e** $\div 4$

f $4x + 3 = 15$; $4x + 3 - 3 = 15 - 3$; $4x = 12$; $\frac{4x}{4} = \frac{12}{4}$; $x = 3$

g Inverse operation is performed to 'undo' the operation used to build the expression on left side. The order is the reversed compared to when the expression is built.

4 **a** **i** -2 **ii** $\times 3$

b
$$\frac{x}{3} + 2 = 7$$
$$\frac{x}{3} + 2 - 2 = 7 - 2$$
$$\frac{x}{3} = 5$$
$$\frac{x}{3} \times 3 = 5 \times 3$$
$$x = 15$$

5 **a** **i** $\div 7$ **ii** $+1$

b
$$7(x - 1) = 14$$
$$\frac{7(x - 1)}{7} = \frac{14}{7}$$
$$x - 1 = 2$$
$$x - 1 + 1 = 2 + 1$$
$$x = 3$$

6 **a** $+2, \div 5$ **b** $-6, \times 4$ **c** $\times 7, +1$ **d** $\div 4, -3$

7 **a** $x = 2$ **b** $x = 16$ **c** $x = 15$ **d** $x = 2$

8 **a** $x = 8$ **b** $x = 1$ **c** $x = 24$ **d** $x = 6$ **e** $x = 18$ **f** $x = 33$ **g** $x = 20$ **h** $x = 2$ **i** $x = 7$ **j** $x = 6$ **k** $x = 1$ **l** $x = 13$

9 **a** $\div 5$ **b** $\div -3$ **c** $\times 8$ **d** $\times -7$ **e** $\div 2.5$ **f** $\div -3.7$

10 **a** Missing values or operations are listed.

i -6 **ii** $\times -3$ **iii** -17

b **i** $-7x = 42$
$$\frac{-7x}{-7} = \frac{42}{-7}$$
$$x = -6$$

ii
$$\frac{x}{-3} = -5$$
$$\frac{x}{-3} \times -3 = -5 \times -3$$
$$x = 15$$

iii $-x = 17$
$$\frac{-x}{-1} = \frac{17}{-1}$$
$$x = -17$$

11 **a** $x = -5$ **b** $x = 3$ **c** $x = -5$ **d** $x = -25$ **e** $x = 21$ **f** $x = -48$ **g** $x = -60$ **h** $x = 0$ **i** $x = 9$ **j** $x = 0$ **k** $x = 0$ **l** $x = -1$

12 **a** $x = 2.5$ **b** $x = 8.7$ **c** $x = -9$ **d** $x = -5$ **e** $x = 4$ **f** $x = 7.2$ **g** $x = -5$ **h** $x = 4.2$ **i** $x = -2$ **j** $x = -6$ **k** $x = 9.6$ **l** $x = 13$

13 **a** $\times 5, \div 3$

b $\frac{3x}{5} = 6$; $\frac{3x}{5} \times 5 = 6 \times 5$; $3x = 30$; $\frac{3x}{3} = \frac{30}{3}$; $x = 10$

c **i** $x = 14$ **ii** $x = 4$ **iii** $x = -6$ **iv** $x = 1.8$

14
$$-2x + 5 = 11$$
$$-2x + 5 - 5 = 11 - 5$$
$$-2x = 6$$
$$\frac{-2x}{-2} = \frac{6}{-2}$$
$$x = -3$$

15 **a** $x = 1$ **b** $x = -2$ **c** $x = 4$ **d** $x = 2$ **e** $x = 15$ **f** $x = 2$ **g** $x = -9$ **h** $x = 10$ **i** $x = -22$ **j** $x = -15$ **k** $x = -1$ **l** $x = 6$

16 **a** $2(x + 4) = 26$; $x = 9$
b $280 + 15x = 1600$; $x = 88$
c $257 + 56x = 649$; $x = 7$
d $\frac{x - 6}{2} = 11$; $x = 28$

17 **a** **i** $+ 7, \div 2, - 4$ **b** **i** $x = 5$
ii $\times 2, + 8, \div 3$ **ii** $x = -4$
iii $\times 5, - 6, \div -1$ **iii** $x = -9$
iv $- 7, \times 4, \div 5$ **iv** $x = 20$

18 **a** $x = 3$ **b** $x = 13$ **c** $x = 9$ **d** $x = -5$
e $x = -4$ **f** $x = 8$ **g** $x = 10$ **h** $x = 14$

5G Solving equations with the unknown on both sides

5G Start thinking!

1 The pronumeral appears on both sides of the equals sign.
2 $x = 3$
3 right side of equation is not a known number
4 **a** scales become unbalanced
b remove $2x$
c two blue boxes are removed from each side; $x + 4 = 7$
d remove 4 g; scales now have one blue box (x) on left side and three orange boxes on right side
e $x = 3$
5
$$\begin{aligned} 3x + 4 &= 2x + 7 \\ 3x + 4 - 2x &= 2x + 7 - 2x \\ x + 4 &= 7 \\ x + 4 - 4 &= 7 - 4 \\ x &= 3 \end{aligned}$$
6 remove pronumeral term from right side

Exercise 5G Solving equations with the unknown on both sides

1 **a** $x = 13$ **b** $x = 4$ **c** $x = 5$
d $x = 2$ **e** $x = -6$ **f** $x = -1$
g $x = -3$ **h** $x = 15$ **i** $x = 2$
j $x = -13$ **k** $x = 8$ **l** $x = 0$

2
$$\begin{aligned} 3x + 5 &= x + 11 \\ 3x + 5 - x &= x + 11 - x \\ 2x + 5 &= 11 \\ 2x + 5 - 5 &= 11 - 5 \\ 2x &= 6 \\ \frac{2x}{2} &= \frac{6}{2} \\ x &= 3 \end{aligned}$$

3 **a** $x = 2$ **b** $x = 4$ **c** $x = 6$ **d** $x = -4$

4
$$\begin{aligned} 2x - 3 &= 6x + 1 \\ 6x + 1 &= 2x - 3 \\ 6x + 1 - 2x &= 2x - 3 - 2x \\ 4x + 1 &= -3 \\ 4x + 1 - 1 &= -3 - 1 \\ 4x &= -4 \\ \frac{4x}{4} &= \frac{-4}{4} \\ x &= -1 \end{aligned}$$

5 **a** $x = 2$ **b** $x = 3$ **c** $x = 9$ **d** $x = 5$

6
$$\begin{aligned} 7x - 9 &= -4x + 13 \\ 7x - 9 + 4x &= -4x + 13 + 4x \\ 11x - 9 &= 13 \\ 11x - 9 + 9 &= 13 + 9 \\ 11x &= 22 \\ \frac{11x}{11} &= \frac{22}{11} \\ x &= 2 \end{aligned}$$

7 **a** $x = 4$ **b** $x = -2$ **c** $x = -1$ **d** $x = 3$
e $x = 2$ **f** $x = 1$ **g** $x = -4$ **h** $x = 5$

8 **a** **i** $- 3x$ **ii** $+ 4x$ **iii** $- 8x$
iv $+ 6x$ **v** $+ x$ **vi** $+ x$
b **i** $x = 17$ **ii** $x = 5$ **iii** $x = 8$
iv $x = -6$ **v** $x = 11$ **vi** $x = 21$

9 **a** $x = 6$ **b** $x = 11$ **c** $x = 4$
d $x = 0$ **e** $x = -3$ **f** $x = 5$
g $x = -15$ **h** $x = 7$ **i** $x = 8$

10 **a** $6x + 4 = 4x + 2$ **b** $x = -1$

11 **a** $x = 3$ **b** $x = 2$ **c** $x = -3$
d $x = 4$ **e** $x = -2$ **f** $x = 13$

12 **a** Andrew: added $2x$ to left side instead of subtracting; Hayley: did not subtract $2x$ from both sides of equation; Alex: simplified right side incorrectly (-13, not $+13$)
b
$$\begin{aligned} 6x + 11 &= 2x - 13 \\ 6x + 11 - 2x &= 2x - 13 - 2x \\ 4x + 11 &= -13 \\ 4x + 11 - 11 &= -13 - 11 \\ 4x &= -24 \\ \frac{4x}{4} &= \frac{-24}{4} \\ x &= -6 \end{aligned}$$

13 **a** B **b** \$6 **14** **a** D **b** 12

15 **a** $3a + 400 = 7a + 160$ **b** 60 cents

16 **a** $2a + 2$ **b** $3a - 4 = 2a + 2$
c $a = 6$; Sean's cousin is 6 years old
d Sean is 14 years old

17 **a** $n + 10$ **b** $3n + 4$ **c** $3n + 4 = n + 10$
d $n = 3$ **e** 3 **f** 9 **g** 13

18 **a** $6g + 27$ **b** $12g + 3$ **c** $12g + 3 = 6g + 27$
d $g = 4$; home team: 8 goals 3 behinds; away team: 4 goals 27 behinds; each team scored 51 points

19 **a** $4t + 8$ **b** $12t + 4$
c $12t + 4 = 2(4t + 8)$ or $12t + 4 = 8t + 16$
d $t = 3$
e home team: 3 tries; away team: 9 tries

20 **a** $5x - 10 = 3x + 24$ **b** $x = 17$

21 **a** $x = 29$ **b** $x = -3$ **c** $x = 1$ **d** $x = 6$
e $x = 25$ **f** $x = -7$ **g** $x = 3$ **h** $x = 0$
i $x = -2$ **j** $x = \frac{17}{18}$ **k** $x = \frac{3}{4}$ **l** $x = 0$

22 $x = 17$

5H Plotting graphs of linear relationships

5H Start thinking!

1 time (shown first) and height

2 a time; shown first in table of values
b height

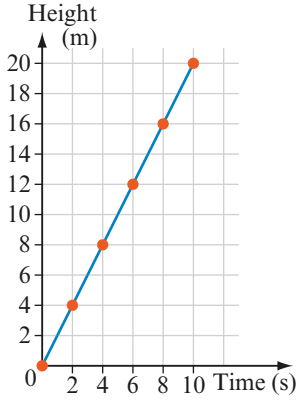
3 a (0, 0), (2, 4), (4, 8), (6, 12), (8, 16), (10, 20)
b See orange dots in graph shown in question 7.

4 a (0, 0), (2, 8), (4, 14), (6, 18), (8, 19), (10, 20)
b See orange dots in graph shown in question 7.

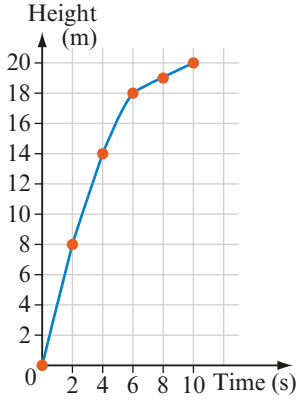
5 yes, height and time are continuous variables

6 Some possible answers are: (1, 2), (3, 6), (0.5, 1).

7 red balloon: straight line (positive relationship)



blue balloon: curved line (positive relationship)



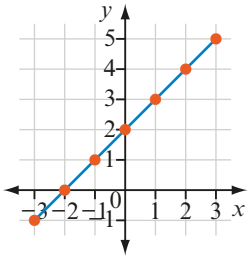
8 red balloon

Exercise 5H Plotting graphs of linear relationships

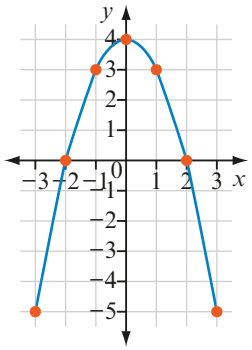
1 a, b, d and f

2 Plotted points form a straight line.

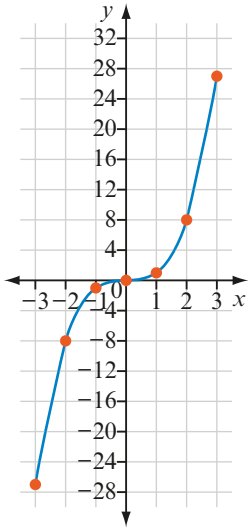
3 a i



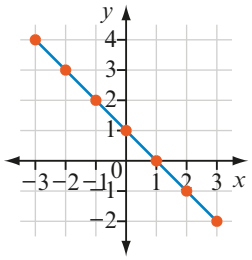
ii



iii



iv

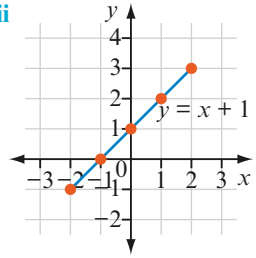


b parts i and iv

4 a i

x	−2	−1	0	1	2
y	−1	0	1	2	3

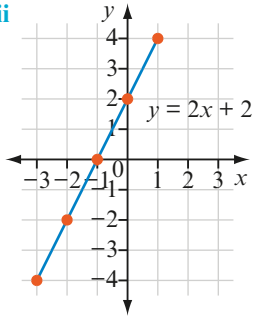
ii



b i

x	−3	−2	−1	0	1
y	−4	−2	0	2	4

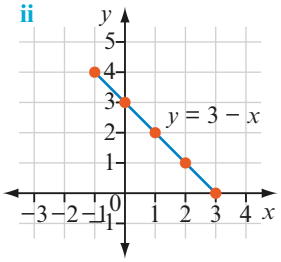
ii



c i

x	−1	0	1	2	3
y	4	3	2	1	0

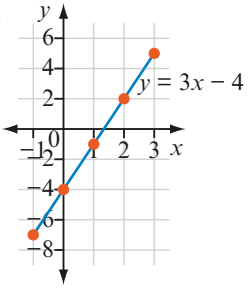
ii



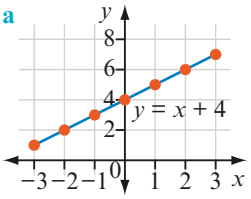
d i

x	−1	0	1	2	3
y	−7	−4	−1	2	5

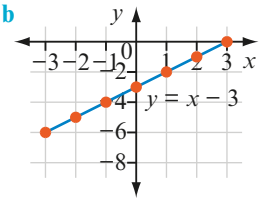
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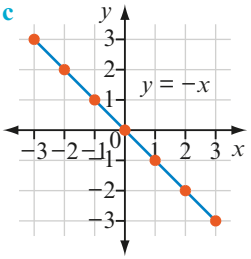
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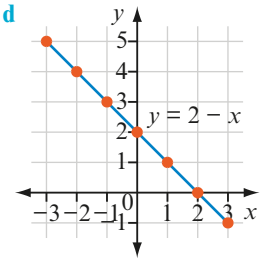
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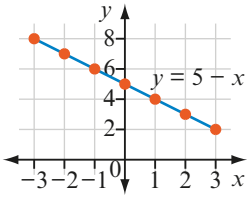
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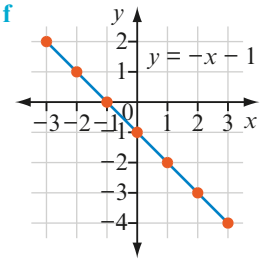
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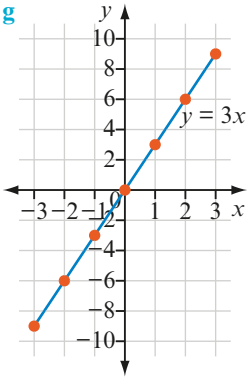
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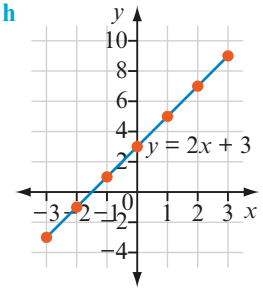
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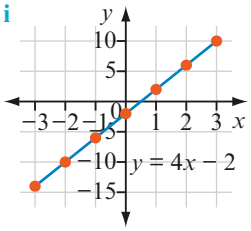
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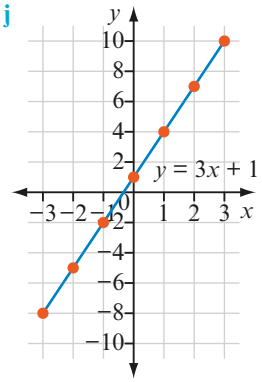
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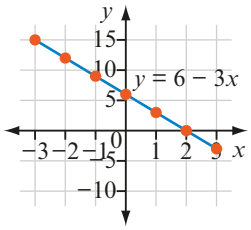
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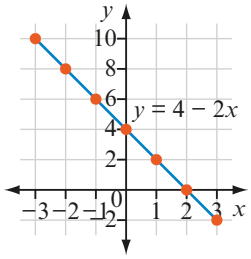
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k

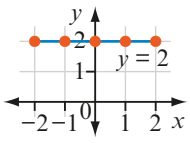


l

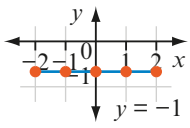


6 a x-intercept: −4; y-intercept: 4
b x-intercept: 3; y-intercept: −3
c x-intercept: 0; y-intercept: 0
d x-intercept: 2; y-intercept: 2
e x-intercept: 5; y-intercept: 5
f x-intercept: −1; y-intercept: −1
g x-intercept: 0; y-intercept: 0
h x-intercept: −1.5; y-intercept: 3
i x-intercept: 0.5; y-intercept: −2
j x-intercept: $-\frac{1}{3}$; y-intercept: 1
k x-intercept: 2; y-intercept: 6
l x-intercept: 2; y-intercept: 4

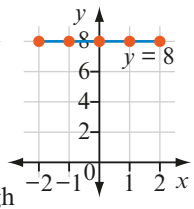
8 a i



ii

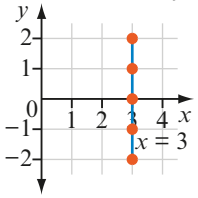


iii

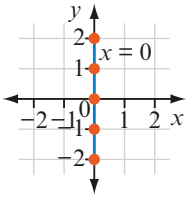


b yes, straight lines
c horizontal line passes through y value described by equation

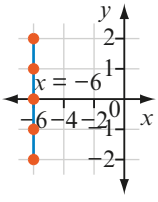
9 a i



ii



iii

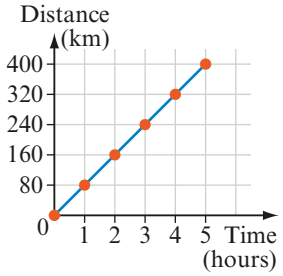


b yes, straight lines
c vertical line passes through x value described by equation

10 **a** vertical line through 2 on x-axis
b horizontal line through 4 on y-axis
c horizontal line through -3 on y-axis
d vertical line through -1 on x-axis
e vertical line through 5 on x-axis
f horizontal line through 0 on y-axis

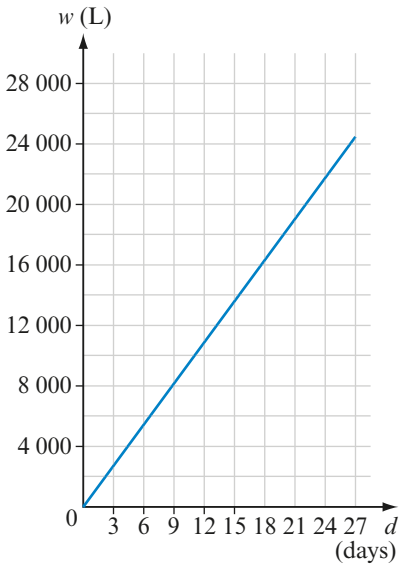
11 **a** **i** 10 m **ii** 18 m **b** **i** 1 s **ii** 3.5 s
c **i** 4 m **ii** 18.5 m **d** **i** 3 s **ii** 5 s
e $h = 2t$

12 **a** time **b**



c yes, as points form a straight line
d 200 km **e** $3\frac{1}{4}$ hours
f 560 km; assume Fabio maintains same speed
g $d = 80t$ **h** $d = 80 \times 7 = 560$

13 **a**

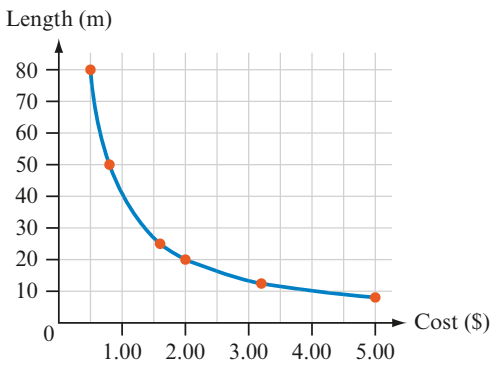


b **i** 6300 L **ii** 18 900 L **iii** 9000 L
c **i** 5 days **ii** 14 days **iii** approx. 22 days

14 **a**

cost ($ per m)	0.50	0.80	1.60	2.00	3.20	5.00
length (m)	80	50	25	20	12.5	8

b



c no, as points do not form straight line
d **i** 16 m **ii** approx. 53 m **iii** approx. 9.5 m
e $1.25 per m or less

15 **a** positive **b** negative **c** zero

16 **i** **a** P **b** P **c** N **d** P
ii **a** P **b** P **c** N **d** N
e N **f** N **g** P **h** P
i P **j** P **k** N **l** N

17 Gradient of a vertical line cannot be calculated, gradient of a horizontal line is zero. Vertical lines have an x-intercept but no y-intercept, horizontal lines have a y-intercept but no x-intercept.

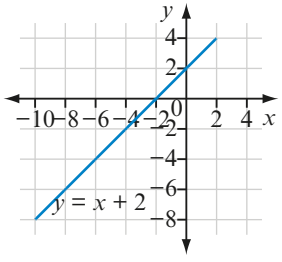
5I Solving linear equations using graphs

5I Start thinking!

1 **a** $y = 4$ **b** $y = 8$ **c** $y = -1$ **d** $y = 3$
2 **a** $x = 1$ **b** $x = 5$ **c** $x = -2$ **d** $x = -5$
3 **a** $x = 2$
b On the line, each point has the relationship $y = x + 3$. For the particular point where y is 5, $x + 3$ must equal 5.
c $x = 2$ **d** they are the same
4 **a** $x + 3 = 4$ **b** $x + 3 = 8$ **c** $x + 3 = 1$
d $x + 3 = -2$
5 **a** $x = 1$ **b** $x = 5$ **c** $x = -2$ **d** $x = -5$
Same values are obtained for x.
6 **a** $x = 3$ **b** $x = -1$ **c** $x = -4$ **d** $x = 0$
7 Since you are looking for the x value that makes a linear equation true, read the x value for a given y value that forms a point on the matching linear graph.

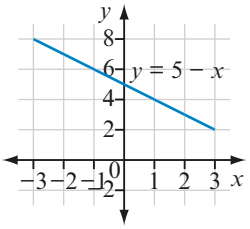
Exercise 5I Solving linear equations using graphs

1 **a** **i** $x = 6$ **ii** $x = 1$ **iii** $x = -2$
iv $x = 2$
b **i** $x = 6$ **ii** $x = 1$ **iii** $x = -2$
iv $x = 2$
c **i** $x = 7$ **ii** $x = 3$ **iii** $x = 8$
iv $x = 5$ **v** $x = 0$ **vi** $x = -4$
2 **a** $x = 2$ **b** $x = -1$ **c** $x = 1$
d $x = 0$ **e** $x = -2$ **f** $x = -3$
3 **a**



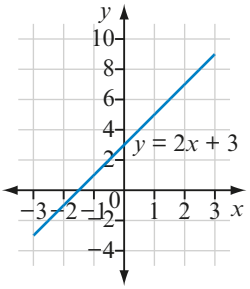
b **i** $x = 1$ **ii** $x = -3$ **iii** $x = 2$
iv $x = -1$ **v** $x = -6$ **vi** $x = -10$

4 a



b **i** $x = 3$ **ii** $x = -2$ **iii** $x = 1$
iv $x = -3$ **v** $x = -1$ **vi** $x = 0$

5 a



b **i** $x = -1$ **ii** $x = -3$ **iii** $x = 1.5$
iv $x = 0$ **v** $x = -2.5$ **vi** $x = 2.5$

6 a $y = x - 3$ **b** $y = 2x + 5$ **c** $y = 6x - 1$
d $y = 7 - x$ **e** $y = 3x - 4$ **f** $y = 2 - 4x$

7 a $y = 3x - 2$
b **i** $y = 7$ **ii** $y = -5$ **iii** $y = -20$
c **i** $x = 3$ **ii** $x = -1$ **iii** $x = -6$

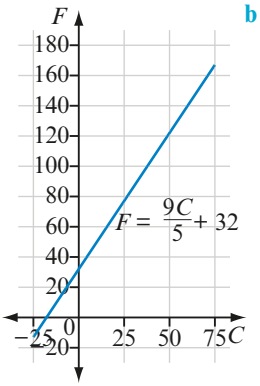
8 a $y = 4x - 5$
b **i** $y = 3$ **ii** $y = -13$ **iii** $y = -7$
c **i** $x = 2$ **ii** $x = -2$ **iii** $x = -0.5$

9 a $x = -1$ **b** $x = 3$ **c** $x = -3$
d $x = 5$ **e** $x = -4$ **f** $x = 2.5$

10 a $x = 1$ **b** $x = 2$ **c** $x = 3$
d $x = 0$ **e** $x = -1$ **f** $x = -0.5$

11 a C **b** **i** $d = 50$ **ii** $d = 150$ **iii** $d = 200$
c **i** $0.72d = 50$ **ii** $d = 69$, \$69
d shirt:
i $0.72d = 72$ **ii** $d = 100$ **iii** \$100
shoes:
i $0.72d = 100$ **ii** $d = 139$ **iii** \$139
jeans:
i $0.72d = 40$ **ii** $d = 56$ **iii** \$56
leather jacket:
i $0.72d = 140$ **ii** $d = 194$ **iii** \$194
mp3 player:
i $0.72d = 29$ **ii** $d = 40$ **iii** \$40

12 a

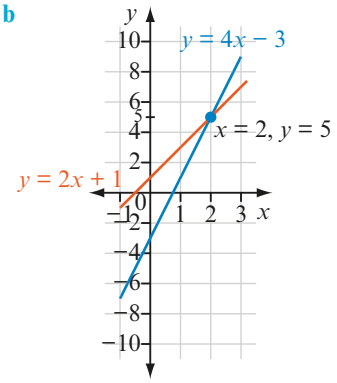


b **i** $C = 4$
ii $C = 40$
iii $C = -10$

c meat:
i $\frac{9}{5}C + 32 = -5$ **ii** $C \approx -21$
dairy products and vegetables:
i $\frac{9}{5}C + 32 = 37$ **ii** $C \approx 3$
tomatoes:
i $\frac{9}{5}C + 32 = 68$ **ii** $C = 20$
iii 'Meat to be frozen should be stored in the freezer at −21°C, dairy products and vegetables stored in the fridge at about 3°C and tomatoes kept at room temperature, ideally around 20°C.'

d $\frac{9}{5}C + 32 = 140$; $C = 60$; 140°F = 60°C

13 a, b



c The point (2, 5), known as the point of intersection, is common to both graphs.

d $x = 2$

14 a **i** $y = x - 2$ and $y = 7 - 2x$ **ii** $x = 3$
b **i** $y = 2x + 4$ and $y = -3x - 6$ **ii** $x = -2$

15 a $x + 2 = 2x - 4$ **b** $y = x + 2$ and $y = 2x - 4$
c $x = 6$ **d** six lamingtons

16 a $x = 1.5$ **b** $x = 0$

5 Chapter review

MULTIPLE-CHOICE

1 C **2** A **3** C **4** A **5** D
6 C **7** B **8** B

SHORT ANSWER

1 a 8 **b** −15 **c** −9 **d** 0
yes, except in part c

2 a $a = 9$ **b** $b = 14$ **c** $c = 3$ **d** $d = 6$

3 a **i** $4x - 2 = 14$ **ii** $x = 4$
iii $4 \times 4 - 2 = 16 - 2 = 14$
b **i** $\frac{x + 5}{5} = 6$ **ii** $x = 25$
iii $(25 + 5) \div 5 = 30 \div 5 = 6$

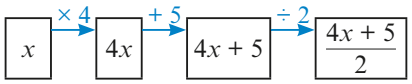
4 a $a = 2$ **b** $b = 6$ **c** $c = 46$ **d** $d = 6.6$

5 a $x \xrightarrow{\times 5} 5x \xrightarrow{-2} 5x - 2$

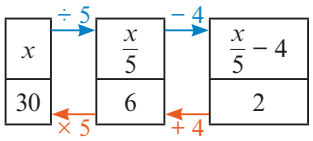
b $x \xrightarrow{\div 3} \frac{x}{3} \xrightarrow{+8} \frac{x}{3} + 8$

c $x \xrightarrow{+9} x + 9 \xrightarrow{\times 7} 7(x + 9)$

d



6 a



b $\frac{x}{5} - 4 = 2$; $x = 30$

7 **a** $a = 19$ **b** $b = 16$ **c** $c = 14$ **d** $d = 2$

8 **a** $x = -2$ **b** $x = -5$ **c** $x = 1.25$ **d** $x = 8$

9 **a** $x + 2 = 14$ **b** $x - 7 = 5$

c $7(x - 4) = 56$ **d** $\frac{x - 4}{2} = 4$

10 **a** $4x + 3 = 23$ **b** $4x - 7 = 13$

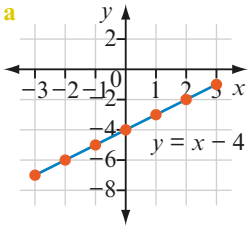
c $24x = 120$ **d** $x = 5$

11 **a** $x = 4$ **b** $x = 19$ **c** $x = 4$ **d** $x = -3$

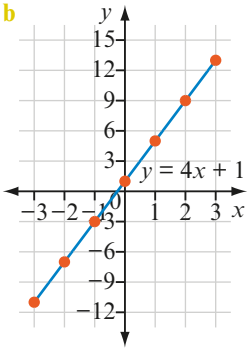
12 **a** $x = -5$ **b** $x = 2$ **c** $x = -6$ **d** $x = -1$

13 $8f + 4 = 2f + 16$ $f = 2$, so cost is \$2

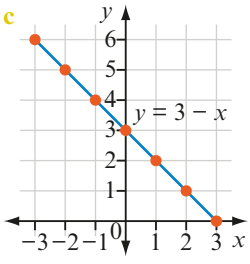
14 **a**



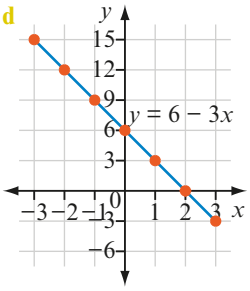
b



c



d



15 **a** $x = 4$ **b** $x = -1$

16 **a** $x = 4$ **b** $x = 1$ **c** $x = 3$ **d** $x = 0$

NAPLAN-STYLE PRACTICE

1 $x = 25$ **2** $x = 5$

3 A number is doubled then 5 is subtracted from this result to give an answer of 25.

4 $\frac{n + 5}{4} = 1$

5 The value of x for the solution must be greater than 15.

6 $3r + 15 = 75$ **7** 20 **8** 6

9 $x = 5$ **10** $\frac{x}{3} - 8 = 12$ **11** $x = 60$

12 $2x = 12$ **13** $4x - 12 = 20$ **14** $x + 8 = 11$

15 $x = -3$ **16** $-3x$ **17** $x = -2$

18 $y = 3x + 2$ **19** -1 **20** $x = 2$

ANALYSIS

a $150 + 20x$ **b** $20x + 150 = 450$

c $x = 15$; 15 people can attend the party, Kacie can invite 14 friends

d For 25 people, cost is \$650.

e no, \$200 over budget

f $200 + 10x$ **g** $10x + 200 = 450$; $x = 25$

h yes **i** five people **j** \$250

5 Connect

For feedback on this open-ended task, see your teacher.

CHAPTER 6 ANGLES AND LOCATION

6 Are you ready?

1 B **2** C **3** D **4** 180°

5 B **6** A **7** 180° **8** B

9 C **10** B **11** D **12** C

13 4 hours and 12 minutes

6A Understanding angles

6A Start thinking!

1 acute, revolution, straight, right, obtuse, reflex

2 acute angle (or reflex angle)

3 **a** right angle (or reflex angle)

b obtuse angle (or reflex angle)

c straight angle

4 One possible answer is given.

a DXA **b** CXG **c** GXH

5 reflex angle

Exercise 6A Understanding angles

1 **a** ∠BDE or ∠EDB, obtuse angle

b ∠PQV or ∠VQP, right angle

c ∠FBY or ∠YBF, reflex angle

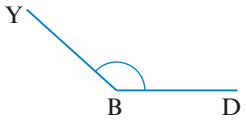
d ∠MYZ or ∠ZYM, acute angle

e ∠IMA or ∠AMI, reflex angle

f ∠DHL or ∠LHD, obtuse angle

2 One possible answer is given.

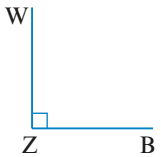
a



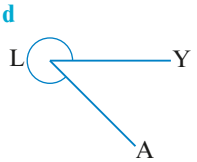
b



c



d



3 acute, right, obtuse, straight, reflex

4 **a** ∠XYZ = 85°, acute

b ∠AOM = 42°, acute

c ∠JLE = 174°, obtuse

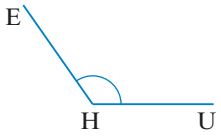
d ∠BWN = 12°, acute

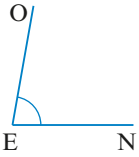
e ∠GCF = 99°, obtuse
f ∠RTM = 151°, obtuse

5 **a** 42° and 138°; 42° correct
b 117° and 63°; 117° correct
c 136° and 44°; 136° correct
d 98° and 82°; 98° correct

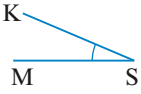
6 Estimating an angle before measuring helps you avoid reading the wrong scale on the protractor.

7 **a** ∠XYZ = 285° **b** ∠CLP = 223°
c ∠LEQ = 270° **d** ∠JDL = 350°
e ∠KAL = 195° **f** ∠CFO = 281°

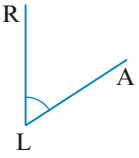
8 **a**


b


c

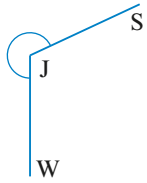

d


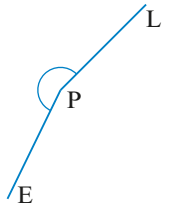
e


f


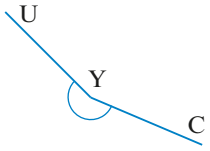
9 Using a 180° protractor, draw the acute or obtuse angle that will give the reflex angle on the other side of the vertex.

10 ∠KCF = 10°, ∠KCL = 18°, ∠KCW = 45° and ∠KCD = 85°

12 **a**


b


c


d


6B Angles and lines

6B Start thinking!

1 **a** ∠CED **b** ∠BEC
c ∠DAE and ∠EAB, ∠ABE and ∠EBC, ∠BCE and ∠ECD, ∠CDE and ∠EDA
d ∠AEB and ∠BEC, ∠BEC and ∠CED, ∠CED and ∠DEA, ∠DEA and ∠AEB

2 **a**, **b**, **c** ∠AJC = 32°, ∠MBK = 58°

3 **a**, **b** ∠BKD = 47°, ∠NCL = 133°
c, **d** The two angles add to 180°.

Exercise 6B Angles and lines

1 **a** $a = 15°$ **b** $b = 15°$ **c** $c = 39°$
d $d = 64°$ **e** $e = 138°$ **f** $f = 55°$

2 **a** $a = 160°$ **b** $a = 79°$, $b = 101°$, $c = 79°$
c $d = 147°$, $e = 33°$, $f = 147°$
d $g = 144°$ **e** $h = 253°$
f $i = 72°$, $j = 69°$, $k = 39°$, $l = 69°$
g $r = 55°$, $s = 35°$, $t = 55°$
h $u = 32°$, $v = 22°$

3 **a** **i** complementary **ii** supplementary
iii vertically opposite **iv** supplementary
b One possible answer is given.
complementary: ∠RMQ and ∠QMP;
supplementary: ∠TMS and ∠SMP; vertically opposite: ∠TMU and ∠QMP

4 **a** $m = 152°$, $n = 68°$ **b** $v = 27°$
c $s = 68°$, $t = 22°$, $u = 68°$
d $a = 7°$, $b = 139°$, $c = 41°$
e $w = 45°$, $x = 38°$, $y = 39°$, $z = 38°$
f $j = 135°$, $k = 45°$, $l = 135°$; $d = 115°$, $e = 65°$, $f = 115°$; $n = 70°$, $g = 110°$, $i = 110°$

5 **a** 60°, 120°, 180° **b** 32°, 148°
c 20°, 90°, 70° **d** 45°, 90°

6 **a** 18°

7 **a** **i** supplementary **ii** 56°
b **i** complementary **ii** 62°
c **i** complementary **ii** 23°
d **i** supplementary **ii** 92°

8 **a** 360°
b **i** 180° **ii** 90° **iii** 270°
c **i** 120° **ii** 330° **iii** 240° **iv** 30°

9 **a** 30°
b **i** 15° **ii** 7.5° **iii** 22.5°
c **i** 165° **ii** 97.5° **iii** 52.5°

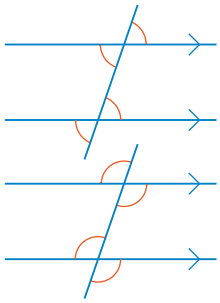
10 **a** 170° **b** 122.5° **c** 72.5° **d** 88.5° **e** 79.5°

6C Angles and parallel lines

6C Start thinking!

1 All acute angles formed are equal.

All obtuse angles formed are equal.



Exercise 6C Angles and parallel lines

1 **a** corresponding **b** alternate
c co-interior **d** alternate
e corresponding **f** co-interior

2 **a** equal **b** 180° **c** equal

3 **a** $a = 63°$ **b** $b = 71°$ **c** $c = 122°$
d $d = 123°$ **e** $e = 82°$ **f** $f = 23°$
g $g = 76°$ **h** $h = 35°$ **i** $i = 49°$
j $j = 25°$

4 **a** corresponding **b** alternate
c co-interior **d** corresponding
e corresponding **f** co-interior
g corresponding **h** alternate
i co-interior **j** corresponding

5 **a** 27° **b** alternate and equal

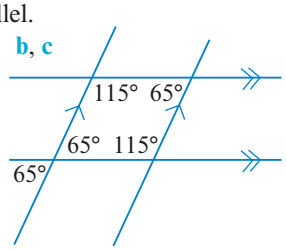
6 **a** $a = 141°$, co-interior
b $b = 147°$, co-interior
c $c = 21°$, alternate
d $d = 66°$, corresponding; $e = 59°$, alternate
e $f = 95°$, co-interior; $g = 85°$, corresponding
f $n = 142°$, co-interior

7 If first pattern has obtuse angle of 135° then its acute angle must be 45° (the angles are supplementary). Second pattern has acute angle 55°, so patterns are different.

9 **a** **i** 70° **ii** 110° **iii** 70° **iv** 70°
b **i** 125° **ii** 55° **iii** 125° **iv** 125°
c One possible answer is given.
i 15° **ii** 165° **iii** 15° **iv** 15°
d One possible answer is given.
i 175° **ii** 5° **iii** 175° **iv** 175°

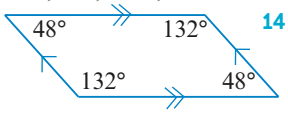
10 **a** Angles corresponding but not equal, so lines not parallel.
b Angles co-interior and supplementary, so lines parallel.
c 99° is supplementary to the angle alternate to 81°, so lines are parallel.

11 **a** parallelogram **b, c**



12 110°, 70°, 110°, 70°

13



14 rectangle

15 **a** c and f; d and e; k and n; l and m, i and d; b and k; f and o; h and m
b a and e; b and f; i and m; j and n; c and g; d and h; k and o; l and p; j and b; i and a; l and d; k and c, n and f; m and e; p and h; o and g
c c and e; d and f; k and m; l and n; i and b; d and k; f and m; o and h

16

	32°	85°	45°	12°	60°
Complementary	58°	5°	45°	78°	30°
Supplementary	148°	95°	135°	168°	120°
Vertically opposite	32°	85°	45°	12°	60°
Co-interior	148°	95°	135°	168°	120°
Alternate	32°	85°	45°	12°	60°
Corresponding	32°	85°	45°	12°	60°

17 $a = 40°$, $b = 50°$, $c = 90°$, $d = 40°$, $e = 50°$, $f = 50°$

18 **a** When a pair of parallel lines is cut by a transversal, angles of only two different sizes are formed. These angles are supplementary. Using an understanding of alternate, corresponding, co-interior and vertically opposite angles or just looking at the angles formed, it is easy to discover which angle size a particular angle is.
b When a pair of parallel lines is cut by another pair of parallel lines, still angles of only two different sizes are formed. It is easy to identify an angle as one size or the other (supplementary) size.

6D Constructions

6D Start thinking!

For feedback on your diagrams, see your teacher.

Exercise 6D Constructions

1 **a** angles 22°, 71°, 87°; sides 4.1 cm, 3.9 cm, 1.5 cm
b angles 43°, 42°, 95°; sides 4.3 cm, 4.4 cm, 6.4 cm
c angles 80°, 70°, 30°; sides 3 cm, 5.9 cm, 5.6 cm

6 **a** SSS **b** SAS **c** ASA
d SAS **e** SSA **f** ASA

6E Bearings

6E Start thinking!

1 north, south, east, west
2 north-east (NE), south-east (SE), south-west (SW), north-west (NW)
5 south: 180°, west: 270°
6 SE: 135°, SW: 225°, NW: 315°
7 **a** 90° **b** 135° **c** 180° **d** 45°

Exercise 6E Bearings

1 **a** **i** N30°E **ii** 030°T
b **i** S55°E **ii** 125°T
c **i** S12°W **ii** 192°T
d **i** S70°W **ii** 250°T
e **i** N55°W **ii** 305°T
f **i** N11°W **ii** 349°T

2 **a** 159°T **b** 043°T **c** 328°T
d 189°T **e** 225°T **f** 100°T
g 358°T **h** 063°T **i** 113°T

3 **a** S30°E **b** N22°E **c** N80°W
d N59°W **e** S83°E **f** S2°E

4 **a** N32°E (032°T) **b** N25°E (025°T)
c S39°W (219°T) **d** N20°W (340°T)

5 **b** True bearings have three digits. 047°T
c Compass bearings always start with N or S. S21°W
e The angle away from north and south in a compass bearing must be an acute angle. S82°W

g True bearings can't be greater than 360°. 020°T
i True bearings have a capital T at the end. 182°T

6 a i S30°E **ii** N65°W
b i N30°W **ii** S65°E
c Angles same but directions opposite.
d 150°T, 295°T, 330°T, 115°T; difference of 180°

7 a 165°T **b** 345°T **c** N79°E **d** The Granites **e** 090°T **f** 135°T **g** 315°T **h** 180°T
i You look at bearing 062°T, your friend looks at bearing 242°T.
j 180°T **k** 275°T **l** N12°E

8 a One possible answer is given.

Start	End	Compass Bearing	True Bearing	Distance
Darwin	Elsey	S40°E	140°T	370 km
Elsey	Willeroo	S80°W	260°T	180 km
Willeroo	Mount Sanford	S28°W	208°T	200 km
Mount Sanford	Banka Banka	S60°E	120°T	400 km
Banka Banka	Alexandria	S83°E	097°T	270 km
Alexandria	Darwin	N41°W	319°T	900 km

b Using distances in part a, total distance is 2320 km.

6F Angles and time zones

6F Start thinking!

1 20°

Exercise 6F Angles and time zones

1 a Rome **b** Berlin **c** Paris **d** Belgrade
2 a 52°N 21°E **b** 52°N 5°E **c** 50°N 14°E **d** 47°N 8°E
3 a USA **b** Canada **c** Greenland **d** Russia
4 a 45°N 76°W **b** 45°N 123°W **c** 64°N 22°W **d** 61°N 150°W
5 The Arctic Circle is approximately 67°N or precisely 66.5622°N
6 a 60°N **b** 110°W **c** 141°W **d** 42°N
7 Lines of longitude are not parallel because they eventually meet up at the north and south poles. As shown on this map, they are further apart in the USA and closer together in Canada. Lines of latitude are always the same distance apart.
8 a 55°N 38°E **b** 32°S 116°E **c** 34°N 118°W **d** 14°N 17°W **e** 13°N 100°E **f** 35°S 58°W

9

City	Latitude and Longitude	City	Latitude and Longitude
Cairo	30°N 31°E	Cape Town	34°S 18°E
Rio de Janiero	23°S 43°W	Hong Kong	22°N 114°E
New Delhi	28°N 77°E	New Orleans	30°N 90°W
Nairobi	1°S 36°E	Sydney	34°S 151°E
Honolulu	21°N 158°W	New York	41°N 74°W
Colombo	7°N 80°E	Beijing	40°N 116°E
Tokyo	36°N 140°E	Panama	9°N 80°W
London	51°N 0°	Jakarta	6°S 107°E

10 a latitude should be first, 24°S 35°E
b maximum movement west is 180°, 42°S 169°E
c maximum movement north is 90°, 70°N 168°W

11 Latitude and longitude split the world into four quarters like a compass: NE, SE, SW and NW (with a world map that has the UK in the middle). If you know the latitude and longitude coordinates of a city you can make a rough guess where it is using this understanding.

12 a Moving from the south pole in a straight line will be heading north.
b i 70°S 76°E **ii** 67°S 65°E **iii** 75°S 26°W **iv** 78°S 166°E
c Australia: 107°, New Zealand: 52°, France: 7°, Norway: 67°, Chile: 36°, United Kingdom: 60°, Argentina: 49°
d i E2 **ii** A2 **iii** Q1 **iv** E2 **v** H1

13 a i Stewart Island **ii** Blenheim **iii** Mount Aspiring **iv** Gore **v** Methven **vi** Kaiapoi
b i 46.0°S (or 45.5°S) 170.5°E
ii 43.5°S 172.5°E **iii** 45.0°S 168.5°E
iv 45.0°S 170.0°E **v** 42.5°S 171.5°E
vi 43.5°S 172.0°E

6G Working with time zones

6G Start thinking!

1 a 360° **b** 360° ÷ 24 = 15°
2 Some possible answers are: Alaska, the Hawaiian islands, North and South America.
3 a 3.00 pm **b** 7.00 pm **c** 12 midnight **d** 9.00 am
4 Time zones enable the time on clocks to fit in with night and day in all countries around the world.

Exercise 6G Working with time zones

1

12-hour time	24-hour time	12-hour time	24-hour time
7.30 am	0730	3.57 pm	1557
9.12 pm	2112	9.04 pm	2104
3.33 pm	1533	11.45 pm	2345

2 a 2 hours **b** 4 hours and 22 minutes
c 6 hours and 30 minutes
d 6 hours and 43 minutes
e 13 hours and 26 minutes
f 9 hours and 42 minutes

3 **a** 10 hours **b** 2 hours **c** 8 hours
4 **a** 12 hours **b** 5 hours **c** 17 hours
d Here, added the two numbers together because New Zealand is ahead of the UK but Peru is behind the UK. In question 3c, subtracted one number from the other because both Hobart and Turkey are ahead of the UK.
5 **a** 5 hours **b** 7 hours **c** same time
d 8 hours **e** 4 hours **f** 13 hours
g 16 hours **h** 8 hours
6 **a** 9 am **b** 7 am **c** 2 pm **d** 6 am **e** 6 pm
f 1 am **g** 10 pm (night before) **h** 6 am
7 **a** 1.25 pm **b** 1530 **c** 11.20 pm
8 One possible answer is given.
a sleeping **b** having breakfast
c eating dinner
9 **a** 5 **b** 1
d To be in same time zone with rest of Europe.
e Some possible answers are:
France, Netherlands, Belgium, parts of Africa, Argentina, Chile.
10 9.00 am the next day **11** 3.00 pm
12 He has time to eat. If it is 5 pm in Paris, it is 4 pm in London.
13 1.00 am
14 Western Australia: 4 pm to 7 pm; South Australia: 5.30 pm to 8.30 pm
15 add, west
16 **a** 15 hours (Peru behind)
b 5 pm (the night before) **c** Monday
d It will be 7.00 am Wednesday in Hobart
e 7.45 am on Tuesday
17 **a** Coordinated Universal Time (Greenwich Mean Time)
b 2 hours **c** 22 hours
d From UTC, moving east across 12 zones is 12 hours ahead. Moving west across 12 zones is 12 hours behind. So there must be one calendar day difference between the places far to the east compared to those far to the west.
e Crossing over the Date Line from east to west you have to move backwards in time as the western parts are behind UTC.
f You go backwards in time, so you subtract a day.
g As you move westward, you are further behind UTC, so the places further west are subtracting time. However, when moving west over the Date Line, you add time as you move into the next day (remembering that places to the east of UTC are ahead in time).
18 So countries can remain in the same time zone.
19 **a** 9.00 pm Saturday
b 7.00 pm Saturday (or 8.00 pm Saturday – depending on where he is in Mexico)
c 11.00 am Sunday **d** 5.00 am Sunday
e 3.00 am Sunday **f** 2.00 am Sunday
g 10.00 am Sunday **h** 4.00 pm Saturday
20 Most times are reasonable; however, for those that require him to call at 2.00 or 3.00 am, he could negotiate with his mother to call in the evening (Queensland time) instead.
21 **a** 9.39 pm Saturday
b 0652 Tuesday (6.52 am Tuesday)
c 6.30 pm Thursday **d** 5.40 am Thursday

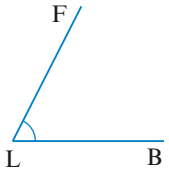
6 Chapter review

MULTIPLE-CHOICE

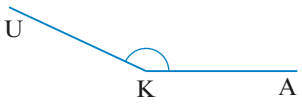
1 C **2** B **3** A **4** A
5 D **6** B **7** B

SHORT ANSWER

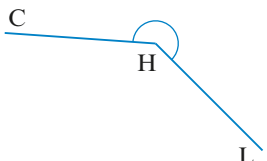
1 **a**



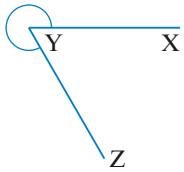
b



c



d



2 **a** 123° **b** 287°
3 **a** $a = 138°$, $b = 42°$, $c = 138°$
b $a = 38°$, $b = 142°$, $c = 52°$
4 **a** d **b** d **c** $c = 108°$ and $d = 72°$
5 **c** Some possible answers are: SSS: 2.9 cm, 1.8 cm, 2.5 cm; SAS: 2.9 cm, 38°, 2.5 cm; ASA: 38°, 2.9 cm, 58°.
6 **a** N58°W; 302°T **b** S58°W; 238°T
c S22°E; 158°T **d** N24°E; 024°T
7 **a** 38°N 123°W **b** 41°N 112°W
c 26°N 80°W **d** 43°N 72°W
e 46°N 122°W **f** 33°N 112°W
8 **a** **i** 9 pm **ii** 1 pm **iii** 5 am or 6 am
b **i** 6 am **ii** 11.30 am **iii** 9 am

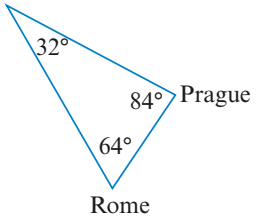
NAPLAN-STYLE PRACTICE

1 ∠DCB **2** reflex angle **3** 210°
4 54° **5** complementary
6 $b = 26°$ **7** 119°
8 Angles are corresponding. **9** 125°
10 SAS **11** 308°T **12** S65°E
13 Line of longitude is 153°E. **14** 40°N 14°E
15 207 **16** 15.5 **17** 6.30 am Sunday
18 11.30 pm Friday **19** 6.45 pm

ANALYSIS

a London: 52°N 0°W; Rome: 42°N 12°E; Prague: 50°N 14°E
b **i** 133°T **ii** 010°T **iii** 285°T

c 12.25 pm
d London
e 9 hours and 52 minutes
f Yes. He arrives back in London at 6.52 pm London time.



6 Connect

For feedback on this open-ended task, see your teacher.

CHAPTER 7 SHAPES AND OBJECTS

7 Are you ready?

1 180° 2 90° 3 A 4 A
5 C 6 D 7 5 8 C
9 a D b C 10 a C b B

7A Classifying triangles

7A Start thinking!

1 180°
2 ∠ABC: $a = b = c = 60°$; ∠LMN: $l = 125°$, $m = 20°$, $n = 35°$; ∠RST: $r = 90°$, $s = t = 45°$
3 180° 4 35° 5 145°

Exercise 7A Classifying triangles

1 a obtuse-angled isosceles
b acute-angled scalene
c acute-angled isosceles
d acute-angled equilateral
e right-angled scalene
f obtuse-angled scalene

2 a 63° b 90° c 41°
d 67° e 106° f 50°

3 a 138° b 141° c 64°
d 111° e 109° f 164°

4 a $a = 60°$, $b = 60°$, $c = 60°$
b $a = 45°$, $b = 45°$ c $a = 142°$, $b = 19°$
d $d = 120°$ e $e = 71°$ f $f = 41°$
g $g = 27°$ h $h = 58°$ i $i = 156°$

5 acute-angled and obtuse-angled scalene triangles

6

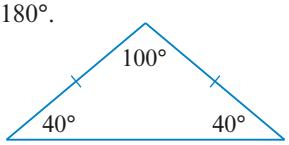
	equilateral	isosceles	scalene
acute-angled	✓	✓	✓
right-angled		✓	✓
obtuse-angled		✓	✓

7 a $x = 30°$ b $x = 10°$ c $x = 10°$
d $x = 10°$ e $x = 12°$ f $x = 20°$

8 a 30°, 60°, 90° b right-angled scalene

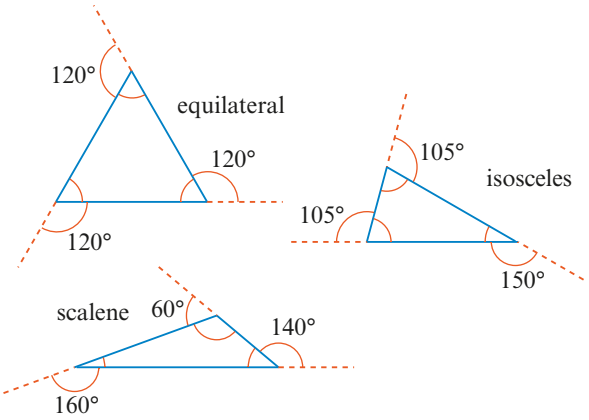
10 acute-angled isosceles

11 a If you follow an 'inside face' around the middle it suddenly becomes an 'outside face', which is not possible.
b i Not possible. The angle sum of a triangle is 180°.
ii



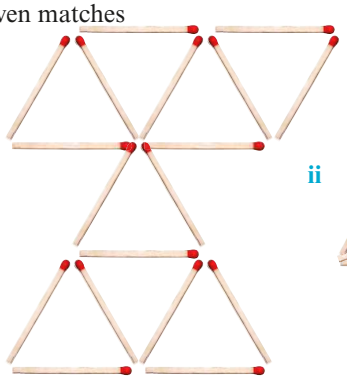
iii Not possible. If base length is 10 cm, the other two sides are not long enough to meet.
iv Not possible: longest side would be longer than 5 cm.
v Not possible: angle sum of a triangle is 180°.
c Angle sum must be 180° and length of two smallest sides, when added, must be greater than largest side length.

12 a–c Some possible answers are given.



d The exterior angles of any triangle add to 360°.

13 a seven matches
b i



ii



ii

7B Quadrilateral properties

7B Start thinking!

1–3

	Parallelogram	Rectangle	Square	Rhombus	Trapezium	Kite
	A	E	B	F	C	D
All angles are equal		✓	✓			
All sides are equal			✓	✓		
Opposite sides are equal	✓	✓	✓	✓		
Opposite sides are parallel	✓	✓	✓	✓		
One pair of opposite sides are parallel					✓	
Opposite angles are equal	✓	✓	✓	✓		
One pair of angles are equal						✓
Diagonals meet at right angles			✓	✓		✓

5 A parallelogram has opposite sides equal in length and parallel. Opposite angles are also equal in size.
6 rectangle, square, rhombus

7 A square always meets the conditions of a parallelogram, so it is a type of parallelogram. A parallelogram does not necessarily meet all conditions of a square (e.g. all angles are equal) so it is not a type of square.

8 A rhombus is a type of square and a type of parallelogram. A square is a type of rectangle. A rectangle is a type of parallelogram.

Exercise 7B Quadrilateral properties

1 **a** kite; two pairs of adjacent sides equal in length, but not parallel
b trapezium; one pair of opposite sides that are parallel
c rhombus; all four sides equal in length and opposite sides parallel, opposite angles equal, but not 90°
d parallelogram; opposite sides are parallel and equal in length, opposite angles are equal
e square; all four sides equal in length and opposite sides parallel, all angles are 90°
f kite; two pairs of adjacent sides equal in length, one pair of angles are equal

2 **a** $x = 76°$ **b** $x = 117°$ **c** $x = 141°$
d 23° **e** 21° **f** 93°

3 **a** parallelogram, $x = 71°$
b rectangle, $x = 90°$ **c** trapezium, $x = 105°$
d kite; $x = 85°$ **e** rhombus; $x = 64°$
f trapezium; $x = 143°$

4 **a** **i** rhombus
ii $a = 72°$, $b = 72°$ and $c = 108°$
b **i** trapezium **ii** $a = 90°$, $b = 112°$
c **i** square **ii** $a = 90°$, $b = 90°$, $c = 90°$
d **i** kite **ii** $a = 130°$, $b = 130°$
e **i** parallelogram
ii $a = 125°$, $b = 55°$, $c = 125°$
f **i** kite **ii** $a = 52°$, $b = 115°$

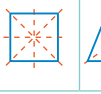
5 trapeziums, parallelograms, kites

6 **a** True
b False. A kite has no parallel sides. It contains two pairs of adjacent sides equal in length, but not parallel.
c False. A rectangle is a type of parallelogram. The reverse is not true as a parallelogram does not have four angles equal to 90°.
d True
e False. A rhombus has two pairs of angles equal in size. When all four angles are equal in size it is called a square.
f False. A rectangle two pairs of equal sides (opposite sides are equal in length).

8 **a** $x = 20°$ **b** $x = 37°$ **c** $x = 80°$
d $x = 20°$ **e** $x = 15°$ **f** $x = 17°$

9 **b** 180° **c** 360°
e, **f** When arranged around a point, the angles from the second quadrilateral will add to 360°.
f The two triangles formed each have an internal angle sum of 180°. The six angles in total represent the angle sum of a quadrilateral; that is, $2 \times 180° = 360°$.
g The triangles each contain an angle formed from the centre of the rectangle. These four angles do not contribute to the angle sum of the quadrilateral.

10

Parallelogram	Rectangle	Square	Rhombus	Trapezium	Kite
A parallelogram does not have to have an axis of symmetry.				A trapezium does not have to have an axis of symmetry.	

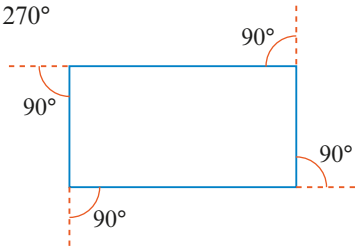
11 A trapezium can have a line of symmetry; for example, an isosceles trapezium has vertical symmetry. Not all trapeziums do, so it is not a rule.

12 **b** A square can only vary the side lengths as all angles must be 90°. A rhombus must have equal side lengths, but angles don't have to be 90° (opposite angles must be equal). Rectangles and parallelograms have many different combinations for the opposite side lengths. The parallelogram has most freedom as the angles don't have the same restrictions as a rectangle.

13 **b** A trapezium only has to have one pair of parallel sides. Internal angles are not restricted as for those within the parallelogram family.

14 **a** Having two internal reflex angles would make the angle sum exceed 360°.
b 270°

15 **a**



b 90° **c** 360° **e** add to 360°
f add to 360°; it is the same

16 **a** **i** 90° **ii**, **iii** square
b **i** 60°, 60°, 120°, 120°
ii, **iii** parallelogram (rhombus is also possible)
c **i** 24°, 48°, 144°, 144° **ii**, **iii** kite

7C 2D shapes and 3D objects

7C Start thinking!

1 A, B

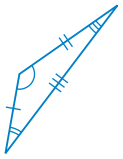
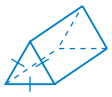
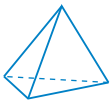
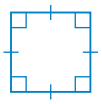
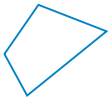
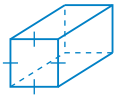
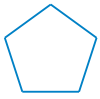
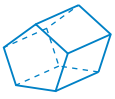
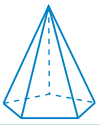
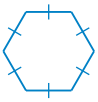
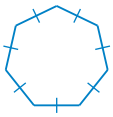
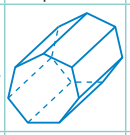
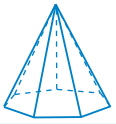
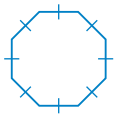
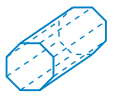
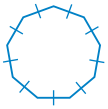
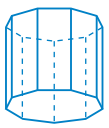
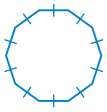
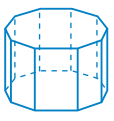
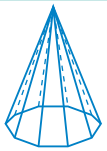
2 A is regular and B is irregular. A regular polygon has sides equal in length and all angles equal in size

3 5

4 Each has at least one face that is a pentagon.

5 C is a prism and D is a pyramid. A prism has ends that are matching polygons and a pyramid meets at a point at the opposite end to the base.

6, 8

		Polygon		3D object	
		Regular	Irregular	Prism	Pyramid
3 sides	Name	equilateral triangle	scalene triangle	triangular prism	triangular-based pyramid
	Image				
4 sides	Name	square	irregular quadrilateral	square-based prism	square-based pyramid
	Image				
5 sides	Name	regular pentagon	irregular pentagon	pentagonal prism	pentagonal pyramid
	Image				
6 sides	Name	regular hexagon	irregular hexagon	hexagonal prism	hexagonal pyramid
	Image				
7 sides	Name	regular heptagon	irregular heptagon	heptagonal prism	heptagonal pyramid
	Image				
8 sides	Name	regular octagon	irregular octagon	octagonal prism	octagonal pyramid
	Image				
9 sides	Name	regular nonagon	irregular nonagon	nonagonal prism	nonagonal pyramid
	Image				
10 sides	Name	regular decagon	irregular decagon	decagonal prism	decagonal pyramid
	Image				
	Name	circle		cylinder	cone
	Image				

7 A circle is not a polygon as it does not have straight sides.

9 Polygons with more than 10 sides (and objects with these polygons as bases), shapes and objects with curved sides, e.g. sphere.

Exercise 7C 2D shapes and 3D objects

1 **a** **i** irregular **ii** convex
b **i** regular **ii** convex
c **i** irregular **ii** concave
d **i** irregular **ii** concave
e **i** regular **ii** convex
f **i** irregular ii concave

2 **a** regular convex hexagon
b regular convex octagon
c irregular convex heptagon
d irregular convex nonagon
e regular convex decagon
f irregular concave hexagon

3 **a** rectangular prism **b** triangular prism
c nonagonal pyramid **d** cylinder
e decagonal prism **f** hexagonal pyramid

4 **a** yes **b** no **c** yes
d yes **e** no **f** yes

5 **a** triangular prism **b** regular octagon
c cylinder

6 **a** semi-regular
d Making sure there are no gaps (angles around a point must sum to 360°).

8 **a** 8 **b** 6 **c** 2
d $6 \times 180° = 1080°$ **e** yes
f To ensure all the triangles are formed, with no overlapping.

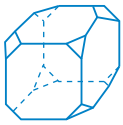
9 **a** **i** 540° **ii** 720° **iii** 900°
iv 1440° **v** 1800°
b Number of triangles always two less than number of sides.

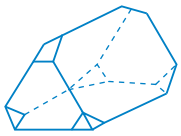
10 **a** **i** five-pointed star **ii** four-pointed star
iii six-pointed star
b **i** concave decagon **ii** concave octagon
iii concave dodecagon
c **i** 1440° **ii** 1080° **iii** 1800°
The internal angles sum follows the same rule as found in question 2. (It holds true for all polygons: regular, irregular, concave or convex).

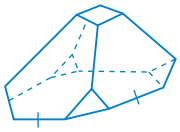
11 **a** 6 **b** 720° **c** $720° \div 6 = 120°$
d **i** 108° **ii** 135° **iii** 144°
e Because all angles are equal in size in a regular polygon, but not an irregular polygon.

12 **a** $x = 140°$ **b** $x = 150°$ **c** $x = 115°$

13 **a** The vertices of the tetrahedron have been cut off (or truncated) to form the truncated tetrahedron.
b Yes, all faces are polygons.
c 12 vertices (three times the number of the tetrahedron)
d 18 edges (an extra 12 edges)

14 **a** 

b 

c 

15 **a** Some possible answers are:
a ring, a beach tyre.

b It has only a single curved face.

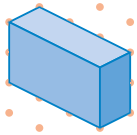
16 **b** It continues along both the inside and outside surfaces.

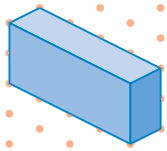
d The alphabet starts on one side of the flat strip and continues on the other side.

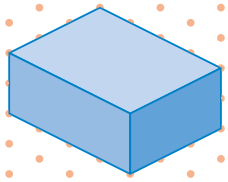
7D Isometric drawings and plans

7D Start thinking!

1 By joining dots to form lines that are angled at 30° to the horizontal.

2 **a** 

b 

c 

3 **a** twice the height

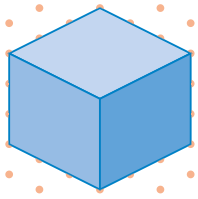
b twice the height and one unit longer

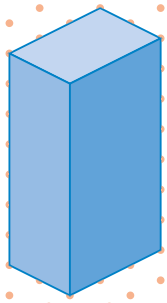
c twice the height, one unit longer and three times the width

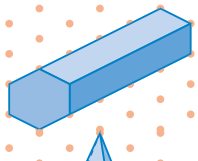
5 three **7** C

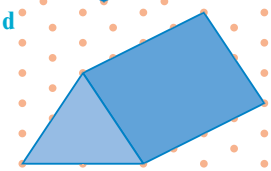
Exercise 7D Isometric drawings and plans

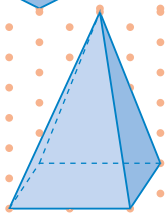
1 A possible answer is shown in each case.

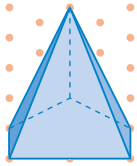
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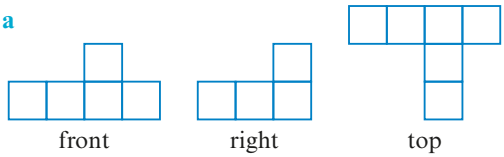
b 

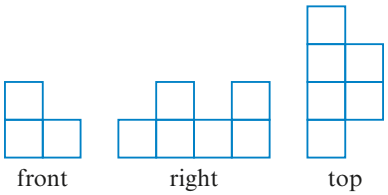
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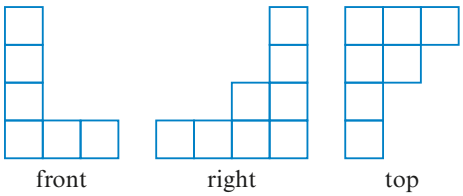
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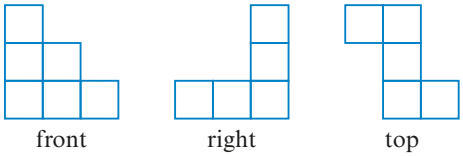
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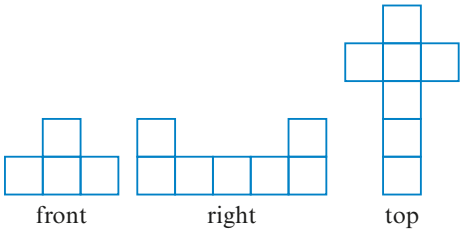
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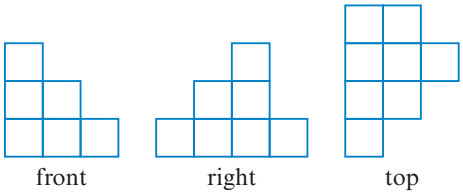
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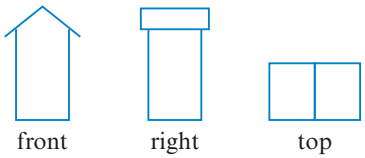
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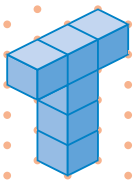
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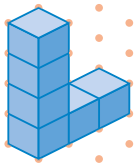
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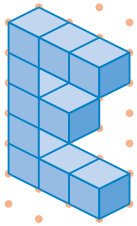
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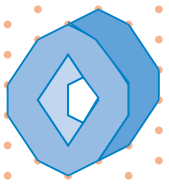
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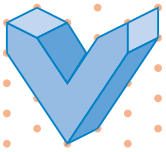
4 **a** T

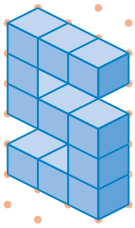
b 

c **i** 

ii 

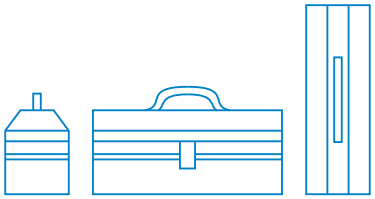
iii 

iv 

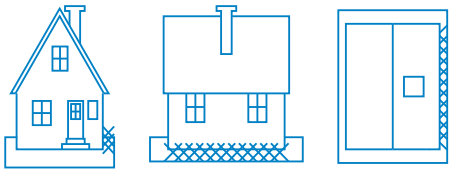
v 

6 a 

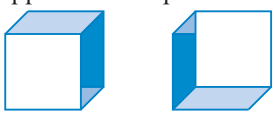
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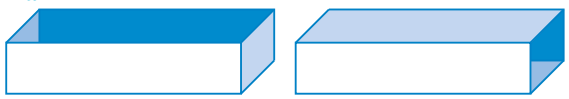
7 It is hard to represent the point in its true perspective.

8 b 

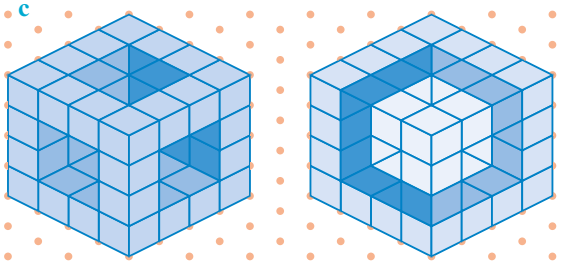
10 a One appears to be open at the top and the other appears to be open at the front.

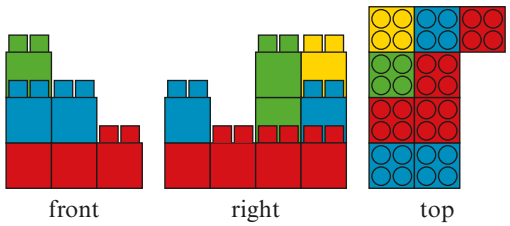
b 

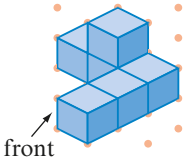
11 Some possible answers are given.

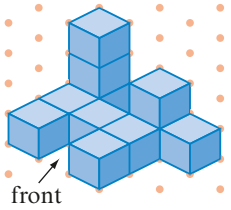
a 

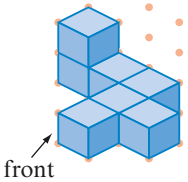
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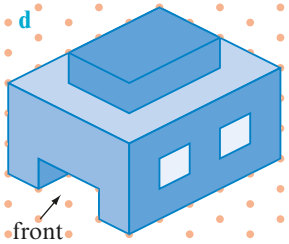
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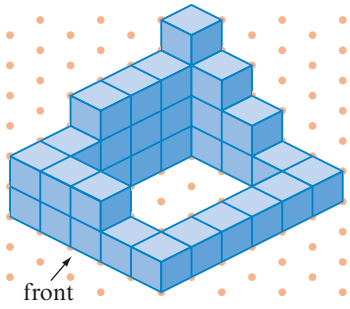
12 e 

13 a 

b 

c 

d 

14 

7E Nets and perspective drawings

7E Start thinking!

1 6; 3

2 B will not fold up to create the 3D object displayed

3 Check that your net folds up to form a rectangular prism by cutting it out and folding along the fold lines.

4 The lines are parallel, although the way they appear in a perspective drawing suggests they are not.

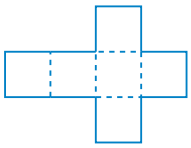
5 a Figure E is not as 'deep' as figure D.

b Figure F has a vanishing point further to the right and it is also deeper than figure E.

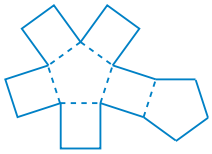
Exercise 7E Nets and perspective drawings

Some possible answers are given for questions 1–4.

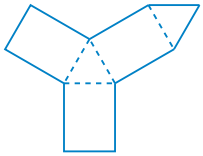
1 a



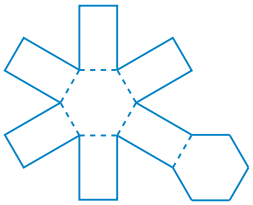
b



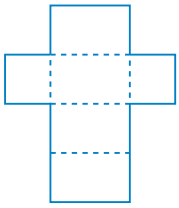
c



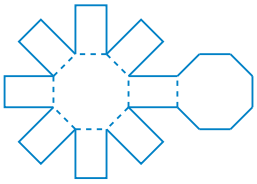
d



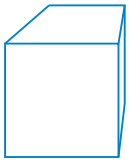
e



f



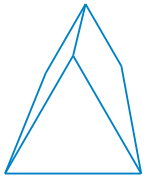
2 a



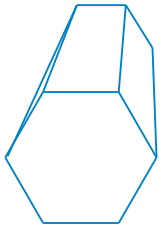
b



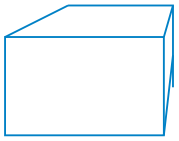
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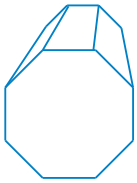
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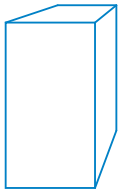
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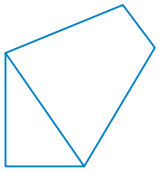
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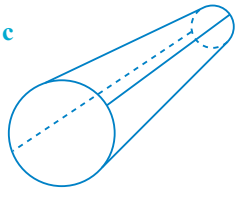
3 a



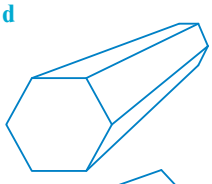
b



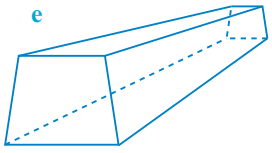
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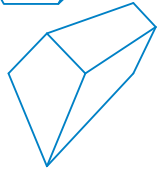
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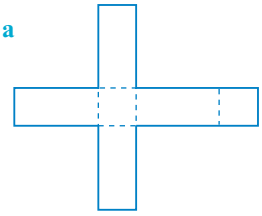
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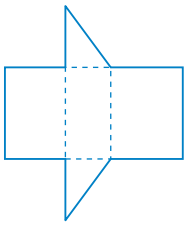
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4 a



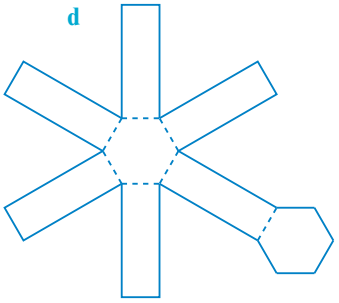
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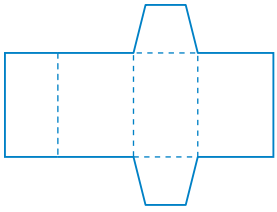
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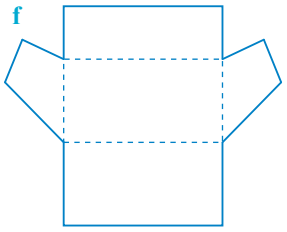
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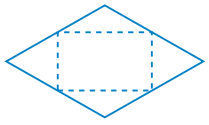


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f

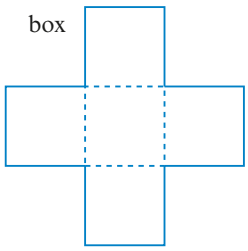




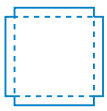
5 One possible answer is shown.

6 Use the Internet to check your answers.

7 a 5 **b** box



lid

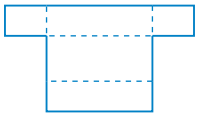


c The lid should be slightly longer and wider than the box, but its sides much shorter than the sides of the box.

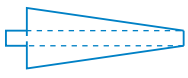
d A pyramid consists of a base with faces that meet at a point (there is no top face). An open box consists of a base and side faces only (with no top face).

8 a rectangular prism **b** triangular prism
c cylinder **d** triangular prism

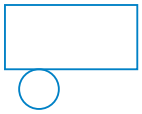
9 a



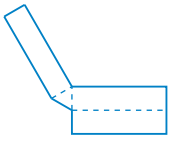
b



c



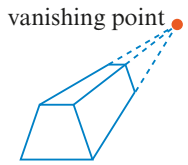
d



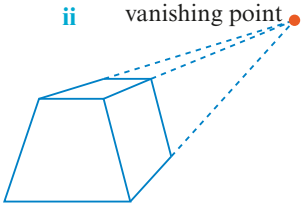
10 a Because the net contains a rectangle (it is curved like a piece of pipe to create the curved part).

11 Because the front face is already sloping backwards. This is unlike a prism, which has a vertical front face that makes it easier to produce accurate perspective drawings

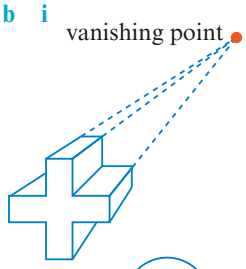
13 **a** **i**



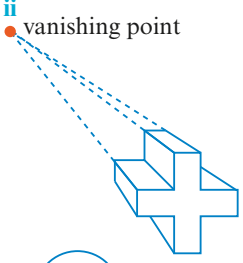
ii



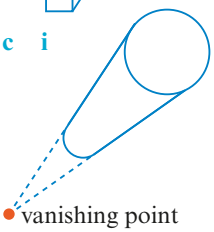
b **i**



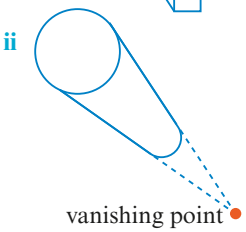
ii



c **i**



ii



14 **a** The perspective is looking up from below.

b



c The building on the left, as it appears to be smaller at the top compared to the bottom, showing it is further away.

15 **a** regular pentagons, regular hexagons

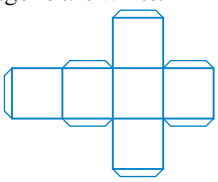
b trapeziums

c Pentagons are black, hexagons are white.

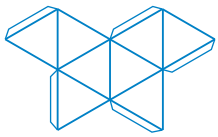
16 **a**



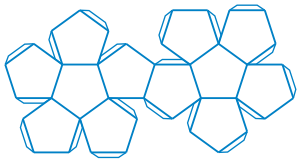
b



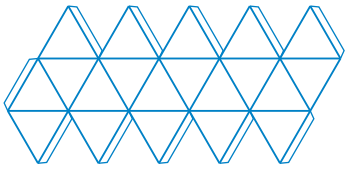
c



d



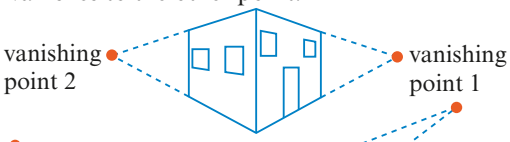
e



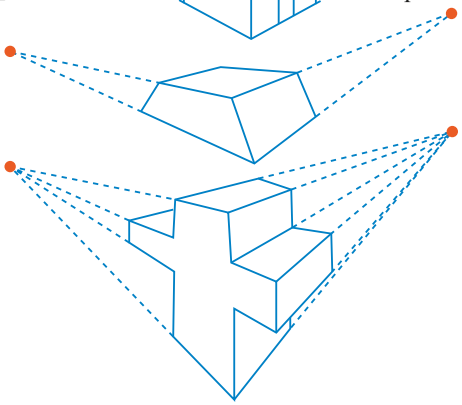
17 **a** There are two vanishing points (one to the left and one to the right).

b Instead of seeing the front face straight on, the object is rotated so you see two faces, one that vanishes to one point and the other that vanishes to the other point.

c



d



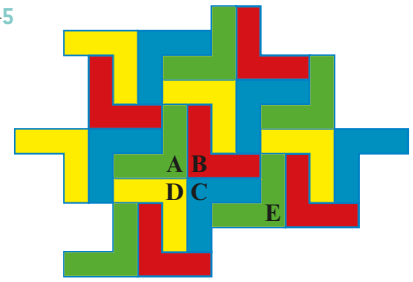
7F Translations, rotations and reflections

7F Start thinking!

1 **a** A **b** C

2 D has been rotated 180° from B, it is a different orientation so it cannot have been translated.

3–5



Exercise 7F Translations, rotations and reflections

1 **a** rotation **b** reflection **c** translation

2 **a** 90° anticlockwise (or 270° clockwise) rotation

b vertical reflection

c translation down and then translation left

3 **a** translation 2 units down, 3 units to the left

b 90° clockwise rotation

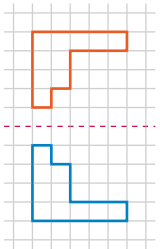
c reflection in a horizontal mirror

d 180° rotation

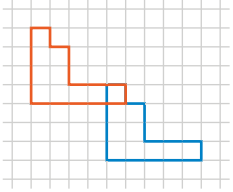
e reflection in a vertical mirror

f translation 3 units up and 2 units to the right

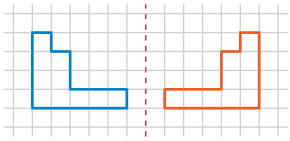
4 **a** A possible answer is:



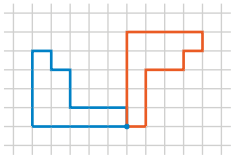
b



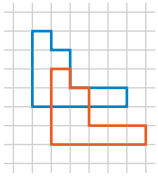
c A possible answer is:



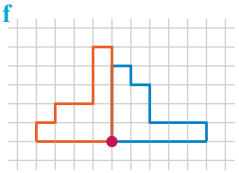
d



e



f



5 Each blade on the pinwheel can be rotated to the next blade position and each pinwheel can be translated to the next pinwheel position.

6 **b** (7, 2)

7 **a**

	Round 1	Round 2	Round 3	Round 4	Round 5
Ollie	(6, 5)	(2, 4)	(3, 8)	(6, 10)	(8, 7)
Sophie	(4, 4)	(2, 1)	(5, 3)	(9, 4)	(8, 8)

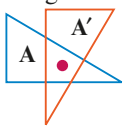
b In Round 5, Sophie got within 1 step of Ollie.

c Sophie is the winner. **d** translation

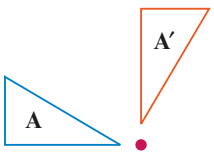
8 **a** 90°

b Each image is in a different position because the rotation occurs about a different point on the original shape.

c, d

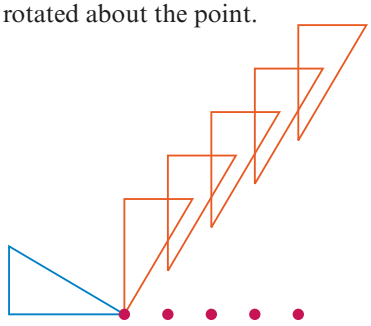


e One possible answer is:



f You need to imagine that the original shape and the point of rotation away from the shape, all form a 'new' shape. This whole part is then rotated about the point.

9 **a**



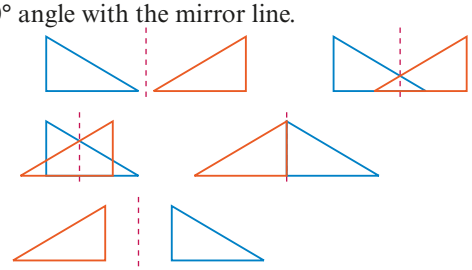
b The further away the point of rotation, the further away the image is located.

c The pattern is the same; however, the images are produced down and to the right.

10 When reflected horizontally, vertically, and horizontally and vertically, the images (or words) appear in three possible locations (like the quadrants of a Cartesian plane). In each position, each image is a different orientation to the original.

When reflected in a diagonal mirror line, the image is again a different orientation to the original. In all cases, corresponding positions on the shape and the image are the same distance from the mirror line and make a 90° angle with the mirror line.

11 **a**



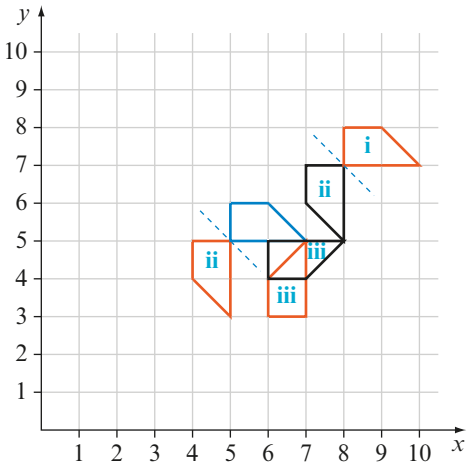
b When the mirror line is next to the object, the image in total appears on the other side of the line. When the mirror line cuts through the object, the image appears on both sides of the line.

c, d A series of isosceles triangles is generated.

e A series of pentagons is generated.

12 Results will be similar to question 11.

13 A possible answer is shown.



c (5, 5), (5, 6), (6, 6), (7, 5); blue shape

e **i** (8, 7), (8, 8), (9, 8), (10, 7); orange shape

ii (5, 5), (4, 5), (4, 4), (5, 3); orange shape

iii (6, 3), (7, 3), (7, 5), (6, 4); orange shape

g (6, 4), (6, 5), (8, 5), (7, 4); black shape iii

h Translations can be added or subtracted to the x- and y-coordinates whether the movement is left, right, up or down.

7G Understanding congruence

7G Start thinking!

1 same size but a mirror image of each other

2, 3 Size and shape of final figure is same as starting figure.

4 They are identical in shape and size so they are congruent.
5 One is same size but not same 'shape' as original two gliders. Other is same 'shape' as original but larger.
6 They are not identical in shape and size.

Exercise 7G Understanding congruence

1 a congruent b not congruent
c not congruent d congruent
2 B and W, C and S, E and N, G and J, H and K, L and X, Q and T
3 A and I, D and H, F and G
4 Corresponding angles (parallel lines) are angles that are in the same relative position (they match in position to the transversal line, but are on a different parallel line) and are of the same size. Corresponding sides (shapes) are sides that are in the same relative position (they match in position on each shape) but do not have to be the same size.
5 a AC and FE, BC and DE
b i LM and QR, KM and PR, LK and QP
ii DE and WT, EF and TU, FG and UV, DG and WV
iii AB and WV, BC and VU, CD and UZ, DE and ZY, EF and YX, AF and WX
iv AB and QR, BC and RS, CD and ST, DE and TM, EF and MN, FG and NO, GH and OP, HA and PQ
6 a not congruent b congruent
c not congruent d not congruent
7 a congruent b congruent
c not congruent d not congruent
8 a congruent: corresponding sides equal length.
b not congruent: corresponding sides not equal length
c not congruent: corresponding angles equal but corresponding sides not equal
d congruent: corresponding sides equal length
9 Piece A: The design contains a series of congruent shapes, but not all are congruent to one another. Emily may think that her friend means that all shapes are congruent, which is not the case.
Piece B: The design consists entirely of congruent shapes, despite being coloured differently. Congruence is about shape and size, not colour.
10 b H and X, B and U, C and Y, D and J, S and M, E and L, F and N, O and Z, Q and V, G and P
c Some possible answers are given. Shape X is a translation of shape H. Shape U is the result of a 90° clockwise rotation then a translation of shape B. Shape Y is the result of a 90° clockwise rotation then a translation of shape C. Shape J is the result of a 90° anticlockwise rotation about the right vertex followed by a translation down of shape D. Shape S is the result of a 180° clockwise rotation, followed by a vertical reflection then a horizontal translation of shape M. Shape N is a translation of shape F. Shape Z is a translation of shape O. Shape V is the result of a 90° anticlockwise rotation about the bottom left vertex followed by a translation of shape Q. Shape P is a translation of shape G. Shape E is the result of a 25° anticlockwise rotation then a translation of shape L.

7H Using congruence

7H Start thinking!

1 A and D are congruent as all corresponding sides are equal in length and all corresponding angles are equal.
2 Yes, congruent triangles must have corresponding sides equal in length.
3 SSS stands for 'side side side', meaning three pairs of sides are equal in length.
4 SAS stands for 'side angle side', meaning that two pairs of sides and the corresponding angles between these two sides are the same.

Exercise 7H Using congruence

1 a congruent b not congruent
c not congruent
2 a congruent b not congruent
c not congruent
3 a not congruent b congruent
c not congruent
4 a red b H for hypotenuse, S for side
c If the right angle is between the two shorter sides, it becomes the condition SAS.
d SSS
e Because SSS is a condition for congruence.
5 a not congruent b congruent
c congruent d not congruent
6 a congruent, SAS (7 cm, 77°, 9 cm)
b not congruent c not congruent
d congruent, AAS (54°, 13 cm, 87°)
e not congruent
f congruent, SSS (9 cm, 13 cm, 14 cm)
7 c yes d yes (37°, 53°, 90°)
e If you draw a triangle using a condition for congruence, you will get a congruent triangle.
8 a i $v = 17$ cm, $w = 80°$, $x = 62°$, $y = 38°$, $z = 38°$
ii $a = 48°$, $b = 25°$, $c = 4$ cm, $d = 25°$, $e = 107°$, $f = 7$ cm
iii $j = 25°$, $k = 7$ cm, $l = 3$ cm, $m = 5$ cm, $n = 115°$
iv $g = 35°$, $h = 35°$, $i = 110°$, $j = 35°$
b i, ii AAS and angle sum of a triangle
iii angle sum of a triangle and SAS
iv isosceles triangle facts and SAS
9 Triangles are different shapes so cannot possibly be congruent.

10 a angle sum of a triangle (three angles sum to 180°)
b Both angle pairs will require the same value to make the angle sum equal 180°.
c Pair 1 supports ASA as the corresponding sides between the angles are equal whereas the equal sides in Pair 2 are not in corresponding positions.
d Once you find the third angle, you have the requirements for AAS (two angles and one corresponding side).

11 a i congruent; angle sum of a triangle is 180°, ASA
ii $n = 40°$, $p = 12$ cm, $q = 66°$, $r = 17$ cm
b i congruent; isosceles triangles, ASA
ii $a = 6$ cm, $b = 72°$, $c = 36°$, $d = 10$ cm, $e = 72°$
c i congruent; isosceles triangles, ASA
ii $d = 20°$, $e = 25$ cm, $f = 140°$, $g = 48$ cm

7I Dilations

7I Start thinking!

1 a identical in shape but not in size.
b A, B

2 a 2 cm square
b Side lengths in D are twice the size of lengths in C.

3 a 3 **b** 4 **c** 6 **d** 10

4 a 2 **b** 3 **c** 4 **d** 6

5 She assumed that the original square had a side length of 1 cm, not 2 cm.

Exercise 7I Dilations

1 a reduction **b** enlargement
c reduction **d** enlargement

2

	Enlargement	Reduction
Two times	SF = 2	SF = $\frac{1}{2}$
Three times	SF = 3	SF = $\frac{1}{3}$
Four times	SF = 4	SF = $\frac{1}{4}$
Five times	SF = 5	SF = $\frac{1}{5}$
Six times	SF = 6	SF = $\frac{1}{6}$
Seven times	SF = 7	SF = $\frac{1}{7}$
Eight times	SF = 8	SF = $\frac{1}{8}$
Nine times	SF = 9	SF = $\frac{1}{9}$
Ten times	SF = 10	SF = $\frac{1}{10}$

3 a i 3 **ii** $\frac{1}{3}$ **b i** $\frac{1}{5}$ **ii** 5
c i $\frac{1}{4}$ **ii** 4 **d i** $\frac{1}{10}$ **ii** 10
e i 8 **ii** $\frac{1}{8}$ **f i** $\frac{1}{20}$ **ii** 20

4 a enlargement **b** reduction
c enlargement **d** enlargement
e reduction **f** reduction

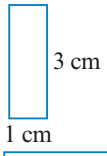
5 9

6 a i reduction **ii** $\frac{1}{2}$
iii dilation by a scale factor of $\frac{1}{2}$
b i enlargement **ii** 3
iii dilation by a scale factor of 3
c i enlargement **ii** 3
iii dilation by a scale factor of 3
d i reduction **ii** $\frac{1}{4}$
iii dilation by a scale factor of $\frac{1}{4}$

7 These dogs are not the same 'shape', so the small dog is not a reduction of the big dog.

8 Check with your teacher.

9 a original is B, image is E
b original is D, image is C
c original is D, image is A

10 a vertical dilation, 2 **b** no
c horizontal dilation, 3 **d** no
e i 

ii 

iii 

iv 

11 a i enlargement
ii vertical scale factor of 2
b i reduction
ii horizontal scale factor of $\frac{1}{2}$
c i reduction **ii** vertical scale factor of $\frac{1}{2}$
d i enlargement
ii vertical scale factor of 3
e i reduction **ii** vertical scale factor of $\frac{1}{5}$
f i reduction
ii horizontal scale factor of $\frac{1}{3}$

12 Check with your teacher.

13 a All side lengths in triangle B are twice the size of the corresponding side lengths in triangle A.
b 6 cm **c** 35°
d Corresponding angle sizes remain the same in all uniform dilations.
e multiply, the same

14 a i 2 **ii** $x = 4$ cm
b i $\frac{1}{2}$ **ii** $x = 1.5$ cm
c i $\frac{1}{3}$ **ii** $x = 11$ cm, $z = 75°$
d i 4 **ii** $x = 12$ cm, $z = 27°$

15 Check with your teacher.

7 Chapter review

MULTIPLE-CHOICE

1 D **2** B **3** D **4** D **5** A **6** D

SHORT ANSWER

1 a $x = 109°$ **b** $x = 66°$, $y = 48°$
c $x = 141°$ **d** $x = 69°$

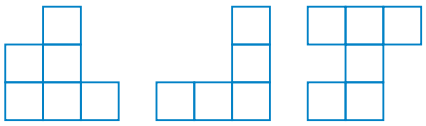
2 a obtuse-angled scalene
b acute-angled isosceles
c acute-angled scalene
d acute-angled scalene

3 a $w = 124°$ b $a = 144°$, $b = 36°$, $c = 144°$
c $m = 129°$, $n = 129°$ d $p = 263°$

4 a trapezium b parallelogram
c kite d irregular quadrilateral

5 a regular convex hexagon
b pentagonal pyramid
c triangular prism
d irregular convex nonagon
e rectangular prism
f irregular concave octagon

6 a



b



7 c

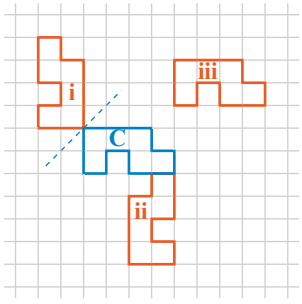


e



8 a B is a vertical reflection of A; C is a 180° rotation about the bottom right vertex; D is a translation 3 units down and 3 units left

b Note that one possible diagonal mirror line is shown for part i.



9 A and E

10 A and E are congruent due to the condition ASA. D and C are not congruent as they do not satisfy RHS. B and F are not congruent, even though they display AAA.

11 a F b reduction
c $\frac{1}{2}$ d dilated by a scale factor of $\frac{1}{2}$

NAPLAN-STYLE PRACTICE

1 isosceles 2 36° 3 kite 4 157°
5 3240° 6 C 7 A 8 A
9 D 10 D 11 D
12 translation of 12 right, 5 down 13 (13, 5)
14 rotation, translation, dilation
15 SSA 16 B 17 AAS 18 $\frac{1}{3}$

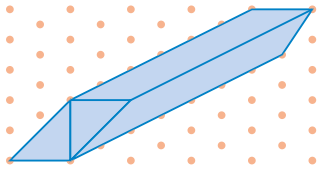
ANALYSIS

a a reflection about the right side; or a horizontal reflection and a series of translations; or a vertical reflection and a series of translations

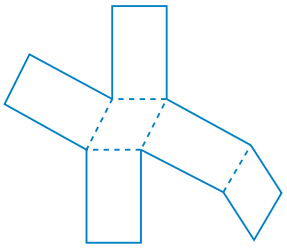
b The front view forms a rhombus, with one pair of opposite angles of 60° and another pair of opposite angles of 120°. Each side length is the same as the original shape side lengths; that is, 2 cm.

c rhombus-based prism

d



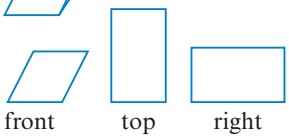
e i



ii



iii



f Triangles B and D; these two satisfy the congruency tests (B: SAS and D: ASA). In triangle A, you can't be certain that the side lengths are each 2 cm. In triangle C, you can't be certain that the triangle is equilateral.

g Only the side lengths are doubled, growing to 4 cm each side. The angle sizes remain at 60° each.

7 Connect

For feedback on this open-ended task, see your teacher.

CHAPTER 8 MEASUREMENT

8 Are you ready?

1 150 2 D 3 15 cm^2 4 B 5 D
6 5 7 A 8 D 9 A 10 D

8A Length and perimeter

8A Start thinking!

1 Length is distance or measurement from one end to other. Perimeter is distance around outside edge of a 2D shape.

4 To make sure measurements are correct and reasonable.

5 A 5-cm ruler must be moved many times to measure the length of a table, which makes error more likely. A trundle wheel is better used for long measurements than short ones and gives a less precise answer.
6 If it is a rectangular table, the missing length will be the same as the opposite side length. If it is a trapezoidal table some estimation may be involved.

Exercise 8A Length and perimeter

1 a 5800 cm b 2700 m c 5.21 m
d 0.398 km e 12 mm f 3.987 km
g 42.3 cm h 0.7 cm
2 a 5340 mm b 395 000 cm c 235 m
d 145 km e 0.023 68 km f 0.287 m
g 193 400 cm h 910 000 mm
3 a 20 cm b 65 cm c 8 m d 168 mm
e 142 mm (14.2 cm) f 43.6 cm (436 mm)
4 a i 1.7 cm, 2.2 cm ii 7.7 cm
b i 1.4 cm, 2.2 cm ii 7.2 cm
c i 2.5 cm, 6 cm ii 15.1 cm
5 Use a piece of string around the shape then measure the string on a ruler. Use a ruler for straight edges.
6 a m b km c cm or mm d cm
7 Some possible answers are given.
a trundle wheel or tape measure
b map or odometer
c small ruler that uses mm d ruler
8 a 14 cm (or 140 mm) b 34 m c 60 cm
d 56 cm e 47 cm f 112 mm
9 a 16 cm b 4.8 km c 7.5 cm d 60 mm
e 50 mm f 14 m
10 a 91.5 m b 549 m c 20.33 m d 122 m
e 488 m
11 4.8 m 12 25.2 km
13 a 50 cm
b Corners of frame are where frame is longer than width and length of photo. Because of this, the sides of frame must be longer than sides of photo.
c 54 cm
d 58 cm; perimeter is longer because the corner lengths are counted twice, horizontally and vertically.
e i length of wood = 58 cm, perimeter = 66 cm
ii length of wood = 62 cm, perimeter = 74 cm
iii length of wood = 74 cm, perimeter = 78 cm
iv length of wood = 94 cm, perimeter = 98 cm
14 20 × 4 + 15 × 4 + 60 = 200 cm 15 9 cm
16 Some possible answers are: 10 cm × 40 cm, 15 cm × 35 cm, 28 cm × 22 cm.
17 There are 15 sets of possible whole-number dimensions: 1 cm × 29 cm, 2 cm × 28 cm, 3 cm × 27 cm …, 14 cm × 16 cm, 15 cm × 15 cm.
19 1.15 m
20 a 36.4 m b length: 0.9 m, width 0.5 m
c length: 7.2%, width: 10.6%
d i length ii width
e 39.2 m f 7.7%
22 51.52 m
23 a Some possible answers are: 6 m × 8 m, 10 m × 4 m, 10.2 m × 3.8 m.
b 8 m longer than garden perimeter: 36 m.
c i 44 m ii 52 m
d Multiply distance by 8 and add to initial perimeter (28 m)
e Rule works for all cases. Start with initial rectangle or inside rectangle. Once you've found this perimeter, the outer perimeter can be found by adding 8 × distance to fence to the inside perimeter.

8B Circumference of a circle

8B Start thinking!

1 A possible answer is to use a piece of string and measure the length of this with a ruler.
8 You should find that the ratio of the circumference to the diameter $(\frac{C}{D})$ is about 3.14.
10 You may make small errors when taking measurements throughout the activity.

Exercise 8B Circumference of a circle

1 a 10 cm b 4 mm c 16 cm
2 a 6 cm b 12 m c 140 mm
d 3 cm e 222 mm f 18 cm
g 24 mm h 5 m i 50 mm
3 a 4 cm b 6 mm c 3.5 m
4 a 5 cm b 2 m c 3 mm
d 15 cm e 2.8 mm f 4.5 m
g 8 cm h 2.5 m i 12 cm
5 a 31.42 cm b 6.28 mm c 23.56 cm
d 47.12 mm e 5.65 m f 131.95 mm
6 a 94.25 cm b 131.95 m c 28.90 m
d 56.55 mm e 40.21 cm f 10.68 m
7 a 81.68 cm b 113.10 cm c 163.36 mm
d 95.82 cm
8 a 7.54 m b 7.85 cm c 20.11 mm
d 2.51 cm e 119.38 cm f 81.68 mm
g 2.36 m h 84.82 cm i 515.22 km
9 a half a circle b 9.42 cm
c 9.42 ÷ 2 = 4.71 cm d 4.71 + 3 = 7.71 cm
10 a 30.85 cm b 5.14 m c 26.28 m
d 81.42 mm e 94.25 cm f 109.98 cm
11 a 60.16 mm. Ring should fit, but the knuckle may pose a problem!
b Knuckle circumference is between 61.45 mm and 62.71 mm.
c 19.76 mm d 62.08 mm
12 a Jack: 172.79 cm, Lily: 100.53 cm
b Jack: 578.74 times, Lily: 994.73 times
c Lily does have to complete more revolutions to cover the same distance.

13 **a** The given distances relate to radius as distance from centre to edge is radius.
b Matt: 25.76 m, Lauren: 33.93 m
c 49.01 m
d Remember that the longer the radius is, the greater the circumference.

14 **a** **i** 99.95 cm **ii** 49.97 cm
b **i** 2 m **ii** 1 m **c** **i** 5 mm **ii** 2.5 mm
d **i** 0.5 cm **ii** 0.25 cm

15 **a** 40 074.16 km
b You would be able to slide yourself under the piece of string. Adding 1 m to a circumference means adding about 32 cm to diameter or 16 cm to radius. Need to consider radius; 16 cm would allow you to slide under.

16 **a** 108 m **b** 113 m **c** 339.29 m
d 84 carriages **e** 3.039 m
f increase of 9.66 m
g Consider how carriages hang on the circular frame and can the required gap be spaced appropriately. Research the measurements of some major Ferris wheels of the world; for example, the London Eye.

8C Area of rectangles and triangles

8C Start thinking!

1 Length is a measurement of distance. Area is the space enclosed by a 2D shape.

2 **i** 5 units **ii** 3 units **iii** 15 square units

3 Multiply length and width and you get the same answer as you would when you count the squares.

4 Diagonal line splits rectangle into two halves, so area of triangle must be half that of area of rectangle.

5 Base and height of triangle are equivalent to length and width of rectangle. Like the rectangle's length and width, the base and height must be at right angles to each other.

6 As b and h in a triangle are equivalent to l and w in a rectangle, the area of a rectangle could be written as $A = b \times h$. The area of the matching triangle would be half of this.

Exercise 8C Area of rectangles and triangles

1 **a** 70 mm^2 **b** 15 cm^2 (or 1500 mm^2)
c 1.02 m^2 (or 10 200 cm^2) **d** 29.4 mm^2
e 9900 cm^2 (or 0.99 m^2)
f 161 mm^2 (or 1.61 cm^2)

2 **a** 22.5 cm^2 **b** 0.96 m^2 (or 9600 cm^2)
c 10.8 cm^2 (or 1080 mm^2) **d** 367.5 cm^2
e 4725 cm^2 (or 0.4725 m^2)
f 33 820 cm^2 (or 3.382 m^2)

3 **a** 14 cm^2 (or 1400 mm^2)
b 20.4 cm^2 (or 2040 mm^2)
c 150 mm^2 (or 1.5 cm^2) **d** 51.84 mm^2
e 1250 mm^2 (or 12.5 cm^2)
f 16.8 cm^2 (or 1680 mm^2) **g** 361 cm^2
h 462 mm^2 (or 4.62 cm^2)
i 1715 cm^2 (or 0.1715 m^2)

4 8704 cm^2

5 They used the slant height (68.5 cm) as the height of each triangle section rather than the perpendicular height (68 cm).

6 **a** 14 cm^2 **b** 14 cm^2 **c** 900 mm^2
d 9 m^2 **e** 90 cm^2 **f** 15 cm^2

7 It does not matter which side is labelled as the length in a rectangle. Once one side is labelled as length, then the side perpendicular to it must be labelled the width.

8 **b** 28 cm^2

9 **a** 92 mm^2 **b** 35 cm^2 **c** 199 m^2 **d** 66 cm^2
e 96 cm^2 **f** 72 cm^2

10 red 1180.675 cm^2; black 136.56 cm^2; yellow 2452.39 cm^2; white 250.36 cm^2; green 4019.99 cm^2

11 **a** rectangle part: $15.3 \times 10.2 = 156.06$ cm^2; top triangle: $\frac{1}{2} \times 15.3 \times 6.6 = 50.49$ cm^2; left and right triangles: $2 \times \frac{1}{2} \times 10.2 \times 10.2 = 104.04$ cm^2; bottom triangle: $\frac{1}{2} \times 15.3 \times 6 = 45.9$ cm^2; total area = 156.06 + 50.49 + 104.04 + 45.9 = 356.49 cm^2
b Remember to ensure that the flaps will overlap when folded. A possible answer is given.
i Decreasing rectangle to 10.3 cm × 5.2 cm, left and right flaps to 5.2 cm, top flap to 3.3 cm and bottom to 3 cm, area is 113.045 cm^2.
ii Adding 0.5 cm to lengths of all given dimensions will result in area of 390.99 cm^2.
iii Increasing length to 25.3 cm and left and right flaps to 15.2 cm, area increases to 572.49 cm^2.

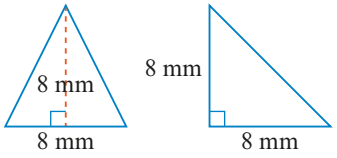
12 Some possible answers are: 34 cm × 2 cm; 17 cm × 4 cm; 13.6 cm × 5 cm.

13 Width is 5 cm. There is only one answer because only one number gives 20 when multiplied by 4.

14 **a** Some possible answers are:



b 8 mm **c** Some possible answers are:



d Because there are many places that perpendicular height can be placed within a triangle.

15 Some possible answers are: 7 m × 8 m (30 m perimeter), 14 m × 4 m (36 m perimeter) and 11.2 m × 5 m (32.4 m perimeter).

16 Some possible rectangles are: 12 cm × 12 cm (144 cm^2); 20 cm × 4 cm (80 cm^2); 16 cm × 8 cm (128 cm^2).

17 A square has equal side lengths. $8 \times 8 = 64$, so dimensions must be 8 mm × 8 mm. Perimeter is 32 mm.
18 a 40 cm b 1400 cm^2
19 a Some possible answers are: 8 m × 4 m, 7 m × 5 m, 6 m × 6 m, 2.4 m × 9.6 m.
b 36 m^2; maximum area corresponds to 6 m square
c Some possible answers are: 2 m × 20 m (20 m side on shed), 4 m × 16 m (16 m side on shed), 5 m × 14 m (14 m side on shed).
d 72 m^2 is the maximum area for a rectangle with dimensions 12 m × 6 m. It is twice the size of the square in part b.

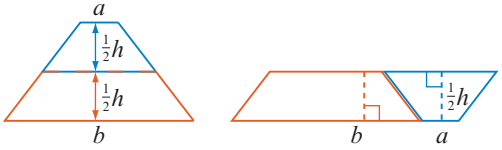
8D Area of other quadrilaterals

8D Start thinking!

1 a rhombus b kite c parallelogram
2 a b is the base and h is the height
b Cut along broken line to separate shaded section. Move shaded section to opposite side of remaining part; combined figure will now be a rectangle.
c Base of parallelogram (b) corresponds to length of rectangle (l) and height (h) of parallelogram corresponds to width of rectangle (w); $A = l \times w$ becomes $A = b \times h$.
3 c Diagonals match dimensions of rectangle: x = width of rectangle and y = length of rectangle.
d Two copies of the kite make up the rectangle.
e The area of this rectangle is $A = x \times y$. As each kite represents only half of the rectangle, $A = \frac{1}{2} \times x \times y$.

Exercise 8D Area of other quadrilaterals

1 a 24 cm^2 b 63 mm^2 c 168 cm^2
2 a 14 cm^2 b 48 mm^2 c 6 m^2
3 a 12 mm^2 b 20 cm^2 c 22.5 mm^2
4 a i rhombus ii $A = bh$ iii 42 cm^2
b i kite ii $A = \frac{1}{2}xy$ iii 3 cm^2
c i parallelogram
ii $A = bh$ iii 120 cm^2
d i kite ii $A = \frac{1}{2}xy$ iii 18 mm^2
e i parallelogram
ii $A = bh$ iii 112 mm^2
f i rhombus ii $A = bh$ iii 20 cm^2
5 a 10.5 cm^2 b 175 mm^2
c 99 cm^2 (or 9900 mm^2) d 90 cm^2
e 44 cm^2 f 56 mm^2
6 a 1.47 m^2 b 29.4 m^2
c Two cans of paint are needed so cost will be $17.98.
d $132.02
7 a 3087.5 cm^2 b 20 cm
8 d $A = bh$ e long diagonal (y)
f other diagonal (x)
g Base (b) is represented by diagonal y and height (h) is represented by half the other diagonal, $\frac{1}{2}x$. $A = b \times h$ becomes $A = y \times \frac{1}{2}x$, which is the same as $A = \frac{1}{2}xy$.
9 a parallelogram b 1806 m^2
c No, as windows on far left or far right sides are not similar to others away from side edges.
d Measure length and widths of rectangular ones and use formula $A = lw$. Use relevant formulas for individual ones that are not rectangular.
11 a Some possible answers are: b = 8 cm, h = 5 cm; b = 10 cm, h = 4 cm; b = 20 cm, h = 2 cm.
b With a rectangle, length and width are at right angles. In a parallelogram, once you have ensured the base length and perpendicular height are accurately done, the sloping sides must also be accurately drawn.
12 a Some possible answers are: b = 6 mm, h = 4 mm; b = 8 mm, h = 3 mm.
b For examples given, perimeters will be 24 mm and 32 mm, respectively. Perimeter is $4 \times b$.
c The areas of squares will always be a square number, as the base and height are always equal in length. In a rhombus that is not a square, the base and height are not the same length, so the result will not be a square number.
13 Some possible answers are: x = 18 m, y = 4 m; x = 9 m, y = 8 m.
15 c parallelogram d $A = bh$
e sum of the trapezium lengths ($a + b$) corresponds to base of parallelogram
f two
g Area of a parallelogram is found using $A = b \times h$. Using dimensions of trapezium, can write this as $A = (a + b) \times h$.
Because two trapeziums make up the area of the parallelogram, the formula for area of a single trapezium is half this:
$A = \frac{1}{2}(a + b) \times h$ or $A = \frac{1}{2}(a + b)h$.
h 19.5 square units
16 a 91 cm^2 b 124 mm^2 c 35 cm^2
17 $731.25
18



8E Area of a circle

8E Start thinking!

1 $C = 2\pi r \div 2$, $C = \pi r$
3 top half of circumference can be found using formula πr
4 10 7 parallelogram 8 $A = b \times h$ or $A = bh$
9 base ≈ 16 cm, height = 5 cm;
area = $16 \times 5 = 80$ cm^2

10 For a parallelogram, $A = bh$. For this parallelogram, $b = \pi r$ and $h = r$ so $A = \pi r \times r = \pi r^2$.

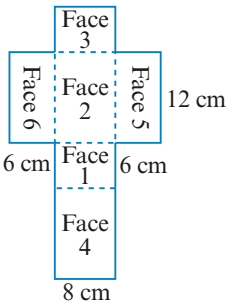
Exercise 8E Area of a circle

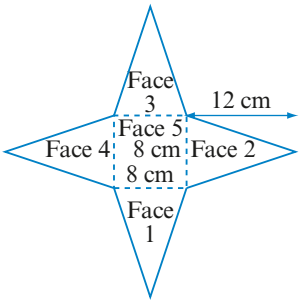
1 a 706.86 cm^2 b 254.47 mm^2 c 78.54 cm^2
2 a 38.48 cm^2 b 176.71 mm^2 c 33.18 m^2
3 a 15.9 cm^2 b 243.28 cm^2 c 153.94 cm^2
d 283.53 cm^2 e 1017.88 mm^2
f 490.87 cm^2
4 a 78.54 cm^2 b 44.18 m^2
c 176.71 mm^2 d 2642.08 mm^2
e 2.01 m^2 f 0.14 cm^2
5 a 87.96 cm^2 b 75.40 cm^2 c 138.23 cm^2
6 a 14.28 cm^2 b 47.52 cm^2 c 48.57 mm^2
d 66.19 cm^2 e 144.31 mm^2 f 81.74 cm^2
7 2375.83 cm^2 8 176.71 m^2
9 452.39 cm^2
10 a 314.16 cm^2 b 2 cm c 706.86 cm^2
d 392.7 cm^2
11 9483.68 mm^2 (or 94.84 cm^2)
12 285.88 mm^2
14 a Divide both sides of the formula $A = \pi r^2$ by π and then take the square root of both sides.
$A = \pi r^2, \frac{A}{\pi} = r^2, \sqrt{\frac{A}{\pi}} = r$
b In finding r^2, you divide A by π, so if the value for A contains π, you can cancel π in the fraction. This makes finding r by taking the square root of (say) 16 easier.
i $\sqrt{\frac{16}{\pi}} \approx 2.26$ ii $\sqrt{16} = 4$
15 a 3 cm b 5 cm c 5.64 cm d 4.55 cm
16 a i 14 cm ii 14π cm
b i 4 m ii 16π m^2
17 There are 28 tiles (assuming dominoes game from 0:0 to 6:6). Total number of dots representing all the numbers is 168. Assuming each dot has diameter 4 mm (r = 2 mm), 168 dots have an area of $168 \times \pi \times 2^2 = 2111.15$ mm^2. Top surface of 28 tiles has an area of 27 104 mm^2.
Required percentage = $\frac{2111.15}{27\,104} \times 100\% \approx 7.8\%$.
18 a 60° b 360° c $\frac{60}{360}$(or $\frac{1}{6}$)
d 4 cm e 50.27 cm^2 f 8.38 cm^2
g A sector is a fraction of a whole circle. So a sector's area is also a fraction of a whole circle's area (with the same radius).
19 a 19.24 cm^2 b 126.71 cm^2 c 230.91 mm^2
20 Each composite shape includes a large sector of a circle: either a half circle (sector size 180°) or quarter circle (sector size 90°). You could calculate the area of the composite shapes using these numbers rather than just finding half or quarter of a circle.

8F Surface area

8F Start thinking!

1 Figure B is the net of figure A.
2





3 The nets all contain shapes that match the faces of the 3D object that they form. The nets contain some matching shapes whereas others are different.
4 a rectangle b triangle c triangle

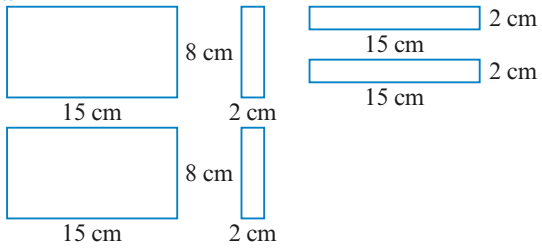
7

	Face 2	Face 3	Face 4	Face 5	Face 6
Rectangular prism	8 cm, 12 cm	6 cm, 8 cm	8 cm, 12 cm	6 cm, 12 cm	6 cm, 12 cm
Triangular prism	6 cm, 10 cm, 8 cm	8 cm, 12 cm	12 cm, 6 cm	12 cm, 10 cm	
Square-based pyramid	12 cm, 8 cm	12 cm, 8 cm	12 cm, 8 cm	8 cm, 8 cm	

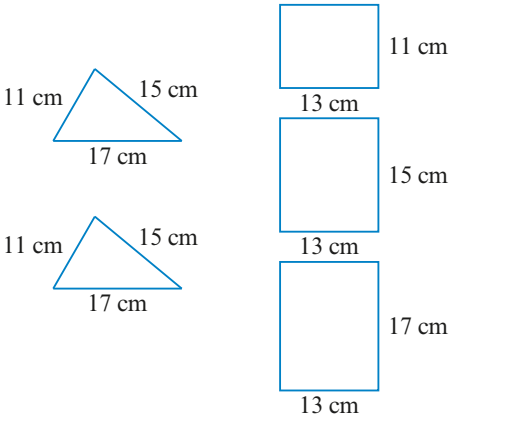
8 Work out the area of each face individually, then add them all up to find total surface area.

Exercise 8F Surface area

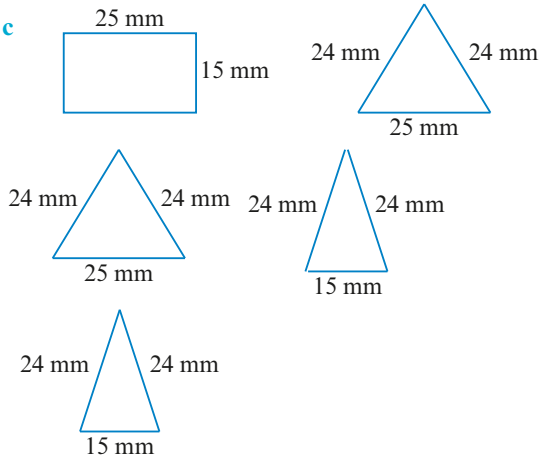
1 a



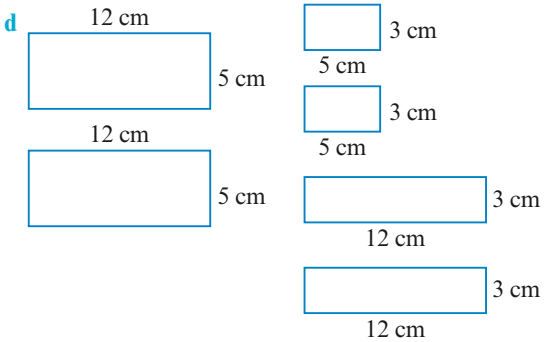
b



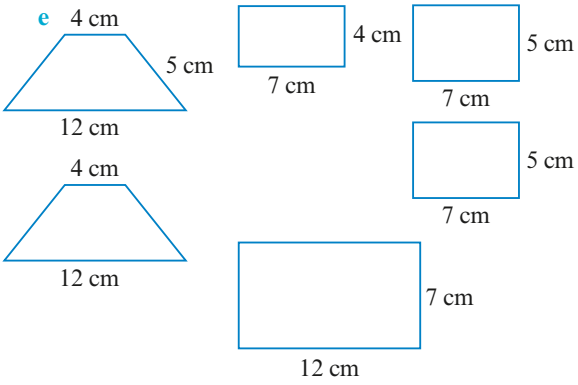
c



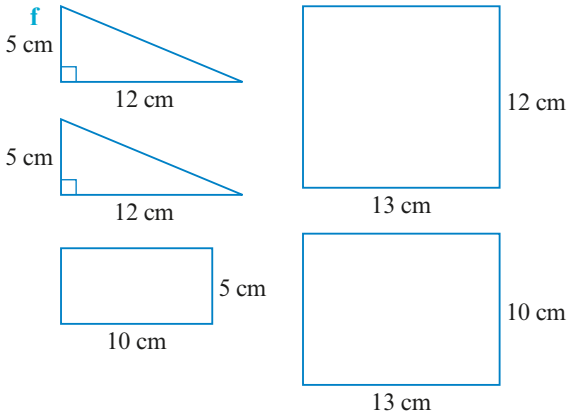
d



e



f



2 **a** 76 cm^2 **b** 72 cm^2 **c** 1120 mm^2
d 160 cm^2 **e** 118 cm^2 **f** 224 cm^2

3 **a** 96 cm^2 **b** 505 cm^2 **c** 94 cm^2
d 1008 cm^2 **e** 372 cm^2 **f** 196 cm^2

4 **a** 150 cm^2 **b** TSA = 6 × area of one face

5 **a** 96 cm^2 **b** 216 m^2 **c** 864 mm^2
d 2400 cm^2

6 **a** 177 cm^2 **b** 235 cm^2 **c** 80 mm^2

7 8.295 m^2 (no material is needed on the base)

8 **a** ten
b Both sides of each face need to be covered.
c need equal amounts of blue and white contact; 5475 cm^2
d 10 950 cm^2

9 **a** 332 cm^2 **b** 262 cm^2 **c** 222 cm^2

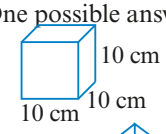
10 **a** 166 cm^2 **b** 131 cm^2 **c** 111 cm^2

11 314 m^2 × \$5 per m^2 = \$1570

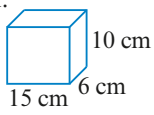
12 16.5 m^2 **13** 417.4 cm^2

14 One possible answer is given.

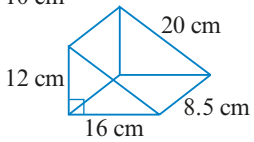
a



b



c



15 Each of the six faces of a cube has the same area, so dividing 600 cm^2 by 6 gives an area of 100 cm^2. Each square face has a side length of 10 cm.

17 **a** 5300 cm^2 (blue: 3750 cm^2, red: 1550 cm^2)
b Some overlap of wrapping paper is needed.

18 approx. 2534 cm^2

19 Find area of one triangle face ($\frac{1}{2}$ × base × height) and multiply it by four.
20 Some possible answers are: volume may be calculated instead; some faces of an object may be forgotten.
21 a 150 cm^2 b 250 cm^2
c i 350 cm^2 ii 450 cm^2 iii 550 cm^2
d Adding a cube to the stack adds 100 cm^2 to the TSA. 17 cubes would have a TSA of 150 cm^2 + 16 × 100 cm^2 = 1750 cm^2.
e A different sized cube would have different results because of its different dimensions. For every cube that is added to a stack, the equivalent of four faces is added to the area (because the top and bottom are already there). By first finding the area of one face, the TSA of any cube stack can be found by multiplying the area of four of these faces by the number of cubes in the stack, and then adding the area of two more faces (top and bottom).
22 Find area of two circular ends and add to area of rectangular face that is wrapped around circles.

8G Volume of prisms

8G Start thinking!

1 rectangle 3 4 cm
4 No, as long as the base and height of a prism are perpendicular to each other.
5 $V = A \times H$ or $V = AH$
6 $A = 3 \times 2 = 6$ cm^2 and $H = 4$ cm
so $V = 6 \times 4 = 24$ cm^3
7 no; $A = 4 \times 2 = 8$ cm^2 and $H = 3$ cm
so $V = 8 \times 3 = 24$ cm^3
8 triangular prism
9 $A = \frac{1}{2}bh = \frac{1}{2} \times 3 \times 2 = 3$ cm^2
10 4 cm 11 $V = 3 \times 4 = 12$ cm^3
12 No, as there is not a dimension given that is at right angles to another base.

Exercise 8G Volume of prisms

1 a 90 cm^3 b 90 cm^3 c 324 mm^3
d 432 cm^3 e 945 cm^3 f 66 mm^3
2 a 346.5 cm^3 b 60 cm^3 c 375 cm^3
d 3828 cm^3 e 4.0375 m^3 f 1512 cm^3
3 a 67.5 cm^3 b 189 mm^3 c 154 m^3
d 33 mm^3 e 480 cm^3 f 1449 mm^3
4 a 60.5 cm^3 b 2673 cm^3 c 100 cm^3
5 a 72 cm^3 b 40 cm^3 c 72 cm^3
d 211.5 cm^3 e 112 mm^3 f 15.54 m^3
6 a 320 cm^3 b 3500 cm^3 c 904.78 cm^3
7 a 2944.62 cm^3 b 123
8 Volume of any prism can be found using formula $V = AH$, that is, multiply area of base by height of prism. It does not matter what shape the base of the prism is.
9 5.2 m^3
10 a 126 m^3 b $3780
11 a $24 b $37.58 c $28.50
12 Sculpture a weighs 36 kg; sculpture b weighs 37.584 kg and sculpture c weighs 35.625 kg. She should buy sculpture c.
13 4 cm
14 a 20 cm^2 b 4 cm
c Some possible answers are: $b = 8$ cm and $h = 5$ cm, $b = 10$ cm and $h = 4$ cm, $b = 20$ cm and $h = 2$ cm.
16 Some possible answers are: $l = 10$ cm, $w = 5$ cm and $H = 10$ cm, $l = 8$ cm, $w = 5$ cm and $H = 12.5$ cm, $l = 20$ cm, $w = 4$ cm and $H = 6.25$ cm.
17 Some possible answers are: $b = 10$ cm, $h = 4$ cm and $H = 25$ cm; $b = 8$ cm, $h = 8$ cm and $H = 15.625$ cm; $b = 25$ cm, $h = 20$ cm and $H = 2$ cm.
18 Some possible answers are: all dimensions may be added together when it is not a rectangular prism; all dimensions may be added together.
19 small block: 10 500 cm^3, large block: 14 250 cm^3
20 Julia forgot to subtract volume of holes in blocks. She will obtain correct answer by subtracting volumes 10 × 15 × 30 cm^3, 25 × 10 × 30 cm^3 and 25 × 10 × 30 cm^3 from the volume that she found.

8H Area and volume conversions

8H Start thinking!

1 a 1 cm × 1 cm b 1 cm^2
2 a 10 mm × 10 mm b 100 mm^2
c It is the same size (just different units used).
d Area of square 10 mm × 10 mm = 100 mm^2. Area of square 1 cm × 1 cm = 1 cm^2. 10 mm is equivalent to 1 cm, so 1 cm^2 must be equivalent to 100 mm^2.
3 b Area of square 100 cm × 100 cm = 10 000 cm^2. Area of square 1 m × 1 m = 1 m^2. 100 cm is equivalent to 1 m, so 1 m^2 must be equivalent to 10 000 cm^2.
4 a 1 cm^3
b cube with dimensions 10 mm × 10 mm × 10 mm
c Volume of cube 10 mm × 10 mm × 10 mm = 1000 mm^3. Volume of cube 1 cm × 1 cm × 1 cm = 1 cm^3. 10 mm is equivalent to 1 cm, so 1 cm^3 must be equivalent to 1000 mm^3.

Exercise 8H Area and volume conversions

1 a 100, 1000
b 10 000; $100^3 = 100 \times 100 \times 100 = 1\,000\,000$
c $1000^2 = 1\,000\,000$;
$1000^3 = 1000 \times 1000 \times 1000 = 1\,000\,000\,000$
2 b 10 000, 40 000 c 100, 9
d 5000 e 1 000 000, 7 000 000
f 1000, 2
3 a 30 000 cm^2 b 1000 mm^2 c 65 000 cm^2
d 1970 mm^2 e 1 600 000 m^2 f 750 000 m^2

4 **a** 2.5 m^2 **b** 7500 cm^2 **c** 4 cm^2
d 0.125 km^2 **e** 0.0675 m^2 **f** 500 km^2

5 **a** 9000 mm^3 **b** 18 000 000 cm^3
c 4 200 000 000 m^3 **d** 4 560 000 cm^3
e 150 000 000 m^3 **f** 6100 mm^3

6 **a** 16 cm^3 **b** 0.00175 km^3
c 4 m^3 **d** 0.845 m^3
e 0.028 cm^3 f 1700 km^3

7 **a** 5600 mm^3 **b** 28 000 cm^2
c 3.752 km^2 **d** 913.458 cm^3
e 0.0186 m^2 **f** 75 000 000 m^3
g 45 600 mm^2 **h** 0.000 000 075 km^3
i 0.000 001 m^3 **j** 1 320 000 m^2
k 555 000 cm^3 **l** 0.16 cm^2

8 11 cm^2 = (11 × 10^2) mm^2 = 1100 mm^2; Ahmed forgot to square the conversion factor.

9 **a** One out of four was correct.
c correct working shown below
i 40 000 cm^2 = (40 000 ÷ 100^2) m^2
(did not divide by 100^2) = 4 m^2
ii 200 cm^2 = (200 × 10^2) mm^2
= 20 000 mm^2 (correct)
iii 7 m^3 = (7 × 100^3) cm^3 = 7 000 000 cm^3
(incorrect multiplication)
iv 3000 mm^3 = (3000 ÷ 10^3) cm^3
(incorrect conversion factor)
= 3 cm^3 (incorrect answer)

10 **a** 27 450 cm^2
b 27 450 cm^2 = 2.745 m^2; total area to paint per box (two coats) = 5.49 m^2
2.7 boxes can be painted from one paint tin (only two whole boxes)

11 **a** 283 500 cm^3 **b** 0.2835 m^3
c l = 1.05 m, w = 0.6 m and H = 0.45 m
$V = l \times w \times H$ = 1.05 × 0.6 × 0.45 = 0.2835 m^3

12 **a** **i** 3200 kg/hectare **ii** 0.32 kg/m^2
iii 320 g/m^2 **iv** 0.032 g/cm^2
b Grams per square metre as they are units people are more familiar with.
c $3 360 000

13 **a** 100 **b** 4000 m^2
c 1 acre = 0.404 69 hectares,
1 hectare = 2.471 05 acres
d Some possible answers are:
1 square yard = 0.836 13 m^2,
1 m^2 = 1.195 99 square yards;
1 square mile = 2.589 99 km^2,
1 km^2 = 0.3861 square miles

14 **a** 1000
b **i** 7500 mL **ii** 0.45 L **iii** 0.9 ML
iv 3750 L

15 **a** **i** 15 cm^3 **ii** 1000 cm^3 **iii** 5000 cm^3
iv 1 000 000 cm^3
b **i** 4 m^3 **ii** 13 m^3 **iii** 0.2 m^3
iv 0.745 m^3
c **i** 26 mL **ii** 5 L **iii** 9 kL **iv** 400 L

16 **a** 82.5 cm^3 **b** 0.0825 L

17 202.5 L **18** 1.2 L

19 One possible answer is 4.8 cm × 10 cm.

20 One possible answer is given.
a 100 mm × 400 mm × 900 mm
b 10 cm × 40 cm × 90 cm
c 0.1 m × 0.4 m × 0.9 m

22 **a** **i** 1 812 175 000 kL **ii** 1 812 175 000 000 L
iii 1 812 175 000 m^3 **iv** 1.812 175 km^3
b Litres is most commonly used unit.
c 851 722 250 kL
d **i** 109 500 L **ii** 109.5 kL
e 438 000 000 kL **f** 2 years
g parks and garden watering, sports ground and golf-course watering, business use, etc.

8 Chapter review

MULTIPLE-CHOICE

1 B **2** D **3** D **4** B **5** D
6 C **7** A **8** C **9** D

SHORT ANSWER

1 **a** 22.6 cm (or 226 mm) **b** 58 mm
2 **a** 31.42 cm **b** 50.27 mm
3 **a** 48 cm^2 **b** 58.5 cm^2 **c** 120 cm^2
d 5.4 cm^2
4 **a** 42 cm^2 **b** 88 cm^2 **c** 80 cm^2 **d** 125 cm^2
5 **a** 78.54 cm^2 **b** 201.06 mm^2
6 **a** 94 cm^2 **b** 168 cm^2 **c** 260 mm^2
d 57 mm^2
7 **a** 640 cm^3 **b** 420 cm^3 **c** 144 cm^3
d 99 cm^3
8 6656.25 cm^3
9 **a** 50 000 cm^2 **b** 0.5 cm^2 **c** 3500 mm^3
d 0.0004 m^3 **e** 7500 mL **f** 845 000 L

NAPLAN-STYLE PRACTICE

1 35 **2** 700 m **3** 0.7 m
4 14.14 cm **5** 20.69 cm **6** 125.66 cm
7 3.87 cm **8** 200 m^2 **9** 136 m^2
10 525 cm^2 **11** 10 cm **12** 1500 cm^2
13 14.16 cm^2 **14** 490.87 cm^2 **15** 202.63 cm^2
16 16 cm **17** 686 cm^2 **18** 949 cm^2
19 336 cm^3 **20** 500 cm^3 **21** 500 mL
22 2000 **23** 35 cm^2

ANALYSIS

a 6 m^2 **b** 10 m **c** 20 m
d 40 m^2 for two coats **e** 825 cm^2
f 20.73 cm **g** 34.21 cm^2 × 6 = 205.27 cm^2
h 6 m^3
i 6 m^3 = 6 000 000 cm^3; 6 000 000 ÷ 150 = 40 000
j Balls will not take up entire space as there will be space between them. The theoretical calculation does not take into account this space.

8 Connect

For feedback on this open-ended task, see your teacher.

CHAPTER 9 STATISTICS

9 Are you ready?

1 B **2** D **3** B **4** 5 **5** A
6 9 m **7** 8 years **8** 42 **9** B
10 45 **11** C **12** 10 **13** 1 **14** B

9A Sampling data

9A Start thinking!

1 You should be able to get an idea of which store is the favourite by asking a smaller number of people.
2 Advantage: you get the opinion of every Year 8 student; disadvantage: extremely time-consuming.
3 This sampling is not fair. People being asked at Wendy's are more likely to say Wendy's is their favourite.
4 people who favour different ice-cream stores
5 The sample only targeted students at one particular ice-cream store (Wendy's). It would be best to ask away from an ice-cream store.
6 The method is still targeting students at only one particular ice-cream store. Ask people away from the ice-cream store.

Exercise 9A Sampling data

1 **a** census **b** sample **c** sample **d** census **e** sample **f** census
2 **a** systematic **b** stratified **c** systematic **d** random **e** random **f** stratified
3 **i** parts a, b, d and f
ii **a** What is your favourite pet?
b From this selection of films, which do you recommend?
d What is your favourite hobby?
f Who do you plan to vote for?
4 **a** **i** sample
ii Biased, as the question does not allow all possible answers. The question could be 'What is your favourite winter sport?'.
b **i** census
ii Fair, as every student has the opportunity to give their opinion.
c **i** sample
ii Biased, as only getting opinions from one store, not across the country. Possible that people at the store share an opinion that is different from a fair sample across the country.
d **i** sample
ii Biased, as only Woolworths customers are targeted. Move to a position away from supermarkets.
e **i** census
ii Biased, as the question does not allow all possible answers. Question could be 'What would you like to do for the Christmas party?'.
f **i** sample
ii Biased, as her friends may not be a good cross-section of members across the school and there are more computer types than just PCs and Macs. Conduct a stratified sample across all year levels (with boys and girls in proportion), and ask question such as 'What is your favourite computer type?'.
5 **a** census **b** sample **c** census **d** sample
6 **a** all students in Year 8 in that town
b the country
c everybody in Western Australia
d the town
e the local area
f all students at school
7 **a** the people in the town
b People at the local mall may have different views on graffiti from wider population, or Lizzie may choose to ask a certain type of person.
c systematic sampling
d Possible that people walking past post office may also have different views on graffiti.
e Some possible answers are: surveying every 100th household according to town records; selecting every 10th person who attends town gathering.
8 Some possible answers are given.
a What is your favourite soft drink? Which soft drink flavour do you recommend?
b How many pets do you have at home? Give the type and total number of pets at home.
c How many hours do you spend using the Internet each week? How much is your weekly download?
d Are you in favour of the new school rule? What are your thoughts about the new school rule?
9 Some possible answers are: males and females; age groups (e.g. 20s, 30s, 40s, 50s); professionals and trades.
10 Some possible answers are: two males and two females; one from each of the age groups (20s, 30s, 40s, 50s); two professionals and two trades.
11 Some possible answers are given.
a Each students is numbered from 1 to the end. Numbers are then randomly selected from a hat.
b Each students is numbered from 1 to the end. Depending on the numbers required for the sample, every 20th (for example) student is selected from the list.
c A sample is selected with fair representation of girls and boys across each year level.
12 **a** **i** Year 8 students at your school
ii One possible answer is to select every fifth student from the Year 8 list of students.
b **i** the people in the town

ii One possible answer is to select every 10th (or agreed number) household from town records.

c i the Australian population

ii One possible answer is to select a sample of families across Australia in proportion to population in each state (targeting families in country as well as cities).

14 Those with a certain opinion of the local issue may be more likely to complete the survey.

9B Collecting data

9B Start thinking!

1

Height (cm)	Frequency	Height (cm)	Frequency
137	1	155	1
138	1	156	0
139	0	157	2
140	0	158	2
141	0	159	0
142	2	160	0
143	0	161	1
144	0	162	0
145	3	163	1
146	1	164	1
147	0	165	0
148	0	166	0
149	0	167	0
150	0	168	0
151	0	169	0
152	0	170	0
153	2	171	1
154	1		

2 The table becomes too large.

3

Height (cm)	Frequency	Height (cm)	Frequency
135–139	2	155–159	5
140–144	2	160–164	3
145–149	4	165–169	0
150–154	3	170–174	1

4 One advantage is shortening the table. One disadvantage is that it doesn't display individual heights. For example, you can see there is one student in the height range 170–174 cm, but you can't tell this student's actual height.

5 The table includes whole number values (discrete values). It appears there is no place for 154.5 cm.

6

Height (cm)	Frequency	Height (cm)	Frequency
135–<140	2	155–<160	5
140–<145	2	160–<165	3
145–<150	4	165–<170	0
150–<155	4	170–<175	1

They allow for fraction and decimal values close to the end of an interval.

Exercise 9B Collecting data

1 a discrete b nominal c continuous
d continuous e nominal f continuous
g ordinal

2 the continuous data statements c, d and f

3 a 10 b 5 c 0.5 d 10

4 a numerical, discrete data

b

Number of text messages sent	Frequency
0–9	2
10–19	5
20–29	5
30–39	4
40–49	3
50–59	3
60–69	0
70–79	1
80–89	1

5 a numerical, discrete data

b 5 (0–4, 5–9, etc.)

c

Class interval	Frequency
0–4	5
5–9	3
10–14	8
15–19	5
20–24	3

6 a

Score	Frequency
0–9	3
10–19	4
20–29	3
30–39	4
40–49	2
50–59	
60–69	1
70–79	
80–89	1

b

Score	Frequency
0–4	5
5–9	4
10–14	3
15–19	2
20–24	2

c

Score	Frequency
0.0–<0.5	1
0.5–<1.0	2
1.0–<1.5	4
1.5–<2.0	2
2.0–<2.5	3
2.5–<3.0	4

d

Score	Frequency
100–109	2
110–119	3
120–129	2
130–139	3
140–149	2
150–159	1
160–169	1

7 a

Favourite Australian animal	Frequency
Sugar glider	4
Echidna	2
Kangaroo	3
Koala	6
Red-eyed tree frog	1
Wombat	4
Dingo	2
Bilby	3
Platypus	2

b koala, 6 c categorical, nominal data

8 a numerical, discrete data

b

Number of times Facebook checked	Frequency
0–4	7
5–9	7
10–14	3
15–19	3
20–24	3
25–29	1

9 a

Number of TVs	Frequency
0	2
1	10
2	11
3	6
4	4
5	3

b 36 **c** 2 **d** 81

10 a favourite sport

b categorical, nominal data

c cricket **d** badminton **e** 63

f

Sport	Frequency	Sport	Frequency
Football	8	Soccer	8
Basketball	10	Rugby	8
Netball	8	Tennis	6
Cricket	12	Badminton	3

11 a categorical, nominal data **b** 5

c pizza, as it is the most popular item

12 a Some possible answers are: the number of items in a pencil case, the absences in a given year level on a series of days.

b

Your title	Frequency	Your title	Frequency
0–4	1	15–19	5
5–9	4	20–24	6
10–14	6	25–29	2

13 a numerical, continuous data

b

Weight of school bag (kg)	Frequency
0–<2	3
2–<4	7
4–<6	7
6–<8	2
8–<10	3
10–<12	2

c A possible answer is that the weights are spread in size from 1.8 kg to 10.7 kg; a wide spread across the same year level.

14 a The class intervals are too large; possible to have five intervals: 0–3, 4–7, 8–11, 12–15, 16–19.

b Each second category has a repeated value; for example, 5 hours appears in the top two intervals.

Number of hours spent exercising per week	Frequency
0–4	9
5–9	6
10–14	3
15–19	1

c Too many intervals have been used. Make the interval sizes larger.

Pocket money received each week	Frequency
0–<4	3
4–<8	5
8–<12	6
12–<16	5
16–<20	4
20–<24	2

d Discrete values have been used in the intervals. Change to continuous intervals. (Note that this type of table is satisfactory if heights have been rounded to the nearest cm).

Height of Year 8 students (cm)	Frequency
140–<150	2
150–<160	4
160–<170	7
170–<180	2
180–<190	1

15 a categorical, ordinal data

b Numerical values are given for place values (and numerical values are often included in different options provided with the ladder; for example, percentage, points for, points against).

c A possible answer is to remind them that ladders represent an order; that is, ordinal data.

17 Possible answers include shoe size and floor numbers.

9C Presenting data in graphs

9C Start thinking!

1 Types of graphs include column, bar, line, picture, pie (sector), scatterplot, dot plot, stem-and-leaf plot, histogram, box plot.

2 a Some possible answers are: column graphs require two scaled axes and columns to display the categories, whereas pie graphs use sectors to display categories.

b column graph; the frequencies relate height of columns

c Frequencies are not usually shown on a pie graph (only rarely are they written with each sector) so a pie graph is less informative than a column graph.

d You have a better chance of determining percentage amounts on a pie graph. For example, 100% is the whole pie graph, 50% is half of the pie graph, etc. It is easier to visualise percentage amounts on a pie graph compared to frequency values.

3 a A column graph has vertical columns for categories whereas a bar graph has horizontal bars.

b When the category labels are long, it is better to represent the data as a bar graph.

4 A line graph is used to represent the change of a single object or event over time. A scatterplot is used to represent a comparison between two variables.

Exercise 9C Presenting data in graphs

1 a comedy, 8 **b** sci-fi, 2

c 6 **d** 27

2 a pie graph **b** types of music lessons

c guitar

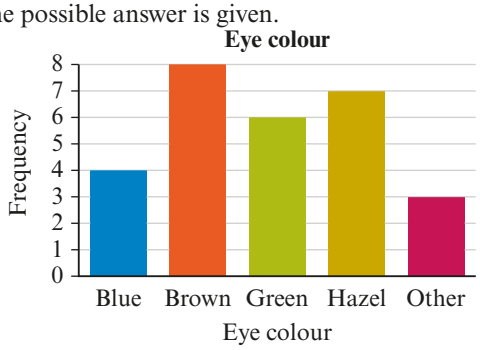
d No. The sector sizes help to determine the largest through to smallest, but as the actual frequency values aren't stated, it is not possible to determine them from this graph.

3 a temperature over a 24-hour period, measured every 2 hours

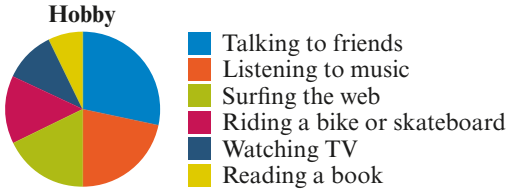
b 11°C **c** 2 pm **d** 5°C, 6 am

4 One possible answer is given.

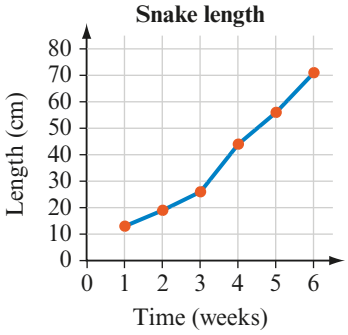
a



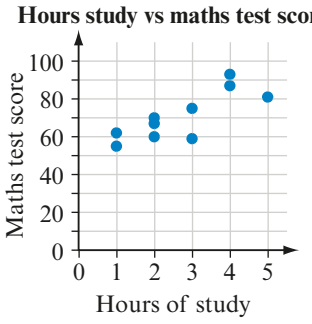
b



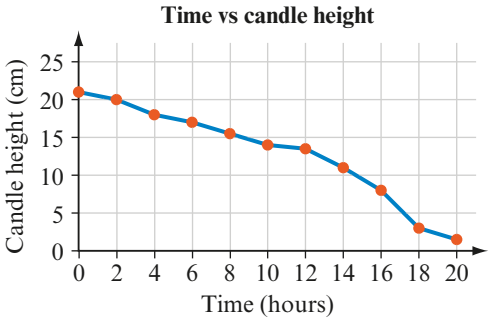
c



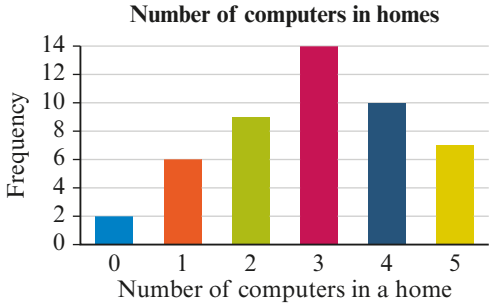
d



e



f

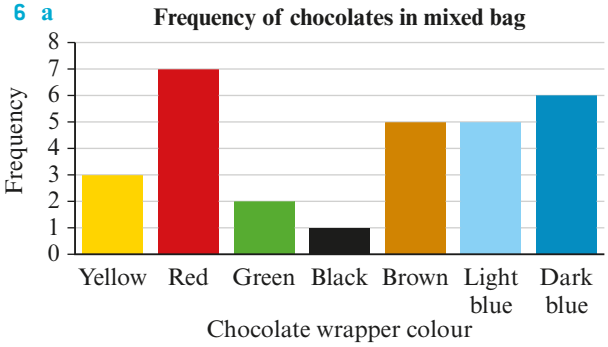


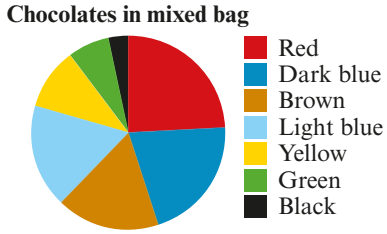
5 a Line graph, because it shows one variable changing over time; scatterplots show the relationship between two variables.



b Negative. As time progresses, the number of visitors is decreasing.

6 a





b red chocolates; 7

c both show the most/least common chocolates

d pie graph does not show actual frequencies

e column graph; it shows more information

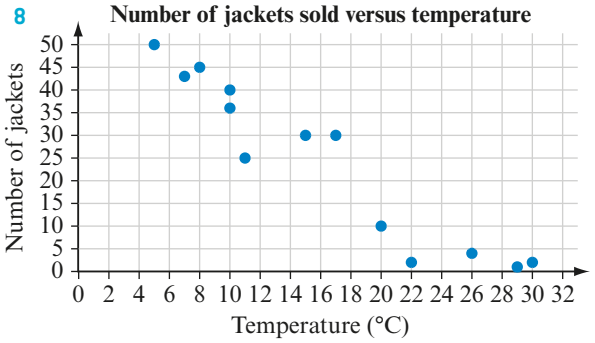
7 a height of people (in metres) and their age (in years)

b positive **c** about 1.6 m **d** 16 years

e about 1.65 m to 1.7 m

f, g Using the scale, you could predict the height to follow a pattern and reach over 2.5 m tall. This is not accurate as people stop growing in their late teenage years or early twenties.

8



9 Table A: D, Table B: C, Table C: B, Table D: A, Table E: E

10 **a** No labels on axes and no graph title; although not essential, scale on vertical axis could have more values.

b No graph title and no legend to explain different sectors.

c No labels on axes and no graph title.

d Scale on vertical axis does not increase in equal intervals; no labels on axes and no graph title.

11 **a** graph C

b A symmetrical graph has a higher proportion of data in the middle of the graph (like graph B). It appears to have matching left and right sides.

c A graph is skewed if it has a higher proportion of data on the right (like graph A) or the left (like graph C).

d Most data could be symmetrically arranged (as in graph B) and there could still be a data piece well to the right or left.

12 The column graph compares six different categories for three years, all displayed on the same graph. All categories except E show an increase over time.

9D Stem-and-leaf plots and dot plots

9D Start thinking!

1 Figure B; labels 'stem' and 'leaf' on graph indicate this.

6 Stem-and-leaf plots; stems act in a similar way to class intervals in frequency tables.

7 Every value from smallest to largest has to be shown on the scale; harder to get a quick 'snapshot' of the data distribution when range is larger.

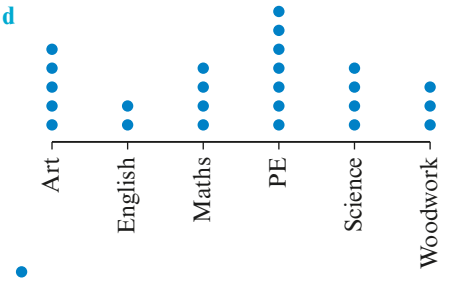
8 Some possible answers are given.

	Stem-and-leaf plot	Dot plot
Advantages	Every data piece represented. Distribution of data is easily seen.	Every data piece represented. Distribution of data is easily seen (unless large range of data).
Disadvantages	Can be slow to construct as each piece of data needs to be entered first and then plot must be redrawn to order data. Can be misinterpreted if no key is provided.	Care must be taken when counting many dots of same value. Not as useful when range of data is large.

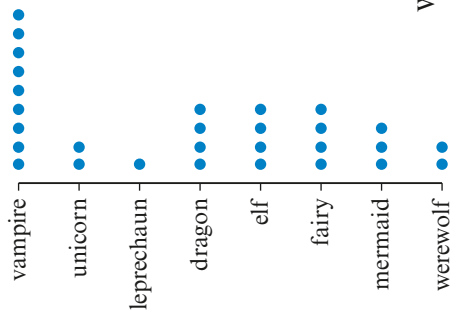
Exercise 9D Stem-and-leaf plots and dot plots

1 **a** PE, 7 **b** English, 2 **c** 23

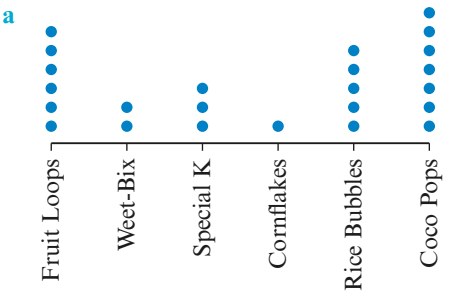
d



2



3 **a**



b results of a survey on favourite breakfast cereals

c Coco Pops **d** 6 **e** 6 **f** 24

4 **a** 11 **b** 61 **c** 3 (30s) **d** 49

e Must include all stem values from smallest to largest.

f 18

5 Key: 1 | 1 = 11

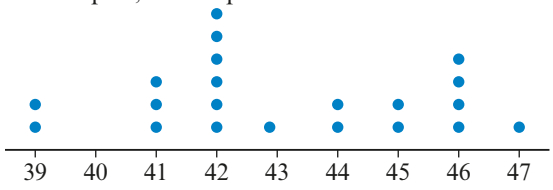
Stem	Leaf
1	1 1 3 7 7 9
2	2 3 5 7 8 9 9
3	3 4 5 6 7 8
4	1 2 6 6 8
5	1 4
6	
7	1

6 a Key: 1 | 1 = 11

Stem	Leaf
0	2 5
1	
2	
3	3 8 9 9 9 9
4	0 2 2 4 6 7 8 9
5	0 5 5 7 9
6	2 3 7
7	1 3 4 5

b 39 **c** 69 **d** 28

7 a Dot plot, as the spread of values is small.



b Stem-and-leaf plot, as the spread of values is large.

Key: 1 | 1 = 11

Stem	Leaf
1	1 2 3 4 5 6 8 8 8 9
2	2 6 7 9
3	0 6 8 8
4	0 1 5
5	
6	6

c Stem-and-leaf plot, as the spread of values is large.

Key: 5 | 3 = 53

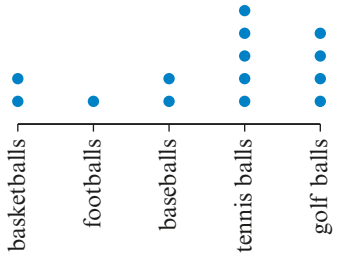
Stem	Leaf
5	3
6	2
7	2 4 4 4 5 5 5 6 6 7 7 7 9
8	0 1 2

d Stem-and-leaf plot as the spread of values is large

Key: 3 | 0 = 30

Stem	Leaf
0	6
1	
2	
3	0 2 2 3 9 9
4	0 1 1 2 2 3 6 6 6 7 9
5	0 3 5 5 5 5 6 8

8 a



b tennis balls **c** football **d** 14

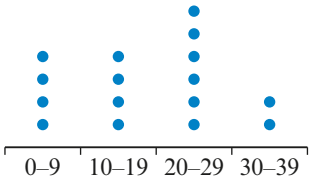
9 Include all stem values from minimum data value (14) to maximum data value (96); that is, 1, 2, 3, 4, 5, 6, 7, 8 and 9. Order leaf values for each stem value.

10 a

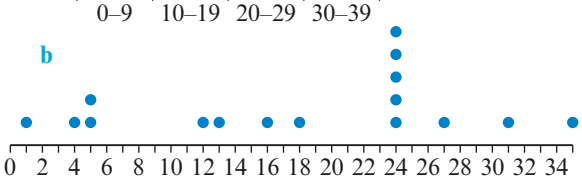
Class interval	Frequency
110–119	4
120–129	7
130–139	7
140–149	4
150–159	1

b Stem-and-leaf plot, as you can see what all the individual data values are.

11 a



b



c Stem-and-leaf plot, as you see the individual data pieces at a quick glance. The dot plot that displays the individual data is too long and in the dot plot that uses each stem as the category, you can't see what number each dot represents.

12 a Two, with both data sets sharing the same stem values.

b Read from right to left instead of left to right.

c 62 **d** 78 at Jay Jay's **e** Jay Jay's, 10

13 a Key: 1 | 7 = 17

Leaf Boys	Stem	Leaf Girls
6 0	0	3 5 7
7 5 5 4 1	1	1 4 7 8
9 0	2	2 2
1	3	0 5
1	4	

14 a Stems represent intervals of 5 rather than 10.

b The numbers 10, 11, 12, 13 and 14 have been placed with stem 1 and the numbers 15, 16, 17, 18 and 19 have been placed with stem 1*. In general, numbers ending in 0, 1, 2, 3 and 4 go with the first listing of the stem and numbers ending in 5, 6, 7, 8 and 9 go with the stem labelled with an asterisk (*).

c 28 **d** 10 **e** 1*

f Key: 1 | 0 = 10

Stem	Leaf
1	0 1 1 4
1*	5 5 7 8 9 9
2	0 0 3 3 4 4
2*	5 6 8

15 Key: 1 | 0 = 10

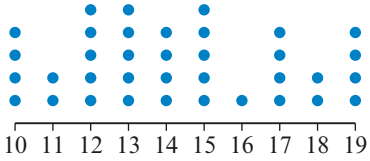
Stem	Leaf
3	0 0 1 2 3 3 3
3*	6 6 7 7 8 8 9 9 9 9
4	0 0 0 0 1 2 2 2 2 2 3 4 4
4*	5 6 8

Most of the data appears to be in the range 36 to 44.

16 a Key: 1 | 0 = 10

Stem	Leaf
1	0 0 0 0 1 1
1*	2 2 2 2 2 3 3 3 3 3
1**	4 4 4 4 5 5 5 5 5
1***	6 7 7 7 7
1****	8 8 9 9 9 9

b



As all the data is in the range 10 to 19, the dot plot gives a better representation.

17 Data sets with large spread of values should always be represented by stem-and-leaf plot rather than dot plot. If spread of values is small but there are a lot of values, split stem-and-leaf plot is better than dot plot. Dot plot should only be used for small data sets that cover a small range of values.

18 a i 11 **ii** 61 **b i** 1.1 **ii** 6.1
c i 1100 **ii** 6100 **d i** 110 **ii** 610

19

Key: 1 | 3 = 13

Leaf Band A	Stem	Leaf Band B
	1*	
33333333222	1**	3
555544444	1***	55555
776	1****	6666666666677777
	1*****	8

9E Presenting grouped data

9E Start thinking!

1 Figure B has no gaps between columns.
2 Figure B
3 The score values are given as a class interval, not an individual value.
4 10

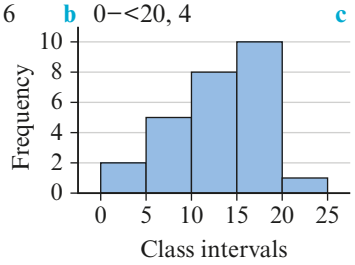
Exercise 9E Presenting grouped data

1 a 30, 50, 60, 80, 90
b 60, 80, 120, 140, 160, 180
c 25, 35, 40, 45, 55, 60

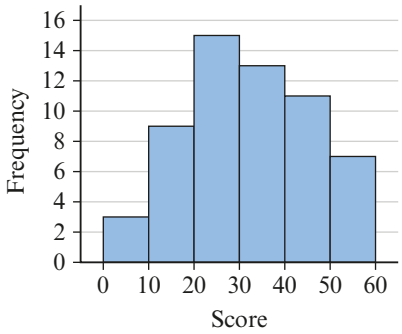
2 a 5 **b** 16 **c** 15−<20, 18 **d** 25−<30

3 a 6 **b** 0−<20, 4 **c** 70

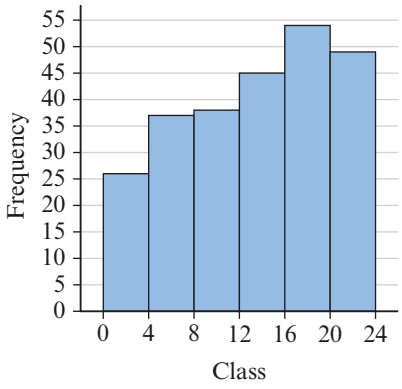
4 a



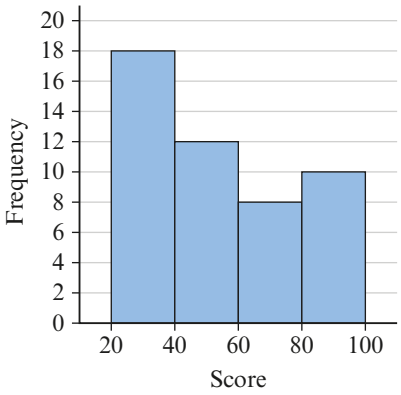
b



c



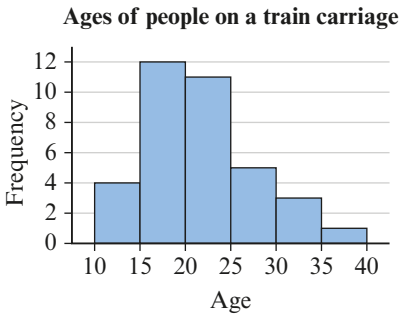
d



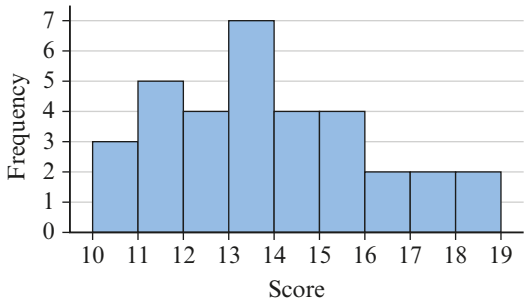
5 a

Ages of people on a train	Frequency
10–<15	4
15–<20	12
20–<25	11
25–<30	5
30–<35	3
35–<40	1

b



6



7 a ages of people at a cinema

b 20–<25, 7 **c** 5

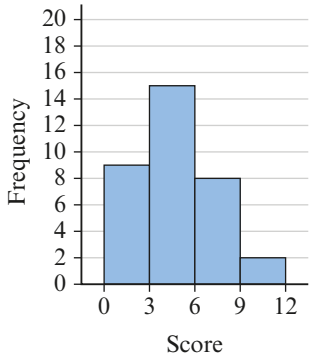
d 25–<30, as interval before this (20–<25) does not include 25

8 a grouped, as category marks are at edges of columns

b 3–<4

c Graph is slightly skewed with higher proportion of data to the left.

d

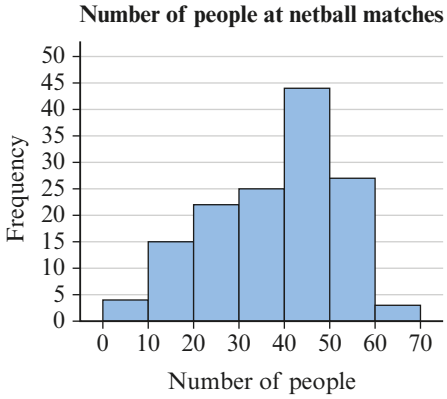


e 3–<6

f Histogram is less skewed, becoming more symmetrical.

9 No label on either axis; scale on both axes is inconsistent; should be a gap between vertical axis and first column.

10



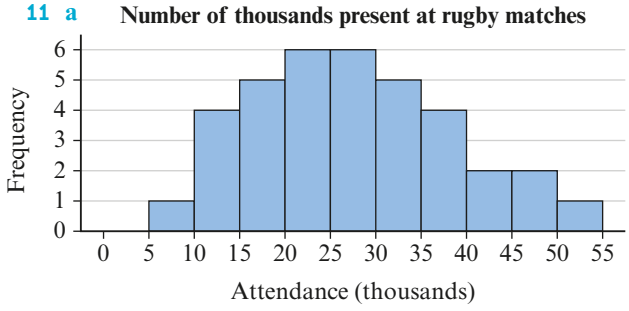
a number of people at netball matches

b 40–<50 or 40–49; 44

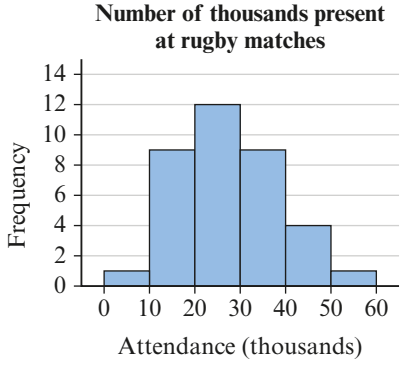
c There were 44 matches where between 40–49 people attended.

d 140

11 a



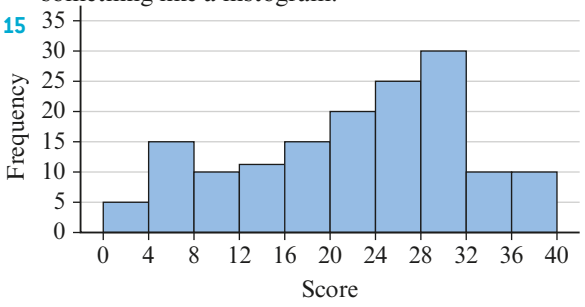
b



c Similar shape for each histogram (roughly symmetrical). Using smaller class intervals has resulted in smaller frequency values. That is, largest frequency is 6 compared with 12.

12 Turning a stem-and-leaf plot on its side with the leaf components running upwards produces something like a histogram.

15



16 **a** 5

b There appears to be no pattern. Too many intervals make it harder to read.

d A pattern should be more visible. Some possible answers are given. Using class intervals of 15 displays a skewed distribution (see table).

Class interval	Frequency
0–<15	19
15–<30	26
30–<45	32
45–<60	36
60–<75	37
75–<90	19
90–<105	10

Using class intervals of 10 displays a more symmetrically-shaped distribution (see table).

Class interval	Frequency
0–<10	13
10–<20	15
20–<30	17
30–<40	23
40–<50	21
50–<60	24
60–<70	22
70–<80	20
80–<90	14
90–<100	10

9F Summary statistics

9F Start thinking!

1 0, 0, 0, 1, 1, 1, 1, 1, 1, 1, 1, 2, 2, 2, 2, 2, 2, 2, 3, 5; 20 pieces

2 mean: 1.5, mode: 1

3 1

4

Score (x)	Frequency (f)
0	3
1	8
2	7
3	1
4	0
5	1
Total	20

Exercise 9F Summary statistics

1 **a** 4, 5, 5, 7, 8, 9, 11

b 7 **c** 7 **d** 5 **e** 7

2 **a** mean = 14.4, median = 12, mode = 12, range = 8

b mean ≈ 4.8, median = 4, mode = 2 and 4, range = 8

c mean ≈ 5.9, median = 7, mode = 7, range = 9

3 **a** 7.1, 8.0, 8.0, 10.0, 10.0, 10.0, 10.0, 10.0, 12.1, 12.1, 12.1, 12.1, 12.1, 14.1, 14.1, 14.1, 14.1, 14.1, 14.1, 14.1, 14.1, 15.1

b ≈ 11.9 **c** 14.1 **d** 12.1 **e** 8.0

4 **a** 2.17 **b** 3.17 **c** 23 **d** 2.84 **e** 14.6 **f** 17.2

5 **a** 2 **b** 3 **c** 20 **d** 3 **e** 14 **f** 20

6 **a** 2 **b** 2 **c** 20 **d** 2 **e** 14 **f** 20

7 **a** 3 **b** 3 **c** 30 **d** 4 **e** 4 **f** 40

8 **a**

Score (x)	Frequency (f)
0	5
1	6
2	5
3	4
4	0
5	2
6	1
7	0
8	0
9	0
10	0
11	0
12	0
13	0
14	0
15	0
16	0
17	0
18	1
Total	24

b 18

c mean ≈ 2.6, median = 2, mode = 1

9 mean ≈ 13.7, median = 14, mode = 14, range = 6

10 mean = 4.5, median = 5, mode = 5, range = 5

11 **a** mean = 158.75 cm, median = 158 cm, mode = 155 cm

b 35 cm

c Not all class members are same height.

12 **a** mean = 160 cm, median = 154.5 cm, mode = 152 cm

b 112 cm

c One piece of data is very large (height of 255 cm). This measurement may have been recorded incorrectly.

13 a An outlier is a value that is far bigger or smaller than most of the values in the group.

b A large outlier value will increase the mean and a low outlier value will decrease the mean.

c Median, as it is the middle value and is not affected by the presence of outliers.

d 255 cm

e Mean ≈ 155.87 cm. There is a 4.13 cm difference in the mean when outlier is disregarded. Result is closer to median value.

f 33 cm

g Disregarding outlier, all statistics indicate Monica's class is taller on average.

14 You can find mode for categorical data as it is the most common piece of data. You cannot calculate median or mean for categorical data as these statistics are based on numerical values.

15 a 41

b **i** mode = 45 **ii** median = 33 **iii** mean ≈ 33.8

c Remember to include stem numbers with each leaf value. Mode is easy to determine as you look for repeating leaf values for each stem. Median is easier to determine by locating middle value when leaf values for each stem are ordered.

16 a mean ≈ 12.78, median = 12, mode = 12 and 16, range = 19

b

Class interval (age in years)	Frequency (f)
0–<5	2
5–<10	5
10–<15	14
15–<20	12
20–<25	3

c 10 – <15 (interval with the highest frequency), mode = 12.

d 10 – <15 (interval containing 18.5th score), median = 12.

e From frequency table, largest value (represented by the largest interval) is 25 and smallest value (represented by the smallest interval) is 0. So range is
25 – 0 = 25, which is different from range obtained using raw data (19).

f 2.5

g–i

Class interval (age)	Midpoint	Frequency (f)	Midpoint × frequency
0–<5	2.5	2	2.5 × 2 = 5
5–<10	7.5	5	7.5 × 5 = 37.5
10–<15	12.5	14	12.5 × 14 = 175
15–<20	17.5	12	17.5 × 12 = 210
20–<25	22.5	3	22.5 × 3 = 67.5
	Total	36	495

j 495 ÷ 36 = 13.75

k Close. Raw data calculation is 12.78 compared with 13.75.

17

Class interval	Midpoint	Frequency (f)	Midpoint × frequency
0–<5	2.5	1	2.5 × 1 = 2.5
5–<10	7.5	8	7.5 × 8 = 60
10–<15	12.5	16	12.5 × 16 = 200
15–<20	17.5	14	17.5 × 14 = 245
20–<25	22.5	10	22.5 × 10 = 225
	Total	49	732.5

mean = 732.5 ÷ 49 = 14.95; median class: 10–<15; modal class: 10–<15; range = 25

18 a

Class interval	Midpoint	Frequency (f)	Midpoint × frequency
0–<5	2.5	1	2.5 × 1 = 2.5
5–<10	7.5	2	7.5 × 2 = 15
10–<15	12.5	5	12.5 × 5 = 62.5
15–<20	17.5	6	17.5 × 6 = 105
20–<25	22.5	8	22.5 × 8 = 180
25–<30	27.5	10	27.5 × 10 = 275
30–<35	32.5	12	32.5 × 12 = 390
35–<40	37.5	6	37.5 × 6 = 225
	Total	50	1255

b 40

c mean = 1255 ÷ 50 = 25.1; median class: 25–<30; modal class: 30–<35

d Range (maximum value – minimum value along horizontal axis) and modal class (tallest column). Mean and median class can be determined from graph by reading number of values from vertical axis and using midpoint between category labels.

19

Class interval	Midpoint	Frequency (f)	Midpoint × frequency
10–19	14.5	3	14.5 × 3 = 43.5
20–29	24.5	4	24.5 × 4 = 98
30–39	34.5	7	34.5 × 7 = 241.5
40–49	44.5	6	44.5 × 6 = 267
50–59	54.5	1	54.5 × 1 = 54.5
	Total	21	704.5

a 49

b modal class: 30–39, median class: 30–39

c mean = 704.5 ÷ 21 ≈ 33.6. Modal class and range are clearly different, but mean is very close to the original.

9G Analysing data

9G Start thinking!

1

Number of people in household (x)	Frequency (f)	$x \times f$
2	6	12
3	19	57
4	38	152
5	20	100
6	14	84
7	2	14
8	1	8
Total	100	427

mean = 4.27, median = 4, mode = 4

2 Population: every Australian household; sample: students Laani knows at her school.

3 100
4 The people she knows are all part of a family. This means that she cannot really include the single-person households that exist in Australia.
5 Spread her sample beyond the school. Choose some systematic method for surveying households across the country; for example, choose random numbers from phone books.
6 It is almost 1.4 people per household less.
7 The sample size (20) is very small. One outlier would affect the mean.
8 The larger the sample, the more reliable the result.
9 Both the median and the mean should be reliable. The mode may become less reliable as there may be many individual values that have a high but similar frequency.

Exercise 9G Analysing data

1 a mean b mode c range (spread)
d mode e mode f range
g mode h median

2 a 100, as a larger sample may be more reliable
b 619, as a larger sample may be more reliable (however, the sample of 500 may also produce similar results).
c Larger sample size may be fairer, but because they are not random (as 20 sample size is), may produce biased results.
d 50 people living in South Australia may have better knowledge of their state's popular sports team that the 100 people.

3 a mean; data does not suggest that outliers are present
b median; data suggests that outliers may be present and the median is unaffected by these
c median; data suggests that outliers may be present and the median is unaffected by these
d mean; data does not suggest that outliers are present

4 a Sample and sampling techniques should not be accepted because, although measures of centre are similar, sample involves your friends from a population of the whole school.
b From a population of Australia, sample appears to be limited to a small area of Canberra. Although measures of centre are similar, range shows a large spread.
c Sample of 20 is too small to allow you to accurately predict characteristics of population of Victoria. High range indicates large spread in responses and mean is much higher than median and mode (may be affected by outlier value).
d High range and higher mean in comparison with median and mode indicate presence of outlier. Sample size and technique appear satisfactory. Ignoring outlier, results may predict population characteristics.
e Measures of centre are similar and range is small. Sampling technique seems fair and sample size is not too small, although could be bigger. These results could be cautiously used to predict population characteristics.

5 Median value of 2 has been a consistent result for all surveys while mean has varied. Since median is not affected by outliers, median result of 2 indicates average number of times a randomly chosen person in population would go to the cinema each month.

6 a mean: 3.6 mobiles, median: 4 mobiles, mode: 4 mobiles
b Consider how survey was conducted and who was in sample.
c Either measure would be satisfactory for a fair survey. However, care needs to be taken in the event of outliers, where median may be the more reliable measure.

7 One possible answer is tennis result may be ignored as survey is biased (asking students at tennis training). The survey of randomly selected students seems fairest as one class (8A) does not represent a fair sample of whole school. Soccer, cricket and netball may be the chosen sports.

8 a Larger sample size may produce more reliable results. Abdullah's larger sample may have included results that are less spread and more centred around the average.
b the larger sample (see explanation in part a)
c Presence of outliers may distort mean value.
d Use a suitable size and take care when outliers are involved in the results.

9 To disregard outlier values. Some houses sell for very high amounts (and others for very low amounts). The median value offers the 'best' average as a result.

10 One possible answer is the use of statistics that benefit the purpose of the article. For example, if a particular real-estate agency has sold expensive homes in a suburb, and wants people to know how much money the houses are selling for on average, they may use the mean value (instead of a 'fairer' median value).

12 a symmetrical b skewed
c skewed d symmetrical, with one outlier

13 a If outlier represents a high value, mean value is increased (compared to mean if value was ignored). If outlier represents a low value, mean value is decreased (compared to mean if value was ignored). As median is middle value, it is not affected by outliers.
b In a symmetrical distribution, the middle is represented by the highest value. Mean, median and mode are all centred around this value.
c Skewed distributions could include outliers and have many values concentrated at the other end.

14 **a** Mean, median, or mode, as distribution is symmetrical.
b Median, as distribution is skewed.
c Median, as distribution is skewed.
d Median, as even though distribution is relatively symmetrical, data contains an outlier.

9 Chapter review

MULTIPLE-CHOICE

1 D **2** A **3** C **4** B **5** A
6 B **7** D

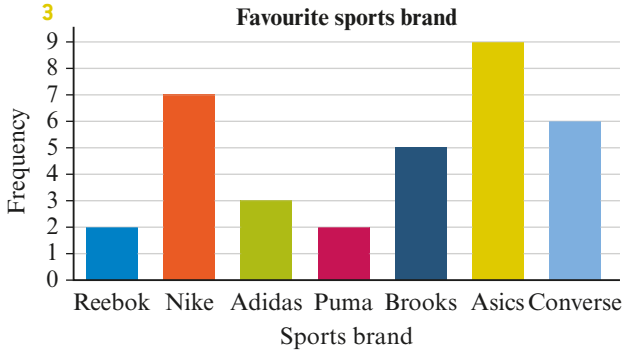
SHORT ANSWER

1 **a** **i** sample
ii biased (limited to a small group); increase the sample size
b **i** sample **ii** fair
c **i** census **ii** fair

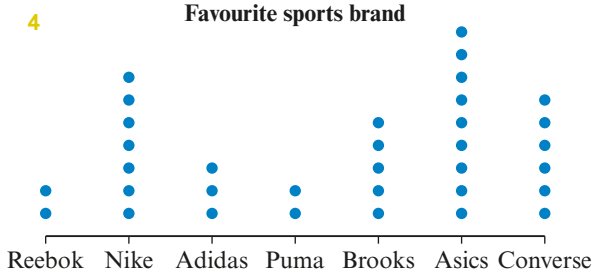
2

Class interval	Frequency
0–4	4
5–9	8
10–14	10
15–19	7
20–24	6
25–29	0
30–34	1

3



4

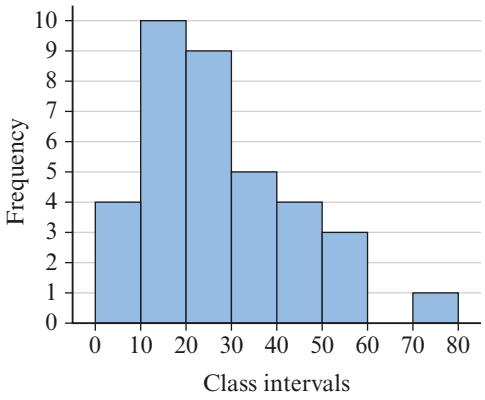


5 **a** Key: 1 | 4 = 14

Stem	Leaf
0	4 6 7 9
1	1 1 1 1 1 3 4 5 6 6
2	2 2 3 4 6 6 8 8 9
3	1 2 2 5 6
4	2 5 5 6
5	0 1 4
6	
7	3

b 11 **c** 10–19

6



7 **a** mean = 3, median = 3, mode = 1 and 4, range = 5
b mean = 12.9, median = 12, mode = 11, range = 19
c mean ≈ 6.67, median = 2, mode = 2, range = 27

8 **a** mean = 13.7, median = 14, mode = 14, range = 6
b Mean value is not very different from median here (the 18 score appears to be an outlier). Mean appears to not be affected, so any measure will be useful.

9 **a** Sample does not include teachers who do not teach Year 8.
b Sample may include people who do not know teachers in school.
c Sample is small and they may not all have experiences with all teachers in school.

10 Sampling technique appears fair though 500 is not a large sample for population. Large range and lower mean in comparison with median and mode indicate presence of outlier. Using median rather than mean, results could be cautiously used to predict population characteristics.

NAPLAN-STYLE PRACTICE

1 systematic sampling
2 asking everyone in your class if the class prefer cats or dogs
3 5 **4** numerical, discrete **5** 15–19
6 F **7** 39 **8** scatterplot
9 The youngest customer is 13. **10** 1|1 = 1100
11 80–100 **12** roughly symmetrical
13 2 **14** 1.7 **15** 4
16 2.48, 3, 3, 6
17 asking 50 random people from the electoral roll
18 mode **19** Surveying technique was biased.

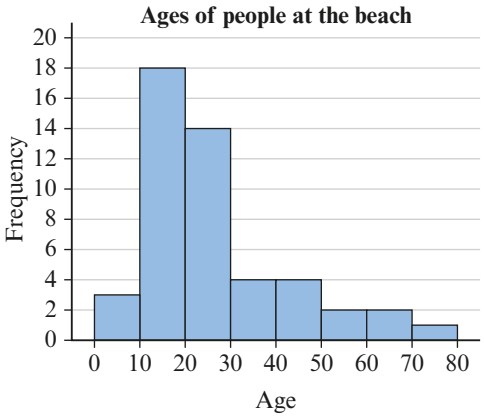
ANALYSIS

a numerical, discrete data
b systematic sample

c

Age	Frequency
0–9	3
10–19	18
20–29	14
30–39	4
40–49	4
50–59	2
60–69	2
70–79	1

d



e Key: 1 | 3 = 13

Stem	Leaf
0	5 8 9
1	3 4 5 5 5 6 6 6 6 7 7 8 8 9 9 9 9 9
2	1 1 1 2 2 3 3 3 3 4 5 6 6 7
3	1 2 5 9
4	3 5 6 9
5	6 9
6	1 3
7	2

f mean ≈ 26.7, median = 21.5, mode = 19, range = 67

g You don't know details such as type of beach, weather on the day, time of year, or if this beach is popular for holiday makers.

h Results indicate a person chosen randomly at the beach is most likely to be between about 13 and 27.

9 Connect

For feedback on this open-ended task, see your teacher.

CHAPTER 10 PROBABILITY

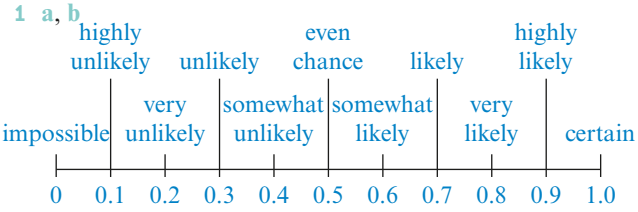
10 Are you ready?

1 B **2** D **3** 6 **4** 4 **5** B
6 A **7** A **8** D **9** A
10 a 7 b 15 c D
11 B **12** B **13** B **14** D

10A Probability

10A Start thinking!

1 a, b



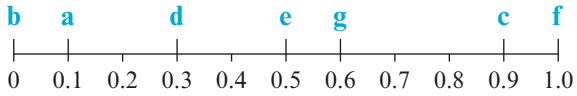
c 0, impossible **d** 1, certain

2 a {1, 2, 3, 4, 5, 6}
b No; five other numbers can be rolled.
c Yes; each individual number has same chance of appearing.
d An even chance of occurring is when result occurs half the time (probability of 0.5), whereas outcomes that are equally likely share the same probability value (but it may not be 0.5).

Exercise 10A Probability

1 a even chance **b** highly unlikely
c very likely **d** impossible
e very likely **f** impossible

2



3 a {1, 2, 3, 4, 5, 6}
b {blue, red, yellow, green}
c {spades, clubs, diamonds, hearts}
d {c, h, o, l, a, t, e} **e** {head, tail}
f {January, February, March, April, May, June, July, August, September, October, November, December}

4 a outcomes equally likely
b outcomes equally likely
c outcomes equally likely
d outcomes not equally likely
e outcomes not equally likely
f outcomes equally likely

5 a i {blue, red, white} ii no
b i {blue, red, yellow, green purple}
ii no
c i {G, R, A, M} ii no
d i {HH, HT, TT, TH}
ii yes, if order is important
e i {E, X, R, C, I, S} ii no
f i {purple, yellow, red, blue, pink}
ii no

6 a outcomes equally likely
b outcomes not equally likely: green most likely and yellow least likely
c outcomes equally likely
d outcomes not equally likely: C or O most likely and H, L, A T and E least likely
e outcomes equally likely

f Outcomes may or may not be equally likely. Use the Internet to research.

7 **a** 2 **b** 6 **c** 5 **d** 6 **e** 7 **f** 8

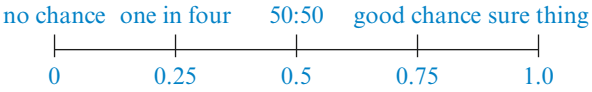
8 **a** 6 **b** 4 **c** 4 **d** 7 **e** 2 **f** 12

9 **b** {blue, blue, blue, red, red, yellow, green, green, green, green}

d {C, H, O, C, O, L, A, T, E}

11 As kittens are all different colours, chance of selecting any kitten is equally likely. Selecting any one kitten is one out of four, fitting in to the 'unlikely' category.

13 One possible answer is:



14 **a** **i** {blue, green, purple, red, yellow}
ii 5 **iii** red
iv It is the largest sector and therefore the most likely outcome.

b **i** {green, yellow, red, blue, pink, purple}
ii 6 **iii** green
iv It has the highest number of sectors and therefore is the most likely outcome.

c **i** {green, red, blue, yellow, purple}
ii 5 **iii** red
iv It takes up the most space in the spinner (its three sectors cover more space than the large blue sector) and therefore is the most likely outcome.

15 One possible answer is given.

a selecting a letter from a list of vowels
b choosing the result of a football game
c selecting a weekday
d selecting a card from a deck of cards and recording the suit
e rolling a number on a ten-sided die (numbered 1, 2, 3, 4, 5, 6, 7, 8, 9, 10) and recording the result
f rolling a die and recording if the outcome is even or odd

16 **a** **i** 5 **ii** very unlikely, 0.2
b **i** 3 **ii** unlikely, 0.3
c **i** 5 **ii** impossible, 0
d **i** 4 **ii** certain, 1
e **i** 10 **ii** highly unlikely, 0.1
f **i** 2 **ii** even chance, 0.5

17 Some possible answers are given.

	Equally likely outcomes	Not equally likely outcomes
b	randomly selecting any letter	selecting a vowel or selecting a consonant
c	selecting a month in spring or selecting a month in summer	selecting a month with 31 days or selecting a month with 30 days
d	selecting a black card or selecting a red card	selecting an ace or selecting a picture card

18 **a** {Monday, Tuesday, Wednesday, Thursday, Friday, Saturday, Sunday}; {weekday, weekend}
One possible answer for each example provided in question 17 is given.

b {A, M, U, S, I, N, G}; {vowel, consonant}
c {January, February, March, April, May, June, July, August, September, October, November, December}; {month with 31 days, month with 30 days, month with 28 days}
d {red card, black card}; {ace, picture card, non-picture card other than an ace}

19 **a** 7, 2 **b** 7, 2 **c** 12, 3 **d** 2, 3

21 It may be highly unlikely that seven tails in a row occurs, but the next flip, the seventh, still has an even chance of heads or tails.

22 One possible answer is:



23



10B Theoretical probability

10B Start thinking!

1 3, 10, $\frac{3}{10}$, 0.3 **2** favourable, total number of

3 No. Outcomes that are not equally likely have a different probability of occurring.

4 $\frac{2}{10}$ or $\frac{1}{5}$ or 0.2 **5** $\frac{8}{10}$ or $\frac{4}{5}$ or 0.8

6 They add to 1.

Exercise 10B Theoretical probability

1 **a** 12 **b** $\frac{1}{12}$ **c** $\frac{2}{12} = \frac{1}{6}$

2 **a** $\frac{1}{6}$ **b** $\frac{1}{4}$ **c** $\frac{1}{5}$ **d** $\frac{2}{10} = \frac{1}{5}$
e $\frac{1}{5}$ **f** $\frac{4}{52} = \frac{1}{13}$

3 **a** 5 **b** $\frac{1}{5}$ **c** $\frac{2}{5}$ **d** $\frac{4}{5}$

4 **a** $\frac{4}{5}$ **b** $\frac{9}{15} = \frac{3}{5}$ **c** $\frac{2}{6} = \frac{1}{3}$ **d** $\frac{12}{52} = \frac{3}{13}$
e $\frac{8}{34} = \frac{4}{17}$ **f** $\frac{5}{6}$

5 **a** {spades, hearts, clubs, diamonds}
b $\frac{1}{4}$ **c** $\frac{3}{4}$

6 **a** 0.6 **b** $\frac{1}{4}$ **c** 0.9 **d** $\frac{1}{2}$ **e** $\frac{7}{8}$ **f** 0.3
g $\frac{1}{3}$ **h** 0.75

7 **a** {1, 2, 3, 4, 5, 6} **b** $\frac{1}{3}$
c probability of rolling neither a 5 nor a 6 (or rolling a 1, 2, 3 or a 4)
d $\frac{2}{3}$

8 **a** $\frac{6}{10}$ or $\frac{3}{5}$ **b** selecting a vowel **c** $\frac{4}{10}$ or $\frac{2}{5}$

9 **a** 15 **b** $\frac{5}{15}$ or $\frac{1}{3}$
c selecting a non-blue button; $\frac{10}{15}$ or $\frac{2}{3}$
d 0 **e** $\frac{7}{15}$
f selecting a button with two holes; $\frac{8}{15}$

10 Because the sample space usually lists every different outcome rather than every possible outcome, these must be equally likely or incorrect answer will be obtained.

11 **a** 52 **b** $\frac{1}{13}$ **c** $\frac{12}{13}$ **d** $\frac{3}{13}$
e ace; there are four aces to choose from whereas there is only one queen of hearts.
f $\frac{9}{52}$ **g** $\frac{43}{52}$ **h** $\frac{5}{13}$

12 **a** 30 **b** $\frac{7}{30}$ **c** $\frac{23}{30}$ **d** $\frac{4}{30}$ or $\frac{2}{15}$
e selecting a Y, E, R, I, G, H, T, S, H, N or V
f 12 **g** $\frac{12}{30}$ or $\frac{2}{5}$
h selecting a consonant; $\frac{18}{30}$ or $\frac{3}{5}$

13 **a** $\frac{2}{6}$ or $\frac{1}{3}$
b Green sector is twice size of other sectors, so it is like having six outcomes but two of these are green.
c $\frac{1}{6}$ **d** not spinning yellow; $\frac{5}{6}$

14 **a** 0.18 **b** 0.37 **c** 0.07

15 One possible answer is shown.
Probability of spinning yellow is $\frac{1}{8}$.



16 **a** $\frac{19}{200}$ **b** $\frac{85}{199}$

17 He forgot the outcome of one head, one tail.

18 **a** $\frac{22}{55}$ or $\frac{2}{5}$ **b** $\frac{8}{55}$
c **i** 54 **ii** 7 **iii** $\frac{7}{54}$
d **i** 53 **ii** 12 **iii** $\frac{21}{53}$

19 **a** $\frac{11}{55}$ or $\frac{1}{5}$ **b** $\frac{39}{55}$
c **i** $\frac{14}{52}$ **ii** $\frac{42}{52}$ or $\frac{21}{26}$
d $\frac{8}{46}$ or $\frac{4}{23}$ **e** yellow, $\frac{10}{46}$ or $\frac{5}{23}$

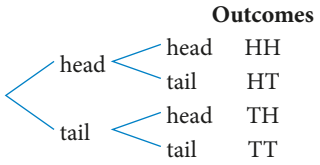
20 blue: $\frac{1}{6} \approx 0.17$, green: $\frac{17}{72} \approx 0.24$,
purple: $\frac{13}{72} \approx 0.18$, red: $\frac{1}{18} \approx 0.06$,
yellow: $\frac{13}{36} \approx 0.36$

21 **a** $\frac{2}{5}$ **b** $\frac{3}{20}$ **c** 20

10C Tree diagrams

10C Start thinking!

1

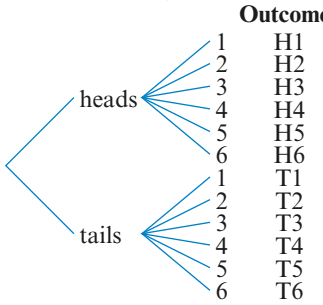


2 4

3 **a** $\frac{1}{4}$ **b** $\frac{1}{2}$

4 Because one head and one tail can come in different orders; HT or TH

5



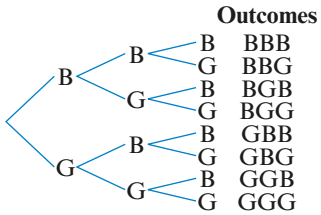
6 **a** $\frac{1}{2}$ **b** $\frac{1}{6}$ **c** $\frac{1}{12}$

7 Because the two experiments are now combined.

Exercise 10C Tree diagrams

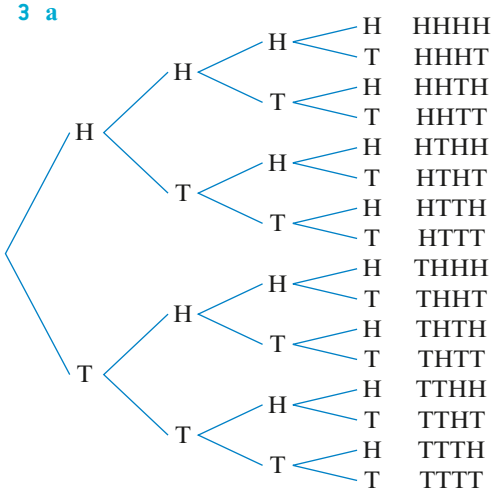
1 **a** Selecting a card from a deck and noting its suit and then flipping a coin.
b 8 **c** **i** 2 **ii** 4 **iii** 5

2 **a**

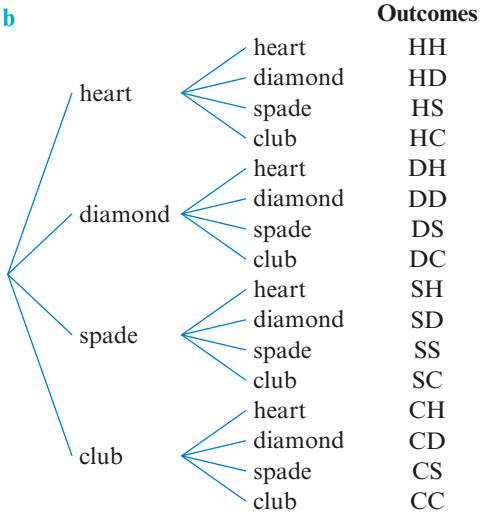


b 8 **c** 7 **d** $\frac{7}{8}$

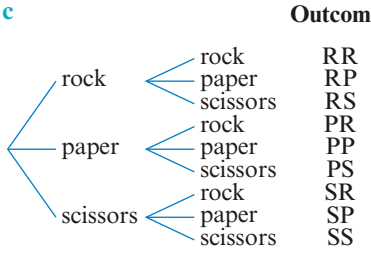
3 **a**



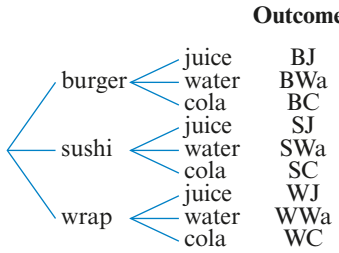
b



c



d

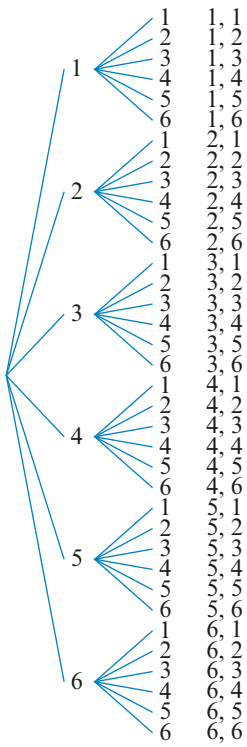


4 **a** $\frac{1}{8}$ **b** $\frac{1}{4}$ **c** $\frac{5}{8}$ **d** $\frac{3}{4}$ **e** $\frac{3}{8}$ **f** $\frac{3}{8}$

5 **a** $\frac{1}{8}$ **b** $\frac{3}{8}$ **c** $\frac{1}{2}$ **d** $\frac{3}{8}$ **e** $\frac{7}{8}$ **f** $\frac{1}{8}$

6 **a** $\frac{1}{16}$ **b** $\frac{6}{16}$ **c** $\frac{11}{16}$ **d** $\frac{4}{16}$ **e** $\frac{5}{16}$ **f** $\frac{15}{16}$

7 **a**



b 36 **c** 6

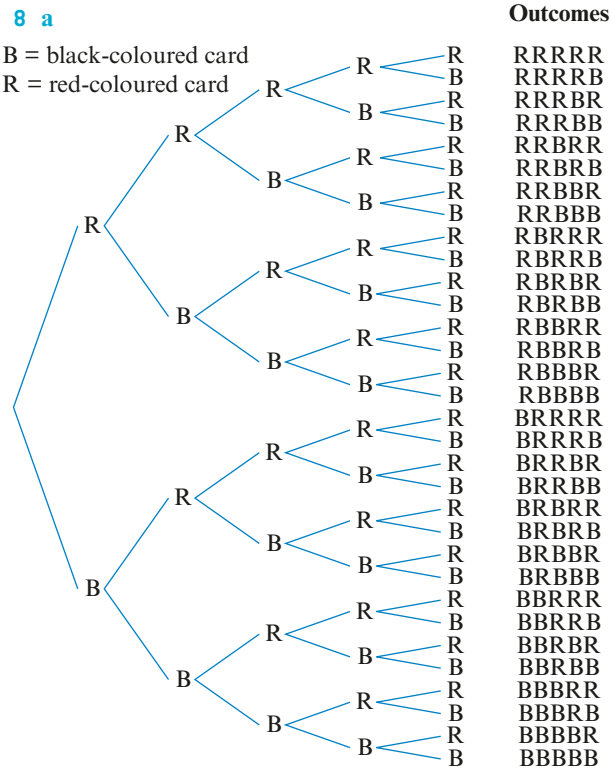
d **i** $\frac{6}{36}$ or $\frac{1}{6}$

ii $\frac{1}{36}$

e Alternative options may have been missed; for example, rolling a 1 then a 3 is different from rolling a 3 then a 1.

8 **a**

B = black-coloured card
R = red-coloured card



b 32

c three red and two black (or three black and two red), $\frac{10}{32}$ or $\frac{5}{16}$

d **i** $\frac{1}{32}$ **ii** $\frac{10}{32}$ or $\frac{5}{16}$ **iii** $\frac{26}{32}$ or $\frac{13}{16}$

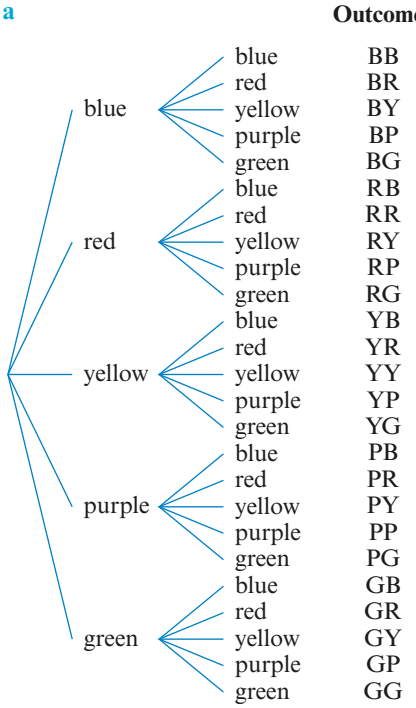
e Part d iii includes two blacks, three blacks, four blacks and five blacks.

f They all add to 1($\frac{32}{32}$).

9 a The first branches should read ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, jack, queen, king. Second branches should be same as first branches.

b $\frac{1}{169}$ **c i** $\frac{9}{169}$ **ii** $\frac{6}{169}$ **d** $\frac{8}{169}$

10 a



b i $\frac{1}{25}$ **ii** $\frac{1}{25}$ **iii** $\frac{2}{25}$

c The order in part b iii did not matter, whereas for part b ii blue had to be first.

d $\frac{9}{25}$

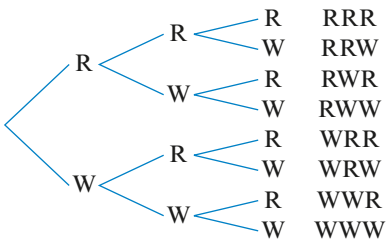
11 a Include an additional two branches showing H and T to each final branch shown in question 10a.

b i $\frac{25}{50}$ or $\frac{1}{2}$ **ii** $\frac{18}{50}$ or $\frac{9}{25}$ **iii** $\frac{9}{50}$

c Part b i (flipping a tail) includes spinning any colour in the first step. Part b ii (spinning blue at least once) includes flipping a head or a tail in the second step. Part b iii is more specific and includes spinning blue at least once in the first step and flipping a tail in the second step.

12 $\frac{14}{16}$ or $\frac{7}{8}$

13 a i



ii $\frac{1}{8}$

iii 1 or 2 correct (both results have the same probability)

b i Each of the three steps has four branches (R, W, W, W).

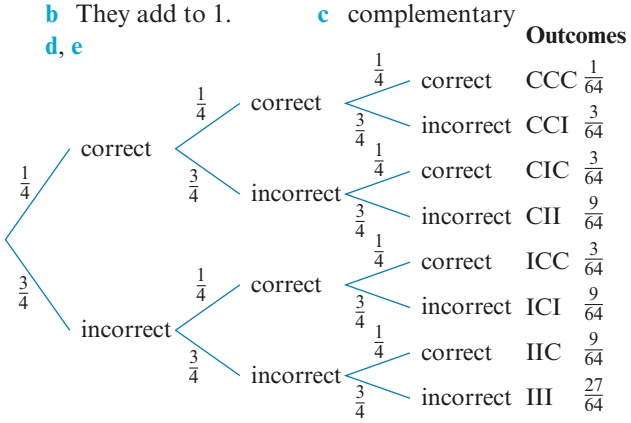
ii $\frac{1}{64}$ **iii** none or one correct

c It is unlikely that you will guess enough questions correctly to pass a test, particularly as the number of questions on a test increase.

14 a There are four options in MULTIPLE-CHOICE and only one can be correct.

b They add to 1. **c** complementary

d, e

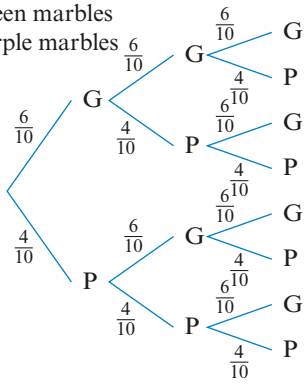


f i $\frac{1}{64}$ **ii** $\frac{27}{64}$ **iii** $\frac{9}{64}$ **iv** $\frac{37}{64}$

g They all add to 1($\frac{64}{64}$).

h If all answers add to 1, all possibilities and probabilities have been included on diagram.

15 a G = green marbles
P = purple marbles



b i $\frac{216}{1000}$ or $\frac{27}{125}$ **ii** $\frac{64}{1000}$ or $\frac{8}{125}$ **iii** $\frac{784}{1000}$ or $\frac{98}{125}$

iv $\frac{96}{1000}$ or $\frac{12}{125}$ **v** $\frac{6}{10}$ or $\frac{3}{5}$

10D Two-way tables

10D Start thinking!

1 They look different but both contain all possibilities for flipping two coins.

2 a girls who play a musical instrument, boys who do not play a musical instrument, girls who do not play a musical instrument

b 4 **c** 14 **d** $\frac{14}{30}$ or $\frac{7}{15}$ **e** $\frac{4}{30}$ or $\frac{2}{15}$

f The four outcomes are not equally likely.

3 The frequency of each outcome can be seen easily.

Exercise 10D Two-way tables

1 a Year 7 students who play sport, Year 8 students who play sport, Year 7 students who don't play sport, Year 8 students who don't play sport

b 50 **c** 17 **d** 23

e Year 7 students who don't play sport

f 22

2 a Children who prefer books, Adults who prefer books, Children who prefer movies, Adults who prefer movies

b 100 **c** 29 **d** 51 **e** 39 **f** 41

3 a

	Group A	Group B	Total
Year 7	9	23	32
Year 8	10	8	18
Total	19	31	50

b

	Cat	Dog	Total
Male	11	48	59
Female	39	2	41
Total	50	50	100

c

	Weekday	Weekend	Total
Volleyball	14	17	31
Tennis	5	31	36
Total	19	48	67

d

	Year 8	Year 9	Total
CD	11	15	26
Digital	18	19	37
Total	29	34	63

4

	Lollies	Not lollies	Total
Red	5	3	8
Not red	4	2	6
Total	9	5	14

5 a 100 **b** 43 **c** $\frac{43}{100}$

6 a 40 **b** 6 **c** 19

d i $\frac{6}{40}$ or $\frac{3}{20}$ **ii** $\frac{19}{40}$ **iii** $\frac{16}{40}$ or $\frac{2}{5}$

7 a $\frac{17}{50}$ **b** $\frac{12}{50}$ or $\frac{6}{25}$ **c** $\frac{23}{50}$

d $\frac{28}{50}$ or $\frac{14}{25}$ **e** $\frac{27}{50}$ **f** $\frac{11}{50}$

8 a $\frac{4}{50}$ or $\frac{2}{25}$ **b** $\frac{33}{50}$ **c** $\frac{27}{50}$

d $\frac{14}{50}$ or $\frac{7}{25}$ **e** $\frac{19}{50}$ **f** $\frac{17}{50}$

9 a

	Year 7	Year 8	Total
M&Ms	16	16	32
Smarties	10	8	18
Total	26	24	50

b i $\frac{18}{50}$ or $\frac{9}{25}$ **ii** $\frac{26}{50}$ or $\frac{13}{25}$ **iii** $\frac{16}{50}$ or $\frac{8}{25}$

c $\frac{16}{24}$ or $\frac{2}{3}$

d Part c asked for students who preferred M&Ms from the Year 8 class only, whereas part b asked who is in Year 8 and prefers M&Ms as a proportion of the whole group.

10 a Preferred football codes across northern and southern parts of Australia

b i $\frac{42}{100}$ or $\frac{21}{50}$ **ii** $\frac{3}{100}$

iii $\frac{58}{100}$ or $\frac{29}{50}$ **iv** $\frac{10}{100}$ or $\frac{1}{10}$

c The northern part has a preference for rugby and the southern part has a preference for AFL.

11 a

	Blond	Brown haired	Total
Blue	16	30	46
Brown	23	31	54
Total	39	61	100

b i $\frac{39}{100}$ **ii** $\frac{46}{100}$ or $\frac{23}{50}$ **iii** $\frac{16}{100}$ or $\frac{4}{25}$

c i 39 **ii** 16 **iii** $\frac{16}{39}$

iv Part b iii asks for the proportion of the entire group who have blonde hair and blue eyes out of the entire group, whereas part c iii asks for the proportion of blondes who have blue eyes.

d i 61 **ii** 30 **iii** $\frac{30}{61}$

12 a i $\frac{30}{50}$ or $\frac{3}{5}$ **ii** $\frac{12}{50}$ or $\frac{6}{25}$

b i $\frac{28}{31}$ **ii** $\frac{12}{42}$ or $\frac{2}{7}$

13

	White bread	Not white bread	Total
Vegemite	40	24	64
Jam	20	16	36
Total	60	40	100

probability(jam on non-white toast) $\frac{16}{100}$ or $\frac{4}{25}$

14 a

	Telstra	Other	Total
Country	0.30	0.10	0.4
City	0.05	0.55	0.6
Total	0.35	0.65	1.0

b

	Telstra	Other	Total
Country	60	20	80
City	10	110	120
Total	70	130	200

c i $\frac{60}{80}$ or $\frac{3}{4}$ **ii** $\frac{10}{120}$ or $\frac{1}{12}$

10E Venn diagrams

10E Start thinking!

1 11 **2** yellow, 3

3 14 **4** blue eyes, but not brown hair, 9

5 12, blue and yellow **6** green, 6 **7** 29

8 The Venn diagram is a different representation of the information in a two-way table.

	Blue eyes	Not blue eyes	Total
Brown hair	3	11	14
Not brown hair	9	6	15
Total	12	17	29

Exercise 10E Venn diagrams

1 a people with brothers

b people with sisters but not brothers

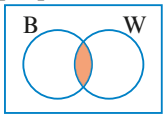
c people without sisters or brothers

d people with sisters and/or brothers

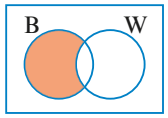
e people with sisters or brothers but not both

f people without sisters

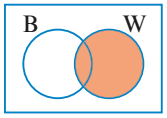
2 a



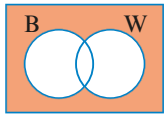
b

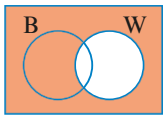


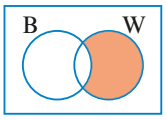
c



d



e 

f 

3 **a** 18 **b** 23 **c** 5 **d** 2

4 **a** 10 **b** 21 **c** 3 **d** 42

5 **a** **i** 31 **ii** 10 **iii** 38 **iv** 2 **v** 48 **vi** 9

b 50

6 **a** **i** 6 **ii** 16 **iii** 16 **iv** 18

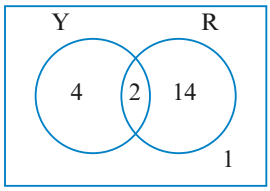
b **i** $\frac{6}{50}$ or $\frac{3}{25}$ **ii** $\frac{16}{50}$ or $\frac{8}{25}$

iii $\frac{16}{50}$ or $\frac{8}{25}$ **iv** $\frac{18}{50}$ or $\frac{9}{25}$

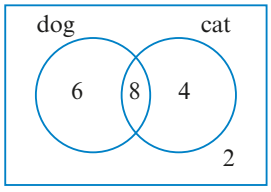
7 **a** $\frac{10}{42}$ or $\frac{5}{21}$ **b** $\frac{18}{42}$ or $\frac{9}{21}$

c $\frac{21}{42}$ or $\frac{1}{2}$ **d** $\frac{3}{42}$ or $\frac{1}{14}$

8 **a** $\frac{7}{50}$ **b** $\frac{41}{50}$ **c** $\frac{31}{50}$ **d** $\frac{48}{50}$ or $\frac{24}{25}$

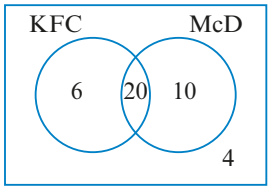
9 

10 **b** 2 **c** 18 **d** 4 **e** 8 **f** 6

g 

h **i** $\frac{14}{20}$ or $\frac{7}{10}$ **ii** $\frac{4}{20}$ or $\frac{1}{5}$

iii $\frac{8}{20}$ or $\frac{2}{5}$ **iv** $\frac{2}{20}$ or $\frac{1}{10}$

11 

12 **a** **i** 19 **ii** 6 **iii** $\frac{6}{19}$

b **i** 21 **ii** 12 **iii** $\frac{12}{21}$

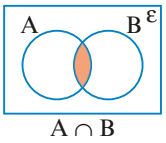
c **i** 18 **ii** 6 **iii** $\frac{6}{18}$ or $\frac{1}{3}$

d The 'total' group is different – in part a the total group is people who like football, but in part c the total group is people who like basketball.

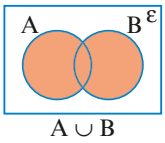
e The 'total' group is different (part a: people who like football, part b: people who don't like football), and the 'target' group is also different (part a: people who like both football and basketball, part b: people who like basketball but not football).

13 **a** $\frac{17}{35}$ **b** $\frac{13}{35}$ **c** $\frac{4}{35}$ **d** $\frac{4}{15}$

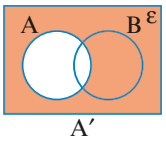
e $\frac{4}{17}$ **f** $\frac{28}{35}$ or $\frac{4}{5}$ **g** $\frac{20}{35}$ or $\frac{4}{7}$

14 **a** 

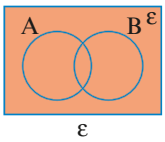
$A \cap B$

b 

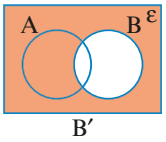
$A \cup B$

c 

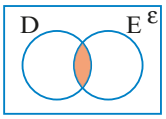
A'

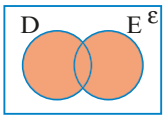
d 

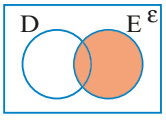
ε

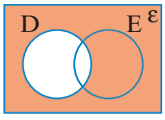
e 

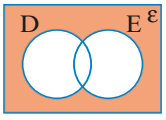
B'

15 **a** 

b 

c 

d 

e 

10F Experimental probability

10F Start thinking!

1 {head, tail} **2** $\frac{1}{2}$ **3** 5, 20

4 Theoretical probability of flipping a tail is $\frac{1}{2}$. This means if you flip the coin 10 times, then $10 \times \frac{1}{2} = 5$.

5 multiply

6 Pr(x); E(x) = expected number where x represents the event, Pr(x) = theoretical probability of the event, n = the number of trials

10 The more trials performed, the closer the experimental results get to the theoretical results.

Exercise 10F Experimental probability

1 **a** $\frac{481}{1000}$ **b** $\frac{5}{60}$ or $\frac{1}{12}$ **c** $\frac{88}{400}$ or $\frac{11}{50}$

d $\frac{62}{200}$ or $\frac{31}{100}$ **e** $\frac{67}{300}$ **f** $\frac{25}{40}$ or $\frac{5}{8}$

2 **a** 100 **b** 25 **c** 100

d 50 **e** 75 **f** 8

3 **a** 200

b $\frac{250}{1200}$ (or $\frac{5}{24}$) ≈ 0.2083.
Theoretical probability is $\frac{200}{1200}$ (or $\frac{1}{6} \approx 0.1667$).
Experimental probability is slightly larger than theoretical probability.

4 **a** 25

b $\frac{17}{100} = 0.17$. Theoretical probability is $\frac{25}{100}$ (or $\frac{1}{4}$) = 0.25. Experimental probability is slightly smaller than theoretical probability.

5 a blue: $\frac{13}{52} = 25\%$; orange: $\frac{12}{52} \approx 23\%$; green: $\frac{9}{52} \approx 17\%$; brown: $\frac{9}{52} \approx 17\%$; yellow: $\frac{3}{52} \approx 6\%$; red: $\frac{6}{52} \approx 12\%$

b With the exception of red and yellow, most of the experimental probabilities are close but slightly greater than the theoretical probabilities. The percentage of yellow M&Ms is a lot less than expected.

c If you sampled many packets you would expect the experimental probabilities to more closely reflect the theoretical probabilities. In particular, you would expect to see a greater percentage of yellow M&Ms.

6 a see Example 10C-2, page 522

b four

c Pr(three heads) = $\frac{1}{8}$; Pr(two heads, one tail) = $\frac{3}{8}$; Pr(one head, two tails) = $\frac{3}{8}$; Pr(three tails) = $\frac{1}{8}$

d E(three heads) = 5; E(two heads, one tail) = 15; E(one head, two tails) = 15; E(three tails) = 5

e They are close to expected values, but differ slightly. Two extra results occurred for three heads and two fewer for three tails. For two heads and a tail, one fewer result was obtained and one extra was obtained for two tails and one head.

f three heads: $\frac{7}{40}$; two heads, one tail: $\frac{14}{40}$ or $\frac{7}{20}$; one head, two tails: $\frac{16}{40}$ or $\frac{2}{5}$; three tails: $\frac{3}{40}$

g In the long term, you would expect results to be closer to theoretical probabilities.

8 a HH: $\frac{3}{20}$; HT: $\frac{6}{20}$ or $\frac{3}{10}$; TH: $\frac{9}{20}$; TT: $\frac{2}{20}$ or $\frac{1}{10}$

b two heads: $\frac{3}{20}$; one head, one tail: $\frac{15}{20}$ or $\frac{3}{4}$; two tails: $\frac{2}{20}$ or $\frac{1}{10}$

c two heads: $\frac{1}{4}$; one head, one tail: $\frac{1}{2}$; two tails: $\frac{1}{4}$

d two heads: $\frac{3}{20} = 0.15$ is smaller than the theoretical probability $\frac{1}{4} = 0.25$
one head, one tail: $\frac{3}{4} = 0.75$ is greater than the theoretical probability $\frac{1}{2} = 0.5$
two tails: $\frac{2}{20} = 0.1$ is smaller than the theoretical probability $\frac{1}{4} = 0.25$

e two heads: 5; one head and one tail:10; two tails: 5. There were two fewer than expected for two heads and three fewer than expected for two tails. There were five extra than expected for one head and one tail.

9 a, b

Outcome	Theoretical probability	Expected number
yellow	$\frac{2}{8} = \frac{1}{4}$	10
green	$\frac{1}{8}$	5
orange	$\frac{1}{8}$	5
red	$\frac{1}{8}$	5
blue	$\frac{3}{8}$	15

c Reasonably close; yellow, red and blue were spun slightly less often than expected and orange was spun more often than expected.

d

Outcome	Frequency	Experimental probability
yellow	12	$\frac{12}{40} = \frac{3}{10}$
green	5	$\frac{5}{40} = \frac{1}{8}$
orange	7	$\frac{7}{40}$
red	3	$\frac{3}{40}$
blue	13	$\frac{13}{40}$

e The experimental probability will more closely match the theoretical probability.

11 a Pr(picture card) = $\frac{3}{13}$; Pr(non-picture card) = $\frac{10}{13}$

b Answers are rounded to nearest whole number.
i 1 **ii** 2 **iii** 5 **iv** 9

e picture card: $\frac{3}{13} \times 40 \times 2 = 18.5$ points
non-picture card: $\frac{10}{13} \times 40 \times 1 = 30.8$ points
Stu has the better deal as he ends up with more points.

f One possible answer is given. Tess gets 10 points for each picture card, Stu gets 3 points for each non-picture card.
picture card: $\frac{3}{13} \times 40 \times 10 = 92.3$ points
non-picture card: $\frac{10}{13} \times 40 \times 3 = 92.3$ points

12 a Pr(picture card) = $\frac{3}{13}$; Pr(ace) = $\frac{1}{13}$; Pr(non-picture card other than ace) = $\frac{9}{13}$

b Answers are rounded to the nearest whole number.

Outcome	4 trials	10 trials	20 trials	40 trials
picture card	1	2	5	9
ace	0	1	2	3
non-picture card other than ace	3	7	14	28

c Tess: $\frac{1}{13} \times 40 \times 10 + \frac{9}{13} \times 40 \times 1 = 30.8 + 27.7 = 58.5$ points
Stu: $\frac{3}{13} \times 40 \times 5 = 46.15$ points

f Not fair; biased in Tess's favour as shown in part c.

14 yellow: sector size = $\frac{10}{40} \times 360° = 90°$
green: sector size = $\frac{5}{40} \times 360° = 45°$
blue: sector size = $\frac{10}{40} \times 360° = 90°$
red: sector size = $\frac{15}{40} \times 360° = 135°$

10G Simulations and long-term trends

10G Start thinking!

5 As the number of trials increases, the difference between experimental probability and theoretical probability decreases. (Experimental probability gets closer to theoretical probability.)

6 expect around 50 tails for 100 trials and 500 tails for 1000 trials

8 As number of trials increases, experimental probability approaches value for theoretical probability.

Exercise 10G Simulations and long-term trends

Devices listed are suggestions only.

1 a i two **ii** flipping a coin
iii heads: true, tails: false
b i four
ii spinner with four equal sections, labelled A, B, C and D
iii The section the spinner lands on determines the group.
c i six **ii** rolling a die
iii Assign each toy to each number on the die.
d i three **ii** rolling a die
iii Assign a colour to pairs of numbers. For example, pink clinker: 1 or 2 on the die, green clinker: 3 or 4 on the die, yellow clinker: 5 or 6 on the die.
e i 2 **ii** flipping a coin
iii tail: left; head: right
f i 7 **ii** spinner with seven sections
iii blue: Mon; yellow: Tue; green: Wed; red: Thu; purple: Fri; pink: Sat; orange: Sun.

2 a Spinning a spinner with eight equally-sized sections; the section the spinner lands on determines the flavour.
b Drawing a card from a deck and recording the suit; clubs: N; diamonds: E; hearts: S; spades: W.
c Flipping a coin; heads: male; tails: female.
d Rolling a 10-sided die, assign each prize to each number on the die.
e Spinning a spinner with twelve equally-sized sections; the section the spinner lands on determines the month.
f Rolling a standard die, assign each prize to each number on the die.

3 a Person flipping coin could influence result, meals may not be equally likely to occur, meals may not be evenly distributed.
b Person drawing card could influence result, babies may be more likely to be born in one or more seasons.
c Person spinning spinner could influence result, people selecting classes may introduce bias, some classes may be bigger and therefore not equally likely, elective choice could influence class placement.
d Person rolling die could influence result, people selecting teams may introduce bias, selection may not be equally likely.
e Person rolling die could influence result, winning is not equally likely and depends on skill of teams.
f Person spinning spinner could influence result.

4 a Rolling a die to represent Chupa Chup flavours; assumes there are equal numbers of each flavour.
b Flipping a coin (heads: boy, tails: girl); doesn't allow for genetics or environment affecting gender result.
c Rolling a four-sided die; assumes equal availability of desserts and that they are all equally well liked.
d Spinning a spinner with five equally-sized sections; assumes that each person has same leadership ability.
e Rolling a die to represent M&M colours; assumes colours are evenly distributed in each packet
f Spinning a spinner with five sections; assumes correct answers are evenly distributed between options A–E.

5 a $25\% = \frac{1}{4}$ and $50\% = \frac{1}{2}$
b flipping two coins
c two heads: plain, two tails: white, one head and one tail: flashy
f Breeding conditions may not be ideal, breeding pattern may be more complex than stated, person flipping coins may introduce some bias by flipping them a certain way.

7 a Winning and not winning are not equally likely outcomes: there is one in six chances of winning and there are five in six chances of not winning.
b You may roll a die and let the number 1 represent chance of winning. The numbers 2, 3, 4, 5 and 6 all represent chance of losing.
c 5
e expect to win about $\frac{1}{6} \times 3000 = 500$ Mars bars
f Winning Mars Bars may be evenly packaged and distributed. The die model may select more than the expected amount (or fewer than the expected amount) on some given trials.

8 a an integer randomly generated from 1 to 6 (inclusive).

9 a i $\frac{7}{10}$ **ii** $\frac{3}{10}$ **b** 30
c A 10-sided die may be used. Assign three numbers to represent meteorite crashing on land and remaining seven numbers to represent meteorite crashing on water.

10 **a** A coin may be used to simulate genders at birth. Flip twice to represent two children, three times for three children and so on. Assign a result to a gender; for example, heads is a boy, tails is a girl.
f 3
11 **a** 6 **b** 18
12 Over the long term, result may approach expected result. However, factors such as gender selection, health or diet may influence gender results.

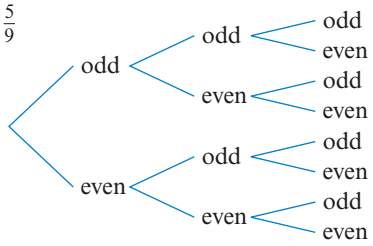
10 Chapter review

MULTIPLE-CHOICE

1 D **2** D **3** D **4** D **5** C
6 A **7** D

SHORT ANSWER

1 **a** **i** {1, 2, 3, 4, 5, 6} **ii** equally likely
b **i** {HH, HT, TH, TT}
ii equally likely (if order is important); flipping one head and one tail (if order is not important).
c **i** {picture card, non-picture card}
ii non-picture card
2 **a** $\frac{2}{9}$ **b** $\frac{4}{9}$
d $\frac{5}{9}$
3 **a**



b **i** $\frac{1}{8}$ **ii** $\frac{3}{8}$ **iii** $\frac{7}{8}$
4 **a** $\frac{115}{200}$ or $\frac{23}{40}$ **b** $\frac{18}{200}$ or $\frac{9}{100}$
c $\frac{56}{200}$ or $\frac{7}{25}$ **d** $\frac{47}{200}$ **e** $\frac{97}{115}$
5 **a** 35 **b** 18
c **i** $\frac{10}{35}$ or $\frac{2}{7}$ **ii** $\frac{8}{35}$
iii $\frac{13}{35}$ **iv** $\frac{10}{18}$ or $\frac{5}{9}$
6 **a, b**

Outcome	Theoretical probability	Expected number
blue	$\frac{1}{2}$	150
yellow	$\frac{1}{6}$	50
red	$\frac{1}{3}$	100

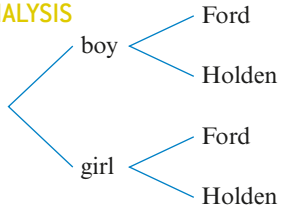
c blue: slight increase (180 compared with 150 expected)
yellow: decrease (30 compared with 50 expected)
red: slight decrease (90 compared with 100 expected)
7 **a** Pr(solid milk chocolate) = $\frac{1}{4}$,
Pr(solid white chocolate) = $\frac{1}{4}$,
Pr(caramel filled milk chocolate) = $\frac{1}{4}$,
Pr(caramel filled white chocolate) = $\frac{1}{4}$
b One possible device is a spinner with four equal sections numbered one to four. Assign a number to each outcome.
c eight
d A spinner may not always produce truly random results.

NAPLAN-STYLE PRACTICE

1 {yellow, green, blue, pink}
2 0.75 or $\frac{3}{4}$ **3** $\frac{1}{2}$ **4** $\frac{1}{4}$
5 yellow, red or blue **6** 0.8 **7** $\frac{8}{20}$ or $\frac{2}{5}$
8 $\frac{9}{20}$ **9** $\frac{7}{9}$ **10** $\frac{3}{8}$ **11** $\frac{18}{25}$ **12** C
13 experimental probability = $\frac{11}{30}$; should decrease with further trials
14 five **15** Table C

ANALYSIS

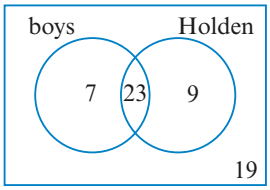
a



b {BF, BH, GF, GH};
B = boy, G = girl,
F = Ford, H = Holden
Each outcome has theoretical probability of 0.25.
c

	Ford	Holden	Total
Boys	7	23	30
Girls	19	9	28
Total	26	32	58

58 people were surveyed.
d Pr(BF) = $\frac{7}{58}$, Pr(BH) = $\frac{23}{58}$, Pr(GF) = $\frac{19}{58}$,
Pr(GH) = $\frac{9}{58}$
e **i** $\frac{32}{58}$ **ii** $\frac{30}{58}$ or $\frac{15}{29}$ **iii** $\frac{23}{30}$
f No; each result would need to have been around 14 or 15.
g Possible limitations include age and location of people surveyed.
h One possible answer is given.



10 Connect

For feedback on this open-ended task, see your teacher.

Glossary

A

acute angle angle greater than 0° but less than 90°.

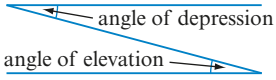


acute-angled describes a triangle where the size of each interior angle is between 0° and 90°.

alternate angles pair of angles not adjacent to (next to) each other on opposite sides of the transversal between two lines.
If the two lines are parallel, the alternate angles are equal.



angle of depression angle between a horizontal line and the line of sight to an object below the horizontal.



angle of elevation angle between a horizontal line and the line of sight to an object above the horizontal.

angle-side-angle *see* ASA

annulus region formed between two circles of different radius but with the same centre.



area amount of space enclosed by a two-dimensional (2D) shape. Common units are mm^2, cm^2, m^2, km^2, ha.

arms lines that meet at a vertex to form an angle.

ASA abbreviation for angle-side-angle. It describes the information needed to draw a triangle and also relates to conditions for congruence. Triangles are congruent if two corresponding angles and the side between them are the same.

ascending order order where a list of numbers is written from smallest to largest.

associative law order in which three or more numbers are added or multiplied is not important.
For example,
$(4 + 5) + 6 = 9 + 6 = 15$ and
$4 + (5 + 6) = 4 + 11 = 15$ and
$(4 + 6) + 5 = 10 + 5 = 15$;
$(4 \times 5) \times 6 = 20 \times 6 = 120$ and
$4 \times (5 \times 6) = 4 \times 30 = 120$ and
$(4 \times 6) \times 5 = 24 \times 5 = 120$.

average for a list of scores, it is the value obtained by adding all of the scores together and dividing by the number of scores.

B

back-to-back stem and leaf plot stem-and-leaf plot that displays two sets of data using the one set of stems.

backtracking method of following a flow chart from the final result back to the starting number using inverse operations.

balance method strategy for solving equations by using inverse operations to make equivalent equations. An equation remains balanced (or equivalent) by performing the same operation on both sides of the equation.

balance model model that demonstrates that two sides of a set of scales (or equation) can be balanced by performing the same operation on each side.

bar graph graph where the frequency of categorical data is presented in bars or columns. Category names are placed underneath the bars if they are drawn vertically or beside the bars if they are drawn horizontally.

base **1** for a value expressed in index form, the base is the number which is repeatedly multiplied. For example, 2^4 has a base of 2 and can be written as $2 \times 2 \times 2 \times 2$.
2 in a 2D shape, it is a nominated side of the shape. The base and height are perpendicular to each other. *See* height for diagram.
3 in a 3D object, it is a nominated face of the object. The base and height are perpendicular to each other.

basic numeral result of a repeated multiplication. For example, $2^4 = 16$ where 16 is the basic numeral.

bearings direction from one position to another.

best buy found by comparing a number of purchase options to find the one that is the best value for money.

biased a biased sample is one where the method of collecting data produces a sample that does not accurately reflect the population.

BIDMAS (BODMAS) word used to remember the correct order of operations: Brackets, Indices (powers and roots), Division and Multiplication, and Addition and Subtraction.

C

capacity amount of liquid that a three-dimensional object (container) can hold. Common units are mL, L, kL, ML.

Cartesian plane number plane or region formed by a pair of horizontal and vertical axes that allows any point to be described.

categorical data data that can be put into categories.

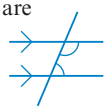
census survey of an entire population.

circumference perimeter of a circle.

class intervals groupings of data into equal-sized classes.

coefficient number that appears in front of a variable showing that the number and the pronumeral are to be multiplied together. For example, the coefficient of $5m$ is 5.

co-interior angle pair of angles on the same side of the transversal between two lines. If the two lines are parallel, the co-interior angles are supplementary (add to 180°).



column graph graph where the frequency of categorical data is presented in columns.

commission payment to an employee calculated by finding a percentage of sales made.

commutative law order in which two numbers are added or multiplied is not important. For example, $3 + 4 = 7$ and $4 + 3 = 7$; $3 \times 4 = 12$ and $4 \times 3 = 12$.

compass bearing directions from one position to another and described by the number of degrees from north or south towards east or west. For example, S43°E represents a direction that is 43° east of due south.

complement the complement of a set is everything in the universal set that is not in the given set. For example, if the universal set contains whole numbers from 1 to 5 ($\varepsilon = \{1, 2, 3, 4, 5\}$) and set A contains even numbers ($A = \{2, 4\}$) then the complement of A contains odd numbers ($A' = \{1, 3, 5\}$).

complementary angles angles that add to make a right angle (90°).

complementary events two probability events are complementary when their probabilities add to 1.

composite shapes shapes made up of more than one type of shape.

concave type of polygon with at least one interior angle that is greater than 180°.

cone 3D object that tapers from a circular base to a point.

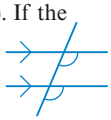
congruent figures that are identical in shape and size but can be in any position or orientation.

constant term numerical value in an algebraic expression or equation. It is a term without any pronumeral component. For example, the constant in the expression $3b - 4c + 10$ is 10.

continuous data type of numerical data that can be measured; for example, height.

convex type of polygon with interior angles that are all less than 180°.

corresponding angles pair of angles on the same side of the transversal and in corresponding positions on the two lines (both above a line or both below a line). If the two lines are parallel, the corresponding angles are equal.



cost price cost of making an item for sale.

cube to raise to the power of three; for example, $2^3 = 8$.

cube root number that, when cubed (raised to the power of 3), results in the original number. For example, the cube root of 8 (or $\sqrt[3]{8}$) is 2.

cumulative frequency running total of frequencies in a frequency table.

cylinder 3D object with a uniform circular cross-section.

D

decimal place position of a digit after the decimal point. Each decimal place represents a different fractional value. The first decimal place is the first digit to the right of the decimal point. The number of digits after the decimal point is the number of decimal places. For example, 5.249 has three decimal places.

denominator bottom number of a fraction. For example, the denominator of $\frac{2}{3}$ is 3.

dependent variable variable that depends on another variable (independent variable) in a relationship. On a Cartesian plane, the dependent variable is shown along the vertical axis.

descending order order where a list of numbers is written from largest to smallest.

diameter line segment from one side of a circle to the other through the centre of the circle. Also defined as the length of this line segment.

dilation transformation that does not produce an identical figure. A dilation produces an enlargement or a reduction of the original figure.

dimensions measurements on a shape or object. For example, the length and width of a rectangle are the dimensions of the rectangle.

directed numbers description of positive and negative numbers, as they have both size and direction.

discount an amount off the selling price. New selling price = original selling price − discount.

discrete data type of numerical data that can be counted; for example, number of siblings.

distributive law same answer is obtained whether you add the numbers inside a pair of brackets first before multiplying by another number or multiply each number inside the brackets by the number outside the brackets before adding. For example, $3(2 + 5) = 3 \times 7 = 21$ and $3(2 + 5) = 3 \times 2 + 3 \times 5 = 6 + 15 = 21$. In algebraic terms, the distributive law can be written as $a(b + c) = ab + ac$.

dot plot graph that presents every piece of data in the data set as a dot above a matching number or category on a horizontal scale. Suitable for discrete numerical data and categorical data.

E

edges line segments that join two faces of a polyhedron. *See* polyhedron.

enlargement larger image produced after a figure has been dilated.

equation collection of two or more algebraic terms separated by mathematical operation symbols and containing an equals sign; for example, $2d + 5 = 10$.

equator circumference of the Earth that runs halfway between the north and south poles. It divides the Earth into the northern hemisphere and the southern hemisphere and has a latitude of 0°.

equilateral triangle triangle with all three sides equal in length. All three angles are also equal in size.



equivalent equations equations that have the same solution.

equivalent fractions different fractions that have the same value. An equivalent fraction is formed by multiplying or dividing the numerator and the denominator of a fraction by the same number. For example, $\frac{24}{36} = \frac{4}{6}$.

equivalent ratio a ratio that can be written as an equivalent fraction when compared to another ratio. An equivalent ratio can be produced by multiplying or dividing all numbers in a ratio by the same value.

estimate make an educated or thoughtful guess.

evaluating calculating the numerical value of a term or expression after substitution. For example, evaluating $3x + 4$ when $x = 2$ gives 10.

event a specified result that may be obtained in a probability experiment.

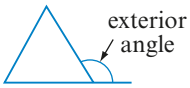
expanded form **1** written as a repeated multiplication. For example, 4^3 written in expanded form is $4 \times 4 \times 4$.
2 an algebraic expression written without brackets (not in factor form).

expected number number of favourable outcomes expected in a probability experiment. It is calculated from theoretical probability; that is, expected number = theoretical probability × number of trials.

experimental probability probability of an event based upon the results of a probability experiment. Also known as relative frequency.

expression collection of two or more algebraic terms separated by mathematical operation symbols like +, −, × and ÷. An expression does not have an equals sign. For example, $3b - 4c + 10$ is an expression.

exterior angle angle formed outside a triangle where a side has been extended. The exterior angle and the interior angle directly adjacent to it are supplementary (add to 180°).



F

faces flat surfaces of a polyhedron. Each face is a polygon. *See* polyhedron.

factor form expression made up of a product of factors.

factorise to write an expression in factor form. For example, $6x + 3$ factorises to $3(2x + 1)$ where the two factors are 3 and $2x + 1$.

favourable outcome outcome within the sample space that we want to occur.

formula relationship or rule between two or more variables. A formula contains an equals sign; for example, $y = 3x + 7$.

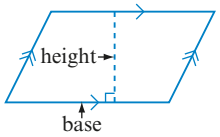
frequency table table organised to show data by recording the frequency of each value in the data set.

G

GMT (Greenwich Mean Time) time zone referenced from Greenwich in London (0° longitude). Also known as Coordinated Universal Time. *See* UTC.

H

height length measurement from the base to the top or end of a shape or object. The height is perpendicular to the base.



highest common factor (HCF) largest factor that is common to two or more given numbers. For example, factors of 8 are 1, 2, 4, 8 and factors of 12 are 1, 2, 3, 4, 6, 12 so HCF of 8 and 12 is 4.

histogram graph used to display the frequency of grouped continuous data where the horizontal scale boundaries are located at the edges of the columns, rather than the centre.

I

IDL (International Date Line) line of longitude that separates two time zones (UTC +12 and UTC −12) where the date changes by one day. If you travel from west to east across the IDL, you gain one day.

independent variable one of two variables in a relationship. On a Cartesian plane, the independent variable is shown along the horizontal axis. It is listed first in a table of values and in pairs of coordinates.

index (power) for a value expressed in index form, the index (or power) indicates the number of times the base is written as a repeated multiplication. For example, 2^4 has an index of 4 and can be written as $2 \times 2 \times 2 \times 2$. (Plural of index is indices.)

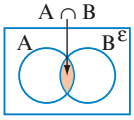
index form (index notation) shorter form of writing a repeated multiplication. A number in index form is written with a base and an index (power). For example, 2^4 is written in index form.

index laws set of laws that make calculations of numbers (or pronumerals) in index form easier to perform.

integers whole numbers that are positive, negative or zero.

International Date Line *see* IDL

intersection the intersection of two sets ($A \cap B$) on a Venn diagram is where the two sets overlap.

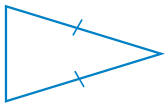


irrational numbers numbers that cannot be written exactly as a fraction. These are non-terminating decimals that are not recurring decimals; for example, $\pi, \sqrt{2}, \sqrt{3}$.

irregular polygon polygon that is not regular. It does not have all sides equal in length or all angles equal in size.

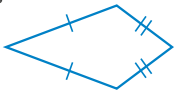
isometric transformation type of transformation (such as translation, rotation or reflection) applied to a shape or object that does not change its shape or size.

isosceles triangle triangle with two sides equal in length. The angles opposite the equal sides are also equal.



K

kite quadrilateral (four-sided shape) with two pairs of adjacent sides equal in length, one pair of opposite angles equal in size and no parallel sides.



L

latitude imaginary lines that run around the Earth, parallel to the equator. Latitude is measured in degrees north and south of the equator (0° to 90°).

leading digit first non-zero digit of a number. For example, the leading digit of 328 is 3.

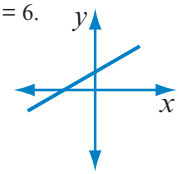
length length measurement from one end to the other of a shape or object. In a rectangle, it is the side measurement that is not defined as the width. Common units are mm, cm, m, km.

like terms algebraic terms that contain exactly the same pronumerals.

line graph graph that displays the relationship between two variables where points are plotted and joined by a line. Often used to show data collected over time (horizontal axis labelled as time).

linear equation an equation that contain a pronumeral term or terms where the highest power is one. For example, $2x + 3 = 6$.

linear graph graph of linear relationship. The graph is a straight line.



linear relationship relationship between two variables where the coordinate points describing this relationship lie in a straight line when plotted on the Cartesian plane.

longitude imaginary lines that run around the Earth through each pole. Longitude is measured in degrees east and west from the prime meridian (0° to 180°).

loss the amount lost when the selling price is less than the original price.

lowest common denominator (LCD) lowest common multiple of the denominators in two or more fractions. For example, $\frac{1}{3}$ and $\frac{1}{2}$ have a LCD of 6.

M

mark-up amount added to an original price. For example, an original price has a 25% mark-up added to obtain the selling price.

mean a measure of the centre of a data set. It is calculated by adding all of the data values together and dividing by the number of values. Also known as the average.

median a measure of the centre of a data set. For data placed in ascending order (smallest to largest), it is the middle number of the data set if there is an odd number of values, and the average of the two middle numbers if there is an even number of values.

meridians lines of longitude. *See* longitude.

midpoint half way point or middle point in an interval.

mode a measure of the centre of a data set. It is the value that occurs most frequently in the data set. There can be more than one mode or no mode.

N

negative numbers numbers that are less than zero; for example, −25, −7.1, $-\frac{3}{4}$.

net two-dimensional plan that can be folded to form a three-dimensional object.

nominal data type of categorical data where the categories are unrelated; for example, eye colour.

non-linear relationship relationship between two variables where the coordinate points describing this relationship do not lie in a straight line when plotted on the Cartesian plane.

non-terminating decimal decimal number with an endless (infinite) number of decimal places; for example, 5.761 238… or 4.266 666….

numerator top number of a fraction. For example, the numerator of $\frac{2}{3}$ is 2.

numerical data data that can be counted or measured.

O

obtuse angle angle greater than 90° but less than 180°.



obtuse-angled describes a triangle where the size of one interior angle is between 90° and 180°.

opposite integers integers that are an equal amount away from a reference point but in the opposite direction. For example, −2 and +2 are opposite integers as they are the same distance from 0 on a number line but on opposite sides.

order of operations specific order in which operations are performed when there is more than one operation in a calculation. The order is: perform operations inside Brackets first, then Indices (powers and roots), then Division and Multiplication (left to right) and finally Addition and Subtraction (left to right). It can also be remembered as BIDMAS (or BODMAS).

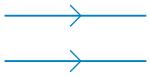
ordinal data type of categorical data that can be placed in categories in a specific order; for example, a rating system from 1–5.

origin point where the x- and y-axes cross on a Cartesian plane. Coordinates are (0, 0).

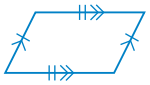
outlier extreme piece of data that is much higher or lower than the rest of the data in the data set.

P

parallel lines (or rays or segments) that are drawn side by side and are the same distance apart so they never touch are parallel. Matching arrowheads are used to indicate that lines are parallel.



parallelogram quadrilateral (four-sided shape) with two pairs of opposite sides that are parallel and equal in length, and two pairs of opposite angles that are equal in size. Rectangles, squares and rhombuses are also classified as parallelograms.



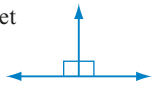
percentage a number expressed as parts out of one hundred.

percentage decrease percentage that an original amount has been decreased by. Found by calculating $\frac{\text{original amount} - \text{new amount}}{\text{original amount}} \times 100\%$.

percentage increase percentage that an original amount has been increased by. Found by calculating $\frac{\text{new amount} - \text{original amount}}{\text{original amount}} \times 100\%$.

perimeter distance around the outside edge or boundary of a two-dimensional (2D) shape. Common units are mm, cm, m, km.

perpendicular two lines (or rays or segments) that meet to form a right angle are perpendicular.



perspective drawings drawings of an image as it is seen by the eye. Objects are drawn smaller as their distance from the observer increases, making use of the vanishing point. The vanishing point is typically located on the horizon.

pi irrational number (3.141 592 653 …) used in calculations involving circumference and area of a circle. It is the ratio of the circumference of a circle to its diameter. The symbol for pi is π.

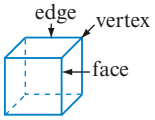
pie graph graph where the size of each sector in a circle represents the frequency of each data category as a fraction of the total number of pieces of data in the data set.

placeholder zeros zeros between non-zero digits in a number. For example, 3.904 has one placeholder zero.

polygon general name for a two-dimensional shape with straight sides.

polyhedra plural of polyhedron. *See* polyhedron.

polyhedron object made up of flat surfaces or faces (each face is a polygon) with vertices and straight edges. For example, a cube is a polyhedron but a cylinder is not.



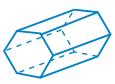
population complete set of individuals or things that we are seeking information about.

positive numbers numbers that are greater than zero; for example, +5, 17, 3.8, $+\frac{1}{2}$.

power (index) *see* index.

prime meridian line of longitude (0°) that runs through Greenwich in London. Other lines of longitude are measured east or west from this reference line.

prism object with two ends that are identical polygons and joined by straight edges.



probability chance of an event occurring. The event can be very likely and have a high probability or not likely and have a low probability. Words or numbers can be used to describe the chance of an event occurring.

probability experiment where trials are performed to obtain data to predict the chances of an event occurring.

probability scale number line from 0 to 1 that can be used to indicate the probability of an event. An impossible event has a probability of 0. A certain event has a probability of 1.

profit amount made when the selling price is greater than the original price.

pronumeral letter or symbol that takes the place of a number.

pyramid object with an end that is a polygon and triangular sides (faces) that meet at a point (vertex).



Q

quadrant one of four different regions on a Cartesian plane formed by the intersection of the x- and y-axes. They are numbered in order from the top right quadrant in an anticlockwise direction.

quadratic equation an equation that contains a pronumeral term or terms where the highest power is 2. For example, $x^2 + 3x - 2 = 10$.

quadrilateral two-dimensional shape with four straight sides.

R

radius line segment from the centre of a circle to the edge of the circle. Also defined as the length of this line segment (and half the diameter of the circle).

random sampling selecting a sample randomly (such as pulling names out of a hat).

range difference between the highest and lowest value in a data set.

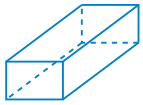
rate comparison of the change in one quantity with respect to another. For example, speed is a rate as it compares the distance over time.

rational numbers numbers that can be written exactly as fractions. This includes whole numbers, fractions, terminating decimals and recurring decimals.

rectangle quadrilateral (four-sided shape) with two pairs of opposite sides that are parallel and equal in length, and four angles of 90°. A rectangle is also classified as a parallelogram.



rectangular prism prism with three pairs of identical faces (total of six faces). A cube is also a rectangular prism.



recurring decimal decimal that has an endless (infinite) number of decimal places and a repeating (recurring) pattern of digits after the decimal point. It can be written in abbreviated form. For example, $0.333\,\overline{33}\ldots = 0.3$ and $5.476\,767\,\dot{6}\ldots = 5.476\dot{7}$.

reduction smaller image produced after a figure has been dilated.

reflection transformation where a shape or object is reflected (flipped) in a mirror line to produce its exact mirror image.

reflex angle angle that is greater than 180° but less than 360°.



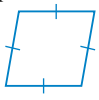
regular polygon polygon with all sides equal in length and all angles equal in size.

relative frequency *see* experimental probability.

revolution angle that is exactly 360°. Also known as a perigon.



rhombus quadrilateral (four-sided shape) with two pairs of opposite sides that are parallel, all four sides equal in length and two pairs of opposite angles that are equal in size. A rhombus can also be classified as a parallelogram.



right angle angle that is exactly 90°. Often represented by a small box drawn where the lines meet.



right-angled describes a triangle where the size of one interior angle is 90°.

rotation transformation where a shape or object is rotated (turned) around a point. Movement is described as an angle of rotation in either a clockwise or anticlockwise direction from a given point.

rounding writing a number as an approximate value by rounding to the leading digit or a given number of decimal places. If the digit to the right of the leading digit or given decimal place is 0, 1, 2, 3 or 4, round down. If it is 5, 6, 7, 8 or 9, round up.

S

sample small selection from a population.

sample space list of all the different outcomes possible for a probability experiment. For example, sample space when rolling a die is {1, 2, 3, 4, 5, 6}.

SAS abbreviation for side-angle-side. It describes the information needed to draw a triangle and also relates to conditions for congruence. Triangles are congruent if two corresponding sides are equal in length and the angle between them is the same.

scale factor indicates how many times larger or smaller an image is after a figure has been dilated. A scale factor larger than 1 produces an enlargement and a scale factor between 0 and 1 produces a reduction.

scalene triangle triangle with three sides of different lengths. All three angles are different in size.



scatterplot graph that displays a relationship between two variables where points are plotted but not joined.

scores pieces of data.

sector region of a circle between two radii. (Plural of radius is radii.)

selling price price for which an item is sold.

set of plans diagrams showing front, right and top view of an object.

sets groups of data. For example, those in your class that have brown hair could be described as belonging to the set of people with brown hair.

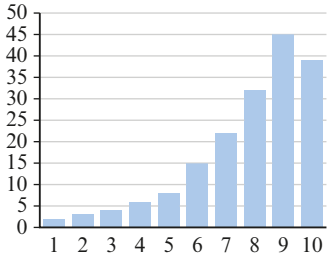
side-angle-side *see* SAS

side-side-side *see* SSS

similar figure an image produced after dilation. The size of the figure has changed but not the shape.

simulation using technology or simple devices like coins or dice to simulate real-life events. Makes performing a large number of trials in a probability experiment easier.

skewed description of a data distribution where there is a higher proportion of the data displayed to the right or to the left of a graph.



solution value of the pronumeral that makes an equation a true statement.

solve find the value of the pronumeral that makes an equation a true statement; that is, the left side and the right side of the equation are equal.

solving by inspection mental strategy to solve simple equations. To inspect or use observation to work out the solution to an equation in your head.

sphere 3D object shaped like a ball.

square **1** quadrilateral (four-sided shape) with all four sides equal in length, two pairs of opposite sides that are parallel and four angles of 90°. A square is also classified as a rectangle.



2 to raise to the power of 2; for example, $3^2 = 9$.

square root number that, when squared (raised to the power of 2), results in the original number. For example, the square root of 9 (or $\sqrt{9}$) is 3.

square-based pyramid pyramid with an end or base that is a square.



SSS abbreviation for side-side-side. It describes the information needed to draw a triangle and also relates to conditions for congruence. Triangles are congruent if all three corresponding sides are equal in length.

stem-and-leaf plot (stem plot) a display of data where each piece of data is split into two parts: the last digit becoming the leaf and the other digits becoming the stem. The stem is written once in the left column of the plot and the leaves are listed in numerical order beside the appropriate stem.

straight angle angle that is exactly 180°.



stratified sampling dividing the population into separate categories (such as gender or age) and then taking a random sample from each category that is proportional to its size.

substitution replacement of a pronumeral with a given number. For example, substituting $x = 2$ in $3x + 4$ gives $3 \times 2 + 4$.

summary statistics information such as the mean, median, mode and range that provide a summary of the data set.

supplementary angles angles that add to make a straight angle (180°).

symmetrical of graphs, having a similar distribution of frequencies either side of a central value.

symmetry property of a shape or object where it can be cut in half and each half is a reflection of the other.

systematic sampling selecting a sample at fixed intervals (such as every third person). The starting point should be random.

T

terminating decimal decimal that 'terminates' or stops after a set number of decimal places; for example, 0.625.

theoretical probability probability calculated by considering the number of favourable outcomes compared to the total number of outcomes; that is, Pr(event) = $\frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}$. Each outcome must be equally likely to occur.

time zones bands on the surface of the Earth where countries experience the same time. Each time zone usually spans a difference in longitude of 15°.

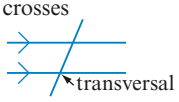
total surface area total area of the surface of an object. Surface area of a prism is the sum of the areas of each face of the prism. Common units are mm^2, cm^2, m^2.

trailing zeros zeros to the right of the last non-zero digit after the decimal point in a decimal number. For example, 3.900 has two trailing zeros.

transformations general name for translations, reflections, rotations and dilations.

translation transformation where a shape or object is translated (moved) in a straight line without turning or changing size. Movement is often described as the number of units up or down and left or right.

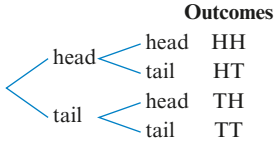
transversal line that crosses (intersects) a pair or set of lines.



trapezium quadrilateral (four-sided shape) with one pair of opposite sides that are parallel.



tree diagram display of outcomes for a multi-step probability experiment. For example, a tree diagram can be used to display the outcomes of flipping two coins.



triangle two-dimensional shape with three straight sides.

triangular prism prism with a triangular base (both ends have the same triangular shape)



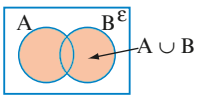
true bearing directions from one position to another and described by the number of degrees from north in a clockwise direction. The angle is written as three digits followed by T (or true). For example, 090°T is due east, 180°T is due south, 270°T is due west.

24-hour time time measured over a 24-hour period from midnight. Usually shown with four digits to indicate the number of hours and minutes from midnight. For example, 1530 means 15 hours and 30 minutes after midnight or 3.30 pm.

two-way table display of outcomes for a two-step probability experiment in a table.

U

union the union of two sets ($A \cup B$) on a Venn diagram is everything contained in both sets.



universal set set of all elements. Everything inside the rectangle of a Venn diagram belongs to the universal set. Symbol for universal set is ε.

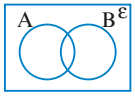
UTC (Coordinated Universal Time) time zone referenced from Greenwich in London at 0° longitude (also known as Greenwich Mean Time). Parts of the world to the east of time zone '0' are ahead of UTC, parts of the world to the west are behind UTC. For example, Melbourne is in the UTC +10 time zone. This means it is 10 hours ahead of UTC.

V

vanishing point point in a perspective drawing to which parallel lines appear to converge and disappear into the horizon.

variables quantities that can have varying or different values. A variable can be represented with words or a symbol.

Venn diagram consists of a number of circles contained in a rectangle to display the relationship between different sets of data. The rectangle represents the universal set, ε, and each set of data within the universal set is represented by a circle.



vertex corner point where three or more edges of a polyhedron meet. *See* polyhedron.

vertically opposite angles pair of equal angles not adjacent to (next to) each other, formed where two lines intersect.



vertices plural of vertex. *See* vertex.

vinculum horizontal line separating the numerator from the denominator. For example, in $\frac{3}{4}$, the vinculum is the horizontal line between 3 and 4. Also represents a division sign.

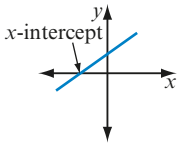
volume amount of space that a three-dimensional object occupies. Common units are mm^3, cm^3, m^3.

W

width length measurement from one end to the other of a shape or object. In a rectangle, it is the side measurement that is not defined as the length. Common units are mm, cm, m, km.

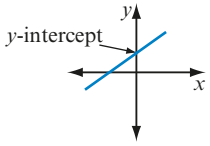
X

***x*-intercept** point where a line crosses the *x*-axis of a Cartesian plane. For example, the *x*-intercept is −3 or the coordinates of the *x*-intercept are (−3, 0).



Y

***y*-intercept** point where a line crosses the *y*-axis of a Cartesian plane. For example, the *y*-intercept is 2 or the coordinates of the *y*-intercept are (0, 2).



Z

zero pair combination of −1 and +1 (two opposite integers) to give zero.

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